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DEBT FORGIVENESS: THE CASE FOR HYPER-INCENTIVE CONTRACTS

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Debt Forgiveness; the Case for Hyper-incentive Contracts[1]

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Abstract

We review two proposals for debt forgiveness; the Highly Indebted Poor Country Initiative (HIPC) and the Jubilee 2000 Coalition Initiative (J2K). We then consider the workhorse model of debt forgiveness (Krugman 1988). We show that the workhorse model solution is a sub-optimal contract, where the incentive parameter is set without regard to the cost of effort. A fully-optimal debt-overhang contract is derived, with an incentive parameter greater than the marginal social benefit of extra effort. The so-named Hyper-Incentive Contract eliminates the effects of moral hazard arising from hidden effort, and provides a fuller rationale for case-by-case debt-overhang contracts.

Key words: Debt overhang; Debt forgiveness; Optimal contracts; Moral hazard

JEL classification: F34, F35

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1. Introduction

When the history of the year 2000 is writ large for future generations, one wonders what event will capture the popular imagination. One contender is the forgiveness of substantial amounts of poor-country foreign debt. At the June 1999 Cologne summit, the G-7 entertained writing off approximately $100 billion, subject to certain conditions. Though progress continues to be slow, the popular and political will now exists to tackle the issue of unredeemable debt.

As evidence of this, we scrutinize the two current proposals for debt relief; the Highly Indebted Poor Country initiative (HIPC) and Jubilee 2000 proposal (J2K). The HIPC proposal provides case-by-case debt relief subject to economic reform conditions. It comes at the issue of unredeemable debt 'from above', at the instigation of private banks, the IMF, the World Bank, and ultimately the US government. In contrast, J2K draws its support from a groundswell of popular support for poor countries, and, from the questioning of free market liberalism, evidenced by the Seattle WTO conference. As one might expect, the J2K coverage is broader and the debt relief deeper than HIPC. However, J2K is burdened with a problematic view of conditionality, and a somewhat dismissive view of orthodox economic development,

Orthodox economics has a role to play in this debate, given that 'the best laid schemes of people and politicians go oft astray'.[2] We therefore examine a model of debt forgiveness

(Krugman 1988), asking if it provides optimal contracts for case-by-case debt forgiveness, as required by both HIPC and J2K. We show that his solution assumes a sub-optimal incentive parameter. The key insight of this paper is that the optimal incentive parameter exceeds the marginal social benefit of extra effort. This *Hyper*-Incentive Contract Pareto dominates Krugman's solution.

In Section 2 we describe the HIPC and J2K proposals. In Section 3, we give Krugman's solution to the debt overhang problem. We outline the intuition of a Hyper-Incentive Contract in Section 4, followed by a rigorous principal/agent treatment in Section 5. Section 6 shows how incentives should be tailored for countries experiencing severe economic hardship, and Section 7 concludes.

2. Proposals for Debt Forgiveness: HIPC and J2K

2.1 HIPC

The stage setting for HIPC goes back to the oil crisis of the 1970s. The Organization of Petroleum Exporting Countries (OPEC) cartel achieved a sizable transfer of wealth from the industrialized world between the time of the first oil price hike in 1974, until the collapse of the oil price in 1985. The so-called petro-dollars were saved by the OPEC nations, and then recycled back through the world financial system to developing countries, many of whom were experiencing balance of payments difficulties due to higher energy prices. In the early 1980s, a high interest rate policy was instigated by the

US Federal Reserve to reduce US inflation. This, together with weakened world demand, adversely impacted upon debtors' debt service costs and export revenues respectively. The international debt crisis emerged in 1982, when Mexico had to reschedule its debts. Capital flows to many developing countries ceased, and a drawn out process of debt rescheduling commenced.

For the poorest countries, most of the debt was sovereign, and owed to OECD governments and international agencies. Rescheduling occurred at the so-called Paris Club of creditors. If the club believed, usually on the say-so of the IMF, that the country was making a significant adjustment effort, the debts would be rescheduled (i.e. payments delayed). However, for many years the Paris Club stuck to the principle that the rescheduling must never occur at concessional rates, implying that the net present value of the debt was maintained. Once the danger to the international financial system passed, there seemed to be little political will to tackle the issue of unredeemable debts. Political-economy considerations (Evans 1999) suggests four obstacles: the absence of a crisis, the insignificance of the countries in the world economy, the reluctance of institutions to acknowledge past mistakes, and the reluctance of the United States, until the Clinton administration, to get involved.

Despite the 1987 introduction of the IMF's much-heralded Enhanced Structural Adjustment Facility (which Evans says was largely a reaction to mounting arrears on IMF loans to the poorest countries), the situation did not change substantially for over a decade. The 1989 Brady plan provided some relief to the Latin American countries (and US banks), but bypassed the poorest countries, most of which are in sub-Saharan Africa.

Over time, the 'no net present value reduction' principle eroded leading, in 1996, to the unveiling of the HIPC initiative by the IMF and the World Bank. Under HIPC, very poor countries could apply for the writing off of sovereign debt provided that they pursued sound economic policies for a sustained period of time (the so-called conditionality requirement). Another innovation was that HIPC aimed to treat each country on a case-by-case basis. The 41 HIPC countries have debts totaling $205 billion. Following the June 1999 Cologne summit, there is a commitment to write off approximately $100 billion, providing conditionality is satisfied.

