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Heterogeneity, Leveling the Playing Field, and Affirmative Action in Contests*

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Abstract

The heterogeneous abilities of the players in various competitive contexts often lead to undesirable outcomes such as low effort provision, lack of diversity, and inequality. A range of policies are implemented to mitigate such issues by enforcing competitive balance, i.e., leveling the playing field. While a number of such policies are aimed at increasing competition, affirmative action (AA) policies are historically practiced in an ethical response to historical discrimination against particular social groups among winners. This survey summarizes the rapidly growing literature of contest theory on AA and other policies that level the playing field. Using a general theoretical structure, we outline research on player and contest designer behavior under a multitude of policy mechanisms; and discuss the theoretical, experimental, and empirical results in relation to some of the common debates surrounding AA.

Keywords: Survey; Affirmative Action; Contest; Heterogeneity.
JEL Codes: A31; C72; D74; D82.

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1 Introduction

> "You do not take a person who, for years, has been hobbled by chains and liberate him, bring him up to the starting line of a race and then say, 'You are free to compete with all the others,' and still justly believe that you have been completely fair."
>
> Lyndon Johnson (36th President of the US; Howard University, 1965)

Various situations around us are characterized by competition. Examples include sports, promotional tournaments, college admissions, competition for market share, grant applications, etc. In many of such competitions, however, the competitors possess heterogeneous abilities. This often leads to undesirable outcomes such as under-representation of a certain section of the community (e.g., minority or women), dwindled representation of the local firms when competing with the multinational corporations, or sports becoming less interesting, to name a few.

Various policies are implemented to address this issue through providing 'competitive balance' (or, as we more often refer to it in this study, by 'leveling the playing field'). Such policies are frequently implemented in sports and other areas of competition. For example, high ranked players are often handicapped in golf, and favorite horses carry extra weights in horse racing in order to make the competition more exciting. Alongside, there are social policies driven by particular ethical concerns. In the US, an in-state student pays less fees compared to an out-state student, and African American students are preferred in college admissions in many states. It is the same for an EU student versus a non-EU student in the UK universities. Gender quotas in political organizations are popular worldwide. In Argentina for example, a quota is set within political parties for the minimum representation of women (Argentine law 24,012 in 1991). In 2014 the European Commission committed to have at least 40% of management positions filled by women; this target was achieved before the end of Jean-Claude Juncker's mandate in 2019 (European Commission, 2019).[1] Such gender quotas are also implemented in India. In addition, a certain percentage of government jobs (and promotion opportunities) in India are reserved for people of the backward castes and tribes. The Australian Government provides financial support specifically to the Aboriginal and Torres Strait Islanders for primary and secondary education, as well as for post-school qualifications and job facilities. Although the main objectives are different, the tools employed in each of the above situations essentially level the playing field in a competition.

A fair share of the policy examples given above can be categorized as 'Affirmative Action' policies. Affirmative Action (AA hereinafter) is a set of ethically driven policies aimed at providing

[1] See Piatti-Crocker (2019) for political quotas for women in Latin America, Bauer and Britton (2006) for policies in Africa, Krook, Lovenduski, and Squires (2009) for policies in Western Europe, North America, Australia, and New Zealand, and other recent studies such as Hughes et al. (2019) and Hughes and Paxton (2019) for global and historical perspectives.

special opportunities to a historically disadvantaged group in order to make the members of this group capable of competing with their privileged counterparts in the society.[2] AA policies "can be distinguished from other antidiscrimination measures by requiring pro-active steps (hence the phrase *affirmative*) to gradually eliminate the differences between women and men, minorities and nonminorities, etc." (Holzer and Neumark, 2000, p. 484). Such policies are also known as equal opportunities (Canada), reservation (India and Nepal), positive action (UK), etc.

The majority of the competitive environments emerging in the socio-economic contexts discussed above can be modeled in economics as contests. Contests refer to a class of games in which players spend irretrievable and costly resources in order to secure some valuable ends, and the individual success probability is an increasing function of own expenditure relative to other players'.[3] As will be argued later, heterogeneity among players can cause incentive problems in contests, and the economic importance (on top of the ethical importance) of AA comes from mitigating such incentive problems by leveling the playing field. Majority of contest models assume that there is a contest designer whose objective is to maximize the total effort from the players. Real-world contests may often have no designer (e.g., war), or a designer with alternative objectives (e.g., the recruiters in a job market are aimed at finding the best deserving candidate). Going with the convention, we limit our discussion to contests that have a designer who chooses the appropriate design elements at the beginning and maximizes total effort in the contest (unless otherwise specified). We then summarize the theoretical and empirical findings from studies that discuss different mechanisms used for implementing competitive balance (including AA) within such a framework. Additionally, we use the contest theoretic findings to shed light on the widely heard debates about productivity vis-à-vis participation, meritocracy vis-à-vis diversity, and preferring internal vis-à-vis external candidates.

1.1 A brief history on the debate around AA

Traces of AA policy can be already found in the Bible. Christians pursue the laws on the Scriptures by individual achievements and, according to these relative achievements, they will be ordered to enter the heaven in a way that might be opposite to their social and economic status ('Many of you who are first will be last, and many who are last will be first' - Matthew 19:30). This is further exemplified by the parable of the laborers in the vineyard (Matthew 20:1-16) where the landowner hires laborers in different times on the same day according to their descending ability. At the end of the day, he decides to reward all laborers by the same amount, a denarius. Moreover, those who

[2]Several sources such as the Cambridge dictionary, the Encyclopaedia Britannica, and the Stanford Encyclopaedia of Philosophy define AA as active effort taken by public or private organizations to improve education or employment opportunities for women and minority groups. Our focus throughout will be on the *leveling the playing field* aspect.

[3]For the sake of terminological consistency, hereinafter we will refer to any resource expenditure as 'effort'.

were hired the last were the first in being paid.

Administrative provision of additional privilege to backward classes has been practiced in India since the beginning of the 20th century,[4] although the particular phrase of 'Affirmative Action' became popular with an executive order issued by the US President John F. Kennedy in 1961. After Lyndon Johnson's executive order of 1965, the earliest implementations of AA in the United States had mostly taken the form of quotas for African Americans. However, such racial quotas attempt to correct past discrimination via the means of present discrimination. This fact was soon pointed out by two cases of white applicants claiming exclusion on the basis of racial discrimination as prohibited under the civil rights act (1974, Louisiana; and 1978, California).[5] One of the arguments against AA policies is therefore related to the non-equal treatment of individuals. These policies, sometimes called discrimination policies, give preferential treatment to preferred groups causing reverse discrimination to non-preferred groups. Other criticisms of AA are mostly related to the potential negative impact of these policies on efficiency; for instance, the decrements in standards and overall effort levels of students, mismatches between skilled workers and jobs, or a negative financial cost-benefit analysis (see Holzer and Neumark, 2000; Sowell, 2004; Fryer Jr and Loury, 2005). Nonetheless, AA is defended under efficiency grounds as well. Loury (1981) highlights that there is a market failure with equally skilled workers being paid unequally, and the intervention via equal opportunities is not enough to eliminate the persistence of economic disparities, especially when the population is segregated by income and race. Niederle, Segal, and Vesterlund (2013) show that while high-performing women may fail to enter competitions, this sub-optimal decision can be corrected by AA without lowering the requirements to win. Moreover, AA could counteract other discriminatory actions against minority groups that harm efficiency, such as the racial discrimination in admitting applicants to Yale School of Medicine in 1935,[6] or the recent case of a Japanese medical university that lowered the entrance results of female applicants (The Guardian, 2018).

Gamson and Modigliani (1994) present a precise account of the framing of AA in different public commentaries between 1965-85 and the corresponding interpretation by popular media. The major arguments against AA, as they point out, relate to reverse discrimination, undeserving advantage, artificial division, and hurting minority sentiments through sympathizing. They

[4]Shahuji, the first Maharaja of the Princely state of Kolhapur, provided educational and employment support for weaker sections of the society as early as 1902. Beginning with the Government of India act of 1909, a few other reforms in the British India ensured political representation of religious and social minorities. Educational quota for scheduled castes and tribes in the independent India was introduced in 1954.

[5]See Fang and Moro (2011, pp. 163-164) for a more detailed account of the history of legal provision of AA in the US.

[6]In 1935 Yale accepted 76 applicants from a pool of 501. About 200 of these applicants were Jewish, and only five got in. The dean's instructions were remarkably precise: "Never admit more than five Jews, take only two Italian Catholics, and take no blacks at all" (Oshinsky, 2005, p. 98).

also classify the associated policy stands - remedial actions, delicate balance, and non-preferential treatment. Based on the empirical findings from two standardized tests used for college admissions in the US (SAT and LSAT), Sturm and Guinier (1996) present an assessment of the policy debate over AA and advocate the 'delicate balance' principle for the role of conventional institutions in treating racial discrimination. Schuck (2014) also reviews this debate from a normative perspective. Overall, criticisms against AA have influenced the policy makers and some states in the US have banned these policies.[7] Kiewiet (2015) exemplifies how controversial is the design of college admissions criteria with the case of UCLA, a university that tries to increase the admissions rate of under-represented minority students without breaking the constitutional laws. Society is divided between the supporters and the detractors of AA, and the policy makers' interest is to implement these policies with the minimum possible compromise in terms of efficiency.

1.2 Related surveys and the scope of the present study

The literature on leveling the playing field and the debate around the same have been assessed from economic (Holzer and Neumark, 2000; Fryer Jr and Loury, 2005; Fang and Moro, 2011; Mealem and Nitzan, 2016), legal (Sturm and Guinier, 1996; Hyman et al., 2012; Somani, 2012; Schuck, 2014), psychological (Yang et al., 2006), and institutional (Murrell and Jones, 1996; Sowell, 2004; Kalev, Dobbin, and Kelly, 2006) perspectives, though most surveys in disciplines outside of Economics focus exclusively on AA.

Murrell and Jones (1996) report existing statistical and literary findings on the impact of AA on employment, education, and business ownership. Then, Holzer and Neumark (2000) assess the effect of AA on performance efficiency in the labor market, education, and business procurement in the US. They observe that the weaker candidates not only derive immediate benefits from AA, but they are often able to improve their subsequent performances as well. In an attempt to evaluate alternative policies for diversity management in US corporate establishments between 1971-2002, Kalev, Dobbin, and Kelly (2006) find that diversity training is useful only when managers are explicitly made responsible for diversity management. Hyman et al. (2012) review institutional attempts towards diversity management under the legislation of the European Union, and Sowell (2004) analyzes the general impact of AA in India, Malaysia, Nigeria, Sri Lanka, and the US.

Within Economics, Fang and Moro (2011) survey the major theoretical models that strive to explain different sources of persistent statistical discrimination, while Fryer Jr and Loury (2005)

[7] Similarly, in the UK AA policies that entail preferential treatment ('positive discrimination') are unlawful. However, other AA policies ('positive action') are allowed. Section 158 in the Equality Act 2010 explains what is not prohibited: enabling or encouraging persons who share a protected characteristic (age, disability, gender reassignment, marriage and civil partnership, pregnancy and maternity, race, religion or belief, sex or sexual orientation) to overcome or minimise that disadvantage, meeting their different needs, and enabling or encouraging their participation in activities.

summarize the economic questions and arguments about AA. Finally, Mealem and Nitzan (2016) review the theoretical literature on the different ways to leveling the playing field (which they term as 'discrimination'). They focus exclusively on the scope of logit-form contests (Tullock, 1980) to compare and contrast the effects of leveling the playing field through (1) modifying players' valuations for the reward ('direct discrimination'), (2) adding some extra effort to their incurred efforts ('head start'), (3) multiplying the incurred effort ('overt discrimination'), and (4) manipulating the intensity return to effort in the probability of winning ('covert discrimination').

In the present study, we consider the economics literature which investigate policies (including AA) for leveling the playing field using contest models (including logit, all-pay auction, and tournaments) and keep the legal, political, and institutional literature out of scope. Although we make relevant references to players' payoffs, our primary focus is on the designer's objectives. Our study is closest to Mealem and Nitzan (2016) in focusing on the different tools that can be used to change the competitors' incentives and induce the desirable results. However, our approach is more general and inclusive on different grounds: (i) we discuss theoretical, experimental, and empirical findings instead of focusing only on formal theoretical literature; (ii) we cover all the major mechanisms in the literature instead of four specific mechanisms; (iii) our discussion comprises of all the logit-form, all-pay auction, and tournament type contests instead of only logit-form contests, and (iv) we consider an n-player set-up instead of 2-player contest.

The primary purpose of this survey is to summarize what research in contests and tournaments has elaborated so far on the implication of AA and related mechanisms on competitive choice and outcomes, while being useful to researchers studying competitive balance in heterogeneous competitions with theoretical or empirical methods. We include literature that studies one of the following topics within a contest framework.

- Desirability of leveling the playing field in a contest framework.
- Effectiveness of one or more leveling mechanisms.
- Optimal choice of parameters or design elements given that the designer has a particular leveling objective or follows a particular leveling mechanism.

In addition, we also mention experimental and empirical studies that do not use an explicit contest theoretic framework but strongly supports or rejects a major finding by another study in the contest theoretic framework.

The remainder of the study is organized as follows. The next section provides the general theoretical structure relevant for the subsequent discussion. Section 3 briefly discusses the implication of heterogeneity for total effort outcome in a contest, and when or why leveling the playing field may be desirable. Section 4 considers different mechanisms for leveling the playing field;

how these mechanisms affect the players' chances, overall performance, and related empirical evidences. Section 5 considers further issues which may arise in deciding leveling mechanisms. Section 6 concludes.

2 Theoretical framework

Consider a contest with a set N of $n \geq 2$ risk-neutral players that exert costly effort to win a prize. Player $i \in N$ exerts effort $e_i \in [0, \bar{e}_i]$ to increase the probability p_i of winning the prize of value $v_i > 0$, where $\bar{e}_i$ is the player's budget or endowment. This budget can be finite or infinite, and a finite budget can be lower than, equal to, or higher than the prize value. The impact of a player's effort on the contest is determined by the impact function $f_i(\cdot)$, such that the final impact of e_i is $x_i = f_i(e_i)$. This distinction is especially important in a contest with heterogeneous players, where the elements of $f_i(\cdot)$ can vary across players. Examples of three popular effort impact functions are described below.

- $x_i = \alpha_i e_i + \theta_i$ as in Gradstein (1995) and Runkel (2006b), where $\alpha_i \geq 0$ and θ_i are multiplicative and additive parameters, respectively, that determine the realized effort.[8]
- $x_i = e_i^r$ as in Tullock (1980), where $r > 0$ measures the sensitivity of p_i to a player's effort. As r increases, the more sensitive the contest outcome is to a player's effort.
- $x_i = e_i + \varepsilon_i$ or $x_i = e_i \varepsilon_i$ as in Lazear and Rosen (1981), where ε_i is a random noise. This type of impact function gives a noisy prediction of a player's performance.

Conventionally, the impact vector of a contest is $\boldsymbol{x} = \{x_1, x_2, \ldots, x_n\}$, and vector $\boldsymbol{x_{-i}} = \{x_j : j \in N \setminus i\}$ contains all players' effort impacts except player i's. The mapping of players' effort impacts into individual winning probabilities, $p_i(\boldsymbol{x}) : \mathbb{R}_+^n \rightarrow [0, 1]$, is called contest success function (CSF). This function specifies the winning probability of player i which increases in own effort impact x_i but decreases in other players' effort impact $x_{j \in N \setminus i}$. Therefore, the CSF must satisfy the following properties: $\frac{\partial p_i}{\partial x_i} \geq 0$, $\frac{\partial p_i}{\partial x_{j \in N \setminus i}} \leq 0$, and $\sum p_i = 1$. Although a player's winning probability increases in own effort, there is a trade-off due to effort being costly. The cost of effort is denoted by c_i.

In the majority of the literature, the CSF $p_i(\boldsymbol{x})$ takes one of the three canonical forms: (1) the logit (Tullock, 1980), (2) the all-pay auction (Hillman and Riley, 1989; Baye, Kovenock, and

[8] In some cases, we will refer to the multiplicative parameter α_i as *ability* and the additive parameter θ_i as the *threshold impact*.

