Essays on the Financial Governance of Firms

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Thesis Abstract

Four essays, or chapters, model the capital structure, governance, and investment decisions as part of a sequential game. Each chapter is separate in its context, assumptions, and conclusions. The titles of the chapters are below. Abstracts of each essay or chapter can be found at the beginning of each chapter. The titles of the chapters or essays are as follows:

I. Managerial Ownership with Rent-Seeking Employees
II. Financing Professional Partnerships
III. Sunk Cost Efficiency with Identical Competitors
IV. Business Stealing and Bankruptcy

With the exception of Chapter III, which is meant to complement Chapter IV, these essays argue that the structure of financial contracts can affect the real behavior of firms. The first chapter argues that financial governance policies affect the behavior of rank-and-file employees. In Chapter II, the governance and capital structure of professional service firms affects clients’ expectations of the firm’s quality. In Chapter IV, the enforcement of financial contracts by bankruptcy courts affects the number of firms that enter and exit the industry.
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I benefited greatly from the many discussions with my thesis supervisor, Kevin Roberts, since returning to Oxford to pursue my D.Phil. in October 2001.

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Introduction

0.0 Introduction

“Essays on the Financial Governance of Firms” argues that financial contracts and governance arrangements can be designed in ways that maximize the value of firms in isolation or maximize social welfare, more broadly defined. In short, the author will discuss cases where financial contracts matter. This thesis is a set of four essays. They have some common themes, but they are distinct. This section highlights two less explored themes in the literature on corporate finance and corporate governance. Below, the author discusses how dynamic inconsistency is especially important to Essays I and II and highlights the fact that overinvestment problems are important in Essays III and IV.¹

0.1 Disclaimer

This introduction aims to orient the reader to a few of the larger problems addressed in this thesis. This introduction is not meant to provide a literature review, but rather to highlight some important points in the essays that follow. This is a thesis of four essays. The individual essays have focused literature reviews that attempt to place the individual chapters in a narrow context of the current boundaries of the discipline. In contrast, this introduction tries to broadly place the set of essays as a group in a much wider context within corporate finance, corporate governance, and the theory of the firm. In addition, each chapter makes arguments that are only

¹ For the purpose of this thesis, the term “Essay” and “Chapter” are synonyms. For example, Essay I is Chapter I and so forth.
applicable to the specific circumstances for which they are developed. Therefore, the reader must absorb the arguments and the assumptions in each of the essays separately.

0.2 Two Less Explored Problems

Much of the research in corporate finance has focused on two problems. The first problem asks, “How do we motivate managers to maximize the value of firms?” This is the agency problem. Agents without proper inducements often cannot be trusted to execute the wishes of their principals. Since principals generally have most of the cash, it is believed that the agency problems can lead to underinvestment problems. The second problem asks, “How do we design cash flow and control rights so that all profitable investments are undertaken?” This is a version of the investment problem or, rather, the underinvestment problem. While the investment problem is considered at the firm level, it is believed to have larger economic implications. It is generally believed that contracting problems contribute to too little investment at the industry level and economy wide.

This view is summarized and reviewed by Shleifer and Vishny (1997, p. 773), who write the following:

“Corporate governance deals with the agency problem: the separation of management and finance. The fundamental question of corporate governance is how to assure financiers that they are going to get a return on their financial investment.”
The author believes these problems are extremely important. Nevertheless, he believes that optimal governance and capital structure arrangements will not always fall under the heading of aligning the incentives of agents with investors. Moreover, there may be cases where increased investment in the sector reduces social welfare. Indeed, in the essays that follow, we will demonstrate that there are cases where dynamic inconsistency and overinvestment may be the most important problem to overcome in a particular firm or in a given industry, respectively. This, of course, is a departure from the more conventional approach of considering agency and underinvestment problems. The author pleads the standard defense for departing from the more popular approaches. He has chosen to highlight the problems of dynamic inconsistency and overinvestment because he was preceded by many very able scholars who have explored agency and underinvestment in great detail. Many authors have made considerable progress in addressing these problems as Shleifer and Vishny (1997) in part have documented. Agency and investment are important problems, but they are not the only problems in corporate finance and corporate governance.

This thesis focuses on two problems that have gotten much less attention from the financial economics community. This thesis focuses on the **dynamic inconsistency problem** in the first couple essays, and this problem plays a role in the fourth essay. Dynamic inconsistency is where a decision maker desires to take actions *ex post* that reduce his *ex ante* welfare. The dynamic inconsistency, or time

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2 For example, if the objectives of shareholders evolve over time, do we want managers aligned with shareholder’s *ex post* or *ex ante* objectives? The essays that follow argue that the expected value of individual firms is maximized by governance structures that obey shareholders’ *ex ante* wishes.

3 For example, Smith (1776)’s critique of joint stock companies is often cited as the earliest mention of the agency problem in the economics literature.
Introduction

consistency, problem is a prominent feature of the first two essays. (Nevertheless, the author does admit that the more standard agency and investment problems will creep into the discussion.) In the third and fourth essays, the overinvestment problem is introduced. In particular, industry-wide investment is discussed. This second problem says that in some cases private investment incentives are excessive from the point-of-view of maximizing social welfare. The presence of this problem means that welfare can be raised by discouraging excessive investment in an industry. If inefficient firms do not crowd out the profits of lower cost firms, social welfare can rise.

Despite the fact that these problems are less pursued than agency and investment problems in the field of corporate finance and corporate governance, this thesis ends up on the wide and well-trodden road. This road leads to the conclusion that, contrary to the propositions in Modigliani and Miller (1958), capital structure affects the value of firms. This thesis sees financial contracts and governance arrangements as an early choice variable used to maximize the value of firms. It finds that the capital structure choice has real effects on the cash flows of firms under specific scenarios. In Essay I, delegation issues affect the optimal level of managerial (CEO) ownership. Further, the debt-versus-equity the hiring decision of the partnership (Essay II), and the after-tax profits and the continuation decision in bankruptcy (Essay IV).

0.2.1 Dynamic Inconsistency
In the corporate governance literature, the agency problem of aligning managers’ incentives with investors has overshadowed the dynamic inconsistency problem. This thesis makes a few small steps to correct this imbalance. When shareholders suffer from dynamic inconsistency, they will want to design governance arrangements that allow them to commit to the \textit{ex ante} optimal course of action. The question of dynamic inconsistency is central to essays one and two, which are entitled “Managerial Ownership with Rent-Seeking Employees” and “Financing Professional Partnerships,” respectively.

Despite the fact that this thesis is focused on very microeconomic questions, it does owe a debt to Kydland and Prescott’s contribution to macroeconomic theory. Kydland and Prescott (1977) introduced most economists to the notion that otherwise rational decision makers could undertake actions that undermined their own welfare when those actions were spread out over time. They argued that constraining the social planner’s ability to act optimally \textit{ex post} may make this rational decision maker better off \textit{ex ante}. This thesis argues that optimal governance structures in firms should control for shareholders’ dynamic inconsistency.

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4 Their argument was that a social planner would be tempted to exploit the Phillips curve’s negative relationship between inflation and unemployment. Yet, because consumers and investors know the social planner’s objectives, they will merely expect to have higher inflation. The ability to trade off the bad of inflation for the greater good of increased employment would lead to only the bad of higher inflation. That is, with discretion, the social planner would be saddled with nothing but the bad of high inflation.

Homer’s story of the Sirens is the earliest and probably the most cited example of the virtues of “tying one’s hands.” The witch Circe advises Odysseus in \textit{The Odyssey}, Book XII, Homer (1944, p. 148):

“First you will come to the Sirens who enchant all who come near them. If any one unwarily draws in too close and hears the singing of the Sirens, his wife and children will never welcome him home again, for they sit in a green field and warble him to death with the sweetness of their song. There is a great heap of dead men's bones lying all around, with the flesh still rotting off them. Therefore pass these Sirens by, and stop your men's ears with wax that none of them may hear; but if you like you can listen yourself, for you may get the men
Delegation was one proposed solution to the dynamic inconsistency problem in monetary policy. Rogoff (1985) argued that conservative central bankers, who have a strong preference for low inflation, are better than strict rules because they can respond to macroeconomic shocks. The central banker in Rogoff (1985) had much stronger preferences for low inflation than the social planner. In other words, principals may optimally delegate to an agent with different objectives than their own.

Essay I explores the optimal level of managerial (CEO) ownership. Since Smith (1776) and more recently Jensen and Meckling (1976), the optimal level of managerial ownership has been thought to be 100 percent under ideal circumstances. (Ideal circumstance would include a risk-neutral manager who had sufficient resources to buy the firm.) In Essay I, under otherwise ideal circumstances for 100 percent managerial ownership, the CEO should optimally be given less than a full share of the profits when employees can engage in rent-seeking.

In the Essay II, “Financing Professional Partnerships,” the dynamic inconsistency problem is revisited. This time founding shareholders would like to commit to be very selective about the employees that they hire. Levin and Tadelis (2005) argue that the partnership organizational form is such a commitment device. Commitment is profitable when many potential clients are uninformed about the firms’ average quality. This essay extends this result and demonstrates that the partnership can optimally adjust its quality threshold by its short run capital structure.
decision. Unlike Levin and Tadelis (2005), which says the partnership will consistently fail to achieve the full monopoly profits, Essay II shows that, by altering the way it pays out its profits, the partnership can earn the full monopoly returns, given that net-debt levels are observed by clients. Nevertheless, Chapter II demonstrates that when there are no financial frictions and net-debt levels are hidden from clients there is no reason to suppose that partnerships are any more selective than corporations. Therefore, while a partnership with transparent finances is only hurt by financial frictions, financial transaction costs can actually make the partnership with opaque financial structure more profitable.

Dynamic inconsistency does play a role in the last chapter, Essay IV. Essay IV, “Business Stealing and Bankruptcy,” has some shareholders suffering from their inability to commit ex post. This essay is concerned with an industry where the incentives to invest are socially excessive. Therefore, welfare-maximizing courts want to discourage entry by raising the taxes of potential entrants. By increasing shareholders’ ex post bargaining position in bankruptcy, these potential entrants will have a lower debt capacity and will pay higher taxes in expectation. In contrast to the literature on CEO pay, where aligning the incentives of top managers to the current wishes of shareholders has been the focus, dynamic inconsistency between debtors and creditors has been well studied.5

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5 The conflicts between shareholders and bondholders is well known. This conflict can be described as a dynamic inconsistency problem. For example, even the classic article on agency conflicts between managers and owners, Jensen and Meckling (1976), illustrates a dynamic inconsistency between current future shareholders in a highly levered firm. Suppose that a 100 percent equity firm would be maximized with some debt in its capital structure. The value of the initial shareholders’ claims in the initial, 100 percent equity firm will be reduced by the fact that future shareholders will wish to take on speculative projects when the firm is highly leveraged. That is, future shareholders will inefficiently shift risk ex post onto bondholders, and this reduces the value of the initial shares that would benefit from the sale of debt.
0.2.2 The Overinvestment Problem

In Chapter’s III and IV the overinvestment problem at the industry level is explored. Welfare is not necessarily maximized by the free entry of firms into an industry. Either the high cost firms enter and crowd out the low cost firms, as in Essay III, or too many firms enter, as in Essay IV.

In Chapter III, we consider how alternative sequencing of firms affect the total level of sunk and fixed costs employed in an industry. In particular, when firms have identical marginal costs but different sunk costs of entry, it explores the circumstances in which only the lowest sunk cost firms will enter the industry. It develops sufficient conditions for only the lowest sunk cost firms to enter. In short, it attempts to find the circumstances where the ordering of firms does not affect the aggregate level of sunk costs in the industry. For example, the entry game in Essay IV is one of those instances where the lowest fixed cost firms enter, regardless of the ordering of entry.

Essay IV is concerned with how bankruptcy courts can alleviate a market failure that leads to overinvestment. Under reasonable assumptions, Mankiw and Whinston (1986) demonstrate that free entry leads to socially excessive duplication of fixed costs. This result holds when firms have identical products, charge markups over marginal cost, and pay no taxes. Overinvestment with free entry is caused by individual firms’ failure to internalize the costs that they incur on their rivals. The
negative externality, called the “business stealing” effect, in this case outweighs the additional benefits that marginal entrants bring to consumers.

Chapter IV considers how bankruptcy courts can raise aggregate welfare in such an industry through the enforcement of debt contracts and liquidation decisions in bankruptcy. It argues that bankruptcy courts can reduce the overinvestment problem by discouraging entry by firms with high investment costs. The main tool for doing this is reducing the debt capacity, and thus raising expected taxes, of potential entrants by redistributing *ex post* returns to equity. Moreover, Essay IV demonstrates that welfare maximizing courts will weakly want to speed the exit of firms with high liquidation values. The author knows of no other study that relates the problem of overinvestment to the objectives of bankruptcy courts.

Unlike Essays I through III, Essay IV has a policy focus. Maximizing aggregate welfare is the assumed objective of the courts. In Chapter IV, this is sometimes in conflict with the objectives of the original owners of firms that land in bankruptcy court. This is in contrast with Essays I and II where maximizing the returns to founding shareholders was often equivalent to maximizing social surplus. Indeed, in the literature on bankruptcy where underinvestment is assumed to be the social problem, the objectives of maximizing the *ex ante* value of firms and the objectives maximizing total welfare are aligned.

The author believes that Essay IV makes an important contribution in illustrating how overinvestment can be a major problem that bankruptcy courts can address. This essay assumes that benevolent courts have perfect information and have no commitment problems. Instead, the biggest constraints that bankruptcy
courts face in Chapter IV are the policy tools that they have at their disposal. For these reasons, “Business Stealing and Bankruptcy” should be considered the upper bound of what courts could achieve in an industry plagued by excessive entry and overinvestment.

0.3 Structure of the Thesis

Four essays, a conclusion, and a general set of references follow this introduction. All essays have their own abstracts, introductions, conclusions, and references. Moreover, all assumptions and conclusions are only relevant to the essay in which they were made or reached, respectively. The four essays are followed by a brief general conclusion. A general list of references common to this introduction and the four essays is given at the end of this thesis.
References


Introduction


Statement of Authorship and Word Count

- I was the sole author of this thesis and the materials contained in this document. I wish to retain all copyrights to the material herein.

  Linus Wilson  
  St. Cross College  
  6 August 2007

- In the chapters that follow, I will often use “we” to refer to the reader and myself.
Chapter I

Managerial Ownership with Rent-Seeking Employees
Managerial Ownership with Rent-Seeking Employees

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Abstract

The traditional agency problem advocates 100 percent share ownership when managers are risk-neutral, and managers either have enough wealth to buy the firm outright or have access to perfect capital markets. This paper says that delegation to the disinterested managers may sometimes explain the separation of ownership and control even before one considers diversification motives or credit market imperfections. High levels of CEO share ownership may induce rent-seeking employees to behave badly. Delegation to disinterested managers, with lower levels of share ownership, makes firms more valuable than retaining CEO-level agents that think like 100 percent owners.

Journal of Economic Literature Classification: D23 & G34

Keywords: CEO compensation, contracts, corporate control, shareholders, rent-seeking, and unions

* Alan Morrison and Antione Faure-Grimaud made many valuable suggestions for paper. I am indebted for their detailed comments. I thank Kevin Roberts many discussions. The author was a guest of Volker Nocke and the Economics department at the University of Pennsylvania and a visitor at the University of Cincinnati’s Department of Finance–Real Estate while writing this paper. The usual disclaimer applies.
1.0 Introduction

“The directors of such [joint stock] companies, however, being the managers rather of other people’s money than of their own, it cannot well be expected that they should watch over it with the same anxious vigilance with which the partners in a private copartnery [partnership] frequently watch over their own.”

--Smith (1776 [1791], Book IV, Chapter 1, Article 3, p. 33)\(^1\)

“…the separation of ownership and control in the large corporation may in some cases not be a bad thing for owners. Indeed, the separation may be essential for the credibility of some threats, promises, and commitments.”

--Vickers (1985, pp. 143-144)

The classic agency problem of Smith (1776) and Jensen and Meckling (1976) advocate aligning the incentives of managers and owners. In effect, managers that think like owners will maximize the value of firms and will seek the most efficient production methods. In a risk neutral world where there are no credit constraints, a top manager should optimally hold all the shares in an enterprise.\(^2\) In contrast, this paper argues that managerial ownership will be driven by delegation motives. In some firms, owners may prefer to have a manager with a minimal ownership stake because the incentives of the manager will affect the rent-seeking motives of the firm’s employees. A disinterested manager will discourage employees from engaging in rent-seeking activities. Owners can increase the value of the firm by eliminating

\(^1\) Brackets, “[ ]” are added by the present author.

\(^2\) Risk neutrality facilitates 100 percent ownership because it allows the manager to be unconcerned about diversification. Lack of credit constraints allows the best manager to buy the firm from its original owners. These assumptions are sufficient for the optimal incentives, 100 percent ownership, to be transferred to the best manager without forcing the original entrepreneur to sell the firm at any discount. (One could also argue that symmetric information would be also necessary for such an ownership sale to be always efficiently consummated.)
rent-seeking costs. They achieve this if they give control to a CEO with a small or non-existent ownership stake.

For all parameter values, the weakly optimal shareholdings of the CEO will be strictly less than 100 percent. These results are entirely caused by the by employee rent-seeking motives. Without employee rent-seeking, it is demonstrated that we will revert to the standard agency problem where 100 percent managerial stakes are optimal.

There is some empirical support for the theoretical results of this study. CEO performance pay declines in the level of unionization according to Anderson, Boylan, and Reeb (2006). Unionized workforces may be susceptible to rank-and-file rent-seeking. Wilson (2006) and the present paper argues this. Anderson, Boylan, and Reeb (2006) find that long term performance pay for CEOs increases as a percentage of total pay by 2.3 percent for every 10 percent increase in the percent of employees that are non-unionized. That is, on average, a completely non-unionized, publicly traded firm in the United States will have 23 percent more of the CEO’s total compensation tied to stock, option, and performance pay related grants. These results are consistent with the delegation story whereby CEO incentives are weaker in order to minimize union member’s incentives towards rent-seeking.

The value of delegation is central to the study of monetary policy, but often goes uncommented when corporate governance is discussed. Alesina and Summers (1993) display how independent central banks correlate with low inflation rates. Their work builds on theory dating back to Kydland and Prescott (1976) and Rogoff (1985) which argues that strict rules or extremely inflation-averse central bankers,
with little interest in low unemployment, best achieve the joint social goals of high employment and low inflation.

Shleifer and Vishny (1997) argue that the agency conflict is the main problem in corporate governance. Yet, Berle and Means (1932) observed that many of the largest corporations are widely held with little in the way of concentrated ownership. Most commentators assume that this occurs in large part because of diversification motives of Markovitz (1952). Concentrated ownership is too costly for many investors with limited wealth. Therefore, small ownership stakes for managers may be seen as response to the problem of the tradeoff between creating efficient incentives and leaving managers with too little diversification. Vickers (1985) and the present paper argue that delegation may be in the interests of shareholders. For the purpose of this paper, “delegation” is the action whereby a principal prefers to give control to an agent with different objectives than the principal. In the present study, delegation to a risk neutral CEO with potentially unlimited wealth but a tiny stake in the enterprise is optimal because it positively affects the incentives of employees working underneath the manager.³

Reducing rent-seeking costs is a rationale for divestiture of declining divisions in Meyer, Milgrom, and Roberts (1992). Declining divisions are characterized by divisional managers who lobby the center for extra resources. Shareholders in these firms can avoid paying for these wasteful lobbying efforts by selling off their underperforming divisions. In Meyer, Milgrom, and Roberts (1992), separation

³ Diversification motives would only strengthen the results of this paper that low levels of CEO ownership are optimal.
eliminates rent-seeking. In contrast, in the present study, delegation to a sufficiently disinterested CEO reduces rent-seeking by rank-and-file employees.

This paper has some common features with Pagano and Volpin (2005), but it has some key differences. That study, like this one, examines managerial ownership and employee relations in the context of a sequential game. Yet, the present paper differs from that study in two respects. First, this paper allows the founder of the firm, the “entrepreneur,” to act as the CEO. Pagano and Volpin (2005) does not. For this reason, and because the manager is credit constrained with limited wealth, the founder in Pagano and Volpin (2005) cannot sell the firm entirely to the manager and will never give away the optimal ownership stake of 100 percent to managers that is advocated by Smith (1776) and Jensen and Meckling (1976). Secondly, this paper, in contrast to Pagano and Volpin (2005), allows employees to affect the manager’s costs of monitoring workers. The latter study takes the manager’s cost of monitoring as exogenous.

There is another closely related recent study. Mueller and Phillipon (2006) complements the present paper. That paper argues that confrontational labor relations leads to concentrated ownership. They find, in countries with less-cooperative labor relations, that ownership and control is more concentrated. In contrast, we argue that confrontational labor relations can be avoided by having managers with small ownership stakes. Here high ownership stakes often lead to confrontational labor relations. In their paper, uncooperative labor relations lead to high ownership stakes. The major difference between our results and theirs is that in their study the level of employee rent-seeking is exogenous. Here the level of employee rent-seeking is
endogenously determined. If rent-seeking—confrontational labor relations—were completely exogenous within the firm, then we would agree with Mueller and Phillipon (2006). In section 3.2, this paper shows that the Smith (1776) and Jensen and Meckling (1976)’s solution of 100 percent managerial ownership is optimal with an exogenous level of employee rent-seeking.\(^4\)

This paper is most closely related to Wilson (2006). Both papers come to similar conclusions about CEO pay. Much like Wilson (2006) this paper predicts that in order to maximize shareholder value CEO incentives should be less sharp and top managers should have lower levels of pay when employees engage in rent-seeking. Nevertheless, that paper does not specifically discuss compensation in terms of share ownership as does this study. Wilson (2006) considers bonus schemes that do not always closely follow the share price. Nevertheless, the clear result in both is that CEOs should receive less share or incentive compensation in unionized firms. This result is confirmed by the empirics of Anderson, Boylan, and Reeb (2006) discussed earlier.

The results differ when we discuss the extent of rent-sharing between shareholders and rent-seeking workers. In the present paper, workers are prevented from sharing in the rents. Here, the top manager’s efforts increase in the monitoring intensity. In Wilson (2006), the CEO’s incentive compatible efforts increase in firm value, as opposed to the monitoring costs of rank-and-file workers in the present

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4Because they assume that there is an increasing cost to holding large blocks of shares, Mueller and Phillipon (2006) do not generally find that 100 percent stakes are optimal. They are assuming that risk-averse managers may have to forgo some diversification benefits by accepting a very large ownership stake. Without this tradeoff, 100 percent ownership stakes would be the first-best, regardless of the exogenous level of employee obstruction.

In addition, that paper’s empirical results are of limited help in illuminating this question because it only considers ownership concentrations and labor relations at the national level, not at the firm level.
model. Shareholders could not afford to give the CEO a minimal merely, individually rational compensation package because in Wilson (2006) this would leave the firm worthless. In the present paper, the entrepreneur can endow the CEO with so little compensation that the union must cease in rent-seeking, or the firm will fall apart. The take-it-or-leave-it result of this paper is in sharp contrast with the rent-sharing between workers and shareholders found in Wilson (2006). This difference stems from the different objective functions of the manager. Wilson (2006) could be viewed as a situation where the manager has many other tasks besides monitoring union members. Giving the CEO too little performance pay in Wilson (2006) will decrease firm value. Yet, in the present paper because the CEO only has one task, monitoring workers, more generous incentive pay is not optimal because low managerial incentives induces union members to curtail rent-seeking behavior.

Grout (1984) and Baldwin (1983) have argued that \textit{ex post} appropriation of surplus by unions, in particular, leads to underinvestment. Rent-sharing in a dynamic context can be seen as a form of the “hold-up” problem, where investors’ returns are expropriated \textit{ex post}. For this reason, the results of this paper should be encouraging because they do not imply, as does Wilson (2006), that the underinvestment associated with rent-sharing, may be an unavoidable result of having a workforce prone to rent-seeking. The present paper argues that rent-sharing, and underinvestment, may be avoided altogether.

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5 Wilson (2006) also does not explicitly consider share contracts as this paper does. Instead it considers non-linear bonuses.
The timeline of the model is as follows:

**Period 0**
Owner-manager sets CEO compensation contract consisting of a fixed wage, $u$, and a share, $\alpha$, in the firm.

**Period 1**
Union chooses the manager’s cost of monitoring, $m$.

**Period 2**
CEO-manager chooses a shirking detection probability, $q$, and a wage, $w$, for workers.

**Period 3**
- Workers and CEO decide to participate or not and how hard to work.
- Output is realized.
- All factors are paid.

**Figure 1: Timeline**
First, an entrepreneur decides on the size of her fixed wage, $u$, and how large an ownership stake, $\alpha$, that she should retain in the firm. Then employees choose how high to set the manager’s cost of detecting slack effort, $m$. This choice can be seen as the level of efficiency-reducing rent-seeking that employees engage in. In period 2, the manager chooses the level of monitoring that maximizes her payoff. Next, in period 3, employees choose effort levels that maximize their payoffs. Then returns are realized, and all factors of production are paid.

In the next section, we describe the timing, technology, and preferences of the actors in more detail. In section 3, we solve the manager’s, employees’, and the entrepreneur’s problems in turn. The optimal level of managerial ownership depends on the costs of rent-seeking to employees, relative to the costliness of monitoring to managers. This study finds that, when employees engage in rent-seeking, the optimal unconstrained level of managerial share ownership will not equal 100 percent as proposed by Smith (1776) and Jensen and Meckling (1976). For all parameter values, smaller managerial ownership stakes at least weakly dominates 100 percent stakes, because lower levels of managerial share ownership induces employees to minimize rent-seeking costs. This is especially true when employee rent-seeking costs are relatively low. Employees have to scale back their rent-seeking when managers are given minimal ownership stakes. Otherwise, production will not take place. The savings from low levels of rent-seeking can be passed on to the entrepreneur when she sells a large stake to outsiders at the beginning of the game.
2.0  Setup

2.1  Production Technologies

The firm can generate revenues of $R \in \{0, \bar{R}\}$. The probability that the firm can generate high revenues is $P\{R = \bar{R}\} = e$, where $e$ is the aggregate effort of workforce normalized to be of size one. Likewise, the probability of low revenues is $P\{R = 0\} = 1 - e$. We can think of the workforce as an interval of identical atomistic agents; their total measure is normalized to one. Each worker can choose an effort level of either $e = 0$ or $e = \bar{e}$. The cost of low effort is normalized to $c(0) = 0$, while the cost of high effort is $c(\bar{e}) = \bar{c}$. High effort work is efficient because the productive benefits of high effort outweigh its costs. Namely, $(\bar{c} - 0)R > \bar{c}$.

2.2  Preferences and Opportunities

Both the entrepreneur and the union members have linear utility in income. Nevertheless, their costs of effort are potentially non-linear. The costs of effort are linearly separable as is the benefit of income. Union members have two costs of effort. They first have the cost of performing productive work $c(e)$, and then they have the cost of obscuring shirking $c(m)$. Low $m$ means that the manager, the CEO,
can detect shirking at little cost. While high $m$ means that the manager must incur great effort to detect slack work by union members.

The opportunity costs to the entrepreneur and the CEO are both $U_M$, that person’s outside option wage. The best alternative investments for the entrepreneur are zero net present value. Further, the union member’s outside option wage is zero.

The CEO and union incur their effort costs and receive their compensation in period 3, the productive period.

For production to be efficient, it must be the case that expected revenues under high effort, $\bar{c}R$, weakly exceed the union member’s effort costs, $\bar{c}$, plus the managers outside option, $U_M$. That is, $\bar{c}R \geq \bar{c} + U_M$. We will assume this is always the case.

Union members are assumed to be credit constrained and have insufficient wealth to buy the firm outright. These credit constraints may stem from the fact that union members have conflicts of interest with outside investors. For example, Grout and Jewitt (1988) demonstrate how, in many instances, employees will be tempted to raise their wages at the expense of other shareholders.\(^6\)

### 2.3 Monitoring Technologies

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\(^6\) Grout and Jewitt (1988) argue that either employees will reduce wages to the efficient level or they will find it optimal to raise wages farther from the efficient level. If the latter is the best response of the workers, post-buyout, then the only way they can win control of the firm is to appropriate the surplus of shareholders who do not sell to the union. By raising wages, they are destroying surplus. Nevertheless, this smaller pie may be large enough to improve the lot of both the workers and the person or persons selling a controlling interest.
• The top manager can detect shirking at the cost to herself of \( c(q) = \frac{mq'}{x} \), where \( x \) is a non-negative, exogenous parameter. That is, \( x \in [0, +\infty) \). Further, the parameter \( m \) is constrained to be non-negative. Namely, \( m \in [0, +\infty) \).

Monitoring is at least weakly costly, \( c'(q) \geq 0 \). The results of the CEO’s monitoring of workers is common knowledge. Further, \( q \) is the probability that low effort work will be detected. Since it is a probability, \( q \) is constrained to \( q \in [0,1] \). The manager sets \( q \) in period 2.

• The union can obstruct monitoring at the variable cost per member

\[
\text{of } c(m) = \frac{km'}{y}.
\]

Both \( k \) and \( y \) are non-negative, exogenous parameters—\( y \in [0, +\infty) \), and \( k \in [0, +\infty) \). Therefore, obstructing monitoring is also weakly costly to union members, \( c'(m) \geq 0 \). The union chooses \( m \) in period 1.

2.4 Contractual Boundaries

• The CEO’s fixed wage, \( u \) cannot exceed the expected gross profits of the firm. That is, \( u \), cannot exceed expected revenues, less total wages to union members, \( u \leq E[R] - w = \bar{R} - w \). Further, we will consider the case where \( u \) is weakly constrained to be less than total revenues. Let us define a parameter \( \phi \geq 0 \), which relates the maximum wage as a function of the CEO’s outside option. By definition, \( u \leq \phi U_M \).
• The shareholder can credibly commit to pay fixed wages of \( u \) and \( w \) in period 3, even if revenues are insufficient to cover expenses. (This will be the case \((1 - \overline{e}) \times 100\) percent of the time even if union members exert high effort.) In effect, we relax the limited liability constraint. Shareholders can have negative payoffs in low output realizations. One simple way to work within the limited liability constraint would to envision shareholders making an upfront, binding, investment to pay wages \( u \) and \( w \) prior to the state of the world being realized. Shareholder’s net payoff is negative in the bad output realizations, but their \textit{ex post} net payoffs are positive in the good states of the world when workers exert high effort.\(^7\)

• \( \alpha \leq 1 \). The share of profits, \( \alpha \), which is granted to the CEO cannot exceed one.

• Union members pay a non-negative penalty if they are caught shirking. This penalty is \( P \geq 0 \). The penalty is an exogenous parameter of the legal environment.

\(^7\) None of the results in this paper rely on investors’ risk aversion or capital market imperfections. Ownership and governance structures are often explained by one or both of these challenges. This simplification distinguishes our results from many studies. Here the optimal governance and ownership structures are set in the context of their strategic impact on the worker’s behavior.

All wages and payoffs are expressed in their expected values before the revenues are realized in period 3.
The first bullet point assumes that the fixed portion of the CEO’s wage has an upper bound. In the first instance, we can think of this upper bound as a budget constraint. The entrepreneur cannot promise to pay herself a fixed wage in excess of the profits generated. It is not hard to imagine that it would be exceedingly hard (impossible) to sell securities to outside investors if this were the case. Further, we have for convenience denoted the upper bound of $u$ to be a multiple of the manager’s outside option. We can think of this as some prohibition against “excessive” self-dealing by the manager. From a practical point of view this may prevent the entrepreneur making take it or leave it offers to the union by paying herself a fixed wage the size of the net rents generated by the firm. Nevertheless, we are allowing for some parameter values where this will be the case. We will discuss this more in the context of the model. In practice, we are saying that the firm can only commit to pay the manager a fixed wage of $\phi U \leq M$.

The second bullet point is a budget constraint. The firm cannot pay out more profits than it generates. This constraint also will become important when the model is solved.

Finally, the third bullet point means that fired workers weakly suffer some sort of penalty for being fired. For example, unemployed workers may suffer some stigma or job search costs. Nevertheless, this is not a choice variable, but instead describes the legal environment in which workers are operating. This will tend to make being caught for shirking less attractive.
3.0 Model

The model will be solved by backwards induction. This will give us closed form solutions for the unconstrained, best-response solutions for monitoring intensity, $q$, monitoring costs, $m$, and ownership stakes, $\alpha$.

The program that is followed is to maximize the returns to the entrepreneur, given the best responses of the Union and the CEO, respectively. That is,

$$\arg \max_{\alpha, x} V_O = (1 - \bar{c})0 + \bar{c}R - w(q(m)) - \frac{mq^x}{x} - U_M$$

The entrepreneur only cares about the expected revenues, less union wages, less her own cost of monitoring when she is the CEO, less her opportunity cost of serving as the CEO.

There are several incentive and rationality constraints of the players that must be satisfied. In particular, the entrepreneur has to worry about the incentive compatibility constraints for the union members in period 1 and herself when she is acting as the CEO in periods 2 and 3. Further, she must satisfy her individual rationality constraints in period 0 as the entrepreneur and in periods 2 and 3 as the manager. In addition, the union members’ participation constraints in period 3 must
also be met. Given that high effort, $\bar{e}$, maximizes the value of the firm, the $IR$ and $IC$ constraints\(^8\) are the following for the entrepreneur,\(^9\) union members, and the CEO:

\[
IR_O: \quad V_O = E(\bar{R}) - w - c(q) - U_M \geq 0
\]
\[
\bar{e}R - w - \frac{mq'}{x} \geq U_M
\]
\[
IR_U: \quad V_U = w - c(e) - c(m) \geq 0
\]
\[
w - e - \frac{km'}{y} \geq 0
\]
\[
IC_U: \quad w - c(e) \geq (1-q)w + q(-P)
\]
\[
w \geq \frac{\bar{e}}{q} - P
\]
\[
IR_M: \quad V_M = \alpha(E(\bar{R}) - w - u) + u - c(q) - U_M \geq 0
\]
\[
\alpha(\bar{e}R - w - u) + u - \frac{mq}{x} \geq U_M
\]
\[
IC_M: \quad \alpha(E(\bar{R}) - w - u) + u - c(q) \geq u
\]
\[
\alpha(\bar{e}R - w - u) - \frac{mq}{x} \geq 0
\]

**Lemma 1**

*If the $IR_M$ constraint is satisfied, then the $IR_O$ constraint must be satisfied.*

This follows from the fact that the CEO’s base wage cannot exceed the expected profits of the firm. Therefore, the entrepreneur has more incentive to participate because she benefits from equity sales of $(1 - \alpha)$ of the firm’s shares. A formal proof of this is left for appendix 5.1. This, of course, simplifies the problem to two incentive and two rationality constraints.

---

\(^8\) The $IC_U$ constraint is much like an efficiency, or non-shirking wage, as proposed by Shapiro and Stiglitz (1984).

\(^9\) The entrepreneur only has a participation constraint, $IR_O$.  

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The union’s program in period 1 is to choose an “m” that both maximizes its utility and satisfies the manager’s and its own rationality and incentive constraints. Let $w(q)$ be the SPE wage schedule, and $q(m)$ be the SPE monitoring intensity.

$$\arg\max_m V_U = w(q(m)) - \bar{c} - \frac{km}{y}$$

The manager’s problem in period 2 is to choose a monitoring intensity, $q$, which maximizes her payoff.

$$\arg\max_q V_M = \alpha[(1-\bar{c})0 + \bar{c}R - w(q(m)) - u] + \frac{mq}{x} - U_M$$

The wage schedule, $w(q(m))$, will depend on whether or not ICU of IRU binds. The discussion begins by considering the CEO’s problem when the union member’s incentive constraint is the most difficult to satisfy. For example, when $P = 0$ and $k = 0$, the satisfaction of ICU implies that IRU is also satisfied. Let us begin by solving the model by backwards induction when the ICU binds and IRU is slack. Backwards induction requires that we first solve the manager’s problem because the CEO moves last after both the entrepreneur, who moves in period 0, and the union, which moves in period 1.

### 3.1 The CEO’s Problem

#### 3.1.1 The manager’s problem when ICU binds
Suppose that the union members’ incentive compatibility constraint implies that working is also individually rational for all union members. Then we can substitute in the non-shirking incentive compatible wage into the CEO’s program.

The CEO’s payoff is the following when the $IC_U$ binds:

\[
V_M = \alpha \left( \bar{c}R - \frac{c}{q} + P - u \right) + u - \frac{mq^x}{x} - U_M \quad (1)
\]

The CEO chooses the shirking detection probability to maximize her payoff:

\[
\arg \max_q V_M = \alpha \left( \bar{c}R - \frac{c}{q} + P - u \right) + u - \frac{mq^*}{x} - U_M
\]

\[
\frac{dV_M}{dq} = \frac{\alpha \bar{c}}{q^2} - mq^{*x-1} = 0
\]

The second order condition is unambiguously negative at $q^*$:

\[
\frac{d^2V_M}{dq^2} \Bigg|_{q=q^*} = -\left\{ (\alpha \bar{c})^{x-2} m^3 \right\}^{\frac{1}{3x+1}} (x+1) < 0
\]

The manager’s payoff is therefore maximized at
\[ q^*(m) = \left( \frac{\alpha c}{m} \right)^{\frac{1}{1+x}}. \]  \hspace{1cm} (2)

Since \( q \) is a probability, there will be many instances where the constraint that \( q \in [0,1] \) will bind. In particular,

\[ q^*(m) = \min \left\{ \left( \frac{\alpha c}{m} \right)^{\frac{1}{1+x}}, 1 \right\}. \]

Since some positive monitoring is needed to ensure the incentive compatibility of effort, it may also be the case that \( q \) is determined by the union member’s incentive compatibility condition, the non-shirking constraint, and not the managers’ unconstrained best response given by equation (2).

### 3.1.2 The Manager’s Problem when the \( IR_U \) binds

The manager’s problem when the \( IR_U \) binds is almost trivial. When the \( IR_U \) binds, it must be the case that the monitoring intensity falls between \( q \in [\overline{c}/(\overline{c} + P), 1] \). Otherwise, the \( JC_U \) will not be satisfied. Further, the union members must be paid at least for their monitoring costs. Therefore, the manager’s problem is as follows:

\[
\max_{w, q} V_M = \alpha(\overline{c}R - w - u) - \frac{mq^*}{x} - U_M,
\]

where \( q \in [\overline{c}/(\overline{c} + P), 1] \) and \( w \in [\overline{c}, \infty) \).  \hspace{1cm} (3)
\[
\frac{dV_m}{dw} = -\alpha \leq 0 \\
\frac{dV_m}{dq} = -mq^{r-1} \leq 0
\]

Therefore, the manager will weakly prefer to both minimize the union members’ wages and minimize the monitoring intensity when the \( IR_U \) constraint binds. The manager will choose the following:

\[
w = c \\
q(m) = \frac{c}{c + P}, \quad \forall \ m > 0, \\
\text{else } q(0) \in \left[ \frac{c}{c + P}, 1 \right]
\]  

Further, the wage paid and the monitoring intensity does not depend on the CEO’s compensation package \( \{\alpha, u\} \).

### 3.2 The Traditional Agency Problem

#### 3.2.1 The Owner’s Problem when the \( IC_U \) binds

We can easily verify that 100 percent ownership is the unconstrained optimum stake when the union cannot affect the marginal costs of monitoring \( m \). We will denote this exogenous level of monitoring costs as \( m^A \). Plugging \( q^*(m) \) from equation (2) into the entrepreneur’s program, we get the following problem:
arg max \( a \& u \) \( V^A_0 = \mathbb{e}R - w^*(q^*) \frac{mq^x}{x} - U_M , \)

where \( V^A_0 = \mathbb{e}R - \left( \frac{m\overline{e}^x}{\alpha} \right)^{\frac{1}{1+\alpha}} + \alpha - \left( \frac{m\overline{e}^x}{\alpha} \right)^{\frac{1}{1+\alpha}} \frac{1}{x} - U_M . \)

Since this function does not depend on the manager’s base wage, the entrepreneur is indifferent over the entire feasible set of possible fixed wages, \( u \).

The first order condition with respect to \( \alpha \) is as follows:

\[
\frac{dV^A_0}{d\alpha} = \left( \frac{m\overline{e}^x}{x+1} \right) \left[ \alpha^{-\frac{1}{1+\alpha}} - \alpha^{-\frac{1}{1+\alpha}} \right] = 0
\]

The stationary point implied above is \( \hat{\alpha} = 1 \).

The second order condition of \( \alpha \) is negative, and we can easily check that this is a maximum:

\[
\frac{d^2V^A_0}{d\alpha^2} = - \left( \frac{m\overline{e}^x}{x+1} \right) < 0
\]

Therefore, the compensation package that maximizes the entrepreneur’s payoff below involves 100 percent share ownership for manager.

\[
\hat{u} \in [0, \varphi U_M ] \quad \hat{\alpha} = 1
\]
Proposition 1

100 percent managerial ownership is optimal when workers cannot engage in rent-seeking.

This interior solution should come as no surprise to the reader. One hundred percent equity ownership is the solution to the classic agency problem result of Jensen and Meckling (1976) and Smith (1776).\textsuperscript{10} The agency problem emerges when less than 100 percent of the equity is owned by the manager. We will find in proposition 3, that 100 percent equity ownership is generally NOT the subgame perfect Nash equilibrium (SPE) level of managerial share ownership when the union engages in rent-seeking activities.

3.2.2 The Agency Problem when the \textit{IRU} binds

With an exogenous level of monitoring costs, $m^A$, the entrepreneur’s payoff when the \textit{IRU} binds is as follows:

$$x^A = \overline{cR} - \overline{c} - \frac{m^A}{x} \left( \frac{\overline{c}}{\overline{c} + P} \right)x - U_M$$

(6)

The entrepreneur’s payoff is the expected wages, $\overline{cR}$, less the \textit{IRU} wage, $\overline{c}$, from equation (4), less the costs of monitoring, less the opportunity cost of managing

\textsuperscript{10} Further, $\alpha = 1$ is not a corner solution because a 100 percent share maximizes the value of the firm in the unconstrained problem. Higher percentages, $\alpha > 1$, even if they would be feasible, would strictly decrease the value of the firm.
the firm, $U_M$. Nevertheless, none of these values depends on the level of managerial share ownership, $\alpha$, or managerial wages, $w$. As long as the CEO gets some positive, though vanishingly small share of the firm it will be strictly in her interests to choose union wages and monitoring intensity given in (4). This will give the entrepreneur a payoff of (6).

3.3 The Union’s Problem

Suppose that the rank-and-file employees can affect the cost to the manager of detecting slack effort. If the $IR_U$ binds, the union will weakly prefer to minimize monitoring costs. If the $IC_U$ binds, the Union may be able to win rents from obstructing monitoring. In the following subsection, section 3.4, it will be shown that the entrepreneur need not share rents and can force the union off the $IC_U$ constraint onto the $IR_U$ constraint. The union’s $IC_U$ strategies are somewhat complex, but they are never credible in equilibrium. For this reason, the unions’ problem when the $IC_U$ constraint binds has been relegated to the appendix sections 6.1.$^{11}$

The unions’ choice of $m$ is $m = 0$ when its wage does not depend on its incentive constraint. The manager will always pay the union for its efforts both productive, $\tau$, and unproductive, $\frac{km^y}{y}$. The union’s payoff in these circumstances is payoff is denoted by $\mathcal{L}_U$ below, where $\mathcal{L}_U = 0$. Obviously, the union’s payoff does not depend on the monitoring costs that it incurs on the manager. Therefore, it will be

$^{11}$Appendices 6.1 through 6.3 are concerned with the non-credible best responses of the union. Those appendices are based on the counterfactual that the $IC_U$ binds and the $IR_U$ is slack and satisfied.
indifferent between any feasible $m \in [0, +\infty)$, when the $IR_U$ binds but the $IC_U$ is satisfied. By assumption, the union will take the action that both maximizes the value of the firm and does not worsen its payoff. That means it will choose $m = m^* = 0$.

### 3.4 The Entrepreneur’s Problem

Here we go to the first step of the game, in period 0, where the owner-manager chooses the optimal wage contract to motivate the manager in period 2 and 3. Suppose that the firm is wholly owned by an entrepreneur. Let us assume that the entrepreneur can tap perfect capital markets where outside investors are price takers. The entrepreneur would like to pick some combination of managerial share ownership and fixed wages such that she maximizes *ex ante* value when she acts *ex post* as the CEO.\(^\text{12}\) The value of the firm to this actor is both her wages from being the CEO, and the returns from selling an equity stake worth $(1 - \alpha)$ of the expected returns.

There are two types of strategies that the entrepreneur can pursue. Deadweight losses drive a wedge between the strategy of minimizing the union’s payoff and maximizing the owner’s payoff with the choice of \(\{u, \alpha\}\). First, the entrepreneur can attempt to minimize the union’s payoff with the choice of the set \(\{u, \alpha\}\). Second, the entrepreneur can maximize her payoff with her choice of the manager’s compensation package.

---

\(^{12}\) It is equivalent to envision the entrepreneur selling the firm to an outside manager. The price of the sale to the outside manager would be $V_{U}$. 
The former strategy, \( \min_{w.r.t.\ u\ &\ a} V_U \), only dominates the latter, \( \max_{w.r.t.\ u\ &\ a} V_O \), if minimizing the union’s payoff means that the \( IR_U \) binds and the union chooses to minimize monitoring costs, \( m = 0 \). This is not always the case when the \( IC_U \) binds. The owner’s \( \min_{w.r.t.\ u\ &\ a} V_U \) strategies when the \( IC_U \) binds are pursued in appendix 6.3.

Yet, the minimization strategy is always effective when the entrepreneur has the CEO operating on the manager’s and union’s \( IR_M \) and \( IR_U \) constraints, respectively. These latter subgame perfect equilibrium (SPE) strategies are pursued in section 3.4.2.

Alternatively, the entrepreneur can attempt to maximize her own payoff. This latter strategy would be pursued if it induces the union to minimize monitoring costs \( m = 0 \), or if the \( \min_{w.r.t.\ u\ &\ a} V_U \) strategy allows the union to earn some rents by choosing an \( m > 0 \). Because the \( \max_{w.r.t.\ u\ &\ a} V_O \) strategy is strictly dominated for many parameter values and weakly dominated for all others, it is relegated to the appendix section 6.2.

We will demonstrate below in section 3.4.2 that one way of minimizing the union’s payoff will always lead to the \( IR_U \) constraint to bind. In particular, it will be shown below that a binding \( IR_M \) constraint is sufficient to induce the union to be passive and minimize monitoring costs. For this reason, it will always be weakly preferred to the \( \max_{w.r.t.\ u\ &\ a} V_O \) strategy.

3.4.1 Passive Workers
In general, $m = m' = 0$ will form a subgame perfect Nash equilibrium (SPE) strategy by the union for a range of $u$’s and $\alpha$’s that the entrepreneur may choose. In such a case, the entrepreneur merely has to choose a combination of $\{\alpha', u'\}$ that causes $V_U(\alpha', u', m') \leq 0$, where $m' > 0$ is defined as the union’s best response positive level of monitoring costs when CEO pay takes the form of $\{\alpha', u'\}$. In this case, the union’s best response will be to minimize monitoring costs and obstruction costs by choosing a $m = m' = 0$. The CEO now bears no monitoring costs. The level of obstruction, detection probability, wages paid, and payoffs to the entrepreneur are as follows:

\[
\begin{align*}
    m &= 0 \\
    q(m) &\in [\bar{c}/(\bar{c} + P), 1] \\
    w(q) &= \bar{c} \\
    V_U(\alpha', u', m) &= 0 \\
    V_o(\alpha', u', m) &= \bar{c}R - \bar{c} - U_M
\end{align*}
\]  

(7)

$IC_U$ will be satisfied for any detection probability high enough such that

\[ w = \bar{c} \geq \bar{c} - P. \]

Nevertheless, even when the union members engage in no obstruction $m = 0$, union members must always be paid for their effort cost of $\bar{c}$ for $IR_U$ to be satisfied. The CEO will weakly prefer to give the union rents of zero, when $\alpha' = 0$. Otherwise, when $\alpha' > 0$, she will strictly prefer to give all the rents to the entrepreneur.

\[ ^{13} \text{The such non-credible best responses of the union are more fully developed in the appendix section 6.3} \]
In general, equation (7) will be a SPE set of best responses and payoffs if there exists any \( \{\alpha', u'\} \) that causes \( V_\nu(\alpha', u', m') \leq 0 \), and is credible in the sense that \( IR_M \) and \( IR_C \) are satisfied. This is summarized in the proposition below:

**Proposition 2**

*When the union’s best response, \( m' > 0 \), leads to a weakly negative payoff for union members, \( V_\nu(\alpha', u', m') \leq 0 \), then the union will minimize the manager’s monitoring costs and earn no rents.*

### 3.4.2 The Subgame Perfect Equilibrium (SPE) of a Binding \( IR_U \) and \( IR_M \) Constraint

One strategy will induce the union to minimize monitoring costs for all sets of parameter values \( \{R, P, \bar{c}, \bar{c}, k, U_M\} \) and also for all \( \{x, y, \phi\} \). By making the CEO’s participation constraint bind when rent-seeking costs, \( m = 0 \), the union is faced with the stark choice of either minimizing monitoring costs or allowing the firm to fall apart.

Suppose, that the manager is not paid enough to cover her opportunity cost. Let us define this wage as

\[
\mu \in [0, U_M)
\]  

(8)

If the \( IC_U \) constraint does not bind, then union members could be paid the \( IR_U \) wage, \( \bar{c} \), and earn no rents. In this case, the absolute lower bound of share ownership
when the manager only gets a fixed wage, $u$. Is given by the following binding $IR_M$ constraint:

$$\alpha(\bar{e}R - \bar{c} - u) + u = U_M \tag{9}$$

The share, $\alpha$, of total surplus without deadweight losses, $\bar{e}R - \bar{c}$, plus net wages paid, $(1 - \alpha)u$, must meet or exceed the manager’s outside option.

$$1 > \alpha = \frac{U_M - u}{\bar{e}R - \bar{c} - u} > 0. \tag{10}$$

The set \{u, $\alpha$\} is sufficient to both minimize the union’s rent-seeking and payoff as well as maximizes the entrepreneur’s payoff. Obviously, $u < U_M$ and $\alpha < \bar{\alpha}$ is a recipe for the firm fall apart. Since the union members’ payoff is zero in alternative employment, the union will weakly prefer to accept a wage of $\bar{c}$ for a net payoff of zero from the manager who is compensated with \{u, $\alpha$\}. Q.E.D.

In the appendices 6.2 and 6.4.5, we discuss alternative strategies that respectively maximize the entrepreneur’s payoff or minimize the union’s payoff when the $IC_{U}$ constraint binds. In most cases, these will be strictly dominated strategies. At best, these other strategies lead to the same payoff for the entrepreneur, which is given in (7), as choosing the compensation package \{u, $\alpha$\}. That is, all alternative strategies for the entrepreneur are weakly dominated by \{u, $\alpha$\}. This discussion leads us to the proposition below.
Proposition 3

In the presence of employee rent-seeking, the entrepreneur will find it to be a weakly dominant strategy to give the CEO ownership stake strictly less than 100 percent and a fixed wage less than the manager’s opportunity cost. This strategy set is \( \{u, \alpha\} \).

Delegation leads to the CEO getting lower levels of share compensation (10) than the agency model where employee rent-seeking costs are exogenous as in proposition 1. In particular, the “agency” model advocates that the CEO gets 100 percent of the shares. The present, “delegation,” model advocates that she gets strictly less than 100 percent of the shares in the enterprise. That is, “delegation” advocates that the manager gets \( \alpha < 1 \) in equation (10).

Further, to the extent that most CEO pay is about creating incentive to work hard, one would tend to think that the CEO’s binding constraint would generally be the incentive compatibility constraint. When both the participation and incentive compatibility constraints are satisfied and the \( IC_M \) binds, it must be the case that the CEO is earning weakly more than her opportunity cost, \( U_M \). Yet, a binding \( IR_M \) means that the manager is just getting her outside option wage, \( U_M \). To the extent that the delegation model makes it more likely that the \( IR_M \) binds while monitoring costs are minimized in equilibrium, one would think that a unionized workforce would also mean that the CEO’s in unionized firms with rent-seeking employees will earn lower wages than in a firm where delegation is less important than motivating the CEO to be a good agent. Such firms are generally where the \( IC_M \) binds, and thus \( u > U_M \), as
in section 3.2. This intuition is confirmed in the following Lemma which is proved in the appendix section 5.2.

**Lemma 2**

*For a given level of monitoring costs and managerial opportunity cost, CEO pay will be strictly lower when the IRM binds and the IC\textsubscript{M} is slack than when the IR\textsubscript{M} is slack and the IC\textsubscript{M} binds.*

It seems reasonable to conclude from equation (10) and section 3.2 that conditions leading to lower CEO pay are more likely to occur in unionized firms with rent-seeking employees. Alternatively, the manager’s incentive constraint is more likely to bind, and CEO pay will be higher, when employees are non-unionized and unlikely to influence the manager’s cost of detecting slack work.

### 3.5 Robustness Checks\(^{14}\)

The model here leads to a corner solution that is a weakly dominant strategy for the entrepreneur regardless of the parameter values. Further, this solution guarantees the entrepreneur all the rents. This occurs because the union’s incentive constraint, which is the source of rents in many contracting problems, never binds. A natural question would be to ask if this solution is peculiar to the setup of the model.

---

\(^{14}\) The author particularly thanks Kevin Roberts, Antoine Faure-Grimaud, and Alan Morrison for their comments on this subsection.
The answer to this question is no. This solution is robust to several different setups for the model.

The moral hazard problem for the union was modeled as an efficiency wage. Workers must exert some effort, $\bar{\varepsilon}$, to do productive effort. This could have been easily modeled as workers receive a private benefit, $B$, of not working. The modeling of private benefits is popular in the corporate finance literature, especially in reference to top executive compensation. The perks or private benefits of control are a feature of many models. Micheal Eisner in his long and well compensated tenure as Chairman and CEO of Disney may have enjoyed substantial private benefits, or perks, from his job. Who would not want to meet with famous movie stars and directors of feature films as part of his or her duties! Further, it might just be nice to be the boss. The Ph.D. textbook in corporate finance, Tirole (2006), uses private benefits instead of “disutility of effort” to model the moral hazard problem, but argues that the private benefit and effort cost interpretations are equivalent, Tirole (2006, 115). Recall the $IC_U$ constraint above Lemma 1. Union members would just get a wage of $w$ if they worked hard. If they shirked, they would get private benefits of not working hard, whether or not they were caught for shirking. They would get a wage, $w$, and private benefits, $B$, $(1 - q)\times 100$ percent of the time if they were not caught for their slack work. Given that union members shirk, they would get private benefits, $B$, and the penalty for unemployment, $-P$, $q\times 100$ percent of the time if they were caught. This can be written below:
\[
IC_U(B) : \quad w \geq (1-q)(w+B) + q(B-P) \\
\quad w \geq \frac{B}{q} - P
\] 

(11)

The \(IC_U(B)\) in equation (11) is identical to the original \(IC_U\), except \(\bar{c}\) has been changed to \(B\). Obviously, this name change has no effect on the equilibrium results.

When the union has private benefits (as when the union has the cost of working hard), the union’s incentive compatibility constraint does not bind because the CEO can push \(q\) as high as she wants to. This occurs because the union is forced to minimize monitoring costs, \(m = 0\). If it does not, the CEO won’t work, the manager’s \(IR_m\) constraint will not be satisfied, and the firm will fall apart. It is the entrepreneur’s ability to set wages of the CEO to such a low level which ultimately makes the union’s incentive constraint slack.

Another robustness check may be altering the ordering of moves. Since the entrepreneur is by definition the founder of the firm, it would be hard to envision a scenario in which she does not move first in period 0. As was mentioned earlier it is the entrepreneur’s ability to set the manager’s pay first that causes the union to earn no rents in equilibrium. If the union could move before the founder of the firm, then the workers could extract substantial rents, assuming the entrepreneur sets managerial compensation to maximize her \(ex \ post\) payoff. Thus, the author would concede that if the union could set monitoring costs prior to the compensation package of the CEO being chosen, then the union could extract some rents, and in this alternative scenario there could be substantial deadweight losses. Nevertheless, this timing seems implausible since we would think that the entrepreneur would be the first mover.
The timing of the CEO’s and the union’s moves could be switched without changing the results. The order of play in figure 1 has the union moving in period 1 to set \(m\) and the CEO moving in period 2 to set wages, \(w\), and monitoring intensities, \(q\). If we reversed this order to have the manager move first in period 1 and the union move second in period 2, then the union would earn no rents as this paper has argued. The manager would simply choose \(q \in [\bar{c}/(\bar{c} + P), 1]\). Then, the manager could easily satisfy the ICU by paying the no-rent IRU wage, \(w = \bar{c}\). Once the manager has committed to a monitoring intensity in period 1, then the union cannot benefit from setting a \(m > 0\). That is because the whole point of setting a high cost of monitoring was to discourage the manager from setting a high level of monitoring, a high \(q\). If the manager sets a \(q\) before the union can move, then the union’s subsequent choice of \(m\) in this scenario would have no effect on \(q\).

If the union and manager moved simultaneously to choose \(q\) and \(m\), then there would also be no benefit to the union from setting an \(m > 0\). Suppose that the manager and the union simultaneously choose \(m\) and \(q\) respectively in period 1 and then the manager sets a wage for workers in period 2. The manager makes her choice of \(q\) without knowing the union’s choice of \(m\). The union’s payoff \(V_U\) would be weakly lower if it chose a positive \(m\) versus \(m = 0\), for any given monitoring intensity, \(q\). That is, when the union would earn a wage determined by the ICU, then its payoff would be strictly declining in \(m\), \(\frac{dV_U}{dm} = -km^{y-1} < 0\), for all \(m\) where the IRU is slack and the ICU binds. Yet, when the IRU binds, when \(q > \bar{c}/(\bar{c} + P)\), or when the monitoring costs are so high that the firm is no longer viable, then in both these
instances \( \frac{dV_u}{dm} = 0 \). In this simultaneous move scenario, the union has a weakly dominant strategy to set \( m = m = 0 \). When \( m = 0 \) and the manager has a non-zero equity stake, she will strictly prefer to choose a monitoring intensity \( q(m) = q(0) \), which is given in equation (7). Therefore, in this simultaneous move setup, it is also the case that the union’s individual rationality constraint binds.

The result that the union minimizes monitoring costs and is paid an individually rational wage is robust to the modeling of private benefits and to different orderings of the moves by the manager and union. It is the entrepreneur’s ability to set CEO pay at the beginning of the game that ultimately dooms the union to earn no rents.
4.0 Conclusion

This paper has demonstrated that the traditional solution of strong incentives proposed by agency theory may not always maximize shareholder value. Getting managers to think like shareholders may be less than optimal when shareholders are attempting to minimize the rent-seeking of the firm’s workforce. Here we have shown that optimal delegation to a risk-neutral CEO involves giving the CEO generally less than 100 percent of the shares. Low levels of managerial shareholding eliminates employees’ incentives to engage in rent-seeking behavior. Fewer shares and weakly lower pay for the CEO avoids deadweight losses and thereby maximizes shareholder value. In short, the delegation model of corporate governance may deserve a second look when employee rent-seeking is a major concern.
References


5.0 Appendix

5.1 Proof of Lemma 1

The proposition says that a satisfied $IR_M$ constraint implies that the $IR_O$ constraint is also satisfied. The right hand side (RHS) of both constraints are identical; therefore, the left hand side (LHS) of $IR_O$ must be greater than or equal to the (LHS) of $IR_M$. If we subtract the two constraints, $IR_O - IR_M \geq 0$.

