A Review of the Engineering Properties

of Soils with Particular Reference to

the Shear Modulus

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List of Symbols

b	$= (\sigma_2' - \sigma_3')/(\sigma_1' - \sigma_3')$
c'	Effective cohesion
Dr	Relative density
e	Voids ratio
e_λ	$= e + \lambda \ln p'$
$e_{c.s.}$	Voids ratio at critical state
E_V, E_H	Young's modulus in vertical and horizontal directions
E_u	Young's modulus in undrained test
F.S.	Factor of safety $= q_{max}/q$
G_{33} G_{50} G_{66}	Secant shear modulus at $q/q_{max} = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ (or $\Delta q/\Delta q_{max} = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ for tests in which the initial stress state is not isotropic).
G_{VH}, G_{HH}	Shear moduli for transversely isotropic material
K_o	Coefficient of earth pressure at rest
L.I.	Liquidity index
N	S.P.T. blow count
OCR	Overconsolidation ratio
p'	Mean effective stress $= \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3')$
p_c'	Preconsolidation pressure
p_o'	p' at start of test
p_f'	p' at failure
p_a	Atmospheric pressure
q	$= (\sigma_1 - \sigma_3)$
q^*	$= \frac{1}{\sqrt{2}}((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)^{\frac{1}{2}}$
q_{max}	Maximum value of q
q_{50}	$q_{max}/2$ or $\Delta q_{max}/2$
r_f	Failure ratio in Duncan and Chang model

s_u Undrained shear strength

$(s_u)_{t.c.}$ s_u in triaxial compression test

$(s_u)_{t.e.}$ s_u in triaxial extension test

$(s_u)_{p.s.}$ s_u in plane strain test

γ Engineering shear strain amplitude in dynamic test

ε $= \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$

ε^* $= \frac{\sqrt{2}}{3}\left((\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right)^{\frac{1}{2}}$

ε_{50} ε at $q = q_{50}$

κ Slope of swelling line in $e - \ln p'$ space

λ Slope of consolidation line in $e - \ln p'$ space

ν_{HV}, ν_{HH} Poisson's ratios for transversely isotropic material

σ_v' Vertical effective stress

σ_{vo}' σ_v' at start of test

σ_{vc}' Maximum previous σ_v'

τ Shear stress

ϕ' Effective angle of shearing resistance

1. INTRODUCTION

Advances in both theoretical and experimental aspects of geotechnical engineering in the last ten years or so, have been such that it is now possible to obtain approximate numerical solutions to complex boundary value problems, and to predict the performance of a foundation both during its construction and under working conditions. As a consequence the emphasis in design of foundations has shifted from exclusive consideration of limit analyses and failure conditions to a combination of limit analyses, and deformation analyses appropriate to working conditions.

An obvious example of this extension and enhancement of the design process is afforded by the case of piles, and in particular the estimate of both the load-displacement performance and the total capacity of driven piles. This problem has been the subject of much recent research, and in March, 1978, a report was submitted to the Exxon Production Research Company entitled 'Stress Changes in Clay due to a Cylindrical Cavity Expansion and Subsequent Consolidation'. The report described the findings of research concerned with the effective stress changes caused in the soil around a driven pile by (i) the driving of the pile (simulated by undrained expansion of a cylindrical cavity) and (ii) the subsequent consolidation.

One of the major conclusions of that report was that if the simulation of the pile driving process is correct and the choice of the Cam-clay model is appropriate, then the load carrying capacity of the shaft of a driven pile in a soil of one particular undrained shear strength is effectively independent of overconsolidation ratio. However, the parametric studies showed that the results were significantly affected by the ratio of elastic shear modulus G to the undrained shear strength s_u .

For all deformation analyses - as opposed to limit analyses - the choice of elastic constants used to represent the behaviour of the soil is a major problem. Values of deformation parameters for soils are notoriously difficult to measure accurately; in most instances, the value of Young's modulus E has been observed without any attempt to measure Poisson's ratio ν , so that G (and the bulk modulus K) cannot be deduced.

In order that existing numerical analyses can be used appropriately, and further parametric studies carried out, it was considered important that a careful review be made of what is presently known about values of the ratio G/s_u , and how this ratio may be related for a given soil to

its overconsolidation ratio, index properties and any other relevant parameters. Consequently a proposal was submitted to the Exxon Production Research Company to carry out such a review, and this was commissioned by a technical service agreement executed on 13 December, 1978.

This report is the outcome of the review which consisted mainly of a comprehensive and critical literature survey for experimental values of elastic moduli (both for clay and sand) and other basic soils data with which these values might be correlated. For reasons fully explained in the report, this review has highlighted the problems in attempting any correlations or establishing any patterns of behaviour, and the need for (i) careful selection of the data, (ii) a distinction between different types of test (e.g. in situ geophysical or laboratory triaxial),(iii) an assessment of the quality of experimental technique, and (iv) an understanding of the importance of the magnitude of strain (or stress) increment that is relevant.

The work described in the report is divided into four main parts as follows:

(a) theoretical considerations,

(b) laboratory static tests (mainly on clay),

(c) in situ static tests and back-analysis of foundation performance, and

(d) laboratory and in situ cyclic and dynamic tests (mainly on sand).

As an entirely separate exercise, research work has been carried out on correlations between index properties and other engineering properties of soils for two separate purposes. The first objective has been to form a framework within which the value of poor quality data can be assessed for its reliability and consistency. The second objective has been to amass information about the basic soil parameters required for the specification of the Cam-clay model. This work has been written up in the form of two recent publications, Wroth and Wood (1978) and Wroth (1979). Copies of these have been included in this report as appendices, rather than editing and reproducing them, and including them in the main text of the report.

2. CHOICE OF SOIL PARAMETERS FOR CORRELATION

In endeavouring to correlate parameters which govern the deformation characteristics of soil, it is important to bear in mind the type of application for which the parameters are to be used. In particular, if the stiffness of a non-linear material such as soil is to be characterised by a single parameter (a secant modulus) then it is necessary to decide over what stress range the stiffness should be measured. In estimating the deformations at working loads of typical structures on piled or raft found-ations, much of the soil undergoes stress changes which are only a small proportion of the available strength. A suitable soil stiffness might then be obtained from a deviator stress range from zero to one third the ultimate deviator stress (see Randolph (1977), where this figure emerged from considering piles embedded in sand whose stress-strain curve was modelled by a hyperbola).

In some applications, very large strains may be expected and the soil modulus should be measured over a greater stress range. In cases where there is contained failure of soil (bearing capacity problems, close to the shaft of a driven pile, or in certain in situ soil tests such as the cone test and the pressuremeter test) Ladanyi (1963) has shown that, if the soil is to be modelled as an elastic/perfectly plastic material, then the elastic shear modulus should be measured over the range zero to half the ultimate shear stress. On the other hand, problems such as the design of retaining walls in overconsolidated clay require, for efficient design, estimates of the likely changes in stress in the soil due to very small deformations. For these cases and also for problems involving dynamic loading, soil stiffness parameters should be measured over relatively small stress ranges.

2.1 Effect of soil type

In addition to the difficulty discussed above of what stress range the soil stiffness should be measured over, the choice of other soil para-meters with which to correlate the stiffness will depend on the type of soil. For clays, the obvious dimensionless ratio to consider is G/s_u , the ratio of the shear modulus of the soil (measured over a suitably chosen stress range) to the undrained shear strength of the soil. This

ratio is likely to vary with the past stress history of the soil - that is not only the current value of the overconsolidation ratio (OCR) but also what unloading-reloading cycles the soil has experienced. Intuitively it would seem logical to take due account of the liquidity index of the soil, since soils of low liquidity index tend to be brittle (high G/s_u) whereas soils of high liquidity index tend to be ductile (low G/s_u). In practice, such a correlation is difficult to adopt due to the large scatter in measurements of all three properties (G, s_u and liquidity index) and individual characteristics of particular clays such as sensitive marine clays where the liquidity index is often greater than unity, and yet the strain to peak strength is often relatively small.

For sands, there is no similar current strength with which to correlate the soil stiffness. Instead, the current mean effective stress p' is often used. There are two drawbacks to this approach: firstly the mean effective stress is not easy to estimate in natural sand deposits; secondly, as p' varies for a particular sand, the ratio G/p' varies. As will be discussed later, the stiffness of sands is a function of the mean effective stress level to some power (i.e. $G = k p'^n$) where the exponent n varies between $\frac{1}{3}$ at low strains and nearly unity at high strains. Power law relationships of this form require some care since the stress p' must be non-dimensionalised before raising to the power n (otherwise the constant k will depend on the units used for p'). The usual technique is to non-dimensionalise the stresses and moduli by the atmospheric pressure p_a to give $G/p_a = k(p'/p_a)^n$. This approach is sufficient to ensure that k is truly constant, and it will be used later in the report.

The equivalent parameter to liquidity index for sands is the relative density D_r. Both the liquidity index and the relative density suffer from a lack of consistency in the tests used to measure the parameters. For clays, the liquid limit may be measured either in the Casagrande apparatus or by the falling cone penetrometer. For sands, the problem is even greater owing to the relatively limited range of voids ratios possible and the lack of consistency in the methods used to estimate e_{max}. These factors considerably reduce the confidence with which correlations between different soil parameters may be made.

2.2 Sources of data

Attempts to correlate parameters such as the stiffness and strength of soil must necessarily take into consideration the type of test used in obtaining the parameters. As discussed above, the level of strain which the soil is subjected to is an important factor and this divides test data into the broad categories of large strain (where the sample is generally taken monotonically to failure) and small strain (for example dynamic tests). There are also inherent differences between laboratory tests, where soil samples are subjected to a relatively uniform stress or strain field, and in situ tests where high stress and strain gradients exist around the testing device. Thus, in the first instance, it is necessary to classify test data according to the four categories of high or low strain, laboratory or in situ tests. Even within single categories, further subdivisions may be necessary. For example, laboratory tests which take a soil sample to failure may be performed under triaxial or plane strain conditions. The triaxial test may be compressive or tensile; the plane strain test may be in the plane containing the axes of consolidation, or at right angles to it, and may be with or without rotation of principal stress axes. All these different tests may lead to variations in the measured soil strength and soil stiffness even for samples which are initially identical.

It is outside the scope of this report to discuss at length the effect of the type of soil test on the measured values of undrained strength, shear modulus or other parameters. In extracting data from the literature, the aim has been to obtain consistency within each type of test and to make general comments on the relationships between data obtained by different testing methods. It has been necessary to discard much information consisting typically of sporadic test results presented as background in a technical paper. Instead, attention has been focus on large bodies of data where one particular type of soil has been investigated in great detail. There is limited circulation of such information - which is mainly to be found in a thesis or an internal research report. Thus to some extent, comprehensiveness of soil type has been sacrificed in order to maintain reliability and consistency of the data presented.

2.3 Summary of contents

The main purpose of the survey carried out, has been to correlate the stiffness of the soil, characterised by a secant shear modulus G, with the strength of the soil. For clays, the strength has been taken as the undrained shear strength s_u at the current moisture content. The relationship between G and s_u (i.e. the ratio G/s_u) has been interpreted in the light of the value of OCR. In laboratory (static) tests, the value of the secant shear modulus at half the ultimate load has been taken as the relevant stiffness. This shear modulus is denoted by a subscript fifty (G_{50} - see Fig.2.3.1). Secant moduli at a percentage n of the ultimate load will be denoted similarly as G_n. The choice of G_{50} as the appropriate secant shear modulus has been adopted in the light of theoretical work, presented in the following section, which investigates the manner in which the soil stiffness varies with the fraction of mobilised shear strength. In most soil tests G is not measured directly (an exception being the simple shear test) and so the shear modulus must be deduced from the appropriate stress-strain curve. Figure 2.3.2 shows typical stress-strain curves and the relationship between the actual secant modulus and the shear modulus G.

For sands, a large body of data exists on the stiffness at low strains under cyclic or dynamic loading. Much of the information comes from investigations conducted by Japanese workers who have studied the variation of shear modulus with strain level. For a particular strain level, the shear modulus is correlated with the mean effective stress p' and with the voids ratio e and relative density D_r.

3. THEORETICAL CONSIDERATIONS

The previous section outlined some of the difficulties encountered when correlating parameters measured in different sorts of soil tests. Even for one particular soil test, differences in the measured parameters will result from variations in the stress level at which the test is conducted. It is possible to use theoretical considerations to examine the sort of differences which may occur, and to provide a framework for the subsequent correlation of results from a variety of sources. Three main effects will be considered below:

(i) the effect of the _mean_ effective stress level on the measured soil strength and stiffness;

(ii) the effect of the deviator stress level on the measured secant shear modulus;

(iii) the effect of the mode of shearing and the orientation of the soil sample on the measured shear strength and shear modulus for clay.

3.1 Normalization of soil properties

The behaviour of a particular clay at a given overconsolidation ratio is found to be similar at different stress levels (e.g. Hvorslev (1937), Henkel (1956), Loudon (1967)). This idea is a central feature of Critical State Soil Mechanics and in particular of the Cam-clay model (Schofield and Wroth (1968)) in which the predicted stress-strain response is identical for geometrically similar stress paths applied to samples having the same overconsolidation ratio. The SHANSEP procedure for testing and design (Ladd and Foott (1974)) is also based on the similarity of normalized stress-strain curves at different stress levels. Both the above approaches to stress-strain behaviour suggest the use of soil parameters normalized with respect to some measure of stress (for example s_u/p_o') where such parameters would be a function only of overconsolidation ratio.

Although these observations would suggest the use of an elastic shear modulus directly proportional to pressure, it has been shown that this leads to unconservative behaviour when linked with the other assumptions of the Cam-clay model (Zytynski, Randolph, Nova and Wroth (1978)). The alternative adoption of a shear modulus proportional to the preconsolidation pressure p_c' satisfies the requirement for a conservative model,

and yet still yields the similarity in shape of stress-strain curves at different stress levels as observed by Ladd and Foott (1974).

If soil is idealized as an assembly of rigid frictional particles, then the stress-strain curves may be expected to be similar in shape at different levels of mean stress; and in particular one may expect $G \propto p'$. However, if soil is considered as an assembly of elastic particles in contact, then the contact theory of Hertz (1881) suggests that the elastic properties would obey a one third power law, i.e. $G \propto p'^{\frac{1}{3}}$.

The observed behaviour lies between these two extremes, and Janbu (1963) suggested that elastic moduli may be expressed in a form similar to:

$$G = a p'^{n}$$, where a and n are constants.

This approach has been used by several workers (e.g. Duncan and Chang (1970), Lade and Duncan (1975)); the value of the exponent n is typically in the range 0.5 to 0.9.

If the whole stress-strain curve is considered then it will be appropriate to use the normalization of properties with respect to stress level (i.e. a modulus proportional to pressure). If, however, the modulus for only a small strain amplitude is required then this procedure will no longer be expected to yield a unique normalized parameter. This variation is amply demonstrated by the variation of the exponent n (in the expression for secant modulus $G = a p'^{n}$) with the strain amplitude over which the modulus is measured (see section 6.2).

The important conclusion is that although strength parameters may be successfully normalized with respect to pressure (e.g. s_u/σ_{vc}' as a function of OCR) this process may not be so successful for elastic properties such as shear modulus. However, since the idea of normalising moduli with respect to some relevant stress has been used frequently, this approach is adopted in certain later sections of this report.

3.2 Normalized stress-strain curves

Many theoretical models for soil take account of the non-linearity of the stress-strain curve. Although these models are based on different concepts (for example non-linear elasticity theory as opposed to plasticity theory) it is possible to derive theoretical stress-strain curves and

hence study the variation of an equivalent secant shear modulus with the proportion of the maximum shear strength mobilised. Figures 3.2.1 and 3.2.2 show the curves derived from several theories normalized with respect to the peak shear stress and the shear strain to 50% of this peak.

A hyperbolic stress strain curve of the form $q = \dfrac{\varepsilon}{a + b\varepsilon}$ was suggested by Kondner (1963)[*] and used for non-linear elastic analysis by Duncan and Chang (1970). In order to provide a better fit to experimental data a limiting deviator stress smaller than the asymptotic value of $q = 1/b$ is found to be necessary; if this limit is given by $q = r_f/b$ then values of r_f between 0.7 and 1.0 are usually appropriate. The range of normalized hyperbolic curves for these values of r_f is shown in Fig.3.2.1. The hyperbolic model has been extensively used for the non-linear elastic modelling of the pre-peak behaviour of both loose and dense sands and also of clays.

Uriel and Merino (1979) found that triaxial stress-strain curves for sand could be fitted well by expressions based on the response curves for critically damped oscillations. A range of possible curves is indicated, and this range is shown (for realistic values of the necessary parameters) in Fig.3.2.1.

The original Cam-clay model (Schofield and Wroth (1968)) attributes all shear strain to plastic deformation and the stress-strain curves are stress path dependent. The narrow band of curves for normally consolidated clays in conventional drained triaxial compression tests (σ_3' = constant) is shown in Fig.3.2.2. For overconsolidated samples, (with the assumption of no elastic shear strain) these curves are modified and lie above the previous curves for strains less that ε_{50} and below the previous curves for higher strains (each curve may be though of as pivoting about the ε_{50} point). However, if an elastic component of shear strain is included, the opposite effect occurs and the first part of the curve is lowered and the latter part raised, so the overall alteration of the curve is uncertain.

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[*] The curves of both Kondner and Uriel and Merino were originally expressed in terms of $(\sigma_1 - \sigma_3)$ and ε_1. They are re-interpreted here in terms of $q = (\sigma_1 - \sigma_3)$ and $\varepsilon = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$.

For a high overconsolidation ratio, with elastic strain included, the curve approximates to the simple linear elastic, perfectly plastic case as shown in Fig.3.2.2.

Zytynski (1979) describes an elastic-plastic model for sands. If the small elastic strains are ignored then the realistic range of normalized stress-strain curves for a conventional drained triaxial compression test is as illustrated in Fig.3.2.2. An additional elastic strain would lower the initial part of the curve (up to ε_{50}) and raise the latter part, the limit again being the linear elastic, perfectly plastic case.

All the above models suggest for conventional drained triaxial tests, curves of approximately the same form, and all lie close to the curve:

$$\frac{\varepsilon}{\varepsilon_{50}} = \frac{3 + 2 \ q/q_{max}}{3 - 2 \ q/q_{max}} \ \frac{q}{q_{max}} \qquad \ldots\ldots(3.2.1)$$

(drawn in Figs.3.2.1 and 3.2.2). An approximate relationship for the secant modulus at any proportion of the shear strength is therefore given by:

$$\frac{G}{G_{50}} = \frac{6 - 4 \ q/q_{max}}{3 + 2 \ q/q_{max}} \ . \qquad \ldots\ldots(3.2.2)$$

It is emphasised that this represents a guideline only as to the variation of an equivalent secant modulus, and is based on an approximation to several theoretical curves which have been used to model the behaviour of soil for triaxial compression tests. For other triaxial stress paths and other tests the variation of G with the mobilised stress ratio may be rather different (see section 3.3).

The variation of G/G_{50} with mobilised stress ratio q/q_{max} is shown in Fig.3.2.3 and with $\varepsilon/\varepsilon_{50}$ (to a logarithmic scale) in Fig.3.2.4. (Note that no values are given beyond $\varepsilon/\varepsilon_{50} = 5$ because by this stage several of the theories indicate a post-peak loss of strength; in any case there can be little confidence in the use of values of G beyond this stage.) Figure 3.2.4 may be compared with the variation of dynamic modulus with shear strain amplitude (Fig.6.2.1) but it is emphasised that Fig.3.2.4 represents a fit to theoretical curves for static tests at relatively large strain amplitudes and in particular that it does not represent adequately the variation of shear modulus at very small strain amplitudes. The formula for the empirical curve in Fig.3.2.4, obtained from eqns.(3.2.1)

and (3.2.2) is

$$\frac{G}{G_{50}} = \frac{2(9+x)}{(3-x)}$$

where

$$x = 2\varepsilon/\varepsilon_{50} - \sqrt{9 + 36\varepsilon/\varepsilon_{50} + 4(\varepsilon/\varepsilon_{50})^2}$$

$$\ldots(3.2.3)$$

Fig.3.2.3 also shows the line for a linear elastic, perfectly plastic material (towards which a highly overconsolidated soil may tend), and the ranges, at selected stress ratios, of the measured values of G/G_{50} for a selection of normally consolidated clays (taken from re-interpretation of Fig. 19 in Ladd et al (1977)). The data of Ladd et al and the elastic perfectly plastic line may be taken as the opposite extremes of soil behaviour and it is seen that eqn.(3.2.2) may represent a reasonable compromise for many soils.

3.3 Effects of mode of shearing on s_u and G_{50}

Although the triaxial compression test is the commonest form of laboratory test for measuring the deformation characteristics of soil, other modes of testing (triaxial extension, plane strain or simple shear) are often more relevant to a particular geotechnical problem. Different forms of test are likely to give different estimates of s_u, G_{50} and the ratio G_{50}/s_u for any given soil sample.

Consider first the measurement of the undrained shear strength s_u. Critical State Soil Mechanics, which was developed from consideration of the behaviour of soil in triaxial compression, suggests that for normally consolidated and lightly overconsolidated soil, there exists a unique relationship for a given soil between the voids ratio and the mean effective stress at failure, given by

$$e = e_{cs} - \lambda \ln p_f' \quad . \qquad \ldots(3.3.1)$$

For this class of soil in more general stress conditions, there is widespread evidence to suggest that failure occurs according to the Mohr-Coulomb failure criterion of

$$(\sigma_1'/\sigma_3')_f = (1 + \sin \phi')/(1 - \sin \phi') \qquad \ldots(3.3.2)$$

where ϕ' is the angle of shearing resistance for the soil and $c' = 0$. Assuming that eqn.(3.3.1) may be extended to stress states other than those existing in triaxial compression (where $\sigma_2' = \sigma_3'$) and combining it with eqn.(3.3.2), it may be shown that the value of s_u, defined as $s_u = (\sigma_1 - \sigma_3)_f /2$, depends on the value of σ_2'. This point is illustrated in Fig.3.3.1 for the three main types of test:

(a) triaxial compression, where $\sigma_2' = \sigma_3'$, $p_f' = \frac{1}{3}(\sigma_1' + 2\sigma_3')_f$,

(b) triaxial extension, where $\sigma_2' = \sigma_1'$, $p_f' = \frac{1}{3}(2\sigma_1' + \sigma_3')_f$, and

(c) plane strain, where σ_2' has been assumed to be close to $(\sigma_1' + \sigma_3')/2$ at failure, giving $p_f' \simeq \frac{1}{2}(\sigma_1' + \sigma_3')_f$.

From the geometry of Fig.3.3.1, it may be shown that

$$\frac{(s_u)_{t.e.}}{(s_u)_{t.c.}} = \frac{3 - \sin\phi'}{3 + \sin\phi'} \qquad \ldots\ldots(3.3.3)$$

and

$$\frac{(s_u)_{p.s.}}{(s_u)_{t.c.}} = \frac{3 - \sin\phi'}{3} \qquad \ldots\ldots(3.3.4)$$

giving ratios of 0.714 and 0.833 respectively for $\phi' = 30^{\circ}$.

It is useful to introduce a parameter b (Bishop (1966)) defined by

$$b = \frac{(\sigma_2' - \sigma_3')}{(\sigma_1' - \sigma_3')} \qquad \ldots\ldots(3.3.5)$$

Thus the value of b relates the value of σ_2' to the major and minor principal stresses, $b = 0$ corresponds to triaxial compression, $b = 1$ corresponds to triaxial extension. At failure in plane strain (including simple shear) b may be expected to be near 0.5 (Stroud (1971) and Sketchley (1973) obtained values of σ_2' at failure in plane strain consistent with $b \sim 0.4$).

Further evidence of the applicability of the Mohr-Coulomb envelope is provided by Pearce (1970) from true-triaxial tests on isotropically normally consolidated samples of clay where the parameter b was held constant during each test. Results are shown in Fig.3.3.2, from which it may be seen that the failure points all lie close to the Mohr-Coulomb failure envelope for $\phi' = 23^{\circ}$. Pearce made a further important observation from his tests which was that contours of equal strain (deviatoric

and volumetric) were circular, centred on the principal stress diagonal. Thus the deviatoric stress * necessary for a given amount of strain was independent of the value of b. This is illustrated in Fig.3.3.2. The observation has a direct bearing on the measurement of G_{50} from different types of test. Since G_{50} is the secant modulus at half the failure stress, it will be evaluated at the deviatoric stress given by the dashed line AB in Fig.3.3.2. The shear strain[†] ε^* is defined as

$$\varepsilon^* = \frac{\sqrt{2}}{3} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{\frac{1}{2}} \quad ...(3.3.6)$$

from which it may be shown that the shear modulus is given by

$$G = \frac{1}{3} \frac{q^*}{\varepsilon^*} \quad .....(3.3.7)$$

For the three cases of $b = 0$, $b = 0.5$ and $b = 1$, the shear modulus G_{50} calculated from Fig.3.3.2 are 735 kN/m², 920 kN/m² and 920 kN/m² respectively. These were drained tests with p' held constant and so the parameter G_{50}/s_u is not relevant. Instead, the ratio of G_{50} to $(\sigma_1' - \sigma_3')/2$ at failure will give some indication of the measured ductility of the clay samples. This ratio has a value of 12 for the case of $b = 0$ compared with 17 and 19 for $b = 0.5$ and $b = 1$ respectively, suggesting that the ratio of shear modulus to shear strength may be as much as 50% higher in plane strain or triaxial extension tests compared with triaxial compression tests.

For drained tests - particularly on sands - the ratio of shear modulus to current mean effective stress is a useful parameter. From the above tests, this ratio takes the values of 5.2, 6.5 and 6.5 for three tests ($p' = 140$ kN/m²). Other true triaxial tests, this time on sand, reported by Green (1969) do not show such a pronounced trend of G_{50}/p' with the

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* Note that this is defined as $q^* = \{\frac{1}{2}[(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2]\}^{\frac{1}{2}}$ $= [1 - b + b^2]^{\frac{1}{2}}(\sigma_1' - \sigma_3')$, which is chosen so that for the triaxial test $q^* = q$. q^* is not equal to the octahedral shear stress τ_{oct} but is related to it by $q^* = \sqrt{\frac{3}{2}} \tau_{oct}$.