De Vries, 1996) and (3) the tournament (Lazear and Rosen, 1981). The specific functional forms of the CSFs for the logit (L) and the all-pay auction (APA) are as given below.

$$p_i^L(\boldsymbol{x}) = \begin{cases} \dfrac{x_i}{\sum_{j \in N} x_j} & \text{if } \sum_{j \in N} x_j > 0 \\ \dfrac{1}{n} & \text{otherwise} \end{cases} \tag{1}$$

$$p_i^{APA}(\boldsymbol{x}) = \begin{cases} 1 & \text{if } x_i > \max\{\boldsymbol{x_{-i}}\} \\ \dfrac{1}{k} & \text{if } x_i = \max\{\boldsymbol{x}\} \text{ and } k = |j \in N : x_j = \max\{\boldsymbol{x}\}| \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Note that the APA can be obtained as a special case of the logit when the effort impact is $x_i = e_i^r$ and $r = \infty$ (Baye and Hoppe, 2003). Another popular form of CSF is the tournament (Lazear and Rosen, 1981). Players' efforts are not directly observable in a tournament setting, but players produce a final output or score which is observable to the contest designer. The precise form of a tournament is

$$p_i^T(\boldsymbol{x}) = \prod_{j \in N \setminus i} \text{prob}(x_i > x_j) = \prod_{j \in N \setminus i} G(e_i - e_j), \tag{3}$$

where $G(\cdot)$ is the cumulative distribution function of $\xi = \varepsilon_j - \varepsilon_i$, and ε is a random noise with a known distribution that has $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma^2$. Therefore, $\xi \sim g(\xi)$, where $g(\cdot)$ is the probability distribution function for $G(\cdot)$, with $E(\xi) = 0$ and $E(\xi^2) = 2\sigma^2$. The tournament CSF can be considered as a special case of the APA when the impact function has the form $x_i = e_i + \varepsilon_i$. The all-pay auction provides a deterministic rule for deciding the contest winner while the other two CSFs, tournament and logit-form, are stochastic.

The cost of effort is modeled as a function $c_i = \gamma_i(e_i)$ with $\gamma_i'(\cdot) > 0$ and $\gamma_i(0) = 0$. This function $\gamma_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ usually takes either a linear $\gamma''(\cdot) = 0$, or a convex $\gamma''(\cdot) > 0$ form. In the latter case, larger effort becomes costlier. Within this setting, a representative risk-neutral player i maximizes the following expected utility,[9]

[9] In this model, CSF can be interpreted either as the probability of winning an indivisible prize when players are uncertain about the designer's selection criteria or as a share of a divisible prize after competing, see Corchón and Dahm (2010) and Chowdhury, Sheremeta, and Turocy (2014). For normative justifications of CSF, see Corchón (2007), Garfinkel and Skaperdas (2007), and Konrad (2009).

$$EU_i = p_i(\boldsymbol{x})v_i - c_i = p_i(f_i(e_i), \boldsymbol{f_{-i}(e_{-i})})v_i - \gamma_i(e_i). \tag{4}$$

We use this general setting and notations to develop the theoretical arguments proposed in the relevant literature.[10] As mentioned earlier, we assume that there is a contest designer who wants to maximize the total effort in the contest, unless otherwise specified. The contest designer can modify the contest setting for achieving the relevant objective(s).[11] Whatever the reasons for modifications may be, we consider any artificial manipulation by the designer as a 'bias' in the contest (and the corresponding act as biasing therefore). We stick to the n-player model wherever possible. The usual implication of higher (lower) heterogeneity in an n-player model is a mean-preserving increase (decrease) in variation of costs or valuations or effort impact. Majority of the findings in the literature, however, is derived in a setting with two players. In all such cases, we assume that player 1 is "stronger" than player 2 due to their features, or backgrounds, or because of historical privileges. The relative advantage of player 1 can be modeled as higher effort impact ($x_1 > x_2$), higher prize valuation ($v_1 > v_2$), lower cost ($c_1 < c_2$), or through a combination of more than one of these elements. We consider complete information unless otherwise specified.

3 Player heterogeneity and the need for competitive balance

There is a sizable discussion in the literature about the desirability of achieving competitive balance from the designer's perspective. We discuss the main propositions regarding the same using the theoretical framework described in Section 2. The equal opportunity approach to leveling the playing field (Roemer, 1998) suggests compensating players for their circumstantial differences; such that success is substantially sensitive to effort, regardless of societal or environmental elements. This perspective considers whether ensuring a close competition can serve the total effort maximizing interest of a designer. This literature thus seeks positive arguments to support leveling-the-playing-field policies which will also be desirable according to the normative arguments presented in Section 1.1.

In sports contests, for example, the designer gains from high audience interest. Audience interest increases with the expected intensity of competition and the consequent uncertainty in outcome. In an empirical study on major league baseball in the '90s, Schmidt and Berri (2001)

[10]See Nitzan (1994), Corchón (2007), Konrad (2009), Vojnović (2015), Corchón and Serena (2018), and Fu and Wu (2019) for comprehensive accounts of the literature on contest theory.

[11]Drugov and Ryvkin (2017) and Serena (2017b) discuss the most common objectives of the designer. Depending on the application context, the designer could be interested in maximizing aggregate effort (e.g., sports), or minimizing aggregate effort (e.g., election). The designer could also be interested in maximizing the winner's effort (e.g., R&D), the players' expected utility (e.g., auctions), or the winning probability of the best candidate (e.g., recruitment).

observe that competitive balance increased in spite of a widening gap between the rich and the poor teams, improving the audience attendance significantly. In a contest where the designer benefits from a close competition between players, Runkel (2006a) shows that increasing the strongest player's (or the favorite's) effort cost improves closeness by reducing the gap in expected winning probabilities, but it also reduces total equilibrium performance. Often these policies may also help the designer to increase aggregate effort when there is a lack of incentives to compete. As pointed out by Rottenberg (1956), "the nature of the industry is such that competitors must be of approximate equal size if any are to be successful." This is because in contests with heterogeneous players, large enough gaps in players' abilities and/or endowments may result in different success probabilities even when players exert similar efforts. Different success probabilities mean different expected payoffs, which may lower effort incentives for all players. This is commonly called the 'discouragement effect'.

The discouragement effect may seem undesirable on the normative grounds of player welfare and distributive justice. However, as we discuss later, whether heterogeneity among players trigger the discouragement effect and thereby results in a lower total effort is debatable.

3.1 Arguments for leveling the playing field

There is an array of empirical studies on the role and importance of competitive balance. Many of those concentrate on sports and their results substantially support the discouragement effect argument. For example, Sunde (2009) shows that, under increasing heterogeneity, weaker players (underdogs) in tennis tournaments are more affected by the discouragement effect. In professional golf tournaments, Brown (2011) finds that average score of players falls in the presence of a superstar like Tiger Woods. In amateur golf tournaments, Franke (2012b) finds that leveling the playing field has a positive impact on performance. Similar results are found in horse racing where handicapping the favorite horses results in an increase in the likelihood of winning of the other horses (Brown and Chowdhury, 2017).

- **Simultaneous contests with heterogeneous players:** A handful of studies (Baik, 1994; Nitzan, 1994; Stein, 2002; Cornes and Hartley, 2005) have theoretically shown that, in the presence of linear costs, aggregate effort in a lottery contest falls due to the discouragement effect from increased heterogeneity. Most of these studies (except Stein, 2002) model heterogeneity in terms of effort impact. The weaker players have lower odds of winning due to lower impact, which in turn may reduce their incentive to exert more effort. Hence, both lower impact and lower incentive reinforce each other. For the favorites on the other hand, higher impact translates into higher winning probability for a given effort level, and thereby results in lower effort incentives. Consequently, both players spend lower effort in the equilibrium compared to a perfectly leveled contest.

The discouragement effect has also been studied using laboratory experiments (for a survey on contest experiments, see Dechenaux, Kovenock, and Sheremeta, 2015). A within-subject experiment in Hart et al. (2015) studies two sources of heterogeneity: budget and prize valuations. They do not find total effort to be statistically different between the homogeneous treatment and the treatment with heterogeneous prize valuations. However, introducing heterogeneous budgets reduces total effort compared to the homogeneous case. March and Sahm (2017) find evidence supporting the discouragement effect in the laboratory, although this effect is not statistically significant for the strong players. The authors explain this result with disappointment aversion. Players experience lower utilities if they receive a pay-off lower than the expected pay-off. At moderate levels of heterogeneity and high prize value, the discouragement effect from heterogeneity and the positive incentive due to anticipated disappointment from losing cancel each other for the strong players so that their final effort does not differ from the symmetric case. For weaker players, the discouragement effect lowers effort for high prize value and the authors suggest the designer not to inform players about large levels of heterogeneity.

- **Dynamic contests with homogeneous players:** Another way the discouragement effect can arise is when initially homogeneous players become eventually heterogeneous in a dynamic contest. This is because an initial success can put one player at an advantageous position which then persists over the course of the contest. The literature on dynamic patent races is relevant in this context (see Grossman and Shapiro, 1985; Harris and Vickers, 1985, 1987; Budd, Harris, and Vickers, 1993). Each player invests effort in each period and the one to reach the finishing line first is the winner. Harris and Vickers (1985) show that the players' prize valuations and effort impacts relative to their distances from the finishing line determine the nature of the competition in a given stage. In particular, these parameter values jointly define ranges in which one of the two players enjoys an absolute strategic advantage and the other drops out, and ranges where both players lack competitive incentives. With a large enough lead, the leading player's effort incentive is high enough to pre-empt the opponent. Both Grossman and Shapiro (1985) and Harris and Vickers (1987) show that the leading player exerts higher effort compared to the lagging player, and efforts increase as the gap between the players decreases. Klumpp and Polborn (2006) and Konrad and Kovenock (2006) consider the discouragement effect in a sequence of simultaneous-move component contests where success requires a player to win a target number of these component contests before the other players meet their targets. Using a logit CSF, Klumpp and Polborn (2006) show that success in the first component contest increases the continuation value for the winner thereby creating asymmetric incentives in the later contests (the "New Hampshire Effect"). Consequently, the first component contest is the most effort intensive. In an APA framework, however, the component contest which gives an absolute advantage to the respective winner (so that he or she can win the remaining contests with no effort) can occur later in the sequence (Konrad and Kovenock, 2006).

If the discouragement effect is strong enough, it can even rationalize artificially biasing an initially homogeneous contest to solve incentive problems arising out of such subsequent heterogeneity. In organizational contexts, Meyer (1992) argues that biasing the final contest (promotion) can encourage effort in the first contest (interim competition). Denter and Sisak (2016) show that, in a two-stage contest with homogeneous players, a biased CSF increases the highest effort without reducing aggregate effort. In a best-of-three contest, Barbieri and Serena (2018) suggest biasing earlier contests to mitigate the incentive problem in the third contest when there might be a clear winner.[12] Malueg and Yates (2010) uses data from professional tennis matches to show that strategic adjustment in effort by players results in equal winning probabilities by the third round, thus supporting the theoretical predictions for dynamic best-of-three contests.

3.2 Arguments against completely leveling the playing field

In contrast to the above arguments in favor of leveling the playing field, there also exist substantial theoretical and empirical evidences showing that some heterogeneity can be desirable in a contest. Drugov and Ryvkin (2017) argue that the overall role of biases in a contest is to equalize the marginal benefit of effort across players. They provide a novel classification of different types of biased CSFs and a general framework for analyzing several possible objectives of the contest designer. They discuss when a biased contest is preferable to an unbiased contest under the different classifications and designer objectives. In a number of scenarios, a biased contest is efficient even when the players are ex-ante symmetric. Some specific instances from the broader literature of when some heterogeneity can be beneficial for maximizing total effort in a contest are as listed below.

- **Flatter marginal costs:** In an n-player contest where individuals face a symmetric logit CSF and quadratic costs, Ryvkin (2013) shows that heterogeneity in terms of prize valuation (or a multiplicative cost parameter) has a positive effect on aggregate effort when effort costs are relatively flat,[13] and a negative effect on aggregate effort when effort costs are very steep. These findings are robust under both complete and incomplete information. Moreover, Ryvkin (2013, footnote 9) also argues that reformulating the model structures of Stein (2002) and Cornes and Hartley (2005) to accommodate heterogeneous costs yields similar results.

[12] For elimination contests, see Cohen et al. (2018) and Fu and Wu (2018a).

[13] The support in Section 4.5 explains the equivalence between heterogeneity in terms of pirze valuations and costs.

- **Designer maximizes the highest effort:** A moderate level of heterogeneity may be desirable in a contest where the designer's objective is to maximize the highest individual effort or to choose the highest ability player as the winner.
  - **Complete information:** In a complete information environment, Seel and Wasser (2014) show that a moderate level of heterogeneity is desirable to a designer who maximizes a weighted sum of the expected highest effort and the expected average effort.
  - **Incomplete information:** If the designer lacks information about the players' types and one of her objectives is to choose the high-ability player, then introducing asymmetry in the contest may be optimal. Cohen, Kaplan, and Sela (2008) consider a set-up in which the players' types are private information and the value of winning depends on the corresponding player's type as well as actual reward-value which is effort-dependent. Due to the effort costs being strictly increasing in effort and strictly decreasing in player-type, the effort-dependent rewards can be considered as a tool to preserve heterogeneity. In a static all-pay auction where both abilities and efforts are privately observable to players, Pérez-Castrillo and Wettstein (2016) claim that a discriminatory contest (a contest where the prize value depends on the winner's identity) strictly dominates a non-discriminatory one when the players' abilities exhibit a concave distribution and the contest designer's objective is to select the high type player. They, in fact, invoke a discriminatory contest even when players are similar to begin with. Kawamura and de Barreda (2014) obtain the same result when players know their own as well as each other's types but the designer does not know the players' types. Gürtler and Gürtler (2015) consider across-firm hiring contests where external employers base their assessments and wage offers on observed career-path of the candidates and the marginal gain from a promotion is higher for a low-ability candidate. A moderate level of heterogeneity increases total effort by enhancing the effort incentives for the low-ability players in such a setting.

In a lab experiment, Fallucchi and Ramalingam (2019) study the impact of heterogeneity stemming from different sources (budget, abilities, prize valuation) on total effort. They design the experiment as a two-player lottery with complete information where the different sources of heterogeneity should have similar impact on aggregate effort. However, they find that the treatment with heterogeneous abilities brings the maximum aggregate effort - even higher than the symmetric treatment. Moreover, the stronger players exert similar efforts in all the heterogeneous and the homogeneous conditions, while the weaker players exert higher (lower) effort in the heterogeneous ability (heterogeneous budget or valuation) condition in comparison to the homogeneous condition.

Overall, there are mixed findings on the impact of heterogeneity on players' incentives to compete. The next section describes different mechanisms to level the playing field and studies whether each mechanism can enhance aggregate effort.

4 Mechanisms for leveling the playing field in contests

Policies to achieve higher competitive balance in a contest can be motivated by revenue concerns, ethical concerns, or both. AA policies, for example, aim to increase the diversity in competitive outcomes. This can be measured by the diversity among the winners, or among the active players. Policy makers manipulate the contest setting in order to increase the win probability of those players who belong to a disadvantaged group. Policies aimed at leveling the playing field may reduce the cost differences by increasing (decreasing) advantaged (disadvantaged) players' marginal effort costs, or balance the gap in success probabilities by attaching higher weights to the disadvantageous players' efforts. Leveling the playing field through introducing a bias in the effort impact functions or modifying the effort cost functions is commonly referred to as 'handicaps' and 'head starts'. The term 'handicap' originates from a game called *hand in cap*, played in the 17th century Britain, where a neutral umpire would determine the odds in an unequal contest between two bettors. By the middle of the 18th century, the term was used in the context of a horse race where an umpire decided the weight to be carried by each horse in order to equalize the chances between superior and inferior horses. A 'handicap' policy in a contest could be any sort of manipulation that curbs the favorite's incentives by reducing her expected payoff. A 'head start', on the other hand, usually refers to a policy of directly favoring the a-priory disadvantaged player. This convention is with historical reference to the early childhood support program launched by the United States Department of Health and Human Services in 1965. The term 'handicap' therefore has a negative connotation while the term 'head start' has a positive one. However, there is no distinction between 'handicap' and 'head start' from a modeling point of view as both policies arguably have the same effect on the incentive structure, ceteris paribus. We return to this point later while discussing experimental results.

