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Harnessing Windfall Revenues:
Optimal policies for resource-rich developing economies

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Abstract

A windfall of natural resource revenue (or foreign aid) faces government with choices of how to manage public debt, investment, and the distribution of funds for consumption, particularly if the windfall is both anticipated and temporary. We show that the permanent income hypothesis prescription of an ever-lasting increase in consumption financed by borrowing ahead of the windfall and then accumulating a Sovereign Wealth Fund (SWF) is not optimal for capital-scarce developing economies. Such countries should accumulate public and private capital to accelerate their development and, only if the windfall is large relative to initial foreign debt, is it optimal to build a SWF. The optimal time profile of consumption is biased towards the near future, as compared to the permanent income hypothesis. Outcomes depend on instruments available to government. We study cases where the government can make lump-sum transfers to consumers; where such transfers are impossible so optimal policy involves cutting distortionary taxation in order to raise investment and wages; and where Ricardian consumers can borrow against future revenues, in which case the policy response to possible over-consumption is a high level of investment in infrastructure.

Keywords: natural resource revenue, windfall public revenues, risk premium on foreign debt, public infrastructure, private investment, credit constraints, optimal fiscal policy, debt management, Sovereign Wealth Fund, asset holding subsidy, developing economies.

JEL codes: E60, F34, F35, F43, H21, H63, O11, Q33

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1. Introduction

Over the period 2000-05 exports of hydro-carbons and minerals accounted for more than 50% of goods exports in 36 countries. In 18 of these, revenues from natural resources contributed more than half of total fiscal revenue (IMF 2007). These earnings figures increased enormously during the commodity boom of 2006-08, before falling back. At the same time new countries have made major resource discoveries – for example oil in Brazil, Ghana, and Uganda. A temporary windfall of natural resource revenues (or foreign aid) poses numerous policy challenges. Should the revenues be used for government investment in public infrastructure to stimulate economic activity? Should the government use the windfall to reduce government debt and thereby lower interest rates and boost private sector investment? Should the extra income be used to provide more education, health care and other public goods to improve the quality of life or transferred directly to citizens through tax cuts or citizen dividends? Alternatively, revenues could be used to transform exhaustible resource assets into interest-earning foreign assets by setting up a Sovereign Wealth Fund (SWF) for future generations. This is a bewildering array of policy options and the most appropriate option depends on what stage of development the economy is and what constraints the economy faces.

The conventional consumption smoothing and debt management guidelines based on the permanent income hypothesis are familiar from the tax smoothing literature (Barro, 1979) or the optimal use of the current account (e.g. Sachs 1981). These arguments underlie much of the advice for the setting up of a SWF proffered by the International Monetary Fund (e.g. Davis et al., 2002; Barnett and Ossowski, 2003; Segura, 2006; Leigh and Olters, 2006; Olters, 2007; Basdevant, 2008). Although the main insights of this advice are sound, they ignore essential problems of developing economies struggling to grow from a low base and in the face of various market failures, and may therefore be only of limited relevance.

Our objective is to provide a rigorous analysis of how to address these policy choices in relatively poor countries which are capital scarce and have less than perfect access to capital markets. We focus on welfare-maximizing government choices between three broad options: using the windfall for private (or public) consumption; spending on public assets that raise income and the marginal productivity of private investment; and altering the country’s foreign asset/ debt position. We look at outcomes with different sets of policy instruments available and in a series of increasingly complex economic environments. These options provide different time-profiles of ultimate consumption benefit and elicit different private sector investment responses. While we focus on responses to windfalls, our analysis of these choices is of more general interest for policy formulation, particularly in developing economies.
Central to our analysis are several features that we think are important in many developing countries. The first is that the country is capital scarce and faces an interest rate premium in borrowing from international credit markets. It therefore has a high domestic interest rate, low capital-labour ratio, little investment in public infrastructure, and low wages and per capita income. We model this by assuming that capital-scarce countries are not price takers in international capital markets, but face an interest premium the size of which depends on the level of foreign debt. A country with low foreign debt has a small risk of default and can borrow on international markets at the world interest rate. But beyond a certain level of foreign indebtedness, it faces an upward-sloping supply schedule of foreign debt. The premium might be a consequence of the perceived likelihood of default, although we do not model this explicitly. In the absence of a foreign exchange windfall, developing economies are on a trajectory of capital accumulation, debt reduction, and rising consumption, and we examine how the windfall can be optimally used to alter this trajectory.

The second feature concerns the behaviour of households in the economy. In many countries households find it hard to borrow against future wage income so Ricardian debt neutrality is unlikely to hold. To capture this, we suppose that households have no access to capital markets, living entirely from current wage income and government transfers. The presence of credit-constrained households means that there is a role for government to smooth consumption by varying taxes paid by or subsidies given to these households. This may involve building a SWF to pay a continuing stream of citizen dividends. In a final section we remove this assumption, and allow households access to capital markets. However, they may not internalise other imperfections in the economy, so government policy has to address possible over-consumption from the resource boom.

The third imperfection arises from the set of policy instruments that government has. We explore a number of different cases, including those where lack of lump-sum transfers or other first best instruments mean that government is obliged to use distortionary taxation or other second best policies.

To focus on the main public finance issues at hand, we abstract from many important elements of the problem. We use a single-sector model in which there are no problems in absorbing expenditure, either from an appreciation of the real exchange rate and its adverse impact on the traded sector (the Dutch disease, Corden and Neary, 1982; van Wijnbergen, 1984; Sachs and Warner, 1997) or from supply bottlenecks in particular domestic sectors such as construction. We abstract from political economy concerns. And most critically, we work in an environment of certainty, so that resource revenue volatility and associated precautionary motives are ignored.

The outline of the paper is as follows. Section 2 first sets up the benchmark for a country in which the home interest rate is pegged to the world interest rate and whose citizens
are unable to smooth consumption but whose government can do it for them. Application of the permanent income hypothesis shows that it is best to use temporary windfall revenue to have an immediate and permanent boost to citizen dividends and private consumption, this being paid for by borrowing ahead of the windfall and then by the interest from a SWF accumulated during the windfall.

Section 3 analyses the best way to harness an anticipated windfall in capital-scarce developing countries which face an interest rate above the world interest rate. Full consumption smoothing is no longer optimal. Instead, an immediate increase in consumption is followed by rapid debt reduction in order to reduce the interest rate. With small windfalls, the economy’s growth path is accelerated, but no SWF is built up. Only if windfalls are large relative to initial debt will it be optimal to build an SWF and associated permanent increase in consumption. In both these cases consumption is relatively more skewed towards current (poorer) generations than is the case with the permanent income hypothesis benchmark. In a capital-scarce country the gains from growing more rapidly towards the long-run level of consumption outweigh those from raising the long-run level through permanent returns on an SWF.

Section 4 develops a richer model of the non-resource economy and of the policy options faced by government, adding investment in public infrastructure and income taxation to the menu of government options. If lump-sum transfers are not possible, the government has to use the distortionary income tax to finance infrastructure. The optimal response to resource revenue is to cut the tax rate and encourage private investment, thereby raising the wage and consumption. This is accompanied by a longer term build up in public infrastructure. Section 5 alters the model to allow domestic consumers access to credit markets. This creates the possibility that, however prudent government may be, Ricardian consumers may over-expand consumption once the windfall is known. Government can respond by an asset holding subsidy or, if this is not available, by committing a higher level of expenditure to public infrastructure rather than putting resource revenue in a SWF.

Section 6 concludes, summarises our guidelines for how to cope with windfall revenue in a developing economy, contrasts them with the advice given by some international bodies, and discusses possible extensions.