† This particular definition of shear strain is used so that for the case of the triaxial test $\varepsilon^* = \varepsilon$. ε^* is not equal to the octahedral shear strain ε_{oct} but is related to it by $\varepsilon^* = \sqrt{\frac{2}{3}} \varepsilon_{oct}$.

value of b . Green's test results show a considerable amount of scatter
with average values of G_{50}/p' for triaxial compression and triaxial
extension of 3 and 3.4 respectively. However, tests conducted at inter-
mediate values of b showed much lower values of G_{50}/p' of between 1
and 1.5. It is not understood why these intermediate tests gave such low
values of G_{50}/p' although the results may reflect shortcomings in the
design of the true triaxial apparatus whereby the corners of the sand
samples were unstressed.

3.4 Effects of anisotropy

No mention has been made so far of the effect of anisotropy on the
measured soil parameters. Anisotropy may arise from two main causes -

(i) structural anisotropy, due to the deposition of soil on well
defined planes (usually horizontal), and

(ii) stress induced anisotropy, due to the differences in normal
stress acting in different directions.
The commonest form of anisotropy encountered in geotechnical engineering
is the so-called transverse isotropy (Love(1928)) with the horizontal plane
being the plane of isotropy. This form of isotropy has independent elastic
soil parameters, E_V , E_H , ν_{HV} , ν_{HH} and G_{VH} . The equations of defor-
mation and restrictions on the parameters are discussed by Gibson (1974)
(see Fig.3.4.1).

Of particular interest is the relationship between $G_{HH} = E_H/(2(1 + \nu_{HH}))$
and G_{VH} . For undrained deformation, it may be shown that $\nu_{HH} = 1 - n/2$
where $n = E_H/E_V$, whence it follows that

$$G_{HH} = \frac{n E_V}{4 - n} \qquad . \qquad \qquad .....(3.4.1)$$

Gibson (1974) has shown, from the test results on London clay reported by
Ward et al (1965) that for London clay, $n \simeq 1.8$ and $G_{VH} \simeq 0.38 E_V$.
From eqn.(3.4.1) $G_{HH} \simeq 0.82 E_V$, that is approximately double G_{VH} .

The parameter G_{HH} gives the shear stiffness for shearing in the
horizontal plane and is thus relevant to the analysis of radial consoli-
dation, interpretation of pressuremeter tests or analysing the effects
of cylindrical cavity expansion due to pile installation. The parameter
G_{VH} , on the other hand, is relevant for all shearing which includes

deformation in a vertical plane such as under a footing, or round an axially loaded pile. It should be noted that this parameter is not easy to measure since it gives the shear strain due to <u>shear</u> stresses acting on the vertical and horizontal planes. It may be estimated from simple shear tests or, alternatively, from triaxial compression tests conducted on samples cut at 45° and at 90° from the vertical by use of the equation

$$G_{VH} = \frac{E_H \cdot E_{45^\circ}}{4E_H - E_{45^\circ}} \qquad \ldots\ldots(3.4.2)$$

where E_{45° is the Young's modulus measured in the diagonally cut sample and E_H is that measured in the horizontal sample (Gibson (1974)).

Little information exists in the literature on the value of n for different soil deposits. However, it is intuitively reasonable to expect that the stiffness of an anisotropic material in a particular direction will in some way be related to the effective in situ stress in that direction. Hence as a first approximation

$$n = \frac{E_H}{E_V} \simeq \frac{\sigma_h'}{\sigma_v'} = K_o \cdot \qquad \ldots\ldots(3.4.3)$$

Most natural clays are at least lightly overconsolidated and thus will seldom have values of K_o less than about 0.8. Thus n is unlikely to be much less than unity. Where anisotropy may become important is in heavily overconsolidated soil deposits where K_o may typically be over 2. Values of n greater than 1.5 may be expected and some care will be needed in choosing the correct type of soil test in order to measure the soil modulus appropriate to the particular problem.

4. LABORATORY STATIC TESTS

In this section an attempt is made to correlate various parameters obtained from laboratory tests. Results from both triaxial tests and plane strain tests are examined to determine the influence of different modes of shearing on elastic parameters. The effects of the type of consolidation, and of sample disturbance are investigated in an attempt to establish the relevance of laboratory data to field conditions.

4.1 Triaxial test correlations

Ladd (1964) reports a thorough investigation into the factors which influence the stress-strain modulus of clay in the triaxial test. He reports the consequences of variations in the type of test, the consolidation history of the sample and the strain rate. Some of his data are represented here, with the notation altered to be consistent with the list of symbols used in this report. The values of undrained Young's modulus, E_u , have been converted to shear modulus, G, by dividing by 3 .

Three dimensionless parameters are examined in this section:

(i)   the ratio of shear strength to initial mean effective pressure (s_u/p_o') ,

(ii) the ratio of shear modulus to initial mean effective pressure (G/p_o') ,

(iii) the ratio of shear modulus to shear strength (G/s_u) .

The approach in each case is to establish if a correlation exists for normally consolidated samples, and then to examine the effects of OCR on any such correlation.

One of the major difficulties in making a satisfactory assessment of laboratory data is the variety of consolidation histories which specimens may have experienced. In most instances, remoulded specimens of soil will have been isotropically consolidated in the triaxial apparatus, whereas 'undisturbed' specimens will have had a one-dimensional consolidation history in the field, followed by an isotropic reconsolidation in the triaxial apparatus. The effect of an original one-dimensional consolidation history will almost certainly make the undisturbed soil anisotropic, with the consequence that the mode of shearing will affect the observed shear modulus. The anisotropy is suppressed if the subsequent laboratory (isotropic) consolidation is taken to a high enough pressure - the work of

Loudon (1967) suggests that a factor of 3 on the preconsolidation pressure is sufficient. For this reason alone, different trends are likely to be observed in triaxial tests on undisturbed as opposed to remoulded specimens.

A further complication arises from the definition of overconsolidation ratio as the maximum vertical effective stress σ'_{vmax} divided by the initial vertical effective stress σ'_{vo}. For a given value of OCR, the stress state, and in particular the value of p'_o will differ for two specimens, one of which has been unloaded isotropically and the other one-dimensionally. Great care needs to be exercised when interpreting the results of any particular set of data in terms of overconsolidation ratio.

The ratio s_u/p'_o

It is well established experimentally that the ratio s_u/σ'_{vo} is constant for a natural deposit of clay which is normally consolidated (in the one-dimensional mode). Skempton (1957) has proposed that this constant ratio is a function of the plasticity index of the soil, given by the expression:

$$\frac{s_u}{\sigma'_{vo}} = 0.11 + 0.0037 \text{ PI} \quad . \qquad \ldots\ldots(4.1.1)$$

It should be noted that the data quoted by Skempton in support of this widely used expression were based on undrained shear strengths measured by the vane .

In relating this to the results of isotropically consolidated triaxial tests, the difference in initial stress state must be accounted for. This means that for one given water content (and hence one undrained shear strength[*]) some assumption has to be made about the respective values of p' on the one-dimensional and isotropic normal consolidation lines.

One approach is to adopt the assumptions embodied in the modified Cam-clay model (Roscoe and Burland (1968)). It can be shown that for the same value of water content the value of σ'_{vo} for one-dimensional normal consolidation is related to the value of p'_o for isotropic normal consoli-

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[*] This is assuming that a sample of clay has one unique undrained shear strength regardless of whether it is tested in the triaxial test or by vane; this is known not to be so.

dation by the expression

$$\frac{p_o'}{\sigma_{vo}'} = \frac{1}{3}(1 + 2K_o)\left[\frac{M^2 + \eta_o^2}{M^2}\right]^{(\lambda-\kappa)/\lambda} \qquad \ldots\ldots(4.1.2)$$

where $\eta_o = \dfrac{3(1 - K_o)}{1 + 2K_o}$ and λ, κ and M are the basic soil parameters describing the modified Cam-clay model.

Assuming $K_o = 1 - \sin\phi'$ for one-dimensionally normally consolidated soil, and adopting typical values of λ, κ and M, the predicted ratio $\dfrac{p_o'}{\sigma_{vo}'}$ is found to vary very little with Plasticity Index, and has a value of about 0.85. Substituting this into Skempton's relationship would suggest that for <u>isotropically</u> normally consolidated clay

$$\frac{s_u}{p_o'} = 0.129 + 0.00435\ PI \qquad \ldots\ldots(4.1.3)$$

(allowing no difference between values of s_u measured by vane or triaxial test).

CLAY	UNDISTURBED			REMOULDED	
	AMUAY	LABUNILLAS	KAWASAKI	BOSTON BLUE	VICKSBURG BUCKSHOT
P.I. %	42	37	34	15	39
p_o' kN/m^2	785	98	294	589	608
s_u MN/m^2	249	31	118	179	166
G_{50} MN/m^2	32.1	5.6	14.7	33.9	24.9
s_u/p_o'	0.317	0.315	0.402	0.302	0.272
s_u/p_o' Eqn.4.1.3	0.312	0.290	0.277	0.194	0.299
G_{50}/p_o'	41	57	50	58	41
G_{50}/s_u	129	180	124	190	150

Table 4.1.1. Triaxial test results on normally consolidated clays (extracted from Ladd (1964))

The results of undrained triaxial tests on five isotropically normally consolidated clays reported by Ladd (1964) are contained in Table 4.1.1. The experimental values of the ratio s_u/p_0' are given in row 5, and those derived from eqn.(4.1.3) are given in row 6. The comparison is very favourable in the case of the Amuay, Lagunillas and Vicksburg Buckshot clays, but not good for the Kawasaki and Boston blue clays, for which the strengths would have been underpredicted by eqn. (4.1.3).

A series of consolidated undrained triaxial tests on undisturbed samples of a soft alluvial clay at Mucking Flats in the estuary of the River Thames has been carried out by Wesley (1975). Figure 4.1.1 is a plot of s_u against p_0', where p_0' is the isotropic pressure to which a sample has been consolidated before being subjected to a conventional undrained triaxial compression test. The samples were carefully trimmed from three large block samples T29, 10 and 13; their respective plasticity indices were 29, 28 and 47%. All samples indicated an isotropic preconsolidation pressure of about 40 kN/m² during reconsolidation in the triaxial apparatus.

The boundary lines given by eqn.(4.1.3) for the PI of 28 and 47% have been included in the diagram. At low consolidation pressures the samples are overconsolidated, and show a consistent pattern in which the ratio s_u/p_0' increases as the OCR increases.

Accepting that s_u/p_0' is a unique function of PI for isotropically normally consolidated (inorganic) clays, the next step is to determine the effect of OCR on this ratio. Considering only isotropic unloading, so that OCR is given directly by p'_{max}/p_0', adoption of the modified Cam-clay model leads to the expression *

$$\frac{s_u}{p_0'} = \frac{M}{2} \left(\frac{p'_{max}}{2p_0'}\right)^{(\lambda-\kappa)/\lambda} \qquad \ldots\ldots(4.1.4)$$

which can be rewritten in the more useful form

$$\left(\frac{s_u}{p_0'}\right) = \left(\frac{s_u}{p_0'}\right)_{n.c.} (OCR)^{(\lambda-\kappa)/\lambda} \qquad \ldots\ldots(4.1.5)$$

---

* This is based on the assumption that undrained failure occurs at the critical state; this overlooks the minor divergence for high values of OCR.

This relationship is of identical form to the one suggested by Ladd et al (1977) - except that their derivation is solely an empirical one, with the power to which OCR is raised being denoted by m. The relationship derived here, as eqn.(4.1.5), has the added advantage that the exponent m is expressed in a rational manner as the ratio $(\lambda-\kappa)/\lambda$ or as $(C_c - C_s)/C_s$ where C_c and C_s are the compression indices in loading and unloading respectively.

Fig.4.1.2 shows data of triaxial compression tests on isotropically consolidated Boston blue clay replotted from Ladd et al (1971). Both the ratio $(s_u/p_o')/(s_u/p_o')_{n.c.}$ and OCR $(= \frac{p'max}{p_o'})$ have been plotted on logarithmic scales, and the resulting straight line has gradient $m = 0.75$. The predicted gradient given by $(\lambda-\kappa)/\lambda$ is 0.80 based on values of $\lambda = 0.15$ and $\kappa = 0.03$ taken from one dimensional consolidation tests. Fig.4.1.3 shows a similar plot for plane strain tests on K_o consolidated Boston blue clay; in this case the ratio $(s_u/\sigma_{vo}')/(s_u/\sigma_{vo}')_{n.c.}$ is used and OCR defined as $\sigma_{vmax}'/\sigma_{vo}'$. The line of slope $m = 0.75$ again represents a good fit to the data, indicating that eqn.(4.1.5) is also applicable for samples with anisotropic consolidation histories.

CLAY	KAWASAKI	REMOULDED BOSTON BLUE CLAY	REMOULDED VICKSBURG BUCKSHOT CLAY
p_o' : kN/m^2	29	98	98
p_c' : kN/m^2	167	589	608
OCR$(= \frac{p_c'}{p_o'})$	5.7	6.0	6.2
s_u : kN/m^2	59	104	94
G_{50} : kN/m^2	1960	7790	7790
s_u/p_o'	2.0	1.1	.96
G_{50}/p_o'	67	79	79
G_{50}/s_u	33	75	83
m	0.92	0.75	0.70

Table 4.1.2. Triaxial test results on overconsolidated clays (from Ladd (1964))

Table 4.1.2 summarises results of tests on three overconsolidated clays reported by Ladd (1964). The corresponding values of the power m are presented, ranging from 0.70 to 0.92.

Further confirmation of the relevance of the form of the relationship of eqn.(4.1.5) is given by Ladd et al (1977) from direct simple shear tests on six one-dimensionally consolidated clays. They claim that a good fit for all clays is given by m = 0.8 'though a better fit is obtained if m is decreased from 0.85 to 0.75 with increasing OCR'.

The ratio G/p_o'

Because of the understandable emphasis on strength rather than stiffness in geotechnical design and soil testing, there is much less information about values of Young's modulus or shear modulus of clays. The ratio G/p_o' can be expected to be constant for one given clay, when normally consolidated, as discussed in section 3.1; but there does not seem to have been any attempt to relate this constant with characteristics of the clay, such as plasticity index.

The values of G_{50}/p_o' quoted in Table 4.1.1 for five normally consolidated clays are in a narrow range of 41 → 48, and there is a general trend for them to decrease with increasing PI.

The value of G_{50}/p_o' for any particular clay increases with OCR, but the manner in which this occurs has not previously been established. The results presented in Fig.3 of Ladd's paper (1964) suggest that the increase is linearly related to the logarithm of OCR. This was also indicated by the analysis by Wroth (1972) of the tests on undisturbed London clay carried out by Webb (1967). In this latter case, however, the overconsolidation ratio is impossible to determine and an alternative parameter e_λ was used. This parameter defined as $e_\lambda = e + \lambda \ln p'$ has the special property that its value is constant along any line which is parallel to the normal consolidation line, in a plot of e against $\ln p'$ as in Fig.4.1.4.

In this idealised plot the gradients of the normal consolidation NM and a swelling line MI are denoted respectively by $-\lambda$ and $-\kappa$. For points I and M, by definition

$$e_{\lambda i} = e_i + \lambda \ln p_i' = e_j$$

$$e_{\lambda m} = e_m + \lambda \ln p_m' = e_n$$

$$\ldots\ldots(4.1.6)$$

From the equation of the swelling line MI

$$e_i - e_m = \kappa \ln (p_m'/p_i') = \kappa \ln (OCR) \qquad \ldots\ldots(4.1.7)$$

Combining these equations

$$e_{\lambda i} = e_n - (\lambda - \kappa)\ln(OCR) \qquad \ldots\ldots(4.1.8)$$

Hence the current value of e_λ for a specimen of soil is linearly
related to the logarithm of OCR: the former parameter can be measured
in circumstances when the latter cannot, as was the case for Webb's data
on undisturbed London clay.

Fig.4.1.4 shows the processed data for London clay where G/p_o' is
plotted against e_λ. The results[*] lie close to a straight line. It
is therefore assumed that the ratio G/p_o' obeys the following relation-
ship:

$$G/p_o' = (G/p_o')_{n.c.} [1 + C \ln(OCR)] \qquad \ldots\ldots(4.1.9)$$

where $(G/p_o')_{n.c.}$ is the value for a normally consolidated specimen,
and C is some (dimensionless) soil constant that would need to be
measured for any particular soil.

In the case of London clay the value of C cannot be deduced because
the actual values of OCR are not known. Eqn.(4.1.9) can be expressed
in the form

$$G/p_o' = (G/p_o')_{n.c.} [1 + C(\frac{e_n - e_\lambda}{\lambda - \kappa})] \qquad \ldots\ldots(4.1.10)$$

but this still does not allow evaluation of C from Fig.4.1.5 because
without a knowledge of OCR a value cannot be assigned to $(G/p_o')_{n.c.}$.

The very limited data available for values of G/p_o' are
only for triaxial tests on _isotropically_ normally consolidated clay.
For one-dimensionally consolidated specimens it seems reasonable to
hypothesise that values of G/p_o' or G/σ_{vo}' would vary with OCR in the
manner suggested by eqn.(4.1.9), but with a different value of C .

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[*] The pre-peak stress strain curves are closely linear so that the values
selected for G are sensibly independent of the strain level.

Further indirect evidence for the relationships proposed above is provided by the next section where consideration is given to values of G/s_u .

The ratio G/s_u

For normally consolidated clays, d'Appolonia, Poulos and Ladd (1971) have concluded that the ratio E_u/s_u decreases with increasing plasticity, and that organic clays may have a lower modulus than inorganic clays of comparable plasticity. A similar trend can be observed for the values of the ratio G_{50}/s_u quoted in Table 4.1.1.

A search of the literature has not produced any good quality data for the variation of G/s_u with OCR from triaxial tests on isotropically consolidated specimens of clay. However, the original MIT research reports by Ladd et al (1971) and Ladd and Edgers (1972) contain good quality data of both plane strain and direct simple shear tests on a number of clays, which are necessarily consolidated one-dimensionally. The pattern of behaviour can be expected to be very similar but not identical to that for triaxial test results on isotropically consolidated specimens.

The data for 3 clays are reproduced in Fig.4.1.6 from the latter report. The values of the secant (undrained) Young's modulus at one third and two-thirds of the peak shear stress (i.e. $3G_{33}$ and $3G_{67}$) normalised by s_u are plotted against OCR.

For any one clay the value of E_u/s_u is seen to decrease with increasing OCR (after showing a small maximum in the case of Boston blue clay) in a well defined manner whose precise form is not immediately clear. However by combining the findings of the two previous sections this pattern of behaviour can be understood and modelled.

The combination of eqns.(4.1.5) and (4.1.9) gives

$$(G/s_u) = (G/s_u)_{n.c.} [1 + C \ln(OCR)](OCR)^{-(\lambda-\kappa)/\lambda} \qquad \ldots (4.1.11)$$

For convenience putting $r = (G/s_u)/(G/s_u)_{n.c.}$ and $\Lambda = (\lambda-\kappa)/\lambda$ this expression can be rewritten in a more concise form as

$$r = [1 + C \ln(OCR)](OCR)^{-\Lambda} \qquad \ldots (4.1.12)$$

For most soils the range of values of κ/λ is between 0 and 0.4 i.e. $1 > \Lambda > 0.6$. It would also seem that a likely range for the unknown soil constant C is between 0 and about 2.

The relationships between r and OCR predicted by eqn.(4.1.12) for various likely combinations of Λ and C are presented in Fig.4.1.7.[*] The general pattern is of the form observed for the experimental data in Fig.4.1.6. In detail, there are some additional interesting features. The mathematical expression (for any one value of Λ and C) has a maximum value of r given by

$$r_{max} = \frac{C}{\Lambda} \exp\left(\frac{\Lambda - C}{C}\right) \qquad \ldots.(4.1.13)$$

which occurs at the overconsolidation ratio

$$OCR = \exp\left(\frac{C - \Lambda}{C\Lambda}\right) \qquad \ldots.(4.1.14)$$

This value of OCR will only be greater than or equal to unity if $C \geq \Lambda$ i.e. for a soil which has $C < \Lambda$, the greatest value of r will be unity which will occur for OCR = 1. The locus of the maxima has been drawn as the dotted curve in each of the three plots in Fig.4.1.7. Note that the maxima occur at relatively small values of OCR (less than 2) i.e. for a lightly overconsolidated condition, and that each maximum is not much greater than the value of unity relevant to an overconsolidation ratio of 1.

An attempt to fit the mathematical expressions to the observed curves of Fig.4.1.5 proves to be disappointing. In the first place the values of λ and κ (and hence Λ) are not known for Maine organic clay or Bangkok clay. Secondly, in the case of Boston blue clay (for which $\Lambda = 0.8$) the predicted curves underestimate the decrease in r with increasing OCR.

But the expressions do provide a coherent pattern of how the ratio G/s_u varies with OCR, which is qualitatively correct. The same expressions can be expected to match the data from (a) other tests, such as plane strain or triaxial and (b) isotropic consolidation, but with different values

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[*] For values of OCR greater than 2.0 the modified Cam-clay model implies a slightly lower value of r than that given by eqn.(4.1.12), this has been taken into account in Fig.4.1.7.

for $(G/s_u)_{n.c.}$ and for C .

Further work is called for to pursue these ideas, and if they prove successful to establish values of $(G/s_u)_{n'.c.}$ and C for different soils. Besides the various difficulties already outlined regarding different tests and different types of consolidation, no account has been taken of the possible effects of the value of the preconsolidation pressure or of the liquidity index. This point is discussed further in section 4.3.

4.2 Effects of Disturbance

Before attempting to use correlations obtained from triaxial test results in field analysis, the effects of sample disturbance on these correlations must be examined.

Ward et al (1959) compared results obtained using two different sampling techniques in London clay. Careful block-sampling techniques were shown to result in less disturbance than more conventional driven tube sampling techniques as shown by the results quoted in Table 4.2.1.

			SITE D	SITE O
BLOCKS	$G_{INITIAL}$	MN/m^2	110	27
	G_{RELOAD}	MN/m^2	151	38
	G_{IN}/s_u		103	79
	G_{REL}/s_u		141	111
TUBES	$G_{INITIAL}$	MN/m^2	32	13
	G_{RELOAD}	MN/m^2	73	26
	G_{IN}/s_u		49	60
	G_{REL}/s_u		111	114
$\dfrac{G_{BLOCK}}{G_{TUBE}}$	(INITIAL)		3.4	2.0
	(RELOAD)		2.1	1.5

Table 4.2.1. Comparison of shear modulus and G/s_u ratio for block and tube samples of London clay (after Ward (1959))

Much of the disturbance in heavily overconsolidated clays results from opening of fissures due to stress relief, and even carefully cut block samples will suffer this disturbance.

Ladd (1964) recommends that consolidated undrained tests rather than unconsolidated undrained tests should be·used to obtain measurements of elastic parameters, as the former test eliminates many of the effects of sample disturbance. However, uncertainty regarding the in situ stress state makes the choice of consolidation pressure difficult, and it is not always clear whether isotropic or anisotropic consolidation is the most appropriate.

Ladd also showed that the modulus of some remoulded clays almost doubled when 60 days were allowed for secondary consolidation in the consolidated undrained traixial test before shearing the sample. Bjerrum and Lo (1963) showed that although the value of s_u was only slightly dependent on time of ageing, the initial modulus, and the strain to failure (and hence the values of all secant moduli) increased considerably with ageing for undisturbed normally consolidated clays.

Therefore, in attempting to eliminate the effects of sample disturbance on measured moduli, consolidated undrained tests, with substantial time allowed for secondary consolidation, offer the best chance of success.

4.3 Plane strain tests on Boston blue clay

Ladd et al (1971) have reported the results of an extensive investigation into the behaviour of Boston blue clay in a laboratory plane strain testing apparatus. The experimental programme consisted of 'active' tests where a vertically consolidated sample was sheared at constant volume by increasing the vertical stress and 'passive' tests in which shear took place by a decrease in the vertical stress. Tests were conducted at overconsolidation ratios of 1,2 and 4.

The tests provide a useful body of data for comparison of:

(i)   the variation of undrained shear strength (normalised by the initial vertical effective stress) with OCR for plane strain active and passive tests and for triaxial compression tests;

(ii) the value of shear modulus, normalised by the initial vertical effective stress, at different values of OCR;

(iii) the ratio G/s_u at different values of OCR.

The first of these comparisons is shown in Fig.4.3.1. The values for plane strain active tests lie just above the line for triaxial compression tests while those for plane strain passive lie below this line. Also marked on the Figure are some undrained strengths deduced from simple shear tests (see later), which are another form of plane strain test. The dashed line in Fig.4.3.1. marks the theoretical plane strain strength given by $(s_u)_{ps} = 0.833(s_u)_{tc}$ (equation (3.3.4) for $\phi' = 30^O$). In general, this line gives a reasonable average for the plane strain tests.

Equations (3.3.3) and (3.3.4) in section 3.3 may be generalised for any value of $b = (\sigma_2' - \sigma_3')/(\sigma_1' - \sigma_3')$ to give

$$\frac{s_u(b)}{s_u(O)} = \frac{3 - \sin\phi'}{3 - \sin\phi' + 2b\sin\phi'} \qquad \ldots\ldots(4.3.1)$$

This relationship is plotted in Fig.4.3.2 for $\phi' = 30^O$, taking $s_u(O)$ as 0.33 times the vertical consolidation stress σ_{vc}'. Test results from normally consolidated samples of Boston blue clay are plotted for comparison. The strength in triaxial extension is surprisingly low by comparison with that for triaxial compression but the general trend of results agrees with the theoretical relationship.

The variation of shear modulus (at half the deviator stress change needed to cause failure) is plotted in Fig.4.3.3 (a) normalised by the vertical effective stress at the start of the test and (b) normalised by the measured undrained shear strength. Some of the results are difficult to understand at first glance particularly the lower value of G_{50}/σ_{vc}' for the passive test at OCR = 1 than for the corresponding active test. However, it must be remembered that the <u>change</u> in deviator stress over which the passive modulus is measured is much larger (by a factor of $4\frac{1}{2}$) than that over which the active modulus is measured.

