

Towards a microscopic construction of flavour vacua from a space–time foam model

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Abstract. The effect on flavour oscillations of simple expanding background space–times, motivated by some D-particle foam models, is calculated for a toy model of bosons with flavour degrees of freedom. The presence of D-particle defects in the space–time, which can interact non-trivially (via ‘particle capture’) with flavoured particles in a flavour non-preserving way, generates mixing in the effective field theory of low-energy string excitations. Moreover, the recoil of the D-particle defect during the capture/scattering process implies Lorentz violation, which, however, may be averaged to zero in isotropic D-particle populations, but implies non-trivial effects in correlators. Both features imply that the flavoured mixed state sees a non-trivial flavour (Fock-space) vacuum of a type introduced earlier by Blasone and Vitiello in a generic context of theories with mixing. We discuss the orthogonality of the flavour vacua to the usual Fock vacua and the effect on flavour oscillations in these backgrounds. Furthermore, we analyse the equation of state of the flavour vacuum, and find that, for slow expansion rates induced by D-particle recoil, it is equivalent to that of a cosmological constant. Some estimates of these novel non-perturbative contributions to the vacuum energy are made. The contribution vanishes if the mass difference and the mixing angle of the flavoured states vanish.

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1. Introduction and motivation

It has been suggested by Blasone and Vitiello and collaborators [1] that in quantum field theoretic systems with *mixing*, such as neutrinos with flavour mixing, for instance, the ‘flavour’ eigenstates can be expressed as a condensate in the Fock space of the mass quanta. Furthermore, it has been shown recently that there is a ‘vacuum energy’ associated with this condensate which would be a contribution to the dark energy. In contrast to the standard mass-eigenstate formalism, where the vacuum expectation value (vev) of the stress tensor vanishes, when one calculates the vev of the stress tensor with respect to the flavour vacuum, the result is non-trivial, indicating spontaneous breaking of the Lorentz symmetry. In fact, computing such vevs for the T_{00} and T_{ii} components of the stress tensor in Robertson–Walker space–times, the authors of [2] have conjectured that there is a non-trivial, and non-perturbative, dark energy contribution to the vacuum, and an equation of state $p = w\rho$ with $w < 0$ close to -1 . These are features that are compatible with recent observations.

In view of the breaking of Lorentz symmetry by the flavour vacuum, characterized by a nonzero vev of the stress tensor, we will examine in this context a microscopic model discussed by Mavromatos and Sarkar [3] where such a feature is accommodated within a specific space–time foam model in the framework of modern brane/string theory. The foam model involves brane worlds which are embedded in higher dimensional bulk space–times, and are punctured by D0-brane (point-like defects) [4]. The interaction of generic stringy matter with such defects involves the discontinuous process of splitting of the matter string by the defect, and subsequent joining, ignoring for the moment constraints from conservation of specific quantum numbers. This yields Lorentz-violating metric distortions [5, 6], $g_{\mu\nu} \propto u_\mu u_\nu$, with u_μ the recoil–velocity of the localized D-particle (D0) defect, due to the ‘topologically non-trivial’ interaction with the stringy matter as well as an induced local expansion. In a situation, where one encounters a statistical population of D0-defects, one may have an average $\langle\langle u_i \rangle\rangle = 0$ (i.e. isotropic models for D-particle foam). Hence, since $u_\mu u^\mu = 1$ for a velocity four-vector, one has

a preferred time-like direction in space–time in this case, leading to induced Lorentz symmetry breaking from non-trivial higher order correlators of u_μ . Such a mechanism for Lorentz symmetry violation is testable since it leads to a quite distinct signature for charge conjugation–parity–time reversal (CPT) violation [7] in meson factories [8], associated with a modification of the Einstein–Podolsky–Rosen (EPR)-type particle correlations. Electric charge conservation implies that charged states cannot undergo such interactions. Moreover, ‘flavour’ need not be conserved in the string splitting process. If D-particle foam contributes to the mixing of flavours then it only operates on the electrically neutral neutrinos (or mesons). The above-mentioned Lorentz-violating ‘flavour-vacua’ appear to be the physically correct ones to characterize the interaction of electrically neutral flavour matter (which for brevity we will refer to as neutrinos) with D0 defects.

It should be pointed out that in the work of [1, 2] the flavour vacuum formalism applies in general to *all* theories with mixing. In the above specific framework of stringy models of D-particle foam, only the *neutral* flavour particles are subject to mixing and contribute to the dark energy. Dark energy, when interpreted as a cosmological constant, gives a de Sitter space–time with an associated horizon. Hence, it is interesting for self-consistency to re-examine and reinterpret the vacuum oscillation phenomena in simple space–times with horizons. In our case, the latter are interpreted as the metric distortions due to D-particle recoil and will be shown to be most important for low energy neutrinos. This is to be contrasted with the approach of [2] where an *a priori* flat space–time formalism was used to discuss the derivation of the dark energy contribution due to mixing. For simplicity, we consider a two-dimensional space–time metric as a physically relevant toy model to illustrate the basic features of the approach. The restriction in space–time dimensionality is plausible if the interaction and recoil of neutrino matter with the D0 brane defects (D-particles), is mainly along the (spatial) direction of the incident neutrino beam.

The paper is organized as follows. We consider the recoil of D-particles after the capture and subsequent emission of stringy matter in terms of suitable vertex operators [5, 6] within a logarithmic field theory [9]. By considering an ensemble of defects, we model the metric induced by the recoil as a simple two-dimensional space–time with a horizon [10]. Finite systems on a lattice with periodic boundary conditions will be considered and the thermodynamic limit will be taken subsequently. This allows a rigorous treatment of the orthogonality (and also of unitary inequivalence [1, 11]) of the flavour and mass vacua. The effect of the space–time expansion on neutrino oscillations is evaluated. In order to check if a dark energy interpretation of the equation of state is allowed, the equation of state of the flavour vacuum (after expansion) is calculated. Given the brane/string motivation for the model, a novel form of normal ordering is introduced which demonstrates a contribution to dark energy. It is important to notice that the type of equation of state obtained by the above normal ordering procedure turns out to be formally independent of the initially assumed form of the scale factor of the Robertson–Walker background space–time, thereby allowing for a self-consistent treatment of the back reaction *non-perturbative* effects of the flavour vacuum onto the space–time. Some technical details of our approach are outlined in the appendix.

2. String inspired toy model

The discovery of new solitonic structures in superstring theory has dramatically changed the understanding of target space structure. These new non-perturbative objects are known as

D-branes and their inclusion leads to a scattering picture of space–time fluctuations. The study of D-brane dynamics has been made possible by Polchinski’s realization that such solitonic string backgrounds can be described in a conformally invariant way in terms of worldsheets with boundaries [12]. On these boundaries, Dirichlet boundary conditions for the collective target-space coordinates of the soliton are imposed. The properties of D-branes in curved backgrounds are not fully developed and so it is necessary to be guided by approximate calculations. In the interaction of low energy matter (e.g. a closed string) with a heavy D-particle, i.e. embedded in a $(d + 1)$ -dimensional space–time the D-particle recoils. Such recoil fluctuations of D0 branes can be described by an effective stochastic space–time. From our point of view, there are two parts to this interaction process: precursor and aftermath of the capture of a string by a D-particle. Firstly, the stringy matter has to approach the D-particle. In a superstring approach, by considering the annulus amplitude for an open string fluctuation between a Dp and Dp' brane which have a nonzero relative velocity it can be shown that there is an attractive potential between them [4]. By specializing to $p = 1$ and $p' = 0$, we can give arguments as to why the low momenta strings are more likely to be captured by the D-particle. Secondly, we have the capture and the re-emission of a string by the D-particle. This involves a change in the background of the string and as such breaks conformal invariance. It is not known how to describe the process of capture for short times. It is the capture and re-emission process which can be responsible for a change in the flavour of the matter string. For long times after the capture and emission process a semi-classical approach is expected to be valid [13]. In the wake of the capture process, the vertex operator of the D-particle is described by an impulse approximation. The breaking of conformal invariance for large times can still be dealt with in terms of conformal data but with a set of recoil vertex operators which satisfy a logarithmic conformal algebra. Such an impulse/recoil event results in general in a time-dependent-induced metric distortion and vacuum energy. It is such a metric distortion that is the motivation for considering the simple model that we will consider.

The timescale of the D-particle matter interaction can be estimated from the perturbative string theory. In the adiabatic approximation for the relative speed u between a p-brane and a D-particle separated by a distance r , it is possible to estimate the potential energy $V(r, u)$ using the perturbative string theory [4]. Typically, the short-range attractive part of the potential \mathcal{V} is given by

$$\mathcal{V}(r, u) \sim -\frac{2\pi\ell_s^2 u^2}{r^3}, \quad (2.1)$$

with ℓ_s being the string scale and is the only relevant piece in the superstring theory. For the case of flavour oscillations with sharp momentum k (whose associated mass eigenstates have masses $m^{(1)}$ and $m^{(2)}$ with $m^{(1)} > m^{(2)}$ in our convention) the timescale τ for the flavour changing capture and subsequent release of a matter particle by a heavy D-particle (when $\hbar = c = 1$) can consequently be estimated by

$$\tau \sim \frac{1}{\Delta\mathcal{V}},$$

where

$$\Delta\mathcal{V} = \frac{2\pi}{\ell_s} \left| \frac{k^2}{m^{(2)2}} - \frac{k^2}{m^{(1)2}} \right| = \frac{2\pi k^2 \Delta m^2}{\ell_s m^{(1)2} m^{(2)2}} \quad \text{and} \quad \Delta m^2 \equiv m^{(1)2} - m^{(2)2}.$$

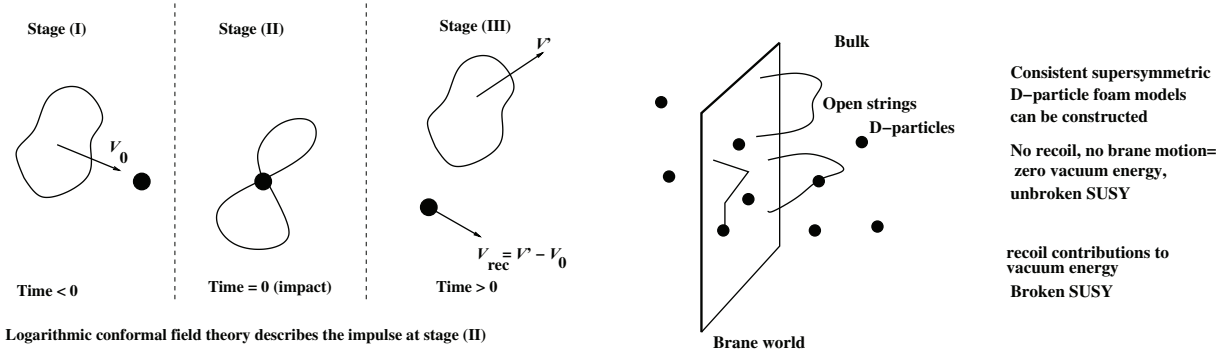


Figure 1. Schematic representation of the capture/recoil process of a string state by a D-particle defect for closed (left) and open (right) string states, in the presence of D-brane world. The presence of a D-brane is essential due to gauge flux conservation, since an isolated D-particle cannot exist. The intermediate composite state at $t = 0$, which has a lifetime within the stringy uncertainty time interval δt , of the order of the string length, and is described by the world-sheet logarithmic conformal field theory, is responsible for the distortion of the surrounding space-time during the scattering, and subsequently leads to Finsler-type (see for instance [14]) induced metrics (depending on both coordinates and momenta of the string state) and modified dispersion relations for the string propagation.

Hence, we deduce that qualitatively

$$\tau \sim \frac{\ell_s}{2\pi} \frac{m^{(1)2} m^{(2)2}}{\Delta m^2} \frac{1}{k^2}. \quad (2.2)$$

We shall then assume that the capture process is enhanced for low k . We can give an estimate for this timescale on considering a typical situation $k = 1 \text{ GeV}$, $m^{(1)} \sim m^{(2)} \sim 10^{-1} \text{ eV}$ and $\Delta m^2 \sim 10^{-5} \text{ eV}^2$; we find $\tau \sim 10^{-60} \text{ s}$ which is effectively instantaneous.

We, for definiteness, consider a D3-brane world with D-particles in the bulk scattering off the brane. The recoil of a D-particle on scattering with a closed (or open) string (see figure 1) leads to a worldsheet deformation given by the semi-classical vertex operator V

$$V = \int_{\partial\Sigma} G_{ij} y^i(X^0) \partial_n X^j, \quad (2.3)$$

where G_{ij} is given by the spatial part of the space-time metric, $\partial\Sigma$ the world sheet boundary, ∂_n is a derivative on the worldsheet normal to the boundary, X^j is a spatial target space field satisfying Dirichlet boundary conditions and X^0 is the time-like target space field satisfying Neumann conditions (in the standard string σ model) and $y^i(X^0)$ is the classical D-particle trajectory which, with a suitable choice of coordinates, vanishes before the time t_0 of the impulse. Let $k_1(k_2)$ be the momentum of the propagating closed-string state before (after) the recoil, y_i be the spatial collective coordinates of the D-particle, $g_s (< 1)$ is the string coupling and M_s is the string scale and $\sqrt{\alpha' \varepsilon^{-1}}$ (α' being the Regge slope) be identified with the target Minkowski time X^0 for $X^0 \gg 0$ after the collision. For an initially Minkowski space-time background, we have [5, 6]:

$$y_i(X^0(z)) = \delta_{ij} y^j(X^0(z)) = \mathcal{C}_i(z) + \mathcal{D}_i(z), \quad (2.4)$$

where $z = e^{-i(\sigma+i\tau)}$, (σ, τ) being the standard worldsheet coordinates, and the operators $\mathcal{C}_i(z)$ and $\mathcal{D}_i(z)$ are given (in terms of the regularized Heavyside function Θ_ε and $u_i \equiv \frac{g_s}{M_s}(k_1 - k_2)_i$) by

$$\mathcal{C}_i(z) = \varepsilon y_i \Theta_\varepsilon \left(X^0(z)/\sqrt{\alpha'} \right) \quad (2.5)$$

and

$$\mathcal{D}_i(z) = u_i X^0(z) \Theta_\varepsilon \left(X^0(z)/\sqrt{\alpha'} \right). \quad (2.6)$$

We define Θ_ε through

$$\Theta_\varepsilon \left(X^0(z)/\sqrt{\alpha'} \right) \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{dq}{q - i\varepsilon} e^{iqX^0(z)/\sqrt{\alpha'}}, \quad \varepsilon \rightarrow 0+. \quad (2.7)$$

We shall from now on choose units with $\alpha' = 1$. These vertex operators have been calculated for non-relativistic branes where u_i is small. Consider now the operators $\mathcal{C} \equiv \mathcal{C}_i \partial_n X^i$ and $\mathcal{D} \equiv \mathcal{D}_i \partial_n X^i$. By calculating the correlators

$$\langle \mathcal{C}(z) \mathcal{C}(w) \rangle \sim O(\varepsilon^2), \quad (2.8)$$

$$\langle \mathcal{C}(z) \mathcal{D}(w) \rangle \sim \frac{1}{|z - w|^2}, \quad (2.9)$$

and

$$\langle \mathcal{D}(z) \mathcal{D}(w) \rangle \sim \log(|z - w|/L) |z - w|^2 \quad (2.10)$$

with respect to the measure of the free bosonic string action, we see that we have a logarithmic conformal field theory [9], *provided* that [5] $\varepsilon^2 \sim (\log \frac{L}{a})^{-2}$, where L is the diameter of the worldsheet disc and a is an ultraviolet worldsheet cut-off. Hence, ε is nonzero for finite L .

