Carroll's Regress, Bradley's Regress, and the Incoherence of Logical Consequence

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Submitted June 14th, 2023

To Michael Della Rocca
Without Whom Not

Foreword

Every morning, I walk past F. H. Bradley's resting place in Holywell Cemetery on my way to New College. In writing this thesis, I have borne in mind what he says in the introduction to *Appearance and Reality*:

> And so, when poetry, art, and religion have ceased wholly to interest, or when they show no longer any tendency to struggle with ultimate problems and to come to an understanding with them; when the sense of mystery and enchantment no longer draws the mind to wander aimlessly and to love it knows not what; when, in short, twilight has no charm — then metaphysics will be worthless.

This thesis is the culmination of many years of study, and I am immensely grateful for all those who have guided me along my way (especially when I did not know where I was headed). Beginning with pure mathematics, taking a detour through mathematical logic, and ending up in metaphysics and the history of philosophy, the last few years of study have been a long journey. I know that each part has played a crucial role in this thesis, but what conclusions we ought to draw from that fact (indeed, whether it even makes sense to say that we *ought* to draw any conclusions at all) is precisely the question.

Abstract

§1 frames Carroll's regress as a problem of metaphysical grounding, i.e. as an instance of Bradley's regress, and attempts to provide it the strongest possible defence. §2 applies this regress to accounts of logical consequence, and argues that all such accounts ultimately fail to explain why modus ponens is valid. §3 argues that rejecting logical consequence as incoherent amounts to rejecting the analytic/synthetic distinction as incoherent, and discusses the devastating consequences of doing so. §4 shows that the self-undermining nature of such a view has its roots in the Principle of Sufficient Reason, and works towards a way of resolving the paradox that is exemplified in §5.

1 The Taught-Us and A-Kill-Ease

> "I know what you're thinking about," said Tweedledum: "but it isn't so, nohow."
>
> "Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."
>
> Lewis Carroll, *Through the Looking Glass*, Chapter IV.

In this section, I will present what I take to be the argument in Lewis Carroll's short story "What the Tortoise said to Achilles". On my reading, the Tortoise demands an explanation for why a paradigmatic modus ponens argument is logically valid — *why* is the conclusion a logical consequence of the premises? — and is answered only by Achilles' infinite regress. Taking inspiration from Bohemian logician Bernard Bolzano, who entertains a similar line of reasoning, I understand Carroll's regress as a regress of grounding with respect to the logical consequence relation.

1.1 Carroll's Formulation

Carroll's dialogue centres around the following argument, which the Tortoise presents to Achilles:

(A): Things that are equal to the same are equal to each other.
(B): The two sides of this triangle are things that are equal to the same.

(Z): The two sides of this triangle are equal to each other.

The Tortoise first asks whether "(Z) follows logically from (A) and (B)", to which Achilles replies "Undoubtedly!"[1] The Tortoise then asks: "if some reader had *not* yet accepted (A) and (B) as true, he might still accept the *sequence* as a *valid* one, I suppose?"[2] Achilles agrees, reasoning that such a reader "might say 'I accept as true that Hypothetical Proposition that, *if* (A) and (B) are true, (Z) must be true; but I *don't* accept (A) and (B) as true.'"[3] Such a reader would take the argument to be valid, but unsound (and, Achilles adds, would do better taking to football).

The Tortoise *then* asks (and here is where the real trouble begins): "might there not *also* be some reader who would say 'I accept (A) and (B) as true, but I *don't* accept the Hypothetical'?" Achilles again agrees. But, the Tortoise claims, this means that "*neither* of these readers ... is *as yet* under any logical necessity to accept (Z) as true."[4] What the argument is missing, the Tortoise claims, is the following premise:

(C): If (A) and (B) are true, (Z) must be true.

[1] Carroll, Lewis. "What the Tortoise said to Achilles." *Mind* 4, no. 14 (1895): 278.
[2] Ibid.
[3] Ibid.
[4] Ibid. p. 279.

Achilles agrees and adds (C) to the premises, unaware of the looming regress. For the very same objection can be made here against the argument with premises (A), (B), *and* (C). The Tortoise asks: "if I failed to see its truth, I might accept (A) and (B) and (C), and *still* not accept (Z), mightn't I?" For reasons parallel to those above, Achilles adds yet another premise, (D),[5] and this continues *ad infinitum*.

At each stage, the Tortoise can be seen as asking: why is (Z) a logical consequence of the premises? And, at each stage, Achilles replies that it is because (Z) must be true if the premises are true. But this is a new claim — "*another* Hypothetical" — which must be added as a premise in the argument. Some months later, the Narrator returns to find Achilles still jotting down premises in his notebook.

I ask you to join me in taking the Tortoise's line of reasoning seriously. Given that logical inferences are seemingly commonplace in philosophy and our daily lives, it is vital to understand what, if anything, goes awry in Carroll's story, just as it is vital to understand what, if anything, goes awry in Zeno's original story about motion.[6] If indeed the Tortoise is to be dismissed, it is only after considering the argument in light of its strongest defence — a defence that I will do my best to provide. If you like, this can be seen as an instance of charity towards our friend, the Tortoise.

1.2 Bolzano's Formulation

On my view, this defence begins six decades prior to the publication of Carroll's story, with the publication of Bolzano's *Theory of Science* in 1837. Bolzano begins by sharply distinguishing between three relations: "the relation between ground and consequence", "the relation between truths wherein some are deducible from others", and "the relation that holds between actual things when one is the cause of the other."[7] The exact details of Bolzano's deducibility relation are complicated (they bear a certain similarity to Tarski's model-theoretic account of logical consequence), but, importantly, it is *not* identical with deducibility in a formal system of logic, e.g. first-order logic.[8] Bolzano writes that

[5]This premise is (D): If (A) and (B) and (C) are true, (Z) must be true.

[6]Carroll considers Zeno's paradox of the Tortoise and Achilles in his first textbook *Symbolic Logic I*. Unlike "What the Tortoise said to Achilles" (which was set to appear in an unpublished *Symbolic Logic II*), it is discussed in the answer key at the back of the book, and Carroll's standard answer makes use of convergent infinite sums.

[7]Bolzano, Bernard, Paul Rusnock, and Rolf George. *Theory of Science.* Volume 2. Oxford Scholarly Editions Online. Oxford, 2015. §198. p. 343.

[8]Such systems would only be developed decades after Bolzano's time. His account of deducibility is a *substitutional* account, whereby one proposition is deducible from others with respect to certain variable ideas if the argument remains truth-preserving on all substitutions of those variable ideas. This in turn is used to give an account of *logical* deducibility by treating all ideas as variable except the "logical ones alone" (§123, p. 392). For a criticism of identifying Bolzano's deducibility relation with the logical consequence relation, see Etchemendy, *The Concept of Logical Consequence*, Ch. 1. One such criticism is that Bolzano's account makes a language's logical consequence relation a *triadic* relation, i.e. relative to the ideas treated as variable.

grounding, deducibility, and causation are often confused because they bear "a great similarity" to one another, but here he is clear that he is most concerned with the first of these relations.[9]

In giving an account of ground and consequence, Bolzano insists that he is investigating not "merely" the "order of *knowing*", but rather the order of metaphysical dependence, which he takes to be "*objective*" in that "this relation holds among truths *in themselves* independent of our recognition."[10] Bolzano understands this notion in much the same way as it is used in contemporary discussions, i.e. a relation that describes what is explanatorily prior or more fundamental.[11] Moreover, Bolzano is careful to distinguish between *complete* and *partial* grounds, since "sometimes it is not just one, but a whole collection of truths, which stand to one or more truths in the relation of ground to consequence."[12] As we will see, Bolzano's concern with understanding a truth's *complete* grounds is something that is particularly admirable about his regress argument.

In "§199: Whether the rule of inference can be considered a partial ground of a true conclusion", Bolzano notes:

> [I]f the rule according to which we want to deduce certain propositions M, N, O, ... from others A, B, C, D, ... is invalid, then it is obvious that we cannot claim that the propositions M, N, O, ... are truths which *follow* from the truths A, B, C, D, ...[13]

I see a similar concern motivating the Tortoise's first questions to Achilles, i.e. when the Tortoise asks him whether someone might accept the premises as true but reject the sequence as invalid. If a rule of inference, which Bolzano defines as "a proposition which indicates what attributes the premises A, B, C, D, ... and the conclusions M, N, O, ..., which follow from them, must have", is invalid, then it is obvious that we cannot claim that the conclusion follows logically from the premises. And this, on my reading, is why the Tortoise asks *why* the rule of inference used by Achilles *is* valid. Bolzano continues:

> Reflecting on this, one could get the idea that the rule, according to which propositions M, N, O, ... are deducible from propositions A, B, C, D, ..., ought to be envisaged as a truth which must be added to the truths A, B, C, D, ... in order to give the *complete* ground of the truths M, N, O, ... On this view, the two truths "Socrates was an Athenian" and "Socrates was a philosopher" do not constitute the complete ground for the third truth, which is deducible from them, namely, "Socrates was an Athenian and a philosopher"; rather, the rule which is employed in this deduction must be added, namely,

[9] Ibid. §198. p. 343.

[10] Bolzano, §198. p. 341.

[11] This being said, Bolzano's example is that the warmth of summer grounds the fact that a thermometer tends to register higher in the summer, and this might instead be seen as a causal relation.

[12] Ibid. §198. p. 340-1.

[13] Ibid. §199. p. 344. Ellipses in original.

> the truth: "If the two propositions 'A has b', and 'A has c' are true, then the proposition 'A has (b + c)' is also true."[14]

Here, Bolzano can be seen as asking: do the premises alone completely ground the truth of the consequence, or is the validity of conjunction-introduction also a partial ground? I see Bolzano's concern with giving the complete grounds of the consequence as highly analogous to the Tortoise's concern with giving the complete proof of the conclusion. In the former case, the conclusion is deducible (in Bolzano's unique sense) from the premises, but he goes on to ask whether the truth of the premises *completely grounds* the truth of the conclusion. And he takes seriously the view that it does not — the view according to which the validity of the rule of inference is *also* a partial ground of the conclusion. After all, Bolzano reasons, the conclusion would not be deducible from the premises if the rule of inference was invalid (just as, for the Tortoise, the conclusion would not be a logical consequence of the premises if (C) was false).

Although Bolzano says he is "quite sympathetic with this view", he rejects it because:

> if it was required in the first place that the complete ground of the truths M, N, O, ... must include, besides the truths A, B, C, D, ... also the rule of their deduction, then it must be required, for the same reason, that the second rule of deduction also be added to the ground on the same footing as the first, since we can also say of the second rule that the truths M, N, O, ... would not follow if it were invalid. We can see at once that this type of argument can be repeated *ad infinitum* and that, if it were legitimate to add one rule of deduction to the ground of the truths M, N, O, ..., an infinite number of them could be claimed to belong to this ground, which seems absurd.[15]

Thus, we are left with something very much like Carroll's regress. If the fact that conjunction-introduction is logically valid partially grounds the truth of the conclusion, then the truth of "if A has b and A has c and [if 'A has b' and 'A has c' are true, then 'A has (b + c)' is also true], then A has (b + c)" would also partially ground the truth of the conclusion, and so on *ad infinitum*. After all, if this sentence were false, then the conclusion would not logically follow from the premises.

Although Bolzano does not present it this way, I will show in the next section that it is not merely the infinitude of the partial grounds, but rather the infinite *regress* of grounds, that is the problem.[16] Nonetheless, Bolzano's concern with the explanatory role of inference rules, as well as his concern with providing *complete* grounds for the conclusion, demonstrate an admirable commitment to

[14] Ibid. Ellipses in original.

[15] Ibid. §199. p. 345. Ellipses in original.

[16] For instance, consider an infinite conjunction whose truth is grounded in the truth of each of its (infinitely many) conjuncts. There is little reason to think that the mere infinitude of grounds poses a problem in this case.

explanation. And unlike Carroll, Bolzano takes an explicit stance on the moral of his regress, namely that the validity of a rule of inference does not partially ground the truth of a conclusion. In what follows, I will use Bolzano's focus on grounding to defend the Tortoise's line of argument in my own voice.

1.3 A Circle

Consider the following two relations:

> Z is a logical consequence of A and B
>
> Z must be true if A and B are true

Are these relations one and the same, for Achilles? There is strong textual evidence that they are. For example, Achilles says that to "accept the *sequence* as a valid one" (i.e. to accept the argument as logically valid) is to "accept as true the Hypothetical Proposition that, if (A) and (B) be true, (Z) must be true."[17] This identity is made clear again when Achilles says: "Because it follows *logically* from them. If (A) and (B) and (C) are true, (Z) must be true."[18] Additionally, the only modality explicitly mentioned in the story is "logical necessity", as when the Tortoise asks whether anyone is "*as yet* under any logical necessity to accept (Z) as true."[19] If this necessity is what characterises the "must" above, then it is trivial that these two relations are identical.

If this is right, then there is an argument that the Tortoise does not consider explicitly in the dialogue, which is that Achilles' line of reasoning is *circular*. In the course of explaining why (Z) is a logical consequence of (A) and (B), Achilles appeals to the truth of (C), which is why the Tortoise suggests he add (C) to the premises. But notice that, on this reading, (C) just *is* the claim that (Z) is a logical consequence of (A) and (B). So, the truth of (C), to which Achilles *appeals*, is exactly what he is trying to *explain*. Achilles explains why (Z) is a logical consequence of (A) and (B) by appealing to the claim that (Z) is a logical consequence of (A) and (B), which is circular. It does not answer the Tortoise's question of *why* (Z) is a logical consequence of (A) and (B).

I am not the first to see this circularity. In fact, one of Carroll's contemporaries, John Cook Wilson, points it out in the course of considering an argument nearly identical to Carroll's regress. He writes that including a rule of inference as a premise "is nugatory, for it turns out to be a mere restatement of the argument itself."[20] It is not a reason *why* the conclusion is a logical consequence of the premises, but "a mere restatement" of the claim *that* the conclusion is a logical consequence of the premises.

[17] Carroll, "What the Tortoise said to Achilles", p. 278-9.
[18] Ibid. p. 279.
[19] Ibid.
[20] Cook Wilson, John, and A. S. L. Farquharson. *Statement and Inference : With Other Philosophical Papers.* Oxford: Clarendon Press, 1926. p. 446.

1.4 A Regress

Given the circularity that comes with the assumption that these relations are one and the same, in this section I will assume that these two relations are distinct. I am doing this, in part, to be charitable to our poor Achilles. Thus, (C) is not the claim that (Z) is a logical consequence of (A) and (B), but rather the claim that (Z) must be true if (A) and (B) are true.[21] And as we saw, this "must" cannot be characterised in terms of logical necessity if we are to avoid circularity.[22]

It will prove helpful to present Carroll's regress in a more formal manner. Although the Tortoise does not present it in these terms, I will be using the language of *grounding*, or metaphysical dependence, to describe the regress. Grounding is a relation that describes what is explanatorily prior or more fundamental,[23] and, on my reading, this language captures very well the Tortoise's commitment to explanation with respect to logical consequence. In a letter where he discusses the story, Carroll takes Achilles to be writing down "the series of Propositions, whose truth is needed to prove" the conclusion.[24] Thus, Carroll is not making the merely epistemological point that that one might know the premises of a valid argument without knowing its conclusion, or the merely psychological point that one might not feel the normative force of logical reasons.[25] Rather, Carroll is demanding a *metaphysical* explanation of how the truth of the premises relates to the truth of the conclusion. That is, I see Carroll's concern with *proving* the conclusion as a concern with *explaining* whether or not it logically follows from the premises. As we have seen, Bolzano understands this regress in terms of grounding as well.

We can also understand this regress by looking to F. H. Bradley, who uses a very similar regress to argue against the coherence of relations and of relational thought. Michael Della Rocca, in his interpretation of Bradley, is also happy to describe his regress in terms of grounding.[26] Presenting the problem in this way will, I hope, dispel such claims as that the Tortoise confuses premises with inference rules, or the object-language with the meta-language — he is enquiring into the grounds of logical validity, whatever they may be. If, for example, it turns out that (C) is rightly conceived of as an inference rule rather than as a

[21] For parallel reasons, none of the additional premises (D), (E), ... can be construed as claiming that (Z) is a logical consequence, either.

[22] I will argue that the upshot of the Tortoise's argument is that there *is no* coherent way to characterise this "must" in a manner that explains why (Z) is a logical consequence of (A) and (B). But this is a *story* (see §5) for §4.

[23] Schaffer, Jonathan. "Grounding, Transitivity, and Contrastivity." In *Metaphysical Grounding: Understanding the Structure of Reality*. Cambridge University Press, 2012. p. 122.

[24] Carroll, Lewis. "Letter 2: From Carroll to Stout, 25 August 1894." *The Carrollian* 28 (2016). p. 10.

[25] Such readings have been popular in recent literature on Carroll's regress. See, for instance, "Carroll's Regress and the Epistemology of Logic" by Philie and "The Philosophical Significance of Carroll's Regress" by Engel. Such readings presuppose the very thing I take Carroll to be calling into question, namely that an argument *is* valid and that there *are* reasons to infer the conclusion.

[26] Della Rocca, Michael. *The Parmenidean Ascent*. Oxford University Press, 2020. Ch. 3.

premise, then, as we will see presently, the question of what grounds the fact that (C) grounds the fact that (Z) is a logical consequence of (A) and (B) will still be perfectly intelligible.[27]

For the Tortoise, (Z) is a logical consequence of (A) and (B) because (Z) must be true if (A) and (B) are true. Thus, the truth of (C) is part of what explains why (Z) is a logical consequence of (A) and (B). To put this another way, the Tortoise claims that the logical validity of the argument is *partially grounded* in (C). And since (C) is not construed in logical terms, this explanation or grounding is not immediately circular.