The G-7 finance ministers stopped short of pledging tax increases to compensate HIPC creditors. Instead, they called on the Multilateral Development Banks (MDBs) to carefully examine any ways of realizing efficiency gains. They also promised to consider giving resources to an expanded HIPC Trust Fund (the HIPC Trust Fund was set up to help all MDBs finance debt forgiveness). This, together with a so-called Millenium Fund (taking private sector contributions), will be key to lowering debt service payments.[3]

In addition to these funding mechanisms, the G-7 agreed to sell up to 10 million ounces of the IMF's gold reserves, using the interest on the proceeds to finance debt relief. Members of the U.S. Congress have come out against the sales, however, and the outcome is in doubt. Some members of Congress oppose the gold sales on the grounds that they will help the IMF's Enhanced Structural Adjustment Facility (ESAF) in its transition to self-sustainability. This self-sustainability will make ESAF less accountable to donor governments. Furthermore, there is concern in Congress, and in some gold-producing developing countries, that the sale of reserves will decrease the price of gold on world markets.

If the sale falls through, the debt initiative will have to be financed directly through government contributions. Absent tax increases, this means that either it will not be financed at all, or it will be financed at the cost of many governments' normal aid budgets (Morrison 1999).

2.2 J2K

In the early 1990s, Non-Government aid agencies (NGOs) campaigned and lobbied for debt relief through the so-called Debt Crisis Network. In 1994, Martin Dent (professor in Keele University UK) linked up with Bill Peters (an ex-Diplomat with experience in Malawi) and Isabel Carter (the Community News Editor of Tearfund) to create Jubilee 2000. In April 1996, the Jubilee 2000 campaign commenced, with funding from three Christian aid organizations: CAFOD (catholic), Christian Aid (ecumenical) and Tearfund (evangelical). In 1987, the campaign was launched in the US and elsewhere. In October of that year, Jubilee 2000 (UK) became a formal coalition of aid agencies.

J2K describes its aims as (a) a one-off cancellation of the unredeemable debts (b) of the world's poorest countries (c) by the year 2000, (d) under a fair and transparent process.

The inspiration for condition (a) allegedly comes from the Jubilee year in the Bible, though the Jubilee year was neither one-off nor confined to unredeemable debts.[4] The one-off condition probably also owes something to game theory, since a one-off cancellation would seem to remove an incentive to refrain from lending in the lead up to a (regular) Jubilee. However, to date J2K has not suggested a pre-commitment technology which would make subsequent forgiveness impossible.

J2K shares aim (b) with HIPC, though it wants more countries covered and more money spent. It has earmarked 52 countries (compared with 41 under HIPC) with debts totaling $350 billion ($205 billion under HIPC) for relief of $200 to $300 billion ($100 billion under HIPC).

Aim (c) is largely directed at marketing, while Aim (d) looks at bankruptcy through the eyes of the poor in debtor nations. The continuance of basic public services in the case of bankrupt local U.S. authorities is enshrined in Chapter 9 of Title 11 (Insolvency) of the US Code (Raffer 1995). J2K argue that this principle of protecting the poor from economic adjustment ought to be upheld in the international arena.

The proposal has received criticism for its ambiguity about conditionality. Stressing 'conditionality from below' (i.e. conditions of a debt relief contract emanating from the debtor country's 'civil society') the coalition refrains from endorsing Western-style economic policies as a means of alleviating poverty. However, granted the centrality of poverty reduction, the more thoughtful wing of the J2K movement would be well advised to address the following questions.

First, the price mechanism is a socially cheap way of providing information and incentives. *In the absence of a well-functioning price mechanism, how is allocation of resources to be achieved?* Second, moderate and stable inflation is necessary for prices to do this job effectively. *How will hyper-inflation help the poor?* Third, unless the fiscal authorities have long term financing credibility (either through a well functioning tax system, or small outlays), the fear of an eventual hyper-inflation tax will prove disruptive. *What does it do to the poor to adjust to government subsidies and benefits only to have them suddenly taken away in a financing crisis, or eroded in a hyperinflation?* Fourth, good supervisory standards are necessary in the financial sector. *How will the poor be affected in a financial sector meltdown?*

This limited economic orthodoxy needs defending, judging from the J2K web page. The silence of the centrally-planned cadavers should not allow us to forget their message.

3. Krugman's Model of Debt Forgiveness

The review of the current debt forgiveness proposals indicates that there is now both a popular and political will to address the issue of unredeemable debt. What economic model of debt rescheduling can guide policy makers?

In an important paper, Krugman (1988) outlined a model that has become something of a workhorse in this area.[5] The following situation was envisaged by Krugman.

insert addition 1

A Creditor wishes to roll over one unit of debt in the hope of minimizing losses. In the first period, the creditor refinances the unit of debt, and the debtor exerts costly effort *C*. The creditor sets *a*, the size of the transfer falling due in period two, and therefore *r* ($r=a-1$). We ignore discounting. Effort *e* is any unobservable adjustment with marginal disutility, such as increased competitiveness, or public sector efficiency. Output *y* depends upon effort, and an error *h* with (expected) constant returns to scale in effort. For Simplicity, $h \sim Uniform(0, s^{2})$.[6]

$$y \quad = \quad e + h$$

Clearly *y* spans *[e-Ö3s, e+Ö3s]* and $E(y) = e$.

