

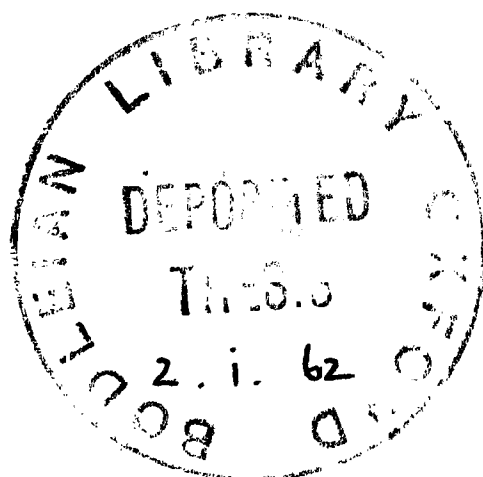
THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
IN THE UNIVERSITY OF OXFORD

A b s t r a c t

of

T H E S O L A R R E D - S H I F T

L. A. Higgs



University College
Oxford
Trinity Term
1961

Shortly after Rowland's fundamental study of the wavelengths of Fraunhofer lines, Jewell in 1896 observed that the solar wavelengths were shifted to the red side of the corresponding wavelengths observed in the laboratory. This shift became known as the red shift of solar lines, and at first led to various erroneous concepts of conditions in the solar atmosphere. Moreover, in 1907, Halm discovered that the solar wavelengths were not themselves constant, but depended upon the radial distance of the point of observation from the centre of the solar disk. This variation of wavelength with the point of observation has now become known as limb effect. If Θ is the angle between the line of sight to the point of observation on the solar surface and the true solar radius at that point, $\Delta(\Theta)$ denotes the difference between the wavelength at that position and that observed at the centre of the disk. In general, $\Delta(\Theta)$ was found to be positive, increasing toward the limb.

Part of the general red shift of the Fraunhofer lines was explained when Einstein published his General Theory of Relativity, in which he predicted the Fraunhofer lines to be shifted to the red by an amount given by

$\frac{\Delta \nu}{\nu} = 2.12 \times 10^{-6}$. In fact, this gravity shift is a consequence of the underlying Principle of Equivalence which has been recently demonstrated in the laboratory. On the other hand, no explanation of limb effect was immediately obvious. The most promising hypothesis was that of St. John, in which limb effect was attributed to the Doppler shifts arising from radial convective motions in the solar atmosphere. At the centre of the disk, these would be predominantly violet shifts while at the limb, the streaming would have no effect. Although satisfying some of the observational data, this interpretation was not wholly satisfactory. In particular, an interferometric determination of $\Delta(\theta)$ by M. G. Adam in 1948 showed that, assuming known photospheric conditions, unreasonably large convective velocities were required for such an interpretation.

Recently, Schroter has elaborated this theory of solar red shifts in which the gravity shift and Doppler shifts arising from unresolved granulation elements combine to give a red shift with limb effect. Again certain characteristics of the observed red shifts are accounted for, but consistent agreement with various observational data is lacking. The most serious difficulty in this

respect was that the observations of M. G. Adam in 1959 indicated that at the extreme limb, the observed red shift exceeded the gravity shift. No theory based on radial convection currents can account for such a super-gravity shift.

In order to submit the radial current theory to a detailed examination and to confirm the existence of a super-gravity shift, the wavelengths and profiles of three solar lines were studied. This investigation forms the subject matter of the thesis. The same three medium strength iron lines at 6300 Å as used by M. G. Adam in her 1959 investigation were chosen, since they are conveniently situated for comparison with terrestrial lines of the α band of atmospheric oxygen. Using the latter as fixed wavelength references, the variation of the solar wavelengths across the disk was determined from micrometer measurements of grating spectra. After correction for the motions of the Earth and Sun and for the effects of scattered light, a mean $\Delta(\theta)$ was determined for the three lines. It was found that $\Delta(\theta)$ increased rapidly for radial distances from the centre of the disk greater than 85 per cent of the disk radius. Expressing the wavelength shifts as equivalent Doppler velocities, it

was found that the value of $\Delta(\theta)$ at the extreme limb was about 0.7 km/sec, in good agreement with the value found by M. G. Adam.

In order to convert these observations to absolute red shifts (referred to terrestrial laboratory measurements), the wavelengths of the iron lines at the centre of the disk as well as in the laboratory were required. The interferometric method of circular channels was adopted for this purpose, and absolute wavelengths (referred to the cadmium line at λ 6438.4696) of the three iron lines and the oxygen reference lines in the spectrum obtained at the centre of the disk were determined. In order to confirm that the effects of local velocity fields in the photosphere had been eliminated from these observations, similar determinations of wavelengths were made using integrated light from the whole solar disk. In addition to the solar observations, the terrestrial wavelengths of the three iron lines have been found using a vacuum iron arc. Combining these measurements with the limb effect observations gave a mean red shift at the centre of the disk of 0.39 km/sec increasing to about 1 km/sec at the limb. Since the gravity shift is 0.64 km/sec, the existence of a super-gravity shift

v

at the limb is confirmed.

Since the interpretation of micrometer measurements of limb effect depends critically on the centre-limb variation of the line profile concerned, in particular, of any asymmetry in the profile, the profiles of the three lines were determined for various disk positions, averaging many observations. It was found that at the centre of the disk the iron lines were slightly asymmetric with stronger violet than red flanks, somewhat as predicted by Schröter. At the limb, however, a large fluctuating red asymmetry was discovered. This is established to be of solar origin and appears to be connected in some manner with nearby chromospheric disturbances. This red asymmetry, occurring predominantly at residual line intensities ~ 0.8 , on occasion corresponded to a shift of the line centre of 25 mÅ and was of sufficient mean magnitude to account for the super-gravity shift (if the micrometer measurements refer to such a residual intensity).

An analysis of these observations of wavelength shifts and line asymmetry at the centre of the disk showed that Doppler shifts due to granulation, such as appear in Schröter's theory, are too small to account for

the observations and, consequently, that the origin of the centre red shifts (actually the deviations from the gravity shift) must lie elsewhere. Moreover, the newly discovered red asymmetry, or alternatively the supergravity shift, at the limb cannot have its origin in streaming motion. Following suggestions by H. H. Plaskett the possibility of this asymmetry being due to the superposition of a chromospheric line on the photospheric line was investigated. Such a superposition can arise from scattering or by chromospheric contributions to the profile in the line of sight. It was found that such a hypothesis could account for the observations if it is assumed that the chromospheric lines are shifted slightly to the violet.

An additional section of the thesis deals with computational problems arising in the reduction of the wavelength and profile observations. Methods were developed for the utilization of the Oxford "Mercury" electronic computer for calculations such as the computation of corrections to observed wavelengths due to motions of the Sun and Earth, the reduction of circular channel interferometric plates, the determination of characteristic

response curves of photographic plates, and the computation of true line profiles. In particular, a study of numerical methods of solving the integral equation arising in the instrumental broadening of spectral lines was made, and programmes for the solution of this equation were written.

From this detailed study of the three medium strength iron lines, it is concluded that the origin of the solar red shift must be more complex than previously assumed. In addition to the gravity shift, small granulation effects, and possible chromospheric effects at the limb, some other, unknown, cause of wavelength shifts remains. Thus a new theoretical treatment of possible mechanisms for the creation of line shifts in the photosphere is required before a comprehensive red shift theory can be attained.

THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
IN THE UNIVERSITY OF OXFORD

THE SOLAR RED-SHIFT

L. A. Higgs

University College
Oxford
Trinity Term
1961

TABLE OF CONTENTS

CHAPTER 1. INTRODUCTION.

PART I. THE GRAVITATIONAL SHIFT.	1
A. The Principle of Equivalence and General Relativity	2
B. Astronomical Tests of Equivalence and General Relativity	3
C. Terrestrial Tests of Equivalence and General Relativity	7
PART II. OBSERVATIONS OF SOLAR WAVELENGTHS.	11
A. Definitions	12
B. Early Wavelength Observations and Interpretations	13
C. Interferometric Determinations of Centre Red Shifts	22
D. Recent Limb Effect Determinations	24
PART III. RECENT THEORETICAL INTERPRETATIONS OF SOLAR WAVELENGTHS.	28
A. Radial Current Theories	29
B. Other Interpretations	41
C. Observations of Asymmetry	42
References for Chapter 1	45

CHAPTER 2. COMPUTATIONAL METHODS. 48

General Introduction 49

PART I. COMPUTATION OF SOLAR COORDINATES AND RADIAL VELOCITY CORRECTIONS.	51
A. Introduction and Notation	52
B. General Method	54

PART II. REDUCTION OF "CIRCULAR CHANNEL" PLATES.	58
A. General Theory	59
B. Measurement of Ring Diameters	62
C. Outline of Computations	66
PART III. DETERMINATION OF PLATE CHARACTERISTIC CURVES.	70
A. Introduction and Method	71
B. Outline of Computations	77
PART IV. SOLUTION OF CERTAIN INTEGRAL EQUATIONS.	81
A. Introduction	82
B. Solution of a System of Linear Equations	83
C. Iterative Methods	86
D. Fourier Transform Solutions	91
PART V. GENERAL REDUCTION OF LINE PROFILES.	94
A. Introduction	95
B. Outline of Computations	96
References for Chapter 2	101
CHAPTER 3. OBSERVATIONS OF WAVELENGTH SHIFTS.	
PART I. LIMB EFFECT FROM MICROMETER MEASUREMENT.	103
A. Introduction and Method	104
B. Solar Spectra	105
C. Centre Measurements	107
D. Limb Measurements	108
E. Accuracy of Measurement	109
F. Relative Velocity Corrections	110
G. Scattering Corrections	111
H. Final Limb Effect	113

PART II. DETERMINATION OF ABSOLUTE WAVELENGTHS AND CENTRE RED SHIFTS.	118
A. Method	119
B. Procedure	120
C. Solar and Arc Wavelengths	122
D. Integrated Disk Wavelengths	123
E. Red Shifts	128
References for Chapter 3	131
 CHAPTER 4. OBSERVATIONS OF LINE PROFILES	
PART I. INSTRUMENTAL EFFECTS ON THE GRATING WAVELENGTHS.	132
PART II. LINE PROFILES AND ASYMMETRY DETERMINATION.	140
A. Introduction and Procedure	141
B. Apparatus Function	145
C. Observed Asymmetries	156
D. Accuracy of Asymmetry Curves	162
E. Examples of Extreme Asymmetry	166
References for Chapter 4	171
 CHAPTER 5. DISCUSSION OF OBSERVATIONS.	
PART I. RELEVANT OBSERVATIONAL DATA.	172
A. New Observations of Red Shifts	173
B. New Observations of Asymmetry	173
C. Existing Granulation Data	174
PART II. COMPARISON WITH EXISTING THEORIES.	176
A. The Path-Length Interpretation	177
B. Schroter's Two Stream Theory	180
PART III. POSSIBLE RED SHIFT MECHANISMS.	184
A. Effects of Granulation on Centre Line Profiles	185
B. Super-Gravity Shift and Asymmetry at the Limb	188

PART IV. CONCLUSIONS.	197
References for Chapter 5	200
APPENDIX A. EFFECTS OF SCATTERED LIGHT.	202
A. Method	203
B. Scattering Function	204
C. Application to Line Profiles	206
APPENDIX B. CENTRE-LINE VARIATION OF LINE PROFILES.	208
APPENDIX C. THE SOLAR RED SHIFT.	212
APPENDIX D. AUTOCORRELATION PROGRAMMES FOR THE "MERCURY" COMPUTER	
PART I. SOLAR COORDINATES AND RADIAL VELOCITY CORRECTIONS.	214
A. Introduction	214
B. Programme OBS 3C - 1	214
C. Other Similar Programmes	223
PART II. REDUCTION OF INTERFEROMETRIC (CIRCULAR CHANNEL) PLATES.	237
A. Introduction	237
B. Programme OBS 4	237
PART III. SOLUTION OF THE FREDHOLM INTEGRAL EQUATION.	266
A. Introduction.	266
B. Programme OBS 11-E	267
C. Other Similar Programmes	282

PART IV. DETERMINATION OF CHARACTERISTIC CURVES.	290
A. Introduction.	297
B. Programme OBS 11-111	290
PART V. REDUCTION OF LINE PROFILES.	311
A. Introduction	311
B. Programme OBS 11	312
ACKNOWLEDGEMENTS.	329

FIGURES

Fig. 1.1	Principle of Equivalence	4
1.2	Limb Effect Observed Before 1950	21
1.3	Recent Limb Effect Curves	26
1.4	Granulation Contrast for Two Stream Model	35
1.5	Line Asymmetry Predicted from Two Stream Model	38
1.6	Variation of Predicted Asymmetry Across the Disk	40
2.1	Measurement of Circular Channel Ring Diameters	64
2.2	Plate Calibration as Recorded by Microphotometer	73
2.3	Determination of Asymmetry Curves	99
3.1	Limb Effect at the East and West Limbs	114
3.2	Mean Limb Effect	116
3.3	Optical Train for Integrated Disk Observations	126
3.4	Observed Red Shift	130
4.1	Effect of Grating Apparatus Function on Profiles	136
4.2	Apparatus Function for Bausch and Lomb Grating	138

4.3	Theoretical Curve of Growth for Atmospheric Oxygen Lines	150
4.4	Derived Apparatus Functions for Babcock Grating	152
4.5	Asymmetry of Oxygen Line Profiles	155
4.6	Asymmetries Determined from East and West-Limb Plates	157
4.7	Asymmetry of Iron lines Observed at Centre	159
4.8	Asymmetry of Iron line λ 6297.8 Observed near the Limb	161
4.9	Asymmetry of Iron line λ 6301.5 Observed near the Limb	163
4.10	Extremely Asymmetric Iron Line Profile	167
4.11	Extreme Observed Asymmetries	169
5.1	The Path-Length Interpretation compared with Observation	179
5.2	The Two Stream Predictions Compared with Observation	182
5.3	The Effects of Chromospheric Superposition	195
1 (Appendix A)	Asymmetry Introduced by Scattering	207 b

Chapter 1

INTRODUCTION

P a r t I

THE GRAVITATIONAL SHIFT

A. The Principle of Equivalence and General Relativity.

Since the gravitational shift of spectral lines is intimately connected with the subject matter of this thesis, the first part of this chapter will be devoted to a discussion of this phenomenon, and the related theory of relativity.

In 1911, Einstein (1) advanced the foundations of his General Theory of Relativity, based on the principle known as the Principle of Equivalence. This latter may be expressed as follows. "In any small region, a gravitational field of force is exactly equivalent to a field of force introduced by a transformation of the coordinates of reference, so that by no possible experiment can we distinguish between them." The observed proportionality of inertial and gravitational masses of material bodies (i.e. in a pendulum) formed the basis of this general hypothesis and it in turn formed the basis of the General Theory. The above principle states, in effect, that a given small region in a four dimensional Euclidean (x,y,z,t) space in which particles experience "gravitational" forces may be transformed into a non-Euclidean (x',y',z',t') space in which no forces act on the particles. The General Theory is then concerned with specifying the required coordinate transformation.

B. Astronomical Tests of Equivalence and General Relativity.

Einstein at once deduced (1) that, if the above principle is a general one and applies to electromagnetic phenomena, light passing through a gravitational field must undergo a change in frequency. This may easily be illustrated for the case of a uniform gravitational field, in which case the General Theory is not required to find the coordinate transformation, a uniform acceleration clearly being the "equivalent" transformation. Following Einstein, consider light passing from A to B, in a frame of reference S (Fig. 1.1), in the gravitational field g . By the Principle of Equivalence, the observations made by an observer at B must be the same as those that would be made if the observer were accelerated with acceleration g in the absence of a gravitational field. If S' is an accelerating frame of reference, so that A' and A coincide and A' is at rest when the light is emitted at A, then the observer at B' will have a velocity of approximately hg/c ($h = |A - B|$) when he observes the light. He will therefore observe it with a Doppler shift

$$\frac{\nu_0 - \nu}{\nu_0} = \frac{hg}{c^2} \quad (1.1)$$

Thus in the frame S , the light must also undergo a frequency shift, due to the gravitational field. This shift is what has become known as the "gravity shift" or "relativistic shift"

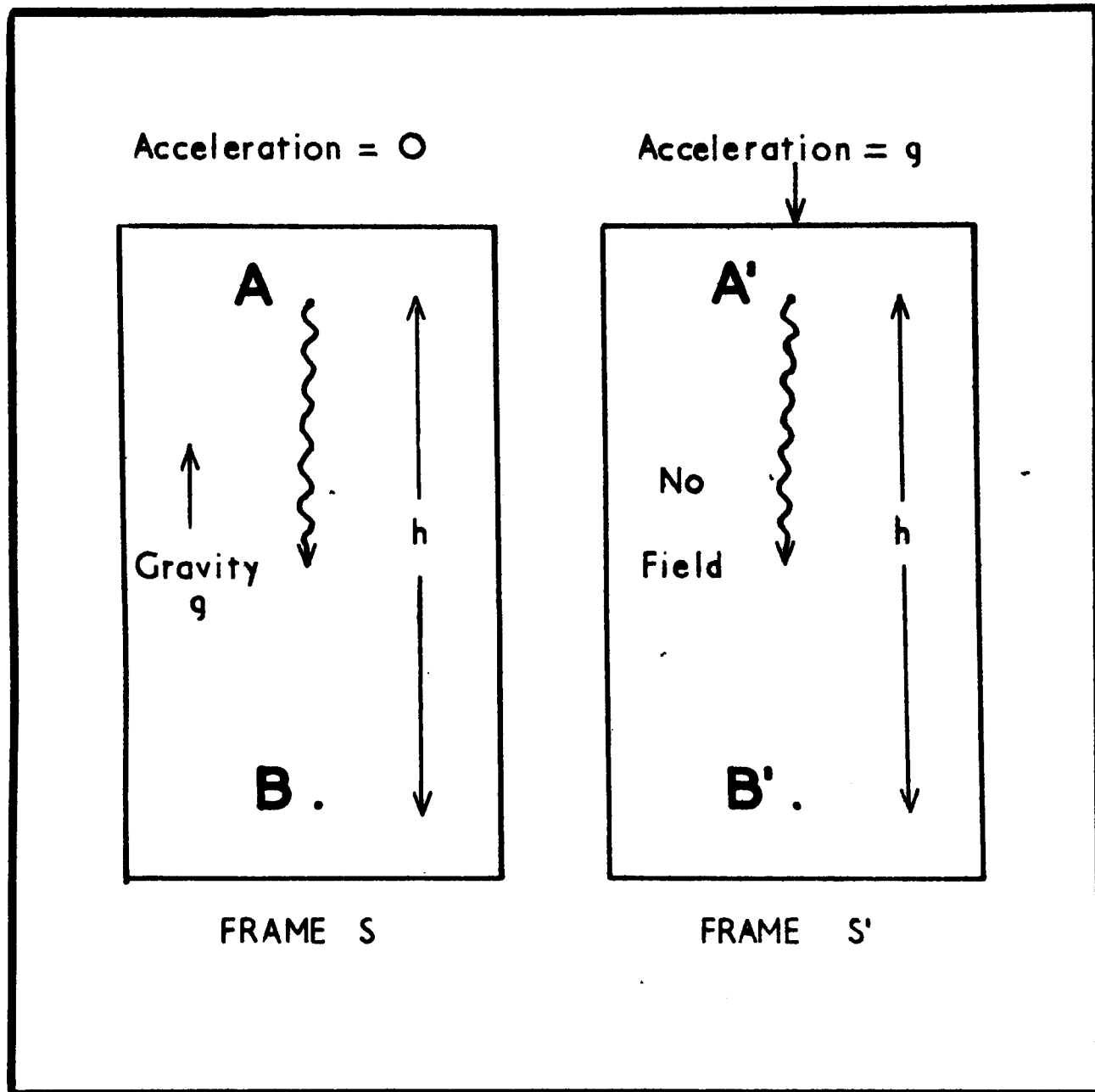


FIG. I.1

THE PRINCIPLE OF EQUIVALENCÉ

of spectral lines.