I say that (C) is a *partial* ground, rather than the *complete* ground, in part because it is evidently the case that (A), (B), and (Z) are also partial grounds of the fact that (Z) is a logical consequence of (A) and (B). This is just one instance of the more general principle that a relation is partially grounded in its relata.[28] The relation depends on its relata to be or be conceived — without the relata, there would be nothing for the relation to relate! So, the logical consequence relation in Carroll's story, which relates premises (A) and (B) with the conclusion (Z), is partially grounded in (A), (B), and (Z) being the particular sentences that they are. To put it another way, the fact that (B) is the sentence "The two sides of this triangle are things that are equal to the same", rather than, say, "The two sides of this triangle are *not* things that are equal to the same", is part of what explains that (Z) is a logical consequence of (A) and (B). Bolzano concurs with this analysis, as he describes each premise of an argument as a partial ground of its conclusion.

On my interpretation, the fact that the logical validity of the argument is partially grounded in (C) makes sense of why the Tortoise insists that (C) be added as a premise. It is natural to want all the partial grounds of an argument's logical validity to be included as premises so that, taken together, the premises constitute the *complete* ground of its logical validity.[29] The premises are, as it were, all that is needed to logically conclude that the conclusion is true. Importantly, though, we need not include (C) as a premise in order to recognise that (C) is a partial ground. If, for example, (C) is not properly conceived of as a premise, this fact has no bearing on the Tortoise's claim that (C) partially grounds the fact that (Z) is a logical consequence of (A) and (B).

The Tortoise does not stop with (C), though. For now it is right to ask: what grounds the fact that (A), (B), and (C), taken together, partially ground the fact that (Z) is a logical consequence of (A) and (B)? After all, (C) does not claim that (Z) is a logical consequence of (A) and (B), but makes some other, non-logical claim. (This is, again, on pain of claiming that the argument is logically valid because it is logically valid.) Achilles replies with (D), i.e. that it is because (Z) must be true if (A) and (B) and (C) are true, where this "must"

[27] Some, following Wittgenstein, maintain that rules of inference are not the kind of thing which can be true or false; rather, they can only be obeyed or disobeyed. Those belonging to this camp can make sense of my claim here by treating (C) as the sentence which is true if and only if the inference rule modus ponens is valid.

[28] Ibid. p. 62.

[29] With the exception, of course, that the conclusion need not be included as a premise.

is not characterised in terms of logical necessity. But again it is right to ask: what grounds the fact that (A), (B), (C), and (D), taken together, completely ground the fact that (Z) is a logical consequence of (A) and (B)? And the regress of grounds is on its way.

One can think of (C), (D), (E), ... as each establishing, in non-logical terms, exactly what criterion must be met in order for an argument to count as logically valid. I see Bolzano characterising them in this way with his definition of a rule of inference, which, as we saw, was "a proposition which indicates what attributes the premises ... and the conclusions ... which follow from them, must have."[30] For Bolzano, a rule of inference describes what "attributes", or properties, the premises and conclusion must have in order so that the conclusion is a logical consequence of the premises. So, Bolzano might ask, why do (C), (D), and so on each give the *right* criterion, or indicate the *right* attributes, that the premises and conclusion must have? In this way, we can see that the Tortoise's line of reasoning pays homage to a much earlier regress, namely Sextus Empiricus' regress of the criterion.[31] Here, the truth for which we are seeking a criterion is the purported fact that an argument is logically valid. On my reading, this is because the regress of the criterion and Carroll's regress are both canonical instances of a more general regress concerning relations owing to F. H. Bradley.

1.5 Bradley's Regress

Consider again the relations from §1.2. As we saw, there does seem to be significant textual evidence that they are one and the same for the Tortoise and Achilles. That is, in the story, it seems that to claim (Z) is a logical consequence of (A) and (B) just is to claim that (Z) must be true if (A) and (B) are true. Because of this, as we saw, Achilles' explanation of the argument's validity does seem to be circular. On my reading, though, the Tortoise may have had another aim in identifying these notions in the story — the goal was to show that, however one characterises the "must" in the claim that (Z) must be true if (A) and (B) are true, an explanatory regress will follow. That is, however one characterises the logical consequence relation, there will be an explanatory regress. In this way, the Tortoise's argument is perfectly general, applying to every purported criterion for logical validity (just as Sextus Empiricus' regress of the criterion applies to every purported criterion for truth).

To see where this generality stems from, it will prove fruitful to consider Bradley's regress, as is not about any one relation in particular (e.g. the relation between a sentence and its constituent terms or the relation between the premises and conclusion of an argument) but rather the very *concept* of a relation. More specifically, Bradley's argument relies only on those properties that a relation must have insofar as it is a relation. It is for this reason that his argument is completely general, applying to all relations. And this, in turn, is why

[30] Bolzano, §199. p. 344.

[31] Bury, R. G. *Sextus Empiricus: Outlines of Pyrrhonism, vol. 1.* Loeb Classical Library 273. Harvard University Press, 1967. II 79.

he is ultimately led to a strict monism according to which there are no relations whatsoever, and, indeed, according to which the very concept of a relation is incoherent.

Bradley begins by making an explanatory demand with respect to relations, one that is similar to the demand I see the Tortoise making with respect to logical consequence. That is, Bradley insists that "somewhere there must be a reason why this and that appear together", i.e. an explanation as to why a relation obtains between this and that.[32] In making this move, he treats relations as dependent entities: there must be grounds for a relation, on which the relation depends in order to obtain between this and that. Like Della Rocca, I see this move in Bradley as consistent with the Principle of Sufficient Reason, according to which each fact has an explanation.[33]

In Della Rocca's reconstruction of Bradley's regress, a relation — insofar as it is a relation — is grounded in its relata. If the relata were, as Della Rocca puts it, "not in place", then there would be nothing for a relation to relate; thus, relations "are, by their nature, dependent at least on their relata."[34] But what is more:

> because standing in a relation of grounding to one or more of its relata is built into the nature of any relation, it follows that [a] relation depends not just on its relata, but also depends on the relation of grounding that it stands in to one or more of its relata.[35]

That is, a relation depends on its relata by its very nature, so it likewise depends, insofar as it is a relation, on the fact that this relation of dependence obtains. Were it not for the relation of partial grounding between a relation and its relata, the relation in question would not obtain. Thus, in addition to its relata, the relation is partially grounded in the relation of partial grounding between itself and its relata.

Let D be the relation of grounding that obtains between a relation C, on the one hand, and its relata A and B, on the other. In at least one case, Bradley describes such a relation D as "the ground, the how and why" of that relation C.[36] Bradley writes, in a famous passage from Chapter III of *Appearance and Reality*:

> The relation C has been admitted different from A and B, and no longer is predicated of them. Something, however, seems to be said of this relation C, and said, again, of A and B. And this something is not to be the ascription of one to the other. If so, it would appear to be another relation, D, in which C, on one side, and, on the other side, A and B, stand. But such a makeshift leads at once to the

[32] Bradley, F. H., *Appearance and Reality*. Clarendon Press, 1897. p. 517.

[33] The commitments to explanation consistent with the Principle of Sufficient Reason which are to be found in Bradley, Carroll, and Quine are worth keeping in mind for §4.2.

[34] Della Rocca, *The Parmenidean Ascent*, p. 62.

[35] Ibid. p. 63.

[36] Bradley, *Appearance and Reality*, p. 521.

> infinite process. The new relation D can be predicated in no way of C, or of A and B; and hence we must have recourse to a fresh relation, E, which comes between D and whatever we had before. But this must lead to another, F; and so on, indefinitely.[37]

There is a relation of partial grounding, D, which holds between C on the one hand and A and B on the other. But this relation of partial grounding, insofar as it is a relation, is partially grounded in *its* relata. That is, there is a relation of partial grounding, E, which holds between D on the one hand and A and B and C on the other. Just as the relation C depends upon the relation D, D in turn depends on the relation E. But because E is a relation, "this must lead to another, F; and so on, indefinitely."[38] For those keeping track at home: C depends on D, which depends on E, which depends on ...

As Della Rocca notes, the issue is not merely one of infinite regress, as there is also a circularity here (much like the circularity we saw in §1.3).[39] For the relation C is partially grounded in D, and D is partially grounded in its relata, one of which is C. So, assuming the transitivity of the partial grounding relation,[40] C is partially grounded in C — the fact that a relation C obtains between A and B partially depends on the fact that a relation C obtains between A and B.

Since the relation of grounding between a relation and its relata cannot be explained, and it pertains to the nature of a relation to be grounded in its relata, it would be premature to conclude that the concept *relation* is intelligible, but that no relations are real; it seems instead that the very notion of a relation is incoherent. We are left with no explanation as to what it means for one thing to be related to another. And, as Della Rocca notes, Bradley at times laments this fact:

> We have found that qualities, taken without relations, have no intelligible meaning. Unfortunately, taken together with them, they are equally unintelligible.[41]

Likewise, one might see Carroll's regress as showing that the logical consequence relation has no intelligible meaning. Foregoing relations, it does not makes sense to say that the premises and conclusion enter into a relation of logical consequence. Unfortunately, making use of relations fares no better, as "we are hurried off into the eddy of a hopeless process, since we are forced to go on finding new relations without end."[42]

[37] Ibid. p. 18.
[38] Ibid.
[39] Della Rocca, *Parmenidean Ascent*, p. 67. Indeed, it was Della Rocca who inspired me to be on the lookout for a circle in Carroll's story.
[40] For a defence, see "On Some Counterexamples to the Transitivity of Grounding" by Litland.
[41] Bradley, *Appearance and Reality*, p. 25.
[42] Ibid. p. 28.

2 The Moral of the Story

> "And how many hours a day did you do lessons?" said Alice, in a hurry to change the subject.
>
> "Ten hours the first day," said the Mock Turtle: "nine the next, and so on."
>
> "What a curious plan!" exclaimed Alice.
>
> "That's the reason they're called lessons," the Gryphon remarked: "because they lessen from day to day."
>
> Lewis Carroll, *Alice's Adventures in Wonderland*, Ch. XI.

Before using Carroll's regress to draw any conclusions about logic, meaning, normativity, etc. (if indeed we *can* draw any conclusions from Carroll's regress)[43], it will prove worthwhile to consider historical responses to Carroll's regress and historical accounts of the logical consequence relation more generally.

2.1 Bolzano and Cook Wilson

The most popular takeaway from Carroll's dialogue is not that the very notion of logical consequence is incoherent.[44] Rather, it is that there is a distinction between *premises* and *inference rules*, a distinction which is not heeded by the Tortoise. The first to draw this conclusion from Carroll's story is his Oxford colleague John Cook Wilson, then the Wykeham Professor in Logic at New College. Cook Wilson writes that it is:

> clearly a fallacy to represent the rule according to which an inference is to be drawn from premisses as one of the premisses themselves. We should anticipate that this must somehow produce an infinite regress, and that this is so can easily be shown.[45]

However, it should be noted that Bolzano independently reached a similar conclusion decades prior, as we saw in §1.2. Cook Wilson, though, is explicit that the issue is not merely the *infinitude* of grounds, but rather the fact that there is a *explanatory regress* involved (which I characterised in §1.4). That is, he sees Carroll's regress as an infinite regress which leaves the validity of the original argument unexplained. Indeed, on my reading, it is *because* he sees this regress as vicious that he is keen to carve a distinction between premises and inference rules.

Is this a successful strategy? That is, can Carroll's regress be avoided by appealing to a distinction between premises and inference rules? As I have formulated the regress, the answer is clearly no. The validity of the argument

[43] See §4.1.

[44] This may be because of the apparent inconsistency or incoherence of such a view, which I will discuss in §4.2.

[45] Cook Wilson, p. 443.

is partially grounded in the truth of (C), and the fact that the validity of the argument is partially grounded in the truth of (C) is itself partially grounded in the truth of (D), and so on. Whether or not (C), (D), etc. are included as premises has no bearing on the regress of grounding.

2.2 Ryle

In contemporary discussions of Carroll's regress, the focus is on Gilbert Ryle's account of the distinction between premises and inference rules, one which takes on a distinctively epistemological character that the previous two lack. Ryle maintains that there is a distinction between knowing-*how* and knowing-*that*, between an ability and propositional knowledge. The distinction between inference rules and premises is just one instance of this more general distinction. To know the premises of an argument is to know-that, because the premises are statements. But on this view, "the actual process of drawing an inference, which is after all at the heart of logic, is something which cannot be represented as a logical formula ... Learning to infer is not just a matter of being taught about explicit logical relations between propositions; it is learning *to do* something."[46] So, when the Tortoise presents the argument to Achilles, Ryle would maintain that it is a mistake to think that (C) is a partial ground of our knowledge *that* (Z) is a logical consequence of (A) and (B). It would be more proper to say that we simply know *how* to infer (Z) from the truth of (A) and (B).

After reading Carroll's story, Ryle's argument for this distinction should sound familiar, since it turns on an infinite regress. He argues that:

> if the intelligence exhibited in any act, practical or theoretical, is to be credited to the occurrence of some ulterior act of intelligently considering regulative propositions, no intelligent act, practical or theoretical, could ever begin.[47]

Again, Ryle is clearly concerned with the order of knowing rather than the order of metaphysical explanation (or, as Bolzano would say, the order of objective grounds). This is clear in Williamson's and Stanley's summary of the structure of Ryle's argument: "If knowledge-how were a species of knowledge-that, then, to engage in any action, one would have to contemplate a proposition. But, the contemplation of a proposition is itself an action, which presumably would itself have to be accompanied by a distinct contemplation of a proposition,"[48] and so on *ad infinitum*, with the original action never occurring. The emphasis is on the impossibility of performing a certain action — drawing a logical inference — rather than the impossibility of explaining a certain fact — there is a relation of logical consequence between the premises and conclusion. Here is Ryle's argument, as I see it:

[46] Winch, Peter. *The Idea of a Social Science and its Relation to Philosophy.* Routledge, 2015. p. 57.

[47] Ryle, Gilbert. "Knowing How and Knowing That: The Presidential Address." *Proceedings of the Aristotelian Society* 46 (1945): 2.

[48] Williamson, Timothy, and Jason Stanley. "Knowing How." *The Journal of Philosophy 98*(8), 2001. p. 413.

(1): If there is no distinction between premises and inference rules, then drawing a logical inference is impossible.
(2): It is possible to draw a logical inference.

---

(3): There is a distinction between premises and inference rules.

The reason Ryle's argument is unresponsive to Carroll's regress is that he takes as a premise the very claim that the Tortoise is calling into question, namely (2), the claim that it is possible to draw a *logical* inference. The Tortoise is asking for an explanation as to why one statement is a logical consequence of another, but Ryle offers nothing of the sort. Rather, he takes an axiomatic stance that it *is* possible to draw a logical inference (and thus that some sentences *are* logical consequences of others), but this is just to give up on the explanatory demand that Carroll's Tortoise is issuing — it is to embrace as a brute fact that we can draw a *logical* inference.

If you like, here is another way to put the same problem, one that will strike closer to home for Ryle (since he is concerned with the possibility of performing certain actions): human beings seem to perform all kinds of practical reasoning. Depending on one's view of logical consequence, some of this reasoning will count as logically valid (e.g. reasoning by modus ponens) and some of it will not (e.g. reasoning by affirming the consequent, or inferring that a scarf is red from the fact that it is scarlet). On my reading, the Tortoise can be seen as asking for an explanation as to why some forms of reasoning that we perform are logically valid and why other forms are not, i.e. for a criterion of logical validity. It does not answer the Tortoise to insist that we just *do* draw logical inferences, or that modus ponens just *is* logically valid.

2.3 Cook Wilson Again

Immediately after his *reductio* argument from §2.1, Cook Wilson goes on to provide another argument for why there must be a distinction between premises and inference rules:

> In the above procedure the rule of inference is made a premiss and a particular inference is represented as deduced from it. But, as we have seen, that is an inversion of the true order of thought. The validity of the general rule of inference can only be apprehended in a particular inference. If we could not see the truth directly in the particular inference, we should never get the general rule at all. Thus it is impossible to deduce the particular inference from the general rule, and the so-called reduction is merely verbal with no corresponding process of thought.[49]

I take Cook Wilson to be arguing that the validity of particular instances of modus ponens reasoning is not explained by the fact that modus ponens, as a general rule, is logically valid. Rather, the fact that modus ponens, as a general

[49] Cook Wilson, p. 445.

rule, is logically valid is is explained *by the validity of particular instances*. Thus, there is something like induction from the particular instances of modus ponens to the general rule. The fact that particular instances are logically valid explains why the rule is logically valid.

Although this suggestion is fascinating, I think there are two lines of thought which make it untenable. The first stems from the Tortoise's demand for explanation — Cook Wilson is assuming that particular instances of modus ponens, like the argument from Carroll's story, are logically valid. But the question at hand is precisely *whether* that instance of modus ponens, with premises (A) and (B) and conclusion (Z), *is* logically valid. It seems that, as with his previous argument, Cook Wilson is taking for granted the very thing Carroll is calling into question.

