Essays in Monetary Policy and International Finance

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Abstract

This dissertation consists of three essays that examine aspects of monetary and exchange rate policy choices in developing and emerging market economies.

The first essay models limited financial market participation in less-developed economies to present a counterpoint to Milton Friedman's well-known case for exchange rate flexibility. I analyze the welfare-maximizing choice of monetary policy in a small open economy with high financial exclusion and nominal rigidities. My findings suggest that the optimality of flexible exchange rates depends on whether agents are able to smooth consumption. When most households in an open economy are unable to hedge against international relative price fluctuations through the financial markets, nominal exchange rate targeting mitigates the resulting consumption volatility and approximates the optimal monetary policy.

The second essay analyzes the appropriate choice of an exchange rate regime in agricultural commodity-exporting economies with limited labor, product, and financial market development. Welfare analysis suggests that flexible exchange rates are preferred in well-functioning domestic markets as they allow for greater relative price fluctuations, which amplify the efficient transmission mechanism of labor re-allocation in response to commodity price volatility. When labor and product markets are inflexible, however, international relative price adjustments exacerbate currency and factor misalignments. Nominal exchange rate targeting, by mitigating relative wage and price fluctuations, is preferred to exchange rate flexibility.

The third essay investigates the effectiveness of restrictions on capital account transactions during periods of nominal exchange rate pressures. Using an event study approach, the paper examines the actions of the Bank of Tanzania which augmented its conventional monetary policy instruments by tightening capital account regulations three times from 2011 to 2015 in an attempt to stabilize the Tanzanian shilling. I show that while initially successful, at least over the short-run, the effectiveness of the controls declined the more they were used, raising concerns about the value of these instruments as part of the monetary policy toolkit.

Contents

[1]This chapter, Iyer (2016), is published as a June 2016 Federal Reserve Bank of Chicago working paper: https://www.chicagofed.org/publications/working-papers/2016/wp2016-06. An earlier version of the study also appears in Iyer (2015), a National Council of Applied Economic Research working paper: http://www.ncaer.org/publication_details.php?pID=253.

[2]This chapter, Iyer (2017), is published as a July 2017 IMF Working Paper as part of a research project on macroeconomic policy in low-income countries supported by the U.K.'s Department for International Development:
https://www.imf.org/en/Publications/WP/Issues/2017/07/10/Exchange-Rate-Choices-with-Inflexible-Markets-and-Costly-Price-Adjustments-44984.

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This dissertation is dedicated to my mother and my father.

Chapter 1

Introduction

Growing up in a developing country, I witnessed severe underdevelopment in Uttar Pradesh, one of the poorest and most populous states in India. My parents were government officials, and their work to promote development, be it providing water and sanitation, or making tax administration more efficient, made for fascinating conversation at the dinner table. These early discussions helped influence my academic interests in the effective design of economic policies in developing countries. I went on to write my undergraduate honours thesis at Duke University, USA on the effective design of public expenditure programs to improve educational outcomes in districts across India. Upon completing my undergraduate studies, I joined the Africa Department of the International Monetary Fund (IMF). I learned a great deal by contributing to the monetary, fiscal, and external sector reports of six countries in Sub-Saharan Africa, and also had the opportunity to witness economic policy-making in developing countries. Work at the IMF motivated my interests in international macroeconomics, after which I began my graduate studies in economics at the University of Oxford, UK, focusing on the macroeconomics of developing countries.

This dissertation is the product of my long-time interest in the effective design of macroeconomic policies to promote economic development. I focus on aspects of monetary and exchange rate policy choices in developing and emerging market economies in three related essays. In the first two essays, I show that certain structural features that are characteristic of less-developed economies, such as financial exclusion, limited financial market integration, and product and labour market imperfections, make exchange rate targeting dominate from a welfare perspective. These features create macroeconomic distortions and lead to costly factor and currency misalignments, which can be mitigated by the Central Bank through stabilizing the nominal exchange rate. My work on the optimality of fixed exchange rates provides a counterpoint to the conventional macroeconomic policy advice arising from Milton Friedman's case for flexible exchange rates (Friedman, 1953). I also provide a rationale for the "fear-of-floating" phenomenon (Calvo and Reinhart, 2002) observed in the data, whereby most developing countries *de facto* target their exchange rates even though many are *de jure* classified as floating exchange rate regimes. The "fear-of-floating" phenomenon can be observed in Tanzania, a low-income country that is structurally similar to the economies modeled in the first two essays, and the subject of my third essay. Though Tanzania is *de jure* classified as a floating exchange rate regime, the Bank of Tanzania *de facto* seeks to stabilize the value of the shilling, and recently through the usage of capital controls. While exchange rate targeting may be optimal in Tanzania, I conclude that capital controls may not be the best instrument to implement this in practice.

The first essay examines optimal monetary policy in a small open economy framework with high financial exclusion. I model a salient empirical fact in developing countries that the majority of households do not have the means to engage in institutional financial transactions, and therefore live largely on a disposable income basis (World Bank, 2015). I investigate whether the Central Bank can play a role in mitigating the unduly costly effects of shocks in the presence of limited financial market participation, and how monetary policy should be designed in an open economy where the majority of agents cannot smooth consumption. To establish benchmark results, I augment the well-known Gali and Monacelli (2005) open economy framework with asset market inequality, and characterize the sources of distortions in the financially segmented economy by deriving the micro-founded welfare loss function, following the analytical approach in Benigno and Woodford (2012). The findings suggest that when most households in an open economy are unable to hedge against international relative price fluctuations through the financial markets, nominal exchange rate targeting mitigates the resulting consumption volatility and approximates the welfare-maximizing choice of monetary policy.

In my second essay, I analyze the appropriate choice of an exchange rate regime in agricultural commodity-exporting economies. In practice, the majority of such economies target their exchange rates (IMF, 2016a). I contribute to the policy debate on monetary policy in commodity-exporters, which has been re-ignited since commodity prices began to fall in 2015, by providing context-specific results on the appropriate exchange rate policy in agricultural commodity-exporting economies. I develop an open economy framework that incorporates key structural characteristics of agricultural commodity-exporting economies including dual labour markets and imperfect financial market integration. The benefits of exchange rate flexibility are shown to depend on the extent of labour and product market development. With developed markets, flexible exchange rates are preferred as they allow for greater relative price fluctuations, which amplify the transmission mechanism of labour re-allocation upon commodity price volatility. When labour and product markets are not well-developed, however, international relative price adjustments exacerbate currency and factor misalignments. An exchange rate peg, by mitigating relative wage and price fluctuations, increases welfare relative to a float.

The third essay narrows in on Tanzania, where the value of the shilling is *de facto* targeted, even though the exchange rate regime is *de jure* classified as a float. The Bank of Tanzania has implemented changes in the regulations concerning capital account transactions at various times in recent years to put 'sand in the wheels' so as to stabilize the Tanzanian shilling. While nominal exchange rate stability might be optimal in Tanzania, which exhibits many of the real and financial market imperfections modeled in the first two essays, I find that capital outflow controls may not be the best instrument to achieve this in practice. Using an event study approach, it is shown that while initially successful, the effectiveness of the outflow controls declined the more they were used, raising concerns about the value of these instruments as part of the monetary policy toolkit. Market anticipation and innovation, as well as the inherently contractionary nature of a tax on financial diversification, may have undermined the effectiveness of the controls. These findings also hold relevance for other similar developing countries in Africa, and would suggest that the Central Banks seek alternate instruments to stabilize the nominal exchange rate. I plan to pursue this line of analysis in future work.

Chapter 2

Optimal Monetary Policy in an Open Emerging Market Economy

> *"In developing countries where financial markets are underdeveloped, the poor do not have access to credit and [are thus restricted to consuming disposable income]... The challenge is how to stabilize output more effectively and reduce the burden on the poor."*
>
> - The 2014 Global Economic Symposium (GES, 2014)

2.1 Introduction[1]

The Global Financial Crisis of 2007-09 led to macroeconomic volatility in many emerging market economies (EMEs) and, in particular, adversely affected poor agents within these countries. One significant factor that contributed to the vulnerability of the poor was their inability to insure their consumption against the crisis-related contraction of incomes by drawing on savings held in the financial markets (World Bank, 2009). The macroeconomic consequences of this are not trivial, as on average, financially-excluded households constitute 60% of households in EMEs, compared to the corresponding average of under 5% in advanced economies (World Bank, 2015; see Figure 2.1).[2] This was highlighted by the crisis, which renewed policy interest in the following interlinked macroe-

[1]This chapter, Iyer (2016), is published as a June 2016 Federal Reserve Bank of Chicago working paper: https://www.chicagofed.org/publications/working-papers/2016/wp2016-06. An earlier version of the study also appears in Iyer (2015), a National Council of Applied Economic Research working paper: http://www.ncaer.org/publication_details.php?pID=253.

[2]Regional abbreviations used in Figure 2.1 are as follows. MENA: Middle East and North Africa. SSA: Sub-Saharan Africa. ASIA: East, South, and Central Asia. LAC: Latin America and the Caribbean. EUR: Europe. CIS: Commonwealth of Independent States.

conomic stabilization questions (Prasad, 2013; IMF, 2015b). How should monetary policy be designed in an open economy where the majority of agents cannot smooth consumption? Relatedly, should the appropriate monetary policy be implemented through a domestic or external nominal anchor? To complement the policy discussions, however, the corresponding theoretical analysis has been fairly limited. I seek to fill in this gap by providing benchmark results on the implications of high financial exclusion for optimal monetary policy, and the corresponding choice of a nominal anchor, in an open economy.

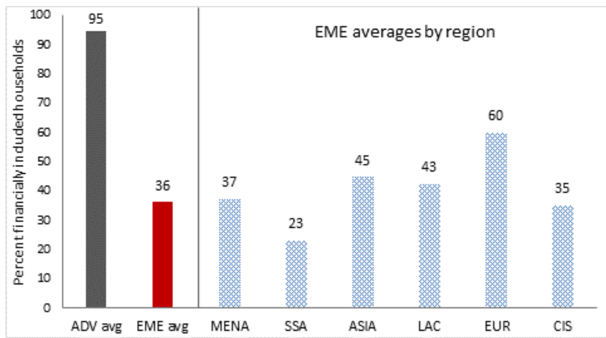

Figure 2.1: Financial Inclusion Across the World
Data: Percent Households with an Account at Any Financial Institution
Sources: World Bank 2014 financial inclusion database (World Bank, 2015); IMF 2015 country classification scheme (IMF, 2015d)

I build an open economy model that incorporates asset market segmentation, with financially-included and financially-excluded agents. While the former can borrow and save, the latter fully consume their income each period, as they do not have the economic means to engage in financial transactions. The Euler Equation does not hold for financially-excluded agents, implying that a fraction of the economy cannot insure against shocks through international risk-sharing arrangements. The framework is that of a small open economy as EMEs are typically price takers for tradable goods (Frankel, 2010). Openness further permits a quantitative comparison of domestic versus external nominal anchors in approximating the optimal monetary policy. I characterize the optimal monetary policy for the model in the presence of distortions caused by monopolistic competition and staggered price setting, and rank simple monetary rules in terms of lowest welfare losses away from the optimum. The implications of different shocks are analyzed, with the focus primarily on adverse supply (or cost-push) shocks. This is of particular interest since over the past few years, cost-push shocks, due to domestic food price volatility (driven largely b by weather conditions) and fuel input price volatility (driven largely by world market conditions), have been a significant source of concern in EMEs (Frankel, 2010).

There are two main results. The first is that with high financial exclusion, it is optimal to smooth the consumption of financially-excluded, or hand-to-mouth, agents by seeking to prevent deep recessions. These households cannot privately smooth consumption or insure themselves through risk-sharing arrangements with foreign agents. Their consumption thus fluctuates with shocks, leading to higher aggregate volatility. Optimal monetary policy, which is designed to reduce macroeconomic volatility, smooths the consumption of financially-excluded agents by stabilizing their income. In its loss function, the Central Bank implements this strategy by increasing the weight on stabilizing the output gap relative to inflation, as both financial exclusion and openness increase.[3] In a more open economy, domestic agents consume more imports relative to domestic goods. This increases the consequences of exchange rate volatility, which adversely affects hand-to-mouth agents as, unlike asset holders, they cannot smooth their consumption against the increasingly volatile price of imported goods. Optimal policy makes up for their inability to insure against exchange rate fluctuations by stabilizing their disposable income.

The second result is that the external anchor of a fixed nominal exchange rate approximates optimal monetary policy. I find that exchange rate stability is desirable in an economy with a large fraction of financially-excluded agents. Upon cost-push shocks, which simultaneously decrease output and increase inflation, thus creating a trade-off for monetary policy, targeting the exchange rate approximates the efficient dynamics through two channels. A fixed nominal exchange rate stabilizes the terms of trade, or the relative price of domestic goods, and hence domestic output and wage income. Mitigating exchange rate fluctuations also directly stabilizes the imported good component of the financially-excluded agents' consumption basket, smoothing their consumption. These results provide a counterpoint to Milton Friedman's long-standing argument for flexible exchange rates, which provides the underlying intuition for the optimality of a float in the more recent complete financial inclusion reference model of Gali and Monacelli (2005).

Friedman advocated a float on the premise that it would allow for more efficient adjustment through international relative prices, given that nominal rigidities constrain real adjustment in practice. A peg would only exacerbate the lack of relative price ad-

[3] A complementary reason is that as financial exclusion increases - the relative weight on stabilizing domestic inflation decreases, since inflation erodes the value of assets, and there are fewer asset holders left to consider. This is suggested in the closed economy model of Bilbiie (2008).

justment and thus lead to greater macroeconomic volatility. However, this argument does not account for high financial exclusion. My results extend Friedman's classic arguments to an economic setting where the majority of agents cannot smooth consumption. Even though a float continues to allow for greater relative price adjustment with nominal rigidities, this actually leads to greater welfare losses in the presence of a large number of financially-excluded agents who cannot optimally hedge against exchange rate volatility. For emerging market economies, however, Friedman advocated a peg due to the gain in credibility by anchoring domestic monetary policy to a stable, advanced country's regime (Hanke, 2008). I corroborate these political economy reasons by providing a new theoretical rationale for the desirability of a peg in EMEs.

Related Literature This paper contributes to two main strands of research.

The first literature is on optimal monetary policy in open economies. There are a number of studies in this vein, reviewed in detail in Corsetti et al. (2010). Central Bank intervention in these papers is required to correct different sources of distortions, for instance, local currency pricing and imperfect risk-sharing. Most papers incorporate nominal rigidities and monopolistic competition. The literature tends to approach optimal monetary policy design through a linear quadratic framework following the perturbation techniques discussed in Benigno and Woodford (2012). Gali and Monacelli (2005) develop a small open economy framework that has become well-cited in the literature, and find that flexible exchange rates allow for efficient adjustment in the presence of staggered price-setting. Papers including Devereux and Engel (2003) and Engel (2011) find that in the case of local currency pricing, or when producers set their prices in the currency of foreign buyers, unrestricted exchange rate movements are undesirable as they diminish efficient risk-sharing. Monacelli (2005) and Farhi and Werning (2012) are able to generate open economy optimal policy conclusions that are not isomorphic to the closed economy cases. A few studies have analyzed the appropriate choice of monetary policy with imperfect risk-sharing (Benigno, 2009; De Paoli, 2009; Corsetti et al., 2010), and find a case for exchange rate targeting as this can redress real exchange rate misalignments. The findings in this literature are based on representative agent models.

This paper contributes to the literature in two main ways. First, I derive the optimal monetary policy in an open economy with financial exclusion, which, to the best of my knowledge, has not been analyzed previously. This fundamental asymmetry in the ability of households to pool risk with foreign agents is new to the literature on the design of optimal policy in open economies, and has striking implications for the exchange rate. The inequality in access to finance of the heterogeneous agents in this study induces financial market incompleteness, complementing related research where a representative agent engages in imperfect risk-sharing. I also contribute in a technical capacity to the optimal policy literature, which has typically used a convenient subsidy to simplify the derivation of the micro-founded loss function (by eliminating the troublesome linear term in an ad-hoc manner). Benigno and Woodford (2012) argue that that approach is not fully correct, and outline perturbation techniques to address the issue. I implement their approach by analytically deriving second-order approximations of the structural equations to replace out for the linear term with the correct quadratic terms.

Related to the open economy general equilibrium model I use for analyzing the optimal monetary policy are other papers where household behaviour is driven by disposable income. These studies, beginning with Mankiw (2000) who introduced the concept of hand-to-mouth agents, pursue a variety of objectives. Galí et al. (2007), Bilbiie (2008), and Ascari et al. (2011) analyze optimal policy and the determinacy properties of ad-hoc monetary rules in a closed economy. Eser (2009) and Boerma (2014) analyze determinacy with hand-to-mouth agents, the former in a monetary union and the latter in an open economy. The literature suggests that active simple rules lead to indeterminacy for a high level of financial exclusion, a point that is not the focus of this study. Iyer (2014) develops an open economy model with hand-to-mouth agents to analyze commodity price shocks in developing countries, a similar version of which is used here with the aim of analytically deriving the optimal monetary policy. There are also a number of other studies that use hand-to-mouth agents in open economy models to analyze issues including the macroeconomic effects of scaling up aid and of public investment (eg. Berg et al., 2010, 2013), or the design of fiscal policy (eg. Kumhof and Laxton, 2013a). This paper contributes by combining hand-to-mouth agents with open economy elements to describe the conduct of welfare-maximizing monetary policy.

This study also contributes to the burgeoning literature on monetary policy choices in emerging market economies. Prasad and Zhang (2015) build a multi-sector model with traded and non-traded sectors. They find that exchange rate targeting benefits households in the tradable sector as it fixes the domestic price of traded goods. Catão and

Chang (2015) analyze monetary policy in a model with imported food, to find that domestic inflation targeting is desirable in response to food price shocks when international risk-sharing is low. Anand et al. (2015) numerically analyze the welfare properties of simple monetary rules in an open economy model with subsistence food, credit-constrained agents, and labour market segmentation, to find that headline inflation targeting is preferred to core inflation targeting as this helps stabilize aggregate demand by more. Portillo et al. (2016) derive the optimal monetary policy for a closed economy model with food to find that core inflation is preferred, attributed to factors including their micro-founding the optimal monetary policy instead of analyzing the welfare properties of alternate simple rules. The authors find that completely stabilizing sticky price non-food inflation also leads to the stabilization of the output gap and flexible price inflation.

This paper contributes by analytically deriving the optimal monetary policy in an open economy with segmented asset markets, and employing rigorous perturbation techniques to micro-found the welfare loss function. This affords a precise characterization of the sources of distortions in the economy. It further shows that financial exclusion has significant implications for the optimal choice of an exchange rate regime, with fixed exchange rates closely approximating the optimal monetary policy. Besides the theoretical models, empirical papers including Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010), have investigated business cycles in EMEs, the former finding that technology shocks could explain fluctuations while the latter questioning this. Frankel (2010) and Holtemöller and Mallick (2016) find that cost-push shocks, due to food and fuel input price fluctuations, are significant drivers of business cycles in EMEs. Calvo and Reinhart (2002) find that most EMEs *de facto* target their exchange rates even though many are *de jure* classified as floats, in a phenomenon they term as "fear-of-floating". This paper adds to the empirical literature on monetary policy and fluctuations in EMEs by investigating the optimal Central Bank response to cost-push and productivity shocks, and as a result providing a rationale for the fear-of-floating phenomenon found in the data.

The rest of the paper proceeds as follows. Section 2.2 develops an open economy model with financial exclusion. Section 2.3 characterizes the optimal monetary policy. Section 2.4 compares optimal policy with simple rules, and considers the choice of an implementable nominal anchor in the presence of financial exclusion. Section 2.5 concludes.

2.2 A Small Open Economy with Financial Exclusion

I develop an open economy general equilibrium framework with nominal rigidities, aggregate uncertainty, and two types of households.[4] The model incorporates financially-excluded agents in addition to the financially-included agents typically considered in the open economy optimal policy literature, and builds upon the small open economy framework of Gali and Monacelli (2005). Section 2.2 is cast in non-linear terms for expositional ease, and the resulting equilibrium is linearized in Section 2.3 to solve for optimal monetary policy using perturbation techniques.

2.2.1 Households

There exists a continuum of households indexed by $l \in [0, 1]$. Fraction $1 - \lambda$ of households participate in the financial markets, which are assumed to be complete both within and across countries. Through a complete set of state-contingent securities available internationally, they are able to share risk with foreign agents. However, a fraction λ of domestic agents do not engage in financial transactions as they do not have the economic means to do so, similar to the "hand-to-mouth" consumers in Mankiw (2000). Their sole source of income is wage income. Thus, although financially-excluded agents can optimize labour supply, they are unable to share risk internationally or smooth consumption. Throughout the paper, financially-included households are denoted with the symbol,$\hat{}$, while financially-excluded households are denoted with the symbol,$\check{}$.

The domestic economy, with its heterogeneous consumers, is of measure zero compared to the world, based on the Gali and Monacelli (2005) model and assumptions. This implies that domestic monetary policy decisions do not have any impact internationally, which consists of a continuum of economies, $j \in [0, 1]$, populated by identical, financially-included households. While different economies are subject to asymmetric shocks, initial net foreign asset positions are symmetrically zero across countries, and preferences and market structures are identical. In each period $t \geq 0$, a stochastic event, s_t, is realized. Let $s^t = (s_0, ..., s_{t-1}, s_t)$ be the history of events until period t, as in Chari et al. (2002).

[4]This model is nested in the small open economy model with financial exclusion to analyze commodity price shocks, derived in my 2014 MPhil Economics thesis (Iyer, 2014). I simplified my existing MPhil model to be able to analytically derive the optimal monetary policy and micro-found the welfare loss function in the presence of financially-excluded agents. I later found that this model was coincidentally also used in Boerma (2014) in his MPhil thesis to look at determinacy. The two sets of derivations for the base framework were done independently.

The unconditional probability, as of period 0, of observing any particular history s^t is $\mu\left(s^t\right)$. The probability of history s^{t+1}, conditional on s^t, is given by $\mu\left(s^{t+1}|s^t\right)$. The initial realization, s_0, is taken as given so that $\mu\left(s_0\right) = 1$ for a particular s_0.

2.2.1.1 Financially-Excluded Agents

Financially-excluded agents, $l \in [0, \lambda]$, consume their income each period and cannot smooth consumption through the financial markets. These agents gain utility from consumption, $\check{C}\left(s^t\right)$, and disutility from hours worked, $\check{N}\left(s^t\right)$. The representative household chooses its period t allocation after the event, s_t, is realized to maximize its utility, $\check{U}(x^t) = \sum_{s^t} \mu(s^t) U\left\{\check{C}\left(s^t\right), \check{N}\left(s^t\right)\right\}$, in the following static optimization problem

$$\underset{\left\{\check{C}(s^t),\check{N}(s^t)\right\}}{Max} \check{U}(s^t) = \sum_{s^t} \mu(s^t) \left\{ \frac{\check{C}\left(s^t\right)^{1-\sigma}}{1-\sigma} - \frac{\check{N}\left(s^t\right)^{1+\phi}}{1+\phi} \right\} \tag{2.1}$$

$$s.t.\ P\left(s^t\right)\check{C}\left(s^t\right) \leq W\left(s^t\right)\check{N}\left(s^t\right) \tag{2.2}$$

where σ is the inverse intertemporal elasticity of substitution (IES), ϕ is the inverse Frisch elasticity of labour supply, $P\left(s^t\right)$ is the consumer price index (CPI), $W\left(s^t\right)$ denotes nominal wages, $\check{N}\left(s^t\right)$ represents hours of labour supplied, and $\check{C}\left(s^t\right)$ is consumption of domestically produced and imported goods from a constant elasticity of substitution (CES) consumption basket. These variables are denominated in domestic currency units. Preferences are locally non-satiated, so that the budget constraint binds. This implies that the consumption of financially-excluded households derives directly from (2.2). As these agents do not hold bonds, they cannot smooth consumption.

Financially-excluded households trade off leisure for consumption goods, through their optimal labour supply condition, (2.3), where $w\left(s^t\right)$ denotes the real wage rate, $w\left(s^t\right) = \frac{W\left(s^t\right)}{P\left(s^t\right)}$. While nominal wages, $W\left(s^t\right)$, are the same for consumers as well as producers, *real* wages faced by households, $w\left(s^t\right)$, differ from the real wages faced by domestic firms, $\frac{W\left(s^t\right)}{P_H\left(s^t\right)}$, where $P_H\left(s^t\right)$ is the domestic (monopolistically set) price index. This is because households take into account the CPI, $P\left(s^t\right)$, which consists of both domestic and imported good prices. The labour supply condition, derived by combining the first-order conditions on consumption and labour, is

$$\omega\left(s^t\right) = \check{N}\left(s^t\right)^{\phi}\check{C}\left(s^t\right)^{\sigma} \tag{2.3}$$

The consumption basket, $\check{C}\left(s^t\right)$, is comprised, as in Gali and Monacelli (2005), of domestic and foreign aggregates, respectively $\check{C}_H(s^t)$ and $\check{C}_F(s^t)$.

$$\check{C}\left(s^t\right) = [(1-\alpha)^{\frac{1}{\varepsilon_I}} \check{C}_H\left(s^t\right)^{\frac{\varepsilon_I - 1}{\varepsilon_I}} + \alpha^{\frac{1}{\varepsilon_I}} \check{C}_F\left(s^t\right)^{\frac{\varepsilon_I - 1}{\varepsilon_I}}]^{\frac{\varepsilon_I}{\varepsilon_I - 1}} \tag{2.4}$$

where $\alpha \in [0,1]$ is the share of imports in the CES basket and represents the degree of openness to international trade in goods (conversely, $1-\alpha$ is the degree of home bias), and ε_I is the elasticity of substitution between domestic and foreign goods. The associated price index is

$$P\left(s^t\right) = \left[(1-\alpha) P_H^{1-\varepsilon_I}\left(s^t\right) + \alpha P_F^{1-\varepsilon_I}\left(s^t\right)\right]^{\frac{1}{1-\varepsilon_I}} \tag{2.5}$$

where $P\left(s^t\right)$ is the consumer price index (CPI), $P_H\left(s^t\right)$ is the price index for domestic goods, and $P_F\left(s^t\right)$ is the import price index. Minimizing expenditure on the CES consumption basket, (2.4), with respect to the CPI, (2.5), gives rise to the following downward-sloping demand functions for domestic and imported aggregates

$$\check{C}_H\left(s^t\right) = (1-\alpha)\left[\frac{P_H\left(s^t\right)}{P\left(s^t\right)}\right]^{-\varepsilon_I} \check{C}\left(s^t\right) \tag{2.6}$$

$$\check{C}_F\left(s^t\right) = \alpha\left[\frac{P_F\left(s^t\right)}{P\left(s^t\right)}\right]^{-\varepsilon_I} \check{C}\left(s^t\right) \tag{2.7}$$

The imported good, $\check{C}_F\left(s^t\right)$, comprises of goods from each foreign country, j, and ε_F denotes the elasticity of substitution between imported goods

$$\check{C}_F\left(s^t\right) = \left[\int_0^1 \check{C}_j\left(s^t\right)^{\frac{\varepsilon_F - 1}{\varepsilon_F}} di\right]^{\frac{\varepsilon_F}{\varepsilon_F - 1}} \tag{2.8}$$

The subcomponents of the domestic and imported good indices measure domestic consumption of individual varieties of goods, i. These varieties are produced at home as well as in the continuum of foreign small open economies, $j \in [0,1]$, so that $\check{C}_H\left(i, s^t\right)$ is the consumption of domestic variety i and $\check{C}\left(i, s^t\right)$ is the consumption of variety i imported from country j

$$\check{C}_H\left(s^t\right) = \left[\int_0^1 \check{C}_H\left(i, s^t\right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \qquad \check{C}_j\left(s^t\right) = \left[\int_0^1 \check{C}_j\left(i, s^t\right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{2.9}$$

where, due to the assumption of identical market structures across countries, ε_p is the elasticity of substitution between individual varieties, i, produced in any country j, including home (Gali and Monacelli, 2005). The price indices associated with the demand for domestic and country j's varieties are

$$P_H\left(s^t\right) = \left[\int_0^1 \check{P}_H\left(i, s^t\right)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}} \qquad P_j\left(s^t\right) = \left[\int_0^1 \check{P}_j\left(i, s^t\right)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}} \tag{2.10}$$

2.2.1.2 Financially-Included Agents

Financially-included agents, $l \in [\lambda, 1]$, gain utility from consumption, $\hat{C}\left(s^t\right)$ and disutility from hours worked, $\hat{N}\left(s^t\right)$. These agents own the domestic firms, and have access to a complete portfolio of state-contingent bonds, which are traded sequentially in spot markets with foreign households in each period, $t \geq 0$, before s_{t+1} occurs (Gali and Monacelli, 2005). The equilibrium that arises from this assumption of sequential trading is equivalent to that which would arise from a time 0 trading of a complete set of history-contingent bonds. Notably, although financial trades are executed before uncertainty realizes, the period t allocation is chosen after event s_t occurs.

I let $B\left(s^t, s_{t+1}\right)$ denote the representative financially-included agent's holdings of a complete vector of state-contingent claims at time t, given history s^t, on time $t+1$ domestic *currency*. One unit of this vector pays off one unit of domestic currency if the particular state s_{t+1} occurs and 0 otherwise (Chari et al., 2002). $B\left(s^t, s_{t+1}\right)$ is denominated in units of domestic currency and each of its units is priced at $Z\left(s^{t+1}|s^t\right)$, where $s^{t+1} = \left(s^t, s_{t+1}\right)$. Taking $Z\left(s^{t+1}|s^t\right)$ as given, the representative household maximizes its utility, $\hat{U}\left(s^t\right) = \sum_{s^t} \mu(s^t) U\left\{\hat{C}\left(s^t\right), \hat{N}\left(s^t\right)\right\}$, subject to a sequence of budget constraints for $t \geq 0$, by solving the following dynamic optimization problem

$$\underset{\left\{\hat{C}(s^t), \hat{N}(s^t), B(s^t, s_{t+1})\right\}}{Max} \sum_{t=0}^{\infty} \beta^t \hat{U}(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} \mu(s^t) \beta^t \left\{\frac{\hat{C}\left(s^t\right)^{1-\sigma}}{1-\sigma} - \frac{\hat{N}\left(s^t\right)^{1+\phi}}{1+\phi}\right\} \tag{2.11}$$

$$s.t. \;\; P\left(s^t\right)\hat{C}\left(s^t\right) + \sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) B\left(s^t, s_{t+1}\right) \leq B\left(s^t\right) + W\left(s^t\right)\hat{N}\left(s^t\right) + \Omega\left(s^t\right) - T\left(s^t\right) \tag{2.12}$$

where $\beta^t \in [0, 1]$ is the subjective discount factor, $\hat{N}\left(s^t\right)$ represents hours of labour supplied, $\hat{C}\left(s^t\right)$ is consumption of domestically produced and imported goods from a CES consumption basket, $\Omega\left(s^t\right)$ denotes profits received from ownership of domestic firms, and $T\left(s^t\right)$ is a nominal lump-sum tax. These variables are denominated in domestic currency units, and the initial condition, $B(s_0)$, is taken as given. The budget constraint binds, and as standard in the optimal policy literature, the lump-sum tax is used to finance a constant wage subsidy, τ^e, to offset the steady state monopolistic distortion

$$T\left(s^t\right) = \tau^e W\left(s^t\right) N\left(s^t\right) \tag{2.13}$$

where $N\left(s^t\right)$ is labour demanded by the monopolistic producers. Each period, financially-included agents trade off leisure for consumption goods through their optimal labour supply condition, (2.14), derived by combining the first-order conditions on consumption and labour

$$\omega\left(s^t\right) = \hat{N}\left(s^t\right)^{\phi} \hat{C}\left(s^t\right)^{\sigma} \tag{2.14}$$

The first-order conditions on state-contingent bonds and consumption can be combined to yield the Euler Equation

$$Z\left(s^{t+1}|s^t\right) = \beta\mu\left(s^{t+1}|s^t\right)\left[\frac{\hat{C}\left(s^{t+1}\right)}{\hat{C}\left(s^t\right)}\right]^{-\sigma}\frac{1}{\Pi\left(s^{t+1}\right)} \tag{2.15}$$

where $\Pi\left(s^{t+1}\right) = \frac{P\left(s^{t+1}\right)}{P\left(s^t\right)}$ is CPI inflation. A similar equilibrium condition with respect to domestic currency state-contingent bonds, adjusted for the presence of the nominal bilateral exchange rate, $e^j\left(s^t\right)$, to ensure price equalization across the world in contingent claims, holds for the representative household in country j

$$Z\left(s^{t+1}|s^t\right) = \beta\mu\left(s^{t+1}|s^t\right)\left[\frac{C^j\left(s^{t+1}\right)}{C^j\left(s^t\right)}\right]^{-\sigma}\frac{e^j\left(s^{t+1}\right)}{e^j\left(s^t\right)}\frac{1}{\Pi^j\left(s^{t+1}\right)} \tag{2.16}$$

where $C^j\left(s^t\right)$, $\Pi^j\left(s^t\right)$, and $\varepsilon^j\left(s^t\right)$ denote the consumption basket, CPI inflation, and bilateral nominal exchange rate of country j.[5] Combining the Euler Equation of the domestic household, (2.15), with that of each foreign household $j \in [0,1]$, (2.16), and integrating across foreign households, $C^*\left(s^t\right) = \int_0^1 C_j\left(s^t\right)dj$, yields the Backus-Smith (Backus and Smith, 1993) international risk-sharing condition from the perspective of the domestic

[5]The representative household in country j faces the following budget constraint in each period $t \geq 0$: $P^j\left(s^t\right)C^j\left(s^t\right) + e^j\left(s^t\right)\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)B^j\left(s^t, s_{t+1}\right) \leq e^j\left(s^t\right)B^j\left(s^t\right) + W^j\left(s^t\right)N^j\left(s^t\right) + \Omega^j\left(s^t\right) - T^j\left(s^t\right)$, where $B^j\left(s^t\right)$ represents the foreign household's holdings of the state-contingent bond denominated in units of domestic currency and $e^j\left(s^t\right)$ is the bilateral nominal exchange rate with respect to the domestic economy ie. the price of domestic currency in units of foreign currency. $e^j\left(s^t\right)$ serves to convert the domestic currency payoffs into foreign currency (Chari et al., 2002).

small open economy

$$\hat{C}(s^t) = vC^*(s^t)Q(s^t)^{\frac{1}{\sigma}} \tag{2.17}$$

which assumes that agents across the world make appropriate ex-ante international insurance transfers through complete financial markets, to ensure that risk is pooled internationally, ie. $v = 1$, and that world consumption is exogenous. The value of the insurance transfers to the domestic economy is given in equation (2.38). International risk-sharing, (2.17), means that the marginal utility of consumption, weighted by the real effective exchange rate, $Q(s^t) = e(s^t)\frac{P^*(s^t)}{P(s^t)}$, is equalized across countries, where $P^*(s^t)$ is the world price index and $e(s^t)$ is the nominal effective exchange rate ie. an index of the prices of foreign currencies $j \in [0, 1]$ in terms of domestic currency.

Efficient risk-sharing implies that demand, by financially-included agents worldwide, is directed at countries where it is cheaper to consume. Risk-sharing is possible because of the timing of financial trades. Financial markets open before monetary policy decisions are made, implying that financially-included agents are able to smooth consumption in the face of uncertainty. Further, this pooling of risk with foreign agents insures financially-included agents in an open economy, compared to the closed economy case (without capital) where it is not possible to insure against aggregate uncertainty.

Asset market arbitrage opportunities do not exist, as these would lead to indeterminacy in the international portfolio allocation problem. This implies that the equilibrium prices - in domestic currency - of risk-free one-period uncontingent nominal bonds at home and in foreign country j, are related to their gross returns as follows

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) = \frac{1}{1+i\left(s^t\right)} \tag{2.18}$$

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) e_j\left(s^{t+1}\right) = \frac{e\left(s^t\right)}{1+i_j\left(s^t\right)} \tag{2.19}$$

where the domestic currency price of a domestic bond, $\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)$, is inversely related to the gross domestic nominal interest rate, $1 + i\left(s^t\right)$, and the domestic currency price of a foreign bond, $\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right) e_j(s^{t+1})$, is inversely related to the gross foreign nominal interest rate, $1 + i_j\left(s^t\right)$, adjusted by the bilateral nominal exchange rate. Domestic monetary policy has direct leverage over $i\left(s^t\right)$. Combining the domestic and foreign bond pricing equations, and aggregating across countries $j \in [0, 1]$, yields the uncovered interest rate parity (UIP) condition

$$\sum_{s_{t+1}} Z\left(s^{t+1}|s^t\right)\left\{(1+i\left(s^t\right)) - (1+i^*\left(s^t\right))\frac{e\left(s^{t+1}\right)}{e\left(s^t\right)}\right\} = 0 \tag{2.20}$$

where $i^*\left(s^t\right) = \int_0^1 i_j\left(s^t\right)dj$ is the world nominal interest rate, taken as given by the domestic economy. UIP implies that the nominal exchange rate is expected to adjust to equalize the domestic currency returns on domestic and foreign contingent bonds.

The consumption basket of the financially-included agents, $\hat{C}(s^t)$, is similar to (2.4), and is comprised of domestic and foreign aggregates, respectively $\hat{C}_H(s^t)$ and $\hat{C}_F(s^t)$

$$\hat{C}(s^t) = [(1-\alpha)^{\frac{1}{\varepsilon_I}}\hat{C}_H(s^t)^{\frac{\varepsilon_I-1}{\varepsilon_I}} + \alpha^{\frac{1}{\varepsilon_I}}\hat{C}_F(s^t)^{\frac{\varepsilon_I-1}{\varepsilon_I}}]^{\frac{\varepsilon_I}{\varepsilon_I-1}} \tag{2.21}$$

Minimizing expenditure on the CES consumption basket, (2.21), with respect to the CPI, (2.5), gives rise to the following downward-sloping demand functions for domestic and imported aggregates for financially-included agents

$$\hat{C}_H(s^t) = (1-\alpha)\left[\frac{P_H\left(s^t\right)}{P\left(s^t\right)}\right]^{-\varepsilon_I}\hat{C}(s^t) \tag{2.22}$$

$$\hat{C}_F(s^t) = \alpha\left[\frac{P_F\left(s^t\right)}{P\left(s^t\right)}\right]^{-\varepsilon_I}\hat{C}(s^t) \tag{2.23}$$

2.2.2 Relative Prices and Exchange Rates

The model equilibrium, following Ferrero and Seneca (2015), is defined in terms of the effective terms of trade, $X\left(s^t\right) = \frac{P_F\left(s^t\right)}{P_H\left(s^t\right)}$, which is an index of the bilateral terms of trade between the domestic economy and all foreign economies $j \in [0,1]$ (Gali and Monacelli, 2005). That is, $X\left(s^t\right) = \left(\int_0^\infty X_{jH}\left(s^t\right)^{1-\varepsilon_F}dj\right)^{\frac{1}{1-\varepsilon_F}}$, where $X_{jH}\left(s^t\right) = P_j\left(s^t\right)/P_H\left(s^t\right)$ denotes the price of country j's goods in terms of domestic goods.

To put prices in terms of $X\left(s^t\right)$, I first normalize all prices by the CPI, (2.5), to define them in relative terms. These are the relative domestic price index, $p_H\left(s^t\right) = P_H\left(s^t\right)/P\left(s^t\right)$, and the relative import price index, $p_F\left(s^t\right) = P_F\left(s^t\right)/P\left(s^t\right)$. As functions of the effective terms of trade, $X\left(s^t\right)$, these are

$$p_H\left(s^t\right)=\left[1-\alpha+\alpha X\left(s^t\right)^{1-\varepsilon_I}\right]^{-\frac{1}{1-\varepsilon_I}} \qquad p_F\left(s^t\right)=X\left(s^t\right)\left[1-\alpha+\alpha X\left(s^t\right)^{1-\varepsilon_I}\right]^{-\frac{1}{1-\varepsilon_I}} \tag{2.24}$$

The effective terms of trade, $X\left(s^t\right)$, is thus a ratio of relative prices

$$X\left(s^t\right)=\frac{p_F\left(s^t\right)}{p_H\left(s^t\right)} \tag{2.25}$$

The nominal effective exchange rate, $e\left(s^t\right) = \left(\int_0^1 e_j\left(s^t\right)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$, is an index of the nominal bilateral exchange rates among foreign countries, $j \in [0,1]$, and the domestic economy. The nominal exchange rate between home and any other country j, $\varepsilon_j\left(s^t\right)$, is the price of foreign currency, j, in units of domestic currency. $P^*\left(s^t\right) = \int_0^1 P^j\left(s^t\right) dj$ is the world price index, where $P^j\left(s^t\right)$ is the CPI in country j. The real effective exchange rate

$$Q\left(s^t\right)=\frac{e\left(s^t\right)P^*\left(s^t\right)}{P\left(s^t\right)}$$

is defined as the domestic currency price of a foreign basket of consumption, $e\left(s^t\right)P^*\left(s^t\right)$, relative to the domestic currency price of a domestic basket of consumption, $P\left(s^t\right)$. $Q\left(s^t\right) = \left(\int_0^1 Q_j\left(s^t\right)^{1-\varepsilon_F} dj\right)^{\frac{1}{1-\varepsilon_F}}$, where $Q_j\left(s^t\right)$ is the real bilateral exchange rate between home and country j. $Q\left(s^t\right)$ can be expressed in terms of $X\left(s^t\right)$. Assuming that law of one price holds in the imported goods market, so that $P_F\left(s^t\right) = e\left(s^t\right)P_F^*\left(s^t\right)$, and that from the perspective of the domestic economy, which is small, the world as a whole behaves like a closed economy, so that $P^*\left(s^t\right) = P_F^*\left(s^t\right)$, I derive

$$Q\left(s^t\right)=\left[(1-\alpha)X\left(s^t\right)^{\varepsilon_I-1}+\alpha\right]^{-\frac{1}{1-\varepsilon_I}} \tag{2.26}$$

Note that the real exchange rate is a function of the effective terms of trade in this framework, so that these two relative prices co-move upon shocks.

2.2.3 Firms

The firm's problem is standard in the literature. Firms, $i \in [0,1]$, are monopolistic and set prices in a staggered fashion. Every period, a fraction $(1-\theta)$ of (randomly selected) firms can re-optimize prices, as in Calvo (1983).[6] Fraction θ of firms cannot re-optimize and instead adjust labour demand to meet changes in output demand upon shocks. Firms that do reset prices upon shocks take into account that the probability of keeping the current period's price k periods ahead is given by θ^k.