To prove that $IR_O - IR_M \geq 0$ is the case, it is sufficient to show that the complement $IR_O - IR_M < 0$ is impossible. The following is a proof by contradiction:

\[
IR_O - IR_M = E\{R\} - w - c(q) - \alpha(E\{R\} - w - u) - u + c(q) < U_M - U_M
\]

\[
IR_O - IR_M = (1 - \alpha)(E\{R\} - w) - (1 - \alpha)u < 0 \tag{12}
\]

The LHS above will be smallest when $u$, the manager’s base wage is the largest. The maximum $u = E\{R\} - w$ from section 2. If we substitute this into (12) above, we get the contradiction, $0 < 0$. This is what we wanted to show. \textit{Q.E.D.}
5.2 Proof of Lemma 2

The compensation of the manager consists of expected share compensation and wages. Suppose there are two identical firms where each manager faces the same total monitoring costs of $C$. In the firm, denoted by the superscript “A”, where the $IC_M$ binds and $IR_M$ is slack, expected share compensation is denoted $S^A$ and the CEO’s fixed wages are $u^A$. (The superscript “A” denotes a firm in which controlling managerial agency costs are the most important concern.) That is,

\begin{align}
IC_M^A & : \quad S^A - C = 0 \\
IR_M^A & : \quad S^A - C > U_M - u^A
\end{align}

Total compensation for the manager is defined as

\[ T^A \equiv S^A + u^A. \]  

Further, substituting the right hand side (RHS) of equation (13) into the left hand side (LHS) of equation (14) and rearranging, it becomes clear that the manager’s wage must exceed her opportunity cost. That is,

\[ u^A > U_M. \]
In contrast, in the firm where \( IR_M \) binds and \( IC_M \) is slack we will denote the share and fixed wage components of compensation by the superscript “\( D \).” This is consistent with the “delegation” solution given by equation (10). The managerial constraints are the following:

\[
IC_M^D : \quad S^D - C > 0 \quad (17)
\]

\[
IR_M^D : \quad S^D - C = U_M - u^D \quad (18)
\]

Total “delegation” compensation for the manager is defined as

\[
T^D \equiv S^D + u^D. \quad (19)
\]

Further, substituting the right hand side (RHS) of equation (18) into the left hand side (LHS) of equation (17) and rearranging, it must be the case that

\[
U_M > u^D. \quad (20)
\]

Subtracting the binding constraint in equation (18) from the binding constraint in equation (13), we get the following:

\[
S^A - S^D = u^D - U_M
\]

Rearranging this relationship, we derive the following relationship:


\[ S^A + U_M = S^D + u^D \]  

(21)

Equation (21) allows us to compare the total compensation when the \( IC_M \) binds given in equation (15) and the total compensation when the \( IR_M \) binds given by equation (19). Since \( u^A > U^M \) according to equation (20), then the LHS of (21) is less than total compensation in equation (15). In short,

\[ T^A \equiv S^A + u^A > S^A + U_M = S^D + u^D \equiv T^D. \]  

(22)

This is what we wanted to show. For identical monitoring costs and identical opportunity costs for each manager, the total compensation, \( T^d \), in a firm where the manager is compensated with a binding \( IC_M \) constraint and a slack \( IR_M \) constraint exceeds the total compensation, \( T^D \), of a manager whose \( IR_M \) constraint binds and the \( IC_M \) constraint is slack. Simply put: equation (22) shows that \( T^d > T^D \). \textit{Q.E.D.}
6.0

Supplemental Appendices

to

Chapter I

Managerial Ownership with Rent-Seeking Employees
6.1 The Union’s Best Response $m$ when the $I_{CU}$ Constraint Binds

This section considers the union’s best responses given that the $I_{CU}$ binds and the $IR_U$ is slack and satisfied. In equilibrium, these two conditions are never met. For this reason, the reader can view this analysis as a counterfactual, because the union never is on the $I_{CU}$ constraint in equilibrium.

The union’s best response depends on first whether or not the $I_{CU}$ binds. Second, given that the $I_{CU}$ does bind, it depends on relative powers of the monitoring costs to the manager, $x$, and the power of the obstruction costs to the union members, $y$. When the power of the obstruction cost technology is relatively large, then the union members may select a finite $m = m^*$ if they are not constrained by the manager’s participation, $IR_M$, or incentive, $IC_M$, constraints, respectively.

When the union’s incentive constraint binds, it picks an $m$ that maximizes its members’ payoff according to the following schedule:

$$
\text{arg max}_m \quad V_U = w^*(q^*) - \bar{c} - \frac{km^*}{y} \\
\text{where} \quad V_U = \frac{\bar{c}}{q^*(m)} - P - \bar{c} - \frac{km^*}{y}
$$

6.1.1 High Costs of Obstruction, $y(x+1) > 1$
The first order condition after plugging in \( q^*(m) \) in equation (2) gives us the following first derivative:

\[
\frac{dV_u}{dm} = m^{\frac{x}{1+x}} \left\{ \left( \frac{1}{1+x} \right) \left( \frac{\bar{c}^x}{\alpha} \right)^{\frac{1}{1+x}} - km^{\frac{y(x+1)-1}{x+1}} \right\}
\]  

(23)

The first order condition at \( m^* \) is

\[
\frac{dV_u}{dm} \bigg|_{m=m^*} = \frac{1}{1+x} \left( \frac{\bar{c}^x}{\alpha} \right)^{\frac{1}{1+x}} - km^{y-1} = 0.
\]

By inspecting equation (23), this is first order condition is only defined when \( y(x+1) \neq 1 \). When \( y(x+1) = 1 \), the payoff to the union is increasing with \( m \) for all \( m \geq 0 \) if the following is greater than zero:

\[
\frac{1}{1+x} \left( \frac{\bar{c}^x}{\alpha} \right)^{\frac{1}{1+x}} - k
\]

(24)

Since the CEO’s share of the profits cannot exceed \( \alpha = 1 \), then there may be many parameter values where (24) is never weakly positive, when \( y(x+1) = 1 \). Therefore, the union will always strictly want to maximize monitoring costs. If there exists any \( \alpha \leq 1 \) that makes (24) weakly negative when \( y(x+1) = 1 \), then the entrepreneur could choose an \( \alpha' \), \( \frac{\bar{c}^x}{[k(x+1)]^{x+1}} \leq \alpha' \leq 1 \), such that it was always strictly or weakly in the
union’s interest to minimize monitoring costs. This follows from setting (24) less than or equal to zero and solving for alpha.

Suppose that the first order condition does in fact hold. Given that the second order condition is in fact negative and the union is unconstrained in its choice of \( m \), this implies that \( m^* \) is the following:

\[
m^* = \left( \frac{1}{k(x+1)} \right)^{1/(y(x+1)-1)} \left[ \frac{\psi^+}{\alpha} \right]^{1/(y(x+1)-1)}
\]  

(25)

For example consider the case where \( x = y = 1 \). In this case, \( m^* = \frac{\psi}{4k^2\alpha} \).

For \( m^* \) to be a finite, local maximum it is necessary and sufficient that \( y(x+1) > 1 \). We can see this from the second order condition evaluated at \( m^* \):

\[
\frac{d^2 V_U}{dm^2} \bigg|_{m=m^*} = -\left[ y(x+1) - 1 \right] \left( \frac{1}{x+1} \right)^2 \left[ k(x+1)^2 + \left( \frac{\psi}{\alpha} \right)^{y(x+1)-2} \right]^{-1}
\]  

(26)

The second order condition above is only negative when \( y(x+1) - 1 > 0 \). Alternatively, the stationary point in equation (25) is a minimum only when \( y(x+1) - 1 < 0 \), and equation (26) is positive at the stationary point. This means that, when \( y(x+1) - 1 < 0 \), the union’s payoff is increasing from this local minimum, given by (25), as we move away from it and approach positive infinity. When \( y(x+1) - 1 = 0 \), there
is no stationary point because the union’s payoff is strictly increasing until \( m \) approaches positive infinity.

While \( m = m^* \) from (25) may be a local solution when \( y(x+1) > 1 \), this need not be the highest utility choice of \( m \) for the union. \( m = m = 0 < m^* \) will sometimes be preferred when \( V_U(m^*, q^*, \alpha^*) \leq 0 \). That is, the union will be passive when behaving in such a way is weakly in its member’s interest. Further, we are assuming that both \( IC_M \) and \( IR_M \) are satisfied when the union chooses \( m^* \). This will not always be the case.

Consider the following proposition:

**Proposition 4**

Supposes that the union’s choice of \( m \) is unconstrained by the manager and owner’s participation, \( IR_M \) and \( IR_O \), respectively, and motivational, \( IC_M \), constraints.

- When the costs of obstructing monitoring are large \( (y(x + 1) − 1 > 0) \) the union will choose a finite \( m > 0 \) if it chooses any \( m > 0 \).
- When the cost of obstructing monitoring are small \( (y(x+1) − 1 < 0) \), the union will choose an \( m \to +\infty \) if it chooses any \( m > 0 \).
- When \( y(x+1) − 1 = 0 \) then the union will minimize monitoring costs if (24) is negative and it will maximize monitoring costs if (24) is positive.

This follows from our discussion and inspection of the first derivative in equation (23), equation (24), and the second order condition in equation (26).
Figure 2:

Union’s Payoffs as a Function of Monitoring Costs for Various Parameter Values

The unconstrained best responses of the union depend on its relative powers of the manager’s cost of detecting shirking, \( x \), and the power of the union’s obstruction technology, \( y \). In cases b. and d. the union will want to maximize the manager’s cost of detecting shirking, \( m \). In case c., the union will minimize, \( m \) to \( m = 0 \), given that the manager has a 100 percent share. In case a., where the power of the union’s obstruction costs are relatively high, its payoff is maximized by choosing an intermediate \( m = m^* \).
6.1.2 Low Costs of Obstruction, $y(x+1) < 1$ \[15\]

When $y(x + 1) - 1 < 0$ and the cost of obstructing monitoring rises slowly, then the union members can potentially gain by pushing $m$ to positive infinity. Recall that equation (26) shows that $m^*$ is a minimum point for the union when $y(x + 1) - 1 < 0$. Indeed, they can gain by pushing $m$ to the maximum.\[16\] Further, when $y(x + 1) - 1 = 0$, there is no stationary point, and the union’s payoff is strictly increasing for all $m > 0$ when equation (24) is positive for all $\alpha < 1$. Both these statements can be verified by inspecting equation (23), (26), and (24) and restate the proposition 4 above. The payoff to the union is increasing from $m^*$ to $+\infty$, in the former case, or from 0 to $+\infty$, in the latter case.

Nevertheless, the firm is not viable when $m$ approaches positive infinity. In this case, the maximum is the highest value of $m$ such that the either the $IC_M$ or $IR_M$ constraint binds. In particular, the union chooses the maximum $m = m^R$ that satisfies $IR_M$ when the maximum fixed wage of the CEO is low and $\phi < 1$. When CEO’s can be paid large fixed wages, $\phi \geq 1$, the $IC_M$ constraint determines the maximum $m$. In this case, then, the union chooses the maximum $m = m^C$, such that the firm is viable

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\[15\] The results of this section also apply to the case where $y(x+1) = 1$ and (24) is strictly positive for all feasible $\alpha$. This is case d. in figure 1 above.

\[16\] Here, too there is a possibility that pushing $m$ to the maximum leads to a negative $V_U$, in which case the union would always choose $m = m = 0$. Over the values of $m$ between 0 and $m^*$ in (25), the union’s payoff is falling. For $m > m^*$ the union’s payoff is rising. Therefore, if the maximum $m$ leads to negative utility, there is no positive $m$ that will give the union a non-negative payoff when $y(x + 1) - 1 < 0$. In this case, the union members can at least get their reservation utility by choosing $m = 0$ and incurring no obstruction cost $c(m) = c(0) = 0$. This is the “passive union” strategy.
under managerial monitoring. In either case, the entrepreneur finds it optimal to set $\alpha$ to the minimum that ensures that production takes place.

6.1.2.1 $u < U_M$

$\phi < 1$ implies that the fixed wage compensation is less than the CEO’s reservation wage. This is because $u \leq \phi U_M$. Therefore, when $\phi < 1$, the manager must earn some share compensations so that she will work in period 2. Otherwise, she does not even earn her reservation wage of $U_M$ with wages, $u$, alone. Further, it may be the case that the entrepreneur will choose a $u < U_M$ even when $\phi \geq 1$. In both cases, some shares are necessary to satisfy the $IR_M$ constraint. Further, consider the following lemma:

**Lemma 3**

*When $u < U_M$, satisfaction of $IR_M$ implies that $IC_M$ is slack and satisfied.*

See the appendix section 6.4.3 for a proof.

The Lemma allows us to focus on only $IR_M$ when $u < U_M$.

Suppose that the Union’s incentive compatibility constraint binds and the CEO chooses a probability of detection given by equation (2). Let us denote the maximum $m$ that will satisfy the $IR_M$ constraint when $u < U_M$ as $m^R$. The superscript “$R$” denotes that the manager’s $IR_M$ constraint binds.
Here we will consider the case in which the CEO’s base wage can exceed her reservation wage. When \( \phi \geq 1 \), it is possible that the manager’s IR\( M \) constraint is slack. In this instance, given that \( u > U_M \), the union can maximize monitoring costs, given production takes place, by choosing the \( m \) that makes the manager’s incentive compatibility constraint, IC\( M \), bind. Let us denote this as \( m^C \). The superscript “C” in \( m^C \) stands for the fact that the manager’s IC\( M \) constraint binds.

\[
m^C = \frac{\alpha}{c^C} \left[ \frac{x}{x+1} \left( \overline{e}R + P - u \right) \right]^{x+1}
\]  

(28)\(^{18}\)

Both equations (27) and (28) depend on the features of the manager’s compensation package. Therefore, as we will see, the entrepreneur in period 0 can potentially reduce the obstruction costs that the union chooses in period 1 by the form of compensation that she uses to motivate herself in period 2.

The maximum, feasible, marginal monitoring costs above may become important even when the union’s costs of obstruction are high, when \( y(x+1) > 1 \). If the union is constrained to choose either (27) or (28) because the manager’s IR\( M \) or

\(^{17}\) This is derived in appendix section 6.4.1.

\(^{18}\) This is derived in appendix section 6.4.2.
$IC_M$ binds, it may be the case that the costs of monitoring are lower than in (25).

More importantly, from the entrepreneur’s perspective, if the she can constrain the union’s choice of $m$ to equations (27) or (28), she may be able to improve her payoff, $V_O$. Therefore, it may be the case that low levels of incentive compensation, shares—$\alpha$, and either low or high levels of managerial wages—$u$, will cause $IR_M$ or $IC_M$ to bind. Therefore, tight managerial rationality or incentive constraints may be used to reduce union rent-seeking and raise the firm’s ex ante value, $V_O$. 
6.2 The $\max_{\alpha \& u} V_o$ strategies when $IC_U$ binds

In this section, we discuss the strategies that the entrepreneur would employ to maximize her payoff when the $IC_U$ binds. None of these strategies can guarantee that the entrepreneur secures all the rents for all values of $\{R, P, \bar{e}, \bar{c}, k, U_M\}$ and set of $\{x, y, \phi\}$. Nevertheless, some strategies will drive the union’s best response payoff to be negative for any positive $m$. Therefore, at best, one or several of these strategies will be weakly preferred to the strategy of paying the CEO $\{\alpha, u\}$ which is presented in section 3.4.2. At worst, all feasible strategies below will be strictly dominated by the entrepreneur choosing the compensation package $\{\alpha, u\}$.

We will explore this problem by exploring two complimentary regions of parameter values. First, we will discuss the case when the costs of obstruction are high $y(x + 1) - 1 > 0$. In the first case, we have seen that the optimal, unconstrained response of the union would be to choose $m = m^*$ as defined in (25). Secondly, we will explore the case when the union would like to push $m$ to positive infinity if the manager’s motivational constraints do not bind. This is the case when the costs of obstruction are low—$y(x + 1) - 1 < 0$.\(^{19}\)

6.2.1 High Costs of Obstruction, $y(x + 1) > 1$

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\(^{19}\) Mid-range obstruction costs, $y(x+1) = 1$ and an equation (24) that strictly positive for all feasible $\alpha$, also leads to a nearly identical best responses for the union as when $y(x + 1) - 1 < 0$, or obstruction costs are low.
Here we are continuing the case where both the manager’s motivational constraints are satisfied, and the cost of obstructing monitoring rises quickly for the union. (“High costs of obstruction” here denotes parameter values where $\gamma(x + 1) - 1 > 0$.) The value of the firm to the entrepreneur, in this case, is made up of the expected value the compensation for managing in period 2. This is the terms in the square brackets. Plus it is the value of selling shares on perfect capital markets, less her opportunity costs of acting as the manager, $U_M$. The value of equity sales is the second set of terms preceded by $(1 - \alpha)$ below:

$$V_o = \left[ \alpha \left( \bar{e}R - \frac{\bar{c}}{q} + P - u \right) + u - \frac{m^* q^*}{x} \right] + (1 - \alpha) \left( \bar{e}R - \frac{\bar{c}}{q} + P - u \right) - U_M$$

This simplifies to the following problem:

$$\arg \max_{\alpha} \quad V_o = \bar{e}R - w^*(q^*) - \frac{m^* q^*}{x} - U_M$$

The manager’s fixed wage, $u$, falls out of the expression above, and thus does not affect the entrepreneur’s payoff in this instance.

The share, $\alpha$, given to the manager does affect the entrepreneur’s payoff. Two of the major costs—the wage bill, $w(q)$, and monitoring costs, $c(q)$—to the entrepreneur are potentially reduced or enlarged by retaining more shares in the firm. When $\gamma(x+1) - 1 > 0$, there exists no level of ownership that gives the entrepreneur the optimal incentives. This is in sharp contrast to the classic agency literature,
Jensen and Meckling (1976), for example, where 100 percent share ownership is optimal when the entrepreneur has sufficient wealth and linear utility. The strategic impact of union members obscuring shirking, low effort work, means that simple share contracts are too soft $\alpha \leq 1$ in this solution. Indeed, the entrepreneur would find $\alpha > 1$ optimal, if it would not break the budget, in these circumstances.

The wage contract for the union members can be solved by combining $w(q)$, $q^*(m)$, and $m^*$.

\[
\begin{equation}
 w^*(q^*(m^*)) = \alpha^{\frac{1}{1+y}} \left( \frac{C^y}{k(x+1)} \right)^{\frac{1}{1+y}} - P \tag{29}
\end{equation}
\]

Note that union members’ wages are falling in the CEO’s share—$\alpha$—when $y(x+1) > 1$.

Total monitoring costs for the CEO are either rising or falling in the CEO’s share depending on the magnitude of the combined power of the CEO’s monitoring technology, $x$, and the power of the union’s evasion technology, $y$.

\[
\begin{equation}
 \frac{m^*q^*}{x} = \frac{\alpha^{(x+1)-1}}{y} \left( \frac{C^y}{k(x+1)} \right)^{\frac{1}{y(x+1)-1}} \tag{30}
\end{equation}
\]

Therefore, the problem for the entrepreneur is
\[
\arg\max_{u, r, \alpha} V_0 = \bar{c}R + P - \alpha^{\frac{x}{y}} \left( \frac{c^{\alpha y}}{k(x+1)} \right)^{\frac{1}{y(x+1)-1}} - \alpha^{\frac{xy-1}{y(x+1)-1}} \left( \frac{\bar{c}^{\alpha y}}{k(x+1)} \right)^{\frac{1}{y(x+1)-1}} - U_M.
\]

As long as the participation and incentive compatibility constraints for the CEO are satisfied, the entrepreneur is indifferent about the \( u \in [0, \phi U_M] \) that is chosen as the CEO’s base wage. This is the case here because \( u \) does not come into the entrepreneurs’ objective function.

There are two cases:

First, when \( xy > 1 \) (and thus \( y(x+1) > 1 \)) total monitoring costs in (30) are rising in the CEO’s share, but union wages in (29) are also falling in \( \alpha \). Therefore, there is a tradeoff between lower wages for rank-and-file workers and monitoring costs. Therefore, in this case, there is some \( \alpha^{**} \) for which the first order condition below is equal to zero.

\[
\left. \frac{dV_0}{d\alpha} \right|_{\alpha=\alpha^{**}, xy>1} = -y\alpha^{**} \left( \frac{x}{y(x+1)-1} \right)^{\frac{x}{y(x+1)-1}} + \left( \frac{xy-1}{x} \right) \alpha^{**} \left( \frac{x}{y(x+1)-1} \right)^{\frac{x}{y(x+1)-1}} = 0
\]

The above condition simplifies considerably. Nevertheless, the stationary point for this problem denoted by the superscript “**” involves giving the CEO a greater than 100 percent share.
\[ \alpha^{**} = \frac{xy}{xy - 1} > 1, \text{ when } xy > 1. \]  (31)

In the second case, total monitoring costs are weakly falling in \( \alpha \) when \( xy \leq 1 \) and \( y(x+1) > 1 \). In this case, \( xy \leq 1 \) and \( y(x+1) \geq 1 \), there can be no interior solution for \( \alpha^{**} \). The entrepreneur will wish that the CEO is given an infinite share because these incentives minimize both union wages and total monitoring costs in (29) and (30), respectively. In this case, \( \alpha \) is a local minimum. We can verify both these observations by evaluating the second order condition at \( \alpha^{**} \) when \( y(x+1) - 1 > 0 \). In this case, the sign of the second order condition depends on sign of \( \{xy - 1\} \). If this sign is positive, \( \alpha > 1 \), and it is a local maximum. Alternatively, if the sign of \( \{xy - 1\} \) is negative, \( \alpha < 0 \), and it constitutes a local minimum. Either way, the entrepreneur finds it constrained-optimal to give herself the maximum of 100 percent of the shares.

\[
\left. \frac{d^2 V_\phi}{d\alpha^2} \right|_{\alpha=\alpha^{**}} = \frac{-1}{x[y(x+1)-1]} \left \{ \frac{xy - 1}{xy} \right \}^{y[y(x+1)-1]} \left( \frac{c^{xy}}{k(x+1)} \right)^{1[y(x+1)-1]} \]  (32)

The second order condition above is negative when \( xy - 1 > 0 \) and positive when \( xy - 1 < 0 \).

Here, union rent-seeking causes the optimal incentive scheme to diverge from giving the entrepreneur a 100 percent share. Indeed, there is no feasible share that
reaches the unconstrained optimum where the manager maximizes her welfare \textit{ex ante}.

Suppose that incentive compatibility implies a slack participation constraint for union members, the manager’s \( IC_M \) and \( IR_M \) constraint are slack, and that \( y(x + 1) - 1 > 0 \). When these conditions are met, we can obtain the following set of solutions denoted by the superscript "*" in terms of the exogenous parameters:

\[
\alpha^* = 1 \\
m^* = \left( \frac{\bar{c}^x}{(k(x+1))^{y+1}} \right)^{\frac{1}{y(x+1)-1}} \\
q^* = \left( [k(x+1)]^{y+1} \right)^{\frac{1}{y(x+1)-1}}
\]  

(33)

Once we have derived these, we can find the deadweight losses, union members’ wages, and the payoffs before opportunity costs to the entrepreneur, the representative union member, and the CEO, respectively. The deadweight losses from union rent-seeking are the following costs incurred by the CEO and the union members respectively:

\[
\frac{m^* q^{**x}}{x} = \frac{1}{x} \left( \frac{\bar{c}^{y}}{k(x+1)} \right)^{\frac{1}{y(x+1)-1}} \\
\frac{km^* y^{**}}{y} = \frac{k}{y} \left( \frac{\bar{c}^{y}}{[k(x+1)]^{y(x+1)}} \right)^{\frac{1}{y(x+1)-1}}
\]  

(34)

The wages paid to union members are as follows:
\[ w'(q^*(m^*(1,u^*))) = \left( \frac{c^0}{k(x+1)} \right)^{-\frac{1}{\gamma(x+1)-1}} - P \]  

(35)

If the union’s payoff is negative, the constrained-optimal solution of \( \alpha^* = 1 \) will lead to a SPE with no rent-seeking. This is when this level of share ownership leads the union to get a negative or zero payoff if the union chooses the best response \( m^* \) in equation (33). The payoff to the union is as follows:

\[ V_U(1,m^*,q^*) = \left( \frac{c^0}{k(x+1)} \right)^{-\frac{1}{\gamma(x+1)-1}} \left( \frac{y(x+1)-1}{y(x+1)} \right) - c - P \]  

(36)

The payoff to the entrepreneur is the wages paid to workers, less her effort and opportunity costs as CEO:

\[ V_O(1,m^*,q^*) = \bar{c}R + P - \left( \frac{x+1}{x} \right) \left( \frac{c^0}{k(x+1)} \right)^{-\frac{1}{\gamma(x+1)-1}} - U_M \]  

(37)

This analysis assumes that the union did not have to worry about satisfying the managers’ incentive and rationality constraints. Yet, if \( \alpha \) is very low \( IC_M \) or \( IR_M \) will bind for a \( m < m^* \). Therefore, it is possible that the solution of \( \alpha^* = 1 \) is dominated by a \( \alpha \) in equation (10) where the manager is indifferent between working in or leaving the firm altogether. Only if \( V_U(1, m^*, q^*) \) in (36) is negative can the choice of \( \alpha^* = 1 \) be part of a subgame perfect Nash equilibrium.

\[ ^{20} \text{This is derived in the section 6.4.4.} \]
\( IR_U \) requires that equation (36) be non-negative. The large, first term is weakly positive because all the parameters are non-negative and \( y(x + 1) > 1 \). Nevertheless, the last two terms may be smaller or larger than this first term. This means that the equation may be negative or zero for many parameter values. Indeed, if this is the case and the \( IR_M \) and \( IC_M \) would be satisfied by the solutions in equation (33), then the union would prefer to set \( m = \underline{m} = 0 \) when \( \alpha = \alpha^* = 1 \), the “passive union” strategy. Generally, under these latter circumstances, there will be a range of \( \alpha < \alpha^* = 1 \), that will that would induce the union to play the passive strategy for a given set of parameter values.

6.2.2 Low Costs of Obstruction, \( y(x + 1) < 1 \) \(^{21}\)

In this subsection we consider the owner’s problem when the union’s cost of obstructing managerial monitoring are low, \( y(x + 1) < 1 \). In this case, the union will push \( m \) to the maximum. The maximum \( m \) is the highest \( m \) such that either the manager’s \( IR_M \) constraint binds when \( u < U_M \), which is given by equation (27), or the manager’s \( IC_M \) constraint binds when \( u > U_M \), which is given by equation (28). In the former case, there must be some positive level of managerial share ownership. Finally, the results in this subsection may be relevant to even the case where the union’s costs of obstructing monitoring are high, \( y(x + 1) > 1 \). In particular, if the entrepreneur can reduce rent-seeking and increase her payoff from (37) by reducing the CEO’s share compensation, she will cause either the \( IR_M \) or \( IC_M \) constraints to

\(^{21}\) As in the complimentary section 0, the results of this section also apply to the case where \( y(x+1) = 1 \) and (24) is strictly positive for all feasible \( \alpha \leq 1 \).
bind. If she can cause the union’s payoff to be negative in either case, such a strategy can weakly dominate choosing a compensation package of \( \{u, \alpha\} \).

**6.2.2.1 \( u < U_M \)**

When the manager must earn some share compensation to satisfy her \( IR_M \) constraint, the union’s best response, would be to push \( m = m^R \) in equation (27).\(^{22}\)

From Lemma 3, we know that \( IC_M \) will be slack. In this case, the best response of the CEO would be to choose a \( q^*(m^R) \). \( q^*(m^R) \) is derived from combining equations (2) and (27).

\[
q^*(m^R) = \min \left\{ \frac{\alpha \bar{c}}{\frac{x}{x+1} \left\{ \frac{\alpha (\bar{c}R + P - u) - (U_M - u)}{\alpha} \right\}}, 1 \right\}
\]

This implies that the \( IC_U \) determined wage would have to be \( w^*(q^R) \) below when \( q^*(m^R) < 1 \).

\[
w^*(q^R) = \left\{ \frac{x}{x + 1} \left\{ \frac{\alpha (\bar{c}R + P - u) - (U_M - u)}{\alpha} \right\} \right\} - P \quad (38)
\]

The manager must bear total monitoring costs of

\(^{22}\) This is only true if the union doesn’t find the “passive” strategy of \( m = 0 \) optimal.
The value of the firm to the entrepreneur, \( V_O \), is the expected revenues, \( \bar{e}R \), less the wages paid to union members in (38), less the monitoring costs incurred by the manager in (39), less her opportunity costs, \( U_M \). The entrepreneur in period 0 selects the compensation package of the manager. In particular, she decides how much she will compensate herself in terms of shares, \( \alpha \), and through a base wage, \( u \).

\[
\arg \max_{w.r.t.u & \alpha} V_O = \bar{e}R + P - \frac{x + \alpha}{\alpha(x + 1)} \left\{ \alpha(\bar{e}R + P - u) - (U_M - u) \right\} - U_M, \quad \text{where } u \leq \phi U_M, \alpha \leq 1, \text{ and } \phi < 1. \tag{40}
\]

If we differentiate (40) with respect to \( u \), we find that the entrepreneur’s payoff is strictly falling in the base wage paid to the CEO when managerial share ownership is less than 100 percent, \( \alpha < 1 \).

\[
\frac{dV_O}{du} = - \frac{(1 - \alpha)(x + \alpha)}{\alpha(x + 1)} \leq 0, \quad \forall \alpha \in [0,1]. \tag{41}
\]

The intuition for (41) is the following. If \( u \) is close to \( U_M \), the union can engage in high levels of obstruction, choose a high \( m \), and still satisfy the \( IR_M \) constraint. Yet, if the entrepreneur chooses a very low fixed wage, it becomes harder to get the manager to participate in period 2. Therefore, low fixed wages encourage
the union to choose lower obstruction costs when the $IR_M$ is the binding constraint for the manager.

While fixed wages always weakly destroy firm value, there are some positive levels of share ownership for the manager in which the expected value of the firm in period 0 is rising.

\[
\frac{dV_\phi}{d\alpha} = \frac{x}{(\alpha^x)^2(x+1)}(U_M - u) - \frac{1}{x+1}(\bar{e}R + P - u)
\]

This leads to the first order condition below.

\[
\left. \frac{dV_\phi}{d\alpha} \right|_{\alpha = \alpha^R} = \frac{x}{(\alpha^x)^2} (U_M - u) - (\bar{e}R + P - u) = 0
\]  
(42)

\(\alpha^R\) is a maximum because the second order condition is unambiguously negative for all \(\alpha > 0\) when \(U_M > u\).

\[
\frac{d^2V_\phi}{d\alpha^2} = -\frac{2}{\alpha^3} \left( \frac{x}{x+1} \right) (U_M - u) < 0
\]

When \(\alpha^R < 1\), equation (41) indicates that the entrepreneur will want to minimize \(u\). When we combine this insight with \(\alpha^R\), which is implied by the first-
order condition in equation (42), we get the following optimal compensation contract for the case where \( \alpha^R < 1 \):

\[
u^R = 0
\]

\[
\alpha^R = \frac{xU_M}{\sqrt{eR + P}} \tag{43}
\]

\( \alpha^R < 1 \) when \( xU_M < eR + P \). If we plug these values into \( m^R \) in equation (27), \( m^R(\alpha^R, u^R) \) is the following:

\[
m^R(\alpha^R, u^R) = \left( \frac{1}{e} \right) \left( \frac{x}{x + 1} \right) \left[ \sqrt{xU_M (eR + P) - U_M} \right]^{1 + 1} \tag{44}^{23}
\]

Here, if \( q^*(m^R(\alpha^R, u^R)) < 1 \), then

\[
q^*(m^R(\alpha^R, u^R)) = \frac{1}{x + 1} \left[ \sqrt{x(eR + P) - xU_M (eR + P)} \right]. \tag{45}
\]

The wage bill paid to workers is

---

\(^{23} m^R \) is only greater than zero when the quantity in square brackets is positive. Therefore, the solution for \( m^R \) in (44) is only positive when \( x(eR + P) \geq U_M \). When \( m^R < 0 \), we have a corner solution where \( m = 0 \) give by equation (7). \( m^R < 0 \) is a sufficient condition for \( \{u^R, \alpha^R\} \) to induce the union into passivity and choose \( m = 0 \). Nevertheless, \( V_U(m^R) < 0 \) (see equation (49)) is a weaker sufficient condition for \( \{u^R, \alpha^R\} \) to compel the union to minimize \( m \). Likewise, a positive payoff, \( V_U \geq 0 \), is sufficient to discard \( \{u^*, \alpha^*\} \) as a possible SPE set of strategies for the entrepreneur.
\[ w^*(\alpha^R, u^R) = \left( \frac{1}{x+1} \right) \left[ x(\overline{c}R + P) - \sqrt{xU_M(\overline{c}R + P)} \right]. \tag{46} \]

Total monitoring costs incurred by the manager are

\[
\begin{align*}
\frac{m^R(\alpha^R, u^R)(q^*(m^R(\alpha^R, u^R))^x}{x} = \\
\left( \frac{1}{x+1} \right) \left[ \sqrt{xU_M(\overline{c}R + P)} - U_M \right]. \tag{47}
\end{align*}
\]

If we subtract (46), (47), and \( U_M \) from expected revenues, then we get the value of the firm when the \( IR_M \) binds and the union pushes \( m \) to the maximum:

\[
V_O(\alpha^R, u^R) = \left( \frac{1}{x+1} \right) \left[ \overline{c}R + P - U_M \right] - U_M \tag{48}
\]

The payoff to union members, \( V_U \), is as follows:

\[
\begin{align*}
V_U &= w^*(m^R(\alpha^R, u^R)) - \overline{c} - \frac{k(m^R(\alpha^R, u^R))^y}{y} \\
&= \left( \frac{1}{x+1} \right) \left[ x(\overline{c}R + P) - \sqrt{xU_M(\overline{c}R + P)} \right] - \overline{c} \\
&- \frac{k}{y} \left( \frac{1}{\sqrt{U_M}} \right)^{xy} \left[ \left( \frac{x}{x+1} \right) \left[ \sqrt{xU_M(\overline{c}R + P)} - U_M \right] \right]^{x+1} \tag{49}
\end{align*}
\]

If equation (49), is negative, which is certainly possible for some combinations of the exogenous variables \( x, y, P, U_M, \overline{c}, \overline{c}, \) and \( R \), then the union will never choose \( m^R(\alpha^R, u^R) > 0 \). Therefore, when equation (49) is negative, the union’s
subgame perfect response will be to set \( m = m = 0 \). This leads to the passive union, “\(_\)”, equilibrium values as in equation (7).

It is necessary that \( V_U < 0 \) for (43) to be a feasible strategy of the entrepreneur. Otherwise, she can induce the union to be passive, \( m = 0 \), by simply choosing the set \( \{u, \alpha\} \). Indeed, given that minimizing the union’s payoff leads weakly dominant strategy for the entrepreneur, \( \{u, \alpha\} \) from section 3.4.2. Maximizing the entrepreneur’s payoff is not the most effective strategy for minimizing the union’s payoff, and inducing the union to minimize monitoring costs, when both \( IR_M \) and \( IC_U \) bind. As we will see in section 6.3.2.

Alternatively, suppose that \( \alpha^R \) in equation (43) exceeds one. Since the manager cannot be given greater than a 100 percent share, the entrepreneur may award the CEO a constrained optimal 100 percent share. In this case the equilibrium values are denoted with the superscript “\( R \)” and the subscript “\( 1 \).” They are as follows:

\[
\begin{align*}
\alpha_i^R &= 1 \\
m_i^R (\alpha_i^R, u_i^R) &= \frac{1}{\alpha^R} \left[ \frac{x}{x+1} \left\{ \varphi R + P - U_M \right\} \right]^{x+1} \\
q^* (m_i^R (\alpha_i^R, u_i^R)) &= \frac{\varphi}{\left[ \frac{x}{x+1} \right]^{\varphi R + P - U_M}}
\end{align*}
\]

(50)
\{\alpha_i^r, u_i^r\}, \quad m^r(\alpha_i^r, u_i^r), \text{ and } q^*(m^r(\alpha_i^r, u_i^r)) \text{ in (50) are the best responses by the entrepreneur, union, and the CEO, respectively. The manager will give union members the minimum } IC_U \text{ minimum wages of}

\[ w^*(q^*(m^r(\alpha_i^r, u_i^r))) = \left( \frac{1}{x+1} \right) \{\varpi R + P - U_M \} - P. \]

Total monitoring costs incurred by the manager are

\[ m^r(\alpha_i^r, u_i^r)(q^*(m^r(\alpha_i^r, u_i^r))) = \left( \frac{1}{x+1} \right) \{\varpi R + P - U_M \} \]

Combining the expected revenues less the union wage bill, the monitoring costs of the manager, and the manager’s opportunity cost, we get the value of the firm to the entrepreneur below:

\[ V_o(\alpha_i^r, u_i^r) = \left( \frac{1}{x+1} \right) \{(x-1)(eR + P) + 2U_M \} - U_M \] (51)

The wage less effort costs and obstruction costs is the payoff to union members. This is,

\[ u_i^{rS} \text{ derivation is left for the appendix 6.4.5.} \]
\begin{align*}
V_U(\alpha^k, \mu^k) &= \left(\frac{1}{x+1}\right)\{\bar{\nu}R + P - U_M\} - P - \bar{\nu} \\
&= \frac{k}{y}\frac{1}{x+1}\left[\left(\frac{x}{x+1}\right)\{\bar{\nu}R + P - U_M\}\right]^{\gamma(x+1)}.
\end{align*}

Of course, equation (52) must be negative for \( IR_U \) to be violated and for this equilibrium set of best responses in equation (50) to hold. Otherwise, this set of values is strictly dominated by \( \{u, \alpha\} \) in equations (8) and (10) that would lead to the equilibrium values in (7), where the union is passive and minimizes monitoring costs.

\textbf{6.2.2.2} \( u \geq U_M \)

In this section, we will show that maximizing the entrepreneur’s payoff when the \( IC_U \) and \( IC_M \) binds leads to rent-sharing and no deadweight losses. Because deadweight losses potentially reduce the union’s payoff, the entrepreneur would be less likely to employ this strategy. (This is because the entrepreneur will only choose strategy sets that induce the union to be passive in equilibrium.) She does not need to share rents with the union! The entrepreneur is strictly better of with the strategy set \( \{u, \alpha\} \) than allowing both the \( IC_U \) and \( IR_U \) to bind.

When \( \gamma(x+1) - 1 < 0 \) or when both \( \gamma(x+1) = 1 \) and (24) is strictly positive for all feasible \( \alpha \), it is the union’s strategy to set \( m \) to the maximum. We have argued that the maximum \( m \), when \( u \geq U_M \), is given by \( m^C \) in equation (28). \( m^C \) is the maximum \( m \) such that the manager’s incentive compatibility constraint, \( IC_M \), binds. (When \( u \geq U_M \) the manager’s \( IR_M \) constraint is satisfied for all non-negative levels of
share ownership.) In this case, the manager’s best response detection probability, given that the union’s $IC_U$ binds, is derived by combining equations (2) and (28). This is

$$q^*(m^c) = \left(\frac{x+1}{x}\right) \frac{\overline{e}}{\overline{e}R + P - u}. \quad (53)$$

Assuming that $q^*(m^c) \leq 1$, the $IC_U$ wage is as follows:

$$w^*(q^*(m^c)) = \left(\frac{x}{x+1}\right)\{\overline{e}R + P - u\} - P \quad (54)$$

Further, the total monitoring costs incurred by the manager are

$$\frac{m^c(q^*(m^c))^x}{x} = \frac{\alpha}{x+1}\{\overline{e}R + P - u\}. \quad (55)$$

The entrepreneur wishes to maximize the expected value of the firm and her expected compensation from her role as manager in period 2. This is $V_O$, which is derived by subtracting (54) and (55) from expected revenues under high effort. The entrepreneur wants to select a compensation package consisting of a fixed wage, $u$, and shares, $\alpha$, which maximizes her payoff, $V_O$. 

70
\[
\arg \max_{w,r,t,u} \mathcal{V}_o = \overline{\mathcal{R}} + P - \frac{x + \alpha}{x + 1} \left( (\overline{\mathcal{R}} + P - u) \right) - \mathcal{U}_M, \quad (56)
\]

where \( u \leq \phi \mathcal{U}_M \) and \( \phi \geq 1 \).

This program is solved by corner solutions. Firstly, the payoff to the entrepreneur is strictly increasing in her fixed wage.

\[
\frac{d\mathcal{V}_o}{du} = \frac{x + \alpha}{x + 1} > 0 \quad \forall \alpha \in [0,1], \quad \& \quad \frac{d^2\mathcal{V}_o}{du^2} = 0. \quad (57)
\]

This is because when the fixed wage increases on the \( \mathcal{I}_M \), there is less money to offer to the manager to undertake the costly monitoring in (55). The union has to respond by lowering monitoring costs to make sure that \( \mathcal{I}_M \) holds. This continues until her base wage is pushed to the maximum, \( \phi \mathcal{U}_M \), or there is no more surplus to promise to the CEO in expectation. (Total surplus before the CEO’s opportunity cost, \( \mathcal{U}_M \), is capped at \( \overline{\mathcal{R}} - \overline{\mathcal{R}} \).)

Secondly, the entrepreneur’s expected payoff is strictly falling in the size of the equity stake that is given to the manager.

\[
\frac{d\mathcal{V}_o}{d\alpha} = - \left( \frac{\overline{\mathcal{R}} + P - u}{x + 1} \right) < 0, \quad \& \quad \frac{d^2\mathcal{V}_o}{d\alpha^2} = 0. \quad (58)
\]
This leads us to the following set of dominated best responses by the entrepreneur, union, and CEO when \( y(x + 1) - 1 < 0 \), \( \phi \geq 1 \), and \( q^*(m^c) \leq 1 \), and the IRU is satisfied given the ICU binds:

\[
\begin{align*}
    u^c &= \min\{\phi U_M, \bar{c}R - \bar{c}\} \\
    \alpha^c &= 0 \\
    m^c &= 0 \\
    q^*(m^c) &= \left(\frac{x+1}{x}\right) \left(\frac{\bar{c}}{\bar{c}R + P - \phi U_M}\right) \leq 1, \text{ by definition.} \tag{59}
\end{align*}
\]

The last condition of the set of best responses denoted by “C” requires that IRU is satisfied when ICU binds.

\[
\begin{align*}
    w^*(q^*(m^c(\alpha^c, u^c))) &= \left(\frac{x}{x+1}\right) \left(\frac{\bar{c}R + P - \phi U_M}{\bar{c}R - \bar{c}}\right) - P \tag{60}
\end{align*}
\]

The union’s payoff is the following given that the ICU binds:

\[
\begin{align*}
    V_U(\alpha^c, u^c) &= w^*(q^*(m^c)) - \frac{k(m^c)^y}{y} \\
    &= \left(\frac{x}{x+1}\right) \left(\frac{\bar{c}R + P - \phi U_M}{\bar{c}R - \bar{c}}\right) - P - \bar{c} \tag{61}
\end{align*}
\]

\(^{25}\) When \( q^*(m^c) > 1 \), there is not a unique CEO compensation package that maximizes the entrepreneur’s payoff. In general, the ICU will no longer bind and IRU will become more important. Namely, \( w(q) = \bar{c} \), as in equation (7).
For these parameter values dominated best response payoff to the entrepreneur is

\[ V_O(\alpha^C, u^C) = \bar{e}R + P - \left( \frac{x}{x+1} \right) \{ \bar{e}R + P - \phi U_M \} - U_M. \] (62)

The attractive part of this equilibrium is that there is no dissipation of rents. If we add up (61) and (62) we get the total economic rents of \( \bar{e}R - \bar{e} - U_M \). This is the case because \( m^C = 0 \), and thus rent-seeking costs go to zero. Nevertheless, this egalitarian split of the surplus will not occur in practice because the entrepreneur can do better.
6.3 \( \min_{u \& \alpha} V_U \) strategies when \( IC_U \) binds

This section explores the various strategies of minimizing the union’s payoff when the union’s incentive compatibility constraint binds. These minimization strategies parallel the strategy of maximizing the owner’s payoff \( \max_{w.r.t. \ u \& \alpha} V_O \).

Minimization of the union’s payoff is not equivalent to maximization of the owner’s payoff because deadweight losses of rent-seeking drive a wedge between what the entrepreneur receives and the union loses. We will consider the three cases which were considered in section 0. These are all contingent on the \( IC_U \) and not the \( IR_U \) constraint binding. Since the latter will be the result of a weakly dominant strategy of the entrepreneur \( \{u, \alpha\} \) the reader can view the discussion in this section like that of section 6.2 as an alternative attempt to drive the union’s payoff from obstruction to zero or below. Only when \( y(x + 1) < 1 \) and a \( \phi \) sufficiently large, can we be sure that the owners manager can drive the union’s payoff to zero for all parameter values of \( \{R, P, \tau, \bar{\tau}, k, U_M\} \) by choosing some compensation set other than \( \{u, \alpha\} \). This latter case is discussed in section 6.3.3 and the proposition 5.

6.3.1 The equivalence of \( \min_{w.r.t. \ u \& \alpha} V_U^{\ast} \) and \( \max_{w.r.t. \ u \& \alpha} V_O^{\ast} \) when \( y(x + 1) > 1 \)

The rent-seeking deadweight losses incurred by the union potentially drives a wedge between a strategy that maximizes the entrepreneur’s payoff and a strategy that minimizes the union’s payoff. If the minimum payoff to the union is not positive, the entrepreneur can induce the union to minimize monitoring costs, in
equation (7), leaving all the rents for the entrepreneur with no deadweight losses. In this appendix section, it is shown that the entrepreneur’s optimal, $\alpha^*$, under $\max V_O^*$ is equivalent to $\alpha^*$. The optimal level of share ownership to maximize the entrepreneur’s payoff is 100 percent. Further, in both strategies there is no unique optimal $u$.

Let us start by defining the function that the entrepreneur will minimize.

$$V_u(\alpha, m^*, q^*) = w(q^*(m^*(\alpha))) - \frac{k}{y}(m^*(\alpha))^y - \bar{e}$$

(63)

From equations (2) and (25),

$$q^*(m^*(\alpha)) = \left(\alpha^* \bar{e}^{-1/y} [k(x+1)]^\frac{1}{y(x+1)-1}\right), \text{ given that } q < 1.$$  

(64)

Combining this with $IC_U$ the union members’ wages are

$$w(\alpha, m^*, q^*) = \left(\frac{\bar{e}^{1/y}}{[k(x+1)]}\right) \frac{1}{y(x+1)-1} \alpha^* - \frac{y}{y(x+1)-1} - P$$

(65)

The union members’ obstruction costs are the following:

$$\frac{km^*}{y} = \frac{k}{y} \left(\frac{\bar{e}^{1/y}}{[k(x+1)]^{1/(x+1)}}\right) \frac{1}{y(x+1)-1} \alpha^* - \frac{y}{y(x+1)-1}$$

(66)

Combining (65) and (66), we get the union’s payoff:
\[ V_U(\alpha, m^*, q^*) = \left( \frac{c^y}{k(x+1)} \right)^{\frac{1}{y(x+1)-1}} \left\{ \frac{y(x+1)-1}{y(x+1)} \right\} \alpha^{-\frac{y}{y(x+1)-1}} - P - \bar{c} \] \tag{67}

(See section 6.2.1 for an analogous derivation of this expression when \( \alpha^* = 1 \).)

Note that the \( \{ \} \) term in (67) is always positive when \( y(x+1) - 1 > 0 \).

If we turn to the minimization problem, we get two simple first derivatives.

\[
\frac{\min}{w.r.t. \quad \alpha \& u} V_U(\alpha, m^*, q^*)
\]

\[
\frac{dV_U(\alpha,m^*,q^*)}{d\alpha} = -\frac{y}{y(x+1)-1} \left( \frac{c^y}{k(x+1)} \right)^{\frac{1}{y(x+1)-1}} \left\{ \frac{y(x+1)-1}{y(x+1)} \right\} \alpha^{-\frac{y}{y(x+1)-1}} < 0, \quad \forall \alpha \in (0,1] \tag{68}
\]

\[
\frac{dV_U(\alpha,m^*,q^*)}{du} = 0 \tag{69}
\]

Equation (68) says that the payoff to the union is a strictly falling in the manager’s positive share in the enterprise, in this case where \( y(x+1) > 1 \). Therefore, the union’s payoff is minimized by \( \alpha^* = 1 \).

Equation (69) is trivial. The union’s payoff is unaffected by the manager’s fixed wage when the manager’s motivational constraints do not bind.
This is what we wanted to show. In short, the \( \min_{\text{w.r.t. } u \& \alpha} V^*_U \) and \( \max_{\text{w.r.t. } u \& \alpha} V^*_O \) strategies lead to the same action for the entrepreneur. She sets \( \alpha^* = \alpha^* = 1 \) and is indifferent about the manager’s fixed wage level \( u \).

### 6.3.2 \( \min_{\text{w.r.t. } u \& \alpha} V^R_U \) strategies

A strategy on the \( IC_U \) constraint, where (27) is the union’s choice of monitoring costs, will not always be able to drive the union to passivity, \( m = 0 \). In particular, consider the following problem:

\[
\begin{align*}
\min_{\text{w.r.t. } u \& \alpha} V^R_U (m^R, \alpha, u) &= \left( \frac{x}{1 + x} \right) \left\{ \alpha (\bar{\epsilon} R + P - u) - (U_M - u) \right\} - P \\
&\quad - \frac{k}{y} \left( \frac{1}{\bar{\epsilon} \alpha} \right)^{x+1} \left[ \left( \frac{x}{1 + x} \right) \left\{ \alpha (\bar{\epsilon} R + P - u) - (U_M - u) \right\} \right]^{x+1}
\end{align*}
\]

(70)

The first order conditions would fill several lines. (Because of this, the author used Mathematica to solve for the stationary points and verify the second order conditions.) The first order conditions of (70) with respect to \( u \) and \( \alpha \) imply the following:
\( \alpha^R = 1 \)

\[
\begin{align*}
\mu^R &= \bar{R} + P - \frac{k \left( \frac{1}{e^\alpha} \right) x^{(x+1)} \left[ \frac{x}{x+1} \right]^{\bar{R} + P - U_M}}{y \left( \frac{1}{e^\alpha} \right) x^{(x+1)} \left[ \frac{x}{x+1} \right]^{\bar{R} + P - U_M} - 1} 
\end{align*}
\]  

\text{(71)}

The strategy of minimizing the union’s payoff when the \( IR_M \) constraint binds yields ambiguous results. There one set of stationary point to this problem in equation (71). This set of points fails the discriminant test for a minimum because

\[
\frac{d^2 V^R_U}{d \alpha^2} (\mu^R, \alpha^R) = 0.
\]

(The values of \( \frac{d^2 V^R_U}{d \alpha^2} (\mu^R, \alpha^R) \) and \( \frac{d^2 V^R_U}{d \alpha d \mu} (\mu^R, \alpha^R) \) are non-zero but are very long. The author was unable to derive a definitive sign for these latter two second order conditions. Fortunately, he does not need to!) The former condition, \( \frac{d^2 V^R_U}{du^2} (\mu^R, \alpha^R) = 0 \), implies that the stationary points \( (\alpha^R, \mu^R) \) lead to a saddle point and neither a local maximum or minimum point. The author cannot declare decisively if this strategy will induce the union to be passive in any general sense.

Yet, the entrepreneur need not worry about whether or not the \( IR_M \) constraint can induce the union to be passive when the \( ICU \) constraint binds, as in (71). (We cannot rule out that this strategy will or will not result in union passivity for some parameter values.) Yet, if she pays the manager a compensation package of \( \{u, \alpha\} \) in equations (8) and (10), we are certain that the entrepreneur can move the union to the \( IR_U \) constraint, and she can win all the rents.
6.3.3 $\min V_C^U$ strategies

Suppose that the entrepreneur concentrates on minimizing the union’s payoff when the $IC_M$ constraint binds, $\min V_C^U$. In this case, 100 percent share ownership will generally minimize the union’s payoff. The effectiveness of this strategy depends on the maximum fixed wage that the CEO can be paid, $\phi U_M$, and the magnitude of whether or not the union faces high or low costs of obstruction. When $\phi U_M > u^C$ and obstruction costs are low, $y(x+1) - 1 < 0$, the union will be forced into passivity. When, $y(x+1) - 1 > 0$ the success of the entrepreneur in inducing the union to minimize monitoring costs depends on the magnitude of the parameter values. Therefore, while the strategy of $\max V_O^C$ may lead to rent-sharing, no managerial ownership, and no deadweight losses; the entrepreneur may maximize her payoff by driving the union’s payoff to zero when the $IC_M$ binds. In this latter case, the passive union equilibrium in (7) may appear where the entrepreneur gets all the rents, managerial ownership stakes are large, and deadweight losses are zero.

The payoff to the union can be derived by combining (28) and (54).

\[
\min_{w.r.t. \quad u,k,\alpha} V_U(\alpha, u, m^C, q^C) \equiv V_U^C = \\
\left( \frac{x}{x+1} \right) \{ \bar{e}R + P - u \} - P - c - \frac{\alpha^* k}{e^\alpha} y \left[ \left( \frac{x}{x+1} \right) \{ \bar{e}R + P - u \} \right]^{y(x+1)}
\] (72)
The first derivative with respect to $\alpha$ is unambiguously negative. The union must incur larger rent-seeking costs when managerial share ownership is higher.

$$\frac{dV^C}{d\alpha} = -\alpha^{-1} \frac{k}{\bar{c}^y} \left[ \left( \frac{x}{x+1} \right)^{\gamma(x+1)} \{\bar{c}R + P - u\} \right] < 0, \quad \forall \alpha \in (0, 1]. \quad (73)$$

(This is in part the case because $\bar{c}R + P - u > 0$.) The other choice variable, the CEOs base wage, has a stationary point. The first order condition is

$$\frac{dV^C}{du} \bigg|_{u = u^C} = \left( \frac{x}{x+1} \right) + \left[ k(x+1) \frac{\alpha^y}{\bar{c}^y} \left( \frac{x}{x+1} \right)^{\gamma(x+1)} \{\bar{c}R + P - u^C\}^{\gamma(x+1)-1} \right] = 0. \quad (74)$$

Given that $\gamma(x+1) \neq 1$,\(^{26}\) The stationary point is

$$u^C(\alpha) = \bar{c}R + P - \left( \frac{x+1}{x} \right) \left[ \frac{\bar{c}^y}{k\alpha^y} \left( \frac{1}{x+1} \right)^{\gamma(x+1)-1} \right]. \quad (75)$$

This point, $u^C$, minimizes the union’s payoff (it is positive) when obstruction costs are low ($\gamma(x+1) < 1$) and maximizes the union’s payoff (it is negative) when obstruction costs are high ($\gamma(x+1) > 1$). We know this because the sign of the second order condition in (76) depends on the quantity in square brackets.

\(^{26}\) If this $\gamma(x+1) - 1 = 0$ there is no stationary point, and the union’s payoff is either monotonically increasing or decreasing in $u$.  

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The union’s payoff is

\[
V_C^e(\alpha^C = 1, u^C) = \left( \frac{e^{xy}}{k(x+1)} \right)^{y(x+1)-1} \left( \frac{y(x+1) - 1}{y(x+1)} \right) < 0,
\]

when \(y(x+1)-1<0\).

Equation (77) only applies to the case where the union’s payoff is negative if it engages in rent-seeking. (The entrepreneur would never choose a local maximum, as is the case when \(y(x+1) > 1\), in a minimization problem.) This is a sufficient condition to guarantee union passivity when both \(y(x+1)-1<0\), and \(u^C\) is attainable.

\(u^C\) is attainable when \(\max\{\phi U_M, u^C\} = \phi U_M\).

**Proposition 5**

*When the manager’s base wage can be set high enough to cover her opportunity cost, \(\phi \geq 1\), and costs of obstruction are low, \(y(x+1) - 1 < 0\), and \(\max\{\phi U_M, u^C\} = \phi U_M\), then the entrepreneur can induce the union to be passive and the equilibrium values are given in (7). This is achieved by having the manager’s incentive constraint bind, \(IC_M\).*
This follows from equation (77) and the second order condition in equation (76).

When the maximum base wage constraint binds and \( y(x+1) < 1 \), this relationship is less clear. That is, when \( \max \{ \phi U_M, u^C \} = u^C \), it cannot be determined whether or not the union can be forced into passivity. (There is no clear sign for the union’s payoff when \( V_\ell (\alpha, u, m^C, q^C) = V_\ell (1, 0, m^C, q^C) \).)

When \( y(x+1) > 1 \), we know that (75) is a maximum, but it is not clear which of the boundary values for \( u, 0 \) and \( \phi U_M \), will minimize the union’s payoff. We know that maximum point in (75) will give the union a strictly positive payoff. This latter insight follows from inspecting equation (77) under the assumption that \( y(x+1) – 1 > 0 \). When minimization of the manager’s wage, \( u = 0 \), induces the union to be passive, it must be the case that giving the CEO a 100 percent share, \( \alpha^C = 1 \), will satisfy the \( IR_M \) constraint regardless of the base wage, \( u \). Therefore, it is reasonable to consider \( u < U_M \). When \( y(x+1) – 1 > 0 \), the payoff to the union is increasing from \( u \in [0, u^C) \), and it is falling for \( u \in (u^C, +\infty) \). Conceivably, the union’s payoff will be minimized at either feasible extreme, 0 or \( \phi U_M \). Yet, it is not clear that the union will be passive with either \( u^C_0 = 0 \) or \( u^C_\phi = \phi U_M \). In short, it is ambiguous whether or not the entrepreneur will attempt to minimize the union’s payoff if the cost of obstruction are large, \( y(x+1) – 1 > 0 \). Further, given that the entrepreneur does choose a minimization strategy here, it is not always clear if she will choose an intermediate
base wage equal to $u^C$, will minimize the manager’s base wage with $u^C_0$, or will maximize the manager’s base wage with $u^C_{\phi}$. 
6.4 Derivations for the Supplemental Appendices

6.4.1 Derivation of equation (27)

Equation (27) is the maximum $m$, denoted $m^R$, that the union can choose when both its incentive constraint, $IC_U$, and the manager’s participation, $IRM$, constraint binds. The manager’s payoff under these circumstances is given by equation (1). Equation (1) is the left hand side of the $IRM$ constraint, which is written below:

$$IRM : \quad V_M = \alpha \left[ eR - \frac{\bar{e}R}{q} + P - u \right] + u - \frac{mq^x}{x} - U_M \geq 0$$  

(78)

As long as the manager’s incentive constraint is satisfied, she will choose a level of monitoring given by equation (2). Combining equations (2) and (78) we get the following:

$$IRM : \quad \alpha \left[ \bar{e}R - \left( \frac{mc^R}{\alpha} \right)^{1/\alpha} + P - u \right] + u - U_M$$

$$-\frac{1}{x} \left( mc^R \alpha^x \right)^{1/\alpha} \geq 0$$

(79)

where

$27$ If equation (2) equals or exceeds unity, then the $IR_U$ binds and the wage $\bar{e}$ not $\frac{\bar{e}}{q} - P$. Therefore, given the $IC_U$ binds $q^*(m^R) < 1$. 

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Below like terms in equation (79) are combined and simplified.

\[ w(q^*(m)) = \left( \frac{mc^x}{\alpha} \right)^{\frac{1}{x+1}} - P \]

\[ \frac{m[q^*(m)]^x}{x} = \frac{1}{x} \left( \frac{mc^x}{\alpha} \right)^{\frac{1}{x+1}}. \]

It is a simple matter to solve for \( m \).

\[ m \leq \left( \frac{1}{c^x} \right)^{x+1} \left[ \frac{x}{x+1} \right] \{ \alpha(\tilde{c}R + P - u) - (U_M - u) \} \]  

\[ \text{(80)} \]

This implies that the maximum \( m \) when both the ICU and IRM constraints bind is the right hand side (RHS) of (80). This is what we wanted to show. The RHS of (80) is defined as \( m^R \) in equation (27).

