



The collective use of the discipline of noticing to inform pedagogic actions

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Abstract

Professional development depends on enriching and expanding the range of professional actions that become available to be enacted either during preparation, or in-the-moment in a session. The Discipline of Noticing was articulated to support this process. Noticing involves both attention and awareness. The Discipline of Noticing provides a method for exploring, researching, or exploiting not only what one is currently sensitised to attend to, but also the nature and form of that attention, together with associated awarenesses which provide access to actions. In this paper, we provide examples of how professionals, in this case mathematics education researchers who are also mathematics teacher educators, located changes in what they were sensitised to notice, and how that became part of their professional repertoire by the process that Gattegno called “educating awareness”. These changes arose and were noticed through joint reciprocal reflections, and the subsequent planning and teaching of future professional development sessions. Among the aspects that contributed to the researchers’ / teacher educators’ development and engagement in the Discipline of Noticing, we emphasise communication with oneself and with others in group discussions.

1 Introduction

The terms ‘noticing’, ‘awareness’, and ‘attention’ are often used in an imprecise and everyday manner, more or less as synonyms. In mathematics education research involving noticing, it is vital that they are used with some technical precision in order not to become superficial or trivial. In this paper, we use ‘attention’ to specifically refer both to ‘where’ one is at any moment (James, 1890/1950) through the act of perception, and also ‘how one is’ (Mason, 1982). Although ‘awareness’ is often used simply to refer to what is being attended to at any given moment, what matters is “educating awareness” which aligns with the way Gattegno (1987) uses the word to include ‘that which enables action’, bringing the action to the surface and available as a choice to enact. When we become aware of such an awareness then we learn and move forward (Gattegno, 1987). We use ‘noticing’ to refer to “a collection of practices both from living in,

and hence learning from, experience and for informing future practice” (Mason, 2002, p. 29). This is one aspect of the Discipline of Noticing (Mason 2002, 2012), which articulates both a research method and a description of how effective professional development can take place. In mathematics education research, ‘noticing’ is often used to describe a sudden shift of focus or form of attention, taking place in a fleeting moment, (Mason, 2003a). However, an important feature of the discipline is the preparation for noticing opportunities to act freshly in the future. The discipline recognises that changing practices is not easy, and may take time. It provides ways of enhancing the possibility of noticing fresh opportunities in the future for making those changes. Teacher noticing in this sense is more than a holistic concept (König et al., 2022), transcending competency-focused uses that concentrate on “specialized ways in which teachers observe and make sense of classroom events and instructional details” (Choy & Dindyal, 2020, p. 1). Our use incorporates an active, dynamic, and evolving access to action that is developed through collective engagement, as well as inter-personal and intra-personal communication, in a disciplined manner for learning about and from professional practice. In this paper, we provide descriptions of an instance of the creation of spaces of communication that support disciplined ways of talking, writing and thinking about

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professional practice, both our own professional practice as researchers and teacher educators, and the professional practice of the teachers we work with. These descriptions serve to exemplify noticing, awareness and attention as they are used in the Discipline of Noticing.

The context for our paper is the preparation of a distance-learning professional development module for use in various countries. Two authors (Jenni and Núria) proposed, as part of the module, the use of extracts from a video recording of a professional development session in England. We report on their reflections as they met with the third author (John) and discussed what they noticed, having watched the video with a view to informing their writing of the module. We record what and how they attended to it, and what this awareness made available in the way of subsequent actions. The asynchronous nature of this professional development requires a re-visiting of the role of teacher educator as facilitator, as the occasions for new awarenesses (Coles, 2018) are more limited than when working directly with mathematics teachers.

Our aim is to capture some details of attention and associated awarenesses which came to the fore as a result of the transformative sharing of noticing among us, and which might not otherwise have emerged had we not been together in the created spaces of communication. This includes the issue of naming and labelling of details of a geometric figure, with implications for ways of thinking and describing pedagogical and mathematical actions, which emerged from the discussions. This attention to naming and labelling connects strongly to the elements of marking and labelling in the Discipline of Noticing. Shared communication (Uptegrove, 2015) of noticing, with oneself and with others, and its potential for disciplined researching and professional practice, are strategically outlined throughout the paper. This shared communication is central to the aspect of validation in the Discipline of Noticing. Shared communication is understood as functional communication (Halliday, 1994), including inner speech and language used with the intention of communicating with others (Pimm, 1994), in either case, mediational of noticing development. We suggest that mathematics education research involving substantive noticing as outlined in the Discipline of Noticing builds on the participant researchers' own disciplining in shared communication by bringing both communication and disciplining to the front.

Underlying our current reflections is the following question: How might shared communication in group discussions contribute to disciplined noticing for learning from and informing future practices as mathematics teacher educators? Alongside illustrating one response to this question, we also intend to advance towards a less disaggregated and more connected view of teacher noticing and of teacher noticing research by showing how apparently distinct aspects are aligned with the understanding and investigation

of noticing as a discipline. Our conceptualisation of noticing as a discipline is, in this way, argued in relationship with some mainstream discourses in teacher noticing research.