In a generalised framework, a contest designer in theory can introduce a variety of biases to affect the competitive outcome in order to enhance total effort, to achieve competitive balance, or to increase efficiency of the contest mechanism by achieving the designer's objective at the minimum cost. However, any of such actions affects all players by modifying the overall incentive structure of the game. Given a player's payoff function (4), the different mechanisms commonly implemented to level the playing field are:

1. The choice of the CSF (p_i).

2. Biasing the contest through the effort impact function (f_i).
3. The choice of the reward valuation (v_i)
4. Modifying the reward structure (changing either p_i or v_i, or both).
5. Modifying the cost of effort (γ_i).
6. Introducing caps on effort (limiting $\bar{e}_i$).
7. Miscellaneous mechanisms.

In what follows, we classify the existing contest literature based on the different mechanisms listed above and discuss the major findings on optimal contest design under heterogeneity. Throughout this survey, optimality refers to total effort maximization by the contest designer, unless otherwise stated. For each mechanism, we summarize the overall findings in one or more 'observations'.[14]

4.1 Mechanism 1: Choice of the CSF

The choice of the contest success function (p_i in equation 4) has been widely researched in the literature. As mentioned in Section 2, the APA can be considered a special case of the logit CSF when $x_i = e_i^r$ and $r \to \infty$. This form of the effort impact function offers full flexibility in terms of how sensitive the contest outcome is to individual effort outlays. As r increases, the contest outcome becomes increasingly sensitive to individual effort discrepancies. The parameter r is often interpreted as the returns to scale from effort (Perez-Castrillo and Verdier, 1992; Baye, Kovenock, and De Vries, 1994; Nti, 1999): $r > 1$ indicating increasing returns and $r < 1$ indicating decreasing returns. We interpret r as the noise in the CSF (Amegashie, 2006; Jia, 2008; Balart, Chowdhury, and Troumpounis, 2017). Setting $r = 0$ implies that the contest outcome is completely random and independent of individual effort levels; when $r = 1$, the contest takes the form of a raffle or lottery, and when $r = \infty$, an all-pay auction is staged. Therefore, as r increases (decreases), the noise in the contest outcome decreases (increases), and the contest becomes more deterministic (stochastic). Therefore, the level of noise can bring different incentives to invest effort and it constitutes a mechanism to level the playing field. In what follow, we consider the general Tullock CSF given by the effort impact function $x_i = e_i^r$ with the logit CSF (1), that is, $p_i(\boldsymbol{x}) = \frac{e_i^r}{\sum_{j \in N} e_j^r}$ (also known as the power CSF). First we focus on the designer's choice between two contest regimes, a lottery ($r = 1$) and an APA ($r \to \infty$). Then, we focus on the optimal choice of continuous noise level $r \in [0, \infty)$.

[14] 'Observations' are numbered according to the respective main mechanisms and sub-mechanisms, in that respective order. For example, the first observation made for sub-mechanism 2 (Quotas) under main mechanism 4 (Reward structure) is numbered as 4.2.1.

4.1.1 Choice of contest regime

Consider n players competing for a prize and players are indexed according to their prize valuations such that $v_1 \geq v_2 \geq \cdots \geq v_n > 0$. The CSF takes the form $p_i(\boldsymbol{x}) = \frac{e_i^r}{\sum_{j \in N} e_j^r}$, and the designer has the flexibility to choose between two contest regimes, $r = 1$ and $r = \infty$, in order to maximize aggregate effort (Fang, 2002). In addition, the designer can choose the optimal set of players according to their prize valuations.

Observation 1.1. *When the contest designer can choose the competing set out of a finite set of players with given prize valuations, then*

(i) excluding some high valuation players can be optimal in an APA but is never optimal in a lottery;

(ii) total effort under the optimal lottery is higher than total effort under the optimal APA if the two highest valuations among the competing players in the APA are sufficiently close but not the same;

(iii) total effort under the optimal lottery is lower than total effort under the optimal APA when players are perfectly homogeneous.

Support: The most extreme form of handicap is the exclusion principle coined by Baye, Kovenock, and De Vries (1993) who show that excluding the player with the highest valuation may generate highest total effort in an APA. The expected total effort in an APA is maximized when the set of active players is given by $\{k^*, k^*+1, ..., n\}$, where k^* is such that

$$\left(1 + \frac{v_{k^*+1}}{v_{k^*}}\right) \frac{v_{k^*+1}}{2} \geq \left(1 + \frac{v_{i+1}}{v_i}\right) \frac{v_{i+1}}{2} \forall i \in N. \tag{5}$$

The expected total effort in an APA with a total effort maximizing designer is given by

$$R^{\text{APA}}(k^*, ..., n) = \left(1 + \frac{v_{k^*+1}}{v_{k*}}\right) \frac{v_{k^*+1}}{2}. \tag{6}$$

The rank order of players' valuations have crucial importance in an APA. Depending on the rank order of their valuations, some players may never spend any positive effort. On the other hand, the total effort maximizing contest designer has an incentive to artificially exclude some players with a higher valuation than v_{k^*} who would otherwise spend positive expected efforts in mixed strategy (Baye, Kovenock, and De Vries, 1993). For $v_1 = v_2 = ... = v_m \geq v_{m+1} \geq ... \geq v_n$, players $m+1$ through n certainly spend zero effort in equilibrium, but the designer does not gain by excluding any player. For $v_1 > v_2 = ... = v_m \geq ... \geq v_n$, however, player 1's expected effort in equilibrium is positive but the contest designer gains from excluding player 1 from the contest. See Baye, Kovenock, and De Vries (1996) for a complete characterization under complete information.

Fang (2002) invalidates the exclusion principle in a lottery and claims superiority of the lottery over the APA by showing that total effort maximization under the lottery does not call for any artificial exclusion and generates higher equilibrium total effort for the designer as well as higher expected total player surplus. The total effort is given by

$$R^L(1, 2, ..., n) = \frac{n^* - 1}{\sum_{i=1}^{n^*} \frac{1}{v_i}} \tag{7}$$

where $N^* = \{1, 2, \cdots, n^*\}$ is the set of players spending positive effort in the equilibrium and is defined as below.

$$N^* = \left\{ i : \frac{i-1}{v_i} < \sum_{j=1}^{i} \frac{1}{v_j}, \quad i = 1, 2, \cdots, n \right\} \quad \text{and } n^* = |N^*| \tag{8}$$

Note that if $m \in N^*$, then it must be that $m - 1 \in N^*$ for all $m = 2, \cdots, n$. All players $i \leq n^*$ spend positive effort and all players $i > n^*$ spend zero effort at the unique pure-strategy equilibrium of the lottery. Among the players spending positive effort in equilibrium, those with higher valuation spend higher effort. Excluding any $i \leq n^*$ reduces total effort, and excluding any $i > n^*$ leaves the total effort unaffected. Hence, exclusion is not optimal in the lottery.[15] In other words, $R^L(1, \cdots, n) \geq R^L(1, \cdots, m)$ for all $1 \leq m < n$. Hence, $R^L(1, \cdots, n) \geq R^L(1, 2)$. But $v_1 \geq v_2 \geq ... \geq v_n$ ensures that $\frac{1}{v_1} + \frac{1}{v_2} \leq \frac{1}{v_{k^*}} + \frac{1}{v_{k^*+1}}$ for all $k^* \in \{1, ..., n-1\}$, so that $R^L(2) = \left(\frac{1}{v_1} + \frac{1}{v_2}\right)^{-1} \geq \left(\frac{1}{v_{k^*}} + \frac{1}{v_{k^*+1}}\right)^{-1} = \frac{v_{k^*} v_{k^*+1}}{v_{k^*} + v_{k^*+1}}$. It is straightforward to show that $R^L(n) - R^{APA}(n) \geq R^L(2) - R^{APA}(n) > 0$ if $\frac{v_{k^*} v_{k^*+1}}{v_{k^*} + v_{k^*+1}} - \left(1 + \frac{v_{k^*+1}}{v_{k^*}}\right) \frac{v_{k^*+1}}{2} > 0$. This condition is true whenever $v_{k^*+1} < (\sqrt{2} - 1) v_{k^*}$. A total effort maximizing contest designer, therefore, prefers the L over the APA as long as the top two players in the optimal APA are not too close in terms of their reward valuations.

Note however, that this conclusion is reversed for a perfectly homogeneous contest with finite number of players; the equilibrium total effort indicates under-dissipation in a lottery ($v_i = v \quad \forall i \in N$ implies $R^L(1, 2, \cdots, n) = (n-1)v/n$ from equation 7) and full dissipation in an APA ($v_i = v \quad \forall i \in N$ implies $R^{APA}(1, 2, \cdots, n) = v$ from equation 6). ■

[15] Cohen and Sela (2005) show that the exclusion principle can be endogenously obtained for a lottery with winner reimbursement, where the strongest player stays out if she/he is not too strong compared to the other players. Matros and Armanios (2009) also consider a lottery with reimbursements and show that total effort is maximized (minimized) with winner (loser) reimbursement.

4.1.2 Continuous choice of noise level in a general stochastic contest

Next we assume that the designer has full flexibility in choosing $r \in [0, \infty)$ in the general Tullock CSF. As the level of noise in a contest decreases (or, r increases), a player spending marginally more effort has a higher increase in his/her probability of winning, ceteris paribus, and this encourages the stronger player to invest more (Nti, 2004). Therefore, tweaking r is equivalent to tweaking the level of noise in the contest outcome. For example, in cricket, the match length can have different formats going from a couple of hours to 5 days, allowing the noise of the outcome decrease with time. Similarly, the winner of a tennis match is determined either by best-of-three or best-of-five set format, allowing the noise of the outcome decrease with the number of sets.

Observation 1.2.1. *Ceteris paribus, in a two-player logit (Tullock) contest*

(i) total effort is maximized under a positive amount of noise in the effort impact function;

(ii) the optimal level of noise decreases with higher levels of heterogeneity.

Support: In a two-player contest with $v_1 \geq v_2$, there exists an $\bar{r} \in (1, 2]$ such that $(v_1/v_2)^{\bar{r}} = 1/(\bar{r} - 1)$, and there is a unique pure strategy equilibrium if and only if $r \in (0, \bar{r}]$ (Nti, 1999, Proposition 3). Further, for $r \in (0, \bar{r}]$, total effort increases with r when heterogeneity is low enough, i.e., $v_1/v_2 \leq 3.57$ (Wang, 2010), and is a concave function of r when heterogeneity is higher. When $r \in [\bar{r}, 2]$, the stronger player always spends a unique effort and the weaker player randomizes between staying inactive and participating with a unique effort (Wang, 2010), and this equilibrium is unique (Ewerhart, 2017b). When $r \geq 2$, there is an APA equilibrium in which both players randomize (Alcalde and Dahm, 2010; Wang, 2010).[16] Figure 1 exemplifies these results through a contest with two players with high heterogeneity ($v_1/v_2 = 6$) or low heterogeneity ($v_1/v_2 = 2$).[17] For the case with high heterogeneity, total effort achieves its maximum value, around 0.146, when $r \approx 0.861 < \bar{r}$. For the case with low heterogeneity, total effort is maximized (with a value around 0.416) when $r = \bar{r} \approx 1.383$. This example provides us with two important intuitions. First, total effort is maximized at some $r \in (0, 2)$ in both cases, indicating that it can be optimal for the contest designer to inject some noise in the contest. Second, depending on the level of heterogeneity among the players, the designer should inject different levels of noise in the

[16] Ewerhart (2017a) shows that for a generic probabilistic contest, the all-pay auction equilibrium is robust when the probabilistic contest is decisive enough, that is, when a player's odds of winning largely increases with a marginal increment in effort. Furthermore, any equilibrium of the probabilistic contest is payoff-equivalent and total effort-equivalent to the corresponding APA equilibrium when the later is unique. Otherwise, there may be multiple payoff-nonequivalent equilibria.

[17] Wang (2010, p. 9) provides the example of high heterogeneity in costs $c_2 = 6c_1$, which can be easily transformed to heterogeneity in prize valuations. Figure 1 provides the same example as in Wang (2010) for high heterogeneity and an additional example for low heterogeneity with $c_2 = 2c_1$ or $v_1 = 2v_2$.

contest to extract the maximum possible effort. ■

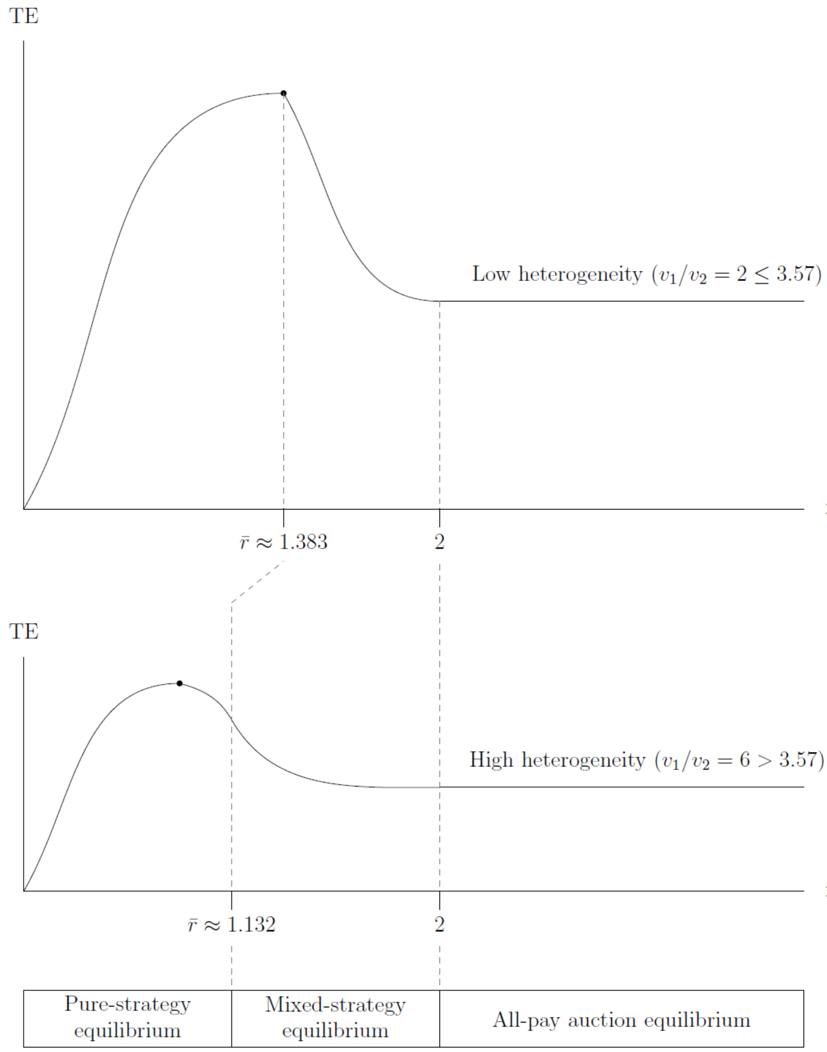

Figure 1: Relationship between total effort (TE) and the accuracy level (r) in a Tullock (1980) contest with 2 players and prize valuations satisfying either $v_1/v_2 = 6$ (high heterogeneity; Wang, 2010) or $v_1/v_2 = 2$ (low heterogeneity).

Observation 1.2.2. *Ceteris paribus, in an n-player logit (Tullock) contest*

(i) participation falls as noise in the effort impact falls;

(ii) the optimal noise depends on the shape of the cost function.

Support: In an n-player setting, Cornes and Hartley (2005) show that for an impact function of the form $x_i = \alpha_i e_i^r$ where α_i captures the natural heterogeneity among the players and the designer can only decide the universally applicable value of r, the number of active players at the equilibrium is bounded above at $r/(r-1)$ when $r > 1$. Therefore, the maximum number of active players falls as r increases.

