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THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
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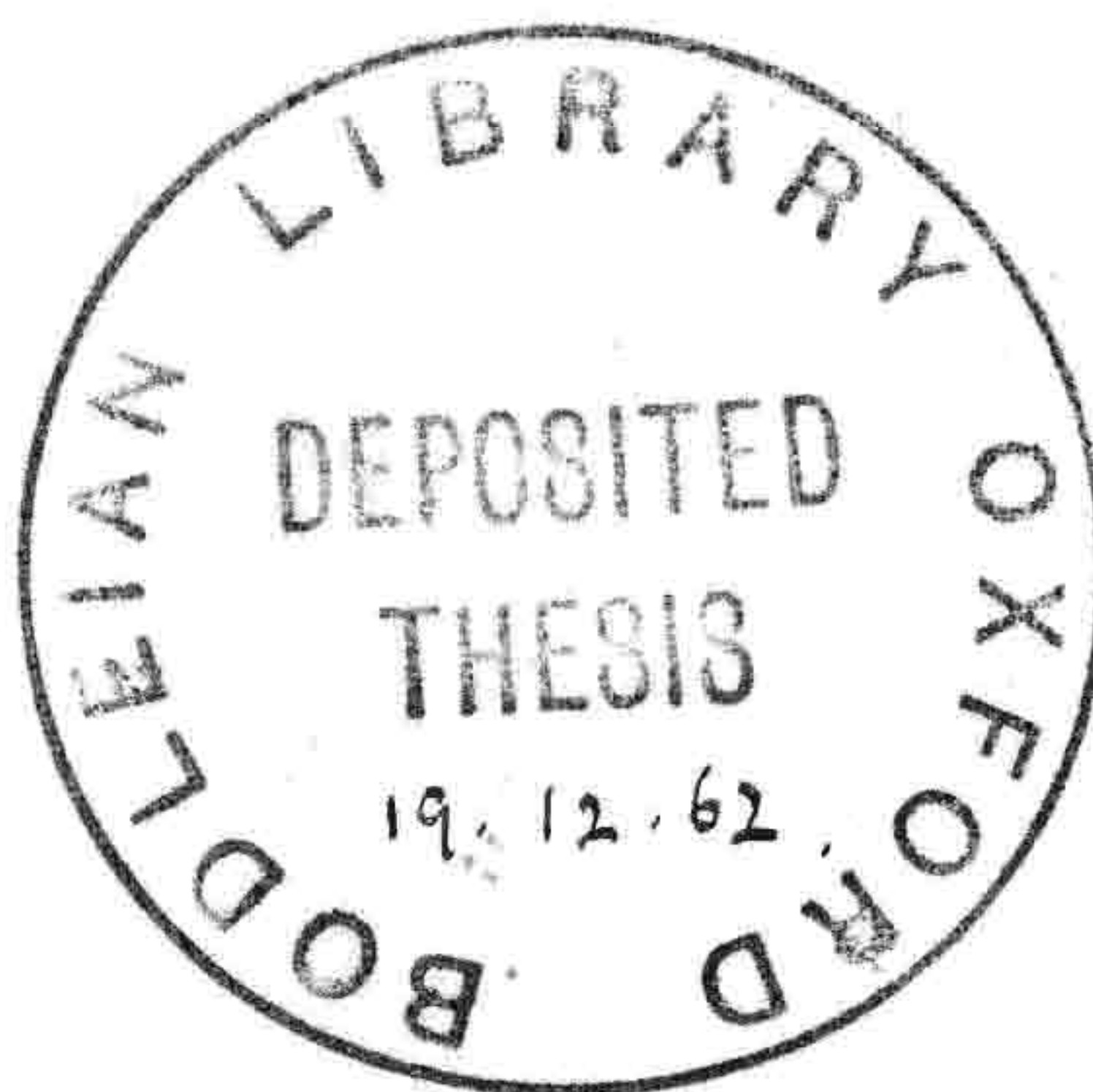
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of

KNOWING AND UNDERSTANDING

(Relations between meaning and truth, meaning and
necessary truth, meaning and synthetic
necessary truth)

A. Sloman



St. Antony's College
Oxford
Trinity Term
1962

KNOWING AND UNDERSTANDING

(Abstract)

The avowed aim of the thesis is to show that there are some synthetic necessary truths, or that synthetic apriori knowledge is possible. This is really a pretext for an investigation into the general connection between meaning and truth, or between understanding and knowing, which, as pointed out in the preface, is really the first stage in a more general enquiry concerning meaning. (Not all kinds of meaning are concerned with truth.) After the preliminaries (chapter one), in which the problem is stated and some methodological remarks made, the investigation proceeds in two stages. First there is a detailed inquiry into the manner in which the meanings or functions of words occurring in a statement help to determine the conditions in which that statement would be true (or false). This prepares the way for the second stage, which is an inquiry concerning the connection between meaning and necessary truth (between understanding and knowing apriori). The first stage occupies Part Two of the thesis, the second stage Part Three. In all this, only a restricted class of statements is discussed, namely those which contain nothing but logical words and descriptive words, such as "Not all round tables are scarlet" and "Every three-sided figure is three-angled". (The reasons for not discussing proper names and other singular definite referring expressions are given in Appendix I.)

Meaning and Truth.

Part Two starts with some general remarks about propositions and meanings. We can answer questions as to what meanings and propositions are, by describing the

criteria for deciding whether words are used with the same meanings or whether sentences are understood to express the same proposition. It turns out that there are various levels at which criteria for identity are required, and various kinds of criteria. (E.g. we need criteria for identifying the functions of statements as opposed to commands or questions, criteria for distinguishing the functions of descriptive words and referring expressions, criteria for identifying or distinguishing the meanings of individual descriptive words.) In our language, and others like it, the existence of a conceptual scheme involving universals (observable properties and relations) is presupposed by the methods used for ^{making} the finest distinctions between meanings of descriptive words. (Section 2.C.)

1) Descriptive words.

After the general remarks in chapter two about criteria for identity of meaning and the existence of universals, chapter three goes on to show in some detail how descriptive words (such as "scarlet", "round", "glossy", "table", and "sticky") can be given their meanings by being correlated with observable properties or combinations of properties. These words can be classified according to how their meanings are "synthesized" from properties. There are logical syntheses and non-logical syntheses, and both kinds may be further subdivided. (In 3.C a tentative answer is given to the question: How does talking about universals, i.e. properties and relations, explain our use of descriptive words.) In this and the next chapter many hidden complexities, including a number of different kinds of indeterminateness (4.A and 4.B) are found even in the meanings of innocent-looking words like "horse" and

"red", but these complexities are taken account of within the framework of a theory which does not assume that correlations between words and universals must be of the simple one-one type. The existence of "borderline cases" is due to the existence of these complexities.

The importance of all this is that it shows how "sharp" criteria may be used for identifying and distinguishing meanings of descriptive words, and helps to explain why the debate about the existence of synthetic necessary truths has gone on for so long: namely, philosophers have unwittingly used loose and fluctuating criteria for identity of meanings. Another cause has, of course, been unclarity about the significance of the terms "analytic", "synthetic", "necessary", etc. These are dealt with later on, their application being illustrated by examples arising out of the discussion of semantic correlations between descriptive words and universals.

ii) Logical words.

Part Two concludes with chapter five, in which the rôle of logical constants in sentences is explained by extending and generalizing some ideas of Frege, Russell and Wittgenstein (in "Tractatus"). The explanation makes use of the concept of what I call a rogator, which, like a function, takes arguments and yields values; the difference is that to a function there corresponds a rule or principle which fully determines its value for any given argument-set, whereas to a rogator there corresponds a principle or technique for finding out the value, the outcome of which may depend on contingent facts, or how things happen to be in the world. So the value of a rogator for a given argument-set is not fully determined

by the rogator and the argument-set, but depends on facts which may have to be discovered by empirical observation, and may change from time to time. The essential thing is that there is a technique, which can be learnt, which, together with the argument-set and the observable facts, determines the value. A special type of rogator is a "logical rogator", which corresponds to the logical form of a proposition and may be represented by sentence-matrices, such as "All P Q's are not R". A logical rogator takes as arguments sets of descriptive words, such as ('round', 'table', 'scarlet') and yields as values the words "true" and "false". Which is the value depends on the meanings of the descriptive words (the properties with which they are correlated) and the facts. (In 5.B.18 a variation on this is mentioned, in which sentences and their negations are taken as values.) In learning to speak, we learn general rules for the use of logical words and constructions, and these are what determine which logical technique (or which logical rogator) corresponds to any sentence. This shows that the commonly held view that the functions of logical words are explicable in purely syntactical terms is either false or vague and superficial. What lies behind it is the fact that the distinguishing feature of logical constants is their topic-neutrality (5.A): they are governed by rules which are so general that from the occurrence of a logical word, e.g. "or", in a sentence one can deduce nothing about the subject-matter, or topic, of which it treats.

Thus, Part Two shows that the meanings of descriptive words are given by correlations with universals, and the meanings or functions of logical words by correlations with logical rogators, or general logical techniques for

finding truth-values, and explains how these meanings or functions determine the conditions in which sentences composed of descriptive words and logical constants express true, or false, propositions.

((Some by-products of this are mentioned in the thesis. Logical relations, such as entailment and incompatibility, are explained as arising out of relations between logical rogators, or, more specifically, between techniques for discovering truth-values. This explains the connection between the geometrical forms of sentences and logical properties of the propositions they express, and shows how formal logic is possible. Secondly, we can clarify the difference between the "implications" of a statement and its "presuppositions", by pointing out that a rogator, like a function, has a limited "domain of definition" and, further, certain empirical conditions may have to be satisfied if its technique is to be applicable to finding out the value corresponding to a given set of arguments. Thus, the presuppositions of a statement are concerned with the conditions which must be satisfied if it is to have a truth-value at all, and its implications are concerned with what must be the case if the techniques are applicable and the truth-value comes out as "true". All this serves to explain why apparently well-formed sentences may be senseless and seems to provide the basis for a simpler and more general theory of types and category rules than that which uses the notion of the "range of significance" of a predicate. This is suggested, but not developed, in 5.E.))

Meaning and Necessary Truth

Part Three explains, in chapter six, how it is possible for a statement to be analytic and then goes on, in chapter seven, to give a more general account of necessarily true statements and show that some are synthetic.

Some uses of the concepts of "possibility" and "necessity" are explained by drawing attention to certain general and fundamental facts, but for which our thought and language and experience could not be as they are, such as the fact that universals (observable properties and relations) are not essentially tied to those particular objects which happen to instantiate them. (The table on which I am writing is brown, but it might have had a different colour, and the colour brown might have had other instances than those which it does actually have, without being a different colour: all this makes use of some of the general remarks about conceptual schemes, in chapter two.) This shows how it makes sense to talk about "what might have been the case but is not", or "what is possible though not actual". It is then noted that although universals are not essentially tied to their actual particular instances, nevertheless they may be essentially tied to one another (or incompatible with one another, etc.). The property of being bounded by four plane surfaces cannot occur without the property of having four vertices. These connections between properties can justify our assertion of some kinds of subjunctive conditional statements, such as "If this had had four sides, then it would have had four angles", and therefore enables us to assert that certain universal statements could not have had any exceptions. This explains a concept of "necessity", in terms of what would be the case in any possible state of this world, where "this world" is a world containing the same universals (observable properties and relations) as our world.

The description of the connection between meaning and

necessary truth follows on naturally from the general description of the connection between meaning and truth. Normally the value of a rogator for a given set of arguments depends on how things are in the world, and has to be discovered by applying the appropriate technique. But in some "freak" cases the value is independent of the facts and may be discovered by examining the technique and the arguments, or relations between the arguments. In particular, the truth-value of a proposition, in "freak" cases, may be discovered by examining the logical technique corresponding to its logical form and noting relations between the meanings of the non-logical words used to express it. Since how things are in the world need not be known, the truth-value would be the same in all possible states of affairs." (But it may be discovered in the normal way, by applying the technique instead of examining it. If one fails to notice that it is necessarily true that every cube has twelve edges one may set out to discover its truth by observing cubes. The fact that empirical enquiries are relevant even where analytic propositions are concerned brings out the defects in most accepted definitions of "analytic".)

So the truth-value of a necessarily true proposition is determined by (a) its logical form, or the logical techniques corresponding to its form and (b) relations between the meanings of non-logical words, or, more specifically, connections between the properties referred to. The notion of a definition or partial definition is examined and found to generate one kind of relation between meanings or properties, called "identifying relations". An "analytic" proposition may then be defined as one whose truth-value is determined only by its logical

form and identifying relations between meanings. This leaves open the question whether there are other sorts of connections between properties, in virtue of which statements may be necessarily true though not analytic. This question is investigated in sections 7.C and 7.D, where it is shown how simple geometrical proofs (using diagrams, for example) may enable one to perceive connections between geometrical properties in a manner which is quite different from the way in which one draws logical conclusions from identifying relations between the meanings of words. This description of the workings of "informal proofs" shows, therefore, how it is possible first of all to identify universals by being acquainted with them and then, by examining them, to have a further "insight" into their interconnections. This helps to answer the question which was left unanswered in chapter five, as to how one can discover that logical rogators are connected in certain ways (and hence that propositions have certain logical properties) by examining their techniques.

All this shows that there are both analytic and synthetic necessary truths. The former are true in virtue of their logical form and identifying relations between the meanings of non-logical words used to express them. The latter are true in virtue of all this, and, in addition, some non-identifying relations between meanings. In order to know the truth-value of an analytic statement, it is enough to know how the logical constants work and that some of the descriptive words stand in certain identifying relations with others, such as that some of them are used as abbreviations for other expressions. But when the statement is synthetic, one must, in addition to knowing that the meanings of the words are identifyingly related

in certain ways, also know what the meanings of some of the descriptive words are, so as to be able to examine the properties referred to and discover the connections between them. ((It is assumed that all these statements have truth-values. This cannot always be discovered apriori. See remarks about applicability-conditions for logical techniques.))

((The discussion of informal proofs is only a beginning, and does not pretend to be conclusive. Complications arising out of indeterminateness of meaning and the fact that neither "absolutely specific" nor "mathematically perfect" properties (e.g. the property of being bounded by four perfectly plane sides) can be described as "observable", are mentioned, but not discussed in detail.))

Chapter eight is a concluding summary. It is followed by appendices. The first explains why nothing has been said about singular definite referring expressions. The second describes some of the confusions which arise out of too much concentration on symbolic logic. The third discusses the notion of "implicit knowledge": knowledge which one may be able to apply without being able to formulate. The fourth makes some remarks about philosophical analysis and suggests some further developments of the thesis. The fifth appendix tentatively suggests that examples of synthetic necessary truths may be found in connection with other than geometrical properties. Finally, the concept "apriori" is discussed, briefly.

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PREFACE

In this thesis I have tried to answer Kant's question: "Are there any synthetic necessary truths?" by developing a theory of meaning within which the question can be stated clearly and given a decisive answer. However, I believe the theory is of more general interest than this, since, although it is formulated so as to deal only with the connection between meaning and truth-conditions, it can be extended quite naturally to include kinds of meaning which have nothing to do with truth. This provides a framework for the classification of types of relations between meanings which treats relations between truth-conditions, and, in particular, logical relations, as a special case. My belief in the wider applicability of what I say in the thesis is what explains the existence of many digressions, not immediately relevant to the main question. Some of these digressions are labelled as such by the word "note", or by their occurrence as footnotes or appendices (especially Appendix IV).

The main factor common to the theory developed within the thesis and its proposed extension is the acceptance of the existence of universals. The only kinds of universals explicitly described as such (chapters two, three and seven) are observable properties of material objects, but essentially the same concept of a universal is implicitly involved in the notion of a "technique" for discovering truth-values, illustrated in chapter five (5.B). A full characterization of this wider concept of a "universal" would require a detailed discussion of the points made by Wittgenstein (in Investigations and R.F.M.) about the concept of "following a rule", in which

he penetratingly criticizes his former beliefs. This thesis could be regarded as a first step in the process of patching up the Tractatus Logico Philosophicus so as to meet some of those criticisms. At any rate, the point I wish to make now is simply that the thesis is incomplete not only insofar as its further developments are hardly explored, but also, and more importantly, insofar as it rests on a basis which still requires a great deal of investigation. (This is hinted at in Appendix IV.8.a.)

It will be clear from what I have said that my main debts are to Kant and Wittgenstein: to the former for formulating the main question and providing what seems to me to be the right sort of answer, and to the latter for providing criticisms of the assumptions on which that answer is based which throw their exact nature into much sharper focus than ever before. (The reader may not find this latter debt evident.)

Now for some practical points. The order of development in the thesis is not the most clear and logical one possible, partly on account of the need for compression, and partly on account of the fact that new ideas kept coming even while the final draft was being written. (For example, a great deal of chapters two and three - especially 2.C - is intended to forestall objections to chapter seven, and ought, ideally, to be preceded first by chapter seven and then the objections. But that would have made the thesis much longer.) For this reason the text is sprinkled with cross-references either in parenthesis or in footnotes, as an aid to clarity. It is hoped, however, that most of them can be ignored, especially when they occur in footnotes, except when the reader has forgotten an earlier definition or argument.

Finally, I should like to thank my supervisor, Mr. D. F. Pears, for showing so much patience, and for criticisms without which this thesis would have been far more confused and obscure than it is.

C o n t e n t s

PART ONE: SOME PRELIMINARIES

Chapter one	<u>Introduction</u>	1
	1.A. The problems	1
	1.B. Methodological remarks	5
	1.C. The programme	13

PART TWO: MEANING AND TRUTH

Chapter two	<u>Propositions and meanings</u>	18
	2.A. Criteria of identity	18
	2.B. General facts about language	24
	2.C. Universals and strict criteria	38
	2.D. The independence of universals	50
Chapter three	<u>Semantic rules</u>	63
	<u>Introduction</u>	63
	3.A. F-words	64
	3.B. Logical syntheses	70
	3.C. How properties explain	83
	3.D. non-logical syntheses	93
	3.E. Concluding remarks and qualifications	102
Chapter four	<u>Semantic rules and living languages</u>	107
	4.A. Indefiniteness	107
	4.B. Ordinary language works	117
	4.C. Purely verbal rules	125
Chapter five	<u>Logical form and logical truth</u>	129
	<u>Introduction</u>	129
	5.A. Logic and syntax	130
	5.B. Logical techniques	144
	5.C. Logical truth	166
	5.D. Some generalizations	176
	5.E. Conclusions and qualifications	181

PART THREE: MEANING AND NECESSARY TRUTH

Chapter six	<u>Analytic propositions</u>	194
	6.A. Introduction	194
	6.B. Some unsatisfactory accounts of the distinction.	199
	6.C. Identifying relations between meanings	217
	6.D. Indefiniteness of meaning	229
	6.E. Knowledge of analytic truth	236
	6.F. Concluding remarks	249
Chapter seven	<u>Kinds of necessary truth</u>	260
	Introduction	260
	7.A. Possibility	261
	7.B. Necessity	272
	7.C. Synthetic necessary connections	283
	7.D. Informal proofs	294
	7.E. Additional remarks	319
Chapter eight	<u>Concluding summary</u>	329

APPENDICES

I.	Singular referring expressions	335
II.	Confusions of formal logicians	340
III.	Implicit knowledge	357
IV.	Philosophical analysis	372
V.	Further examples	381
VI.	Apriori knowledge	386

Bibliography	389
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P a r t O n e

S O M E P R E L I M I N A R I E S

Chapter One

INTRODUCTION

1.A. The problems

1.A.1. In order to know that some statement is true, one must understand that statement, one must know what it means. Sometimes understanding seems to be enough. For example, I know that the statement "All bachelors are unmarried" is true simply because I understand it, in particular because I know the meaning of the word "bachelor". In general, however, understanding is not enough: one must do more than learn the meaning of a statement in order to discover whether it is true or false.

In the simplest cases one must, in addition to understanding, also carry out some sort of observation of facts, or rely on the reports of others who have done this. In these cases, the meaning of a statement does not, on its own, suffice to determine whether it is true or not, for facts, that is, the way things happen to be in the world, may also be relevant.

The main aim of this thesis is to enquire whether the truth or falsity of a statement depends on how things happen to be in the world in all cases where the meaning does not suffice to determine this. Where understanding a statement does not on its own enable one to know that it is true, is some empirical observation of contingent facts always necessary? In short, the thesis is concerned with two old philosophical questions, first clearly formulated by Kant, namely:

Are there any synthetic necessary truths?
Is synthetic a priori knowledge possible?

1.A.2. These problems generate a whole family of problems, some of which will be tackled in this thesis. First, the terms in which the questions are expressed must be explained, and also many related terms, such as "analytic", "contingent", "empirical", "factual", "meaning", "definition", "concept", "proposition", and so on.

Although often used by philosophers, these words have no precisely defined standard meanings. A large part of this thesis will, therefore, be concerned with their clarification.

1.A.3. In the course of this process of clarification, a wide range of further problems will arise. Exactly how is the meaning of a statement ever relevant to whether it is true or not? What happens when one learns to understand a statement, and what is the connection between this and what happens when one comes to know that it is true? How is it possible for a statement to be true simply in virtue of what it means? How is it possible for a statement to be necessarily true, or to be known to be true without empirical investigation? Are all necessary truths analytic? Can all necessary truths be known a priori? Are there different sorts of necessary truths worth distinguishing from one another, even if the distinctions are not the same as the analytic-synthetic distinction? Are there different ways in which a statement can be true in virtue of what it means? When a proposition is true by definition in what sense can we describe it as "true"?

1.A.4. Even if we start with questions about language or about linguistic entities, as some of these appear to be, we soon find ourselves dealing with problems about other entities, such as the persons who use words, or the things and properties to which they intend their words to refer. We shall find that it is impossible to answer questions in logic without discussing a much wider range of topics. Logic seems to be inseparable from metaphysics, from philosophical psychology, and from epistemology.

It is concerned with concepts and meanings and propositions and truth, so it cannot be divorced from metaphysics, the general study of the kinds of things which can fall under concepts, which can be referred to in propositions, and which can make statements true or false.

We cannot discuss problems about meanings or concepts or propositions without mentioning thinkers or speakers, the persons who use words with meanings, who understand or intend propositions to be expressed by sentences. So a philosopher of logic should be prepared to discuss various mental states or activities, such as meaning, intending, thinking, or paying attention. This is why I say that logic cannot be divorced from philosophical psychology, the study of ordinary psychological concepts, or at least those connected with our use of language. (It may sometimes look as if logic can be done by talking only about symbols, but there must always be some implicit reference to persons who use those symbols or could use them, for symbols have meanings only insofar as they are taken to have meanings by some person or group of persons. They do not have meanings in themselves.)

Finally, the discussion of concepts and propositions leads us into the discussion of ways of coming to know that objects fall under concepts, or that propositions are true

or false, and this shows that there are connections between logic and epistemology.

All this should help to explain the fact that although this is primarily a logical investigation, a very wide range of problems and topics must be dealt with. Unfortunately, there will not be space to deal with all the problems which are raised in the discussion.

1.A.5. Although the problems to be discussed were first raised so long ago, they still seem to be of some interest. Indeed, it is sometimes suggested that the question of synthetic necessary truth is one of the most important of the philosophical problems which remain unsolved, for connected with it are problems about philosophical method which ought to be of interest to philosophers engaged in conceptual analysis, if they wish to be clear about what sort of thing they are trying to do. Are they merely producing reports on linguistic usage, or uttering analytic statements, or demonstrating truths which are not analytic but nevertheless necessary, or what?

It has recently been pointed out that there are difficulties in saying precisely what is meant by such words as "analytic", "synonymous", "necessary" (mainly by Quine), and in consequence it seems to have become rather unfashionable to employ them in philosophical discussions. Certainly a survey of recent publications in philosophical journals shows that people are not at all clear as to what the analytic-synthetic and necessary-contingent distinctions are. But they cannot really get along without them, and end up talking about "absurdity", "logical impossibility", "contradiction", "nonsense", "inconsistent usage", "conflict with ordinary language", "inference-licences", "rules of grammar", etc., often

unaware that, in a groping sort of way, they are making use of Kant's distinctions. It seems to me to be time they faced up to this fact and tried to be clear about the distinctions, taking seriously some of the problems connected with them. This is what I shall try to do.

1.A.6. In the remainder of this chapter some remarks will be made about the procedure to be followed in the rest of the thesis.

1.B. Methodological remarks.

1.B.1. I have undertaken to explain the meanings of certain terms and to answer some of the questions expressed in these terms. In order to do this without circularity I should have to avoid using words such as "necessary", "contingent" and "analytic" until after explaining their meanings, but this would increase the length of the thesis considerably. My excuse for using the words before showing that they correspond to real distinctions, apart from the fact that it makes a great deal of compression possible, is that philosophers and others do seem to have some sort of intuitive understanding of them, and their usage seems often to be in accord with the definitions which will be given later on, even though those philosophers give very different definitions from mine when they try to say precisely what such words mean. (This is an illustration of the familiar fact, to be discussed in the Appendix on "Implicit Knowledge", that one may perfectly well know how to use a word, without being able to say how it is used.)

When the meanings of these words are finally explained, this will not be done by giving an explicit definition.

Instead, the explanation is more nearly a process of drawing attention to those aspects of our thought and experience and our use of language which make it possible for the words to be used. This process in some ways resembles ostensive definition, except that here the "pointing" is done with words.

1.B.2. The description of those aspects of our thought and language and experience which make it possible to distinguish analytic from synthetic propositions, and necessary truths from contingent ones, will proceed from a certain point of view, which I shall now try to describe, in order to reduce the possibility of misunderstanding.

Not everything that can be said about thinkers and speakers has a content which can be exhausted by descriptions from the point of view of experimental psychologists or physiologists, or anthropologists who study human beings as if they were only one kind of animal, to be observed in a scientific way. The reason why statements about persons cannot always be translated into the statements of scientific observers is that they answer different kinds of questions, they serve different kinds of purposes, they are made and listened to by persons with different sorts of interests and different kinds of curiosity.

When I make a statement about what a person thinks or feels, or what he intends, or when I try to explain his behaviour in terms of what he wants or how he reacts to what he sees, or thinks he sees, then I may be trying to say something which enables other persons to know what it would have been like to be in his place: and this is not the same thing as describing the physical state of his brain or his dispositions to produce certain publicly observable noises or movements in specified situations -

though I may be talking about these things as well.

1.B.3. In order to understand descriptions or explanations which refer to mental states or processes, it is not enough to have observed their outward manifestations or the concomitant physical or physiological processes (the latter is not even necessary, let alone sufficient: people are able to understand such statements as "I jumped because I saw a face at the window", without knowing anything about electromagnetic waves or what goes on in the brain). In order to understand completely, one must have had experiences sufficiently similar to those described. One must be the same kind of being. For example, unless I know what it is to act for reasons, or perhaps to act because I want something, I cannot fully understand a statement such as the following: "I climbed on the chair because I wanted one of the biscuits on the top shelf". Such "ignorance" would not, however, prevent my coming to understand a scientific explanation (e.g., at a physiological level) of this kind of behaviour.

All this should be remembered in connection with chapters three, six and seven, below (especially section 3.C). For example, when it is asserted that a property, such as a shape or a colour, can explain how a person uses a descriptive word, or how he groups things together, this is not an assertion which might be made by an experimental psychologist or a physiologist, but the sort of statement which can be understood (fully) only by a person, by one who knows what it is like to select objects on the basis of their colour or shape or some other visible property.

1.B.4. It is sometimes suggested that the reason why statements about conscious persons cannot be translated into statements about publicly observable physical events or states of affairs is that the two sorts of statements describe things at different levels. For example, statements about individual persons are at a different "level" from statements about crowds or nations, and statements about the positions of a moving point of light are at a different "level" from statements about its velocity, the radius of curvature of its path, and so on. (Wittgenstein's comparison, on p.179 of "Philosophical Investigations" might suggest that this was his view. But see below: 1.B.6.(note).) This makes it look as if it were just a matter of a difference in degree of complexity, or a difference in which facts are counted as relevant, or a difference in the ways in which the facts or objects described are "organized" or "structured". The impossibility of translation, and the failure of attempts to find logical connections between classes of statements at the two levels might then be explained in terms of the "open texture" of the concepts at a higher level. However, if there were just a difference in level, then there would surely be some logical implications from one level to the other despite the "open texture". For example: a complete description of the positions of a dot at all times would logically entail statements about its velocity, etc. In some cases a complete description of the behaviour of millions of individual persons would entail a statement about a nation, such as that the nation was at war with another. But no description of physical and physiological states, however exhaustive, can entail the statement that a person has a toothache, or that he wants a drink. Something is always

left out, namely a description of what is going on from his point of view, how it feels to him.

1.B.5. The differences between statements to which I am trying to draw attention, therefore, are not merely differences in "level", or in the method of organization of facts, but differences in the point of view from which the descriptions are given. This is what seems to me to be one of the most important reasons for the failure of reductive programmes.

This question cannot be dealt with in detail here. I shall simply assume that there is a difference and that it can be characterized thus: when describing things from the "rational" or "personal" point of view, one assumes that it makes sense to wonder what it would be like to be in the position of the person being described, whereas, from the scientific (or "tough-minded") point of view one tries only to describe what could be observed by anyone at all, and seeks causal explanations for human behaviour. For example, the experimental psychologist investigating threshold levels is concerned only with the subject's responses to stimuli, he is no more concerned with what it would be like to be in the subject's position, than a physicist wonders what it would be like to be in the position of a magnet which attracts iron filings or an electron which is deflected by a magnetic field. (In the early stages of development of a science, of course, people may be concerned with what it feels like to be set in motion by an external force, for example. Here an intermediate point of view is adopted.)

1.B.6. The descriptions and explanations in the chapters

which follow are given from this personal or rational point of view, which means that I shall have to rely heavily on the reader's ability to reflect on his own experience while I try to draw attention to certain aspects of it. I am not writing from the point of view of an anthropologist or psychologist or physiologist of the sort who simply observes people "from the outside" and records correlations between stimuli and responses. (Compare the point of view adopted in Quine's book: "Word and Object".) I am writing from the point of view of a person who thinks and speaks and has experiences, and I am trying to describe certain general features of his thought and language and experience from that point of view. This may be described, therefore, as a "phenomenological" essay.

When, on occasion, I describe anything from the point of view of a person who observes other persons, then it is important to remember that the observer is the same kind of being as the ones which he is observing, otherwise he can only observe them, he cannot understand them. That is, he can only record that they produce certain marks on paper, or certain noises, or that they respond in certain ways when vibrations in the air stimulate their ear-drums, or when electromagnetic waves ^{reach their} retinas. He will not be able to record that they are saying anything, let alone what they are saying, or that they hear sounds or see colours; or, if he can record these things, then his records will be only short-hand descriptions of patterns of observable behaviour. (Remember that we do not see wavelengths nor hear vibrations when we see colours and hear sounds.)

(Note. Wittgenstein tried - in "Philosophical Investigations" - to describe things from an intermediate point of view: he talked from the point of view of one who uses language, who is a person and can communicate with other persons, but he concentrated on publicly observable phenomena, on the kind of evidence which makes people assume that they are communicating successfully, on the publicly observable social aspects of our use of language. It seems to me that he omitted a great deal that can be said about our thought and language from the point of view of the one who thinks and speaks and knows what he means, and I have tried to fill that gap, or part of it.)

1.B.7. The point of view from which this thesis is written is only one among several different possible points of view. It should not be thought to be more correct or more important than any other (though perhaps it might be argued that it is the only one which a philosopher can adopt without laying himself open to the charge that he is trying to be an arm-chair scientist!). There are many different sorts of interest which one may have in the world, and there is no reason why only one of these interests should be fed, to the exclusion of all others.

This seems sometimes to be denied by so called "tough-minded" philosophers. For example, Quine wrote, in "Word and Object" (p. 264):

If there is a case for mental events and mental states, it must be just that the positing of them, like the positing of molecules, has some indirect systematic efficacy in the development of theory. But if a certain organization is achieved by thus positing mental states and events behind physical behaviour, surely as much organization could be achieved by positing merely certain correlative physiological states and events instead.

But "organization" and "systematic efficacy in the development of theory" (presumably scientific theory)

need not be the only aims of a philosophical enterprise. There is another point of view, the one which I have tried to characterize. It has not been arbitrarily invented by me, nor is it anything mysterious or unfamiliar, for we are constantly asking people how they feel, asking them what their intentions are, what they want, why they behave as they do; and we are not merely requesting information which could be supplied by any sufficiently clever and well-informed physiologist. If we were to abandon this point of view in our dealings with other people (e.g., when we ask "Is your toothache very bad?"), our whole attitude to life and personal relations would have to change. There would be no more scope for such utterances as "I cannot understand why he thinks his plan has the slightest chance of working."

1.B.8. In what follows, it will therefore be taken for granted that there is a point of view of the sort which I have tried, somewhat too briefly, to characterize. From this point of view, the point of view of a conscious person who talks and thinks about the things he sees about him, I believe that a coherent description can be given of much of our language, thought and experience, which shows that there is room for a distinction between propositions which are analytic and propositions which are synthetic. (Cf. Quine: "No systematic experimental (sic) sense is to be made of a distinction between usage due to meaning and usage due to generally shared collateral information". Op.Cit., p.43.) Within this general framework it will be shown that there is a clear and interesting sense in which some statements which are not analytic are nevertheless necessarily true.

1.C. The programme

1.C.1. So far the main problem has been stated, some subsidiary problems mentioned, and remarks made about the general framework within which these problems will be treated. Now I shall briefly describe the programme to be followed in succeeding chapters.

After these preliminaries, the remainder of the thesis will be divided into two parts. The first is concerned with the general connection between meaning and truth, the second with the connection between meaning and necessary truth. (Some side issues and further developments will be dealt with in appendices at the end.)

Part Two, on meaning and truth, is divided into four chapters. First I shall try to explain, in a general way, how we can talk about meanings and propositions, bringing out some presuppositions of such talk. This lays the foundation for most of the remainder of the thesis. Secondly, in chapter three, a detailed description will be given of some ways in which descriptive words and expressions (e.g. adjectives) can have the meanings which they do have, in virtue of their being intended to refer to universals (i.e., observable properties and relations). This will involve a great deal of oversimplification, and in chapter four some of the complications in our ordinary use of words will be described which are overlooked in chapter three. Finally, chapter five describes our use of logical words and constructions (pointing out that attempts to reduce logic to syntax are quite misguided), and shows how the way in which logical constants and descriptive words are combined to form a sentence determines the class of possible states

of affairs in which that sentence would express a true proposition. This shows how the truth of a statement can depend both on what it means and ^{on} what the facts are, and prepares the way for a description of cases in which the truth depends only on what the statement means.

Part Three, which is concerned with necessary truth, starts with the explanation in chapter six of how it is possible for a statement to be analytic (true by definition), and how we can know that such statements are true without knowing how things happen to be in the world. Chapter seven seeks to explain the meanings of "necessary" and "contingent", and to show that there are good reasons for saying that the necessary-contingent distinction is different from the analytic-synthetic distinction. It is hoped that some examples taken from elementary geometry will illustrate the claim that there are some necessary truths which are not analytic. (For example, is it analytic that no solid object is bounded completely by three plane surfaces? Further examples are given in an appendix.)

In all these chapters some general and sweeping statements are made, followed by attempts to show how they are oversimplified and ignore complications. In most cases, however, there will be no room to go into these qualifications in detail.

1.C.2. It will be noticed that no account is to be given of our use of proper names and other singular definite referring expressions which refer to particular material objects. The reasons for this exclusion are stated in Appendix I. The whole discussion will be restricted to relatively simple statements about material objects and their properties, such as "All red things are round", "No

green things are glossy and pink" and "If a rectilinear figure is three-sided, then it is three-angled".

These statements contain only logical constants and descriptive words referring to observable features or properties or combinations of properties (little will be said about relations), and they are all universal in form: that is, they contain no definite referring expressions which presuppose the existence of particular material objects.

It is important to stick to relatively simple cases at first, as it is much more difficult to avoid confusions if one tries to discuss things in a completely general way right from the start, so as to take account of even the most complex examples. It should not simply be taken for granted, as it often is by philosophers, that it obviously makes sense, or any clear sense, to apply the analytic-synthetic distinction to almost every sort of utterance. A failure to be clear about the conditions in which the distinction can be applied leads to great confusion. Some limitations on the applicability of the distinction will be described later on, based on the facts pointed out in chapter four.

1.C.3. The main conclusion of this essay, that there exist synthetic necessary truths, or synthetic necessary connections between concepts and propositions, was first put forward by Kant. But my aim is not exegesis. I shall not be concerned with whether Kant really was trying to say the sort of thing which I shall be saying, or whether he would have approved, and there will not be as much in the way of reference to and comment on his texts as there might have been in a more scholarly work. (Nevertheless, it appears to me that most of my main arguments can be

found in Kant's "Critique of Pure Reason", though often expressed in an obscure and muddled fashion. He did not, after all, have the benefit of advances in clarity and insight achieved by philosophers during this century and the last.)

The reader who desires an historical account of the development of the problems discussed here, or criticism of the views put forward by other philosophers, is referred to "Semantics and Necessary Truth", by Arthur Pap. In order to save space I shall refer to the writings of other philosophers only when I think that this will help to clarify what I am saying. (Many of my debts will have to go unacknowledged.)

1.C.4. To summarize, I shall try, making use of the assumptions and methods described in the previous section, to describe the general connection between the meanings of certain sorts of statements and the conditions in which they are true, and then show how it is possible for a proposition to be true solely in virtue of what it means, that is, to be analytic. The question will then be raised whether the class of analytic truths includes all necessary truths, and the negative answer will be illustrated by the description of examples of necessary truths which are synthetic. I hope that in the course of all this it will become clear why other philosophers have reached different conclusions, the most important reason being, I think, that they have used much looser (and fluctuating) criteria for identity of meanings and propositions than I use. Failing to make fine discriminations, they fail to notice interesting relationships. (See section 2.C.)

It is hoped that there will be something of interest in the general picture which will be painted, even if the details are neither new nor very interesting in themselves.

P a r t T w o

M E A N I N G A N D T R U T H

Chapter Two

PROPOSITIONS AND MEANINGS

2.A. Criteria of identity

2.A.1. Before we can explain how the analytic-synthetic distinction and the necessary-contingent distinction are to be applied, and discuss the question whether they divide things up differently or not, we must be sure we know what sorts of things they are meant to distinguish. This applies also to the true-false distinction. Sometimes it is not clear whether philosophers think these distinctions apply to sentences or to statements or to ways of knowing, or something else, (c.f. section 6.A) and this leads them into ambiguity and confusion. I shall apply the distinctions to statements or propositions, which are expressed by sentences. When I talk about statements, I am talking about sentences together with the meanings they are understood or intended to have. When I talk about propositions, I shall be talking about the meanings of sentences (as understood by some person or group of persons). I shall often use the words "statement" and "proposition" interchangeably, as the difference between them is important only in contexts in which we are concerned about the actual form of words used to express a proposition.

But this leaves unanswered the question: What is the meaning of a sentence, or the meaning which it is taken by some person or persons to have? The only way to answer this question is to describe the ways in which words and sentences can be used with meanings or understood

with meanings, and to say clearly how to tell whether two words or sentences are used or understood with the same meaning or not. That is to say, we must describe criteria for identity of meanings and propositions. I shall show presently how the failure to do this may lead to confusion and the begging of questions.

It will not be possible to answer all questions about identity of meanings in this chapter. A few rather vague remarks, concerning very general facts about languages, will be made in section 2.B. Section 2.C explains why it is necessary to use physical properties to provide criteria for identity of meanings of descriptive words, and section 2.D attempts to show that this is not a circular procedure, nor completely trivial. But first of all a few general remarks about criteria of identity will help to explain why all this discussion is necessary.

(It should be noted that most of the general remarks of this chapter will be presupposed in all that follows.)

2.A.2. Why should we talk about criteria for identity of meanings? Talk about true or false statements, or about meanings or propositions, is not merely talk about sentences or words, for these are merely signs, and cannot, as such, be true or false, or uniquely identify the sense which has been given to them. We cannot tell simply by looking at the shape of a mark which someone has drawn, or by listening to the sound he utters, what he means by it, or how others will understand it. For one hearer or reader may understand it in one way, while another understands it differently, and both may have failed to understand what the author meant by it. What is more, one and the same person may understand different

tokens of the same type of word or sentence differently on different occasions, or in different contexts. (It is sometimes suggested or implied that this is entirely due to ambiguities in words or expressions which refer to particular objects, expressions such as "John", "the tree on the corner", "you", and so on, but it is important not to forget that descriptive words may also be ambiguous, though perhaps less systematically.) Talking about meanings presupposes that we know what it means to talk about the absence of ambiguity, that we have some way of telling when words or sentences are understood or intended to have the same meanings. So we need some way of identifying the meanings with which words are used and the propositions which they are intended, or taken, to express. How can this be done?

2.A.3. There must be an answer to this question, for we are quite used to talking about meanings: we can ask what a word means in German, whether two words mean the same in English, and whether two persons mean the same by the word "tadpole". In learning to speak we implicitly learn the answers to questions about identification. We learn to apply tests for telling whether two persons mean the same by a word, whether two words mean the same in a language, and so on. We learn how to pick out the occasions when we are using words inconsistently (i.e. with changing meanings) and the consequences of doing this. (We do not need to be given some philosopher's criterion for synonymy. So we are not troubled by the impossibility of breaking out of Quine's "circle of intensional words". See "Two Dogmas of Empiricism".)

Having learnt to apply tests, and having acquired

much skill in applying them over the years, we can go right ahead and say such things as: "The word 'red' refers to the hue of that object over there", or "The English word 'red' means the same as the German word 'rot'", without offering any further explanation. We learn to say and understand things like "I said that he had taken the money, but I did not mean that he was a thief. You obviously misunderstood me." In using such familiar language about meaning and referring and translating, we presuppose the answers to many questions about identity of meaning and make use of very general facts about language and words and sentences. So in order to state answers to those questions, we must make explicit the general knowledge which is presupposed and used in this way, and this involves making explicit some of the things we learn when we learn to talk.

2.A.4. By explaining how we ordinarily tell whether two sentences are taken to express the same proposition, or whether two persons take the same proposition to be expressed by some sentence, and so on, we remove much of the obscurity which is involved in talking about meanings and propositions. People sometimes object to talk about propositions (and other intensional entities, such as properties) because they do not wish to populate the world with such mysterious things. But meanings and propositions are not mysterious, if adequate criteria of identity are available, and they do not "populate the world" in the sense in which material objects do, any more than directions or numbers do. We can talk about the number of things in a class of material objects without mystery, and straight lines can have directions, because there are tests for identity of directions and

numbers.

It may be objected that there are no universally acceptable criteria for identity of propositions. But are there universally applicable and acceptable criteria for identity of physical objects, or shadows, or events, or persons? (Think of the paradox of the twice-mended axe, or paradoxes connected with immortality and re-incarnation.) In general, the suitability of criteria for identity depends on our purposes in identifying, and, if purposes vary, then what counts as adequate criteria of identity may vary. What counts as "the same colour" for the purposes of the editor of a cheap glossy magazine may not count as "the same colour" for the purposes of an artist or a fashion expert. What counts as "the same length" for a civil engineer may not do for the physicist, or the mechanical engineer.

Almost any set of criteria may be shown to break down in some conceivable situation or other. That is, criteria may come into conflict with one another, or may yield no answer, or an unsatisfactory answer to the question "Are they the same?" That, however, need not make us say that the things which these criteria serve to identify do not exist, or that they are in any way mysterious entities. Material objects, colours, shadows and lengths all exist. But some sets of criteria are more stable and widely accepted, because they are more useful, than other sets of criteria. Criteria for identifying material objects are simply of more general applicability than criteria for identifying propositions, or meanings, or shadows.

2.A.5. It is a matter of fact that there are ordinarily

accepted criteria for identifying propositions and meanings, on which we rely when we talk about ambiguities or the correctness of translations. But they are not infallible: some kinds of ambiguities and misunderstandings are very difficult to discover and to eliminate (see note at end of 2.B). In addition, it should be noted that, as remarked above, which criteria are employed may depend on the purposes for which judgments of identity are made. This may be illustrated by fluctuations in the criteria ordinarily adopted for eliminating ambiguities.

Thus, if one is interested only in testing for and eliminating flagrant ambiguities, the kinds which matter for purposes of ordinary conversation about the weather, about one's latest illness, or about Mrs Jones' son who insists on bringing tadpoles into the house, then one may employ fairly loose criteria. Conversations on such topics may usually be reported in a wide variety of ways without the charge of misrepresentation being incurred.

On the other hand, if one is discussing the weather, in an airport control-tower, or if one is a doctor recommending a patient for treatment, or a zoologist writing about the breeding habits of frogs, one may have to be more careful about what one means: one must look not only for obvious ambiguities, but for more subtle ones too. Someone reporting what is said in such cases has to be more careful about the words which he uses. Here stricter criteria of identity for meanings are employed, not necessarily because the words used are different, but because the purposes served by their utterance are different. When engaged in logical enquiries, one may use still stricter criteria for identity: a logician may regard two propositions as different if he wishes to

investigate the logical relation between them, such as mutual entailment, although even a careful scientist would regard them as one proposition.

We shall find (in Section 2.C) that the only way to avoid begging questions by ignoring subtle ambiguities is to use the strictest possible criteria for identity of meanings. This means that we shall have to be more careful than most logicians have been, and look for ambiguities even where they would be quite unimportant for most philosophical or non-philosophical purposes. Our motto will have to be the following remark made by Kant while discussing the role in philosophy of appeals to common ideas (in the introduction to "Prolegomena"): "Chisels and hammers may suffice to work a piece of wood, but for steel-engraving we require an engraver's needle." (Cf. 2.C.9.).

2.A.6. All this shows that we must not expect any very simple general answers to questions about meanings. There are various ways of comparing and distinguishing meanings, none of them intrinsically correct, each suitable for some purpose or other. But there is another complication, which arises out of the fact that tests for identifying meanings operate at several different levels. This will come out in the next section, where I shall discuss some of the general presuppositions of statements about meanings.

2.B. General facts about language

2.B.1. So far I have merely said that it is important to be clear as to what we mean by talking about "the same meaning", or "the same proposition", if we are to be clear

about applying the analytic-synthetic distinction and related distinctions. A thorough treatment of the subject would require a detailed investigation of what goes on when children learn to speak, when a child or adult learns a foreign language, or when we look for and find ambiguities in our own familiar language. This is the only way to answer all the questions raised in the previous section. There is no room for such a detailed investigation here, so, in this section I shall try only to indicate some of the sorts of things which would probably come out of it, by making a few rather vague generalizations.

2.B.2. First of all, it will turn out that even in employing simple tests for discovering the meanings with which ordinary descriptive words are employed, we make use of very general presuppositions about language and linguistic activities. For example, knowing how to tell whether two persons mean the same by some word or sentence presupposes a knowledge of what kind of thing a language or linguistic utterance is. For otherwise we should not be able to tell whether the noises people were making, or the things they were "writing" were part of a game, or a religious ritual, for example. This is not a trivial problem of identification, to be solved by looking to see what the marks they write down look like, or listening to the sound of the noises they produce, since it is quite possible for the same marks or noises to be produced as moves in a complicated game, even where making such a move is not using a language. Without presupposing an answer to the question whether a person is using a language, we cannot find out what he means by what he says. It follows that there must be some means of identifying kinds of

linguistic behaviour as such, and these methods must be learnt, at least implicitly, by a child when it learns to talk, and applied, explicitly or implicitly, by anthropologists when they first decide that the grunts and clicks and other noises produced by the members of some newly-discovered tribe are linguistic utterances.

2.B.3. This is not all. Not only must we know what it is for behaviour to be linguistic behaviour, in addition we must know what sorts of things various kinds of linguistic activities are, if we wish to talk about the meanings of words and sentences. We must, for example, know what it is to make a statement, and how this differs from asking a question, giving a command, exclaiming, or expressing jubilation, and so on. Knowing what statements, questions, commands, etc., are cannot be simply a matter of knowing the appropriate forms of words, for trying to teach someone what a statement is, is not just a matter of teaching him which forms of words are called "statement-making sentences". We have to teach him what can be done with these forms of words. We must know how to tell whether he is doing the right sorts of things with them or not. Hence, being a statement, a command, a question, etc., cannot be merely a matter of having certain syntactical properties. It is a matter of being correlated somehow with certain purposes and activities.

There are rules for the correct use of various forms of sentences, and they differ from language to language. So the important thing in common between statements in one language and statements in another, which they do not share with questions in either, cannot be a type of form of words, but rather a type of use to which such a form may be put, for example. Knowing what this use is

involves knowing what it is for a statement to give information, to be true or false, to be believed or disbelieved, to be contradicted or agreed with. Knowing what a command is, involves knowing what it is to want to get something done. It also involves knowing what it is for a command to be obeyed or disobeyed. Perhaps it involves knowing what it is to have authority. Such things, and many more, must be learnt when we learn to speak.

All this knowledge is presupposed even by simple statements about the meanings of simple words, since we cannot understand what a descriptive word like "smooth" means without knowing what it would be for that word to be used with that meaning in a statement, or a command - such as, "Bring me the smooth block of wood". We have therefore found two different levels at which tests are required for identifying meanings:

- I. There must be criteria for identifying activities as linguistic.
- II. There must be tests for identifying and distinguishing different kinds of linguistic activity.

(We could put this by saying: I. we must know what sort of thing a language is, and II, we must know what sorts of things various kinds of linguistic utterances are, if we wish to talk about meanings.)

2.B.4. It is time now to be a little more specific. Knowing what it is for a statement to be true involves knowing what sorts of things a statement can be about, and how the words in the statement determine which particular things are referred to in the statement and what is said about them. What sorts of things a statement can be about will depend on the particular language in

question, and different statements in the same language may be about quite different sorts of things. Compare a statement about the weather in Oxford, a statement about a mathematical theorem, and a statement about the morally best course of action in some situation.

But even if we restrict ourselves to the class of statements to be discussed later, which may vaguely be characterized as being "about the perceptible world", we may find that in some languages a greater variety of things may be said than in others. Or different languages may involve quite different ways of looking at the world, or, what comes to the same thing almost, quite different ways of talking about it. In particular, they may employ quite different conceptual schemes. Thus, one language may treat the world as consisting of arrangements of enduring physical objects, as English does, and permit the making of statements which say things about the qualities or properties of such objects, or their mutual relations, or the changes in such properties and relations, whereas another language does not use the concept of an enduring physical object, permitting only statements to the effect that the speaker is aware of certain features (hardness, roundness, furriness, and so on) in his environment.

Unless we are sure that two persons do not use languages which differ in this way, unless we are sure that they employ the same sort of conceptual scheme, we cannot be sure that they mean the same by the statements they make, even if their statements would be true in the same situations. For when statements employ different conceptual schemes, there may be no way of translating one into the other, or there may be several different systems of translation, all equally satisfactory or equally

unsatisfactory, there being no question of one translation being better than others. In such a case, there can be no clear sense to the question whether two persons who use these different conceptual schemes, mean the same by the words or sentences they utter. We can give it a sense by selecting criteria for identity of meaning, but in doing this we alter the sense of the words "mean the same".

To sum up: knowing how any particular language works involves knowing what sorts of conceptual schemes it employs, and questions about identity of meaning presuppose identity of conceptual schemes. So:

III. There must be tests for identity of conceptual schemes. ✓

2.B.5. As I am not trying to give a detailed account of all the criteria for identity of meanings and propositions, I shall not explain how we compare and distinguish conceptual schemes, but will take it for granted that we can, which is not unreasonable, since we can and do successfully compare and distinguish the meanings with which various persons use their words. For example, we are reasonably sure about the translation of "red" into German. This shows that Quine must be mistaken in his assertion that we can never discover with certainty that one conceptual scheme rather than another is employed by some person or group of persons. (See "Word and Object", sections 12, 15, 16, etc.) He must be wrong in any case, for unless we had some method of identifying conceptual schemes, we could not have learnt to understand the words with which he describes the various conceptual schemes, which he says we cannot distinguish! The method of making such distinctions must be embodied in the way in which we

learn to use words like "property", "unobserved", "the same" (applied to persisting material objects at different times), and so on. (We may have to rely on our shared natural reactions to some extent.)

If we had no inkling of the kind of conceptual scheme employed by certain people, then we could not ask or answer questions about the meanings of individual words or sentences in their language: indeed, we could not even be sure that it was a language. We could, at most, learn, by empirical observation, the conditions in which they produced certain noises, the situations in which they gave the appearance of "assenting" to statements, and perhaps some idea of the causal connections between their noise-producing habits and the smooth running of their society. But to know all this is not to understand. We could not say that we had understood them until we knew to what aspects of their environment they were referring, or whether they were referring to anything at all, when they produced their noises as predicted. (Cf. 1.B.6 above).

2.B.6. So, on the assumption that we can compare and distinguish conceptual schemes, I shall restrict the discussion to a language which is like our own in allowing talk about particulars and universals. Particulars are material objects, events, persons and other things which are spatio-temporally located. Universals are the properties which may be possessed and shared by these particulars, and the relations in which they may stand to one another. (I shall talk only about observable properties and relations.) So knowing how a language of this sort works, involves knowing what sorts of things material objects and other particulars are, and what sorts of things their properties and relations are. It involves

knowing how words and combinations of words may be correlated with such things and combined, perhaps with other words, to form sentences which can be understood as making statements, asking questions, and so on. The assumption that there are observable properties and relations, to which words can refer, does not seem to be a very implausible assumption. The next section but one (2.D) will be devoted to an explanation of what the assumption means, and what justifies it. At present I shall say only that knowing what sort of thing a property is, involves knowing what it would be like for that property to occur in other objects than the ones in which it does in fact occur. Universals are not essentially tied to those particular objects which happen to instantiate them, and one who does not see this has not fully mastered the conceptual scheme about which I am talking. Much will be made of this in the sequel (Cf. 2.D.9, 3.C.4, 3.E.5, and chapter seven). (Very little will be said about particulars, for reasons explained in Appendix I.)

It should be noted that a language may be over-determined as regards its conceptual schemes; for example, our language may have some other conceptual scheme built into it in addition to the one which I have described. If so, then there are two or more quite different ways in which we are able to look at the world or talk about it. Perhaps, for example, in addition to seeing it as made up of things and their properties, we can see it as made up of facts, or instantaneous events, etc. But we shall ignore such complications and problems, and concentrate only on (a) the fact that identification of meanings of words which can occur in statements about

the world presupposes the identification of conceptual schemes, and (b) the fact that we have at least the conceptual scheme which allows the existence of material objects and their observable properties and relations. These are the only particulars and universals I shall mention.

2.B.7. We have seen that in order to find out what a person means by his words and statements we must find out what sort (or sorts) of conceptual scheme he employs, or what sorts of entities he thinks of as making up the world and how. In addition, we must understand how he thinks of words as making up his sentences. For there will not only be words which refer to the entities for which there is room in his scheme (e.g. words referring to particular material objects, or descriptive words which refer to properties), but also other kinds of words and types of logical and grammatical constructions which help to determine what sort of statement is being made about the things referred to. Examples are: the word "is" in "My pencil is round", and the structure which this statement shares with "Tom's hat is brown". Neither the word nor the structure refers to any material object or a property or relation, or anything else which might be described as an observable entity, an object of experience. But they help to determine the meanings of sentences, and so we must know how they work if we are to understand statements which employ them.

For example, we must understand the difference between subjects and predicates if we are fully to understand the statements quoted above. It is also necessary for an understanding of the difference in meaning between "round" and "roundness", which do have different meanings

despite the fact that they refer to the same property, owing to the fact that they have different roles in the language. There might have been a language in which the same word was capable of occurring in both sorts of contexts, the difference in role being indicated by something other than a difference in the word. In that case it would not have quite the same meaning as either "round" or "roundness". (Think of our word "red".) In order fully to know the meaning of a word it is not enough to know what things it refers to; one must know also what kind of word it is meant to be, and how it can be combined with other words to form sentences of various kinds. So when we compare and distinguish the meanings of individual words we take for granted a whole system of logical and grammatical constructions, and if we are to be able to identify the meanings of statements we must know what sorts of logical systems are employed, and what the functions are of individual logical words and constructions. In chapter five I shall discuss the ways in which these "logical constants" help to determine the meanings of sentences in which they occur, by determining the conditions in which statements are true, commands are obeyed or disobeyed, and so on.

2.B.3. From all this we can see that giving a full account of tests for identity of meanings and propositions would involve describing a great many different sorts of criteria, operating at many different levels, and also criteria for distinguishing things at the same level. To sum up: there must be (I) tests for telling whether certain behaviour should be described as linguistic, (II) tests for identifying and comparing various kinds of linguistic activity (statement making, questioning,

commanding, etc.), (III) tests for identifying various kinds of conceptual schemes, (IV) tests for identifying various kinds of logical and grammatical functions of words and constructions, and (V) tests for identifying and distinguishing (if the language is like English) the particular material objects or properties or relations to which individual words may refer.

Our knowledge of how to apply all these tests is presupposed not only when we talk about words and their meanings, but also when we use words, in thinking or talking. For we cannot use words without knowing what we mean, and this involves knowing, for example, what it would be like to mean the same or something different at another time, or to be understood or misunderstood by other persons. This requires some knowledge of how to apply criteria of identity. Moreover, the existence of criteria at all the levels described above is presupposed, even when we use familiar "low-level" words, as pointed out at the end of 2.B.3. (All this is very much oversimplified. Some qualifications are made in a note at the end of the section.)

2.B.9. To give a detailed account of the criteria required for identifying meanings at all these levels would be a very complicated and lengthy task. I shall take most of the answers for granted, concentrating explicitly only on those aspects of meaning which are directly relevant to my main problem, the problem of clarifying and justifying the assertion of the existence of synthetic necessary truths. Thus, several restrictions will be imposed on the discussion.

I shall not, for example, try to say how we recognize linguistic behaviour as such.

Neither shall I try to describe the differences between statements, commands, questions, exclamations and other kinds of linguistic utterances, but will concentrate only on statements, with the further restriction to statements containing only logical constants and descriptive expressions referring to properties or relations. (Cf. 1.C.2, above.) This eliminates the need to discuss aspects of meaning which are not concerned with the conditions in which statements are true or false. For example, we need not discuss the conditions in which it is appropriate to say "I advise you to leave home" rather than "Please leave home", or "If you leave home you will be happy". I use the notion of an appropriateness-condition to cover a wide variety of cases, including the conditions in which it is appropriate to say "Ouch!" or "Alas!" or "Why?", or the conditions in which it is appropriate to use statement-forms of sentence rather than question-forms, and so on. The identification of appropriateness-conditions presupposes not only the identification of conceptual schemes and logical systems (see III, and IV, above) but also the identification of certain kinds of social institutions and "forms of life". (Note, for example, how the use of expressions such as "I advise ..", "You may ...", "Please ...", "You ought ...", etc., presupposes the existence of ways of life. Words whose use presupposes the existence of social institutions and patterns of social behaviour may, of course, be relevant to determining the truth-conditions of statements in which they occur. ✓ The statement that someone made a promise, or gave advice, or asked a question, may be true or false.) The rules determining appropriateness-conditions for the utterance of various forms of expression (e.g., questions

or commands) may generate some so-called "pragmatic" implications, such as the "implication" from "P is the case", or "I assert that P is the case", to "I believe that P is the case". I shall not go into this sort of question, but it might be relevant when attempts are made to generalize my account of the analytic-synthetic distinction. (See chapter six).

There is far more to meanings of words and statements and other utterances than can be taken account of by considering truth-conditions (see 5.A.11, for example), but these other sorts of meanings can be ignored in a discussion of analyticity or necessity, for this is a matter of ways of being true or false.

In the next two chapters, three and four, I shall discuss ways of identifying these aspects of the meanings of descriptive words and expressions which help to determine truth-conditions of statements in which they occur. Then I shall proceed to discuss the ways in which logical words and constructions determine truth-conditions. (Nothing will be said about proper names and other expressions referring to particulars, for reasons given in appendix I.) All this will prepare the way for a discussion of statements which are true in all conditions.

First, however, I shall try, in the remaining sections of this chapter, to explain why we have to take account of the existence of universals (observable properties and relations) and then to explain what their existence amounts to.

Note on section 2.B

In this section I have made many oversimplifying assumptions, and now I should like to suggest a few qualifications to my remarks. I asserted that in order to

understand talk about meanings of words, or even in order to be able to use words with definite meanings, we presuppose a large amount of general knowledge about language, and, in particular, rely on the existence of criteria for identity at various levels. It must not be thought, however, that all this knowledge is explicit, that we should be able to formulate it or describe the criteria for identity which we presuppose. There is much that we can do without being able to say how we do it. (See Appendix III on "Implicit Knowledge".)

