

Pattern understanding: Relationships with arithmetic and reading development

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Abstract

Pattern understanding (patterning) is commonly taught in early years classrooms. However, the relationship between patterning and academic attainment is not well understood. Studies investigating children's pattern understanding are reviewed. There is some evidence to suggest that pattern understanding is causally related to acquiring math and reading skills. However, much of the evidence is weak and these conclusions remain tentative. Future studies of the relationship between patterning and other skills need to use psychometrically robust measures and analytic techniques that control for the effects of measurement error. The suggestion from current studies that training patterning skills in young children can improve math and reading attainments is tantalizing but larger, methodologically more robust, Randomized Controlled Trials are badly needed to confirm such claims.

Keywords: pattern understanding, reading, mathematics, cognitive development

Pattern understanding, or patterning, is the ability to detect regularities in a set of units: for example, detecting that a sequence of units (colours, shapes, objects, letters or numbers) repeats or increases in a predictable way. This is reminiscent of processes involved in series completion tasks, where the relations between a series of stimuli (e.g., a number sequence) need to be determined before a rule can be induced and used to extrapolate the sequence (1, 2). Activities designed to support children's patterning skills are commonly used in early years classrooms (3) and early years learning standards both in the UK and Australia (4, 5) feature aspects of pattern understanding. In the U.S., however, the National Mathematics Advisory Panel (6) questioned the importance of patterning instruction and pattern knowledge is not included in recent U.S. content standards for schools (7). Although there is a lack of consensus regarding the educational value of teaching children about patterns, there is strong interest in the possible role of patterning as a predictor of, and a possible causal influence on, the development of arithmetic and reading. This review examines the methods that have been used to study children's patterning abilities, goes on to consider the relationship between patterning ability and attainment in arithmetic and reading, and ends with some proposals for how future studies can advance understanding in this area.

Children's pattern understanding

One of the first types of patterns children become familiar with are linear repeating patterns formed of objects, colours and/or shapes (e.g. blue-green-blue-green). A series of studies by Rittle-Johnson and colleagues has shown a developmental sequence for patterning ability, coupled with substantial variability (8-10). Early in the pre-school year, most 4-year old children (around 75%) can duplicate (copy) repeating patterns, and around 50% can extend them. Fewer than 33% children are able to abstract patterns (i.e. recreate a pattern using different materials) or

identify the unit of repeat (10). With the exception of unit identification, these skills show growth across the pre-school year, though the sequence of task difficulty remains the same (9).

The ability to copy and continue patterns develops earlier than pattern abstraction and unit identification, and these tasks appear to depend on different strategies to solve them (11). Basic (copy and continue) pattern tasks can be solved using visual matching strategies. However, pattern abstraction tasks require children to go beyond the surface features of the pattern to identify the underlying structure (8, 12) and depend on more complex forms of mental representation, abstraction and manipulation. Indeed, it has been argued that the ability to abstract patterns represents an early form of relational reasoning which is fundamental for cognitive development (13).

If pattern understanding is important for cognitive development and educational attainment, how should we teach it? Teachers and parents alike report routinely engaging in patterning activities with their preschool children but these are typically basic pattern tasks which involve modelling and having children create, duplicate and extend patterns with objects and sounds (9). Learning standards that feature patterning are also limited to copying, continuing and creating patterns (e.g. 5). In contrast, studies by Rittle-Johnson and colleagues (e.g., 9, 10) suggest it may be more effective to focus instruction on more complex tasks such as pattern abstraction which highlight the importance of identifying the underlying structure of patterns.

Studies of patterning in older children have largely centred on the early years of formal education (6-7 years of age; though see 14 for a study of 10-year-old children). These studies have used more complex and challenging patterning tasks that involve sequences of units which increase/decrease, or rotate, according to an implicit rule (e.g. an increasing number pattern such

as 1-2-4-7). Tasks commonly include patterns formed of alphanumeric (letters and numbers) and non-alphanumeric stimuli (objects, shapes and/or colours). Typically, children are shown a 5- or 7-item linear pattern in which an item is missing, and are asked to choose one of a number of alternatives to complete the pattern. A small number of studies have also assessed patterning using function machine tasks, which require children to identify the rule governing related pairs of inputs and outputs (14, 15).