2. Benchmark: the permanent income hypothesis

We first consider a small open economy that can borrow or lend unlimited amounts at the world interest rate. This economy has exogenous and constant non-resource output $Y$. Consumers receive a lump-sum transfer or citizen dividend $T$ from the government so their consumption is given by $C = Y + T$. The government is the only agent in the economy that
has access to the international capital market, so foreign debt $F$ corresponds to public debt. It chooses transfers $T$ and public consumption $G$ to maximise utility of its citizens,

$$U \equiv \int_{0}^{\infty} \left( \frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma} \right) \exp(-\rho t) dt,$$

where $\psi \geq 0$ is the weight given to public consumption, $\sigma$ is the elasticity of intertemporal substitution and the rate of time preference is $\rho$. Maximisation is subject to the budget constraint $\dot{F} = r^* F + G + T - N = r^* F + G + C - Y - N$, with fixed initial debt $F_0$ and exogenous world interest rate $r^*$, assumed to equal the rate of time preference, $\rho = r^*$. $N$ stands for the flow of windfall revenue from the sale of resource or foreign aid, all of which accrues to government.

The conditions for optimal government policy are familiar. The intertemporal efficiency condition states that consumption of government and its citizens are smoothed over time, $\hat{G} = \hat{C} = 0$, and that lump-sum transfers adjust to achieve this. The intratemporal efficiency condition demands that public and private consumption move up and down together, $G = \psi C$. Combining these conditions with the present-value budget constraint gives

$$(1) \quad T(t) = \frac{1}{1+\psi} \left[ N^p(t) + Y - r^* F(t) \right], \quad G(t) = \frac{\psi}{1+\psi} \left[ N^p(t) + Y - r^* F(t) \right],$$

and $\dot{F}(t) = N^p(t) - N(t)$,

where $N^p(t)$ is the permanent level of resource revenue at each date, defined as the amortised value of the stream of future revenues, $N^p(t) = r^* \int_{t}^{\infty} \exp\left(-r^* (z-t)\right) N(z) dz$. Debt accumulation/decumulation equal to the difference between the permanent level and current flow of resource wealth, $\dot{F}(t) = N^p(t) - N(t)$ ensures that $N^p(t) - r^* F(t)$ is held constant, as can be seen by differentiating the definition of permanent resource revenue to give $\dot{N}^p(t) = r^* \left( N^p(t) - N(t) \right)$, and hence $\dot{N}^p(t) - r^* \dot{F}(t) = 0$. Aggregate spending equals permanent resource revenues plus production income minus interest on foreign debt, i.e., $C(t) + G(t) = N^p(t) + Y - r^* F(t)$, and the division between private and public spending depends on the relative weight given to public consumption. The non-resource primary deficit must simply equal the permanent level of resource revenues.

Figure 2.1 summarises the responses derived in (1) to an anticipated temporary windfall with revenue flow given by the step function $N$. This flow of revenue is announced
(A) at date $t = 0$; it starts to flow at some date at or beyond $t = 0$ which we refer to as the date of extraction (E) and ceases flowing, or is depleted (D), at some later date. The vertical axis measures income flows, so an economy with initial debt $F_0$ (illustrated with $F_0 > 0$) has an associated income flow (negative debt service) of $-r^*F_0$. Permanent resource revenue is given by curve $N^p$. This rises as the date of extraction comes closer, and falls once extraction begins, going to zero when the windfall ends. There is a permanent rise in public spending equal to the permanent value on discovery of the windfall; prior to the discovery $T + G = r^*F_0$ and following the discovery this jumps to $T + G = r^*F_0 + N^p(0)$, as illustrated by the horizontal line $T + G$. Ahead of the windfall revenue, the country borrows abroad, with the path of debt illustrated by $-r^*F$, this mirroring the shape of $N^p$ as $T + G = N^p - r^*F$.

Debt rises until the windfall revenue comes in, at which date the country starts paying off debt and eventually builds up assets abroad sufficient to sustain the permanent increase in $T + G$.

**Figure 2.1: Use of Sovereign Wealth Fund to manage temporary windfall revenue according to the permanent income hypothesis**

In summary, the permanent income hypothesis suggests that the country runs a current-account deficit when it anticipates future windfall revenue, then pays off foreign debt and builds a SWF by running a current-account surplus during the temporary windfall. Total spending (public consumption plus citizen dividends) rises immediately upon news of the windfall, one-for-one with the permanent level of windfall revenue. The foreign assets that are built up at the end of the windfall generate just sufficient interest revenue to finance the permanent rise in public spending. This policy of borrowing, then saving and finally living off the return on the SWF thus transforms an anticipated, temporary windfall revenue into a
permanent increase in aggregate spending, where the amount devoted to citizen dividends and private consumption decreases with the weight given to public consumption.

3. Developing economies: departure from the permanent income hypothesis

The benchmark of using debt to smooth consumption may be applicable for countries able to borrow or lend unlimited amounts at a given world interest rate. Yet most developing economies are capital scarce and have high domestic interest rates. They are unable to remedy this by international borrowing, as they are likely to face a high and increasing interest premium on such borrowing. We capture this with a supply schedule of foreign debt, where for low values of foreign indebtedness the home interest rate equals the world interest rate and for high levels of indebtedness the home interest rate rises above the world rate.

The empirical rationale for this is illustrated in Figure 3.1(a) which shows a positive relationship between interest rate spreads and the ratio of public and publicly guaranteed debt to GNI.

**Figure 3.1: Interest rate spreads and public and publicly guaranteed debt**

* The slope coefficient corresponding to the unconditional correlation for the pooled regression with N=165 and 25 countries is 2.270 with standard error 0.250, which is significantly different from zero at the 1% level, and the adjusted R² is 0.332.

** The slope coefficient corresponding to the conditional correlation after controlling for country and time fixed effects, reserves/GDP, ln(inflation), output gap, in-default dummy and regional spread is somewhat smaller, namely 1.855 with standard error 0.536 which is also significant at the 1% level. Within-R² = 0.732.
Following Akitobi and Stratmann (2008), we estimate the interest rate spreads after controlling for country and fixed effects and other relevant fiscal and monetary variables.\(^1\) Figure 3.1(b) shows that interest rate spreads still react strongly to debt. We also find that higher foreign reserves and a higher output gap have a negative impact on interest rate spreads while a higher probability of default has a positive impact on interest rate spreads.

To capture this relationship in our model we assume that the domestic interest rate, \(r\), is determined by:

\[
(2) \quad r = r^* \text{ for } F \leq \bar{F} \quad \text{and} \quad r = r^* + \Pi(F) > r^* \text{ for } F > \bar{F} \geq 0, \tag{2}
\]

where \(\Pi(F)\) is the interest rate premium and \(\bar{F}\) the debt threshold below which the country is price taker at the world rate of interest, so \(\Pi(\bar{F}) = \Pi'(\bar{F}) = 0\) and \(\Pi'(F) > 0\), \(F > \bar{F} \geq 0\).\(^2\) Figure 3.2 portrays this supply schedule. One can interpret \(\Pi(F)\) as an international premium on foreign debt to capture the risk of default, but we do not model that. In the macroeconomics literature (e.g. Turnovsky, 1997, section 2.6), it is common to close small open economy models by specifying a supply schedule of foreign debt which slopes upwards for all \(F\).\(^3\) Although this is analytically convenient, it has the unattractive feature of implying a unique steady-state value of \(F\) at which the domestic interest rate equals the world rate; this level is independent of windfall revenue. This is in contrast to the permanent income hypothesis under which, as we saw in section 2, countries choose their steady-state value of \(F\) by, for example, building a SWF. It is to capture both the interest premium and the endogeneity of the steady-state value of \(F\) that we suppose that economies face a premium, \(\Pi(F) > 0\), above some threshold level of indebtedness \(\bar{F}\), while below that level countries are price takers at \(r^*\).