One of the surprising features of the measured soil moduli is the marked decrease in G_{50}/s_u with the value of OCR(which has already been demonstrated for CIU triaxial tests). This contrasts with the values of G/s_u for natural deposits where heavily overconsolidated materials like London clay are generally fairly brittle (i.e. low strain to failure and thus high G/s_u : typical values around 150) while lightly overconsolidated deposits are more ductile, leading to lower values for G/s_u (typically 50).

The tests reported by Ladd et al were all consolidated to approximately
the same vertical effective stress. The overconsolidated samples were
then allowed to swell back from this value of vertical effective stress.
However, natural deposits have been overconsolidated in a manner such
that they finish at the same vertical effective stress. Thus in general,
the liquidity index of naturally overconsolidated deposits is much lower
than that for normally consolidated deposits. By contrast, the liquidity
index of the overconsolidated deposits tested by Ladd et al is higher than
for the normally consolidated deposits (by about 10%) and this may have
led to the more ductile behaviour during shear and thus lower values of
G/s_u . The possible influence of the liquidity index emphasises the
importance of investigating the effect of OCR by means of samples which
are consolidated to the same final vertical effective stress. In partic-
ular, a relationship between G/s_u and liquidity index would imply that
the value of G/s_u for a normally consolidated clay would not be unique
but would increase with the magnitude of the consolidation stress.

Fig.4.3.4 shows the variation of G_{50}/p' and G_{50}/p'_{max} with OCR.
The first of these follows exactly the pattern already established from
triaxial test data on undisturbed London clay in Fig.4.1.4, in which
G_{50}/p' varies linearly with the logarithm of OCR as expressed by
eqn.(4.1.9). (For the active case the appropriate value of the constant
C is 0.258.)

The variation of G_{50}/p'_{max} is also linear with logarithm of OCR, but
with a negative constant. As mentioned in section 3.1 there are theore-
tical reasons for preferring the use of a shear modulus proportional to
maximum consolidation pressure rather than to the current mean pressure.

In addition to the plane strain tests on Boston blue clay discussed
above, Ladd and Edgers (1972) have reported the results of simple shear
tests on the same clay. Simple shear tests are a form of plane strain
test where the directions of major and minor principal stress axes rotate
gradually throughout the test. The principal stresses are not measured
directly which complicates the interpretation of the test results. The
strength of the sample must be deduced from the value of the shear stress
and normal stress acting on the top face of the sample at failure. The
shear stress at failure may be converted to an undrained shear strength
by following the assumption proposed by de Josselin de Jong (1971) that

rupture takes place on the most convenient zero extension planes (see Fig.4.3.5). For a sample normally consolidated by a vertical stress, these will be the <u>vertical</u> planes, whereas for a heavily overconsolidated sample rupture will be assumed to take place on horizontal planes. For these two extremes, the ratio of measured shear stress at failure to the undrained strength of the soil is equal to $\cos \phi'$ (see Fig.4.3.5).

At intermediate values of K_o it may be shown that the ratio of $(\tau_{VH})_f$ to s_u is given by

$$(\tau_{VH})_f / s_u = [1 - \frac{|1 - K_o|}{1 + K_o}]^{\frac{1}{2}} \qquad \ldots (4.3.2)$$

$$\geq \cos \phi' \ .$$

Making use of this expression, the values of $(\tau_{VH})_f / \sigma'_{vc}$ measured by Ladd and Edgers have been converted to values of s_u / σ'_{vc} and the results plotted on Fig.4.3.1. The agreement with the theoretical plane strain strength line is very good. In the same document, Ladd and Edgers report the results of simple shear tests on six other undisturbed clays all of which are normally or lightly overconsolidated. In comparing the peak value of τ_{VH} with strengths measured in triaxial compression, they come to the conclusion that $(\tau_{VH})_f / (s_u)_{tc}$ varies from 0.65 for soils of low plasticity ($10 < PI < 25$) up to 0.80 for soils of high plasticity ($30 < PI < 85$). Combining eqn.(3.3.4) in section 3.1 and eqn.(4.3.2) above, the theoretical ratio $(\tau_{VH})_f / (s_u)_{tc}$ is given by

$$(\tau_{VH})_f / (s_u)_{tc} = \frac{(3 - \sin \phi') \cos \phi'}{3} \qquad \ldots (4.3.3)$$

This ratio varies from 0.66 for $\phi' = 35^o$ up to 0.83 for $\phi' = 20^o$. Thus the theoretical values agree well with those deduced from tests on a number of different soil types.

Returning to the simple shear tests on Boston blue clay conducted by Ladd and Edgers' the variation of shear modulus G_{50} with OCR is shown on Fig.4.3.3. There is a certain amount of scatter and the low value of shear modulus for the normally consolidated sample must be suspect. However, the results generally fit well with those obtained from plane strain tests. Again, the samples were prepared by consolidating to the same maximum

vertical effective stress rather than the same final effective stress. Thus the measured trend of G_{50}/s_u with OCR may not be consistent with the trend for naturally overconsolidated deposits of soil.

5. MEASUREMENT OF STATIC SOIL PROPERTIES IN SITU

In general, it is to be expected that the testing of soil in situ will give the best estimates of the deformation properties for use in design, since the disturbance caused by obtaining a sample of soil is avoided. Inevitably there will still be some disturbance, caused either by the insertion of the testing instrument or by the stress relief which ensues when the soil to be tested is exposed (for example in a plate loading test). Many in situ soil tests are aimed solely at obtaining an estimate of the soil strength (e.g. vane, cone, SPT) and will not be considered here where prime interest is centred on the soil stiffness.

The two commonest tests which yield values of soil stiffness as well as strength are the pressuremeter test and the plate loading test. Of these, the pressuremeter test is the more satisfactory in that the test is readily amenable to sophisticated analysis (e.g. Palmer,(1972), Hughes et al (1977)) which leads to direct values of shear modulus G and shear strength. The plate loading test on the other hand must be interpreted in a semi-analytical fashion. The value of shear modulus is usually obtained by assuming that the material beneath the plate is homogeneous and isotropic and the plate deforms according to the solution for a rigid punch resting on an elastic half space (with a correction factor for the depth of the test below ground level). Thus the shear modulus is defined as

$$G_{secant} = \frac{\Delta q}{\Delta \rho} \frac{\pi}{8} B(1 - \nu)f(z) \qquad .....(5.1.1)$$

where B is the diameter of the plate and f(z) varies from unity, for a surface test, down to ~0.85 for a test conducted at the base of a deep borehole. It is customary to measure the secant modulus over a range of bearing pressure (Δq) from the in situ effective overburden up to half the ultimate bearing pressure (Marsland and Randolph (1977)). It should be pointed out that the strain distribution beneath a loaded plate is seldom consistent with the assumption of homogeneity (Burland and Lord (1969); Marsland and Eason (1973); Moore (1975)) and thus the estimated value of shear modulus is likely to be lower than the true undisturbed value for the soil. However, in many cases (in the design of underreamed bored piles for example) the 'measured' shear modulus will be particularly appropriate for the calculation of likely settlements.

The shear strength of the soil may be estimated from a plate loading test by the use of a suitable bearing capacity factor. For clays, the ultimate average bearing pressure on the plate is usually divided by a factor close to 9 (for deep tests) to obtain the estimate of s_u. The meaning of this deduced shear strength is open to question although good correlations have been made with results from 100mm diameter triaxial tests (Marsland (1971a)). Again, however, the value may be used with some confidence in the estimation of bearing capacity for deep foundations.

The following section compares values of soil stiffness and strength, as measured in pressuremeter and plate loading tests, with results from laboratory tests. In addition, a section is included comparing values of soil stiffness deduced from the performance of actual foundations. Back-analysis of foundation behaviour seldom furnishes an estimate of soil strength (it is to be hoped) but it is common practice to interpret the estimates of stiffness as ratios of E/s_u or G/s_u. The s_u used is normally that measured in triaxial compression (or sometimes field vane) tests and so must be viewed with some caution.

5.1 In situ static tests

Two types of in situ static tests are considered here : the pressure-meter test and the plate bearing test. Shear modulus values can be obtained from both tests, though more directly from the former than from the latter test. (See Marsland and Randolph (1977) for a review of the methods of deriving soil parameters from these tests.)

The advent of self-boring pressuremeters (Windle and Wroth (1977), Amar et al (1975)) has resulted in an upgrading of the pressuremeter test, both on account of the minimal disturbance and in that the greater accuracy of the strain-measuring systems in these instruments has resulted in better quality tests for determining shear modulus.

In comparing deformation parameters obtained from different tests, the orientation of the planes of maximum shear must be considered. The pressuremeter measures G_{HH}, while the plate bearing test measures a combination of both G_{HH} and G_{VH}. Furthermore, in situ tests measure the properties of a relatively large body of soil in comparison to laboratory tests, and this will be important in soils with fabric and in stiff fissured clays, especially if the fissure spacing is of the same

order as the size of the sample tested.

Amar et al (1975) report a series of tests using the French self-boring pressuremeter (P.A.F.) in a soft clay at Cran. They compare strength and modulus values obtained from these tests with those obtained from triaxial tests on samples from the same site. Table 5.1.1 shows typical results.

	P.A.F.			TRIAXIAL		
DEPTH (m)	s_u (kN/m^2)	G_{100} (kN/m^2)	G/s_u	s_u (kN/m^2)	G_{100} (kN/m^2)	G/s_u
5	34	1300	39	28	300	12
10.5	62	3700	59	43	300	7

Note: G_{100} : secant modulus q_o to q_{max}

Table 5.1.1. Pressuremeter test results compared with triaxial test results at Cran (after Amar et al (1975))

Windle and Wroth (1977) report tests using the Cambridge self-boring pressuremeter in soft clay at Canvey Island and Mucking Flats in the estuary of the River Thames. Typically, at Mucking Flats, they show the pressuremeter shear modulus to be in the range 2,000 kN/m^2 to 3,000 kN/m^2, while Wesley (1975) shows the shear modulus from unconsolidated undrained triaxial tests on samples from the same site to be in the range 700 kN/m^2 to 1200 kN/m^2.

An extensive series of plate bearing tests has been carried out by Marsland (1971b) in heavily overconsolidated London clay. He has compared his plate bearing test results with triaxial test results on different sized samples, and with Ménard pressuremeter results. Windle has conducted tests using the Cambridge self-boring pressuremeter at a number of the sites investigated by Marsland.

Figure 5.1.1 shows results obtained by Windle and by Marsland at Hendon. Moduli obtained from the unload-reload loops of triaxial tests on 38mm and 98mm diameter samples are shown by lines A and B respectively. Line C represents the moduli obtained from plate bearing tests, and the individual

points are Windle's pressuremeter results.

Selecting typical shear strength results from the same tests, a large variation in s_u , G and G/s_u is obtained.

	$s_u (kN/m^2)$	$G (kN/m^2)$	G/s_u
CAMKOMETER	120	27,000	220
PLATE TEST	95	27,000	381
TRIAXIAL TEST	95	5,000	56

Table 5.1.2. Test results from Hendon - ~6m depth

Tests by Windle in the Gault clay near Cambridge show a similar trend, this clay is also heavily overconsolidated and in many respects is very like the London clay.

		CAMKOMETER	
DEPTH	$s_u (kN/m^2)$	$G (kN/m^2)$	G/s_u
2m	80	11,000	133
6m	160	51,000	317

Table 5.1.3

The strongest and stiffest triaxial test reported by him gives $s_u = 150$ kN/m^2, $G_{50} = 3800$ kN/m^2 and $G/s_u = 25$.

In the plate bearing test, secant moduli are determined over a bearing pressure range from zero to a half of the ultimate bearing pressure, so the shear stress increment is the same order as that used in determining G_{50} from triaxial tests.

Marsland and Randolph (1977) conclude that deformation parameters measured using a Ménard pressuremeter are substantially lower than those from plate tests (and hence from Camkometer tests), but Burgess and Eisenstein (1977) have shown that carefully carried out Ménard pressure-meter tests yield values approximately double those obtained from consoli-

dated undrained triaxial tests. Having used their pressuremeter moduli to make reasonable predictions of immediate settlements, they conclude that the pressuremeter moduli are appropriate for such predictions. Calhoon (1972) also shows that immediate settlement calculations are best carried out using pressuremeter moduli.

Values of shear modulus determined from the pressuremeter test agree well with those from the plate bearing test; both tests give values which are usually much greater than those measured in the triaxial test. Unless appropriate correction factors are applied, moduli obtained from triaxial tests are likely to be underestimated and to lead to over-predictions of settlements, and other deformations.

5.2 Back-analysis of the behaviour of foundations

The previous section looked at the wide discrepancies which may arise between deformation moduli measured from laboratory tests and those measured in situ. Discrepancies tend to be larger in cases where the soil has a well defined macro-structure such as is the case with heavily over-consolidated clay deposits. Inevitably, the laboratoy test (usually a triaxial compression test) must test intact specimens of soil, often not containing any of the fissures which dominate the in situ behaviour. In stiff clays, or soft rocks such as chalk, even in situ tests may cause sufficient local disturbance such that measured stiffness values are much lower than values back-analysed from the behaviour of a real foundation - the ultimate form of in situ test. In general, soil stiffness values back-analysed from large raft foundations are higher than those deduced from the settlement of a pile foundation (Burland and Lord (1969)). In the former case, any disturbance in constructing the foundation is confined to a layer close to the underside of the raft while the pressure bulb from the foundation will extend considerably deeper. In the pile foundation however, the stress relief on the sides and base (particularly for underreamed piles) of bored piles may cause considerable swelling and opening of any fissures present. This will reduce the overall stiffness of the foundation. Additionally, in chalk for example, installation of driven piles tends to dislodge blocks of material breaking up the close jointed structure.

In this section, in order to gain a grasp of the relative values of

stiffness measured by laboratory, in situ or back-analysis methods, the behaviour of foundations in different types of material will be studied. In particular, in view of the extensive amount of data readily available, stiffness moduli of London clay deduced from building settlement will be compared to the data from laboratory and in situ tests already reviewed in this report. Also the results of different methods of assessing the stiffness of chalk, a soft rock which may vary in stiffness by an order of magnitude depending on the degree of weathering, will be compared to values deduced from the behaviour of actual foundations.

In comparing modulus values derived from laboratory static tests with those back-analysed from observed deformations of the ground, it must be realised that the methods of back-analysis do not take into account the fact that modulus varies with strain amplitude. The shear strain amplitude to which the soil under a structure is subjected will vary from point to point. In some areas, yielding will have occurred, while in others, only very small shear strains will have been generated. Though the factor of safety of the structure against failure may be known, this does not mean that the back-analysis yields the modulus corresponding to this factor of safety.

For normally consolidated and lightly overconsolidated clays, Simons (1974) has reviewed suggested correlations between E_u and s_u from back-analysis of immediate settlement of structures. He shows that the value of G/s_u can range from 13 to 1000, though some of the higher values are for quick clays in Norway.

In a similar review by d'Appolonia, Poulos and Ladd (1971), values of G obtained from back-analysis of published settlement records are compared with values of s_u obtained from field vane and laboratory triaxial tests (see Table 5.2.1). They conclude that the value of G/s_u depends on sensitivity, OCR, plasticity index and organic content. They recommend $G/s_u = 330$ to 500 ($E_u/s_u = 1000$ to 1500) for lean inorganic clays of mean to high sensitivity, and G/s_u considerably lower (130 to 230 - i.e. E_u/s_u 400 to 800) for highly plastic clays and for organic clays. These values of G are much higher than the values measured in triaxial unconsolidated undrained tests (UU) on undisturbed samples, and are somewhat higher than the values measured in triaxial consolidated undrained tests in the case of lean, sensitive clays. In heavily overconsolidated clays

(e.g. London clay) it should be easier to obtain a representative value of shear modulus from back analysis of ground movements, as the stress-strain behaviour of such clays can more correctly be described as linear-elastic over a considerable range of shear stress.

For London clay, various E/s_u correlations have been suggested. Skempton (1957) suggested $G/s_u \sim 47$ ($E/s_u = 140$) while Ward et al (1959) showed that G/s_u is very much influenced by sampling methods. They showed G/s_u values as high as 140 for reloading tests on block samples (cf. Table 4.2.1).

Using data of Hooper (1973), Butler (1974) derived correlations between G and s_u for London clay at the Hyde Park Cavalry Barracks. He reported G/s_u ranging from 100 to 160 from back-analysis of deformations at this site.

Serota and Jennings (1959) determined that the results of triaxial compression tests on London clay from near St. Paul's Cathedral gave the relationship $G_i/s_u = 42$ ($E_i/s_u = 125$), but that the triaxial values underestimated the values of modulus obtained by back-analysis by at least a factor of 3. However, in discussing this result, Skempton (1959) stated that triaxial extension (i.e. unloading) tests should be used where heave during excavation is of interest; he quoted his (Skempton (1957)) results which showed that triaxial extension moduli can be two to four times greater than compression moduli, though the strength measured in the two tests can be the same.

In chalk and other soft rocks deformation moduli are heavily influenced by the structure of the rock. . Laboratory tests on chalk tend to measure the stiffness of the intact material. While this stiffness may be close to that for high grade (unweathered) chalk, it may well be an order of magnitude higher than the relevant stiffness for chalk nearer the ground surface. A large amount of data was collected on the deformation moduli of chalk at the time of the geotechnical investigation of the proposed site for a large proton accelerator (Ward et al (1968)). Adopting the system of grading described by Ward et al - ranging from highly weathered struc-tureless chalk (Grade V) up to hard, brittle,close packed block structure (Grade I) - it is possible to compare modulus values measured

    (i)   from triaxial compression tests on intact specimens,

    (ii)   from 0.865 diameter plate loading tests,

(iii) from measurement of vertical strains beneath a 18.3 diameter
test loading tank.

The laboratory tests generally gave shear modulus values of about
2,500 MN/m^2 which compare with moduli of Grade II chalk (the highest
grade enountered by Ward et al) from methods (ii) and (iii) above of
1,000 MN/m^2 and 1,800 MN/m^2 respectively. Table 5.2.2 summarises the
values of G in the various grades of chalk (data from Ward et al (1968)
and from Burland and Lord (1969)).

SHEAR MODULI MN/m^2

Grade of chalk	(i) From 0.865m diameter plate tests	(ii) From 18.3m diameter test tank	$G_{(ii)}/G_{lab}$
I	3500	–	(~1)
II	1000	1800	0.6 – 0.8
III	800	800	0.2 – 0.4
IV	400	400	0.1 – 0.2
V	200	150	< 0.1

Table 5.2.2. Comparison of Shear Modulus Values for Chalk

Wakeling (1969) has correlated shear modulus values in chalk with SPT
value and shown an approximately linear correlation of $\log_e$ G with N.
However, he showed also (see Burland and Lord (1969)) that the shear
modulus decreases by an order of magnitude or more (for the higher Grades
of chalk) if the structure of the chalk is disturbed. Thus, whereas the
appropriate shear moduli for calculating the settlement of a large raft
foundation might best be estimated from method (iii) above, the values
for the deformation of a piled foundation may be much lower. In partic-
ular, back-analysis of six tests on driven cast in situ piles in chalk
reported by Lord (1976) show that the relevant shear modulus for what
is described as Grade I to Grade II at the level of the pile bases, is
only about 36 MN/m^2 (see Randolph and Wroth (1978)).

In general therefore, great care must be taken when calculating design
settlements for structures on a soil with a well defined macro-structure.

It appears that the wide variation in the reported stiffness of such material is due not to variations in the quality of the undisturbed material, but due to disturbance caused at the time of testing or construction of the foundation. A common feature of all the analyses of structures on soft rock (Burland and Lord (1969); Burland et al (1973); Moore (1974); Randolph and Wroth (1978)) is that the soil is best modelled as a material whose shear stiffness is proportional to depth. This model takes account of the very low stiffness of the highly weathered material near the ground surface and the gradual transition to unweathered rock-like material at depth, see Fig.5.2.1.

Recently, attempts have been made to obtain a truly undisturbed value for the shear modulus of chalk by geophysical investigations (Abbis (1979)). Results have shown very good agreement with the model of a soil whose stiffness is proportional to depth. The dynamic values of shear modulus measured were consistently almost exactly twice the static values deduced from the strain distribution beneath the test loading tank reported by Ward et al (1968) and by Burland and Lord (1969). The factor of two between dynamic and static stiffness values is not confined to the site on chalk but has been found to hold for other soil deposits. This factor may be deduced from theoretical considerations by modelling the soil response as a viscoelastic material (Abbis (1979)).

In this section an attempt has been made to review the in situ measurement of deformation properties of soils by so call 'static' methods. The two main direct methods available at present are the pressuremeter test and the plate bearing test. There are few sites where a comprehensive programme of testing has been carried out so that valid comparisons can be made of the shear moduli measured on one soil deposit by different test methods. Both for the case of several soft clays and the case of the much-studied London clay, the values of shear moduli measured in the field are generally two to three times the values in the laboratory even when great care is taken in sampling and reconsolidating the specimen to the correct effective stress state before testing.

An alternative approach is the back-analysis of carefully observed measurements of deformation of the ground, or settlement of structures. However it is difficult to draw any sensible guidelines, because the interpretation of the results is obscured by anisotropy, non-homogeneity

and the varying strain amplitude to which different elements of the
ground are subjected.

6. DYNAMIC TESTING

The static shear moduli discussed in previous sections are relevant to situations where the soil undergoes monotonic loading. In other situations, where the soil is subjected to cyclic loading, a modulus measured by a method which imposes cyclic loading on a sample is more appropriate. Such moduli may be measured either by dynamic tests, or by cyclic 'static' tests. The only essential difference between the two types of tests is the frequency at which the cycling is carried out. For soils, the frequency affects two features of behaviour: (a) viscous and (b) the amount of drainage and dissipation of excess pore-pressure than can occur (presuming drainage is permitted). No distinction is made in the following sections between the two types of tests, and they are both considered to be dynamic in nature.

6.1 Measurement of dynamic modulus

In the laboratory the main method used to measure the dynamic modulus is the resonant column test. The details of the apparatus used varies widely between research establishments, but in all cases a high frequency cycle of small strain amplitude is applied to the sample. The resonant frequency of the sample is detected and the shear modulus calculated from the shear wave velocity v_s :

$$G = \rho v_s^2 \qquad . \qquad\qquad\qquad \dots\dots(6.1.1)$$

Since the cycles are at high frequency (typically 2 kHz) the behaviour is essentially 'undrained' and Poisson's ratio will be close to 0.5 for saturated soils (although considerably lower values for unsaturated soils may be expected).

In the field all techniques involve high frequency low amplitude cycles and are directly comparable with the above method. The conditions are again undrained, but Poisson's ratio may be measured as lower than 0.5, and for very stiff materials may fall as low as 0.4 (Ohsaki and Iwasaki (1973)). This is to be expected since for stiff materials the shear modulus ($G \sim 500$ MN/m^2) and hence the bulk modulus of the soil skeleton are comparable to the bulk modulus of water ($K_w \approx 2200$ MN/m^2)

so that volumetric strains are no longer very small compared to shear strains, and a reduction of Poisson's ratio from 0.5 occurs.

For laboratory tests at higher strain amplitudes it is common practice to use cyclic 'static' tests such as torsional shear or cyclic triaxial tests, with stress and strain being measured directly to give the modulus. Under these conditions it is possible to distinguish between 'drained' and 'undrained' tests since the cycles are usually sufficiently slow (e.g. 0.1 Hz) that drainage may be allowed. (Note that although the modulus is no longer measured dynamically the results are directly comparable with the dynamic tests and so are included here.)

It is found that good agreement between shear moduli measured in different devices (e.g. resonant column and cyclic triaxial) is achieved in the range of strain amplitudes available to both machines (see Hardin (1978)). Thus a continuous study of the variation of modulus with shear strain amplitudes from $\gamma = 10^{-6}$ to $\gamma = 10^{-2}$ is possible. At the high amplitude end of this scale the moduli may be expected to be comparable to the G_{50} considered for static tests (since γ_{50} may be of the order of 1%).

In passing, it should be remembered that the relationship depicted in Fig.3.2.3 is a consequence of the shape of the stress strain curve at large strains, and does not give detailed information about the trends in secant moduli at very small strains.

The measured modulus in a dynamic test depends on the particular variation of the amplitude of strain with time. Although there is little quantitative information available, Yong et al (1977) present data indicating that for small random cycles at low stress levels (30 kN/m^2) the modulus may be about 20% higher than for sinusoidal cycles but that it shows virtually no difference at high stress levels (300 kN/m^2). It should be noted that most laboratory tests involve sinusoidal cycles and field tests random cycles.

6.2 Dynamic laboratory testing

Dynamic laboratory testing is commonly used to obtain a secant shear modulus for soils. A variety of systems is available, and a review of several is given by Skoglund, Marcuson and Cunny (1976). No comments are offered here of the merits of the alternative methods, nor of the usual

problems of laboratory testing, such as sample disturbance. Only values of shear modulus are considered; the damping ratio, which gives a measure of the non-linearity of response is not discussed. Most of the published data on this subject relate to sands, but some less detailed information is also available for clays.

The secant shear modulus depends on many factors, the most important being shear strain amplitude, mean effective pressure and density. Of less importance are grading and angularity of the soil particles. An empirical correlation which includes the effects of the major variables is described later.

The secant modulus during cyclic loading is strongly dependent on the strain amplitude of the cycle. Below strains of about 3×10^{-6} there is little increase in the modulus, and the value for a sand in a given state approaches a maximum. No theoretical relation has yet been suggested for the dependence of modulus on strain amplitude, but an empirical fit has been derived to the extensive data on Toyoura sand (Iwasaki et al (1978)) and to more limited data on a wide variety of clean and natural sands (Iwasaki and Tatsuoka (1977)).

The dependence of shear modulus on mean effective stress may be expressed by a power law, but the exponent of the pressure term varies with strain amplitude. At low strain amplitudes the exponent approaches a value slightly higher than the value of $\frac{1}{3}$ which would be predicted by the theory of particle contact due to Hertz (1881). At high amplitudes the exponent approaches unity, indicating that the stress strain curve is then dominated by frictional behaviour. An empirical relationship fitted to the available data has been used to express the variation of this exponent with strain amplitude, and is presented later in this section.

There is evidence (Hardin and Black (1966)) that the modulus (at least for small amplitudes) does not depend significantly on the ambient deviator stress (i.e. the deviator stress about which the cycles occur) so that the effect of stress level may be accounted for by the hydrostatic component of the pressure.