Patterns based on non-alphanumeric stimuli do not depend on prior knowledge, such as counting skills or knowledge of the alphabet or number sequence, whereas patterns formed of letters and numbers clearly do. Few studies have explicitly examined whether the use of alphanumeric versus non-alphanumeric stimuli affects patterning ability, though some evidence suggests that alphanumeric and non-alphanumeric patterning tasks do not differ in difficulty (16, 17). These studies, however, focus on children aged 6-7 years and the same might not be true in younger children with less secure alphanumeric knowledge. Furthermore, scores on the patterning tasks used in these studies are often at low levels (e.g. 16, 17, 18) which may obscure differences.

It seems logical to expect that alphanumeric patterning tasks may relate more closely to attainment in arithmetic and reading than non-alphanumeric patterning tasks. Lee et al. (15) directly compared performance on patterns which require numerical knowledge (numerical function machine task) with two tasks assessing geometric pattern understanding (a pattern completion task and a geometric function machine task) in children aged 6-7 years ($N=163$). The different pattern tasks correlated weakly with each other ($r's = .23-.32$) and as might be expected arithmetic proficiency did correlate more strongly with numerical patterning ($r = .46$) than with the geometric pattern tasks ($r's = .14-.25$).

Summary: Studies of patterning have largely focused on children in the pre-school years and the early stages of formal education. There is evidence of a developmental sequence of patterning ability in pre-school children, but little is known about how patterning develops in older children. Furthermore, possible differences between alphanumeric versus non-alphanumeric patterning tasks and the strength of their correlation with measures of academic attainment are not well understood.

Relationships between patterning, reading and arithmetic

We understand little about the cognitive mechanisms underlying patterning or the mechanisms by which it might contribute to the development of arithmetic or reading. Nonetheless, some have claimed that patterning is central to early mathematical thinking, particularly pre-algebraic reasoning (19-23). Many early math concepts, such as counting, involve predictable sequences (counting in 5's for example involves a simple alternating pattern of 0's and 5's). Plausibly, the ability to identify and generalise from the underlying relationship among a diverse set of items may support the acquisition of early math concepts and provide a foundation for understanding algebra.

It is less clear how patterning might relate to learning to read. One suggestion is that understanding sequences enables children to identify relationships between spoken and written language, and make predictions based on the rules underlying letter-sound relationships and word order (grammar; 17, 18). It has also been suggested that the ability to identify and make predictions based on patterns may be part of the development of broader cognitive abilities, such as fluid intelligence, analogical reasoning or relational thinking (13, 17, 18). This implies that patterning instruction might lead to improvements in fluid intelligence, and in turn, enable children to develop reading and arithmetic skills more effectively (18).

Correlational studies

Patterning is a concurrent correlate of math ability (14, 15, 24-26; r 's = .25-.68). A small number of studies also demonstrate longitudinal relationships between patterning and math (25, 27, 28). The most convincing evidence for a longitudinal relationship comes from a large-scale ($n=517$) study by Rittle-Johnson et al. (28), who found that patterning at age 5, along with understanding of non-symbolic quantity, contributed unique variance to mathematical ability at age 11, over and above other numerical and cognitive skills. The bivariate correlations between patterning in Grade 1 and the different math measures in Grade 5 were moderate ($r = .31$ to $.38$) which in the face of the patterning measure's low reliability in this study ($\alpha = .56$) is quite impressive.

Fewer studies have examined the relationship between patterning and reading. There is some evidence that patterning correlates concurrently with measures of early reading ($r = .29$; Bock et al., 29) and reading efficiency ($r = .33$, Schmerold, 24; $r = .20$, Pasnak et al., 27). However, any relationship with reading appears to be weaker than that with maths and does not extend to measures of reading comprehension (30). A longitudinal study of 6- to 7-year-old children by Pasnak et al. (27) reported a correlation ($r = .23$) between children's performance on a letter sequence pattern task and word reading efficiency measured 7 months later. However, this relationship was bidirectional (reading efficiency and later letter sequence knowledge correlated $r = .28$) which the authors suggest reflects mutual causal effects or a potential third variable. However, this correlation between patterning (as assessed using a letter-sequence task) and reading is inherently ambiguous and may just reflect the fact that letter knowledge is one of the critical determinants of reading ability (31, 32).

Intervention studies

The teaching of patterning in early school curricula implies a causal relationship – that training patterning skills should lead to improvements in other skills, particularly arithmetic and reading. This has been examined in several studies (17, 18, 20, 23, 33-35). Here we focus on those studies which provide the most robust evidence i.e. studies that have used random assignment to groups receiving patterning training or an alternative treatment control group. We identified four such studies (17, 18, 33, 35). Table 1 shows the estimated effect sizes for the three studies where we were able to derive such estimates (17, 18, 35): the study by Hendricks et al. (33) is excluded from Table 1 as it only reports pretest to posttest gain scores making it impossible to calculate effect sizes.

