The Role of Consumer Leverage in Financial Crises

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Thesis Abstract

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This thesis demonstrates that consumer leverage can contribute to financial crises such as the subprime mortgage crisis characterised by increased bankruptcy prospects and tightened credit access. A recession may follow even when the leveraged sector is not a production sector and can be triggered by seeming positive events such as a technological innovation and a relaxation of borrowing conditions. The first preliminary chapter updates the Bernanke, Gertler and Gilchrist (1999) approach with financial frictions in the production sector to a two-sector model with consumption and housing. It shows that credit frictions in the capital financing decisions of housing firms are not sufficient to capture the negative consumer experience with falling housing prices and relaxed credit access during the recession. The second chapter brings the model closer to the subprime mortgage crisis by shifting credit constraints to the consumer mortgage market. Increased supply of houses lowers asset prices and reduces the value of the real estate collateral used in the mortgage which in turn worsens the leverage of indebted consumers. A relaxation of borrowing conditions turns credit-constrained households into a potential source of disturbances themselves when market optimism allows them to raise their leverage with little downpayment. Both cases demonstrate that although households are not production agents, their worsening debt levels can trigger a lasting financial downturn. The third chapter develops a chained mortgage contracts model where both homeowner consumers and the financial institutions that securitize their mortgage loan are credit-constrained. Adding credit constraints to the financial sector that provides housing mortgages creates opportunities for risk sharing where banks shift some of the downturn onto indebted consumers in order to hasten their own recovery. This consequence is especially evident in the case of relaxed credit access for banks. Financial institutions repair their debt position relatively fast at the expense of consumers whose borrowing ability is squeezed for a long period despite the fact that they may not be the source of the disturbance. The result mirrors the recent subprime mortgage crisis characterised by a sharp but brief decline for banks and a protracted recovery for mortgaged households.

Thesis Summary

This thesis focuses on the role of consumer mortgage leverage in downturns such as the subprime housing crisis. It develops a general equilibrium model of a two-sector economy that produces regular consumption as well as housing and embeds credit constraints in the mortgage decisions of consumers. The thesis demonstrates that in an economy that produces more than one good, positive disturbances can have a negative effect on the debt position of leveraged agents. Furthermore, production firms and financial institutions are not the only sectors that can be leveraged. Household debt can also contribute to precipitating a recession. Finally, a complex risk sharing phenomenon arises when there are two or more loan contracts linked by intermediaries such as financial institutions. Such risk sharing creates opportunities for the intermediaries to shift a considerable burden of the recession onto the final borrowers, the consumers.

The credit constraints of this model are based on the financial accelerator model of Bernanke, Gertler and Gilchrist (1999). The financial frictions in the one-sector Bernanke, Gertler and Gilchrist (1999) economy augment the effect of economic disturbances because positive shocks relax credit constraints that in turn allow more borrowing and negative shocks have the opposite effect. While interesting, the one-sector model offers a one-dimensional study of financial constraints. The model in this thesis exploits the microfounded nature of the financial accelerator approach but embeds credit constraints in a more elaborate framework in order to capture the subprime experience of mortgaged households. It expands the one-sector model to a two-sector model, introduces credit constraints in the household sector and finally, adds financial intermediaries.

Chapter one is a preliminary work that extends the familiar one-sector model of financial frictions to two sectors that produce a generic consumption good and housing. The introduction of housing is motivated by the fact that the residential real estate market was a major contributor to the recent recession. The housing sector is constrained in its ability to finance its capital acquisition. An innovation that raises productivity in the housing sector increases housing output but also reduces the equilibrium price creating “Fisher-type” price effects. Since the price decrease is relatively less than the output rise, the returns of housing manufacturers improve. Higher returns increase the demand for capital raising

the return to capital and improving the value of housing capital held as collateral by credit-constrained firms so their net worth improves. A higher net worth implies lower indebtedness that attracts a lower risk premium. When the credit-constrained sector experiences a temporary easing of credit conditions, it can purchase capital with less downpayment and is charged a lower risk premium. The relaxation of borrowing conditions permits the financially-constrained housing producers to take larger loans thus excessively leveraging with debt. The rise in their indebtedness, coupled with the temporary nature of the shock, implies an increase in their bankruptcy prospects which signals the end of lax credit. The risk premium rises, tightening access to capital financing. While the two-sector model offers richer financial frictions dynamics than the one-sector version, it does not capture the experience of house-purchasing consumers who in the version are outside of the financial market.

The next chapter introduces homeowner mortgages by shifting credit constraints from the producer to the consumer sector. A fraction of consumers are credit-constrained since they do not have sufficient funds to purchase a house and need a mortgage to finance their housing acquisition. This chapter demonstrates two phenomena associated with the subprime mortgage crisis. First, non-production agents such as consumers are just as likely to experience a downturn as firms when they are leveraged. Second, a subprime recession can be easily triggered when seemingly positive events such as excess housing supply and credit easing reduce the equilibrium price of housing and permit mortgage-holders to raise their leverage. An oversupply of housing modelled as a technological innovation in the production of real estate reduces the equilibrium price of housing as well as the value of the housing already held as collateral by credit-constrained households. The result is an increase in their indebtedness and a corresponding rise in the risk premium charged on mortgages. An improvement in the borrowing conditions also causes adverse consequences for borrowing households by allowing them to increase leverage using relatively little collateral. When as a result of the leverage deterioration, the previously reduced lending rate adjusts upwards, these households end up with subprime mortgages that they cannot service and are more likely to become bankrupt. Although the relaxation of borrowing conditions is brief, the resulting leverage buildup drags credit-constrained borrowers into a protracted financial downturn.

The third chapter extends the consumer mortgage contract of Chapter 2 to include credit-constrained financial institutions. Leveraged banks borrow from investors and in turn lend to mortgage-buying consumers acting as financial intermediaries. Since banks are the intermediaries of this chained loan contract, they can ration lending to consumers in such a way that in times of a downturn, households bear some of the negative consequences. Banks resort to risk sharing both following housing oversupply and following a relaxation of credit access for consumers and for themselves. Owing to their high initial indebtedness, banks experience a significant worsening in their indebtedness. They recover their solvency by transferring some of the balance sheet deterioration to consumers who have no financial recourse outside of the mortgage contract. The leverage of consumers deteriorates more than in Chapter 2 and their credit crunch is more prolonged than that of financial institutions. The risk sharing is especially pronounced in the case with relaxed borrowing constraints for financial institutions. This case also closely approximates the subprime mortgage crisis. Although consumers are not the originators of the disturbance, they take a long time to recover from the bank-induced recession while financial institutions experience a sharp but relatively short downturn.

Contents

List of Figures

List of Tables

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Statement of Publication and Word Count

This thesis is respectfully submitted for the degree of D.Phil. in Economics at the University of Oxford, UK. An abridged version of Chapters 2 and 3 has been converted into a working paper as University of Oxford Department of Economics Discussion Paper No. 631. This work has been done solely by me, after discussion with my supervisor. All parts of this document are the product of my own work. All errors are mine alone.

The word count of this thesis is 58,669 words.

Introduction

In 2004, Edmund Andrews, an economic reporter for *The New York Times*, joined millions of American home-buyers in purchasing a house at the peak of the real estate price bubble (Andrews, 2009). He had regularly reported from the Federal Reserve, covered the Asian financial crisis of 1997, the Russian meltdown of 1998 and the dot-com collapse of 2000 but was not quite prepared for what was in store at the subprime mortgage party. "I had just come up with almost a half-million dollars, and I had barely lifted a finger. It had been so easy and fast," Andrews said of obtaining his mortgage despite having modest disposable income and putting down very little downpayment.

Andrews' experience with easy loans along with that of many other homeowners was far from positive and came to be known as the subprime mortgage crisis. Prior to the crisis consumer leverage was not considered sizable and significant enough to contribute to a recession. The confluence of easy access to loans and cheap housing during the subprime mortgage crisis implies that household indebtedness could be a significant source of economy-wide distress. This thesis focuses on the role of consumer leverage in contributing to financial downturns. It demonstrates that although homeowners may not be as highly indebted as financial institutions and they are not production agents, their mortgage indebtedness could precipitate a recession.

The subprime mortgage crisis differed from previous recessions because it was mostly concentrated in one sector of the economy, the housing market, at the heart of it were consumers rather than production firms or entrepreneurs, and it was not triggered by outright adverse circumstances. Two seemingly positive factors - increased housing supply and easy refinancing conditions - allowed borrowers to excessively leverage with debt to disastrous consequences. The lax loans were also securitized by financial institutions which at the peak of the recession transferred some of the leverage worsening onto consumers in order to repair their own debt positions thus prolonging the credit crunch for home-buyers. Andrews' cautionary subprime tale provides the empirical motivations to study the buildup and consequences of consumer leverage within a rigorous theoretical framework.

This thesis focuses on the role of consumer mortgage leverage in downturns such as the subprime housing crisis. It shows that positive shocks can have a recession-inducing role, that producers and financial institutions are not the only agents who can be credit-constrained and that risk sharing can affect the intensity and the duration of the recession for mortgage-holders. The three chapters of the thesis expand the Bernanke, Gertler and Gilchrist (1999) financial frictions model to a two-sector model with housing and borrowing constraints in the consumer sector. This microfounded approach to credit constraints is especially suitable to studying subprime loans since it does not impose an exogenous loan limit like collateral constraint models such as Kiyotaki and Moore (1997) and Aoki, Proudman and Vlieghe (2004). Instead, like most subprime loans, it allows borrowers to obtain credit with limited downpayment but at a higher repayment rate that is determined endogenously and varies with unexpected disturbances. The thesis demonstrates that in an economy that produces more than one good, positive disturbances can have a negative effect on the debt position of leveraged agents. Furthermore, production firms and financial institutions are not the only sectors that can be leveraged. Household debt can also contribute to precipitating a recession. Finally, a complex risk sharing phenomenon arises when there are two or more loan contracts linked by intermediaries such as financial institutions. Such risk sharing creates opportunities for the intermediaries to shift a considerable burden of the recession onto the final borrowers, the consumers.

The financial frictions of this model are based on the seminal paper of Bernanke, Gertler and Gilchrist (1999). This paper introduced the concept of the financial accelerator by modelling microfounded financial frictions in the capital purchasing decisions of final good firms which Bernanke, Gertler and Gilchrist (1999) call the "entrepreneurs." Entrepreneurs are credit-constrained so they must obtain external funding from investors to finance the purchase of capital as one of the production inputs. Their borrowing is complicated by the fact that they possess an idiosyncratic risk of default that is known to them but is unknown to lenders (Townsend, 1979). To learn this probability of default, lenders must pay an auditing fee which is passed to entrepreneurs as costlier capital financing. In order to offset expenses associated with monitoring, investors charge borrowers a premium known as an external finance (risk) premium above the risk-free interest rate. The model is microfounded in the sense that the risk premium is endogenous to the loan contract and responds to changes in the bankruptcy prospects of borrowers. This approach is especially well-suited to studying the consequences of financial recessions since unlike collateral constraint models such as Kiyotaki and Moore (1997), there is no exogenous debt ceiling to limit the volatility that the model can generate.

The financial frictions in Bernanke, Gertler and Gilchrist (1999) augment the effect of economic disturbances because positive shocks relax credit constraints that in turn allow more borrowing and negative shocks have the opposite effect. The authors demonstrate this with a positive productivity shock in the production of the final good. A technological innovation raises the output of the final good and improves the returns of entrepreneur producers. The higher profits afford entrepreneurs increased capital holdings so their collateral (also known as net worth) improves relative to the value of the loan. A lower debt to net worth ratio (also known as leverage) implies a lower probability of bankruptcy which attracts a reduced risk premium. The risk premium in Bernanke, Gertler and Gilchrist (1999) is strictly anti-cyclical. Positive shocks reduce the risk premium allowing entrepreneurs to borrow even more and to expand their production even further. The financial frictions in essence generate an accelerator effect that amplifies the impact of disturbances. Since Bernanke, Gertler and Gilchrist (1999) were looking precisely to explain this amplification mechanism, they developed a simple one-sector model with one type of borrowers that produced only the desired anti-cyclical risk premium.

The model in this thesis exploits the microfounded nature of the approach developed by Bernanke, Gertler and Gilchrist (1999) and its ability to produce an endogenous risk premium but embeds the credit constraints in a more elaborate framework. The main objective of the thesis is to demonstrate how consumer leverage and positive shocks contributed to financial downturns such as the subprime mortgage crisis. In order to do so, the general stochastic model of financial frictions is augmented so that it can demonstrate how initially beneficial innovations can contribute to a downturn. This is not just of theoretical interest but also helps explain how the confluence of easy credit and falling housing prices helped trigger the subprime recession. The thesis achieves this by developing a two-sector model of credit constraints within the household sector. This is accomplished in three stages. The first chapter is a preliminary step that extends the one-sector model of Bernanke, Gertler and Gilchrist (1999) to two sectors with credit constraints in one of the production sectors and introduces housing. The second chapter continues with the two-sector approach but moves financial frictions from the producer to the consumer sector and describes the mortgage purchase of a share of households. Finally, the third chapter adds an indebted intermediary financial sector in addition to borrowing households and highlights the role of risk sharing during downturns.

Chapter one is preliminary work that extends the familiar one-sector model of financial frictions to two sectors, the first one of which produces generic consumption and the second one - housing. The introduction of housing is motivated by the fact that the residential real estate market was a major contributor to the recent recession. The placement of credit constraints and the production functions are the same as in Bernanke, Gertler and Gilchrist (1999). Like the original paper on the financial accelerator, the producers of one final good, housing, are constrained in their ability to finance their capital acquisition. The major difference is that this model has an unconstrained sector in addition to the credit-constrained sector unlike the one-sector version of Bernanke, Gertler and Gilchrist (1999). The chapter first considers a version where housing is a one-period good (i.e. the two goods are identical in their production functions) and then transforms housing into a multi-period durable good where new production is a fraction of total available housing (Iacoviello and Neri, 2010). The two versions demonstrate that the effect of shocks is largely influenced by “Fisher-type” price effects where relative price changes increase or erode the net wealth of credit-constrained firms and which are absent in a one-sector model. Chapter one considers the effect of a technological innovation in housing production and a shock to the financial system.

An innovation that raises total factor productivity in the housing sector increases housing output but also reduces the equilibrium price (due to the Cobb-Douglas utility function) leaving profits unchanged and so unlike the one-sector model, the two-sector version does not experience commensurate movements in the financial market. However, if the price decrease is relatively less than the output rise as in the case with multi-period housing where new production is only a fraction of total available housing stock, the returns of housing manufacturers improve. Higher returns increase the demand for capital raising the return to capital. Both of these work to raise the value of housing capital held as collateral by credit-constrained firms so their net worth improves. A higher net worth implies lower indebtedness and signals decreased probability of default that in turn attracts a lower risk premium. Housing producers benefit from the technological expansion both in the real and in the financial market. The simulations confirm the findings of Bernanke, Gertler and Gilchrist (1999) that positive shocks tend to improve the financial position of borrowers and the lower price of housing (following a housing sector innovation) benefits consumers. This is at odds with the subprime mortgage observations where a lower housing price worsened the house financing abilities of consumers.

A disturbance directly to the credit sector such as a credit easing yields results closer to the financial downturn. When the credit-constrained sector experiences a temporary easing of credit conditions, it can purchase capital with less downpayment and is charged a lower risk premium. The relaxation of borrowing conditions permits the financially-constrained housing producers to take larger loans than prior to the easing thus excessively leveraging with debt. The rise in their indebtedness, coupled with the temporary nature of the shock, implies an increase in their bankruptcy prospects which signals the end of lax credit. The risk premium rises, tightening access to capital financing. Although the relaxation of borrowing conditions is brief, the resulting leverage buildup drags credit-constrained borrowers into a protracted financial downturn during which they struggle to repair their debt position. While this positive financial disturbance results in a downturn for the credit-constrained sector, consumers, who are outside of the financial market in this model, are largely unaffected by the resulting credit tightening. While the two-sector model offers a richer understanding of the role of financial frictions, it cannot accurately portray the contribution of positive shocks to generating the subprime crisis and the experience of mortgage-buying consumers. Chapter 2 upgrades the two-sector model by adding credit constraints in the household sector to demonstrate how falling housing prices and a credit easing contributed to the subprime crisis for mortgage-holders.

In an effort to bring the financial frictions model closer to recent downturns such as the subprime mortgage crisis, Chapter 2 makes two major changes to the existing model.

It splits consumers into two types and introduces homeowner mortgages. The housing loans essentially shift credit constraints from the producer to the consumer sector. In this model, consumers face credit-constraints when purchasing a home. In order to facilitate the lending process in the household sector, the model separates households into Ricardian and credit-constrained consumers. Ricardian consumers receive non-wage income from owning shares in production firms in addition to labour wages. They have sufficient resources to purchase housing without resorting to borrowing and have savings. Credit-constrained consumers have only wage income and need outside funding in the form of a mortgage to finance their housing acquisition. They borrow from the savings of Ricardian consumers facilitated by risk-neutral investors.

This chapter demonstrates two phenomena associated with the subprime mortgage crisis. First, non-production agents such as consumers are just as likely to experience a downturn as firms when they are leveraged. Second, a subprime recession can be easily triggered when seemingly positive events such as housing oversupply and credit easing reduce the equilibrium price of housing and permit mortgage-holders to raise their leverage. While in Chapter 1 unconstrained consumers are able to afford more housing when the equilibrium price falls, in Chapter 2 the lower price affects their net worth of credit-constrained households negatively. An oversupply of housing modelled as a technological innovation in the production of real estate reduces the equilibrium price of housing as well as the value of the housing already held as collateral by credit-constrained households. The result is an increase in their indebtedness and a corresponding rise in the risk premium charged on mortgages. An improvement in the borrowing conditions also causes adverse consequences for borrowing households who are now part of the financial market. It demonstrates that the credit-constrained sector can be the source of a crippling downturn even if it is not a production sector. A reduction in the volatility of the mortgage contract can impact indebted consumers directly by allowing them to increase leverage using relatively little collateral. When as a result of the leverage deterioration, the previously reduced lending rate adjusts upwards, these households end up with subprime mortgages that they cannot service and are more likely to become bankrupt.

The third chapter extends the consumer mortgage contract of Chapter 2 to include credit-constrained financial institutions. Leveraged banks borrow from investors and in turn lend to mortgage-buying consumers acting as financial intermediaries. The model first considers frictions only in the financial sector where banks borrow in a loan contract but consumers do not face constraints and have a fixed share arrangement with banks. Finally, the model puts credit-constrained financial institutions and credit-constrained households together. In this scenario, both banks and credit-constrained households are subject to aggregate uncertainty. The result is two chained mortgage loan contracts, similar to the ones discussed in Hirakata, Sudo and Ueda (2009). This version of the model is closest to real financial markets and can also demonstrate the opportunities for risk sharing between banks and indebted households. Since banks are the intermediaries of the chained loan contract, they can ration lending to consumers in such a way that in times of a downturn, households bear some of the negative consequences. In contrast, when financial institutions are the sole credit-constrained agent, their share arrangement with consumers gives both parties a fixed return. The share purchase protects households from downturns concentrating the leverage worsening in the financial sector. This is not the case when both consumers and banks are credit-constrained. In this case households have a loan arrangement where financial institutions where their repayment to lenders is contingent on changes in overall default risk and leverage. This allows banks to extract a larger return from consumers in times of downturn to mitigate the worsening of their own leverage while in the process delay the recovery of consumers.

Banks resort to risk sharing both following an innovation in the production of housing that reduces the price of houses and following a relaxation of credit access for consumers and for themselves. Owing to their high initial indebtedness, banks experience a significant worsening in their indebtedness following any of the disturbances. They recover their solvency by transferring some of the balance sheet deterioration to consumers who have no financial recourse outside of the chained mortgage contracts. The leverage of consumers deteriorates more than in the share arrangement and their credit crunch is more prolonged that that of financial institutions. As monopolists in the financial intermediation, banks can extract higher loan repayments from households to repair their balance sheets relatively fast while consumers suffer a protracted recession. The risk sharing is especially pronounced in the case with relaxed borrowing constraints for financial institutions. This case also closely approximates the subprime mortgage crisis. Although consumers are not the originators of the disturbance, they take a long time to recover from the bank-induced recession while financial institutions experience a sharp but relatively short downturn.

The thesis is organized as follows: Chapter 1 describes the preliminary two-sector model with financial frictions in one of the production sectors. Chapter 2 introduces credit-constrained consumers and their mortgage-buying decisions. Chapter 3 extends the model of Chapter 2 by adding leveraged financial institutions and discusses the occurrence of risk sharing between the two types of indebted agents. The Conclusion summarizes and points to directions of future research.

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Chapter 1

Financial Frictions in a Two-Sector Economy

Abstract

This chapter extends the one-sector financial frictions model of Bernanke, Gertler and Gilchrist (1999) to two sectors that produce generic consumption and housing. It demonstrates that a two-sector approach with credit constraints in one of the producer sectors is not sufficient to capture the effect of falling housing prices and relaxed access to borrowing in generating recessions like the subprime crisis. Price effects may negate any output gains from a productivity shock and leave the financial market unchanged. Where a positive technological shock does influence the debt position of borrowing firms, it largely improves the value of their collateral and eases their leverage, resulting in more beneficial financial terms. Shocks directly to the financial system can have detrimental effects on the credit-constrained sector since a temporary easing of borrowing conditions permits loans secured with less downpayment. Borrowers respond by raising their debt and increasing their leverage. The resulting higher indebtedness implies a reversal in the lax lending practises that subjects borrowers to a lasting credit tightening. In both cases, house-purchasing consumers, many of who suffered considerably in the subprime crisis, are unaffected by the disturbances since they are outside of the financial market. While the two-sector model offers a richer understanding of the role of financial frictions, it is not sufficient to explain the recession and fails to capture the experience of homeowner consumers. Chapter 2 moves financial frictions from the producer to the consumer sector to considerably improve the performance of the model.

1.1 Introduction

Edmund Andrews' mortgage seemed like a good deal when easy financing was the norm and asset prices were expected to keep increasing. The recent financial crisis demonstrated, however, that credit imperfections can cause seemingly positive events like falling asset prices and relaxed borrowing terms to induce a downturn for house-buyers. These phenomena are at odds with the existing literature where credit constraints amplify the beneficial effects of a positive shock. This chapter extends the one-sector model of Bernanke, Gertler and Gilchrist (1999) to a two-sector model that produces consumption and housing with credit constraints in the housing sector to study the role of positive disturbances on the debt position of borrowers and on house-purchasing consumers. The model has the potential to match empirical observation better since in a two-sector economy the response of financial constraints may be influenced by adjustments in the relative price of the two goods that are absent in a one-sector model. However, this chapter demonstrates that simply extending the one-sector model to two sectors while keeping financial frictions in one of the production sectors is not sufficient to capture the subprime downturn and fails to replicate the experience of homeowner consumers. The subsequent chapter moves financial frictions from the producer to the consumer sector to improve the performance of the model.

This chapter develops a two-sector model with credit constraints in the production of one of the final goods to study the ability of positive disturbances to generate financial downturns. In doing so, it serves as a bridge between the one-sector models of financial frictions that dominate the literature such as Bernanke, Gertler and Gilchrist (1999) and the two-sector model with credit constraints in the household sector developed sector in Chapter 2. In Bernanke, Gertler and Gilchrist (1999) positive innovations have strictly beneficial consequences for the credit-constrained production sector. In contrast, the model in Chapter 2 demonstrates that when the credit-constrained sector is the household sector, positive shocks can have negative effects on their indebtedness. This chapter straddles the two models by extending the existing one-sector studies of credit constraints to a two-sector economy but without yet introducing frictions in the household sector. While the approach developed here does not fully serve to explain the emergence of downturns such as the subprime mortgage crisis and consumer experience with it, it offers valuable lessons about the behaviour of financial constraints and justifies the need for more advanced models such as the one in Chapter 2.

The phenomenon of credit constraints amplifying the effect of disturbances is well-documented in the literature. In a perfect world, there is complete information so borrowing is costless. In reality, lenders often do not know the trustworthiness of borrowers or the likelihood of return of the investment, both of which may lead to default. Since bankruptcy is costly for lenders, access to outside funds is often more expensive than access to own funds for borrowers. The extra cost takes the form of an external finance premium or risk premium that lenders charge to borrowers to offset costs associated with a potential default. The probability of default grows with the value of the loan relative to the collateral of borrowers so the risk premium usually depends on the financial position of borrowers. In the seminal paper of Bernanke, Gertler and Gilchrist (1999) the risk premium moves strictly anti-cyclically to the disturbance. A positive technological innovation improves the value of the collateral of loan-holders thus raising their equity (net worth) relative to the debt held. This improvement in the debt position of credit-constrained agents reduces their likelihood of default that in turn attracts a lower risk premium. Improved financing allows further expansion beyond that generated by the initial disturbance creating what Bernanke, Gertler and Gilchrist (1999) call a “financial accelerator” mechanism.

The presence of two sectors offers a more nuanced insight into the role of credit imperfections since in a two-good economy their cumulative effect on borrowers depends on “Fisher-type” price effects where relative price changes increase or erode the net worth of credit-constrained firms. In contrast, most of the existing literature of financial frictions (discussed in detail in the subsequent section), including the seminal papers by Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997) and the recent works they spawned, notably Luk and Vines (2011), are based on general stochastic equilibrium models with one final good. In these models, the presence of only one consumer good eliminates relative price tradeoffs so a positive disturbance generates an anti-cyclical risk premium that has predictable beneficial effects on the financial position of borrowing agents. Even Kiyotaki and Moore (1997)’s two-sector model does not exhibit price effects since the two goods are linked as an input and an output so there is no relative price adjustment. Monacelli (2009) is one of the few papers with two separate sectors but it focuses on monetary shocks that affect both sectors strictly pro-cyclically. The model described here involves two final goods where equilibrium price movements open the door for Fisher-type price effects that moderate the effect of the disturbance. It also demonstrates that nominal rigidities as in Fernández-Villaverde (2010) are not necessary to generate these Fisher-type price effects on the leverage of indebted borrowers.

The two-sector model developed here offers a richer understanding of how credit frictions affect indebted borrowers. It updates the original paper of Bernanke, Gertler and Gilchrist (1999) with credit constraints in a production sector and adds another unconstrained final good sector. This ensures comparability of this paper with the existing works while at the same time highlights the fact that shock effects may depend on relative price adjustments. Nevertheless, the extended model presented here is not sufficient to explain the occurrence of downturns such as the subprime crisis and consumers' experience with it. However, it paves the road to the more empirically relevant model developed in Chapter 2. The subsequent chapter elaborates the two-sector approach enabling it to model downturns like the subprime mortgage crisis. It achieves that by shifting financial frictions from the producer to the consumer sector. This allows it to demonstrate that positive shocks can have negative consequences for household borrowers and that consumer leverage can be just as recession-inducing as financial or firm leverage.

This paper develops a dynamic stochastic general equilibrium model (DSGE) with two goods: consumption and housing. Both sectors borrow from risk-neutral investors to finance their capital purchase. While the consumption sector can borrow at the risk-free rate, the housing sector is credit-constrained in its borrowing decisions. Housing manufacturers have an idiosyncratic risk of default that is known to them but is unknown to lenders (Townsend, 1979). In order to learn this probability of default, lenders must pay an auditing fee which makes capital financing costlier for housing manufacturers. The model follows the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999) where credit constraints take the form of costly state verification. In this loan contract, there is no mismatch of default perceptions between borrowers and lenders as in Cúrdia (2007).

Once investors pay the fee, they can observe the bankruptcy prospects of borrowers truthfully. Furthermore, bankruptcy is involuntary so unlike Cúrdia and Woodford (2009), there are no strategic defaults. Although borrowers cannot declare bankruptcy by choice, the probability of default is not purely exogenous. It depends on the ratio of the loan value relative to the collateral held by borrowers known as leverage. The presence of the default probability makes lending to housing manufacturers costlier and riskier than lending to consumption producers so investors charge housing firms a risk premium above the risk-free rate. The risk premium exists to offset the expenses associated with eventual bankruptcy and is not a source of profits. Since the costs of bankruptcy depend on the probability of default, the risk premium also varies with it. In this model of financial constraints, there is no upper limit on the amount of borrowing as in collateral constraint models like Kiyotaki and Moore (1997). Such exogenous upper limits restrict the volatility that disturbances can generate. Instead, the risk premium grows with the leverage ratio endogenously limiting access to credit.

This chapter serves a dual purpose. First, it compares a two-sector economy with financial imperfections with the Bernanke, Gertler and Gilchrist (1999) one-sector version. Second, it lays the foundation for the more advanced model presented in Chapter 2. There are two versions of housing production, one where housing is a simple one-period good and one where it is durable multi-period good. In the two-sector model, the nature of housing production determines the response of the relative housing price and ultimately the magnitude of the disturbance. This chapter simulates two shocks: a positive technological innovation in the credit-constrained sector that reduces the equilibrium price of the final good and a relaxation of credit conditions in the financial market that permits more borrowing.

A positive technological innovation in the production of housing raises output in the housing sector but when housing is a one-period good, the price of houses diminishes to leave firm profits unchanged. In this version, credit constrains have no effect on borrowing producers since the downward adjustment in the equilibrium price offsets the increase in production. In contrast, when housing is a multi-period good, new production is only a fraction of total available housing so the fall in price is relatively less than the gain in output. The profits of housing producers increase driving up their capital demand and raising their net worth. Borrowing firms experience an improvement in their debt position which reduces the risk premium that they face. A positive technological innovation has strictly positive consequences for the credit-constrained sector. The lower price of housing also improves the house purchasing ability of consumers who are not constrained in their house-purchasing abilities. This is at odds with the subprime mortgage observations where a lower housing price worsened the house financing abilities of consumers.

Simulating a disturbance in the credit sector itself yields results closer to the financial downturn. When the credit-constrained sector experiences a temporary easing of credit conditions, it can purchase capital with less downpayment and is charged a lower risk premium. The relaxation of borrowing conditions permits the financially-constrained housing producers to take larger loans than prior to the easing, excessively leveraging with debt. The rise in their indebtedness, coupled with the temporary nature of the shock, implies an increase in their bankruptcy prospects which signals the end of lax credit. The risk premium rises, tightening access to capital financing. Although the relaxation of borrowing conditions is brief, the resulting leverage buildup drags credit-constrained borrowers into a protracted financial downturn during which they struggle to repair their debt position. While this positive financial disturbance results in a downturn for the credit-constrained sector, consumers, who are outside of the financial market in this model, are largely unaffected by the resulting credit tightening.

While the two-sector model developed here offers a richer understanding of the role of financial frictions, it cannot accurately portray the contribution of positive shocks to generating the subprime crisis and the experience of mortgage-buying consumers. Chapter 2 upgrades the existing model by introducing credit constraints in the housing sector. The improved version shows that a technological innovation that reduces housing prices and a credit easing can bring lasting negative consequences for mortgage holders.

Section 1.2 traces the evolution of the literature of financial frictions describing major breakthroughs in modelling financial markets. Section 1.3 describes the two-sector credit-constrained model. Section 1.4 presents the complete model where housing is a one-period good. Section 1.5 discusses the results of the model where housing is a one-period good. The model is then extended by modelling housing as a durable multi-period good in Section 1.6 while Section 1.7 compares the outcomes to these of the previous version. Finally, the Conclusion summarizes the results and points to the upgrades in Chapter 2.

1.2 The Evolution of Financial Frictions Models

This chapter studies the role of financial frictions in a two-sector model. The present section compares the various approaches to modelling credit constraints and explains the reasons a microfounded model of financial imperfections is the most suitable for the model in this thesis.

The standard DSGE models assume that representative economic agents operate in perfect financial markets and have immediate access to an unlimited amount of funding. Such models, however, cannot explain the financial downturns that may accompany real economic disturbances. The literature has attempted to address those shortcomings in modelling of financial markets by incorporating either rigidities or constraints that limit perfect accessibility to funding. This survey focuses on the literature of financial constraints to which this paper also belongs. The financial constraints fall broadly in several categories: credit constraints, collateral constraints, external funds limits and financial regulations. Credit constraints emerge when firms are not able to raise enough capital from their internal funds and have to use external funds which are usually more costly. Expenditures related to raising external capital lead to an increase in the cost of investment decision-making in relation to expected profits. Collateral constraints play a significant role when firms with high leverage ratios face deterioration in the conditions for raising additional external funds. Constraints on the supply of external funding are present when the possibility of financial intermediaries screening the results of investment projects leads to the emergence of an external finance premium that allows only a limited amount of external capital to be obtained. Financial regulations approaches take a macroprudential view of financial imperfections where exogenously imposed restrictions on lending limit the availability of funding. Generally, the aim of these studies is to identify the mechanism for the transformation of observed shocks into fluctuations with properties corresponding to the observed properties of business cycles.

This literature overview focuses mostly on works on credit constraints, the category of which this model belongs to, and explains why they are better suited to modelling downturns such as the recent subprime crisis than collateral constraint approaches. Other methodologies such as investment screening and financial regulations are types of exogenously imposed restrictions which are incompatible with the microfounded approach to credit imperfections undertaken in this thesis. There are two microfounded mechanisms that transform sudden and short-lived disturbances in the financial markets such as a change in leverage into sharp and prolonged fluctuations that may impact the real economy and are known as financial accelerator mechanisms. Such models attribute the presence of a financial accelerator mechanism either to the existence of an external finance premium known as the credit channel which limits the supply of external resources, or to the collateralization of debt known as the collateral constraint channel.

The precursor to the credit channel methodology is Bernanke and Gertler (1989). It first introduced financial constraints in a model of overlapping generations. Entrepreneurs (producing firms) who produce capital borrow outside funds using their existing assets which constitute their collateral or net worth. They use financial contracts to transform their assets together with borrowed funds through investment projects into capital goods that produce consumer goods. In this model, only entrepreneurs can see the results of their individual projects. Lenders have to pay monitoring costs to verify the results of investment projects. Monitoring costs combined with uncertainty about the outcome of the project lead to a positive premium on external resources which restricts loans to the producer of capital. Bernanke and Gertler (1989) demonstrate that a positive shock to firm productivity raises the price of capital leading to increased profits for financial agents. The higher profits of entrepreneurs increase their wealth, resulting in a downward adjustment of the risk premium. As a consequence of the positive shock to productivity and the increase in net worth, it is cheaper to acquire external financing for the implementation of investment projects. This stimulates further growth of investment and capital production which promotes future economic growth. Bernanke and Gertler (1989) called this type of amplification a "financial accelerator" effect on investment income.

After the development of the overlapping generations model, Bernanke, Gertler and Gilchrist (1999) presented an advanced model on credit constraints with a financial accelerator mechanism. The model places producers of capital who face an external finance premium in a New-Keynesian DSGE model which is characterised by the presence of price rigidities. In this model, entrepreneurs face random termination (a mechanism capturing the ongoing exit of firms). This assumption of termination allows abstracting from incorporating investment reputation in financial decisions. Also, bankruptcy of entrepreneurs prevents them from accumulating enough internal funds to achieve independence from external sources of funds. In this dynamic stochastic general equilibrium model, only one good is produced by entrepreneurs and the economy is populated by households (lenders), firms, retailers and government. The model of Bernanke, Gertler and Gilchrist (1999) allows to analyse the response of the economy to a technology shock, a monetary policy shock, and a shock to government expenditures. The production sector is made up of heterogeneous firms set in a perfect competition environment which use capital and labour to produce consumption goods. The net worth of companies originates from investing their capital and labour. Firms borrow against their net worth to purchase the necessary capital to produce consumption goods from a financial intermediary that lends household savings. The financial contracts used minimize the expected costs of financial intermediation. Bernanke, Gertler and Gilchrist (1999) assume that only one-period contracts between borrowers and lenders are feasible but Carlstrom and Fuerst (1997) demonstrate that longer-term contracts do not affect the basic results. The form of the financial contract and the assumption of production with constant returns to scale generate a linear relationship between demand for capital and entrepreneurial net worth. Firms raise capital using the optimal financial contract based on net worth. Using this advanced model, Bernanke, Gertler and Gilchrist (1999) show that the financial contract provides a monotonically increasing linear relationship between the capital-to-net worth ratio (leverage ratio) and the risk premium on external funds. The emergence of positive shocks reduces demand for external funding because an increase in profits increases the net worth of good producers. The net worth of entrepreneurs responds strictly pro-cyclically generating an anti-cyclical external finance premium in this model that narrows following a positive disturbance.

Bernanke, Gertler and Gilchrist (1999) expand their analysis of business cycles by comparing models with and without a financial accelerator mechanism. They note that the presence of a financial accelerator may explain the extent and persistence of fluctuations which are a response to monetary policy and to demand and supply shocks.

Although the one sector model presents only a one-directional response of the risk premium, the microfounded nature of the model lends itself to variations and upgrades that make the credit approach especially suited to studying a variety of disturbances and consequences. Shocks can be either originating in the real sector as in the original Bernanke, Gertler and Gilchrist (1999) or could have a financial origin. The introduction of more than one final good sector allows price effects to determine the magnitude of financial response as evidenced in this chapter. The rich potential of microfounded credit constraints models is the primary reason they are the favoured approach in this thesis.

An early extension of the framework of Bernanke, Gertler and Gilchrist (1999) is presented by Christiano, Motto and Rostagno (2003) in their analysis of the Great Depression. Christiano, Motto and Rostagno (2003) introduce perceptions of risk, disturbances to labour demand and to capacity utilization and wage frictions to capture the key forces of economic cycles. Using the extended model, Christiano, Motto and Rostagno (2003) replicate the important features of the data observed during the Great Depression. In this period, they also identify an increase in households' preferences for holding money and a shift away from the formation of savings. According to the authors, the reduction of savings led to the emergence of a restriction on the amount of external funds available for capital creation and to a growth in the external finance premium. The authors conclude that the financial accelerator mechanism strengthens the liquidity preference shock and affects the rest of the economy. The model of Aoki, Proudman, and Vlieghe (2004) borrows from Bernanke, Gertler and Gilchrist (1999) to include credit constraints in the purchase of housing bringing the model closer to the present thesis. In Aoki, Proudman, and Vlieghe (2004), producers of capital are replaced by a realty sector that undertakes investment projects that result in the provision of residential services to households. The realty sector faces external finance premium when searching for capital for their purchases of real estate. The motivation for this paper is based on the strong pro-cyclical nature of residential investment and house prices because residential investment costs depend on the structure of household wealth. Also, the demand for houses depends on the return on housing services provision and the expected marginal cost of financing. The model of Aoki, Proudman, and Vlieghe (2004) is able to replicate the characteristics of the data (housing investment, housing prices, and consumption) and highlights the importance of the presence of an external finance premium to account for these properties. However, since the realty operates essentially as a final good sector and consumers are renters, the model is not a big step further from Bernanke, Gertler and Gilchrist (1999).

The implications of including an external finance premium in a DSGE framework in order to match the data characteristics are generally based on the comparison of different models. Meier and Müller (2006) compare the model with a financial accelerator and the model with increasing capital adjustment costs. They focus on the monetary policy transmission mechanism so they primarily examine the responses to a monetary policy shock. They argue that both models are able to replicate the characteristics of the observed data on investment so do not consider the external financing mechanism to be more important than the mechanism of costly investment for description of the properties of the transmission mechanism. Since most financial contracts are specified in nominal terms, Christiano, Motto and Rostagno (2010) developed a financial accelerator model that allows producers to raise capital through nominal contracts. By comparing the model with and without the financial accelerator mechanism they identified only very small differences resulting from the inclusion of the external finance premium. Due to the use of nominal contracts, the authors attribute only minor importance to the financial accelerator mechanism in explaining the observed fluctuations in economic cycles as compared to the aforementioned studies. Similarly to Christiano, Motto and Rostagno (2010), Christensen and Dib (2008) extend the model of Bernanke, Gertler and Gilchrist (1999) by writing debt contracts in terms of the nominal interest rate. The authors conclude that the inclusion of the financial accelerator mechanism leads to a better fit of the observed data properties and that the inclusion of the external finance premium allows the propagation of investment shocks to the rest of the economy. Moreover, Christensen and Dib (2008) focus on changes in the impulse responses to demand shocks which show that introducing the financial accelerator greatly amplifies and propagates the effects of demand shocks on investment while dampening those of supply shocks. Increased household demand for consumption reduces savings and increases the premium on external financing of capital formation. This increase in premium restricts the availability of capital required for the production of consumer goods for households that in turn limits the ability of producers to absorb the increased demand.

Models with an external finance premium do not contain limits on the availability of the external funds needed by the indebted agents. This makes credit constrains models especially useful in studying downturns such as the subprime crisis where borrowers can quickly amass a significant amount of debt. The only limiting factor is the price which is increasing in the amount of external funding needed from the lender. An alternative approach to modelling the financial accelerator mechanism is to incorporate a limit on the amount of funds available, the collateral constraint approach. The limitation is motivated by the creditworthiness of the borrower. The larger the debtor's assets the less likely the borrower is to default so loans are capped up to a limit that is a fraction of the assets (collateral) used to secure them. Collateral constraint models are not very suitable for modelling financial downturns for reasons described below.

In models using a collateral constraint there are limits on the availability of funding. The limits are placed to restrict the exposure of lenders to risk associated with bankruptcy on the part of borrowers. In the event of bankruptcy of an investment project, creditors may recover the debt only if the loans are fully secured. Loans are collateralized by the debtor's durable assets such as land, property and long-term capital. These assets are transferred to the lender in the event of bankruptcy of the debtor's investment project. When the firm is hit by an adverse shock or by a reduction in its income, the response depends on the size of the shock. A small negative shock can be absorbed either by reducing the amount of financial capital or by reducing the consumption of resources, or by a combination of the two. In the case of a large negative shock, the financial capital may be depleted. The firm then has to adjust the scope of its activities by reducing the number of production facilities. In their seminal work, Kiyotaki and Moore (1997) construct a theoretical model in which firms use contracts with fully secured debt to obtain capital resources. They show how small-scale and temporary shocks to the productivity of firms can lead to significant fluctuations that spread throughout the economy. The assets of firms are not only used for the production of consumer goods but also as collateral for raising capital. The source of the financial accelerator effect and of the propagation of technology shocks to the rest of the economy is the interaction of asset prices for debt securitization with credit limits.

In the model of Kiyotaki and Moore (1997) there are farmers (generally understood as firms) and gatherers (commonly households) who are characterised by different degrees of patience for consumption. Impatient farmers have a lower subjective discount factor that patient gatherers that in equilibrium generates an incentive to borrow. Hence, the ex-ante heterogeneity induces credit flows between the impatient farmers and the patient gatherers. Also, farmers differ in their credit limits arising from the presence of debt collateralization. Farmers own capital (land) that can be used either for the production of goods or as collateral when searching for external funds. The rest of the model is a fairly standard model of real business cycles. Kiyotaki and Moore (1997) demonstrate that even small and short-term shocks to productivity or income distribution can cause prolonged changes in production, consumption, and prices of capital. A key mechanism in the amplification and propagation of shocks in the model with debt collateralization is changes in the value of collateral and their reflection in credit limits. A negative shock to the price of collateral used to secure debt leads to a decrease in the net worth of firms. Producers that become constrained by the credit limit are forced to reduce their demand for investment, including investment in land. The decline in demand for investment leads to a decrease in expected future output and income, thereby reducing expected demand in subsequent periods. The expected decline in demand for collateral by credit-constrained firms and the clearing of markets (to maintain equilibrium in the markets) forces unconstrained firms to demand more land. This leads to a decrease in the opportunity cost of holding land which gives rise to a fall in the price of collateral in the period of the shock. Due to the expected and actual decline in the collateral price, the shock hurts the credit-constrained firms a second time and forces them to make deeper cuts in their investment in land. This self-reinforcing feedback is what amplifies economic fluctuations in the model.

Kiyotaki and Moore (1997) argue that the securitization of debt plays a significant role in amplifying the fluctuations of economic cycles. However, Kocherlakota (2000), using his theoretical models with secured debt, demonstrates that the mechanism of securing debt alone is not capable of delivering a strong financial accelerator effect (credit constraints with full securitization do not generate enough volatility and asymmetry). He shows that the strength of the financial accelerator mechanism increases with an increasing share in the production function of factors whose alternative function is debt securitization. Cordoba and Ripoll (2004) assess the assumptions made by Kiyotaki and Moore (1997) and demonstrate that the significant role of contracts with debt collateralization in providing debt contracts is too specific. Furthermore, Cordoba and Ripoll (2004) conclude that the financial accelerator effects in Kiyotaki and Moore (1997) are mainly due to constant returns to scale in the production of investment projects (linearity of the production function) and to the willingness of lenders to support a project of any size (risk-neutrality of lenders). Cordoba and Ripoll (2004) opt to use more realistic assumptions (decreasing returns to scale, risk-averse players) and they demonstrate the insignificance of the financial accelerator effect for the amplification of responses. As their simulations indicate, large amplification can be obtained merely with the "right" combination of a low elasticity of intertemporal substitution, a large, but not too-close-to-one capital share and a sizable but not too-close-to-one share of constrained agents. They conclude that unless one has this right combination of parameters, collateral constraints can generate amplification when compared with perfect-market models but this amplification is small.

A significant shortcoming of collateral constraint models is the fact that they employ an upper limit on borrowing that restricts the post-shock volatility and that the magnitude of response depends on the specific calibration. Exogenous restrictions on borrowing prevent a significant buildup of debt which is common during financial recessions. Hence this approach is not well-suited for studying credit downturns which is the goal of the present paper. In contrast, the credit constraint approach of Bernanke, Gertler and Gilchrist (1999) places no exogenous upper limits on borrowing. Instead, the variable external finance premium endogenously restricts financing by making it costlier as the indebtedness-to-net worth ratio increases. This microfounded approach gives borrowers more control over their financial position especially when faced with a credit easing (one of the circumstances that led to the subprime crisis). Since the objective of this model is to study how the leverage of borrowers responds to real and financial disturbances and how volatile their debt position can become during downturns, the microfounded credit constraint is the preferred approach to financial frictions in this thesis.

1.3 The Financial Frictions Model

This section sets up the financial contract in a partial equilibrium framework and takes as given the price of inputs and the risk-free rate of interest. The subsequent section endogenises these variables as part of a full general equilibrium solution.

1.3.1 Description of the Financial Friction Contract

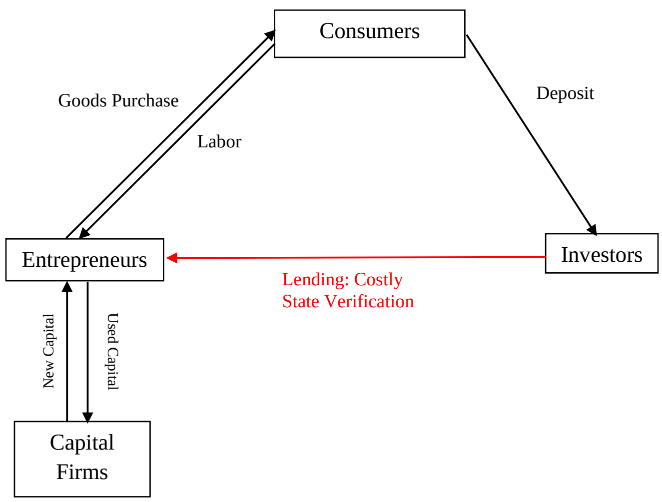


Figure 1.1: The One-Sector Model with Financial Frictions (BGG)

This paper extends the one-sector model of financial frictions of Bernanke, Gertler and Gilchrist (1999) to two sectors to demonstrate that in an economy with more than one final good, price effects may determine how credit constraints respond to aggregate disturbances. In Bernanke, Gertler and Gilchrist (1999) there is only one production sector called "entrepreneurs" (Figure 1.1). Entrepreneurs are final good firms that purchase capital from a capital producing sector and using labour and own technology manufactures a good that is purchased by consumers. There is a capital sector that supplies entrepreneurs with capital and uses as investment the output of the final good firms. Consumers purchase the final good and supply labour to production. They also make deposits with risk-neutral investors which in turn lend to credit-constrained entrepreneurs.

The two-sector economy consists of two sectors: one that produces generic consumption and one that manufactures housing (Figure 1.2). The consumption sector produces its output using sector-specific capital, labour and own technology as inputs. The housing sector is modelled in two steps. In the first version that is a direct extension of Bernanke, Gertler and Gilchrist (1999), housing is produced using a production function identical to that of consumption. Housing manufacturers use housing capital, labour and sector-specific technology to produce their output. In this version, housing exists for one period only and completely depreciates at the end of the period. While this approach allows for direct comparisons with Bernanke, Gertler and Gilchrist (1999), the modelling of housing is not empirically realistic. The next step borrows from Iacoviello and Neri (2010) in modelling housing as a durable multi-period good. The production of housing still uses housing capital, labour and sector-specific technology but it also utilises land, a finite resource. Additionally, each period only a fraction of housing deteriorates. The rest survives for the subsequent period. In each period, new housing production only replaces the depreciated housing. This approach to housing as a multi-period good brings a more staggered response to disturbances since newly produced output is only a fraction of total housing on the market.

In addition to the two final goods, there are capital producing firms that supply final good manufacturers with sector-specific capital. Investment in both types of capital is the output of the consumption sector. The relationship between capital firms and final goods producers is a two-way one in the sense that capital firms buy used capital from goods producers and along with investment transform it into new capital. Consumers supply undifferentiated labour to both sectors and purchase the output of these firms, i.e. buy consumption and housing. Households own capital-producing firms and absorb their profits or losses. Consumers also have savings which they deposit with risk-neutral investors. The investors use these deposits to finance the capital acquisition of final good firms.

Consumption producing firms face no risk and they can borrow funds for capital financing at the risk-free rate as part of a fixed share arrangement. The housing sector, however, is financially constrained and possesses an intrinsic idiosyncratic risk of default that is not observable to outside parties. In this sense, housing firms essentially correspond to the entrepreneurs in Bernanke, Gertler and Gilchrist (1999). Due to the unobservable probability of bankruptcy, the investors that provide funds to housing manufacturers need to pay an auditing fee in order to learn their probability of default. This gives rise to a costly state verification problem that makes lending to housing firms more expensive than to risk-free agents. When indebted housing manufacturers go bankrupt, investors pay the monitoring fee and take possession of all of the borrower's remaining assets. These auditing costs could include the costs of legal proceedings to the value the borrower's assets and the administrative costs of selling their remaining capital stock to realise its collateral value. The presence of these financial frictions motivates the need for a loan contract between investors and housing producers as opposed to a share contract that usually takes place in the absence of idiosyncratic uncertainty.

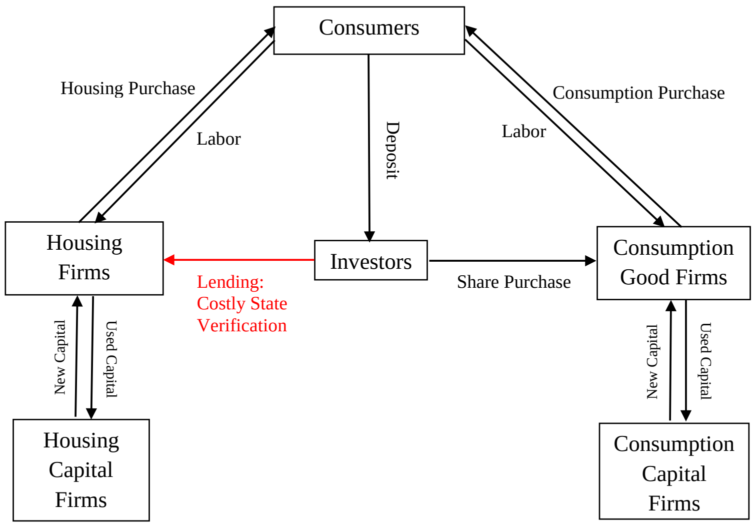


Figure 1.2: The Two-Sector Model with Financial Frictions in the Housing Sector

The loan contract must guarantee a safe return to investors on their funds regardless of the realization of the idiosyncratic probability of default. In order to ensure that, the rate at which the loan is repaid includes a risk premium that is related to the probability of bankruptcy. The higher the default prospects, the larger the repayment rate relative to the risk-free rate. Hence with idiosyncratic uncertainty, the repayment rate is no longer the risk-free rate but is a state-contingent markup rate that must account for the probability of default. The difference between the loan repayment rate and the risk-free rate is the markup known as the external finance (risk) premium. It exists in order to offset the expenses associated with a potential bankruptcy of borrowers. The greater the default prospect, the more likely the lender will have to pay the auditing cost so the risk premium will be higher. Hence the external finance premium is a function of the default rate, the collateral (net worth) of credit-constrained housing producers and the value of the loan. Investors are risk-neutral and do not make any profit on the loan to housing manufactures. The risk premium is exactly sufficient to offset the costs associated with a potential borrower default.

The two-sector model is real and abstracts from nominal rigidities. In a one-sector model such as Fernández-Villaverde (2010) a nominal setup allows for Fisher-type price effects to influence the net worth and the leverage of indebted agents. The two-sector economy described here produces two goods whose relative price allow for such price effects to arise naturally without the need to consider nominal rigidities.

1.3.2 The Nature of Financial Frictions

In a frictionless world, there is no information asymmetry. Both borrowers and lenders know the return of the investment and the likelihood of the loan repayment. In such an ideal scenario, project financing yields a fixed return and a firm is able to raise capital from external sources in exchange for a share in its future stream of profits. Such contracts in a frictionless economy are known as share contracts. They are the topic of the following subsection.

