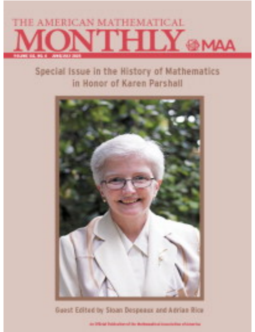

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Abraham Robertson, the Binomial Theorem, and the Nature of Plagiarism in Mathematics

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Abstract. We examine a charge made against the Oxford mathematician Abraham Robertson in 1807 that in publishing what he claimed to be a new proof of the Binomial Theorem, he had in fact plagiarized a method used by Leonhard Euler thirty years earlier. We place Robertson and his work into their historical context, before looking at the details of the proof itself and considering the nature of the charges against him. We use the case to reflect on the nature of plagiarism in mathematics.

1. INTRODUCTION. Some of the most often repeated tales from the history of mathematics concern accusations of plagiarism of one mathematician by another. Most famous perhaps is the dispute between Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716) over priority in the development of the calculus, which also tapped into wider political and religious disagreements [1]. In another example from around the same time, nationalistic considerations motivated John Wallis (1616–1703) to assert that René Descartes (1596–1650) has stolen the algebraic ideas of the English mathematician Thomas Harriot (*c.*1560–1621) [2]. Showing up the jealousies and rivalries of historical mathematicians in this way serves perhaps to humanize these figures, if in a rather negative way.

The fact that such disputes over the priority of mathematical ideas can be so long-running points to the fact that identifying plagiarism in mathematics is by no means an easy task. We are usually dealing not with the direct reproduction of words and sentences on a page, but with the duplication of ideas, which can be nebulous in their definition, and which can certainly arise independently in a number of different contexts. This is compounded by some of the referencing practices of mathematicians, which simply name-check other authors without necessarily giving formal citations, or omit references entirely when an idea is deemed to be part of the common currency of mathematics. This causes problems in particular in university systems, such as that in the UK, where specialist mathematics students with little prior experience of prose writing are often expected to produce rigorously referenced dissertations in their final year. Such students can struggle to grasp what plagiarism actually means. The questions of what they can or cannot include in their writing, and which points must be referenced properly cause anxiety. A university might provide a formal definition of academic plagiarism, but this goes only so far in aiding understanding. What students need is a hard-and-fast, foolproof, and practical guide to what plagiarism is and how to avoid it. Unfortunately, no such easy guide is forthcoming, and yet we can gain insight into the nature of plagiarism in mathematics via the study of individual cases. This is precisely what we propose to do in the present paper.

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At the beginning of the nineteenth century, the Oxford mathematician and astronomer Abraham Robertson (1751–1826) published what he claimed to be a new and elementary proof of the Binomial Theorem (supplementing a paper of the mid-1790s in which he had dealt only with positive powers). Within the year, however, he found himself accused of plagiarizing a proof that had been given by Leonhard Euler (1707–1783) thirty years earlier. Defending himself strenuously, he acknowledged that his paper did indeed feature some of the same ideas as Euler's, but that in order to make the link it was necessary to translate the language and notation used in one into that of the other. To his accuser, however, this process of translation was irrelevant: it was the ideas in common that mattered, not the forms in which they were presented.

We begin the paper with a brief biography of Robertson, before turning to the details of his proof of the Binomial Theorem and considering the accusations against him. We do not study Robertson's proof for its place in the story of the Binomial Theorem,[1] but as a means of providing insight into the nature of plagiarism in mathematics, upon which we make a few brief comments at the end of the paper.