Secondly, it should not be assumed that all the criteria for identity to which I have referred are commonly applied, even when we are explicitly talking about meanings. We take a great deal for granted in our dealings with other persons (and ourselves). If I want to teach someone how to ask a question in French, I may simply assume that he knows what questions are, and say: "This form of words and symbols is used for asking questions". Similarly, when I am not sure whether someone is asking a question or making a statement, I do not apply direct criteria, usually, but simply assume that he knows how to apply them, and ask: "Are you asking me or telling me?" In fact, we hardly ever apply criteria for identity at the higher levels, since the things we take for granted do not often lead us into trouble, owing to great regularities in human behaviour: we cross our bridges only when we come to them, and we don't often come to them. Nevertheless, it seems that it makes sense to talk about meanings only because it is possible to test our assumptions by applying criteria, even at the highest levels. (But they are not infallible. See 2.A.5.)

Finally, many of my remarks must be modified in

order to apply to a person whose linguistic training is incomplete, owing either to an unfortunate environment and bad teaching, or to his own constitutional inability to pick up concepts, or his young age. Children who cannot yet form sentences and make statements on their own initiative may be able to respond with correct answers to questions like "Is this red?" or "Is this round?", saying "Yes" or "No". But nothing very definite is likely to come out if we apply tests to find out exactly how they understand the question (e.g. in the sense of "Is this object red?" or "Is redness here?"). Their conceptual schemes may be still too underdeveloped. The process of development and elimination of indefiniteness continues even in later life. (See 4.B.4, below.)

2.C. Universals and strict criteria

2.C.1. I have stated that all mystery can be removed from talk about meanings and propositions by making criteria for identity of meanings explicit. (In 2.A.) I went on to describe some general presuppositions of talk about meanings, showing how criteria for identity had to apply at several different levels. In particular, the identification of meanings of individual words presupposes the identification of some conceptual scheme. In this section, taking for granted the existence of a conceptual scheme in which words may refer to universals or particulars (cf. 2.B.6, above), I shall try to show how observable properties and relations (i.e., universals) can provide sharp criteria for identity of meanings of descriptive words, at least from the point of view of a person who uses such words. Unless we use sharp criteria for identity of meanings we are likely to find ourselves confusing issues and begging questions when we try to

apply the analytic-synthetic distinction. This will be illustrated with the aid of some controversial examples, about which more will be said in chapter seven. These and other examples help to demonstrate that criteria for identity of meaning which are normally employed are too loose for our purposes. (It may be recalled that there are no "correct" sets of criteria: their adequacy depends on the purposes for which they are chosen. (2.A.5)).

Many complications in our ordinary language will be ignored, at present, attempts being made in chapters three and four to remedy this deficiency.

2.C.2. What are we to make of the statement that two descriptive words mean the same, or are taken to have the same meaning, by some person or group of persons? How are we to answer questions about synonymy? Philosophers are sometimes inclined to deal with such questions not by looking to see how we in fact decide whether to say that two words or two sentences mean the same or not, but by proposing neat tests, like "substitutability salva veritate". In search of a slogan they ignore our everyday practice. When they have found that their slogans will not work, they give up, demanding that the notion of synonymy be rejected, or they turn to nominalism, or some such thing. Of course we do not and need not decide whether we mean the same by two words by substituting one for another in all possible sentences and seeing whether the truth-value of the statement expressed is changed by the substitution. How could we possibly apply such a test? How could we know that the truth-value would or would not be changed unless we knew whether the words had the same meanings?

Surely in order to settle questions about the meanings

of words we cannot merely talk about relations between words and sentences, without ever mentioning the things to which those words refer? I shall try to show that talking about the properties to which words are understood to refer may help to explain talk about synonymy of descriptive words.

2.0.3. Let us look at some ordinary ways of eliminating ambiguities. If it is possible for a sequence of sounds or marks produced by some person to be taken as either a sentence in English or a sentence in French, then we can find out which of two possible meanings he intends his utterance to have by asking which of the two languages he was intending to speak. But this may still leave some questions unanswered, for ambiguities may persist within a language.

In some cases, remaining ambiguities may be eliminated by simple re-interpretation, by saying in other less ambiguous words precisely what was meant. Thus we may ask: "When you said 'I saw three tadpoles yesterday' did you mean you saw three of the army's new amphibious craft, or were you talking about animals which are the larvae of frogs?" Alternatively, it may be possible to eliminate the ambiguity by pointing to objects which the words are supposed to describe. For example, by pointing to frog-larvae and saying "I was talking about those things", one may enable others to identify one's meaning. But this eliminates only flagrant ambiguities. There may be remaining ambiguities which are more subtle, as is brought out by the question: "Did you intend the word 'tadpole' to mean 'animals with this shape colour and habitat', or did you intend it to mean 'animals which are the larvae of frogs', or did you mean the conjunction of the two?"

In most cases, there will be no definite answer to this question, and for normal purposes it does not matter (for reasons which will be explained below, and in chapter four), but it may matter for our purposes, for example if we want to know whether the statement "All tadpoles are frog-larvae" is analytic or synthetic.

2.C.4. Normally the difference between "I saw three frog-larvae" and "I saw three of the army's new amphibious craft" is important because it is very likely that when one is true the other is false. Similarly, the difference between the corresponding two senses of the word "tadpole" will be important because it makes a difference to whether a particular object is correctly described by that word or not. But the difference between "I saw three frog-larvae" and "I saw three animals with the shape colour and habitat of tadpoles" does not matter for ordinary purposes since it is true (or let us so assume for the sake of illustration), and generally believed, that whenever one of these statements would be true the other would be true too. The assertion of either would enable the hearer to know what had been seen by the speaker. So, to report "I saw three tadpoles" as "He said he saw three frog-larvae" is to report accurately enough for normal purposes, and it would be equally accurate to say "He said he saw three animals of such and such an appearance and habitat." In other words, we are often quite content to use extensional criteria for identity of meanings of descriptive words: where it can be taken for granted and is true that the same objects are correctly describable by two words, then, for many normal purposes, those words are synonymous. There is often no point in distinguishing their meanings, or in objecting to reports such as the

first one above, by saying: "You are misrepresenting me, as I merely intended to say I saw three animals with a certain appearance, without implying anything about their parentage". Such an objection would often provoke complete bafflement.

This helps to explain why Malcolm wrote (in Mind, 1940, p.339, et.seq.) that if two persons would take the same states of affairs as verifying what they take to be expressed by certain sentences, then we should say that they understand the same thing by those sentences, that they take them to express the same proposition. It also gives some support for Frege's decision to take identity in extension as a criterion for identity of concepts: "... coincidence in extension is a necessary and sufficient criterion for the occurrence between concepts of the relation corresponding to identity between objects." (See "Translations", p.80.)

2.0.5. Despite all this, identity in extension is not a universally acceptable criterion for identity of meanings. For it is possible that there might be an object which had the appearance and habitat of a tadpole which was not in fact the larva of a frog, and such an object, if it existed, would be describable by the word "tadpole" in one of the two senses explained above, but not in the other. So although there would not normally be a point in making the objection mentioned above to the report of the statement "I saw three tadpoles", there would be an objection if the speaker thought that this possibility should be taken seriously, and did not want to assume that animals were frog-larvae just because they had certain recognizable features. So the mere possibility of a state of affairs

in which with one sense a sentence would express a falsehood while with the other sense it expressed a truth can count against the identification of the two senses, for some purposes, despite the fact that the possibility is not actually realized. We acknowledge this when we ask the question "What would you have said if such and such had been the case?" in order to discover exactly what a person means by some word or sentence. So extensional criteria of identity may be too loose to pick out relatively subtle ambiguities of a kind which do not matter for ordinary purposes, but might matter. We apply a sharper criterion when we talk about possibilities.

2.C.6. But how do we apply the sharper criterion? How do we tell that there might have been an animal with the characteristic appearance of a tadpole which was not the larva of a frog? How do we perform the activity of considering possibilities? The only possible states of affairs which we can observe and examine in order to find out which statements would be true and which false if they were actual states of affairs are those which are actual states of affairs. We cannot perceive the set of all possible worlds, we can perceive only the actual one. So when we decide that two meanings of a word or sentence should be distinguished on account of the possibility of its making a difference to whether the word correctly describes something or whether the statement is true or false, possibilities cannot be the fundamental explanation of our decision. I shall argue in chapter seven that it is by paying attention to observable properties that we are able to think and talk about unactualized possibilities. Although in fact the extension of the word "W" is the same whether it has the

meaning M_1 , or the meaning M_2 we can tell nevertheless that it is possible for "W" correctly to describe an object when it has one meaning while it does not describe it when it has the other meaning, because the word refers to different properties when it has these different meanings, and the properties may "come apart". We can examine the characteristic appearance of a tadpole to see if there is anything involved in the possession of this property (i.e., the appearance) which necessitates being the off-spring of frogs, and find that there is not: the properties may come apart, an animal could have one property without the other. So we are able to distinguish two (or more) possible meanings of "tadpole", even though there would usually be no point in distinguishing them, for normal non-philosophical purposes.

2.C.7. It might be thought that we could avoid talking about the properties to which descriptive words are intended to refer, if we concentrated our attention instead on the process in which people learn to use those words. Then we might discover whether two persons meant the same by "tadpole" or not, by finding out whether they had learnt its use in the same way. But this is open to two objections.

First of all, it would be a difficult test to apply, since the process in which we learn to speak is very gradual and extended, and subject to an enormous amount of possible variation. This means that it would not be easy to say what counted as "learning in the same way", or to discover whether two persons had learnt some word in the same way. Secondly, even if two persons are given exactly the same instructions, they may understand them differently. Two children may both be shown tadpoles and told that they

are produced by frogs and will, if they survive, themselves grow into frogs. One may take this as merely an additional fact about those animals which are describable as "tadpoles", while the other regards it as a necessary condition for being correctly so describable. The latter regards being a frog-larva as part of what is meant by being a tadpole, the former does not. The facts being what they are, this difference in the way they understand may never happen to come out (though it could do so).

So even a careful examination of the teaching process may fail to reveal subtle ambiguities, unless we take into account what goes on in the pupil's mind, and this means asking which properties he takes the word to refer to, or which properties were drawn to his attention by the teaching process. ✓

2.3.8. But there is another more important reason why we must look to properties rather than mere possibilities for a criterion for identity of meanings of descriptive words. Suppose the word "tetrahedral" to mean "solid figure bounded by planes and having four vertices", while the word "tetralateral" means "bounded by four plane surfaces". Since it is impossible for a plane-sided object to have four vertices without being a plane-sided object with four faces and vice versa, it is impossible for either of the following statements to be true unless the other is (in the same circumstances): namely (1) "The paperweight on the table is tetrahedral" and (2) "The paperweight on the table is tetralateral".

Despite this equivalence, it seems intelligible to say that they are different statements, that they have different meanings, since the words "tetrahedral" and

"tetralateral" refer to different properties. (In the terminology of chapter three, their meanings are "synthesized" differently.) It is possible to notice, ~~to~~ attend to, ~~we~~ have in mind, ~~we~~ think about, or ~~we~~ recognize one of these properties without being aware of the existence of the other - so why can one not intend a word to refer to one of them and not the other? After all, if to understand what a person means by describing something as "tetrahedral" is to know what it would be like to do so for the same reason as he does (Cf. 1.B.6 and 3.C.5), then we shall not have understood if he intends the word to refer to the property of having four vertices, etc., and we think it refers to the other property, despite the impossibility of there being any object which is correctly described by the word in one sense and not in the other.

This brings out another reason why the teaching process cannot serve infallibly as a criterion for identity of meaning, for once again the same method of teaching may give two different pupils two different concepts, if one of them happens to notice one aspect of the illustrative examples, while the other notices another aspect. And here it is even less likely that the ambiguity will be detected than in the case of "tadpole" - though it is also less likely that it will matter, for most normal purposes.

This example shows also that yet another criterion for identity of statements will not always do, though it is sometimes appealed to, namely that two sentences express the same statement if the statement made by each of them entails the statement made by the other, or if each entails and is entailed by the same statements as the other. This is too loose because it would fail to distinguish the statements (1) and (2) above, since, owing to the

impossibility that either of them is false while the other is true, each entails the other.

2.C.9. Now it may be objected that there is a good reason for saying that the words "tetrahedral" and "tetralateral" refer to one and the same property, since they are analytically equivalent, in a sense to be explained below. This may be so, but it should not be taken as obvious that there is no point in distinguishing the meanings of the words, or the properties to which they refer, especially when we are discussing the question whether there are any necessary connections which are not analytic. If we wish to take this question seriously, we must be prepared to distinguish meanings where for most other purposes meanings do not need to be distinguished, and this means looking for the sharpest possible criteria for identity of meanings. (We must use Kant's engraver's needle. Cf. 2.A.5.) For otherwise we shall be in danger of begging questions.

If we do not distinguish meanings as finely as possible, we may fail to separate two propositions, or two concepts, thinking there is only one. Hence we may fail to notice the relation between these propositions, or concepts, and may simply overlook a possible candidate for the title of "synthetic necessary connection", or we may fail to distinguish some necessarily true proposition which is analytic from one which is synthetic, if there are such things. In this way we settle the question at issue merely by selecting such criteria for identity of meanings as ensure that analytic connections between meanings cannot be distinguished from necessary ones and that no proposition can be necessarily true without

being identical with an analytic one. In order to avoid this question-begging procedure we must look for the sharpest possible criteria. Only then can we hope to understand what people mean, or think they mean, when they assert that there are synthetic necessary truths. It seems very likely that the apparently irresolvable disagreement amongst philosophers on this topic can be traced to a failure to make explicit the sets of criteria for identity which are implicitly used, so that one lot uses sharp criteria while the others use looser ones, and arguments proceed at cross purposes, without any hope of agreement. I do not wish to imply that any one set of criteria is correct. (See section 2.A.) It all depends on the purposes for which they are chosen. But for the purposes of this discussion we must, for the time being, use what seem to be the sharpest criteria for identity of meanings, and say that "tetrahedral" and "tetralateral" have different meanings, for the reasons given in 2.C.8.

2.C.10. We shall therefore reject as superficial and question-begging, arguments such as the following: "It is analytic that every figure bounded by three straight lines has exactly three vertices, for the concept of a triangle can be defined either in terms of being bounded by three sides, or in terms of having three angles. So the statement 'Every figure bounded by three straight sides has exactly three vertices' is the same as the statement 'Every triangle has exactly three vertices', which is analytic, by definition of 'triangle'. Q.E.D." (Cf. 3.C.10.)

I have put the argument in a very crude form, but

(see 7.C.6.)

it can appear in more subtle guise. [^] The essential thing to note in it is the use of the phrase "can be defined ..." with the implication that one and the same word can be given its meaning in more than one way. This is not uncommon usage in mathematics (see, for example, how many different "definitions" mathematicians may use of the property of being a conic section), but it presupposes the use of relatively loose criteria for identity of meanings, too loose for our purposes. For we are not concerned with whether words can be so defined as to make certain statements analytic, but whether they have to be. We wish to ask whether they can be distinguished, as having different meanings, and yet be necessarily connected.

2.C.11. All this should be recalled if it appears that some of the techniques employed in later chapters for describing and distinguishing meanings are too nice and artificial. We cannot avoid them, if we are to use the sharpest possible criteria for identity of meanings of descriptive words.

We have seen that various more or less familiar tests for identity of meaning are not sufficiently stringent, for our purposes, such as comparing extensions, comparing methods of instruction, and so on. I have suggested that stricter tests are possible if we ask always to which properties descriptive words are taken to refer. We shall see, in chapter four, that there are sometimes no answers to such questions: our ordinary statements are not made with sufficiently definite meanings. This will have the consequence that it is not possible to apply the analytic-synthetic distinction to all ordinary

statements, but that need not trouble us, so long as it can be applied in some cases, at least in principle.

It may be objected that talk about properties and other universals does not help, because they cannot be used to explain anything, since they do not exist as "complete and independent entities" (see Price, "Thinking and Experience"), and in any case their existence depends on the existence of words with meanings, so they cannot be used to identify and distinguish meanings, without circularity. I shall try to answer this in the next section, by showing how, at least from one point of view (cf. 1.B.2), the existence of universals is independent of language, and so may be used to explain our use of descriptive words. The sense in which it explains will become clearer in chapter three.

2.D. The independence of universals

2.D.1. Meanings, concepts and propositions are not things which exist in their own right as objects of experience, for we cannot see them, hear them, feel them, or in any other way perceive their existence. We might say that talk about meanings, concepts or propositions is a sort of circumlocution for talk about entities of other kinds, such as particulars referred to by proper names, or properties, qualities or relations referred to by descriptive words, the types of situations and verbal contexts in which the use of a word is appropriate, the purposes with which sentences are uttered, and so on. In short: talk about meanings, etc., is a circumlocution for talk about the things which we examine in applying criteria for identity of meanings.

On the other hand, universals, that is observable properties and relations, do exist in their own right as objects of experience, for they can be seen, attended to, thought about, imagined or referred to, without the mediation of any other kinds of things out of which they have to be "logically constructed". That is why they can be said to explain our use of descriptive words. It is only because observable properties and relations exist that we can use descriptive words as we do, just as it is only because their referents exist that we can use proper names and other definite referring expressions as we do.

Not only the use of referring expressions, but also the use of descriptive words and expressions requires that conditions of existence and identifiability be satisfied. The mere fact that a word occurs in a predicate-position in an utterance does not guarantee that it has a meaning, that it predicates successfully. What more is required? In some cases at least, what is required is that there should be some property or combination of properties to which it refers. So, if universals are the things referred to by such words and expressions, then their existence cannot be reduced to the existence of words in a language, since their existence is presupposed by the use of those words. This will now be amplified.

2.D.2. Descriptive words may occur in sentences expressing statements, questions, commands, etc., and knowing their meanings involves knowing how they contribute to the meanings of these sentences. (See section 2.B.)

A descriptive word contributes to the meaning of a statement, by helping to determine the conditions in which that statement would be true. The conditions in which

the statement would be true depend on the conditions in which particular objects are correctly describable by the word in question. So knowing in general how to tell whether statements including the word "P" are true or false requires the ability to tell whether simple statements of the form "That is a P" or "That is P" are true, for example, "That is a cube" or "That is red". So understanding a descriptive word involves knowing how to tell which objects fall within its extension and which do not. How does one tell?

In general the answer is very complicated, as will be seen in the next two chapters, but in the simplest cases one tells whether an object is correctly describable by a word "P" by looking to see whether that object has a property which one has learnt to correlate with "P", or not. (Here we have a sense in which the meaning of a word can determine an application "in advance".) In many cases the examination is visual, but it needn't be: consider how we tell whether an object is describable as "sticky" or "cool".

I shall try to show that what determines the describability of particular objects by descriptive words and expressions may, at least in some cases, be correlations between words and recognizable properties. I call these semantic correlations, because they correlate words with non-linguistic entities. They may also be described as "semantic rules" since, in virtue of them, some descriptions are correct and some are incorrect. They are to be distinguished from what I shall describe as "purely verbal rules", which merely correlate words with other words. By providing us with "assertion licences", that is, by fixing the conditions in which assertions may be made truthfully, semantic correlations help to ensure that

concepts have boundaries, or rather that the extensions of concepts have boundaries, for they enable us to decide whether objects fall under those concepts or not. (I have been talking about observable properties, but similar remarks could be made about relations.)

2.D.3. There must be these correlations, or something similar, if words are to have meanings, if concepts are to have boundaries, if there is to be a difference between describing something correctly and describing it incorrectly. It may look as if we do not need anything more than correlations between words and other words, such as definitions of words in terms of other words, since we can sometimes teach the meaning of a word by giving a definition, or discover whether two persons mean the same by their words by asking them for definitions. But this assumes that the meanings of the words used in those definitions have been taught and understood. The process of defining words in terms of other words must start somewhere, with the setting up of correlations between words and other things. Words alone will not do. For example, to say that two words are to be incompatible descriptions, or synonyms, does not even begin to tell us which things are and which things are not describable by either of them unless the meaning of the other is already known. It seems that Hampshire overlooked this when he wrote: "In all cases, clarifying the use of a descriptive word or phrase is a process of drawing attention to its established links with other descriptions." (In Philosophy 1950, p.243.)

Correlations between words and other words may, of course, enable us to decide that some statements are necessarily true and that others are necessarily false, and

they may license us to make inferences, or to substitute one expression for another in a true statement.

(Cf. 4.C.3.) But they cannot, of their own accord, enable us to decide in which circumstances contingent statements are true or false. They may enable us to assert "Nothing is both round and square", but they cannot tell us when we may say things like "There is a square piece of paper on my typewriter". For such "assertion licences" we require not just rules relating words and words, but semantic rules correlating words and non-linguistic entities, such as properties. We need something more than mere verbal rules, and when we have the "something more", it may turn out that we do not need correlations between words and words as well: Hampshire's "links between descriptive expressions" may turn out to be superfluous.¹ (Cf. 2.D.4 & 3.B.4.c.)

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1. I said that correlations between words and words may tell us which statements are necessarily true or necessarily false, though they cannot yield assertion-licences for contingent statements. But this needs qualification. For if there are only verbal rules, and no rules correlating words with non-linguistic entities, how can a statement be about anything? And if it is not about anything, how can it be a statement? How can it be true, or false? The mere fact that someone is uttering sounds according to rules which permit some sounds and not others does not guarantee that he is using a language, or that he is saying anything which is true or false, or which conveys any information. This shows that when philosophers talk about truth in connection with formal systems, or when they describe them as "languages", they are just muddled and talking well-disguised nonsense. They are muddled, because, although formulae in such a system may represent certain aspects of statements for purposes of classification, they are not themselves statements, and cannot be true or false, for the symbols of a formal system cannot be used to make contingent statements about anything. (Some further remarks are made on this topic in Appendix II, and in section 5.A.)

2.D.4. In "Analytic/Synthetic I" (Analysis, Dec. 1949), Waismann pointed out that certain kinds of linguistic rules, namely explicit definitions, or what I call "verbal rules", serve as "substitution licences", which enable us to make inferences from one proposition to another (see pp. 33 and 37). Later on (in "Analytic/Synthetic II", p. 31) he remarked that ostensive definitions do not serve as substitution licences. We may now point out that ostensive definitions are procedures for teaching the semantic rules which govern the use of descriptive words, and therefore, although they do not directly set up substitution licences, they do serve as assertion licences: that is, they help to determine whether contingent statements are true or false. But we may add also that, in some cases, when words have been correlated with properties it may turn out that two words or expressions are thereby rendered synonymous. For example, in virtue of the semantic correlations it may turn out that "gleen" is synonymous with "glossy and green". In that case, even semantic rules, as taught ostensively, may, indirectly, provide us with substitution licences, or inference licences. (This is one of the ways in which Hampshire's "links between descriptive expressions", mentioned above, may turn out to be superfluous, once semantic correlations have been taken into account.)

2.D.5. All this may seem obvious and trivial. But it is not yet quite clear what my assertion that universals exist in their own right and can explain our use of descriptive words, comes to. The best way for me to clarify this further is to say where I disagree with other philosophers.

First there are those who say that the existence of properties is merely a matter of the existence of their instances. (It would be foolish to deny outright that there are properties, for we are all aware that there are shapes, colours, sizes, textures, kinds of feel, and so on.) This seems to me to be quite wrong, for, as remarked above (2.B.6.) properties are not essentially tied to their actual instances. For example, I can assert that a certain complicated shape is not the shape of my table, and in doing so I presuppose that there is such a shape, but I certainly do not presuppose that anything ever did have or will have that shape, since I know quite well which shape I mean without thinking about any particular object or objects with that shape. The existence of a property does not imply the existence of actual instances, only that it is possible for instances to exist. There may be many complicated shapes which never have been or will be instantiated, but they nevertheless exist. Perhaps there are colours which would be instantiated for the first time, if only someone would put the correct combination of chemicals together. Perhaps someone with a sufficiently good imagination can "think up" a colour which has hitherto not been instantiated, and even decide to associate a word with it.

Anyone who disagrees with me and thinks that the existence of properties involves the existence of particular instances, will, of course, deny that correlation with properties can explain the use of descriptive words, since such a word can be used, as remarked above, without presupposing that it refers to a property which is instantiated. Thus confusion about the sense in which universals exist can lead to confusion as to whether they explain.

2.D.5. (Note). Strawson seems to think that when we assert that a universal exists all we mean is to imply that it has instances,

"... as when one says, for example, that saintliness exists, or that there is such a thing as saintliness, and means by this the same as we mean by saying that there exist, or that there are, saintly people ..." ("Individuals", p. 241.)

I do not think we often do mean this sort of thing, any more than when one denies that there is such a thing as saintliness he means simply to deny that there are have been or will be saintly people. (To say "There is no such thing as X-ness" would normally be to imply that there couldn't be anything which was an X.)

Strawson does, of course, allow that there is another sense in which existence may be ascribed to universals (p.239-241), but this is a purely formal sense, and seems to imply only that there are formal analogies between the word "saintliness" and other substantive words. I want to say that even the use of "red" in contexts like "My notebook is red", where it is not a substantive, presupposes the existence of a property, and does so in a "metaphysically charged" way (op. cit. p.239), since the existence of such a property is a fact about the world, which must usually be learnt through experience. It is a fact about the world because there might have been a world in which the property did not exist, a world in which nothing could be red. (It is clear from his remarks on pp. 183, 184, 185, 186, 193, 238, 239-241, that Strawson did not consider this way of looking at the existence of universals. Had he done so I cannot see how he could have tried to relate the grammatical subject-predicate distinction to the categorial particular-universal distinction, via a distinction between expressions

which do and expressions which do not presuppose facts about the world.)

2.D.6. This explains how I disagree with those who would reduce the existence of universals to the existence of their instances. Secondly, I disagree with philosophers who say that universals depend for their existence on language, or imply that we must know what a language is in order to know what universals are, or deny that universals can explain any aspects of our use of language. Usually, such philosophers really mean only to reject bad theories of universals, which oversimplify things. For example, they assume that universals can explain our use of words only if there is exactly one universal for every descriptive word, such as a single property common to all the objects describable by that word. But this "one-one" model leads people into grave difficulties, when they try to find some one thing common to all the objects describable by a word with a complicated meaning. [Their failure to find a common property leads them to say, for example, that universals are intangible to the senses, being apprehended only in thought (Cf. Lazerowitz in Mind 1946, p.1.), or that universals are "partial realizations of the specific forms, existing only as the thought of them" (Cf. Blanshard, "The Nature of Thought", Vol.I, p.609). Or they may be led to say that universals cannot be sensed because things sensed are many and different where there is only one universal. (Cf. Austin, in P.A.S.Supp, 1939, p.85).]

However, when the intolerable implications of the "one-one" model lead philosophers to give up universals

altogether, they give up too much, for then they are left only with words and no way of explaining how words describe or have meanings. By taking into account complexities in our use of descriptive words, as in the next two chapters, we can preserve a theory of universals while rejecting the one-one model. (If we insist on looking for one property corresponding to every descriptive word, then some must be "improper" properties, constructed or synthesized out of other "proper" properties which are tangible to the senses. (Cf. 3.B.5.))

2.D.7. A stubborn insistence on the one-one model is not the only thing which accounts for the refusal to acknowledge the existence of universals as independent entities. Another is the fact that which properties a person sees in the world, and the way in which he classifies things as having something in common, may depend to some extent on the society in which he has been reared, and the classifications made in its language. (See, for example, Waismann, in "Verifiability", p.137-9, etc.) This may provide good reason for saying that the existence of a property is a fact about people not a fact about the world, but only if we take up a different point of view from the one adopted in this essay. (See chapter one, section 1.B.) For from the point of view of a person who can see properties, they certainly exist in their own right, as things which he can perceive, attend to, recognize, bear in mind, etc., without having to think about people or language. What is more, from his point of view, or the point of view of a person who understands him, the existence of these properties which he can see explains how he classifies things and what he says about objects in his environment (as will be shown

in more detail in section 3.C.) This, however, leaves open the possibility of explaining his behaviour from some other point of view, such as a psychological or anthropological point of view, a possibility with which we are not concerned, since we are looking for rational explanations, not causal ones.

2.D.8. From the point of view of one who talks and can see properties, therefore, we must disagree with the attitudes underlying remarks made by philosophers to the effect that universals are somehow generated by language, or that their existence is to be explained by talking about language. Here are some examples:

- (i) "To say that a property exists is to say that a general word has been or could be introduced to characterize the things which possess it."
(Quinton, in "Properties and Classes", P.A.S., 1957-8.)
- (ii) "Saying what meaning a symbol has involves describing the relevant experiences, and this brings us back into the realm of symbols."
(Ayer, in "Thinking and Meaning", p.27.)
- (iii) "And in the end the kind of similarity which is meant can be specified only by a backward reference to the name". (Pears, in LL.II, p.56-7.)
- (iv) "It is fatally easy to talk carelessly about things in a way which suggests that they stand out there already labelled in a way which indicates their properties One refers airily to THE TWO classes as if one could say WHICH TWO classes without using the words." (Pears, LL.II, p.116.)
- (v) "The sense of a predicate expression (e.g. 'is a rose') generates its referent ('rosehood') if there is one. It could not fail to refer."
(J. R. Searle, D. Phil, thesis, p.208.)

- (vi) "The notion that an entity stands to a predicate as an object stands to a singular referring expression must be finally abandoned." (Searle, p.188.) "Universals ... do not lie in the world" (p.192) and "... propositions asserting their existence are tautologies" (p.191). (See also Strawson's "Individuals"; p.184: "But now we no longer have an empirical proposition, a fact about the world. We have a tautology It is a fact about language.")

Concerning (ii), (iii) and (iv), consider the following question: When I say what person or thing a proper name refers to, by using words, does this bring us back to the "realm of symbols"? Is this a "backward reference" to a name?

Concerning (v) and (vi), recall the muddles, e.g., about "substance" into which philosophers have been led by thinking that the existence of proper names and other referring expressions somehow suffices to guarantee the existence of the particulars referred to. (Cf. Wittgenstein, "Tractatus", 2.02 to 2.03, especially 2.0211 and 2.024.)

2.D.9. I have so far been labouring the point that if our words and sentences are to have any meaning, to be capable of being used to make statements which can be true or false, then there must be semantic rules correlating words with non-linguistic entities, and I have tried to show that universals, that is, observable properties and relations, are suitable non-linguistic entities. Not only are they non-linguistic, but their existence cannot be reduced to the existence of sets of objects which resemble one another, since a universal (such as a shape) can exist even though it has no instances.

This means that anyone who tries to explain our use of descriptive words such as "red", "round" or "smooth", in terms of actual particular objects or sets of particular objects whose existence is presupposed by the use of these words, has gone wrong somewhere. (Cf. A. Pap, in S.N.T., chapters 9 and 13. See also Körner, in "Conceptual Thinking", where he talks about sets of "exemplars".) Properties are not essentially tied to actual instances, and can be thought about or referred to independently of their instances. (See circa 3.C.4 & 3.E.5.)

It may be objected that it is still not clear in what sense the existence of universals can explain our use of descriptive words. It explains because, by memorizing the properties (etc.) correlated with descriptive words we can learn to distinguish states of affairs in which statements using those words are true from those in which they are false. This will be made clearer in the next chapter, which describes some of the ways in which we may correlate descriptive words with observable properties. In addition, it should show in detail how we can make sharp distinctions between the meanings of words by taking into account the properties to which they refer, as required by the programme adopted in the previous section. (See also the motto at the end of 2.A.5.) In order to avoid digressions into philosophical psychology, I shall not deal with questions as to how we can tell which properties or things a particular person takes or intends his words to refer to, but will assume, for the time being, that we can. At any rate, each of us knows what he means his words to refer to, in most cases. This omission means that, from a certain point of view, my account of how to apply sharp criteria for identity of meanings is essentially incomplete.

Chapter Three

SEMANTIC RULES

Introduction

Chapter two contained an argument to show that in order to avoid begging questions we must look for the sharpest possible criteria for identity of meanings, and it was suggested that only by taking note of the universals (i.e. observable properties and relations) to which words are intended to refer could we find sufficiently sharp criteria. (See 2.C.) The way had been prepared for this in section 2.B., where it was shown how conceptual schemes were important in connection with identification of meanings, and how our own conceptual scheme had provision for a distinction between material objects and the universals which they instantiate. Section 2.D contained arguments to show that talk about universals can explain since their existence is a fact about the world, independent of the existence of instances or of our use of language. In this chapter an attempt will be made to show in more detail how properties may be used to give descriptive words their meanings, and how we may compare and distinguish meanings by examining the ways in which words refer to properties. This will provide many interesting examples to which the analytic-synthetic distinction may be applied later on.

The programme for the chapter will be roughly as follows. First of all the simplest type of correlation between words and properties will be discussed, and then it will be shown how more complicated correlations are possible, firstly by means of logical syntheses of concepts and secondly by means of non-logical syntheses.

This will help to justify my claim that universals explain our use of descriptive words.

There will be many oversimplifications in this chapter, since it ignores the fact that words are ordinarily used with relatively indefinite meanings, but it is hoped that this will be compensated for by the discussion in chapter four. In addition, this chapter will be concerned only to show how we decide whether or not a particular object is describable by some word. In order to explain how descriptive words can contribute to the meanings of whole sentences, we must wait for the discussion of logical words and constructions in chapter five.

Finally, notice that although the discussion is restricted to words which refer to properties, nevertheless similar remarks could be made about words referring to observable relations.

3.A. F-words

3.A.1. The simplest sort of semantic rule, though by no means the only sort, is one which correlates a descriptive word with only one property, which must be possessed by objects correctly describable by that word. I describe this sort of word as an "f-word" (or feature-word), and shall say that it is governed by an f-rule. Such words describe objects in virtue of something which they have in common, some respect in which they are all alike. If, for example, the word "scarlet" refers to a specific shade of colour, then we may say that it is an f-word, and all the things which it describes, since they have exactly the same shade of colour, are alike in some respect.

The word "red", as used by normal persons, also refers to one property, not a shade, but a hue, which may be common to objects of different shades. When we look at the white light spectrum (or a rainbow), we see a continuous range of continuously varying shades of colour. Yet despite this continuity, the spectrum is divided into fairly definite bands, each containing a range of specific shades which are different from one another, yet have something in common. All the shades in the red band, for example, have something in common which they do not share with shades in the orange band, or the yellow band, despite the possibility that shades of red and shades of orange may resemble one another closely, if they are near the red-orange boundary.

Hampshire wrote, in "Thought and Action", on p.35: "there are a definite number of discriminable shades, to each one of which a definite name can be allotted". He must surely have meant hues rather than specific shades, for there seem to be indefinitely many different specific shades. Nevertheless his remarks illustrate what I mean by an f-word. I shall ignore for the time being, the fact that the boundaries between bands may be more or less indeterminate, and the fact that different persons may see their bands in different places. (Contrast what I have said with Wittgenstein's remarks, in the "Blue and Brown Books", p.133-5.)

3.A.2. Just as normal persons can learn to see the hue common to objects with different shades of red, and associate it with the word "red", so can most normal persons learn to perceive the property common to objects

which are all triangular, even though they have different specific triangular shapes. (Such persons may adopt an f-rule, correlating the word "triangular" with that common property. In addition, each of the many different specific triangular shapes may be memorized and correlated with a descriptive word by an f-rule. (E.g., the shape of an equilateral triangle, or a triangle whose sides meet at angles of 90° , 60° and 30° .)

It should be noticed that I am not talking about so-called "perfect" triangles. I am talking about shapes which we can all recognize and which a child can learn to distinguish long before it learns to prove geometrical theorems or talk about "perfectly" straight lines. We all know how to distinguish triangular pieces of cardboard, or diagrams, from round or square ones, for example. In chapter seven something will be said about "perfect" geometrical concepts and other idealized concepts, such as the concept of a perfectly specific shade of colour. But this chapter is not concerned with such things.

3.A.3. The examples "triangle" and "red", illustrate an ambiguity in talking about a word which is correlated with just one property. This does not mean that there may not be a whole range of different properties which correspond to the word. For example, there are very many different shades of red which may be possessed by red objects, and different triangular shapes which may be possessed by triangles. Nevertheless, in each case, if the word is an f-word, then there is only one property in virtue of which all those objects are correctly describable by it. (Cf. 3.C.5.)

Neither do I wish to rule out the possibility that there may be other less specific properties common to all the objects described by an f-word. For example, even if the word "triangular" refers to only one property, there are nevertheless several other properties common to all objects which it describes. For example, all are bounded by straight lines, may be inscribed in circles, and have no reflex angles. These properties may be possessed by other objects too, such as square or hexagonal objects. But there are other properties common only to triangles, such as the property of being rectilinear and having angles which add up to a straight line.

3.A.4. It may be objected that there is not just one feature or property associated with the word "triangular" since a definition can be given in terms of simpler notions. But anyone who talks about the possibility of analysing such a concept in terms of simpler ones, or about criteria for telling whether an object has the property or not, must at least admit that at some stage we simply have to recognize something, be it a criterion or one of the "simpler" properties. Then a word could be correlated with that "something" by means of an f-rule and would illustrate what I am talking about. However, since triangularity is a feature which most of us can perceive and take in at a glance, why not allow that the word "triangular" can be used as an f-word, if there are f-words at all? I do not wish to settle this here.

(One person may regard some property as simple or unanalysable, while another regards it as built out of simpler properties. Are there two properties, or only one? Cf. "tetrahedral" example in 2.C.8.)

3.A.5. F-words need not describe only continuously existing material objects. A sound which starts, lasts a few minutes, then stops is a particular, and may be described as a sort of physical object with physical properties. It can be located in time, and sometimes in space too. It may be a sound of a definite pitch, and this property may be shared with other sounds. Or it may have a definite timbre, such as the tone of a flute, or clarinet, or electronic organ, and share this property with other sounds quite different in pitch. It may be the sound of a major chord, and share this property with other sounds in different keys, or with different dynamic distributions (e.g. the tonic may be louder than the dominant in one, but not the other). Each of these properties common to different sounds can be memorized, associated with a descriptive f-word, and recognized again later on.

A sound may also change. If it changes in pitch, then the pattern of changes may be recognizable, and we can speak of a "tune", and other sounds may have the same tune. Some persons may be able to memorize the sound of a whole symphony, and associate that property with an f-word. Less fortunate beings can merely recognize parts of symphonies, or the styles in which they are written, such as Beethoven's style, or Hindemith's. These are properties of enduring objects or events, and have to be perceived during an interval of time. But they may all be correlated with descriptive f-words, by means of f-rules.

3.A.6. The important thing about all the examples is that they involve properties which can be perceived by

means of the senses, memorized, and recognized in new instances. A property which is not observable by means of the senses, such as the property of being magnetized, or of having a certain electrical resistivity, cannot be correlated thus with a descriptive word and provide a rational explanation of our use of the word. Words may, of course, refer to such "inferred" properties (e.g. "dispositional" properties), but not in the same way. (There may be some intermediate cases.)

3.A.7. These observable properties are the basic entities out of which the meanings of many kinds of descriptive words are constructed. I have so far described only the very simplest kind of descriptive word, governed by the very simplest kind of semantic rule, namely a rule which correlates one property with one word.

It is commonly denied that descriptive words correspond to single entities which are their meanings, or account for their having meanings (see, for example, remarks in 2.D.6 and 7, etc., to the effect that the "one-one" model will not do). Unfortunately, this denial is usually much too vague to be of use to anyone. By showing that there are other kinds of descriptive words than f-words, and why they fail to fit the "one-one" model, I shall be describing one clear sense in which the denial is justified, though relatively trivial. But it is important to distinguish the thesis that the one-one model is inadequate to account for most of our descriptive words from the thesis that descriptive words do not refer to properties or other universals which can explain their use. It is very easy to confuse these theses. (I think Wittgenstein's discussion of the notion of "following a rule" in "Philosophical Investigations"

was intended to support something like the latter thesis. I shall not explicitly argue against him, but my account can be construed as an attempt to show that an alternative picture can be coherently constructed.)

The time has now come to turn to more complicated types of semantic correlations.

3.B. Logical syntheses

3.B.1. Some one-one correlations between descriptive words and properties have been described, and now we must see how more complicated correlations are possible if new semantic correlations are constructed out of the simplest ones. Three methods of construction will be described in this section, namely disjunction, conjunction and negation. These correspond to the use of the logical connectives "or", "and" and "not" in explicit definitions. They may be thought of not only as propositional connectives, but also as meaning-functions, which take words as arguments and yield expressions whose meanings are simple functions of the meanings of the arguments. I shall simply assume that we understand these logical words, and will not try to explain how they work. (See chapter five.)

The construction of new semantic correlations of the sorts about to be described may be called a process of "logical synthesis". Later, we shall contrast it with processes of "non-logical synthesis".

3.B.2. D-words

The first sort of rule which does not fit the simple one-one model is a semantic rule which correlates a word with more than one property, disjunctively. I shall call

such a rule a d-rule, and the word it governs a d-word. For example, the word "ored" may be correlated with the two hues, red and orange, so that the word describes an object if and only if it has one or other of these two properties. If the words "red" and "orange" are f-words which refer to these two properties, then the word "ored" means the same as "red or orange".

A more interesting kind of disjunctive rule is one which correlates a word with a whole range of properties, such as a range of specific shades of colour. The word "red" may be used as a d-word of this sort, instead of as an f-word. For there may be persons who can see and discriminate and memorize specific shades of colour, though quite unable to see hues in the way in which most normal persons can, as described in 3.A.1, above. Such a person will see the spectrum as a single band of continuously varying shades of colour, much as we see one of the bands of the spectrum. This hue-blind (but not colour-blind) person will not see the spectrum divided up into different bands, so he cannot learn to use the word "red" in the normal way. If presented with pieces of coloured paper all of different shades, and instructed to arrange them in groups with a common feature, he will be unable to do so, even if there are several red pieces, several yellow pieces, and so on. To him they all simply look different. (They look different to normal persons too, but they also have respects of similarity, which is why we can group them.) Though unable to learn to use the word "red" in the normal way, such a hue-blind person may learn to use it as a d-word, by memorizing all the different shades in the spectrum which lie in the red band, and then describing an object as "red" if and only if it has one of the specific shades of colour which he

has learnt to associate with the word. Similarly, a person who is not hue-blind, but sees the spectrum divided up differently from the way we do (his "hues" are different because he sees bands in different places) may learn to use our word "red" as a d-word, by memorizing specific shades of colour. All we require of such persons is that they agree with normal persons as to whether objects are exactly the same shade of colour or not.

3.B.2.a. In the same way, there may be a person who is unable to see anything common to all those shapes which are triangular, although he can see and discriminate specific shapes and tell, for example, whether two objects are both equilaterally triangular, or not. Perhaps he is unable to count up to three - but the explanation of his inability to perceive triangularity need not concern us. Such a person cannot use the words "triangle", "quadrilateral", etc., as f-words, for he cannot see any common property with which they may be correlated. But if he can see and memorize specific triangular shapes, such as the shape of a right-angled isosceles triangle, and distinguish them from other specific shapes, such as the shape of a square or a regular pentagon, then he can memorize a whole range of specific triangular shapes and adopt a d-rule correlating them with the word "triangular". He then uses the word to describe objects if and only if they have one of the many shapes which he has memorized, as in the case of "ored" or the d-word "red". (As before, I am not talking about "perfect" mathematical shapes, but shapes which we can all learn to recognize and discriminate with greater and lesser degrees of accuracy.)

Of course, these examples are highly artificial, since there are indefinitely many different specific shades of red, and indefinitely many specific triangular shapes and nobody could memorize them all. But the essential point could as well be illustrated by a person who merely memorized very many different shades of red, or triangular shapes, enough to get by with in most ordinary circumstances. (Later, a procedure for picking out a whole range of properties without memorizing them all will be described.) Notice that a person who memorizes a set of properties and correlates them with a word need not have a name for each of them. His d-word need not, therefore, be definable in his vocabulary.

3.B.3. C-words

The next type of semantic rule is one which correlates a word with a combination of properties. This is a c-word, and refers to a set of properties conjunctively. For example, the word "gleen" might be defined so as to refer to the combination of the hue, green, and the surface-property, glossiness. It would then describe objects which possessed both of these properties, and would be synonymous with the expression "green and glossy". (As before, someone might learn to use a c-word to refer to a combination of properties without being taught names for those different properties. Then, in his vocabulary, the word would be indefinable, despite the possibility of defining it in a richer vocabulary.)

We may think of such c-rules, like d-rules, as being logically constructed out of f-rules, just as we can think of the meanings of c-words and d-words as logically constructed out of the meanings of f-words (although, of

course, the language in which they are used need not include the required f-words, for the reason just stated).

We need not restrict the notion to combinations of only two properties. A word might describe a sequence of sounds if and only if it possessed the three properties of being in the key of E-major, of being the sound of a piano, and of being in the style of Beethoven. This would then be a c-word referring conjunctively to three properties.

3.B.4. N-rules

Semantic correlations involving negation can be very confusing as there are several different ways in which negation may come in, and it is important to be clear about them.

I shall describe a strong n-rule as a rule which correlates a word "W" with a property P negatively, as follows: the word "W" does not describe an object if that object has the property P. In such a case, the possession of the property is a sufficient condition for not being describable by the word, and the absence of the property is a necessary condition for being describable. Whether it is also a sufficiently condition, will depend on the other rules, if any, with which the n-rule is conjoined. Thus, the expression "scalene-triangle" is correlated negatively with the property of symmetry, and requires the absence of that property in objects which it describes. But the absence of the property is not sufficient, for in addition the object must be triangular. Usually there are other rules and absence of the negatively correlated property is not sufficient to ensure describability.

3.B.4.a. These "strong" n-rules, specify inapplicability-conditions for words. They are to be distinguished from "weak" n-rules, which merely limit the applicability-conditions of words, thereby helping to make the meanings of indefinite words more definite. The difference may be illustrated by means of an example.

I have hitherto ignored doubts which may arise over the possession or non-possession of a property by an object, but it is sometimes difficult to decide whether an object possesses some property or not, where this is not an empirical difficulty arising out of the difficulty of seeing the object clearly or the difficulty of remembering what the property looked like. I may have plenty of red objects around to remind me of the hue associated with the word "red" (an f-word) and be able to see an object quite clearly in a good light, and yet be undecided as to whether it has the same hue as the other red objects or not. In this case I am undecided about the redness of the object, though I may be able to see its specific shade quite clearly and recognize it again in other objects. We may say that the word "red" refers to an indefinite property, and that its extension has an indefinite boundary. (Many more kinds of indeterminateness will be described in chapter four.)

In such a case, the indefiniteness may be eliminated, or at least reduced, by the adoption of an additional rule. Suppose we call the difficult shade of colour, of the doubtful object, "redange" (if it is on the red-orange boundary). Then we may decide to adopt an additional rule correlating the word "red" with the shade redange positively, or an n-rule correlating it negatively. In either case the decision would make the word more definite. In the former case, we should have a new

word "RED", say, governed by a disjunctive rule: it describes objects if they definitely have the hue redness, or if they have the specific shade, redange. In the latter case we should have a new word "RED" which does not refer to the shade redange, and means, roughly, "red and not redange".

Now, however, there is an important ambiguity to be noticed. Does this new n-rule specify that not being redange is a necessary condition for being RED, or does it merely specify that being redange is not a sufficient condition for being RED? In the former case, the n-rule is a strong one, in the latter case we have a weak n-rule.

3.B.4.b. The weak n-rule, unlike the strong one, leaves open the question whether objects which are redange in colour may not have some other feature in virtue of which they are RED. That is, the strong rule takes "not-redange" to be part of the meaning of "RED", while the weak n-rule merely specifies that "redange" is not part of the meaning of "RED". Something else may make it impossible for any object which is redange also to be RED, such as the impossibility of its having some other specific shade of colour which is definitely a shade of red. But the impossibility does not have its origin solely in the weak n-rule. Indeed, the weak rule leaves open the possibility that the word "RED" is conjunctively correlated with the property of being glossy, in which case a glossy and redange object would definitely be RED, despite the weak negative correlation between "RED" and the shade, redange.

The weak rule specifies a sort of irrelevance condition: being redange is irrelevant to being RED, and

other factors must settle the matter. If there are definitely no other factors, then the object which is redange is definitely not to be described as "RED": this is how even a weak n-rule may help to eliminate borderline cases and so reduce indefiniteness.

3.B.4.c. It might be thought that weak n-rules were always necessary to specify that words are neither incompatible nor stand in a relation of entailment, but this is not so. We can learn to correlate the word "red" with a recognizable hue, and the word "glossy" with a recognizable property of surfaces, without the need for any explicit rule to the effect that the property referred to by one of them is irrelevant to describability by the other. This is because we can tell whether an object is red, or glossy, without ever having to notice whether it has the other property or not. We can therefore learn to understand either word without having to be told anything about its connection with the property referred to by the other, since each refers to a property which is sufficiently definite without any rule correlating it with the other. The mere fact that a thing is glossy does not, on its own, raise the slightest doubt as to whether it is red or not, so there is no doubt to resolve by adopting an n-rule, even a weak one. Only where there is some kind of indefiniteness, as in the case of redange objects, can there be a point in adopting a weak n-rule (and even then there is a point only insofar as there is a point in removing the indefiniteness: see chapter four). This is another illustration of the remark made in 2.D.3. and 2.D.4 to the effect that "links between descriptive expressions" may be rendered

superfluous by semantic correlations between descriptive expressions and properties.

3.B.4.d. The importance of all this is that it shows that sometimes correlations between words and properties are enough to determine the uses of the words without the aid of additional correlations between words and words. This shows that when people argue that the incompatibility of determinates in the same range of determinables is due to linguistic rules which make descriptive expressions incompatible, then this must be defended by an argument to show that such rules are necessary. Perhaps correlations between words and properties can suffice to give the words the meanings they have, and the incompatibilities are due to something other than the rules which fix their meanings. What is more, even if weak n-rules are required, in order to remove certain kinds of indefiniteness, the argument shows that these n-rules do not on their own make descriptions incompatible: strong n-rules are needed for that. But philosophers who so blithely say that it is analytic that nothing can be red and yellow all over at the same time owing to linguistic rules which make the words "red" and "yellow" incompatible descriptions, are not usually even aware of the difference between weak and strong n-rules, and so do not notice that an argument in support of the need for weak n-rules does not establish that we need strong n-rules too. More will be said about this below. (All this helps to illustrate the application of sharp criteria for identifying and distinguishing meanings.)

3.B.4.e. It might be argued that there is no difference

between f-rules and strong n-rules since every f-rule correlating a word positively with a property is equivalent to a strong n-rule correlating that word negatively with the absence of that property. Thus, the f-word "red" would be correlated negatively with the property of not having the hue, red. This is irrelevant to our purposes, since the important thing is that given a word and a property with which it is correlated we must know whether it is positively correlated with the property if we are to know its meaning, and it doesn't matter if we find out the answer by discovering whether the word is negatively correlated with the absence of the property. In any case, it is unreasonable to argue that in general there is a symmetry between the possession of a property and the non-possession of a property, since the perception and identification of, for example, redness, is quite different from the perception and identification of the "property" of not being red. For example, when I look at the surface of an object, I see one colour, but if the absence of a colour is also a perceptible property, then I perceive indefinitely many different properties of this sort in any one object. (There are, however, intermediate cases. For example, some rectilinear shapes are regular and some are irregular: which is the perceptible property and which the absence of a perceptible property, regularity or irregularity? It doesn't matter.)

3.B.5. Reiterated constructions

It should not be thought that the logical operations of disjunction, conjunction and negation can be applied only to f-rules. For the process of constructing new

semantic rules out of old ones is a process which can be reiterated, like the process of constructing new propositions out of old ones, using truth-functional connectives. So not only f-rules can be disjoined, conjoined or negated, but also d-rules, c-rules and n-rules.

For example, if P, Q and R are three different properties, and S represents the range of properties (S_1, S_2, \dots), then a word may be governed by the following semantic rule: The word "W" describes an object correctly if and only if the object either has the property P and not the combination of properties Q and R, or it has the property R and not one of the properties in the range S, or it has the property Q and not the property P. The word therefore refers to the following complex property, which is logically synthesized out of simpler properties:

$$P \ \& \ \text{not}-(Q \& R) \ .v. \ R \ \& \ \text{not}-(S_1 \vee S_2 \vee \dots) \ .v. \ Q \ \& \ \text{not}-P$$

There is clearly not just one property correlated with the word "W". Nevertheless the correlation between the word and the observable properties mentioned serves to explain how the word means what it does: it determines the boundaries of the extension of the word. So we see that universals can explain even if the one-one model is rejected.

There is no need to say that there is one property to which such a word refers, or that there is any one thing common to all the objects which it describes, to be discovered by abstracting from their specific differences. What on earth could abstraction yield in the case of objects describable by a word like "W"? We may, if we wish, make it true by definition of "property" that

there is a property correlated with such a word, but then we should have to distinguish some properties (or universals) as "improper" properties,¹ as they are not perceptible objects of experience, but mere logical constructions out of other perceptible properties. (This should be clear even from a consideration of simple d-rules. One word may be disjunctively correlated with two or more properties which have absolutely nothing to do with one another. What point could there be in saying that this created a new property common to all the objects it describes? The point would be merely verbal.)

Another thing shown by this example, is that demonstrating that the possession of a property by an object is neither a necessary nor a sufficient condition for describability of that object by some word, does not establish that there is no definite semantic correlation between the word and the property. For neither possession of the property P, nor possession of S_2 is either a necessary or a sufficient condition for describability of an object by "W". This is sometimes overlooked by philosophers who try to show that there is no logical connection between concepts by showing that there are neither necessary nor sufficient connections. (Cf. "Goodness and Choice", by Mrs Foot, in P.A.S.Supp. 1961. See also all the talk about necessary and sufficient conditions in Hart's essay: "The Ascription of Responsibility and Rights" in L.L.I.)

3.B.6. We have seen how words may be correlated by means

1. Cf. 2.D.6.

of f-rules with single properties, and how repeated application of logical methods of construction may yield more and more complicated kinds of semantic correlations. When words whose meanings are synthesized in these ways occur in a proposition, then it is possible to analyse that proposition into a truth-functional complex constructed out of simpler propositions, in the manner of Wittgenstein's "Tractatus".

There are, however, more complicated kinds of logical synthesis than those mentioned so far, since quantifiers may be used too. For example, I might define a word to mean the same as the expression "as big as the biggest of the mammals", and this would involve a sort of logical synthesis of a new descriptive word in terms of old ones, namely "mammal", and "big". Or, to take a slightly more complicated example, someone might use the word "lawnmower" as a synonym for "machine which has most of the properties common to things which can cut grass". Here we have a complicated logical synthesis involving quantification over properties. A full discussion of such complicated cases would require us to go into the "Ramified theory of types" of "Principia Mathematica", which would really be unnecessary for the main purposes of this essay.

In addition to these more complicated types of logical synthesis, there are also non-logical methods of synthesising meanings of descriptive words, some of which will be discussed presently. But first we must see what light all of this sheds on my claim that talk about universals can explain our use of descriptive words.

3.C. How properties explain

3.C.1. The discussion of logically synthesized semantic rules in the previous section puts us in a position to see how talking about observable properties can account for our use of descriptive words. We have already noticed (in 3.B.5) that it explains because the possession and non-possession of properties may determine whether objects are or are not describable by a word, and this is an explanation because, as pointed out in section 2.D, universals are independent entities, not essentially tied to their actual particular instances. This is why pointing to an observable property common to a set of objects can explain why they are classified together or why the extension of a concept has the boundaries which it does have. As shown by the examples of the previous section, talk about universals can explain, even when correlations are not of the simple one-one type. In such cases, it is not the complex, logically synthesized "improper" property which explains, but the observable ones out of which it is synthesized, as will be shown also by the discussion of this section. (See 3.C.6, for example.)

3.C.2. In this section, I wish to try to show how talking about universals can provide explanations of the sort which were described in chapter one as "rational", or "personal", explanations of linguistic behaviour such as describing and classifying. By describing a person's behaviour from his own point of view, we can explain what it would be like to be in his position (to act for his reasons) and this can remove certain kinds of puzzlement. (The description may also serve, partly, as a

causal explanation, from a slightly different point of view: explanations from different points of view may overlap to some extent.)

It should be noted that talking about observable properties can explain not only linguistic matters, but other things too, such as how one recognizes a person, for example by the sound of his voice, the shape of his head, the colour of his hair or some other feature or combination of features. It may explain one's reaction to a work of art: "It is not so much because of the pattern of shapes that I like the painting, as because of the distribution of colours." Similarly, mention of an observable property explains how a person recognizes an object as being of a certain kind, and the fact that he intends a word to refer to that property explains why he describes the object as he does.

Of course, we do not feel puzzled or curious concerning many familiar types of behaviour, since we know what it is like to produce that kind of behaviour. Hence we do not feel the need for explanations of the kind which I am talking about. But this is only because of the familiarity of the situations, which ensures that we already possess the necessary explanations. Talk about universals can make explicit the reasons for which we do not regard certain things as puzzling, as well as providing explanations which remove puzzlement.

3.0.3. We must now be more precise. Talk about properties can explain because we can have them in mind without having any of their instances in mind. We can perceive properties (they are tangible to the senses); we can pay attention to them or draw attention to them ("Look at the colour of her dress!"); we can bear them

in mind ("Think of the colour of our wall-paper when you choose the curtains"); we can think about them and imagine them in the absence of any instances ("Try to imagine a shape for a suitable frame for this picture"); we can memorize them and recognize them again in new instances, or in new contexts ("Look, that roof is the same shape as ours!" or "I'll never forget the sound of his walk, I'll always be able to recognize his approach by the sound of his footsteps").

All these are ways of having a property in mind, and it should be noted that this need not involve having any sort of "mental image" of the same sort as after-images. I can think of the way a tune goes without actually hearing it in my head, or remember what someone's face looks like without actually having a visual image. Of course, in a sense I hear the tune or see the face "in my mind" (e.g. "in my mind's eye"), but this may be quite different from, for example, seeing ghost pictures. In addition, one may be quite sure, and correctly so, that one can recognize a face or a tune or the sound of a word when one next meets it, even though one cannot at present remember how it goes. ("I've got his name on the tip of my tongue ...")

3.C.4. Since I can have a universal in mind, I can decide to associate a word with it, so that the word describes objects if and only if they are instances of it. Or I may associate two universals with the word, so that it describes anything which is an instance of at least one of them. Or I may decide that the word is to describe only objects which instantiate both of them. And so on. Alternatively, instead of deciding, I may

simply acquire the habit of making the association, as a result of my environment or education.

If we say that the correlation of a descriptive word with a property or set of properties, in one of these ways, gives the word its meaning, then, since it is possible to have a universal in mind without having any particular instances in mind, there is a clear sense in which the meaning which is given to the word determines the way in which it is to be applied in particular cases. The decision to associate a word with a universal or set of universals does not require the word to be correlated with any actual particular instances, so this decision is independent of and distinct from any later decision to say that some particular object is correctly described by the word. (E.g. it may be an object which one has never previously seen.) Nevertheless, the later decision is justified, or explained, or determined, by the earlier one, together with the fact that the object has such and such observable properties.

This shows that there is room for a distinction between the intended use of a word, or its meaning, and its actual use. The meaning, or intended use, explains the actual use and is therefore not constituted by it. (It seemed that Hampshire wished to deny this, in "Scepticism and Meaning". See also Bennett: "On Being Forced to a Conclusion", in P.A.S.Supp., 1961.) Here is the main reason why arguments from paradigm cases are likely to be fallacious. Of course, since the meaning of a word may be indefinite in some respects, it need not fully determine the whole use, as unexpected borderline cases may turn up. (This fact seems to have obsessed Wittgenstein, in his discussion of "following a rule", in "Investigations", so that he overlooked the

fact that part of the use may be determined "in advance" by the meaning of a word.)

3.0.5. We can see more clearly in what sense the properties correlated with a word can determine its application or explain our use of the word, by going back to some of the examples of the previous section.

It was shown that the word "red" could be used either as an f-word, referring to a single property, the observable hue, redness, or as a d-word, disjunctively correlated with many specific shades of red. Let us distinguish these two cases by talking about "f-red" and "d-red". A person who can see and memorize hues, may learn to use the word "f-red", while a hue-blind person, who can only see specific shades, has to use the word "d-red". Now suppose each of them comes across an object with a specific shade which he has never previously seen, though it is a shade of red. The first person, who can see the hue exhibited by the object, is able to recognize it as being describable as "f-red". The hue-blind person, however, since he cannot see the hue, and does not recognize the specific shade, will say that the object is not d-red. Here it is clear that the different ways in which they correlate the word "red" with properties can explain the difference in their behaviour in the case described. In the same way, the way in which each of them correlates words and properties explains his behaviour in cases where their classifications do not diverge. The behaviour is the same in such cases, but the explanations are not, and the difference is a real one even if, as a matter of fact, no shade of colour ever happens to turn up which would show up the difference.