By considering the operator product of these operators with the stress energy tensor, we find an anomalous dimension $\Delta = \frac{\varepsilon^2}{2}$. In order to restore conformal invariance, we need the Liouville dressing with a dilaton field. For simplicity, if we restrict the slow recoil to the x -direction (on the brane), we obtain the vertex operator $V_L^{(x)}$

$$V_L^{(x)} = \int_{\Sigma} d^2z e^{\alpha_{0x}\phi} \partial_\alpha (u_x X^0(z) \Theta_\varepsilon(X^0(z)) \partial^\alpha X^x(z)), \quad (2.11)$$

where ϕ is the dilaton field. From considerations associated with conformal invariance of the bulk worldsheet of the dressed theory [15]

$$\alpha_{0x} = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + \frac{\varepsilon^2}{2}}, \quad (2.12)$$

where Q is the central charge deficit of the σ -model describing the stringy excitations of the recoiling D-particle. We remind the reader that the restoration of the conformal invariance by the Liouville ϕ system implies

$$c_\phi = 1 + 3Q^2, \quad (2.13)$$

where c_ϕ is the central charge of the Liouville part of the σ -model. As discussed in detail in [5, 6], one may argue that, in our case, Q^2 is $\mathcal{O}(\varepsilon^4)$, assuming a critical bulk space-time dimension (cf figure 1); hence, to leading order in $\varepsilon \rightarrow 0^+$, its contributions can be ignored

in comparison to the anomalous dimension term $\varepsilon^2/2$ in (2.12). Thus, we have $\alpha_{0x} \sim \frac{\varepsilon}{\sqrt{2}}$ (for $u \ll 1$) and from (2.11) we obtain:

$$V_L^{(x)} \approx \int_{\Sigma} e(\varepsilon/\sqrt{2}) \phi \partial_{\alpha} X^0(z) \partial^{\alpha} X^x(z) u_x \Theta_{\varepsilon}(X^0(z)) + \dots, \quad (2.14)$$

where the \dots denote terms that do not contribute to physical correlators. Such terms are proportional to either $X^0 \delta(X^0)$ (obtained by the action of ∂_{α} on the Heavyside operator $\Theta_{\varepsilon}(X^0)$) or $\partial_{\alpha} \partial^{\alpha} X^i$ (which vanish in the conformal gauge on the worldsheet).

In the recoil case, the string is slightly *supercritical*, i.e. $Q^2 > 0$. In such a case, the zero mode $\phi_{(0)}$ of ϕ can be interpreted as a second time in addition to X^0 [16]. From (2.14), we then observe that the effect of $V_L^{(x)}$ in the action can be reinterpreted as an induced target space metric in the extended $D+1$ -dimensional space-time $(\phi_{(0)}, X^0, X^i)$

$$G_{\phi\phi} = 1, \quad G_{xx} = -1, \quad G_{00} = -1, \quad G_{0x} = e(\varepsilon/\sqrt{2}) \phi u_x \Theta_{\varepsilon}(X^0), \quad (2.15)$$

where, following [13], we have assumed euclideanized X^0 , as required for convergence of the worldsheet path integral.

At first sight, it appears that this metric is characterized by two times, $\phi_{(0)}$ and X^0 . However, it has been argued [13] that the open string excitations on the brane world are such that they live on a D -dimensional hyper-surface of the extended $D+1$ -dimensional two-time space-time; the Liouville mode is proportional to $\sum_{\mu=0}^3 u_{\mu} X^{\mu}$, with u_{μ} the four-velocity on the D3 brane for the recoiling D-particle and u_{μ} can be written as $\gamma(1, \vec{u}_i)$, where $\gamma(1 = \sqrt{1 - u_i^2})$ is the Lorentz factor.

We shall demonstrate this structure in an explicit example in which the three-brane world is obtained by compactification on magnetized tori, characterized by a magnetic field with intensity H . This magnetic field is not a real one, but a ‘statistical’ field characterizing the compactification. An open string excitation coupled to such a magnetic field, has its ends attached to the compactified three-brane world. An encounter of the string with a D-particle is described by the addition of the following deformation to a free open-string σ -model action:

$$\int_{\partial\Sigma} H \Theta_{\varepsilon}(u_{\mu} X^{\mu}) X^4 \partial^{\tau} X^5, \quad (2.16)$$

where ∂^{τ} denotes tangential derivative on the worldsheet boundary. This corresponds to the vertex operator for a gauge potential corresponding to a uniform magnetic field in the compactified toroidal dimensions (X^4, X^5) that appears suddenly at a time $\sum_{\mu=0}^3 u_{\mu} X^{\mu} = 0$, in the co-moving frame of the recoiling D-particle, which has captured the open string excitation. The reader should notice that the specific calculation takes place in this co-moving frame; the argument of the Heavyside operator is the rest-frame time of the recoiling massive D-particle, which in terms of the original coordinates is expressed as $u_{\mu} X^{\mu} = \gamma(X^0 + u_i X^i)$.

It can be easily seen [5] that the operator (2.16) has anomalous dimension $-\frac{\varepsilon^2}{2} u_{\mu} u^{\mu} = -\frac{\varepsilon^2}{2}$, since the four velocity satisfies by definition $u_{\mu} u^{\mu} = 1$, with respect to the background Minkowski metric. This implies a Liouville dressing similar to (2.11), with the corresponding Liouville anomalous dimension $\alpha \sim \frac{\varepsilon}{\sqrt{2}}$,

$$\int_{\partial\Sigma} e(\varepsilon/\sqrt{2}) \phi H \Theta_{\varepsilon}(u_{\mu} X^{\mu}) X^4 \partial^{\tau} X^5. \quad (2.17)$$

As shown in [13], the open superstring excitations that couple to such a magnetic field will exhibit Zeeman-like supersymmetric mass splittings between fermions and bosons of the form:

$$\delta m^2 \sim H e(\varepsilon/\sqrt{2}) \phi H \Theta_\varepsilon(u_\mu X^\mu) \Sigma_{45} \quad (2.18)$$

with Σ_{45} the spin operator given by $\frac{i}{4}[\gamma_4, \gamma_5]$, where γ_μ ($\mu = 0, \dots, 5$) are the Dirac matrices relevant for six-dimensional space-time. However, masses need to be time (or equivalently ε) independent for stability. Representing the Heavyside operator for positive arguments as $\Theta_\varepsilon(u_\mu X^\mu) \sim e - \varepsilon u_\mu X^\mu$, δm^2 is independent of the value of ε if [13]

$$\frac{\phi_{(0)}}{\sqrt{2}} - \sum_{\mu=0}^3 u_\mu X^\mu = \text{constant}, \quad (2.19)$$

where the constant can be taken to zero without loss of generality. Since the above-mentioned mass splittings can be tuned by adjusting appropriately the intensity of the ‘magnetic’ field H , in a way independent of the size of the four-dimensional vacuum energy contribution [13], the above example is sufficiently generic and so of interest for phenomenological applications of superstring theories.

The constraint (2.19), which is consistent with general coordinate invariance on the brane world, implies that the dynamics of the open string excitations can effectively be considered on a D -dimensional space-time (hypersurface), with a *single* time. Although above we have argued in favour of (2.19) in the context of a specific superstring situation, nevertheless we may assume its validity in more general frameworks of supercritical strings [17].

From (2.19) and (2.15), we then obtain for the induced D -dimensional target space-time metric (with $X^0 \equiv t$):

$$ds^2 = d\phi^2 - dt^2 + u_x dX dt - (dX)^2 = 2\gamma^2(dt + u_x dX)^2 - dt^2 + u_x dX dt - (dX)^2. \quad (2.20)$$

We next consider the case of a statistically significant population of D-particles on the brane world (cf figure 1) (‘D-particle foam’). We may assume a random (Gaussian) D-particle foam, such that [3]:

$$\langle\langle u_x \rangle\rangle = 0, \quad \langle\langle u_x u_x \rangle\rangle = \sigma^2 \quad (2.21)$$

and we assume slowly moving D-particles, so that cubic or higher powers of the recoil velocities are ignored. The average of the metric (2.20), then, yields (upon expanding appropriately the Lorentz factors in powers of u_x^2):

$$\langle\langle ds^2 \rangle\rangle = (1 + 2\sigma^2) dt^2 - (1 - 2\sigma^2) dX^2 + dY^2 + dZ^2. \quad (2.22)$$

Notice the existence of *horizons* in this metric, the ‘xx’ metric component vanishes for $\sigma^2 = 1/2$ (which is consistent with the fact that the velocities are subluminal $|u_\mu| < 1$). The metric (2.22) has a *global* character, as a result of averaging over populations of D-particles that cross the *entire* D-brane world.

We next notice that the quantities σ^2 are in principle time dependent, denoted by $\sigma^2(t)$, since the density of the D-particle defects may depend on time. This can happen, for instance, if the bulk density of the D-particles is not uniform. The nature of the time dependence of $\sigma^2(t)$ is therefore dependent on a microscopic model. If the *density* of the D-particles that cross our

brane world (cf figure 1) decreases with time, we then have an expanding Robertson–Walker-type universe in the (t, X) -directions, and a static universe in the transverse directions (Y, Z) :

$$\langle\langle ds^2 \rangle\rangle = d\zeta^2 - a^2(\zeta)(dX)^2 + \dots \quad (2.23)$$

where the ‘cosmic time’ ζ is defined as $d\zeta^2 = (1 + 2\sigma^2(t)) dt^2$ and the \dots denote the static directions which will not be of relevance to us here. It is *a priori* natural to consider an isotropic D-particle foam, in which the D-particles recoil in random directions so that $\langle\langle u_i \rangle\rangle = 0$ and $\langle\langle u_i u_j \rangle\rangle = \sigma^2 \delta_{ij}$. Asymptotically $a(\zeta)$ monotonically tends to a constant with ζ . In such a case one obtains an isotropic Robertson–Walker-like universe. For simplicity in this work, we concentrate on effective two-dimensional universes, since such an assumption will not affect the qualitative features of our approach. In this sense, the interaction of D-particle foam with stringy matter on brane worlds, leads to an effective space–time metric, ‘felt’ by the matter, which, depending on properties of the foam, can have the form of homogeneous and isotropic Robertson–Walker cosmology.

3. Effective toy model field theory of bosons with mixing

We will use the string theory considerations, leading to (2.23), as a motivation for studying the possible consequences of our D-particle/string hypothesis for flavour mixing within the context of a simple field theoretic 1 + 1-dimensional model with a horizon. Before starting, we mention that flavour need not be conserved during the capture process of figure 1, in the sense that the re-emitted string, after the capture, may have a different flavour (and mass) from the incident one. However, the spatial momentum is conserved in the process. In this sense, our D-particle foam may be considered as a medium of inducing flavour *mixing* [3, 18], which might be a feature of quantum gravity foam ζ in general.

As we shall discuss below, the effects of flavour changing on the space–time are drastic, and result in a ‘new Fock-space vacuum’ state of the form suggested in [1] for generic theories with mixing (for the purposes of this section the third reference of [1], where the mixing of bosons was first treated, is particularly useful). As we shall discuss in section 5, this results in non-trivial contributions to the vacuum energy, of cosmological constant type, which come *mostly* from the infrared modes of the flavoured particles, of momenta less than the representative scale of the order of the sum of the masses. These constant (in time) contributions imply that, eventually, the flavour-changing process *dominates* the simple Robertson–Walker metric structure (2.22) and (2.23) arising from the D-particle recoil distortions, resulting in de Sitter-type universes.

3.1. Formalism in an expanding universe

Let us consider the metric in 1 + 1 dimensions [10]

$$ds^2 = dt^2 - a^2(t) dx^2 \quad (3.1)$$

which in conformal time coordinates has the form

$$ds^2 = C(\eta)(d\eta^2 - dx^2). \quad (3.2)$$

According to our discussion above, we first consider Robertson–Walker expanding universes, interpolating *smoothly* between asymptotically flat space–times. In our D-particle-foam picture, leading to (2.22) and (2.23) this can be achieved by considering models of foam in which the

density of the D-particles in the bulk is not uniform but varies appropriately in such a way that, as the D3-brane world moves through the population of D-particles (cf figure 1), the effective density of the D-particles crossing the D3 brane varies smoothly with time so as to lead to the above situation of smoothly expanding isotropic universes. Moreover, we assume for simplicity the case in which the fluctuation parameter $\sigma^2(t)$ never reaches the horizon scale $1/2$, but is always kept sufficiently small. This is only for simplicity, since the essential features of our expanding universe that would be relevant for our discussion in this work, namely particle production, can already be captured in such a horizon-free situation. In any case, as we have already mentioned, and shall discuss in section 5, non-perturbative effects of our flavour vacuum will produce de Sitter space-times with asymptotic future horizons.

A simple choice for $C(\eta)$, capturing the above features is

$$C(\eta) = A + B \tanh(\rho\eta) \quad (3.3)$$

with $A > B > 0$ and $\rho > 0$ and has the feature of $C(\eta)$ asymptotically tending to a constant as $\eta \rightarrow \infty$. This space-time is asymptotically accelerating, for sufficiently negative $\rho\eta \rightarrow -\infty$, and decelerating for positive $\rho\eta \rightarrow \infty$; hence there are no asymptotic future cosmic horizons. This is consistent with the absence of such horizons in the perturbative string theory [19], employed in our worldsheet approach to D-particle foam, which is based on well-defined scattering-matrix elements and asymptotic states. In the D-particle picture given above, if we assume that asymptotically in cosmic conformal time $\eta \rightarrow \infty$, the recoil fluctuations σ^2 tend to a constant value, $\tilde{\sigma}^2$, we can identify $A \simeq 1 - 4\tilde{\sigma}^2$ and $B = 2\tilde{\sigma}^2$. Of course, there is also a flat space limit when $B \rightarrow 0$, i.e. vanishing D-particle recoil. The space-time (3.3) is considered as a *perturbative* stringy background on which we shall discuss flavour changing process resulting from the stringy, topologically non-trivial processes of capture in our foam model, depicted in figure 1. As mentioned above, such a flavour changing process will lead to a new vacuum state, which in turn will imply contributions to the vacuum energy of the universe of de Sitter-type, that will eventually dominate the space-time (3.3), leading to eternal acceleration and asymptotic horizons. This should be considered as a *non-perturbative* contribution of the flavour-changing non-trivial vacuum structure of our model.