With production function $Y(i,s^t) = A\left(s^t\right)N\left(i,s^t\right)$, each reoptimizing firm i sets its optimal reset price as a markup over current and expected marginal costs, where $MC\left(i,s^t\right) = W\left(s^t\right)/A\left(s^t\right)$, giving rise to domestic inflation. Noting that a firm that reoptimizes in period t will choose the price $P_H^*\left(i,s^t\right)$ that maximizes current and future expected discounted profits until period $t+k$ while this price remains effective, so that $P_H^*\left(i,s^{t+k}\right) = P_H^*\left(i,s^t\right)$ for $k = 0,...,\infty$, the optimal reset price at time t solves the following problem (Chari et al., 2002)

$$\underset{P_H^*(i,s^t)}{Max}\sum_{k=0}^{\infty}\sum_{s^{t+k}} Z\left(s^{t+k}|s^t\right)\theta^k\left\{(1+\tau^e)P_H^*\left(i,s^t\right)Y\left(i,s^{t+k}\right) - W\left(s^{t+k}\right)N\left(i,s^{t+k}\right)\right\}$$

$$s.t.\;\; Y\left(i,s^{t+k}\right) = \left(\frac{P_H^*\left(i,s^t\right)}{P_H\left(s^t\right)}\right)^{-\varepsilon_p}\left(C_H\left(i,s^{t+k}\right) + \int_0^1 C_H^j\left(i,s^{t+k}\right)dj\right) \tag{2.27}$$

where $Z\left(s^{t+k}|s^t\right)$ is the stochastic discount factor (as households own the firms) in period $t+k$ given history s^t (recall that $s^{t+k} = \left(s^t, s_{t+k}\right)$), τ^e is a steady state wage subsidy, $Y\left(i,s^{t+k}\right)$ and $W\left(s^{t+k}\right)N\left(i,s^{t+k}\right)$ are respectively the output and total cost in period $t+k$ for a firm that last reset its price in period t, and $C_H\left(i,s^{t+k}\right)$ and $\int_0^1 C_H^j\left(i,s^{t+k}\right)dj$ represent demand for good i in period $t+k$ respectively by domestic consumers and foreign consumers in countries j. In a symmetric equilibrium, found in Chari et al. (2002) and Galí (2007), the same price is chosen by all firms that can re-optimize so that $P_H^*\left(i,s^t\right) = P_H^*\left(s^t\right)\forall i$. The first-order condition is

$$\sum_{k=0}^{\infty}\sum_{s^{t+k}} Z\left(s^{t+k}|s^t\right)\theta^k Y\left(i,s^{t+k}\right)\left[P_H^*\left(s^t\right) - \frac{1}{1+\tau^e}\frac{\varepsilon_P}{\varepsilon_P - 1}\frac{W\left(s^{t+k}\right)}{A\left(s^{t+k}\right)}\right] = 0 \tag{2.28}$$

where productivity shocks, $A\left(s^t\right)$, are determined relative to their steady state value A and follow the following stationary autoregressive process

$$Ln\left(1+A\left(s^t\right)\right) - Ln(1+A) = \rho_A Ln\left(1+A\left(s^{t-1}\right)\right) - Ln(1+A) + s_t^A$$

[6] This setup assumes that agents have preferences over intermediate goods. A mathematically equivalent approach is that agents have preferences over final goods, with the final goods being produced from the intermediate goods Galí (2007).

with $\rho_A \in (0,1)$ and $s_t^A \sim N(0,\sigma_A^2)$. The optimal reset price relates to the domestic inflation rate as follows

$$\frac{P_H^*\left(s^t\right)}{P_H\left(s^t\right)} = \left(\frac{1-\theta\Pi_H\left(s^t\right)^{\varepsilon_P-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon_P}} \tag{2.29}$$

where $\Pi_H\left(s^t\right) = \frac{P_H\left(s^t\right)}{P_H\left(s^{t-1}\right)}$.[7] Combining, as in Ferrero and Seneca (2015), the constraint (3.25), the labor-market clearing condition $N_t = \int_0^1 N\left(i,s^t\right)di$, the price index associated with monopolistic goods $P_H\left(s^t\right) = \left[\int_0^1 P_H\left(i,s^t\right)^{1-\varepsilon_p}di\right]^{\frac{1}{1-\varepsilon_P}}$, and the definition of price dispersion, $\Delta\left(s^t\right) \equiv \int_0^1\left(\frac{P_H\left(i,s^t\right)}{P_H\left(s^t\right)}\right)^{-\varepsilon_p}di$, which follows law of motion

$$\Delta\left(s^t\right)^{\frac{\varepsilon_P-1}{\varepsilon_P}} = \theta\pi_H\left(s^t\right)^{\varepsilon_P-1}\Delta\left(s^{t-1}\right) + (1-\theta)\frac{1-\theta\Pi_H\left(s^t\right)^{\varepsilon_P-1}}{1-\theta} \tag{2.30}$$

the aggregate production function, (2.31), is

$$Y\left(s^t\right)\Delta\left(s^t\right) = A\left(s^t\right)N\left(s^t\right) \tag{2.31}$$

The monopolistic sector faces exogenous cost-push shocks which directly increase domestic inflation without excess aggregate demand pressures. These disturbances represent unanticipated food price or fuel input price shocks that arise in markets outside the authority's control. As standard in the literature, and discussed further in Sutherland (2005), $V\left(s^t\right)$ is the net monopolistic markup, $\frac{\varepsilon_P}{\varepsilon_P-1}\frac{1}{1+\tau^e}$. Markup shocks, s_t^V, are assumed to arise from random changes in the production subsidy or the degree of monopoly power. Cost-push/ markup shocks are determined relative to their steady state value V and follow the following stationary autoregressive process

$$Ln\left(1+V\left(s^t\right)\right) - Ln(1+V) = \rho_V Ln\left(1+V\left(s^{t-1}\right)\right) - Ln(1+V) + s_t^V$$

where $\rho_V \in (0,1)$ and $s_t^V \sim N(0,\sigma_V^2)$.

[7]The linearized version of the optimal price-setting rule, (2.28), is a standard result and can be found in chapter 3 of Galí (2007): $\pi_{H,t} = \beta\pi_{H,t+1} + \xi mc_t + v_t$, where mc_t is marginal cost, $\xi = \frac{(1-\beta\theta)(1-\theta)}{\theta}$, and v_t is a cost-push shock that is appended on the Phillips Curve, as standard in the literature.

2.2.4 Central Bank

Monetary policy is set according to either simple rules or an optimal targeting rule. The strict simple rules (in linearized terms) are domestic inflation targeting (DIT), $\pi_H\left(s^t\right) = 0$, CPI inflation targeting (DIT), $\pi\left(s^t\right) = 0$, and a fixed exchange rate, $\Delta e\left(s^t\right) = 0$, and the flexible ones are Taylor-type rules where the nominal interest rate responds to a measure of inflation and the output gap.

2.2.5 Market-Clearing and Accounting

The demand for each monopolistic good $i \in [0,1]$, is

$$Y_t\left(i, s^t\right) = C_H\left(i, s^t\right) + \int_0^1 C_H^j\left(i, s^t\right) dj$$

where $C_H\left(i, s^t\right)$ is consumption of home good i by domestic consumers and $C_H^j\left(i, s^t\right)$ denotes consumption of home good i by country j. Replacing in the expressions for the consumption of individual varieties, i, international risk-sharing, and the definitions of the bilateral and effective terms of trade

$$Y\left(s^t\right) = \left(\frac{P_H\left(s^t\right)}{P\left(s^t\right)}\right)^{-\varepsilon_I}\left[(1-\alpha)C\left(s^t\right) + \alpha\hat{C}\left(s^t\right)\int_0^1 Q_j\left(s^t\right)^{\varepsilon_I - \frac{1}{\sigma}}\left(X_j\left(s^t\right)X^j\left(s^t\right)\right)^{\varepsilon_F - \varepsilon_I} dj\right] \tag{2.32}$$

where $X_j\left(s^t\right)$ and $X_j\left(s^t\right)$ denote bilateral variables for the domestic economy, and $X^j\left(s^t\right)$ denotes the effective terms of trade for country j. The labour market is Walrasian, with the real wage, $w\left(s^t\right)$, moving instantly to clear demand and supply imbalances

$$\lambda\check{N}\left(s^t\right) + (1-\lambda)\hat{N}\left(s^t\right) = N\left(s^t\right) = \int_0^1 N\left(i, s^t\right) di \tag{2.33}$$

where $\int_0^1 N_t\left(i, s^t\right))di$ is the demand for labour by each firm, i. Aggregate consumption as in Galí et al. (2007) and Bilbiie (2008) is a weighted average, with weights given by λ, of consumption by financially-included and financially-excluded agents

$$C\left(s^t\right) = \lambda\check{C}\left(s^t\right) + (1-\lambda)\hat{C}\left(s^t\right) \tag{2.34}$$

Aggregate consumption of domestic and imported foreign aggregates is likewise

$$\begin{aligned} C_H\left(s^t\right) &= \lambda\check{C}_H\left(s^t\right) + (1-\lambda)\hat{C}_H\left(s^t\right) && (2.35) \\ C_F\left(s^t\right) &= \lambda\check{C}_F\left(s^t\right) + (1-\lambda)\hat{C}_F\left(s^t\right) && (2.36) \end{aligned}$$

The real trade balance, in terms of the CPI, $P\left(s^t\right)$, is defined as the difference between domestic production and consumption, and is given by

$$NX\left(s^t\right) = p_H\left(s^t\right) Y\left(s^t\right) - \left(\lambda\check{C}\left(s^t\right) + (1-\lambda)\hat{C}\left(s^t\right)\right) \quad (2.37)$$

The trade balance is non-zero, unlike Gali and Monacelli (2005), which is nested in (2.37) when $\lambda = 0$ in this framework. This point is independent of calibration, but is seen more easily for $\sigma = \varepsilon_I = \varepsilon_F = 1$, as in the welfare analysis, so that $NX\left(s^t\right)$ boils down to $NX\left(s^t\right) = \alpha\lambda\left(\hat{C}\left(s^t\right) - \check{C}\left(s^t\right)\right)$ upon replacing out for (2.32) in (2.37). The fact that $NX\left(s^t\right)$ is away from zero is driven by $\lambda > 0$, ie. a positive degree of financial exclusion, and has implications for the domestic net foreign asset position.

Net foreign assets, $NFA\left(s^t\right)$ are non-zero upon shocks (they are zero in the initial steady state), and quantify the net present value of insurance transfers to the domestic financially-included households when uncertainty realizes. In each period, the stock of $NFA\left(s^t\right)$, normalized by the CPI, $P\left(s^t\right)$, derived by iterating the financially included household's budget constraint, is given by

$$\begin{aligned} NFA\left(s^t\right) = & -\sum_{k=0}^{\infty}\sum_{s^{t+k}} Z\left(s^{t+k}|s^t\right)\Big[NX\left(s^{t+k}\right) && (2.38) \\ & +\left(C\left(s^{t+k}\right) - \hat{C}\left(s^{t+k}\right)\right) - w\left(s^{t+k}\right)\left(N\left(s^{t+k}\right) - \hat{N}\left(s^{t+k}\right)\right)\Big] \end{aligned}$$

Equation (2.38) is a useful summary of the difference between closed economies with financially-excluded agents, for example, Galí et al. (2007) and Bilbiie (2008), and an open economy. It arises because of (i) international risk-sharing and the associated insurance transfers, possible only in an open economy and (ii) the fact that the trade balance is not zero with $\lambda > 0$. If $\lambda = 0$, then $NFA\left(s^t\right) = 0$ in each period, as in Gali and Monacelli (2005), and the open economy model is isomorphic to the closed case.

In the current framework, however, as only a fraction of domestic agents can share risk with foreign agents, open economy elements do not affect the two types of agents in a symmetric fashion. In particular, financially-included agents can share risk and smooth consumption upon exchange rate fluctuations, whereas financially-excluded agents cannot. This fundamental asymmetry, connected with international risk-sharing, is why $NFA\left(s^t\right) \neq 0$ and optimal policy in an open economy model with financially-excluded agents is not isomorphic to the closed economy case.

2.2.6 Equilibrium

For a particular specification of monetary policy (which determines the nominal interest rate, $i\left(s^t\right)$), an equilibrium for the model is a state-contingent sequence of prices

$$\{X\left(s^t\right), Z\left(s^{t+1}|s^t\right), \Pi_H\left(s^t\right), \Pi\left(s^t\right), MC\left(s^t\right), \Delta\left(s^t\right), e\left(s^t\right)\}_{t=0}^{\infty}$$

and quantities

$$\{\check{C}\left(s^t\right), \check{C}_H\left(s^t\right), \check{C}_F\left(s^t\right), \hat{C}\left(s^t\right), \hat{C}_H\left(s^t\right), \hat{C}_F\left(s^t\right),$$
$$\check{N}\left(s^t\right), \hat{N}\left(s^t\right), N\left(s^t\right), C\left(s^t\right), C_H\left(s^t\right), C_F\left(s^t\right), Y\left(s^t\right)\}_{t=0}^{\infty}$$

such that

- Financially-excluded agents optimize: (2.3), (2.4), (2.6), and (2.7)
- Financially-included agents optimize: (2.14), (2.15), (2.21), (2.22), and (2.23)
- International-risk sharing and no-arbitrage conditions hold: (2.17), (2.18), and (2.20)
- Aggregate consumption is given by (2.34), (2.35), and (2.36)
- Goods, (2.32), and labour, (2.33), markets clear
- Firms optimize: (2.29), (2.30), and (2.31)

taking as given initial conditions, $B\left(s^0\right)$, $\Delta\left(s^0\right)$, $X\left(s^0\right)$, and exogenous processes for shocks and foreign variables $\{A\left(s^t\right), V\left(s^t\right), i^*\left(s^t\right), P^*\left(s^t\right)\}_{t=0}^{\infty}$. The effective terms of trade, $X\left(s^t\right)$, given by (2.25), is the only relative price that matters for the characterization of equilibrium.

2.3 Optimal Monetary Policy

I proceed to characterize the optimal monetary policy. I provide a generalized analysis of the optimum, before specifically analyzing cost-push shocks and ranking simple rules. Some salient results emerge. These include the insurance role played by optimal monetary policy in the presence of nominal rigidities and financial exclusion, and the desirability of stabilizing the nominal exchange rate in these circumstances. I focus on the Cole-Obstfeld parameterization (Cole and Obstfeld, 1991), $\sigma = \varepsilon_I = \varepsilon_F = 1$, to keep the analysis tractable. Two flexible price allocations are characterized, to serve as references, before turning to the constrained efficient case where monetary policy plays a role.

2.3.1 Flexible Prices

I begin by describing the efficient allocation of economic resources in the absence of market imperfections. This is the allocation away from which optimal monetary policy, in the presence of real and nominal rigidities, seeks to minimize deviations (Gali and Monacelli, 2005). The efficient allocation corresponds to the solution of a Planner's problem, that in this study maximizes a weighted sum of household utilities and faces the flexible price and perfectly competitive versions of the constraints in the optimal monetary policy problem. The efficient allocation can be decentralized through a wage subsidy, funded through lump-sum taxes on the financially-included household, (2.13).

I will show that the efficient allocation does not coincide with the natural allocation, flexible price version of the model, sometimes called the natural allocation, implying that the flexible price business cycle and steady state of the model are inefficient. I choose an appropriate wage subsidy, τ^e, to implement the efficient allocation at the steady state. To derive the efficient allocation, the Planner maximizes a weighted sum of household utilities, $U\left(s^t\right) = \lambda \check{U}\left(s^t\right) + (1-\lambda)\hat{U}\left(s^t\right)$, with weights given by λ.

The optimization problem is below. The first two constraints in the optimization problem are aggregate consumption and labour, the third is the production function, the fourth and fifth are the optimal labour supply conditions of the financially-excluded and financially-included agents respectively, the sixth is the financially-excluded agents' consumption function through their budget constraint, the seventh is the international risk-sharing condition of financially-included agents where exogenous world consumption is constant, and the eighth is the goods-market clearing condition. $\chi_1\left(s^t\right) - \chi_8\left(s^t\right)$

are Lagrange multipliers attached to the constraints.

$$\underset{\Sigma(s^t)}{Max}\, U\left(s^t\right) = \lambda\left(Ln\check{C}^e\left(s^t\right) - \frac{\check{N}^e\left(s^t\right)^{1+\phi}}{1+\phi}\right) + (1-\lambda)\left(Ln\hat{C}^e\left(s^t\right) - \frac{\hat{N}^e\left(s^t\right)^{1+\phi}}{1+\phi}\right)$$

$$\begin{array}{llrcl}
s.t. & \left[\chi_1\left(s^t\right)\right] & C^e\left(s^t\right) & = & \lambda\check{C}^e\left(s^t\right) + (1-\lambda)\hat{C}^e\left(s^t\right) \\
 & \left[\chi_2\left(s^t\right)\right] & N^e\left(s^t\right) & = & \lambda\check{N}^e\left(s^t\right) + (1-\lambda)\hat{N}^e\left(s^t\right) \\
 & \left[\chi_3\left(s^t\right)\right] & Y^e\left(s^t\right) & = & A\left(s^t\right)N^e\left(s^t\right) \\
 & \left[\chi_4\left(s^t\right)\right] & w^e\left(s^t\right) & = & \check{C}^e\left(s^t\right)\check{N}^e\left(s^t\right)^{\phi} \\
 & \left[\chi_5\left(s^t\right)\right] & w^e\left(s^t\right) & = & \hat{C}^e\left(s^t\right)\hat{N}^e\left(s^t\right)^{\phi} \\
 & \left[\chi_6\left(s^t\right)\right] & \check{C}^e\left(s^t\right) & = & w^e\left(s^t\right)\check{N}^e\left(s^t\right) \\
 & \left[\chi_7\left(s^t\right)\right] & \hat{C}^e\left(s^t\right) & = & X^e\left(s^t\right)^{1-\alpha} \\
 & \left[\chi_8\left(s^t\right)\right] & Y^e\left(s^t\right) & = & \alpha X^e\left(s^t\right)^{\alpha}\hat{C}^e\left(s^t\right) + (1-\alpha)X^e\left(s^t\right)^{\alpha}C^e\left(s^t\right)
\end{array}$$

where $\Sigma\left(s^t\right) = \left\{\hat{C}^e\left(s^t\right), \check{C}^e\left(s^t\right), C^e\left(s^t\right), \hat{N}^e\left(s^t\right), \check{N}^e\left(s^t\right), N^e\left(s^t\right), \omega^e\left(s^t\right), Y^e\left(s^t\right), X^e\left(s^t\right)\right\}$.

The optimum derives as a non-linear system of seventeen endogenous variables, which solve an equilibrium with nine first-order conditions and eight constraints. Further to some manipulations, it is possible to summarize the equilibrium in a more compact system of three equations with three unknowns: $X^e\left(s^t\right)$, $\hat{N}^e\left(s^t\right)$, and the shadow value of the resource constraint, $\chi_8\left(s^t\right)$.

$$\begin{array}{rcl}
X^e\left(s^t\right) - A\left(s^t\right)\hat{N}^{e-\phi}\left(s^t\right) & = & 0 \\
\dfrac{1}{X^e\left(s^t\right)} - \chi_8\left(s^t\right)\left(\lambda\hat{N}^e\left(s^t\right)^{\phi} + (1-\lambda) + \dfrac{\alpha}{1-\alpha}\right) & = & 0 \\
\dfrac{\lambda\phi\hat{N}^e\left(s^t\right)^{-1} - (1-\lambda)\hat{N}^e\left(s^t\right)^{\phi} - \chi_8\left(s^t\right)A\left(s^t\right)(1-\lambda)}{X^e\left(s^t\right)(1-\alpha)\phi\lambda} - \chi_8\left(s^t\right) & = & 0
\end{array}$$

Proposition 1 gives the closed-form expressions for the efficient labour allocation, $N^e\left(s^t\right)$, the financially-included agent's consumption, $\hat{C}^e\left(s^t\right)$, and the financially-excluded agent's consumption, $\check{C}^e\left(s^t\right)$.

Proposition 1. (Efficient Allocation) *In the first-best allocation for the economy when prices are flexible and firms operate in perfect competition, employment is given by*

$$\lambda\phi\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{-1}-(1-\lambda)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{\phi}$$
$$=\frac{(1-\alpha)\phi\lambda-(1-\lambda)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{\phi}}{\lambda\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{\phi}+(1-\lambda)+\frac{\alpha}{1-\alpha}} \tag{2.39}$$

and financially-included and financially-excluded consumption are functions of aggregate employment

$$\begin{aligned}\check{C}^e\left(s^t\right) &= A\left(s^t\right)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{\phi\alpha}\\ \hat{C}^e\left(s^t\right) &= A\left(s^t\right)\left(\frac{N^e\left(s^t\right)}{1-\lambda}-\frac{\lambda}{1-\lambda}\right)^{-\phi(1-\alpha)}\end{aligned}$$

Closed-form solutions for the other endogenous variables, as well as the Lagrange multipliers, are backed out from the remaining constraints and optimality conditions.

The solution to the Planner's problem with heterogeneous agents, when $\lambda = 0$, nests Gali and Monacelli (2005), where $N^e\left(s^t\right) = (1-\alpha)^{\frac{1}{1+\phi}}$. Furthermore, note that the consumption of financially-included and financially-excluded agents differ as only $1-\lambda$ fraction of the economy shares risk internationally. This is in contrast to the closed-economy case of Bilbiie (2008), where the consumption of both agents are limited to be identical in the flexible price allocations. The competitive allocation for the flexible price economy derives from the definition of equilibrium provided in the previous section, and the solutions are provided in Proposition 2.

Proposition 2. (Natural Allocation) *In the second-best allocation with flexible prices and monopolistic competition, employment is*

$$N^n\left(s^t\right)^{\phi}\left(N^n\left(s^t\right)-(1-\alpha)\lambda\frac{\varepsilon_P-1}{\varepsilon_P}\right)=\frac{\varepsilon_P-1}{\varepsilon_P}(1-\lambda)^{\phi}\left((1-\alpha)(1-\lambda)+\alpha\right) \tag{2.40}$$

Other variables are backed out in closed-form from the decentralized equilibrium conditions. The subsidy τ^e that restores steady state efficiency is of size $\tau^e = 1-\frac{\varepsilon_P-1}{\varepsilon_P}/(1-\alpha)$.

Monetary policy does not play a stabilization role in either of the flexible price allocations since prices can instantly jump. When prices are sticky, however, the relative price distortions that arise due to sluggish real adjustment justify policy intervention. Upon trade-off creating shocks, welfare with sticky prices is strictly lower than with flexible prices, and for the purposes of comparison, it will be useful to express the allocation with nominal rigidities in log-deviations, $\tilde{x}_t$, from the flexible price reference allocations.

2.3.2 Sticky Prices

It is convenient to work with a linearized version of the model hereafter, as the non-linear equilibrium is a complicated system of stochastic difference equations. Computing analytical solutions to the non-linear problem with nominal rigidities becomes intractable. This approach is also followed by much of the analytical literature on optimal monetary policy, for eg. Gali and Monacelli (2005) and Farhi and Werning (2012). I characterize the constrained efficient allocation away from a symmetric deterministic steady state. This is done by taking a linear approximation of the constraints and a quadratic approximation of the welfare loss function. The optimal targeting rule that results is internally consistent and a locally linear approximation of the non-linear optimal policy. These perturbation techniques are described further in Benigno and Woodford (2012).

2.3.2.1 Constraints

The constraints in the Central Bank optimization problem are the first-order linear approximations, in log-deviation or "gap" terms, of the equilibrium conditions defined in section 2.2.6. A linearized variable, $f\left(s^t\right)$, is related to its non-linear value and steady state, $F\left(s^t\right)$ and F, approximately as $f\left(s^t\right) \approx \frac{F\left(s^t\right)-F}{F}$.[8] A variable in log-deviation terms, $\tilde{f}\left(s^t\right)$ is defined as the sticky price linearized variable, $f\left(s^t\right)$, in deviation from the efficient linearized variable, $f\left(s^t\right)^e$, so that $\tilde{f}\left(s^t\right) = f\left(s^t\right) - f\left(s^t\right)^e$. Note that each linearized variable is measured in deviations from its *efficient* version, since the first-best case is the relevant welfare benchmark. Henceforth, I refer to $\tilde{f}\left(s^t\right)$ as an "efficient gap". I also use $\tilde{f}\left(s^t\right) \equiv \tilde{f}_t$, and the expectations operator, $E_t\left\{.\right\}$, to save on notation.

[8] At the symmetric and efficient steady state with zero inflation, shocks $\{A, V, C^*, i^*\}$ are normalized to 1. Quantities and relative prices are endogenous. As financial assets and profits are zero in the steady state, the budget constraints of both households coincide. Then due to the same functional form for preferences, $C = \hat{C} = \check{C}$ and $N = \hat{N} = \check{N}$. It is possible to derive that $X = 1$ by combining the resource constraint, $Y = CX^{1-\alpha}$, risk-sharing condition, $C = X^{1-\alpha}$, production function, $Y = N$, marginal cost condition, $1 = wX^{\alpha}$, and labour supply condition, $w = CN^{\phi}$. The remaining variables are directly backed out.

It is possible to summarize the constraints in demand (dynamic IS Equation) and supply blocks (New Keynesian Phillips Curve, or NKPC). Notably, in the presence of financially-excluded agents, it is still possible to derive a condition that resembles the canonical IS Equation for the small open economy. This is done by substituting the resource constraint, (2.32), into the aggregate Euler Equation. The latter combines the consumption of financially-excluded agents, (2.2), and the intertemporal optimality condition of financially-included agents, (2.15), so thus summarizes the joint consumption evolution of the heterogeneous households in the model

$$\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\tilde{y}_t - \alpha\tilde{x}_t = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}E_t\tilde{y}_{t+1} - \alpha E_t\tilde{x}_{t+1} - (i_t - E_t\pi_{t+1})$$

The Phillips Curve is derived by combining the evolution of domestic inflation resulting from firm optimization, $\pi_{Ht} = \beta\pi_{Ht+1} + \xi\tilde{mc}_t$, where $\xi = \frac{(1-\beta\theta)(1-\theta)}{\theta}$, and $\tilde{mc}_t = \frac{\phi}{1-\lambda}\tilde{y}_t + \tilde{x}_t$, which is a version of the marginal cost condition derived by replacing the resource constraint, (2.32), risk-sharing condition (2.17), financially-excluded consumption, (2.2), and the optimal labour supply conditions, (2.3) and (2.14), into real marginal costs given by $\tilde{mc}_t = \tilde{\omega}_t + \alpha\tilde{x}_t$. In the NKPC, I also make use of the relation between the terms of trade and output, $\tilde{x}_t = \Upsilon\tilde{y}_t$, which is derived by combining the resource constraint with risk-sharing. Proposition 3 formalizes the constraints in the optimal policy problem.

Proposition 3. (Canonical Equilibrium) *For a particular specification of monetary policy, the equilibrium with inflexible prices is*

$$\begin{aligned}
\tilde{y}_t &= E_t\tilde{y}_{t+1} - \frac{1}{\Upsilon}\left(i_t - E_t\pi_{Ht+1} - r_t^e\right) && (2.41)\\
r_t^e &= -\Upsilon\left(1-\rho_a\right)a_t && (2.42)\\
\pi_{H,t} &= \beta E_t\pi_{H,t+1} + \frac{\xi}{\Lambda}\tilde{y}_t + v_t && (2.43)
\end{aligned}$$

where $\Upsilon = \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}$, $\xi = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\Lambda = \frac{1-\lambda}{1-\lambda-\lambda\phi(1-\alpha)+\phi}$, v_t *is an exogenous cost-push shock in the NKPC, and* r_t^e *is the equilibrium efficient rate of interest.*

These constraints nest Gali and Monacelli (2005) when $\lambda = 0$. Note, however, that α enters the canonical equilibrium when $\lambda > 0$. In the presence of financial exclusion, openness, α (or conversely home bias, $1-\alpha$), plays an explicit role in linking inflation and output. As financially-excluded agents cannot smooth consumption, they are increasingly adversely affected by exchange rate volatility as the economy opens up.

The property of Divine Coincidence (Blanchard and Galí, 2007) is present in the model, implying that the planner can simultaneously stabilize domestic inflation and the *efficient* output gap. In the absence of cost-push shocks, setting $\pi_{H,t} = 0$ closes the *natural* output gap, $y_t - y_t^n$, when $\tilde{f}_t = f_t - f_t^n$ in Proposition 3. But natural and efficient output are scalar functions of each other, so that strict domestic inflation targeting (DIT), $\pi_{H,t} = 0$, is able to achieve the first-best allocation of $\pi_{H,t} = y_t - y_t^e = 0$ in the absence of cost-push shocks. This has implications for the optimal monetary policy response to productivity shocks, as discussed further in Section 2.4.2. Productivity shocks in these types of models do not create any meaningful trade-offs, unlike cost-push shocks that are typically considered for the optimal design of monetary policy (Clarida et al., 2001).

The equilibrium is determinate as long as the optimal targeting rule, (2.49), is implemented. Optimal policy is the focus of this study. For ad-hoc rules, the equilibrium is indeterminate for high values of λ, as is typical of DSGE models with hand-to-mouth agents, for instance in Galí et al. (2007).[9] This threshold is given by $\lambda < \lambda^* = \frac{1}{1+\phi(1-\alpha)}$, a result shown in Bilbiie (2008) and Boerma (2014). However, even with ad-hoc rules, it is possible to have λ^* as high as 0.8 with a standard range of calibrations for ϕ and α, so that the threshold value does not affect any of the results. The percentage of financially excluded agents, λ, in most emerging market economies, is less than 80%.

2.3.2.2 Central Bank Loss Function

The objective of optimal monetary policy is to maximize the expected utility of households. The Central Bank loss function is a second-order approximation of expected utility, as described in Benigno and Woodford (2012) who show how to derive an approximation to household welfare that takes the form of a discounted quadratic loss function with terms including those in inflation and the output gap. The advantage of using this method to micro-found the monetary policy objective is that is affords an internally consistent and precise characterization of which terms appear in the loss function, with relative weights that are contingent on the specific distortions and monetary transmission

[9] When $\lambda < \lambda^*$, the elasticity of output with respect to the nominal interest rate, $-\frac{1}{Y}$, from the IS Equation, (2.41), is negative so that, ceteris paribus, contractionary monetary policy would lead to a real contraction as one would expect. However, when $\lambda \geq \lambda^*$, the interest elasticity of output is positive, ie. $-\frac{1}{Y} > 0$, which leads to indeterminacy (discussed further in Bilbiie, 2008).

mechanism in the model considered.

In this paper, I extend the closed-economy, representative agent approach of Benigno and Woodford (2012) to an open economy, heterogeneous agent setting. The Central Bank maximizes aggregate welfare, W_t, through the *weighted sum* of household expected utilities, as in Bilbiie (2008), with the weight, λ, depending on the degree of financial exclusion

$$W_t = E_t \sum_{k=0}^{\infty} \beta^k \left\{ \lambda \check{U}_{t+k} + (1-\lambda) \hat{U}_{t+k} \right\} \tag{2.44}$$

A second-order approximation of $\lambda \check{U}_{t+k} + (1-\lambda)\hat{U}_{t+k}$ initially yields some non-zero linear terms, where $t.i.p.$ stand for "terms independent of policy" (constants and functions of disturbances), and $o(3)$ contains terms of third-order and higher (to place a bound on the amplitude of the perturbations)

$$\mathscr{L}_t = -E_t \sum_{k=0}^{\infty} \beta^k \left\{ \frac{1}{2} \frac{\varepsilon}{\xi} \pi^2_{H,t+k} + \frac{U_N}{U_C} \frac{N}{C} \tilde{y}_{t+k} + \tilde{c}_{t+k} \right\} + t.i.p. + o(3)$$

To be able to evaluate optimal policy upto second-order, the linear terms in $\tilde{y}_{t+k}$ and $\tilde{c}_{t+k}$ need to be eliminated. I follow the method of Benigno and Woodford (2005) to eliminate $\tilde{y}_{t+k}$ and $\tilde{c}_{t+k}$ through the analytical approach of replacing for these terms through second-order approximations of the following non-linear equilibrium conditions: the consumption of financially-excluded agents, (2.2), labour supply by both agents, (2.3) and (2.14), risk-sharing, (2.17), evolution of price dispersion, (2.30), the aggregate production function, (2.31), the optimal price-setting equation, (2.28), the resource constraint, (2.32), and finally aggregate labour and consumption, (2.33) and (2.34). The resulting loss function is an expression with *purely quadratic* terms, as described in Proposition 4.

Proposition 4. (Welfare Loss Function) *Central Bank preferences in a small open economy with financially-excluded agents, with $\tilde{x}_t = x_t - x_t^e$ and $\tilde{y}_t = y_t - y_t^e$, are represented as*

$$\mathscr{L}_t = -\frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k \left\{ \Psi_\pi \pi_{H,t+k}^2 + \Psi_y \tilde{y}_{t+k}^2 + \Psi_x \tilde{x}_{t+k}^2 \right\} \tag{2.45}$$

where the weights on domestic inflation, output gap, and terms of trade gap, are

$$\begin{aligned}
\Psi_\pi &= \frac{\varepsilon_P}{\xi} \left(\varXi \varLambda \left(1+\nu\right) - \left(1 + (1-\alpha)\varPhi\right) \nu\lambda \right) - \frac{\varepsilon_P}{\xi} \frac{U_N}{U_C} \frac{N}{C} \\
\Psi_y &= \lambda(1-\alpha)^2\nu^2 - (1-\alpha) - \lambda(1-\lambda)\nu^2 - \varXi\varLambda\lambda(1-\alpha)\nu^2 \\
&\quad + \varXi \left(\varLambda^{-1} + \varLambda \right) - \frac{1+\phi}{1-\lambda} \frac{U_N}{U_C} \frac{N}{C} \\
\Psi_x &= (1-\alpha)\left(1-2\alpha\right) + 2(1-\alpha)\omega\varUpsilon - \varXi\varLambda\left(1-2\alpha\right) - 2\varXi\varLambda\omega\varUpsilon
\end{aligned}$$

and the composite parameters are

$$\begin{aligned}
\varUpsilon &= \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda} \\
\varLambda &= \frac{1-\lambda}{1-\lambda-\lambda\phi(1-\alpha)+\phi} \\
\omega &= \frac{\lambda\phi(1-\alpha)^2 + \alpha(1-\lambda)}{1-\lambda} \\
\nu &= \frac{\phi}{1-\lambda} \\
\varXi &= (1-\alpha)\varUpsilon + \lambda\nu + \frac{U_N}{U_C}\frac{N}{C} \\
\varPhi &= \varXi\varLambda - (1-\alpha)
\end{aligned}$$

with the efficient variables x_t^e and y_t^e as functions of structural parameters and shocks.

The loss function in an open economy model with financial exclusion features a quadratic term in the terms of trade beyond what is usually found in models where all households are financially-included. When $\lambda = 0$, the criterion collapses to that in Gali and Monacelli (2005) where $\tilde{x}_t^2 = \tilde{y}_t^2$: $\Psi_\pi|_{\lambda=0} = \Psi_\pi^{GM} = \frac{\varepsilon_P}{\xi}\frac{U_N}{U_C}\frac{N}{C}$ and $\Psi_y|_{\lambda=0} + \Psi_x|_{\lambda=0} = \Psi_y^{GM} = (1+\phi)\frac{U_N}{U_C}\frac{N}{C}$. I give the loss function below with $\lambda = 0$, ie. the nested Gali and Monacelli (2005) case, as this serves as a useful point of comparison for some of the results. (2.46) requires that $\frac{U_N}{U_C}\frac{N}{C} = -(1-\alpha)$, ie. the steady state is rendered efficient.

$$\mathscr{L}_t|_{\lambda=0} = \mathscr{L}_t^{GM} = -\frac{1-\alpha}{2} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\varepsilon_p}{\xi} \pi_{H,t+s}^2 + (1+\phi) \tilde{y}_{t+s}^2 \right\} \tag{2.46}$$

The key driver for the differences in Central Bank preferences with financial exclusion is the asymmetry in the ability of domestic households to share risk. Recall that financially-included households can pool risk with foreign agents, (2.17), by receiving or making appropriate international insurance transfers, (2.38). This smooths their consumption upon fluctuations in their income. In sharp contrast, financially-excluded agents must fully consume their income, (2.2). This implies that, unlike the $\lambda = 0$ case, (2.46), the stabilization of the terms of trade gap, $\tilde{x}_t^2$, is required in (2.45) to minimize welfare losses.

Result 1. *Terms of trade fluctuations have first-order distortionary effects in the presence of financial exclusion.*

In an economy where all agents can smooth consumption, shocks to purchasing power due to exchange rate fluctuations are offset through international risk-sharing arrangements. For instance, import price volatility, caused by exchange rate movements, does not affect financially-included agents by much as they are able to adjust their financial assets in a manner that consumption is smoothed. However, financially-excluded agents cannot similarly hedge against exchange rate volatility. Their inability to smooth their consumption against the volatility in international relative prices increases macroeconomic volatility, leading to aggregate welfare losses. This incentivizes the Central Bank to stabilize the terms of trade.

Policy Trade-offs Result 1 is one dimension of the loss-minimizing objective, but what do Central Bank preferences look like overall? It is useful to provide more intuition on the micro-founded objective, (2.45), as this drives most of the optimal monetary policy results. However, the weights in (2.45) are complicated functions of structural parameters, so that their implications are not transparent. This can be addressed by re-writing the loss function in terms of only domestic inflation, $\pi_{H,t}$, and the output gap, $\tilde{y}_t$, as (2.47), by using the proportionality between the terms of trade gap and the output gap, $\tilde{x}_t = \Upsilon \tilde{y}_t$. The optimal policy trade-offs that arise are then captured by the relative weight, (2.48).[10]

$$\mathscr{L}_t = -\frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^s \left\{ \pi_{H,t+k}^2 + \Phi^{\lambda} \tilde{y}_{t+k}^2 \right\} \tag{2.47}$$

where the relative weight on $\tilde{y}_t$, Φ^{λ}, is given by

$$\Phi^{\lambda} = \frac{\Xi \left(\Lambda^{-1} + \Lambda\right) - (1-2\alpha)\, \Phi \Upsilon^2 - \left((1-\lambda) + (1-\alpha)\Phi\right) \lambda \nu^2 - 2\Phi\omega\Upsilon - (1-\alpha) - \frac{1+\phi}{1-\lambda} \frac{U_N}{U_C} \frac{N}{C}}{\frac{\varepsilon}{\xi} \left(\Xi \Lambda \left(1+\nu\right) - \left(1 + (1-\alpha)\Phi\right) \nu\lambda\right) - \frac{\varepsilon_P}{\xi} \frac{U_N}{U_C} \frac{N}{C}} \tag{2.48}$$

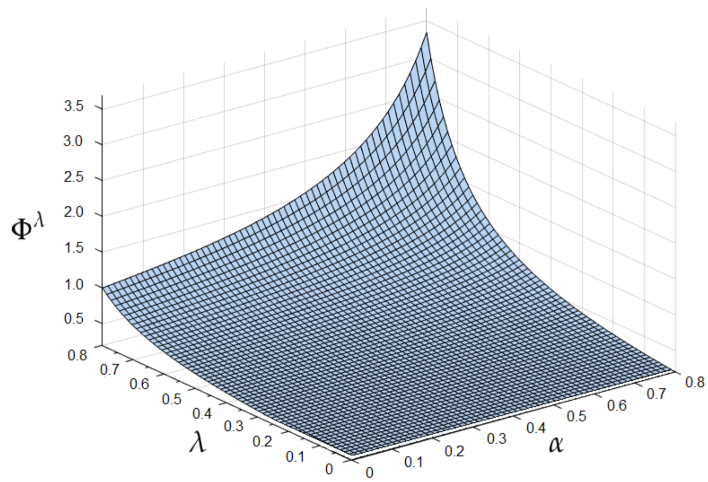

Figure 2.2: Optimal Relative Weight on Output Gap

Result 2. *The relative weight on output gap stabilization, Φ^{λ}, is increasing in α and λ.*

It is instructive to compare the relative weight with limited financial market participation, Φ^{λ}, with the Gali and Monacelli (2005) complete financial inclusion case, where the relative weight, $\Phi^{GM} = \frac{1+\phi}{\frac{\varepsilon}{\xi}}$, is invariant with respect to openness, α. In contrast, Φ^{λ} in this model increases with both α and λ, as shown in Figure 2.2.

Greater openness implies that domestic agents consume more imports relative to domestic goods. This increases the consequences of exchange rate fluctuations, which adversely affects financially-excluded agents as unlike asset holders, they cannot smooth

[10] A numerical check yields that the relative weight, Φ^{λ}, is strictly positive when the structural parameters in the loss function, $\alpha, \lambda, \phi, \varepsilon, \beta, \theta$, jointly satisfy the following system of inequalities: $0 < \alpha < 0.95$, $0 < \lambda < 0.95$, $0 < \theta < 1$, $0 < \beta < 1$, $\varepsilon_P > 0$, $\phi > 0$. The positive weight implies that the quadratic loss function is convex so that the second-order sufficient conditions are satisfied. Note that convexity ensues for a wide range of parameter values in this model due to the logarithm and isoelastic functional forms in utility, as also in Benigno and Woodford (2005).

their consumption against the increasingly volatile price of imported goods. The resulting consumption volatility leads to higher aggregate welfare losses, incentivizing the Central Bank to smooth the consumption of hand-to-mouth agents by stabilizing their income through the output gap. $\frac{\partial \Phi^{\lambda}}{\partial \alpha} \neq 0$ in the model is thus contingent on $\lambda > 0$, ie. a fraction of agents unable to insure against risk. Similarly, Φ^{λ} increases with λ to minimize the greater losses as progressively more agents cannot smooth consumption.

Result 3. *The closed economy case is not isomorphic for $\lambda > 0$.*

This point is of independent interest to the question at hand, but merits a mention. Optimal policy in the open economy model with financially-excluded agents does not necessarily converge to the closed economy case when $\alpha \to 0$. Usually considered desirable from a modeling perspective, other instances can be found in Monacelli (2005) and Farhi and Werning (2012). There is an irreversible open economy asymmetry in this model compared to the closed economy version of Bilbiie (2008), as only a fraction of domestic consumers can share risk with foreign agents as the economy opens up. Exchange rate movements affect the heterogeneous agents in completely different ways, implying that consumption patterns fundamentally diverge over the business cycle, overriding the type of closed economy isomorphism that characterizes Gali and Monacelli (2005).

The argument for this is similar to Farhi and Werning (2012). In the closed economy limit ie. as $\alpha \to 0$, a fixed nominal exchange rate, $e_t - e_{t-1} = 0$ implies a fixed nominal interest rate (equal to the exogenous foreign interest rate, i_t^*) through the UIP condition, $i_t = i_t^* + E_t e_{t+1} - e_t$. However, it is known that a fixed nominal interest rate could imply equilibrium indeterminacy in a closed economy (Galí, 2009). I numerically find that a fixed nominal interest rate, $i_t = 0$, also results in indeterminacy in my model when $\alpha = 0$, and indeed for all α. When $\alpha > 0$, however, an exchange rate peg, $e_t - e_{t-1} = 0$, ensures a unique equilibrium and moreover approximates the optimal policy, as will be discussed in Section 2.4. This result is also found in Monacelli (2005) and Farhi and Werning (2012), and would suggest caution in using closed economy models to approximate the open.