Morgan, Tumlinson, and Vardy (2019) interpret the noise in performance ranking as the level of meritocracy in a contest. In other words, the lower the noise in effort outcomes, the more meritocratic the contest is. In an n-player Tullock contest, they show that the highest level of r for which there exists an equilibrium in pure strategies (again, let's denote this as $\bar{r}$) uniquely maximizes total effort when the effort costs are convex. For linear costs, total effort is maximized at any r above $\bar{r}$. They also show that this optimal r strictly decreases in the number of players, n. The authors obtain comparable results in an n-player tournament model, where they assume that the distribution of the noise in the effort impact, ε, is scaled down by a factor σ, where $\sigma \to 0$ implies perfect meritocracy. The optimal σ is unique (i.e., total output is maximized at $\sigma = \bar{\sigma}$) under convex costs, and there exists a continuum of values of $\sigma \in (0, \bar{\sigma}]$ which maximizes output and produces Pareto efficient outcomes when effort costs are linear. Participation is also at the maximal level at this optimal σ and players start dropping out as σ falls further below. In this framework, perfect meritocracy turns out to be sub-optimal for both homogeneous or sufficiently heterogeneous contests, but can be optimal for a moderate level of heterogeneity. ■

Another interesting possibility is that players, who are otherwise similar, may differ in terms of r_i as considered by Cornes and Hartley (2005). The individual best response for $r_i \leq 1$ is strictly positive (Perez-Castrillo and Verdier, 1992), whereas multiple possibilities exist for $r_i > 1$: player i has a strictly positive best response, two best responses $e_i^{BR} = \{0, (r_i - 1/r_i)v\}$, or zero best response depending on whether the total effort impact from other players ($\sum_{j \in N \setminus i} e_j^{r_j}$) is less than, equal to, or greater than the threshold value $(r_i - 1)^{r_i - 1}(v/r_i)^{r_i}$ (Perez-Castrillo and Verdier, 1992; Cornes and Hartley, 2005). The set of active players at the equilibrium may not be unique but there is a unique equilibrium for any active set.

4.2 Mechanism 2: Bias in the effort impact function

One of the most widely modeled tool for leveling the playing field is to introduce a bias in the effort impact function of the players. This can be best understood with reference to the effort impact function $x_i = \alpha_i e_i + \theta_i$ used by Gradstein (1995) and Runkel (2006b), as described in Section 2. The contest designer can bias the contest in favor of one of the players (or, a group of players) by assigning different parameter values to different players' impact functions. A multiplicative bias is modeled as $\alpha_i \neq \alpha_j$ and an additive bias is modeled as $\theta_i \neq \theta_j$. Recent findings show that the multiplicative and additive biases drive different incentives, and this has an impact on aggregate effort. Below we explain these differences and their interaction with the CSF.

4.2.1 Multiplicative bias

Under the sole application of multiplicative bias (i.e., normalizing the additive parameter to zero), the players' effort impact functions take the form $x_i = \alpha_i e_i$.[18] Suppose that the contest designer can affect players' effort impact functions by assigning a vector of weights $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n) \in (0, \infty)^n$, where α_i is the weight assigned to player i's effort. Accordingly modifying the CSFs for the logit (1) and the APA (2), we get the following.

$$p_i^L(\boldsymbol{\alpha}, \boldsymbol{e}) = \begin{cases} \frac{\alpha_i e_i}{\sum_{j \in N} \alpha_j e_j} & \text{if } \sum_{j \in N} \alpha_j e_j > 0 \\ \frac{1}{n} & \text{if } \sum_{j \in N} \alpha_j e_j = 0 \end{cases} \tag{9}$$

$$p_i^{APA}(\boldsymbol{\alpha}, \boldsymbol{e}) = \begin{cases} 1 & \text{if } \alpha_i e_i > \max\{\boldsymbol{\alpha}_{-\boldsymbol{i}}^T \boldsymbol{e}_{-\boldsymbol{i}}\} \\ \frac{1}{k} & \text{if } \alpha_i e_i = \max\{\boldsymbol{\alpha}^T \boldsymbol{e}\} \text{ and } k = \left| j \in N : \alpha_j e_j = \max\{\boldsymbol{\alpha}^T \boldsymbol{e}\} \right| \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

With the optimal multiplicative bias, total effort under the APA dominates the total effort under the lottery. Franke et al. (2014) study the optimal multiplicative bias to maximize total effort. They show that the conditional superiority of the lottery to the APA (Fang, 2002), as demonstrated in Subsection 4.1, is unconditionally reversed with the introduction of optimal multiplicative bias in the effort impact function.

The choice problem faced by the contest designer can be formulated as a three stage game - in the first stage the designer chooses between an APA and a lottery, in the second stage (s)he chooses the agent specific biases $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \cdots, \alpha_n\} \in (0, \infty)^n$, and in the final stage, the players choose their efforts. Standard backward induction then gives the Subgame Perfect Equilibrium.

Under this modified contest, the payoff function (4) can be written as $\pi_i(\boldsymbol{\alpha}, \boldsymbol{e}) = p_i(\boldsymbol{\alpha}, \boldsymbol{e}) v_i - e_i$ for all $i \in N$. Multiplying both sides by α_i gives the expression $\alpha_i \pi_i(\boldsymbol{\alpha}, \boldsymbol{e}) = p_i(\boldsymbol{\alpha}, \boldsymbol{e}) \alpha_i v_i - \alpha_i e_i$. This is equivalent to a standard contest with transformed efforts $\alpha_i e_i$ and transformed valuations $\alpha_i v_i$ for all $i \in N$. For heterogeneous biases, an ordered arrangement of these transformed valuations may be different from the order of the original valuations. Drawing a contrast with Section 4.1, in what follows, we consider the APA and the lottery under these transformed valuations which follows from introducing bias in the effort impacts.

[18] Clark and Riis (1998b) axiomatize contests with multiplicative bias.

Observation 2.1. *When the contest designer can choose the optimal bias vector for a finite set of players with given valuations under any contest regime, then*

(i) total effort in the optimally biased APA, which usually has two active players competing fiercely, is higher than total effort in the unbiased APA,

(ii) total effort and participation in the optimally biased lottery is at least as large as the same in the unbiased lottery and some heterogeneity is preserved, and

(iii) total effort in the optimally biased APA is higher than total effort in the optimally biased lottery.

Support: To analyze the APA under these transformed valuations, let us first consider an arbitrary bias where the weights are assigned so as to keep the order of the valuations same after the transformation, i.e. given $v_1 \geq v_2 \geq ... \geq v_n$, α_i for all i are chosen such that $\alpha_1 v_1 \geq \alpha_2 v_2 \geq ... \geq \alpha_n v_n$. For example, a vector $\boldsymbol{\alpha}$ with $\alpha_1 = \frac{1}{v_1}, \alpha_2 = \frac{1}{v_2}$ and $\alpha_i = \frac{1}{2v_3}$ $\forall\ i > 2$ satisfies this condition because the corresponding transformed valuations are then $\alpha_1 v_1 = \alpha_2 v_2 = 1$ and $\alpha_i v_i \leq \frac{1}{2} < 1$ for all $i > 2$. According to the standard properties of the APA (Theorem 1 in Baye, Kovenock, and De Vries, 1996), the transformed APA has a unique Nash equilibrium, where $(\alpha_1 e_1)^* = (\alpha_2 e_2)^* = \frac{1}{2}$ and $\alpha_i e_i = 0$ for all $i > 2$. The total effort is $\frac{1}{\alpha_1}(\alpha_1 e_1)^* + \frac{1}{\alpha_2}(\alpha_2 e_2)^* = \frac{1}{2}(v_1 + v_2)$. Hence, total effort under the optimal bias must be at least as much as this. On the other hand, arranging the transformed valuations in a decreasing order and applying the solution concept from Baye, Kovenock, and De Vries (1996) one can show that $\frac{1}{2}(v_1 + v_2)$ is also the upper bound to the maximum effort (see Franke et al., 2014, Proposition 3.3). Hence, the equilibrium total effort under the optimal bias vector $\boldsymbol{\alpha}^*$ is

$$R^{APA}(\alpha^*) = \frac{1}{2}(v_1 + v_2), \tag{11}$$

which is greater than v_2, the maximum total effort obtainable under unbiased APA for $v_1 \geq v_2 \geq \cdots \geq v_n$. The optimally biased APA therefore generates a higher total effort than an unbiased APA.[19]

Next, to consider a lottery under these transformed valuations, let $\tilde{N} = (\tilde{1}, \tilde{2}, \cdots, \tilde{n})$ denote the permutation of the player indices on N such that $\alpha_{\tilde{1}} v_{\tilde{1}} \geq \alpha_{\tilde{2}} v_{\tilde{2}} \geq \cdots \geq \alpha_{\tilde{n}} v_{\tilde{n}}$. Using (8), the

[19] This may not be true under incomplete information. For example, in a two-player auction with private valuation where one player enjoys a multiplicative bias, Walzl, Feess, and Muehlheusser (2002) show that a disadvantaged player with lower valuation does not win with positive probability. Therefore, the only possibility for inefficient allocation is when an advantaged player with lower valuation wins in the equilibrium. They obtain the expected welfare loss due to such inefficient allocation and show that such loss increases in the magnitude of the bias.

set of players exerting positive effort in equilibrium can be denoted as

$$\tilde{N}^* = \left\{ i : \frac{i-1}{\alpha_i v_i} < \sum_{j=1}^{i} \frac{1}{\alpha_j v_j}, \quad i = \tilde{1}, \tilde{2}, \cdots, \tilde{n} \right\} \quad \text{with } \tilde{n}^* = |\tilde{N}^*|. \tag{12}$$

Franke et al. (2013, Theorem 4.2(d)) show that an optimal bias in L yields a total effort of

$$R^L(\alpha^*) = \frac{1}{4}\left[\sum_{i \in \tilde{N}^*} v_i - \frac{(\tilde{n}^* - 2)^2}{\sum_{i \in \tilde{N}^*} \frac{1}{v_i}} \right]. \tag{13}$$

Since the complete set of possible biases also include the neutral policy $\boldsymbol{\alpha} = (1, 1, \cdots, 1)$, one can conclude that the total effort in the optimally biased lottery is at least as large as the total effort in the unbiased lottery. The optimal bias thus increases participation from weaker players at a relative disadvantage for the stronger players. However, this relative disadvantage should only be enough to induce both weaker and stronger players to exert higher effort. Franke et al. (2013) show that the natural ordering of players are preserved under the optimal bias, and the set of active players in the optimally biased lottery is at least as large as the set of active players in the unbiased lottery. The optimal bias cannot be determined uniquely, however.

Now let us consider a sufficiently small increase in the valuation of any player $i \in N$. Given (12), if i is strictly inactive (i.e., $i \notin \tilde{N}^*$) prior to this marginal change in valuation, then this small increase does not make him active. On the other hand, if i is indifferent between being active and not, then a marginal increase in valuation induces him/her to be active with an increase in total effort (Franke et al., 2014, Lemma 4.2.). Similarly, if i is strictly active prior to this marginal increase, then a marginal increase in his/her valuation further increases the total effort. Hence, total effort in a lottery with player valuations $(v_1, v_2, v_2, v_4, ..., v_n)$ is greater than the total effort in a lottery with $(v_1, v_2, v_3, ..., v_n)$. By induction, total effort with $(v_1, v_2, v_2, ..., v_2)$ is greater than the total effort with $(v_1, v_2, v_3, ..., v_n)$. Using the expressions for total efforts under the optimally biased APA (11) and the optimally biased lottery (13),

$$\begin{aligned} R^{APA}(\alpha^*, (v_1, v_2, ..., v_2)) - R^L(\alpha^*, (v_1, v_2, ..., v_2)) &= \frac{v_1 + v_2}{2} - \frac{1}{4}\left(v_1 + (n-1)v_2 - \frac{(n-2)^2}{\frac{1}{v_1} + \frac{n-1}{v_2}} \right) \\ &= (n-1)v_1^2 + 2v_1 v_2 - (n-3)v_2^2 \\ &= (n-1)(v_1^2 - v_2^2) + 2v_2(v_1 + v_2) > 0. \end{aligned}$$

Consequently, $R^{APA}(\alpha^*) > R^L(\alpha^*)$ must be true for $v_1 \geq v_2 \geq ... \geq v_n$.

It was noted earlier that the APA is optimal over the lottery when players are homogeneous, and the lottery is preferable over the APA when players are only sufficiently heterogeneous. The sub-optimality of APA under moderate heterogeneity comes from the potential exclusion of high-valuation players. In the biased APA, however, total effort is maximized without excluding the high-valuation player(s). On the other hand, the optimally biased lottery also encourages higher participation compared to the unbiased lottery. Referring back to the above analysis with the transformed valuations, it is apparent that the optimal bias in APA equalizes the stakes of the two highest-valuation players while the optimally biased lottery does not completely eliminate the heterogeneity. The optimal bias in APA thus ensures fierce competition between the two highest valuation players, and this competition effect outweighs the participation effect under the optimally biased lottery. ■

However, Epstein, Mealem, and Nitzan (2013) invalidate this unconditional superiority of the biased APA over the biased lottery in a two-player contest where the contest designer can choose both the effort weights ($\boldsymbol{\alpha}$) and the level of noise (r) in the contest. In such a framework, the expected total effort under the optimal logit contest equals the expected total effort from the optimal APA, i.e., $R^L(r^*, \alpha^*) = R^{APA}(\alpha^*)$. Fu and Wu (2018b) take a different approach to identify the optimal weights to maximize efforts. They consider that players can be heterogeneous not only in valuations but also in the level of noise in their individual effort impact r_i (this latter possibility has also been considered by Cornes and Hartley (2005), as discussed in Section 4.1.2). Assuming high amount of noise in the impact functions (i.e., $r_i \leq 1$ for all $i \in N$), they find the equilibrium winning probabilities that maximize total effort and the optimal biases that induce such probabilities.[20] It turns out that, for settings with more than two players, the optimal bias does not have to be monotone with the players' valuations. Given the number of players n, the distribution of their valuations and noise levels, the most favored players will be the ones who respond more sensitively to extra favoritism. Therefore, the optimal biases can favor the stronger players, the weaker players or even players in intermediate positions.

4.2.2 Additive bias

Next, let us consider contests where the players' impact functions have the form $x_i = e_i + \theta_i$, with θ_i being an additive bias. Such an impact function is often used to model head-start advantages.

Observation 2.2. *An optimally chosen additive bias increases total effort under both an APA and a tournament but is ineffective in a lottery.*

Support: Franke et al. (2014) and Liu and Lu (2017) analyze a lottery and an APA with additive bias and, similarly to Section 4.2.1, they find that total effort under an optimally chosen bias vector

[20] Moreover, Fu and Wu (2018b) consider maximizing the winner's expected effort as an alternative objective function for the designer.

is larger in the APA ($r = \infty$) than in the lottery ($r = 1$). In the lottery, the additive bias becomes a perfect substitute for individual effort and, thus, it is optimal for the designer to avoid the use of it. However, under APA, the optimal additive bias can increase total effort. In this case, total effort increases with the difference in valuations between the strongest player and the rest of the players.

Seel and Wasser (2014) consider additive biases or head starts in an APA with uniformly distributed iid private values. Here the contest designer maximizes a weighted sum of the expected average and the expected highest effort. They establish uniqueness of optimal head start which is positive only when the weight on the highest effort is large enough. Siegel (2014) also considers additive biases in both single-prize and multi-prize APAs. He assumes identical valuation but heterogeneous biases across players. If a unique equilibrium exists in a contest with m prizes and players are ordered such that $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$, then all players $i \in \{i : \theta_i \geq \theta_{m+1} + v\} \bigcup \{m+2, \cdots, n\}$ choose an effort equal to their respective head starts in that equilibrium.