In other words, the difference may be of a kind which can be described only by saying what it would be like to be in the position of each of them, to be deciding in the same way as they do how to describe the things they see. They may describe the same objects with the same words, but in virtue of different facts. (Cf. "tetrahedral" example, in 2.C.8.)

3.C.6. It should be noted that it is possible for these two persons to agree in their behaviour even when they come across new shades of colour, for, in such a case, the hue-blind person may somehow simply guess that people who can see hues would describe the object as "red". Or he may simply "decide" to enlarge the scope of his d-word "red" by including the new shade as one of the properties henceforth to be disjunctively correlated with the word. Or he may simply happen to call the object "red" without even noticing that its shade is not familiar to him, though if asked why he used the word he is baffled. In all these cases the explanation of what he does, if there is one, is quite different from that of the person who sees the hue. The hue-blind person is not, from his own point of view, using the word rationally, or intelligently, or according to a rule in the cases described. His decision to describe the object as "red" is quite arbitrary, as it is not explained or justified in any way by the meaning with which he understands the word, despite the fact that other persons may be able to see a justification for classifying the object with those which he calls "red". Here is a case where meaning does not determine use.

There may, of course, be a causal explanation for this non-rational behaviour, such as a psychological

or physiological explanation. But this does not make his decision any the less arbitrary, from his point of view. Talk about what goes on in his brain does not give a rational explanation. It does not describe from his point of view what it would be like to act as he does, for he is as unaware of anything going on in his brain as we are.

(In some cases, it may be difficult to tell whether a decision is arbitrary or not (determined by a meaning or not).)

3.C.7. This difference between a person who uses a word like "red" as an f-word referring to a single property and a person who uses it as a d-word referring to a whole range of properties disjunctively, gives some point to the assertion that they do not really understand each other, despite the fact that they describe things in the same way in general. For similar reasons we may say that persons who are totally blind or colour-blind cannot fully understand what others mean by colour-words, even though they can, in a way, use the words; for example, when a blind man asks whether the sky is red because he wants to know whether it is likely to be raining the next day or not. A blind person cannot use the word "red" in the same way as one who can see, for he cannot discover in the same way whether things are red or not. This inability is explained by the fact that he cannot see the property or properties to which the word refers. By contrast, the sighted person's use of the word is explained by the fact that he can see the property.

If everything that explained our use of descriptive words were in the "realm of symbols" (see 2.D.8), then

the blind or colour-blind person would be able to use colour words in the same way as normal persons, since they have as much access to symbols.

3.0.8. Of course, talk about universals does not explain everything, it does not answer all questions. For example, we cannot explain the fact that the hue-blind person chooses just this range of specific shades of colour to correlate with his d-word "red" in terms of properties which he can see. Similarly, the fact that words such as "horse" and "rod" refer to those ranges of specific shapes to which they do refer is not explainable in terms of some common visible property. (See next chapter: 4.A.4.a & 4.A.6.) Quite a different level of explanation is required. It may be an historical explanation, referring to some arbitrary decision made in the past, or there may be a sociological or anthropological explanation, in terms of our environment, or in terms of certain natural reactions which we all share, or in terms of our purposes in classifying things. One way of looking for these explanations is to consider factors which could cause changes in usage.

But even if universals do not explain how words have the meanings which they do have, nevertheless, they explain how their having these meanings determines what we say about the world.

3.0.9. It should not be thought that all this talk about the way in which properties can explain our use of descriptive words is irrelevant to the main purpose of this essay. For the difference in the explanations of the two kinds of use of the word "red" gives us good reason

to say that the word has different meanings when it is used as an f-word and when it is used as a d-word, even if the difference does not affect the class of objects which the word happens to describe correctly. Thus, the d-word is synonymous with a long disjunction (or would be if there were enough names for specific shades of colour in the vocabulary), whereas the f-word is not. But here we are obviously applying very sharp criteria for identity of meanings, for it is obvious that for most normal purposes the difference would not matter in the least and the words would be regarded as synonymous (see chapter two, especially 2.C and 2.A.). Similarly, when "triangular" is used as a d-word correlated with a range of specific shapes it does not have the same meaning as when it is used as an f-word correlated with a single property common to all triangular objects, if sharp criteria of identity are employed. In 3.B.4.b, etc., we saw that the word "RED" might be negatively correlated with the specific shade redange, by either a strong n-rule or a weak n-rule. Here again, for similar reasons, we must say that these rules give the word two different meanings, if we are to use the strictest possible criteria for identity of meanings. (Cf. example in 2.C.8.)

So all these examples help to illustrate the claim that by considering correlations between words and properties, we can apply strict criteria for identity.

3.C.10. The importance of all this for the purpose of this essay is that it provides us with a whole host of potential candidates for the title of "synthetic necessary truth", which we should not be able to discuss if we used more familiar loose criteria. Thus, since the meanings

of "d-red" and "f-red" are so different (they are correlated with different properties), it seems unlikely that it is analytic that all d-red objects are f-red. But is it necessarily true? Is it necessarily true that anything which has one of the specific shades correlated with "d-red" has the hue correlated with "f-red"? And what about the converse? We are armed against the slick, question-begging argument which demonstrates that the necessity is analytic by sliding from one meaning of "red" to another: our sharp criteria will not let this go undetected. (Cf. 2.C.10.) Similarly, we can open an interesting question of the form: Is it analytic that nothing is RED and redange? (Cf. 3.B.4.d.)

3.C.11. It may be objected that my account is incomplete, since I have failed to describe how we can tell which sort of rule is being followed in some of these examples. This is, of course, only one aspect of a general problem: How do we tell which properties are the objects of a person's mental acts? How can we be sure which property a person is thinking about, or looking at, or trying to draw attention to, or surprised by? I think I have given the beginnings of an answer to this by describing how in some cases correlating words with properties in different ways may lead persons to behave in different ways. But I do not wish to solve these problems of mind and body here. This is a phenomenological enquiry, and I am trying only to describe the use of words and sentences from the point of view of the person who uses them, and from his point of view there is certainly a difference between being able to see the redness common to all red things and having to memorize a whole range of different

specific shades of red, no matter how difficult it may be for other persons to detect the difference in him.

The fact that we have words in English which enable us to describe the difference is strong evidence for the existence of ways of detecting it. But I shall not look for them.

3.C.12. I have so far illustrated the application of sharp criteria for identity of meanings only by comparing and distinguishing different methods of logical synthesis. But, as already remarked, there are other ways in which the meanings of descriptive words may be synthesized, and they also yield interesting examples of connections between concepts which are apparently not analytically related. Some of these will now be described.

3.D. Non-logical syntheses

3.D.1. So far, only logical methods of constructing new concepts out of old ones have been described. In this section it will be shown that there are other types of construction, which involve more or less complicated procedures for picking out properties or for deciding whether an object is describable by a word. Where such a procedure is involved in the application of a word, I shall refer to it as a "p-word", and say that it is governed by a "p-rule".

3.D.2. We have seen that a person who is hue-blind, but can see and memorize specific shades of colour, may learn to use the word "red" as a d-word, disjunctively correlated with a range of specific shades. If he finds it difficult to memorize so many specific properties, such

a person may adopt a procedure for picking out the right shades.

For this we require that he should be able to arrange specific shades of colour in the order in which they occur in the white-light spectrum, or to tell whether three given shades are in the right order or not. There are many different ways of doing this. For example, he may simply be able to see which of three given shades lies between the other two. Or he may simply memorize the order in which the shades occur in the spectrum (though this would again raise the memory difficulty). Or he may simply memorize the appearance of the whole spectrum and then tell whether the shade of colour of one object lies between the shades of two others by looking to see whether it does so in the spectrum as he remembers it. Perhaps he can arrange bits of coloured paper in the right order by experimenting with them until their colours vary in the least discontinuous way along the row. The differences between these various ways of judging the order of shades of colour may, as before, be detectable in cases where new shades turn up. Which of these methods is employed will make a difference to the procedure I am about to describe, but that need not concern us.

If a person is capable of making judgements of the sort "This object has a shade of colour between the shades of those two", then he can learn to use the word "red" as a p-word by memorizing two shades which lie as near as possible to the boundaries of the red band of the spectrum and then applying the word to objects if they have shades lying between the two which he has memorized.

In this way he avoids having to memorize all the individual shades, though he must, of course, memorize the

two boundary shades and the procedure to be employed. So, unlike the person who simply follows a d-rule, he can deal with specific shades of red which he has never seen before. (See 3.C.5.)

3.D.2. (Note). There are, of course, far more complicated and indirect procedures which may be used for applying colour words. Thus, a person who is completely colour-blind, and cannot even distinguish specific shades of colour, may have to employ a spectroscope, and make use of a correlation between spectroscopic readings and colour-words, in order to decide how to describe objects. Or instead he may take a "colour slave" around with him, that is someone who can perceive colours and has been trained to give the right answers to questions about colours of objects. (See Smart, in Philosophy, April and July, 1961, circa p.140.) Or he may simply ask other people, without bothering to acquire a slave.

A person who can distinguish shades, but cannot memorize them easily may have to carry a colour-chart around with him, for comparison. Even so, he must memorize the correct procedure for using the chart, such as what it is about the samples that he has to compare with the objects he wishes to describe. This may be compared with our use of metre rules and standard weights, which we require on account of our inability to memorize lengths and weights accurately. I call these standard particulars. Of course, they aid not only our memory, but also our rather limited powers of discrimination and comparison (rulers and pieces of string help us to compare lengths, weights and balances help us to compare weights accurately). That is to say, one and the same thing may

serve both as a standardized particular and also as an instrument (e.g. graduated rulers, spring balances). Notice that even with these aids, there is still always some point at which something or other has to be perceived and recognized by the observer, even if they are only numbers or letters flashed on a dial.

3.D.3. Procedures may be used in connection with shape concepts too. For example, the person who is unable to perceive and recognize the feature common to all triangles, and has trouble memorizing all the specific triangular shapes, may learn to use a procedure for picking out triangular objects.

Suppose, for example, that such a person is able to tell, by examining two objects, whether it is possible to deform the shape of one of them into the other by using only stretches and shears. Such a deformation will turn triangles into triangles, quadrilaterals into quadrilaterals, and so on, since it preserves straightness of lines and does not turn corners into straight angles. Since any triangular shape can be turned into any other triangular shape by two stretches and a shear (one stretch to get the base right, another to get the height right, and then a shear to get the vertex in the right position relative to the base) it is possible for the partially shape-blind person to memorize just one specific triangular shape, and then decide whether objects are triangular or not by seeing whether their shapes are deformable into the one which he has memorized by a succession of stretches and shears. In this way he could use the word "triangular" according to a procedure, as a p-word. Similar sorts of procedures could be used for

words like "rectangle", "quadrilateral", "parallelogram", if suitable kinds of deformations are allowed.

3.D.4. There are, of course, other sorts of procedures which might be used for applying the word "triangular", by a person who could neither see triangularity, nor discriminate and memorize specific triangular shapes. He might apply the word to objects by looking to see if they had an outline bounded by straight lines, and then uttering the sounds "bing" "bang" "bong" in sequence as he pointed to each side in turn. If he could do this without leaving out any side, and without pointing to any side more than once or uttering any of the sounds more than once, then he would describe the object as "triangular". He need not know how to count, or read anything more into the ceremony than I have described in it. (Compare also, Nicod: "Foundations of Geometry and Induction", Part III.)

3.D.5. Still more geometrical examples are available. The word "star" may be used to describe rectilinear plane figures in which alternate angles are reflex and acute, and the word "starlike" to describe objects with this shape. A person who could not perceive and memorize this sort of shape might pick out objects to be described by the word, by seeing whether all the sides were straight and the angles came in the order: bending in, bending out, bending in, bending out ... etc., as he ran his attention round the boundary.

3.D.6. In each of these cases, a new geometrical concept is synthesized out of other geometrical concepts by means of a geometrical construction. It is not a logical



construction, since, for example, the procedure for picking out "starlike" objects does not involve looking to see whether objects have certain combinations of properties, or whether certain properties are absent, etc. The notion of a shape built up by adding straight lines one after the other, bending first one way then another and finally closing up, is different from the notion of a shape which is a certain combination of shapes or other properties. We do not, in employing this sort of procedure, look to see which properties an object has and then apply truth-tables. We use notions which do not correspond to properties of the object as a whole, in order to build up a property of the object as a whole. (So we have a kind of complexity which cannot be analysed truth-functionally, in the manner of the "Tractatus". Compare 3.B.6, above, and 3.D.9, below.)

3.D.7. There are also many musical examples. A person who can listen to a triad (sound made up of three notes) and tell whether it is a major chord or not just by its sound, can use the expression "major chord" according to an f-rule correlating it with a single property. A person who cannot do this can nevertheless use the word according to a p-rule provided that he can hear the three notes separately (some can do this, some cannot), and can sing, aloud or "to himself" a major scale starting on any given note. (One may be able to recognize the sound of a major scale without being able to recognize the sound of a major chord.) The following procedure could then be used for picking out major chords: sing the major scale starting on each of the three notes in the triad, and if one of the scales is such that the

other two notes occur as the third and the fifth notes of the scale, then the triad is a major chord.

It is conceivable that a person may be able to recognize the sound of a major chord as a whole without being able to hear the three notes separately, in which case he could not apply this procedure. Thus we should have two different concepts of "major chord", and familiar questions would arise about the relation between them. (See 3.C.9, etc.) (Compare: most of us can recognize the characteristic timbre of a flute without hearing the harmonics separately. Perhaps some persons can recognize the sound only by listening for harmonics and seeing how they are distributed.)

Here again, the synthesis is non-logical, because the object (a sound) has the synthesized property not in virtue of having or not having several different properties, but in virtue of the fact that its various "parts" stand in some non-logical relation to one another, or some of its properties stand in non-logical relations to other properties.

3.D.8. Just as logical operations can be applied recursively, so as to construct complex semantic correlations, so also is it possible to apply logical operations to p-rules, yielding "mixed" rules. For example, a p-rule may be conjoined with a d-rule, and the whole may be negated and conjoined with a c-rule. Or a procedure may start with a property which has already been synthesized to some extent. (See 3.B.5.)

All this helps to illustrate the way in which the one-one model for semantic correlations is inadequate. We could, as pointed out above (3.B.5.) say that every descriptive word referred to one property (or, more

generally, one universal), but then not all properties could be used to explain the use of descriptive words, since some of them would be "improper" properties constructed logically or otherwise out of simpler properties, and only the simpler observable properties can explain (e.g. by explaining in detail how the procedure for applying a word works).

3.D.9. Remarks

There are several points to notice about these examples. First of all, although a procedure may help someone to pick out something which he cannot perceive or memorize, it is always necessary for him to be able to perceive some features or properties of the objects which he wishes to describe, and he must be able to memorize something, including the type of procedure to be employed (which is, of course, a complicated universal).

Secondly, as before, we have found that two different p-rules, or a p-rule and an f-rule may both give a word very similar uses (e.g. the extension may be the same in both cases). Once more we can apply sharp criteria for identity and say that they then have different meanings (though they are the same for normal purposes), thereby leaving open interesting questions about synthetic necessary connections.

Thirdly, it should be noted that a person may learn to follow a procedure without being able to describe it in words, for one reason or another. (See appendix on "implicit knowledge".) Hence he may have difficulty in saying what he means by a word, though he can use it as a matter of course. This helps to account for the fact that people may fail to notice how all these different kinds of rules may lie behind one and the same familiar

word, such as "red".

Words which do not refer to observable properties, such as "magnetic", cannot be used according to f-rules, but must be governed by p-rules or rules constructed in a complicated way out of f-rules. These words may be described as referring to "inferred" properties.

3.D.10. Finally, it may be remarked that the point in describing the syntheses of the present chapter as non-logical is that in each case the type of synthesis is restricted in its applicability to special kinds of features or properties, whereas logical methods of synthesis are quite generally applicable. They are topic-neutral. (In chapter five, topic-neutrality will be selected as the main distinguishing characteristic of logical constants, as opposed to non-logical words.) For example, when someone picks out specific shades corresponding to the word "red" by seeing whether they lie between two shades which he has memorized, the relation between the shade picked out and the ones memorized is not a logical relation: it is a relation which holds specifically between shades of colour, and in order to know what the relation is one must be acquainted with colours and know what it is like for one shade to be between two others. Acquaintance with this kind of property is required. (In Kant's terminology: an appeal to intuition is required. See chapter seven.) Contrast this with knowing what it is like for one "property" to be the combination or disjunction of two or more others: here we have a very general kind of knowledge, for the relation in question can hold between any sorts of properties, so acquaintance with no particular kind of

property is presupposed.

(Problem: could these examples of procedures be reduced to a kind of logical synthesis by talking about "properties" which are logically synthesized in a complicated way out of both properties and relations? E.g. the property p-redness is synthesized out of the two boundary shades of colour and the relation of "betweenness" holding between shades of colours, the synthesis being logical. This does not matter much for my purposes, as the main aim was to show how to distinguish different concepts where they are not normally distinguished owing to the use of loose criteria for identity of meanings.)

Where necessary, we may describe these non-logical types of synthesis as "geometrical synthesis", "musical synthesis", and so on.

3.E. Concluding remarks and qualifications

3.E.1. It may be thought odd that most of my examples to illustrate the various kinds of semantic rules described in this chapter should be so contrived and artificial, and that in several cases I had to invent new words to illustrate a point, instead of using words we all know. This is because I have oversimplified many features of our use of descriptive words in order to illustrate the principles which are to be employed for making sharp distinctions between meanings. It is only to be expected that there should be some oversimplification in the early stages of the description of any system of classification. But because most of our ordinary concepts are very complicated, in ways which will be described

presently, they cannot be used without modification to illustrate oversimplified schemes of classification.

It is necessary to oversimplify at first, in the interests of clarity. Normally people start right off talking about complicated cases, and then they fail to sort out all the various complexities, having nothing with which to contrast them, and this, I think, helps to account for the fact that controversies concerning the analytic-synthetic distinction and related distinctions have gone on for so long, without any progress being made.

Thus, the importance of these oversimplified examples, as will appear presently, is that they show that it is possible for concepts to stand in definite relations, even if, owing to the complexities which we have so far ignored, and will discuss in chapter four, most of our ordinary concepts do not, a fact which sometimes leads philosophers to think that there is no clear distinction between analytic propositions and synthetic propositions.

3.E.2. One way in which my descriptions oversimplify what goes on when we ordinarily use descriptive words has been by disregarding some of the complexities in the ways in which various rules may be synthesized. For example, the ordinary word "red" is probably used partly as an f-word, by those who can see hues, partly as a d-word, by those who can memorize shades of colour, partly as a p-word, by those who can memorize boundary shades and tell whether a given colour lies between them or not, partly as a word correlated with a scientific procedure for measuring wave-lengths of light, and so on;

and all these different kinds of rules or concepts may be "superimposed" in one concept "red" without being combined definitely as a conjunction, or a disjunction, or a disjunction of conjunctions, or anything as simple as these. (This sort of thing helps to account for so called "open texture".) The meanings with which we use words are far less definite than has been suggested by the descriptions of this chapter. (This is connected with the fact that, for normal purposes, there is no need to apply strict criteria for identity of meanings. See section 2.C.) Some of these oversimplifications will be eliminated in the next chapter.

3.E.3. In addition to ignoring complications in the way in which we correlate words with universals, I have oversimplified other matters. For example, I have assumed that colours vary only in one dimension, so that all shades can be arranged along a continuous spectrum in a definite order. I have failed to take note of the difficulties in saying that the same colour (whether a hue, or a specific shade) may be present in ordinary opaque objects, in transparent objects (solid or liquid), in objects with various sorts of surface textures, or even in phosphorescent objects and neon signs. Is it the same property in all these cases? I have ignored the fact that there may be limits to our powers of discriminating specific shades of colour, or specific shapes. (Something will be said about this in chapter seven).

It is not possible for all problems to be solved at once. Many of my remarks are idealizations which require qualifications of one form or another, but the qualifications do not usually affect the main argument.

3.E.4. One of the main points of the discussion has been to show how people who oversimplify things even more than I have done may be led to adopt intolerably obscure and confused theories of meaning and universals, or to make sweeping generalizations in rejecting such bad theories, so that they overlook the element of truth behind them. The main oversimplification is to ignore the possibility that the use of a descriptive word may be explained in terms of complex correlations between words and universals. Insistence on the one-one model, or a determination to say that one property or universal corresponds to each word, leads people to say such things as that universals are "intangible to the senses, apprehended in thought alone, the potentiality of their differentiations, the identity to be found in variety", etc. (See 2.D.6.) Or it prevents their seeing clearly how talk about universals can explain.

3.E.5. In showing how talk about properties and other universals can explain, we stressed the fact that the ability to use words presupposes the ability to perceive and attend to features or properties, to memorize them and recognize them again later. This explains how we can learn the use of a word from examples and then go on and use it in quite different contexts. It explains how, having learnt to use the word, we can understand its use even in false statements which ascribe properties to objects when those objects do not have those properties. All this is possible because in memorizing a property one need not bear in mind any particular object or objects having that property (cf. section 2.D) (The particulars used in teaching the meaning of a word do not thereafter have any special role in connection with it: after they

have provided the required illustration they drop out as irrelevant and may change their properties or relations without this having any effect on the meaning with which the word is understood.)

This loose connection between universals and their actual instances, or between descriptive words and the particulars which they describe, which has been so important in our explanations so far, will turn out to be very important once again in chapter seven, where it will provide the basis for an explanation of the meanings of "possible" and "necessary".

3.E.6. Despite all the oversimplifications, the discussion of this chapter has shown in a general way how correlations between descriptive words and properties can help to determine which objects are correctly describable by which words, or at least the conditions in which objects are describable by words. In chapter five, the discussion of logical constants will show how descriptive words may be combined with other words to form sentences expressing statements. So this chapter and chapter five will together have shown how correlations between descriptive words and properties can help to fix the conditions in which statements are true. The importance of this for our main problem is that it helps to explain how the analytic synthetic distinction works by showing how it is possible for a statement to be analytic, or true in virtue of its meaning. We shall see that analytic statements form merely a special case of the class of all statements which are true in virtue of both what they mean and what the facts are.

Chapter Four

SEMANTIC RULES AND LIVING LANGUAGES

4.A. Indefiniteness

4.A.1. In chapter three an attempt was made to describe various ways in which descriptive words may be correlated with universals by semantic rules. It was pointed out in section 3.E that our ordinary use of words is much more complex than the uses described in that chapter, and the purpose of this chapter is to describe some of those complexities.

There are many respects in which the description of semantic correlations and logical and non-logical syntheses of meanings provided an oversimplified model. For example, it took no account of descriptive words which refer to tendencies or dispositions or unobservable properties or theoretical notions of the sciences, or those words, such as "angry", "hopes", "intends" which may be used to talk about conscious beings. However, even if we leave out these complicated concepts, and concern ourselves only with words which are correlated with observable properties in something like the manner described in the previous chapter, we shall find complications which have not been accounted for, though very briefly mentioned near the end.

Philosophers sometimes draw attention to these complications by saying that ordinary empirical concepts have "open texture", are vague, have indefinite boundaries, or admit of difficult borderline cases. Sometimes the point is made by saying that concepts do not stand in exact logical relations to one another, or that it is

impossible to make a clear distinction between usage due to meaning and usage due to generally shared collateral beliefs. (Cf. Quine: "Word and Object", p.43). Sometimes they are carried away by all this and say that words in a language are not used according to rules, or that logical laws do not apply to ordinary languages, or that there is no clear distinction between analytic and synthetic statements, or between necessary and contingent truths.

Unfortunately, no one seems to have given a very clear and systematic account of all these complications and difficulties, nor explained how a language is able to work despite many kinds of indeterminateness in it. I suspect that this is because people have no clear notion of what it would be like for these indeterminacies to be absent: so they do not have any model for the missing simplicity with which to contrast the actual complexity and provide a basis for systematic discussion. I believe that the account of semantic rules in the previous chapter provides at least part of such a model and hope to illustrate this by contrasting some of the complications in ordinary usage with its relative simplicity.

4.A.2. The kinds of indeterminateness which will be described in this chapter fall into two main classes: (i) those due to indefiniteness of properties themselves and (ii) those due to indefinite semantic correlations between words and properties. It will not be possible to describe all possible cases: there is room for only an incomplete and somewhat condensed sketch.

4.A.3. Indefiniteness of properties.

It is sometimes remarked that properties can be indefinite, as if it were perfectly clear what this means or how it is possible. It is certainly not clear to me. The point seems to be that when we try to decide which objects have and which have not got some property, we may come across a borderline case where it is difficult to decide. (Compare 3.B.4.a.) There seem to be several different cases in which one may have this sort of doubt about an object in one's field of perception.

1) The doubt may be empirical, and due to abnormal circumstances. For example, the light may be bad, or one may be too far away to see clearly, or one may be temporarily unable to concentrate, owing to tiredness, a headache or emotional problem. Or one may simply have forgotten what the property looked like. Such doubts can be eliminated by eliminating the abnormal circumstances, and are of little interest.

2) The doubt may be due to permanent psychological or physiological limitations, such as an inability to make fine discriminations. This can cause doubt whether two visible objects are exactly alike in some respect (e.g. shape, or colour), or whether an object which is present has exactly the same shade of colour as the shade which one has in mind (e.g. a shade seen on the previous day). Other examples are: a permanent inability to memorize fine shades of colour, or an inability to "take in" or "survey" complicated properties, like the property of being a figure bounded by 629 sides. (In some of these cases, the use of procedures, such as counting, may help to resolve the doubt. This raises interesting problems as to which doubt it resolves.)

3) It is possible that there is a third sort of doubt

to be described as being due to indefiniteness of a property. An object and all its properties can be seen plainly, and yet one may be in doubt as to whether its hue is red or not despite the presence of many red objects to ensure that memory is not at fault. This does not seem to be an empirical doubt, to be settled by closer examination in a better light, for example, or under a spectroscope. (If I let the spectroscopic readings settle the question, then I have taken a decision to use the word "red" in a new way.) For what the spectroscope tells me cannot remove any doubt about how the object looks to me. Notice that although the doubt concerns the way the object looks, nevertheless there is a sense in which I am in no doubt as to how it looks, for I can memorize its appearance, and recognize other objects as having exactly the same shade of colour. This is what suggests that it is a doubt about a property: is that hue (red) present in this shade (e.g. redange - see 3.B.4.a.)?

It is not at all clear to me that there is a difference between cases 2) and 3). Perhaps it depends on whether there is only one person who is unable to decide or whether everyone is unable to decide. At any rate, I shall be content to leave this and carry on with discussion of indeterminate semantic correlations.

4.A.4. Indefinite rules

Not only may the properties to which words refer be indefinite, but in addition the way in which they are referred to may be indefinite, or indeterminate. (Though it is not clear that these two cases can always be distinguished.) Here again, there are various ways in which doubt as to the describability of an object by a word may arise.

First consider an f-rule, correlating a word with one observable property. It may be unclear which is the property with which the word is meant to be correlated, and this, of course, may cause doubt in the application of the word. No more need be said, as this is just a special case of the next sort of doubt.

Secondly, if a word is correlated with a set or range of properties disjunctively, by a d-rule (see 3.B.2,ff.), then the boundaries of the set of properties may be indefinitely specified. This is one of the things which may be meant by the word "vagueness". Notice that although it is a range of properties which has indeterminate boundaries, this may have the consequence that the class of objects with those properties has indeterminate boundaries, in which case the word has an extension with indeterminate boundaries. (Part of the indeterminateness may be due to indefiniteness of the individual properties, of course.) The previous case is clearly an example of this, for it involves a unit-set of properties, with indeterminate boundaries, so that it is not clear which is the property in that set.

4.A.4.a. It is difficult to illustrate this by means of an example, because our ordinary words are too complicated and illustrate too many things at once. But we can come close to seeing what sort of thing is meant by noting that the word "rod" is correlated with a whole range of shapes, each more or less straight, with a fairly uniform cross-section, and a length somewhat greater than its diameter. But how much greater? How much longer must a rod be than it is wide? It should be clear that the range of shapes correlated with the word "rod" has somewhat indeterminate boundaries, for although the ratio of length

to diameter must not be too small or too great (else the word "disc" or "filament" or "wire" may be more appropriate), nevertheless there are no definite limits. So there is a range of shapes which may definitely be possessed by rods, and a range which may definitely not be possessed by rods, but there are no determinate boundaries between them. (Compare also the words "heap", "few", "small", "many", "giant", etc. As with "rod", caution is required, since these words illustrate more than one kind of indeterminateness.)

4.A.4.B. Next we have indefinite conjunctive correlations. Two or (usually) more properties may be conjointly referred to by a descriptive word in an indefinite way. (Cf. 3.B.3.) For example, if the properties are as a matter of fact always found together and never separately, then the word "W" may be used to describe objects which have all the properties without its ever being decided whether possession of all of them is necessary for describability by the word, or whether only some subset need be possessed, and if the possession of only a subset is sufficient, in that case. We can describe this sort of indefiniteness in terms of the last, as follows. Form the set of all possible combinations of one two or more of the properties in question. Then we regard the word "W" as correlated disjunctively with some subset of this set of complex properties, the boundaries of the subset being indeterminate. When, in such a case, the original set of properties (and so also the set of possible combinations of those properties) is infinite, or at least indefinitely extensible, we seem to have an illustration of what Waismann called "open texture". (In "Verifiability".) I shall postpone illustrations till later, for the reason already mentioned: ordinary

words are too complex and illustrate too many different things.

4.A.4.c. The next kind of indefiniteness involves n-rules, as described in 3.B.4,ff. It was shown that words might be negatively correlated with properties by either strong or weak n-rules. The correlation may be indefinite in some cases, where for example, it is not certain whether the n-rule actually does govern the word or not. Thus, before the discovery of black swans, there might have been an indeterminate negative correlation between "swan" and blackness. Perhaps the more interesting case is that in which the indefiniteness is due to its not being clearly specified whether a weak or a strong n-rule correlates some word with a property (see 3.B.4.b.). This has the consequence that it is not definitely analytic nor definitely synthetic that nothing with the property is correctly describable by that word.

4.A.4.d. Not only logical syntheses, but non-logical ones also may be indefinite. Thus a word governed by a p-rule (see section 3.D) may have a meaning which is indefinite in a way analogous to that discussed above, in connection with d-rules. For example, in 3.D.2. we described a procedure for using the word "red". Two boundary-shades are selected and memorized, and then the word is applied to an object if and only if it has a specific shade of colour lying between the two boundary-shades. If, however, there is something indeterminate in the selection of the boundaries, or in the decision whether specific shades lie between the boundary shades or not, then the procedure considered as a whole will

be partly indeterminate. For example, the word "red" may be correlated with a set of properties with indeterminate boundaries by such a procedure.

4.A.5. We have seen how each of the types of synthesis described in chapter three may be indeterminate, giving rise to concepts whose boundaries are not clearly defined. If we recall that all these operations for constructing new semantic rules can be reiterated, to yield very complicated correlations between words and properties, involving both logical and non-logical syntheses, we see that the final product may be indeterminate in many different ways all at once, and even more so if we allow that properties themselves may be indefinite (see 4.A.3.).

In 3.B.5, we saw that a word may be correlated with the following combination of properties:

$$P \ \& \ \text{not}-(Q \& R) \ .v. \ R \ \& \ \text{not}-(S_1 \vee S_2 \vee \dots) \ .v. \ Q \ \& \ \text{not}-P.$$

In such a case, each of the properties P, Q and R may be indefinite in the manner of 4.A.3, the range S may have indeterminate boundaries in the manner of 4.A.4 or 4.A.4.d, and it may not be certain that any one of the main disjuncts is a sufficient condition for the applicability of the word, though any two together do definitely provide a sufficient condition. This illustrates the way in which several different kinds of indefiniteness may simultaneously contribute to the indeterminateness of the boundary of the extension of a word.

It should be stressed that we must distinguish borderline cases due to difficulty in deciding whether certain particular objects do or do not exhibit certain properties, and those which arise out of indecision as to whether those properties which are quite evidently possessed

or not possessed by objects are correlated with a word in one way or in another. This shows that concepts may have indeterminate boundaries in two quite different senses: it may mean that the extension, the set of particular objects falling under the concept, has indeterminate boundaries, or it may mean that the set of properties, or combinations of properties, sufficient to guarantee inclusion in the extension may have indefinite boundaries. In either case borderline cases are possible, that is, particular objects which are neither definitely describable, nor definitely not describable, by some word. We may say that in these cases the application of the word is not determined by or explained by the meaning of the word, or by the universals correlated with it. (See 3.C.4, 2.D.2.)

4.A.6. I have remarked that it is difficult to find words in a living language which illustrate only one kind of indeterminateness. It is much easier to find words which simultaneously illustrate several kinds. The word "horse" is a familiar example. There is a range of shapes which may be possessed by horses, but the range has no definite boundaries, for the shape of a horse may change continuously into that of an elephant or giraffe without definitely ceasing to be a possible shape for a horse at some one point.

Similarly, there is a range of possible colours for horses, and here it is not even clear whether there is any boundary at all, since whether a colour is possible may depend on other factors, such as whether the horse has been painted that colour. If some "horse" were born bright blue and produced off-spring with red white and blue stripes, we might not be sure whether to say that it

was a horse after all. Similar remarks may be made about the textures of the skins of horses. They must not be metallic, but there is no definite boundary.

In addition, it is likely that even within the ranges of permissible shapes colours and textures, there are some which must not occur together. Some odd colours may be allowed, but not if the animal also has too odd a shape and texture too. However, there is surely no definite limit to the kinds of combinations which we should allow in objects correctly describable as "horses". Further, we may allow the possibility that biological investigations will provide an "explanation" of the existence of freaks and so persuade us once more to call them "horses". It is very likely that no explanation at all would redeem some cases, yet there is surely no clear boundary between those cases which may be explained away and those which may not. Investigation would doubtless reveal further complexities here.

4.A.7. These remarks help to illustrate the claim that the account given in chapter three was hopelessly inadequate to explain the use of all kinds of descriptive words. But there are still many kinds of complexity and indeterminateness which have not been mentioned. For example, we discover empirical regularities in the world, and construct scientific (or non-scientific) theories based on these regularities. Then in some cases the theories may be built into the definitions of some of the words used to state them. This may occur in an indeterminate way so that, for example, it is not clear whether it is a matter of definition that gases at constant pressure have a linear coefficient of increase in volume with rise in temperature, or a contingent fact. The

correspondence between mercury column readings, and gas-thermometer readings is, of course, a matter of experience, not a matter of definition. The indeterminateness consists in the fact that it would not be clear how to describe the situation in which the correspondence broke down. (In some cases, further investigation might make it clear, by yielding explanations in terms of accepted theories.) So we can say that increase in length of a mercury column is neither definitely merely evidence nor a defining criterion for the applicability of the expression "a rise in temperature".

But there is no room for a detailed discussion of all kinds of indefiniteness. Many cases are already familiar (see, for example, the chapter on "Reduction and Open Concepts", in "Semantics and Necessary Truth", by Pap). I shall leave the description and classification of examples now, and make some general remarks about indefiniteness.

4.B. Ordinary language works

4.B.1. The previous section showed how it is possible to take account of various sorts of indefiniteness within the framework of a theory which attempts to explain our use of descriptive words in terms of correlations with observable properties and other universals. It brings out more clearly than ever some of the inadequacies of the one-one model, which assumes that there is one universal correlated with each descriptive word, and simultaneously shows why there is no need to give up talking about universals altogether just because the one-one model will not work.

For example, one sort of objection to talking about meanings as explained by properties, is that the properties to which words seem to correspond may change over the years while there is no need to say that their meanings change, or that the concepts corresponding to them change: concepts may have a history. But we can easily take account of this, for, owing to the complexities in the correlations between words and properties, it is possible for small changes to take place while most of the correlations remain unaltered. A concept may become more or less definite in some respect, a boundary may shift along a range of properties without becoming more definite, and so on. But, for normal purposes, or when people talk about the "history of ideas", the fact that so much remains unaltered while these changes take place is regarded as a sufficient reason for talking about "the same concept", or "the same meaning".

Here loose criteria for identity of meanings are used, and work as well as loose criteria of other sorts, as when we talk about "the same car", despite the minor repairs and replacements which it has suffered. In either case, where changes are too drastic, we may be unsure whether to say they are "the same" or not: any system of criteria for identity may work well in some circumstances and break down in others (see section 2.A).

In short, the complexity of semantic correlations, their indeterminateness, and the fact that in normal parlance we use looser criteria for identity of meanings or concepts than for identity of properties explains why oversimplified theories of universals will not work. But the argument of 2.D shows that there must be semantic correlations if there is to be anything definite about

the meanings of words, if concepts are to have any boundaries at all, however indeterminate, and that is why we need some explanation of the sort which I have tried to give, of various ways in which descriptive words may be correlated with observable properties.

4.B.2. The question now arises: how can we get away with so indefinite and imprecise a use of words? How can our ordinary language be "in order as it is"? (Cf. "Tractatus", 5.5563, and "Philosophical Investigations", I.98). To answer this, we must examine the purposes for which we speak and write, and the circumstances in which we do ^{so} ~~that~~, and this will show why it does not matter for our normal purposes that what we mean by our words is not perfectly precise and definite, no matter how distasteful logicians may find it. Of course, when our purposes change, or the circumstances change, (e.g. black swans are discovered, or unexpected counter-examples turn up to accepted mathematical theorems), then we may have to eliminate some of the indeterminateness. New scientific discoveries, or new purposes, may reveal to us previously unnoticed indeterminateness and force us to eliminate it. (How to eliminate it may be a matter for arbitrary decision, or it may be determined by questions of convenience.) The indeterminateness does not matter because until the new possibilities turn up (such as horses which produce giraffe-shaped off-spring) we need not consider how to describe them: indeed it would not only be futile, but quite impossible, to consider all such possibilities and adopt definite linguistic rules for dealing with them. (Read Wittgenstein's "Philosophical Investigations", I.65-88, 97-100, inclusive.)

4.B.3. It may help to explain why the meanings of our words contain so much indefiniteness, if we notice that our desires and intentions may be indefinite in the same sort of way. When a farmer instructs his employee: "Go to the market and buy me a horse", can we say that there is a definite kind of thing that he wants? Can he say honestly that there are definite limits to what would satisfy him? Could he be expected to take seriously the request for a precise specification of the range of possible shapes, colours, textures and kinds of behaviour the animal may have? Must he have an answer to the question: "Will you mind if the only horse I can find has a neck as long as a giraffe's?" If we can see why, in normal circumstances, this would be a stupid question for the employee to ask before setting out, then we shall see also why it would be stupid to expect people to use words with more precise meanings for normal purposes than they do in fact. (It is important that so little of our linguistic behaviour consists in merely making statements.)

If something works, then this is a justification for using it. Of course, something else may serve the same purposes better: but that is a question to be settled by experience, not by appeals to logical ideals of exactness and rigour.

4.B.4. All this may explain why it doesn't matter for normal purposes that words and sentences are used with indefinite meanings, but it does not really explain how this comes about. That can be seen by examining the conditions in which we first learn to speak.

Our initial lessons in the use of language must give us all the apparatus described in section 2.B. We

must learn what a language is, what it is to ask a question, make a statement, give a command, etc. We must learn what it is for statements to be true or false, for a word to describe an object correctly or incorrectly. We must learn to employ a conceptual scheme in which the world is seen as made up of enduring physical objects with shared properties and relations. General knowledge at all these different levels has to be picked up in addition to more specific knowledge of the rules correlating individual words with things and properties. A child cannot learn to use words with definite meanings without learning all these things. But he cannot acquire the general knowledge without first having learnt to speak: so, many different things must be learnt simultaneously, in a gradual and complicated process.

Progress must be made not only at different levels simultaneously, but also along a wide front. We do not first learn the precise meaning of one word, then the precise meaning of another, as if we could already speak and were learning the vocabulary of a new language by memorizing a dictionary. The clarity and definiteness of our understanding of many different words increases in one slow process. Only when we have already acquired a fair vocabulary and a considerable mastery of linguistic techniques, are we able to use meta-linguistic concepts (such as "meaning", "refers to", etc.) and thereby give our words relatively precise, simple and definite meanings. Not until an advanced stage has been reached can people intend their words to be governed by definite semantic rules of the kinds described in chapter three. Even so, the process of making our meanings more definite and

precise does not proceed beyond the point at which it serves our purposes to do so. We have already remarked (in 2.A.5.) that persons with special purposes, such as scientists, lawyers, or philosophers, may give ordinary words meanings more precise than in ordinary speech. The process can go on almost indefinitely. (See note at end of 2.B.)

4.B.5. Two reasons have been found why words are first of all used with indefinite meanings and only later on made more precise, namely because first of all a higher-level conceptual apparatus is required for making meanings precise, and secondly because many words are learnt simultaneously in a gradual process. But there is another reason, which is important if we wish to understand some of the things said by philosophers about the analytic-synthetic distinction.

The third reason is that much of the child's vocabulary is picked up from things said by people around him such as "Here comes Daddy", "Look, there's a kitten", "Would you like some more jam?" "Don't splash your milk", and these are complete statements using words, not definitions explaining them. The child does not hear things like: "The word 'red' is a descriptive word referring to the hue common to those three objects". The child learns which statements are true and which are false: but this cannot teach him the precise meanings of the words occurring in those statements. Knowing the meanings involves more than knowing that statements are true in certain conditions: it requires a knowledge of why the statements are true in those conditions, what it is about those circumstances which makes the statements true, or whether, for example, the statement would be true in

any circumstances at all.

For example, when a child hears someone utter a statement that is analytic or necessarily true, all that the child can learn is that it is true. More has to be said before he can learn that it is analytic, and usually not enough is said for the child to be able to answer all questions about precise meanings of words (not that he formulates these questions, of course). How much evidence is available to the child for him to learn whether "tadpole" is used to refer to the property of being a frog-larva, so that it is analytic that all tadpoles are frog-larvae, or whether it is merely a contingent, generally accepted fact?

In view of all this, it is perhaps remarkable that children do learn to use words with meanings at all. It is certain that they must take many a leap into the dark, extrapolating beyond what they can learn from the linguistic utterances of their elders. (Of course, they do not do this consciously.)

4.B.6. In view of all this, and also the fact that our powers of discrimination, etc., are limited in the way described in 4.A.3., it is not at all surprising that people learn to associate only relatively indeterminate meanings with words. In order to understand all the words in a statement in a definite way, one would have to know how to decide in all possible conditions whether that statement would be true or false (cf. 2.C.5.). But the child can only observe the use of words in actually existing conditions, and so it understands things in an indefinite way, until new experiences force it to remove some of the indefiniteness. As already pointed out, some

kinds of indefiniteness are never removed, and this does not matter for normal purposes (which is why loose criteria for identity of meanings are employed). But we shall see later on that it does matter when we try to apply the analytic-synthetic distinction.

4.B.7. The indefiniteness of which I have been speaking may manifest itself in many ways. There may be fluctuations in usage which can be observed in a whole society at any one time, or over a period of time. It may manifest itself in fluctuations in the usage of only one person over a period of time. Even at one time, the meaning with which a person understands a statement may be indefinite in any of the ways described, as is easily shown by asking someone how much sand is required in order to amount to a heap, or how long a cylinder has to be relative to its diameter in order to be describable as a "rod". ✓

4.B.8. We can now summarize the main points made so far.

(a) Indeterminateness, or indefiniteness in meaning, or the existence of borderline cases, may be a consequence of indefiniteness of the properties with which words are correlated, or it may be due to indeterminateness in the semantic rules correlating words with observable properties, or features.

(b) It may not always be possible clearly to distinguish these two causes of indefiniteness from each other, or from indeterminateness in usage which is due to limitations in our abilities to make fine discriminations, to survey complicated patterns or structures, or to memorize very specific features or complex properties. But in some cases the distinctions can be made.

(c) As we shall see later on, the indefiniteness of the meanings with which ordinary words are used, may make it impossible to apply sharp criteria for identity of meanings to ordinary statements, and so impossible to apply the analytic-synthetic distinction to some of those statements.

(d) Finally, we have seen how, within the framework of a theory of universals, to take account of complexities which could not be coped with by the one-one model. Universals can explain our use of descriptive words, and the existence of boundaries to empirical concepts, but not in a simple way.

4.C. Purely verbal rules

4.C.1. The description of various sorts of indefiniteness in the meanings with which words are normally understood, has helped to bring out one of the ways in which the description in chapter three gave an oversimplified picture of correlations between words and universals. But there is another oversimplification, which goes back to section 2.D, where it was argued (see 2.D.3.) that there was something wrong with Hampshire's claim that "In all cases, clarifying the use of a descriptive word or phrase is a process of drawing attention to its established links with other descriptions." This claim ignored the fact that if words are to have meanings, if they are to be able to occur in statements which are about anything, then they must be correlated not just with other words, but with non-linguistic entities. However, there is some truth in the claim, for it is possible for some aspects of the use of words to be determined by rules which merely

correlate words with other words, and this has been overlooked so far, engrossed as we have been with semantic correlations. This oversimplification must now be eliminated.

4.C.2. In many cases, setting up a correlation between words and other words has the effect of setting up a semantic correlation, between one word and the properties referred to by the other words, for example. Thus, if I rule that the word "gleen" is to describe objects if and only if they are correctly describable by the English words "glossy" and "green", then this correlates the word "gleen" with the combination of the properties referred to by the other two words, namely glossiness and greenness. Similarly, as shown in 3.B.4.a-b, the adoption of an incompatibility rule, correlating the word "RED", which primarily refers to a hue, with the word "redange", which refers to a specific shade of colour on the red-orange boundary, may help to make the meaning of "RED" less indefinite than it would be otherwise, by correlating it negatively with a specific shade.

Sometimes, however, we may adopt a purely verbal rule, which merely correlates a word with another word, in cases where indeterminateness of meaning gives rise to indeterminateness of relations between descriptive words. Thus, suppose the relation between the words "red" and "orange" is indeterminate, owing to the fact that there are difficult borderline cases which are neither definitely red nor definitely not red, and at the same time neither definitely orange nor definitely not orange. (It does not matter for the illustration, whether this is due to indefiniteness of the hues referred to by the two words, or to indeterminateness of the

boundaries of ranges of specific shades, if they are used as d-words.) In such a case a rule may be adopted to the effect that "red" and "orange" are incompatible descriptions. We may call this a purely verbal (strong) n-rule.

4.C.3. Such a rule, unlike the one correlating "RED" and "redange", leaves the two concepts it correlates as indefinite as they were without it, for it does not say which objects are to be described as "red" and which as "orange": it leaves borderline cases as undecided as ever. But it does mean that if one makes a decision about the use of the word "red" in some of these cases, then one may be committed to a decision concerning "orange": the two lots of problems about borderline cases may not be settled quite independently. So it rules out the possibility of the truth of some statements, such as "This box is red and orange all over" and the falsity of some others such as "Nothing is red and orange all over at the same time". This will turn out to be important when we come to look for a definition of "analytic", for no definition will do which does not make this last statement analytic in cases where the words are governed by the incompatibility rule under discussion.

4.C.4. Since such a rule does not provide more determinate boundaries to the extension of either word, it might be thought to be a completely useless sort of rule, and so it almost is, for no situation can be described any more precisely after it has been adopted than before. But it does have the advantage of making a "second-order"

concept more definite: the concept of a hue. If we have no way of telling what sorts of things hues are, except by saying that hues are referred to by hue-words, then, if we adopt the linguistic convention that hue-words must be incompatible with one another, this helps to make the concept of a hue more definite, since it has the effect that no two hues may occur together, despite the fact that it does not make any of the individual hue-concepts any more definite. (Compare this with Dummett's example, in Phil.Rev., July, 1959, p.328.) All this may be of some use in enabling us to formulate some scientific theory or other kind of theory about hues in a simple or elegant way, without fear of embarrassing counter-examples in borderline cases. At present I am not concerned to show what the point of such an incompatibility convention might be, so much as to show that there might be such a convention, and thus to illustrate the fact that there is some truth in the claim that "links between descriptive expressions" may have to be described if the full meanings of some words are to be described.

4.0.5. This completes my account of ways in which chapter three contained an oversimplified description of the linguistic conventions giving our descriptive words their meanings. Not all oversimplifications have been pointed out and the description contains many omissions, but it will have to do.

The next chapter attempts to show how it is that once the semantic rules have been adopted, which determine which objects are describable by which words, those words may be combined with other words to form statements which may be true or false.

Chapter Five

LOGICAL FORM AND LOGICAL TRUTH

Introduction

We are now ready to set out upon the last lap of Part Two, in which our main aim has been to explain how certain kinds of words and sentences can have the meanings they do have, and how their having these meanings helps to determine the conditions in which propositions which they are used to express are true. This explanation serves two important purposes. First of all, it provides an answer to the question: What sorts of things are propositions, the entities to which the analytic-synthetic and necessary-contingent distinctions are to be applied? (Cf. 2.A.1.) Secondly it helps to display the general connection between truth and meaning, between knowing and understanding, at least in a certain class of cases. This prepares the way for the discussion of some more restricted kinds of connection, in Part Three. (Part of that discussion will be anticipated in the present chapter.)

So far, except for a few rather vague and general remarks in chapter two, we have been concerned only with descriptive words, and have seen how semantic correlations between them and universals (observable properties and relations) can determine which particular objects they describe correctly, and which they do not describe correctly, depending on whether those objects are or are not instances of the universals referred to. This, however, is not the full story of what happens when such words are put together with other words to form sentences expressing propositions. In addition, we have to describe the

functions of the other words. Even if there were no other sorts of words, even if it were possible to make statements just by combining descriptive words, we should have to discuss the way in which the method of construction of a sentence contributed towards its having a certain meaning in cases where the meaning depends on how the words are arranged. In short, we must explain what the logical form of a proposition is, and how logical words and constructions work. This will now be done.

First of all, an attempt will be made to say what logical words and constructions are, that is, to characterize the notion of a "logical constant". Then the way in which the logical constants occurring in a statement help to determine its truth-conditions will be described. Finally a discussion of what makes a proposition a formal truth (i.e. true in virtue of its logical form) will serve as an introduction to some of the problems of Part Three.

5.A. Logic and syntax

5.A.1. In the sentences "Fido is black", "All cubes have plane faces", we seem to be able to distinguish words which refer to entities, whether particular objects or universals, and words which do not. Among the former are "Fido", "black", "cubes". "All" and "is" are in the latter class. The latter are commonly described as "logical" words, or "logical constants", and in this section I wish to discuss the rationale behind our selection of some words to be described as "logical", while others are "non-logical". What is so special about the words "all", "is", "not", "some", "and", etc.?

One answer which is sometimes given to this question

is that these logical words are governed by linguistic rules which are purely syntactical. That is to say, unlike the semantic rules which correlate "Fido" and "black" with non-linguistic entities (material objects or properties), the rules for the use of logical words merely correlate words with other expressions, never with non-linguistic entities. After all, such words can occur in statements which can be seen to be true merely by examining their structure, that is, merely by examining the way in which logical words occur in them, for they can occur in formal truths, such as "It is raining or it is not raining", i.e. in statements which are true in virtue of their logical form. It is claimed that all that is relevant to their being true is their syntax, or their verbal structure, whence it follows that the linguistic rules which give the logical words their meanings, since they permit structure to generate truth, must surely be rules which do not refer to anything other than verbal structure. That is, they must be purely syntactical rules.

In addition, it is sometimes argued that formal systems of axioms and rules of inference, such as any standard formulation of the propositional calculus, serve to define the primitive symbols occurring in the axioms and rules, and that these primitive symbols are our ordinary logical words. Since the axioms and rules of a formal system are concerned only with symbols and relations between symbols, no mention being made of anything non-linguistic, it appears that the rules which "define" logical words are purely syntactical.

Despite all this, I think it can be shown that the assertion that logical words are governed by purely syntactical rules is either false or so vague as to be

quite misleading.

5.A.2. Let us now see what is wrong with saying that the rules governing logical words are purely syntactical.

When we learn to use the truth-functional connectives, such as "or", and other logical words, we do not learn to use them only in logically true propositions, such as "It is raining or it is not raining", for they may occur also in sentences like "My book is on the table or you have moved it" and "Dawn is breaking or the moon is still shining". Now, how can the meaning of "or" contribute to the meanings of these sentences? How do we learn the principles according to which logical words work? This is something which has often worried people. For example, Pap, who was sure that logical words could not be ostensively defined, wrote:

"The analogy between interpretation of descriptive constants and interpretation of logical constants seems to break down: in the case of descriptive constants we can, after having reached the primitives, go on to ostensive definition, since there is something 'in the world' which they designate. But what would it be like to show the semantic meaning of the primitive LOGICAL constants of a natural language, such as the English word 'or'?"
 ("Semantics and Necessary Truth", p.364. Cf. p.366.)

Faced with this problem, some philosophers have been driven to talk about subjective feelings of "hesitation" or "indecision" which are correlated with such logical words. Others, rightly rejecting this, have gone to the other extreme and abandoned the search for anything which can be correlated with such words, taking refuge in the thesis that logic can be reduced to syntax.

Surely the correct answer is that learning the meaning or function of a logical word involves learning

how to recognize the states of affairs in which statements using the word are true or false, just as learning the meaning of a descriptive word involves learning to recognize the states of affairs in which sentences employing it are true or false?

5.A.3. Consider the word "is", in the statement "Fido is black". In order to understand how it works one must, in effect, learn the following: "A sentence may be made up of a referring expression (i.e. an expression referring to a particular object), the word 'is' and a descriptive expression, in that sequence. In order to discover whether the statement expressed by the sentence is true or false, examine the particular object referred to and see whether it has the property (or combination of properties) correlated with the descriptive word." This rule does not correlate the word "is" with any one entity, but it certainly is concerned with non-linguistic entities, though in a very general way. In order to discover whether statements using the word are true, it is not enough to examine the structure of the sentences expressing those statements. The same goes for contingent statements using the word "or". In order to understand its role in a sentence such as "Dawn is breaking or the moon is still shining", one must (at least implicitly) learn the following rule: "If S and S' are sentences expressing statements, then a new statement may be expressed by the sentence consisting of S followed by the word 'or' followed by S'. In order to discover whether the new statement is true or false, examine the facts (i.e. look to see how things are in the world) and see whether a state of affairs obtains in which at least one of the statements expressed by S and S' is true or

whether both are false". (An understanding of the two sentences S and S' is, of course, presupposed.)

In this latter case, as in the former, whether the statement is true or false does not depend merely on the structure of the sentence expressing it, and the rule for the use of "or" does not refer only to syntactical properties of sentences, but also to states of affairs, which are non-linguistic entities. It is concerned with how things are in the world, with the facts in virtue of which the disjuncts express true or false propositions. What, therefore, is left of the assertion that the rules for the use of logical words are purely syntactical?

5.A.4. We can see what has happened here. Rules for the use of logical constants are extremely general. The rule for "is" does not correlate it with any particular object or set of objects, nor with any particular property or set of properties (or set of describability conditions). The rule for "or" does not correlate it with any specific state of affairs, but with all kinds of states of affairs. The rules are highly non-specific: they concern objects, but no specific kinds of objects; properties, but no specific kinds of properties; and states of affairs, but no specific kinds of states of affairs. The rules are "topic-neutral". They allow logical constants to occur in statements which are about anything at all: they are not restricted to statements concerning certain topics. The word "or" has the same function in "That table is wet or highly polished" and in "She is unhappy or unwell". Since the rules governing the use of logical words are topic-neutral, one cannot discover anything about the specific subject-matter of a

statement from the fact that such a word occurs in it, as one could if the word "table" occurred in it. This makes it seem that such words are not correlated with anything non-linguistic, that they are governed by purely syntactical rules.

5.A.5. It was argued that the rules for the use of logical constants must be purely syntactical since they had the consequence that statements like "It is raining or it is not raining" can be seen to be true without examining anything non-linguistic. But in order to know that such a sentence expresses a truth, it is not enough to see the marks of which it is made up: one must know also how they contribute to the meaning of the sentence as a whole, and this involves knowing in general when a sentence containing these words expresses a truth, including cases where the truth is contingent. It is not enough to know the visible structure of the sentence: one must know the functions of the various things which make up this structure. The function of a symbol is not a syntactical property, even when it is as general a function as that of the word "or". I shall show later on how a knowledge of these general functions may enable us to discover truth-values without empirical enquiry, in "freak" cases.

5.A.6. We can now see also that the argument in terms of definability of logical words by means of formal systems (see 5.A.1.) falls away, for, since a formal system cannot define the use of words in contingent statements, it cannot fully define the function of symbols like "or". (It cannot define the use of words in logically true statements either. See 2.D.3, and

2.D.3. note. Compare Appendix II.) Truth-tables can, of course, be used to explain the use of some logical words, but only for someone who knows what truth is, and, as already remarked, since propositions are, in general, true or false in virtue of how things are in the world, truth-table definitions do not merely correlate symbols with other symbols.

5.A.7. It may be objected that I have missed the point of the assertion that logic can be reduced to syntax. It is true that I have in fact used the very arguments employed by some people to defend the assertion. But then the word "syntax" is used in a sense which must be clearly defined if it is not to lead to confusion. For example, when Wittgenstein argued (in "Tractatus") that the truth of a logical proposition can be perceived in the symbol alone (6.126, 6.113), he did not regard a symbol as just a sign (e.g. a mark on paper or a sound), but a sign with a use, i.e. a sign standing in a "projective relation to the world". (See 3.12, 3.32, 3.321, 3.327, 3.262.) So when he talked about "logical syntax", he was not concerned only with what is now often meant by "syntax", namely something concerned only with combinatorial geometrical properties of signs. This, unfortunately, is not realized by some who think that the "Tractatus" supports their belief that logic can be reduced to syntax.

5.A.8. If I explain what it would be like for a logical constant to be defined by purely syntactical rules, this may make it clear exactly what I am denying. A "purely syntactical" rule for the use of a word, as I understand

the term, would specify that the truth or falsity of statements making use of that word could in all cases be determined entirely from a consideration of its verbal form, without considering the meanings of any of the words in it which were correlated with non-linguistic entities, and without considering how things are in the world.

The following rule introduces into the English language two new words, "plit" and "plat". Each word may occur at most once in any sentence, right at the beginning or right at the end. If "plit" occurs at the beginning, or "plat" at the end, the sentence expresses a truth. If "plit" occurs at the end, or "plat" at the beginning, the sentence expresses a false proposition. So "Plit the sun is shining", "It is dark plat" and "Plit it is raining plat" all expresses true propositions, and "It is raining plit" and "Plat all men are mortal" both express false propositions. In all cases this can be seen merely by examining the structures of the sentences.

We can tell whether the propositions expressed by such sentences are true or false: but what propositions are they? Do such sentences say anything? Is there any difference in meaning between "Plit the sun is shining" and "It is dark plat"? Do these sentences say the same thing or something different? The mere fact that the words "true" and "false" are used in formulating rules for the use of "plit" and "plat" is not enough to guarantee that they have anything to do with true or false propositions. Similar remarks may be made about theorems in a formal system, considered simply as theorems in a formal system: what has being a theorem, that is a formula derivable according to fixed rules from a specified

combination of symbols, to do with truth or falsehood? What can mere syntactical properties have to do with truth?

5.A.9. This example shows that although there may be words whose "use" is governed by purely syntactical rules, our ordinary logical words, such as "not", "and", "is", "or", etc., are not like that, for, unlike "plit" and "plat", they can occur in sentences expressing contingent propositions, whose truth has to be established by observation. They can occur in formal truths or formal falsehoods, whose truth-value can be discovered without considering how things are in the world, but only because they have a more general employment can we speak of truths or propositions in such cases. We shall see later on, that such formal truths are merely degenerate cases and that empirical enquiries can be used to show that they are true, despite the fact that they are not necessary. (See 5.0.7.) Thus, if normal observations show that dawn is breaking, then this establishes the truth of "Dawn is breaking or dawn is not breaking", though it could be established otherwise. How could empirical observation even be relevant if such a statement were true in virtue of syntactical properties? (Compare: "Concepts which occur in 'necessary' propositions must also occur and have a meaning in non-necessary ones." Wittgenstein, "Remarks on the Foundations of Mathematics", part IV, 41. Compare also "Tractatus" 6.124.)

5.A.10. All this arose out of the question: "What is special about the logical words, that distinguishes them from descriptive words or proper names?" We have found

that it is not true in any easily intelligible sense that they are distinguished by being governed by purely syntactical rules.