In addition to these factors the modulus depends both on the density of the sand and its grading. The effect of the density may be accounted for by including a term related to the voids ratio, and the function

$\dfrac{(2.17-e)^2}{1+e}$ proposed by Hardin and Black (1966) has frequently been used. It is then found that the grading of the sand has an influence on the modulus yet to be accounted for. Clean sands show a higher modulus than either artificial sands containing fines or natural sands. Some improvement can be made by using a function of relative density rather than voids ratio, but the same trend of natural sands showing a lower modulus is still observed.

The suggested correlation to take account of the above factors is:

$$\frac{G}{p_a} = \frac{710}{1+1600\gamma} \, (P/p_a)^{(0.765-0.33e^{-3000\gamma})} \times 1.23 \times (0.9 + \frac{D_r}{500}) \qquad (6.2.1)$$

if relative density (measured by the Yoshimi-Tohno method) is used, and

$$\frac{G}{p_a} = \frac{710}{1+1600\gamma} \, (P/p_a)^{(0.765-0.33e^{-3000\gamma})} \times \frac{(2.17-e)^2}{(1+e)} \qquad (6.2.2)$$

if voids ratio is used. The symbol p_a denotes atmospheric pressure.

In the latter expression the factor $\dfrac{(2.17-e)^2}{(1+e)}$, widely used in the literature, has been retained. However, the data of Iwasaki and Tatsuoka (1977) suggest that an improved correlation is obtained if a further factor of $(0.1 + 1.2e)$ is included.

The predicted variation of modulus with strain is shown in Fig.6.2.1. The ordinate is the factor G/G^* where G^* is as defined by Iwasaki et al (1978)

$$G^* = 700 \, \frac{(2.17-e)^2}{1+e} \, (P/p_a)^{0.5} \qquad \ldots\ldots (6.2.3)$$

Thus, eqn.(6.2.2) leads to

$$G/G^* = \frac{1.014}{1+1600\gamma} \, (P/p_a)^{(0.265-0.33e^{-3000\gamma})} \qquad \ldots\ldots (6.2.4)$$

The measured values at various strains and stress levels are for Toyoura sand taken from Iwasaki et al (1978), and it is seen that the theoretical curves show the correct trend but underpredict the numerical value by

about 30%. It should be recalled that higher modulus values are usually observed for clean sands such as that from Toyoura.

The same data are represented in Fig.6.2.2 where the variation with pressure is indicated. Again the correct trends are shown, but with absolute values underpredicted.

γ		10^{-5}	10^{-4}	10^{-3}	10^{-2}
$G/G_{\gamma = 10^{-6}}$	Measured	.95	.74	.34	.07
	Calculated	.99	.86	.39	.06

Table 6.2.1. Measured and Calculated Variation of $G/G_{\gamma = 10^{-6}}$ (data from Tatsuoka et al)

A more detailed comparison is made in Table 6.2.1 of the measured and predicted variation of the modulus with strain (at a fixed pressure). In this case the modulus is non-dimensionalised with respect to the value at $\gamma = 10^{-6}$ and the measured data are from the average for all sands given by Iwasaki et al (1978). The table shows that the details of the variation are not correctly given, but the overall trend is right.

It is emphasised that no term in either of the equations 6.2.1 and 6.2.2 has any theoretical basis, the expressions being derived entirely from curve fitting. These is no particular significance in the algebraic form which has been used. The expressions do not represent a rigorously established statistical best fit to the data. The fit is derived from data in the following ranges:-

$$10^{-6} < \gamma < 10^{-2}$$
$$0.25 < p'/p_a < 2$$
$$20 < D_r < 120$$
$$0.3 < e < 0.9$$

The curve fit is based on laboratory tests on a variety of sands, and a wide scatter is observed. An improved estimate of G may be

obtained by increasing the predicted value by say 20% for clean sands
and reducing it by 20% for natural sands.

The data studied (256 data points) fall entirely within a factor of
two either side of the expressions given above.

For small strains the maximum moduli may be derived as

$$\frac{G}{P_a} = 710 \frac{p}{P_a}^{(0.435)} (0.9 + \frac{Dr}{500})1.23 \qquad \ldots\ldots(6.2.5)$$

or

$$\frac{G}{P_a} = 710 \frac{p}{P_a}^{(0.435)} \frac{(2.17 - e)^2}{(1 + e)} \qquad \ldots\ldots(6.2.6)$$

It may be expected that the shear modulus for clays will vary in a
similar manner to that for sands. Kovacs et al (1971) give data for
the variation of shear modulus with strain amplitude, which shows the
same trends as for the sands. A direct numerical comparison cannot be
made since the effective confining pressure is unknown.

For clays there is in addition an effect due to time on the shear
modulus. Stokoe and Lodde (1978) report that the dynamic modulus increases
slightly (and approximately linearly with Log (time)) during primary
consolidation, and then at a higher rate with Log (time) during secondary
consolidation. The secondary effect is more marked if the previous
preconsolidation pressure is exceeded.

Afifi and Richart (1973) give data for the variation of $\Delta G/G_{1000}$
(where ΔG is the increase of small amplitude G per Log_{10} cycle of
time and G_{1000} is the small strain modulus at 1000 minutes) with both
overconsolidation ratio and with D_{50} for the soil. The value of
$\Delta G/G_{1000}$ may be as high as 15% for a normally consolidated clay, falling
to about 5% for a silt and about 1% for a fine sand. For overconsolidation
ratios of up to about 3 this factor is reduced slightly. Although these
data are presented as a proportional increase, Stokoe and Lodde (1978)
indicate that at a given stress level the increase with Log (time) is
approximately the same arithmetic quantity at different strain amplitudes.

If a sample is subjected to a series of cycles of high shear strain
amplitude, the small strain modulus subsequently measured is lower than
the value measured before the high amplitude cycling (by say 10% for

cycling at $\gamma = 10^{-3}$). The original value of the small strain modulus is regained with time, with the time for recovery increasing with both the amplitude and number of high amplitude cycles.

The effects of stress level and voids ratio are similar to those for sands, but there are insufficient data to establish numerical expressions independently for cohesive materials. However, Yong et al (1977) give data for shear modulus values of three materials (Grundite Soil, Sensitive Clay and Silica Sand). Using the best estimates of the voids ratio which can be deduced from their data, Table 6.2.2 gives a comparison of the measured small strain modulus (for sinusoidal vibration) with the prediction from eqn.(6.2.6). The agreement is remarkable good for all three materials.

		Shear Modulus * (MN/m^2)			
Soil	Water Content %	$p = 35$ kN/m^2 Meas.	Calc.	$p = 350$ kN/m^2 Meas.	Calc.
Grundite	20.9	67	50	137	141
	29.4	38	34	64	96
Remould. Sensit. Clay	19.6	75	52	280	147
	30.7	34	33	74	94
Silica Sand	8.6	32	49	134	133

* Shear modulus is calculated assuming that the maximum water content corresponds to maximum recorded voids ratio; and similarly for the minimum. For the Silica Sand the mean voids ratio is used.

Table 6.2.2. Measured and Calculated Shear Modulus (Data from Yong et al (1977))

6.3 Dynamic In Situ Testing

The small strain shear modulus for a soil may be measured by a
variety of in situ techniques, using measurements either of naturally
generated microtremours or of artificial impulses. No discussion is
made here of the different techniques for the measurement of shear
modulus in situ, but it may be expected that the different methods
will lead to some variation in calculated moduli. The in situ shear
modulus is derived from velocity and density measurements. The value
of the density used may be a source of error, and several of the
correlations discussed below are based only on the density values
assumed by Ohsaki and Iwasaki (1973).

The magnitude of strain amplitide may have some effect on the
magnitude of the modulus, but data quoted by Ohsaki and Iwasaki (1973)
from Hara (1972) indicate amplitudes of less than 3×10^{-6} for shear
waves generated by hammering a plate on the ground surface. The measured
modulus therefore corresponds to the small strain value. The strain
amplitude decreases rapidly with depth to a value of approximately
10^{-7} at 20 metres depth.*

Hara (1973) compared the moduli measured by seismic exploration and
those measured in dynamic triaxial tests at a shear strain amplitude of
10^{-4}; a close agreement between the two methods is found. Cunny and
Fry (1973) also find reasonable agreement, and attribute any variation
mainly to the difficulty in re-establishing the field stresses in the
laboratory.

The shear modulus may be correlated with the blow count N of the
Standard penetration test. The SPT is not a fully standardized test
and so the results show a considerable variation due to different
operators. The SPT is also influenced by many factors other than the
shear modulus (e.g. soil type, depth) so a close correlation cannot be
expected.

Fig.6.4.1 shows several correlations for shear modulus with the standard

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* Some doubt is attached to this conclusion, however, since Hara et al
(1974) requote apparently the same data, but indicate all strain ampli-
tudes as increased by a factor of about 5. At depths less than ten
metres the measured modulus may therefore be rather less than the small
strain modulus.

penetration test. The very low shear modulus values for sand reported by Kanai et al (1966) are not confirmed by later authors. Imai and Yoshimura (1970) and Ohba and Toruimi (1970) give correlations for shear wave velocities, which were converted to moduli by Ohsaki and Iwasaki (1973), who also derived their own correlation. These correlations, and also those of Ohta et al (1972) and Hara et al (1974) show substantial agreement. Kovacs (1975) reports a rather different trend in the variation, but the numerical values are quite close in the most common range of SPT values (say 5 to 50). The suggested correlation is in agreement with the majority of the above, and the limits fall close to the limits of scatter of the data of Ohta et al (1972) and Ohsaki and Iwasaki (1973). Although some authors have studied the variation in the correlation for different materials and indicate that clays give higher moduli than sands at a given N value, the difference is not substantial compared with the considerable scatter, and so no attempt to distinguish between materials is made here.

Ohsaki and Iwasaki (1973) collect the correlations from various sources and obtain an approximate correlation which may be written:

$$\frac{G}{P_a} = 120 \ N^{0.8} \quad . \qquad \qquad \dots \dots (6.3.1)$$

Taking account of addition data given by Hara et al (1974) and Kovacs (1975) the suggested correlation is:

$$\frac{G}{P_a} = 120 \ N^{0.77} \quad . \qquad \qquad \dots \dots (6.3.2)$$

With the minimum and maximum limits of the data being

$$60 \ N^{0.71} \ < \ G/P_a \ < \ 300 \ N^{0.8}$$

giving a range of about one order of magnitude. The correlation is based on several hundred tests having N values in the range 1 to 100. The correlation above is given, together with other correlations in Fig.6.3.1.

The in situ shear modulus was correlated with the undrained shear strength by Hara et al (1974), who obtained the relationship:

$$\frac{G}{p_a} = 516 \left(\frac{s_u}{p_a}\right)^{1.012} \qquad \ldots\ldots(6.3.3)$$

This correlation applies for s_u/p_a values between 0.1 and 10. Since the exponent is close to unity this suggests an approximately constant value of $G/s_u = 500$. Widely differing average values of G/s_u have been obtained by workers using different laboratory and field techniques. Although the variation of shear modulus may be the main cause of this scatter, it is emphasised that the value of undrained strength also depends considerably on the method of measurement used. Even allowing for these sources of scatter a single correlation between G and s_u is not to be expected to apply for all material types, and is certainly not supported by the findings presented earlier in this report.

Seed and Idriss (1971) study the variation of G/s_u with shear strain amplitude (applying "correction" factors to some of the data) and although there is a wide scatter in the data the indication is that G/s_u falls from a value in the range 1000 to 3000 at small strain to about 30 to 300 at a strain of 1%. Lower values of G/s_u are reported by Kovacs et al (1971) for undrained cycling of soft clay.

The above correlations between G and N and between G and s_u may be combined to give a relationship between s_u and N, resulting in

$$s_u/p_a = .234 \, N^{0.761} \qquad \ldots\ldots(6.3.4)$$

if eqn.(6.3.3) is used and

$$s_u/p_a = .24 \, N^{0.77} \qquad \ldots\ldots(6.3.5)$$

if the constant value of $G/s_u = 500$ is used. These correlations may be compared with

$$s_u/p_a = 0.297 \, N^{0.72} \qquad \ldots\ldots(6.3.6)$$

as derived by Hara et al (1974) directly from the same data as used for the previous correlation. Figure 6.3.2 shows this comparison as well as the range of s_u/N values indicated in the Navfac Manual (1971) and the s_u against N correlation suggested by Terzaghi and Peck (1948).

A comparison may also be made with the shear modulus as calculated using the formulae in section 6.2. Estimates of vertical pressure and SPT values allow a value of relative density D_r to be estimated using the charts given in the Navfac Manual (1971).

For example, consider a saturated coarse sand at depth 14m having $\phi = 35^O$ and $N = 30$. Estimate $K_o = 0.43$, $\sigma_v' = 150$ kN/m^2, and $p' = 93$ kN/m^2. Using the curve for saturated coarse sand, $D_r = 91$. The small strain value given by eqn.(6.2.5) is

$$G/p_a = 800 \left(\frac{93}{100}\right)^{0.435} (0.9 + \frac{91}{500})$$

$$= 839 .$$

From the direction correlation with N

$$G/p_a = 120 \times 30^{0.8}$$

$$= 1823 .$$

The factor of 2.2 between the two estimates of shear modulus is within the range of accuracy one may expect from the above approximate correlations.

6.4 A comparison of field observations of shear moduli in sand

A unique opportunity occurred recently for a detailed comparison to be made of various field methods of measuring the shear modulus in a dense sand deposit. The site in question was that of a proposed nuclear reactor in the U.K., for which one design requirement is the specification of a small allowable settlement, and the consequent need for an accurate assessment of the stiffness of the ground.

The methods used were (a) plate bearing tests, (b) self boring pressuremeter tests with the Camkometer, (c) seismic tests, and (d) back-analysis of the observed settlement of the reactor. The results are presented in Fig.6.4.1 as shear modulus plotted against the best estimate of shear strain amplitude (on a logarithmic scale); they are compared with values calculated by the methods recommended by Hardin and Drnevich (1972) and by Sherif and Ishibashi (1976)*.

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* The correlations by Hardin and Drnevich and by Sherif and Ishibashi give different expressions for shear modulus, accounting for shear strain amplitude, stress level and soil type. The results are qualitatively very similar to those of eqn.(6.2.2) or (6.2.3) and the numerical values derived are only slightly different

The results of the plate bearing tests and the back-analysis of the settlement of the reactor are shown as bars, because of the range of strain to which different elements of sand near the load undergo. Although there is a rapidly varying strain field immediately under the load, which provides the major proportion of the settlement, the results are interpreted by the theory of elasticity to give one value of shear modulus independent of the amplitude of shear strain. It is significant that the two tests (at very different scales) give two very different values of shear modulus each of which is appropriate to the relevant (average) level of strain imposed.

The seismic tests, carried out by a team from the University College of North Wales, used the method of direct measurement of horizontal shear wave velocities.

The Camkometer tests were from two series. The first, conducted by Windle (1976), was subject to a greater degree of disturbance caused during the insertion of the pressuremeter; the two full circles represent initial values of tangent moduli. The second series conducted by Jewell and Fahey (1979) were carried out with a much improved drilling technique resulting in a higher value of the initial tangent modulus. Around the pressuremeter there is a varying strain field, and the value of strain chosen for plotting is that experienced by the sand in contact with the membrane. The truly elastic behaviour of the sand observed with very accurate measurements during an unloading-reloading cycle gives a modulus which is the same as that for the very small strain amplitude imposed by the seismic tests. However, some caution should be exercised when comparing dynamic and static moduli, particularly for materials where the effect of strain rate may be important.

The field observations agree encouragingly well with the values calculated from the theoretical expressions derived by Hardin and Drnevich (1972) and by Sherif and Ishibashi (1976) from laboratory tests. They confirm that the relevant value of the modulus is critically dependent on the strain amplitude for this particular sand. The results should give added confidence to the use of such relationships in design calculations, and the prediction of deformations in foundation engineering.

7. CONCLUSIONS AND RECOMMENDATIONS

The main purpose of this report has been to review what is currently known about the shear stiffness of soils and to attempt to draw this together and establish as far as possible whether there is a coherent pattern of how the stiffness of a soil element is influenced by such factors as the ambient effective stress state, the consolidation history, the index properties, the particle size distribution and the magnitude of the stress (or strain) increment.

It has been taken for granted that although the stress-strain relationships for nearly all soils are non-linear, that for a particular increment of stress (or strain) a suitable equivalent elastic response can be ascribed to a given soil. In most instances this will probably be limited to perfect elasticity, and it is assumed that (i) a proper distinction is made between the values of the elastic constants in terms of total stresses and in terms of effective stresses and (ii) the superiority of working in terms of the shear and bulk moduli G and K over Young's modulus E and Poisson's ratio ν is recognised.

This does not mean that the use of elasticity will necessarily be appropriate for any deformation analysis: in many circumstances it may well be desirable to use an elastic/plastic model to simulate the soil behaviour. The choice is an essential one in the design process, with the outcome depending on such factors as the magnitude and importance of the construction, the funds available for design and whether the control of deformations is crucial or not. In any event whether the ground is characterised by a purely elastic or by an elastic-plastic model, an elastic modulus will have to be selected for use in the calculations.

In reviewing the data available in the literature a careful and critical selection is necessary for a variety of reasons. There is of course the major distinction to be made between cohesive soil and cohesionless soil and what parameters can be used accordingly for purposes of correlation.

In the case of clays the first and foremost parameter is the undrained shear strength - and at once there is the difficulty that this is not a unique property of a particular soil element, but that it is dependent on the type of stress increment or test such as axially symmetric, plane strain (with or without rotation of principal axes) or truly three-

dimensional.

Apart from the type of test, the ambient stress state and consolidation history are important and care must be taken to differentiate between isotropic and one-dimensional consolidation. For each type of consolidation and type of test a clear pattern has been shown to exist between the dimensionless ratio G/s_u and OCR for samples originally consolidated to the same maximum stress level. In the case of some heavily overconsolidated soils, it may not be possible to determine a value of OCR and it is suggested that the alternative parameter e_λ might be used with advantage.

The non-linear behaviour of soils entails that the size of the increment for which a secant shear modulus is measured, is of great importance: the magnitude of engineering strain may vary from 10^{-6} in dynamic tests to 10^{-2} in 'static' laboratory tests.

Another important factor is the influence of sample disturbance and the well-established discrepancy between moduli measured in the laboratory and those obtained from in situ tests, such as plate bearing tests or pressuremeter tests or from back-analysis of the behaviour of full scale structures.

For the case of sands, the undrained shear strength must be replaced by the mean effective pressure p', or by the atmospheric pressure p_a as the parameter of stress for sensible correlations to be made. Likewise overconsolidation ratio has to be replaced by voids ratio or relative density, or failing either, by the blow count of the (non-standard) Standard Penetration Test.

The specific correlations and recommended procedures are discussed in the next section.

7.1 Detailed comments and recommendations

Throughout the report, an attempt has been made to correlate data in the literature with expressions derived from theoretical considerations. Where no theoretical basis as yet exists, empirically derived expressions have been introduced which, used with due caution, may serve as a framework to assess and possibly supplement scanty field data. Important examples of the latter type of expression are those used to relate the secant modulus

of soil to the stress or strain level over which the modulus is measured.
In section 3.2 it was shown that for stress-strain curves in triaxial
compression from a number of theoretical soil models, the secant modulus
at different stages of the test closely followed the relationship

$$\frac{G}{G_{50}} = \frac{6 - 4 \, q/q_{max}}{3 + 2 \, q/q_{max}} \qquad \ldots(3.2.2 \text{ bis})$$

This expression highlights the typical range of secant modulus (from $2\,G_{50}$
at the start of the test down to $0.4\,G_{50}$ near failure) to be expected for
soil stress-strain curves.

Inspite of the variation in the value of the undrained shear strength,
s_u , measured in different forms of test, it is widely accepted that, for
a particular form of test, the ratio of s_u to the relevant effective stress
(either σ_{vo}' for one-dimensional consolidation or p_o' for isotropic
consolidation) follows a definite pattern depending on the degree of over-
consolidation. Theoretical reasoning for this pattern has been obtained in
terms of the Cam-clay model which leads to a similar form for the relation-
ship (see eqn. 4.3.1) between s_u/p_o' (for normally consolidated soil) and
plasticity index to Skempton's empirical relationship. The theoretical
model also leads to the more general expression for s_u/p_o' for overconsoli-
dated soil of

$$\left(\frac{s_u}{p_o'}\right) = \left(\frac{s_u}{p_o'}\right)_{n.c.} \cdot (OCR)^{(\lambda-\kappa)/\lambda} \qquad \ldots(4.1.5 \text{ bis})$$

An empirical relationship for the variation of G/p_o' with OCR (based on
theoretical considerations) has been presented which enables the key ratio
of G/s_u to be calculated in terms of $(G/s_u)_{n.c.}$ as the degree of over-
consolidation is varied. The expression is

$$r = \frac{G/s_u}{(G/s_u)_{n.c.}} = [1 + C \, \ln(OCR)](OCR)^{-(\lambda-\kappa)/\lambda} \qquad (4.1.12 \text{ bis})$$

The form of this expression (Fig.4.1.7) compares well with laboratory
data from three clays (Fig.4.1.6), where, for each clay, the samples have
all been taken to the same maximum consolidation stress, and thus will have

the same value of $(G/s_u)_{n.c.}$. The trend of G/s_u decreasing with increasing OCR is in contrast to recommended values of ε_{50} (proportional to the <u>inverse</u> of G_{50}/s_u) in the literature (e.g. Sullivan et al (1979)). These recommendations show ε_{50} decreasing (i.e. G_{50}/s_u increasing) the stiffer and more overconsolidated the clay. The key to this apparent contradiction lies in the variation of $(G/s_u)_{n.c.}$ with the effective stress level for which sufficient data do not yet exist in the literature. Intuitively it seems that the ratio of $(G/s_u)_{n.c.}$ may well be linked to the liquidity index of the soil, increasing as the soil is consolidated from its liquid limit up to its plastic limit. There is a pressing need for further experimental research in this area.

The problem of measuring values of G and s_u from standard soil samples was discussed in section 4.2. In order to obtain reasonable estimates of the in situ stiffness of the soil, it is necessary to reconsolidate the soil samples in the laboratory back to the in situ stress state allowing time for any secondary consolidation to occur. There is a growing practice in testing laboratories to overcome sampling disturbance by means of reconsolidating soil samples to much higher effective stresses than they have ever experienced and then allowing them to swell back to the same value of OCR as exists in the field. An approach such as SHANSEP is then used to estimate the true field strengths from the measured laboratory strengths (which will be higher due to the higher effective stress level). However, the considerations of the previous paragraph indicate that deformation moduli estimated in the same manner may be overestimated since the liquidity index of the laboratory samples will be lower than in the field, leading to higher values of G/s_u.

In general, deformation moduli obtained from triaxial samples are considerably lower than those determined from in situ tests such as the pressuremeter or plate loading test. Results quoted in section 5.1 show factors varying from 3 to 8 between values of G/s_u from triaxial tests and from self-boring pressuremeter tests for both soft and stiff clay deposits. When choosing appropriate values of stiffness, the evidence obtained from the back-analysis of observed foundation behaviour is invaluable. For London clay, back-analysed values of G/s_u are of the order of 120 to 150; the only way to obtain such values from triaxial samples is from <u>reloading</u> loops on samples obtained from carefully excavated blocks

of clay (Table 4.2.1). In the other extreme, laboratory determination of the stiffness of chalk is liable to overestimate the stiffness relevant for an actual foundation (Table 5.2.2). While it is difficult to draw firm conclusions it is clear that the effects of disturbance, either in obtaining samples or in actual construction of a foundation, play a major role in determining the relevant stiffness of the soil.

The importance of moduli determined from small strain amplitudes in dynamic tests on soil is gradually becoming recognised. Not only is the small strain often relevant to the strain level for the actual foundation (see section 6.4 and Fig.6.4.1) but it provides an opportunity to investigate how the truly elastic properties of a soil depend on its structure and on the stress level. In section 6.2, an attempt has been made to fit a large amount of data from dynamic laboratory tests on sand into a simple framework, enabling the shear modulus G to be correlated with stress level, density of the sand and the strain amplitude over which the sand was cycled. Eqns.6.2.1 and 6.2.6 present expressions giving the correlations, the accuracy and usefulness of which are demonstrated in Tables 6.2.1 and 6.2.2. In dynamic field (seismic) measurements of the shear modulus of sand, the density and stress level are often difficult to determine. As such, many of the data from seismic tests have been compared with results from SPT tests. The different correlations of G and s_u with blow count N are discussed in section 6.3.

As a parallel exercise to that described in this report, research work has been carried out on correlations between index properties and other engineering properties of soils. This work has appeared as two publications which are included in this report as appendices.

The main hypotheses forming the basis of the work are that the index tests are essentially shear tests conducted on remoulded soil in which the water content is changed until the soil has a specific consistency or strength. It is shown that within a small variation the undrained shear strength of a clay at its liquid limit is approximately 1.7 kN/m^2, and at its plastic limit is 100 times the value.

Two direct consequences are (i) that a simple relationship exists between the liquidity index of a soil and its (remoulded) undrained strength and (ii) that a linear relationship exists between the plasticity index and the compression index. Hence a knowledge of the index properties, and

the natural water content of a specimen allows (conservative) predictions to be made of its strength and compressibility.

In the second paper, the concepts are developed in terms of effective stresses to establish that the normal consolidation line and the critical state line for an insensitive soil are essentially unique in a plot of liquidity index against logarithm of the effective overburden pressure. This allows an estimate of the overconsolidation ratio of a specimen to be made solely from a knowledge of its liquidity index and its depth.

The findings form a powerful framework within which poor quality data can be assessed for their reliability and consistency. It must be emphasised, however, that this approach is only an adjunct to site investigation and soil testing, and is in no way intended as a substitute.

7.2 Summary

An attempt has been made to pick out key observations on engineering properties of soils from diverse published data and to interpret these observations in a rational manner. The data are of widely differing quality and there is a limited number of soils which have been reliably tested. In particular, data have been drawn from soil tests on Boston blue clay, London clay and on Japanese sands. The theoretical and empirical correlations which have been proposed are intended, not as a substitute for thorough site investigation, but to provide a framework of soil parameters as an aid when test data are not available and when choosing parameters for theoretical models of soil-structure interaction.