However, in the real world agents face certain restrictions when searching for external sources of capital. These restrictions occur due to the limited supply of credit (credit rationing) or the unattractiveness of the lending conditions. The limitations on borrowing and the conditions of lending originate from information asymmetry between the borrower and the lender. The cost of external financial resources is therefore a premium (termed an external finance premium or a risk premium) which a debtor must pay to creditors, generally in the form of interest rate markup above the risk-free rate. The costs of external sources can vary considerably and depend on many characteristics such as company risk, net worth and the return on the proposed investment project. The external finance premium is defined as the difference between the cost to the borrower of raising capital in the financial markets (external funding) and the opportunity cost of the borrower's use of internal resources. Empirical data (Banerjee, 2002; De Graeve, 2008) show that external funding (equity borrowed from creditors) is almost always more expensive than using internal resources (capital raised from own profits). This is due to the increase in the costs of lenders who have to evaluate the prospects of success of the investment projects and monitor the borrower's conduct. Hence for a better fit of reality, models of financial markets account for the observed positive premium on external funds.

Due to the costs of obtaining external funding, the external finance premium is almost always positive and dependent on the financial situation of the borrower. A debtor is financially assessed in terms of their net worth and the actual amount of liquidity and the expected income stream of their investment projects. The size of the external finance premium decreases with increasing net worth or liquidity. A debtor with a better financial position is rewarded with a lower cost of external sources of capital. The external finance premium in the literature is usually anti-cyclical in nature (Besley et al. 2008) because a firm's income and net worth generally increase in periods of economic growth and this improves its financial position (pro-cyclicality of profits and asset prices). This thesis demonstrates that it is possible to generate a pro-cyclical risk premium when there are more than one final goods the borrowers are not production firms.

The financial accelerator models receive their name from the fundamental property of imperfect financial markets to propagate shocks to the rest of the economy and magnify their effect (Bernanke, Gertler and Gilchrist, 1999). A shock that increases the external finance premium raises the cost of obtaining external funding and worsens the financial position of the debtor. The further deterioration in financial position leads to a further increase in the premium and a further reduction in demand for funding. The decrease in the utilization of resources leads to a reduction in investment projects and thus reduces economic output and the future stream of profits from projects. The less profitable projects lead to an increase in dependence on external funding and a decrease in net worth, leading to an increase in the premium. As a result, a small shock can, via the financial accelerator mechanism, significantly affect the whole economy for a long time.

One of the sources of the imperfect functioning of financial markets is the information asymmetry that arises between the borrower and the lender. Lenders providing funding for investment projects face information asymmetry because they have less, or more imprecise, information about the status of investment projects or the repayment ability of debtors than borrowers. Therefore, a significant share of the costs of external financing of investment projects can be attributed to this asymmetry. Due to the presence of asymmetric information, financial contracts often include the possibility of costly monitoring of the status of the investment project (costly state verification) by the creditor because the control delivered by only monitoring the financial market is considerably simplified. Monitoring of project results, as assumed by Diamond (1965), involves expenses related to obtaining information which increases the project implementation costs. Akerlof (1970) shows that the presence of information asymmetry between the parties in a contract may result in a restriction of market functioning or, in extreme cases, in the establishment of barriers to the existence of a market.

The types of contracts designed for the presence of asymmetric information and the possibility of monitoring have been described in Townsend (1979). The theoretical microfoundations of these contracts assume the existence of credit-constrained firms to which lenders lend their funds for the purchase of a manufacturing input for a pre-agreed profit. The borrowers constantly face the possibility of bankruptcy which they can observe free of charge. Creditors, however, have to pay for monitoring when trying to find the actual likelihood of default of the borrower. The lender faces the dilemma of when to opt for monitoring since it is not cost-effective to monitor at all times. However, not monitoring raises the possibility of loss of the investment. Hence monitoring occurs but its costs are reflected in the premium paid by debtors. Townsend (1979) specifies a contract type able to handle this information asymmetry and describes the properties of this contract. The optimal contract is defined by a function describing the returns on the contract when bankruptcy or success occurs. Also, the optimal contract defines the reported income threshold. The optimal contract is specified as follows: if a project posts returns higher than or equal to the threshold, the lender does not require auditing of the project result; if the reported profit from the project is lower than the monitoring threshold, the lender always requires an audit of the project result. The monitoring threshold is a decreasing function of the amount of the borrower's net worth, because wealthier borrowers can pay an agreed yield despite greater losses in the event of an adverse project result. The monitoring threshold is an increasing function of the risk-free return which serves as an alternative to investing in the project. The difference between the monitoring threshold and the risk-free yield serves as an impulse for the emergence of financial agents, i.e. investors, who mediate the investment contract.

The following subsections develop the share contract of consumption firms and the loan contract of credit-constrained housing firms. While a share contract yields a fixed return to both borrowers and lenders, the return of a loan contract is contingent on the realization of the information asymmetry.

1.3.3 The Share Contract of the Consumption Sector

Consumption producing firms have no aggregate information asymmetry so they borrow as part of a fixed share contract arrangement. They buy consumption capital K_t^F at price x_t^F one period in advance to be used for the production of their final good in the subsequent period. Consumption firms finance part of the purchase with their own equity and borrow the rest. Each firm i in the consumption sector earns positive profits and amasses equity (net worth) denoted $N_{i,t}^F$. The net worth of firms is not sufficient to finance their capital acquisition so they borrow the difference $x_t^F K_{i,t}^F - N_{i,t}^F$ from investors. The loan is repaid in the subsequent period according to the borrowing terms.

Consumption producers are subject to an idiosyncratic shock to their return denoted ω^F. Firms draw the idiosyncratic shocks from the same distribution and there is no asymmetric information regarding the realization of ω^F - both lenders and borrowers know the value of the firm-specific shock upfront. Hence there is no idiosyncratic uncertainty about the realization of ω^F. The absence of idiosyncratic uncertainty implies that lenders do not need to monitor the contracts with consumption firms so loans take the form of a share contract. As equity holders of producers in the consumption sector, investors finance the capital purchase and absorb the profits and losses of the firms. Consumption producers and investors split the revenue according to the share of their investment, regardless of the realization of ω^F. To diversify away from the firm-specific idiosyncratic risk, each lender invests in an infinite number of final good firms.

Since lenders are not exposed to any idiosyncratic risk from an unexpected realization of the idiosyncratic shock, the contract has no uncertainty about the return of the shares. This allows producers to borrow at the (predetermined) risk-free rate R_t. Each period, lenders receive a share $\Gamma^F_{i,t}$ from their loan $x^F_t K^F_{i,t} - N^F_{i,t}$ to consumption firm i disbursed in the previous period:

$$\Gamma^F_{i,t} R_t x^F_t K^F_{i,t} = \frac{x^F_t K^F_{i,t} - N^F_{i,t}}{x^F_t K^F_{i,t}} \int_0^\infty \omega^F R_t x^F_t K^F_{i,t} dF(\omega^F)$$

The share is defined as the ratio of the borrowed funds to the total capital purchase times the probability of return given the firm-specific shock ω^F. Since lenders can diversify over an infinite number of consumption producers and the share contract does not depend on ω^F, then the probability of return is unity, i.e. $\int_0^\infty \omega^F dF(\omega^F) = 1$ This implies that the share is independent of the idiosyncratic shock ω^F:

$$\Gamma^F_{i,t} = \frac{x^F_t K^F_{i,t} - N^F_{i,t}}{x^F_t K^F_{i,t}}$$

The share Γ^F_i is independent of the idiosyncratic shock ω^F because there is no idiosyncratic uncertainty to influence the share return. Since the firms in the consumption sector draw the idiosyncratic shocks from the same distribution, every lender receives the same average share of profits. Let Γ^F be the aggregate share of profits in the share contracts that goes to investors, then $\Gamma^F_{i,t} = \Gamma^F_t$. Hence, the aggregate share Γ^F_t can be expressed as:

$$\Gamma^F_t = \frac{x^F_t K^F_t - N^F_t}{x^F_t K^F_t} \tag{1.3.1}$$

where $K^F_t = \sum K^F_{i,t}$ and $N^F_t = \sum N^F_{i,t}$

Firms in the consumption sector earn profits which they split with investors according to their share contract. In any given period, the equity of consumption manufacturers V^F_t

is their part of the share agreement in the previous period:

$$V_t^F = \left(1 - \Gamma_t^F\right) R_{t-1} x_{t-1}^F K_{t-1}^F = R_{t-1} N_{t-1}^F \tag{1.3.2}$$

In order to prevent consumption firms from growing out of their financial constraints, the model assumes that at the end of every period a constant fraction of consumption producers $1 - \nu^F$ exit the market. When they exit, they immediately spend their remaining equity on their own output. The consumption on exit of consumption firms $C_{E,t}^F$ is:

$$C_{E,t}^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F \tag{1.3.3}$$

Furthermore, firms need to start off with some net worth so this paper follows Bernanke, Gertler and Gilchrist (1999) in assuming that they provide inelastically one unit of labour to the production of their respective output. Their income from labour is w_t^F. The weight of the producers labour supply in the total labour in each sector is small so that it does not distort the labour supply of households.

The evolution of the net worth of consumption firms is the sum of the equity of surviving firms plus their income from work:

$$N_t^F = \nu^F V_t^F + w_t^F \tag{1.3.4}$$

1.3.4 The Loan Contract of the Credit-Constrained Housing Sector

The presence of credit constraints implies that there is uncertainty associated with the returns of housing manufacturers. Namely, there is an idiosyncratic shock to their return that is known to them but unknown to lenders. Therefore, their capital purchase cannot be financed by a simple share purchase like consumption firms. It must be specified as part of a loan contact where the returns on lending insure investors against fluctuations in the realization of the idiosyncratic shock of housing producers.

Like consumption capital, housing capital must be purchased one period in advance so firms are exposed to fluctuations in capital prices that affect their net worth and borrowing possibilities. The net worth of housing manufacturers is not enough to meet their capital needs so they need to obtain external financing. At time t, housing firm j purchases capital $K^H_{j,t}$ for use at time $t+1$. In order to do so, it borrows external funds from investors at time t to be repaid at the time capital is used in $t+1$. The relationship between investors and housing manufacturers is a loan contract. This setup follows the financial accelerator approach developed by Bernanke, Gertler and Gilchrist (1999) that assumes there is a costly state verification problem of monitoring on the lender's part as a way of motivating financial frictions. Housing manufacturers face idiosyncratic shocks to their return to capital. In each period, a firm draws from a distribution of ω^H where ω^H follows a log-normal distribution with mean one and variance σ^2. The realization of the firm ω^H_j is known to the borrower but unknown to lenders. If the investors want to observe the shock for a specific firm, they need to pay a monitoring cost μ which is a fraction of the firm's total capital stock. Due to the monitoring cost, it would be too costly for the investors to monitor every contract. Therefore, there is a cutoff value of the shock $\bar{\omega}^H$ above which investors do not monitor and below which the investors do. Above the cutoff, housing manufacturers get a return sufficient to pay the investors a fixed rate of interest Z^H and keep what is left for themselves; investors have no incentive to monitor housing manufacturers' true return. Below the cutoff, housing producers are bankrupt and investors pay the monitoring cost and take the firm's entire wealth.

The idiosyncratic shock ω^H_j is independent across time and across credit-constrained firms. Lenders can diversify away the idiosyncratic risk of the borrowers by investing in many borrowers. The ex-post gross return on capital is $\omega^H_j R^H$, where ω^H_j is an idiosyncratic disturbance to firm j's return and R^H is the realised aggregate return to capital to housing producers. The rate R^H is a markup above the risk-free rate R and this markup is known as the external finance (risk) premium. This external finance premium insures investors against the volatility of the return to capital faced by housing producers and is a central part of the loan contract. The even return is achieved by making R^H contingent on the realization of any disturbances that happen at the time of loan repayment.

Consider a housing firm j repaying its debt at time t whose idiosyncratic shock is exactly the cutoff value $\bar{\omega}^H_j$. The returns to this producer are exactly equal to the contractual rate of interest Z_j^H charged on the contract between the investors and that firm, times the amount borrowed in the previous period which is the value of capital stock minus the firm's net worth $x^H K_j^H - N_j^H$. The return of this housing producer in this period is the value of capital stock $x^H K_j^H$ times the R^H and the firm's risk $\bar{\omega}_j^H$.

The investors making loans to housing manufacturers are risk-neutral and the opportunity cost of their funds is the risk-free interest rate. Hence in order for them to agree to lend to the particular housing firm j whose idiosyncratic shock is at the cutoff point, they must break even on their investment. This implies that the interest rate charged on the loan must be such so that its expected gross return must be equal to the opportunity cost of lending:

$$\bar{\omega}_j^H R^H x^H K_j^H = Z_j^H (x^H K_j^H - N_j^H) \tag{1.3.5}$$

The left hand side is the gross revenue from capital to the firm whose risk is just at the cutoff point. The right hand side is the amount the housing manufacturer needs to repay the investor. The marginal housing producer is just able to repay the loan at the contractual rate and does not retain any revenue. The equation above also demonstrates that a rise in the contractual rate Z_j^H increases the probability of default of the firm and so $\bar{\omega}_j^H$ rises.

The idiosyncratic shock is continuous and has a cumulative distribution function $F^H(\omega^H)$ (discussed in more detail in Appendix 1.F). To be perfectly insured against the idiosyncratic shocks of borrowers, each investor signs contracts with an infinite number of firms. The solution of the loan contract defines how the investment revenue is split between the lenders (investors) and the borrowers (housing manufacturers). Each lender gets a net share $\Gamma^H(\bar{\omega}_j^H)$ from lending to borrower j. The net share is equal to the sum of the gross share (before monitoring) from housing manufacturers which default and the fixed interest payment from the non-defaulting debtors:

$$\Gamma^H(\bar{\omega}_j^H) R^H x^H K_j^H = \int_0^{\bar{\omega}_j^H} \omega_j^H dF^H(\omega_j^H) R^H x^H K_j^H + (1 - F^H(\bar{\omega}_j^H)) Z_j^H (x^H K_j^H - N_j^H)$$

Using the fact that the expected gross return for investors on a loan must be equal to their opportunity cost of lending as determined in equation (1.3.5), the previous equation can be expressed as:

$$\Gamma^H(\bar{\omega}_j^H)R^H x^H K_j^H = \int_0^{\bar{\omega}_j^H} \omega^H dF^H(\omega^H)R^H x^H K_j^H + (1 - F^H(\bar{\omega}_j^H))\bar{\omega}_j^H R^H x^H K_j^H$$

The left hand side is the net return from the loan contract. The first term on the right hand side captures the return from defaulting debtors whose default probability is below $\bar{\omega}_j^H$. The second term is the fixed repayment of non-defaulting borrowers. The total repayment lenders receive is the loan amount $R^H \left(x^H K_j^H - N_j^H\right)$ times the probability of non-defaulting that is $(1 - F^H(\omega_j^H))$.

Let the share of the returns from defaulting firms be denoted $G^H(\bar{\omega}_j^H) = \int_0^{\bar{\omega}_j^H} \omega^H dF^H(\omega^H)$. Then the gross return is the receipts from defaulting borrowers $G^H(\bar{\omega}_j^H)R^H x^H K_j^H$ and the fixed payment from non-defaulting debtors $(1 - F^H(\bar{\omega}_j^H))\bar{\omega}_j^H R^H x^H K_j^H$:

$$\begin{aligned}\Gamma^H(\bar{\omega}_j^H)R^H x^H K_j^H &= \int_0^{\bar{\omega}_j^H} \omega^H dF^H(\omega^H)R^H x^H K_j^H + (1 - F^H(\bar{\omega}_j^H))\bar{\omega}_j^H R^H x^H K_j^H \\ &= G^H(\bar{\omega}_j^H)R^H x^H K_j^H + (1 - F^H(\bar{\omega}_j^H))\bar{\omega}_j^H R^H x^H K_j^H\end{aligned}$$

Subtracting $R^H x^H K_j^H$ from both sides gives the share of investors only in terms of the default probability $\bar{\omega}_j^H$:

$$\Gamma^H(\bar{\omega}_j^H) = G^H(\bar{\omega}_j^H) + (1 - F^H(\bar{\omega}_j^H))\bar{\omega}_j^H$$

This equation describes the investors' gross share (before monitoring) $\Gamma^H(\bar{\omega}_j^H)$ as the sum of the share received from defaulting firms $G^H(\bar{\omega}_j^H)$ and the fixed share of the profits of the non-defaulting firms where the probability of repayment is $1 - F^H(\bar{\omega}_j^H)$. The investors' return is graphically represented in Figure 1.3. Area A is the share received from defaulting borrowers $G^H(\bar{\omega}_j^H)$ and area B is the share repaid from non-bankrupt firms $(1 - F^H(\bar{\omega}_j^H))\bar{\omega}_j^H$. The combined area under the solid line represents the loan returns retained by investors. Borrowers receive $1 - \Gamma^H(\bar{\omega}_j^H)$ which is area C. The first derivative of $\Gamma^H(\bar{\omega}_j^H)$ with respect to $\bar{\omega}_j^H$ is positive $\Gamma_\omega^H(\bar{\omega}_j^H) > 0$ implying that the higher the cutoff value, the larger the share that goes to the lender.

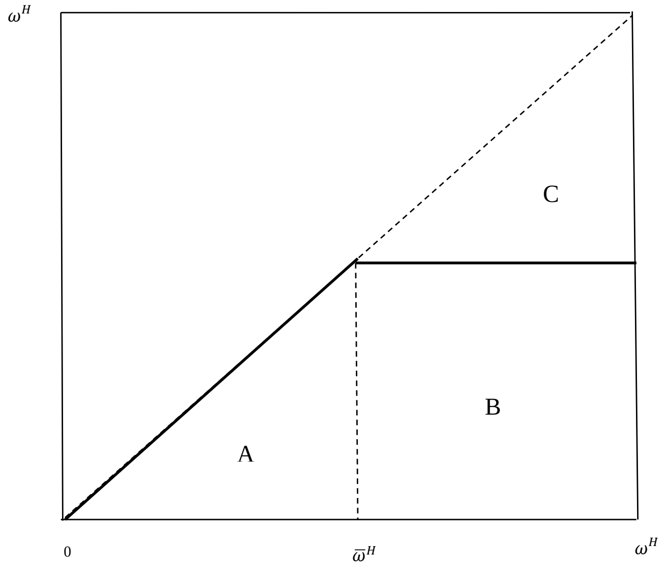


Figure 1.3: The Return of the Loan Contract

Taking into account the monitoring cost μ, the net share that goes to the lender is:

$$\Gamma^H(\bar{\omega}_j^H) - \mu G^H(\bar{\omega}_j^H) \tag{1.3.6}$$

where $\mu G^H(\bar{\omega}_j^H)$ is the share from defaulting borrowers times the fraction of return μ spent on monitoring.

By assumption, the investors make zero profits on their loans to housing manufacturers. As the loan risk is totally diversifiable for the investors, the relevant opportunity cost to investors is the risk-free rate R. Hence the zero profit condition for each lender for a loan disbursed at time t to be repaid at time $t+1$ is:

$$E_t\left(\left(\Gamma^H(\bar{\omega}_{j,t+1}^H) - \mu G^H(\bar{\omega}_{j,t+1}^H)\right) R_{t+1}^H\right) x_t^H K_{j,t}^H = R_t\left(x_t^H K_{j,t}^H - N_{j,t}^H\right) \tag{1.3.7}$$

The left hand side is the net return on the loan to housing firm j (the gross return $E_t(1-\Gamma(\bar{\omega}_{j,t+1}^H))$ minus the monitoring cost $\mu^H E_t G(\bar{\omega}_{j,t+1}^H))$ at the time $t+1$ of the repayment. The right side is the opportunity cost of the investor funds (the value of capital $x_t^H K_{j,t}^H$ minus the borrower's net worth $N_{j,t}^H$) valued elsewhere at the risk-free rate R_t that prevailed at the time t when the loan was agreed. Since the price of capital, the risk-free interest rate and the housing producer's net worth $N_{j,t}^H$ are predetermined, the borrower gets to choose a pair $(\bar{\omega}_j^H,\ K_j^H)$ according to the zero profit condition. Given the other variables, this is equivalent to the lenders offering a schedule of loans B^H and a non-default interest rate R to the credit-constrained firm.

In this partial equilibrium setting, the borrowing housing producer j is faced with a menu of amount of loan $E_t B^H_{j,t+1}$ and interest rate $E_t R^H_{t+1}$ where both the loan and the interest rate are related by the participation constraint of investors. Every period at time t, borrowers choose the optimal pair of capital and the expected cutoff risk $(K^H_{j,t},\ E_t\bar{\omega}^H_{j,t+1})$ to maximise their expected share of the loan at the time $t+1$ of the loan repayment:

$$max\ E_t\left(\left(1-\Gamma^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t K^H_{j,t} \tag{1.3.8}$$

subject to the zero profit condition of investors.

A contract agreed in period t is repaid back in period $t+1$ at the realised return R^H_{t+1} determined at time $t+1$. Hence the ex-post interest rate R^H_{t+1} at which the loan is repaid incorporates all updated information about any unexpected shock occurring at $t+1$, whereas the expected interest rate $E_t R^H_{t+1}$ is predetermined at t and does not reflect any information from time $t+1$. When firms optimise the loan contract at time t which is to be repaid at time $t+1$, they only know $E_t R^H_{t+1}$ and so the maximization problem is written in terms of $E_t R^H_{t+1}$. However, the repayment is always at the realised rate R^H_{t+1} which forms at $t+1$ and reflects any unexpected disturbances.

The Lagrangian that borrowing firms maximise is:

$$\begin{aligned} L_t &= E_t\left(\left(1-\Gamma^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t K^H_{j,t} + \\ &\quad +\lambda_t\left[E_t\left(\left(\Gamma^H(\bar{\omega}^H_{j,t+1})-\mu G^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t K^H_{j,t} - R_t\left(x^H_t K^H_{j,t} - N^H_{j,t}\right)\right] \end{aligned}$$

The first order conditions for the optimal cutoff expected risk $E_t\bar{\omega}^H_{j,t+1}$ is:

$$\begin{aligned} &-E_t\left(\Gamma^H_\omega(\bar{\omega}^H_{j,t+1})R^H_{t+1}\right)x^H_t K^H_{j,t} + \\ &+\lambda_t\left[\left(\left(\Gamma^H_\omega(\bar{\omega}^H_{j,t+1})-\mu G^H_\omega(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t K^H_{j,t}\right] = 0 \end{aligned}$$

where the subscript ωdenotes that the derivative is with respect to $\bar{\omega}^H_{j,t+1}$.

The first order condition for the optimal capital $K^H_{j,t}$ is:

$$E_t\left(\left(1-\Gamma^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t+$$
$$+\lambda_t\left[E_t\left(\left(\Gamma^H(\bar{\omega}^H_{j,t+1})-\mu G^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t-R_tx^H_t\right]=0$$

The two first order conditions solve for an optimal cutoff $E_t\bar{\omega}^H_{j,t+1}$. The Lagrange multiplier λ_t measures the shadow value of an extra unit of net worth to borrowers. The shadow price is than unity since the loan repayment rate $E_tR^H_{t+1}$ exceeds the risk-free rate R_t. This can be best seen in the expression for the Lagrange multiplier:

$$\lambda_t(\bar{\omega}^H_{j,t+1})=E_t\left(\frac{\Gamma^H_\omega(\bar{\omega}^H_{j,t+1})}{\Gamma^H_\omega(\bar{\omega}^H_{j,t+1})-\mu G^H_\omega(\bar{\omega}^H_{j,t+1})}\right)>1 \tag{1.3.9}$$

It is obvious that the shadow value of own funds is larger than 1 unless the monitoring cost is zero.

The first order condition for capital $K^H_{j,t}$ states that the revenue of buying a marginal unit of capital $K^H_{j,t}$ to the firm, $E_t\left(\left(1-\Gamma^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t$, equals the fall in lenders' profit multiplied by the shadow value of own funds λ_t. The profit of the housing manufacturer falls because, ceteris paribus, the marginal unit of lending earns the lender's share of the returns on capital $E_t\left(\left(\Gamma^H(\bar{\omega}^H_{j,t+1})-\mu G^H(\bar{\omega}^H_{j,t+1})\right)R^H_{t+1}\right)x^H_t$ which is smaller than the cost of purchasing capital $E_tR^H_{t+1}x^H_t$. To compensate for the monitoring cost, investors require the aggregate return to the investment R^H to be bigger than the risk-free rate R, as suggested by the other first order condition:

$$\frac{E_tR^H_{t+1}}{R_t}=E_t\left(\frac{\lambda_t(\bar{\omega}^H_{j,t+1})}{\left(1-\Gamma^H(\bar{\omega}^H_{j,t+1})\right)+\lambda_t(\bar{\omega}^H_{j,t+1})\left(\Gamma^H(\bar{\omega}^H_{j,t+1})-\mu G^H(\bar{\omega}^H_{j,t+1})\right)}\right) \tag{1.3.10}$$

The combined first order condition is independent of the stock of housing capital $K^H_{j,t}$ since the monitoring fee μ is a constant fraction of the investors' return from the loan contract. As a result, both the marginal rate of substitution in the constant expected profits line (the first order condition for $E_t\bar{\omega}^H_{j,t+1}$) and the slope of the participation constraint are linear in the desired expenditure on capital. A rise in the capital stock increases the slopes of these equations in an offsetting way, so the expected cutoff value is independent of the desired expenditure on capital. Hence a higher repayment rate $E_tR^H_{t+1}$ implies a larger probability of default for the same value of the capital stock.

Eliminating the Lagrange multiplier λ_t, the risk premium defined as the ratio of the loan borrowing rate and the risk-free rate $\frac{E_t R^H_{t+1}}{R_t}$ becomes:

$$\frac{E_t R^H_{t+1}}{R_t} = E_t \left(\frac{\frac{\Gamma^H_\omega(\bar{\omega}^H_{j,t+1})}{\Gamma^H_\omega(\bar{\omega}^H_{j,t+1}) - \mu G^H_\omega(\bar{\omega}^H_{j,t+1})}}{\left(1 - \Gamma^H(\bar{\omega}^H_{j,t+1})\right) + \frac{\Gamma^H_\omega(\bar{\omega}^H_{j,t+1})}{\Gamma^H_\omega(\bar{\omega}^H_{j,t+1}) - \mu G^H_\omega(\bar{\omega}^H_{j,t+1})} \left(\Gamma^H(\bar{\omega}^H_{j,t+1}) - \mu G^H(\bar{\omega}^H_{j,t+1})\right)} \right) \tag{1.3.11}$$

This is the first order equation that determines the optimal level of the cutoff risk $\bar{\omega}^H_j$. Again, the right hand side of the first order condition will be greater than 1 unless there is no monitoring fee. The presence of a positive monitoring fee implies that the cost of external financing $E_t R^H_{t+1}$ is higher in the presence of credit constraints than in their absence where it is R_t.

Since equation (1.3.11) holds for every borrower, it is possible to aggregate across all housing producers. The external finance premium on the right hand side is determined macroeconomically. This equation implies that every borrower will choose the same expected cutoff value, $E_t \bar{\omega}^H_{j,t+1}$, regardless of its net worth so that $E_t \bar{\omega}^H_{j,t+1} = E_t \bar{\omega}^H_{t+1}$, for all j. Hence (1.3.11) holds on the aggregate level as well and can be replaced by the following equation for the economy as a whole:

$$\frac{E_t R^H_{t+1}}{R_t} = E_t \left(\frac{\frac{\Gamma^H_\omega(\bar{\omega}^H_{t+1})}{\Gamma^H_\omega(\bar{\omega}^H_{t+1}) - \mu G^H_\omega(\bar{\omega}^H_{t+1})}}{\left(1 - \Gamma^H(\bar{\omega}^H_{t+1})\right) + \frac{\Gamma^H_\omega(\bar{\omega}^H_{t+1})}{\Gamma^H_\omega(\bar{\omega}^H_{t+1}) - \mu G^H_\omega(\bar{\omega}^H_{t+1})} \left(\Gamma^H(\bar{\omega}^H_{t+1}) - \mu G^H(\bar{\omega}^H_{t+1})\right)} \right) \tag{1.3.12}$$

Similarly, the aggregate amount of housing manufacturers' net worth in the economy is $N^H_t = \sum N^H_{j,t}$ and the aggregate amount of capital purchased by housing producers is $K^H_t = \sum K^H_{j,t}$.

Note that although firms maximise the loan contract subject to the expected participation constraint of investors (1.3.7), the model implements the realised participation constraint:

$$\left(\Gamma^H(\bar{\omega}^H_t) - \mu^H G^H(\bar{\omega}^H_t)\right) R^H_t x^H_{t-1} K^H_{t-1} = R_{t-1} \left(x^H_{t-1} K^H_{t-1} - N^H_{t-1}\right) \tag{1.3.13}$$

The first order condition (1.3.12) and the participation constraint (1.3.7) hold at any point in time in which there had not been any unanticipated shocks. However, even after a shock, the amount borrowed from investors is already fixed so lenders must continue to receive the risk-free rate R_t. Hence the realised participation constraint along with the first order condition determine the realised cutoff value $E_t\bar{\omega}^H_{t+1}$.

A positive productivity shock leads to a reduction in the price of housing p_t. If the price decrease is proportionally less than the output expansion, the profits of credit-constrained producers rise. Higher profits imply increased capital demand so the price of housing capital x^H_t increases. A rise in the asset price increases the net worth of housing manufacturers. The higher price of capital also raises the realised ex post return on capital R^H_{t+1} relative to the expected rate $E_t R^H_{t+1}$formed in the previous period. Since investors get repayment from borrowers at the same realised rate R^H_{t+1}, they see the return on their loan increasing. A higher return would imply that lenders receive a higher share relative to the risk they are exposed to which would violate the participation constraint that requires that in any possible state investors get a return equal to the risk-free return of the same investment elsewhere. Hence to satisfy the participation constraint, the investors increase lending to housing manufacturers thus shifting the participation constraint to the right (Figure 1.4). However, in the first period when the unexpected shock occurs, the supply of capital is predetermined so the equilibrium moves from point A to point B in Figure 1.4. At this point, the housing manufacturers adjust to a lower cutoff risk $\bar{\omega}^H_t$ in order to satisfy the new participation constraint of investors. In the periods following the shock, capital demand gradually increases and the equilibrium moves up along the participation constraint to the long-run equilibrium at point C. The adjustment completes when housing manufacturers return to the pre-shock cutoff risk $\bar{\omega}^H_t$ but with a higher profit-maximizing level of capital.

In this loan contract, it is the borrowers who optimise subject to satisfying the participation constraint of lenders. This is due to the fact that there is informational asymmetry regarding the idiosyncratic shock: borrowers know their shock realization but lenders do not observe it freely. Borrowers have superior information about their own financial status since they can observe the idiosyncratic shock costlessly, while the lenders have to pay a cost μ to learn it. Due to this informational superiority, the borrowers can earn positive profits, pushing the lenders, who have inferior information, to their participation constraint. In other words, borrowers choose the optimal combination of $(K^H_t,\ E_t\bar{\omega}^H_{t+1})$

while lenders only determine the state-contingent repayment rate $E_t R^H_{t+1}$ that corresponds to the choice of borrowers.

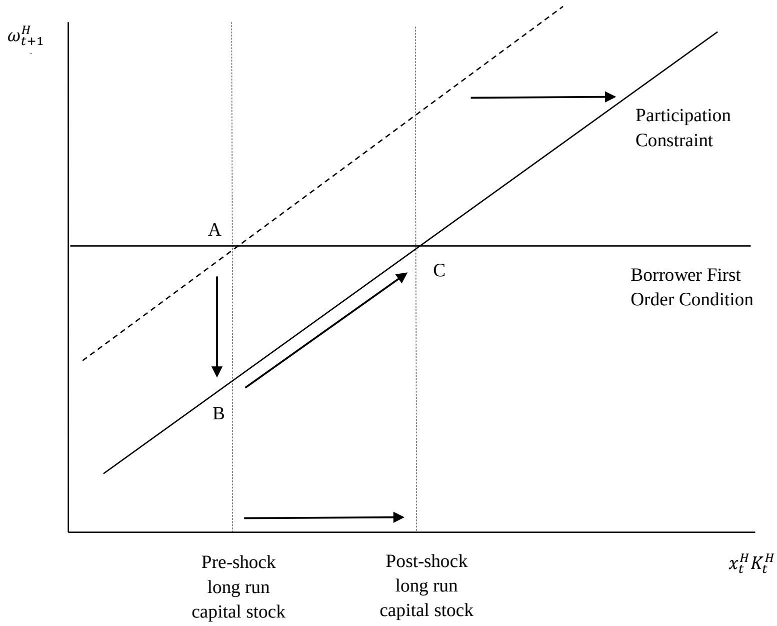


Figure 1.4: Shock Adjustment in the Loan Contract

It is worthwhile to note that although at the time of the optimization t lenders and borrowers only know $E_t R^H_{t+1}$, the rate at which the loan is repaid is the realised return on housing capital R^H_{t+1} at time $t+1$. This is due to the fact that the credit contract is an incentive contract. The cutoff rate is state-contingent and depends on the realization of the return on capital. Suppose firms buy capital in period t at the price x^H_t. If a negative productivity shock occurs in the beginning of period $t+1$, it affects the marginal product of capital and the price of capital in the same period. The expectation of the return on capital $E_t(R^H_{t+1})$ has been formed in the previous period and it does not reflect the changed price of capital. If the loan were repaid at the expected return on capital, then the repayment amount would not reflect the fact that the realised return on assets has changed and that the leverage of borrowers $\frac{x^H_t K^H_t}{N^H_t}$ has responded as well. Suppose the new leverage is higher reflecting lower net worth N^H_t. A higher leverage raises the probability of default and investors would require a higher risk premium to compensate for it. The expected return on capital would not reflect this higher risk premium in the period of the shock since it is predetermined so investors will get a repayment that falls short of the new higher risk of lending. The ex-post return R^H_{t+1}, on the other hand, will reflect the higher indebtedness and the new lower net worth so it will jump to widen the risk premium gap. Hence, if the debt is repaid at the expected return on capital, then lenders get a repayment rate that does not match the higher ex-post cutoff risk $\bar{\omega}^H_{t+1}$ they are exposed to. In order to safeguard against this outcome, lenders make their loan contingent on the realised return on capital that reflects the productivity shock at time $t+1$ and is known only at time $t+1$. Neither borrowers nor lenders know what the ex-post return R^H_{t+1} will be at the time the loan is negotiated. Once the unexpected shock realises, then R^H_{t+1} is formed reflecting all respective changes to the value of the collateral and the net worth. Namely, R^H_{t+1} jumps downward following a decrease in the price of capital while $E_t(R^H_{t+1})$ does not reflect this information since the expectation is formed at time t. Hence, lenders must make the loan contingent on the ex-post return on capital R^H_{t+1} in order to break even on their investment and to safeguard against disturbances at the time of the loan repayment that impact the probability of repayment.

When there is no idiosyncratic uncertainty, the expected and actual returns are one and the same. In the case of the share purchase contract between consumption firms and households, there is no idiosyncratic uncertainty about the realization of the idiosyncratic shock to affect the return on capital so the share contract can be negotiated at the expected risk-free return which is also the realised one. On the other hand, when there is uncertainty about the realization of $\bar{\omega}^H$ that cannot be diversified away, lenders require a state-contingent incentive loan contract to ensure a guaranteed return even in cases of default on the part of the borrower. This incentive loan contract is at the heart of the financial frictions setup that sets it apart from a riskless share financing arrangement. Since borrowers bear the risk of the loan return, disturbances that affect the value of their net worth or of their capital stock lead to changes in their leverage that impact their borrowing ability. Hence credit-constrained firms experience a shock both directly through its effect on demand or on output, and indirectly via its impact on the realised return of the loan contract.

Housing firms also earn positive profits and amass net worth. However, their equity reflects the fact that they are charged the markup realised return on their borrowing and that their share of the loan contract depends of the realization of $\bar{\omega}^H$. In any given period, their equity V_t^H is:

$$V_t^H = \left(1 - \Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H \tag{1.3.14}$$

In order to prevent housing firms from growing out of their financial constraints, the model assumes that at the end of every period a constant fraction of housing producers $1 - \nu^H$ exit the market. When they exit, they immediately spend their remaining equity on their own output. Their consumption $C_{E,t}^H$ on exit is:

$$C_{E,t}^H = \left(1 - \nu^H\right)\left(1 - \Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H \tag{1.3.15}$$

Housing firms also need to start off with some net worth so they provide inelastically one unit of labour to the production of their respective output. Their income from labour is w_t^H. The weight of the producers labour supply in the total labour in each sector is small so that it does not distort the labour supply of households. The net worth of housing firms is the sum of the equity of surviving housing producers and their income from labour:

$$N_t^H = \nu^H V_t^H + w_t^H \tag{1.3.16}$$

1.4 Complete Model with Housing as a One-Period Good

This chapter describes the rest of the economy in which the financial contracts take place. The economy consists of two production sectors: consumption and housing. Both sectors have capital firms that manufacture sector-specific capital used in the production of the final good. Final goods firms in each sector use labour, sector-specific capital and own technology to produce their output. Households consume both consumption and housing.

1.4.1 Consumption Capital Sector

Firms that produce capital K_t^F for the consumption sector own technology that converts goods into capital. They use depreciated capital which they purchase from final good firms in the consumption sector and make investments to produce new capital. The investment I_t^F is consumption. The newly produced capital is sold back to consumption producing firms.

There are standard quadratic adjustment costs to producing capital. These capital adjustment costs are:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F \tag{1.4.1}$$

The function J is such that $J' > 0$ and $J" < 0$. New capital is produced within the period and sold to final good producing firms at the price x_t^F. The profit maximization problem of capital producing firms is:

$$max\; x_t^F K_t^F - (1-\delta)x_t^F K_{t-1}^F - I_t^F = J\left(\frac{I_t^F}{K_{t-1}^F}\right)x_t^F K_{t-1}^F - I_t^F$$

Solving the profit maximization gives the optimal condition for investment:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1 \tag{1.4.2}$$

1.4.2 Consumption Producers

Firms in the consumption sector use capital K_t^F, labour L_t^F and sector-specific technology A_t^F to produce their output. Consumption producing firms have to buy capital one period earlier. They borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. In order to do so, these firms issue claims to consumers (via investors who facilitate the share arrangement) at the prevailing price of consumption capital x_t^F. At the end of each production period, they sell the remaining capital minus depreciation back to capital producing firms. The production function of consumption firms is:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_t^F)^{(1-\alpha_F)} \tag{1.4.3}$$

According to the share purchase setup, consumption firms supply inelastically one unit of labour to their own production in order to start the accumulation of their net worth. Factoring in this labour supply in the production function, the total labour supply in the consumption sector is:

$$L_t^F = (L_{F,t}^F)^{\Omega_F}(L_{H,t}^F)^{(1-\Omega_F)} = (L_{H,t}^F)^{(1-\Omega_F)} \tag{1.4.4}$$

where $L_{H,t}^F$ is the regular labour supply by consumers and $L_{F,t}^F$ is the labour supply by consumption producers.

Recasting the production function only in terms of household labour yields:

$$Y_t^F = A_t^F(K_{t-1}^F)^{\alpha_F}(L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)} \tag{1.4.5}$$

The firms in the sector are perfectly competitive so they maximise profits subject to input costs. The price of consumption is normalized to 1. The Lagrangian is:

$$\mathcal{L}_t^F = A_t^F(K_{t-1}^F)^{\alpha_F}(L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)} + (1-\delta)x_t^F K_{t-1}^F - w_t L_{H,t}^F - R_{t-1}x_{t-1}^F K_{t-1}^F$$

The two first-order conditions for optimal capital and labour are:

$$w_t = (1-\alpha_F)(1-\Omega_F)\left(\frac{Y_t^F}{L_{H,t}^F}\right) \tag{1.4.6}$$

$$R_t = \frac{E_t\left(\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F\right)}{x_t^F} \tag{1.4.7}$$

The wage consumption firms receive for their labour supply is:

$$w_t^F = (1-\alpha_F)\,\Omega_F Y_t^F \tag{1.4.8}$$

1.4.3 Housing Capital Sector

Firms that produce capital K_t^H for the housing sector own technology that converts final goods into capital. They use depreciated capital which they purchase from housing firms and make investments to produce new capital. The investment I_t^H is consumption. The newly produced capital is sold back to housing producers.

Housing capital is subject to the same adjustment costs as consumption capital. The housing capital production equation is:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H \tag{1.4.9}$$

New capital is produced within the period and sold to final good producing firms at the price x_t^H. The profit maximization problem of housing capital firms is:

$$max\ x_t^H K_t^H - (1-\delta)x_t^H K_{t-1}^H - I_t^H = J\left(\frac{I_t^H}{K_{t-1}^H}\right)x_t^H K_{t-1}^H - I_t^H$$

The profit maximization problem of capital producing firms solves for the first-order condition for investment in housing capital:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1 \tag{1.4.10}$$

1.4.4 Housing Producers

Housing manufacturers use housing capital K_t^H along with undifferentiated labour L_t^H and sector-specific technology A_t^H to produce housing. The equilibrium price of housing is p_t. Housing manufacturers must buy capital one period earlier. They borrow funds for the purchase of capital at the markup borrowing rate R_t^H since they are credit-constrained. At the end of each production period, they sell the remaining capital minus depreciation back to capital producers. The production function of housing firms is:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_t^H)^{(1-\alpha_H)} \tag{1.4.11}$$

Factoring in that fact that housing producers supply inelastically labour to their own production, the total labour supply in the production of housing is:

$$L_t^H = (L_{F,t}^H)^{\Omega_H} (L_{H,t}^H)^{(1-\Omega_H)} = (L_{H,t}^H)^{(1-\Omega_H)} \tag{1.4.12}$$

where $L_{H,t}^H$ is the labour supply by consumers and $L_{F,t}^H$ is the labour supply by housing producers.

The production function in terms of household labour becomes:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H)(1-\Omega_H)} \tag{1.4.13}$$

The firms maximise their profits subject to the input costs. The Lagrangian is:

$$\mathcal{L}_t^H = p_t^H A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H)(1-\Omega_H)} + (1-\delta) x_t^H K_{t-1}^H - R_t^H x_{t-1}^H K_{t-1}^H - w_t L_{H,t}^H$$

The first-order conditions for housing manufacturers yield:

$$w_t = (1-\alpha_H)(1-\Omega_H) \left(\frac{p_t Y_t^H}{L_{H,t}^H} \right) \tag{1.4.14}$$

$$R_t^H = \frac{\alpha_H \frac{p_t Y_t^H}{K_{t-1}^H} + (1-\delta) x_t^H}{x_{t-1}^H} \tag{1.4.15}$$

It is obvious from equation (1.4.15) that the cost of capital is higher in an economy with financial frictions than in one without. The loan contract already established that due to the monitoring fee, the cost of external financing R_t^H in an economy with credit constraints is higher than in a one without, where it is the risk-free rate R_t. Furthermore, as K_t^H rises, R_t^H increases at a higher rate than R_t since the risk premium $\frac{E_t R_{t+1}^H}{R_t}$ reflects an increase in the probability of default $\bar{\omega}^H$. Hence in an economy with idiosyncratic uncertainty about the realization of $\bar{\omega}^H$, the capital financing line is steeper and to the left of the one in an economy with no uncertainty about the realization of the idiosyncratic shock (Figure 1.5). Ceteris paribus, an economy with financial frictions has less capital that is financed at a higher price than an economy without idiosyncratic risk. This is is also easy to see algebraically by reworking the first order condition for capital (1.4.15) in terms of K_t^H:

$$K_{t-1}^H = \frac{\alpha_H p_t Y_t^H}{R_t^H x_{t-1}^H - (1-\delta) x_t^H} < \frac{\alpha_H p_t Y_t^H}{R_{t-1} x_{t-1}^H - (1-\delta) x_t^H}$$

For a given housing price p_t, output Y_t^H and price of capital x_t^H, a higher loan repayment rate R_t^H implies a lower equilibrium capital stock K_{t-1}^H. If borrowers want to raise their capital stock, they must pay a higher contractual interest rate to investors and incur a higher value of $\bar{\omega}_t^H$.

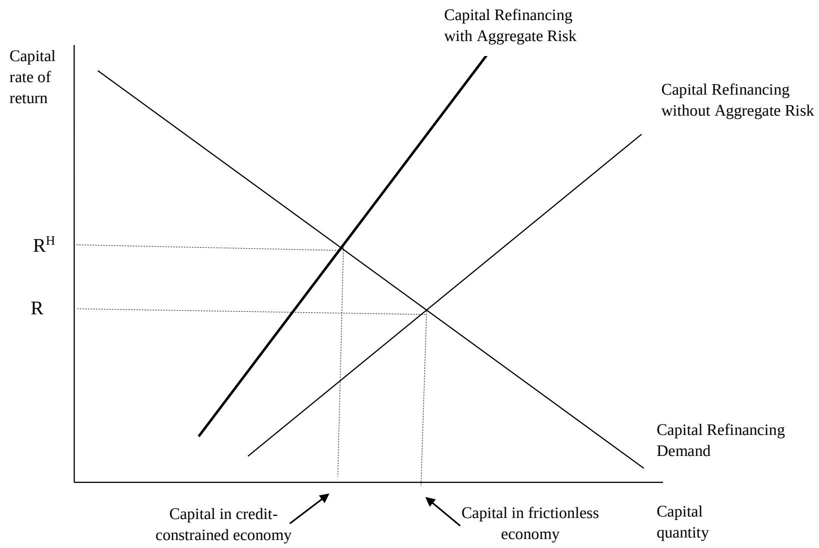


Figure 1.5: Steady State Capital in a Riskless and a Credit-Constrained Economy

In the first order condition for housing capital the price of capital enters with a lag. This is due to the fact that the return rate of capital is state-contingent, as discussed in the loan contract section, and the capital purchase is based on the post-shock realization of R_t^H. The corresponding equation for consumption producers has no lags since they borrow at the risk-free rate which is non-state-contingent. Since capital is optimised intertemporally, the return on capital R_t^H depends on the purchasing price of capital at the time of loan agreement and on the selling price of capital at the time of the loan repayment. In this context, the realised return on capital R_t^H depends on the past purchasing price x_{t-1}^H and on the current selling price x_t^H of capital. In contrast, the expected return on capital $E_t(R_{t+1}^H)$ depends on the current purchasing price of capital and on the expected future selling price which is unknown at the time of decision-making:

$$E_t(R_{t+1}^H) = \frac{\alpha_C p_{t+1} \frac{Y_{t+1}^H}{K_t^H} + (1-\delta)x_{t+1}^H}{x_t^H} \tag{1.4.16}$$

However, the loan contract is state-contingent so the expected return on capital has no bearing on the capital purchasing decisions of housing manufacturers. Equation (1.4.15) that relates the asset purchase to the ex-post return on capital is the one used in a credit-constrained economy.

Finally, the wage housing producers receive for their labour services is:

$$w_t^H = (1 - \alpha_H)\, \Omega_H p_t Y_t^H \tag{1.4.17}$$

1.4.5 Consumers

Households choose consumption C_t, housing H_t and labour L_t subject to their budget constraint. Each household seeks to maximise its lifetime expected utility:

$$U = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, H_t, L_t) \tag{1.4.18}$$

Households have a period utility function given by:

$$U(C_t,\ H_t,\ L_t) = logC_t + \kappa logH_t - \gamma \frac{L_t^{(1+\varphi)}}{1+\varphi} \tag{1.4.19}$$

The period utility function is separable in consumption C_t, housing H_t and labour L_t. Housing enters the utility function additively along with consumption. This allows to study the role of housing demand on consumer behaviour directly and not as a part of a consumption aggregator. There is a taste parameter κ that reflects the relative preference for consumption and housing.

Consumers purchase consumption and housing. Each period, they lend an amount B_t to investors at the risk-free rate. Their lending essentially provides funds for the loan to credit-constrained housing firms as well as finances the capital purchase of consumption firms. Consumers also absorb the profits Π_t of both capital sectors. The budget constraint of households is:

$$C_t + p_t H_t + B_t = w_t L_t + R_{t-1} B_{t-1} + \Pi_t \tag{1.4.20}$$

Consumers maximise their utility function subject to this budget constraint. The left hand side reflects their consumption and housing purchases as well as their lending, while the right hand side represents their income from wages and from reselling the non-depreciated housing from the previous period on the market as well as their returns from lending and from capital firms profits. The Lagrangian for consumers is:

$$\begin{aligned}\mathcal{L}_t &= E_t \sum_{t=0}^{\infty} \beta^t \left[logC_t + \kappa logH_t - \gamma \frac{L_t^{(1+\varphi)}}{1+\varphi} \right] + \\ &+ \lambda_t \left(w_t L_t + R_{t-1} B_{t-1} + \Pi_t - C_t - p_t H_t - B_t \right)\end{aligned}$$

The optimization problem yields three first-order conditions for consumption, housing and leisure. The first-order condition for the consumption-labour tradeoff is fairly standard:

$$\gamma L_t{}^{\varphi} C_t = w_t \tag{1.4.21}$$

The tradeoff between consumption and housing depends on the equilibrium price of housing and the taste parameter:

$$p_t H_t = \kappa C_t \tag{1.4.22}$$

Consumers also have a standard Euler equation:

$$E_t \left(\frac{C_{t+1}}{C_t} \right) = \beta R_t \tag{1.4.23}$$

1.4.6 Market Clearing

Market clearing requires that the output of consumption must cover household consumption and consumption by conventional good producers on exit as well as investment in the two sectors:

$$Y_t^F = C_t + C_{E,t}^F + C_{E,t}^C + I_t^F + I_t^H \tag{1.4.24}$$

Housing production must meet the housing needs of consumers as well as the consumption of housing firms on exit and the monitoring costs of the loan contract:

$$p_t Y_t^H = p_t H_t + C_{E,t}^H + \mu^H G^H(\bar{\omega}_t^H) R_t^H x_{t-1}^H K_{t-1}^H \tag{1.4.25}$$

Household labour supply equals the demand by housing and consumption firms:

$$L_{H,t}^F + L_{H,t}^H = L_t \tag{1.4.26}$$

Finally, consumer lending must equal the loan to housing firms as well as the share purchase of consumption firms:

$$B_t = x_t^H K_t^H - N_t^H + x_t^F K_t^F - N_t^F \tag{1.4.27}$$

The equations for the complete model are listed in Appendix 1.A. The results provide a comparison of the outcomes of the two-sector model with credit constraints and the one-sector version of Bernanke, Gertler and Gilchrist (1999). The one sector model is described in detail in Appendix 1.E.

1.4.7 Model Calibration

The calibrated model is quarterly so four periods correspond to a year. The model parameters are chosen to be as close as possible to standard literature values while reflecting a number of key model features. The parameters governing the loan contract are calibrated to match those in Bernanke, Gertler and Gilchrist (1999) and satisfy several steady state conditions:

1. The steady state rate of the external risk premium is 0.5%. (Bernanke, Gertler and Gilchrist, 1999).

2. The steady state housing firm leverage, i.e. value of housing capital stock to net worth ratio is $\frac{x^H K^H}{N^H} = 2$ (Bernanke, Gertler and Gilchrist, 1999).

3. The failure rate of housing manufacturers $F^H(\bar{\omega}^H)$ (i.e. the number of firms that exit the market each period) is 2% (Bernanke, Gertler and Gilchrist, 1999).

The remaining model parameters are chosen to be as close as possible to a representative two-sector economy. (Table 1.1). Both sectors are identical in their production function. Since housing and consumption enter the utility function additively, the parameter κ reflects consumer relative preferences for the two good as well as calibration choices (in this case, both sectors are of equal size to eliminate any sector dominance). Housing depreciates at the same rate as capital, δ. For the temporary volatility shock, the persistence parameter ϱ has a value of 0.95, a conventional choice for the frequency of the model.

Table 1.1: General Parameters in the Consumer Mortgage Contract Economy

Parameter	α_F	α_H	β	γ	φ	κ	J"	δ	Ω_i
Value	0.35	0.35	0.99	0.03	0.33	2.04	-10	0.0025	0.01

To obtain the deterministic steady state, the loan contract is first solved so that it satisfies the aforementioned three conditions. The solution of the first order condition of housing manufacturers, the participation constraint and the cumulative distribution function $F^H(\bar{\omega}^H)$ solve for the monitoring fee μ and the fraction of exiting housing firms ν^H as well as the steady state cutoff value of the idiosyncratic shock $\bar{\omega}^H$. Then the rest of the model is solved for the steady state taking the loan contract solution as given.

The credit contract parameters and steady state values are given in Table 1.2. The steady state values of the macroeconomic variables for the two-sector credit-constrained model are in Table 1.3. For comparison reasons, the results are presented along with the classic one-sector model of Bernanke, Gertler and Gilchrist (1999). This one-sector model follows all specifications from the original paper except that the only production sector, housing, is calibrated to match the size of the housing sector in the two-sector economy (Table 1.4). The production function of housing is identical, but in the one-sector model investment is the own good, so in the Bernanke, Gertler and Gilchrist (1999) version households consume less housing than the two-sector economy.

Table 1.2: Steady State Values of the Two-Sector Economy Loan Contract

Variable	R	R^H	$\bar{\omega}^H$	σ	μ					
Value	1.01	1.015	0.50	0.314	0.06					
Variable	Γ^H	G^H	Γ^H_ω	G^H_ω	Γ^H_σ	G^H_σ	$\Gamma^H_{\omega\omega}$	$G^H_{\omega\omega}$	$\Gamma^H_{\omega\sigma}$	$G^H_{\omega\sigma}$
Value	0.49	0.009	0.98	0.154	-0.02	0.16	-0.31	2.02	-0.36	1.89

Table 1.3: Steady State Values in the Two-Sector Economy with Financial Frictions

Variable	Y^F	Y^H	C	H	K^F	K^H	L^F	L^H	I^F	I^H
Value	100	100	46.5	90.6	997	915	29.9	31.4	24.9	22.9
Variable	N^H	C_E^H	N^F	C_E^F	R	R^H	w	p	ν^H	ν^F
Value	457	9.41	498	5.68	1.01	1.015	2.15	1.05	0.98	0.99

Table 1.4: Steady State Values in the One-Sector Economy with Financial Frictions

Variable	Y^H	H	K^H	L^H	I^H	N^H	C_E^H	w	ν^H	R^H
Value	100	68.8	873	32.2	21.8	436	8.98	2.00	0.98	1.015

The log linearization of the two-sector model is described in Appendix 1.B. The one-sector model is described in Appendix 1.E. The derivations of the log-normal distribution of $\bar{\omega}^H$ are in Appendix 1.G.