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\(^1\) Apart from the public and publicly guaranteed debt and GNI variables which we obtained from World Bank Development Indicators (April 2008), we use exactly the same years and sample of countries and the same explanatory variables as Akitobi and Stratmann (2008).

\(^2\) \(F\Pi'(F)\) is the terms of trade loss from a marginal increase in \(F\). We assume \(\bar{F} \geq 0\) and thus \(F\Pi'(F) \geq 0, \forall F\), so that there is never a terms of trade gain from reducing \(F\).

\(^3\) Most small open economy models with incomplete asset markets have steady states that depend on initial conditions and furthermore have equilibrium dynamics with a random walk component. To ensure stationarity and a unique steady state, one often postulates an upward-sloping supply schedule of foreign debt. Alternatives are to have an endogenous discount rate, convex portfolio adjustment costs or asset markets with a complete menu of state-contingent claims (Schmitt-Grohé and Uribe, 2003), but we do not explore these as a debt-elastic risk premium seems relevant for developing economies. The alternative of portfolio adjustment costs when asset holdings are different from some long-run level gives qualitatively similar conclusions.
As in section 2, the government maximises utility of its citizens subject to its budget constraint $\dot{F} = [r^* + \Pi(F)]F + T + G - N, \quad F(0) = F_0$. The intratemporal efficiency condition $G = \psi^\sigma C$ can be used to write the current-account dynamics as

$$\dot{F} = [r^* + \Pi(F)]F + (1 + \psi^\sigma)C - Y - N, \quad F(0) = F_0.$$  

(3)

Perfect consumption smoothing is no longer optimal, since the marginal cost of borrowing is not equal to the pure time preference rate. The intertemporal efficiency condition (the first-order condition for the optimal consumption path) is

$$\dot{C} = \sigma C [r^* + \Pi(F) + F\Pi'(F) - \rho].$$  

(4)

Consumption is low – and therefore rising – if the marginal cost of foreign borrowing (or marginal return to accumulating foreign assets) exceeds the rate of pure time preference. The marginal cost of foreign borrowing now includes the premium $\Pi(F)$ and the value of any change in the premium, $F\Pi'(F)$. At this higher rate a country with $F > F$ has an incentive to postpone consumption and save.

Figure 3.3 portrays the phase-plane diagram corresponding to (3)-(4). Looking first at the lower part of the figure, the $\dot{F} = 0$ locus slopes downwards; above it consumption is high and foreign debt increases and below it foreign debt declines over time. $\dot{C} = 0$ is not a line, but the set of all values of $F \leq F$. Countries with substantial foreign debt $F > F$ face high domestic interest rates and have rising consumption, while countries with $F \leq F$ have $r = \rho$.
and constant consumption. This system has a set of stationary points, given by the line S-S. For an economy which finds itself with $F \leq \bar{F}$, this line segment is unstable, so $Q$ must jump to S-S; this is precisely the permanent income hypothesis. For an economy with $F > \bar{F}$ there is a unique saddlepath, illustrated by the dashed line.

Our focus is on a developing country, which is initially indebted and which starts out at point $E_0$ on figure 3.3. This economy faces relatively high interest rates and is gradually converging to its final steady state at point S. Along the saddlepath (dashed line), the economy saves a lot and consumption grows. As it pays off its foreign debt the domestic interest rate falls so that the propensity to save and the growth rate decline. In the long run the economy has paid off its foreign debt ($F = \bar{F}$), the domestic interest rate has fallen to the world interest rate, and private and public consumption have risen to their steady-state values.

Figure 3.3: Consumption and investment response to a permanent windfall

3.1. A permanent windfall
We look first at a permanent and constant flow of resource revenue. The effect of this is to shift the $\dot{F} = 0$ locus on figure 3.3 upwards, moving the steady state from S-S to $S'-S'$ with dynamics based around this new steady state. If extraction occurs from the date of announcement, then consumption immediately jumps, $C$ going from $E_0$ to point A/E; the economy then converges along the new saddlepath towards the new steady state $S'$. 
What if the permanent increase in foreign windfall revenue is anticipated some time ahead of extraction? At announcement, consumption immediately increases to point A. During the interval between announcement and extraction, the government borrows abroad and thus pushes up foreign debt and interest rates at home as illustrated by the movement up and to the right. Once extraction commences, the economy must be on the new saddlepath at point E. The government stops borrowing and starts to pay off the foreign debt. As the economy moves along the saddlepath to S', the interest rate gradually falls to the world interest rate, economic growth tapers off and consumption rises to the new steady-state value. Anticipated resource revenues thus imply that a debt-ridden country adds to its debt before the revenue comes on stream, and this finances higher consumption. Debt is eventually paid off, but – as a direct consequence of the resource windfall being permanent and constant – the economy does not accumulate a SWF.

3.2. A temporary windfall

The more interesting case is that in which the revenue flows from the windfall are known to be temporary, addressed in figure 3.4. We assume that revenues follow a step function (as in figure 2.1), so the system is under the influence of the stationary at S' - S' only during the period during which the resource is extracted. For simplicity in construction of this figure we assume that extraction and revenue flow commences at the date of announcement. The analysis now depends critically on whether the windfall is 'small' or 'large'. When the windfall is small the economy jumps from E₀ to E₅ as the government immediately raises transfers to its citizens and boosts private and public consumption. Under the dynamics associated with S' the government pays off debt relatively fast; the jump in consumption is less than the flow of revenue. The marginal cost of capital remains above the rate of time preference so consumption continues to rise. At the date when the revenue flow stops the economy has to hit the original saddlepath (point D₅) and it then continues up this path to the steady state S. The windfall raises consumption at all dates, but does not raise the long run level of consumption to which the economy asymptotically converges. In contrast to the permanent income hypothesis there is no SWF, since the economy’s initial low level of consumption means that it is optimal to use the windfall proportionately more on the current generation.

4 Notice that we have not allowed the announcement of the windfall to have any direct effect on the 'creditworthiness' of the economy. Such a direct effect would shift the Π(F) function indicating the extent to which foreign lenders are prepared to take the present discounted value of future resource revenues into account. We do not pursue this idea, and would argue that such a direct effect is not rational. Along an optimal path the government will spend all the revenue, so any improvement in creditworthiness is illusory.
Figure 3.4: Temporary windfall and permanent spending

A larger windfall is associated with a larger initial jump in consumption, and with the establishment of a SWF to finance permanently higher consumption. This is illustrated in figure 3.4 by the jump to point $E^L$. As with the small-windfall case, during the period of extraction consumption is higher and debt is being repaid, but now it is optimal to repay all debt (reach $F^*$) before the resource is depleted. Consumption therefore increases until this point is reached and the marginal cost of capital reaches the rate of time preference. Once consumption has reached its permanent value, foreign assets continue to be built up to the level sufficient to sustain this consumption once resource revenues cease. At the date the resource runs out (point $D^L$), the economy becomes stationary. The economy has to reach the original stationary, $S$–$S$, at the date the resource is depleted and the windfall ceases, this determining the size of the jump to $E^L$. The size of the long-run SWF is now endogenously determined. For example, a higher initial debt or a smaller and less protracted windfall will reduce the size of the terminal SWF. The boundary between the ‘large’ and ‘small’ windfall is when points $D^S$ and $D^L$ coincide at foreign debt level $F^*$.  