The procedure in these three studies is essentially the same. The effect of patterning instruction on reading and math skills was evaluated in 6 to 7-year-old children (N 's = 120-246) who scored poorly on a screening measure of patterning (a task which required children to select the missing item in a 5-item sequence of letters, numbers, clock faces, or rotated objects). Children were randomly allocated (in pairs, within each classroom) to one of four instructional programmes: patterning, reading, math, or social studies. Children in the patterning group received repeated practice in identifying the missing item in patterns (including symmetrical, increasing, rotation and random repeating patterns using a wide range of stimuli) supported through teacher-explanation. Children were also taught to use objects to create, complete and extend patterns.

Each form of instruction was delivered by class teachers to pairs of students in 15 minute sessions, 3 times a week over 6-7 months. Children were assessed following instruction on

standardized measures of reading and math. Although all studies report attrition, only Hendricks et al. (33) reported rates of attrition for each group.

In interpreting Table 1 it is important to emphasize that two of the studies have small samples and hence low power and that effect sizes taken from the same study are not statistically independent. Although effect sizes across studies and measures are inconsistent it appears that patterning training yields improvements in scores on several measures of arithmetic and reading. The effect sizes on arithmetic appear larger than for reading although it is worth emphasizing that three of these effect sizes for arithmetic are implausibly large (the large effect sizes in Kidd et al., 2014 for the two WJ arithmetic measures reflect very small standard deviations on these measures at posttest, likely because of ceiling effects).

Although these studies offer the most robust evidence to date, a number of design issues limit the conclusions that can be reached. The studies by Kidd et al. (17, 35) and Pasnak et al. (18) only assessed math and reading outcome measures at posttest (not at pretest). This is suboptimal because it makes it impossible to take account of imbalances at pretest which are likely to occur with simple random assignment using modest sample sizes; not accounting for baseline imbalance reduces the precision of estimates of the size of any intervention effect (36). It is also worth noting that scores on patterning tasks are low and remain so even after lengthy periods of instruction which suggests there may be more effective methods (e.g. using abstract language to describe patterns; 13) by which to improve patterning than those evaluated here.

In the study by Hendricks et al. (33), children in an experimental group ($N=33$ after attrition) received patterning training for approximately 1 hour per week over a 6-month period (more precise details of dosage are lacking). A control group ($N=29$ after attrition) received a different form of teaching ('academic instruction') which was tailored to children's needs by

teachers and was therefore highly variable. There was a large imbalance in IQ (measured at posttest only) between the groups which raises doubts about the adequacy of the randomization procedure used. However, after using IQ as a covariate, they report that patterning training produced improvements in mathematics and written language measures from the Diagnostic Achievement Battery, with effect sizes (presumably Cohen's *d*) of .18 and .49.

The studies reviewed above focus on children in Grade 1 (6-7 year old children). The only study to evaluate patterning instruction for pre-school children (aged 3-5 years; is reported in 20 and 37) found gains in early number skills following a programme which combined individualised and curriculum-embedded instruction in patterning; however, non-random allocation to groups and high rates of attrition limits the conclusions that can be drawn this study.

Conclusions and Future Directions

In recent years there has been an upsurge in work concerned with children's pattern understanding. Our review has focused on two interrelated issues: 1. The nature of patterning, and 2. The relationship between patterning and the development of reading and mathematics.

We believe that patterning merits further research. One major limitation of research to date is that we do not yet have a clear understanding of the cognitive mechanisms involved in patterning. Patterning shows small to moderate correlations with aspects of executive functioning (including cognitive flexibility, working memory and inhibition; 8, 10, 29, 38), relational knowledge (8, 11) and fluid intelligence (14, 15). This evidence suggests that patterning ability is related to general cognitive skills, though such relationships are often inconsistent across studies and in some cases are compromised by overlap in the tasks used to assess them. It remains unclear to what extent patterning predicts unique variance in reading and arithmetic beyond other key constructs (e.g. executive function, fluid intelligence) with which it correlates.

