A Strategic Analysis of Prepayments in Financial Credit Networks

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Abstract

In financial credit networks, *prepayments* enable a firm to settle its debt obligations ahead of an agreed-upon due date. Prepayments have a transformative impact on the structure of networks, influencing the financial well-being (utility) of individual firms. This study investigates prepayments from both theoretical and empirical perspectives. We first establish the computational complexity of finding prepayments that maximize welfare, assuming global coordination among firms in the financial network. Subsequently, our focus shifts to understanding the strategic behavior of individual firms in the presence of prepayments. We introduce a *prepayment game* where firms strategically make prepayments, delineating the existence of pure strategy Nash equilibria and analyzing the price of anarchy (stability) within this game. Recognizing the computational challenges associated with determining Nash equilibria in prepayment games, we use a simulation-based approach, known as empirical game-theoretic analysis (EGTA). Through EGTA, we are able to find Nash equilibria among a carefully-chosen set of heuristic strategies. By examining the equilibrium behavior of firms, we outline the characteristics of high-performing strategies for strategic prepayments and establish connections between our empirical and theoretical findings.

1 Introduction

A financial system can be understood as a credit network, in which nodes represent financial institutions such as firms engaging in financial transactions, and directed edges capture bilateral financial obligations between firms, with the weight on edges denoting the amount of the corresponding debt or liability. A firm's *total assets* encompass exogenous external assets (e.g., cash or claims on entities outside the network) and its future incoming payments due within the system; *equity* measures the amount of remaining assets after payments (total assets minus liabilities) if this is positive, and zero otherwise (in which case the firm is bankrupt/insolvent). Firms employ their total assets to fulfill their liabilities by making payments to lenders. Should a firm's assets prove insufficient to cover its liabilities, the firm enters default, leading to a reduction in the value of its assets, often through costly liquidation. The extent of this decrease is denoted by *default costs*, signifying that the firm will have only a portion of its total assets available for payments. On liquidation day, "clearing" takes place, with the calculation and execution of actual payments in the financial network. These payments adhere to three fundamental principles of bankruptcy law, as outlined by Eisenberg and Noe [2001]: i) *absolute priority*: firms must first settle their liabilities in full to attain positive equity; ii) *limited liability*: firms cannot pay more than their total assets; iii) *proportionality*: in the event of default, payments to lenders are made in proportion to the respective liability.

Prepayments generally refer to transactions in which a borrower opts to make repayments to lenders ahead of the debt's maturity. A common example of individual-level prepayment is observed in mortgage security markets, where some borrowers might pay off their mortgage loans before the designated maturity date [Patruno, 1994]. Prepayments are also prevalent in the financial sector, notably in the commercial and industrial (C&I) business loan market, which constituted approximately two-thirds of short-term credit in the US in 2022 [Gallegati *et al.*, 2022]. C&I loans provide companies with funds for working capital or capital expenditures (e.g., machinery purchases). Penalty-free prepayments, often permitted in C&I loan agreements, offer borrowing firms the flexibility to prepay without subsequent penalties[1]. Lenders universally accept such prepayments, so our work assumes such consistent acceptance (i.e., prepayments are always accepted by the lenders [Eckbo *et al.*, 2022]). Motivations for prepayments can vary among firms in practice. Some borrowers prepay loans to lower the firm's leverage ratio (e.g., debt-to-asset ratio), while others may aim to reduce total interest costs or mitigate interest rate risk.

In our context, prepayments specify the early utilization of a firm's external assets to fulfill its liabilities before the clearing process for a given financial system. An individual firm in a financial network can have incentives to prepay with the fol-

[1]Borrowers typically incur an upfront fee for the penalty-free prepayment option during the loan commitment drafting phase [Eckbo *et al.*, 2022].

lowing underlying rationale: in the event of firm bankruptcy during clearing, a firm can only employ a fraction of its original total assets to settle its liabilities due to the presence of default costs; if the firm decides to prepay before the anticipated clearing day, prior to the realization of default costs, then it can make full use of its external assets. These undiscounted prepayments could potentially prevent other firms from facing bankruptcy, yielding augmented cash flow throughout the network during clearing. In return, the augmented cash flow could benefit the firm initiating the prepayments when it flows back. Given that making prepayments leads to an immediate loss in a firm's external assets (and, consequently, in utility), strategic decisions regarding whether to prepay and whom to prepay should be contingent on whether the resulting benefit can offset the immediate loss. Our study explores the impacts of strategic prepayments in financial credit networks, addressing relevant questions through a blend of theoretical and empirical analyses.

Contributions. Our work has three main contributions:

1. We establish prepayment games, where the act of making prepayments is a strategic decision;
2. Using our game model, we offer a comprehensive theoretical understanding of the existence, (in)efficiency, and computational aspects of equilibria in prepayment games; and
3. We scrutinize the equilibrium behavior of firms in prepayment games through the application of empirical game-theoretic analysis (EGTA), involving game-theoretic reasoning via extensive simulation.