In the case of light passing through a non-uniform field, such as from the Sun to the Earth, the above principle applies only in each infinitesimal region of space. For such a case, we may, however, consider a very large number of "local" frames of reference along the light path, for each of which the field may be considered uniform. Then in each frame, from equation (1.1)

$$\begin{aligned}\frac{\Delta\nu}{\nu_0(r)} &= -\frac{g\Delta r}{c^2} \\ &= -\frac{GM_\odot\Delta r}{c^2 r^2}\end{aligned}\tag{1.2}$$

since $g = GM_\odot/r^2$. The total shift in passing from $R_\odot$ to R_E is given by:

$$\begin{aligned}\frac{\Delta\nu}{\nu_0} &= \int_{R_\odot}^{R_E} \frac{d\nu}{\nu_0(r)} \\ &= -GM_\odot \left(\frac{1}{c^2 R_\odot} - \frac{1}{c^2 R_E} \right)\end{aligned}\tag{1.3}$$

assuming $\nu_0(r)$ to be constant. Thus we have the equation

$$\nu = \nu_0 \left(1 - \frac{GM_\odot}{c^2 R_\odot} + \frac{GM_\odot}{c^2 R_E} \right)\tag{1.4}$$

If we neglect the third term, since $R_E \gg R_\odot$, and insert the values of $M_\odot$ and $R_\odot$, we arrive at the expression for the solar red shift due to gravity:

$$\frac{\Delta\nu}{\nu} = -2.12 \times 10^{-6}\tag{1.5}$$

It has been proposed that the observation of such a shift would constitute a test of the General Theory, although as we have seen, it is actually a test of the more fundamental Principle of Equivalence. At the time when observational verification of the theory was first sought, the only bodies having gravitational fields of sufficient magnitude to yield observable shifts were stars. It was therefore in astronomical observations that the verification was sought. These fell into two groups, observations of the solar spectrum, and observations of the spectra of massive stars, in particular white dwarfs and some O and B stars. For the former, although the shift is small, modern high dispersion spectra would allow its measurement. The latter objects, because of the large value of M/R , would yield larger shifts, but the observation of these is compensated by the limitations placed upon the dispersion that may be used, owing to the faintness of these stars. It has been found, however, that neither of these are satisfactory methods of testing equation (1.4), principally due to a lack of knowledge of the exact physical conditions accompanying the emission of the light in the stellar atmospheres. In addition, the masses of individual O and B stars are not known with any accuracy. The great difficulties in the interpretation of solar spectral observations will be discussed

in the next part of this chapter and in fact is the main theme of this thesis. A recent discussion of these astronomical tests of the gravity shift is given by M. G. Adam (2), in which these problems are illustrated.

In addition to the gravity shift of spectral lines, two other tests of the General Theory were originally proposed (3). These are the deflection of light rays passing near a massive body, e.g. the Sun, and the advance of the perihelion of minor planets, e.g. Mercury. These are both deduced from the geometrical structure of General Relativity, and observations of such effects are genuine tests of this theory (although, as Whittaker (4) points out, such effects may be predicted by other gravitational field theories). The astronomical observations of the first of these effects are quite inconclusive, and it seems, therefore, that the only astronomical verification of General Relativity to date is the advance of the perihelion of Mercury.

C. Terrestrial Tests of Equivalence and General Relativity.

It has recently become possible to test the validity of equation (1.1), and hence the Principle of Equivalence as applied to photons, in a terrestrial laboratory. This was made possible by the discovery, by Mössbauer (5), that

emission of certain γ -rays from the Fe^{57} nucleus occurs in such a manner that the emitted photons have a very precisely defined frequency. These are γ -rays emitted in the de-excitation of the 14.4 keV state of this nucleus. For a large fraction of emissions in this transition, the photon momentum is absorbed by the crystal structure, and this has the effect that most of the Doppler and recoil broadening of the line is removed. From very elementary considerations, it can be shown that in such an emission, the emitted photon has a frequency given by:

$$\frac{\nu_0 - \nu}{\nu_0} \approx \frac{P^2}{2m_0^2 c^2} + \frac{h\nu_0}{2Mc^2} \quad (1.6)$$

where ν_0 is the natural frequency of the transition, P is the momentum of the emitting nucleus, m_0 is its rest mass, and M is the "effective" mass of the crystal. These two terms are small, and hence the line width is very small, Pound and Rebka (6) finding a width given by $\frac{\Delta\nu}{\nu} = 1.13 \times 10^{-12}$. Aside from contributing to the line width, the above terms give rise to a shift in the emitted frequency, which is temperature dependent and also depends on the crystal structure.

Pound and Rebka have used this very narrow line to measure the frequency shift of a photon falling in the Earth's gravitational field. The shift expected from equation (1.1), for $h = 22.5$ metres, is $\frac{\Delta\nu_{\text{rel}}}{\nu} = 4.92 \times 10^{-15}$,

so that it was required to measure a shift of $1/200$ of the line width. By using long counting times and by carefully eliminating any residual shifts due to the two terms in equation (1.6), they obtained a final value of the observed shift, $\Delta\nu_{\text{obs}}$, given by $\Delta\nu_{\text{obs}} / \Delta\nu_{\text{rel}} = 1.05 \pm 0.10$.

A similar experiment had been carried out by Cranshaw et al. (7) at A.E.R.E. Harwell, using a fall of 12.5 metres. No corrections were made for possible residual shifts, and consequently their results have a much greater uncertainty. Their final value, however, agreed well with that above, being $\Delta\nu_{\text{obs}} / \Delta\nu_{\text{rel}} = 0.96 \pm 0.45$

It thus appears that the general Principle of Equivalence is valid, and with improved techniques expected to reduce the uncertainty in the above determinations, it seems that astronomical observations can no longer be expected to give the best verification. In fact it would seem that the emphasis must now change from an investigation of the validity of equivalence to an investigation of the observed deviations from its predictions.

As yet, no terrestrial test of General Relativity has been carried out, although such a test has recently been suggested. Schiff (8) has proposed that it may soon be possible to measure the precession of a spinning body in a terrestrial laboratory, that is predicted by the theory.

This precession is due to the distortion in the metric produced by the Earth's rotation and depends upon its off-diagonal elements. It is therefore a genuine test of the complete structure of General Relativity. Although the predicted precession is very small, about 6×10^{-9} radians/day, it may be measurable using a super-conducting gyroscope.

P a r t I I

OBSERVATIONS OF SOLAR WAVELENGTH

A. Definitions.

The many investigations of solar wavelengths in the past fifty years have given rise to different terminologies for the description of the observations. For the rest of this chapter, in which these observations will be discussed, and for the new observations to be presented, the following terminology will be adopted. A point of observation on the solar disk will be specified by two coordinates, ϕ and ρ/ρ_0 . The former is a position angle measured from a specified direction, and ρ/ρ_0 is the distance from the centre, expressed as a fraction of the radius of the disk. It is customary to also express distance from the centre by the angle Θ , where this is the angle between the line of sight and a true solar radius at the point of observation on the Sun. These two distance coordinates are related (to a high degree of approximation) by $\sin \Theta = \rho/\rho_0$.

Any variation of the wavelengths of spectral lines across the disk ($\cos \Theta = 1.00$ to 0.00) will be designated as "limb effect". Its value, denoted by $\Lambda(\Theta)$, will give the wavelength difference between the observed point and the centre of the disk. The term "red shift" will denote the difference between the wavelength of a line at a given point

on the Sun and the corresponding wavelength obtained in a terrestrial laboratory for similar physical conditions.

It should be noted that the Principle of Equivalence requires that all lines have a red shift of $\frac{\Delta \nu}{\nu} = - 2.12 \times 10^{-6}$ at all disk positions and thus predicts no limb effect. Since the predicted gravity shift can be expressed as an equivalent Doppler velocity of 0.636 km/sec, red shifts and limb-effect values are sometimes expressed in terms of their equivalent Doppler velocities for comparison.

B. Early Wavelength Observations and Interpretations.

In order to view recent observations of solar wavelengths in proper perspective, the trend of thought concerning the physical conditions in the solar atmosphere, over the last sixty years, must be considered. This is clearly reflected in the changing interpretations of the observations of wavelengths in the solar spectrum.

The study of solar wavelengths began with the observations of Fraunhofer in 1823, but it was not until 1896 that any differences between the solar wavelengths and corresponding terrestrial wavelengths were noticed. In that year, Jewell (9) concluded that the wavelengths of lines in the solar spectrum were generally longer than those in an ordinary arc spectrum. Although observations of granulation had indicated

that motions existed in the solar atmosphere, Jewell discounted the possibility that the observed difference in wavelength could be due to a Doppler effect and suggested that it was a pressure shift. From this assumption, the existence of large pressures in the solar atmosphere was deduced. Unfortunately, the physical conditions in the solar atmosphere were largely unknown at this time, and this concept of a pressure shift dominated the interpretation of solar spectra for nearly twenty years. One must remember that the gravity shift had not yet been predicted.

The existence of a limb effect was discovered in 1907 by Halm (10), when he found that the wavelengths of two iron lines (6300 Å) were 12 mÅ longer at the limb than at the centre of the disk. He attributed this effect also to a pressure shift, dependent upon the geometric path of the light through the solar atmosphere. According to this interpretation, the lower, higher pressure, layers of the atmosphere contributed more to the absorption of the line at the limb than at the centre of the disk. Buisson and Fabry, from an interferometric investigation of solar wavelengths (11,12), confirmed this discovery and supported Halm's interpretation. They found the lines at the limb to be shifted 5mÅ to the red and interpreted this to be due to a red asymmetry in the lines, produced by the greater effective pressure.

In 1910, Adams (13) undertook an extensive survey of limb effect for lines ranging in wavelength from 3800 Å to 6400 Å. He found that the limb effect varied from line to line and in some cases was even negative, obtaining values ranging from -4mÅ to 14 mÅ . An analysis of these results indicated that the lines originating at the lowest levels in the atmosphere had the largest limb effects and therefore he too supported the pressure shift interpretation.

After an analysis of new determinations of red shifts and limb effect, Evershed and co-workers (14,15,16,17) discarded the pressure interpretation. Instead they ascribed the observations to a general motion of the solar gases away from the Earth, the magnitude of the motion decreasing with depth in the atmosphere. The component in the line of sight of this motion was assumed to increase toward the limb, thus accounting for limb effect. This hypothesis became known as the "objectionable Earth hypothesis" and was seriously investigated, Evershed seemingly convinced of its validity. On the other hand, it was in one of this series of investigations (15), that the gravity shift proposed by Einstein was first considered as an explanation of the red shifts at the centre of the disk. These were of the right magnitude, but the gravity shift was discarded because of its inability to explain limb effect.

Another interpretation of solar wavelengths was advanced by Julius (18) who proposed that the shifts were due to anomalous dispersion in the solar atmosphere. A long and lengthy debate arose over this question (19), leading finally to the abandonment of the theory.

Evershed's concept of a motion of the atmospheric matter away from the observer was soon replaced by the concept of radial convection currents in the atmosphere. In order to explain the observations, however, St. John and Babcock (20) had to assume that in general the motion was downward, except for the very faint lines for which the motion was upward, i.e. the convective motions were depth dependent. The red shifts at the centre were successfully explained in this manner, but the problem of limb effect was left unsolved. Their observations indicated that the convective motions were quite steady, the centre wavelengths showing no time variation. At the limb, however, they found considerable fluctuations in the wavelengths.

It was in 1928 that the first physically reasonable theory of limb effect and centre red shifts appeared. According to the theory proposed by St. John (21), the observed effects were due to the superposition of the gravity shift and the Doppler effects of radial convection currents. From observations of 1537 lines at the centre of the disk,

he found a mean red shift of 8.3 mÅ with red shifts of individual lines varying from 6 mÅ to 12 mÅ. The deviations of these red shifts from the gravity shift of 9 mÅ were attributed to radial currents, inward at high atmospheric levels, and outward at lower levels. At the limb, observations of 133 lines gave a mean red shift exceeding the gravity shift by only 1.5 mÅ, whereas his theory would predict a red shift equal to the gravity shift. The occurrence of this slight "super-gravity" shift at the limb was only one of the two difficulties presented by the radial current theory. The other difficulty, more fundamental, was that it required a change in the direction of the convection currents with depth, which gives rise to an accumulation of matter at a certain level. The fundamental concept that photospheric convection currents play a role in the interpretation of solar wavelengths has, however, formed the basis of most subsequent theories of red shift and limb effect.

In the period from 1930 to 1940, several new determinations of limb effect and red shifts were made in order to verify or disprove the radial current-gravity shift hypothesis. In 1930, Freundlich, Brunn and Brück (22) published the results of a limb effect determination for nine lines of Fe at 4420 Å. From observations along four

radii of the 130 mm image (including polar and equatorial radii), they found similar limb effect curves. The mean $\Delta(\theta)$ so obtained increased rapidly near the limb ($\sin \theta > 0.8$) and they concluded that the variation with $\cos \theta$ did not fit St. John's predictions (requiring $\Delta(\theta)$ to be linear in $\cos \theta$). They also discounted the possibility that Compton scattering at the limb could have any effect.

On the other hand, Evershed (23) found that an analysis of his earlier (1916) limb effect results indicated that $\Delta(\theta)$ was linear in $\cos \theta$. His observations of several lines (including H and K) at the extreme limb, however, indicated shifts exceeding the gravity shift by as much as 7 mÅ. In a later paper (24), he finds even greater excesses at the limb, in some cases amounting to 12 mÅ. This latter figure corresponds to twice the gravity shift at the limb, and Evershed concluded that some modification of St. John's theory was required. Observations of similar super-gravity shifts in the wavelengths of lines originating in prominences convinced him that such a modification cannot be dependent on the path length of the light through the photosphere.

A survey of these early wavelength studies has been presented by Hunter (25), who concluded that no theoretical explanation of the various observations had yet been entirely satisfactory. In particular, no explanation had been found

for shifts at the limb exceeding the gravity shift, although the radial current theory of limb effect was generally favoured. Even in this respect, contradictions were to be found in the observations of various researchers. Although Evershed supported the $\cos \theta$ variation, Freundlich et al. did not and in 1939, Babcock (26) reported that he found a limb effect that was linear in $\sin \theta$. For $\sin \theta > 0.58$, $\Delta(\theta)$ increased from zero to 6 mÅ at $\sin \theta = 0.99$.

It is difficult to see why these observations obtained from an image 175 mm in diameter disagreed so much with those of Freundlich et al. On the other hand, the similarity of the limb effect for polar and equatorial radii found by the latter observers was confirmed by Babcock. It was in this unsatisfactory, contradictory position that the study and interpretation of solar wavelengths remained until 1948.

In that year, the first of a long series of investigations of solar wavelengths at Oxford was published. M. G. Adam (27) presented the results of absolute wavelength determinations for several points along a polar radius of a 180 mm image. These indicated that the limb effect was only appreciable for $\rho/\rho_0 > 0.9$, after which it increased very rapidly, giving a shift near the limb ($\cos \theta = 0.18$) approximately equal to the gravity shift. The form of the limb effect curve agreed well with the determination of

Freundlich et al., being non-linear in $\cos \Theta$, as is shown in Fig. 1.2. Nevertheless, it was shown that if radial convection currents do exist, the sight line velocity should vary, not linearly, but as a polynomial in $\cos \Theta$, thus these observations did not exclude a radial current interpretation. In this discussion, allowance was made for slight red shifts due to collisional damping as predicted by Lindholm (28). Because of its fundamental impossibility, she discarded the concept of predominantly upward currents in the lower photosphere and downward currents at upper levels, replacing these with a micro-circulation. In this interpretation, the derived sight line velocity is a mean of the upward and downward circulation velocities. Unfortunately, such an interpretation led to circulation velocities that were impossibly large (assuming a granular contrast of 10%). It was therefore concluded that even a modified radial current theory was physically incompatible with the observations and that a new theoretical treatment of the problem was required. In addition, better observations were needed near the limb where the limb effect varied rapidly.

These were the conclusions that prompted the intense investigations into limb effect and red shifts at the centre of the disk, of which the work presented in this thesis forms a part. In the next two sections, several interfero-

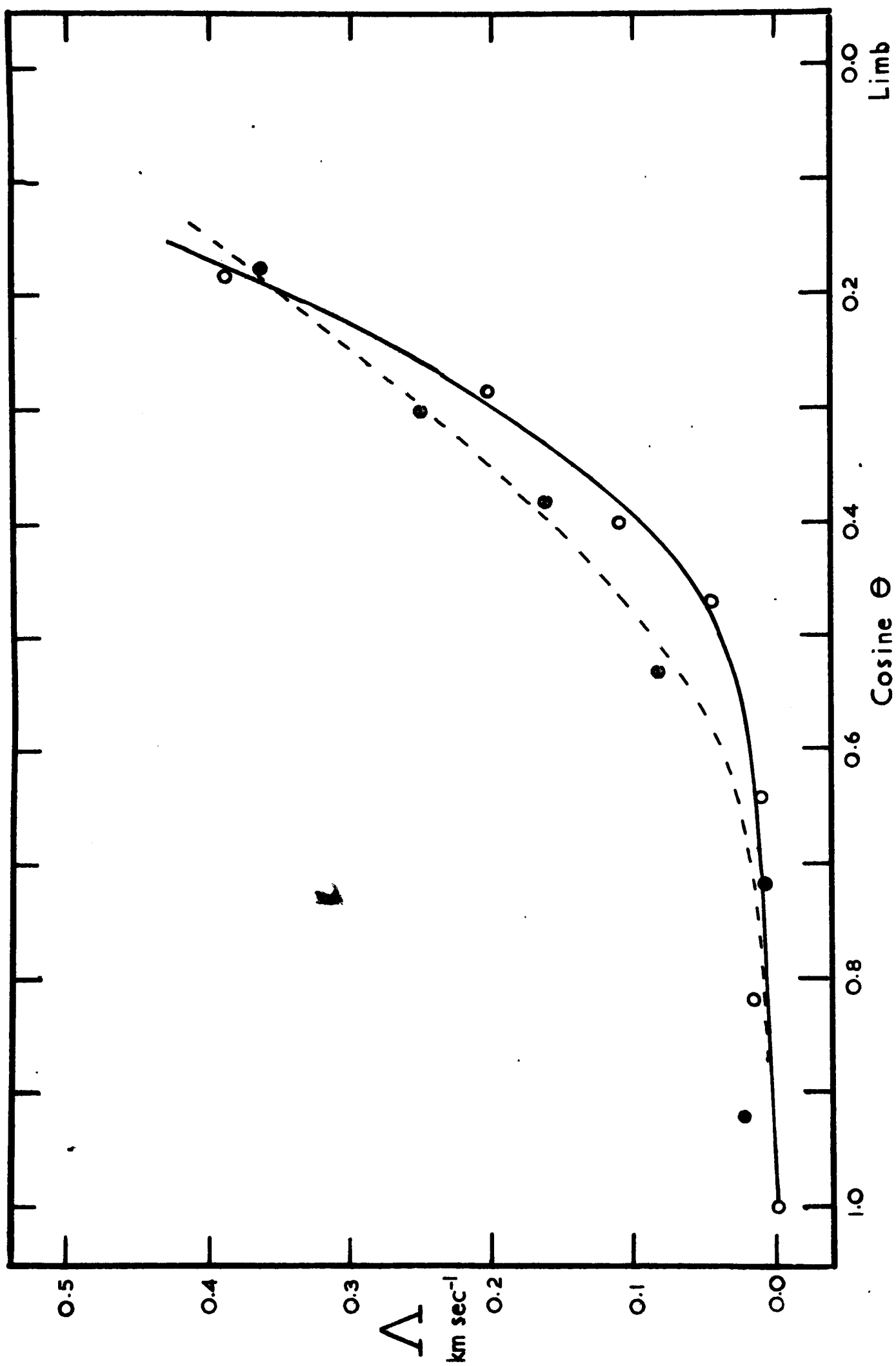


FIGURE 1.2 LIMB EFFECT OBSERVED BEFORE 1950
M.G. Adam 1948 $\circ$ — $\circ$ 14 lines 6100 Å
E.F. Freundlich et al. 1930 $\bullet$ --- $\bullet$ 9 lines 4420 Å

metric investigations of red shifts at the centre of the disk, and recent limb effect determinations will be briefly discussed.