For simplicity, we follow Fernandez-Ruiz (1996) in specifying quadratic costs.

$$C \quad = \quad l\, e^2/2$$

In the second period, the benefits for the debtor are non-linear, depending on the outcomes for y. If s/he is lucky (meaning $y > a$) s/he keeps the excess of output over the loan repayment. If unlucky, s/he just sends all the output the creditor's way. Algebraically, the benefit to the debtor (B) is:

$$B = \begin{cases} y - a & \text{If } y > a \\ 0 & \text{If } y \text{ £ } a \end{cases} \qquad (1)$$

[7]

$$E(B) = \int_a^{e+\sqrt{3}s} (y-a)\left(\frac{1}{2\sqrt{3}s}\right)dy \qquad if \quad e-\sqrt{3}s < a < e+\sqrt{3}s$$

For simplicity, Krugman defines debtor utility[8] as:

$$E(Ud) = E(B) - C \qquad (2)$$

To find the optimal e, we set the marginal expected private benefit equal to marginal cost.

$$\frac{(e+\sqrt{3}s - a)}{2\sqrt{3}s} = MEB = MC = l\ e$$

$$\Rightarrow \qquad e = \frac{\sqrt{3}s - a}{2\sqrt{3}l\,s - 1} \qquad (3)$$

with the following second order condition.

$$1-2\sqrt{3}l\,s\ <0$$

The equation for e can be interpreted as the debtor reaction function. For a given a, the effort choice maximizes debtor utility. Krugman's important insight is that this is downward sloping in a x e space. Creditors face a tradeoff; more forgiveness (lower a) increases effort and the chance of collecting a, but a is also a cap on what can be collected.

The optimal e is substituted into expected creditor utility

$$\begin{aligned} Uc &= y - B - 1 \\ E(Uc) &= e - E(B) - 1 \end{aligned} \tag{4}$$

and differentiated to yield an optimal a, which is a function of l. This Krugman (debt forgiveness) Contract provides micro-foundations for case-by-case debt relief, since λ may differ across countries.[9] Graphically,

insert addition 2

The straight line is the debtor's reaction function. *eud* is one of a family of debtor iso-utility curves. These curves are increasing in utility as we move down the page, and the locus of their maxima define the debtor reaction function. *euc* is one of a family of creditor iso-utility curves, increasing in utility as we move North-East. Given the reaction function, the Krugman optimum is the *(a,e)* tuple *K*. Upon substitution into *(2)* and *(4)*, we obtain:

$$\mathrm{E}(Ud) = \frac{-1+2\sqrt{3}s\,l}{24l^3 s^2} \tag{5}$$

$$E(Uc) = -1+\frac{1}{4\sqrt{3}l^2 s} \tag{6}$$

Now it is clear from Diagram 1 that *K* is an inefficient bargain of the kind encountered in labor market analysis (McDonald and Solow 1981). There are Pareto improvements to be made in the lens South-East of *K*. Algebraically, we find the intersection of the creditor and debtor utility curves insisting on only one solution (thus finding the tangency). We use *eud* and *euc* to denote shift parameters of the iso-utility curves. From *(2)* and *(4)* we have:

$$euc + eud \quad = \quad e - \frac{l}{2}e^2 - 1$$

$$\Rightarrow e \quad = \quad \frac{1 \pm \sqrt{1-2l\ (euc+eud+1)}}{l}$$

A single solution (i.e. a tangency) implies two conditions.

$$e = \frac{1}{l}$$

$$euc + eud = \frac{1}{2l} - 1$$

From the first condition, the contract curve is a vertical straight line at $e=1/l$. One Pareto-improved point is *P* where the debtor is held to his utility at *K*. At *P*, the creditor utility is found by substituting *(5)* for *eud* in the second condition. The resultant expression can be shown to be greater than the value of creditor utility at *K* (given by equation *(6)*).

$$euc = \frac{1-2ls(\sqrt{3}+6ls(2l-1))}{24l^3s^2} \qquad (7)$$

Unfortunately, the debtor and creditor cannot bargain over an unobservable *e*. There is a moral hazard that the debtor will commit to $1/l$, but then choose a lower *e* from the reaction function. Therefore *P* is infeasible with a Krugman Contract, due to the inefficiency arising from hidden effort.

4 The Intuitive Case for Hyper-Incentive-Contracts[10]

It was no accident that the contract curve in Diagram 1 was defined by $e=1/l$. When the utilities of the creditor and debtor are combined we obtain:

$$cake \quad = \quad E(Uc) + E(Ud) \quad = \quad e - \frac{l}{2}e^2 - 1$$

from which it is obvious that the expected marginal *social* benefit of an increase in e is unity. Setting this equal to the marginal cost (le) we obtain a social optimum at $e=1/l$. However, the Krugman Contract can never make marginal private benefit (i.e. the benefit to the debtor) equal to marginal social benefit. As a result, effort will always be too low.

insert addition 3

Therefore a Krugman Contract produces too little effort. The marginal private benefit will always be less than unity - the marginal social benefit.[11]

When the problem is stated this way, the solution is clear. We have to design a contract where the debtor receives more than the increase in *y* in the upward sloping portion of the benefit schedule. That is, if the debtor can pay back the creditor fully, s/he gets a bonus above and beyond being able to keep the extra output. Only in this way can the *expected* private marginal benefit be made equal to the expected social benefit (here unity). This is called a Hyper-Incentive Contract.

5 The Formal Case for Hyper-Incentive Contracts

More formally, we rename the creditor the principal and the debtor the agent.[12] Instead of equation *(1)*, we propose a 'wage' for the debtor, and make a notional transfer of the output to the creditor. We think of the creditor as then choosing to 'pay' a multiple *b* of it back to the debtor.[13]

w	=	*-a*	+	*by*	*If y > a/b*	
	=	*0*			*If y £ a/b*	(8)

$$E(w) \;=\int_{a/b}^{e+\sqrt{3}s}(-a+by)\left(\tfrac{1}{2\sqrt{3}s}\right)dy \qquad if \quad e-\sqrt{3}s<\frac{a}{b}<e+\sqrt{3}s \qquad (9)$$

insert addition 4

It should be clear from the above diagrams how a change in *a* or *b* will effect the expected wage function. From the new *a/b*, measure *Ö3 s* either side, and draw a convex portion between these two values.