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DEPTH (m)	P.A.F.			TRIAXIAL		
	s_u (kN/m^2)	G_{100} (kN/m^2)	G/s_u	s_u (kN/m^2)	G_{100} (kN/m^2)	G/s_u
5	34	1300	39	28	300	12
10.5	62	3700	59	43	300	7

Note: G_{100} : secant modulus q_0 to q_{max}

Table 5.1.1.   Pressuremeter test results compared with triaxial test results at Cran   (after Amar et al (1975))

	s_u (kN/m^2)	G (kN/m^2)	G/s_u
CAMKOMETER	120	27,000	220
PLATE TEST	95	27,000	381
TRIAXIAL TEST	95	5,000	56

Table 5.1.2.   Test results from Hendon – ~6m depth

CAMKOMETER			
DEPTH	s_u (kN/m^2)	G (kN/m^2)	G/s_u
2m	80	11,000	133
6m	160	51,000	317

Table 5.1.3

Case number (1)	Location and structure (2)	Clay Properties			G (Field) (kN/m^2) (6)	G/s_u (7)	Source of S_u (8)	Reference (9)
		Pl, as a percentage (3)	S_t (4)	OCR (5)				
1	Oslo—nine story bldg.	15	2	3.5	76,000	400	CIU	Simons (1963)
2	Asrum I—circular load test	16	100	2.5	9,900	330 / 400	Field Vane / CIU	Höeg, et al. (1969)
3	Asrum II—circular load test	14	100	1.7	8,800	330 / 370	Field Vane / CIU	Höeg, et al. (1969)
4	Mastemyr—circular load test	14		1.5	13,000	400 / 570	Field Vane / Bearing Capacity	Clausen (1969)
5	Portsmouth—highway embankment	15	10	1.3	30,000	670 / 570	Field Vane / Bearing Capacity	Haley and Aldrich, Inc. / Ladd (1969)
6	Boston—highway embankment	24	5	1.5	100,000	530 / 400	Field Vane / CK_0U	MIT
				1.0	130,000	830 / 500	Field Vane / CK_0U	
7	Drammen—circular load test	28	10	1.4	32,000	470 / 370	Field Vane / CK_0U	NGI
8	Kawasaki—circular load test	38	6 ± 3	1.0	22,000	130	Field Vane / CIU	MIT
9	Venezuala—oil tanks	37	8 ± 2	1.0	50,000	230	CIU	Lambe (1962)
10	Maine—rectangular load test	33 ± 2	4	1.5-4.5	1,000-2,000	27-53	UU and Bearing Capacity	Ladd, et al. (1969)

Table 5.2.1. Field data on G and s_u (after d'Appolonia, Poulos and Ladd (1971))

$$\underline{\text{SHEAR MODULI}} \quad MN/m^2$$

Grade of chalk	(i) From 0.865m diameter plate tests	(ii) From 18.3m diameter test tank	$G_{(ii)}/G_{lab}$
I	3500	–	(~1)
II	1000	1800	0.6 – 0.8
III	800	800	0.2 – 0.4
IV	400	400	0.1 – 0.2
V	200	150	< 0.1

Table 5.2.2. Comparison of Shear Modulus Values for Chalk

γ		10^{-5}	10^{-4}	10^{-3}	10^{-2}
$G/G_{\gamma=10^{-6}}$	Measured	.95	.74	.34	.07
	Calculated	.99	.86	.39	.06

Table 6.2.1.   Measured and Calculated Variation of $G/G_{\gamma=10^{-6}}$ (data from Tatsuoka et al)

		Shear Modulus [*] (MN/m^2)			
Soil	Water Content %	$p = 35 \ kN/m^2$		$p = 350 \ kN/m^2$	
		Meas.	Calc.	Meas.	Calc.
Grundite	20.9	67	50	137	141
	29.4	38	34	64	96
Remould. Sensit. Clay	19.6	75	52	280	147
	30.7	34	33	74	94
Silica Sand	8.6	32	49	134	133

[*]   Shear modulus is calculated assuming that the maximum water content corresponds to maximum recorded voids ratio, and similarly for the minimum.   For the Silica Sand the mean voids ratio is used.

Table 6.2.2.   Measured and Calculated Shear Modulus (Data from Yong et al (1977))

Fig.2.3.1.   Definition of G_{50}
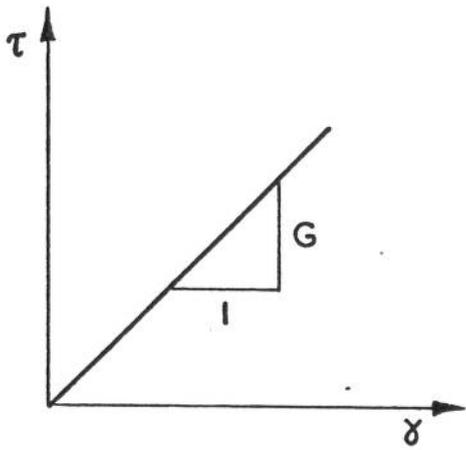
(a) Simple shear test
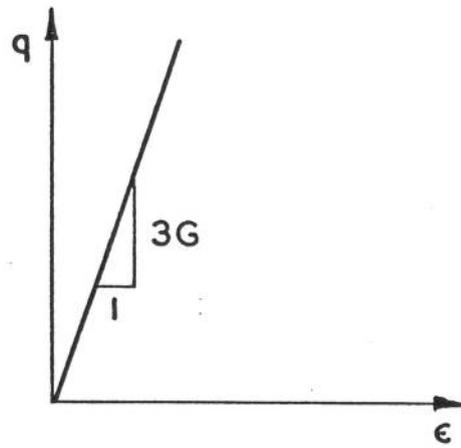
(b) Triaxial test
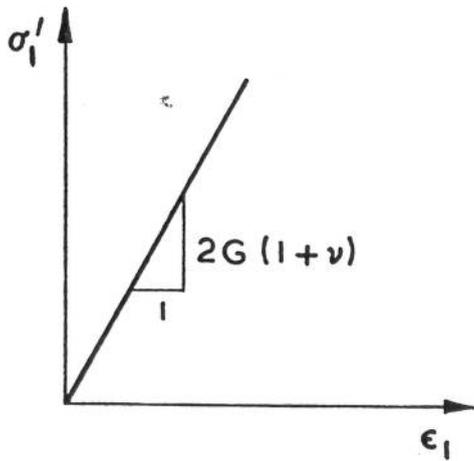
(c) Standard triaxial test $(\Delta\sigma'_3 = 0)$
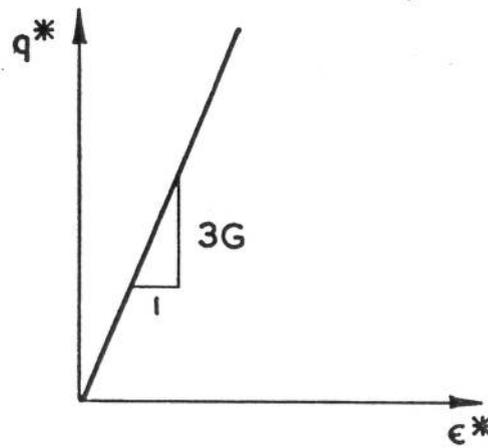
(d) General test (e.g. plane strain)

Fig.2.3.2.   Relationship of gradients of stress-strain
curves to the shear modulus G
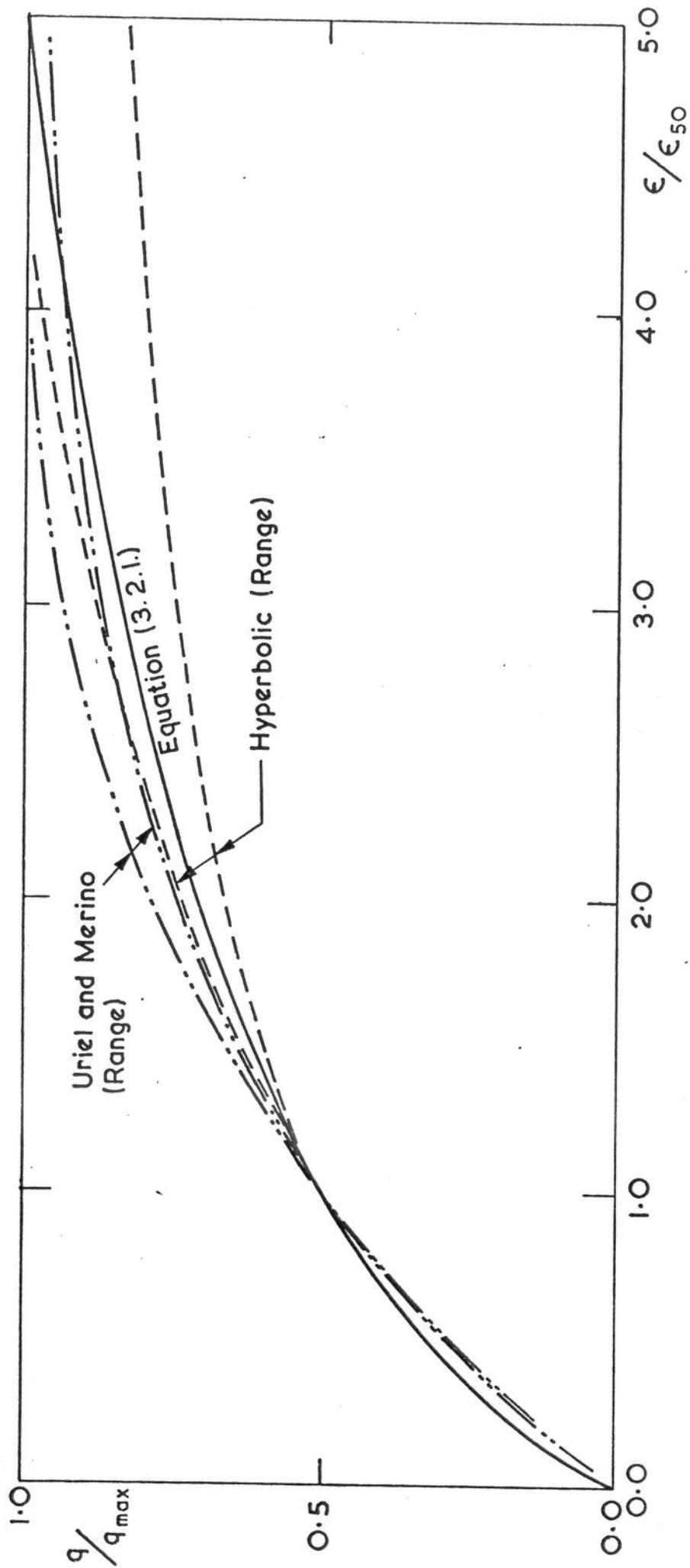
Fig.3.2.1. Non-dimensional stress-strain curves for hyperbolic and Uriel models for drained triaxial compression tests
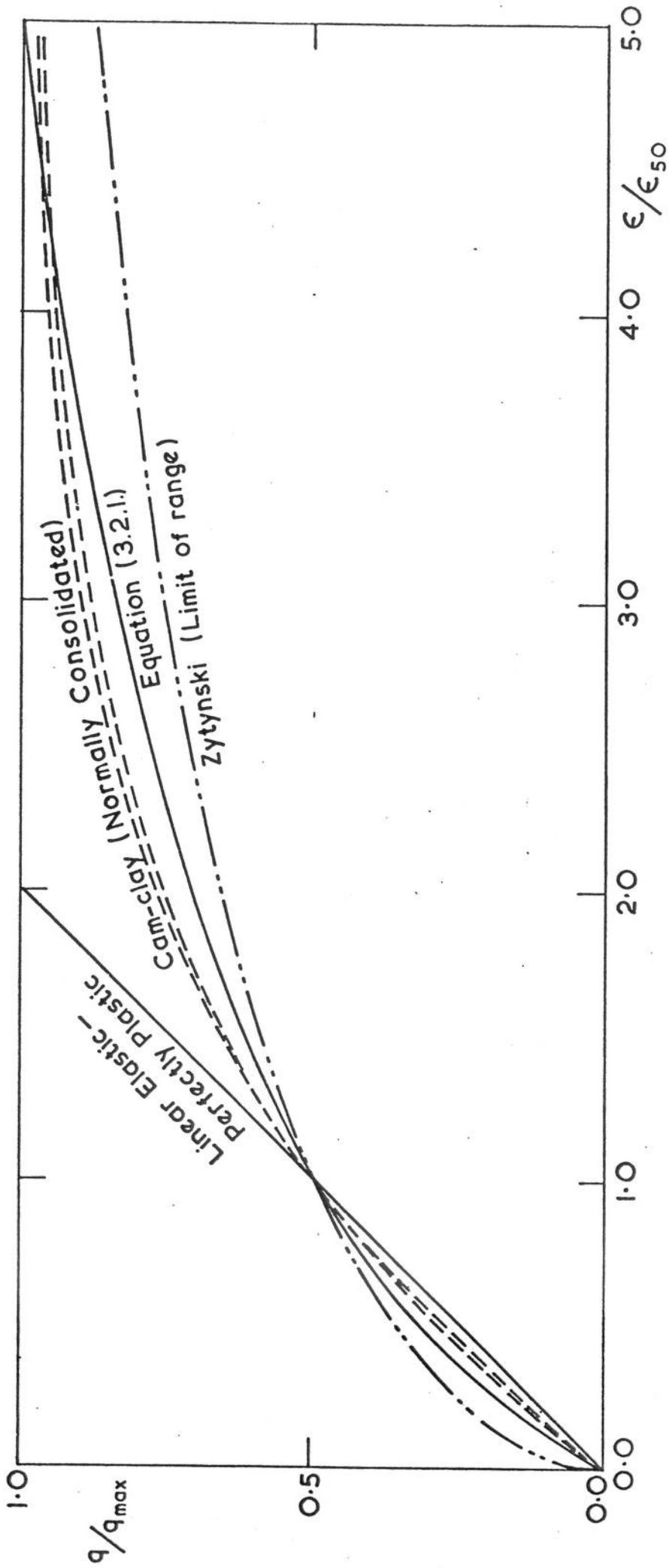
Fig.3.2.2. Non-dimensional stress-strain curves for Cam-clay and Zytynski models for drained triaxial compression tests
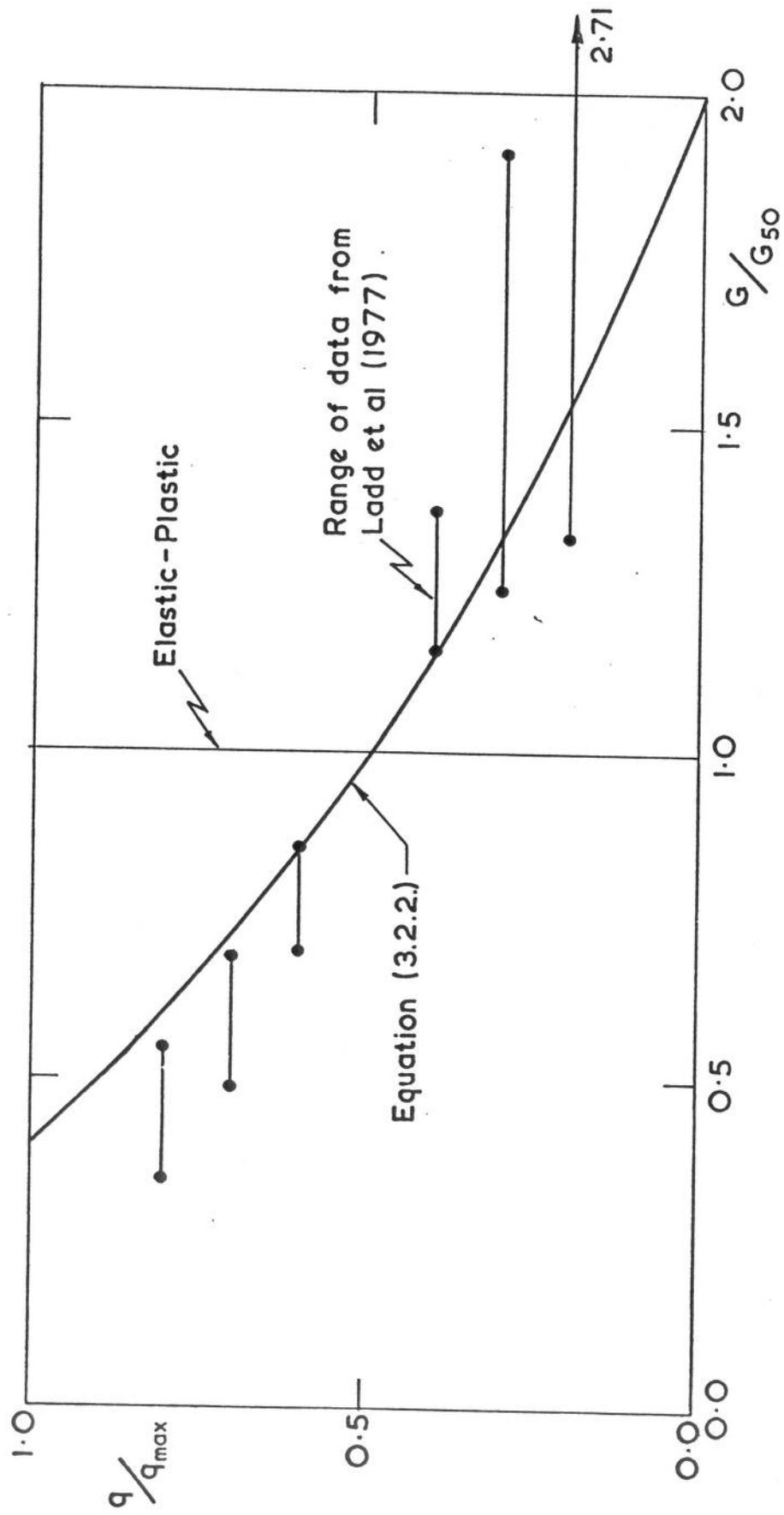
Fig.3.2.3. Approximate variation of secant shear modulus with proportion of shear strength mobilised (in drained triaxial compression tests)
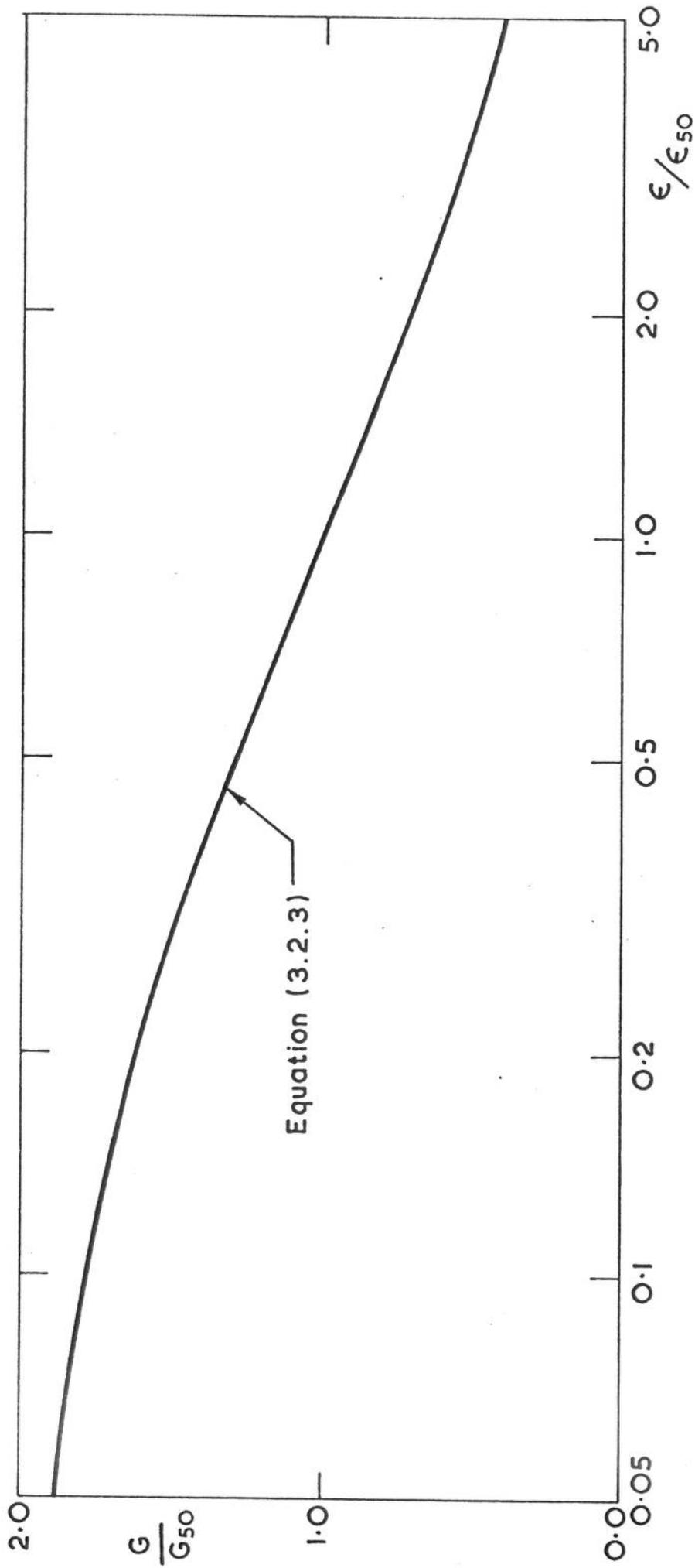
Fig.3.2.4. Approximate variation of secant shear modulus with shear strain (in drained triaxial compression tests)

$$p_f' = \tfrac{1}{3}(\sigma_1' + 2\sigma_3')$$

(a) Triaxial compression

$$p_f' = \tfrac{1}{3}(2\sigma_1' + \sigma_3')$$

(b) Triaxial extension

$$p_f' = \tfrac{1}{2}(\sigma_1' + \sigma_3')$$

(c) Plane strain
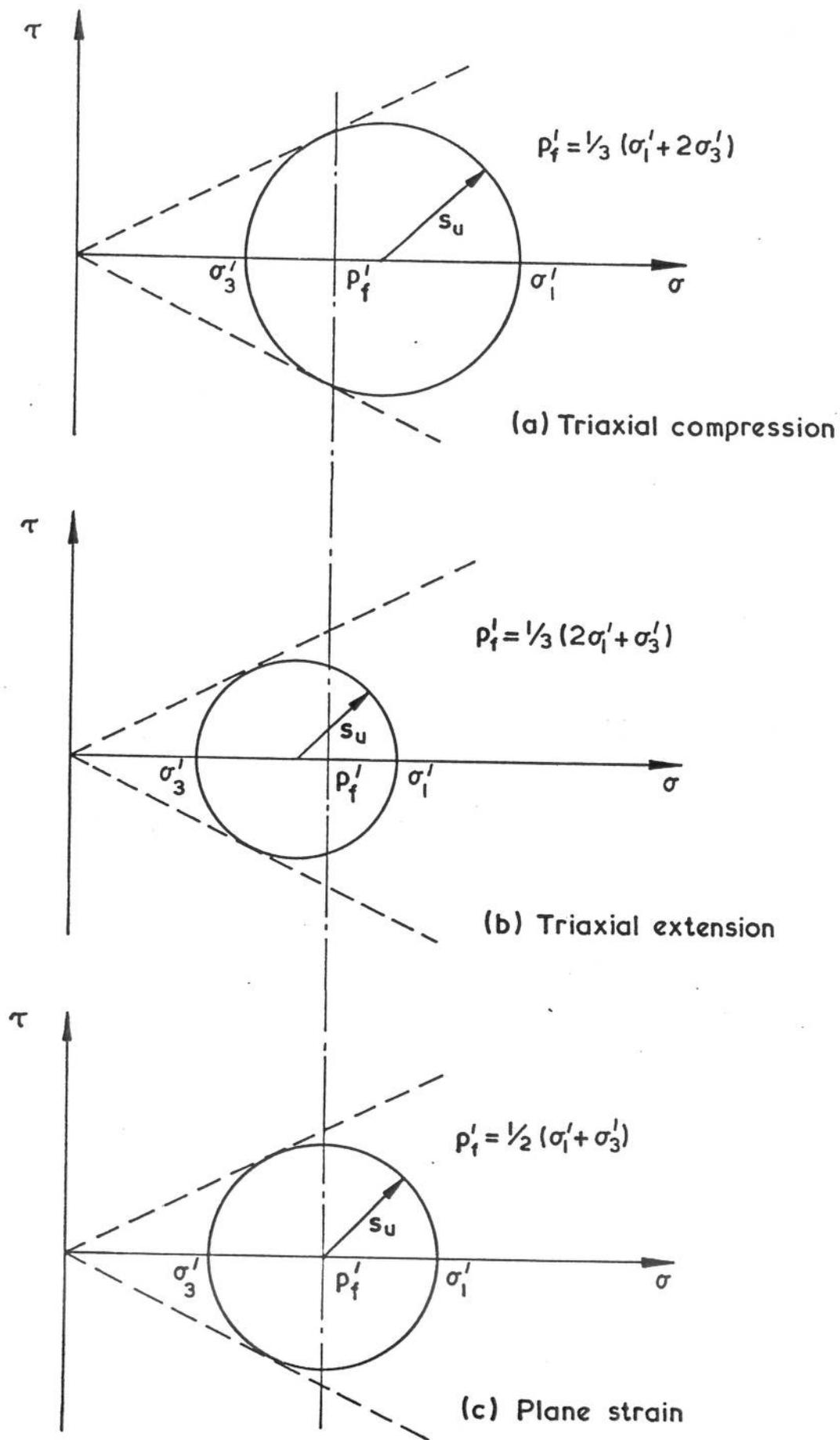
Fig.3.3.1. Variation of undrained shear strength
in different tests
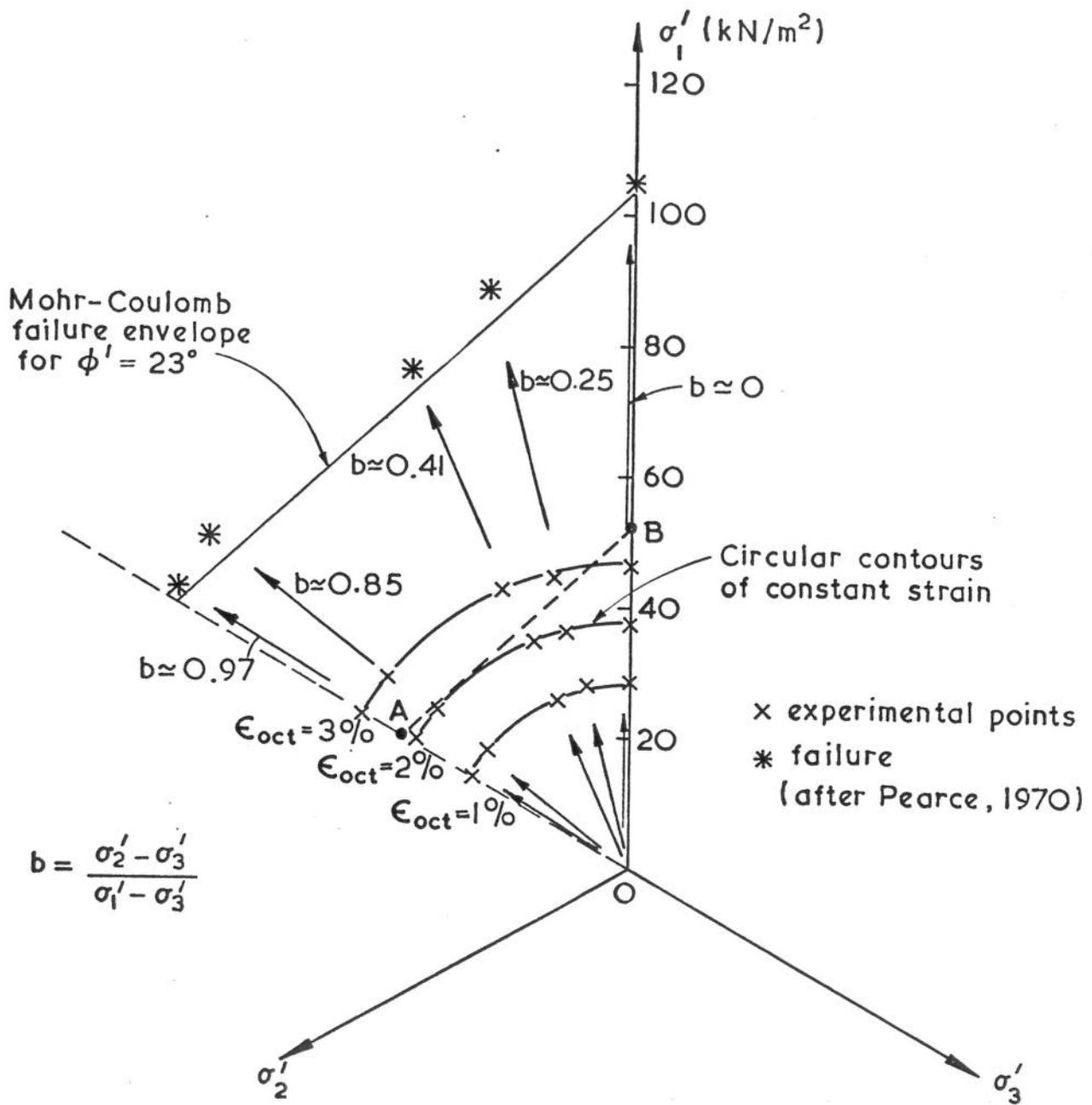
Fig.3.3.2.   Results from constant b tests in true
             triaxial apparatus (after Pearce (1970))