C. Interferometric Determinations of Centre Red Shifts.

Early interferometric determinations of wavelengths, both in the solar spectrum and in the vacuum arc, were carried out by the Allegheny Observatory in conjunction with the Bureau of Standards (29,30,31) for many lines in the wavelength range of 3600-7000 Å. Integrated solar light was used for these observations, but it is now known that such a procedure yields wavelengths differing little from centre wavelengths. These indicated, as had other investigations, that the centre red shifts were dependent upon the intensity and wavelength of the lines, at this period, a theoretically inexplicable phenomenon. It later became, however, a fundamental basis of St. John's radial current hypothesis.

As mentioned above, it was suggested in 1948 that collisional shifts might occur in the photosphere. In fact, Spitzer (32) proposed that all the observed centre shifts were due to the combination of collisional shifts and the gravity shift. In order to investigate the above

intensity effect and the possibility of collisional shifts, a series of interferometric observations were conducted at Oxford. Utilizing the new interferometric method of "circular channels", M. G. Adam (33) showed that Spitzer's theory was not compatible with her observations, even if small convective motions were allowed. Moreover, it was found (34) that the red shift observed for the weak calcium resonance line at 6572.8 Å was very small, not exceeding the gravity shift as predicted by Spitzer. Later solar and laboratory observations (35,36) with reference to the 4227 Å resonance line of calcium indicated that collisional shifts in the solar atmosphere could not account for the observed red shifts.

From her observations, M. G. Adam concluded that there was a strong correlation between red shifts and the logarithms of equivalent widths of the lines, expressed as "absolute" widths ($\log W/\lambda$), confirming the earlier observed dependence on line intensity. A theoretical explanation of this relation was proposed by Schröter (37), arising from a new theory of line formation in an inhomogeneous photosphere (see Part III). A detailed theory based on the assumption of a granular circulation in the photosphere, Schröter's two-stream hypothesis predicted a red shift at the centre, dependent on the line strength, and a limb effect agreeing

fairly well with the early Oxford (27) and Potsdam (22) determinations. In a detailed comparison of this theory with her red shift and limb effect observations, M. G. Adam (35) concluded that the predicted and observed limb effects were not in agreement if one adopted centre red shifts consistent with both the theory and observations. In a statistical analysis of the red shift observations, she also found that a predicted correlation with excitation potential was in fact negligible. Moreover, an analysis of any correlation between the red shifts and wavelength proved to be inconclusive. Unless one chooses lines originating at exactly the same level for such an analysis, however, a linear relation is not a necessary consequence of a radial current theory. Other observations by Nichols and Clube (38) brought the number of red shifts determined at Oxford to 51, and extended the red shift - line strength relation. From these investigations, it appeared that the only observational evidence in support of the two stream theory was this correlation, and that in almost all other respects, theory and observation were in disagreement.

D. Recent Limb Effect Determinations.

In the last five years, three new investigations of limb effect have been carried out with particular emphasis

on the testing of the two stream theory. Since most wavelength shifts due to physical causes increase with wavelength, L. Hersberg (39,40) has studied the limb effect of several infrared solar lines. In general, limb effects (in km/sec) similar to those found by M. G. Adam (27) at 6100 Å were obtained, although the ionized calcium lines at 8500 Å showed a strong negative limb effect. It was suggested that this was due to the high level of formation of these lines.

In 1959, M. G. Adam (41) published the results of detailed measurements of limb effect, using three iron lines at 6300 Å and with a solar image 330 mm in diameter. This investigation was the most exact in existence when the research discussed in this thesis was begun, and supported her earlier 1948 observations. Moreover, it indicated a red shift at the extreme limb exceeding the gravity shift by 50%. Thus the question of the interpretation of a super-gravity shift again arose. Certainly no radial current theory can account for such shifts.

More recently, Schröter himself has obtained data for the determination of the limb effect of a large number of lines, using a new photographic technique to determine the "centre" of a spectral line. Preliminary results of this survey (42) indicate a limb effect for medium strength

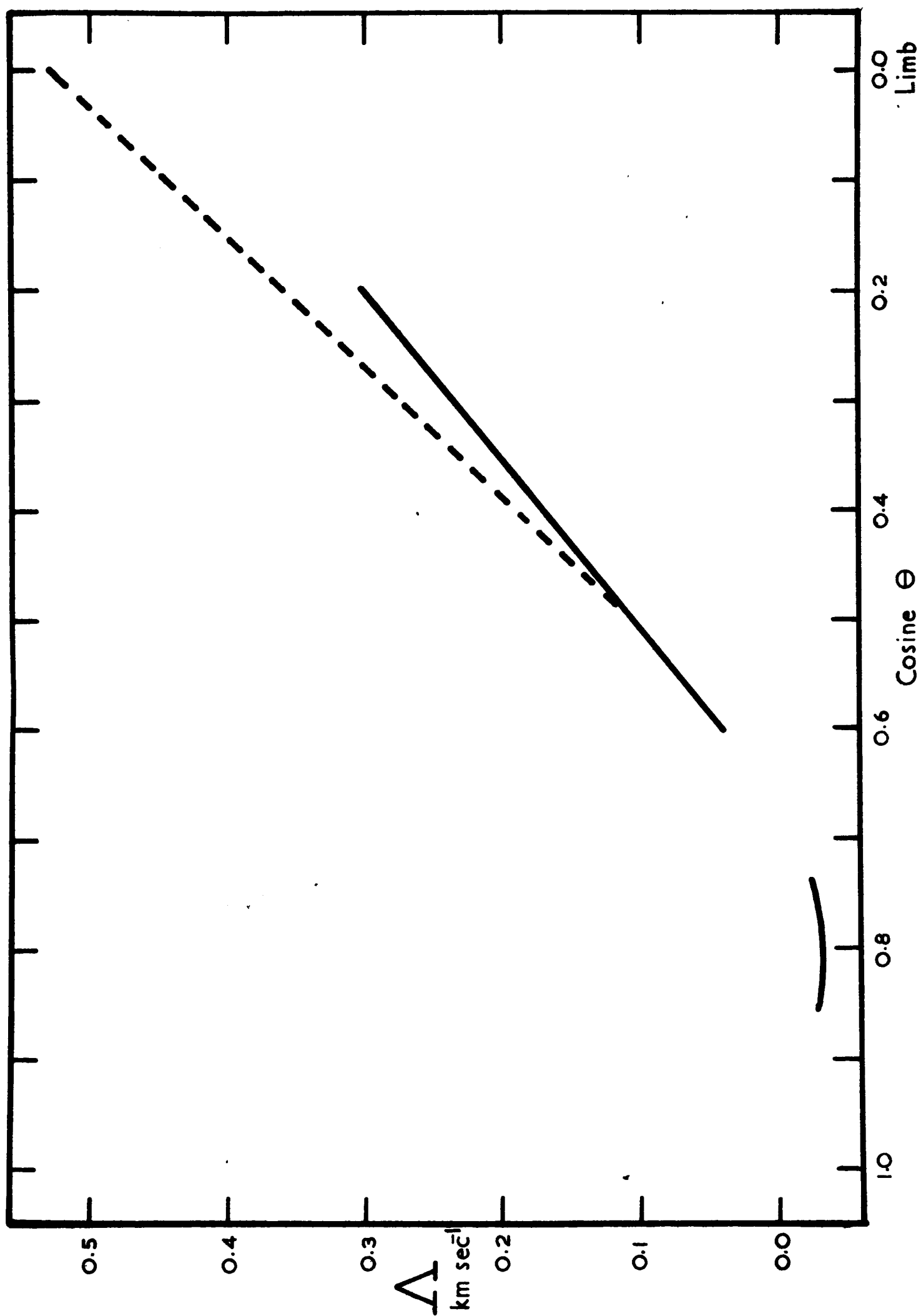


FIGURE 1.3 RECENT LIMB EFFECT CURVES

M.G. Adam 1959 ----- 3 lines 6300 Å
 E.H. Schröter 1959 ————— 9 lines 6400 Å

lines as given in Fig. 1.3. Also in this figure are the results of the 1959 determination at Oxford. The most notable features of these two curves are the approximate linearity of the curves in $\cos \theta$, (for $\cos \theta < 0.5$) and the negative limb effect found by Schröter at $\cos \theta = 0.8$. In addition, he obtained interferometric solar wavelengths at the centre of the disk which confirmed the relation between line strength and red shift. If δ represents the difference between the observed wavelength shift and the gravity shift, then the correlation he found was

$$\frac{\delta}{\lambda} \times 10^6 = -2.46 + 1.32 \log \left(\frac{W}{\lambda} \times 10^6 \right)$$
 This may be compared with the similar correlation found for the Oxford data (38) (over a greater range of equivalent widths) of

$$\frac{\delta}{\lambda} \times 10^6 = -1.28 + 0.60 \log \left(\frac{W}{\lambda} \times 10^6 \right)$$

This concludes the survey of observational data to date and some of the various interpretations proposed. Although much of the early observations were contradictory, suffering from large errors owing to equipment and technique, the recent data should be reliable. As a consequence, we must conclude that no theory at present is consistent with the data as a whole. The most acceptable theory at present is, however, that based on a convective photospheric circulation and this topic will be discussed in the next part of this chapter, along with two other recent theories of limb effect.

P a r t I I I

RECENT THEORETICAL INTERPRETATIONS OF SOLAR WAVELENGTHS

A. Radial Current Theories.

We have seen in the preceding part of this chapter that the early theories of pressure shifts, anomalous dispersion effects, the "objectionable" Earth, and finally the recent theory of collisional shifts have all been discarded in the light of more detailed observations and increased knowledge of the physics of the photosphere.

On the other hand, observations of granulation have indicated that turbulent motions exist owing to some convective instability in the photosphere. In fact, recent observations (43,44) indicate that there exists a semi-stationary convective circulation in the photosphere rather than a purely turbulent motion. It is therefore plausible that these convective motions will influence the profiles of spectral lines. This effect, and the gravity shift, which we have seen is now an effect demonstrable in the laboratory, must enter into any theoretical discussion of the problem. This does not, however, exclude other effects of unknown origin.

Aside from the early theory of St. John, detailed consideration of the effects of a photospheric circulation on wavelength observations began with recognition of the fact that such a circulation does not lead to a limb effect that is linear in $\cos \theta$. McCrea and Mitra (45) pointed out

that the equation of continuity,

$$\text{div} (\rho \vec{v}) = 0 \quad (1.7)$$

requires the circulation velocity to be depth dependent.

In such a case, Mitra (46) then showed that the mean observed velocity at an angle θ is given by:

$$\bar{v}(\theta) = \frac{\int_0^{\infty} v(\tau) e^{-\tau \sec \theta} \cos \theta \, d\tau}{\int_0^{\infty} e^{-\tau \sec \theta} \, d\tau} \quad (1.8)$$

The weighting factors in this equation result in a polynomial dependence of $\bar{v}$ on $\cos \theta$.

As mentioned in the previous discussion of observations, these results were combined into an elementary theory of a photospheric micro-circulation suggested by H.H. Plaskett and applied by M. G. Adam (27). She assumed that the observed velocities (assuming the centre red shifts to be due to a combination of Doppler shifts and the gravity shift) were means of the upward and downward velocities of the circulating elements. These means were weighted by the intensities of the two "streams" so that

$$\bar{v} = \frac{J_1 v_1 - J_2 v_2}{J_1 + J_2} \quad (1.9)$$

where the upward moving elements have a continuum intensity J_1 and a velocity v_1 , similarly for the downward moving elements. Using then available granulation theory, it

was found that the streaming velocities required by the observations were impossibly high. On the other hand, Thiessen (47) showed that this simplified approach to the problem was not inconsistent with the physics of the photosphere if a higher figure is used for the granulation contrast.

More recently, Schröter (37) has pursued this line of thought in detail, using a model photosphere consisting of granules streaming upwards through a downward sinking medium. A consequence of this model is that the Fraunhofer lines should be slightly asymmetrical. Schröter then uses this asymmetry and its variation across the disk to explain the red shifts at the centre and the limb effect. Since some of his results will be required for the discussion of the new observational material to be presented in following chapters, his "two stream" theory will be described in some detail. It should here be noted that a similar "three stream" theory has been developed by Voigt (48), which leads to similar asymmetry effects. No application, however, was made toward the interpretation of red shifts and limb effect.

Since no detailed criticism of Schröter's theory will be made in this chapter (see Chapter 5), the three basic assumptions underlying the theory should be pointed out: 1) the mean size of the granules (equal in area to the sinking elements) is taken to be $0.4''$; 2) the granular

contrast is taken to be 1.45 at the centre of the disk, increasing to a maximum at $\cos \Theta = 0.3$, and thereafter decreasing rapidly to unity; and 3) the Fraunhofer lines are formed by true absorption.

The temperatures and velocities of the two streams in Schröter's theory are adjusted so that the "integrated" photosphere is identical with an adopted homogeneous model. This is accomplished by satisfying the two relations:

$$P_g = P_b = P_h \quad (1.10)$$

$$\frac{1}{2} (\sigma T_g^4 + \sigma T_b^4) = \sigma T_h^4 \quad (1.11)$$

In these, the subscripts g, b, and h refer respectively to the granules, the intergranular background, and the homogeneous model. In addition, the equation of continuity requires that

$$\rho_g v_g = \rho_b v_b \quad (1.12)$$

The combination of this equation and equation (1.10) results in

$$\frac{v_g}{v_b} = \frac{T_h + \Delta T_g}{T_h - \Delta T_b} \quad (1.13)$$

where the ΔT 's represent the temperature deviations from the homogeneous model. The inequality of the two ΔT 's follows from elementary convection theory. Adopting

Prandl's concept of "mixing length" it can be shown that the mean velocity at a given level is related to the temperature deviation (from a static background) at that level by

$$\overline{V^2} = \frac{g d \Delta T}{2 T} \quad (1.14)$$

where d is the mixing length and g is the acceleration due to gravity. If β_g is used to express $\Delta T_g/T_h$, etc. then equations (1.13) and (1.14) may be combined into

$$\frac{1 + 2\beta_g + \beta_g^2}{1 - 2\beta_b + \beta_b^2} = \frac{\beta_g}{\beta_b} \quad (1.15)$$

or

$$\beta_g \approx \frac{\alpha}{1 - 2\alpha} \quad (1.16a)$$

from (1.15)

$$\beta_b \approx \frac{\alpha}{1 + 2\alpha} \quad (1.16b)$$

where α is a function depending on the level in the photosphere. To this point, the derivation of the model is standard, and only requires the adoption of some numerical values. Schröter does this by assuming a dependence of α on depth such that the predicted centre-limb variation of the granular contrast agrees with observations. The emergent continuum intensities for the two streams are readily found from:

$$J_g(0, \theta) = \int_0^1 B_g(\tau) d e^{-\tau \sec \theta} \quad (1.17a)$$

$$J_b(0, \theta) = \int_0^1 B_b(\tau) d e^{-\tau \sec \theta} \quad (1.17b)$$

when the depth dependence of the Planck function B (which

depends upon α) is known. The trial and error fitting of α resulted in the theoretical granulation contrast given in Fig. 1.4, which agrees with the observational data adopted by Schröter.