The second stems from the use of induction in Cook Wilson's argument here. It seems that proceeding by induction on individual instances of modus ponens opens him up skepticism about rule-following, as in Saul Kripke's *Wittgenstein on Rules and Private Language*. A finite number of instances of modus ponens reasoning will underdetermine what the logically valid general rule *is*. Indeed, some have taken Carroll's story to be an argument along the lines of Kripke's argument for the incoherence of rule-following, but in the specific case of *rules* of inference.[50] While I do believe that skepticism about rule-following, Carroll's regress, and Bradley's regress are deeply connected (see §4.2), it is highly unlikely that Carroll had Kripke-style rule-following skepticism in mind when he wrote his story.[51]

Cook Wilson might reply to this second objection by claiming that his argument is better understood as a universal generalisation on the basis of an arbitrary individual case. Such reasoning is commonplace in mathematics — by assuming that an arbitrary element in set X must belong to a set Y, one can prove that *every* element of X is also an element of Y, i.e. that X is a subset of Y. Under the assumption that only a few expressions, the so-called "logical constants", contribute to logical validity, the logical validity of one argument can be used to show the logical validity of any argument with identical logical form. Putting aside worries about selecting suitable logical constants, which I will address in §2.5, the main objection here is precisely the same as before: Cook Wilson is assuming that that a particular instance of modus ponens, like the argument from Carroll's story, *is* logically valid. But this is precisely the question at hand.

2.4 Conventionalism

We may be tempted to say that we just posit rules conventionally, or by fiat. For example, we may say to those who accept (A) and (B) but deny (Z) that

[50] Engel, p. 6.

[51] Engel agrees that "it is hard to suppose that Carroll could have anticipated Wittgenstein or Kripke" (6).

"You use the words *if..., then...* differently."[52] If one knew the meanings of these words, the thought goes, then one would see that (Z) *is* a logical consequence of (A) and (B). Similarly, one might point to someone's holding true a sentence S and a sentence $\neg S$ as evidence that they do not mean the same thing by "$\neg$" as we do.[53] However, Willard van Orman Quine famously uses Carroll's regress to show that such a position is untenable.

In "Truth by Convention", Quine notes that conventionalists about logic cannot go through each logical sentence one by one in order to declare by fiat whether they are true or false. This is due to the infinitude of logical sentences that the conventionalist wants to make declarations about. As a result, conventionalists must affirm conventions that determine "an infinite class of expressions all of which, under ordinary usage, are true statements".[54] For example, the conventionalist might hold that all results of putting a statement for 'p', a statement for 'q', and a statement for 'r' in "If if p then q then if if q then r then if p then r" are logical truths.[55] Since there are an infinitude of sentences that can be substituted in this way, the above convention covers infinitely many cases of logical truth. However, Quine notes that moving from the claim that a particular sentence is an instance of a general convention (like the one above) to the conclusion that it is a logical truth requires making a logical inference:

> Each of these conventions is general, announcing the truth of every one of an infinity of statements conforming to a certain description; derivation of the truth of any sufficient statement from the general convention thus requires a logical inference, and this involves an infinite regress.[56]

Quine goes on to cite Carroll's "What the Tortoise said to Achilles" explicitly.[57]

For conventionalists, the criterion for logical validity is non-circular because it is characterised in non-logical terms, i.e. in terms of linguistic convention. But, for this self-same reason, we are led on an explanatory regress when it comes to explaining why (Z) is a logical consequence of (A) and (B). It seems that the suggestion is to replace (C) with:

> **(Convention-C)**: if (A) and (B) are true, then (Z) is true by fiat.

But, for consistency, when it comes to explaining why (Convention-C) grounds the fact that (Z) is a logical consequence of (A) and (B), the conventionalist must say:

> **(Convention-D)**: if (A) and (B) and (Convention-C) are true, then (Z) is true by fiat.

[52] Quine, W. V. "Truth by Convention." In *Quintessence*, Harvard University Press, 2008. p. 15.
[53] Ibid. p. 21.
[54] Ibid. pp. 16-17.
[55] Ibid.
[56] Ibid. p. 26.
[57] Ibid. p. 27.

And so on. This is precisely the regress that Quine is pointing out. And he shows great insight in noting that conventionalism fails to explain the validity of modus ponens. However, I will try to show that this regress is not unique to conventionalism — every other account of logical consequence faces it as well. And, as a forewarning, the "view" of logic to which Quine is committed after rejecting the analytic/synthetic distinction, and thus rejecting the idea that a sentence can be true by linguistic convention alone, will be the subject of §3.

2.5 Tarski

All of the above might just be taken as showing that it is futile to explain the inference rules of a formal deductive system using the system itself. This might lead one to consider a model-theoretic account, as Bartley suggests:

> The validity of an argument, as opposed to the truth of the conclusion, must be defended metalinguistically. In order to defend a derivation, one would have to show metalinguistically that no counter-example exists for any interpretation of the inference which is form-preserving and whose premises are true.[58]

The view I have in mind was outlined by Alfred Tarski in "On the Concept of Logical Consequence", and has seen many technical modifications since then. Tarski sought to capture the "common concept" of logical consequence in a mathematically precise way, namely through the use of models.[59] Given an argument, such as the Tortoise's, Tarski devised a corresponding *argument form* in which so-called "logical" words such as "or" and "not" are held constant and the remaining terms are treated as variables (with like terms corresponding to like variables). An argument is declared logically valid if and only if it is truth-preserving on all semantically well-behaved reinterpretations of the variables, i.e. truth-preserving in all models. Carroll's argument would have the corresponding argument form:

$$\frac{\begin{array}{c} P \to Q \\ P \end{array}}{Q}$$

No matter how we interpret the declarative sentences P and Q, the argument will be truth-preserving. For this reason, Tarski's model-theoretic account declares Carroll's argument logically valid.

In practice, proving that an argument is truth-preserving in all models might involve making use of a variety of logical inferences in the meta-language. However, what is presently under consideration is the question of what *grounds* the

[58] Bartley, William Warren. "Editor's Note on 'What the Tortoise Said'". In *Lewis Carroll's Symbolic Logic*. Crown Publishers, 1977. p. 468.

[59] Tarski, Alfred. "On The Concept of Logical Consequence." *Logic, Semantics, Metamathematics: Papers from 1923 to 1938*. Hackett Publishing, 1983. p. 409.

logical validity of an argument. For the model-theorist, the fact that an argument is truth-preserving in all models (on some selection of logical constants) is what grounds the fact that it is valid. Whether, in the order of knowing, we employ logical inferences to realise that the argument is truth-preserving in all models is, for our purposes, besides the point.

The major problem with this and similar views is that there is no explanation as to why possessing a logical form which is truth-preserving in all models grounds the fact that an argument is logically valid. In other words, an explanation is wanting as to why the model-theoretic account actually captures the logical consequence relation — even if the argument is truth-preserving in every model (on some selection of logical constants), *why* does this mean that the argument is logically valid? Etchemendy raises precisely this point in *The Concept of Logical Consequence*, where he argues that one variation of the model-theoretic account[60] happens to be coextensive with the deductive system in the first-order case only because of the strength of the set-theoretic assumptions at work (e.g. the assumption that there are infinitely many objects in the domain) and the simplicity of the first-order language.[61] However, in general, there is no explanation as to why truth-preservation in models is related to logical consequence, at least when models range over *interpretations* of the variables (as Tarski envisaged).

If we instead employ representational semantics (where models range over possible worlds), then at first blush it seems that truth-in-models is conceptually related to logical consequence, but now the account is only as correct as the assumptions about which worlds are logically possible. However, *logical possibility*, e.g. whether it logically possible that (A) and (B) are true but (Z) is false, is exactly what the Tortoise is trying to understand! So this representational model-theoretic account threatens to be unenlightening.

On my reading, Etchemendy's criticism is, in effect, the force of Carroll's regress when the model-theoretic explanation is is substituted for (C). Tarski's criterion for logical validity is non-circular because it is characterised in non-logical terms, i.e. in terms of the normal, run-of-the-mill truth of a universal generalisation. But, for this self-same reason, we are led on an explanatory regress when it comes to explaining why this criterion is the right one for *logical* validity. It seems that Tarski's suggestion is to replace (C) with

> **(C-Tarski)**: If the argument form <(A),(B);(Z)> is truth-preserving in all models, then (Z) is a logical consequence of (A) and (B).

Let us suppose that the argument form <(A),(B);(Z)> *is* truth-preserving in all models. We are still left with the question: what grounds the fact that (C-Tarski) grounds the fact that (Z) is a logical consequence of (A) and (B)? Perhaps the model-theorist, for the sake of consistency, would say

[60] This is the modification where the domain of quantification is varied.

[61] Etchemendy, John. *The Concept of Logical Consequence.* Harvard University Press, 1990. Ch. 7.

> **(D-Tarski)**: If the argument form <(A),(B),(C-Tarski);(Z)> is truth-preserving in all models, then (Z) is a logical consequence of (A), (B), and (C-Tarski).

Since (C-Tarski) is a sentence in the meta-language, the model-theorist may have to find a suitable selection of logical constants for that language in order to consider the models of <(A),(B),(C-Tarski);(Z)>. However, this is not the main problem. The main problem, of course, is that we are off on a vicious regress of grounding. It may be tempting for the model theorist to treat (D-Tarski) differently from (C-Tarski), but it is unclear how there could be a principled reason for doing so. To claim that we must treat these cases differently because, otherwise, (Z) would not be a logical consequence of (A) and (B) would be to assume the very fact that we are trying to explain, namely that (Z) *is* a logical consequence of (A) and (B).

2.6 Deductive Systems

One attempt to explain why (Z) is a logical consequence of (A) and (B) proceeds as follows: when we translate the argument into a formal system such as first-order logic, the conclusion is deducible from the premises. This is fine as far as it goes, but it does not go very far. For, assuming that the translation into the formal system is adequate, the validity of the argument now turns on whether the formal system in question is logically sound, i.e. whether every theorem of the system is a logical truth. Even if the conclusion is deducible from the premises in some formal system, why does this mean that the conclusion is a logical consequence of the premises?

Supporters of formal deductive systems might point to the fact that the system in question, e.g. first-order logic, enjoys a soundness theorem. Indeed, those who reject the view that logical consequence is identical with derivability in a formal system do so not because they deny that the system's theorems are logically true, but because they think there are logical truths which are *not* theorems, i.e. that the system is missing some logical truths. In other words, derivability in a formal system with a soundness theorem is widely taken to be sufficient, but not necessary, for being a logical truth. Gerald Massey has argued, for example, that this creates an asymmetry between proving an argument is valid and proving an argument is invalid: to prove an argument is valid, it suffices to prove it is a theorem of some formal system with a soundness theorem; to prove an argument is invalid, one must show that, in principle, no formal system with a soundness theorem could produce it as a theorem.[62] Etchemendy makes a similar point:

> [A]t best we might mean by "consequence" derivability in some *sound* deductive system. But the notion of soundness brings us

[62]Massey, Gerald J. "Are There Any Good Arguments that Bad Arguments are Bad?" *Philosophy in Context* 4 (1975): 62.

> straight back to the intuitive notion of consequence: a deductive system is sound if it allows us to prove only genuinely valid arguments, those whose conclusions follow logically from their premises.[63]

The logical soundness of a formal system, in the robust sense meant by Etchemendy above, is not what is explained by a "soundness theorem" — the purpose of such a theorem is not to explain what grounds the logical validity of an argument. Rather, its purpose is to show that every theorem of a syntactic system is likewise a theorem of a corresponding semantic system. In cases where there is also a completeness theorem, the two theorems together prove that the syntactic and semantic systems are coextensive: every theorem of one is a theorem of the other. Etchemendy argues that the "completeness theorem" is a misnomer, as it does not show that the deductive system in question is *logically complete* in the sense that every logical truth of the language is a theorem, but rather that every theorem of the semantic system is likewise a theorem of the syntactic system.[64] Likewise, I maintain that "the soundness theorem" is a misnomer, as it does not show that the deductive system in question is *logically sound* in the sense that every theorem is a logical truth of the language, but rather that every theorem of the syntactic system is likewise a theorem of the semantic system.

The reason that "soundness" theorems do not prove that a formal system is sound in the robust sense of *logically sound* is that it assumes that the semantic system in question captures the logical consequence relation. But, as with Tarski's system above, there is no reason to suspect that this is the case; each of the "semantic" systems in question succumbs to Carroll's regress.

Another way to see this is through Dummett's discussion of three different ways in which one could ask for a justification of deduction.[65] (Importantly, Dummett is interested in justification, which is an epistemological notion, rather than explanation, which is a metaphysical notion; nevertheless, we can translate his insight into the metaphysical problem that I take Carroll to be concerned with). The first way is, for a given theorem of a formal deductive system, to ask for the series of steps that constitute its proof. The second is, for a given formal deductive system, to ask for a soundness theorem of the kind just described. "But there is yet a third, deeper, level" says Dummett (and notice how he switches from talk of justification to talk of explanation): "we require an explanation, not of why we should accept certain forms of argument or canons for judging forms of argument, but of how deductive argument is possible at all."[66] This is the question that I take Carroll to be concerned with in the story

[63] Etchemendy, *The Concept of Logical Consequence*, pp. 2-3.

[64] Ibid. Ch. 11.

[65] Dummett, Michael. "The Justification of Deduction". In *Truth and Other Enigmas*. Harvard University Press, 1978.

[66] Ibid. p. 297. Dummett's worry above is with a problem that traces back at least to John Stuart Mill, namely the no-new-knowledge problem of deductive inference. In brief, the worry is that, on the one hand, we seem to learn something new by deducing conclusions from premises, but, on the other hand, all the knowledge contained in the conclusion is already latent within the premises. For more, see Ch. 2. of "Mill, John Stuart (1806–73)"

— how deducing (Z) from (A) and (B) is possible at all, i.e. what it would mean to say that one sentence is a logical consequence of another. And this is a question which is unanswered by an appeal to a "soundness" theorem.

2.7 Etchemendy

In the course of rejecting Tarski's account of logical consequence, Etchemendy identifies the notions of logical consequence and analytic consequence. For Etchemendy, an argument is logically valid if it holds true simply in virtue of what the conclusion means and what the premises mean. Contra Tarski, Etchemendy maintains that the meanings of *all* constituent expressions can count towards the logical validity of an argument, not just the expressions deemed the "logical constants" of the language. Thus, Etchemendy rejects the view that logically valid arguments must have a certain logical form — instances of modus ponens are logically valid, but so are arguments such as "the scarf is scarlet; therefore, the scarf is red".[67] Indeed, for Etchemendy, even things like maps and graphs can have logical consequences.[68] With respect to Carroll's story, Etchemendy would say that, given the meaning of "if..., then...", the conclusion (Z) is a logical consequence of the premises (A) and (B).

Again, the criterion for logical validity is non-circular because it is characterised in non-logical terms, i.e. in terms of the meanings of individual expressions such as "or" and "scarlet". But, for this self-same reason, we are led on an explanatory regress when it comes to explaining why the meaning of "if..., then..." is such that (Z) is a logical consequence of (A) and (B). Why are the meanings of (A), (B), and (Z) such that the argument is logically valid?[69] Proceeding in this way — demanding an explanation for why the meanings of (A), (B), (C), ... ground the fact that the argument is logically valid — is, I will argue, another way to challenge the analytic/synthetic distinction, and thus to challenge the very concept of a proposition. For this regress argument applies not only to so-called "logical" words like "or", but also to words like "scarlet". When we ask why the meaning of the word "scarlet" is such that anything scarlet is red, we are likewise led on an explanatory regress.

If one adopts Etchemendy's account of consequence, though, there is room for criticism in the reverse direction as well: the incoherence of the analytic/synthetic distinction amounts to the incoherence of logical consequence. For if the concept *logical truth* and the concept *analytic truth* are one and the same, then if the latter is incoherent, so too is the former. An argument against the analytic/synthetic distinction, such as Quine's in §6 of "Two Dogmas of Empiricism", translates directly into an argument against the logical/non-logical

by Skorupski. This is related to problems with the concept of a proposition that we will encounter in §3.

[67] Etchemendy, "Reflections on Consequence", *New Essays on Tarski and Philosophy* (2008): 296.

[68] Ibid. p. 284.

[69] If (D) is the claim that the meanings of (A), (B), (C) and (Z) are such that the argument with premises (A), (B), and (C) and conclusion (Z) is logically valid, then it is easy to see how this leads to an infinite regress. See §2.4.

distinction. As I have been stressing with the concept *logical consequence*, the upshot of Quine's argument is not merely that the concept *analytic truth* is coherent, but has empty extension; rather, the very notion is incoherent, and it makes no sense to speak of a sentence that is true by meaning alone.

3 Logic and Meaning

> When *I* use a word it means just what I choose it to mean — neither more nor less.
>
> Humpty Dumpty, *Through the Looking Glass*, Chapter VI.

A conclusion that one might draw from Carroll's regress, as pointed out in §1.5 and perhaps suggested by the failures of the accounts considered in §2, is to reject the logical consequence relation as incoherent. Given the regress of grounding that arises when it comes to explaining why (Z) is a logical consequence of (A) and (B), and given that this is a paradigmatic instance of modus ponens, it might seem that logical consequence in general cannot be explained. That is, because the relation of logical consequence ultimately receives no metaphysical explanation, one could argue that we are left with no coherent concept of logic whatsoever.