One peculiarity which has turned up (see 5.A.4.) is that their rules are topic-neutral, that is, so general as not to mention any specific kind of subject-matter. For example, rules for the use of truth-functional connectives are concerned only with whether something or other is the case, without being concerned with what kind of thing is the case. Other logical words also exhibit this topic-neutrality. From the mere fact that the word "is" occurs in the sentence "My car is green", one cannot discover what the sentence is about, or what sort of thing it is about. One can tell only that if it states a truth then the particular object referred to in it satisfies the describability-conditions for the descriptive word occurring in it. This tells us nothing about which object it is, nor what kinds of properties are referred to by the descriptive word.

Topic-neutrality is not only displayed by logical words. For example, in the sentence "All red horses are red", the fact that the same word occurs in the second and fifth places is a topic-neutral aspect of the sentence which helps to specify its meaning. The fact that the second word is a descriptive expression rather than a referring expression is likewise a topic-neutral aspect. We may say that all these topic-neutral aspects of propositions are logical constants.

5.A.11. I shall take topic-neutrality of words or constructions as a necessary condition for being a logical

constant. But it is not sufficient. For example, there is a sense in which the word "good" is topic-neutral, but I do not wish to say that it is a logical constant in "He has a good appetite". In addition, there are words, such as "alas", which may occur in sentences without restricting their subject-matter in any way, but which will not be described as logical constants, since they are not relevant to the question of truth or falsity of the propositions expressed. As already remarked (in 2.B.9), the study of truth-conditions is only one aspect of the study of meanings.¹

5.A.12. The following, therefore, will serve as our definition of "logical constant":

The expression "logical constant" describes any word or feature or aspect of a sentence which helps to determine whether the sentence expresses a true or a false proposition, and which is topic-neutral, so that from the fact that it occurs in the sentence one cannot discover what things or kinds of things that statement is about.

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1. There are many expressions which are vaguely like logical constants, at least in so far as they are topic-neutral, but are not immediately relevant to questions of truth and falsity. Examples are "incidentally", "however", "moreover", "perhaps", "probably", "nevertheless", "it seems that ...", "of course," "obviously", etc. These all have some kind of "pragmatic" meaning. That is, they concern some relation between the speaker or hearer and what is being said (e.g. surprise, or absence of surprise, hesitancy, etc.). They are of the same general nature as the following: "As you would expect ...", "Much to my surprise", "It is clear to me that", "In my opinion, for what it's worth", "I am inclined to think that", "The evidence available to me seems to show that", "I confidently predict that ...", and so on. (See remark about appropriateness-conditions in 2.B.9.)

(It should be recalled that I am talking only about relatively simple statements using words which can refer to or describe material objects in virtue of their observable properties and relations. Cf. 1.C.2.)

In so far as two or more statements have logical constants in common, we can describe them as having a common "logical form". The logical form of a proposition, therefore, is determined by the topic-neutral words or constructions used to express it which help to determine the conditions in which it is true or false. (Cf. Section 5.B, and 5.E.4.)

In general, the sentences which we use to make statements or ask questions about things in the material world contain some parts or features which determine the particular things and properties talked about, and others which do not limit the subject-matter in any way, but merely help to determine the kind of thing which is being said, or the way in which it is said. The latter are the logical constants and comprise the logical form of the statements. Thus, the sentences "All books are red", "Not all books are red", and "Some books are red" say different kinds of things though the subject-matter is the same: they refer to the same things and properties. Similarly, in each of the following pairs of sentences the same kind of statement is made, but about different topics:

- (1) "All books are red" and "All horses are four-legged."
- (2) "Fido is not black" and "Socrates is not triangular".¹

1. We could generalize this and show how logical constants determine whether a statement, question or command is expressed. Cf. 2.B.3.)

5.A.13. We see therefore, that the logical forms of propositions correspond to common "structures" of sentences (where the occurrence of logical words and other topic-neutral aspects counts as part of the structure.) The logical form common to a pair of propositions may be represented symbolically in the usual way, by removing all non-logical words and replacing them by symbols called "variables" which indicate the kinds of words whose places they take, and whether the same word occupies two or more places in the sentence. These symbols may be described as "sentence-matrices", such as "All P's are Q", or "x is not P". Different kinds of letters may be used to indicate positions of different kinds of non-logical words, such as capitals for descriptive words or expressions, and small letters for expressions referring to particulars. Such a sentence-matrix represents the structure common to a family of sentences obtained by replacing the variable-letters by suitable non-logical words. It therefore also represents the logical form corresponding to that structure. (The symbol is not itself the structure, any more than a blue-print is the structure of the houses whose common structure it represents. The structure is a property of statements. It is neither a symbol nor a physical object, and you will not find it left behind when the things which are not the structure are removed from a sentence, any more than the structure of a house is left behind when the bricks and other materials are removed. This point can cause confusion and lead to talk about "unsaturated" entities, which cannot exist on their own, etc.)

5.A.14. It is possible to study the logical properties

and relations of propositions by classifying them according to their logical forms (for reasons which will become evident later on). Symbols are usually used to represent those logical forms in the manner just described, and systems of symbols represent sets of propositions. In Appendix II, I shall try to show briefly how a concentration on the geometrical properties of such systems of symbolic representations can lead to muddles about logic. Even where it does not lead to muddles, this concentration on geometrical (or syntactical) properties of sentences or symbols, cannot, unless accompanied by a study of the functions of words and symbols in determining the meanings of sentences, explain anything. It can, at best, lead to description and classification of logical properties of propositions and inferences. I shall try to explain, leaving the classification to others.

5.A.15. To summarize. I have tried to show that the distinguishing feature of logical constants is not that they are governed by syntactical rules, but the fact that their rules are topic-neutral. I have not yet described in any detail the ways in which such words and constructions contribute to the meanings of sentences which include them, and this will be attempted in the following sections. I hope eventually to provide an explanation of the fact that some propositions are formal truths (i.e. true in virtue of their logical form) and the fact that some inferences are formally valid (i.e. valid in virtue of their logical form), by showing how this comes about.

Once we understand what sorts of functions logical words can have, we can see what is involved in using a

logical word with one meaning rather than another, and can apply criteria for identity of meanings of logical words. This explains how it makes sense to say that the English word "and" means the same as the German word "und", and could take us one step further in the programme of chapter two. (See 2.A.) But I shall not go into this aspect in any detail, since ambiguity of logical constants does not cause as much trouble in connection with the analytic-synthetic distinction as ambiguity of descriptive words. (See 2.C.)

5.B. Logical techniques

5.B.1. So far I have explained in a vague sort of way how to pick out those parts or aspects of sentences which are purely logical, namely by seeing whether they are topic-neutral. I have not yet said how they work, how their occurrence in sentences contributes to the meanings of statements which they express, but will do so now. The explanation will be extended in the next section to show how it is possible for a statement to be true in virtue of its logical form. Later on, the account will be generalized to show how it is possible for a statement to be analytic.

My descriptions of the functions of logical words will have to be greatly oversimplified, and it will not be possible to make more than a few qualifications near the end, in 5.E.

5.B.2. If, as pointed out in the previous section, it will not do to say that the functions of logical words, such as "or" and "not" are defined by the recursive rules of a formal system, then how can we explain what their

functions are? What are the topic-neutral rules which enable them to occur significantly in sentences expressing contingently true or false propositions? The answer, as suggested in 5.A.2, seems to be that the rules help to specify the conditions in which statements employing those words are true, and conditions in which they are false. Learning to use logical words and constructions in sentences involves learning general principles for recognizing conditions in which statements are true or false. The rest of the chapter will simply be an amplification of this statement.

(It is notorious that no matter how much one says about what words mean, it is always possible for the objection to be made that the account is either circular, since it presupposes what it explains, or incomplete, since it presupposes something else - one of the facts which seems to have led to the doctrine of the "unsayable" in Wittgenstein's "Tractatus". So the most that I can hope to do is draw attention to certain features of what we all know about our use of logical words, in the hope that this will remove some puzzles. When I say "we all know", I do not wish to imply that we can explicitly formulate the knowledge. See Appendix III on "Implicit Knowledge".)

5.B.3. Frege, Russell and Wittgenstein each tried at some stage to make use of the notion of a function (not in the sense of a "role", but in the sense defined below) to explain our use of logical and other words. I shall use a slightly different notion, the notion of what I call a "rogator", which will be explained presently. First I shall say what is meant by a function. (The account will be brief, as the notion is familiar.) Then I shall

show why talking about such things seems to help, and after that I shall offer my own variation on the theme.

5.B.3.a. The notion of a "function" may be defined as follows. A function is a rule, or a principle or a mapping which correlates entities, called "arguments", with other entities, called "values". More precisely, a function correlates sets of arguments with values, one value to each argument-set. A given function will have a restricted domain of definition, so that not every set of objects can be an argument-set with which the function correlates a value. The class of argument-sets for which a function has a value is called its "domain", or "domain of definition". A function is defined, or set up, by specifying a domain of definition, and by stipulating either some generally applicable principle or technique, or method of calculation, which enables the value of the function to be discovered for every argument-set of its domain, or simply by enumerating the arguments and the values correlated with them. (A function "has" or "yields" a value for a given argument-set, viz. the one which it correlates with the set. The argument-set "yields" a value for that function. The function may be said to be "applied" to its arguments, or to argument-sets, to yield its values.) Normally the value of any function for an argument-set depends on the order of the arguments in the set, and if we restrict ourselves to functions whose domains contain only ordered sets with the same number of elements, say n , then we can speak of the function as "having n argument places" or as an "n-ary function", and can speak of an argument as occurring "in the i -th place" in some argument set. The number of argument-places is usually indicated in a name or sign

for the function by a string of so called "variable-letters", one for each place. (Sometimes the letters are called simply "variables".)

5.B.3.b. If words or signs referring to arguments are substituted for each of the variable-letters in the sign for the function, and if the ordered set of objects corresponding to the ordered set of argument-signs lies in the domain of definition of the function, then the new sign thus obtained is taken as referring to the value of the function for that argument-set (or, more simply, for those arguments). Thus, the sign " $x + y$ " is a sign for the arithmetical function, addition, and substitution of the numerals "2" and "3" for the variables yields the sign " $2 + 3$ " which refers to the number which is the value of the function for the set of numbers (2,3), namely, the number 5. A set of names or signs for arguments which form an argument-set, is called an "argument-name-set", or, more shortly "name-set".

5.B.3.c. When the domain of definition is restricted so that some argument-places may be taken only by objects of one kind, and some argument-places only by objects of another kind, this may be indicated by a convention using special kinds of variable-letters to indicate kinds of arguments. (Cf. 5.A.13.)

Where the objects which are taken as values of a function can all occur as members of the argument-sets for which the function is defined, the function is called an "operator," and its application may be "re-iterated". (E.g. re-iterated application of addition is symbolized thus: " $x + (y + z)$ ".) Where there is a family of functions, such that the values of some may occur

as the arguments of others, and the values of the latter may be arguments for still others, etc., the functions may be applied successively, as is customarily indicated by such notations as " $F(x, g(y, h(z, w)), u)$ " for the successive application of the functions $F(x, y, z)$, $g(x, y)$ and $h(x, y)$. In this way more and more complex functions may be built up by successive application. The rules or techniques for finding values of such complex functions are derived from or constructed out of the rules or techniques of the component functions.

5.B.3.d. An example of a kind of function which is often an operator is what I shall call the "name-function" of a function. If " $F(x, y, z)$ " is the sign for a function, then, as shown by the above remarks, there will always be another function, called the "name-function" for $F(x, y, z)$, which takes as its argument-sets name-sets¹ for the function F , and yields as its values the signs obtained by substituting names of arguments for variables in the sign for F . As the sign for the name-function, I write " $/F/$ " or " $/F(x, y, z,)/$ ", enclosing the name of the function between strokes. So, in this case, if " a ", " b ", " c " are names of arguments, then (a, b, c) is an argument-set for $F(x, y, z,)$, and $(\text{"a"}, \text{"b"}, \text{"c"})$, being the corresponding name-set, is an argument-set for $/F(x, y, z)/$, and its value, viz. $/F(\text{"a"}, \text{"b"}, \text{"c"})/$, is the name " $F(a, b, c)$ ". [There is a whole hierarchy of name-functions, since to the function $/F(x, y, z)/$, there corresponds the name-function $//F(x, y, z)//$, taking for its values such signs as $/F(\text{"a"}, \text{"b"}, \text{"c"})/$, which, as just

1. See 5.B.3.b, above.

indicated, is a name for the name " $F(a,b,c)$ ".] Mathematicians and philosophers often confuse functions and their corresponding name-functions, and so also arguments and names of arguments, values and names of values. Sometimes this does not matter as the context makes clear what is meant. But it does matter when attempts are made to explain what propositions are in terms of the notion of a function. (Note: the name-function is not a sign for the name of a function. It is the sign for a rule which is applied by substituting names of arguments for variable-letters in the name of a function.)

5.B.4. Now let us return to the question: how do logical constants contribute towards the meanings of statements which employ them?

We may recall the fact, mentioned in 5.A.13, that the logical form of a statement, i.e. the way in which logical words and constructions occur in it, can be represented symbolically by sentence-matrix in which the non-logical words of the sentence expressing the statement are replaced by variable-letters. Thus, starting with the statement "Fido is a dog, and all dogs are four-legged", we obtain the matrix: " x is a P , and all P 's are Q ". We have here something strongly reminiscent of the notation for functions, and this tempted Frege, for example, to say that the original sentence must be the name of a value of a function. He wished to say that the thing named by the sentence (i.e. the value of the function for Fido, etc., as arguments), was a truth-value, the True or the False. This seemed odd, because what the sentence was a name of, i.e. its truth-value,

must depend on how things are in the world, whereas what we understand by the sentence (or by a name usually) does not depend on the facts, such as whether Fido really is a dog. Frege, of course, was not faced with this sort of difficulty, since he, unfortunately for logic, was interested mainly in mathematical propositions, whose truth-value does not depend on contingent facts.

5.B.5. One way out of the difficulty, would be to say that the sentence is the name of a proposition, which is the value of the function "x is a P, and all P's are Q" for the arguments (Fido, dog, dog, four-legged), but this would be of no use for our programme, since we are concerned to explain what propositions are, and so must not assume a knowledge of what they are.

I believe that we could regard a sentence as naming a class of possible states of affairs (possible states of the world). But I think there is a more illuminating way of looking at things, which makes it easier to explain how a statement may be true in virtue of its logical form, or in virtue of what it means. We may allow that a sentence corresponds to a truth value. But the meaning of the sentence, what is understood by it, is a method for discovering truth-values.

Meanwhile we may notice one thing. To every sentence-matrix, no matter what function or other entity it represents, there corresponds a name-function (see 5.B.3.d), which takes non-logical expressions as arguments, such as "Fido", "Willie", "The animal under the table", "dog", "horse", "four-legged", "hungry", etc., and yields sentences as values. Thus, the function /"x is a P, and all P's are Q"/ may take as a value the sentence "The

animal under the table is a horse and all horses are hungry". Sentences, therefore, may be regarded as values of name-functions.

5.B.6. Now in order to show how logical constants contribute towards the meaning of a sentence, I wish to introduce a new concept, the concept of a rogator, which is something like the concept of a function, but not quite. A function is a rule or principle which yields a value for an argument-set, the value being determined by the rule and the argument-set, whereas a rogator is something which does not fully determine the value but to which there corresponds a method or technique for discovering the value, which (i.e., the value) may depend on contingent facts having nothing to do with the rogator itself, or the principle on which it works.

A simple example of a rogator is the following, $R(x)$, which takes bottles for its arguments and the sun, the moon or the earth for its values. In learning how to find out the value of the rogator for any particular argument, i.e. any particular bottle, one must learn to apply the following technique:

Examine the bottle to see whether it is empty or contains liquid, and, depending on whether it is empty, less than half full of some liquid, or half full or more, write down, respectively, "the sun", "the moon" or "the earth". What has been written down is then the name of the value of the rogator $R(x)$ for the bottle in question (at the time of observation).

In this example, as in general, in order to know what the value of a rogator is for a given argument, one must know what the argument is (i.e. which object it is), one must know the general technique for determining the value, and one must know certain facts, or have performed

experiments. The argument and technique alone do not determine the value, for that depends also on what the facts are, and the value of a rogator for given arguments may change from time to time. (E.g. emptying a bottle may change its value for $R(x)$ from the earth to the sun.) A time-dependent rogator can always be turned into one which is not time-dependent by adding an argument-place for a time, or time indicator. It is important to notice that the technique for applying a rogator (for determining its values) may be learnt by example, and memorized, without the aid of an explicit description of the way in which it is applied. (See Appendix III.)

5.B.6. (note). Frege did not need to talk about rogators since he was concerned with mathematics, in which the values of functions are determined by general principles, independently of empirical facts. Of course, from a certain point of view, which takes account only of the way things actually are in the world, and not of what might have been the case, the notion of a rogator collapses into that of a function. But one cannot develop a complete theory of meaning without taking into account possibilities as well as actual states of affairs, since to understand a sentence is not merely to know whether it is true or false. (See 2.C.5 and 4.B.6.) This is why "extensional" systems of logic are of limited interest.

5.B.7. I wish now to describe a more interesting kind of rogator, illustrated by a game played with the aid of arithmetical symbols, and in particular the symbols for addition and multiplication, $(x + y)$ and $(x.y)$.

The game is played as follows. A machine, or some

person (God) continually churns out little boxes, in each of which is a slip of paper with a numeral, the name of a positive or negative integer, such as "3", "-27", "3862". (0 is taken to be a positive integer.) On each box is written a letter or other sign, which is described as its "name", e.g. "A", "B", etc., there being no principle connecting the name on the box with the numeral inside it.

The players make their "moves" in turn, by selecting a name-function of some arithmetical function compounded by successive application (see 5.B.3.c.) of addition and multiplication [e.g. the function $x.(x.(x.x))$, or $(5.x + y^2.z).(x + 3.w)$, and so on]. This name-function is then applied to an argument-set consisting of an ordered set of names of boxes. Thus if a player selects the name-function $/x + y.z/$, and the names, "A", "B" and "C", then he will make his move by reading out "A + B.C" or "A plus B times C".

Each move is awarded a tick or a cross, as follows. The boxes corresponding to the selected names are examined and the value of the arithmetical function worked out for the numbers referred to in those boxes as arguments. Thus, if the numerals in the boxes named are found to be "5", "-2" and "3", then the value of the function in the case illustrated will be -1, that is, $5 + (-2).3$. A tick is awarded if the value is positive, a cross if it is negative.

The next player then makes his move, in the same way, by selecting a function and set of names and "applying" the function to the names, being awarded a tick or a cross, depending on the results of examining the boxes so named and calculating the value of the function. (The player with most ticks is said to be "winning".)

5.B.3. This game provides us with a new kind of rogator, which takes boxes or names of boxes as arguments and ticks and crosses, or perhaps the words "tick" and "cross", as values. Though derived from arithmetical functions these rogators have different domains of definition, and different domains of values, from arithmetical functions, and they do not fully determine their values for given sets of arguments (for that depends on which numerals happen to be in which boxes).

Learning to play the game involves learning certain techniques, such as the technique of calculating the values of arithmetical functions for particular sets of numerical arguments. But this is not all. One must know how to decide whether a "move" is to be awarded a tick or a cross, and this involves knowing how, given the ordered set of names used in the move, and the function employed, to select the appropriate boxes, look at the numerals inside them, calculate the value of the function, and then say "tick" or "cross", depending on what comes out of the calculation.

We see therefore that a complicated technique must be mastered by anyone who wishes to play the game. It is a general or uniform technique, since new boxes are continually being produced, with new names on them and new numerals inside them, and one must know how to deal with whatever turns up, and not just how to work with the first twenty boxes which appear (e.g. by memorizing the numerals inside them, and their values for a certain set of functions). We can learn to apply such general techniques quite easily, for example by watching others and being given instruction in elementary arithmetic. We need not, however, either hear, nor be able to formulate, any explicit description of the techniques.

Thus, knowing which rogator is involved in a move, means knowing how to apply a general technique for awarding a tick or a cross, given a set of names. We may represent these rogators by means of symbols, such as " $P + Q.R$ ", or " $5.P^2$ ", etc., where " P " and " Q " etc., are variable-letters which indicate that argument-places are to be filled by names of boxes, and the whole symbol indicates which technique is to be applied for working out the value of the rogator.

To each rogator there corresponds a name-function (5.B.3.d.) very like that which corresponds to the arithmetical function from which it is derived except that one takes names of boxes (or names of names of boxes) as arguments, and the other takes names of numbers.

5.B.9. It should be clear now what I am getting at. Instead of regarding the symbols (sentence-matrices) which represent the logical forms of propositions as corresponding to functions (see 5.B.4), I shall regard them as corresponding to rogators, which will be described as "logical rogators". They may be represented by such symbols as " x is P ", " $\text{All } P \text{ } Q\text{'s are } R$ ", " x is a Q and there are no R 's which are S ", etc. Corresponding to them are ^{also} name-functions which take sentences as values. (See end of 5.B.5.)

5.B.10. We can regard rogators as taking either things or words which refer to them as arguments. For reasons of convenience of exposition I shall describe logical rogators as taking meaningful non-logical descriptive words and referring expressions as arguments, their

argument-places being represented by higher-case and lower-case variable-letters, respectively. We could, instead, talk about the things referred to as the arguments. (A rogator, may, like a function, have a restricted domain of definition. See 5.B.3.a and 5.B.3.c.) For the time being I shall take the words "true" and "false" to be the values of logical rogators. (But see 5.B.18, below.)

To each rogator there corresponds a general technique, which I shall describe as a "logical technique" for determining its values, given an argument-set. The technique involves looking at non-linguistic entities and then deciding to award the word "true" or the word "false" for the "move" in language which is (or would be) made by uttering the sentence obtained by replacing variable-letters in the sign for a logical rogator by suitable arguments for that rogator.

5.B.11. For example, for applying the logical rogator "All P Q's are R", (derived from the logical form of statements like "All red boxes are square"), we might use the following technique. Given an argument-set of three descriptive words, seek out the objects having the properties referred to by the first two words, examine each of them to see whether it has the property referred to by the third descriptive word, writing down a tick if it has, or otherwise a cross. When finished, look to see if there is a cross amongst the things written down; if not the value is "true" and otherwise "false".

In 5.A.3, we have already described the technique for the use of the copula, in the rogator "x is P".

It is not essential that the techniques should be described in these ways. There may be other techniques

with the same effect, and there may be various ways of describing the same technique. The important thing is that there are techniques which can be learnt, and which enable one, given a knowledge of the things (particulars or universals) referred to by non-logical words, to examine "the facts" or "the way things are in the world", and award truth-values to statements.

5.B.12. As before (see 5.B.8) generally applicable techniques correspond to each logical rogator. For in learning to use the logical form "x is a Q and there are no Q's which are R", it is not enough to learn to determine the truth-value when one of the words "Tom", "Dick" or "Harry" is taken as argument in the place of "x", and the other arguments are "man" and "happy". Nor is it enough to know how to find a truth-value for an argument-set as things actually are. One must know how to determine it for all suitable arguments in all possible circumstances, otherwise one does not fully understand. (Cf. 5.B.6.note.)

The techniques are, in fact, so general that it does not matter to which particular material objects the referring expressions correspond, nor to which properties (or "improper" properties) the descriptive words correspond. The techniques are topic-neutral. (See 5.A.3-4.) (We shall see later on, in Section 5.E, that this must be qualified.)

5.B.13. We are now almost in a position to say explicitly what must be learnt when one learns to use logical words and constructions. This requires a slight extension of the notion of a logical rogator and the corresponding name-function. (See 5.B.3.d.)

So far we have considered only name-functions which take sentences as values (see end of 5.B.5). But there are name-functions which take referring expressions or descriptive expressions as values. In addition, we must allow not only descriptive expressions and referring expressions as arguments, but also whole sentences. Thus, the name-function /"the R of x"/ takes relation words and referring expressions as arguments and yields referring expressions as values, such as "the father of Napoleon". The function /"P and Q"/ takes descriptive expressions both as arguments and as values, as in "red and round". The function /"Ø and Ø"/ takes sentences as arguments and yields sentences as values, such as "Fido is a dog and all dogs are four-footed".

Since the value of a name-function may be a sentence or a referring expression or a descriptive word, it may occur as the argument of another name-function. Thus, the function /"as P as x"/ may take as arguments "tall" and the expression mentioned above, and yield as a value the descriptive expression "as tall as the father of Napoleon".

Thus, by successive application of name-functions we can construct more and more complicated name-functions, just as in our arithmetical game more and more complex arithmetical functions could be constructed out of addition and multiplication, by successive application. (See 5.B.3.c.) For example, we get the name-function /"x is as Q as the R of z"/ by successive application of the functions /"x is P"/, /"as Q as z"/ and /"the R of y"/. Similarly, the logical form of a proposition may be regarded, often, as constructed by successive application of logical rogators to form a new, more complex rogator.

5.B.14. We have already noticed that a name-function which takes non-logical words as arguments and yields sentences expressing statements as its values may be thought of as corresponding to a logical rogator (a technique for determining truth-values). In addition, any of the name-functions described in 5.B.13 can be thought of as corresponding to a logical rogator, which takes as its arguments linguistic expressions of various sorts, and as its values either a particular object, or a property (proper or improper) or a truth-value, depending on whether the values of the name-function are referring expressions, descriptive expressions or sentences. For example, the logical rogator "the R of x" takes for its values particulars, such as the father of Napoleon, which are referred to by the values of the name-function /"the R of x"/. To each such logical rogator there corresponds a technique which must somehow be learnt for finding out which thing, property or truth-value (etc.) is the value of the rogator, given a set of arguments and their meanings. The technique, as before, must be generally applicable. (See 5.B.12.) ✓

5.B.15. Now the role of logical words and constructions can be described explicitly: learning to use them involves learning to apply name-functions to non-logical words and expressions as arguments, obtaining referring expressions, descriptive expressions and sentences as values. (Formation-rules must be learnt.) Secondly, it involves learning how the meanings of the resulting combinations of words depend on the meanings of the expressions taken as arguments, or, more specifically, if the resulting expression is a referring one, one must know how to tell which is the object to which it refers; if

it is a descriptive expression, one must know to which properties it refers (or, more generally, how to recognize objects which it describes); if it is a full sentence, one must know which logical techniques to apply to the material objects, properties or other non-linguistic entities referred to by the expressions taken as arguments, in order to arrive at a truth-value. (I.e. one must learn which logical rogators correspond to which logical forms.)

The principles and techniques which must be picked up if one is to use logical constants may be very complicated and difficult to formulate explicitly.

5.B.16. I shall not try to describe in detail or classify the various rules for the use of specific logical constants (i.e. logical words and constructions) in all contexts. That is the task of the formal logician, and in any case it would be very complex since there is an enormous variety of cases and many intricacies would have to be taken account of, such as the fact that one and the same English word can correspond to functions and rogators taking various sorts of arguments and values. (E.g. "or" as a function of descriptive expressions in "red or round", and as a function of sentences in "That is red or that is round"; "is" of identity and "is" as a copula.) Moreover, the logical form of a proposition (or the corresponding logical rogator) may not be fully determined by its geometrical or syntactical form: other things, such as the context of utterance, or the type of entity referred to by one of the non-logical words may have to be taken into account. (Compare, for example, "I want that cake" and "I made that cake"; or "Fido is black", and "The dog you heard is Fido".) In consequence, difficulties arise if we try to represent logical form in

the usual way simply by removing non-logical words and replacing them by means of variable-letters.¹

5.B.17. I shall henceforth ignore the rules for the individual logical constants, discussing only the results of combining them with non-logical words and expressions to form whole sentences. So I shall discuss only the logical rogators or logical techniques which correspond to complete sentence-matrices (5.A.13, 5.B.9-10). All we need notice in connection with individual logical constants is that learning to use them involves learning very general principles for dealing with the non-logical words with which they may be combined, or which are taken as arguments for logical rogators. (See 5.B.12.) This generality, or topic-neutrality helps to account for the fact that we can cope with newly invented descriptive words (e.g. the name for the colour of a new synthetic dye) without formulating new rules for the use of logical constants in

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1. Some of these complexities are described briefly by Holloway, in "Language and Intelligence" (see pp.144 to 152). He seems to think that pointing out these irregularities demonstrates that signs in a calculus do not work like words in a language (p.144). But I have argued that there is a more fundamental reason, namely, that signs in a calculus simply cannot occur in true or false statements about anything. (See circa 5.A.6, above.) What the irregularities show, however, is that certain types of calculus do not provide adequate symbolic representations of forms of propositions, but this can surely be remedied at the cost of a loss of elegance by the choice of a calculus with suitably complex formation rules, rules of derivation, etc. (There would have to be different sorts of variables.) No such remedy can, however, turn a calculus into a language, for a language needs semantic rules as well as syntactical ones. (See section 2.D.)

sentences including them. It helps to explain how we are able to construct sentences to deal with totally new and unexpected situations. (However, see qualifications in 5.E.6,ff, below.) It explains how we can successively apply name-functions to form more and more complicated sentences, expressing more and more complicated propositions.

These general principles for dealing with new combinations of words, and for discovering truth-values in new conditions, all have to be memorized in learning to talk. (This does not mean that formulations of the principles have to be memorized.) The fact that they can be memorized is all that we need to explain the possibility of learning and teaching the use of logical words. (See quotation from Pap, in 5.A.2.) We certainly do not have to postulate the existence of any "if-feelings" or other peculiar subjective entities correlated with logical words.

(Though I do not plan to formulate principles for the use of individual logical words, I have already described explicitly the logical techniques which correspond to certain logical forms of complete propositions, such as "x is P" and "All P Q's are R", in 5.A.3 and 5.B.11.)

5.B.18. I have so far regarded the words "true" and "false" as the values of logical rogators corresponding to certain logical forms of statement, or sentence-matrices. This, however, does not explain how we can make statements to convey information, except by asserting something like: "The sentence 'The sun is shining' corresponds to 'true'". It is possible to modify my account of logical rogators by regarding them as taking

for their values, not truth-values, but sentences, namely the sentences obtained by applying their name-functions to argument-name-sets, in cases where the value would be true, or slightly different sentences including the word "not" where the value would be "false". (Compare: in our arithmetical game, instead of awarding ticks or crosses for "moves", the players might simply ~~award~~ write down whatever is read out by a person in making a move in cases where it would be awarded a tick. E.g. they write down " $A + B.C$ " when that move is awarded a tick. If it merits a cross (i.e. if the value of the arithmetical function turns out negative) they write down, instead, " $(-1).(A + B.C)$ ". The reader may fill in details of how this works in the game and what its point is.)

So in learning to talk we learn to utter sentences themselves in cases where our investigations and applications of logical techniques would yield the value "true". In cases where the value would be "false", we learn to prefix the sentence with the words "It is not the case that" or something similar, or, more commonly, to utter another sentence which would correspond to the value "true" (which, in many cases can be derived from the original one by suitable insertion or removal of the word "not"). Others, who have learnt to speak the same language, then, if they hear us and trust us, know what to expect when they look at the facts we have observed in applying the logical techniques.¹

1. The rule might have been the other way round. We might have learnt to play the game in such a way that utterance of the original sentence took the place of utterance of the word "false", and utterance of the amended sentence took the place of the word "true", everything else being the same. In that case, sentences in that language would simply mean what their negations mean in ours. To utter a sentence in that language would still, of course, be to say what is the case, though we should take it to say what is not the case, since we should not understand.

5.B.19. We now see that whether a sentence *S* expresses a true or a false proposition depends on three things:

- (a) the logical form of *S*, which determines a logical rogator or general technique for determining truth-values, or for determining when *S* may be uttered and when not;
- (b) the meanings of the non-logical words and expressions taken as arguments (i.e. their semantic correlations with things and properties); and
- (c) the way things are in the world, which is, in general, discovered by carrying out observations in the course of applying logical techniques.

(The fact that there is this third element shows that there is something right in correspondence theories of truth. Only by talking about rogators instead of functions can we bring this out.)

These three elements are all illustrated by our arithmetical game. For example, to three elements in the claim to know that some statement is true correspond three elements in the claim to know that a move in the game deserves a tick, thus:

- (a) Knowing which boxes correspond to which "names" may be compared with knowing the meanings of non-logical words.
- (b) Knowing how to look into appropriate boxes and apply the technique for deciding whether to award a tick or a cross corresponds with knowing the general principle or logical rogator determined by the logical form of a proposition.
- (c) Looking at numerals actually written out in the boxes corresponding to names used in a move in the game, is like observing the facts which determine whether a sentence expresses a true or a false proposition.

The analogy should not be taken too seriously. I have tried to find a comparison which is close enough to serve my purposes without being too close to be illuminating: very likely an impossible requirement!

5.B.20. I have tried to describe the role of logical constants in giving sentences their meanings by saying that the way they occur in sentences determines which logical rogators correspond to the propositions expressed by those sentences. Thus they help to determine the kind of technique which must be employed in discovering whether a proposition happens to be true or not. So they help to determine which sorts of facts can count as verifying the proposition, or in which possible states of affairs the proposition would be true. Logical constants do not indicate which entities or kinds of entities a proposition is about (they are topic-neutral - see 5.A), but they do indicate how things must be with those entities if the proposition is true. (I have ignored the role of logical constants in questions, commands, and so on, but I think this could easily be taken account of.)

It might be thought that this is an unnecessarily long-winded and round-about way of describing how propositions work. Certainly there are alternatives. For example, we could regard sets of non-logical words as determining functions and the logical words as the arguments to which they are applied. (In general, there is a symmetry between argument-sets and functions.) Or we might try to avoid mentioning rogators and manage with functions alone, as Frege and Wittgenstein and Russell did. Notice, however, that my method eliminates many obscurities in their accounts. Thus, there is no need to discuss Frege's "unsaturated" entities of which nothing can be predicated. Nor need we talk about "unsayable" facts which merely "show" themselves, though, as will appear in chapter seven, in connection with knowledge of necessity,

this is only a temporary advantage. The price which we have had to pay for explicitness is, of course, circularity. But I hope the circle is so wide that this does not matter.

5.B.21. This completes the account of the way in which logical constants help to determine the truth-conditions of propositions which they are used to express. (All this shows, incidentally, that the employment of verbs is not essential for the activity of statement-making. Verbs have a special function which need not be described here.) Now it remains to show how this sheds light on the existence of logical properties of propositions or logical relations between them. We must try to understand, for example, how it is possible for propositions to be true, or inferences to be valid, in virtue of their logical form. Once again, the analogy of the arithmetical game will be useful. ✓

5.C. Logical truth

5.C.1. Logical properties of propositions and logical relations between propositions can be shown to be due to relations between the logical rogators involved in their construction. I shall first of all illustrate, using the example of the arithmetical game (see 5.B.7.), the general way in which rogators may be related and then turn back to logical rogators and propositions.

5.C.2. It will be recalled that "moves" are made in the game by applying rogators, derived from arithmetical functions, to names of boxes as arguments, ticks or crosses being awarded according as the arithmetical functions have

positive or negative values for the numerals in the boxes named. The first thing to notice is that relations of "entailment" may hold between moves in the game.

Consider moves made with the two functions $f(x,y)$ and $g(x,y)$, where the former is $x^2 + (-3).x.y$ and the latter is $x^2 + (-3).x.y. + 27$. The rogators derived from these functions are $f(P,Q)$ and $g(P,Q)$, taking names of boxes as arguments. To each rogator there corresponds a general technique for working out its value for any argument-set, the value being "tick" or "cross" depending on which numerals are found in the boxes referred to. (See 5.B.3). Now, since the value of the function $g(x,y)$ is always greater than the value of $f(x,y)$ by 27, for the same arguments, owing to a relation between the techniques for calculating their values, it follows that the former is positive whenever the latter is. Hence, if any move made with $f(x,y)$ and names "A" and "B" is awarded a tick, then, in those circumstances, a move made with $g(x,y)$ and the same names would also merit a tick.

Owing, therefore, to a relation between the techniques for finding their values, the two rogators $f(P,Q)$ and $g(P,Q)$ are themselves related so that the value of the latter for a pair of names of boxes as arguments must be "tick" whenever the value of the former is, for the same arguments (in the same circumstances). Of course, the value of $f(P,Q)$ might be "cross" when the value of $g(P,Q)$ was "tick", but the converse could not happen. We can tell, merely by examining the techniques corresponding to the two rogators, without looking to see what is in the boxes, that if the value of f is "tick", then the value of g is also "tick". We may say of a

pair of moves made with these rogators, such as " $A^2 + (-3).A.B$ " and " $A^2 + (-3).A.B. + 27$ ", that the former "entails" the latter.

5.C.3. Now we must notice what happens when complex rogators are constructed out of rogators between which relations hold. Since the value of the arithmetical function g is greater than the value of f for the same arguments no matter what they are, it follows that the arithmetical function $g(x,y) + (-1).f(x,y)$ is positive for all values of the arguments. Call this function $h(x,y)$. To it, as usual, there corresponds a rogator $h(P,Q)$, taking names of boxes as arguments, but with the peculiarity that the value of the rogator for all arguments is "tick" in all circumstances. The general technique for discovering the value ensures that all moves in the game made with h and any pair of names of boxes must be awarded a tick, no matter what numerals are in the boxes, and this may be discovered simply by examining the technique for working out values of this rogator.

Of course, this is not the only way in which a one-valued rogator can be constructed. Other examples are the rogators derived from the following arithmetical functions:

$$x.x, \quad x.y.x.y, \quad \text{and} \quad x.x + y.x + y.y$$

each of which has positive values for all arguments.

5.C.4. All this shows that although in general the value of a rogator for a given set of arguments has to be discovered by looking at the facts and applying the general technique for determining its value, there are, nevertheless, some cases where a complex rogator, compounded out of simpler ones (see 5.B.3.c, 5.B.13) by

successive applications, is a "freak" in that one can discover its value merely by examining the technique for working out its value, or by examining the techniques of the simpler rogators out of which it is compounded. Similarly, by examining the techniques for working out the values of a pair of rogators, one may discover that they stand in some "internal relation" so that knowing the value of one of them may enable one to work out the value of the other without consulting the facts or actually applying its general technique.

5.C.5. In some cases we can look at the way in which the rogator determines its value independently of the facts slightly differently. For example, in the arithmetical game, the use of the function $u.v + w.x + y.z$ in a move may result in a tick or a cross being awarded, depending on which names are used and which numerals are in the boxes corresponding to those names. But if we take the argument-set consisting of the names "A", "A", "B", "A", "B", "B", then we obtain the move "A.A + B.A + B.B", which gets a tick no matter what is in the boxes, since $x^2 + y.x + y^2$ is positive for all values of x and y . This shows that in some cases we can look at the value of a rogator as determined not only by properties of the technique for working out its values, but also by the "structure" of the argument-set. In these cases, by examining the technique for determining values of the rogator and the structure of the argument-set, we can find a value which it must take for all arguments^{-set} with that structure, no matter what the facts are, although in general the result of applying the techniques corresponding to the rogators out of which the "freak" is constructed does depend on the facts, that is, on how things

happen to be in the world. (See 5.B.6.)

5.C.6. All this applies to logical rogators as well as the ones which we have been discussing: logical rogators may also generate "freaks". We saw (in section 5.B) that every sentence can be thought of as the result of applying a logical name-function to a set of non-logical words, the logical rogator corresponding to that name function being what determines the conditions in which the sentence so obtained expresses a true proposition. As in the cases discussed above, it may be possible, by examining the general techniques for determining the value of a logical rogator, to discover its values for all arguments, or for argument-sets with certain structures, without consulting the facts at all. So we can determine the truth-values of propositions constructed with the aid of such rogators merely by examining the logical techniques for discovering whether those propositions are true or false, i.e. without applying the techniques.

For example, the sentence "All red horses are red" is a value of the logical name-function/" All P Q's are R"/, for the argument set ("red", "horse", "red"). To it there corresponds a logical rogator and a technique for determining truth-values. By examining that technique, and the structure of the argument-set, we can discover the truth-value in question without actually applying the technique (which would involve examining all red horses to see whether they are red). We can discover that the proposition expressed by the sentence is true independently of the facts, and independently of the actual meanings of the words in the argument-set (as long as the structure is ("A", "B", "A")). We say that it is

a "formal" truth, true in virtue of its logical form.

5.C.7. It is extremely important to notice, in all these cases, that where the value of a rogator is determined independently of the facts, this has to be discovered by examining the technique, not by applying it. But since one may have mastered a technique without ever examining it (see appendix on "Implicit Knowledge"), one may fail to notice that a rogator is a "freak" whose values are determined independently of the way things are, and go on as usual to find out its value by applying the technique. (It should be recalled that the techniques are generally applicable: they work for all argument-sets, even those whose structure restricts the possible outcome of applying the techniques. See 5.B.8, 5.B.12, 5.B.17.)

So, in the case of the game, the players may fail to notice that a move such as "A.A + B.A + B.B" would merit a tick no matter what numbers were found in the boxes referred to, and go on in the usual way to look into the boxes, calculate the values of the arithmetical functions, and base their decision whether to award a tick or a cross on the result of applying this technique. Similarly, one may fail to notice that some proposition is true in virtue of its logical form, and apply the usual logical techniques for determining its truth-value by observation. This is possible because the techniques are generally applicable. Thus I can discover that "All the red horses in this room are red" expresses a truth by examining the red horses in the room (cf. 5.B.11), but there is no need to, since I can see what the outcome would be merely by thinking about the method which I should have to apply.

(See 5.A.9 for another example.) The importance of this will emerge in section 6.E on "Knowledge of analytic truth".

5.C.8. All this can be extended to explain the existence of logical relations between propositions, arising from relations between their structures. For example, we may learn that one proposition entails another in the same sort of way as we found in 5.C.2. that one move in the game could "entail" another. We may find, by examining the logical techniques for discovering truth-values of the two propositions, and the structures of their argument-sets, that no matter how things are in the world, if the outcome of applying one of these techniques is "true", then so will the other be. E.g. If "All black horses are hungry" expresses a true proposition, then so does "All big black horses are hungry". In such a case, we may speak of "formal entailment". The inference from one proposition to the other will be "formally valid", or valid in virtue of its logical form. (The logical form of an inference can be represented by substituting variables for non-logical words, in much the same way as the logical form of a proposition. (Cf. 5.A.13).) Similarly, propositions may be formally contradictory, or formally incompatible. As with formal truth, such logical relations may pass unnoticed by persons who apply logical techniques without examining them. (We cannot give a general definition of "entails" until after chapter seven.)

5.C.9. I wish to stress the (by now obvious) point that these logical properties and relations of propositions are not due merely to geometrical relations between symbols,

but primarily to properties of and relations between techniques which have to be learnt for doing things with these symbols. Admittedly, in most languages the rules for the use of symbols are probably so chosen that to certain geometrical relations there correspond relations between techniques (as implied by my remarks in 5.B.14-15 about the connections between name-functions and logical rogators). This is indispensable if there are to be general principles for constructing more and more complicated types of propositions out of a small set of symbols without continually introducing new ad hoc grammatical and logical rules of construction: to this extent logic may be connected with syntax, though it is never reducible to it. However, as remarked in 5.B.17(note), not all rules of formation of sentences are quite like this, so the connections between logical techniques and geometrical forms are not absolutely indispensable and, in any case, it is not enough to notice the connection between geometrical relations and logical relations. Indeed, noticing this may blind philosophers to the intermediary in virtue of which they are connected, with unfortunate results, as I shall try to show in Appendix II.

Part of the explanation of the tendency of philosophers of logic to ignore these logical techniques, is the fact that we can learn to use symbols and apply the corresponding techniques, and sometimes even draw consequences from the interconnections between these techniques, without fully realizing what we are doing. We need not even be aware of the existence of the techniques. This is an illustration of a general point that one may have knowledge which one cannot formulate, or one may know that something is so without being quite

aware of the reasons why they are so or how it is that one knows this. One may claim, with perfect justification, to know that "If anything is red then it is red" expresses a truth, and yet be completely inarticulate when there is any question of justifying the claim. (This sort of thing is discussed in Appendix III, and in 6.E.6.) I have been trying to make the missing justification explicit, or at least to describe it in general terms.

5.C.10. I have not, however, fully explained how we can draw conclusions about the outcome of applying certain techniques merely by examining those techniques, without actually applying them. I have not explained what goes on when one has the kind of logical insight which is involved in perceiving that two logical techniques are related in certain ways, or that a logical technique has certain properties, apart from relating it to the general way in which one may discover properties of or connections between rogators.

Some would probably try to reduce logical insight to a matter of seeing that a certain sequence of formulae or symbols have certain syntactical properties, but this would leave unexplained the kind of insight one has when one sees that this is the case, which is a sort of mathematical insight into the connections between geometrical forms. In any case, we have already argued against attempts to reduce logic to syntax. (See section 5.A and Appendix II.) (For some reason it was only after the discovery of Gödel's famous incompleteness theorem that some logicians began vaguely to appreciate this point. Gödel expressed it as follows, in his contribution to

"The Philosophy of Bertrand Russell", p.127-8: "It has turned out that ... the solution of certain arithmetical problems requires the use of assumptions essentially transcending arithmetic i.e. the domain of the kind of elementary indisputable evidence that may be most fittingly compared with sense perception." I do not wish to say that this transcends arithmetic: I should rather say that it turns out that arithmetical knowledge requires more than was once thought by some logicians to be required. It transcends their conception of arithmetic.)

5.C.10.a. It cannot be argued that when we have this sort of insight or draw the sorts of conclusions under discussion, by examining the structures of argument-sets and the techniques for determining the values of logical rogators, what goes on is that we consider statements describing these techniques and structures and then apply some formally valid procedure for inferring, via formal entailments, that certain statements have certain logical properties or stand in logical relations. That would clearly be circular, since it is the nature of formal validity that we are trying to explain by talking about these properties of logical techniques. We cannot without circularity explain this by assuming that their having these properties is merely a formal consequence of other facts. (Cf. 7.D.9,ff.)

5.C.10.b. It seems to me that what goes on when we have this sort of insight, and in general when we discover facts about the application of techniques by examining them instead of applying them, is essentially the same sort of thing as goes on when we discover necessary

connections between, for example, geometrical structures or properties, by examining those structures or properties and perhaps constructing informal proofs. The difference lies in the degree of generality. (Logic is topic-neutral: see section 5.A.) This sort of thing will be discussed in more detail in the section on "Informal proofs", in chapter seven.

5.D. Some generalizations

5.D.1. Let us leave aside questions about what goes on when we examine logical and other techniques instead of applying them, and consider how some of the remarks of the previous section may be generalized, so as to prepare the way for the discussion of propositions which are analytic, or true by definition.

We have seen that although in general the value of a rogator for any argument-set depends on how things are in the world, nevertheless there are some "freak" cases where the value can be discovered independently of observing facts and applying the technique for that rogator. In some cases we found that what determined the value independently of facts was an interaction between the general technique for discovering values, and the "structure" of the argument-set. (See 5.C.5.) Let us look a little more closely at this sort of case.

It is clear that when the logical form "All P things are Q" is applied to the argument-set ("red", "red"), the basic reason why the proposition expressed by the sentence so obtained is true independently of what is the case in the world, is not the fact that the two words in the argument-set are identical, but that they have the same meaning, that they refer to the same property.

This is seen from the fact that if we define a new word, "rot", say, to refer to the same property (proper or "improper" property¹) as "red", then applying the logical form to the argument-set ("red", "rot") will yield the statement "All red things are rot", which is true independently of the facts for much the same reason as the original one. So our description of the original case was not sufficiently general. The fixed truth-value of a statement like "All red things are red" is not essentially due to the fact that it is obtained from an argument-set with the structure ("P", "P"), but to the fact that it is obtained from a set of two arguments standing in a certain relation (in our example the relation is synonymy: the words refer to the same property).

5.D.2. We see therefore, that when the value of a rogator for certain arguments is determined independently of the facts, this may be due to (a) the general technique for discovering its values (e.g. the way the rogator is constructed out of other rogators), (b) the structure of the argument-set, and (c) relations between the arguments. (The second is really a special case of the third.) In this sort of case, the value of the rogator is always the same for a set of arguments standing in the appropriate relations, no matter what the arguments are, and no matter how things are in the world. (As before (see 5.C.7), a person may fail to notice this interaction between a rogator and an argument-set, and work out the value in the usual way by applying the technique as if its outcome could depend on the facts.)

5.D.3. Once again we can find an illustration in our arithmetical game. (5.B.7.) If it is known that no

1. 3.B.5, 3.D.8.

numeral occurs in more than one box in the game and that every numeral will eventually occur in at least one of the boxes, then we might, in the course of playing the game, introduce new names for boxes, as follows. If "B" is known to be the name of a box, then we say: "Let 'A' be the name of whichever box contains the numeral obtained by adding three to the number referred to in 'B'". We shall not, of course, know which box is the one referred to by the name "A", but we do know that whichever it turns out to be, the numeral in it will be in the stated relation to the numeral in B, whatever that may be. By considering this fact, and by examining the technique for deciding whether moves are to be awarded ticks or crosses, we can tell without looking into boxes that the move $A + C.C + (-1).B + C$ must be awarded a tick since the value of the arithmetical function $x + y.y + (-1).z + w$ must be positive for all argument-sets in which the second and fourth arguments are the same, and the first argument is greater than the third.

5.D.4. This example illustrates the same sort of thing as may happen when we apply a logical form to descriptive words whose meanings stand in some complicated relation owing to the fact that they have been logically synthesized in the manner described in section 3.B. Relations between meanings of words in virtue of which the value of a rogator is independent of the facts need not be as simple as in the example of 5.D.1, where the relation was synonymy. For example, if the word "U" refers to the property P, and the word "V" refers to the combination of properties Q and R, while the word "W" describes objects if and only if they have the property P or do not have the property Q, then the result of applying the name-function /"No F are G

and not H"/ to the argument-set ("W", "V", "U") is the sentence "No W are V and not U" which can be seen to express a proposition whose truth is independent of how things are in the world, owing to (a) facts about the logical rogator corresponding to its logical form, and (b) relations between the meanings of the words.

Since the proposition is not true merely in virtue of its logical form (not all propositions with that logical form are truths), but also in virtue of relations between the meanings of some of the non-logical words, the proposition is not a "formal" truth. (See 5.C.6.) (We could alter customary philosophical usage and extend the notion of the "logical form" of a proposition to include such facts about the logical relations between meanings of non-logical words, and this would be a good thing insofar as it drew attention away from syntactical properties of sentences, but I shall not do so.)

5.D.5. We can now see that the fact that if words are given meanings standing in certain relations then this may have the consequence that some sentences in which they occur express propositions whose truth-values are independent of the way things are in the world, is just a special case of a more general fact about rogators, namely that relations between arguments in an argument-set together with properties of the general technique for discovering values of the rogator may in some cases suffice to determine the value for that argument-set, though in general it does not, since facts are relevant too. This does not mean that the general technique cannot be applied in order to discover the value in such freak cases, but that it need not be. (Cf. 5.C.7.)

(We shall see later on that even an analytic proposition

can be verified by empirical observations, though it need not be.)

5.D.6. It should be noted that when there are relations between the meanings of descriptive words, although this does not enable us to discover the truth-values of all propositions which they may be used to express, there will certainly be a great many whose truth-values are determined. Thus, from the fact that the word "red" refers to the same property as "rot" we can infer not only that "All red things are rot" expresses a true proposition (See 5.D.1.), but also that each of the following does: "Nothing is both rot and not red", "If anything is not red then it is not rot", "All rot and round things are red or the moon is made of green cheese", etc. In addition, the following are false independently of the facts: "Some red things are not rot", "All round things are red and not rot and there is at least one round thing", etc. (To any one relation between the meanings of descriptive words, there corresponds a whole family of analytic propositions. See 6.F.5.)

In all these cases we can discover the truth-value in essentially the same way as before: namely by studying the general logical techniques which would normally be applied in finding out the truth-values of statements made with these logical words and constructions.

In addition, we could discover, by examining the logical techniques and relations between meanings, that certain relations such as entailment and incompatibility hold between some propositions. (This is just an extension of the remarks in 5.C.8.)

5.E. Conclusions and qualifications

5.E.1. The time has come to summarize what has been done in this chapter, and show how it fits into the general programme of this thesis.

The aim of Part Two, which this concludes, was to describe the general connection between meaning and truth, in order to prepare the way for a description of the connection between meaning and necessary truth, in Part Three. (We have been mainly concerned with statements containing only logical words and descriptive words referring to universals, but many of the remarks of 5.B apply also to statements in which particulars are mentioned.)

The general connection can be summed up thus: learning the meanings of words or sentences containing them involves learning to recognize or pick out states of affairs in which to utter such sentences is to make true statements. I have tried to isolate out two aspects of this learning process. First we have to learn semantic correlations between non-logical words and non-linguistic entities, and secondly we must learn the use of logical words and constructions. (1) Semantic correlations between descriptive words and universals (observable properties and relations) were described in chapters three and four. Learning these involves learning to recognize the particular objects which may be correctly described by such words. (2) Learning to use logical words and constructions involves learning to tell which logical rogator (which generally applicable logical technique for determining truth-values) corresponds to the way in which logical words and constructions occur in a sentence. It is in virtue of this correspondence

that the occurrence of such logical constants helps to determine the conditions in which the proposition expressed by the sentence is or would be true (or false).

All this showed how the truth-value of a proposition expressed by a sentence containing logical words and descriptive words depended on (a) the meanings of the descriptive words, (b) the logical techniques corresponding to the logical form and (c) the facts, i.e. how things are in the world. (5.B.6, 5.B.19.) We saw that this was just one instance of the general fact that the value of a rogator depends on (a) the objects taken as arguments (b) the general technique (or rule, or principle) for discovering values, and (c) the way things are in the world.

This completed the account of the general connection between meaning and truth.

5.B.2. Further investigation showed that the existence of formal truths and formally valid inferences could be explained in terms of properties of and relations between rogators. (See 5.C.6, 5.C.8.) This eliminated the need for making obscure, misleading or false remarks about the connection between logic and syntax. (See section 5.A.) As remarked in the previous paragraph, the value of a rogator depends, in general, on three things, but we found some "freak" cases in which the third element dropped out. In these cases, although the value could be discovered by applying the general technique and investigating facts or conducting experiments, nevertheless this was not necessary, since the value could be determined a priori.

We distinguished three types of freak case. (1) In the most general case, the value depended on both of the

first two factors (namely (a) and (b) above), and could be discovered by examining the argument-set and the technique for discovering values of the rogator. The value was determined by relations between the arguments together with properties of the general technique, independently of the facts. It did not matter which particular objects were taken as arguments: the value was always the same, provided that they were related in a certain way. (Section 5.D.) (2) A simpler type of freak rogator was one whose value was the same for all argument-sets with a certain structure. Here the value could be discovered by examining the structure of the argument-set and the general technique for finding values of the rogator, independently of how things were in the world, or which particular objects were taken as arguments. (3) In the simplest sort of case, the value was completely independent of which objects were taken as arguments, and was fully determined by the general technique. Here both the first and the third factors dropped out, leaving only the second (b).

The second and third type of freak rogator sufficed to explain the existence of propositions true in virtue of their logical form, since such propositions corresponded to logical rogators constructed in such a way that their values for some or all arguments might be determined independently of the facts. The first type will be used to explain the more general fact that there are propositions which are analytic, that is true in virtue of the meanings of words, or true by definition.

A slight modification, taking account of relations between rogators, due to relations between their general techniques and their argument-sets, serves to account for logical relations between propositions, such as entailment,

incompatibility or logical equivalence.

5.E.3. We see from all this that it is possible to give an account of logically true propositions which arises naturally out of a description of the general connection between meaning and truth, covering contingent propositions too. There is no need at all to explain away logical truths as not being truths at all, or logically true propositions as not being propositions at all, but rules or conventions or expressions of acceptance of conventions, etc. They are propositions, and their truth-values may be discovered empirically by applying the general logical techniques for discovering the truth-values of contingent propositions expressed by sentences including the same sorts of words and constructions. Their peculiarity is only that their truth-values may also be discovered in the other way which I have described. (Cf. 6.E.1,ff.)

5.E.4. It is probably obvious that what I have said is closely related to Wittgenstein's explanation of logical truth in "Tractatus Logico Philosophicus". (He too was not content to classify and describe logical properties of propositions and relations between them, but tried to explain them.) His account, however, seems to me to have involved some unnecessary obscurity, and was certainly not sufficiently general. I have tried to give a more general account of logical form (of propositions containing logical words, descriptive words correlated with observable properties, and words referring to particular material objects). The logical form of a proposition corresponds to the way in which the truth-conditions of the proposition are related to the entities,

such as material objects or properties, mentioned or referred to in the proposition. Knowing the logical form involves knowing in general how to tell whether propositions with that logical form are true or false, no matter what entities are referred to, and no matter how things are in the world. (But see qualifications below.)

This is what people are talking about when they refer to the "real" logical form of a proposition, contrasting it with the "apparent" logical form suggested by the verbal form of a sentence. Of course, every intelligible sentence must fully determine the real logical form (otherwise it could not be understood correctly: we should not be able to recognize its truth-conditions). It is only when instead of applying the logical techniques we reflect on the logical form that the form of the sentence can suggest anything misleading to us. This, however, is a failing on our part, due to our not thinking clearly about what, in a way, we know quite well (see appendix on "Implicit Knowledge"), and does not mean that there is anything inaccurate or imprecise about the sentence. (E.g. though we know quite well the difference between the copula and "is" of identity we may get muddled when talking about it.)

5.E.5. It should be emphasized that I am not talking about a "perfect" language, or any one particular language. I have been trying to bring out general facts about any language which can be used for making true or false statements about material things and their properties, about the way things are in the publicly observable world. Neither do I restrict my remarks to sentences in some

special notation or "canonical form": what I say is intended to apply to all sorts of statements using all sorts of logical constructions, provided that it is possible to think of the statements as built up out of parts which have a general use in statements. (A language in which there was just one sound, which had to be learnt separately, corresponding to each statement, would be very different from ours. There would be no way of talking about possibilities, or of teaching the meanings of false statements - and my remarks would probably not apply in that case.)

5.B.6. Despite its generality, my account of logical form has involved a number of over-simplifications, which must now be eliminated. The first oversimplification is concerned with presuppositions. I have continually stressed the fact that to every rogator discussed so far there corresponds a generally applicable technique for discovering its value for all permissible argument-sets, which works in all possible circumstances (Cf. 5.B.8 and 5.B.12). This must now be qualified, for there may be some techniques which are applicable only when certain conditions are satisfied. For example, we described an arithmetical game (in 5.B.7) which involved techniques for working out the values of rogators taking names of boxes as arguments. Those techniques involved looking into boxes and working out the values of arithmetical functions taking the numerals in the boxes as arguments. If, however, a box were found to contain no numeral (e.g. there might be an apple in it instead) or two different numerals, then the technique could not be applied, and there was nothing in the rules of the game to say how to deal with this case. The rules (as I described them) did not say whether a

move using the name of a box with an apple in it should be awarded a tick or a cross or what. They merely took it for granted that the question would not arise.

5.E.6.a. Similarly, the applicability of logical rogators presupposes the satisfaction of certain conditions. Thus, the logical technique corresponding to the logical form "All P Q's are R" as described in 5.B.11 presupposes that there are no objects which are borderline cases for the descriptive words taken as arguments. (Recall the various sorts of indefiniteness of meaning of descriptive words described in chapter four.) If some boxes turn up which are neither definitely scarlet, nor definitely not scarlet, then that technique provides us with no way of assigning a truth-value to the proposition expressed by "All scarlet boxes are red". It should be noticed that the technique does not even provide us with a value for "All scarlet boxes are scarlet" in this case. Of course, an examination of the logical technique and the structure of the argument-set would, as described above, lead us to say that the truth-value must come out to be "true". But this presupposes that the technique yields a value at all, that its applicability-conditions are satisfied. (Compare the case where a player makes the move "A.A + 3", and the box corresponding to "A" has only an apple in it: he gets no tick although $x^2 + 3$ is always positive.)

So, had we been more precise and explicit, we should continually have had to make qualifications of the form: "... provided that the applicability-conditions of the technique are satisfied". These were omitted in the interests of clarity and simplicity (see 5.B.12.).

5.E.6.b. One kind of presupposition which has drawn some attention is that if a non-logical word or expression occupies an argument-place intended for an expression which refers to some one entity of a certain kind, then there is exactly one entity referred to and it is of the correct sort. For example, the use of the logical form "x is P" presupposes that the expression taking the place of "x" refers to a particular object, and if there is not exactly one to which it refers, then the technique (described in 5.A.3) for determining a truth-value lacks application. Hence that technique does not provide us with a truth-value. (Section 2.D was concerned to show that the same applies to descriptive expressions substituted for "P".)