Targeting Rule I derive the flexible inflation targeting rule for the model under dynamically consistent timeless commitment, where the optimal plan set in later periods is the same as the one that would have been set initially (Woodford, 2011). This requires minimizing the quadratic loss function (2.45) with respect to the linear aggregate supply relation, (2.43). The IS Equation does not bind since the nominal interest rate is unconstrained. I attach the Lagrange multiplier, $\chi_{\pi,t}$, on the NKPC and take first-order conditions with respect to domestic inflation, $\pi_{H,t}$, and the output gap, $\tilde{y}_t$

$$\begin{aligned} 2\Psi_\pi \pi_{H,t} + \chi_{\pi,t} - \chi_{\pi,t-1} &= 0 \\ 2\Psi_y \tilde{y}_t - \xi \Lambda^{-1} \chi_{\pi,t} &= 0 \end{aligned}$$

These can be combined to yield the flexible targeting rule, which is a locally linear approximation to optimal policy, assuming that the shocks are sufficiently small in amplitude.

Proposition 5. (Optimal Policy) *The optimal plan under timeless commitment is implemented as follows*

$$\pi_{H,t} = -\frac{\Lambda}{\xi}\Phi^\lambda \left(\tilde{y}_t - \tilde{y}_{t-1}\right) \tag{2.49}$$

The optimal plan, (2.49), implies the classic "leaning against the wind" analogy of contracting the output gap to bring down domestic inflation, whenever the latter is inefficiently high. The targeting rule holds only if prices are slow to adjust, $\xi \neq \infty$, as it would be redundant in a flexible price environment where inflation creates no real distortions.

The equilibrium is unique with optimal policy for all $\lambda \in [0, 1)$, corroborating the closed economy analog in Bilbiie (2008). The targeting rule, (2.49), is combined with the NKPC, (2.43), to get the following second-order difference equation

$$E_t \tilde{y_{t+1}} = \left[1 + \frac{1}{\beta}\left(1 + \frac{\xi^2}{\Lambda^2}\frac{1}{\Phi^\lambda}\right)\right]\tilde{y}_t - \frac{1}{\beta}\tilde{y}_{t-1} + \frac{1}{\beta}\frac{\xi}{\Lambda}\frac{1}{\Phi^\lambda}v_t$$

whose roots are numerically found to be on the opposite sides of the unit circle as required (Blanchard and Kahn, 1980; Neusser, 2016).

2.3.2.3 Optimal Policy Dynamics

I proceed to analyze how the optimal targeting rule, (2.49), is employed to mitigate welfare losses in response to unexpected disturbances. The focus is primarily on cost-push shocks as these have been a significant concern in emerging market economies, and create a meaningful trade-off between stabilizing inflation and output (Frankel, 2010). I also analyze productivity shocks. Dynamics are analyzed based on a calibrated version of the framework. Calibration is a challenging task, as the required micro-level data is scarce for EMEs. I thus select parameters from the existing open economy literature, and pair this with extensive sensitivity analysis. The fraction of randomly chosen monopolistic producers that can reset prices, θ, is set at 0.75, which implies an average period of one year between price adjustments, as in Gali and Monacelli (2005).

The household discount factor β equals 0.99, which implies a steady state real interest rate of around four percent. The elasticity of substitution between differentiated monopolistic goods is set at $\varepsilon_P = 6$, which implies a steady state markup of around 20%. The degree of openness, α, is set at 0.5 and the inverse Frisch elasticity of labour supply, ϕ, at 1. The persistence of shocks, ie. ρ_j in the stationary autoregressive process $j_t = \rho_j j_{t-1} + s_{j,t}$, where $j_t = \{v_t, a_t, i_t^*\}_{t=0}^{\infty}$, is set at 0.9, consistent with the evidence for emerging market economies provided in Aguiar and Gopinath (2007). In most cases, optimal policy with complete financial market participation, $\lambda = 0$, is contrasted with $\lambda = 0.6$, which is around the EME financial exclusion average (World Bank, 2015).

Figure 2.3 provides dynamics for varying degrees of financial exclusion upon a unit positive cost-push shock. There is an immediate rise in domestic inflation that arises independently from variations in domestic demand. The Central Bank contracts output to control inflation, and a recession ensues. The rise in domestic prices leads to a fall in aggregate consumption, while also causing a terms of trade appreciation. The variations in dynamics, depending on λ, are based on the trade-off embodied in the optimal targeting rule, (2.49). With higher financial exclusion, the Central Bank is incentivized to stabilize the output gap by more to smooth the disposable income of hand-to-mouth agents. As these households are unable to insure their consumption against the rise in prices, their response would otherwise be very volatile and lead to aggregate welfare losses. As per the optimal plan, domestic inflation is allowed to rise by a bit more so that a deep recession can be prevented. To support this optimal trade-off, the nominal exchange rate appreciates to put downward pressure on inflation when financial inclusion is high. The exchange rate appreciates by less as exclusion increases to stabilize the terms of trade, and hence the demand for tradable output, as per the optimal plan, (2.49).

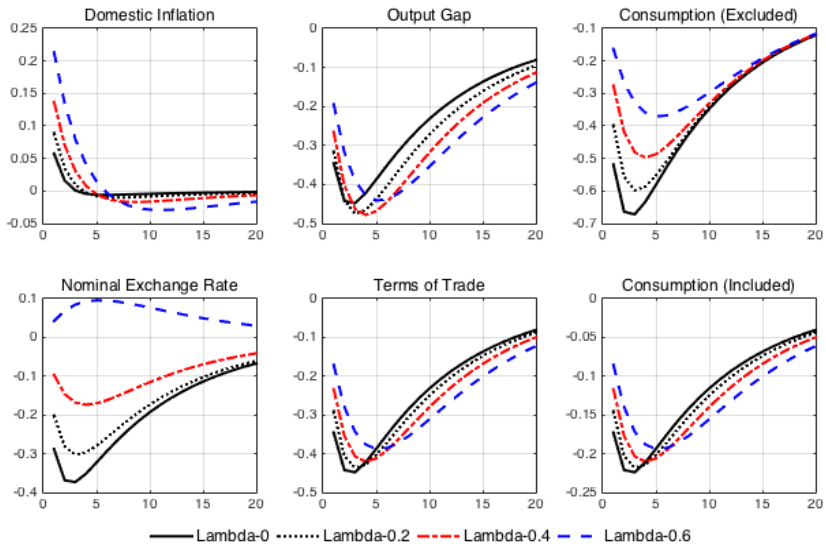

Figure 2.3: 1% Cost-Push Shock, Optimal Policy when Varying Financial Exclusion

Result 4. *Monetary policy plays an additional insurance role in the presence of financial exclusion.*

For $\lambda > 0$, optimal monetary policy takes into account the inability to share risk and hence the more volatile consumption of financially-excluded agents. Implementation of the optimal plan, (2.49), results in disposable income being stabilized by more as financial exclusion increases. This can be interpreted as provision of insurance by monetary policy to agents who cannot privately smooth consumption, and is a required transmission channel in the efficient plan. Thus, besides macroeconomic stabilization, monetary policy plays an additional insurance role when $\lambda > 0$. The extent of insurance varies depending on the degrees of openness and price stickiness, as shown next.

2.3.2.4 Sensitivity Analysis

I analyze the implications of the optimal plan, (2.49), for $\alpha = \{0.1, 0.4, 0.7\}$ and $\theta = \{0.7, 0.75, 0.8\}$. The financial exclusion case ($\lambda = 0.6$) while varying α is depicted below in Figure 2.4 (corresponding $\lambda = 0$ case is in the appendix). The dynamics suggest that the insurance provided by monetary policy is increasing in openness and price flexibility.

Openness

When financial inclusion is complete, $\lambda = 0$, the trade-off is independent of openness as changes to real income from exchange rate fluctuations are offset through international risk-sharing. This independence is broken in the presence of financial exclusion, $\lambda > 0$. Here, exchange rate volatility causes equivalent fluctuations in the consumption of financially-excluded agents (and by a greater amount as $\alpha \to 1$ due to the increased consumption of imports), unlike that of financial asset holders. To mitigate the resulting greater macroeconomic volatility, optimal monetary policy smooths the consumption of hand-to-mouth agents by more. The optimal plan thus places greater relative weight on stabilizing output in response to cost-push shocks. To support this, the nominal exchange rate depreciates by more to mitigate the terms of trade appreciation, hence relatively stabilizing the demand for output as per the optimal trade-off, (2.48). This helps smoothen the disposable income of hand-to-mouth households as the economy opens up.

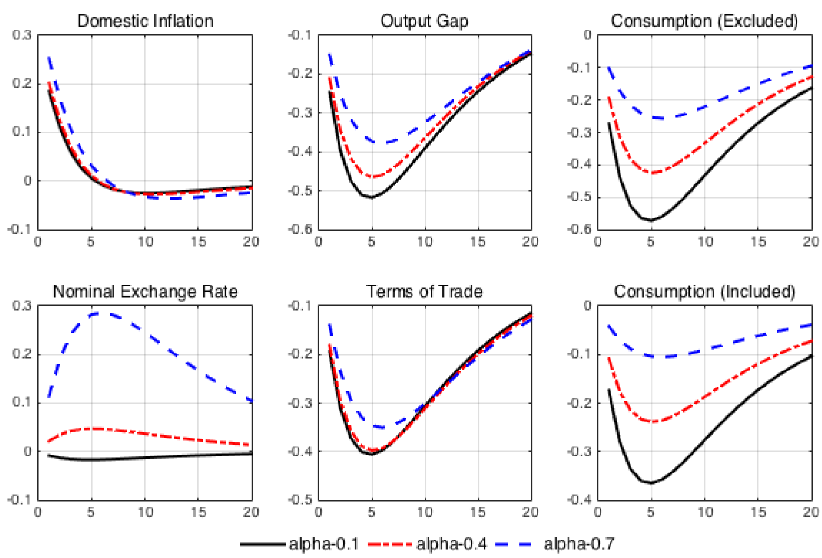

Figure 2.4: 1% Cost-Push Shock, Optimal Policy with $\lambda = 0.6$, Varying Openness

Nominal Rigidities

Stickier prices, $\theta \to 1$, imply that fewer randomly chosen price-setters re-optimize each period. Thus, upon a cost-push shock, the required downward re-optimization of prices (in anticipation of future high domestic inflation) is hindered. This leads to more domestic inflation volatility, requiring a greater contraction of the output gap as per the optimal plan. The resulting steeper recession dampens labour demand by more. Consequently, lower real wages clear the market, implying that hand-to-mouth consumption decreases by more. Nominal rigidities thus hamper the provision of insurance by optimal monetary policy. These dynamics are found in the appendix.

2.4 Nominal Anchor: Peg or Float?

I analyze the appropriate choice of a nominal anchor in an economy with financial exclusion. To do so, simple and implementable rules are ranked according to the loss function, (2.47), in terms of the lowest welfare losses away from the optimum. Simple rules as approximations of optimal monetary policy are in particular useful, since the optimum can sometimes be cast as a complicated and unintuitive function of model parameters. Optimal policy is often thus not as transparent and implementable in practice as a simple rule. I show that targeting the exchange rate is an implementable way to internalize the insurance properties of the optimal plan characterized in the previous section.

Simple Rule	Financial Inclusion	Financial Exclusion
Strict CPI IT (CIT)		
$\pi_t = 0$	**0.019**	0.024
Strict domestic IT (DIT)		
$\pi_{H,t} = 0$	0.032	0.143
Fixed exchange rate (PEG)		
$\Delta e_t = 0$	0.139	**0.001**

Table 2.1: Relative Welfare Losses of Simple Rules in % Units of Steady State Consumption

Table 2.1 provides the relative welfare loss numbers of some standard simple rules compared to the benchmark optimal policy, upon trade-off creating cost-push shocks. There are two columns: the first reports welfare losses in the case of complete financial inclusion, $\lambda = 0$, whereas the second does likewise with high financial exclusion, $\lambda = 0.6$.[11] All entries in Table 2.1 are, as in Gali and Monacelli (2005), percentage units of steady-state consumption and in deviation from the first-best case of optimal monetary policy. It can be seen that while strict CPI Inflation Targeting, $\pi_t = 0$, best approximates

[11] Many variations of simple rules were compared, including flexible Taylor-type rules with inflation and the output gap, but the ranking remains the same ie. CIT leads to lowest welfare losses with high financial inclusion and PEG leads to lowest welfare losses with high financial exclusion. Also, while the table here discusses the cases of $\lambda = 0$ versus $\lambda = 0.6$, CIT is appropriate till around a threshold of 40% financial inclusion ($\lambda = 0.4$), after which PEG is preferred.

the optimal policy when all agents can smooth consumption, a fixed exchange rate, $\Delta e_t = 0$, is least suboptimal when financial exclusion is high.

Figure 2.5 connects the optimal monetary policy, described in (2.45) and (2.49), with the implementation of simple monetary rules. Complementing Table 2.1 that reports whether a peg or float is preferred for $\lambda = 0, 0.6$ with cost-push shocks, Figure 2.5 displays the appropriate choice of a nominal anchor for $\lambda, \alpha \in \{0, 0.0.8\}$. I reproduce the optimal relative weight on output gap stabilization, Figure 2.2, to show that while flexible exchange rates (CIT) are preferred for lower values of λ and α, exchange rate targeting (PEG) approximates the optimal plan with higher financial exclusion and openness.

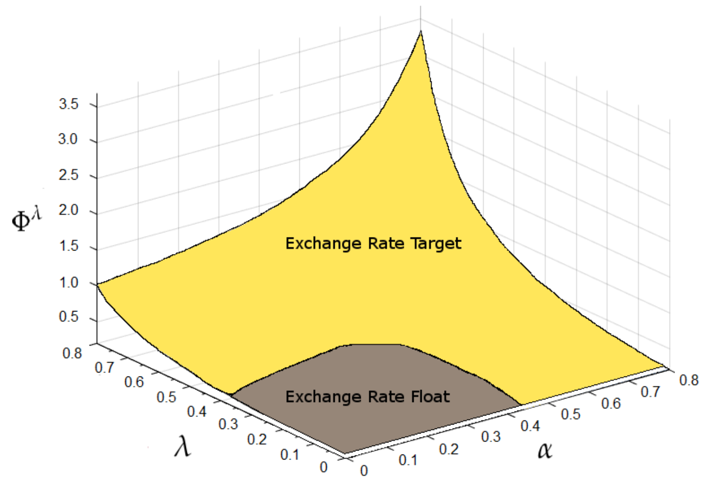

Figure 2.5: Optimal Relative Weight on Output Gap: Implemented by Fixed (PEG) versus Flexible (CIT) Exchange Rates with a 1% Cost-Push Shock

Result 5. *A nominal exchange rate peg is least suboptimal with high financial exclusion.*

Table 2.1 indicates that fixed exchange rates are preferred to flexible exchange rates when the majority of households in the economy are financially-excluded. Targeting the nominal exchange rate mitigates fluctuations in the relative price of domestic goods, which stabilizes output and disposable income similar to the optimal plan, (2.49). This policy is desirable in the presence of high asset market inequality, when a large fraction of agents cannot smooth consumption. A fixed exchange rate also stabilizes the fluctuations in imported good prices, smoothing hand-to-mouth consumption with cost-push shocks. Notably, dynamics under a fixed exchange rate almost completely mirror those under optimal policy, as shown in Figure 2.6, which compares PEG with the optimal targeting rule, (2.49), CIT (least suboptimal policy when $\lambda = 0$ with cost-push shocks), and DIT (optimal policy in this model with productivity shocks).

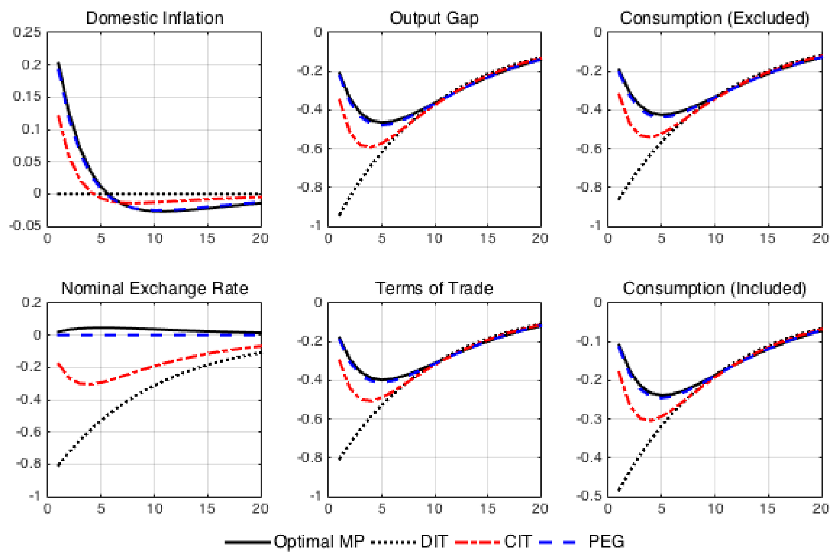

Figure 2.6: 1% Cost-Push Shock, Optimal vs Simple Rules, Financial Exclusion ($\lambda = 0.6$)

Result 6. *CPI Inflation Targeting is appropriate with high financial inclusion.*

When financial inclusion is close to complete, domestic inflation volatility leads to greater aggregate welfare losses compared to when financial market participation is limited. Thus, optimal policy places greater weight on stabilizing domestic inflation. With fixed exchange rates, the muted real appreciation implies that marginal costs, and hence domestic inflation, increase by too much. Thus, PEG is suboptimal. DIT, on the other hand, leads to extreme output gap volatility (which is still penalized, albeit by less) in fully stabilizing domestic inflation. CIT strikes the appropriate in-between as it results in less output gap volatility than DIT (it stabilizes *both* domestic and imported good inflation, and the output gap responds to *domestic* inflation only as per the optimal targeting rule, (2.49), and less domestic inflation volatility than PEG due to greater real appreciation. These dynamics are found in the appendix.

2.4.1 Role of Openness

I vary the degrees of openness and nominal rigidities, to analyze whether CIT and PEG remain robust. I find that they do. Price stickiness matters to the extent that the simple rules approximate optimal policy better as nominal rigidities lessen ie. $\theta \to 0$ (because the role of optimal policy diminishes with increased price flexibility); however, varying α yields an interesting result.

Result 7. *When $\alpha > 0$, CIT and PEG work better with lower degrees of openness.*

CIT, $\pi_t = 0$, approximates the $\lambda = 0$ optimal monetary policy better, and PEG, $\Delta e_t = 0$, does likewise for the $\lambda = 0.6$ optimal policy, as $\alpha \to 0$. Note, however, that this does *not* imply that $\pi_{H,t} = 0$ and $i_t = 0$ (the closed economy analogs of CIT and PEG) perform well when $\alpha = 0$. In the former case, $\pi_{H,t} = 0$ is achieved at the expense of high output gap volatility, which is suboptimal - CIT does not require this. Furthermore, $i_t = 0$ results in an indeterminate equilibrium for $\alpha = 0$. Intuition for the better CIT and PEG approximations with low $\alpha > 0$ is as follows.

Consider first the $\lambda = 0$ case. It is useful to note first the following: (i) lower α implies that more domestic goods are consumed relative to imported goods, (ii) CIT targets both domestic and import price inflation, and (iii) upon a cost-push shock, while relative domestic prices increase, relative import prices *decrease* due to an expenditure-switching effect. Now, as $\alpha \to 0$, CIT implies the stabilization of progressively more *domestic* inflation (which has increased), which requires higher real appreciation (preventing import deflation would, in contrast, require real *depreciation*). This higher real appreciation as α decreases matches that under optimal policy for the complete financial inclusion case. PEG does not provide the required high real appreciation for $\lambda = 0$.

Now, consider the case when $\lambda = 0.6$, where the weight on output gap stabilization increases with α to smooth the consumption of financially-excluded agents. This requires progressively less real appreciation, as discussed in section 2.3.2.4. With very high α, the optimal real appreciation is required to be so low that although a nominal peg comes closest to engendering this, even it cannot completely mirror the optimal policy (a different simple rule, perhaps targeting the *real* exchange rate itself, might be appropriate here). However, for low ($\alpha = 0.1$) and moderate ($\alpha = 0.4$) degrees of openness, the real appreciation under the peg is appropriate. In contrast, CIT is inefficient for all α with financial exclusion, since it requires suboptimally high real appreciation. The corresponding dynamics for $\lambda = 0$ and $\lambda = 0.6$ are found in the appendix.

2.4.2 Productivity Shocks

Result 8. *A fixed nominal exchange rate in a high financial exclusion economy is appropriate only if cost-push shocks predominate.*

It is useful to understand the circumstances under which a fixed exchange rate provides the most efficient stabilization. I find that the type of shock matters for the choice of domestic versus external nominal anchor. As shown in Proposition 3, optimal policy with productivity shocks leads to perfect stabilization and is strict domestic inflation targeting (DIT) with any degree of financial exclusion, as in Gali and Monacelli (2005). This can be seen by setting $\pi_{H,t} = 0$ in the system of equations, (2.41), (2.42), and (2.43), in the absence of cost-push shocks, v_t, so that the output gap is also perfectly stabilized, $\tilde{y}_t = 0$. This DIT policy requires considerable nominal exchange rate volatility. In the presence of financial exclusion, exchange rate stability is appropriate only in the presence of shocks that create a meaningful trade-off for monetary policy.

2.5 Conclusion

This study analyzed optimal monetary policy and the corresponding choice of an appropriate nominal anchor in a small open economy with financial exclusion. There are two main findings. The first result is that that optimal policy seeks to smooth the consumption of financially-excluded agents who cannot privately do so themselves, by increasing the relative weight on the output gap in the micro-founded welfare loss function as financial exclusion and openness increase. The second result is that a fixed nominal exchange rate implements the insurance provision properties of the optimal plan when the majority of households do not have financial market access. The desirability of exchange rate stability provides a counterpoint to Milton Friedman's long-standing argument for a float. This paper sought to establish benchmark analytical results, and lends itself to some extensions. It might be interesting to interact imperfect risk-sharing with the asset market segmentation considered in this study, and I leave this for future work.

Chapter 3

Exchange Rate Choices with Inflexible Markets and Costly Price Adjustments

"All main commodity price indices are expected to fall in 2016... Low prices for commodities are likely to be with us for some time... Commodity exporters are feeling the pain right away."

-World Bank, Press Release, January 26, 2016

3.1 Introduction[1]

This paper analyzes the appropriate choice of an exchange rate regime in developing economies that are significantly dependent on labour-intensive agricultural commodity exports.[2] In practice, the majority of such economies have exchange rate anchors.[3] This poses a puzzle in light of the conventional macroeconomic policy advice since Friedman (1953) that flexible exchange rate have superior stabilization properties. Falling agricultural commodity prices of late (Figure 3.1), alongside a persistent downward trend and volatility in such prices (World Bank, 2017) have re-ignited discussions across interna-

[1]This chapter, Iyer (2017), is published as a July 2017 IMF Working Paper as part of a research project on macroeconomic policy in low-income countries supported by the U.K.'s Department for International Development:
https://www.imf.org/en/Publications/WP/Issues/2017/07/10/Exchange-Rate-Choices-with-Inflexible-Markets-and-Costly-Price-Adjustments-44984.

[2]Commodity-exporting economies are defined as those economies where commodity export revenues constitute more than 35% of overall export revenues (IMF, 2015a).

[3]See Table 3.2 in Section 3.2 for details.

tional policy circles on exchange rate choices in commodity exporters (IMF, 2016). The policy recommendations to float have not taken into account the low level of development that influences monetary policy in labour-intensive commodity exporters. This paper seeks to reconcile the exchange rate puzzle in the data and contribute to the ongoing policy debate by providing benchmark theoretical results on the appropriate choice of an exchange rate regime in agricultural commodity-exporting economies.

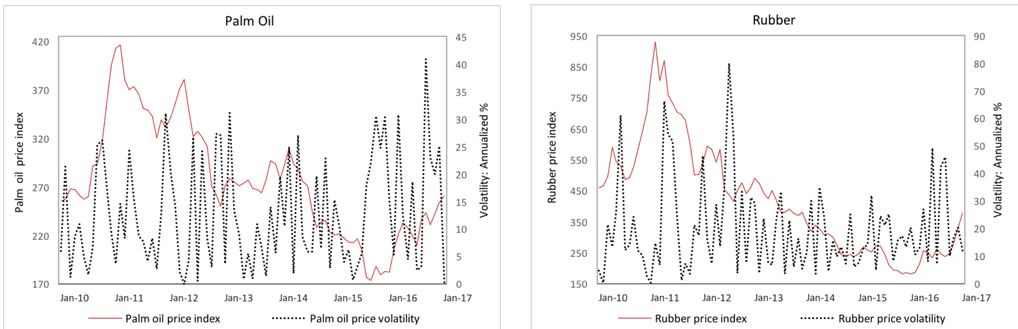

Figure 3.1: Agricultural Commodity Price Indices (2000=100) and Annualized Volatility Numbers
Source: UNCTAD 2016 Statistics Database (UNCTAD, 2016)

I develop an open economy dynamic stochastic general equilibrium (DSGE) framework that incorporates key structural characteristics of agricultural commodity-exporting economies. The model is able to match stylized facts on the macroeconomic response to commodity export price volatility (discussed further in Section 3.2). The economy has a heterogeneous production structure with commodity and non-commodity firms. Each sector employs workers to capture the labour-intensive production structure in agricultural commodity-exporting economies (UNCTAD, 2013). Labour market rigidity exists in the form of imperfect labour mobility across sectors, implying that wage rates are different across the economy. Both sectors are tradable, with commodities being fully exported and non-commodity goods being supplied to domestic and foreign agents. The commodity sector takes the world price of agricultural goods as given. The non-commodity sector is monopolistic with sticky prices, implying a role for monetary policy. There is limited financial integration with the world and international risk-sharing is imperfect, as representative of developing economies (Kose et al., 2006).

In a novel set of results, I show that the appropriate choice of an exchange rate regime in agricultural commodity exporters depends on the degree of flexibility of domestic labour and product markets. Inflexible labour markets in the model prevent workers from re-allocating hours worked across sectors in response to wage differentials. In developing countries, labour markets are fairly rigid due to factors including sector-specific skills and institutional regulations (Fields, 2009; Artuc et al., 2013). Inflexible product markets in the model are represented by a low Armington (1969) trade elasticity, or the elasticity of substitution between domestic and imported goods. The Armington elasticity is considered low in developing economies as in contrast to industrialized nations, domestic products are typically of poorer quality than foreign goods (Hummels and Klenow, 2005; Henn et al., 2013). With inflexible labour and product markets, a fixed exchange rate leads to higher welfare than a float. Intuitively, with inflexible real markets, and limited financial opportunities to hedge against country-specific risk, relative wage and price fluctuations exacerbate currency and factor misalignments. Instead of allowing for more efficient economic adjustment, price fluctuations in inflexible markets lead to the misallocation of domestic resources. The Central Bank prefers to mitigate international relative price fluctuations, and correspondingly relative wage fluctuations through labour re-allocation, by targeting the exchange rate.

As domestic labour and product markets become more developed, agents can more efficiently respond to wage and price differentials. Flexible labour markets allow households to re-allocate hours worked across sectors worked more efficiently, and thus enjoy higher wages for any given amount of labour effort. Flexible product markets allow households to re-allocate consumption expenditure toward relatively cheaper domestic or foreign goods. In fact, while it is easier said than done in developing economies, when either domestic labour *or* product markets become more flexible, relative price fluctuations become desirable. An exchange rate float leads to higher welfare than a peg in this case. Intuitively, upon a negative shock to the commodity sector, flexible exchange rates allow for greater real depreciation. This increases aggregate demand for tradable non-commodity goods, amplifying the macroeconomic adjustment mechanism of greater wage differentials through labour re-allocation away from the lagging commodity sector. Table 3.1 summarizes these results, which (i) reconcile the exchange rate puzzle by offering a theory as to why it might be appropriate for less-developed agricultural commodity exporters to target their exchange rates and (ii) these economies should transition to flexible exchange rates, as per conventional wisdom, as they develop over time.

	Rigid Labour Markets	Flexible Labour Markets
Rigid Product Markets	**Peg**	Float
Flexible Product Markets	Float	Float

Table 3.1: Appropriate Exchange Rate Regime in Agricultural Commodity Exporters

Related Literature This paper contributes to three areas of the literature.

First, several studies have analyzed macroeconomic policy options to deal with commodity revenue volatility. The literature has tended to focus on the design of fiscal frameworks that promote macro-fiscal stability, fiscal sustainability, and the accumulation of precautionary savings (reviewed in Baunsgaard et al., 2012), and a countercyclical fiscal stance (Frankel et al., 2013).[4] Other papers have studied the stabilization properties of various short- and medium-term fiscal rules as an alternative to discretionary fiscal policy (Bi and Kumhof, 2011; C. Garcia and Tanner, 2011; Kumhof and Laxton, 2013b; Snudden, 2016). Monetary policy choices have been analyzed in single-sector models with frictionless complete financial markets (Wills, 2013; Ferrero and Seneca, 2015).[5] This does not account for the several distinguishing factors that influence monetary policy in commodity exporters, including factor re-allocation dynamics or that their financial markets are not deep (UNCTAD, 2013; IMF, 2016). I advance the research agenda by incorporating heterogeneous production, factor re-allocation, and financial market incompleteness. In contrast to the previous literature's prescription of inflation targeting that relies on complete markets, I show that exchange rate targeting, instituted in practice by over 70% of commodity exporters, is preferred with inflexible real and financial markets.

My work is also related to the open economy literature on exchange rate choices with incomplete financial markets. This literature refutes the existence of complete financial markets in the data, and shows that imperfect risk-sharing in open economy models is generally required to match relative price dynamics (Backus and Smith, 1993; Chari et al., 2002; Corsetti et al., 2008). While the majority of New Keynesian models nevertheless continue to use complete markets, a few studies have analyzed the appropriate choice of monetary policy with imperfect risk-sharing, albeit in single-sector models (Benigno, 2009; De Paoli, 2009; Corsetti et al., 2010). These papers find a case for exchange targeting

[4]In related analyses, studies that examine the growth effects of scaling-up public investment out of resource revenues include Berg et al. (2013) and Alter et al. (2015).

[5]The literature on commodity prices and exchange rates includes proposals to target the domestic price of commodities (Frankel, 2011), as well as incorporating commodity prices to improve the forecasting ability of exchange rate models (Cashin et al., 2004) in a separate but related set of questions.

under certain parameter configurations, as it can redress inefficient cross-country demand imbalances. I contribute to this literature by incorporating dual labour markets in an incomplete financial market world, and showing that the desirability of exchange rate targeting is contingent on the efficiency of factor re-allocation. My results are relevant to developing economies with export-intensive structures and inflexible markets. Finally, my paper adds to the burgeoning literature on monetary policy and factor re-allocation, which has previously focused on closed economy models with labour (Petrella and Santoro, 2011) and capital (Bouakez et al., 2009). I advance this research agenda by adding an open economy dimension, as well as incomplete financial markets.

The rest of the paper is structured as follows. Section 3.2 provides stylized facts that motivate the theoretical analysis. Section 3.3 develops an open economy model that incorporates key structural features of agricultural commodity exporters. Section 3.4 calibrates the framework. Section 3.5 assesses the role of labour and product market rigidity in influencing the appropriate choice of an exchange rate regime. Section 3.6 concludes, discusses extensions, and offers policy recommendations.

3.2 Stylized Facts

Background Commodity exporters, defined as those economies where commodity export revenues constitute 35% or more of total export revenues, are uniquely vulnerable to the high volatility in international commodity prices (IMF, 2015a). Economic progress in these countries is tightly linked to unstable foreign exchange revenues, making it paramount to consider exchange rate regimes that mitigate the resulting domestic macroeconomic volatility (IMF, 2015c). There are around 40 commodity-exporting economies that produce labour-intensive agricultural commodities, a large number of which are in Africa (UNCTAD, 2013). Other commodity exporters extract metals and oil using capital-intensive technology, and the challenges facing those economies are quite different (for an overview, see IMF, 2015a). This paper focuses on the agricultural commodity exporters in Table 3.2. Most of these are developing economies, and commodity export employment generally constitutes a high fraction of aggregate employment. Moreover, in contrast to the conventional wisdom on the superiority of flexible exchange rates, over 70% of agricultural commodity exporters have exchange rate anchors.

Structural Features The following key structural characteristics of agricultural commodity exporters are important to take into account in a DSGE model (IMF, 2016; UNCTAD, 2013). Agricultural commodity exporters are typically small and open economies, which take the volatile international price of commodities as given. Production activities are typically heterogeneous, with commodity and non-commodity sectors. These commodities include sugar, rubber, coffee, and rice, and harvesting them is highly labour-intensive with limited machinery. Non-commodity products are typically of lower quality than in advanced economies (Hummels and Klenow, 2005; Henn et al., 2013). Domestic households supply labour, but cannot respond efficiently to inter-sectoral wage differentials as labour markets are not flexible in developing economies (Fields, 2009; Artuc et al., 2013).

Asset markets are typically thin and imperfectly integrated with the rest of the world, making it difficult for households to insure against aggregate risk (Kose et al., 2006). This can be modeled in a DSGE framework through incomplete international financial markets, which implies suboptimal international risk-sharing. Agricultural production is subject to decreasing returns to scale, and export revenues generally accrue to domestic households who account for almost all of commodity production (UNCTAD, 2013). Commodities are also typically almost fully exported. In practice, a very small fraction of agricultural exports is consumed domestically, but for simplicity, all commodities are fully exported in the framework. All the structural characteristics discussed above are explicitly modeled, and allow the framework to replicate key macroeconomic dynamics upon a commodity price fall, as discussed next.

A Commodity Price Fall The dual labour markets structure allows the model to replicate certain stylized facts on macroeconomic fluctuations in commodity exporters. Empirical evidence from Kose and Riezman (2001) and IMF (2015a) suggests that most macroeconomic aggregates are procyclical in response to commodity price volatility. Upon a negative shock to agricultural commodity export revenues, aggregate output and consumption fall. Labour tends to move out of the lagging commodity export sector, and aggregate employment falls due to the substitution effect arising from the fall in wages. The real exchange rate depreciates. These features of macroeconomic fluctuations are qualitatively matched by my model, especially with the monetary regime of exchange rate targeting (see Figure 3.2 in Section 3.5). Further details on the framework and its calibration are provided in Sections 2.3 and 2.4.

Table 3.2: Exchange Rate Regimes and Employment in Agricultural Commodity Exporters

Country	Agri Exports/ Total Exports	% Agri Employment	Exchange Rate Regime
Guinea-Bissau	91%	53%	Exch rate anchor (soft peg)
Rwanda	84%	59%	Exch rate anchor (soft peg)
Malawi	80%	53%	Exch rate float (money target)
Paraguay	76%	17%	Exch rate float (inflation target)
Ethiopia	75%	51%	Exch rate anchor (soft peg)
Nicaragua	70%	10%	Exch rate anchor (soft peg)
Gambia	69%	51%	Exch rate anchor (soft peg)
Djibouti	69%	49%	Exch rate anchor (hard peg)
Burundi	67%	59%	Exch rate float (money target)
Burkina Faso	65%	61%	Exch rate anchor (soft peg)
Afghanistan	65%	40%	Exch rate float (money target)
Belize	62%	16%	Exch rate anchor (soft peg)
Sao Tome and Principe	60%	37%	Exch rate anchor (soft peg)
Vanuatu	60%	20%	Exch rate anchor (soft peg)
Timor-Leste	59%	53%	Exch rate anchor (hard peg)
Somalia	58%	44%	Exch rate float (inflation target)
Kenya	56%	5%	Exch rate float (money target)
Uganda	55%	50%	Exch rate float (inflation target)
Tonga	53%	18%	Exch rate anchor (soft peg)
Côte d'Ivoire	53%	25%	Exch rate anchor (soft peg)
Argentina	52%	5%	Exch rate anchor (soft peg)
New Zealand	52%	5%	Exch rate float (inflation target)
Uruguay	51%	7%	Exch rate float (money target)
Benin	50%	29%	Exch rate anchor (soft peg)
Saint Vincent	50%	13%	Exch rate anchor (hard peg)
Dominican Republic	49%	7%	Exch rate anchor (soft peg)
Eritrea	49%	49%	Exch rate anchor (soft peg)
Samoa	49%	19%	Exch rate anchor (soft peg)
Comoros	48%	47%	Exch rate anchor (soft peg)
Honduras	47%	16%	Exch rate anchor (soft peg)
Fiji	43%	24%	Exch rate anchor (soft peg)
Panama	41%	11%	Exch rate anchor (hard peg)
Guatemala	40%	25%	Exch rate anchor (soft peg)
Liberia	40%	41%	Exch rate anchor (hard peg)
Dominica	39%	14%	Exch rate anchor (hard peg)
Guyana	39%	10%	Exch rate anchor (soft peg)
Moldova	38%	10%	Exch rate float (inflation target)
Average	**57%**	**30%**	**Exch rate anchor = 73%**

Sources: FAO 2010 Yearbook (FAO, 2010) and IMF 2014 Exchange Rates Report (IMF, 2014)
Note: Benin, Burkina Faso, Cote de Ivoire, Guinea-Bissau are in the WAEMU monetary union, and Dominica is in the ECCU currency board. *Exchange rate anchor* refers to (i) *soft pegs* (conventional pegs including monetary unions, crawl-like arrangements, stabilized arrangements, horizontal bands) and (ii) *hard pegs* (currency boards, no separate legal tender) as per *IMF (2014)*. I approximate these through a fixed nominal exchange rate in this paper.

3.3 A Small Open Commodity-Exporting Economy

I develop an open economy framework, based on Benigno (2009), that incorporates key structural characteristics in agricultural commodity-exporting countries. The economy has a dual production structure, and each of the two sectors employs workers, to capture the labour-intensive nature of agricultural production. Wage rates are different across the economy due to barriers to labour mobility, deviating from the assumption of homogeneous labour markets typically used in general equilibrium models. Financial markets are not well-developed in the economy, and international risk-sharing is imperfect. This implies that agents are unable to insure consumption from income fluctuations, commonly found in developing countries (Kose et al., 2006).

3.3.1 Households

There exists a continuum of identical, infinitely-lived households indexed by $l \in [0, 1]$ in the domestic small open economy, which is of measure zero compared to the rest of the world. Thus, external conditions are taken as given and domestic policy decisions do not impact foreign agents. Initial net foreign asset positions are zero across all countries. The representative domestic household derives utility from consumption of domestic non-commodity and imported goods and disutility from labour supplied to the commodity and non-commodity sectors. As more consumption is desirable, but the marginal utility from each additional unit increases at a decreasing rate, it is modeled through an isoelastic functional form which is concave and continuously differentiable. Disutility from labour supply is strictly increasing in hours worked, and is measured by an isoelastic functional form that is convex and continuously differentiable. At each date $t > 0$, a stochastic event, x_t, is realized with probability $\mu(x_t)$. The initial realization, x_0, is given so that $\mu(x_0) = 1$. The history of events until period $t + s$ is given by $x^{t+s} = \{x_t, x_{t+1}, ..., x_{t+s}\}$. Subject to discounting, β^s, over time $t + s$, $s = 0, 1, 2, ...$, expected household utility at time t, U_t, is given by

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{N_{t+s}^{1+\phi}}{1+\phi} \right] \tag{3.1}$$

where $\sigma > 0$ is the inverse intertemporal elasticity of substitution, $\phi > 0$ is the inverse Frisch elasticity of labour supply, and $E_t(.) = \sum_{x_{t+s+1}} \mu\left(x^{t+s+1} | x^{t+s}\right)$ is the expectations operator over all possible future states of nature conditional on history x^{t+s} (Ljungqvist and Sargent, 2012). Hours worked, N_t, are supplied to the commodity and non-commodity sectors according to a constant elasticity of substitution (CES) labour supply index

$$N_t = \left[\eta^{-\frac{1}{\lambda}} N_{C,t}^{\frac{1+\lambda}{\lambda}} + (1-\eta)^{-\frac{1}{\lambda}} N_{H,t}^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{1+\lambda}} \tag{3.2}$$

where $\lambda > 0$ is the CES elasticity of substitution in hours worked across sectors and captures the extent of labour market rigidity, $N_{C,t}$ is labour demanded by the commodity sector, $N_{H,t}$ is labour demanded by the non-commodity sector, and η is the share of commodity sector labour in the steady state (ie. $\eta = N_{C,t}/N_t$). Labour market rigidity, which restricts inter-sectoral re-allocation of hours worked in response to relative wage fluctuations, is inversely proportional to λ.

For $\lambda = 0$, labour markets are fully segmented and workers cannot migrate across sectors even in the presence of wage differentials. As $\lambda \to \infty$, labour markets become flexible and workers devote all their time to the sector paying the higher wage. For $\lambda < \infty$, hours worked are imperfect substitutes, and labour is supplied to both sectors. (3.2) is similar to the CES aggregator used by Petrella and Santoro (2011) in that it allows for a full employment steady state where sectoral employment shares to sum up to 1, consistent with all workers being employed over the business cycle. C_t is a CES consumption index given by

$$C_t = [(1-\alpha)^{\frac{1}{\varepsilon}} C_{H,t}{}^{\frac{\varepsilon-1}{\varepsilon}} + \alpha^{\frac{1}{\varepsilon}} C_{F,t}{}^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.3}$$

where $C_{H,t}$ denotes consumption of non-commodity goods produced domestically, $C_{F,t}$ is consumption of goods imported from the rest of the world, $\alpha \in [0,1]$ is the share of imported goods in the consumption basket, C_t (conversely, $1-\alpha$ measures the extent of home bias in consumption), and ε is the Armington (1969) trade elasticity, or the elasticity of substitution between home and foreign goods. The associated price index is

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\varepsilon} + \alpha P_{F,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.4}$$

where P_t is the consumer price index (CPI), $P_{H,t}$ is the price index for domestic goods, and $P_{F,t}$ is the import price index. The following downward-sloping demand functions for domestic and imported goods are derived from minimizing expenditure on the consumption basket (3.3) as in Galí (2007)

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{Pt}\right)^{-\varepsilon} C_t \tag{3.5}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{Pt}\right)^{-\varepsilon} C_t \tag{3.6}$$

The subcomponents of the domestic and imported good indices, $C_{H,t}$ and $C_{F,t}$, measure domestic consumption of individual varieties of goods, i, with elasticity of substitution ν between varieties

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}} \qquad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}} \tag{3.7}$$

where $C_{H,t}(i)$ is the consumption of variety i produced in the domestic small open economy, and $C_{F,t}(i)$ is the consumption of variety i imported from the rest of the world.