This may sound like too radical of a conclusion, one that requires too many alterations to our common sense understanding of the world. After all, it really does seem that some sentences are logically true, and that some sentences logically follow from others. And many have argued that conservatism is a philosophical virtue.[70] However, I will argue that such a view is actually logically equivalent to a popular view of meaning put forth by W. V. Quine early in his philosophical career, namely the meaning holism of "Two Dogmas of Empiricism". In doing so, I hope to shed light on both the consequences of rejecting the analytic/synthetic distinction as incoherent *and* the consequences of rejecting logical consequence as incoherent.

3.1 Come What May

To begin with a rough characterisation, analytic truths are variously thought to be those sentences which one must hold true come what may; which are immune to revision; which are true by meaning alone; which do not "impose constraints on the world" that the world may or may not meet.[71] By contrast, synthetic truths are variously thought to be those sentences whose truth depends on both meaning and fact, or which one need not hold true come what may.

[70] Lewis, David. "Elusive Knowledge". *Australasian Journal of Philosophy* 74, no. 4 (1996): 559.

[71] Williamson, Timothy. *The Philosophy of Philosophy.* Second ed. Blackwell / Brown Lectures in Philosophy. Hoboken, 2022. p. 54.

To start, I would like to motivate Quine's rejection of the analytic/synthetic distinction by appealing, as Quine does, to the "law of sufficient reason".[72] I completely agree with Della Rocca's analysis:

> I regard Quine as making a rationalist move in so rejecting the analytic/synthetic distinction: that distinction is to be rejected because there is no principled way to draw it, no reason for drawing the distinction here rather than there.[73]

Note that, for Della Rocca, a "rationalist move" is one which demonstrates a commitment to explanation compatible with the Principle of Sufficient Reason. In this respect, then, Quine's argument can be seen to have a lot in common with both Bradley's and Carroll's regress, in that all three are motivated by explanatory demands. Whereas Bradley is concerned with explaining why relations in general hold, and Carroll is concerned with explaining why the logical consequence relation in particular holds, Quine is concerned with explaining why a word has the particular meaning that it does, and thus why some sentences involving that word, but not others, are true by meaning alone.

I also see Quine's worry here to be about the arbitrariness that is involved in claiming that a *sentence* is what is true or false. There is no principled way to delineate the extension of a predicate term, as any such delineation can be accommodated "if we make drastic enough adjustments elsewhere in the system"; thus, privileging one such delineation over another can only be done in an arbitrary or brute manner.[74] Likewise, when it comes to the referent of a subject term, *any* object can be treated as the referent come what may. Thus, on my reading, Quine's desire to avoid arbitrariness is what leads him to conclude that words and sentences are not what is meaningful, and not what is true or false. What is meaningful and true is "the whole of science".[75]

It would be helpful to look at an example. While considering the meaning of the set-theoretic epsilon relation, '$\in$', Quine writes that:

> [I]t is difficult to conceive of how to be other than democratic towards the truths of set theory. In exposition we may select some of these truths as so-called postulates and deduce others from them, but this is subjective discrimination, variable at will, expository and not set-theoretic.[76]

I do not take this to be a claim that is somehow unique to the domain of set theory, but rather something that applies more broadly across science. To introduce a postulate, e.g. about the extension of the predicate "red", in order to conclude that the sentence "scarlet is red" is true by meaning alone would amount to subjective discrimination, as there is no principled, non-arbitrary

[72] Quine, "Carnap and Logical Truth". In *Quintessence*, Harvard University Press, 2008. p. 70. See §4 and Della Rocca, *The Parmenidean Ascent*, Ch. 11.
[73] Della Rocca, p. 171.
[74] Quine, "Two Dogmas of Empiricism", p. 51.
[75] Ibid. p. 50.
[76] Quine, "Carnap and Logical Truth", p. 70.

way to introduce such a postulate. Nothing explains why the meaning of "red" is such that one collection of objects must be regarded as red come what may, or (what for Quine is the same thing) why the extension of "red" must contain one collection of objects rather than another, in order to remain faithful to *the* meaning of red. And because there is no such explanation, it does not make sense to pick out a subset of truths which one must hold true come what may. Likewise, it does not make sense to speak of *the* meaning of "red". Two sentences later, Quine himself explicitly ties his rejection of the analytic/synthetic distinction in set theory to a demand for explanation, writing: "Given this democratic outlook, finally, the law of sufficient reason leads us" to reject the view that some set-theoretic claims, but not others, must be held true come what may in order to remain faithful to the meaning of "$\in$".[77]

3.2 Equivalence

In this section, I will argue that rejecting the analytic/synthetic distinction is tantamount to rejecting the logical/non-logical distinction. From the outset, though, it is worth noting an important similarity between the logical/non-logical distinction and the analytic/synthetic distinction: in both cases, to reject the distinction is not merely to claim that the distinction makes sense, but that there are simply zero of one or the other types of truth. As Putnam notes, "giving up the analytic-synthetic dichotomy does not mean — that is, *should not mean* — thinking of all our beliefs as empirical."[78] Rather, in rejecting the distinction, we are rejecting it *as* incoherent. It does not even make sense to speak of a sentence as being analytic as opposed to synthetic, or vice versa. We see Quine making this move in "Carnap and Logical Truth" when he writes that: "we at present lack any tenable general suggestion, either rough and practical or remotely theoretic, as to what it is to be an analytic sentence."[79] In other words, the concept *analytic truth* is incoherent, as there is no explanation as to why some sentences, but not others, are immune to revision. As Quine expounds in "Two Dogmas":

> Taken collectively, science has its double dependence upon language and experience; but this duality is not significantly traceable into the statements of science taken one by one.[80]

I take Quine's concern with significance as further evidence that he rejects the analytic/synthetic distinction as incoherent, and that he does so on the basis of the fact that there is no explanation as to why the purported distinction between analytic and synthetic truths is drawn in one way rather than another. Given that *any* sentence can be held as true come what may, there is no basis for

[77] Ibid.

[78] Putnam, Hilary. "Rethinking Mathematical Necessity". In Crary, Alice, and Rupert J. Read. *The New Wittgenstein.* London; New York: Routledge, 2000. p. 221.

[79] Quine, "Carnap and Logical Truth", p. 84. Emphasis mine. See Della Rocca, *The Parmenidean Ascent*, p. 278.

[80] Ibid. "Two Dogmas of Empiricism", p. 50. Emphasis mine.

tracing down any one collection of sentences as comprising the analytic truths. As a result, the coherence, or significance, of the concept *analyticity* is what is rejected.

3.2.1 Backwards

First, consider the backwards direction: if the logical/non-logical distinction is incoherent, then the analytic/synthetic distinction is incoherent. By assumption, it does not make sense to say that some sentences are logical truths while others are non-logical, run-of-the-mill truths. And, by extension, it does not make sense to say that some sentences are logical consequences of other sentences. For if (Z) is a logical consequence of (A) and (B), then the sentence "If (A) and (B), then (Z)" will count among the logical truths.

Now, our task is to prove that the analytic/synthetic distinction is incoherent under such an assumption. The crucial claim is this:

Light Inferentialism: if a declarative sentence S has meaning, then S enters into logical relationships (i.e. can feature in the premises or conclusion of a logically valid argument)

I contrast this with Heavy Inferentialism, the view that the meaning of a sentence *just is* the role it plays in logical relationships with other sentences.[81] Presently, we need not so strong of an assumption as that there are no other, non-logical factors that contribute to the meaning of a sentence. If Light Inferentialism is true, then a meaningful sentence enters into logical consequence relations. But logical consequence relations are exactly what are ruled out as incoherent by our assumption. So, it does not make sense to speak of the meaning of a sentence. But if this is right, then it also does not make sense to speak of a sentence being true by its meaning alone. Therefore, the incoherence of the logical/non-logical distinction implies the incoherence of the analytic/synthetic distinction. (The trouble we may encounter in claiming that the incoherence of logical consequence *logically implies* anything will be addressed in §4).

To see why I find Light Inferentialism compelling, consider the classical argument that Socrates is mortal. Suppose that Socrates is a man and that all men are mortal. If it does not logically follow that Socrates is mortal, then it may seem absurd to believe that the sentences "Socrates is a man" and "All men are mortal" have the meanings we ordinarily take them to have, i.e. express that Socrates is a man and that all men are mortal. For if they *did* have these meanings, the thought goes, then it *would* logically follow that Socrates is mortal. In this way, skepticism about logical consequence leads to skepticism about meaning — a skepticism that is shared by the analytic/synthetic distinction.

Another way to see this backwards direction of the biconditional is that Carroll's regress applies to the notion of analytic consequence as well as to the notion of logical consequence. That is, a slight variation of §1 can be directed at the analytic consequence relation rather than the logical consequence relation.

[81] See *A Theory of Concepts*, by Peacocke.

If the relation of logical consequence between the premises and conclusion of an argument cannot be explained, then, for parallel reasons, any inference involving analyticity, e.g. "John is a bachelor; therefore, John is unmarried", likewise cannot be explained.[82] In many ways, such an inference is *more* suspect than the strict logical inferences involving terms such as "and" and "or".[83] In making this move, both analytic consequence and logical consequence would would be rejected as incoherent in the same stroke.

3.2.2 Forwards

The more difficult direction is to prove that if the analytic/synthetic distinction is incoherent, then the logical/non-logical distinction is incoherent. One might think that this is the easy direction, since logical truth is often conceived as a species of analytic truth. That is, while it is commonly thought that not all analytic truths are logical truths (e.g. the truth that every bachelor is unmarried), it *is* commonly thought that all logical truths are analytic truths. In particular, it is thought that logical truths are those analytic truths which hold true simply in virtue of the meanings of so-called "logical" vocabulary, such as "and" and "¬". So, if the concept of an analytic truth is incoherent, then logical truth, being a species of analytic truth, is likewise incoherent.

However, the story is not so simple. First, some reject the analytic/synthetic distinction as incoherent while still holding onto the concept of logical truth, such as Timothy Williamson and late-Quine. For them, logical truths are not analytic truths, but they are truths nonetheless. Second, some maintain that the web of belief — the picture that early-Quine thinks we are left with after rejecting the analytic/synthetic distinction as incoherent — *requires* the concept of logical consequence. Here I have in mind Graham Priest, for whom Quine's argument shows only that the first dogma — reductionism — should be axed.[84] The second dogma — the analytic/synthetic distinction — must be preserved on pain of generating Carroll's regress. I will argue that both of these objections do not salvage the coherence of logical consequence.

3.3 Logic without Analyticity

3.3.1 Which Logic?

Those like Williamson and late-Quine who reject the view that logical truth is a species of analytic truth must find some other explanation for the logical/non-logical distinction. As it happens, both of them choose a variation of Tarski's model-theoretic account of logical consequence, but it need not have been this

[82]Indeed, we did just this in §1.7 when considering Etchemendy's account of logical consequence.

[83]Many logicians, especially followers of Tarski, regard such an inference as non-logical because there are interpretations of "bachelor" and "unmarried" on which the argument is not truth-preserving.

[84]Priest, Graham. "Two Dogmas of Quineanism." *The Philosophical Quarterly* 29, no. 117 (1979): 289-301.

way — one can easily imagine a position which rejects the analytic/synthetic distinction as incoherent and identifies the theorems of propositional calculus or first-order logic with the logical truths. In any case, it is worth pointing out from the start that I have already addressed a great many of the popular explanations of logical consequence. In each case, including for Tarski's account, I have argued that the system in question fails to explain why the conclusion is a logical consequence of the premises, either because it helps itself to the notion of logical consequence that we are seeking to explain, or else because it generates a vicious regress of grounding analogous to the regress of the criterion. If one adopts an explanation similar to ones I have addressed, it will have to contend with those objections.

For Williamson, the way that we can draw the line between logical and non-logical truths is given by Tarski's model-theoretic account. But what explains why a logical truth is *true* is not simply the meanings of its logical vocabulary. Rather, logical truths are true for the same sorts of reasons as run-of-the-mill truths are true. And, as a consequence, Williamson maintains that logical truths need not have the modal properties that are often ascribed to them. For instance, he writes that sentences of the form "P iff actually P" are logically true (because they are true in all models when "iff" and "actually" are treated as logical constants) even though such sentences do not always express necessary truths — "it is not necessary for me to be my actual height".[85] The fact that Tarski's account has such consequences has been used by Etchemendy to argue that it does not capture our pre-theoretic notion of logical consequence, which Etchemendy claims *does* have certain modal properties. However, Williamson is keen to point out that our pre-theoretic conception of logical consequence need not be regarded as totally coherent — something that I, too, am happy to point out!

Before we proceed with Williamson's account, it is worth noting that Tarski's model-theoretic account is perfectly compatible with the view that logical truths are true by meaning alone. The logical truth of a sentence depends on the truth of a corresponding universal generalisation, and it does not matter whether this generalisation is true by meaning alone or not. However, since Tarski's account ranges over infinitely many interpretations of a sentence's non-logical vocabulary, one could argue that, in order to make use of Tarski's account, one must appeal only to the meanings of the logical vocabulary. For example, when we use Tarski's account to explain the logical truth of "$\forall x\ x = x$", we are not going around checking that, in fact, the coffee mug *is* identical with the coffee mug, the empty set *is* identical with the empty set, and so on. We could not even do this in principle, assuming that there are infinitely many objects. Rather, we look only to the meanings of "$\forall$" and "$=$" and conclude that, no matter how we interpret the non-logical constants or vary the domain of quantification, the resulting statement will be true. This seems to be a highly plausible picture for how a Tarskian comes to know which of the sentences their model-theoretic

[85] Williamson, *Philosophy of Philosophy*, p. 67.

account declares logically true.[86] But this picture is not available to Williamson, for whom it does not make sense to speak of a sentence like "the coffee mug is the coffee mug" as being true by meaning alone.

3.3.2 Vixens, Bachelors, and Logic, Oh My!

So, how does Williamson square his adoption of Tarski's model-theoretic account of logical consequence with his rejection of the analytic/synthetic distinction? He writes:

> In virtue of what are vixens female foxes? ... An appeal to semantic or other facts about the words "vixen," "female" and "fox" in answer to those questions would confuse use and mention. Vixens would have been female foxes no matter how we had used words. Presumably, vixens are female foxes in virtue of whatever female foxes are female foxes in virtue of.[87]

Williamson quotes early-Quine approvingly when Quine entertains the idea that the logical truth of "everything is self-identical" depends for its truth "on an obvious trait, viz., self-identity, of its subject matter, viz., everything."[88] On this picture, then, the truth of "the coffee mug = the coffee mug" depends not merely on the meaning of "=". Rather, it depends on the fact that the coffee mug *is*, in fact, identical with the coffee mug.

Although Williamson quotes the above passage with approval, Quine himself was entertaining the idea only in the course of showing it to be incoherent. Indeed, for Quine it is just as incoherent as the idea that "the coffee mug is coffee mug" is true by meaning alone. In the same breath, Quine goes on to say:

> I have been using the vaguely psychological word "obvious" non-technically, assigning it no explanatory value. My suggestion is merely that the linguistic doctrine of elementary logical truth likewise leaves explanation unbegun. I do not suggest that the linguistic doctrine is false and some doctrine of ultimate and inexplicable insight into the obvious traits of reality is true, but only that there is no real difference between these two pseudo-doctrines.[89]

I take Williamson's position to be the one which Quine lambasts as a "pseudo-doctrine", although Quine obviously did not have Williamson in mind in 1954. One way to couch Quine's objection is as follows: if it does not make sense to

[86] On Tarski's original construction of the model-theoretic account, the domain of quantification is not varied, so the synthetic-looking sentence "$\exists x \exists y\ x \neq y$" (which says, roughly, that there are at least two distinct things) turn out to be logically true if and only if there *are* at least two distinct things. Williamson would argue that sentences like this are not true by meaning alone, despite being logical truths. However, such sentences are not widely regarded as logical truths, not least because they fail to be theorems of first-order logic.

[87] Williamson, p. 61.

[88] Quine, "Carnap and Logical Truth", p. 69. Emphasis mine.

[89] Ibid. Emphasis mine.

speak of a sentence as an analytic truth, then it likewise does not make sense to speak of a sentence as a synthetic truth. The concept *synthetic truth*, just like the concept *analytic truth*, is incoherent. As Putnam argued in a passage quoted previously, it would be wrong to conclude from "Two Dogmas" that all grammatical sentences are either synthetic truths or synthetic falsehoods. Indeed, the above passage from Quine can be seen as evidence for that reading. But it seems that what Williamson is doing here is treating "everything is self-identical" as a synthetic truth, hence why Quine criticises the view as leaving explanation unbegun, as relying on inexplicable insight about reality, and as amounting to an appeal to analyticity.

Take a sentence with less logical flavour, e.g. "the coffee mug is on the desk". Williamson is keen to treat this sentence in precisely the same way as "the coffee mug is the coffee mug". Indeed, one of the great virtues of Williamson's account is the egalitarian manner in which he treats all sentences — he does not brutely divide sentences into distinct categories in order to treat them differently. For Williamson, "the coffee mug is on the desk" is true if and only if the coffee mug is on the desk, and, indeed, the coffee mug *is* on the desk; thus "the coffee mug is on the desk" is true. This, for Williamson, is strictly parallel to the reasons why "the coffee mug is the coffee mug" is true — "the coffee mug is the coffee mug" is true if and only if the coffee mug is the coffee mug, and, indeed, the coffee mug *is* the coffee mug; thus "the coffee mug is the coffee mug" is true. So, Williamson would implore, since it is obviously non-objectionable to use such reasoning in the case of the coffee mug being on the table, it is likewise non-objectionable to use it in the case of the coffee mug being the coffee mug.