1.5 Single-Period Housing Model Results

This section demonstrates the effect of two disturbances - a permanent technological innovation in the production of housing and a temporary improvement in the borrowing conditions for the credit-constrained sector. The oversupply of housing is modelled as a 10% technological innovation in the production of housing. This stylistically simple setup allows generating excess supply of houses without unnecessarily complicating the model with discussions of the preceding housing bubble. Nevertheless, this approach successfully captures the housing inventory buildup that amassed during the boom. This disturbance is simulated as a permanent shock since the excess housing availability began at the peak of the asset bubble and continued during the subsequent recession as the housing demand shrank (Duke, 2012; Madigan, 2012).

The relaxation of borrowing conditions is introduced as a 10% reduction in the volatility of the default risk that made loans seem safer to lenders. The easing permitted borrowers to secure loans with less collateral and as a result to increase their debt holdings. The reduction in financial volatility is a temporary shock corresponding to the brief period of lax lending practises that prevailed on the cusp of the subprime crisis. There are empirical and theoretical underpinnings for the temporary nature of the disturbance. Empirically, the lax access to credit was quickly reversed as the proliferation of defaults signalled the beginning of an economic downturn (Dennis, 2010; Duke, 2012). Theoretically, the model assumes that lenders update the loan terms after observing the borrower risk. A permanent shock would be at odds with the microfounded nature of the loan contract and would imply that the loan terms remain relaxed despite the worsened bankruptcy prospects of borrowers following a leverage buildup.

1.5.1 The Housing Oversupply

A positive technological innovation raises output in the housing sector but the equilibrium price falls to accommodate the excess supply leaving the profits of firms unchanged. In the absence of changes in firm returns, their capital demand remains the same and the financial position of borrowers is unaffected. This is in contrast to the one-sector model where the output expansion raises the returns of housing firms and improves their financial position. In a two-sector model, price effects may render credit constraints irrelevant for the performance of the economy. Consumers, however, can enjoy housing at a lower price.

A permanent 10% positive technological shock in the production of housing augments total productivity in the housing sector (Figure 1.6). The innovation raises the output of the housing industry for the same amount of inputs (Figure 1.6.2, solid line). Due to the Cobb-Douglas nature of the utility function, household spending on housing is unchanged. Hence consumers do not immediately absorb the higher output unless housing becomes more affordable. The price of housing falls to accommodate excess supply (Figure 1.6.14). The price decrease exactly offsets the output rise so the profits of the housing producers remain unchanged. This is a feature of the unity elasticity of demand exhibited by the Cobb-Douglas utility function. If the elasticity of demand were greater of 1, then the price of housing may fall proportionally less that the increase in supply affecting the returns of housing firms. This is not, however, an argument against the Cobb-Douglas utility function. It is standard in DGSE models and replacing it with a different utility function may lead to fringe results. Furthermore, frictions caused by the utility function are too small to be the driver of aggregate disturbances. Finally, as the version with multi-period housing and Chapter 2 demonstrate, the Cobb-Douglas utility function does not prevent credit constraints from affecting the economy in more sophisticated models.

Since in this version housing firms see no change in their returns, their demand for housing capital does not respond to the technological shock (Figure 1.6.6). Capital demand remains unchanged so both the risk-free rate R_t and the markup interest rate R_t^H are unaffected (Figures 1.6.16 and 1.6.21). The debt position of the housing firms remains the same and the leverage does not respond. In this two-sector version of financial constraints, a positive technological shock has no consequences for the returns of indebted firms and for their debt position. The response to the innovation is entirely contained in the housing sector without affecting the final good producers. The only beneficiaries to the shock are consumers who now enjoy more housing at a lower price (Figure 1.6.4).

The response of a two-sector economy to a technological advancement is very different from that of a one-sector economy. In the Bernanke, Gertler and Gilchrist (1999) version of the model, there is no offsetting price adjustments since there is only one final good so producers experience a one-for-one rise in profits when output increases (Figure 1.6.2, dotted line). Higher returns stimulate the demand for capital, which in turn raises the price of capital and the return to capital, the markup rate R_t^H (Figures 1.6.6, 1.6.13 and 1.6.21). The value of the loan increases by the capital price x_t^H but the net worth of borrowing firms goes up by both the price of capital and the return to capital R_t^H. The improvement in the net worth of housing firms is proportionally larger than the value of their housing stock so their leverage diminishes (Figures 1.6.17 and 1.6.23). Lower leverage signals reduced bankruptcy prospects so lenders require a lower risk premium on new loans thus implying the risk premium moves anti-cyclically (Figures 1.6.22 and 1.6.24). The lower risk premium further stimulates capital acquisition resulting in a larger output expansion than generated only by the technological innovation (Figure 1.6.2). This is the "financial accelerator" effect documented by Bernanke, Gertler and Gilchrist (1999).

The reason that the same welfare improvement is not observed in the two-sector version is that price effects negate any gains from the innovation. In the economy of Bernanke, Gertler and Gilchrist (1999) there is only one good so firm profits do not depend on price adjustments. The producers see a one-for-one improvement in their returns which prompt them to raise their capital demand. In contrast, their two-sector counterparts see any potential gains from the output increase obliterated by the drop in the equilibrium price that leaves profits unchanged. Hence the amplifying effect of financial frictions documented by Bernanke, Gertler and Gilchrist (1999) disappears in the two-sector model due to Fisher-type price effects.

The effects of this disturbance do not come to approximate the consequences of the housing recession, especially not for consumers. Not only does the shock fail to trigger any changes in the financial market, it also leaves consumers better off enjoying more housing at a lower price. In contrast, during the subprime crisis consumers experienced a devaluation of their housing that constrained their mortgage-financing abilities.

1.5.2 The Temporary Financial Relaxation

A temporary credit easing in the capital financing market allows borrowing housing firms to obtain larger loans with the same collateral so their leverage worsens. Their indebtedness rises implying increased default likelihood so subsequent loans attract higher risk premium. Borrowing conditions quickly deteriorate leaving housing manufacturers with reduced access to credit for a prolonged time despite the temporary nature of the initial easing.

The relaxation of borrowing conditions is modelled as a negative temporary shock to σ^2, the variance of the borrower default risk ω^H. A lower variance implies less volatility in the bankruptcy prospects, ceteris paribus, and can signal safer loans (Figure 1.7). Improved access to credit allows borrowers to secure loans with less collateral or to repay their debt at a lower risk premium (Figure 1.7.24). Credit-constrained housing manufacturers experience the lower risk premium as a decreased loan repayment rate R_t^H (Figure 1.7.21). The effects are the same both for the two-sector model and the one-sector Bernanke, Gertler and Gilchrist (1999) version since the disturbance originates in the financial market and is not influenced by price effects.

Relaxed credit conditions allow borrowers to obtain larger loans that they would do otherwise and essentially to increase their leverage. The housing sector raises its demand for capital (Figure 1.7.6), driving up its price (Figure 1.7.13). The higher capital price increases the value of their capital stock and improves their net worth immediately after the shock (Figure 1.7.17). The reduction in the markup rate implies that the increase in the net worth is smaller than the growth in borrowing so the leverage of housing manufacturers gradually rises after the initial period (Figures 1.7.21 and 1.7.23). Higher leverage implies increased probability of bankruptcy despite the initial improvement in the financial position of borrowers (Figure 1.7.22). Seeing the default prospects of debtors rise, lenders require a higher risk premium. The temporary nature of the shock also implies a reversal of the lower external finance premium after the initial disturbance. Both events combine to signal a tightening of the risk premium (Figure 1.7.24) that puts an end to the brief period of easy access to credit.

With their financing access restricted, credit-constrained firms attempt to improve their debt position in order to repair their ability to acquire capital financing. They lower their capital demand after the initial increase and the depressed demand leads to a reduction in the price of capital (Figures 1.7.6 and 1.7.13). The lower value of the capital stock implies a further decrease in the net worth of borrowers so their leverage continues to rise reaching a peak 25 quarters (6 years) after the initial credit easing (Figures 1.7.17 and 1.7.23). The staggered buildup of the indebtedness of housing producers reflects the continued tightening of the risk premium. The risk premium widens because the temporary shock is wearing out and because firm leverage is now higher. Both of these effects imply that the new indebtedness level is even higher relative to the now tighter risk premium. This credit deterioration prompts borrowers to deleverage. However, the deleveraging reduces the value of their net worth due to diminished capital demand. These effects combine to imply a persistently high leverage relative to the newly tightened financing conditions. The vicious circle of deleveraging due to significant indebtedness and the ensuing fall in net worth value takes a long time to unwind despite the short-lived initial credit easing.

Consumers, however, are largely unaffected by the downturn in the housing sector. Since they are only lenders (via investors) in the financial market, they are not vulnerable to the credit easing and they hold no debt. Their demand for housing marginally improves following a fall in the price of housing (Figures 1.7.4 and 1.7.14). The financial recession that befalls housing firms is entirely contained in that sector and unlike the subprime crisis, does not to spread to the consumer sector.

Unlike a productivity shock, an easing of borrowing conditions produces negative outcomes for the credit-constrained sector. The resulting increased leverage and tightened credit access is conductive to a recession for the housing producers. However, consumers are spared from the downturn and even can afford slightly more housing since the equilibrium price goes marginally down due to the credit tightening in the housing sector. Both simulations confirm that the two-sector model with credit constraints does not perform well in approximating the subprime mortgage crisis and especially its consequences for house-buyers. The subsequent section evolves the production of housing closer to empirical observations in an attempt to improve the performance of the model.

Figure 1.6: Positive Technological Innovation in a Two-Sector Model with Financial Frictions and One-Period Housing

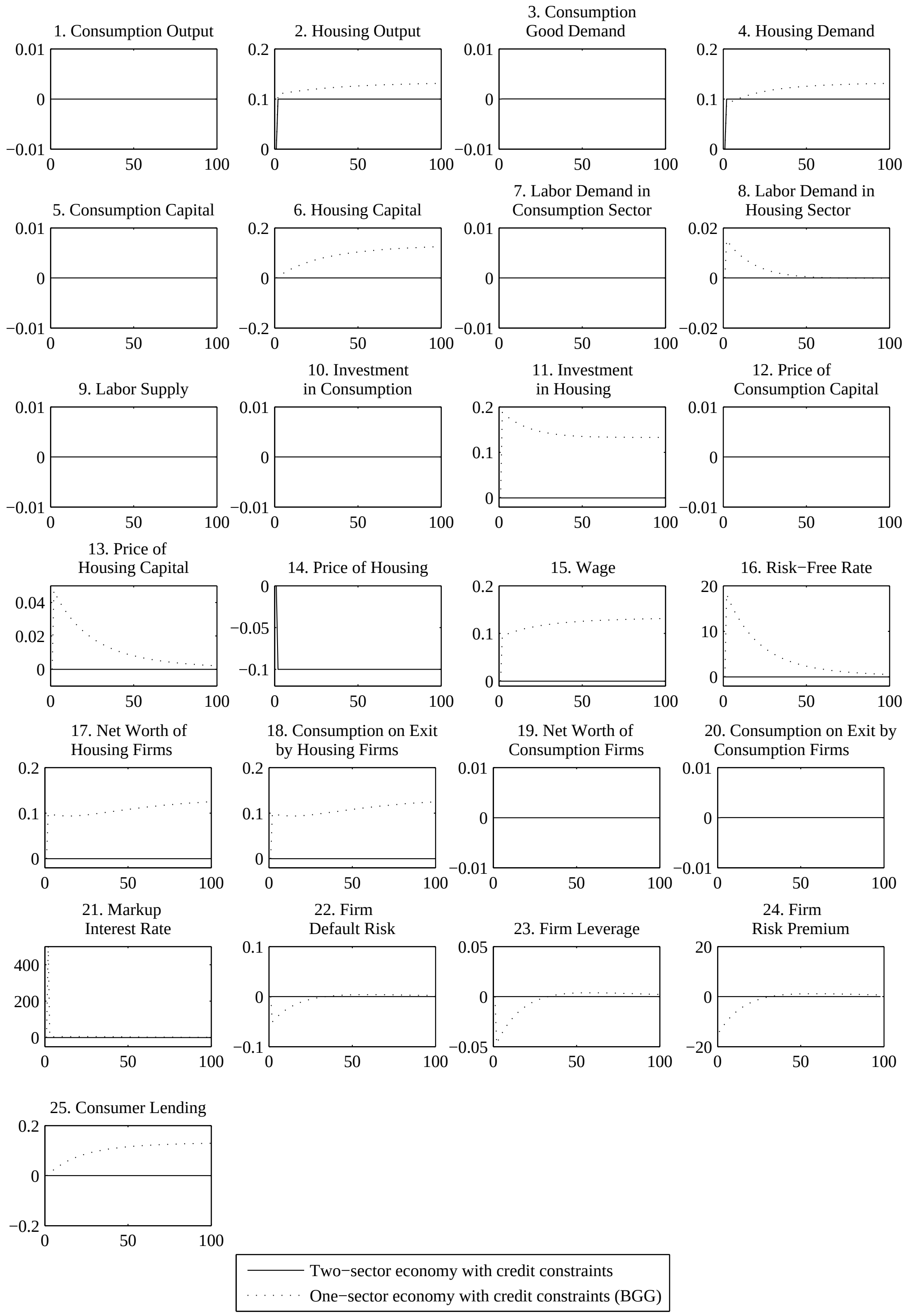
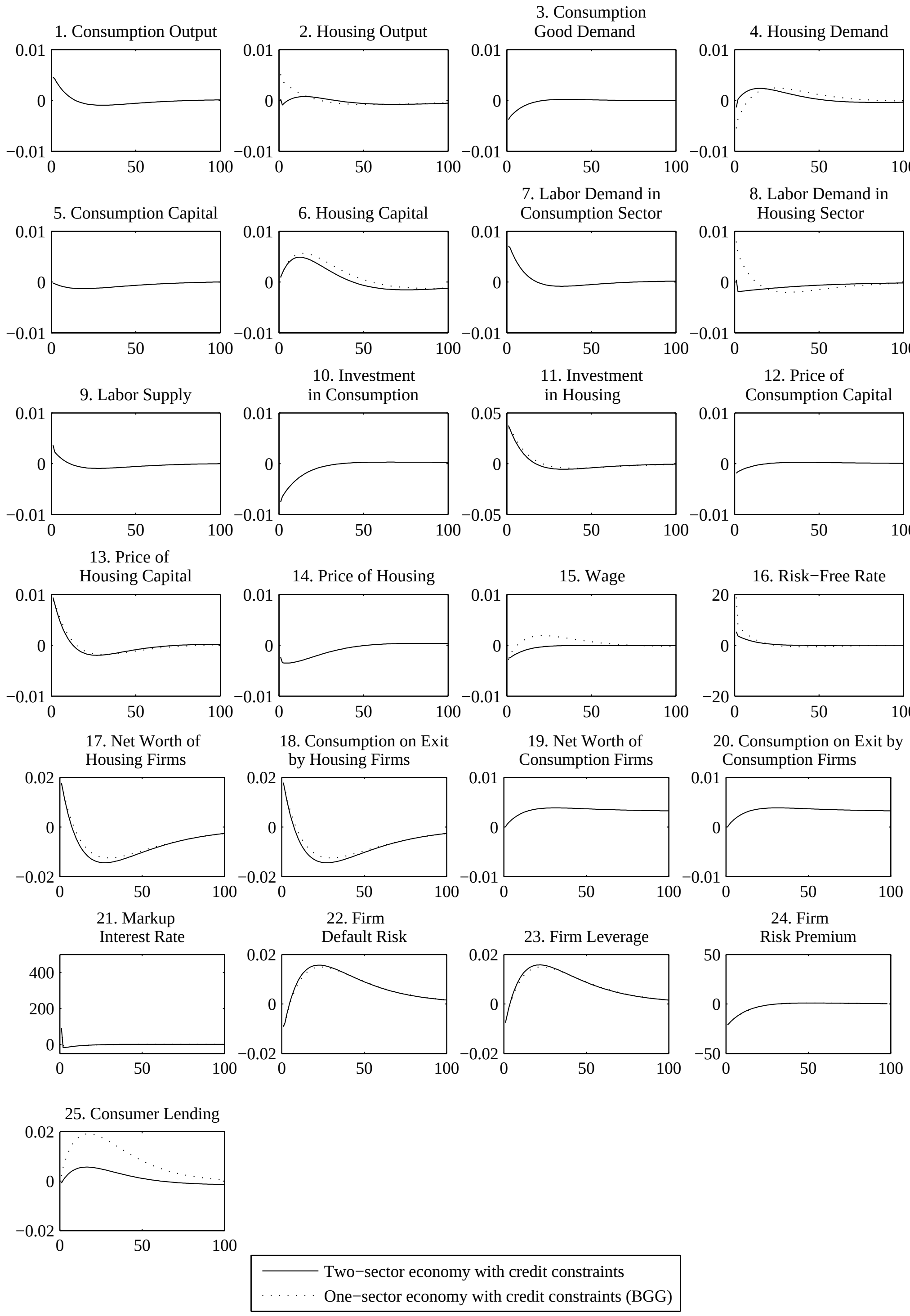


Figure 1.7: Reduced Default Volatility in a Two-Sector Model with Financial Frictions and One-Period Housing



1.6 Complete Model with Housing as a Multi-Period Good

In this version, housing is no longer a simple one-period good but is a durable multi-period good that uses land as well as capital and labour in production. This version follows Iacoviello and Neri (2010) in setting up housing production and brings the model closer to real life real estate development. It also creates opportunities for the financial market to respond to a technological innovation since the fall in the housing price is less than in the previous model and producers see their profits increase thereby stimulating their capital demand. The production of capital as well as the consumption sector remains the same.

1.6.1 Housing Producers

Housing is a multi-period good that survives for more than one period, unlike consumption which is not durable beyond the period in which it is produced. Each period a fraction δ of housing depreciates. The remaining housing is available for use in the subsequent period. Housing manufacture only replaces depreciated housing within the period. The production of housing follows closely Iacoviello and Neri (2010). Housing producers use capital K_t^H, labour L_t^H, land X_t and sector-specific technology A_t^H to produce new houses. Housing firms buy capital one period earlier. They borrow funds for the purchase of capital at the markup borrowing rate R_t^H since they are credit-constrained. At the end of each production period, they sell the remaining capital back to capital firms. The production function of housing firms is:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (X_{t-1})^{\varepsilon} (L_t^H)^{(1-\alpha_H-\varepsilon)} \tag{1.6.1}$$

The amount of land is fixed and normalized to one. Furthermore, the loan contract of credit-constrained housing firms assumed that producers supply inelastically one unit of labour in order to start the accumulation of their net worth. Factoring in that labour supply, the total labour supply in the production of housing by origin is:

$$L_t^H = (L_{F,t}^H)^{\Omega_H} (L_{H,t}^H)^{(1-\Omega_H)} = (L_{H,t}^H)^{(1-\Omega_H)} \tag{1.6.2}$$

Where $L_{H,t}^H$ is the labour supply of consumers and $L_{F,t}^H$ is the labour supply of housing producers.

Recasting the production function only in terms of household labour and factoring in the fixed supply of land yields:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H)} \tag{1.6.3}$$

The price of housing is p_t. Housing firms maximise the following Lagrangian:

$$\mathcal{L}_t^H = p_t A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H)} + (1-\delta) x_t^H K_{t-1}^H - R_t^H x_{t-1}^H K_{t-1}^H - w_t L_{H,t}^H$$

obtaining the optimal conditions for housing capital and labour:

$$w_t = (1-\Omega_H)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right) \tag{1.6.4}$$

$$R_t^H = \frac{\alpha_H \frac{p_t Y_t^H}{K_{t-1}^H} + (1-\delta) x_t^H}{x_{t-1}^H} \tag{1.6.5}$$

The wage housing producers receive for their labour services is:

$$w_t^H = (1-\alpha_H-\varepsilon)\,\Omega_H p_t Y_t^H \tag{1.6.6}$$

1.6.2 Consumers

Like in the previous version, households choose consumption C_t, housing H_t and labour L_t subject to their budget constraint. The utility function remains the same except for the fact that housing is purchased one period in advance and consumed in the subsequent period. This approach matches empirical reality better where acquiring a house involves transactional delays that involve search time, time spent with real estate agents and time to process escrow, payment and home insurance. In Chapter 2 where housing is purchased as part of a mortgage, the transaction period also involves loan approval and mortgage processing. The utility function of consumers is:

$$U = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, H_{t-1}, L_t) \tag{1.6.7}$$

Households have a period utility function given by:

$$U(C_t,\ H_{t-1},\ L_t) = logC_t + \kappa log H_{t-1} - \gamma \frac{L_t^{(1+\varphi)}}{1+\varphi} \tag{1.6.8}$$

The period utility function is separable in consumption C_t, housing H_{t-1} and labour L_t. Housing is purchased at the beginning of the period and consumed in the following period. At the end of the period, the remaining housing minus depreciation is sold back on the market.

Consumers purchase consumption and housing. Each period, they lend an amount B_t to investors at the risk-free rate. Their lending essentially provides funds for the loan to credit-constrained housing firms as well as finances the capital purchase of consumption firms. Consumers also absorb the profits Π_t of both capital sectors. The budget constraint of households is:

$$C_t + p_t H_t + B_t = w_t L_t + (1-\delta) p_t H_{t-1} + R_{t-1} B_{t-1} + \Pi_t \tag{1.6.9}$$

Consumers maximise their utility function subject to this budget constraint. The left hand side reflects their consumption and housing purchases as well as their lending, while the right hand side represents their income from wages and from reselling the non-depreciated housing from the previous period as well as loan repayment and capital firms profits. The Lagrangian for consumers is:

$$\begin{aligned} \mathcal{L}_t &= E_t \sum_{t=0}^{\infty} \beta^t \left[logC_t + \kappa log H_{t-1} - \gamma \frac{L_t^{(1+\varphi)}}{1+\varphi} \right] + \\ &\quad + \lambda_t \left(w_t L_t + (1-\delta) p_t H_{t-1} + R_{t-1} B_{t-1} + \Pi_t - C_t - p_t H_t - B_t \right) \end{aligned}$$

The optimization problem yields three first-order conditions for consumption, housing and leisure. The first-order condition for the consumption-labour tradeoff remains the same:

$$\gamma {L_t}^{\varphi} C_t = w_t \tag{1.6.10}$$

The relationship between consumption and housing is now intertemporal since housing is a multi-period good. It includes both the current and the future price of housing as well as the intertemporal consumption substitution and the depreciation rate of housing:

$$E_t\left(\frac{p_t}{\beta}\left(\frac{C_{t+1}}{C_t}\right) - (1-\delta)p_{t+1}\right)H_t = \kappa E_t C_{t+1} \tag{1.6.11}$$

The standard Euler equation remains unchanged:

$$E_t\left(\frac{C_{t+1}}{C_t}\right) = \beta R_t \tag{1.6.12}$$

Combing the last two equations yields a simpler expression for the consumption-housing tradeoff. Higher expected future price of housing raises demand for housing in the present period and vice versa:

$$E_t\left(R_t p_t - (1-\delta)p_{t+1}\right)H_t = \kappa E_t C_{t+1} \tag{1.6.13}$$

1.6.3 Market Clearing

The market clearing conditions for consumption and for labour supply remain the same. The only notable change is in the housing market. Housing is a multi-period good and each period a fraction δ of the housing available on the market depreciates. The remaining non-depreciated housing along with new production constitutes the available housing in the subsequent period. In each period, the available housing stock must satisfy the housing needs of consumers as well as the consumption of housing firms on exit and the monitoring costs of the loan contract:

$$p_t Y_t^H + (1-\delta)p_t H_{t-1} = p_t H_t + C_{E,t}^H + \mu^H G^H(\bar{\omega}_t^H) R_t^H x_{t-1}^H K_{t-1}^H \tag{1.6.14}$$

The equations for the complete model are listed in Appendix 1.C.

1.6.4 Model Calibration

The model with multi-period housing has the same key parameters and the loan contract satisfies identical conditions as outlined in the version with single-period housing (Tables 1.1 and 1.2). The only notable difference is that housing production is now much smaller since it only replaces the depreciated housing in the period. Furthermore, the housing sector is less capital-intensive than the consumption sector, but unlike the consumption sector, it also utilises land, which is fixed at unity (Iacoviello and Neri, 2010). Housing continues to depreciate at the same rate as capital, δ. The one-sector model still has the same housing production function as in Bernanke, Gilchrist and Gertler (1999) that is discussed in Section 1.4 and Appendix 1.E. The steady state values of the macroeconomic variables for the two-sector model with multi-period housing are in Table 1.5.

Table 1.5: Steady State Values in the Two-Sector Economy with Multi-Period Housing

Variable	Y^F	Y^H	C	H	K^F	K^H	L^F	L^H	I^F	I^H
Value	100	10	66.5	372	997	117	29.9	5.63	24.9	2.93
Variable	N^F	C_E^F	N^H	C_E^H	R	R^H	w	p	ν^H	ν^F
Value	498	5.69	58.5	1.24	1.01	1.015	2.15	1.88	0.98	0.99

The log linearization of the two-sector financial frictions model with multi-period housing is described in Appendix 1.D.

1.7 Multi-Period Housing Model Results

This section simulates again a permanent productivity shock in the housing sector and a temporary relaxation of credit access.

1.7.1 The Housing Oversupply

An innovation in the housing sector raises the production of new houses that add to the existing housing stock. However, since new production is only a fraction of total housing availability, unlike in the version with single-period housing, the fall in the price is less than the rise in output since only new production expands. The profits of housing manufacturers improve stimulating their capital demand and improving their net worth relative to their debt. A lower indebtedness attracts a reduced risk premium that implies more favourable borrowing terms. The technological shock improves the financial position of credit-constrained housing firms and affords consumers more housing at a lower equilibrium price.

A technological advancement in the production of multi-period housing presents more interesting results than when housing is a perishable one-period good (Figure 1.8). In the previous version, each period the total available housing on the market was equal to the housing produced that period. Now new production makes only a fraction of total housing availability. Housing production again rises (Figure 1.8.2, solid line). However, since new output is only a fraction of the total available housing stock, the equilibrium price of all housing falls but by a smaller amount because the increase is only in new production. As a result, the drop in the housing price is less than the corresponding rise in production (Figures 1.8.2 and 1.8.14) so the profits of housing firms improve accordingly. Higher returns signal an increase in the demand for both labour and housing capital (Figures 1.8.6 and 1.8.8). Consumers now demand more of the relatively cheaper housing than consumption so output in the consumption sector diminishes slightly (Figures 1.8.1, 1.8.3 and 1.8.4).

The higher profits of housing manufacturers stimulate the demand for capital so both the price of capital x_t^H and the return to capital, which is the markup loan rate R_t^H, overshoots (Figure 1.8.21). As a result of the increase in the price and in the return to capital, both the value of the capital stock held by credit-constrained borrowers and their net worth improve (Figures, 1.8.6, 1.8.13 and 1.8.17). However, the value of the capital stock rises only by the improvement in the capital stock while the net worth is augmented by both the rise in the capital stock and the overshooting of the realised return to capital R_t^H. The twin effects of improved return to capital and higher value of firm equity imply that the rise in the net worth is proportionally larger than the increase in the capital stock so the leverage of housing manufactures improves (Figure 1.8.23). Lower indebtedness of housing manufacturers signals reduced bankruptcy prospects so investors demand a lower risk premium (Figures 1.8.22 and 1.8.24). This decrease in the risk premium allows credit-constrained firms to borrow even more thus contributing to the amplification effects of financial frictions. The capital stock expands beyond the rise generated by the initial shock further, stimulated by the improved debt position of borrowers (Figure 1.8.6).

The one-sector economy experiences the same effects but on a different scale. Output rises only gradually above the level generated by the technological expansion (Figure 1.8.2, dotted line). A key difference lies in the fact that while in the two-sector model investment in housing is consumption, in the one-sector version it is the own output. Hence in the two-sector model, investment in housing is eventually constrained by the relatively scarce consumption so it diminishes after an initial spike (Figure 1.8.11). In the one-sector version, there is a virtuous cycle of higher housing output generating even more investment that stimulates capital demand beyond that in the two-sector economy and ever rising housing production (Figures 1.8.2, 1.8.6 and 1.8.11). Consumers also enjoy more of the only good, housing, whereas in the two-sector version they are less willing to substitute between consumption and housing due to the Cobb-Douglas utility function (Figures 1.8.3 and 1.8.4).

The introduction of multi-period housing improves the performance of the model but financial frictions continue to contribute to a strictly positive expansion in the credit-constrained sector. The experience of consumers is also positive as they benefit from the lower lousing price.

1.7.2 The Temporary Financial Relaxation

The relaxation of borrowing conditions in the model with multi-period housing produces identical results to these with one-period housing (Figure 1.9). The relaxed risk premium is sufficient to allow credit-constrained housing manufacturers to take additional debt that worsens their leverage and deteriorates their bankruptcy prospects (Figures 1.9.22, 1.9.23 and 1.9.24). This prompts a tightening of borrowing terms as the risk premium creeps up and the ability of firms to purchase new capital is constrained (Figures 1.9.6 and 1.9.24). The recovery is again staggered despite the temporary nature of the initial easing. The effects of the financial relaxation do not depend on the way housing is modelled since the disturbance impacts the financial sector directly without first passing through the real economy. Hence the magnitude of the deterioration of the debt position of borrowers is the same regardless of whether housing depreciates completely or only partially within a single period.

The introduction of multi-period housing fails to improve the performance of the model since the consequences of a financial easing remain contained in the credit-constrained sector and do not affect consumers, unlike the subprime crisis. Although the modelling of housing as a durable good is more empirically accurate, a model with credit constraints in the production sector is not sufficient to capture the recent recession. In order to portray the negative experience of consumers with housing mortgages amidst falling housing prices and credit easing, it is necessary to take the model to the next level by moving credit constraints from the producer to the consumer sector. This is the primary objective of Chapter 2.

Figure 1.8: Positive Technological Innovation in a Two-Sector Model with Financial Frictions and Multi-Period Housing

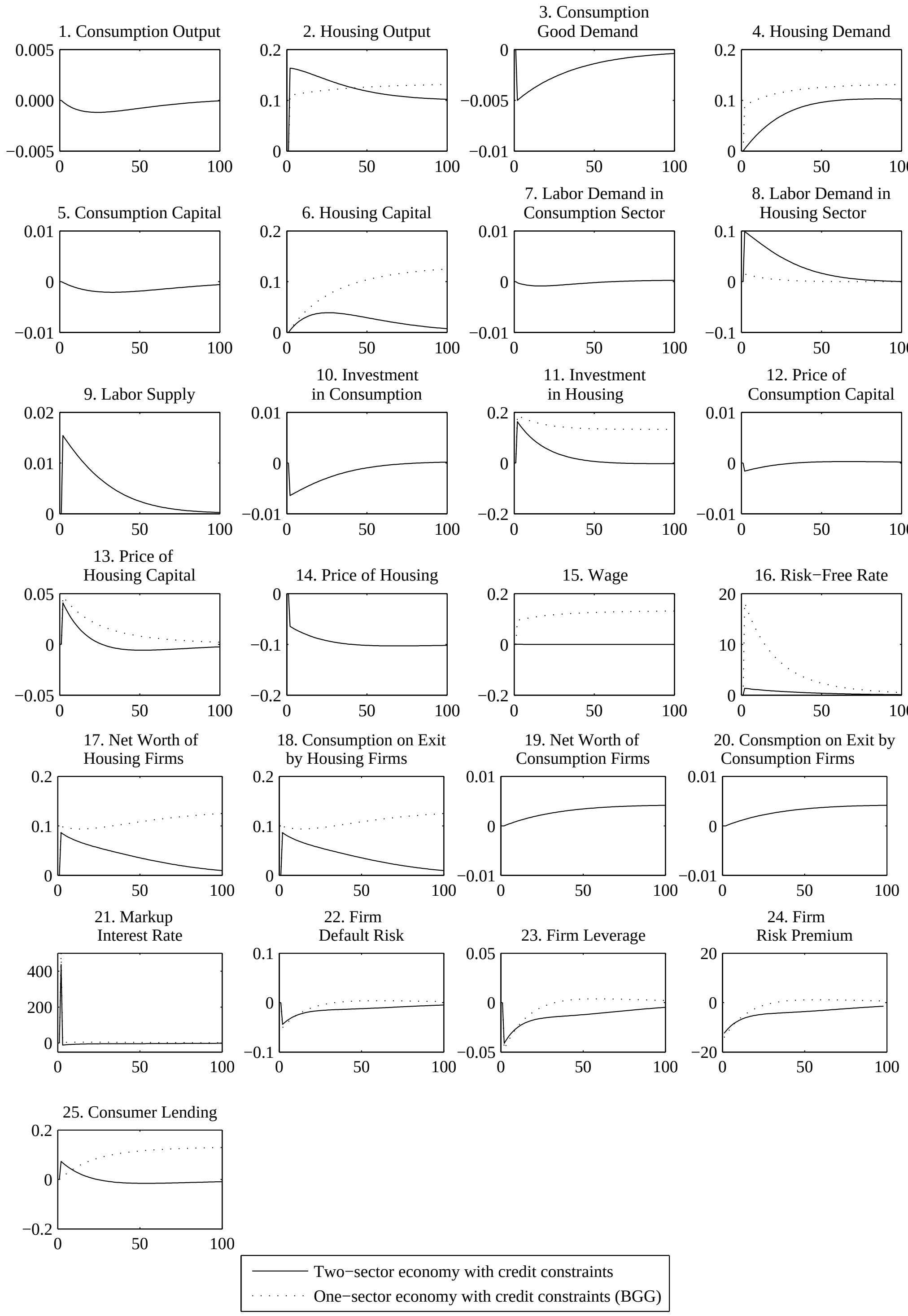
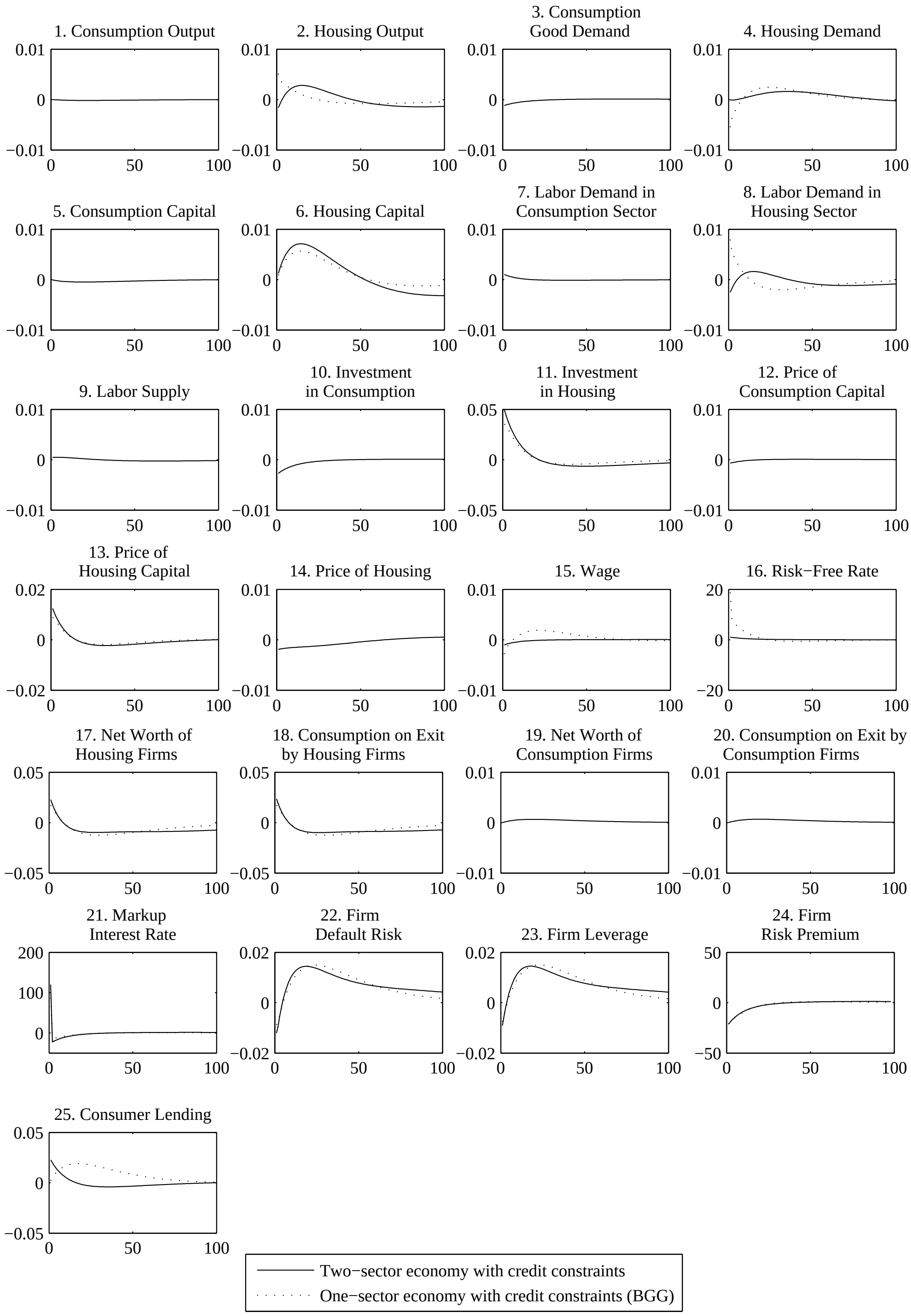


Figure 1.9: Reduced Default Volatility in a Two-Sector Model with Financial Frictions and Multi-Period Housing



1.8 Conclusion

The subprime mortgage crisis established the precedent of positive shocks having a negative influence on house-purchasing consumers. This is at odds with existing studies. This chapter attempts to expand the one-sector models with financial frictions to two sectors in an effort to reconcile the literature and empirical observations. The model developed here offers a more nuanced understanding on financial constraints. In an economy with more than one final good, Fisher-type price effects can largely influence the propagation of the shock and the accelerator effect of financial frictions documented first in Bernanke, Gertler and Gilchrist (1999) may be completely absent. In this extended version of a financial frictions model, there are two final goods, consumption and housing and the housing sector is credit-constrained. A positive technological innovation in the housing sector fails to trigger the accelerator role of financial constraints when housing is a one-period good because the drop in housing price negates the gain from the expansion in output leaving profits in the housing sector unchanged. When housing is a multi-period good, the price drop is not very pronounced since new output is only a fraction of the existing housing stock so housing producers see an increase in profits and raise their demand for capital. The return to capital improves reducing the leverage of indebted firms. In the second version, credit constraints magnify the expansion in production but on a smaller scale than in a one-sector model. In both cases, the house purchasing ability of homeowners is boosted from the reduced housing price. These results largely confirm the findings in Bernanke, Gertler and Gilchrist (1999) but are at odds with the recent downturn where a positive disturbance causes negative consequences for borrowers.

Less positive are the consequences of a temporary relaxation of borrowing conditions that allows credit-constrained housing producers to raise their leverage. A credit easing is transmitted to borrowers in the form of a lower risk premium that allows them to borrow more with the same amount of equity. The consequences, however, are far from beneficial. The improved access to credit allows credit-constrained borrowers to amass more debt. A rise in their indebtedness implies an increase bankruptcy prospects that makes future lending costlier. As a result, the risk premium tightens from its previously relaxed position reversing the credit easing. Although this disturbance is brief, the initial leverage buildup of indebted housing firms implies that they face a protracted recovery process. Unlike the previous shock where the capital financing of producers depends on their profits determined by the real sector, this disturbance arises directly in the financial sector and is not influenced by price effects. This study brings the model closer to the recent recession but fails to explain the experience of home-buying consumers during the subprime mortgage crisis. Since households are outside of the financial market, they are not affected by the downturn.

The model described here captures the more nuanced behaviour of financial constraints in a two-sector economy but fail to replicate the experience during the subprime mortgage crisis. Hence the two-sector model is elaborated on further in Chapter 2. Moving credit frictions from the producer to the household sector captures the negative experience of mortgage-buying consumers after both a technological expansion that reduced house values and after a relaxation of borrowing. The model is then further enriched in Chapter 3 with the addition of a credit-constrained banking sector.

The straightforward two-sector model with financial frictions in this chapter is elaborated on in the subsequent chapter. Nevertheless, it can be extended further on its own. For one, it does not include nominal rigidities and omits the role of government policy. In view of the arguments about the relevance of bailout policies, it would be interesting to consider whether government intervention can mitigate the downturn for the credit-constrained sector and whether this policy would have any undesirable spillover effects on other sectors. Modifying the model to incorporate sluggish adjustment of housing prices would afford to see to what extent the downturn could be drawn out and whether the initial impact would be more muted. Another interesting avenue of research may consider a two-country model and investigate whether a change in consumer preferences for foreign versus domestic housing would yield any qualitatively different effect. These effects would likely depend on the size of the trade between the two economies and whether both or only one of them is credit-constrained. The conclusions, in any case, may be relevant to the interconnected global economy today.

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Appendix 1.A

Complete Two-Sector Financial Frictions Model with Housing as One-Period Good

The complete model for the economy with financial friction includes the loans contract of housing manufacturers, the share purchase of consumers, the production of both sectors and the consumption decisions of households.

The participation constraint for investors:

$$\left(\Gamma^H(\bar{\omega}_t^H) - \mu G^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H = R_{t-1}\left(x_{t-1}^H K_{t-1}^H - N_{t-1}^H\right)$$

The first order condition of housing manufacturers' profit maximization:

$$\frac{E_t R_{t+1}^H}{R_t} = E_t\left(\frac{\frac{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)}{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)-\mu G_\omega^H(\bar{\omega}_{t+1}^H)}}{\left(1-\Gamma^H(\bar{\omega}_{t+1}^H)\right) + \frac{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)}{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)-\mu G_\omega^H(\bar{\omega}_{t+1}^H)}\left(\Gamma^H(\bar{\omega}_{t+1}^H) - \mu G^H(\bar{\omega}_{t+1}^H)\right)}\right)$$

The consumption on exit by housing producers:

$$C_{E,t}^H = \left(1-\nu^H\right)\left(1-\Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H$$

The law of motion of the net worth of housing manufacturers:

$$N_t^H = \nu^H\left(1-\Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H + (1-\alpha_H)(1-\Omega_H) p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{F,t}^E = \left(1-\nu^F\right) R_{t-1} N_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$N_t^F = \nu^F R_{t-1} N_{t-1}^F + (1-\alpha_F)(1-\Omega_F) Y_t^F$$

The consumption capital accumulation equation:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F$$

Investment demand by consumption capital firms:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1$$

The consumption production function:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)\Omega_F}$$

The labour demand by consumption firms:

$$w_t = (1-\alpha_F)\Omega_F\left(\frac{Y_t^F}{L_{H,t}^F}\right)$$

The capital demand by consumption firms:

$$R_t = \frac{E_t\left(\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F\right)}{x_t^F}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H)\Omega_H}$$

The labour demand by housing manufacturers:

$$w_t = (1-\alpha_H)\Omega_H \left(\frac{p_t Y_t^H}{L_{H,t}^H} \right)$$

The capital demand by housing manufacturers:

$$R_t^H = \frac{\alpha_H \frac{p_t Y_t^H}{K_{t-1}^H} + (1-\delta) x_t^H}{x_{t-1}^H}$$

The labour supply of consumers:

$$\gamma {L_t}^{\varphi} C_t = w_t$$

The consumption-housing tradeoff:

$$p_t H_t = \kappa C_t$$

The Euler equation of consumers:

$$E_t \left(\frac{C_{t+1}}{C_t} \right) = \beta R_t$$

Market clearing in the housing sector:

$$p_t Y_t^H = p_t H_t + C_{E,t}^H + \mu^H G^H(\bar{\omega}_t^H) R_t^H x_{t-1}^H K_{t-1}^H$$

Market clearing in the consumption sector:

$$Y_t^F = C_t + C_{E,t}^F + I_t^F + I_t^H$$

Market clearing in the labour market:

$$L_{H,t}^F + L_{H,t}^H = L_t$$

Appendix 1.B

Log-Linearization of the Two-Sector Financial Frictions Model with Housing as One-Period Good

The equations of the model are log-linearized below. The steady state value for each variable appears without time subscript and hats denote the percentage deviation of the variable from its steady state such that $\hat{Z}_t = \frac{Z_t - Z}{Z}$. The log-linearized equations for the credit-constrained economy are below.

The participation constraint for investors:

$$\begin{aligned}\hat{R}_t^H &= \hat{R}_{t-1} + \frac{N^H}{K^H - N^H}\left(\hat{x}_{t-1}^H + \hat{K}_{t-1}^H - \hat{N}_{t-1}^H\right) - \\ &\quad -\frac{\Gamma_\omega^H(\bar{\omega}^H) - \mu G_\omega^H(\bar{\omega}^H)}{\Gamma^H(\bar{\omega}^H) - \mu G^H(\bar{\omega}^H)}\bar{\omega}^H\hat{\bar{\omega}}_t^H - \frac{\Gamma_\sigma^H(\bar{\omega}^H) - \mu G_\sigma^H(\bar{\omega}^H)}{\Gamma^H(\bar{\omega}^H) - \mu G^H(\bar{\omega}^H)}\sigma^H\hat{\sigma}_{t-1}^H\end{aligned}$$

The first order condition of housing manufacturers' profit maximization:

$$\begin{aligned}&(E_t\hat{R}_{t+1}^H - \hat{R}_t)\frac{\Gamma_\omega^H(\bar{\omega}^H)}{\Gamma_\omega^H(\bar{\omega}^H) - \mu G_\omega^H(\bar{\omega}^H)} = \\ &= (1 - \Gamma^H(\bar{\omega}^H))\frac{R^H}{R}\left(\frac{\Gamma_{\omega\omega}^H(\bar{\omega}^H)}{\Gamma_\omega^H(\bar{\omega}^H)} - \frac{\Gamma_{\omega\omega}^H(\bar{\omega}^H) - \mu G_{\omega\omega}^H(\bar{\omega}^H)}{\Gamma_\omega^H(\bar{\omega}^H) - \mu G_\omega^H(\bar{\omega}^H)}\right)\bar{\omega}^H E_t\hat{\bar{\omega}}_{t+1}^H + \\ &+ \frac{R^H}{R}\left(-\Gamma_\sigma^H(\bar{\omega}^H) + \frac{\Gamma_\omega^H(\bar{\omega}^H)\left(\Gamma_\sigma^H(\bar{\omega}^H) - \mu^H G_\sigma^H(\bar{\omega}^H)\right)}{\Gamma_\omega^H(\bar{\omega}^H) - \mu G_\omega^H(\bar{\omega}^H)}\right)\sigma^H\hat{\sigma}_t^H - \\ &- \frac{R^H}{R}\left((1 - \Gamma^H(\bar{\omega}^H))\left(\frac{\Gamma_{\omega\sigma}^H(\bar{\omega}^H)}{\Gamma_\omega^H(\bar{\omega}^H)} - \frac{\Gamma_{\omega\sigma}^H(\bar{\omega}^H) - \mu^H G_{\omega\sigma}^H(\bar{\omega}^H)}{\Gamma_\omega^H(\bar{\omega}^H) - \mu G_\omega^H(\bar{\omega}^H)}\right)\right)\sigma^H\hat{\sigma}_t^H\end{aligned}$$

The consumption on exit by housing producers:

$$\hat{C}_{E,t}^H = \hat{R}_t^H + \hat{x}_{t-1}^H + \hat{K}_{t-1}^H - \frac{\Gamma_\omega^H(\bar{\omega}^H)}{1 - \Gamma^H(\bar{\omega}^H)}\bar{\omega}^H\hat{\bar{\omega}}_t^H - \frac{\Gamma_\sigma^H(\bar{\omega}^H)}{1 - \Gamma^H(\bar{\omega}^H)}\sigma^H\hat{\sigma}_{t-1}^H$$

The law of motion of the net worth of housing manufacturers:

$$\begin{aligned}\hat{N}_t^H &= \nu^H(1 - \Gamma^H(\bar{\omega}^H))\frac{R^H K^H}{N^H}(\hat{R}_t^H + \hat{x}_{t-1}^H + \hat{K}_{t-1}^H) - \nu^H\frac{R^H K^H}{N^H}\Gamma_\omega^H(\bar{\omega}^H)\bar{\omega}^H\hat{\bar{\omega}}_t^H - \\ &\quad -\nu^H\frac{R^H K^H}{N^H}\Gamma_\sigma^H(\bar{\omega}^H)\sigma^H\hat{\sigma}_{t-1}^H + (1 - \alpha_H)(1 - \Omega_H)\frac{pY^H}{N^H}(\hat{p}_t + \hat{Y}_t^H)\end{aligned}$$

The consumption on exit by consumption firms:

$$\hat{C}^F_{E,t} = \hat{R}_{t-1} + \hat{N}^F_{t-1}$$

The law of motion of the net worth of consumption firms:

$$N^F_t = \nu^F R(\hat{R}_{t-1} + \hat{N}^F_{t-1}) + (1-\alpha_F)(1-\Omega_F)\frac{Y^F}{N^F}\hat{Y}^F_t$$

The consumption capital accumulation equation:

$$\hat{K}^F_t = \hat{K}^F_{t-1} + \delta\left(\hat{I}^F_t - \hat{K}^F_{t-1}\right)$$

Investment demand by capital firms:

$$\hat{x}^F_t = J"\delta\left(\hat{K}^F_{t-1} - \hat{I}^F_t\right)$$

The consumption production function:

$$\hat{Y}^F_t = \hat{A}^F_t + \alpha_F\hat{K}^F_{t-1} + (1-\alpha_F)\Omega_F\hat{L}^F_{H,t}$$

The labour demand by consumption firms:

$$\hat{w}_t = \hat{Y}^F_t - \hat{L}^F_{H,t}$$

The capital demand by consumption firms:

$$\hat{R}_t + \hat{x}^F_t = \frac{R-(1-\delta)}{R}(\hat{Y}^F_{t+1} - \hat{K}^F_t) + \frac{(1-\delta)}{R}\hat{x}^F_{t+1}$$

The housing capital accumulation equation:

$$\hat{K}^H_t = \hat{K}^H_{t-1} + \delta\left(\hat{I}^H_t - \hat{K}^H_{t-1}\right)$$

Investment demand by housing manufacturers:

$$\hat{x}^H_t = J"\delta\left(\hat{K}^H_{t-1} - \hat{I}^H_t\right)$$

The housing production function:

$$\hat{Y}_t^H = \hat{A}_t^H + \alpha_C \hat{K}_{t-1}^H + (1-\alpha_H)\Omega_H \hat{L}_{H,t}^H$$

The labour demand by housing manufacturers:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_t^H - \hat{L}_{H,t}^H$$

The capital demand by housing manufacturers:

$$\hat{R}_t^H + \hat{x}_{t-1}^H = \frac{R^H - (1-\delta)}{R^H}(\hat{p}_t + \hat{Y}_t^H - \hat{K}_{t-1}^H) + \frac{(1-\delta)}{R^H}\hat{x}_t^H$$

The labour supply of consumers:

$$\varphi \hat{L}_t + \hat{C}_t = \hat{w}_t$$

The consumption-housing tradeoff:

$$\hat{p}_t + \hat{H}_t = \hat{C}_t$$

The Euler equation:

$$E_t\left(\hat{C}_{t+1}\right) - \hat{C}_t = \hat{R}_t$$

Market clearing in the housing sector:

$$\begin{aligned}\hat{p}_t + \hat{Y}_t^H \quad = \quad & \frac{H}{Y^H}(\hat{p}_t + \hat{H}_t) + \frac{C_E^H}{pY^H}\hat{C}_{E,t}^H + \mu G^H(\bar{\omega}^H)\frac{R^H K^H}{pY^H}\left(\hat{R}_t^H + \hat{x}_{t-1}^H + \hat{K}_{t-1}^H\right) + \\ & + \mu G_\omega^H \bar{\omega}^H \hat{\bar{\omega}}_t^H \frac{R^H K^H}{pY^H} + \mu G_\sigma^H \sigma^H \hat{\sigma}_{t-1}^H \frac{R^H K^H}{pY^H}\end{aligned}$$

Market clearing in the consumption sector:

$$\hat{Y}_t^F = \frac{C^F}{Y^F}\hat{C}_t^F + \frac{C_E^F}{Y^F}\hat{C}_{E,t}^F + \frac{I^F}{Y^F}\hat{I}_t^F + \frac{I^H}{Y^F}\hat{I}_t^H$$

Market clearing in the labour market:

$$\hat{L}_t = \frac{L_H^F}{L}\hat{L}_{H,t}^F + \frac{L_H^H}{L}\hat{L}_{H,t}^H$$

Appendix 1.C

Complete Two-Sector Financial Frictions Model with Housing as Multi-Period Good

The participation constraint for investors:

$$\left(\Gamma^H(\bar{\omega}_t^H) - \mu G^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H = R_{t-1}\left(x_{t-1}^H K_{t-1}^H - N_{t-1}^H\right)$$

The first order condition of housing manufacturers' profit maximization:

$$\frac{E_t R_{t+1}^H}{R_t} = E_t\left(\frac{\frac{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)}{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)-\mu G_\omega^H(\bar{\omega}_{t+1}^H)}}{\left(1-\Gamma^H(\bar{\omega}_{t+1}^H)\right)+\frac{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)}{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)-\mu G_\omega^H(\bar{\omega}_{t+1}^H)}\left(\Gamma^H(\bar{\omega}_{t+1}^H)-\mu G^H(\bar{\omega}_{t+1}^H)\right)}\right)$$

The consumption on exit by housing producers:

$$C_{E,t}^H = \left(1-\nu^H\right)\left(1-\Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H$$

The law of motion of the net worth of housing manufacturers:

$$N_t^H = \nu^H\left(1-\Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H + (1-\alpha_H)(1-\Omega_H) p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{F,t}^E = \left(1-\nu^F\right) R_{t-1} N_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$N_t^F = \nu^F R_{t-1} N_{t-1}^F + (1-\alpha_F)(1-\Omega_F) Y_t^F$$

The consumption capital accumulation equation:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F$$

Investment demand by consumption capital firms:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1$$

The consumption production function:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)\Omega_F}$$

The labour demand by consumption firms:

$$w_t = (1-\alpha_F)\Omega_F \left(\frac{Y_t^F}{L_{H,t}^F}\right)$$

The capital demand by consumption firms:

$$R_t = \frac{E_t\left(\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F\right)}{x_t^F}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right) K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H)}$$

The labour demand by housing manufacturers:

$$w_t = (1-\Omega_H)\,(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing manufacturers:

$$R_t^H = \frac{\alpha_H \frac{p_t Y_t^H}{K_{t-1}^H} + (1-\delta)x_t^H}{x_{t-1}^H}$$

The labour supply of consumers:

$$\gamma L_t{}^{\varphi} C_t = w_t$$

The consumption-housing tradeoff:

$$E_t \left(R_t p_t - (1-\delta) p_{t+1} \right) H_t = \kappa E_t C_{t+1}$$

The Euler equation of consumers:

$$E_t \left(\frac{C_{t+1}}{C_t} \right) = \beta R_t$$

Market clearing in the housing sector:

$$p_t Y_t^H + (1-\delta) p_t H_{t-1} = p_t H_t + C_{E,t}^H + \mu G^H(\bar{\omega}_t^H) R_t^H x_{t-1}^H K_{t-1}^H$$

Market clearing in the consumption sector:

$$Y_t^F = C_t + C_{E,t}^F + I_t^F + I_t^H$$

Market clearing in the labour market:

$$L_{H,t}^F + L_{H,t}^H = L_t$$

Appendix 1.D

Log-Linearization of the Two-Sector Financial Frictions Model with Housing as Multi-Period Good

The additional equations that differ from those in Appendix 1.B are log-linearized below.