Agents across the world have access to a one-period risk-free nominal bond, denominated in units of foreign currency. Unlike foreign households, domestic agents face adjustment costs on changing their holdings of this bond away from its steady state level. This ensures that the equilibrium in an open economy with incomplete markets is stationary as the marginal cost of engaging in asset market transactions is increasing in the quantity of bonds purchased. Domestic households also have access to a one-period uncontingent domestic currency risk-free bond. This is in zero net supply as idiosyncratic risk is pooled among domestic households so that, in effect, only foreign currency bonds are traded in equilibrium. At time t, utility, (3.1), is maximized in a dynamic optimization problem, subject to the following sequence of nominal budget constraints

$$\begin{aligned}&\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1\int_0^1 P_{j,t}(i)C_{j,t}(i)didj + e_t\left[B_{F,t} + \Gamma\left(B_{F,t}\right)\right] + B_{H,t} \\ &\leq e_t\left(1+i^*_{t-1}\right)B_{F,t-1} + \left(1+i_{t-1}\right)B_{H,t-1} + W_{C,t}N_{C,t} + W_{H,t}N_{H,t} + \Omega_{C,t} + \Omega_{H,t}\end{aligned} \tag{3.8}$$

where $P_{H,t}(i)$ is the price of domestic variety i, $P_{j,t}(i)$ is the price of variety i imported from country j, $W_{C,t}$ is the nominal wage rate per unit of hours worked in the commodity sector, $W_{H,t}$ is the nominal wage rate per unit of hours worked in the non-commodity sector, $\Omega_{C,t}$ denotes nominal dividends from ownership of commodity firms, and $\Omega_{H,t}$ denotes nominal dividends from ownership of non-commodity firms. $B_{F,t}$ is a foreign currency bond, with $\Gamma\left(.\right)$ as adjustment costs and e_t as the nominal exchange rate ie. the price of foreign currency in units of domestic currency. The price of the foreign bond is inversely proportional to the gross foreign nominal interest rate, $1+i_t^*$. $B_{H,t}$ is a domestic currency bond, with its price inversely proportional to the gross domestic nominal interest rate, $1+i_t$.

Domestic households take the quadratic adjustment cost, $\Gamma\left(B_{F,t}\right)=\frac{\kappa}{2}\left(B_{F,t}-B_{F,0}\right)^2$, as given when choosing their optimal holdings of the foreign bond. This cost is not needed for the characterization of incomplete financial markets in a stochastic open economy, but addresses the associated issue of equilibrium non-stationarity as discussed further in, for example, Schmitt-Grohé and Uribe (2003). $\Gamma\left(B_{F,t}\right)$ induces stationarity by linking consumption growth to asset holdings (which are prevented from growing or shrinking explosively over time due to adjustment costs) in the Euler Equation. I impose some restrictions on the function $\Gamma\left(.\right)$. It is differentiable and increasing in the aggregate level of foreign debt, $\Gamma'\left(.\right)>0$. Furthermore, varying $\kappa\in[0,\infty)$ is a convenient way of accounting for the different degrees of international risk-sharing arising from financial market imperfections, as capital mobility from the domestic small open economy becomes increasingly restricted as $\kappa\to\infty$.

Using the optimal demand functions for the domestic and imported good baskets, (3.5) and (3.6), as well as the CPI, (3.4), total consumption expenditures by the domestic household are given by $P_tC_t=P_{H,t}C_{H,t}+P_{F,t}C_{F,t}$. The budget constraint reduces to

$$\begin{aligned}&P_tC_t+e_t\left[B_{F,t}+\Gamma\left(B_{F,t}\right)\right]+B_{H,t}\\&\leq e_t\left(1+i_{t-1}^*\right)B_{F,t-1}+\left(1+i_{t-1}\right)B_{H,t-1}+W_{C,t}N_{C,t}+W_{H,t}N_{H,t}+\Omega_{C,t}+\Omega_{H,t}\end{aligned}\qquad(3.9)$$

The budget constraint binds as preferences are locally non-satiated. The first-order conditions on consumption and sector-specific labour supply can be combined to yield the following set of optimal labour supply conditions for the representative household

$$\eta^{-\frac{1}{\lambda}}N_t^{\frac{\phi(1+\lambda)-1}{1+\lambda}}N_{C,t}^{\frac{1}{\lambda}}C_t^{\sigma}=w_{C,t}\qquad(3.10)$$

$$(1-\eta)^{-\frac{1}{\lambda}}N_t^{\frac{\phi(1+\lambda)-1}{1+\lambda}}N_{H,t}^{\frac{1}{\lambda}}C_t^{\sigma}=w_{H,t}\qquad(3.11)$$

where $w_{C,t}=\frac{W_{C,t}}{P_t}$ is the real commodity sector wage, and $w_{H,t}=\frac{W_{H,t}}{P_t}$ is the real non-commodity sector wage. It is useful to note that, due to the dual labour market structure in this framework, compared to the majority of DSGE models that assume a single labour market, there are two optimal labour supply conditions instead of one. This distinction will play a crucial role in some of the results. Hours supplied by workers respond to two different wage rates, with the substitutability of hours worked across sectors governed by λ. The first-order condition on consumption can be combined with the optimal holdings of domestic and foreign bonds respectively, to yield the following set of Euler Equations

$$1 = (1+i_t)\,\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \tag{3.12}$$

$$1 = (1+i_t^*)\left(\frac{1}{1+\kappa b_{F,t}}\right) \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{Q_{t+1}}{Q_t} \right\} \tag{3.13}$$

where $Q_t = \frac{e_t P^*}{P_t}$ is the real exchange rate, $b_{F,t} = \frac{B_{F,t}}{P^*}$ are real foreign bond holdings, and P^* is the constant foreign CPI, which is taken as given by the domestic small open economy and normalized to one as in Portillo and Zanna (2015). I now discuss the risk-sharing and no-arbitrage conditions with incomplete financial markets. To do so, it is useful to consider the problem of the representative foreign household, which faces the following budget constant each period

$$P^* C_t^* + B_{F,t}^* \leq (1+i_{t-1}^*)\, B_{F,t-1}^* + W_t^* N_t^* + \Omega_t^*$$

where $C_t^*, W_t^*, N_t^*, \Omega_t^*$ are foreign consumption, wages, labour supply, and profits. $B_{F,t}^*$ denotes foreign holdings of the foreign currency bond. As foreigners have the same functional form for preferences as domestic households, the foreign Euler Equation is

$$1 = (1+i_t^*)\,\beta E_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{1}{\Pi^*} \right\} \tag{3.14}$$

Combining the intertemporal optimality conditions of the domestic and foreign households with respect to foreign bonds, (3.13) and (3.14), yields the Backus-Smith (Backus and Smith, 1993) international risk-sharing condition with incomplete financial markets

$$E_t \left\{ \frac{C_{t+1}}{C_t} \right\} = E_t \left\{ \frac{C_{t+1}^*}{C_t^*} \left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{1}{\sigma}} \right\} \left(\frac{1}{1+\kappa b_{F,t}}\right)^{\frac{1}{\sigma}} \tag{3.15}$$

The risk-sharing condition holds in expected future changes in relative consumption, $E_t\left(\frac{C_{t+1}}{C_t} / \frac{C_{t+1}*}{C_t^*}\right)$, and the real exchange rate, $E_t\left(\frac{Q_{t+1}}{Q_t}\right)$. (3.15) formalizes the limited scope for sharing risk internationally in a bond only world. Agents cannot hedge against asymmetric country-specific shocks, so that the smoothing of consumption is limited.

As asset market arbitrage opportunities do not exist, domestic investors are indifferent between holding domestic currency bonds and foreign currency bonds (taking into account adjustment costs on the latter). This makes it possible to derive an equilibrium condition linking the nominal interest and exchange rates by combining (3.12) and (3.13) to yield a version of the uncovered interest rate parity (UIP) condition with incomplete financial markets

$$1 + i_t = (1 + i_t^*)\, E_t \left(\frac{e_{t+1}}{e_t} \right) \left(\frac{1}{1 + \kappa b_{F,t}} \right) \tag{3.16}$$

Note that bond adjustment costs drive a wedge between the expected change in nominal exchange rates and the spread in nominal interest rates. (3.16) implies that the nominal exchange rate is expected to adjust upon shocks to equalize the domestic currency returns on domestic and foreign bonds.

3.3.2 Relative Prices

Domestic and international prices are defined in relative terms and normalized by the CPI, P_t, as in Ferrero and Seneca (2015). Key relative prices are the relative domestic non-commodity price index, $p_{H,t} = \frac{P_{H,t}}{P_t}$, the relative import price index, $p_{F,t} = \frac{P_{F,t}}{P_t}$, and the relative domestic commodity price index, $p_{C,t} = \frac{P_{C,t}}{P_t}$. It is possible to retrieve all relative prices upon determining $S_{H,t}$, the non-commodity terms of trade, defined as the relative price of imported to domestic goods

$$S_{H,t} = \frac{p_{F,t}}{p_{H,t}} \tag{3.17}$$

Noting that the CPI, (3.4), can be written as a function of the terms of trade as $P_t^{1-\varepsilon} = P_{H,t} \left[1 - \alpha + \alpha S_{H,t}^{1-\varepsilon} \right]$, relative domestic non-commodity and import prices are given by

$$p_{H,t} = \left[1 - \alpha + \alpha S_{H,t}^{1-\varepsilon} \right]^{-\frac{1}{1-\varepsilon}} \qquad p_{F,t} = S_{H,t} \left[1 - \alpha + \alpha S_{H,t}^{1-\varepsilon} \right]^{-\frac{1}{1-\varepsilon}} \tag{3.18}$$

The real CPI exchange rate, $Q_t = \frac{e_t P^*}{P_t}$, is defined as the domestic currency price of a foreign basket of consumption, $e_t P^*$, relative to that of a domestic basket of consumption, P_t, where e_t is the nominal exchange rate ie. the price of foreign currency in units of domestic currency and P^* is the constant world CPI Gali and Monacelli (2005). Q_t can be expressed in terms of $S_{H,t}$. Assuming that purchasing power parity holds in the imported goods market, so that $P_{F,t} = e_t P_F^*$, and that from the perspective of the domestic economy, which is small, the world as a whole behaves like a closed economy, so that $P^* = P_F^*$

$$Q_t = \left[(1-\alpha)S_{H,t}^{\varepsilon-1} + \alpha\right]^{\frac{1}{\varepsilon-1}} \tag{3.19}$$

As the domestic economy is small, the international price of commodities, $P_{C,t}^*$, is taken as given. Further, purchasing power parity (PPP) holds for commodities traded worldwide, so that the domestic price of commodities, $P_{C,t}$, is related to the exogenous international price as $P_{C,t} = e_t P_{C,t}^*$. Using the definition of the real exchange rate, (3.19), and the fact that the relative international price, $p_{C,t}^* = \frac{P_{C,t}^*}{P^*}$, is exogenous, the relative domestic price of commodities is given by

$$p_{C,t} = Q_t p_{C,t}^* \tag{3.20}$$

The commodity terms of trade, $S_{C,t}$, is a ratio of relative prices, $S_{C,t} = \frac{p_{F,t}}{p_{C,t}}$. Noting however that PPP holds for imported and commodity goods, the commodity terms of trade fluctuates exogenously, ie. $S_{C,t} = \frac{p_F^*}{p_{C,t}^*}$, as is approximately the case in small, commodity-exporting economies (Cashin et al., 2004).

3.3.3 Firms

As commodity exporters rely heavily on commodity exports as primary sources of revenue, the framework incorporates explicit macroeconomic dependence on commodities. Heterogeneous production activities in the economy are undertaken by (i) commodity firms and (ii) non-commodity firms. Both are owned by domestic households. While commodities are fully exported, non-commodity goods are supplied to both domestic and foreign agents. Further, as the economy in question is small and open, the international price of commodities is taken as given. The domestic commodity price, however, is influenced by exchange rate and sectoral dynamics. Non-commodity firms are monopolistic and set prices in a staggered manner, which gives rise to inefficient price dispersion.

3.3.3.1 Commodity Sector

Commodity firms, $h \in [0,1]$ are perfectly competitive and are subject to an industry-wide decreasing returns to scale technology. Commodity goods, $Y_{C,t}$, produced in this sector are demanded by foreign agents. An individual firm h employs the following production function

$$Y_{C,t}(h) = A_{C,t}\bar{K}N_{C,t}^{1-\psi}(h) \tag{3.21}$$

where $A_{C,t}$ is overall productivity in the non-commodity sector, $N_{C,t}(h)$ is labour demanded by commodity firm h, $\bar{K}$ is industry-wide available capital and land used in the production of commodities that is fixed, and $\psi \in [0,1]$ measures the extent of deviation from constant returns to scale. $\psi > 0$, common across the sector, implies positive profits for firm h. The cost incurred by commodity firms in hiring workers is the commodity sector wage, $W_{C,t}$. Noting that $P_{C,t}$ is the domestic currency price of commodities, the profit maximization problem faced by firm h is given by

$$\underset{\tilde{P}_{C,t}(h)}{Max}\ \tilde{P}_{C,t}(h)A_{C,t}\bar{K}N_{C,t}^{1-\psi}(h) - W_{C,t}N_{C,t}(h) \tag{3.22}$$

The aggregate production function in this sector is given by $Y_{C,t} = A_{C,t}\bar{K}N_{C,t}^{1-\psi}$, which is derived by imposing the labour market clearing condition, $N_{C,t} = \int_0^1 N_{C,t}(h)dh$, and the condition that the supply of all individual commodities are met by foreign demand. In a symmetric equilibrium, the same optimal price, $\tilde{P}_{C,t}$, is chosen by all commodity firms, so that aggregate employment in this sector is given by

$$N_{C,t}^{\psi} = (1-\psi)\left(\frac{W_{C,t}}{\tilde{P}_{C,t}(h)}\right)^{-1}\bar{K}A_{C,t} \tag{3.23}$$

Commodity sector profits are correspondingly

$$\Omega_{C,t} = \psi\tilde{P}_{C,t}Y_{C,t} \tag{3.24}$$

3.3.3.2 Non-Commodity Sector

In a standard problem, non-commodity firms, $i \in [0,1]$, are monopolistic and set prices in a staggered fashion according to the Calvo (1983) pricing scheme. Every period, a fraction $(1-\theta)$ of firms can re-optimize prices. Fraction θ of firms cannot re-optimize. Labour demand is instead adjusted to meet changes in output demand upon shocks. Firms that do reset prices take into account that the probability of keeping this period's price k periods ahead is given by θ^k.

With production function $Y_{H,t}(i) = A_{H,t}N_{H,t}\,(i)$, each reoptimizing firm i sets its optimal reset price as a markup over current and expected future marginal costs, where $MC_{H,t}(i) = W_{H,t}/A_{H,t}P_{H,t}(i)$, giving rise to domestic inflation. Noting that a firm that reoptimizes in period t will choose the price $\tilde{P}_{H,t}(i)$ that maximizes current and future discounted profits until period $t+k$ while this price remains effective, so that $\tilde{P}_{H,t+k}(i) = \tilde{P}_{H,t}(i)$ for $k = 0, ..., \infty$, the optimal reset price at time t solves

$$\underset{\tilde{P}_{H,t}(i)}{Max}\, E_t \sum_{k=0}^{\infty} \theta^k Z_{t,t+k} \left\{ \tilde{P}_{H,t}(i) Y_{H,t+k}(i) - W_{H,t+k} N_{H,t+k}(i) \right\}$$

$$s.t.\;\; Y_{H,t+k}(i) = \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t}} \right)^{-\nu} \left(C_{H,t+k}(i) + C^{*}_{H,t+k}(i) \right) \tag{3.25}$$

where $Y_{H,t+k}(i)$ and $W_{H,t+k}N_{H,t+k}(i)$ are respectively the output and total cost in period $t+k$ for a firm that last reset its price in period t, $E_t Z_{t,t+k} = (1+i_t)^{-1}$ is the price of the domestic currency bond, and $C_{H,t+k}(i)$ and $C^{*}_{H,t+k}(i)$ represent demand for good i in period $t+k$ respectively by domestic and foreign consumers. In a symmetric equilibrium, the same price is chosen by all firms that can re-optimize so that $\tilde{P}_{H,t}(i) = \tilde{P}_{H,t} \forall i$. As shown in Benigno (2009), the first-order condition is

$$E_t \sum_{k=0}^{\infty} \theta^k Z_{t,t+k} Y_{H,t+k}(i) \left[\tilde{P}_{H,t} - \frac{\nu}{\nu-1} \frac{W_{H,t+k}}{A_{H,t+k}} \right] = 0 \tag{3.26}$$

Combining (3.25), the labor-market clearing condition, $N_{H,t} = \int_0^1 N_{H,t}(i) di$, the price index associated with the demand for monopolistic goods $P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\nu} di \right]^{\frac{1}{1-\nu}}$, and the definition of price dispersion, $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\nu} di$, which follows law of motion

$$\Delta_t^{\frac{\nu-1}{\nu}} = \rho \Pi_{H,t}{}^{\nu-1} \Delta_{t-1} + (1-\theta) \frac{1-\theta \Pi_{H,t}{}^{\nu-1}}{1-\theta} \tag{3.27}$$

we get the aggregate production function

$$Y_{H,t} \Delta_t = A_{H,t} N_{H,t} \tag{3.28}$$

Using the price index and definition of price dispersion as in Galí (2007), the optimal reset price, (3.26), relates to the domestic inflation rate, $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$, as follows

$$\frac{\tilde{P}_{H,t}}{P_{H,t}} = \left(\frac{1-\theta \Pi_{H,t}{}^{\nu-1}}{1-\theta} \right)^{\frac{1}{1-\nu}} \tag{3.29}$$

3.3.4 Central Bank

The Central Bank follows a monetary policy rule of the following form

$$\left(\frac{1+i_t}{1+r_0}\right) = \left(\frac{1+\pi_{H,t}}{1+\pi_{H,0}}\right)^{\phi_\pi} \left(\frac{e_t}{e_0}\right)^{\phi_e} \tag{3.30}$$

where r_0 is the steady state real interest rate, and $\pi_{H,0}$ and e_0 are the steady state inflation rate and nominal exchange rate respectively. In the model simulations, exchange rate targeting will be represented by $\phi_e > 1, \phi_\pi = 0$, and flexible exchange rates will be represented by $\phi_e = 0, \phi_\pi > 1$.

3.3.5 Market-Clearing and Accounting

3.3.5.1 Goods, Labour, National Accounts

The demand for each non-commodity good $i \in [0, 1]$, ie. $Y_{H,t}(i)$, is

$$Y_{H,t}(i) = C_{H,t}(i) + C^*_{H,t}(i) \tag{3.31}$$

The demand for commodities exported by the domestic economy, $Y_{C,t}$, is fully met internationally so that $Y_{C,t}(h) = Y^*_{C,t}(h)$ for each individual variety, $h \in [0, 1]$. Hours worked by domestic households are demanded by the commodity and non-commodity sectors as follows

$$N_{C,t} = \int_0^1 N_{C,t}(h)dh \qquad N_{H,t} = \int_0^1 N_{H,t}(i)di \tag{3.32}$$

Real gross domestic product (GDP), adjusted by the CPI, is

$$GDP_t = p_{C,t}Y_{C,t} + p_{H,t}Y_{H,t} \tag{3.33}$$

which can also be written as the sum of domestic consumption, C_t, and net exports, NX_t, so that $GDP_t = C_t + NX_t$, where net exports are defined as the difference between domestic production and consumption. Adjusted by the CPI, real net exports are

$$NX_t = p_{C,t}Y_{C,t} + p_{H,t}Y_{H,t} - C_t \tag{3.34}$$

3.3.5.2 Assets

Domestic currency bonds, $B_{H,t}$, are in zero net supply domestically, and the market for foreign currency bonds clears internationally so that

$$B_{F,t} + B^*_{F,t} = 0 \tag{3.35}$$

The evolution of net foreign assets held by the domestic economy is required to characterize the equilibrium as adjustment costs, $\Gamma(.)$, depend on the level of foreign bond holdings and affect the ability of domestic households to smooth consumption through the risk-sharing condition, (3.15). The evolution of foreign bonds, adjusted by the foreign CPI, is derived from the representative domestic household's budget constraint (3.8) by replacing in for profits, to yield

$$Q_t\left[b_{F,t} + \Gamma\left(b_{F,t}\right)\right] = NX_t + \frac{1}{\beta}Q_t b_{F,t-1} \tag{3.36}$$

where $b_{F,t} = \frac{B_{F,t}}{P^*}$ with $P^* = 1$ and $\beta = \frac{1}{1+i^*_t}$.

3.3.6 Equilibrium

I define an equilibrium from the perspective of the domestic economy, assuming that the no-Ponzi and transversality conditions are satisfied, and that the initial net foreign asset position is zero. For any specification of monetary policy, (3.30), which determines the nominal interest rate, i_t, the equilibrium is a sequence of prices

$$\{S_{H,t}, Z_{t,t+1}, \Pi_{H,t}, \Pi_t, p_{C,t}, MC_t, W_{C,t}, \Delta_t, e_t\}_{t=0}^{\infty}$$

and quantities

$$\{C_t, C_{H,t}, C_{F,t}, N_t, N_{H,t}, N_{C,t}, Y_{H,t}, Y_{C,t}, B_{F,t}\}_{t=0}^{\infty}$$

such that

- Households optimize: (3.10), (3.11), (3.13), (3.3), (3.5), and (3.6)
- International risk-sharing and UIP conditions hold: (3.15) and (3.16)
- Commodity firms optimize: (3.21) and (3.23)

- The relative price of commodities is given by (3.20)
- Non-commodity firms optimize: (3.27), (3.28), and (3.29)
- Goods, (3.31), labour, (3.32), and asset, (3.35), markets clear
- Net foreign assets evolve according to (3.36)

taking as given initial conditions, $B_{F,0}$, Δ_0, and $S_{H,0}$, and exogenous processes for shocks and foreign variables $\left\{p^*_{C,t}, A_t, X_t, C^*_t, i^*_t\right\}_{t=0}^{\infty}$.[6] $S_{H,t}$, given by (3.17), is the only relative price required for the characterization of equilibrium.

3.4 Calibration

I analyze equilibrium dynamics based on a calibrated version of the framework for a representative agricultural commodity-exporting economy. The degree of labour market flexibility, or the elasticity of substitution in hours worked across sectors, λ, is calibrated at 0.8. I set λ to be lower than the estimate of 1 for a developed economy (the United States) from Horvath (2000), as a specific value for λ is not available for developing countries where labour markets are typically far more rigid (Fields, 2009; Artuc et al., 2013). The degree of product market flexibility, or the Armington (1969) trade elasticity of substitution between domestic non-commodity goods and imports, ε, is calibrated at 0.8. While a specific estimate of ε is also not available for developing economies, this parameter has been estimated to be a little over 1 on average for advanced economies in a range of studies summarized by Feenstra et al. (2014). It is likely that ε is much lower in developing countries where domestically produced goods are typically of much lower quality than imports, implying that home and imported products are poor substitutes (Hummels and Klenow, 2005; Henn et al., 2013).

I pair the baseline calibration of λ and ε with extensive sensitivity analysis to assess the implications of rigid versus flexible markets for the appropriate choice of an exchange rate regime. The rest of the baseline parameterization is as follows. Following Kose et al. (2006), limited financial integration captured by the parameter on the adjustment costs for the household's holding of foreign bonds, κ, is set at 0.1, which is 10 times higher than the

[6] The exogenous variables are determined according to stationary autoregressive processes of the following functional form: $B_t = B_0^{1-\rho_B} B_{t-1}^{\rho_B} \exp\left\{\epsilon_B\right\}$, where $B_t \equiv \left\{p^*_{C,t}, A_t, X_t, C^*_t, i^*_t\right\}$, B_0 is the steady state value of B_t, and ϵ_B is a shock.

analogous calibration used for developed countries (Benigno, 2009). Calibration of the other structural parameters for small, commodity-exporting economies is challenging, as there is low availability of micro-level data. The parameters are hence selected from existing open economy studies with respect to developing countries, and paired with sensitivity analysis. Consistent with the estimates in Berg et al. (2013), the intertemporal elasticity, σ, equals 2. I set the extent of openness to trade, α, at 0.5 and the inverse Frisch elasticity, ϕ, at 5, which implies fairly inelastic labour supply. I calibrate the returns to scale in the commodity sector, ψ, at 0.1, consistent with the micro-level evidence on diminishing returns to agricultural production (FAO, 2001).

The steady state share of employment in the commodity sector, η, is set at 0.3, which is the average value across the agricultural commodity exporters reported in Table 3.2. This value for the steady state is supported by the choice of calibration of the structural parameters. I consider strict exchange rate and inflation targeting (sufficiently high weights on ϕ_e and ϕ_π respectively in the monetary rule, (3.30), and pair this with sensitivity on the weights. Following standard values in the literature and pairing these with sensitivity analysis, price stickiness, θ, is set at 0.75, implying that the price is reset on average every year. I set the discount factor, β, at 0.99. The elasticity of substitution, ν, is calibrated at 4, implying a markup of around 30% at steady state. The persistence of shocks, ie. ρ_b in the autoregression $b_t = \rho_b b_{t-1} + \epsilon_{b,t}$, where $b_t \equiv \{\hat{p}^*_{C,t}, a_t, x_t, c^*_t, i^*_t\}$, is set at 0.9, as in Aguiar and Gopinath (2007).

3.5 Exchange Rate Choices in Inflexible Markets

This section assesses whether fixed or flexible exchange rates are more appropriate in agricultural commodity exporters. Section 3.5.1 describes how the economy responds to a negative shock to the commodity export price, differentiating between peg versus float (domestic inflation targeting). Sensitivity analysis confirms that the dynamics hold robust with alternative calibration schemes. Sections 3.5.2 and 3.5.3 discuss how the macroeconomic response through relative price and wage adjustments is affected by labour and product market rigidity. Sections 3.5.4 and 3.5.5 assess the welfare implications of fixed versus flexible exchange rate regimes, contingent on the degree of flexibility of domestic markets.

3.5.1 Dynamics under Alternate Exchange Rate Regimes

Figure 3.2 simulates the economy with a 5% unexpected fall in the international price of agricultural commodities. I solve for the equilibrium dynamics in Section 3.3.6 using second-order perturbation techniques around a deterministic steady state (described in the appendix), and calibrate the parameters based on the discussion in Section 3.4. I contrast dynamics with an exchange rate peg versus float. As commodity export revenues fall, labour demand in the commodity sector contracts and wages fall. Consumption falls with the decline in income, as households cannot hedge against the adverse shock due to the lack of financial deepening and asset market insurance. Due to the fall in consumption demand for non-commodity goods, non-commodity labour demand decreases and wages fall. Correspondingly, non-commodity prices decline, and real depreciation ensues.

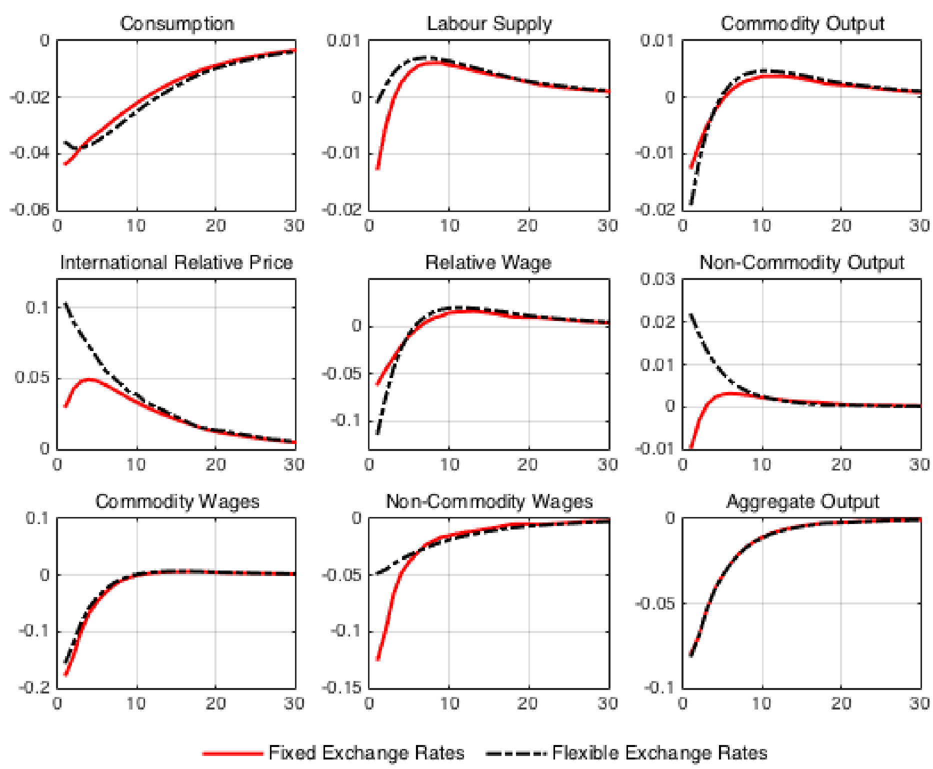

Figure 3.2: 5% Negative Shock to the International Price of Commodities: Fixed versus Flexible Exchange Rate Regimes

The key difference between fixed versus flexible exchange rates lies in the extent of real depreciation. While non-commodity production falls under fixed exchange rates due to the decrease in domestic demand, it increases under a float as greater real depreciation allows the impact of relatively higher foreign purchasing power to carry through. Correspondingly, non-commodity wages do not fall by as much under a float. Commodity output declines further, however, as labour re-allocates to the non-commodity sector driven by the relative increase in marginal product there. Commodity sector wages also decline by less with flexible exchange rates due to the outflow of workers. Consumption is more stable under a float due to the higher wage income compared to a peg.

The welfare properties of peg versus float with underdeveloped markets, discussed in Section 3.5.4 depend on how these regimes affect *relative* price and relative wage fluctuations. Result 1 discusses the relative price and wage effects of exchange rate flexibility.

Result 1. *Exchange rate flexibility amplifies relative wage and price fluctuations.*

Flexible exchange rates allow for a greater fall in the price of non-commodity goods relative to imports. This increases the international price differential, acting as a "shock absorber" by allowing for more real depreciation in the face of restricted domestic price adjustment. The wage differential, which measures the value of commodity wages relative to non-commodity wages, also increases. Commodity wages initially fall by more than non-commodity wages, as they are directly affected by the negative shock to commodity export revenues. As the greater foreign purchasing power under a float incentivizes non-commodity production compared to a peg, there is a further relative increase in the marginal product of non-commodity labour. This increases the wage differential.

Sensitivity The equilibrium response to a negative commodity price shock is analyzed for the following empirically relevant range of parameters: openness, $\alpha \in [0.2, 0.8]$, bond adjustment costs, $\kappa \in [0.01, 100]$, price stickiness, $\theta \in [0.4, 0.8]$, inverse elasticity of intertemporal substitution, $\sigma \in [0.5, 5]$, inverse elasticity of labour supply, $\phi \in [1, 10]$, decreasing returns to scale, $\psi \in [0.1, 0.6]$, elasticity of substitution between individual varieties, $\nu \in [4, 8]$, shock persistence, $\rho_b \in [0.5, 0.9]$, monetary rule flexibility, $\phi_e, \phi_\pi \in [1.5, \infty]$, elasticity of labour supply between sectors, $\lambda \in [0.5, \infty]$, and elasticity of substitution between domestic and foreign aggregates, $\varepsilon \in [0.5, 5]$. The dynamics do not differ in terms of direction, or in relative terms between the exchange rate regimes; however, the following parameter changes affect quantitative magnitudes in interesting ways.

An increase in α implies that households consume a higher fraction of relatively more expensive imports compared to domestic non-commodity goods. For any given change in the international relative price, consumption thus declines by more. Non-commodity output is higher as greater openness increases the consequences of greater foreign purchasing power upon the shock, and commodity output is lower as labour re-allocates away. The wage differential increases due to higher non-commodity wages. An increase in bond holding costs, κ, implies that there is less international financial integration and dynamics become more volatile. For any given change in the international relative price, consumption decreases more, and non-commodity output follows suit. Commodity output falls by less due to labour re-allocation, and the wage differential decreases due to the fall in non-commodity output. As $\theta \to 0$ (prices become more flexible), the dynamics under peg and float converge.

A decrease in ϕ (more elastic labour supply) implies that labour supplied by households is allowed to fall by more upon the shock. In response, output across the economy falls but aggregate wages are pulled up due to the shortfall in labour supply. This smooths consumption, leading to lower real depreciation. As σ decreases (more elastic response of consumption), consumption falls by more, and non-commodity output follows suit. This eases the downward pressure on non-commodity prices, leading to less real depreciation. The lower depreciation results in a greater fall in commodity output. Note that the above sensitivity analysis holds for both peg and float. Small adjustments to the commodity returns to scale, the elasticity of substitution between varieties, or monetary rule flexibility, do not produce significant changes in dynamics. The roles of greater labour and product market flexibility, reflected in higher values of λ and ε, are the focus of this paper and highlighted next.

3.5.2 Labour Market Rigidity

The baseline calibration, with $\lambda = 0.8$, reflects the under-developed labour markets in agricultural commodity exporters. An increase in labour market flexibility, ie. as $\lambda \to \infty$, implies that for any given wage differential, $\frac{w_{C,t}}{w_{H,t}}$, relative hours supplied, $\frac{N_{C,t}}{N_{H,t}}$, adjust by more. This has opposing effects on household consumption and the monetary transmission mechanism. On the one hand, workers are freer to migrate to the sector with relatively higher wages, which leads to relatively *higher* consumption for any given amount of labour effort. On the other hand, there is a greater outflow of workers from the lagging commodity sector toward the non-commodity sector. This puts additional downward pressure on non-commodity wages, contributing to relatively *lower* consumption for any given amount of labour effort. The first effect generally dominates so that consumption is smoother, and regardless of exchange rate regime, labour market flexibility has the following impact on relative wages and prices.

Result 2. *Labour market flexibility stabilizes relative wages but increases price differentials.*

Unlike the case of exchange rate flexibility, which amplifies relative wage and price fluctuations, labour market flexibility serves to narrow the wage differential between sectors. As labour becomes more mobile, sectoral output dynamics are amplified with more workers migrating away from the lagging commodity sector toward the non-commodity sector. The influx of labour in the non-commodity sector and corresponding surge in production puts downward pressure on non-commodity prices, increasing the international price differential. However, the outflow of workers from the commodity sector in search of a higher marginal product puts downward pressure on non-commodity wages while easing the fall in commodity wages. This stabilizes relative wages.

3.5.3 Product Market Rigidity

The baseline calibration, with $\varepsilon = 0.8$, reflects the limited ability of agents to substitute domestic goods for better-quality imports in agricultural commodity exporters. An increase in product market flexibility increases opportunities to smooth consumption by re-allocating expenditure toward relatively cheaper goods. Consumption remains more stabilized with higher ε upon the commodity price shock, with either peg or float and regardless of the degree of labour market flexibility.

Result 3. *Product market flexibility stabilizes relative prices but increases wage differentials.*

While labour market flexibility narrows the wage differential between sectors while exacerbating relative price fluctuations, product market flexibility does the opposite. An increase in ε allows agents to consume more, which mitigates real depreciation and narrows the international price differential. Recall that non-commodity wages decrease *more* with flexible labour markets, as workers face a lower cost of labour re-allocation and can thus be compensated less. However, non-commodity wages decrease *less* with flexible product markets as the non-commodity sector can afford to pay higher wages due to higher consumption demand. This increases the wage differential.

3.5.4 Welfare

Table 3.3 reports whether fixed or flexible exchange rates are preferred for different degrees of labour and product market flexibility The corresponding loss numbers are also provided. The losses are measured in consumption equivalent units, ζ, as in Gali and Monacelli (2005). The consumption equivalent is a useful measure of welfare costs as utility, (3.1), is not cardinal. ζ is defined as the constant fraction of consumption households have to give up each period starting from the steady state, to equate the present discounted value of welfare under a peg with the present discounted value of welfare under a float. ζ solves

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{[(1-\zeta)C_{t,e}]^{1-\sigma}}{1-\sigma} - \frac{N_{t,e}^{1+\phi}}{1+\phi} \right] \right\} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{t,\pi}^{1-\sigma}}{1-\sigma} - \frac{N_{t,\pi}^{1+\phi}}{1+\phi} \right] \right\} \tag{3.37}$$

where $C_{t,e}$ and $N_{t,e}$ are consumption and labour supply under a peg, and $C_{t,\pi}$ and $N_{t,\pi}$ are consumption and labour supply under a float. Note that $\zeta > 0$ when labour and product markets are rigid (implying that "Peg" is appropriate), but the consumption equivalent becomes negative as markets becomes more flexible (implying that "Float" is appropriate). This brings us to a key insight of this study in Result 4 (robust to sensitivity analysis), which draws upon Results 1-3 that greater flexibility in exchange rates and markets amplifies relative wage and/or relative price fluctuations.

Result 4. *Exchange rate targeting leads to higher welfare than a float in agricultural commodity exporters.*

As agricultural commodity exporters tend to be developing economies, their labour and product markets are typically more rigid (Hummels and Klenow, 2005; Fields, 2009; Artuc et al., 2013; Henn et al., 2013). With inflexible real markets, and limited financial opportunities to insure against country-specific risk, relative wage and price fluctuations exacerbate currency and factor misalignments. Instead of allowing for more efficient economic adjustment, price fluctuations in inflexible markets lead to the misallocation of consumption and employment. The Central Bank prefers to mitigate international relative price fluctuations, and correspondingly relative wage fluctuations through labour re-allocation, by targeting the nominal exchange rate. As real markets become more flexible, agents are better able to respond to relative wage and price fluctuations. In this case, flexible exchange rates are preferred as they increase international price and wage differentials, which amplifies the adjustment mechanism. This result provides a novel rationale as to why over 70% of agricultural commodity exporters have exchange rate anchors, in contrast to conventional arguments in favor of flexible exchange rates.

Welfare Ranking of Fixed vs Flexible Regimes						
	Product Markets					
Labour Markets		$\varepsilon = 0.4$	$\varepsilon = 0.6$	$\varepsilon = 0.8$	$\varepsilon = 1.0$	$\varepsilon = 1.6$
	$\lambda = 0.4$	**Fixed**	**Fixed**	**Fixed**	Flex	Flex
	$\lambda = 0.6$	**Fixed**	**Fixed**	**Fixed**	Flex	Flex
	$\lambda = 0.8$	**Fixed**	**Fixed**	**Fixed**	Flex	Flex
	$\lambda = 1.0$	**Fixed**	**Fixed**	**Fixed**	Flex	Flex
	$\lambda = 1.6$	**Fixed**	**Fixed**	Flex	Flex	Flex

Welfare Loss (% Consumption Equivalent)						
	Product Markets					
Labour Markets		$\varepsilon = 0.4$	$\varepsilon = 0.6$	$\varepsilon = 0.8$	$\varepsilon = 1.0$	$\varepsilon = 1.6$
	$\lambda = 0.4$	**8.21**	**2.35**	**0.29**	-0.06	-0.15
	$\lambda = 0.6$	**6.07**	**1.37**	**0.19**	-0.10	-0.19
	$\lambda = 0.8$	**3.81**	**0.91**	**0.11**	-0.13	-0.23
	$\lambda = 1.0$	**2.35**	**0.65**	**0.06**	-0.16	-0.26
	$\lambda = 1.6$	**0.93**	**0.28**	-0.06	-0.23	-0.35

Table 3.3: Welfare Rankings (in terms of ζ) with a 5% Commodity Price Fall, Fixed versus Flexible Exchange Rate Regimes

3.5.5 Robustness

I find that Result 4 is robust to alternative calibration schemes. To assess how the welfare properties of fixed versus flexible exchange rates are affected with a range of parameters, I do sensitivity analysis as described in Section 3.5.1. There are some interesting findings. The welfare rankings of peg versus float in Table 3.3 are robust to changes in calibration of most parameters. However, varying the degree of trade openness, α, and the degree of international capital mobility, κ, makes a significant difference. These results are reported in Tables 3.4 and 3.5. Greater trade openness and more limited international financial integration, characteristic of developing commodity-exporting economies, *increase* the case for fixed exchange rates, thus strengthening the results.

Welfare Ranking of Fixed vs Flexible Regimes: Higher Openness ($\alpha = 0.8$)						
		Product Markets				
		$\varepsilon = 0.4$	$\varepsilon = 0.6$	$\varepsilon = 0.8$	$\varepsilon = 1.0$	$\varepsilon = 1.6$
Labour Markets	$\lambda = 0.4$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	**Fixed**
	$\lambda = 0.6$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	**Fixed**
	$\lambda = 0.8$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	**Fixed**
	$\lambda = 1.0$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	**Fixed**
	$\lambda = 1.6$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	**Fixed**

Welfare Loss (% Consumption Equivalent)						
		Product Markets				
		$\varepsilon = 0.4$	$\varepsilon = 0.6$	$\varepsilon = 0.8$	$\varepsilon = 1.0$	$\varepsilon = 1.6$
Labour Markets	$\lambda = 0.4$	**15.23**	**13.20**	**4.10**	**1.46**	**0.29**
	$\lambda = 0.6$	**9.18**	**8.17**	**3.11**	**1.38**	**0.25**
	$\lambda = 0.8$	**6.54**	**5.14**	**2.48**	**1.26**	**0.23**
	$\lambda = 1.0$	**5.91**	**3.74**	**2.07**	**1.15**	**0.22**
	$\lambda = 1.6$	**3.05**	**2.13**	**1.42**	**0.91**	**0.21**

Table 3.4: Welfare Properties (in terms of ζ) with a 5% Commodity Price Fall, Fixed versus Flexible Exchange Rate Regimes, Higher Openness

The Central Bank faces a trade-off between mitigating the price dispersion distortion (increasing the case for inflation targeting and flexible exchange rates) and mitigating costly real exchange rate misalignments due to the incomplete financial market distortion (increasing the case for exchange rate targeting). In a more open economy ($\alpha = 0.8$), more imported products are consumed relative to domestic goods. This implies that real exchange rate misalignments lead to a greater misallocation of domestic resources. To mitigate this, a peg is preferred even with flexible labour and product markets. With low international financial integration ($\kappa = 100$), real exchange rate misalignments are also more costly. This is because agents are restricted from efficiently responding to shocks through adjustments in their asset portfolios. Greater flexibility in labour and product markets cannot fully overcome the limited ability to share risk internationally, increasing the case for exchange rate targeting.

Welfare Ranking of Fixed vs Flexible Regimes: Lower financial integration ($\kappa = 100$)						
	Product Markets					
Labour Markets		$\varepsilon = 0.4$	$\varepsilon = 0.6$	$\varepsilon = 0.8$	$\varepsilon = 1.0$	$\varepsilon = 1.6$
	$\lambda = 0.4$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	Flex
	$\lambda = 0.6$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	Flex
	$\lambda = 0.8$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	Flex
	$\lambda = 1.0$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	Flex
	$\lambda = 1.6$	**Fixed**	**Fixed**	**Fixed**	**Fixed**	Flex

Welfare Loss (% Consumption Equivalent)						
	Product Markets					
Labour Markets		$\varepsilon = 0.4$	$\varepsilon = 0.6$	$\varepsilon = 0.8$	$\varepsilon = 1.0$	$\varepsilon = 1.6$
	$\lambda = 0.4$	**9.40**	**7.62**	**1.41**	**0.05**	-0.47
	$\lambda = 0.6$	**6.31**	**4.28**	**1.05**	**0.05**	-0.46
	$\lambda = 0.8$	**3.22**	**2.88**	**0.84**	**0.04**	-0.44
	$\lambda = 1.0$	**2.82**	**2.14**	**0.70**	**0.03**	-0.43
	$\lambda = 1.6$	**2.02**	**1.41**	**0.48**	**0.02**	-0.32

Table 3.5: Welfare Properties (in terms of ζ) with a 5% Commodity Price Fall, Fixed versus Flexible Exchange Rate Regimes, Lower Financial Integration

3.6 Conclusion

In light of the significant economic dependence of agricultural commodity-exporting economies on volatile and downward-trending commodity prices, this study sought to further our understanding on the appropriate choice of an exchange rate regime. A small open economy framework incorporating structural characteristics of agricultural commodity exporters was developed, and the roles of labour and product market flexibility were analyzed in influencing exchange rate choices. The results provide novel insights into whether exchange rate targeting, as adopted by over 70% of agricultural commodity exporters in practice, might be appropriate. In a counterpoint to the conventional wisdom since Friedman (1953) on the optimality of flexible exchange rates, it is shown that fixed exchange rates are desirable in underdeveloped markets as they mitigate costly relative price and wage adjustments.