We now replace *E(B)* by *E(w)* in equation *(2)*. The optimal *e* is then obtained by differentiation.

$$e \;=\; \frac{b\sqrt{3}s\;-a}{2\sqrt{3}l\,s\;-b} \qquad (10)$$

Note that the optimal *e* depends upon both *a* and *b*. As before, it is decreasing in *a*. Since *e* is increasing in *b*, we call the latter the incentive parameter. The sign of the partial with respect to *b* follows from the second order condition.

$$b - 2\sqrt{3}\,l\,s \;< 0$$

We assert that the participation constraint binds (proved in the Appendix) and we therefore differentiate the sum of the utilities ((*2*) and (*4*)) with respect to b.

$$cake \quad = e - \frac{l}{2}e^2 - 1$$

$$\frac{\partial cake}{\partial b} = e'(1 - le) = 0 \qquad \Rightarrow \qquad e = \frac{1}{l}$$

The second order condition holds at the optimum.

$$\frac{\partial^2 cake}{\partial b^2} = -l\,(e')^2$$

This problem is more complex than the Krugman problem because the optimal e, which is a function of b, must be made to equal the value $1/l$. In other words, b must be chosen by the principal to make it privately optimal for the agent to set e equal to $1/l$.

However, if that were the only choice variable, there would be no guarantee that the value of b so chosen would satisfy the participation constraint.[14] Fortunately, choosing a provides a way of satisfying that constraint. The full solution thus involves equating the two expressions for optimal e

$$\frac{1}{l} \quad = \quad e \quad = \quad \frac{b\sqrt{3}s - a}{2\sqrt{3}l s - b}$$

meeting the participation constraint with equality

$$\frac{(-a + b(e + \sqrt{3}s))^2}{4b\sqrt{3}s} - \frac{l}{2}e^2 = \text{umin} \qquad (\text{umin} \geq 0)$$

and solving.

$$\left.\begin{aligned} a &= \frac{2\sqrt{3}l s (\sqrt{3}s - 2\text{umin})}{1 + 2\text{umin}\ l} \\ b &= \frac{2\sqrt{3}l s}{1 + 2\text{umin}\ l} \end{aligned}\right\} \qquad (11)$$

Finally, we assume that the country can always obtain HIPC/J2K forgiveness (*b=1*), which we stylize as being guaranteed access to a Krugman Contract.[15] We can therefore substitute equation *(5)* in for *umin*, giving:

$$\left.\begin{aligned} a &= \frac{2s (\sqrt{3} - 6 l s + 36 l^3 s^3)}{-1 + 2l s (\sqrt{3} + 6 l s)} \\ b &= \frac{24\sqrt{3}l^3 s^3}{-1 + 2 l s (\sqrt{3} + 6 l s)} \end{aligned}\right\} \qquad (12)$$

By construction, when these values for *a* and *b* are used to calculate *E(w),* and then *E(Ud),* the debtor utility at point *K* in Diagram 1 is obtained.

The Hyper-Incentive Contract exactly replicates the infeasible Pareto-improved point *P* in Diagram 1. When optimal *a* and *b* are used to calculate *E(Uc),* it is equivalent to *(7)* -

creditor utility at P - and effort is $1/l$. From the diagram, a is lower at P, implying (slightly) more forgiveness in the Hyper-Incentive Contract.[16]

Of course, we could have made the participation constraint bind on the creditor, and insisted that a Hyper-Incentive Contract make the creditor no worse off. In that case, the contract would replicate the Pareto-improved point at the bottom of the lens in Diagram 1. Considerations of equity, justice or compassion - which have not been modeled - may dictate this course of action.[17]

The solution can also be visualized with the aid of a diagram.

insert addition 5

The above diagram can be used to justify the 'Hyper' prefix, by showing that the incentive parameter is greater than the marginal expected social benefit (unity here). The optimum must occur when the slope of the expected wage function equals the slope of cost function. This, in turn, is set equal to unity by the principal as s/he maximizes the cake. We therefore have:

$$\frac{\partial E(w)}{\partial e} = l \;\; e \;\; = 1$$

But since there is always a chance of a bad state of nature, we are always on the convex portion of the *E(w)* curve, with a slope strictly below b.[18] Q.E.D.

6 Hyper-Incentive Contracts for Countries Experiencing Severe Difficulties[19]

Countries slated for HIPC/J2K debt forgiveness ($b=1$) are generally those experiencing severe economic problems. We model the onset of a crisis as an upward shift in l; countries in crisis find high effort very costly. How should creditors optimally respond to this? We first note that a/b

$$a/b = Ö3\ s - 2\ umin$$

is invariant to l. Furthermore, b in increasing in l. The rescheduling contract therefore has a steeper slope through an unchanged kink point.

insert addition 6

Therefore it is optimal to for the creditor to offer even greater incentives in the (upward sloping) range where the debt has been cleared. To provide the intuition, consider a version of Diagram 5 showing the increase in l.

insert addition 7

7 Conclusion

We began this paper by asserting that orthodox economics has a role to play in the debt forgiveness debate; case-by-case debt relief needs firm micro-foundations. We have found that the workhorse model of debt forgiveness (Krugman 1988), while providing a rationale for case-by-case debt write-downs, is not a fully optimal solution. Put simply, to talk in terms of a standard debt-overhang contract implicitly assumes the incentive parameter is unity - without justification. When the incentive parameter *b* is chosen optimally, two things happen. First, the problem of moral hazard due to hidden effort is solved. Second, a new way of making case-by-case contracts comes into being; *b* will larger for countries facing severe economic difficulties.

