



PERSPECTIVE

Causality gets entangled

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Abstract

Understanding the role of causality in quantum theory is a growing research direction in quantum information and the foundations of quantum theory. One particular area is to understand generalizations of quantum theory where there is an indefinite causal order between various operations. Building on recent work developing the process matrix formalism, Araújo *et al* (2015 *New J. Phys.* **17** 102001) give formal tools to analyse how causally indefinite processes can be by drawing inspiration from entanglement theory. This approach draws together concepts in quantum information with more speculative ideas in the foundations of quantum theory.

Physics has long sought to understand cause and effect. In spite of this, our two main physical theories, quantum theory and general relativity, have contrasting perspectives on the role of causality in physics. In quantum physics, experimental processes are described operationally with respect to a fixed notion of causality: preparations of quantum states come before measurements on them. Therefore, in standard quantum theory there is always a well-defined causal order. The story can be different in general relativity where there is no fixed causal structure in advance. Lucien Hardy highlighted this contrast between the two theories and suggesting that in reconciling quantum theory with general relativity, definite causal order may have to be relaxed in quantum theory [1].

Inspired by these ideas, Ognjan Oreshkov, Fabio Costa and Časlav Brukner developed a method of extending quantum theory to allow for indefinite causal order [2]. The basic premise is to imagine that there are various laboratories and in each laboratory, systems abide by standard quantum mechanics but there is no definite causal relation between each laboratory in a larger spacetime. This is in contrast to scenarios within quantum information where in order to communicate or process quantum information, laboratories are assumed to be space-like or time-like separated thus fixing a causal order between them. Figuratively, the framework of Oreshkov *et al* can exhibit now have a *quantum superposition* of causal order between laboratories just as a quantum system can be in a superposition of, say, energy levels; the causal structure, in some sense, is now quantum and thus prone to quantum uncertainty.

The main theoretical tool developed by Oreshkov *et al* was that of the *process matrix* which is analogous to a quantum state where the latter encodes information about the properties of a system, the former encodes information about the causal order between laboratories. Just as quantum states can dictate the uncertainty of certain measurement outcomes, the process matrix can be associated with an indefinite causal order. Only when each laboratory performs its particular quantum operation can a definite causal order be established.

In quantum information, entanglement is seen as a vital resource for many protocols and, as a result, has received a great deal of attention [3]. Entanglement emerges when two systems cannot be described by a quantum state that is described as a product of the two parties individual quantum states: the sum is greater than its parts. Quantum states that are not entangled are referred to as *separable*. A consequence of entanglement is that each party's local knowledge of their system can be very noisy and uninformative but the two parties can have complete knowledge of their quantum state only when the two parties perform some global operation between the two of them. However, it turns out that for any two-party quantum state, it is very difficult to check whether it is separable or otherwise [4]. Despite these challenges there is a strong mathematical foundation for detecting entanglement through so-called *entanglement witnesses*: expressions which if violated indicate the presence of entanglement.

Given the importance of entanglement for quantum states, is there a natural analogue of entanglement for process matrices? The work of Araújo *et al* aims to address this question by studying *causal inseparability* which is now the inability to describe a process matrix in terms of process matrices that have a definite causal structure [5]. This fascinating work highlights both the similarities as well as dissimilarities with quantum entanglement and provides rigorous tools for future study. One main technical contribution of this work is to develop the theory of *causal witnesses* which are the natural analogues of entanglement witnesses. On the other hand, it is shown that causal witnesses can be easily constructed and thus one can efficiently detect whether a process matrix is causally separable, in stark contrast to the theory of entanglement.

A natural question to ask is whether experiments with indefinite causal order can ever be constructed? Also inspired by the work of Hardy, Giulio Chiribella and collaborators discussed the *quantum switch* in relation to quantum computations without a definite causal structure [6]. Loosely speaking, quantum circuits consist of quantum operations performed in some fixed order but Chiribella *et al* discussed the possibility of performing two different computations (with differing causal orders) in quantum superposition. It was later shown that one can devise a quantum optical set-up to reproduce the action of this quantum switch [7]. Since it can be implemented experimentally, then perhaps there is a traditional causal explanation to its workings? Araújo *et al* show that this is not the case since the process matrix corresponding to the quantum switch violates a causal witness and thus has indefinite causal order [5]. That is, witnessing causal inseparability is a very practical possibility and motivates the devising of new experiments. Within the foundations of quantum physics, there is a long-established method for detecting quantum entanglement: the violation of a Bell inequality [8]. This test for entanglement is far more stringent than the violation of an entanglement witness and not all entangled quantum states can violate a Bell inequality [9]. In the work by Oreshkov *et al*, the *causal inequality* was devised as the causal separability analogue of the Bell inequality in entanglement theory: violation of a causal inequality demonstrates indefinite causal order. Araújo *et al* demonstrate the true depth of this analogy with entanglement theory by showing that the quantum switch will never violate a causal inequality despite not being causally separable, also demonstrated in [10].

This tantalizing analogy between entanglement and causal inseparability offers new tools for studying relaxations of quantum theory that allow indefinite causal order. We are a long way from a theory of quantum gravity and these in-depth analyses of causality in quantum theory can offer new perspectives on why we are so far away.

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