z axis is vertical

Planes of isotropy are horizontal
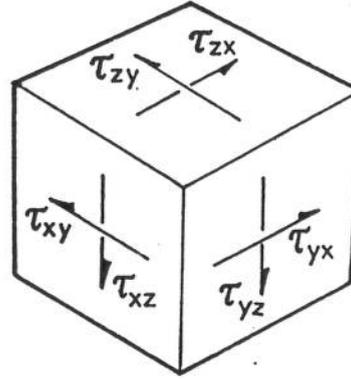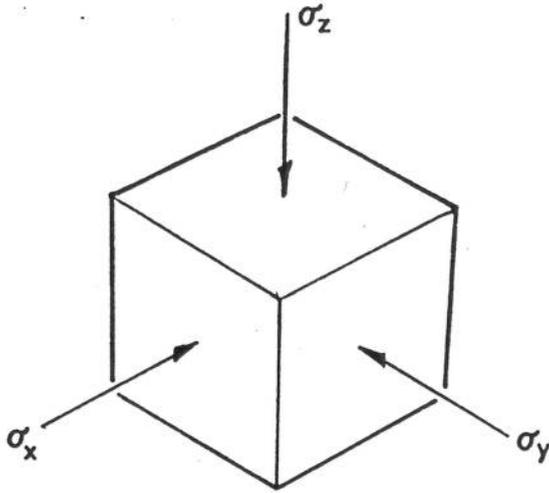
$$\varepsilon_x = \frac{1}{E_H} [\sigma_x - \nu_{HH}\sigma_y - \nu_{VH}\sigma_z]$$

$$\gamma_{xy} = \frac{1}{G_{HH}} \tau_{xy}$$

$$\varepsilon_y = \frac{1}{E_H} [-\nu_{HH}\sigma_x + \sigma_y - \nu_{HV}\sigma_z]$$

$$\gamma_{yz} = \frac{1}{G_{HV}} \tau_{yz}$$

$$\varepsilon_z = \frac{1}{E_V} [-\nu_{VH}\sigma_x - \nu_{VH}\sigma_y + \sigma_z]$$

$$\gamma_{zx} = \frac{1}{G_{HV}} \tau_{zx}$$

$$\nu_{VH} = \nu_{HV} \cdot \frac{E_V}{E_H}$$

$$G_{HH} = \frac{E_H}{2(1 + \nu_{HH})}$$

Fig.3.4.1. Definition of elastic parameters for transversely isotropic material
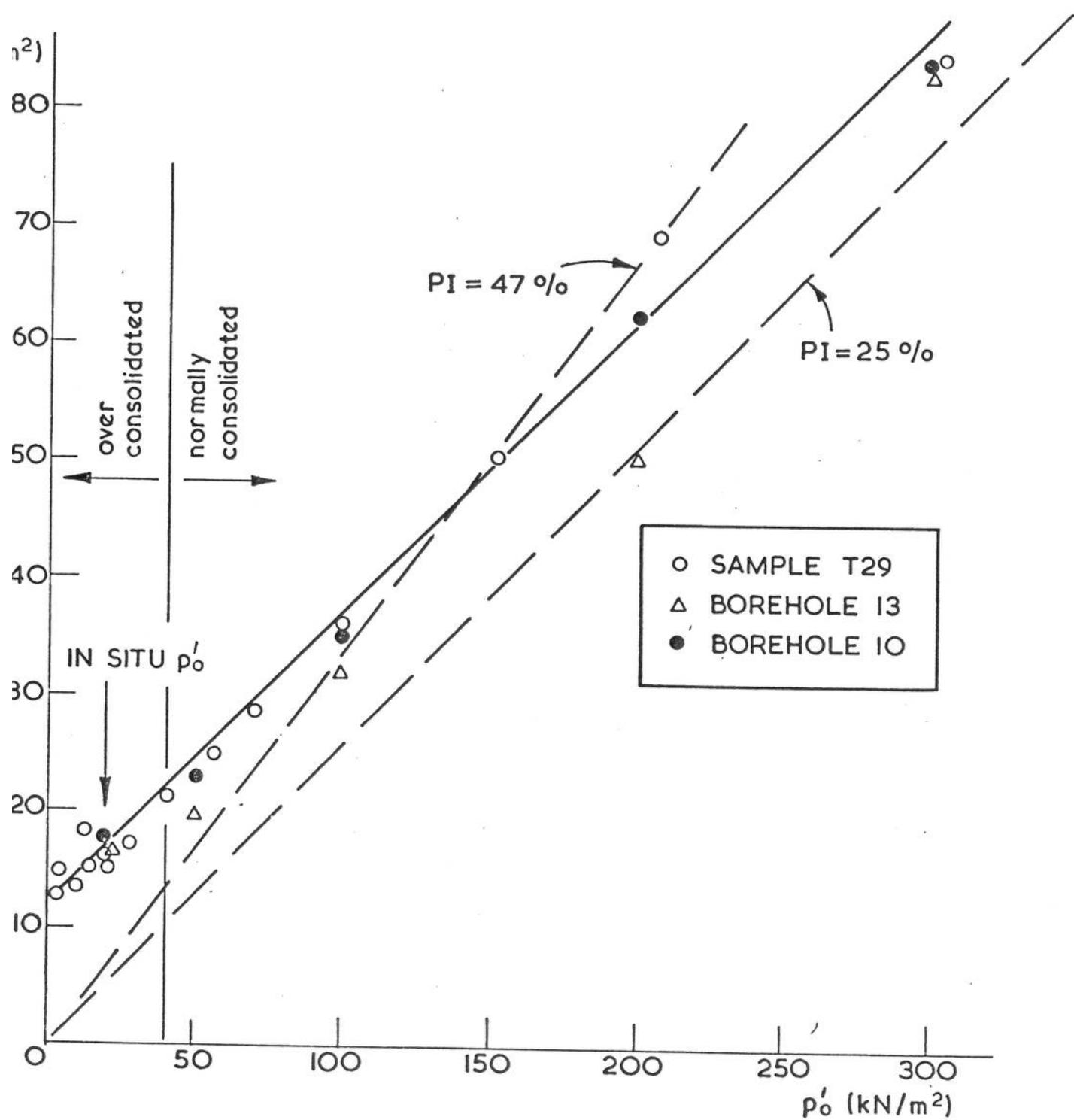
Fig.4.1.1. Values of undrained shear strength from CIU tests on clay from Mucking Flats (after Wesley (1975))
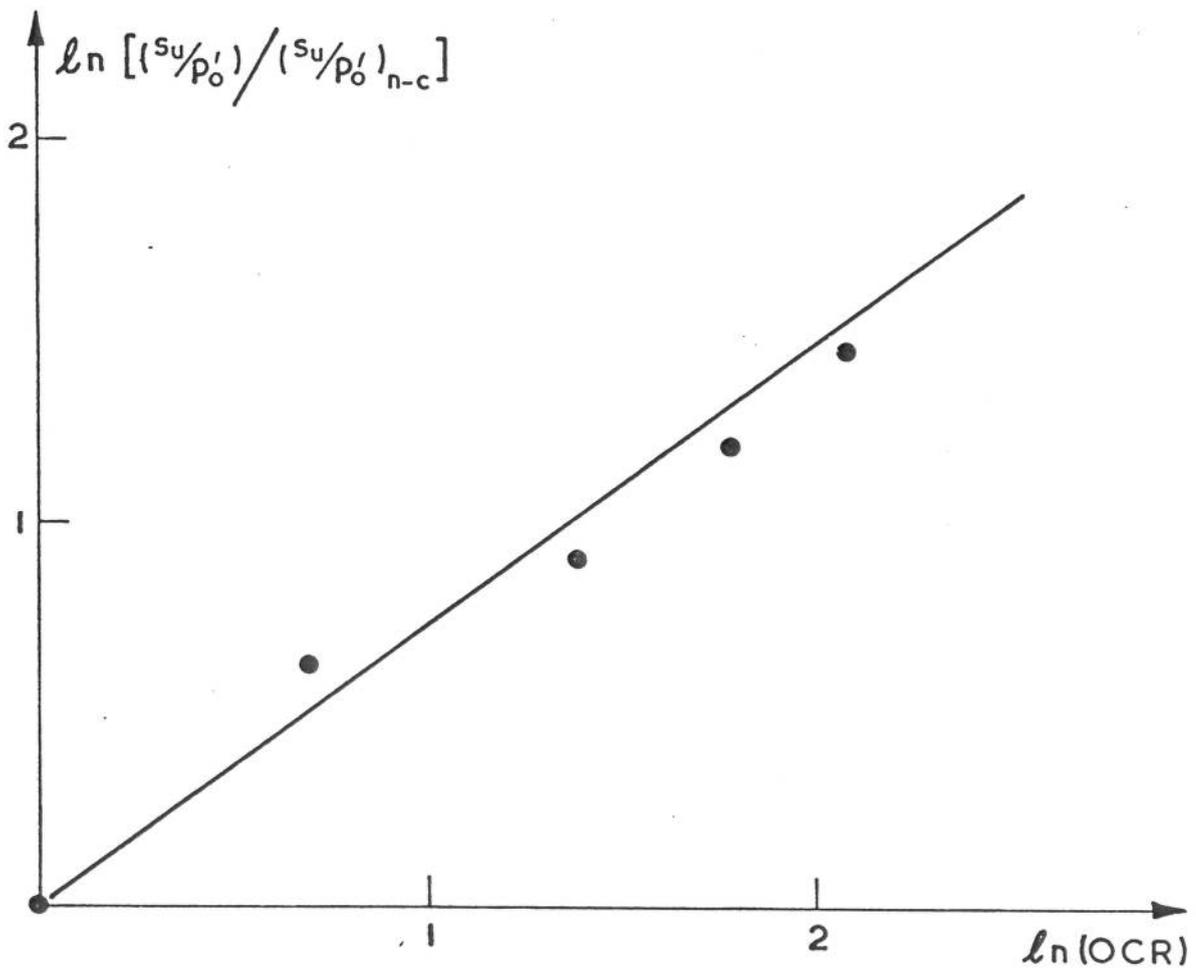
Fig.4.1.2.   Normalised values of s_u/p_o' plotted against
            OCR for CIU tests on remoulded Boston blue clay
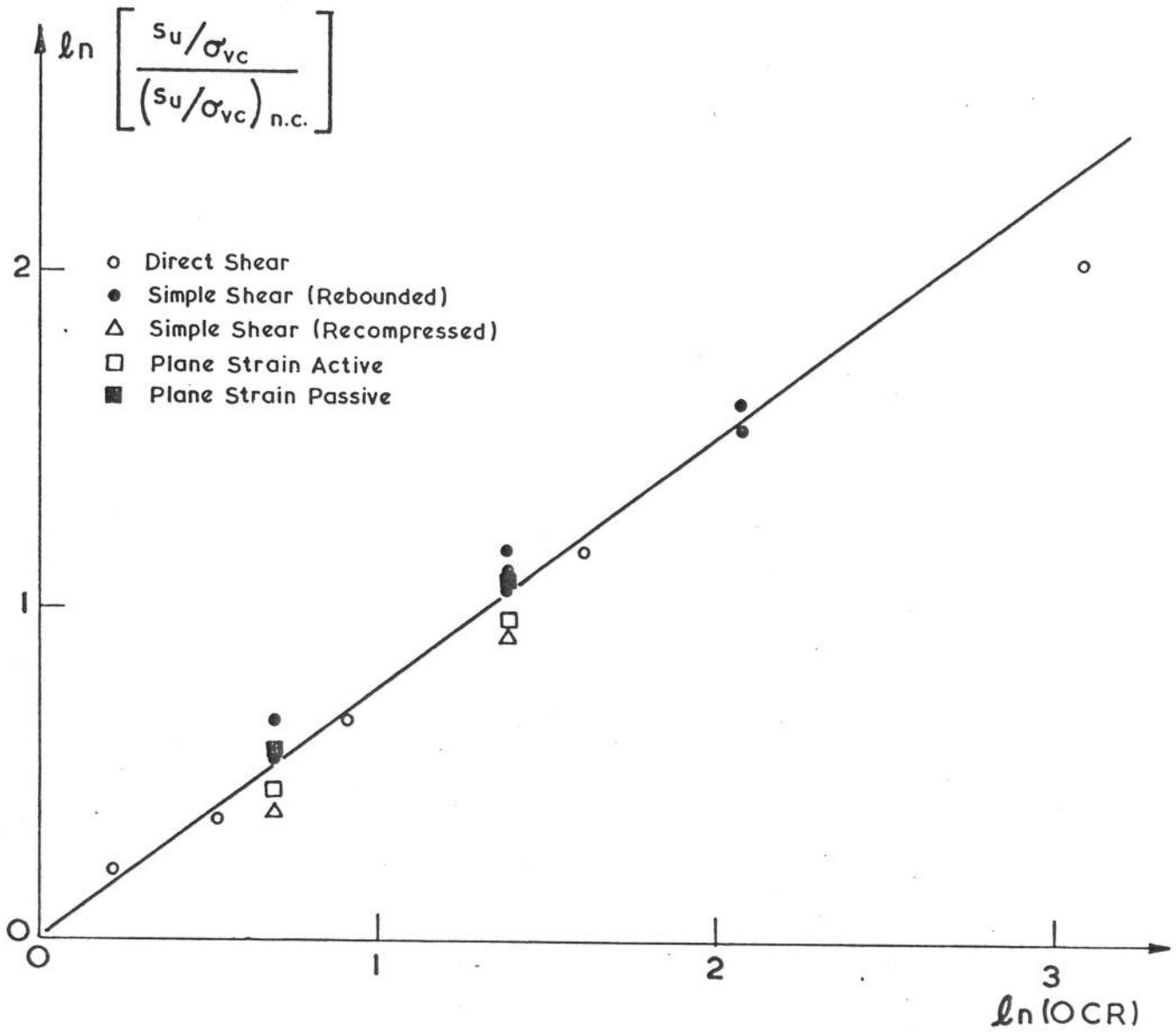
Fig.4.1.3. Normalised values of s_u/σ'_{vc} plotted against OCR for plane strain tests on remoulded Boston blue clay.
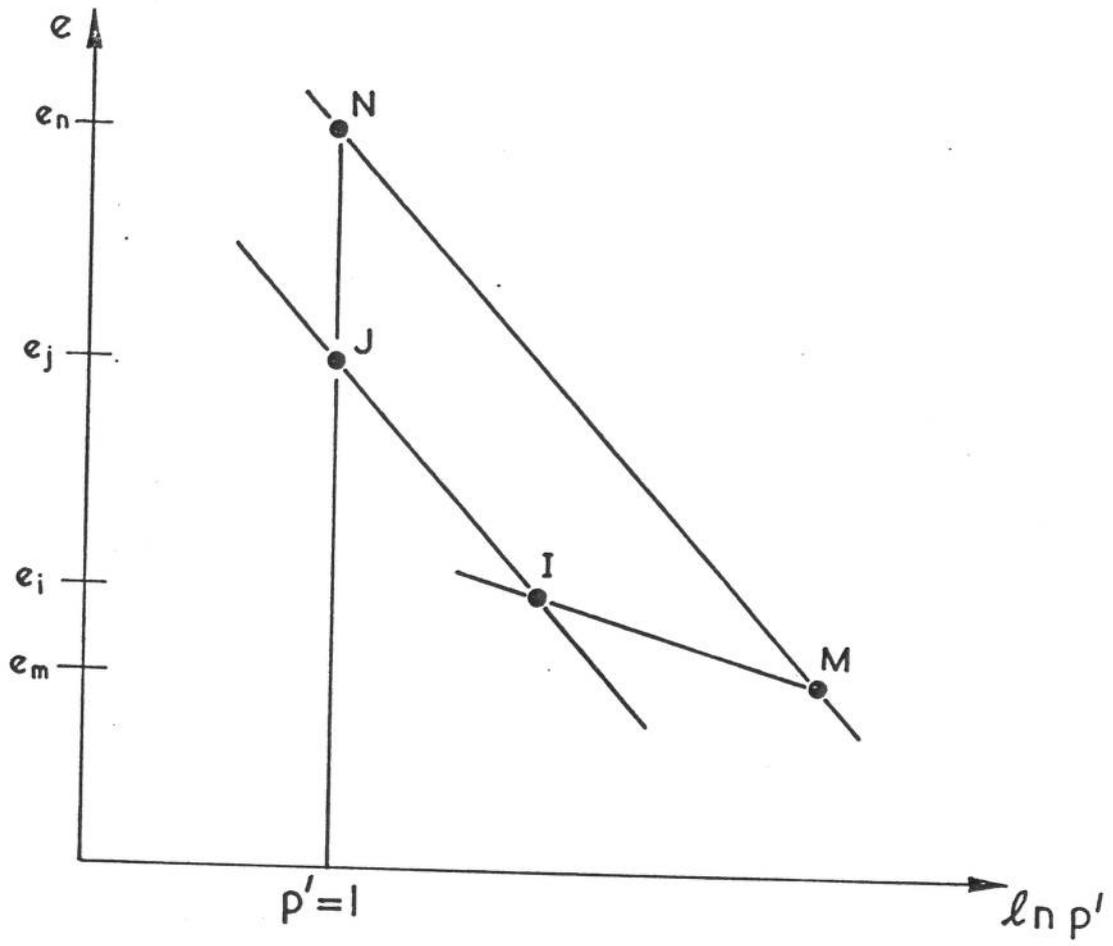
Fig.4.1.4. Idealised consolidation plot for definition of e_λ
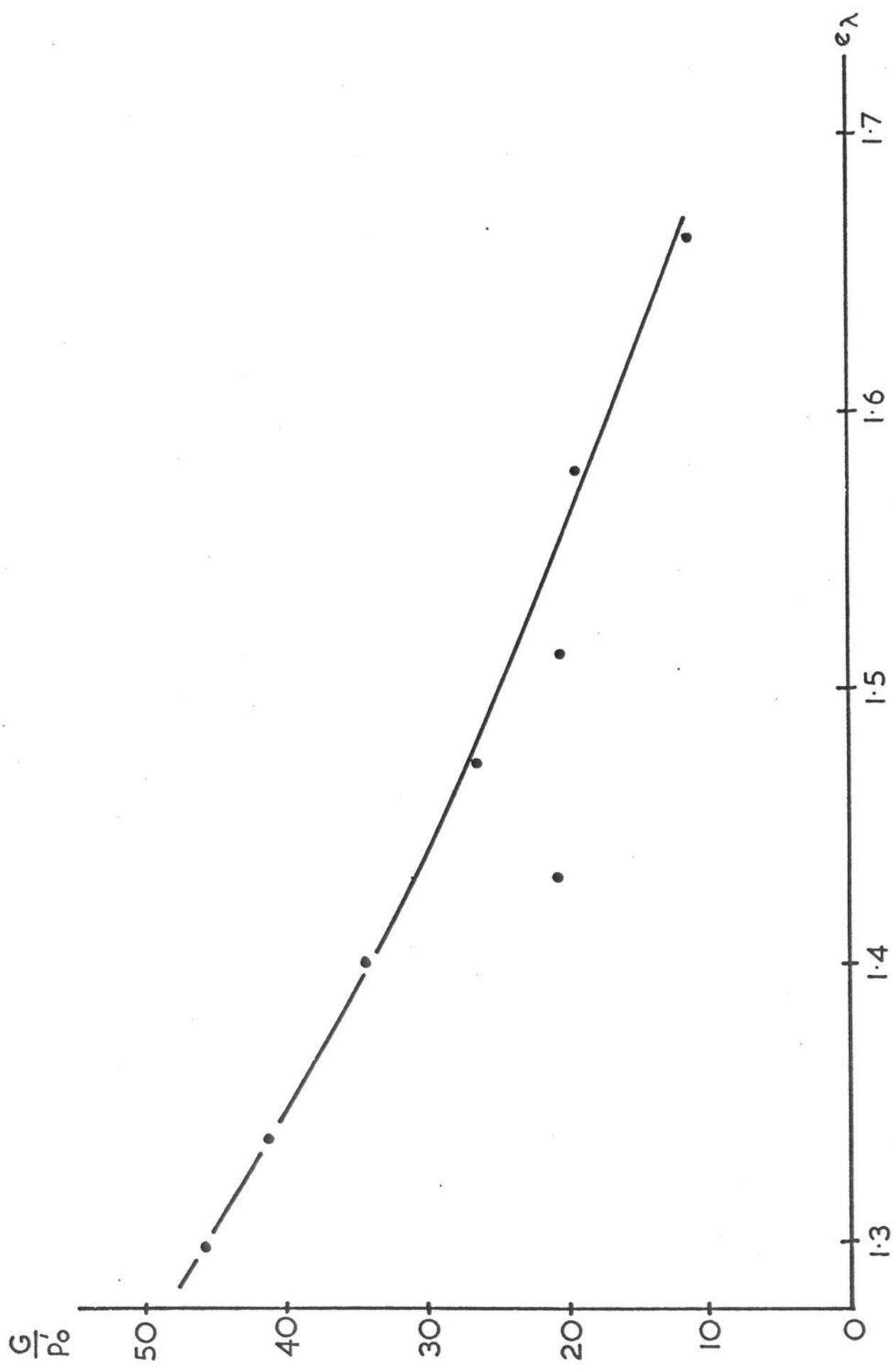
Fig.4.1.5. Values of G/p_o' plotted against e_λ for undrained triaxial tests on samples of London clay (after Webb (1967))