To assess the relationship between patterning and other skills requires large-scale correlational studies with multiple measures to assess a range of seemingly closely related constructs (i.e. patterning, fluid intelligence, executive function, working memory, language). Research on patterning has not done the necessary psychometric ground work to establish the structure of patterning ability, and how it relates to other constructs. Do multiple measures of patterning correlate strongly with each other and define a meaningful unitary construct? If a unitary patterning factor can be identified and measured reliably, how does such a construct relate to other constructs?

Evidence from correlational studies provides some support for the claim that patterning is related to reading and mathematics. Furthermore, there is limited evidence that training patterning skills can improve children's attainments in math and (to a lesser extent) reading. Such findings are promising, though it is clear that further research is needed to confirm them. It would be important for future studies to use more robust methodology, including: 1. larger samples; 2. pretest-posttest design; 3. using the CONSORT guidelines for reporting (39); 4. using active control interventions that are theoretically motivated (and evidence-based) and follow a standard model of delivery; and 5. preferably use minimization to reduce baseline imbalance for key outcome measures which will increase power (40). If patterning is a meaningful and distinct cognitive construct which can be trained, and if such training translates into gains in math and reading outcomes, that would be a finding of great theoretical and practical importance.

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Table 1 Effect sizes for math (patterning group vs reading group) and reading (patterning group vs math group) outcomes

Study	N	Math Outcomes	Cohen's d [95% CI]	Reading Outcomes	Cohen's d [95% CI]
Kidd et al. (35)	35 vs 35	WJ 10 Applied Problems	0.33 [-0.15, 0.80]	WJ Letter-Word Reading	-0.03 [-0.50, 0.44]
		WJ 18A Quantitative Concepts	0.77 [0.28, 1.25]	WJ Fluency	0.00 [-0.47, 0.46]
		WJ 18B Quantitative Concepts	0.34 [-0.13, 0.81]	WJ Comprehension	-0.01 [-0.48, 0.46]
Kidd et al. (17)	30 vs 30	WJ 18A Quantitative Concepts	2.28 [1.62, 2.93]	TOWRE Word	1.25 [0.69, 1.80]
		WJ 18B Quantitative Concepts	2.06 [1.43, 2.69]	TOWRE Phonemics	0.29 [-0.22, 0.80]
		Key Math 3 Numeration	0.79 [0.26, 1.32]	GORT	0.58 [0.06, 1.10]
		Key Math 3 Addition	0.97 [0.43, 1.50]	TERA Meaning	1.10 [0.55, 1.64]
		Key Math 3 Algebra	0.64 [0.12, 1.16]		
		Key Math 3 Measurement	0.63 [0.11, 1.15]		
		Key Math 3 Foundations	0.85 [0.32, 1.37]		
		Key Math 3 Computation	0.60 [0.08, 1.11]		
		Key Math 3 Data Problems	0.53 [0.01, 1.04]		
		Key Math 3 Geometry	0.33 [-0.18, 0.84]		
		Key Math 3 Multiplication	0.47 [-0.04, 0.99]		
		Key Math 3 Applied Problems	0.52 [0.00, 1.03]		
Pasnak et al. (18)	62 vs 62	WJ 18A Quantitative Concepts	2.17 [1.72, 2.61]	TOWRE Word	0.63 [0.26, 0.99]
		WJ 18B Quantitative Concepts	1.49 [1.09, 1.89]	TOWRE Phonemics	0.17 [-0.19, 0.52]
		Key Math 3 Algebra	0.66 [0.30, 1.02]	GORT	0.15 [-0.20, 0.51]
		Key Math 3 Foundations	0.54 [0.18, 0.90]	TERA Meaning	0.65 [0.29, 1.01]
		Key Math 3 Addition/Subtraction	0.41 [0.05, 0.76]		
		Key Math 3 Measurement	0.37 [0.01, 0.72]		
		Key Math 3 Computation	0.34 [-0.02, 0.69]		
		Key Math 3 Data Problems	0.24 [-0.11, 0.59]		
		Key Math 3 Numeration	0.38 [0.02, 0.74]		

Note: WJ = Woodcock Johnson Tests of Achievement III; TOWRE = Test of Word Reading Efficiency; GORT = Gray Oral Reading Test; TERA = Test of Early Reading Ability-3. For the purposes of calculating effect sizes we have assumed uniform rates of attrition between groups. Effect sizes for math outcomes are calculated from differences between children in the patterning and reading groups; for reading outcomes, we compare the patterning group with those who have received instruction in math. Effect sizes are Cohen's d (mean differences at posttest divided by the pooled standard deviation at posttest)