Specifically, when assuming firms have complete information of the credit network, we prove that computing a prepayment profile that maximizes the sum of total assets or total equity is **NP**-complete if there are non-trivial default costs. Then we shift our focus to understanding the strategic behavior of individual firms in the presence of prepayments. We first introduce prepayment games where firms strategically decide which lender to prepay. Then we explore the existence, quality, and computational complexity of pure-strategy Nash equilibria (PSNE) under two different utility models; total assets and equity. We observe that PSNE may not exist when firms aim to maximize their total assets in the presence of default costs, whereas they always exist when the goal is to maximize equity, irrespective of default costs. By assessing the quality of equilibria, we find that, when firms' utility is total assets, even the best PSNE can have arbitrarily bad social welfare (sum of firms' utilities) relative to the optimal subset of prepayments. In contrast, when utility is defined in terms of equity, the best PSNE is socially optimal. However, the worst PSNE remains arbitrarily inefficient. Furthermore, we emphasize the intractability of determining PSNE existence and computing the best response in the total-asset-based setting, whereas these problems are trivially solvable in the equity-based setting. The computation of the best PSNE remains **NP**-hard in the equity-based setting.

Finally, we complement our theoretical results with an empirical analysis through EGTA, with a realistic setting where firms only have local information about the network. We discover the existence of mixed Nash equilibria with prepayment strategies in the equilibrium support, demonstrating higher social welfare compared to abstaining from prepayments. This underscores the benefits of prepayments in financial systems. Additionally, we perform a systematic ablation study, unveiling the strengths and weaknesses of different prepayment strategies and providing insights for making beneficial prepayments.

Related work. Our model is grounded in the influential work of Eisenberg and Noe [2001] in which firms are nodes that are connected via debt contracts, with proportional payments in the case of insolvency. Subsequent work extended the basic model, incorporating features such as default costs [Rogers and Veraart, 2013], cross-ownership relations [Elliott *et al.*, 2014; Vitali *et al.*, 2011], and credit default swaps [Schuldenzucker *et al.*, 2020; Papp and Wattenhofer, 2020]. Rogers and Veraart [2013] prove the existence of maximal clearing payments in the presence of default costs and provide an efficient algorithm to compute them. Schuldenzucker et al. [2017] demonstrate the intractability of finding clearing payments with credit default swaps. Ioannidis et al. [2022] study the clearing problem from the point of view of irrationality of solutions and strength of approximation, while Ioannidis et al. [2023] investigate the complexity of clearing problems in networks with derivatives and lender priorities.

A game-theoretic approach to financial networks is introduced by Bertschinger et al. [2024]. They relax the principle of proportionality [Eisenberg and Noe, 2001] and model payments under bankruptcy as a decision-making process. In particular, they propose two different payment schemes – coin-ranking and edge-ranking – and present a range of results on the existence and quality of equilibria arising in games corresponding to each payment scheme. Kanellopoulos et al. [2021] extend this line of research by examining priority-proportional payment schemes. Furthermore, Papp and Wattenhofer [2020] analyze firms' incentives for redefining liabilities and donating external assets. In subsequent work, Papp and Wattenhofer [2021] investigate the impact of debt swaps in risk mitigation. In recent contributions, Hoefer and Wilhelmi [2022] investigate clearing games with varying seniorities, while Bertschinger et al. [2023] study equilibria in a game that models fire sales, analyzing the convergence of best-response dynamics. Kanellopoulos et al. [2022] study edge-removal games where each bank wants to maximize its total assets by strategically removing a part of incoming edges, and provide results about the properties of resulting equilibria. More recently, the same authors investigate debt transfer games, where banks can be strategic about whether or not to transfer their debt claims, and complement their theoretical study with an empirical game analysis [Kanellopoulos *et al.*, 2023]. To the best of our knowledge, our work is the first to study a model for financial networks with prepayment.

Extensive empirical research on financial network topologies indicates that very different structures may arise, ranging from centralized networks [Müller, 2006] to core-periphery structures [Fricke and Lux, 2015; Li and Schürhoff, 2019] and scale-free structures [Boss *et al.*, 2004; Cont *et al.*, 2010].

Additionally, numerous studies, including [Gai and Kapadia, 2010; Elliott *et al.*, 2014; Leventides *et al.*, 2019], simulate default contagion within random networks. A comparable random financial network has been employed by Mayo and Wellman [2021] to simulate strategic behavior from an empirical game-theoretic perspective.