The housing production function:

$$\hat{Y}_t^H = \hat{A}_t^H + \alpha_C \hat{K}_{t-1}^H + (1-\alpha_H-\varepsilon)\Omega_H \hat{L}_{H,t}^H$$

The labour demand by housing manufacturers:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_t^H - \hat{L}_{H,t}^H$$

The capital demand by housing manufacturers:

$$\hat{R}_t^H + \hat{x}_{t-1}^H = \frac{R^H-(1-\delta)}{R^H}(\hat{p}_t + \hat{Y}_t^H - \hat{K}_{t-1}^H) + \frac{(1-\delta)}{R^H}\hat{x}_t^H$$

The consumption-housing tradeoff:

$$\hat{C}_{t+1} = \hat{H}_t + \hat{p}_{t+1} + \frac{R}{(R-1+\delta)}\left(\hat{R}_t + \hat{p}_t - \hat{p}_{t+1}\right)$$

Market clearing in the housing sector:

$$\begin{aligned}\hat{Y}_t^H \quad = \quad & \frac{\delta H - Y^H}{Y^H}\hat{p}_t + \frac{H}{Y^H}\hat{H}_t - \frac{(1-\delta)}{Y^H}H\hat{H}_{t-1} + \frac{C_E^H}{pY^H}\hat{C}_{E,t}^H + \\ & +\mu G^H(\bar{\omega}^H)\frac{R^H K^H}{pY^H}\left(\hat{R}_t^H + \hat{x}_{t-1}^H + \hat{K}_{t-1}^H\right) + \mu G_\omega^H \bar{\omega}^H \hat{\bar{\omega}}_t^H \frac{R^H K^H}{pY^H} + \mu G_\sigma^H \sigma^H \hat{\sigma}_{t-1}^H \frac{R^H K^H}{pY^H}\end{aligned}$$

Appendix 1.E

Complete One-Sector Financial Frictions Model (Bernanke, Gertler and Gilchrist, 1999)

The participation constraint for investors:

$$\left(\Gamma^H(\bar{\omega}_t^H) - \mu G^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H = R_{t-1}\left(x_{t-1}^H K_{t-1}^H - N_{t-1}^H\right)$$

The first order condition of housing manufacturers' profit maximization:

$$\frac{E_t R_{t+1}^H}{R_t} = E_t\left(\frac{\frac{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)}{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)-\mu G_\omega^H(\bar{\omega}_{t+1}^H)}}{\left(1-\Gamma^H(\bar{\omega}_{t+1}^H)\right) + \frac{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)}{\Gamma_\omega^H(\bar{\omega}_{t+1}^H)-\mu G_\omega^H(\bar{\omega}_{t+1}^H)}\left(\Gamma^H(\bar{\omega}_{t+1}^H) - \mu G^H(\bar{\omega}_{t+1}^H)\right)}\right)$$

The consumption on exit by housing producers:

$$C_{E,t}^H = \left(1-\nu^H\right)\left(1-\Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H$$

The law of motion of the net worth of housing manufacturers:

$$N_t^H = \nu^H\left(1-\Gamma^H(\bar{\omega}_t^H)\right) R_t^H x_{t-1}^H K_{t-1}^H + (1-\alpha_H)(1-\Omega_H)Y_t^H$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H}(L_{H,t}^H)^{(1-\alpha_H)\Omega_H}$$

The labour demand by housing manufacturers:

$$w_t = (1-\alpha_H)\Omega_H \left(\frac{Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing manufacturers:

$$R_t^H = \frac{\alpha_H \frac{Y_t^H}{K_{t-1}^H} + (1-\delta)x_t^H}{x_{t-1}^H}$$

The labour supply of consumers:

$$\gamma {L_t}^{\varphi} H_t = w_t$$

The Euler equation of consumers:

$$E_t\left(\frac{H_{t+1}}{H_t}\right) = \beta R_t$$

Market clearing in the housing sector:

$$Y_t^H = H_t + C_{E,t}^H + I_t^H + \mu^H G^H(\bar{\omega}_t^H) R_t^H x_{t-1}^H K_{t-1}^H$$

Appendix 1.F

Derivatives of the Log-Normal Distribution

The distribution of $\bar{\omega}$ is log-normal such that $ln\bar{\omega} \sim N\left(\frac{1}{2}\sigma^2,\ \sigma^2\right)$ follows the Normal distribution and where σ^2 is the variance of the cutoff idiosyncratic risk. The cumulative distribution function $F(\bar{\omega})$, the investors' share of the loan contract from non-defaulting borrowers $\Gamma\left(\bar{\omega}\right)$ and from defaulting debtors $G\left(\bar{\omega}\right)$ are defined as:

$$F\left(\bar{\omega}\right) = \Phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)$$

$$G\left(\bar{\omega}\right) = \Phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)$$

$$\Gamma\left(\bar{\omega}\right) = \Phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) + \left[1 - \Phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\right]\bar{\omega}$$

The derivatives of the distribution terms, namely F_ω, F_σ, Γ_ω, Γ_σ, G_ω, G_σ, $\Gamma_{\omega\omega}$, $\Gamma_{\omega\sigma}$, $G_{\omega\omega}$ and $G_{\omega\sigma}$ are derived below. The derivatives use the normal distribution property that $\phi'(x) = -x\phi(x)$ where Φ and ϕ are the cumulative distribution function and the probability distribution function of the standard normal distribution.

$$F_\omega = \frac{1}{\sigma\bar{\omega}}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$F_\sigma = -\frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$G_\omega = \frac{1}{\sigma\bar{\omega}}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$G_\sigma = -\frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$\Gamma_\omega = 1 - F\left(\bar{\omega}\right) = 1 - \Phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) > 0$$

$$\begin{aligned}\Gamma_\sigma &= G_\sigma - \bar{\omega}F_\sigma = \\ &= -\frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega}^C - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) + \frac{\bar{\omega}}{\sigma}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\end{aligned}$$

$$\Gamma_{\omega\omega} = -F_\omega = -\frac{1}{\sigma\bar{\omega}}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right) < 0$$

$$\Gamma_{\omega\sigma} = -F_\sigma = \frac{1}{\sigma}\phi\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) < 0$$

$$\begin{aligned}G_{\omega\omega} &= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) + \left(\frac{1}{\sigma\bar{\omega}}\right)^2\phi'\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) = \\ &= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) - \left(\frac{1}{\sigma\bar{\omega}}\right)\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\end{aligned}$$

$$\begin{aligned}G_{\omega\sigma} &= -\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) - \frac{1}{\sigma\bar{\omega}}\phi'\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma^2}\right) = \\ &= -\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right) + \frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} - \frac{1}{2}\sigma^2}{\sigma}\right)\left(\frac{ln\bar{\omega} + \frac{1}{2}\sigma^2}{\sigma}\right)\end{aligned}$$

Chapter 2

Consumer Mortgage Leverage and Subprime Mortgage Crises

Abstract

Consumer leverage can contribute to financial crises, such as the subprime mortgage crisis, that are characterised by increased bankruptcy prospects and tightened credit access for borrowers. This chapter embeds financial frictions in the mortgage contracts of home-buyers within a two-sector economy to show that although households are not production agents and may not start as highly leveraged as financial institutions, their worsening debt levels can trigger a lasting financial downturn. The model demonstrates this using two seemingly positive disturbances that contributed to the subprime mortgage crisis - increased housing supply and relaxed borrowing conditions. The surplus of available housing leads to lower asset prices that in turn reduce the value of the mortgaged houses relative to the loan held. This worsens the leverage of indebted consumers and raises their bankruptcy prospects. A relaxation of borrowing conditions turns credit-constrained households into a potential source of disturbance themselves when market optimism allows them to increase their indebtedness with relatively little downpayment. When the previously relaxed borrowing terms begin to tighten as a result of the higher leverage, consumers face a protracted period of debt reduction. In both cases, the increased indebtedness, along with higher repayment rates due to the larger default likelihood, impairs mortgage-holders' access to credit and plunges them into a lasting recession.

2.1 Introduction

Edmund Andrews, whose story was detailed in the Introduction, had a mortgage that was less generous than he had thought. In fact, it was a classic subprime loan. The monthly payment first jumped from \$2,500 to \$3,700. If he kept the mortgage for two years, the interest rate could reach as high as 11.5 percent and the monthly payments could ratchet up to \$4,500 (Andrews, 2009). After his wife lost her job, he fell behind on all payments from the mortgage to the electricity bill. The default on his mortgage was inevitable.

Andrews' story is just one of the hundreds of recent foreclosure experiences that characterise the mortgaged homeowner's experience with subprime loans. The subprime mortgage crisis was a seemingly unusual one since it was not precipitated by production firms or entrepreneurs and it was not triggered by adverse circumstances. The unravelling of the mortgage market began when two seemingly positive factors - increased housing supply and easy financing conditions - allowed borrowers to excessively leverage with debt to disastrous consequences. At the heart of the downturn were not production agents but heavily indebted consumers and the financial institutions that securitized their mortgages. Nevertheless, the collapse of the highly leveraged mortgage market was sufficient to trigger a significant downturn.

The asset bubble that preceded the subprime mortgage crisis increased housing inventories to record high numbers (Coleman, LaCour-Little and Vandell, 2008). After the bubble burst, the oversupply of real estate reduced both the sale price of real estate and the value of the houses held by homeowners (Duca, Muellbauer and Murphy, 2010; Ellis, 2010). As a result, many mortgage-holders owed more on their housing loan than their residence was worth. Furthermore, at the peak of the asset bubble, many investors eager to tap into mortgage profits underwrote housing loans secured with very little downpayment and with a variable repayment rate, the so-called "subprime" loans (Duca, Muellbauer, and Murphy, 2009a and 2009b). These lax borrowing conditions allowed consumers to hold significant amounts of debt and to be at the mercy of the adjustable interest rate. Once the first signs of trouble raised the repayment rate, their risk premiums rose worsening their indebtedness (Demirgüç-Kunt, Evanoff and Kaufman, 2011; Laeven, Igan and

Dell'Ariccia, 2008; Mian and Sufi, 2009). This caused a flurry of short sales and defaults as mortgage-buyers attempted to deleverage or declared bankruptcy. The result was a depressed housing market with even lower asset prices and sharply tightened credit access (Dennis, 2010; Duke, 2012; Madigan, 2012). The housing market downturn also exported the worsened financial conditions beyond the affected sector and depressed demand for goods in the rest of the economy.

Glick and Lansing (2010) demonstrate that household leverage in the United States and in many industrial countries increased dramatically in the decade prior to 2007. Countries with the largest increases in household leverage underwent the fastest rise in house prices over the same period. These same countries tended to experience the biggest decline in household consumption once house prices started falling. Household leverage growth and the dependence on borrowing also explain a large fraction of the overall consumer default, house price, unemployment, residential investment and durable consumption patterns during the recession (Mian and Sufi, 2010). Kim and Isaac (2010) show that consumer debt can affect macroeconomic dynamics and can contribute to financial fragility as strongly as corporate leverage. The authors demonstrate that looser consumer credit can also make the system more vulnerable to changes in the state of confidence, the interest rate and the saving propensity of renters. Papers like these motivate a microfounded modelling of the consequences of sharp decline in house prices and of improvements in borrowing confidence on the leverage of indebted households.

This chapter develops a model of financial frictions in the mortgage market to demonstrate that an increase in the supply of housing and relaxed access to credit can lead to lower return on housing and greater indebtedness for mortgaged consumers causing bankruptcy and downturn. For this purpose, the paper updates the two-sector model of credit constraints from Chapter 1 by shifting financial frictions from producers to homeowner consumers. In Chapter 1, housing manufacturers faced credit frictions in their financing of capital acquisition. Here there are credit-constrained consumers who are subject to an idiosyncratic probability of default so their only recourse for the purchase of a house is costly external financing. In order to facilitate the borrowing process, the previously homogenous consumer sector is split into Ricardian consumers who can purchase housing freely and credit-constrained consumers who need to borrow to acquire real estate. The loan contract now takes the form of a mortgage contract that provides funding for financially constrained households to buy housing. Using credit constraints in the mortgage-buying decisions of consumers, this chapter establishes that a downturn such as the subprime crisis can be triggered relatively easily when there are financial frictions even if the borrowers are not production agents or financial institutions.

The proposed model with credit constraints in the household sector demonstrates the two precedents observed in the subprime mortgage crisis, namely, that it was precipitated by non-production agents (credit-constrained consumers) and that it was triggered by positive events (increased supply of housing and relaxed access to credit). The two-sector model with credit constraints in the production sector in Chapter 1 failed to explain how positive disturbances can have negative consequences for borrowers. In Chapter 1, indebted firms largely benefited from an innovation in their own sector due to a rise in profits that stimulated their capital demand and improved their debt position. This is at odds with the observations of the subprime mortgage crisis. Only a financial easing had negative consequences for borrowers since it permits excessive leverage but households were outside of the financial market. Since consumers did not borrow so they only experienced a lower housing price that afforded them better access to housing.

In this chapter, on the other hand, home-buyers obtain a loan mortgage where their collateral is their existing housing stock. Since credit-constrained consumers have an idiosyncratic risk of default, they cannot borrow unless they pay a risk premium that would cover expenses associated with potential bankruptcy. This risk premium is a function of the default risk that in turn depends on the size of the collateral and the value of the mortgage. So a shock that reduces housing prices makes real estate more affordable but also creates a Fisher-type effect where the price decrease worsens the value of the housing collateral of indebted households and increases their leverage. The higher leverage triggers an increase in the adjustable mortgage interest rate and the risk premium widens. A credit relaxation on the other hand impacts the leverage of mortgage-buyers directly by creating the incentive to borrow excessively. Hence, with credit constraints in the consumer sector, both a technological expansion in the housing sector and a financial easing imply a downturn for mortgaged consumers thus bringing the model closer to the subprime crisis.

It order to describe the behaviour of leveraged consumers, the paper departs from the traditional modelling of house purchasing decisions such as Kiyotaki and Moore (1997), Aoki, Proudman and Vlieghe (2004) and Iacoviello (2005) in favour of a microfounded model of consumer credit constraints. The housing model of Aoki, Proudman and Vlieghe (2004), which belongs to the Kiyotaki and Moore (1997) collateral constraint type of models, imposes an exogenously determined limit on borrowing depending on the availability of collateral. Such models were a fairly common way of modelling housing loans prior to the financial crisis. However, the emergence of subprime loans that require little to no collateral and instead have an adjustable repayment rate makes such approaches unsuitable for modelling the subprime recession. To capture better the subprime mortgages, the model developed here avoids exogenously imposed loan-to-value ratios that cap borrowing and limit the resulting volatility. Instead, this chapter continues to use the Bernanke, Gilchrist and Gertler (1999) microfounded loan contract to model frictions in the mortgage market. Such a setup imposes no upper restrictions on borrowing but introduces an endogenous adjustable mortgage rate that makes loans costlier as the leverage increases. A subprime loan within this framework could be secured with modest downpayment but at a higher interest rate that adjusts upwards following unfavourable macroeconomic disturbances. This approach allows a robust study of the role of leveraged consumers in the subprime crisis.

Most of the works on financial constraints overlook the role of consumer leverage in the recession since households are generally not as leveraged as financial institutions. Bernanke, Gilchrist and Gertler (1999) and Luk and Vines (2011) show that high firm or financial institutions leverage can be instrumental in precipitating a financial downturn. Similarly, Fernández-Villaverde (2010) and Gertler and Kiyotaki (2010) demonstrate that a disruption to financial intermediation can induce a crisis that affects real economic activity. The subsequent section looks at the literature of housing mortgages and consumer leverage in detail but the majority of models either omits the role of consumer leverage or only attempts to fit models to empirical data. One of the few studies on consumer mortgage decisions, Mian and Sufi (2009) establishes that consumers can experience a worsening in their leverage as a result of deteriorating lending practises but does not study the consequences of this increase in indebtedness for their borrowing and the economy. This paper augments our understanding of credit frictions by considering explicitly the role of household leverage in creating and propagating downturns. It also demonstrates that although households do not start very indebted and do not produce output, changes in their leverage can have lasting consequences for their credit access and for the economy as a whole as mentioned in Kim and Isaac (2010).

At the heart of the financial frictions setup is the inability of credit-constrained consumers to fully finance their housing purchase so they need to borrow external funds from risk-neutral investors. This borrowing is complicated by the presence of an idiosyncratic risk of default on the part of mortgage-buyers that is known to them but is unknown to lenders (Townsend, 1979). Unlike Cúrdia and Woodford (2009) where borrowers can choose whether to default, here bankruptcy is involuntary but depends on factors both endogenous (the size of the mortgage) and exogenous (macroeconomic shocks) to consumer decisions. If credit-constrained households default, investors must pay an auditing fee to assume possession of any remaining assets. Since investors cannot fully diversify away this risk, they charge borrowers a risk premium that would offset the expenses associated with eventual bankruptcy. Investors obtain their funds from the deposits of Ricardian consumers who have significant non-wage income from firm profits and share ownership. In contrast, credit-constrained consumers earn only labour income and have no savings. The economy is completed by two production sectors: one that produces conventional consumption and one that manufactures housing. Housing is a multi-period durable good whose manufacture follows the setup of Iacoviello and Neri (2010). Both production sectors are perfectly competitive and there is no idiosyncratic uncertainty in their returns.

The model demonstrates the effects of two shocks that contributed to precipitating the subprime mortgage crisis: the oversupply of housing and the relaxation of borrowing conditions. The overproduction of housing depresses the price of housing and lowers the return on housing. As a result, the value of the collateral of credit-constrained consumers, primarily composed of their housing equity, decreases by both the price and the return to housing. The drop in their housing equity value is proportionally larger than the fall in the mortgage value which falls only due to the price reduction so the leverage of mortgaged home-buyers increases. The higher leverage signals increased risk of default since lenders that provide loans to credit-constrained consumers are exposed to bigger risks. To compensate for this, investors require a higher return from indebted households so the risk premium rises. The higher risk premium further depresses credit access that can remain lower for a long time while consumers attempt to repair their debt position by gradually improving their real estate holding.

A lower housing price is not the only disturbance that can induce a credit downturn for indebted consumers. A relaxation of credit access improves borrowing conditions so mortgage loans attract a lower risk premium and require a smaller downpayment to secure (Duca, Muellbauer, and Murphy, 2009a and 2009b; Mian and Sufi, 2009). The lax credit access prior to the crisis reduced interest rates and lured many potential homeowners into obtaining mortgages with little collateral. However, the lower collateral implies higher indebtedness for mortgage-buyers that can easily lead to higher probability of default. Borrowers attempt to amass more collateral in order to improve their mortgage position but since they cannot achieve that overnight, their increased bankruptcy prospects prompt investors to raise the repayment interest rate. The higher repayment rate makes improving housing equity even harder for indebted consumers. As a result, households experience a lasting recession with tight credit access.

The next section provides an overview of the literature of housing mortgages and on consumer borrowing decisions. Section 2.3 describes the mortgage loan contract and Section 2.4 embeds it within a general equilibrium model. Section 2.5 looks in detail at the simulations of the two disturbances - an innovation in the production of housing and a relaxation of borrowing conditions. The Conclusion wraps up.

2.2 The Literature of Housing Mortgages and Household Leverage

This chapter focuses on the role of consumer leverage and its potential to generate downturns so the section discusses the attention housing mortgages and household indebtedness have received in the literature. It argues that despite the recent interest in the housing market and house financing spawned by the subprime recession, there is a lack of a microfounded approach to consumer mortgages as part of a dynamic stochastic general equilibrium (DSGE) framework. Studies have also generally overlooked consumers as a significant source of leverage in favour of more highly indebted financial institutions. This chapter attempts to address the gap in the literature by developing a robust DSGE model with endogenous credit constraints embedded in the consumer mortgage market.

Housing has received special interest in the literature of business cycles since house prices have been traditionally considered an indicator of the health of the economy and since housing by itself is a sizable investment that figures prominently in financial markets. The recent subprime crisis has led to resurgence in the interest toward modelling housing and house financing decisions. As a production good, housing is unlike any other good. It is durable and lasts far longer than regular consumption. It is also relatively expensive so residential housing is a major investment for homeowners. Most households cannot purchase housing directly but do so as part of a mortgage contract where they borrow a large share of the value of the house against some collateral. Since housing is primarily financed by lending where borrowers are exposed to potential default, modelling house purchasing decisions frequently involves financial markets and credit imperfections.

The majority of papers that involves housing either does not explicitly model it as part of a DSGE framework or abstracts from frictions in house financing. Of those who study the role of housing borrowing within models with incomplete markets, most combine a multi-sector structure with housing and non-housing goods with financing frictions in the household sector. Gervais (2002) uses a utility function where housing enters additively along with consumption. This permits to study the role of housing demand on consumer behaviour directly and not as a part of a consumption aggregator. Consumers can either purchase or rent houses and a house purchase is financed partly with collateral and partly with borrowing against the value of the house. The interest rate charged on borrowing is fixed but the minimum downpayment varies with the value of the house. This loan contract is a version of the Kiyotaki and Moore (1997) collateral constraint type of financial frictions. Households choose between borrowing and lending depending on the cost of borrowed funds. The model finds that the main housing tax provisions currently in place in the U.S. make the return on housing capital larger than that on business capital so consumers have a bias toward home ownership. The author, however, focuses solely on tax distortions and does not include aggregate disturbances that may affect the demand for housing.

Peterson (2006) also uses a utility function where housing enters separately from consumption. House financing requires costly intermediation that involves a risk premium as in Bernanke, Gilchrist and Gertler (1999) but unlike Bernanke, Gilchrist and Gertler (1999), in this model the uncertainty pertains to non-housing assets and individual earnings. The model tests the hypothesis that a demand shock for housing leads the business cycle. The paper proposes one possible mechanism that can generate such shifts in the demand for housing. The mechanism is a transaction cost in the housing market such as broker fees, closing costs, uninsurable shocks to individual earnings and anti-cyclical shocks to individual earnings. These three components make a potential home buyer reluctant to buy a home at the start of a recession due to the greater likelihood of a bad earnings shock causing them to have to re-sell a home just purchased at a substantial loss due to the transaction cost. The model establishes that there are large increases (decreases) in housing demand at the start of an expansion (recession) and those fluctuations help explain the leading behaviour of housing investment. However, it does not explore the effect housing demand has on business cycles and especially in triggering downturns.

Other papers establish the need for collateral and that costly financing may delay house acquisition. In Díaz and Luengo-Prado (2010), homeowners cannot borrow if they do not own residential assets. This constraint implies that the maximum debt a household can incur is a fraction of the household's residential stock. There are also transaction costs when changing houses which results in infrequent changes of the residential stock. The authors look for theoretical underpinnings to the observation that housing as a fraction of total wealth decreases with the level of wealth. Such calibration is achieved with very persistent shocks to earnings, frictions in the housing market and the presence of a rental market. Ortalo-Magné and Rady (2006) show that first-time home-buyers delay their house purchase due to financing costs which also force other households to buy a home smaller than the one they would like. The financing costs take the form of collateral constraints where agents are allowed to borrow at a fixed interest rate but their end-of-period non-housing wealth is not allowed to fall below a fraction of the housing value. The model establishes a primary driver of the housing market - the ability of young households to afford the downpayment on a starter home that is determined by their income. Changes in income may yield housing price overshooting, with prices of upgrade homes displaying the most volatility, and create a positive correlation between housing prices and transactions.

While these papers explicitly model housing and its financing process, they abstract from aggregate shocks. Iacoviello and Neri (2010) are the first to use a rich set of aggregate disturbances within a framework that captures the main housing characteristics and reflects fluctuations in household consumption. The model features two sectors of production - housing and consumption. Consumption is a one-period good but housing is a multi-period good and new housing production only replaces depreciated housing. The housing sector utilises capital, land (in fixed supply) and labour to provide housing services to households. Consumption goods and business capital are produced in the non-housing sector by utilizing labour and capital. To bring the model closer to reality, the authors introduce nominal rigidities. Price rigidity in the consumption sector takes the form of a differentiation between competitive flexible price/wholesale firms that produce wholesale consumption goods and housing using two technologies on one hand and a final good firm that operates in the consumption sector under monopolistic competition on the other hand.

In this model, motivated by the recent developments in the U.S. economy, there are financial frictions in the consumer sector where households use their homes to secure debt. As an extension to the collateral constraint model of Kiyotaki and Moore (1997), Iacoviello and Neri (2010) consider two types of households: patient (lenders) households and impatient (borrowers) households. Patient households work, consume, accumulate housing, supply capital to firms and supply funds to impatient households. Impatient households work, consume and accumulate housing but they only accumulate the net worth required to finance the downpayment on their home. Due to an inability to fully enforce the repayments of debt, impatient agents have to use the housing stock they own as collateral. Like in the previous studies mentioned, the housing stock is considered as a separate argument of households' utility function, in addition to consumption and leisure. Since housing is not part of a consumption aggregator, it is possible to directly observe its effect on consumer decisions. Furthermore, since housing and consumption goods are produced using different technologies, the model generates endogenous dynamics both in residential investment vis-à-vis business investment and in the price of housing. At the same time, fluctuations in house prices affect the borrowing capacity of a fraction of households, on the one hand and the relative portability of producing new homes, on the other. Both of these mechanisms generate feedback effects for the expenditure of households and firms.

Iacoviello and Neri (2010) incorporate an array of aggregate shocks to explain the trends in real housing prices and investment over the last four decades. The results support the importance of the mechanism of collateralized debt. In doing so, the authors do not need to commit to a particular view of sources of business cycle fluctuations. The rich set of shocks also allows them to better explain the interactions between housing and the economy. The model shows that the presence of collateral constraints amplifies the financial accelerator effect and propagates shocks from the housing market to the rest of the model economy. Negative price shocks in the housing market, via the collateral constraint, reduce the amount of household loans used to facilitate household consumption. The relationship between house prices and household consumption allows the model to capture the extent of fluctuations in the growth of household consumption as observed in the data. Despite the advancement of Iacoviello and Neri (2010) in modelling consumer house purchasing decisions, the main goal of the study is to explain trends in housing prices, not the buildup of consumer leverage. The model also relies on Kiyotaki and Moore (1997), a non-microfounded approach to representing credit constraints.

Calza, Monacelli and Stracca (2008) go one step further than Iacoviello and Neri (2010) by considering both fixed rate as well as variable rate mortgages. The model has two sectors, one sector produces consumption, business investment and intermediate goods (using capital and labour), while the other sector produces new homes (using capital, labour, land and intermediate goods). There are two types of households. Patient households work, consume, buy homes, rent capital and land to firms, and lend to impatient households. Impatient (and hence assumed to be credit-constrained) households work, consume, buy homes and borrow against the value of their home. The trend in technological progress differs across sectors and there are sticky prices in the non-housing sector (with Calvo-style price rigidity and indexation). Technology and preferences are subject to random shocks. Wages are sticky in both sectors. Real frictions include habits in consumption, imperfect labour mobility, adjustment costs for capital and variable capital utilization which all help generate persistence in the real variables. The central bank runs monetary policy. Like in Iacoviello and Neri (2010), there is a separability of housing and consumption in the utility function. The model works like a real business cycle model with sticky prices (though not for housing) and some real frictions. Housing collateral then generates wealth effects on aggregate consumption from fluctuations in house prices. In this model, the economy is closed and there is no financial sector. Furthermore, in lending to the impatient households, the patient households apply a maximum loan-to-value ratio which constitutes the financial frictions of the model. While the introduction of variable rates improve the empirical appeal of the model, the collateral constraints imply non-microfounded frictions which limit the volatility of the model as impatient households are assumed always to be borrowing the maximum allowable.

The model described in this chapter follows Iacoviello and Neri (2010) in separating consumers into two groups and in adding housing additively in the utility function. However, the similarities end here. Iacoviello and Neri (2010) use nominal rigidities to match empirical observations in housing price movements. The model in this chapter does not focus on replicating the data and gets its price volatility from the presence of more than one production sector, not from nominal rigidities. This paper also departs from Iacoviello and Neri (2010) in its setup of financial constraints. The primary goal of the aforementioned paper is to bring the model on housing to the observed data so the financial constraints are mostly empirically-inspired but not quite microfounded. In particular, house financing is part of a loan contract with an exogenously-imposed constraint (upper limit on borrowing or lower minimum of downpayment). Such financial constraints lack micro foundations and have limited ability to generate large volatility in the financial position of homeowners. Also problematic is the fixed interest rate consumers are charged on loans (standard feature of collateral constraint models) that fails to capture the reality of subprime adjustable rates. Furthermore, most of the models discussed so far abstract from defaults on loans and therefore there is no default risk. In this sense, mortgage foreclosures are outside the scope of such models. The model developed here instead uses the microfounded financial frictions approach with a default probability of Bernanke, Gertler and Gilchrist (1999). This approach imposes no limits on borrowing and produces an endogenous risk premium that depends on the loan characteristics. It is also better suited to the objective of this chapter to demonstrate the extent of the downturn that households can experience following exogenous disturbances that either affect the price of housing or relax access to mortgage financing.

A different set of studies has focused on credit market liberalization that has improved access to house financing. It has made it possible for existing homeowners to make more use of their housing collateral than before and so has raised the spendability of housing wealth. It also contributed to the subprime mortgage crisis so this issue received renewed interest after the recession. The easier access to home equity loans was emphasised first by Muellbauer and Murphy (1990, 1997) and by the recent literature of the household credit channel, notably Aoki, Proudman and Vlieghe (2004), Iacoviello (2005) and Disney, Bridges and Gathergood (2006). However, all of these were written prior to the recent financial depression. For example, Aoki, Proudman, and Vlieghe (2004), discussed in more detail in Chapter 1, have credit constraints in the homeowner sector that rents out residential services to households. There are heterogeneous consumers as in Iacoviello and Neri (2010) and in this paper but the financial frictions are contained in a housing intermediary sector that functions akin to realty and not within the consumer sector itself. One of the first papers to focus on the issue post-subprime crisis is Muellbauer (2007). It considers the implications of easier credit for the influence of higher house prices on consumer expenditure. It argues that with poorly developed credit markets, the effect is likely to be negative, but becomes positive as access to housing collateral increases and downpayments for first time home buyers fall in relation to house values. The paper uses the permanent income hypothesis to model life cycle income taking a long-run approach. However, it abstracts from microfounded financial rigidities and forgoes a general equilibrium model in favour of a partial equilibrium approach that focuses on consumer decisions.

Recent works such as Favilukis, Ludvigson and Van Nieuwerburgh (2009), Kiyotaki, Michaelides, and Nikolov (2008), and Rios-Rull and Sanchez (2009) have expanded the existing framework of housing and financial frictions in a number of directions. Rios-Rull and Sanchez (2009) argue that under rational expectations, the multi-sectoral DSGE models, even with housing added, will find it very difficult to generate the persistence and volatility of house prices, residential investment and transactions volumes actually observed in recent data. They argue that other assumptions about the generation of expectations are needed, particularly for house price expectations. Favilukis, Ludvigson and Van Nieuwerburgh (2009) take a different direction by emphasizing shifts in risk premiums. They remove the closed economy restriction and introduce a major role both for the (assumed exogenous) rise in loan-to-value ratios for U.S. first-time home buyers (as documented in Duca, Muellbauer and Murphy, 2009) and for a reduction in transactions costs for housing. These changes, they argue, sharply lower the risk premium in both housing and equity markets. Their model can generate substantial variability in national house price-rent ratios, both because they fluctuate endogenously with the state of the economy and because they rise in response to a relaxation of credit constraints and a decline in housing transaction costs. They find that in a calibrated model, the relaxation of the loan-to-value ratio constraint, plus lower transactions costs and an infusion of foreign capital (calibrated to match the increase in foreign ownership of U.S. Treasuries for the period 2000–2007) can explain much of the rise in U.S. house prices relative to rents in this period. The rise leads to a short-run boom in aggregate consumption but also crowd out investment. In some ways their findings are in line with the analysis of the U.S. house price boom in Duca, Muellbauer and Murphy (2009) and the consumption implications modelled in Aron et al. (2009). Aron et al. (2009) use an updated version of an Ando-Modigliani style consumption function in a model that incorporates income growth expectations, income uncertainty, housing collateral and other credit effects. The Aron et al. (2009) version, however, places more emphasis on the overshooting due to extrapolative expectations that is consistent with a more general role for the relaxation of credit constraints, as opposed to a restriction that only affects the risk premium.

Many of the latest works on housing focus explicitly on reproducing the volatility and imperfections produced by the subprime mortgage crisis. Muellbauer (2010) makes another attempt to incorporate many of the liquidity and financial accelerator mechanisms revealed in the global financial crisis that began in 2007. In DSGE models without financial frictions, asset prices are merely a proxy for income growth expectations and play no separate role. However, Muellbauer (2010) uses an Ando-Modigliani consumption function that includes a role for liquidity, uncertainty, time varying credit conditions, wealth and housing collateral effects, as well as income expectations that explains the data well. The author presents new evidence on the striking rejection by aggregate data of the standard consumption Euler equation central to all DSGE models. The paper concludes that, as expected, credit-constrained consumers cannot intertemporally optimise their housing purchase. The model developed in this chapter confirms that borrowing households have difficulties with intertemporal optimization and do not have a standard Euler equation. However, this chapter goes beyond this implication to demonstrate the role of consumers in generating subprime-like downturns since they are subject to external constraints.

Indeed, Johnson and Li (2011) show that subprime borrowers are more likely to be subject to borrowing constraints that hinder their ability to smooth consumption. Subprime mortgages were built on an initial low interest rate which subsequently adjusted upwards. Therefore, the subprime product allowed payments at the teaser rate essentially to help the borrower build up equity, but once the loan reset into the indexed rate, payment obligations increased (Sengupta and Tam, 2010). This was done to reduce the lenders' exposure to a high-risk borrower over a long horizon and essentially force a refinancing of the mortgage. The borrower was prevented from refinancing early by including a penalty for prepayment on the mortgage. The survival of subprime lending rested on increasing house prices that kept housing equity above water (Brueckner, Calem and Nakamura, 2011). In a period of rising house prices, borrowers could build up equity in their homes and, in the eyes of the lender, become less of a risk on subsequent mortgages. This allowed them to refinance at a lower rate (on the subsequent mortgage) which also reduced their likelihood of default. Housing prices were vital for the sustainability of mortgages especially subprime ones. In a regime of rising house prices, borrowers could avoid default by refinancing the mortgage or selling the property (Sengupta and Tam, 2010). If the house price appreciation was sufficiently large, a borrower could recover the costs of refinancing and even choose to extract equity. However, when house prices started on a downward spiral, borrower defaults began to increase sharply. The present paper takes this idea further demonstrating it in a microfounded model. Since falling housing prices were a considerable part of the subprime mortgage crisis, they have an active role in worsening consumer leverage and in the increased likelihood of bankruptcies of indebted home-buyers.

Yet some works such as Duca, Muellbauer and Murphy (2010) point to the fact that the rise and fall of subprime mortgages has many of the classic characteristics of financial bubbles. The bubble starts when new innovations initially work well yielding above normal returns. Over-optimism about subsequent returns then fuels an over-investment in the new product initially spurring large increases in asset prices. These factors alone are usually insufficient, as some source of funding is needed to sustain the build-up, typically in the form of increased leverage and liquidity in the financial system. If there is a perception that the innovation is fundamentally changing the structure of a market ("this time it's different") or agents use the recent past as a guide, the excess investment and the asset price appreciation will be amplified. The combination of these factors gives rise to an asset price bubble. A flurry of momentum trading ensues along the lines described in Piazzesi and Schneider (2008) and Scheinkman and Xiong (2003) building the subprime mortgage bubble through the interaction between housing market inefficiency and expectation formation. In the U.S., there is evidence that easier credit standards raised permissible loan-to-value ratios for home purchase mortgages and increased the ability of households to borrow against housing equity (Duca, Muellbauer, and Murphy, 2009a and 2009b). Then an event sparks a downward reassessment of the new asset and price expectations which in turn unwinds the over-optimism and triggers the bursting of a bubble. This paper also studies the disastrous consequences of investor over-optimism but in a microfounded DSGE model rather than a descriptive approach. It shows that lax lending is the other main trigger of housing downturns that, along with falling housing prices, that can induce a lasting recession.

2.3 The Consumer Mortgage Contract

2.3.1 Description of the Consumer Mortgage Contract

This section develops the consumer mortgage contract in a partial equilibrium framework, taking as given the price of the collateral and the risk-free rate of interest. The subsequent section endogenises these variables as part of a general equilibrium solution. The model builds on the one in Chapter 1 with two notable elaborations. First, financial frictions move from the production sector to the consumer sector and second, households are split into two groups to facilitate the borrowing process.

In order to motivate credit frictions in the household sector, this model formally separates consumers into two types following Iacoviello and Neri (2010) (Figure 2.1). Ricardian consumers possess no intrinsic risk of default and can borrow at the risk-free rate. They have significant non-wage income in the form of revenue from owning shares in final good firms (through their deposits with investors) and from absorbing the profits of capital producers. Their wealth allows them to finance their own housing purchase entirely. Ricardians save any unused income as deposits that are lent to investors and in turn to final good firms and to credit-constrained consumers. Credit-constrained consumers, on the other hand, receive only wage income so their net worth is not enough to finance their housing acquisition. Therefore, they must obtain external financing from investors.

The housing purchase of credit-constrained consumers is financed by a mortgage contract similar to the firm loan contract in Chapter 1. The mortgage contract is necessitated by the existence of an agency problem that makes external borrowing for house purchase more expensive than own funds. The reason for this discrepancy is that credit-constrained consumers, similar to producers in Chapter 1, possess an inherent risk of default that is known to them but unobservable to lenders. In this case, investors cannot perfectly observe the borrower's ability to repay and must pay an auditing cost in order to learn the bankruptcy prospect of the mortgage-buyer. When indebted consumers go bankrupt, investors pay the monitoring fee and take possession of all of the borrower's remaining assets. In the household context, these auditing costs can be interpreted as the costs of legal proceedings to the value the borrower's assets and the administrative costs of selling the house to realise its collateral value. The presence of these financial frictions motivates the need for a loan (mortgage) contract as opposed to a share contract that usually takes place in the absence of idiosyncratic uncertainty. Additionally, the idiosyncratic uncertainty implies that credit-constrained consumers cannot borrow directly from their Ricardian counterparts. The intermediary services of investors are necessary not only to disburse the loan but also to conduct the necessary monitoring.

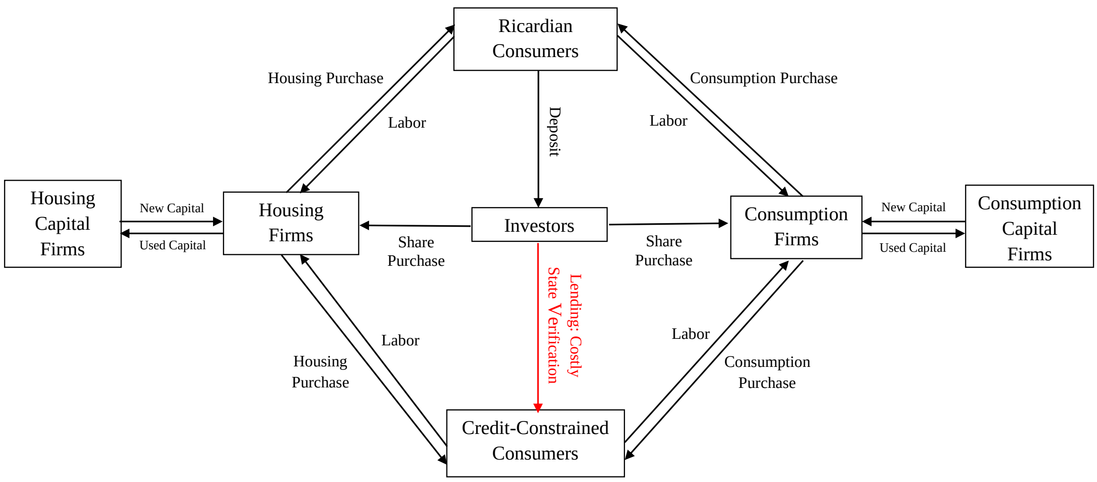


Figure 2.1: Model with Financial Frictions in the Consumer Sector

Credit-constrained households obtain a loan from investors who pool the deposits of Ricardian consumers, paying them the risk-free rate, and lend to credit-constrained households at a markup consumer interest rate. The difference between the borrowing and the lending rates, known as the external finance (risk) premium, exists in order to offset the expenses associated with a potential bankruptcy of mortgage-buyers. The greater the default prospect, the more likely the lender will have to pay the auditing cost so the risk premium will be higher. Hence the external finance premium is a function of the default rate, the collateral (net worth) of credit-constrained consumers and the value of the mortgage. Investors do not make any profit on the loan to indebted households. The risk premium is exactly sufficient to offset the costs associated with a potential borrower default.

The rest of the economy remains the same. There are two sectors that produce a perishable consumption good and durable housing. The two final good sectors are supported by sector-specific capital manufacturers. Both production sectors face no risks so they finance their capital acquisition by the means of share arrangements with investors.

2.3.2 The Loan Contract of Credit-Constrained Households

The setup for the mortgage contract is fairly similar to the loan contract for firms described in detail in Chapter 1 so this section just outlines the main characteristics. The mortgage loan contract begins when at time t, credit-constrained household i obtains a mortgage from investors to purchase housing at price p_t for use at $t+1$. To improve the model tractability, the paper assumes that mortgage contacts last one period only and are renegotiated every period. This is not a significant departure from reality since it captures the adjustable interest rate phenomenon of the subprime downturn. In this sense, this loan contract can be thought as a multi-period mortgage with an adjustable rate. A multi-period mortgage with a fixed rate would not be properly microfounded since it will not update according to the most recent information on the value of the housing stock and the default risk and hence would not provide a fair return to lenders.

The quantity of housing purchased is denoted $H_{i,t}^C$ with the subscript denoting the household that obtains it and the period in which the housing is purchased and the superscript indicating the type of consumer that purchases it (C denotes credit-constrained households and R refers to Ricardian consumers). Each credit-constrained household faces an idiosyncratic shock ω_i^C to their return to housing. In each period, homeowner i draws from a distribution of ω_i^C where ω_i^C follows a log-normal distribution with mean one and variance $\left(\sigma^C\right)^2$. The variance $\left(\sigma^C\right)^2$ can also be thought as a measurement of the volatility of the default probability ω_i^C - a lower variance would imply safer loan returns. The ex-post gross return on housing is $\omega_i^C R^C$, where ω_i^C is an idiosyncratic disturbance to household i's return and R^C is the ex-post aggregate return to housing for credit-constrained consumers (the markup consumer interest rate paid on the mortgage). The idiosyncratic shock ω_i^C is also independent across time and across credit-constrained households. Lenders can diversify away the idiosyncratic risk of the borrowers by investing in a lot of borrowers.

This setup follows the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999) described in detail in Chapter 1. This approach assumes that there is costly state verification on the lender's part that motivates financial frictions. If investors want to observe the idiosyncratic shock ω_i^C for a specific homeowner, they need to pay a monitoring cost which is a fraction μ^C of the homeowner's total housing stock. Due to the monitoring cost, it would be too costly to monitor every contract. Therefore, there is a cutoff value of the shock $\bar{\omega}_i^C$ above which investors do not monitor and below which they do. Above the cutoff, the credit-constrained homeowner gets a return sufficient to pay investors a fixed rate of interest Z^C and keeps what is left for themselves; investors have no incentive to monitor the homeowner' true return. Below the cutoff, credit-constrained households are bankrupt, and the lenders pay the monitoring cost and take their entire wealth.

The investors who make loans to credit-constrained households are risk-neutral and the opportunity cost of their funds is the risk-free interest rate. They will agree to lend to credit-constrained households only if they can break even on their investment. This implies that at the time of the loan repayment, the interest rate R^C charged to borrowers must be such that the expected gross return on the loan must equal the opportunity cost of lending:

$$\bar{\omega}_i^C R^C p H_i^C = Z_i^C (p H_i^C - N_i^C)$$

The left hand side is the gross revenue from housing to credit-constrained homeowner i whose risk $\bar{\omega}_i^C$ is just at the cutoff point. The right hand side is the amount the consumer needs to repay the investors: the total value of the loan pH_i^C minus the net worth of the borrower N_i^C. The household is just able to repay the loan at the contractual rate.

The idiosyncratic shock is continuous and has a cumulative distribution function $F^C(\omega^C)$. To be perfectly insured against the idiosyncratic shocks of households, each investor signs contracts with an infinite number of households. The solution of the loan contract (explained in detail in Chapter 1) defines how the investment revenue is split between the lenders (investors) and the borrowers (credit-constrained households). Each lender gets a gross share $\Gamma^C(\bar{\omega}_i^C)$ that is the sum of the share (before monitoring) in case the household defaults $\int_0^{\bar{\omega}_i^C} \omega^C dF^C(\omega^C)$ and the fixed interest payment the lender gets if the debtor does not default with probability $(1 - F^C(\bar{\omega}_i^C))$. Let the defaulting share $\int_0^{\bar{\omega}_i^C} \omega^C dF^C(\omega^C)$ be denoted $G^C(\bar{\omega}_i^C)$, then the gross share $\Gamma^C(\bar{\omega}_i^C)$ is:

$$\Gamma^C(\bar{\omega}_i^C) = \int_0^{\bar{\omega}_i^C} \omega^C dF^C(\omega^C) + (1 - F^C(\bar{\omega}_i^C))\bar{\omega}_i^C = G^C(\bar{\omega}_i^C) + (1 - F^C(\bar{\omega}_i^C))\bar{\omega}_i^C$$

The borrower receives the remaining share $(1 - \Gamma^C(\bar{\omega}_i^C))$. After taking into account the monitoring cost $\mu^C G(\bar{\omega}_i^C)$, the net share that goes to the lender is:

$$\Gamma^C(\bar{\omega}_i^C) - \mu^C G^C(\bar{\omega}_i^C)$$

By assumption, the investors make a zero profit on their loans to credit-constrained households. Investors can diversify away the loan risk so their opportunity cost is the risk-free rate R_t. Hence the zero-profit condition for lending to a particular borrower at time t to be repaid at time $t+1$ is:

$$E_t \left(\left(\Gamma^C(\bar{\omega}_{i,t+1}^C) - \mu^C G^C(\bar{\omega}_{i,t+1}^C) \right) R_{t+1}^C \right) p_t H_{i,t}^C = R_t \left(p_t H_{i,t}^C - N_{i,t}^C \right) \tag{2.3.1}$$

The left hand side is the net return on the loan to investors (the gross return $E_t \Gamma^C(\bar{\omega}_{i,t+1}^C)$ minus the monitoring cost $\mu^C E_t G^C(\bar{\omega}_{i,t+1}^C)$) at the time of the repayment. The right side is the opportunity cost of lending the funds (the value of the housing purchase $p_t H_{i,t}^C$ minus the consumer's net worth $N_{i,t}^C$) valued elsewhere at the risk-free rate R_t that prevails at the time of the loan agreement.

Since the price of housing, the risk-free interest rate and the homeowner's net worth $N_{i,t}^C$ are predetermined, at time t the credit-constrained household gets to choose a pair of $(E_t \bar{\omega}_{i,t+1}^C,\ H_{i,t}^C)$ according to the zero-profit condition. Given the other variables, this is equivalent to the lenders offering a schedule of loans $E_t B_{i,t+1}^C$ and a non-default interest rate R_t to the household.

In this partial equilibrium setting, the credit-constrained homeowner i is faced with a menu of loan $E_t B_{i,t+1}^C$ and interest rate $E_t R_{t+1}^C$ where both the loan and the interest rate are related by the participation constraint of the investors. At time t, borrowers choose the optimal pair of housing and the expected cutoff risk $(H_{i,t}^C,\ E_t \bar{\omega}_{i,t+1}^C)$ to maximise their expected share of the loan at time $t+1$ of the loan repayment:

$$max\ E_t\left(\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t} \tag{2.3.2}$$

subject to the zero profit condition of investors (2.3.1).

Like in Chapter 1, households optimise their mortgage from the perspective of time t in terms of $E_t R^C_{t+1}$ but repay it at time $t+1$ at the realised rate R^C_{t+1}. The mortgage is state-contingent so its return depends on the realization of the ex-post return on housing R^C_{t+1}. It is at this rate that the loan contract is repaid, rather than at the expected return on housing $E_t R^C_{t+1}$. The expected return on housing $E_t R^C_{t+1}$ is all that is known at the time the contract is negotiated but it does not account for unexpected shocks that can affect the repayment ability of credit-constrained consumers at the mortgage maturity date. Hence to ensure a fair return, investors negotiate the loan contact at $E_t R^C_{t+1}$ but require repayment at the realised ex-post return R^C_{t+1} that is formed only after all disturbances have occurred.

The Lagrangian of credit-constrained consumers is:

$$\begin{aligned} L^C_t \ &= \ E_t\left(\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t} + \\ &\quad +\lambda_t\left[E_t\left(\left(\Gamma^C(\bar{\omega}^C_{i,t+1})-\mu^C G^C(\bar{\omega}^C_{i,t+1}\right)R^C_{t+1}\right)p_t H^C_{i,t} - R_t\left(p_t H^C_{i,t} - N^C_{i,t}\right)\right] \end{aligned}$$

The first order conditions are:

$$\begin{aligned} &E_t\bar{\omega}^C_{i,t+1}: \ \ -E_t\left(\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})R^C_{t+1}\right)p_t H^C_{i,t} + \\ &+\lambda_t\left[E_t\left(\left(\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t}\right] = \end{aligned}$$

$$\begin{aligned} &H^C_{i,t}: \ \ E_t\left(\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t H^C_{i,t} + \\ &+\lambda_t\left[E_t\left(\left(\Gamma^C(\bar{\omega}^C_{i,t+1})-\mu^C G^C(\bar{\omega}^C_{i,t+1})\right)R^C_{t+1}\right)p_t - R_t p_t\right] = 0 \end{aligned}$$

Solving the maximization problem gives the external finance (risk) premium defined as the relationship between the risk-free rate and the markup consumer interest rate $\frac{E_t R^C_{t+1}}{R_t}$

as a function of the cutoff default risk $E_t\bar{\omega}^C_{i,t+1}$:

$$\frac{E_tR^C_{t+1}}{R_t} = E_t\left(\frac{\frac{\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{i,t+1})\right)}}{\left(1-\Gamma^C(\bar{\omega}^C_{i,t+1})\right)+\frac{\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{i,t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{i,t+1})\right)}\left(\Gamma^C(\bar{\omega}^C_{i,t+1})-\mu^C G^C(\bar{\omega}^C_{i,t+1})\right)}\right) \tag{2.3.3}$$

This equation implies that unless the monitoring cost μ^C is zero, investors would require the aggregate return to the their investment $E_tR^C_{t+1}$ to be larger than the risk-free rate R_t.

An interesting feature of the risk premium $\frac{E_tR^C_{t+1}}{R_t}$ is that it does not depend on the amount of the loan since the monitoring fee μ^C is scale independent. It depends only on the cutoff risk $E_t\bar{\omega}^C_{i,t+1}$. This implies that a higher cutoff risk will adjust the consumer repayment rate $E_tR^C_{t+1}$ upward for a given risk-free rate. Similarly, the interest rates ratio is proportional to the variance $\left(\sigma^C\right)^2$ (the volatility parameter) of $E_t\bar{\omega}^C_{i,t+1}$ so a lower risk volatility would reduce the gap between the consumer rate and risk-free interest rate.