Summing up, with an interest premium on foreign borrowing, perfect smoothing of public and private consumption is no longer optimal. Instead of raising long-run consumption, optimal policy accelerates progress towards this long-run value. While consumption will jump up at the date of announcement (this involving borrowing if announcement precedes the revenue flow), consumption does not jump the whole way to its steady-state value because the marginal cost of debt exceeds the rate of pure time preference.
Whereas the permanent income hypothesis suggests that a SWF should be built up in response to a temporary windfall, this is no longer true, unless the windfall is so large that it moves the economy out of the regime in which it faces a premium on its foreign debt.  

4. Public infrastructure and domestic production with foreign-owned capital

The previous section made the point that, in an economy with interest rate greater than the rate of time preference and growing consumption, it is optimal to use revenue to accelerate the growth of consumption towards its steady-state value, rather than to increase that value through investment in a SWF. However, the government could invest only in foreign assets, either by debt reduction or construction of an SWF. We now turn to the next question. If there are domestic assets – private and public capital stock – as well as foreign, how should optimal policy combine current consumption, debt reduction, public investment, and incentives to private investment, and what are the implications for the consequent growth of non-resource national income?

To answer these questions we make non-resource output endogenous by including private capital and public infrastructure. Non-resource domestic income is given by a production function with constant returns to scale with respect to private capital and labour, expressed as \( Y = f(K, S) \) where the labour force is normalised at unity, \( K \) denotes the private capital stock and \( S \) is the stock of public infrastructure. Given the exogenous supply of labour, the function \( f(.) \) exhibits decreasing returns in \( K \) and \( S \) together, to rule out ever-increasing growth. Infrastructure can be thought of as consisting not only of seaports, airports, roads and railroads, but also of education, health or any other public investment that boosts the productivity of private production.

We retain for the moment the assumption that there are no private domestic asset holders. Public infrastructure is owned by government, while private capital is rented from abroad from foreign owners who face the world interest rate, \( r^* \). They are subject to host country income taxation at a proportional rate \( \tau \). Profit maximisation requires that the after-tax marginal product of capital, net of depreciation \( \delta K \), equals the world interest rate, so that

\[
(1 - \tau) f_s(K, S) = r^* + \delta K
\]

5 Flight capital as a policy choice (e.g., formation of a SWF) has been discussed before in a different context (Collier, Hoeffler and Pattillo, 2001; Collier and Gunning, 2005).
The equilibrium capital stock follows from (5), and can be written as $K = K(S, \tau)$, this giving wages $W(S, \tau) = (1-\tau)[f(K(S, \tau), S) - K(S, \tau)f_K(K(S, \tau), S)]$ and production income $Y(S, \tau) = f(K(S, \tau), S)$. Capital stock, wages and income are all increasing in $S$ and decreasing in $\tau$ and, differentiating (5) and $W(S, \tau)$, the effect of a tax change on the wage rate, $W_\tau$, satisfies $W_\tau = -Y$.

In this structure the only domestic debt is that of government, $D$. It is held entirely by foreigners, and the interest rate becomes $r = r^* + \Pi(D)$. The dynamics of government debt come from the government budget constraint in the familiar way:

$$\dot{D} = rD + G + I_S + T - \tau Y - N$$

where $I_S$ is spending on infrastructure investment and the final terms are lump-sum transfers to consumers, income taxation and resource revenues. The stock of infrastructure evolves according to $\dot{S} = I_S - \delta_S S$, with $\delta_S$ is the depreciation rate. Analysis is simplified by working with net government assets defined as the stock of public infrastructure minus government debt, $B = S - D$. The budget constraint is then

$$\dot{B} = r(B - S) + N + \tau Y - G - T - \delta_S S, \quad B(0) = B_0$$

with the initial value of net government assets fixed at $B_0$. The no-Ponzi game condition must be satisfied, $\lim_{t \to \infty} B(t) \exp\left(-\int_0^t r(v)dv\right) = 0$, so that initial net government assets plus the present value of the stream of future income taxes and resource revenue must cover the present value of the stream of future spending on public consumption, government transfers and infrastructure services.

The government’s problem is now to choose the public capital stock $S$, public consumption $G$, together with the rate of income taxation $\tau$ and transfers to households $T$, where households’ consumption is $C = W + T$. Its objective is social welfare, as before,

$$U = \int_0^\infty \left( \frac{C^{1-\sigma} + \psi G^{1-\sigma}}{1-1/\sigma} \right) \exp(-\rho t)dt$$

and the constraints are the budget equation (6'), together with initial conditions, the no-Ponzi condition and equilibrium levels of private

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6 Section 5 looks at the case where there are domestic asset holders, and the spread is determined by both public and private liabilities and sets the cost of borrowing for both private and public capital.

7 Asset market equilibrium implies that the private and public capital stocks that are not owned by the government are owned by foreigners, so that foreign liabilities are given by $F = K + S - B$. 
capital (and hence wages and income) as captured by \( K(S, \tau), W(S, \tau) \) and \( Y(S, \tau) \). The Pontryagin function is defined as:

\[
H(\tau, T, G, S, \mu) = \left[ \frac{W(S, \tau) + T}{1 - 1/\sigma} + \psi G^{1/\sigma} \right]
+ \mu \left[ \{ r^* + \Pi(S - B) \} (B - S) + N + \tau Y(S, \tau) - G - T - \delta_S \right]
\]

with co-state for net government assets \( \mu \). This yields the optimality conditions

(7.1) \( H_\tau = C^{-1/\sigma} W_\tau + \mu (Y + \tau Y_\tau) = 0 \),

(7.2) \( H_T = C^{-1/\sigma} - \mu = 0 \)

(7.3) \( H_G = \psi G^{-1/\sigma} - \mu = 0 \)

(7.4) \( H_S = C^{-1/\sigma} W_S + \mu \left[ \tau Y_S - \{ r^* + \Pi(D) + D \Pi'(D) + \delta_S \} \right] \)

(7.5) \( r^* \mu - \bar{\mu} = H_B = \mu \left[ r^* + \Pi(D) + D \Pi'(D) \right] \)

and the transversality condition

(7.6) \( \lim_{t \to \infty} \left[ \exp(-r^* t) \mu(t) B(t) \right] = 0 \).

We analyse this system in two stages, first looking at the case in which lump-sum transfers – the instrument \( T \) – are possible, and then in section 4.2 removing this instrument.

4.1. Policy with lump-sum taxes/transfers

If government can make lump-sum transfers to consumers, then it is optimal to set the income tax rate at zero (from (7.1) and (7.2) together with \( W_\tau = -Y \)). The optimal level of public consumption follows from (7.2) and (7.3) as \( G = \psi C \). Infrastructure is set optimally to satisfy

(7.4') \( W_S(S, 0) = r^* + \Pi(D) + D \Pi'(D) + \delta_S \).

The marginal value of infrastructure is simply its effect on national income which, absent income taxation, is its effect on the wage. Its marginal cost is the full marginal cost of public borrowing including the marginal cost of the interest premium, so that the optimal level of
infrastructure will be lower in a capital-scarce economy. The optimal path of consumption is as in section (3); using (7.2) in (7.5),

\[
(7.5') \quad \dot{C} = \sigma C \left[ \Pi(D) + D \Pi'(D) \right] \quad \text{for} \quad D > \overline{D}, \quad \text{and} \quad \dot{C} = 0 \quad \text{otherwise.}
\]

Hence, in a capital-scarce economy the optimal consumption paths are tilted towards the future to redeem government debt, lower the interest rate, boost public and private capital accumulation and thus spur the process of economic development.