On the other hand, if one would like to use an exactly soluble model, without future horizons, interpolating between a de Sitter space-time at a certain era in the past and a flat Minkowski space-time at the future asymptotic end of cosmic time, then one could consider as an instructive example the following form of the scale factor²:

$$C(\eta) = A + B \frac{\eta}{\sqrt{\eta^2 + (1/\rho^2)}} \quad (3.4)$$

with the model incorporating the de Sitter phase in the far past, $\rho\eta \ll -1$ for $A = B > 0$ and $\rho > 0$. Indeed, for sufficiently negative conformal times, such that $(\rho\eta)^2 \gg 1$, the model implies inflationary evolution, since in that range of parameters

$$C(\eta) \simeq \frac{A}{2} \frac{1}{(\eta\rho)^2}. \quad (3.5)$$

On the other hand, in the asymptotic future $\eta\rho \gg 1$, the model asymptotes to flat Minkowski space-time, with $C(\eta) \rightarrow 2A$.

² This model has been used (in its four-dimensional version) as an instructive example of particle (string) production at the end of inflation in local field (string/brane) theories, see for instance: [20].

In what follows, we shall not discuss explicitly the space–time (3.4) further, but concentrate instead on (3.3), since for our qualitative purposes in this section of demonstrating the effect of expansion insofar as particle production is concerned, the two space–times (3.4) and (3.3) are equivalent. However, we note that, on anticipating a small Hubble parameter of our D-foam-induced-de Sitter space–time (cf (5.34)), we are essentially interested in the small ρ regime in which both space–times (3.3) and (3.4) undergo linear expansion with the (conformal) time. This limit of slow expansion, $\rho \rightarrow 0$ will be understood in what follows.

It goes without saying that the above issues are far from being considered as resolved, especially in a string theory context, where the issues of an inflationary phase, and in particular a smooth (‘graceful’) exit from it, are far from being understood analytically. In critical string theory, there are recent attempts towards a construction of exact solutions, incorporating inflation, by making use of the properties of the compact Calabi–Yau spaces in concrete, phenomenologically semi-realistic (as far as incorporation of the Standard Model is concerned) brane world models (see for instance, [21] and reference therein). However, the exit phase from the de Sitter era, together with attaining the very small value for the vacuum energy observed today, are still not understood, in our opinion. Some attempts towards these latter issues do exist, however, within the context of non-critical (Liouville) string framework, which the present D-particle foam model belongs to. For further details, we refer readers to the literature [22].

In the present paper, we shall certainly not attempt to offer any quantitative resolution of these problems, given that our considerations below will be based on effective field theories. Nevertheless, we think that our findings are sufficiently interesting to probe further studies along this direction, which might yield useful contributions to the quest for a proper understanding of quantum gravity and its connection with other unresolved issues in fundamental physics, such as the flavour-mixing problem.

After these necessary explanatory remarks, we now proceed with our analysis. In conformal coordinates our expanding universe background space–time is written as

$$g_{\mu\nu} = \begin{pmatrix} C(\eta) & 0 \\ 0 & -C(\eta) \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} C^{-1}(\eta) & 0 \\ 0 & -C^{-1}(\eta) \end{pmatrix},$$

with $|\det g_{\mu\nu}| \equiv g = C^2(\eta)$. We will consider the dynamics of a real scalar free field ϕ (appropriate for a neutral meson) of mass m in this background:

$$C^{-1}(\eta) \partial_\mu (C(\eta) g^{\mu\nu} \partial_\nu \phi) + m^2 \phi = 0. \quad (3.6)$$

This simplifies to

$$(\partial_\eta^2 - \partial_x^2) \phi + m^2 C(\eta) \phi = 0. \quad (3.7)$$

On writing [10] $\phi(\eta, x) = U(\eta) V(x)$, we can find a complete basis set for expanding fields from

$$\frac{d^2}{dx^2} V + k^2 V = 0 \quad (3.8)$$

and

$$\frac{d^2}{d\eta^2} U + (k^2 + m^2 C(\eta)) U = 0. \quad (3.9)$$

We have $V(x) = e^{ikx}$ and $U(\eta) = c_1 U_1(\eta) + c_2 U_2(\eta)$, where $U_1(\eta)$ and $U_2(\eta)$ are given in terms of the hypergeometric function

$$F(a, b; c, z) \left(= \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!} \right).$$

It will be helpful to introduce the notation

$$\omega_{\text{in},k} = \sqrt{k^2 + m^2(A - B)}, \quad (3.10)$$

$$\omega_{\text{out},k} = \sqrt{k^2 + m^2(A + B)} \quad (3.11)$$

and

$$\omega_{\pm,k} = \frac{1}{2} (\omega_{\text{out},k} \pm \omega_{\text{in},k}). \quad (3.12)$$

In terms of these variables it can be shown that [10]

$$U_1(\eta) \propto (1 + \tanh(\rho\eta))^{\alpha_1 - \frac{1}{2}} (1 - \tanh(\rho\eta))^{\beta_1 - \frac{1}{2}} \\ \times F\left(\frac{i}{\rho}\omega_+, 1 + \frac{i}{\rho}\omega_+; 1 + i\frac{\omega_{\text{in}}}{\rho}, \frac{1}{2}(1 + \tanh(\rho\eta))\right), \quad (3.13)$$

where

$$\alpha_1 = \frac{1}{2} \left(1 + \frac{i\omega_{\text{in}}}{\rho} \right), \quad \text{and} \quad \beta_1 = \frac{1}{2} \left(1 + \frac{i\omega_{\text{out}}}{\rho} \right).$$

We can find another independent solution $U_2(\eta)$ on making the change $\eta \rightarrow -\eta$ and $B \rightarrow -B$ (i.e. $\omega_{\text{in}} \longleftrightarrow \omega_{\text{out}}$) and so

$$U_2(\eta) \propto (1 - \tanh(\rho\eta))^{\alpha_1 - (1/2)} (1 + \tanh(\rho\eta))^{\beta_1 - (1/2)} \\ \times F\left(\frac{i}{\rho}\omega_+, 1 + \frac{i}{\rho}\omega_+; 1 + i\frac{\omega_{\text{out}}}{\rho}, \frac{1}{2}(1 - \tanh(\rho\eta))\right). \quad (3.14)$$

We should note that complex conjugation of $U_i(\eta)$ is also a solution of (3.9) and this is effected by taking the complex conjugations of the arguments of F and also the exponents in (3.13) and (3.14). In terms of these solutions, the appropriate ‘in’ and ‘out’ solutions can be taken to be

$$u_k^{\text{in}}(\eta, x) = e^{ikx} U_1^*(\eta) \quad (3.15)$$

and

$$u_k^{\text{out}}(\eta, x) = e^{ikx} U_2(\eta), \quad (3.16)$$

where the constants of proportionality in (3.13) and (3.14) are chosen on the grounds of normalization. It is known that there are Bogolubov coefficients α_k and β_k such that [10]

$$u_k^{\text{in}}(\eta, x) = \alpha_k u_k^{\text{out}}(\eta, x) + \beta_k u_{-k}^{\text{out}*}(\eta, x) \quad (3.17)$$

and

$$u_k^{\text{out}}(\eta, x) = \alpha_k^* u_k^{\text{in}}(\eta, x) + \beta_k u_{-k}^{\text{in}*}(\eta, x) \quad (3.18)$$

with

$$\alpha_k = \sqrt{\frac{\omega_{\text{out},k}}{\omega_{\text{in},k}}} \frac{\Gamma(1 - (i\omega_{\text{in},k}/\rho))\Gamma(-i\omega_{\text{out},k}/\rho)}{\Gamma(-i\omega_{+,k}/\rho)\Gamma(1 - (i\omega_{+,k}/\rho))} \quad (3.19)$$

and

$$\beta_k = \sqrt{\frac{\omega_{\text{out},k}}{\omega_{\text{in},k}}} \frac{\Gamma(1 - (i\omega_{\text{in},k}/\rho))\Gamma(i\omega_{\text{out},k}/\rho)}{\Gamma(i\omega_{-,k}/\rho)\Gamma(1 + (i\omega_{-,k}/\rho))}. \quad (3.20)$$

(N.B. the limit of no expansion is given by $B \rightarrow 0$ with $\alpha_k \rightarrow 1$ and $\beta_k \rightarrow 0$ as required.) This formalism requires a slight generalization when there are two distinct mass eigenstates. We will need in the obvious way $\omega_{\text{in}}^{(i)}$, $\omega_{\text{out}}^{(i)}$ and $m^{(i)}$ with $i = 1, 2$ with m replaced by $m^{(i)}$. In terms of creation $\{a^\dagger\}$ and annihilation $\{a\}$ operators for massive particles in the ‘in’ and ‘out’ Hilbert spaces, we have the relations

$$a_k^{\text{in}(i)} = \alpha_k^{(i)*} a_k^{\text{out}(i)} - \beta_k^{(i)*} a_{-k}^{\text{out}(i)\dagger}, \quad (3.21)$$

$$a_k^{\text{out}(i)} = \alpha_k^{(i)} a_k^{\text{in}(i)} + \beta_k^{(i)*} a_{-k}^{\text{in}(i)\dagger}, \quad (3.22)$$

where for $i = 1, 2$

$$\alpha_k^{(i)} = \sqrt{\frac{\omega_{\text{out},k}^{(i)}}{\omega_{\text{in},k}^{(i)}}} \frac{\Gamma(1 - (i\omega_{\text{in},k}^{(i)}/\rho))\Gamma(-i\omega_{\text{out},k}^{(i)}/\rho)}{\Gamma(-i\omega_{+,k}^{(i)}/\rho)\Gamma(1 - (i\omega_{+,k}^{(i)}/\rho))} \quad (3.23)$$

$$\beta_k^{(i)} = \sqrt{\frac{\omega_{\text{out},k}^{(i)}}{\omega_{\text{in},k}^{(i)}}} \frac{\Gamma(1 - (i\omega_{\text{in},k}^{(i)}/\rho))\Gamma(i\omega_{\text{out},k}^{(i)}/\rho)}{\Gamma(i\omega_{-,k}^{(i)}/\rho)\Gamma(1 + (i\omega_{-,k}^{(i)}/\rho))}. \quad (3.24)$$

The corresponding massive fields have the standard nonzero commutator

$$[\phi^{(i)}(\eta, x), \partial_t \phi^{(j)}(\eta, y)] = i\delta(x - y)\delta_{ij}. \quad (3.25)$$

It will be useful when discussing issues of orthogonality for condensate states that we put our spatial system in a box of length L with periodic boundary conditions i.e. with $k = \frac{2\pi j}{L}$; in natural units

$$\phi^{(i)}(\eta, x) = \sum_{j=-\infty}^{\infty} \frac{1}{\sqrt{2L\omega_j^{(i)}}} \left[a_j^{(i)}(\eta) e^{-i(2\pi j/L)x} + a_j^{(i)\dagger}(\eta) e^{i(2\pi j/L)x} \right] \quad (3.26)$$

with $\omega_j^{(i)} = \sqrt{(\frac{2\pi j}{L})^2 + m^{(i)2}}$ and so

$$[\phi^{(i)}(\eta, x), \partial_t \phi^{(i)}(\eta, x')] = i \sum_j \frac{1}{L} e^{i(2\pi j/L)(x-x')} \equiv i\Delta_L(x - x'). \quad (3.27)$$

We also introduce a short distance cut-off δx (or lattice spacing) so that we have a discrete lattice structure: $N\delta x = \frac{L}{2}$, where N is an integer. There would then be a maximum $|j| = N$ ($= \frac{L}{2\delta x}$) and so $-N \leq j \leq N$. Similarly with momentum $-\frac{\pi}{\delta x}$ and so $-N \leq j \leq N$. Space points become discrete: $x \rightarrow n\delta x$ and $x' \rightarrow n'\delta x$ so that $-N \leq n, n' \leq N$. We can then define

$\Delta_{L,\delta x}(x-x') \equiv \frac{1}{L} \sum_{j=-N}^N e^{i(x-x')(2\pi j/L)}$ as a lattice analogue of $\Delta_L(x-x')$. Consequently on the lattice

$$\phi^{(i)}(n\delta x) = \sum_{j=-N}^N \frac{1}{\sqrt{2L\omega_j^{(i)}}} \left(a_j^{(i)} e^{i(\pi j n/N)} e^{-i\omega_j^{(i)} \eta} + a_j^{(i)\dagger} e^{-i(\pi j n/N)} e^{i\omega_j^{(i)} \eta} \right) \quad (3.28)$$

with $-N \leq n \leq N$.

3.2. Flavour mixing and vacuum

Although it would be desirable to rigorously demonstrate string matter flavour changes due to D-particle capture, this requires non-perturbative string field theory or M-theory. Since such a theory is not available, we will consider a field theoretic treatment where the mixing angle θ is put in by hand. It is later when we consider the vacuum energy contribution due to D-particles that we will require a correlation between θ and D-particle recoil. Following [1] flavour mixings are generated by $\mathcal{G}(\theta)$

$$\mathcal{G}(\theta) = \exp(i\theta \mathcal{S}), \quad (3.29)$$

where \mathcal{S} is given by

$$\mathcal{S} = \frac{i}{2} \sum_{j=0}^N \mathfrak{S}_j, \quad (3.30)$$

where

$$\begin{aligned} \mathfrak{S}_j = & \gamma_{j-} \left(\tilde{a}_j^{(1)} \tilde{a}_{-j}^{(2)} + \tilde{a}_{-j}^{(1)} \tilde{a}_j^{(2)} \right) - \gamma_{j-} \left(\tilde{a}_j^{(1)\dagger} \tilde{a}_{-j}^{(2)\dagger} + \tilde{a}_{-j}^{(1)\dagger} \tilde{a}_j^{(2)\dagger} \right) \\ & + \gamma_{j+} \left(\tilde{a}_j^{(1)} \tilde{a}_j^{(2)\dagger} + \tilde{a}_{-j}^{(1)} \tilde{a}_{-j}^{(2)\dagger} \right) - \gamma_{j+} \left(\tilde{a}_j^{(1)\dagger} \tilde{a}_j^{(2)} + \tilde{a}_{-j}^{(1)\dagger} \tilde{a}_{-j}^{(2)} \right), \end{aligned} \quad (3.31)$$

with

$$\gamma_{j\pm} \equiv \sqrt{\frac{\omega_j^{(1)}}{\omega_j^{(2)}}} \pm \sqrt{\frac{\omega_j^{(2)}}{\omega_j^{(1)}}} \quad (3.32)$$

(and so $\gamma_{-j\pm} = \gamma_{j\pm}$) and $\tilde{a}_j^{(i)} = a_j^{(i)} e^{-i\omega_j^{(i)} t}$. There are of course additional superscripts associated with the ‘in’ and ‘out’ Hilbert spaces. Clearly, we have the standard commutation relations $[\tilde{a}_j^{(i)}, \tilde{a}_{j'}^{(i')\dagger}] = \delta_{jj'} \delta_{ii'}$ and $[\tilde{a}_j^{(i)}, \tilde{a}_{j'}^{(i')}] = [\tilde{a}_j^{(i)\dagger}, \tilde{a}_{j'}^{(i')\dagger}] = 0$. Hence

$$[\mathfrak{S}_j, \mathfrak{S}_{j'}] = 0, \quad (3.33)$$

$$\mathcal{G}(\theta) = \prod_{j=1}^N \exp\left(-\frac{\theta}{2} \mathfrak{S}_j\right) \quad (3.34)$$

and

$$\mathcal{G}^{-1}(\theta) = \prod_{j=1}^N \exp\left(\frac{\theta}{2} \mathfrak{S}_j\right). \quad (3.35)$$