2 Perspectives of teacher noticing in the mainstream research discourse

Recent survey papers have identified four dominant perspectives within mathematics education research concerned with teacher noticing, each with roots which can be associated with different theoretical traditions. König et al. (2022, p. 2) summarise these as:

“a cognitive-psychological perspective”, originally framed by van Es and Sherin (2002) around *what* teachers attend to and make sense of when focusing on specific mental processes;

“a socio-cultural perspective”, rooted in the notion of professional vision by Goodwin (1994) and interested in the social and discursive nature of noticing;

an apparently “discipline-specific perspective”, originally framed by Mason (2002) to refer to the research and learning practices of mathematics teachers, which inform and are part of their professional practices;

and “an expertise-related perspective”, interested in the distinction between novices and experts and what constitutes expertise in the profession, following Berliner (2001).

The aspects stressed within each perspective actually overlap, indicating the variety of differing elements and emphases involved in researching and enriching preparation-for and in-the-moment practice of teaching. From the perspective of professional vision, for example, we can see several connections with the other perspectives described in König et al. (2022). The practices of coding, highlighting, and producing and articulating material representations, investigated by Goodwin (1994) for building professional vision, can be interpreted in relation to the research-led ways of promoting attention to specific elements of teaching through investigations with teachers. The elements of teaching that are chosen and marked for focused attention and related focused interpretation, following van Es and Sherin (2002), are relevant objects of knowledge for some mathematics education researchers and hence result in coding and highlighting different aspects which shape the discourse of teachers and educators. The material representations whose production and articulation are in focus, such as lesson plans, student learning trajectories or series of tasks, themselves constitute a statement about what the professional experts are able to examine carefully and meaningfully, following Berliner (2001), compared to novices or casual observers. This concerns what participants and colleagues are sensitised to notice, and what actions are accessed by what they become

aware of. Attention to and engagement in these practices, as well as awareness of their importance and criticality, point to the complexity of the teaching profession, and to the existence of several other practices which need to be seen and understood so that we (teachers and teacher educators) can learn from them and become more expert professionals. Conceptualising teacher noticing as a discipline relates to continually developing expertise.

The four perspectives outlined in König et al. (2022) are also connected in their common emphasis on the criticality of noticing with respect to the choice of acting differently and hence change. Since noticing, both noticing a possible action, and then noticing opportunities to enact that action, are the key ingredients for changing and developing practice, what is of interest to researchers are disjunctions between what teachers are sensitised to notice already, and what the educator-researcher wants them to notice, coupled with mathematical and pedagogical actions which might then be appropriate. These disjunctions refer to a range of possibilities of what teachers notice and of what educators-researchers notice of what teachers notice, and what remains invisible, unspoken or submerged. It is not possible not to notice (Mason, 2002), but in some research it is strategic to position signalling the participants' observable noticing in ways that fit into the core aspects of the particular research or professional development. So, noticing becomes 'noticing assessed in the context of the research'. All this raises questions about how noticing can be supported, developed and noticed, given the role of the social and cultural context and of the immediate situation (Louie, 2018, but also Goodwin, 1994) in how links are made with relevant mathematical and pedagogical actions (Mason, 2008). This is the thrust of the Discipline of Noticing (Mason 2002, 2012).

In the next section, we present the theoretical framing of noticing as a discipline, with special emphasis on how this conceptualisation can gain additional significance in a historical momentum of strong growth in mathematics teacher education research on noticing. The porous borders and connections of "the discipline-specific perspective" with the other perspectives described in König et al. (2022) make it possible to adopt a conceptualisation of noticing as a discipline which reflects both ancient and modern views of human psychology, and which in turn recognises and brings together the relevance of discursive and communication practices in professional vision development, of focused attention and interpretation in mental processes, and of the features that distinguish professional expertise, amongst others. Our conceptualisation of teacher noticing hence lies in the need for continuous and active study, for participation in the discipline and its co-construction. This study develops and manifests in a diversity of ways and occasions, sometimes including university researchers with some established agenda and foci in the approach to the school teachers' practices, experiences and knowledge.

3 Conceptualising noticing as a discipline

Conceptualising noticing as a discipline does not imply providing a map of connected notions, because participation in the Discipline of Noticing is dynamic and requires highlighting practices and experiences over notions. Moreover, this conceptualisation can be strengthened further by putting it in relation to some meanings from the current mainstream research discourse involving teacher noticing at present. For example, awareness is often used to refer to a moment of attention being focused on some particular pedagogic or mathematical detail, such as 'high-potential instances' of students' mathematical thinking (Stockero et al., 2020) or the substance of student thinking (van Es & Sherin, 2021) in research drawing on a cognitive-psychological approach. These particular details may be described as worthy or relevant, for example, and are indicative of the political dimension of noticing (Coles & Helliwell, 2025) as there arise questions about what is worthy or relevant and for whom. Yet, this may not be stable or long-lasting. When it becomes established as a repeated experience, it usually involves an action which is then available to be enacted, what Gattegno (1987) referred to as 'educated awareness'. It is no longer simply a diffuse form of noticing, below the surface of consciousness. It informs both perspective and action. For example, on the one hand, one can be vaguely aware of someone else's presence in the room and, on the other, one can be acutely aware of a possible pedagogic action to enact in a specific classroom situation. In other words, for us awarenesses are what afford access to actions, so the point is to educate those awarenesses so as to internalise appropriate actions. Hence, Gattegno's memorable claim that "only awareness is (what is) educable". Drawing on an image from the Upanishads (Radhakrishnan, 1953, p. 623; Mason, 2008), this assertion can be augmented to include 'Only behaviour is (what is) trainable' and 'Only emotion is (what is) harnessable' (provides the energy needed to notice and act). The three assertions together can act as stimuli to provoke reflection and action. These three assertions underpin the design of the professional development courses being developed by the first two authors.