The dynamic system in $B$ and $C$ is qualitatively similar to that in $F$ and $C$ illustrated in figures 3.3 and 3.4, although $S, K, Y$ and $W$ are now all changing along the optimal path. We therefore choose to illustrate results by simulation of an example presented in the panels of figure 4.1. Time is on the horizontal axis, and scaling of the vertical axes is achieved by having the long-run stationary value of income equal to unity. Production is Cobb-Douglas, $Y = AK^\alpha L^{1-\alpha} S^\gamma$, where $A$ scales long-run output to unity, and parameters are set to $\alpha = 0.4, \gamma = 0.25, \rho = r^* = 0.05, \sigma = 0.75, \psi = 0$ and $\delta_K = \delta_S = 0.05$. In figures 4.1 and 4.2 $\Pi = \rho(F_0^2$).

Simulations are done with a reverse multiple shooting algorithm with a horizon of $t^* = 130$ and using the computer package GAUSS. We assume that $D_F = 0$ and that the time dimension is scaled such that the horizontal axis can be (loosely) interpreted as years.

The solid lines in figures 4.1 and 4.2 give the path of an economy which starts out with national wealth, $B_0$, set at half its long-run value, and which experiences no shocks. The economy has positive initial foreign debt and converges smoothly to its stationary value with accumulation of assets, decumulation of foreign debt, falling interest rates, and rising income and consumption.

The effect of a temporary anticipated ‘small’ windfall is given by the dashed lines. Initial asset values are as in the base case, but at year zero a flow of resource revenue between years 16 and 35 is announced. The flow is equal to 8% of long-run stationary non-oil income.

At the date of announcement it is optimal to increase consumption, and there is a (small) upward jump in transfers $T$ and consumption $C$. Transfers are then on a steeply rising path during the period prior to resource revenue flow. (Notice that overall lump-sum transfers to households are negative, because of the need to finance public infrastructure). Additional transfers in the interval before resource revenues start to flow have to be financed by foreign borrowing, and the downwards path of $D$ flattens, leaving debt above what it otherwise would have been. Higher foreign debt translates into higher $r$ and lower $S, K$ and non-resource income, $Y$.

Once the revenue flow comes on stream debt $D$ is paid off more rapidly than was the case absent the windfall, with the associated rapid fall in $r$ and increase in $K, S, Y$ and $W$. 
Rapidly rising wages mean that consumption growth can be maintained with $T$ falling back. All the variables describing the production side of the economy cross their non-windfall path during the period of revenue flow, including the public capital stock. At the date when the windfall revenue ceases ($t = 35$, at the kink) domestic capital stocks are something over 10% higher than they otherwise would have been, and foreign debt at half the level. At this date the economy reverts to its previous path, but earlier than would otherwise have been the case. Thus, the dashed lines converge to the same value as the solid ones, but are shifted to the left by some 30 time periods. The bottom right panel of figure 4.1 compares incremental consumption with the resource revenue flow (step function) and illustrates clearly the periods of borrowing, saving, and then higher consumption, this because the economy’s development has been brought forward, and not because of perpetual income from an SWF.\(^8\)

**Figure 4.1: Optimal development with lump-sum transfers**

![Graphs showing economic variables over time](image)

**Key:** Solid lines without a windfall and dashed lines with an anticipated temporary windfall.

In summary, optimal use of the windfall involves increased consumption from the date at which the resource is discovered, and faster asset accumulation (debt decumulation) from the date windfall revenue flows. Higher public and private investment brings forward the economy’s development path, but does not lead to the formation of a SWF.

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\(^8\) If the windfall was large enough for the interest rate to fall to $r^*$ during the period of revenue flow then the economy would commence construction of a SWF, as in section 3.4.
4.2. Second best: No lump-sum taxes and subsidies

The possibility of lump-sum transfers makes it easy for government to control the level of private consumption. Without such transfers consumption can only be controlled indirectly via the wage rate, and two instruments affect this. More public infrastructure raises wages directly and also by attracting private investment; lower distortionary taxation attracts private investment and raises wages. Of course, these instruments are linked by the budget constraint. Resource revenues relax this constraint, and the ensuing second-best optimal policy response is outlined below.

The optimal policy is found from the first-order conditions above, but with \( T = 0 \), so instead of first order condition (7.2) we have simply \( C = W(S, \tau) \). It is helpful to define the marginal cost of public funds as the shadow price of public funds relative to the marginal utility of private consumption \( \phi = \mu / C^{1/\sigma} \). In the preceding subsection \( \phi = 1 \), but the fact that the government now has to raise funds by distortionary taxes means that \( \phi > 1 \). The relationship between the optimal income tax and the marginal cost of public funds is given by equation (7.1) (using (7.2) and \( W_\tau = -Y / \tau \)) as

\[
\tau = \left(1 - \frac{1}{\phi}\right) \left(\frac{Y}{Y_\tau}\right) \quad \text{or} \quad \phi = \frac{1}{1 + \tau Y_\tau / Y}.
\]

Since \( Y_\tau < 0 \) a positive tax rate is associated with marginal cost of funds greater than unity. A higher cost of funds depresses the demand for public relative to private consumption:

\[
G = \left(\frac{\psi}{\phi}\right)^\sigma W(S, \tau).
\]

The first-order condition for infrastructure becomes

\[
W_S(S, \tau) = \phi \left[ r^* + \Pi(D) + D\Pi'(D) + \delta_s - \tau Y_s \right].
\]

Thus, \( \phi \) greater than unity raises the cost of capital which tends to reduce the optimal level of public infrastructure, although this may be offset as an increase in infrastructure raises income and tax revenue, \( \tau Y_s > 0 \).

We once again illustrate the optimal development paths, with and without resource revenue, by numerical example. Figure 4.2 describes the same economy as figure 4.1, but with this restricted set of instruments. Government finance of infrastructure requires distortionary taxation (\( \tau > 0 \)) and hence a shadow premium on public funds (\( \phi > 1 \)). Along the development path without resources there is steady pay back of debt, increasing capital stock and rising income and consumption. This is accompanied by a declining cost of public
funds and rate of income tax as accumulation of infrastructure reduces the flow of infrastructure investment (relative to the size of the economy) to be financed. The presence of the distortion means that income and consumption are lower at all dates (including the long run) than they are when lump-sum transfers can be used.

Discovery of the resource revenue causes an immediate decrease in the marginal cost of public funds, as would be expected. There is also an immediate (small) jump in consumption, which is then on an accelerating path. Consumption is equal to the wage rate, so this jump is engineered by a lower income tax rate which attracts private capital and raises income. However, the lower tax requires government borrowing which raises the cost of funds and causes public infrastructure investment to fall below its previous path. It is only once resource revenues flow that the government is able to afford a lower tax rate, higher level of public infrastructure, and sharply falling level of debt. All of these things put income and wages on a rapid growth path.

It is interesting to note that the government could have increased output and wages either by cutting income tax or by increasing the public capital stock. In this example the stock of infrastructure initially falls back as the interest rate rises before it starts to rise steeply. This is a consequence of the fact that a Cobb-Douglas technology (as used in the simulation) implies that neither the tax rate nor the marginal cost of public funds have a direct impact on the optimal stock of infrastructure. Equations (7.1’) and (7.4”’) are particularly simple in this case, namely

\[
\phi = \frac{1}{1 - \left(\frac{\alpha}{1 - \alpha}\right)\left(\frac{\tau}{1 - \tau}\right)} \quad \text{and} \quad \frac{S}{Y} = \frac{\gamma}{r^* + \Pi(D) + D\Pi(D) + \delta_s}
\]

where \(0 < \alpha < 1\) denotes the elasticity of output with respect to \(K\) and \(0 < \gamma < 1\) the elasticity with respect to \(S\). Hence, the optimal public capital/production ratio depends only on parameters and the interest rate and not on the cost of funds or the tax rate.
Figure 4.2: Second-best optimal development.