It is then straightforward to check that

$$\mathcal{G}^{-1}(\theta) \tilde{a}_{\pm j}^{(1)} \mathcal{G}(\theta) = \cos \theta \tilde{a}_{\pm j}^{(1)} + \frac{1}{2} \sin \theta \left(\gamma_{j-} \tilde{a}_{\mp j}^{(2)} + \gamma_{j+} \tilde{a}_{\pm j}^{(2)} \right) \quad (3.36)$$

and so

$$\mathcal{G}^{-1}(\theta) a_{\pm j}^{(1)} \mathcal{G}(\theta) = \cos \theta a_{\pm j}^{(1)} + \frac{1}{2} \sin \theta \left(\gamma_{j-} a_{\mp j}^{(2)\dagger} e^{i(\omega_j^{(1)} + \omega_{-j}^{(2)})t} + \gamma_{j+} a_{\pm j}^{(2)} e^{i(\omega_j^{(1)} - \omega_j^{(2)})t} \right). \quad (3.37)$$

Similarly, we have (with $j > 0$):

$$\mathcal{G}^{-1}(\theta) \tilde{a}_{\pm j}^{(2)} \mathcal{G}(\theta) = \cos \theta \tilde{a}_{\pm j}^{(2)} + \frac{1}{2} \sin \theta \left(\gamma_{j-} \tilde{a}_{\mp j}^{(1)\dagger} - \gamma_{j+} \tilde{a}_{\pm j}^{(1)} \right). \quad (3.38)$$

These relations for creation and annihilation operators are consistent with the following mixing relations for the corresponding massive fields $\phi^{(i)}$:

$$\mathcal{G}^{-1}(\theta) \phi^{(1)}(\eta, x) \mathcal{G}(\theta) = \cos \theta \phi^{(1)}(\eta, x) + \sin \theta \phi^{(2)}(\eta, x), \quad (3.39)$$

$$\mathcal{G}^{-1}(\theta) \phi^{(2)}(\eta, x) \mathcal{G}(\theta) = \cos \theta \phi^{(2)}(\eta, x) - \sin \theta \phi^{(1)}(\eta, x), \quad (3.40)$$

the right-hand sides of which can be identified with $\Phi_\alpha(\eta, x)$ and $\Phi_\beta(\eta, x)$ fields with flavour quantum numbers α and β [1]. The annihilation operators $a_{j,\alpha}$ and $a_{j,\beta}$ for particles with flavour α and β can be identified with $\mathcal{G}^{-1}(\theta) a_j^{(1)} \mathcal{G}(\theta)$ and $\mathcal{G}^{-1}(\theta) a_j^{(2)} \mathcal{G}(\theta)$, respectively. Since we are interested in the particular contributions of D-particles to mixing, the above definition of flavour vacua $|0\rangle_{\alpha,\beta}$ will be restricted to $k < k_{\max} = \frac{2\pi N^*}{L}$

$$a_{j,\alpha} |0\rangle_{\alpha,\beta} = 0, \quad (3.41)$$

$$a_{j,\beta} |0\rangle_{\alpha,\beta} = 0 \quad (3.42)$$

on assuming that a D-particle mechanism is responsible for particle mixing. For values of k higher than the cut-off $|0\rangle_{\alpha,\beta}$ will coincide with the massive vacuum, i.e. $a_{\pm j}^{(i)} |0\rangle_{\alpha,\beta} = 0$ for $i = 1, 2$. This is an important difference between our model and that given in [1]. Hence, we can deduce that

$$\mathcal{G}_*(\theta) |0\rangle_{\alpha,\beta} = |0\rangle_{1,2}, \quad (3.43)$$

where

$$\mathcal{G}_*(\theta) = \prod_{j=1}^{N^*} \exp \left(-\frac{\theta}{2} \mathfrak{S}_j \right)$$

and so

$$|0\rangle_{\alpha,\beta} = \mathcal{G}_*^{-1}(\theta) |0\rangle_{1,2}. \quad (3.44)$$

For finite N^* the flavour and massive particle vacua have a nonzero overlap. In the thermodynamic limit $N^* \rightarrow \infty$ this ceases to be the case as will be discussed in the next section.

3.3. Relations between flavour and mass-eigenstate vacua

We will be interested in the N dependence of the overlap $f(\theta)$ between $|0\rangle_{1,2}$ and $|0\rangle_{\alpha,\beta}$ viz.

$$f(\theta) = {}_{1,2}\langle 0 | \mathcal{G}_*^{-1}(\theta) | 0 \rangle_{1,2}, \quad (3.45)$$

which has a factorized structure since $\mathcal{G}_*^{-1}(\theta) = \prod_{j=1}^{N^*} \mathcal{G}_j^{-1}(\theta)$, where $\mathcal{G}_j^{-1}(\theta) = \exp \left(\frac{\theta}{2} \mathfrak{S}_j \right)$. Hence, it is sufficient to consider

$$f_j(\theta) = {}_{1,2}\langle 0 | \mathcal{G}_j^{-1}(\theta) | 0 \rangle_{1,2}. \quad (3.46)$$

Now

$$\begin{aligned}\frac{d}{d\theta} f_j(\theta) &= \frac{1}{2} {}_{1,2} \langle 0 | \mathfrak{S}_j \mathcal{G}_j^{-1}(\theta) | 0 \rangle_{1,2} \\ &= \frac{1}{2} {}_{1,2} \langle 0 | \gamma_{j-} \left(\tilde{a}_j^{(1)} \tilde{a}_{-j}^{(2)} + \tilde{a}_{-j}^{(1)} \tilde{a}_j^{(2)} \right) \mathcal{G}_j^{-1}(\theta) | 0 \rangle_{1,2}\end{aligned}\quad (3.47)$$

and

$$\left(\tilde{a}_j^{(1)} \tilde{a}_{-j}^{(2)} + \tilde{a}_{-j}^{(1)} \tilde{a}_j^{(2)} \right) \mathcal{G}_j^{-1}(\theta) = \mathcal{G}_j^{-1}(\theta) \left[-\frac{1}{2} \sin 2\theta \gamma_{j-} + \frac{1}{4} \sin^2 \theta \gamma_{j-}^2 \left(\tilde{a}_j^{(1)\dagger} \tilde{a}_{-j}^{(2)\dagger} + \tilde{a}_{-j}^{(1)\dagger} \tilde{a}_j^{(2)\dagger} \right) \right]. \quad (3.48)$$

Also, since $[\mathfrak{S}_j, \mathcal{G}_j^{-1}(\theta)] = 0$,

$$\begin{aligned}\frac{d}{d\theta} f_j(\theta) &= \frac{1}{2} {}_{1,2} \langle 0 | \mathcal{G}_j^{-1}(\theta) \mathfrak{S}_j | 0 \rangle_{1,2} \\ &= -\frac{1}{2} \gamma_{j-1,2} {}_{1,2} \langle 0 | \mathcal{G}_j^{-1}(\theta) \left(\tilde{a}_j^{(1)\dagger} \tilde{a}_{-j}^{(2)\dagger} + \tilde{a}_{-j}^{(1)\dagger} \tilde{a}_j^{(2)\dagger} \right) | 0 \rangle_{1,2}.\end{aligned}\quad (3.49)$$

From (3.49) and (3.47), we deduce that

$$\left(1 + \frac{\gamma_{j-}^2}{4} \sin^2 \theta \right) \frac{d}{d\theta} f_j(\theta) = -\frac{\gamma_{j-}^2}{4} \sin[2\theta] f_j(\theta), \quad (3.50)$$

which has a solution

$$f_j(\theta) \propto \left(1 + \frac{\gamma_{j-}^2}{4} \sin^2(\theta) \right)^{-1}. \quad (3.51)$$

Because $(m^{(1)2} - m^{(2)2})$ is small within the context of our model, we can still choose a sufficiently large j (say, >5) such that

$$\gamma_{j-}^2 \simeq \frac{(m^{(1)2} - m^{(2)2}) L^2}{4\pi^2 j^2} \quad (3.52)$$

and so

$$\prod_{j=0}^{N^*} f_j(\theta) \simeq \prod_{j=N'+1}^{N^*} \frac{1}{1 + ((m^{(1)2} - m^{(2)2}) L^2 / 4\pi^2 j^2) \sin^2 \theta} \times \prod_{j=0}^{N'} \frac{1}{1 + (\gamma_{j-}^2 / 4) \sin^2 \theta}, \quad (3.53)$$

where $N' \sim \frac{m^{(1)} m^{(2)} L}{\sqrt{2\pi(m^{(1)} + m^{(2)})}}$.

The factor

$$\prod_{j=0}^{N'} \frac{1}{1 + (\gamma_{j-}^2 / 4) \sin^2 \theta} \equiv F_{N'}$$

satisfies the bound

$$F_{N'} < \prod_{j=1}^{N'} \frac{1}{1 + a [1 - (j^2 / N'^2)]}, \quad (3.54)$$

where $a \equiv \frac{1}{4} \sin^2 \theta \frac{(m^{(1)} - m^{(2)})^2}{m^{(1)} m^{(2)}}$. The other factor in (3.50) can be written as

$$\prod_{j=N'+1}^{N^*} \frac{1}{1 + (b/j^2)},$$

where $b \equiv \frac{(m^{(1)2} - m^{(2)2}) L^2 \sin^2 \theta}{4\pi^2}$. Now we can define

$$\mathfrak{h}(b, n) = \prod_{j=1}^n \frac{1}{(1 + (b/j^2))} \quad (3.55)$$

and so

$$\prod_{j=N'+1}^{N^*} \frac{1}{1 + (b/j^2)} = \frac{\mathfrak{h}(b, N^*)}{\mathfrak{h}(b, N')}. \quad (3.56)$$

As $N^* \rightarrow \infty$, also $N' \rightarrow \infty$ and so we examine the asymptotic behaviour of $\mathfrak{h}(b, n)$. It can be shown that

$$\mathfrak{h}(b, n) = \frac{\Gamma^2(n+1) \Gamma(1 - i\sqrt{b}) \Gamma(1 + i\sqrt{b})}{\Gamma(1 - i\sqrt{b} + n) \Gamma(1 + i\sqrt{b} + n)}. \quad (3.57)$$

From Stirling's formula as $n \rightarrow \infty$

$$\frac{\Gamma^2(n+1)}{\Gamma(1 - i\sqrt{b} + n) \Gamma(1 + i\sqrt{b} + n)} \sim \left(1 - \frac{i\sqrt{b}}{n}\right)^{-1/2 + i\sqrt{b}} \left(1 + \frac{i\sqrt{b}}{n}\right)^{-1/2 - i\sqrt{b}} \sim 1.$$

Thus,

$$\prod_{j=N'+1}^{N^*} \frac{1}{1 + b/j^2}$$

does not contribute to any orthogonality of the flavour and mass vacua. An upper bound estimate $\mathfrak{F}_{N'}$ for $F_{N'}$ given in (3.54), is

$$\mathfrak{F}_{N'} = (1+a)^{-N'} \prod_{j=1}^{N'} \frac{1}{1 - (a/1+a)(j^2/N^2)}. \quad (3.58)$$

Now it can be shown that

$$\log \prod_{j=1}^{N'} \frac{1}{1 - (a/1+a)(j^2/N^2)} < N' \left[2 + \log(1+a) - 2\sqrt{\frac{a+1}{a}} \tanh^{-1} \left(\sqrt{\frac{a}{1+a}} \right) \right] \quad (3.59)$$

and since a can reasonably be assumed to be very small, and so

$$\log \prod_{j=1}^{N'} \frac{1}{1 - (a/1+a)(j^2/N^2)} < N' \left[\frac{1}{3}a - \frac{7}{30}a^2 + O\left(a^{\frac{5}{2}}\right) \right]. \quad (3.60)$$

From (3.58)

$$|\mathfrak{F}_{N'}| < \exp \left[-\frac{2a N'}{3} \right] \quad (3.61)$$

and so $\mathfrak{F}_{N'} \rightarrow 0$ as $N' \rightarrow \infty$. This demonstrates that the flavour and mass ground states are orthogonal separately in the asymptotic ‘in’ and ‘out’ Hilbert spaces provided a is nonzero (i.e. for nonzero $\sin^2 \theta$ and $(m^{(1)} - m^{(2)})^2$). This orthogonality is actually robust to modifications of the energy–momentum relation of the form $E^2 = \frac{p^2 + m^2}{1 - \eta((E/M_p))^n}$ due to possible quantum gravity effects, where n is a positive integer [23].