3.3.3 Quine's Objection

But, once we reject the analytic/synthetic distinction as incoherent, is it *really* non-objection-able to use such reasoning in *either* case? I contend that Williamson is underestimating the destructive consequences of rejecting the analytic/synthetic distinction as incoherent. Quine and Putnam are right: it does not make sense to classify a sentence as synthetic. I hope to share Williamson's virtue of treating "the coffee mug is on the table" and "the coffee mug is the coffee mug" the same way — in *neither* case does it make sense to speak of the sentence as true or as false, as meaning and truth are not properties of individual sentences.

In Chapter 6 of *The Parmenidean Ascent*, Della Rocca gives a compelling argument as to why it does not make sense to speak of the meaning of a word or the meaning of a sentence once we have rejected the analytic/synthetic distinction as incoherent. I will elaborate on his argument below, which proceeds by way of a *reductio ad absurdum*:

> One might think that, if terms generally do have meaning, then there is always the possibility that terms can be combined in such a way that the resulting statement is analytically true, i.e. true simply by virtue of these meanings. But if there are no analytic statements

> and no analytic-synthetic distinction, then it seems that terms and statements do not genuinely have meaning.[90]

This line of reasoning is what inspired the Light Inferentialism of the previous section. Just as, if a sentence has meaning, then it enters into logical relationships, so too, if a word has meaning, then it can enter into analytic relationships. If "vixen" means "female fox", then we can combine these terms in such a way as to generate a statement that is true by meaning alone, e.g. "vixens are foxes". But if it does not make sense to speak of a sentence as true by meaning alone, then it likewise does not make sense to speak of the meaning of a word, e.g. "vixen". Or, as Della Rocca stresses, "there is no way to speak meaningfully of *the* meaning of the terms".[91] The same goes for sentences — a sentence has meaning, then it can enter into analytic consequence relations by virtue of what it means. But if it does not make sense to speak of one sentence as an analytic consequence of another, then it likewise does not make sense to speak of *the* meaning of a sentence. And if it does not make sense to speak of the meaning of a sentence, then it likewise does not make sense to speak of a sentence being true or false, since a sentence is true or false partially due to what it means.

But what of Williamson's synthetic treatment of each sentence? What prevents us from gathering together all the vixens and confirming that, in fact, they *are* all foxes? The worry here is very similar to the worry that analytic conceptions of logical consequence, such as Etchemendy's, face due to Caroll's regress. There, the worry was: why is the meaning of "vixen" such that if something is a vixen, then it is a fox? Here the worry is: why does the word "vixen" pick out the creatures you have just gathered, as opposed to some other collection of creatures? This ties into Williamson's epistemicist view of vagueness: there is a fact of the matter about whether a creature is a vixen, we just do not know one way or another in borderline cases. But the worry about arbitrariness remains unanswered: the fact that a creature is a vixen, i.e. falls under the extension of the word "vixen", is taken for granted and not explained. The epistemicist view of vagueness says that every predicate has sharp boundaries, but it does not explain why a given predicate has the extension that it does. And, as a result, the doctrine "leaves explanation unbegun".[92]

Let's take stock: the fact that there is no explanation, on Williamson's account, for why a term has the extension that it does is why Quine criticises it for being a "doctrine of ultimate and inexplicable insight".[93] Thus, appealing to linguistic meaning (Etchemendy) and appealing to empirical facts (Williamson) both fail to explain why a sentence is true, and this is why I see Quine saying that there is "no real difference between these two pseudo-doctrines."[94] In both cases, there is ultimately no explanation as to why a *sentence*, e.g. "vixens are foxes", is what is true or false. And, as a result, Quine rejects that a sentence

[90] Della Rocca, *The Parmenidean Ascent*, p. 172.
[91] Ibid.
[92] Quine, "Carnap and Logical Truth", p. 69.
[93] Ibid. "Carnap and Logical Truth", p. 69.
[94] Ibid.

is what is true or false — the unit of significance, and thus of truth or falsity, is the whole of science.

3.4 Saving a Dogma

What is Quine left with after rejecting the analytic/synthetic distinction as incoherent? In "Two Dogmas of Empiricism", he variously describes our scientific theory as "a man-made fabric which impinges on experience only along the edges" and as "a field of force whose boundary conditions are experience."[95] These metaphors get at the point that we can accommodate an observation in our scientific theory in many different ways, so long as we make "compensatory adjustments" within the theory as a whole.[96] But once we reject the analytic/synthetic distinction, what compensatory adjustments are called for, if any, by a change in the theory?

Suppose we expel a sentence S from our theory and introduce $\neg S$ in its stead. On one picture — call it the *web of belief picture* — while we may still hold $S \vee \neg S$ true, we must, for example, stop holding $S \wedge T$ true (where T is some other sentence of the theory). This is because the switch from S to $\neg S$ in the theory calls for us to hold true those sentences which are logical consequences of $\neg S$ and hold false those sentences which contradict $\neg S$. As Quine puts it:

> A conflict with experience at the periphery occasions readjustments in the interior of the field. Truth values have to be redistributed over some of our statements. Reëvaluation of some statements entails reëvaluation of others, because of their logical interconnections. . . [97]

The key words in the web of belief picture that call out for explanation are "calls for", "occasions", and "have to". This is because, as it stands, the picture Quine paints above misses Quine's own point that *any* sentence can be held true come what may. In particular, we can hold that (A) and (B) are true, but that (Z) is false. Indeed, we can hold that (A) and (B) *and (C)* are true, but that (Z) is false. And so on. Luckily, though, Quine realises this in the very same sentence:

> Reëvaluation of some statements entails reëvaluation of others, because of their logical interconnections — the logical laws being in turn further elements of the field. Having reëvaluated one statement we must reëvaluate some others, which may be statements logically connected with the first or may be the statements of logical connections themselves.[98]

Even here, though, there is some evidence that Quine is conflicted. On the one hand, Quine suggests that logical laws (e.g. modus ponens) can be abandoned, as claims about their validity are simply elements in the web of belief. They

[95] Ibid. "Two Dogmas of Empiricism". p. 50.
[96] Ibid. "Ontological Relativity", *The Journal of Philosophy* 65, no. 7 (1968): 190.
[97] Ibid. "Two Dogmas of Empiricism". p. 50.
[98] Ibid. Emphasis mine.

are sentences in the theory like any other; they do not enjoy a privileged status, and do not need to be held as true come what may. Call this the *egalitarian picture*. On the other hand, Quine says that "having reëvaluated one statement, we must reëvaluate some others", which might suggest the web of belief picture in which we cannot hold some sentences as true, e.g. (A) and (B), without also holding other sentences as true, e.g. (Z).[99] The logical relationships between sentences in the theory *must* be obeyed (come what may...), such that if we expel S from our theory and introduce $\neg S$ in its stead, we *must* also accept all the logical consequences of $\neg S$.

In "Two Dogmas of Quineanism", Graham Priest points out these competing strands in Quine's thought, and, in an attempt to salvage the intelligibility of speaking of *the* meaning of a term, argues that Quine must not do away with the analytic/synthetic distinction. For Priest, in order to preserve the web of belief picture in which sentences have meaning and in which there are logical relationships between sentences, Quine must hold onto the notion of analyticity and say that there are logical sentences which one must hold true come what may. Essentially, Priest argues that making sense of logical relationships between sentences requires making sense of the analytic/synthetic distinction. And I could not agree more. Priest and I agree on all the facts: on a Quinean holist picture, one can hold (A) and (B) as true, but (Z) as false. Indeed, on such a picture, one can hold (A), (B), (C), etc. as true but (Z) as false. And so on. Where we differ, though, is that Priest takes these facts to show that there is a distinction between statements and rules of inference that Quine does not heed (see §2.2). Indeed, he laments the fact that, as he sees it, Quine is aware of Carroll's Regress in "Truth by Convention" (see §2.4) but fails to see its bite in "Two Dogmas of Empiricism".

I agree with Priest that there is no room for logical necessity on my preferred, egalitarian reading of Quine. If we take Quine's rejection of the analytic/synthetic distinction seriously, then it does not make sense to say that "having reëvaluated one statement we must reëvaluate some others" — Quine's come-what-may argument and Carroll's regress each show that there is no coherent way to characterise this "must". Thus, Priest is perfectly right to say that we cannot hold onto the web of belief picture of Quine's holism once we reject the analytic/synthetic distinction as incoherent. On the web of belief picture, as Priest notes, "beliefs may provide the content of the web of belief but the rules [of inference] provide the structure."[100] However, when we reject the analytic/synthetic distinction and treat all sentences the same, there is no room for a content/structure distinction. Such a distinction relies on the idea that, given what a sentence S means, if we hold S true then there are other sentences that we must hold true come what may. But this is precisely what Quine rejects as inexplicable, for (A), (B), *and* $\neg$(Z) can all be held true come what may.

Given the close connection between rejecting the analytic/synthetic distinc-

[99] Again, the Tortoise might ask, what is meant by this "must" and this "cannot"?
[100] Priest, pp. 291-2.

tion and rejecting the logical/non-logical distinction, I hope to show that Quine did not simply forget about Carroll's regress between these two landmark papers. On my reading, Quine's reasons for rejecting conventionalism about logic are wholly compatible with, and reinforced by, his reasons for rejecting the analytic/synthetic distinction.

In "Truth by Convention", Carroll's regress shows that, in positing logical rules by fiat, one is involved in an infinite regress — to explain why an argument is logically valid using the fact that it is an instance of a given convention, we must appeal to another linguistic convention, and so on *ad infinitum* (see §2.4). Ultimately, we never arrive at a non-circular explanation for why (Z) is a logical consequence of (A) and (B), and thus the very concept of logical consequence is never explained by the conventionalist account. In "Two Dogmas of Empiricism", Quine (on my preferred, egalitarian reading) shows that the analytic/synthetic distinction is incoherent, which is just to say that the logical/non-logical distinction is incoherent (as I argued in §3.1). To put it another way: Quine used Carroll's regress to argue against *one* way of establishing the logical/non-logical distinction (namely, by linguistic convention), but Carroll's regress can be applied to *any* way of establishing that distinction, of rendering that distinction intelligible by explaining why it obtains. Thus, Quine's reasoning can also be used more generally to argue against the coherence of the logical/non-logical distinction, and thus the analytic/synthetic distinction.

This is one way to think about how Quine's argument actually proceeds. On a broadly analytic account of logical consequence, such as Etchemendy's, it is a logical consequence of the fact that some animal is a vixen that that animal is a fox. Quine demands an explanation as to why the meaning of "vixen" is such that "vixens are foxes" must be held true come what may, and he *rejects* that it must because there is no explanation to be found. There is no principled way to say that the extension of the word "vixen" is one collection of objects rather than another — indeed, this is the kind of argument for which Quine invokes the "law of sufficient reason" explicitly. I am suggesting that Quine, for highly analogous reasons, is committed to rejecting the notion of logical consequence as well, and in his more egalitarian moods, he *does*: he demands an explanation as to why the meanings of "$\wedge$", "$\vee$", and "$\rightarrow$" are such that the distributive law must be held true come what may, and *rejects* that it must because there is no explanation to be found.[101] The privileged status of logical sentences is incompatible with Quine's explanatory demands about meaning.

Of course, there are times when Quine finds himself in less egalitarian moods, especially in his later writings. In "Two Dogmas in Retrospect", for example, Quine admits: "I give logic a special status: logical implication is the link between theory and experiment."[102] Similarly, he at times suggests that we *must* interpret other speakers as operating with the same logical rules as we

[101]The distributive law is "$[P\wedge(Q\vee R)]\rightarrow[(P\wedge Q)\vee(P\wedge R)]$. Quine uses it as an example in "Two Dogmas of Empiricism" in the course of arguing that observations about quantum mechanics may cause us to alter which logical sentences we hold true.

[102]Quine, "Two Dogmas in Retrospect". In *Quintessence*, Harvard University Press, 2008. p. 58.

are, and that "disagreement as to a logical truth is evidence of deviation in usage (or meanings) of words."[103] What I hope to highlight, though, is that, in making such commitments to the special status of logical truths and to talk of *the* meanings of logical words, Quine is not really rejecting the analytic/synthetic distinction as incoherent.

3.5 Carroll's Diagnosis

Rejecting the analytic/synthetic distinction as incoherent amounts to rejecting the logical/non-logical distinction as incoherent. In both cases, it does not make sense to speak of *the* meaning of a word or *the* meaning of a sentence. Carroll's regress thus poses a threat to talk of the meanings and referents of terms, as well as to talk of the truth or falsity of sentences. Here, I would like to point out a line of thought in Carroll's own writing which suggests that he was well aware of the connections between meaning and logical consequence. Although Carroll himself did not, in all likelihood, have a clear moral in mind when he wrote his story — as we will see in §4.3, he does not draw any conclusions about what to make of his paradox — it certainly occurred to him that the very concept of a proposition takes centre stage.

Carroll's story was published in the philosophy quarterly *Mind* in April of 1895, almost a year after he submitted it in August of 1894. In the interim, Carroll engaged in a brief correspondence with G. F. Stout, the publication's editor at the time. Stout wrote to him on August 24th, 1894 to express his confusion with Carroll's dialogue, since he mistakenly believed it turned on a difference "between affirming A and affirming *the truth* of A."[104] Carroll replied the next day to clarify that "of course affirming a Proposition, & affirming that the Proposition is *true*, are the same thing,"[105] and he proceeded to give his own diagnosis of the regress. He writes:

> My paradox does *not* attempt to draw any distinction between these two processes: it turns on the fact that, in a Hypothetical, the *truth* of the Protasis, the *truth* of the Apodosis, & the *validity of the sequence*, are *3* distinct Propositions.[106]

Here, the "Protasis" is the conjunction of premises and the "Apodosis" is the conclusion. Carroll's point, in effect, is that the regress occurs because of the assumption that the premise (A)∧(B), the conclusion (Z), and the hypotheticals (C), (D), ... all express distinct propositions.

I make sense of Carroll's concern with propositions as follows: under the assumption that these sentences express distinct propositions, an explanation is needed as to why the conclusion *must* be true if the premises are true. After all,

[103]Ibid. "Carnap and Logical Truth". p. 68.
[104]Stout, G. F. "Letter 1: From Stout to Carroll, 24 August 1984." *The Carrollian* 28 (2016). p. 9. Emphasis in original.
[105]Carroll, Lewis. "Letter 2: From Carroll to Stout, 25 August 1894." p. 10. Emphasis in original.
[106]Ibid. Emphasis in original.

Carroll says, the conclusion (Z) means something different than (A)∧(B). But when we try to explain, as Achilles does, why this relationship of logical consequence holds between distinct propositions, we are led on an infinite regress. To speak of propositions in this fine-grained way is to introduce a distinction between the meaning of the premises and the meaning of the conclusion, yet the logical consequence relation seeks to create a tight-knit bond between sentences with these disparate meanings: the conclusion *must* be true if the premises are true. But because this claim, (C), in turn expresses a distinct proposition (as Carroll himself notes), we are led on an infinite regress when it comes to explaining why the conclusion *must* be true if (A), (B), and (C) are true.

Read in this way, there is a strand of Carroll's thought that may be more Bradleyan and Quinean than most interpreters would like to admit. For if the Protasis, Apodosis, and validity of the sequence are *not* distinct propositions — indeed, if it does not make sense to speak of propositions, to speak of *sentences* being true or false — then we are not faced with Carroll's regress.[107] Indeed, rejecting assumptions about meaning that underlie Carroll's regress is precisely what I take Quine to be doing. Carroll's regress turns on the assumption that the premises and conclusion express distinct propositions, but the idea of a proposition is rejected as incoherent when we reject the analytic/synthetic distinction as incoherent.

In sum, I see Carroll to be making the observation that his regress turns on a commitment to propositions. In effect, this is Carroll's way of recognising that the analytic/synthetic distinction and the logical/non-logical distinction go hand in hand — to reject the one as incoherent is to reject the other as incoherent (see §3.1). Although he sees that his regress turns on the assumption that, as Quine would say, the unit of significance is the sentence, he does not draw any conclusions one way or the other about what to make of this fact. That is, he does not say that his regress is benign, and that the premises (A) and (B) and (Z) do express propositions. Nor does say that his regress is vicious, and that it does not make sense to speak of *the* meanings of (A), (B), and (Z). Indeed, after presenting the regress in both the story and the letter to the editor of *Mind*, he draws no conclusions about it. I will argue in §4.3 that this is fitting for a story in which the ability to draw conclusions is called into question.

3.6 Propositions and Salt: What's the Grain?

This leads me to a potential response to Carroll's regress, one that Carroll himself certainly did not have in mind. Carroll's insight is that his regress turns on the fact that (A), (B), (C), ... and (Z) express distinct propositions. However, one could argue that this claim relies on a fine-grained account of propositions that ought to be rejected. Consider, instead, a coarse-grained view of propositions like the ones defended by Williamson and David Lewis.[108] Roughly, on such an account, the proposition expressed by a sentence is identified with a set

[107] What we *are* faced with will be the subject of §4.

[108] Lewis, David. "General Semantics." In *Montague Grammar*, Academic Press, 1976. pp. 1-50.

of possible worlds, namely the possible worlds in which the sentence is true. For example, the proposition expressed by the sentence "it is always cold on Thursday" is the set of all possible worlds in which it is always cold on Thursday.

A coarse-grained account of propositions can be leveraged into a criticism of Carroll in the following way: Carroll's assumption that the proposition expressed by the conjunction of the premises and the proposition expressed by the conclusion are distinct is not always true. There may be cases where they are, in fact, one and the same proposition — cases where the possible worlds in which the premises are true are the very same possible worlds in which the conclusion is true. Such is the case when, for example, the premises and conclusion are both necessary truths, meaning they both hold true in *every* possible world.