### 6.4.2 Derivation of equation (28)

Equation (28) is the maximum marginal monitoring cost both that the manager’s and the union’s incentive compatibility constraint binds. The payoff to the manager is given by (1) and after substituting in equation (2), the manager’s best
response level of monitoring, the manager’s incentive compatibility constraint is as follows:

\[
IR_c : \alpha \left( \bar{e}R - \left( \frac{m\bar{c}^x}{\alpha} \right)^\frac{1}{x+1} + P - u \right) - \frac{1}{x} \left( m\bar{c}^x \alpha^x \right)^\frac{1}{x+1} \geq 0
\]  

(81)

It is relatively straightforward to solve for \( m \) in this case.

\[
m \leq \left( \frac{\alpha}{\bar{c}^x} \right) \left( \frac{x}{x+1} \right) \left( \bar{e}R + P - u \right)^\frac{1}{x+1}
\]

(82)

In other words, the maximum \( m \) that can be chosen when both the \( IC_m \) and \( IC_u \) constraints bind is given by the right hand side (RHS) of equation (82). This is what we wanted to derive. This maximum monitoring cost parameter, the RHS of (82), is denoted by \( m^R \) and is rewritten in equation (28).

6.4.3 Proof for Lemma 3

We want to show that the \( IC_m \) is always slack and satisfied given the \( IR_m \) and \( u < U_M \). Consider the constraints below.

\[
IR_m : V_M \geq 0 \\
IC_m : V_M \geq u - U_M
\]
The left hand sides (LHS) of each is identical, but the right hand side (RHS) differs. If one constraint binds and one is slack, the binding constraint must have a RHS greater than the slack constraint. The $IR_M$, with a RHS of zero, must bind because the RHS of the $IC_M$ is negative when $u < U_M$. Q.E.D.

6.4.4 Derivation of equation (36)

If we combine the union’s wages in equation (35) with the obstruction costs in equation (34), and the costs of effort for union members, $\bar{\tau}$, then we get the following expression for the union members’ individual and aggregate payoffs:

$$V_U(1, m^*, q^*) = w^*(q^*(1, u^*)) - \bar{\tau} - \frac{km_{xy}^*}{y} =$$

$$\left(\frac{\bar{\tau}_{xy}}{k(x+1)}\right)^{\frac{1}{y(x+1)-1}} - P - \bar{\tau} - \frac{k}{y}\left(\frac{\bar{\tau}_{xy}}{(k(x+1))^{y(x+1)}}\right)^{\frac{1}{y(x+1)-1}}$$

This is equivalent to

$$V_U(1, m^*, q^*) = \left(\frac{\bar{\tau}_{xy}}{k(x+1)}\right)^{\frac{1}{y(x+1)-1}} \left(1 - \frac{k}{y}\left(\frac{1}{(k(x+1))^{y(x+1)}}\right)^{\frac{1}{y(x+1)-1}}\right) - P - \bar{\tau}.$$  

(84) simplifies to the following:
\[ V_U(1, m^*, u^*) = \left( \frac{\bar{c}^{\infty}}{k(x+1)} \right) \frac{1}{y(x+1)^{-1}} \left( 1 - \frac{1}{y(x+1)} \right) - P - \bar{c} = \]

\[ \left( \frac{\bar{c}^{\infty}}{k(x+1)} \right) \frac{1}{y(x+1)^{-1}} \left( \frac{y(x+1) - 1}{y(x+1)} \right) - P - \bar{c}, \]

which is what we wanted to show.

### 6.4.5 Derivation of \( u_i^R \) in equation (50)

Recall the first-order condition in equation (42). If we solve for \( \alpha_R(u) \), we get the following expression:

\[ \alpha^R(u) = \frac{x(U_M - u)}{\sqrt{\bar{c}R + P - u}} \quad (86) \]

We want to find the maximum \( u \) such that \( \alpha^R(u) = 1 \). When \( \alpha^R(u) = 1 \) equation (41) implies that the entrepreneur is indifferent about the level of \( u \). Setting the right hand side of (86) greater than 1, implies that

\[ \max \{ u_i^R \} = \frac{1}{x - 1} \{ xU_M - \bar{c}R - P \} > 0, \quad \text{if} \quad \frac{1}{x - 1} \{ xU_M - \bar{c}R - P \} < \phi U_M. \quad (87) \]

Equation (87) must be positive. This is the case because both \( xU_M - \bar{c}R - P > 0 \), and \( x - 1 > 0 \) when \( \alpha_i^R \) is chosen by the entrepreneur. \( \alpha_i^R = 1 \) is the (constrained)
optimal, in terms of \( \max_{u \in \mathcal{U}} V_o \), level of share ownership, from the entrepreneur’s perspective, only when \( \alpha^R \) in equation (43) exceeds unity. This is only the case when \( x U_M > \bar{e} R + P \). Therefore, the sum in the “{}” brackets is positive.

Further, we know that \( x - 1 > 0 \). This must be the case here because the manager’s outside option, \( U_M \), must be less than expected revenues, \( \bar{e} R \). (We know this because production is assumed weakly efficient and thus \( \bar{e} R \geq \bar{e} + U_M \), where \( \bar{e} > 0 \).) When \( U_M < \bar{e} R \), and \( P \geq 0 \) then \( \alpha^R > 1 \) in equation (43) implies that \( x > 1 \).

Since equation (41) implies that the entrepreneur is indifferent between any feasible value of the \( u \) when \( \alpha = 1 \), then \( u^*_i \) can take on any value in the range:

\[
0 < u^*_i \leq \min\left\{ \left( \frac{1}{x-1} \right) \left( x U_M - \bar{e} R - P \right), \phi U_M \right\}
\]

(88)

This is what we wanted to show.
Chapter II

Financing Professional Partnerships
Financing Professional Partnerships a

by

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Abstract

This paper finds that increases in net-debt obligations of profit sharing partnerships give these organizations a strong incentive to expand. Professional partnerships are generally more profitable and selective than corporations when their capital structure changes are transparent to clients. When capital structure changes are secret, then partnerships are nearly as profitable and as selective as corporations. Levin and Tadelis (2005) predicts that professional service firms with fewer informed clients will tend to choose the partnership form. The present paper demonstrates that this prediction holds when financial frictions are present, and net-debt levels are observed by clients.

Keywords: capital structure, information, partnerships, profit sharing

JEL Classifications: L15, L2, G32

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This is the result of major periods of work at three institutions. This was begun while the author served as a college lecturer at Keble College, Oxford. Major revisions were done while the author was visiting the University of Pennsylvania (economics). Finally, most recent draft was written while the author was supported by the University of Cincinnati (Finance-Real Estate).
1.0 Introduction

This paper shows how financial structure and net-debt levels, in particular, affect the behaviour of professional service partnerships. (Net-debt is defined as debt obligations minus cash on hand.) The professional service firm that adopts a partnership structure chooses to tie employment to equity stakes and control rights. In contrast, the professional service firm can adopt a more traditional employment relationship in which control and residual claims are separated from employment contracts. Here we argue that the organizational form that maximizes shareholder value depends on clients’ ability to observe the quality of professionals employed, and clients’ knowledge of the firm’s capital structure. In addition to the informational barriers that clients face, the magnitude of financial frictions will play a large role in a professional service firm’s decision to adopt a partnership or corporate structure.

This paper builds on an idea of Levin and Tadelis (2005). That paper argues that, when clients only detect the average ability of employees a small fraction of the time, the equal profit share-maximizing partnership will be more profitable and supply higher quality than the profit maximizing corporation. Yet, for high levels of market monitoring, Levin and Tadelis (2005) argue that the corporation is more profitable. This observation is used to explain why partnerships are clustered in the relatively opaque professional services where clients find it difficult to judge the quality of the firm’s employees.
Unlike that study, here we argue here that partnerships can always achieve the full monopoly profits when capital structure is transparent to potential clients.

When all clients can observe these capital structure changes, they can infer the partnership’s hiring decision from the partnership’s choice of net-debt. Therefore, even if clients cannot observe the quality of the firm’s professionals directly, they can infer employee ability from the partnership’s fully informative capital structure signals.

When some clients are uniformed about the abilities of the professionals they employ, corporations may deviate from the profit maximizing output and quality level. Further, net-debt levels, capital structure, does not affect the corporation’s hiring decision. Therefore, even if the corporation had a transparent capital structure, this could not credibly signal its hiring intentions to uniformed clients.

Ward (1958) recognized that a partnership’s equilibrium size is a function of its fixed costs or debt obligations. In contrast to Ward (1958), which never envisioned a need for outside equity stakes in a static setting, the present paper has a role for outside equity. This essay argues that, when the partnership is too large with existing levels of fixed costs, it should sell non-voting equity claims to pay down its fixed obligations. When the partnership is too small, it can sell debt to increase its fixed costs and expand its equilibrium size. This topic is explored in section 3.2 and in Proposition 1, in particular. The solution in Ward (1958) involved having debt obligations equal to total profits. With uninformed clients as in Levin and Tadelis (2005) such a prescription would lead to the professional partnership, which is a type of worker cooperative, to hire too many professionals. Therefore, unlike Ward
(1958), the present paper has a role for both outside equity and debt obligations in a static model.

The profitability and the selectivity of the partnership in the present paper depend on two factors. First, the partnership’s profitability and selectivity depend on the financial transparency of its capital structure choices. Secondly, they depend on the ease with which the partnership can adjust its financial structure. Transparent capital structure, $\rho = 1$, and frictionless financial markets, $\theta = 0$, unambiguously make the partnership form more profitable than the corporate form. This is the topic of section 4.1. Opaque finances, $\rho = 0$, and frictionless financial markets, $\theta = 0$, reduce the partnership’s profitability and selectivity to that of the corporation. This is discussed in section 4.2. Costly financial adjustments ($\theta \neq 0$) are discussed in section 5.0.

The transparency of a professional partnership’s finances has a big impact on partnership’s hiring behaviour. This is especially true when no financial frictions are present as in section 4.0. If clients can readily observe capital structure changes, $\rho = 1$, as in section 4.1, then they can infer the optimal hiring decisions of the partnership for the observed level of net-debt. This transparency will encourage the partnership to only take on a level of debt or cash on hand that would induce the partnership to hire at the full-information, profit maximizing level. Financial transparency allows the partnership to be more profitable and more selective than the corporation. If, on the other hand, capital structure is hidden from clients, $\rho = 0$, as in section 4.2, then they can infer that the partnership will hire exactly like a corporation. The choice of the partnership organizational form has no affect on clients’ beliefs when there are no
financial frictions. Clients will rightly assume that the partnership will pursue financial policies which cause the partnership to exploit its information advantage and over-hire, relative to the full information optimum. Therefore, the curse of this lack of the financial transparency is that it encourages the partnership to behave just like a corporation.

Financial frictions would only serve to lessen the profitability of the partnership when it has transparent finances. This is the topic of section 5.1. Yet, when its capital structure is opaque to clients, financial frictions can actually make the opaque partnership more profitable and selective than the corporation as we demonstrate in section 5.2. The present paper finds that financial frictions aid the opaque partnership when net-debt levels are low but positive. Large financial frictions are more consistent with Levin and Tadelis (2005)’s results where the partnership does not alter its capital structure.

With transparent finances and financial frictions, as assumed in section 5.1, we are able to support the dichotomy advanced by Levin and Tadelis (2005). That is, the present paper supports Levin and Tadelis (2005)’s proposition that the partnership organizational form will be the preferred business structure when many clients are uninformed. Likewise, the corporate form will be the more preferred mode of organization when most clients directly observe the quality of the professionals that they hire. This support for Levin and Tadelis (2005, p. 142)’s “central comparative static result” only applies when finances are transparent and financial adjustments are costly.
There are several recent papers trying to explain the weaknesses and strengths of professional partnerships. Huddart and Liang (2003) argue that free rider problems in the monitoring of the effort of partners limits the partnership’s size. In contrast, Morrison and Wilhelm (2004) argue that partnerships overcome free rider problems in the mentoring of associates. In that paper, partners mentor so that they will be able to sell their shares in the firm to the next generation of partners. Morrison and Wilhelm (2007) argues that investment banks went public as the development of human capital became less important, relative to the raising and management of financial capital. Bar-Isaak (2004) also discusses the virtues of mentoring, but that paper says that mentoring can solve the moral hazard problems of the firm’s more senior members. That paper argues that partners with good reputations have little incentive to exert effort unless they are residual claimants on part of the future reputations of their associates. Garicano and Santos (2004) say that profit sharing in partnerships gives partners incentives to make efficient referrals to their customers. This present paper builds on the idea of Levin and Tadelis (2005) that the partnership is a quality commitment that resolves the problem of over-hiring. Unlike these recent papers, in the present paper, debt levels play a prominent role in the partnership’s ability to profitably serve its clients.

The present paper differs from Wilson (2004) [old Chapter II], which it means to replace. Both rely on Levin and Tadelis (2005)’s assumption about uninformed clients. Further, Wilson (2004) and the present paper show that net-debt levels are crucial to partnerships’ hiring decision, but do not play a role in corporations’ hiring
decision. Both papers show that partnerships will want to alter their financial structure to take advantage of profit opportunities.

One major difference between the two papers deals with clients’ ability to observe net-debt levels. Wilson (2004) assumes that clients can observe net-debt levels therefore its results are only comparable to sections of the present paper that attempt to solve the model with that assumption. Sections 4.1 and 5.1 in the main text in particular make the assumption that net-debt levels are observed by all clients. The equilibrium actions of partnership with hidden net-debt levels as in sections 4.2 and 5.2 were not explored in Wilson (2004).

Another major difference between the two papers is that Wilson (2004) focuses on how conflicts between more and less able partners endogenously raise the cost of finance in the partnership. In contrast, here we focus on how exogenous costs of finance, such as equity floatation costs, affect the partnership’s profitability. This simplification better allows the present draft to focus on how the non-neutrality of finance on the partnership’s hiring decision can be a benefit and liability when professionals share profits.

Finally, the present game is meant to have a simpler structure. There is no uncertainty about the state of demand as in Wilson (2004). Further, potential hires or partners have the same opportunity cost in the present paper while in Wilson (2004) conflicts are modelled as having risen out of the partner’s different opportunity costs. While both these features are interesting extensions, they could distract the reader from the main message of this paper. That is, net-debt levels affect the hiring decision of the professional partnership. Further, the ability to impact future hiring
decisions in profit sharing partnerships with net-debt levels affects the relative desirability of adopting the partnership or corporate form.
Nature selects the accuracy of market monitoring, $0 < \mu \leq 1$, and the transparency of partnership finances ($\rho = 0$ or $\rho = 1$).

<table>
<thead>
<tr>
<th>Period = 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partnership or corporation is formed. That is, $\gamma = 1$ or $\gamma = 0$, respectively.</td>
<td>Capital structure is chosen. Therefore, the financial variables $\alpha$ and $F$ are determined.</td>
<td>$N$ partners or employees are hired by firm.</td>
<td>• Consumers make bids. • Services are rendered to winning bidders. • All factors are paid.</td>
<td></td>
</tr>
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Figure 1: The Sequence of Events
2.0 Model

Consider a firm that employs a one-to-one technology, $f(N) = N$, where $N$ is the firm’s size and output. The firm must pay an endogenous fixed cost, $K > 0$, to operate. Further, it has market power. This market power comes from its ability to vary the expected quality of its workforce.

Suppose that the firm can hire employees who have abilities distributed over a continuous, smooth probability density function $g(a) > 0$, which is defined on the support $a \in [a, \bar{a}]$. The continuous distribution of employee abilities is defined as $G(\bar{a}) = \int_a^{\bar{a}} g(a)da$. Because the firm is unable to affect the distribution of abilities of its employees, the average quality of the firm’s workforce declines in its size. Appendix 7.1.1 proves this property.

Let all employees vary in their ability, but they all have the same reservation wage $w \geq 0$. We will assume that all potential employees and investors have linear preferences for consumption. There is no cost to employees beyond their opportunity cost $w$ or providing productive work. The risk free discount rate between time periods is zero. Therefore, investors and employees have no preference for receiving certain compensation in period 2 or period 4, for example.

Let us assume that market price, $p(N)$, equals the average quality of the firm’s employees, $q(N)$, times a constant, $x$. $p(N) = xq(N)$. Therefore, the price, $p(N)$, that the firm can charge to informed consumers declines in output. This means that price
falls in output or equivalently employment, $p_N < 0$, where subscripts denote partial derivatives.

There are two types of organizational forms that the firm can adopt in period 1. The first organizational form is the corporation. The corporation has equity stakes that are freely tradable that are not tied to employment in the firm. Secondly, the partnership organizational form can be adopted. The partnership is composed of employees, partners, who are given an equal share of the firm’s voting-equity in exchange for their labors. These shares cannot be traded. We assume partners pay nothing for their membership in the partnership and they cannot sell their membership share.

Dow (1986) proves that, when membership markets are as competitive as stock markets, labor-managed firms (LMF)—partnerships here—will have the same objective functions as capital-managed firms (CMF)—corporations here. Therefore, LMFs need not be associated with different hiring objectives than CMFs. Ward (1958) and Levin and Tadelis (2005) assume that there are no membership markets. In the present paper, employees do not buy their way into the firm. Nevertheless, all that is necessary for this paper’s results to hold is that membership shares trade for less than their full net present value.

Indeed, Dow (2001; 2003 pp. 156-161) argues that there are many good theoretical reasons for membership shares in a LMF to trade at discount relative to shares in a CMF. One of the obstacles to efficient membership markets highlighted by Dow (2001; 2003 p. 160) is particularly relevant to the present paper. The highest bidder for the LMF’s shares may not be the most able employee. Therefore,
membership shares cannot be awarded merely to the highest bidder if that employee is ill suited for the job. The quality of the employees hired is at the core of the discussion in the present paper and Levin and Tadelis (2005).

While partners endorse policies that maximize their payoff from the perspective of period 3, the partnership maximizes the value of voting-equity per partner in its period 3 hiring decision. Just as Levin and Tadelis (2005) assumes, we assume that the partnership has an equal profit sharing structure. This means that the decisive voter in the partnership has the same preferences as all other voters. Each voting equity member would like to increase profits by hiring a new partner, but hiring a new partner leads them to dilute their individual share of the profit. Therefore, the period 3 partners maximize their payoff. Yet, the way that this is achieved in equilibrium is to only hire partners that increase the average payoff of partners. This is the problem first introduced by Ward (1958).

The present paper does not assume that partnerships maximize average profits. All that is assumed is that they maximize their payoff. Indeed, the game is structured so that partnerships get the opportunity to maximize total profits. Namely, in period 2, capital structure is chosen by the partnership, and corporation for that matter, in a way that maximizes total profits. Yet, after capital structure is chosen in period 2, in period 3 partners maximize their payoffs by maximizing profits per partner or average profits.

We will assume that all dividends paid out to period 2 partners cannot be forcibly returned to the partnership in subsequent periods. For example, if the firm had unlimited liability, some dividends paid out in period 2 could be an asset of the
firm if the partners had not squandered them. A sufficient condition to guarantee that dividends paid out are not an asset of the firm would be to assume that partners have limited liability. This paper will assume that partners have limited liability. Levin and Tadelis (2005) argue that it is not uncommon for professional partnerships in the United States to have such a limited liability structure today. Yet, unlimited liability characterized professional partnerships of the 19th century, according to Morrison and Wilhelm (2007, 268).

Let us define an indicator variable, $\gamma$, that only takes on the value 0 or 1. If the firm adopts the corporate form in period 1, the indicator variable $\gamma = 0$. Alternatively, if the firm adopts the partnership form in period 1, then $\gamma = 1$. This is a choice variable of the firm’s founding shareholders.

Let us consider three types of liabilities that must be paid in period 4 after production takes place—net-debt, non-voting equity, and voting equity. These financial contracts are selected by the firm’s controlling shareholders in period 2. Net-debt, $F$, can be positive if the firm has debt obligations due in period 4. Alternatively, net-debt will be negative if the firm has extra cash to pay out to shareholders in period 4. Non-voting equity holders have a claim to the residual profits of the firm. Voting equity holders control the firm’s hiring decisions and get a residual profit share. Let $\pi$ be revenues minus variable costs. That is, it is profit before fixed or financial costs, $\pi(N) = p(N)N - wN$. Further, let $\alpha$ be the fraction of

---

1 $\pi$ “$\pi$” is not the same in this paper as in Levin and Tadelis (2005). In Levin and Tadelis “$\pi$” denotes total profits or revenues minus variable and fixed costs. The notation here is consistent with the other essays in this thesis.
residual profits, \( \pi - K - F \), promised to non-voting equity holders, where \( 0 \leq \alpha \leq 1 \). Therefore, voting equity holders split claims worth \((1 - \alpha)(\pi - K - F)\).

We will define an indicator variable, \( \rho \), to indicate whether or not clients can observe the net-debt levels, \( F \), of the partnership in period 3 or 4. This variable is chosen by nature and only takes on two values, 0 or 1. If the part of capital structure is visible to all clients, then the indicator variable \( \rho = 1 \). If no clients can see the net-debt levels of the firm, then \( \rho = 0 \).

While disclosure, \( \rho \), may seem to be a discretionary activity, it may be more a function of the environment in which the firm operates. Certainly, the government could mandate disclosure of net-debt levels and other financial arrangements. Yet, in the United States, for example, there is no such public disclosure requirement for privately held partnerships. Credible, public disclosure (\( \rho = 1 \)) of net-debt levels, capital structure, may be difficult to implement in professional partnerships.

Often there is great reluctance to disclose financial information on the part of individuals. Many partners may feel uncomfortable with disclosing detailed firm financial information. One reason for this is an individual partner’s compensation is very closely correlated to the firm’s financial performance. Disclosing capital structure may be \textit{de facto} disclosure of partners’ compensation.

Further, it may be difficult to credibly disclose the true capital structure to clients. Financial contracts have a tendency to cancel each other out. Incomplete information like pension obligations to retired partners or lease arrangements could have a big impact on the level of net-debt. Partners in Goldman Sachs and Arthur Andersen appeared to have both equity and debt claims before the former went public.
and the latter disintegrated, according to Endlich (2002, p. 369) and Harris (2002), respectively. For example, retained equity accounts of current partners can add to the fixed obligations, or net-debt, of the partnership if those obligations must be returned to those partners. In short, full disclosure of the partnership’s finances, $\rho = 1$, may not be possible even if it is somewhat desirable.

It is often the case that credible revelation of favourable, asymmetric information is prohibitively costly. The owner of a “plum (or ”good“) used car in Akerlof (1970) would surely want to signal that her car is indeed a good one and not a “lemon,” a bad used car. Yet, she may find credible revelation of its quality prohibitive. Namely, the cost of finding credible certification would be greater than the difference between the symmetric information value of the plum, less her own valuation of it. The fact that these costs of certification, offering warranties or the like, are very high is an exogenous feature of the used car market. These costs of certification are not a choice variable of the used car seller. Here, too the costs to the partnership of revealing its net-debt ratio to clients may exceed the benefits of doing so. For this reason, $\rho$ is exogenous choice of nature, not an endogenous choice of the partnership.

We will assume that there are costs to raising outside finance. One way of measuring financing costs for public and newly public firms is the underwriter’s spread. For example, the underwriter’s spread is the difference between what the issuing firm receives, the net proceeds, from the sale of the security and the price that the investment banking underwriter sells the security to the public, the gross proceeds. The underwriter’s spread is usually measured as a percent of gross
proceeds. Kim, Palia, and Saunders (2003) report from 1970 to 2000 the median underwriter’s spread for debt and new equity issues were 0.750 percent and 7.00 percent, respectively. Public firms often have more favourable disclosure and control arrangements when compared to privately held firms. Therefore, these numbers should be viewed as closer to the lower bound of financial frictions faced by the professional partnership.

We assume that the financing cost function is piecewise defined to reflect the fact that the costs of raising equity exceed the costs of debt. Namely, \((\theta_d)^2 < (\theta_e)^2\). In addition, we will assume that costs are a constant fraction of the total amount raised. Further, let \(1 > \theta_d \geq 0\) and \(-1 < \theta_e \leq 0\). This reflects the fact that, when equity must be raised, new net-debt is negative. Further fees cannot be equal to or exceed 100 percent of the amount raised, \(\theta_d \leq 1\) and \(\theta_e \geq -1\). The costs of finance \(c(F)\) is defined below:

\[
c(F) = \begin{cases} 
\theta_d F, & \text{where } F > 0 \text{ and } 1 > \theta_d \geq 0 \\
0, & \text{where } F = 0 \\
\theta_e F, & \text{where } F < 0 \text{ and } -1 < \theta_e \leq 0 
\end{cases}
\]

Let us denote the parameter theta, \(\theta_i\), by the general subscript “\(i\),” which can take on the values \(d\) or \(e\). That is, \(i = d\) or \(e\). “\(d\)” denotes that the cost parameter is for debt financing. “\(e\)” denotes that the cost parameter refers to equity financing costs.

As with Levin and Tadelis (2005), we assume that the expected price that the firm will receive is weighted by the fraction of clients, \(\mu\), who detect the size of the
firm, \( N \), and thus the average quality, \( q(N) \), of the firm’s professionals, where \( 0 \leq \mu \leq 1 \). The fraction of informed, \( \mu \), and uninformed clients, \( 1 - \mu \), is common knowledge.

As with Levin and Tadelis (2005) we will assume that the distribution of employees from which the firm draws is common knowledge. As long as there is some positive cost to employing low quality workers, lower prices, and workers have the same opportunity cost, \( w \), then the firm will only select the highest quality \( N \) employees. Therefore, given that a client knows that the number of workers employed, \( N \), then this client can deduce the average quality of the firm.

Let \( N^e(r) = N^e \) be the uninformed clients’ expectation of the firm’s size. We will focus on the case where the market has rational expectations about firm size. That is, the \((1 - \mu)\times100\%\) uninformed clients compute the profit maximizing size of the firm based on common knowledge variables. \( r \) is a row vector of endogenous variables—\( \gamma \) and \( F \)—, exogenous variables—\( \rho, \theta, \mu, x, w, K \)—, and the distribution of abilities—\( G(a) \). \( r = [\gamma, \rho F, \theta, \mu, x, w, K, G(a)] \).

\( N^e(r) \) does not depend on \( N \). Nor is the ability, \( a \), of any of the employees observed when forming expectations. (Only the distribution of abilities is known.) This is because the uninformed clients do not observe firm size or employee ability. Further, \( F \) is not in the rational expectations vector if \( \rho = 0 \).

Price is

\[
p(N, N^e, \mu) = \mu p(N) + (1 - \mu) p(N^e(r)).
\] (2)
Informed and uninformed, clients bid for services without observing the price. All winning bidders pay the uniform price that clears the market. In the rational expectations equilibrium, expectations of the uninformed are accurate because they have enough information to accurately infer the firm’s hiring decision. Namely, uninformed buyers know both the firm’s organizational objectives and the distribution of abilities from which the firm draws its workforce. We can also think of the firm’s equilibrium objectives as being driven by these endogenous and exogenous variables. That is, \( N = N(r) \). Therefore, in equilibrium the rational expectations are accurate,

\[
N^e(r) - N(r) = 0. \tag{3}
\]

Let us define \( r_k \) to be a generic element of the vector of \( r \). Suppose that \( r_k \) is non-zero element which is a continuously defined parameter over for a small change. In this case,

\[
\frac{\partial N^e(r)}{\partial r_k} - \frac{\partial N(r)}{\partial r_k} = 0. \tag{4}
\]

\(^2\) For example, when \( \rho = 0 \), the derivative is not continuously defined for a small change in net-debt or the second element in the \( r \) vector. In this case, expectations will not move with the actual level of net-debt chosen. The reason is that debt choices are not observed. \( \gamma \) only takes on the value of zero or one; therefore, it does not satisfy the criteria for (4) to hold. \( G(a) \) would always be a non-zero element. Further, it could be that parameters of the distribution \( G(a) \) could be continuous and satisfy the criteria. For example, the upper or lower bound of talent could shift. That is, movement in either \( a \) or \( g \), respectively, could lead to a continuous shift in expectations. Yet, if the distribution of talent moved from uniformly distributed to normally distributed, then (4) would not be well defined.
While expectations move to the actual, in equilibrium, the firm will treat expectations as given when choosing output. The reason for this is $N'(r)$ does not depend on $N$. Therefore, the firm is unable to move expectations through its choice of $N$ in period 3. Yet, it may be able to move expectations through its choice of $\gamma$ and $F$ in periods 1 and 2, respectively.
3.0 The Non-Neutrality of Net-Debt on the Partnership’s Period 3 Hiring Decision

In this section, we consider the period 3 hiring problem where capital structure—
\{\alpha, F\}—are taken as given by the corporation and the partnership, respectively. We find that both the corporation and the partnership will hire more employees as market monitoring falls. In subsection 3.1, we find that the corporation’s hiring decision does not depend on the firm’s capital structure—\{\alpha, F\}. In contrast, the number of employees demanded by the period 3 partnership is shown to depend on net-debt levels, \(F\), in subsection 3.2. Yet, the period 3 partnership’s hiring decision is not directly affected by non-voting equity stakes, \(\alpha\). In proposition 1, we explain how the period 2 partnership can structure financial contracts to alter the period 3 partnership’s hiring decision.

3.1 Imperfect Market Monitoring and the Irrelevance of Financial Structure in the Corporation

Let us define profit before fixed and financial costs as the following:

\[
\pi(N, N^c; \mu) = \mu p(N) + (1-\mu)p(N^c(r)) \cdot N - wN
\]  (5)
Generally, in the textbook theory of the firm we think of the firm as maximizing total profits. In this context, financial claims are irrelevant to the hiring and output decisions of a firm controlled by a fraction of shareholders \((1 - \alpha)\), where \(0 \leq \alpha \leq 1\). Let us assume that there are fixed payments or inflows of a size \(F\) in period 4. A positive \(F\) means that the firm has bonds or loans that it must pay. A negative \(F\) means that the firm has reduced its fixed costs in period 4. Neither \(\alpha\) nor \(F\) affects the neoclassical maximization problem below. Let \(V^C\) be the total value of the shares held by voting equity holders in a corporation.

Note that if we cast the corporation’s problem as maximizing the returns to voting equity holders, the objective function would be the following:

\[
\arg\max\limits_{\mu} V^C = (1 - \alpha)(\pi(N, N^r; \mu, \theta) - K - F(1 + \theta)) \\
= (1 - \alpha)(\{\mu p(N) + (1 - \mu) p(N^r(\mu))\} N - wN - K - F(1 + \theta))
\]

Let us use the superscript “\(C\)” to denote the choice of \(N\) that would be chosen by the corporation that maximizes returns to voting equity holders.

\[
\frac{\partial V^C}{\partial N}\bigg|_{N=N^C} = \mu p_N(N^C) N^C + \{\mu p(N^C) + (1 - \mu) p(N^r(\mu))\} - w = 0
\]

Suppose that the firm was 100% inside equity controlled with zero net-debt then the objective of the firm would be to maximize total profit. In this case, \(\alpha = F = 0\).
arg max \( \{ \pi(N, N^\ast; \mu) - K \} = \{ \mu p(N) + (1 - \mu) p(N^\ast(\mathbf{r}))\} N - wN - K \) \hspace{1cm} (8)

The first order condition is the following where the corporation’s optimum size is \( N^C \):

\[
\left. \frac{\partial \{ \pi - K \} }{\partial N} \right|_{N=N^C} = \mu p_N(N^C) N^C + \{ \mu p(N^C) + (1 - \mu) p(N^\ast(\mathbf{r}))\} - w = 0 \hspace{1cm} (9)
\]

Not surprisingly, the first order conditions in (7) and (9) are identical. The financial variables, \( F \) and \( \alpha \), drop out of the optimization problems. That is, for the corporation’s voting equity holders the financial structure does not affect their hiring decision in this basic setup. The second order condition is the following:

\[
\frac{\partial^2 \mathcal{V}^C}{\partial N^2} = \left. \left. \frac{\partial^2 \{ \pi - K \} }{\partial N^2} \right|_{N=N^C} \right. = \mu \{ 2 p_N(N^C) + N^C p_{NN}(N^C) \} \hspace{1cm} (10)
\]

Let the superscript “C” denote the corporation’s optimal choice. The second order condition must be negative for \( N^C \) to be a maximum. Therefore, we will confine ourselves to discussing the non-trivial cases where equation (10) is negative. Further, in equilibrium, expectations are accurate. That is, \( N^\ast = N^C \). When expectations are accurate, then the first order conditions in (7) and (9) become the following:
\[
\frac{\partial V^C}{\partial N} \bigg|_{N=N^c} = \frac{\partial \{\pi - K\}}{\partial N} \bigg|_{N=N^c} = \mu \ p_N(N^c) N^c + p(N^c) - w = 0
\] (11)

It is clear from the first order condition, that the corporation is more selective in its hiring when market monitoring is higher. The comparative static is

\[
\frac{\partial N^c}{\partial \mu} = -\frac{N^c p_N(N^c)}{\mu \{2 p_N(N^c) + N^c p_{NN}(N^c)\} + (1 - \mu) p_N(N^c)} < 0.
\] (12)

This is derived in the appendix 7.1.2. Equation (12) tells us that selectivity (hiring) is rising (falling) in \(\mu\).

We can use the envelope theorem to determine the relationship between profits and market monitoring levels in the corporation. Equation (13) below is derived in appendix 7.1.3. Total equilibrium profits in the corporation are unambiguously increasing in the fraction of clients who observe the firm’s true size, and, thus, average quality.

\[
\frac{\partial \{\pi(N^c; \mu) - K\}}{\partial \mu} = (1 - \mu) p_N(N^c) \frac{\partial N^c}{\partial \mu} N^c > 0.
\] (13)

Another lens to look at the impact of market monitoring on hiring is the inverse elasticity rule of microeconomic textbooks. Rearranging equation (11), we get the following:
\[ p(N^C) \left( 1 + \mu \frac{\partial p}{\partial N} \frac{N^C}{p(N^C)} \right) = w, \]

Let \( \varepsilon \) stand for the price elasticity of demand where

\[ \varepsilon = -\frac{dN}{dp(N)} \frac{p(N)}{N}. \]

If we substitute in the \( \varepsilon \), then we have equation (14) below. The first-order condition for the corporation’s profit maximization problem can be written in terms of the inverse elasticity rule below:

\[ p(N^C) \left( 1 - \frac{\mu}{|\varepsilon^c|} \right) = w \quad (14) \]

The first-best level of output, or equivalently hiring, for the monopolist occurs when \( \mu = 1 \) and marginal revenue equals the wage, or marginal cost. When \( \mu < 1 \), the firm will over-hire because it is unable to affect the expectations of \((1 - \mu) \times 100\) percent of its clients. When \( \mu = 0 \) the firm will behave as if its output and hiring decisions have no effect on the price.
3.2 Imperfect Market Monitoring and the Importance of Net-Debt on the Partnership’s Hiring

Let us consider the partnership’s problem. Let us assume that partners control the firm to maximize their residual claims. Each partner is guaranteed his reservation wage $w$ plus some profit share, $S$, equal to profits less the fixed payment (or inflow), $F$, times the partners’ share of residual profits divided by the number of partners, $N$.

In period 3, partners maximize profits by choosing a partnership size. The decisive voter in the partnership is the median partner. That median “controlling” partner will want the firm to be controlled to maximize his or her share. Under an equal sharing rule where partners are identical in abilities, there will be unanimous agreement about the size that maximizes all partnerships shares. The objective of the controlling partner is the following:

$$\arg \max_N S(N; F, \alpha; \mu, N^e) = (1 - \alpha) \left( \frac{\pi(N; \mu, N^e)}{N} - \frac{K + F(1 + \theta)}{N} \right)$$

The first order condition is the following:

$$\frac{\partial S}{\partial N} \bigg|_{N=N^e-N^*} = \mu \cdot p_N(N^e) + \frac{K + F(1 + \theta)}{(N^e)^2} = 0$$
Note that the percent of non-voting equity predictably has no effect on the decision problem. Similar to equation (14), the first-order condition can be rewritten in terms of the inverse elasticity rule; but, in contrast to equation (14), fixed costs and debt service, $K$ and $F$, determine the optimal output and hiring for the partnership:

$$p(N^p) \left(1 - \frac{\mu}{\epsilon^p}\right) = p(N^p) - \frac{K + F(1 + \theta)}{N^p}$$

(17)

The derivation of equation (17) is shown in the appendix section 7.1.4.

Once we use the implicit function theorem, it is clear that the partnership’s size is increasing in the fixed payments due in period 3.

$$\frac{dN}{dF}_{N=N^p} = -\frac{S_{NF}}{S_{NN}} = -\frac{\mu p_{NN}(N^p) - 2(K + F(1 + \theta))}{\mu p_{NN}(N^p) - 2(K + F(1 + \theta))} > 0$$

(18)

The comparative static in equation (18) is derived in appendix section 7.1.5.

The comparative static in (18) raises the possibility that financial structure does indeed matter for the hiring decisions of the partnership. This is what Ward (1958) found. Yet, these results indicate that only certain kinds of financial policies affect the partnership. Namely, higher net-debt levels cause the partnership to expand. Yet, selling an outside equity stake, $\alpha$, as we found in the first order

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3 By definition, if $N^p$ is a profit maximizing point and an interior solution of the smooth profit function, the second order condition must be negative. Therefore, the denominator in (18) must be negative.
condition in equation (16) does not affect the equilibrium size of the partnership. Therefore, financial changes in period 2 can have an impact on period 3 hiring.

In an idealized setting capital structure is seen as irrelevant. In equation (18) we show that it can be relevant for the employment policy in professional partnerships. For Modigliani and Miller (1958) the size of the pie is not affected by how it is sliced. Without taxes and financial frictions ($\theta_i = 0$) such as underwriting costs, the debt-to-equity ratio does not matter holding investment policy as given. Modigliani and Miller (1963) and Miller (1977) comment on financial policy as affecting value through either through corporate and personal income taxes.

Underwriting costs, as documented in Chen and Ritter (2000), can also be significant leakages of value. Yet, financial structure is seen as only affecting employment indirectly through investment policy. For example, risk shifting caused by excessive debt as in Jensen and Meckling (1976) destroys value through leading to investment in speculative negative net present value projects. That paper argues that capital structure can also affect agency problems in the firm. Myers (1977) argues that too much debt may lead to debt overhang, which leads the firm to pass up positive net present value projects.

Here, capital structure affects firm value for another reason not pursued by the often cited studies mentioned in the previous paragraph. Ward (1958) and the present paper find that debt can affect the employment policy directly in worker cooperatives where equity stakes given to new employees. Therefore, according to Ward (1958), Modigliani and Miller (1958) will be violated if we assume away taxes, financial,
frictions, or changes to the firm’s investment policy. Ward (1958)’s result is confirmed here.

Nevertheless, raising debt by itself does not affect the period 3 hiring decision. Suppose that financing costs are $\theta_i = \theta_0 = 0$. In particular, if partnership raised debt in period 2 and let the proceeds sit within the firm’s coffers until period 3, then there would be no effect on the period 3 partnership’s hiring decision. They could use the proceeds from the debt sale $F > 0$ to pay off the period 4 liability of $-F$. In effect, raising debt had no effect on net-debt if the proceeds from the debt sale sit inside the firm. It is the distribution of the net-debt that affects the hiring here. To change the period 3 decision problem of the partnership, the debt proceeds must be spent. The best way to do this is give a dividend to the owners worth $F$ in period 2.

Alternatively, selling an outside equity stake leads to a cash inflow that does affect the period 3 hiring without any period 2 dividends. Suppose that the firm raises $-F > 0$ from selling an outside equity stake, $\alpha$. If the partners do nothing, and let this cash sit in the firm until period 3, this has a real impact on the firms’ output and hiring! This increase in cash reduces the number of partners hired according to equation (18). Yet, the sale of the outside equity stake, $\alpha$, will have no effect on the hiring decision if the partners distribute the proceeds from the sale of equity in period 2 prior to the period 3 hiring decision.

Therefore, if the period 2 partners want to affect the equilibrium hiring decision of the period 3 partners, there are things two things they can do which are summarized in the proposition below.

---

4 We will more fully discuss the impact of financing costs in section 5.0.
Proposition 1

• If the period 2 partnership wants the period 3 partnership to be less selective, the former will issue debt worth $F > 0$ and distribute the proceeds in period 2.

• If the period 2 partnership wants the period 3 partnership to be more selective, the former will raise negative net-debt ($F < 0$) with a non-voting equity stake, $\alpha$. The proceeds from this sale of equity will be allowed to remain in the firm until period 4.

The upshot of this recipe, which follows from our discussion above and equations (16) and (18), is that timing of the partners’ compensation also depends on whether they raise debt or equity. With debt, period 2 partners get at least some of their compensation in period 2. This is because they want to make sure that proceeds of the debt raised cannot be used to pay off the debt in period 4. Alternatively, if the firm raises $F < 0$ with outside equity, $\alpha$, partners will get all of their compensation in period 4. This is because the negative net-debt raised from an equity stake must stay in the firm for it to have an effect on the period 3 hiring decision.

Finally, it is worth commenting on the effect of financial frictions on the period 3 partnership’s hiring. To find this comparative static, we combine both the second order condition and the partial of the first order condition in (16) with respect to $\theta_i$. 

119
\[
\frac{dN}{d\theta_d\big|_{f_N-F}} = -\frac{S_{NN}}{S_{NN}} = \frac{-F/(N^p)^2}{\mu P_{NN}(N^p) - 2(K + F(1 + \theta_d))/(N^p)^2} < 0
\] (19)

We cannot sign this because net-debt can be either positive or negative. Yet, we can sign \(\theta_d\) and \(\theta_e\) separately because \(F > 0\) when debt is raised and \(F < 0\) when equity has been raised.

\[
\frac{dN}{d\theta_d\big|_{f_N-F}} > 0, \quad \& \quad \frac{dN}{d\theta_e\big|_{f_N-F}} < 0,
\] (20)

This means that as the costs of debt rise, \(\theta_d\), the period 3 partnership is less selective. When the cost of equity falls, or equivalently \(\theta_e\) approaches zero, the period 3 partnership becomes more selective. In short, rising financial frictions, as measured by the squared distance of the cost parameter from zero \((\theta)^2\), tend to make the period 3 partnerships less selective for any given level of net-debt, \(F\). Higher costs of raising debt magnify the effect of having debt obligations on hiring. In contrast, as \((\theta_e)^2\) increases or the costs of equity rises, the effect of cash infusions are blunted by the leakage of cash due to financing costs. As the costs of equity finance rise, which means that \(\theta_e\) approaches –1, when \(F < 0\), then the partnership becomes less selective.

It is important to underscore that the comparative statics in (18) and (20) only apply to the period 3 partnership that takes financial structure as given. For the period 3 partnership \(F\) is exogenous. As we will see in subsequent sections, given a
partnership does form ($\gamma = 1$), there is a joint size, $N$, and capital structure decision, $F$, which the period 2 partnership makes. For the period 2 partnership, net-debt levels are endogenous best responses. We will derive these best responses under four different exogenous environments in sections 4.0 and 5.0.
4.0 Costless Finance ($\theta = 0$) and the Selectivity of the Partnership

In contrast to the previous section we allow the partnership to vary its financial structure prior to the period 3 hiring decision. The last section discovered in equation (18) that hiring in the period 3 partnership is increasing in the level of net-debt due at the end of period 4. Therefore, the period 2 partnership can shape the hiring decision of its future self, the period 3 partnership, by choosing some net-debt that maximizes total profits. In this section, we consider the period 2 partnership’s problem when there are no costs to raising outside finance, $\theta_i = 0$. In section 4.1, the best responses of the transparent partnership in period 2 are derived. In section 4.2, we find the best responses for the partnership with hidden net-debt levels.

4.1 Transparent ($\rho = 1$) and Costless ($\theta = 0$) Net-Debt and the Dominance of The Partnership Form ($\gamma^* = 1$)

In this subsection we show that the partnership can commit to hiring the profit maximizing number of partners through its choice of capital structure in period 2.

The notation of this subsection uses the superscript “*” to denote the solution for the monopoly partnership that has transparent capital structure, $\rho = 1$. It happens to be that these are the equilibrium choices that induce the partnership to earn the full monopoly profits. Therefore, the reader would not be too misled if he believed that the superscript “*” denotes the first-best choices for the firm. In subsequent sections,
sections 5.1 and 5.2, we will introduce subscripts to denote the presence of costly
finance, $\theta_i \neq 0$. Therefore, the lack of a subscript on the endogenous variables $\{\gamma^*, F^*, N^*\}$ denotes that the partnership can raise finance without transaction costs, $\theta_i = 0$.

The partnership in period 1 is formed to maximize total profits in period 4. That means choosing a capital structure in period 2 that induces the partnership to hire the optimum number of partners in period 3. Suppose that $\pi^* = \pi(N^*)$ is the perfect information ($\mu = 1$) maximum profits before fixed costs. This is implied by differentiating the full information profit function and solving for the $N^*$ that maximizes profits.

$$\begin{align*}
\arg \max_N \{\pi(N;1) - K\} &= \arg \max_N \{p(N)N - wN - K\} \\
\frac{\partial \{\pi(N;1) - K\}}{\partial N} \bigg|_{N=N^*} &= p_N(N^*)N^* + p(N^*) - w = 0 \quad (21) \\
\frac{\partial^2 \{\pi(N;1) - K\}}{\partial N^2} \bigg|_{N=N^*} &= p_{NN}(N^*) + 2p_N(N^*) < 0
\end{align*}$$

The optimal $F = F^*$ induces the firm to choose $N^p = N^*$ defined by equation (21). This $F$ causes the partnership to maximize both profits per partner and total profits. That is, partners are changing their future decision problem by their choice of $F^*$, which is a function of $N^*$ defined by equation (21).

$$F^*(\mu) = -K - N^{*2} \mu p_N(N^*) \quad (22)$$
Equation (22) is derived by finding the $F^\ast(\mu)$ that will satisfy the first order condition of the partnership’s problem in equation (16) when $\theta_i = 0$. If we rearrange the full information first order condition in (21) we can find that

\[-N^\ast \pi(N^\ast) = p(N^\ast)N^\ast - wN^\ast = \pi(N^\ast;1).\] (23)

That is, equation (23), $\pi(N^\ast;1)$, is the equilibrium profits before fixed costs when $N^\ast$ employees are hired and market monitoring is $\mu = 1$. Substituting in (23) into (22) we get the following relationship:

\[F^\ast(\mu) = \mu \pi(N^\ast;1) - K\] (24)

In short, the partnership will maximize profits in period 2 by selecting the net-debt level, $F(N)$, which will cause the first order conditions in (16) and (21) to be satisfied at $N^\ast$. By observing that the partnership has taken on net-debt levels of $F^\ast(\mu)$, clients know the partnership’s hiring incentives in period 3. Therefore, they do not need to observe the quality of employees directly. They can infer it accurately regardless of whether or not they are in the fraction, $\mu$, of potential clients who directly observe the size and, thus, the average quality of the partnership’s members.

It is clear that the level of net-debt in equation (24) is rising in the accuracy of market monitoring:
Ward (1958) observed that worker cooperatives would hire like corporations if they had financial commitments that drove them to make zero profits. That paper only considered the perfect information case where \( \mu = 1 \). When \( \mu < 1 \), the optimal level of debt will leave the partnership earning some profits net of financial payments or credits in period 4. Indeed, \( F^*(\mu) \) could be a cash inflow \( (F^* < 0) \) if market monitoring is sufficiently close to zero. The partnership chooses some \( F^* \) in period 2, which is payable in period 3, where \(-K \leq F^*(\mu) \leq \pi(N^*; 1) - K\).

### Proposition 2

When capital structure is transparent, \( \rho = 1 \), and there are no extra costs associated with raising debt or equity, \( \theta_i = 0 \), then the partnership will choose net-debt payments of \( F^*(\mu) = \mu \pi(N^*; 1) - K \), and will earn the full-information monopoly profits.

Suppose that the partnership would be too small even in the presence of imperfect market monitoring. Founding partners could borrow \( F^* \), where \( \pi(N^*; 1) \geq F^* > 0 \), in period 1 and immediately distribute the proceeds among themselves. Then, in period 2, partners would decide on the equilibrium size of the firm. Therefore, in total, partners receive their reservation wages plus first-best profits, \( \pi(N^*; 1) \).
Alternatively, suppose that the partnership would be too large in absence of capital structure changes. That is, the optimal hiring rule requires that \(-K \leq F^*(\mu) < 0\). In period 1, \(F^*(\mu)\) is needed to pay down some fixed costs, \(K\). When \(F^*\) is negative, it must be financed by an equity claim to provide the right incentives to the partnership. Recall that non-voting equity claims do not affect the firm’s production decisions. Therefore, a fraction, \(\alpha^*(\mu)\), of the total profits less the reduction in fixed costs will be promised to non-voting equity holders who receive a total share \(-F^*(\mu) = \alpha^*(\mu) (\pi(N^*;1) - K - F^*(\mu)) > 0\), where \(F^*(\mu) < 0\). Plugging equation (24) into this relationship, we get the following formula for the optimal level of non-voting equity.

\[
\alpha^*(\mu) = \begin{cases} 
0, & \text{when } \mu \pi(N^*;1) - K \geq 0, \\
\frac{K - \mu \pi(N^*;1)}{(1 - \mu) \pi(N^*;1)}, & \text{otherwise.} 
\end{cases}
\]  

(26)

Our discussion begs the question, “Why are partnerships so rare outside of professional services?” Whenever there is a \(\mu < 1\), we would expect all firms to adopt the objective of maximizing profits per employee. When partnerships can achieve the full monopoly profits through their choice of \(F^*\), the imperfect market monitoring explanation of Levin and Tadelis (2005) only explains why firms adopt the partnership form, but it does not explain why most firms are organized as corporations. We will see that that paper’s predictions will fare better when make financial adjustments costly in section 5.1.
4.2 Opaque ($\rho = 0$) and Costless ($\beta = 0$) Financial Adjustments and the Irrelevance of Organizational Form ($\gamma = 0$ or $1$)

The conclusions of subsection 4.1 are blunted considerably when we relax the extreme assumption of transparent capital structure. When capital structure is secret, those results no longer hold. Partnerships are not obligated to report their finances to the public. Indeed, they often do not. The finances of professional partnerships are not often circulated to potential clients or anyone else for that matter.

There are a few notable exceptions of disclosures of partnerships’ finances. Anecdotal descriptions of Goldman Sachs’ finances prior to its going public in 1999 can be found in Endlich (2002). In addition, partnerships preparing for initial public offerings would have to disclose their finances. Further, Ernst & Young’s 1999 and 2000 financial results were disclosed as part of a divorce proceeding. These financial results were reported on in the New York Times by Johnston and Glater (2003). With the exception of Ernst & Young’s disclosures, the other cases could not possibly be used by clients to predict hiring intentions. Endlich (2002)’s descriptions of partnership finances were delayed by many years and incomplete at best. Further, net-debt levels will not affect the hiring behaviour of a professional service firm after it becomes a public corporation.

The transparent partnership ($\rho = 1$) of the previous subsection is effective in signalling its hiring intentions by its capital structure choices in period 2. If its capital structure is effectively hidden from clients ($\rho = 0$), as we will explore in the present sub-section, then it cannot credibly signal its hiring intentions to clients. In this case,
the partnership falls into the same trap that the corporation faces. Nevertheless, the
mechanism by which the partnership achieves over-hiring, relative to perfect market
monitoring, $\mu = 1$, involves its capital structure decision. In particular, the period 2,
opaque partnership will choose some level of debt, $F^C(\mu)$, that causes the period 3,
opaque partnership to hire the corporation’s imperfect market monitoring level of
employment, $N^C$.

The equilibrium choices of the firm, when capital structure is opaque to
clients, $\rho = 1$, and there are no financial frictions, will be denoted by the superscript
“$C$.” When financing entails transaction cost, we will add a subscript, as in section
5.2; but the lack of a subscript will denote that there are no transaction costs to raising
net-debt. As we will show, the superscript “$C$” happens to match up well with how a
partnership hires when capital structure is secret and there are no financial leakages.
We will see that the partnership hires exactly like the corporation in section 3.1.

As we found earlier in equation (7), capital structure does not matter for the
corporation. Therefore, any assumption about the observation of capital structure by
clients is only relevant for the discussion of the hiring decision of the partnership.
Capital structure choices in period 2 cannot be changed in period 3. Therefore, the
partnership’s hiring incentives are signalled by the period 2 capital structure. Yet,
when clients cannot observe capital structure, then the hiring incentives of the
partnership are hidden.

When clients cannot observe capital structure and the associated hiring
incentives for the partnership, then they must assume that capital structure will be
chosen in such a way to maximize total profits for the firm. In this case, maximizing
total profits in period 2 involves endowing the partnership with hiring incentives that take advantage of the informational asymmetries between clients and the firm. The partnership, just like the corporation, will want to exploit these informational asymmetries in the absence of a commitment to do otherwise. In effect, choosing the partnership form, \( \gamma^C = 1 \), with zero financing costs, \( \theta_i = 0 \), and opaque finances, \( \rho = 0 \), is a completely uninformative signal about the firm’s period 3 hiring. It does not signal any changes in equilibrium hiring by the firm.

The partnership will attempt to hire \( N^C(\mu) \) partners. To do this it will choose a level of net-debt that causes the first order conditions in both (16) and (11) to be satisfied at \( N^C(\mu) \). Let us rearrange equation (11) and multiply all terms by \( N^C \).

\[
-\mu p_N(N^C)(N^C)^2 = (p(N^C) - w)N^C = \pi(N^C; \mu)
\]  

(27)

Further, for the right level of net-debt, which we will denote \( F^C(\mu) \), the partnership’s first order condition in (16) will be satisfied when \( N^C \) employees are hired. The first order condition in equation (16) when \( \theta_i = 0 \) can be rearranged to yield the following:

\[
F^C(\mu) = -K - (N^C)^2 \mu p_N(N^C)
\]  

(28)

If we substitute in the right hand side of (27) for \( -\mu p_N(N^C)(N^C)^2 \) in equation (28), then the level of debt chosen by the partnership with opaque capital structure is the following:
With rational expectations that are formed without reference to the firm’s hiring decision, net-debt and profits before fixed costs are rising in the fraction of informed clients, $\mu$.

\[
\frac{\partial F^C(\mu)}{\partial \mu} = \frac{\partial \pi(N^C; \mu)}{\partial \mu} > 0.
\]  

This follows from equation (13) discussed earlier.

Further, since $F^C(\mu)$ always equals the equilibrium profits generated by the firm the net-debt-to-value ratio must be 100 percent for the partnership to achieve the full corporate profits. It is also safe to conclude that no firm with either a partnership or corporate form will come into being unless profits are non-negative. Therefore, the net-debt in $F^C(\mu)$ must also always be positive.

**Proposition 3**

Suppose that the partnership’s capital structure cannot be observed by clients, $\rho = 0$, and there are no costs to adjusting financial structure, $\theta_i = 0$. In this case, the partnership will choose a level of debt equal to its profits, and it will make the same hiring decision as a corporation.

This follows from equation (29) and our discussion above.

Our equilibrium levels of net-debt in equation (24), where the partnership earns the full information first-best profits and equation (29) where it over-hires
provides support for the idea that partners should not pay for the full value of their shares. For example, after its founders’, James O. McKinsey’s, death in 1937, Marvin Bower is often credited with reviving and building the management consulting firm McKinsey into the franchise it is today. In 1963 Bower had a large stake in the enterprise:

“When he turned 60, he sold back his shares at book value. Selling them back at their real value would have forced the firm into debt… He ordered that no partner could accumulate more than a small percentage of the company’s shares and older partners had to start selling their shares to younger partners long before they retired… The precedent of selling shares back set by Bower continues today at McKinsey & Company.” Schleler (2000)

The analysis here says that taking on debt would have made the partnership less selective and quality could have suffered as a result. We have argued that if debt drives the firm to make zero economic profits in period 4, then the firm will hire exactly like a corporation.

Further, if McKinsey’s shares were bought back by the firm at close to the fair market value, it is likely that the partnership would begin to behave more like a corporation because economic profits in the firm would be driven close to zero. Therefore, the over-hiring problems of the corporation would be present. Partnership shares that trade for close to the value of the discounted cash flows that they generate would encourage partners to operate the firm like a corporation. (This is what we have been trying to avoid!) Dow (2003, p. 154) argues that fully tradable membership shares will induce labour managed firms—partnerships in our context—to make hiring decisions more like corporations. Because membership shares are
sold at a discount at McKinsey, the firm may be enforcing a commitment to its customers of only retaining high quality partners.

It seems reasonable that financial structure in partnerships is somewhat hidden from clients and debt ratios are much less than 100 percent. For this reason, the incentives to adopt such a capital structure must be blunted by the fact that capital markets are imperfect. There is a cost to raising outside finance. The impact of financial frictions on the perfect Bayesian equilibrium set of $\{\gamma, N, F\}$ are discussed in the next section.
5.0 Costly Financial Adjustments ($\theta \neq 0$) and the Selectivity of the Partnership

So far we have seen that the partnership can earn the full monopoly profits when capital structure is transparent, $\rho = 1$. Yet, the partnership becomes no more profitable than the corporation when clients cannot view capital structure, $\rho = 0$. Both these observations are altered when we introduce costs to raising finance.

Levin and Tadelis (2005) implicitly assumed that the partnership faced very large to infinite financial adjustment costs. That is, the net costs of adjusting the partnership’s capital structure exceeded any possible benefits. Such an assumption, though not explicitly stated, would allow Levin and Tadelis (2005) to analyse the partnership’s hiring problem in the absence of financial adjustments.

In this section, we relax both the extreme assumptions of costless finance, $\theta = 0$, of section 4.0 and the implicit assumption of prohibitively costly finance of Levin and Tadelis (2005). In section 5.1, we consider how costly adjustments affect a partnership with a transparent capital structure. In general, financial adjustment costs make the transparent partnership less profitable than the corporate form for high levels of market monitoring. This is because higher costs lower the transparent partnership’s value through a pure “cost of finance effect.” In section 5.2, we consider the effect of financial adjustment costs on an opaque partnership ($\rho = 0$). The opaque partnership benefits from greater selectivity when the costs of debt rise. This is referred to here as the “selectivity effect.” At the same time, it suffers from a
“cost of finance effect” when $\theta_d$ goes up. In proposition 7, we find that when target debt levels are approaching zero, an increase in the cost of debt unambiguously raises the value of the opaque partnership. When debt costs are almost zero, the “selectivity effect” dominates.

In general, the results of sub-section 5.1 and proposition 5 support the dichotomy developed in Levin and Tadelis (2005). When financial frictions are present ($\theta_i \neq 0$) and capital structure is transparent ($\rho = 1$), proposition 5 finds that low market monitoring organizations will tend to adopt the partnership form. In addition, high market monitoring organizations will be more profitable if they adopt the corporate form. This is in sharp contrast to the previous section. Without financial adjustment costs, as we discussed in section 4.0, we found that the level of market monitoring was irrelevant to the decision whether to organize as a partnership or corporation. When net-debt levels were transparent as in section 4.1, then the partnership would be strictly more profitable than a corporation for all $\mu \in [0, 1)$, and it would be just as profitable as a corporation when $\mu = 1$. If $\rho = 0$ and $\theta_i = 0$ as in section 4.2, then it does not matter what percent of clients are informed. In that case, the partnership hires exactly like a corporation for all levels of $\mu$. In contrast to the case where there are no financing costs in section 4.0, the main results of Levin and Tadelis (2005) are largely supported in section 5.1 when there are financial frictions and firms have a transparent capital structure.