Additive biases are also used in tournaments. Schotter and Weigelt (1992) were the first to study the players' behavior in a tournament setting with an additive bias in favor of the weaker player. In their model, players belong to either a cost advantaged group or a cost disadvantaged group. Their experimental findings show that total tournament effort is increased by the additive bias only when the cost difference between groups is severe; without the additive bias, disadvantaged players drop out of the competition. Their experimental finding, however, was contradictory to their theoretical result. Fain (2009) points out a shortcoming in the theoretical model in Schotter and Weigelt (1992) and presents a more complete argument of the effects of leveling the playing field in a tournament. In Fain (2009)'s two-player tournament model, the players are assumed to have the same valuation for the contest reward but different costs of effort. For the sake of consistency, we consider that the winner receives a common value v and the loser gets nothing. Assuming player 2 to be the cost disadvantaged player, the payoff functions of the two players are given by $\pi_1 = p_1 v - ce_1^2 \quad \text{and} \quad \pi_2 = p_2 v - \beta ce_2^2$, with $\beta > 1$ and $c < 1$. Under an additive bias α in favor of the weaker (cost disadvantaged) player, player 1 receives v if $x_1 > x_2 + \alpha$ and zero otherwise. Following equation 3, player 1's winning probability can be expressed as $F(e_1 - e_2 - \alpha)$, where $F(\cdot)$ is the CDF of $(\epsilon_2 - \epsilon_1)$. The payoff functions of the two players can therefore be written as $\pi_1 = F(e_1 - e_2 - \alpha)v - ce_1^2 \quad \text{and} \quad \pi_2 = (1 - F(e_1 - e_2 - \alpha))v - \beta ce_2^2$. The first order condition for payoff maximization can be written as $f(e_1 - e_2 - \alpha)v = 2ce_1 = 2\beta ce_2$. This can be solved to obtain the optimal efforts for given values of α, β, c and v. Further, using the implicit function theorem one can find $\partial e_2^* / \partial \alpha$ and setting this equal to zero, the optimal α can be found. Since $e_1 = \beta e_2$, the maximum value of e_2 ensures maximum total effort. If $F(\cdot)$ follows a normal distribution with zero mean and positive variance, then $f'(\cdot)$ is zero only when the functional argument takes a zero value. In the present context, that means $(\beta - 1)e_2 = \alpha$ or $e_2 = \alpha/(\beta - 1)$. But at $(\beta - 1)e_2 = \alpha$, the probability of winning of either player is $f(0) = 0.5$ implying complete leveling of the playing field. If, on the other hand, $F(\cdot)$ follows a uniform distribution, then the optimal efforts depend on the relative value of the effort difference $(e_1 - e_2)$ and α. Total effort as well as individual efforts are maximized at the mode of the distribution, where α is positive and the success probabilities of the two players are again one-half. Fain (2009) also shows that the total effort at this level is greater than the total effort with $\alpha = 0$. ■

The empirical observations by Estevan, Gall, and Morin (2019) about the effects of an AA policy in the form of an additive bias implemented by a Brazilian public university is worth a mention in this respect. The particular benefit is in the form of 30 bonus points added to the college admission scores of the public high school students, with a smaller additional bonus of 10 points for those who come from disadvantaged backgrounds. Building on the theoretical model of Stein (2002), they predict an increase (a decrease) in the performance and success probability of the receivers (non-receivers) of the bonus. The results indicate significant increase in the admission probability and college participation rate of public school students irrespective of their socio-economic background but there were no significant improvement in their test performance. In fact, the test score of the private school applicants exhibited a marginal improvement.

4.2.3 Additive and multiplicative biases

The multiplicative bias affects the marginal return on effort while the additive bias does not, and this may lead to different strategies and outcomes. A few example scenarios considered in the literature where the additive and multiplicative biases have been compared are as follows:

- **Simultaneous contest:** In simultaneous contests, Li and Yu (2012) study an APA with two players, where the contest designer decides the optimal bias towards the weaker player so as to maximize the contest revenue by completely eliminating the heterogeneity between the two players. They find that aggregate effort is larger under the optimal additive bias compared to the optimal multiplicative bias. Kirkegaard (2012) also considers a two-player APA with both additive and multiplicative biases. He shows that an additive bias favoring the weaker player always increases aggregate effort while a multiplicative bias that handicaps the stronger player may have mixed effects. He advises a combination of the two biases if there is a high level of heterogeneity between the two players' abilities. See Dahm and Porteiro (2008) for an assessment of multiplicative and additive biases in a rent-seeking context with a lottery CSF.

- **Sequential contest:** Segev and Sela (2014) analyze the effect of an additive versus a multiplicative bias in a sequential APA with two players. A multiplicative bias is likely to incentivize the first-mover to invest more effort but may cause the second-mover to withdraw under certain circumstances. An additive bias on the other hand, induces the second-mover to invest effort equal to the amount of the bias while the first-mover invest zero effort under most circumstances. For maximizing the total expected effort, they invoke a combination of additive and multiplicative biases.

- **Appropriation contest:** Another interesting two-period setting is considered by Konrad (2002), where the first period comprises of investment by the period 1 incumbent and the appropriation contest over the fruits of this investment takes place in period 2. The value of the prize in the second period contest is determined by the amount invested by the period 1 incumbent. Modeling incumbency advantage as a possible combination of an additive (head start advantage) and a multiplicative (productivity advantage) bias (i.e., the incumbent wins the contest whenever his/her effort falls short of the opponent's effort by no more than the bias size), the author shows that these biases set a critical threshold to the investment amount for obtaining positive appropriation efforts in period 2.

- **CES impact function:** Esteve-González (2016) analyzes contests through an impact function that is a Constant Elasticity of Substitution function. In that case, an incumbent's multiplicative (additive) bias is interpreted as a perfect complement (substitute) of effort to solve a moral hazard problem. It turns out that aggregate effort is maximized when the bias and the contest effort are not too complementary and, actually, they should be more substitutes when effort cost decreases.

While these examples can guide a designer choosing between either type of bias, the obvious question that follows is whether it is optimal to use both types of biases (as has been prescribed by Kirkegaard, 2012; Segev and Sela, 2014) when the designer have the flexibility to do so. The following observation presents the findings in the literature on the optimal bias when the effort impact function can have both types of biases at the same time, i.e., $x_i = \alpha_i e_i + \theta_i$.

Observation 2.3. *A combination of additive and multiplicative biases increases total effort in an APA. However, the optimal level of additive bias in a lottery remains zero even when the contest designer can combine the two types of biases.*

Support: Nti (2004) and Runkel (2006b) show that the total effort in the 2-player lottery is maximized when the effort of the low-valuation player is boosted up with a multiplicative bias equal to the ratio of the high value to the low value. Both of these studies consider effort impact functions with simultaneous application of additive and multiplicative biases. When all players must be active, the optimally biased contest has no additive bias and the optimal multiplicative bias equalizes the players' valuations such that $\alpha_1 v_1 = \alpha_2 v_2$. Nti (2004) further shows that when the designer enjoys full autonomy in the contest architecture, the optimal bias excludes the weaker player ($\alpha_2 = \theta_2 = 0$ following the notation in Section 2); and for the stronger player, there is a combination of additive and multiplicative biases such that $\theta_1 = -\alpha_1 v_1$. In this equilibrium the stronger player bids v_1 and the weaker player remains inactive. Franke, Leininger, and Wasser (2018) extend this result to more than 2 players, and compare the expected aggregate effort between a lottery and an APA. It turns out that multiplicative biases are more effective in lotteries because more participation is induced.[21] However, additive biases are never optimal in a lottery but are highly effective for maximizing effort in the APA. The authors generalize the total effort dominance result of Franke et al. (2014), and find that total effort is maximum if the designer combines multiplicative and additive biases under the APA. Aggregate effort in the lottery and the APA under the optimal additive and multiplicative biases can be unambiguously ranked as below. ■

$$\begin{aligned}
\text{If } v_1 = v_2, \text{ then } & \sum x_{(\alpha=\alpha^*,\theta=\theta^*,APA)} = \sum x_{(\alpha=1,\theta=\theta^*,APA)} = \sum x_{(\alpha=\alpha^*,\theta=0,APA)} \\
& > \sum x_{(\alpha=\alpha^*,\theta=0,L)} = \sum x_{(\alpha=\alpha^*,\theta=\theta^*,L)} \geq \sum x_{(\alpha=1,\theta=\theta^*,L)} \\
\text{If } v_1 > v_2, \text{ then } & \sum x_{(\alpha=\alpha^*,\theta=\theta^*,APA)} > \sum x_{(\alpha=1,\theta=\theta^*,APA)} > \sum x_{(\alpha=\alpha^*,\theta=0,APA)} \\
& > \sum x_{(\alpha=\alpha^*,\theta=0,L)} = \sum x_{(\alpha=\alpha^*,\theta=\theta^*,L)} > \sum x_{(\alpha=1,\theta=\theta^*,L)}
\end{aligned}$$

4.3 Mechanism 3: Reward valuations

Heterogeneity among players is often modeled with heterogeneity in reward valuation, and the convention is to model the stronger player as the high-value player and the weaker one as the low-value player. However, Epstein, Mealem, and Nitzan (2011) point out that whether the higher value represents the stronger player depends on the contest scenario. Higher valuation for a monetary prize may indicate relative poverty, whereas higher valuation for an environmental or trade policy may be typical of the wealthier class. Similarly, Kräkel (2012) argues that in a given contest, the disadvantaged (advantaged) player(s) may have higher (lower) valuation due to their lower (higher) outside options, which increases (decreases) their effort incentives and the resulting winning probability. When the contest designer cares about both total effort and player welfare, higher concern for players' welfare will bias the contest in favor of the high-value player when the CSF has a logit form. Bias is independent of the relative concern for player welfare if the contest is an APA. Thus, if high valuation represents lower income, then the socially optimal policy is to favor the low-income player. For high valuation representing the wealthier segment, favoring the

[21] In the Tullock (1980) setting, Ewerhart (2017b) shows that, in contests with 2 heterogeneous players, total effort with the optimal multiplicative bias strictly increases with r until $r = 2$ and is constant for $r \geq 2$.

wealthier player is socially optimal for a welfare maximising government. Reward values can be manipulated by taxing or subsidising the final rewards depending on the winner's identity.

Observation 3.1. *An optimally taxed APA generates at least as much total effort as an optimally taxed Tullock contest. An optimally taxed APA completely eliminates the heterogeneity in reward valuations, while an optimally taxed lottery still preserves some heterogeneity.*

Support: Taxation leaves the CSF intact as in an unbiased contest and, instead, it changes the reward values through post-contest taxation. In a two-player contest through optimal taxation of the reward, Mealem and Nitzan (2014) find support for Epstein, Mealem, and Nitzan (2013)'s conclusion about the total effort equivalence between an optimally biased lottery and an optimally biased APA (see Section 4.2.1). The realized reward value to player i upon player i's success is $v_i + \tau_i$. Player i therefore maximizes $\pi_i = p_i(e_i, e_j)(v_i + \tau_i) - e_i$, where $p_i(e_i, e_j)$ can be any contest technology. Using (11) with valuations $(v_1 + \tau_1, v_2 + \tau_2)$, the expected total effort in APA is $(v_2 + \tau_2)(v_1 + \tau_1 + v_2 + \tau_2)/2(v_1 + \tau_1)$. For the general Tullock CSF, the expected total effort in the pure strategy Nash equilibrium is $r(v_1 + \tau_1)^r(v_2 + \tau_2)^r(v_1 + \tau_1 + v_2 + \tau_2)/(v_1^r + v_2^r)^2$ for any $r \in (0, 2)$. Alcalde and Dahm (2010) show that there exists an equilibrium in mixed strategies for Tullock contests with $r > 2$ which is comparable to the APA equilibrium. Under either regime, the contest designer chooses (τ_1, τ_2) in order to maximize the expected total effort subject to the balanced budget constraint $p_1\tau_1 + p_2\tau_2 = 0$. Mealem and Nitzan (2014) find that the optimal taxation under APA is $(\tau_1^*, \tau_2^*) = (-0.5(v_1 - v_2), 0.5(v_1 - v_2))$, which equalizes the two players' valuations from winning. However, the optimal taxation under the Tullock technology with $r \in (0, 2)$ is such that $v_1 + \tau_1^* \geq v_2 + \tau_2^*$. Further, with the aid of Alcalde and Dahm (2010)'s neutrality result for $r > 2$, they show that expected total effort under the optimally taxed APA is at least as large as the expected total effort under Tullock. Further, in support of Franke et al. (2014), they show that the total effort maximizing taxation scheme under APA is to equalize the two players' final stakes whereas in a lottery, the optimal scheme (reduces but) still preserves inequality between the final stakes. ■

Another relevant study is by Gürtler and Kräkel (2010), who consider a two-player rank-order tournament and show that individual prizes allow the designer to fully extract the rent from the high-ability player by imposing a handicap. They also show that individual or identity-dependent prizes do not do worse than a uniform prize even when a negative handicap is not possible. While individual prizes still mean that the prize value to be obtained by a player is fixed ex-ante and ex-post, making the prize value a function of contest outcome or performance can also be useful.

Observation 3.2. *A reward scheme that distributes the reward according to players' effort contribution can be preferable in providing balanced incentives in a heterogeneous contest.*

Support: Jönsson and Schmutzler (2013) argue that the ratio of expected highest effort to expected total effort is higher in an all-pay auction with endogenous or effort-dependent rewards than in an all-pay auction with fixed reward amounts. Endogenous rewards are therefore suitable for designers interested in obtaining high expected highest effort while also limiting effort wastage. Another related study is Palomino and Sákovics (2004), that rationalizes the performance-based revenue sharing scheme of the European Leagues by the underlying structure of competition. There being multiple national leagues in Europe competing for the TV broadcast revenue, it is in the interest of each national league to incentivize the domestic teams to bid high prices for the star players. The only way the leagues can affect the teams' bidding incentives is through the TV revenue sharing scheme. Palomino and Sákovics (2004) argue that the leagues can do better with a performance-based revenue sharing as compared to an egalitarian sharing or a winner-take-all scheme.[22] This is because an egalitarian scheme will fail to incentivize the teams to bid high for the star players, and on the other hand, a winner-take-all scheme will lower the level of competition by creating a large incentive gap between the teams that get the star players and other teams who do not get a star player. However, if the league-winner has a higher revenue share, then all teams have incentive to play well whether or not they get a star player but also to bid high for the star players who increase their winning probability ex-ante. ■

4.4 Mechanism 4: Reward structure

A popular policy tool affecting competitive balance is the reward structure, also referred to as contest architecture. A designer with a given reward budget can decide how many prizes to award in a contest, and how to allocate the budget among the different prizes when there are more than one prize. We consider two different policy mechanisms: manipulating the number of prizes, and reserving some prizes for the disadvantaged players (or, quotas). Both mechanisms modify the level of competitiveness in a contest.

4.4.1 Number of Prizes

Moldovanu and Sela (2001) and Moldovanu and Sela (2006) show that this mechanism depends on the effort cost of the players; in particular, the optimality of single or multiple prizes depend on the shape of the cost functions. [23] When the CSF is an APA and each player exerts effort only once, it can be optimal for the contest designer to provide multiple prizes as long as both players

[22] Chang and Sanders (2009) show that the pool revenue sharing scheme, as practised in the Major League Baseball for example, works as a winning tax and a loser subsidy thereby injecting a discouragement effect which negatively affect total effort in the league. Each team under this scheme contributes a certain percentage of their locally generated revenues into a common pool which is then redistributed equally to all the participating teams.

[23] Sisak (2009) reviews the literature on multiple-prize contests.

are risk neutral and their effort cost is convex (or, players are risk-averse and their effort costs are linear). When the CSF has a logit form, total effort can be increased as long as the positive effect of the participation enhancement (an increment of the weaker players' effort) overcomes the discouragement effect on the stronger players.

Observation 4.1. *Splitting the designer's budget into multiple prizes increases total effort in a sufficiently heterogeneous contest. The optimal number of prizes and budget allocation among the different prizes depend on the level of heterogeneity among players.*

Support: There are several papers that study the optimality of multiple-prize contests under particular distribution of player types. See, for example, Szymanski and Valletti (2005) for L, Krishna and Morgan (1998) for tournaments and, for APA, Barut and Kovenock (1998),Glazer and Hassin (1988), Clark and Riis (1998a), Moldovanu and Sela (2001) and Cohen and Sela (2008).[24] Szymanski and Valletti (2005) argue that the effectiveness of increasing incentives via increasing the number of winning prizes depends on the distribution of strong and weak players. The authors consider a general Tullock CSF with 3 players and two prizes, the first prize and the second prize. When there are 2 strong players and one weak player, a single prize is optimal. However, when there is one strong player and two weak players, it is optimal to split the single prize into two prizes with the first prize being $(2+r)/(2+2r)$ fraction of the total budget. Note that this fraction decreases as the noise level r increases; at the limit case ($r = \infty$) of an APA, there is a single prize. Moreover, in their result, the second prize should never be larger than the first prize. Liu and Lu (2017) study the impact of the number of homogeneous and indivisible prizes on players' efforts. They consider an APA where players have private information on their costs. It turns out that the optimal number of prizes for maximizing the total effort and the expected highest effort are both larger than one but have an upper bound. Further, expected highest effort maximization requires fewer prizes to be awarded in comparison to total effort maximization.