One problem here, though, is that, even on this course-grained view of propositions, the conclusion and the conjunction of the premises do not always express one and the same proposition. This is because, for proponents of the coarse-grained view, all that is required is that the proposition expressed by the sentence (A)∧(B) is a subset of the proposition expressed by (Z). In other words, there is no possible world in which the premises (A) and (B) are true but the conclusion (Z) is false, but there are perhaps worlds in which (Z) is true but (A)∧(B) is false. To deny this is akin to affirming the consequent.

However, this is not the main problem for the advocate of coarse-grained propositions. The question we must ask is: what is the precise sense of "possible" that we are using to characterise these possible worlds? If we are using *logical possibility*, then our account of the meaning of a sentence will only be as accurate as our assumptions about which worlds are logically possible. But, as discussed in §2.5, the question of whether there is a possible world in which (A) and (B) are true but (Z) is false is precisely the question at hand. To say that (Z) is a logical consequence of (A)∧(B) because every logically possible world in which (A)∧(B) is true is a world in which (Z) is also true would be hopelessly circular (see also §1.3).

What happens, then, if we characterise possibility in some non-logical way? There are two potential paths forwards, here. First, one could characterise possibility in a way that is *broader* than logical possibility. Recent literature about Descartes has suggested that, for him, the laws of logic may only be *contingently* necessary because God could have willed that the laws of logic be different (but still necessary).[109] This is not the place to discuss Descartes' view in depth, but it suffices to say that if this broader kind of necessity (something like *divine necessity*) is what supporters of coarse-grained propositions have in mind, then there is no reason to think that (Z) must be true if (A) and (B) are true. Just as God could have "brought it about from eternity that it was not true that twice four make eight" and that contradictories were true together, so too could he have brought it about from eternity that (A) and (B) were true and (Z) false.[110]

[109]Conant, James. "The Search for Logically Alien Thought: Descartes, Kant, Frege, and the Tractatus". In *The Logical Alien: Conant and His Critics.* Harvard University Press, 2020. pp. 31-43.

[110]Descartes, René. *The Philosophical Writings of Descartes.* Cambridge University Press,

The second, more intuitive path forwards for the defender of coarse-grained propositions is to characterise possibility in some way that is *narrower* than logical possibility. Take *physical possibility* as an example — to put it crudely, a world is physically possible if it obeys the same laws of physics as the actual world. But to do this is, in effect, to generate Carroll's regress. For now an explanation is wanted as to why the truth of

> **(Physical-C)**: every physically possible world in which (A) and (B) are true is a world in which (Z) is true

grounds the fact that (Z) is a logical consequence of (A) and (B). In other words, why does physical possibility have any bearing on logical possibility? In this way, it is clear that little progress is made by appealing to coarse-grained propositions. The problem is not that we are treating propositions more finely than they ought to be treated, but rather that we are holding on to the very notion of a proposition.

3.7 A Hitherto Unknown Kind of Madness

I have argued that Quine is at his strongest when, like Williamson, he does not introduce an arbitrary division in treatment between so-called "logical" and "non-logical" sentences — when he treats all sentences of the theory the same, as in the egalitarian picture. Just as "there is a male vixen in the field" can be held as true come what may, so too can "(A) and (B) are true, but (Z) is false", "(A) and (B) and (C) are true, but (Z) is false", etc. This is because the reasoning in Quine's come-what-may argument does not seem to call for an exception to be carved out for the "logical" sentences. As Quine notes elsewhere:

> It is only the objective of ending up with our mother tongue that dissuades us from marking both '- - -' and '¬ - - -,' and this objective would dissuade us also from from marking 'It is always cold on Thursday'.[111]

Here, Quine's egalitarian treatment of sentences is clear as day. But just as we *can* hold "It is always cold on Thursday" true, (and, in so doing, deviate from how English speakers ordinarily speak), so too can we hold true $S \wedge \neg S$. Thus, it does not make sense to speak of *the* meaning of "¬" or *the* meaning of "∧" (perhaps as given by the standard truth-table semantics). If it did, the thought goes, then we *would* have to hold $S \wedge \neg S$ false come what may.

Famously, though, many have called into question the coherence of denying a logical truth. Kant writes that logic "contains the absolutely necessary rules of thinking, without which no use of the understanding takes place."[112] James Conant makes sense of this by arguing that, for Kant, the principles of logic

1984. 2:294.

[111] Quine, "Truth by Convention", p. 21.

[112] Kant, Immanuel, Paul Guyer, and Allen W. Wood. *Critique of Pure Reason*. 1992. Cambridge: Cambridge University Press, 1998. A52/B76.

are "constitutive of the possibility of thought",[113] by which he means that "the ostensible idea of their failing to hold in fact represents no intelligible possibility whatsoever, but the mere illusion of a thought."[114] Frege, following Kant, calls it "a hitherto unknown kind of madness" to suppose that there could be beings "whose laws of thought directly contradicted ours".[115] Wittgenstein, following Frege, says in the *Tractatus* that "It is as impossible to represent in language anything that 'contradicts logic' as it is in geometry to represent by its coordinates a figure that contradicts the laws of space, or to give the coordinates of a point that does not exist."[116] And Putnam, also following Frege, argues against Quine explicitly that there is at least one *a priori* truth, namely that not every statement is both true and false.[117] He later claims that he "cannot even imagine *finding out* that" this claim is false.[118] In this way, each of these thinkers calls into question the coherence of of a view which even so much as entertains the idea that (A) and (B) could be true while (Z) is false.

Without focusing too much on the important differences between these four landmark views in the philosophy of logic, at first blush there is something extremely plausible underlying each of them: it just does not make sense to deny a logical truth, or to hold true a logical falsehood. What thought would one even be having, in such a purported case? However, I will argue that Quine is able to account for this incoherence. Given the extremely close relationship between logic and meaning, and given the extremely close relationship between meaning and thought, mental representations, aboutness, etc., it should come as no surprise that Quine's criticisms of meaning spill over into both the possibility of drawing a logical inference and the possibility of forming a coherent thought, or as Kant would say, of *understanding*.[119] Logic may be constitutive of the possibility of understanding, but this just means that the very idea of understanding — of forming a coherent thought which stands in logical consequence relations to other thoughts — is itself incoherent.

3.7.1 Against Determinacy

The best way to see this is by investigating Quine's deeply skeptical view on propositional attitudes. On my reading, the best way to make sense of this view is as stemming from his deeply skeptical view on propositions. As Quine

[113]Conant, p. 57.

[114]Boyle, Matthew. "Kant on Logic and the Laws of the Understanding". In *The Logical Alien: Conant and His Critics.* Harvard University Press, 2020. p. 118.

[115]Frege, Gottlob. *Basic Laws of Arithmetic. Derived Using Concept-Script* (1893). Ed. and trans. Philip Ebert, Marcus Rossberg and Crispin Wright. Oxford: Oxford University Press, 2013. p. xvi.

[116]Wittgenstein, Ludwig, David Pears, and Brian. McGuinness. *Tractatus Logico-Philosophicus.* London: Routledge, 2001. 3.032.

[117]Putnam, Hilary. "There is At Least One A Priori Truth." *Erkenntnis* (1978): 155. Emphasis mine.

[118]Putnam, "Rethinking Mathematical Necessity". p. 250. Emphasis mine.

[119]Watkins, Eric. "Kant on the Distinction between Sensibility and Understanding." In *Kant's Critique of Pure Reason: A Critical Guide.* Cambridge: Cambridge University Press, 2017. p. 9.

makes clear in *Word and Object* and "Ontological Relativity", once we reject the analytic/synthetic distinction as incoherent, we are left with many ways to attribute propositional attitudes to others that are equally compatible with their behaviour.[120] The famous example has to do with rabbits, undetached rabbit parts, and rabbit time-slices — which of these best approximates the meaning of "gavagai"? — but I will not rehearse the entire argument here. The important part is: because there is no reason to prefer one translation over others (as any translation can be ascribed to the speaker come what may, by making suitable adjustments to the other meanings and propositional attitudes we ascribe to them), we are left with an indeterminacy of meaning. It is not that the speaker has a determinate thought, but we just do not know which one it is — whether their thought is about a rabbit or about many undetached rabbit parts, for example. That is, for Quine, the problem is not merely one of ignorance. Rather, there *is no* fact of the matter about what they think: "the inscrutability of reference is not the inscrutability of a fact; there is no fact of the matter."[121]

Quine is fully aware of the consequences of his argument, concluding that, because there is no fact of the matter about what the meanings of others' words, there is likewise no fact of the matter about the meanings of one's own words.[122] On my view, this marks a high point in Quine's philosophy, on a par with §6 of "Two Dogmas of Empiricism". Through his commitment to explanation, Quine knocks down, in quick succession, the determinacy of meaning and the determinacy of reference, and he does so for both oneself and for others. The kicker is: all of this was brought on by his rejection of the analytic/synthetic distinction, a rejection which, itself, was motivated by an explanatory demand about meaning — why is the extension of a predicate one collection of objects, rather than another?

A corollary of this is that there is no fact of the matter about what anyone's propositional attitudes are. If there is no fact of the matter about what "gavagai" means, and thus no fact of the matter about what "gavagai" refers to, then there is ultimately no fact of the matter about what the speaker's beliefs, wishes, fears, and so on *are*. Do they believe that there are undetached rabbit parts nearby, or do they believe that there are rabbit stages nearby? There is no fact of the matter. And, following Quine, if there is no fact of the matter about what someone else's propositional attitudes are, then there is likewise no fact of the matter about what one's own propositional attitudes are, as there is nothing to explain such a difference. In sum, if it does not make sense to speak of propositions, then it likewise does not make sense to speak of propositional attitudes.

But there is another way to see how rejecting the analytic/synthetic distinction as incoherent leads to the indeterminacy of propositional attitudes. In Quine's argument for the indeterminacy of translation, the reason why we can interpret a speaker's utterances in many different ways is because we can

[120] Quine. *Word and Object.* Massachusetts Institute of Technology, 1960. Ch. 2.
[121] Quine, "Ontological Relativity". pp. 199-200.
[122] Ibid.

make compensatory adjustments to the propositional attitudes that we ascribe to them. So, just as the this accommodating process yields many different *meanings* that are equally compatible with their behaviour, so too does it yield many different *propositional attitudes* that are equally compatible with their behaviour.[123] There being no reason to prefer one of these to any other, Quine would conclude here, too, that there is no fact of the matter about the content of someone's beliefs.

3.7.2 Against Thought

All of this indeterminacy precipitates incoherence. Indeed, as shown in the paragraph above, the come-what-may reasoning that leads to the conclusion that there is no fact of the matter about what a word means directly mirrors Quine's reasoning in §6 of "Two Dogmas of Empiricism" for the conclusion that it does not even make sense to speak of *the* meaning of a term. If it makes sense to speak of the meaning of "gavagai", then there is some fact of the matter about what "gavagai" means, and, moreover, we can use this fact of the matter to construct a sentence which is true by virtue of its meaning alone. Likewise, if it makes sense to speak of *the* belief of a speaker, then there is some fact of the matter about what the content of that belief is. But this is precisely what Quine must give up. That is, the possibility of forming a coherent thought is what we must give up when we give up the analytic/synthetic distinction. Indeed, to reject the analytic/synthetic distinction as incoherent is to reject the very notion of *a coherent thought* as incoherent, as it relies a notion of mental content which cannot be explained.

Quine considers propositional attitudes explicitly in "Quantifiers and Propositional Attitudes", where his main concern is not with indeterminacy, but with another problem that has come to be known as Frege's Puzzle.[124] As with gavagai, you already know the drill: Hesperus is Phosphorus, but if Fred is ignorant of this fact, then it seems natural to say both that Fred believes that Hesperus does not rise in the morning *and* that Fred believes that Phosphorus does rise in the morning. Because Hesperus is Phosphorus, we are forced to either assign Fred contradictory beliefs (because he believes that Phosphorus both rises and does not rise in the morning) or else come up with some other story about Fred's propositional attitudes. Many would like to avoid the first option, as it seems odd to say that Fred believes that Hesperus rises in the morning — if you asked him "Does Hesperus rise in the morning" he would say "No, but Phosphorus does".

Quine correctly points out that Frege's puzzle turns on assumptions about meanings and beliefs that Quine himself rejects, namely that it makes to speak of *the* intension of a sentence and *the* belief of a speaker. But, in this paper, he tries to salvage something that is sort of similar to propositional attitudes

[123] For a discussion of the close-knit relationship between meanings and attitudes, see "Belief and the Basis of Meaning" and "Radical Interpretation" by Davidson.

[124] Quine, "Quantifiers and Propositional Attitudes". In *Quintessence*, Harvard University Press, 2008.

— something that tells us something about the contents of the representational mental states of others — but which has the benefit of "dodging the intensions".[125] Quine replaces *believing a proposition* with *believing-true a sentence.*

I contend that Quine's solution to Frege's puzzle does not salvage the coherence of mental representations. Quine rightly points out that

> if anyone does approve of speaking of belief of a proposition at all and of speaking of a proposition in turn as meant by a sentence, then certainly he cannot object to our semantical reformulation 'w believes-true S' on any special grounds of obscurity; for, 'w believes-true S' is explicitly definable in his terms as 'w believes the proposition meant by S.'[126]

However, this does not in turn make Quine's extensionalist account successful — it only shows that his extensionalist account of meaning is no more obscure than intensionalist accounts. One problem for Quine, which has been discussed often, is that, without appealing to the meaning of a sentence, we are in no position to know whether some speaker holds it true. This is true especially if they speak a different language, or no language at all (as in Quine's own example of a mouse, which might believe-true the sentence "cheese is tasty").[127] To appeal to the meaning of a sentence would be to rely on the intensions that Quine derides as "creatures of darkness".[128] This is an epistemic problem concerning our ability to know which sentences a speaker does, in fact, hold true.

There is a more drastic problem, though, which is that it does not even make sense to speak of a sentence being held true once we reject the analytic/synthetic distinction. This was discussed at length in §3.2, but another way to see this is via the familiar principle that intension determines extension: it is (at least in part) due to what a sentence means that it is true or else false. For sentences, to reject intensions as incoherent is just to reject extensions as incoherent, but the extension of a grammatical declarative sentence are, on this picture, either truth or falsity. Thus, to reject that sentences have an intension — indeed, to reject the very idea of an intension as incoherent — is to reject that that sentences are what is true or false — indeed, to reject the very idea of a sentence's being true or false as incoherent.

In Quine's defence, it might be thought that, when a speaker believes-true a sentence, what they are really doing is declaring that the sentence in question is one of the many sentences that will compose their entire theory. So, even though the sentence in question is not strictly speaking *true*, it is part of a collection of sentences — a *theory* — which is true. For example, if a speaker believes-true "what goes up must come down", this is to be understood as part of a larger theory which includes, among other things, equations for the laws of gravity. The problem here, though, is the indeterminacy that is part and parcel of rejecting the analytic/synthetic distinction. Indeed, one way to see

[125] Ibid. p. 346.
[126] Ibid. p. 347.
[127] For a similar debate, see "Reply to Foster" by Davidson.
[128] Ibid. p. 341.

the problem of the "gavagai" case is as a problem of deciding precisely which sentence is part of the foreign speaker's theory — does their theory include the sentence "Lo, a rabbit!" or the sentence "Undetached rabbit parts are near"? Quine does not say that one of these sentences is in the theory, but we just do not know which — his concern is not merely the epistemic problem of the previous paragraph. Rather, Quine says that there is *no fact of the matter* about which sentence is in the theory. And this is devastating, not only in the translation case, but in the homophonic case as well — in both cases, the theory itself is indeterminate.

It turns out, then, that Quine cannot even help himself to talk of theories being true or false, either. For it might be thought that the truth of a theory depends on facts about its constituent claims — a theory, understood as some (potentially infinite) collection of sentences, is true if its sentences, taken as a corporate body, are true.[129] But if there is no fact of the matter about what the sentences of a theory *are*, then we cannot use facts about a theory's sentences (e.g. what their truth-values are, or what the relationships *between* different sentences are) to draw conclusions about whether the theory as a whole is true. Speaking properly, then, it is wrong to say, as Quine does, that the unit of significance is "the whole of science."[130] It would be more proper to say that there is no unit of significance.

3.8 The Upshot

To take stock, §3 has sought to establish an equivalency between two views, namely the view that the analytic/synthetic distinction is incoherent and the view that the logical/non-logical distinction is incoherent. We have considered Williamson's objection to this equivalency, and rejected it on the grounds that Williamson does not really *give up* the analytic/synthetic distinction, does not really reject it as *incoherent* like Quine sometimes does. Rather, for Williamson, all statements are synthetic, as their truth depends on a combination of meaning and fact. As a result, he disagrees with Quine's view that it does not make sense to speak of a *sentence* as true or as false. On this disagreement, I have argued that Quine's arguments demonstrate a deeper commitment to explanation.

Quine's rejection of the analytic/synthetic distinction as incoherent leads him to reject the very notion of a proposition as incoherent. If it made sense to speak of *the* meaning of a sentence, then it would make sense to speak of analytic truth. Similarly, a rejection of the logical/non-logical distinction as incoherent leads one to reject the very notion of a proposition as incoherent. If it made sense to speak of *the* meaning of a sentence, then it would make sense to speak of logical truth, as a sentence would enter into logical consequence relations by virtue of what it means. Thus, Priest is totally correct to say that the web-of-belief view of meaning holism, in which there are logical consequence

[129] Whether they are true by virtue of *cohering together*, or else by virtue of *corresponding to the world*, is an important question. There is some evidence that Quine prefers the former picture, but both pictures are jeopardised by indeterminacy.