Key to the analysis has been the observation that contracts must be offered where the slope of the upward sloping portion of the benefit (or 'wage') schedule *exceeds* the marginal social expected benefit of increased effort. Where the marginal social expected benefit is unity, as in Krugman's set-up, this *hyper*-incentive raises operational questions. In the real world, where debtors actually own *y*, how will they be paid more than their marginal output? One possibility is that the creditor could buy their output at a higher price and then on-sell it, or pay them a production fee.

Finally, we note that this whole analysis depends upon the assumption that effort is unobservable. Where this is false, the debtor can credibly commit to the socially optimal effort level, striking an efficient bargain of the kind advocated by McDonald and Solow for the labor market. In this case our analysis - together with Krugman's - is bypassed.

However, if effort is not observable the case for a Hyper-incentive contract is a strong one. We have shown in this paper that such a contract Pareto dominates the Krugman Contract, precisely by mimicking the McDonald-Solow movement away from the reaction function. That being the case, it may be a valuable tool to apply to an undeniably significant problem.

Appendix

Does the Participation Constraint Bind? [20]

The creditor maximizes his utility subject to the incentive compatibility and the participation constraints. We start with the simplified expression for creditor utility, with optimal *e* substituted in, (i.e. incentive compatibility is already satisfied).

$$E(Uc) = -1 + \frac{\sqrt{3}bs - a}{2\sqrt{3}ls - b} - \frac{\sqrt{3}s\,l^2}{b}\left(\frac{\sqrt{3}bs - a}{2\sqrt{3}ls - b}\right)^2 \qquad (A1)$$

We want to choose *a* and *b* to maximize this subject to the following constraints:

$$2\sqrt{3}ls - b > 0 \qquad (A2)$$

$$\sqrt{3}bs - a > 0 \qquad (A3)$$

$$b > 0 \qquad (A4)$$

$$\frac{l}{2b}\frac{(\sqrt{3}bs - a)^2}{(2\sqrt{3}ls - b)} \geq \text{u min} \qquad (A5)$$

Equation (*A2*) is the second order condition. Equation (*A3*) then follows automatically since optimal *e* is positive. Equation (*A4*) limits the solutions to those of economic interest. Equation (*A5*) is the participation constraint, with optimal *e* substituted in, and simplified.

The solution given in the text assumes that (*A5*) holds with equality - that is, that (*A5*) 'binds'. To verify this, we maximize (*A1*), subject to (*A2*), (*A3*) and (*A4*) (but not by (*A5*)).

The unconstrained optimum is not within the region specified by (*A5*). Therefore, in a constrained optimum, (*A5*) must bind.

We make the following transformation to simplify the analysis:

$$y = 2\sqrt{3}l\,s - b \qquad \Rightarrow \qquad b = 2\sqrt{3}l\,s - y$$
$$x = \sqrt{3}b\,s - a$$

We therefore want to maximize:

$$E(Uc) = -1 + \frac{x}{y} - \frac{\sqrt{3}s\,l^2}{(2\sqrt{3}l\,s - y)}\frac{x^2}{y^2} \qquad (A6)$$

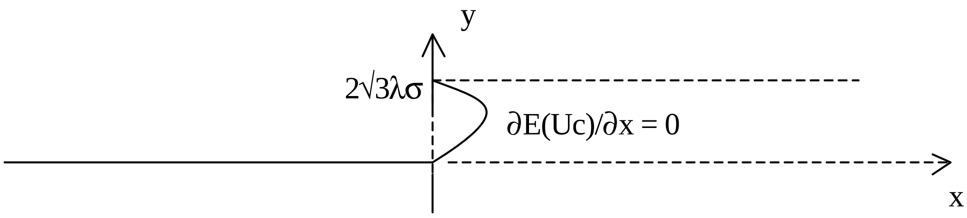
where $x \in R^+, y \in (0, 2\sqrt{3}\lambda\sigma)$

We differentiate *E(Uc)* with respect to *x* to obtain the maximum values of *x* for given *y*.

$$\frac{\partial E(Uc)}{\partial x} = \frac{1}{y} - \frac{2\sqrt{3}l^2 s\;x}{(2\sqrt{3}l\,s - y)y^2} = 0$$

$$\Rightarrow x \quad = \frac{y(2\sqrt{3}l\,s - y)}{2\sqrt{3}l^2 s} \qquad (A7)$$

This ridge is the maximum of the function in the *x* direction (i.e. for given *y's*), viewed from above in the following diagram. The dashed lines show the boundaries.



We now differentiate *E(Uc)* with respect to *y*, substituting in (*A7*). This gives the change in *E(Uc)* in the *y* direction along the ridge. Straightforward algebra reveals:

$$\frac{\partial E(Uc)}{\partial y} = -\frac{1}{4\sqrt{3}l^2 s} < 0 \qquad \text{(using } \frac{\partial E(Uc)}{\partial x} = 0\text{)}$$

This implies that the function is always *increasing* as *y* approaches zero along the ridge, and that there appears to be no interior maximum.