2 Preliminaries

We examine a *financial credit network* model proposed by Rogers and Veraart [2013]. Specifically, we analyze a network G where nodes represent firms, and edges symbolize debt contracts. Each firm v_i initially holds non-negative *external assets* e_i, representing liquid assets received from entities external to the financial system. Moreover, firms are connected by directed edges that represent *liabilities*. The directed edge (v_i, v_j) signifies a financial relationship between firms v_i and v_j, with $l_{ij} \geq 0$ denoting the liability that firm v_i (the borrower) has to firm v_j (the lender). If no directed edge connects firms v_i and v_j, then v_i has no liability to v_j, and $l_{ij} = 0$. The graph G is irreflexive, meaning no firm can be liable to itself, resulting in a directed and positively weighted graph without self-loops. It is worth noting that both $l_{ij} > 0$ and $l_{ji} > 0$ might coexist. The total liabilities of v_i are denoted by $L_i = \sum_j l_{ij}$. Firms capable of fully meeting their obligations are considered *solvent*, while those unable to do so are labeled *in default* or *insolvent* interchangeably.

Let p_{ij} denote the actual payment made by firm v_i to firm v_j, where p_{ij} may not be equal to the liability l_{ij}, and $p_{ii} = 0$. We denote by $\mathbf{P} = (p_{ij})$ for $i, j \in [n]$ the *induced payment matrix*, where $[n] := \{1, \ldots, n\}$. The total outgoing payments of v_i are represented by $p_i = \sum_{j \in [n]} p_{ij}$. In the case of insolvency, a firm may need to liquidate its assets or make payments beyond the financial system, such as salary disbursements. Consequently, a defaulting firm can only utilize an α fraction of its external assets and incoming payments. When $\alpha = 1$, it means an absence of default costs. Payments $\mathbf{P}$ aligned with the three principles discussed in the introduction are termed *clearing payments*. Specifically, a solvent firm must fully pay all its obligations, while a firm in default can only pay partially but should repay as much of its debt as possible, based on its total assets affected by the default costs. A partial payment to a lender should be proportional to its liability to the same lender. Mathematically, the clearing payment from v_i to v_j with $i, j \in [n]$ satisfies $p_{ij} = l_{ij}$ when v_i is solvent, and $p_{ij} = \alpha \cdot \left(e_i + \sum_{j=1}^{n} x_{ji}\right) \cdot \frac{l_{ij}}{L_i}$ when v_i is in default. It is important to note that clearing payments are not necessarily unique; however, we focus on maximal clearing payments—those that point-wise maximize all corresponding payments. Such maximal clearing payments are known to exist and unique, and can be computed in polynomial time [Rogers and Veraart, 2013].

Strategic prepayments. Prepayments in our context are the settlement of debt obligations ahead of clearing using external assets. For a given financial network G, a single prepayment from borrower v_i to lender v_j results in a direct reduction of v_i's external assets e_i by an amount equal to the liability l_{ij} (i.e., $e'_i = e_i - l_{ij}$); meanwhile, there is an increase of l_{ij} to v_j's external assets (i.e., $e'_j = e_j + l_{ij}$), accompanied by a cancellation of the liability l_{ij}.

Each firm v_i has the option to select a set of lenders to prepay, and consequently, a strategy determines the chosen set of lenders for prepayment. Assuming that all firms strategically engage in prepayments to enhance their individual financial well-being, this naturally leads to the formulation of a *prepayment game* in strategic form. Let D_i and C_i represent the sets of borrowers and creditors for v_i, respectively. Consequently, each *pure strategy* of firm v_i constitutes a subset $C'_i \subseteq C_i$ of lenders to prepay. A *mixed strategy* for firm v_i is defined as a discrete probability distribution over the power set of its lenders C_i. A *pure strategy profile*, denoted by $\mathbf{s} = (s_1, \ldots, s_n)$ or $\mathbf{s} = (s_i, \mathbf{s}_{-i})$, specifies a pure strategy for each firm. Here, the pure strategy profile for all firms except v_i is denoted by $\mathbf{s}_{-i}$.

Given a prepayment profile[2], $\mathbf{s}$, and the corresponding clearing payments $\mathbf{P}$, we explore two definitions of a firm's utility. The *total assets* of firm v_i encompass the updated external assets post-prepayments, along with its incoming clearing payments, expressed as $a_i(\mathbf{P}) = e'_i + \sum_{j \in [n]} p_{ji}$, while its *equity* is defined as $E_i(\mathbf{P}) = \max\{0,\ a_i(\mathbf{P}) - L'_i\}$. Here, e'_i and L'_i represent v_i's updated external assets and total liabilities, respectively, under the strategy profile $\mathbf{s}$. Specifically, e'_i is computed as $e_i + \sum_{j \in D'_i} l_{ji} - \sum_{j \in C'_i} l_{ij}$, and L'_i is determined by $L_i - \sum_{j \in C'_i} l_{ij}$, where D'_i denotes the set of v_i's borrowers choosing to prepay v_i under $\mathbf{s}$. When $\mathbf{P}$ is clear from the context, the notation a_i and E_i will be used. The social welfare $\mathcal{SW}(\mathbf{P})$ is the sum of the firms' utilities, with the specific utility notion (total assets or equity) clarified by the context. The optimal social welfare (chosen globally, disregarding strategic considerations) is denoted by $\mathcal{OPT}$.