In our approach to the discipline of noticing, we include in 'behaviour' practices such as the internalisation of technical terms and phrases, the recording of reasoning, and specific mathematical actions enacted. Behaviour is not, therefore, extractable from contexts and not understandable in isolation from a variety of practices linked to the profession. Stressing behaviour alone, and the practices involved, leads to valuing performance in specific procedures that have become routine but without concern for meaning; stressing noticing alone focuses on 'knowledge about', but without effective action in-the-moment; stressing awareness (in the

technical sense) engenders flexibility and creativity as it enables non-routine actions to become available and modifiable. Combining the awareness and associated actions can lead to comprehension and appreciation, or what is often referred to as ‘understanding’. This focus on stressing awareness echoes the focus on educating perception that Goodwin (1994) proposes through the notion of professional vision (Mason, 2003b).

Gattegno’s (1987) approach suggests discerning at least three levels of awarenesses involved in professional development, which Mason (1998) labelled *awareness-in-action*, *awareness-in-discipline*, and *awareness-in-counsel*. Awareness-in-action refers to having one or more mathematical or pedagogical actions become available to be enacted, for example, knowing to find a common denominator in order to add two fractions. Awareness-in-discipline refers to being aware of a variety of actions which as a teacher or teacher educator one hopes or intends learners to internalise in order to enrich their mathematical or pedagogical thinking, for example, the various actions appropriate to manipulation of fractions, as things that need to be learned. Awareness-in-counsel refers to being aware of ways of prompting and provoking teachers to enrich their awareness-in-discipline so as to make their teaching more effective, for example, constructing task-exercises through which prospective and serving teachers may encounter significant mathematical themes or the use of their own natural powers in a mathematical context. These levels of awarenesses, and particularly in teacher education the awareness of noticing being an active research task for the teacher, often receive little attention in current research involving mathematics teacher noticing, with the exception of the work focusing on teachers adopting of an inquiry stance described by van Es and Sherin (2021). In the context of mathematics teacher education, teacher educators need to become aware of the teachers they are working with awarenesses of the awarenesses that teachers have concerning their students (Brown et al., 2021). Consequently, we, the authors, are working with our awarenesses of the awarenesses of teaching mathematics.

Noticing nonetheless means more than awareness. It means actually attending to something that might be unexpected or a surprise, but also ‘expected’ or ‘exceptional’ in the sense described by van Es and Sherin (2002) of attending to “what is noteworthy about a particular situation” (p. 573). Tripp (1993) gives multiple examples of how significance of what is noticed develops. This expectedness and exceptionality begs the meaning and scope of noteworthiness: noteworthy to whom and for what purpose? It means at the very least, gazing or holding a whole, but it can also mean discerning details, recognising relationships (among details), perceiving properties (as being instantiated as relationships), and using properties as warrants, to generate

reasoning on the basis of agreed properties (Mason, 2001). All these meanings contribute to the porosity of distinctions between what is noticed.

As well as simply not noticing something, there are different intensities or energies of noticing, some of which echo with the development of professional expertise described by Berliner (2001). There is the intensity of noticing which enables someone to make a spontaneous remark about what was noticed. A less intense mode brings about recognition of having noticed something only when someone else makes a remark about it. Hence the importance and value to the authors of this paper in working jointly in created spaces of communication so as to amplify the intensity of noticing alongside validating this noticing with others. It is important that noticing sometimes reaches an intensity in which there is sufficient energy and commitment to make some written or other communication record (Mason, 1998). Varying intensities mirror the energy available to activate awareness and desire so as to effect action.

Within this conceptualisation, moments of communication can be seen as potential moments of activating awarenesses, attention and noticing. Both interpersonal communication and intrapersonal communication can have such a function, where communication with others implies communication with oneself and one’s consciousness, and communication with oneself and one’s consciousness may be expressed in joint reciprocal reflection (Pimm, 1987/2017; Planas & Pimm, 2024). Some challenges and peculiarities of noticing can thus represent challenges and peculiarities of the communication moment as well. When a teacher and learners are attending to different things in-the-moment of the classroom practice, for example, then communication between them is likely to be impeded, at best (Ingram, 2014). Furthermore, even when people are attending to the same things at the same time, they may be attending in different ways because, for example, they may be making different connections “between specific events and broader principles of teaching and learning” (van Es & Sherin, p. 574). In these cases, communication is likely to be impoverished and the potential intensity of the noticing that takes place in shared work is likely to be poorly realised. Overall, attending to something and the shifts in the form or nature of attention can happen in moments of shared communication with others and with oneself (Planas et al., 2024) in ways that can either amplify or reduce the intensity and sustainability of attention.