In section 4.0, we have assumed that the partnership can raise debt or equity at no costs. Here in section 5.0 we will relax that assumption. In particular, in equation (1) we assume that the net cost, $c(F)$, of outside finance is a fraction of the debt or
equity raised. It would be inconvenient to analyse the hiring or quality decision of the partnership in terms of the amount of debt, $F$ when $F > 0$, or equity, $F$ when $F < 0$, raised. We have analysed this problem in previous sections by the number, $N$, of partners hired. The partnership actually chooses a target level of hiring though its choice of period 2 capital structure. Since taking on non-zero levels of net-debt is costly, there are costs to taking on the new net-debt, $c^{-1}(N)$. The financing costs of the particular level of hiring in the partnership can be derived by combining the first order condition in equation (16) and the costs of debt and equity defined in equation (1). This gives us the inverse cost of finance function below

\[
c^{-1}(N) = \theta_i F = -\frac{\theta_i}{1 + \theta_i} (\mu N^2 p\lambda (N) + K) > 0
\]

when $\theta_i \neq 0$ & $F \neq 0$.  

This function is only well defined for positive financing costs.

5.1 Costly Financial Adjustments ($\theta_i \neq 0$) and Transparent Financial Structure ($\rho = 1$) in the Partnership

First, let us explain the notation for the partnership when both $\rho = 1$ and $\theta_i \neq 0$. Let us denote the best response choices of the transparent partnership by a superscript and a subscript. The superscript is the same for the transparent partnership in section 4.1. This superscript is the asterix “*.” It denotes that the
transparent partnership achieves the first best or nearly the first best profits. To distinguish the best responses of the partnership with transparent net-debt level and costly finance, $\theta_i \neq 0$, of the present section and the best responses of the transparent partnership with costless finance, $\theta_i = 0$, of section 4.1 we have added a subscript in the present section. The subscript “$i$” is used to denote that finance is costly for the period 2 partnership. The subscript “$i$” can be represented in its generic form in which it does not specify whether or not net-debt is positive or negative. The subscript will equal “$d$” when the best response refers to the case where net-debt is positive and debt must be raised and distributed in period 2 to meet the firm’s hiring targets. In contrast, when the subscript “$e$” appears non-voting equity must be raised to increase the cash inside the firm until period 4. (Recall the prescription for altering the hiring of the partnership in proposition 1.) These subscripts are used to denote that debt and equity are costly to raise when $\theta_i \neq 0$.

The transparent partnership always has the option of taking on zero net-debt, $F_i^* = 0$. In this case, the period 2 partnership is choosing to not affect the period 3 partnerships’ hiring incentives. When this is the case, the equilibrium hiring will be given by equation (16) where $F_i^* = F = 0$. The presence of financial transaction costs, $\theta_i \neq 0$, makes the zero net-debt option non-trivial.

Let us assume that the period 2 partnership takes on a non-zero level of net-debt. That is, $F_i^* \neq 0$. Further let us suppose that this level of net-debt $F_i^* = c^{-1}(N_i^*; \mu, \theta_i)$ satisfies the condition set out in equation (1) that $F_d^* > 0$ and $F_e^* < 0$. That is, net-debt is positive when the firm raises debt and has a cost of debt
\( \theta_d > 0 \) and net-debt is negative when the firm raises equity and has a cost of negative net-debt \( \theta_e < 0 \). When these conditions are met, the cost of finance for any choice of the transparent partnership will be strictly positive, \( c^{-1}(N^*_i; \mu, \theta_i) > 0 \). The optimal hiring level for the partnership with financial adjustment costs is the level of hiring that satisfies the following maximization problem:

\[
V^*_i = \arg \max_N \left\{ \pi(N;1) - K - c^{-1}(N; \mu, \theta_i) \right\}
\]

\[
= \arg \max_N \left\{ p(N)N - wN - K + \frac{\theta_i}{1+\theta_i}(\mu N^2 p_N(N) + K) \right\},
\]

where \( c^{-1}(N^*_i; \mu, \theta_i) > 0 \).

The first order condition for this problem is the following:

\[
\left. \frac{\partial V^*_i}{\partial N} \right|_{N=N^*_i} = p_N(N^*_i)N^*_i + p(N^*_i) - w - \frac{dc^{-1}(N^*_i)}{dN} = 0,
\]

where \( c^{-1}(N^*_i; \mu, \theta_i) > 0 \).

Given that the stationary point is a maximum, the partnership can choose the level of net-debt \( F^*_i(N^*_i; \mu, \theta_i) \) consistent with the hiring level \( N^*_i(\theta_i) = N^*_i \). This is the

\[
F^*_i = \frac{1}{1+\theta_i} \left[ \mu \left( \pi(N^*_i;1) - \frac{dc^{-1}(N^*_i)}{dN} N^*_i \right) - K \right]
\]
The derivation of (34) involves several steps, but is not complex. This derivation can be found in appendix section 7.1.6.

There is an interval where the partnership finds zero net-debt optimal. It is where the net-debt in (34) implied by the cost of equity parameter, $\theta_e \leq 0$, is positive and the net-debt implied by the cost of debt, $\theta_d \geq 0$, is negative. Both signs are contradictions indicating that the first order condition (33) no longer holds because $c^{-1}(N) < 0$. Let us implicitly define the points were net-debt is equal to zero for the debt and equity cases, $i = d$ or $e$, as the following:

$$
\phi_i \equiv \frac{K}{\pi(N^*_i;1)} - \frac{\partial c^{-1}(N^*_i)}{\partial N} N^*_i
$$

A special case of equation (35) occurs when $\theta_i = 0$, and thus $\frac{\partial c^{-1}(N^*_i)}{\partial N} = 0$ and $N^* = N^*_i$. We know from section 4.1 that there is a $\mu$ where no net-debt is needed in the partnership. When the partnership raises no net-debt, financial costs are zero. From equation (24) the $\mu$ implied by zero net-debt is

$$
\mu^o \equiv \frac{K}{\pi(N^*;1)}
$$
This $\mu^p$ conforms to the notation of Levin and Tadelis (2005, p. 165). In that paper the partnership cannot adjust its financial structure; therefore, the partnership is most profitable at this level of market monitoring. The same is true here when we allow the partnership to adjust its financial structure when $\rho = 1$ and $\theta_i \neq 0$. This is in sharp contrast to section 4.1 where the $\rho = 1$ but financial costs were zero. In that case, all values of $\mu \in [0, 1]$ allowed the partnership to achieve first-best profits, $\pi(N^*;1) - K$.

In this section, there is only one such level of market monitoring, $\mu^p$. As the actual $\mu$ moves further from $\mu^p$ the financial costs of hitting the optimal hiring targets implied by (34) will rise, and profits will fall from the first-best, $\pi(N^*;1) - K$.

For a low enough level of market monitoring, a low $\mu$, the level of net-debt could be negative. (By inspecting equation (34), for example, if $\mu = 0$ the level of net-debt must be negative.) If (34) is negative, then the partnership will have to sell an equity stake that generates proceeds of $-F^*_e$. That is,

$$-F^*_e = \alpha^*_e[\pi(N^*;1) - K - F^*_e(1 + \theta_e)], \text{ when } F^*_e < 0. \tag{37}$$

If we rearrange (37) to solve for $\alpha^*_e$ and substitute in $F^*_e$, then we can represent the fraction of total profits awarded to outside equity as the following:

---

5 Our definition of lower case pi, “$\pi$,” differs from Levin and Tadelis (2005)’s definition of uppercase pi, “$\Pi$.” In the present paper, $\pi$ is profits before fixed costs. In Levin and Tadelis (2005), $\Pi$ stands for profits after fixed costs. Therefore, the slight difference in the denominator of $\mu^p$ in Levin and Tadelis (2005, p. 165) reflects this slightly different definition of uppercase pi versus lowercase pi in the present paper. The definitions of $\mu^p$ are identical in both papers.
\[
\alpha^*_e = \begin{cases} 
0, & \text{if } F^*_d \geq 0 \\
\frac{1}{1 + \theta_e} \left( K - \mu \pi(N_e^*;1) + \mu \frac{\partial c^{-1}(N_e^*)}{\partial N} N_e^* \right), & \text{if } F^*_e < 0.
\end{cases}
\]

(38)

It is conceivable that the desired level of negative net-debt will break the budget constraint and require the firm to sell greater than a 100 percent stake. See the appendix discussion for a derivation of the sign. Intuitively, \( \alpha_e^* \) can exceed 1 because the term in square brackets “[]” will weakly exceed 1. This is because \(-1 < \theta_e \leq 0\). By inspection, a corner solution is more likely when the costs of raising equity are very high or \(-1 < \theta_e <= 0\). In such a case, the first best profits after fixed and financing costs for the partnership are never positive. Therefore, if there is a solution where a professional service firm forms when \( \alpha_e^* > 1\), it must be dominated by one in which the firm’s founders either choose no financing \( F_i^* = 0 \) and the partnership form \( \gamma_i^* = 1 \), or the corporate form, \( \gamma_i^* = 0 \), in period 1.

Before we continue, let us consider figure 2 on the following page. This illustrates that the first order condition in (33) is not well defined for many parameter values. There is a range of market monitoring, \( \mu \in [0.1942, 0.2016] \), where the partnership’s hiring is nearly optimal with zero net-debt. For these values, no financial adjustments are made. On the horizontal axis we vary the fraction of clients, \( \mu \), who observe firm quality directly. On the vertical axis is profit after fixed and financial costs. In this example, first-best monopoly profit is 40. The values are derived in the worked out example in appendix section 7.2. The profit function for
the transparent partnership is given in equation (137) while the profit function for the partnership that does not adjust its capital structure is found in equation (115). The relevant parameters of the model that describe the inverse demand function (the distribution of employee ability), wages, and fixed costs are given in equation (116). The optimal envelope of profits as $\mu$ varies is denoted by the thick black line. For every dollar of equity raised 7 cents are lost due to transaction costs, $\theta_e = -0.07$.

Likewise, when debt is raised, 2 percent of the debt’s value is lost, $\theta_d = 0.02$. The optimal profit function is piecewise defined. To the left it is characterized by the $N^*_e$ implied by (33), $\pi(N^*_e; \mu, -0.07) - K - c^{-1}(N^*_e; \mu, -0.07)$. In this region net-debt is negative. That is, $F^*_{e}(\mu) < 0$. In the centre region, the first order condition in (33) violates the condition that financing costs are strictly positive.

From $\mu \in [0.1942, 0.2016]$ the partnership chooses zero net-debt and hiring is governed by the first order condition in equation (16) with $F^*_{i}(\mu) = 0$. This profit function is part of the envelope near its peak, $\mu^* = 0.2$. Yet, is dominated outside the domain $\mu \in [0.1942, 0.2016]$. The profit function with zero net-debt lies below the highest feasible profits for the transparent partnership at the extremes when $\mu \not\in [0.1942, 0.2016]$.

The level of market monitoring denoted $\mu^*_e = 0.1942$ is the level of market monitoring where the $c^{-1}(N^*_e; \mu^*_e, -0.07) = 0$. $\mu^*_e$ is defined in equation (35).

Indeed, the thin lines extending above the thick black envelope are not well defined and are infeasible points.
Consider higher levels of market monitoring $\mu > \mu^{\ast}_{0} = 0.2016$. $\mu^{\ast}_{0}$ is defined in equation (35). These are the points that lie to the left of $\mu^{\ast}_{0}$ on the maximum profit envelope, the dark black line. Here the maximum feasible profits are once again determined by the first order condition in (33). Yet, for this part of the envelope, positive net-debt is taken on, $F_{d}^{\ast}(\mu) > 0$. The partnership does reach the first-best profit of 40 in the middle region where $\mu^{\ast} = 0.2$, which is defined in (36). This occurs when zero net-debt is taken on; and, therefore, no financing costs are incurred.
\( \rho = 1, \ \theta_d = 0.02, \ \theta_e = -0.07 \)

\[
\begin{align*}
\pi(N_e^*; \mu) - K &= -c^{-1}(N_e^*; \mu, -0.07) \\
\pi(N_d^*; \mu) - K &= \pi(N_d^*; \mu, 0.02) - c^{-1}(N_d^*; \mu, 0.02) \\
\pi(N_p^*; \mu) - K &= \end{align*}
\]

Figure 2
Let us turn to how financing cost affect the transparent partnership. In the
appendix section 7.1.8 we use the envelope theorem to derive the following partial
derivative of the equilibrium net-debt:

\[
\frac{\partial V_i^*(N_i^*)}{\partial \theta_i} = \begin{cases} 
\frac{\partial V_e^*(N_e^*)}{\partial \theta_i} & = -\frac{1}{1+\theta_e} F_e^*(N_e^*; \mu, \theta_e) \geq 0, \\
\frac{\partial V_d^*(N_d^*)}{\partial \theta_d} & = -\frac{1}{1+\theta_d} F_d^*(N_d^*; \mu, \theta_d) \leq 0.
\end{cases}
\] (39)

The sign of the comparative statics depends on whether or not net-debt, \(F_i^*\), is
positive or negative. \(F_i^* = F_e^* < 0\), when the firm raises cash with equity, and
\(F_i^* = F_d^* > 0\), when the firm adds to its obligations with debt. Further, the costs of
equity are rising as \(\theta_e\) falls or becomes more negative. The top partial derivative
shows that as \(\theta_e < 0\) approaches zero the transparent partnership is becoming more
valuable. This supports the proposition. Further, the bottom partial shows that as the
cost of debt rises, that is as \(\theta_d > 0\) gets further from zero, the value of the firm
declines. This is what we wanted to show for debt costs. Therefore, firm value is at
least weakly declining in the absolute value of \(\theta_i\).

**Proposition 4**

The profitability of the transparent partnership, \(\rho = 1\), is weakly, monotonically

\(\text{declining in the cost of outside finance, } |\theta_i|\).
The proposition follows from our discussion of equation (39) above and the derivation of that equation in the appendix section 7.1.8.

This proposition is very intuitive. Because the transparent partnership without financial costs discussed in section 4.1 achieves the first best level of profits, any additional costs will make the organization less profitable. Further, the transparent partnership must adjust its capital structure to achieve its hiring targets, but a corporation does not. Therefore, the corporate form will be more attractive as financial costs rise. That is, as \( \theta \) gets further from zero, the corporation will become relatively more attractive than the partnership.

It is also interesting to note that the sign of the comparative static in equation (39) depends on the magnitude of net-debt costs. A rise in the costs of net-debt, an increase in \( |\theta| \), means that the transparent partnership’s total financing bill is going up. We will contrast the comparative static in (39) with the one in equation (51) in the next subsection. The transparent partnership only suffers from this negative “cost of finance effect.” Yet, we will see that the opaque partnership, because it over-hires when financing costs are zero, potentially benefits from a “selectivity effect” that may mean that it becomes more selective and profitable as the costs of debt, \( \theta_d \), rise.

Levin and Tadelis (2005) argues that partnerships become relatively more attractive as market monitoring, \( \mu \)—the fraction of clients that observe the true quality of the firm—falls. Before we introduced financing costs, there was no such dichotomy. For all but the knife’s edge case where \( \mu = 1 \), the transparent partnership would be more profitable and thus preferred to corporation. We denote the
equilibrium choices by the superscript “*” for the transparent, \( \rho = 1 \), partnership with no financing costs. That is, \( \gamma^* = 1 \) when \( \mu < 1 \) and \( \rho = 1 \). Yet, when we add financing costs, Levin and Tadelis (2005)’s predictions perform better. As financing costs rise and \( \theta_i \) becomes more distant from zero, proposition 4 tells us that total profits are falling in the transparent partnership. The corporation does not need to adjust its financial structure to achieve its hiring targets and thus does not have to incur the financial expenses of the partnership.

In contrast, total profits are rising as market monitoring rises for the corporation. We know this from equation (30) in section 4.2. Therefore, conceivably there can be some combination \( \{\mu, \theta_i\} \) where both organizations are equally profitable.

While we know that the corporation benefits from a rise in market monitoring, does the partnership also? The answer is that it depends on whether the partnership needs to raise debt or equity. Consider the effect of a rise in market monitoring on equilibrium profits for the transparent partnership. The envelope theorem allows us to differentiate the objective function in equation (32) directly.

\[
\frac{\partial V^*_i}{\partial \mu} = \frac{\theta_i}{1 + \theta_i} (N^*_i)^2 p_s(N^*_i) \tag{40}
\]

The sign in equation (40) depends on whether or not the transparent partnership takes on debt or equity.
The value of the transparent partnership is falling in market monitoring when the firm must raise positive net-debt and it is rising when the firm needs to raise negative net-debt with equity.

**Proposition 5**

Suppose that \( \pi(N) \) is smooth and concave in \( N \). If any organization form is profitable for a given \( \mu \), the transparent partnership, \( \rho = 1 \), with non-zero financing costs, \( \theta_i \neq 0 \), will be, at least, weakly more profitable than a corporation for \( \mu \in [0, \hat{\mu}_i^*) \).

For \( \mu \in [\hat{\mu}_i^*, 1] \), a corporation will be weakly more profitable than a partnership.

The proof is in the appendix 7.1.9. \( \hat{\mu}_i^* \) will be defined more fully in equation (42) on the next page.

The intuition for this proposition is that the partnership is always more selective than the corporation for positive levels of profit. This result was first discovered by Ward (1958). With imperfect market monitoring, \( \mu < 1 \), this extra selectivity is a virtue for low levels of \( \mu \) according to Levin and Tadelis (2005). In contrast, Ward (1958) only viewed this extra selectivity of workers cooperatives as a vice. In the present section, the transparent partnership’s extra selectivity can only be overcome by issuing positive levels of net-debt for high levels of market monitoring.
Yet, these positive levels of net-debt are costly and cause total profits after fixed and
financial costs to decline as the partnership moves farther away from $\mu_P$ in (36). We
know this from the sign of $\frac{\partial V^*}{\partial \mu} < 0$, which is given in equation (41).

Therefore, with financing costs and transparent net-debt, the relationship
between market monitoring and the optimal organizational form generally confirms
the “central comparative static result” of Levin and Tadelis (2005, p.142). That is,
when $\rho = 1$ and $\theta_i \neq 0$, here we find that high levels of market monitoring tend to
favour the corporate form and low levels of market monitoring favour the partnership.

There are several interesting values of $\mu$ that we will define below and
discuss in figure 3. The cut off levels of $\mu$, $\mu^*$, and $\mu^*$, for this subsection and Levin
and Tadelis (2005), respectively, are defined mathematically below:

$$
\pi(N^*_j; \mu^*_j, \theta_j) - K - c^{-1}(N^*_j; \mu^*_j, \theta_j) \equiv \pi(N^C; \mu^*) - K
$$

$$
\pi(N^P, F = 0; \mu) - K = \pi(N^C; \mu) - K
$$

The hiring levels $N^*_j$, $N^C$, and $N^P$ when $F = 0$ are defined by the first order conditions
in equations (33), (11), and (16), respectively.

Let us define one more level of market monitoring which features
prominently in Levin and Tadelis (2005). This is the level of market monitoring, $\mu$, 
where the partnership in section 3.2 with zero net-debt breaks even.

$$
\pi(N^P, 0; \mu) - K = 0
$$
While this general result is largely confirmed when $\rho = 1$ and $\theta_1 \neq 0$, not all of the predictions of Levin and Tadelis (2005) are fully supported when net-debt is costly and is observed by clients. For example, with very low financing costs there need not be a lower level of market monitoring $\mu > 0$ that puts all organizations out of business. If historic fixed costs, $K$, are very low relative to first-best profits before fixed and financial costs, $\pi(N^*)$, the transparent partnership will have no trouble raising net-debt nearly equal to $-K$. When capital structure adjustments are transparent and costly, the selectivity and the profitability of the partnership may not suffer enough to forgo entry into the business even for very low levels of market monitoring. In contrast, Levin and Tadelis (2005)’s proposition 2 does predict that there will be a $\mu = \hat{\mu}$ in which both the partnership and the corporation make zero profits.

There is another difference between proposition 5 above and proposition 3 in Levin and Tadelis (2005). The minimum levels of market monitoring where the corporation is at least as profitable as the partnership differ in this section and in Levin and Tadelis (2005). $\hat{\mu}$, as defined in Levin and Tadelis (2005) and here in equation (42), applies only to a partnership that cannot adjust its net-debt levels. In this paper, the cut off level of market monitoring where the corporation is more profitable than the partnership, $\hat{\mu}^*$, will be weakly higher for a transparent partnership with financing costs than the cut off level in Levin and Tadelis (2005), $\hat{\mu}$. That is, $1 > \hat{\mu}^* \geq \hat{\mu}$. This follows from proposition 5 above.
In figure 3, we continue the example in figure 2, but we expand out the graph for all feasible levels of market monitoring—$1 \geq \mu \geq 0$. Further, we compare the profitability of the transparent partnership with financing costs to the transparent partnership to the profits of a partnership that cannot adjust its net-debt levels. The curve that peaks at 0.2 and crosses the horizontal axis at $\mu \approx 0.0557$, which is defined in (43) above, is the profit function for the partnership that cannot adjust its net-debt levels. This curve is the profit function of the partnership in Levin and Tadelis (2005). The other curve that touches the horizontal axis at $\mu \approx 0.0557$ is the corporation’s profit function. Its profits are strictly rising in $\mu$. The corporation's profit peaks as $\mu$ approaches 1. The transparent partnership’s total profits are the curve that forms the top of the graph until it intersects the corporation’s profit function at $\mu = \hat{\mu}_d \approx 0.8054$.

In this example, there is no minimum $\mu$ where the transparent partnership goes out of business. Even when $\mu$ approaches 0, the period 2 partnership can pay down all the fixed obligations to prevent the period 3 partnership from hiring too many partners. Further, the transparent partnership will be profitable—more profitable than the corporation—for weakly larger ranges of $\mu$ than Levin and Tadelis (2005) predicts. The partnership of Levin and Tadelis (2005) would be the dominant organizational form for levels of market monitoring from $\mu \in (\underline{\mu}, \hat{\mu})$, where $\underline{\mu} \approx 0.0557$ and $\hat{\mu} \approx 0.4780$, and that paper would predict that corporations would dominate for $\mu \in (\hat{\mu}, 1]$. Yet, the present paper predicts that, in this example, the transparent partnership with financing costs would be the most profitable
organizational form for market monitoring parameters $\mu \in [0, \hat{\mu}_d^*)$, where $\hat{\mu}_d^* \approx 0.8054$. For $\mu \in (\hat{\mu}_d^*, 1]$, the corporate form will be more profitable and preferred. That is, the perfect Bayesian equilibrium (PBE) organizational strategy would be $\gamma_i^* = 1$ when $\mu \in [0, \hat{\mu}_d^*)$, $\gamma_i^* = 0$ or 1 at $\hat{\mu}_d^* \approx 0.8054$, and $\gamma_i^* = 0$ when $\mu \in (\hat{\mu}_d^*, 1]$.
$\rho = 1, \theta_d = 0.02, \theta_e = -0.07$

$$\pi(N_i^*; \mu, \theta) - K - c^{-1}(N_i^*; \mu, \theta)$$

Figure 3
5.2 Costly Adjustments ($\theta_i \neq 0$) and Opaque Financial Structure ($\rho = 0$) in the Partnership

In this section, we will denote endogenous choices by a combination of superscripts and subscripts. The superscript “$C$” denotes that net-debt is not visible to clients. This is similar to section 4.2. In addition, the subscript is introduced to denote the presence of financial adjustment costs, $\theta_i \neq 0$. As in section 5.1, the subscript can be the generic “$i$” or take on the more specific value “$d$” or “$e$.” The latter two subscripts denote positive or negative net-debt, respectively. (The firm must raise debt to increase its net-debt or sell outside equity to raise negative net-debt.)

A partnership that has an opaque capital structure and costless financial adjustments will be tempted to set net-debt levels $F^C(\mu)$ in equation (29) so that it hires like a corporation. This was discussed in section 4.2. Nevertheless, when it is costly to obtain outside finance, then it is costly to hire like a corporation. This is because this will involve taking on a non-zero level of net-debt, $F^C_i$. This financing involves leakages of value outside the firm. In contrast, a corporation can adopt the profit maximizing hiring level for a given amount of market monitoring without adjusting its capital structure. (Capital structure is irrelevant for the corporation, but not the partnership.)

As in section 5.1, we will solve this problem by substituting in $c'(N)$ in equation (31) for $\theta_i F$. This is the major difference with “corporation’s” problem in equation (6). (The “corporation’s” problem actually was both the problem facing the
corporation and the period 2, opaque, \( \rho = 0 \), partnership with zero financing costs, \( \theta_i = 0 \). Here the period 2 maximization problem for the partnership takes into account the non-zero financing costs:

\[
\arg\max_N V_i^C = \pi(N; N^C, \mu, \theta) - K - c^{-1}(N; \mu, \theta) \\
= \{ \mu p(N) + (1 - \mu) p(N^c(r)) \} N - wN - K + \frac{\theta_i}{1 + \theta_i} (\mu N^2 p_N(N) + K) 
\]

(44)

The first order condition is below.

\[
\frac{\partial V_i^C}{\partial N} \bigg|_{N=N_C} = \mu N_i^C p_N(N_i^C) + \{ \mu p(N_i^C) + (1 - \mu) p(N^C(r)) \} - w \\
+ \frac{\theta_i}{1 + \theta_i} \mu (2 N_i^C p_N(N_i^C) + (N_i^C)^2 p_{NN}(N_i^C)) = 0 
\]

(45)

The second order condition the following

\[
\frac{\partial^2 V_i^C}{\partial N^2} \bigg|_{N=N_C} = \mu \{ N_i^C p_{NN}(N_i^C) + 2 p_N(N_i^C) \} \\
+ \frac{\theta_i}{1 + \theta_i} \mu (2 p_N(N_i^C) + 4 N_i^C p_{NN}(N_i^C) + (N_i^C)^2 p_{NNN}(N_i^C)) . 
\]

(46)

If \( N_i^C \) is to be a non-trivial stationary point, it must be the case that it is a maximum point. (That is, the period 2 partnership would not choose a minimum profit point!) Therefore, in the non-trivial case equation (46) must have a negative sign.
Let us turn to the equilibrium level of net-debt chosen when $\theta \neq 0$ and $\rho = 1$.

This can be expressed as a function of the profits before financing costs. To do this we substitute back the period 3 partnership’s first order condition into the period 3 partnership’s first order condition in (45). This is derivation is left for the appendix section 7.1.10. When we are done we are left with

$$F^C_i = \frac{1}{1 + \theta_i} \left( \pi(N^C_i; \mu) - K - N^C_i \frac{dc^{-1}(N^C_i)}{dN} \right) > 0, \quad (47)$$

where $(p(N^C_i) - w)N^C_i \equiv \pi(N^C_i; \mu)$.

We could use the zero profit condition to examine the sign of (47). While we know that the partnership will only form if equilibrium profits less fixed and financial costs are non-negative, it is harder to determine if the quantity in brackets in equation (47) is non-negative. In short, given that a partnership has formed in the presence of opaque financial structure and costly financial adjustments, then

$$\pi(N^C_i; \mu, \theta_i) - K - N^C_i \frac{dc^{-1}(N^C_i)}{dN} > \pi(N^C_i; \mu, \theta_i) - K - c^{-1}(N^C_i) \geq 0. \quad (48)$$

Yet, it seems reasonable to conclude that there is less scope for equity, negative net-debt, in the opaque partnership with costly financial adjustments than in the transparent partnership. This is because the incentives to expand are much greater for the opaque partnership, relative to the transparent partnership. Equity, or negative
net-debt, shrinks the size of the partnership and is more attractive to partnerships with transparent financial structure.

If we make the additional assumption about the shape of the inverse demand function in equation (49), we can rule out the possibility that the firm will take on negative net-debt. A derivation of this observation is left for the appendix section 7.1.11.

Assumption 5.2

\[ N_i^C p_{NN}(N_i^C) + 2 p_N(N_i^C) \leq 0 \]  

(49)

What is really interesting when we add positive financing costs is the relationship between the costs of finance and the overall profitability of the opaque partnership. When \( \theta = 0 \) the partnership mirrors the corporation in both profitability and selectivity. This one-to-one correspondence is broken when the cost of finance becomes non-zero. To do this we can employ the envelope theorem to differentiate the value function in equation (44) with respect to \( \theta \). Yet, unlike the transparent partnership that can credibly signal its hiring objective, the opaque partnership’s value is affected by shifting expectations \( \frac{\partial N^e(r)}{\partial \theta} \).

\[
\left. \frac{\partial V^C}{\partial \theta} \right|_{N=N_i^C} = (1-\mu) p_N(N_i^C) \frac{\partial N^e(r)}{\partial \theta} N_i^C \\
+ \frac{1}{(1+\theta)^2} \mu \{(\mu(N_i^C)^2 p_N(N_i^C) + K \}
\]  

(50)
Rational expectations will converge to the actual choices of the partnership, according to equation (4). Further, we can substitute in the definition $F(N)$ in equation (31) into (50) where $F(N_t^C) = F_t^C$. This leaves us with

$$
\frac{\partial V^C}{\partial \theta_i} \bigg|_{N = N^c = N^C} = (1 - \mu)p_N(N_t^C) \frac{\partial N_t^C}{\partial \theta_i} N_t^C - \frac{1}{1 + \theta_i} f_t^C < 0
$$

(51)

The sign in (51) is ambiguous. That is, the opaque partnership may find its value rising as the costs of debt rises. The proposition below follows from equation (51) above.

**Proposition 6**

*The partnership with hidden net-debt levels, $\rho = 0$, may or may not become more profitable as the absolute value of the cost of finance parameter, $|\theta|$, rises.*

If we make the additional assumption 5.2 in equation (49) about the shape of the inverse demand curve, we can comment more generally about the sign of (51). The assumption in equation (49) above is very similar to the second order condition in equation (10) when financing costs are zero. Therefore, since the second order condition must be negative at an interior optimum, it is a sufficient condition for this assumption 5.2 to hold when $\theta_t = 0$ and there is no corner solution. The reader can verify this by inspecting the second order condition in equation (46). It is an
assumption since the second order condition in this case, equation (46), does not necessarily have the same sign as equation (49) when $\theta_i \neq 0$.

Finding the sign of an increase in financing costs becomes easier if we accept assumption 5.2 in equation (49). Armed with this assumption, we can derive the sign of the comparative static for $\frac{\partial N_i^C}{\partial \theta_i} \leq 0$. A derivation of this partial is left for the appendix section 7.1.12. Since price is declining in the number of employees, $p_N < 0$ the first term in (51) is weakly positive. That is, $(1 - \mu) p_N(N_i^C) \frac{\partial N_i^C}{\partial \theta_i} N_i^C \geq 0$. The second term is easier to sign, but its sign depends on the sign of the net-debt. Fortunately, since we have proved in appendix 7.1.11 that the opaque partnership with financing costs will never take on negative net-debt when the assumption in equation (49) holds, we do not have to worry about negative net-debt. Because, we have ruled out negative net-debt, the second term, $-\frac{1}{1 + \theta_d} F_d^C < 0$, must be negative.

The sign of (51) is ambiguous when both equation (49) holds and the partnership raises positive net-debt $F_d^C > 0$. When debt is zero or very low but positive, then the opaque partnership is becoming weakly more profitable as the cost of debt parameter $\theta_d$ rises. This is because the “selectivity effect”

$$(1 - \mu) p_N(N_d^C) \frac{\partial N_d^C}{\partial \theta_d} N_d^C \geq 0$$

dominates. Yet, as the level of debt becomes large, $F_d^C > 0$, the increase in the cost of debt financing will outweigh the increase in
revenues from greater selectivity. Therefore, for high levels of debt, the “cost of finance effect,” \[ \frac{1}{1+\theta_d} F^C_d < 0, \] will dominate.

**Proposition 7**

*When the equilibrium level of net-debt, \( F^C_d \), approaches 0+, the opaque partnership will become weakly more profitable as the cost of debt parameter \( \theta_d \) rises.*

This is fairly straightforward from our discussion above. To find out if the profitability of the opaque partnership, taking the limit of equation (51)

\[
\lim_{F^2_d \to 0^+} \left( \frac{\partial P^C_d}{\partial \theta_d} \right)_{N^d = N^C_d} \\
= \lim_{F^2_d \to 0^+} \left( (1-\mu)p_N(N^C_d) \frac{\partial N^C_d}{\partial \theta_d} N^C_d - \frac{1}{1+\theta_d} F^C_d \right) \\
= (1-\mu)p_N(N^C_d) \frac{\partial N^C_d}{\partial \theta_d} N^C_d \geq 0.
\]

We know that (52) is positive because \( p_N < 0 \) and \( \frac{\partial N^C_i}{\partial \theta_i} \leq 0 \) when condition (49) holds. *Q.E.D.*
6.0 Conclusion

This paper has considered the effect of net-debt levels on the selectivity of professional service firms. When some clients are uninformed about the firm’s true quality as in Levin and Tadelis (2005), the choice of organizational form by itself does not cause a professional partnership to signal higher hiring thresholds. Here we have argued that the partnership can get around its profit sharing constitution and the hiring distortions that this profit sharing creates by adjusting its net-debt levels.

This ability to adjust net-debt levels can be a fully informative signal of the firm’s hiring intentions when capital structure is transparent. This ability to signal hiring intentions makes the partnership form more profitable. Yet, if net-debt levels are secret and financial adjustment costs are minimal, clients will have little reason to believe that choosing a partnership versus a corporate organizational form will lead to radically different hiring objectives.

When capital structure is transparent but financial transaction costs are positive, this paper confirms “the central comparative static result” of Levin and Tadelis (2005 p.142). Namely, firms facing mostly informed clients will organize as corporations while firms facing mostly uninformed clients will form profit sharing partnerships. While transparent partnerships suffer from financial transaction costs, the present paper shows that it is ambiguous whether an opaque partnership becomes more profitable as financial leakages increase. Higher financing costs make it more
costly for the opaque partnership to expand. In such a case, increased selectivity in the opaque partnership may totally offset an increase in financing costs.
References


7.0 Appendix

7.1 Derivations and Proofs

7.1.1 The Inverse Relationship Between Size and Quality

Here we want to show that there is an inverse relationship between the size and the average quality of the workforce. To do this, first, we will show that firm size falls as the ability of the minimum ability employee rises. Then, we will demonstrate that average quality rises as the cut off ability level rises. Therefore, if the firm lowers (raises) its ability threshold, size increases (falls) and quality falls (rises).

Firm size is the fraction of the distribution of employees selected times the number of employees in the distribution. \( \sigma \) is a parameter that stands for the number of potential employees in the distribution \( G(a) \). For example, if the firm hired everyone, then firm size would be \( \sigma = \sigma[G(\bar{a}) - G(\bar{a})] \). The firm will only select the highest ability subset of employees because all employees have the same wage. Therefore,

\[
N(\bar{a}) = \sigma \int_{\bar{a}}^{\pi} g(a) da = \sigma(1 - G(\bar{a})) \tag{53}
\]

If we differentiate (53) by the ability of the marginal, lowest ability employee, then given that \( g(a) > 0 \) inside its support
$$\frac{\partial N(\bar{a})}{\partial \bar{a}} = -g(\bar{a}) < 0, \quad \forall a \in [\underline{a}, \bar{a}]. \quad (54)$$

This is the first thing that we wanted to show. \textit{Q.E.D.}

Secondly, let us turn to the average quality of the workforce.

$$q(N(\bar{a})) = q(\bar{a}) = \frac{\int_a^\pi a g(a) da}{G(\bar{a}) - G(\bar{a})} \quad (55)$$

Differentiating (55) with respect to \( \bar{a} \), we are left with the following:

$$\frac{\partial q(\bar{a})}{\partial \bar{a}} = \frac{g(\bar{a})}{G(\bar{a}) - G(\bar{a})} \left[ \frac{1}{G(\bar{a}) - G(\bar{a})} \right]^{\pi} \left[ a g(a) da \right] - \bar{a} \quad (56)$$

If we substitute (55) into (56) it becomes easier to see why quality must rise in \( \bar{a} \)

$$\frac{\partial q(\bar{a})}{\partial \bar{a}} = \frac{g(\bar{a})}{1 - G(\bar{a})} [q(\bar{a}) - \bar{a}] > 0, \quad \forall a \in [\underline{a}, \bar{a}]. \quad (57)$$

The change in average quality as the lowest quality rises is the hazard rate times the difference between average quality and the lowest quality. The lowest ability employee, with ability \( \bar{a} \), must have a lower ability than the average ability employee
hired. Therefore, the square-bracketed term must be positive. Further, \( g(\bar{a}) > 0 \). The sign in (57) must be positive, which is the second thing we wanted to demonstrate.

Q.E.D.

As the firm chooses a higher ability threshold, \( \bar{a} \), for the lowest ability employee, size falls and quality rises.

### 7.1.2 Derivation of the Comparative Static, \( \frac{\partial N^C}{\partial \mu} < 0 \), in Equation (12)

The endogenous \( N^C(\mu) \) is entirely determined by the first order condition (FOC) in (9). We can use the first order condition to sign \( \frac{\partial N^C(\mu)}{\partial \mu} \). We can use the FOC to derive the comparative static using the implicit function rule. The implicit function rule requires us to find the cross partial of \( N \) and \( \mu \) and the second order condition (SOC), respectively.

\[
m_{\mu} \equiv \frac{\partial^2 \{\pi - K\}}{\partial N \partial \mu}_{N=N^C} = p_N(N^C)N^C + p(N^C) - p(N^C(\mu)) + (1-\mu)p_N(N^C)\frac{\partial N^C(\mu)}{\partial \mu} \tag{58}
\]

\[
m_{\mu} \equiv \frac{\partial^2 \{\pi - K\}}{\partial N \partial \mu}_{N^N_{\mu}=N^C} = p_N(N^C)N^C + (1-\mu)p_N(N^C)\frac{\partial N^C(\mu)}{\partial \mu} \tag{59}
\]

\[
m_{\pi} \equiv \frac{\partial^2 \{\pi - K\}}{\partial N^2} \bigg|_{N=N^N_{\mu}=N^C} = \mu\{p_N(N^C)N^C + 2p_N(N^C)\} < 0
\]
While we cannot immediately sign (58), some algebra allows us to isolate

$$\frac{\partial N^C(\mu)}{\partial \mu}.$$  

$$\frac{\partial N(\mu)}{\partial \mu} = -\frac{m_p}{m_N} \bigg|_{N^C=N^C} = \left( p_N(N^C)N^C + (1-\mu)p_N(N^C) \frac{\partial N^C(\mu)}{\partial \mu} \right)$$  

$$\frac{\partial N(\mu)}{\partial \mu} = -\frac{p_N(N^C)N^C}{\mu \{p_N(N^C)N^C + 2p_N(N^C)\} + (1-\mu)p_N(N^C)} < 0 \quad (60)$$  

The sign in (60) follows from the negative sign for the second order condition in (59) and the assumption that $p_N < 0$.

7.1.3 Derivation of Equation (13)

We want to find how the value of the firm’s equilibrium profit changes with a change in market monitoring. The envelope theorem allows us to differentiate the objective function without reference to the endogenous $N^C(\mu)$. Consider the optimization problem in (8), evaluated at the optimum. If we differentiate it by $\mu$.

$$\frac{\partial \{\pi - K\}}{\partial \mu} \bigg|_{N^C=N^C} + \frac{\partial \{\pi - K\}}{\partial N} \bigg|_{N^C=N^C} + \frac{\partial N^C(\mu)}{\partial \mu} = 0 \quad (61)$$  

$$\frac{\partial \{\pi - K\}}{\partial \mu} \bigg|_{N^C=N^C} = \frac{\partial \{\pi - K\}}{\partial N} \bigg|_{N^C=N^C} = 0 \quad (61)$$
This is a restatement of the envelope theorem. Nevertheless, this problem is slightly complicated by the fact that we cannot ignore the $\mu$ contained in market expectations $N^c(r)$. Let us differentiate equation (8):

$$
\frac{\partial \{\pi - K\}}{\partial \mu} \bigg|_{N=N^c} = p(N^c)N^c - p(N^c)N^c + (1-\mu)p_N(N^c) \frac{\partial N^c(r)}{\partial \mu} N^c
$$

The second line relies on rational expectations shifting in response to exogenous stimuli in step with the endogenous $N^c$ according to the assumption in (4). Armed with the sign in equation (60), we now can conclude that equation (62) is, in fact, positive. That is,

$$
\frac{\partial \{\pi - K\}}{\partial \mu} \bigg|_{N=N^c} = (1-\mu)p_N(N^c) \frac{\partial N^c(\mu)}{\partial \mu} N^c > 0.
$$

This relationship is rewritten in equation (13).

**7.1.4 Derivation of Equation (17)**

Equation (16) can be rearranged to the following

$$
\mu p_N(N^p)N^p = -\frac{K + F}{N^p}
$$
This brings us to the inverse elasticity rule for the partnership given in equation (17).

### 7.1.5 Derivation of Equation (18)

From (16), the first-order condition, we derive the comparative statics below:

\[
p(N_p^p) + \mu p_N(N_p^p)N_p^p = p(N_p^p) - \frac{K + F}{N_p^p} \tag{65}
\]

\[
p(N_p^p) \left(1 + \mu \frac{\partial p}{\partial N} \frac{N_p^p}{p(N_p^p)}\right) = p(N_p^p) - \frac{K + F}{N_p^p} \tag{66}
\]

The equation above is negative because it is the second order condition at a maximum point \(N_p^p\).

\[
S_{NN} \bigg|_{N=N_p^p-N_p^p} = \mu p_{NN}(N_p^p) - \frac{2(K + F(1 + \theta_i))}{(N_p^p)^3} < 0 \tag{67}
\]

\[
S_{NF} \bigg|_{N=N_p^p-N_p^p} = \frac{1 + \theta_i}{(N_p^p)^2} > 0 \tag{68}
\]
The total differential of the FOC is \( \partial(S) = \partial S_{xN} \partial N + \partial S_{xF} \partial F = 0 \). This equation above can be rearranged to solve for \( \frac{\partial N}{\partial F} \), which is given in equation (4).

The sign of \( \frac{\partial N}{\partial F} \) is unambiguously positive.

As mentioned in the main text these comparative statics are only relevant to the period 3 partnership, which takes net-debt as given, exogenous. \( F \) is a choice, endogenous, variable of the period 2 partnership.

7.1.6 Derivation of Equation (34)

Equation (34) is found by rearranging and combining the first order conditions in (16) and (33). Suppose that both first order conditions are satisfied by \( N_i^* \) when a level of net-debt, \( F_i^* \neq 0 \) is chosen. First equation (16) can be rearranged so that

\[
p_n(N_i^*)(N_i^*)^2 = -\frac{1}{\mu}(K + F_i^*(1 + \theta_i))
\](69)

If we multiply \( N_i^* \) by equation (33), then the first order condition can be rewritten as

\[
N_i^* \left. \frac{\partial V^*}{\partial N} \right|_{N=N_i^*} = p_n(N_i^*)(N_i^*)^2 + p(N_i^*)N_i^* - wN_i^* - \frac{dc^{-1}(N_i^*)}{dN} N_i^* = 0.
\](70)
Substituting the left hand side of equation (69) into (70), we get the following relationship:

\begin{equation}
\frac{1}{\mu} (K + F_i^* (1+ \theta_i)) + p(N_i^*)N_i^* - wN_i^* - \frac{dc^{-1}(N_i^*)}{dN} N_i^* = 0
\end{equation}  \quad (71)

Equation (71) can be rewritten as a function of $F_i^*$.

\begin{equation}
F_i^* = \frac{1}{1+\theta_i} \left[ \mu \left( p(N_i^*)N_i^* - wN_i^* - \frac{dc^{-1}(N_i^*)}{dN} N_i^* \right) - K \right]
\end{equation}  \quad (72)

Profits before fixed and financial costs when market monitoring is perfect, but costly finance must be raised to hit hiring targets is

\begin{equation}
p(N_i^*)N_i^* - wN_i^* = \pi(N_i^*; 1)
\end{equation}  \quad (73)

If (73) is inserted into (72), then we have a restatement of equation (34). Since this is what we wanted to derive, we are done.

7.1.7 The Sign and Magnitude of Equation (38)

In section 5.1 we argued the interior solution for the non-voting equity stake in equation (38) exceeded 0 and could even exceed one.
First, we will show that the outside equity share is non-negative. The numerator in curly brackets “{}” is unambiguously positive, given that net-debt, $F_e$ in (34), is negative. Therefore,

$$K - \mu \pi(N_e^*;1) + \mu N_e^* \frac{\partial c^{-1}(N_e^*)}{N} > 0. \quad (74)$$

For any firm to consider opening for business, it must be the case that it makes non-negative profits before financing costs when market monitoring is unity. That is,

$$\pi(N_e^*;1) - K \geq 0 \quad (75)$$

If we subtract can $\mu \pi(N_e^*;1)$, add $K$, and add $\mu N_e^* \frac{\partial c^{-1}(N_e^*)}{N}$ to both sides of (75)we get equation (76) below. We know from (74) that the left hand side below exceeds zero.

$$(1 - \mu)\pi(N_e^*;1) + \mu N_e^* \frac{\partial c^{-1}(N_e^*)}{N} \geq K - \mu \pi(N_e^*;1) + \mu N_e^* \frac{\partial c^{-1}(N_e^*)}{N} > 0 \quad (76)$$

Further, rearranging this inequality, it becomes clear that the quantity in curly brackets, {}, in equation (38) must be between 0 and 1. That is,
\[
1 \geq \frac{\frac{\partial c^{-1}(N_e^*)}{N} N_e^*}{(1 - \mu) \pi(N_e^*; 1) + \mu N_e^* \frac{\partial c^{-1}(N_e^*)}{N}} > 0
\]

(77)

Yet, \( \alpha_e^* \) can exceed 1 or 100 percent in equation (38) because \(-1 < \theta_e \leq 0\).

Therefore, the term in square brackets, “[]”

\[
\frac{1}{1 + \theta_e} \in [1, +\infty)
\]

(78)

Therefore, for some very low values of \( \theta_e \), in which \( \theta_e \) is relatively close to -1, equation (77) multiplied by the right hand side of (78) will exceed unity.

This is what we wanted to show.

7.1.8 Proof of Proposition 4

To prove this proposition we will differentiate the profit function at its maximum point with respect to the cost parameter \( \theta_i \). That is, we will sign \( \frac{\partial V_i^*(N_e^*)}{\partial \theta_i} \).

To do this we will use the envelope theorem. The objective function in (32) only depends on \( \theta_i \) through the indirect financing cost equation \( c^{-1}(N; \mu, \theta) \). Therefore, using the definition of \( c^{-1}(N; \mu, \theta) \) in equation (31)
\[
\frac{\partial V_i^*(N)}{\partial \theta} = \frac{\partial c^{-1}(N; \mu, \theta)}{\partial \theta} = \frac{1}{(1+\theta)^2} \{\mu N^2 p_N(N) + K\}.
\] (79)

The envelope theorem allows us to translate to this to the optimal \( N = N_i^* \). The optimal level of net-debt is given by equation (31) evaluated at \( N_i^* \). This means that equation (79) can be rewritten as

\[
\frac{\partial V_i^*(N_i^*)}{\partial \theta} = \frac{\partial c^{-1}(N_i^*; \mu, \theta)}{\partial \theta} = \frac{1}{1+\theta} F_i^*(N_i^*; \mu, \theta).
\] (80)

This is what we wanted to derive. \( Q.E.D. \)

An interpretation of this can be found in section 5.1 near equation (39).

### 7.1.9 Proof of Proposition 5

There are two pieces to this proof. The first piece is to show that there must be a \( \mu \) which we will denote, \( \hat{\mu_d} \), defined in equation (42). For all \( \mu > \hat{\mu_d} \), the corporation will be weakly more profitable than the transparent partnership. For all \( \mu \in [\mu^\rho, \hat{\mu_d}^*] \) the partnership must be more profitable than the corporation. The second piece involves showing that there is no \( \mu \) at or below \( \mu^\rho \) where the corporation can earn non-zero profits higher than the transparent partnership.

First, let us show that the when financial adjustments are costly but visible to clients that the corporate form must be weakly preferred for some \( 1 \leq \mu < \hat{\mu_d} \). If the
\( \pi(N) \) is concave and the corporation’s profits are maximized at \( \mu = 1 \), then it must be the case that the corporation is becoming increasingly profitable as \( \mu \) rises. (This is what we found in equation (13) without the global concavity assumption.) In contrast, the comparative static result in (41) says that the value of the transparent partnership is strictly declining in \( \mu > \mu_0 \), which is defined in equation (35), when it takes on some positive level of net-debt. Therefore, given that the partnership finds it optimal to take on some positive net-debt \( F_0^*(\mu) > 0 \), there must be some range of \( \mu \), \( 1 \leq \mu < \mu_0 \), where the corporation is more profitable than the transparent partnership.

Alternatively, suppose that the transparent partnership optimally takes on no debt for all \( \mu \in [\mu^c, 1] \). For the purposes of this proof, the \( \pi(N) \) is strictly concave in \( N \). We know that profits in the partnership with no net-debt are maximized at \( N^d(\mu^c) \) where \( \mu^c \) is defined in (36). Therefore, any movement away from \( N^d(\mu^c) = N^* \) will lead to a fall in profits for the partnership with zero net-debt. A sufficient condition for total profits in the zero net-debt partnership to be falling from \( \mu \in (\mu^c, 1] \) is

\[
\frac{\partial N^d}{\partial \mu} < 0.
\]

The implicit function rule allows us to derive the comparative static from the first order condition for the period 3 partnership in equation (16). When net-debt is zero, the comparative static is

\[
\frac{\partial N^d}{\partial \mu} = -\frac{\partial^2 S}{\partial N \partial \mu} \frac{\partial N}{\partial S} \bigg|_{N=N^d} = \frac{-p_s(N^d)}{\mu p_{NN}(N^d) - 2K/(N^d)^3} < 0
\]

(81)
This is unambiguously negative since $p_N < 0$ and the denominator is the second order condition of a maximum point. Therefore, the zero net-debt partnership will never return to the optimal hiring point as long as $\mu \in (\mu^\rho, 1]$. In contrast, we know that the corporation’s profits are strictly rising in $\mu$. Therefore, there must be some $\hat{\mu}_d = \hat{\mu} \in [\mu^\rho, 1]$, where $\hat{\mu}$ is defined in equation (42), at which corporate profits equal partnership profits. If there exist any $1 \geq \mu > \hat{\mu}_d^\ast$, then corporate profits must strictly exceed partnership profits for those value of $\mu$.

Let us turn to the second part of the proof. Namely, partnership profits are always weakly higher than corporate profits from $\mu \in [0, \hat{\mu}_d^\ast]$. We have already shown that the transparent partnership’s profits must be higher than the corporation from $\mu \in [\mu^\rho, \hat{\mu}_d^\ast)$. We still need to prove that the transparent partnership with costly financial adjustments is weakly more profitable than the corporation from $\mu \in [0, \mu^\rho)$. To do this we consider a partnership with zero net-debt. (The transparent partnership can always choose to have zero net-debt and behave like a partnership that cannot adjust its capital structure.)

The second part of the proof relies more heavily on Levin and Tadelis (2005)’s propositions 2 and 3 and Ward (1958). (Nevertheless, we do not attempt to confirm the assertion in both propositions 2 and 3 of Levin and Tadelis (2005) that there exists a lower bound level of informed clients, $\mu^\ast$, defined in equation (43), where the transparent partnership no longer makes positive profits. We do provide a counter example in section 5.1 of the present paper, showing that there is no lower bound, $\mu^\ast$, where the transparent partnership is no longer profitable. See figure 3,
which illustrates that the transparent partnership with financing costs can profitably operate for $\mu < \mu_0$. The partnership that cannot alter its capital structure—the partnership in Levin and Tadelis (2005) and Ward (1958) serves as a lower bound of profitability for the transparent, $\rho = 1$, partnership. This is why the second part of the proof is partially attributed to these papers.

The partnership that does not adjust its capital structure, $F_i^* = 0$, always is more selective than a corporation given that it makes positive profits. If there exists some $\mu$ in which neither organization is viable, they both will make zero profits by not operating. The transparent partnership always has the option to not adjust its capital structure, $F_i^* = 0$, and mimic these payoffs.

We need to show that the partnership is always more profitable than the corporation for all $\mu < \mu^\rho$, in which either organization makes positive profits.

Consider the first order condition in equation (16) for the period 3 partnership when $F = 0$.

$$\mu P_N(N^\rho) + \frac{K}{(N^\rho)^2} = 0$$

(82)

We can rearrange this by and multiplying by $N^\rho$ and then adding $p(N^\rho)$ to both sides.

$$\mu P_N(N^\rho)N^\rho + p(N^\rho) = p(N^\rho) - \frac{K}{N^\rho}$$

(83)
In contrast, let us consider the first order condition of the corporation in equation (11)

\[ \mu N p \mu p(N) N + p(N) = w \]  \hspace{1cm} (84)

On the left hand sides (LHS) are the rational expectations marginal revenues, which we will denote \( MR(N; \mu) = \mu p(N) N + p(N) \), of the partnership and the corporation, respectively. On the right hand side (RHS) is the marginal cost of an employee to each organization. Suppose that the RHS of (83) is at least as large as the RHS of (84).

\[ p(N) - \frac{K}{N} \geq w \]  \hspace{1cm} (85)

\[ \pi(N^p, 0; \mu) - K = p(N^p) N^p - wN^p - K \geq 0 \]

This implies that the LHS of (83), the marginal revenue of the partnership, must meet or exceed the LHS of (84), the marginal revenue of the corporation whenever the partnership with zero net-debt makes non-negative profits. Namely,

\[ MR(N^p; \mu) \geq MR(N^c; \mu) \]

\[ \forall \mu \text{ where } \pi(N^p, 0; \mu) - K \geq 0. \]  \hspace{1cm} (86)

Therefore, (86) implies that the partnership is weakly more selective than the corporation for all \( \mu \) where the partnership earns non-negative profits. Since \( \pi(N) \) is concave and the maximum point the for the partnership does not occur until \( N^p(\mu^p) = \)
$N^*$, this weakly greater selectivity implied by equation (86) indicates that the partnership must also be weakly more profitable than the corporation for all $\mu \in [\underline{\mu}, \mu^\rho ]$, where $\mu$ is defined in (43) as the point where the partnership makes zero profits.

Finally, we must show that the corporation does not operate for $\mu \in [0, \underline{\mu})$. That is, when the partnership with zero net-debt is out of business, the corporation is also out of business. Consider when $\mu = 0$, then the first-order condition for the corporation is (84) is

$$p(N^C; 0) = w.$$  
$$\Rightarrow \pi(N^C; 0) - K = p(N^C; 0)N^C (0) - wN^C (0) - K < 0,$$

$$\therefore K > 0.$$

Both the concavity of $\pi(N)$ assumption and equation (13) implies that that the corporation’s profits are strictly increasing from $\mu = 0$ to $\mu = \mu^\rho$. When the LHS of (83) and (84) are equal, profits for both the corporation and the partnership must be identically zero. By definition in equation (43), the $\mu$ at this level of profits is $\underline{\mu}$. In short, we can conclude that the corporation does not operate from $\mu \in [0, \underline{\mu})$.

This is what we wanted to show. Q.E.D.

In summary, there must be a crossing point $0 < \hat{\mu}_d^* < 1$, defined in equation (42), where the transparent partnership with financing costs and the corporation make the same profits. When $\mu \in [\hat{\mu}_d^*, 1]$ the corporation is the weakly more profitable
organization. For all \( \mu \in [0, \hat{\mu}_d^\ast) \), the transparent partnership is the weakly most profitable organization, given that any organization is profitable.

### 7.1.10 Derivation of Equation (47)

When expectations converge to equilibrium hiring, \( N^r(r) = N_i^C \), and we rename the bottom term in equation (45) as \( \frac{\partial c^{-1}(N_i^C)}{\partial N} \), then we can get the following simpler first order condition.

\[
\frac{\partial V_i^C}{\partial N} \bigg|_{N=N^r-N_i^C} = \mu N_i^C p_N(N_i^C) + p(N_i^C) - w - \frac{dc^{-1}(N_i^C)}{dN} = 0 \quad (88)
\]

Given that net-debt of \( F_i^C \) is chosen by the period 2 partnership, the period 3 partnership will hire \( N^p = N_i^C \) partners in period 3. We can rearrange equation (16), the first order condition of the period 3 partnership, so that

\[
\mu N_i^C p_N(N_i^C) = - \frac{K + (1 + \theta)F_i^C}{N_i^C} \quad (89)
\]

We can substitute (89) this into (88). After this substitution we can rearrange the first order condition so that
This is rewritten in equation (47).

7.1.11 On the Impossibility of Negative Net-Debt, \( F^C_i < 0 \), in the Opaque Partnership, \( \rho = 0 \), when the Inequality in (49) Holds

Here we will attempt to show that it cannot be the case that both \( F^C_i < 0 \), and \( N_i C_{p_N}(N_i^C) + 2 p_N(N_i^C) \leq 0 \). The later condition is an additional assumption, which may not always be true.

When signing equation (47) what is in dispute is the sign of the \( N_i C \frac{dc^{-1}(N_i^C)}{dN} \) term. Non-negative profits economic profits at the optimum implies that

\[
\pi(N_i^C, \mu, \theta) - K - c^{-1}(N_i^C) \geq \pi(N_i^C, \mu, \theta) - K \geq 0.
\]

Let us differentiate the inverse cost function \( c^{-1}(N) \) in equation (31).

\[
\frac{\partial c^{-1}(N)}{\partial N} = \theta F_{\theta} = -\frac{\theta}{1 + \theta_i} \mu N \{ 2 p_N(N) + N p_{NN}(N) \}
\]
Given that $-1 < \theta_i < 1$, our conclusion about the sign of (92) depends on the sign of $\theta_i$ and the sign of the quantity in curly brackets “{}". In this section, we have taken up assumption 5.2 in equation (49) which says that the latter term’s sign is negative.

Further, we know that $\theta_e \leq 0$ and $\theta_d \geq 0$ from equation (1). Therefore, the marginal financing cost of an additional employee is weakly positive for equity and weakly negative for debt. That is,

$$
\frac{\partial c^{-1}(N; \theta_e)}{\partial N} = \theta_e F_N = -\frac{\theta_e}{1+\theta_e} \mu N \{2 p_N(N) + N p_{NN}(N)\} \leq 0, \&
$$

$$
\frac{\partial c^{-1}(N; \theta_d)}{\partial N} = \theta_d F_N = -\frac{\theta_d}{1+\theta_d} \mu N \{2 p_N(N) + N p_{NN}(N)\} \geq 0,
$$

when $2 p_N(N) + N p_{NN}(N) \leq 0$. (93)

If we combine the top sign in equation (93) with our net-debt equation in (47),

$$
F_c^e = \frac{1}{1+\theta_e} \left( \pi(N_e^C; \mu) - K - N_e^C \frac{dc^{-1}(N_e^C)}{dN} \right) \geq 0. \tag{94}
$$

Equation (94) is a contradiction because equilibrium net-debt must be negative for the firm to take on outside equity.

The second inequality in equation (93) leaves open the possibility that net-debt could be negative when the firm takes on debt. This is also a contradiction. If the partnership raises positive debt at a percent of $\theta_d * 100$ percent of the value raised, it will not have negative net-debt.
Therefore, under these assumptions about the shape of the inverse demand curve at the optimum, net-debt cannot be negative for the opaque partnership with positive financing costs. That is,

\[
F_t^C = F_d^C = \max \left\{ \frac{1}{1 + \theta_d} \left( \pi (N_d^C; \mu) - K - N_d^C \frac{dc^{-1}(N_d^C)}{dN} \right), 0 \right\},
\]

when \( N_d^C p_{NN} (N_d^C) + 2 p_N (N_d^C) \leq 0. \)

This is what we wanted to demonstrate.