Key: Solid lines without a windfall and dashed lines with an anticipated temporary windfall.

In summary, inability to raise consumption via lump-sum taxes means that the government must instead increase wages by a combination of lower income tax and higher public infrastructure. The initial response is to lower the tax rate, but at the date when the windfall revenue ceases ($t = 35$, at the kink) domestic capital stocks (public and private) are approximately 20% higher than they otherwise would have been, indicating increased reliance on the non-resource economy to deliver additional consumption. Comparing the bottom right panels of figures 4.1 and 4.2, it is noteworthy that the resource discovery is associated with a larger increase in consumption at all dates in the second case. The reason is that windfall revenue to government allows it to reduce other distortions that are present in the economy, specifically the rate of income tax.

5. Using public infrastructure to avoid the Ricardian curse

Up to this point government has been the only agent making choices about the intertemporal profile of consumption. In practise there may also be forward-looking private agents who own assets and adjust savings and consumption decisions in response to current and future resource revenues. This raises the possibility that Ricardian consumers may, in some sense, negate the effect of government policy. They fully anticipate their future shares in resource
revenues and adjust consumption accordingly, but not necessarily optimally from a social perspective, a ‘Ricardian curse’. Thus, even if the government is seeking to save a substantial share of resource revenues, policy may be undermined by a private consumption boom fuelled by private borrowing, as has happened in some countries. How should government react to this?

To model this, we now assume that households are no longer credit constrained but may own private assets. Aggregate private wealth is denoted by $A$ and can be held in either domestic equity or government bonds which we assume to be perfect substitutes. Thus, the physical assets in the economy, $K + S$, are owned by foreigners (foreign debt $F$), government (net assets $B$), and households (wealth $A$), so $K + S = F + B + A$ (or $K = F - D + A$ where $D = S - B$ is government debt). Foreign liabilities $F$ thus defined correspond to the excess of public debt over private bond holdings plus net import of capital. We will assume that the interest premium now depends on the asset position of private asset holders as well as that of the government, so $r = r^* + \pi(F) = r^* + \pi(K + S - A - B)$.

The production side of the economy is as before, except that we now assume that all investors face the domestic interest $r$ (inclusive of the premium $\pi(F)$). For simplicity we ignore income taxation, so profit maximisation implies that the marginal product of capital equals the user cost of capital:

$$f_d(K, S) = r^* + \pi(K + S - A - B) + \delta K.$$  

This implicitly defines $K = K(S, A + B)$ and correspondingly production $Y(S, A + B)$, wages $W(S, A + B)$ and domestic interest rate $r(S, A + B)$. Capital stock, wages and production are increasing in $S$ and in $A + B$. The interest rate is increasing in $S$ and decreasing in $A + B$. For future reference we note that $W_A + Kr_A = 0$ (see appendix). Effects with respect to $S$ hold with strict equality, but effects with respect to $A + B$ occur only if the economy is highly indebted so $F > F$. The responses are intuitive. A higher stock of assets owned by domestic households, $A$, or the government, $B$, corresponds to lower foreign liabilities and thus pushes down the premium and the domestic interest rate. Consequently, capital, wage income and output increase. A higher level of public infrastructure boosts the marginal productivity of capital and of labour, hence increases the demand for capital and boosts output. As a result, the domestic interest rate and wage rate rise. For given $A + B$, a higher public infrastructure also increases foreign liabilities and pushes up the domestic rate of interest. Full details of the Cobb-Douglas case are given in the appendix.

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9 Notably Kazakhstan where public saving has been offset by private borrowing (Esanov and Kuralbeyeva, 2009).
Private households have access to domestic capital markets and can smooth their consumption $C$. They are subject to two government instruments, an asset holding subsidy at rate $\tau_A$ and a lump-sum transfer of $T_A$. Their budget constraint is therefore

$$\dot{A} = (r + \tau_A)A + W + T_A - C.$$  

The privately optimal growth in consumption is proportional to the gap between the interest rate (inclusive of the asset holding subsidy rate) and households’ rate of pure time preference $\rho$, so that

$$\frac{\dot{C}}{C} = \sigma (r + \tau_A - \rho).$$  \hspace{1cm} (10)

The government borrows and issues debt $D = (S - B)$ at rate of interest $r$. The government budget constraint is thus,

$$\dot{B} = r(B - S) + N - G - T_A - \delta_s = B(0) = B_0.$$

Ricardian equivalence implies that the intertemporal profile of government transfers $T_A$ does not affect private consumption, so we may as well use the consolidated private and public budget constraint:

$$A(0) + B(0) = A_0 + B_0.$$  \hspace{1cm} (11)

This budget constraint, together with $F = K + S - A - B$ and $Y = W + (r + \delta_K)K$ imply that the trade deficit (the excess of public and private spending over production plus windfall revenue) plus interest on foreign liabilities equals the increase in indebtedness of the nation,

$$\dot{F} = rF + C + G + I_K + I_S - Y - N, \quad F(0) = S(0) + K(0) - A_0 - B_0.$$  

The no-Ponzi condition implies that the present discounted value of net exports of goods and services minus windfall revenue exports must cover initial foreign liabilities. These liabilities jump on impact if the government borrows for infrastructure or firms import capital.

Social welfare depends on consumption by households and government, as before, and we maximise with respect to the asset holding subsidy rate $\tau_A$, public consumption $G$, and public infrastructure $S$. Notice that transfers or citizen dividends, $T_A$, do not enter explicitly; Ricardian consumers know the combined budget constraint (11) and hence the implicit value of these payments. Maximisation is subject to (11) and (10) for the state variables $A + B$ and $C$, with respective co-states $\mu$ and $\lambda$. Given that the Pontryagin function is defined by

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10 Although noting that the asset holding tax still affects private consumption, as in (10).
\[
H(\tau_\lambda, G, S, \lambda, \mu) \equiv \left(\frac{C^{1-\lambda} + \psi G^{1-1/\sigma}}{1-1/\sigma}\right) + \lambda \sigma \left[r(S, A + B) + \tau_\lambda - \rho\right]C + \\
\mu \left[r(S, A + B)(A + B - S) + N + W(S, A + B) - G - C - \delta S\right].
\]

The first-order conditions are:

(12.1) \[ H_{\lambda} = \sigma C \lambda = 0 \]
(12.2) \[ H_G = \psi G^{-1/\sigma} - \mu = 0 \]
(12.3) \[ H_S = \left[ W_S + (A + B - S)r_S - r - \delta S\right] + \lambda \sigma Cr_S = 0 \]
(12.4) \[ \rho \mu - \mu = H_{\lambda + B} = \left[ r + W_A + (A + B - S)r_A\right] + \lambda \sigma Cr_A \]
(12.5) \[ \rho \lambda - \dot{\lambda} = H_C = C^{-1/\sigma} - \mu + \sigma \left(r + \tau_\lambda - \rho\right) \lambda \]

with the transversality conditions

(12.6) \[ \lim_{t \to +\infty} \exp(-\rho t) \mu(t) \left( A(t) + B(t) \right) = 0 \text{ and } \lim_{t \to +\infty} \exp(-\rho t) \dot{\lambda}(t) C(t) = 0. \]

5.1. Lump-sum taxes and optimal asset holding subsidy

We first look at the case in which government can control private consumption growth by using the asset holding subsidy rate \( \tau_\lambda \). The first-order condition (12.1) then implies that \( \lambda = 0 \), and hence from (12.2) and (12.5) \( \mu = C^{-1/\sigma} = \psi G^{-1/\sigma} \). Consumption paths follow from (12.4) as

(13.1) \[ \dot{C}/\sigma C = \dot{G}/\sigma G = \left[ r + W_A + (A + B - S)r_A - \rho\right] \text{ and } G = \psi^\sigma C. \]

The optimal level of public infrastructure is implicitly given in (12.3),

(13.2) \[ W_S = r + \delta S - (A + B - S)r_S, \]

indicating that \( S \) increases with the stock of private plus public assets, via an effect on the domestic rate of interest. Analogous to previous cases the optimal path of the economy is described by differential equations for assets, \((A + B, \text{equation (11)})\) and for \( C \) (equation (13.1)), with values of \( G, S, \) and hence \( K = K(S, A + B), Y(S, A + B), W(S, A + B), r = r(S, A + B) \) being computed at each instant. Notice that expressions (13.1) and (13.2) for consumption
growth and infrastructure are more complex than the analogous equations in the previous section, (7.4′) and (7.5′), because debt now affects the rate at which the private sector borrows, giving rise to the dependence of $K$, $Y$, $r$ and $W$ on $(A+B)$.