The results from this study suggest that the exchange rate anchors adopted by the majority of agricultural commodity-exporting economies are appropriate given their current low stages of development. The analysis further indicates, however, that these countries should transition to flexible exchange rates and inflation targeting as their economies mature and become more flexible over time. The study complements recent empirical evidence that inflation targeting could lead to higher macroeconomic volatility in developing economies (Samarina et al., 2014), and cautions against recommending exchange rate flexibility to countries without taking into account the degree of rigidity of their labour and product markets. In future work, it would be useful to extend the analysis to oil and metal commodity-exporting economies with capital-intensive production.

Chapter 4

Capital Controls in Tanzania: An Investigation

"Temptation: the fiend at my elbow." - William Shakespeare

4.1 Introduction

This study examines whether capital control measures, employed several times over recent years in Tanzania, achieved their purpose of stabilizing the economy's exchange rate.[1] Tanzania maintains a *de jure* flexible exchange rate regime, under which the Central Bank does not pursue an explicit target value for the official exchange rate (Calvo and Reinhart, 2002; IMF, 2016a). However, the Bank of Tanzania *de facto* intervenes in the interbank foreign exchange rate market (IFEM) to pursue a limited number of objectives, including avoiding destabilizing short-run movements in the IFEM exchange rate.[2] To complement its conventional monetary policy and intervention instruments, the Bank of Tanzania has implemented changes in the regulations concerning capital account transactions at various times in recent years. The centrepiece of these measures consisted of a progressive reduction in Tanzanian commercial banks' permitted net open (i.e. uncovered) foreign exchange position. The central net open position limits were reduced three times between October 2011 and April 2015 (Kessy et al., 2016).

[1]The proprietary data for this chapter was given by the Bank of Tanzania to my supervisor, Christopher Adam in his role as Lead Academic for Tanzania at the International Growth Centre. This chapter is based on a suggestion by Chris, who has generously shared his deep expertise on Tanzania and provided several inputs throughout the study.

[2]See Section 4.2 for an overview of monetary policy in Tanzania.

The declared rationale for these complementary capital control measures is to put 'sand in the wheels' so as to stabilize the Tanzanian shilling by limiting its (short-run) rate of depreciation. While this might be a tempting option, there is little evidence to back the success of these measures, which would support the continued usage of such capital outflow controls in the future. This study provides suggestive evidence that the effectiveness of the controls declined the more they were used, raising concerns about the value of these instruments as part of the Bank of Tanzania monetary policy toolkit.

The efficacy of capital controls is evaluated using an event study approach. This methodology is typically used to analyze issues such as the impact of policy interventions and announcements (reviewed in MacKinlay, 1997 and Neely, 2005). Events in this paper comprise of identified specific changes in capital controls, and the behavior of the exchange rate is analyzed during the days surrounding each event. Particular attention is paid to the direction and magnitude of ex-post abnormal returns, which represent the deviation of the observed exchange rate returns from appropriate counterfactuals. Significant abnormal returns, tested through parametric and non-parametric evaluation techniques, would indicate the presence of event-induced price discontinuities. Generating appropriate counterfactuals is crucial, as this allows for the cleaner identification of the impact of each event. The counterfactual analysis is based on a monetary policy framework specific to Tanzania, with theory drawn from the exchange rate forecasting literature. The Bank of Tanzania has provided proprietary data, so that I am able to control for key variables in the Central Bank's information set.

There are some interesting results. I find that capital outflow controls, defined as decreases in the permitted net open position (NOP) of Tanzanian commercial banks, were initially effective. The first decrease in the NOP from 20% to 10% on 18th October 2011 stabilized the exchange rate by reversing, at least in the short-run, the depreciation of the shilling, and producing a statistically significant appreciation of 3.5% over one month relative to the counterfactual. The two subsequent reductions in the NOP (10% to 7.5%, and 7.5% to 5%), however, were not as effective, and may have even been counterproductive. The declining efficacy of capital controls over time in Tanzania adds to the evidence on historical experiments with capital outflow controls, summarized in Magud et al. (2011). Save the classic cases of Malaysia and Chile where capital controls were effective, the findings are mixed for other countries but veer toward lower success compared to the two poster children. Reasons include anticipation and innovation by the markets in response to the controls Habermeier et al. (2011). In Tanzania, anticipation, innovation, and the inherently contractionary nature of a tax on financial diversification, could have contributed to the declining efficacy of reductions in the net open position. The analysis suggests caution in continuing the usage of capital outflow controls in Tanzania.

Related Literature This paper contributes to two main areas of the literature.

The first is the literature on the efficacy of capital controls. Over the past few decades, developing countries have become more integrated into the global financial markets. The ensuring volume and volatility of capital flows in and out of their economies have led to prudential concerns such as credit booms and busts, as well as macroeconomic concerns including exchange rate pressures (Ostry et al., 2011). Given the increasing exposure of these economies to global business cycles, an influential school of thought contends that the classic Mundell-Fleming Trilemma boils down to a Dilemma in practice (initiated by Rey, 2015). In other words, Central Banks in small open economies have only two choices - either they leave their capital accounts open or use capital controls. With an open capital account, however, the sheer volume and volatility of capital flows implies that monetary policy cannot be independent regardless of fixed or floating exchange rates. Hélène Rey suggests that monetary policy effectiveness, while mitigating prudential and macroeconomic concerns, can only be achieved through the usage of capital controls.

But what does the data say? Magud et al. (2011) conduct a meta-analysis that synthesizes the results from an empirical literature that spans the past few decades. They differentiate between inflow versus outflow controls, and take into account that the effectiveness of capital controls is measured through different objectives depending on the country in question and research methodology. Success criteria include whether controls reduced the volume of capital flows, changed the composition of capital flows, made monetary policy more independent, and reduced exchange rate pressures. Key takeaways include the conclusion that controls on capital inflows are more effective than those on capital outflows. Furthermore, while the overall volume and volatility of capital flows is not affected much by capital controls (see for eg. the multi-country studies by Binici et al., 2010 and Forbes and Warnock, 2012), the composition and maturity of capital flows can change significantly (for eg. Jittrapanun and Prasartset, 2009 and Levy Yeyati et al., 2009). In terms of influencing the exchange rate, the results have been fairly mixed, but the meta-study of Magud et al. (2011) favors this conclusion.

Habermeier et al. (2011) and Engel et al. (2012) contend that the literature has not reached a consensus on whether capital controls are indeed beneficial in practice. It is also not understood why controls work in some cases but not in others. Furthermore, when controls are initially successful, their effectiveness typically diminishes over time. Habermeier et al. (2011) suggest that this could be because markets begin to anticipate the controls and find ways to circumvent them. To evaluate the effectiveness of controls, methodologies vary (reviewed in detail in Magud et al., 2011). Many papers base their conclusions on the qualitative analysis of the time series of the main variables (for eg. Reinhart and Smith, 1998, Thaicharoen et al., 2008, and Mohan and Kapur, 2009). To address endogeneity, other papers pair OLS with IV regression models, or use structural vector autoregressions (SVARs). The more rigorous studies also include hypothesis testing. I deviate from much of the literature by using an event-study methodology, which is theory-free, so that the estimates are free from the model uncertainty implicit in other econometric methods. A rigorous hypothesis testing procedure is also followed, based on the event study framework outlined in MacKinlay (1997).

Related to the econometric approach used in this paper are other event studies on the effects of policy intervention on the exchange rate. Reviewed in Neely (2005), papers in this literature include Humpage (2000), Fatum and Hutchison (2003), Fatum and Hutchison (2006), and Fratzscher (2008). These studies tend to define forex intervention *clusters* as single events, where the size of each cluster varies and can range from a few days to weeks depending on the researchers' judgment. One difficulty with this definition of an event is the possibility of other non-event related news arriving during the clusters.[3] In comparison, I am able to more precisely identify the capital control events, each of which happens on a single day. Instead of being calculated a few weeks apart as with forex intervention clusters, the ex-ante and ex-post returns are calculated on consecutive days. Also, related event studies do not typically consider counterfactual analysis due to reasons including data unavailability. This could lower the precision with which they identify the impact of each event. On the other hand, I condition the efficacy of capital

[3]For instance, in Fatum and Hutchison (2006), an event is defined as a cluster of several days (or weeks) of daily intervention, with upto 15 continuous non-intervention days between days of intervention. Each event comprises of a different number of intervention and non-intervention days, and can extend over weeks.

outflow controls on a counterfactual model that is specific to the Tanzanian economy and draws upon key variables in the Central Bank's information set.

This paper also contributes to the literature on exchange rate forecasting. Ever since the seminal Meese and Rogoff (1983) result that a random walk model forecasts the exchange rate better than any economic model, an extensive literature has attempted to reverse this finding. Several facts on the predictability of exchange rates have been established, and have been reviewed in meta-studies by Frankel and Rose (1995), Engel et al. (2007), Melvin et al. (2013), and Rossi (2013). The predictive power of exchange rate models depends on a number of factors including the forecast horizon, sample period, and choice of predictors. Furthermore, in-sample fit is not necessarily correlated with out-of-sample predictive ability, and there is no one model with the greatest predictive accuracy across all sample periods and forecast horizons. Exchange rates, however, are more predictable in general over shorter forecast horizons. Textbook frameworks such as the flexible and sticky price monetary models, or models based on purchasing power parity (PPP) and uncovered interest rate parity (UIP) do not fare well. Econometric models such as VARs do well sometimes. This paper contributes by modeling and forecasting the exchange rate in Tanzania, a country for which, akin to many other low-income countries, scant such empirical evidence exists. This study draws upon the lessons from the literature while accounting for the specific monetary policy framework in Tanzania.

The rest of the study proceeds as follows. Section 4.2 provides an overview of monetary policy in Tanzania. Section 4.3 describes the event study based econometric framework employed to evaluate the effectiveness of capital controls. Section 4.4 constructs an exchange rate forecasting model, an important step in the event study approach. Section 4.5 conducts the event study and discusses the empirical findings. Section 4.6 concludes and offers policy recommendations.

4.2 Monetary Policy in Tanzania

Tanzania, a low-income country in Sub-Saharan Africa, is in transition from a money targeting (MT) to an inflation targeting (IT) regime (see Kessy et al., 2016, on which this section is based). While IT has enjoyed varying degrees of success in several developing and emerging market economies, Tanzania has faced institutional constraints, lack of financial depth, and a data-scarce environment, that precluded the transition till a few years ago. To target inflation under the regime of IT, the Bank of Tanzania (BoT) adjusts the official interest rate ie. the interbank cash market overnight rate. To operationalize the framework of money targeting, the BoT targets a broad money aggregate by setting a target for the growth of reserve money. To affect the path of reserve money, the BoT buys and sell quantities of reserve money by issuing government paper to mop up or inject reserve money into the system. The effectiveness of money targeting is conditioned on an assumption on the high-powered money multiplier. Also, for a stable velocity, broad money is correlated with prices. The implementation of monetary policy, which can currently be described as a hybrid approach, is subject to the BoT's primary mandate of price stability, following from the Bank of Tanzania Act of 1995. A secondary mandate of the BoT is to support financial development.

While Tanzania is currently classified as a money targeting regime (IMF, 2016a), the Bank of Tanzania plans to transition to inflation targeting by 2023. Although money targeting has served the BoT well, four considerations have increased the case for inflation targeting: (i) shocks to velocity and the money multiplier in Tanzania, (ii) the relative lack of transparency of money targeting, (iii) the possibility that quantity targets could hinder financial development, and (iv) the convergence by 2023 to the East African Monetary Union (EAMU) which will likely adopt a union-wide system of inflation targeting (for further discussion on these factors, see Kessy et al., 2016). To model monetary policy in Tanzania, econometric frameworks are developed for money targeting, inflation targeting, as well as a hybrid of the two monetary regimes. In particular, I take into account that while Tanzania maintains a *de jure* floating exchange rate regime, broadly similar to that of its neighbours, Kenya and Uganda, in the East African Community (EAC), comprised of Kenya, Uganda, Tanzania, Rwanda, Burundi and South Sudan, the Bank of Tanzania *de facto* manages the Tsh-USD bilateral exchange rate.

The BoT intervenes in the interbank foreign exchange rate market (IFEM) to buy and sell dollar reserves to stabilize the Tsh-USD exchange rate, the official relative price in IFEM transactions. As can be seen in Figure 4.1, the Bank of Tanzania has been intervening constantly in the IFEM and net foreign exchange intervention is positive, with a few daily exceptions. This would suggest that they are managing the exchange rate, or that the exchange rate is not a pure float. The left-hand side panel of the figure indicates that

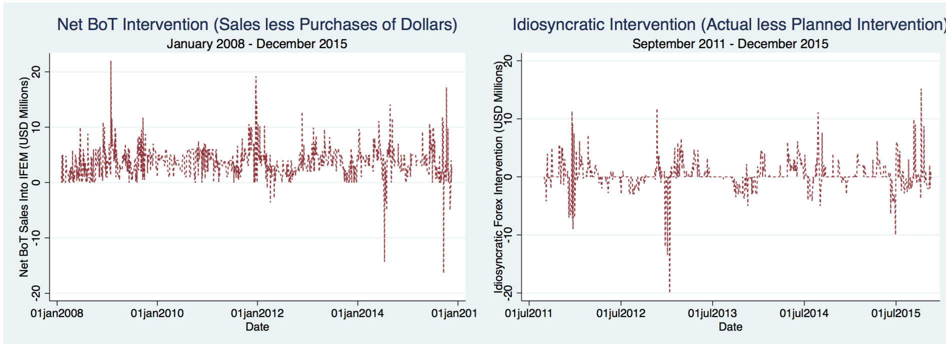

Figure 4.1: Foreign Exchange Intervention by the Bank of Tanzania
Source: Bank of Tanzania Daily Monetary Indicators Data

while overall the intervention is positive and consistent, the right-hand side panel indicates that the BoT can also deviate from its planned foreign intervention path on any given day based on incoming news. This idiosyncratic component of net forex intervention (actual less planned net intervention) is of mean zero, and daily numbers are depicted in Figure 4.1 from Sept 2011 - Dec 2015 (based on data availability), along with net foreign exchange intervention from Jan 2008 - Dec 2015.

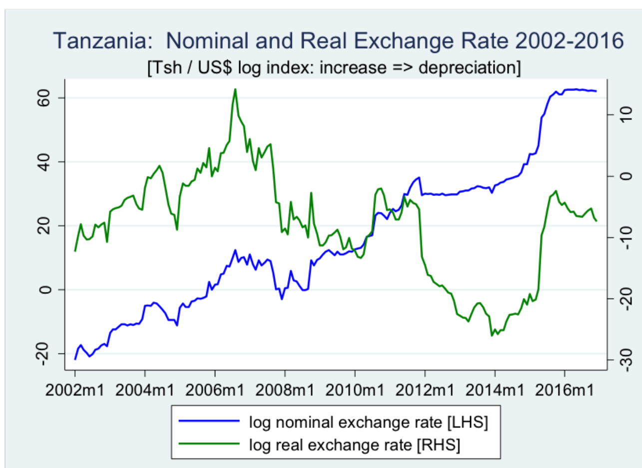

Figure 4.2: Nominal and Real Bilateral Tsh/USD Exchange Rates
Source: Bank of Tanzania Monthly Monetary Indicators Data

Figure 4.2 provides suggestive evidence that the BoT has put 'sand in the wheels' so as to stabilize the Tanzanian shilling.[4] It shows a period of surprising nominal exchange rate stability from late 2011 to mid-2015. The figure depicts the nominal and real Tanzania-US bilateral exchange rates from 2002 till the end of 2015. Inflation differentials have

[4]The data in Figure 4.2 is of monthly frequency, as there is no daily data available on prices. Also, the exchange rates have been converted to indices (where 2005m12=1). Taking logs give the log index = 0 in this month so that the scale on the plot is in percentage deviation from December 2012.

been the primary driver for the depreciation of the shilling vis-à-vis the dollar since the early 2000s (Adam et al., 2016). At various times in recent years – particularly at times of exchange rate turbulence following the ramp-up in inflation in 2011 (due mainly to the spike in food and fuel inflation) through the period of exchange rate instability in mid-2015 – the Bank of Tanzania has implemented changes in the regulations concerning capital account transactions as adjunct measures to complement conventional monetary policy and intervention instruments. This is the period where policy seems to be geared toward keeping the nominal exchange rate stable.

From late-2011 till mid-2015, while the rate of depreciation of the nominal bilateral exchange rate seems to have slowed down dramatically, this is associated with a sharp appreciation of the real bilateral exchange rate in Figure 4.2. The real exchange rate appreciated during 2011-15 because the nominal exchange rate did not compensate for the inflation differential. Normally in a float, market forces would force the nominal adjustment towards the rate required to close the inflation differential. The fact that it doesn't suggests that monetary policy might have prevented this depreciation.

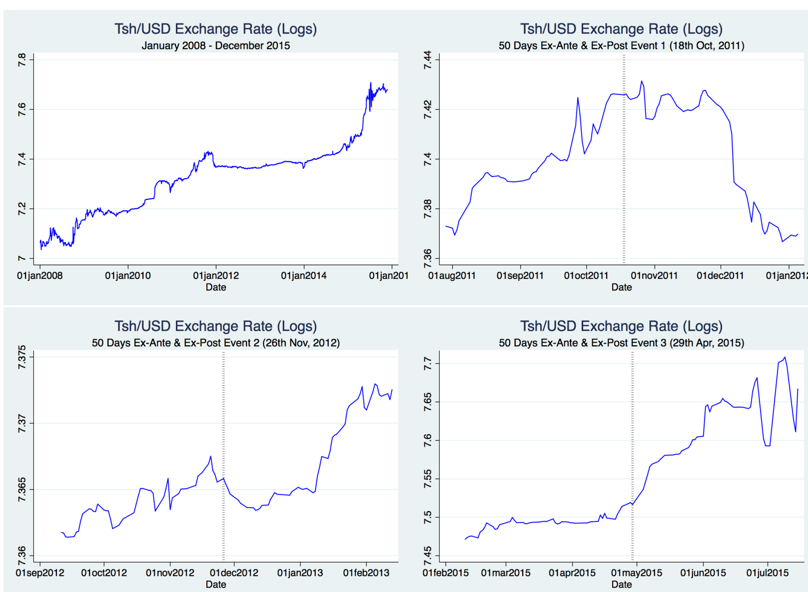

Figure 4.3: The Tsh/USD Exchange Rate from 2008-15 & around the Three Events
Source: Bank of Tanzania Daily Monetary Indicators Data

While the BOT continued the usage of its conventional monetary policy instruments through this period, requiring us to control for this in the econometric analysis, capital control measures, i.e. reductions in the permitted net open position (NOP) limits of Tanzanian commercial banks, were also being deployed. The NOP was reduced three time between October 2011 and April 2015 (from 20% to 10% on 18th October, 2011, from 10% to 7.5% on 26th November, 2012, and from 7.5% to 5% on 29th April, 2015). Figure 4.3 depicts the daily Tsh-USD exchange rate (in logs) from Jan 2008 - Dec 2015, as well as before and after each reduction in the net open position.

4.3 Econometric Approach

I employ an event study framework to examine the impact of capital controls on the exchange rate in Tanzania. This approach is theory-free and free from the model uncertainty innate in other econometric methods. Event studies are typically used to analyze issues like the market's reaction to events such as announcements or policy interventions. Pioneered almost a century ago by Dolley (1933), detailed reviews can be found in MacKinlay (1997) and Neely (2005). The event study methodology is prevalent in the finance literature to answer questions such as the effects of a firm's earning announcements on asset prices. Event studies have also been regularly used to analyze the impact of policy interventions in international finance studies. While this latter literature tends to define each event as an intervention cluster that varies in length and can span several days or weeks, compared to this study's sharper definition of each event happening on a single day, papers that evaluate the impact of sterilized intervention on the exchange rate include Fatum and Hutchison (2006) and Fratzscher (2008).

The impact of events is generally assessed by examining the behavior of asset returns before and after the events, as well as ex-post "abnormal returns". Abnormal returns measure the difference between the observed asset return following an event less the "normal return" or counterfactual. Significant abnormal returns would suggest the presence of price discontinuities associated with an event. Relative to related event studies on the effects of policy intervention, a more rigorous evaluation procedure is employed based on a counterfactual model that accounts for the monetary policy framework in Tanzania. The event study is structured around the following steps: (i) definition of event, including event window length, (ii) measurement of cumulative abnormal returns, (ii) testing and evaluation procedure, (iv) empirical results, and (v) interpretation. The remainder of this section discusses the first three steps, and the specific choices made with respect to the Tanzanian context.

4.3.1 Event Definition

I first identify the events and the time horizons over which the exchange rate is analyzed. Each event in this study constitutes an announcement by the Bank of Tanzania on instituting capital outflow controls. Controls are defined as the reduction in the net open position (NOP) of banks trading on the interbank foreign exchange rate market (IFEM). Three such events have taken place in Tanzania's recent monetary history: the NOP was reduced from 20% to 10% on 18th October, 2011, from 10% to 7.5% on 26th November, 2012, and from 7.5% to 5% on 29th April, 2015. To accompany these announcements, the actual implementation took place on either the same day (latter two events) or a month after (21 November, 2011 in the case of the first event). It is assumed that the market priced the capital controls upon their announcements, though this assumption is tested by allowing also for the lagged effects of news as well as information leakages.

The lengths of the pre-event and post-event windows are set around each event. It is useful to define some notation, as in MacKinlay (1997), to facilitate the analysis of the exchange rate in the days surrounding each event. Working with daily data, let returns be indexed by t. Defining $t = 0$ as the event date, $t \in [T_0, T_1]$ constitutes the pre-event window and $t \in [T_1 + 1, T_2]$ constitutes the post-event window. The lengths of these windows are then respectively $L_1 = T_1 - T_0 + 1$ and $L_2 = T_2 - T_1$. I set the pre-event window, L_1, to be six weeks, and also pair this choice of window length with sensitivity analysis. L_1 is varied from 6 weeks to 10 weeks. The pre-event window is used to generate appropriate counterfactuals, to be compared with the ex-post exchange rate returns.

The post-event window, L_2, is set to be one week for the baseline analysis, though it is varied from one week to four weeks. Related literature has typically veered toward even shorter window sizes of just a few days, with an upper bound of three weeks (Fatum and Hutchison, 2003; Fratzscher, 2008), but the immediate as well as the short-term progressive impact of capital controls is of interest. In the baseline analysis, the post-event window includes the day the capital control measure was announced. The lagged effects of news are tested by adjusting the post-event window so that it starts at $t > 0$, and for information leakages by starting the post-event window at $t < 0$. Notably, there is a trade-off in determining the appropriate length of the post-event window, L_2. On the one hand, if L_2 is too long, there is a risk of non-event related news also impacting the exchange rate. But too short a post-event window may not capture the progressive effects of the capital control announcements. Further, in a very thinly traded financial market such as the IFEM in Tanzania, where information acquisition is costly, the costs of too short a post-event window may be atypically high.

4.3.2 Cumulative Abnormal Returns

I analyze the ex-post cumulative abnormal returns (CARs), as these quantities indicate the direction as well as the magnitude of the event's impact. The CARs comprise of observed, normal, and abnormal exchange rate returns. An exchange rate return is defined as the difference in logs of the Tsh-USD exchange rate, and measures the change in the local currency price of a dollar. Normal returns (NRs) are the counterfactual estimates of the exchange rate returns, and abnormal returns (ARs) measure the distance between the ex-post observed returns and the normal returns. The CARs, which are the sum of the ARs, control for the magnitude and direction of the abnormal returns induced by each event. As in MacKinlay (1997), the cumulative abnormal return from $t = \tau_1$ to $t = \tau_2$, with $T_1 < \tau_1 \leq \tau_2 \leq T_2$, is defined as

$$CAR\left(\tau_1, \tau_2\right) = \sum_{t=\tau_1}^{\tau_2} AR_t \tag{4.1}$$

where the abnormal return for each period t, with $T_1 < t \leq T_2$, is

$$AR_t = R_t - NR_t \tag{4.2}$$

and R_t and NR_t are respectively the observed and normal returns. The framework used to generate normal returns in the case of the Tanzanian monetary policy framework and foreign exchange rate market is developed further in Section 4.4.

4.3.3 Testing and Evaluation

To assess the impact of capital controls on the path of the exchange rate, the statistical tests compare the ex-post returns with the counterfactual estimates. As in MacKinlay (1997), an event is considered successful if the $CARs\left(\tau_1, \tau_2\right)$s following the event for $T_1 < \tau_1 \leq \tau_2 \leq T_2$ are negative and statistically significant. Negative CARs over the post-event window, L_2, would indicate that the exchange rate strengthened relative to its counterfactual path. To assess the direction and magnitude of $CAR\left(\tau_1, \tau_2\right)$, I use a parametric t-test as well as a non-parametric Wilcoxon (1945) sign-rank test. The former has higher power (though it makes distributional assumptions), and the latter is a complementary approach that does not make specific assumptions about the distribution of abnormal returns. The Wilcoxon test is conducted by ranking the observed differences between the cumulative abnormal returns and zero, the latter of which would imply that the event had no impact. Under the null hypothesis that each $CAR(\tau_1, \tau_2) = 0$, there would be a similar number of positive and negative ranks. The Wilcoxon null hypothesis is tested for significance using the standard normal distribution.

For the t-test, certain assumptions are made on the distribution of $CAR(\tau_1, \tau_2)$. Let the disturbance term of the model estimated in L_1 (used to generate normal returns) be denoted as e_t, where $E(e_t) = 0$ and $Var(e_t) = \sigma_{e_t}^2$. MacKinlay (1997) shows that as each AR_t in the window L_2 is the out-of-sample disturbance term of the model estimated in L_1, $Var(AR_t)$ equals $\sigma_{e_t}^2$ for a large enough window size of L_1. Noting that $CAR(\tau_1, \tau_2)$ is the sum of the AR_t from $t = \tau_1$ to $t = \tau_2$, the variance of each cumulative abnormal return in (4.1) is given by

$$\sigma_t^2(\tau_1, \tau_2) = (\tau_2 - \tau_1 + 1)\, \sigma_{e_t}^2 \tag{4.3}$$

Given the null hypothesis that the cumulative abnormal returns are zero following each capital control announcement, $CAR(\tau_1, \tau_2)$ is distributed as

$$CAR(\tau_1, \tau_2) \sim N\left(0, \sigma_t^2(\tau_1, \tau_2)\right) \tag{4.4}$$

To estimate $\sigma_t^2(\tau_1, \tau_2)$, which is unknown, the sample variance measure of $\sigma_{e_t}^2$ is used.

The counterfactual analysis and hypothesis testing would place this study among the more rigorous analyses of capital controls in the literature (econometric rigor is discussed in Habermeier et al., 2011; Magud et al., 2011). While some papers base their conclusions on the qualitative analysis of key time series (eg. Thaicharoen et al., 2008; Mohan and Kapur, 2009), others do not compare the ex-post exchange rate returns with counterfactual estimates, due in part to data unavailability (eg. Humpage 2000; Fatum and Hutchison 2006). Instead, this study specifies counterfactual estimates that account for several key drivers of the exchange rate in Tanzania. The Bank of Tanzania has provided proprietary data on variables in its information set including net foreign exchange rate intervention, idiosyncratic foreign exchange rate intervention, money market operations, and interest rate usage. Controlling for these conventional monetary policy instruments allows for the better isolation of the impact of the more unconventional usage of capital controls.

4.4 Normal Returns

Normal exchange rate returns are constructed by forecasting from an appropriate model fitted in the pre-event window, L_1. The normal returns, or counterfactuals, are compared to the ex-post observed returns in the post-event window, L_2, to assess whether the capital controls significantly influenced the exchange rate. Constructing appropriate counterfactuals that specify the path of the exchange rate in the absence of capital controls is crucial, as this allows for a cleaner identification of the impact of each event. For the purposes of counterfactual model selection, I draw upon the tools, hypothesis tests, and other insights from the literature on exchange rate forecasting (reviewed in detail in Frankel and Rose, 1995, Engel et al., 2007, Melvin et al., 2013, Rossi, 2013) while conducting a context-specific analysis. The forecasting framework accounts for monetary policy in Tanzania, including that the Bank of Tanzania is transitioning from money to inflation targeting, and that it frequently engages in foreign exchange rate interventions.

The literature suggests, building on a point originally made by Meese and Rogoff (1983), that it is difficult to forecast better than a random walk model of the exchange rate, which results in a no-change forecast (NCF henceforth). The predictive content of exchange rate models also depends on the frequency of the data available, the types of models and predictors used, and the timespan and countries under consideration. Takeaways from the literature include that textbook exchange rate frameworks such as uncovered interest rate parity (UIP), sticky and flexible price monetary models, and purchasing power parity (PPP), typically forecast no better than the NCF. Structural models such as structural vector autoregression (SVARs) and dynamic stochastic general equilibrium (DSGE) models also do not have much success, and often make stringent and unrealistic assumptions on the data. Building on these insights while accounting for monetary policy in Tanzania, the approach in this study is to let the data "speak" as much as possible, through appropriately specified vector autoregressions that model the interplay between the instruments and targets of the Bank of Tanzania.

4.4.1 Multivariate Forecasting Framework

To forecast the exchange rate, I consider the monetary policy framework in Tanzania as well as the availability of daily data. Unsal et al. (2010) suggest that reserve money is an important variable to account for in developing countries that target both money supply and inflation. Money growth is included in the forecasting model as the BoT is currently in transition from money targeting toward greater usage of the interest rate. The baseline VAR is in the endogenous variables Δs_t, i'_t, Δm_t, χ_t, and ω_t, the latter three of which reflect proprietary data. These variables are constructed as follows. Δs_t is the exchange rate return or the daily percentage depreciation ie. the difference in logs of the Tsh-USD exchange rate, which is the weighted average of the previous day's trades on the IFEM. $i'_t = i_t - i^*_t$ is the daily Tsh-US official interest rate spread. Δm_t is the daily growth in reserve money in Tanzania. χ_t denotes the difference between daily foreign exchange (US dollar) sales and purchases by the BoT in the interbank foreign exchange rate market (IFEM). ω_t is the difference between daily actual and planned foreign exchange interventions. Reflecting the idiosyncratic component of BoT interventions in the IFEM, ω_t controls for relevant economic and political news.

I define some notation for the purposes of forecasting (as in Zivot and Wang, 2010). Consider a vector, Y_t, of n jointly endogenous variables, $Y_t = [Y_{1,t}, Y_{2,t}, ..., Y_{n,t}]'$. Y_t can be modeled as a function of n constants, r prior values of Y_t, k exogenous variables, and n random disturbances

$$Y_t = \mu + \Phi_1 Y_{t-1} + \cdots + \Phi_p Y_{t-r} + \Psi X_t + u_t \tag{4.5}$$

where μ is an n-element vector of constant terms, $\mu = [\mu_1, \mu_2, ..., \mu_n]'$, X_t is an n-dimensional matrix of k exogenous variables, $X_t = [X_{1,t}, X_{2,t}, ..., X_{k,t}]'$, and u_t is an n-element vector of error terms, $u = [u_{1,t}, u_{2,t}, ..., u_{n,t}]'$. I assume that the covariance matrix of error terms is positive-definite, that each element of u_t has an expectation of zero, and that there is no serial correlation. It is possible to add lagged values of the exogenous variables in (4.5), where $\Psi_l X_{t-l}$ denotes the l^{th} lag. The matrices of coefficients on the endogenous variables, Φ_i, are

$$\Phi_i = \begin{bmatrix} \Phi_{i,11} & \Phi_{i,12} & \cdots & \Phi_{i,1n} \\ \Phi_{i,21} & \Phi_{i,22} & \cdots & \Phi_{i,2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{i,n1} & \Phi_{i,n2} & \cdots & \Phi_{i,nn} \end{bmatrix}$$

The r^{th}-order VAR, (4.5), can be written more compactly in the lag operator form as

$$\Phi(L)\, Y_t = \mu + \Psi X_t + u_t \tag{4.6}$$

where $\Phi(L)$ is a matrix polynomial in the lag operator, L, i.e. $\Phi(L) = 1 - \Phi_1(L) - \cdots - \Phi_r(L)$. The VAR, (4.6), is required to be stationary for the purposes of forecasting, or equivalently, all the roots of $|\Phi(L)| = 0$ lie outside the unit circle (Zivot and Wang, 2010). Another condition that required before forecasting is that the estimated residuals of the VAR are white noise, tested through a portmanteau test with the null hypothesis that the residuals are homoskedastic and not serially correlated. At time period t, the h-step-ahead forecast is

$$Y_{t+h|t} = \mu + \Phi_1 Y_{t+h-1|t} + \cdots + \Phi_p Y_{t+h-p|t} + \Psi X_{t+h} \tag{4.7}$$

for $h = 1, 2, ..., T$, where h is the forecast horizon. The vector of corresponding h-step-ahead forecast errors, $\dot{\varepsilon}_{t+h|t}$, can be expressed as

$$Y_{t+h} - Y_{t+h|t} = \dot{\varepsilon}_{t+h|t}$$

where $\dot{\varepsilon}_{t+h|t}$ is a convolution of the white noise disturbances, i.e. $\dot{\varepsilon}_{t+h|t} = \sum_{m=0}^{h-1} \phi_m u_{t+h-m}$, $\phi_m = \sum_{b=1}^{j-1} \phi_{m-b} \Phi_b$, $\Phi_b = 0$ for $i > j$ (Zivot and Wang, 2010). (4.7) produces an unbiased forecast as the errors, u_t, have expectation zero. To compare the predictive ability of inflation targeting, money targeting, and a hybrid of these two monetary regimes, several variants of the vector autoregression, (4.5), are estimated, where $Y_t = [\Delta s_t, i'_t, \Delta m_t, \chi_t, \omega_t]'$. I evaluate twelve baseline exchange rate models, $Y_{A,t} - Y_{L,t}$, described in Table 4.1.

Model	Description
$Y_{A,t} = [\Delta s_t, i'_t]'$	Joint modeling of the exch and interest rates
$Y_{B,t} = [\Delta s_t, i'_t, \omega_t]'$	Joint modeling of the exch rate, interest rate, idiosyncratic forex intervention
$Y_{C,t} = [\Delta s_t, i'_t, \chi_t]'$	Joint modeling of the exch rate, interest rate, and net forex intervention
$Y_{D,t} = [\Delta s_t, i'_t, \omega_t, \chi_t]'$	Joint modeling of the exch rate, interest rate, idiosyncratic and net intervention
$Y_{E,t} = [\Delta s_t, \Delta m_t]'$	Joint modeling of the exch rate and reserve money supply
$Y_{F,t} = [\Delta s_t, \Delta m_t, \omega_t]'$	Joint modeling of the exch rate, reserve money, and idiosyncratic intervention
$Y_{G,t} = [\Delta s_t, \Delta m_t, \chi_t]'$	Joint modeling of the exch rate, reserve money, and net forex intervention
$Y_{H,t} = [\Delta s_t, \Delta m_t, \omega_t, \chi_t]'$	Joint modeling of the exch rate, reserve money, idiosyncratic and net intervention
$Y_{I,t} = [\Delta s_t, i'_t, \Delta m_t]'$	Hybrid monetary regime: Incorporating reserve money supply growth in $Y_{A,t}$
$Y_{J,t} = [\Delta s_t, i'_t, \Delta m_t, \omega_t]'$	Hybrid monetary regime: Incorporating reserve money supply growth in $Y_{B,t}$
$Y_{K,t} = [\Delta s_t, i'_t, \Delta m_t, \chi_t]'$	Hybrid monetary regime: Incorporating reserve money supply growth in $Y_{C,t}$
$Y_{L,t} = [\Delta s_t, i'_t, \Delta m_t, \omega_t, \chi_t]$	Hybrid monetary regime: Incorporating reserve money supply growth in $Y_{D,t}$

Table 4.1: Description of Exchange Rate Forecasting Models

The first model, $Y_{A,t}$, allows the Tsh-USD exchange rate and interest rate spread to influence each other. This is a multivariate extension of uncovered interest rate parity, and corrects for the endogeneity inherent in the univariate version. The second model, $Y_{B,t}$, adds idiosyncratic forex intervention, to capture the effects of news shocks on the exchange rate. The third model, $Y_{C,t}$, adds net forex intervention to $Y_{A,t}$, to control for the managed float exchange rate regime, while $Y_{D,t}$ incorporates both net and idiosyncratic intervention. Models $Y_{E,t} - Y_{H,t}$ replace the interest rate spread with reserve money to check whether money targeting might add more information than inflation targeting. Models $Y_{I,t} - Y_{L,t}$ allow for the additional influence of reserve money in the first four models, $Y_{A,t} - Y_{D,t}$. This reflects Tanzania's ongoing transition from a money targeting to an inflation targeting framework. Exogenous variables in the VARs include daily dummies, global and emerging market VIX indices, and spot and future oil price indices. These control respectively for daily trading effects, the impact of global and/or emerging market risk aversion, and terms of trade fluctuations.

As in the exchange rate forecasting literature, the VAR forecasts of the exchange rate, s_t, are compared to the exchange rate forecast from a random walk model: $s_t = s_{t-1} + u_t$. The random walk model yields a no-change forecast (NCF) as follows

$$s_{t+h|t} - s_t = 0 \tag{4.8}$$

where the h-step-ahead forecast error, $\check{\varepsilon}_{t+h|t}$, is defined as $s_{t+h} - s_t = \check{\varepsilon}_{t+h|t}$.

4.4.2 Univariate Forecasting Framework

To complement the multivariate analysis, I also check the forecasting properties of univariate models of the exchange rate. Univariate, or $ARIMA(p,d,q)$, models seek to describe the data generating process for a particular time series by specifying its order of integration (d), as well as its appropriate autoregressive (p) and moving-average (q) components. As the exchange rate is typically integrated of order one in the data, one works in practice with $ARIMA(p,1,q)$ models written as $\left(1-\phi_1 L_1 - ... - \phi_p L_p\right)\Delta s_t = \left(1-\theta_1 L_1 - ... - \theta_p L_p\right) u_t$, where u_t is a white noise term. The Box-Jenkins (Box et al., 2015) procedure for selecting the optimal $ARIMA(p,d,q)$ model is followed, with the aim of achieving significant coefficients and white noise residuals. Further, as more ARMA terms do not necessarily imply greater informational content, another goal of the Box-Jenkins approach is to keep the optimal $ARIMA(p,d,q)$ model parsimonious.

To implement the Box-Jenkins approach, first the data series is transformed to stabilize its variance (in this case the exchange rate is log-transformed). Second, the appropriate order of integration is determined through a unit root test (such as the Dickey and Fuller (1979) test with the null hypothesis that the series is not stationary) and the series is differenced accordingly. Third, the stationary series is plotted along with its autocorrelation and partial autocorrelation functions (ACFs and PACFs). Fourth, a preliminary $ARIMA(p,d,q)$ model is estimated based on the results of the ACF and PACF plots and the unit root test. If the ARMA terms are not significant, or if the residuals are not white noise, then the model is re-specified. Evidence of white noise is checked through ACF and PACF plots of the residuals, as well as a portmanteau Box-Pierce test with the null hypothesis that the residuals are white noise. Selecting the optimal $ARIMA(p,d,q)$ model is an iterative procedure. The h-step-ahead forecast error of the model, $\ddot{\varepsilon}_{t+h|t}$, is defined as $x_{t+h} - x_t = \ddot{\varepsilon}_{t+h|t}$, for x_t as the $ARIMA(p,d,q)$ representation of s_t.

4.4.3 Forecast Evaluation

I evaluate each model's predictive ability through a series of steps. First, the pre-event window, L_1, is split into two portions - the estimation period and the validation period. The estimation period consists of observations $[T_0, K]$. In the estimation period, each model is estimated and used to generate forecasts for $K+h$, where $h = 1, 2, ..., T_1$ is the forecast horizon. In the validation period, consisting of observations $[K+1, T_1]$, I evaluate the predictive accuracy of different models. While there is no rule of thumb for splitting a sample into estimation and validation periods, I follow the literature in reserving around a fourth of the sample for validation purposes Rossi (2013). Sensitivity analysis is also conducted by varying the breakpoint, K. To evaluate each forecast, the root mean squared forecast error (RMSFE) of each model is analyzed, where $RMSFE = \frac{1}{T_1}\sum_{h=1}^{T_1}\left(\varepsilon_{t+h|t}\right)^2$ for ε_t as any model's forecast error, and conduct the Diebold and Mariano (2002) test of equal predictive accuracy.

For a model to beat the random walk, the RMSFE of that model is required to be lower than that of a random walk or, equivalently, that the *relative* root mean squared forecast error (RRMSFE) is less than one (Rossi, 2013). It is additionally required that the forecast from the specified model is significantly different than a NCF. Define the loss differential between the two models as $\sigma_t = g\left(\varepsilon_t\right) - g\left(\check{\varepsilon}_t\right)$, where ε_t is the forecast error of a multivariate or univariate exchange rate model, and $\check{\varepsilon}_t$ is the forecast error of a random walk. The function $g\left(.\right)$ is typically the squared error loss i.e. $g\left(\varepsilon_t\right) = \varepsilon_t^2$, though it could also be the absolute error loss $g\left(\varepsilon_t\right) = \left|\varepsilon_t\right|$. The null hypothesis of the Diebold and Mariano (2002) test is that the two models have equal predictive accuracy. The null hypothesis, $H_0 : E\left(\sigma_t\right) = 0\ \forall t$, is tested versus the alternate hypothesis $H_A : E\left(\sigma_t\right) \neq 0$, using the standard normal distribution.

When choosing between models, I account for an important takeaway from the literature on exchange rate forecasting. The in-sample fit of a model is not necessarily correlated with its out-of-sample predictive accuracy (Rossi, 2013). Even if a particular model fits the data better than any others, it may still fare poorly when used to predict future values. Thus, when the informational content of different exchange rate models is evaluated, the analysis focuses on their abilities to generate good out-of-sample forecasts. Models with good in-sample fit but poor forecasting accuracy are not useful for predictive purposes. That said, the models are still required to satisfy certain in-sample criteria before assessing their predictive abilities. In particular, it is required that the estimated models have white noise residuals - ie. residuals that are not heteroskedastic or serially correlated, and also, in the multivariate vector autoregression case, be stable.