4. Vacua and the effect of expansion

4.1. Orthogonality

The effect of expansion is well known to result in particle production for co-moving observers. Consequently we shall consider its effect on the orthogonality of the flavour and massive vacua. The relation between ‘in’ and ‘out’ operators (cf (3.21) and (3.22)) connected by Bogoliubov coefficients can be effected in terms of an operator B . In terms of

$$\mathcal{S}(v_j^{(i)}) = \exp \left[v_j^{(i)} a_{-j}^{(i)\dagger} a_j^{(i)\dagger} - v_j^{(i)*} a_j^{(i)} a_{-j}^{(i)} \right] \quad (4.1)$$

and

$$P(\phi_j^{(i)}) = \exp(-i\varphi_j^{(i)} [a_j^{(i)\dagger} a_j^{(i)} + a_{-j}^{(i)\dagger} a_{-j}^{(i)}]), \quad (4.2)$$

$B(\varphi_j^{(i)}, v_j^{(i)})$ can be written as $\mathcal{S}(v_j^{(i)})P(\varphi_j^{(i)})$. It can be shown that

$$\begin{aligned} a_j^{(i)\text{in}} &= B^\dagger \left(\varphi_j^{(i)\text{out}}, v_j^{(i)\text{out}} \right) a_j^{(i)\text{out}} B \left(\varphi_j^{(i)\text{out}}, v_j^{(i)\text{out}} \right) \\ &= \frac{v_j^{(i)\text{out}} e^{i\varphi_j^{(i)\text{out}}}}{|v_j^{(i)\text{out}}|} \sinh \left(\left| v_j^{(i)\text{out}} \right| \right) a_{-j}^{(i)\text{out}\dagger} + \cosh \left(\left| v_j^{(i)\text{out}} \right| \right) e^{-i\varphi_j^{(i)\text{out}}} a_j^{(i)\text{out}} \end{aligned} \quad (4.3)$$

on identifying

$$\frac{\tilde{v}_j^{(i)\text{out}}}{|\tilde{v}_j^{(i)\text{out}}|} \sinh \left(\left| \tilde{v}_j^{(i)\text{out}} \right| \right) = -\beta_j^{*(i)}, \quad (4.4)$$

$$\cosh \left(\left| \tilde{v}_j^{(i)\text{out}} \right| \right) e^{-i\varphi_j^{(i)\text{out}}} = \alpha_j^{(i)*}, \quad (4.5)$$

where $\tilde{v}_j^{(i)\text{out}} \equiv v_j^{(i)\text{out}} e^{i\varphi_j^{(i)\text{out}}}$. Analogously

$$a_j^{(i)\text{out}} = B^\dagger \left(\varphi_j^{(i)\text{in}}, v_j^{(i)\text{in}} \right) a_j^{(i)\text{in}} B \left(\varphi_j^{(i)\text{in}}, v_j^{(i)\text{in}} \right) \quad (4.6)$$

with

$$\left| \tilde{v}_j^{(i)\text{in}} \right| = \left| \tilde{v}_j^{(i)\text{out}} \right| = \tanh^{-1} \left[\frac{\sinh \left((\pi \omega_{-,j}^{(i)} / \rho) \right)}{\sinh \left((\pi \omega_{+,j}^{(i)} / \rho) \right)} \right], \quad (4.7)$$

where we have used the natural generalizations of (3.10), (3.11) and (3.12) defined by

$$\omega_{\text{in},j}^{(i)} = \sqrt{\frac{4\pi^2 j^2}{L^2} + m^{(i)2}(A - B)}, \quad (4.8)$$

$$\omega_{\text{out},j}^{(i)} = \sqrt{\frac{4\pi^2 j^2}{L^2} + m^{(i)2}(A + B)} \quad (4.9)$$

and

$$\omega_{\pm,j}^{(i)} = \frac{1}{2} \left(\omega_{\text{out},j}^{(i)} \pm \omega_{\text{in},j}^{(i)} \right). \quad (4.10)$$

Also

$$e^{i\varphi_j^{(i)\text{out}}} = \sqrt{\frac{\omega_{\text{out},j}^{(i)} \Gamma\left(1 + (i\omega_{\text{in},j}^{(i)}/\rho)\right) \Gamma\left(i\omega_{\text{out},j}^{(i)}/\rho\right)}{\omega_{\text{in},j}^{(i)} \Gamma\left(i\omega_{+,j}^{(i)}/\rho\right) \Gamma\left(1 + (i\omega_{+,j}^{(i)}/\rho)\right) \cosh\left(\left|\tilde{v}_j^{(i)\text{in}}\right|\right)}} \quad (4.11)$$

Moreover $\tilde{v}_j^{(i)\text{in}} = -\tilde{v}_j^{(i)\text{out}}$ and $\varphi_j^{(i)\text{in}} = -\varphi_j^{(i)\text{out}}$ follows straightforwardly.

We have now the basic operators to calculate the overlap amplitude

$$\mathfrak{M}(1, 2; \alpha, \beta) = {}_{1,2,\text{in}}\langle 0|0\rangle_{\text{out},\alpha,\beta}, \quad (4.12)$$

which is equivalent to

$$\mathfrak{M}(1, 2; \alpha, \beta) = {}_{1,2,\text{in}}\langle 0|\mathcal{G}_*^{-1}(\theta)|0\rangle_{\text{out},1,2}. \quad (4.13)$$

The relation between ‘in’ and ‘out’ massive vacuum states ($|0\rangle_{\text{in},1,2}$ and $|0\rangle_{\text{out},1,2}$) can be deduced from

$$a_j^{(i)\text{in}}|0\rangle_{\text{in},1,2} = 0 = B^\dagger\left(\varphi_j^{(i)\text{out}}, v_j^{(i)\text{out}}\right) a_j^{(i)\text{out}} B\left(\varphi_j^{(i)\text{out}}, v_k^{(i)\text{out}}\right) |0\rangle_{\text{in},1,2} \quad (4.14)$$

since we require

$$a_j^{(i)\text{out}}|0\rangle_{\text{out},1,2} = 0 \quad \forall j. \quad (4.15)$$

Hence

$$|0\rangle_{\text{in},1,2} = \prod_{i=1}^2 \prod_{j \geq 0} B^\dagger\left(\varphi_j^{(i)\text{out}}, v_j^{(i)\text{out}}\right) |0\rangle_{\text{out},1,2}. \quad (4.16)$$

Similarly

$$|0\rangle_{\text{out},1,2} = \prod_{i=1}^2 \prod_{j \geq 0} B^\dagger\left(\varphi_j^{(i)\text{in}}, v_j^{(i)\text{in}}\right) |0\rangle_{\text{in},1,2}. \quad (4.17)$$

\therefore

$$\mathfrak{M}(1, 2; \alpha, \beta) = {}_{1,2,\text{out}}\langle 0|\prod_{i=1}^2 \prod_{j \geq 0} B\left(\varphi_j^{(i)\text{out}}, v_j^{(i)\text{out}}\right) \mathcal{G}_{*\text{out}}^{-1}(\theta)|0\rangle_{\text{out},1,2},$$

where $\mathcal{G}_{*\text{out}}^{-1}(\theta)$ is defined analogously as before (cf (3.29)).

$$\mathcal{F}_j(\theta) = {}_{1,2,\text{out}}\langle 0|\left\{ \prod_{i=1}^2 B\left(\varphi_j^{(i)\text{out}}, v_j^{(i)\text{out}}\right) \right\} \mathcal{G}_{\text{out},j}^{-1}(\theta)|0\rangle_{\text{out},1,2} \quad (4.18)$$

in terms of which

$$\mathfrak{M}(1, 2; \alpha, \beta) = \prod_{k>0} \mathcal{F}_k(\theta). \quad (4.19)$$

In the small θ approximation (appropriate for the size of effects generated by quantum gravity)

$$\mathcal{F}_k(\theta) \simeq_{1,2,\text{out}} \langle 0 | \left\{ \prod_{j=1}^2 \mathcal{S} \left(v_k^{(j)\text{out}} \right) \tilde{P} \left(\phi_k^{(j)\text{out}} \right) \right\} \left(1 + \frac{\theta}{2} \mathfrak{S}_{\text{out},k} + \frac{\theta^2}{8} \mathfrak{S}_{\text{out},k}^2 \right) | 0 \rangle_{\text{out},1,2}. \quad (4.20)$$

In terms of the operators

$$b_{+,j}^{(i)} \equiv \frac{a_{-j}^{(i)} + a_j^{(i)}}{\sqrt{2}}, \quad (4.21)$$

$$b_{-,j}^{(i)} \equiv \frac{a_{-j}^{(i)} - a_j^{(i)}}{\sqrt{2}}, \quad (\text{with } j > 0) \quad (4.22)$$

$$\mathcal{S}(v_j^{(i)}) = \exp \left[\frac{1}{2} \left(v_j^{(i)} b_{+,j}^{(i)\dagger 2} - v_k^{(j)*} b_{+,j}^{(i)2} \right) \right] \exp \left[-\frac{1}{2} \left(v_j^{(i)} b_{-,j}^{(i)\dagger 2} - v_j^{(i)*} b_{-,j}^{(i)2} \right) \right], \quad (4.23)$$

which has the form of a product of standard squeezing creation operators $\mathbb{S}(\xi) = \exp \left[\frac{1}{2} (\xi a^{\dagger 2} - \xi^* a^2) \right]$ which generate a Fock space representation of squeezed states

$$\mathbb{S}(\xi) | 0 \rangle = (1 - |\zeta|^2)^{\frac{1}{4}} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} \zeta^n | 2n \rangle, \quad (4.24)$$

where

$$\zeta = \frac{\xi}{|\xi|} \tanh(|\xi|). \quad (4.25)$$

On using this property, in terms of

$$\zeta_k^{(1)\text{out}} \equiv \frac{v_k^{(1)\text{out}}}{|v_k^{(1)\text{out}}|} \tanh \left(\left| v_k^{(1)\text{out}} \right| \right) \quad \text{and} \quad \zeta_k^{(2)\text{out}} \equiv \frac{v_k^{(2)\text{out}}}{|v_k^{(2)\text{out}}|} \tanh \left(\left| v_k^{(2)\text{out}} \right| \right),$$

it can be shown that

$$_{1,2,\text{out}} \langle 0 | \left\{ \prod_{j=1}^2 \mathcal{S} \left(v_k^{(j)\text{out}} \right) \tilde{P} \left(\phi_k^{(j)\text{out}} \right) \right\} | 0 \rangle_{\text{out},1,2} = \sqrt{1 - |\zeta_k^{(1)\text{out}}|^2} \sqrt{1 - |\zeta_k^{(2)\text{out}}|^2}. \quad (4.26)$$

Let us now compute the remaining terms in (4.20). We can show that

$$_{1,2,\text{out}} \langle 0 | \left\{ \prod_{i=1}^2 \mathcal{S} \left(v_j^{(i)\text{out}} \right) P \left(\phi_j^{(i)\text{out}} \right) \mathfrak{S}_{\text{out},j} \right\} | 0 \rangle_{\text{out},1,2} = 0. \quad (4.27)$$

The second-order term in θ gives

$${}_{1,2,\text{out}}\langle 0 | \left\{ \prod_{i=1}^2 \mathcal{S} \left(v_j^{(i)\text{out}} \right) P \left(\phi_j^{(i)\text{out}} \right) \mathfrak{S}_{\text{out},j}^2 \right\} | 0 \rangle_{\text{out},1,2} \\ = -2\gamma_{j,-}^2 \sqrt{1 - |\zeta_j^{(1)\text{out}}|^2} \sqrt{1 - |\zeta_j^{(2)\text{out}}|^2} \left[1 - \frac{3}{4} e^{-2i\eta_{\text{out},j}} \left(\zeta_j^{(1)\text{out}*} \zeta_j^{(2)\text{out}*} \right)^2 \right] \quad (4.28)$$

and so

$$\mathcal{F}_j(\theta) \simeq \sqrt{1 - |\zeta_j^{(1)\text{out}}|^2} \sqrt{1 - |\zeta_j^{(2)\text{out}}|^2} \left(1 - \frac{\theta^2}{4} \gamma_{j,-}^2 \left[1 - \frac{3}{4} e^{-2i\eta_{\text{out},j}} \left(\zeta_j^{(1)\text{out}*} \zeta_j^{(2)\text{out}*} \right)^2 \right] \right). \quad (4.29)$$

If the small θ expansion had not been made, the total expression would have been periodic in θ and so, to be compatible with the earlier calculation (in the absence of expansion), we can rewrite (4.29) as

$$\mathcal{F}_j(\theta) \simeq \frac{\sqrt{1 - |\zeta_j^{(1)\text{out}}|^2} \sqrt{1 - |\zeta_j^{(2)\text{out}}|^2}}{1 + \gamma_{j,-}^2 (\sin^2 \theta / 4) \left(1 - (3/4) e^{-2i\eta_{\text{out},j}} \left(\zeta_j^{(1)\text{out}*} \zeta_j^{(2)\text{out}*} \right)^2 \right)}, \quad (4.30)$$

which is again valid for small θ . Now the denominator can be bounded

$$\left| 1 + \gamma_{j,-}^2 \frac{\sin^2 \theta}{4} \left(1 - \frac{3}{4} e^{-2i\eta_{\text{out},j}} \left(\zeta_j^{(1)\text{out}*} \zeta_j^{(2)\text{out}*} \right)^2 \right) \right| > 1 + \gamma_{j,-}^2 \frac{\sin^2 \theta}{16} \left\{ 4 - 3 |\zeta_j^{(1)\text{out}}|^2 |\zeta_j^{(2)\text{out}}|^2 \right\}.$$

Hence

$$|\mathcal{F}_j(\theta)|^2 < \frac{\left(1 - |\zeta_j^{(1)\text{out}}|^2 \right) \left(1 - |\zeta_j^{(2)\text{out}}|^2 \right)}{\left(1 + \gamma_{j,-}^2 (\sin^2 \theta / 16) \left\{ 4 - 3 |\zeta_j^{(1)\text{out}}|^2 |\zeta_j^{(2)\text{out}}|^2 \right\} \right)^2}. \quad (4.31)$$

For large j ,

$$|\zeta_j^{(i)\text{out}}| \simeq \frac{m^{(i)2} L B}{2\pi j \rho} e^{-(2\pi^2 j / \rho L)} \quad (4.32)$$

which is a very sharp fall off with j . To leading order in B for small j and for small ρ (i.e. slow expansion)

$$|\zeta_j^{(i)}| \simeq \exp \left(-\frac{\pi}{2\rho A^{1/2}} \left[(2A - B) m^{(i)} + \frac{4\pi^2 j^2}{m^{(i)} L^2} \left(1 + \frac{B^2}{2A} \right) \right] \right). \quad (4.33)$$

This behaviour for $|\zeta_j^{(i)}|$ does not change the previous arguments for orthogonality and so the flavour vacuum (after expansion) and the original massive vacuum in the thermodynamic limit are orthogonal. In [1], it was asserted that this orthogonality implies that the two vacua are *unitarily inequivalent*, i.e. there is no unitary transformation that connects them, and that the Fock-space states constructed from one vacuum are orthogonal to the Fock-space states constructed from the other. The lack of a unitary transformation connecting the two vacua can be explicitly seen [11] in our approach by observing that the flavour vacuum is *not* normalized to 1, as should be the case if this vacuum were unitarily equivalent to the standard mass eigenstate vacuum.