Now (*A6*) implies that *E(Uc)* takes on the following values close to the boundaries.

$$\lim_{y \downarrow 0, x>0} E(Uc) = -\infty$$
$$\lim_{x \downarrow 0, y>0} E(Uc) = -1$$
$$\lim_{y \uparrow 2\sqrt{3}s\,l} E(Uc) = -\infty$$
$$\lim_{x \downarrow 0, y \downarrow 0} E(Uc) = \textit{undefined}$$

But *E(Uc)* >-1 can be solved for sufficiently small *x* (from *A6*), thus ruling out the first three boundary values as maxima. We therefore have that the function achieves a maximum approaching, the undefined boundary *(0,0)* along the ¶*E(Uc)*/¶*x = 0* ridge. We conclude that (absent (*A5*)) the maximizing values of *x* and *y* both approach zero.

It now remains to be proven that the participation constraint prevents the attainment of the unconstrained optimum. For a given small y equal to e, (A7) dictates the unrestricted maximizing value of x.

$$x = \frac{e(2\sqrt{3}ls - e)}{2\sqrt{3}l^2 s}$$

For the participation (*A5*) constraint

$$\frac{l}{2(2\sqrt{3}ls - y)}\frac{x^2}{y} \geq u\min \quad \Rightarrow \quad x \geq \sqrt{\frac{2y(u\min)(2\sqrt{3}ls - y)}{l}}$$

to bind, the unrestricted maximizing value of x must be less than or equal to the value dictated by the participation constraint. This will be the case if:

$$\frac{e(2\sqrt{3}ls - e)}{2\sqrt{3}l^2 s} \leq \sqrt{\frac{2e(u\min)(2\sqrt{3}ls - e)}{l}}$$

implying

$$e(2\sqrt{3}ls - e) \leq 24l^3 s^2 (u\min)$$

The last condition will be true for sufficiently small e. If e is not small enough for this to be true, then the participation constraint is not stopping the creditor reducing e further, which is optimal. The creditor continues to make e smaller until the inequality is true.

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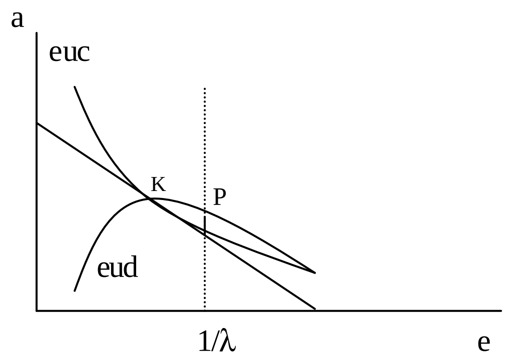
addition 1

	Creditor		***Debtor***	
Period	***y> a***	***y£ a***	***y> a***	***y£a***
1.	-1	-1	- *C*	- *C*
2.	a	y	y - a	0
1. and 2. combined	a-1	y-1	y - *C* - a	- *C*

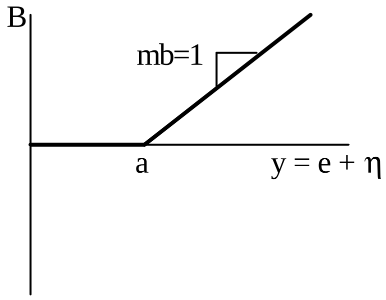
addition 2

Diagram 1: Krugman Optimum



addition 3

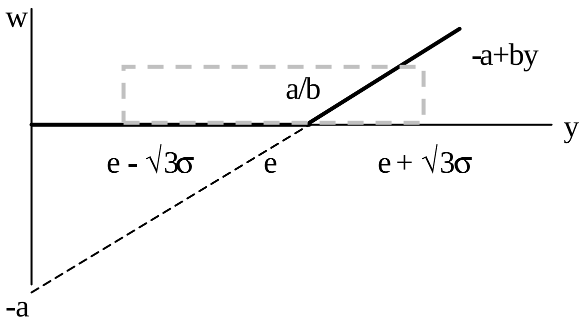
Diagram 2: The Krugman Contract



For our exposition, suppose that h is volatile enough that all y values are possible for any e in the diagram. The benefits for these values of y are shown in a diagrammatic representation of equation *(1)*. Given h, the marginal private benefit of an increase in e will be unity in a good state of nature (high h), and zero in a bad state of nature (low h). The expected marginal benefit, being a convex combination of the two, must be less than unity.

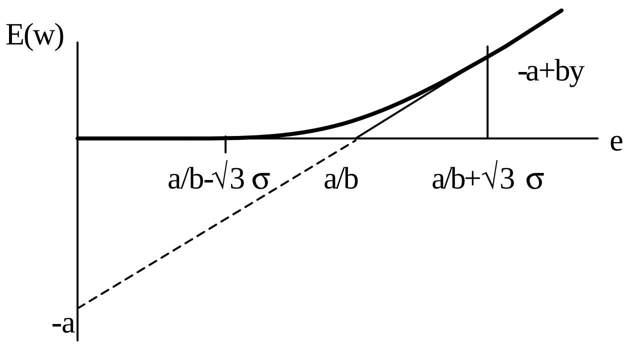
addition 4

Diagram 3: Wage Function, and Uniform Output
(wage function in bold)



The expected value of the wage, as a function of e, is obtained by integrating over the above rectangle, centered on a moving *e*. When the right-hand edge of the rectangle is at *a/b*, implying that *e* is at *a/b-Ö3s*, the integral is zero. When the left hand edge of the rectangle is at *a/b*, implying that *e* is at *a/b+Ö3s*, the integral is just the value of the straight line. Thus the expected wage, as a function of e, takes on a convex shape between these two extremes.