4.5 Empirical Analysis

Using the framework developed in Sections 4.3 and 4.4, I conduct the empirical analysis. This section progresses in four stages: (i) data description, (ii) generation of appropriate normal returns, (iii) event study results, and (iv) discussion of results.

4.5.1 Data Description

The proprietary dataset on monetary policy for this study has been provided by the Bank of Tanzania. The dataset is of daily frequency and ranges from the beginning of 2008 till the end of 2015. Five trading days constitute each week. The analysis accounts for key variables in the Central Bank's information set, which allows for a cleaner identification of the impact of each capital control announcement. It is possible to control for the conventional monetary policy instruments and targets of the Bank of Tanzania including net foreign exchange rate intervention, idiosyncratic intervention, money market operations, and interest rate usage. The bilateral Tanzanian-Dollar exchange rate, determined through three daily foreign exchange trades and foreign exchange interventions in the interbank foreign exchange rate market (IFEM), is a key relative price that affects the Bank of Tanzania operations. This is used to construct the exchange rate return, Δs_t, or the daily percentage depreciation ie. the difference in logs of the Tsh-USD exchange rate, which is the weighted average of the previous day's trades on the IFEM. There is also data on the daily Tanzanian and US overnight interbank interest rates, i_t and i_t^*, allowing for the construction of a daily bilateral interest rate spread, $i_t' = i_t - i_t^*$.

Daily data is also available on reserve money growth, foreign exchange (USD) sales and purchases by the Bank of Tanzania on the IFEM, as well as the BoT's planned and actual components of foreign exchange rate intervention. The former variable, Δm_t, is defined as the difference in logs of daily reserve money supply. This allows the analysis to account for the regime of money targeting, where broad money is targeted by setting a path for reserve money. Δm_t remains highly relevant in ongoing transition to inflation targeting, as the monetary regime in Tanzania could currently be thought of as a hybrid approach using both i_t and Δm_t. The variable, χ_t, or net daily foreign exchange rate intervention, is constructed as the difference between daily US dollar sales and purchases by the Bank of Tanzania. I also construct a variable that controls for the effects of news shocks, ω_t, or idiosyncratic forex intervention. ω_t is the difference between actual and planned intervention on any given day. Other variables include daily dummies to control for daily trading effects. Daily data is retrieved from the Federal Reserve Bank of St. Louis on the VIX and emerging market VIX indices (St. Louis Fed, 2017) to control for risk aversion, as well as spot and future oil prices from the US Energy Information Administration (EIA, 2017) to control for terms of trade effects.

4.5.2 Normal Returns

To generate the normal returns, I forecast from exchange rate models estimated in the pre-event windows. Normal, or counterfactual, returns are critical in better identifying the impact of each event. Price discontinuities associated with the capital controls are assessed by analyzing the deviation of the ex-post observed returns from the normal returns. The forecasting approach is based on multivariate reduced-form VAR models of the exchange rate that control for the conventional monetary policy instruments of the Bank of Tanzania, in order to better isolate the unconventional usage of capital controls. The data on the information set of the Central Bank allows for the construction of such an exchange rate framework. The exchange rate is modeled over the entire dataset from January 2008 - December 2015, as well as the windows before each of the three events. Event 1 constitutes a decrease in the NOP from 20% to 10% on 18th October, 2011. Subsequently, Events 2 and 3 represent reductions in the NOP from 10% to 7.5% on 26th November, 2011, and from 7.5% to 5% on 29th April, 2015. I estimate 12 models of the exchange rate, as described in Table 4.1 in Section 4.1, that account for inflation targeting, money targeting, as well as a hybrid of these two monetary regimes.

For the purposes of estimation, the following steps are taken. First, each series is tested for the presence of nonstationarity through the augmented Dickey and Fuller (1979) test. The null hypothesis of a unit root is rejected if the test statistic is sufficiently negative. Next, for variables that are integrated of the same order, the Johansen (1991) test is used to check for the presence of co-integrating relations. If co-integration exists, then the VAR can be estimated in levels with the appropriate long-run equilibrium relations specified. Otherwise, the VAR is estimated in stationary variables, through appropriately differencing variables with unit roots. Based on the unit root test results over the pre-event windows, the interest rate spread as well as the oil price and VIX series are first-differenced. The exchange rate return, money supply growth, idiosyncratic forex intervention, and net forex intervention, are stationary. There are no co-integrating relations. Over the entire sample, the VIX and oil price, as well as the endogenous variables, Δs_t, Δm_t, $i_t^{'}$, χ_t, and ω_t, are all stationary.

An appropriate lag length is selected based on the Akaike and Bayesian information criteria (Hamilton, 1994). To preserve degrees of freedom in the pre-event windows, the lag length is kept as parsimonious as possible based on the information criteria, and I choose one lag. Finally, as a pre-condition for forecasting accurately, all estimated models are required to have white noise residuals. The best $ARIMA(p,d,q)$ model of the exchange rate is selected following the iterative Box-Jenkins methodology (Box et al., 2015) described in Section 4.4. In the multivariate case, it is further required that each model is stable, i.e. all the roots of $|\Phi(L)| = 0$ lie outside the unit circle. If possible, for the purposes of in-sample fit in the VAR case, I would also like jointly significant lags (by rejecting the Wald test null hypothesis of jointly zero endogenous variables at a given lag). Predictive accuracy is, however, of primary interest. Noting that fit is not necessarily correlated with predictive ability, a stable model with white noise residuals is ultimately chosen based on the forecast accuracy tests through the relative root mean squared forecast error (RRMFE) and Diebold-Mariano statistics.

Table 4.2 summarizes the estimation and forecasting results for Event 1 (the results for Events 2 and 3 are presented in the appendix). In the table, models A to D account for inflation targeting, with model A in the exchange rate return, Δs_t, and the interest rate spread, i_t', models B and C adding net forex intervention, χ_t, and news shocks, ω_t, respectively to A, and model D in all four variables: Δs_t, $i_t^{'}$, χ_t, and ω_t. Models E to H replace the interest rate spread with reserve money growth, Δm_t, to check whether money targeting adds more information than inflation targeting. Models I to L allow for the additional influence of reserve money growth, Δm_t, in the first four models, A to D, to account for Tanzania's ongoing transition from money targeting to inflation targeting. Each model is first estimated in the jointly endogenous variables, before adding the exogenous variables. The second column for each of the twelve models, A to L, adds the VIX index and spot oil price (the EME VIX and future oil price yield similar results) to control for the impact of global risk aversion and terms of trade fluctuations, and the third column additionally incorporates daily dummies to control for daily trading effects.

It is found that for Event 1, Model H is optimal. This money targeting framework passes the pre-event window criteria of stability and white noise residuals, with jointly significant lags. It forecasts significantly differently than the random walk as per the results of the Diebold-Mariano test, with a low RRMSFE of 0.60. There are some other frameworks with lower RRMSFEs, especially those with a hybrid regime, but these either do not pass the pre-condition of white noise residuals ($J2$ and $K2$) or their forecasts are not significantly different than a random walk ($I2$ and $L2$). The version of Model H that performs the best controls for the oil price and VIX indices, and has one lag. Model H with two lags has similar properties, but one lag is chosen to preserve degrees of freedom and increase estimation accuracy in the pre-event window. Single-equation exchange rate models were also estimated, despite the endogeneity issues, to find that the predictive accuracy of the linear regressions is beaten by that of the VARs. The best univariate model of an $ARIMA(2,1,0)$ on the exchange rate, with a relative root mean square forecast error of 0.78, is also beaten by almost all the vector autoregressions.

The exchange rate models for Event 1 are generally able to forecast better than the random walk, especially those that allow for the influence of reserve money. For Events 2 and 3, as for the sample as a whole, however, the predictive accuracy of the univariate and multivariate models is not significantly different than that of a random walk. This result is not surprising in light of the literature (reviewed in Rossi, 2013), which suggests that the (i) random walk tends to perform the best over longer forecast horizons, and (ii) there is no consistent forecasting model that explains an entire range of data. For the sake of consistency as well as to control for the effects of conventional monetary policy, the money targeting model, H, is used to generate normal returns for all three events. This is paired with sensitivity analysis by also estimating the inflation targeting, hybrid regime, and random walk models, to generate normal exchange rate returns. I now turn to the results of the event study.

Table 4.2: Event 1 Forecast Model Selection Criteria Under Inflation Targeting, Money Targeting, and Hybrid Regimes

I. Inflation Targeting	**Model A**			**Model B**			**Model C**			**Model D**		
	A1	A2	A3	B1	B2	B3	C1	C2	C3	D1	D2	D3
Model Adequacy												
WN (p-value)	0.00	0.00	0.22	0.12	0.02	0.30	0.09	0.02	0.20	0.32	0.12	0.33
Wald (p-value)	0.05	0.05	0.01	0.13	0.17	0.10	0.77	0.67	0.24	0.23	0.21	0.17
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.06	0.07	0.05	0.06	0.06	0.05	0.06	0.07	0.04	0.07	0.10	0.04
Relative RMSFE	0.82	0.74	0.86	0.82	0.74	0.86	0.83	0.70	0.87	0.82	0.67	0.87
II. Money Targeting	**Model E**			**Model F**			**Model G**			**Model H**		
	E1	E2	E3	F1	F2	F3	G1	G2	G3	H1	H2	H3
Model Adequacy												
WN (p-value)	0.30	0.14	0.27	0.18	0.27	0.14	0.88	0.49	0.62	0.27	0.61	0.40
Wald (p-value)	0.00	0.00	0.00	0.04	0.03	0.02	0.69	0.59	0.46	0.07	0.05	0.05
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.08	0.12	0.05	0.08	0.10	0.06	0.08	0.12	0.05	0.07	0.08	0.06
Relative RMSFE	0.78	0.60	0.75	0.78	0.60	0.75	0.76	0.60	0.79	0.75	0.60	0.76
III. Hybrid Regime	**Model I**			**Model J**			**Model K**			**Model L**		
	I1	I2	I3	J1	J2	J3	K1	K2	K3	L1	L2	L3
Model Adequacy												
WN (p-value)	0.13	0.69	0.64	0.19	0.08	0.10	0.20	0.09	0.28	0.12	0.10	0.17
Wald (p-value)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.13	0.28	0.06	0.12	0.04	0.06	0.14	0.50	0.05	0.13	0.17	0.05
Relative RMSFE	0.67	0.51	0.66	0.67	0.52	0.67	0.68	0.48	0.68	0.68	0.46	0.66

Model Ai, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread

Model Bi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Idiosyncratic Forex Intervention

Model Ci, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Net Forex Intervention

Model Di, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Idiosyncratic Intervention, Net Intervention

Model Ei, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth

Model Fi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth, Idiosyncratic Forex Intervention

Model Gi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth Net Forex Intervention

Model Hi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth, Idiosyncratic Intervention, Net Intervention

Model Ii, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Reserve Money Growth

Model Ji, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Int Rate Spread, Reserve Money Growth, Idiosyncratic Forex Intervention

Model Ki, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Reserve Money Growth, Net Forex Intervention

Model Li, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Int Rate Spread, Reserve Money Growth, Idiosyncratic & Net Intervention

For $k \in \{A, ..., L\}$, $k1$ is the baseline VAR, $k2$ controls for the VIX and oil price, $k3$ controls for the VIX, oil price, and daily dummies

* WN (p-value) refers to the significance level of a Portmanteau white noise test with $H0$: Residuals are white noise.

* Stability refers to whether the eigenvalues of the VAR lie inside the unit circle.

* Wald (p-value) refers to the significance level of the Wald lag exclusion test with $H0$: Endogenous variables at a given lag are jointly zero.

* DM (p-value) refers to the significance level of the Diebold-Mariano test with $H0$: VAR forecast is no different than a NCF from a random walk.

* Relative RMSFE compares the RMSFEs of the VAR and a random walk. A RRMSFE<1 implies that the VAR forecast is better than a NCF.

4.5.3 Event Study Results

To conduct the event study, I begin by estimating the appropriate model for the exchange rate, as discussed in Section 4.5.2, over 6-week pre-event windows. The model is used to generate normal returns for post-event window sizes ranging from one to four weeks. Abnormal returns are computed to measure the difference between the observed and normal returns. Cumulative abnormal returns (CARs) control for the direction and magnitude of the abnormal returns, and allow for the measurement of the impact of each event. For each event, the CARs are reported for up to 4 weeks (20 days) after the event. To test whether the capital outflow controls stabilized the exchange rate in Tanzania, the impact of each event is evaluated using parametric t and non-parametric sign rank tests. Figure 4.4 shows how the CARs evolved over the four-week horizon after each event. Table 4.3 provides the values of the cumulative abnormal returns for 20 days after each event as well as the p-values on the t and Wilcoxon test statistics. There are several interesting results.

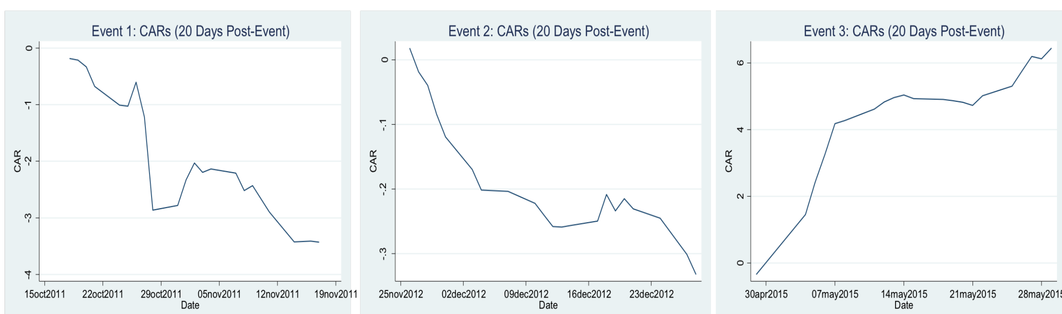

Figure 4.4: Evolution of the Cumulative Abnormal Returns for Events 1,2,3

I find that Events 1 and 3 (reductions in the NOP by 20% to 10% on 18th October, 2011, and 7.5% to 5% on 29th April, 2015 respectively) had immediate impacts, and this was only slightly delayed in Event 2 (reduction in the NOP from 10% to 7.5% on 26th November, 2012). The cumulative abnormal returns are generally very significant, with the Wilcoxon test p-values corroborating those from the t-test, despite the lower power of the former non-parametric test. Furthermore, the effectiveness of capital controls appears to have deteriorated over time. Event 1 led to a 1% appreciation of the Tsh-USD exchange rate over the first week relative to the counterfactual. This increased to 3.5% by the end of the fourth week. Event 2 was less effective but also led to a slight strengthening of the exchange rate relative to the counterfactual from 0.17% by the end of the first week to 0.32% by the end of the fourth week. Event 3, however, was counterproductive. The exchange rate actually depreciated by 6.5% relative to the counterfactual by the end of the fourth week. The empirical evidence indicates that attempts by the Bank of Tanzania to lean against the depreciation of the shilling became progressively more ineffective.

Days	Event 1			Event 2			Event 3		
Ex-Post	CAR (%)	p (par)	p (non)	CAR (%)	p (par)	p (non)	CAR (%)	p (par)	p (non)
1	-0.212	0.003	0.180	-0.018	0.126	0.655	1.448	0.000	0.623
2	-0.331	0.001	0.109	-0.040	0.026	0.285	2.430	0.000	0.214
3	-0.679	0.000	0.068	-0.084	0.000	0.144	3.280	0.000	0.139
4	-1.010	0.000	0.043	-0.120	0.000	0.080	4.179	0.000	0.060
5	-1.028	0.000	0.028	-0.170	0.000	0.046	4.276	0.000	0.046
6	-0.603	0.001	0.018	-0.202	0.000	0.028	4.617	0.000	0.024
7	-1.222	0.000	0.012	-0.204	0.000	0.017	4.826	0.000	0.015
8	-2.864	0.000	0.008	-0.222	0.000	0.011	4.966	0.000	0.012
9	-2.780	0.000	0.005	-0.240	0.000	0.007	5.040	0.000	0.007
10	-2.328	0.000	0.003	-0.258	0.000	0.004	4.935	0.000	0.004
11	-2.032	0.000	0.002	-0.259	0.000	0.002	4.905	0.000	0.003
12	-2.198	0.000	0.001	-0.257	0.000	0.002	4.872	0.000	0.002
13	-2.134	0.000	0.001	-0.250	0.000	0.001	4.824	0.000	0.001
14	-2.210	0.000	0.001	-0.209	0.000	0.001	4.729	0.000	0.000
15	-2.517	0.000	0.000	-0.234	0.000	0.000	5.018	0.000	0.000
16	-2.433	0.000	0.000	-0.215	0.000	0.000	5.308	0.000	0.000
17	-2.896	0.000	0.000	-0.231	0.000	0.000	5.750	0.000	0.000
18	-3.423	0.000	0.000	-0.245	0.000	0.000	6.200	0.000	0.000
19	-3.411	0.000	0.000	-0.301	0.000	0.000	6.126	0.000	0.000
20	-3.428	0.000	0.000	-0.332	0.000	0.000	6.448	0.000	0.000

* CAR (%) is the cumulative abnormal depreciation rate, and p(par) & p(non) and the p-values for the parametric and non-parametric tests.

Table 4.3: CAR Test Statistics - 20 Days After Events 1, 2, 3

Sensitivity Sensitivity analysis is conducted to see whether the empirical results hold away from the baseline assumptions. I vary the pre-event window length, L_1, away from 6 weeks to 10 weeks. The lagged effects of news and as well as information leakages are also allowed for by adjusting the post-event window accordingly. Lagged news effects are captured by starting the post-event window at $t = 1$ and $t = 2$, while information leakage effects are captured by starting the post-event window at $t = -1$ and $t = -2$. The baseline VIX index is replaced with emerging market economy (EME) and China VIX indices to check whether the VAR forecasts are robust to the growing influence of EMEs in African stock markets. I check whether the Event 2 and 3 findings hold robust to the usage of random walk, inflation targeting, and hybrid regime models to generate normal returns. It is found that the baseline results hold robust to all of these modifications. I report results from the random walk model and a 10-week L_1 in the appendix. Note that the random walk model shows a slight mean reversion relative to the counterfactual two weeks after Event 2, and the CARs are much higher in Event 3. While the baseline conclusions continue to hold, this underscores the importance of controlling for the information set of the Central Bank. The results from the 50-day pre-event window are very similar to those with a 30-day pre-event window, and the same conclusions can be extrapolated.

4.5.4 Interpretation

The analysis suggests that capital controls declined in efficacy over time in Tanzania. Their effectiveness weakened between the first and second time the net open position was reduced (decrease in the NOP from 20% to 10% on 18th October, 2011, and from 10% to 7.5% on 26th November, 2012), and reversed the third time (decrease in the NOP from 7.5% to 5% on 29th April, 2015). There are several interpretations for these findings.

First, while Event 1 was a shock to the markets, the subsequent Events 2 and 3 may have been anticipated as policy responses. Second and relatedly, innovation and circumvention by the interbank foreign exchange rate market participants could have contributed to the waning effectiveness of the controls. The literature suggests that anticipation and circumvention played no small role in undermining the effectiveness of capital controls across time in countries such as Brazil, Columbia, and Thailand, although the specific channels have been difficult to disentangle due to confounding factors and measurement difficulties (Habermeier et al., 2011). One popular interpretation offered in the literature is that firms were able to innovate through sophisticated derivatives markets, but this is more plausible in financially-developed countries such as Brazil. In Tanzania's case, it is plausible that evasion could have occurred through the East African Community (EAC), comprised of Kenya, Uganda, Tanzania, Rwanda, Burundi and South Sudan. Kenya and especially Uganda have far more liberal capital account regimes so firms operating across the EAC are able to, through intra-firm trading and invoicing, and transfer pricing, shift assets and liabilities across their EAC accounts and then acquire dollar assets through the Uganda foreign exchange market.

A third interpretation is based on the contractionary nature of the capital controls. Preventing banks from taking open positions is a tax on financial diversification and intermediation. Expecting medium-term depreciation, the banks could have been instigated to take advantage of the current value of the shilling and bring forward their IFEM transactions by hedging through the current account. In other words, banks, on behalf of their corporate clients, could have seen the controls compromising their ability to hold dollar financial assets against the prospect of a shilling depreciation. Instead of holding financial assets (via the capital account) they could have acquired 'dollar assets' in the form of imported consumer durables through the current account in the expectation that they could sell them into the domestic market for a shilling profit at some point in the future. As the market built capital outflow controls into its expectations set after the first event, this current account hedging behavior could have caused exchange rate overshooting, especially as the ability of banks to hedge through holding net open positions was closed down toward zero the third time.

Fourth, mid-2015 saw the end of the commodity supercycle and a plunge in commodity prices. The third reduction in the net open position took place around then in April 2015. As an exporter of gold and other heavy metals (despite being a net oil importer), Tanzania faced depreciation pressure (IMF, 2016b). Recognizing that it could not lean against the unusual depreciation pressure, while noting the fact that the NOP was nearing the zero lower bound, the capital account restrictions put in place by the Bank of Tanzania could have been at least partially politically motivated at that point. The BoT was also being advised by the IMF to let the exchange rate adjust as commodity prices fell, and that exchange rate flexibility was required to eliminate real exchange rate misalignments. This policy advice could have prompted an unobserved change in the Central Bank's stance that is not picked up by the normal model. Though the Bank of Tanzania put in place the capital controls, it was not prepared to intervene any more aggressively to stop the exchange rate from depreciating, leading to a sharp depreciation after Event 3 relative to the counterfactual.

A few additional points are of note. First, besides examining the impact of the capital controls on the direction and magnitude of the exchange rate, it was also sought to analyze whether the controls affected exchange rate volatility. However, ARCH effects, which are typically required for volatility modeling and forecasting (Erdemlioglu et al., 2013), were not found for the most part in the Tanzanian data, and realized volatility (defined as the square root of the realized daily variance ie. the sum of squared intraday returns) is too noisy with only one exchange rate return statistic available per day. Thus, the numbers on abnormal volatility ie. realized minus forecasted volatility, are not reliable. Second, disentangling the channels is not possible. This is, not, however, at odds with the capital controls literature, and the analysis faces the additional constraint of a data-scarce environment. Surveys and other measures of market expectations are unavailable in Tanzania, and there is no daily data on international trade and capital flows. Despite this, as the study is able to control for the Bank of Tanzania's conventional monetary policy instruments, the counterfactual analysis and hypothesis testing would place this paper among the more rigorous analyses on capital controls (econometric rigor is discussed in Habermeier et al., 2011; Magud et al., 2011).

Finally, it is interesting to connect the declining efficacy of capital controls in Tanzania to that of quantitative easing (QE). QE, an unconventional monetary policy instrument designed to stimulate the economy by reducing long-term interest rates, is thought to have become progressively more ineffective in the UK and elsewhere (eg. Breedon et al., 2012; Brunnermeier and Koby, 2016; Blanchard, 2016). Interpretations offered include the impetus provided by QE to banks to take more risk, which can be detrimental after a point, as well as the squeeze on profits of banks that hold long-term bonds, the contractionary implications of which relate to those of capital outflow controls.

4.6 Conclusion

A pressing monetary policy question in Tanzania is whether capital controls should be part of the regular Central Bank toolkit in the future. Outflow controls in the form of reductions in the net open position of banks have been employed three times in Tanzania's recent monetary policy history: on 18th October, 2011, 26th November, 2012, and 29th April, 2015. This study provides the first formal evaluation of whether the capital controls achieved their purpose of stabilizing the exchange rate through an event study approach buttressed with counterfactual analysis. Empirical findings are documented that will be of interest to policymakers in Tanzania as well as other small economies considering capital controls. These measures were successful in strengthening the Tsh-USD exchange rate the first time they were used. However, the controls lost effectiveness over time and may have even become counterproductive. Market anticipation and innovation, as well as the inherently contractionary nature of a tax on financial diversification, may have undermined the effectiveness of the controls. The findings indicate that capital outflow controls be used with caution in Tanzania in the future, if at all.

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Appendix A

Appendix to Chapter 2

I work with the Gali and Monacelli (2005) framework, and incorporate hand-to-mouth agents, with the objective of deriving the micro-founded optimal monetary policy.

Households Financially-excluded households maximize (2.1) subject to (2.2). I attach the Lagrange multiplier, λ_t, on the budget constraint, (2.2). The first-order conditions (FOCs) on consumption and labour supply are

$$\check{C}_t^{-\sigma} - \lambda_t P_t = 0 \tag{A.1}$$

$$-\check{N}_t^{\phi} + W_t \lambda_t = 0 \tag{A.2}$$

The FOCs on consumption and labour, (A.2) and (A.1), are combined to yield (2.3).

Financially-included households maximize (2.11) subject to (2.12). Attaching the Lagrange multiplier, λ_t, on the budget constraint, (2.12), the FOCs are

$$\hat{C}_t^{-\sigma} - \lambda_t P_t = 0 \tag{A.3}$$

$$-\hat{N}_t^{\phi} + W_t \lambda_t = 0 \tag{A.4}$$

$$\lambda_t Z_{t,t+1} + \lambda_{t+1} = 0 \tag{A.5}$$

The FOCs on consumption and labour, (A.3) and (A.4), are combined to yield (2.14). The FOCs on consumption and bonds, (A.3) and (A.5), are combined to yield (2.15). The international risk-sharing condition is derived by combining the Euler Equations of domestic financially-included and foreign agents with respect to foreign bonds, summing over foreign economies, and iterating backwards to period 0 (Gali and Monacelli, 2005).

The derivations for the CES consumption basket with corresponding domestic and foreign aggregates, as well as the firm's Calvo price setting problem can be found in the Chapter 1-3 appendices of Galí (2007) as well as Bergholt (2012).

Market-clearing To derive the goods market-clearing condition, start with the demand for each monopolistic good $i \in (0,1)$

$$Y_t(i) = C_{H,t}(i) + \int_0^1 C^j_{H,t}(i)\, dj$$

where $C_{H,t}(i)$ is consumption of home goods i by domestic consumers and $C^j_{H,t}(i)$ denotes consumption of home good i by country j. Using that (Gali and Monacelli, 2005)

$$C_{H,t}(i) = (1-\alpha)\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p}\left(\frac{P_{H,t}}{P_t}\right)^{-\varepsilon_I} C_t,\ \ C^j_{H,t}(i) = \alpha\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p}\left(\frac{P_{H,t}}{e_{j,t}P^j_{F,t}}\right)^{-\varepsilon_F}\left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\varepsilon_I} C_t$$

the market clearing condition is

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon_p}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\varepsilon_I} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{e_{j,t}P^j_{F,t}}\right)^{-\varepsilon_F}\left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\varepsilon_I} C^j_t dj\right]$$

where $e_{j,t}$ is the bilateral nominal exchange rate of Home with respect to country j, and $P^j_{F,t}$, P^j_t, C^j_t are respectively the import price index, CPI and consumption bundle for country j (Galí, 2007). As in Galí (2007), Bergholt (2012), Boerma (2014), Iyer (2014), summing up over all goods i and using the definition of aggregate output $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\varepsilon_I} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{e_{j,t}P^j_{F,t}}\right)^{-\varepsilon_F}\left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\varepsilon_I} C^j_t dj$$

Or (noting that $\hat{C}_t = C^j_t Q_{jt}^{\frac{1}{\sigma}} \rightarrow C^j_t = \hat{C}_t Q_{jt}^{-\frac{1}{\sigma}}$)

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\varepsilon_H}\left[(1-\alpha)C_t + \alpha\hat{C}_t \int_0^1 Q_{j,t}^{\varepsilon_I - \frac{1}{\sigma}}\left(X_{j,t}X^j_t\right)^{\varepsilon_F - \varepsilon_I} dj\right]$$

where $X_{j,t}$ are the bilateral real exchange rate and terms of trade of Home with respect to country j, and X^j_t is the effective terms of trade for country j. For $\sigma = \varepsilon_H = \varepsilon_F = 1$, this becomes $Y_t = X^\alpha_t\left[(1-\alpha)C_t + \alpha\hat{C}_t\right]$.

Optimal Monetary Policy

I focus on the Cole and Obstfeld (1991) case, $\sigma = \varepsilon_I = \varepsilon_F = 1$, as typical in the literature.

Proposition 1: Efficient Equilibrium

There are 9 unknowns in the optimization problem

$$\underset{\Sigma_t}{Max}\, U_t = \lambda \left(Ln\check{C}_t^e - \frac{\check{N}_t^{e1+\phi}}{1+\phi} \right) + (1-\lambda) \left(Ln\hat{C}_t^e - \frac{\hat{N}_t^{e1+\phi}}{1+\phi} \right)$$

$$\begin{array}{llrcl} s.t. & [\chi_{1,t}] & C_t^e & = & \lambda \check{C}_t^e + (1-\lambda)\hat{C}_t^e \\ & [\chi_{2,t}] & N_t^e & = & \lambda \check{N}_t^e + (1-\lambda)\hat{N}_t^e \\ & [\chi_{3,t}] & Y_t^e & = & A_t N_t^e \\ & [\chi_{4,t}] & w_t^e & = & \check{C}_t^e \check{N}_t^{e\phi} \\ & [\chi_{5,t}] & w_t^e & = & \hat{C}_t^e \hat{N}_t^{e\phi} \\ & [\chi_{6,t}] & \check{C}_{,t}^e & = & w_t^e \check{N}_t^e \\ & [\chi_{7,t}] & \hat{C}^e & = & X_t^{e1-\alpha} \\ & [\chi_{8,t}] & Y_t^e & = & \alpha X_t^{e\alpha} \hat{C}_t{}^e + (1-\alpha) X_t^{e\alpha} C_t^e \end{array}$$

where $\Sigma\left(s^t\right) = \left\{ \hat{C}_t^e, \check{C}_t{}^e, C_T^e, \hat{N}_t^e, \check{N}_t^e, N_t^e, \omega_t^e, Y_t^e, X_t^e \right\}$.

It is possible to reduce the optimization problem to 4 unknowns, $\hat{N}_t^e, N_t^e, Y_t^e, X_t^e$, through appropriate manipulations of the constraints as follows.

Wages are given from the asset holder labour supply condition

$$\begin{aligned} w_t &= \hat{C}_t^e \hat{N}_t^{e\phi} \\ &= X_t^{e1-\alpha} \hat{N}_t^{e\phi} \end{aligned}$$

The terms of trade can be derived from the marginal cost condition, noting that $\frac{P_{H,t}}{P_t} = p_{H,t} = X_t^{-\alpha}$ from the relations $P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}$ and $X_t = \frac{P_{F,t}}{P_{H,t}}$, and also that $MC_t^e = 1$ in the efficient equilibrium with competitive firms and flexible prices.

$$
\begin{aligned}
MC_t^e &= \frac{w_t^e}{p_{Ht}^e A_t} \\
1 &= \frac{w_t^e X_t^{e\alpha}}{A_t} \\
A_t &= w_t^e X_t^{e\alpha} \\
A_t &= X_t^{e1-\alpha} \hat{N}_t^{e\phi} X_t^{e\alpha} \\
A_t &= X_t^e \hat{N}_t^{e\phi} \\
X_t^e &= A_t \hat{N}_t^{e-\phi} \\
X_t^e &= A_t \left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{-\phi}
\end{aligned}
$$

The consumption of financially-included and financially-excluded agents, and aggregate consumption are given respectively by

$$
\begin{aligned}
\hat{C}_t^e &= X_t^{e1-\alpha} \\
\hat{C}_t^e &= A_t \left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{-\phi(1-\alpha)}
\end{aligned}
$$

$$
\begin{aligned}
\check{C}_t^e &= w_t \\
&= X_t^{e1-\alpha} \hat{N}_t^{e\phi} \\
&= A_t \left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{-\phi(1-\alpha)} \left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{\phi} \\
\check{C}_t^e &= A_t \left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda} \right)^{\phi\alpha}
\end{aligned}
$$

$$
C_t^e = \lambda \left(X_t^{e1-\alpha} \hat{N}_t^{e\phi} \right) + (1-\lambda) X_t^{e1-\alpha}
$$

The resource constraint is

$$
\begin{aligned}
Y_t^e &= X_t^{e\alpha} \left((1-\alpha) C_t^e + \alpha X_t^{e1-\alpha} \right) \\
&= X_t^{e\alpha} \left((1-\alpha) \left(\lambda \left(X_t^{e1-\alpha} \hat{N}_t^{e\phi} \right) + (1-\lambda) X_t^{e1-\alpha} \right) + \alpha X_t^{e1-\alpha} \right) \\
&= (1-\alpha)\lambda X_t^{e\alpha} \left(X_t^{e1-\alpha} \hat{N}_t^{e\phi} \right) + X_t^{e\alpha}(1-\alpha)(1-\lambda) X_t^{e1-\alpha} + X_t^{e\alpha} \alpha X_t^{e1-\alpha} \\
&= (1-\alpha)\lambda X_t^e \hat{N}_t^{e\phi} + (1-\alpha)(1-\lambda) X_t^e + \alpha X_t^e \\
&= X_t^e \left((1-\alpha)\lambda \hat{N}_t^{e\phi} + (1-\lambda)(1-\alpha) + \alpha \right)
\end{aligned}
$$

The reduced problem is therefore

$$\underset{\{\hat{N}_t^e, N_t^e, Y_t^e, X_t^e\}}{Max} U_t = \lambda \left(Ln \left(X_t^{e1-\alpha} \hat{N}_t^{e\phi} \right) - \frac{1^{1+\phi}}{1+\phi} \right) + (1-\lambda) \left(Ln \left(X_t^{e1-\alpha} \right) - \frac{\hat{N}_t^{e1+\phi}}{1+\phi} \right)$$

$$\begin{array}{lrcl} s.t. & & & \\ (\mu_2) & N_t^e & = & \lambda + (1-\lambda)\hat{N}_t^e \\ (\mu_3) & Y_t^e & = & X_t^e \left((1-\alpha)\lambda \hat{N}_t^{e\phi} + (1-\lambda)(1-\alpha) + \alpha \right) \\ (\mu_8) & Y_t^e & = & A_t N_t^e \end{array}$$

The FOCs on $\hat{N}_t^e, N_t^e, Y_t^e, X_t^e$ are

$$\begin{aligned} \mu_2 - \mu_8 A_t &= 0 \\ \mu_8 &= \frac{\mu_2}{A_t} \end{aligned} \tag{A.6}$$

$$\begin{aligned} \mu_3 + \mu_8 &= 0 \\ \mu_8 &= -\mu_3 \end{aligned} \tag{A.7}$$

$$\begin{aligned} \lambda \frac{(1-\alpha) X_t^{e-\alpha} \hat{N}_t^{e\phi}}{X_t^{e1-\alpha} \hat{N}_t^{e\phi}} + (1-\lambda) \frac{(1-\alpha) X_t^{e-\alpha}}{X_t^{e1-\alpha}} - \mu_3 \left((1-\alpha)\lambda \hat{N}_t^{e\phi} + (1-\lambda)(1-\alpha) + \alpha \right) &= 0 \\ \lambda \frac{(1-\alpha)}{X_t^e} + (1-\lambda) \frac{(1-\alpha)}{X_t^e} - \mu_3 \left((1-\alpha)\lambda \hat{N}_t^{e\phi} + (1-\lambda)(1-\alpha) + \alpha \right) &= 0 \\ \frac{1}{X_t^e} - \mu_3 \left(\lambda \hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha} \right) &= 0 \\ \frac{1}{X_t^e \left(\lambda \hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha} \right)} &= \mu_3 \end{aligned} \tag{A.8}$$

$$\begin{aligned} \lambda \frac{\phi X_t^{e1-\alpha} \hat{N}_t^{e\phi-1}}{X_t^{e1-\alpha} \hat{N}_t^{e\phi}} - (1-\lambda)\hat{N}_t^{e\phi} - \mu_2(1-\lambda) - \mu_3 X_t^e (1-\alpha)\phi\lambda \hat{N}_t^{e\phi-1} &= 0 \\ \lambda\phi - (1-\lambda)\hat{N}_t^{e(1+\phi)} - \mu_2(1-\lambda)\hat{N}_t^{e\phi} - \mu_3 X_t^e (1-\alpha)\phi\lambda \hat{N}_t^{e\phi} &= 0 \\ \lambda\phi - (1-\lambda)\hat{N}_t^{e(1+\phi)} - \mu_2(1-\lambda)\hat{N}_t^{e\phi} &= \mu_3 X_t^e (1-\alpha)\phi\lambda \hat{N}_t^{e\phi} \\ \frac{\lambda\phi \hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi} - \mu_2(1-\lambda)}{X_t^e (1-\alpha)\phi\lambda} &= \mu_3 \end{aligned} \tag{A.9}$$

Use (A.6) to replace out for μ_2

$$\begin{aligned}
\mu_8 &= \frac{\mu_2}{A_t} \\
\mu_2 &= \mu_8 A_t \\
&= -\mu_3 A_t && \text{(A.10)}
\end{aligned}$$

Replace (A.10) in (A.8)

$$\begin{aligned}
\frac{\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi} - \mu_2(1-\lambda)}{X_t^e(1-\alpha)\phi\lambda} &= \mu_3 \\
\frac{\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}}{X_t^e(1-\alpha)\phi\lambda} + \frac{\mu_3 A_t(1-\lambda)}{X_t^e(1-\alpha)\phi\lambda} &= \mu_3 \\
\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi} + \mu_3 A_t(1-\lambda) &= \mu_3\left(X_t^e(1-\alpha)\phi\lambda\right) \\
\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi} &= \mu_3\left(X_t^e(1-\alpha)\phi\lambda - A_t(1-\lambda)\right) \\
\frac{\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}}{X_t^e(1-\alpha)\phi\lambda - A_t(1-\lambda)} &= \mu_3 && \text{(A.11)}
\end{aligned}$$

Equate (A.9) and (A.11)

$$\begin{aligned}
\frac{\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}}{X_t^e(1-\alpha)\phi\lambda - A_t(1-\lambda)} &= \frac{1}{X_t^e\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right)} \\
\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi} &= \frac{X_t^e(1-\alpha)\phi\lambda - A_t(1-\lambda)}{X_t^e\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right)} \\
\left(\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}\right)\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) &= \frac{X_t^e(1-\alpha)\phi\lambda - A_t(1-\lambda)}{X_t^e} \\
\left(\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}\right)\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) &= (1-\alpha)\phi\lambda - \frac{A_t(1-\lambda)}{X_t^e}
\end{aligned}$$

The marginal cost condition implies that $X_t^e = A_t\hat{N}_t^{e-\phi}$, so that

$$\begin{aligned}
\left(\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}\right)\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) &= (1-\alpha)\phi\lambda - \frac{A_t(1-\lambda)}{X_t^e} \\
\left(\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}\right)\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) &= (1-\alpha)\phi\lambda - \frac{A_t(1-\lambda)}{A_t\hat{N}_t^{e-\phi}} \\
\left(\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}\right)\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) &= (1-\alpha)\phi\lambda - (1-\lambda)\hat{N}_t^{e\phi} \\
\left(\lambda\phi\hat{N}_t^{e-1} - (1-\lambda)\hat{N}_t^{e\phi}\right)\left(\lambda\hat{N}_t^{e\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right) + (1-\lambda)\hat{N}_t^{e\phi} &= (1-\alpha)\phi\lambda
\end{aligned}$$

Since $\hat{N}_t^e = \frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda}$

$$\left(\lambda\phi\left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda}\right)^{-1} - (1-\lambda)\left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda}\right)^{\phi}\right)\left(\lambda\left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda}\right)^{\phi} + (1-\lambda) + \frac{\alpha}{1-\alpha}\right)$$
$$= (1-\alpha)\phi\lambda - (1-\lambda)\left(\frac{N_t^e}{1-\lambda} - \frac{\lambda}{1-\lambda}\right)^{\phi}$$

leading to Proposition 1. For $\lambda = 0$, the above condition reduces to efficient employment in Gali and Monacelli (2005), which is $N_t^e = (1-\alpha)^{\frac{1}{1+\phi}}$.

Proposition 2: Natural Equilibrium

Combine the optimal labour supply condition, $\omega_t^n = \hat{C}_t^n \hat{N}_t^{n\phi}$, and the marginal cost condition, $\omega_t^n = \frac{\varepsilon_P - 1}{\varepsilon_P} X_t^{n-\alpha} A_t$

$$\begin{aligned}
\hat{C}_t^n \hat{N}_t^{n\phi} &= \frac{\varepsilon_P - 1}{\varepsilon_P} X_t^{n-\alpha} A_t \\
X_t^{n1-\alpha}\left(\frac{N_t^n}{1-\lambda}\right)^{\phi} &= \frac{\varepsilon_P - 1}{\varepsilon_P} X_t^{n-\alpha} A_t \\
X_t^n A_t N_t^{n\phi}\left(\frac{1}{1-\lambda}\right)^{\phi} &= \frac{\varepsilon_P - 1}{\varepsilon_P} \\
X_t^n &= N_t^{n-\phi}(1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P} A_t
\end{aligned}$$

Replace this in the goods market-clearing condition

$$\begin{aligned}
Y_t^n &= X_t^{n\alpha}\left((1-\alpha)C_t^n + \alpha\hat{C}_t^n\right) \\
A_t N_t^n &= X_t^{n\alpha}\left((1-\alpha)\left(\lambda\check{C}_t^n + (1-\lambda)\hat{C}_t^n\right) + \alpha\hat{C}_t^n\right) \\
&= X_t^{n\alpha}\left((1-\alpha)\left(\lambda\omega_t^n + (1-\lambda)X_t^{n1-\alpha}\right) + \alpha X_t^{n1-\alpha}\right) \\
&= X_t^{n\alpha}\left((1-\alpha)\lambda\frac{\varepsilon_P - 1}{\varepsilon_P} X_t^{n-\alpha} A_t + (1-\alpha)(1-\lambda)X_t^{n1-\alpha} + \alpha X_t^{n1-\alpha}\right) \\
&= (1-\alpha)\lambda\frac{\varepsilon_P - 1}{\varepsilon_P} A_t + X_t^n\left((1-\alpha)(1-\lambda) + \alpha\right) \\
A_t N_t^n &= (1-\alpha)\lambda\frac{\varepsilon_P - 1}{\varepsilon_P} A_t + \left(\frac{A_t}{N_t^{n\phi}}(1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P}\right)\left((1-\alpha)(1-\lambda) + \alpha\right) \\
N_t^{n1+\phi} &= N_t^{n\phi}(1-\alpha)\lambda\frac{\varepsilon_P - 1}{\varepsilon_P} + (1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P}\left((1-\alpha)(1-\lambda) + \alpha\right) \\
N_t^{n\phi}\left(N_t^n - (1-\alpha)\lambda\frac{\varepsilon_P - 1}{\varepsilon_P}\right) &= \frac{\varepsilon_P - 1}{\varepsilon_P}(1-\lambda)^{\phi}\left((1-\alpha)(1-\lambda) + \alpha\right)
\end{aligned}$$

leading to Proposition 2. For $\lambda = 0$, the above condition reduces to natural employment in Gali and Monacelli (2005), which is $N_t^n = \left(\frac{\varepsilon_P - 1}{\varepsilon_P}\right)^{\frac{1}{1+\phi}}$.