We consider *Nash equilibrium* (NE) as our strategic solution concept for prepayment games. A NE is a strategy profile in which no firm has an incentive to unilaterally deviate (and improve its utility); if such a profile is pure, we call it a *pure-strategy Nash equilibrium*. Let $\mathbf{P_{eq}}$ be the set of clearing payments consistent with all pure NE strategy profiles. The *Price of Anarchy* and *Price of Stability* in a game are characterized by the worst-case and best-case ratios, respectively, of the optimal social welfare over the achieved welfare at any equilibrium, considering all potential networks. Formally, PoA $= \max_G \max_{\mathbf{P} \in \mathbf{P_{eq}}} \frac{\mathcal{OPT}}{\mathcal{SW}(\mathbf{P})}$, and PoS $= \max_G \min_{\mathbf{P} \in \mathbf{P_{eq}}} \frac{\mathcal{OPT}}{\mathcal{SW}(\mathbf{P})}$, respectively. At times, a broader class of equilibria is considered, (e.g., mixed or correlated equilibria), but we focus on PSNE in this work.

An illustrative example. Figure 1 presents a specific financial network that exemplifies an individual firm's incentive to prepay, illustrating some key concepts. If v_1 chooses to prepay v_2, where the liability l_{12} is 4, v_1's external assets directly decrease from 4 to 0. Simultaneously, v_2's external assets become 4, leading to the removal of the edge (v_1, v_2). The clearing payments before and after this prepayment are outlined below.

[2]Throughout this paper, the terms 'strategy profile' and 'prepayment profile' are used interchangeably.

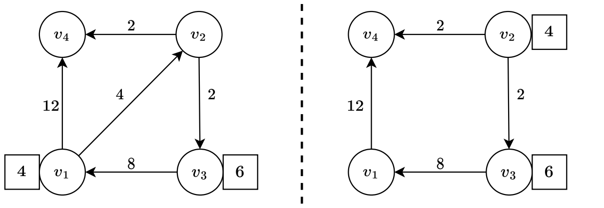


Figure 1: The figure (left) shows the initial network, while the other figure (right) shows the network after the prepayment by v_1. Nodes correspond to firms, edges are labeled with the respective liabilities, while external assets appear in a rectangle near the relevant firm.

In this example, we assume a default cost of $\alpha = \frac{1}{2}$. In the initial network (without prepayments), firm v_1 defaults since its potential maximum total assets of 12 fall short of total liabilities of 16. This triggers a default contagion, rendering v_2 and v_3 insolvent sequentially. Consequently, the clearing payments are $p_{12} = \frac{8}{9}$, $p_{14} = \frac{24}{9}$, $p_{23} = p_{24} = \frac{2}{9}$, and $p_{31} = \frac{28}{9}$ with total assets $a_1 = \frac{64}{9}$, $a_2 = \frac{8}{9}$, $a_3 = \frac{56}{9}$, $a_4 = \frac{26}{9}$, and equities $E_1 = E_2 = E_3 = 0$, and $E_4 = \frac{26}{9}$.

However, if v_1 strategically prepays v_2, both v_2 and v_3 become solvent, avoiding asset loss due to default costs. This results in more payments on the existing edges, including those flowing back to v_1. The clearing payments in this case are $p'_{14} = 4$, $p'_{23} = p'_{24} = 2$, $p'_{31} = 8$ with $a'_1 = 8$, $a'_2 = 4$, $a'_3 = 8$, $a'_4 = 6$, and equities $E'_1 = E'_2 = E'_3 = 0$, and $E'_4 = 6$, respectively. These values can be easily verified by running the clearing algorithm of Rogers and Veraart [2013]. We can conclude that v_1 is better off after the prepayment in terms of total assets, i.e., $a'_1 > a_1$, although it remains insolvent with zero equity.

3 Computing Optimal Prepayments

We start with the centralized setting, where as motivation one can imagine a financial regulator that possesses the authority to govern each firm. We present computational hardness results associated with achieving specific objectives related to the financial well-being of the entire system. With this in mind, we define three decision problems. The first is PARTITION, a well-known **NP**-hard problem, whilst the other two decision problems correspond to the maximization of total assets and equity in financial networks. All missing proofs are included in the full version.

Decision Problem 1 (PARTITION). *Given an instance $\mathcal{I}$ that consists of a set X of positive integers $\{x_1, x_2, \ldots, x_k\}$, determine whether there exists a subset X' of X such that $\sum_{i\in X'} x_i = \sum_{i\notin X'} x_i = \frac{1}{2}\sum_{i\in X} x_i$.*

Decision Problem 2 (MAX-SUM-TOTAL-ASSETS). *Given a financial credit network G and a positive constant μ, determine whether there exists a prepayment profile such that the sum of total assets is greater than or equal to μ.*

Decision Problem 3 (MAX-SUM-EQUITY). *Given a financial credit network G and a positive constant μ, determine whether there exists a prepayment profile such that the total equity is greater than or equal to μ.*

Theorem 1. MAX-SUM-TOTAL-ASSETS *is* **NP**-*complete.*

Note that this result holds for any $\alpha \in [0, 1]$. Next, we consider the objective of maximizing total equity.