Findings from a randomized control trial (Singh and Masters, 2018) carried out among salaried childcare workers in India is relevant in this context. It is found that instead of rewarding only the top performers, dividing the reward budget among all workers in proportion to the measured gains in their service outcome improves overall childcare outcome. These gains in overall outcome were led by better performance among the lower-ranked workers. Interestingly, this result contrasts with the finding in Cason, Masters, and Sheremeta (2020) where winner-take-all contests generate higher total effort compared to proportional-prize-contests in a laboratory contest experiment with homogeneous players. In a heterogeneous set-up, increased chances of receiving a reward helps eliminating the discouragement effect among lower-ranked players. ■

[24]For an analysis of the optimal prize structure in contests, see as well Myerson (1981), Moldovanu and Sela (2006), Azmat and Möller (2009), Fu and Lu (2009), Möller (2012), and Chowdhury and Kim (2017) among others.

In line with the literature on rationing, too many prizes in a contest discourage the stronger players while too few prizes discourage the weaker ones (see Dechenaux and Kovenock, 2011; Faravelli and Stanca, 2012). Instead of making all players compete for all the available prizes, the designer can reserve some prizes for the weaker players only, thus increasing the chance of the weaker players' representation among the winners. Examples include job positions for differently able candidates, additional grant for women scientists, or best paper award for graduate students in an international conference. Such a provision is equivalent to a winning quota.

4.4.2 Quotas

Quotas are arguably the most commonly practiced form of AA policy tools, especially in the contexts of educational subsidization and employee hiring. Yet, compared to other tools of affirmative action, theoretical analysis of quota policies are not so common in the contest literature.

Observation 4.2.1. *Quotas, as extra prizes reserved for the disadvantaged players, can enhance total effort given that there is competition among the disadvantaged players for the extra prize.*

Support: Dahm and Esteve-González (2018) study a lottery where total effort can be enhanced by putting aside a part of the total prize budget as an extra prize accessible by the disadvantaged players only. Then, similar to a quota, the contest must have at least one winner from the disadvantaged group. This particular prize structure thus excludes advantaged players from a part of the prize (partial exclusion principle), and has the potential of increasing total effort when the heterogeneity level is intermediate and there are at least 2 disadvantaged players competing for the extra prize. Fallucchi and Quercia (2018) analyze this model in the lab and find that the benefits of this AA policy can be undermined when participants have the possibility of retaliation. A similar prize structure in an APA is considered by Dahm (2018) and it turns out that, when there are 2 disadvantaged players and one advantaged player, it is always optimal to introduce an extra prize independently of the level of heterogeneity between groups. This result contrasts with the one found by Szymanski and Valletti (2005) which suggests that an extra prize brings different incentives than second prizes. However, complete exclusion of the advantaged player is never optimal. Ip, Leibbrandt, and Vecci (2020) study how organizational response to gender-quotas for managerial positions vary depending on whether women are actually discriminated against or not. In a laboratory experiment, they find that total effort in the quota treatment falls when female candidates are equally suited or less suited to their male counterparts on average but do not face discrimination in the selection process. However, if they are equally suited to the male candidates and yet face discrimination in the selection process, then average effort increases in the quota treatment. ■

In a loosely related set-up, Epstein and Mealem (2015) consider a situation where the contest designer can control the size of one of the two groups with opposite interests. They argue that limiting the size of one group is total effort maximizing under L if the limited group is weakly dominating. The result also holds if the limited group is dominated by the other group in stake size but there are sufficiently large number of players.

Observation 4.2.2. *Quotas (especially, gender-based quotas) enhance participation of the disadvantaged players (women) without a detrimental effect on total effort.*

Support: Several experimental studies find positive impact of quotas, especially gender-based quotas, in laboratory task environments. Beaurain and Masclet (2016) randomly assign subjects in the roles of employers and prospective employees and find that hiring of women significantly improved under mandatory AA without any compromise in team performance.[25] A standard experimental design for studying willingness to compete makes all subjects perform a real-effort task first under a piece-rate, then under a tournament, and finally asking the winners of the second stage tournament to choose between the two schemes in a third stage. A group of studies implement this design to study the potential gains in women's competitiveness under a gender-based quota. Based on the findings of an experiment with this design framework applied to teenager subjects, Sutter et al. (2016) show that performance does not deteriorate under a gender-quota and suggest quotas (and preferential treatments) from an early age to eliminate gender gaps in confidence and performance later on. Czibor and Dominguez-Martinez (2019) find that a quota at the intermediate stage of a dynamic tournament significantly increases female representation without harming efficiency. They find that women shy away from competition in absence of a quota, but performance gap between men and women disapears when half the winner positions are reserved for women. Consequently, the quota also results in a significant increase in women's selection into the tournament. Maggian, Montinari, and Nicolò (2020) also use a similar design to study the optimal implementation phase of a quota in a dynamic tournament. Based on their findings, a quota implemented only in the first stage does not increase women's willingness to compete but a quota exclusively for the second stage or for both stages increases competitiveness without efficiency loss in performance. In both Czibor and Dominguez-Martinez (2019) and Maggian, Montinari, and Nicolò (2020), however, the choice between piece rate and tournament is preceded by feedback on first stage tournament outcome and cannot be claimed to be purely led by quota provision. Participants in Kölle (2017)'s experiment experience a team-incentive in addition to piece-rate and tournament, and they choose between the piece-rate and the team-incentive in the final stage. The gender quota is implemented to the treatment group in the tournament stage. However, they find effort and concealment, as well as the selection into teams, to be independent of gender and quota

[25] The employers are penalized if they fail to respect the quota requirements.

implementation. ■

4.4.3 Prize allocation in dynamic contests

We know that dynamic contests may create asymmetric incentives even for players who are homogeneous to begin with, as has been previously discussed in Subsection 3.1. An interim success of a player may increase her continuation value (expected value from continuing to play) relative to the other player(s). If the higher continuation value is due to a higher probability of winning the final contest, then it sets on a 'strategic momentum'. If, on the other hand, the higher continuation value is simply due to a higher confidence level, then it's called a 'psychological momentum'. Mago, Sheremeta, and Yates (2013) distinguishes between strategic momentum and psychological momentum in a best-of-three contest. The strategic momentum results from equilibrium play, while the psychological momentum implies higher effort in the following round by the winner (loser) of a given round. In a laboratory implementation of the best-of-three lottery, they introduce an intermediate prize to capture the effect of the two momenta, and vary the noise in effort impact. They report evidence against the psychological momentum, and concludes that subsequent heterogeneity in effort is primarily driven by the strategic momentum.

Observation 4.3. *A balanced redistribution of the prize budget over the course of a dynamic contest can mitigate the asymmetric incentives arising out of intermediate outcomes.*

Support: When a dynamic contest is organised as a sequence of simultaneous-move component contests and winning the entire contest requires a certain number of component contests before the other players, the discouragement effect can make some of the later contests trivial (see Subsection 3.1). Introducing separate intermediate prizes for each of the component contests can make them non-trivial (Konrad and Kovenock, 2006). The laggard can then catch up with a positive probability. However, in the special case with ex-ante homogeneous players and constant prize value across all component contests, the laggard never has a higher expected effort than the leader. Clark and Nilssen (2018) shows that the opposite is possible when the dynamic structure of the contest is such that the net number of wins does not affect the continuation values but works as a head-start. This ensuing head-start may reduce effort over time due to a discouragement effect on both the leader and the laggard. In a simple two-period tournament, Casas-Arce and Martínez-Jerez (2009) show that effort in the second period takes an inverse u-shape relationship with the first period output. They mention that this "∩-shape is exaggerated with heterogeneity because confidence of leaders and demotivation of laggards arise earlier" (Casas-Arce and Martínez-Jerez, 2009, p. 1313). Based on empirical analysis of the retailers' performance in dynamic sales contests in a commodities company, they argue that interim rankings in a tournament may serve as important signals of heterogeneity even when initial achievement targets are set according to the size of the respective retailers (preferential treatment). They show that the leading retailers significantly reduce their sales efforts and lagging retailers increase their sales efforts unless they are far behind in terms of interim performance. Later, in a model of dynamic tournaments, Klein and Schmutzler (2017) agree that a larger gap in the interim performance reduces effort in the second period. To mitigate this problem, they suggest a redistribution of the entire prize value to the second period and increasing weight on first-period performance for final evaluation.

Players can also be asymmetric to begin with, in which case the contest outcome in the earlier stages of the dynamic contest may reinforce such heterogeneity or become pivotal depending on the winner's identity. Clark and Nilssen (2019) model a two-period all-pay auction where one player has a head start to begin (asymmetric θ_i in the effort impact function) with and the first-period winner gets a head start in the second period. The authors show that the optimal policy in such a scenario is to allocate the entire reward budget to the first period, effectively canceling the second period contest. They reconsider this problem of allocating a fixed reward budget over several rounds in Clark and Nilssen (2020). Here the players differ in their ability (asymmetric α_i in the effort impact function) and how ability increases further due to an early win. The optimal allocation of the reward budget for the two-period contest depends on the level of ex-ante heterogeneity. For large heterogeneity the optimal policy is to allocate the entire budget to the first period, just like the scenario with head-start advantage in Clark and Nilssen (2019). For intermediate heterogeneity, total effort is maximized when the entire reward budget is allocated to the second period. In case of small ex-ante heterogeneity, an equilibrium total effort as high as the reward value (full rent dissipation) is achievable when the competition can be organized over a longer sequence of contests. This result, however, also requires that the increment in ability consequent to an early win be sufficiently higher for the ex-ante disadvantaged player. ■

4.5 Mechanism 5: Cost of effort

Modifying the cost structure to create handicaps and head starts is another common practice for leveling the playing field. A decrease (increase) in the weaker (stronger) player's marginal cost can be interpreted as a head start (handicap). For example, in public procurement, providing subsidies (Ayres and Cramton, 1995; Brannman and Froeb, 2000) or bid discounts (Krasnokutskaya and Seim, 2011) to small and medium firms can be interpreted as head starts (see Section 5.2.1 for a more detailed discussion). In many sports (e.g., sports car, chess, golf), handicapping is used to standardize the outcomes of heterogeneous players by mapping their scores according to their individual abilities. In particular, these rules aim to equalize win probabilities of players when they incur the same effort costs, whether the effort spent by them is equal or not. Handicap rules have been analyzed in the literature in the context of golf tournaments (Franke, 2012b) and horse racing (Brown and Chowdhury, 2017).

To further understand the implications of this kind of handicaps, Franke (2012a) considers a contest with 2 or more players who differ in their linear costs $c_i = \gamma_i e_i$ and introduces a multiplicative bias $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_i, \cdots, \alpha_n)$ in the impact function of the general Tullock CSF (as in equation 9). The author analyzes the players' incentives under an equal treatment policy, i.e., $\alpha_i = \alpha_j = \alpha^{ET}$, and under an AA policy given by $\alpha_i^{AA} = \gamma_i^r$. In this AA policy, the ratio of the weights attached to the two players' efforts is proportional to the ratio of their marginal costs. It turns out that both the total and the individual efforts are higher under AA than under equal treatment in a two-player contest. But, in an n-player contest with $n > 2$, this result may not always hold. When all players are treated equally, there is a positive probability that only a subset of players $M \in N$ are active, thus incentivizing the stronger players to exert higher effort. Under the AA policy, on the other hand, all players are always active. This participation effect of the AA, in addition to the lower effort impact of the stronger players, can imply that AA drives a lower total effort compared to the equal treatment. The author concludes that, in the n-player game, AA is likely to generate a higher total effort compared to ET when the effort costs are not too heterogenous to begin with. Dahm and Esteve-González (2018) obtain a similar conclusion although considering a completely different AA mechanism that entails modifications on the reward valuation and structure (see Section 4.4.2).

Observation 5. *The impact of heterogeneity in terms of effort cost is analytically equivalent to the impact of heterogeneity in terms of prize valuation and effort impact, and can therefore be corrected by similar policy measures.*

Support: Heterogeneity through cost functions can be easily transformed to heterogeneity through prize valuations or impact functions under risk neutrality. Runkel (2006a) explains the analytical equivalence between a heterogeneous cost framework and a heterogeneous valuation framework in a two-player contest where $EU_i = p_i(e_i, e_j)v - e_i(c_i + \theta_i)$, and θ_i is the designer's tool for modifying a player's cost function. By multiplying the expected utility by $1/(c_i + \theta_i)$, one can obtain the transformed expected utility $p_i(e_i, e_j)v_i - e_i$ where $v_i = v/(c_i + \theta_i)$. Analytical equivalence between effort cost and impact function has been considered by Kirkegaard (2012), for linear effort costs, and Alcalde and Dahm (2019), for general cost functions. For policy makers, however, introducing a change in the cost structure is often easier to implement than a change in the players' reward valuations or their effort impact functions. ■

4.6 Mechanism 6: Budget and caps

The use of caps on effort to level the playing field can be interpreted as limiting the budget $\bar{e}_i$. This is common especially in electoral contests in which the expenditure on campaign is often capped. However, Fang (2002) shows that caps never result in a higher aggregate expenditure in a L compared to no-intervention. The reason behind this is that although limiting any arbitrary player's effort improves the winning probability of the weaker player, it reduces total effort by depressing the efforts of all the stronger players.

Observation 6. *An effort cap does not increase total effort in a L, but can increase total effort in an APA if the players are sufficiently heterogeneous and the cap is not too restrictive.*

Support: Che and Gale (1998) show that, contrary to common intuition, caps on expenditure may increase aggregate expenditure. In particular, if the cap is no less than half the valuation of the low-value player, then both players in an APA have mass points at the cap. Similar results were obtained by Gavious, Moldovanu, and Sela (2002) for a two-player APA with incomplete information and convex costs. Kaplan et al. (2002) and Kaplan, Luski, and Wettstein (2003) consider APA with incomplete and complete information, respectively, where caps can be exceeded at a higher cost and it turns out that a cap always lowers the aggregated bids. Kaplan and Wettstein (2006) show that such flexible enforcement invalidates Che and Gale (1998)'s result, but their argument was refuted by Che and Gale (2006) who explain that caps may indeed increase expenditure when players have heterogeneous costs of lobbying.

Pastine and Pastine (2010) consider a lobbying contest where the concerned politician has a preference ordering over the two competing policies. In an APA framework, they show that a binding cap always reduces lobbying expenditure, though initial imposition increases lobbying expenditure if the politician slightly prefers the low-value lobbyist. Szech (2015) shows that maximization of aggregate expenditure calls for a less restrictive cap and a rather deterministic tie-breaking rule favoring the low-valuation player replacing the symmetric tie-breaking rule of Che and Gale (1998).

Surprisingly, there is very little applied research, with either laboratory or field data on the effectiveness of caps. An exception is Llorente-Saguer et al. (2018) who study in the lab the impact of bid caps and tie-breaking rules on effort in a two-player APA. The authors find that the optimal combination of these two policies specially encourages the weaker player (more than predicted by the theory). In addition, aggregate effort increases more under mild caps than under strict caps independently of the tie-breaking rule. In a recent experimental study, Baik, Chowdhury, and Ramalingam (2020) argue that in symmetric contests, overall effort shows a concave response to players' budget. They explain this concave shape with the diminishing marginal utility of winning induced by a high budget. This may imply that a contest designer can be better off by introducing a cap on effort even in a symmetric contest, when the players are sufficiently wealthy.

4.7 Mechanism 7: Miscellaneous mechanisms

Each of the above mechanisms consider modifying one element in the expected utility (equation 4) of the player. However, one element can mean more than one parameters. For example, modifying the reward structure is possible either by perturbing the CSF p_i, or by revising v_i. In practice, it still amounts to playing only with the reward aspect, without affecting players' effort cost or abilities. Nevertheless, it is possible for the contest designer to simultaneously modify more than one element of the contest. In Subsection 4.7.1, we summarise the studies which compare between two or more of the above discussed mechanisms. There also are ways to level the playing field without modifying one of the parameters in the equation 4. A most prominent one is manipulating information in a contest, which is discussed in Subsection 4.7.2.