As a research paradigm, the Discipline of Noticing involves colleagues reporting on (hence communicating among and with themselves) the ways in which what they have noticed enables them to act in the future. Validity lies not with some comparison between control groups and subjects, but with whether what people notice (perhaps more than once, and on more than one occasion) contributes to

sensitising them to notice opportunities to enact some action which had not previously been available to them. In other words, whether through noticing situations in which particular actions might have been effective (becoming aware of alternative actions), and through internalising those actions, those actions become available to enact when deemed appropriate. Thus, the mark of successful use of the Discipline of Noticing is finding yourself with a fresh or unusual choice whether to enact a fresh action, or to enact one of several actions that have become available.

4 Example of a context for educating awareness

As predictive organisms, human beings prepare subconsciously for imminent action on the basis of past habits and experiences, tempered by aspects of the current situation and context. In order to develop professionally as a teacher, it is necessary to become aware of habitual actions (discerning details and recognising relationships between aspects of situations and enactable actions). This is not easy because we often are not aware of behaviours and actions that have become automatised in our professional practice. Discerning alternative actions (and valuing these) is then a step towards noticing a situation in which a fresh action might usefully displace a habit. More is required than simply noticing, however. Something has to trigger access to a fresh enactable action in the moment, before habit kicks in. This distinction between habit and fresh action, and the research of fresh actions, is a central feature in the conceptualisation of noticing as a discipline, which does not feature prominently in other perspectives on teacher noticing in the research discourse.

Despite the distinction between the expert and the novice (Berliner, 2001), experienced and novice educators are both subjected to habits of the profession in specific contexts which are difficult to kick or interrogate. It can actually be more difficult to make some habits become visible and discussed for those that have followed the habits over more years of profession. What an experienced educator notices when attending a professional development session is likely to overlap with, but in some ways differ from, what participating teachers notice, and of course different educators, like different teachers, will be differently sensitised in what they notice and the habits that they are ready to interrogate. What these teachers or educators attend to, and in what ways, is necessarily influenced by the situation context and the intentions of their current project. In our case, the project of creating a module of professional development for teachers shapes the context in which three experienced teacher educators/researchers communicate in group discussions what they are differently sensitised to notice and how they bring up additional remarks to what each of them noticed. The

contrasting histories, experiences and contexts of the three authors also supported the process of making some habits visible and fresh actions to become available. Yet we also have a shared history between us, and over time our noticings might become closer (Coles & Helliwell, 2025). This closeness is what Amador et al. (2023) describes as collective noticing, which may be useful in explaining how mathematics teachers learn from each other. For a participant in a professional development session, the core question is whether they can imagine themselves in the future enacting some fresh mathematical or pedagogic action, or perhaps enacting some familiar action but freshly. The Discipline of Noticing offers ways of supporting choices of action in-the-moment, in contrast to the activation of habitual re-actions. For a professional researching the development of their own practice, it is necessary to be more disciplined by making records of instances when they notice (often in retrospect) missed opportunities to act freshly, or when and why a particular choice was made, and to use these to further support sensitising themselves to appropriate situations so that a possible action becomes available in the moment in the future. This is an example of educating awareness. The important dimension which justifies the use of the term ‘research’ is to seek evidence of validity, which in this research paradigm means fashioning task-exercises for use with colleagues to see whether they too notice what the researcher has been noticing, and whether they too find that their practice is ultimately augmented and informed by an increased sensitivity to notice and a propensity for enacting fresh actions. By sharing our own records of instances of noticing opportunities and missed opportunities to act freshly both when reflecting back on the video and when looking forward to the development of the online professional development, we illustrate a core aspect of this discipline.

As a participant in a professional development session, attention is likely to be on discerning fresh pedagogic or discipline-specific actions, including new connections between the mathematics and the focus of the professional development, and recalling situations in which those actions could have been effective and so might be effective in the future. As someone researching the process of professional development, interest is not only in what actions are discerned by participants, but in getting a sense of the sorts of situations in which these might be effective. This involves attending to discerning details of observable and possible actions and recognising relationships between pedagogic situations and appropriate pedagogic actions. As teacher educators, in addition to discerning details of observable and possible actions, and recognising relationships between particular situations and appropriate actions, it is necessary to perceive properties of those situations and actions so as to be able to direct participant attention appropriately. This complexity is an instance of interaction between awareness-in-action, awareness-in-discipline and awareness-in-counsel

(Mason, 1998), which in Gattegno's terms would be awareness, awareness of awareness and awareness of awareness of awareness. The disciplining experience reported in this paper involves all these related kinds of awarenesses.

5 Methods in the creation of a communication space for disciplining our noticing

The Discipline of Noticing is a practical approach to becoming more methodological and systematic in developing your own practice, that can also be used to improve research or as a research paradigm (Mason, 2002). Disciplined noticing involves keeping and using accounts, developing sensitivities, recognising choices preparing to notice in the moment and labelling. The experiences being reported on here are that of three mathematics educators undertaking to prepare some distance learning materials for in-service secondary-school teachers. These experiences are presented as accounts to exemplify the different aspects of using accounts in the discipline: creating, working on, working with, working with accounts arising from other sources, and using accounts with others.