Lemma 1. ([Schuldenzucker *et al.*, 2020]). *In the networks without default costs, the total equity always equals the sum of external assets, that is, $\sum_i E_i = \sum_i e_i$.*

An implication of Lemma 1 is that any prepayment profile maximizes the total equity. The situation changes significantly, however, in the presence of non-trivial default costs. We formally present these remarks in the following result.

Theorem 2. MAX-SUM-EQUITY *is*

a) trivial when $\alpha = 1$;

b) **NP**-*complete when $\alpha \in [0, 1)$.*

Proof Sketch. We only present a proof sketch here. For case a) the statement holds by Lemma 1. In case b), we demonstrate that MAX-SUM-EQUITY is in **NP** by employing the maximal cleaning payments algorithm, which runs in polynomial time [Rogers and Veraart, 2013], to verify if a proposed prepayment profile achieves the total equity at least value μ. Thereupon, we establish the **NP**-hardness of MAX-SUM-EQUITY by reducing it from the PARTITION problem. The reduction works as follows. Given an instance $\mathcal{I}$ of PARTITION problem, we construct an instance $\mathcal{I}'$ of MAX-SUM-EQUITY. We add firms v_i for each element $x_i \in X$; we also allocate an external asset of x_i to v_i. Additionally, we include four additional firms S, S', T and T' and allocate an external asset of $\frac{1}{2}\sum_{i\in X} x_i$ to both S and T. Each firm v_i has a liability of x_i to S and T, while S and T have a liability of $\sum_{i\in X} x_i$ to S' and T' respectively; see also Figure 2. Clearly, the reduction requires polynomial time.

By minimizing default costs through prepayments and ensuring prepayments to firm S and T, the total equity is equal to $2\sum_{i\in X} x_i$, reaching its maximum value. Finally, we prove that $\mathcal{I}$ is a yes-instance for PARTITION if and only if the prepayment profile has total equity of $2\sum_{i\in X} x_i$. $\square$

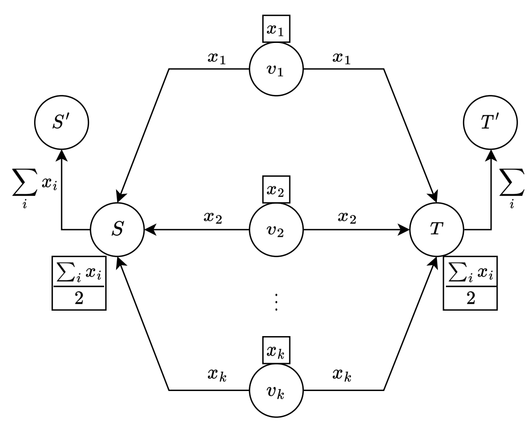


Figure 2: Illustrating the reduction from the proof of Theorem 2.

Note that the hardness of decision problems 2 and 3 immediately imply that the corresponding optimization problems are computationally intractable unless **NP = P**.

4 Prepayment Games

We turn our attention to the scenario where firms strategically determine prepayments to maximize their utility. Specifically, we investigate two variations of prepayment games, in which the utility of a firm is defined by total assets and equity respectively. For each variant, we examine the existence, quality, and computational complexity of finding PSNE. In both the total asset and equity setting, we assume that the firms know the financial network G.

4.1 Maximizing Total Assets

We first consider prepayment games where firms attempt to maximize their total assets and investigate the existence of PSNE. The following result establishes the existence of a unique PSNE in prepayment games without default costs.

Theorem 3. *In the total assets setting, for $\alpha = 1$, the prepayment profile where no firm prepays is the unique PSNE.*

The existence of pure equilibria, however, is not guaranteed when non-trivial default costs apply.

Theorem 4. *In the total assets setting, there is a game such that for all $\alpha \in (0, 1)$ there is no PSNE.*

Next, we investigate the quality of pure equilibria and observe that even the best pure equilibria can be highly undesirable in terms of social welfare.

Theorem 5. *In the total assets setting, for all $\alpha \in [0, 1]$, the Price of Stability is unbounded.*

We conclude with our results on computational complexity for the setting with default costs. These findings are quite negative and mean that computing pure equilibria with total assets is not tractable, which essentially serves as a foundational motivation for the subsequent exploration of EGTA in Section 5.

Theorem 6. *In the total assets setting, for all $\alpha \in (0, 1)$, the following problems are* **NP***-hard:*

a) verifying if a given strategy profile is a PSNE even when PSNE are guaranteed to exist;

b) computing the best response strategy;

c) deciding if there exists a PSNE.