5.E.6.c. To sum up: just as functions and rogators may have restricted domains of definition, that is the classes of argument-sets for which they yield values may have certain limitations, so may there be restrictions on the class of states of affairs in which the techniques for determining their values can be applied. The domain of applicability-conditions may be restricted. In many cases, whether the technique yields a value or not, i.e. whether one of its applicability-conditions obtains or not, will depend on how things happen to be in the world (e.g. on whether there happen to be any borderline cases of instances of the colour scarlet, or whether there happens to be no king of France). But there are probably some cases where, from the way in which a rogator is constructed, and from facts about the things taken as arguments, one can discover without trying to apply the technique that it cannot yield a value for those arguments. That is to say, it may be impossible for the applicability-

conditions of some complex rogator to be satisfied when certain things are taken as arguments. A detailed investigation of such things as limitations on domains of definition and restrictions on applicability-conditions of the techniques corresponding to logical rogators, would, I think, shed a great deal of light on the subject of so-called "category mistakes", and, in particular, show how they differ from straightforward contradictions.

5.E.7. We see therefore that there are various ways in which even a logically well-formed sentence may fail to express a true proposition. The logical technique corresponding to it may yield the value "false", or it may yield no value, for any of several different reasons. This makes it look as if in some cases it is correct to say that the proposition expressed by the sentence is neither true nor false, or that no proposition is expressed at all. But things are not quite as simple as this, for, just as the semantic correlations between descriptive words and properties may be indeterminate, giving rise to difficult borderline cases, so may the rules governing the use of logical forms and the principles for deciding on truth-values be indeterminate, giving rise to difficult borderline cases.

5.E.7.a. For example, superimposed on the principle for determining the value of a logical rogator for given argument-sets, may be a general principle of the form: "When the technique does not (definitely) yield the value 'true', then the value is 'false'." If this principle is added to the original one (for example to the rules for "is", "or" and "all" described in 5.A.3 and 5.A.11), then, when the applicability-conditions of the original technique

are not satisfied, this new general rule ensures that the truth-value is false. But in that case trouble arises from the fact that normally when the truth-value of a proposition is "false", there is a proposition derived from it by the insertion of the word "not" into the sentence expressing it, namely, its negation, or one of its contraries, which has the truth-value "true". And usually the applicability-conditions for the technique corresponding to the new proposition are the same as for the old one. Hence there is a conflict in cases where applicability-conditions are not satisfied, between what these rules lead to and what we should normally expect, and there may be no rules which are definitely part of the language to specify what is to be said in such cases.

This can be summed up by saying that there may be more than sufficient rules for the use of logical forms, which work in most cases but may come into conflict in others, and then there may be no definite answer to the question: "Is this proposition true or false?".

Various rules for the use of logical constants are superimposed in an indeterminate way (Cf. 7.D.11, note.). Failing to see that this is a case of indeterminateness of linguistic rules, philosophers may argue in vain that one or other answer or some third one is correct.

(See 6.D.4.) (Such controversies are not, of course, completely useless, since they help us to see various ways in which the principles governing the use of logical operators can be made more definite. See Appendix IV.)

5.E.7.b. Other kinds of indeterminateness in the principles governing the use of logical forms arise out of the fact that as a language develops, different ways may be found for saying the same thing, and this may involve extending

or changing the functions of logical and other words. For example, instead of saying "All P things are Q", we may learn to say "The class of things which are P is included in the class of things which are Q", or "The property P-ness is always accompanied by the property Q-ness" or even "P-ness is possessed only by things which have Q-ness". In this sort of way abstract substantives referring to universals are allowed to enter into sentences as if they had the same grammatical roles as words referring to particulars. We learn to say things like "Red is a colour", "Idleness is annoying", and these sentences strongly resemble "Fido is a dog" and "My table is brown", whose logical form is represented by "x is P".

It looks therefore as if the domain of definition of the logical rogator corresponding to this form has been extended so as to include new argument-sets. We cannot, however, simply say "Let there be an extension", for the technique for determining truth-values must be extended too. This extension enables us to apply the form "x is P" to argument-sets like ("my table", "brown") and ("brown", "a colour") or ("brown", "attractive"). This may lead us to think that we have extended the technique to cope with argument-sets like ("brown", "brown") even when we have not done so. Again, there may be conflicts between rules, with nothing definite in the language to settle them. We can, of course, extend the technique to cope with this kind of argument-set if we wish to do so, for we can give any form of words a use if we wish, but we are tempted to think that we have extended the technique and the domain of definition before we have done so in fact, or that we are compelled to extend it in a certain way, and this may lead us into

difficulties such as Russell's paradox, and others. (See what happens when people forget that division by zero is not defined in arithmetic. We could extend the notion of division to include division by zero, but if we do not wish our rules to lead into conflict some other changes may have to be made. (Rules lead to conflict when, for example, they enable a rogator to have more than one value - e.g. both "true" and "false" - for the same argument-set.))

5.E.7.c. A similar sort of indeterminateness arises when we use logical constants in talking about quite new kinds of things, such as infinite sets, thinking that their use is fully determined in these contexts by the rules for their use in other contexts. People may even disagree about the way in which the use is determined in the new contexts, failing to notice that this is a case of indeterminateness, where some new convention must be adopted if the matter is to be settled. (Cf. Section 4.C and 7.D.10,ff.) So philosophers of mathematics may disagree as to how the logical constants are to be used in connection with statements about infinite sets, without realizing that there are alternative ways of using them, either of which may be freely chosen. (Or it may not definitely be the case or not the case that either can be freely chosen: the rules of the language in question may not definitely leave the matter quite undetermined.)

5.E.8. These rather brief remarks give a rough indication of some of the qualifications which must be made to all my assertions about the general applicability of the

techniques for determining truth-values, and about the possibility of discovering values of rogators by examining general techniques without applying them. They also enable us to explain away many apparent counter examples to the so-called "Laws of Logic", as being cases where the applicability-conditions for logical rogators are not satisfied, or where arguments are taken from outside the domains of definition of rogators. (Question: does this approach have any advantages over the "ranges of significance" approach, for a theory of types? Cf. 5.E.6.c.)

This concludes my account of the workings of logical constants, and also my account of the general connection between meaning and truth. We may now proceed to discuss meaning and necessary truth, and in particular to distinguish various ways in which relations between arguments may determine the values of rogators.

Part Three

MEANING AND NECESSARY TRUTH

Chapter Six

ANALYTIC PROPOSITIONS

6.A. Introduction

6.A.1. The main stream in Part Three will be a continuation of the attempt to describe the various factors which can determine or help to determine the truth-value of a proposition. This will provide illustrations for my explanation of the meanings of "analytic", "necessary", "possible", and related words, which will proceed at the same time. It will be shown that there are several different ways in which a proposition may be necessarily true, corresponding to a number of different ways in which its truth-value may be discovered. In particular, it will be argued that, in the sense of "analytic" which is to be defined in this chapter, not all necessary truths are analytic. This is because there are some properties which are necessarily connected, although they can be completely identified independently of each other. Hence their necessary connection is not an identifying relation or a logical consequence of an identifying relation. (What this means will be explained presently.)

6.A.2. The first problem must be to get clear about the meaning of "analytic". There are at least two ways in which people can be unclear about the analytic-synthetic distinction. The first is to be unclear as to what the things are which it distinguishes. Thus, philosophers are often confused about the sorts of things "propositions"

or "statements" are, the entities to which they apply the distinction. Even when they try to say explicitly what it is that they are talking about, their usage often conflicts with their explanations.

Sometimes it looks as if they are talking about sentences, but a sentence considered simply as a sequence of signs, or marks on paper, or sounds, cannot, as such, either be analytic or fail to be analytic, any more than it can be true or false. (Cf. 2.A.2, 2.D.3 (note), 5.A.8-9.) Before a sentence can be described as true or false, or analytic or synthetic, it must be thought of as a sequence of signs with a meaning or linguistic function, and the meaning must be fairly definitely specified. If a sentence is taken simply to have its meaning in English, for example, then there may be no answer to the question whether it is analytic or not, owing to the ambiguities of the English language. (An obvious example is provided by the sentence "All mothers bore their children". More subtle examples were discussed in section 2.C.) Neither can the distinction usefully be applied to formulae of a formal system, since, as argued in the previous chapter (section 5.A) and appendix II, a language is quite a different sort of thing from a formal system. All this may seem obvious, but, as will appear in a moment, it appears not to have been noticed by some philosophers who try to define "analytic". (Such as Carnap.)

We must apply the distinction to sentences only if they are taken to have meanings, and we must know which meanings they are taken to have. This involves knowing what counts as "the same meaning". I have tried to show how the properties referred to by descriptive words, and the logical techniques corresponding to logical constants

can be used to provide criteria for identity of meanings. (Section 2.C, and chapters three and five.)

6.A.3. The second kind of unclarity about the analytic-synthetic distinction involves the way in which it is applied. The word "analytic" is fairly common in philosophical writings, and most philosophers have a rough idea of what it means, but their usage is nevertheless bedevilled with confusions, obscurities, ambiguities and errors. As pointed out by Mario Bunge, in Mind, April 1961 (p.239), the word is used with an unrecognized, or at least unacknowledged multiplicity of meanings.

For many philosophers (see for example p.21 of Strawson's "Introduction to Logical Theory") the word is apparently synonymous with "necessarily true", which is, of course, a question-begging usage when the possibility of synthetic necessary truths is being discussed. Others offer "pragmatic" definitions, so that analyticity admits of degrees. Quine's definition of "analytic" as meaning "definitionally derivable from a logical truth", was accepted with modification by Waismann in his famous series of articles on the subject (in Analysis, Dec. 1949, etc.: this will be dealt with in detail below). Some (following Frege?) turn the Quine-Waismann definition around, and instead of talking about derivability from logical truth by substitution from definitions, they talk about logical derivability from definitions or "meaning-postulates". (It is not usually noticed that these two definitions are not equivalent.)

Sometimes followers of Carnap define the word "analytic" in terms of "state-descriptions" and the rules

of a so-called "language", L, which is admittedly precise, but also quite useless, since it applies only to formulae in a formal system, and not to statements in a language. Such people are inclined to regard the analytic-synthetic distinction as system-relative, so that whether a proposition is analytic or not depends on the "system" in which it occurs. (See, for example, Mario Bunge, *op.cit.*, p.239,ff.) I have never been quite sure what these "systems" are supposed to be. My guess is that philosophers who talk in this way are making the mistake (see section 5.A and Appendix II) of confusing formal systems and languages. Neither can I understand why they regard propositions occurring in different "systems" as the same proposition. Why not say that they are different propositions, for then there will be no need to regard the distinction as system-relative? There appears to be some confusion as to whether they are talking merely about sentences, which, admittedly, may have different meanings in different languages, or about propositions. (See 6.A.2.) Other philosophers talk as if a proposition either is or is not analytic, no relation to a system being regarded as relevant.

Often the word "analytic" is defined rather vaguely, as "true in virtue of meanings", or "proposition which cannot be denied without contradiction", or "proposition which cannot intelligibly be denied". Sometimes it is suggested or implied that we can decide to make a proposition analytic, whereas others will allow only that we can decide to let a sentence express a proposition which already happens to be analytic, independently of our choice. (Cf. 6.F.1, below.)

I shall not describe in detail, nor criticize most of these accounts of the distinction. For detailed

exposition and criticism the reader is referred to "Semantics and Necessary Truth", by Arthur Pap. A few critical remarks will be made by the way in some of the discussion.

6.A.4. Behind all this chaos and confusion there seems to lie a fairly simple concept, familiar even in non-philosophical contexts, for people often speak of something or other as being "true by definition", or use the expression "by definition" to preface a remark in order to indicate the sort of justification which they would be prepared to offer for accepting it as true. All my attempts to define the word "analytic" aim at trying to clarify and make precise something like this ordinary notion of a proposition which is true by definition. The word does not correspond only to a technical distinction invented for philosophical purposes.

6.A.5. A small point to be cleared up is that I shall use all three expressions "analytic", "analytically true" and "analytically false" to describe statements or propositions. The latter two expressions are unambiguous, whereas the former is ambiguous. It may mean either "analytically true", or "analytically true or analytically false". This ambiguity is customary, and should cause no confusion, as exactly what is meant will be clear always from the context.

6.A.6. It should be noted that the distinction between analytic and synthetic propositions is not the same as the distinction between propositions which are verbal (or merely conventional), and those which are non-verbal

(or non-conventional, or independent of conventions of any particular language). The verbal/non-verbal distinction works at a different level from the analytic/synthetic distinction, as is shown by the fact that philosophers seem to be trying to say something significant when they say that all analytic propositions are merely verbal, but the distinction is obscure. It is very difficult to see what could be meant by saying this sort of thing. I think that only Wittgenstein has come close to being clear about it, but there will not be space to discuss his view (in R.F.M.), as it can be made intelligible only in the context of an account of his general theory of meaning. Even if I manage to demonstrate that some necessary truths are synthetic, this will not settle the question whether all necessary truths are verbal, or conventional. (See R.F.M. III.42).

6.A.7. I shall now turn to a more detailed discussion of some unsatisfactory accounts of the analytic-synthetic distinction, in order to lead up to my own account.

6.B. Some unsatisfactory accounts of the distinction

6.B.1. In this section I shall describe a number of attempts to explain what the analytic-synthetic distinction is, picking on some of their weak points in order to contrast them with my own definition later on. Its purpose is purely introductory, and it should not be taken too seriously, as it may be somewhat unfair to some of the philosophers mentioned.

6.B.2. Kant's explanations of the distinction are not very clear, though it is fairly easy to understand, at least in a vague way, the sort of thing he is getting at.

For example, in A.6, B.11 ("Critique of Pure Reason") he says:

"Either the predicate B belongs to the subject A, as something which is (covertly) contained in this concept A; or B lies outside the concept A, although it does indeed stand in connection with it. In the one case I entitle the judgement analytic, in the other synthetic."

This is not very helpful, and seems to be too narrow a definition for his purposes, especially as it applies only to propositions in subject-predicate form (or apparent subject-predicate form, such as "All A's are B's"). (This, incidentally, illustrates the sort of lack of clarity which can follow on too much concentration on "canonical forms" of propositions. Cf. Appendix II. 11,ff.)

Kant's explanation is not made much clearer when we are told, in A.716, B.744, that analytic knowledge is obtained merely by meditating on concepts, or that synthetic knowledge involves going beyond concepts in an appeal to intuition (see A.721, B.749). (The notion of an "appeal to intuition" will be clarified below. Cf. G.C.11, and sections 7.C and 7.D.) I think that what Kant was getting at in these passages will be illustrated by my discussion of "identifying relations" between meanings, below.

Most modern attempts to explain the distinction are probably related to Kant's assertion that a judgement is analytic if "its truth can always be adequately known in accordance with the principle of contradiction" (A.151, B.191).

6.B.3. Finding Kant's attempt to characterize the distinction unsatisfactory, some philosophers have tried to define the class of analytic propositions to be those whose truth follows from the meanings of the words occurring in them. This, however, is also difficult to understand, as Waismann pointed out in Analysis, Dec., 1949. He asks (p.27): "What can be meant by saying that a statement follows from the very meaning of its terms?"

The attempt to elucidate this by saying that what is meant is a statement which follows from the definitions of its terms, provokes another of Waismann's questions: "If an analytic statement is characterized as one that follows from mere definitions, why is it not itself a definition? ... Why is it that what follows from a definition is not, as one would expect, a definition, but an analytic judgement?" (p.29.)

Quine also found it incomprehensible that definitions should be available for founding truths. (See "Truth by Convention" in Feigl & Sellars, p.259). However, he allowed that they might be used to transform truths, and this is echoed by Waismann: "Definitions are substitution licences of a particular sort, ... and every substitution licence can be re-written as an equivalence." (op.cit. p.39).

6.B.4. Having noticed that definitions could be thought of as rules or licences permitting substitutions of synonymous expressions without change of truth-value, Quine, and later Waismann, decided to define "analytic proposition" to mean "logical truth definitionally abbreviated" ("Truth by Convention", p.251), thereby removing the difficulty of explaining how a definition

could make true a proposition which was not itself a definition. This presupposes the notion of a "logical truth", which seems to be the notion of a proposition which is true in virtue of its logical form, or, in my terminology (see 5.A.3 and section 5.6), a "formal truth". Waismann's version of the definition of "analytic" was as follows: "A statement is analytic if it can, by means of mere definitions, be transformed into a truth of logic." (op.cit. p.31.)

For example, on this view, "All bachelors are unmarried men" is analytic, since, by definitional substitutions, it can be transformed into "All unmarried men are unmarried men", which is a formal truth. Any proposition which cannot in this way be transformed into a formal truth would, according to this definition, be synthetic, not analytic. Pap, on p.5 of "Semantics and Necessary Truth", seems to indicate his acceptance of this definition, when he writes: "One may be inclined to characterize as synthetic, necessary statements whose descriptive terms occur essentially yet cannot be eliminated through analysis." (I.e. analysis of meanings, on the basis of which one can replace defined symbols with their definienda.)

6.B.5. Now, there does seem to be a close connection between propositions which are analytic and propositions which are true in virtue of their logical form, as will appear later on when it is shown that formal truths are just a particular kind of analytic truth. But the connection cannot be that the class of analytic propositions is defined as suggested above in terms of derivability from formal truths, for this definition does not seem to be wide enough. This is because there are propositions

which are true in virtue of partial definitions, of the sorts discussed above (in section 4.0), and these propositions cannot be derived from formal truths by means of synonymy substitutions.

The examples discussed there were all concerned with incompatibility relations between colour words (such as "red" and "orange"), but similar remarks might be made about relations between certain sound-concepts. Consider, for example, the expressions which refer to the kind of feature of a sound which we call its "timbre", such as "flute-timbre" or "the sound of a bassoon". It seems that the meanings of these expressions might be taught ostensively, in such a way as to leave them indeterminate in some respects so that the question: "Can a sound have two timbres at the same time?" and the question: "Can a sound be the sound of a flute and the sound of a bassoon at the same time?" would not have definite answers. (For sources of indeterminateness see chapter four, section A.)

Consider the sound produced by the loudspeaker when a (monophonic) recording of a duet for flute and bassoon is being played on a gramophone: here it is possible clearly to "hear" both instruments in the one sound coming out of the loudspeaker. But is it really one sound, or is it two sounds? If it is one sound, does it have two different timbres at the same time? That is, does it have the flute-timbre and the bassoon-timbre or does it have no timbre at all? Or does it have a third timbre different from that of the sound produced by either a solo flute or a solo bassoon? I believe that as far as the English language is concerned, there is no definite answer one way or the other to these questions. At any rate, if this sort of case is not produced during

the process of teaching someone to use the word "timbre", then there may be nothing in what the pupil understands by the word to settle these questions: the meaning which he associates with the word does not determine "in advance" what he should say about this sort of phenomenon. (See 2.D.2, 3.C.4, etc.)

In that case, each of the questions could be settled one way or another by the adoption of a linguistic convention for the use of the words describing sounds, which might have the consequence that certain statements were "true by definition" though not definitionally derivable from formal truths.

6.B.5.a. We might, for example, adopt the convention that the sound in question was to be described as "one sound with two timbres". Or we might adopt a rule to the effect that "flute-timbre" and "bassoon-timbre" were to be incompatible descriptions, which would rule out the possibility of describing the sound of the duet as a sound with both timbres. (It would not, however, tell us whether the sound was to be described as having either of the two timbres alone, or as having no timbre at all, etc. The rule leaves each of the individual concepts as indefinite as it was without the rule: see 4.C.3-4.)

Such an incompatibility convention is an arbitrarily chosen linguistic rule which helps to remove certain kinds of indeterminateness of meaning (4.C.4). It serves as a partial definition of the word "timbre", say. It does not define any expression as being synonymous with any other, it sets up no synonymy relations, but it does have the consequence that sentences like "No sound has a

flute-timbre and a bassoon-timbre at the same time" express true propositions, owing to the incompatibility between the two descriptive expressions.

We have therefore found a statement which is true in virtue of the fact that certain words have certain meanings or are governed by certain linguistic rules, but which is not derivable from a formal truth by substitution of synonyms. (Notice, incidentally, that this partial definition does not rule out any kind of experience as impossible: it does not make the experience of hearing the duet impossible, but merely rules out the possibility of describing it in a certain way. This should be borne in mind when the incompatibility of colours is under discussion. Compare 3.B.4.d.)

6.B.5.b. Another sort of proposition which is, in an obvious sense, true by definition though not derivable from formal truths by substitution of synonyms is provided by an ostensively-defined relational expression with an added verbal rule. The ostensive teaching of an expression like "to the left of" might show that expressions of the form: "X is to the left of Y" are applicable to a whole range of cases, including pairs of objects at various distances apart, without specifying the use in connection with just one object. The indefiniteness might then be removed by the adoption of an arbitrary convention, giving one or other answer to questions like "Can an object be to the left of itself?" or "In the expression 'X is to the left of Y', can 'X' and 'Y' refer to the same thing?" For example, it might be decided that the expression was to be irreflexive, in which case "X is to the left of X" would express a false

proposition no matter what referring expression took the place of "X", and the statement "Nothing is to the left of itself" would be true by definition.

Once more, we have an example of an analytic proposition which is not definitionally derivable from a formal truth, since the linguistic convention in virtue of which it is true does not generate any synonymy-relations: it is, as before, a partial definition.

6.B.6. In these examples of propositions which are true by definition without being definitionally abbreviated logical truths, descriptive terms, such as ("flute-timbre", "to the left of", etc.) occur essentially yet cannot be eliminated through analysis (see end of 6.B.4). Since they are obvious candidates for the title of "analytic" propositions, the Quine-Waismann definition of "analytic" in terms of derivability from formal truth cannot be wide enough.

These examples show that Quine was wrong when he wrote (in "Truth by Convention", p. 258) that "... definitions are available only for transforming truths, not for founding them." We seem to have discovered propositions whose truth is founded in linguistic conventions. At any rate, they are not merely derived from some other truths by some kind of transformation.

6.B.7. However, even if we were wrong about these examples, the question would arise: What is it for a statement to be true in virtue of its logical form? Surely only that the statement is true in virtue of the meanings or functions of the logical words and constructions employed in it. But to say that formally true statements are true in virtue of the meanings of logical constants surely cannot mean

that they are true in virtue of being derivable from formal truths by substitution of synonyms: this would be circular, or lead to a vicious infinite regress. So there must be some other sense in which a proposition's truth may follow from the fact that its words are governed by certain rules, than the one suggested by Quine and Waismann. If there is this other way in which a proposition may be true in virtue of meanings or linguistic conventions, why should it be restricted to formal truths, why should it not also explain the sense in which other analytic propositions are true by definition? We must try, therefore, to find a wider definition of "analytic" than the Quine-Waismann definition, which avoids this last objection, but before doing so let us see what Frege had to say.

6.B.8. Frege, as we shall see, did not limit the role of definitions to that of "substitution licences". According to him, the question whether a judgement is analytic or not, is a question not about the content of the judgement, but about "the justification for making the judgement". ("Grundlagen", p.3.) The judgement that some proposition is analytic is not concerned with its being true, or with what it means, but is "a judgement about the ultimate ground upon which rests the justification for holding it to be true."

Now notice how Frege goes on. In order to discover whether a proposition is analytic or not, we have to find the proof of the proposition and then follow it right back to the primitive truths on which it is based. "If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an

analytic one, bearing in mind that we must take account also of all propositions upon which the admissibility of any of the definitions depends." ("Grundl." p.4.)

(We need not worry about the fact that different persons may justify a proposition in different ways: what Frege clearly means is that a proposition is analytic if there is some justification resting ultimately only on general logical laws and definitions. As will be shown later on, an analytic proposition may be justified empirically too. So Frege ought not really to talk about "the ultimate justification" or "the ultimate ground", as if there could be only one.)

6.B.8.a. This may seem clear at first, but it becomes mysterious as soon as we try to find out what Frege means by a "definition" or how he thinks a proof can rest on definitions.

From what is said in "Grundlagen", and in his essays "On the Foundations of Geometry" (Phil.Rev. 1960), it is clear that he thinks (or did at least once think) of a definition as some kind of proposition, which first of all "lays down the meaning of a symbol" and then "transforms itself into a judgement ... (which) ... no longer introduces the object, (but) is exactly on a level with other assertions made about it." ("Grundl." p.78.)

Admittedly, he says (Phil.Rev. p.4.): "Although definitions which have been made into (sic) statements formally play the part of basic propositions, they are not really such", but it is apparent from the context that he does not mean to deny that they are propositions on a level with other propositions in the proof, but only that they are basic propositions, i.e. statements of general logical laws or axioms, and that they "extend our knowledge"

(Phil.Rev. p.5.)

He also regards a definition as a means of "determining a reference of a word or symbol" (Phil.Rev. p.4) or as something which "lays down the meaning of a symbol" ("Grundl", p.78.). How can a definition both lay down meanings and serve as a proposition "on a level with other propositions"?

6.B.8.b. In order to understand what lies behind all this, we must remember that Frege required a rigorous proof to satisfy certain conditions. "All propositions used without proof should be expressly mentioned as such, so that we can see distinctly what the whole construction rests upon" and "all the methods of inference used must be specified in advance. Otherwise it is impossible to ensure satisfying the first demand." (See "Translations", p.137.) But the principles of inference specified "in advance" by Frege permit inferences to be drawn only in cases where the premises and conclusion stand in a certain kind of formal or logical relationship (having nothing to do with their content), which requires that the premises should be propositions "exactly on a level with" the conclusions.

So if a definition is to serve as a premiss in such an inference, then it must have the same general form as the other kinds of propositions which may serve as premisses. Now we can see why Frege requires his definitions to lead a double life: first they must somehow or other specify that certain words or symbols have certain meanings, and secondly, in order to serve as premisses for inferences, they must be propositions "exactly on a level with other assertions" ("Grundl", p.78). Thus we find Frege talking about definitions as propositions which first do one thing,

and then "transform themselves" into something else. (Mario Bunge, in Mind, 1961, p.140-141, manifests the same confusion in talking about "linguistic conventions taking the form of propositions, not of proposals".)

6.B.3.c. But how can definitions lead this kind of double life? How can anything which works like an ordinary proposition do what a definition is supposed to do, namely lay down the meaning of a symbol? How can a definition first assign a meaning to a symbol and thereupon "transform itself" into a proposition?

Frege seems not to have realized that in order to define a word one must mention it, or somehow indicate that one is making a statement about words and their meanings, and that such a statement is not "on a level with" other statements which use those words.

One might try to defend him by saying that a proposition using a proper name can say what its reference is, as in "The number five is the number of fingers on a normal human hand". This certainly tells us which number is the one referred to by "the number five", but it does not tell us the sense of the expression, since it leaves open the question whether it is a matter of definition or a matter of fact that the number of fingers on the normal human hand is what is referred to by "the number five". Similarly, a proposition using a descriptive word can tell us something about the extension of that word, but it does not tell us the meaning of the word. Thus "A square is a rectangle in which the adjacent sides are equal" tells us something about the extension of "square" if we happen to know the meanings of the other words, it tells us which objects happen to be described by the word, but it does not say why it describes

them. (Cf. 2.C.3.) The statement does not say what the meaning of "square" is, it merely describes a property which can be found in squares, leaving open the possibility that this is an accident. Frege, however, regards this statement as a definition (see p.145 of "Translations").

6.B.3.d. Of course, if I say "That vase on the top shelf is turquoise in colour", the person I am talking to may guess that I mean to tell him what the word "turquoise" means, and he may guess the meaning correctly. But I have not told him the meaning, for what I say leaves open the possibility that "turquoise in colour" means the same as "on the top shelf"! "Tomatoes are red in colour" could be used to teach the meaning of "red": does that make it a definition? An explicit definition must not merely be something which enables a meaning to be guessed: it must say what the meaning is. It is not enough to state a fact which happens (though this is not asserted) to be true by definition. So in order to state a definition of "gleen" I must say something like "The word 'gleen' means the same as 'glossy and green'" or "By definition of 'gleen', a thing is gleen if and only if it is glossy and green", and not merely "All gleen things are glossy and green and all glossy and green things are gleen".

Thus Frege's device (in "Grundgesetze") of adding a vertical stroke to a formula to indicate that it is a definition is not enough, for it leaves unsettled exactly which symbol is being defined and exactly what is being said about it: its only effect is to assert that a formula expresses a statement which is true in virtue of some definition, but it does not say which word is defined

by it nor what the definition is.

6.B.9. I have been trying to force Frege into a dilemma of the following sort. When he talks of definitions as propositions which may occur in a completely explicit proof without themselves being proved, then either he means to refer to definitions proper, that is statements which are explicitly about words and their meanings, or he wishes to refer to statements (in the "material mode") which do not mention words or meanings but are nevertheless true in virtue of the fact that certain words have certain meanings.

In the former case, some kind of explanation is required of how the definitions can be used as premisses for logical inferences to propositions which use the words defined.

In the latter case, Frege is already making use of the notion of an analytic proposition when he talks about "definitions", and so he has not begun to explain how to tell that a proposition is analytic in the first place: he has, at most, shown us how we can tell that a proposition is analytic if we already know that other propositions are analytic, namely by seeing if it can be derived from them using purely logical (formal) truths and formally valid inferences.

In either case he has left unanswered Waismann's question: "Why is it that what follows from a definition is not, as one would expect, a definition, but an analytic judgement?" (See 6.B.3.) He has not shown us how to get from explicit definitions to statements which are true in virtue of those definitions.

6.B.10. To sum up: there is a gap to be bridged between

statements which are about meanings of words and statements which are true in virtue of meanings. Frege eliminated the gap by failing to distinguish two different kinds of statement properly, and talking instead about one thing which could do two kinds of jobs. Waismann and Quine, on the other hand, tried to eliminate the gap by giving up the idea of inferring true propositions from definitions. Instead they regarded definitions as substitution licences, which permit analytic statements to be derived from formal truths. But we have seen that this is not sufficiently general, for it does not take account of statements true in virtue of partial definitions. (6.B.5,ff.)

It is clear that what is needed here is a new explanation of the notion of a definition or linguistic convention, and a new description of principles according to which from definitions or statements about meanings one can infer that certain sentences express true propositions. In short: the notion of a "definition" must be clarified, and the notion of a logically valid inference must be generalized.

6.B.11. It is at this stage that we must turn back to what was said in chapter five, especially sections C, D and E (5.E.1 - 5.E.4). We have seen that although the value of a regator for an argument-set will usually depend on how things happen to be in the world, which may affect the outcome of applying the technique for discovering values, nevertheless, there are some "freak" cases where the value is determined independently of the facts and may be discovered by examining the argument-set and general technique, without applying the technique. This showed us how it is possible to discover that some

sentences express true propositions merely by examining the non-logical words occurring in them (taking their meanings into account, of course) and the logical techniques for discovering the truth-values of the propositions expressed by those sentences.

Thus, by examining the argument-set ("red", "rot") and the logical technique corresponding to the logical form "All P things are Q", we could see that the proposition expressed by "All red things are rot" must be true, without actually applying the technique and examining all red things. (See 5.D.1. The word "rot" was defined to refer to the same property as "red".)

So we have already seen that it is possible to infer from facts about the meanings of words and the functions of logical constants that a sentence expresses a true proposition. When that happens, we know that we have the right to assert that proposition, since in general we know that we can utter a sentence when the logical rogator corresponding to its logical form yields the value "true" (see 5.B.13). Thus we have an answer to Waismann's questions quoted in 6.B.3: "What can be meant by saying that a statement follows from the very meaning of its terms?" and "Why is it that what follows from a definition is not, as one would expect, a definition, but an analytic judgement?" The answer is simply that there are ways of drawing conclusions from the fact that certain descriptive words and logical constants have certain meanings or functions, such as the conclusion that a sentence including them expresses a true proposition.

6.B.12. This shows that Waismann's conception of a logically valid inference was too narrow, for he failed

to see that the inference from a truth about meanings of words to a proposition using those words can, in a sense, be logically valid even though it is not formally valid, like the inference from "All roses attract bees" to "All red roses attract bees", which is valid in virtue of its logical form. It should not surprise us that an inference from a statement about words to a statement using those words should be logically valid. After all, the inference from (1) "The sentence 'Plato was precocious' expresses a true proposition" to (2) "Plato was precocious", is surely logically valid?

I describe such inferences as logically valid since they do not depend on special facts about the subject-matter referred to by the words and sentences mentioned. They depend on very general facts about the logical techniques for determining truth-values and the conditions in which it is appropriate to utter a statement. All the essential features of the examples of propositions whose truth-values could be determined independently of the facts (in 5.C and 5.D), were topic-neutral features. This is why I call an inference from the fact that a proposition possesses those features to the fact that the proposition is a true logical inference.

6.B.12.a. We might say that we have discovered logical theorems which could be formulated in some such manner as the following:

If a word "P" refers to the same property as the word "R", "P" "Q" and "R" being descriptive words, and if the relevant logical constants are used as described in 5.B.11, then the sentence "All P Q's are R" expresses a true proposition.

This statement is certainly not a formal truth, like "If all red things are rot, then all red boxes are rot",

for it is not true in virtue of its logical form in the same way. (Content is relevant as well as structure in deciding that the theorem is true: metalinguistic words and expressions occur essentially. Its truth has to be established by considering the things it is about, and in particular by investigating the logical techniques corresponding to the logical constants mentioned.)

Such logical theorems may be described as "non-formal truths of logic". They state the facts which justify the (non-formal) inferences which we make in deciding that analytic propositions are true. It should be noted that although such truths and inferences are not formally valid, this in no way implies that they are lacking in rigour, though we must remember that, as pointed out at the end of chapter five, they may presuppose that certain conditions are satisfied. There will be a more general discussion of non-formal proof in chapter seven. (Though these non-formal truths of logic are often appealed to implicitly, and sometimes stated explicitly, logicians appear not to have taken them into account when explaining what they mean by talking about "logical truths" or "logically true propositions". Almost always they seem to think they are talking only about propositions which are true in virtue of their logical form, the formal truths described in 5.C.)

6.B.13. So much for the (non-formal) logical theorems and logical inferences which are implicitly employed when we discover that some sentence is true in virtue of what it means. However, we are not yet quite ready to offer a definition of "analytic", for we must first turn our attention to facts about meanings, in order to clarify

the notion of a "definition". We shall then be able to follow Frege in defining analytic propositions to be those whose truth-values can be discovered merely by examining facts about the meanings of words used to express them, and making inferences justified on general logical grounds (of a topic-neutral kind). For this we require the concept of an "identifying fact about meanings".

6.C. Identifying relations between meanings

6.C.1. The discussion of chapter five brought out three factors which, in general, determine the truth-value corresponding to a sentence (at any time), namely (a) the meanings of the non-logical words, (b) the logical techniques corresponding to the logical form and (c) the way things happen to be in the world. (See 5.B.1.) In some "freak" cases, we found that the third factor dropped out as ineffective (though one might fail to notice this and take the facts into account in the usual way in finding out the truth-value). In these freak cases, the truth-value of the proposition expressed by a sentence could be discovered by examining (i) the logical techniques corresponding to its logical form, (ii) the "structure" of the argument-set (set of descriptive words) to which the logical form was applied and (iii) relations between the arguments, or, more specifically, relations between the non-logical words. Thus, from facts about the meanings of the words we were able to infer that combining words in certain ways produced sentences expressing true propositions.

We must now show how it is possible to pick out a

class of facts about the meanings of words which correspond to definitions in virtue of which analytic propositions may be true. Let us describe them as "identifying" facts about meanings. We may also talk about identifying relations between meanings, or analytic relations between meanings. Statements about meanings which do not state identifying facts about meanings state non-identifying facts, or describe synthetic relations between meanings. A definition is then a statement of an identifying fact about the meanings of words. What does all this mean?

6.C.2. It was argued in 6.B.8.c-d that a statement can be a definition only if it mentions words. But not every statement which mentions words is a definition, even if from the fact that it is true we can infer that some sentence using those words expresses a truth. For example, it may be the case that

- (1) the class of objects with the property referred to by the word "red" and the class of objects with the property referred to by the word "mat" are mutually exclusive,

in which case we can infer (logically, but not formally) that

- (2) the sentence: "No red things are mat" expresses a true proposition.

From the relation between the words in the argument-set ("red", "mat") described in (1), we can infer, by considering the appropriate logical techniques for determining truth-values, that applying the logical form "No F things are G" to that argument-set yields a true proposition (as described in 5.D). But the relation in question may hold simply because of the contingent fact that nothing which is red happens to have a surface with a mat texture. The statement (1) is not a definition, for one could fully

understand the words it mentions without knowing that they stand in the relation it describes. In order to know that they stand in that relation one must not only know the meanings of the words, but also have carried out some empirical observation of the class of red things or the class of objects with mat surfaces. So (2), though inferred from a statement about words, is not inferred from a definition, or from the statement of an identifying fact about meanings.

6.C.3. When is a statement about words a definition? How do we discover whether a statement states an identifying fact about meanings? What must a person ask himself when he asks whether a statement defines the meanings which he associates with the words which it mentions?

The answer seems to be suggested by the example just mentioned. The fact (if it is a fact) that the words "red" and "mat" have mutually exclusive extensions is not an identifying fact about their meanings because it is possible fully to specify what they mean, by indicating the properties to which they refer, without mentioning the fact or anything which logically implies it. One could successfully draw a person's attention to either property in a object without mentioning or getting him to think about or attend to the other property in any way. The relation of incompatibility is not an identifying relation because to assert that the relation holds is not essential to a full specification of the meanings of the words. Their having the meanings which they do have is not even partly constituted by their being incompatible descriptions, as it would be if there

were an n-rule to the effect that they could not both describe the same object at the same time.

In short, to discover whether a statement identifies meanings one must ask "Is it possible to know exactly what I mean by these words without knowing the fact stated by this statement or any other logically equivalent to it?" (This last clause is included because there is no need for us to say that words have different meanings when their definitions are logically equivalent: our criteria for identity of meaning need not be quite as sharp as that - see sections 2.A, 2.C - though for some purposes it might be necessary to discriminate between different forms of definitions.)

6.C.4. An identifying statement about the meanings with which words are used states something which must be known if one is to know what those meanings are, in the sense of knowing how to use the words with those meanings. One need not, however, know that the statement is true. For the statement may employ metalinguistic concepts without which it cannot be understood, though one can perfectly well use the words it mentions without having them. I can use a word to refer to a property, and yet not understand the expression "refer to a property". I may know that a word, such as "gleen" refers to a combination of two properties, without knowing that the words "glossy" and "green" refer to those two properties separately. Nevertheless, the statement that "gleen" refers to the same property as "glossy and green" states an identifying fact about the meaning of "gleen", since it correctly describes the way "gleen" is used by anyone who uses it correctly, who knows what it means. In such cases I say that the identifying fact about the meanings

is one which must be known at least implicitly by anyone who knows the meanings of the words in question. (For more detailed remarks on "implicit" knowledge, see Appendix III.)

What is required for the implicit knowledge to become explicit may be the acquisition of new meta-linguistic concepts, or the acquisition of a new vocabulary, or any of the other sorts of things described in Appendix III (see III.5). In these cases one is not learning the meanings of the words, but merely learning to say what, in a way, one already knows about the meanings of the words, since one knows how to use them. One does not have to carry out observations of the objects described by those words, nor examine the properties referred to by the words in order to discover new aspects (Cf. 7.D.) That is to say, for making implicit knowledge of meanings explicit, neither experience nor insight is required. If, in addition to knowing the meanings of words, one must have some experience or insight, in order to see that a statement is true, then it does not state an identifying fact about meanings. (Cf. 6.C.10, below.)

6.C.5. If a statement states an identifying fact about the meanings of certain words, then this means that unless that fact is (at least implicitly) known, the full meanings of the words will not be known. But this leaves open the possibility that part of their meanings may be known. For example, in section 4.C it was shown that a partial definition might be adopted according to which the two hue-words "red" and "orange" were to be incompatible descriptions. A person who had been taught

their meanings ostensively, without being told about this incompatibility convention, would know something about their meanings, for he would be able to decide correctly in most cases whether objects were describable by these words, but he would not fully understand the words as used by persons who followed the incompatibility rule. He would not know that certain descriptions of borderline cases were excluded, such as "both red and orange".

So in some cases one may know part of what a person means by a word without knowing all the identifying facts about the meaning: but then the partial meaning is likely to be less determinate than the full meaning.

6.C.6. Some identifying facts about meanings are "purely verbal" and some are not. For example, suppose the word "V" to be semantically correlated with the property P and the word "W" with the combination of properties P and Q (cf. 3.B.3). Then either of these correlations may be set up without the other. For example, a person may use the word "V" to refer to P, while another person uses the word "W" to refer to the combination of P and Q, though neither of them has a word synonymous with that used by the other. Nevertheless, the statement that the word "W" refers to a combination of the property referred to by "V" with another property states an identifying fact about the meanings of the words: the relation it describes is an identifying relation between their meanings. On the other hand, if two words, such as "red" and "orange", are related by an incompatibility rule of the sort described in 4.C.2, then it is impossible to say exactly what either of the words

means without mentioning the other word, unlike the previous case, where one can fully explain the meaning of either "W" or "V" without mentioning the other.

In the one case we have a purely verbal identifying relation, which holds merely in virtue of a rule relating two words, whereas in the other case we have an identifying relation which is not purely verbal because it holds primarily on account of rules correlating words and properties.¹

6.0.7. If an identifying relation holds between entities, then at least one of them is not capable of existing on its own. Certainly the hue redness (i.e. the observable property) may exist on its own (since one may be fully acquainted with it without being acquainted with the hue orangeness). But if the words "red" and "orange" are related by a defining incompatibility rule,

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1. Here we see one of the things which may be meant by the distinction between "real" definitions and "nominal" definitions. The rule relating "red" and "orange" is purely nominal, whereas the definition of "W" in terms of "V" and some other word would be "real" since it would be a correct definition in virtue of the prior correlation of these words with properties. There is a sense in which one of the definitions is quite arbitrary, since there is nothing to justify the statement that the words are related by an incompatibility rule, except the fact that unless they were so related they would not be the same words (or at any rate they would have different meanings), whereas the other is non-arbitrary, since the statement of the relation between words is justified by the fact that those words are correlated with certain properties, though, of course, this correlation is itself as arbitrary as any linguistic convention. But this is a digression.

then the properties to which they refer are "improper" properties (cf. 2.D.6, 3.B.5), not objects of experience, and neither could exist alone. If the property referred to by the word "gleen" is the combination of the properties referred to by "glossy" and "green", then the former ("improper") property cannot exist unless the latter two do, though either of the latter two may, of course, exist independently of the other.

In general, when several entities stand in some identifying relation, at least one of them is a thing which could not exist unless the others did: for otherwise it would not stand in the relation in question, and so it would not be what it is. This brings out the fact that an identifying fact about meanings may be essential only to the full specification of the meaning of only one of the words mentioned. But if a statement states an identifying fact about meanings then there must be at least one word whose meaning could not be fully specified without it.

6.C.8. As we have seen, the statement (1) of 6.C.2. does not state an identifying fact about the meanings of the words "red" and "mat". There are some kinds of statements which can only state identifying facts about meanings, if they are true. Examples are statements of the form:

- (1) The word "U" refers to the same property as "V".
- (2) The word "U" refers to the combination of the properties referred to by "V" and "W".
- (3) "U" means the same as "V".

On the other hand, the following may either be logical consequences of identifying facts, or they may describe synthetic (i.e. non-identifying) relations between the

meanings of words.

- (4) The words "U" and "V" have the same extension.
- (5) The extension of the word "U" is the intersection of the extensions of "V" and "W".
- (6) The property referred to by "U" is possessed by all objects which have the property referred to by "V".

In general, if the meaning of a word is logically synthesized out of properties (see section 3.B), then the relation which holds between the meaning of this word and words referring to the properties from which its meaning is synthesized must be an identifying relation, since one cannot know its meaning without knowing (at least implicitly) that this relation holds. On the other hand, a statement about the classes of objects correctly describable by certain words may fail to state an identifying relation between the meanings of those words. (Cf. 6.C.2.)

6.C.9. In addition, it should be noted that it is possible for words whose meanings are non-logically synthesized to stand in identifying relations. For example, if the word "red" is governed by a p-rule of the sort described in 3.D.2, and if the words "red-inf" and "red-ult" refer to the two specific shades taken as boundaries, then the statement "The word 'red' refers to shades of colours lying between the shades referred to by 'red-inf' and 'red-ult'" states an identifying fact about the meaning of "red", and the relation it describes between the three words mentioned is an identifying relation between their meanings, if they are used in the way specified.

Similarly, the statement "The word 'tetralateral'

refers to the property of being bounded by four plane surfaces" states an identifying fact about the meaning of the word "tetralateral", introduced in 2.C.8.

However, the difference between these statements of identifying facts, or identifying relations between meanings, and those discussed previously is, as pointed out in 3.D.10, that the relations between words which hold in virtue of non-logical syntheses are not logical relations, that is to say, they are not relations which are describable in quite general topic-neutral terms, but are relations which can hold only between meanings of words referring to special kinds of properties. Similarly, we may say that these identifying facts are not facts which can be described in purely logical, or topic-neutral terms. This will be important when we come to discuss ways in which one can infer that sentences express true propositions from the fact that certain words occurring in them are identifyingly related.

6.C.10. We can now use the concept of an "identifying relation between meanings" to complete the definition of "analytic" which was begun at the end of section 6.B.

We define "analytic" so that a statement S, obtained by applying a logical form F to an argument-set A (i.e. an ordered set of non-logical descriptive words) is analytic if it is possible to determine the truth-value of that statement merely by examining some or all of the following: (i) the logical technique corresponding to F, (ii) the "structure" of the argument-set A and (iii) identifying relations between the meanings of the words in A, provided that only purely logical (i.e. topic-neutral) considerations are relevant in the inference.

Thus, if a statement S is analytic, then any other statement with the same logical form will also be analytic, if its non-logical words stand in the same sort of identifying relation as the non-logical words of S, no matter what the topic with which they are concerned (e.g. no matter which properties they refer to: this is the force of the underlined part of the definition).

For example, the statement "All gleen things are glossy and green" can be seen to be true by considering the general logical technique corresponding to the logical form "All P things are Q and R", and by taking note of the fact that the words in the argument-set ("gleen", "glossy", "green") stand in the following identifying relation: the property referred to by the first is the combination of the properties referred to by the other two. Since all one needs to know is that the words stand in this relation, without knowing what sorts of properties they refer to, or what sort of topic they are concerned with (i.e. only purely logical topic-neutral considerations are relevant), one can conclude that any other statement of the same logical form whose non-logical words stand in the same identifying relation is true, and therefore analytically true, no matter what those non-logical words mean. (See also the example in 5.D.1.)

The proviso that only general logical considerations and identifying relations between meanings can be relevant rules out the cases where the truth-value of a statement has to be discovered either by applying the technique corresponding to its logical form and carrying out empirical enquiries concerning the things referred to by the non-logical words, or by examining any such

things as the properties referred to by these words: the latter would not be a topic-neutral enquiry since it would presuppose an acquaintance with these properties. (Cf. 6.C.4.)

6.C.11. This account of the analytic-synthetic distinction will become clearer later on, when analytic propositions are contrasted with synthetic propositions which are necessarily true. Meanwhile it may be noted that much of what I have said could be construed as an attempt to clarify some of Kant's remarks about the distinction.

For example, when he considers the possibility that "... the predicate B belongs to the subject A as something which is (covertly) contained in this concept A" (see 6.B.2.), it seems that he is considering just one of the kinds of facts which I should describe as an identifying fact about the meanings of words. This is in the same spirit as his remarks that analytic knowledge is obtained merely by meditating on concepts.

When he says that synthetic judgement involves going beyond concepts in an appeal to intuition, he seems to have in mind the same sort of thing as I have when I rule that some statements about meanings do not state identifying facts since in order to know that they are true it is not sufficient merely to know the meanings of the words mentioned: in addition one must examine the things referred to, either by carrying out empirical investigations, or by examining properties to discover that they are synthetically related (see chapter seven). Appeals to "intuition" are also ruled out by the condition that knowledge of the truth of analytic propositions must be based only on logical considerations, which do not presuppose

acquaintance with any special kind of property or object (cf. 3.D.10, 7.C.2, 7.D.3,ff). Kant's account simply happens to be less general than mine, and is not as detailed, since it does not explain how it is possible for a statement to be analytic. (Cf. Sections 5.C & 5.D.)

In addition, I think that my definition of "analytic" brings out what people are getting at when they describe some statement as "true by definition", or say such things as "If that's what you mean by so and so then you must admit that" or "You cannot really believe that, unless you mean it in an unusual sense", etc.

6.C.12. It should be noted that in my definition of "analytic", the phrase occurred "... if it is possible to determine the truth-value ...". The point of this is the fact that one may fail to notice that a statement is analytic, and then discover whether it is true or not in the usual way, by carrying out empirical enquiries. I shall explain the significance of this presently.

But first we must take note of the complexities in our ordinary use of words which were described in chapter four.

6.D. Indefiniteness of meaning

6.D.1. It should not be thought that every time anyone makes a statement that statement is either analytic or synthetic, even if it falls within the class of statements we have selected for discussion (in 1.C.2). For there may be some fact about the words used to express the statement which is not clearly an identifying fact about the meanings of those words nor clearly not an identifying

fact about the meanings of those words. If it is possible to infer from this fact that the statement is true, then it may not be clearly analytic or clearly not analytic, since it is not clear whether the truth of the statement can be logically inferred from an identifying fact or not.

The reason why a fact about words may be neither definitely an identifying fact nor definitely not an identifying fact is simply that words may have meanings which are indeterminate in any of the ways described in chapter four. This indeterminateness has the effect that sharp criteria for identity of meanings cannot be applied. As pointed out in section 2.C and section 4.B, this does not matter much for normal purposes, but it does matter when philosophers are discussing the analytic-synthetic distinction, and say such things as "A particular speaker on a particular occasion who uttered 'All phosphorus melts at 44°C ' without knowing whether he was following a rule which made this analytic or not, would be cheating" (Pears, in Mind, 1950, p.204). To use a word with an indeterminate meaning is to cheat only when one implies that one means something perfectly definite by it, that one would know in all cases what would count as settling questions about the truth-value of a statement using the word, but one need not imply this when one uses a word.

6.D.2. For example, if the word "U" is correlated by an indeterminate d-rule with a range of properties with indeterminate boundaries (4.A.4, ff.), and the word "V" refers to one of the borderline properties, then the statement "Possession of the property referred to by 'V'"

is a sufficient condition for being correctly describable by the word 'U' is neither definitely an identifying statement about the meaning of "U" nor definitely not an identifying statement about the meaning of "U", even if it is generally believed that all things which are V are also U, for some such reason as that the property referred to by V happens always to accompany some other property which is taken as a sufficient condition for being describable by "U". (Far more complex and interesting cases are possible.) In such a case, the statement "All things which are V are U" is neither definitely analytic nor definitely synthetic, even if it is definitely true. In this case the proposition would cease to be definitely true if someone produced an object which had the property referred to by "V" without definitely having anything else which sufficed to ensure correct describability by "U".

More interesting cases, from the point of view of this essay, concern necessarily connected properties. If what was said in 2.C.8 and following paragraphs is correct, then the property of being bounded by three straight sides is different from the property of being rectilinear and having three vertices. But if we ask whether the English word "triangle" refers to one of these two properties, or to the other, or to both conjunctively, or to both disjunctively, then there will surely be no answer, for since these two properties are always found together it makes no difference for practical purposes (e.g. when a man orders a table with a triangular top) to which of them or which combination of them the word refers. This helps to account for our use of the expression "can be defined ..." in such contexts as "The word 'triangle' can be defined as referring to the

property of being bounded by three straight lines". (Cf. 2.C.10, where it was shown how this could lead to a question-begging argument.) The word can be defined in this way for normal purposes, since a word so defined will, for normal purposes, do exactly the same job as the ordinary English word "triangle": loose criteria for identity of meanings are all we need. Since it is not definitely the case that the word has to be defined this way, or that it has to be defined as referring to the property of having three angles, we cannot say definitely that the statement "All triangles are bounded by three sides" is analytic, as understood in English, or that it is definitely synthetic.

6.D.3. When a word or sentence is used with an indefinite meaning, it is often possible to make the meaning more definite in one way or another by adopting an additional linguistic convention. This was illustrated by the n-rule correlating the words "RED" and "redange" in 3.B.4,ff. Sometimes one way of making meanings more definite makes a statement analytic, while another way of making the meanings of the same words more definite makes the statement synthetic. In such a case the original statement is neither analytic nor synthetic: its meaning is too indeterminate for the distinction to apply. When a sentence does not definitely express this or that proposition, we cannot always ask whether the proposition which it expresses is analytic or synthetic.

Normally there are several different ways in which the meaning of an ordinary sentence could be made more determinate: for example, if one of the words in it is correlated with a range of properties whose boundaries

are indeterminate, then there must be many different possible determinate boundaries. To each way of making a concept or meaning more determinate there corresponds what may be described as a (relatively) "sharply identified" concept or meaning, since sharp criteria for identity can be used for distinguishing these new concepts. (Cf. 2.C, and 3.C.9-10). All of these different sharply identified meanings may be thought of as somehow "superimposed" in the old meaning, to produce the indeterminateness, (as if several different but similar faces were superimposed to produce a blurred photograph). (In 3.E.2, it was remarked that several different concepts of the sorts distinguished in chapter three were superimposed in our ordinary concept "red". Cf. 7.D.11.note.)

As pointed out in 4.B.7, this indeterminateness can occur at different levels in a language and manifest itself in different ways.

6.D.4. This sort of thing is sometimes ignored by philosophers when they are engaged in "conceptual analysis". For example, they may argue over the question whether it is part of the meaning of "daughter" that a good daughter behaves in a certain way, or whether it is part of the meaning of "table" that the word describes objects which are used for certain purposes. One side will argue that it is part of the meaning, and another will argue that it is not, that it is just a fact which is generally taken for granted in one way or another, and both may fail to consider the possibility that as far as ordinary use goes the concept may be too indeterminate for either side to be correct, or perhaps as some people understand the word it has one sort of meaning, and as

others understand it it has the other. In such a case, the argument can only be pointless. (But see 5.E.7.a.)

6.D.5. It should not be assumed that the analytic-synthetic distinction can never be applied to ordinary statements using words which have indefinite meanings. If two areas have fuzzy boundaries it may not be clear whether they overlap or not, but this doesn't rule out the possibility that a circle with fuzzy boundaries may be completely inside another, or completely outside another, or that they may definitely overlap. Similarly a concept may stand in a definite relation to some other concept even though each of them is indeterminate in some way. For example, no matter how indefinite the concept "horse" may be it is certain that the statement "Every horse chews everything it eats at least five times" is synthetic, whether it is true or false. Some ordinary statements may be definitely synthetic despite the indefiniteness of some of the concepts employed in them.

If all statements in some language were to be either analytic or synthetic then even the most subtle ambiguities would have to be eliminated (which need not be done for ordinary purposes). But as soon as the most flagrant ambiguities have been eliminated it is possible for some statements to be definitely analytic or definitely synthetic.

6.D.6. All this shows that there is no need to say that there is no such distinction as the distinction between analytic and synthetic statements just because some statements fail to fall on one side or the other of the distinction. (It is often difficult to tell whether an

utterance is meant as a question or as a statement: should we therefore abandon the distinction between questions and statements? Indeterminateness can make a statement neither definitely true nor definitely false, where borderline cases turn up: should we say that there is no distinction between true and false statements?) Certainly we cannot assume that the distinction can be unambiguously applied to everything which can be described as a statement: if that was Kant's assumption, then it is an assumption he should not have had. We could try to make the distinction cover all cases by making it a "pragmatic" distinction and talking about degrees of analyticity, as suggested by Pap in "Semantics and Necessary Truth" (p.352, etc.), but that would be of interest only in answering empirical questions about how statements as understood by certain persons ought to be classified, and would certainly not be relevant to the problems discussed by Kant (and in the next chapter of this thesis). The alternative of adopting a "system-relative" distinction (suggested, for example, by M. Bunge in Mind April, 1961) is quite unintelligible to me unless it is simply an obscure and confused version of what I have said, namely that we must be clear as to what meanings (definite or indefinite) are associated with words before we can ask whether the statements they express are analytic or synthetic.

6.D.7. To sum up: the fact that words may be used with indeterminate meanings has the consequence that, as understood in some language, or by some person, or group of persons, a sentence may express a statement which is neither definitely analytic nor definitely not

analytic. This is because the relations which hold between the meanings of some of the words occurring in such sentences may not be definitely identifying relations, nor definitely not identifying relations. This possibility is sometimes ignored by those who believe that there is a distinction. But others who are aware of the possibility go to the opposite extreme and ignore the fact that the distinction can be applied in a clear way in some cases. My description of the way logical and descriptive words work has the advantage of being able to take account of or explain both of these facts, namely the fact that the distinction can be applied in some cases, and the fact that it cannot be applied in all.

6.E. Knowledge of analytic truth

6.E.1. We are now able to see the resemblances and differences between analytic propositions and synthetic ones. Both kinds of proposition are expressed by sentences built up out of descriptive words and logical words (and constructions) which have a perfectly general application in statements about the world. So in both cases it is possible to discover truth-values by taking account of meanings of non-logical words and applying logical techniques corresponding to the logical forms of propositions, i.e. by investigating facts. The difference is that in the case of analytic propositions there is another way of discovering truth-values, namely by examining the meanings of non-logical words and the logical techniques which have to be applied in discovering the truth-value in the other way.

This shows that the notion that "the way we get to know the truth of necessary propositions is by inspecting them" is not quite as misleading as Malcolm suggested in Mind, 1940 (p.192). For it is one way in which we can get to know the truth of some kinds of propositions.

But it is not the only way in which one can come to know that they are true. For, as repeatedly pointed out, analytic propositions are the same sorts of things as all other propositions, insofar as they are expressed by sentences which refer to non-linguistic entities, and whose logical form determines what sorts of observations count as verifying them. That is to say: analytic propositions merely form a subclass of the class of propositions which can be verified empirically. (Cf. 5.A.9, 5.C.7, 5.D.5, 5.E.3.)

6.E.2. For example, a person may fail to notice that the proposition "All gleen things are glossy" is analytic ("gleen" as I have defined it means "glossy and green"), and look to see whether it is true or not by examining all the things which are gleen to see whether they are glossy or not. (Cf. 5.B.11). We know that he must find that they all are, but he may simply fail to notice that one of the steps in recognizing that something is gleen is recognizing that it is glossy, and so he may fail to see that his search is superfluous. Perhaps he divides the investigation into two stages: he first examines all gleen things and finds that they are ellipsoidal in shape, and then he examines all ellipsoidal things and finds that they are glossy. In either case, on the basis of what he has observed, of what he knows about the meanings of "gleen", "glossy", etc., and of what he knows about

the general technique for verifying statements of the form "All P's are Q's" he is justified in asserting: "All gleen things are glossy".

(This shows, incidentally, that although only an analytic proposition can be formally entailed by an analytic proposition, nevertheless, any kind of proposition, whether analytic or not, can entail an analytic proposition. For example the proposition (1) "All bachelors are happy men and all happy men are unmarried", which is certainly synthetic, and may perhaps be false, formally entails the proposition (2) "All bachelors are unmarried," which is analytic. Some people - such as P. Long in Mind, April 1961, pp. 190-191 especially - find this sort of fact surprising. But the fact that a synthetic proposition can entail an analytic proposition is no more surprising than the fact that a true proposition may be entailed by a false one. Even a purely formal truth of the form "All P are P" may be entailed by a false contingent proposition of the form "All P are Q and all Q are R and all R are P".)

This fact that analytic propositions have in common with synthetic propositions the possibility of being verified empirically helps to show how wrong people are when they say that analytic propositions are not really propositions but rules, or when they say that empirical facts are irrelevant to their truth-values. (They are not irrelevant but ineffective.)

6.E.3. When knowledge of an analytic truth is based on empirical enquiries, all three elements in the justification of the proposition are involved (see 5.E.1). But we have seen that the third element (observation of the facts) need not be involved in a justification for asserting

the proposition. This, however, does not mean that the other two elements can, on their own, provide a justification in the same way as they did when accompanied by the third. For when all three are involved, the logical technique corresponding to the form of the proposition has to be applied, and the technique can only be applied when facts are observed. When we dispense with empirical observation, we no longer apply our knowledge of meanings and logical techniques: instead we study them, which is quite a different matter.