Our main focus is on the asset holding subsidy, since it is this that controls the time profile of private consumption. The implied optimal asset holding subsidy rate follows from comparison of (13.1) with (10)

\[
\tau_d = \left[ W_A + (A + B - S) r_A \right] = Fr_A \geq 0.
\]

Where the equality comes from the definition of $F$ and $W_A + Kr_A = 0$. This is positive, since the sign in (13.3) holds with strict inequality for $F > F \geq 0$ and equality if debt is below this threshold. The intuition is that an asset holding subsidy is required because of a terms-of-trade effect. By saving and raising $A$, private agents reduce the interest rate premium that the economy has to pay on its foreign debt, an effect that not internalised by individual price-taking asset holders. This interest rate change benefits the economy in aggregate (if $F > 0$), while raising wages ($W_A > 0$) and reducing returns to domestic asset holders ($r_A < 0$). The asset holding subsidy therefore starts relatively high and, without the windfall, falls monotonically to zero. The effect of a resource windfall is to initially increase the asset holding subsidy, followed by a fall in the subsidy rate to below its level absent the resource. This exactly mirrors the path of outstanding debt (as in figures 4.1 or 4.2), since it is this that drives the terms-of-trade effect.

In summary, the asset holding subsidy fully corrects the distortion that arises from households’ failure to internalise the adverse effect of their consumption on the interest rate at which the economy borrows. The government funds the optimal level of public consumption and infrastructure and transfers other revenue to households through citizen dividends. Since private agents are Ricardian consumers facing the same cost of capital as the public sector, the timing of transfers is immaterial. However, this requires all consumers to have access to capital markets and any macro-economic impacts of debt on interest rates to be internalised by the asset holding tax such that citizen dividends are optimal.

5.2. Absence of an optimal asset holding subsidy

Time-varying asset holding subsidies may be difficult to implement and are seldom seen in practice. We therefore turn to the case in which this instrument is unavailable and $\tau_d = 0$. Since the return to saving is reduced there is a tendency for households to be on a consumption path that is too flat, involving too much consumption in the early years and too
little saving. What is the optimal response to this situation? Since (12.1) no longer applies, necessary conditions are, from (10) with (12.2) – (12.5):

\[
\begin{align*}
(14.1) \quad \dot{C}/C & = \sigma(r - \rho) \\
(14.2) \quad \dot{G}/G & = \sigma \left[ r - \rho + W_A + r_s \left( A + B - S + \left( \frac{\sigma \lambda C}{\psi G^{-1/\sigma}} \right) \right) \right] \\
(14.3) \quad W_s = r + \delta_s - r_s \left( A + B - S + \left( \frac{\sigma \lambda C}{\psi G^{-1/\sigma}} \right) \right) \\
(14.4) \quad \lambda = \left[ \rho - \sigma(r - \rho) \right] \lambda + \psi G^{-1/\sigma} - C^{-1/\sigma}.
\end{align*}
\]

The dynamic system (11), (14.1), (14.2) and (14.4) is solved with aggregate assets as a predetermined variable (i.e., \( A(0) + B(0) = A_0 + B_0 \)) and private and public consumption as jump variables (\( C(0) \) and \( G(0) \) free to jump), where at each point of time the interest rate is given by \( r(S, A+B) \) and the level of public infrastructure follows from (14.3). Since the government has no access to an asset holding subsidy or lump-sum transfers, it is unable to control the initial level of private consumption\(^{11}\) and consequently the initial marginal social value of private consumption is free to jump at time zero (i.e., \( \lambda(0) \) free). The dynamic system can thus be solved with a standard reverse multiple shooting algorithm. Due to the initial bias towards over-consumption, the social value of a marginal reduction in initial private consumption must be positive. Since \( C \) is a forward-looking variable, this implies that its initial co-state must satisfy \( \lambda(0) > 0 \).\(^{12}\) We can show that the steady-state value of \( \lambda \) is zero if \( F = 0 \).\(^{13}\) The government’s inability to use the asset holding subsidy thus means that \( \lambda \) is initially positive and then declines to the limiting value of zero. In general \( \lambda \) need not converge to zero.

The consequences of \( \lambda > 0 \) are seen by inspection of equations (14.2) and (14.3) for public consumption and infrastructure. From (14.2), \( \lambda > 0 \) implies slower growth of public consumption \( G \) (since \( r_s < 0 \)) and thus a higher value of initial public consumption \( G(0) \). And

\(^{11}\) However, the government can by varying \( G \) and \( S \) control the present value of private consumption \( PV(C) = A + B + PV(W^+N) - PV(G+(r+\delta)S) \), where \( PV(\cdot) \) indicates the present value using the market rate of interest. Raising public infrastructure also boosts wages, which offsets the downward adjustment of the private consumption path. Private agents also realise that, if government does not spend the windfall, the windfall is going to accrue to them and thus spend accordingly.

\(^{12}\) We use the result that at the optimum \( \partial U/\partial C(0) = \lambda(0) < 0 \) while \( \partial U/\partial[A(0)+B(0)] = \mu(0) > 0 \).

\(^{13}\) In steady state (14.2) gives \( W_s + (A + B - S)r_s + \left( \frac{\sigma \lambda C}{\psi G^{-1/\sigma}} \right) r_s = 0 \), which implies that \( \lambda \Pi' = \Pi' F/\psi G^{-1/\sigma} / \sigma C \). Since steady state \( \Pi' = 0 \) we use a limiting argument; suppose that \( \Pi = \Pi(F+\epsilon) \), \( \epsilon > 0 \), and let \( \epsilon \rightarrow 0 \) to yield steady-state value of \( \lambda = 0 \) if \( F = 0 \).
from (14.3), \( \lambda > 0 \) implies a higher value of \( S \); since \( r_S < 0 \), the effect is like a lower cost of capital. The only way that the government can, in the absence of the asset holding subsidy, dampen the consumption of Ricardian households is by itself raising spending on public consumption and infrastructure. This spending has a negative income effect on private households, and also causes foreign debt to be larger than it otherwise would be, increasing the interest rate and thereby increasing private saving.

This is illustrated in figure 5.1.14 The top panel of this figure looks at two cases where there are no resource revenues. The dashed line is if \( \tau_A \) is set optimally, and the solid line if \( \tau_A \) is constrained to be zero. In line with the discussion above, public infrastructure is larger – a full 20% larger – if the asset holding subsidy is not available. This has the effect of increasing indebtedness and hence interest rates. As a consequence the paths of consumption and private capital stock (not illustrated) are very similar in the two cases. Essentially, the government commits to public investment, debt and higher interest rates, in order to prevent over-consumption by the private sector.