### 7.1.12 Deriving the Sign of \( \frac{\partial N_l^C}{\partial \theta_l} \)

Let us define the first order condition in equation (45) as the function “\( h \).” The first order condition evaluated at \( N_l^C \) is equal to zero. Therefore, \( h = 0 \). This allows us to perform comparative statics on this condition such that

\[
h_N \partial N_l^C + h_{\theta_i} \partial \theta_i = 0,
\]

where the partial derivatives of the “\( h \)” function are denoted by subscripts. This implies that

\[
\frac{\partial N_l^C}{\partial \theta_i} = -\frac{h_{\theta_i}}{h_N}.
\]
The second order condition in (46) is simply the partial derivative $h_N$, which we know is negative at the optimum. Let us differentiate the first order condition in equation (45) with respect to $\theta_i$

\[
\frac{\partial^2 V_i^C}{\partial N\partial \theta_i}_{_{N=N_F}} = (1-\mu)p_N(N^C(r))\frac{\partial N^C(r)}{\partial \theta_i} + \frac{1}{(1+\theta)}\mu N_i^C \{2p_N(N_i^C) + N_i^C p_{NN}(N_i^C)\} \tag{97}
\]

Inserting the rational expectation conditions in equations (3) and (4) we can rewrite equation (97) as

\[
h_\theta = \frac{\partial^2 V_i^C}{\partial N\partial \theta_i}_{_{N=N'}-N_F} = (1-\mu)p_N(N_i^C)\frac{\partial N_i^C}{\partial \theta_i} + \frac{1}{(1+\theta)}\mu N_i^C \{2p_N(N_i^C) + N_i^C p_{NN}(N_i^C)\} \tag{98}
\]

Inserting equation (98) into equation (96) and solving for $\frac{\partial N_i^C}{\partial \theta_i}$ we get the following:

\[
\frac{\partial N_i^C}{\partial \theta_i} = \frac{-\mu N_i^C \{2p_N(N_i^C) + N_i^C p_{NN}(N_i^C)\}}{(1+\theta)h_\theta + (1-\mu)p_N(N_i^C)} \leq 0, \tag{99}
\]

when $2p_N(N_i^C) + N_i^C p_{NN}(N_i^C) \leq 0$
The numerator is weakly positive given our assumption in equation (49). In contrast the denominator is negative because the second order condition is negative, 

\[ h_N < 0, \]

and price is falling in output or hiring, \( p_N < 0 \).

This is what we wanted to derive. \textit{Q.E.D.}
7.2 Numerical Examples

Here we derive the basis for the figures 2 and 3 in the main text. More importantly, we also explore the perfect Bayesian equilibrium best responses and payoffs when $\mu = 0.8$ or $0.1$, $\rho = 0$ or $1$, and $\theta_l = 0$ or $\theta_r = 0.02$ and $\theta_e = -0.07$. In all, we explore $2^3 = 8$ scenarios summarized in the table at the end of the text. These examples are derived from a uniform distribution of employee ability. Explicit solutions for possible equilibrium choices and payoffs are derived for all the eight scenarios. The author believes that some readers may even enjoy the specific cases discussed here more than the general cases explored in the main text!

Supposes that ability, $a$, is distributed uniformly on the continuous interval $[a, \bar{a}]$, where $\bar{a}$ is the talent of the highest ability individual in the distribution of potential employees. That is $a \sim U(a, \bar{a})$. The firm, which observes ability directly, will select professionals from the top of the talent distribution. That is, the firm will select individuals of abilities on the interval $a \in [\bar{a}, \bar{a}]$. The probability density function for the uniform distribution is as follows:

$$
g(a) = \begin{cases} 
0, & \text{when } a < a \\
\frac{1}{\bar{a} - a}, & \text{when } a \leq a \leq \bar{a} \\
0, & \text{when } a > \bar{a}
\end{cases}
$$

A firm that only hires workers of ability $a \geq \bar{a}$ has an average ability of
Clients are risk neutral price takers who are randomly assigned employees from the pool of workers in the firm. They do not observe the workers’ ability directly. Instead they are willing to pay for the expected quality of workers in the firm. Suppose that the price that consumers are willing to pay is a multiple of the average quality, \( q(\tilde{a}) \). That is,

\[
p(\tilde{a}) = xq(\tilde{a}),
\]

where \( x \) is a parameter measuring clients’ willingness to pay for ability.

If the size of the potential workforce is given by the parameter \( \sigma > 0 \), then the size of the firm is as follows:

\[
N(\tilde{a}) = \sigma \left( \frac{\bar{a} - \tilde{a}}{\tilde{a} - a} \right), \quad \text{when } a \sim U(\tilde{a}, \bar{a})
\]

If we rearrange (102), we can solve for the ability cut off as a function of the size of the firm.

\[
\tilde{a}(N) = \bar{a} - \frac{N}{\sigma} (\bar{a} - a)
\]
If we combine equations (100) and (103) and equations (100), (101), and (103), we can solve for average quality and price, respectively, as a function of firm size, $N$.

\[
q(N) = \bar{a} - \frac{N}{2\sigma} (\bar{a} - \bar{q})
\]

\[
p(N) = x\bar{q}(N) = x\left(\bar{a} - \frac{N}{2\sigma} (\bar{a} - \bar{q})\right)
\]

\[
p_{\bar{q}}(N) = -\frac{x}{2\sigma} (\bar{a} - \bar{q})
\]

The first order condition from equation (21) can be combined with the price function in equation (104).

\[
\frac{\partial \{\pi(N;1) - K\}}{\partial N} \bigg|_{N = N^*} = x\left(\bar{a} - \frac{N^*}{2\sigma} (\bar{a} - \bar{q})\right) - \frac{N^* x}{2\sigma} (\bar{a} - \bar{q}) - w = 0
\]

The implied first-best hiring rule is

\[
N^* = \frac{\sigma}{x} \left(\frac{x\bar{a} - w}{\bar{a} - \bar{q}}\right)
\]

Further, combining (106) and (104) the first best price is

\[
p(N^*) = \frac{1}{2} (x\bar{a} + w)
\]
If we insert equation (106) into our definition of profits before fixed costs, we can solve for this quantity. In this example, first-best profits before fixed costs is

$$\pi(N^*) = \frac{\sigma(x\bar{a} - w)^2}{2x(\bar{a} - a)}.$$  \hfill (108)

On the other hand, a corporation will hire according to the first order condition in (11), which can be rewritten for this example as the following:

$$\left. \frac{\partial V^C}{\partial N} \right|_{N = N^* - N^C} = \left. \frac{\partial \{\pi - K\}}{\partial N} \right|_{N = N^* - N^C} = x\left( \frac{\bar{a} - N^C}{2\sigma} - \frac{\mu x N^C}{2\sigma} \right) = 0$$

This implies that the corporation will hire

$$N^C = \frac{\sigma}{x} \left( \frac{x\bar{a} - w}{\bar{a} - a} \right) \left( \frac{2}{1 + \mu} \right).$$  \hfill (110)

The equilibrium price for the corporation will be

$$p(N^C) = \frac{1}{1 + \mu} (\mu x\bar{a} + w).$$  \hfill (111)

The profits before fixed costs will be
\[ \pi(N^c) = \frac{2\mu \sigma}{x(a - a)} \left( \frac{x\bar{a} - w}{1 + \mu} \right)^2. \] (112)

Let us consider the hiring decisions of the partnership. The FOC for the partnership is given by equation (16). Inserting in the values from equation (104), the first order condition becomes

\[ \frac{\partial S}{\partial N_{[N_1, N']}} = -\frac{\mu x}{2\sigma}(\bar{a} - a) + \frac{K + F(1 + \theta)}{(N')^2} = 0 \]

\[ \Rightarrow N^p = \sqrt{\frac{2\sigma(K + F(1 + \theta))}{\mu x(\bar{a} - a)}} \] (113)

The equilibrium price for the partnership is

\[ p(N^p) = x\bar{a} - \sqrt{\frac{x(\bar{a} - a)(K + F(1 + \theta))}{2\sigma \mu}}. \] (114)

The profit after fixed costs of the partnership as a whole is

\[ \pi(N^p) - K = (x\bar{a} - w) \sqrt{\frac{2\sigma(K + F(1 + \theta))}{\mu x(\bar{a} - a)}} - \frac{K + F(1 + \theta)}{\mu} - K \] (115)

Suppose that the firm has the following parameter values:
\[ \bar{\alpha} = 1 \]
\[ a = 0 \]
\[ \sigma = 100 \]
\[ x = 4 \]
\[ w = 2 \]
\[ K = 10 \]  \hspace{1cm} (116)

The preceding parameter values mean that the maximum size of the firm is 100; the choke price where demand is zero is 4, and the firm cannot cover its variable costs if the price falls below the wage of 2.

In the first-best, 100 percent market monitoring, benchmark case

\[ N^* = 50 \]
\[ p(N^*) = 3 \]
\[ \pi(N^*) = 50 \]
\[ \pi(N^*) - K = 40 \]  \hspace{1cm} (117)

We will consider the optimal capital structure decisions when market monitoring is high, \( \mu_H = 0.8 \), and when market monitoring is low, \( \mu_L = 0.1 \).

### 7.2.1 Transparent Finances (\( \rho = 1 \)), No Financing Costs (\( \theta = 0 \)), and High Levels of Market Monitoring (\( \mu_H = 0.8 \))

A partnership with a transparent capital structure will hire the same number of employees, be able to charge the same price, and will realize the same profits before
and after fixed costs as the first-best in equation (117). Nevertheless, to achieve this feat, the partnership will have to take on some debt.

Recall the formula for the optimal level of net-debt for the transparent partnership ($\rho = 1$) in the partnership in equation (24). We can combine this with our formula for first-best profits before investment costs in equation (108) to get the following relationship:

$$2 \ast \left(\left(\frac{\sigma(z-a-w)^2}{2x(a-g)}\right) - K\right) = \sigma(\frac{z}{a})$$

In this case this will be

$$F^\ast(\mu) = F^\ast(0.8) = 30$$

Therefore, given that first-best profits are 40, the net-debt-to-value ratio for the partnership will be $30/40 = 75\%$ for a relatively high level of market monitoring.

The transparent partnership earns the first-best profits in equation (117) compared to the corporate profits which we will derive subsequently in equation (120). Since $40 > 39.38$, the partnership organization will be selected in period 1, given that $\rho = 1$. That is,

$$\gamma^\ast(0.8) = 1.$$
7.2.2 Opaque Finances ($\rho = 0$), No Financing Costs ($\theta = 0$), and High Levels of Market Monitoring ($\mu_H = 0.8$)

The corporation will systematically over-hire relative to first-best profit maximization. The same is true for partnerships which have opaque capital structures. They will find hiring at $N^C$ to be optimal. For this reason, the founders of the firm will be indifferent between choosing the corporate or partnership form when capital structure is hidden, $\rho = 0$, and finance costs are zero. This means that

$$\gamma^C(0.8) = 0 \text{ or } 1.$$ 

For these parameter values and market monitoring of $\mu = 0.8$, the equilibrium output, price, profit before and after fixed costs for the opaque partnership or the corporation are

\[ N^C(\mu) = N^C(0.8) = 55.55 \]
\[ p(N^C;0.8) = 2.88 \]
\[ \pi(N^C;0.8) \approx 49.38 \]
\[ \pi(N^C;0.8) - K \approx 39.38 \]  

From equation (29) we know that the opaque partnership will take on net-debt equal to $\pi(N^C; \mu) - K$. In equation (112), we calculated profits before fixed costs for
the opaque partnership. Therefore, for the opaque partnership equilibrium net-debt will be

\[ F^C(\mu) = \pi(N^C; \mu) - K = \frac{2\mu \sigma}{x(\bar{a} - x)} \left( \frac{x\bar{a} - w}{1 + \mu} \right)^2 - K. \] (121)

To do this it would have to take on debt of

\[ F^C(\mu) = F^C(0.8) = \pi(N^C; 0.8) - K \approx 39.38. \] (122)

The partnership with an opaque capital structure, takes on a 100 percent net-debt-to-value ratio. This is because it attempts to mimic the profit maximization of the corporation. This is the prescription of Ward (1958). That article advocated that Yugoslavian worker cooperatives have 100 percent debt-to-value ratios so that they would behave like normal corporations.

Consider the diagrams that follow. In the top diagram, we plot the profit functions when capital structure is transparent, \( \rho = 1 \), and opaque, \( \rho = 0 \). We assume rational expectations. The net-debt function intersects the opaque partnership’s profit function at its peak. Yet, it intersects the \( \mu \pi(N, N^e = N^*) - K \) at \( N^* = 50 \) to denote the optimal level of net-debt under perfect information. Note the thin black curve to the left with a peak above all the other curves. This curve indicates that the partnership, by employing 50 partners, is deviating from \textit{ex post} (period 3) profit maximization if clients have expectations of \( N^e = N^* = 50 \). Yet, these expectations are rational if the
equal profit sharing, partnership constitution, \( \gamma^* = 1 \), is adopted and the level of net-debt is visible to clients, \( \rho = 1 \). For the opaque partnership, given that \( N^c = 55.6 \) the firm can do no better than employ \( N^c(0.8) = 55.6 \) partners or employees.

The diagram immediately below the one we just described shows the period 4 profit-per-partner \( S(N) \) functions of the transparent and opaque partnerships. The period 3 partnership will choose the hiring level that leads to the peak in each curve. Yet, the profits per partner are dependent on the amount of net-debt distributed in period 2. It is clear that the period 4 profits-per-partner are positive when \( \rho = 1 \), but they are zero when \( \rho = 0 \).
\[ \theta_i = 0, \mu = 0.8 \]

\[ \pi(N), F(N) \]

\[ \pi(N^*, 1) - K = 40 \]

\[ \pi(N^C; 0.8) - K = 39.38 \]

\[ F^*(0.8) = 30 \]

\[ \theta_i = 0, \mu = 0.8 \]

\[ S(N) \]

\[ S(N; F^*(0.8) = 30, \mu = 0.8, N^* = N^C = 50) \]

• The top curve shows profits per partners in period 4 when capital structure is transparent, \( \rho = 1 \). Proceeds from the sale debt, 30, was distributed in period 2. Therefore, total firm profits are \( 30 + 0.2 \times 50 = 40 \).

• The bottom curve shows profits per partner in period 4 when capital structure is hidden, \( \rho = 0 \). Zero profit is earned in period 4, but debt of value of approximately 39.4 was distributed in period 2.
7.2.3 Transparent Net-Debt ($\rho = 1$), No Financing Costs ($\theta = 0$), and Low Levels of Market Monitoring ($\mu_L = 0.1$)

The partnership in which clients observe the capital structure choices of the partnership can achieve the first-best profits in equation (117). Nevertheless, to do so when $\mu = \mu_L = 0.1$ it must actually raise some cash by way of outside equity if it hopes to only hire $N^* = 50$ partners. By combining the general formula for the outside equity stake in equation (26) with our knowledge of first-best profits before fixed costs in (108) for this example, we can obtain the formula for a non-voting equity stake below:

$$
\alpha^*(\mu) = \begin{cases} 
K - \mu \left( \frac{\sigma(x\bar{a} - w)^2}{2x(\bar{a} - a)} \right), & \text{if } F^*(\mu) \geq 0 \\
(1 - \mu) \left( \frac{\sigma(x\bar{a} - w)^2}{2x(\bar{a} - a)} \right), & \text{otherwise.}
\end{cases}
$$

(123)

The capital structure choices obtained from combining equations (116), (118), (123), and the level of market monitoring, $\mu_L = 0.1$, are

$$
F^*(\mu) = F^*(0.1) = -5
$$

$$
\alpha^*(\mu) = \alpha^*(0.1) = 11.1\%
$$

(124)
The negative $F^*(0.1)$ means that net-debt is optimally negative in the partnership in this case. The net-debt-to-value ratio for this partnership is $-5/40 = -12.5\%$. Low levels of market monitoring allow the partnership with transparent capital structure to have lower net-debt-to-value ratios relative to higher levels of market monitoring, higher $\mu$. Indeed, in this case, the net-debt-to-value ratio is negative if the partnership is to achieve the first-best.

The transparent partnership, $\rho = 1$, facing no financing cost, $\theta_i = 0$, is unambiguously more profitable than the corporation. (The corporation’s profits are reported in equation (125) below. $40 > 6.53$.) Therefore, its founders will choose the partnership form or set

$$\gamma^*(0.1) = 1.$$ 

### 7.2.4 Opaque Net-Debt ($\rho = 0$), No Financing Costs ($\theta_i = 0$), and Low Levels of Market Monitoring ($\mu_L = 0.1$)

Consider the problem of the corporation and the problem of the partnership with a hidden capital structure, $\rho = 0$. For these parameter values and market monitoring of $\mu = 0.1$, the equilibrium output, price, profit before and after fixed costs are
\[ N^C(\mu) = N^C(0.1) \approx 90.91 \]
\[ \rho(N^C; 0.1) \approx 2.16 \]
\[ \pi(N^C; 0.1) \approx 16.53 \]
\[ \pi(N^C; 0.1) - K \approx 6.53. \]  

The corporation can achieve these outcomes with any level of debt, but the partnership will maximize profits by taking on 100 percent net-debt-to-value ratio in period 2. That means that the opaque partnership will take on debt payments of

\[ F^C(\mu) = F^C(0.1) = \pi(N^C; 0.1) - K \approx 6.53. \]  

Checking the first-best profits in equation (117) we can verify that the corporation and the partnership with a hidden capital structure generate only 6.53/40 = 16.32\% of first-best profits.

The expected profits in an opaque partnership, $\rho = 0$, and in a corporation are identical when financing costs are zero. In period 1 the founders of the firm will be indifferent between either organisational form. Therefore,

\[ \gamma^C(0.1) = 0 \text{ or } 1. \]

Consider figures 6 and 7 which follow. The top diagram plots the profit functions and the net-debt functions for the transparent and opaque partnership. The bottom diagram plots per partner profits.
Let us first turn to the top diagram. The thin black line that extends above all other curves is the profit function when expectations are fixed at $N^e = N^* = 50$. The firm could make substantial profits, which rise above the range of the graph, if the firm could exploit these expectations and push hiring far beyond 50. The only way that such expectations could be rational would be if the firm could commit through transparent capital structure $\rho = 1$ and a partnership constitution $\gamma^* = 1$. The thick gray curve that intersects the $\pi(N, N^*) - K$ curve at $N = 50$ is relevant to the case where such commitments are possible. The optimal level of net-debt to achieved this is determined by the intersection of the $F(N)$ and $\mu_1 \pi(N;1) - K = 0.1 \mu_1 \pi(N;1) - K$ functions at $N^* = 50$ and $F^*(0.1) = -5$.

The thick black hill towards the bottom of the plot is the opaque partnership’s profit function. Alternatively, it is also the corporations’ profit function. Because neither the corporation nor the opaque ($\rho = 0$) partnership can credibly signal its hiring intentions, clients expect the worst. Rational expectations with $\rho = 0$ leads $N^e = N^c(0.1) \approx 90.9$. When the partnership form is selected, $\gamma^c = 1$, then the partnership sells net-debt worth about 6.5. This value occurs at the intersection of the two thick black curves at $N^c(0.1) \approx 90.9$. The $F(N)$ curve intersects $\pi(N, 90.9) - K$ at its peak; therefore, all period 4 profits are sold to debt-holders.

Let us turn to the bottom graph. This plots the period 4, per-partner profits as a function of $N$. With a cash infusion of 5 from equity sales, the transparent partnership will maximize per partner profits at $N^* = 50$. This is the top curve. The opaque partnership’s per partner, period 4 profits are weighed down by the debt proceeds of about 6.5 that they distributed in period 2. Therefore, they can just break
even by hiring 90.9 partners.
\[ \theta_i = 0, \mu = 0.1 \]

\[ \pi(N), F(N) \]

\[ \pi(N^*; 1) - K = 40 \]

\[ \pi(N^*; 1) - K = 40 \]

\[ \pi(N^*; 1) - K = 40 \]

\[ F^*(0.1) = -5 \]

\[ N^* = 50 \]

\[ 0.1 \pi(N, N^*) - K \]

\[ N^C(0.1) = 90.9 \]

**Figure 6**

\[ \theta_i = 0, \mu = 0.1 \]

\[ S(N) \]

\[ S(N; F^*(0.1) = -5, \mu = 0.1, N^e = N^* = 50) \]

\[ S(N; F^*(0.1) = 6.5, \mu = 0.1, N^e = N^C = 90.9) \]

**Figure 7**

- The top curve shows profits per partners in period 4 when capital structure is transparent, \( \rho = 1 \). Proceeds from the sale of a 11.1\% equity stake, net-debt of -5, are used to reduce the fixed obligations of partners in period 4. Per-partner profits are 0.8 split between 50 partners. Therefore, total profits are 0.8*50 = 40. Yet, this is only 88.9\% of profits distributed in period 4. Non-voting equity holders get back 5 or a 11.1\% stake of the 45 proceeds in period 4.

- The bottom curve shows profits per partner in period 4 when capital structure is hidden, \( \rho = 0 \). Zero profit is earned in period 4, but debt of value of approximately 6.5 was distributed in period 2.
7.2.5 Costly Financial Adjustments \((\theta_i \neq 0)\)

If we invert the period 2 partnership’s demand for employees as a function of net-debt, \(N^{\mu}(F; \mu, \theta_i)\), in equation (113) to the function \(F(N; \mu, \theta_i)\) we are left with

\[
F(N) = \frac{1}{1 + \theta_i} \frac{\mu x(\bar{a} - a)}{2\sigma} (N)^2 - \frac{1}{1 + \theta_i} K.
\]  \hspace{1cm} (127)

The inverse cost of net-debt function is

\[
c^{-1}(N) = \theta_i F = \frac{\theta_i}{1 + \theta_i} \frac{\mu x(\bar{a} - a)}{2\sigma} (N)^2 - \frac{\theta_i}{1 + \theta_i} K.
\]  \hspace{1cm} (128)

Let us assume that, in this example, the net value leakage as a percent of the amount raised is 2% for debt and 7% for outside equity. That is,

\[
\theta_d = 0.02 \quad \theta_e = -0.07
\]  \hspace{1cm} (129)

The parameters in (129) are consistent with underwriting spreads in the United States. For the United States, the debt parameter would be on the high side and the equity parameter would be at the median. Therefore, the dichotomy between the cost of debt versus the cost of equity is probably greater than the parameters in...
A seven percent gross underwriting spread is the median reported by Chen and Ritter (2000) and Ritter (2006) for equity initial public offerings (IPOs). Kim, Palia, and Saunders (2003) reports that the 5th and 95th percentile underwriting spreads from 1970 to 2000 were 0.250 and 3.829 percent for debt and 6.00 and 10.00 percent and equity IPOs, respectively. That study reports the median for debt issues at 0.750 percent.

7.2.6 Transparent Capital Structure ($\rho = 1$) and Costly Financial Adjustments

($\theta_i \neq 0$)

Given that net-debt is transparent, the period 2 partnership can achieve the first best profits before financial costs by choosing a net-debt level that would induce the period 3 partnership to hire $N^* = 50$. Yet, we saw in the previous sections that this means raising outside finance. When there are financial frictions, the partnership has to weigh the gains from a more profitable employment policy against the financing costs, given in equation (128). The problem for the period 2 partnership that is attempting to determine the optimal hiring level with costly finance is in equation (130) below.

$$
\arg \max_{w.r.t. \ N} \ V_i^* = (p(N) - w)N - K - c^{-1}(N)
$$

$$
= \left( x\bar{a} - \frac{N}{2\sigma}(\bar{a} - q) - w \right)N - K - \theta_i \left( \frac{\mu x(\bar{a} - q) - \sigma^2}{2\sigma^2} (N)^2 - \frac{\theta \mu}{1 + \theta_i} \right)
$$

(130)
Equation (130) can be obtained by combining the objective function in equation (32), the definition of $c^{-1}(N)$ in either equation (31) or (128), and the inverse demand function for this example in equation (104). Differentiating this function with respect to $N$, we can solve for the optimal hiring level of the partnership which can credibly signal its capital structure.

$$\frac{\partial V^*_i}{\partial N} = (x\bar{a} - w) - N^*_i \frac{x}{\sigma} (\bar{a} - a) - N^*_i \left( \frac{\theta_i}{1 + \theta_i} \right) \frac{\mu x}{\sigma} (\bar{a} - a) = 0$$

(131)

Since the second order condition above is unambiguously negative, we can conclude that the stationary point $N^*_i$ below is a maximum.

$$N^*_i = \frac{\sigma (x\bar{a} - w)}{x(\bar{a} - a)} \left( \frac{1 + \theta_i}{1 + \theta_i (1 + \mu)} \right)$$

(132)

If we compare this to the case where financing costs were zero in equation (106), we can verify that zero financial costs is a special case of equation (132). The hiring level in (132) is identical to equation (106) when $\theta_i = 0$.

Combining $p(N)$ in equation (104) and $N^*_i$ from (132) we have the equilibrium price,
Equation (133) is identical to equation (107) when $\theta = 0$.

The profits before fixed and financing costs are

$$\pi(N^*_i) = \frac{\sigma(1+\theta_i)}{2x(a-a)} \left( \frac{x\bar{a} - w}{1+\theta_i(1+\mu)} \right)^2 (1+\theta_i(1+2\mu)).$$  \hspace{1cm} (134)

Financial cost are obtained by combining equations (128) and (132) below

$$c^{-1}(N^*_i) = \frac{\mu\sigma\theta_i(1+\theta_i)}{2x(a-a)} \left( \frac{x\bar{a} - w}{1+\theta_i(1+\mu)} \right)^2 - \frac{\theta_i}{1+\theta_i} K$$  \hspace{1cm} (135)

Net-debt raised can be obtained by dividing equation (135) by $\theta_i$.

$$F^*_i = c^{-1}(N^*_i) / \theta_i = \frac{1}{1+\theta_i} \left[ \mu \left( \pi(N^*_i) - \frac{dc^{-1}(N^*_i)}{dN} N^*_i \right) - K \right]$$

$$= \frac{\mu\sigma(1+\theta_i)}{2x(a-a)} \left( \frac{x\bar{a} - w}{1+\theta_i(1+\mu)} \right)^2 - \frac{K}{1+\theta_i}$$  \hspace{1cm} (136)

The equilibrium value of the firm is obtained by combining equation (134), $K$, and the financing costs in equation (135) into the equation below:
\[ V_i^* = \pi(N_i^*) - K - c^{-1}(N_i^*) \]
\[ = \frac{\sigma(1 + \theta_i)(x\bar{a} - w)^2}{2x(\bar{a} - a)1 + \theta_i(1 + \mu)} - \frac{K}{1 + \theta_i} \]  

(137)

The marginal financing cost of an employee multiplied by the number of employees, which figures prominently in the derivations of \( F_i^* \) and \( \alpha_i^* \) in equations (34) and (38) is

\[ \frac{\partial c^{-1}(N_i^*)}{\partial N} N_i^* = \frac{\mu \sigma \theta_i (1 + \theta_i)}{x(\bar{a} - a)} \left( \frac{x\bar{a} - w}{1 + \theta_i (1 + \mu)} \right)^2. \]  

(138)

In terms of parameters, the outside equity stake in (38) can be obtained by combining (134) and (138).

\[ \alpha_e^* = \frac{K}{1 + \theta_i} - \frac{\mu \sigma (1 + \theta_i)}{2x(\bar{a} - a)} \left( \frac{x\bar{a} - w}{1 + \theta_i (1 + \mu)} \right)^2, \]

(139)

when \( F_i^* < 0 \).

7.2.7 **Transparent Capital Structure (\( \rho = 1 \)), Costly Financial Adjustments (\( \theta_d \)) = 0.02 and \( \theta_e = -0.07 \), and High Levels of Market Monitoring (\( m_H = 0.8 \))**
Let us solve the model for this set of parameter values. The equilibrium values for the set of parameter values specified in equation (116) and costs of outside finance specified in equation (129) are as follows. These values are obtained by inserting the numerical values in equations (132) to (137). When market monitoring is relatively high, $\mu_H = 0.8$, these are approximately

$$
N^*_d(\mu, \theta_d) = N^*_d(0.8, 0.02) \approx 49.23 \\
p(N^*_d; \mu, \theta_d) = p(N^*_d; 0.8, 0.02) \approx 3.02 \\
\pi(N^*_d; \mu, \theta_d) = \pi(N^*_d; 0.8, 0.02) \approx 49.99 \\
F^*_d(0.8, 0.02) = F(N^*_d; 0.8, 0.02) \approx 28.21 \\
\alpha^*_d(0.8, 0.02) = 0 \\
\theta^*_d F^*_d(0.8, 0.02) \approx 0.56 \\
\pi(N^*_d; 0.8, 0.02) - K - \theta^*_d F(N^*_d; 0.8, 0.02) \approx 39.42 \\
\gamma^*_d(0.8, 0.02) = 1.
$$

These values are very similar to equations (117) and (119) that characterise the equilibrium values for when capital structure is transparent and financing costs are zero. In general, the partnership is more selective with costly debt. It hires 49.23 employees when $\theta_d = 0.02$ versus 50 when $\theta_i = 0$. Raising and distributing the proceeds of costly debt, $F^*_d(0.8, 0.02) \approx 28.21$, in period 2 is necessary to induce the period 3 partnership to take the profit maximizing hiring decision. Yet, the costs of raising debt and the reduced hiring because of it reduce the overall profitability after fixed and investment costs of the partnership from 40 to 39.42.

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*Exact solutions are available. The results have been rounded for convenience.*

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Let us consider the diagram on the following page. In the figure below, we consider the profit and net-debt function of the transparent partnership ($\rho = 1$) and the profit function of a corporation. The presence of costly debt makes the transparent partnership nearly as profitable as the corporation. Therefore, the partnership form is adopted in equilibrium, $\gamma^* = 1$. Recall from equation (120) that the corporation could expect 39.38 in profits. The partnership facing financing costs optimally loses 0.56 to financing costs and net of financing cost and only makes 39.42 in profits net of fixed and financing costs according to equation (140). The intersection of the lower gray hill and the thick black $F(N; 0.02)$ shows us the optimal level of net-debt, 28.2, for its target output, $N^*_d = 49.2$. The higher gray hill to the left is the partnership’s net profit function. The black hump in the top right is the corporation’s profit function, which is unaffected by financing costs.
The transparent partnership organizational form is slightly more profitable than the corporate form. This is despite the fact that the partnership must raise costly debt. Under these parameter values, it is a perfect Bayesian equilibrium that the partnership form will be selected.

**Figure 8**
7.2.8 Transparent Capital Structure ($\rho = 1$), Costly Financial Adjustments ($\theta_d = 0.02$ and $\theta_e = -0.07$), and Low Levels of Market Monitoring ($\mu_L = 0.1$)

Consider the case were market monitoring is low. Let us plug in the parameter values in equations (116) and (129) and $\mu = \mu_L = 0.1$ into equations (132) to (137) and equation (139). The following are the equilibrium values for the transparent partnership:

\[
\begin{align*}
N_e^* (\mu, \theta_e) &= N_e^* (0.1, -0.07) \approx 50.38 \\
p(N^*_e; \mu, \theta_e) &= p(N^*_e; 0.1, -0.07) \approx 2.99 \\
\pi(N^*_e; \mu, \theta_e) &= \pi(N^*_e; 0.1, -0.07) \approx 50.00 \\
F_e^* (0.8, 0.02) &= F(N_e^*; 0.1, -0.07) = -5.29 \\
\alpha_e^* (0.1, -0.07) &\approx 11.79\
\theta_e F_e^* (0.1, -0.07) &\approx 0.37 \\
\pi(N^*_e; 0.1, -0.07) - K - \theta_e F(N_e^*; 0.1, -0.07) &\approx 39.63 \\
\gamma_e^* (0.1, -0.07) &= 1
\end{align*}
\]

The profitability of the partnership is reduced primarily by the additional financing costs of raising non-voting equity, relative to the equilibrium values reported in equations (117) and (124). Net-debt is negative, $F_e^* (0.1, -0.07) = -5.29$, in this case and the partnership over-hires relative to profit maximization $50.38 > 50$. Nevertheless, only $93\% = (1 - \theta_e) * 100\% = (1-0.07) * 100\%$ of the net-debt, $-4.93$, reduces the fixed costs of the period 3 partnership. The remainder of the net-debt raised goes to its financing costs.
Consider the figure on the following page. The transparent partnership achieves its hiring objective by choosing the debt level where the black upward sloping $F(N; -0.07)$ function and lower gray hill-shaped function intersect. This is the aforementioned net-debt level of -5.29 and the hiring level of about 50.4. The partnership is much more profitable than a corporation. The corporate profit function with rational expectations fixed at $N^C(0.1) = N^e = 90.91$ peaks at a profit of merely 6.53. This was calculated in equation (125). Compare this with the profit function including financing costs for the partnership which peaks at 39.63. Since the partnership is the more profitable organizational form, it is the one that is selected in period 1. Therefore, $\gamma^*_e = 1$. 
The transparent partnership organizational form is greatly more profitable than the corporate form when market monitoring is low, \( \mu = 0.1 \). This is despite the fact that partnership must adjust its hiring incentives with costly non-voting equity. Under these parameter values, it is a perfect Bayesian equilibrium that the partnership form will be selected.

Figure 9
7.2.9 Opaque Capital Structure \((\rho = 0)\) and Costly Financial Adjustments

\[(\theta_i \neq 0)\]

When capital structure is hidden, the partnership will want to choose a hiring level that both takes into account the informational asymmetries between the clients and the firm and the financing costs of achieving that target level of hiring. The hiring level consistent with this objective is denoted \(N_i^C\). Capital structure in the opaque partnership is chosen to maximize the objective of maximizing total profits after financing costs. The objective function in equation (142) below is obtained from combining the objective function in equation (44) with the inverse demand function for this example in equation (104). That is,

\[
V_i^C = \pi(N, N^e; \mu) - c^{-1}(N) = \left\{ (1 - \mu)p(N^e) + \mu x \bar{a} - \frac{\mu x N}{2\sigma} (\bar{a} - \bar{a}) \right\} N - wN - K
- \frac{\theta_i}{1 + \theta_i} \left( \frac{\mu x (\bar{a} - \bar{a})}{2\sigma} \right) N^2 + \frac{\theta_i}{1 + \theta_i} K
\]

We can rule out the possibility that the firm will ever take on outside equity because the inverse demand function in equation (104) satisfies assumption 5.2 or the inequality in equation (49). Namely,
Therefore, according to our derivation in appendix section 7.1.11, this opaque partnership will never take on negative net-debt. This allows us to only focus on the case in which the opaque partnership raises positive net-debt, or debt. The equilibrium values below will have the subscript “\(d\)” to reflect the fact that the partnership only will take on debt.

The first order condition after expectations converge on the actual, \(N^c = N^C_d\), is the following:

\[
\frac{\partial V^c_d}{\partial N} \bigg|_{N^c = N^C_d} = x\bar{a} - w - \frac{1 + \theta_d}{2\sigma(1 + \theta_d)} x(\bar{a} - \bar{a})N^C_d = 0
\]  

(144)

The second order condition must be negative for this stationary point, \(N^C_d\), to be a maximum. It is unambiguously negative for all \(N\).

\[
\frac{\partial^2 V^c_d}{\partial N^2} = -\frac{1 + 2\theta_d}{1 + \theta_d} \mu x(\bar{a} - \bar{a}) < 0.
\]  

(145)

The stationary point, \(N^C_d\), implied by equation (144) is
The price for this level of output is

\[
p(N^C_d) = \frac{x\bar{a} - w}{(1 + \theta_d) + \mu(1 + 3\theta_d)}. \quad (147)
\]

Total profits before fixed costs and finance costs are

\[
\pi(N^C_d; \mu) = \frac{2\sigma \mu(1 + 3\theta_d)(1 + \theta_d)}{x(\bar{a} - a)} \left( \frac{x\bar{a} - w}{(1 + \theta_d) + \mu(1 + 3\theta_d)} \right)^2. \quad (148)
\]

The financing cost can be derived from combining equations (128) and (146).

\[
c^{-1}(N^C_d) = \frac{2\sigma \mu \theta_d(1 + \theta_d)}{x(\bar{a} - a)} \left( \frac{x\bar{a} - w}{(1 + \theta_d) + \mu(1 + 3\theta_d)} \right)^2 - \frac{\theta_d}{1 + \theta_d} K. \quad (149)
\]

Further, the choice of optimal capital structure depends on

\[
\frac{\partial c^{-1}(N^C_d)}{\partial N} N^C_d = \frac{2\sigma \mu \theta_d(1 + \theta_d)}{x(\bar{a} - a)} \left( \frac{z\bar{a} - w}{(1 + \theta_d) + \mu(1 + 3\theta_d)} \right)^2. \quad (150)
\]
The partnership with opaque capital structure will need to raise debt \( F_d^C \) equal to total profits after finance costs. Total profits after finance costs are

\[
F_d^C (\mu) = c^{-1} \left( N_p^C \right) / \theta_d = \\
\frac{2\sigma_\mu(1+\theta_d)}{x(\bar{\alpha} - w)} \left( \frac{x(\bar{\alpha} - w)}{(1+\theta_d) + \mu(1+3\theta_d)} \right)^2 - \frac{1}{1+\theta_d} K
\]

\[(151)\]

### 7.2.10 Opaque Capital Structure \((\rho = 0)\), Costly Debt \((\theta_d = 0.02)\), and High Levels of Market Monitoring \((\mu_H = 0.8)\)

Let us plug in the parameter values into equations (146), (147), (148), (149), and (151). When market monitoring is high the opaque partnership is not as profitable as a corporation. (To verify this compare profits after fixed and financial costs in equation (120) and (152).)

\[
N_d^C (\mu, \theta_d) = N_d^C (0.8, 0.02) \approx 54.60
\]
\[
p(N_d^C ; \mu, \theta_d) = p(N_d^C ; 0.8, 0.02) \approx 2.91
\]
\[
\pi(N_d^C ; \mu, \theta_d) = \pi(N_d^C ; 0.8, 0.02) \approx 49.58
\]
\[
F_d^C (0.8, 0.02) = F(N_d^C ; 0.8, 0.02) \approx 36.97
\]
\[
\alpha_d^C (0.8, 0.02) = 0
\]
\[
\theta_d F_d^C (0.8, 0.02) \approx 0.74
\]
\[
\pi(N_d^C ; 0.8, 0.02) - K - \theta_d F(N_d^C ; 0.8, 0.02) \approx 38.84
\]
\[
\gamma_d^C (0.8, 0.02) = 0
\]

\[(152)\]
Consider the following diagram that represents the profit and net-debt functions of the partnership and corporation, respectively. When market monitoring is high ($\mu = \mu_H = 0.8$) but financial structure is opaque ($\rho = 0$), the partnership cannot be more profitable than the corporation. The partnership would be more selective if formed, but is a perfect Bayesian equilibrium that it will not form, $\gamma_d^C = 0$. The black hill-shaped function represents the corporation’s profit function. It lies above the upper gray hill, which represents the partnership’s profit function net of financing costs. The profit maximizing hiring point for the opaque partnership is at the intersection of the lower gray hill and the $F(N; 0.02)$ function. This intersection determines the partnership’s optimal level of net-debt, 36.7, for its hiring target of 49.2.
The partnership organizational form is less profitable with costly debt than the corporate form. Under these parameter values it is a perfect Bayesian equilibrium that the corporate form will be selected.

**Figure 10**
7.2.11 Opaque Capital Structure ($\rho = 0$), Costly Debt ($\theta_d = 0.02$), and Low Levels of Market Monitoring ($\mu_L = 0.1$)

The situation is somewhat different when market monitoring is low, $\mu = \mu_L = 0.1$. In this case, the opaque partnership is more profitable than the corporation. The equilibrium values from equations (146), (147), (148), (149), and (151) are as follows

$$N^C_d(\mu, \theta_d) = N^C_d(0.1, 0.02) \approx 90.58$$
$$p(N^C_d; \mu, \theta_d) = p(N^C_d; 0.1, 0.02) \approx 2.19$$
$$\pi(N^C_d; \mu, \theta_d) = \pi(N^C_d; 0.1, 0.02) \approx 17.06$$
$$F^C_d(0.1, 0.02) = F(N^C_d; 0.1, 0.02) \approx 6.29$$
$$(\mu, \theta_d) = (0.1, 0.02) \approx 0.13$$
$$\gamma^C_d(0.1, 0.02) = 1.$$  \hspace{1cm} (153)

The opaque partnership is more profitable than the corporation when it must raise costly debt. Compare the profit after fixed and financing costs in equation (153) to the corporations’ profits from equation (120). Because $6.93 > 6.53$, then it is a perfect Bayesian equilibrium that partnership form will be selected over the corporate form, $\gamma^C_d(0.1) = 1$.

Consider the figure on the following page. They are obtained by combining the parameter values in (116) and (129) and the derivation of the inverse demand function in equation (104). The intersection of the black $F(N; 0.02)$ function, which is given by equation (127), and the lower gray function which defined in equation...
(47) where $N$ is allowed to vary, gives us the optimal level of net-debt of about 6.3. This level will induce the firm to maximize profits by employing 90.6 partners. The upper gray hill is the profit after fixed and financing costs for the partnership. This clearly lies above the black hill, which represents the corporation’s profit function. As in the previous variations of this example where $\mu = \mu_e = 0.1$, the corporation’s profit function peaks at 90.9 employees.
The partnership organizational form is more profitable with costly debt than the corporate form. Under these parameter values, it is a perfect Bayesian equilibrium that the partnership form will be selected.

Figure 11
Table 1 summarises the perfect Bayesian equilibrium outcomes for the eight different scenarios. Each scenario discussed in this example has a different set of \( \{\rho, \theta, \mu\} \). Variations in the transparency of finances in the partnership, the partnership’s cost of finance, and the level of market monitoring lead to substantial variation in the organizational form selected, the number of employees hired, the firm’s capital structure, and the firm’s profits.
Table 1: Summary of the Perfect Bayesian Equilibrium (PBE) Strategies and Payoffs for a Given Set of Parameter Values \({\{\rho, \theta, \mu}\} \)
Chapter III

Sunk Cost Efficiency with Identical Competitors
Sunk Cost Efficiency with Identical Competitors

by

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Abstract

When entrants only differ in their exogenous entry costs, the order in which potential firms enter does not affect industry size. In contrast, the entry ordering can affect the level of sunk costs in the industry when firm size is positive. A couple of discrete examples are pursued to illustrate these results. When firm size approaches zero, it is shown that, for all possible entry and exit orderings, only the lowest fixed cost firms will enter and only the highest opportunity cost firms will exit.

Journal of Economic Literature Classifications: L11 & L13

Keywords: Sunk Costs, Entry, Market Structure
1.0 Introduction

In an entry game, the sequence by which potential firms are given the opportunity to enter can potentially affect aggregate welfare, output and the cost structure of the industry. In this paper, we consider a sequential game where firms have identical variable cost functions. Actual and potential competitors only differ in their exogenous, sunk entry costs. Potential competitors play a free entry game where the sequence of potential entrants is common knowledge. Each firm is given the opportunity to pay their entry cost or stay out of the industry forever. Then the firms compete and collect their profits before investment costs in the last stage.

We will see that the sequencing of entry decisions does not affect the size of the industry. Yet, the ordering of entry potentially affects the magnitude of fixed costs incurred. This paper demonstrates that the entry ordering in which the lowest fixed cost competitors are allowed to enter first is always weakly the most efficient. (Efficiency means here that total entry costs are minimized.) Sufficient conditions for efficiency are established. Further, it is shown that, when firm size approaches zero, all entry orderings will minimize sunk costs. Random shocks to profits and the possibility of exit do not change the result that when firm size approaches zero the lowest sunk cost firms enter. Further, it is shown with a continuum of firms that that the highest opportunity cost firms also exit the industry, leaving only the most efficient competitors when the profits from competition are low.
The author knows of no other model of this type. While the structure is relatively simple, it appears that this setup has been ignored for more complex endogenous sunk cost models of entry. Models that have exogenous sunk costs for potential entrants usually assume that entrants have identical sunk costs. Sutton (1991, pp. 28-37) and Mankiw and Whinston (1986) are examples of such games. The ordering of entry does not affect welfare in these models because all firms have identical sunk entry costs. In contrast, here potential entrants differ in their exogenous sunk entry costs. Therefore, some firms can enter at lower cost than their rivals.

Further, there are many papers exploring entry models where firms endogenously determine entry costs. Under the latter setup, the entrant, for example, chooses to sink some investment, which can be potentially used to deter subsequent entrants, Spence (1977) and Dixit (1980) on capacity commitments are examples of this approach. These endogenous sunk cost models, as Sutton (1991) characterizes them, generally focus on the strategic choices of firms, as opposed to rivals’ innate capabilities. Indeed, there are many variants on the theme of endogenous sunk costs which affect the entry decisions of rivals and/or the intensity of competition in the final stage of the sequential game. Shapiro (1989), for example, provides a review of the large endogenous entry cost literature.

Roberts has a similar model to the discrete entry models in this chapter. Yet, Roberts (2007) has some key differences. First, that paper endogenizes the opportunity cost of entry while the entry cost is exogenously given in the present chapter. Second, the model of Roberts (2007) assumes that entry decisions are taken
simultaneously. In contrast, this paper assumes that entry decisions are sequential. Many of the inefficiencies in Roberts (2007) disappear when sequential entry decisions are made in this chapter. The relative efficiency of the sequential results here when compared to the results in Roberts (2007)’s simultaneous move game is not surprising. Mixed strategy equilibria found in simultaneous move games often lead to inefficiencies even when players wish to behave cooperatively.

Chapter IV as in the present paper has a model of exogenous entry where competitors differ in the sunk entry costs. Yet, the game in Chapter IV only considers one entry ordering and only considers the limiting case of the present model. In this paper, we consider all possible entry orderings and both the discrete and limiting (or continuous) cases and attempt to make statements about all feasible entry orderings. This paper confirms that the results in Chapter IV are robust to alternative entry orderings.

Here the exogenous heterogeneity in sunk costs of entry does not improve a firm’s ability to compete in the final stage of the game. Instead, lower setup costs makes entry more attractive (and socially efficient) for some firms. This allows some firms to benefit from exogenous comparative advantages over their rivals. This paper asks, “To what extent can we expect that the talented, low entry cost firms will drive away the less efficient, high entry cost firms?” This paper attempts to find out under what circumstances path dependence—the ordering of entry—can affect the efficiency of the industry (the industry’s aggregate sunk costs.)

The paper proceeds as follows. In section 2, we describe the model. In section 3, sufficient conditions for all entry orderings to be efficient are found for the
discrete case. In section 4, a couple of Cournot examples are pursued to illustrate when the ordering of entry does and does not affect sunk cost efficiency. In section 5, we consider the continuous case. When firm size approaches zero, we prove that only the lowest sunk cost competitors enter. This continuous case with infinitesimal competitors is generalized to include random demand and the possibility of exit. It is shown in this more general setup that the industry that entry, and exit, will be sunk cost efficient. In section 6, the paper concludes.
<table>
<thead>
<tr>
<th>Period -1</th>
<th>Period 0</th>
<th>Period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordering of potential entrants is determined and becomes common knowledge.</td>
<td>• Firms sequentially choose to enter. • Entrants pay the sunk costs of entry.</td>
<td>$N^<em>$ entrants each earn a payoff of $\pi(N^</em>)$.</td>
</tr>
</tbody>
</table>

Figure 1: Sequence of Events
2.0 Model

Let us begin by describing the game outlined in the figure above. The order by which firms of a given rank are allowed to enter is known by all firms in period -1. Further, all previous entry decisions are common knowledge as they occur in period 0. In period 1, entrants compete without regard to their exogenous sunk costs. In period 1, all entrants receive an identical payoff before entry costs, which is only a function of the $N^e$ firms that enter. Let us define $\pi(N^e)$ as the payoff before entry costs for a unit-sized firm.

A firm that enters pays a sunk entry cost, $K_i$, which depends on the firm’s rank $i$, where $i \in F = \{1, 2, 3, \ldots, n-1, n\}$. The set of firms ranks $F \subset \mathbb{N}$. The number of elements of this set is $n(F) = n$, where $n \in \mathbb{N}$. The investment costs, $K_i$, are increasing in the firm’s index number, $i$. That is, $K_1 < K_2 < K_3 < \ldots < K_n$. Let us assume that per firm payoff, $\pi(N^e)$, declines in the number of entrants. That is, $\pi(1) > \pi(2) > \ldots > \pi(n-1) > \pi(n)$.\(^2\)

To ensure that it is subgame perfect Nash equilibrium (SPE) that at least one firm enters let us assume that it is profitable for the lowest cost firm to always enter.

$$\pi(1) - K_1 \geq 0$$ (1)

---

\(^1\) Because entry costs are sunk, this paper precludes the possibility of the hit-and-run equilibria of contestable markets described by Baumol (1982).

\(^2\) An example of a game with these properties would be a Cournot game where all entrants had identical variable cost functions. This example is pursued in section 4 and the appendix in section 7. (Competition is often a negative sum game from the perspective of firms in an industry.) Any zero or negative sum partnership game would have the property that payoffs per player decline in the number of players.
Further, if all firms enter, the highest cost firm in the set $F$ will find entry unprofitable.

$$\pi(n) - K_n < 0. \tag{2}$$

Suppose that the ordering of entry is a permutation of the set $F$. The entry set, $E$, consists of $n(E) = N^e$ firms. If there is a potential entrant of rank $i$, it must be the case that $i \in F$. Further, if the firm of rank $i$ enters, $i \in E$. Since both $N^e$ and $n$ are integers, the set $E$ can consist of any ordering of the firms in set $F$ such that the $E \subseteq F$. There is an upper bound of

$$M(n, N^e) = \frac{n!}{(n-N^e)!} \tag{3}$$

possible permutations of entry orderings. Nevertheless, there will only be at most $C(n, N^e)$ different combinations of equilibrium entry sets for any random ordering of entry.

$$C(n, N^e) = \frac{n!}{N^e!(n-N^e)!} \tag{4}$$

Yet, there will only be one (unordered) entry set that is part of the subgame perfect Nash equilibrium (SPE) for any entry sequence.
3.0 Analysis

In this section we will prove three propositions regarding free entry and sunk cost efficiency when firms are of identical but discrete size. First, we demonstrate that the industry size does not depend on the entry ordering. Then, we will derive sufficient conditions to achieve sunk cost efficiency in the industry. In particular, the lowest fixed costs firms will enter when these lowest ranked firms have the opportunity to enter first—proposition 2—or when the higher fixed cost firms will not credibly enter or cannot block lower fixed cost firms from entering—proposition 3.

The implication of the inequalities in (1) and (2) is that some firms enter and some firms stay out of the industry. Let us define the free entry number of firms by the following set of conditions:

\[
\pi(N^e) - K_{N^e} \geq 0 \\
\pi(N^e + 1) - K_{N^e + 1} < 0
\]  \hspace{1cm} (5)

This set of entry conditions is similar to Sutton (1997). That paper argues that first firms will only enter if they expect to break even. Second, there will be no arbitrage opportunities where firms stay out of the industry when they could profitably enter.

Proposition 1
The ordering of potential firms will not affect the number of entrants.

From equation (5), we know that all firms with index numbers \(0 \leq i \leq N^e\) will enter if no one else does. Further, these low index number firms, \(i \leq N^e\), might even enter if the industry has greater than \(N^e\) firms. To get more than \(N^e\) entrants, it is necessary that the firm of rank \(N^e + 1\) or some higher ranked firm enters also. The free entry condition in (5) precludes this possibility because \(\pi(N^e + 1) < K_{N^e+1}\). If the firm with the index number \(N^e + 1\) will not enter an industry which will have \(N^e + 1\) entrants, then no other higher index number firm with an index number \(N > N^e + 1\) will enter under such circumstances. Q.E.D.

For any given equilibrium industry size, \(N^e\), efficiency minimizes the entry costs incurred to reach that industry size. Therefore, efficiency under free entry depends on the identity of who enters not just on the number of firms entering, \(N^e\). Total sunk cost of entry are \(\sum_{i \in N} K_i\). Since sunk costs are increasing in firms index numbers, sunk cost efficiency means that only the \(N^e\) firms with the lowest index numbers, \(i \leq N^e\), enter.

One way to achieve sunk cost efficiency is summarized in the proposition below:

**Proposition 2**
Consider all the entry permutations in which firms with index numbers \( i \leq N^e \) are the first \( N^e \) firms allowed to enter. These permutations will always lead to the most efficient entry set, where \( E = \{ \| 0 \leq i \leq N^e \} \).

First, let us show that welfare will be the highest if lowest ranked \( N^e \) firms enter. Let us define the set \( E = \{ \| 0 \leq i \leq N^e \} \). That is, firms with index numbers 1, 2, 3 \ldots, \( N^e - 1 \), and \( N^e \) enter. Total investment costs are \( \sum_{i \in E} K_i \). We need to show that if we replaced any firm with rank \( i \leq N^e \) with any firm of rank \( i > N^e \) that the total entry costs would unambiguously rise. Suppose that we replaced the highest ranked firm of the first \( N^e \) firms to enter, firm \( N^e \), with the next highest ranked firm, firm \( N^e + 1 \).

Total investment costs prior to the replacement would be

\[
\sum_{i \in E} K_i + K_{N^e} - K_{N^e}.
\] (6)

After the replacement, total investment costs will be

\[
\sum_{i \in E} K_i + K_{N^e+1} - K_{N^e}.
\] (7)

Clearly, equation (6) is less than equation (7) because \( K_{N^e} < K_{N^e+1} \). Since the firm ranked \( N^e + 1 \) is the lowest cost firm with an index number \( i > N^e \) it must be the case
that only the firms with index numbers at or below $N^e$ must minimize fixed costs.

This is what we wanted to show in the first instance.

Second, we need to demonstrate that all the entry permutations where the first $N^e$ firms are allowed to enter lead to all entrants having index numbers $i \leq N^e$. Firms, 1, 2, ..., $N^e$ first decided to enter or stay out of the industry. (They need not be given the opportunity to enter in any particular order.) Recall the definition of the free entry in equation (5). All these firms will have investment costs below the per firm profits, $\pi(N^e) - K_i \geq \pi(N^e) - K_{N^e} \geq 0$, because $i \leq N^e$ or $\forall i \in E$. In addition, after the first $N^e$ competitors have entered, no more firms will enter. Using second inequality in equation (5), we know that $\pi(N^e + 1) - K_{N^e+1} \leq \pi(N^e + 1) - K_{N^e+1} < 0$ when $i > N^e$
or $\forall i \notin E$. All firms with index numbers $i > N^e$ will find further entry unprofitable. 

Q.E.D.

**Proposition 3**

All possible orderings of entry will be efficient if one of the two bullet points holds:

- For a given number of entrants, $N^e$, it is sufficient that social welfare in this entry game will always be at its maximum if $K_{N^e+1} > \pi(N^e)$.

- A weaker sufficient set of conditions for sunk cost efficiency requires that $K_{N^e+1} \leq \pi(N^e)$ and $\pi(N^e + 1) \geq K_{N^e}$.

Consider the first bullet point. We know from the free entry condition in (5) that the $N^e$-th firm (and thus all lower index number firms) will enter unless it is (or
they are) blocked by another firm. Yet, even if the \((N^e + 1)\)-th firm could block the 
\(N^e\)-th firm, this firm will not want to do so because \(K_{N^e+1} > \pi(N^e)\). Because \(K_{N^e+1} < \ K_{N^e+2} < \ldots < K_n\), this must also be true for all firms of rank \(N > N^e + 1\), as well. Therefore, only firms with index numbers of \(i\), where \(0 < i \leq N^e\), will enter. \(Q.E.D.\)

Now, we can consider the second bullet point. The \((N^e + 1)\)-th will find entry profitable if it can deter a firm of lower rank from entering. This is necessary for \((N^e + 1)\)-th firm to enter because the second condition in equation (5) specifies that \(\pi(N^e + 1) < K_{N^e+1}\). Consider all the firms of rank \(i \leq N^e\) that the \((N^e + 1)\)-th firm could discourage from entering the industry. Firm \(N^e + 1\) will have the best chance of deterring the \(N^e\)-th firm from entry. Yet, if firm \(N^e + 1\) enters before firm \(N^e\), it cannot deter firm \(N^e\) from entering by its entry alone if firm \(N^e\) is profitable in an industry of \(N^e + 1\) firms. Yet, if the firm with the index number \(N^e + 1\) enters, it knows that the firm ranked \(N^e\) and all lower ranked, and thus lower fixed cost, firms will enter also. An industry of size \(N^e + 1\) with \(N^e + 1\) firms will lead to strictly negative profits for the \((N^e + 1)\)-th firm if it enters. Therefore, the \((N^e + 1)\)-th firm will stay out if \(K_{N^e} \leq \pi(N^e + 1)\), regardless of the entry permutation. All higher \(i\) firms, \(i > N^e + 1\), will also stay out regardless, of the entry ordering because the \(\pi(N^e + 1) < K_{N^e+1} < K_{N^e+2} < \ldots \ Q.E.D.\)
4.0 Examples

Here we will explore two numerical examples that illustrate the concept that ordering does not affect the number of entrants when competitors have identical payoffs after entry, \( \pi(N^e) \). Nevertheless, ordering does potentially affect welfare in the discrete case as higher fixed cost competitors may enter and preclude lower fixed cost firms from entering. In each example, the set of potential entrants is \( F = \{1, 2, 3\} \). Therefore, we have three firms, \( n(F) = n = 3 \), that are contemplating entry. In both examples, we will find that two firms, \( N^e = 2 \), will enter the industry in equilibrium. Nevertheless, welfare would always be weakly higher under free entry if the entry order was such that the lowest fixed cost firms moved first.

In both examples, two firms will find entry optimal. Yet, in the first example, there are \( C(n, N^e) = C(3, 2) = 3 \) potential subgame perfect Nash equilibrium (SPE) entry combinations—\( E_1 = \{1, 2\}, E_2 = \{2, 3\}, \) and \( E_3 = \{1, 3\} \). In contrast, in the second example, the 3rd ranked firm cannot deter the entry of the 2nd ranked firm. Therefore, as in proposition 3, the most efficient free entry equilibrium combination—

\[ E = E_1 = \{1, 2\} \]

is the SPE, regardless of entry ordering.

The Nash equilibrium outputs, producer surplus, consumer surplus and welfare for a given number of entrants are derived in the appendix. Firms are assumed to play a simultaneous Cournot game in period 1.
Under the parameters suggested in equation (26) in the appendix and the formula for the per firm payoff before entry costs in equation (21) in the appendix, we know that per firm producer surplus as a function of the number of entrants will be

\[ \pi(1) = 25 \]
\[ \pi(2) = 11 \frac{1}{9} \]
\[ \pi(3) = 6.25. \]  \hspace{1cm} (8)

Inserting in the parameter values in (26) into equation (24) in the appendix, consumer surplus is the following, depending on the number of entrants:

\[ CS(1) = 12.5 \]
\[ CS(2) = 22 \frac{2}{9} \]
\[ CS(3) = 28.125 \]  \hspace{1cm} (9)

4.1 Example 1

\( K_1 = 7 \), \( K_2 = 8 \), and \( K_3 = 9 \).

Here free entry dictates that only two firms enter. \( N^e = 2 \) as defined by equation (5). That is, \( K_2 = 8 < \pi(2) = 11.\bar{1} \), and \( K_3 = 9 > \pi(3) = 6.25 \). Yet, the most efficient entry set is not the only possible SPE when all entry orderings are possible. We know this because \( K_3 = 9 < \pi(2) = 11.\bar{1} \), and \( \pi(3) = 6.25 < K_2 = 8 \). Therefore, neither bullet point in proposition 3 is satisfied. Indeed, there is no ordering where
the 3rd ranked firm is given the first or second opportunity to enter where the last firm, of either rank 1 or 2, will credibly enter. That is, both $K_1 = 7 > \pi(3) = 6.25$ and $K_2 = 8 > \pi(3) = 6.25$. Therefore, if entry sequences are independent and identically distributed (i.i.d.) one-third of the time the first-best free entry set, $E = E_1 = \{1, 2\}$, will enter, generating social welfare of 29.4; one-third of the time, the second-best set of firms will enter, $E_3 = \{1, 3\}$, generating social welfare of 28.4; and one-third of the time the worst set of firms will enter, $E_2 = \{2, 3\}$, generating social welfare of 27.4. Therefore, if entry orderings are i.i.d., then expected welfare is 28.4, which is lower than 29.4, the social welfare if the lowest ranked firms entered first.3

4.2 Example 2

$K_1 = 5$, $K_2 = 6$, and $K_3 = 10$.