4.2 Maximizing Equity

We now turn our attention to prepayment games where utility is defined by equity. In contrast to the negative results considering total assets, we illustrate that prepayment games with equity-based utilities possess several desirable properties. To begin, we establish the existence of PSNE, even in the presence of default costs.

Theorem 7. *In the equity setting, for all $\alpha \in [0, 1]$, any prepayment profile is a PSNE.*

According to Theorem 7, the strategy profile with the maximum total equity is a PSNE, which implies that:

Corollary 1. *In the equity setting, for all $\alpha \in [0, 1]$, the Price of Stability is* 1*.*

Although the best NE consistently yields the global optima, we then show that the worst pure equilibria can deviate significantly from the optimal scenario in terms of total equity.

Theorem 8. *In the equity setting, for all $\alpha \in [0, 1)$, the Price of Anarchy is unbounded.*

Regarding the computational aspects, Theorem 7 ensures that it is trivial to compute the best response for a given strategy profile and determine whether pure equilibria exist or not. Nevertheless, the problem of computing the best PSNE remains **NP**-hard – a corollary of Theorem 2.

Corollary 2. *In the equity setting, for all $\alpha \in [0, 1)$, computing the best PSNE is* **NP***-hard.*

5 Empirical Game-Theoretic Analysis

We now present our empirical analysis of prepayments on synthetic networks. Due to lack of space some parts have been omitted, the complete analysis will appear in the full version.

5.1 Method

Given the computational challenges in determining NE in prepayment games involving multiple firms, as indicated in Theorem 6, we employ EGTA, a process that engages in game-theoretic reasoning through extensive simulation. Our approach begins by proposing a set of heuristic prepayment strategies, which, in contrast to the theoretical results in Section 4, *rely solely on local information* that would definitely be available to firms, such as claims (expected incoming payments), liabilities, and external assets. This is *realistic* and mirrors real-world financial scenarios where firms typically lack awareness of the external assets and liabilities of other market participants. Consequently, each strategy becomes a mapping from the local information to the set of lenders to prepay. Additionally, each strategy is inherently designed to adhere to feasibility constraints. The individual strategies are outlined below:

No Operation ($\mathbf{h_1}$): Make no prepayment.

Random ($\mathbf{h_2}$): Uniformly prepay a subset of lenders.

Max Claim ($\mathbf{h_3}$): Prepay the lender who has highest claim to itself.

Max Claim Greedy ($\mathbf{h_4}$): Prepay all lenders, ordered by the claims, until the external assets are exhausted.

Heuristic Belief ($\mathbf{h_5}$): Estimate the external assets of lenders (details in the full version). If the sum of the prepayment to a lender and its asset estimate is larger than the claim from the lender *and* this does not hold under their estimated payment without prepayment, then prepay the lender.

Random with Solvency Check ($\mathbf{h_6}$): If the liability is greater than the sum of claims and the external assets, employ the random strategy h_2. Otherwise, take no-operation strategy h_1.

Max Claim with Solvency Check ($\mathbf{h_7}$): If the liability is greater than the sum of claims and the external assets, employ the max-claim-greedy strategy h_3. Otherwise, take no-operation strategy h_1.

Max Claim Greedy with Solvency Check ($\mathbf{h_8}$): If the liability is greater than the sum of claims and the external assets, employ the max-claim-greedy strategy h_4. Otherwise, take no-operation strategy h_1.

Heuristic Belief with Solvency Check ($\mathbf{h_9}$): If the liability is greater than the sum of claims and the external assets, employ the heuristic-belief strategy h_5. Otherwise, take no-operation strategy h_1.

Among these strategies, the "no-operation" strategy and the "random" strategy correspond to scenarios without prepayment and those involving random behaviors, respectively. The "max-claim" strategy and the "heuristic-belief" strategy consider whether a firm could benefit from prepayments. In essence, a firm should only engage in prepayments to lenders within a debt circle that leads back to itself, preventing the insolvency of lenders. Due to the limited local observation of individual firms, it is challenging to identify lenders on this cycle (i.e., which lenders to prepay). Therefore, heuristics become crucial in making such decisions.

The "max-claim" strategy opts to prepay the lender with the highest claim (i.e., the firm owing the most). In contrast, the "heuristic-belief" strategy estimates other firms' external assets (e.g., based on annual financial reports) and prepays only lenders with insufficient estimated external assets to cover the claim. Essentially, the "heuristic-belief" strategy estimates whether prepayments would make a difference.

We then refine these strategies (h_2-h_5) and introduce strategies h_6-h_9 by incorporating an additional condition that checks the solvency of the firm engaging in prepayment. Since a firm can only observe the claims rather than the actual incoming payments, which could be less than the claims, a firm may still turn out to be insolvent even if the solvency check passes. However, if the solvency check fails, a firm will undoubtedly be insolvent. Adding a solvency check is rooted in the fact that prepayments, when made by a firm believing itself to be insolvent, are not discounted by α. Consequently, engaging in prepayments under these circumstances injects more cash flow into the network, potentially benefiting the entire financial system and the firm itself.