Schröter assumed that the granules were small ($0.4''$) and generally unresolved, so that the continuum intensity actually observed was $J(0, \theta) = \frac{1}{2}(J_g(0, \theta) + J_b(0, \theta))$. In order to determine the effects of this unresolved structure on line profiles, Schröter first determined the line profiles that would be observed if the lines were formed in the sinking background only, and in the rising granules only. Such profiles were derived for both a fictitious medium-strength iron line and the strong iron line at 4045.8 Å. The computation of these was as follows. For a given stream, the profile is given by:

$$1 - r_\nu(0, \theta) = \frac{J(0, \theta) - J_\nu(0, \theta)}{J(0, \theta)} \\ = \int_0^\infty g(\tau, \theta) \frac{k_\nu(\tau)}{k(\tau)} e^{-\tau_\nu(\tau) \sec \theta} d\tau \quad (1.18)$$

where τ is the optical depth in the continuum, τ_ν is the selective optical depth in the line, k_ν and k are the selective and continuous absorption coefficients, and the $g(\tau)$ is the weighting coefficient for true absorption. The values of τ_ν , k_ν and k as functions of τ were found

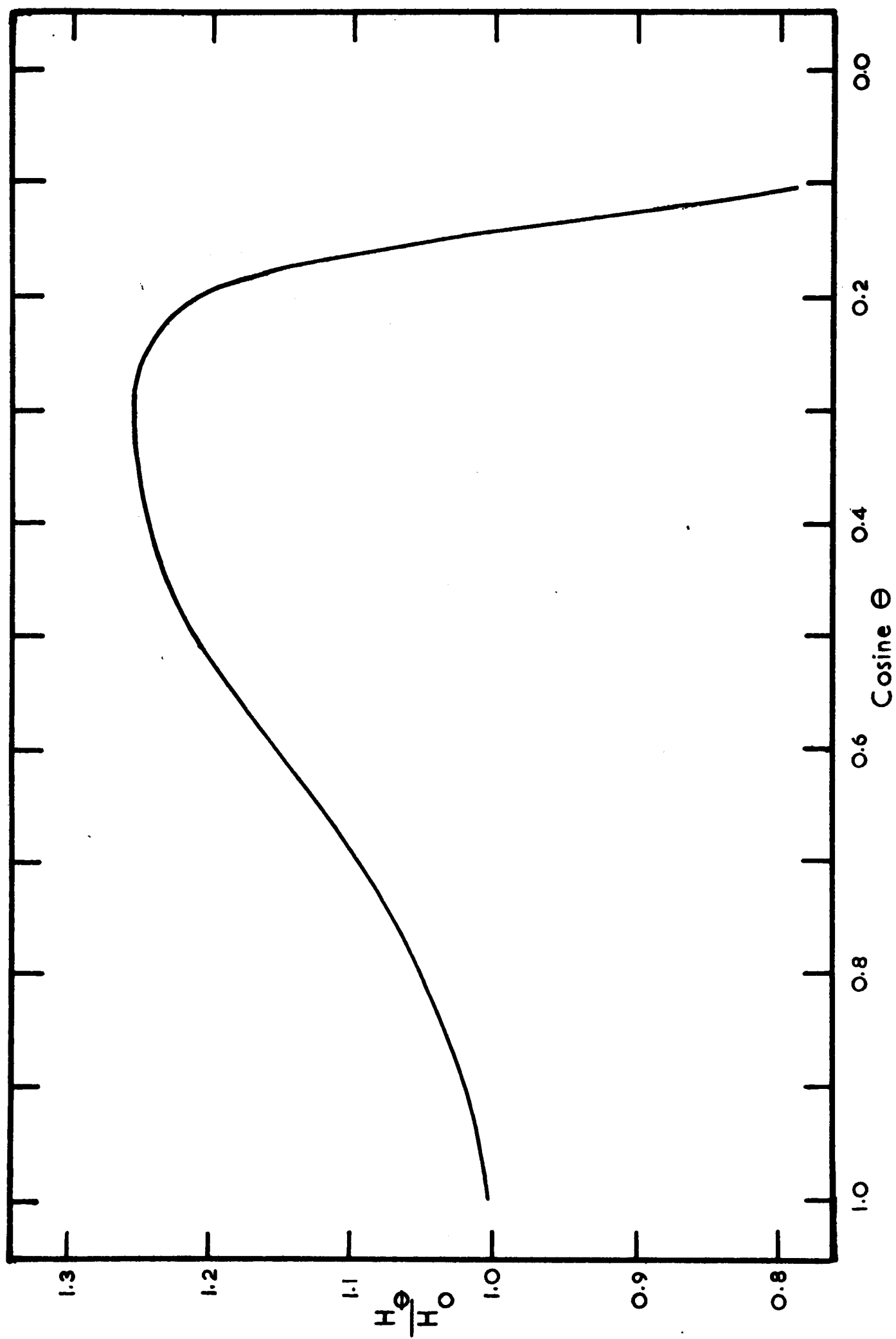


FIGURE 1.4 GRANULATION CONTRAST FOR TWO STREAM MODEL

$$H_\theta = j_g(\theta)/j_f(\theta)$$

from the two stream model, assuming a Voigt profile for the absorption coefficient. At the centre of the line, where weighting functions cannot be used, the value of $J_\nu(0, \theta)$ was chosen equal to $B(\tau^*)$ where $\tau^* + \tau_\nu = 1$.

In order to obtain the mean profile resulting from a superposition of the profiles from the two streams, the Doppler shifts due to the stream velocities had to be allowed for. For a given stream profile, the Doppler effects vary with the frequency since different parts of the profile refer to different depths in the photosphere. Therefore at each frequency, a mean velocity was computed from

$$\overline{V}_\nu = \frac{\int_0^\infty \frac{k_\nu}{k} g(\tau, \theta) e^{-\tau_\nu(\tau) \sec \theta} v(\tau) d\tau}{\int_0^\infty \frac{k_\nu}{k} g(\tau, \theta) e^{-\tau_\nu(\tau) \sec \theta} d\tau} \quad (1.19)$$

where the stream velocity $v(\tau)$ was found from the two stream model. The profiles found for each stream were then modified at each wavelength by a Doppler shift Δ given by $\Delta = \overline{V}_\nu \lambda / c$. Naturally the effect of this was to give most parts of the "rising" profile a violet shift, and most parts of the "sinking" profile a red shift. The addition of the two profiles then gave the resulting observed profile, the asymmetry of which depends upon the magnitude of the Doppler effects for the two streams, and the strength of

the line in the two streams. The asymmetry determined for the two iron lines (with equivalent widths of 150 and 1500 mÅ) is shown in Fig. 1.5, the curves denoting the centre lines of the corresponding profiles. It can be seen from this that the profile of the stronger line is affected most by the sinking elements. Schröter points out that this is due to the smaller temperature gradient and consequently greater effective absorption in the cooler elements. The results presented in the figure are the basis of the dependence of the centre red shifts on line strength, according to this theory, since the stronger the line is, the greater red asymmetry it has. In a wavelength determination, this is then interpreted as a wavelength shift. In addition, this theory predicts a dependence of the asymmetry on the excitation potential of the lower state, because lines of higher excitation are formed predominantly in the hotter elements.

If one designates to which level in a line profile a wavelength measurement refers (say $r_{\lambda} = 0.2$ to 0.8) then an effective shift, $\bar{\Delta}_{\text{eff}}$, for the centre can be deduced from Fig. 1.5. The observed wavelength is then given by:

$$\lambda = \lambda_0 + \Delta\lambda_{\text{gravity}} + \bar{\Delta}_{\text{eff}} \quad (1.20)$$

This expresses the red shift-line strength relation since

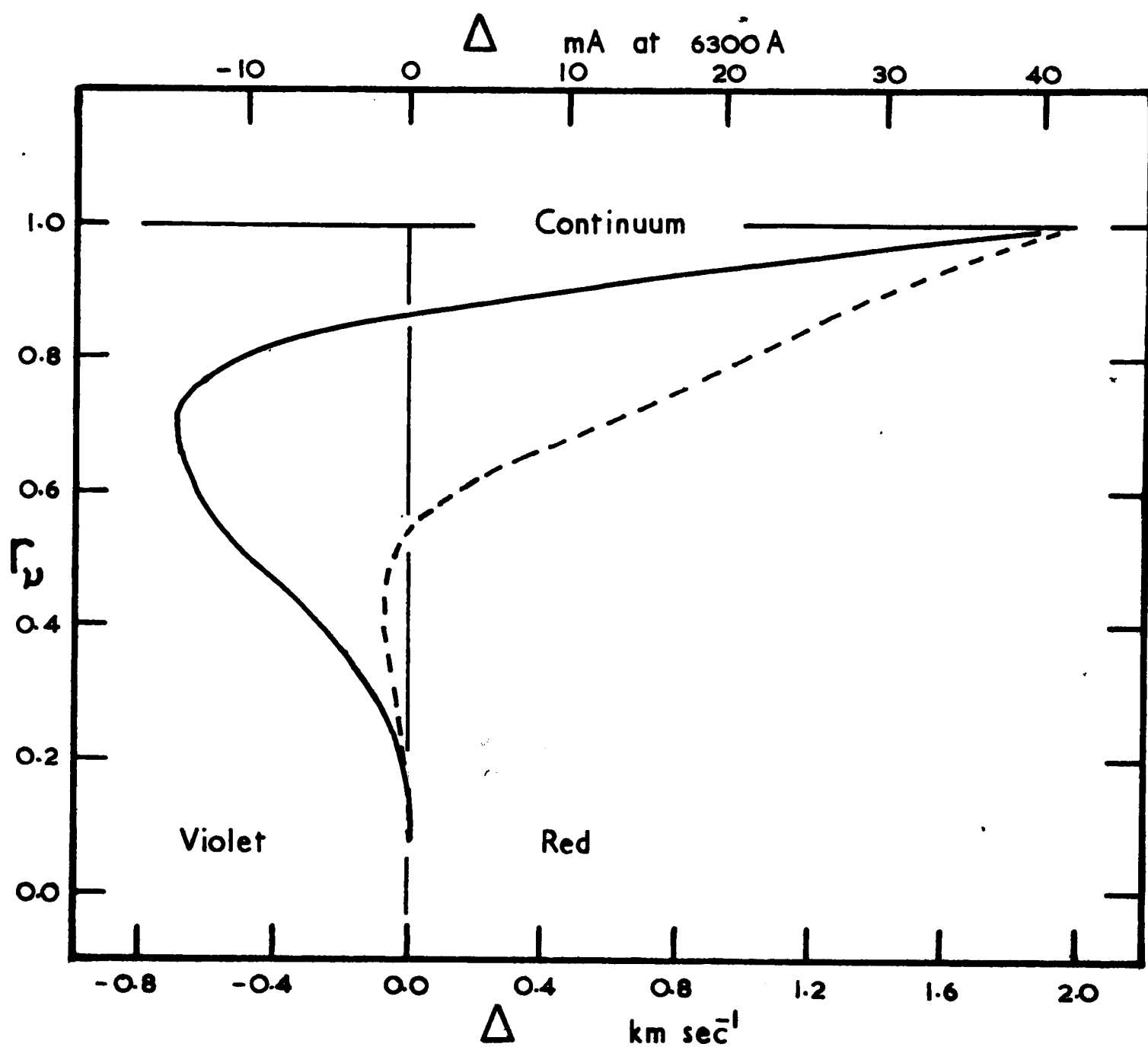


FIGURE 1.5 LINE ASYMMETRY PREDICTED
FROM TWO STREAM MODEL
(denoted as in Fig.4.1(a))

Medium Strength Iron Line ——— E.W. = 150 mÅ
Strong Iron Line (4045.8 Å) ----- E.W. = 1.5 Å

$\overline{\Delta}_{\text{eff}}$ is negative for weak lines and positive for strong lines.

Since $\overline{\Delta}_{\text{eff}}$ is a function of θ , equation (1.20) also expresses the limb effect. In order to find the dependence of $\overline{\Delta}_{\text{eff}}$ on θ , Schröter repeated the above computations of profiles for various disk positions. He found that its variation was determined mainly by the granulation contrast, and it is given in Fig. 1.6, where

$$\beta(\theta) = \frac{\overline{\Delta}_{\text{eff}}(\theta)}{\overline{\Delta}_{\text{eff}}(0)}$$

Since it is known from the observations that granular contrast varies little with wavelength, this may be considered to be fairly independent of wavelength. The observed red shift, due to the variation of $\overline{\Delta}_{\text{eff}}$ is then given by:

$$\delta\lambda(\theta) = \Delta\lambda_{\text{gravity}} + \overline{\Delta}_{\text{eff}}(0) \beta(\theta) \cos \theta \quad (1.21)$$

As discussed in the preceding section, Schröter's theory is satisfactory in that it predicts the line strength dependence of red shifts at the centre, but does not give a limb effect consistent with recent observations. Further discussion of this point will be given in Chapter 5, in the light of the new data. Clearly this theory is in need of modification before it can be considered a satisfactory explanation of the observations.

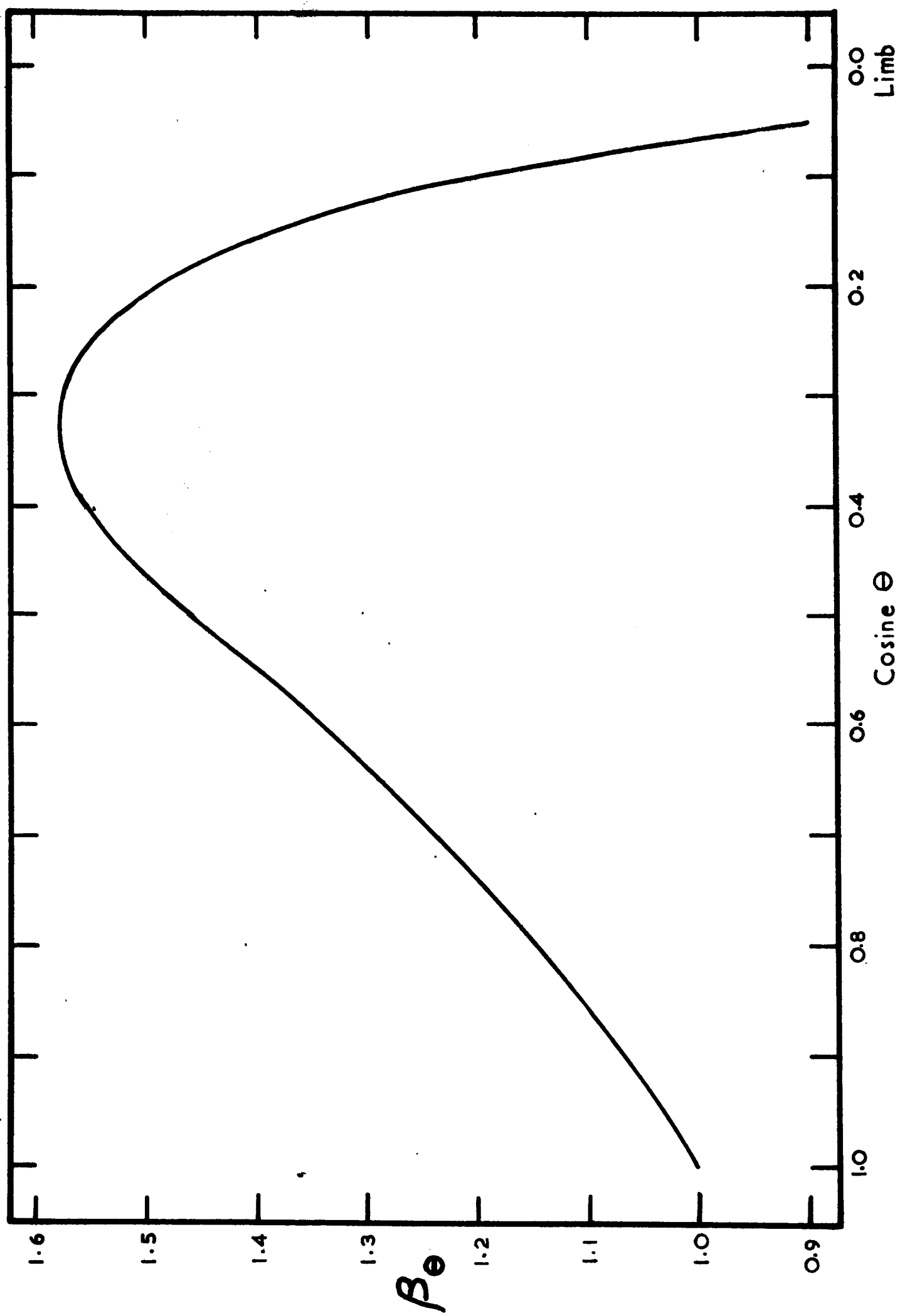


FIGURE 1.6 VARIATION OF PREDICTED ASYMMETRY
ACROSS THE DISK

B. Other Interpretations.

Before concluding this survey of theoretical considerations, two other proposals must be briefly mentioned. It has been suggested by Finlay-Freundlich and Forbes (49,50) that the observed limb effect is due to a path length interaction between the emitted photons and the solar radiation field. This was proposed as an alternative to the gravity shift-Doppler effect combination favoured by most researchers. It results in a red shift equation of the form

$$\delta\lambda = X + Y \cdot f(\theta, R/l_0) \quad (1.22)$$

where $f(\theta, R/l_0)$ is a function approaching $\sec \theta$ for $R \rightarrow \infty$, and X and Y are unknown coefficients. Such an approach, although lacking in physical basis, can give a super-gravity shift at the limb. Also, according to its supporters, it fitted the early Potsdam (22) and Oxford (27) observations very well. On the other hand, it offers no explanation of the residual shifts represented by the X 's, or of a negative limb effect as observed by Hertzberg (40). It cannot be claimed, therefore, to fit the observations any better than Schröter's theory and is certainly far less plausible.

Another effect has recently been proposed by Rudkjøbing (51) which depends upon a general form of perturbation of the

atoms in the photosphere. If the atoms are perturbed so that the energy of the state emitting light with its $\mathbf{B}$ vector in a radial direction is not the same as that of the state emitting light with its $\mathbf{E}$ vector perpendicular to the radial direction, then the wavelengths observed at the centre and the limb will be different. This arises from the fact that at the centre, only light from the second state would be observed while at the limb, a mixture of light from both states would be observed. If the perturbation introduces a wavelength shift $\Delta\lambda$ between the two emitting states, then at any disk position two lines will be observed, with wavelengths λ and $\lambda + \Delta\lambda$, and intensities $K(1 + \cos^2\theta)$ and $K \sin^2\theta$. If the centre of gravity of the two lines is taken as the centre of the "composite" line, then a limb effect given by $\Delta = \frac{\Delta\lambda \sin^2\theta}{2}$

results. In addition, at the limb these two lines will be linearly polarized in perpendicular directions, the one with the longer wavelength being polarized with the $\mathbf{E}$ vector radial. It can easily be seen that such a limb effect curve does not fit the Oxford data (27,41) at all, nevertheless polarization measurements at the limb could distinguish an effect such as this.

C. Observations of Asymmetry.

We have seen that a fundamental result of the two stream

theory of Schröter is that lines at the centre of the disk are, in general, asymmetric. From his similar theory, Voigt (48) had suspected such an effect, and expected a line of high excitation potential to exhibit a measurable asymmetry. From observations using a dispersion of 5.5 mm/A, he found such an asymmetry for the fairly weak lines of the oxygen triplet at 7770A. He denoted asymmetry by $(h_v - h_r)/(h_v + h_r)$ where h_v and h_r are the widths at half intensity from the line centre to the violet and red flanks. At the centre, the observed asymmetry amounted to 9%, decreasing to zero at $\cos \theta = 0.1$. Further observations were made of the potassium resonance line at 7699 A. This line arises at high levels in the photosphere and consequently little asymmetry could be expected. As predicted, none was observed. Another high excitation line, the carbon line at 5380 A, was observed in the hope of finding a large asymmetry as for the oxygen lines. None was found, however, indicating that perhaps the line was shifted as a whole to the violet. Indeed, Voigt's theory predicted such a shift, maintaining the line symmetry, for such a weak line. Other observations (52), of the nickel lines at 7790 A, also failed to reveal any line asymmetry, but the amount predicted for these lines was very small in any case.

Recently, Voigt's observations of the oxygen lines

have been questioned by Delbouille, de Jager and Neven (53) who find asymmetries of less than 3% for these lines using photoelectric observations. No explanation has yet been given for the discrepancy between the two.

Aside from these measurements, no reliable asymmetry determinations have been made, primarily due to the difficulties involved in removing instrumental effects from the observed profiles. In any case, there is insufficient data available on line asymmetries for any conclusion to be drawn concerning the validity of one theory or another.

This concludes the introductory chapter on the history of the study of the solar red shift. The next chapter is concerned solely with computational methods required in the observational programme to be discussed in the following chapters. It can therefore be omitted without breaking the continuity of the development of the work on red shifts.