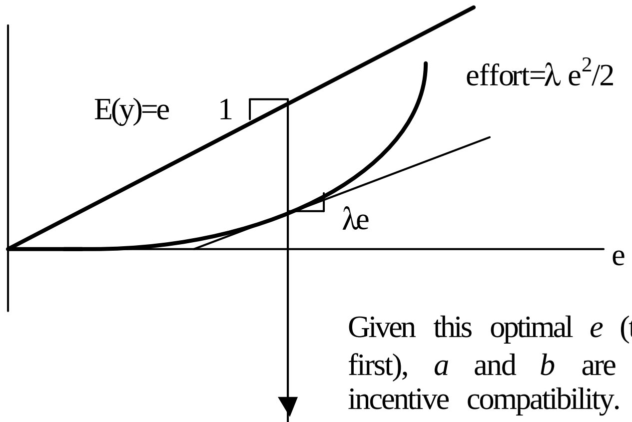
Diagram 4: The Expected Wage as a Function of Effort



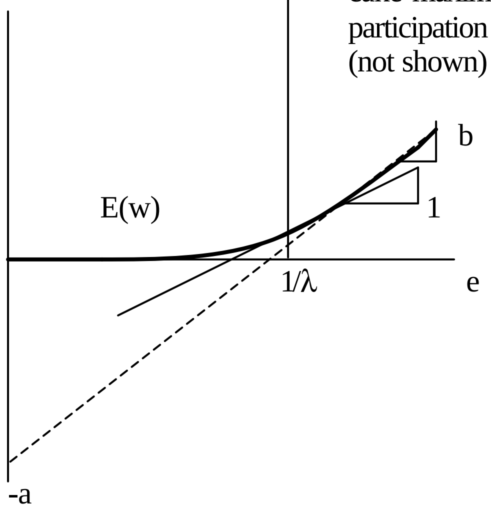
addition 5

Diagram 5: Optimal e Determines a and b

The sum of the utilities, *cake*, is maximized when the slope of expected output (marginal expected social benefit) *e* equals the slope of effort $\lambda e^2/2$. That is, when $1=\lambda e$, or $e=1/\lambda$.

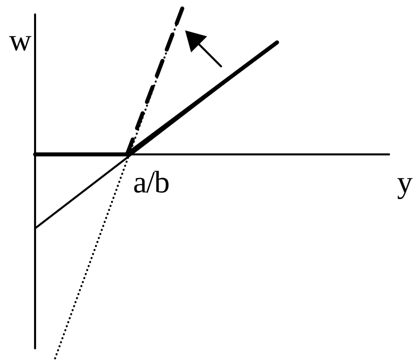


Given this optimal *e* (the arrow signifies that *e* is calculated first), *a* and *b* are adjusted by the principal to ensure incentive compatibility. That is, the principal chooses *a* and *b* simultaneously to make the slope of the expected wage equal to the marginal social benefit (unity) at the cake-maximizing level of effort, and, to satisfy the participation constraint. As shown earlier, moving *a* and *b* (not shown) shifts the convex portion of the *E(w)* curve. This happens until there is a tangency at $1/\lambda$.



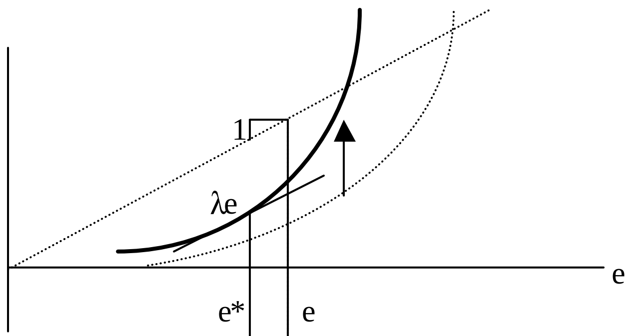
addition 6

Diagram 6: Response to a l Shock

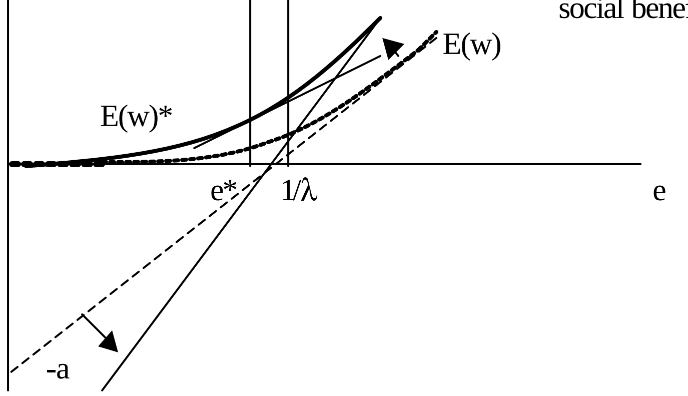


addition 7

Diagram 7: Response to a λ Shock



At the *original* effort level *e* the marginal cost (λe) rises. Therefore the social optimum *e** must be below the original *e*. However, the current incentive structure makes it impossible to *both* make the debtor reduce *e* and set the marginal private benefit equal to unity. A lower *e** will lead to a lower marginal private benefit as we mover along the (convex) original private expected benefit schedule (*E(w)*). Pivoting the wage contract by increasing *a* and *b* (as shown in Diagram 6) increases the marginal private benefit for all effort[21] allowing marginal private benefit to equal marginal social benefit at the lower *e** along *E(w)**.