Here, too, free entry dictates that only two firms enter. That is, $K_2 = 6 < \pi(2) = 11.1$, and $K_3 = 10 > \pi(3) = 6.25$. The 3rd ranked firm with fixed costs $K_3 = 10$ would want to enter. Yet, it would lose money if three firms entered. The 3rd ranked firm cannot deter entry by the lower ranked firms. With these fixed costs for potential entrants, only the most efficient free entry equilibrium is possible. That is, the entry set $E = E_1 = \{1, 2\}$ is the SPE, regardless of entry ordering. We know this is the case because the sufficient condition from bullet point two in proposition 3 is met.

3 Mankiw and Whinston (1986) argue that sometimes free entry leads to excessive entry. This is the case here. If entry was regulated by a social planner in this example, then only the 1st ranked firm would be allowed to enter and social welfare would be 30.5. This is higher than the first-best free entry equilibrium, which generated welfare of 29.4.
Namely, $K_3 = 10 < \pi(2) = 11.\bar{1}$. Yet the 3rd firm cannot deter entry of the second firm because $\pi(3) = 6.25 > K_3 = 6$. In this case, social welfare is $33.\bar{4}$ under free entry or when entry is regulated by a social planner.
5.0 Epsilon Competitors and the Continuous Case

We will consider a general model where the size of competitors can be quite small, or large, relative to the markups in the industry. Both Mankiw and Whinston (1986) and Chapter IV consider a continuous model where infinitely small firms may enter. (While Mankiw and Whinston (1986) has all firms incurring identical sunk costs, Chapter IV indexes the firms with sunk costs increasing in a firm’s index number as we do in this paper.)

The number entrants need not be the only factor determining the intensity of competition. The intensity of competition may not be identical across different industries which have identical numbers of entrants. Consider two industries with ten and two identical firms, respectively. The Herfindahl index (HHI) for these two industries would be 0.5 and 0.1 respectively. Industries with higher HHI’s are generally viewed to be less competitive. Nevertheless, the HHI measures industry concentration, not industry markups over marginal costs. In our example of the industries with ten and two competitors, respectively, the markups over marginal

\[ \text{HHI} = \sum_{i=1}^{Z} s_i^2 \]

4 The Herfiundahl index is calculated as the sum of the squares of the market shares of all the firms in the industry. Let \( s_i \) = market share of total industry sales and \( Z \) = the total number of firms in the industry of the \( i \)-th firm. The HHI = \( \sum_{i=1}^{Z} s_i^2 \). In the first case, the HHI = 2*(1/2)^2 = 0.5. In the second case, HHI = 10*(1/10)^2 = 0.1. The Herfindahl index (HHI) cannot exceed 1 and cannot fall below 0. Higher Herfindahl indexes are supposed to be associated with less competitive industries with few competitors. The U.S. Department of Justice considers any industry with a Herfindahl index higher than 0.18 to be concentrated. Further, any proposed merger that raises the industry’s HHI by 0.01 would “raise antitrust concerns” according to their website http://www.usdoj.gov/atr/public/testimony/hhi.htm. This is codified in the U.S. Department of Justice’s Merger Guidelines section 1.5 available at http://www.usdoj.gov/atr/public/guidelines/horiz_book/15.html.
costs in the two industries may be identical for a variety of reasons that are too difficult to put into a parsimonious model.

Further, many insights that can be easily gained from calculus are more involved and less transparent if the number of firms must be an integer. Calculus requires that the variable of interest be continuous. In this case, that would mean that entrants are infinitesimally small. In such a case, the Herfindahl index (HHI) would equal zero. Since at least Frank (1965) it has been recognized that the Cournot model predicts that, as number of firms approaches infinity, prices converge to marginal cost. It would be useful to find a way to allow us to use calculus to characterize entry games (as Mankiw and Whinston (1986) does) without having to say this implies that price equals marginal cost. For this reason, in this section we will have two different measures of the number of entrants. The first measure is the number of unit-sized competitors \( N_e \).

What is unique to this section is that we have a second measure. This is the number of epsilon, \( \epsilon \geq 0 \), competitors \( N^\epsilon / \epsilon \), where \( \epsilon \in [0, N^\epsilon] \). While the intensity of competition and the markups over marginal cost are determined by the number of unit-sized competitors, the actual number of firms entering is the number of epsilon competitors \( N^\epsilon / \epsilon \). Total producer surplus in such an industry is

\[
\sum_{i=1}^{N^\epsilon / \epsilon} \epsilon_\pi(N^\epsilon) = \frac{N^\epsilon}{\epsilon} \epsilon_\pi(N^\epsilon) = N^\epsilon \pi(N^\epsilon). \tag{10}
\]

The Cournot model can be seen as a special case of the model presented here where \( \epsilon = 1 \). A unit-sized competitor is equivalent to \( 1/\epsilon \) epsilon competitors. One
interpretation of this, when \( N^\varepsilon \) is an integer, would be to view the \( 1/\varepsilon \), firms as a collusive group of firms that jointly agree upon output. For example, if there were 10 firms for an industry of size \( N^\varepsilon = 2 \) then \( \varepsilon \) must be 0.2. In this case, two groups consisting of five firms \((1 ÷ 0.2 = 5)\) would behave as a collusive group. Each group of five firms would jointly determine output. Suppose that the two groups of five then would play a Cournot game. The resulting output would be identical to the case where there were two firms playing a Cournot game. Thus, the Cournot model can be seen as a special case of the model presented here where \( \varepsilon = 1 \). In the limit, as the size of each epsilon firm approaches zero, the number of epsilon competitors approaches infinity, \( \lim_{\varepsilon \to +\infty} N^\varepsilon / \varepsilon = +\infty \). This may be a reasonable approximation when the number of competitors is very large in an industry, but prices do not converge to marginal cost.\(^5\)

When \( \varepsilon < 1 \), the industry has large markups over marginal costs, but firms have small shares of total revenues. When \( \varepsilon > 1 \) and \( \varepsilon \leq N^\varepsilon \), then industry is more competitive than the standard Cournot model would predict. Suppose that \( \varepsilon = 2 \) and \( N^\varepsilon = 2 \) then there would only be one firm, but it would produce the output of two Cournot duopolists.

To be more consistent with the notation of Chapter IV and to prepare the way for discussing the continuous case, let us define the entry cost as a function of a firms’ rank, \( N \). A firm that enters pays a sunk entry cost, \( \varepsilon I(N) \), which depends on

\(^5\)The author thanks Antoine Faure-Grimaud and Alan Morrison for this analogy. Nevertheless, this analogy of collusive groups is not meant as an explanation of why markups may exceed what is suggested by a Cournot-Nash equilibrium. That is, the author does not believe collusive groups are common. The collusive group analogy is used to explain how epsilon competition behaves, but it is not an explanation of why markups are as high or low as they are in equilibrium.
the firm’s rank, \( N \). (This replaces \( K_i \) of the discrete case discussed in the previous sections.) The investment costs, \( \varepsilon I(N) \), are increasing in the firm’s index number, or rank, \( N \). That is, \( \frac{\partial I(N)}{\partial N} > 0 \).

### 5.1 Entry in the Continuous Case

Let us consider the limiting case. If we divide through by \( \varepsilon \) and take the limit as \( \varepsilon \to 0 \), then the free entry conditions will converge to the single condition below:

\[
\pi(N^e) - I(N^e) = 0.
\]

**Proposition 4**

*In the limit \( \varepsilon \to 0 \), there is only one feasible set of entrants, regardless of entry ordering. This feasible set is characterized by all firms with index numbers on the interval \( N \in [0, N^e] \) entering and all firms with index numbers \( N > N^e \) staying out.*

Suppose that a firm has \( N > N^e \). This firm has fixed costs in excess of per-firm profits under free entry. We know this because both \( \frac{dI(N)}{dN} > 0 \), and equation (11) holds with equality. Further, since firms are infinitesimal, they cannot affect the size of the industry. Further, this means that per-unit-sized-firm producer surplus, \( \pi(N^e) \), is unaffected by a single epsilon’s entry. That is, \( \pi(N^e) = \pi(N^e + \varepsilon) \). Therefore, any firm with an index number \( N > N^e \) will stay out. Alternatively, because both
\[
\frac{dI(N)}{dN} > 0, \text{ and equation (11) holds with equality, all firms with index numbers } N \leq N^e \text{ will find entry at least weakly profitable. } Q.E.D.
\]

As firms become smaller, the per-firm profits, \( \pi(N) \), move less as a new firm is added. Because of this, it becomes harder for a single higher ranked epsilon to block any lower ranked epsilon from entering. At the same time, higher ranked firms with \( N > N^e \) have entry costs that are strictly increasing in their rank. Therefore, in the limit higher ranked firms only have higher entry costs, but cannot affect per firm payoffs in period 1. This means that, in the limit, only the lowest cost epsilon firms enter.

If we move away from the limiting case of proposition 4 and if the sufficient conditions for both propositions 2 and 3 are not satisfied, then it is possible that some entry orderings will lead to higher fixed cost entrants entering and lower fixed cost entrants staying out.
Ordering of potential entrants is determined and becomes common knowledge.

- Firms sequentially choose to enter.
- Entrants pay the sunk costs of entry.

- State of profits are revealed.
- Ordering of exit is determined.
- Firms sequentially exit.
- Exiting firms receive a fraction of their entry cost $\gamma$.

**Figure 2: Sequence of Entry and Exit with a Continuum of Firms**
5.2 Exit in the Continuous Case with Random Payoffs

The continuous case also ensures that the firms with the highest opportunity costs of staying in the industry when demand is low. Suppose that we slightly alter the timeline in figure 1 at the beginning of this essay to the timeline in figure 2 above.

This is a more general game than is in Chapter IV, which has the lowest fixed cost firms enter and exit first. Yet, this timeline is similar to the one in Chapter IV because firms in both have the opportunity to exit after the state of demand is revealed. The results of this subsection will show that the game in Chapter IV is robust to alternative entry orderings and alternative distributions for the state of demand.

In this game the payoffs are random. In the preceding sections of the present chapter the payoffs for any given number of firms competing in period 1 was deterministic. Let us define \( s \) as the state and \( N^i \) as the number of unit sized firms competing in period 1. For any given number of competitors in period 1, \( N^i \), the per-firm payoff before entry costs, \( \varepsilon\pi(s, N^i) \), is random when firms must decide to enter in period 0. In period \( \frac{1}{2} \) firms learn their payoffs, \( \varepsilon\pi(s) \), for a given number of firms competing in period 1, \( N^i \). We have added an extra exit period of period \( \frac{1}{2} \) after firms learn the state of the world. Once they learn what the payoffs, \( \varepsilon\pi(s, N^i) \) for all values of \( N^i \), then firms have the opportunity to exit.

Let us suppose, when that the state space is smooth and continuous, it is characterized by the probability density function \( g(s) \) with the support \([\bar{s}, \tilde{s}]\).
States, $s$, are ordered such that profits before entry costs are increasing in the state. Higher states, $s$, are associated with higher producer surplus for any given $N^i$. If the distribution of states is continuous, this means that $\pi_s(s, N^i) = \frac{\partial \pi(s, N^i)}{\partial s} > 0$.

For a given number of firms competing in period 1, $N^i$, the net payoff after entry costs for the $N$-th epsilon competitor is $\epsilon \pi(s, N^i) - \epsilon I(N^i)$. An epsilon competitor can recover a fraction of its entry cost $\gamma \in [0,1)$. That is, the $N$-th exiting epsilon competitor receives $\epsilon \gamma I(N)$ for exiting. Any competitor will weight the opportunity cost of exiting with the expected benefit of staying in the industry. Dividing difference between staying in and exiting by $\epsilon$, the following condition determines the number of firms staying in the industry.

$$\pi(s, N^i) - \gamma I(N^i) = 0.$$ (12)

Alternatively, suppose that the state space is discrete. (Chapter IV has a discrete distribution of states. There are two states the high state and the low state.) Let $T$ be an integer greater than zero. There are $T$ total states. A given state is denoted by the subscript $j$. We will order the states, $s_j$, by the subscript $j = 1, 2, 3, \ldots T$. As in the continuous case, higher states are associated with higher per firm producer surplus. Higher index numbers, higher $j$’s, indicate higher states. That is, $s_1 < s_2 < s_3 < \ldots < s_T$. Per (unit-sized) firm producer surplus is increasing in the index number of the state, $j$, for any given number of firms, $N^*$, operating in period 1. That is, $\pi(s_1, N^*) < \pi(s_1, N^*) < \pi(s_3, N^*) \ldots \pi(s_T, N^*)$. The probability of the $j$-th state is $q_j$. 

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and \( \sum_{j=1}^{r} q_j = 1 \). When the states are discrete, the free exit condition is a single equality.

\[ \pi(s_j, N^i) - \gamma I(N^i) = 0 \]  

(13)

This proof is similar to proposition 4. Let us show that only the lowest ranked firms with \( N \in [0, N^1] \) will stay in the industry while the \( N > N^i \) indexed firm will prefer to exit in period \( \frac{1}{2} \). For a given industry size \( N^i \), all firms with rank \( N \leq N^i \) would weakly prefer to remain in the industry, because

\[ \pi(s, N^i) - \gamma I(N) \geq 0 \quad \forall N \leq N^i, \]  

when the states are continuously distributed, and

\[ \pi(s, N^i) - \gamma I(N) \geq 0 \quad \forall N \leq N^i, \]  

when the states are discretely distributed. Further, infinitesimally small firms cannot block the entry of other firms because \( \pi(N^i) = \pi(N^i + \varepsilon) \) when \( \varepsilon \to 0 \). Yet, firms of rank \( N > N^i \) have higher opportunity costs of staying in because \( \frac{dI(N)}{dN} > 0 \). Since both equations (12) and (13) bind with equality, higher opportunity costs \( \gamma d(N) > \gamma d(N^i) \) implies that all firms with rank \( N > N^i \) will exit. This ensures that the industry is only populated by the lowest opportunity cost competitors.  

Q.E.D.

**Proposition 5**

*In the limit \( \varepsilon \to 0 \), there is only one feasible set of remaining in the industry, after the state of profits has been revealed, regardless of exit ordering. This feasible set is*
characterized by all firms with index numbers on the interval \( N \in [0, N^1] \) entering and all firms with index numbers \( N > N^1 \) staying out.

This also implies that expected profits in period 0 are invariant to exit orderings revealed in period \( \frac{1}{2} \). Instead, expected profits prior to the state being revealed in period \( \frac{1}{2} \) only depend the distribution of the profit states. Therefore, we can characterize the number of firms in period 1 as a function of the state \( N^1(s) \) with a smooth distribution of states or \( N^1(s_j) \) with a discrete state space.

Let us define expected profits for the \( N \)-th entrant when the distribution of states is continuous. Let state \( s^* \) be the lowest state in which the highest ranked entrant, the \( N^e \)-th competitor, will stay in the industry in period \( \frac{1}{2} \) because \( N^1(s^*) = N^e \). At states \( s > s^* \) all firms will choose to remain in the industry and not exit. Expected profits for the \( N \)-th epsilon competitor given that it enters when the distribution of states is continuous is \( E\{\Pi_c(N)\} \).

\[
E\{\Pi_c(N)\} = \mathbb{E}\left[ \int_{s^*}^{s} g(s)\pi(s; N^1)ds + \gamma I(N) \int_{s}^{\infty} g(s)ds - I(N) \right] 
\]

(14)

Expected profits conditional on entry are declining in the firm’s rank, \( N \):

\[
\frac{\partial E\{\Pi_c(N)\}}{\partial N} = -\epsilon(1 - \gamma G(s^*)) \frac{\partial I(N)}{\partial N} < 0
\]

(15)
Because $G(s') \leq 1$ and $\gamma < 1$ and $\frac{\partial I(N)}{\partial N} > 0$, the sign of (15) is unambiguously negative. This means expected profits are declining in the firm’s rank $N$.

Expected profits are also declining in the firm’s rank if there is a discrete distribution of states. Let the highest state in which the $N$-th firm will exit is denoted by $1 \leq X(N) \leq T$. Let $X(N)$ be defined by the following set of inequalities:

$$
\pi(s_{X(N)+1}; N^1) - \gamma I(N) \geq 0
$$
$$
\pi(s_{X(N)}; N^1) - \gamma I(N) < 0
$$

(16)

In this discrete states case, expected profits for the $N$-th competitor are given by

$$
E\{\Pi_D(N)\} \text{ below:}
$$

$$
E\{\Pi_D(N)\} = E\left[ \sum_{j=1}^{X(N)} q_j \pi(s_j; N^1) + \gamma I(N) \sum_{j=1}^{X(N)} q_j - I(N) \right]
$$

(17)

We want to show that firms of higher rank, higher $N$, have lower expected profits than lower $N$ firms. There are two cases. In the first case, a firm with higher $N$ will pay a higher entry cost, but it will compete in all the same states that a lower $N$ firm. It is easy to show and very intuitive that in this first case expected profits are declining in rank. The higher rank firm pays a higher entry cost, but does not benefit any more than a lower ranked firm from its greater liquidation value. In the second case, the lower ranked firm competes in more states than the higher ranked firm because the higher ranked firm liquidates in at least one state in which the lower
ranked firm competes. Here, too, the higher ranked firm makes lower expected profits, but the proof is somewhat more involved. The intuition is that the higher ranked firm’s gains from liquidating in some low profit states is outweighed by the losses incurred, regardless of the state of having higher entry costs. A formal proof that expected profits decline with firm rank with a discrete state space is left for the appendix.

Since a single competitor cannot affect industry profits by his entry or exit decision when \( \epsilon \to 0 \), sunk cost efficiency can be proven if only the lowest ranked, lowest sunk cost firms, will enter. Equation (15) shows that expected profits decline in firm rank, \( N \), when the distribution of states is continuous; and, in the appendix section 7.2 equations (28) and (31), is shown that expected profits decline in firm rank when the states are finite and discrete. Therefore, only the lowest sunk cost firms, with a rank \( 0 < N < N^e \), will enter when there are random payoffs, exit, and infinitesimal firms. \( Q.E.D. \)

**Proposition 6**

*With random payoffs from competing in period 1 and infinitesimal competitors, \( \epsilon \to 0 \), only the lowest entry cost competitors will enter, regardless of entry ordering.*

This generalization of proposition 4 for the stochastic case with exit follows from expected profits declining in firm’s entry costs \( \epsilon I(N) \).

These results imply that the subgame perfect Nash equilibrium (SPE) level of sunk costs incurred in the entry game in Chapter IV is invariant to entry and exit.
ordering. This is because Chapter IV considers the limiting case where \( \varepsilon \rightarrow 0 \).

Therefore, the game in Chapter IV will always have the lowest sunk cost entrants entering for a given industry size \( N^\varepsilon \).
6.0 Conclusion

This paper has considered an entry game in which firms have identical payoffs upon entry but differ in their sunk entry costs. In this game, both the exogenously given cost functions and entry orderings are common knowledge. Under these conditions, entry ordering cannot affect the size of the industry or the number of competitors. Yet, entry orderings can sometimes affect the sunk costs in an industry. When firms have a positive size, the discrete case, higher fixed cost entrants can potentially block lower fixed cost potential competitors from entering. Sufficient conditions for all entry orderings to minimize fixed costs are derived when firms have a positive size. This paper has developed a couple of discrete examples that have illustrated when entry ordering does and does not matter.

Finally, when firms have a mass of zero, the continuous case, no firm can block more efficient competitors from entering. Therefore, in this limiting case, only the lowest sunk cost competitors enter the industry, regardless of entry ordering. This result holds when payoffs are random and entrants are allowed to exit after the state of demand is revealed.
References


7.0 Appendix

7.1 Deriving the Linear Cournot Model

Here we derive the linear Cournot model which is used to analyze the examples in section 4. \( q^e \equiv q(N^e) \) is per-firm output, which is a function of the number of entrants, \( N^e \). Total industry output is \( N^e q(N^e) \equiv Q(N^e) \equiv Q \). Inverse demand is defined as price as function of industry output, \( P(Q) \). Suppose that all firms are identical Cournot competitors who face a linear inverse demand curve \( P(Q) = a - bQ \). Further, all competitors have identical cost functions \( c(q) = c(q(N^e)) \), where \( c \geq 0 \) is the marginal cost parameter. Firms are assumed to play a simultaneous move Cournot game in period 1.

The Nash equilibrium per firm output for an industry with \( N^e \) identical competitors is

\[
q(N^e) = \frac{a - c}{b(N^e + 1)}.
\] (18)

Total industry output in equilibrium is

\[
Q(N^e) \equiv N^e q(N^e) = \frac{N^e}{N^e + 1} \frac{a - c}{b}.
\] (19)
The equilibrium price is

\[ P(N^e) = \frac{a + N^e c}{N^e + 1}. \]  

(20)

Per firm producer surplus for all entrants is

\[ \pi(N^e) = \left( \frac{a - c}{N^e + 1} \right)^2 \frac{1}{b}. \]  

(21)

If the \( N \)-th firm enters, its profits after sunk costs are

\[ \pi(N^e) - K_i = \left( \frac{a - c}{N^e + 1} \right)^2 \frac{1}{b} - K_i. \]  

(22)

Total profits for the industry are industry producer surplus, or payoff before sunk costs, \( \Pi(N^e) \), less total investment costs, \( \sum_{\forall i \in E} K_i \). That is, industry profits are

\[ \Pi(N^e) - \sum_{\forall i \in E} K_i = \left( \frac{a - c}{N^e + 1} \right)^2 \frac{N^e}{b} - \sum_{\forall i \in E} K_i. \]  

(23)

Total consumer surplus, \( CS(N^e) \), generated by this industry is
\[ CS(N^e) = \frac{bQ^2}{2} = \left( \frac{N^e(a-c)}{N^e+1} \right)^2 \frac{1}{2b}. \] (24)

Therefore total welfare, \( W(N^e; E) \), which is a function of both the size of the industry, \( N^e \), and the entry set, \( E \), is

\[
W(N^e; E) \equiv \Pi(N^e) + CS(N^e) - \sum_{\forall i \in E} K_i \\
= \left( \frac{a-c}{N^e+1} \right)^2 \frac{N^e}{b} \left( 1 + \frac{N^e}{2} \right) - \sum_{\forall i \in E} K_i. \] (25)

For the examples in section 4.0, the inverse demand intercept, \( a \), inverse demand slope, \( b \), and the marginal cost, \( c \), parameters for both examples are

\[
\begin{align*}
a &= 10 \\
b &= 1 \\
c &= 0.
\end{align*} \] (26)
7.2 Proof that Expected Profits Decline in Entry Costs when the States are Discrete

To prove that expected profits decline in entry costs when states are discrete, we will begin by showing that no firm will enter if it will find it optimal to exit in every state. Suppose that there is a firm of rank $N'$. If there is continuous distribution of states and if $\pi(s, N'(s)) < I(N') \forall s$, then the $N'$-th firm will exit regardless of the state of profits, $s$. With a discrete distribution of states, if $\pi(s_j, N'(s_j)) < I(N')$ for all $j$, then $N'$-th firm will exit in every state given that it enters. In both cases, if it enters, it will have expected profits of $-(1-\gamma)I(N') < 0$. Therefore, no firm that exits in every state will enter in equilibrium. This is because entering when exit is certain will guarantee negative profits. Q.E.D.

From now on we will confine ourselves to firms that will not exit in every state of demand, because such firms are the only firms that will enter in equilibrium.

Let us define a few terms. $X(N') \equiv k$, where $k$ is a non-negative integer.

That is, there are $k$ ordered states. For example, there are four or more states that the $N$-th firm will exit they are $j = 1, 2, 3, .. k$ where the $N^e$-th firm will exit. Let $X(N') \equiv k + h$, where $h$ is a non-negative integer, be the number of states in which the $N'$-th firm will exit. The firm ranked $N'$ exits in $h$ higher states than $N^e$-th firm. This is due to the fact that $N' > N^e$ and $I(N') > I(N^e)$, and thus the recovery value of the $N'$-th firms assets are higher, $\gamma I(N') > \gamma I(N^e)$. 

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Let us consider the case where \( h = 0 \) and both firms compete in the same number of states

\[
E\{\Pi_D(N^c)\} = \varepsilon \left[ \sum_{j=k+1}^{T} q_j \pi(s_j; N^l(s_j)) + \gamma I(N^c) \sum_{j=1}^{k} q_j - I(N^c) \right]
\]

\[
E\{\Pi_D(N')\} = \varepsilon \left[ \sum_{j=k+1}^{T} q_j \pi(s_j; N^l(s_j)) + \gamma I(N') \sum_{j=1}^{k} q_j - I(N') \right] \tag{27}
\]

when \( h = 0 \).

We want to show that the difference between these two expected values

\[
E\{\Pi_D(N^c)\} - E\{\Pi_D(N')\}
\]

is positive. If this difference is positive, then this implies that the \( N^c \)-th firm has a higher expected value from entering than the \( N' \)-th firm. To simply the expression of this difference, let us define the following parameter used in equation (28) below.

\[
\chi \equiv \sum_{j=1}^{k} q_j < 1 \]

\( \chi \) is less than 1 in the subgame perfect Nash equilibrium (SPE) because, as we demonstrated earlier, no firm would enter if it would exit in every state. The difference between the expected value of the lower ranked firm and the expected value of the higher ranked firm is positive in equation (28) because \( I(N') - I(N^c) > 0 \) and \( (1 - \chi) > 0 \).

\[
E\{\Pi_D(N^c)\} - E\{\Pi_D(N')\} = \varepsilon \left[ (1 - \chi \gamma)(I(N') - I(N^c)) \right] > 0 \tag{28}
\]

when \( h = 0 \).
Because the marginal entrant makes zero expected profits, equation (28) allows us to conclude that firms with higher rank than the marginal entrant will not enter if they operate in the same states in which the marginal entrant operates. \( Q.E.D. \)

Now we need to prove that higher ranked firms than the marginal entrant will not enter even if they liquidate in at least one state where the marginal entrant operates. Firms that liquidate in a given state are worth more than firms that stay in operation; therefore, conceivably, they could have higher expected value from entering. We will prove this is not the case in the second half of this proof. Suppose that the \( N' \)-th firm finds it optimal to liquidate in \( h > 0 \) states in which the \( N \)-th firm finds it optimal to operate. The expected value of entering the industry are as follows:

\[
E\{\Pi_d(N')\} = E \left[ \sum_{j=h+1}^{N'} q_j \pi(s_j; N^i(s_j)) + \gamma I(N') \sum_{j=1}^{h} q_j - I(N') \right]
\]

\[
E\{\Pi_d(N)\} = E \left[ \sum_{j=h+1}^{N} q_j \pi(s_j; N^i(s_j)) + \gamma I(N') \sum_{j=1}^{k+h} q_j - I(N') \right]
\]

when \( h > 0 \).

The difference between these two terms are as follows:

\[
E\{\Pi_d(N')\} - E\{\Pi_d(N)\} = E \left[ \sum_{j=h+1}^{k+h} q_j \pi(s_j; N^i(s_j)) + \gamma I(N') \sum_{j=1}^{h} q_j - \gamma I(N') \sum_{j=1}^{k+h} q_j - I(N') + I(N') \right]
\]

when \( h > 0 \).
In the equation that follows, we add and subtract \( \gamma I(N^e) \sum_{j=k+1}^{k+h} q_j \) to the right hand side of (30). Further, let us replace the summation of the probabilities of all the states in which the higher ranked firm is liquidated with the parameter \( \lambda \equiv \sum_{j=1}^{k+h} q_j < 1 \).

(We know that \( \lambda < 1 \) because no firm in equilibrium will enter if they are liquidated 100 percent of the time in period \( \frac{1}{2} \).) With these rearrangements it is clear that the marginal entrant of rank \( N^e \) has a higher expected value than any higher ranked firm that would be liquidated in \( h > 0 \) more states than the marginal entrant.

\[
E\{\Pi_p(N^e)\} - E\{\Pi_p(N')\} = \\
\varepsilon\left\{ (1 - \gamma \lambda) [I(N') - I(N^e)] + \sum_{j=k+1}^{k+h} q_j [\pi(s_{j}, N^i(s_{j})) - \gamma I(N^e)] \right\} > 0 \quad (31)
\]
when \( h > 0 \).

\( \pi(s_{j}, N^i(s_{j})) - \gamma I(N^e) > 0 \) for all state where \( j = k + 1, \ldots k + h \) because these are by definition states where the \( N^e \)-th firm finds it profitable to operate.

\( I(N') - I(N^e) > 0 \). Further, \( 0 \leq \gamma < 1 \) and \( 0 \leq \lambda < 1 \). Combining all these properties, we can conclude that the marginal entrant is more valuable in expectation than any higher ranked firm that liquidates itself in some higher states.

Equations (28) and (31) show that all firms of rank higher than \( N^e \) have a strictly lower expected value than the \( N^e \)-th firm in period zero. This is what we wanted to show. Q.E.D.
Chapter IV

Business Stealing and Bankruptcy
Business Stealing and Bankruptcy

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Abstract

Increased competition does not necessarily raise welfare when firms incur entry costs. Indeed, under fairly innocuous assumptions, discouraging entry in homogenous goods industries raises welfare. Because entry is usually determined by the willingness of investors to supply funds, bankruptcy courts have a role in discouraging overinvestment by their enforcement of financial contracts. The courts can move such an industry closer to the social optimum by lowering the debt capacity and thereby increasing the taxable income of potential entrants. This paper shows that firms have weakly insufficient incentives to exit in the homogenous goods case. In this context, bankruptcy courts can sometimes increase social surplus by shutting down positive net present value firms.

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Journal of Economic Literature classifications: G32, G33, K2, L11, L50

Keywords: bankruptcy, debt, entry, exit, investment, judges, taxes
1.0 Introduction

Increased competition does not always raise welfare. Competition gives rise to two externalities. A given firm raises its own profits by taking away some of the profits of rival firms. This is the “business stealing effect.” Generally, competition is seen as beneficial because it entails another externality. Competition raises consumer surplus. The latter externality always is more potent than the first when firms have no fixed costs. Yet, when setup costs are positive, there is no guarantee that increased competition raises total welfare. Indeed, under very bland assumptions, Mankiw and Whinston (1986) show that free entry of firms unambiguously reduces welfare when producers have homogenous products and face positive entry costs.¹

There is some empirical support for the phenomenon of excessive entry. Berry and Waldfogel (1999) argue that the free entry in radio markets in the United States is excessive. They find that entry is excessive if one only counts the benefits to firms in a given market—radio stations—and the benefits to direct consumers—advertisers. The listening public enjoys a free—but not commercial free—externality of radio programming. Berry and Waldfogel (1999) estimate that entry is excessive

¹ Spence (1976a) and Spence (1976b) considered this problem for the more general cases where firms can produce differentiated products. When one moves from strictly homogenous products to diverse product lines all with their unique fixed costs of entry, it becomes ambiguous whether or not free entry produces too much entry (when products are identical) as in Mankiw and Whinston (1986) or too little entry (when products are differ slightly in the mind of consumers). The present paper only considers the unambiguous case of identical products.
in that industry if radio listeners valued their listening at less than $0.15/hour.\textsuperscript{2} Hsieh and Moretti (2003) exploit the fact that real estate broker’s commissions are a fixed 6 percent of the selling price of a house. They find that real estate broker incomes vary little across communities. In cities with high real estate prices, and high commissions per house sold, there are many more brokers as a percent of the houses sold or the urban population. These brokers in communities with high prices for real estate sell fewer houses. Further, buyers find it no easier to buy a house. They point to these observations to argue that excessive entry drives out profits for brokers without raising consumer surplus. Additionally, Hortacsu and Syverson (2004) point out that in 2000 there were 82 different retail S&P 500 index funds. Compounding potential deadweight losses from the duplication of these nearly identical investment vehicles, they find a high degree of price dispersion among these funds. They estimate that, after the factoring in the consumer search costs, the duplicated management fees, and the potential gains from product variety, that welfare could be substantially improved by monopoly provision of the S&P 500 index fund!

Not only is entry excessive, but here we demonstrate that there is weakly too little exit. Amir and Lambson (2003) in an infinite period model find that exit is also insufficient when exogenous shocks to demand or input prices make the industry less profitable. Here we show that there is weakly too little exit in an entry game with one productive period. In this paper, the real option to wait for conditions to improve has

\textsuperscript{2} This is estimate is for 1993 dollars.
no value because conditions will not improve. Therefore, the insufficient exit result here is independent of any concerns about sunk costs and hysteresis.³

Implicit in the objective of maximizing the going concern surplus of bankrupt firms is the idea that financial contracts should be enforced in a way that encourages investment. For example, Schwartz (2005) argues that bankruptcy institutions should be designed to minimize firms’ cost of capital. Firms that face lower costs of capital face lower costs of investment. This leads to increased investment. Crucial in this argument is the notion that increased investment unambiguously increases welfare.

Here we point out that increased, positive net present value (NPV) investment may actually reduce overall welfare. Suppose that competition and entry is unregulated except for the bankruptcy institutions in place. Under these circumstances, bankruptcy courts that increase the cost of borrowing may raise welfare both ex post and ex ante. When firms produce homogenous goods and must incur investments prior to competition, bankruptcy judges that attempt to maximize the ex post value of firms will encourage overinvestment in the industry. Ex ante such objectives will lead to the excessive duplication of fixed costs and decrease welfare in expectation.

There is a large literature on how capital structure and the enforcement of financial contracts affects the level of competition in an industry.⁴ In contrast to the present paper, these studies have generally taken the number of competitors as given.

³ Hysteresis is defined by Dixit (1989, p. 622) as “the failure of an event to reverse itself after the underlying cause has reversed.” Dixit (1989) explains why uncertainty and sunk costs will bias firms towards waiting before exiting an industry.
⁴ Unfortunately, the present discussion of this literature is not exhaustive.
Theoretical studies such as Brander and Lewis (1986), Brander and Lewis (1988), and Maksimovic (1988) argue that debt increases product market competition. In contrast, empirical work by Chevalier (1995a), Chevalier (1995b), Phillips (1995), Opler and Titman (1995), and Kovenock and Phillips (1997) finds that highly leveraged competitors are slower to expand output and invest. In contrast to the aforementioned work on competition and capital structure and the present study, several papers have focused on informational asymmetries between creditors and debtors. In Fundenberg and Tirole (1986), Bolton and Scharfstein (1990), and Faure-Grimaud (2000) informational asymmetries mean that low current profits or postponed interest payments will lead to credit constraints for the debtor. Because the debtor must pass up positive net present value projects, these studies predict that highly levered firms will be less aggressive competitors relative to one hundred percent equity-financed firms.

Allen (2000) is also consistent with the empirical studies of capital structure and product market competition because that paper argues that highly leveraged firms are less aggressive competitors. Both that study and the present paper argue that highly levered competitors trade off the tax advantages of debt with the cost of forgone profit opportunities. In Allen (2000), there are two identical Cournot competitors in the first round of competition. In equilibrium, for some parameter values, one firm chooses the tax benefits of leverage while the other firm chooses more efficient investment decisions. The highly leveraged firm becomes a

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5 Debt unambiguously makes firms produce more in Brander and Lewis (1988) when there is a large fixed cost associated with entering bankruptcy. Whether or not more debt makes firms tougher competitors is less clear when bankruptcy costs are proportional to the size of the default. These predictions are not borne out well by the empirical studies mentioned above.
Stackelberg follower if it ends up in bankruptcy at the end of the first round. If reinvestment needs are large enough, the bankrupt firm will be liquidated.

There are some important differences between the present study and Allen (2000). While Allen (2000) takes the number of competitors at the beginning of the game as given, the present paper says that entry is endogenously determined. The number of entrants here is a function of the expected costs and benefits of leverage. Moreover, not only do the modeling assumptions differ, but also both papers differ in their focus. Allen (2000) explains why a highly leveraged firm would be at a competitive disadvantage. In contrast here, we wish to explore how the enforcement of financial contracts by bankruptcy courts can shape the social efficiency of investment at the industry level.

As mentioned earlier, the major difference between the earlier theoretical studies of competition and capital structure and the present paper is that the former generally take the number of potential competitors as given. In contrast, here the number of entrants is endogenous and determined by more-or-less free entry. Moreover, here there are no assumed barriers to entry. In this study, debt reduces taxes and makes entry more attractive, but, for some liquidation values, this leads to some highly leveraged, high-liquidation-value competitors exiting when they would earn higher returns from staying in the industry. Therefore, it is largely consistent with the aforementioned empirical studies, which say that highly leveraged firms are more likely to contract output. Here highly leveraged “risky debt” firms will
sometimes be liquidated with low demand realizations. In equilibrium, “safe debt” firms will never be liquidated and thus will always compete.⁶

In many ways, this study has more in common with Cestone and White (2003) and many recent empirical studies of how financial intermediaries affect entry and thus the level of competition in the industry.⁷ Like these studies we argue that entry is regulated by how many firms can finance their entry into the market. In contrast, here it is the enforcement ex post of financial contracts through the bankruptcy courts, not competition among the suppliers of credit, banks and equity investors, which affects the equilibrium entry and exit in the industry. Further, unlike these studies that generally assume that more vigorous product market competition increases welfare, here we focus on the case where the duplication of fixed costs is more harmful than the gains from increased competition.

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⁶ Brander and Lewis (1986), Brander and Lewis (1988), Glazer (1994) and Faure-Grimaud (2000), for example, assume that firms choose outputs before the state of the world is revealed. Therefore, in the Brander and Lewis (1986) paper firms will engage in the “risk-shifting” as described by Jensen and Meckling (1976). This is when equity has an incentive to increase the risk of the cash flows by a more aggressive output stance because it only gets paid in high demand realizations. The present paper assumes that output decisions are made after the state of demand is realized. Without uncertainty, equity-controlled firms cannot benefit from shifting risk onto creditors. Because of this, capital structure does not affect the output decisions of firms in the market. Yet, this paper shows that courts can affect how many competitors enter and stay in the industry by the enforcement of debt contracts. Therefore, capital structure in this paper only has an indirect effect on competition by its influence on the number of competitors entering and staying in the market.

⁷ There are several recent empirical studies of how financial intermediaries affect the entry and competition in product markets. Strahan and Cetorelli (2006) find that concentrated bank ownership correlates with less competitive product markets. Bertrand, Schoar, and Thesmar (2005) argue that deregulation in the French banking sector both increased entry into product markets and accelerated the exit of inefficient firms. Cetorelli (2004) finds non-financial firms are smaller in countries where the banking sector is more competitive. Bonaccorsi di Patti and Dell’Arricca (2004) argue that competitive banking sectors only increase entry when firms products are well known, but, when the firms are more opaque, bank credit declines with the competitiveness of the financial sector.

In the present model, we assume that suppliers of capital are price takers and that there are no informational asymmetries. The only suppliers of capital that earn extra-normal returns here are the original asset owners—the “founding shareholders.” Other authors would call the “founding shareholders” the firm’s “entrepreneur.”
If courts aim to maximize social welfare, we argue that firms will trade off the potential costs of “inefficient liquidation”\(^8\) with potential benefits of reduced taxes. Debt saves money on taxes. Modigliani and Miller (1963) is the most cited example of this argument. The tax rationale for debt is so prevalent, and so easy to calculate, that it is programmed into the spreadsheets of most cost of capital calculations. Therefore, shareholders would trade the tax advantages of debt against the bankruptcy cost, which is endogenous to our argument here.\(^9\)

The extra bankruptcy cost imposed by courts may reduce investment, but it increases before tax expected profits and total welfare generated by the industry. We assume that entrants only differ in the magnitude of their initial investment. Courts that liquidate economically profitable firms may increase total welfare \textit{ex post} and improve \textit{ex ante} investment incentives. Moreover, absolute priority rule (APR) violations may indirectly induce firms to pay more taxes in expectation, reducing the marginal incentive to invest. In these ways, the courts can help correct the overinvestment problems in a homogeneous goods industry.

The role of bankruptcy courts in inducing efficient investment is a far more common theme than the effect of bankruptcy on industry-wide competition. A recent interesting study by Bernhardt and Nosal (2004), for example, argues that courts that do not maximize the \textit{ex post} going concern surplus may raise \textit{ex ante} welfare. Nevertheless, that study like most studies on bankruptcy considers the effects of

\(^8\)“Inefficient liquidation” as used here is inefficient from the perspective of investors in the insolvent firm. We will argue that “inefficient liquidation” sometimes improves welfare and is thus socially efficient.

\(^9\)Yet, because we use this model, we do not attempt to generate reasonable predictions of the capital structure of firms. (Modigliani and Miller (1963) predicts that value maximizing firms will be 100 percent levered when there are no financial distress costs.) Instead we are attempting to show how bankruptcy courts can shape investment, entry, and exit in the industry when debt has tax advantages.
continuation decisions and creditor rights in the context of a single firm. Here we are arguing that the firm produces externalities that affect consumers and producers in the market and investment outside the industry. Bernardt and Nosal (2004) consider maximizing the expected *ex ante* value of the firm as an objective of the courts. Here the objective of the courts is to maximize *ex ante* social welfare. When investment is excessive, there is a conflict between the objectives of maximizing the *ex ante* value of firms and *ex ante* welfare.

The author knows of no other paper that has attempted to relate the problem of excessive entry to the objectives of bankruptcy courts. This paper finds that not only is entry excessive, but also there is weakly too little exit relative to the social optimum. Finally, the author knows of no study that has considered how debt and taxes can affect this excessive entry equilibrium.

The paper proceeds as follows. We take up Mankiw and Whinston (1986)’s argument that free entry leads to overinvestment in a homogenous goods industry. We show that total welfare will be improved by sometimes forcing economically profitable firms to liquidate in the low demand state. In equilibrium, firms that make larger investments have more durable assets and are targets for liquidation in the low demand state. If the bankruptcy courts engage in *ex post* regulation of the industry and make the liquidation decisions in light of overall welfare, *ex post* and *ex ante* welfare will be weakly improved.

There are limits to how much entry can be regulated by liquidating economically profitable firms. Increasing the exit rate of firms probably will not

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10 In contrast, when investment incentives are insufficient or perfectly aligned with maximizing welfare, the courts can raise welfare by taking actions that maximize the *ex ante* value of firms. Therefore, our analysis is only valid for those cases where entry is socially excessive.
totally solve the excessive entry problem by itself. Therefore, we show that APR violations that give equity investors returns in bankruptcy can lead to the first-best level of investment as long as the corporate tax rate is sufficiently high. We then discuss some limitations of the analysis and discuss possible extensions.
<table>
<thead>
<tr>
<th>Period 0: Entry and Capital Structure</th>
<th>Period 1: Bankruptcy and Reorganization</th>
<th>Period 2: Production</th>
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</thead>
<tbody>
<tr>
<td>• Founding shareholders decide to either enter or stay out of the industry.</td>
<td>• State of demand is revealed as either high or low to all players.</td>
<td>• Firms compete.</td>
</tr>
<tr>
<td>• A continuum of firms ordered from ([0, N^0]) sequentially enter and choose their debt-to-equity ratio.</td>
<td>• All past actions are common knowledge. Further, liquidation and continuation decisions are taken sequentially for firms ordered from ([0, N^0]).</td>
<td>• Profits are returned to investors.</td>
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<tr>
<td>• High (N) firms observe whether lower (N) firms enter. Further, all lower (N) entrants’ capital structure is known to the higher (N) firms.</td>
<td>• Current shareholders decide whether or not to enter bankruptcy court or pay creditors in full in period 2.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Bankruptcy court decides whether or not to liquidate the bankrupt firm. If the firm is allowed to operate, then the court reallocates cash flow rights between current shareholders and creditors.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Current shareholders decide whether or not to liquidate the firm or operate in period 2.</td>
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Figure 1: The Sequence of Events
2.0 Model

Firms are identical, but they differ in their initial investment to enter the industry. Firms are ordered on a continuous interval from \([0, \infty)\). Firms enter in order from the 0-th to the \(N\)-th firm. The \(N^0\)-th firm is defined as the last firm to enter the industry. The \(N\)-th firm must invest \(I(N)\) in period 0 to enter the market and compete in the second period of the game. The investment is continuously increasing in the firm’s rank, \(N\). That is, \(\frac{dI(N)}{dN} > 0\). Let \(0 \leq N \leq N^0\). Further, \(I(N) \geq 0\) for all \(N\).\(^{11}\)

Entry decisions in period 0 are sequential. The firm ranked \(N'\), for example, gets to observe whether all firms with a lower rank, \(N < N'\), entered the industry. Further, the founding shareholders of firm \(N'\) observe the capital structure choices of all firms of rank \(N < N'\). At the time of entering, the founding shareholders can determine the new firm’s debt-to-equity ratio. The founding, period 0, shareholders decide on how the claims to period 2 cash flows are to be divided between debt and equity holders in period 2. By assumption, these claims are only altered if the firm

\(^{11}\) For the purposes of using calculus, we will treat \(N\) as a continuous variable. Nevertheless, it is often useful to think of the number of firms as a non-negative integer. Amir and Lambson (2003) for example, solves a game with a non-negative integer number of firms. That is, the number of firms in the industry in that paper takes on the values 0, 1, 2, …

Chapter IV considers alternative entry sequences. It finds that, as the size of entrants becomes very small, the ordering of entry does not affect the ranks of entrants into the industry. In particular, only the lowest rank firms enter regardless of entry ordering.
ends up in bankruptcy court in period 1. That is, capital structure decisions are sticky, and renegotiation of ownership claims is prohibitively costly outside of bankruptcy court. In summary, the entry and capital structure decisions of lower ranked firms can be taken as given by higher ranked firms.\(^{12}\)

At the start of period 1, all players find out whether or not the state of demand is high or low. Then, in that same period, firms sequentially choose to exit or enter bankruptcy court and choose to liquidate or are forced to liquidate. The 0-th firm enters bankruptcy (or not) and is liquidated (or not) before all higher \(N\) firms. The firm ranked \(N'\), for example, gets to observe whether all firms with a lower rank, \(N < N'\), exited the industry.

It is assumed that firms are controlled to maximize the returns to the current generation of shareholders. Therefore, future generations of shareholders may find it optimal to take actions that strictly lower \textit{ex ante} value if it leads to higher returns \textit{ex post}. In this model, there is one action that is dynamically inconsistent from the perspective of period 0 shareholders. Namely, period 1 shareholders can choose to enter bankruptcy court to renegotiate their claims with bondholders. (Renegotiations are assumed to be impossible outside of bankruptcy court.) Therefore, shareholders suffer from the inability to commit not to renegotiate inside bankruptcy.

In contrast, we assume that it is the courts’ objective to maximize total \textit{ex ante} welfare. We also assume that courts can commit to maximize \textit{ex ante} welfare even though they must act \textit{ex post}. In practice, this latter assumption is not as crucial as one would suspect initially because we will find that there is little conflict between \textit{ex}

\(^{12}\) Suppose that the bankruptcy court’s strategy would be to liquidate the firm \(N^*\) if it entered bankruptcy court. The sticky capital structure choice is a commitment to stay in the industry in a given state if firm \(N^*\) does not take on too much debt or to exit the industry if firm \(N^*\) takes on a lot of debt.
post and ex ante welfare. In the context of homogeneous goods industries, we will show that this means discouraging the duplication of fixed costs—$I(N)$, encouraging the redeployment of assets—$\gamma I(N)$—outside the industry, and increasing before-tax profits in the industry.

The life of the industry is compressed into period 2 where firms operate from time 0 beginning in the start of period 2 to time infinity at the end of period 2. This infinite period structure allows the firm to deduct all payments to bondholders. If the debt is perpetual, all payments can be interest, which is tax deductible.

For simplicity, bankruptcy occurs when the state of demand is revealed in period 1, prior to production taking place. This way, the industry benefits from firms exiting. In practice, bankruptcy is often postponed until default is almost imminent. Therefore, there may be a span of time where “zombie” competitors drive down prices in the industry prior to exiting. Moreover, demand evolves over time. Therefore, firms that have entered may be reluctant to exit because some fixed costs are sunk. We avoid discussions of hysteresis as described in Dixit (1989), for example, because all entry decisions in this game are “now or never” and all exit decisions are made after uncertainty is resolved. This allows us to separate out the pure exit incentives in the presence of the consumer surplus and business stealing externalities versus the better understood problems of sunk costs and hysteresis.

Firms can recover a fraction, $\gamma$, of their period 0 investment, $I(N)$, where $0 < \gamma < 1$, if they shut down in period 2.\footnote{By setting the parameter $\gamma < 1$, we are assuming that the investment is at least is partially sunk and does not appreciate in value with use. Alternatively, because $\gamma > 0$, we are assuming that the resale value of the capital exceeds the costs of disposing of it. For example, firms are sometimes required to} Therefore, since the $N^{th}$ firm made the largest
investment, it has the greatest total value of non-specific assets, \( \gamma(N^0) \). Furthermore, \( \gamma(N^0) > \gamma(N^*) \), if and only if \( 0 < N^* < N^0 \). This seems to be a reasonable assumption. In general the new, or marginal, entrants have newer vintages of capital. This capital could have a higher cost and higher resale value. In this model, the marginal \( N^0 \)-th entrant has the least to gain from entering the industry and the most to gain from exiting it. This will become very important as we discuss capital structure choice.

Suppose that there are two states, high and low. That is, \( s = H \) or \( L \). The probability of the high demand state is \( h \) where \( 0 < h < 1 \). In the high demand state, consumers are willing to pay a higher price for every quantity sold to the market. The inverse demand schedule given by \( P(s, Q^*) \) is a function of the state, \( s \), and of aggregate equilibrium output in state \( s \), \( Q^* \). The inverse demand is increasing in the state. That is, \( P(H, Q^*) > P(L, Q^*) \) for a given \( Q^* \). In addition, the market price is falling in aggregate output \( \frac{\partial P}{\partial Q} < 0 \).

Further, the number of firms in operation is given by the superscript \( s \). That is \( N^s \) firms operate in period 2 in the state of demand \( s \). By assumption, no firms are able to enter after the state is revealed because there are lags between the initial investment and a firm’s ability to bring its product to market. In particular, no new firms can enter after period 0.

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14 In part, because we have chosen just two demand states for reasons of simplicity, this model does not generate reasonable debt-to-equity ratios. This paper does not attempt to mimic real-world debt-to-equity ratios, but to demonstrate how courts can move the level of entry and exit in the industry closer to the optimum. Instead, this paper is the first to suggest that overinvestment problems can be partially cured by courts’ enforcement of financial contracts in bankruptcy. It is not the first to argue that firms may have an optimal capital structure or the first to argue that capital structure may affect the equilibrium output in the industry.
After the state is revealed and liquidation decisions are made, all firms are identical competitors. Firms are identical after entry and liquidation decisions because they have symmetric variable cost functions \( c(q') \). Further, all firms weakly have diseconomies of scale after they have entered the industry. That is, \( c(0) = 0, c'(\bullet) \geq 0, \) and \( c''(\bullet) \geq 0 \). Aggregate industry output is just the outputs of the \( N^* \) identical firms producing an individual output \( q' = q(s, N^*) \). That is, \( Q' = N^* q' \).

These remaining firms produce the same output \( q(s, N^*) \) in equilibrium. Further, it is robust to assume that an individual firm’s output is increasing in the state. That is, \( q(H, N^*) > q(L, N^*) \) for any given \( N^* \).

Let us denote profits in period 2 as a function of the state. What is important for our purposes is the number of firms in the productive period. Output only occurs in period 2. Output is a function of the number of firms and the state. The discounted profits before investment costs, revenues minus variable costs, for an infinitely-lived, single firm that is in operation in a given state is the following:

\[
\pi(s, N^*) = P(s, Q')q(s, N^*) - c(q')
\]  
(1) \(^{15} \)

We will assume that per firm producer surplus is increasing in the state. That is, the equilibrium output response leads to rising per firm producer surplus for a given industry size. That is, for any given industry size \( N^* \), \( \pi(H, N^*) > \pi(L, N^*) \). \(^{16} \)

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\(^{15} \) \( \pi \) stands for per firm producer surplus, not profit. \( \pi \) —is only revenues minus variable costs without the fixed costs of investment.

\(^{16} \) All the output, price, and per firm producer surplus assumptions are consistent with an industry composed of Cournot competitors facing a linear inverse demand curve.
We can combine our entry assumption and our profit assumption to conclude the following:

**Assertion**

*No firms exit in the high demand state. Further, the number of firms in the high demand state weakly exceeds the number of firms in the low demand state. That is, \( N^0 = N^H \geq N^L \).*

A proof is left for the appendix.

In this paper, it is assumed that there is no regulator controlling entry, but the courts may be able to speed the exit of firms and may be able to affect the enforcement of financial contracts. This is an assumption, but it is motivated by the idea that judicial regulation is preferable to all other feasible regulatory regimes. In the appendix, we more formally consider the idea that judicial regulation dominates an entry restriction.

This role for courts is at odds with the economics and legal scholarship at present. It is assumed that either courts objectives should be to either strictly enforce the priority of asset claims, absolute priority, or to maximize the going concern surplus of the firm. Aghion, Hart, and Moore (1992), for example, attempts to design a non-cash options procedure that reconciles potential conflicts between respecting priority and maximizing the continuing value of bankrupt firms. The latter goal of
maximizing the going concern surplus of firms is tied to the idea that encouraging investment and greater competition unambiguously raises welfare. We will show this is not the case.
3.0 Business Stealing, Taxes, and APR violations

Under fairly weak assumptions there is excessive entry in homogenous product industries. We show that entry is excessive in the absence of taxation. Here we will argue that if the courts liquidate some economically profitable firms, they will raise welfare *ex post* and *ex ante*. In this section, founding shareholders trade off the risks of inefficient liquidation (costly bankruptcy) for lower taxation of corporate profits. The higher, before-tax, industry profits generated by liquidating economically profitable firms will more than offset the losses to consumers and the losses to investors in the liquidated firms. Moreover, by invalidating some of shareholders’ promises to creditors in bankruptcy by means of absolute priority rule (APR) violations, courts can lower the debt capacity of firms entering the industry. Lower debt capacities mean that firms pay more taxes in expectation, which further discourages entry. These policies may improve investment incentives; nevertheless, there is no guarantee that they will lead to the optimal level of entry and exit.

3.1 Business Stealing

Suppose that entry into the market is unregulated. That is, the government cannot sell licenses or impose additional taxes on the industry in question. In
practice, this seems reasonable. With the exception of taxi services or liquor retailing in some states and municipalities, most markets have more or less free entry.\textsuperscript{17} Moreover, corporate tax rates are a function of a firm’s income, but not the line of business it is in.

Following Mankiw and Whinston (1986) we will make 3 more assumptions. That paper proved that these fairly innocuous conditions will guarantee excess entry in expectation:

3.0 Total industry output is rising in the number of firms, $\frac{\partial Q^s}{\partial N^s} > 0$.

4.0 Individual firms’ outputs are falling in the number of firms, $\frac{\partial q^s}{\partial N^s} < 0$.

5.0 All firms price at or above marginal cost, $P(s, Q^s) - c'(q^s) \geq 0$.

These assumptions seem reasonable when firms weakly face diseconomies of scale and are producing homogenous goods.

Our presentation here differs from Mankiw and Whinston (1986). We are also concerned with how the exit behavior affects the investment incentives of firms. Mankiw and Whinston (1986) only considers entry behavior since there is only one state. Since firms usually exit industries through bankruptcy courts, we focus here on how these firms end up in default and how forcefully the courts push the firms towards liquidation. Moreover, unlike the previous study, we discuss the financial

\textsuperscript{17} Other examples of restricted entry markets are some professions where there is occupational licensure. These restrictions are not enforced by the government \textit{per se}, but by the medical or dental professions, for example. See Kleiner (2000) for a non-technical discussion.
contracts, which are used to fund investment. In addition, we differ slightly in their presentation by introducing asymmetric entry costs, $I(N)$.

For the moment we will assume that firms pay no taxes. Ideally, if financial distress and bankruptcy were costless, firms could avoid all their tax obligations with an all debt capital structure.\(^{18}\) The social welfare function is the following:

\[
W(N^H, N^L) = h \left[ \int_0^{N^H} P(H, v)dv - N^H c(q^H) \right]
+ (1-h) \left[ \int_0^{N^L} P(L, v)dv - N^L c(q^L) + \gamma \int_{N^L}^{N^H} I(u)du \right]
- \int_0^{N^H} I(u)du,
\]

where $q^H = q(H, N^H)$, $q^L = q(L, N^L)$.

The top term is the expected total surplus in the high demand state. This is the price that consumers are willing to pay, less the total variable costs of producing the output. The second term is the expected total surplus generated in the low demand state plus the expected scrap value of the firms that are liquidated.\(^{19}\) Finally, the last term is the total investment costs for the industry. Let us differentiate this with

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\(^{18}\) In the presentation that follows, we endogenize the costs of bankruptcy as the cost of inefficient liquidation. These costs give firms a reason to not take on an all debt capital structure. We are not arguing that the inefficiencies or biases of the courts are the only source of bankruptcy related costs. Instead, we are arguing that the additional costs that are imposed by the courts will affect the total welfare generated by the industry.

\(^{19}\) We have assumed in equation (2) that a non-zero interval of firms of ranks between $(N^H, N^L)$ or $[N^H, N^L]$ is being liquidated. Otherwise, all firms would take the low state profits and not take the liquidation value. This alternative scenario is especially a possibility when $\gamma$ is low and liquidations are voluntary. See the appendix equation (51) for this alternative scenario.
respect to the initial number of firms in the market, $N^H$. After rearranging some terms, the first derivative can be expressed as the following:

\[
\frac{dW}{dN^H} = \left[ h(P(H,Q^H) q^H - c(q^H)) + (1 - h)\gamma I(N^H) - I(N^H) \right] \\
+ h N^H \frac{dq(H,N^H)}{dN}(P(H,Q^H) - c'(q^H)),
\]

(3)

Under free entry the first term in square brackets should equal zero. The marginal entrant’s expected returns in both the high and low state minus its cost of investment should just equal that firm’s cost of investment. If this were not the case, another higher $N$ firm would want to enter. Let us define the free entry equilibrium (FE) number of firms as

\[
h(P(H,Q^{FE}) q^{FE} - c(q^{FE})) + (1 - h)\gamma I(N^{FE}) - I(N^{FE}) \equiv 0
\]

(4)

or

\[
h\pi(H,N^{FE}) + (1 - h)\gamma I(N^{FE}) - I(N^{FE}) \equiv 0
\]

where the superscript “FE” signifies the initial number of firms entering the unregulated market.

The free entry (FE) condition in (4) implies that the first-order condition in equation (3) can be simplified considerably. The first-order condition must be weakly negative, and unregulated industry has excessive entry:

\[
\left. \frac{dW}{dN^H} \right|_{N^H = N^{FE}} = h N^{FE} \frac{dq(H,N^{FE})}{dN^{FE}}(P(H,Q^{FE}) - c'(q^{FE})) \leq 0
\]

(5)
We know that $dq/dN^* < 0$ from assumption 2. The term in brackets is the difference between price and marginal cost for an individual firm. Price must weakly exceed marginal cost if firms are profit maximizing. Therefore, the whole quantity must be weakly negative. If the first order condition is weakly negative, then this implies that entry is weakly excessive. When markups are strictly positive, free entry is strictly excessive.\(^{20}\)

\[
\left. \frac{dW}{dN^*} \right|_{q'' - N^*} = \left[ h(P(H, Q''^*)q''^* - c(q''^*)) + (1 - h)\gamma I(N''^*) - I(N''^*) \right] + h N''^* \frac{dq(H, N''^*)}{dN^*} (P(H, Q''^*) - c'(q''^*)) = 0
\]

(6)

When markups are positive, we know that the bottom term is negative. This implies that the top term on the left-hand side must be positive. This is impossible under free entry (FE). Firms cannot make strictly positive expected profits without encouraging more firms to enter. Since we assumed that entry was unregulated, (6) is not achievable. Later on we will consider if taxes and bankruptcy costs can move us closer to the optimum, $N^{\text{H*}}$. Yet, for now, this discussion leads us to the first proposition, which follows from equations (5) and (6):

**Proposition 1**

*Free entry leads to excess entry when firms have positive markups, produce homogenous products, and must pay entry costs.*

\(^{20}\) Positive markups are a necessary condition for firms to make zero profits with positive entry costs.
This is a restatement of Mankiw and Whinston (1986)'s result.