The other variables can be backed out as follows

$$
\begin{aligned}
Y_t^n &= N_t^n \\
X_t^n &= N_t^{n-\phi}(1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P} \\
\omega_t^n &= \frac{\varepsilon_P - 1}{\varepsilon_P}\left(N_t^{n-\phi}(1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P}\right)^{-\alpha} \\
\hat{N}_t^n &= \frac{N_t^n}{1-\lambda} \\
\check{N}_t^n &= 1 \\
\hat{C}_t &= \left(N_t^{n-\phi}(1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P}\right)^{1-\alpha} \\
\check{C}_t &= \frac{\varepsilon_P - 1}{\varepsilon_P}\left(N_t^{n-\phi}(1-\lambda)^{\phi}\frac{\varepsilon_P - 1}{\varepsilon_P}\right)^{-\alpha}
\end{aligned}
$$

Proposition 3: Canonical Equilibrium

I linearize the equilibrium using the transformation $X_t = Xe^{x_t}$ ($X_t = X\frac{X_t}{X} = Xe^{Ln\frac{X_t}{X}} = Xe^{x_t}$) for any variable X_t, and around a steady state where $x_t = 0$ (ie. a steady state where $X_t = X \rightarrow LnX_t = LnX \rightarrow LnX_t - LnX = 0 \rightarrow x_t = 0$). x_t is the linearized version of X_t. I also use log-deviations, $\tilde{x}_t$, where $\tilde{x}_t = x_t - x_t^e$.

The firm's price-setting decisions are standard and a linearized approximation is derived in Bergholt (2012) $\pi_{Ht} = \beta\pi_{Ht+1} + \xi mc_t$ with $\xi = \frac{(1-\theta)(1-\theta\beta)}{\theta}$. In log-deviation terms, this is $\pi_{Ht} = \beta\pi_{Ht+1} + \xi \tilde{mc}_t$. The canonical equilibrium is nested in Iyer (2014) and also found in Boerma (2014). To derive the New Keynesian Phillips Curve (NKPC), start with the marginal cost condition, $\tilde{mc}_t = \tilde{\omega}_t + \alpha\tilde{x}_t$, and combine this with the international risk-sharing condition, $\tilde{\hat{c}}_t = (1-\alpha)\tilde{x}_t$, asset holder intratemporal condition, $\tilde{\omega}_t = \phi\tilde{\hat{n}}_t + \tilde{\hat{c}}_t$, hand-to-mouth consumption function, $\tilde{\check{c}}_t = \tilde{\omega}_t$, labour market-clearing, $\tilde{n}_t = (1-\lambda)\tilde{\hat{n}}_t$, goods-market clearing, $\tilde{y}_t = \alpha\tilde{x}_t + (1-\alpha)\left(\lambda\tilde{\check{c}}_t + (1-\lambda)\tilde{\hat{c}}_t\right) + \alpha\tilde{\hat{c}}_t$, and aggregate consumption, $\tilde{c}_t = \lambda\tilde{\check{c}}_t + (1-\lambda)\tilde{\hat{c}}_t$, to find that

$$\tilde{mc}_t = \Lambda^{-1}\tilde{y}_t$$

where $\Lambda^{-1} = \frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}$. Replace this in $\pi_{Ht} = \beta\pi_{Ht+1} + \xi\tilde{mc}_t$ to derive the NKPC as in equation (2.43). To derive the IS Equation, start with aggregate consumption, $\tilde{c}_t = \lambda\tilde{\check{c}}_t + (1-\lambda)\tilde{\hat{c}}_t$, and combine this with the hand-to-mouth consumption function, $\tilde{\check{c}}_t = \tilde{\omega}_t$, asset holder Euler Equation, $\tilde{\hat{c}}_t = \tilde{\hat{c}}_{t+1} - (i_t - E_t\pi_{t+1})$, asset holder intratemporal condition, $\tilde{\omega}_t = \phi\tilde{\hat{n}}_t + \tilde{\hat{c}}_t$, goods market-clearing condition, $\tilde{y}_t = \alpha\tilde{x}_t + (1-\alpha)\left(\lambda\tilde{\check{c}}_t + (1-\lambda)\tilde{\hat{c}}_t\right) + \alpha\tilde{\hat{c}}_t$, and the relation between CPI inflation, domestic inflation, and terms of trade, $E_t\pi_{t+1} = E_t\pi_{H,t+1} + \alpha\Delta E_t\tilde{x}_{t+1}$ to get equation (2.41).

Micro-Founding the Central Bank's Welfare Loss Function

Proposition 4: Welfare Loss Function

The Social Planner maximizes a weighted combination of individual agents' utilities, with weights given by λ, the fraction of financially-included consumers in the economy. Denoting financially-included households with the symbol,$\hat{}$, and financially-excluded households with the symbol,$\check{}$, the aggregate utility function, as in Bilbiie (2008), is

$$U_t = \lambda\check{U}_t + (1-\lambda)\hat{U}_t$$

where $U^m = U(C_t^m, N_t^m) = LnC_t^m - \frac{N_t^{m1+\phi}}{1+\phi})$ for each agent $m = \{\hat{}, \check{}\}$. We take a second-order approximation of U_t^m (m superscript omitted from the right-hand side for clarity, for the time-being). In these derivations, $t.i.p.$ represents "terms independent of policy" such as flexible price variables, and $o(3)$ contains Taylor approximations that are higher than second-order.

$$U_t^m = U^m + \left[U_C(C_t - C) + \frac{U_{CC}}{2}(C_t - C)^2\right] + \left[U_N(N_t - N) + \frac{U_{NN}}{2}(N_t - N)^2\right] + t.i.p. + o(3)$$

$$U_t^m - U^m = \left[U_C C\left(\frac{C_t - C}{C}\right) + \frac{U_{CC}C^2}{2}\left(\frac{C_t - C}{C}\right)^2\right] + \left[U_N N\left(\frac{N_t - N}{N}\right) + \frac{U_{NN}N^2}{2}\left(\frac{N_t - N}{N}\right)^2\right] + t.i.p. +$$

$$U_t^m - U^m = \left[U_C C\,(c_t) + \left(\frac{U_C C + U_{CC}C^2}{2}\right)c_t^2\right] + \left[U_N N\,(n_t) + \left(\frac{U_N N + U_{NN}N^2}{2}\right)n_t^2\right] + t.i.p. + o(3)$$

$$\frac{U_t^m - U^m}{U_C C^m} = \left(c_t + \frac{1 + \frac{U_{CC}C}{U_C}}{2}c_t^2\right) + \frac{U_N}{U_C}\frac{N}{C}\left(n_t + \frac{1 + \frac{U_{NN}N}{U_N}}{2}n_t^2\right) + t.i.p. + o(3)$$

Since $U = LnC - \frac{N^{1+\phi}}{1+\phi}$, $U_C = \frac{1}{C}$, $U_{CC} = -\frac{1}{C^2}$, $U_N = -N^{\phi}$, $U_{NN} = -\phi N^{\phi-1}$

$$\frac{U^{m_t} - U^m}{U_C C^m} = \left(c_t + \frac{1 + \frac{CC}{-C^2}}{2}c_t^2\right) + \frac{U_N}{U_C}\frac{N}{C}\left(n_t + \frac{1 + \frac{-\phi N^{\phi-1}N}{-N^{\phi}}}{2}n_t^2\right) + t.i.p. + o(3)$$

$$\frac{U_t^m - U^m}{U_C C^m} = c_t + \frac{U_N}{U_C}\frac{N}{C}\left(n_t + \frac{1+\phi}{2}n_t^2\right) + t.i.p. + o(3)$$

Note the following relation between variables in log-deviation terms (deviation of a variable from its efficient level), sticky price log-linearized variables, and flexible price log-linearized variables: $\tilde{x}_t = x_t - x_t^e \rightarrow x_t = \tilde{x}_t + x_t^e$

$$\frac{U_t^m - U^m}{U_C C^m} = (\tilde{c}_t + c_t^e) + \frac{U_N}{U_C}\frac{N}{C}\left((\tilde{n}_t + n_t^e) + \frac{1+\phi}{2}(\tilde{n}_t + n_t^e)^2\right) + t.i.p. + o(3)$$

Noting that terms c_t^e , $\frac{U_N}{U_C}\frac{N}{C}n_t^e$, and $\frac{U_N}{U_C}\frac{N}{C}\frac{1+\phi}{2}n_t^{e2}$ go into $t.i.p$ and expanding out the quadratic expressions

$$\frac{U_t^m - U^m}{U_C C^m} = \tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t + \frac{U_N}{U_C}\frac{N}{C}(1+\phi)\tilde{n}_t n_t^e + \frac{U_N}{U_C}\frac{N}{C}\frac{1+\phi}{2}\tilde{n}_t^2 + t.i.p. + o(3)$$

Then, since $U_t = \lambda\check{U}_t + (1-\lambda)\hat{U}_t$ (and noting that $\hat{C} = \check{C} = C$ as steady state profits and assets are 0, and $\hat{N} = \check{N} = N$ as preferences are identical)

$$\begin{aligned}\frac{U_t - U}{U_C C} \;&=\; \tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t + \frac{U_N}{U_C}\frac{N}{C}(1+\phi)\left(\lambda\tilde{\check{n}}_t\check{n}_t^e + (1-\lambda)\tilde{\hat{n}}_t\hat{n}_t^e\right)\\ &\quad + \frac{U_N}{U_C}\frac{N}{C}\frac{1+\phi}{2}\left(\lambda\tilde{\check{n}}_t^2 + (1-\lambda)\tilde{\hat{n}}_t^2\right) + t.i.p. + o(3)\end{aligned}$$

Noting that $\check{n}_t^e = 0$ and $\tilde{\check{n}}_t = 0$ (as hand-to-mouth labour supply is constant over the business cycle due to the special case of log utility), and also $\hat{n}_t^e = 0$ in both flexible price and efficient equilibriums, $\tilde{\hat{n}}_t = \frac{\tilde{n}_t}{1-\lambda}$, and $\tilde{\hat{n}}_t^2 = \frac{\tilde{n}_t^2}{(1-\lambda)^2}$

$$\frac{U_t - U}{U_C C} = \tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t + \frac{U_N}{U_C}\frac{N}{C}\frac{(1+\phi)(1-\lambda)}{2(1-\lambda)^2}\tilde{n}_t^2 + t.i.p. + o(3)$$

Separate the above into linear and quadratic terms

$$\frac{U_t - U}{U_C C} = \underbrace{\tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t}_{linear} + \underbrace{\frac{U_N}{U_C}\frac{N}{C}\frac{(1+\phi)}{2(1-\lambda)}\tilde{n}_t^2}_{quadratic} + t.i.p. + o(3)$$

The quadratic term is

$$\frac{U_N}{U_C}\frac{N}{C}\frac{(1+\phi)}{2(1-\lambda)}\tilde{n}_t^2 + t.i.p + o(3)$$

Since $\tilde{n}_t^2 = (\tilde{y}_t + \Delta)^2 + t.i.p + o(3) \rightarrow \tilde{n}_t^2 = \tilde{y}_t^2 + t.i.p + o(3)$

$$\frac{U_N}{U_C}\frac{N}{C}\frac{1}{2}\left(\frac{1+\phi}{1-\lambda}\right)\tilde{y}_t^2 + t.i.p + o(3)$$

Consider now the linear term $\tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t$. Note from the (exactly log-linear) production function that $\tilde{n}_t = \tilde{y}_t + \Delta_t$, so that

$$\tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\left(\tilde{y}_t + \Delta_t\right) + t.i.p + o(3)$$

The loss function is therefore

$$\frac{U_t - U}{U_C C} = \tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\left(\tilde{y}_t + \Delta_t\right) + \frac{U_N}{U_C}\frac{N}{C}\frac{1}{2}\left(\frac{1+\phi}{1-\lambda}\right)\tilde{y}_t^2 + t.i.p. + o(3) \tag{A.12}$$

In Gali and Monacelli (2005), $\tilde{c}_t = (1-\alpha)\tilde{x}_t$ and $\tilde{y}_t = \tilde{x}_t$ and $\frac{U_N}{U_C}\frac{N}{C} = -(1-\alpha)$ so that the linear terms become $-(1-\alpha)\Delta_t$ and the loss function derivation is completed at this point. In Bilbiie (2008), its even simpler since $\tilde{c}_t = \tilde{y}_t$ and $\frac{U_N}{U_C}\frac{N}{C} = -1$ and the loss function derivation is also completed at this point. In the present case, however, deriving a quadratic loss function is far more complicated, due to the fundamental asymmetry in the ability of households to share risk in an open economy.

To replace out for the linear terms, second-order approximations of all the equilibrium conditions are required. These derivations proceed as follows.

Aggregate consumption

Aggregate consumption is a weighted average of the consumption of the heterogeneous agents: $C_t = (1-\lambda)\hat{C}_t + \lambda\check{C}_t$. Defining each variable X_t as $X_t = Xe^{x_t}$ where $x_t = LnX_t - LnX$), this is

$$Ce^{c_t} = (1-\lambda)\hat{C}e^{\hat{c}_t} + (\lambda)\check{C}e^{\check{c}_t} + t.i.p + o(3)$$

Taking a Taylor Expansion around $c = \hat{c} = \check{c} = 0$

$$\begin{aligned} Ce^{c_t}|_{c_t=0} + Ce^{c_t}|_{c_t=0}(c_t - 0) + Ce^{c_t}|_{c_t=0}\frac{(c_t-0)^2}{2} \\ = \\ (1-\lambda)\hat{C}e^{\hat{c}_t}|_{\hat{c}_t=0}\ldots \\ +(1-\lambda)\hat{C}e^{\hat{c}_t}|_{\hat{c}_t=0}(\hat{c}_t - 0) + (1-\lambda)\hat{C}e^{\hat{c}_t}|_{\hat{c}_t=0}\frac{(\hat{c}_t-0)^2}{2}\ldots \\ +\lambda\check{C}e^{\check{c}_t}|_{\check{c}_t=0} + \lambda\check{C}e^{\check{c}_t}|_{\check{c}_t=0}(\check{c}_t - 0) + \lambda\check{C}e^{\check{c}_t}|_{\check{c}_t=0}\frac{(\check{c}_t-0)^2}{2} + t.i.p + o(3) \end{aligned}$$

Simplifying

$$C + Cc_t + C\frac{c_t^2}{2} = ((1-\lambda)\hat{C} + \lambda\check{C}) + (1-\lambda)\hat{C}\hat{c}_t + (1-\lambda)\hat{C}\frac{\hat{c}_t^2}{2}...$$
$$+\lambda\check{C}\check{c}_t + \lambda\check{C}\frac{\check{c}_t^2}{2} + t.i.p + o(3)$$

Noting that $C = (1-\lambda)\hat{C} + \lambda\check{C}$ and that $C = \hat{C} = \check{C}$ at steady state

$$c_t + \frac{c_t^2}{2} = (1-\lambda)\hat{c}_t + (1-\lambda)\frac{\hat{c}_t^2}{2} + \lambda\check{c}_t + \lambda\frac{\check{c}_t^2}{2} + t.i.p + o(3)$$

The first-order approximation is thus

$$c_t = (1-\lambda)\hat{c}_t + \lambda\check{c}_t + t.i.p + o(3) \tag{A.13}$$

In log-deviation terms, noting that for a variable $x_t \to x_t = \tilde{x}_t + x_t^e$, the above is

$$(\tilde{c}_t + c_t^e) = (1-\lambda)\left(\tilde{\hat{c}}_t + \hat{c}_t^e\right) + \lambda\left(\tilde{\check{c}} + \check{c}_t^e\right) + t.i.p + o(3)$$

Since flexible price variables go into $t.i.p$, the first-order approximation is

$$\tilde{c}_t = (1-\lambda)\tilde{\hat{c}}_t + \lambda\tilde{\check{c}} + t.i.p + o(3)$$

The second-order approximation is

$$\left(c_t + \frac{1}{2}c_t^2\right) = (1-\lambda)\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \lambda\left(\check{c}_t + \frac{1}{2}\check{c}_t^2\right) + t.i.p + o(3)$$

In log-deviation terms, where $\tilde{x}_t = x_t - x_t^e$, the above is

$$(\tilde{c}_t + c_t^e) + \frac{1}{2}(\tilde{c}_t + c_t^e)^2 = (1-\lambda)\left(\left(\tilde{\hat{c}}_t + \hat{c}_t^e\right) + \frac{1}{2}\left(\tilde{\hat{c}}_t + \hat{c}_t^e\right)^2\right)$$
$$+\lambda\left(\left(\tilde{\check{c}}_t + \check{c}_t^e\right) + \frac{1}{2}\left(\tilde{\check{c}} + \check{c}_t^e\right)^2\right) + t.i.p + o(3)$$

Or

$$\tilde{c}_t + \frac{1}{2}\tilde{c}_t{}^2 + \tilde{c}_t c_t^e = (1-\lambda)\left(\tilde{\hat{c}}_t + \frac{1}{2}\tilde{\hat{c}}_t^2 + \tilde{\hat{c}}_t\hat{c}_t^e\right) + \lambda\left(\tilde{\check{c}} + \frac{1}{2}\tilde{\check{c}}^2 + \tilde{\check{c}}\check{c}_t^e\right) + t.i.p + o(3)$$

Noting that $c_t^e = \hat{c}_t^e = \check{c}_t^e$ in the linearized efficient equilibrium, the second-order approximation is

$$\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t{}^2\right) = (1-\lambda)\left(\tilde{\tilde{c}}_t + \frac{1}{2}\tilde{\tilde{c}}_t^2\right) + \lambda\left(\tilde{\tilde{c}} + \frac{1}{2}\tilde{\tilde{c}}^2\right) + t.i.p + o(3) \tag{A.14}$$

The square of the first-order approximation is

$$\tilde{c}_t{}^2 = (1-\lambda)^2\tilde{\tilde{c}}_t^2 + \lambda^2\tilde{\tilde{c}}^2 + 2\lambda(1-\lambda)\tilde{\tilde{c}}_t\tilde{\tilde{c}} + t.i.p + o(3) \tag{A.15}$$

Resource constraint

The resource constraint, $Y_t X_t^{-\alpha} = \left((1-\alpha)C_t + \alpha\hat{C}_t\right)$, can be written as

$$Ye^{y_t}\left(Xe^{x_t}\right)^{-\alpha} = \left((1-\alpha)Ce^{c_t} + (\alpha)\,\hat{C}e^{\hat{c}_t}\right) + t.i.p + o(3)$$

A Taylor Expansion around $y = x = c = \hat{c}_t = 0$ yields

$$\begin{aligned}
&YX^{-\alpha}e^{y_t}e^{-\alpha x_t}|_{y_t=0,x_t=0} + YX^{-\alpha}e^{y_t}e^{-\alpha x_t}|_{y_t=0,x_t=0}(y_t-0) + YX^{-\alpha}e^{y_t}e^{-\alpha x_t}|_{y_t=0,x_t=0}\frac{(y_t-0)^2}{2}... \\
&\quad -\alpha YX - \alpha e^{y_t}e^{-\alpha x_t}|_{y_t=0,s_t=0}(x_t-0) + \alpha^2 YX^{-\alpha}e^{y_t}e^{-\alpha x_t}|_{y_t=0,x_t=0}\frac{(x_t-0)^2}{2}... \\
&\quad -\alpha YX^{-\alpha}e^{y_t}e^{-\alpha x_t}|_{y_t=0,x_t=0}(x_t-0)(y_t-0)... \\
&= \\
&\left((1-\alpha)Ce^{c_t} + (\alpha)\,C^o e^{\hat{c}_t}\right)|_{c_t=0,\hat{c}_t=0} + (1-\alpha)Ce^{c_t}|_{c_t=0}(c_t-0) + (1-\alpha)Ce^{c_t}|_{c_t=0}\frac{(c_t-0)^2}{2}... \\
&\quad +\alpha\hat{C}e^{\hat{c}_t}|_{c_t=0}(\hat{c}_t-0) + \alpha\hat{C}e^{\hat{c}_t}|_{\hat{c}_t=0}\frac{(\hat{c}_t-0)^2}{2} + t.i.p + o(3)
\end{aligned}$$

Simplifying the above

$$\begin{aligned}
YX^{-\alpha} + YX^{-\alpha}y_t + YX^{-\alpha}\frac{y_t^2}{2} - \alpha YX^{-\alpha}x_t + \alpha^2 YX^{-\alpha}\frac{x_t{}^2}{2} - \alpha YX^{-\alpha}x_t y_t \quad &= \\
\left((1-\alpha)C + \alpha\hat{C}\right) + (1-\alpha)Cc_t + (1-\alpha)C\frac{c_t^2}{2} + \alpha\hat{C}\hat{c}_t + \alpha\hat{C}\frac{\hat{c}_t^2}{2} + t.i.p + o(3)
\end{aligned}$$

Noting that $X = 1$ and $C = \hat{C}$, the resource constraint at steady state implies that $Y = (1-\alpha)C + \alpha\hat{C}$ or $Y = C$

$$y_t + \frac{y_t^2}{2} - \alpha x_t + \alpha^2\frac{x_t{}^2}{2} - \alpha x_t y_t = (1-\alpha)c_t + (1-\alpha)\frac{c_t^2}{2} + \alpha\hat{c}_t + \alpha\frac{\hat{c}_t^2}{2} + t.i.p + o(3)$$

Since $\hat{c}_t = (1-\alpha)x_t$ and $\hat{c}_t^2 = (1-\alpha)^2 x_t^2$, $\alpha\hat{c}_t + \alpha x_t = \alpha(2-\alpha)x_t$. Similarly, $\alpha\frac{\hat{c}_t^2}{2} - \alpha^2\frac{x_t{}^2}{2} =$

$\frac{\alpha^3-3\alpha^2+\alpha}{2}x_t^2$. We can thus rewrite the above as

$$y_t + \frac{y_t^2}{2} = (1-\alpha)c_t + \frac{(1-\alpha)}{2}c_t^2 + \alpha(2-\alpha)x_t + \frac{\alpha^3-3\alpha^2+\alpha}{2}x_t^2 + \alpha x_t y_t + t.i.p + o(3)$$

The first-order approximation in log-deviation terms is

$$(\tilde{y}_t + y_t^e) = (1-\alpha)\left(\tilde{c}_t + c_t^e\right) + \alpha(2-\alpha)\left(\tilde{x}_t + x_t^e\right) + t.i.p + o(3)$$

Since flexible price variables go into $t.i.p$, the first-order approximation is

$$\tilde{y}_t = (1-\alpha)\tilde{c}_t + \alpha(2-\alpha)\tilde{x}_t + t.i.p + o(3) \tag{A.16}$$

The second-order approximation in log-deviation terms is

$$\begin{aligned}(\tilde{y}_t + y_t^e) + \frac{(\tilde{y}_t + y_t^e)^2}{2} \quad &= \quad (1-\alpha)\left(\tilde{c}_t + c_t^e\right) + \frac{(1-\alpha)}{2}\left(\tilde{c}_t + c_t^e\right)^2 ... \\ &+\alpha(2-\alpha)\left(\tilde{x}_t + x_t^e\right) + \frac{\alpha^3-3\alpha^2+\alpha}{2}\left(\tilde{x}_t + x_t^e\right)^2 ... \\ &+\alpha\left(\tilde{x}_t + x_t^n\right)\left(\tilde{y}_t + y_t^e\right) + t.i.p + o(3)\end{aligned}$$

Noting that $x_t^e = y_t^e$ and $c_t^e = (1-\alpha)y_t^e$ in the flexible price equilibrium, the above can be reduced to

$$\tilde{y}_t + \frac{\tilde{y}_t^2}{2} = (1-\alpha)\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2\right) + \alpha(2-\alpha)\tilde{x}_t + \frac{(\alpha^3-3\alpha^2+\alpha)}{2}\tilde{x}_t^2 + \alpha\tilde{x}_t\tilde{y}_t + t.i.p + o(3) \tag{A.17}$$

Squaring the first-order approximation gives

$$\tilde{y}_t^2 = (1-\alpha)^2\tilde{c}_t^2 + \alpha^2(2-\alpha)^2\tilde{x}_t^2 + 2\alpha(1-\alpha)(2-\alpha)\tilde{c}_t\tilde{x}_t + t.i.p + o(3) \tag{A.18}$$

Multiplicative (exactly log-linear) equilibrium conditions

The international risk-sharing condition, $\hat{C}_t = X_t^{1-\alpha}$, in log-deviation terms is

$$\tilde{\hat{c}}_t = (1-\alpha)\tilde{x}_t + t.i.p + o(3) \tag{A.19}$$

The production function, $Y_t = \Delta_t N_t$, in log-deviation terms is

$$\tilde{y}_t = \tilde{n}_t - \Delta_t + t.i.p + o(3) \tag{A.20}$$

The hand-to-mouth consumption function, $\check{C}_t = w_t$, in log-deviation terms is

$$\tilde{\check{c}} = \tilde{\omega}_t + t.i.p + o(3) \tag{A.21}$$

The asset holder optimal labour supply condition, $\omega\left(s^t\right) = \hat{N}_t{}^{\phi}\hat{C}_t{}^{\sigma}$, in log-deviation terms is

$$\tilde{\omega}_t = \tilde{\hat{c}}_t + \phi\tilde{\hat{n}}_t + t.i.p + o(3) \tag{A.22}$$

I now begin to replace out for the linear terms in $\tilde{c}_t$ and $\tilde{y}_t$.

Putting $\tilde{c}_t$ in terms of $\tilde{y}_t$ and $\tilde{x}_t$

We first re-write the consumption function of hand-to-mouth agents as follows

$$\begin{aligned}
\tilde{\check{c}}_t &= \tilde{w}_t + t.i.p + o(3) \\
&= \tilde{\hat{c}}_t + \phi\tilde{\hat{n}}_t + t.i.p + o(3) \\
&= \tilde{\hat{c}}_t + \frac{\phi}{1-\lambda}\tilde{n}_t + t.i.p + o(3) \\
&= (1-\alpha)\tilde{x}_t + \frac{\phi}{1-\lambda}\tilde{y}_t + \frac{\phi}{1-\lambda}\Delta_t + t.i.p + o(3)
\end{aligned}$$

Squaring the above

$$\tilde{\check{c}}_t^2 = (1-\alpha)^2\tilde{x}_t^2 + \left(\frac{\phi}{1-\lambda}\right)^2 \tilde{y}_t{}^2 + 2(1-\alpha)\left(\frac{\phi}{1-\lambda}\right)\tilde{x}_t\tilde{y}_t$$

Aggregate consumption is then

$$\begin{aligned}
\tilde{c}_t &= (1-\lambda)\tilde{\hat{c}}_t + \lambda\tilde{\check{c}}_t + t.i.p + o(3) \\
&= (1-\lambda)(1-\alpha)\tilde{x}_t + \lambda\left((1-\alpha)\tilde{x}_t + \frac{\phi}{1-\lambda}\tilde{y}_t + \frac{\phi}{1-\lambda}\Delta_t\right) + t.i.p + o(3) \\
&= (1-\alpha)\tilde{x}_t + \frac{\lambda\phi}{1-\lambda}\tilde{y}_t + \frac{\lambda\phi}{1-\lambda}\Delta_t + t.i.p + o(3)
\end{aligned}$$

Squaring the above

$$\tilde{c_t}^2 = (1-\alpha)^2\tilde{x}_t^2 + \left(\frac{\lambda\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + 2(1-\alpha)\left(\frac{\lambda\phi}{1-\lambda}\right)\tilde{x}_t\tilde{y}_t + t.i.p + o(3)$$

The second-order Taylor approximation of the consumption accounting equation is

$$\left(\tilde{c}_t + \frac{1}{2}\tilde{c_t}^2\right) = (1-\lambda)\left(\tilde{\hat{c}}_t + \frac{1}{2}\tilde{\hat{c}}_t^2\right) + \lambda\left(\tilde{\check{c}} + \frac{1}{2}\tilde{\check{c}}^2\right) + t.i.p + o(3)$$

Replacing in for the appropriate consumption definitions

$$\tilde{c}_t + \frac{1}{2}\left((1-\alpha)^2\tilde{x}_t^2 + \left(\frac{\lambda\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + 2(1-\alpha)\left(\frac{\lambda\phi}{1-\lambda}\right)\tilde{x}_t\tilde{y}_t\right)$$
$$=$$
$$(1-\lambda)(1-\alpha)\tilde{x}_t + \frac{1}{2}(1-\lambda)(1-\alpha)^2\tilde{x}_t^2 + \lambda\left((1-\alpha)\tilde{x}_t + \frac{\phi}{1-\lambda}\tilde{y}_t + \frac{\phi}{1-\lambda}\Delta_t\right)...$$
$$+\frac{1}{2}\lambda\left((1-\alpha)^2\tilde{x}_t^2 + \left(\frac{\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + 2(1-\alpha)\left(\frac{\phi}{1-\lambda}\right)\tilde{x}_t\tilde{y}_t\right) + t.i.p + o(3)$$

This reduces to

$$\tilde{c}_t = (1-\alpha)\tilde{x}_t + \frac{\lambda\phi}{1-\lambda}\tilde{y}_t + \frac{\lambda\phi}{1-\lambda}\Delta_t + \frac{\lambda(1-\lambda)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 \tag{A.23}$$

Squaring the above

$$\tilde{c}_t^2 = (1-\alpha)^2\tilde{x}_t^2 + \left(\frac{\lambda\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + 2(1-\alpha)\left(\frac{\lambda\phi}{1-\lambda}\right)\tilde{x}_t\tilde{y}_t \tag{A.24}$$

Putting $\tilde{x}_t$ in terms of $\tilde{y}_t$

The second- order approximation of the resource constraint is

$$\tilde{y}_t + \frac{\tilde{y}_t^2}{2} = (1-\alpha)\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2\right) + \alpha(2-\alpha)\tilde{x}_t + \frac{(\alpha^3 - 3\alpha^2 + \alpha)}{2}\tilde{x}_t^2 + \alpha\tilde{x}_t\tilde{y}_t + t.i.p + o(3)$$

Replacing for (A.23) and (A.24)

$$\begin{aligned}\left(\tilde{y}_t + \frac{1}{2}\tilde{y}_t{}^2\right) &= (1-\alpha)\left((1-\alpha)\tilde{x}_t + \frac{(1-\alpha)^2}{2}\tilde{x}_t{}^2 + \frac{\lambda\phi}{1-\lambda}\left(\tilde{y}_t + \Delta_t\right)\right)... \\ &\quad +(1-\alpha)\left(\frac{\lambda}{2}\left(\frac{\phi}{1-\lambda}\right)^2\tilde{y}_t^2 + \frac{\lambda\phi(1-\alpha)}{1-\lambda}\tilde{y}_t\tilde{x}_t\right)... \\ &\quad +\alpha(2-\alpha)\tilde{x}_t + \frac{\alpha^3-3\alpha^2+\alpha}{2}\tilde{x}_t^2 + \alpha\tilde{x}_t\tilde{y}_t + t.i.p + o(3)\end{aligned}$$

Or

$$\begin{aligned}\tilde{y}_t - \frac{\lambda\phi(1-\alpha)}{1-\lambda}\tilde{y}_t &= (1-2\alpha+\alpha^2+2\alpha-\alpha^2)\tilde{x}_t + \frac{(1-\alpha)^3+\alpha^3-3\alpha^2+\alpha}{2}\tilde{x}_t^2... \\ &\quad +\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\tilde{y}_t^2 - \frac{1}{2}\tilde{y}_t{}^2... \\ &\quad +\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\tilde{y}_t\tilde{x}_t + \frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t + t.i.p + o(3)\end{aligned}$$

Noting that $(1-\alpha)^3 = 1-3\alpha+3\alpha^2-\alpha^3$, the above can be reduced to

$$\begin{aligned}\tilde{y}_t\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right) &= \tilde{x}_t + \frac{1-2\alpha}{2}\tilde{x}_t^2 + \left(\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 - \frac{1}{2}\right)\tilde{y}_t^2... \\ &\quad +\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\tilde{y}_t\tilde{x}_t... \qquad \text{(A.25)} \\ &\quad +\frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t + t.i.p + o(3) \qquad \text{(A.26)}\end{aligned}$$

Squaring the above yields

$$\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{(1-\lambda)}\right)^2\tilde{y}_t^2 = \tilde{x}_t{}^2 + t.i.p + o(3) \qquad \text{(A.27)}$$

To put $\tilde{x}_t$ in terms of $\tilde{y}_t$, manipulate (A.25) so that

$$\tilde{x}_t = \left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\tilde{y}_t - \left(\frac{1-2\alpha}{2}\right)\tilde{x}_t^2 - \left(\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 - \frac{1}{2}\right)\tilde{y}_t^2 - ...$$

$$-\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\tilde{y}_t\tilde{x}_t - \frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t + t.i.p + o(3)$$

Use the second-order approximation of the resource constraint, (A.17), to replace out for

$\tilde{x}_t$ in $\frac{\lambda\phi(1-\alpha)^2-\alpha(1-\lambda)}{1-\lambda}\tilde{y}_t\tilde{x}_t$

$$\begin{aligned}
\tilde{x}_t &= \left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\tilde{y}_t - \left(\frac{1-2\alpha}{2}\right)\tilde{x}_t^2 - \left(\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 - \frac{1}{2}\right)\tilde{y}_t^2 ... \\
&\quad - \frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\tilde{y}_t{}^2 ... \\
&\quad - \frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t + t.i.p + o(3)
\end{aligned}$$

Replace out for $\tilde{x}_t{}^2$ with (A.27) to get

$$\begin{aligned}
\tilde{x}_t &= \left(\frac{1}{2} - \frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 - \left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{y}_t^2 ... \\
&\quad - \left(\left(\frac{1-2\alpha}{2}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{(1-\lambda)}\right)^2\right)\tilde{y}_t^2 + \left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\tilde{y}_t ... \\
&\quad - \frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t + t.i.p + o(3) \qquad \text{(A.28)}
\end{aligned}$$

Aggregate supply

I take a second-order approximation of marginal costs in the model

$$\begin{aligned}
\tilde{mc}_t &= \tilde{\omega}_t + \alpha\tilde{x}_t + t.i.p + o(3) \\
&= \tilde{\tilde{c}}_t + \phi\tilde{\tilde{n}}_t + \alpha\tilde{x}_t + t.i.p + o(3) \\
&= (1-\alpha)\tilde{x}_t + \frac{\phi}{1-\lambda}\tilde{n}_t + \alpha\tilde{x}_t + t.i.p + o(3) \\
&= x_t + \frac{\phi}{1-\lambda}\tilde{y}_t + \frac{\phi}{1-\lambda}\Delta_t + t.i.p + o(3)
\end{aligned}$$

Replace out for $\tilde{x}_t$ from (A.28) to get

$$\begin{aligned}
\tilde{mc}_t &= \left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\tilde{y}_t - \frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t + \left(\frac{1}{2} - \frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2\right) ... \\
&\quad - \left(\frac{1-2\alpha}{2} + \left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)x_t^2 ... \\
&\quad + \frac{\phi}{1-\lambda}\tilde{y}_t + \frac{\phi}{1-\lambda}\Delta_t + t.i.p + o(3)
\end{aligned}$$

Or

$$\begin{aligned}\tilde{mc}_t &= \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)\tilde{y}_t + \left(\frac{1}{2}-\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 \ldots \\ &\quad -\left(\frac{1-2\alpha}{2}+\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\ &\quad +\left(\frac{\phi-\lambda\phi(1-\alpha)}{1-\lambda}\right)\Delta_t + t.i.p + o(3)\end{aligned} \tag{A.29}$$

Squaring the above equation

$$\tilde{mc}_t{}^2 = \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^2 \tilde{y}_t^2 + t.i.p + o(3) \tag{A.30}$$

Equate the expression for marginal costs, (A.29), with the second-order approximation of the Phillips Curve, $\tilde{mc}_t = V_o - \frac{1}{2}\tilde{mc}_t{}^2 - \frac{1}{2}\frac{\varepsilon}{\xi}\pi_{Ht}^2$ (Wills, 2013), where V_0 is predetermined and independent of policy when considered from a timeless perspective (Benigno and Woodford, 2005), so that

$$\begin{aligned}V_o - \frac{1}{2}\tilde{mc}_t{}^2 - \frac{1}{2}\frac{\varepsilon}{\xi}\pi_{Ht}^2 &= \left(\frac{1}{2}-\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 + \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)\tilde{y}_t \ldots \\ &\quad -\left(\frac{1-2\alpha}{2}+\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\ &\quad +\left(\frac{\phi-\lambda\phi(1-\alpha)}{1-\lambda}\right)\Delta_t + t.i.p + o(3)\end{aligned}$$

Rearranging

$$\begin{aligned}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)\tilde{y}_t &= \left(\frac{1-2\alpha}{2}+\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\ &\quad +V_o - \frac{1}{2}\tilde{mc}_t{}^2 - \frac{1}{2}\frac{\varepsilon}{\xi}\pi_{Ht}^2 - \left(\frac{\phi-\lambda\phi(1-\alpha)}{1-\lambda}\right)\Delta_t \ldots \\ &\quad -\left(\frac{1}{2}-\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 + t.i.p + o(3)\end{aligned}$$

Replace for $\tilde{mc}_t{}^2$ from (A.30) to get

$$\begin{aligned}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)\tilde{y}_t &= V_o - \frac{1}{2}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^2\tilde{y}_t^2 - \frac{1}{2}\frac{\varepsilon}{\xi}\pi_{Ht}^2 \ldots \\ &\quad +\left(\frac{1-2\alpha}{2}+\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\ &\quad -\left(\frac{1}{2}-\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 - \left(\frac{\phi-\lambda\phi(1-\alpha)}{1-\lambda}\right)\Delta_t + t.i.p + o(3)\end{aligned}$$

Simplifying this further, an expression for $\tilde{y}_t$ is derived in terms of purely quadratic terms

$$\begin{aligned}
\tilde{y}_t \quad = \quad & \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1} V_o - \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1} \left(\frac{1}{2}\frac{\varepsilon}{\xi}\left(1+\frac{\phi-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\pi_{Ht}^2 \ldots \\
& + \left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1} \frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 \ldots \\
& - \left(\frac{1}{2}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right) - \frac{1}{2}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\right)\tilde{y}_t^2 \\
& + \left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \\
& + \left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\left(\frac{1-2\alpha}{2}\right)\right)\tilde{x}_t^2 + t.i.p + o(3) \qquad \text{(A.31)}
\end{aligned}$$

Replacing out for the linear terms in the welfare loss function

The linear terms are

$$\tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\tilde{n}_t + t.i.p + o(3)$$

Or

$$\tilde{c}_t + \frac{U_N}{U_C}\frac{N}{C}\left(\tilde{y}_t + \Delta_t\right) + t.i.p + o(3)$$

Use (A.23) to replace out for $\tilde{c}_t$

$$(1-\alpha)\tilde{x}_t + \frac{\lambda\phi}{1-\lambda}\tilde{y}_t + \frac{\lambda\phi}{1-\lambda}\Delta_t + \frac{\lambda(1-\lambda)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + \frac{U_N}{U_C}\frac{N}{C}\left(\tilde{y}_t + \Delta_t\right) + t.i.p + o(3)$$

Or

$$(1-\alpha)\tilde{x}_t + \frac{\lambda(1-\lambda)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + \left(\frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)\tilde{y}_t + \left(\frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)\Delta_t + t.i.p + o(3)$$

Use (A.28) to replace out for $\tilde{x}_t$

$$\begin{aligned}
(1-\alpha)&\left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\tilde{y}_t + \left(\frac{1}{2} - \frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 - \frac{\lambda\phi(1-\alpha)}{1-\lambda}\Delta_t\right)\ldots \\
&\qquad -(1-\alpha)\left(\frac{1-2\alpha}{2} + \left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\
+&\frac{\lambda(1-\lambda)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 \tilde{y_t}^2 + \left(\frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)\tilde{y}_t + \left(\frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)\Delta_t + t.i.p + o(3)
\end{aligned}$$

Reduce this further, noting that $\Delta_t = \frac{1}{2}\frac{\varepsilon}{\xi}\pi_{Ht}^2$ (Woodford, 2011), to get

$$\begin{aligned}
&\left(\frac{(1-\alpha)}{2} - \frac{\lambda(1-\alpha)^2}{2}\left(\frac{\phi}{1-\lambda}\right)^2 + \frac{\lambda(1-\lambda)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 \ldots \\
-&\left(\frac{(1-\alpha)(1-2\alpha)}{2} + (1-\alpha)\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\
&+\left((1-\alpha)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right) + \frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)\tilde{y}_t \ldots \\
&+\frac{1}{2}\frac{\varepsilon}{\xi}\left(\frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C} - \frac{\lambda\phi(1-\alpha)^2}{1-\lambda}\right)\pi_{Ht}^2 + t.i.p + o(3)
\end{aligned}$$

Use (A.31) to replace out for $\tilde{y}_t$, defining $\Xi = \left((1-\alpha)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right) + \frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C}\right)$

$$\begin{aligned}
&\left(\frac{(1-\alpha)}{2} - \frac{\lambda(1-\alpha)^2}{2}\left(\frac{\phi}{1-\lambda}\right)^2 + \frac{\lambda(1-\lambda)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 \ldots \\
-&\left(\frac{(1-\alpha)(1-2\alpha)}{2} + (1-\alpha)\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\
&+\frac{1}{2}\frac{\varepsilon}{\xi}\left(\frac{\lambda\phi}{1-\lambda} + \frac{U_N}{U_C}\frac{N}{C} - \frac{\lambda\phi(1-\alpha)^2}{1-\lambda}\right)\pi_{Ht}^2 + \Xi\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1} V_o \ldots \\
&-\Xi\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\left(\frac{1}{2}\frac{\varepsilon}{\xi}\left(1+\frac{\phi-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\pi_{Ht}^2 \ldots \\
&+\Xi\left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2 \ldots \\
&-\Xi\left(\frac{1}{2}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right) + \frac{1}{2}\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\right)\tilde{y}_t^2 \ldots \\
+&\left(\Xi\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\left(\frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}\right)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)\right)\tilde{x}_t^2 \ldots \\
&+\Xi\left(\left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}\left(\frac{1-2\alpha}{2}\right)\right)\tilde{x}_t^2 + t.i.p + o(3)
\end{aligned}$$