Endnotes

[1] I wish to thank David Vines and Donald Hay for their invaluable insights as supervisors, Meg Meyer for her help with optimal contracts and Francis Chigunta for providing detailed comments on the draft. Jens Rittscher and Andrew Dancer provided me with the mathematical tools to prove the result in the appendix. Hugh Evans helped to clarify the political economy of HIPC. Ian Harper first stimulated my interest in debt forgiveness. None of the above are responsible for the inadequacies of this paper.

[2] Apologies to Robert Burns, 'The best-laid schemes o' mice and men Gang aft a-gley' *To a Mouse*, and, apologies to those who correctly point out economists' schemes sometimes do likewise.

[3] Debt principal relief does not necessarily imply debt service relief. Although a reduction in the debt stock should translate roughly into a proportional cut in the service repayments, it is not so for the HIPCs. Much of the debt service is not being paid anyway, so there is a point up to which debt forgiveness will not reduce interest payments.

[4] In Deuteronomy 15, the Israelite people were commanded to cancel fellow-Israelite debts and free Israelite slaves every seven years (though it is not called a Jubilee in this passage). Leviticus 25 is the actual Jubilee legislation. Every 50 (or 49) years, on the so-called day of the atonement, the people were commanded to 'proclaim Liberty'. All land was to return to its original family, redefining all land sales as leases (with the exception of houses in walled cities, under some circumstances). In the same passage, charging interest, or selling food to those in need at a profit, is prohibited.

[5] The model continues to be used, and adapted. In a recent paper, Fernandez-Ruiz used a very elegant version of it to illustrate a dynamic interaction between debtors and creditors (Fernandez-Ruiz 1996).

[6] Simple distributions are commonplace. Fernandez-Ruiz (1996) use a Bernoulli trial with the probability of success equal to effort. The normal distribution is unsuitable because it cannot be integrated.

[7] Krugman implicitly assumes that the debtor already has subsistence consumption provided for. Thus, it is feasible to accept a benefit of zero in a bad state of nature. One way to think of this is to identify output with output in the market economy. In many developing economies, this is a small share of total output.

[8] The inclusion of variance leads to mathematical intractability. As we shall see presently, the non-linear benefit function leads to polynomials of degree two in effort. The inclusion of variance would lead to polynomials of degree four. Furthermore, the writers in the field may have been influenced by a tentative theoretical argument in favor of its exclusion. The debtor is already insured, in an ex post sense. Given the floor in the benefit function, the maximum loss for the debtor is $-l\ e^2/2$. It is conceivable that maximal loss is as good a risk-metric as variance. Fernandez-Ruiz (1996) uses the utility function in the text without explanation.

[9] To calculate the rate of forgiveness on the face value, recall that $a-1=r$. To calculate the market discount, substitute optimal e and a into $E(Uc)$ and compare this to unity, the original rescheduled amount.

[10] I am particularly grateful to David Vines for helping me develop the intuition of this section.

[11] This is true within the confines of the Krugman effort technology, which is (expected) constant returns to scale. In contrast, if $E(y)=\phi e$, and $\phi<1$, it is possible, though unlikely, that a Krugman Contract will be fully optimal. However, the point still stands that b ought to be chosen optimally rather than imposed at unity. Furthermore, the contract still requires hyper-incentives in the sense that the slope of the upward portion of the contract curve must still exceed the marginal social expected benefit of an increase in e.

[12] The framework chosen is appropriate, because the creditor desires the debtor to perform an unobserved action, leading potentially to moral hazard. However, since wage variance is omitted from the utility function, it is not to be understood in terms of the usual insurance vs. incentive tradeoff.

[13] If the contract breaks down after the output is produced, but prior to repayment of the loan, then clearly it matters who really owns the output. Our analysis sits within the class of models that assume that debtors and creditors can credibly commit themselves to honor the terms of the contract (Calvo and Kaminsky 1991), perhaps because reneging in one relationship creates problems in other relationships (Cole and Kehoe 1998).

[14] The same issue may be relevant for Krugman (1988). There, an optimal a is derived without reference to a participation constraint. The analysis there may have become more difficult had there been two targets, given the one choice variable.

[15] To do so, we also must assume the Krugman second order condition *1-2Ö3 s l<0*.

[16] The extra forgiveness is contingent upon more (costly) effort. The decline in a - measured as vertical distance in Diagram 1 - may be slight because we are moving from an optimum of a smooth function.

[17] Counter-arguments typically appeal to moral hazard. For a discussion strongly critical of the IMF, see the J2K web page article 'dictators and debt' (Hanlon 1998). For example, 'the issue of moral hazard in the case of Rwanda has been raised. IMF officials in Rwanda explained that the conflict should not be `rewarded' and that rapid debt cancellation would be seen to reward genocide'

[18] The Krugman second order condition *1-2Ö3s l<0* guarantees this. Here is the logic: To have a chance of a bad state of nature means that optimal *e (*i.e. *1/l) < a/b+Ö3s*. Substituting optimal *a/b* from equations *(13)*, and simplifying, we find that the second order condition guarantees the truth of this inequality. A formal proof could thus start from the second order condition, work backwards, to reach the conclusion that optimal *e* must be in the convex portion of the expected wage curve.

[19] This section treats *umin* as a constant. Otherwise, the fall in *umin* as *l* changes clouds the intuition.

[20] Meg Meyer first alerted me to the importance of this question. Jens Rittscher and Andrew Dancer provided important insights for this Appendix.

[21] The proof notes that optimal *a/b=y* (a constant). Therefore *a=by*, and marginal benefit *(-a+bÖ3s)/(2Ö3s)+be/(2Ö3s)* >0 = *b [(-y +Ö3s)/(2Ö3s)+e/(2Ö3s)]*, which is increasing in *b*.