This collaboration is part of a broader project, coordinated by the first author, aimed at working with mathematics teachers in England to support professional development on the language-responsive teaching of angle properties, probability and linear equations (see, e.g., Ingram et al., 2024, for the origins and background of the broader project). For one of the modules, it is proposed to make use of a video from a workshop involving six teachers talking about their work on three geometry tasks centred on comprehending and appreciating the use of angle properties. In the video, teachers explain their reasoning in response to the task, other participants comment on what they noticed about the presentation of reasoning, and the presenter comments on what their attention was thereby directed to. A key part of the workshop activity was, therefore, to pay attention to the other participants' methods of solution and reasoning, and to discuss the mathematical practices proposed and carried out. The developing professional development would then focus on experiences, stimuli and prompts that both resonate with teachers' habitual actions whilst also educating their awareness so that they find themselves with a fresh or unusual choice whether to enact a fresh action, or to enact one of several actions that have become available, a mark of the successful use of the Discipline of Noticing.

Figure 1 shows three slides with the visual-and-verbal tasks presented to the teachers in the workshop, in the order from left to right. The three tasks pose the question of finding the size of the angle labelled in the respective diagram with the letter a . The intended focus of the module was to be on labelling and naming in mathematics,

whether in teaching, learning, or doing, building on the work of Adler (2021), Mwaadzaangati and Adler (2024), Planas and Alfonso (2023), and Planas et al. (2024) on these mathematical-linguistic practices. Like coding in Goodwin (1994), labelling and naming play a crucial role in any profession and, particularly, in the thinking, preparation and development of pedagogical and mathematical actions in teaching. In developing these professional development materials we are also marking the practice of naming and labelling.

Linguistically, word names and symbolic labels are signs used to communicate meanings in a discursive context, for example, a triangle can be labelled with a name that highlights a quality (e.g., the isosceles triangle, the acute triangle), its vertices (e.g., ABC), using the labels of the line segments that make up its sides (e.g., abc), by its visual appearance (e.g., the red triangle), or by some relationship with another triangle taken as reference (e.g., the congruent triangle). The tasks, displayed in Fig. 1 and used in the workshop, were intended to draw attention to the importance of the linguistic practices of mathematical naming and labelling of the lines and angles in the three diagrams, to ways in which this can be done, and to implicit ambiguities of labels and labelling. As it turned out, the teacher educators leading the workshop did not address any of these explicitly. They anticipated that the chosen task, and the activity in working on the task, would together result in addressing labelling explicitly and that participants would notice such practices as intrinsic to the reasoning for finding the angle size.

The video was viewed multiple times by the authors of this paper, as researchers and as mathematics educators preparing a professional development module for use by yet other teachers. Our data consists of written accounts of (as distinct from accounting-for what we noticed) what struck us (the three authors) during the session, while reviewing the video, what arose from our group discussions (what we noticed in shared communication), and how our sensitivities to notice were influenced by other recent experiences, due to our different research foci, by our educator roles, and by our attending to what the other two in the discussions noticed. We then report on how what we noticed influenced the choices that we were aware of as being available, and the choices actually made, in preparing tasks for the professional development module. Thus, we offer elements of three aspects of awareness, and actions made available by those awarenesses.

These data illustrate a communication space where each author engages in the discipline, both individually and together. It is distinct from notions of joint, shared or collective noticing in that the focus is on our own attention and awarenesses and the actions available to each of us, rather than the development or establishment of similarities in our attention, awarenesses or noticing. Yet our time together

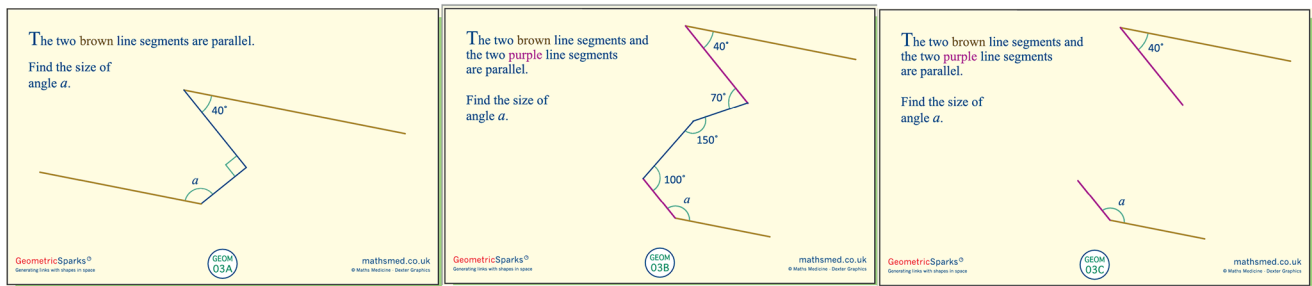


Fig. 1 Taken from Küchemann (2023) *Geometric Sparks*. ATM. Reprinted with permission

did lead to “noticing and perceiving similar things” (Coles & Helliwell, 2025, p. 7). This complex interaction demonstrates how communication was importantly involved in our noticing through group discussions and written accounts. The discussions were synchronous online, with a total of six hours distributed over three meetings. This process involved systematic reflection by each of the authors, both individually and collectively. Notes of these reflections were made using the recorded conversations and individual accounts of important moments or connections that arose while watching the video of the workshop with the teachers, including retrospective connections to previous accounts and conversations. Each group meeting of the three authors involved validating with others (Mason, 2002) but also further opportunities to identify, mark, and distinguish threads, choices and possibilities. In the case of the two authors involved in the development of the online module for in-service mathematics teachers, this process also involved the refining of the module materials to highlight the issues in focus.