2. ABRAHAM ROBERTSON. Abraham, or sometimes Abram, Robertson was born into a humble background in Berwickshire in Scotland in November 1751 [**6**, **7**]. The early part of his life was spent in the town of Duns, whose grammar school he attended. Robertson seems—at least according to an account published four decades after his death—to have shown early promise in mathematics: "he showed the bent of his genius by a remarkable aptitude for making difficult calculations, and solving abstruse questions in algebra" [**8**, p.153]. Robertson began his working life as a peddler of small wares in Duns, but around the age of 24, he left home to seek his fortune in London. Accounts of the next phase of his life are somewhat contradictory, though it appears that his time in London was unprofitable. Moving next to Oxford, Robertson seems to have had some initial success working as a private mathematical tutor, but an attempt to found an evening school failed, and he instead took a job as an assistant to an apothecary. It was perhaps in the apothecary's shop that he first met his fellow Scot, John Smith (1721–1797), who was at that time the University of Oxford's Savilian Professor of Geometry.

Founded by Sir Henry Savile (1549–1622) in the early seventeenth century, Oxford's two Savilian Professorships, one of Geometry, the other of Astronomy, were established to provide centralized teaching in mathematics and astronomy within the university [**9**]. Over the four centuries of its existence, however, the Savilian Professorship of Geometry has often succumbed to the whims of the patronage system that has at times governed British public life, particularly in the Georgian period: university chairs could be given to people who were deemed worthy of the honor, even if they had no credentials in the nominal discipline of the chair. Thus we find the physician Smith occupying the Savilian Chair of Geometry during the final three decades of the eighteenth century [**10**]. After receiving his medical doctorate in 1757, Smith certainly lectured in anatomy and chemistry within the university, but there is no evidence that he ever attempted to teach mathematics. Some individuals who held university chairs as sinecures would appoint a deputy, or "sub-lecturer," to carry out the associated teaching, and Smith was conscientious enough to do so. By 1784, when he left Oxford for private medical practice in Cheltenham, Smith's deputy was Abraham Robertson.

The precise mechanism by which Robertson became Smith's sub-lecturer is unknown, but Smith certainly found an effective deputy in Robertson. It was perhaps

[1] Indeed, Craik [**3**, p.258, n. 22] is dismissive of Robertson's proof, which he notes in passing within a short discussion of other proofs of the theorem that were available (particularly in Britain) at the beginning of the nineteenth century. On the history of the Binomial Theorem more generally, see [**4**, **5**].

under Smith's patronage, moreover, that Robertson was formally admitted as a member of the university, obtaining his BA in 1779 and his MA in 1782; in the latter year, he was also appointed a chaplain of Christ Church. An obituary notice for Robertson would later praise his style of teaching and commend his readiness to aid students [**6**, p.176]. The conscientiousness with which Robertson pursued his teaching duties may perhaps also be measured by the published works that derive from them (such as [**11**] and [**12**]). Indeed, these works were cited when Robertson was elected to Fellowship of the Royal Society in 1795.[2] By the mid-1790s, Robertson's standing in Oxford (academic and otherwise) was such that it was entirely natural that he should succeed Smith as Savilian Professor of Geometry—a significant achievement for one from such a simple background.

Over the following years, Robertson also cultivated an interest in astronomy. Thus in 1810, when the long-standing Savilian Professor of Astronomy, Thomas Hornsby (1733–1810), died, Robertson applied successfully to take the sideways step from the Geometry Chair to that of Astronomy. This gave him oversight of Oxford's Radcliffe Observatory, and also use of the house that had been specially built for the Observer. Robertson certainly engaged in astronomical observations, and his publications hereafter became considerably more astronomical than purely mathematical. There are also suggestions that he was a visible public scientist of the time: as well as being a member of the Board of Longitude, he was consulted, for instance, on the possible replacement of London Bridge, and also on proposed changes to the system of coinage in Britain [**7**]. Robertson died at the Observatory in December 1826.

3. ROBERTSON'S PROOF OF THE BINOMIAL THEOREM. Robertson's first paper on the Binomial Theorem was read at a meeting of the Royal Society on May 21st, 1795, which was also the meeting at which his election to Fellowship was confirmed by ballot. The paper was published in the Society's *Philosophical Transactions* later that year [**13**].[3]

Robertson's motivation for producing a new proof of the Binomial Theorem appears to have been pedagogical, as he was keen to stress the simplicity and obviousness of his proof.[4] He began his paper by listing the various different approaches taken in previous proofs of the Binomial Theorem: inductive, combinatorial, fluxional, etc. He held, however, that a proof based upon "the obvious principles of multiplication was still wanted, much to be desired, and also attainable" [**13**, p.298]. In his view, the Binomial Theorem is an essentially multiplicative result, and therefore ought not to be proved by means of "any doctrine foreign to the subject" [**13**, p.299].