Taking an ex-ante perspective, we model the prepayment game as a symmetric game, where firms share the same strategy set and utility function. The utility function is based on total assets rather than equity, as every profile serves as a PSNE with equity as the utility function, as per Theorem 7. In this model, a firm aims to maximize its expected utility with respect to a network generator that defines the joint distribution of liabilities and external assets. We give details about the network generator in the full version.

In a symmetric game, it is sufficient to outline a strategy profile by the number of players employing each strategy. In our EGTA approach, given a strategy profile, expected payoffs are calculated by averaging the payoffs over individual instances sampled from the network generator. For each individual instance, the payoffs are determined by the clearing of a corresponding stable network—meaning no firm would make further prepayments to others. This stability is achieved by executing the strategies in the profile simultaneously and continuously.

As the number of firms and strategies in a game grows, conducting a thorough game-theoretic analysis becomes computationally impractical. To address this challenge, we employ an aggregation technique, known as deviation-preserving reduction (DPR) [Wiedenbeck and Wellman, 2012], to approximate complex many-player games as simpler ones with fewer participants. DPR has been widely applied in the analysis of various AI applications, including for financial systems [Wah *et al.*, 2017; Wang and Wellman, 2017; Wright and Wellman, 2018]. We identify symmetric NE of the reduced game, in lieu of being able to find NE of the original game.

5.2 Experimental Results

We begin by calculating the NE for the prepayment games generated by the network generator with external assets ranging from 40 to 70. Our findings reveal the existence of a mixed-strategy NE, wherein all firms decide between the random strategy h_1 and the heuristic-belief with solvency check strategy h_9. As we varied the default cost discount α, we observed a reduction in the probability mass on h_9 as α approaches 1. This trend is attributed to the diminishing benefits of prepayments as α nears 1. As α becomes closer to 1, firms are less inclined to choose prepayment, as the immediate loss incurred outweighs the damage caused by insolvency. The explanation of this immediate loss will be provided later in this section. In the extreme case where α equals 1, we observed that refraining from prepayment becomes a PSNE, and the probability of playing h_9 drops to zero, as per Theorem 3.

Figure 3 illustrates the equilibrium measures across varying αs. The considered measures encompass PoA, PoS, *Effect of Anarchy* (EoA), and *Effect of Stability* (EoS), as defined by Kanellopoulos et al. [2022]. EoA and EoS represent the ratio of the initial-state social welfare to the social welfare of the worst, and best, respectively, NE. They serve as metrics for evaluating the performance of NE in comparison to making no prepayment. Upon examination of Figure 3, we initially observed that both EoA and EoS are below 1. This indicates that making prepayments enhances social welfare compared to refraining from prepayments. Subsequently, all curves converge to a horizontal line with a value of 1 as α increases. This convergence is attributed to the diminishing benefits of making prepayments. Ultimately, abstaining from prepayment becomes the social optimum.

Through our experiments, we observed that opting not to make prepayments emerges as a robust strategy when compared to other heuristic prepayment approaches. This strength stems from the fact that making prepayments incurs an immediate loss in a firm's external assets, consequently impacting its utility. The viability of prepayments depends on their substantial influence on the well-being of firms within a debt circle that includes the concerned firm, wherein the immediate loss could be offset by the resulting benefit. Given the stringent conditions required for prepayments to be effective (e.g., the existence of debt circles, prepayments making a significant difference to lenders, and the benefit of a favorable prepayment outweighing the costs of all other non-beneficial ones), choosing not to make prepayments can easily outperform simplistic heuristics.

To assess the relative effectiveness of all strategies except h_1, we conduct an ablation study with $\alpha = 0.5$, as depicted

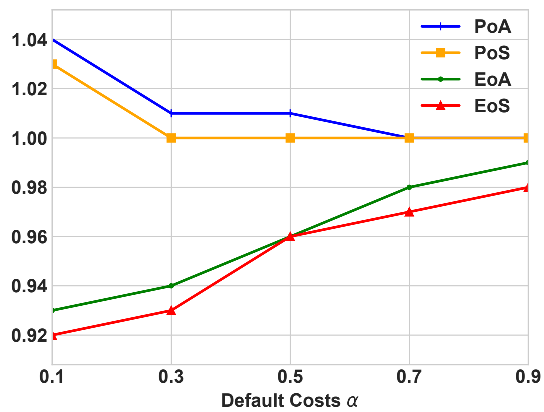
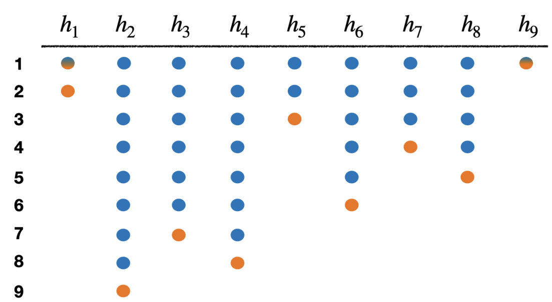