Throughout the process, which took us about six months, we were aware that teaching (and teacher education) is a curious but caring profession. We are working for change in student (and teacher) behaviour, affect, and cognition, yet we know that we cannot change others directly. As teacher educators, however, we can influence those indirectly who are working to change or develop ourselves, since all interaction is necessarily reflexive, though to varying extent depending on sensitivity (Mason, 2011). For this paper, we focus on data evidence of noticing that takes the form of extracts from our individual written accounts, where each of us reflects on which aspects of the shared communication with the other two, and with ourselves, are alluded and responded to. These accounts are, in this way, evidence of and reflections on the influence of the group discussions on our awarenesses and sensitivities, as well as means for us to become aware of what we had noticed in that context. We particularly select fragments of a written task with evidence in support of the influence of interpersonal communication on the disciplining of our noticing in researching the process of mathematics teachers’ professional development.

5.1 Written intra-inter-personal records of our noticing

In this section, we provide three written records of noticing, which we claim were enacted by and generated out of the group discussions and the shared communication that came with these discussions. We suggest that our respective records of noticing interacted with our processes and practices during the multiple re-viewing of the videoed workshop, and what we noticed and communicated in the online group discussions. As reported below, the three of us experienced different intensities of noticing and sorts of awarenesses in relation to: (i) mathematical and pedagogical actions available in the session with the teachers (awareness-in-action); (ii) a variety of actions which as teacher educators we wanted the teachers to pay attention to in order to enrich their mathematical and pedagogical thinking (their awareness-in-discipline); and (iii) a variety of ways of enacting and enriching our awareness-in-discipline so as to make our teacher education work more effective (awareness-in-counsel). All these awarenesses co-developed in hybrid intra- and inter-personal communication. They were documented after having watched individually the workshop video, having met on three occasions and engaging with numerous emails conversations, and having spent time to conduct the task of systematically writing what we had noticed in the video that was mathematically and pedagogically relevant for informing the preparation of the distance learning module. This process was not sequential and each of us re-watched the video and re-viewed their written records of noticing as much as we felt necessary. For example, following an online meeting, some of us might decide that we wanted to write an email to the others with some reflections and then to watch the video once more. These email communications could begin with reflections such as: “Woke up this morning having finally realised what I kept overlooking in the videos” (John), or “I might need to reflect more on the distinction between naming and labelling, and on some evidence of what the teachers in England did to support this distinction” (Núria).

5.1.1 Record of noticing (Jenni)

In my first re-viewing of the video, I was struck by the difference in my focus. I moved from focusing on whether the original aim of the task was achieved, that is, *whether* a need for names or labels for features, relationships, and generalities arising from the diagrams was generated, to questions about *when* and *how* names or labels were used by the different participants, and similarities and differences between these (as well as the distinction between names and labels). The task was used in a way that allowed individuals to annotate the diagrams, but when sharing their reasoning, this had to be done verbally, and hence there was a need to use names and labels for the different components and aspects of the diagram. This re-viewing of the video made me aware of different possible actions through noticing the actions of the teachers in the video, reflecting on my own planning of the task, and labelling and marking these actions as associated with the mathematical discourse practice of naming and labelling. These actions became a focus for the development of the tasks for the module, an instance of an awareness being educated through being associated with an enactable action.

There were different ways of responding to the task(s) that needed to be communicated so we could all ‘see’ the reasoning taking place. Some were recognised as similar approaches to my own, but others brought different features, different perspectives and different choices of names and labels to the fore. Do these differences matter mathematically and/or pedagogically? One effect of this re-viewing is to bring to the surface implicit assumptions about participants’ sensitivities and awarenesses.

These differences during the session itself led me to focus on different solution approaches, and in particular, differences between static approaches, where lines were added or extended, and dynamic approaches, where lines were translated. Note the use of the labels ‘static’ and ‘dynamic’ for a distinction worth incorporating into our module. The re-viewing led me to focus on similarities across the approaches and the tasks. What was particularly notable was the fact that similarities (and differences) seemed missing from the video itself, thus bringing the distinction to my awareness associated with some actions to include in the module preparation. The focus on naming and labelling in the session drove this focus for me on comparing and contrasting the different approaches and the decision I made in-the-moment of the session to not generalise across the tasks meant that the pedagogic focus of naming and labelling was emphasised over the mathematical focus.

Soon after our second meeting together, I needed to draft an early version of the online module. Here, my thoughts were dominated by what to stress and make explicit so as to enable teachers using the module not only to see the

need for labels but also to reflect on when, what and how to label in ways that support their students, which consider the possibility of choosing between a static and a dynamic approach in geometric thinking generally. These decisions are both mathematical and pedagogical; they may converge, but they can also be in tension. For me, there are also so many choices about what to comment upon, when to comment, and when not to comment. These choices lie alongside choices about whether that should be in the module based on these geometric tasks, or in other places, or left for another time.

Following our third meeting together for this paper, I was struck by John’s comment that when teachers label something, it is because, for them, it is worth marking and remarking on (different intensities of noticing). It signals a landmark or pivot point in the education of their awareness. The labels of angles, vertices and the labels of relationships between angles and lines used by the teachers in the video reveal something of what they are attending to in their reasoning. But is what the teacher says and does sufficient to focus learner attention similarly? As a teacher educator, it feels important for me to notice where this might differ from what I might label or remark upon so that the revising of the professional development materials enable teachers with different ways of viewing and working on the tasks, make sense of this reasoning.