Robertson began his own exposition with a brief outline of the structure of polynomials, building them up iteratively as products of linear factors. Suppose, for instance, that we have arrived at a cubic equation by successive multiplication of the linear equations $x + a = p$, $x + b = q$ and $x + c = r$, all of the constant quantities being assumed implicitly to be real. Robertson typeset the resulting cubic as follows:

$$x^3 \begin{array}{l} + \; a \\ + \; b \\ + \; c \end{array}\left.\vphantom{\begin{array}{l} a \\ b \\ c \end{array}}\right\} x^2 \begin{array}{l} + \; ab \\ + \; ac \\ + \; bc \end{array}\left.\vphantom{\begin{array}{l} a \\ b \\ c \end{array}}\right\} x + abc = pqr.$$

[2] Royal Society Archives: Certificates of election and candidature for Fellowship of the Royal Society, EC/1795/14.

[3] The manuscript version is preserved in the Royal Society Archives as L&P/10/121 (available online at https://makingscience.royalsociety.org/items/l-and-p_10_121); this does not differ substantially from the published version.

[4] His published paper closely follows a proof of the Binomial Theorem that Robertson gave in his Oxford lectures (Bodleian Library: MS Rigaud 16).

If we next multiply this by $x + d = s$ to obtain a quartic (or biquadratic) equation, then certain of the coefficients remain unchanged, namely in that part of the resulting quartic that is obtained by multiplying the cubic by x: the coefficients survive, but are now attached to higher powers of x. When we multiply the cubic by d, however, each part of the cubic must be "drawn one term back" [**13**, p.304], that is to say, when the first term of the cubic is multiplied by d, the result must be placed into the second column of the quartic, and so on:

$$x^4 \left.\begin{matrix} + & a \\ + & b \\ + & c \\ + & d \end{matrix}\right\} x^3 \left.\begin{matrix} + & ab \\ + & ac \\ + & bc \\ + & ad \\ + & bd \\ + & cd \end{matrix}\right\} x^2 \left.\begin{matrix} + & abc \\ + & abd \\ + & acd \\ + & bcd \end{matrix}\right\} x + abcd = pqrs.$$

This iterative approach provided the key to Robertson's proof of the Binomial Theorem for integral powers, via the observation that in each coefficient there are as many summands (which Robertson referred to as the "members" of the coefficient) that feature the last-introduced constant quantity as there are summands in the preceding coefficient that do not. Thus in the example given here, the coefficient of x contains three members that involve d, and the coefficient of x^2 contains three members that do not. Similarly, the latter coefficient features d three times, and there are three members in the coefficient of x^3 that do not contain d.

Robertson next introduced some notation. Let n denote the number of quantities $a, b, c, \ldots$ (essentially, the degree of the polynomial under consideration), and let s be the number of times that each of these appears in a given coefficient (say the kth). Let p be the number of quantities in each member of the coefficient. Thus the total number of quantities in the coefficient (including repetitions) is sn, and the number of members in the coefficient is $\frac{sn}{p}$. It follows that the number of times that a given quantity *does not appear* in the coefficient is $\frac{sn}{p} - s$. By the above reasoning, $\frac{sn}{p} - s$ is then the number of times that each of the quantities $a, b, c, \ldots$ *does appear* in the $(k+1)$st coefficient. The number of quantities (with repetitions) in this next coefficient is therefore $(\frac{sn}{p} - s)n$, and since each member of the $(k+1)$st coefficient will contain $p+1$ quantities, the number of members of this coefficient is, after a little rearrangement, $\frac{sn}{p} \times \frac{n-p}{p+1}$. The proof of the Binomial Theorem for positive integral powers then follows quite easily. If the quantities $a, b, c, \ldots$ are set equal to each other, then the polynomial becomes $(x+a)^n$, and each member of a given coefficient is the same power of a, hence the coefficient is that power of a prefixed by a numerical value that is the number of members of the coefficient. If this numerical value is m and the corresponding exponent of a is p, then the numerical value affixed to the next term is $m(\frac{n-p}{p+1})$. With this observation, together with knowledge of what the first two coefficients look like, Robertson concluded that