CLAY	W_L %	P. I. %	STRESS RATIO q/s_u	
			1/3	2/3
Boston Blue	41	21	—○—	--●--
Maine Organic	65	34	—△—	--▲--
Bangkok	65	41	—□—	--■--
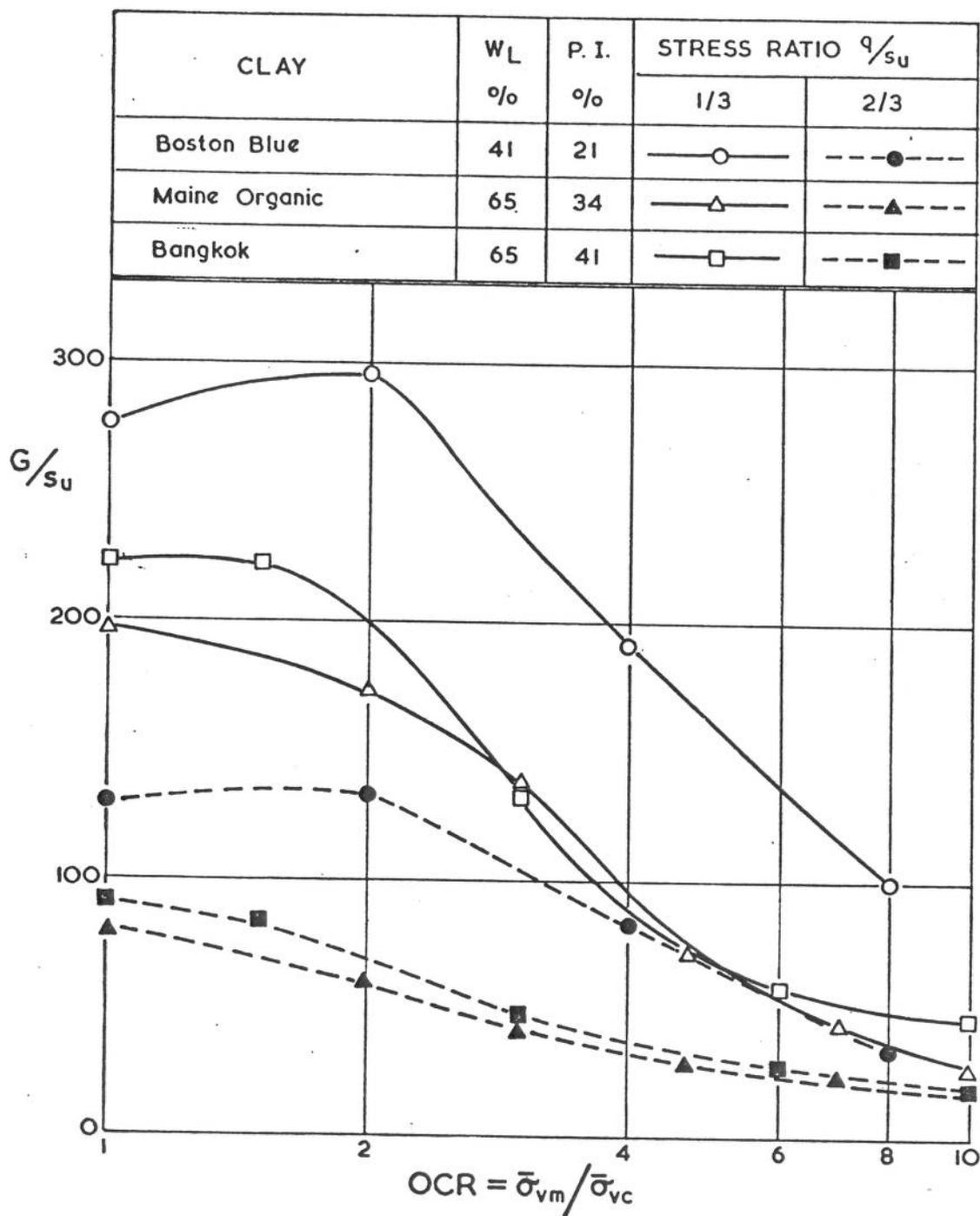
Fig.4.1.6. Values of G/s_u plotted against OCR from CK_O U DSS tests on three clays (after Ladd and Edgers (1972))
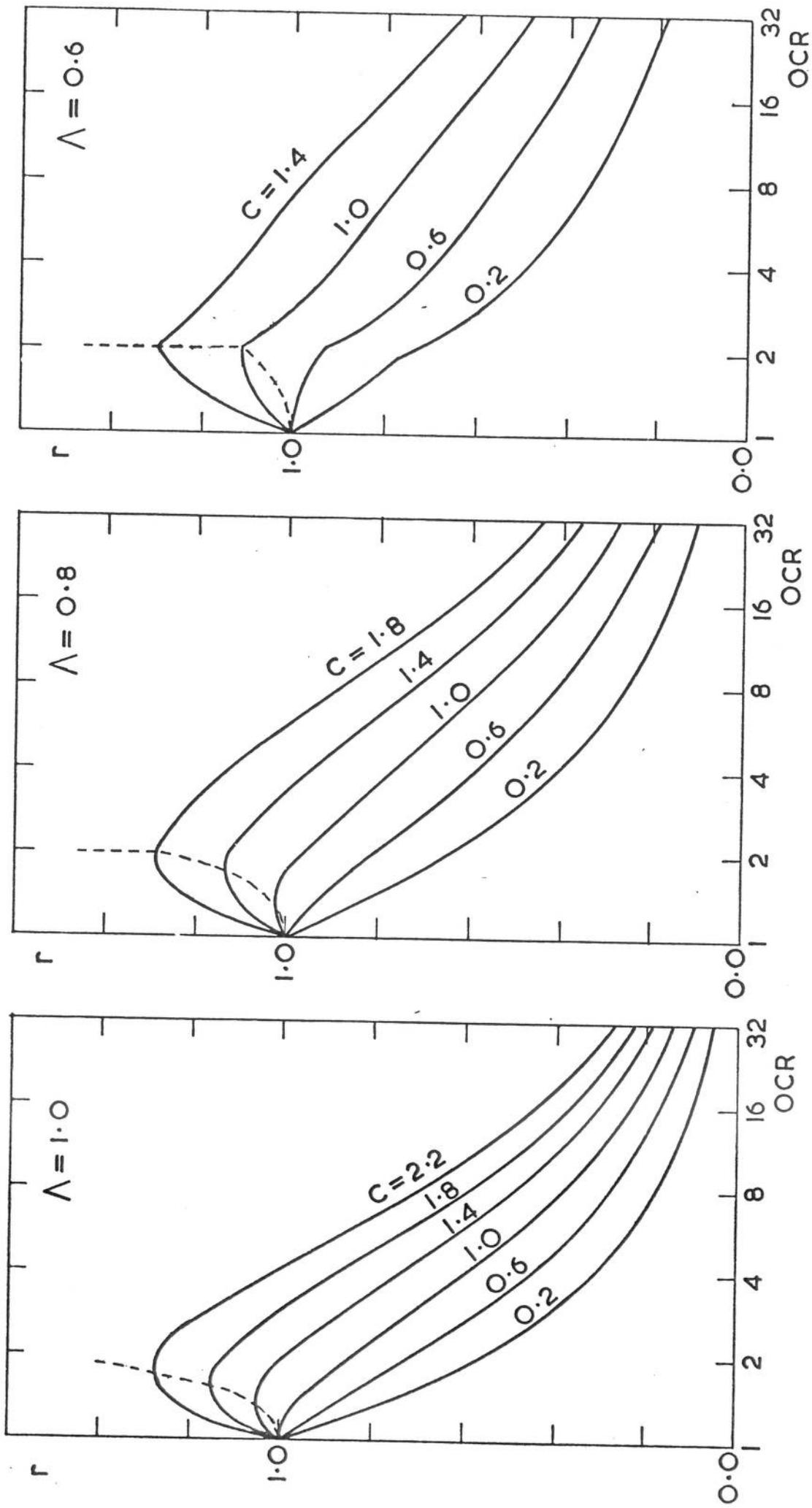
Fig.4.1.7. Theoretical variations of G/s_u with OCR
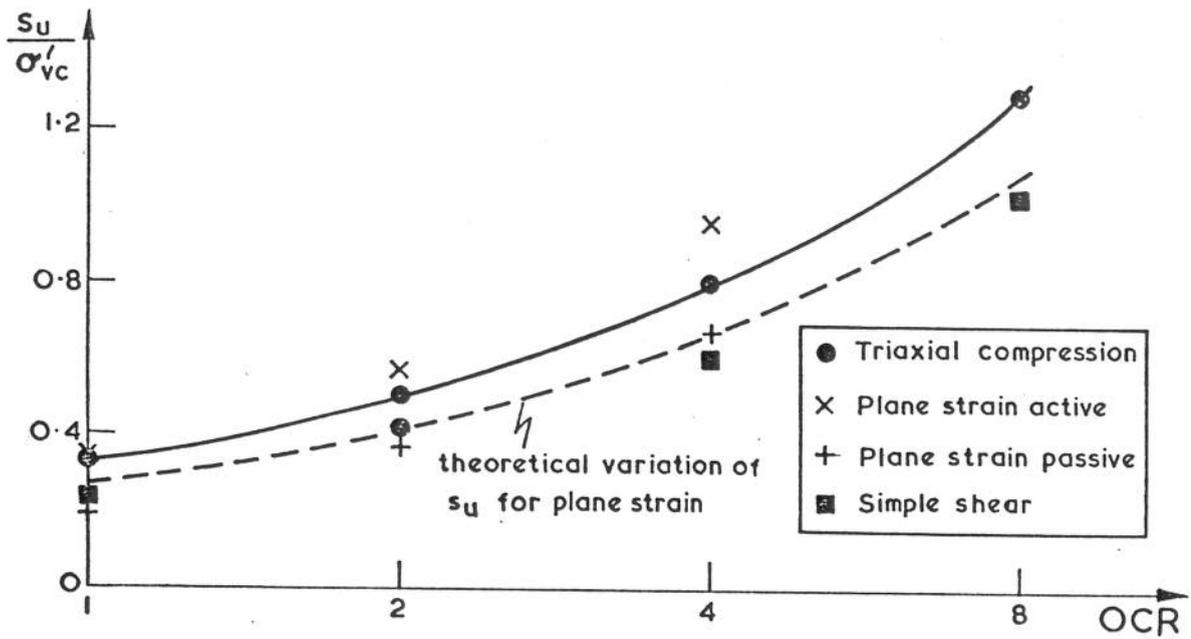
Fig.4.3.1. Variation of undrained shear strength with OCR
for Boston blue clay (data from Ladd et al
(1971) and Ladd and Edgers (1972))
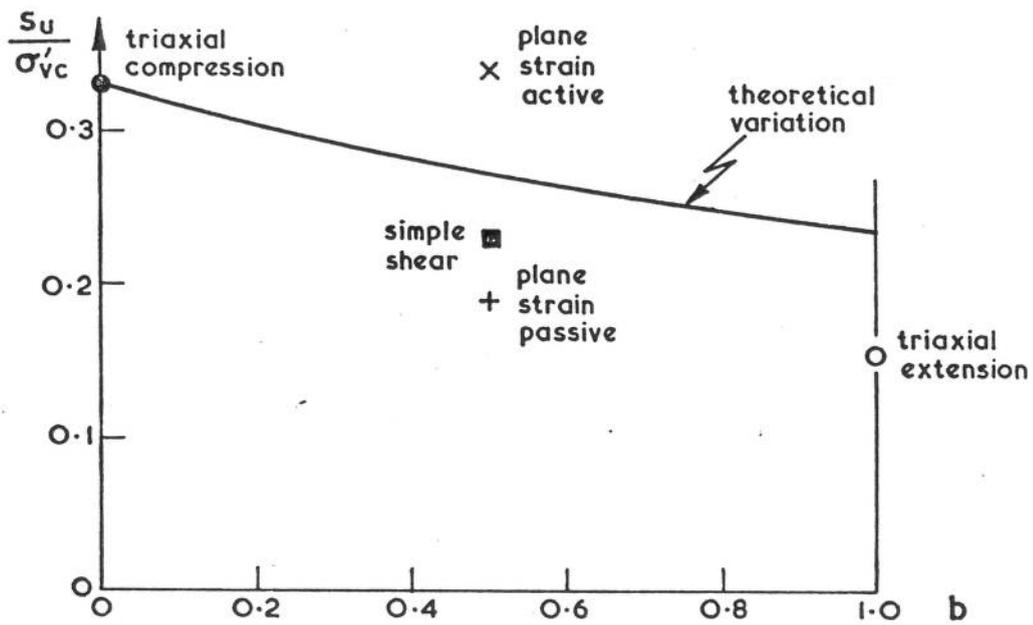
Fig.4.3.2.   Variation of undrained shear strength with
             intermediate stress for normally consolidated
             tests on Boston blue clay (data from Ladd et al
             (1971) and Ladd and Edgers (1972))
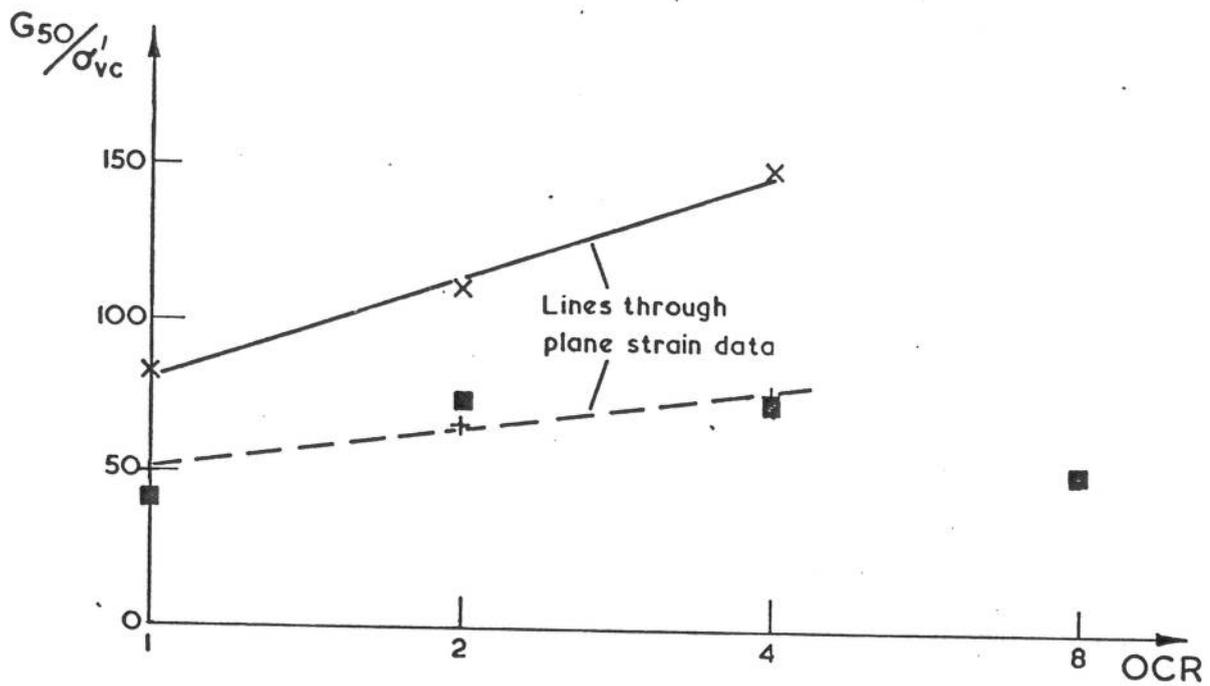
(a) G_{50} normalised by vertical effective stress at start of test

×	plane strain active
+	plane strain passive
◼	simple shear
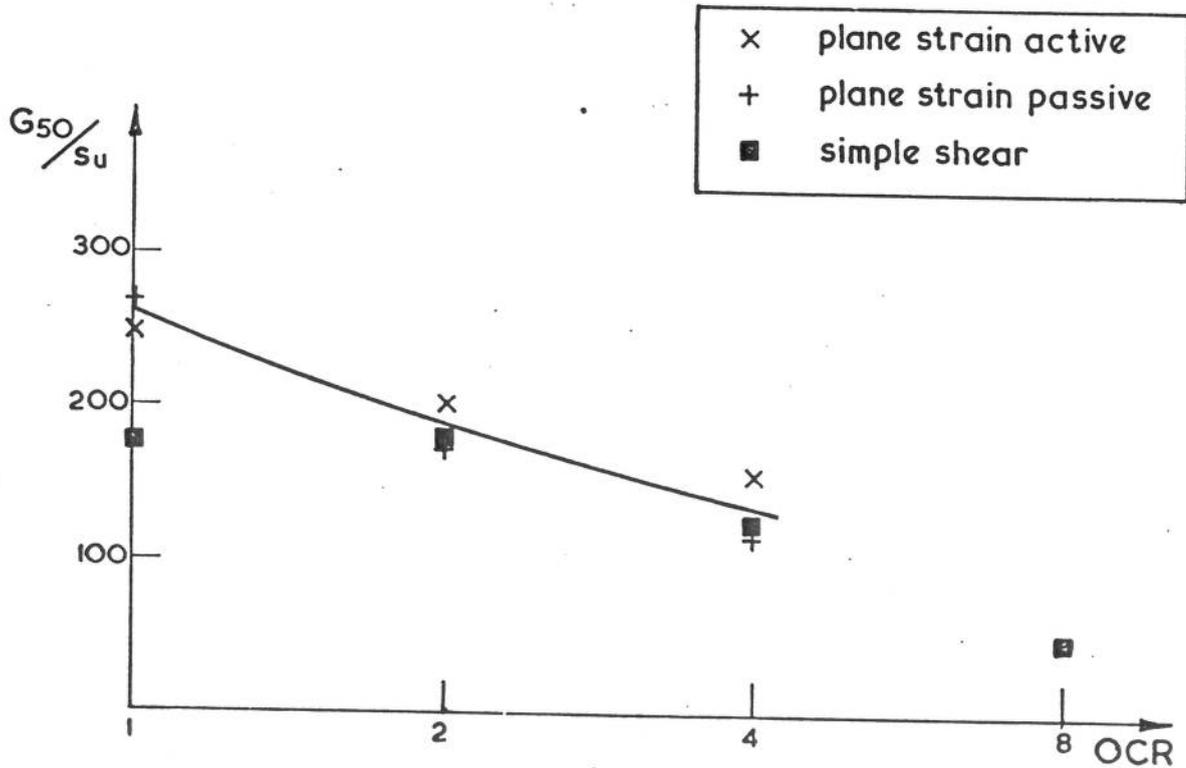
(b) G_{50} normalised by the undrained shear strength