References

1. A. Einstein, Annal d. Physik 35, 898, 1911.
2. M. G. Adam, The New Scientist, Vol. 8, 863, 1960.
3. A. Einstein, Annal. d. Physik 49, 769, 1916.
4. E. Whittaker, "History of the Theories of Aether and Electricity", Vol. 2, p. 148, Thomas Nelson and Sons Ltd., 1953.
5. R. L. Mossbauer, Zs. f. Phys. 151, 124, 1958.
6. R. V. Pound and G. A. Rebka Jr., Phys. Rev. Letters 4, 337, 1960.
7. T. E. Cranshaw, J. P. Schiffer and A. B. Whitehead, Phys. Rev. Letters 4, 163, 1960.
8. L. I. Schiff, Phys. Rev. Letters 4, 215, 1960.
9. L. E. Jewell, Ap. J. 3, 89, 1896.
10. J. Halm, Astron. Nachrichten 173, 273, 1907.
11. H. Buisson and C. Fabry, Comptes Rendus 148, 1741, 1909.
12. C. Fabry and H. Buisson, Ap. J. 31, 97, 1910.
13. W. S. Adams, Ap. J. 31, 30, 1910.
14. J. Evershed, Kodaikanal Obs. Bull. No. 36, 1913.
15. J. Evershed and T. Royds, Kodaikanal Obs. Bull. No. 39, 1914.
16. J. Evershed and A. A. Narayana Ayyar, Kodaikanal Obs. Bull. No. 46, 1915.
17. J. Evershed and T. Royds, Kodaikanal Obs. Bull. No. 49, 1916.
18. W. H. Julius, Ap. J. 31, 419, 1910.

19. C. E. St. John, Ap. J. 44, 311, 1916.
20. C. E. St. John and H. D. Babcock, Ap. J. 55, 36, 1922.
21. C. E. St. John, Ap. J. 67, 195, 1928.
22. E. F. Freundlich, A. v. Brunn, and H. Brück, Zs. f. Ap. 1, 43, 1930.
23. J. Evershed, M.N. 91, 260, 1931.
24. J. Evershed, M.N. 96, 152, 1936.
25. A. Hunter, M.N. 94, 594, 1934.
26. H. D. Babcock, P.A.S.P. 51, 222, 1939.
27. M. G. Adam, M.N. 108, 446, 1948; Comm. Univ. Obs., Oxford, No. 26.
28. E. Lindholm, Arkiv. f. Mat., Astr. och Fys., 28 B, No. 3, 1941.
29. K. Burns, Publ. Alleg. Obs. VI, 141, 1927.
30. K. Burns and C. C. Kiess, Publ. Alleg. Obs. VI, 125, 1927.
31. K. Burns and W. F. Meggers, Publ. Alleg. Obs. VI, 105, 1927.
32. L. Spitzer, M.N. 110, 216, 1950.
33. M. G. Adam, M.N. 112, 149, 1952; Comm. Univ. Obs., Oxford, No. 36.
34. M. G. Adam, M.N. 115, 367, 1955; Comm. Univ. Obs., Oxford, No. 49.
35. M. G. Adam, M.N. 118, 61, 1958; Comm. Univ. Obs., Oxford, No. 66.
36. W. R. Hindmarsh, M.N. 119, 81, 1959; Comm. Univ. Obs., Oxford, No. 68.
37. E. H. Schröter, Zs. f. Ap. 41, 141, 1957.
38. S. Nichols and S. V. M. Clube, M.N. 118, 73, 1958; Comm. Univ. Obs., Oxford, No. 67.

39. L. Herzberg, Can. Jour. Phys. 35, 766, 1957.
40. L. Herzberg, Can. Jour. Phys. 38, 853, 1960.
41. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.
42. E. H. Schröter, Mitt. Astrophys. Obs. Potsdam, Nr. 83, 1959; Monatsberichte d. Deutsche Akad. d. Wiss., Bd. 1, Hft. 12, 738, 1959.
43. D. E. Blackwell, D. W. Dewhirst, and A. Dollfus, M.N. 119, 98, 1959.
44. M. Schwarzschild, Ap. J. 130, 345, 1959.
45. W. H. McGrea and K. K. Mitra, Obs. 57, 379, 1934.
46. K. K. Mitra, Obs. 59, 160, 1936.
47. G. Thiessen, Zs. f. Ap. 35, 237, 1955.
48. H. H. Voigt, Zs. f. Ap. 40, 157, 1956.
49. E. Finlay-Freundlich and E. G. Forbes, Annal. d'Ap. 19, 183, 1956.
50. E. Finlay-Freundlich and E. G. Forbes, Annal. d'Ap. 19, 215, 1956.
51. M. Rudkjøbing, Annal. d'Ap. 23, 802, 1960.
52. H. H. Voigt, Zs. f. Ap. 47, 144, 1959.
53. L. Delbouille, C. de Jager, and L. Neven, Annal d'Ap. 23, 949, 1960.

Chapter 2

COMPUTATIONAL METHODS

General Introduction.

In the next two chapters, new determinations of limb effect, red shifts, and solar line profiles will be discussed. These new observations depend upon the derivation of wavelengths and line profiles from solar spectra. Much of the time used in the analysis of solar spectra is spent on certain simple yet laborious calculations, the tedious reduction of interferometric plates for absolute wavelength determinations being such a case. Since a modern electronic computer was available at Oxford, an attempt was made to do these computations on such a machine. Calculations of the type just mentioned are characterized by the fact that the numerical procedure is the same whether done by electronic machine or by desk calculator. On the other hand, several other problems, of a different type, arose for which machine methods were also developed. These are problems that are usually solved graphically or perhaps only solved approximately, if at all. These require special numerical methods for machine computation, and present difficulties peculiar to this type of problem. This arises from the fact that ordinarily in the solution of such problems, the astronomer applies considerable intuition and experience, both of which are qualities difficult to obtain from an electronic computer. An example of such a problem is the

determination of the characteristic response curve of a photographic plate.

In this chapter, several problems for which programmes have been developed will be discussed. All the programmes have been written in the simplified coding system of "Autocode" (1), and the computations were made on the Ferranti "Mercury" computer of the Oxford University Computing Laboratory. Five main problems were investigated and about ten programmes were written for their solution.

The first two problems, namely the determination of wavelength shifts in solar spectra due to the observer's motion and solar rotation, and the reduction of interferometric plates, are of the simple straight-forward type, and presented few difficulties. The remaining three problems were of the second type mentioned above, however, and the machine methods adopted are not entirely satisfactory. These three problems were the determination of characteristic curves, the solution of certain integral equations, and the general reduction of solar line profiles. In the following, the methods adopted for their solution will be discussed.

P a r t I

**COMPUTATION OF SOLAR COORDINATES AND RADIAL VELOCITY
CORRECTIONS**

A. Introduction and Notation.

The observed wavelength of a solar spectral line requires two corrections before a true wavelength may be obtained. The first is due to the motion of the observer (sometimes designated Earth's motion correction) which includes the effect of the Earth's rotation and the Earth's orbital motion. Secondly a correction must be applied to allow for solar rotation. These corrections depend upon the point of observation on the solar disk and the date and time of observation. Several programmes were written for the computation of both of these corrections as well as for the computation of the coordinates of the solar disk at the time of observation. For the interpretation of solar wavelengths, it is also useful to know the direction cosines of the line of sight in a photospheric frame of reference, so that these too are given by the computations. In the following description of the computational procedure, the notation will be as follows:

θ The angle ($\leq \pi/2$) between the line of sight and a true solar radius at the point of observation

θ^* The angle ($< \pi/2$) between the true solar radius to the sub-terrestrial point and the true solar radius to the point of observation.

S	The apparent angular radius of the solar disk (semi-diameter).
ρ_0	The linear radius of the solar image.
ρ	The linear radius to the point of observation on the solar image.
P	The position angle of the solar north pole measured eastward from the north point of the disk.
ϕ	The position angle of the point of observation measured eastward from the north point of the disk.
ψ	The position angle of the point of observation measured westward from the solar north pole.
B_0	The heliographic latitude of the sub-terrestrial point.
L_0	The heliographic longitude of the sub-terrestrial point.
δ_0	The apparent declination of the sub-terrestrial point.
α_0	The apparent right ascension of the sub-terrestrial point.
H_0	The hour angle of the sub-terrestrial point.

(These last five symbols without subscripts refer to the point of observation.)

X, Y, Z	The geocentric equatorial rectangular coordinates of the Sun (arbitrary equinox).
$\dot{X}, \dot{Y}, \dot{Z}$	The time derivatives of the rectangular coordinates of the Sun (equinox same as X, Y, Z).

$\cos \gamma_1$
 $\cos \gamma_2$
 $\cos \gamma_3$

The direction cosines of the line of sight, directed away from the observer, in a right-handed system of rectangular axes at the point of observation. The x axis is in the direction of solar rotation, the y axis is along the local meridian, and the z axis is radially outward.

R_0

The solar equatorial velocity of rotation (km/sec).

B. General Method.

In order to compute the wavelength corrections for various points of observation, the coordinates of the sub-terrestrial point, the equatorial rectangular coordinates of the Sun and their derivatives, and the parameters P , S , B_0 , and L_0 must all be known for the time of observation. In the machine computations, these are interpolated from short tables, taken directly from the Astronomical Ephemeris, which are entered as data along with the U.T.'s of the observations.

The right ascension and declination of the sub-terrestrial point, and the equatorial rectangular coordinates, X, Y, Z , of the Sun are interpolated using the Stirling interpolation formula:

$$f(x) = f(0) + \frac{x}{2} \left(\delta_{-\frac{1}{2}}^1 + \delta_{\frac{1}{2}}^1 \right) + \frac{x^2}{2} \delta_0^2 + \frac{(x+1)x(x-1)}{12} \left(\delta_{-\frac{1}{2}}^3 + \delta_{\frac{1}{2}}^3 \right) \quad (2.1)$$

The values of the equation of time, P , S , B_0 and L_0 are interpolated using the above equation to second differences. The hour angle, H_0 (hours), of the sub-terrestrial point is then found from:

$H_0 = \text{U.T.} - \text{Longitude of Oxford} + \text{Equation of Time} + 12$
 where the longitude of Oxford is taken to be 5 min. (West).
 The derivatives of X , Y , and Z at 0 hrs. U.T. are then found using the Stirling formula:

$$\dot{f}(0) = \frac{1}{2} \left(\delta_{-\frac{1}{2}}^1 + \delta_{\frac{1}{2}}^1 \right) - \frac{1}{12} \left(\delta_{-\frac{1}{2}}^3 + \delta_{\frac{1}{2}}^3 \right) \quad (2.2)$$

From these values, the derivatives at the time of observation are interpolated using the interpolation formula given above.

For some observations, the angle ϕ may be determined from H_0 , in which case $\phi = H_0 + \text{constant}$. In such a case, the machine also computes ϕ for the time of observation.

Each observed point on the solar disk is specified by the two coordinates, ρ/ρ_0 and ϕ (the latter may be a computed quantity) and these are entered as data for the second part of the computations. Following the procedure given by Plaskett (2), from these data and the above computed coordinates, the wavelength corrections (more exactly, the equivalent Doppler velocity corrections), the solar latitude,

and the three direction cosines of the line of sight are computed for each observed point.

The corrections to the equivalent Doppler velocities for the Earth's rotation, orbital motion, and the Sun's rotation have been called C_1 , C_2 , and C_3 respectively, and are given (in km/sec) by the expressions:

$$C_1 = -0.2885 \cos \delta \sin H \quad (2.3)$$

$$C_2 = -1732.34 (\dot{X} \cos \delta' \cos \alpha' + \dot{Y} \cos \delta' \sin \alpha' + \dot{Z} \sin \delta') \quad (2.4)$$

$$C_3 = -f.R_0 \quad (2.5)$$

where $\delta = \delta_0 + \rho/\rho_0 S \cos \phi \quad (2.6)$

$$H = H_0 - \rho/\rho_0 S \sec \delta_0 \sin \phi \quad (2.7)$$

$$\alpha' = \arctan Y/X + \rho/\rho_0 S \sec \delta_0 \sin \phi \quad (2.8)$$

$$\delta' = \arctan \frac{Z}{\sqrt{X^2 + Y^2}} + \rho/\rho_0 S \cos \phi \quad (2.9)$$

$$f = (1 - 0.229 \sin^2 B) \rho/\rho_0 \cos B_0 \sin \psi \quad (2.10)$$

and $\dot{X}$, $\dot{Y}$, and $\dot{Z}$ are in A.U./solar day

The expression obtained for C_3 depends upon the adoption of the law of solar rotation used by M. G. Adam (3):

$$R_B = R_0(1 - 0.229 \sin^2 B) \cos B \quad (2.11)$$

The signs of these three corrections have been chosen so that true equivalent Doppler velocity is equal to the observed velocity plus the corrections. In the machine computations, the sum of C_1 and C_2 is given as the Earth's motion correction, and the factor f is given for the solar rotation correction.

The solar coordinates of the point of observation are given by the two equations:

$$\sin B = \cos \theta^* \sin B_0 + \sin \theta^* \cos B_0 \cos \psi \quad (2.12)$$

$$\sin(L-L_0) = \sin \theta \sin \psi / \cos B \quad (2.13)$$

where $\theta^* \approx \theta - \rho/\rho_0 S$, $\theta \approx \arcsin \rho/\rho_0$ and $\psi = P - \phi$

Of these, the solar latitude only was explicitly computed for each point of observation.

The direction cosines at the point of observation, in the frame of reference given in the previous section, are given by:

$$\cos \gamma_1 = \sin(L-L_0) \cos B_0 \sin \theta \operatorname{cosec} \theta^* \quad (2.14)$$

$$\cos \gamma_2 = \left[\sin B \cos B_0 \cos(L-L_0) - \cos B \sin B_0 \right] \times \sin \theta \operatorname{cosec} \theta^* \quad (2.15)$$

$$\cos \gamma_3 = -\cos \theta \quad (2.16)$$

Several programmes were written for these computations, many of them dealing with specific aspects of the calculations, but including one general programme for the complete set of calculations as outlined above. This is discussed in detail in Appendix D, where short descriptions of the other programmes are also given.

P a r t I I

REDUCTION OF "CIRCULAR CHANNEL" PLATES

A. General Theory.

The method of circular channels (4,5,6,7), which has been developed at Oxford for the interferometric determination of wavelengths has the disadvantage that the computations involved are lengthy and tedious. In order to render the measurement of many interferometric plates possible in a reasonable period of time, a programme was written for the computations involved in the reduction.

The fundamental relation involved in the reduction of the circular channel plates is the interferometric relation giving the angular semi-diameter, Θ , of an observed interference ring:

$$m\lambda = 2t \cos \Theta \quad (2.17)$$

where m is the integral order of interference, λ is the wavelength of the light, and t is the optical thickness of the etalon. At the centre of the interference pattern, we define the fractional order of interference, ϵ , by:

$$(p+\epsilon)\lambda = 2t \quad (2.18)$$

Since for the n^{th} ring, counting outwards from the centre, we have $(p-n+1)\lambda = 2t \cos \Theta$, we obtain the relation:

$$\epsilon + n - 1 = (1 - \cos \Theta) \frac{2t}{\lambda} \quad (2.19)$$

From equation (2.19), we have that the increment in the cosine of the angle subtended by successive rings is:

$$\Delta \cos \theta = - \frac{\lambda}{2t} \quad (2.20)$$

As was shown by M. G. Adam (5), we also have the approximate relation:

$$\Delta \cos \theta \approx - \frac{\Delta D^{*2}}{8f^2} \quad (2.21)$$

where
$$D_n^{*2} = D_n^2 \left(1 - \frac{3}{16} \frac{D_n^2}{f^2} \right) \quad (2.22)$$

In these equations, D_n is the diameter of the n^{th} ring, D_n^* is the "effective" diameter of the ring, ΔD^{*2} is the increment in the square of the effective diameters of two successive rings, and f is the focal length of the camera imaging the interference rings. For the 18 inch Cooke lens used in the investigation to be discussed, $D_n^{*2} = D_n^2 (1 - 8.4868 \times 10^{-7} D_n^2)$. From equations (2.20) and (2.21), we have:

$$\Delta D^{*2} = \frac{4f\lambda}{t} \quad (2.23)$$

Since $\frac{D_n^{*2}}{8f^2} = 1 - \cos \theta$ from equation (2.19) we have:

$$D_n^{*2} = D_1^{*2} + (n-1) \Delta D^{*2} \quad (2.24)$$

and
$$\epsilon = \frac{D_1^{*2}}{\Delta D^{*2}} \quad (2.25)$$

Thus from the observed effective ring diameters of the interference pattern of a given wavelength, ΔD^{*2} may be found

from a linear least square fit to equation (2.24). In addition, the fractional order of interference may then be found using (2.25).

In the method of circular channels, the interference patterns of the unknown wavelengths are directly compared with that of the standard cadmium red line (λ 6438.4696). If the fractional orders of interference at the centre of the pattern are known for the unknown and standard wavelengths, the unknown wavelength may be calculated from:

$$\lambda_1 = \frac{\lambda_{cd} (p_{cd} + \epsilon_{cd})}{p_1 + \epsilon_1 - k_1} \quad (2.26)$$

where the subscript 1 refers to the unknown wavelength. The factor k_1 is a slight correction due to the different effective thickness of the etalon for the two wavelengths (phase change correction). In addition, the values of p_{cd} and p_1 must be known, but these are readily found by a trial and error process, using the approximate wavelengths of the lines..

It is found that the ring diameters for the unknown wavelengths cannot be measured sufficiently accurately to determine ϵ from equation (2.25), so the factor ΔD^{*2} for the unknown wavelength must be determined from the relation (2.23); that is:

$$\Delta D_1^{*2} = \Delta D_{cd}^{*2} \frac{F_1 \lambda_1}{\lambda_{cd}} \quad (2.27)$$

where F_1 is an empirical factor related to the chromatic variation of the camera focal length, and is found, for the unknown wavelength, from a previous investigation (8). Using this value of ΔD^{*2} , a fractional order of interference (at the centre of the pattern) is found for each measured diameter from:

$$\frac{D_n^{*2}}{\Delta D^{*2}} = \varepsilon_n + n - 1 \quad (2.28)$$

The values for all the measured diameters are then averaged to give a mean ε for the line of unknown wavelength.

Another slight correction must be applied to the value of ε_{Cd} found from the cadmium interference pattern for accurate wavelength determinations. Since different parts of the etalon plates may be used by the cadmium light and the light of unknown wavelength, each may use different effective etalon thicknesses. In practice, this is allowed for by using two pseudo-lines of cadmium λ 6438.4696 at either side of the plate to determine corrections to the ε_{Cd} for each line of unknown wavelength (7).

B. Measurement of Ring Diameters.

The interference pattern of any absorption (or emission) line in the method of circular channels consists of a series of ring segments (or arcs) formed by taking a chordal

section of the concentric interference rings at the position of the line in the grating spectrum of the region. By a system of diaphragms, two perpendicular diametral sections of the cadmium interference pattern are photographed plus the two pseudo-lines, bracketing the lines of unknown wavelength. This cadmium "cross" is positioned so that the spectral lines are perpendicular to one of its arms, which will be designated the "a-c" arm. With this arm horizontal, the four sides of the photographic plate are now labelled "a", "b", "c" and "d", clockwise from the left. We may also specify an x,y coordinate system centred on the cadmium interference pattern, with the x-axis in the "c" direction and the y-axis in the "b" direction. In conformity with the above scheme, the two pseudo-lines of cadmium are designated Cadmium "a" and Cadmium "c". The arcs of a typical line are indicated in Fig. 2.1 (the arcs of the cadmium pattern having been omitted for clarity).