$$(x+a)^n = x^n + nax^{n-1} + n\left(\frac{n-1}{2}\right)a^2x^{n-2} + \cdots . \tag{1}$$

It remained now for Robertson to extend his result to positive fractional powers, and this is where he returned to his emphasis on multiplication, for his proof relied upon the multiplication of two infinite series (in a manner that looks somewhat naive to modern eyes). Robertson expressed this more general case of the theorem in a form that was deliberately similar to (1):

$$(x+z)^{\frac{n}{r}} = x^{\frac{n}{r}} + \left(\frac{n}{r}\right)zx^{\frac{n}{r}-1} + \left(\frac{n}{r}\right)\left(\frac{\frac{n}{r}-1}{2}\right)z^2x^{\frac{n}{r}-2} + \cdots , \tag{2}$$

and his initial approach to it was reminiscent of the way in which he had built up successive polynomials via multiplication. He asserted that the general Binomial Theorem for positive powers would "clearly appear" [**13**, p.309] once it had been proved that the product of the right-hand side of (2) with the series

$$x^{\frac{1}{r}} + \left(\frac{1}{r}\right) z x^{\frac{1}{r}-1} + \left(\frac{1}{r}\right)\left(\frac{\frac{1}{r}-1}{2}\right) z^2 x^{\frac{1}{r}-2} + \cdots \tag{3}$$

is equal to

$$x^{\frac{n+1}{r}} + \left(\frac{n+1}{r}\right) z x^{\frac{n+1}{r}-1} + \left(\frac{n+1}{r}\right)\left(\frac{\frac{n+1}{r}-1}{2}\right) z^2 x^{\frac{n+1}{r}-2} + \cdots . \tag{4}$$

Robertson's method thus has the flavor of an inductive proof, though he never put it explicitly into those terms. He considered the term-by-term multiplication of the two series, arranging his findings in columns labeled by powers of x, carefully establishing "the laws of arrangement upon clear and general principles" [**13**, p.310]. We omit most of the details here, since Robertson's method very quickly became notationally heavy. The essence of his approach was to find the general laws governing the form of the members of each coefficient, and to exploit these to determine the sum of those members. After several pages of algebra, Robertson had established the required product to his satisfaction. By way of rounding off the proof, he next used what he had just established in order to determine successive powers of (3), concluding that the nth power of (3) is in fact (2), and moreover that (3) is equal to $(x+z)^{\frac{1}{r}}$. The general case of the Binomial Theorem, in the form stated by Robertson, then follows.

4. RESPONSES TO ROBERTSON'S PROOF. Following its publication, a few brief comments were made on Robertson's paper by an anonymous author in *The Monthly Review*, a periodical that routinely published summaries of papers in the *Philosophical Transactions*, as well as reviews of mathematical books (see, for example, [**14**]). This author, identified only by the monicker "Re_s," found Robertson's method to be "very intelligible to persons accustomed to mathematical processes" and declared it "satisfactory," but entered into few specifics [**15**, p.296]. A reviewer for a different periodical, *The Critical Review*, was much less impressed, however, finding Robertson's method for fractional powers "too tedious and too intricate for the generality of readers" [**16**, p.316]. But this seems to have been criticism born of misconception: the reviewer found Robertson's method "of little or no use whatever" for practical calculation, something for which it was never in fact intended.