Since every credit-constrained consumer solves the same loan contract, they choose the same expected cutoff risk $E_t\bar{\omega}^C_{i,t+1}$ to maximise their share of the loan. Each household is essentially a representative home-buyer so equation (2.3.3) holds for every mortgage-buyer. Hence it is possible to aggregate across all credit-constrained households so that this equation is valid for the whole economy. The ratio of the realised return to housing to the expected return on the right hand side is determined macroeconomically, and given that every borrower chooses the same expected cutoff default risk, then the aggregate cutoff risk is $E_t\bar{\omega}^C_{i,t+1} = E_t\bar{\omega}^C_{t+1}$ Hence equation (2.3.3) holds on the aggregate level and determines the macroeconomic cutoff risk $E_t\bar{\omega}^C_{t+1}$:

$$\frac{E_tR^C_{t+1}}{R_t} = E_t\left(\frac{\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}}{\left(1-\Gamma^C(\bar{\omega}^C_{t+1})\right)+\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}\left(\Gamma^C(\bar{\omega}^C_{t+1})-\mu^C G^C(\bar{\omega}^C_{t+1})\right)}\right) \tag{2.3.4}$$

Similarly, the aggregate amount of net worth held by indebted households in the economy is $N^C_t = \sum N^C_{i,t}$ and the aggregate amount of housing purchased by credit-constrained consumers is $H^C_t = \sum H^C_{i,t}$.

The first order condition (2.3.4) and the participation constraint (2.3.1) hold at any point in time in which there have not been any unanticipated shocks. However, even after a shock occurs at time $t+1$, the borrowed share $p_t H_t^C - N_t^C$ is already fixed so investors must continue to receive the risk-free rate R_t. Taken together, these two equations determine the realised cutoff value $\bar{\omega}_{t+1}^C$ that adjusts after an unexpected disturbance at $t+1$. Hence although consumers optimise using the expected participation constraint of lenders, the model utilises the realised participation constraint that incorporates the realised default risk along with the expected first order condition:

$$\left(\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C = R_{t-1}\left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right) \tag{2.3.5}$$

A positive productivity shock reduces the price of housing p_t. This causes the net worth of indebted households to decrease. To satisfy their participation constraint, the investors will either reduce lending or raise the interest rate they charge to the households or both which implies that the participation constraint contracts. However, in the general equilibrium context, the supply of housing is predetermined. To maintain equilibrium with the original level of housing, the first-order condition moves to a higher level. This means that the ratio of the realised return to housing to the risk-free rate increases raising the external finance premium. Observing this, households choose a higher cutoff risk $\bar{\omega}^C$ and an unchanged profit-maximizing level of housing. This is not the long-run equilibrium. Over time, consumers reduce their housing stock and return to the previous profit-maximizing level of $\bar{\omega}^C$.

In order to complete the partial equilibrium setting, it is necessary to determine the evolution of the credit-constrained households' net worth. In any given period, the equity of the credit-constrained households, V_t^C, is the remaining share of the mortgage after replaying back investors:

$$V_t^C = \left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C \tag{2.3.6}$$

Consumers can spend this dividend income on new housing. When house prices fall - and therefore the equity of the households V_t^C - households face the following decision problem. If they decrease housing demand today, current household utility would fall. But, if demand were kept constant, net worth would decrease, increasing the future external finance premium. Thus households face a trade-off between current housing purchase and future borrowing.

It is also necessary to make sure that credit-constrained consumers do not eventually grow out of their financial constraints. This paper assumes that every period a constant fraction $1-\nu^C$ of households retire. When they retire, they spend their remaining equity on consumption. The retirement consumption $C^C_{E,t}$ of credit-constrained consumers is:

$$C^C_{E,t} = \left(1-\nu^C\right)\left(1-\Gamma^C(\bar{\omega}^C_t)\right) R^C_t p_{t-1} H^C_{t-1} \tag{2.3.7}$$

Over time, the number of credit-constrained consumers decreases but their individual net worth increases as they cycle through many periods of mortgage contracts. Hence macroeconomically, the value of the net worth remains the same.

Credit-constrained consumers need to get started on their net worth with some income not devoted to purchasing consumption goods and housing. This is equivalent to establishing a savings account dedicated to the initial mortgage downpayment. The model assumes they provide one unit of labour inelastically to the production of housing that generates a wage w^C_t. This labour supply is solely for the purposes of starting their net worth accumulation and is weighted heavily so that it does not distort the overall labour supply. Having determined the period equity and the start-up net worth of credit-constrained consumers, is it easy to describe the evolution of their net worth. The evolution of the credit-constrained households' net worth is the sum of equity of non-retiring households plus their income from work:

$$N^C_t = \nu^C V^C_t + w^C_t \tag{2.3.8}$$

2.3.3 The Share Contracts of the Consumption and Housing Sectors

The contracts between investors on one side and producers of consumption and of housing on the other hand are share contracts. As equity holders of final goods firms in both sectors, investors finance their capital purchase and absorb their profits and losses. No monitoring takes place. The manufacturers and investors split the revenue according to the shares of their investments, regardless of the idiosyncratic shocks to consumption producers ω^F and to housing producers ω^H. To diversify away from the firm-specific idiosyncratic risk, each consumer will invest in an infinite number of firms.

Investors finance the capital purchase $x_t^F K_t^F$ of consumption firms, which occurs one period prior to production. The aggregate return to capital purchased by the consumption firms is the risk-free rate R_t. In the following period, the share of consumption firms' revenue that goes to investors is Γ_t^F.

$$\Gamma_t^F R_t x_t^F K_t^F = \frac{x_t^F K_t^F - N_t^F}{x_t^F K_t^F} \int_0^\infty \omega^F R_t x_t^F K_t^F dF(\omega^F)$$

Since lenders can diversify over an infinite number of food producers and there is no upper cutoff amount of ω^F, $\int_0^\infty \omega^F dF(\omega^F) = 1$ This implies that the share is independent of the idiosyncratic shock ω^F:

$$\Gamma_t^F = \frac{x_t^F K_t^F - N_t^F}{x_t^F K_t^F} \tag{2.3.9}$$

Firms in the consumption sector accrue profits which they split with investors according to their share contract. In any given period, the equity of consumption firms is their part of the share agreement in the previous period:

$$V_t^F = \left(1 - \Gamma_t^F\right) R_{t-1} x_{t-1}^F K_{t-1}^F = R_{t-1} N_{t-1}^F \tag{2.3.10}$$

In order to prevent consumption firms from growing out of the financial constraints, the model assumes that at the end of every period, a constant fraction $1-\nu^F$ exits the market. Exiting firms consume immediately their remaining equity. In this case consumption producers' consumption $C_{E,t}^F$ on exit is:

$$C_{E,t}^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F \tag{2.3.11}$$

Similarly to consumers, firms need to start off with some net worth so this setup assumes they provide inelastically one unit of labour to the production of their output for which they receive a wage w_t^F. The labour contribution of consumption producers is significantly discounted to avoid crowding out regular household labour supply. The evolution of firms' net worth is the sum of equity of surviving firms plus their income from work:

$$N_t^F = \nu^F V_t^F + w_t^F \tag{2.3.12}$$

Investors also finance the capital purchase $x_t^H K_t^H$ of housing firms. Similarly to the consumption sector, there is no idiosyncratic risk in the housing sector and firms can borrow at the risk-free rate. The share purchase in the housing sector Γ_t^H equals:

$$\Gamma_t^H = \frac{x_t^H K_t^H - N_t^H}{x_t^H K_t^H} \tag{2.3.13}$$

Housing producers also accrue profits which they split with investors according to their share contract. In any given period, the equity of housing firms is:

$$V_t^H = \left(1 - \Gamma_t^H\right) R_{t-1} x_{t-1}^H K_{t-1}^H = R_{t-1} N_{t-1}^H \tag{2.3.14}$$

Housing producers' consumption $C_{E,t}^H$ on exit is:

$$C_{E,t}^H = \left(1 - \nu^H\right) R_{t-1} N_{t-1}^H \tag{2.3.15}$$

Housing firms also provide inelastically one unit of labour to the production of their output for which they receive a wage w_t^H. The labour contribution of housing producers is also discounted. The evolution of firms' net worth is the sum of equity of surviving firms plus their income from work:

$$N_t^H = \nu^H V_t^H + w_t^H \tag{2.3.16}$$

2.4 The Complete Model with Consumer Mortgage

This section embeds the partial equilibrium of the loan contract derived in the previous section into a general equilibrium framework that endogenises the risk-free rate R_t and the price of housing p_t. The economy consists of two production sectors: consumption and housing. Capital producers supply sector-specific capital to both types of final good firms. Households consume both consumption and housing.

2.4.1 Consumption Capital Sector

Firms that produce capital K_t^F for the consumption sector own technology that converts final goods into capital. They use depreciated capital which they purchase from final good firms in the consumption sector and make investments to produce new capital. The investment I_t^F is consumption. The newly produced capital is sold back to consumption producing firms.

There are standard quadratic adjustment costs to producing capital. The capital adjustment costs for the consumption capital are:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F \tag{2.4.1}$$

The function J is such that $J' > 0$ and $J" < 0$. New capital is produced within the period and sold to final good producing firms at the price x_t^F. The profit maximization problem of capital producing firms is:

$$max\ x_t^F K_t^F - (1-\delta)x_t^F K_{t-1}^F - I_t^F = J\left(\frac{I_t^F}{K_{t-1}^F}\right)x_t^F K_{t-1}^F - I_t^F$$

The profit maximization problem of capital producing firms solves for the optimal condition for investment:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1 \tag{2.4.2}$$

2.4.2 Consumption Producers

Firms in the consumption sector use capital K_t^F, labour L_t^F and sector-specific technology A_t^F to produce their output. Consumption producing firms buy capital one period in advance. They borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. At the end of each production period, they sell the remaining capital back minus depreciation to capital producing firms. The production function of consumption firms is:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_t^F)^{(1-\alpha_F)} \tag{2.4.3}$$

According to the share purchase setup, consumption firms supply inelastically one unit of labour to their own production in order to start the accumulation of their net worth. Factoring in this labour supply in the production function, the total labour supply in the consumption sector is:

$$L_t^F = (L_{F,t}^F)^{\Omega_F} (L_{H,t}^F)^{(1-\Omega_F)} = (L_{H,t}^F)^{(1-\Omega_F)} \tag{2.4.4}$$

where $L_{H,t}^F$ is the regular labour supply by both types of consumers and $L_{F,t}^F$ is the labour supply by consumption producers.

Recasting the production function only in terms of household labour yields:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)} \tag{2.4.5}$$

The firms in the sector are perfectly competitive so they maximise profits subject to input costs. The price of consumption is normalized to 1. The Lagrangian is:

$$\mathcal{L}_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)} + (1-\delta) x_t^F K_{t-1}^F - w_t L_{H,t}^F - R_{t-1} x_{t-1}^F K_{t-1}^F$$

The first-order conditions for optimal capital and labour are:

$$w_t = (1-\alpha_F)(1-\Omega_F) \left(\frac{Y_t^F}{L_{H,t}^F} \right) \tag{2.4.6}$$

$$R_t = \frac{E_t \left(\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta) x_{t+1}^F \right)}{x_t^F} \tag{2.4.7}$$

The wage consumption firms receive for their labour supply is:

$$w_t^F = (1-\alpha_F)\, \Omega_F Y_t^F \tag{2.4.8}$$

2.4.3 Housing Capital Sector

Firms that produce capital K_t^H for the housing sector own technology that converts goods into capital. They use depreciated capital which they purchase from housing firms and make investments to produce new capital. The investment I_t^H is consumption. The newly produced capital is sold back to housing producers.

Housing capital is subject to the same adjustment costs as consumption capital. The housing capital production equation is:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H \tag{2.4.9}$$

New capital is produced within the period and sold to final good producing firms at the price x_t^H. The profit maximization problem of housing capital firms is:

$$max\ x_t^H K_t^H - (1-\delta)x_t^H K_{t-1}^H - I_t^H = J\left(\frac{I_t^H}{K_{t-1}^H}\right)x_t^H K_{t-1}^H - I_t^H$$

The profit maximization problem of capital producing firms gives the first-order condition for investment in housing capital:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1 \tag{2.4.10}$$

2.4.4 Housing Producers

Housing is a multi-period good (as described in the second version of the model in Chapter 1) that survives for more than one period, unlike consumption which is not durable beyond the period in which it is produced. The production of housing follows closely Iacoviello and Neri (2010). Housing producers use capital K_t^H, labour L_t^H, land X_t and sector-specific technology A_t^H to produce new houses. Housing firms also buy capital one period earlier. In this version, housing firms have no aggregate uncertainty so they can borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. At the end of each production period, they sell the remaining capital minus depreciation back to housing firms. The production function of housing firms is:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (X_{t-1})^{\varepsilon} (L_t^H)^{(1-\alpha_H-\varepsilon)} \tag{2.4.11}$$

The amount of land is fixed and normalized to one. Furthermore, both the share purchase setup and the loan contract of credit-constrained consumers assumed that housing producers and indebted households supply inelastically one unit of labour in order to start the accumulation of their respective net worth. Factoring in that labour supply, the total labour supply in the production of housing by origin is:

$$L_t^H = (L_{F,t}^H)^{\Omega_H} (L_{CC,t}^H)^{\Omega_C} (L_{H,t}^H)^{(1-\Omega_H-\Omega_C)} = (L_{H,t}^H)^{(1-\Omega_H-\Omega_C)} \tag{2.4.12}$$

Where $L_{H,t}^H$ is the labour supply of both types of consumers for the purpose of financing their regular consumption, $L_{CC,t}^H$ is the labour supply of credit-constrained consumers in order to start their net worth accumulation and $L_{F,t}^H$ is the labour supply of housing producers.

Recasting the production function only in terms of household labour and factoring in the fixed supply of land yields:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)} \tag{2.4.13}$$

The price of housing is p_t. In this chapter, housing firms in the sector are perfectly competitive so they maximise profits subject to input costs. Their Lagrangian is:

$$\mathcal{L}_t^H = p_t A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)} + (1-\delta) x_t^H K_{t-1}^H - R_{t-1} x_{t-1}^H K_{t-1}^H - w_t L_{H,t}^H$$

The Lagrangian solves for the following optimal conditions for housing capital and labour:

$$w_t = (1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right) \tag{2.4.14}$$

$$R_t = \frac{E_t\left(\alpha_H \frac{p_{t+1} Y_{t+1}^H}{K_t^H} + (1-\delta) x_{t+1}^H\right)}{x_t^H} \tag{2.4.15}$$

The wage housing firms receive for their labour supply is:

$$w_t^H = (1 - \alpha_H - \varepsilon)\, \Omega_H p_t Y_t^H \tag{2.4.16}$$

The wage credit-constrained consumers receive for the purpose of starting their net worth is:

$$w_t^C = (1 - \alpha_H - \varepsilon)\, \Omega_C p_t Y_t^H \tag{2.4.17}$$

2.4.5 Consumers

Both Ricardian and credit-constrained consumers have the same preferences. Households choose consumption C_t^i, housing H_t^i and labour L_t^i subject to their respective budget constraints. Here the superscript i denotes the type of consumers: R for Ricardian and C for credit-constrained. As in the second model developed in Chapter 1, housing is purchased one period in advance. In addition to the transactional delays explained in Chapter 1, there are financial motivations for the advance purchase. Since credit-constrained consumers purchase housing in advance of using it and they do not have non-wage income like Ricardians, they need the mortgage arrangement to facilitate the housing acquisition. Furthermore, the mortgage is intertemporal where consumers optimise their expected share of the loan at the time of the housing purchase but repay it only in the subsequent period after using the housing. As a result, they are exposed to unexpected shocks at the time of repayment so the mortgage is an inherently risky undertaking.

Each household seeks to maximise its lifetime expected utility:

$$U = E_t \sum_{t=0}^{\infty} \beta^t U(C_t^i, H_{t-1}^i, L_t^i) \tag{2.4.18}$$

The period utility of each household is given by:

$$U(C_t^i,\ H_{t-1}^i,\ L_t^i) = log(C_t^i) + \kappa log(H_{t-1}^i) - \gamma \frac{L_t^{i(1+\varphi)}}{1+\varphi} \tag{2.4.19}$$

The period utility function is separable in consumption C_t^i, housing H_{t-1}^i and labour

L_t^i. Housing is purchased one period in advance and consumed the following period. At the end of the period, the remaining housing minus depreciation is sold back on the market. Following Iacoviello and Neri (2010), housing enters the utility function additively, rather than as part of a consumption aggregator in order to demonstrate its direct effect on consumer decisions. The additive nature of the utility function also facilitates housing to be be purchased both directly (by Ricardians) and via a mortgage (by credit-constrained consumers). There is a taste parameter κ that reflects the relative preference for consumption and housing.

2.4.5.1 Ricardian Consumers

Ricardian consumers purchase consumption and housing. Each period, they lend an amount B_t at the risk-free rate. Their lending covers the mortgage loan to credit-constrained households as well as finances the capital purchase of consumption firms and housing producers through their respective share arrangements. Ricardian consumers also absorb the profits Π_t of both capital sectors. The budget constraint of Ricardian households is:

$$C_t^R + p_t H_t^R + B_t = w_t L_t^R + (1-\delta) p_t H_{t-1}^R + R_{t-1} B_{t-1} + \Pi_t \tag{2.4.20}$$

Ricardian consumers maximise their utility function subject to this budget constraint. The left hand side reflects their consumption and housing purchase as well as their lending, while the right hand side represents their income from wages and from reselling the non-depreciated housing from the previous period as well as their returns from lending and from capital firms profits. The Lagrangian for Ricardian consumers is:

$$\begin{aligned} \mathcal{L}_t^R &= E_t \sum_{t=0}^{\infty} \beta^t \left[log(C_t^R) + \kappa log(H_{t-1}^R) - \gamma \frac{L_t^{R(1+\varphi)}}{1+\varphi} \right] + \\ & +\lambda_t \left(w_t L_t^R + (1-\delta) p_t H_{t-1}^R + R_{t-1} B_{t-1} + \Pi_t - C_t^R - p_t H_t^R - B_t \right) \end{aligned}$$

The optimization problem yields three first-order conditions for consumption, housing and leisure. The first-order condition for the consumption-labour tradeoff is fairly standard:

$$\gamma(L_t^R)^\varphi C_t^R = w_t \tag{2.4.21}$$

The relationship between consumption and housing reflects the fact that housing is purchased one period in advance so the tradeoff between housing and consumption depends on both the current and on the future price of housing as well as on the intertemporal consumption substitution and the depreciation rate of housing:

$$E_t\left(\frac{p_t}{\beta}\left(\frac{C_{t+1}^R}{C_t^R}\right) - (1-\delta)p_{t+1}\right)H_t^R = \kappa E_t C_{t+1}^R \tag{2.4.22}$$

Ricardian consumers also have a standard Euler equation:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t \tag{2.4.23}$$

Combining the last two equations yields a simpler expression for the consumption-housing substitution:

$$E_t\left(R_t p_t - (1-\delta)p_{t+1}\right)H_t^R = \kappa E_t C_{t+1}^R \tag{2.4.24}$$

2.4.5.2 Credit-Constrained Consumers

Just like Ricardian households, credit-constrained households also consume both consumption and housing. They earn income only from labour and do not own any shares. Since their income is not sufficient to allow them to purchase housing in full, they must obtain a mortgage from investors. Their mortgage is subject to an idiosyncratic risk of default so their borrowing is not riskless. Due to this probability of bankruptcy, they cannot borrow at the risk-free rate R_t and can do so only at the consumer interest rate R_t^C. As a result, buying a house is costly for them. Credit-constrained households also cannot optimise intertemporally their purchase of consumption since they have no access to risk-free financing. Hence their consumption needs must be met solely with their wages after their mortgage is repaid. The budget equation for credit-constrained households is:

$$C_t^C + R_t^C p_{t-1} H_{t-1}^C = w_t L_t^C + (1-\delta) p_t H_{t-1}^C \tag{2.4.25}$$

The left hand side reflects their consumption purchase as well as their housing mortgage, while the right hand side represents their income from wages and returns from reselling the non-depreciated housing from the previous period. Credit-constrained consumers maximise their utility function subject to this budget constraint. The Lagrangian of credit-constrained consumers is:

$$\begin{aligned} \mathcal{L}_t^C &= E_t \sum_{t=0}^{\infty} \beta^t \left[log(C_t^C) + \kappa log(H_{t-1}^C) - \gamma \frac{L_t^{C(1+\varphi)}}{1+\varphi} \right] + \\ & +\lambda_t \left(w_t L_t^C + (1-\delta) p_t H_{t-1}^C - C_t^C - R_t^C p_{t-1} H_{t-1}^C \right) \end{aligned}$$

The optimization problem yields two first-order conditions for housing and leisure. The first-order condition for consumption-labour tradeoff is identical to that of Ricardian households:

$$\gamma (L_t^C)^{\varphi} C_t^C = w_t \tag{2.4.26}$$

The relationship between consumption and housing however depends on the consumer interest rate R_t^C instead of the risk-free rate R_t:

$$\left(R_t^C p_{t-1} - (1-\delta) p_t \right) H_{t-1}^C = \kappa C_t^C \tag{2.4.27}$$

The first-order condition for housing-consumption tradeoff for credit-constrained consumers is lagged, unlike that for Ricardian households which is forward-looking. This is due to the fact that the return rate at which credit-constrained consumers repay their mortgage is state-contingent and depends on the realization of the ex-post return on housing R_t^C that incorporates all shocks at the time of the repayment. The realised return on housing R_t^C depends on the past purchasing price p_{t-1} and the current selling price p_t of housing. It does not depend on the expected future price of housing which is unknown at the time of the loan contract. Hence equation (2.4.27) is a function of the ex-post return on housing R_t^C, while the corresponding equation for Ricardian households (2.4.24) is not lagged since Ricardians borrow at the risk-free rate which is not state-contingent.

Finally, credit-constrained consumers cannot optimise intertemporally their consumption purchase since they cannot borrow at the risk-free rate R_t. Their demand for consumption must be met by their income once all housing loans are repaid:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1}) H_{t-1}^C \tag{2.4.28}$$

Any disturbances that can increase the mortgage repayment, would crowd out regular consumption and the resulting diminished demand might impact negatively the consumption sector.

2.4.6 Market Clearing

Market clearing requires that the output of consumption must cover household consumption, consumption by conventional good producers on exit and consumption by credit-constrained consumers on retirement as well as investment in the two sectors:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + I_t^F + I_t^H \tag{2.4.29}$$

Housing is a multi-period good and each period a fraction δ of the housing available on the market depreciates. The remaining non-depreciated housing along with new production constitutes the available housing in the subsequent period. In each period, the sum of new production and leftover housing must meet the housing needs of consumers as well as the consumption of housing firms on exit and the monitoring costs of the mortgage contract:

$$p_t Y_t^H + (1-\delta) p_t (H_{t-1}^R + H_{t-1}^C) = p_t (H_t^R + H_t^C) + C_{E,t}^H + \mu^C G^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{2.4.30}$$

The labour that both types of households supply equals the demand by housing and consumption firms:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C \tag{2.4.31}$$

Finally, Ricardian consumer lending must equal the loan to credit-constrained households as well as the share purchase of consumption firms and of housing firms:

$$B_t = p_t H_t^C - N_t^C + x_t^F K_t^F - N_t^F + x_t^H K_t^H - N_t^H \qquad (2.4.32)$$

All the equations for this model are in Appendix 2.A.

2.4.7 Model Calibration

The calibrated model is quarterly so four periods correspond to a year. Most of the parameters are identical to the ones in Chapter 1. The only notable differences pertain to the calibration of the consumer utility function (Table 2.1). The labour supply of credit-constrained consumers, consumption producers, housing producers and financial institutions for the purpose of starting their net worth is heavily weighted by Ω_i where $i = C,\ F,\ H$ so that it does not crowd out regular labour supply by consumers. For the temporary volatility shock, the persistence parameter ϱ has a value of 0.95, a conventional choice for the frequency of the model (Fernández-Villaverde, 2010).

Table 2.1: General Parameters in the Consumer Mortgage Contract Economy

Parameter	α_F	α_H	ε	β	γ	φ	κ	J"	δ	Ω_i
Value	0.35	0.25	0.10	0.99	0.03	0.33	0.41	-10	0.0025	0.01

The conditions governing the mortgage contract are identical to the ones for the firm loan contract in Chapter 1:

1. The steady state rate of the external risk premium is 0.5% (Bernanke, Gilchrist and Gertler 1999).

2. The steady state leverage of credit-constrained households, i.e. value of housing stock to net worth ratio is $\frac{pH^C}{N^C} = 2$ (Bernanke, Gilchrist and Gertler 1999).

3. The failure rate of credit-constrained consumers $F^C(\bar{\omega}^C)$ (i.e. the number of credit-constrained consumers who retire each period) is 2% (Bernanke, Gilchrist and Gertler 1999).

Solving the first-order condition of credit-constrained consumers, the participation constraint of investors and the cumulative distribution function $F^C(\bar{\omega}^C)$ in the steady state gives the parameters μ^C, ν^C , ν^H, ν^F, the steady state value of the cutoff risk $\bar{\omega}^C$ and the standard deviation σ^C. The credit contract parameters are given in Table 2.2.

Table 2.2: Loan Contract Parameters in the Consumer Mortgage Contract Economy

Parameter	ν^F	ν^H	ν^C	μ^C
Value	0.99	0.99	0.98	0.06

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The steady state values of the model variables are in Table 2.3.

Table 2.3: Steady State Variables in the Consumer Mortgage Contract Economy

Variable	Y^F	Y^H	C^R	C^C	H^R	H^C	L^R	L^C	I^F	I^H
Value	100	10	32.8	30.2	210	169	15.5	19.9	24.9	3.28
Variable	K^F	K^H	L^F	L^H	p	w	R	R^C	N^F	C_E^F
Value	997	131	29.9	5.48	1.84	2.15	1.01	1.015	498	5.68
Variable			N^H	C_E^H	N^C	C_E^C	$\bar{\omega}^C$	σ^C		
Value			65.7	0.78	156	3.10	0.5	0.31		
Variable	Γ^C	G^C	Γ_ω^C	G_ω^C	Γ_σ^C	G_σ^C	$\Gamma_{\omega\omega}^C$	$G_{\omega\omega}^C$	$\Gamma_{\omega\sigma}^C$	$G_{\omega\sigma}^C$
Value	0.50	0.01	0.98	-0.02	-0.02	0.16	-0.31	2.02	-0.36	1.89

Appendix 2.B outlines the log-linearization of the complete model. The derivations of the log-normal distribution of $\bar{\omega}^C$ are the same as in Appendix 1.F in Chapter 1.

2.5 The Consumer Mortgage Model Results

The chapter simulates the same two shocks as in Chapter 1. This chapter, however, embeds them in a more empirically-relevant framework that brings the model closer to approximating the subprime mortgage crisis. The two disturbances were the major contributors to the subprime mortgage crisis - the oversupply of housing depressed the return on housing and the financial easing permitted credit-constrained consumers to amass too much debt.

Taken together, the two disturbances demonstrate that positive shocks can have negative consequences for credit-constrained consumer borrowers and that indebted agents can experience a downturn even if they are not production firms or financial institutions.

2.5.1 The Housing Oversupply

Chapter 1 demonstrated that a technological innovation shock in the production of housing improves the welfare of credit-constrained housing firms when housing is a multi-period good. Higher returns from housing production raised the ability of manufacturers to finance new capital acquisition thus improving their net worth and reducing their leverage. Consumers also benefited in terms of cheaper housing. These results were at odds with the subprime crisis evidence so this chapter moved credit constraints from the producer to the consumer sector in order to improve the performance of the model. With financial frictions in the household sector, a permanent expansion of the housing stock depresses the price of all houses on the market including the value of the housing collateral held by indebted consumers. Since the value of their housing stock falls relative to their mortgage, households enter a recession characterised by a worsening in their leverage and an increased likelihood of default. Seeing this, lenders require a higher risk premium restricting household access to house financing for a prolonged period of time.

The oversupply of housing affects the leverage of indebted consumers by reducing the value of the mortgage collateral (Figure 2.2). A permanent positive technological shock in the production of housing raises the supply of houses (Figure 2.2) and creates excess real estate inventory. In the absence of concurrent changes in housing demand, the equilibrium price of housing decreases (Figure 2.2.17) to accommodate the extra supply. The lower price of housing, however, applies not only to newly produced houses, but also to the existing housing stock that includes the housing equity held by credit-constrained consumers. The housing equity of borrowers consists of their share of housing bought with the loan contract after repaying investors. It depends both on the price of housing and on the realised return on housing that is the mortgage borrowing rate R_t^C. The unanticipated fall in the housing price reduces not only the value of the housing stock but also the return to housing in the period in which the shock occurs (Figure 2.2.20), i.e. the adjustable rate of the subprime-type loans decreases. Hence the lower price of housing affects the net worth of credit-constrained consumers both directly through the consumer share of real estate holdings and indirectly via the ex-post return on housing R_t^C. In contrast, the value of the loan diminishes only by the drop in the housing price. As a result, the net worth of indebted households (Figure 2.2.21) falls proportionally more than the value of the mortgage so the leverage of borrowers increases (Figure 2.2.24). Higher leverage signals increased probability of default for credit-constrained consumers (Figure 2.2.23).

The aggregate net worth of credit-constrained consumers falls not only because the realised return of housing is low relative to the expected return evaluated one period before, but also because a higher cutoff default risk increases bankruptcy prospects and reduces the share of the mortgage returns that goes to credit-constrained consumers in the loan contract. The investors are now exposed to bigger risks of default on the part of indebted households. To compensate for the rise in the monitoring costs, the investors require a higher return from borrowers, forcing up the external finance premium (Figure 2.2.29), i.e. the adjustable mortgage rate increases. Consequentially, the falling equilibrium price of housing generates a pro-cyclical risk premium unlike the anti-cyclical premium of Chapter 1. In both cases, the shock reduces the equilibrium price of housing. However, in Chapter 1 credit-constrained firms make their capital financing decisions based on their profits which increase since the fall in the housing price is outweighed by the output expansion. Borrowing consumers on the other hand, observe only the price of the housing collateral with no corresponding rise in their housing stock so they experience a worsening of their bankruptcy prospects.

The worsened indebtedness levels prompt mortgaged households to attempt to repair their net worth but the higher risk premium implies that borrowers can increase their housing stock very gradually. The inability of credit-constrained consumers to easily afford housing after the deterioration of their leverage is the reason housing output makes a dip from its initially high levels (Figure 2.2.2). The innovation in the housing sector also affects demand for consumption so output decreases (Figure 2.2.1). Due to the lower price of housing, Ricardian consumers switch away from consumption towards real estate (Figure 2.2.3). Indebted households initially enjoy a lower repayment rate on their existing loan so they can initially afford slightly more consumption (Figure 2.2.5). This rise, however, is a token amount so it is not sufficient to compensate for the reduced demand by Ricardians. Furthermore, the higher consumption demand by credit-constrained households is short-lived. The negative effects of the worsening leverage and the widening risk premium soon catch up with them and credit-constrained consumers have to switch away from consumption and toward repairing their net worth (Figure 2.2.21) and their borrowing abilities. Over time, indebted consumers gradually increase their housing stock (Figure 2.2.6) which translates into improving net worth (Figure 2.2.21) and decreasing default prospects (Figure 2.2.23).

The leverage and probability of default of credit-constrained consumers eventually returns to pre-shock levels owing to their diverting resources away from consumption toward increasing their housing stock (Figures 2.2.21 and 2.2.23). However, the recovery of household leverage is a protracted process. Their improvement is relatively slow and takes indebted households about 100 quarters (25 years) to return to their pre-crisis financial position. The simulations demonstrate that the interim period is characterised by a subprime-like recession with higher leverage and increased bankruptcy prospects for credit-constrained households.

2.5.2 The Temporary Financial Relaxation

A credit easing caused a financial downturn for credit-constrained housing firms in Chapter 1 when the temporarily relaxed interest rate allowed them to increase their borrowing and consequentially their leverage. Consumers, however, were unaffected since they were not part of the financial market. This contradicts the subprime experience where housebuyers were the primary targets of easy financing terms. Making homeowners part of the borrowing arrangements captures their experience with subprime loans. Temporarily easing access to house financing for credit-constrained consumers allows them to obtain a mortgage with less downpayment. As a result, their debt holding are higher implying a rise in their bankruptcy prospects. Since the external finance premium reflects the solvency of borrowers, it rises with the increased risk. A higher risk premium tightens the previously relaxed access to credit plunging consumers into a recession despite the short-lived nature of the initial shock.

Leading up to the subprime mortgage crisis, the housing market was characterised by exceptionally favourable borrowing conditions with low interest rates and little downpayment requirements. In the model, this lax lending atmosphere is captured by a negative temporary shock on $\left(\sigma^C\right)^2$, the variance of the consumer default risk ω^C. A lower variance implies narrower swings in the bankruptcy prospects ceteris paribus and signals safer loans (Figure 2.3). For borrowers, it can translate into a smaller risk premium on loans, less required collateral, or both. The reduction in volatility is modelled as an autoregressive temporary disturbance with moderate persistence to reflect that this phenomenon was relatively short-lived and to demonstrate the worsening of refinancing possibilities as the initial shock dies off. A permanent credit easing, on the other hand, would imply that borrowing terms remain permanently favourable although consumers have raised their leverage, a contradiction to the microfounded nature of the model. The temporary shock reflects that relaxed borrowing conditions prompt mortgagers to raise their leverage leading to a deterioration of their debt position as credit access tightens subsequently.

Reduced financial volatility implies safer loan contracts that require a smaller downpayment and that attract a reduced risk premium (Figure 2.3.29). The lower risk premium is passed on to credit-constrained consumers as a decreased mortgage interest rate R_t^C (Figure 2.3.20). Faced with a more favourable borrowing rate, indebted households increase their housing demand (Figure 2.3.6), driving marginally up the price of housing (Figure 2.3.17). Housing output also increases being demand driven by consumers (Figure 2.3.2). The higher housing price raises the value of their housing stock and positively influences their net worth immediately after the shock. However, the downward effect of the reduced consumer rate is quantitatively larger than the upward influence of the housing price so the net worth of borrowers eventually diminishes below pre-shock levels (Figure 2.3.21). The falling net worth, coupled with the subsequent credit tightening as the temporary shock wears out, worsen the leverage of credit-constrained households (Figure 2.3.24). Higher leverage signals increased probability of bankruptcy despite the improved financial conditions (Figure 2.3.23).

Seeing their indebtedness increase, credit-constrained consumers attempt to deleverage. They reduce their housing demand after the initial increase in an attempt to improve their net worth (Figures 2.3.6 and 2.3.21). The lower demand for housing reduces housing production shortly after the initial increase (Figure 2.3.2). It takes 100 quarters (25 years) of gradual improvement for their debt position to recover and for their bankruptcy prospects to return to pre-shock levels (Figures 2.3.23 and 2.3.24). Like in Chapter 1, the recovery is staggered since the higher refinancing rate restricts the ability of debtors to easily improve their housing stock which in turn maintains a higher refinancing rate. Overall, the effects of reduced default risk volatility are far from beneficial to indebted consumers. The initial benefits of relaxed borrowing conditions are quickly overshadowed by the tightening of credit access as the shock wears out. The result is rising consumer leverage and increased bankruptcy prospects. Even a temporary easing of lending has long-lasting consequences for the debt position of credit-constrained households. The volatility shock, just as the production shock, shows that positive disturbances can have negative effects on the indebtedness of households since they permit them to increase their leverage. It also demonstrates that non-production agents, such as consumers, can be the source of financial disturbances when they are leveraged.

Figure 2.2: Housing Oversupply in Economy with Credit-Constrained Consumers

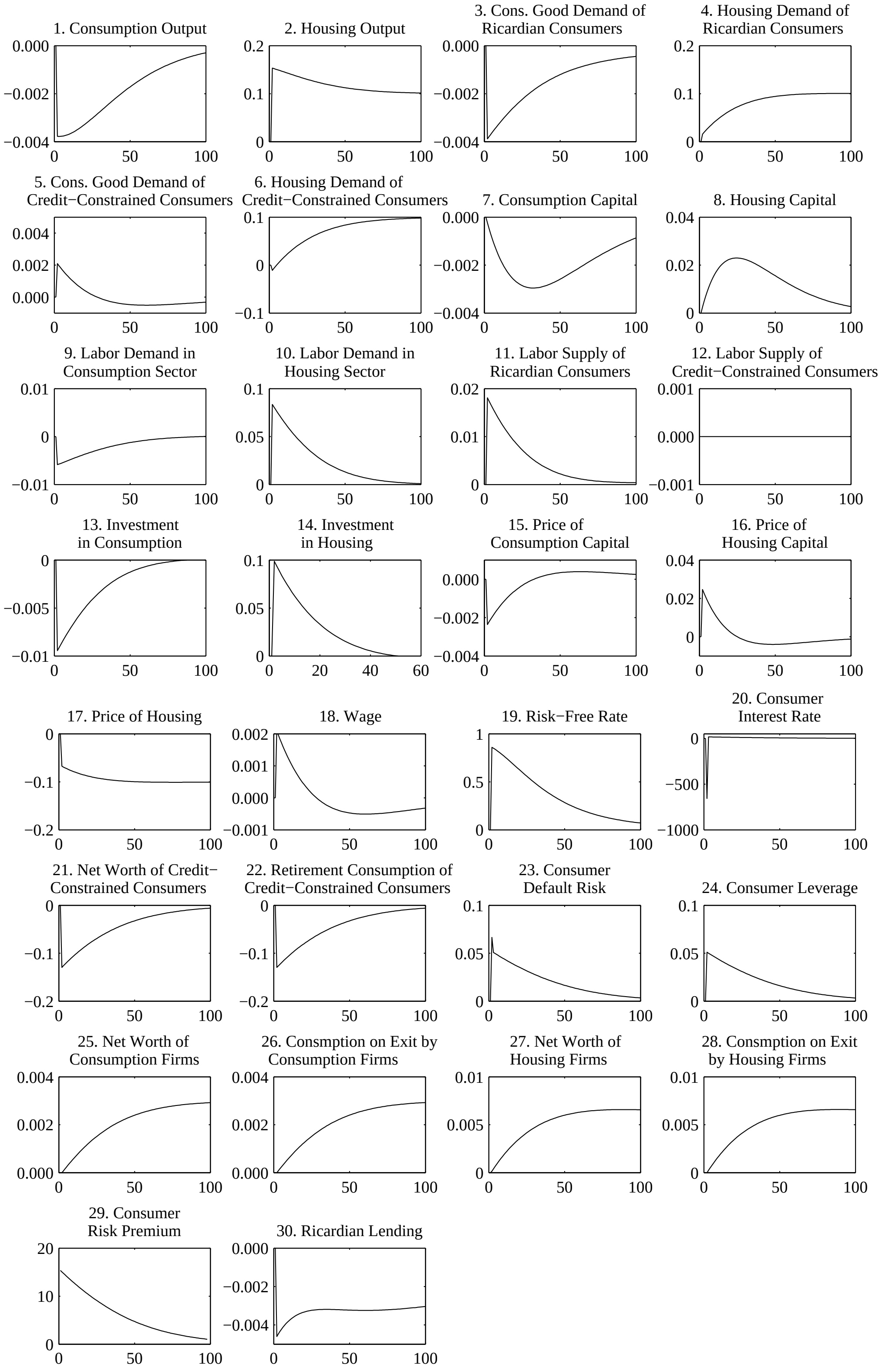
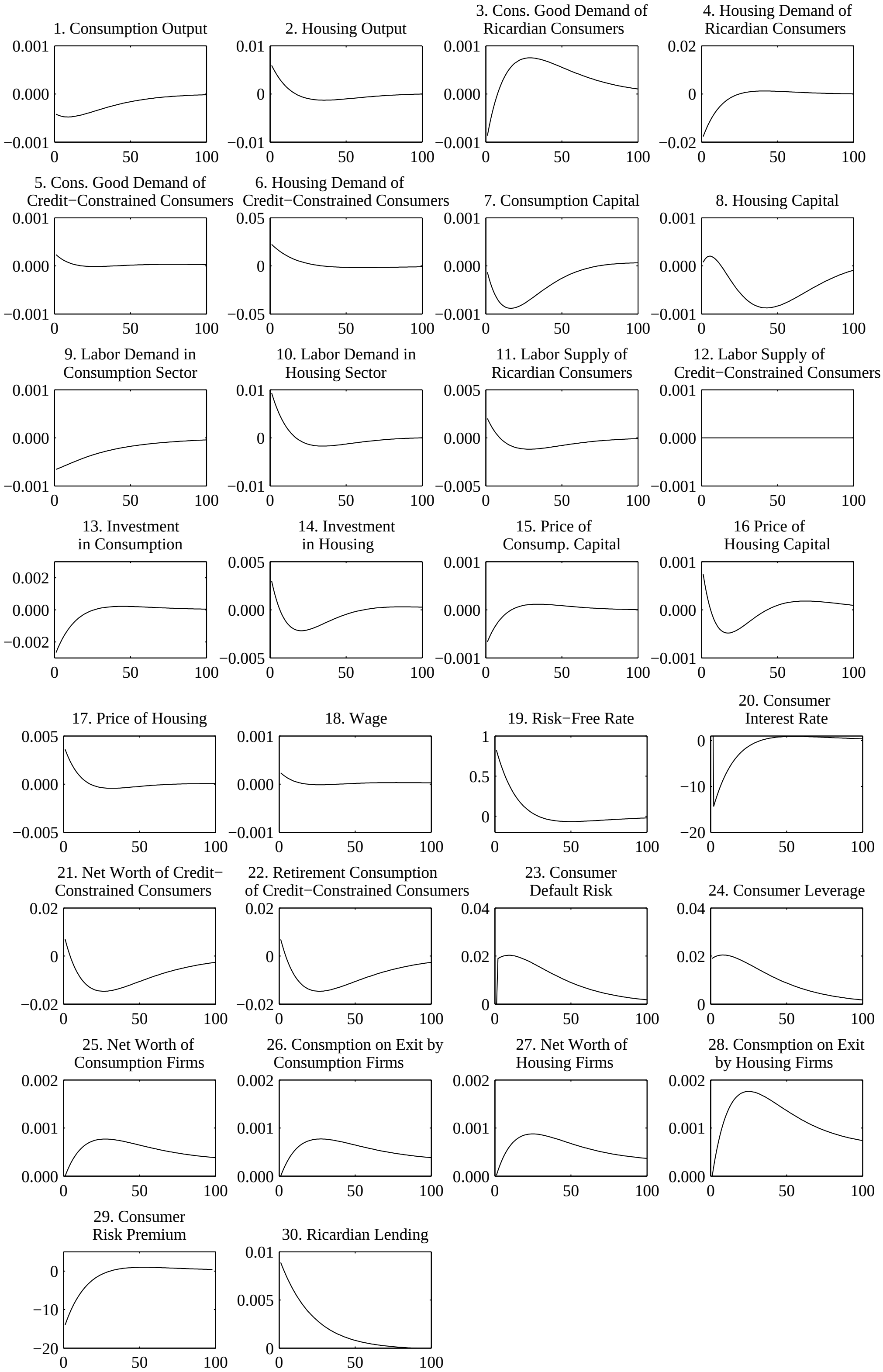


Figure 2.3: Reduced Volatility in Economy with Credit-Constrained Consumers



2.6 Conclusion

The subprime mortgage crisis was an unusual one since at its core were not producing firms but homeowner consumers and financial institutions. While financial leverage has seen an extensive treatment in the economy literature, the contribution of homeowner leverage to the downturn has attracted little attention since mortgage-buyers are usually not as highly leveraged as financial institutions. This paper demonstrates that even without banks, consumer leverage is sufficient to induce a lasting recession.

The model described in this paper updates the two-sector model with credit constraints from Chapter 1 by shifting financial frictions from the producer to the household sector in order to demonstrate the role of consumer leverage in the subprime recession. Using credit constraints in the mortgage-buying decisions of consumers, this chapter establishes that a downturn such as the subprime crisis can be triggered relatively easily when there are financial frictions even if the borrowers are not production agents or financial institutions. The two-sector economy with housing demonstrates that a positive housing supply shock can have negative repercussions for mortgage-holders since it reduces the value of the good used as collateral in the loan contract. A lower value of the collateral implies higher leverage for indebted home-buyers, increased default prospects and a higher risk premium. In this chapter, a lower price of housing output leads to a downturn for consumers whereas in Chapter 1, the lower price, coupled with higher output, implied increased profits and an improvement for housing firms. An improvement in the borrowing conditions also causes adverse consequences for leveraged agents and reinforces the idea that the credit-constrained sector can be the source of a crippling downturn even if it is not a production sector. A reduction in the riskiness attached to the mortgage contract can impact indebted consumers directly by allowing them to excessively leverage using relatively little collateral. When as a result of the excessive leverage the lending rate adjusts upwards, these households end up with subprime mortgages that they cannot service and are more likely to become bankrupt.

This chapter captures the essential aspects of the role of consumer leverage in triggering a downturn such as the subprime mortgage crisis but its ability to portray consumer experience with risk and debt can be further enriched. One notable addition that brings the model closer to reality is the addition of financial institutions that serve as intermediaries in the mortgage contract. This extension is developed in Chapter 3. Another important contribution could be to model the role of heterogeneous consumer expectations. It may be especially interesting to consider to what extent departures from the representative agent theory could explain the collective failure of agents to foresee the subprime mortgage crisis. Furthermore, the use of more sophisticated discounting such as hyperbolic or quasi-hyperbolic discounting could explain why consumers tend to leverage beyond some optimal point and thus make themselves more vulnerable to sudden shocks.

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Appendix 2.A

Complete Consumer Mortgage Contract Model

The participation constraint for investors:

$$\left(\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C = R_{t-1}\left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right)$$

The first order condition of credit-constrained households' maximization:

$$\frac{E_t R_{t+1}^C}{R_t} = E_t\left(\frac{\frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(\Gamma_\omega^C(\bar{\omega}_{t+1}^C) - \mu^C G_\omega^C(\bar{\omega}_{t+1}^C)\right)}}{\left(1 - \Gamma^C(\bar{\omega}_{t+1}^C)\right) + \frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(\Gamma_\omega^C(\bar{\omega}_{t+1}^C) - \mu^C G_\omega^C(\bar{\omega}_{t+1}^C)\right)}\left(\Gamma^C(\bar{\omega}_{t+1}^C) - \mu^C G^C(\bar{\omega}_{t+1}^C)\right)}\right)$$

The consumption of credit-constrained households on retirement:

$$C_{E,t}^C = \left(1 - \nu^C\right)\left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$N_t^C = \nu^C\left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C + (1 - \alpha_H - \varepsilon)\Omega_C p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{E,t}^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$N_t^F = \nu^F R_{t-1} N_{t-1}^F + (1 - \alpha_F)\Omega_F Y_t^F$$

The consumption on exit by housing firms:

$$C_{E,t}^H = \left(1 - \nu^H\right) R_{t-1} N_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$N_t^H = \nu^H R_{t-1} N_{t-1}^H + (1 - \alpha_H - \varepsilon)\Omega_H p_t Y_t^H$$

The consumption capital accumulation equation:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F$$

Investment demand by consumption capital firms:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1$$

The consumption production function:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)}$$

The labour demand by consumption firms:

$$w_t = (1-\alpha_F)(1-\Omega_F)\left(\frac{Y_t^F}{L_{H,t}^F}\right)$$

The capital demand by consumption firms:

$$R_t = \frac{E_t\left(\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F\right)}{x_t^F}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C)}$$

The labour demand by housing firms:

$$w_t = (1 - \Omega_H - \Omega_C)(1 - \alpha_H - \varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing firms:

$$R_t = \frac{E_t\left(\alpha_H \frac{p_{t+1} Y_{t+1}^H}{K_t^H} + (1-\delta) x_{t+1}^H\right)}{x_t^H}$$

The labour supply of Ricardian consumers:

$$\gamma (L_t^R)^{\varphi} C_t^R = w_t$$

The consumption housing tradeoff of Ricardian consumers:

$$E_t\left(R_t p_t - (1-\delta) p_{t+1}\right) H_t^R = \kappa E_t C_{t+1}^R$$

The Euler equation of Ricardian consumers:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t$$

The labour supply of credit-constrained consumers:

$$\gamma (L_t^C)^{\varphi} C_t^C = w_t$$

The consumption-housing tradeoff of credit-constrained consumers:

$$\left(R_t^C p_{t-1} - (1-\delta) p_t\right) H_{t-1}^C = \kappa C_t^C$$

The consumption purchase equation of credit-constrained consumers:

$$C_t^C = w_t L_t^C + ((1-\delta) p_t - R_t^C p_{t-1}) H_{t-1}^C$$

Market clearing in the consumption sector:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + I_t^F + I_t^H$$

Market clearing in the housing sector:

$$p_t Y_t^H + (1-\delta) p_t (H_{t-1}^R + H_{t-1}^C) = p_t (H_t^R + H_t^C) + C_{E,t}^H + \mu^C G^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C$$

Market clearing in the labour market:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C$$

Appendix 2.B

Log-Linearization of Consumer Mortgage Contract Model

This appendix derives the log-linearized form of the model equations as used in the simulations. The steady state value for each variable appears without time subscript and hats denote the percentage deviation of the variable from its steady state such that $\hat{Z}_t = \frac{Z_t - Z}{Z}$.