5.1.2 Record of noticing (Núria)

My first re-viewing of the video reminded me of how much in-the-moment attention I had been paying, during the session in England, to tracing the teachers’ mentions and uses of the diagrams in the resolution of the tasks. I had been expecting that they would attend together to the diagrams, and that they would explain the diagrams or pose questions around how to interpret them. For me, in the context of those tasks, any mathematical reasoning had to be done ‘with’ some careful reading of the diagrams and their visual annotations. Re-viewing the video was like a reminder and confirmation of the mathematical and pedagogical actions that I had considered to be available, and those that I still missed in the shared thinking. The teachers had been jointly and openly attentive to transformations and strategies of extending the lines, moving the lines without changing the relative positions, adding lines... but they had not talked about how changes in the diagrams might need changes in the labelling of the elements to be considered in the reasoning around the angle size. The diagrams were labelled diagrams!

None of the teachers had explicitly related the thinking of the three tasks to the search for equivalent or alternative diagrams for reasoning about the size of the angle. If some mathematical reasoning can be supported by finding equivalent diagrams, the inclusion of an attentional focus on

explaining and teaching diagrams might be relevant for the distance-learning module. Moreover, these teachers did not seem to think the three tasks related as a sequence in which adding and removing lines was important. I had joined the first meeting with John and Jenni thinking that the module in preparation could not be ‘international’ because the experiences of the teachers in England with school geometry seemed very different to the experiences of the teachers in my country. At the end of the third meeting, however, I could imagine an ‘international’ module also for teachers from Spain, with some good knowledge of the English mathematical register and similar challenges in geometry work with labelled diagrams. Thus, my general awareness became more focused and educated as possible actions became associated with what I had noticed. For example, extending the ‘purple line’ of the original diagram to ‘show’ parallelism might require a few seconds for naming the ‘extended purple line’.

For some minutes in one of the discussions of the video with Jenni and John, who are familiar with the specificities of mathematics teaching in England, we were all attending to features of the labelled diagrams and how these could be interpreted differently or seen as unconventional. Does a label refer to a vertex, or to an angle at that vertex, or to the measure of that angle, or perhaps even to all of these at different times? The three of us seemed to interpret labelled diagrams as challenging and the idea of creating equivalent diagrams by adding and removing lines as a helpful action, thus educating my awareness. This sharing made me think that my noticing during the day of the session and the individual viewing of the video might inform some actions of utility in a module for teachers in England and perhaps elsewhere.

Now I was thinking of materials for the module with some labelled diagrams that could be considered equivalent, and some others which could not, in the context of verbal-and-visual tasks. In this way, the module could enhance practices of visualising, labelling and naming diagrams for thinking angles and their properties. What about making teachers’ thinking start with the mathematical structure underlying the diagram by first asking them to draw equivalent diagrams in which the letter a represents the same angle size?

5.1.3 Record of noticing (John)

It was only on my second viewing of the video that I realised that the three tasks were different versions of the same task. That made me reflect on the three tasks as possible variants of one task, or as three phases of a single task, so that learners would encounter the same diagram and similar reasoning in slightly different forms. This would “keep them in the situation long enough” (Nick James, personal communication,

1985) to enhance the possibility of their noticing and becoming aware of the varieties of angle-reasoning. This in itself is a notable pedagogic action, and not one I have fully integrated into my functioning. Only then did I realise that I had not noticed that Jenni and Núria had already been talking as if the tasks were variants of a single task.

I was struck by how some speakers concentrated on extending straight lines, sometimes getting into a bit of a tangle in trying to label or otherwise describe which segment they were extending, while others concentrated on angles or on translations of line-segments and angles.

I was also struck by the absence of any indications that participants were aware of the scope of generality illustrated by the three tasks. Although ‘absence of evidence is not evidence of absence’, it surprised me that no-one explicitly observed that the reasoning did not depend on the specific angle measures used in the task. Experience with one or more tasks is in my experience much the richer when the mathematical actions used are expressed as generalities rather than remaining lodged in particular values. I would anticipate stressing this in the creation of the distance-learning module, which is to be based on these videos, among others.

During the videos I noticed that when someone has introduced labels for things so that they can refer to them, they tend to use the labels with alacrity. The label and what is labelled become fused, whereas for the listener, this fusion takes time. Being mindful of different forms of attention might suggest that when a label is used, a detail has been discerned, which means that on hearing the label, the audience has to discern the label and then what it labels before they can attend to relationships. Especially in geometric diagrams, labels often come in clumps (e.g., “Triangle ABC”), which requires discernment of the individual labels, and only then recognition of the relationship which the clumping indicates, in order to make sense of what the speaker is saying. In other words, mindfulness of forms of attention might recommend the use of pausing between uses of labels so as to enable listeners to keep up.

It was only in our discussion that the potential ambiguity of labels really became salient for me. Is it the vertex or the angle or the angle measure being labelled, and more generally, is it a specific action, or the action in relation to other actions as part of a framework that is being labelled pedagogically, and are there potential ambiguities for learners?