Conscious of the fact that his first paper on the Binomial Theorem had only considered positive powers, Robertson returned to the subject over a decade later with a further paper in which he also addressed negative powers [**17**].[5] This later paper recounted the proofs of the earlier one almost verbatim, but with some streamlining and additional commentary, such as the explicit remark that certain parts of the algebra remain valid when we replace quantities previously supposed to be integers by fractions [**17**, p.318]. With the Binomial Theorem for positive exponents easily established, Robertson turned next to the question of negative integral powers. Simplifying his expressions slightly, Robertson wrote down one for the product of polynomials $(1+x)^m$ and $(1+x)^n$ for m, n positive integers. He then used this to derive by inspection an infinite series for the quotient $\frac{(1+x)^m}{(1+x)^n}$. The Binomial Theorem for negative

[5] The original manuscript version of this paper survives in the Royal Society Archives as L&P/12/136 (https://makingscience.royalsociety.org/items/l-and-p_12_136/). The manuscript consists in part of annotated copies of some of the printed pages of Robertson's 1795 paper.

integral powers arose upon setting $m = 0$. Although Robertson claimed at the start of the paper also to have proved the result for negative fractional powers, he said little about this—the reader was probably expected to combine the other results to obtain this one, and Robertson appears to have been quite confident that the algebra involving integral powers was equally valid for fractional powers, as we shall see below.

Upon finding its way to the reviewing journals, this 1806 paper received a rather harsh treatment compared to the 1795 paper. The reviewer for *The Critical Review*, for instance, found it "unnecessarily prolix and dilated" [**18**, p.255]. The reviewer for *The Monthly Review*, on the other hand, though less sharp in tone, offered what was perhaps the more damning critique. Summarizing the early parts of Robertson's method, the reviewer noted the easy derivation of expressions for $(1+v)^m$ and $(1+v)^{m'}$ for integers m, m', and that if these be multiplied together, then the result is $1 + Av + A'v^2 + A''v^3 + \cdots$, where

$$A = m + m', \quad A' = \frac{(m+m')(m+m'-1)}{2}, \tag{5}$$

$$A'' = \frac{(m+m')(m+m'-1)(m+m'-2)}{1 \cdot 2 \cdot 3}, \tag{6}$$

and so on. The reviewer also recorded Robertson's observation that the algebra proceeds in exactly the same manner regardless of whether m, m' are integers or fractions. To the reviewer, this observation was key:

> Now, if this demonstration of the binomial has any claim to originality, it must be in the remark relative to the property of the coefficients A, A', A'', &c. remaining the same in regard to form, whatever be the indices. [**19**, p.164]

The problem, however, was that the reviewer claimed to have found precisely this observation in a 1775 paper by Leonhard Euler, published in the journal of the St Petersburg Academy [**20**]. In the paper in question, Euler had provided a proof of the general Binomial Theorem via manipulations of infinite series that were in principle somewhat similar to Robertson's. Along the way, Euler had considered the expression $[n] = (1+x)^n$ and had shown that $[m][n] = (1+x)^{m+n}$, in connection with which he had noted that

> the nature of this composition does not depend on the nature of the letters m and n, but will be the same whether these letters m and n denote whole numbers or any other numbers whatsoever.[6]

The reviewer was thus quite confident in having proved "beyond the possibility of a doubt" that this central principle of Robertson's paper was the same as one adopted by Euler thirty years earlier. This was not intended as an attack on Robertson, however:

> It is not our wish, on the sameness of the principle and the simplicity of the two demonstrations, to found a charge of plagiarism against Mr. R.: but he must abandon all claim to originality or priority of invention; which must be awarded to Euler. [**19**, p.165]

It was probably of small comfort to Robertson that, having granted priority to Euler, the reviewer nevertheless concluded with criticism of the indirect nature of some of Euler's (and by extension, Robertson's) methods: that he had first written down a plausible

[6]"[...] hic autem imprimis observari convenit, hanc compositiones rationem non ab indole literarum m et n pendere, sed perinde se esse habituram, sive hae literae m et n denotent numeros integros sive alios numeros quoscunque." [**20**, pp. 108–109]

series for a fractional power and then proved that this was indeed correct, rather than deriving the series outright.