The participation constraint for investors:

$$\begin{aligned}\hat{R}_t^C &= \hat{R}_{t-1} + \frac{N^C}{pH^C - N^C}\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C - \hat{N}_{t-1}^C\right) - \\ &\quad - \frac{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}{\Gamma^C(\bar{\omega}^C) - \mu^C G^C(\bar{\omega}^C)}\bar{\omega}^C \hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C) - \mu^C G_\sigma^C(\bar{\omega}^C)}{\Gamma^C(\bar{\omega}^C) - \mu^C G^C(\bar{\omega}^C)}\sigma^C \hat{\sigma}_{t-1}^C\end{aligned}$$

The first-order condition of credit-constrained households' maximization:

$$\begin{aligned}0 &= \frac{\Gamma_\omega^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}(E_t \hat{R}_{t+1}^C - \hat{R}_t) - \\ &\quad -(1 - \Gamma^C(\bar{\omega}^C))\frac{R^C}{R}\left(\frac{\Gamma_{\omega\omega}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\Gamma_{\omega\omega}^C(\bar{\omega}^C) - \mu^C G_{\omega\omega}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}\right)\bar{\omega}^C E_t \hat{\bar{\omega}}_{t+1}^C + \\ &\quad + \frac{R^C}{R}\left(-\Gamma_\sigma^C(\bar{\omega}^C) + \frac{\Gamma_\omega^C(\bar{\omega}^C)\left(\Gamma_\sigma^C(\bar{\omega}^C) - \mu^C G_\sigma^C(\bar{\omega}^C)\right)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}\right)\sigma^C \hat{\sigma}_t^C - \\ &\quad - \frac{R^C}{R}\left((1 - \Gamma^C(\bar{\omega}^C))\left(\frac{\Gamma_{\omega\sigma}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\Gamma_{\omega\sigma}^C(\bar{\omega}^C) - \mu^C G_{\omega\sigma}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C) - \mu^C G_\omega^C(\bar{\omega}^C)}\right)\right)\sigma^C \hat{\sigma}_t^C\end{aligned}$$

The consumption of credit-constrained households on retirement:

$$\hat{C}_{E,t}^C = \hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^C(\bar{\omega}^C)}{1 - \Gamma^C(\bar{\omega}^C)}\bar{\omega}^C \hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C)}{1 - \Gamma^C(\bar{\omega}^C)}\sigma^C \hat{\sigma}_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$\begin{aligned}\hat{N}_t^C &= \nu^C(1 - \Gamma^C(\bar{\omega}^C))\frac{R^C pH^C}{N^C}(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C) - \nu^C \frac{R^C pH^C}{N^C}\Gamma_\omega^C(\bar{\omega}^C)\bar{\omega}^C \hat{\bar{\omega}}_t^C - \\ &\quad - \nu^C \frac{R^C pH^C}{N^C}\Gamma_\sigma^C(\bar{\omega}^C)\sigma^C \hat{\sigma}_{t-1}^C + (1 - \alpha_H - \varepsilon)\Omega_C \frac{pY^H}{N^C}\left(\hat{p}_t + \hat{Y}_t^H\right)\end{aligned}$$

The consumption on exit by consumption firms:

$$\hat{C}_{E,t}^F = \hat{R}_{t-1} + \hat{N}_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$\hat{N}_t^F = \nu^F R(\hat{R}_{t-1} + \hat{N}_{t-1}^F) + (1-\alpha_F)\Omega_F \frac{Y^F}{N^F}\hat{Y}_t^F$$

The consumption on exit by housing firms:

$$\hat{C}_{E,t}^H = \hat{R}_{t-1} + \hat{N}_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$\hat{N}_t^H = \nu^H R(\hat{R}_{t-1} + \hat{N}_{t-1}^H) + (1-\alpha_H-\varepsilon)\Omega_H \frac{pY^H}{N^H}\left(\hat{p}_t + \hat{Y}_t^H\right)$$

The consumption capital accumulation equation:

$$\hat{K}_t^F = \hat{K}_{t-1}^F + \delta\left(\hat{I}_t^F - \hat{K}_{t-1}^F\right)$$

Investment demand by consumption capital firms:

$$\hat{x}_t^F = J\text{''}\delta\left(\hat{K}_{t-1}^F - \hat{I}_t^F\right)$$

The consumption production function:

$$\hat{Y}_t^F = \hat{A}_t^F + \alpha_F \hat{K}_{t-1}^F + (1-\alpha_F)(1-\Omega_F)\hat{L}_{H,t}^F$$

The labour demand by consumption firms:

$$\hat{w}_t = \hat{Y}_t^F - \hat{L}_{H,t}^F$$

The capital demand by consumption firms:

$$\hat{R}_t + \hat{x}_t^F = \frac{R-(1-\delta)}{R}(\hat{Y}_{t+1}^F - \hat{K}_t^F) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^F$$

The housing capital accumulation equation:

$$\hat{K}_t^H = \hat{K}_{t-1}^H + \delta\left(\hat{I}_t^H - \hat{K}_{t-1}^H\right)$$

Investment demand by housing capital firms:

$$\hat{x}_t^H = J\text{"}\delta\left(\hat{K}_{t-1}^H - \hat{I}_t^H\right)$$

The housing production function:

$$\hat{Y}_t^H = \hat{A}_t^H + \alpha_H \hat{K}_{t-1}^H + (1 - \Omega_H - \Omega_C)(1 - \alpha_H - \varepsilon)\hat{L}_{H,t}^H$$

The labour demand by housing firms:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_t^H - \hat{L}_{H,t}^H$$

The capital demand by housing firms:

$$\hat{R}_t + \hat{x}_t^H = \frac{R - (1-\delta)}{R}(\hat{p}_{t+1} + \hat{Y}_{t+1}^H - \hat{K}_t^H) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^H$$

The labour supply of Ricardian consumers:

$$\varphi \hat{L}_t^R + \hat{C}_t^R = \hat{w}_t$$

The consumption housing tradeoff of Ricardian consumers:

$$\hat{C}_{t+1}^R = \hat{H}_t^R + \hat{p}_{t+1} + \frac{R}{(R - 1 + \delta)}\left(\hat{R}_t + \hat{p}_t - \hat{p}_{t+1}\right)$$

The Euler equation of Ricardian consumers:

$$E_t\hat{C}_{t+1}^R - \hat{C}_t^R = \hat{R}_t$$

The labour supply of credit-constrained consumers:

$$\varphi \hat{L}_t^C + \hat{C}_t^C = \hat{w}_t$$

The consumption housing tradeoff of credit-constrained consumers:

$$\hat{C}_t^C = \hat{H}_{t-1}^C + \hat{p}_t + \frac{R^C}{(R^C - 1 + \delta)}\left(\hat{R}_t^C + \hat{p}_{t-1} - \hat{p}_t\right)$$

The consumption purchase equation of credit-constrained consumers:

$$\hat{C}_t^C = \frac{wL^C}{C^C}(\hat{w}_t + \hat{L}_t^C) + \frac{(1-\delta)pH^C}{C^C}\hat{p}_t - \frac{R^C pH^C}{C^C}(\hat{R}_t^C + \hat{p}_{t-1}) + \frac{(1-\delta-R^C)pH^C}{C^C}\hat{H}_{t-1}^C$$

Market clearing in the consumption sector:

$$\hat{Y}_t^F = \frac{C^R}{Y^F}\hat{C}_t^R + \frac{C^C}{Y^F}\hat{C}_t^C + \frac{C_E^F}{Y^F}\hat{C}_{E,t}^F + \frac{C_E^C}{Y^F}\hat{C}_{E,t}^C + \frac{I^F}{Y^F}\hat{I}_t^F + \frac{I^H}{Y^F}\hat{I}_t^H$$

Market clearing in the housing sector:

$$\begin{aligned}\hat{Y}_t^H \quad = \quad & \frac{\delta(H^R + H^C) - Y^H}{Y^H}\hat{p}_t + \frac{H^R}{Y^H}\hat{H}_t^R + \frac{H^C}{Y^H}\hat{H}_t^C - \frac{(1-\delta)}{Y^H}(H^R\hat{H}_{t-1}^R + H^C\hat{H}_{t-1}^C) + \\ & + \frac{C_E^H}{pY^H}\hat{C}_{E,t}^H + \mu^C G^C(\bar{\omega}^C)\frac{R^C H^C}{Y^H}\left(\hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C\right) + \\ & + \mu^C G_\omega^C \bar{\omega}^C \hat{\bar{\omega}}_t^C \frac{R^C H^C}{Y^H} + \mu^C G_\sigma^C \sigma^C \hat{\sigma}_{t-1}^C \frac{R^C H^C}{Y^H}\end{aligned}$$

Market clearing in the labour market:

$$L_H^F \hat{L}_{H,t}^F + L_H^H \hat{L}_{H,t}^H = L^R \hat{L}_t^R + L^C \hat{L}_t^C$$

Chapter 3

Risk Sharing in an Economy with Multiple Credit Constraints

Abstract

This chapter develops a chained mortgage contracts model where both homeowner consumers and the financial institutions that securitize their mortgage loan are credit-constrained. The presence of financial frictions in both the consumer and the financial sectors brings the model closer to real financial markets and demonstrates the role of risk sharing between leveraged borrowers. Highly leveraged banks act as financial intermediaries in the mortgage contract so when a negative disturbance worsens their indebtedness, they transfer some of the downturn to indebted consumers who have no financial recourse outside of the chained loan contracts. The leverage of homeowner households experiences a smaller worsening than in the single loan contract model since households share the deterioration of the debt position with banks but their downturn is more protracted. This consequence is observed both after a housing oversupply shock and a relaxation of borrowing conditions but is especially evident following relaxed credit access for banks. After initially raising their leverage in response to the lax borrowing conditions, financial institutions repair their debt position relatively fast at the expense of consumers whose borrowing ability is squeezed although they are not the source of the disturbance. The result mirrors the recent subprime mortgage crisis characterised by a sharp but brief decline for banks and a protracted recovery for mortgaged households.

3.1 Introduction

When Edmund Andrews finally defaulted on his home mortgage, he was far from being the only one to declare bankruptcy. In fact, he had outlived two of his three mortgage lenders. The first one collapsed overnight when the financial markets froze up at the beginning of the downturn in August 2007 and the second one was forced out of the mortgage business by federal regulators. Andrews' personal mortgage crisis would not have unfolded if highly leveraged financial institutions did not extend subprime loans under easy financing conditions in pursuit of higher returns. At the first tightening of the financial market, banks attempted to salvage their own balance sheets by increasing the repayment rate on loans like his (Andrews, 2009). With the downturn threatening their solvency, many highly leveraged financial institutions resorted to risk sharing in which they shifted some of the burden of their deteriorating indebtedness onto borrowing consumers. Raising the interest rates for mortgage-buyers and restricting their credit access were among the actions that banks undertook affording themselves a swift recovery at the expense of consumers who were faced with a protracted recession. Understanding these complex interactions and the risk sharing that happens requires a deeper knowledge of linked loan contracts.

This chapter extends the consumer mortgage model of Chapter 2 with the addition of credit-constrained financial institutions. This step brings the model closer to reality and demonstrates that risk sharing between indebted agents makes the ensuing recession deeper and more prolonged. The model of Chapter 2 captured the recession of mortgage-buying households but abstracted from active financial intermediation where leveraged banks borrow from investors and in turn lend to consumers. Investors were passive intermediaries who only facilitated the lending process and monitored the loans without earning profits and being indebted themselves. The previous version also modelled financial frictions but in only one sector so there are no opportunities for risk sharing. This chapter develops a model of financial frictions both in the consumer and financial sectors. There are mortgage-buying homeowners just as in Chapter 2. The important addition is credit-constrained financial institutions (i.e. banks) that also possess idiosyncratic uncertainty. The presence of more than one indebted sectors raises the combined leverage in the economy resulting in more volatility.

The introduction of banks is in addition to risk-neutral investors. While investors have no intrinsic risk of default and exist purely to facilitate the lending of Ricardian consumers, financial institutions are leveraged and contribute to the aggregate volatility of the financial market. Banks borrow from investors and in turn finance the housing purchase of credit-constrained consumers. They participate in two loan contracts: one with lender investors and with final borrower credit-constrained consumers. Since financial institutions participate in both contracts, they optimise the two transactions together resulting in a chained contracts model similar to Hirakata, Sudo and Ueda (2009). However, unlike Hirakata, Sudo and Ueda (2009), the final borrowers are not entrepreneurs but credit-constrained households.

Before the financial crisis, the financial sector did not receive much attention in the literature of credit frictions. Papers like Kiyotaki and Moore (1997) and Iacoviello (2005) focus primarily on the demand side of credit and abstract from an active role of financial intermediation. The first models to consider the role of banks are Goodfriend and McCallum (2007) and Christiano, Trabandt and Walentin (2007). Both of these estimate the quantitative importance of the banking sector for central bank policy and for business cycles. Christiano, Motto and Rostagno (2010), Gerali, Neri, Sessa and Signoretti (2009), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2011) and Luk and Vines (2011) develop a financial sector to explain specific precedents of the financial crisis such as excessive volatility, the proliferation of risk accumulation and the popularity of unconventional monetary policy. The contributions to the literature of the financial sector are discussed in detail in the next section. While these recent additions to the theoretical body of financial frictions augment the general knowledge of the financial crisis, they overwhelmingly focus on credit frictions in only one sector, namely the banking sector, and abstract from constraints in other sectors of the economy, including households. This makes them unsuitable to studying the extent of risk sharing that can occur with more than one type of financially-constrained agents. Only Hirakata, Sudo and Ueda (2009) introduce more than one leveraged sector modelling constraints in the production sector along with the financial sector but they also overlook the role of indebted consumers. This chapter models the chained contracts along the lines of Hirakata, Sudo and Ueda (2009) but the final borrowers are not entrepreneurs but credit-constrained households. It highlights the opportunities for risk sharing between both types of indebted agents and contrasts them with an alternative version similar to Luk and Vines (2011) where consumers have a fixed share arrangement with banks.

When there are two types of credit-constrained agents, the chained loan contracts create opportunities for risk sharing that exacerbate the ensuing recession. As intermediaries of the two mortgage agreements, financial institutions have the ability to transfer some of the burden of a downturn to final borrower consumers. Furthermore, banks are often more leveraged so they experience a proportionally larger deterioration of their leverage ratio which is an added impetus for them to unload the leverage fast. The higher initial leverage and the sharper subsequent deterioration necessitate quick deleveraging in order to maintain solvency. That is the primary reason why banks resort to shifting part of the burden of financial tightening onto households. Two characteristics of the financial system allow them to do so. First, credit-constrained consumers have no recourse to alternative funding so they will participate in the mortgage contract even if the terms deteriorate. Second, the arrangement between banks and mortgage-buying households is such that the return from borrowing to consumers varies with the realization of the idiosyncratic uncertainty. When the uncertainty increases in times of a downturn, lenders can extract a larger share from indebted households. Banks can channel the increased proceeds toward repairing their own balance sheets during a deterioration of financial conditions. Consequentially, they can recover relatively fast at the cost of inducing a lasting recession for mortgaged households.

Before discussing the implications of the chained loan contract, the chapter considers an economy where only banks are leveraged but consumers possess no idiosyncratic risk. This version borrows from Luk and Vines (2011) but final borrowers are consumers and not firms. This preliminary scenario is not a reflection of reality but provides the opportunity to contrast the experience of financial institutions in a setting where they can engage in risk sharing with one where they cannot. When banks are the only indebted agents, they have a share arrangement with mortgage-buying households that provides a fixed return regardless of the realization of the idiosyncratic shock. Due to this fixed return that the share contract provides, financial institutions cannot extract extra equity from their lending to consumers. This forces them to bear the full force of the resulting downturn while consumers are protected by their guaranteed return share contract. However, when consumers participate in the mortgage contract, banks transfer some of the leverage worsening onto them. The experience of homeowner mortgage-buyers in the chained contract is not as severe as when they are the sole credit-constrained agents as in Chapter 2, but risk sharing still creates a considerable burden for them. This chapter demonstrates the consequences of risk sharing on the indebtedness of credit-constrained consumers following both an oversupply of housing and a relaxation of borrowing conditions for consumers as well as for financial institutions. Out of three disturbances considered (an oversupply of housing, a relaxation of borrowing conditions for consumers and a relaxation of borrowing conditions for financial institutions), the scenario with a relaxation of borrowing conditions for banks comes closest to the consequences of the subprime mortgage crisis. Although consumers are not the originators of the disturbance, they experience the most protracted recession of all scenarios.

The next section discusses the literature of the banking sector, especially after the financial crisis. Section 3.3 describes the loan contract of the financial institutions in the preliminary setup where banks are credit-constrained but consumers have a fixed share arrangement with them. Section 3.4 explains the changes to the complete model and the results of this version are in Section 3.5. Section 3.6 develops the chained loan contracts model where both financial institutions and households possess idiosyncratic risk and Section 3.7 describes changes to the complete model. Section 3.8 outlines the results and compares them with the the previous versions of financial frictions (the one in Chapter 2 and in Section 3.3 of this chapter) and explains the risk sharing that occurs when there is more than one credit-constrained sector. Finally, the Conclusion concludes and points to directions of future research.

3.2 The Role of the Financial Sector in the Literature

This chapter discusses the opportunities for risk sharing when there are credit-constrained financial institutions in addition to indebted households. This literature review describes the role of banks in financial frictions models focusing especially on post-crisis works and on works that include more than one type of borrower. It argues that there has been a dearth of attention to multiple credit-constrained sectors and that the risk sharing between borrowing agents is not sufficiently explored.

The financial accelerator mechanism of Bernanke, Gertler and Gilchrist (1999) and the collateralized debt of Kiyotaki and Moore (1997) and of Iacoviello (2005) are some of the most well-known models that introduce financial rigidities into dynamic stochastic general equilibrium (DSGE) models. However, these models do not consider an explicit role for banks and thus the focus is primarily on the demand side of credit. Most of the literature introducing a banking sector into DSGE models has been motivated mainly by the aim to explain specific features of the financial crisis. This section focuses on the literature of financial constraints and discusses how the combination of frictions both in the household and the banking sector are better suited to capturing the main characteristics of the subprime recession.

The literature presented in this section extends the financial frictions framework by adding agents and tailoring credit constraints in order to study the role of bank capital in business cycle fluctuations. The financial crisis has emphasised the importance of systemic risk and sudden changes in that risk. Thus, the studies below search for foundations for the existence of risky portfolios. The liquidity crisis after the Lehman collapse triggered the development of models focusing on the function of bank capital requirement in the process of loan creation. The pioneering model introducing banks into the DSGE framework is by Goodfriend and McCallum (2007). The model builds on the methodology of Bernanke, Gertler and Gilchrist (1999) but like the model in this chapter, the risk premium works pro-cyclically after a monetary shock. Goodfriend and McCallum (2007) develop a banking sector in order to describe the interaction and differences between various types of interest rates to determine the extent to which the central bank is misled in its decisions by relying on a standard model without a banking sector. There are both credit and balance sheet channels in the banking sector. The banking sector is set up similarly as a firm's optimization problem. Loan creation in the competitive banking sector is introduced by a production function and a cash-in-advance assumption. Loan production depends on collateral and loan-monitoring costs. Government bonds serve as collateral. Due to this additional function, risk-free securities have to be more expensive. The authors assume that capital has lower quality as collateral; hence it has to provide a higher yield in equilibrium to its holder than government bonds. The no-arbitrage condition between the loan market and the asset market determines the interbank rate. The loan rate must account for the cost of loan monitoring and collateral costs. Therefore it has to be higher than the interbank rate in order to avoid an arbitrage opportunity.

On the perfectly competitive banking market deposits must pay the interbank rate. However, because banks are forced to hold reserves that do not bear interest, the deposit interest rate is lower than the interbank rate. This competitive banking market setup creates two opposite external finance premium effects. The first is called the "banking attenuator" effect which dampens the effect of monetary policy shocks. The second effect amplifies monetary policy actions as in Bernanke, Gertler and Gilchrist (1999). The spread between the reference rate and the loan rate is pro-cyclical. This means that the banking sector attenuates a monetary policy shock because the external finance premium grows in booms and drops in recessions. An expansionary monetary shock drives consumption growth in the model economy. The higher consumption generates a proportional increase in the demand for bank deposits that in turn raises the monitoring costs. The higher costs of lending given by the increased spread between the loan rate and the reference rate discourage demand for loans and further dampen consumption growth. On the other hand, the "banking accelerator" effect arises from the fact that the monetary policy shock raises the opportunity cost of investment, therefore the marginal product of capital and the price of capital have to increase. The higher price of capital increases the collateral value and hence implicitly decreases the monitoring costs. The increased demand for deposits is compensated by a lower marginal cost of loan production. Goodfriend and McCallum (2007) argue that for reasonable parameter values the "attenuator" effect is stronger and the external finance premium may be pro-cyclical. The implications, however, apply only to monetary shocks and require special calibration. The model in this chapter achieves the same risk premium effect within a more general framework and without the need for special calibration.

Christiano, Trabandt and Walentin (2007) pose the important question whether banking, and the financial sector in general, is quantitatively important for the business cycle. Their paper extends the model of Bernanke, Gertler and Gilchrist (1999) by introducing a financial sector. The banking sector is integrated into the model using the methodology of Chari, Christiano and Eichenbaum (1995). The modelling approach to banking is in the spirit of Goodfriend and McCallum (2007) in the way liquidity is supplied. Instead of a loan production function, however, they use a deposit production function. Christiano, Trabandt and Walentin (2007) argue that the Bernanke, Gertler and Gilchrist (1999) type of financial frictions improves the data fit but the banking sector plays a small role. Explanation of the stylized facts of the global financial crisis motivate Christiano, Motto and Rostagno (2010) to further extend their earlier model (Christiano, Trabandt and Walentin, 2007) in a way that can explain the main elements of the crisis. They find that liquidity constraints and changing risk perceptions are the main determinants of economic fluctuations. External risk perceptions as captured by a credit easing play an important role in this chapter as well but here the focus is on risk sharing rather than systemic risk that creates contagion. In Christiano, Motto and Rostagno (2010), financial frictions in the credit supply that turn market risk into systemic risk contribute importantly to the model's empirical fit. The ability of the central bank to supply liquidity when the supply of credit in the banking sector is low has significant smoothing effects on the business cycle.

The next important contribution to the literature comes from Cúrdia and Woodford (2009). They concentrate on similar questions as Goodfriend and McCallum (2007). The important novelty is given by the specific model framework. Cúrdia and Woodford (2009) develop a stylized model with a banking sector where the basic New-Keynesian model is its special case. They differ in their modelling approach by assuming heterogeneous households that are able to change their type. This implies that the credit spread is a function of the markup in the intermediary sector and the costs of the loan (monitoring and origination) may be exogenous or endogenous. The modified model delivers an economy with financial intermediation realised between households rather than households and firms. Like the model in Chapter 2, half of households are lenders and the other half borrowers. Savers discount the future less than borrowers so the optimality conditions of the model contain two discount factors. There is a credit spread between the deposit rate and the borrowing rate. This spread is time-varying in response to the total volume of lending and Cúrdia and Woodford (2009) consider the spread to be determined by exogenous as well as endogenous factors. The endogeneity of the credit spread comes from the markup in the intermediary sector. Furthermore, the cost of loans has the character of intermediation technology as in Goodfriend and McCallum (2007) where there are monitoring costs to the collateral value.

Cúrdia and Woodford (2009) introduce insurance against both the idiosyncratic risk and the risk associated with a change of household type. Without insurance, the distribution of the marginal utility of income would be too dispersed because of the histories of each individual type of household and the model would not have a stable solution. The fact that the change of household type is random, together with the insurance, enables the authors to perform aggregation and derive a stationary equilibrium. The heterogeneous agent approach allows the authors to abstract from the assumption that households need to hold deposits before purchasing goods. The main implication of the paper is that including the credit channel in the standard New-Keynesian model does not fundamentally alter the effect of financial shocks. The appeal of Cúrdia and Woodford (2009) lies in the fact that they build on the broadly known baseline New-Keynesian model. A major drawback, however, is that the credit spread, the main ingredient of the model's credit market, is exogenously introduced into the model. It is not derived from agents' behaviour and is therefore not microfounded. The spread also abstracts from the threat of default while default is one of the main factors determining the spread between interest rates.

Gerali, Neri, Sessa and Signoretti (2009) introduce a banking sector into the DSGE model to study the role of banking intermediation, and in particular, the implication of tightening credit conditions and their transmission for the real economy. They merge the model of Christiano, Trabandt and Walentin (2007) with that of Iacoviello (2005) and extend it to include an imperfectly competitive banking sector in both the deposit market and the loan market. This setup is mainly geared toward explaining the role of financial frictions in the banking sector for monetary policy. The paper finds that the monopolistic power of banks over the loan and deposit rate changes the pass-through of central bank policy rate which is not transmitted fully and instantaneously into households' and firms' decisions. The monopolistic power of banks is partly reflected in the present model but consumers have a more active role in the financial market than simply being recipients of policy.

Another sub-class of models featuring a banking sector concentrates on analysis of the Federal Reserve Bank's (Fed) unconventional balance sheet operations in reaction to the consequences of the financial crisis. Gertler and Karadi (2011) use a model with an agency problem between intermediaries and their depositors to produce endogenous constraints on intermediary leverage ratios. A specific feature of the model is that the central bank acts as an intermediary. The monetary authority can borrow funds from savers and then lend them to investors. The distinction from private banks lies in the fact that the central bank does not face constraints on its leverage ratio. The central bank borrows against collateral in the form of government bonds. By buying its own debt, the central bank avoids agency problems. Gertler and Karadi (2011) show that when the policy rate hits its zero lower bound the net benefits of unconventional monetary policy are significant. This approach is elaborated on in Gertler and Kiyotaki (2010) with the introduction of an interbank market. The paper models the situation where banks are not willing to lend to one another. Banks face an idiosyncratic liquidity shock which creates a deficit or surplus of funds across financial institutions. Together with agency problems, the disruption to the inter-bank market affects real activity. These models are used to illustrate the fact that various credit market interventions may mitigate the negative effects of financial frictions at times of crisis. The present paper undertakes a more two-way approach in which both real and financial activity can cause disruption on one another via credit constraints.

The most recent contributions to the literature dealing with the banking sector in DSGE models go back to the roots of the financial crisis and search for the underlying rationality of building risky portfolios. In this sense, they go beyond the scope of the present chapter and seek to find how macroprudential and other policy can limit such risk buildup. Gertler, Kiyotaki and Queralto (2011) focus on explaining the motivations of banks for taking excessive risks. The aim in their very recent working paper is not just to match the banks' vulnerability to risk, but also to explain why banks tend to build risky balance sheets. The modelling framework builds on Gertler and Karadi (2011) and on Gertler and Kiyotaki (2010). The endogenous choice of a risky balance sheet is incorporated by a trade-off between short-term debt and equity. The authors conclude that an appropriately designed macroprudential policy can mitigate moral hazard costs. Incentives for risk taking reduce the benefits of credit policies aimed at stabilizing the financial markets.

The model in Dib (2010) stands out by offering a synthesis of much of what can be found in the studies discussed previously. Dib (2010) unifies many of the ingredients of previous research and adds an analysis of a bank's capital requirement condition and its effects on business cycle fluctuations. The economy is populated by workers and bankers maximizing their utility functions. The difference compared to the heterogeneous households in Cúrdia and Woodford (2009) is that the share of workers and bankers is fixed and they do not change their types. In addition, the utility function is derived from different variables in each group. Bankers are assumed to be owners of both saving and lending banks. The role of bankers is to create a link between changes in bank capital and changes in the real economy. This link rejects the short-run efficient market hypothesis and highlights the default risk of the lending bank.

Dib (2010) assumes that two types of banks, saving and lending, operate in the heterogeneous intermediary sector. Saving banks collect deposits from workers. The lending bank may make use of injections of liquidity from the central bank, facilitating modelling of a quantitative monetary easing. The lending bank is also allowed to swap part of its loans for bonds in order to simulate a quantitative easing by the Fed. Loans are produced using interbank borrowing and bank capital subject to a regulatory capital requirement. Banks have monopoly power, set nominal deposit and prime lending rates, choose their leverage ratio and their portfolio composition, and can endogenously default on a fraction of their interbank borrowing. The model also includes two unconventional monetary policies (quantitative and qualitative easing) that reduce the negative impacts of financial crises. Because it is costly to raise capital to satisfy the regulatory capital requirement, the banking sector attenuates the real effects of financial shocks as in Goodfriend and McCallum (2007), reduces macroeconomic volatilities, and helps stabilize the economy. Overall, financial shocks affect real variables much less in the model with a banking sector than in models with an external finance premium and collateralized debt.

Models that focus on financial frictions in the banking sector usually abstract from credit imperfections in other sectors or model them rather crudely. Hirakata, Sudo and Ueda (2009) explicitly model frictions in both the banking and the producer sectors. Their model extends the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999) by adding credit-constrained financial institutions to the existing setup of credit-constrained entrepreneurs. In this version, both banks and entrepreneurs need to borrow. Entrepreneurs finance their investment project using their own net worth and borrowings from the financial institutions. Financial institutions in turn borrow from risk-neutral investors and lend to credit-constrained firms. Thus financial intermediation takes place through chained credit contracts where financial institutions make loans to the entrepreneurs, but the loans are financed by borrowing from the market, in addition to the financial institutions' own net worth. Agency problems exist in both borrowing contracts so the external finance premium depends on financial institutions' net worth and entrepreneurial net worth.

The authors study how the net worth of financial institutions and entrepreneurs affect the two credit contracts and aggregate investment decisions. Because the credit contracts are chained, the agency problems in the two credit contracts work complimentary. The supply of funds to entrepreneurs is squeezed if either of the two contracts is severely constrained. Next, they study the implication on the model dynamics, based on calibration to the U.S. economy. Because the credit contracts are chained, the model propagates and amplifies the adverse shocks more than a model in which only entrepreneurial credit frictions exist. The paper also compares how a shock to the financial sector and a shock to the entrepreneurial sector are propagated to the aggregate economy. It finds that a sectoral shock to financial institutions is propagated more than a shock to the entrepreneurs. This reflects the fact that the agency problem in the financial institution sector and the entrepreneurial sector are asymmetric. Financial institutions usually have smaller net worth than entrepreneurs and the cost associated with financial institutions' bankruptcy is higher than that of entrepreneurs resulting in the former facing a more severe agency problem than the latter. Consequently, a shock to the financial sector has a larger impact on aggregate variables. Finally, the authors examine how the financial accelerator effect associated with the aggregate shock can change when the net worth distribution is altered. They find that the financial accelerator effect can be reduced if an appropriate amount of net worth is distributed from entrepreneurs to financial institutions. This model is this chapter is closest to Hirakata, Sudo and Ueda (2009) in including two credit-constrained sectors. However, the approach developed here maintains frictions in the consumer sector, unlike the entrepreneurial sector of Hirakata, Sudo and Ueda (2009), in addition to constraints in the banking sector. Its primary focus is to highlight risk sharing between borrowing agents rather than compare shocks.

Luk and Vines (2011) offer a simpler version of Hirakata, Sudo and Ueda (2009) which they believe is better suited to an analysis of the recent financial crisis. Instead of considering credit constraints in both the banking and production sectors, the authors develop a model where banks face financial frictions when borrowing from depositors while firms are not subject to credit constrains. Financial institutions can diversify away the risk from idiosyncratic shocks affecting the producers so firms can obtain financing from banks by issuing risk-free shares. By contrast financial institutions are leveraged and because they are also subject to idiosyncratic shocks they can only obtain their funds by paying a risk premium on their loan. The authors demonstrate that their setup produces a stronger amplification of shocks than Bernanke, Gertler and Gilchrist (1999) since the financial sector is more leveraged than the production sector, a fact that reflects reality. Their model also leads to a greater amplification of shocks than Hirakata, Sudo and Ueda (2009) since the Hirakata, Sudo and Ueda (2009) assume that financial institutions lend money to entrepreneurs, rather than buying their shares. The presence of this loan arrangement dampens the amplification effect by engaging in risk sharing. It occurs only when there is more than one credit-constrained sector and both sectors can share the positive or negative effects of the disturbance. Risk sharing is not observable in Luk and Vines (2011) since the entrepreneurs are not credit-constrained. Hence the amplification effect is strongest.

The model in this chapter incorporates a banking sector but the focus continues to be on consumers as in Chapter 2 and specifically on how banks engage in risk sharing with mortgage-buying households. The modelling of the financial sector follows Hirakata, Sudo and Ueda (2009) but does not model the two opposing external finance premium effects as in Goodfriend and McCallum (2007) which complicate the setup without adding to the analysis. However, like Goodfriend and McCallum (2007), this paper still generates a pro-cyclical risk premium due to Fisher-type price effects of the collateral. Most of the papers that incorporate a financial sector focus either on its quantitative importance (Christiano, Trabandt and Walentin, 2007) or on the policy transmission (Gerali, Neri, Sessa and Signoretti, 2009; Gertler and Karadi, 2011). Cúrdia and Woodford (2009) introduce heterogeneous consumers as in this paper, but their focus is on facilitating the credit channel and not on consumer borrowing decisions themselves. With the exception of Hirakata, Sudo and Ueda (2009), none of the papers uses financial constraints in more than one sectors and none incorporates the role of housing mortgages. This chapter considers both credit-constrained financial institutions and consumers and focuses on the role of risk sharing between indebted agents when purchasing housing mortgages. The model described here is closest to Hirakata, Sudo and Ueda (2009) in its treatment of financial institutions and the dual financial frictions. However, unlike Hirakata, Sudo and Ueda (2009), the final borrowers are not firms but credit-constrained consumers. The paper also develops a version of the loan-share model proposed by Luk and Vines (2011) where credit-constrained consumers replace entrepreneurs in the share contract. This interim version does not reflect real economy fundamentals but is introduced in order to highlight the opportunities for risk sharing that exist in a chained loan contract setup but are absent in loan-share contract version.

3.3 The Financial Institutions Loan Contract

3.3.1 Description of the Financial Institutions Loan Contract

This section considers the role of credit frictions only in the financial sector. Rather than reflect real life mortgage markets, it provides a basis for comparison with the more realistic chained contracts model discussed later. It demonstrates that when households are not subject to financial constraints, they are relatively protected from downturns which are instead concentrated in the indebted financial intermediary sector.

In this version, financial institutions act as intermediaries that borrow funds from investors and in turn lend to credit-constrained households. Banks are exposed to an idiosyncratic risk of default while credit-constrained consumers possess no idiosyncratic uncertainty. This version is similar to Luk and Vines (2011) except that here the final borrowers are households instead of production firms. The only change in the model concerns the financial sector. The rest of the economy, notably the behaviour of consumers and the functioning of the productions sectors, remains as in Chapter 2.

This approach considers financial institutions that have an active intermediary role in the loan contract. Unlike investors whose sole role in the model is to facilitate the lending of Ricardian consumers and who do not possess intrinsic risk, banks have an inherent probability of default just as credit-constrained consumers had before (Figure 3.1). The financial institutions in this setup are leveraged and often much more than the indebted households of Chapter 2. The distinction between investors and banks is not purely a model complication but mirrors real life financial markets. Investors can be characterised as representing "safe" mutual funds institutions that possess no idiosyncratic risk, while financial institutions correspond to investment banks that are highly leveraged. The emergence of such highly leveraged banks may be traced back to the desire for larger profits from riskier investments which safe financial agents such as the investors in this model would be unwilling to finance directly. The presence of financial institutions that possess idiosyncratic risk on their own allows comparing the effects of an increase in financial leverage with a rise in consumer indebtedness on households and on the economy as a whole.

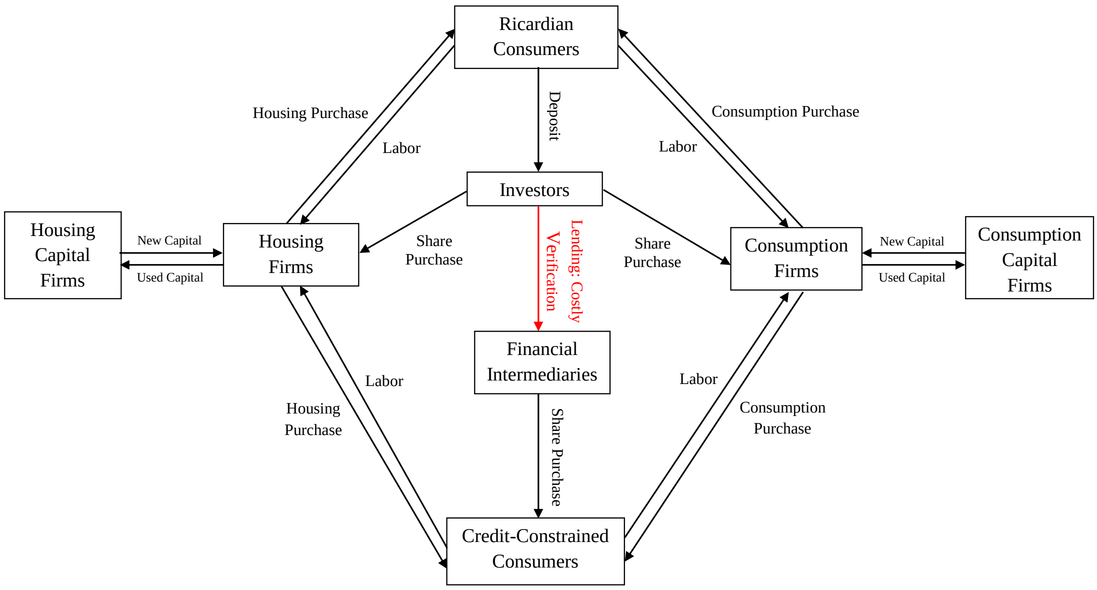


Figure 3.1: Model with Financial Frictions in the Financial Sector

In this version, borrowing consumers possess no idiosyncratic uncertainty. This is not a reflection of reality but serves to contrast the experience of households with the subsequent scenario where both consumers and banks are leveraged. Since mortgage-buying households have no idiosyncratic risk of default, their arrangement with financial institutions is a share contract, not a loan agreement as in Chapter 2. There is no asymmetric information in the return of consumers so they offer a guaranteed return on their mortgage. In this scenario, the financial frictions are entirely contained in the intermediary financial sector. Borrowing households are no longer "credit-constrained" in the strict sense of the word as used in Chapter 2 since they have no unknown probability of bankruptcy. However, this section will continue to call them so to distinguish them from non-borrowing Ricardian consumers. The rest of the economy remains the same. The two production sectors that manufacture consumption and housing as well as the capital producing firms operate as described in Chapter 2.

3.3.2 The Loan Contract of Financial Institutions

Credit-constrained financial institutions borrow from investors and lend to mortgage-buying consumers. There are financial frictions in the contract between investors and financial institutions motivated by bankruptcy prospects inherent to banks. The loan contract of financial institutions is very similar to the mortgage contract of credit-constrained consumers detailed in Chapter 2 so this section only briefly outlines how banks borrow from investors. In this model, consumers possess no idiosyncratic risk so they do not borrow as part of the loan contract. The contract between investors and banks is a loan contract, while the one between banks and credit-constrained households is a share contract.

In this version, there is an idiosyncratic shock ω^B associated with lending to financial institutions. Similar to the credit-constrained consumers' setup in Chapter 2, investors have to pay an auditing fee μ^B to learn the realization of ω^B. This makes lending to banks risky so financial institutions have to pay a premium on external funds. The interest rate charged to financial institutions is R^B.

The contract between borrowing consumers and financial institutions is a share contract. This is akin to banks purchasing "shares" in the value of the mortgage of consumers. Credit-constrained households purchase housing at the value of $p_t H_t^C$. Part of the purchase is covered by their own net worth N_t^C. The difference $p_t H_t^C - N_t^C$ is the amount consumers borrow from financial institutions.

Since the individual optimization problem of each bank can be aggregated to hold for the whole economy in the same way as the loan problem of credit-constrained consumers in the previous model, the financial institutions' loan contract setup proceeds directly on the aggregate level. Financial institutions purchase $p_t H_t^C - N_t^C$ of these shares. Let the net worth of banks be N_t^B. Assuming costless financial intermediation, banks will need to borrow $p_t H_t^C - N_t^C - N_t^B$ from investors in order to finance the mortgage purchase of credit-constrained consumers. The returns to financial institutions will be exactly equal to the contractual rate of interest Z_t^B (the fixed interest rate paid to investors) in the contract between banks and investors, times the amount borrowed from its investors $p_t H_t^C - N_t^C - N_t^B$. Let the state-contingent interest rate charged to financial institutions be $E_t R_{t+1}^B$. This is the rate at which financial institutions maximise their loan return but repay at the realised rate R_{t+1}^B. Then at the loan repayment next period, the expected returns to banks from the loan extended today are exactly $E_t \left(\bar{\omega}_{t+1}^B R_{t+1}^B\right) \left(p_t H_t^C - N_t^C\right)$ which must equal the repayment to investors:

$$E_t\left(\bar{\omega}_{t+1}^B R_{t+1}^B\right)\left(p_t H_t^C - N_t^C\right) = Z_t^B\left(p_t H_t^C - N_t^C - N_t^B\right)$$

Investors want to insure themselves against the idiosyncratic shocks of financial institutions in all possible states so they diversify by signing contracts with an infinite number of banks. The loan contract specifies how the investment revenue is split between the lenders (investors) and the borrowers (financial institutions) under all possible realizations of the default probability of banks $\bar{\omega}^B$. Let $\Gamma^B(\bar{\omega}^B)$ be the gross share of the loan contract that goes to lenders (defined in a similar way as $\Gamma^C(\bar{\omega}_i^C)$ in Chapter 2). Investors have to pay an auditing fee μ^B on the fraction of contracts that default denoted $G^B(\bar{\omega}^B)$ (explained in detail in the consumer mortgage contract setup in Chapter 2). Hence the net share that goes to lenders is $E_t\left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right)$. Investors are perfectly competitive, making zero profit. As the idiosyncratic risk is totally diversifiable for the investors, the required return to investors is the risk-free rate R_t. The participation constraint for investors requires that tomorrow's earnings of investors on their loans made today $E_t\left(\left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right) R_{t+1}^B\right)\left(p_t H_t^C - N_t^C\right)$ are exactly equal to the risk-free rate (the opportunity cost of the deposits of Ricardian consumers) times the amount they are lending to financial institutions $p_t H_t^C - N_t^C - N_t^B$. The participation constraint of investors in this loan contract is:

$$E_t\left(\left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right) R_{t+1}^B\left(p_t H_t^C - N_t^C\right)\right) = R_t\left(p_t H_t^C - N_t^C - N_t^B\right) \quad (3.3.1)$$

Banks borrow funds from investors and in turn lend to credit-constrained households. Every period, they choose the optimal pair of cutoff risk $\bar{\omega}^B$ and housing H^C to maximise their next period expected share of the loan $E_t\left(1 - \Gamma(\bar{\omega}_{t+1}^B)\right)$ of the total value of the contract that consists of the housing stock $p_t H_t^C$ minus the net worth of credit-constrained consumers N_t^C valued at the expected bank interest rate $E_t R_{t+1}^B$:

$$max\ E_t\left(\left(1 - \Gamma^B(\bar{\omega}_{t+1}^B)\right) R_{t+1}^B\right)\left(p_t H_t^C - N_t^C\right) \quad (3.3.2)$$

subject to the zero profit condition of investors (3.3.1). The Lagrangian is:

$$
\begin{aligned}
L_t^B \;=\; & E_t\left(\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)R_{t+1}^B\right)\left(p_tH_t^C-N_t^C\right)+\\
& +\lambda_t\left[E_t\left(\left(\Gamma^B(\bar{\omega}_{t+1}^B)-\mu^BG^B(\bar{\omega}_{t+1}^B)\right)R_{t+1}^B\right)\left(p_tH_t^C-N_t^C\right)-R_t\left(p_tH_t^C-N_t^C-N_t^B\right)\right]
\end{aligned}
$$

The first order conditions are:

$$
\begin{aligned}
E_t\bar{\omega}_{t+1}^B:\;\; & -E_t\left(\Gamma_\omega^B(\bar{\omega}_{t+1}^B)R_{t+1}^B\right)\left(p_tH_t^C-N_t^C\right)\\
& +\lambda_t\left[E_t\left(\left(\Gamma_\omega^B(\bar{\omega}_{t+1}^B)-\mu^BG_\omega^B(\bar{\omega}_{t+1}^B)\right)R_{t+1}^B\right)\left(p_tH_t^C-N_t^C\right)\right]=0
\end{aligned}
$$

$$
H_t^C:\;\; E_t\left[\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)R_{t+1}^Bp_t+\lambda_t\left[\left(\Gamma^B(\bar{\omega}_{t+1}^B)-\mu^BG^B(\bar{\omega}_{t+1}^B)\right)R_{t+1}^Bp_t-R_tp_t\right]\right]=0
$$

The optimization problem determines the risk premium of financial institutions $\frac{E_tR_{t+1}^B}{R_t}$ as a function of the cutoff bank default risk $\bar{\omega}^B$:

$$
\frac{E_tR_{t+1}^B}{R_t}=E_t\left[\frac{\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(\Gamma_\omega^B(\bar{\omega}_{t+1}^B)-\mu^BG_\omega^B(\bar{\omega}_{t+1}^B)\right)}}{\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)+\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(\Gamma_\omega^B(\bar{\omega}_{t+1}^B)-\mu^BG_\omega^B(\bar{\omega}_{t+1}^B)\right)}\left(\Gamma^B(\bar{\omega}_{t+1}^B)-\mu^BG^B(\bar{\omega}_{t+1}^B)\right)}\right] \tag{3.3.3}
$$

The equation for the external finance premium facing banks is similar to that for the risk premium of indebted households in Chapter 2. In both cases, the risk premium depends on the bankruptcy prospects of credit-constrained agents.

Like consumers in Chapter 2, financial institutions earn positive returns from the mortgage loan which they share with investors. The equity of financial institutions is their share of the return on the consumer loan:

$$
V_t^B=\Gamma^C\left(1-\Gamma^B(\bar{\omega}_t^B)\right)R_t^Bp_{t-1}H_{t-1}^C=\left(1-\Gamma^B(\bar{\omega}_t^B)\right)R_t^B\left(p_{t-1}H_{t-1}^C-N_{t-1}^C\right) \tag{3.3.4}
$$

Additionally, every period a constant fraction $1-\nu^B$ of financial institutions exit the market. When they exit, they spend their remaining equity on consumption. In this case the consumption of banks $C_{E,t}^B$ on exit is:

$$
C_{E,t}^B=\left(1-\nu^B\right)\left(1-\Gamma^B(\bar{\omega}_t^B)\right)R_t^B\left(p_{t-1}H_{t-1}^C-N_{t-1}^C\right) \tag{3.3.5}
$$

In order to start the accumulation of their net worth, financial institutions supply inelastically one unit of labour to the housing sector that goes toward starting their net worth. For their labour supply, banks earn a fixed wage w_t^B.

The net worth of banks N_t^B evolves according to:

$$N_t^B = \nu^B V_t^B + w_t^B \tag{3.3.6}$$

3.3.3 The Share Contract of Credit-Constrained Consumers

The contract between financial institutions and credit-constrained consumers is a share contract. Banks finance the housing mortgage and absorb any resulting gains and losses. In this version, there is no asymmetric information problem on the part of consumers so no monitoring takes place. The financial institutions and the households split the proceeds of the mortgage according to the shares of their investments, regardless of the idiosyncratic shock to credit-constrained consumers $\bar{\omega}^C$. As intermediaries, banks finance a mortgage worth of $p_t H_t^C - N_t^C$ to borrowing households. The expected return from the mortgage for consumers is $E_t R_{t+1}^C$. The average share of profits from the share contracts that goes to banks is Γ^C:

$$\Gamma_t^C E_t R_{t+1}^C p_t H_t^C = \frac{p_t H_t^C - N_t^C}{p_t H_t^C} \int_0^\infty \omega^C R_{t+1}^C p_t H_t^C dF(\omega^C)$$

Since intermediaries can diversify over an infinite number of households and the share contract does not depend on ω^C, then $\int_0^\infty \omega^C dF(\omega^C) = 1$. Hence the share Γ^C is independent of the idiosyncratic shock ω^C because there is no asymmetric information in the share contract:

$$\Gamma_t^C = \frac{p_t H_t^C - N_t^C}{p_t H_t^C} \tag{3.3.7}$$

The equation above implies that the share agreement provides both consumer borrowers and intermediaries with a fixed return regardless of the realization of ω^C and ω^B. Hence when an adverse shock changes borrowing conditions for financial institutions by raising the risk premium and as a result, the portion of the loan that goes to investors, banks cannot readjust the terms of the share contract to extract more equity and improve their balance sheets. The rigidity of the share contract protects consumers against potential downturns in the financial sector and forces banks to bear most of the negative consequences of tighter mortgage financing.

Finally, the realised return to financial institutions' investment must equal the realised return on the consumer mortgage so $R_t^C = R_t^B$.

Both financial institutions and credit-constrained consumers earn positive returns from the mortgage. The period equity of indebted households is:

$$V_t^C = \left(1 - \Gamma^C\right) R_t^C p_{t-1} H_{t-1}^C = R_t^C N_{t-1}^C \tag{3.3.8}$$

Like in Chapter 2, each period a fraction $1 - \nu^C$ of consumers retire. When they retire, they spend their remaining equity on consumption. The consumption on retirement of credit-constrained consumers is:

$$C_{E,t}^C = \left(1 - \nu^C\right) R_t^C N_{t-1}^C \tag{3.3.9}$$

In order to start the accumulation of their net worth, credit-constrained households supply inelastically one unit of labour to the housing sector that goes toward starting their net worth for which their receive a wage w_t^C.

The net worth of credit-constrained consumers N_t^C is:

$$N_t^C = \nu^C V_t^C + w_t^C \tag{3.3.10}$$

3.4 The Complete Model with Financial Institutions Loan Contract

3.4.1 Complete Model Changes

The production sectors and consumer behaviour are the same as in Chapter 2. The introduction of financial institutions necessitates some changes to accommodate their presence. These involve the labour supply of banks in the housing sector and the market clearing conditions for consumption and housing. The production functions in both sectors, the capital development and the goods demand by consumers remain unchanged. These are described in detail in Chapter 2.

The labour supply equation in the housing sector must account for the unit labour supply by financial institutions denoted $L^H_{B,t}$. Factoring in that labour supply, the total labour supply in the production of housing by origin is:

$$L^H_t = (L^H_{F,t})^{\Omega_H}(L^H_{CC,t})^{\Omega_C}(L^H_{B,t})^{\Omega_B}(L^H_{H,t})^{(1-\Omega_H-\Omega_C-\Omega_B)} = (L^H_{H,t})^{(1-\Omega_H-\Omega_C-\Omega_B)} \quad (3.4.1)$$

where $L^H_{F,t}$ is the labour supply by housing firms, $L^H_{CC,t}$ the labour supply by credit-constrained consumers dedicated to starting their net worth, $L^H_{B,t}$. The labour supply by financial institutions and $L^H_{H,t}$ is regular household labour supply.

The production function for housing becomes:

$$Y^H_t = A^H_t(K^H_{t-1})^{\alpha_H}(L^H_{H,t})^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C-\Omega_B)} \quad (3.4.2)$$

The optimal condition for housing capital remains the same while that for labour becomes:

$$w_t = (1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y^H_t}{L^H_{H,t}}\right) \quad (3.4.3)$$

The market clearing condition for consumption includes the consumption of financial institutions on exit $C^B_{E,t}$ in addition to that of consumers $C^C_{E,t}$:

$$Y^F_t = C^R_t + C^C_t + C^F_{E,t} + C^C_{E,t} + C^B_{E,t} + I^F_t + I^H_t \quad (3.4.4)$$

The market clearing condition for housing is augmented by the monitoring fee paid by investors on their loan to banks $\mu^B G^B(\bar{\omega}^B_t)R^B_t\left(p_{t-1}H^C_{t-1} - N^C_{t-1}\right)$ which replaces the one investors paid on their lending to credit-constrained consumers:

$$\begin{aligned} &p_t Y^H_t + (1-\delta)p_t(H^R_{t-1} + H^C_{t-1}) = \\ &= p_t(H^R_t + H^C_t) + C^H_{E,t} + \mu^B G^B(\bar{\omega}^B_t)R^B_t\left(p_{t-1}H^C_{t-1} - N^C_{t-1}\right) \end{aligned} \quad (3.4.5)$$

A complete list of the model equations is in Appendix 3.A.

3.4.2 Model Calibration

The parameters that govern the general equilibrium for the banks' loan contract model are the same as in Chapter 2 to ensure comparability of the two models. The presence of financial institutions in the loan contract setup is calibrated to satisfy the following requirements:

1. The steady state housing value to net worth of financial institutions is $\frac{pH^C}{N^B} = 10$. This implies that the leverage of financial institutions is $\frac{pH^C - N^C}{N^B} = 5$ (the steady state leverage of credit-constrained consumers remains 2) (Hirakata, Sudo and Ueda, 2009).

2. The failure rate of financial institutions $F^B(\bar{\omega}^B)$ is 2% (same as that of credit-constrained consumers).

The loan parameters for this version of the model are in Table 3.1.

Table 3.1: Loan Contract Parameters in the Financial Institutions Loan Contract Economy

Parameter	ν^F	ν^H	ν^C	ν^B	μ^B
Value	0.99	0.99	0.98	0.97	0.43

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The steady state values of the model variables are in Table 3.2.

Table 3.2: Steady State Variables in the Financial Institutions Loan Contract Economy

Variable	Y^F	Y^H	C^R	C^C	H^R	H^C	L^R	L^C	I^F	I^H
Value	100	10	32.4	29.9	211	170	15.6	19.9	24.9	3.35
Variable	K^F	K^H	L^F	L^H	R	R^B	N^F	C_E^F	N^H	C_E^H
Value	997	134	29.9	5.53	1.01	1.015	498	5.68	67.0	0.80
Variable	w	p	N^C	C_E^C	N^B	C_E^B	$\bar{\omega}^B$	σ^B		
Value	2.15	1.88	86.4	2.54	32.0	1.13	0.79	0.11		
Variable	Γ^B	G^B	Γ_ω^B	G_ω^B	Γ_σ^B	G_σ^B	$\Gamma_{\omega\omega}^B$	$G_{\omega\omega}^B$	$\Gamma_{\omega\sigma}^B$	$G_{\omega\sigma}^B$
Value	0.79	0.02	0.98	0.45	-0.04	0.74	-0.57	10.8	-0.97	14.4

The log-linearization of the model is in Appendix 3.B.

3.5 The Financial Institutions Loan Contract Results

This section simulates the role of an oversupply of housing and of a reduction in financial volatility in an economy where financial institutions are subject to credit frictions and compares it to the results from the consumer loan contract model described in Chapter 2.

3.5.1 The Housing Oversupply

In Chapter 2 leveraged consumers experienced the full impact of the depressed housing price as a financial downturn characterised by restricted access to credit and higher indebtedness. When consumers have a share contract with banks, they are relatively shielded from a shock in the housing sector. However, financial institutions bear most of the downturn. Being more highly leveraged to begin with, banks see their leverage deteriorate more than that of households in the model discussed in Chapter 2 plunging them into a deep recession.

A permanent technological improvement in the production of housing in an economy where the borrowing constraints are in the financial sector does not produce quantitatively different results for households and producers compared to the same disturbance in the consumer mortgage contract economy from Chapter 2 (Figure 3.2). The oversupply of housing triggers a reduction in the housing price as the economy absorbs the excess real estate. The net worth of leveraged borrowers still declines but this time it is the financial sector which experiences the contraction (Figure 3.2.22). The indebtedness of banks deteriorates leading to an upward adjustment in the risk premium that reflects new higher default probabilities of banks (Figures 3.2.25, 3.2.26 and 3.2.28). The experience of financial institutions does not just mirror that of households in the model described in Chapter 2 but is quantitatively worse.

Banks experience a more profound worsening of their balance sheet than consumers in Chapter 2 due to their higher initial leverage (the steady state housing stock value is twice the net worth of credit-constrained consumers in Chapter 2 while the steady state housing stock value is ten times the net worth of banks). This results in a more volatile position for financial institutions since they have very few assets (net worth) relative to their liabilities (mortgage value) to insulate them against adverse shocks. Leveraged financial institutions experience a worsening in their net worth that is almost four times the corresponding decrease in the net worth of indebted consumers (Figures 3.2.22, solid line and 3.2.21, dashed line). As a result, the indebtedness of banks deteriorates five-fold compared to that of mortgage-buyers in the previous model (Figures 3.2.26, solid line and 3.2.27, dashed line). The default risk of banks suffers a percentage worsening similar to that of consumers in the first scenario but from a much higher initial level that reflects the higher starting leverage of financial institutions (Figure 3.2.25). The increase of the already high default risk of banks prompts investors to raise the risk premium above what they charged to consumers in their downturn discussed in Chapter 2 (Figure 3.2.28).

In this version, households have no idiosyncratic uncertainty in their returns so the share contract does not involve risk sharing between consumers and financial institutions. The lack of risk sharing implies households are relatively shielded from the downturn that befalls banks. Their net worth experiences no deterioration (Figure 3.2.21). Since credit-constrained consumers have no idiosyncratic risk, their return from the share contract is not contingent on the shock realization. Their leverage not only does not deteriorate (unlike the case when they were the ones subject to idiosyncratic risk) but experiences a relative improvement that corresponds to their post-shock increased housing stock (Figure 3.2.27). This is at the expense of financial institutions which have to honour their obligations to credit-constrained consumers and must provide them with a fixed return regardless of the realization of their own idiosyncratic shock and according to the share agreement. Since once agreed, the share contract cannot be renegotiated to improve the net worth of banks in times of recessions, financial institutions bear the full force of the worsened financial position.