Fig.4.3.3. Variation of shear modulus with OCR for Boston blue clay from plane strain tests after Ladd et al (1971) and from simple shear tests after Ladd and Edgers (1972)
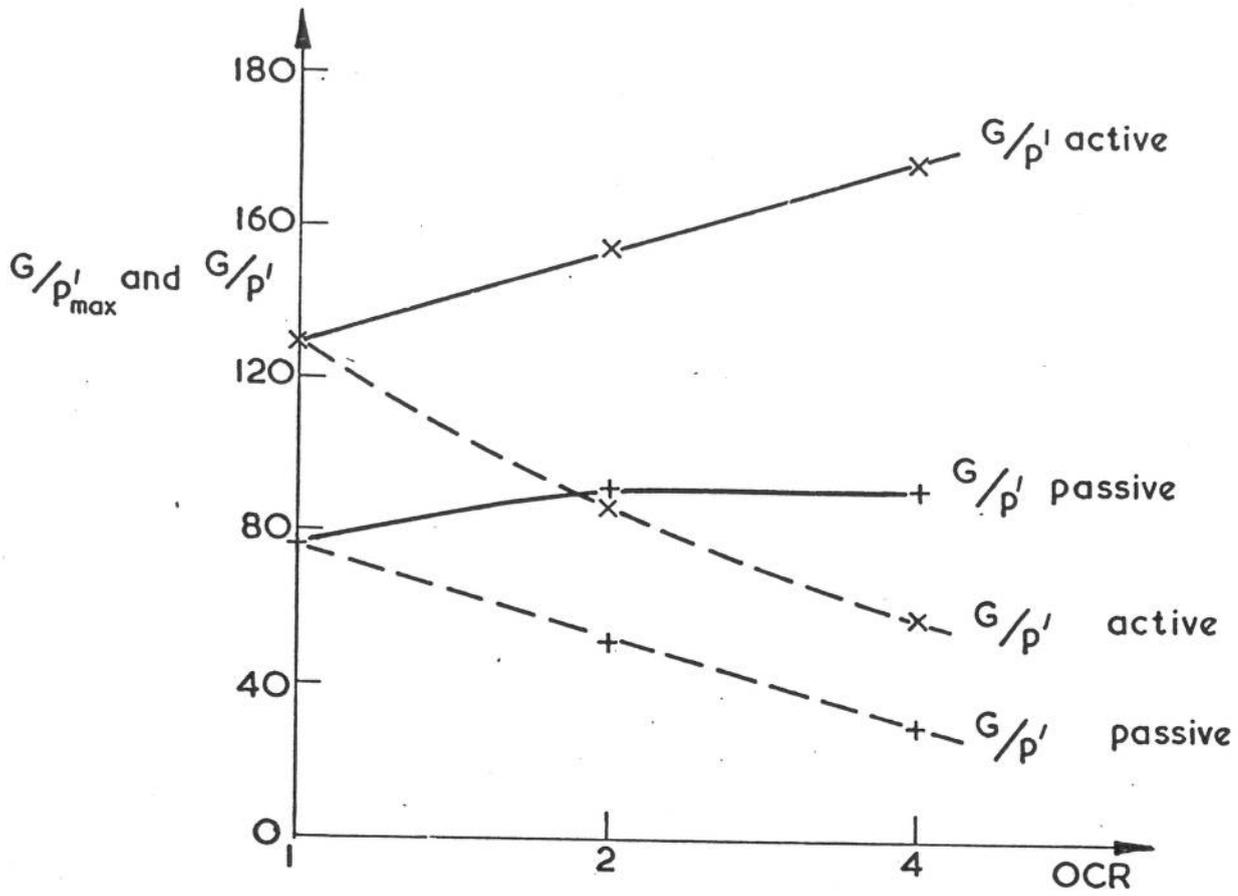
Fig.4.3.4. Variation of G/p'_{max} and G/p'_o with OCR from plane strain tests on Boston blue clay (data from Ladd et al (1971))
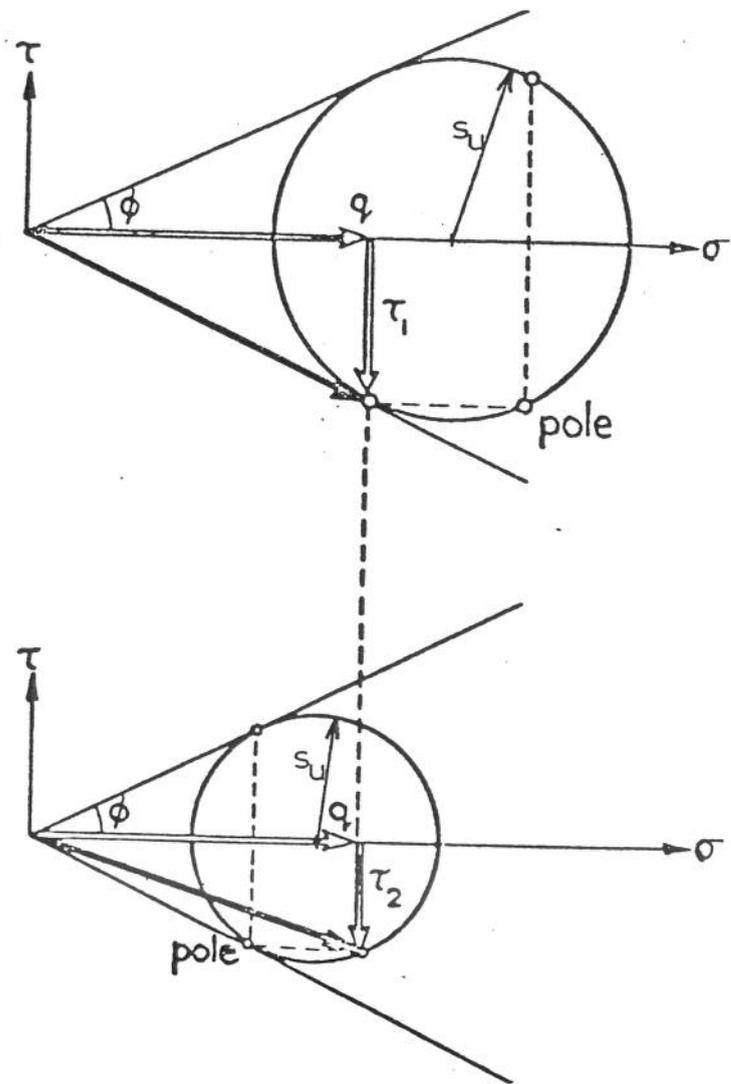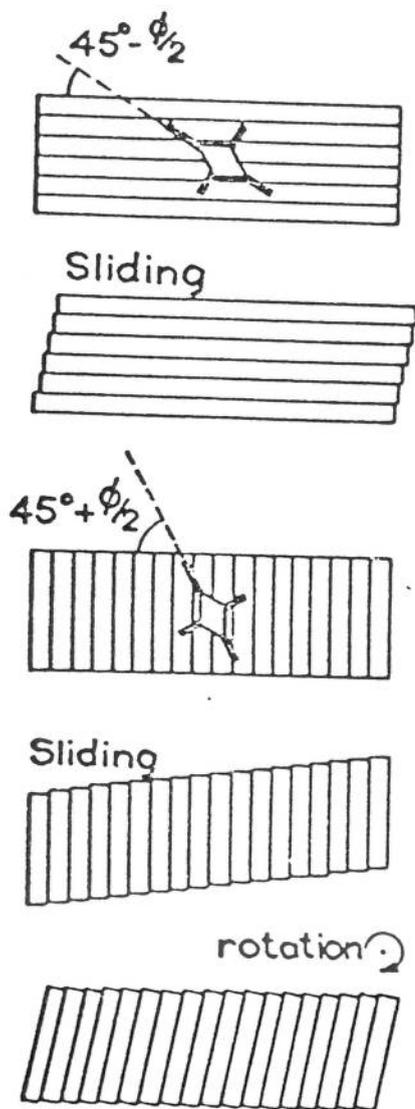
Fig.4.3.5. Possible planes of failure in a simple shear
test (after de Josselin de Jong (1971))
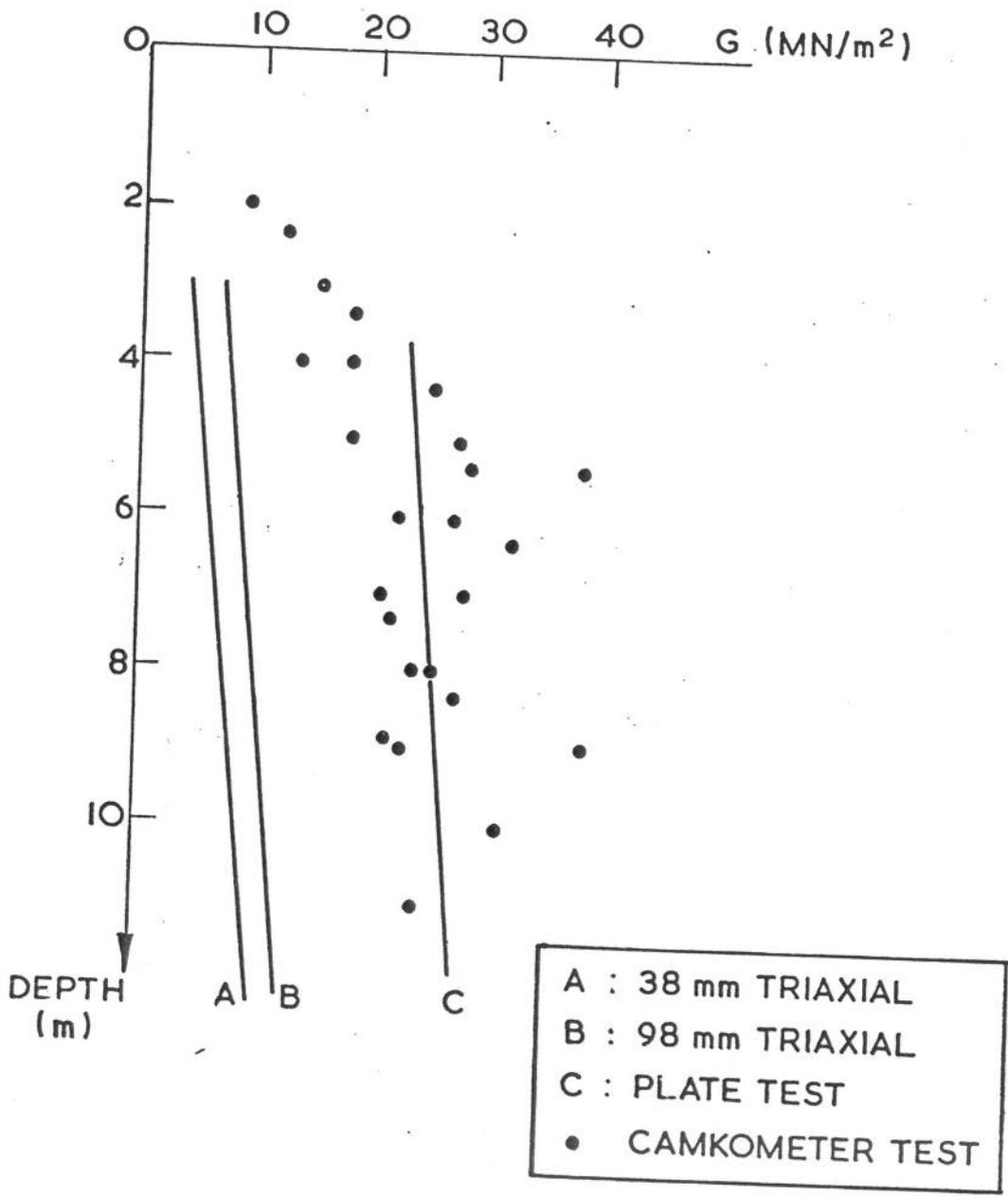
Fig.5.1.1. Values of shear moduli measured by various tests in London clay at Hendon (data from Marsland (1971b) and Windle (1976))
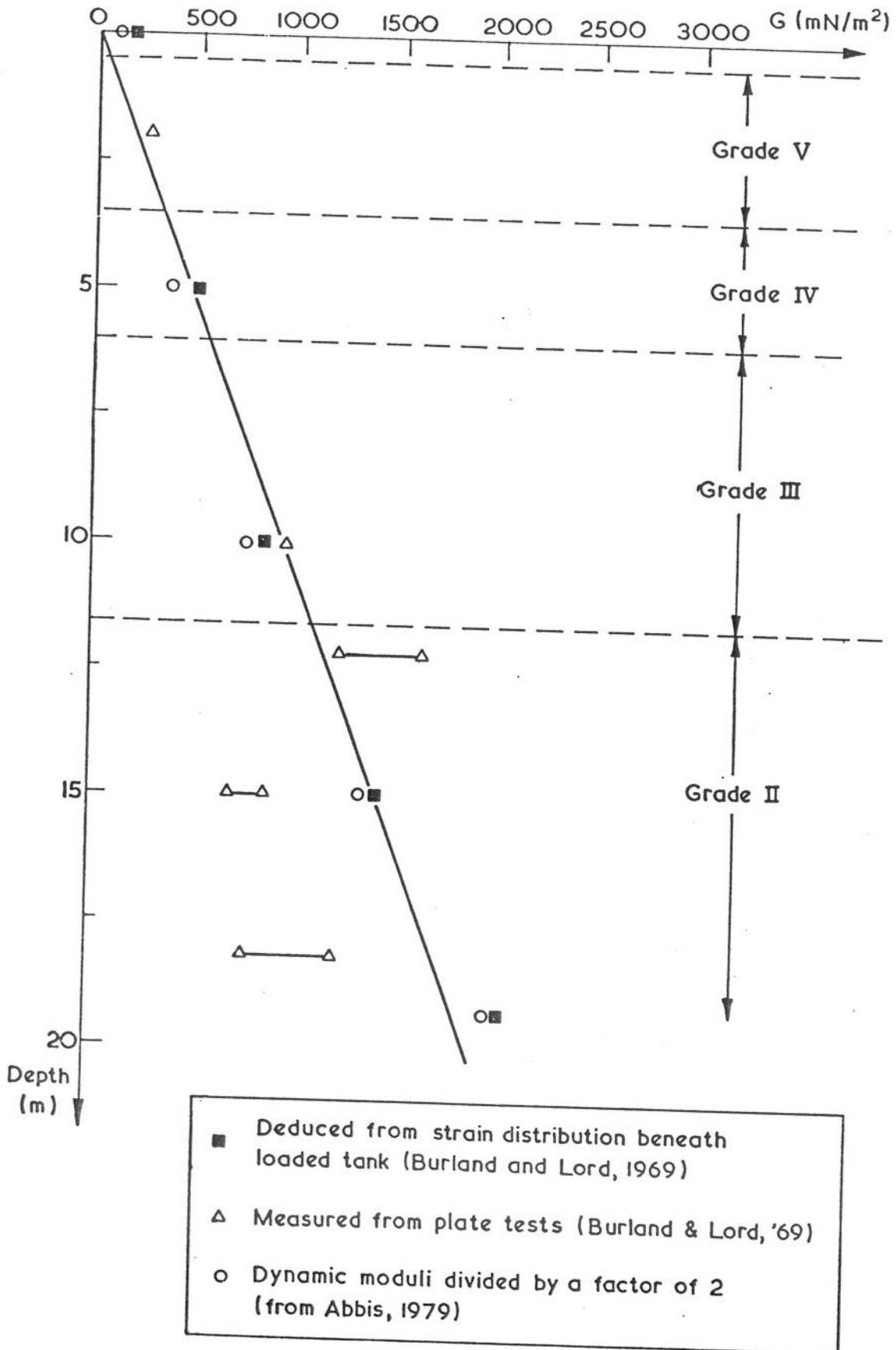
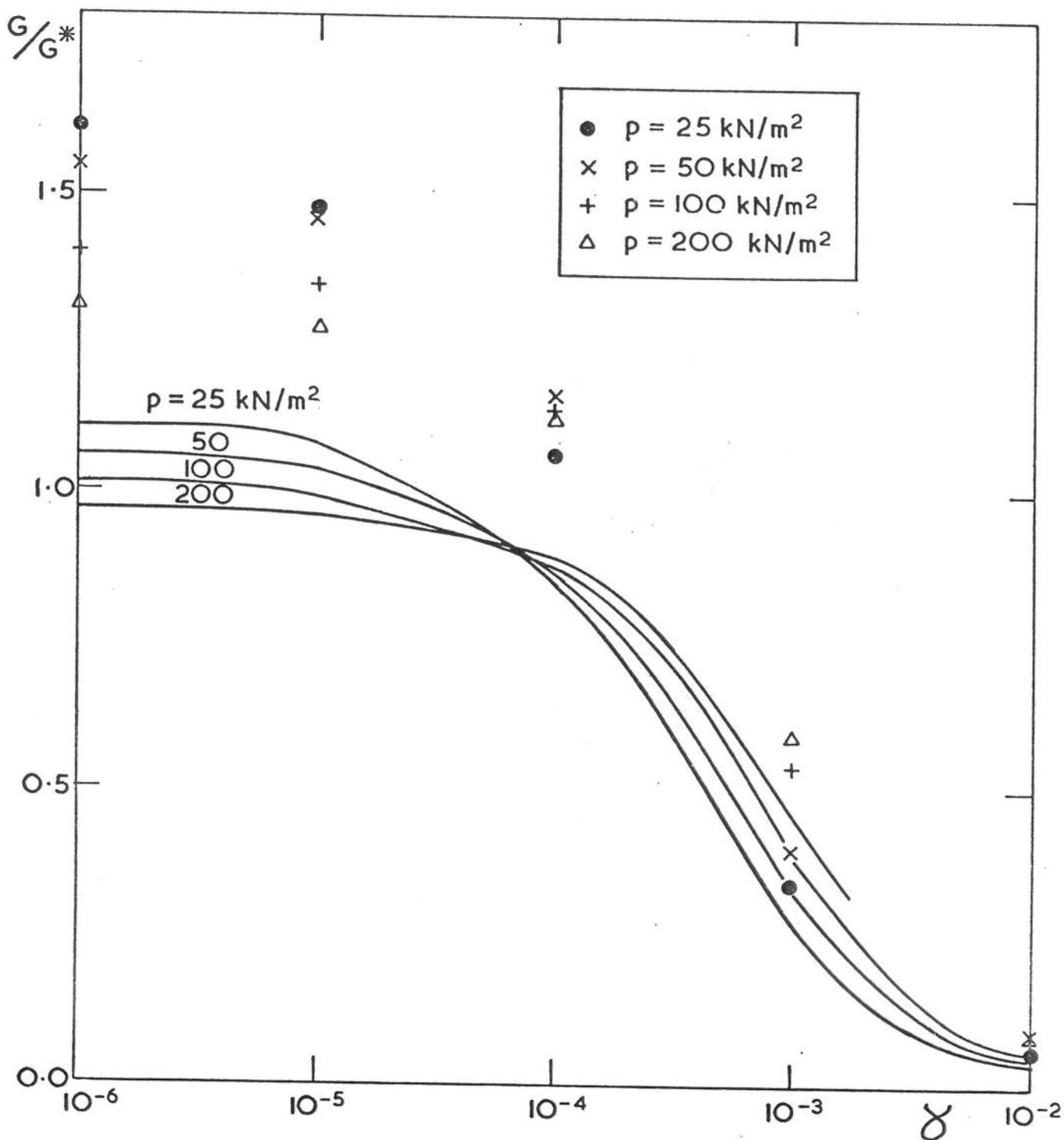Fig.5.2.1. Variation of shear modulus with depth for typical soft rock deposit (chalk at Mundford, Norfolk)

Fig.6.2.1. Variation of dynamic shear modulus with amplitude of shear strain (data from Tatsuoka et al (1978))
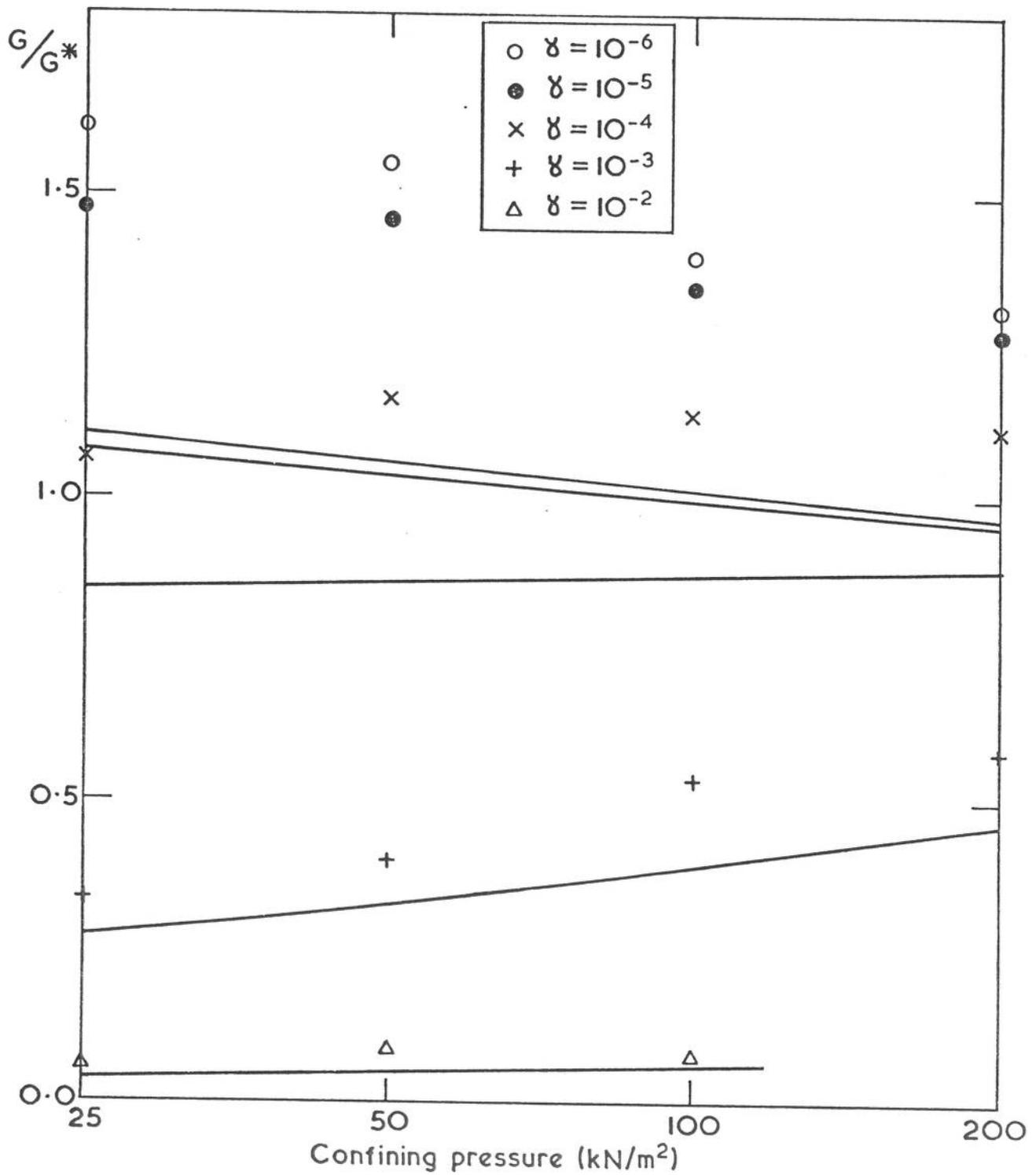
Fig.6.2.2. Variation of dynamic shear modulus with
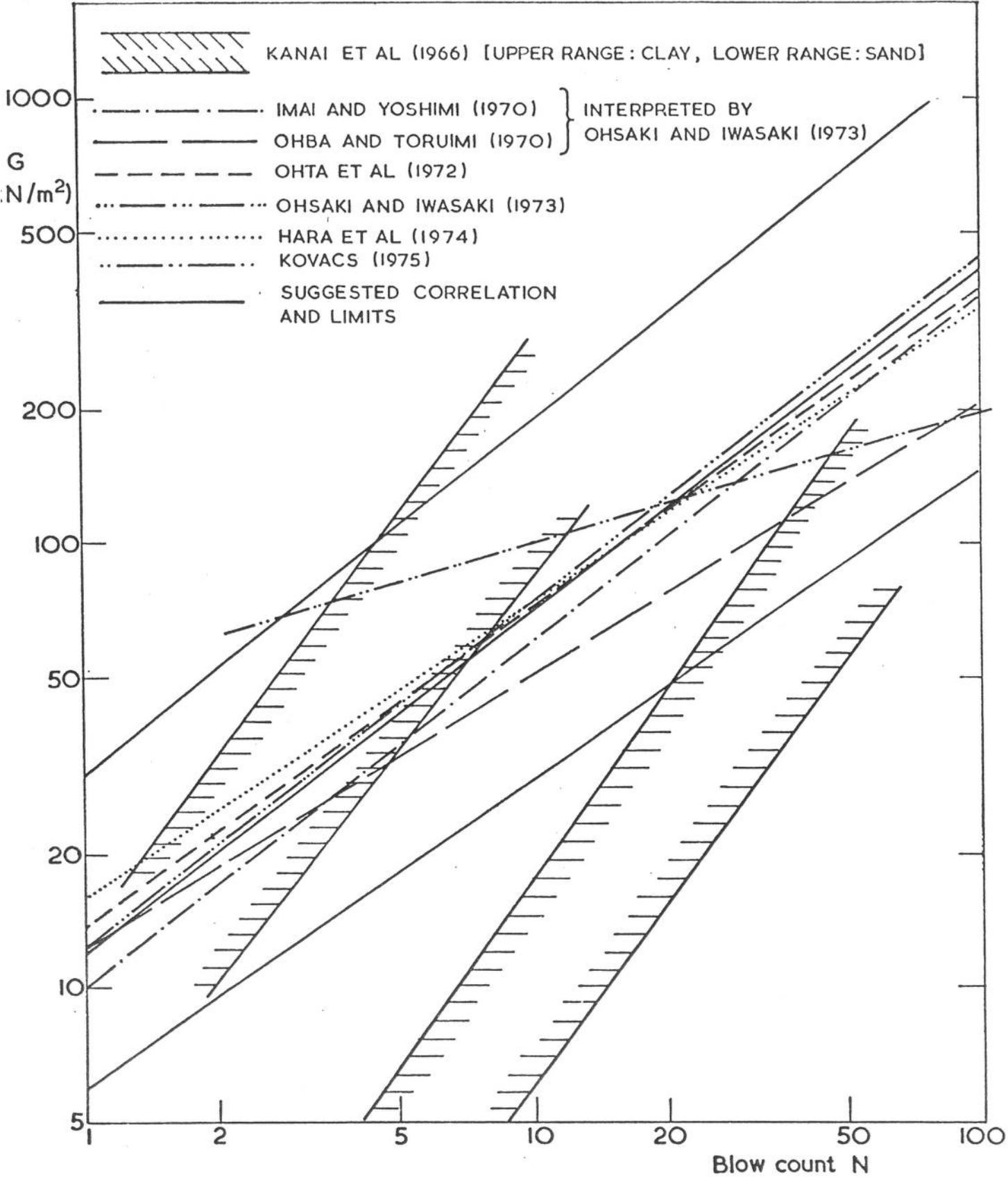confining pressure (data from Tatsuoka
et al (1978))

Fig.6.3.1. Correlations of dynamic shear modulus
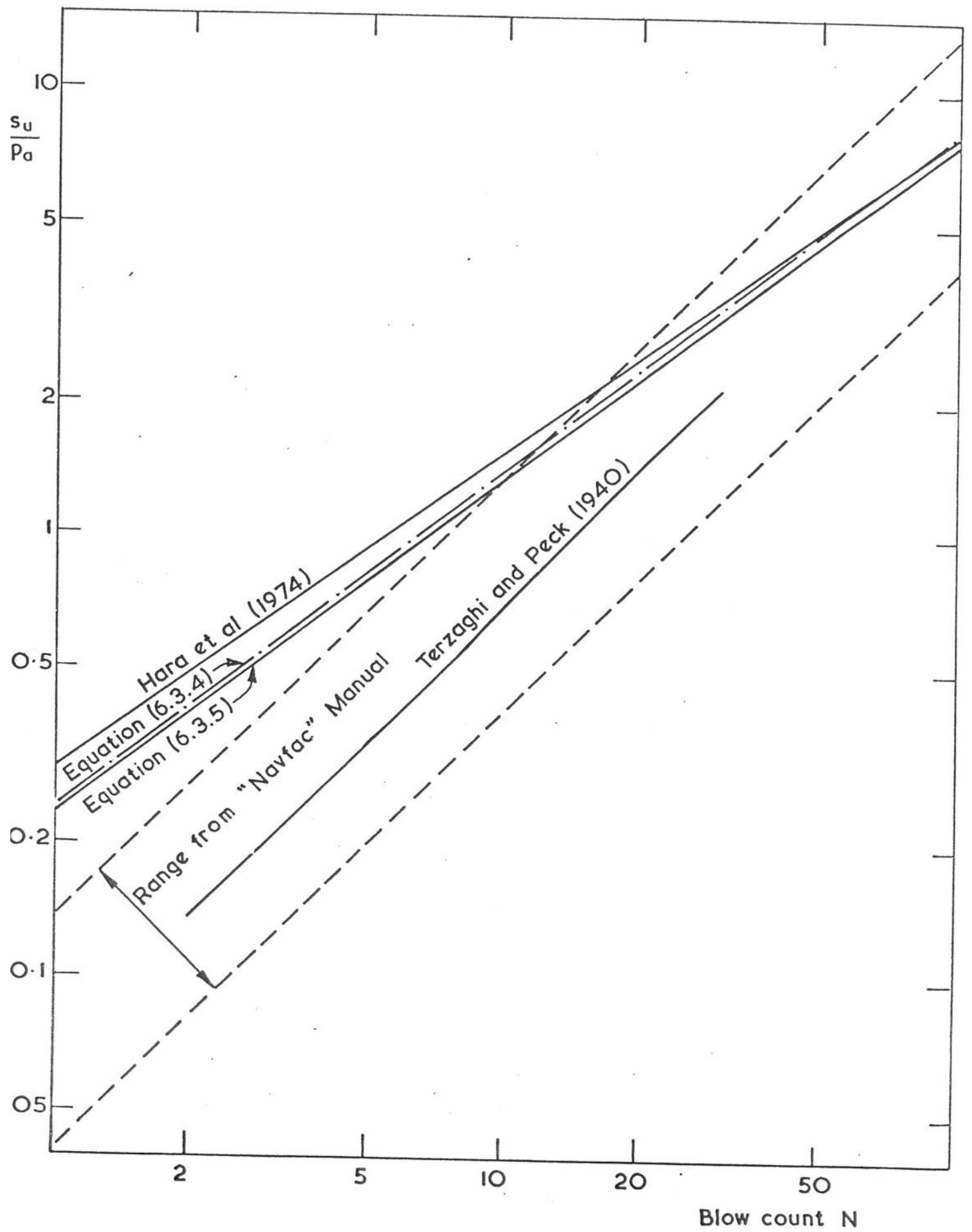with standard penetration test

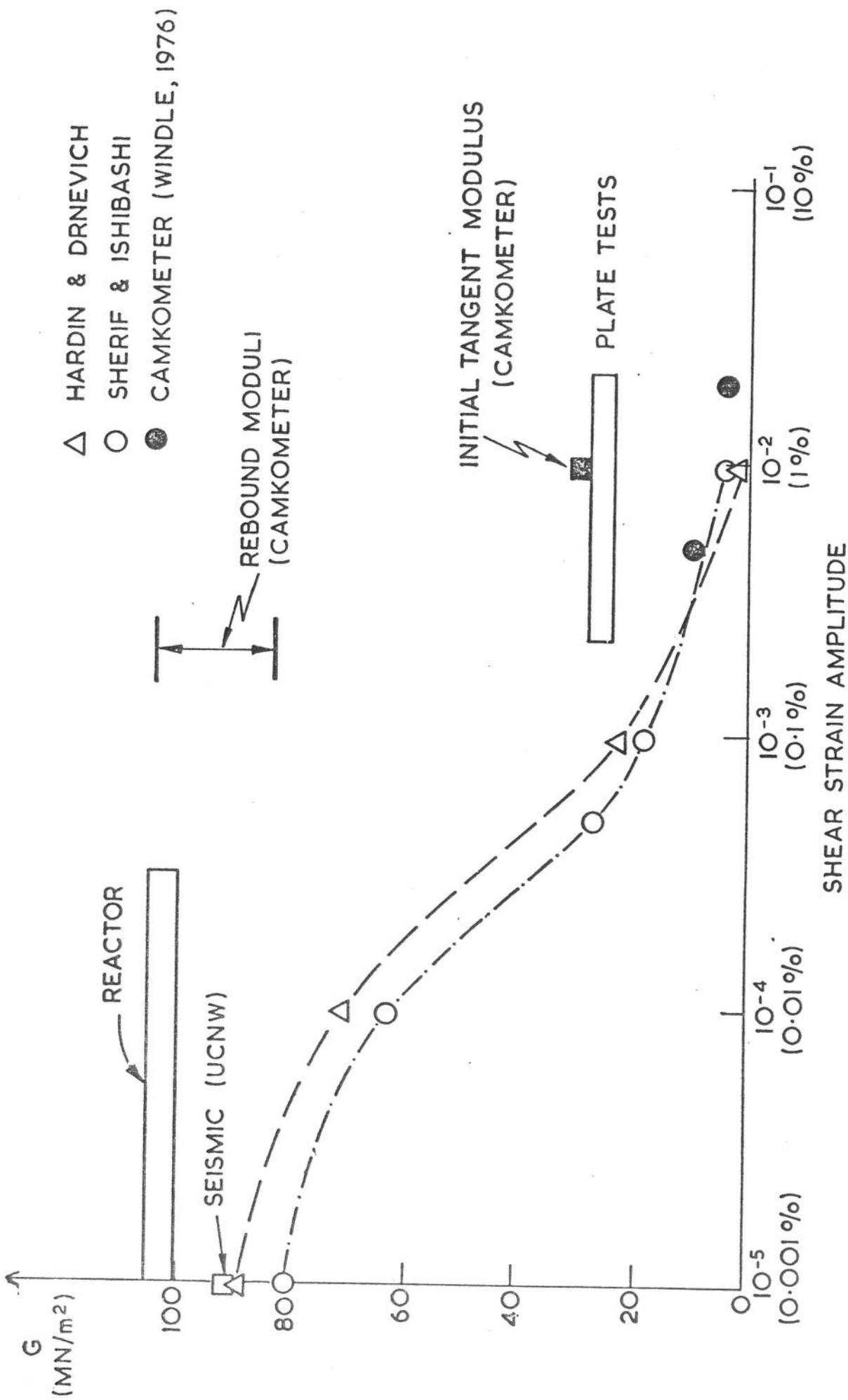Fig.6.3.2. Correlations of undrained shear strength with standard penetration test

Fig. 6.4.1   Values of shear modulus measured in the field in sand (after the Central Electricity Generating Board, U.K.)

This technical report had, when first produced in 1979, two appendices. These appendices are papers which have been published and are readily available.   The references are given here.

Appendix A

Wroth, C. P. and Wood, D. M. (1978)

The correlation of index properties with some basic engineering properties of soils.

Canadian Geotechnical Journal $\underline{15}$, 2, 137-145.

Appendix B

Wroth, C.P. (1979)

Correlations of some engineering properties of soils.

Proc. 2nd Int. Conf. on behaviour of Off-Shore Structures,

London $\underline{1}$ 121-132.