Since only segments of the interference rings of each line are available, a scheme of measurement was devised from which accurate values of the ring diameters could be determined (4), using a measuring micrometer with a rotating plate stage. Instead of measuring the true diameter, D , one obtains the two distances S_1 and S_2 (see Fig. 2.1). These are determined by setting the micrometer on the arc in question, obtaining a reading R , and

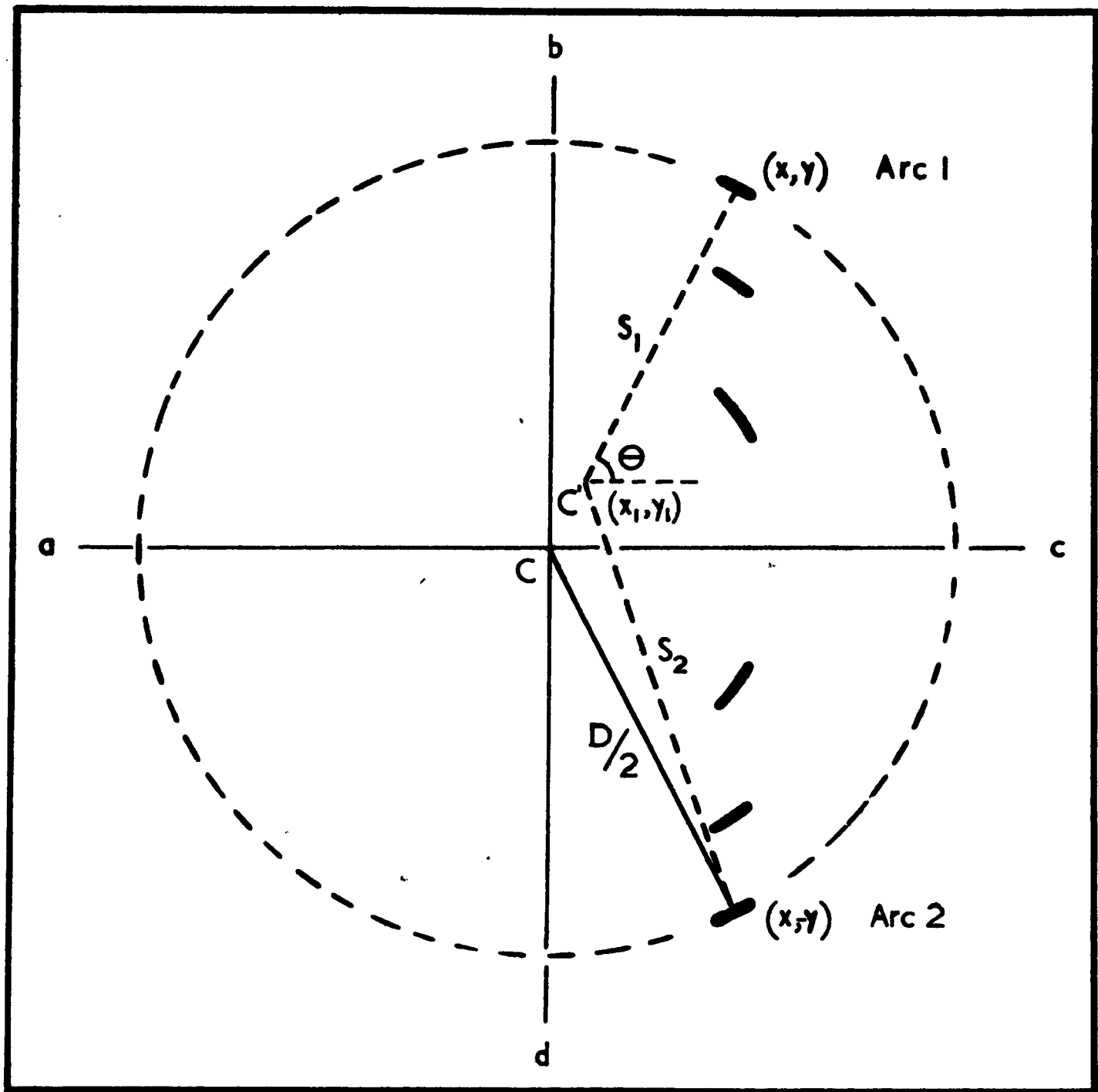


FIGURE 2.1 MEASUREMENT OF CIRCULAR
CHANNEL RING DIAMETERS

then rotating the plate through 180° and re-setting on the same arc to obtain a reading R' . Then $2S = |R - R'|$.

If the centre of rotation of the plate is specified by coordinates (x_1, y_1) in the above system, the ring diameter is given by:

$$D^2 = 2(S_1^2 + S_2^2) + 8x \cdot x_1 - 4\rho^2 \quad (2.29)$$

where $\rho = \sqrt{x_1^2 + y_1^2}$. If the quantity ρ is very small, equation (2.29) reduces to:

$$D^2 \approx 4 S_1 S_2 + 4x \cdot 2x_1 \quad (2.30)$$

The value of x for the ring in question may be found from $S \cos \theta = x$, $S \cos \theta$ being an average value from the two arcs, 1 and 2. The value of $2x_1$ may be obtained to a high degree of accuracy from the readings obtained for the arcs of the "a-c" cadmium arm. In this case, $S_a - S_c \approx 2x_1$, so that a mean value of $S_a - S_c$ for all the rings gives a good approximation to $2x_1$.

In order for the programme to be used most effectively, the arc measurements should be recorded in the same order that they must enter the computer as data. This is specified in Appendix D where the programme is discussed in detail. The general outline of the calculations will, however, be briefly discussed in the next section.

C. Outline of Computations.

Aside from the actual arc measurements, certain general input data are required by the computer. These are numbers specifying the number and positioning on the plate of the unknown lines, and approximate wavelengths, phase change and camera focal length correction factors for these lines. Following these are two numbers specifying the procedure for determining integral orders of interference.

From the arc measurements for the "a-c" arm of the cadmium cross, the quantity $2x_1$ is found, and using all the arcs of the cross, quantities specifying the maximum value of ρ are computed and given in the output. The ring diameters are then found, and, from a least square fit of equation (2.24), ΔD^2 is computed for the cadmium pattern. Using this value, fractional orders of interference are found for each ring in the three pseudo-lines, Cadmium "a", Cadmium "c", and the "b-d" arm of the cross, using equation (2.28). For each line, the ε_n 's are averaged to yield a mean ε . In such a determination of a mean fractional part from individual fractional parts, one must be careful to average values such as 0.99 and 0.03 to obtain 0.01, not 0.51. This is one of the cases in which one must programme "intuition" into the machine. In this case,

the following criterion has been adopted. If the mean computed ε differs from any of the individual ε_n 's by more than 0.3, the ε_n 's less than 0.5 are increased by one and a new mean is computed. Using the ε 's obtained from each of the pseudo-lines, an ε_{cd} is interpolated for each of the lines whose wavelength is to be determined, assuming a linear dependence of ε_{cd} on x .

For each of the lines of undetermined wavelength, the true ring diameters are found from equation (2.30). Using the ΔD^{*2} determined from equation (2.27), these then give ε_n 's. Following the above criterion for averaging, a mean ε is then found for each line and this is corrected for the chromatic change of phase. The procedure used to find the integral orders of interference for the cadmium light is a trial and error method, and as such requires almost as much computing time as the rest of the calculations (about 15 sec). Two methods are included in the programme, one of which is selected by the specification in the input data.

In the first method, starting at a cadmium integral order of interference given in the data, the expected ε for each of the lines of undetermined wavelength is found from equation (2.26) using its approximate wavelength. The difference between this and the observed value is

determined, and a mean value of the square of this difference is found using all the lines. Again, care must be taken to give values such as 0.99 and 0.01 a difference of 0.02. If this mean square difference is less than a number specified in the data, the integral order used for the cadmium line is adopted. If not, it is increased by one and the process repeated. This is continued, if necessary for 100 trial integral orders of interference, after which, if no satisfactory value has been found, the computations are stopped and data for another plate is read.

In the second method, specified by setting the desired mean square difference to zero, the above is carried out for 100 values of p_{Cd} starting with the one entered in the data. The one that yields the minimum mean square difference between the trial and observed ϵ 's is then adopted. Using the value of p_{Cd} obtained by either method, p_1 is then found for each line using equation (2.26). Again the ambiguity in ϵ must be allowed for. (If a trial order of interference of 10321.998 was compared with an observed ϵ of 0.001, the adopted p_1 must be 10322.) The accurate wavelength of each line is then found from the observed ϵ 's and computed p 's. The mean error in this value is also estimated, using the computed standard deviations in ϵ_{Cd} and ϵ_1 .

In addition to the accurate wavelengths and their errors,

several other quantities are given in the output, including the fractional orders of interference for each line. These are given in the hope that any errors that might possibly arise due to ambiguity in the interpretation of $\mathcal{E}$'s may be detected. In the many computations involved in the investigation given in the next chapter, however, no such errors occurred. The use of this programme for the plate reduction has reduced the computation time for a single plate from over an hour to twenty minutes (the time required to transfer data to tape form) and has practically eliminated personal errors in the reduction. A detailed description of the programme and its use is given in Appendix D.

P a r t I I I

DETERMINATION OF PLATE CHARACTERISTIC CURVES

A. Introduction and Method.

The reduction of photometric observations obtained from photographic plates, i.e. the determination of line profiles, requires a knowledge of the characteristic response curve of the plate used. This is normally determined by measuring the plate densities corresponding to a number of known exposures. For the investigation of line profiles to be presented in Chapter 4, this was done using a step wedge, allowing seven exposures of equal duration but different intensities to be taken simultaneously. From the measurement of the plate density, obtained using a microphotometer, and the pre-calibrated intensities transmitted by the wedge, a smooth characteristic curve can be estimated. Although curve fitting by hand for so few points (seven) is not a difficult process, a machine method was developed for the determination of an analytical form of the characteristic curve (over the region of the observations), as part of a general programme for the reduction of spectral observations.

In order to present the method adopted for the derivation of the analytical form of the characteristic curve, a brief summary must be given of the standard process used for its determination. The wedge used consists of seven strips

of rhodium coating between two glass plates, the transmission coefficient for each strip having been accurately determined (9). On each plate for which the response curve is required, two exposures of the spectrum, of equal duration, are taken, one through the wedge, and one through only the glass plates of the wedge. For the latter, a filter is used to obtain a medium density on the plate. Microphotometer tracings through each spectrum, perpendicular to the dispersion, then give a record of the corresponding plate densities. In addition, measures of the density of the unexposed plate and of a completely "black" plate are included on the microphotometer records. A diagram of such a microphotometer record is shown in Fig. 2.2. Usually several such recordings are made for each plate.

If we now adopt a scale for determining the microphotometer deflection, x , so that the unexposed plate reading is x_{cf} and the "black" reading is x_0 , we may specify any other degree of deflection by a y/y_0 where $y = x - x_0$ and $y_0 = x_{cf} - x_0$. The characteristic curve may then be defined as the relation between the plate density, y/y_0 , and the intensity, I , incident on the plate. Since the intensity incident on the plate is given by tI_0 where t is the transmission coefficient of the wedge step and I_0

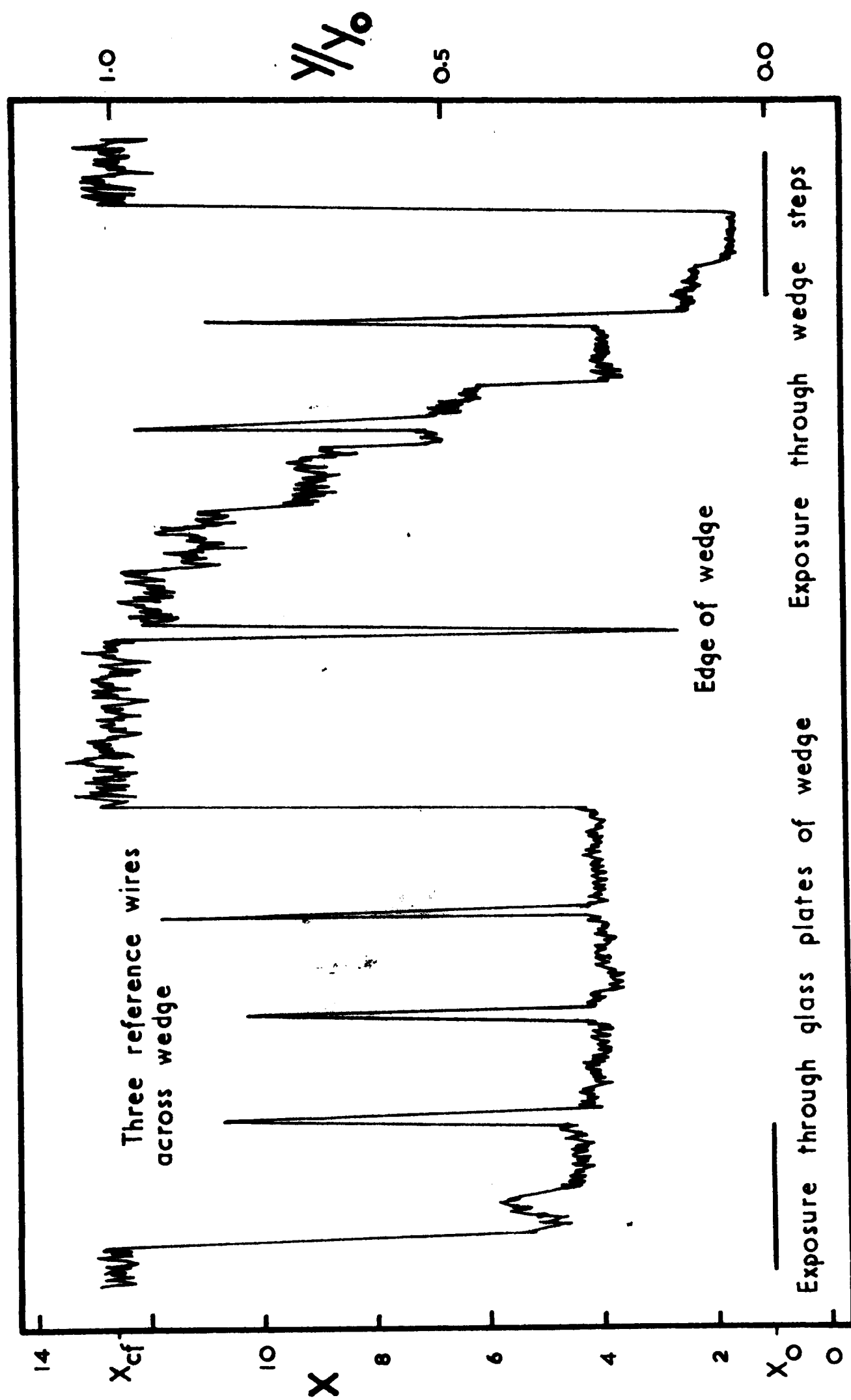


FIGURE 2.2 PLATE CALIBRATION AS RECORDED BY MICROPHOTOMETER

is the intensity incident on the wedge, the characteristic curve is usually expressed as the relation between y/y_0 and $\log t$. This allows the comparison of different determinations (made with different I_0 's) by a change of the origin of the $\log t$ axis.

In the determination of the relation between y/y_0 and $\log t$, the intensities, I_0 , incident on each wedge step must be the same. Unfortunately this is not usually the case, and corrections must be determined from the density recording of the spectrum taken through the glass plates of the wedge. A first approximation to the characteristic curve is found using the measured y/y_0 's for each wedge step and their known transmission coefficients. The measured y/y_0 's for the glass plates, using this approximate curve, then give the variation in I_0 across the wedge steps. A reference I_0 is chosen and the transmission coefficients of the wedge steps are corrected to (t') 's to correspond to an incident intensity equal to the reference intensity, I_0' . This is given by

$$t'/t = I_0/I_0' \quad (2.31)$$

Using these new transmission coefficients and the measured y/y_0 's for each step, a second approximation to the curve is obtained. If the y/y_0 's obtained for the glass readings fall on the linear portion of the characteristic curve,

this second approximation is a good one. If not, it may be considerably in error; however, such cases are assumed to be eliminated by the choice of exposures, and the second approximation is always adopted.

In the machine computations, the same procedure is followed, but at each stage at which a curve is drawn by hand, an analytical expression is fitted to the points by a least-square solution. Since the characteristic curve resembles the curve:

$$y = \frac{1}{1 + Ax^b} \quad (2.32)$$

where $y = y/y_0$ and $x = I/I_0$, this was adopted as a first approximation. A better approximation is given by the polynomial expression:

$$\ln \frac{I}{I_0} = \sum_{i=0}^n a_i \left\{ \ln \left(\frac{y_0}{y} - 1 \right) \right\}^i \quad (2.33)$$

which reduces to (2.32) for $n = 1$. Curves of the form given by (2.33) were fitted to the observed points for values of n from one to four, and it was found that the extra terms (for which $a_1 \ll a_0$) allowed a very good fit. It was therefore decided to use equation (2.33) as the analytical form of the characteristic curve, choosing the value of n that gave the best fit.

Equation (2.33) has the fault, however, that for certain values of a_n and n , it does not have the proper physical convergence properties as $y/y_0 \rightarrow 1$ and 0. This occurs when n is odd and a_n is negative, and when n is even. It was found that for most plate emulsions, a_n was positive, so this difficulty usually arises only when n is even. In this case, equation (2.33) does not behave properly for $y/y_0 \rightarrow 1$. These deviations from the correct asymptotic forms of the characteristic curve do not occur within the range of y/y_0 's observed for the wedge steps and so do not present any problems in the application of the analytical form of the curve (since any extrapolation beyond this range is not justified by the observations). Since, however, the case when a_n is positive and n is even arises frequently, provision has been made in the programme to preserve the asymptotic form of the curve, in a somewhat arbitrary manner, for $y/y_0 \rightarrow 1$. For y/y_0 's less than the greatest observed y/y_0 , equation (2.33) is used, while for y/y_0 's greater than this value, the analytical form is taken to be:

$$\ln I/I_0 = A + B \ln(1 - y/y_0) \quad (2.34)$$

The constants A and B are chosen so that the curves (2.33) and (2.34) match (up to the second derivatives) at the maximum observed y/y_0 .

In fitting equation (2.33) to the data, the coefficients a_1 are computed using a least-square solution for polynomials. This is provided by the Ferranti library programme No. 523. In this sub-programme, the variance $(\sum_{i=1}^m v_i^2 / (m-n))$, where v_i is the difference between the computed $\ln I/I_0$ and the observed value, and m is the number of observed points) is also computed. From this, the percentage uncertainty in a single determination of I may be found.