Because welfare is a function two variables if there is an interior optimum, it will be a stationary point where the first derivative of welfare with respect to both the number of firms entering, \( N^H \), and the number of firms remaining in the low state, \( N^L \), are equal to zero.

Let us differentiate welfare in equation (2) with respect to \( N^L \).

\[
\frac{dW}{dN^L} = (1-h)[P(Q^L)q^L - c(q^L) - \gamma I(N^L)] + (1-h) \frac{d}{dN^L} N^L \left[ P(Q^L) - c'(q^L) \right],
\]

where \( Q^L \equiv N^L q^L \). (7)

If firms are free to exit per the wishes of their investors, they will do so when their revenues less variable costs are less than or equal to the recovery value of their assets. Because the marginal firm to exit—firm \( N^L \)—has the lowest recovery value—\( \gamma I(N^L) \)—of all the exiting firms, it must be indifferent between staying in and exiting the industry. The free exit (FX) equilibrium number of firms is defined as the number of firms, \( N^{fx} \), where the following condition is met:

\[
P(L, Q^{FX})q^{FX} - c(q^{FX}) - \gamma I(N^{FX}) \equiv 0
\]

or

\[
\pi(L, N^{FX}) - \gamma I(N^{FX}) \equiv 0
\]

(8)
If there is free exit (FX), we can simplify the first-order condition. That is, by inserting equation (8) into equation (7) above, the FOC with respect to $N^L$ reduces to the following:

\[ \left. \frac{dW}{dN^L} \right|_{N^L = N^{ex}} = (1 - h) \frac{dq}{dN} N^{FX} \left[ P(Q^{FX}) - c'(q^{FX}) \right] \leq 0. \tag{9} \]

We know this is weakly negative because per firm output falls in the number of firms, markups are weakly positive, and the probability of the low state occurring—$1 - h$—is non-negative. Therefore, equation (9) implies that weakly too few firms, $N^H - N^L$, exit in the low demand state. This is what Amir and Lambson (2003) found. In every state, there are too few firms exiting. That paper considers an infinite horizon model in which the number of firms is always a non-negative integer. In contrast, we have just slightly altered the setup of Mankiw and Whinston (1986) to generate this result of insufficient exit. (The present paper has merely added liquidation values, $\gamma I(N)$, and two demand states to Mankiw and Whinston (1986)'s welfare function.)

Evaluated at the optimum, equation (7) reduces to the following expression:

\[ \left. \frac{dW}{dN^L} \right|_{N^L = N^{\ast}} = \left[ P(Q^{L^*})q^{L^*} - c(q^{L^*}) - \gamma I(N^{L^*}) \right] + \frac{dq}{dN} N^{L^*} \left[ P(Q^{L^*}) - c'(q^{L^*}) \right] = 0. \tag{10} \]
We know that the second term, \( \frac{dq}{dN} N^{L*} [P(Q^{L*}) - c'(q^{L*})] \), is negative when markups are positive. Therefore, an interior optimum implies that there is a tendency for too few firms to exit. That is, positive markups imply that
\[
P(Q^{L*})q^{L*} - c(q^{L*}) - \gamma I(N^{L*}) = \pi(L, N^{L*}) - \gamma I(N^{L*}) > 0,
\]
at the optimum.

Further, the first-order condition in (10) does not depend on number of firms entering, \( N^H \). The only limit to this is that \( N^L \leq N^H \). \( N^{L*} \) may sometimes exceed \( N^H \), especially when recovery values are very low—\( \gamma \) is close to zero. In this case, if \( \pi(L, N^H) \geq \gamma I(N^H) \), the \( N^H \)-th firm might do the socially optimal thing by not exiting in the low demand state. In this case, no firms will exit and this will weakly coincide with the social optimum because technological constraints prevent further entry in period 1. This anomaly comes from the fact that welfare could be improved if firms could enter the industry at the liquidation value. Therefore, the constraint that \( N^L \) cannot exceed \( N^H \) must bind and \( N^H = N^H \) when \( N^{L*} > N^H \). Since zero firms optimally exit in this case, then we can say that sometimes exit is socially optimal. In particular, free exit is socially optimal when \( N^{EX} \) implied by equation (8) is greater than or equal to \( N^H \). Yet, it still must be the case that \( N^{L*} < N^{EX} \). Despite the technical constraints, we can make the following statement from our analysis above:

**Proposition 2**

There are weakly too few firms that exit in the low demand state.
The proposition above follows from equation (9) and the preceding discussion.

3.2 Taxes and Capital Structure

Firms sometimes trade off the expected tax advantages of debt with the expected costs of inefficient liquidation. Welfare maximizing judges will sometimes liquidate bankrupt firms even when they are economically viable. Therefore, firms can use debt to save money on taxes, but they may risk inefficient liquidation by judges. In this subsection, we analyze the financial structure decisions of the $N$-th firm, holding the number of firms in the high and the low demand states, $N_{H}$ and $N_{L}$, respectively, fixed.

In this presentation, we rely on Gertner and Scharfstein (1991)’s observation that debt restructurings often fail due to the holdout problem. We assume that all out-of-court restructurings fail. In the appendix section 6.3, we show that refusing an

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21 We do NOT allow for renegotiation of debt covenants outside of bankruptcy. An interesting extension of the model would allow for renegotiation of debt to avoid bankruptcy. If we relaxed this assumption and debt could be swapped for equity at no cost, then involuntary liquidation by the courts could always be avoided. In such a world APR violations could discourage investment, but involuntary liquidation would have no role in reducing investment and reducing competition in the low demand state. Even with costless renegotiation, the courts would still have a role in reducing entry by violating priority and lowering the debt capacity of firms which are subject to corporate income taxes on their earnings.

Renegotiation of debt claims may be costly. In practice troubled firms suffer from several problems when pursuing exchange offers where debt claims are swapped for equity to avoid a bankruptcy filing. Gertner and Scharfstein (1991) point out that from 1977 to 1990 only 73 of 156 financially distressed junk bond issuers successfully completed exchange offers. They say that one reason for this is the “holdout” problem. The holdout problem stems from the fact that creditors who do not swap their claims for equity see the value of their claims rise as costly bankruptcy is avoided. The holdout problem means that creditors who fail to tender free ride off those who do. This problem may be especially acute the United States, for example. The Trust Indenture Act requires that there must be unanimous consent before the interest, principal, or maturity of any public debt issue is changed. This means that each creditor must agree individually for any exchange where the creditor receives equity for debt, making the free rider problem almost unavoidable.
exchange offer is a Nash equilibrium strategy for small bondholders who take other bondholder’s tendering decisions as given. If we relaxed this assumption, firms would choose higher levels of debt on average, and inefficient liquidations by the courts would become more unlikely as the probability that troubled firms could restructure their debt out-of-court increases. This would hamper the courts’ ability to speed exit in the industry. Nevertheless, the courts could still shape the industry and discourage entry by way of absolute priority rule (APR) violations that reduce the debt capacity of firms and thus increase these firms’ tax burdens. This later observation is the subject of the next subsection 3.3. For this subsection, we assume that creditors of the $N$-th firm get all the ex post returns when the $N$-th firm enters bankruptcy protection.

A firm will only enter an industry if it can recover the expected cost of its investment after corporate taxes. The marginal corporate tax rate is $\tau$, where $0 \leq \tau \leq 1$. (Nevertheless, this tax rate is assumed to be constant in this analysis for all positive levels of corporate profits. This is done for simplicity.) In addition, we will assume that investors pay no personal taxes. Therefore, total returns to the founding shareholders are the period 0 cash flows from issuing debt and the period 2 profits after taxes and debt payments.

The firm is owned by founding shareholders who set capital structure in period 0 to maximize the value of their shares. All investors are risk neutral, and the risk free rate of interest is $\rho > 0$. Debt holders are price takers and will supply funds if they can merely recover the cost of their investment in expectation. The debt pays a coupon interest of $C \geq 0$. The debt is perpetual. Interest payments are tax
deductible. If all promised payments are made, the discounted value of the debt is denoted \( F = C/\rho \). We will call this the promised value of the debt. Thus, the maximum allowable tax deduction that a firm can make will equal the *ex post* investment returns of that firm in a given state. Therefore, if \( F \geq \gamma(N) \), when the firm is liquidated, or if \( F \geq \pi(s, N') \), when the firm operates in state \( s \), then the \( N\)-th firm gets to deduct up to \( \tau\gamma(N) \) or \( \tau\pi(s, N') \), respectively, from its tax bill.

We will assume that there is only one class of bondholders. Bondholders’ claims are always senior to shareholders’ claims. Control is not automatically given to bondholders in default. Control of the firm rests with shareholders prior to default and with the court in period 1 when default is certain to occur in period 2. All uncertainty is resolved in period 1. If the promised value of the debt is high enough, then default will be certain. Bondholders are only given control of the firm at the court’s discretion after the firm has entered bankruptcy.

Shares have limited liability. Therefore, the minimum payoff in period 2 is zero. Taxes due are reduced by the payments to creditors. The expected payments to creditors in period 0 is denoted \( B \) or \( B^\delta \). The promised value, \( F \) or \( F^\delta \), is due in period 2 after production takes place. Let the superscript \( \delta \) take on two values \( \delta = f \) or \( r \).22

“Risk free debt” has no default risk. We will denote risk free debt by the superscript “\( f \)” Safe debt issued by the \( N\)-th firm can take on any value between zero and \( \max\{\pi(L, N^d), \gamma(N)\} \). With risk-free debt, the promised value and the value of the

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22 We present the choice of debt as discrete with the designation \( d = f \) or \( r \). In fact, the founding shareholders have a continuum of capital structure choices, but in equilibrium they choose to set their capital structure to maximize the value of either the safe or risky debt. The type of debt that is chosen depends on the parameter values of the industry in question and the rank, \( N \), of the firm.
debt is equal to $F^f = B^f$, regardless of the period or the state. That is, shareholders are free to choose risk free debt worth:

$$0 \leq B^f = F^f \leq \max\{\pi(L, N^L), \gamma I(N)\} \quad (11)$$

The founding shareholders can also sell risky debt. With risk free debt, shareholders can always optimally control whether or not the firm is liquidated in the low demand state. With risky debt the courts have the discretion to liquidate the $N$-th firm. Let us define a variable, $\theta(N)$, to indicate if they $N$-th firm is liquidated by either current shareholders or the courts in the low state. Let $\theta(N) = 1$ if the firm is liquidated in the low demand state, and let $\theta(N) = 0$ if it is not.

“Risky debt,” denoted by the superscript “$r$,” pays $\min\{F^r, \pi(H, N^H)\}$ in the high state and $(1 - \theta(N)) \pi(L, N^L) + \theta(N) \gamma I(N)$ the low demand state. Further, it always has a higher promised value than safe debt, $F^r > F^f$.

As long as the period 2 debt payment does not exceed the high state profits, the courts cannot liquidate solvent firms in the high state. It is trivial to show that all firms will find it weakly in their interest to set the promised value of the debt no higher than the profits in the high state. Firms can always reap the full tax advantages of risky debt by setting $F^r = \pi(H, N^H)$. If $F^r > \pi(H, N^H)$, on the other hand, then this may lead to inefficient liquidation for some firms that the courts would prefer to liquidate even in the high demand state. This presentation does not consider the dominated equilibrium where there are costly high state liquidations by
the courts because shareholders can always structure debt claims to avoid this
eventuality at no \textit{ex ante} cost.

The value of risky debt is

\[ B' = hF' + (1-h)[(1-\theta(N))\pi(L, N^L) + \theta(N)\gamma I(N)], \]

where \( \pi(H, N^{H}) \geq F' > \max\{\pi(L, N^L), \gamma I(N)\} \).

(12)

The objective of judges and holders of risky debt are in conflict when \( \gamma I(N) < \pi(L, N^L) \). That is, the judge liquidates economically profitable firms where \( \pi(L, N^L) > \gamma I(N) \). This is the “interesting case” with which we are most concerned. When judges liquidate a firm with \( \pi(s, N^s) \) in excess of the firm’s liquidation value, \( \gamma I(N) \), then we say that firm \( N \) has been inefficiently liquidated in state \( s \).\(^{23}\)

If founding shareholders of the \( N\)-th firm intend to enter, they will also structure debt claims in a way that maximizes the value of their shares. Suppose that investors in the \( N\)-th firm expect the court to inefficiently liquidate it if default is imminent. If \( \gamma I(N) < \pi(L, N^L) \), in equilibrium, and courts pursue inefficient liquidation, then the value function jumps at the point where debt becomes risky because the low state returns are less than if the firm had no risky debt.

In Figure 2, we have plotted the expected value of the debt in period 0, \( B \), as a function of its promised value in period 2. As the promised value increases to \( \pi(L, N^L) \), the value of the debt is increasing on a one-to-one basis. After the promised

\(^{23}\) In practice, founding shareholders will always want to avoid inefficient liquidation in the high demand state if inefficient liquidation in that state carries a positive probability. They can easily do this by not setting the promised value of the debt too high.
value of debt exceeds $\pi(L, N^\dagger)$, the debt becomes risky. Increasing the promised value only increases the monies paid to bondholders in the high state. In the low state, payments in excess of $\pi(L, N^\dagger)$ will not be paid. Therefore, every one-unit increase in the promised value where $F > \pi(L, N^\dagger)$ linearly increases the value of the debt by $h$ units, where $h$ is the probability of the high state.

The line segment $\overline{XY}$ is not feasible when the courts’ strategy is to liquidate this firm when it enters bankruptcy. Liquidating this firm is inefficient from the viewpoint of maximizing the value of the firm. Yet, given that firm $N$ faces inefficient liquidation in bankruptcy, the courts find that liquidating firm $N$ will be ex ante efficient from the viewpoint of social welfare. Note that the value of the debt jumps when we go from $F < \pi(L, N^\dagger)$ to $F > \pi(L, N^\dagger)$. The line segment $\overline{X'Y'}$ plots the feasible values of the bond and its promised value when $\pi(L, N^\dagger) < F^* \leq \pi(H, N^\dagger)$. This is because the court liquidates economically profitable firms where $\pi(L, N^\dagger) > \gamma(N)$. The diagram is drawn from the perspective of a safe debt firm. Not only is $\pi(L, N^\dagger) > \gamma(N)$, but also the maximum value of the risky debt is less than the maximum value of the safe debt, $B^F* < \max B^S*$. 
Expected bankruptcy cost
\[ = (1 - h)[\pi(L, N^L) - \gamma I(N)] \]

Figure 2: Safe v. Risky Debt with Inefficient Liquidation
The value of the $N$-th firm with safe debt is denoted by $V^\theta(N, F)$. $\theta = 0$ or 1 indicates if the firm continues to operate or is liquidated in the low demand state, respectively. The first argument denotes the rank of the firm. The second argument, in part, denotes both the size of the promised value, and whether or not the debt is safe or risky. Let us find out what is the maximum value of the firm given that it takes on safe debt:

$$\arg \max_{w.r.t. F^f} V^\theta(N, F^f), \quad \text{where}$$

$$V^\theta(N, F^f) = \tau F^f + (1 - \tau) h \pi(H, N^H)$$

$$+ (1 - \tau)(1 - h) \{ (1 - \theta(N)) \pi(L, N^L) + \theta(N) \gamma I(N) \} - I(N),$$

and $F^f \leq (1 - \theta(N)) \pi(L, N^L) + \theta(N) \gamma I(N)$. \hfill (13)

Because $\frac{dV^\theta}{dF^f} = \tau > 0$ for all $F^f$ the upper bound of the problem’s constraint binds. Equation (13) should be familiar. It is a restatement of Modigliani and Miller (1963)’s conclusion that the value of the firm is increasing linearly with the value of the debt. Their analysis rests on the assumption that bankruptcy costs are zero.

When all debt is risk free, there is a zero probability of bankruptcy; and, thus, the expected cost of bankruptcy is zero. The upper bound of the constraint is maximized by $\theta(N) = 0$ when $\pi(L, N^d) > \gamma I(N)$ or by $\theta(N) = 1$ when $\pi(L, N^d) \leq \gamma I(N)$. In other words, the firm’s value is maximized with safe debt when $F^* = \max\{\pi(L, N^L), \gamma I(N)\}$.

The maximum value of the safe debt firm that is not liquidated is
\[ V^0(N, F^{r*}) = \tau \pi(L, N^L) + (1 - \tau)[h \pi(H, N^H) + (1 - h)\pi(L, N^L)] - I(N). \] (14)

The maximum value of the safe debt firm that is liquidated is

\[ V^1(N, F^{r*}) = \tau \gamma I(N) + (1 - \tau)[h \pi(H, N^H) + (1 - h)\gamma I(N)] - I(N). \] (15)

We can perform a similar analysis to conclude that the maximum value of a risky debt firm is obtained by setting the promised value of the risky debt to \( F^{r*} = \pi(H, N^H) \).\(^{24}\) This means that, with \( F^{r*} \), the risky debt firm will be 100 percent debt. If \( \theta(N) = 0 \), and the \( N \)-th firm continues to operate in the low demand state, the \( N \)-th firm’s value is the following:

\[ V^0(N, F^{r*}) = h \pi(H, N^H) + (1 - h)\pi(L, N^H) - I(N). \] (16)

Moreover the maximum value of a risky debt firm that is liquidated in the low demand state is the following:

\[ V^1(N, F^{r*}) = h \pi(H, N^H) + (1 - h)\gamma I(N) - I(N). \] (17)

It is useful here to distinguish between liquidation decisions made by courts and liquidation decisions made by the managers of the debtor firm. The indicator

\(^{24}\) If the court will not liquidate the risky debt firm in the high demand state, founding shareholders could choose a \( F^{r*} > \pi(H, N^H) \).
variable \( \theta(N) \), which denotes whether or not the firm is liquidated in the low demand state, can be rewritten as the following:

\[
\theta(N) = \max \{ \theta(c, N), \theta(d, N) \},
\]

(18)

Above “\( c \)” denotes the liquidation decision taken by the courts and “\( d \)” denotes the liquidation decision made by controlling investors of the debtor firm. For example, when the firm does not enter bankruptcy court in the low demand state, \( \theta(c, N) = 0 \).

**Proposition 3**

- Safe debt is only strictly preferred by the N-th firm’s founding shareholders if the strategies of the firm’s period 1 shareholders and the bankruptcy court are \( \theta(d, N) = 0 \) and \( \theta(c, N) = 1 \), respectively.
- Otherwise, risky debt (\( \delta = r \)) is weakly preferred by the N-th firm’s founding shareholders.

A proof is in the appendix.

Intuitively, the firm will maximize tax shields if there is no cost to such a policy. Yet, when judges will liquidate profitable firms, \( \theta(c, N) = 1 \), to lessen the business stealing effect, a firm will only take on risky debt when the expected tax
advantages of risky debt, \( \varrho[H, N^H] - \pi(L, N^L) \), exceed the expected losses from inefficient liquidation, \( (1 - h)[\pi(L, N^L) - \gamma(N)] \). Namely,

\[
V_0^0(N, F^F_r^r) - V_1^0(N, F^r^r) = (1 - h)[\pi(L, N^L) - \gamma I(N)] - \tau h[\pi(H, N^H) - \pi(L, N^L)] \leq 0.
\]

Consider the following lemma.

**Lemma 1**

If the marginal, \( N^H \), firm voluntarily liquidates itself in the low state, there exists a firm of rank \( \hat{N} < N^H \) that both strictly prefers to operate in the low demand state and is liquidated by the bankruptcy court.

See the appendix for a proof.

Indeed, if \( \gamma I(N^H) > \pi(L, N^L) \), there must be firm, which we will denote as the \( \hat{N} \)-th firm, that will be strictly better off with risky debt when it faces inefficient liquidation in bankruptcy—\( \pi(L, \hat{N}) > \gamma I(\hat{N}) \).

The expected value of the marginal entrant in time zero that takes on risky debt when \( \theta(c, N) = 1 \), is the following:

\[
V_1^1(N^H, F^r^r) = h\pi(H, N^H) + (1 - h)\gamma I(N^H) - I(N^H) \equiv 0
\]
The first term is the value of the debt-financed payout to shareholders. The second term is the expected after-tax profits returned to shareholders in period 2. The marginal risky debt firm is unaffected by the higher industry profits in the low state due to the court-ordered liquidations. Yet, all the surviving firms with \( N < N^d \leq N^H \) in the low state are better off from those liquidations. Note that (20) is identical to equation (4)—the free entry (FE) equilibrium definition. In the free entry equilibrium, we assumed that taxes were zero. Here we have found that the marginal entrant can escape taxes by taking on 100 percent debt.

This leads us to the following lemma:

**Lemma 2**

*Given that the marginal entrant chooses risky debt and is liquidated in the low demand state, its entry decision is unaffected by the profits in the low-demand state.*

This explains why there is often no conflict between maximizing welfare in period 0 and maximizing welfare in period 2. The marginal entrant does not care about the low state operating profits because it only operates in the high demand state.
3.3 Absolute Priority Rule (APR) Violations

We have argued that the courts should pursue a liquidation policy that moves the industry as close as possible to \textit{ex post} efficiency. Yet, there is no guarantee that this would move the industry all the way to \textit{ex ante} efficiency or even \textit{ex post} efficiency. The $N^L$-th firm only loses $(1 - h) \max \{ \pi(L, N^L) - \gamma(N), 0 \}$ of expected profits. If this loss is positive, it will mean that fewer firms enter, but the new level of entry may not necessarily move the industry to the point where welfare is maximized. Further, courts can only force liquidations on firms that end up in bankruptcy court. Therefore, \textit{ex post} efficiency is dependent on firms’ willingness to take on risky debt. Here we argue that if founding shareholders expect deviations from the absolute priority ranking of claims, bankruptcy courts may be able to move the industry closer to the first-best optimum.

The courts could reduce entry by facilitating absolute priority rule (APR) violations. APR violations lower the debt capacity of the marginal entrant and increase the taxes that must be paid in expectation. Suppose that the courts only are willing to pay non-equity investors a fraction of the returns in a given state. Let $0 \leq \alpha^L \leq 1$ and $0 \leq \alpha^H \leq 1$, where $\alpha$ is the maximum fraction of revenues minus variable costs that the courts will award to holders of debt in a given state. In addition, let us assume that the APR violation depends on the type, $N$, of the firm. That is $\alpha^L = \alpha^L(N)$ and $\alpha^H = \alpha^H(N)$. A fall in alpha improves equity’s \textit{ex post} bargaining position with
respect to its lenders in a given state. Therefore, equity and its agents will always extract a fraction of the returns, \(1 - \alpha(N)\), by undertaking strategic default.

In fact, courts will find it difficult to violate the priority of secured lenders. Lenders who have been awarded durable assets in the event of default can reasonably expect to recover most, if not all, of their claims.\(^{25}\) Therefore, the minimum percent of assets that courts will award \textit{ex post} to bondholders, as opposed to shareholders, must be in the following ranges, depending on the state and on the equilibrium liquidation decisions:

\[
1 \geq \alpha^L(N) \geq (1 - \theta(N)) \frac{\gamma I(N)}{\pi(L, N^L)} + \theta(N) \quad (21)\]

\[
1 \geq \alpha^H(N) \geq \frac{\gamma I(N)}{\pi(H, N^H)} \quad (22)
\]

The scrap value of the firm, \(\gamma I(N)\), may or may not exceed the going concern value of the firm in the low demand state, \(\pi(L, N^L)\). Therefore, the courts may or may not be able to violate priority in the low demand state. Nevertheless, for any given

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\(^{25}\) Here we are arguing that the lower bound of the courts’ discretion to violate priority is increasing in the proportion of assets that are durable. This seems reasonable, but sometimes courts even dissipate or divert the value of secured claims. For example, secured creditors are stayed from seizing assets in U.S. Chapter 11, which are deemed essential for the running of the business by the bankruptcy court. Eastern Airlines’ bankruptcy may be an extreme example of how this can be abused. According to Weiss and Wruck (1998), only 38 percent of asset sales were distributed to debt holders during and after Eastern Airlines’ prolonged stay in Chapter 11 bankruptcy and its eventual shutdown on January 19, 1991. The assumption that secured claims are 100 percent secure as in (21) and (22) may be extreme. For example, Bris, Welch, and Zhu (2006) in a study of small and large bankruptcies in Arizona and New York state, find that secured creditors lose assets to unsecured creditors in roughly 20 percent of the cases. It may be more reasonable to conclude that the courts’ discretion to violate priority is decreasing in the percent of assets that are durable, \(\gamma\).

\(^{26}\) Voluntary liquidations always occur when \(\gamma I(N) > \alpha(L, N^L)\).
entrant, it is impossible that the scrap value exceeds the going concern value of the firm in the high demand state. Otherwise, it would be unprofitable for that firm to enter. Therefore, the courts always weakly have the scope to violate priority in the high demand state.

The potential for APR violations in the high demand state makes “strategic default” profitable to shareholders *ex post*. Here, strategic default occurs when the equity-controlled firm enters bankruptcy protection in period 1 to avoid paying creditors the full promised value of the debt in period 2. In particular, default is “strategic” in the sense that the firm would be able to pay debt holders in full in that state of demand. The potential for strategic default reduces the value of the debt that the firm can issue. Moreover, given that courts commit not to liquidate firms in the high demand state, APR violations reduce the maximum returns that bond holders can expect in the high state by \((1 - \alpha^H(N)) \times 100\) percent. In order that safe debt is truly safe, founding shareholders will also reduce the promised value of safe debt by the percent of low state returns that shareholders can win by strategic default in the low state.

The value of the firm is still linearly increasing in the debt it can issue. The maximum promised value of risky debt that will be paid to bondholders in the high state is the following:

\[
F^a(N) \equiv \alpha^H(N)\pi(H, N^H)
\]  

(23)

27 If the firm is liquidated in the high state, then the shareholders get nothing, but if the firm is allowed to continue, they get \(1 - \alpha^H(N)\) of the returns. By committing to allow the firm to continue debt capacity is reduced. If high state involuntary liquidations in bankruptcy would occur, period 1 shareholders would have no incentive for strategic default.
If the firm issues safe debt, the promised value of safe debt will be a function of the absolute priority violations, $\alpha^L(N)$ and $\alpha^H(N)$, and whether or not the firm is (optimally) liquidated, $\theta(d, N) = 1$, in the low demand state:

$$F^\alpha(N) = B^\alpha(N) = \min\{\alpha^L(N)(1 - \theta(d, N))\pi(L, N^L) + \theta(d, N)\gamma I(N), \alpha^H(N)\pi(H, N^H)\}.$$ \hspace{1cm} (24)

Suppose that the $N$-th firm is a risky debt firm. The debt tax shields will be reduced by $\tau(1 - \alpha^H(N))\pi(H, N^H)$ in the high state and $\tau(1 - \theta(N))(1 - \alpha^L(N))\pi(L, N^L)$ in the low state if the firm continues. (From (21), the courts cannot reduce tax shield in the low demand state if the firm is liquidated in the low demand state.)

Ideally, in a risky debt firm, shareholders receive their whole payoff in period 0. This allows them to avoid corporate taxes. Yet, if they get part of the returns in period 2, these earnings are taxable. Therefore, the firm’s expected value is reduced by the extra taxes levied on equity investors. Let us denote the value of the risky debt firm of rank $N$, which will be liquidated in the low state, as $V^1(N, F'^\alpha; \alpha^L, \alpha^H)$.

$$V^1(N, F'^\alpha; \alpha^L, \alpha^H) = h(1 - \tau + \tau\alpha^H(N))\pi(H, N^H) + (1 - h)\gamma I(N) - I(N).$$ \hspace{1cm} (25)

Likewise, when the firm optimally chooses safe debt, the expected value of the firm is reduced by the increased taxes caused by the lower expected value of the
debt in the low state. It is at least weakly in period 0 shareholders’ interest to only sell debt with a promised value equal to (24). Any higher promised value would put the firm at the mercy of the courts. Given that safe debt strictly increases the *ex ante* value of the firm, liquidation in the low demand state is more costly than continuation, \( \gamma(N) < \pi(L, N^L) \). Otherwise, the firm would have chosen the higher tax shields of risky debt. This is just a restatement of proposition 3. Founding shareholders in period 0 will want to prevent period 1 shareholders from triggering a strategic default (and court mandated liquidation). Therefore, founding shareholders need to ensure that the payoffs to the current shareholders in a given state will always weakly exceed the gains from strategic default in that state. In this case the maximum value of the safe debt firm that continues to operate in the low demand state is

\[
V^0(N, F^{sa}; \alpha^L, \alpha^H) = \\
\tau \min[\alpha^L(N)\pi(L, N^L), \alpha^H(N)\pi(H, N^H)] \\
+ \{(1 - \tau)[h\pi(H, N^H) + (1 - h)\pi(L, N^L)] - I(N)\}.
\] (26)

The terms in \{\} in (26) is the value of the unlevered firm. The first term on the LHS of (26) is the value of leverage, which is tax rate times the value of the risk free bonds. This should be very familiar as an application of Modigliani and Miller (1963).

To see if an individual firm will favor risky or safe debt or *vice versa*, we can subtract equation (25) from equation (26). If the relationship is positive, the \(N\)-th firm will prefer safe debt. If the relationship is negative, it will prefer risky debt. This lax...
enforcement of the absolute priority of claims reduces the \textit{ex ante} value of debt by the before-tax value of returns that are awarded to equity. Yet, when equity gets returns in bankruptcy, these returns are taxed. On balance, the value of the firm is reduced. Moreover, the firm will have a greater equity-to-value ratio after bonds are sold, relative to strict APR enforcement of debt covenants.

We can think of the problem of selecting the levels of APR violations as a constrained optimization problem. The social planner operating though the courts will want to adjust the size of the industry at two points. First, the number of firms entering in period 0 should only be $N^H\ast$. Second, in the low state ideally $N^L\ast$ firms will remain after liquidations. We know from equation (10) and proposition 2 that some of those liquidations will have to be involuntary if $N^H\ast > N^L\ast$. If the courts are going to be able to achieve the second objective, two things must happen. First, only $N^H\ast$ must voluntarily enter the industry. Second, firms of index numbers $N \geq N^L\ast$ must voluntarily choose to take on risky debt. In practice, it may be very hard to get a firm of rank $N \geq N^L\ast$ to voluntarily risk inefficient liquidation.

The social planner attempting to reach the optimal level of entry and exit through the courts faces the following problem:

$$
\text{arg max}_{N^H, N^L} W(N^H, N^L) \quad \text{w.r.t.} \quad N^H \& N^L
$$

Welfare is defined in equation (2). The optimal $N^H\ast$ and $N^L\ast$ are implied by the first-order conditions in equations (6) and (10). The courts don’t control entry
and exit directly, but they do control APR violations and liquidation decisions in bankruptcy, $\theta(c, N)$. Therefore, the courts’ problem is to maximize the objective function in (27) with respect to $N^H$ and $N^L$ subject to three constraints on $N^H$ and $N^L$. These constraints are given by equations (28), (29), and (32) below. Let us define a column vector $a_N$ that denotes the level of priority that the courts will choose for a given firm rank ($N$), state ($H$ or $L$), and type of debt ($\delta = f$ or $r$). That is, $a_N = \{a^L(N, f), a^H(N, f), a^L(N, r), a^L(N, f)\}$. Further, their control over $a_N \forall N \in [0, N^H]$ are given by equations (21) and (22).

The following be true of any equilibrium:

\begin{align*}
(1) \quad N^H &\geq N^L, \quad (28) \\
\arg\max_{\delta, \theta(d, N)} V^\theta(N, F^{\delta \alpha}; a^L, a^H) &\geq 0 \quad \forall N \in [0, N^H], \\
\quad (2) \quad E^\theta(a^L, a^H) &= \arg\max_{\delta, \theta(d, N)} V^\theta(N^H, F^{\delta \alpha}; a^L, a^H) = 0, \quad (29) \\
\arg\max_{\delta, \theta(d, N)} V^\theta(N, F^{\delta \alpha}; a^L, a^H) &< 0 \quad \forall N \in (N^H, \infty). \\
\end{align*}

The set of conditions above says firstly the number of firms in the low demand state cannot exceed the number of entrants. Secondly, (28) says that all entrants must be making positive, after-tax profits, given their capital structure, $\delta$, and liquidation decisions, $\theta(d, N)$, are optimally chosen. Then, the marginal entrant must
make at maximum exactly zero after-tax net returns. That is, the entry constraint,
\( E^0(\alpha^L, \alpha^H) \), in (29) must equal zero. Finally, no \( N > N^H \) firms must enter.

Let us further define the constraint by whether or not the firm is liquidated,

\[
E^1(\alpha^L, \alpha^H) = \arg \max_{\delta} V^1(N^H, F^{\delta}; \alpha^L, \alpha^H) = 0,
\]

(30)

or continues

\[
E^0(\alpha^L, \alpha^H) = \arg \max_{\delta} V^0(N^H, F^{\delta}; \alpha^L, \alpha^H) = 0
\]

(31)
in the low demand state.

In many cases, the marginal entrant will have safe debt, \( \delta = f \), because the courts commit to the maximum absolute priority violations. That is \( B^f = F^f = gA(N^H) \). Nevertheless, the firms will optimally be liquidated in the low demand state. That is, \( \delta(d, N) = 1 \) because \( \pi(L, N^f) < gA(N^H) \). Therefore, when the courts can violate priority, in many instances the marginal entrant will be forced to have safe debt.

APR violations make the firm pay more corporate taxes in expectation.

Lower APR violations or higher alphas make the firm more valuable in expectation and make more firms enter. If possible, \( \alpha'(N) \) would be chosen such that either \( N^{H*} \) firms enter. Yet, this is only possible if the constraints 1 and 2 in (28) and (29) are met given the constraints on \( \alpha'(N) \). This is the case where the first order condition of
welfare with respect to \( N^{H} \) equals zero. Alternatively, the maximum set of APR violations is chosen for the \( N^{H} \)-th firm. That is, \( \{ \alpha^{L}(N^{H}), \alpha^{H}(N^{H}) \} \) will be the lower bounds of equations (21) and (22) if entry is excessive. If first-best entry is achievable, for example, the courts will want to choose a set of alphas—\( \{ \alpha^{L}(N^{H*}), \alpha^{H}(N^{H*}) \} \)—for the marginal entrant such that (29) binds for firm \( N^{H*} \).

Let us consider a third constraint, the “risky debt constraint.” It says that all firms that will be liquidated in the low state with risky debt must prefer risky debt over safe debt and no liquidation. This is a particularly interesting problem when \( \theta(d, N) = 0 \). Otherwise, it is easily satisfied. The risky debt constraint is the following in the interesting case where firms may not want to be liquidated in the low demand state:

\[
0 \leq \alpha^{H}(N^{H}) - \alpha^{L}(N^{H}) \leq 0,
\]

Taking taxes as given, equation (32) will most likely be satisfied when the courts select the lower bounds of equations (21) and (22) when a firm takes on safe debt and continues to operate in period 2.

\[
\arg\min_{\alpha^{L}(N), \alpha^{H}(N)} V^{0} (N, F^{L}, \alpha^{L}, \alpha^{H}) = \tau \gamma I(N) + \{(1 - \tau)[h \pi(H, N^{H}) + (1 - h)\pi(L, N^{L})] - I(N)\},
\]

---

\[\text{Since the marginal entrant is of size zero, we allow it to be safe or risky debt, depending on what maximizes social welfare. Being infinitesimally small, the marginal entrant alone does not affect the size of the industry, whether or not it stays or exits in the low demand state. That is, the interval } (N^{H}, N^{L}) \text{ is of length zero.}\]
Further, by completely respecting priority and thus selecting the upper bound of (22), the courts can make risky debt and inefficient liquidation more attractive.

\[
\arg \max_{\alpha^H, \alpha^L} \left\{ V^1(N, F^m; \alpha^L, \alpha^H) \left| \Theta(c, N) = 1 \right. \right\}
\]

\[
= h\pi(H, N^H) + (1 - h)\gamma I(N) - I(N).
\]

Generally, treating different firms differently will best accomplish the goals of minimizing entry and encouraging welfare-increasing exit. Suppose that \(N^H > N^{H*} > N^{d*}\). The marginal entrant and all higher \(N\) firms can face the maximum high state APR violation. This minimizes entry. This is most easily accomplished when the marginal entrant chooses risky debt even when the lower bounds of (21) and (22) are selected by the courts. If the marginal entrant chooses risky debt, then it will not care about the fact that profits are higher in the low demand state due to forced liquidations. Yet, this only can occur when the marginal entrant is more valuable if it liquidates itself in the low demand state. This is the case when \(\pi(L, N^d) < \gamma(N^{dH})\), and \(\Theta(d, N) = 0\).

To minimize entry in this case, the courts can encourage as many firms as possible to take on risky debt. To best accomplish this, the courts can minimize the value of shareholders’ choices in period 0 and period 1 of \(\delta = f\) and \(\Theta(d, N) = 0\), respectively. This is the minimization problem in equation (33). This is
accomplished by choosing \( \alpha^L(N, f) = \frac{\gamma I(N)}{\pi(L, N^L)} \), and \( \alpha^H(N, f) = \frac{\gamma I(N)}{\pi(H, N^H)} \).

Further, the courts can maximize the value of being a risky debt firm that is liquidated as in equation (34). They do this by setting \( \alpha^H(N, r) = 1 \). The courts need to commit to these policies for all firms on the interval \((N^L, N^H)\).

In some instances, there may be a conflict between discouraging entry and encouraging the marginal firm to take on risky debt. Suppose that the court cannot convince the marginal entrant to take on risky debt when APR violations are at their maximum as in the lower bound of equations (21) and (22). Further suppose that entry is excessive \( N^H > N^{H*} \). In this case, it may be possible to convince the non-marginal entrants of rank \( N < N^H \) to take on risky debt. Nevertheless, encouraging the non-marginal entrants to take on risky debt and to be involuntarily liquidated makes entry more attractive as profits in the low state rise with fewer \( N^L \). We show in appendix section 6.3 that, when the marginal entrant chooses safe debt, it is ambiguous whether or not welfare maximizing courts will choose to liquidate any firms in the low demand state.

Consider the following proposition.

**Proposition 4**

*Given the corporate tax rate, \( \tau \), is sufficiently high and the proportion of durable investments, \( \gamma \), is sufficiently low, APR violations can move the homogenous product industry to the efficient level of investment.*
Suppose that the corporate tax rate, \( \tau \), is 100 percent. The maximum APR violations is bounded by the proportion of durable investments in equations (21) and (22). Therefore, when \( \gamma = 0 \), the APR violation can be 100 percent of the returns in a given state. The net loss of tax shields can equal the total increase in returns to shareholders \textit{ex post} if the APR violation is 100 percent and the corporate tax rate is 100 percent. In this case, no firm would enter if investment costs \( I(N) > 0 \). Since we proved in proposition 1 that entry is excessive in absence of taxes, there must exist some intermediate fractions \( \alpha_l(NH^*) \) and \( \alpha_h(NH^*) \) that induces the \textit{ex ante} efficient level of investment in the industry. \textit{Q.E.D.}

Further, it seems reasonable to conclude that when \( NH^* > NL^* \) the courts can encourage the efficient level of exit when firms are very dependent on tax shields. When taxes are 100 percent and the courts are unconstrained in violating priority \textit{ex post}, the courts can select a vector \( a_N = \{ \alpha_l(N, f), \alpha_h(N, f), \alpha_l(N, r), \alpha_h(N, f) \} \) for each and every firm on the interval \( (NL^*, NH^*) \) such that these firms’ founding shareholders will prefer inefficient liquidation in the low demand state to having lower tax shields in the high demand state.
4.0 Limitations and Possible Extensions

This paper attempts to demonstrate that welfare is not always improved by encouraging investment. This insight seems to be missing from the discussion of bankruptcy in the law and economics literature. Here we have argued that bankruptcy courts have a couple of tools at their disposal to discourage investment. First, they can liquidate profitable but insolvent firms. Second, courts can limit the debt capacity of firms by regularly awarding returns to shareholders that rightly belong to bondholders. Whether or not welfare can be raised by such policies in practice needs to be explored with further study. In this subsection, we point out some of the pitfalls that could arise if the courts make bankruptcy less efficient.

First, not all industries produce homogenous products. Mankiw and Whinston (1986) point out that as product diversity increases, it becomes hard to tell if free entry leads to too much or too little entry. Long distance phone services may be very close substitutes; but, in other industries, it may be more difficult to conclude if goods are sufficiently homogenous.

Airlines in particular may seem likely candidates for excessive entry. There is little difference between a seat on one major carrier or another. Yet, even in the case of airlines, there is some product differentiation. For example, airline Y may fly to Tampa from Philadelphia. Yet, if Y’s flight connects through Atlanta, it may not be a good substitute for a direct flight from Philadelphia on airline Z. Therefore, to the
extent that Z’s route structure is not closely duplicated by other airlines, there may be a case for there being too little entry into the industry.

Our analysis here has also assumed that labor markets are perfect. We assumed that all discharged employees could find employment at the same wage immediately at no cost. This is often not a good approximation. Let us continue the example of the airline industry in the U.S. Most legacy carriers organized prior to deregulation are heavily unionized. If employees are earning rents, then welfare ex post may not be raised by liquidating these carriers when the value of the firm to investors drops below the firm’s liquidation value.29 Schleifer and Summers (1988) point out that hostile takeovers may be privately profitable, but they may decrease welfare because shareholders and raiders do not take into account employment rents. Here, if the courts do not take into account the above-market wages of employees when making liquidation decisions, some welfare-improving firms may be prematurely shut down.

If firms are not 100 percent levered, they will pay taxes. It may be the case that the taxes that are paid are enough of a wedge to completely reverse the result of excess entry. Indeed, current tax collections may already lead to underinvestment. We assumed that there was only one source of bankruptcy costs. That cost was

29 In fact, these legacy carriers are partially prevented from liquidation by explicit or implicit government subsidies. Many legacy carriers qualified for federal loan guarantees after September 11, 2001. In addition, airlines have recently avoided paying under-funded pension obligations through Chapter 11, Carey (2004).

The canceling of pension plans by distressed firms may not be too costly in terms of contracting with employees because pension obligations are partially insured by Pension Benefit Guarantee Corporation (http://www.pbgc.gov), which insures defined benefit pension plans. Defined benefit plans guarantee a specified monthly payment on retirement. The PBGC will take on the obligations of employers that would otherwise be shut down if they were forced to honor their commitments. Ippolito (1985) provides some discussion of under-funded pension plans. Copeland, Weston, and Shastri (2005, p. 737) discusses some of the moral hazard problems of PBGC insurance.
inefficient liquidation by the courts. In absence of that cost, we would be in
Modigliani and Miller (1963) world where firms would have a 100 percent debt-to-
value ratio. With 100 percent debt, firms would pay no taxes. We know that this is
not the case. Corporations do not completely avoid taxes through debt. This is
partially true because there are other costs of bankruptcy than those imposed by the
courts. Let us consider a couple of canonical examples of bankruptcy costs. Jensen
and Meckling (1976) point out that, when default is imminent, shareholders may want
the firm to take on high-variance, negative-NPV projects at the expense of
bondholders. Alternatively, Myers (1977) says that bankruptcy can be especially
costly if it leads to forgone growth opportunities.

In addition, courts do not assess the prospects of firms in an environment of
stable demand and industry conditions. They face considerable uncertainty. If one
combines uncertainty and sunk costs, there is a case to be made that the option of
waiting may be valuable. In particular, firms that are making economic losses if
present trends continued may have value if there is a non-zero probability that
conditions will improve. In our analysis, judges in period 1 faced no uncertainty
about demand conditions. Baird and Morrison (2001) model the liquidation decision
by judges as a real option. Morrison (2007) argues that judges in the Northern
District of Illinois were much more likely to make the shutdown decision as
information came in and uncertainty was resolved. The irreversible aspect of
shutdown decisions should bias courts against shutdown, relative to the model in this
paper. Nevertheless, in environments with very little uncertainty, this bias towards
waiting will be small.
Moreover, the industry may be already be plagued by underinvestment relative to the idealized free entry equilibrium of Mankiw and Whinston (1986). If problems of asymmetric information or moral hazard mean that many profitable projects go unfunded, then a model where firms enter until they make zero profits in expectation may not be appropriate. Therefore, if firms or entrepreneurs already face high costs of capital, discouraging investment through the courts may be unnecessary and inappropriate, even in industries where firms produce products that are perfect substitutes.

There are other important extensions. Firms may produce goods in several different markets. In some of the markets, there may be insufficient entry. In other markets, entry will be excessive. It may be impossible to force such a multi-product firm out of one market, but into another. Moreover, firms may differ in their productive efficiency. In other words, some firms will have lower marginal costs than their competitors. Yet, these same firms may also have high liquidation values. In this case, welfare will not be maximized by simply forcing out the firms with the highest liquidation values. Moreover, it is not unreasonable to assume that the firms with highest initial investments, those firms with the highest fixed costs, may also have the lowest marginal costs. A social planner may face a tradeoff between curing the problem of excessive entry and the goal of reducing production costs in the industry.

Finally, we have not modeled a bankruptcy regime that exists in practice. Instead, we have considered a bankruptcy courts that behave optimally with the tools that they have. An interesting extension would see how well the bankruptcy regimes
in the United States and the United Kingdom cure or exacerbate the overinvestment problems in homogenous goods industries. Acharya, Sundaram, and John (2004) among others have contrasted the United States’ Chapter 11 with the Receivership in the United Kingdom. U.S. Chapter 11 regime is considered friendly towards equity and may lead too many economically distressed firms to continue. Compare this to Receivership in the U.K., which is friendly towards debt, and may allow creditors to liquidate economically viable firms. The extent to which these regimes lessen competition and possibly raise welfare *ex post* and *ex ante* would be an interesting question to be pursued in future work.
5.0 Conclusion

There are good reasons to believe that many industries are plagued by overinvestment, not underinvestment. Potential firms ignore both the positive externality to consumers of increased competition and the negative externality of lower industry profits. The latter externality is called the “business stealing” effect. If firms face setup costs and produce identical goods, Mankiw and Whinston (1986) illustrate that duplicated fixed costs and business stealing will outweigh the benefits from increased consumer surplus.

When entry is excessive, courts can have a role in pushing the industry to the welfare maximizing level of investment. Here we argue that bankruptcy courts can shape the industry through their enforcement of financial contracts. In this context, actions that discourage investment and push profitable firms to exit may actually raise aggregate welfare.

There are two ways that courts can discourage investment. First, they can increase bankruptcy costs by liquidating economically profitable firms. Second, if courts can reduce the debt capacity of potential entrants, they can increase firms’ tax burdens and lower firms’ after-tax returns. Debt capacity is reduced not only by increasing the costs of bankruptcy but also by predictable absolute priority rule (APR) violations that would lead to shareholders appropriating returns from bondholders ex post. Less debt capacity means higher taxes paid and lower after-tax returns.
Courts may not be able to move the industry to its welfare maximizing size. They are limited by how much they can reduce debt tax shields through APR violations. Further, they can only liquidate firms that choose to take on risky debt. In general, higher corporate tax rates aid the courts in reducing investment because they make tax shields and risky debt more important. The first-best may not be achievable, but the courts sometimes can make Pareto improvements by liquidating economically profitable firms and giving shareholders a slice of the returns when the firm defaults.

The author believes this is the first study to relate the problem of excessive entry to judicial decisions in bankruptcy. In practice, entry into most industries is not regulated by the government directly. Instead, entry is regulated by the willingness of investors to supply investment funds. Creditors, both in principle and in practice, are rarely in control of the firms under which their bonds are written. These creditors are dependent on public and private enforcement of contracts. When bankruptcy judges are likely to ignore some contracts, this tendency will make potential lenders much less likely to supply funds.

We have argued here that sometimes a little inefficiency may be a good thing. Nevertheless, careful attention would have to be paid to the unique nature of the industry before we can conclude that welfare will be raised by courts that actively try to discourage investment and or liquidate profitable firms.
References


6.0 Appendix

6.1 Proofs

Proof of $N^0 = N^H \geq N^L$:

Let us prove this by contradiction. Suppose that $N^H < N^0$ and $N^L > N^0$.

It must be true that in the high demand state the marginal entrant, with the highest liquidation value of all firms, finds it optimal to liquidate itself. That is, $\pi(H, N^0) < \gamma I(N^0)$. Moreover, we assumed that $\pi(H, N^0) > \pi(L, N^0)$. Therefore, $\pi(L, N^0) < \pi(H, N^0) < \gamma I(N^0)$. Given the $N^0$-th firm is optimally run, the marginal entrant will be liquidated in both states. Expected returns are $\gamma I(N^0) - I(N^0) < 0$ because $0 < \gamma < 1$. The marginal entrant must make non-negative profits, when it is optimally run, or else it will not enter. Therefore, this is a contradiction. Further, we assumed that no entry will be permitted in period 1 after the state is revealed. Therefore, it is impossible that $N^L > N^0$. Q.E.D.

Proof of Proposition 3:

To see this let us compare (14), (15), (16), and (17). It is clear that $V^0(N, F^r^*) > V^0(N, F^f^*)$ and $V^j(N, F^r^*) > V^j(N, F^f^*)$ when $\tau > 0$. Further, since creditors can always liquidate the firm after it emerges from bankruptcy court, it cannot be the case that both $\theta(N) = \max \{ \theta(c, N), \theta(d, N) \} = 0$ and $\pi(L, N^L) < \gamma I(N)$. Therefore, while it is possible that $V^j(N, F^r^*) > V^0(N, F^r^*)$, choosing both $\{ \theta(c, N) = 1 | \delta = f \}$ and $\{ \theta(c, N) = 0 | \delta = r \}$ cannot be an equilibrium strategy for the $N$-th firm’s investors. This leaves $V^0(N, F^r^*) >$
$V^i(N, F^*)$. Subtracting equation (17) from equation (14), we get the following relationship:

\[
V^0(N, F^*) - V^1(N, F^*) = (1 - \theta)(\pi(L, N_L^H) - \gamma I(N)) - \tau h[\pi(H, N_H^H) - \pi(L, N_L^H)] > 0.
\]

(35)

It seems possible that (35) is satisfied, while we have ruled out that safe debt could be preferred to risky debt under any other scenario.

Let us turn to the second part of the proposition, which says that safe debt is only preferred when the courts pursue a strategy of inefficient liquidation of the $N$-th firm. For (35) to be satisfied it is necessary that $\pi(L, N_L^I) > \gamma I(N)$. Further, if $\pi(L, N_L^I) > \gamma I(N)$, this implies that the risky debt firm’s liquidation was forced by the bankruptcy courts.

Alternatively, if $\pi(L, N_L^I) < \gamma I(N)$, both the low state returns are lower with safe debt, and the safe debt firm pays more taxes. This would mean that (35) is negative, which is a contradiction. Therefore, we can conclude that inefficient liquidation (both $\theta(N, c) = 1$ and $\theta(N, d) = 0$) must occur if the $N$-th firm prefers safe debt. Q.E.D.

Proof of Lemma 1:

To prove this we need to show that there is a firm that would not only be hurt from liquidation but also would take on risky debt. Suppose that $\pi(L, \tilde{N}) = \gamma I(\tilde{N}) + \epsilon$, where $\epsilon > 0$ but is arbitrarily small. Therefore, the expected losses from inefficient liquidation, $\epsilon$, are approximately zero. Yet, the gains from (19) are lower than the expected taxes in the high state. These gains are non-zero because $\pi(H, N_H^H)$ >
\( \pi(L, \hat{N}) \) and \( \tau \) and \( h > 0 \). We know that high state profits are higher because higher index firms \( N > \hat{N} \) voluntarily liquidated themselves for \( \gamma(N) < I(N) \). Since \( \gamma < 1 \), these firms liquidated themselves for less than the cost of entering the industry. Therefore, it must be the case that the profits in the high state exceed profits in the low state \( \pi(H, N^h) > \pi(L, \hat{N}) \). This allows us to conclude that risky debt is profitable for the \( \hat{N} \)-th firm.

Secondly, we need to show that \textit{ex post} welfare would be improved by forced liquidation of the \( \hat{N} \)-th firm. This follows from equation (9). Welfare is strictly increasing at the point of “free exit,” which is approximately where the \( \hat{N} \)-th firm is located. \textit{Q.E.D.}

This generalizes to the case where we allow absolute priority violations. The courts are indifferent about the profitability of non-marginal entrants. Yet, they are not indifferent about the number of firms in the low demand state. Therefore, they will be willing to set absolute priority violations such that risky debt is attractive to non-marginal entrants if further liquidations raise \textit{ex ante} welfare.
6.2 Judicial Regulation v. Entry Restrictions

Here we argue that regulating the industry by means of bankruptcy courts, “judicial regulation,” may be preferable to setting entry restrictions. Judicial regulation benefits from the fact actions are taken *ex post*. This means that much of the uncertainty concerning industry conditions has been resolved. There are two types of uncertainty. First, there is uncertainty about the period 0 expected welfare generated by the industry. Let us call this $\sigma$-uncertainty. Second, there is uncertainty about the evolution of the industry in period 1. Let us call this $s$-uncertainty. Judicial regulation has the benefit of acting after both these uncertainties are resolved. Nevertheless, bankruptcy courts cannot affect entry directly. Therefore, entry restrictions may raise welfare relative to judicial regulation when there is little uncertainty, taxes are low, and courts have little scope to give *ex post* returns to equity.

When bankruptcy judges shape the industry they have the benefit of history. It is said that “hindsight is 20/20.” While this may not always be the case, at least hindsight generally benefits from more information than forecasting. When regulators try to determine entry initially, they must make decisions with a great deal of uncertainty. They must make regulatory decisions before the features of the market and the profitability of individual firms are known. Bankruptcy courts, because they are looking back in time, have a better idea of the production conditions governing the industry than a

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30 For example, astronomers hoped to look far back into the history of the universe by deploying the very powerful Hubble telescope. When one looks at very distant, dim, stars one is looking at the distant past because it takes the light so long to travel to the Earth. Nevertheless, the first images that came back were out of focus and had to be repaired over the course of several space walks. Our views of history, even recent history, are fuzzy, but our glimpses into the future are even more opaque.
regulator who is peering into the future. Suppose that bankruptcy courts operate in period 1, but entry restrictions must be set in period –1 before firms enter. From the perspective of period 1, the bankruptcy courts will be in a better position to calculate period 0 expected welfare than a regulator who is trying to forecast demand and production conditions in a period –1.

Further, bankruptcy courts regularly are asked to make judgments about the value of distressed firms and to make decisions that may impede or speed up the liquidation of these over levered firms. This paper shows that there will be weakly too little exit in the low demand state. Entry restrictions cannot encourage firms to exit in the low demand state, but bankruptcy courts can push firms to liquidate.

Suppose there is a period –1 where the social planner can either choose to regulate the industry through a period –1, *ex ante* entry restriction or through period 1, *ex post* regulation by bankruptcy courts. The entry restriction chooses a number of entrants, \( N^R \), which is *ex ante* optimal from the perspective of period –1. Alternatively, she can organize the scheme of judicial regulation as described here. The social planner has beliefs about the expected welfare, \( W_\sigma(N^H, N^L) \) generated by the industry. Expected welfare in period –1 is a function of \( \sigma \), which is drawn from a continuous support \([u, d]\), and the density function, \( f(\sigma) > 0 \), about the period 0 welfare function generated by the industry. In period 1, both \( \sigma \) and \( s \) are known to the social planner and to the bankruptcy courts acting on her behalf. That is, bankruptcy courts can benefit from the option to “wait and see” how demand and industry conditions evolve up to the point where a firm lands up in court. The entry restriction, in contrast, must be set before the realization of both \( \sigma \) and \( s \) are known. By choosing a regime of judicial regulation, one gets a set of
high and low state industry sizes \( \{N_{j_{\sigma}}^{H}, N_{j_{\sigma}}^{L}\} \forall \sigma \), where \( N_{j_{\sigma}}^{*} \) is the subgame perfect Nash equilibrium size of the industry under judicial regulation, subscript “J”, for a given set of states \( \{\sigma, s\} \). Alternatively, with an entry restriction, one gets a set of high and low state industry sizes \( \{\min(N^{R}, N_{U_{\sigma}}^{H}), \min(N^{R}, N_{U_{\sigma}}^{L})\} \forall \sigma \), where \( N_{U_{\sigma}}^{*} \) is the subgame perfect Nash equilibrium industry size in an unregulated industry, subscript “U,” for the given set of states \( \{\sigma, s\} \). Judicial regulation will be chosen when

\[
\int_{d} f(\sigma)W_{\sigma}(N_{j_{\sigma}}^{H}, N_{j_{\sigma}}^{H})d\sigma \geq \int_{d} f(\sigma)W_{\sigma}(\min\{N^{R}, N_{U_{\sigma}}^{H}\}, \min\{N^{R}, N_{U_{\sigma}}^{L}\})d\sigma.
\]

Judicial regulation by benevolent courts will weakly dominate an entry restriction for all states where the entry restriction did not bind. This is when \( N^{R} > N_{U_{\sigma}}^{*} \). Moreover, it seems that judicial regulation would be preferred to an entry restriction as the density function, \( f(\sigma) \), became more diffuse. Indeed, judicial regulation may be preferred when there is no period \(-1\), \( \sigma\)-uncertainty! This is because judicial regulation also gets to adjust for period \(1\), \( s\)-uncertainty. If we drop the sigma notation, this is the case when

\[
W(N_{j_{\sigma}}^{H}, N_{j_{\sigma}}^{H}) \geq W(\min\{N^{R}, N_{U_{\sigma}}^{H}\}, \min\{N^{R}, N_{U_{\sigma}}^{L}\})
\]

Yet, entry restrictions may dominate judicial regulation when the industry conditions are stable and judges are tightly constrained. (For example when there is no
uncertainty, entry restrictions will be first-best. \( N^R = N^{H^*} = N^{K^*} \). In section 3, we find that bankruptcy courts have the least power to shape the industry when assets are durable (\( \gamma \) is close to one) and the marginal corporate tax rate is low (\( \tau \) is close to zero.) Therefore, if the tools of bankruptcy courts are weak, then judicial regulation, with all its informational advantages, may be dominated by a regime of entry restrictions.
6.3 The Failure of Exchange Offers

In this appendix, we will show that the Nash equilibrium strategy for a bondholder is to not tender in an exchange offer that would prevent inefficient liquidation by the bankruptcy court. Gertner and Scarfstein (1991) and Roe (1987) argue that in the United States that the Trust Indenture Act’s section 316(b) makes it very difficult if not impossible under many circumstances to restructure bond covenants outside of bankruptcy court. In particular all bondholders must agree if the interest, principle, or maturity. This is sometimes referred to the “unanimous consent” requirement.

Suppose that the firm faces inefficient liquidation by the courts given that it enters bankruptcy. The debtor firm’s and the court’s strategies are both \( \theta(d, N) = 0 \) and \( \theta(c, N) = 1 \), respectively. Further, debt is risky. The promised payments exceed the low state cash flows \( F^r > \pi(L, N^L) \). If the firm enters bankruptcy court, bondholders receive \( \gamma(N) < \pi(L, N^L) \) upon the dissolution of the firm. Suppose that a fraction of the bondholders, \( 0 < \beta < 1 \), exchange their old debt for new debt that has promised payments that are a fraction, \( \varepsilon \), of the promised value of the debt, \( F^r \). For the exchange offer to be a success, it must be the case that the following relationship holds.

\[
(1 - \beta)F^r + \beta \varepsilon F^r \leq \pi(L, N^L)
\]  

(36)

Since both \( F^r > \pi(L, N^L) \) and \( \beta \leq 1 \) rearranging (36) it becomes clear that \( \varepsilon < 1 \).