4.7.1 Comparison and combination of multiple mechanisms

Mealem and Nitzan (2016) review the literature on contests with 2 heterogeneous players and a designer who can use three of the mechanisms seen so far when deciding the winner: changing the prize valuations through a tax, choosing the accuracy level r, and biasing the effort impact function either multiplicatively or additively. If the designer can choose only one mechanism to maximize total effort, modifying the prize valuations through an additive tax is optimal when the level of heterogeneity is high enough. With low heterogeneity, choosing the optimal noise level r is the designer's preferred tool. When the designer is allowed to combine the two mechanisms, there are two different optimal combinations which allow to increase effort up to $\frac{1}{2}(v_1 + v_2)$. The first one combines the optimal multiplicative bias with the optimal level of noise: either $r = 2$ or $r = \infty$ (Epstein, Mealem, and Nitzan, 2011, 2013). The second one combines the optimal modification of valuations through an additive tax and the optimal level of noise: either $r = 2$ or $r = \infty$ (Mealem and Nitzan, 2014). Franke et al. (2014) conclude that total effort could reach v_1 by combining the optimal level of noise $r = \infty$ and an optimal combination of multiplicative and additive bias.

Balafoutas and Sutter (2012) study the effectiveness of quotas in comparison to a weak and a strong additive bias in a real effort contest experiment. They find quotas to be equivalent to the weaker additive bias. They also consider the policy of repeating the competition until a player from the disadvantaged group wins. This results in a weaker effect. In a similar task environment, Balafoutas, Davis, and Sutter (2016) measure the within-subject effectiveness of gender-based versus arbitrarily allocated quotas. They also elicit subjects' preferences for these two types of quotas. In line with the theoretical expectations, they find that majority of the advantaged (disadvantaged) subjects vote against (for) quotas when voting is cost-less. In contrast, the majority of subjects from both groups abstain from voting if voting is costly. Individual performance in the real effort task is not affected significantly under the gender quota, but the advantaged (disadvantaged) group increases (decreases) its performance under the random assignment quotas.

4.7.2 Information manipulation

The information available to players about different aspects of the contest has a relevant impact on their incentives to compete. There are not very many theoretical studies on how availability of different types of information regarding the contest may affect player strategies. Drugov and Ryvkin (2017) present their findings for both private and public information settings. In most real-life situations, however, participants lack full information about one or other elements of the contest. Below we summarise the findings about availability of information regarding players' abilities, number of winners, and players' identities.

- **Players' abilities:** Bastani, Giebe, and Gürtler (2019) show that the discouragement effect does not always hold when there is uncertainty about the ability of players. Players may not be fully aware of their own abilities or that of other players'. Under such conditions, increasing heterogeneity by manipulating the ability distribution can drive higher individual efforts. In addition, when players are expected to have the same ability, the contest designer can enhance effort by increasing uncertainty. Fu, Jiao, and Lu (2014) consider a multi-prize APA where players' ability (modeled as lower cost for higher ability and vice-versa) is privately observed. Only the contest designer learns about all abilities. The contest designer decides whether to disclose or to conceal the ability of players before the abilities are realized. It turns out that concealing such information results in higher expected total effort regardless of the distribution of abilities. However, in a two-player contest with a single prize and a logit CSF, Serena (2017a) finds that full disclosure brings higher expected total effort than full concealment if the distribution of types is skewed toward strong-types. When the designer has flexibility to disclose partial information, the optimal policy is to disclose the realization of types only when both the players are strong.

- **Number of winners:** Balafoutas and Sutter (2019) experimentally compare two environments, one with uncertainty about the number of winners and another with ambiguity about the number of winners, to a full information baseline. Subjects in their experiment exhibit higher willingness to compete under uncertainty. Moreover, men achieve a higher success rate than women under both treatment environments. Gee (2019) report contrasting evidence based on the data from a very large field experiment where the intervention determines whether or not the job-applicants observe the number of applicants for a given job opening.

It turns out that when applicants have this information, they are more likely to complete an application.

- **Players' identities:**

  Identity is often a strong trigger in competitive situations. Chowdhury, Jeon, and Ramalingam (2016) present experimental evidence that efforts increase significantly when identity of the opposition group is revealed in a contest between Chinese and Caucasian subjects. Such increase in the intensity of competition may reinforce the existing discrepancies in achievement. Players' identity information can also bias the contest organizer's preferences and the umpire's or the judge's verdict. A blind AA conceals sensitive information (gender, race, nationality, etc.) about players in competitive settings.[26] Thus, when deciding the winner(s), the evaluator(s) can only observe their merits relevant to the competition. This encourages meritocracy and there are supporting empirical evidences. On the other hand, it can also encourage diversity by eliminating taste-based discrimination (Becker, 1957).[27] However, this conclusion may not apply in contexts in which the minorities are disadvantaged for reasons other than taste-based discrimination. Below we summarize the literature where the contest designer implements a blind AA policy with the aim of increasing diversity.

  In a hiring process with statistical discrimination, Lundberg (1991) compares a blind AA policy with an alternative AA policy that imposes equal compensations to workers with the same observed score. It turns out that there is a trade-off between the efficient allocation of high skilled workers to top jobs and the efficient investment in human capital. While workers' investment is more efficient under the blind AA policy, the efficient allocation of jobs requires more information and this is achieved by the alternative AA policy. Chan and Eyster (2003) obtain similar results for a college admissions office that values both students' academic qualifications and diversity. When an identity-based AA is banned and there is a blind AA policy, the admissions office partially ignores the candidates' qualifications in order to obtain the same level of diversity as with identity-based AA. However, this less meritocratic rule fails in selecting the best candidates of both groups. Fryer Jr, Loury, and Yuret (2008) obtain the same conclusion with an empirical analysis of matriculates at seven elite colleges in the US in 1989. Similarly, Fryer and Loury (2005) consider the desirability of ex-ante and ex-post subsidy for an identity-based and a blind AA policy. Note that ex-ante subsidy can be interpreted as a head start while ex-post subsidy will be more comparable to a handicap. They find that a handicap (ex-post subsidy) is preferable in a identity-based AA,

[26] It is often coined as 'color blind' in the popular media due to the use of race in the US policies.

[27] For example, in auditions for candidates to join symphony orchestras, the use of physical screens to conceal candidates' identities has increased significantly the probability of female musicians to obtain positions (Goldin and Rouse, 2000).

while a blind AA should involve a head start (ex-ante subsidy) that allows a large proportion of players to exert zero effort.

Finally, Sethi and Somanathan (2018) consider a social planner whose objective is to select the highest performers, regardless of diversity. Whereas players' performance depend on ability and costly training, the social planner only observes training and player identity. It turns out in such situation, a blind policy is not always optimal. When training is heavily resource dependent, the optimal policy has different training thresholds for different groups.

5 Additional discussions on policies for leveling the playing field

The discussions in Section 4 indicate the crucial role played by several key ingredients of the competitive setting behind the effectiveness of AA. Different AA tools can bring similar incentives and outcomes. For implementation purposes, however, it is important to consider the interaction of the AA mechanism with the given competitive setting. Politicians, for example, would not want to lose the confidence of the majority voters due to implementation of race or caste-based quotas. Employers may fear increased sabotage and lower cooperation among employees due to a certain section of employees receiving some preferential treatment. Though head starts and handicaps have similar theoretical implications, those may not be equivalent from a procedural justice perspective. This may have additional behavioural implication. In specific, for loss-averse players, handicapping can have a strong discouragement effect.

There is a dearth of literature on estimating the effects of an introduction or repeal of various AA policies on effort and other outcomes (see Subsection 5.3 for the impact of AA on unethical behavior). Koch and Chizmar (1976) find that an introduction of gender-based AA helped erasing salary discrimination against female faculties in the Illinois State University. Girard (2018) studies the effectiveness of caste-based quotas in political representation in India. She finds that although it brings significant economic gains to the marginalized areas, the effect of the quotas does not persist beyond the term of representation (e.g., when a caste based quota is repealed in India, the lower castes' access to public properties goes back to the level it was prior to the implementation of the quota).[28] In an artefactual field experiment exploiting the same social context, Banerjee, Gupta, and Villeval (2018) also support this evaporation of AA gains on competitiveness once the action is revoked. Neither of the two studies find any spillover effect of quotas on competitiveness and aspiration of the disadvantaged class, nor a significant change in the pre-existing bias among

[28] See Hannagan and Larimer (2010) for a conceptual framework, supported by experimental evidence, of how participation of disadvantaged-group members affect group-decisions and policy-making through difference in process strategies.

the advantaged class (unless their private payoffs are compromised due to the quota policies, in which case there is a marginal increase in aggression as found by the second study). However, AA policies targeting at skill development can have long-term positive impact on the disadvantaged group's performance (Leonard, 1984).

Hence, different AA mechanisms that level the playing field can be justified on the grounds of efficiency in addition to equity. However, these different mechanisms bring different incentives that may drive further questions depending on the particular competitive environment. This section considers relevant discussions in the literature related to the implementation of the AA tools.[29]

5.1 Does increasing participation compromise total effort?

Any policy focused on competitive balance, especially AA, involves a potential conflict between optimality and diversity. AA encourages the participation of the disadvantaged players by increasing their expected utility (larger prize, higher marginal benefits of effort, lower marginal cost, etc.). However, there is also the concern that the discouragement effect can affect the stronger players and overall effort. The proponents of AA tend to focus more on the moral justifications of AA while one of the major arguments against AA rests in its implication of encouraging mediocrity over meritocracy (see Section 1.1). A suitable counter-argument to this objection ought to address the incentive structures. The literature has investigated incentives mostly in the context of education, employment, and public procurement.

AA policies in education have been justified for social mobility reasons. A handful of studies apply two-player contests to look at the incentives of particular AA policies in education. For example, Fu (2006) models college admission processes as a two-player APA between an advantaged and a disadvantaged players. It turns out that it is optimal for the college to favor the minority candidate even when the college authority maximizes expected quality without regard for diversity. The expected quality is maximized under a multiplicative bias to the disadvantaged player's effort. The optimal bias equals the ratio of the advantaged player's valuation to the disadvantaged player's valuation. This bias maximizes the expected payoffs to both the candidates, and equalizes their chance of getting admitted. However, even though attaching higher weight to the disadvantaged player extracts maximal effort from both candidates and equalizes their expected winning probabilities, the advantaged candidate becomes more aggressive resulting in a higher performance gap in the equilibrium.

Pastine and Pastine (2012) reconsider Fu (2006)'s model but with an additive AA bias.[30] They

[29]In this section we consider only multiplicative bias, mainly for the sake of juxtaposition of some seminal findings. The main conclusion does not prevail under additive bias, as summarised in Section 4.2.2.

[30]See Fu (2006) and Pastine and Pastine (2012) respectively for a comparison of multiplicative and additive biases in otherwise similar settings.

find a larger gap in performance and, unlike Fu (2006), the admission probability of the minority student does not improve. The AA bias required to ensure diversity is higher, indicating even higher gap in performance. AA in the form of additional coaching or scholarship is needed to balance this gap. Thus, a multiplicative advantage is preferable to an additive advantage from an incentive point of view. Lee (2013) finds positive impact of AA on total effort in a tournament with several prizes. He concludes that an additive bias favoring the disadvantaged group improves overall effort as long as the players' performance is informative enough about their efforts. In this case, a rise in the disadvantageous group's performance exceeds the fall in the advantaged group's performance, and equalizes the winning opportunities of both groups.

Bodoh-Creed and Hickman (2018) use the contest setting of large contests from Olszewski and Siegel (2016). They model a structural competition between a continuum of heterogeneous students with unobservable cost types for enrolment in a continuum of heterogeneous colleges. They find that both quotas and admission preferences (or additive biases) for minority groups achieve equivalent outcomes in terms of diversity in colleges and the students' effort (human capital) decisions. However, their most interesting finding is the impact on the final distribution of efforts. The highest ability players within the disadvantaged (advantaged) group reduce (increase) their effort, and the intermediate and low ability players increase (decrease) the same. Not applying any AA policy induce pre-college minority students to reduce effort and, consequently, they are allocated to the worst colleges.

Related empirical and experimental literature obtain mixed results. For example, in a lab-in-the-field experiment among school children, Calsamiglia, Franke, and Rey-Biel (2013) find that AA (additive or multiplicative bias) increases the average performance of disadvantaged students (in this case the disadvantaged are the ones with lower task capacity) and also increases their representation among the winners. Cotton, Hickman, and Price (2014) study pre-college human capital investment with a field experiment that mimics university admissions in the US. A sample of middle school and junior high students are monitored by their access to a website with practice materials; it turns out that disadvantaged students (based on race) use more than twice this website under the quotas treatment than under the color-blind treatment. On average, the test scores improve among the disadvantaged group although students with lower learning cost shirk in their effort provision. In a set of natural recruitment experiments conducted in Colombia, Ibañez and Riener (2018) shows that the gender gap in application closes under an announced AA policy (in comparison to an AA policy that is disclosed only after the application stage). Moreover, the AA policy increases (reduces) applications from women (men) in areas with high (low) gender-wage gaps. The robustness in their design comes from the ex-post equality of information for all subjects, that is all subjects faced the same incentive structure once already applied for the job but only subjects in the treatment group knew about it before deciding whether to apply or not. There is no empirical evidence, to our knowledge, that advantaged players reduce their effort or deliver worse performance because of quotas, but some studies report negligible (Holzer and Neumark, 1999) or detrimental (Coate and Loury, 1993) impact of AA among the disadvantaged players in employment contexts. Roy (2018) compares affirmative action policies targeting financially disadvantaged vis-á-vis socially disadvantaged students in a field experiment conducted among university students in India. The results indicate an improvement in performance when AA is based on social disadvantage, but the discouragement effect sets in if AA is implemented based on financial disadvantage.

There is also a growing body of empirical and experimental literature that analyzes particular AA policies implemented in public procurement. On the one hand, there are both lab-based (Corns and Schotter, 1999) and empirical (Ayres and Cramton, 1995) evidences showing that AA in auctions can potentially lead to lower costs or higher revenues (respectively). But on the other hand, there is also evidence on the inefficiency of subsidizing small firms in public procurement (Marion, 2007; Athey, Coey, and Levin, 2013). Although such subsidies can increase small firms' participation (Athey, Coey, and Levin, 2013), sometimes at the cost of lower participation from larger firms (Marion, 2007), the impact on overall effort is negative or only negligibly positive (Brannman and Froeb, 2000).

5.2 Who should be the target of preferential action?

As explained in Section 4.2, policies aimed at leveling the playing field can be either head starts or handicaps depending on whether they are targeted at the disadvantaged or advantaged player(s). It is of interest whether such targeting choices have any effects on contest outcomes. For example, Che and Gale (2003) show that when firms participating in an innovation contest have heterogeneous abilities, the expected profit of the contest designer is maximized by handicapping the most efficient firm with a cap on the prize. Clark and Riis (2000) model a bribery game where the bribee discriminates against the bigger briber who affords to bribe more. However, not all studies invoke unconditionally handicapping the more capable player. Kirkegaard (2013) considers a three-player APA with heterogeneous bidders and shows that handicapping the strong player may result in the medium player becoming more aggressive and the weak player getting hurt even to the extent of staying out of the contest. Kitahara and Ogawa (2010) construct the Bayesian-Nash equilibrium for an APA with individual handicaps and claim that the total effort falls in the maximum handicap when players' valuations follow a uniform distribution. We identify two well-discussed targeting dilemmas, and present the related findings in the remainder of this subsection. The first dilemma arises from an organizational perspective - whether to prefer internal candidates over external ones for a higher position within the organization. The second one addresses the question of whether to prefer the winner or loser in a dynamic competitive environment - this is relevant for both sports tournaments and promotional contests.

5.2.1 Preferring the internal candidates vs. encouraging the external candidates

Chan (1996) considers handicap policies in the context of a promotional tournament for positions within a firm. The author argues that the optimal handicap policy depends on the threat from external candidates. If external candidates are much better capable relative to the existing employees, then a preferential treatment favoring the internal candidates will incentivize them to work harder. On the other hand, if external threat is sufficiently low, then handicapping the internal candidates will improve the firm's current performance. These conclusions are supported empirically by Chan (2006), who argues that the probability of external recruitment at a US financial company is significantly higher at lower level of job hierarchy due to a higher threat from the external candidates. He further shows that a handicap on external candidates results in external candidates of higher ability being taken in, which is evident from the external recruits outperforming the internally promoted ones during subsequent promotions. Analyzing personnel data from a German company, Pfeifer (2011) shows that outsiders' promotion advantage over the insiders' is mostly explained by outsiders possessing higher educational qualification and experience. Tsoulouhas, Knoeber, and Agrawal (2007) consider a similar problem in a two period model, where the firm faces a trade-off between maximizing current output by incentivizing insiders and obtaining a higher future output by choosing the most efficient manager. This trade-off results in an optimal strategy of handicapping insiders when outsiders are significantly more efficient. The conclusions reached by Chan (1996) and Tsoulouhas, Knoeber, and Agrawal (2007) are therefore contradictory, due to the dynamic structure of the later model compared to the static nature of the former.