4.2. Oscillation probability

Particle production during the expansion will lead to a modified oscillation and we shall examine the signature of this modification within this model. In the presence of expansion, the transition amplitude $\mathfrak{M}_{\text{osc}}(\alpha, k'; \beta, k)$ for the oscillation of flavour α (momentum $k' > 0$) to β (momentum $k > 0$) is

$$\mathfrak{M}_{\text{osc}}(\alpha, k'; \beta, k, \eta) = {}_{k', \text{in}} \langle \alpha | \beta(\eta) \rangle_{\text{out}, k}, \quad (4.34)$$

where (by definition) $|\iota(\eta)\rangle_{\text{out}} = a_{\dagger, \eta}^{\text{out}} |0\rangle_{\text{out}, \alpha, \beta}$ for $\iota = \alpha, \beta$ and similar definitions apply for ‘in’ states. (Previous formulae with discrete momentum labels j , when required, translate readily to the continuous k label.) Hence, in terms of the massive Fock space,

$$\begin{aligned} \mathfrak{M}_{\text{osc}}(\alpha, k'; \beta, k, \eta) = {}_{1,2, \text{in}} \langle 0 | & \left\{ \cos(\theta) a_{k'}^{(1)\text{in}} + \frac{\sin \theta}{2} \left(\gamma_{k', -} a_{-k'}^{\dagger(2)\text{in}} + \gamma_{k', +} a_{k'}^{(2)\text{in}} \right) \right\} \\ & \times \left\{ \cos \theta \tilde{a}_k^{(2)\text{out}\dagger} + \frac{\sin \theta}{2} \left(\gamma_{k, -} \tilde{a}_{-k}^{(1)\text{out}} - \gamma_{k, +} \tilde{a}_k^{(1)\text{out}\dagger} \right) \right\} |0\rangle_{\text{out}, 1,2}. \end{aligned} \quad (4.35)$$

It can be shown that

$$\begin{aligned} |0\rangle_{\text{out}, 1,2} = \prod_{k \geq 0} & \left(\sqrt{1 - |\zeta_k^{(1)\text{in}}|^2} \sqrt{1 - |\zeta_k^{(2)\text{in}}|^2} \sum_{n_{+,1}=0}^{\infty} \sum_{n_{+,2}=0}^{\infty} \sum_{n_{-,1}=0}^{\infty} \sum_{n_{-,2}=0}^{\infty} \mathfrak{g}_k^{\text{in}} \right. \\ & \left. \times (2n_{+,1}, 2n_{+,2}, 2n_{-,1}, 2n_{-,2}) |2n_{+,1}, 2n_{+,2}, 2n_{-,1}, 2n_{-,2}\rangle_{\text{in}} \right), \end{aligned} \quad (4.36)$$

where

$$\begin{aligned} \mathfrak{g}_k^{\text{in}}(2n_{+,1}, 2n_{+,2}, 2n_{-,1}, 2n_{-,2}) \equiv & \exp \left(2i \left\{ \varphi_k^{(1)\text{in}} [n_{+,1} + n_{-,1}] + \varphi_k^{(2)\text{in}} [n_{+,2} + n_{-,2}] \right\} \right) \\ & \times \zeta_k^{(1)\text{in}} [n_{+,1} + n_{-,1}] \zeta_k^{(2)\text{in}} [n_{+,2} + n_{-,2}] \frac{\sqrt{(2n_{+,1})! (2n_{+,2})! (2n_{-,1})! (2n_{-,2})!}}{2^{n_{+,1} + n_{+,2} + n_{-,1} + n_{-,2}} (n_{+,1}! n_{+,2}! n_{-,1}! n_{-,2}!)}. \end{aligned} \quad (4.37)$$

Hence

$$\begin{aligned} \mathfrak{M}_{\text{osc}}(\alpha, k'; \beta, k, \eta) = & \left\{ \prod_{k'' \geq 0} \sqrt{\left(1 - |\zeta_{k''}^{(1)}|^2\right) \left(1 - |\zeta_{k''}^{(2)}|^2\right)} \right\} \\ & \times \delta_{k'k} \{ \varkappa_{1,k} \nu_{1,k}(\eta) + \varkappa_{2,k} \nu_{2,k}(\eta) + \varkappa_{3,k} \nu_{3,k}(\eta) + \varkappa_{4,k} \nu_{4,k}(\eta) \} \end{aligned} \quad (4.38)$$

with $\varkappa_{1,k} = \frac{1}{\sqrt{2}} \cos \theta = -\varkappa_{2,k}$, $\varkappa_{3,k} = \frac{1}{2^{3/2}} \sin \theta$, $\gamma_{k,+} = -\varkappa_{4,k}$ and

$$\nu_{1,k}(\eta) = \frac{1}{2^{3/2}} \sin \theta \left(\gamma_{k,-} \beta_{-k}^{(1)*} e^{-i\omega_k^{(1)\text{out}} \eta} - \gamma_{k,+} \alpha_k^{(1)*} e^{i\omega_k^{(1)\text{out}} \eta} \right) = -\nu_{2,k}(\eta)$$

and

$$\nu_{3,k}(\eta) = \frac{1}{\sqrt{2}} \cos \theta \alpha_k^{(2)*} e^{i\omega_k^{(2)\text{out}} \eta} = -\nu_{4,k}(\eta).$$

Equation (4.38) simplifies to

$$\mathfrak{M}_{\text{osc}}(\alpha, \mathbf{k}'; \beta, \mathbf{k}, \eta) = \frac{1}{4} \delta_{\mathbf{k}'\mathbf{k}} \left\{ \prod_{k'' \geq 0} \sqrt{\left(1 - |\zeta_{k''}^{(1)}|^2\right) \left(1 - |\zeta_{k''}^{(2)}|^2\right)} \right\} \\ \times \sin 2\theta \left[\gamma_{k,+} \left(\alpha_k^{(2)*} e^{i\omega_k^{(2)\text{out}}\eta} - \alpha_k^{(1)*} e^{i\omega_k^{(1)\text{out}}\eta} \right) + \gamma_{k,-} \beta_{-k}^{(1)*} e^{-i\omega_k^{(1)\text{out}}\eta} \right] \quad (4.39)$$

and can be readily interpreted as a modification of the usual formula for oscillations with the extra momentum dependence (due to the field theoretic treatment); now we have the *additional* modification due to expansion which is encoded in both the square root factor as well as the α and β coefficients. There is even a further effect in the oscillation frequency due to the renormalization of the frequency represented by ω^{out} . We can readily estimate the values of α and β for both large and small $|k|$; for large $|k|$ we find $\alpha_k \sim e^{-\frac{\pi|k|}{2\rho}} \left(\frac{\rho}{|k|}\right)^{1/2}$ and $\beta_k \sim -2i\pi^2 \frac{m^2 B}{\rho|k|} e^{-(\pi|k|/\rho)}$, where the mass index has been suppressed.

5. The equation of state for the flavour vacuum

The flavour vacuum as we have seen is orthogonal to the vacuum in terms of mass eigenstates in the thermodynamic limit even in the presence of expansion. Arguments based on D-particle scattering of stringy matter give support to the flavour vacuum as the correct vacuum. By considering the expectation value of the stress-energy tensor, we can see to what extent it might play the role of dark energy. This calculation, unlike previous ones, will consider a non-flat metric which is of course necessary if there is an expanding universe. Our aim is to investigate the theoretical possibility of a dark energy contribution and the issues of D-particle capture and recoil, rather than phenomenological relevance. Both early and late time expectation values are calculated.

The generic stress-energy tensor $T_{\mu\nu}$ for our scalar theory is formally given by

$$T_{\mu\nu} = \frac{1}{2} (\phi_{,\mu} \phi_{,\nu} + \phi_{,\nu} \phi_{,\mu}) - g_{\mu\nu} \frac{L}{\sqrt{-g}}, \quad (5.1)$$

where L is the Lagrangian density, $g = \det(g_{\mu\nu})$ and $\phi_{,\mu} = \frac{\partial \phi}{\partial x^\mu}$ (the x^μ are the generic space-time coordinates, 1+1-dimensional in our case). There are of course operator ordering ambiguities which we shall address later. In our model

$$T_{00} = \sum_{i=1}^2 \left\{ \frac{1}{2} \left(\frac{\partial \phi^{(i)}}{\partial \eta} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi^{(i)}}{\partial x} \right)^2 + \frac{m^{(i)2} C(\eta)}{2} \phi^{(i)2} \right\}, \quad (5.2)$$

$$T_{11} = \sum_{i=1}^2 \left\{ \frac{1}{2} \left(\frac{\partial \phi^{(i)}}{\partial x} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi^{(i)}}{\partial \eta} \right)^2 - \frac{m^{(i)2} C(\eta)}{2} \phi^{(i)2} \right\}, \quad (5.3)$$

$$T_{01} = \frac{1}{2} \sum_{i=1}^2 \left(\frac{\partial \phi^{(i)}}{\partial \eta} \frac{\partial \phi^{(i)}}{\partial x} + \frac{\partial \phi^{(i)}}{\partial x} \frac{\partial \phi^{(i)}}{\partial \eta} \right). \quad (5.4)$$

We shall consider expectation values of $T_{\mu\nu}$ for the asymptotic early and late times. For reasons of rigour, we will work in the discrete momentum basis. In this basis (suppressing ‘in’ and ‘out’ indices), let us define

$$b_j^{(i)}(\eta, x) \equiv -a_j^{(i)} e^{i(2\pi j/Lx - \omega_j^{(i)}\eta)} + a_j^{(i)\dagger} e^{-i(2\pi j/Lx - \omega_j^{(i)}\eta)} \quad (5.5)$$

and

$$d_j^{(i)}(\eta, x) \equiv a_j^{(i)} e^{i((2\pi j/L)x - \omega_j^{(i)}\eta)} + a_j^{(i)\dagger} e^{-i((2\pi j/L)x - \omega_j^{(i)}\eta)}. \quad (5.6)$$

In terms of these operators

$$T_{00} = \frac{1}{2} T_{00}^{(1)} + \frac{1}{2} T_{00}^{(2)} + \frac{C(\eta)}{2} T_{00}^{(3)}, \quad (5.7)$$

$$T_{11} = \frac{1}{2} T_{00}^{(2)} + \frac{1}{2} T_{00}^{(1)} - \frac{C(\eta)}{2} T_{00}^{(3)}, \quad (5.8)$$

$$T_{01} = \pi \sum_{i=1}^2 \sum_{j,j'} v_{jj'}^{(i)} b_j^{(i)}(\eta, x) b_{j'}^{(i)}(\eta, x), \quad (5.9)$$

where

$$T_{00}^{(1)} = - \sum_{i=1}^2 \left(\sum_j \sqrt{\frac{\omega_j^{(i)}}{2L}} b_j^{(i)}(\eta, x) \right)^2, \quad (5.10)$$

$$T_{00}^{(2)} = -4\pi^2 \sum_{i=1}^2 \left(\sum_j \frac{j}{\sqrt{2L^3 \omega_j^{(i)}}} b_j^{(i)}(\eta, x) \right)^2, \quad (5.11)$$

$$T_{00}^{(3)} = - \sum_{i=1}^2 \left(\sum_j \frac{m^{(i)}}{\sqrt{2L \omega_j^{(i)}}} d_j^{(i)}(\eta, x) \right)^2, \quad (5.12)$$

and

$$v_{jj'}^{(i)} = \frac{1}{2L^2} \left(\sqrt{\frac{\omega_j^{(i)}}{\omega_{j'}}} j' + \sqrt{\frac{\omega_{j'}^{(i)}}{\omega_j}} j \right).$$

We will now calculate ${}_{\text{in},\alpha,\beta} \langle 0 | T_{\mu\nu}^{\text{out}} | 0 \rangle_{\alpha,\beta,\text{in}}$. It is straightforward to show that

$${}_{\text{in},\alpha,\beta} \langle 0 | b_j^{(i)\text{out}}(\eta, x) b_{j'}^{(i)\text{out}}(\eta, x) | 0 \rangle_{\alpha,\beta,\text{in}} = \delta_{jj'} f_{j,i}^{(+)} + \delta_{j-j'} f_{j,i}^{(-)} \quad (5.13)$$

and

$${}_{\text{in},\alpha,\beta} \langle 0 | d_j^{(i)\text{out}}(\eta, x) d_{j'}^{(i)\text{out}}(\eta, x) | 0 \rangle_{\alpha,\beta,\text{in}} = -\delta_{jj'} f_{j,i}^{(+)} + \delta_{j-j'} f_{j,i}^{(-)}. \quad (5.14)$$

The expressions for $f_{j,i}^{(\pm)}$ are given in the appendix. There are infinities in the expectation value of the stress tensor which have conventionally been removed by renormalization. In flat space-time, this has been achieved by a suitable normal ordering. Certainly, if we consider our model conventionally as a quantum field theory in a time-dependent metric then we would have to renormalize [10]. This would involve state-dependent counter terms which, in a covariant procedure, can be tensorially constructed from the metric tensor. However, our model, as we have seen, is motivated from D-particle capture of stringy matter and so its interpretation cannot be completely that of a conventional field theory. We will look at both the low- and high-momenta behaviour of $f_{j,i}^{(\pm)}$. The high-momenta behaviour will indicate the necessity of

‘conventional’ renormalization since the flavour vacuum is indistinguishable from the standard massive vacuum, while the low-momenta behaviour will give the contribution adapted to the D-particle capture approach.

The results can be summarized as:

$${}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(1)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} = -\frac{1}{2L} \left\{ \sum_j \sum_{i=1}^2 \omega_j^{(i)} \left(f_{j,i}^{(+)} + f_{j,i}^{(-)} \right) \right\}, \quad (5.15)$$

$${}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(2)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} = -4\pi^2 \sum_j \sum_{i=1}^2 \frac{j^2}{2L^3 \omega_j^{(i)}} \left\{ f_{j,i}^{(+)} - f_{j,i}^{(-)} \right\}, \quad (5.16)$$

$${}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(3)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} = \frac{1}{2L} \sum_j \sum_{i=1}^2 \frac{m^{(i)2}}{\omega_j^{(i)}} \left(-f_{j,i}^{(-)} + f_{j,i}^{(+)} \right) \quad (5.17)$$

and

$${}_{\text{in},\alpha,\beta}\langle 0|T_{01}^{\text{out}}|0\rangle_{\alpha,\beta,\text{in}} = 0. \quad (5.18)$$

Hence

$$\begin{aligned} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{\text{out}}|0\rangle_{\alpha,\beta,\text{in}} &= \frac{1}{2} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(1)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} + \frac{1}{2} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(2)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} \\ &\quad + \frac{C(\eta)}{2} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(3)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} \\ &= -\frac{1}{4L} \sum_j \sum_{i=1}^2 \omega_j^{(i)} \left(f_{j,i}^{(+)} + f_{j,i}^{(-)} \right) - 2\pi^2 \sum_j \sum_{i=1}^2 \frac{j^2}{2L^3 \omega_j^{(i)}} \left\{ f_{j,i}^{(+)} - f_{j,i}^{(-)} \right\} \\ &\quad + \frac{C(\eta)}{4L} \sum_j \sum_{i=1}^2 \frac{m^{(i)2}}{\omega_j^{(i)}} \left(f_{j,i}^{(+)} - f_{j,i}^{(-)} \right) \end{aligned} \quad (5.19)$$

and

$$\begin{aligned} {}_{\text{in},\alpha,\beta}\langle 0|T_{11}^{\text{out}}|0\rangle_{\alpha,\beta,\text{in}} &= \frac{1}{2} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(2)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} + \frac{1}{2} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(1)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} \\ &\quad - \frac{C(\eta)}{2} {}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{(3)\text{out}}|0\rangle_{\alpha,\beta,\text{in}} \\ &= -2\pi^2 \sum_j \sum_{i=1}^2 \frac{j^2}{2L^3 \omega_j^{(i)}} \left\{ f_{j,i}^{(+)} - f_{j,i}^{(-)} \right\} - \frac{1}{4L} \left\{ \sum_j \sum_{i=1}^2 \omega_j^{(i)} \left(f_{j,i}^{(+)} + f_{j,i}^{(-)} \right) \right\} \\ &\quad - \frac{C(\eta)}{4L} \sum_j \sum_{i=1}^2 \frac{m^{(i)2}}{\omega_j^{(i)}} \left(-f_{j,i}^{(-)} + f_{j,i}^{(+)} \right). \end{aligned} \quad (5.20)$$

The expressions in (5.19) and (5.20) can be normal-ordered with respect to a suitable vacuum. Unlike the elementary field theory, there is more than one obvious choice. The equation of state will vary depending on this choice and can give physical insight into the fluid-like properties of the unusual flavour vacuum in the presence of an expanding universe; so a normal-ordered expectation can be defined as

$${}_{\text{in},\alpha,\beta}\langle 0| : T_{11}^{\text{out}} : |0\rangle_{\alpha,\beta,\text{in}} = {}_{\text{in},\alpha,\beta}\langle 0|T_{11}^{\text{out}}|0\rangle_{\alpha,\beta,\text{in}} - \langle \Psi|T_{11}^{\text{out}}|\Psi\rangle, \quad (5.21)$$

where $|\Psi\rangle$ is a suitable ‘vacuum’ state. We should remark that various types of ‘normal’ orderings can be accommodated through the choice of $|\Psi\rangle$. The usual one in Minkowski space–time reads:

$$:b_j^{(i)}(\eta, x)b_{j'}^{(i)}(\eta, x): = b_j^{(i)}(\eta, x)b_{j'}^{(i)}(\eta, x) + \delta_{jj'}, \quad (5.22)$$

$$:d_j^{(i)}(\eta, x)d_{j'}^{(i)}(\eta, x): = d_j^{(i)}(\eta, x)d_{j'}^{(i)}(\eta, x) - \delta_{jj'}. \quad (5.23)$$

In our D-particle foam model, which motivates our use of the ‘flavour’ vacuum, the physically correct normal ordering is dictated by the underlying microscopic physics of the foamy space–time at Planck scales, and the non-trivial interactions with it of *some* of the flavour momentum modes, specifically the low-energy ones. A cloud of D-particles induces a metric which is proportional to σ^2 (cf (2.22) and (2.23)). In our picture, the vacuum contribution to the expectation value of the stress tensor is due to the D-particle recoil-velocity (statistical) Gaussian fluctuations (2.21), i.e. we subtract away the σ independent contribution, following our earlier treatment of the induced stochastic light-cone fluctuations due to the recoil-velocity fluctuations [6]. This is consistent with the recovery of the Minkowski, Lorentz-invariant situation (cf (2.22)) in the absence of D-particle recoil-velocity effects, i.e. the vanishing of the vacuum expectation value of the matter stress-energy tensor in the limit $\sigma^2 \rightarrow 0$.