5. ROBERTSON'S DEFENSE. It is possible that by the end of 1807, Abraham Robertson felt like a scholar under siege. Not only had his proof of the Binomial Theorem received the criticism that we have already outlined, but a further paper of his had also come under attack on very similar grounds. The paper in question concerned the precession of the equinoxes [21], and a very lengthy critique in *The Monthly Review* [22, p.6–16], written by the same reviewer as before, asserted that Robertson had borrowed rather too heavily from the writings of Thomas Simpson [23, 24]. We will not go into the details of this latter case here, for lack of space, but note instead that the two criticisms, coming in close succession, prompted Robertson to compose a vigorous defense of both of his papers.

Robertson's defense appeared in pamphlet form in early 1808 [25], printed by the University Press in Oxford, but distributed also in London and Cambridge—Robertson evidently wanted his side of the argument to be read as widely as possible within the English scholarly circles of the time. The tone of the pamphlet is rather acerbic from the outset, with Robertson attacking the "illiberal expressions" and "falsehoods" [25, p.1] of the reviews. Robertson noted that he had not planned to respond to the criticism of his paper on the Binomial Theorem, intending instead to let the mathematics speak for itself, but that the "violence of hostility" shown by the reviewer of his paper on the precession of the equinoxes had prompted him to action [25, p.2]. For Robertson, a wider issue was at stake: the fact that "those who publish on subjects of abstract science [. . .] are much at the mercy of the reviewer, if he is determined to lower or stifle their reputation" [25, p.3]. In Robertson's view, the "misrepresentations" of such reviewers are "boldly imposed on general readers as fair criticism" [25, p.3].

With regard to his proof of the Binomial Theorem, Robertson began by professing his surprise at the "mutilated" form in which he had found his results represented by his reviewers [25, p.4], and so he proceeded to outline his method in his original notation. Melodramatic language notwithstanding, it is certainly true that the reviewers had quite liberally rewritten Robertson's notation. The expressions recorded here as (5) and (6), for example, do not appear explicitly in Robertson's paper (nor do the symbols v, A, A', A'', . . .), though the ideas that they embody are present. Moreover, Robertson was at great pains in his defense to assert the *direct* nature of his methods, contrary to the reviewer's criticism.

Understandably, however, Robertson devoted the most attention to addressing the charge of plagiarizing Euler; he found the reviewer's claim that the accusation was not intended somewhat disingenuous [25, p.14]. Here in particular, Robertson accused the reviewer of "torturing" his formulas and having rewritten them in "uncouth symbols" [25, p.7]. So that the reader might more easily compare his methods with Euler's, Robertson provided an extensive quotation from the latter's paper (which Robertson claimed not to have seen until after the publication of his own), and set this beside the passage from his own paper in which the reviewer had claimed to identify similarities. Cosmetically, at least, the two do appear quite different. Robertson identified two key points upon which the accusation of plagiarism seemed to him to be founded. First, that he and Euler had developed comparable schemes for multiplying (infinite) binomial series, and second, that each had written down similar expressions and had asserted their truth both for integral and fractional exponents. Robertson dismissed the first point very quickly: if such multiplication is to be considered evidence of plagiarism, then "almost every writer on Algebra" must be judged guilty [25, p.15]. As to the second point, Robertson regarded this as "frivolous in the highest degree" [25, p.15].