5.2.2 Preferring the winner vs. encouraging the loser

A reconciliation between the contrasting conclusions from Chan (1996) and Tsoulouhas, Knoeber, and Agrawal (2007) can be possible by studying the context of dynamic contests. Consider a two-period two-player contest where the organizer can decide to favor one of the players in the later period. The crux of the problem is the prevalence of incomplete information, which complicates the determination of the optimal target of AA. Harbaugh and Ridlon (2011) show that in a dynamic APA with incomplete information, the decision to favor the winner or loser of a penultimate stage depends on the objective of the contest designer. Favoring the loser is advisable if the goal is total effort maximization, but favoring the winner is advisable if the goal is to identify the most efficient player. In a lottery, Ridlon and Shin (2013) show that under large ability differences, handicapping the first-period winner maximizes total efforts while handicapping the first-period loser is optimal when the abilities are similar. This is because favoring the first-period loser reduces effort incentives in the first period (ratcheting), while favoring the first period winner lowers incentives in the second period (moral hazard). Even when the players are ex-ante symmetric, Meyer (1992) suggests biasing the contest in favor of the first period winner in order to increase total effort. As there is a trade-off between the size of this additive bias and aggregate effort, the optimal bias size has a lower and upper bound. The lower bound ensures that there is enough competition in the first period, and the upper bound mitigates the discouragement effect on the first stage loser in the second stage.[31] However, the results differ when the players are ex-ante heterogeneous.

Assuming the two players can be ex-ante heterogeneous in terms of both valuation and efficiency, the organizer can affect the ability ratio α_2/α_1 by multiplying it with an AA bias λ such that $\lambda = \alpha_1 v_1/\alpha_2 v_2$. If players have identical valuations, then the optimal AA multiplier under the APA is equal to α_1/α_2, and in the lottery it is less than α_1/α_2. However, the implementation of this tool is not possible if the organizer cannot identify the weaker player from the stronger. Suppose, the organizer still knows the common valuation v and ability ratio α_1/α_2 but doesn't know which player is weaker. $\alpha_2 < \alpha_1$ implies that a correctly targeted policy will alter the ability ratio to $\lambda\alpha_2/\alpha_1$ while an incorrectly targeted action will alter it to $\alpha_2/\alpha_1\lambda$.

In a static contest, the optimal λ will approach α_2/α_1 as the probability of incorrect targeting becomes larger, shifting the optimal bias away from the weaker to the stronger player. In a dynamic contest, the first period outcome serves as a signal and the winner in the first period contest is identified as the stronger player. Therefore, if player 1 wins the first period contest, she/he will be correctly identified as the stronger player; however, if player 2 wins the first period contest then he/she will be incorrectly identified as the stronger player. The direction of the AA will be different under the two circumstances. The players' implicit values of the first period contest are determined by taking both possibilities into account. The equilibrium total effort is then a function of the ability ratio and the designer's bias.

Harbaugh and Ridlon (2011) show that the expected total effort under a policy of favoring the loser is strictly higher than under favoring the winner in an APA. However, Ridlon and Shin (2013) argue that, in a lottery, the optimal policy depends on the value of α_2/α_1 because it affects the implicit valuations of the first contest. A policy in favor of the loser less than fully compensates for the ability difference in a lottery. Hence, the disadvantage from winning the first contest causes the implicit valuations to fall below the true values for both players but more for the stronger one. This causes a stronger incentive for the weaker player only when abilities are sufficiently different. Under a reverse policy, however, the weaker player's success probability is lower than the stronger

[31]The trade-off between the size of the bias and aggregate effort in dynamic models is also present in other contexts such as public procurement (Esteve-González, 2016), conflicts (Beviá and Corchón, 2013), electoral campaigns (Kovenock and Roberson, 2009) and other competitive environments with learning effects (Möller, 2012) or ex-ante commitment (Melkonyan, 2013; Siegel, 2010).

player's, even when the former wins the first contest.

5.3 Can affirmative action induce negative reactions?

Favoring one group over another can sometimes induce the members of the unfavored group to engage in retaliation in the form of sabotage. Sabotage refers to foul play or costly actions beyond the rules of the game that reduces the probability of success of one's opponent(s) (Chowdhury and Gürtler, 2015). In sports, sabotage is considered an unsportsmanlike behavior and is usually penalized. Balafoutas, Lindner, and Sutter (2012) study the impact of reducing the cost of committing sabotage in the Judo world championship; the authors find that the cost reduction increases the observed sabotage and, in addition, stronger players are the most common target among saboteurs.

Brown and Chowdhury (2017) find, theoretically and empirically, that the increment of total effort under policies for leveling the playing field can be counterproductive because sabotage increases in presence of these policies. An even more disturbing consequence has been reported by Girard (2017). Using a difference-in-difference analysis, the author shows that the number of murder attempts against the lower-caste representatives in India significantly increases in the aftermath of the implementation of a caste-based electoral quota. Similarly, Iyer et al. (2012) find that increased female representation in local governments in India induces a significant increase in crimes against women. Both studies use state-level variation in the implementation of political reforms as their identification strategy and run robustness checks to show that these increased attack rates are not out of general envy but in reaction to affirmative action policies.

Chowdhury, Danilov, and Kocher (2020) experimentally study the impact of the introduction and repeal of a head start and a handicap policy on the provision of effort and sabotage. A high-ability and a low-ability subject are matched to compete against each other for a prize in a real effort task. An additive bias reducing the performance gap between them is either introduced only in the second half of the experiment, or introduced at the very beginning of the experiment and repealed after the first half. They find that AA has no positive effect on effort provision. However, the effect of the AA on sabotage activities depends on whether the subjects have experienced the environment without the AA in the past - if the subjects start competing in an environment where AA exists to begin with, then there is less sabotage in treatments with AA than without. The introduction of AA in the middle of the experiment induces an increase in the sabotage exerted by the low-ability subjects under handicap, whereas the repeal of the AA (especially head start) increases the sabotage by both high-ability and low-ability types. Based on experimental implementation of quotas in both field and laboratory (Chowdhury and Gürtler, 2015; Fallucchi and Quercia, 2018; Leibbrandt, Wang, and Foo, 2018; Petters and Schröder, 2018; Banerjee, Gupta, and Villeval, 2018), there is mixed evidence about the impact of quotas on sabotage, spite and unethical behavior targeted at the affirmed subjects. Maggian and Montinari (2017) fail to find any significant effect of gender quotas on unethical behavior in a laboratory real effort task.

6 Conclusion

Contests are important features of life. In various contests, however, engaged parties may have heterogeneous abilities, making the contest less competitive and the outcome more favorable towards the stronger players. Various polices are employed to counter this situation and to provide competitive balance in the contest. A very specific type of such policies are Affirmative Action policies, which are based on ethical reasons. In this survey we covered the contest literature on mechanisms aimed at leveling the playing field, with specific focus on the AA policies.

AA policies favor certain demographic groups, commonly called minority or disadvantaged players, who are under-represented in the top socio-economic strands. The members of such minority groups may bear a disadvantage that weaken them in competitive environments and cause their under-representation. The aim of AA policies is to enhance diversity in the competition outcomes and to equalize opportunities among all the members of the society. These policies are widely implemented around the world. However, their effect on the competition incentives is still unclear, which harms public support for these policies.

The literature indicates that too much asymmetry among players causes incentive problems that reduce overall effort (Section 3). Weaker players have lower expected pay-offs and lower incentives to invest effort in the contest. If players are highly heterogeneous, even the strongest player is discouraged because of the decline in the expected intensity of competition. This dual incentive problem to compete is known as the discouragement effect that can be mitigated by policies that level the playing field. Such policies, including AA, can enhance competition by equalizing the ex-ante success probabilities across the heterogeneous players - either through weakening the advantaged players (handicap), or through strengthening the disadvantaged players (head start).

A major purpose of this survey is to compare and contrast such different mechanisms to implement policies including AA that aim to enhance diversity while not harming overall effort. In particular, we have examined how a designer can level the playing field by selecting the contest rule (Section 4.1), introducing an additive or multiplicative bias to the effort impact function (Section 4.2), biasing the players' valuations of the prize (Section 4.3), modifying the reward structure (Section 4.4), modifying the effort costs (Section 4.5), restricting effort with a cap (Section 4.6), or by adopting other miscellaneous mechanisms (Section4.7) such as manipulating information. The summarized theoretical results characterize the optimal conditions under which all these AA mechanisms can be used by policy makers to enhance both diversity and total effort. AA can enhance overall effort more easily when the rule to select the winner(s) has an intermediate level of noise. That is, when the contest rule, as stated through the CSF, is deterministic enough. This kind of CSF enhances competition by having only a few active players, who compete fiercely, because AA completely eliminates any heterogeneity among them. A less deterministic CSF enhances competition by reducing the heterogeneity between players while preserving the relative advantages. This increases the participation of disadvantaged players. But at the same time, increased competition reduces individual probability of winning.

Overall, there is a trade-off between maximizing total effort and minimizing the number of inactive players that usually belong to the minority group (Section 5.1). Moreover, the distribution of the heterogeneous characteristics (valuations, costs, effort endowments, etc.) is crucial in determining the success of AA policies. When this distribution is particularly steeper on the top, i.e., there are larger differences between the strongest players, the benefits of leveling the playing field can be larger to mitigate the discouragement effect. However, several authors highlight that the gap between the advantaged and the disadvantaged players need to be intermediate. In such a case, the AA policy provides enough incentives to the disadvantaged players to compete (and avoid lowering the requirements to win) while not discouraging the advantaged players too much (and mitigate the prospect of reduction in the highest individual effort). The distribution of players and the available information about this distribution can be crucial in selecting who will be treated with AA. While the common targets of AA are the weaker competitors, there is mixed theoretical findings about the optimality of favoring the winner from a penultimate stage in a dynamic contest (Section 5.2).

Although this survey gives an assessment to policy makers on how to implement a wide range of AA mechanisms, it does not contemplate a thorough analysis of other important factors that may influence the success or failure of such policies. For example, information about players' ability and progress can be concealed to mitigate discouragement effect (Section 3). Sensitive information about players can be concealed when selecting the winner (Section 4.7.2), and role-models can provide signals to high-ability minority players about their expected probabilities to succeed (Chung, 2000). Another relevant factor that may arise with AA is the incentive to sabotage advantaged opponents (5.3).

After revising the theoretical and empirical literature on AA and related mechanisms, we have detected several future research questions to shed further light into the debate. As a general note, heterogeneity within the very population of weaker players is understudied. The related issue of ethnically aimed policies, in a situation where stronger players may exist within the weaker ethnicity, is not thoroughly investigated either. One tricky question, for example, is which collectives should be prioritized for AA in multi-ethnic and multiracial populations. As Sowell (2004) highlights, there are different disadvantaged groups competing for the benefits of AA, and there is the risk that the most disadvantaged members of the society does not receive such benefits. Among the black student population in the US, for example, students from African and West Indian origins have a significantly higher representation compared to the descendants of the African American slaves. Massey et al. (2007) find that the immigrant and the native black students do not differ significantly in terms of grades and performance once admitted. However, the immigrant students showed higher drive to get in and had higher parental qualification. Uniformly applying a color-based AA policy can therefore lead to a meritocracy within the colored population. Fryer Jr and Loury (2005) point that, under certain circumstances, AA can yield even inferior outcomes for many of the disadvantaged players than a situation without the AA. In addition, there is heterogeneity not only between groups but also within groups and this can be used to target different beneficiaries of AA.[32] We are not aware of any model in contest theory considering the interaction of more than two population groups. However, it would be reasonable that different collectives have different optimal mechanisms to enhance their representation. Moreover, competition among ethnic groups are usually multi-dimensional. The majority and the minorities in a society, for example, engage in unequal competition on multiple fronts including educational performance, political participation, rights over practising religion and languages, and so on. Avrahami and Kareev (2009) show that the weaker players, in a multi-battle contest between heterogeneous players, give up on many fronts so that they can make optimal use of their limited resources on the remaining fronts.

Another scope for future research is the long-run evaluation of AA policies and the influence of policy makers over the duration of AA policies. Majority of AA policies seem to be permanent instead of temporary. Sowell (2004) suggests that AA feeds and is fed by conflicts between social groups specially in countries where political parties are perfectly classified by ethnic groups. Although there are some dynamic models that study AA, more research is needed to evaluate their long-run effects (Bodoh-Creed and Hickman, 2018), and the requisite political conditions to sustain AA policies in case they have positive long-run effects. As far as we are aware, only Chan and Eyster (2009) consider political preferences over AA in their analysis on how both income inequality and the level of competition for college admissions influence the median voter's support for diversity and AA policies in education. To our knowledge, only (Chowdhury, Danilov, and Kocher, 2020) investigate the effects of implementation and removal of AA head-start and handicap policies at different periods of time.

There is also a scope for future work to investigate the response to AA mechanisms that level the playing field with different impacts on participation and aggregate effort. The theoretical find-

[32]For example, Espenshade, Chung, and Walling (2004) points out that US universities do not use only SAT scores and race to admit students, but also other characteristics such as their athletic ability and children of alumni. They find empirical evidence showing that admission bonuses for athletes and legacies can have the same size of preferences for minority applicants or even being larger.

ings clearly suggest that AA mechanisms in more deterministic contests seem to achieve the maximum levels of aggregate effort by reducing participation to the minimum (2 or even only 1 player). Aggregate effort in less deterministic contests, on the other hand, is driven by a wider participation. This contrasting outcomes can be studied experimentally and empirically. Understanding the effectiveness of leveling the playing field in dynamic contests can also benefit from experimental and empirical studies. Empirical studies about dynamically structured contests mostly use data from different sports tournaments. Studies evaluating the effect of affirmative action policies on college admission and performance are based on cross-section time-series data. Such data look at a sequence of contests among different representatives from the same social groups. On the other hand, same players often face each other in a sequence of contests with different structure and different reward values. These can be interesting topics for future research. See Konrad (2009, Ch.8) for further topics in dynamic contests.

The final concern is the (legal and logistic) implementability of the AA mechanisms in terms of concrete policies. For example, it may be easy to implement an AA policy when ability differs strictly along some social identity dimension (e.g., gender, race, or nationality) due to historical reasons. For the case of female underrepresentation, for example, Niederle and Vesterlund (2011) review empirical and laboratory studies on women's lower probability of entering competitions and compares different AA policies that target women.[33] However, their conclusions may not be generalized to other minority groups with different distribution of characteristics, different shares of the population, and different determinants of their under-representation. When heterogeneity in ability do not correspond sufficiently to a social identity demarcation, implementing AA with the objective of enhancing diversity may appear logistically challenging. Identifying the target group is harder. Ethical and political justifications of the policy decisions may also be less obvious. There is a dearth of investigation in this area, with two prominent exceptions (although with non-standard contest models) by Fryer Jr and Loury (2013) and Krishna and Tarasov (2016).[34] Whereas literature outside the contest research try to progress in this area, the framework of contests could complement these findings and pursue other research questions on AA that are still to be answered.

[33] Niederle and Vesterlund (2011) explain some AA policies not studied in our survey such as the role of feedback when information is incomplete, the stereotypical-female and -male efforts, the gender composition in team competitions, and the role of risk aversion. Another AA policy not analyzed in this survey is the gender composition of the decision panel (Bagues, Sylos-Labini, and Zinovyeva, 2017).

[34] Fryer Jr and Loury (2013) consider a two stage game in which players first invest on own skill, and then compete in for a job. When identity based AA is possible, then implementing AA in the recruitment process is efficient. But when such identity based AA is not possible and a majority of the players are of lower skill, support in skill development is more efficient. Krishna and Tarasov (2016), however, consider ability to be a combination of nature and nurture, i.e., it comes both by birth as well as by further investments in improving ability. The players are interested in exerting effort that, combined with their ability, will help them achieve a cut off score. With heterogeneity in in-born ability, the authors show that when the cost of skill accumulation is high, then an AA policy may be more socially efficient than when it is low.

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