Adding the quadratic expression $\frac{U_N}{U_C}\frac{N}{C}\frac{1}{2}\left(\frac{1+\phi}{1-\lambda}\right)\tilde{y}_t^2$, and defining $\Upsilon = \left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)$, $\Lambda = \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1}$, $\omega = \frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda}$, $\nu = \frac{\phi}{1-\lambda}$, a further set of manipulations yields that

$$\left(\frac{U_N}{U_C}\frac{N}{C}\frac{1}{2}\left(\frac{1+\phi}{1-\lambda}\right)+\frac{(1-\alpha)}{2}-\frac{\lambda(1-\alpha)^2}{2}\nu^2+\frac{\lambda(1-\lambda)}{2}\nu^2\right)\tilde{y}_t^2\ldots$$
$$+\Xi\left(\Lambda\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\right)\tilde{y}_t^2-\Xi\left(\frac{1}{2}\Lambda^{-1}+\frac{1}{2}\Lambda\right)\tilde{y}_t^2\ldots$$
$$+\left(\frac{1}{2}\frac{\varepsilon}{\xi}\left(\nu\lambda-\nu\lambda(1-\alpha)^2\right)+\frac{1}{2}\frac{\varepsilon}{\xi}\frac{U_N}{U_C}\frac{N}{C}-\Xi\Lambda\frac{1}{2}\frac{\varepsilon}{\xi}\left(1+\nu-\nu\lambda(1-\alpha)\right)\right)\pi_{Ht}^2+\Xi\Lambda V_o\ldots$$
$$-\left(\frac{(1-\alpha)\,(1-2\alpha)}{2}+(1-\alpha)\omega\Upsilon\right)\tilde{x}_t^2+\Xi\left(\Lambda\left(\frac{1-2\alpha}{2}\right)\right)\tilde{x}_t^2+\left(\Xi\Lambda\omega\Upsilon\right)\tilde{x}_t^2+t.i.p+o(3)$$

Reducing further, and defining $\Phi=(\Xi\Lambda-(1-\alpha))$

$$\left(\frac{U_N}{U_C}\frac{N}{C}\frac{1}{2}\left(\frac{1+\phi}{1-\lambda}\right)+\frac{(1-\alpha)}{2}-\frac{\lambda(1-\alpha)^2}{2}\nu^2+\frac{\lambda(1-\lambda)}{2}\nu^2\right)\tilde{y}_t^2\ldots$$
$$+\left(\Xi\Lambda\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2-\Xi\left(\frac{1}{2}\Lambda^{-1}+\frac{1}{2}\Lambda\right)\right)\tilde{y}_t^2$$
$$+\left(\frac{1}{2}\frac{\varepsilon}{\xi}\nu\lambda(1-(1-\alpha)^2+\Xi\Lambda(1-\alpha))+\frac{1}{2}\frac{\varepsilon}{\xi}\frac{U_N}{U_C}\frac{N}{C}-\Xi\Lambda\frac{1}{2}\frac{\varepsilon}{\xi}\left(1+\nu\right)\right)\pi_{Ht}^2+\Xi\Lambda V_o$$
$$+\left(\Xi\Lambda\left(\frac{1-2\alpha}{2}\right)+\Xi\Lambda\omega\Upsilon-\frac{(1-\alpha)\,(1-2\alpha)}{2}-(1-\alpha)\omega\Upsilon\right)\tilde{x}_t^2+t.i.p+o(3)$$

This yields Proposition 4 where the final second-order approximation is (noting that V_o is a transitory component and thus predetermined/ independent of policy when considered from a timeless perspective)

$$\mathscr{L}_t=E_t\sum_{t=0}^{\infty}\beta^s\left\{\Psi_\pi\pi_{H,t+s}^2+\Psi_y\tilde{y}_{t+s}^2+\Psi_s\tilde{x}_{t+s}^2\right\}$$

with weights on the output gap, terms of trade gap, and inflation as

$$\begin{aligned}\Psi_y &= \frac{(1-\alpha)}{2}-\frac{\lambda(1-\alpha)^2}{2}\nu^2+\frac{\lambda(1-\lambda)}{2}\nu^2+\Xi\Lambda\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2\\ &\quad -\Xi\left(\frac{1}{2}\Lambda^{-1}+\frac{1}{2}\Lambda\right)+\frac{1}{2}\frac{1+\phi}{1-\lambda}\frac{U_N}{U_C}\frac{N}{C}\\ \Psi_x &= \Xi\Lambda\left(\frac{1-2\alpha}{2}\right)+\Xi\Lambda\omega\Upsilon-\frac{(1-\alpha)\,(1-2\alpha)}{2}-(1-\alpha)\omega\Upsilon\\ \Psi_\pi &= \frac{1}{2}\frac{\varepsilon}{\xi}\left(\left(1+(1-\alpha)\Phi\right)\nu\lambda-\Xi\Lambda\left(1+\nu\right)\right)+\frac{1}{2}\frac{\varepsilon}{\xi}\frac{U_N}{U_C}\frac{N}{C}\end{aligned}$$

where

$$\begin{aligned}
\Upsilon &= \frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda} \\
\Lambda &= \left(\frac{1-\lambda-\lambda\phi(1-\alpha)+\phi}{1-\lambda}\right)^{-1} \\
\nu &= \frac{\phi}{1-\lambda} \\
\omega &= \frac{\lambda\phi(1-\alpha)^2+\alpha(1-\lambda)}{1-\lambda} \\
\Xi &= (1-\alpha)\left(\frac{1-\lambda-\lambda\phi(1-\alpha)}{1-\lambda}\right)+\frac{\lambda\phi}{1-\lambda}+\frac{U_N}{U_C}\frac{N}{C} \\
\Phi &= (\Xi\Lambda-(1-\alpha))
\end{aligned}$$

The welfare loss function nests Gali and Monacelli (2005) when $\lambda = 0$. To see this, set $\lambda = 0$ in the above to reduce the weights

$$\begin{aligned}
\Psi_\pi &= \frac{1}{2}\frac{\varepsilon}{\xi}\left((1+(1-\alpha)\Phi)\,\nu\lambda-\Xi\Lambda\,(1+\nu)\right)+\frac{1}{2}\frac{\varepsilon}{\xi}\frac{U_N}{U_C}\frac{N}{C} \\
&= \frac{1}{2}\frac{\varepsilon}{\xi}\left((1+(1-\alpha)\Phi)\,\nu*0-0*\Lambda\,(1+\nu)\right)+\frac{1}{2}\frac{\varepsilon}{\xi}\frac{U_N}{U_C}\frac{N}{C} \\
&= \frac{1}{2}\frac{\varepsilon}{\xi}\frac{U_N}{U_C}\frac{N}{C} \\
&= -\frac{1-\alpha}{2}\frac{\varepsilon}{\xi}
\end{aligned}$$

which is the Gali and Monacelli (2005) loss function's weight on domestic inflation. Now

$$\begin{aligned}
\Psi_y &= \frac{(1-\alpha)}{2}-\frac{\lambda(1-\alpha)^2}{2}\nu^2+\frac{\lambda(1-\lambda)}{2}\nu^2+\Xi\Lambda\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 \\
&\quad -\Xi\left(\frac{1}{2}\Lambda^{-1}+\frac{1}{2}\Lambda\right)+\frac{1}{2}\frac{1+\phi}{1-\lambda}\frac{U_N}{U_C}\frac{N}{C} \\
&= \frac{(1-\alpha)}{2}-\frac{0*(1-\alpha)^2}{2}\nu^2+\frac{0*(1-\lambda)}{2}\nu^2+0*\Lambda\frac{\lambda(1-\alpha)}{2}\left(\frac{\phi}{1-\lambda}\right)^2 \\
&\quad -0*\left(\frac{1}{2}\Lambda^{-1}+\frac{1}{2}\Lambda\right)+\frac{1}{2}\frac{1+\phi}{1-\lambda}\frac{U_N}{U_C}\frac{N}{C} \\
&= \frac{(1-\alpha)}{2}+\frac{1}{2}\frac{1+\phi}{1-\lambda}\frac{U_N}{U_C}\frac{N}{C} \\
&= \frac{(1-\alpha)}{2}-\frac{(1-\alpha)\,(1+\phi)}{2}
\end{aligned}$$

$$
\begin{aligned}
\Psi_x &= \Xi\Lambda\left(\frac{1-2\alpha}{2}\right)+\Xi\Lambda\omega\Upsilon\frac{(1-\alpha)\,(1-2\alpha)}{2}-(1-\alpha)\omega\Upsilon \\
&= 0*\Lambda\left(\frac{1-2\alpha}{2}\right)+0*\Lambda\omega\Upsilon-\frac{(1-\alpha)\,(1-2\alpha)}{2}-(1-\alpha)\alpha \\
&= -\frac{(1-2\alpha-\alpha+2\alpha^2)}{2}-\alpha+\alpha^2 \\
&= -\frac{1}{2}+\alpha+\frac{\alpha}{2}-\alpha^2-\alpha+\alpha^2 \\
&= -\frac{(1-\alpha)}{2}
\end{aligned}
$$

Noting that $\tilde{x}_t^2=\tilde{y}_t^2$ in Gali and Monacelli (2005) , the weights in the loss function reduce further to

$$
\begin{aligned}
\Psi_y\tilde{y}_t^2+\Psi_x\tilde{x}_t^2 &= \left(\frac{(1-\alpha)}{2}-\frac{(1-\alpha)\,(1+\phi)}{2}\right)\tilde{y}_t^2-\frac{(1-\alpha)}{2}\tilde{x}_t^2 \\
&= \left(\frac{(1-\alpha)}{2}-\frac{(1-\alpha)\,(1+\phi)}{2}\right)\tilde{y}_t^2-\frac{(1-\alpha)}{2}\tilde{y}_t^2 \\
&= \frac{(1-\alpha)\,(1+\phi)}{2}\tilde{y}_t^2
\end{aligned}
$$

which is exactly the weight on the output gap in Gali and Monacelli (2005).

Proposition 5: Targeting Rule

Under commitment from a timeless perspective, the first order conditions on domestic inflation and the output gap are

$$2\pi_{H,t}+\mu_{\pi,t}-\mu_{\pi,t-1} = 0 \tag{A.32}$$

$$2\Phi^{\lambda}\tilde{y}_t-\xi\Lambda^{-1}\mu_{\pi,t} = 0 \tag{A.33}$$

Taking (A.33) one period back

$$\mu_{\pi,t-1}=\frac{2\Phi^{\lambda}}{\xi\Lambda^{-1}}\tilde{y_{t-1}}$$

Replacing this in (A.32)

$$2\pi_{H,t}=-\mu_{\pi,t}+\mu_{\pi,t-1}$$

which yields the result in (2.49).

Figures

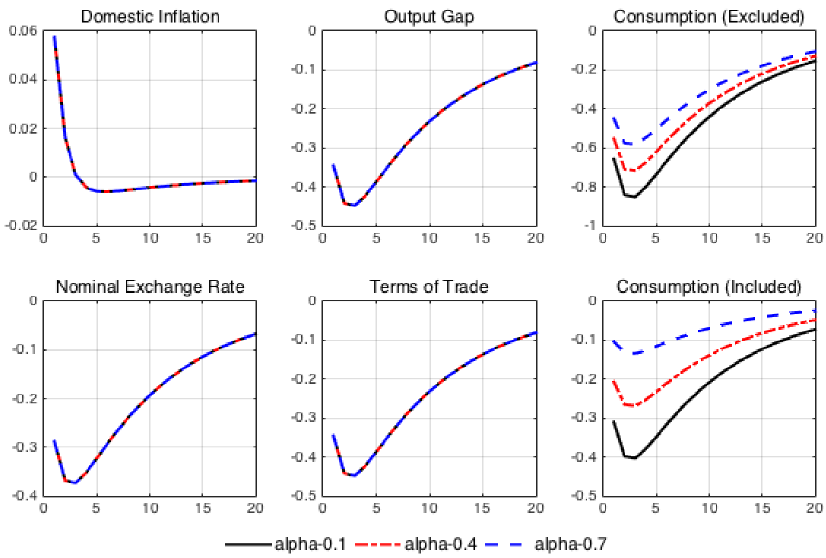

Figure A.1: 1% Cost-Push Shock, Optimal Policy with Complete Financial Inclusion: Varying Openness

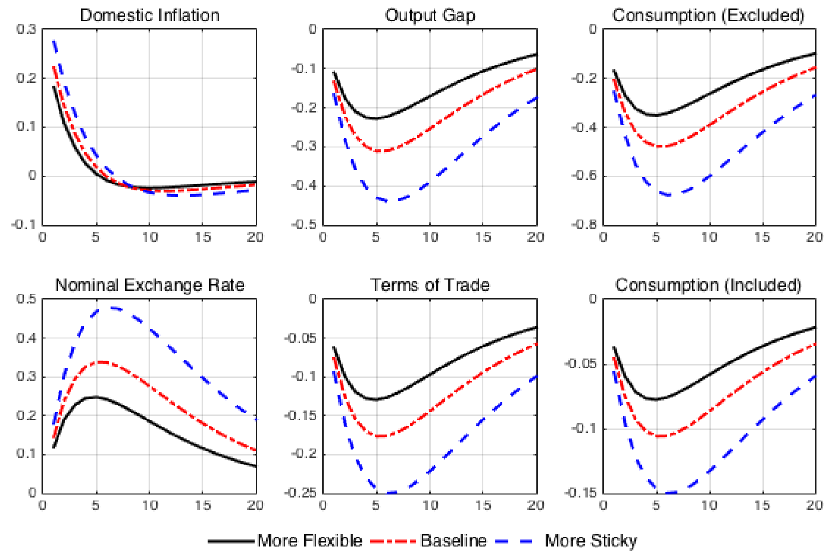

Figure A.2: 1% Cost-Push Shock, Optimal Policy with Financial Exclusion: Varying Stickiness

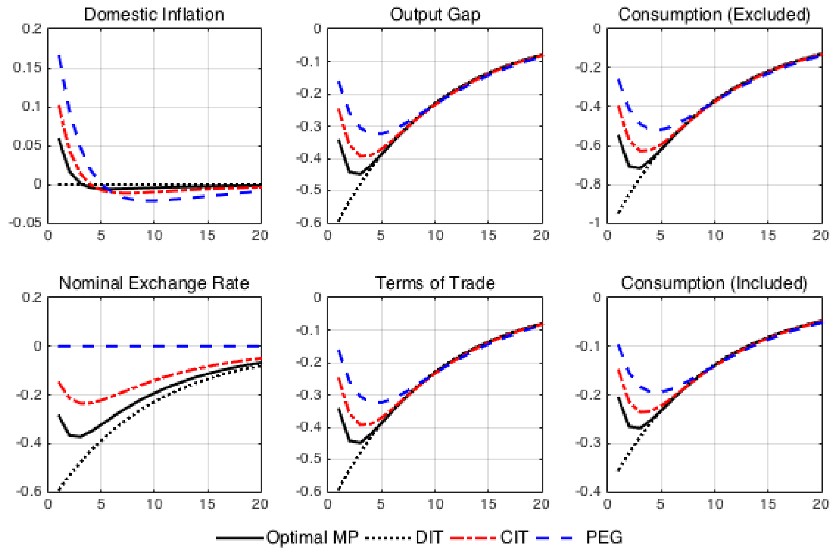

Figure A.3: 1% Cost-Push Shock, Optimal Policy Versus Simple Rules: Complete Financial Inclusion

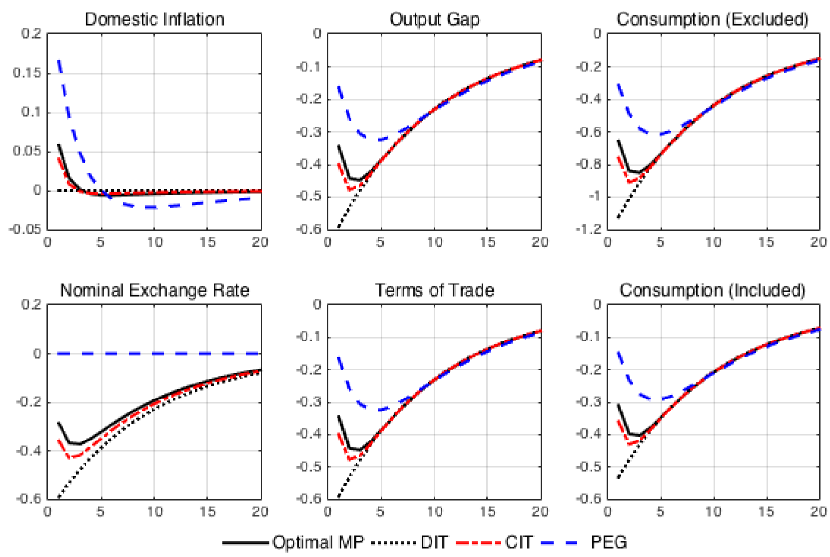

Figure A.4: 1% Cost-Push Shock, Optimal Policy vs Simple Rules: Complete Financial Inclusion, Low Openness

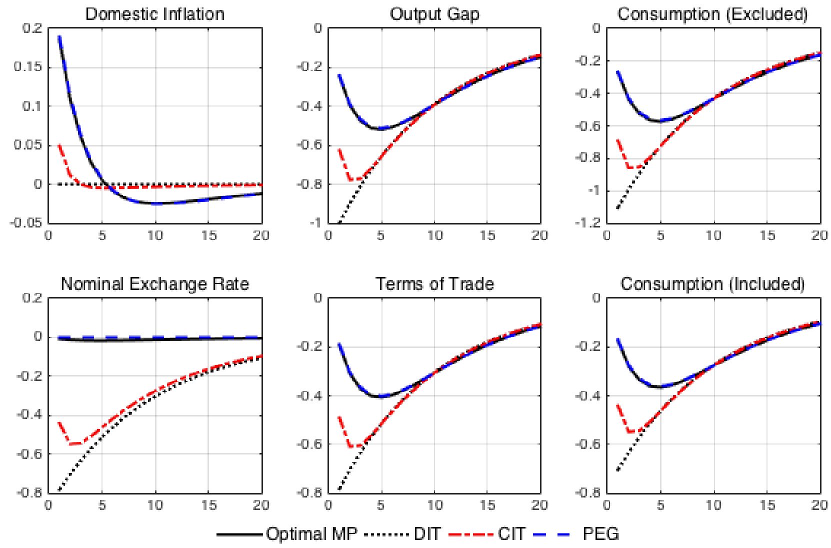

Figure A.5: 1% Cost-Push Shock, Optimal Policy Versus Simple Rules: Financial Exclusion, Low Openness

Appendix B

Appendix to Chapter 3

Households maximize (3.1) subject to (3.9). I attach the Lagrange multiplier, λ_t, on the budget constraint, (3.9). First-order conditions (FOCs) on consumption, C_t, commodity sector labour supply, $N_{C,t}$, non-commodity sector labour supply, $N_{H,t}$, domestic bond holdings, $B_{H,t}$, and foreign bond holdings, $B_{F,t}$, are

$$\begin{aligned}
C_t^{-\sigma} - \lambda_t P_t &= 0 && \text{(B.1)}\\
W_{C,t}\lambda_t - \frac{\lambda}{1+\lambda} N_t^{\phi} N_t^{\frac{\lambda-1-\lambda}{1+\lambda}} \eta^{-\frac{1}{\lambda}} \frac{1+\lambda}{\lambda} N_{C,t}^{\frac{1+\lambda}{\lambda}-1} &= 0 && \text{(B.2)}\\
W_{H,t}\lambda_t - \frac{\lambda}{1+\lambda} N_t^{\phi} N_t^{\frac{\lambda-1-\lambda}{1+\lambda}} (1-\eta)^{-\frac{1}{\lambda}} \frac{1+\lambda}{\lambda} N_{H,t}^{\frac{1+\lambda}{\lambda}-1} &= 0 && \text{(B.3)}\\
-\lambda_t + \lambda_{t+1}(1+i_t) &= 0 && \text{(B.4)}\\
-\lambda_t \varepsilon_t (1+\nu B_{F,t}) + \lambda_{t+1}\varepsilon_{t+1}(1+i_t^*) &= 0 && \text{(B.5)}
\end{aligned}$$

The FOCs on consumption, (B.1), and (B.2) and (B.3) are combined to yield (B.2) and (B.3) respectively. The FOCs on consumption, (B.1), and (B.4) and (B.5) are combined to yield (3.12) and (3.13) respectively. The derivation of the subcomponents of the CES consumption basket, (3.5) and (3.6), are standard and follow the Chapter 3 appendix of Galí (2007). The non-commodity monopolistic firm problem is standard in the literature, and derived, for example, in Benigno (2009). Commodity firms solve the profit maximization problem (3.22). The first-order condition is

$$\begin{aligned}
0 &= \tilde{P}_{C,t}(h) A_{C,t} \bar{K} (1-\psi) N_{C,t}^{-\psi}(h) - W_{C,t}\\
\rightarrow N_{C,t}^{\psi} &= (1-\psi)\left(\frac{W_{C,t}}{\tilde{P}_{C,t}(h)}\right)^{-1} \bar{K} A_{C,t}
\end{aligned}$$

To derive the current account, start with the household's budget constraint

$$
\begin{aligned}
& P_t C_t + e_t \left[B_{F,t} + \Gamma \left(B_{F,t} \right) \right] + B_{H,t} \\
& \leq e_t \left(1 + i^*_{t-1} \right) B_{F,t-1} + \left(1 + i_{t-1} \right) B_{H,t-1} + W_{C,t} N_{C,t} + W_{H,t} N_{H,t} + \Omega_{C,t} + \Omega_{H,t}
\end{aligned}
$$

Note that domestic bonds $B_{H,t}$ are in zero net supply at equilibrium, the budget constraint binds, and that profits for each sector are respectively $\Omega_{C,t} = P_{C,t} Y_{C,t} - W_{C,t} N_{C,t}$ and $\Omega_{H,t} = P_{H,t} Y_{H,t} - W_{H,t} N_{H,t}$. This implies that

$$
\begin{aligned}
& P_t C_t + e_t \left[B_{F,t} + \Gamma \left(B_{F,t} \right) \right] - e_t \left(1 + i^*_{t-1} \right) B_{F,t-1} \\
& \leq W_{C,t} N_{C,t} + W_{H,t} N_{H,t} + P_{C,t} Y_{C,t} - W_{C,t} N_{C,t} + P_{H,t} Y_{H,t} - W_{H,t} N_{H,t}
\end{aligned}
$$

Or

$$
P_t C_t + e_t \left[B_{F,t} + \Gamma \left(B_{F,t} \right) \right] = e_t \left(1 + i^*_{t-1} \right) B_{F,t-1} + P_{C,t} Y_{C,t} + P_{H,t} Y_{H,t}
$$

Collecting terms and noting that $\frac{1}{\beta} = 1 + i^*_{t-1}$

$$
e_t \left[B_{F,t} + \Gamma \left(B_{F,t} \right) \right] = e_t \frac{1}{\beta} B_{F,t-1} + P_{C,t} Y_{C,t} + P_{H,t} Y_{H,t} - P_t C_t
$$

Dividing through by P_t

$$
\frac{e_t}{P_t} \left[B_{F,t} + \Gamma \left(B_{F,t} \right) \right] = \frac{e_t}{P_t} \frac{1}{\beta} B_{F,t-1} + \underbrace{p_{C,t} Y_{C,t} + p_{H,t} Y_{H,t} - C_t}_{= NX_t}
$$

Note that $Q_t = \frac{e_t P^*}{P_t}$, where P^* is normalized to 1, and defining $b_{F,t} = \frac{B_{F,t}}{P^*}$, the evolution of bond holdings is

$$
Q_t \left[b_{F,t} + \Gamma \left(b_{F,t} \right) \right] = \frac{1}{\beta} Q_t b_{F,t-1} + NX_t
$$

The equilibrium conditions are solved numerically in Dynare and Matlab around a deterministic steady state where net foreign assets are zero, commodity sector technology, A_C, is normalized to 1, and as in Petrella and Santoro (2011), the CES labour supply basket at steady state, $N = \left[\frac{N_C}{N}^{-\frac{1}{\lambda}} N_C^{\frac{1+\lambda}{\lambda}} + (1 - \frac{N_C}{N})^{-\frac{1}{\lambda}} N_H^{\frac{1+\lambda}{\lambda}} \right]^{\frac{\lambda}{1+\lambda}}$ implies that $N = N_C + N_H$, so that steady state employment shares add to 1 for accounting consistency i.e. $1 = \frac{N_C}{N} + \frac{N_H}{N}$. Solving for the steady state proceeds as follows

Marginal costs in the non-commodity sector, MC_H, are defined as the non-commodity wages in units of productivity

$$MC_H = \frac{w_H}{p_H A_H} \tag{B.6}$$

Due to the assumption of symmetry across small open economies, relative prices are 1. This can be seen by combining the CPI, $1 = (1-\alpha)\, p_H^{1-\varepsilon} + \alpha p_F^{1-\varepsilon}$, the terms of trade, $S_H = \frac{p_H}{p_F}$, and the relative domestic and import price indices, $p_H^{-1-\varepsilon} = \left(1-\alpha+\alpha S_H^{1-\varepsilon}\right)$ and $p_H^{-1-\varepsilon} = S_H\left(1-\alpha+\alpha S_H^{1-\varepsilon}\right)$. This implies that (B.6) is

$$MC_H = \frac{w_H}{A_H} \tag{B.7}$$

The non-commodity sector production function is

$$y_H = A_H N_H \tag{B.8}$$

Use (B.6) in (B.8) to get an expression for N_H as below

$$\begin{aligned} Y_H &= A_H N_H \\ &= \frac{w_H}{MC_H} N_H \\ N_H &= \frac{Y_H MC_H}{w_H} \end{aligned}$$

Replacing for steady state marginal costs $MC_H = \frac{\varepsilon_P - 1}{\varepsilon_p}$ in the above

$$N_H = \frac{Y_H}{w_H}\frac{\varepsilon_P - 1}{\varepsilon_p} \tag{B.9}$$

The goods market-clearing condition is

$$Y_H = C_H + C_H^*$$

The demand functions for domestic and foreign aggregates are $C_H = (1-\alpha)\, p_H^{\varepsilon} C \rightarrow C_H = (1-\alpha)\, C$ and $C_H^* = \alpha p_H^{*-\varepsilon} \rightarrow C_H^* = \alpha$. The goods market-clearing condition can be reduced to

$$Y_H = (1-\alpha)\, C + \alpha \tag{B.10}$$

The current account, based on a zero initial net foreign asset position, and using that relative prices in steady state are 1, is

$$\begin{aligned} NX &= 0 \\ p_C Y_C + p_H Y_{H,t} - C &= 0 \\ C &= Y_C + Y_H \end{aligned} \tag{B.11}$$

Using (B.11) in (B.10)

$$\begin{aligned} Y_H &= (1-\alpha)\, C + \alpha \\ Y_H &= (1-\alpha)\,(Y_C + Y_H) + \alpha \\ Y_H\,(1-1+\alpha) &= (1-\alpha)\, Y_C + \alpha \\ Y_H &= 1 + \left(\frac{1-\alpha}{\alpha}\right) Y_C \end{aligned} \tag{B.12}$$

The commodity sector production function is

$$Y_C = N_C^{1-\psi} \tag{B.13}$$

The closed-form solution for commodity sector employment is

$$N_C = \left(\frac{1-\psi}{w_C}\right)^{\frac{1}{\psi}} \tag{B.14}$$

Using (B.12), (B.13), and (B.14) in (B.9), non-commodity sector employment is

$$\begin{aligned} N_H &= \frac{Y_H}{w_H}\frac{\varepsilon_P - 1}{\varepsilon_p} \\ &= \frac{1}{w_H}\left(1 + \left(\frac{1-\alpha}{\alpha}\right) Y_C\right)\frac{\varepsilon_P - 1}{\varepsilon_p} \\ &= \frac{1}{w_H}\left(1 + \left(\frac{1-\alpha}{\alpha}\right) N_C^{1-\psi}\right)\frac{\varepsilon_P - 1}{\varepsilon_p} \end{aligned} \tag{B.15}$$

From the household's two labour supply conditions

$$\frac{N_C}{N}^{-\frac{1}{\lambda}} N^{\frac{\phi(1+\lambda)-1}{1+\lambda}} N_C^{\frac{1}{\lambda}} C_t^{\sigma} = w_C \tag{B.16}$$

$$\frac{N_H}{N}^{-\frac{1}{\lambda}} N^{\frac{\phi(1+\lambda)-1}{1+\lambda}} N_H^{\frac{1}{\lambda}} C^{\sigma} = w_H \tag{B.17}$$

It follows from (B.16) and (B.17) that wages are equalized at steady state

$$w_C = w_H$$

Use (B.11), (B.12), and (B.13) in (B.17) to get an expression for N

$$\begin{aligned}
\frac{N_H}{N}^{-\frac{1}{\lambda}} N^{\frac{\phi(1+\lambda)-1}{1+\lambda}} N_H^{\frac{1}{\lambda}} C^{\sigma} &= w_H \\
N^{\frac{1}{\lambda}} N^{\frac{\phi(1+\lambda)-1}{1+\lambda}} C^{\sigma} &= w_H \\
N^{\frac{1}{\lambda}+\frac{\phi(1+\lambda)-1}{1+\lambda}} (Y_C + Y_H)^{\sigma} &= w_H \\
N^{\frac{1}{\lambda}+\frac{\phi(1+\lambda)-1}{1+\lambda}} \left(Y_C + 1 + \left(\frac{1-\alpha}{\alpha}\right) Y_C\right)^{\sigma} &= w_H \\
N^{\frac{1}{\lambda}+\frac{\phi(1+\lambda)-1}{1+\lambda}} \left(1 + \frac{1}{\alpha} Y_C\right)^{\sigma} &= w_H \\
N^{\frac{1}{\lambda}+\frac{\phi(1+\lambda)-1}{1+\lambda}} \left(1 + \frac{1}{\alpha} N_C^{1-\psi}\right)^{\sigma} &= w_H \qquad \text{(B.18)}
\end{aligned}$$

The steady state can be reduced to a system of five equations in five unknowns, w_C, w_H, N_C, N_H, and N, which are numerically solved in Matlab

$$\begin{aligned}
N_C &= \left(\frac{1-\psi}{w_C}\right)^{\frac{1}{\psi}} \\
N_H &= \frac{1}{w_H}\left(1 + \left(\frac{1-\alpha}{\alpha}\right) N_C^{1-\psi}\right) \frac{\varepsilon_P - 1}{\varepsilon_p} \\
w_C &= w_H \\
w_H &= N^{\frac{1}{\lambda}+\frac{\phi(1+\lambda)-1}{1+\lambda}} \left(1 + \frac{1}{\alpha} N_C^{1-\psi}\right)^{\sigma} \\
N &= N_C + N_H
\end{aligned}$$

The other variables are retrieved as follows: C from the current account, (B.11), Y_H from the goods-market clearing condition, (B.12), Y_C from the commodity sector production function, (B.13), A_H from the non-commodity sector production function, (B.8). Relative prices are 1. For the purposes of computing welfare, $V_t = U_t + \beta E_t V_{t+1}$, given by $V = \frac{1}{1-\beta} U$ at steady state, the steady state value of utility is $U = \frac{C^{1-\sigma}}{1-\sigma} + \frac{N^{1+\phi}}{1+\phi}$.

Appendix C

Appendix to Chapter 4

Table C.1: Event 2 Forecast Model Selection Criteria Under Inflation Targeting, Money Targeting, and Hybrid Regimes

I. Inflation Targeting	**Model A**			**Model B**			**Model C**			**Model D**		
	A1	A2	A3	B1	B2	B3	C1	C2	C3	D1	D2	D3
Model Adequacy												
WN (p-value)	0.70	0.85	0.35	0.16	0.05	0.21	0.43	0.28	0.30	0.48	0.07	0.28
Wald (p-value)	0.00	0.00	0.00	0.52	0.81	0.69	0.00	0.00	0.00	0.61	0.89	0.83
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.62	0.07	0.06	0.83	0.50	0.13	0.62	0.08	0.06	0.72	0.47	0.15
Relative RMSFE	0.96	1.12	1.21	1.05	1.09	1.39	0.96	1.11	1.21	1.03	1.08	1.39
II. Money Targeting	**Model E**			**Model F**			**Model G**			**Model H**		
	E1	E2	E3	F1	F2	F3	G1	G2	G3	H1	H2	H3
Model Adequacy												
WN (p-value)	0.65	0.78	0.85	0.22	0.38	0.31	0.55	0.68	0.16	0.17	0.48	0.22
Wald (p-value)	0.73	0.67	0.80	1.00	0.97	0.69	0.00	0.00	0.00	0.97	0.98	0.83
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.60	0.19	0.00	0.74	0.26	0.00	0.61	0.19	0.00	0.64	0.33	0.00
Relative RMSFE	1.15	1.42	1.62	1.10	1.33	1.76	1.15	1.41	1.62	1.08	1.31	1.76
III. Hybrid Regime	**Model I**			**Model J**			**Model K**			**Model L**		
	I1	I2	I3	J1	J2	J3	K1	K2	K3	L1	L2	L3
Model Adequacy												
WN (p-value)	0.72	0.78	0.47	0.92	0.36	0.47	0.31	0.25	0.38	0.64	0.28	0.60
Wald (p-value)	0.78	0.63	0.59	0.87	0.71	0.52	0.55	0.50	0.47	0.67	0.57	0.45
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.68	0.20	0.07	0.67	0.89	0.34	0.67	0.19	0.29	0.50	0.79	0.66
Relative RMSFE	0.96	1.06	1,21	1.08	1.07	1.54	0.96	1.07	1.21	1.06	1.08	1.56

Note: Further details on the models and tests can be found in the notes under Appendix Table C.2.

Table C.2: Event 3 Forecast Model Selection Criteria Under Inflation Targeting, Money Targeting, and Hybrid Regimes

I. Inflation Targeting	**Model A**			**Model B**			**Model C**			**Model D**		
	A1	A2	A3	B1	B2	B3	C1	C2	C3	D1	D2	D3
Model Adequacy												
WN (p-value)	0.32	0.02	0.09	0.49	0.25	0.54	0.49	0.32	0.56	0.76	0.55	0.57
Wald (p-value)	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.78	0.91	0.44	0.73	0.43	0.78	0.70	0.22	0.53	0.69	0.22	0.60
Relative RMSFE	1.21	1.30	1.21	1.40	1.55	1.48	1.45	1.64	1.65	1.43	1.63	1.63
II. Money Targeting	**Model E**			**Model F**			**Model G**			**Model H**		
	E1	E2	E3	F1	F2	F3	G1	G2	G3	H1	H2	H3
Model Adequacy												
WN (p-value)	0.92	0.70	0.43	0.77	0.89	0.73	0.68	0.85	0.66	0.96	1.00	0.89
Wald (p-value)	0.04	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.57	0.65	0.17	0.76	0.86	0.58	0.49	0.84	0.94	0.62	0.76	0.91
Relative RMSFE	1.15	1.17	1.08	1.30	1.35	1.28	1.41	1.47	1.50	1.38	1.45	1.49
III. Hybrid Regime	**Model I**			**Model J**			**Model K**			**Model L**		
	I1	I2	I3	J1	J2	J3	K1	K2	K3	L1	L2	L3
Model Adequacy												
WN (p-value)	0.97	0.33	0.54	0.94	0.67	0.82	0.77	0.38	0.72	0.95	0.66	0.90
Wald (p-value)	0.93	0.89	0.89	0.97	0.96	0.93	0.97	0.95	0.96	0.90	0.93	0.87
Stability	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Forecast Evaluation												
DM (p-value)	0.58	0.78	0.57	0.96	0.42	0.73	0.91	0.23	0.47	0.96	0.18	0.50
Relative RMSFE	1.23	1.32	1.22	1.39	1.54	1.46	1.46	1.64	1.64	1.43	1.62	1.62

Model Ai, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread

Model Bi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Idiosyncratic Forex Intervention

Model Ci, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Net Forex Intervention

Model Di, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Idiosyncratic Intervention, Net Intervention

Model Ei, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth

Model Fi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth, Idiosyncratic Forex Intervention

Model Gi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth Net Forex Intervention

Model Hi, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Reserve Money Growth, Idiosyncratic Intervention, Net Intervention

Model Ii, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Reserve Money Growth

Model Ji, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Int Rate Spread, Reserve Money Growth, Idiosyncratic Forex Intervention

Model Ki, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Interest Rate Spread, Reserve Money Growth, Net Forex Intervention

Model Li, $i \in \{1,2,3\}$: VAR in the Tsh-USD Exch Rate Return, Tsh-US Int Rate Spread, Reserve Money Growth, Idiosyncratic & Net Intervention

For $k \in \{A, ..., L\}$, $k1$ is the baseline VAR, $k2$ controls for the VIX and oil price, $k3$ controls for the VIX, oil price, and daily dummies

* WN (p-value) refers to the significance level of a Portmanteau white noise test with $H0$: Residuals are white noise.

* Stability refers to whether the eigenvalues of the VAR lie inside the unit circle.

* Wald (p-value) refers to the significance level of the Wald lag exclusion test with $H0$: Endogenous variables at a given lag are jointly zero.

* DM (p-value) refers to the significance level of the Diebold-Mariano test with $H0$: VAR forecast is no different than a NCF from a random walk.

* Relative RMSFE compares the RMSFEs of the VAR and a random walk. A RRMSFE<1 implies that the VAR forecast is better than a NCF.

Days	Event 2			Event 3		
Ex-Post	CAR (%)	p (par)	p (non)	CAR (%)	p (par)	p (non)
1	-0.014	0.292	0.655	1.724	0.000	0.655
2	-0.048	0.010	0.593	2.682	0.000	0.285
3	-0.089	0.000	0.273	3.617	0.000	0.144
4	-0.102	0.000	0.138	4.652	0.000	0.080
5	-0.137	0.000	0.075	4.989	0.000	0.046
6	-0.161	0.000	0.043	5.345	0.000	0.028
7	-0.192	0.000	0.025	5.653	0.000	0.017
8	-0.194	0.000	0.015	5.905	0.000	0.011
9	-0.213	0.000	0.009	6.153	0.000	0.007
10	-0.209	0.000	0.006	6.154	0.000	0.004
11	-0.199	0.000	0.004	6.201	0.000	0.003
12	-0.176	0.000	0.002	6.289	0.000	0.002
13	-0.173	0.000	0.002	6.289	0.000	0.001
14	-0.132	0.008	0.001	6.352	0.000	0.001
15	-0.106	0.038	0.001	6.748	0.000	0.001
16	-0.079	0.133	0.000	7.141	0.000	0.000
17	-0.092	0.091	0.000	7.548	0.000	0.000
18	-0.096	0.087	0.000	8.173	0.000	0.000
19	-0.099	0.084	0.000	8.173	0.000	0.000
20	-0.073	0.213	0.000	8.533	0.000	0.000

* CAR (%) is the cumulative abnormal depreciation rate

*p(par) & p(non) and the p-values for the parametric and non-parametric tests.

Table C.3: Cumulative Abnormal Return Test Statistics - Random Walk Forecasting Model for Events 2 and 3

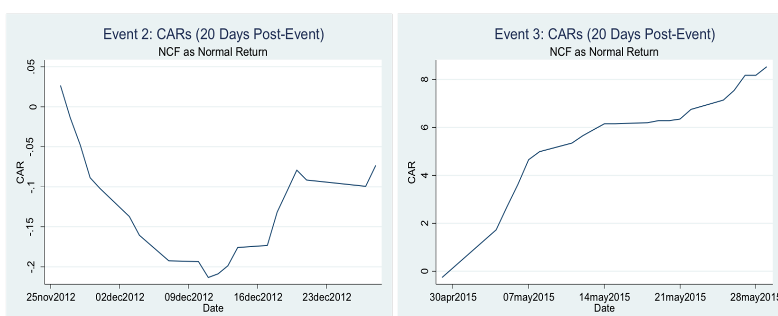

Figure C.1: Evolution of the Cumulative Abnormal Returns: Random Walk Forecasting Model for Events 2 and 3

Days	Event 1			Event 2			Event 3		
Ex-Post	CAR (%)	p (par)	p (non)	CAR (%)	p (par)	p (non)	CAR (%)	p (par)	p (non)
1	-0.239	0.000	0.180	-0.006	0.464	0.655	1.372	0.000	0.655
2	-0.506	0.000	0.109	-0.027	0.019	0.465	2.303	0.000	0.285
3	-0.747	0.000	0.068	-0.061	0.000	0.214	3.152	0.000	0.144
4	-0.840	0.000	0.043	-0.089	0.000	0.144	4.069	0.000	0.079
5	-0.844	0.000	0.028	-0.138	0.000	0.116	4.211	0.000	0.046
6	-0.443	0.000	0.018	-0.171	0.000	0.063	4.535	0.000	0.028
7	-0.818	0.000	0.012	-0.181	0.000	0.036	4.738	0.000	0.017
8	-2.288	0.000	0.008	-0.189	0.000	0.021	4.880	0.000	0.011
9	-2.418	0.000	0.005	-0.205	0.000	0.013	4.980	0.000	0.007
10	-2.305	0.000	0.003	-0.214	0.000	0.008	4.880	0.000	0.004
11	-2.011	0.000	0.002	-0.216	0.000	0.005	4.850	0.000	0.003
12	-1.952	0.000	0.001	-0.208	0.000	0.003	4.838	0.000	0.002
13	-1.774	0.000	0.001	-0.200	0.000	0.002	4.765	0.000	0.001
14	-1.813	0.000	0.001	-0.153	0.000	0.001	4.688	0.000	0.001
15	-1.986	0.000	0.000	-0.165	0.000	0.001	4.982	0.000	0.000
16	-2.163	0.000	0.000	-0.149	0.000	0.001	5.280	0.000	0.000
17	-2.516	0.000	0.000	-0.168	0.000	0.000	5.690	0.000	0.000
18	-2.877	0.000	0.000	-0.178	0.000	0.000	6.166	0.000	0.000
19	-2.851	0.000	0.000	-0.220	0.000	0.000	6.083	0.000	0.000
20	-2.949	0.000	0.000	-0.254	0.000	0.000	6.376	0.000	0.000

* CAR (%) is the cumulative abnormal depreciation rate, and p(par) & p(non) and the p-values for the parametric and non-parametric tests.

Table C.4: Cumulative Abnormal Return Test Statistics - Baseline Forecasting Model with Sensitivity on Window Size (10-Week Pre-Event Window)

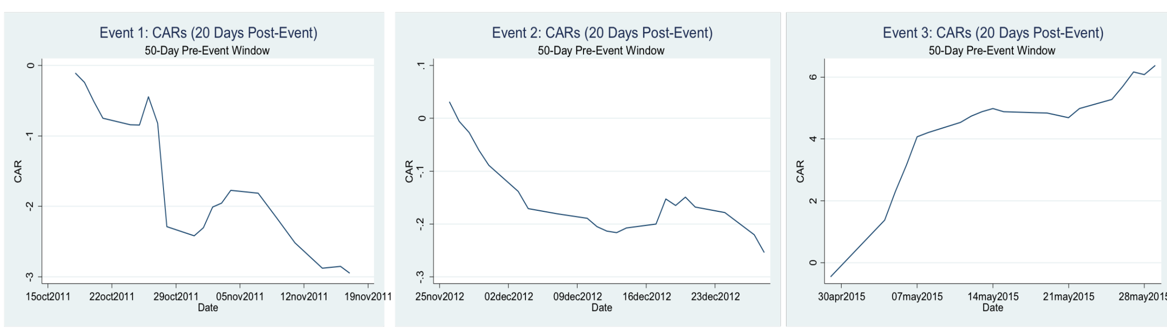

Figure C.2: Evolution of the Cumulative Abnormal Returns: Baseline Forecasting Model with Sensitivity on Window Size (10-Week Pre-Event Window)