We should recall here that for a relativistic perfect fluid

$$T_{\mu\nu} = -pg_{\mu\nu} + (p + \rho)v_\mu v_\nu,$$

where v_μ is the 4-velocity of the observer. From (5.15), (5.16) and (5.17) and (5.7), (5.8), (5.9) we have the form of the expectation values of the various components of the stress-energy tensor, and from those we can conclude about the equation of state of our boson gas and check whether $p = w\rho$ with $w = -1$, the expected form for cosmological constant. In order to obtain finite expressions for p and ρ , we would need some form of momentum cut-off for the low k modes. However, on identifying (in a co-moving cosmological frame) $p = {}_{\text{in},\alpha,\beta} \langle 0 | : T_{11}^{\text{out}} : | 0 \rangle_{\alpha,\beta,\text{in}}$ and $\rho = {}_{\text{in},\alpha,\beta} \langle 0 | : T_{00}^{\text{out}} : | 0 \rangle_{\alpha,\beta,\text{in}}$ and on adopting the σ independent subtraction procedure, we find interestingly (cf (5.19) and (5.20)) that

$$w = -1. \quad (5.24)$$

It is important to notice that (5.24) has been obtained by a subtraction procedure which was adopted *independently* of the particular choice of the expanding universe conformal factor $C(\eta)$. This is important, in that it allows the back reaction effects of the flavour vacuum onto the background space–time to be incorporated self-consistently in our approach.

The determination of p and ρ , which are vacuum expectation values, involves a tacit averaging over the time-scale τ in our vacuum. We earlier estimated τ to be extremely short and so a time average over it is equivalent to putting $\eta = 0$ in the expressions. The contribution to the cosmological constant from this vacuum is given by ρ .

It must be noted at this stage that the above calculations have been performed in the regime in which the expansion rate of the universe is slow, and the space–time is almost a Minkowskian one. Hence, the cosmological constant like-equation of state that characterizes our Lorentz-violating vacuum pertains only to late eras of the universe. In general, one might encounter situations with a time-dependent equation of state and a time varying dark energy (i.e. quintessence).

We have earlier argued that the capture process is likely to be most efficient for stringy matter with small k . In order to get an estimate for the momentum cut-off of the capture process, and thus of the flavoured-vacuum contribution to the cosmological constant, we can adopt an argument in [24] by considering the scale inherent in the k dependence of the single particle momentum distributions in the flavour vacuum

$$n_{\iota}(k, \eta) \equiv {}_{\alpha, \beta} \langle 0 | \tilde{a}_{\iota, k}^{\dagger}(\eta, x) \tilde{a}_{\iota, k}(\eta, x) | 0 \rangle_{\alpha, \beta} \quad (5.25)$$

for $\iota = \alpha, \beta$. It can be shown that [11]

$$n_{\alpha, k}(\eta) = \gamma_{k, -}^2 \left[\frac{\sin^4 \theta}{4} \gamma_{k, +}^2 \sin^2 \omega_{\text{in}, k}^{(2)} \eta + \frac{\sin^2 2\theta}{4} \sin^2 \left(\omega_{\text{in}, k}^{(1)} + \omega_{\text{in}, k}^{(2)} \right) \eta \right] \quad (5.26)$$

and

$$n_{\beta, k}(\eta) = \gamma_{k, -}^2 \left[\frac{\sin^4 \theta}{4} \gamma_{k, +}^2 \sin^2 \omega_{\text{in}, k}^{(1)} \eta + \frac{\sin^2 2\theta}{4} \sin^2 \left(\omega_{\text{in}, k}^{(1)} + \omega_{\text{in}, k}^{(2)} \right) \eta \right] \quad (5.27)$$

(and we also have the identity $\gamma_{k, +}^2 - \gamma_{k, -}^2 = 4$). The k dependence is dominated by the behaviour of $\gamma_{k, -}$. Now for large k

$$\gamma_{k, -} \sim \frac{1}{2} \frac{m^{(1)2} - m^{(2)2}}{k^2} - \frac{1}{4k^4} (m^{(1)4} - m^{(2)4}) \quad (5.28)$$

and so there is a scale (determined by the ratio of the two terms in the above)

$$k_0 \sim \frac{1}{\sqrt{2}} \sqrt{(m^{(1)2} + m^{(2)2})}, \quad (5.29)$$

which is a plausible cut-off scale in k . We note that $k_0 \sim \bar{m} + \frac{1}{8} \frac{\delta m^2}{\bar{m}}$, where $m^{(1)} = \bar{m} + \frac{1}{2} \delta m$ and $m^{(2)} = \bar{m} - \frac{1}{2} \delta m$. Hence, there is a fall off for $n_{\alpha, k}(t)$ with k determined by $\gamma_{k, -}^2$, which is qualitatively similar to the four-dimensional fermion case of [24] and to an earlier conjectured cut-off $\frac{m_1 + m_2}{2}$ [18] for $m_1 \sim m_2$.

From these considerations, we obtain the flavour-vacuum contributions to the cosmological constant by integrating over the extremely small timescales of capture, which according to our previous discussion is qualitatively equivalent to setting the time η -dependence to $\eta = 0$. After taking appropriately the continuum limit, $\frac{j}{L} \rightarrow \frac{k}{2\pi}$, $\frac{1}{L} \sum_j \rightarrow \frac{1}{2\pi} \int dk \rightarrow \frac{1}{2\pi} \int_0^{k_0} dk$, with the cut-off k_0 given by (5.29), we can obtain an estimate for the cosmological constant contribution due to the flavour vacuum by considering the case of small expansion and expansion rate. Returning to the expression for ${}_{\text{in}, \alpha, \beta} \langle 0 | T_{00}^{\text{out}} | 0 \rangle_{\alpha, \beta, \text{in}}$ in (5.19), we renormalize the vacuum expectation so as to leave the contribution from the string capture of the stringy matter. One part of this renormalization is the C-independent subtraction which leads to

$${}_{\text{in}, \alpha, \beta} \langle 0 | T_{00}^{\text{out}} | 0 \rangle_{\alpha, \beta, \text{in}} = \frac{C(\eta)}{8\pi} \sum_{i=1}^2 \int_0^{k_0} dk \frac{m^{(i)2}}{\omega_k^{(i)}} \left(f_{k,i}^{(+)} - f_{k,i}^{(-)} \right) \quad (5.30)$$

and

$${}_{\text{in}, \alpha, \beta} \langle 0 | T_{11}^{\text{out}} | 0 \rangle_{\alpha, \beta, \text{in}} = -\frac{C(\eta)}{8\pi} \sum_{i=1}^2 \int_0^{k_0} dk \frac{m^{(i)2}}{\omega_k^{(i)}} \left(f_{k,i}^{(+)} - f_{k,i}^{(-)} \right). \quad (5.31)$$

As noted earlier, we can already see the D-particle contribution to the flavour vacuum giving $w = -1$. We further renormalize $f_{k,i}^{(\pm)}$ to only include the $\gamma_{k, -}^{\text{in}}$ dependent terms since it is only

such terms which distinguish it from the mass vacuum state. Details are given in the appendix. To leading order in δm , on averaging over the uncertainty of the capture time η (which is effectively instantaneous) and owing to (A.20) and (A.21)

$${}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{\text{out}}|0\rangle_{\alpha,\beta,\text{in}} = 0. \quad (5.32)$$

In the next-to-leading order in δm

$${}_{\text{in},\alpha,\beta}\langle 0|T_{00}^{\text{out}}|0\rangle_{\alpha,\beta,\text{in}} \sim -\frac{\sin^2 \theta}{24\pi}(\delta m)^2 + \frac{(\delta m)^2}{8\pi} \sin^2 \theta (1 - C(\eta_0)), \quad (5.33)$$

where η_0 is the capture time which in our model can be taken to be $\eta_0 = 0$. For small $\tilde{\sigma}$, we have $C(\eta_0) \sim 1 - 4\tilde{\sigma}^2$ (cf (2.22) and (2.23)). Hence, finally making the subtraction of the $\tilde{\sigma}$ independent terms we get

$${}_{\text{in},\alpha,\beta}\langle 0|:T_{00}^{\text{out}}:|0\rangle_{\alpha,\beta,\text{in}} \sim \frac{(\delta m)^2}{2\pi} \sin^2 \theta \tilde{\sigma}^2. \quad (5.34)$$

This shows the contribution to the dark energy from the flavour changing D-brane-recoil (statistical) fluctuations.

The reader should notice that, within our effective field theory framework, the (small) fluctuation parameter $\tilde{\sigma}^2$ should be considered as *phenomenological*. In order to determine its real order of magnitude, one needs detailed microscopic stringy models of such D-particle foam, along the lines proposed in [4], in the context of brane world cosmologies (cf figure 1). This issue is still far from being complete, given that realistic three-space-dimensional brane world models of D-particle foam require appropriate compactification, consistent with phenomenologically realistic supersymmetry breaking scenarios. We postpone a discussion of such issues for future work.

A related comment concerns the asymptotic in time nature of the flavour-oscillation-induced de Sitter vacuum energy (5.34). The reader should recall that this result was derived on the assumption that, asymptotically in cosmic time, one has a uniform *constant* density of defects, and the space–time approaches the flat, but not standard Minkowski (i.e. $\tilde{\sigma}^2$ -dependent), limit (2.22). This will cause eternal acceleration, as the situation is equivalent to that of a cosmological constant, and perturbative scattering matrix and asymptotic states will not be well defined. This may have consequences on a quantum field theory of matter defined in such space–times, especially from the point of view of an *ill*-defined CPT operator, a situation which might arise in such a case due to decoherence [25] associated with the existence of future horizons (cosmologically induced *intrinsic* CPT violation (see for instance [26]), in particular section 2.8).

On the other hand, in cases where the asymptotic density of defects crossing our brane world vanishes, it is evident that the flavour-changing induced vacuum energy (5.34) (and also pressure, in view of (5.24)) will also *vanish* asymptotically. In such a case, the situation will resemble the one illustrated in the example of the space–time (3.4) in section 3, where the de Sitter phase characterizes only a certain era of the universe, relaxing asymptotically to a standard flat Minkowski space–time. In the latter case, the perturbative scattering matrix and asymptotic states are then well defined and the associated quantum field theory of matter fields on the brane world is CPT invariant, at least from a cosmological view point.

6. Conclusions and outlook

We have discussed here an approach to the flavour mixing problem and its consequences to the vacuum energy in a toy model for bosons, taking into account the effects of universe expansion.

We have attempted to argue that the flavour (Fock space) vacuum introduced by Blasone and Vitiello in order to describe canonical quantization of theories with mixing characterizes certain string models entailing Lorentz violation, in particular D-particle foam models. In such models, the interaction of certain (electrically neutral) string excitations with the D-particle defects cause back reaction effects onto the space–time, due to the capture of the string state by the D-particle and subsequent recoil of the space–time defect. When a statistically significant population of such defects is present in our brane world, then their effects are global, and may result in an expanding Robertson–Walker universe, depending on the details of the time profile of the D-particle density. We have provided arguments in favour of such a behaviour in the context of specific brane models propagating in a bulk space–time, punctured by D-particles.

During the capture/recoil process, flavour may not be conserved, and it is in this sense that the D-foam background provides a way for flavour mixing. The recoil of the defects causes Lorentz violation, as a result of the induced momentum-dependent Finsler-type [14] metric in the process. Such a Lorentz violation may be averaged to zero in D-foam models, but leaves traceable effects on quantum fluctuations.

In such an expanding universe, with flavour mixing, the vacuum felt by the string states is not the normal vacuum but the Fock-space flavoured vacuum introduced by Blasone and Vitiello in theories with mixing. However, in contrast to their generic considerations, in our approach the introduction of the flavour vacuum is a result of a specific underlying microscopic model of space–time background with defects interacting with some but not all matter excitations.

The details of the D-particle foam played an important role in determining the characteristic timescales involved in the problem (especially the (small) duration of the capture of the string state by the defect), and as a consequence the pertinent time intervals over which one has to average in order to obtain the contribution to the vacuum energy.

We have calculated the effects of the universe expansion in the flavour-vacuum formalism for a toy model of bosons. By taking into account the details of our specific model involving D-particle defects, we have been able to determine the physically relevant subtraction scheme in the calculation of the flavour-vacuum expectation value of the matter stress-energy tensor. In this way, we have obtained an equation of state of cosmological constant-type for (late) eras of the universe, where the expansion rate is small. This is consistent with the constant contributions to the vacuum energy obtained by averaging over the relevant (small) timescales in the problem. Although our considerations are toy, as referring to two-dimensional target space–time, and hence one cannot make a direct connection with dark-energy observations, nevertheless, the order of magnitude of this non-perturbative contribution (5.34) to the cosmological constant can be made phenomenologically consistent with the small value suggested by observations today, provided the order of magnitude of the recoil-velocity (statistical) fluctuations σ^2 , which it depends upon, is sufficiently small. Moreover, the contribution vanishes if the mass difference or the mixing angle vanish, as expected in the generic approach of Blasone and Vitiello.