Namely,
\[ \varepsilon \leq \frac{\pi(L, N_L^t)}{F'} + 1 - \frac{1}{\beta} < 1 \]  

(37)

That is, tendering bondholders must accept a face value less than non-tendering bondholders if the exchange offer will be a success. If we denote $\bar{\varepsilon}$ as the upper bound, which is given by equation (37), of the fraction of face value that can be promised for a given $F'$, $\pi(L, N_L^t)$, and $\beta$, then it is clear that the maximum promised payments $\bar{\varepsilon} F'$ to tendering bondholders are increasing in the fraction of bondholders who tender their shares, $\beta$.

\[ \frac{d \bar{\varepsilon}}{d \beta} = \frac{1}{\beta^2} > 0. \]

Since $\varepsilon < 1$ we can conclude that given an exchange offer is a success a bondholder loses out by accepting the exchange. Next, we want to show that the bondholder that tenders also loses out if the exchange offer fails. Given that the exchanged securities cannot have higher priority than the original bonds this is a fairly simple task.

Suppose that the tendered and non-tendered bonds have equal priority in bankruptcy. They will split the value of the liquidated firm in proportion to their promised values. That is, the tendering bondholders will receive
In contrast, holdouts will receive in aggregate

\[ B_{\beta} = \frac{\beta \epsilon}{(1 - \beta) + \beta \epsilon} \gamma I(N) . \]

Likewise on a per dollar of face value, \( F^r \), basis, a bondholder accepting the exchange receives

\[ \frac{B_{\beta}}{\beta F^r} = \frac{\epsilon}{(1 - \beta) + \beta \epsilon} \frac{\gamma I(N)}{F^r} . \]

Likewise on a per dollar of face value, \( F^r \), basis, a bondholder rejecting the exchange receives

\[ \frac{B_{\beta}}{(1 - \beta)F^r} = \frac{1}{(1 - \beta) + \beta \epsilon} \frac{\gamma I(N)}{F^r} . \]

Since equation (41) exceeds equation (40) because \( \epsilon < 1 \), it must be the case that a bondholder is worse off tendering given the exchange offer fails. This is what we wanted to show in the second instance.
In summary, a bondholder is worse off exchanging their debt if either the exchange offer succeeds or fails. Therefore, the dominant strategy of all bondholders will be to not tender. Therefore, the Nash equilibrium will be for all bondholders to reject the exchange offer. *Q.E.D.*
6.4 Conflicts Between Entry and Exit

Sometimes, entry will be minimized when the marginal entrant takes on safe debt. Yet, welfare may be raised by liquidating some of the high $N$ firms, which are of rank $N < N^H$. In this case, there is a tradeoff between encouraging more entry by improving welfare in the low demand state and liquidating high-$N$, bankrupt firms. Lemma 2 no longer holds in the case where the marginal entrant is a safe debt firm. Here we show that it is generally ambiguous if the strategy of liquidating firms in the low demand state will raise welfare when the marginal firm prefers safe debt.

There are three main things that must occur for the welfare maximizing courts to want and be able to trade increased entry for less competition in the low demand state:

1. Entry is minimized and welfare is maximized when the marginal entrant chooses safe debt. For this to be true, it is a necessary condition that risky debt constraint below is violated given that APR violations are maximized for both the safe debt firm that continues and the risky debt firm that is liquidated. Namely,

$$ R\left[N^H, \left\{\frac{\gamma I(N^H_H)}{\pi(L, N^L)}, \frac{\gamma I(N^H_H)}{\pi(H, N^H)}, 1, \frac{\gamma I(N^H_H)}{\pi(H, N^H)}\right\}\right] > 0. \tag{42} $$

This is only can happen when $\pi(L, N^H) > \gamma I(N^H_H)$. The only way that the courts can encourage the marginal entrant to take on risky debt is to raise the returns to risky debt by
increasingly respecting priority in the high state. In other words, courts have to choose a higher $\alpha^H(N^H, r)$ to convert the marginal entrant to risky debt.

It also must be the case that $N^H \geq N^{hr}$, when both APR violations are at a maximum and there are too many firms in the low demand state under free exit. In other words, the courts must face some cost of encouraging more firms to enter. Otherwise, there would be no tradeoff between efficient entry and efficient exit for some positive interval of court ordered liquidations in the low demand state.

2. There must also exist a positive interval of firms $N \in [N^{LR}, N^H)$, where the risky debt constraint in (32) can be satisfied with the maximum low state APR violation and the minimum high state APR violation. Let $N^{LR}$ be implied by the $N = N^L = N^{LR}$ that would cause the risky debt constraint to bind. More generally, it must be true that

$$R\left(N; \left\{\frac{\gamma I(N^H)}{\pi(L, N^L)}, \frac{\gamma I(N^H)}{\pi(H, N^H)}, 1, 1\right\}\right) \leq 0, \forall N \in [N^{LR}, N^H).$$

(43)

$N^{LR}$ may be greater than or less than the optimal number of firms, $N^{lr}$, operating in the low demand state when there is no tradeoff between entry and exit. If $N^{LR} < N^{lr}$, then the courts would never want to move to only $N^{LR}$ firms.

3. Welfare must be falling in the number of firms in the low demand state. When the number of entrants is determined by the binding entry constraint of the safe debt marginal firm, welfare is only a function of the number of firms in the low demand state.
Therefore, with this substitution, welfare simplifies to a function of one variable $N^L$. It must be the case that

$$\frac{dW(N^H(N^L), N^L)}{dN^L} \leq 0,$$

for some $N^L < N^H$. 

(44)

Therefore, a necessary condition for (44) to be satisfied is that $N^H > N^L$. (Since $N^L$ is independent of the number of entrants, there will be many cases where $N^L$ is determined by the constraint $N^H \geq N^L$. When $N^H < N^H = N^L < N^L$, courts will never trade off low state liquidations with increased entry, $N^L$. We illustrate this in the worked out example.)

We asserted that when the marginal entrant is a safe debt firm, then the entry constraint can be used to solve for $N^H$ in terms of $N^L$. If entry is minimized and welfare is maximized by the maximum APR violations, then the safe debt marginal entrant is a function of the following entry constraint:

$$E^Q = E^I \left( \frac{\gamma I(N^H)}{\pi(L, N^L)}, \frac{\gamma I(N^H)}{\pi(H, N^H)} \right) = \tau \gamma I(N^H) + (1 - \tau)[h \pi(H, N^H) + (1 - h)\pi(L, N^L)] - I(N^H) = 0$$

(45)

This can be rearranged such that pre-tax returns as a ratio of total investment are increasing in the percent, $\gamma$, of the initial investment, $I(N^H)$ that can be returned before taxes.
No investment distortions occur for individual firms, as opposed to the market, when this ratio equals one and thus $\gamma = 1$ or $\tau = 0$.\(^{31}\) (This is the Mankiw and Whinston (1986) world where there are no taxes.) We want to raise the ratio above one because the incentives for individual firms are out of line with social welfare. That is, without taxes firms will over invest.

We need to show that the number of firms entering, $N^H$, is falling in the number of firms that remain in the low demand state, $N^L$. (We can violate Lemma 2 because the marginal entrant chooses safe debt.) To do this, let us totally differentiate equation (45) and solve for:

\[
\frac{\partial N^H}{\partial N^L} = \frac{(1-h)(1-\pi L, N^L)}{(1-\pi L, N^L)-(1-\tau \gamma)I_N(N^H)} < 0
\]  

It is easy to sign (47) because $\pi_N < 0$, $I_N > 0$, and $h$, $\tau$, $\gamma < 1$.

With the knowledge of the interaction between the number of firms in the low demand state and the number of entrants, we can differentiate the welfare function with respect to the number of firms in the low demand state. (The courts only directly can choose $N^L$, not $N^H$.) If welfare is falling in the number of firms in the low demand state

\(^{31}\) When courts pursue the strategy that minimizes entry—choosing the maximum APR violations in both states, the value of the marginal entrant is maximized by selling safe debt worth $\pi(N^L)$. This means that only $\tau$ percent of the original investment returns are shielded from taxes. This means that entry for the marginal firm must be a positive, before-tax net present value investment.
after solving for \( N^H \) in terms of \( N^L \) and substituting this into the welfare function, then some liquidations in the low demand state will be optimal.\(^{32}\)

Unfortunately, the sign of this first order condition is ambiguous and therefore depends on the entry and welfare functions that characterize a particular industry.

\[
\frac{dW(N^H(N^L), N^L)}{dN^L} \bigg|_{E' = 0, R(N) < 0 \forall \in (N^L, N^H), N'' \geq N^L} = \\
\left\{ h \frac{dN^H}{dN^L} \pi(H, N^H) + (1 - h) \frac{dq(L, N^L)}{dN}(P(L, Q^H) - c'(q^H)) \right\} \\
+ \left[ h \frac{dN^H}{dN^L} \frac{dq(H, N^H)}{dN}(P(H, Q^H) - c'(q^H)) \right] \\
+(1 - h)(\pi(L, N^L) - \gamma I(N^L)) - \frac{dN^H}{dN^L} (1 - \gamma(1 - h)) I(N^H),
\]

(48)

where \( q^H = q(H, N^H), q^L = q(L, N^L), Q^H = N^H q(H, N^H), \\
Q^L = N^L q(H, N^L), \pi(H, N^H) = P(H, Q^H) - c(q^H), \\
& \pi(L, N^L) = P(L, Q^L) - c(q^L).\)

Equation (48) has an ambiguous sign. The terms in \{\} have a negative sign while the terms in [ ] have a positive sign. Thus, welfare may be increasing or decreasing with \( N^L \) on the values of \( N^H \) and \( N^L \), satisfying the entry condition where the risky debt constraint is slack and \( N^H \geq N^L \). This equation cannot even be signed at the point where

\(^{32}\) If the opposite is true, we cannot automatically conclude that no liquidations will be second-best optimal, \( N^H = N^L \). For example, the welfare function may have several local maxima, and we just happen to be on the down slope where welfare is rising in \( N^L \). The maximum of this constrained problem may lie on some feasible set of points \( (N^L, N^H) \), where \( N^H > N^L \). In general, we have to inspect the boundaries where \( N^H = N^L = N^L_{max} \) and the entry constraint binds and where the risky debt constraint binds for the \( N^L_{th} \) firm—firm \( N^L_{min} \). In addition, we have to find all local maxima, if they exist, on \( N^L \) in between \( N^L_{min} \) and \( N^L_{max} \) where the first order condition of welfare with respect to \( N^L \) is equal to zero after plugging in the entry constraint. In most cases, this will have to be approximated as the entry constraint is unlikely to be linear. It is likely that approximate solutions will be needed to keep the welfare function on the entry constraint.
the first liquidation would be made at $N^{d^l} = N^l$. Therefore, it is ambiguous whether or not courts would want to liquidate firms in the low demand state if the marginal firm continues to operate in the low demand state.
6.5 Comparative Statics

Here we derive the comparative statics for the equilibrium entry and exit. For a given set of parameter values there will be a unique subgame perfect (SPE) set of equilibrium conditions for the number of firms entering and the number of firms remaining in the low demand state. Therefore, we only need concern ourselves with the binding first order conditions or constraints that characterize a given equilibrium. In all, there are nine equilibrium conditions describing entry and exit. Some equilibrium conditions only contain only one endogenous variable as is the case for $FHQ$, $EQ$, and $FLQ$. The other six have both $N^H$ and $N^L$ in them. These conditions are formed from various combinations of the constraints (21), (22), (28), (29), and (32) and first order conditions of the welfare function with or without some constraints substituted into optimization problem.

There are nine pairs of equilibrium conditions analyzed here. Any equilibrium involving any of the other six equilibrium conditions will be jointly determined by the entry and exit conditions. In all, there are two separable equilibria $\{FHQ, FLQ\}$ and $\{ER, FLQ\}$ and seven jointly determined equilibria $\{FHQ, XQ\}$, $\{EQ, XQ\}$, $\{EQ, XC\}$, $\{FHC, XE\}$, $\{EC, XQ\}$, $\{EC, XE\}$, and $\{EC, XC\}$. Nevertheless, all the joint equilibria involving $XE$—$\{FHC, XE\}$ and $\{EC, XE\}$—reduce to one variable comparative statics since $XE$ implies that $N^H = N^L$.

There are four entry conditions and five exit conditions. Let us turn to the four entry conditions first.
E1. Entry is optimal and the marginal entrant is liquidated in the low state. In this case entry is determined by the first order condition of welfare with respect to the number of entrants:

$$\frac{dW}{dN^H} \bigg|_{N^H-N_{H^*}} = $$

$$ = \left[ h(P(H, Q^{H^*})q^{H^*} - c(q^{H^*}))) + (1-h)\gamma I(N^{H^*}) - I(N^{H^*}) \right] $$

$$ + h N^{H^*} \frac{dq(H, N^{H^*})}{dN^{H^*}} (P(H, Q^{H^*}) - c'(q^{H^*})) = 0$$

(E1)

E2. Entry excessive and the marginal firm is liquidated in the low state:

$$EQ = E \left( \frac{1}{\pi(H, N^H)} \right) = $$

$$ = \tau \gamma I(N^{H^*}) + (1-\tau)[h\pi(H, N^{H^*}) + (1-h)\gamma I(N^{H^*})] - I(N^{H^*}) = 0$$

(E2)

This is a restatement of (45).

E3. Entry is determined by the first-order condition of welfare with respect to $N^H$ where no firms exit. This is the case where all firms take on safe debt and are not liquidated. The welfare function in this case is the following:
\[
W(N^H) = h \left( \int_0^{N^H} P(H,v)dv - N^H c(q^H) \right) + (1-h) \left( \int_0^{N^H} P(L,v)dv - N^H c(q^L) \right) - \int_0^{N^H} I(u)du,
\]
where \( q^H \equiv q(H,N^H), \ q^L \equiv q(L,N^H) \)

The first-order condition with respect to welfare is the following:

\[
FHC \equiv \frac{dW(N^H)}{dN^H} = \left[ h\pi(H,N^H) + (1-h)\pi(L,N^H) - I(N^H) \right] + hN^H \frac{dq(H,N^H)}{dN}(P(H,Q^H) - c'(q^H)) + (1-h)N^H \frac{dq(L,N^H)}{dN}(P(L,Q^L) - c'(q^L)) = 0,
\]
where \( \pi(H,N^H) \equiv P(H,N^H)q(H,N^H) - c(q^H), \)
\( \pi(L,N^H) \equiv P(L,N^H)q(L,N^H) - c(q^L), \)
\( q^L \equiv q(L,N^H), \ \text{and} \ Q^L \equiv N^H q^L. \)

We will look at the comparative statics where entry and exit are determined by this constraint or equivalently by this constraint and \( (X3) \).

E4. Entry is excessive, and the marginal entrant continues to operate in the low demand state:

\[
EC = E^0 \left( \frac{\gamma I(N^H)}{\pi(L,N^L)}, \frac{\gamma I(N^H)}{\pi(H,N^H)} \right) = (E4)
\]
\[ \tau \gamma I(N^H) + (1-\tau)[h\pi(H,N^H) + (1-h)\pi(L,N^H)] - I(N^H) = 0 \]
In addition, there are four exit conditions. Exit or rather the number of firms remaining in the low demand state, $N^L$, is governed by the following three equalities:

X1. The number of firms in the low demand state is optimally determined, $N^L = N^L^*$, by first order condition of welfare with respect to the number of firms in the low demand state:

$$FLQ = \frac{dW}{dN^L} \bigg|_{N^L = N^L^*} = [P(Q^L^*)q^L - c(q^L) - \gamma I(N^L^*)]$$

$$+ \frac{dq(L, N^L^*)}{dN} N^L^*[P(Q^L^*) - c'(q^L^*)] = 0$$

(X1)

X2. $N^L$ is partially determined by the binding risky debt constraint below and its entry condition.

$$XQ = -\tau h \pi(H, N^H) + (1 - \tau)(1 - h)\pi(L, N^L) + (\tau - 1 + h)\gamma I(N^L) = 0$$

(X2)

This is derived by subtracting (34) from (33) and setting the quantity equal to zero.

X3. The number of firms in the low demand state is identical to the number of entrants below. This is the case where no firms are liquidated. We will treat these cases as one variable problems.

$$XE \equiv N^H - N^L = 0$$

(X3)
For (X3) we can use the comparative statics for the entry condition for those equilibria to see the immediate effect of an exogenous shock on $N^d$. Therefore, we conserve on calculus by not differentiating (X3).

X4. The number of firms is jointly determined by the intersection of the entry constraint $EC$ given in equation (E4) and the first order condition in (48) evaluated at zero.

$$
FLC = \frac{dW(N^H(N^L), N^L)}{dN^L} = 0
$$

(X4)

This is the case where both welfare is rising by liquidating some non-marginal firms and the marginal firm, firm $N^H$, prefers to continue in the low demand state when APR violations are maximized as in (E2) and (E4). That is, $\pi(L, N^d) > \gamma(N^d)$.

X5. Entry is minimized when absolute priority violations are maximized. This is given by $EQ$ and $EC$, respectively. At some point, there begins to be a tradeoff between low demand state liquidations and entry. This is where $EQ$ and $EC$ intersect. At this point, it may be the case that the tradeoff between liquidating further firms leads to too much entry on the margin to be justified. Prior to this point, the marginal entrant would have chosen risky debt. Therefore, there was no tradeoff. After this point there is a tradeoff as the marginal entrant takes on safe debt and will be in operation in the low demand state. If we subtract $EQ$ from $EC$ given by equations (E2) and (E4), respectively we get the $XC$ condition for exit.
\( XC = EC - EQ = \pi(L, N^L) - \gamma I(N^H) = 0 \) \hspace{1cm} (X5)

This condition, by definition, must intersect \( EC \) if it intersects \( EQ \) and *vice versa.*

Nevertheless, we only derive the comparative statics for \( \{EQ, XC\} \). Nevertheless, we can conclude that the signs of the exogenous changes are the same, regardless of whether we think of \( EQ \) and \( XC \) intersecting or if we imagine \( EC \) and \( XC \) intersecting.

Not all of the entry and exit conditions are compatible with one another except for at a single point or in a rare combination of parameter values. In particular, we avoid discussing intersections where more than one entry or exit condition happens to be present. For example, \( EQ \) and \( FHQ \) may be identical in a rare combination of parameter values. In this rare case, \( EQ \) and \( FHQ \) would have all the same exit conditions. We only analyze the equilibria where both the entry and exit conditions are not repeated by another entry or exit condition. Repeated entry and exit conditions primarily only occur at a single point. (Except in the case where two conditions depend on a single variable as in \( EQ \) and \( FHQ \). In this case, if they intersect, they are identical lines on the \( N^H \cdot N^L \) plane.)

In the table 1 below, we summarize the entry and exit conditions that are compatible with one another. We have numbered the non-trivial sets of equilibrium conditions so that we can easily reference the comparative statics results.
For ease of reference we have numbered the equilibrium pairs. Each pair stands for a potential set of entry and exit conditions that characterize a subgame perfect Nash equilibrium (SPE) for more than one set of parameter values. “No” means that pair of conditions could not characterize a set of SPE outcomes except for a single set of exogenous parameters.

The exception to this rule is \{EC, XC\}. It is a possible set of equilibrium conditions that holds for many parameter values. The intersection of EC and XC occurs at the same points as equilibrium 5 where EQ = XC = 0. To avoid duplication of identical conditions, we only find comparative statics for the latter set of entry and exit conditions. (XC is exceptional because it is defined as the intersections of EC and EQ.)

Table 1: Entry and Exit Pairs

Table 2 summarizes these results for increases in the probability of the high demand state, \(h\); the corporate tax rate, \(\tau\); the percent of assets, which are recoverable, \(\gamma\); and an increase in the costs of initial investment, \(\iota\). The last parameter is defined by the relationship

\[
I(N) = \iota f(N),
\]

where \(\iota > 0\), \(f(\cdot) > 0\), and \(f'_{N} > 0\).
The signs of the comparative statics are dependent on the equilibrium conditions governing the industry, but, in general, the comparative statics for entry, $N^H$, are generally very intuitive. A higher chance of the high demand state, lower tax rates, higher liquidation values, and lower investment costs tend to increase the number of initial entrants. Taxes do not affect entry when $N^H$ is determined by the first order conditions $FHQ$ or $FHC$. In addition, when $N^L$ is determined by the first order condition $FLC$, taxes play no role in exit. Fewer firms remain in the low demand state when investment costs, and thus liquidation values increase. These results are summarized in table 2, and derived below.

6.5.1 *FHQ* Determines Entry (Entry in Equilibria 1 and 2)

To find the comparative statics for optimal entry, first we can find the second order condition evaluated at the social optimum. From (E1),

$$ FHQ_{N^H} = h \left[ P_N q_N^H + \left( 2q_N + q_N N^H \right) (P - c') + N^H q_N \left( P_N - c^* q_N \right) \right] $$

$$ - (1 - (1 - h) \gamma) I_N < 0, $$

where $P_N = P_0 (N q_N + q)$. (60)

Since this is a second order condition of an interior optimum, it must be negative.

Now let us differentiate the FOC with respect to the exogenous variables—$h$, $\tau$, $\gamma$, and $\iota$. 

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\( FHQ_h = (\pi(H, N^{II*}) - \gamma I(N^{II*})) - N^{II*} q_N (P - c') > 0, \)
\( FHQ_\tau = 0, \)
\( FHQ_\gamma = (1 - h)I(N^{II*}) > 0, \& \)
\( FHQ_t = -(1 - (1 - h)\gamma) f(N^{II*}) < 0. \)

The top term \( FHQ_h \) is positive because the first and second terms are positive.

High state profits must exceed recoverable investments. Further, \( q_N < 0 \), and price will meet or exceed marginal cost if they are to break even. These are Mankiw and Whinston (1986)’s assumptions 2 and 3. Therefore, the second term and all of \( W_{nh} \) must be positive. The other signs are more obvious.

Combining the results from equations (60) and (61) we get the following comparative statics results:

\[ \frac{\partial N^{II*}}{\partial h} = -\frac{FHQ_h}{FHQ_{N^{II*}}} > 0, \quad \frac{\partial N^{II*}}{\partial \tau} = -\frac{FHQ_\tau}{FHQ_{N^{II*}}} = 0, \]
\[ \frac{\partial N^{II*}}{\partial \gamma} = -\frac{FHQ_\gamma}{FHQ_{N^{II*}}} > 0, \& \quad \frac{\partial N^{II*}}{\partial t} = -\frac{FHQ_t}{FHQ_{N^{II*}}} < 0. \]

In summary, we can conclude several things from the equations above. The optimal number of entrants is increasing in the probability of the high demand state, \( h \), and the ease with which investments can be redeployed for other purposes, \( \gamma \). Taxes have no effect on the optimal number of firms. Yet, when investment costs increase, there should optimally be fewer entrants.
These are the comparative statics for the intersections of \{FHQ, FLQ\} and \{FHQ, XQ\}.

### 6.5.2 EQ Determines Entry (Entry in Equilibria 3, 4, and 5)

\[
EQ_{H} = (1 - \tau)h\pi_{N}(H, N^{H}) - [1 - \gamma(1 - h(1 - \tau))]I_{N}(N^{H}) < 0
\]  \hspace{1cm} (63)

The partials for the exogenous variables are the following:

\[
EQ_{h} = (1 - \tau)[\pi(H, N^{H}) - \gamma I(N^{H})] > 0
\]
\[
EQ_{\tau} = -h[\pi(H, N^{H}) - \gamma I(N^{H})] < 0
\]
\[
EQ_{\gamma} = (1 - h(1 - \tau))I(N^{H}) > 0
\]
\[
EQ_{f} = -f(N^{H})[1 - \gamma(1 - h(1 - \tau))] < 0
\]  \hspace{1cm} (64)

The fact that most of the parameters take on values between zero and one make the partial derivatives of \(EQ\) easy to sign. By combining equations (63) and (64) in equation (65), we can conclude that the number of entrants is increasing in the probability of the high demand state. Yet, \(N^{H}\) is falling in the corporate tax rate, increasing in the durability of investments, and falling as investment costs rise. These signs are summarized below.

\[
\begin{align*}
\frac{\partial N^{H}}{\partial h} &= \frac{EQ_{h}}{EQ_{N^{H}}} > 0, \quad \frac{\partial N^{H}}{\partial \tau} = -\frac{EQ_{\tau}}{EQ_{N^{H}}} < 0, \\
\frac{\partial N^{H}}{\partial \gamma} &= \frac{EQ_{\gamma}}{EQ_{N^{H}}} > 0, \quad &\frac{\partial N^{H}}{\partial t} = -\frac{EQ_{f}}{EQ_{N^{H}}} < 0.
\end{align*}
\]  \hspace{1cm} (65)
These are the comparative statics for $N^H$ when $N^H$ and $N^L$ are determined by the intersection of \{EQ, FLQ\}, \{EQ, XQ\}, or \{EQ, XC\}.

### 6.5.3 FHC Determines Entry (Entry and Exit in Equilibrium 6)

FHC generally maximizes welfare when $N^L > N^H$. This is the case in the numerical example in section 6.5. In that example, FHC is the feasible point that leads to the highest welfare.

The second order condition evaluated at the optimum must be negative. That is, $FHF_{N^H} < 0$. To find the signs of the comparative statics, all we must do is take the partials of the first order condition with respect to the exogenous parameters. They are

\[
FHC_b = [\pi(H, N^H) - \pi(L, N^H)] + N^H \frac{dq(H, N^H)}{dN} [P(H, Q^H) - c'(q^H)]
\]

\[
- N^H \frac{dq(L, N^H)}{dN} [P(L, Q^L) - c'(q^L)] = 0,
\]

where $Q^L \equiv q(L, N^H) N^H$ & $q^L \equiv q(L, N^H)$

\[
FHC_c = 0, \\
FHC_d = 0, \text{ and} \\
FHC_e = -f(N^H) < 0.
\]
We cannot sign $FHC_h$. It seems reasonable that entry would increase with the probability of the high demand state, but we would have to make further assumptions about the markups as a function of the state and $q_N$ as a function of the state. The rest of the partials of the first order condition are not ambiguous. The tax rate, $\tau$, does not play a role in the optimum entry condition. There is no exit. Therefore, the proportion of assets that are recoverable is also irrelevant. Higher costs of entry, higher $\iota$ parameters, do make more entry less attractive. The comparative statics are summarized below and in table 1.

$$
\frac{dN^H}{dh} = -\frac{FHF_h}{FHF_{N^H}} > 0, \quad \frac{dN^H}{d\tau} = -\frac{FHF_I}{FHF_{N^H}} = 0, \\
\frac{dN^H}{d\gamma} = -\frac{FHF_{\gamma}}{FHF_{N^H}} = 0, \quad & \frac{dN^H}{d\iota} = -\frac{FHF_{\iota}}{FHF_{N^H}} < 0.
$$

(67)

These comparative statics are relevant for the intersection of \{FHQ, XE\}.

### 6.5.4 EC Binds (Entry in Equilibria 7, 8, and 9)

This is the case where the marginal entrant prefers to take on safe debt and avoid inefficient liquidation in the low demand state. Further, in this scenario, the courts, despite their best efforts to deter entry, cannot make entry optimal. Therefore, welfare must be strictly decreasing in the number of entrants. Output is jointly determined by the
entry and exit conditions. So we will have to find the partials for the exit conditions $XQ$, $XE$, and $FLC$ in turn.

It is easiest to set up this problem in the form $Jy = a$, where $J$ is a 2 by 2, “Jacobin” matrix of the partials of the constraints with respect to the endogenous variables. $y$ is the column vector of comparative statics of the endogenous variables with respect to the exogenous variable. We want to solve for $y$. $A$ is column vector of negative partials of the constraints with respect to the exogenous variable.

Suppose that a generic exit constraint is $jX$, where $j = 1, 2, 3$, and $1X = XQ$, $2X = XE$, and $3X = FHC$. If we differentiate the constraints for entry, $EC$, and exit, $jX$, with respect to an endogenous variable $i\epsilon$, where $i = h, 2, 3, 4$, and $1\epsilon = h$, $2\epsilon = \tau$, $3\epsilon = \gamma$, and $4\epsilon = \iota$.

This system can be rewritten as

$$
EC_{N^H} \frac{\partial N^H}{\partial j\epsilon} + EC_{N^L} \frac{\partial N^L}{\partial j\epsilon} + EC_{j\epsilon} = 0
$$

$$
\begin{aligned}
\begin{bmatrix}
EC_{N^H} & EC_{N^L} \\
\frac{\partial N^H}{\partial j\epsilon} & \frac{\partial N^L}{\partial j\epsilon}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial N^H}{\partial j\epsilon} \\
\frac{\partial N^L}{\partial j\epsilon}
\end{bmatrix}
\end{aligned}
\begin{aligned}
= -EC_{j\epsilon}
\end{aligned}
$$

or $J_{ij}y = ij a$. 

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The subscript $j$ preceding the matrix $J$ and the subscripts $i$ and $j$ preceding the column vectors $y$ and $a$ denote the $i$-th exogenous variable for the $j$-th set of equilibrium conditions.

Cramer’s rule allows us to solve for the components of $i_j a$.

$$\frac{\partial N^H}{\partial \varepsilon} = \left|\begin{array}{c} i_j J_{N^H} \\ J \end{array}\right|,$$

where

$$\left|\begin{array}{c} i_j J_{N^H} \\ J \end{array}\right| = \left|\begin{array}{cc} -EC_{i\varepsilon} & EC_{N^H_{i\varepsilon}} \\ -fX_{i\varepsilon} & fX_{N^H_{i\varepsilon}} \end{array}\right|,$$

and

$$\frac{\partial N^L}{\partial \varepsilon} = \left|\begin{array}{c} j_i J_{N^L} \\ J \end{array}\right|,$$

where

$$\left|\begin{array}{c} j_i J_{N^L} \\ J \end{array}\right| = \left|\begin{array}{cc} EC_{N^L_{i\varepsilon}} & -EC_{i\varepsilon} \\ fX_{N^L_{i\varepsilon}} & -fX_{i\varepsilon} \end{array}\right|.$$
\[ EC_{N^L} = (1-\tau)(1-h)\pi_N(L,N^L) < 0 \] (69)

The assumptions about the functional forms and parameter values allows us to sign (68) and (69) easily. Per firm profits \( \pi(s,N) \) are falling in the number of firms. \( I_N \geq 0 \) for all \( N \). Moreover, \( h, \gamma \) and \( \tau \) are defined as between zero and one.

The partial derivatives of \( EC \) with respect to the four exogenous variables are the following:

\[
EC_h = (1-\tau)[\pi(H,N^H) - \pi(L,N^L)] > 0 \\
EC_r = -[h\pi(H,N^H) + (1-h)\pi(L,N^L) - \gamma I(N^H)] < 0 \\
EC_\gamma = \tau I(N^H) > 0 \\
EC_\tau = -\tau (N^H) < 0
\] (70)

6.5.4.1 \( EC = XQ = 0 \) (entry and exit in equilibrium 7)

First, we will solve for the partial derivatives of the \( XQ \) constraint. Then we will use these components of the matrix \( \mathbf{J} \) and the vector \( \mathbf{a} \) to solve for the \( i = 1, \ldots, 4 \) comparative statics with these equilibrium conditions.

The partial derivatives of \( XQ \) with respect to the endogenous variable \( N^H \) and \( N^L \) are the following:

\[
XQ_{N^H} = -\tau h \pi_N(H,N^H) > 0
\] (71)
There is only one term in equation (72) that creates problems for signing the whole expression. The first term on the LHS is negative, but the second term has an ambiguous sign because we cannot sign \((\tau - 1 + h)\). To proceed with any meaningful comparative statics, equation (72) needs a sign. Unfortunately, there is no theory for why the corporate tax rate, \(\tau\), should be bigger or smaller than the probability of the low demand state, \(1 - h\).

Because we cannot sign \((\tau - 1 + h)\) it is also impossible to determine the signs of a couple of the partials with respect to \(XQ\) below.

\[
XQ_{\pi} = (1 - h)(1 - \tau)\pi_{\pi}(L, N^L) + (\tau - 1 + h)\gamma I_{\pi}(N^L) = 0
\]

(72)

By combining our knowledge of the signs of equations (68), (69), (71), and (72), the Jacobin determinant for this intersection of entry and exit conditions is the following:
\[ |J| = \begin{vmatrix} EC_{\alpha} & EC_{\gamma} \\ XE_{\alpha} & XE_{\gamma} \end{vmatrix} = 0 \]

(74)

Without a sign of the Jacobin determinant there is no point in proceeding further.

6.5.4.2 \( EC = XE = 0 \) (entry and exit in equilibrium 8)

In the other sections, we solved this as a one-variable problem by substituting in \( N^H \) in place of \( N^E \). Here, we will use Cramer’s rule to solve for the comparative statics vectors \( i,j,y \). To do this we need to find the partials for the \( XE \) constraint. They are given below:

\[
\begin{align*}
XE_{\alpha} &= 1 \\
XE_{\gamma} &= -1 \\
XE_{\beta} &= XE_{r} = XE_{\gamma} = XE_{i} = 0
\end{align*}
\]

(75)

Next, the \( 2 \textbf{J} \) matrix’s determinant is given below.

\[
|2 \textbf{J}| = \begin{vmatrix} EC_{\alpha} & EC_{\gamma} \\ XE_{\alpha} & XE_{\gamma} \end{vmatrix} = \begin{vmatrix} EC_{\alpha} & EC_{\gamma} \\ 1 & -1 \end{vmatrix} > 0
\]

(76)

The positive sign for \( \det(2 \textbf{J}) \) follows from (68), (69), and (75).

Now we can solve for the various components of the vectors vectors \( i,j,y \).
The number of entrants is rising in the probability of the high demand state and falling in the marginal tax rate and the investment costs. The number of entrants is also rising in the proportion of assets that are recoverable upon liquidation. This latter observation is somewhat counterintuitive since we said that no firm will shut down in this equilibrium with safe debt. \( \gamma \) comes into the picture because the maximum amount that shareholders can credibly promise to bondholders in the presence of courts using absolute priority violations to discourage entry is determined by the percent of investments that are recoverable, \( \gamma \). Therefore, the value of the debt is determined by the recoverable assets. More valuable debt means more tax savings and more entry as \( \frac{\partial N}{\partial \gamma} \) states.

\[\text{6.5.4.3 } EC = FLC = 0 \text{ (entry and exit in equilibrium 9)}\]
The signs of the comparative statics for this equilibrium cannot be found without further specifying the parameter values of the problem. The signs for the comparative statics cannot be found for the general case because a crucial partial derivative of the FLC constraint has an ambiguous sign. Namely, the sign of $FLC_{N^H}$ cannot be deduced without specifying demand and production functions and the parameter values and or further characterizing the competitive framework. Without the knowledge of $FLC_{N^H}$’s sign, it is impossible to sign the determinant of the Jacobin matrix, $3J$, for the general case. Further, this implies that the comparative statics for $N^H$ and $N^L$ with respect to the exogenous variables cannot be found.

The partial derivative of FLC with respect to $N^H$ is a follows:

$$FLC_{N^H} = h \frac{dN^H}{dN^L} d^2 q(H, N^H) \left( P(H, Q^H) - c'(q^H) \right) + h \frac{dN^H}{dN^L} \pi_N (H, N^L) +$$

$$h \left[ \frac{dN^H}{dN^L} dq(H, N^H) \left( P_N (H, Q^H) \frac{dQ^H}{dN} - c''(q^H) \frac{dq(H, N^H)}{dN} \right) \right] = 0$$

The bottom term in square brackets is unambiguously positive while the top term on the left-hand side has an ambiguous sign. $\frac{dN^H}{dN^L}, \pi_N, \frac{dQ}{dN}, \frac{dq}{dN} < 0$. $c''(q) \geq 0$. If we assume that $q_{NN}(s, N) > 0$, as is the case with the linear Cournot model, the top term on
the LHS has a negative sign. This does not help us when we want to sign (81). Largely because we cannot sign (81), our comparative statics are all of ambiguous signs.

The $\det(3J)$ takes the form:

$$
\begin{vmatrix}
EC_{N^H} & EC_{N^L} \\
FLC_{N^H} & FLC_{N^L}
\end{vmatrix} = 0
$$

(82)

If $EC_{N^L} = 0$, then we could sign $\det(3J)$. Unfortunately, this is not the case. From equation (69) we know that $EC_{N^L} < 0$. Therefore, the only way we could sign the $\det(3J)$ would be to sign all the combined terms of the determinant in (82). This latter challenge is a feat that we cannot accomplish. All the comparative statics depend on the sign of the $\det(3J)$. Therefore, when $N^H$ and $N^L$ are determined by $EC = FLC = 0$, we can find definitive signs for the initial movements of $N^H$ and $N^L$ when any of the exogenous variables changes by a small amount. Proceeding further with the partial derivatives with respect to the $FLC$ constraint becomes pointless.

6.5.5 Exit is determined by $FLQ$ (exit in equilibria 1 and 3)

It may also be the case that the ex post social optimum is achieved in the low demand state. That is, $N = N^{d^*}$ as defined by equation (10). Let $FLQ_{N^{d^*}}$ denote the second order condition with respect to welfare evaluated at $N^{d^*}$, the optimum number of firms in the low demand state. This is,
\[ FLQ_{N^*} = (P_N q - \gamma l_N) + (P - c')[2q_N + q_{NN} N] + q_N N[P_N - c''q_N] < 0, \]  
where \( P_N = P_{\phi}(L, Q^{\ell^*})Q_{\phi}(L, N^{\ell^*}) \).

Once again, since this is by definition an interior optimum, we have the luxury of concluding its sign is negative.

We can find the partial derivatives with respect to the first order condition in (10):

\[
\begin{align*}
FLQ_h &= 0, \\
FLQ_{\tau} &= 0, \\
FLQ_{\gamma} &= -I(N^{\ell^*}) < 0, & \\
FLQ_t &= -\gamma f(N^{\ell^*}) < 0.
\end{align*}
\]  

Combining the results in (83) and (84) we get the set of comparative statics below:

\[
\begin{align*}
\frac{\partial N^{\ell^*}}{\partial h} &= -\frac{FLQ_h}{FLQ_{N^{\ell^*}}} = 0, & \frac{\partial N^{\ell^*}}{\partial \tau} &= -\frac{FLQ_{\tau}}{FLQ_{N^{\ell^*}}} = 0, \\
\frac{\partial N^{\ell^*}}{\partial \gamma} &= -\frac{FLQ_{\gamma}}{FLQ_{N^{\ell^*}}} < 0, & \frac{\partial N^{\ell^*}}{\partial t} &= -\frac{FLQ_t}{FLQ_{N^{\ell^*}}} < 0.
\end{align*}
\]  

Let us briefly interpret these results. The high state’s period 0 probability is irrelevant, after the state has been revealed in period 1. Further, taxes do not directly factor into welfare. An increase in either the durability parameter, \( \gamma \), or the parameter tracking the magnitude of investments, \( \imath \), means that the opportunity cost of operating the
the low demand state.

This comparative statics are relevant for parameter values where the SPE $N^H$ and $N^L$ are given by $\{FHQ, FLQ\}$ or $\{EQ, FLQ\}$. In both cases, $N^L$ is solely determined by the $FLQ$ condition, which is invariant to $N^H$.

6.5.6 \textit{XQ binds (exit in equilibria 2, 4, and 7)}

When the marginal entrant takes on risky debt—when entry is either determined by $FHQ$ or $EQ$—the social planner would like to liquidate all firms in which the first order condition of welfare with respect to the number of firm’s remaining $N^L$ is equal to zero. That is, welfare maximizing courts would want $N^L$ to be determined by $FLQ$.

Nevertheless, they have to convince $N^H$ to $N^L$ firms to take on risky debt. If $XQ > 0$ for the $N^L^{*-th}$ firm, the courts may have to settle for a marginally liquidated firm of rank $N^L > N^L^*$, where $N^L$ is determined by $XQ = 0$ and the entry condition evaluated at zero. There is a tradeoff between entry and exit in the low demand state when the marginal firm is a safe debt firm. In this case, the $EC$ constraint helps to determine entry and exit. The courts may find that welfare is rising until $XQ = 0$ intersects with $EC = 0$. Therefore, in all these instances, $XQ$ will help us determine how the low state industry size changes, for $\{FHQ, XQ\}$, $\{EQ, XQ\}$, and $\{EC, XQ\}$, and it may possibly help to determine entry, for $\{EC, XQ\}$.

To perform the comparative statics for a system of equations involving $XQ$ it is useful to set up the problem in vector matrix form.
\[
\begin{pmatrix}
  _k E_{N^k} & _k E_{N^k} \\
  XQ_{N^k} & XQ_{N^k}
\end{pmatrix}
\begin{pmatrix}
  \frac{\partial N^i}{\partial E} \\
  \frac{\partial N^l}{\partial E}
\end{pmatrix}
= \begin{pmatrix}
  -_k E_{i, \varepsilon} \\
  -XQ_{i, \varepsilon}
\end{pmatrix}
\]

or \( _k K_{i,j} y = _k b \).

Let \( k = 1, 2, \) or \( 3 \) where \( _i E \equiv FHQ, _j E \equiv EQ, \) and \( _3 E \equiv EC \). Since we have already considered the system involving \( EC \) above, we can focus on only the problems involving \( FHQ \) and \( EQ \). (We could not sign the Jacobin determinant of this latter system. Therefore, we do not have signs for the comparative statics of the equilibrium \( EC = XQ = 0 \).)

6.5.6.1 \( FHQ = XQ = 0 \) (entry and exit in equilibrium 2) Let us solve for the comparative statics. We have calculated all the \( FHQ \) partial derivatives that we need, except for \( FHQ_N^L \). The signs of these partial derivatives are given in equations (60) and (61). The only missing partial is

\[ FHQ_N^L = 0, \]  \( \text{Eq. (86)} \)

which is trivial because this condition does not depend on \( N^L \).

The Jacobin determinant for this system is
As with \( EC = XQ = 0 \), we cannot sign the Jacobin determinant of this system, because we don’t know the sign of \( XQN^L \) because we cannot sign \( (\tau - 1 + h) \), in particular. Therefore, there are no unambiguous comparative statics results for this system.

6.5.6.2 \( EQ = XQ = 0 \) (entry and exit in equilibrium 4)

The partials for \( EQ \) with respect to \( N^H \) and the exogenous variables have already been solved in equations (63) and (64), respectively. There is one more partial that has not been calculated:

\[
EQ_{N^L} = 0. \\
(88)
\]

Using equations (71), (72), (63), and (88) we can construct the Jacobin determinant for this system.
For the same reasons as above, there is no point in proceeding any further.

6.5.7  \( XC \) binds (entry and exit in equilibrium 5)

This is really the intersection of the two entry conditions \( EQ \) and \( EC \). Yet, we treat this as an exit condition, because at this point there begins to be a tradeoff between low state liquidations and entry. When \( N^H \) and \( N^L \) are determined by \( XC = EQ = 0 \), it may be that the marginal cost of encouraging more entry exceeds the benefit of pushing firms to exit in the low demand state.

The partials of \( XC \) are as follows:

\[
\begin{align*}
XC_{N^H} &= -\gamma I_N(N^H) < 0, \\
XC_{N^L} &= \pi_N(L, N^L) < 0, \\
XC_h &= 0, \\
XC_{\tau} &= 0, \\
XC_{\gamma} &= -I(N^H) < 0, & \\
XC_f &= -\gamma f(N^H) < 0.
\end{align*}
\]

\( EQ \) is invariant to \( N^L \):

\[
EQ_{N^L} = 0.
\]

\( 376 \)
To find the comparative statics we can combine the partials for $EQ$ in equations (63), (64), and (91) and the partials for $XC$ in (90). First, the comparative statics for each exogenous, $i\varepsilon$, variables can be represented by the linear system below:

\[
\begin{pmatrix}
EQ_{N_H}^i & EQ_{N_L}^i \\
XC_{N_H}^i & XC_{N_L}^i
\end{pmatrix}
\begin{pmatrix}
\frac{\partial N_{H}}{\partial \varepsilon} \\
\frac{\partial N_{L}}{\partial \varepsilon}
\end{pmatrix}
= \begin{pmatrix}
-EQ_{\varepsilon} \\
-XC_{\varepsilon}
\end{pmatrix}
\]  

(92)

or $M_n = r$.

The Jacobin determinant is

\[
|M| = \begin{vmatrix}
EQ_{N_H}^i & EQ_{N_L}^i \\
XC_{N_H}^i & XC_{N_L}^i
\end{vmatrix} > 0.
\]  

(93)

The comparative statics are

\[
\frac{\partial N_{H}}{\partial \tau} = \begin{vmatrix}
-EQ_{\tau} & EQ_{N_H}^i \\
-XC_{\tau} & XC_{N_H}^i
\end{vmatrix} < 0, \quad \frac{\partial N_{L}}{\partial \tau} = \begin{vmatrix}
EQ_{N_H}^i - EQ_{\tau} \\
XC_{N_H}^i - XC_{\tau}
\end{vmatrix} < 0,
\]  

(94)

\[
\frac{\partial N_{H}}{\partial \varepsilon} = \begin{vmatrix}
-EQ_{\varepsilon} & EQ_{N_H}^i \\
-XC_{\varepsilon} & XC_{N_H}^i
\end{vmatrix} < 0, \quad \frac{\partial N_{L}}{\partial \varepsilon} = \begin{vmatrix}
EQ_{N_H}^i - EQ_{\varepsilon} \\
XC_{N_H}^i - XC_{\varepsilon}
\end{vmatrix} > 0,
\]  

(95)
\[
\frac{\partial N^H}{\partial \gamma} = \frac{\begin{vmatrix} \overline{E Q} & \overline{E Q}_{N^H} \\ \overline{X C} & \overline{X C}_{N^H} \end{vmatrix}}{\mathbf{M}} > 0, \quad \frac{\partial N^L}{\partial \gamma} = \frac{\begin{vmatrix} \overline{E Q}_{N^L} & \overline{E Q} \\ \overline{X C}_{N^L} & \overline{X C}_L \end{vmatrix}}{\mathbf{M}} < 0, \quad (96)
\]

\[
\frac{\partial N^H}{\partial t} = \frac{\begin{vmatrix} \overline{E Q} & \overline{E Q}_{N^H} \\ \overline{X C} & \overline{X C}_{N^H} \end{vmatrix}}{\mathbf{M}} < 0,
\]

\[
\frac{\partial N^L}{\partial t} = \frac{\begin{vmatrix} \overline{E Q}_{N^L} - \overline{E Q} \\ \overline{X C}_{N^L} - \overline{X C}_L \end{vmatrix}}{\mathbf{M}} = \gamma (1 - \tau h) \pi \nu (H, N^H) f (N^H) < 0.
\]
Exogenous increase $h \tau \gamma \iota$

<table>
<thead>
<tr>
<th>Equilibrium conditions</th>
<th>No. of firms Entering, $N^H$</th>
<th>No. of firms remaining in the low demand state, $N^L$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $FHQ = FLC = 0$</td>
<td>$+$ $0$ $+$ $-$</td>
<td>$0$ $0$ $-$ $-$</td>
</tr>
<tr>
<td>3. $EQ = FLC = 0$</td>
<td>$+$ $-$ $+$ $-$</td>
<td>$0$ $0$ $-$ $-$</td>
</tr>
<tr>
<td>5. $EQ = XC = 0$</td>
<td>$+$ $-$ $+$ $-$</td>
<td>$-$ $+$ $-$ $-$</td>
</tr>
<tr>
<td>6. $FHC = XE = 0$</td>
<td>$?$ $0$ $0$ $-$</td>
<td>$?$ $0$ $0$ $-$</td>
</tr>
<tr>
<td>8. $EC = XE = 0$</td>
<td>$+$ $-$ $-$ $-$</td>
<td>$+$ $-$ $-$ $-$</td>
</tr>
</tbody>
</table>

$h \in [0,1]$ is the probability of the high-demand state; $\tau \in [0,1]$ is the corporate tax rate; $\gamma \in [0,1)$ is the proportion of the original investment that is recoverable upon liquidation; and $\iota > 0$ is an exogenous increase in investment costs as defined by equation (59). “$+$” means an increase in the exogenous parameter unambiguously causes the endogenous variable to increase. “$-$” means an increase in the exogenous parameter unambiguously causes the endogenous variable to decrease. “?” signifies that the relationship cannot be determined without more restrictions on the problem.

Table 2: Comparative Statics
6.6 Numerical Example

Figure 3: Feasibility in the Numerical Example
Figure 4: Entry and Exit Conditions and Welfare Contours for the Numerical Example
Suppose that the industry is characterized by Cournot competition. It faces a linear demand curve, $P(s, Q) = a' - bQ$, and there are two states $s = H$ and $L$. We will assume for convenience that the number of firms in the industry does not have to be an integer. This allows us to use calculus. Let us assume that all firms have a linear cost function $c(q) = cq$. In equilibrium all firms will produce the following identical output

$$q(s, N) = \frac{a' - c}{b(N + 1)}.$$  \hspace{1cm} (98)

Industry output is just $N$ times the symmetric firm output. Per firm profits before investment in a given state is just the square of (98) times $b$.

We will assume that the investment is increasing with a firm’s rank in the industry, $N$:

$$I(N) = 0.005 N.$$  

Further, the following parameters characterize the industry:

$$\gamma = 0.1$$

$$\{h, 1-h\} = \{0.6, 0.4\}$$

$$\{a^H, a^L\} = \{2, 1\}$$
\[ b = 1 \]
\[ c = 0.2 \]
\[ \tau = 0.4 \]

In this case, setting up the unconstrained welfare maximization problem as in equation (2) and substituting in the Cournot equilibrium outputs, we get an impossible solution.

Here we refer to the equilibria in the comparative statics problem. While finding the subgame perfect Nash equilibrium here is equivalent to solving a constrained optimization problem, most of the constraints are non-linear. This means solutions have to be numerical and approximate. Here we use the equilibrium approach pursued in the comparative statics section, section 6.4, to find the constrained optimum and explain how the parameter values led to the SPE of equilibrium 8 in figures 3 and 4 above.

Figures 3 and 4, plot the entry and exit conditions from equations (E1) to (E4) and (X1) to (X5) from the comparative statics section. The equilibria numbered 1 to 9 from table 1 are plotted. Most equilibria are either technically or strategically impossible for the courts to reach. In figure 3, we identify two shaded regions as “technically impossible” or “strategically infeasible.” “Technically impossible” equilibria are all those equilibria defined in the comparative statics section where \( N^h > N^d \). This includes equilibria 1, 5 (not pictured), and 9. These points all lie above the 45-degree line in this diagram. This line is also the \( XE \) constraint. “Strategically infeasible” points are those points where the courts cannot force the industry to a given \( \{N^d, N^h\} \) pair.

A given set of points \( \{N^d, N^h\} \) are strategically infeasible if they do not satisfy any one of the constraints (1), (2), or (3), which are given by equations (28), (29), and
Equilibria 7 and 8 are achievable through judicial regulation. Nevertheless, equilibrium 7 is strictly dominated by equilibrium 8. We know this must be the case because both points lie below the FLC curve. The FLC curve gives the minimum \( N^l \) that the courts would want to have stay in the industry in the low demand state, given that entry is determined by the EC constraint. Both points lie below the point where the FLC curve intersects the EC constraint. This intersection lies in the technically infeasible region. This tells us that there are too few firms in the low demand state. This is the case because liquidation values are very low. Yet, the anomaly in the first order condition comes from the discontinuity that new entrants must enter at the price, \( I(N) = 0.005 \), that is higher than the price at which old firms are liquidated, \( \gamma I(N) = 0.1 \times 0.005 \).
Further liquidations along the EC curve will lead to falling, not rising, *ex ante* welfare. As an extra check, we can see on figure 4 that equilibrium 7 falls on a lower indifference curve than equilibrium 8.

The global optimum, labeled as equilibrium 1 in the constraints diagram above, number of entrants and competitors in the low demand state is \( \{N^H, N^L\} = \{3.76, 5.25\} \). The determinant of the Hessian matrix is positive and both second order conditions at this point are negative, indicating that this is a global maximum. Visual inspection of the welfare function in 3-D below, as opposed to the 2-D level curves in figure 4 above, helps to further verify that this stationary point is a maximum:

Unfortunately, this global optimum is not technically feasible. Essentially, if firms could enter in the low demand state for \( \gamma I(N) \), one-tenth of the entry charge, welfare would be improved by having 5.25 – 3.76 = 1.49 firms enter even in the low demand state. This clearly is not possible, because all firms must pay the full entry charge to

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33 All reported numbers are rounded to three significant digits in scientific notation, but the calculations were taken to many digits in Excel and Mathematica.
enter. Further, no entry is permitted in period 1 when liquidations occur. Equilibrium 1, where $FHQ = FLQ = 0$, falls into the solid gray, “technically impossible” region in figure 3.

Let us concern ourselves with points that are technically feasible. These are all the points below the $XE$ constraint.

In the “first-best,” technically feasible equilibrium, entry is determined by the first order condition $FHC$ or entry condition (E3). The technically feasible, number of firms is $\{N^H, N^L\} = \{3.85, 3.85\}$. The welfare at this point will be designated as “first-best” welfare. This is denoted by the point 3, which lies on the $XE, N^H = N^L$, constraint in the diagram above. This may be technically feasible, unlike the global optimum, but it is not feasible by judicial regulation. That is, it is strategically infeasible.

With judicial regulation, the minimum number of entrants is given by equilibrium number 8 at the point $\{5.85, 5.85\}$. This point is determined by the intersection of $XE$ given by (X3) and $EQ$ given by (E4). Here, absolute priority violations are maximized and the marginal entrant will prefer safe debt. You can see that at $N^H = N^L = 5.85$ the risky debt constraint with the maximal APR violations, $EQ$, would lead to less entry than $EC$. Nevertheless, individual firms get to choose between risky or safe debt, and the marginal entrant will choose safe debt. Equilibrium 8 is the subgame perfect Nash equilibrium (SPE) because it is the highest-welfare, strategically feasible point. We will denote welfare at this point as “second-best” welfare.

Table 3 below reports the welfare and pre-tax profits as a percent of first-best welfare at point 3. “First-best” welfare is not achievable by the courts, but it is a useful
and a technically-feasible benchmark to compare the “second-best” (SPE) under judicial regulation to other entry regimes.

All the numbered equilibrium in figure 3 above represent possible SPE that could occur under different sets of parameter values. Below we discuss some results for some possible regimes that are less than optimal from the perspective of maximizing *ex ante* welfare:

“Entry deterrence and *ex post* welfare” is where the courts pursue the optimal number of low state liquidations, and does its best to deter entry. This is the intersection of *FLC* and *EC* in the constraints diagram. This behavior is consistent with the courts having lexographic preferences favoring welfare *ex post* over *ex ante* welfare. This court attempts to minimize entry, but only to the extent that minimizing entry does not interfere with *ex post* welfare maximization, which entails only having 5.25 firms in the low demand state. The table below shows that expected pre-tax profits are almost identical to the third-best under this regime, but welfare is slightly lower.

“Free entry and free exit” has courts completely respecting priority and never liquidating profitable firms. Entry is determined by the free entry condition in equation (4). All firms that enter are making positive economic profits in the low demand state. Therefore, no firms want to exit because liquidation values are so low. That is, the left hand side of the free exit condition in equation (8) is positive. This implies that number of firms entering and in the low demand state are identical, 6.95. All firms escape taxation with one hundred percent leverage, because bankruptcy is not costly.
Under the “free entry and forced exit” we have assumed that the courts do not attempt to discourage entry directly. Yet, the courts do attempt to maximize ex post welfare in the low demand state. Therefore, the courts attempt to have $N^* = 5.25$ firms in operation in the low demand state. This is given by the first-order condition also defined as the $FLQ$ constraint in (X1). The marginal firm faces no absolute priority violations. Rather, the marginal firm chooses the maximum value capital structure. Either, it can choose risky debt and face inefficient liquidation in the low demand state, but save more on taxes; or it can choose safe debt and have higher returns in the low demand state. The courts encourage non-marginal firms to take on risky debt as in $XQ$ (X2). We can check that $XQ$ is negative for the 5.25-th firm. To find the marginal entrant, we must find the $N^H$ that satisfies the following condition:

$$N^H = \max \{N', N^f\}$$

where 

$$h\pi(H, N^f) + (1-h)\pi(L, 5.25) + \tau\pi(L, 5.25) - I(N^f) = 0$$

$$h\pi(H, N^f) + (1-h)\gamma I(N^f) - I(N^f) = 0$$

That is, the marginal entrant can either choose risky or safe debt, respectively. $N^f$ is the $N$ that makes the marginal entrant break even if they choose to issue the maximum value of safe debt. The firm can always make $\pi(L, 5.25)$ in either state if it is not liquidated, and this is the maximum promised value of the safe debt. For this set of parameter values, the marginal entrant chooses risky debt and $N^H = 6.75$ as implied by (99). Because the firm’s liquidation value is strictly less than its continuation value in the low demand state, forced liquidation lowers the marginal entrant’s expected profits. This causes entry to
fall from 6.95 in the unregulated “free entry and free exit” case to 6.75 in the “free entry forced exit regime.” In this example, this regime is an improvement in both ex post and ex ante terms to “free entry and free exit.”

“All equity” denotes the equilibrium where firms choose all equity financing and there are no tax deductions for debt. That is, firms are free to enter and exit, but tax collections are high because there are no tax deductions. Namely, $N^H$ is implied by:

\[
(1 - \tau)[h\pi(H, N^0) + (1 - h)\max\{\pi(L, N^0), \gamma I(N^0)\}] - I(N^0) = 0
\]  

$N^L$ is given by

\[
N^L = \min\{N^H, N^X\}
\]

where \(\pi(L, N^X) = \gamma I(N^X)\).

That is, firms freely exit as in equation (8). If it is not profitable for any firms to exit, then $N^L = N^H$. In this case \(\{5.77, 5.77\} = \{N^H, N^L\}\). This deters entry even from what activist courts could achieve in equilibrium 8 in figure 3. Welfare is also higher under this regime than the second-best equilibrium. Welfare is higher under all equity compared to the SPE of the game in this paper, because there was an upper bound of APR violations that the courts could pursue. Because courts lacked the power to award all returns to equity ex post, the SPE had the marginal entrant shielding some returns.
from taxes. In the “all equity” regime, all firms pay taxes on all profits, and this deters entry relative to even the second-best SPE for the game pursued in this paper.

Table 3 summarizes the results concerning $N^H, N^L$, pre-tax industry profits, and total welfare for equilibria 6 and 8 and the various alternative regimes.
### Table 3: Numerical Example with Low Liquidation Values

<table>
<thead>
<tr>
<th>Regime</th>
<th># of entrants</th>
<th># of firms operating in the low state</th>
<th>Pre-tax, expected industry profits as a % of first-best welfare</th>
<th>Expected welfare as % of second-best expected welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-best, equilibrium 6</td>
<td>3.85</td>
<td>3.85</td>
<td>31.8%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Second-best, equilibrium 8 (SPE)</td>
<td>5.85</td>
<td>5.85</td>
<td>18.5%</td>
<td>97.5%</td>
</tr>
<tr>
<td>Entry deterrence and <em>ex post</em> welfare</td>
<td>5.91</td>
<td>5.25</td>
<td>18.5%</td>
<td>97.4%</td>
</tr>
<tr>
<td>Free entry and free exit</td>
<td>6.95</td>
<td>6.95</td>
<td>11.9%</td>
<td>94.6%</td>
</tr>
<tr>
<td>Free entry and forced exit</td>
<td>6.75</td>
<td>5.25</td>
<td>13.9%</td>
<td>95.3%</td>
</tr>
<tr>
<td>All equity</td>
<td>5.77</td>
<td>5.77</td>
<td>19.1%</td>
<td>97.7%</td>
</tr>
</tbody>
</table>
Conclusion

“Essays on the Financial Governance of Firms” has viewed governance arrangements, capital structure, and investment decisions as an early choice by shareholders in a sequential game with their workers, their clients, their competitors, and the courts, respectively. Most of these essays have argued that the structure and enforcement of governance arrangements and financial contracts can affect the cash flows of firms and their ultimate value. Each essay has a different set of assumptions and conclusions that must be considered in the context that they were made.
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