As far as he was concerned, anyone who understands algebra can derive the relevant formulas and observe their validity both for integral and fractional values—this simplicity of extension had been the motivation behind his publishing this proof in the first place. That some of his statements were similar to Euler's stemmed from the fact that their "enunciation [. . .] admits so little variation" [25, p.15]. In concluding his refutation, Robertson offered a firm opinion of his reviewer:

> I consider his review of my demonstration as a bungling and gross misrepresentation from the beginning to the end; and his charge of plagiarism as more becoming a boy, with a smattering of Algebra, than an intelligent and candid critic. [25, p.16]

Robertson's pamphlet turned next to an equally detailed refutation of the claims made against his paper on the precession of the equinoxes, which we will not go into here. At the very end of the pamphlet, however, Robertson noted another factor that he thought might be at play in the background. Observing that hostile critiques of published papers were by no means unusual within the pages of *The Monthly Review*, he had examined back-issues of the periodical and had concluded that

> it has been the general practice of the mathematical writer, or writers, to give as favourable accounts as possible of the productions of Cambridge authors; and as generally unfavourable accounts of other authors of this country. [25, p.34]

By way of evidence, he contrasted the way in which *The Monthly Review* had treated two works on the construction of arches, published several years earlier by the Cambridge mathematician George Atwood (1745–1807) and by Charles Hutton (1737–1823), Professor of Mathematics at the Royal Military Academy at Woolwich and a non-Cambridge man of humble origins similar to Robertson's. In a manner that Robertson deemed to be "ridiculous and unjust", Atwood's work had been exalted and Hutton's condemned [25, p.35]. Robertson went on to offer some sly speculation as to the identity of the mathematical reviewer for *The Monthly Review*, concluding from the internal evidence of the reviews that it must be a "friend" of the mathematician Robert Woodhouse (1773–1827), Fellow of Caius College, Cambridge. Robertson was perhaps avoiding an accusation of libel here, for it was an "open secret" that Woodhouse himself was by this time the mathematical reviewer, in which role he had been particularly critical of Hutton's works [26, p.128 and letter 87]. Little else is known about these circumstances. The question of whether Robertson's charge of Cantabrigian bias holds water is one that I hope to investigate more fully elsewhere.

6. CONCLUDING REMARKS: PLAGIARISM IN MATHEMATICS. There is a coda to Robertson's dispute with his ostensibly anonymous reviewer: the defensive pamphlet was itself reviewed at length in *The Monthly Review* in June 1808, evidently by the same author [27]. As we might expect, the reviewer was in no way chastened by Robertson's counterattack, and was rather critical of the tone of Robertson's "intemperate and precipitate reply" [27, p.145], while condescendingly considering Robertson to be deserving of "pity rather than anger" [27, p.144]. Playing Robertson at his own game, the reviewer reproduced lengthy passages from the original papers, with Euler's now translated into English. Working step by step through the respective methods, the reviewer once again made a convincing case for the presence of the same ideas in both, but now insinuated more strongly that the reproduction had been deliberate.

The fact that both parties could present exactly the same evidence but expect the reader to draw entirely contrary conclusions tells us something about the nature of mathematical ideas and mathematical writing. It is abundantly clear that the two proofs rest upon the same ideas: the multiplication of series and the assumption that certain formulas may be extended in their application from integers to fractions. Either method could be translated into the notation of the other, as the reviewer made clear. It is not our purpose here to attempt to reach a definitive judgment on the charge of plagiarism, but we suggest that there was nothing so specialized in this proof that Robertson could not have arrived at it independently of Euler. Why else would his method feature a redundant step (pointed out by the reviewer [**27**, p.143]) not taken by Euler? Moreover, much of Euler's proof is an outline only, whereas Robertson supplied most of the necessary algebra. The balance of evidence seems to be in Robertson's favor.

This, however, is beside the point for our present purposes. As we noted at the beginning of the paper, the history of mathematics abounds with accusations of plagiarism, justified or otherwise. These are accusations that seem to be particularly easy to make in mathematics, because it is rarely the case that one mathematician is accused of directly cribbing the work of another. Rather, it is the underlying ideas that are seen to be reproduced, often with little regard for how they are presented, thus affording a great deal of latitude for interpretation. If the ideas in question are radically new ones, then a charge of plagiarism might be justified, but in most cases, it will simply be a matter of common inspiration or shared mathematical culture. Indeed, the assumption made by both Euler and Robertson that their formulas could be extended automatically to fractional values is a reflection of a widely applied method of proof at the time. This can be found, for instance (and perhaps ironically), in Woodhouse's writings [**28**, p.8–9], and it would later be formalized by George Peacock (1791–1858) as the "principle of the permanence of equivalent forms" [**29**, p.xvii–xviii] (see [**30**]). In other instances, a similarity of methods might be because there is essentially only one good and direct way to achieve something—what constitutes "good and direct" will usually be determined by the shared values of a mathematical community, thereby increasing the likelihood of two independently developed methods looking the same.