[130] Ibid. "Two Dogmas of Empiricism". p. 50.

relations between sentences, is incompatible with Quine's rejection of the analytic/synthetic distinction. Although Quine teeter-totters between egalitarian and web-of-belief views of meaning holism, there is still evidence that, in his more explanation-driven strands of thought, he sees the tight connection between rejecting the analytic/synthetic distinction as incoherent and rejecting logical consequence as incoherent:

> Commonly logical equivalence is adopted as the principle of individuation of intensions. ... But the relevant concept of logical equivalence raises serious questions in turn.[131]

In the passage above, he cites "Two Dogmas of Empiricism" and "Carnap and Logical Truth" explicitly. And, in the latter paper, he explicitly invokes Carroll's regress.

Carroll himself is totally correct to say that his regress turns on the assumption that (A), (B), and (Z) express different propositions. But if it does not make sense to talk about propositions, as Quine suggests in both his earlier and later work,[132] I have argued that it likewise does not make sense to speak of propositional attitudes, or of the content of a representational mental state. The very notions of *meaning*, *reference*, and *aboutness* are to be rejected as incoherent. Thus, Quine is left with something like F. H. Bradley's relation-less monism with respect to meaning; he is left with nihilism about meaning.

4 Incoherence and Explanation

> The executioner's argument was, that you couldn't cut off a head unless there was a body to cut it off from: that he had never had to do such a thing before, and he wasn't going to begin at *his* time of life.
>
> The King's argument was, that anything that had a head could be beheaded, and that you weren't to talk nonsense.
>
> The Queen's argument was, that if something wasn't done about it in less than no time she'd have everybody executed, all round.
>
> Lewis Carroll, *Alice's Adventures in Wonderland*, Chapter VIII.

I know what you've been thinking. (I've been thinking it, too). There must have been times (like just now, or perhaps all throughout §1) when you were just about ready to jump out of your seat or grab me by the collar. "Come on!" you must saying. "Isn't it clear that all of this is completely self-undermining? You're using logic in the course of arguing for the conclusion that logic is incoherent. The skepticism is self-defeating on its face, and we shouldn't pay it any mind."

[131] Quine, "Quantifiers and Propositional Attitudes", p. 346.

[132] Quine, *Philosophy of Logic*. 2nd ed. Harvard University Press, 1986. §1.1 is titled "Objection to propositions".

Bear with me. It will prove fruitful to see in precisely what ways Carroll's regress is self-undermining. And besides, if we're to draw any conclusions, the time to do so is at the end of our investigation, not the beginning. As Sextus Empiricus (whose regress, as we have seen, bears more than a passing resemblance to Carroll's and Bradley's) says, "the Sceptics keep on searching".[133]

4.1 Self-Refutation?

J. L. Mackie provides a formal account of self-refutation.[134] In it, he distinguishes between several ways in which an argument can be self-undermining. The first is *pragmatic* self-refutation. For example, if I write "I am not writing anything", then the sentence, read literally, must be false as it is being written. As Mackie notes, it is not the proposition that is self-refuting, as "at another time I may well not be writing", so the proposition may very often be true.[135] Rather, it is the actual operation, i.e. my writing that I am not writing, which is self-refuting. This is not what concerns us here; more relevant for our purposes is what Mackie calls *absolute* self-refutation. Given a truth-entailing operation, e.g. "it is true that" or "it is provable that", we can construct a sentence that *must* be false:

> For example, if it can be proved that nothing can be proved, then nothing can be proved; but equally if it can be proved that nothing can be proved, it is not the case that nothing can be proved; therefore it cannot be the case that it can be proved that nothing can be proved.[136]

There is a very natural way to use Carroll's regress to construct an absolutely self-refuting argument. It goes like this, by way of modus ponens:

> (P1): If an argument is logically valid, then there is an explanation for why the relation of logical consequence holds between its premises and conclusion.
>
> (P2): However, because of Carroll's regress, there is ultimately no explanation for why the relation of logical consequence holds between the premises and conclusion of an argument.
>
> ---
>
> Therefore, there are no logically valid arguments.

The above argument, when taken as a logically valid argument whose conclusion is that there are no logically valid arguments, is absolutely self-refuting. For if the argument above is logically valid, then it is not true that there are no logically valid arguments.

[133] Bury, R. G. *Sextus Empiricus: Outlines of Pyrrhonism, vol. 1.* I 3.
[134] Mackie, John L. "Self-Refutation — A Formal Analysis." *The Philosophical Quarterly* 14, no. 56 (1964): 193-203.
[135] Ibid. p. 194.
[136] Ibid. p. 195.

Hold on, though. It is not as simple as saying that the argument above is a valid argument for the claim that there are no valid arguments. To do so would be, at the end, to hold onto the concept *logically valid argument* as coherent, and to claim that there are simply zero of them. We have seen this before in the difference between Quine's rejection and Williamson's "rejection" of the analytic/synthetic distinction. To reject that distinction as incoherent is not simply to claim that there are zero analytic truths, but rather to claim that the very idea of a sentence being true by meaning alone does not make sense. Similarly, to reject logical consequence as incoherent need not commit one to such claims as, e.g. that there is a coherent notion of logical validity, but it has null extension. In any case, there is certainly something strange going on, as the notion of logical consequence is relied upon in the course of reaching a conclusion which denies the intelligibility of that very notion.

It is extremely tempting to take the argument above as showing that we must have gone wrong somewhere along the way. We might want to conclude, for example, that the truths of logic must just be true for no reason, or that they must not stand in need of metaphysical explanation. Doing so is, in effect, is to reject (P1) — the claim that if an argument is logically valid, then there is a reason why. Likewise, it is very tempting to conclude that Carroll's regress must not be vicious, or that we must have been relying on false assumptions about logic or grounding.[137] Doing so is, in effect, to deny (P2) — the claim that Carroll's regress shows that there is ultimately no explanation for why the relation of logical consequence holds. What the Tortoise would say, though, is that modus tollens stands in just as much need of explanation as modus ponens — Carroll's regress applies to any *reductio ad absurdum* argument that one might wish to give.

4.2 The Role of Explanatory Demands

Using Carroll's regress to reject the coherence of logical consequence stems from an explanatory demand, as is evident from (P1). Thus, it is, in part, an explanatory demand which leads to the strange kind of self-undermining showcased above, as well as to the incoherence showcased in §3. But this should come as no surprise, since explanatory demands are *also* responsible for Quine's rejection of the analytic/synthetic distinction as incoherent, as well as Bradley's rejection of relational thought as incoherent (see §1.5). The Principle of Sufficient Reason, which states that every fact has an explanation, has been found at the scene of the crime three times over.

4.2.1 The Smoking Gun

Quine can be seen as presenting arguments (e.g. the come-what-may argument) whose premises, taken together, have as a logical consequence that there is no analytic/synthetic distinction, and, indeed, that such a distinction is incoherent

[137] The difficulty, of course, is pointing out exactly where things went awry (see §2).

because it cannot be explained — "we at present lack any tenable general suggestion, either rough and practical or remotely theoretic, as to what it is to be an analytic sentence."[138] But, as we have seen in §3, if the analytic/synthetic distinction is incoherent, then it does not even make sense to speak of the premises and conclusion as expressing distinct propositions, or as being true or false. Indeed, if the analytic/synthetic distinction is incoherent, then it does not even make sense to say that this fact (viz. that the analytic/synthetic distinction is incoherent) logically follows from the premises of Quine's come-what-may argument.

There are other examples, too. In Parmenides' epic poem *On Nature*, he can be seen as using the Principle of Sufficient Reason to reach the conclusion that "non-being is not",[139] and thus that non-being "cannot be said nor thought".[140] In effect, Parmenides denies the coherence of negation, i.e. denies that it makes sense to say that one thing is *not* another, or that there are relations of distinctness. However, among other things, making sense of the very notion of *denial* requires negation.

Sextus Empiricus, a Pyrrhonian skeptic, can be seen as using the Principle of Sufficient Reason in his argument for the regress of the criterion, but the regress of the criterion applies just as much to the Principle of Sufficient Reason as to other statements. He compares Pyrrhonian expressions (e.g. "for every statement I withhold judgement, including this very statement") to "aperient drugs", which "do not merely eliminate the humours from the body, but also expel themselves along with the humours."[141] It is no surprise, then, that the ladder-throwing analogy was used by Sextus Empiricus centuries before it was used by Wittgenstein.[142]

A final, more recent example is Saul Kripke (or, at least, the Saul Kripke of §2 of *Wittgenstein on Rules and Private Language*).[143] There, Kripke presents a skeptical challenge to rule-following that can be seen as motivated by the Principle of Sufficient Reason: nothing explains why I am following one rule (e.g. addition) rather than another (e.g. quaddition), so there is no fact of the matter about which rule I am following. In effect, Kripke's argument challenges the coherence of the very idea of a rule as something distinct from the world that purports to be governed by the rule. But it might be thought that, in order to even articulate his skeptical problem, Kripke helps himself to the grammatical rules of English, for example. Indeed, as we saw with Quine, he helps himself

[138]Quine, "Carnap and Logical Truth", p. 84.

[139]Parmenides, *On Nature.* Trans. Laks, André, and Glenn W. Most. *Early Greek Philosophy, Volume V: Western Greek Thinkers, Part 2.* Loeb Classical Library; 528. Cambridge, Massachusetts, 2016. B6.2.

[140]Ibid. B8.8. For a reading of Parmenides as motivated by the Principle of Sufficient Reason, see p. 91 of Sattler's *The Concept of Motion in Ancient Greek Thought* and Della Rocca, Ch. 1.

[141]Bury, R. G. *Sextus Empiricus: Outlines of Pyrrhonism, vol. 1.* I 206-7.

[142]Ibid. *Sextus Empiricus. Against Logicians.* Loeb Classical Library 291. Harvard University Press, 1935. II 481.

[143]Kripke, Saul A. *Wittgenstein on Rules and Private Language: An Elementary Exposition.* Harvard University Press, 1982.

to logical rules, like modus ponens, in the course of his argument.

I am certainly not the first to see this tight connection between the Principle of Sufficient Reason and incoherence. In *The Parmenidean Ascent*, Della Rocca, who explicitly endorses the principle, argues that it leads to Bradley's regress, and he goes on to endorse Bradley's conclusion that relational thought is incoherent. In rejecting the very concept of a relation as incoherent on the grounds that this concept cannot be explained, Della Rocca recognises that the very notion of metaphysical explanation, or grounding, that is employed in his Bradleyan argument is a relational, and therefore incoherent, notion.[144] That is, the Principle of Sufficient Reason claims that every fact has an explanation, but the very notion of explanation — of one fact explaining another — is relational, and so (because of Bradley's regress) cannot be explained. For Della Rocca and Bradley, no relation is explicable — not even the relation of explanation.

Della Rocca is forthright about the self-undermining nature of his view:

> If, as I've just argued, differential metaphysical explanation is unintelligible and thus ruled out, then we are not entitled to the explanation *of* one thing *in terms of* something ... [M]y overall arguments — in relying on notions that I come to see, by virtue of those very arguments, as illusory — are self-undermining ... My own arguments lead to the conclusion that I can't even state my own position (and, of course, that I can't even state that I can't state my own position, etc.).[145]

Likewise, Della Rocca would use Carroll's regress to conclude that the concept of logical consequence is incoherent. If this is right, then it does not even make sense to conclude that the concept of logical consequence is incoherent; and it does not even make sense to conclude that it does not even make sense to conclude that the concept of logical consequence is incoherent; and...

Kripke makes a very similar observation about the difficulty of articulating his rule-following paradox. Beginning with the assumption that it makes sense to speak of the rule for addition, Kripke constructs uses this rule to construct a deviant *quaddition* rule.[146] However, this rule for quaddition is then used to show that nothing explains the purported fact that it made sense *or makes sense* to speak of *the* rule for addition:

> [U]ltimately, if the sceptic is right, the concepts of meaning and of intending one function rather than another will make no sense. ... But before we pull the rug out from under our feet, we begin by speaking as if the notion that at at present we mean a certain function by 'plus' is unquestioned and unquestionable. Only *past* usages

[144] Della Rocca, Ch. 8.

[145] Ibid. p. 219.

[146] Quaddition is defined to be identical with the addition function except when one of the addends is greater than 57, in which case the output of the function is 5. If, in the finitely many additions one has performed, neither addend was greater than 57, Kripke argues that there will be no evidence, behavioural or otherwise, that supports the claim that one has been adding instead of quadding.

> are to be questioned. Otherwise, we will be unable to *formulate* our problem.[147]

Della Rocca takes an interesting stance on this kind of ladder-throwing, rug-pulling, and aperient-swallowing move — he praises it for demonstrating a laudable commitment to explanation. As we have seen, all of these thinkers begin by making an explanatory demand, and this commitment to explanation (in some cases) leads them to reject the very notion of explanation itself (on the grounds that this relational notion cannot be explained). In turn, this leads to a view that cannot even be coherently articulated or thought. Della Rocca cites Bradley's discussion of the "happy suicide" of relational thought,[148] and affirms that "the self-undermining in this case thus shows that we — Parmenides and Bradley and I and other fellow travellers — are on the right track."[149] Indeed, he sees explanatory demands as part and parcel of philosophical enquiry. Of course (and Della Rocca would be the first to admit this), he is in no position to conclude that "this self-undermining is a virtue, not a vice",[150] not least because such a claim relies on a distinction between virtue and vice that he rejects. As we have seen, Della Rocca is in no position to draw any conclusions whatsoever, logical consequence being a relation that has been rejected as incoherent on the basis of Carroll's regress. Perhaps, then, it makes sense that the penultimate chapter of his book is a blank page.[151]

4.2.2 Against Quietism

But *does it* make sense? It may sound strange, but the worry is that Della Rocca, in fact, does not go far *enough*. The fact that the penultimate chapter is a blank page (coupled, perhaps, with the fact that the book's cover is completely white) might suggest that Della Rocca ultimately defends a kind of quietism — "What we cannot speak about we must pass over in silence."[152] On such a view, because it does not make sense to say that one sentence is a logical consequence of another, we should respond by drawing no conclusions whatsoever. If one understands quietism as a kind of withdrawal from the world, then, as Della Rocca makes clear,[153] quietism would have to be rejected insofar as it relies on realist assumptions — assumptions about the distinctness between one's thoughts and the world that is thought about.

A better way for Della Rocca to end the book would have been to follow in the footsteps of Parmenides himself, who, after presenting explanation-driven arguments for his conclusion that there are no distinctions, goes on to say "many things about the earth, the heavens, the sun, the moon and the heavenly bodies,

[147] Kripke, pp. 13-4.
[148] Bradley, p. 150-2.
[149] Della Rocca, p. 222.
[150] Ibid.
[151] Ibid. Ch. 12. "Tractatus Parmenideo-Philosophicus".
[152] Wittgenstein, §7.
[153] Della Rocca, Ch. 7.

and ... the origin of human beings."[154] Indeed, there is evidence that the so-called "Way of Opinion", in which Parmenides discusses these things, took up vastly more space in the poem than the "Way of Truth", in which he rejects non-being as incoherent, did.[155]

Some find it odd, or even contradictory, that Parmenides would end his poem with the "Way of Opinion". Palmer questions "why Parmenides should have bothered providing the poem's elaborate cosmology" given that "he has demonstrated that plurality and change are impossible."[156] And Schofield admits that it "remains a mystery" why the "Way of Opinion" was included.[157] On my view, though, it makes perfect sense — if one is to reject distinctions as incoherent, then, in particular, one must reject the distinction between truth and opinion as incoherent. To only include the "Way of Truth" would be to give primacy to a relation-less reality as something distinct from, and more fundamental than, the world of appearances (with its heavenly bodies, human beings, and the like). But in the end, for Parmenides, it does not make sense to speak of such a distinction, so he includes the second part as well. If there were no second part, the form of the work would contradict its content — since it does not make sense to speak of appearance as something distinct from reality, one must have a place in the work if the other does. But if this is right, it also follows that the first and second "parts" are not distinct parts at all, so the work is not truly bifurcated — it is one cohesive text.

As Parmenides writes, in my favourite passage from his poem:

> ...the helplessness in their
>
> Breast directs their wandering thought; and they are borne along,
>
> Deaf and likewise blind, stupefied, without judgement,
>
> Who suppose that to be and not to be are the same
>
> And not the same, and that for all the path is backward-turning.[158]

On my reading, Parmenides places himself in the company of those who are helpless, blind, stupefied, and without judgement. For, on his view, being and non being *are* the same and not the same, as there is ultimately no distinction between sameness and difference and ultimately no distinction between being and non-being. This, on my reading, explains why he goes on to give a cosmogony — there being no distinction between the opinions of mortals and well-convincing truth, he goes on to give an account of well-convincing truth *by* giving an account of the opinions of mortals. This is also why Parmenides

[154] Plutarch, *Against Colotes.* Trans. Laks, André, and Glenn W. Most. *Early Greek Philosophy, Volume V: Western Greek Thinkers, Part 2.* Loeb Classical Library; 528. Cambridge, Massachusetts, 2016. D9.

[155] However, most surviving fragments from Parmenides' poem are from the "Way of Truth".