5.2 Joint reflection informing the preparation of the module

While there is variety in the three reflections, this variety is powerful in educating our own various awarenesses. We collectively became sharply aware of the role of noticing and attention in this. We also noticed the educating of our awarenesses (In action) in the form of more actions becoming

available as choices to include in our general work as teacher educators (awareness-in-counsel), and in our design of the materials being developed: actions such as taking time over the introduction of labels; being mindful of what aspects or details are being labelled, differences between static and dynamic extensions of diagrams and the mental work required, and so on. The power of the Discipline of Noticing in developing our own practice lay in the synergy of the different aspects of noticing, awareness and attention. These different aspects are foregrounded in different ways in our individual accounts and in our meetings together. Importantly, these different accounts highlight how the four perspectives identified by König et al. (2022) are all aspects of the functioning of the Discipline of Noticing when contributing to our future practice. Fracturing and fragmenting noticing into different components, such as specific facets and specific processes, and reducing what is ‘noteworthy’ in a situation of teaching and learning to what is ‘noteworthy’ that fits into a research agenda, might have the danger of limiting the potential to influence future practice. Fresh actions and openness in future practice are also endangered by how our theoretical orientations can privilege some interpretations, and by doing this, do not help sensitising us about the limits of our interpretations and noticing, and reduce the possibility of acting differently in the future.

Making records of our individual noticing when watching the videos, followed by collective discussion, brought to the surface pedagogic principles, constructs and actions, which were then incorporated into the draft of the professional development module. Our differing sensitivities enabled us each to enrich our personal noticing and contributed to educating our awarenesses through linking noticing with future action. Some final observations

An important observation is that the three of us attended to and reflected on the function of labelling and naming in mathematics teaching, whereas the teachers in the workshop in England generally did not appear to. We noticed ourselves progressively deepening our appreciation of the role of labels both in mathematics and in mathematical pedagogy as a result of the shared communication in our group meetings and email exchanges, and the writing tasks about our noticing that we set ourselves. This highlights a difference between the role of goals in the Discipline of Noticing and much of the other research on teacher noticing. We did not ‘learn to notice’ in any particular direction pre-established by any of us, but together developed a sense of labelling and naming as being decisive actions in the thinking of the mathematical tasks with the diagrams and of the teaching based on these tasks. This is an example of awarenesses-in-action becoming awarenesses-in-counsel. Our goal was not *a priori* to notice aspects of labelling and naming in relation to the mathematical task and the teachers’ response to the task, rather our goal was to share our accounts of noticing in

response to the video and to educate our own awarenesses in response to both the sharing of these accounts and our own re-viewings of the video. Thus, our appreciation of the role of names and labels results from our shared (Coles & Helliwell, 2025), joint (Planas et al., 2024) and collective (Amador et al., 2023) noticing.

Our different experiences and sensitivities interacted to make the distinction between the visual and the verbal in the mathematical tasks or between the pedagogical and the mathematical, for example, more salient. We tried to share or make available what we noticed, rather than imposing our own noticing or making evaluations of what each other noticed. One striking feature of some of the literature is that it often talks about learning to notice (*soto voce*, learning to notice what the educator has noticed), which sometimes leads to assertions and evaluations being made about people on the basis of what they appear to notice in relation to what the educator or researcher thinks others should notice. This is completely different to educating awareness. We were trying to construct tasks which would result in participants noticing for themselves. There is resonance here with the distinction we make between teacher training and teacher education. Teachers can already notice - what educators are trying to do is change or influence what/how they notice, just as teachers are trying to influence what and how learners notice about mathematics or mathematical actions. None of us failed to notice, indeed our noticing was fairly detailed and specific, similar to that reported in van Es and Sherin (2009). The differences between what we noticed contributed to a richness which, in turn, influenced the educating of our awarenesses.

Another feature of the literature is the dominance of work that focuses on novice and expert noticing, which in some cases makes a distinction based on supposed quality - experts are assumed to be better at noticing student thinking, etc. - whereas their expertise includes more extensive experience of recognising more possibilities and having these linked to a wider range of possible actions than is available to a novice. The expert is likely to have greater and more extensive sensitivities, more threads and links via labels accumulated over time. If we apply this distinction between the novice and the expert to our team of three, with one of us (John) having a more dilated professional trajectory, there is nothing that seems to change substantially in terms of how we disciplined our noticing together. Noticing is relational rather than vertical or just individual. The more experienced educators or researchers also need communication with others (less or differently experienced) to make sense of what they attend to and to understand, for example, some of the consequences of what they notice for others.

Noticing for us is about informing and improving future practice. In particular, the educating of awareness of aspects of the functions that labelling and naming has in the teaching

and learning of mathematics was foregrounded in the subsequent drafting of the module. The process of jointly drafting and redrafting, viewing and reviewing, listening and responding, educated our own awarenesses, in interaction with others, because others prompt us to pay attention to things we were not previously aware of, as well as developing our noticing of aspects within our awareness in slightly different, often fresh ways. As teacher educators and researchers, we gained a more nuanced understanding of the mathematical and pedagogical aspects of labelling and naming mathematical elements, and of the pedagogical role in assisting teachers in displacing the common habit of relating these practices to mere knowledge about and teaching of conventions in school mathematics.

Declarations

Competing Interests The authors declare no competing interests.

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