**Figure 5.1: Counteracting the Ricardian curse**

![Graphs showing public capital stock, rate of interest, and consumption](image)

**Key:** Top panel gives simulations without windfall; dashed lines give case with optimal asset holding subsidy and solid lines without asset holding subsidy. The bottom panel gives simulations without the asset holding subsidy; dashed lines are with the windfall, and solid lines without.

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14 Parameters and production function are as in section 4 except that figure 5.1 uses \( \Pi = 0.75 \rho (F_t)^2 \).
The bottom panel of figure 5.1 has $\tau_i = 0$, and compares variables with (dashed line) and without (solid line) the anticipated temporary windfall revenue. We see that the effect of the windfall is to cut the stock of public infrastructure. In this example infrastructure investment goes to zero for a short period and the downward-sloping section of the infrastructure stock schedule arises as existing infrastructure depreciates. This perverse effect occurs because the windfall moves the economy closer to development and thereby reduces the magnitude of the distortion that we have built into the system. In particular, resource wealth reduces indebtedness, and so reduces the terms-of-trade effect of private consumption on debt service obligations. Consumption is therefore closer to its first best optimal path and there is less need to control it indirectly though high spending on infrastructure.

Two further points are noteworthy. First, the reduction in infrastructure spending when the resource is discovered occurs because government is assumed to be implementing the second-best optimal policy prior to the windfall; such very high rates of infrastructure investment are not observed in most developing countries. Second, the only reason for private sector over-consumption in this model is the terms-of-trade effect of changing the interest rate. Other distortions affecting the time profile of private consumption (domestic capital market imperfections, high spreads and low returns to private saving) might not be mitigated by the windfall.

6. Concluding remarks

We have established four main results. The first is that a developing economy which is capital scarce with rate of interest greater than the rate of time preference and growing consumption should not follow the prescriptions of the permanent income hypothesis and devote the proceeds of a resource windfall to construction of a SWF. It should instead invest to raise the rate of growth of consumption; in a poor country the gains from reaching the long-run level of consumption sooner outweigh those from raising the long-run level through permanent returns on an SWF.

The second concerns the composition of spending. There should be some immediate increase in consumption (by transfer payments, if these are available), accompanied by investment in a combination of public infrastructure and debt reduction, the latter bringing lower interest rates and higher private investment. If there is a substantial time lag before windfall revenues flow, then the immediate increase in consumption remains optimal, although it has the effect of increasing indebtedness and pushing further into the future the date at which public infrastructure and debt reduction takes place.

Third, if direct transfers to consumers are difficult to implement, then more of the windfall revenue should be devoted to public investment and to tax measures that increase
private investment. This is because consumption can be increased only by raising wages in the economy, and higher investment is the means to achieve this.

Finally, the prescription of citizen dividends is optimal only if households have access to capital markets and any tendency to private over- (or under-)consumption can be corrected by a time-varying asset holding subsidy (tax). Access to capital markets means that consumption can be separated from the date at which transfers are made, but private smoothing need not be socially optimal. This can be corrected by a time varying asset holding subsidy. If this instrument is not available, then government will need to correct the over (or under) consumption of Ricardian consumers by other means, such as varying the level of investment in public infrastructure.

These results challenge aspects of the standard advice for handling windfall revenues, for example the recommendation that revenues should be used to build an SWF and, according to some, consumption limited to the interest on this fund (e.g., Barnett and Ossowski, 2003). Developing countries have both an urgent need both for consumption to reduce poverty, and high-return domestic investment opportunities. Our analysis shows how these factors make it optimal to use revenues to grow the domestic economy.

Of course, the analysis abstracts from many important elements that will be the subject of future research. First, if windfall revenue directly impacts creditworthiness, there may be a danger of over-borrowing (e.g. Mansoorian, 1991; Manzano and Rigobon, 2001). Second, the economy may have difficulty in absorbing additional expenditure. At the macro-economic level there may be an appreciation of the real exchange rate and decline of the traded sector (e.g., Corden and Neary, 1982). At the micro-economic level maintaining and raising the efficiency of public expenditure is essential. Third, it is important to allow for endogenous optimal resource depletion and examine how the well-known Hotelling (1931) rule should be modified when the government faces the tough public-finance dilemmas we have highlighted. For example, does it still make sense to have a current-account surplus matching the Hotelling rents? Fourth, the government may be myopic for political reasons or due to competing fractions and the voracity effect (Tornell and Lane, 1999) in which case the government brings forward public spending and postpones taxation. Furthermore, an incumbent, worried about being removed from office by a political rival with preference for a different type of public goods, typically issues too much debt and spends too much on its own pet projects (Alesina and Tabellini, 1990). These political distortions are exacerbated if the incumbent uses the windfall to opportunistically pacify the electorate. On the other hand, governments may prefer to invest in public infrastructure rather than a SWF as the former is more difficult to be raided by future political rivals. Resource-rich countries may also get addicted to high public spending and find it difficult to kick the habit once resource revenues dry up (e.g., Leigh and Olters, 2006; Olters, 2007).
Perhaps most importantly, resource revenues are not only uneven through time, as we have modelled, but also in many cases highly uncertain due to the notorious volatility of commodity prices and uncertainty about future extraction costs. This creates a case for accumulating precautionary buffers in a Sovereign Liquidity Fund to smooth shocks. However, it remains important that, as we have argued in the context of certainty, revenues are used to grow the domestic economy and raise consumption in the short to medium term, and are not simply deposited abroad.

References


**Appendix: Comparative statics of production (section 5).**

Profit maximisation implies  \( f_k(K, S) - \delta_K = r = r^* + \Pi(K + S - A - B) \), this implicitly defining \( K(S, A+B) \). Hence \( r(S, A+B) = f_k(K(S, A+B), S) - \delta_K \) and \( W(S, A+B) = f(K(S, A+B), S) - K(S, A+B)f_k(K(S, A+B), S) \). Comparative statics are:

\[
K_A = \Pi' / (\Pi' - f'_{KK}), \quad r_A = \Pi' f_{KK} / (\Pi' - f'_{KK}), \quad W_A = -\Pi' Kf_{KK} / (\Pi' - f'_{KK}), \text{ so } W_A + Kr_A = 0.
\]

For the Cobb-Douglas case: \( \alpha K^{-1}S^\alpha - \delta_K = r = r^* + \Pi(K + S - A - B) \) and \( W = (1 - \alpha)K^\alpha S^\alpha \).
\[ K_A = \frac{\pi' K}{(1-\alpha)(r + \delta_k) + \pi' K} \], \quad K_S = \frac{-\pi' K + \gamma(r + \delta_k)K / S}{(1-\alpha)(r + \delta_k) + \pi' K} \]

\[ r_A = \frac{-\pi'(1-\alpha)(r + \delta_k)}{(1-\alpha)(r + \delta_k) + \pi' K} \], \quad r_S = \frac{\pi'(r + \delta_k)(1-\alpha + \gamma K / S)}{(1-\alpha)(r + \delta_k) + \pi' K} \]

\[ W_A = \frac{\pi' \alpha W}{(1-\alpha)(r + \delta_k) + \pi' K} \] and \[ W_S = \frac{W}{S} \left[ \frac{\pi'(\gamma K - \alpha S) + \gamma(r + \delta_k)}{(1-\alpha)(r + \delta_k) + \pi' K} \right] \].

Also, \[ Y = W(S, A+B)/(1-\alpha) \equiv Y(S, A+B) \] with \[ Y_{\gamma} = -\alpha Y \left[ \frac{r + \delta_k}{(1-\alpha)(r + \delta_k) + \pi' K} \right] < 0. \]