Figure 3: Curves of equilibrium measures.

in Figure 4. Specifically, we iteratively eliminate equilibrium strategies, starting from h_9, from the current strategy set and recompute NE. In Figure 4, the orange dots signify NE, the blue dots denote strategies outside the NE support, and the absence of a dot indicates the removal of a strategy. Each row in the figure represents a game obtained by removing the NE strategy (pure NE exists in row 2 to row 9) from the preceding row. The earlier a strategy appears in an NE, the stronger its strategic position.



$$h_9, h_1 \succ h_5 \succ h_7 \succ h_8 \succ h_6 \succ h_3 \succ h_4 \succ h_2$$

Figure 4: An ablation study for the performance of strategies. The orange circles represent NE and the blue ones indicate strategies outside the NE support. Gradient colors identify the mixed strategies. Each row represents a game given by removing NE strategy from the previous row. The strength of strategies is ordered.

Our initial observation underscores the significance of heuristics in determining which lender to prepay, as evidenced by the superior performance of h_3, h_4, and h_5 compared to the random strategy h_2. This trend persists even when applying the solvency check, as strategies h_7, h_8, and h_9 all outperform the random strategy h_6. Upon comparing the performance of h_3 and h_4, we discerned that making one prepayment at a time might be more advantageous than making multiple ones, primarily due to the potential loss incurred by non-beneficial prepayments. Additionally, it becomes apparent that strategies h_7 and h_8 do not participate in an NE if h_9 remains in the strategy set. This underscores the importance of considering the estimated external assets of other firms in the decision-making process.

Moreover, we observed that strategies incorporating the solvency check outperform those without. This finding aligns with intuition, as only insolvent firms stand to increase their utilities through prepayments. The solvency check effectively prevents a solvent firm from engaging in prepayments, a scenario that would otherwise result in an immediate loss of external assets for the firm.

Finally, we made more aggressive adjustments to the configurations of network generators, enabling either insufficient or excessive external assets. In both scenarios, we observed a lack of prepayment. When external assets are insufficient, no firm has the capacity to engage in prepayments. Similarly, when the entire network is solvent with an ample amount of external assets, firms have no incentive to make prepayments.

To sum up, the crux of making advantageous prepayments lies in the fact that prepayments made by insolvent firms are not discounted by α. This implies that a greater amount of money flows into the network, potentially benefiting other firms. The payoff from prepayments occurs when there is a path for the money to circulate back, covering the immediate loss. Consequently, making prepayments is meaningful only if the firm engaging in prepayment is insolvent, and appropriate debt circles exist.

6 Conclusion

We examine the characteristics and impacts of prepayments in financial credit networks through both theoretical analysis and empirical study. We evaluate the computational complexity of identifying prepayments that maximize welfare from a centralized standpoint. Additionally, we delve into the behavior of individual firms from a game-theoretic perspective, outlining the strengths and weaknesses of different heuristics. Our findings indicate that prepayments can be advantageous for financial systems, particularly in situations where the insolvency of firms happens.

Building upon the existing model, our analysis leaves a number of compelling questions unresolved. For instance, there remains a need for a conclusive answer regarding the existence of PSNE when firms seek to maximize their total assets under the condition $\alpha = 0$. Additionally, our discrete treatment of prepayments, wherein firms either fully prepay or refrain entirely, prompts an interesting trajectory for further investigation. Extending the prepayment game model to encompass fractional prepayments will make our analysis more realistic. Furthermore, while our theoretical analysis utilizes the PSNE solution concept, an unexplored avenue entails the examination of prepayment games under the mixed Nash equilibria. The limitations and structural assumptions inherent in our analysis give rise to additional avenues for future exploration. In particular, since in our model firms act in a solitary manner, a trajectory for future work involves exploring scenarios where a firm enhances its utility by incentivizing another firm to prepay. This injects the prepayment games into the area of cooperative game theory and gives rise to many intriguing research questions such as the outcome of the interaction and the formation of coalitions.

Contribution Statement

Hao Zhou and Yongzhao Wang contributed equally.

Acknowledgements

This research was supported by a UKRI AI World Leading Researcher Fellowship awarded to M. Wooldridge (grant EP/W002949/1). **MW**, **AC** and **HZ** acknowledge funding from Trustworthy AI - Integrating Learning, Optimisation and Reasoning (TAILOR), a project funded by European Union Horizon2020 research and innovation program under Grant Agreement 952215. **RS**, **KV**, and **YW** thank the Alan Turing Institute for its support via the "Automated Analysis of Strategic Interactions" project. **RS** would also like to acknowledge the support of EPSRC (grant EP/W014750/1).

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