(Similarly, in showing that when numbers standing in certain relations are taken as arguments the value of some arithmetical function must be positive, no matter what those numbers are, we do not apply the calculating techniques for determining the value of the function: we study them. They can only be applied when actual numbers are taken as arguments. This step to a higher level is concealed in normal mathematical procedure owing to the technical devices used for proving general theorems with the aid of variables, which helps to give the misleading impression that techniques are being applied (e.g. to entities of a special kind known as "variable numbers"), owing to a misleading formal analogy. In proving the algebraic theorem normally stated thus " $(a + b)^2 = a^2 + 2ab + b^2$ " we do not add or multiply: we study the general effects of adding and multiplying in certain ways.)

This is important, because it explains why people may fail to see that a proposition is analytic even though they understand it perfectly well. For they may understand it perfectly in the sense of knowing the meanings or functions of all the words and constructions involved

and knowing how to tell whether statements expressed by sentences using them are true or false, and yet fail to notice the aspects of these meanings or functions in virtue of which some of these statements are analytic.

(Cf. 5.C.7.) People may be quite good at applying a technique automatically without being able to think clearly about it. It may never even occur to them to study it. (See appendix on "Implicit Knowledge".)

Having failed to see that a statement is analytic, one may also fail to notice that it is true.

By describing several different ways in which one may come, mistakenly, to think a proposition false when it is analytically true, I shall now try to bring out the inadequacy of the definition of an analytic proposition as a proposition which cannot be intelligibly denied, or which one must know to be true if one knows what it means, or which is such that when a person denies it this is a sufficient justification for saying that he does not understand the meanings of the words used to express it.

6.E.4. First of all, we may get out of the way a whole series of cases which are rather puzzling from a certain point of view, but need not be discussed here: the cases where a person seriously denies something simply because he is temporarily muddled, or confused, or absent-minded. We must simply take these for granted as possibilities which can explain completely why a person denies or affirms anything at all, whether it is true or false or analytic or synthetic.

Secondly, we must notice that a person who has failed to see that a proposition is analytically true may then deny it on account of mistakes of the same sort as could lead him to deny a true synthetic proposition which he has

tried to verify by making empirical enquiries or observations. For example, he may have asked someone and been given the wrong answer. Or he may have misunderstood one or more of the individual words or constructions, or failed to take in the structure of the sentence expressing the proposition, so that he thinks it expresses some other proposition than the one which it does express, in which case he may take certain facts as falsifying it when they do not really do so. Failing to take in the structure of the sentence properly is a different matter from failing to understand one or more of the descriptive or logical words. One may know perfectly well what the elements of a sentence are, and how they work, but simply be mistaken as to the way in which the sentence is built out of them, or mistaken as to the way in which the verifying technique corresponding to its logical form is in this case determined by the rules for the individual logical words and constructions (see circa 5.B.15).

So there are many different ways in which one can get the wrong idea, and not all can be described simply as "failing to understand the meanings of the words". (Notice, incidentally, that instead of getting the wrong idea one may, as a result of imperfectly grasping the structure of a sentence, not have any clear idea at all of what it means, even, in some cases, without realizing that one does not have a clear idea.)

In addition to all these factors, which may account for a person's making the wrong enquiries, or drawing the wrong conclusions from what he observes, there is also the possibility of mistakes of observation. For example, someone who wishes to find out whether all gleen things are glossy may try to collect all gleen things

together and, having done so, look them over and mistakenly think he sees one which is not glossy. Or he may make a mistake which has nothing to do with special features of the proposition in question, such as establishing the two premises "All bachelors are happy men" and "Not all happy men are unmarried" and mistakenly concluding that "Not all bachelors are unmarried". Obviously still more complex mistakes may explain a person's denying some proposition which is analytic.

6.E.5. But all this presupposes the possibility of failing to notice that the proposition is analytic. What sorts of things can account for this failure? Once again, there are many distinct possibilities.

A person who knows perfectly well how to tell whether any particular object is or is not correctly described by some word may fail to notice what he is doing in doing this, and so fail to observe that there is some identifying fact about the meaning of that word: he knows the meaning of the word in a practical way, since he can use it, he can apply his knowledge, but he is not explicitly aware of its connections with other words. In the same way one may be quite good at counting, and be able to decide which numeral follows any given numeral, and yet be unable to formulate the general principle on which one constructs the new numerals. (E.g. one may never have thought about it.) Failing to notice the relations between the meanings of words one may fail to see a consequence of the fact that they stand in these relations.

6.E.5.a. In a similar way one may fail to notice some general feature of the logical technique corresponding to

the logical form of a proposition, despite the fact that one can apply it in particular cases in deciding whether statements are true or not. Such a person fails to take explicit note of the fact that in deciding that the proposition "All gleen things are glossy" is true he examines each thing which has the combination of properties referred to by "gleen" to see whether it also has the property referred to by "glossy". Failing to notice the general procedure corresponding to the logical form of the proposition, he fails to notice facts about the application of that procedure which determine its outcome.

6.E.5.b. Alternatively, one may be perfectly well aware of the relations between the non-logical words in a sentence, and be able to describe, in a general way, the logical technique corresponding to the way logical constants occur in that sentence, and yet fail to notice how all this applies to the particular case in question, for some reason. (It is possible to fail to notice something which is well within one's field of view.)

6.E.5.c. Each of these, and perhaps other possibilities, may explain a person's failing to notice that some proposition is true independently of the facts, even though in a clear sense he understands the proposition, since he understands its parts, he knows how they are put together, and he knows what counts as the proposition's being true. We could say that in such a case he doesn't fully understand it unless he notices that empirical enquiries are unnecessary, but then we are adopting a new terminology, giving a new sense to the notion of "full understanding", and to say that a person who sincerely

denies an analytic proposition cannot fully understand it is correct and not misleading only if the new terminology has been made clear, and even then it is not very informative, since it slurs over the differences between possible explanations of a failure to notice the truth of an analytic proposition.

6.E.6. All this may suggest that in order to notice that a proposition is true by definition, that it has to be true on account of its meaning, one has to be an expert logician who can formulate facts about meanings and logical constants and draw conclusions from them, or perhaps formulate non-formal truths of logic of the sort described in 6.B.12.a. But this is not so, for one may know these facts implicitly (without being able to formulate them) and see, perhaps in a "dim" sort of way, what they imply. One may know something, and be fully justified in asserting it and yet be unable to say what justifies the assertion.

This can be illustrated by much of the ordinary person's knowledge of arithmetical facts. It is fairly easy for someone who knows what a recursive definition is to say explicitly how the series of numerals used in counting is generated (for example either in Arabic notation or in English words). But a person who knows perfectly well how to go on producing new numerals when counting may be quite unable to formulate the general principle which he is following, and may even be unable to recognize a correct formulation suggested by someone else. Nevertheless, he knows the principle, since he can apply it and distinguish incorrect from correct moves in accordance with it. We may say that he knows it

"implicitly". (Cf. Appendix III.) Now this implicit knowledge may give him the right to assert with confidence some general statement about numerals such as that numerals ending in "6" are always closer to numerals ending in "3" than they are to numerals ending in "0". ("Closer" = "separated by fewer numerals".) In particular, it may justify his asserting that between "0" and "100" there is no numeral ending in "6" which is closer to one ending in "0" than to any ending in "3". He is justified by his knowledge of the general principle for constructing numerals, properties of which he can see more or less clearly; so there is no need for him to justify the assertion by writing out the sequence from "0" to "100", though he could do this.

Similarly a person who says "It is true and has to be true that all gleen things are glossy" may be perfectly justified in making this claim, on account of his implicit knowledge of the techniques for working out the truth-values of such propositions. He may be quite inarticulate about the reasons why it "has to be true", yet what he says is correct, and he is justified in saying it. He need not have seen a logician's proof (or anyone else's).

6.E.7. We see therefore, that a person may see, correctly, that a proposition is analytic, that on account of what it means it has to be true no matter how things happen to be in the world, without fully understanding the reasons why it is true, or why it would be true in all possible states of affairs. He does not fully understand what makes it true, because he has not noticed that it is true on account of a general feature which the proposition shares with other propositions expressed by

sentences whose meanings are related in the same way.

Most philosophers have, of course, hitherto been in this position, which is why they have not been able accurately to characterize the class of analytic propositions, or propositions true by definition. Their misunderstanding is shown, for example, when they say that the difference between statements which are true in virtue of their logical form (such as "All bachelors are bachelors") and analytic statements which are not formal truths (like "All bachelors are unmarried") is that in the sentences expressing the latter, some non-logical words occur "essentially". This is mistaken, since it is not in virtue of any special property of the words "bachelor", "unmarried", that the sentence "All bachelors are unmarried" expresses an analytic proposition, any more than it is an essential property of the word "red" which accounts for the truth of the proposition "All red things are red". All that is essential is that the words occupying their positions in the sentences should have meanings which stand in certain identifying relations. That is to say, an analytic proposition is not true in virtue of the fact that the non-logical words have the special meanings which they do have, but in virtue of the fact that those meanings, whatever they may be, stand in certain relations, and other propositions, including words with quite different meanings, may be true for the same reason. One need not know the meaning of a sentence in order to know that it expresses an analytic proposition, one need only know certain facts which must be known implicitly by anyone who knows the meaning. (Though if one knows only these facts one will not, of course, know which proposition the sentence expresses.)

This shows how the class of formal truths is merely

a subclass of the class of analytic truths. (Compare 5.D.1.)

6.E.8. One thing that is clearly brought out by all these examples, which is sometimes overlooked, is that there is a difference between knowing that a proposition is true and knowing that it is analytic. This was pointed out as long ago as 4.B.5, in connection with the more general fact that there is a difference between knowing that the propositions expressed by certain sentences are true, and knowing what those sentences mean. (This does not contradict the thesis that knowledge of meanings can often be explained as knowledge of the general way in which the various words and constructions used in sentences contribute towards determining the conditions in which the propositions expressed are true, or false.) Knowledge that an analytic proposition is true can be based on observations in the same way as knowledge of the truth of any empirical proposition, whereas the knowledge that it is analytic has to be based on a priori considerations of facts about meanings and the techniques for determining truth-values. (Cf. 5.C.7, 6.E.3.)

6.E.9. It is possible to get into a muddle by failing to distinguish knowledge that some proposition is analytic from knowledge that a sentence expresses an analytic proposition as understood by certain people. The latter presupposes knowledge of empirical facts about the way the people concerned use words, the former does not. We may put this by saying that knowledge that some proposition is analytic is knowledge that if certain words are used with certain kinds of meanings, then the proposition which

they express when combined in a certain way is analytic.

In order to discover this one need not know anything about the way in which anybody actually uses words, though one will not be able to report one's discovery unless one knows which language is understood by the people to whom one is reporting it. Of course, one will not be able to think about such matters without having learnt a language, and one learns a language by seeing and hearing what happens when others speak it (and trying to speak it oneself), which involves picking up some empirical knowledge. But that is irrelevant. I believe Malcolm must have been muddled about all this when he wrote (in Mind 1940) that a child learns necessary truths by observation of the way people talk (p. 193) and that we answer questions about entailments "by finding out certain empirical facts about the way we use words" (p.195). What one learns by this sort of observation is how people understand sentences. The discovery that the propositions expressed by those sentences are analytic, or stand in relations of entailment, requires a further step which is quite different from empirical observation. (Pointing to the way people use the words "triangle", "line", etc., is not relevant in the proof of a theorem about triangles. Neither would it be relevant to establishing an empirical fact about triangles.)

6.E.10. To sum up: I have tried to bring out various factors involved in the justification of the claim to know that some proposition is true when that proposition is analytic. One way of bringing them out is to describe ways in which one may fail to see that such a proposition is true. This also reveals the superficiality of some commonly accepted definitions of "analytic".

One of the most important points to be stressed is that analytic propositions can be verified empirically like other propositions using the same empirical concepts. (This suggests that the empirical-nonempirical distinction should be applied to ways of knowing, not to kinds of proposition.) The importance of this is that it shows in what sense they are propositions, capable of being true or false.

6.F. Concluding remarks

6.F.1. The contents of this chapter may now be summarized. I have tried to show how it is possible for propositions to be analytic, by showing how the truth-value of the proposition expressed by some sentence may be determined by identifying relations between the meanings of non-logical words in that sentence together with facts about the logical technique corresponding to its logical form. This is equivalent to saying that there is something which is essential to the proposition's being the one which it is, from which it follows logically (but not formally: cf. 6.B.12) that the proposition has the truth-value which it does have. It could not have any other truth-value unless the descriptive words expressing it had different meanings or the logical words and constructions involved in it had different functions, in which case it would have been a different proposition (since we are using strict criteria for identity of meanings and propositions - see section 2.C.). It follows that anyone who talks about "declaring a proposition to be analytic" is muddled or using loose criteria for identity, since one cannot simply declare

a proposition to be analytic without risk of changing the proposition. The most one can do is declare that words are to be used in a certain way, which may have the consequence that a certain combination of these words expresses an analytic proposition, but that proposition is analytic whether those words are used to express it or not. (Cf. Waismann in "Analytic-Synthetic II", p.25. See 6.A.3, above.)

6.F.2. The identifying facts about meanings, which must be known if it is to be known what those meanings are, are what correspond to the notion of a "definition" in the crude account of an analytic proposition as one whose truth follows logically from definitions. The fact that metalinguistic concepts are required for stating these identifying facts, shows why Frege was wrong to think of definitions as occurring on "the same level" as other propositions. (Cf. 6.B.8.a,ff.) The fact that we can, in a non-formal (but perfectly rigorous) way, draw logical conclusions from facts about meanings, shows how Quine and Waismann were wrong in denying that "definitions" could be used to found truths, and in asserting that they were merely substitution licences for transforming truths. (6.B.3-4,ff.)

6.F.3. We found, in section 6.B, that the Quine-Waismann definition of "analytic proposition" as "proposition derived by substitution for synonyms in formal truths" was not sufficiently wide. The question now arises whether propositions which are analytic in their sense are also analytic in the sense defined in 6.C.10: is our new definition simply wider than the old one, or is it completely different?

We can show that propositions which are analytic in the old sense are also analytic in the new sense, as follows. A formal truth is one kind of analytic proposition, for its truth-value is determined in a purely logical (topic-neutral) way by (i) its logical form and (ii) the structure of the argument-set of non-logical words to which that form is applied. But the example of 5.D.1 shows clearly that replacing a word or expression by another word or expression referring to the same property or properties, in some sentence, cannot alter in any essential way the factors which determine the truth-value of the proposition expressed by that sentence, since the actual shapes of the words used in the sentence do not matter: it is what they refer to or what their functions are that matters. Hence, if general logical considerations about logical form and meaning suffice for a determination of the truth-value of a proposition P , and if P' is derived from it by substituting synonyms for some of the non-logical words used to express it, then P' can be shown to be true by general logical considerations of the same sort as before, taking into account also some new identifying facts about meanings. Hence, if P is analytic, then so is P' , and if P is a formal truth then P' is analytic, since formal truths are analytic.

This shows that the Quine-Waismann definition is included under our definition, which simply happens to be more general, since it allows us to take account of identifying relations other than synonymy, as shown in 6.B.

6.F.4. We have just proved a logical "theorem" by

non-formal considerations, namely, the theorem that analyticity is preserved by synonymy substitutions. We could prove, in a more general way, that analyticity is preserved by any substitution of one descriptive expression for another which is such that those two expressions are logically equivalent, in the sense that identifying facts about meanings, together with purely logical considerations, can show that if anything is correctly described by one of these expressions then it is also correctly described by the other, no matter how things happen to be in the world.

In addition, we can prove that if a proposition is analytic, then any other proposition formally entailed by it (5.C.8) is also analytic.

For example, suppose that by general logical considerations we are able to deduce from some identifying fact about the meanings of "A", "B" and "C" that the logical regator corresponding to "All P things are Q and all Q things are R" takes the value "true" for the argument-set ("A", "B", "C"), in which case the proposition expressed by the sentence "All A things are B and all B things are C" must be analytic. In addition, we know that this proposition formally entails the one expressed by "All A things are C", since we can discover, from a consideration of the logical techniques corresponding to their logical forms, that the latter must be true whenever the former is, no matter how things happen to be in the world. (See 5.C.8). Now add these considerations to the previous ones which showed that the former sentence expressed an analytic truth, and we find that we have a way of showing, by consideration of identifying facts about meanings and general logical principles, that the latter sentence

expresses a proposition which must be true independently of what is the case in the world.

This example should show that a general theorem could be proved to the effect that analyticity, like truth, is preserved by formal entailment. (The reader who wishes to test his grasp of my definition of "analytic", "formal", etc., may try to write out the general proof in detail.) This is a theorem of non-formal logic. (I.e. it is not just a formula in a formal system derivable from axioms using rules of inference, nor is it a proposition about what is derivable in formal systems.)

6.F.5. The fact that any analytic proposition formally entails many other propositions implies that if there is one proposition which is true in virtue of some identifying fact about the meanings of non-logical words, then there will be many others which are true in virtue of the same fact (though different logical considerations may be required in order to establish their analyticity). We are reminded, therefore, of the fact pointed out in 5.D.6, namely that to any identifying fact about meanings of non-logical words, there corresponds a whole family of analytic propositions whose truth it helps to guarantee. This fact is sometimes made use of when people try to define "analytic". Hare's definition, in "The Language of Morals" (pp. 41-42) went as follows, for example:

"A sentence is analytic if, and only if, either (1) the fact that a person dissents from it is a sufficient criterion for saying that he has misunderstood the speaker's meaning or (2) it is entailed by some sentence which is analytic in sense (1)."

We have shown how (2) is not part of the definition of

"analytic", in our wide sense, but is a theorem about analytic propositions. The importance of this is that we do not need to say that some propositions are analytic in one sense and others in another: the class of analytic propositions is homogeneous on our definition.

6.F.6. The homogeneity of the class of analytic propositions is due to the fact that definitions or statements of identifying facts about meanings must be thought of as being on a different level from propositions using the words they mention. So the "definitions" in virtue of which analytic propositions are true are not themselves included in the class of analytic propositions. There are not some analytic propositions, or propositions true by definition, which are themselves definitions, or "registers" of linguistic conventions, or direct expressions of linguistic conventions, or linguistic "proposals" (see end of 6.B.3.b), while the remainder are analytic only in virtue of being logical consequences of these propositions. Instead we must say that no analytic proposition is the definition in virtue of which it is true, and all analytic propositions can be shown to be true in the same way, namely directly from a consideration of meanings and logical (topic-neutral) facts, without the mediation of other analytic propositions.

This homogeneity relieves us of the task of discovering which of the analytic propositions are the definitions and which are merely entailed by these "definitions", a task which could be quite embarrassing. In addition, this homogeneity enables my definition to escape the objection to the Quine-Waismann definition of "analytic" which I raised in 6.B.7.

6.F.7. For reasons given in 1.C.2, and Appendix I, I have deliberately restricted the discussion of the analytic-synthetic distinction to a small class of propositions, namely those which are universal in form (i.e. mention no particular objects) and include only fairly simple descriptive words in addition to logical constants. The main reason for this restriction is that any attempt to define "analytic" straight off for all kinds of propositions seems to lead to muddle and confusion. However, now that we have taken the first steps in the clarification of the notion, the question arises whether it may not be generalized.

The first and most obvious generalization would be to take account of relational expressions, which have not been mentioned since 6.B.5.b, where it was pointed out that the irreflexiveness of the relation "to the left of" might be due to a purely linguistic convention, in which case "Nothing is to the left of itself" must express an analytic proposition. The extension of the notion of an identifying fact about meanings to take account of relational expressions, and the extension of the notion of a logical rogator to allow relational expressions as arguments, can be easily accomplished. I shall say no more about this.

6.F.8. Secondly, one might try to extend the notion of an analytic proposition to include those using proper names and other singular referring expressions, by extending the notion of an identifying fact to include facts about the meanings of proper names and other referring expressions. For example, it might be said to be an identifying fact about the meaning of the word "Socrates" that the thing referred to by the word is a

human being and a man. This assumes that we can talk about knowing the meaning of a referring expression in a sense which is different from knowing which thing is its referent (for I might very well know which material object was called "Socrates" without having the faintest idea whether that object was a human being: for example I might not have the concept of a human being).

Whether or not it is possible to extend the notion of an analytic proposition to include propositions like "Socrates is human", "Tom's bachelor uncle is unmarried", etc., need not concern us now, since we are mainly interested in necessary connections between universals. (See Appendix I. For an attempt - not very successful in my view - to show that proper names may occur in analytic statements of the form "x is P", see Searle's D.Phil. thesis.)

6.F.9. The next possible generalization concerns words or expressions for which there are "appropriateness-conditions" instead of just truth-conditions. (See 2.D.9.) In general, rules laying down appropriateness-conditions for utterances (such as "Alas!") are concerned not with observable states of affairs which make propositions true, but with such things as the purposes which may be served by utterances, the contexts of utterance, the things which are to be expected of the person producing the utterance if he is not to be said to have been deceitful or changed his mind, and so on.

There is an enormous variety of cases, and it is not to be expected that we can deal with them all at once, except in a very vague way, as follows: it may be possible to extend the notion of an identifying fact

about meanings to include identifying facts about the functions of words or expressions governed wholly or partly by appropriateness-rules. For example, it may be that the conditions in which it is appropriate to say "Ouch!" are identifyingly related to the conditions in which it is appropriate or true to say "Something just hurt me". Perhaps there are identifying relations between the conditions in which it is appropriate to utter sentences expressing statements and the conditions in which it is appropriate to utter the words "I believe that" followed by such sentences.

(Thus identifying relations between appropriateness conditions may help to account for so-called "pragmatic" implications, and also the notion of "logical oddness". See 2.B.9.)

Perhaps there are identifying relations between the conditions in which expressions of moral judgements are appropriate and the conditions in which statements to the effect that one has decided to do something are appropriate, though we must be prepared to find that such relations are extremely complicated and difficult to describe, if there are any (we are prepared for complicated identifying relations by the example of 3.B.5, and comments thereon).

6.F.10. In general, we may talk about an identifying fact about the meanings or functions of words (or perhaps an identifying fact about concepts), wherever there is some fact which must be known if the meanings or functions of those words (or expressions, or constructions) are to be known. (Cf. 6.C.3-4.) But it is doubtful whether this extension of the notion of an identifying fact to include facts about appropriateness-conditions, or, more

generally, meanings or functions of words, always leads to an extension of the notion of an analytic proposition, or analytic utterance. For the connection between analyticity and identifying facts was explained in terms of the ways in which truth-values of propositions might be determined, and the whole point about appropriateness-conditions, for example, is that they need not be concerned with truth. For this reason, I regard with suspicion the use of the terms "analytic" and "synthetic" in connection with imperatives, or moral judgements, or aesthetic judgements, unless it is made clear that a special new terminology is being used. I should prefer to talk about analytic and synthetic connections between meanings or functions.

6.F.11. This concludes my account of analytic truth. I have tried to show in what way analytic statements are true, and why they are true independently of facts, that is, independently of how things happen to be in the world. It should be clear that their being true, or even necessarily true, does not rule out any states of affairs as actual or even as possible states of affairs, since their being true is fully determined by matters which have nothing to do with observable states of affairs, though this may be concealed by the fact that they can be verified by observation in the usual way. In addition, they have the appearance of saying something, they have a meaning, they seem to state facts. ("It is a fact that all bachelors are unmarried, just as it is a fact that pieces of wood fall to the ground when dropped, only the former fact couldn't have been otherwise.") However, their having a meaning comes only to this: they are constructed out of words and expressions which have

meanings and can occur in statements which are not analytic. To say of any statement that it states a fact can be very misleading. Certainly if it is true it is true in virtue of some fact, which may be described as the fact which it states, but if it is false we have to talk about "possible facts". We might try saying that the fact stated is the one which if it actually existed, would make the proposition true: but in general there are indefinitely many different possible states of affairs in which a proposition would be true (e.g. the truth of "Tibs is on the mat" leaves open the possibility of many different arrangements of the cat and the mat). Is the fact stated by the proposition a collection of all these possible states of affairs, or is it something common to them all, or what? Until questions of this sort have been answered, it is not at all clear what the significance is of the assertion that even analytic statements state facts, unless it simply means that carrying out the normal procedure for discovering their truth-values by making empirical observations will always yield the result "true". But that should not surprise us, since we selected this as the characteristic property of analytic propositions.

What we must now ask is whether there is any other way in which a proposition can be true in all possible states of affairs, or any other way in which the truth-value of a proposition can be discovered independently of applying the normal logical techniques. Are there any other ways than those which we have described, in which the truth-value of a proposition may be due to facts about the meanings of the words and constructions used to express it? Is there any other way in which a proposition can be a necessary truth than by being analytic?

The meanings of these questions will be clarified, and answers suggested, in the next chapter.

Chapter Seven

KINDS OF NECESSARY TRUTH

Introduction

In chapter six I explained what is meant by saying that a proposition is analytic, and showed how it is possible to know that such a proposition is true independently any observation of facts. The features of an analytic proposition in virtue of which it is true ensure that it would be true in all possible states of affairs, so we can say that it could not possibly be false, that it must be true, that it is necessarily true, and so on. All these truth-guaranteeing features are topic-neutral and can be described in purely logical terms, such as that the proposition is made up of certain logical words in a certain order, with non-logical words whose meanings stand in certain identifying relations. This chapter will be concerned with the question whether there is any other way in which a proposition can be necessarily true.

In order to give this question a clear sense I must explain what is meant by "necessary", that is, give an account of the way in which the necessary-contingent distinction is to be applied. I shall start off by talking about the meaning of "possible". The next section will attempt to explain the meaning of "necessary". The rest of the chapter will be concerned to describe and distinguish kinds of necessary truths, and ways in which a proposition may be known to be true independently of observation of contingent facts.

(Throughout the chapter it must be remembered that

this thesis is written from the point of view described in section 1.B.)

7.A. Possibility

7.A.1. We have reached the stage at which it is not enough to have only a rough intuitive grasp of the necessary-contingent distinction. If we are to make any further progress with the problem of synthetic necessary truth we must try to see clearly exactly what this distinction, or family of distinctions, comes to.

It is often pointed out that there is a close connection between the notion of necessity and the notion of possibility. A statement is necessarily true if it would be true in all possible states of affairs, or if it is not possible that it should be false. This is sometimes put by saying that necessity is definable in terms of possibility and negation. I do not think the connection is quite as simple as some logicians would have us believe (See 7.B.1, 7.B.10). There certainly is a close connection between the two notions, however, so I shall try, in this section, to explain how we can understand talk about possibility, or about "what might have been the case".

In order to do this, I shall make use of the very general facts which, in chapter two, I argued to be presupposed by statements about the meanings of words in English and similar languages. (See section 2.B, especially 2.B.6.) These are facts such as that our sentences describe states of affairs which can be thought of as made up of material objects possessing observable properties and standing in observable relations. More specifically, I shall rely on some of the arguments in

2.D to the effect that in this conceptual scheme universals (properties and relations) are not essentially tied to the particular objects which happen to instantiate them. (Cf. 2.D.5,ff., 3.C.3,ff.)

7.A.2. It is worth noting that the notions of necessity and possibility are not merely technical notions invented by philosophers, for we are all able to use the following words and expressions: "necessary", "necessarily", "possible", "impossible", "must", "had to happen", "couldn't have happened", "cause", "if so and so had happened", "if only I had done so and so ...", etc. Think of the words of the popular song: "That's the way it's got to be"!

Despite their familiarity, these notions are puzzling because they are "non-empirical" in a strange way. We can point to what is the case, but we cannot point to what isn't the case and might have been. Worse still, we cannot point to what is not the case and could not have been. At any rate, we cannot produce examples to be looked at, in the way in which we can produce or point to actual observable states of affairs, or events. How then do we learn to understand these kinds of expressions in the first place? The clue seems to be provided by a fact pointed out in 2.C.6, namely that in order to decide that something or other is possible, we have to consider properties, or, more generally, properties and relations and ask whether they are connected or not. (There are really many different kinds of necessity and possibility: I shall not discuss them all.)

7.A.3. Let us consider some examples. The piece of

paper in front of me is not blue and square, but it might have been, or at any rate there might have been a blue and square piece of paper in front of me. The piece of paper which is in front of me is white and oblong, but it might have been different. There is a cardboard box on my table; it has a lid which is neither white nor oblong, though it might have been both. There is no paper on the floor near my chair, but there might have been, and it might have been either white and oblong or blue and square (or it might have had other shapes and colours).

What lies behind all this, is simply the fact, to which I have already drawn attention,¹ that universals are not essentially tied to those particular objects which happen to instantiate them. Universals are not extensional entities, they exist independently of the classes of objects which actually possess them. As remarked previously, one can have a property in mind, think about it, attend to it, recall it, associate a word with it, talk about it, etc., without thinking about any actual particular object which has that property. Neither the property of being blue and square nor the property of being white and oblong is essentially tied to the particular material objects which actually have them. Nor are they essentially tied to the times and places at which, as a matter of fact, they can be observed. When we see the properties of objects, or the relations in which they stand, we can see that they are not the sorts of things which have to occur where they do occur.

1. (2.B.6, 2.D.5, 3.C.3, etc.)

7.A.4. This possibility of recurrence is, after all, what makes us describe properties and relations as universals, and contrast them with particulars. We can isolate out three aspects of their universality.

First of all there is actual recurrence. The whiteness of the piece of paper on which I am typing is a property which it shares with many other objects existing at the same time and at different times. Secondly, other objects exist which, although they are not instances of the property, might have been. The box on my table is not white, but it might have been. Thirdly, there might have existed objects which do not in fact exist (there might have been a piece of paper on the floor next to my chair), and if they had existed, then they might have had these properties. If there had been a piece of paper on the floor next to my chair, it might have been white.

Some philosophers would explain the universality of properties in terms of the first of these three aspects, namely actual recurrence, but this will not do, for there are probably properties, such as very complicated shapes, which are, as a matter of fact, instantiated by exactly one object, or possibly by no objects at all.

7.A.5. Universals can recur, even when they do not in fact do so. Now how do we know this?

When I look at an object and pay attention to one of its properties which does not have any other instances, how do I tell that that property is the sort of thing which could occur elsewhere, even though it does not. Is it simply a generalization from experience? Is it because I have seen many objects which share properties that I come to believe that this specific property at

which I am looking is also the sort of thing which could be shared by several objects? If it were an empirical generalization, then I should have to leave open the possibility of an empirical refutation, or at least counter-evidence, but there does not seem to be any such possibility. Apart from the fact that I do not know what sort of experience would count as a refutation or as counter-evidence, the suggestion seems to be nonsensical because the sort of doubt which is appropriate to an empirical generalization does not seem to be appropriate here. (Indeed, the possibility of recurrence of universals is presupposed by any empirical generalization.)

It seems that when I look at the shape or the colour of an object, I can see then and there that what I am looking at is the sort of thing which can recur, since there is nothing about it which ties it essentially to this object, or this place and time.

7.A.6. When I look at a colour and see that it is the sort of thing which could occur in other objects at other places and times, I do this by abstracting from the particular circumstances of its occurrence, such as the fact that it is possessed by this piece of paper here and now, is being looked at by this person, can be found to be two feet away from that particular table, and so on.

I believe that this sort of abstraction is often confused with another kind, namely abstraction from specific features, for example in Kant's remark (in C.P.R., A.713, B.741):

"The single figure which we draw is empirical, and yet it serves to express the concept without impairing its universality ... for in this empirical intuition we consider only the act whereby we construct the concept, and abstract from the many determinations

(for instance, the magnitude of the sides and of the angles), which are quite indifferent as not altering the concept 'triangle'."

I think Kant confuses two things which are very often confused, but which must be very carefully distinguished, namely universality and generality, both of which may be involved in one universal (or concept) "expressed" by a particular material object. Universality is common in the same way to all properties, but some properties are more general (or less specific) than others. The universality of a property consists in the possibility of its occurring in other objects than those which actually instantiate it. The generality of a property (or concept) consists in the possibility of its occurring in several different objects in different forms. I do not know whether there is a sense in which properties can be thought of as general or non-specific in any absolute way, but there is certainly a relative general-specific distinction. The property of being a triangle is more general, or less specific, than the property of being an isosceles triangle. A specific shade of red is more specific, or less general, than the hue, redness. (Cf. 3.A.1.)

In order to perceive the (relative) generality of a property we have to abstract from the specific features of an object which has that property. In Kant's example, we have to abstract from the specific ratios between the sides of a triangle and the specific sizes of the angles, its specific orientation, and so on, in order to perceive the generality of the property of being triangular. But this sort of abstraction is not what concerns us at present: we are interested not in abstraction from specific features, but in abstraction from particular

circumstances: that is what occurs when we see a property as the sort of thing which can recur, whether it actually does so or not, that is, as a universal. It is presupposed by the other kind of abstraction, for only if it makes sense to talk of a property as being possessed by other objects does it make sense to talk of other objects as possessing other determinate (specific) forms of this property.

7.A.7. It should be clear that I am not trying to define the notion of possibility or "what might have been" in terms of what can be conceived or imagined: I am not saying "P is possible" means "P can be imagined". Imaginability is not a criterion for possibility nor vice versa. There may be things which are possible though no human being can imagine them, either owing to lack of experience, or owing to complexity. There may be colours which have never yet been seen and cannot be imagined at present, or shapes too complicated to be taken in. Worse still, people have imagined or conceived of things which later turned out to be impossible.

For example, one has to be a rather sophisticated mathematician to be unable to imagine trisecting an angle with ruler and compasses, and in the sense in which most of us can imagine that, it is surely possible for someone who is still more unsophisticated than we are to imagine seeing a round square. (Someone might draw a straight line and say that it was a picture of a round square seen from the edge!) So it will certainly not do merely to say that what is possible is what can be imagined, or that what is necessary is what could not be conceived to be otherwise: it will not do to offer this as a definition.

What I am saying is rather this: look at what goes on when you imagine what it would be like for something to be the case, and then you will see more clearly, from a philosophical point of view, what it is to describe a state of affairs as "possible". The important thing is that the various properties (and relations) which we can see in the world need not be arranged as they are, in the instances which they happen to have, and we acknowledge and make use of this fact when we imagine non-existent states of affairs, or when we talk about them or write stories about them or wish for them, or draw pictures of them. In short, that which makes imaginability possible in some cases, is what explains how states of affairs may be possible though not actual, namely, the loose connection between universals and their actual instances.

7.A.8. All this may be used to explain the notion of the set of "truth-conditions" of a proposition. We have seen (cf. 5.E.1) that in general whether the proposition expressed by a sentence *S* is true or not depends on three things:

- (a) the non-logical words in *S* and their meanings,
- (b) the logical form, and corresponding logical technique
- (c) facts, or the way things happen to be in the world.

This shows that when a logical form, and a set of non-logical words are combined to form a sentence expressing a proposition, the linguistic roles of the logical constants, and the semantic correlations governing the non-logical words together determine a set of possible states of affairs in which to utter the sentence would be to make a true statement. These are the "conditions" in which

applying the logical technique for determining its truth-value would yield the result "true". There is not usually just one truth-condition, as pointed out in 6.F.11. Every statement ignores some aspects of the states of affairs in which it would be true. Variations in these "irrelevant" aspects help to increase the size of the class of possible states of affairs in which such a statement would be true. Whether it is true or not depends on whether one of these possible states of affairs actually obtains, i.e. is a fact. (This can be generalized. If R is any rogator, and T is one of the values which it can take, and if A is an argument-set for that rogator, then R and A together determine a class of possible states of affairs, or T-conditions, namely those in which applying the technique for determining the value of R for A would yield the result T. "Rogator" was defined in 5.B.6.)

7.A.9. By taking note of the fact that universals can recur, that is by abstracting from the particular circumstances in which we see shapes, colours, and other properties, we are able to learn such things as that this book might have been the colour of that one, there might have been a box on the floor the same shape as the one on the table, there might have been pennies in my right-hand pocket instead of in my left-hand pocket. It should be stressed that there is nothing mysterious in this: apprehending the universality of a shape or colour or other property, does not involve making use of "inner-eyes" or other occult faculties: it is just a matter of using ordinary intelligence and ordinary eyes and imagination. We thereby take note of very general facts but for which our language, thought, and experience could not have been

the sorts of things which they are. (See chapter two, section B.)

7.A.10. All this shows that there is a non-linguistic kind of possibility. By this I mean merely that when we talk about possibilities we are not talking about combinations of words which are permitted by the rules of some language. Contrast this with Schlick's remark (Feigl and Sellars, p.154): "I call a fact or process 'logically possible' if it can be described, i.e. if the sentence which is supposed to describe it obeys the rules of grammar we have stipulated for our language."

The class of possible states of affairs is much more complex and numerous than the class of sentences formulable in any language. Sentences are discrete and individually describable, and, at any one time, either finite in number or able to be arranged in a fairly simple sequence, unlike possible states of affairs, which shade continuously into one another in many different dimensions. (Austin: "Fact is richer than diction.")

7.A.11. Further, it should be noted that the concept of possibility cannot be reduced to that of logical possibility or analytic possibility. To say that a proposition is not a formal or analytic falsehood is to say that one cannot show it to be false merely by considering the meanings of words and the logical techniques of verification corresponding to its logical form. This simply means that observation of the facts may be relevant to determining its truth-value. It does not imply that any state of affairs is a possible one, or that there is

a non-empty class of possible states of affairs corresponding to it as truth-conditions. For the question whether, for some other reason, the truth-value would come out as "false" in all possible states of affairs is still left open. At any rate, some argument is required to show that it is not left open: and that shows that there are different concepts of possibility here.

To sum up, knowing what possibility is, is not a matter of knowing the laws of logic and seeing which descriptions of possible states of affairs do not contradict them, neither is it a matter of knowing which combinations of words are permitted by the rules of some language. It is a matter of knowing that the world is made up of material things and their properties and relations, and knowing that these properties and relations are not essentially tied to those material things which actually instantiate them, that they need not occur in the arrangements in which they do occur, or at the places and times at which they do occur. Other factors might have been taken into account, such as loose ties between particulars and the actual places and times at which they exist. (This table is here now, but it might have been next door.) These factors will be ignored, not being relevant to our main problem.

(Note. This conception of possibility can be used to solve some philosophical problems. For example, it makes puzzles about the identity of indiscernibles disappear. If "indiscernible" means "could not possibly be different in some respect", then the principle that indiscernibles are identical is true. If "indiscernible" means "is not actually different in some respect", then the principle is false. A sphere in an otherwise empty universe will have two halves, despite its symmetry, because one of them could be a different colour from the

other, even if it is not. If the principle were correct in its second sense, then, although an unsymmetrical object could exist alone in the universe, it could not be gradually transformed into a sphere, for on becoming a sphere it would consist of two (or indefinitely many - if we take account of sectors) parts which have all their properties and relations in common. What would happen to it then? It is clear that the principle is absurd.)

7.B. Necessity

7.B.1. The concept of possibility has been shown to have an application on account of the loose tie between universals and actual instances.

But understanding talk about possibilities is not enough for an understanding of talk about necessity. For that, one must know what is meant by "the range of all possibilities", or "what is not possible". The use of negation or the word "all" has to be defined afresh in these modal contexts, and corresponding to different ways in which its use is explained there may be different kinds of ranges of possibility, different kinds of necessity, different kinds of impossibility. We must therefore proceed with caution.

7.B.2. We have seen that the concept of "analytic" possibility is not very substantial (7.A.11). Similarly, the kind of necessary truth which we have found in analytic propositions does not seem to affect the range of all possible states of affairs, since the necessary truth of such a proposition is merely a matter of its having certain general features which ensure that it comes out as true, no matter what particular things or kinds of things it is about, and no matter how things

happen to be in the world. So its necessary truth is not due to anything at all specific which has to be the case in all possible states of affairs. Hence its being necessarily true imposes no special restrictions on what may be the case: it does not seem to limit the range of all possible states of affairs in any way.

Let us try to find a more substantial and more general concept of necessity by following up what Kant said in the "Critique of Pure Reason" (B.4), namely that a statement is necessarily true if

"it is thought with strict universality, that is in such a manner that no exception is allowed as possible, it is not derived from experience."

(The last clause, "not derived from experience", will be ignored for the time being. See Appendix VI.)

7.B.3. What is strict universality? How can no exception be allowed as possible?

Suppose the following were true: (1) "All triangles are red". It would then be a universal truth with no exceptions, but it would not be strictly universal, since it is clear that triangles do not have to be red: even if all triangles happen to be red, I can see, just by looking at a triangle that although it is not green it might have been, while still a triangle. Although there are no exceptions, nevertheless exceptions are allowed as possible. By contrast, the proposition (2) "All squares have exactly four angles" is not only a universal truth without exceptions, but it is strictly universal, since there could not be any exceptions: even if there were any other squares than the ones which there actually are, they would all have exactly four angles. (2) is a necessary truth, whereas (1) would not be necessary, even if it were true.

7.B.4. This can be expressed more generally if it is tied up with some of the remarks in 7.A about possibility and properties. First of all, let us consider propositions of the form "All P's are Q's", where "P" and "Q" are descriptive words referring to observable properties. Now recall that the universality of a property has three aspects (7.A.4). Firstly, the property may occur in several different actual particular objects. Secondly, it might have been instantiated in some of those particular objects which are not in fact instances. Finally, there might have been objects, which if they had existed, might have had the property. We may therefore say that the property referred to by "P" is necessarily connected with the property referred to by "Q", and the proposition "All P's are Q's" is necessarily true, if, and only if, all the objects in the first class, namely all those which actually have the property referred to by "P", also have the property referred to by "Q"; all the objects in the second class, that is all those which might have had the property P, would, if they had had it, also have had the property Q; and, finally, all the objects in the third class, namely those which might have existed though they do not, would, if they had existed and had the property P, also have had the property Q. In short, there are three sorts of potential counter-examples to a proposition of the form in question, namely those objects which have the property P, those which do not, but might have had it, and those which, if they had existed, might have had it; and to say that no exceptions are allowed as possible, is to say that none of these objects, if it had (existed and) had the property P would have been without the property Q.

7.B.4.a. We can generalize this further if we recall that only propositions are being discussed which are universal in form and mention no particular objects (see Appendix I). For a sentence made up only of logical constants and descriptive words referring to properties is true if and only if certain relations hold between the classes of objects possessing certain properties, relations such as inclusion, or mutual exclusion. Such a proposition is necessarily true, then, if all the classes of objects with the specified properties do in fact stand in the specified relations, and, in addition, they would do so even if other objects had the properties in question than the ones which actually do so, even if there were other objects in existence than the ones which there actually are. In such a case not only are there no exceptions, but, in addition, no exceptions are possible.

This could be generalized a stage further to include propositions referring not only to properties, but also to relations, such as "two feet away from", "brighter than", "inside", and so on, but I shall leave that to the reader.

It should be noted how this definition differs from the definition of an analytic proposition: here we make no mention of "identifying relations" between meanings, nor do we restrict the sources of necessity to topic-neutral features of propositions. Thus, it is so far an open question whether all necessary truths are analytic.

7.B.5. All this may suffice for a definition of "necessity", but it is not very helpful, since it does not explain how it comes about that any statement is necessarily true, or how we can ever tell that it is.

What is missing is an explanation of how we can tell whether a counter-factual conditional statement asserting that no exceptions are allowed as possible is true, that is, how we can tell what would have been the case if certain objects had had certain properties. How do I tell that if the piece of paper on which I am now typing had been square then it would have had four angles? How do I tell that if there had been a tetralateral block of wood on my table then it would also have been tetrahedral? (Cf. 2.0.8.) It should not be assumed that simply because I know how to tell that something or other might have been the case, I know how to tell what else would then have been the case, or that I know how to tell what would be the case in all possible states of affairs. (See 7.B.1.)

There seems to be so complicated a range of possible worlds and possible states of affairs that it is hard to see how anything at all could be excluded from the range. There might be worlds in which space had five dimensions, or only two. There might have been a world in which there were only sounds, and no space or spatial objects or spatial properties (see Strawson's "Individuals", chapter two). There might have been worlds in which properties and relations existed which were quite unlike anything we can imagine. Or might there?

It seems clear that there is a tangled and complicated question here, which is not really relevant to such problems as concern us, for example, the problem whether it is both necessary and synthetic that two properties which actually do exist always occur together (such as the property of being four-sided and the property of having four angles). The source of the trouble is that there

are different concepts of necessity, and different kinds of ranges of possibilities.

7.B.6. But our definition of "necessary truth" was restricted in such a way that we need not take account of all these complexities, for it is concerned only with classes of objects possessing properties which actually do exist in our world. We therefore have no need to talk about all possible worlds, since we can limit ourselves to talking about all possible states or configurations of this world, where "this world" describes a world in which the same observable properties and relations exist as exist in our world. (It should be recalled that the existence of universals need not involve actual existence of instances. See section 2.D.) Thus, since we are talking only about states of this world, we need not consider worlds without space and time, or five-dimensional worlds. (Compare what Kant says about his Copernican Revolution in the Preface to the second edition of C.P.R. B.xvi-xvii, etc.) (See note at end of this section.)

7.B.7. Now we may return to an explanation of how it is possible to tell that a statement is necessarily true. Once again we shall make use of general facts about universals, that is, observable properties and relations.

It has already been pointed out (7.A.4,ff) that a property exhibited by an object is the sort of thing which can recur. Now we must notice further that one object may possess more than one property at the same time. A material object may be both red and round. It may be cubical and transparent. It may be cigar-shaped, glossy and green.

When two or more properties are exhibited by an object, we may be able to see that some of them have no connection with the others. For example, the fact that a box is cube-shaped has nothing to do with the fact that it is red. Not only could the property of being cube-shaped occur in other objects, in addition it could occur in other objects without the colour which accompanies it in this one. Even if neither of these properties did occur without the other (which, of course, is not the case), we could still nevertheless see that there might have been an object which was cubical without being red, or red without being cubical. One need not have seen either of the properties actually exhibited without the other in order to see that they are capable of occurring separately, any more than one must have seen the shape or the colour in another object at another time or place in order to see that it can have other instances. (Cf. 7.A.5.) All we need is our eyes and intelligence, and the knowledge of what it is to be cube-shaped and of what it is to be red, and then one can see that it is possible to recognize either property in an object without its mattering whether the other is there or not.

Similarly, where there are two properties which we have never, as a matter of fact, seen in the same object at the same time, we may be able to tell that there could be an object with both of them. I have never, as far as I know, seen an object which is both cigar-shaped and blue, but there is nothing in either property, insofar as it is an observable property, which excludes the presence of the other. I know what it would be like to recognize both properties in one and the same object.

Thus, even if the two statements "All cubes are red" and "Nothing cigar-shaped is blue" are true, that is have no exceptions, nevertheless they do allow exceptions as possible. We can see, by examining the properties concerned, that they are not necessarily connected. It is by contrast with this sort of case that I shall explain how statements can be necessarily true.

7.B.8. We have added a refinement to our concept of possibility by taking note of the fact that not only are universals not essentially tied to their actual instances, so that they can be instantiated in other places and times than they are in fact, but, in addition, they are not essentially tied to one another, so that they can occur in different combinations from those in which they in fact occur. Universals are unfettered by their instances, and also, sometimes, by one another. Not always, however, and limitations on this second sort of freedom generate the kind of necessity which is of interest to us. There may be something in the constitution of a property which ties it to another property, or which prevents its occurring with another property. If so, this may have the consequence that a statement using words which refer to those properties is not only true, but necessarily true, since no exceptions are allowed as possible. (Exceptions would be objects in which these tied properties occurred separately, or in which incompatible properties occurred together.)

If there are any such relations between properties which are not identifying relations, then they will provide us with a new class of relations between descriptive words referring to properties so related, and here, as in

the case of analytic propositions, the relations between descriptive words, together with features of the logical technique for discovering truth-values of a statement, may determine the outcome of applying the technique in any possible state of affairs (cf. 6.C.1.). So if there is some way of knowing that the properties referred to by words stand in such relations, then we may be able to determine the outcome of an empirical investigation to discover the truth-value of a proposition, without actually making that investigation. If this is so, then we shall have found a new type of illustration of the fact pointed out in 5.E.1 and 5.E.2, that although in general the value of a rogator depends on (a) the arguments, (b) the technique for discovering values and (c) the facts, nevertheless there are cases where without knowing any facts (i.e. without having any empirical knowledge of how things happen to be in the world) we can discover the value by taking note of relations between the arguments and examining the general technique for determining values. We shall have found a way of telling, without knowing which particular objects there are in the world, nor what properties and relations they instantiate, that none of them is an exception to what is asserted by some statement. That is, we shall have found a new kind of a priori knowledge of the truth of a statement. (See end of 7.B.2.)

7.B.9. If there are these connections between properties, and if we can know that they exist, for example by examining the properties in question, then this will explain how we can be in a position to assert such statements as "If this had been square then it would have had

four angles", or "If this had been turquoise, then it would not have been scarlet". Thus, by talking about properties, and their ties with or independence of one another and their instances, we are able to explain some uses of the words "necessary" and "possible", and counter-factual conditional statements.

To sum up: since properties are not tied to their actual instances we can talk about what might have been the case in the world, and since they may be tied to one another (this includes incompatibility), we can talk about what would have been the case if so and so had been the case. Hence we can talk about statements to which no exceptions are possible, that is, statements which are necessarily true.

7.B.10. All this should show that the concept of necessity is far more complicated than the concept of possibility. (See 7.A.1, 7.B.1.) Only the latter is required if we are to be able to use our ordinary language to describe new situations, to ask questions about unknown facts, to understand false statements. We need only understand that the range of things which might have been the case is wider than the range of things which are the case. The concept of necessity is required when we grow more sophisticated, when we wish to do more than simply describe what we see or ask questions about what is to be seen in the world. It comes in when we wish to draw inferences, when we wish to know about the properties of all things of a certain kind without examining them all, when we do mathematics or philosophy, or try to explain what "makes" things happen as they do, or when we ask whether happenings are avoidable or not. It comes in also when we try to justify the assertion of a counterfactual

conditional statement, about what would have happened if something or other had been the case. In order to understand talk about possibility, one need only see that states of affairs are possible which are not actual, whereas in order to understand talk about necessity, one must, in addition to understanding talk about possibility, also see the reasons why the range of possibilities is limited in certain ways. The former requires only a perception of the loose tie between all universals and their actual instances (by abstraction from particular circumstances), the latter requires perception of the strong ties between some universals and other universals.

(7.B.note. It should not be forgotten that in all this we are talking only about the kind of necessity which arises out of limitations on the possible states of this world, in which objects have properties and stand in relations only of the sorts which are capable of having instances in our world. There may, however, be other kinds of necessity, other kinds of limitations on what may be the case in the observable world. (Compare 7.B.6.)

For example, there may be limitations on the range of possible states of affairs - or possible worlds - which can be talked about in a language using a distinction between subjects and predicates. Or there may be limitations on what can be the case in states of affairs which are observable by beings with senses of any kind. (E.g. a sense which enables them to perceive magnetic and electrical properties directly.) Perhaps there is some other kind of necessity, called "natural necessity" by Kant, which is operative when types of events or states of affairs stand in causal connections.

Kant talked also about a kind of necessity which involves particular objects, such as the necessity in the synthesis of an experience of a particular object (corresponding to the "form" of the object), but this sort of thing need not concern us. We have decided to ignore statements mentioning particulars - see Appendix I - and in any case the ascription of necessity to such statements

can usually be explained in terms of their being instances of some universal statement which is necessarily true, as when we say "Tom's bachelor uncle must be unmarried".

There is no space here to discuss a sufficiently wide concept of necessity to allow us to take account of all these cases and such questions as whether it is necessary that space is three-dimensional. It is not clear to me that there is a perfectly general and absolute concept of necessity. For example: if a statement is necessarily true, then it is not obvious that it makes sense to ask whether it is necessarily true. See end of Appendix I. There may be only a relative concept, operating at different levels, each level being characterized by the type of thing which can count as the reason why a statement is necessarily true. At the level which concerns us, the reason must be that there is a perceptible connection between observable properties or relations.)

7.C. Synthetic necessary connections

7.C.1. This chapter has so far shown, by drawing attention to certain features of the conceptual framework which we presuppose in using descriptive words and referring expressions (cf. 2.B.4-6), how we can understand talk about possibility, and, in a vague way, what is meant by saying that some statements are necessarily true. A statement of the sort under discussion (using only descriptive words and logical constants) is necessarily true if there are connections between the properties referred to by the descriptive words, which ensure that no particular object could be a counter-example to the statement, since certain combinations of those properties in one object are ruled out by the connections between them. Now we must ask whether all such connections between properties are identifying connections (see 6.C) or whether some non-identifying or synthetic connections

between properties can ensure the necessary truth of a statement.

7.C.2. Let us be clear about what we must look for. If knowing the meanings of words (sharply identified meanings, that is, see 6.D.3 and section 2.C), suffices, on its own or together with purely logical (topic-neutral) considerations, to justify the claim to know that properties are related in some way, then that is an identifying relation, not a synthetic relation. For relations between properties to be synthetic, knowledge of them must require something more than the knowledge of which properties they are, and the "something more" must not be purely logical. But what more could there be? Is there some way of examining properties themselves (a non-logical enquiry, since it presupposes actual acquaintance with a special kind of subject-matter) in order to discover that there is a connection between them, a connection which need not be known in order to know which properties they are? We must now investigate some examples, and see whether this sort of insight is possible. If any such insight is possible, it will explain Kant's talk about "appeals to intuition". (See 6.B.2, 6.C.11.)

7.C.3. The most interesting examples come from geometry, though there are many other kinds which cannot be described here for reasons of space. (More examples will be found in Appendix V. See also 2.C.8, 3.C.10 and 3.D.10).

In 2.C.8. we defined the words "tetralateral" and "tetrahedral", the former referring to the geometrical property of being bounded by four plane faces, the latter

to the property of being bounded by plane faces and having four vertices. I argued that the two words refer to two different properties which can be identified independently of each other, since one can notice either property, attend to it, think about it, or talk about it, without being aware of the existence of the other. So in order to know that they are inseparable it is not enough to know which properties they are: in addition one must carry out some sort of construction, either in imagination or with sheets of cardboard, or with diagrams, or somehow examine the two properties, in order to be sure that all possible ways of putting four plane surfaces together to bound a closed space must result in there being exactly four vertices, and that no other number of plane surfaces can yield exactly four vertices. This examination presupposes acquaintance with a special kind of property, and cannot take a topic-neutral form. It does not, therefore, involve drawing conclusions in a purely logical way, so cannot account for knowledge of an analytic truth, according to the definition of 6.C.10.

I call such a construction, carried out for the purpose of enabling oneself or someone else to perceive the connection between two or more properties (or relations), an "informal proof". (For more detailed remarks see next section.)

7.C.4. It seems, therefore, that since an informal proof of a non-logical kind is required, in addition to a specification of the meanings of "tetrahedral" or "tetralateral", for a justification of the assertion (1) "All tetralateral objects are tetrahedral", this must be a

synthetic statement. Its justification is quite different from the justification of (2) "All gleen things are glossy", which proceeds by specifying that the word "gleen" refers to a combination of the property referred to by "glossy" with another property (that referred to by "green", say), and then taking account of purely logical properties of the technique for verifying statements of the form "All P things are Q". There is no identifying relation between the meanings of "tetralateral" and "tetrahedral", from which a logical proof of (1) could proceed.

7.C.5. There are many more examples of this sort of connection between properties, some more problematic than others. Here are a few. (In most cases "improper" or synthesized properties are involved.)

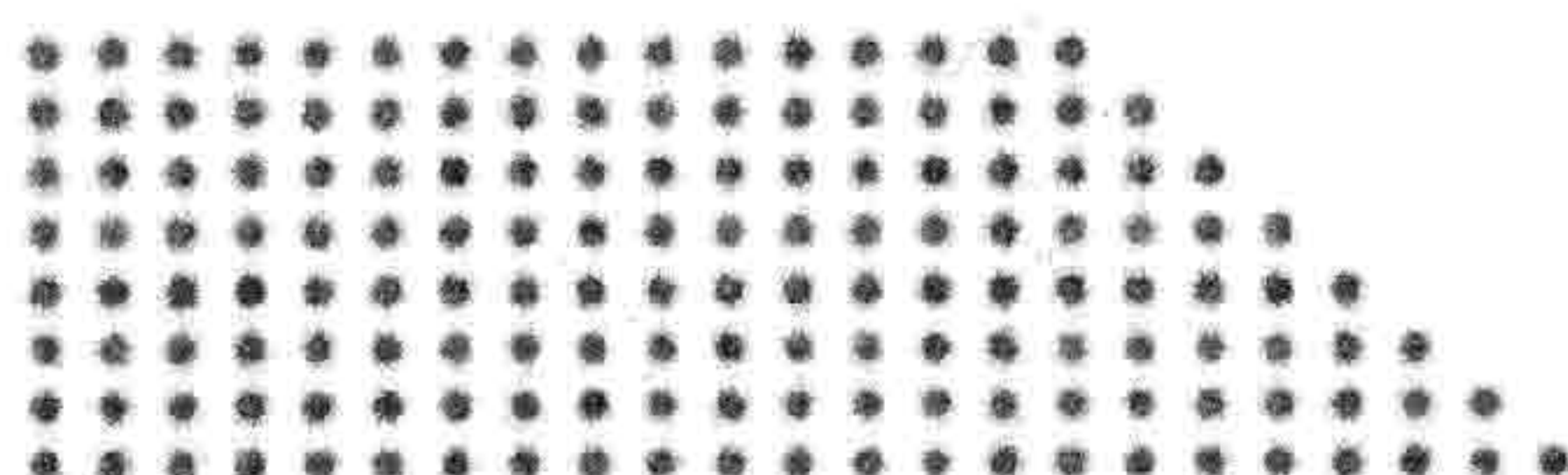
(a) The property of being bounded by three straight lines is necessarily connected with the property of being a plane figure with three vertices.

(b) In 3.D.3 and 3.D.4 we described two different "procedures" for picking out triangular shapes, of which the first involved memorizing one triangular shape and picking out others on account of their deformability into it, while the second involved pointing at sides in turn and reciting "Bing bang bong". Here are two synthesized properties which seem to be necessarily connected. Can the connection be shown, by purely logical considerations, to follow from identifying relations?

(c) No closed space is bounded by three planes. Is the incompatibility between the property of being a closed space and the property of being bounded by three plane surfaces analytic?

(d) If a cube is inside a sphere and a piece of wire is inside the cube, then the wire is inside the sphere. Is the transitivity of the relation "inside" due to some identifying fact, or can one know which relation it is without being aware of its transitivity, or anything which logically entails its transitivity?

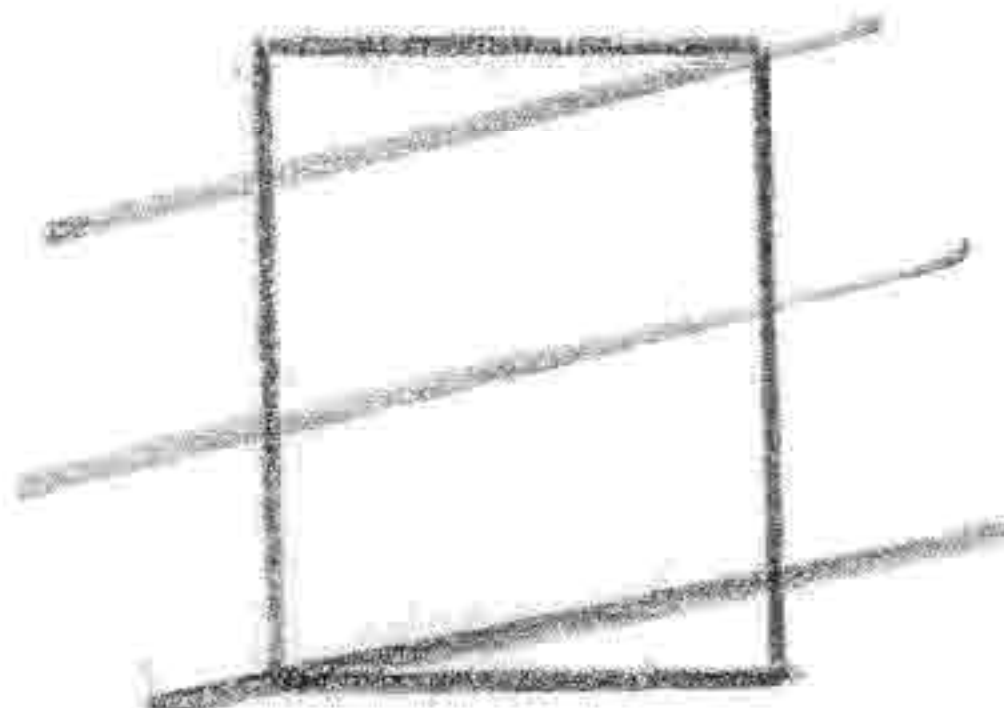
(e) Any pattern made up of regularly spaced rows of regularly spaced dots is also made up of a sequence of regularly spaced columns of dots.