Although it is difficult, if not impossible, to determine whether he was correct, Robertson raised an additional reason for the accusation having been made against him: the personal animosity, for whatever reason, of the reviewer.[7] Since this had little to do with the details of the mathematics, it would probably not have been possible for Robertson to have guarded against it. As argued above, it seems fair to grant Robertson the benefit of the doubt and to believe that he had not simply lifted his ideas from Euler's work. Indeed, this was probably the view taken by his peers, since this episode does not appear to have had any negative impact on his subsequent career.[8] Even if his proof contained techniques similar to those of Euler, it would perhaps have been quite easy, particularly for those who knew him, to believe that Robertson had nevertheless acted in good faith. In the end, this is probably the most effective defense against inadvertent plagiarism, both for academic papers and for student dissertations.

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[7]Robertson's entry in the *Dictionary of Living Authors* (1816) suggests envy of his achievements as a motivation for the criticism [**31**, p.296–297].

[8]At least one comment appeared in print in Robertson's favor in connection with [**25**]; in a piece headed 'Comparative criticism', the periodical *The Satirist* for November 1809 paired an extract from [**27**] with the following quotation, which it attributed to *The Eclectic Review*: "This injured writer has succeeded in demonstrating the unfairness and malignity of his judge" [**32**, p.513]. Unfortunately, I have not been able to identify the original article from which this quotation was drawn.

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100 Years Ago This Month in *The American Mathematical Monthly*
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The THIRD PAN-AMERICAN SCIENTIFIC CONGRESS was held in Lima, Peru, from December 20, 1924, to January 6, 1925, immediately following the elaborate celebration of the hundredth anniversary of the Battle of Ayacucho (by which the independence of the Latin-American republics was finally secured from Spain).

All the republics in the western hemisphere were represented (except Chile), 138 members coming from Peru and 131 from foreign countries, including 30 from the United States, 25 from Argentina, 15 from Cuba, 9 from Venezuela, 8 from Colombia, 7 from Uruguay, 6 from Brazil, 5 from Ecuador, 5 from Mexico, etc.

Of the thirty delegates from the United States, ten were official representatives of the United States Government, under the chairmanship of Dr. L. S. ROWE, Director-General of the Pan-American Union in Washington, and Honorary President of the Congress. [...]

On the scientific side, the Congress was divided into nine sections, as follows: (1) Anthropology and History. (2) Physical and Mathematical Sciences (including Geology and Geography). (3) Mining and Metallurgy. (4) Engineering. (5) Medicine and Sanitation. (6) Biology and Agriculture. (7) Law. (8) Economics and Sociology. (9) Education.

The subject of Pure Mathematics was assigned to Subsection 1 under Section 2. This subsection held one meeting for the reading of papers (December 28), at which about fifteen members were present, and the following three papers were read:

"Conjugate ordinates and their geometrical applications," by Dr. FLORENCIO D. JAIME, President of the recently organized Argentinian Mathematical Society, and delegate from the Ministry of Public Instruction of Argentina.

"Elementary types of order," by Dr. E. V. HUNTINGTON, vice-president of the American Mathematical Society, and delegate from the American Mathematical Society, the Mathematical Association of America, and the American Academy of Arts and Sciences.

"On the descriptive geometry of the sphere," by Dr. ALEJANDRO GUEVARA, Honorary Professor of the School of Engineers of Lima.

—Excerpted from "Notes and News" (1925). 32(6): 322–328.

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