B. Outline of Computations.

The input data for the machine computations consist of numbers specifying the number of characteristic curve determinations to be averaged together, certain control instructions, and the observed densities for the wedge steps, the corresponding wedge positions exposed through the glass only, and for the "black" and clear plate, for each determination.

For each determination, from the densities observed for the wedge steps, y/y_0 's are found and the polynomial (2.33) is fitted to the data. Using the y/y_0 's found for the glass exposure, the variation of I_0 across the wedge is found and a second approximation to the characteristic

curve is found as outlined above. Again, curves for each value of n are found and the best one chosen. Using this curve, the values of $\ln I/I_0$ are found for six values of y/y_0 between 0.25 and 0.75.

The analytical curves obtained for each determination are then "averaged" together. In order to do this, the y/y_0 and $\ln I/I_0$ values used to determine each of the individual curves must be adjusted to a common $\ln I/I_0$ axis. The additive adjustment for each curve is given by the mean difference between the six computed $\ln I/I_0$'s for the central part of the curve and those found from the curve obtained in the first determination. Now using all the adjusted y/y_0 and $\ln I/I_0$ values, for all the determinations, a new polynomial (2.33) is fitted to the data, using the optimum n . This is adopted as the final analytical form of the characteristic curve in the region of the observations, and the coefficients a_1 are given in the output. In addition, the variance of the final fit and the percentage uncertainty in a determination of I are given, along with the maximum and minimum observed y/y_0 .

If the final curve is one where n is even and a_n is positive, a curve (2.34) is also calculated for a rough extrapolation for $y/y_0 \rightarrow 1$. The constants A and B are found from the relations:

$$B = \left(\frac{d \ln I/I_0}{d y/y_0} \right)_{y_m} (y_m^{-1}) \quad (2.35)$$

$$A = (\ln I/I_0)_{y_m} B \ln (1 - y_m) \quad (2.36)$$

where y_m is the maximum observed value of y/y_0 . The derivative is approximated by $\frac{\Delta \ln I/I_0}{\Delta y/y_0}$ where $\Delta y/y_0 = 0.01$.

Using equations (2.33) and (2.34), if applicable, a table of $\ln I/I_0$ for thirteen values of y/y_0 is given in the output, and the coefficients in the expansions are stored for future use.

It was found that the accuracy of the determination depended critically upon the measured points with y/y_0 's nearly 1 or 0. Since these are transformed to points far from the origin by the transformation which gives the polynomial expansion (2.33), they are liable to contribute greatly to the calculated variance. Thus the computed percentage uncertainty in a determination of I refers primarily to the two extremes of the response curve, not to its linear portion, where the uncertainty is much less. In practice, it was found that most determinations gave computed uncertainties of the order of 4%. Care must be taken to include readings for at least five wedge steps

in the data, and preferably six, since for five or less observed points, the programme will yield an exact fit to all points. One thus loses all statistical degrees of freedom in the fitting. Even in this case, however, the polynomial chosen will be the one where $n = m-1$, m being the number of observed wedge steps, so that it will still yield a smooth interpolation.

P a r t I V

SOLUTION OF CERTAIN INTEGRAL EQUATIONS

A. Introduction.

For a detailed interpretation of spectral line profiles, it is advantageous to correct the profiles for the apparatus function of the spectrograph. This function, $S(x)$, is defined by the integral equation:

$$O(x_0) = \int_0^{\infty} T(x) S(x-x_0) dx \quad (2.37)$$

where $T(x)$ is the true line profile that would be obtained by an "ideal" spectrograph, $O(x)$ is the line profile actually observed, and x is a coordinate related to λ . Since $S(x-x_0) \rightarrow 0$ for $|x-x_0| \rightarrow \infty$, we may replace the lower limit of integration by $-\infty$; although in practice, the limits of integration may be considered to be finite, $T(x)$ being zero outside the region of integration. Equation (2.37) is a Fredholm equation of the first kind, for which numerical solutions are difficult. In the following, several numerical methods tried for an approximate solution of $T(x)$, knowing $O(x)$ and $S(x)$, will be discussed. A suitable machine method for the routine reduction of line profiles was sought and the one that was chosen for the investigation presented in Chapter 4 is described in some detail. Other methods, not discussed here, may be found in a survey by Van de Hulst (10,11).

The solution of the problem is formally the same whether

$T(x)$ is the profile of an emission or absorption line, since the apparatus function is normalized so that $\int_{-\infty}^{\infty} S(x) dx \equiv 1$. In the following, we will therefore treat the problem for the case when $T(x) \rightarrow 0$ as $|x| \rightarrow \infty$.

B. Solution of a System of Linear Equations.

If we approximate the integral in (2.37) by a finite summation, as we must in any numerical treatment, we have (assuming a very simple quadrature formula):

$$O(x_i) = T(x_j) S(x_j - x_i) \Delta x \quad j = 0, \dots, n \quad (2.38)$$

where the summation is over $n+1$ values of x , at equal intervals, Δx . The $n + 1$ equations given by (2.38) for $i = 0, \dots, n$ constitute a set of linear equations in the unknowns $T(x_j)$. (For simplicity we may set $\Delta x = 1$). Thus one might hope to find the values of $T(x)$ at the $n + 1$ pivotal points from a direct solution of these equations. If we write (2.38) in matrix form,

$$O = S T \quad (2.39)$$

where O and T are column vectors and S is the apparatus function matrix, the solution is given by:

$$T = S^{-1} O \quad (2.40)$$

Such a solution depends upon the existence of an inverse of S , i.e. a non-singular S . It has been shown by Fox and Goodwin (14), that if the kernel, $S(x)$, can be expressed

as a polynomial of order p , $\det |S_{ij}| = 0$ for $n \geq p$. Thus the direct solution of the equations to yield a unique solution is impossible. For an arbitrary $S(x)$, one would therefore expect that $\det |S_{ij}| \rightarrow 0$ as n increases. The errors introduced into the solution by replacing the integral by a finite summation, although decreased by increasing n , are magnified by the large elements in S^{-1} and a large error is introduced into the determination of $T(x)$. For a small number of pivotal points these errors are usually not too critical, if the functions $O(x)$ and $S(x)$ are exactly known, however this is not the case for observed line profiles. It was found that application of this method to observed data yielded an error in $T(x)$ that was an oscillatory function of x , of short period and large amplitude.

This problem of large amplitude, oscillatory errors in $T(x)$ is fundamental in all exact solutions where $O(x)$ and $S(x)$ are subject to experimental errors. Suppose that the true observed function should be $O^*(x)$, whereas that actually observed is $O(x)$. Then

$$O^*(x) = O(x) + \delta(x) \quad (2.41)$$

where $\delta(x)$ is the observational error function. The true integral equation to be solved should then be:

$$O^*(x_0) = \int_{-\infty}^{\infty} T^*(x) S(x-x_0) dx \quad (2.42)$$

whereas the equation actually used for the determination of the true profile is (2.37). Taking the difference between these, we have

$$\delta(x_0) = \int_{-\infty}^{\infty} \{T^*(x) - T(x)\} S(x-x_0) dx \quad (2.43)$$

If we denote $T^*(x) - T(x)$ by $\Delta(x)$, the error in the determination of $T(x)$, we see that a large amplitude, oscillatory function, $\Delta(x)$, will give only a small $\delta(x)$ provided that $S(x-x_0)$ is fairly constant over one oscillation of $\Delta(x)$. From this, one might expect small observational errors in $O(x)$ to yield large amplitude errors in $T(x)$. This is discussed in slightly more detail by Kreisel. (15)

One must therefore find an approximate solution of the problem that preserves "smoothness" in the derived function $T(x)$. Two iterative procedures of obtaining approximate solutions of the equations (2.38) were therefore investigated. Naturally, these procedures still suffer from the difficulty that successive approximations must converge to a solution with large amplitude errors in it. One may hope, however, that a reasonable solution can be obtained after few iterations before the large errors appear. The problem then becomes one of determining the optimum number of iterations to be made.

C. Iterative Methods.

The first method investigated was that of Gauss and Seidel, as described in (16). An initial trial vector, $T(x_1)$ is chosen and the components of this are successively modified by solving the i^{th} equation of (2.38) for the value of $T(x_i)$. This is repeated until all the $n + 1$ equations have been used, to give a second approximation to the vector $T(x_1)$. This cycle is repeated until a trial vector is found that gives an "observed" function fitting the actual observed function within a certain limit. This method is known to converge if the matrix S is symmetric and positive definite with dominating diagonal elements. Since in most cases, the apparatus function is very nearly symmetric, rapid convergence might be expected; and smoothness could be introduced by using the observed function for the initial trial solution. It was found, however, that the convergence was slow, requiring about 15-20 cycles for a satisfactory solution to be obtained (in a typical case arising from the investigation in Chapter 4). This resulted in the complete obliteration of the smoothness introduced by the choice of initial solution, and the method was discarded.

The second method investigated was that due to Burger and van Cittert (12,13), which has been adopted in the past (17)

for the derivation of true line profiles. Again the observed function is adopted as a first approximation to the true function, and corrections to this approximation are successively derived. The process is best described using a slightly different notation. We write equation (2.37) in operator form:

$$O = ST \quad (\text{cf. equation (2.39)}) \quad (2.44)$$

where ST denotes $\int_{-\infty}^{\infty} S(x-x_0) T(x) dx$. The observed function, O , is then adopted as a first approximation to T , i.e.

$T_0 = O$. Further approximations to T are given by:

$$T_1 = T_0 + (O - ST_0) = T_0 + (1-S)T_0 \quad (2.45)$$

$$T_2 = T_0 + (1-S)T_1$$

etc. where 1 represents the unity operator, giving for the n^{th} approximation:

$$T_n = \left\{ \sum_{i=0}^n (1-S)^i \right\} T_0 \quad (2.46)$$

Since $1 - (1-x)^n = x \sum_{i=0}^{n-1} (1-x)^i$ we have:

$$T_n = \left\{ \frac{1 - (1-S)^{n+1}}{S} \right\} T_0 \quad (2.47)$$

It may easily be shown that this ^{usually} converges to the solution of (2.37) by taking Fourier transforms of equation (2.37).

We then have

$$O(t) = S(t) T(t) \quad (2.48)$$

where the symbols now refer to the Fourier transforms of $O(x)$, $S(x)$ and $T(x)$. The Fourier transform of the solution is then given by:

$$T(t) = O(t)/S(t) \quad (2.49)$$

From the formal similarity of equations (2.48) and (2.44), we see that the Fourier transform of the n^{th} approximation to $T(x)$ is given by equation (2.47) when $T_n(x)$ is replaced by $T_n(t)$, etc. This converges to (2.49) if $|1 - S(t)| < 1$ which is easily shown ^{usually} to be true from the normalization of $S(x)$.*

It was found that this method converged rapidly to an acceptable solution; a solution, T_n , being judged acceptable when it yielded an "observed" function fitting $O(x)$ within a preset degree of accuracy. For typical profiles obtained in the investigation to be discussed in Chapter 4, it was found that only 2-7 iterations were required to obtain a solution fitting the data to within a maximum error of 1% of the continuum intensity for iron lines, and 4% for the sharper oxygen lines. These figures do not, however, refer to the accuracies of the solutions themselves. Experiment has shown that the accuracies of solutions so adopted are about twice these values (for 2-7 iterations). In this small number of iterations, the large amplitude errors inherent in the solution had not reached an

* For very asymmetric apparatus functions, $|1-S(t)|$ may exceed unity although $|S(t)| \leq 1$ because of normalization. In such a case there is no convergence.

appreciable magnitude, thus the approximate solution obtained was usually smooth (see Section D). Any irregularities, however, that do happen to be present in $O(x)$ are enhanced in $T_n(x)$, and troublesome irregularities can arise. (In particular, see Section D, Part II, Chapter 4). The question of the optimum number of iterations, i.e. choice of the error limit, depends upon the value of Δx in equation (2.38), the functions $O(x)$ and $S(x)$, and the observational errors in these.

The convergence rate can be slightly increased if $O(x)$ is really a smooth function by a better choice of initial approximation. The method follows that given by Eddington (18). If $S(x)$ is Gaussian and is given by:

$$S(x) = \frac{1}{\alpha\sqrt{2\pi}} e^{-\frac{x^2}{2\alpha^2}} \quad (2.50)$$

then equation (2.37) may be written as:

$$O(x) = \frac{1}{\alpha\sqrt{2\pi}} \int_{-\infty}^{\infty} T(x+y) e^{-\frac{y^2}{2\alpha^2}} dy \quad (2.51)$$

If we then expand $T(x+y)$ by a symbolic Taylor expansion:

$$T(x+y) = e^{y \frac{d}{dx}} T(x) \quad (2.52)$$

then equation (2.51) becomes:

$$\begin{aligned} O(x) &= \frac{1}{\alpha\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{y \frac{d}{dx}} - \frac{y^2}{2\alpha^2} T(x) dy \\ &= \left(e^{-\frac{\alpha^2}{2} \frac{d^2}{dx^2}} \right) T(x) \end{aligned} \quad (2.53)$$

treating the differential operator as a constant. Thus we have

$$\begin{aligned} T(x) &= \left(e^{-\frac{\alpha^2}{2} \frac{d^2}{dx^2}} \right) O(x) \\ &= O(x) - \frac{\alpha^2}{2} \frac{d^2 O(x)}{dx^2} + \dots \end{aligned} \quad (2.54)$$

If $S(x)$ is not quite Gaussian, one would still expect

$$T_0 = O(x) - \frac{\alpha^2}{2} \frac{d^2 O(x)}{dx^2} \quad (2.55)$$

to be a good starting approximation for the Burger van Cittert method. This requires, however, that the second derivative of $O(x)$ also be fairly smooth, or else irregularities will be introduced in the first approximation. Experiment showed that this initial approximation, (2.55) decreased the number of iterations required by one or two.

Owing to its rapid convergence, the Burger van Cittert method was therefore adopted and a programme was written for its rapid execution. The function $O(x)$, tabulated at up to 180 pivotal points, is taken as a first approximation to $T(x)$ and successive approximations are computed. The required integrations are replaced by finite summations, $O(x)$ being taken as zero outside the region of tabulation. If $O(x)$ is sufficiently smooth, equation (2.55) may be used for the first approximation, the second derivatives at each

point being approximated by the corresponding second differences. The actual value of the second difference adopted at each point is determined from a quadratic least-square fit to the second differences in the neighbourhood of the point, and the value of α is estimated from the half-width of the apparatus function. For each trial function, $T_n(x)$, obtained, the quantity

$$E_i = \sum \{ S(x_j - x_i) T_n(x_j) \} - O(x_i) \quad (2.56)$$

is found for each point. If the mean value of E is less than a preset value ε , or if $E_{\max.}$ is less than 5ε , the approximation is accepted; if not, a new approximation is found using the E_i 's. The solution finally obtained is given in the output at as many points as $O(x)$ was given. If the mean values of E found for successive n 's begin to increase, the computations stop and the best solution is given. Such a precaution would not really be necessary for the Burger van Cittert method if rounding errors could not arise, since it must converge to a solution. The computations for each trial solution require 30 seconds, so that most profiles require 2-3 minutes for their reduction.

D. Fourier Transform Solutions.

Although the above method yields fairly smooth solutions,

for certain cases the convergence is too rapid. For profiles that are very broad compared with the apparatus function, it was found that $T_n(x)$ might under-fit the data, having E considerably greater than ϵ , while $T_{n+1}(x)$ might over-fit the data, with E considerably less than ϵ . Thus neither of these would be satisfactory solutions, the first being inaccurate and the second containing enhanced observational errors. From the Fourier transform equation (2.49), we see that large amplitude, short period, oscillatory errors in $T(x)$ can arise from observational errors that make $O(t)$ too large or $S(t)$ too small. For a well-behaved solution, we require that $O(t) \rightarrow 0$ faster than $S(t)$, as $t \rightarrow \infty$.

In the Burger van Cittert method, we see from (2.47) that as $S(t) \rightarrow 0$, $T_n(t) \rightarrow (n+1) \{1-S(t)\}^n O(t)$. Thus, if $O(t)$ has a finite value due to observational errors while $S(t)$ is very small, $T_n(t)$ will grow nearly in proportion to the number of iterations, and a corresponding error of frequency t will be introduced. This is a slow growth, however, and accounts for the relative stability of the Burger van Cittert method.

It has been suggested by J. S. Rollett (19) that the Fourier transform solution given in equation (2.49) would be a faster machine method than that of Burger and van Cittert, if some means could be found to eliminate the high frequency

errors in $T(x)$ arising from observational errors in $O(x)$ and $S(x)$. He has written a programme in which the estimated standard deviations of the tabulated values of $O(x)$ and $S(x)$ are considered. Using these, the complex Fourier transforms (computed numerically) of these two functions are modified, by adjusting their amplitudes, so that there is a 1000:1 probability against any $T(t)$ being too large because of experimental errors making $S(t)$ too small, and 20:1 against it being too large because of errors making $O(t)$ too large. This programme was found to give results in good agreement with those obtained by the Burger-van Cittert method but much smoother, in times equivalent to 3 iterations in the latter method. The method of modifying $O(t)$ and $S(t)$ is not entirely satisfactory, however, and further investigation into this promising method is required.

P a r t V

GENERAL REDUCTION OF LINE PROFILES

A. Introduction.

In the preceding two parts of this chapter, machine programmes have been discussed which allow the correction of line profiles, as observed on a microphotometer tracing, for the response curve of the photographic plate and the spectrograph apparatus function. These were derived as part of a general programme for the reduction of line profiles. For the purposes of the investigation given in Chapter 4, a programme was desired that would use profile data taken directly from microphotometer tracings to compute 1) an observed, true intensity profile allowing for the characteristic curve of the photographic plate, 2) the corresponding true line profile allowing for the apparatus function, and 3) a function giving the asymmetry of the resulting profile. Such a programme was developed and the methods used will be outlined in the next section.

B. Outline of Computations.

The data required for the execution of the reductions consist of two sections. The first are data that are the same for several profile determinations, consisting of a tabulation of the apparatus function, at equal intervals Δx , and the data required for the determination of the

characteristic curve. Secondly, the data for the line profile to be reduced must be given. This is in the form of up to 54 (x,y) points taken from the microphotometer tracing, where x is a coordinate parallel to the dispersion and y is a coordinate denoting the microphotometer deflection. In addition, y readings for the continuum, the "clear" plate and the "black" plate must be entered. The relation between the unit of the x coordinates and Δx , along with several control parameters complete the input data.

Using the characteristic curve data (see Part III), the observed (x,y) points are converted to (x, r) points, where r is the residual intensity in the profile. (If any observed points have residual intensities > 1 , the continuum level is adjusted to make them equal to 1.) Values of the observed profile are then interpolated at intervals of Δx treating the line profile as that of an emission line. For this purpose, a quadratic interpolation procedure was adopted. If the value of $O(x)$, the observed profile, is required at x_0 , the two observed points on either side of x_0 , four in all, are used to obtain a least-square quadratic fit to $O(x)$ in that region (the two central points being given twice the weight of the outer two). In this manner, up to 180 values of $O(x)$ are interpolated. At this stage in the computations, the equivalent width

of the observed profile is estimated (by a summation over the interpolated values).