It also consists of an array of diagonal rows of dots. (Diagonal rows inclined at various angles may be seen in the array.) All these several aspects of one pattern seem to be necessarily connected: the presence of some of them can be seen to entail the presence of others. Are these identifying connections between the aspects? Are they purely logical consequences of identifying connections?

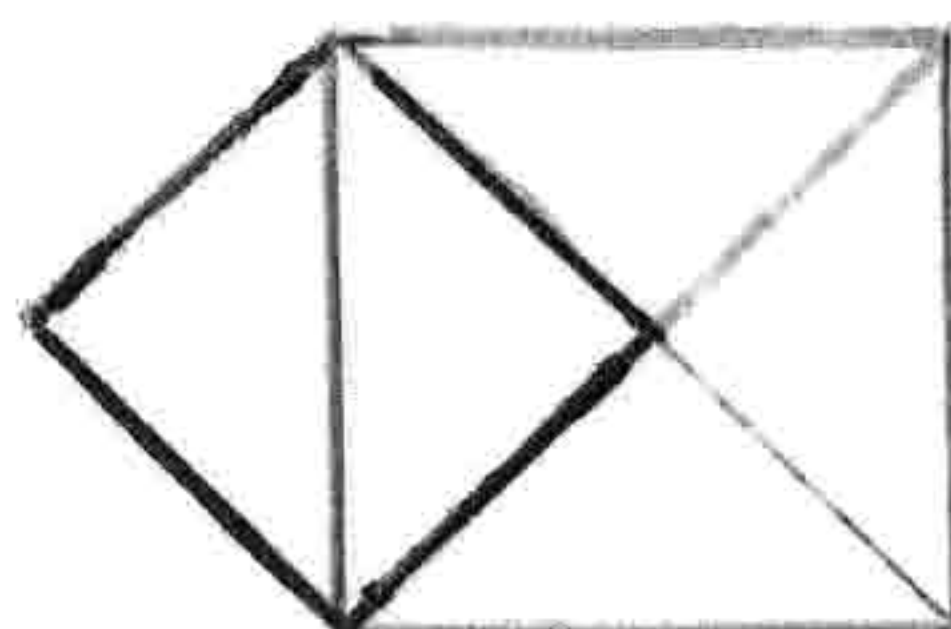
(f) Consider Kant's example; no left-handed helix may be superimposed on a right-handed helix.

(g) Consider Wittgenstein's example (R.F.M., Part I, 50.): any rectangle can be divided into two parallelograms and two triangles (by a pair of parallel straight lines passing through opposite corners, and a third parallel line between them). Is this due to some identifying fact about the meaning of "rectangle"?



(h) Any object with the property of having a shape which occupies the space common to three cylinders equal in diameter whose axes pass through one point at right angles to one another, has also the property of being bounded by twelve equal foursided faces, each of which is part of a cylindrical surface. (This is the shape obtained by pushing a hollow cylindrical cutter through a potato three times in mutually perpendicular directions in a symmetrical way.)

(1) If the side of one square is the diagonal of another, then the former can be divided into pieces which, on being rearranged, form two squares congruent with the latter.



In all these cases try, seriously, to say what the linguistic conventions are on which we must be relying implicitly when we perceive these necessary connections. (See Wittgenstein's "Remarks on the Foundations of Mathematics" for a serious attempt to meet this challenge. Cf. 7.D.10,ff., below.)

7.C.6. To all this the following objection may be made: "Of course there is no simple identifying relation between the meanings of the words 'tetralateral' and 'tetrahedral', and between the other words used in your examples, but this does not mean that the necessary connection between them is not a logical consequence of identifying relations between meanings. For the meanings of the words must be explained in terms of more fundamental geometrical words, such as 'plane', 'line', 'point', 'intersection', etc., and the meanings of these words stand in identifying relations, from which the connections to which you have drawn attention can be deduced by purely logical considerations." The objector will thereupon produce some axiomatic system of geometry, in which his "more fundamental" words occur as primitive or undefined terms, which he will use to define the words which interest us, and then, triumphantly, he will deduce from the axioms of his system, together with his definitions, using only logically valid inferences, that such statements as "All tetrahedral objects are tetralateral" are theorems.

But this is not enough. He must show first of all that his definitions do not simply take as identifying relations, relations which can be regarded as synthetic necessary connections. That is to say, it is not enough for him to show that words like "tetrahedral" can be defined as he suggests, he must also show that they have to be so defined, that it is impossible to understand them in some other way (e.g. by associating them with immediately recognizable properties), otherwise he will be arguing in the manner criticized in 2.C.10 and 3.C.10 (i.e. trading on ambiguities, and using loose criteria for identity of meanings.) In short, he must show that his definitions are definitions of the words they purport to define. Secondly, he must show that his "axioms" are in some sense true by definition, that they state or are logical consequences of identifying facts about the meanings of the primitive terms, and that they are not themselves statements which are necessarily true and synthetic. We could, of course, adopt additional verbal rules of the sort described in section 4.C to make the sentences expressing his axioms into expressions of analytic propositions, but he must show that only if such rules are adopted can the words in these sentences be understood as referring to those geometrical features to which they do refer. Once again: it is not enough for the objector to show that words can be defined in such a way as to make certain sentences express analytic propositions. He must show that as ordinarily understood they have to be so defined, or at least that unless they are so defined they cannot refer to properties which are necessarily connected.

7.C.7. It is far from obvious that there must be identifying relations of the sorts which could correspond to the axioms of an axiomatic system of geometry. (See remarks about superfluous "links between descriptive expressions", in 2.D.3 and 2.D.4.)

After all, we are not concerned with an abstract system of lifeless symbols having no empirical use, but with words which describe properties or aspects of physical objects which we can perceive and learn to recognize. (See 3.A.2) These words occur in ordinary everyday statements, such as "Here's a table with a square top", or "These three edges of this block of wood meet in a point", expressing contingent propositions which may be true or false. But (as pointed out in 2.D.2 note, and again in section 5.A), no system of axioms can suffice to give words meanings of this sort, for, in addition, semantic rules are required, correlating the words with non-linguistic entities such as observable properties. If we must have such semantic correlations, is it not conceivable that they may, on their own, suffice to give words their meanings, and determine their use and their relations to one another, without the aid of any further "axioms" or "linguistic conventions"? If so, it is surely an open question, requiring further investigation, whether all such relations are either identifying relations between meanings or logical consequences thereof. As remarked in 3.A.4. (cf. 3.D.9), in order to be able to use a descriptive word one must be able to recognize some universal immediately, so it is an open question whether some of these immediately recognized properties or features stand in relations with others, of a kind which must be discovered by examining them: why should the only things we can see

using our eyes and intelligence be facts about particulars?

7.C.7 (note). It is very common nowadays to think that any geometrical proof must start from axioms which are all arbitrarily selected, serving as expressions of linguistic conventions of some kind, specifying the meanings of the geometrical words involved in them. The reason why people think this is that different systems of axioms may all be internally consistent, as is usually pointed out in connection with systems which do not include Euclid's parallel axiom. Consider Hempel's assertion (in Feigl and Sellars, p.243):

"The fact that these different types of geometry have been developed in modern mathematics shows clearly that mathematics cannot be said to assert the truth of any particular set of geometrical postulates; all that pure mathematics is interested in, and all that it can establish, is the deductive consequences " (Compare Russell's definition of pure mathematics at the beginning of "The Principles of Mathematics." 2nd. Ed, p.3.)

All this, however, presupposes that internal consistency and deductive consequences are all that interest us, but it need not be all, if geometrical theorems are intended to state facts about observable geometrical properties. In that case, the axioms include words which refer to non-linguistic entities, and may be true or false, as well as consistent or inconsistent with one another. They are then not definitions, since the words in them are given their meanings independently, by correlations with different properties. It is not an accident that the kind of geometrical proof which involves not logical deductions from axioms, but the construction of diagrams, with construction-lines and sides and angles marked as

equal, etc., occurs in a branch of what mathematicians call "pure" mathematics. Some philosophers (unlike Frege and Kant) give the impression that they are quite unaware of this, as is shown by the quotation from Hempel and many remarks which I have heard in discussions.

Perhaps it is wrong to think that by examining geometrical concepts or properties we can "discover" that the parallel axiom is true, but that is a very special case, since it concerns infinitely long lines (and therefore not ordinary observable properties), and it does not follow that other axioms are also merely matters of convention. For example, the "theorem" that every rectangle is divisible into two triangles and two parallelograms is on quite a different footing from the assertion that two parallel lines never intersect. It is not a mere defining postulate, and neither is it a contingent fact. (See 7.E for more on this.)

7.C.8. The argument so far may be summarized thus: at some point in the explanation of the meanings of descriptive words we must point to objects of experience with which they are correlated (i.e. we have to appeal to what is "given in intuition"). But then it is an open question whether these non-linguistic entities (properties) are so related as to ensure that some of the statements using words which refer to them are necessarily true, or whether all such relations must be identifying relations between meanings. This is not a question which can be settled merely by pointing to a set of axioms or linguistic conventions which could set up identifying relations and make statements analytic, for to say that they can do the job of making statements necessarily true is not to say that they are indispensable.

And to say that anything else which does the job must give the words the same meanings anyway, is to base a question-begging argument on the use of loose criteria for identity of meanings. (2.C.10, 3.C.10.)

I am trying to show that some very superficial and slipshod thinking lies behind many denials of the existence of synthetic necessary truths.

7.C.8. note. We are not interested in the question whether some statement in some actual language is or is not analytic, or whether the relation between certain sets of words in some actual language is an identifying relation. (Cf. 6.E.9.) This sort of question is of little philosophical interest and has to be based on an empirical enquiry in order to discover exactly what people mean by the words they use, and the discussion of chapter four and section 6.D shows clearly that there may be no definite answer to such a question, or there may be answers which can be summarized only in a statistical form. (Cf. 4.B.7.) We are concerned only with the question whether certain sorts of statements have to be analytic, or whether it is possible to give their words meanings which are identified independently of one another, and then discover, by examining the properties referred to, that the outcome of applying the logical techniques for determining the truth-values of such statements can yield only the result "true". Even if statements referring to such properties are analytic in English, or in some axiomatic geometrical system, owing to the fact that auxiliary rules have been adopted, setting up identifying relations between meanings, that does not prove anything, for the rules may be superfluous.

Failure to appreciate this point can cause people to argue at cross-purposes, for example over the question whether it is analytic that nothing can be red and green all over at the same time.

7.C.9. All this should at least show that the question whether some necessary truths are synthetic, on account of being true in virtue of relations between universals which are neither identifying relations nor logical consequences of identifying relations, is an open question. It has to be settled by a closer investigation of what goes on when one examines a pair of properties, such as the property of being tetralateral (bounded by four plane surfaces) and the property of being tetrahedral (having four vertices), and discovers, possibly with the aid of an informal proof, that there is some unbreakable connection between those properties. That is, we must look at what goes on when a person discovers, perhaps with the aid of an informal proof, that, owing to the relation between some properties (relations such as entailment or incompatibility), a sentence expresses a universal proposition which not only has no exceptions in fact, but which allows no exceptions as possible. (See 7.B.2, 7.B.4.ff.)

7.D. Informal proofs

7.D.1. So far, I have tried to show that just as we can see (using our eyes and ordinary intelligence, cf. 7.B.7) that the redness of a round and red object is able to occur elsewhere without the roundness, even if as a matter of fact it does not, so can we see that some

properties are unable to free themselves from certain others, with which they are always found, or unable to cohabit with some with which they are never found. For example, I have argued that by examining the appropriate properties and discovering their relations we can detect the necessary truth of such statements as "All tetrahedral objects are tetralateral" or "No closed spaces are bounded by three plane surfaces." This shows that not only are we able to see that the actual state of affairs in this world¹ is not the only possible one, by seeing that universals are not essentially tied to actual particular instances nor to one another, but, in addition, we can see that there are some limitations on the ways in which these universals can be instantiated, some limits to what may be found in a possible state of the world. (This can be used to explain Kant's distinction between "form" and "content", in some contexts, and also, since it is concerned only with connections between properties and relations "tangible" to the senses, why he talked about "the form of sensible intuition". See C.P.R. A.20, B.34,ff; A.45, B.62; B.457 n.)

Section 7.C was concerned to establish that the necessary truths discovered in this way are not analytic, since first of all their necessity is due to non-identifying relations between properties, and, secondly, in order to become aware of them, one requires some kind of insight which is not purely logical, since it presupposes acquaintance with a specific kind of subject matter and is therefore not topic-neutral. For the first step I relied on arguments very like those of

1. (7.B.6.)

3.C.9 and 3.C.5-7, to show that the properties are independently identifiable, and for the second I relied on the fact that the insight into the connections between properties always requires some kind of examination of those properties. In this section I wish to say a little more about what goes on when one examines properties, by talking about informal proofs. (7.C.3.)

7.D.2. What happens when I construct an informal proof to enable someone (possibly myself) to see that properties are related in some way? There is a very great variety of cases. For example:

(a) I might enable someone to see that nothing can be both circular and square by drawing a circle on transparent paper and getting him to try to draw a square on which it can be superimposed, in the hope that he will perceive the incompatibility of the two properties. I might point to a curved bit of the circle and a straight bit of a square and say: "This sort of thing can never fit onto that sort of thing".

(b) To show someone that if a triangle has two equal sides then it has two equal angles, I may point out that if it is picked up, turned over, and laid down in its former position with the sides interchanged, it must exactly fit the position it occupied previously since each of the two equal sides lies where the other was, and the angle between them does not change by being reversed. Hence each of the two angles which have changed places fits exactly on the position occupied formerly by the other, so the angles are equal.

(c) To show someone that if a figure is bounded by four plane surfaces then it must have four vertices, I may hold up sheets of cardboard and show that the only way to get four of them to enclose a space is first to form an angle with two of them, then to form a "corner" or pyramid without base, by adding a third, then to complete the pyramid by adding the fourth. He can then count the number of vertices, or corners. (This also helps to show why three planes cannot enclose a space.)

(d) To enable a person to see that if anything has both the property of being a "kite" (four-sided figure with a diagonal axis of symmetry) and the property of being a rectangle, then its shape is square, I may draw a kite and show that a pair of adjacent sides must be equal if it is symmetrical about a diagonal, and remind him that a rectangle with a pair of adjacent sides equal must be square.

(e) To enable a person to see that any rectangle has the property of being divisible into two triangles and two parallelograms, I may simply draw a rectangle, and then draw three parallel lines obliquely across it so that each of the two outer ones passes through one of a pair of opposite corners. (See 7.C.5, example (g).)

Owing to the enormous variety of cases, I shall be able to make only a very few rather vague and general remarks. (See Wittgenstein's "Remarks on the Foundations of Mathematics", for a detailed discussion of many more examples.)

7.D.3. First of all, I claim that each of these proofs is perfectly rigorous, and having once seen and understood it I am perfectly justified in asserting and believing the general statement which it is supposed to prove, such as "Nothing is both circular and square", or "Every rectangular figure is divisible into two triangles and two parallelograms". (The claim that such a proof is valid is a mathematical claim, not a philosophical one, since it is to be tested mathematically by trying to construct counter-examples: more on this presently.)

What happens when I see such a proof as a proof, when I see, as a result of going through the proof, that two properties are connected, and a universal statement necessarily true? The answer seems to be that I pay attention to a property, and notice that although it can

be abstracted from the particular circumstances in which it is instantiated (see 7.A.6), so that it could occur elsewhere, and be recognized, even if it does not actually do so, nevertheless, it cannot be abstracted from the fact that it occurs in an object which has some other property (or from which some other property is absent). In particular, the construction of the proof may show me how, once I have found any other object which has the first property, I can repeat the method of construction of this proof in order to demonstrate that the other object has (or has not, as the case may be) the other property. The proof gives me a general principle for going from one property or aspect of an object to another, thereby showing me the reason why no exceptions to the proved general statement are "allowed as possible" (7.B.2,ff. 7.B. ff.) In Wittgenstein's terminology: the proof serves as the "picture of an experiment" (see R.F.M. I.36.) It may be better to say: the proof serves as a picture of a proof.

7.D.4. Perhaps we can see more clearly what goes on by distinguishing token-proofs from type-proofs. A token-proof is the particular event or set of marks on paper etc., spatio-temporally located, observed by you or me. The type-proof is a new universal, a property common to all token-proofs which use the same method of proof. The function of the token-proof is to exhibit the common property, the type-proof (a pattern); and to have grasped the proof, to have seen "how it goes", is to have seen this new universal. Now, the essential thing about the new property is that it is made up of various parts connected together (compare: the shape of a cube is a property made up of various parts connected together,

such as the several faces - see section 3.D on non-logical synthesis). We may think of it as a "bridge-property" which connects one or more of its parts with others. Thus, when I start with a rectangle, draw three parallel lines, and end with a figure divided up into two parallelograms and two triangles, I have exhibited a bridge-property which starts from the property of being rectangular and goes to the property of being divided up in a certain way. This bridge-property is a temporal property, like the tune common to two occurrences of sequences of sounds (cf. 3.A.5.): it has to be exhibited by an "enduring particular". What the token-proof shows me, when it shows me that the property P is connected with the property Q, is that any object with the property P is capable of being used in a token-proof of the same type, since P is the starting point of a bridge-property which leads to Q. Thus the proof makes evident the connection between two properties by exhibiting them as parts of a new property. The token-proof shows how P and Q both "fit" into the type-proof.

This reveals a relation between words which are semantically correlated with those properties. In virtue of this relation some propositions using those words are necessarily true (Cf. 5.E.2, 6.C.1.).

7.D.5. But how do we discover the connection between the initial property and the bridge-property, or the connection between the bridge-property and the final property? How do we see that any object with the property P must be capable of being used in a proof of this type?

This is a crucial question. Consider a particular

instance: how does the proof that every rectangle is capable of being divided into two triangles and two parallelograms show me that every rectangle is capable of being the starting point of such a proof? Do we need another bridge-property here, starting from the property P and ending in the former bridge-property? Obviously not. Then why not? The answer seems to be simply that a proof must start somewhere, and wherever it starts there must be something which is taken as not needing proof, namely that the first steps of the proof are possible. The reason why this needs no proof is that it may be discovered simply by inspecting the original property. Just by inspecting the property of being a rectangle I can see that if anything has the property then a line traversing it obliquely may be drawn through one of its corners. A person who cannot see even this will not follow the proof in question.

In other words, the account of the function of a proof in terms of type-proofs, or bridge-properties, is incomplete, since it leaves out the essential fact that at every stage of the proof something just has to be seen by examining a universal, namely that the next stage may proceed from there: this must be something which requires no proof. The whole point of a proof is to bring out a connection which is not evident. Where a connection is evident no proof is required, and this kind of connection which displays itself must be found at every "step" in a successful proof.

7.D.6. By pointing to a particular object I may draw someone's attention to some property or other universal instantiated in that object, but I cannot force him to see it. Similarly, by drawing his attention to a

property or pair of properties I may succeed in drawing his attention to a connection between those properties in virtue of which one cannot occur without the other, or in virtue of which they are incompatible, but I cannot force him to see it. In some cases I can help him to see it by constructing a proof, by drawing his attention to a new property, a bridge-property of which the other two are somehow parts or constituents. But I cannot force him to see the bridge-property (I cannot force him to see what the type-proof is so that he could recognize it again in another instance; I cannot force him to see how the proof goes), and I cannot force him to see how it reveals a connection between the two properties in virtue of their connection with it.

This is very vague, and ignores differences between different kinds of proof. Perhaps it will be made a little clearer by the replies to some objections.

7.D.7. The first objection is that all my talk about "seeing" properties and connections between properties is far too psychological to serve as an account of what a proof is.

It is important to be clear about the sense in which the account is psychological and the sense in which it is not. Certainly the fact that someone takes a proof as valid is not what makes it valid (cf. 7.A.7, where a similar objection was raised to my account of possibility). But no account of what a proof is can avoid using psychological concepts such as "belief", "certainty", "understanding", "conviction"; for what is a proof supposed to do? It certainly cannot make the proposition true which it is supposed to prove. It cannot make it necessarily true either. The necessary truth of "Nothing is both round and square" in no way

depends on the fact that anyone has ever proved it. Perhaps its necessary truth might be said to depend on the possibility of constructing a proof. But what makes this a possibility is a connection, or set of connections, between properties - which is precisely what is shown by the proof. The existence of the proof (token-proof) does not bring the connections between properties into existence, for the proof depends upon them for its own existence, and they are capable of ensuring the necessity of the proved proposition without the intermediary of the proof.

The proof neither makes the proposition true, nor makes it necessarily true. Rather, it brings out the reason why it is necessarily true. "Bringing out" can only mean "making evident to someone or other", for the reason is there, doing its job faithfully, whether any proof is constructed or not. So the role of proof is to enable someone to see that a statement is true, or necessarily true, and it follows that psychological concepts must be employed in a description of what proof does and how it does it.

7.D.8. The error in the objection is to assume that because psychological terms are used to explain what a proof is, the validity of proofs is a psychological matter. But this is not so. (Necessity is not defined in terms of inconceivability, but both are explained together. See 7.A.7.)

A person cannot simply turn up and say that he knows that it is possible for a round square to exist because he has seen and been convinced by a proof of its possibility. If he has seen a proof, and followed it, then

he has become acquainted with a new universal (the type-proof: 7.D.4), and, as pointed out in 7.A, a universal is the sort of thing which can recur, so he must be able to reproduce the proof and point out its relevant features to us. He cannot get by with the remark that he remembers how the proof goes, and can imagine it, but cannot produce it for us, for what can be imagined by him proves nothing unless it is the sort of thing which could be drawn on paper, or otherwise concretely reconstructed and subjected to scrutiny. Neither is it enough for him to draw a straight line and say that it is a picture of a round square seen from the end, for he must explain in virtue of what this can serve as a picture of a round square, i.e. how it exhibits the roundness and the squareness of the thing it is meant to represent.

7.D.8.a. Of course, a person may produce a perfectly valid proof which, for the time being, no one else can follow, on account of its complexity. But this does not make its validity a subjective matter, any more than the possession by an object of a complex property (e.g. a shape, or other structure) is a subjective matter in cases where only one person happens to be able to "take in" the property. We all know, at least in a vague way, what would count as an objective refutation of the proposition alleged to be necessarily true, for if we understand the proposition then we are able to recognize counter-examples, should they turn up. (This must be modified to take account of existence-proofs, or proofs of possibility.)

Even if the proposition happens to be necessarily true, but not validly proven, we know what would count

as a public demonstration of the invalidity of the proof, for there must be something about the proof in virtue of which it is supposed to establish the connection in question (i.e. there must be type of which it is a token), and it could be shown to be invalid by a construction based on the same principles (a token of the same type) which leads to a proposition which has demonstrable counter-examples.

(A full account would describe and classify various ways in which one may fail to see the validity of a valid proof, or come to think an invalid proof is valid. E.g. one may think one has seen a property which one has not seen. One may have seen a bridge-property, but not one which does quite what it is taken to do, as when one notices a connection which works in most cases without seeing that there is a special class of counter-examples.)

7.D.9. Next it may be objected that what I say is just wrong, since what really goes on in a proof is that, in a "dim" way, we are shown how a formalized proof would go, starting from identifying relations between concepts and drawing purely logical conclusions, without any need for such things as "showing" the connections between properties. Since it looks as if I am not doing any such thing when I use an informal proof to enable someone to see that all tetralateral objects are tetrahedral, it is up to the objector to say what the formalized proof is that I am unwittingly presenting, e.g. by showing me the identifying relations or definitions from which he thinks I am drawing purely logical conclusions, whereupon the arguments of 7.C.6 will come into play.

Secondly, the objector will find himself in difficulties

as soon as we ask how the conclusion follows from those statements of identifying facts which he claims to be implicit premises in the proof. For, as pointed out in 5.C.9 and 5.C.10, etc., in order to see that some inference is logically valid, it is necessary to perceive properties of general logical techniques corresponding to the logical forms of propositions, or connections between such techniques, and this is just the sort of insight into the connections between universals which is provided by the informal proofs whose existence the objector wishes to deny. (The difference is only that logical techniques are topic-neutral, whereas we are discussing informal proofs concerned with special kinds of observable properties.)

7.D.9.a. This last point is important, because one of the strong motivating forces behind the desire to establish that all necessary truths are analytic, or that all apriori knowledge is knowledge of analytic truth, is the desire to eliminate the need to talk about special kinds of "insight" into the relations between universals. It is apparently thought that if all necessary truth and apriori knowledge can be shown to be derived by purely logical considerations from definitions then that need will be eliminated, but the remarks of the previous paragraph, and 5.C.9-10, show that this merely shifts the problem.

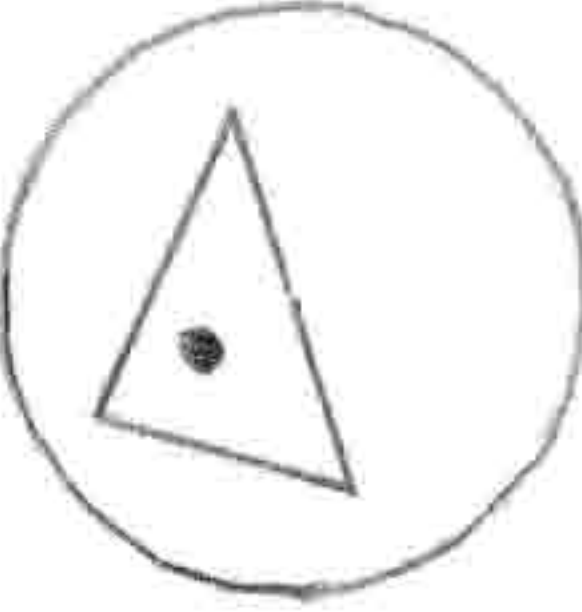
The amazing thing is that some philosophers thought this problem could be avoided by explaining all logical connections and all perception of logical connections in terms of formal systems and derivability of theorems from specified axioms according to rules of derivation specified in advance. It is amazing for two reasons, firstly because

it is hard to see how anyone was ever able to think that merely talking about rules for manipulating symbols could explain logical properties and relations of statements (see appendix II), and secondly because even if talk about proofs in formal systems did explain logical connections and our knowledge of them, this would be at the cost of reducing logic to geometry, and there would remain the problem of explaining what sort of insight was involved in perceiving that strings of symbols stood in certain geometrical (or syntactical) relations to others. For, after all, the formal logician is not trying to establish the merely contingent fact that he can here and now derive (or has here and now derived) this particular set of marks from that particular set of marks (all tokens) while trying to follow certain rules: he wishes to show that a relation holds between types of marks, or, in other words, that geometrical properties (or patterns) stand in a connection of the kind which we have been discussing and he thinks he can explain away.

7.D.9.b. Sometimes it is argued that this question about the justification for asserting that a formula has a formal property need not arise (e.g. the property of being a theorem) since the rules of the formal system are so devised that even a moron, or a machine, could be instructed to check a proof to see that it went in accord with the rules. But this misses the point, for it relies on the very fact to which attention was drawn above (7.D.5), namely that each step in a proof must depend on connections between (e.g.) properties which are so evident as to need no proof.

If I "prove" that the statement "If a triangle is

inside a circle, and a dot is inside the triangle, then the dot is inside the circle" is necessarily true, by drawing a diagram, namely a circle surrounding a triangle with a dot in the middle, or if a moron comes upon the diagram and utters the statement, then there is no more doubt about mistakes here than when a moron asserts that some pattern of formulae satisfies a recursive definition of "proof-sequence".



In any case, pointing out that a moron can apply some test for picking out certain sequences of sentences does not answer the question why sequences of sentences picked out in this way are valid proofs. Are morons supposed to be able to tell that any sequence of statements, no matter how complicated, which satisfies some recursive test constitutes a valid proof?

7.D.9.c. The assertion that an informal proof is really a formal proof in disguise does not seem to be of any help at all. It would be truer to say that a formal proof is an informal proof in disguise. The formulae in a formal proof represent propositions, or the logical forms of propositions, and serve the purpose of drawing our attention to logical relations between those propositions, in an indirect way. For the symbols are so chosen, that geometrical relations between them represent relations between the logical techniques corresponding to the logical forms of propositions (see 5.C.9). So, when we look at the logician's symbols, our attention is drawn (half consciously) to the relations between these logical techniques for discovering truth-values, and we are thereby enabled to see the relations between truth-values of propositions (i.e. relations between the outcomes of applying the logical techniques). It is not essential

to this process that the symbols which draw our attention to logical relations between propositions should constitute a "proof-sequence" in some formal system.

(Herein lies the answer to Wittgenstein's puzzle in R.F.M. Part II sections 38 and 43: a Russellian proof is cogent only insofar as it has geometrical cogency, yet one may "accept" it without ever noticing the geometrical application. This is because the geometrical application is a consequence of the contingent fact that the rules of our language correlate logical forms of propositions, or logical techniques of verification, with geometrical forms of sentences in a uniform way.)

A fully explicit logical proof would draw attention directly to logical techniques and their interrelations, in much the same way as the informal proofs under discussion draw attention to connections between geometrical properties. Such an informal logical proof would help someone to perceive logical properties of or relations between propositions, which is what a formal proof does only indirectly and implicitly. (Compare Appendix II.)

7.D.10. We have now dealt with two objections, first that the account of informal proof was too psychological, and secondly that there is a formal proof, proceeding from definitions or identifying facts about meanings, underlying every informal proof. The first objection was met by asking what a proof is supposed to do, the second by showing that only an informal proof can do this, even if it is purely logical. But it is still open to someone who wishes to deny the existence of synthetic necessary truths to admit that informal proofs are possible, while asserting that they can only start from identifying relations between meanings or properties and proceed

logically, so that at every stage only topic-neutral considerations are relevant. Such a proof could only demonstrate the truth of an analytic proposition, according to the definition of 6.C.10. This assertion might be based on the argument that only an identifying relation between meanings can guarantee that there will not be an exception or counter-example to the proposition proved.

How can I be sure, simply because I have seen one rectangular figure divided up into a pair of triangles and a pair of parallelograms, that there will never be a rectangle which cannot be divided up in this way? How can I be sure that I shall never see a figure which has both the property of being round and the property of being square? The suggestion is that I can be sure only if I adopt a linguistic convention ruling out the possibility of describing any unexpected object as a counter-example to the proved proposition. That this suggestion has a point is shown by the history of mathematics. For it has happened more than once that a proof has been accepted as valid, and then later shown to be invalid. (Dr I. Lakatos has investigated such cases.) Many of Wittgenstein's remarks in R.F.M. seem to be directed towards showing that the possibility of an unexpected counter-example can never be eliminated except by a convention relating the meanings of words, in something like the way in which the "purely verbal" rules described in section 4.C were able to rule out the possibility of a counter-example to statements like "No red thing is orange" (see 4.C.3-4). (Similar, though more vague, arguments were used by von Wright on p.38 of "Logical Problem of Induction", 2nd Ed.)

7.D.10.a. Suppose, for example, that a (token) proof connecting a property P with a property Q purports to show that any object with P can serve as the starting point for another (token) proof of the same type. If so, the object must also provide an instance of the "bridge-property" starting with P and ending with Q, so it must be an instance of Q, and thus no counter-example to "All P things are Q" is possible. But suppose the proof does not work: an object turns up which has the property P, but from which the bridge-property cannot start, that is, an object which has P but cannot enter into a token-proof of the required type (the construction cannot be carried out). Then the only way to save the theorem proved is to adopt a new linguistic convention. E.g. we may say that being the starting point for the bridge-property is one of the defining criteria for having the property P, and so the new object does not really have P and is not really a counter-example. Now the argument we are considering claims that even before any counter-example has turned up, the only way we can guarantee that none will do so is by adopting this sort of new defining criterion for having some property, and what the informal proof does is show us how the new criterion works by displaying the bridge-property which we thereafter take to be identifyingly related to the property P. Similarly, we may have to take the connection between the bridge-property and the property Q as an identifying fact about the meaning of the word which formerly referred to Q, in order to ensure that no counter-example may turn up to break the link at the other end. (This applies only to one, relatively simple, kind of proof, of course.) By

adopting these new rules we have given a more determinate meaning to the words expressing the theorem which was meant to be proved, and we have also set up identifying relations between their meanings from which it follows logically (see 6.B.11,ff.) that the sentence expressing the theorem in question must correspond to the truth-value "true". In Wittgenstein's terminology: "in the proof I have won through to a decision" (R.F.M. Part II, 27.)

7.D.10.b. Now it is very likely that this sort of thing happens sometimes in mathematics: we may think that we have completely identified some complicated geometrical or arithmetical property when in fact it is indeterminate in some respects (see section 4.A), and borderline cases could occur to provide potential counter-examples to some theorem about that property. (Cf. end of 7.B.4.) Then the construction used in a proof of that theorem may show us a new way of defining the meanings of the words used to express the theorem so as to rule out these counter-examples. But it is important to notice the differences between borderline cases which produce the following reactions (a) "I had not thought of that possibility, so I was wrong after all", (b) "I had not considered that possibility, but it does not matter as it is not the kind of thing that I was talking about" and (c) "I had not considered that possibility, and now I don't know whether to say it is the kind of thing I meant to refer to or not." Case (a) is an admission that the proof was invalid and the theorem wrong after all. (b) rejects the borderline case as not providing a counter-example. Only (c) leaves room for a new decision to

adopt a convention as to how to describe the borderline case in order to save the theorem. The thesis that every proof is covertly a logical proof from identifying relations between concepts which we implicitly accept in accepting the proof requires that every geometrical concept be indeterminate in such a way that there is room for this sort of decision.

It is not possible to go into the question whether every concept is indeterminate in this way without embarking on a general enquiry concerning meaning and universals, and all the topics raised by Wittgenstein in his discussion of the notion of "following a rule" in "Philosophical Investigations". I shall say only that the fact that in some cases mathematicians have failed to see the possibility of counter-examples to propositions which they believed to have been proved, does not in the least convince me that no non-logical informal proof is secure, and neither does the fact that in some cases a property or a proof may be so complex that it cannot be "surveyed" properly and has some indeterminate aspects: there are other cases where properties are sufficiently simple for their connections to be quite perspicuous, leaving no room for any doubt that something will go wrong. I am perfectly certain that if anyone brings me an object which is alleged to be bounded by four plane surfaces, and not to have four vertices, then it will turn out that either the four planes do not bound the object, or there are not exactly four of them, or they are not planes, or he has miscounted the vertices, or (Why should I specify in advance all the mistakes which could possibly be made?)

7.D.10.c. I conclude that there is little reason to doubt

that there are some connections between properties of such a kind as to prevent their occurring in certain combinations in particular objects, that these connections need not be identifying connections, and that they may either be quite evident, or sometimes made evident by an informal proof, which enables us therefore to see that some proposition states a necessary truth. This does not deny that there are cases where indeterminateness of meaning makes it necessary to adopt purely verbal rules (4.C) to rule out the possibility of counter-examples, neither does it imply that we are infallible and can never wrongly think we see connections between complicated properties.

It is worth noting that the difference between a doubtful borderline case of an instance of a property, and other objects which clearly are or clearly are not instances of it cannot be merely a numerical difference between them. There must be a difference in kind between borderline and non-borderline cases, they must exhibit different properties (e.g. an object with a borderline shade of red looks different from one which is bright scarlet). So even where a new verbal rule is required to ensure that borderline cases do not provide counter-examples to theorems proved by an informal proof, the verbal rule has to be applied only in some kinds of situations, involving objects which differ in certain respects from those which are not borderline cases. In other sorts of instances of the properties referred to, the connections between the properties are as shown in the informal proof, so no verbal rule is required to ensure that no counter-example to the theorem can arise amongst them, that is amongst objects which do not have

the properties peculiar to the borderline cases. This shows that even if it is true that some verbal rule is always required to make it certain that no unexpected borderline cases can provide counter-examples to a theorem proved in the manner under discussion; that is, even if every necessary truth has an analytic aspect, nevertheless there is a synthetic aspect, brought out by the informal proof, which shows a necessary connection in at least a limited range of cases. ("No rectangle can look just like this one and fail to be divisible into two triangles and two parallelograms, and I do not need to adopt any convention to ensure that, for it is evident to anyone who examines the shape of this rectangle".)

7.D.11. Now our persistent objector may argue that even if it cannot be shown that what goes on in an informal proof is always implicit logical deduction from implicitly acknowledged identifying relations between meanings or properties, nevertheless it remains for me to demonstrate that the "connections" revealed by such proofs are not breakable, that they are necessary connections, allowing no exceptional particular objects as possible. How do we know that the constant conjunction of these two immediately perceptible features, or their failure to occur together, as the case may be, is not just a contingent fact? Certainly it is up to me to show that the propositions in question are necessarily true, but I do not do this by means of a philosophical argument, I do so by means of the proof which we are discussing!

If the objector cannot see for himself by examining properties (either in particular instances, or in his imagination if he is well acquainted with them), or by

going through an informal proof, that nothing can be both round and square, that every tetralateral object is tetrahedral and no exceptions are possible, if he cannot see the necessary connections between these independently identifiable properties, then he cannot be forced to see them, as already pointed out, and he must forever remain in doubt as to whether, perhaps in the depths of darkest Africa, there lies hidden somewhere a slab of some hitherto unknown material, whose boundary can be seen to be at once both square and circular, or perhaps a little pyramid, completely enclosed by exactly four flat sides, but with only three vertices, or five.

The objector can surely not expect me to convince him by offering a proof which starts from definitions, and then draws purely logical inferences, for the whole point of this section and the last is to show that only by altering the meaning of the statement proved can one replace a non-logical informal proof by a logical one. (To show that the statements in question are necessarily true is not a philosophical task.)

7.D.11.note. All this must be qualified by the remark that our ordinary geometrical concepts ("round", "square", "straight", "flat", etc.) are extremely complicated in a way which makes it very difficult to describe clearly what goes on when a normal person is confronted with an informal proof of the sort used in school geometry. I am referring to the fact that where we have one word, such as "ellipse", there are usually very many concepts superimposed in its meaning, in a quite indeterminate way, and this is not noticed, owing to our use of loose criteria for identity of meaning, which cannot distinguish these different superimposed concepts. Consider each of the various definitions of the notion of an ellipse which might be given by a mathematician, and blur its edges a little. Add the semantic correlation between the word and the visual property or range of visual properties which we associate with it. Load all these meanings onto one word in an indeterminate way: and then try to explain

what goes on in the mind of someone who uses the word in this overdetermined fashion when he sees a proof of a theorem about ellipses! (For other examples of superimposed concepts see 3.E.2 and 6.D.3. Compare also 5.E.7.a. I believe that our ordinary arithmetical concepts are indeterminate and overdetermined in a similar fashion, which is why different philosophies of mathematics have all been able to claim some plausibility - logicism, formalism, intuitionism, empiricist theories. Each has picked out one aspect of the truth, while making the mistake of claiming it to be the whole truth. There is no time now to show how a unifying theory could be developed.)

7.D.12. Some of the things said by Kant about synthetic apriori knowledge are explained by this discussion, in particular that it requires an "appeal to intuition". This is firstly because without the intuition (acquaintance with properties, etc.) one cannot know which concepts are involved or that there are any empirical concepts involved which can be applied to observable objects; and secondly an appeal to intuition is required since without it one cannot come to see how the concepts (or properties) are connected. (See, for example, C.P.R., A.239-40, B.298,ff; B.308-9; A.716,B.744). The fact that looking at a diagram (real or imagined) can play an essential part in perceiving the connections between properties shows that in doing so one is not merely drawing logical inferences whose validity depends on topic-neutral principles. (This might also be put by saying that the type-proof, or bridge-property, mentioned in 7.D.4, is not logically synthesized out of the properties whose connection it is supposed to reveal: the synthesis in the proof - i.e. the way in which it is constructed - is non-logical, for reasons of the sort given in 3.D.6,ff.)

I think my account of informal proof helps also to

explain why Frege believed that one could have synthetic apriori knowledge concerning geometry. (See sections 14 and 90 of "Foundations of Arithmetic".) It also explains some of the talk of Intuitionists about "mental constructions" (see Heyting "Intuitionism", e.g. p.6,ff), though my account would have to be modified to take account of proofs of theorems about the properties of infinite sets, since these are not perceptible properties.

7.D.13. Whether this brief and highly condensed sketch is correct or not, one thing should be clear: informal proofs certainly do something, and what they do is different from what is done by a proof of an analytic proposition starting from identifying facts about meanings and proceeding logically. This shows that if the "proved" statements are necessarily true in both cases, then it is very likely that there are at least two kinds of necessity worth distinguishing. My suggestion is that the way to distinguish them is to notice that in both cases the propositions exemplify the notion of a "freak" case of a rogator whose value happens to be determined by relations between its arguments together with facts about the technique for working out its values, the difference being that in the one case the arguments are identifyingly related, and all arguments standing in the same relation must yield the same truth-value, whereas in the other case the relations between the arguments are not identifying, and they are not relations of a sort which could hold between any sorts of entities at all (they are not topic-neutral).

However, it is open to anyone who does not like talking about synthetic necessary truths or synthetic apriori knowledge to reject my terminology and say that in both cases the propositions are analytic since their truth is

determined, however indirectly, by what they mean. But then "true in virtue of meanings" seems to be synonymous with "necessarily true" in this terminology, and the assertion that all necessary truths are analytic says nothing and says it in a redundant terminology which fails to take account of distinctions which are of some interest.

7.D.14. It should not be thought that the assertion that there are synthetic necessary truths has any great metaphysical significance, or that it justifies any claim to have perceived with the inner light of reason, or any other mysterious faculty, moral or theological truths, or truths of a transcendent nature. (See 7.D.8). So far, the assertion has been justified only by a discussion of ways of perceiving connections between simple empirical statements (i.e. between the techniques corresponding to logical rogators). If it can be extended to cover other cases, such as the principle of causality, then this has to be shown by detailed investigation.

I claim only to have given an informal proof of the existence of synthetic necessary truths of a simple and uninteresting kind, or at least to have shown that there is a distinction to be made between different sorts of necessary truth. But the topic is difficult and complex, and I have been unable to do much more than provide an introduction to it by showing how its problems are related to and can arise out of general considerations about thought and language and experience. [I am very dissatisfied with the discussion of this section, though I believe it to be a first step in the right direction. I have included it for the sake of completeness.]

7.E. Additional remarks

7.E.1. This chapter may now be summarized. Chapter six had explained how the truth of a statement may be a purely logical consequence of identifying facts about the meanings of the words used to express it. Such a statement would be true in any possible state of the world because its truth-value is determined independently of how the world happens to be. In this chapter I have tried to explain why it makes sense to contrast the way the world happens to be with ways it might have been, so as to give a fairly clear sense to the question: Are there any statements which would be true in all possible states of the world, besides those described in chapter six? That is: are there any non-analytic necessary truths? I was able to give a sense to this question, by making use of some of the very general facts about our language which were pointed out in chapter two, especially 2.B and 2.D, namely the fact that we use a conceptual scheme with provision for a distinction between universals (observable properties and relations) and particulars, and the fact that universals are not essentially tied to actual particulars. The question then became: Are there any limitations on the distribution of universals to be found in any actual or possible state of the world, apart from purely logical limitations, which are in no way concerned with anything special about specific properties but are topic-neutral? [This was the fundamental question, but in order to take account of "improper" or "synthesized" universals (see chapter three), we asked the question in the following form: Are there any connections between universals (i.e. restrictions on the ways in which they may be instantiated)

which are not due simply to (a) identifying facts about those universals and (b) purely logical or topic-neutral restrictions?]

7.E.2. I tried to answer this question by drawing attention to observable connections between observable properties, where (a) the properties can be identified independently of each other and (b) logical considerations alone do not account for the connections between them, since the properties themselves must be examined for the connection to be perceived. Thus, a slightly more general account was available of the way in which relations between the arguments of a logical rogator might help to determine its value, than the account given in chapter six. In short, we saw that a sentence may express a proposition which would be true in all possible states of the world, though it is not analytic. The reason why no exceptions to such a synthetic necessary truth are possible is that exceptions would have to be objects in which properties were combined in ways which are excluded by the connections between those properties.

7.E.3. This also explained why one might have the right to make such statements as "If this had been P then it would also have been Q" or "If this had been P then it would not have been R". The connections between properties which make some statements necessarily true, also give a sense to counterfactual conditional statements, by giving us the right to assert some of them as true.

(Note: We could generalize this slightly to explain the concept of entailment. The proposition p entails the proposition q if something ensures that if p were true

then q would be. This can be put more precisely:

The proposition p entails the proposition q if and only if there is some relation R satisfying the following conditions:

- 1) The relation holds between p and q .
- 2) The relation holds between some propositions which are neither necessarily true nor necessarily false.
- 3) If the relation holds between two propositions ϕ and ψ , then this ensures that in any possible state of the world in which ϕ would be true ψ would also be true.

(The relation may be purely formal - i.e. it may be a topic-neutral relation, or it may be concerned with the content of the two propositions.)

I suspect that our ordinary expressions of the form "If ϕ then ψ " are more like assertions of entailment than like assertions of material implication, though probably much more complex than either, as can be seen by examining the sorts of things which are normally regarded as justifying such assertions. In consequence, it is not obvious that what I said in chapter five about logical forms of propositions, and the logical rogators which correspond to them, applies without modification to conditional statements.)

7.E.4. The discussion of informal proofs was intended to explain how we can become aware of connections between properties of the sorts which ensure the necessary truth of some synthetic propositions. It also provided a partial answer to the question raised in section 5.C about the manner in which one can become aware of the relations between logical techniques in virtue of which propositions whose logical forms corresponded to those techniques might have logical properties or stand in

logical relations. The answer was very vague, namely that perceiving connections between logical techniques is the same sort of thing as perceiving connections between (say) geometrical properties.

It is clear that there is a lot more work to be done on the subject of informal proof, as I have talked only about some very simple cases, and left many questions unanswered.

7.E.5. For example, there is a puzzling fact which I have hardly mentioned, except in 6.E.6-7, and without an explanation of which it is impossible to give a complete account of the way in which informal proofs work, or the way in which we normally come to have knowledge of necessary truth, namely the fact that a person may be justified in claiming or believing something, and for the right reasons, without his being able to see clearly or say clearly what the reasons are.

This is exemplified by a layman's assertion of an analytic proposition which he correctly justifies by saying that it is "true by definition", even though he may be quite unable to explain what it is for a proposition to be true by definition. Similarly, he may have seen an informal proof, and so be quite justified in asserting the proposition which it proves, saying "It must be so", and yet be quite incapable of saying how the proof proves the proposition. This is connected with some of the remarks in the appendix on "Implicit Knowledge".

It is pretty certain that if ever a philosopher does manage to give a clear and accurate account of how informal proofs work and why we are justified in asserting the propositions which such proofs are taken to justify, we shall not be able to retort to him that we knew it all

before, just as the person who cannot see that some statement is true until he has studied a proof cannot claim to have known it all before, even though the proof does lead him on from things which he did know before. Perhaps there is an analogy here between what happens when a mathematician convinces us of the truth of some surprising theorem by drawing construction-lines and what happens when a philosopher solves some kinds of problem: the philosopher draws "construction-lines" of a different sort in order to bring out connections between concepts, such as the connection between the concept of a diagram used in a geometrical proof and the concept of necessity. (Was I drawing philosophical "construction-lines" when I talked about rogators in chapter five, in order to give an account of logical form and explain the connection between formal properties of sentences and logical properties of propositions?) This suggests that if a mathematical proof can enable one to see that some synthetic proposition is necessarily true, then perhaps philosophical investigations may also reveal synthetic necessary truths. This is something which requires detailed investigation. (For some remarks on philosophical analysis, see Appendix IV.)

7.E.6. Another subject which requires investigation is the relation between our ordinary empirical concepts of shape and colour, and the idealized concepts which, at times, it may have appeared that I was discussing. (See the disclaimers in 3.A.2. and 7.C.7.) Idealized concepts are somehow extrapolated from our ordinary concepts. Examples are the concept of an "absolutely specific" shade of red, or an "absolutely specific" triangular shape, and the concept of a "perfect" geometrical shape, such as

the shape of a perfect cube. Philosophers sometimes suggest that there is no connection between these idealized concepts and our ordinary empirical ones. At any rate they are usually unclear as to how they are derived from our empirical concepts, perhaps because they fail to see that two sorts of idealization are involved.

7.E.6.a. First there is the idealization towards perfect specificity, which explains our use of expressions like "exactly the same shade of colour as" or "exactly the same shape as". We see pairs of objects which are more and more alike in some respect, and then extrapolate to the limit, on the assumption that it makes sense to do so, even though we are not able to discriminate properties finely enough to base the notion of "exact likeness" directly in experience. This kind of exact likeness is supposed to be transitive, unlike perceptual likeness. So the absolutely specific shade of colour (for example) of my table is a property common to all objects exactly like it in respect of colour. (There can be no borderline cases.) Perhaps some argument can be given to justify the assumption that it makes sense to talk like this. I do not know.

7.E.6.b. The second sort of idealization is quite different, and helps to explain such concepts as the concept of a "perfectly straight" line, or a "perfectly smooth" curve, or a "perfectly plane" surface, or a "perfectly perpendicular" pair of lines. It may also be connected with such notions as a "perfectly pure" shade of red, or a "perfectly pure tone", or a "perfect" musical octave. For the purposes of this sort of idealizations it is first of all assumed that it makes

sense to speak of absolutely specific properties (shapes, colours, tones, etc.) and then use is made of the fact that in some cases the properties can be arranged in a series, apparently tending towards a limit. Thus one line looks straighter (smoother, more nearly circular, etc.) than another, and a third looks straighter (etc.) than the first, and so on: so we extrapolate and assume that it makes sense to talk about the perfectly straight (smooth, circular, etc.) line which lies at the end of the series and is straighter (smoother, etc.) than the others. A similar process may account for the concept of an infinitely long line, or the concept of perfectly parallel lines, or the concept of infinity in arithmetic or set theory. Similarly, one colour looks a purer blue than another, and so on, so we assume that there could be a perfectly pure shade of blue. In some cases there may be more than one route by which the limit is approached.

7.E.6.c. It is taken for granted that such methods of extrapolation fully define the "perfect" concepts which they generate, and that different methods of extrapolation may define the same limit. And it used to be thought that facts about these perfect concepts could be discovered with the aid of old-style Euclidean proofs. But it is more likely that although the method of generation of such idealized concepts fully determines some things about them (thus, the relation "inside", applied to perfect squares, triangles, circles, etc., is transitive), nevertheless in order fully to define them it may be necessary arbitrarily to stipulate that certain relations hold between them, or that certain statements about them are true (such as Euclid's parallel axiom). Since such

a stipulation is arbitrary (there is nothing in virtue of which it is "correct"), we could adopt alternative "axioms" and complete the definitions in another way. This is the tiny grain of truth which lies behind current opinions of the sort which I criticised in 7.C.7(note).

7.E.6.d. It is also sometimes not noticed that the process of idealization does not remove all empirical elements from these "perfect" concepts. Hence it is assumed that geometrical proofs which are concerned with them have nothing to do with objects of experience. This is why philosophers sometimes talk as if a perfectly sharp distinction can be made between "pure mathematics" and "applied mathematics", the latter being regarded as an empirical science, perhaps a branch of physics. There is no space here to explain in detail why this is muddled.

7.E.7. A failure to understand the nature of these "perfect" mathematical concepts, or to see the difference between those "axioms" which served the purpose of completing the definitions of concepts and the "theorems" whose truth in no way depended on arbitrary stipulations of identifying conventions, left people unable to cope with the shock of the discovery of alternative internally consistent axiom-systems for geometry. The notion of a proof as something which served to establish the truth of a theorem was therefore undermined, and philosophers tried to salvage what was left by treating proofs as nothing more than methods of deducing consequences from arbitrary hypotheses or postulates. This at least seemed to be secure: for, by means of formalized systems of logic one could at least give fool-proof criteria for the

validity of a proof. Criticisms of this conception of proof have been made elsewhere (in 5.C.10,ff, 7.D.9,ff and Appendix II). It seems not to have been realized that such a conception severs the concept of "proof" completely from the concept of "truth". It seems not to have been realized that if proofs are intended to serve the purposes described in section 7.D, namely, to enable people to perceive the truth of propositions, to bring out the reasons why propositions are true, then the search for a fool-proof criterion of validity is futile: for, no matter what criterion is adopted, questions remain about the justification for accepting proofs which satisfy that criterion, and the justification for the statement that any particular type of proof satisfied the criterion. If a justification is offered, then its validity cannot be constituted by satisfaction of the criterion in question - that would be circular. The only way to avoid a circle is to give up talking about criteria of validity, and either follow Wittgenstein in his talk about conventions (in R.F.M.) or try to explain how we can simply see necessary connections between properties and other universals by examining them, perhaps with the aid of informal proofs. (Are these really distinct alternatives?)

7.E.8. Finally, the reader is reminded that although an informal proof enables one to discover that a proposition like "All tetralaterals are tetrahedrals" is true without discovering how things happen to be in the world (i.e. without looking to see which particular objects exist where, and what properties they have, etc.), nevertheless it is possible to verify such a proposition empirically,

just as (cf. section 6.E) it is possible to verify an analytic proposition empirically. Thus, one might carry out a survey of all objects bounded by four plane sides in order to discover whether they also possess the property of having four vertices. Such an empirical justification for the assertion of the proposition is adequate, despite the fact that it is unnecessary.

Chapter Eight

CONCLUDING SUMMARY

This thesis may be concluded with a brief summary, which may be supplemented by section C of chapter one. (See especially 1.C.2, concerning the limitations on the discussion.)

Meaning and truth.

The description of the general connection between meaning and truth (between understanding and knowing) began with some general remarks about the presuppositions of talk about meanings and propositions, and criteria for identity of meanings, preparing the way for much of what followed.

We saw that descriptive words have their meanings in virtue of semantic correlations with combinations of observable properties (or relations), which one must learn to recognize in learning to use the words. A system for classifying such words on the basis of the ways in which they are correlated with universals was described, which enabled us to give an account of many hidden complexities in the meanings of simple-looking adjectives and common nouns.

The role of logical words in sentences was described by developing some ideas of Frege and Wittgenstein. It is possible to regard the logical form of a proposition as corresponding to a rogator, which takes descriptive words and expressions as arguments and takes as values the words "true" and "false": to each logical rogator there corresponds a "logical technique" for determining the value given the meanings of the non-logical words taken as arguments,

the outcome of which generally depends on how things happen to be in the world. In learning the use of logical words and constructions, we learn how their occurrence in sentences determines the logical form of the propositions expressed, by determining which logical techniques or which rogators correspond to those sentences.

Thus, the semantic correlations between descriptive words and universals, together with the correlations between "logical forms" of sentences and logical techniques, determine the conditions in which sentences containing descriptive and logical words express true or false propositions. This is how meanings of statements are determined by the meanings or functions of individual words.

(This inquiry was not without by-products. We found reason to reject the reduction of logic to syntax. We were able to clarify the difference between "presuppositions" of a statement and its "implications", by talking about the conditions in which logical techniques are applicable. This, and the notion of the "domain of definition" of a rogator, looked like a suitable basis for a doctrine of "types" and "category rules" slightly more general and less arbitrary-looking than theories based on "ranges of significance" of predicates. See end of chapter five.)

Meaning and necessary truth.

Making use of some of the early general remarks about meanings and propositions, especially the remarks about conceptual schemes, we analysed some aspects of the concepts of "possibility" and "necessity" by drawing attention to general and fundamental facts of experience, but for which our language and thought could not be as they are, such as the fact that universals are not essentially tied

to their actual particular instances. Necessity was explained in terms of connections between universals which limit the possible ways in which they might have occurred in other instances than those which actually possess them. Such connections between universals may also explain our use of subjunctive conditional statements in some contexts.

The description of the connection between meaning and necessary truth (between understanding and knowing apriori) followed on naturally from the earlier description of the general connection between meaning and truth. Though the value of a logical rogator for a set of arguments normally depends on how things are in the world, and has to be discovered by applying the appropriate technique, nevertheless there are "freak" cases where the truth-value may be discovered by examining the technique and relations between the meanings of non-logical words taken as arguments. (Though even here it may be discovered also in the normal way, by applying the technique.) Relations between the meanings of descriptive words, which may help to determine the truth-value of a statement in all possible states of the world, may either be identifying relations, corresponding to definitions or partial definitions, or non-identifying relations, corresponding to connections between universals (observable properties and relations). Thus there are two sorts of propositions which are necessarily true, namely those which are analytic and those which are synthetic. The discovery of the relations which may make a synthetic proposition necessarily true is made by examining observable properties or relations, possibly with the aid of an informal proof.

(It is assumed throughout that the statements under discussion do have truth-values, that the applicability-conditions for rogators are satisfied. This may not always be discoverable apriori. See 5.E.6, ff.)

All this showed that there were four types of true or false statements using only descriptive and logical words.

- 1) Formal truths and falsehoods, whose truth-values are determined by their logical form alone.
- 2) Analytic, but non-formal, truths or falsehoods, whose truth-values are determined by both their logical form and identifying relations between meanings of some non-logical words.
- 3) Synthetic necessary truths and falsehoods, whose truth-values are determined by the factors mentioned so far, together with synthetic or non-identifying relations between the meanings of some of the non-logical words.
- 4) Synthetic contingent statements, whose truth-values depend on their logical form, on the meanings of their non-logical words, and on how things happen to be in the world (i.e. on which particular objects have which properties, etc.).

In order to know the truth-value of the first kind, it is enough to know how logical constants (topic-neutral words and constructions) work, and perceive properties of the corresponding logical techniques. Of the other words one need know nothing except that they are descriptive words referring to properties.

For knowledge of the truth-value of the second kind, something must, in addition, be known about the descriptive words, such as that some of them are used as abbreviations for other expressions, or that the meanings stand in certain identifying relations. What the meanings are need not be known.

Knowledge of the truth-value of the third kind of proposition requires, in addition to the factors so far mentioned, a complete understanding of at least some of the descriptive words. One must know which properties are referred to, in order to be able to examine them and discover the connections between them.

Finally, not only is complete understanding required for knowledge of the truth-value of propositions of the fourth kind, but also an empirical enquiry to find out how things stand with the particular objects which have (or do not have) the properties referred to. Here knowledge of meanings and logical techniques is applied, where in the other cases it was only examined.

The discussion of section 2.C showed that the failure of many philosophers to see all this could be explained not only by their confused understanding of the terms "synthetic", "necessary", etc., but also by their unwitting use of loose and fluctuating criteria for identity of meanings. They have failed to use Kant's "engraver's needle", partly on account of not having noticed that a theory of universals (properties and relations) need not rely on the oversimplified "one-one" model. (Cf. 2.D.6-7, 3.B.5, 4.B.1, etc.).

This concludes my answer to the main question raised in section 1.1. Many subsidiary questions have been raised which could not be answered in the limited space available - some of these are dealt with briefly in the appendices. I claim to have shown that Kant was justified in describing some kinds of knowledge as both synthetic and a priori,¹ and, which is perhaps more important, to

1. See Appendix VI.

have revealed some relations between very general concepts, such as "property", "meaning", "truth", "proof", "possibility" and "necessity".

A P P E N D I C E S

I.	Singular referring expressions	335
II.	Confusions of formal logicians	340
III.	Implicit knowledge	357
IV.	Philosophical analysis	372
V.	Further examples	381
VI.	Apriori knowledge	386

Appendix I

SINGULAR REFERRING EXPRESSIONS

The discussion in the thesis has hardly been concerned at all with singular referring expressions, that is, proper names, definite descriptions and other words or pronouns which, in one way or another, refer to actual particular objects. The main reason is that in order to illustrate the existence of synthetic necessary truths it is enough to discuss sentences containing words which do not refer to anything besides universals.

There is another reason why it is most helpful to define the analytic-synthetic and necessary-contingent distinctions in such a way as not to apply to propositions which mention particulars, namely to avoid difficulties with such propositions as the following:

- i) The Queen of England is a woman.
- ii) I am speaking now.
- iii) John's uncle is a man.
- iv) He is a male.
- v) Socrates is human. (Searle, D.Phil. thesis, pp. 132-7.)

The difficulty is that in each case there are reasons for regarding the proposition as analytic and reasons for regarding it as synthetic. The reason for regarding each of them as analytic is that its negation looks self-contradictory in some sense. "Queen" surely means "female monarch", so the Queen of England could not fail to be a woman. (Etc.) The reason for regarding each of them as synthetic is that it presupposes the existence of some particular, which it mentions, and so one cannot discover

that it is true merely by investigating the meanings of the words and the logical form: one must know that the particular object exists, as the result of some empirical enquiry.