We should extend the above analysis to include fermions, and also we should consider higher-dimensional isotropic universe situations. Moreover, despite the motivations from string theory, our considerations have been restricted so far to the context of effective low-energy

field theories. It would be interesting to extend these results to pure stringy considerations and attempt to estimate microscopically the order of the recoil velocity fluctuations, σ^2 , in realistic models. Such a task is non-trivial, as it requires first an understanding of the details of the compactification procedure, consistent with supersymmetry breaking in target space. Flux compactification has been briefly used in our stringy model in the arguments in favour of the identification of the Liouville mode of our supercritical recoil string model with a function of target time. It is our belief that such compactifications provide a way of resolving the problem of the smallness of the dark energy of the universe today consistently with phenomenologically realistic values of supersymmetry-breaking mass scales. We hope to start tackling such issues in the near future.

A final comment we would like to make concerns the global effect of the induced de Sitter space-time on photons. According to the analysis in [27], in four-dimensional de Sitter space-times, vacuum polarization of charged scalar fields, such as charged Higgs in supersymmetric theories, implies mass for the long wavelength photon modes that cross the horizon H^{-1} of the (locally) de Sitter space-time. Indeed, let \vec{k} denote the momentum of such modes, with $c|\vec{k}| \gg H$ initially at some (conformal) time $\eta_0 = -H^{-1}$, but whose actual (physical) magnitude is diminished significantly due to the universe expansion at a later time η , after a long period of inflation, $|\vec{k}|/a(\eta)$, with $a(\eta)$ the scale factor such that $a(t) \gg c|\vec{k}| \gg H$. Then the induced mass m_γ of the photon is, to leading order in the appropriate expansions [27]:

$$m_\gamma \sim \sqrt{\alpha} c^{-2} \hbar H \left[\frac{2}{\pi} \ln \left(\frac{c|\vec{k}|}{H} \right) + \mathcal{O}(1) \right]^{1/2}, \quad (6.1)$$

where α is the fine structure constant. In our case (5.34), the flavour-vacuum-induced Hubble parameter is extremely small (at late eras of the universe), of order $|(\delta m)| \sin^{1/2} \theta \sqrt{\tilde{\sigma}^2}$, and thus only extremely long-wavelength (far infrared) photon modes would acquire a (very small) mass via the vacuum-polarization mechanism of [27]. However, in the early universe, our statistical D-particle fluctuations could be much larger, according to the models of D-foam discussed above, and thus one does not exclude the possibility of significant photon masses (in supersymmetric theories, where charged scalar fields occur naturally) via this mechanism. As discussed in [27], this may produce weak, but of cosmological-scale relevance, *seed* magnetic fields, which could be amplified to produce the micro-Gauss magnetic fields observed in galaxies and galactic clusters. It would be interesting to examine such scenarios in the context of our flavour-vacuum induced de Sitter space-time, as such studies may provide indirect tests of (and impose constraints on) space-time foam models from novel view points. We hope to tackle such issues in a forthcoming publication.

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Appendix

In this appendix we calculate the quantities $f_{k,i}^{(\pm)}$, appearing in the appropriately normal-ordered (subtracted) expressions (5.30) and (5.31) for the boson stress-energy tensor flavour-vacuum

expectation value, for slow universe-expansion rate and small momenta k (with respect to the boson mass).

It is straightforward to demonstrate that

$$f_{k,i}^{(+)} = - \left[\left| \mathfrak{A}_{k,i}^{(1)} \right|^2 + \left| \mathfrak{A}_{k,i}^{(2)} \right|^2 + \left| \tilde{\mathfrak{A}}_{-k,i}^{(1)} \right|^2 + \left| \tilde{\mathfrak{A}}_{-k,i}^{(2)} \right|^2 \right], \quad (\text{A.1})$$

$$f_{k,i}^{(-)} = - \left[\mathfrak{A}_{k,i}^{(1)} \tilde{\mathfrak{A}}_{k,i}^{(1)*} + \mathfrak{A}_{k,i}^{(2)} \tilde{\mathfrak{A}}_{k,i}^{(2)*} + \tilde{\mathfrak{A}}_{-k,i}^{(1)} \mathfrak{A}_{-k,i}^{(1)*} + \tilde{\mathfrak{A}}_{-k,i}^{(2)} \mathfrak{A}_{-k,i}^{(2)*} \right], \quad (\text{A.2})$$

where

$$\mathfrak{A}_{k,i}^{(1)} = -\delta_{i1} \alpha_k^{(1)} e^{i(kx - \omega_k^{(1)\text{out}} \eta)} \cos \theta + \frac{1}{2} \delta_{i2} e^{i(kx - \omega_k^{(2)\text{out}} \eta)} \left[-\alpha_k^{(2)} \gamma_{k,+}^{\text{in}} e^{i(\omega_k^{(2)\text{in}} - \omega_k^{(1)\text{in}}) \eta} \right. \\ \left. + \beta_k^{(2)*} \gamma_{k,-}^{\text{in}} e^{-i(\omega_k^{(2)\text{in}} + \omega_k^{(1)\text{in}}) \eta} \right] \sin \theta, \quad (\text{A.3})$$

$$\mathfrak{A}_{k,i}^{(2)} = \frac{1}{2} \delta_{i1} e^{i(kx - \omega_k^{(1)\text{out}} \eta)} \sin \theta \left\{ \gamma_{k,+}^{\text{in}} \alpha_k^{(1)} e^{-i(\omega_k^{(2)\text{in}} - \omega_k^{(1)\text{in}}) \eta} + \gamma_{k,-}^{\text{in}} \beta_k^{(1)*} e^{-i(\omega_k^{(2)\text{in}} + \omega_k^{(1)\text{in}}) \eta} \right\} \\ - \delta_{i2} e^{i(kx - \omega_k^{(2)\text{out}} \eta)} \alpha_k^{(2)} \cos \theta, \quad (\text{A.4})$$

$$\tilde{\mathfrak{A}}_{-k,i}^{(1)} = \delta_{i1} e^{-i(kx - \omega_k^{(1)\text{out}} \eta)} \beta_j^{(1)} \cos \theta + \frac{1}{2} \delta_{i2} e^{-i(kx - \omega_k^{(2)\text{out}} \eta)} \sin \theta \\ \times \left\{ \beta_k^{(2)} \gamma_{k,+}^{\text{in}} e^{-i(-\omega_k^{(2)\text{in}} + \omega_k^{(1)\text{in}}) \eta} - \alpha_k^{(2)*} \gamma_{k,-}^{\text{in}} e^{-i(\omega_k^{(2)\text{in}} + \omega_k^{(1)\text{in}}) \eta} \right\} \quad (\text{A.5})$$

and

$$\tilde{\mathfrak{A}}_{-k,i}^{(2)} = -\frac{1}{2} \delta_{i1} e^{-i(kx - \omega_k^{(1)\text{out}} \eta)} \sin \theta \left\{ \alpha_k^{(1)*} \gamma_{k,-}^{\text{in}} e^{-i(\omega_k^{(2)\text{in}} + \omega_k^{(1)\text{in}}) \eta} + \beta_k^{(1)} \gamma_{k,+}^{\text{in}} e^{-i(\omega_k^{(2)\text{in}} - \omega_k^{(1)\text{in}}) \eta} \right\} \\ + \delta_{i2} e^{-i(kx - \omega_k^{(2)\text{out}} \eta)} \beta_k^{(2)} \cos \theta. \quad (\text{A.6})$$

We note the general identity $\gamma_{k,+}^2 = 4 + \gamma_{k,-}^2$ and that, for small k ,

$$\gamma_{k,-}^{\text{in}} \simeq \frac{\delta m}{\sqrt{m^{(1)} m^{(2)}}} - \frac{2 \Delta m^2 m}{4 \lambda^2 (m^{(1)} m^{(2)})^{\frac{5}{2}}} k^2 \quad (\text{A.7})$$

where $A = \frac{1+\lambda^2}{2}$, $B = \frac{1-\lambda^2}{2}$ and $\Delta m^2 \equiv m^{(1)2} - m^{(2)2}$. In terms of the D-particle model $\lambda = 1 - 4\sigma^2$ when σ is small. The in flavour vacuum differs from the mass eigenstate vacuum through terms proportional to $\gamma_{k,-}^{\text{in}}$ and so the effect of the D-particle capture which we regard as responsible for the mixing in this model is present through such terms in $f_{k,i}^{(+)}$ and $f_{k,i}^{(-)}$. Consequently we write for $s = 1, 2$

$$\mathfrak{A}_{k,i}^{(s)} = \mathfrak{g}_{k,i}^{(s,0)} + \mathfrak{g}_{k,i}^{(s,1)} \gamma_{k,-}^{\text{in}} + O(\gamma_{k,-}^{\text{in}2}) \quad (\text{A.8})$$

and

$$\tilde{\mathfrak{A}}_{-k,i}^{(s)} = \tilde{\mathfrak{g}}_{-k,i}^{(s,0)} + \tilde{\mathfrak{g}}_{-k,i}^{(s,1)} \gamma_{k,-}^{\text{in}} + O(\gamma_{k,-}^{\text{in}2}), \quad (\text{A.9})$$

where

$$\mathfrak{g}_{k,i}^{(s,0)} = -\delta_{is} \alpha_k^{(s)} e^{i(kx - \omega_k^{(s)\text{out}} \eta)} \cos \theta \\ + (-1)^s \delta_{i[s+1]_2} \alpha_k^{([s+1]_2)} e^{i(kx - (\omega_k^{([s+1]_2)\text{out}} - \omega_k^{([s+1]_2)\text{in}} + \omega_k^{(s)\text{in}}) \eta)} \sin \theta, \quad (\text{A.10})$$

$$\mathfrak{g}_{k,i}^{(s,1)} = \frac{1}{2} \delta_{i[s+1]_2} e^{i(kx - (\omega_k^{([s+1]_2)\text{out}} + \omega_k^{([s+1]_2)\text{in}} + \omega_k^{(s)\text{in}})\eta)} \beta_k^{([s+1]_2)*}, \quad (\text{A.11})$$

$$\begin{aligned} \tilde{\mathfrak{g}}_{-k,i}^{(s,0)} &= \delta_{is} e^{-i(kx - \omega_k^{(s)\text{out}})\eta)} \beta_k^{(s)} \cos \theta \\ &\quad - (-1)^s \delta_{i[s+1]_2} e^{-i(kx - (\omega_k^{([s+1]_2)\text{out}} + \omega_k^{([s+1]_2)\text{in}} - \omega_k^{(s)\text{in}})\eta)} \beta_k^{([s+1]_2)} \sin \theta, \end{aligned} \quad (\text{A.12})$$

and

$$\tilde{\mathfrak{g}}_{-k,i}^{(s,1)} = -\frac{1}{2} \delta_{i[s+1]_2} \alpha_k^{([s+1]_2)*} e^{-i(kx - (\omega_k^{([s+1]_2)\text{out}} - \omega_k^{([s+1]_2)\text{in}} - \omega_k^{(s)\text{in}})\eta)} \sin \theta, \quad (\text{A.13})$$

with $[1]_2 = 1$, $[2]_2 = 2$ and $[3]_2 = 1$. The leading contributions to the renormalized $: \mathfrak{f}_{k,i}^{(-)} :$ and $: \mathfrak{f}_{k,i}^{(+)} :$ are

$$: \mathfrak{f}_{k,i}^{(-)} : = - \sum_{s=1}^2 \left(\mathfrak{g}_{k,i}^{(s,1)} \tilde{\mathfrak{g}}_{k,i}^{(s,0)*} + \mathfrak{g}_{k,i}^{(s,0)} \tilde{\mathfrak{g}}_{k,i}^{(s,1)*} + \mathfrak{g}_{-k,i}^{(s,0)*} \tilde{\mathfrak{g}}_{-k,i}^{(s,1)} + \mathfrak{g}_{-k,i}^{(s,1)*} \tilde{\mathfrak{g}}_{-k,i}^{(s,0)} \right) \gamma_{k,-}^{\text{in}} \quad (\text{A.14})$$

and

$$: \mathfrak{f}_{k,i}^{(+)} : = - \sum_{s=1}^2 \left(\mathfrak{g}_{k,i}^{(s,0)} \mathfrak{g}_{k,i}^{(s,1)*} + \mathfrak{g}_{k,i}^{(s,1)} \mathfrak{g}_{k,i}^{(s,0)*} + \tilde{\mathfrak{g}}_{-k,i}^{(s,0)} \tilde{\mathfrak{g}}_{-k,i}^{(s,1)*} + \tilde{\mathfrak{g}}_{-k,i}^{(s,1)} \tilde{\mathfrak{g}}_{-k,i}^{(s,0)*} \right) \gamma_{k,-}^{\text{in}}. \quad (\text{A.15})$$

Now on noting, for real y , that $\Gamma(iy)\Gamma(-iy) = \frac{\pi}{y \sinh(\pi y)}$ and $\arg \Gamma(iy) \sim y \log y - y - \frac{\pi}{4} + O\left(\frac{1}{y}\right)$ as $y \rightarrow \infty$ we can show that for small $|1 - \lambda|$ and ρ that

$$\alpha_k^{(i)} \sim \exp \left[-\frac{i}{4\rho} \frac{m^{(i)4}(\lambda - 1)^2}{(k^2 + m^{(i)2})^{3/2}} \right] \quad (\text{A.16})$$

and

$$\beta_k^{(i)} \sim -i \exp \left[-\frac{\pi}{\rho} \left(\lambda m^{(i)} + \frac{k^2}{2\lambda m^{(i)}} \right) \right]. \quad (\text{A.17})$$

This leads to

$$\begin{aligned} : \mathfrak{f}_{k,i}^{(+)} : &= 2\gamma_{k,-}^{\text{in}} \sin^2 \theta \sum_{s=1}^2 (-1)^s \delta_{i[s+1]_2} \exp \left(-\frac{\pi}{\rho} \left[\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}} \right] \right) \\ &\quad \times \sin \left(\frac{m^{([s+1]_2)4}(\lambda - 1)^2}{4\rho (k^2 + m^{([s+1]_2)2})^{\frac{3}{2}}} \right) \end{aligned} \quad (\text{A.18})$$

and

$$: \mathfrak{f}_{k,i}^{(-)} : = \gamma^{\text{in}} \sin^2 \theta (-1)^{s+1} \delta_{i[s+1]_2} \left\{ \begin{aligned} &\cos \left(\frac{m^{([s+1]_2)4}(\lambda - 1)^2}{2\rho (k^2 + m^{([s+1]_2)2})^{\frac{3}{2}}} \right) \\ &- \exp \left(-\frac{2\pi}{\rho} \left(\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}} \right) \right) \end{aligned} \right\}. \quad (\text{A.19})$$

To lowest order in δm

$$: \mathfrak{f}_{k,2}^{(+)} : = - : \mathfrak{f}_{k,1}^{(+)} :, \quad (\text{A.20})$$

$$: f_{k,2}^{(-)} : = - : f_{k,1}^{(-)} : \quad (\text{A.21})$$

and (cf (3.32))

$$\gamma_{k,-}^{\text{in}} = \frac{\delta m}{m}, \quad (\text{A.22})$$

which we use in (5.34) to estimate the leading contribution to the dark energy coming from the flavour-changing D-brane-recoil fluctuations in our model.

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