Comparing the subsequent recovery of optimizing agents in both versions of the model, it is obvious that the leverage of financial institutions returns to its pre-shock position relatively faster than that of credit-constrained consumers. The higher initial leverage of banks, coupled with the bigger subsequent worsening, implies that financial institutions must repair their balance sheets relatively quickly in order to maintain their solvency and to continue to provide the mortgage funding to consumers (Figure 3.2.26). The leverage of financial institutions takes about 60 quarters (15 years) to return to pre-shock levels while that of credit-constrained consumers took almost 100 quarters (25 years) to reach its steady state value in Chapter 2. The default risk mirrors the respective leverage evolution of the two types of credit-constrained agents (Figures 3.2.25). The higher leverage of banks implies financial intuitions tend to experience a sharp but relatively short downturns, whereas consumers suffer a milder but a relatively protracted recession.

3.5.2 The Temporary Financial Relaxation

Consumers experienced a leverage worsening following easy borrowing conditions in Chapter 2. In this version, they are not indebted but financial institutions are instead. An improved access to borrowing permits financial institutions to increase their debt holdings thereby raising their leverage. As the risk premium tightens in response to higher bankruptcy prospects, their downturn is more severe than that of consumers in the mortgage contract due to their higher initial indebtedness. Households, on the other hand, have a fixed share contract with banks so their indebtedness experiences a modest and brief impact.

Relaxing the borrowing constraints for financial institutions yields qualitatively similar results as the case with a credit easing for indebted consumers described in Chapter 2 (Figure 3.3). The decreased bank volatility reduces the risk premium of banks much in the same way as for indebted households (Figure 3.3.28). Since the initial leverage of financial institutions is higher than that of consumers, banks take a better advantage of the lower risk premium and amass more liabilities. As a result, the leverage of indebted banks rises more than twice that of indebted households (Figures 3.3.26, solid line and 3.3.27, dashed line). The manner in which the indebtedness level increases is also different. Moderately leveraged consumers are able to immediately increase their mortgage holdings, while highly indebted financial institutions achieve their peak indebtedness sometime after the initial relaxation of borrowing conditions. This is due to the fact that owing to their high initial indebtedness, banks take more time after the credit easing to amass additional liabilities thus stretching their net worth even further (Figure 3.3.22). In contrast, the moderately indebted consumers could ratchet up their leverage relatively faster post-shock (Figure 3.3.27, dashed line).

Since the leverage of financial institutions rises more than that of credit-constrained consumers and from a higher initial level, their probability of default increases (Figure 3.3.25). Banks are now more likely to go bankrupt so as the shock wears out, lenders raise the interest rate they charge financial institutions and the risk premium begins to tighten (Figures 3.3.26 and 3.3.28). The reversal of the risk premium signals the end of the brief period of relaxed borrowing for financial institutions. Although the risk premium reverts to its steady state fairly quickly, the improvement in the leverage ratio is a long and protracted process. It takes financial institutions considerable time of austerity to unwind their high indebtedness.

The effect of the temporarily relaxed credit access on the balance sheet of financial institutions is not much different from the balance sheet of credit-constrained consumers except for the considerable worsening of their leverage. Since both types of indebted agents have different initial leverage, although the reduction in volatility is identical, the ensuing downturn is deeper for banks. In contrast to indebted financial institutions, in this version credit-constrained consumers experience only a mild increase in their leverage followed by a speedy recovery. This is again a feature of the share contract between financial institutions and borrowing households which is unlike the loan contract described in Chapter 2 A loan arrangement for households would involve a participation constraint (equation (2.3.1) in Chapter 2) that reflects a rise in their own risk of default following the shock. The participation constraint would shift to the left implying a degree of risk sharing between financial institutions and consumers. However, a share contract is based on fixed returns regardless of the realization of the idiosyncratic shock for both lenders and borrowers so consumers experience none of the dampening effects of risk sharing. If financial institutions were to attempt to transfer some of the worsening in financial conditions onto consumers, consumers would be better off borrowing from elsewhere since they have no idiosyncratic uncertainty. Hence banks bear the full effect of the credit tightening while credit-constrained consumers are relatively shielded.

Figure 3.2: Housing Oversupply in a Financial Institutions Loan Contract

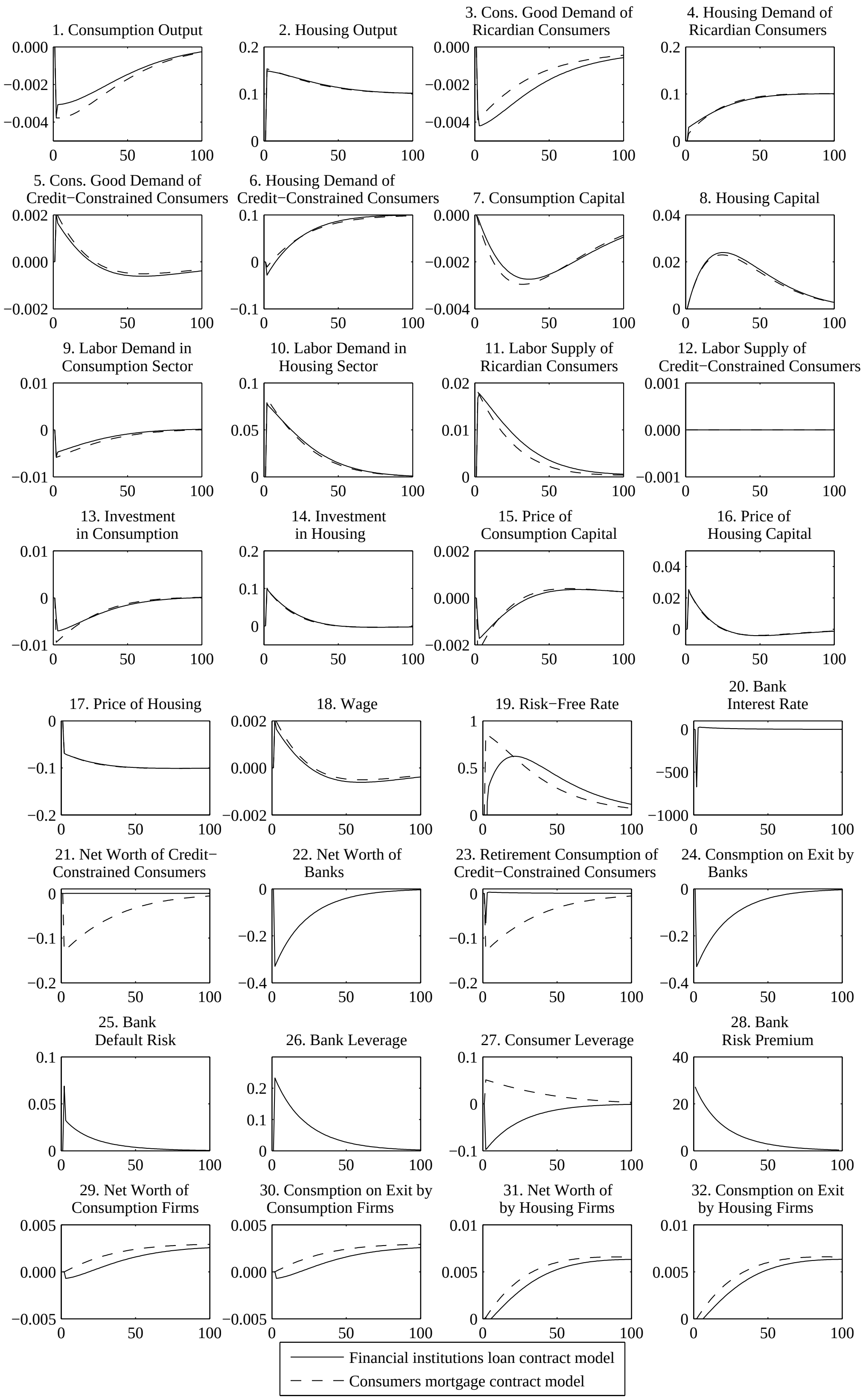
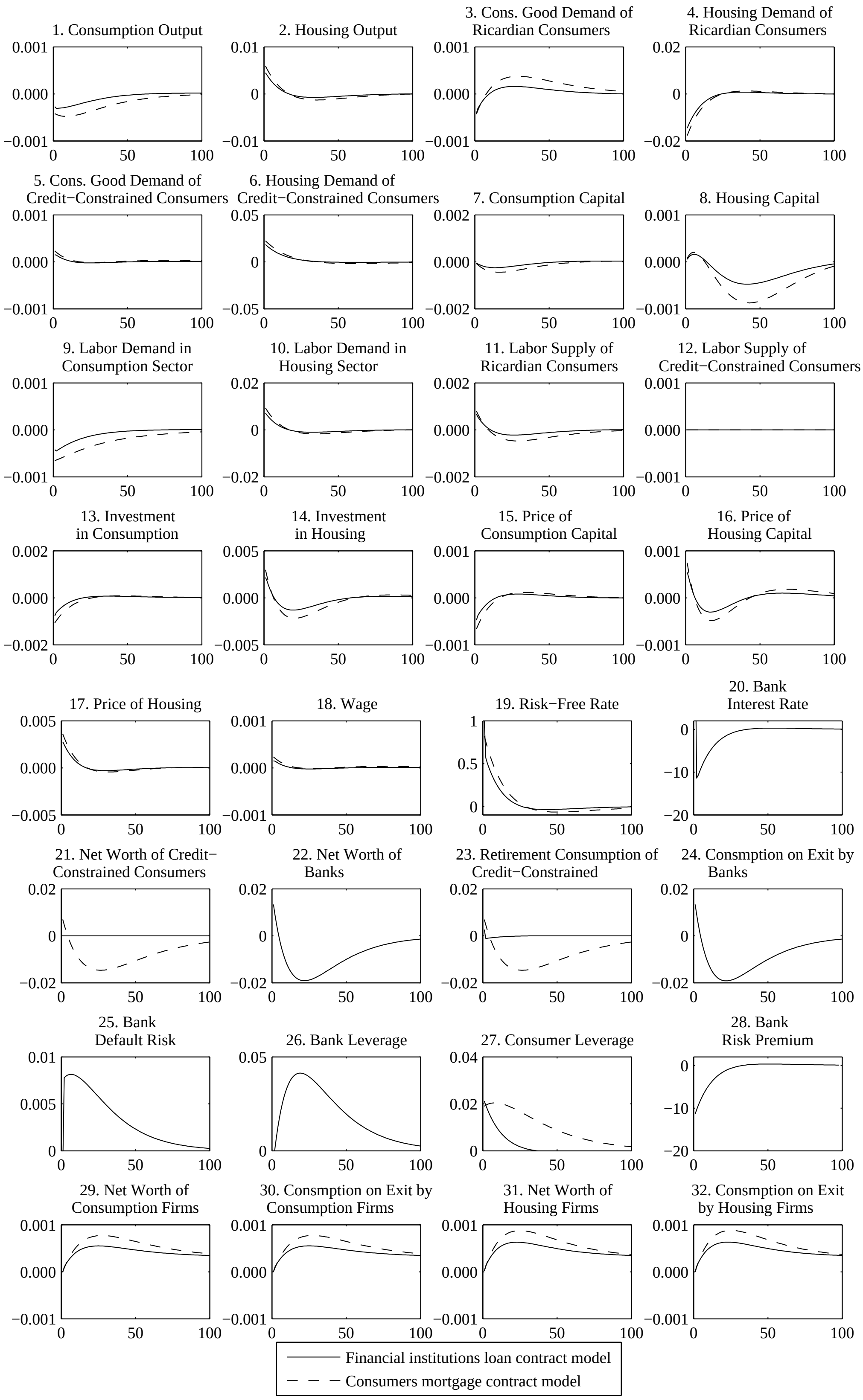


Figure 3.3: Reduced Bank Volatility in a Financial Institutions Loan Contract



3.6 The Chained Loan Contracts

3.6.1 Description of the Chained Loan Contracts

The model with credit constraints both in the consumer and in the financial sector highlights opportunities for risk sharing following a disturbance that negatively impacts the leverage of indebted agents. When consumers are no longer protected by a fixed share return and instead participate in the chained loan contracts, financial institutions can transfer some of the downturn onto them in order to hasten their own recovery. The results mirror the subprime crisis with a sudden but brief downturn for banks and a prolonged recession for credit-constrained consumers.

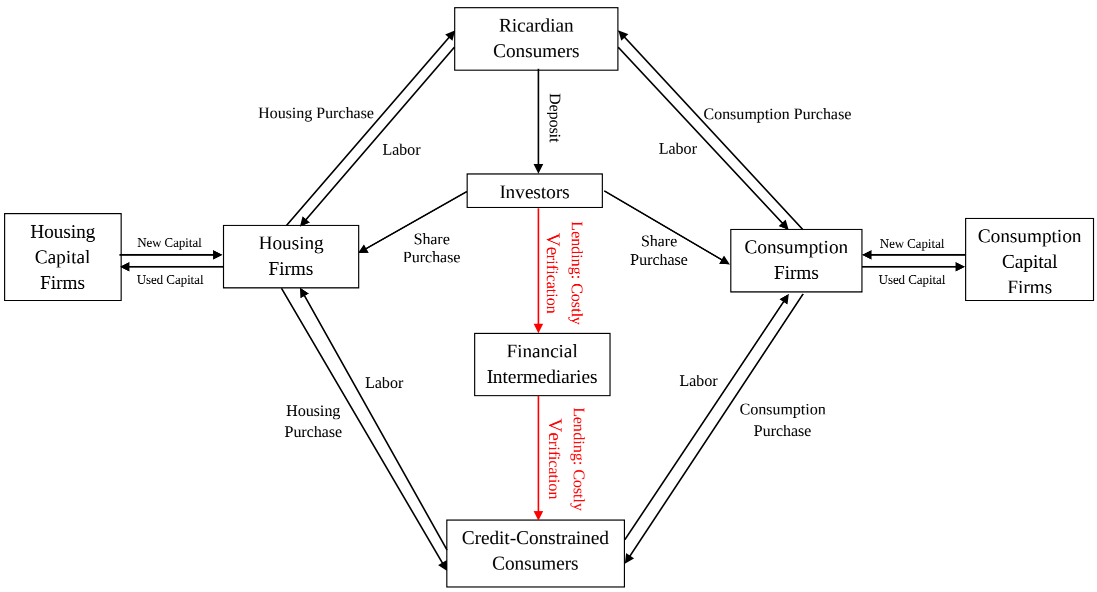


Figure 3.4: Model with Financial Frictions in the Consumer and Financial Sectors

The model in the previous section considered credit frictions in the loan contract between investors and financial institutions while the model of Chapter 2 discussed financial constraints in the loan contract between investors and credit-constrained consumers. This section puts the two together in what is known as the chained loan contracts model (Figure 3.4). The model assumes that financial institutions act as intermediaries that borrow funds from investors and in turn lend to credit-constrained households. Both banks and borrowing consumers are subject to idiosyncratic uncertainty and participate in loan contracts. Taken together, the two transactions constitute two joined mortgage contracts similar to the ones in Hirakata, Sudo and Ueda (2009).

In this version, banks borrow from investors and in turn lend to households. Both financial institutions and credit-constrained consumers possess idiosyncratic risk of default that is distinct from one another. The presence of idiosyncratic uncertainty in the returns of banks and households necessitates two loan contracts that provide state-contingent returns to investors. At the beginning of the financial intermediation, investors lend to financial institutions in a loan arrangement that ensures them a fair return regardless of the realization of the idiosyncratic shock of banks. At the tail end of the financial intermediation, credit-constrained consumers borrow from financial institutions as part of another loan contract that ensures their participation. Banks are the intermediaries that participate in both loan contracts so they optimise both arrangements at the same time. This is also consistent with empirical observations of real life banks. Hence the two loan contracts must be chained in the sense that they are linked in the same optimization problem.

3.6.2 The Chained Loan Contracts

Financial institutions (i.e. banks) borrow from investors one period in advance and lend to mortgage-buying consumers to finance their housing purchase. Both banks and credit-constrained consumers are borrowing-constrained in the sense that they each have a distinct probability of default that is known to them but unknown to other participants in the financial market who know only the distribution of the bankruptcy likelihood. Hence there are financial frictions both in the contract between investors and financial institutions and in the contract between banks and credit-constrained households. The presence of credit constraints necessitates that both arrangements are loan contracts where the returns are contingent on the realization of the two idiosyncratic shocks. Since financial institutions are intermediaries and participate in both contacts, the contracts are chained so that the returns from the first contract can provide sufficient funds to cover the lending in the second one.

In this version, there is an idiosyncratic shock ω^B associated with lending to financial institutions and a distinct idiosyncratic shock ω^C associated with lending to credit-constrained consumers. As with the financial institutions' loan contract, investors have to pay an auditing fee μ^B to learn the realization of ω^B. This makes lending to banks risky so financial institutions have to pay a premium on external funds. Financial institutions on their part, must pay a monitoring cost μ^C to learn the realization of the consumer default probability ω^C.

Since the individual optimization problem of each bank can be aggregated to hold for the whole economy in the same way as before, the chained contracts setup proceeds directly on the aggregate level. Banks borrow funds from investors and in turn lend to credit-constrained households. Every period, they choose the optimal pair of cutoff risk $\bar{\omega}^B$ and housing H^C to maximise their next period expected share $1 - \Gamma^B(\bar{\omega}^B_{t+1})$ of the total value of the contact that consists of the housing stock $p_t H^C_t$ minus the net worth of credit-constrained consumers N^C_t:

$$max\ E_t \left(\left(1 - \Gamma^B(\bar{\omega}^B_{t+1})\right) R^B_{t+1} \right) \left(p_t H^C_t - N^C_t \right) \tag{3.6.1}$$

Banks lend to households at a state contingent markup rate $E_t R^C_{t+1}$ that is different from the rate $E_t R^B_{t+1}$ at which they repay their loans. The difference accounts for the distinct probabilities of default of both types of indebted agents. The expected earnings of financial institutions from lending to credit-constrained consumers equal the share $E_t \Gamma^C(\bar{\omega}^C_{t+1})$ they will receive tomorrow from the loan made today to households minus the auditing fee on insolvent consumer loans $\mu^C E_t G^C(\bar{\omega}^C_{t+1})$:

$$E_t \left(\left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1}) \right) R^C_{t+1} \right) p_t H^C_t$$

The earnings of banks must equal the opportunity cost of lending to credit-constrained consumers, which is the value of the loan to banks at their markup interest rate $E_t R^B_{t+1}$. Hence the gross return on the banks' loan to credit-constrained consumers is:

$$E_t \left(\left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1}) \right) R^C_{t+1} \right) p_t H^C_t = E_t R^B_{t+1} \left(p_t H^C_t - N^C_t \right) \tag{3.6.2}$$

The left hand side is the banks' share of the loan to borrowing households after monitoring and the right hand side is the gross return (the value of the housing purchase $p_t H^C_t$ minus the consumers' net worth N^C_t valued at the bank interest rate $E_t R^B_{t+1}$) from the housing purchase to financial institutions.

Credit-constrained households will participate in the chained loan contracts only if their participation constraint is met. Instead of taking part in the chained loan contracts, credit-constrained consumers can purchase housing using their own net worth N_t^C. In this alternative case, the ex-post return to their investments equals $E_t R_{t+1}^C N_t^C$. Hence credit-constrained consumers will participate in the chained contracts only if their share of the loan is at least equal to the value of their net worth:

$$E_t \left(\left(1 - \Gamma^C(\bar{\omega}_{t+1}^C)\right) R_{t+1}^C \right) p_t H_t^C \geq E_t R_{t+1}^C N_t^C \tag{3.6.3}$$

The first part of the chained contracts consists of investors lending to banks. Financial institutions split the gross profit from their loan to credit-constrained consumers with investors. This contract has the same costly state verification structure as the single loan contract. Banks own the net worth N_t^B and invest the amount $p_t H_t^C - N_t^C$ in the loan to credit-constrained consumers. They borrow the rest $p_t H_t^C - N_t^C - N_t^B$ from investors and repay the loan using their profits from lending to credit-constrained households. Investors receive a net share of the loan to banks $E_t \left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right)$ that includes the monitoring fees paid on failed banks. Financial institutions are subject to an idiosyncratic bankruptcy shock ω^B and their ex-post gross return on the loans to credit-constrained consumers is $\omega^B R^B$. Investors would participate in the chained contracts only if they get a fair return on their lending to financial institutions. Like in the single contract model, the lender's share of the profit in the contract with financial institutions $E_t \left(\Gamma^B(\bar{\omega}_{t+1}^B) R_{t+1}^B\right) \left(p_t H_t^C - N_t^C\right)$ after paying the monitoring fee $\mu^B E_t G^B(\bar{\omega}_{t+1}^B)$ must at least equal the opportunity cost of the investors' funds $R_t \left(p_t H_t^C - N_t^C - N_t^B\right)$ valued at the risk-free rate R_t. Hence the zero profit participation constraint for investors must specify the amount of funds that banks borrow from investors $pH^C - N^C - N^B$, the cut-off value of the idiosyncratic shock $\bar{\omega}^B$ and the return rate of the loan to non-defaulting banks R^B. Since banks borrow at time t and repay the funds at $t+1$, the participation constraint of investors is:

$$E_t \left(\left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right) R_{t+1}^B \right) \left(p_t H_t^C - N_t^C\right) \geq R_t \left(p_t H_t^C - N_t^C - N_t^B\right) \tag{3.6.4}$$

Lenders sign contracts with a lot of banks, to diversify away the idiosyncratic risk of financial institutions.

Substituting equation (3.6.2) into (3.6.4) and noting that both equations (3.6.3) and (3.6.4) bind at the optimum, eliminates the bank interest rate $E_t R^B_{t+1}$ and reduces the conditions that financial institutions must satisfy to two:

$$E_t \left(\left(\Gamma^B(\bar{\omega}^B_{t+1}) - \mu^B G^B(\bar{\omega}^B_{t+1}) \right) \left(\Gamma^C(\bar{\omega}^C_{t+1}) - \mu^C G^C(\bar{\omega}^C_{t+1}) \right) R^C_{t+1} \right) p_t H^C_t = \\ = R_t \left(p_t H^C_t - N^C_t - N^B_t \right) \tag{3.6.5}$$

$$E_t \left(1 - \Gamma^C(\bar{\omega}^C_{t+1}) \right) p_t H^C_t = N^C_t \tag{3.6.6}$$

In the two previous single loan contracts, the borrowing agents maximized the arrangement - credit-constrained consumers in the consumer mortgage contract and banks in the financial institutions loan contract. In both cases, the superior knowledge of borrowers about their own possibility of default allows them to push lenders to their participation constraint and extract maximum returns from the loan. The chained loan contracts, however, are linked by the presence of financial institutions. Hence it is the intermediaries (financial institutions) which maximise their profits subject to satisfying the participation constraints of both credit-constrained consumers and of investors. In this case banks, which are borrowers in one part of the contract and lenders in the other part, optimise the two sides of the contract. The reason for banks maximizing their profits is twofold. First, it is empirical fact that financial institutions are often the ones which dictate both the lending and the borrowing terms. The reason for that may be informational asymmetry. Banks use monitoring technology to collect information on borrowers and lenders that would reduce the agency cost associated with lending. This process is costly and location-specific (a bank may not be willing to lend outside of its area of expertise). On the other hand, investors, who are geographically dispersed and consumers who lack the means find the cost of this monitoring technology prohibitive. Second, financial institutions, as the intermediaries that participate in both parts of the loan contract, have a transactional advantage. It is easier for them to optimise the two loan contracts together rather than optimizing only their borrowing arrangements with investors and leaving the second loan contract to consumers. The subsequent analysis demonstrates that the results from the chained contracts model are not analytically different from those of two separate loan arrangements.

In the chained contracts model, the presence of two sectors with idiosyncratic uncertainty has important implications for risk sharing between the indebted banks and households. As intermediaries in the two mortgage contracts, financial institutions are the first to experience a deterioration in their balance sheet. However, since they also maximise the chained contracts, they can shift some of the burden of the increased risk of default onto credit-constrained consumers. As end participants in the mortgage arrangement, households have no control over the leverage distribution between indebted sectors. In times of a downturn characterised by worsened leverage for financial institutions, they can improve their own position by forcing consumers to bear a share of the deteriorating leverage. Banks achieve this by extracting a larger return from the loan contract with households in times when the default probabilities of both borrowers are higher. The participation constraint of households, unlike the share agreement in the single loan contract of financial institutions, depends on changes in bankruptcy prospects. The increased bankruptcy prospects allow financial institutions to demand a higher repayment thus shifting the participation constraint (3.6.6) of consumers to the left. This can work because, in contrast to the share contract, consumers in the chained mortgage contracts have no recourse to funds outside of the borrowing arrangement due to their idiosyncratic uncertainty. Hence credit-constrained consumers have no choice but to bear some of the deterioration in indebtedness. Often, this affords banks a speedier recovery at the expense of indebted households who are faced with a protracted recession.

The chained loan optimization problem involves banks simultaneously maximizing both loan arrangements. Let for simplicity of expression $\Gamma^B(\bar{\omega}_t^B) - \mu^B G^B(\bar{\omega}_t^B) = \Psi^B(\bar{\omega}_t^B)$ and $\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C) = \Psi^C(\bar{\omega}_t^C)$. Banks choose the optimal level of $\bar{\omega}^C$, $\bar{\omega}^B$ and H^C by solving the following Lagrangian:

$$\begin{aligned} L_t^B &= E_t\left(\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right)p_tH_t^C + \\ &\quad +\lambda_{1,t}\left[E_t\left(\Psi^B(\bar{\omega}_{t+1}^B)\Psi^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right)p_tH_t^C - R_t\left(p_tH_t^C - N_t^C - N_t^B\right)\right] \\ &\quad +\lambda_{2,t}\left[E_t\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)p_tH_t^C - N_t^C\right] \end{aligned}$$

The first order conditions are:

$$\begin{aligned} &E_t\bar{\omega}_{t+1}^C: \quad E_t\left(\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi_\omega^C(\bar{\omega}_{t+1}^C)\right)R_{t+1}^C + \\ &+\lambda_{1,t}E_t\left(\Psi^B(\bar{\omega}_{t+1}^B)\Psi_\omega^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right) - \lambda_{2,t}E_t\Gamma_\omega^C(\bar{\omega}_{t+1}^C) = 0 \end{aligned}$$

$$E_t\bar{\omega}^B_{t+1}: \quad -E_t\Gamma^B_\omega(\bar{\omega}^B_{t+1}) + \lambda_{1,t}E_t\Psi^B_\omega(\bar{\omega}^B_{t+1}) = 0$$

$$\begin{aligned} &H^C_t: \quad E_t\left(\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right)\Psi^C(\bar{\omega}^C_{t+1})R^C_{t+1}\right) + \\ &+\lambda_{1,t}\left[E_t\left(\Psi^B(\bar{\omega}^B_{t+1})\Psi^C(\bar{\omega}^C_{t+1})R^C_{t+1}\right) - R_t\right] + \lambda_{2.t}\left(1 - E_t\Gamma^C(\bar{\omega}^C_{t+1})\right) = 0 \end{aligned}$$

Solving for the Lagrange multipliers yields:

$$\lambda_{1,t} = E_t\left(\frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\Psi^B_\omega(\bar{\omega}^B_{t+1})}\right) \tag{3.6.7}$$

$$\lambda_{2,t} = E_t\left[\left[\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right) + \frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})\Psi^B(\bar{\omega}^B_{t+1})}{\Psi^B_\omega(\bar{\omega}^B_{t+1})}\right] R^C_{t+1}\frac{\Psi^C_\omega(\bar{\omega}^C_{t+1})}{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}\right] \tag{3.6.8}$$

Here $\lambda_{1,t}$ is the marginal value of the internal funds to financial institutions, and $\lambda_{2,t}$ is the marginal increase in the profits of financial institutions per unit increase in the net worth of credit-constrained consumers.

Substituting both into the first order condition for H^C_t and rearranging gives the consumer risk premium $\frac{E_tR^C_{t+1}}{R_t}$:

$$\begin{aligned} \frac{E_tR^C_{t+1}}{R_t} \quad = \quad & E_t\left(\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(1-\Gamma^C(\bar{\omega}^C_{t+1})\right)\Psi^C_\omega(\bar{\omega}^C_{t+1}) + \Gamma^C_\omega(\bar{\omega}^C_{t+1})\Psi^C(\bar{\omega}^C_{t+1})}\right)\times \\ & E_t\left(\frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right)\Psi^B_\omega(\bar{\omega}^B_{t+1}) + \Gamma^B_\omega(\bar{\omega}^B_{t+1})\Psi^B(\bar{\omega}^B_{t+1})}\right) \end{aligned} \tag{3.6.9}$$

Consider rather than the chained contracts, there are two separate loan contacts. In the first one, financial institutions borrow from investors just as in Section 3.3 of this chapter. In the second arrangement, consumers borrow from banks. The first contract is characterised by maximizing the share of financial institutions (3.3.2) subject to the participation constraint (3.3.1) of investors producing the following first order condition (reproduced from Section 3.3):

$$\frac{E_tR^B_{t+1}}{R_t} = E_t\left(\frac{\frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\left(\Gamma^B_\omega(\bar{\omega}^B_{t+1})-\mu^BG^B_\omega(\bar{\omega}^B_{t+1})\right)}}{\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right) + \frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\left(\Gamma^B_\omega(\bar{\omega}^B_{t+1})-\mu^BG^B_\omega(\bar{\omega}^B_{t+1})\right)}\left(\Gamma^B(\bar{\omega}^B_{t+1}) - \mu^BG^B(\bar{\omega}^B_{t+1})\right)}\right)$$

In the second arrangement, consumers maximise their returns (equation (2.3.2) in Chapter 2) subject to the participation constraint of financial institutions (equation (2.3.1) in Chapter 2). The result is the first order condition (2.3.4) from Chapter 2 except for the fact that the risk-free rate R_t is replaced by the bank interest rate $E_t R^B_{t+1}$:

$$E_t\left(\frac{R^C_{t+1}}{R^B_{t+1}}\right) = E_t\left(\frac{\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}}{\left(1-\Gamma^C(\bar{\omega}^C_{t+1})\right)+\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}\left(\Gamma^C(\bar{\omega}^C_{t+1})-\mu^C G^C(\bar{\omega}^C_{t+1})\right)}\right)$$

Combining these two first order conditions produces:

$$\begin{aligned}\frac{E_t R^C_{t+1}}{E_t R^B_{t+1}}\frac{E_t R^B_{t+1}}{R_t} &= E_t\left(\frac{\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}}{\left(1-\Gamma^C(\bar{\omega}^C_{t+1})\right)+\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(\Gamma^C_\omega(\bar{\omega}^C_{t+1})-\mu^C G^C_\omega(\bar{\omega}^C_{t+1})\right)}\left(\Gamma^C(\bar{\omega}^C_{t+1})-\mu^C G^C(\bar{\omega}^C_{t+1})\right)}\right)\times \\ & E_t\left(\frac{\frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\left(\Gamma^B_\omega(\bar{\omega}^B_{t+1})-\mu^B G^B_\omega(\bar{\omega}^B_{t+1})\right)}}{\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right)+\frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\left(\Gamma^B_\omega(\bar{\omega}^B_{t+1})-\mu^B G^B_\omega(\bar{\omega}^B_{t+1})\right)}\left(\Gamma^B(\bar{\omega}^B_{t+1})-\mu^B G^B(\bar{\omega}^B_{t+1})\right)}\right)\end{aligned}$$

Which collapses to the familiar equation (3.6.9):

$$\begin{aligned}\frac{E_t R^C_{t+1}}{R_t} &= E_t\left(\frac{\Gamma^C_\omega(\bar{\omega}^C_{t+1})}{\left(1-\Gamma^C(\bar{\omega}^C_{t+1})\right)\Psi^C(\bar{\omega}^C_t)+\Gamma^C_\omega(\bar{\omega}^C_{t+1})\Psi^C(\bar{\omega}^C_t)}\right)\times \\ & E_t\left(\frac{\Gamma^B_\omega(\bar{\omega}^B_{t+1})}{\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right)\Psi^B(\bar{\omega}^B_t)+\Gamma^B_\omega(\bar{\omega}^B_{t+1})\Psi^B(\bar{\omega}^B_t)}\right)\end{aligned}$$

This proves that the chained contracts feature is not just based on ad hoc assumptions and empirical observations. It is is also a mathematical and transactional simplification that provides results identical to the ones achieved with two separate loan contracts.

Equation (3.6.9) along with the two participation constraints determines the realised default probabilities $\bar{\omega}^B$ and $\bar{\omega}^C$. Even after a shock occurs, investors continue to receive the risk-free rate R_t so the model implements the realised participation constraints:

$$\begin{aligned}&\left(\Gamma^B(\bar{\omega}^B_t)-\mu^B G^B(\bar{\omega}^B_t)\right)\left(\Gamma^C(\bar{\omega}^C_t)-\mu^C G^C(\bar{\omega}^C_t)\right)R^C_t p_{t-1}H^C_{t-1} = \\ &= R_{t-1}\left(p_{t-1}H^C_{t-1}-N^C_{t-1}-N^B_{t-1}\right)\end{aligned} \tag{3.6.10}$$

$$\left(1-\Gamma^C(\bar{\omega}^C_t)\right)p_{t-1}H^C_{t-1} = N^C_{t-1} \tag{3.6.11}$$

along with the first order condition that contains the expectation of what will occur at $t+1$.

The period equity of credit-constrained consumers remains the same as in the consumer mortgage contract. The period equity of financial institutions is amended to include the return to banks $\Psi^C(\bar{\omega}_t^C)$ from lending to households:

$$\begin{aligned} V_t^B &= \left(1-\Gamma^B(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1}H_{t-1}^C - N_{t-1}^C\right) = \\ &= \left(1-\Gamma^B(\bar{\omega}_t^B)\right) \Psi^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \end{aligned} \tag{3.6.12}$$

The consumption on retirement of borrowing households remains the same as in the consumer mortgage contract. However, the consumption of banks $C_{E,t}^B$ on exit becomes:

$$C_{E,t}^B = \left(1-\nu^B\right)\left(1-\Gamma^B(\bar{\omega}_t^B)\right) \Psi^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{3.6.13}$$

3.7 The Complete Model with Chained Loan Contracts

3.7.1 Complete Model Changes

The chained contracts model has the same labour supply by banks as in the financial institutions' single loan contract version. Therefore, equations (3.4.1-3.4.3) remain unchanged. The only new modification involves the market clearing condition for housing.

The market clearing condition for housing is augmented by the two monitoring fees: one paid by investors on their loan to banks and one paid by financial institutions on their lending to credit-constrained consumers:

$$\begin{aligned} p_t Y_t^H + (1-\delta) p_t (H_{t-1}^R + H_{t-1}^C) &= p_t(H_t^R + H_t^C) + C_{E,t}^H + \\ &+ \left[\mu^C G^C(\bar{\omega}_t^C) + \mu^B G^B(\bar{\omega}_t^B) \Psi^C(\bar{\omega}_t^C)\right] R_t^C p_{t-1} H_{t-1}^C \end{aligned} \tag{3.7.1}$$

A complete list of the model equations is in Appendix 3.C.

3.7.2 Model Calibration

The parameters that govern the general equilibrium for the chained contracts model are the same as those in the consumer mortgage contract in Chapter 2 and in the financial institutions loan contract in Section 3.4. The chained contracts model satisfies the same steady state requirements as the respective single loan contracts for consumers and for financial institutions:

1. The steady state rate of the external consumer risk premium is 0.5% (Bernanke, Gertler and Gilchrist, 1999).

2. The steady state consumer leverage, i.e. value of housing stock to net worth ratio is $\frac{pH^C}{N^C} = 2$ (Bernanke, Gertler and Gilchrist, 1999).

3. The steady state leverage of financial institutions is $\frac{pH^C - N^C}{N^B} = 5$ (Hirakata, Sudo and Ueda, 2009).

4. The failure rate of both credit-constrained consumers $F^C(\bar{\omega}^C)$ and financial institutions $F^B(\bar{\omega}^B)$ is 2% (Bernanke, Gertler and Gilchrist, 1999).

The loan parameters that satisfy the chained contracts model are in Table 3.3.

Table 3.3: Loan Contract Parameters in the Chained Contracts Economy

Parameter	ν^F	ν^H	ν^C	ν^B	μ^C	μ^B
Value	0.99	0.99	0.98	0.97	0.02	0.03

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The steady state values of the model variables are in Table 3.4.

Table 3.4: Steady State Variables in the Chained Contracts Economy

Variable	Y^F	Y^H	C^R	C^C	H^R	H^C	L^R	L^C	I^F	I^H
Value	100	10	32.5	29.9	210	169	15.6	19.9	24.9	3.35
Variable	K^F	K^H	L^F	L^H	R^C	R^B	N^F	C_E^F	N^H	C_E^H
Value	997	134	30.0	5.53	1.015	1.0147	498	5.68	67.0	0.80
Variable	N^C	C_E^C	N^B	C_E^B	$\bar{\omega}^C$	σ^C	Γ^C	G^C	Γ_ω^C	G_ω^C
Value	159	2.53	31.9	1.11	0.5	0.31	0.50	0.01	0.98	0.15
Variable	Γ_σ^C	G_σ^C	$\Gamma_{\omega\omega}^C$	$G_{\omega\omega}^C$	$\Gamma_{\omega\sigma}^C$	$G_{\omega\sigma}^C$	$\bar{\omega}^B$	σ^B	Γ^B	G^B
Value	-0.02	0.16	-0.31	2.02	-0.36	1.89	0.79	0.11	0.79	0.02
Variable	Γ_ω^B	G_ω^B	Γ_σ^B	G_σ^B	$\Gamma_{\omega\omega}^B$	$G_{\omega\omega}^B$	$\Gamma_{\omega\sigma}^B$	$G_{\omega\sigma}^B$	p	w
Value	0.98	0.45	-0.04	0.74	-0.57	10.8	-0.97	14.4	1.88	2.15
Variable	Ψ^B	Ψ_ω^B	Ψ_σ^B	$\Psi_{\omega\omega}^B$	$\Psi_{\omega\sigma}^B$	Ψ^C	Ψ_ω^C	Ψ_σ^C	$\Psi_{\omega\omega}^C$	$\Psi_{\omega\sigma}^C$
Value	0.79	0.97	-0.05	-0.83	-1.32	0.50	0.98	-0.02	-0.36	-0.41

The log-linearization of the additional equations is in Appendix 3.D.

3.8 The Chained Loan Contracts Results

This section compares the effect of an oversupply of housing and of a reduction in financial volatility in a chained contracts economy with both single contract versions where either credit-constrained consumers or financial institutions are the subject to credit constraints.

3.8.1 The Housing Oversupply

In Chapter 2 consumers experienced a downturn following an increase in housing inventory that reduced the market price of housing whereas in Section 3.5 of this chapter they were relatively shielded by their share arrangement with financial institutions. Chained loan contracts provide less security to credit-constrained consumers than a share contract but more than a single loan contract when they are the sole indebted borrowers. As a result, a disturbance that reduces the value of the housing stock generates a smaller improvement in their leverage than in the share contract version but nevertheless puts them in a more favourable position than in Chapter 2. When consumers participate in the chained loan contracts, intermediary financial institutions transfer some of their leverage worsening onto households in order to repair their own debt position faster.

Like the two single loan contract models (the consumer mortgage contract in Chapter 2 and the financial institutions loan contract in Section 3.5 of this chapter), this version simulates a permanent technological improvement in the production of housing. However, since both types of agents are credit-constrained the combined leverage in the economy is larger contributing to more volatility. The presence of credit frictions in both the financial sector and the household sector does not change the directional impact of the housing innovation shock on households and on producers but creates opportunities for risk sharing driven by the larger joint leverage (Figure 3.5). The oversupply of housing still triggers a reduction in the housing price that raises consumer demand for housing (Figures 3.5.4 and 3.5.6). The return to housing decreases and the value of the housing stock held as collateral falls (Figures 3.5.17 and 3.5.18). However, the chained model version yields qualitatively and quantitatively different effects on the leverage of indebted agents. Credit-constrained consumers experience a mild improvement in their debt position while leveraged financial institutions enter a considerable recession (Figures 3.5.25 and 3.5.26). The diverging experiences can be attributed to the relative position in the chained contracts of both borrowers and their initial leverage ratios. As optimisers of the chained contracts, banks must meet the participation constraint of both investors and households so they absorb most of the adverse effects of the shock. Their leverage deteriorates by as much as in their single loan contract.

Consumers know that banks will honour their participation constraint so they are relatively shielded from the adverse effects of the falling housing price on their equity. Their net worth still diminishes but by less than in the single loan contract when credit-constrained consumers had to meet the participation constraint of investors. Overall, the decrease in the net worth of credit-constrained consumers (Figure 3.5.19) is proportionally less than the fall in the value of their housing stock so households experience a small improvement in their leverage position. The improvement is smaller than the one observed in the case with the financial institutions loan contract where consumers had no idiosyncratic risk. Since now consumers are part of the chained loan contract, their participation constraint is more accommodating to risk than the share agreement which yielded a fixed return regardless of the realization of the default probability. The participation constraint, unlike the share contract, does not guarantee a fixed return and is responsive to the risk premium. It shifts to the left following an increase in the external finance premium thus dampening the improvement in their leverage. The reduced leverage, however, is sufficient to imply lower bankruptcy prospects despite the higher risk premium (Figure 3.5.23). Hence the chained contracts model creates opportunities for risk sharing between indebted agents. Consumers are not as insulated from the downturn as when they had a share contract. Nevertheless, they fare far better than when they are the sole borrowers and bear the increase in the risk premium alone.

Financial institutions, on the other hand, experience the same deterioration in their leverage as in their single loan contract. Although the combined worsening of consumer and bank net worth is larger in the chained contracts model than in the single contract version, banks do not fare worse than when they are the sole borrowers since they shift some of the burden of the downturn onto consumers. This is due to the fact that as maximisers of the chained contracts, financial institutions can force consumers to the break-even point of their participation constraint and extract a higher return from them to ensure that their indebtedness is no worse than in the single contract version. Despite this, they still must satisfy the participation constraints of both lenders (investors) and final borrowers (credit-constrained consumers) so their ability to share risk is limited. As a result, the net worth of banks bears most of the devaluation in the housing equity and their leverage ratio deteriorates accordingly (Figures 3.5.20 and 3.5.26). Unlike in the single loan contract where financial institutions are exposed only to their own idiosyncratic risk, here they are exposed both to their own risk of the default and that of credit-constrained households. Hence their bankruptcy prospects deteriorate more than in the single contract (Figure 3.5.24). However, due to the risk sharing, banks see a smaller rise in their risk premium. This implies that despite the dual risks in the chained contracts, the worsening in the leverage position of financial institutions is not proportionally larger (Figure 3.5.26). The opportunity for risk sharing allows banks to dampen their recession at the expense of mortgaged households.

Comparing the subsequent recovery of the optimizing agents in the three versions, it is clear that financial institutions return to their pre-shock position faster than consumers. The improvement in the financial position of indebted agents follows the same path as in their respective single loan contracts. Banks start with a higher leverage which necessitates a speedy recovery in order to maintain their solvency. Furthermore, they are the ones that maximise the loan contracts so they can recover faster than mortgaged households, who have no say in the loan arrangement and no recourse to outside funds (Figures 3.5.26 and 3.5.25). The leverage of financial institutions in the chained contracts version takes about 60 quarters (15 years) to return to pre-shock levels while that of credit-constrained consumers in the single contract (described in Chapter 2) remains above its steady state value for almost 100 quarters (25 years). The default risk mirrors the respective leverage evolution of the two credit-constrained agents (Figures 3.5.24 and 3.5.23).

The single loan contract models and the chained contracts version offer insight into the contrasting experiences of credit-constrained households. In the single consumer mortgage model of Chapter 2 they maximise the loan contract; in the bank loan contract version they split fixed shares with financial institutions, and in the chained contracts extension they are final borrowers whose participation constraint must be satisfied by financial institutions. As maximisers in the first version of the model, indebted households bear fully the negative consequences of the oversupply of houses, while in the second and third cases they are relatively shielded by the binding of their participation condition or the share agreement with financial institutions. Credit-constrained households fare by far the best in the financial loan contract model where they have no idiosyncratic risk and the share contract guarantees them a fixed return regardless of the realization of the idiosyncratic shock. As result, they experience an improvement in their indebtedness. In the chained contracts version, they are part of the joint mortgage contract where they are exposed to bankruptcy prospects but the binding of their participation constraint still guarantees them a degree of protection from the recession, albeit on a smaller scare than in the financial institutions' loan contract. Barring the case where consumers have no risk of default, the chained contracts model is more favourable toward credit-constrained consumers than the single mortgage version. However, it is not preferable since the worsening of the debt position of banks is quantitatively larger making the economy as a whole more volatile. Its only redeeming quality is that it results in a relatively short-lived downturn compared to the single loan contract economy since financial institutions use their role as optimisers to repair their leverage position relatively fast. What used to be a moderate but protracted credit crunch in the consumer loan setup of Chapter 2 becomes a steep but short-lived financial downturn in the chained loans version. Nevertheless, in both scenarios the falling housing price can act as a vehicle of shock propagation that turns a positive innovation in the housing sector into the main cause of debt worsening for credit-constrained agents that characterises a downturn such as the subprime mortgage crisis.

The models described so far, although, theoretical in nature, have important policy implications. The three versions of credit frictions demonstrate the importance of sound financial regulation. In order to protect consumers from an unduly high burden of risk sharing from the financial system, policymakers should consider regulating the maximum permissible leverage ratio for borrowers. However, mandating a debt-to-income cap only for indebted households may be of limited usefulness when banks are also leveraged. As intermediaries in the mortgage contract, financial institutions usually enter the loan arrangement with much less equity and experience a more volatile leverage than households. Their excessive indebtedness in the chained contracts version causes a far larger drag on economic performance than that of consumers in the single mortgage model so it is important that there are sufficient loan loss provisions and reserve requirements in place to guarantee the liquidity of the financial system in a downturn.

3.8.2 The Temporary Financial Relaxation

A financial relaxation was detrimental to the debt position of borrowing consumers in Chapter 2 but left them unaffected in the share arrangement in Section 3.5 of this chapter. In the chained contracts setting, both consumers and financial institutions can experience a credit easing. Relaxing borrowing access for households spreads the resulting leverage worsening across both types of credit-constrained agents but the combined downturn is deeper than that of households in their sole borrowing arrangement in Chapter 2 due to the larger combined leverage. However, a financial relaxation for banks implies stronger risk sharing that is more tilted toward consumers. Banks struggle to repair their debt positions after the credit easing in their own sector so they shift a large share of the downturn onto households. Credit-constrained consumers suffer the negative consequences of a reduction in bank volatility with the same magnitude as following a decrease in their own volatility, although in this case they are not the originators of the disturbance. While the downturn is sharp but brief for banks, consumers are drawn into a prolonged recession.

Unlike the single contract models that demonstrate the consequences of a temporary improvement in credit access for either indebted households or financial institutions, the chained contracts setup allows to study both versions of a relaxation in borrowing conditions - one that targets credit-constrained consumers and one that pertains to financial institutions - within the same model. A reduction in the volatility of consumer borrowing in the chained contracts model yields qualitatively different results than the same disturbance in the single contract version (Figure 3.6). The decreased consumer volatility in the chained contracts version reduces not only the household risk premium but also the risk premium of banks since the two loan contracts are linked in the same maximization problem (Figures 3.6.27 and 3.6.28). The reduction in risk premium is passed from consumers to financial institutions. As a result, households experience a smaller reduction in the external finance premium than in the consumer loan contract.

The improvements in consumer credit access also affect financial institutions which, as intermediaries in the loan contract, need less downpayment to secure funding from investors on behalf of consumers (Figure 3.6.20). Less net worth implies higher indebtedness both for financial institutions and for households. However, as optimizing agents in the chained loan contract, financial institutions experience most of the consequences of the collateral reduction on their own debt position (Figure 3.6.26). Their indebtedness rises more than that of consumers but less than the corresponding increase in the single loan contract version. The presence of two chained loan contracts allows them to share the downturn with consumers. The leverage of credit-constrained consumers rises as well but by only half as much as in their own mortgage arrangement described in Chapter 2

(Figure 3.6.25). Overall, the risk sharing in the chained loan contract dampens the effect of the shock, spreading it over both types of indebted agents while at the same time concentrating a larger share in the intermediary sector that optimises the dual contracts. The default probability of both agents also increases but by less than in the respective single contract models (Figures 3.6.23 and 3.6.24).

Taken together, the worsening in the leverage of both credit-constrained consumers and banks is larger than that of only consumers in the single loan contract of Chapter 2. This is due to the higher initial indebtedness of banks which reflects on both agents. Despite the large combined downturn in the chained contracts model, the debt position of financial institutions recovers relatively fast and about 70 quarters (17.5 years) after the shock, their leverage ratio and bankruptcy prospects return to pre-shock levels (Figure 3.6.26). Bank recovery, however, is at the expense of the revival of the borrowing ability of households. As maximisers of the chained contracts, financial institutions can push consumers to their participation constraint so most of the subsequent improvements in housing equity benefit them at the cost of delaying the recovery of mortgaged households. The indebtedness of credit-constrained consumers unwinds slowly and remains above its pre-shock level for close to 100 quarters (25 years) (Figure 3.6.25). Risk sharing implies that the net worth of banks and their leverage position improve relatively fast, while consumers are faced with a protracted recovery.

A model with relaxed credit access for financial institutions produces the deepest recession of all scenarios discussed so far (Figure 3.7). Following a reduction in their own volatility, financial institutions see a smaller increase in their leverage than when they are the only borrowers since some of the increase is transferred onto consumers (Figures 3.7.25 and 3.7.26). Unlike the single loan contract version where the arrangement between consumers and financial institutions is a share one, here households participate in the chained contract creating opportunities for risk sharing. Consumers have no ability to borrow outside of the loan contract due to their idiosyncratic uncertainty so banks can afford to transfer some of the downturn onto them without turning them away from the mortgage purchase. Hence households are forced to absorb some of the deterioration in the balance sheets of banks although they are not the source of the disturbance. Following a shock to the volatility of banks, the leverage of mortgaged households rises by as much as when their own volatility is reduced in the same chained contracts setting (Figure 3.7.25).

The leverage of banks, however, rises more than in the previous case with consumer credit easing. Since the default probability of financial institutions is initially much higher than that of households, a reduction in their own volatility leads to a more significant tightening of their financial conditions. In order to mitigate this threat to their own solvency, financial institutions shift a considerable share of the burden onto households. Despite the risk sharing, the higher pre-shock indebtedness of financial institutions implies that their leverage worsens slightly more than when they are faced with a reduction in consumer volatility. As a result, the combined worsening of financial positions of both credit-constrained agents is the most severe of all considered scenarios. The subsequent recovery again benefits financial institutions more than credit-constrained consumers. As optimisers of the chained loan contract, financial institutions can repair their debt position faster at the expense of consumers who experience a smaller leverage worsening and despite not being the source of the disturbance, undergo a slower recovery.

When it comes to relaxing credit access, the single consumer loan contract produces the sharpest deterioration in the leverage of credit-constrained households since there is no risk sharing. The two chained contract simulations demonstrate that consumers share some of the burden of higher indebtedness with financial institutions so their leverage does not deteriorate as much as before. Nevertheless, their subsequent recovery could be more protracted since banks can use their optimizing position to extract more equity from consumers in order to improve their own debt position. This is especially evident in the case with a decrease in financial volatility. Since consumers are not the source of the disturbance, they should be relatively protected by their participation constraint. However, the deterioration in the balance sheets of banks is so considerable that financial institutions resort to extracting as much equity from households as possible in order to repair their own debt position. Credit-constrained consumers suffer the negative consequences of a reduction in bank volatility with the same magnitude as following a decrease in their own volatility. The financial troubles of banks draw them into a prolonged downturn and their leverage remains above its steady state value for more than 100 quarters (25 years). The last scenario describing a reduction in bank volatility in a chained contracts setup offers the closest cautionary tale to the subprime recession. Leading up to the subprime crisis, the balance sheets of banks were deemed sufficiently low risk and fairly stable allowing them to accumulate excessive debt. The resulting downturn was passed on to consumers who experienced significant worsening in their debt positions that led to a prolonged tightening of credit access. Nevertheless, all three scenarios in the single and chained contracts context demonstrate the perils of higher leverage that a temporary relaxation in borrowing conditions can create.

The policy recommendations for the scenario with decreased financial volatility point in the same direction as these for housing oversupply. In order to safeguard against such sudden relaxations in borrowing conditions that reflect animal spirits and are not supported by macroeconomic fundamentals, it is important to have loan-to-value requirements and debt-to-income caps in place for mortgage-buyers. Both would ensure that homeowners would not raise their leverage significantly or secure loans with little downpayment. As in the case with housing oversupply, macroprudential regulations should be aimed at financial institutions as well to ensure that they hold sufficient reserves to weather unexpected downturns that would reduce the incentive to shift the burden of recessions onto consumers. The scenario with a reduction in bank volatility also implies that in times of bank distress due to excessive leverage, consumers should not be forced to share the burden of repairing the balance sheets of financial institutions and may need alternative sources of credit.