[156] Palmer, John. *Parmenides and Presocratic Philosophy*, Oxford University Press, 2010, p. 31.

[157] Kirk, G. S., J. E. Raven, and M. Schofield. *The Presocratic Philosophers: A Critical History with a Selection of Texts.* 2nd ed. Cambridge: Cambridge University Press, 1983, p. 262.

[158] Parmenides, D8.7-13.

begins by saying "It matters not where I begin, for there I shall return again"[159] — his own path is back-turning.

The same holds in *Appearance and Reality* — Bradley reaches his conclusion about the incoherence of relational thought, but then goes on to say:

> Whatever is rejected as appearance is, for that very reason, no mere nonentity. It cannot bodily be shelved and merely got rid of, and, therefore, since it must fall somewhere, it must belong to reality. To take it as existing somehow and somewhere in the unreal, would surely be quite meaningless. For reality must own and cannot be less than appearance...[160]

There is a strand of Della Rocca's thought from which this anti-quietist point might be developed. He stresses that "the distinctions, the differentiation, the relations that we leave behind are not real."[161] And so, in rejecting them, we are not losing anything real, and, in particular, nothing has changed. Whatever problems we faced before rejecting relational thought as incoherent are problems that we face afterwards. It does not make sense to conclude that there is a distinction between a view that embraces logic and a "view" that rejects logic as incoherent. So, there is no need to reject Della Rocca, Bradley, and Parmenides for having a strange and self-undermining "view", for we can say that nothing would be different if what they said is true.

What happens when you throw the ladder away? You fall back down to the ground, where you started.

4.3 Carroll's Moral

In §3.5, we saw how Lewis Carroll diagnosed what he calls his "paradox".[162] In a letter to the editor of *Mind*, he says that "it turns on the fact that, in a Hypothetical, the *truth* of the Protasis, the *truth* of the Apodosis, & the *validity of the sequence*, are *3* distinct Propositions."[163] In effect, he is remarking on the connection between logic and meaning — a connection that, as I have argued, he was right to point out. However, one might be surprised to learn that, between the story and his surviving letters, we lack any clear evidence of what Carroll took the *moral* of his story to be. For example, he does not make any claims about whether the regress in the story is vicious, and, if so, what conclusions we should draw from this fact. I hope to show that this is, in fact, not surprising at all.

4.3.1 Carroll's Thoughts

It will be helpful to begin by discussing the evidence we *do* have. Of course, there is the story itself. Technically, there are three characters in the story:

[159] Ibid. D5.
[160] Bradley, p. 119
[161] Della Rocca, p. 225.
[162] Carroll, Lewis. "Letter 2: From Carroll to Stout, 25 August 1894." p. 10.
[163] Ibid. Emphasis in original.

the Tortoise, Achilles, and the Narrator. At the end of the story, the Narrator leaves because he has "pressing business at the Bank", but later returns to find Achilles still seated on the Tortoise, jotting down premises in his notebook.[164] Here is what the Narrator does *not* do: he does not come out and say that the pair ought to have stopped a long time ago, or that they should be ashamed of themselves for confusing premises and inference rules, or any other such normative claim. Historically, there are many fables in which the narrator *does* come right out and say this — not so in this fable. Rather, I take it that the Narrator's job is to show that the Tortoise and Achilles are dealing with an *infinite* regress: after an arbitrary length of time has elapsed, they are still sitting there jotting down premises.

Besides the Narrator, there is no reason to suspect that the other characters are impartial, or that they represent Carroll's own view. Towards the beginning, the Tortoise claims that explaining why (Z) is a logical consequence of (A) and (B) is like "a race-course, that most people fancy they can get to the end of in two or three steps, while it *really* consists of an infinite number of distances, each one longer than the previous one."[165] This, of course, foreshadows the infinite regress of the story. But it also suggests that, for the Tortoise, explaining why one proposition is a logical consequence of another is an impossible task — one cannot complete an infinite race-course, and one cannot prove that (Z) is a logical consequence of (A) and (B). Nonetheless, it would be unwise to ascribe the conclusions drawn by a talking tortoise to the man Lewis Carroll.

It is far more likely that the Tortoise is merely Carroll's vehicle for presenting the problem. The same goes for Achilles — without a representative of common sense, there would be no one for the Tortoise to question, but there is no reason to think that Carroll thinks Achilles' strategy for defending the validity of modus ponens is successful. On top of this, we do not leave the story with any particularly strong conviction that one of the characters is right and the other wrong. If anything, we are left with the conviction that *both* are foolish: the Tortoise for incessantly asking for explanations, Achilles for incessantly providing them. One is left with the Sisyphean impression that they are making no progress no matter how many premises they add — whatever the problem is, it will not be solved by adding another premise.

What about outside the story? In his letter to Stout, Carroll summarises the regress by "[writing] out, in full, some of the series of Propositions, whose truth is needed to prove 'Socrates is mortal.'"[166] These propositions are functionally identical to what the Tortoise says in the story. Again, though, Carroll does not draw any conclusions about the moral of his story. This fact is most evident in Lewis Carroll's unpublished *Symbolic Logic, Part II, Advanced*, where "What the Tortoise said to Achilles" is included in Book XXI, "Logical Puzzles", Chapter III, "Other Puzzles", §8.[167] Another one of the puzzles in this

[164]Carroll. "What the Tortoise said to Achilles." p. 280.
[165]Ibid. p. 278
[166]Carroll. "Letter 2: From Carroll to Stout, 25 August 1894." p. 10.
[167]Carroll, Lewis and William Warren Bartley III. *Lewis Carroll's Symbolic Logic : Part I, Elementary, 1896, Fifth Edition, Part II, Advanced, Never Previously Published : Together*

chapter is Carroll's famous barber-shop paradox, which was also published in *Mind*.[168] However, in Chapter V, "Solutions to Other Puzzles", there is no solution written for "What the Tortoise said to Achilles". This is remarkable, as the only other solutions omitted from *any* of the chapters in Book XXI were for two problems that Carroll likely thought to be extremely simple. By contrast, Carroll goes on at length about his barber-shop paradox.

All of this led Bartley, a biographer of Carroll, to conclude that "it seems most plausible to understand ["What the Tortoise said to Achilles"] as an attempt on Carroll's part to express some difficulties he felt but could not adequately explain".[169] And I agree — Carroll had the insight that there is some difficulty when it comes to explaining the logical consequence relation, but did not know what conclusions to draw from this fact. Indeed, this may have been what motivated him to submit his story to *Mind* — he wanted other people to think about the problem, too, so that he might learn what conclusions they drew from it. In this respect, Carroll embodied the Pyrrhonian ideal: he earnestly kept on searching.

4.3.2 A Note on Form and Content

Carroll must have thought about Carroll's regress before writing "What the Tortoise said to Achilles". The regress may have occurred to him as he was writing his logic textbook, a work in which he, among other things, categorises propositions and develops a diagrammatic method for representing syllogisms. If Carroll wanted to, he could have included a chapter in his textbook detailing the regress, analysing possible avenues of resolving it, including practice problems, etc. That is, one can imagine a world in which Carroll, rather than writing a story, presented this regress in a drier, more prosaic manner — such a rigorous treatment is more along the lines of Bolzano's and Cook Wilson's discussions of the regress. But this is not what Carroll did. Instead, he wrote a funny story with the characters from Zeno's paradox that ends without an obvious moral — the story ends with a pun, where the characters call each other "Taught-Us" and "A-Kill-Ease". What are we to make of this?

In *Love's Knowledge*, Martha Nussbaum argues that the form of a philosophical work should suit the content of that work. While her focus is on moral philosophy, her reasoning is applicable to other areas of philosophy as well, including metaphysics and logic. Indeed, she cites Part I of Spinoza's *Ethics* — which establishes his metaphysical views — as a prime example of a work whose form suits its content, writing that "Spinoza cultivates the intellectual joy of which he will speak" by employing the geometric method.[170] In the domain of ethics, Nussbaum argues that the form of a novel suits the philosophical

with Letters from Lewis Carroll to Eminent Nineteenth-century Logicians and to His "logical Sister," and Eight Versions of the Barber-Shop Paradox. Crown Publishers, 1977. pp. 431-4.

[168] Carroll, "A Logical Paradox." *Mind* 3, no. 11 (1894): 436-438.

[169] Ibid. "Editor's Note on 'What the Tortoise Said'. pp. 467-8.

[170] Nussbaum, Martha C. *Love's Knowledge: Essays on Philosophy and Literature.* Oxford University Press, 1992. p. 35.

content of the work, e.g. by enabling authors to better relay their values and set up situations of greater moral complexity.

Nussbaum argues that if there is a disparity between a work's form and its content, then there is reason to doubt whether the content is presented with assertoric force, or else whether the author has thought through the implications of their own view. Both of these reflect poorly on the content of the work. As Nussbaum writes, "there is, with respect to any text carefully written and fully imagined, an organic connection between its form and its content",[171] and when this connection is lacking, "it seems at least *prima facie* plausible to suppose that the author has either been oddly inattentive or is actually ambivalent about the view in question."[172] I suggest that Carroll was neither inattentive nor ambivalent, and that a fable was the perfect way for him to express the thoughts he was having. By writing a story, rather than a logical treatise, Carroll was able to present his idea that it might be difficult to draw a logical conclusion without the burden of having to actually draw any logical conclusions. That is, he does not have to perform the action (i.e. drawing a logical inference) that his story suggests cannot be performed. And in this way, there is an organic connection between the form and content of the work.

4.4 Normativa-what?

Drawing a conclusion is burdensome, in part, because of the normative component of logical consequence. It is thought that one *should* accept the conclusion of a logically valid argument if one accepts the premises, as the conclusion *must* be true if the premises are true. If Carroll had been presenting the premises of an argument, as in a logic textbook, he would have felt compelled to draw a particular conclusion based on whether those premises were true or false. Likewise, if he concluded with an overall moral to the story, then he simply *would not* have expressed his idea that there is a problem here that needs solving, but which he does not know how to solve. It is here, in the assumption that there is a problem to be solved, where Carroll and I part ways.

Regarding the normativity of logic, it is worth briefly considering Harman's challenge. Harman introduces a distinction between logic and theories of reasoning, writing that "it is one thing to say 'A, B, and C imply D'. It is quite another thing to say, 'If you believe A, B, and C, you should or may infer D'."[173] His motivation for doing so is, in part, to separate the Tarskian model-theoretic account of logic he prefers from the theories of reasoning which that account evidently does not capture. For example, Harman rejects that "the scarf is scarlet" implies "the scarf is red" (because there are interpretations of "scarlet" and "red" in which the argument fails to be truth-preserving), but would accept that you may infer the latter from the former. However, without an indepen-

[171] Ibid. p. 4.
[172] Ibid. p. 7.
[173] Harman, Gilbert. "Internal Critique: A Logic is not a Theory of Reasoning and a Theory of Reasoning is not a Logic." In *Studies in Logic and Practical Reasoning*, vol. 1, Elsevier, 2002. p. 173.

dent reason for accepting Tarski's model-theoretic account, there is no basis for Harman's distinction (see §2.5). Although my focus has been on Etchemendy in this regard, I see Priest making the very same point:

> [T]he notion of validity that comes out of the orthodox account is a strangely perverse one according to which any rule whose conclusion is a logical truth is valid and, conversely, any rule whose premises contain a contradiction is valid. By a process that does not fall short of indoctrination most logicians have now had their sensibilities dulled to these glaring anomalies. However, this is possible only because logicians have also forgotten that logic is a normative subject: it is supposed to provide an account of correct reasoning.[174]

There is one way to see the normativity of logic that even Harman would accept: suppose that P is true and that P implies Q; in such a case, one *should* accept Q. In Carroll's story, for example, it seems that one would be acting not just foolishly, but *irrationally*, if one accepted (A) and (B) but denied (Z). One *should* infer (Z) from (A) and (B), and the failure to do so shows that one's reasoning is defective in some way.

The normative component of logic is also evident in Carroll's story:

> [The Tortoise says] "I accept (A) and (B) and (C) and (D). Suppose I *still* refused to accept (Z)!"
>
> "Then Logic would take you by the throat, and *force* you to do it!" Achilles triumphantly replied. "Logic would tell you 'You ca'n't help yourself. Now that you've accepted (A) and (B) and (C) and (D), you *must* accept (Z)!' So you've no choice, you see."[175]

To reject logic as incoherent is to claim that this normativity is incoherent, i.e. that there is no explanation for the normative force of this "*must*". Of course, Logic will not literally take you by the throat, but the fact that Achilles resorts to this metaphor shows that we are still lacking an explanation of what it *means* to say that one must accept (Z) if one accepts (A) and (B). I see the same strand of thought in Quine's come-what-may argument, where he can be seen as questioning why one *must* hold a given sentence true come what may.

Della Rocca argues that, in "freeing ourselves from metaphysical confusions of relationality," we gain "a freedom to speak, to think, to play, and simply to be in the same way and also in new ways."[176] We act the way we do and draw conclusions that we do not because we *must*, i.e. not because Logic takes us by the throat and *forces* us to. Similarly, we act the way we do and draw the conclusions that we do not because we *may*, i.e. not because Logic (or a theory of reasoning) allows us or permits us to. We simply do act in certain ways and draw conclusions in certain ways. His discussion of freedom is reminiscent

[174] Priest, p. 297.
[175] Carroll, "What the Tortoise said to Achilles", pp. 279-80.
[176] Della Rocca, p. 224.

of Dummett's conclusions about Quine's rejection of the analytic/synthetic distinction as incoherent. I quote the passage in full, as it encapsulates the freedom of which Della Rocca speaks:

> Holism, however, removes all desire to ask for a justification. We speak as we choose to speak, and our practice, in respect of the whole of our language, determines the meaning of each sentence belonging to it. Forms of deductive inference do not need to be faithful to the individual contents of the sentences which figure in the inference, because there is no individual content other than that determined by the language as a whole, of which those forms of inference are a feature. It is not, therefore, that there is something which must hold good of deductive inference, if it is to be justified, but which, because we should thereby be trapped in a vicious circle, we are unable to demonstrate, but must simply assume: rather, there is no condition whatever which a form of inference can be required to satisfy, and therefore nothing to be shown.[177]

On this view, there is nothing about logical validity that stands in need of explanation. The meanings of words and sentences do not constrain us or enable us — they do not force us or allow us to draw some conclusions but not others. Rather, there is nothing to which we must be faithful, and no condition that an inference must satisfy to count as logical.

Of course, Della Rocca's conclusion that, in rejecting relational thought, one rejects the coherence of normative claims might be regarded as self-undermining for the very same reasons that rejecting the coherence of logical consequence is self-undermining (insofar as logic has a normative component). In effect, he is saying that if his premises are correct, you *should* draw a certain conclusion, namely that one gains a certain kind of freedom by rejecting relational metaphysical notions.

However, I suggest that there is another way of reading Della Rocca's argument that does not rely on the coherence of normative claims. He can be seen as simply concluding what he concludes (viz. that normative claims are incoherent), not because he should and not because he may; he simply does. Similarly, if someone rejects Della Rocca's argument, e.g. because it is self-undermining and conflicts with common-sense beliefs about the world, Della Rocca would not chastise them for drawing the conclusions that they do — he would not say that if you accept his premises, then you *must* accept his conclusion. Indeed, he would not even say that you *may* draw whatever conclusions you like — you simply draw the conclusions that you do. No course of action is prescribed and no course of action is permitted; as Dummett says, there is nothing to be shown. And this is the freedom of which Della Rocca speaks.

[177] Dummett, p. 304. Emphasis mine.

4.5 The Thesis of this Thesis

In an attempt to understand what it would mean to reject logical consequence as incoherent, I investigated Quine's rejection of the analytic/synthetic distinction as incoherent. But, if I am right that these two views are one and the same, it turns out that Quine's "view" is just as difficult to understand. (Can an incoherent view — a view which cannot be articulated or thought — properly be called a "view" at all?). By looking to the history of philosophy, in which there are many who present arguments against the intelligibility of relational thought, I learned that we are in no position to conclude that anything changes, or that any problem is solved, by rejecting logical consequence as incoherent. That being said, there was never any problem to begin with — we simply conclude what we conclude, believe what we believe, and do what we do, not because we may and not because we must — we just do. Consequently, I maintain that all the arguments in this thesis (including this one) "should" be read in the same, non-normative way as Della Rocca's argument about freedom (though how you read them is, of course, up to you).

5 What The Tortoise said to Achilles

[An Alternate Ending]

A few days later, the Tortoise was *running* some errands (about as fast as a tortoise can, at any rate) when he bumped into Achilles at the cheesemonger's shop.

"My my!" said the Tortoise with a smirk. "Funny seeing you here! Have you finished proving that beautiful First Proposition of Euclid?"

"I'm afraid my pencil wore down to the end" replied Achilles. "And I was on my way home to fetch a new one, but . . . "

"But you knew it was *futile* to keep going!" interjected the Tortoise.

"Not at all!" replied Achilles. "I certainly didn't draw *that* conclusion. You see, I was on my way home to fetch a new pencil when I bumped into my old friend Patroclus, and we decided to go play some football in the field nearby. It was such a lovely day, and after so much running around I suppose I had forgotten all about your puzzle. We're actually hosting a dinner party tonight if you'd like to come — he makes a great halibut."

"Well if you're getting cheese for the halibut, then you must go for a block of sharp cheddar" said the Tortoise.

A thought passed through Achilles' head — and why *must* I? — and he let out a hearty chuckle.

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