The same sort of difficulty arises when we consider inferences to propositions mentioning particulars. For example, from (a) "If anyone is an uncle then he is a man" we may apparently infer (b) "If Tom is an uncle then he is a man", and the inference looks as if it is logically valid: certainly it is normally regarded as valid. But then, if (a) is analytic, why should (b) not be analytic too, since anything logically implied by an analytic proposition is itself analytic? One way out would be to say that the inference is not purely logical, since (b) presupposes something which (a) does not, namely that there is a person referred to by the word "Tom", and only one, and this cannot be guaranteed by logic.

Now consider the inference from (b) to (c) "John's uncle, Tom, is a man". Again, the inference looks as if it is logically valid, and is normally treated as if it were, but, as before, the conclusion presupposes something which is not presupposed by the premiss, namely that there is exactly one person who is referred to by "John", that he has exactly one uncle, and that that uncle is named "Tom".

Inferences of this sort are normally regarded as valid because we take their presuppositions for granted, and when this can be done logic suffices to ensure that the conclusion will be true if the premiss is. Nevertheless, the fact that there is the additional presupposition in each case seems to provide a good reason for saying that the conclusion does not state a necessary truth, and is therefore not analytic.

Whether we describe statements like (b) or (c) as "analytic" or not does not matter very much. What is usually of philosophical interest is whether statements of the form of (a) are analytic, or necessarily true. In other cases we can talk about the connections between the meanings of words as being analytic, or, in the terminology of chapter six, identifying relations, without asserting that the propositions which they express are analytic.

There is a further reason for withholding the necessary-contingent distinction from statements mentioning particular objects. Necessity is defined, in chapter seven, in terms of what would be the case in all possible states of affairs; a statement is necessarily true if no exceptions are "allowed as possible". Thus, in order to ask whether the statement "This round box is not square" was necessarily true, we should have to ask whether it would be true in all possible states of affairs, or whether one of them would provide an exception. But the normal criteria for identity of material objects simply do not extend far enough to enable us to decide, in all possible states of affairs, which box, if any, was the same box as the one referred to in the statement in question. Hence we cannot decide in every possible state of affairs whether anything is "this box", and so cannot decide whether the statement would be true in all possible states of affairs. (Indeed, in some possible states of affairs there would be nothing which was "this round box".)

The criteria for identity work up to a point, but only if we try to identify objects in different possible states of affairs which are not too different: there must be a

certain amount in common between them. This explains why we can intelligibly say: "This box might have been red", "If that wall had been painted yesterday it would have been dry by now", and so on. We can talk like this because we do not imagine other things to be very different, which is why criteria for identity work. But they could break down. Suppose the house had been bombed twenty years ago, and the wall rebuilt as before. Would it still have been this wall? And if it had been built of a different material? Or with a different thickness? Or in a slightly different position? ...

Of course, we can, if we wish, treat singular statements such as "This round box is not square" as necessary truths, simply because they are substitution-instances of universal statements which are necessarily true. This is the fact which underlies our use of such idioms as "Tom's uncle has to be a man", "If the box is square then it must be rectangular". More commonly, perhaps, the phrase "not necessarily" is used to deny the existence of a universal statement which is necessarily true of which some statement would be a substitution instance: "Her aunt is not necessarily a spinster". But I shall not apply the necessary-contingent distinction to statements mentioning particulars, for the reasons already given. This does not mean that singular statements are contingent, but that the question simply will not arise. It is of no interest for our purposes, as all interesting questions about necessity can be reduced to questions about necessary connections between universals. (See 7.B.)

It should be noted that I have defined "necessary" in such a way that it cannot always be applied to pro-

positions which mention universals. Thus, the statement "Roundness is incompatible with squareness" is neither necessary nor contingent according to my definition, since it is not clear what could count as an exception to it, unless it means "Nothing is both round and square". The definition of "necessary" in chapter seven is concerned only with what would be the case in all possible states of this world (any world in which the same universals exist as in our world). We can then ask whether in some possible state of affairs two properties would exist in the same object, and if the answer is in the negative, then the properties are incompatible. But there is no further intelligible question whether in some possible state of affairs the answer would be positive! Hence there is no clear sense to the question whether the properties would be compatible in some possible state of affairs. We may be able to invent some sense for the question whether there might have been a world in which these properties were not incompatible, but it would certainly not be a question about all possible states of this world. This illustrates the general fact that there is no question of truths being necessarily necessary or only contingently necessary, in my terminology.

Appendix II

CONFUSIONS OF FORMAL LOGICIANS

II.1. In chapter five I tried to illustrate a way of studying logic by explaining the logical properties of propositions and relations between propositions. In this appendix I wish to show how people may be led astray as a result of a concentration on methods of classifying and describing propositions according to their logical properties and relations to one another. I shall exaggerate some features of this "formal" approach to logic, in the interests of brevity and clarity: it should not be regarded as an accurate historical survey. There is probably no philosopher who has consistently made all the mistakes which I shall describe.

II.2. Many philosophers have thought that logic consists in the study of propositions which are true in virtue of their logical form and inferences which are valid in virtue of their logical form. (See, for example, p. 10 of Russell's "The Principles of Mathematics".) They have not often been quite clear about the distinguishing characteristics of logically true propositions and logically valid inferences, but they do seem to mean to refer to what I call "formal truths" and "formally valid inferences". (See 5.C.6, 5.C.8.)

The first step in such a study seems to be to describe and classify propositions according to their logical properties, or, more commonly, to classify inferences. As shown in chapter five, a convenient way of classifying them according to their logical form is to replace all non-

logical words in the sentences concerned by variable-letters. For example, "All P's are Q's" represents the logical form of "All horses are animals", and the logical form of the inference from "All stallions are horses and all horses are animals" to "All stallions are animals" is represented by "All P's are Q's and all Q's are R's = ergo = All P's are R's", or some such symbol. In this way all propositions or inferences with the same logical form may be represented by the same symbol, and the geometrical or typographical properties of the symbol are supposed to show or represent the logical properties and relations of the propositions and inferences represented by the symbols. (Instead of such symbols, we could, of course, use such words as "disjunctive", "conditional", "universal affirmative", etc., to describe logical forms for purposes of classification.)

II.3. For a long time logicians apparently did little more than give these various forms (or at least the ones they had bothered to symbolize) names, and list those which corresponded to logically valid inferences, or logically true statements, and gradually they came to think they could forget about the propositions and inferences with which they had started and concentrate entirely on the symbols representing their logical forms. They even went so far as to mistake symbols for propositions, with dire consequences, as will be seen.

II.4. Eventually came the discovery that a system of symbols representing logical forms could be regarded as something like an algebraic system, as follows. A few of the (simpler) symbols representing logically true

propositions could be taken as "axioms", and a few, or perhaps even only one, of the symbols representing logically valid inferences could be interpreted as expressing a rule for deriving new symbols from given ones, in such a way that the class of "theorems", that is symbols derivable from the axioms by successive applications of the rule(s) of derivation, constituted the class of symbols representing propositions true in virtue of their logical form.

The discovery of such "formal systems" amounted to the discovery of a recursive method of characterizing a class of symbols which represent propositions true in virtue of their logical form: for the axioms and rules of derivation provide a recursive definition for the predicate "is a theorem".

II.5. Extensive mathematical investigations were carried out of the various ways of characterizing such recursive systems and comparisons were made between different methods of defining the same class of "theorems", and between quite different systems, many entertaining mathematical results being obtained.

Unfortunately, some people mistook this mathematical study for a philosophical one: thus, to show that some symbol was a theorem in a formal system was thought of as a sufficient explanation of the fact that propositions with the logical form represented by that symbol were true. For example, Waismann, in his article "Analytic-Synthetic" in Analysis, December 1949 (pp. 31, 33, 36), implied that correspondence with a theorem of 'Principia Mathematica' or some other text-book of logic was a necessary and sufficient condition for being a logical truth.

Sometimes the construction of derivations of theorems in such a system was thought of as constituting a proof of the propositions whose forms were represented by those theorems. Logicians apparently failed to notice that any class of symbols can be represented by a suitably chosen formal system (though in some cases there may be no rules of derivation and all theorems may have to be taken as axioms), and so they thought that there was something significant about the fact that logical truths could be represented by such a system. (There is something significant, namely that our rules for the use of logical words enable more and more complex sentences to be built up using those words, their meanings being determined by the way they are constructed. But by this time philosophers had forgotten about meanings, having been mesmerized by symbols.)

II.6. The trouble was that the possibility of representing properties of propositions or relations between propositions by symbols in a formal system, together with the geometrical resemblance between such symbols and sentences in a language, led philosophers to think of such a formal system as a kind of language, or even as constituting a part of our own language. At any rate, those parts of symbols which looked like or represented our logical words, such as "and", "or", "not", etc., were identified with our logical words. A formal system was somehow thought of as providing a framework for our language, in which non-logical words or concepts could be embedded. (This attitude is clearly expressed in the writings of Carnap.) Sometimes it was hinted that insofar as any language fell short of this ideal it must be deficient.

II.7. Thus, a language came to be thought of as made up of a set of logical words, whose functions were fully defined by the rules of some formal system, together with extralogical words governed by rules of some other kind. But, since the rules of a formal system are concerned only with the symbols in that system and their geometrical relationships, since they mention nothing extralinguistic, it looked as if the rules for the use of logical words must themselves mention no non-linguistic entities. Thus it was claimed that the rules were purely syntactical. Similarly, since the property of being a theorem in the system was a recursively defined syntactical property of symbols, and since theorems represented logically true propositions, it was claimed that logical properties were purely syntactical properties of sentences. Thus it was thought that logic could be reduced to syntax. I have already argued against this in section 5.A: if logical constants were governed by purely syntactical rules, then they could never have any essential function in sentences expressing contingent truths or falsehoods.

II. 8. If formal systems provide frameworks for languages, and if formal systems have axioms and rules of inference, then surely languages must have them too? So languages came to be talked of as "systems" with axioms and rules of inference of their own. (I have argued that these concepts do not intelligibly apply to real languages, in "Rules of Inference or Suppressed Premisses?" which should appear in Mind soon.) Since different formal systems may have different sets of axioms and rules, and even different classes of theorems, it seemed that languages too might have different systems of logic, and so people quite

happily talked about "the logic" of a given language, or talked of logical truth as in some way relative to a system. (See H. Bunge, in Mind, 1961.)

II.9. The discovery of formal systems, and the possibility of giving recursive characterizations of the class of symbols representing propositions in the system (by "formation rules") in addition to the possibility of recursively characterizing the class of "theorems", sparked off a number of reductive programmes. It was found that the set of logical constants employed in a formal system could be decreased in size without effectively diminishing the number of theorems, since all theorems containing other logical constants than those chosen as primitive could be reintroduced merely by giving "definitions" for the eliminated constants in terms of the constants taken as primitive, and substituting "synonymous" symbols.

Thus the class of theorems and the class of logical constants were "reduced" to subclasses by definitional elimination, and it was thought that a similar procedure could be employed for reducing the concepts and theorems of arithmetic to those of logic. Since logic was thought of by some as merely a matter of syntax, and since it was thought that arithmetic could be reduced to logic, it seemed that arithmetic should be thought of as a syntactical science concerned with the manipulation of symbols in a formal system. Frege brought powerful arguments to bear against such views, but it is not quite clear whether he realized that they could be used against Formalist philosophies of logic as effectively as against Formalist philosophies of mathematics.

II.10. One of the very strong motivations behind the search for formal systems was the desire to find some absolutely rigorous and explicit method of proof:

hardheaded philosophers did not like to talk about "self-evidence". The discovery of formal systems made it look as if the notion of logical proof or justification could be "reduced" to formal derivation from a fixed set of axioms and definitions by means of predetermined rules of inference. The production of proof-sequences and proofs of theorems in formal logic came to be thought of as the paradigm of rigorous argument. Only deduction in a logical calculus could be regarded as strictly valid reasoning.

II.10a. Eventually, some thought that the only way of proving anything was by producing a sequence of statements starting from axioms and definitions and proceeding according to the rules of inference which were supposed to be among the rules of our language. (See II.8, above.) No notice was taken of the fact that there must be some other way of being justified in accepting something as logically true, or as logically following from something else: the question "What right have we to accept the axioms or to follow the rules of inference? How do we know that we shall not be led to affirm false statements?" was not given a proper examination. Thus people tried to "analyse" the actual processes of reasoning which we follow when we are not involved in logical investigations by looking for "suppressed premises" and "rules of inference". They regarded informal proofs, such as the proofs employing diagrams which are used in geometry, as somehow inferior or inadequate: such proofs had to be replaced by something formalized. (Cf. 7.D. ff.)

The logical conclusion of this line of thought is that unless one has worked through a proof of some formal system

such as "Principia Mathematica", one is really not fully justified in believing that three plus two equals five, or in believing that if all wooden boxes are red, and not all wooden boxes are round, then not all red boxes are round, or in believing that nothing can be round and square at the same time. But people are justified in believing these and other things, and completely justified, without having gone through formalized proofs, and this shows once again that there must be some other kind of justification than that given by a formal proof. (I have tried to describe this other justification in section 7.D on informal proofs.) But perhaps, if we have this other kind of justification, then formalized proofs may be superfluous?

II.10b. Even if proof-sequences gave some sort of justification for acceptance of statements corresponding to the formulae terminating them, this justification could not be complete. The point of asserting an arithmetical or logically true proposition cannot simply be to announce that the sentence expressing it is derivable in a formal system from other symbols according to fixed rules. For then asserting such a proposition would be analogous to displaying a chess-board with the pieces arranged in a certain way in order to announce that they are in a position obtainable from the "starting position" via moves in accord with the rules of the game. If asserting the proposition has some further point, then a justification is required for assuming that mere derivability in some system guarantees that it is true, if asserted with this further point: a justification is required for regarding the axioms as true and for regarding the rules of derivation as truth-preserving. (It is forgotten that we knew how

to select logical truths and logically valid inferences before we constructed formal systems of symbols representing them.)

All that a formalized proof can do, is "show" that a sentence has a certain geometrical or syntactical structure, or that its structure is related to other structures in some way. But this is not enough for logic: in addition one must know that when sentences have certain structures, or when structures are related in certain ways, then the propositions expressed by those sentences have certain logical properties or stand in logical relations, and one can learn this only by taking account of the roles in the language of various aspects or parts of sentences, not by looking at formal systems.

II.10c. I do not wish to imply that the construction of a formalized proof can never serve any useful purpose. It can be used to demonstrate that certain syntactical relations (i.e. geometrical relations) hold between sentences expressing certain propositions. That is to say, a formalized proof rigorously, but informally, proves a theorem of combinatorial geometry.

A formalized proof can demonstrate that if logical relations correspond to certain syntactical relations, then those logical relations hold between certain propositions. (It does not show that logical relations correspond to these syntactical relations, nor does it show why they do.) In addition, just as formulae in a calculus can be thought of as providing a useful symbolic representation or "map" of propositions in a language (cf. II.2 and II.4), so can proof-sequences, which pick out certain sorts of "routes" along such a map, usefully

represent patterns of formally valid arguments, for purposes of classification or summary, for example. A formal proof can be a useful guide when one is trying to understand an argument to see whether it is valid or not, but the proof does not make understanding superfluous.

I am not trying to show that such proofs are quite useless so much as to draw attention to some mistaken views as to the purposes which they can serve, and to show how they are connected with other mistakes which arise out of a concentration on the symbols which represent forms of logical truth or logically valid inferences. The argument has been aptly summarized by Frege; (in "Translations" p. 201):

"Apparently we are being tacitly referred to our knowledge of meaningful arithmetic. But if we have a knowledge of meaningful arithmetic, we have no need of formal arithmetic."

Replace the word "arithmetic" by "logic" in this statement, and it will serve as a summary of all my remarks.

II.10d. All this seems to show that there is something fundamentally misguided in the attempt to produce absolutely explicit proofs, except perhaps as an exercise in a branch of mathematics - combinatorial geometry. It shows that there is something wrong with the Leibnizian dream of an ideal language which somehow has its meaning written on its face, so that one can settle questions about truth and falsity by mechanical manipulations alone. It is misguided because, no matter how much is written on the face of a symbol, there will always be something left out: an explanation of what the "writing" means, a description of its function in the language. Semantics cannot be reduced to syntax.

(This does not mean, of course, that there is no such thing as a rigorous proof. It merely means that one kind of analysis of rigour is wrong. See chapter seven, section D.)

II.11. One of the sources of an oversimplified view of logic (logic = syntax) is the selection of a class of "canonical" forms for study. It is obvious that if all possible symbols corresponding to the logical forms of propositions and inferences were constructed in the usual manner by replacing non-logical words in sentences by variable-letters, then many more different sorts of symbols would be obtained than have ever been encompassed within the class of symbolic forms discussed in any one text-book of logic. For example, the following are not usually listed separately by logicians:

- (1) All A's are B's.
- (2) Every A is a B.
- (3) If a thing is an A then it is a B.
- (4) Only B's are A's.

Instead, they represent the whole lot by one symbol, such as

(5) Whatever x may be, if x is an A then x is a B, which is then described as a "canonical" form. (This is similar to the old mistake of thinking that logicians need consider only propositions in subject-predicate form.)

II.11a. Normally the selection of canonical forms is done as a matter of course, following a philosophical tradition originating with some philosopher's (understandably) limited survey. But sometimes an attempt is made to justify the failure to discuss statements or inferences

not in canonical form. Several different sorts of reasons may be offered.

i) It is obviously more convenient to classify only a small class of logical forms than to take all varieties into account.

ii) The omission of forms like (1) - (4) above may be defended by the assertion that any proposition with one of those forms "obviously" means the same as a proposition with the canonical form (5), and may therefore be replaced by it without any loss of generality. (See for example, Quine's remark in "Word and Object", p. 228: "... Such a canonical idiom can be abstracted and then adhered to in the statement of one's scientific theory. The doctrine is that all traits of reality worthy of the name can be set down in an idiom of this austere form if in any idiom.") It may be added that statements in canonical form are clear and precise, whereas other statements are vague, ambiguous, or unclear. Moreover, if anyone wishes to use one of the other forms with a clear meaning he may do so by redefining it in terms of the symbols and constructions employed in the canonical forms. (Cf. "Word and Object", p. 188.)

iii) Finally, it seems to be thought, sometimes, that certain canonical forms are most suitable for representing logical properties of propositions. For, if written in these forms the sentences may more effectively "show" their logical properties and relations. ("... the inner connection becomes obvious ...": Wittgenstein, "Tractatus" 5.1311.)

II.11b. There is certainly no reason why, if we find it convenient, and if only some facts of logic interest us, we should not select only a subclass of the whole class of

logical forms for purposes of study and systematic representation in symbols. But the choice of such canonical forms must always be, at least to some extent, arbitrary, depending on such subjective factors as what interests us, or what we find "obvious" and therefore not worth recording in our symbolism.

Why should we regard the following as different forms, whose logical equivalence is worth recording,

(5) Whatever x may be, if x is an A then x is a B ,

(6) There is no x such that x is an A and x is not a B ,
while the equivalence between (5) and, for example,

(4) Only B 's are A 's,

is regarded as "trivial" or "obvious", or "merely linguistic", or "merely a matter of meaning"?

Surely the equivalence between (5) and (4) is as much in need of study and explanation as the equivalence between (5) and (6)? What should we say if someone turned up who claimed to find the latter "trivial, obvious, and merely a matter of meaning" while the former equivalence was tremendously important for him? If some proposition of the form (5) turns out to be true in virtue of its logical form (e.g. if the same predicate is substituted for both " A " and " B ") then why should this be any more interesting or important than the fact that the corresponding proposition of the form (4) is a logical truth?

II.11c. What I am driving at is this: if any explanation of the logical truth of, or logical relations between, propositions makes use of the fact that they are expressed by sentences in canonical form, then the explanation points to something inessential, for logical properties and relations of the same kind are found amongst propositions

expressed by other sentences. If there is some notation which renders certain logical connections perspicuous, a symbolism in which the logical properties and relations of propositions "show" themselves any more obviously than they do in other notations, then this is merely an interesting fact about that notation and its effect on us, and does not reveal any general truth about the logical properties of propositions.

After all, propositions expressed in this notation do not as obviously "show" their logical connections with propositions expressed in other notations, nor the connections between those other propositions. This is because the function of a sign is not generally shown by that sign, though if we know the functions, then, in some cases, we may more easily be able to see the consequences of their having these functions than in other cases.

II.11d. Whether propositions are expressed by sentences in canonical form or not, their logical properties and relations are due to the fact that they are built up in certain ways with logical words and constructions which have been given functions of the sort described in section 5.B. The connection between logical properties and relations of propositions and geometrical or syntactical properties and relations of sentences is a consequence of the fact that the logical constants have the functions which they do have in determining the conditions in which propositions are true or false. It may help someone to see that a proposition has certain logical properties or stands in certain logical relations to other propositions by "rewriting" it in canonical form, but pointing to the geometrical features of the new sentence does not explain

the logical properties of the proposition expressed by the old one: that has to be done by talking about their functions.

II.11e. It is a failure to see this that sometimes leads philosophers to talk about the "real" logical form of a proposition as apposed to its apparent logical form, shown by the grammatical form of a sentence. But the logical form of a proposition is the way in which its truth-conditions are determined by the meanings of the non-logical words, and that must be quite correctly shown by the sentence itself, for otherwise we could not understand it properly, and we do understand our ordinary sentences whether they are in canonical form or not. The reason why we find some types of sentences misleading is that we are philosophers who have swallowed a short-sighted traditional philosophical doctrine and fail to see a counter-example to that doctrine for what it is. (Or if we are not philosophers, then we find certain forms misleading only because we fail to think clearly and allow ourselves to be convinced that because an analogy or comparison works in some cases it must work in all.)

II.11f. Finally, in connection with canonical forms it should be noted that the tendency to regard formalized proofs as having some kind of exalted status is quite analogous to the tendency to regard some forms of propositions as somehow "superior" from the logical point of view. We think of these special kinds of proofs, or deductions, as having a "canonical form" in which logical relations are most efficiently demonstrated. Having noticed that these proofs convince us, we fail to ask why

they should do so, or why they alone should do so.

II.12. I have tried, in this Appendix, to carry out a very brief survey of some of the mistakes and confusions which arise when philosophers restrict their attention to the forms of sentences and neglect the functions of words and constructions. Partial description is mistaken for complete explanation, largely because a formal system, which is a device for representing certain features of propositions, is thought of as containing propositions, owing to the physical resemblance between its formulae and sentences in a language. The study of methods of representing facts of logic, and classifying them, leads to a mathematical study of various methods of recursively defining a class of combinations of symbols, and this study, which is really a branch of Geometry, is mistaken for a philosophical study of logic or truth or inference. The concepts invented for the purposes of such mathematical studies are mistakenly assumed to have some philosophical application: geometrical concepts referring to shapes of symbols and their interrelations are employed by philosophers when they should be talking about the functions of symbols and their interrelations. (Analogous mistakes are sometimes made by physicists, when they assume that concepts which apply only to mathematical models also have application to the reality which these models are supposed to represent. But such mistakes are less frequent because there is, fortunately, no physical resemblance between the symbols used by mathematicians and the things which physicists take them to represent.)

I think that I have been drawing attention to some of

the facts which led Wittgenstein to complain:

'Mathematical logic' has completely deformed the thinking of mathematicians and of philosophers, by setting up a superficial interpretation of the forms of our everyday language as an analysis of the structures of facts. Of course, in this it has only continued to build on the Aristotelian logic.
("Remarks on the Foundations of Mathematics",
IV-48.)

The discussion of this appendix and chapter five (especially sections 5.A and 5.B) may be construed as an attempt to sort out the geometrical from the philosophical questions.

Appendix III

IMPLICIT KNOWLEDGE

III.1. Throughout the thesis I have been making remarks about things which must be known by persons who use words to make statements. But I have often qualified them by saying that such knowledge need not be explicit. In this Appendix I wish to describe some examples of what I call "implicit" knowledge, and explain why it is possible to talk about knowledge in such cases. I shall not be able to deal with the subject thoroughly or systematically, and will content myself with a few disorganized remarks. It is important to clarify the notion of implicit knowledge if we are to be clear about philosophical analysis and the nature of analytic propositions.

III.2. First I shall give a list of examples of the sort of thing I mean to talk about.

(a) In his article "Philosophical Discoveries" (in Mind, April 1960) Hare talked about some persons who all know how to do a certain kind of dance but are unable to be sure about the correct description of the way the dance goes until they actually try to do it, and he compares this with knowing what a word or expression means without being able to say what it means or how it is used.

(b) Another example is provided by a person who wishes to mention the fact that he has recently seen someone, but cannot, for the moment, recall his name. He may say: "Of course I know it - it's on the tip of my tongue."

(c) I know a tune very well, and can recognize it as

soon as I hear it, but try as I will, I cannot, for the moment, sing it or even imagine how it goes. (But if you sing the first two bars, I may be able to carry on from there.)

(d) I know a tune and can recognize it on hearing it, but if someone writes it out I may not be sure whether he has written it out correctly, until I play what he has written on the piano.

(e) I am familiar with a face, or a style of painting or musical composition, yet quite unable to say how I recognize it. I cannot say what it is about the face, or style, in virtue of which I recognize it and distinguish it from others. Even when confronted with the face, or an example of the style, I may be unable to describe the distinguishing characteristics.

(f) A person who can type very easily, even when blindfold, may find it very difficult to describe from memory the relative positions of the keys on the typewriter.

(g) A person tries to describe everything in a room he has just left, and is sure he has left out nothing. Then someone asks: "Was there a carpet?" He replies: "How silly of me! Of course there was a carpet, and I knew that very well. I don't know why I didn't think of it."

(h) We can all count, and can tell, given any numeral written out in English or in Arabic notation, which is the next one in the series. But most persons who can do this cannot give a general formulation of the principle for going from one to the next, despite their ability to apply the principle. (Cf. 6.E.5, 6.E.6.) Even if someone else offers a formulation, they may not be able to think clearly

enough to tell whether it is correct or not.

III.2.a. This should be compared with some of the following facts mentioned in the thesis.

1) I asserted that talk about meanings presupposes the existence of criteria for identity of meanings, at all levels, in section 2.B; then, in 2.B(note), I allowed that people who talk about meanings need not explicitly know which criteria they are relying on.

2) In chapters three and four I described various kinds of correlations between words and properties which explain how we use descriptive words. But one need not be able to formulate explicitly the principle on which one decides whether to call objects "horses" or not. One may use a word according to a complicated procedure, and yet not know in an explicit way what that procedure is. (Cf. 3.D.9.) One may use a word according to several different rules superimposed in an indeterminate way, without realizing this until (e.g.) one starts thinking about difficult borderline cases. (Cf. 3.E.2, 7.D.11. note, and 4.B.2.)

3) In 5.A.3 and 5.A.11 I described techniques which we have to learn to use for discovering whether statements using the words "is", "or" and "all" are true or not. A person must know what the technique is in order to understand sentences using the words: but he need not know in an explicit way, for he may be unable to distinguish between correct and incorrect formulations of the rules for the words. The techniques may be learnt by example and memorized, without any explicit description ever being formulated by pupil or teacher. (5.A.6, 5.B.8, 5.C.9.)

III.3. Each of these examples is puzzling. In each case we want to say that there is something a person knows all the time, or really knows, despite his inability to give a correct answer to a question about it. He knows what he is doing, that something is the case, how to do something, what something is, etc., and yet, without deliberately deceiving, gives the impression of not knowing. What do we mean by saying that he really knows? What explains his inability to answer correctly in these cases?

I believe that the answer to these questions is given by the fact that there are a great many different tests for knowing any one thing, and passing any one of them counts as a sufficient justification for the claim to know, or the assertion that someone knows, provided that there is no reason to think that success in the test can be explained as an accident or some kind of lucky guess.

The fact that I can type correctly without looking justifies my claim to know the relative positions of the keys on a typewriter, despite the mistakes in my attempted description of their positions. I know where my pen is because, as soon as I need it, I go straight to the right place, despite the fact that if someone had asked me where it was I might not have been able to answer correctly. I know how the features on a face are arranged because I can recognize the face and distinguish it from other faces despite my inability to describe the peculiarities in virtue of which I recognize it. I know what the technique is for deciding whether a statement of the form "x is P" is true or not, despite my inability to formulate the technique, since, when confronted with such statements, and told the meanings of the non-logical words, I am able to decide whether they are true or not.

III.4. There are many different tests for a person's knowing any one thing, and his passing any one of them counts as strong evidence that he knows. But there is no one of them that he must pass in order to show that he knows: when he fails one of the tests this need not count as strong evidence for his not knowing, since it may often be assumed that there is an explanation for his not passing the test. (E.g. for his giving the wrong answer or not being able to think of the right answer.) Some of these explanations of an apparent lack of knowledge are the following:

First of all there is a whole family of cases which need not be discussed in detail. A person may quite sincerely give a wrong answer despite his knowledge of the correct one, simply because of a slip of the tongue, or on account of his being absentminded, or preoccupied with something else, or because he has misheard the question, or because giving the correct answer requires concentration and he has a headache. All these are cases of temporary muddle or confusion, and can usually be detected by asking the person again, in a suitable tone of voice!

III.4.a. Next there is the relatively uninteresting case where the person is unable to express his knowledge in words simply because he does not know any words which could express it adequately, either because he hasn't learnt any or because he cannot think of them at the moment. A person who knows the difference between the sound of a clarinet and the sound of a flute may simply not think of saying that the former has more upper harmonics, or that the latter is "purer" or "more naive", or "less reedy".

Connected with this is the case where a person is

not able to give the correct answer to a question simply because he has not thought of that answer as a possible one. But as soon as it is suggested he recognizes it as correct, and if several are suggested he can pick out the correct one from among them. His passing this test (perhaps in addition to his displaying his knowledge in his behaviour) shows that he "really knows", despite his failing the more difficult test of having to think up the right answer for himself. (Compare: in philosophy the difficulty often lies in thinking up the correct answer, not in seeing that it is correct, once stated.) (Compare example (g) in III.2.)

III.4.b. In some cases, a person may be unsure whether a description is correct or not, not because he has not noticed the possibility, but because it describes from a new point of view. I may know how to recognize a tune by its sound, and yet be unsure about recognizing it when written down, despite the fact that I can read music. (See example (d).) I may, in addition, be unable to recognize it when played backwards or upside down. I may recognize the feel of something (e.g. a familiar chair) and yet be unable to distinguish it from others by the way it looks: and for this reason I may be unsure about the correct description of the way it looks. I believe this may apply to one's knowledge of how to do something, such as a dance, or touch-typing. (See examples (a) and (f), of III.2.) A person who is able to tell whether he is doing a dance correctly or not (he knows which movements he is supposed to make, and he knows which movements he is making), may be quite unable to be sure whether other persons are doing it correctly when he watches them. Similarly, if he is given a description of the

dance from the point of view of a person watching its performance he may not be sure whether it is correct or not. He may find out by making the movements and looking to see whether they fit the suggested description or not. In that case he is observing himself in two ways at once, or, more accurately, he knows in two different ways what he is doing.

III.4.c. A slightly different case is the following: a person may know how something is done in the sense that he can do it in an unthinking way when he has to, and yet when he stops and thinks about it he may not be sure. For example, an experienced flautist may be able to play a descending chromatic scale at high speed on the flute, but if his fingers are placed in the position for one note and he is asked to indicate which must be moved up and which must be moved down for the next note down the scale, he may have to pause and think for some time, especially if he is not actually holding a flute. He may find it even more difficult to say which fingers must move if he is not first permitted to move them. This fact, that he cannot be sure about the individual steps of a routine which he knows how to apply quickly and unthinkingly, may help to explain his difficulty in telling a pupil which fingers to move and when, in addition to the factor already pointed out, namely that he may be able to recognize the correct movements from the point of view of an agent while being unsure about them from the point of view of an external observer.

III.4.d. Yet another possible explanation of a failure to give the correct answer is the fact that one may be able to apply a technique which is very complicated,

insofar as it has several stages, or insofar as exactly how it goes at any stage may depend on other things. For despite one's ability always to go on from one stage to the next correctly, even where how one goes on depends on what the technique is being applied to, one may not be able to think about all the stages, or to recall all the possible variants, when one is trying to describe the technique in the abstract. For example: a chemist who has been trained to identify samples of some substance, which can occur in several varieties, for which the tests are slightly different, may make a mistake in describing all the tests, despite the fact that he never errs in performing them. Similarly, we may find it difficult simply to sit back and describe all the observable factors which we have learnt to take into account in deciding whether an object is a horse or not (or in deciding whether a person has an intention to do something or not), despite our ability to make the decision when necessary.

In the previous case (III.4.c.) a person who could recognize correct applications of a complete routine might be in doubt about individual stages of the routine. In this new case a person who is sure about any one of the stages when asked may be in doubt if asked simply to describe them all. The complexity of the routine explains his failure.

III.5. I said that in each of these cases a person may be described as knowing something or other despite the fact that he fails some test for knowing. The reason why we do not take the failure as a criterion for his not knowing, is that we are able to explain the failure in some other way than by saying that he doesn't know. This

is supported by the fact that in most cases he may be able to pass the test later on without having gone through a process of acquiring the relevant knowledge inbetween.

For example, the person who gives the wrong answer because he is absent-minded simply has to think again. He need not learn again.

When I have a person's name "on the tip of my tongue", and correctly pick it out from among several suggestions offered to me, all I needed was to hear the name to bring it back to mind: in such a case I do not learn that that is his name.

The person who cannot describe the way a dance goes until he does the dance does not thereby learn how the dance goes. What he learns is how to describe the dance from a new point of view.

The person who cannot describe something with which he is perfectly familiar because he must first learn the appropriate vocabulary is not thereby acquiring the knowledge which he is able later on to express in his new vocabulary. Learning the name of a colour is not the same thing as learning that that is the colour of my table, even though it may enable me for the first time to say correctly what the colour of my table is.

To summarize: many different sorts of things may enable a person to know in an explicit way something which he previously knew only implicitly. We say in such cases that he nevertheless "knew" previously because the process in which he learns to express his knowledge or to pass one of the tests which he previously failed, is not the same sort of process as is required for acquiring the knowledge. (He does not make the relevant empirical observations, or

examine properties to discover their inter-relationships.) He is all the time potentially able to pass the test, and the evidence for this, apart from his later success, is the fact that he was previously able to pass some other test which displayed his knowledge (cf. III.3.). (I shall not discuss the question how passing one test counts as evidence of ability to pass others.)

III.6. All this may help to explain why I was able to describe some of the things which people know when they know how to talk, without fear of being contradicted by the fact that my descriptions would probably come as news to many people who know how to talk! These people know how to talk, but they do not know explicitly that their descriptive words have the meanings which they do have in virtue of being correlated with observable properties as described in chapters three and four, and neither do they know explicitly that to the logical form of a proposition there corresponds a technique for determining truth-values for sets of non-logical words by examining the way things happen to be in the world. Their knowledge of all this is implicit, and to say that they know it all is to say that it accurately describes what they are doing when they decide whether to use some word to describe an object, or whether a sentence expresses a true proposition, or whether two persons understand some sentence in the same way. Their not knowing explicitly may be explained by factors of the sorts described already.

But there is one sort of factor which can be very important, and which I have not yet discussed; I shall do so now.

III.7. A possible explanation of a person's not knowing that in doing A he is doing B, C, D ... may be the fact that he does not have the concepts which would enable him to think about his activity in this way. More specifically, he may not have the metalinguistic concepts which would enable him to think about and describe the way he uses words. For example, the child who can use the word "cat", but does not have the concept "word", can hardly be expected to know explicitly how it uses the word "cat". Similarly, the explanation of a person's inability to say how he uses the word "horse", for example, may not only be that the procedure for picking out horses is too complex (see III.4.d and also 3.B.5 and 4.A.6), but in addition that he does not have the concepts "meaning", "property", "semantic correlation", "disjunctive range", etc.: he does not have the concepts which I have used in my description of what people learn when they learn to talk. (Similarly, philosophers have hitherto been unable to give a correct explicit account of analytic propositions, despite their implicit knowledge of what it is for a proposition to be true by definition, on account of not having something like the concept of a "rogator" [see section 5.B], or so it seems to me.) A person may acquire the concepts which enable him to know explicitly that he uses his logical words or descriptive words in a certain way, without actually being taught that he uses them in that way. He thereby learns to say what the words mean, but he does not learn what they mean - he knew that all along, since he could use and understand them.

III.7.a. Now we can see how to cope with the difficulty mentioned in 6.C.4. I defined "identifying fact about the meaning of words" to mean "fact which must be known

by anyone who knows the meanings of those words". The difficulty was that someone might learn to use the word "gleen" to refer to the combination of the properties which are referred to by the English words "glossy" and "green", without knowing explicitly that the property referred to by "gleen" was the combination of the properties referred to by "glossy" and "green". He might not know the meanings of the words "glossy" and "green", and he might not have the metalinguistic concepts which would enable him to understand a statement about the meanings of descriptive words. Still, he knows the fact in question implicitly, since he makes use of it in employing the word "gleen". He decides whether to describe an object as "gleen" or not by looking to see whether it has the two properties in question. If he decides in any other way he does not understand the word "gleen" as I have described it: if he does not know, even implicitly, that it is correlated with those two properties, then he does not know its meaning. (This is still not quite clear: a complete discussion would require an investigation and comparison of the following expressions: "Knowing what the word 'W' means", "Knowing how to use the word", "Knowing the meaning of the word", "Knowing that the word 'W' means ...", "Being able to understand the word 'W'", and so on.)

III.7.b. It is important to distinguish the acquisition of new metalinguistic concepts, which enable one to express one's knowledge of facts about the meanings of words, from the acquisition of other sorts of concepts, which enable one to discover new facts about the meanings of words, facts which one did not know previously even though one understood those words. Consider, for example, the concept of a "starlike" figure, introduced in 3.D.5. A

starlike figure is one which is bounded by straight lines meeting alternately in reflex and acute angles. Now a person may be able to use the word "square" to refer to the usual recognizable property, without having the concept "starlike". I may teach him the new concept by giving a definition, or showing him examples, without mentioning squares at all. Having acquired the new concept he may then notice, for the first time, that no square is starlike. He discovers a new fact about the property referred to by the word "square", and thereby learns that the words "square" and "starlike" are incompatible descriptions. But he does not thereby learn an identifying fact about the meaning of the words, for in order to acquire this knowledge it was not enough for him to acquire new metalinguistic concepts.

Consider another example: I may know how to use the expression "Daisy-daisy" as the name of a tune, and be able to recognize the tune on hearing it, without knowing, even implicitly, that the first and second intervals of the tune are thirds (first a descending minor third, then a descending major third), or that the first three notes form a major chord, on account of not having the concept of a musical interval, or a major chord. Suppose someone teaches me to pick out musical intervals and name them, and then one day I hear the tune I knew previously, and notice immediately that the first two intervals are both thirds. Have I acquired explicit knowledge of something I knew previously in an implicit way? Surely not. It seems much more reasonable to say that I have discovered a new aspect of the tune; I had not previously noticed the possibility of looking at a tune as a sequence of musical intervals, and I in no way made use of the

possibility, implicitly or otherwise, for I could only have done so if I had had the appropriate concepts. (How can a person who is unable to tell whether two intervals are the same or not ever make use of the sameness of two intervals? It should not be forgotten that this is a phenomenological essay: I am not interested in what would be given as a causal explanation of how he recognizes the tune. See section 1.B.)

III.7.c. This difference between acquiring metalinguistic concepts which enable one to say explicitly how one had previously been using words, and acquiring other sorts of concepts which enable one to discover new facts about the meanings of one's words, or about the properties to which they refer, is one of the factors which lies behind the distinction between an identifying (or analytic) fact about meanings and a non-identifying (or synthetic) fact about meanings, of which so much use was made in chapter seven. This is what justifies talk about synthetic necessary truths (whose necessity has to be discovered by examining properties, perhaps with the aid of informal proofs - see 7.D), and shows that the term "synthetic a priori" is not just an old label with little explanatory force, as averred by Hare, on p. 145 and p. 153 of "Philosophical Discoveries".

III.8. To sum up: there are many kinds of things which one may know implicitly without being able to express the knowledge in words, or to answer questions about it. This inability may be explained in any one of a number of different ways. It may also be removed in a number of different ways, none of which involves actually acquiring

the knowledge in question: they all involve merely learning to express the knowledge in a new way. We say, in such cases, that one knows, despite the inability to express the knowledge, because one is able to use it, in applying a technique, in carrying out a routine, in making allowance for facts, etc.

I have tried to distinguish cases where implicit knowledge of how words are used is made explicit, from cases where a new discovery is made, where something new is discovered about previously familiar meanings, namely a connection with other meanings or some other previously unnoticed aspect.

(Some more remarks about implicit knowledge will be found in Appendix IV on "Philosophical Analysis". See also remark about "implicit justification" in 7.E.5.)

Appendix IV

PHILOSOPHICAL ANALYSIS

IV.1. It was suggested in 1.A.5 that clarification of the analytic-synthetic and necessary-contingent distinctions might help to solve problems about the nature of philosophical analysis, and perhaps lead to methodological advances. Not much has been said on this in the thesis, and in this appendix I shall make a few vague remarks, in the hope of suggesting lines for more detailed investigation.

IV.2. Philosophical analysis, which may also be described as "conceptual analysis", is essentially the search for identifying relations between the meanings or functions of words or sentences or types of linguistic constructions. (See section 6.C, and 6.F.7, ff.) This is what seems to lie behind such questions as:

- (a) Can one be pleased without being pleased at or about anything?
- (b) Is there something queer in the assertion "I have definitely decided to go, but I am sure I shall not"?
- (c) Is it part of the meaning of 'table' that tables are used in certain ways, or have certain functions, or is it just an additional fact about tables?
- (d) When a person asserts that something is the case, does he imply that he believes that it is the case?

In answering questions like these one is presumably drawing attention to connections between concepts or features of concepts, and this means drawing attention to identifying facts about the meanings or functions or uses of words or other expressions. From the discussion of section 2.B

and 6.F.7,ff., it should be clear that there are many different "levels" at which meanings or functions can be related, any of which can explain the existence of implications between utterances, or the queerness or oddness of certain utterances (self-contradiction, or analytic falsity, is just one sort of case). It would seem to be important to devise some principle for systematically classifying such identifying relations, if philosophical analysis is not to look like the piecemeal collection of linguistic oddities.

IV.3. For example, there are connections between the describability-conditions of descriptive words, between the techniques of verification corresponding to logical forms of sentences, between the purposes served by the utterance of statements, between the preconditions (existence of social habits or institutions or empirical regularities in our physical environment) for the efficacy of certain sorts of utterances. In all these cases there may be identifying relations at one level, or relations between one level and another. Thus, if one of the conventions of a language is that the utterance of a statement of the form "I intend to do X" primarily serves the purpose of giving people the assurance that X will be done, then identifying relations between appropriateness-conditions for this sort of utterance and truth-conditions for the utterance of statements of the form "I believe that I shall not do X" may generate a kind of queerness manifested in the utterance of "I intend to do X but I believe that I shall not do it". The queerness is to be accounted for, not by conducting empirical observations of what people can believe and intend, but by examining

the knowledge of meanings which we normally apply when we talk, just as we account for the necessary falsity of analytically false statements. (See section 6.E.)

The difference is merely that in the latter case we are concerned only with truth-conditions.

I shall not now try to describe a system for classifying such identifying relations between meanings and functions and the consequences which they can have.

(It may turn out that we must also allow for the possibility of non-identifying relations between meanings, analogous to those relations between universals which were described in sections 7.C and 7.D.)

IV.4. Now it would certainly be of some interest to see what sort of system could be used for classifying various sorts of connections between meanings, and the consequences of such connections. This would amount first of all to an extension of the Aristotelian classification of forms of inference, of far greater significance than the mere extension to take account of more varieties of logically valid inference (the much-vaunted achievement of modern symbolic logic), and secondly to an extension of my explanation (in chapters five and six) of the existence of logical relations and properties of propositions. But this interest in principles of classification can certainly not explain why philosophers should debate with such great interest questions of the kind illustrated in IV.2.

(System-building is, after all, supposed to be out of fashion.) Why are they interested in finding out whether it is actually part of the meaning of "table" that tables have certain functions, and whether the English words "red" and "green" are actually analytically incompatible, instead of merely noting that there are these possible linguistic

conventions, and that adopting them would have certain consequences?

IV.5. Perhaps some philosophers are simply interested in empirical questions about how people use words, and these questions are not entirely trivial, despite the fact that we may already know how the words are used. For our knowledge may be implicit, and, as pointed out in the previous appendix, there may be some difficulty in expressing such knowledge explicitly, especially if superficial analogies lead us mistakenly to expect that the answers will be of a certain kind. (Cf. App. II.11.e, above.) But the best way to serve this sort of interest in empirical questions is to carry out empirical surveys (e.g. using statistical methods), and it would be important to allow for the possibility that in general there will be no definite answers to such questions, since how a word is understood may vary from person to person, and even an individual may understand it in an indefinite way. (See the discussion of indeterminateness in chapter four, and also 3.E.2, 5.E.7.a, 6.D.3, 7.D.11.(note).)

IV.6. However, most of those who indulge in conceptual analysis are not inclined to use the methods of popularity pollsters, and this is not merely a matter of laziness: they are not really interested in how people actually talk, though their words often seem to belie this. What else are they trying to do then? What sorts of non-empirical questions can they be trying to answer?

One kind of non-empirical question is the question whether a philosophical distinction is vacuous, or whether a philosophical system of classification works in this way

or that, or not at all. Thus, a philosopher (Kant) who has begun to describe a system of classification of the sort envisaged above, might try to illustrate its application by assigning particular examples to their place in it, and his opponents might dispute that his principles of classification have been correctly applied in these particular cases. He says the statement S has certain features in virtue of which it satisfies the conditions which he has laid down for being synthetic. They say that it does not. But now there comes a confusion between the question whether S has those features as it is in fact understood, the question whether having those features entails satisfaction of the conditions for being synthetic and the question whether it is possible for any statement to satisfy those conditions. Compare the following case. A mathematician tries to prove that any geometrical figure which has the property P also has the property Q, and he does so by drawing a diagram with construction-lines, etc. (See section 7.D.) Now if the case is sufficiently complex there may be a debate as to whether the figure which he has drawn actually has the property P, or whether it actually has the property Q, or whether the construction-lines as he has drawn them actually serve the purpose they are meant to serve. These (semi-empirical) questions about the particular diagram may then be confused with other questions about properties, such as whether they are necessarily connected, or whether it is possible for objects to possess them at all, etc.

Thus, what really underlies an interest in an apparently empirical question about how words are actually used is an interest in a non-empirical question as to whether and how a system of classifying possible ways of using words may be applied. But the failure to distinguish

the empirical from the non-empirical questions may lead philosophers into endless disputes about "What we really mean" - endless because what we really mean is too indeterminate for either side of the dispute to be correct. (This seems to me to be clearly illustrated by disputes as to whether "I know I am in pain" is odd, disputes as to the connection between expressions of intentions and predictions of one's future actions, and disputes as to whether it is part of the meaning of "good" that certain types of men are good men or whether it is part of the meaning of "good" that believing certain types of men to be good men is connected with being inclined to behave in certain ways. In the last case, not only are empirical and non-empirical disputes confused, but, in addition, practical disputes about how we ought to use the word "good" are mixed in too.)

The existence of this sort of confusion is what led me, in 2.C.10 and 7.C.8, to stress the fact that even if it is established that in English the statement "All triangles have three sides" is analytic, this does not close the question whether it is possible to use the word "triangle" to refer to the property of having three angles in such a way as to make the sentence "All triangles have three sides" express a synthetic necessary truth. We were not concerned with the question whether some statement in English is actually synthetic, but with the question whether it is possible for certain sorts of sentences to express synthetic propositions.

IV.7. We have so far found that conceptual analysis can serve the following purposes: 1) It can provide empirical reports on linguistic usage, by making our implicit knowledge of how we talk explicit, though this purpose might

be better served by conducting statistical surveys.

2) It can be a disguised account of the workings of some system of classification of kinds of relations between meanings and functions of linguistic items, and their consequences. 3) The discussion of the way in which particular words are actually used may serve to illustrate or clarify or provide counter-examples to general statements about systems of classification (or about possible ways of using words) in much the same way as the particular diagram used in an informal proof can enable one to see the truth of some general statement about geometrical properties, or provide a counter-example.

IV.8. However, this may make it look as if philosophical analysis is concerned only with language and theories about language, but that is not so. For a conceptual analysis, in drawing attention to previously unnoticed facts about the ways in which we actually use words, serves also to draw attention to a possible way of classifying the things referred to or described by these words: types of material objects, states of mind, kinds of behaviour, etc. It may draw our attention to a system of classification with which we are all familiar in one way, since we employ it all the time, though in another way it has general features of which we are unaware, for the sorts of reasons described in the previous appendix. ("So that's what I've been doing all the time - I'd never have guessed" may be the expression of having made a philosophical discovery!)

IV.8.a. What interests the philosopher, however, is not so much the fact that we do classify things in this way or in that way, as the fact that it is possible to

classify them in one way or another. (This may be important in dispelling philosophical prejudice as to what it means for consciousness to exist, for example.) The interest in possible ways of classifying things need not be fed only by analysis of concepts which we actually employ: the scientist or philosopher may draw our attention to new possible ways of classifying things, perhaps by teaching us to use new concepts.

(A very interesting and difficult question, which underlies much of Wittgenstein's discussion in "Philosophical Investigations" is the question whether and to what extent the possibility of adopting certain systems of classification depends on what is actually the case in the world. In a more specific form, this becomes the question whether I have been right in saying that the existence of observable properties does not depend on which particular objects actually have those properties. (See 2.D and 7.A.) Would it make sense to talk of the size or shape of objects if everything were constantly changing in size and shape? Colours? Etc.)

IV.8.b. The wish to understand the world, in a philosophical way, is, at least partly, constituted by the wish to know how things in the world may be classified. (Can we divide the world up into material entities and mental entities, or are there only material entities, some of which have certain observable properties while others look and act differently? Are there electrons and other subatomic particles out of agglomerations of which the other things in the world are formed, or must the place for electrons in our scheme of things be explained in terms of ways of classifying observable macroscopic phenomena

such as flashes on fluorescent screens and clicks in geiger counters?) The wish to know how things may be classified may be satisfied by finding out the answers to questions of the form "Can we say that ...?" "If such and such had been the case, might we have said that ...?" "Can the word so and so be applied in such and such a sort of case without changing its meaning?" Indeed, this may be the only way of making satisfactory progress. But the fact that these questions are explicitly about words disguises the fact that they are implicitly about things. This is why the criticism that so-called "linguistic philosophy" is just a diletantish enquiry into empirical facts about linguistic conventions, merely misses the point. (This misunderstanding is excusable, however, since its practitioners often miss the point themselves. This, like the rejection of the possibility of synthetic apriori knowledge, is one of the manifestations of the neurotic fear of doing anything resembling old-style metaphysics.)

IV.9. To summarize: in addition to the purposes listed in IV.7, above, philosophical analysis can also, in an indirect way, help to provide answers to very general questions about the world of our experience, which, according to the Pocket Oxford Dictionary, is the business of philosophy.

Appendix V

FURTHER EXAMPLES

V.1. In section 7.C the notion of a non-identifying relation between the meanings of words was illustrated by means of geometrical examples. Geometrical properties are not the only ones which may stand in synthetic relations, but they give the assertion of the existence of synthetic necessary truths its strongest support. Some additional problematic examples will be mentioned now, but not discussed in any detail.

V.2. Our first examples involve colour-concepts. Discussions of the relations between colours or between colour-words can be very confused if the distinctions made in chapter three are not taken seriously. Thus, we have seen that one and the same colour-word "red" may have different sorts of meanings depending on whether it is an f-word directly correlated with a hue (see 3.A.1), a d-word disjunctively correlated with a range of specific shades of colour (see 3.B.2) a p-word correlated with a range of specific shades picked out by some procedure (see 3.D.2), or a word correlated with the disposition of normal persons to say "red" (see 3.D.2.note). The words given their meanings in these various ways may have the same extension. So our first set of problematic examples come from questions of the form: Is it analytic that everything which is red in one of these senses is red in another of these senses? (This is a question which cannot be asked in ordinary English, since the word "red" as ordinarily understood has neither one of these meanings nor another, and our

ordinary vocabulary does not make provision for distinguishing these several senses easily. See 3.E.2.)

V.2.a. Now suppose that the word "scarlet" is an f-word, referring to just one specific shade (a shade of red). Then we can ask whether the sentence "All scarlet things are red" expresses an analytic proposition. Owing to the indeterminateness of the meanings with which words are normally understood, this question probably has no answer (see section 6.D). By distinguishing various possible (sharply identified) meanings of "red" we can understand the question in such a way that it has an answer. Thus, if "red" is a d-word disjunctively correlated with a range of specific shades, of which the shade referred to by "scarlet" is one, then the sentence in question expresses an analytic proposition.

On the other hand, if "red" is an f-word simply correlated with the hue redness (3.A.1), then the meaning of "red" and the meaning of "scarlet" can probably be identified independently of each other: one could learn to recognize the specific shade without being able to see the hue, and one could learn to recognize the hue without ever having seen the specific shade. It follows that if there is a necessary connection between the hue and the shade this must be discovered by examining the two properties, in which case the proposition that everything which has the shade in question has the hue in question must be synthetic and necessarily true.

V.2.b. People sometimes say that it is merely analytic that nothing can be two colours at the same time. Certainly, we could adopt n-rules to ensure the incompatibility

of colour words (see section 4.C), and it may be the case that we do. But perhaps we do not need to: perhaps there are ways of understanding colour-words so that the connections between them are synthetic. Exactly what sort of relation holds between a pair of colour words will, of course, depend on the sorts of meanings they have.

Thus, if "red" and "yellow" are both d-words disjunctively correlated with ranges of specific shades of colour (with no overlap between the ranges), then the question of the incompatibility of "red" and "yellow" is logically equivalent to the question of the incompatibility of different specific shades of colour.

On the other hand, if "red" and "yellow" are both f-words, directly correlated with hues, then their incompatibility depends not only on the impossibility of finding an object with two different specific shades of colour, but also on the impossibility of finding an object which has a shade of colour which is simultaneously a shade of red and a shade of yellow, that is an object which has two hues at once, though only one shade of colour.

If one is a d-word correlated with a range of specific shades of colour, and the other is an f-word correlated with a hue, then the incompatibility will depend on the impossibility (for example) of finding an object with one of the specific shades correlated with "red" and the hue correlated with "yellow". Is it analytic that nothing which is scarlet in shade can be yellow in hue?

If both words are p-words, correlated with properties by means of procedures, of the sort described in section 3.D, then the relation between them may be still more complex and problematic.

V.3. Examples are also forthcoming when we consider properties of sounds.

What sort of fact is it that if two musical intervals with the characteristic sounds of a fifth and a fourth are added together, then the outer interval will have the characteristic sound of an octave?

What sort of fact is it that only if the three notes of a chord are separated by two of the following intervals: a third, a minor third, and a fourth, can they form a triad with the characteristic sound of a major chord? Can all these musical properties be identified independently, or can they be identified only by specifying their relations? Or are the relations contingent?

V.4. Another example: no surface is both glossy and mat at the same time. Analytic or synthetic? Necessary or contingent?

V.5. There seem also to be relations between mechanical properties, which are really the same sorts of things as the geometrical properties already discussed, except that motion comes in too.

For example, if a rod remains straight and its midpoint remains fixed while one end moves down, then the other end moves up.

If two gear wheels are meshed, their shapes and distance apart remaining constant, then if one of them turns about its axis, and neither penetrates the other, then the second one will move too. (This is the sort of thing which enables us to predict what will happen when one part of a machine starts moving, if none of the parts bends or disintegrates or penetrates the others.) Is this analytic?

V.6. I believe that more examples can be found by considering relations between numbers. First of all it should be noted that among the several different concepts superimposed in our ordinary arithmetical concepts are "perceptible" numerical concepts. For it is possible to learn to recognize the number of objects in small discrete collections just by looking, and without counting or otherwise correlating the objects with anything else. Similarly, one might learn, by being shown examples, to recognize simple operations performed on such sets, such as addition and subtraction, nothing being allowed to come into or out of existence or merge with anything else during such an operation. Then, by examining these observable numerical properties and operations, perhaps with the aid of informal proofs of the sorts described in section 7.D, we may be able to see connections which justify the assertion of such statements as "A two-set added to another two-set with which it is disjoint yields a four-set". "A five-set can be divided into a two-set and a three-set."

It is arguable that such statements, if understood in one way, are both synthetic and necessarily true. But I shall not go into the argument.

V.7. All these examples rely on the fact that there are properties which are independently identifiable. The sort of thing that is meant by saying that such properties can be identified independently of one another was illustrated by the discussion in section 3.C of the way in which properties can explain our use of descriptive words.

In addition, the examples rely on the fact that in order to perceive the necessary truth of the statements which are alleged to be both synthetic and necessarily true, it is necessary to be acquainted with the specific kinds of properties referred to: purely logical, or topic-neutral, enquiries will not suffice.

Appendix VI

APRIORI KNOWLEDGE

VI.1. I think the arguments of chapter seven show that the common tendency to confuse the terms "analytic" and "necessary" ought to be resisted. Similarly it is tempting to confuse the terms "necessary" and "apriori". I shall now try to show briefly that this is undesirable.

VI.2. Kant asserted that one of the marks of apriori knowledge was necessity, and there is something in this. If a statement is contingently true, then it would be false in some possible state of affairs, so there are no reasons why it should be true which can be known without discovering which possible state of the world is the actual one. Hence the truth of a contingently true statement has to be ascertained by empirical observation of the facts to make sure that there are no counter-instances. Thus no statement which is not a necessary truth can be known apriori to be true, from which it follows logically that if a statement can be known apriori to be true then it is necessarily true.

But the converse does not follow. There may be statements which are necessarily true which are not known to be true at all, let alone known apriori. Some necessarily true statements may be known to be true only on the basis of empirical enquiry: a person who fails to realize that every object bounded by four plane faces must have four vertices may establish the truth of "Every object bounded by four plane faces has four vertices" by carrying out a survey of objects bounded by four plane faces and counting their vertices. Indeed, it seems likely that there

are some truths which cannot be known apriori by any human being despite their necessity, owing to the complexity of the connections between universals in virtue of which they are necessarily true. Perhaps there are kinds of connections between properties which simply cannot be discovered by examining those properties: causal connections may be like this, in which case there are necessary truths which cannot be known a priori at all. At any rate, it is clear that the concepts "necessary" and "known apriori" are distinct, though there is a connection between them.

VI.3. This helps to show that the term "apriori" should not be applied to statements or truths: it applies to kinds of knowledge or ways of knowing. I have not defined the expressions "apriori" and "empirical", and it is probable that their use in philosophical discussion is even more confused than the use of the terms "analytic" and "necessary". One way in which the aprior-empirical distinction may be applied is as follows. Where the truth-value of a statement depends on whether or not certain particular objects have certain properties or stand in certain relations, it is, in general, necessary to ascertain the truth-value by carrying out observations to see which particular objects exist, and which relations and properties they instantiate. This is an empirical way of acquiring knowledge: one observes particular contingent facts. However, as shown in the thesis, there are some cases where it is not necessary to carry out such an investigation, since its outcome can be discovered merely by examining the properties and relations concerned and perceiving connections between them. This is an apriori way of acquiring knowledge: one does not discover by empirical observation how things happen to be in the world which

might have been otherwise (see section 7.A). This is the distinction which I have had in mind whenever I used the term "apriori", namely the distinction between knowledge obtained by observation of particular facts and knowledge obtained without observation of particular facts.

VI.4. It is sometimes suggested that apriori knowledge, and knowledge of necessary truth, is not derived from or dependent on experience. But it is not at all clear what this means, unless it is an obscure version of what I have just said.

For there are two ways in which the examples of apriori knowledge of necessary truth mentioned above are derived from or at least dependent on experience. First of all, each of the necessarily true statements which I have discussed employs descriptive words referring to observable properties, and, in general, in order to have the requisite concepts and understand the statements it is necessary to have had some experience of objects with these properties, or properties similar to them. At any rate, this is how we do in fact usually acquire such concepts. Secondly, experience comes in when one perceives the connections between such properties. For example, one may have the required insight while looking at a particular diagram which exhibits the properties in question. So in this sense apriori knowledge may be based on particular experiences (rather than on detached and lofty exertions of "Pure Reason"). But it is still not empirical in the sense defined above. Whether there is knowledge of a sort which is completely independent of all experience in some sense is dubious: what could it be knowledge of?

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(Abbreviations used in the thesis are placed between square brackets.)

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