Figure 3.5: Housing Oversupply in a Chained Contracts Model

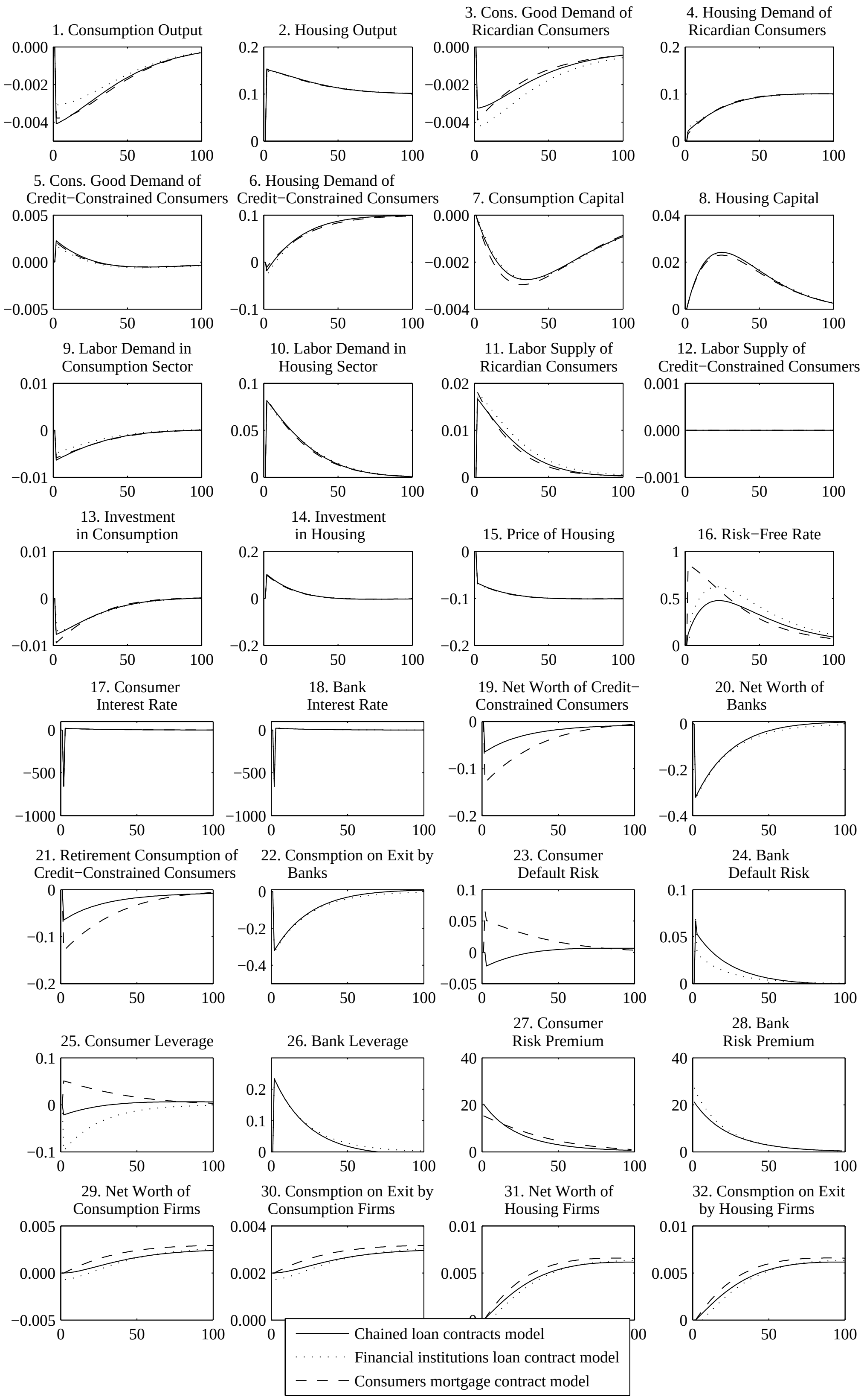


Figure 3.6: Reduced Consumer Volatility in a Chained Contracts Model

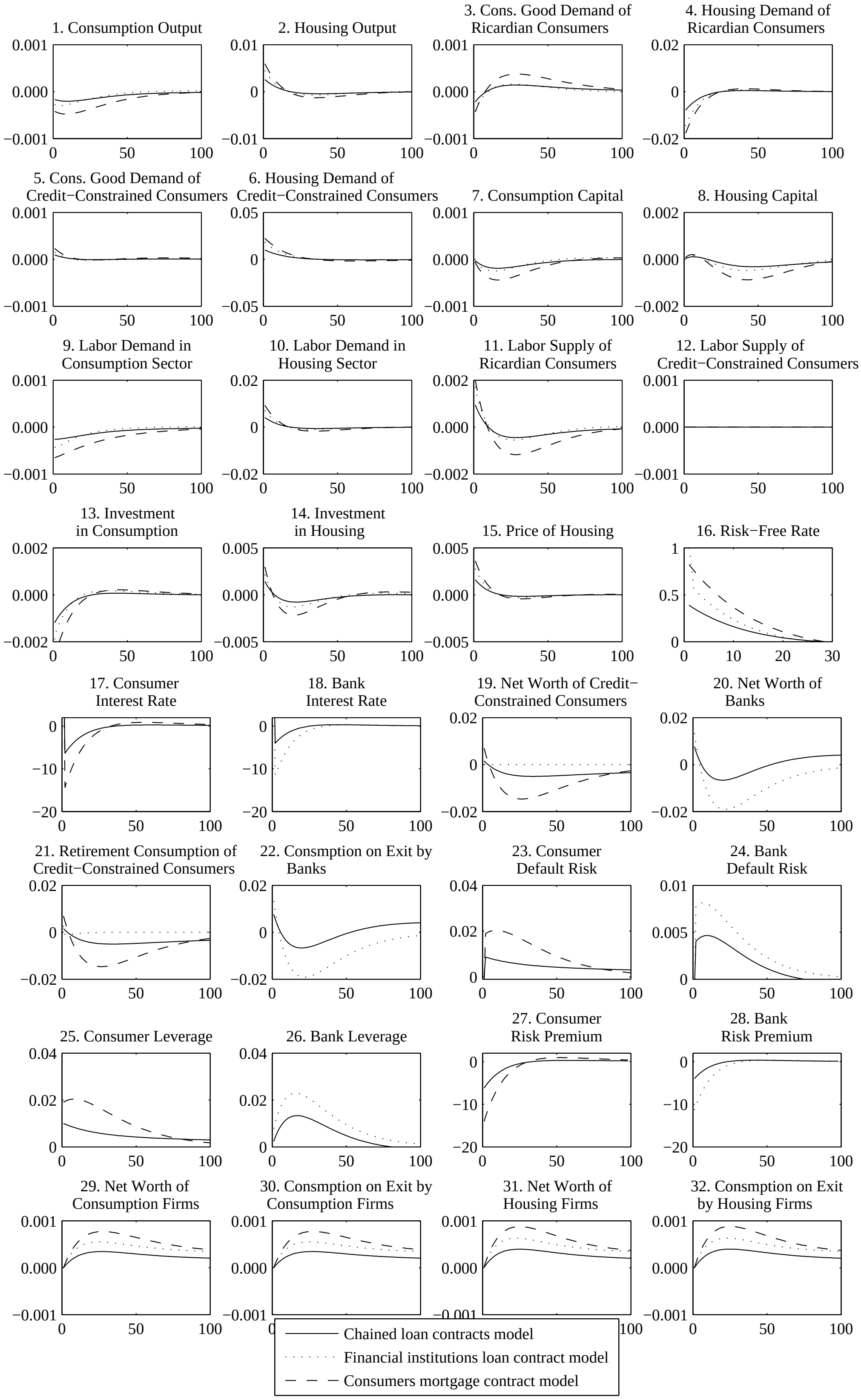
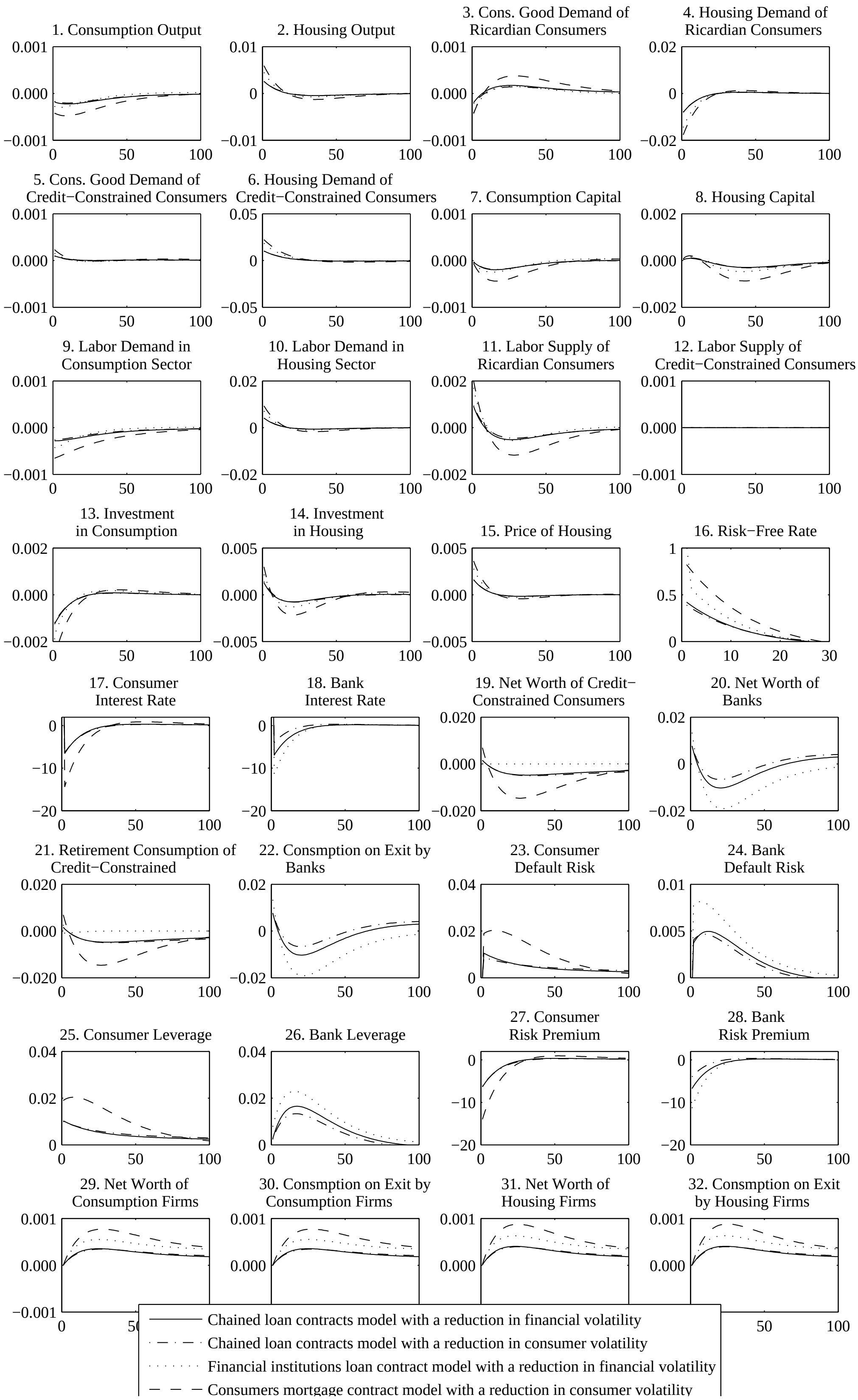


Figure 3.7: Reduced Bank Volatility in a Chained Contracts Model



3.9 Conclusion

The subprime mortgage crisis dashed the hopes of many for home ownership and set off a deep recession. Loan applicants saw their financing prospects reduced for years ahead as cash-strapped banks struggled to improve their balance sheet positions. Many financial institutions were brought to the brink of collapse by excessive leverage and saved by the "too big to fail" policy only to push through their recovery at the expense of credit-squeezed consumers. The consequences of excessive leverage and excessive lending in the form of subprime loans had many calling for more stringent supervision of borrowing transactions. An improved regulation framework cannot emerge without understanding how a mortgage crisis can happen.

This chapter extends the consumer mortgage contract of Chapter 2 to include financial institutions acting as intermediaries between investors and credit-constrained consumers. The result is two chained mortgage contracts in which both banks and credit-constrained households are subject to idiosyncratic uncertainty. The model not only has the benefit of depicting financial markets more accurately but also demonstrates that there are opportunities for risk sharing between financial institutions and credit-constrained consumers. Since banks participate in both loan arrangements - they borrow from investors and in turn lend to mortgage-buying consumers - they maximise the chained loan contracts. Their position as optimisers allows them to shift some of the negative consequences of a disturbance onto households. When financial institutions are the sole credit-constrained agents as in their single loan contract, they have a share arrangement with consumers which gives both parties a fixed return regardless of the realization of the idiosyncratic shock. The share purchase protects households from downturns concentrating the leverage worsening in the financial sector. In contrast, when both consumers and banks are credit-constrained, households have loan arrangements where financial institutions where the return is responsive to changes in the default risk and leverage of both types of agents. Hence in the chained contracts model banks can engage in risk sharing where they extract a larger return from consumers in order to mitigate the negative effects on their leverage and thereby prolonging the downturn for consumers.

Banks resort to risk sharing both following an innovation in the production of housing that reduces the price of houses and following a relaxation of credit access for consumers and for themselves. Owing to their high initial indebtedness, any of the disturbances causes a more significant worsening in the indebtedness of banks than in the leverage of consumers. Banks recover their solvency by transferring some of the balance sheet deterioration onto consumers who have no financial resort outside of the chained mortgage contracts. In each case, the leverage of consumers worsens more than when they have a share arrangement with financial institutions but less than when households are the sole borrowers as in Chapter 2. Although the terms of the loan contract prevent excessive risk transfer and the consumer downturn is milder than that of banks, the recession of credit-constrained households is prolonged. As optimisers of the financial intermediation, banks can extract higher loan repayments from households to repair their balance sheets relatively fast while consumers suffer a protracted recession. The risk sharing is especially pronounced in the case with relaxed borrowing constraints for financial institutions. Although consumers are not the originators of the disturbance, they take a long time to recover from the bank-induced recession while financial institutions experience a sharp but relatively short downturn. The results of this case closely approximate the consequences of the subprime mortgage crisis.

While the presence of idiosyncratic default probabilities implies that financial frictions cannot be easily eliminated, the consequences of excessive leverage hint at the type of macroprudential regulation needed to safeguard a sound financial market (Dimova, Kongsamut and Vandenbussche, 2014). Debt-to-income caps and loan-to-value requirements for consumers would ensure that households obtain sensible mortgages that they can service even in a downturn and would prevent banks from unduly transferring risk onto mortgaged consumers. Reserve requirements and loan loss provisions for financial institutions would guarantee sufficient liquidity in times of distress. Taken together, these measures would prevent a potential leverage worsening that can cripple credit access for a prolonged time and can spill out of the mortgage market and distress the whole economy.

The model, while aptly demonstrating the drawbacks of risk sharing in a financial downturn, could be enriched further to offer a deeper understanding of credit mechanisms.

When the crisis started, many banks attempted to salvage their debt position by recalling loans to other financial institutions in addition to transferring some of the increased leverage to consumers. The model can be augmented by adding inter-bank relationships and loan networks that may trigger a domino-like effect of rising default risk. Furthermore, the potential role of a bailout policy could be explored by adding government to the existing setup. It may also be a cautionary tale to consider the possibility of endogenous steady state leverage ratios for both banks and consumers. As history leading to the crisis demonstrates, leverage limits were poorly regulated and enforced prior to the subprime mortgage crisis allowing instead financial institutions to reach dangerously high debt-to-equity ratios. Letting the steady state leverage ratio be endogenously determined would shed light on how banks ratchet up their indebtedness in the absence of regulation and thus precipitate crises. Finally, incorporating various macroprudential regulations in the model would allow to study how effective they are in preventing leverage buildup and whether regulations in the mortgage market would not cause a shift of lending to other less regulated financial entities.

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Appendix 3.A

Complete Financial Institutions Loan Contract Model

The participation constraint for investors:

$$\left(\Gamma^B(\bar{\omega}_t^B) - \mu^B G^B(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right) = R_{t-1}\left(p_{t-1} H_{t-1}^C - N_{t-1}^C - N_{t-1}^B\right)$$

The first order condition of financial institutions' maximization:

$$\frac{E_t R_{t+1}^B}{R_t} = E_t \left(\frac{\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(\Gamma_\omega^B(\bar{\omega}_{t+1}^B) - \mu^B G_\omega^B(\bar{\omega}_{t+1}^B)\right)}}{\left(1 - \Gamma^B(\bar{\omega}_{t+1}^B)\right) + \frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(\Gamma_\omega^B(\bar{\omega}_{t+1}^B) - \mu^B G_\omega^B(\bar{\omega}_{t+1}^B)\right)} \left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right)} \right)$$

The returns on holding shares:

$$R_t^C = R_t^B$$

The consumption on exit by banks:

$$C_{E,t}^B = \left(1 - \nu^B\right)\left(1 - \Gamma^B(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right)$$

The law of motion of the net worth of banks:

$$N_t^B = \nu^B \left(1 - \Gamma^B(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1} H_{t-1}^C - N_{t-1}^C\right) + (1 - \alpha_H - \varepsilon)\Omega_B p_t Y_t^H$$

The consumption of credit-constrained households on retirement:

$$C_{E,t}^C = \left(1 - \nu^C\right) R_t^C N_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$N_t^C = \nu^C R_t^C N_{t-1}^C + (1 - \alpha_H - \varepsilon)\Omega_C p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{E,t}^F = \left(1 - \nu^F\right) R_{t-1} N_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$N_t^F = \nu^F R_{t-1} N_{t-1}^F + (1-\alpha_F)\Omega_F Y_t^F$$

The consumption on exit by housing firms:

$$C_{E,t}^H = \left(1-\nu^H\right) R_{t-1} N_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$N_t^H = \nu^H R_{t-1} N_{t-1}^H + (1-\alpha_H-\varepsilon)\Omega_H p_t Y_t^H$$

The consumption capital accumulation equation:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right)K_{t-1}^F$$

Investment demand by consumption capital firms:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1$$

The consumption production function:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)}$$

The labour demand by consumption firms:

$$w_t = (1-\alpha_F)\left(1-\Omega_F\right)\left(\frac{Y_t^F}{L_{H,t}^F}\right)$$

The capital demand by consumption firms:

$$R_t = \frac{E_t\left(\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta)x_{t+1}^F\right)}{x_t^F}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C-\Omega_B)}$$

The labour demand by housing firms:

$$w_t = (1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing firms:

$$R_t = \frac{E_t\left(\alpha_H \frac{p_{t+1}Y_{t+1}^H}{K_t^H} + (1-\delta)x_{t+1}^H\right)}{x_t^H}$$

The labour supply of Ricardian consumers:

$$\gamma(L_t^R)^{\varphi} C_t^R = w_t$$

The consumption housing tradeoff of Ricardian consumers:

$$E_t\left(R_t p_t - (1-\delta)p_{t+1}\right) H_t^R = \kappa E_t C_{t+1}^R$$

The Euler equation of Ricardian consumers:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t$$

The labour supply of credit-constrained consumers:

$$\gamma(L_t^C)^{\varphi} C_t^C = w_t$$

The consumption-housing tradeoff of credit-constrained consumers:

$$\left(R_t^C p_{t-1} - (1-\delta)p_t\right) H_{t-1}^C = \kappa C_t^C$$

The consumption purchase equation of credit-constrained consumers:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C$$

Market clearing in the consumption sector:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + C_{E,t}^B + I_t^F + I_t^H$$

Market clearing in the housing sector:

$$\begin{aligned} &p_t Y_t^H + (1-\delta)p_t(H_{t-1}^R + H_{t-1}^C) = \\ &= p_t(H_t^R + H_t^C) + C_{E,t}^H + \mu^B G^B(\bar{\omega}_t^B) R_t^B \left(p_{t-1}H_{t-1}^C - N_{t-1}^C\right) \end{aligned}$$

Market clearing in the labour market:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C$$

Appendix 3.B

Log-Linearization of the Financial Institutions Loan Contract Model

This appendix derives the log-linearized form of the model equations as used in the simulations. The steady state value for each variable appears without time subscript and hats denote the percentage deviation of the variable from its steady state such that $\hat{Z}_t = \frac{Z_t - Z}{Z}$.

The participation constraint for investors:

$$
\begin{aligned}
\hat{R}_t^B &= \hat{R}_{t-1} + \left[\frac{1}{pH^C - N^C - N^B} - \frac{1}{pH^C - N^C}\right]\left(pH^C\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C\right) - N^C \hat{N}_{t-1}^C\right) - \\
&\quad - \frac{N^B}{pH^C - N^C - N^B}\hat{N}_{t-1}^B - \frac{\Gamma_\omega^B(\bar{\omega}^B) - \mu^B G_\omega^B(\bar{\omega}^B)}{\Gamma^B(\bar{\omega}^B) - \mu^B G^B(\bar{\omega}^B)}\bar{\omega}^B \hat{\bar{\omega}}_t^B - \frac{\Gamma_\sigma^B(\bar{\omega}^B) - \mu^B G_\sigma^B(\bar{\omega}^B)}{\Gamma^B(\bar{\omega}^B) - \mu^B G^B(\bar{\omega}^B)}\sigma^B \hat{\sigma}_{t-1}^B
\end{aligned}
$$

The first-order condition of financial institutions' maximization:

$$
\begin{aligned}
0 &= \frac{\Gamma_\omega^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B) - \mu^B G_\omega^B(\bar{\omega}^B)}(E_t \hat{R}_{t+1}^B - \hat{R}_t) - \\
&\quad -(1 - \Gamma^B(\bar{\omega}^B))\frac{R^B}{R}\left(\frac{\Gamma_{\omega\omega}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B)} - \frac{\Gamma_{\omega\omega}^B(\bar{\omega}^B) - \mu^B G_{\omega\omega}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B) - \mu^B G_\omega^B(\bar{\omega}^B)}\right)\bar{\omega}^B E_t \hat{\bar{\omega}}_{t+1}^B + \\
&\quad + \frac{R^B}{R}\left(-\Gamma_\sigma^B(\bar{\omega}^B) + \frac{\Gamma_\omega^B(\bar{\omega}^B)\left(\Gamma_\sigma^B(\bar{\omega}^B) - \mu^B G_\sigma^B(\bar{\omega}^B)\right)}{\Gamma_\omega^B(\bar{\omega}^B) - \mu^B G_\omega^B(\bar{\omega}^B)}\right)\sigma^B \hat{\sigma}_t^B - \\
&\quad - \frac{R^B}{R}\left((1 - \Gamma^B(\bar{\omega}^B))\left(\frac{\Gamma_{\omega\sigma}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B)} - \frac{\Gamma_{\omega\sigma}^B(\bar{\omega}^B) - \mu^B G_{\omega\sigma}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B) - \mu^B G_\omega^B(\bar{\omega}^B)}\right)\right)\sigma^B \hat{\sigma}_t^B
\end{aligned}
$$

The returns on holding shares:

$$\hat{R}_t^C = \hat{R}_t^B$$

The consumption on exit of financial institutions:

$$
\begin{aligned}
\hat{C}_{E,t}^B &= \hat{R}_t^B + \frac{pH^C}{pH^C - N^C}\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C\right) - \\
&\quad - \frac{N^C}{pH^C - N^C}\hat{N}_{t-1}^C - \frac{\Gamma_\omega^B(\bar{\omega}^B)}{1 - \Gamma^B(\bar{\omega}^B)}\bar{\omega}^B \hat{\bar{\omega}}_t^B - \frac{\Gamma_\sigma^B(\bar{\omega}^B)}{1 - \Gamma^B(\bar{\omega}^B)}\sigma^B \hat{\sigma}_{t-1}^B
\end{aligned}
$$

The law of motion of the net worth of financial institutions:

$$\begin{aligned}\hat{N}_t^B &= \nu^B(1-\Gamma^B(\bar{\omega}^B))\frac{R^B}{N^B}\left(\left(pH^C-N^C\right)\hat{R}_t^B+pH^C\left(\hat{p}_{t-1}+\hat{H}_{t-1}^C\right)-N^C\hat{N}_{t-1}^C\right)+\\ &+(1-\alpha_H-\varepsilon)\Omega_H\frac{pY^H}{N^B}\left(\hat{p}_t+\hat{Y}_t^H\right)-\\ &-\nu^B\frac{R^B\left(pH^C-N^C\right)}{N^B}\Gamma_\omega^B(\bar{\omega}^B)\bar{\omega}^B\hat{\bar{\omega}}_t^B-\nu^B\frac{R^Bp\left(H^C-N^C\right)}{N^B}\Gamma_\sigma^B(\bar{\omega}^B)\sigma^B\hat{\sigma}_{t-1}^B\end{aligned}$$

The consumption of credit-constrained households on retirement:

$$\hat{C}_{E,t}^C=\hat{R}_t^C+\hat{N}_{t-1}^C \tag{3.9.1}$$

The law of motion of the net worth of credit-constrained households:

$$\hat{N}_t^C=\nu^CR^C(\hat{R}_t^C+\hat{N}_{t-1}^C)+(1-\alpha_H-\varepsilon)\Omega_C\frac{pY^H}{N^C}\left(\hat{p}_t+\hat{Y}_t^H\right)$$

The consumption on exit by consumption firms:

$$\hat{C}_{E,t}^F=\hat{R}_{t-1}+\hat{N}_{t-1}^F$$

The law of motion of the net worth of consumption firms:

$$\hat{N}_t^F=\nu^FR(\hat{R}_{t-1}+\hat{N}_{t-1}^F)+(1-\alpha_F)\Omega_F\frac{Y^F}{N^F}\hat{Y}_t^F$$

The consumption on exit by housing firms:

$$\hat{C}_{E,t}^H=\hat{R}_{t-1}+\hat{N}_{t-1}^H$$

The law of motion of the net worth of housing firms:

$$\hat{N}_t^H=\nu^HR(\hat{R}_{t-1}+\hat{N}_{t-1}^H)+(1-\alpha_H-\varepsilon)\Omega_H\frac{pY^H}{N^H}\left(\hat{p}_t+\hat{Y}_t^H\right)$$

The consumption capital accumulation equation:

$$\hat{K}_t^F=\hat{K}_{t-1}^F+\delta\left(\hat{I}_t^F-\hat{K}_{t-1}^F\right)$$

Investment demand by consumption capital firms:

$$\hat{x}_t^F = J\text{"}\delta\left(\hat{K}_{t-1}^F - \hat{I}_t^F\right)$$

The consumption production function:

$$\hat{Y}_t^F = \hat{A}_t^F + \alpha_F \hat{K}_{t-1}^F + (1-\alpha_F)(1-\Omega_F)\hat{L}_{H,t}^F$$

The labour demand by consumption firms:

$$\hat{w}_t = \hat{Y}_t^F - \hat{L}_{H,t}^F$$

The capital demand by consumption firms:

$$\hat{R}_t + \hat{x}_t^F = \frac{R-(1-\delta)}{R}(\hat{Y}_{t+1}^F - \hat{K}_t^F) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^F$$

The housing capital accumulation equation:

$$\hat{K}_t^H = \hat{K}_{t-1}^H + \delta\left(\hat{I}_t^H - \hat{K}_{t-1}^H\right)$$

Investment demand by housing capital firms:

$$\hat{x}_t^H = J\text{"}\delta\left(\hat{K}_{t-1}^H - \hat{I}_t^H\right)$$

The housing production function:

$$\hat{Y}_t^H = \hat{A}_t^H + \alpha_H \hat{K}_{t-1}^H + (1-\Omega_H-\Omega_C-\Omega_B)\left(1-\alpha_H-\varepsilon\right)\hat{L}_{H,t}^H$$

The labour demand by housing firms:

$$\hat{w}_t = \hat{p}_t + \hat{Y}_t^H - \hat{L}_{H,t}^H$$

The capital demand by housing firms:

$$\hat{R}_t + \hat{x}_t^H = \frac{R-(1-\delta)}{R}(\hat{p}_{t+1} + \hat{Y}_{t+1}^H - \hat{K}_t^H) + \frac{(1-\delta)}{R}\hat{x}_{t+1}^H$$

The labour supply of Ricardian consumers:

$$\varphi \hat{L}_t^R + \hat{C}_t^R = \hat{w}_t$$

The consumption housing tradeoff of Ricardian consumers:

$$\hat{C}_{t+1}^R = \hat{H}_t^R + \hat{p}_{t+1} + \frac{R}{(R-1+\delta)}\left(\hat{R}_t + \hat{p}_t - \hat{p}_{t+1}\right)$$

The Euler equation of Ricardian consumers:

$$E_t \hat{C}_{t+1}^R - \hat{C}_t^R = \hat{R}_t$$

The labour supply of credit-constrained consumers:

$$\varphi \hat{L}_t^C + \hat{C}_t^C = \hat{w}_t$$

The consumption housing tradeoff of credit-constrained consumers:

$$\hat{C}_t^C = \hat{H}_{t-1}^C + \hat{p}_t + \frac{R^C}{(R^C-1+\delta)}\left(\hat{R}_t^C + \hat{p}_{t-1} - \hat{p}_t\right)$$

The consumption purchase equation of credit-constrained consumers:

$$\hat{C}_t^C = \frac{wL^C}{C^C}(\hat{w}_t + \hat{L}_t^C) + \frac{(1-\delta)pH^C}{C^C}\hat{p}_t - \frac{R^C pH^C}{C^C}(\hat{R}_t^C + \hat{p}_{t-1}) + \frac{(1-\delta-R^C)pH^C}{C^C}\hat{H}_{t-1}^C$$

Market clearing in the consumption sector:

$$\hat{Y}_t^F = \frac{C^R}{Y^F}\hat{C}_t^R + \frac{C^C}{Y^F}\hat{C}_t^C + \frac{C_E^F}{Y^F}\hat{C}_{E,t}^F + \frac{C_E^C}{Y^F}\hat{C}_{E,t}^C + \frac{C_E^B}{Y^F}\hat{C}_{E,t}^B + \frac{I^F}{Y^F}\hat{I}_t^F + \frac{I^H}{Y^F}\hat{I}_t^H$$

Market clearing in the housing sector:

$$\begin{aligned}\hat{Y}_t^H \;=\; & \frac{\delta(H^R+H^C)-Y^H}{Y^H}\hat{p}_t + \frac{H^R}{Y^H}\hat{H}_t^R + \frac{H^C}{Y^H}\hat{H}_t^C - \frac{(1-\delta)}{Y^H}(H^R\hat{H}_{t-1}^R + H^C\hat{H}_{t-1}^C) + \\ & + \frac{C_E^H}{pY^H}\hat{C}_{E,t}^H + \mu^B G(\bar{\omega}^B)\frac{R^B}{Y^H}\left(\left(pH^C - N^C\right)\hat{R}_t^B + pH^C\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C\right) - N^C\hat{N}_{t-1}^C\right) + \\ & + \mu^B G_\omega^B \bar{\omega}^B \hat{\bar{\omega}}_t^B \frac{R^B\left(pH^C - N^C\right)}{Y^H} + \mu^B G_\sigma^B \sigma^B \hat{\sigma}_{t-1}^B \frac{R^B\left(pH^C - N^C\right)}{Y^H}\end{aligned}$$

Market clearing in the labour market:

$$L_H^F \hat{L}_{H,t}^F + L_H^H \hat{L}_{H,t}^H = L^R \hat{L}_t^R + L^C \hat{L}_t^C$$

Appendix 3.C

Complete Chained Contracts Model

The participation constraint for investors:

$$\Psi^B(\bar{\omega}_t^B)\Psi^C(\bar{\omega}_t^C)R_t^C p_{t-1}H_{t-1}^C = R_{t-1}\left(p_{t-1}H_{t-1}^C - N_{t-1}^C - N_{t-1}^B\right)$$

The participation constraint for credit-constrained consumers:

$$N_{t-1}^C = \left(1 - \Gamma^C(\bar{\omega}_t^C)\right) p_{t-1}H_{t-1}^C$$

The first order condition of financial institutions' maximization:

$$\begin{aligned}\frac{E_t R_{t+1}^C}{R_t} &= E_t\left(\frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)\Psi_\omega^C(\bar{\omega}_{t+1}^C) + \Gamma_\omega^C(\bar{\omega}_{t+1}^C)\Psi^C(\bar{\omega}_{t+1}^C)}\right) \times \\ & \quad E_t\left(\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi_\omega^B(\bar{\omega}_{t+1}^B) + \Gamma_\omega^B(\bar{\omega}_{t+1}^B)\Psi^B(\bar{\omega}_{t+1}^B)}\right)\end{aligned}$$

The consumption on exit by banks:

$$C_{E,t}^B = \left(1-\nu^B\right)\left(1-\Gamma^B(\bar{\omega}_t^B)\right)\Psi^C(\bar{\omega}_t^C)R_t^C p_{t-1}H_{t-1}^C$$

The law of motion of the net worth of banks:

$$N_t^B = \nu^B\left(1-\Gamma^B(\bar{\omega}_t^B)\right)\Psi^C(\bar{\omega}_t^C)R_t^C p_{t-1}H_{t-1}^C + (1-\alpha_H-\varepsilon)\Omega_B p_t Y_t^H$$

The consumption of credit-constrained households on retirement:

$$C_{E,t}^C = \left(1-\nu^C\right)\left(1-\Gamma^C(\bar{\omega}_t^C)\right)R_t^C p_{t-1}H_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$N_t^C = \nu^C\left(1-\Gamma^C(\bar{\omega}_t^C)\right)R_t^C p_{t-1}H_{t-1}^C + (1-\alpha_H-\varepsilon)\Omega_C p_t Y_t^H$$

The consumption on exit by consumption firms:

$$C_{E,t}^{F} = \left(1 - \nu^{F}\right) R_{t-1} N_{t-1}^{F}$$

The law of motion of the net worth of consumption firms:

$$N_{t}^{F} = \nu^{F} R_{t-1} N_{t-1}^{F} + (1 - \alpha_{F}) \Omega_{F} Y_{t}^{F}$$

The consumption on exit by housing firms:

$$C_{t}^{H} = \left(1 - \nu^{H}\right) R_{t-1} N_{t-1}^{H}$$

The law of motion of the net worth of housing firms:

$$N_{t}^{H} = \nu^{H} R_{t-1} N_{t-1}^{H} + (1 - \alpha_{H} - \varepsilon) \Omega_{H} p_{t} Y_{t}^{H}$$

The consumption capital accumulation equation:

$$K_{t}^{F} = (1 - \delta) K_{t-1}^{F} + J\left(\frac{I_{t}^{F}}{K_{t-1}^{F}}\right) K_{t-1}^{F}$$

Investment demand by consumption capital firms:

$$x_{t}^{F} J'\left(\frac{I_{t}^{F}}{K_{t-1}^{F}}\right) = 1$$

The consumption production function:

$$Y_{t}^{F} = A_{t}^{F} (K_{t-1}^{F})^{\alpha_{F}} (L_{H,t}^{F})^{(1-\alpha_{F})(1-\Omega_{F})}$$

The labour demand by consumption firms:

$$w_{t} = (1 - \alpha_{F}) (1 - \Omega_{F}) \left(\frac{Y_{t}^{F}}{L_{H,t}^{F}}\right)$$

The capital demand by consumption firms:

$$R_{t} = \frac{E_{t}\left(\alpha_{F} \frac{Y_{t+1}^{F}}{K_{t}^{F}} + (1 - \delta) x_{t+1}^{F}\right)}{x_{t}^{F}}$$

The housing capital accumulation equation:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H$$

Investment demand by housing capital firms:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1$$

The housing production function:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C-\Omega_B)}$$

The labour demand by housing firms:

$$w_t = (1-\Omega_H-\Omega_C-\Omega_B)(1-\alpha_H-\varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right)$$

The capital demand by housing firms:

$$R_t = \frac{E_t\left(\alpha_H \frac{p_{t+1}Y_{t+1}^H}{K_t^H} + (1-\delta)x_{t+1}^H\right)}{x_t^H}$$

The labour supply of Ricardian consumers:

$$\gamma(L_t^R)^{\varphi} C_t^R = w_t$$

The consumption housing tradeoff of Ricardian consumers:

$$E_t\left(R_t p_t - (1-\delta)p_{t+1}\right) H_t^R = \kappa E_t C_{t+1}^R$$

The Euler equation of Ricardian consumers:

$$E_t\left(\frac{C_{t+1}^R}{C_t^R}\right) = \beta R_t$$

The labour supply of credit-constrained consumers:

$$\gamma(L_t^C)^{\varphi} C_t^C = w_t$$

The consumption-housing tradeoff of credit-constrained consumers:

$$\left(R_t^C p_{t-1} - (1-\delta)p_t\right) H_{t-1}^C = \kappa C_t^C$$

The consumption purchase equation of credit-constrained consumers:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C$$

Market clearing in the consumption sector:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + C_{E,t}^B + I_t^F + I_t^H$$

Market clearing in the housing sector:

$$\begin{aligned} p_t Y_t^H + (1-\delta)p_t(H_{t-1}^R + H_{t-1}^C) &= p_t(H_t^R + H_t^C) + C_{E,t}^H + \\ &+ \left[\mu^C G^C(\bar{\omega}_t^C) + \mu^B G^B(\bar{\omega}_t^B)\Psi^C(\bar{\omega}_t^C)\right] R_t^C p_{t-1} H_{t-1}^C \end{aligned}$$

Market clearing in the labour market:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C$$

Appendix 3.D

Log-Linearization of the Chained Contracts Model

This appendix lig-linearizes the additional equations for the chained contracts model.

The participation constraint for investors:

$$\begin{aligned}\hat{R}_t^C &= \hat{R}_{t-1} + \frac{1}{pH^C - N^C - N^B}\left(\left(N^C + N^B\right)\left(\hat{p}_{t-1} + \hat{H}_{t-1}^C\right) - N^C\hat{N}_{t-1}^C - N^B\hat{N}_{t-1}^B\right) - \\ &- \frac{\Psi_\omega^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Psi_\omega^B(\bar{\omega}^B)}{\Psi^B(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_t^B - \frac{\Psi_\sigma^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C - \frac{\Psi_\sigma^B(\bar{\omega}^B)}{\Psi^B(\bar{\omega}^B)}\sigma^B\hat{\sigma}_{t-1}^B\end{aligned}$$

The first-order condition of financial institutions' maximization of the chained contracts:

$$\begin{aligned}&E_t\hat{R}_{t+1}^C - \hat{R}_t = \\ &= \left[\frac{\Gamma_{\omega\omega}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_{\omega\omega}^C(\bar{\omega}^C) + \Gamma_{\omega\omega}^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_\omega^C(\bar{\omega}^C) + \Gamma_\omega^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}\right]\bar{\omega}^C E_t\hat{\bar{\omega}}_{t+1}^C + \\ &+ \left[\frac{\Gamma_{\omega\sigma}^C(\bar{\omega}^C)}{\Gamma_\omega^C(\bar{\omega}^C)} - \frac{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_{\omega\sigma}^C(\bar{\omega}^C) - \Gamma_\sigma^C(\bar{\omega}^C)\Psi_\omega^C(\bar{\omega}^C) + \Gamma_\omega^C(\bar{\omega}^C)\Psi_\sigma^C(\bar{\omega}^C) + \Gamma_{\omega\sigma}^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}{\left(1-\Gamma^C(\bar{\omega}^C)\right)\Psi_\omega^C(\bar{\omega}^C) + \Gamma_\omega^C(\bar{\omega}^C)\Psi^C(\bar{\omega}^C)}\right]\sigma^C\hat{\sigma}_t^C + \\ &+ \left[\frac{\Gamma_{\omega\omega}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B)} - \frac{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_{\omega\omega}^B(\bar{\omega}^B) + \Gamma_{\omega\omega}^B(\bar{\omega}^B)\Psi^B(\bar{\omega}^B)}{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_\omega^B(\bar{\omega}^B) + \Gamma_\omega^B(\bar{\omega}^B)\Psi^C(\bar{\omega}^B)}\right]\bar{\omega}^B E_t\hat{\bar{\omega}}_{t+1}^B + \\ &+ \left[\frac{\Gamma_{\omega\sigma}^B(\bar{\omega}^B)}{\Gamma_\omega^B(\bar{\omega}^B)} - \frac{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_{\omega\sigma}^B(\bar{\omega}^B) - \Gamma_\sigma^B(\bar{\omega}^B)\Psi_\omega^B(\bar{\omega}^B) + \Gamma_\omega^B(\bar{\omega}^B)\Psi_\sigma^B(\bar{\omega}^B) + \Gamma_{\omega\sigma}^B(\bar{\omega}^B)\Psi^B(\bar{\omega}^B)}{\left(1-\Gamma^B(\bar{\omega}^B)\right)\Psi_\omega^B(\bar{\omega}^B) + \Gamma_\omega^B(\bar{\omega}^B)\Psi^B(\bar{\omega}^B)}\right]\sigma^B\hat{\sigma}_t^B\end{aligned}$$

The financial institutions' interest rate:

$$\hat{R}_t^B = \hat{R}_t^C + \frac{N^C}{pH^C - N^C}\left(\hat{N}_{t-1}^C - \hat{p}_{t-1} - \hat{H}_{t-1}^C\right) + \frac{\Psi_\omega^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C + \frac{\Psi_\sigma^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The participation constraint of credit-constrained consumers:

$$\hat{N}_{t-1}^C = \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C - \frac{\Gamma_\sigma^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The consumption on exit of financial institutions:

$$\hat{C}_{E,t}^B = \hat{R}_t^C + \hat{p}_{t-1} + \hat{H}_{t-1}^C - \frac{\Gamma_\omega^B(\bar{\omega}^B)}{1-\Gamma^B(\bar{\omega}^B)}\bar{\omega}^B\hat{\bar{\omega}}_t^B - \frac{\Gamma_\sigma^B(\bar{\omega}^B)}{1-\Gamma^B(\bar{\omega}^B)}\sigma^B\hat{\sigma}_{t-1}^B +$$

$$+\frac{\Psi_\omega^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C+\frac{\Psi_\sigma^C(\bar{\omega}^C)}{\Psi^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The law of motion of the net worth of financial institutions:

$$\begin{aligned}\hat{N}_t^B &= \nu^B(1-\Gamma^B(\bar{\omega}^B))\Psi^C(\bar{\omega}^C)\frac{R^C pH^C}{N^B}(\hat{R}_t^C+\hat{p}_{t-1}+\hat{H}_{t-1}^C)+(1-\alpha_H-\varepsilon)\Omega_H\frac{pY^H}{N^B}\left(\hat{p}_t+\hat{Y}_t^H\right)+\\ &-\nu^B\frac{R^C pH^C}{N^B}\Psi^C(\bar{\omega}^C)\left(\Gamma_\omega^B(\bar{\omega}^B)\bar{\omega}^B\hat{\bar{\omega}}_t^B+\Gamma_\sigma^B(\bar{\omega}^B)\sigma^B\hat{\sigma}_{t-1}^B\right)+\\ &+\nu^B\frac{R^C pH^C}{N^B}\left(1-\Gamma^B(\bar{\omega}^B)\right)\left(\Psi_\omega^C\bar{\omega}^C\hat{\bar{\omega}}_t^C+\Psi_\sigma^C\sigma^C\hat{\sigma}_{t-1}^C\right)\end{aligned}$$

The consumption of credit-constrained households on retirement:

$$\hat{C}_{E,t}^C=\hat{R}_t^C+\hat{p}_{t-1}+\hat{H}_{t-1}^C-\frac{\Gamma_\omega^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\bar{\omega}^C\hat{\bar{\omega}}_t^C-\frac{\Gamma_\sigma^C(\bar{\omega}^C)}{1-\Gamma^C(\bar{\omega}^C)}\sigma^C\hat{\sigma}_{t-1}^C$$

The law of motion of the net worth of credit-constrained households:

$$\begin{aligned}\hat{N}_t^C &= \nu^C(1-\Gamma^C(\bar{\omega}^C))\frac{R^C pH^C}{N^C}(\hat{R}_t^C+\hat{p}_{t-1}+\hat{H}_{t-1}^C)-\nu^C\frac{R^C pH^C}{N^C}\Gamma_\omega^C(\bar{\omega}^C)\bar{\omega}^C\hat{\bar{\omega}}_t^C-\\ &-\nu^C\frac{R^C pH^C}{N^C}\Gamma_\sigma^C(\bar{\omega}^C)\sigma^C\hat{\sigma}_{t-1}^C+(1-\alpha_H-\varepsilon)\Omega_C\frac{pY^H}{N^C}\left(\hat{p}_t+\hat{Y}_t^H\right)\end{aligned}$$

Market clearing in the housing sector:

$$\begin{aligned}\hat{Y}_t^H &= \frac{\delta(H^R+H^C)-Y^H}{Y^H}\hat{p}_t+\frac{H^R}{Y^H}\hat{H}_t^R+\frac{H^C}{Y^H}\hat{H}_t^C-\frac{(1-\delta)}{Y^H}(H^R\hat{H}_{t-1}^R+H^C\hat{H}_{t-1}^C)+\\ &+\frac{C_E^H}{pY^H}\hat{C}_{E,t}^H+\left[\mu^C G^C(\bar{\omega}^C)+\mu^B G^B(\bar{\omega}^B)\Psi^C(\bar{\omega}^C)\right]\frac{R^C H^C}{Y^H}\left(\hat{R}_t^C+\hat{p}_{t-1}+\hat{H}_{t-1}^C\right)+\\ &+\frac{R^C H^C}{Y^H}\left[\mu^C\left(G_\omega^C(\bar{\omega}^C)\bar{\omega}^C\hat{\bar{\omega}}_t^C+G_\sigma^C(\bar{\omega}^C)\sigma^C\hat{\sigma}_{t-1}^C\right)+\mu^B G(\bar{\omega}^B)\left(\Psi_\omega^C\bar{\omega}^C\hat{\bar{\omega}}_t^C+\Psi_\sigma^C\sigma^C\hat{\sigma}_{t-1}^C\right)\right]+\\ &+\mu^B\Psi(\bar{\omega}^C)\frac{R^C H^C}{Y^H}\left[G_\omega^B\bar{\omega}^B\hat{\bar{\omega}}_t^B+G_\sigma^B\sigma^B\hat{\sigma}_{t-1}^B\right]\end{aligned}$$

Conclusion and Extensions

Edmund Andrews, the journalist from *The New York Times* who defaulted on his subprime mortgage, was far from the only one - in fact, many other borrowers facing soaring repayment rates on their house mortgages ended up in foreclosure. However, prior to the crisis, it was not common knowledge that financial frictions, coupled with seemingly beneficial events like falling collateral prices and lax access to funding, can exacerbate the leverage of borrowers to disastrous consequences. Most of the works on credit constraints, including the seminal papers of Kiyotaki and Moore (1997) and of Bernanke, Gertler and Gilchrist (1999) and the subsequent literature they spawned, assumed that positive shocks can only offer advantages for indebted borrowers. They also failed to consider the role of debt held by non-production agents such as mortgage-buying consumers and overlooked the role of financial disturbances.

This thesis updated the literature of financial frictions by considering the role of credit-constrained consumers and focusing on the possibility of positive shocks generating negative disturbances. The model developed in this thesis continued the microfounded financial accelerator approach of Bernanke, Gertler and Gilchrist (1999). There are credit-constrained agents who need to obtain external funding from investors to finance the purchase of a good. Their borrowing is complicated by the fact that they possess an idiosyncratic risk of default that is known to them but is unknown to lenders. Lenders must pay an auditing fee to learn this probability of default. In order to offset expenses associated with monitoring, investors charge borrowers a premium known as an external finance (risk) premium above the risk-free interest rate. The premium increases with the likelihood of default since a higher probability of bankruptcy implies that there is a greater chance the fee will be paid.

While this thesis shared the microfounded feature of Bernanke, Gertler and Gilchrist

(1999), it went further by bringing the model closer to the subprime mortgage crisis scenario. In Bernanke, Gertler and Gilchrist (1999), there is only one sector and the financial constraints are in the borrowing decisions of producer firms. This creates a one-dimensional economy where positive disturbances have strictly positive outcomes. There is no discussion of the role of shocks to the financial system and no other credit-constrained agents besides production firms are considered. This thesis rectified these shortcomings in three chapters. The first chapter extended the one sector model to a two-sector version to study the role of positive innovations on the debt position of borrowers. The second chapter made the crucial addition of credit-constrained consumers who purchase a mortgage to finance their housing acquisition. The third chapter added an intermediary financial sector and demonstrated that there is risk sharing between indebted agents in times of downturns.

Chapter one was essentially a preliminary study that extended the one-sector model of Bernanke, Gertler and Gilchrist (1999) to two sectors. It preserved the placement of financial frictions in a production sector, namely house manufacturing, and added another unconstrained sector that produced generic consumption. The two-sector model studied a production shock in the credit-constrained sector and demonstrated that price effects can minimize the pass-through of the real shocks to the financial market. Where the positive technological shock does influence the debt position of borrowing firms, it largely improves the value of their collateral and eased their leverage, resulting in more beneficial financial terms. This is at odds with the credit worsening experienced curing the subprime recession. Shocks directly to the financial system such as a credit easing disturbance, however, have a negative effect on the credit-constrained sector. A temporary relaxation of borrowing conditions allows borrowers to indulge in excess borrowing and amass more debt. The higher leverage and the temporary nature of the shock imply that the risk premium does not remain relaxed for long. Borrowers enter a protracted downturn despite the temporary nature of the initial easing. In both cases, however, consumers are better off enjoying more housing at a lower price and remaining unaffected by the recession that befalls indebted firms. This positive experience of households contradicts the worsening home-buying consumers experienced after falling housing prices and relaxed borrowing at the peak of the subprime recession. The inability of the simple two-sector model to match real life consumer experience necessitated the update introduced in Chapter 2.

The second chapter took the model even closer to a subprime mortgage setting by shifting credit constraints from the producer to the consumer sector. In order to motivate financial frictions in the consumer sector, the model separated households into Ricardian and credit-constrained consumers. Ricardian consumers have assets in addition to labour wages so they can purchase housing without resorting to borrowing and have savings. Credit-constrained consumers have only wage income and need outside funding in the form of a mortgage to finance their housing acquisition. This chapter simulated the same two shocks as in Chapter 1 and established that positive shocks can have negative consequences for credit-constrained households. More importantly, it demonstrated that non-production agents such as consumers as just as likely to trigger a downturn as firms when they are leveraged.

In this chapter, two positive shocks - excess housing supply and credit easing - contribute to the leverage buildup of mortgage-holders to create a subprime-like recession. An oversupply of housing modelled as a technological innovation in the production of real estate reduces the equilibrium price of housing as well as the value of the housing already held as equity by credit-constrained households. The result is an increase in their indebtedness and a corresponding rise in the risk premium charged on mortgages. An improvement in the borrowing conditions also causes adverse consequences for borrowing households and demonstrates that the credit-constrained sector can be the source of a crippling downturn even if it is not a production sector. A reduction in the volatility of the mortgage contract can impact indebted consumers directly by allowing them to excessively leverage using relatively little collateral. When the previously reduced lending rate adjusts upwards, these households end up with subprime mortgages that they cannot service and are more likely to become bankrupt. In both cases, these seemingly positive shocks create a lasting recession for indebted consumers.

The third chapter brought the consumer mortgage contract of Chapter 2 even closer to real life mortgage markets by adding credit-constrained financial institutions in addition to credit-constrained households. Leveraged banks participate in two loan contracts, one with investors and one with mortgage-buying households, essentially providing a financial intermediary role. The two contacts are linked by the presence of financial institutions which optimise the two loan arrangements similarly to the chained contracts of Hirakata, Sudo and Ueda (2009). In this more realistic financial environment, both banks and credit-constrained households are subject to aggregate uncertainty. However, banks are the intermediaries of the chained loan contracts which allow them to share risk with consumers so that in times of a downturn households bear some of the negative consequences. Banks resort to risk sharing both following an innovation in the production of housing that reduces the price of houses and following a relaxation of credit access for consumers and for themselves. Owing to their high initial indebtedness, banks experience a significant worsening in their indebtedness so they repair their debt position by transferring some of the balance sheet deterioration to consumers. The leverage worsening of households is less than when they are the sole borrowers as in Chapter 2 but it can be more protracted. As monopolists in the financial intermediation, banks can extract higher loan repayments from households to repair their balance sheets relatively fast while consumers suffer a long recession. The risk sharing is especially pronounced in the case with relaxed borrowing constraints for financial institutions. The last scenario also closely approximates the subprime mortgage crisis. Although consumers were not the originators of the disturbance, they took a long time to recover from the bank-induced recession while financial institutions experienced a sharp but relatively short downturn.

While this thesis focused on the theoretical underpinnings of the subprime mortgage crisis, it also offered relevant policy recommendations. The recommendations concern primarily macroprudential regulation that would help avoid the unnecessary buildup of risk prior to recessions. Debt-to-income caps and loan-to-value requirements for consumers would ensure that households obtain sensible mortgages that they can service even in a downturn and would prevent banks from unduly transferring risk to mortgaged consumers. Reserve requirements and loan loss provisions for financial institutions would guarantee sufficient liquidity in times of distress. These measures should be wide-reaching and not just apply to the mortgage market to avoid the mere transference of risk to other sectors of the economy. Taken together, these macroprudential steps would prevent the concentration of debt that opens the door to potential downturns to the distress the whole financial market.

The three versions of the model developed in this thesis capture the experience with leverage deterioration that characterises subprime downturns. However, their relevance can be improved with several extensions. The addition of bank competition could offer opportunities to study the domino-like effect of the rising default probability of one bank affecting other banks as financial institutions restrict lending to one another during credit tightening. The present model assumes, along with Bernanke, Gertler and Gilchrist (1999), that the initial leverage ratio is exogenous. Allowing for endogenous steady state leverage ratios for both banks and consumers would shed light on whether borrowers tend to amass excessive leverage in the absence of regulation and could determine optimal debt-to-equity ratios. Making the model an open economy model could investigate whether internationally linked financial markets could export a recession from one country to another essentially creating contagion. Perhaps one of the most interesting avenues of research would be to consider the role of heterogeneous consumer expectations. Departures from the representative agent theory may explain the collective failure of agents to foresee the subprime mortgage crisis and more sophisticated discounting such as hyperbolic or quasi-hyperbolic discounting could explain why borrowers develop appetite for high risk and are willing to tolerate excessive indebtedness. More nuanced understanding of borrower behaviour may help shed light on why people like Edmund Andrews found the subprime mortgages alluring and how to prevent such disasters in the future.

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