The function $O(x)$ is then corrected for the apparatus function using the Burger-van Cittert method discussed in Part IV. Since the interpolation of $O(x)$ tends to introduce a discontinuous second derivative, the method of subtracting second differences to speed up convergence could not be used. A trial solution, $T_n(x)$, was accepted when it fitted the observed $O(x)$ within a preset limit, as discussed in Part IV, or when it fitted $O(x)$ within the mean error arising from the quadratic interpolation if this exceeded the preset limit. In most reductions, it was the practice to let the second limit (a mean error of 0.4 - 0.8% of the continuum intensity) determine the number of iterations. From the maximum value of the adopted $T(x)$, the true central intensity of the profile is then found.

It was decided to denote asymmetry by the curve giving the midpoint of the profile as a function of residual intensity. This curve is computed from $T(x)$ by an inverse quadratic interpolation procedure as follows. At a given intensity I , ($I = 1 - r$), the function $T(x)$ is analysed to find the values of x at which $T(x) - I$ changes sign. The exact value of x at which this occurs is determined by finding a least-square quadratic fit to $T(x)$ in this region

(as in the interpolation of $O(x)$). The solution of the resulting quadratic equation for $T(x)=I$ then gives the desired value of x . Great care must be taken in this solution since two values of x are usually found, one of which must be selected, or both roots may be complex. Certain "intuitive" criteria are included in the programme to allow for such cases. If there exist n separate peaks in the profile at the intensity level in question, then there must be $2n$ points at which $T(x)=I$. If x_0 and x_1 denote the two such points for each peak, the midpoint of the profile is assumed to be given by:

$$\bar{x} = \frac{\sum_{i=1}^n (x_1 - x_0)_i \left(\frac{x_1 + x_0}{2} \right)_i}{\sum_{i=1}^n (x_1 - x_0)_i} \quad (2.57)$$

This is illustrated in Fig. 2.3. It was found that only at large values of r , did more than one peak occur. This is closely related to the question of error "ripples" in the solution of the integral equation. The midpoint is found for up to 50 values of r , at intervals of 0.02, and the resulting curve of $\bar{x}$ as a function of $1-r$, is given in the output. The origin of the coordinate x is directly related to a reference line on the original tracing, so that the $\bar{x}$ curve may be given an absolute interpretation.

The whole programme is very flexible, and by various

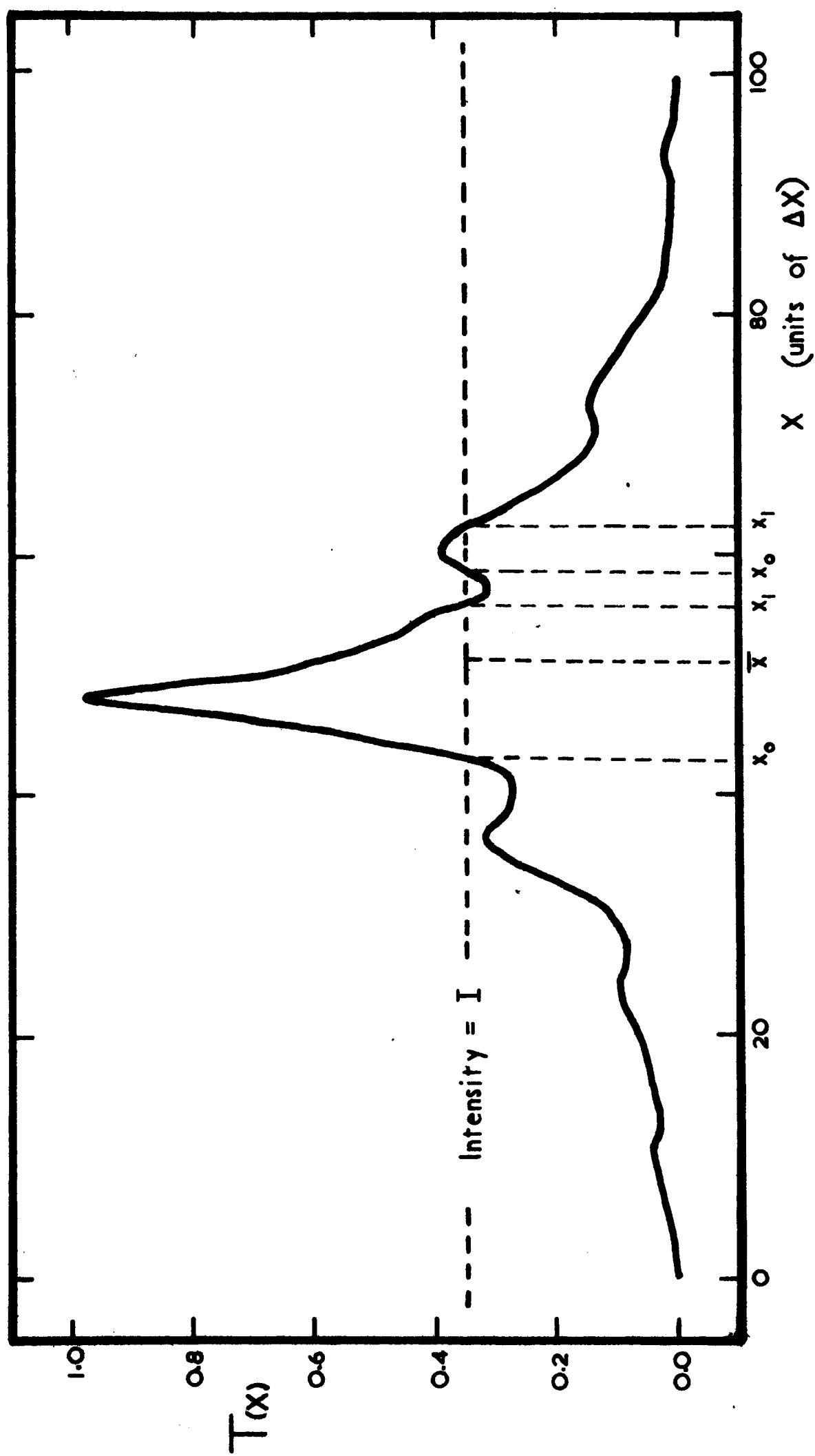


FIGURE 2.3 DETERMINATION OF ASYMMETRY CURVES
Asymmetry is denoted by plot of midpoint $\bar{x}$ vs. I

choices of the control parameters one can omit various sections of the computations, e.g. the solution of the integral equation or the determination of asymmetry. In addition, most of the computed functions may be printed out or suppressed at request, e.g. the observed $O(x)$ points, the interpolated $O(x)$ curve, the adopted $T(x)$ curve and the asymmetry curve. In order to conserve output time, the curve $T(x)$ may be printed out at intervals that are any integral multiple of Δx . The complete reduction of a typical profile requires about 3-4 minutes, much of which is required for the punching onto tape of the various curves, and as many profiles as desired can be reduced one after the other. About 10 hours of machine time were used in the reduction of the data for Chapter 4 (some 140 profiles), which is considerably faster than the corresponding reduction by hand (considering even the 30 hours required for the preparation of data tapes). Moreover, it is an objective method of determination of very small asymmetries. A detailed discussion of the programme and operating instructions is given in Appendix D.

References.

1. R. A. Brookes, B. Richards, E. Berg, and R. H. Kerr, "The Manchester Mercury Autocode System", Computing Machine Laboratory, Manchester, 1959.
2. H. H. Plaskett, M.N. 112, 117, 1952; Comm. Univ. Obs., Oxford, No. 34.
3. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.
4. P. J. Treanor, S. J., M.N. 109, 37, 1949; Comm. Univ. Obs., Oxford, No. 28.
5. M. G. Adam, M.N. 112, 149, 1952; Comm. Univ. Obs., Oxford, No. 36.
6. M. G. Adam, M.N. 115, 367, 1955; Comm. Univ. Obs., Oxford, No. 49.
7. M. G. Adam, M.N. 115, 385, 1955; Comm. Univ. Obs., Oxford, No. 50.
8. M. G. Adam and S. Nichols, M.N. 118, 51, 1958; Comm. Univ. Obs., Oxford, No. 65.
9. R. J. Bray, M.N. 114, 318, 1954; Comm. Univ. Obs., Oxford, No. 45.
10. H. C. van de Hulst, B.A.N. 9, 225, 1941.
11. H. C. van de Hulst, B.A.N. 10, 75, 1946.
12. H. C. Burger and P. H. van Cittert, Za. f. Phys., 79, 722, 1932.
13. H. C. Burger and P. H. van Cittert, Zs. f. Phys., 81, 428, 1933.
14. L. Fox and E. T. Goodwin, Phil. Trans. Roy. Soc. A, 245, 501, 1953.
15. G. Kreisel, Proc. Roy. Soc. A, 197, 160, 1949.

16. A. Ralston and H. S. Wilf, "Mathematical Methods for Digital Computers", p. 56, John Wiley and Sons, 1960.
17. R. J. Bray, M.N. 116, 471, 1956; Comm. Univ. Obs., Oxford, No. 57.
18. A. S. Eddington, M.N. 73, 359, 1913.
19. J. S. Rollett, Oxford University Computing Laboratory, private communication.

Chapter 3

OBSERVATIONS OF WAVELENGTH SHIFTS

P a r t I

LIMB EFFECT FROM MICROMETER MEASUREMENT

A. Introduction and Method.

Because of the great theoretical importance of the exact form of the limb-effect curve and also the magnitude of the wavelength shift at the limb, a new determination of limb effect and red shifts was undertaken. The method adopted by M. G. Adam (1) was used, and the same three iron lines were investigated. The new determination, however, was made at intermediate solar latitudes ($\sim \pm 55^\circ$) with a view to the detection of a latitude effect if such exists.

The method is a differential one in which the positions of the solar lines on a spectral plate are compared with those of a set of reference lines, in this case terrestrial oxygen lines, whose wavelengths are assumed constant. Since observations were confined to the part of the disk near the limb and the centre of the disk, such a method requires two essentially different sets of measurements. First, a set of standard positions of the solar lines at the centre of the disk with respect to the reference lines is required, and secondly, a set of measurements of the positions of the solar lines at various disk positions with respect to those of the reference lines is needed. Then the difference between the two gives the limb effect at various disk positions, when corrections have been made

for the effects of the rotation of the Sun and motion of the observer. These shifts can then be converted to a limb effect expressed either in wavelength units or, as is customary, in equivalent Doppler velocities. For such a procedure to be valid, the dispersion used in the two sets of measurements must be identical, otherwise corrections must be applied, all measurements being reduced to one standard dispersion.

B. Solar Spectra.

The spectral plates used in the determination were some of a series obtained in 1957 (1), others of which were used in the 1959 limb-effect study. They were taken using the 35m solar telescope with its associated spectrograph, the Bausch and Lomb grating giving a dispersion of 1.4 mm/A in the first order. The plates used were all Ilford panchromatic type R50 and were developed in Parkhurst two-solution developer.

Ten plates taken at the centre of the disk were used for the determination of standard positions and six plates of the region near the limb were used. For the limb plates, the solar image was guided so that the spectrograph slit was radial, the limb falling upon the slit. These plates were divided equally between the east and west limbs. The

sixteen plates used are listed in Table 1.

Table 1. Details of Measured Plates.

<u>Centre Plates</u>		<u>Limb Plates</u>	
<u>Date U.T. 1957</u>		<u>Date U.T. 1957</u>	<u>Solar Latitude</u>
May 21.3812		May 24.3929	West -50° to -59°
22.6342			
27.4411		24.3979	West -48.7 to -57.8
28.4674			
31.4592		27.3944	East 48.1 to 60.0
June 12.4850		June 16.4084	West -50.6 to -63.7
13.5754			
13.6131		16.4141	East 51.0 to 61.8
14.4505			
14.6085		16.4233	East 48.9 to 58.5

The spectral lines are the same as those used by M. G. Adam and are listed in Table 2, with their I.A.U. wavelengths. There is no I.A.U. wavelength for the third oxygen line and the value given is an estimated one (1). Table 2 also gives their Rowland intensities, and for the solar lines, the excitation potentials of the lower levels and the transitions involved. The oxygen lines are associated with the α band of atmospheric oxygen.

Table 2.Measured Lines

<u>I.A.U. Wavelength</u>	<u>Element</u>	<u>Rowland Intensity</u>	<u>Excitation Potential</u>	<u>Transition</u>
6295.960 A	Atm. O ₂	3	2.21 volts	$a^5P - y^5D^o$
6297.799	Fe	5		
6299.228	Atm. O ₂	3	3.64	$z^5P^o - e^5D$
6301.508	Fe	7		
(6302.000)	Atm. O ₂	2	3.67	$z^5P^o - e^5D$
6302.499	Fe	5		
6302.764	Atm. O ₂	2		

C. Centre Measurements.

The spectra were measured on a Hilger photomeasuring micrometer fitted with a transverse micrometer motion. A diaphragm in the focal plane of the eyepiece limited the field of view to a strip of spectrum 0.145 mm wide. This was positioned parallel to the plate dispersion so that only a small section of each line was visible. The plates were measured in the forward and reverse directions at the slit height corresponding to the centre of the disk and average line positions were found. These were referred to the first oxygen line after corrections for the observer's motion had been applied to the solar lines, the dispersions for the three lines being 33.99, 33.96, and 33.95 km/sec/mm. The average positions of the seven lines were then found from the ten plates. These were

adopted as the standard centre positions and the oxygen positions were used to indicate the standard dispersion. These standard centre positions are listed in Table 1, Chapter 4, where they are compared with the standard positions used by M. G. Adam (1).

D. Limb Measurements.

Each limb plate was measured at about forty slit heights, each corresponding to a different radial position (different ρ/ρ_0). The positions of measurement were so chosen that they fell at approximately equal intervals in $\cos \theta$. In order to compute the value of ρ/ρ_0 for each "spectrum" measured, the limb plates were marked perpendicular to the dispersion at one millimeter intervals from the inner edge of the spectrum. The positions of each of the measured "spectra" were referred to these marks, and the position of the limb was determined visually with respect to them at both ends of the plate. The image radius having been determined for the times of observation, a ρ/ρ_0 was calculated for each set of measurements.

The measurements were made in the same manner as for the centre plates and mean positions of the seven lines

were determined at each value of ρ/ρ_0 . Using the observed oxygen positions, the mean positions were corrected to the standard dispersion, assuming a linear correction factor. The difference between the corrected iron line positions and the standard iron positions then gave the wavelength shift, uncorrected for the observer's motion and solar rotation, for the corresponding value of ρ/ρ_0 . The shifts so found for the three lines were then converted to equivalent Doppler velocities and averaged to give a mean shift for that value of ρ/ρ_0 .

E. Accuracy of Measurement.

It was found that the mean velocity shifts determined as a function of ρ/ρ_0 from a single plate exhibited large fluctuations about a systematic trend, with amplitudes up to 0.2 km/sec. From an analysis of the micrometer settings in the spectra, it was concluded that the uncertainty in an individual reading of the position of an oxygen line was ± 0.05 km/sec, and for an iron line, ± 0.07 km/sec. at the centre, and ± 0.06 km/sec. at the limb. From these figures, it is found that the uncertainty in the mean shift of all three solar lines at a given disk position due to measurement alone is ± 0.05 km/sec. Since this uncertainty is much smaller than the observed fluctuations, the latter

are attributed to the effects of local velocity fields on the Sun (2).

From an analysis of micrometer measurements of the limb position with respect to the millimetre marks, it was concluded that the values of ρ/ρ_0 are uncertain by about ± 0.0003 . At the limb, this corresponds to ± 0.02 in $\cos \theta$.

F. Relative Velocity Corrections.

The corrections to be applied to the mean shifts obtained for each value of ρ/ρ_0 due to the motion of the observer and solar rotation were computed using the programmes discussed in Chapter 2. The solar equatorial velocity, R_0 , which is needed in these corrections was regarded as an undetermined parameter, to be supplied from the observations themselves.

The mean shifts at each disk position were then corrected for the observer's motion, giving at each point a residual shift

$$\Delta(\theta) = R_0 f(\theta) + \Lambda(\theta)$$

where $R_0 f(\theta)$ is the correction for solar rotation, $f(\theta)$ being the computed correction, and $\Lambda(\theta)$ is the limb effect.

This gives essentially 249 equations in R_0 and the $\Lambda(\theta)$. The 1948 limb effect curve obtained by M. G. Adam (3) showed, however, that $\Lambda(\theta)$ varies only slowly with θ except very near the limb. Thus the observations were split into thirteen groups, each covering a small range in $\cos \theta$ over which $\Lambda(\theta)$ could be considered approximately constant. For each group of observations, R_0 and $\bar{\Lambda}(\theta)$ were then determined by a least squares solution. As in the 1959 determination, the values of R_0 were fairly constant for the inner groups but decreased rapidly as the limb ^{was} approached. This was attributed to the effect of scattered light (1). From the four inner groups of observations, little affected by scattering, a mean value of 1.992 km/sec. was determined for R_0 . Using this, the residual shifts were corrected for solar rotation, leaving values of $\Lambda(\theta)$.

G. Scattering Corrections.

Since the positions of the observed points near the limb were uncertain by ± 0.02 in $\cos \theta$, and scattering effects depend primarily on the limb being observed (east or west), the shifts obtained from the three east limb plates were averaged together in intervals of 0.02 in $\cos \theta$, and similarly for the west limb. These results

are shown in the top half of Figure 3.1. The large scatter in the observed mean shifts is attributed to local velocity-field effects (2), and it is seen that the west limb values are lower near the limb than those for the east limb. This is as would be expected, since scattering effects are far greater for the west limb where solar rotation and true limb effect combine to give a rapid variation of wavelength near the limb.

The scattering effect was assumed to be represented mathematically by the simplified expression

$$\overline{u(x_0)} = \frac{\int_{-1}^1 I(x) u(x) S(x-x_0) dx}{\int_{-1}^1 I(x) S(x-x_0) dx} \quad (3.1)$$

where $u(x)$ is the true velocity shift (in the line of sight) at $\rho/\rho_0 = |x|$, $I(x)$ is the intensity at x , $S(x-x_0)$ is the scattering function and $\overline{u(x)}$ is the observed velocity shift. The limb darkening curve, $I(x)$, used by M. G. Adam was chosen, and, in the absence of any other "scattered light" exposures of the sky spectrum, the scattering function used by her was adopted (1).

To calculate corrections for scattering, mean observed velocity-shift curves were obtained for the two limbs. Each was then smoothed and a true velocity shift curve

was approximated by iteration using the above expression for the scattering. From this, a scattering correction factor as a function of $\cos \theta$ was determined for each limb. Using this, the observed limb-effect plots were then corrected, and the final values for the two limbs are given in the lower half of Fig. 3.1. Owing to limitations in the iteration process, reliable scattering corrections could not be found for points nearer the limb than $\cos \theta = 0.05$, and hence the limb-effect results terminate at this point.

H. Final Limb Effect.

The east and west limb results were then averaged together to give a final limb-effect curve. This is given in Fig. 3.2. It is evident that local velocity-field effects still persist in the final curve despite the averaging of six plates. A smooth curve (a parabolic formula in $\cos \theta$ fitted by least squares) through the observed points gives the values tabulated in Table 3 as a function of ρ/ρ_0 .

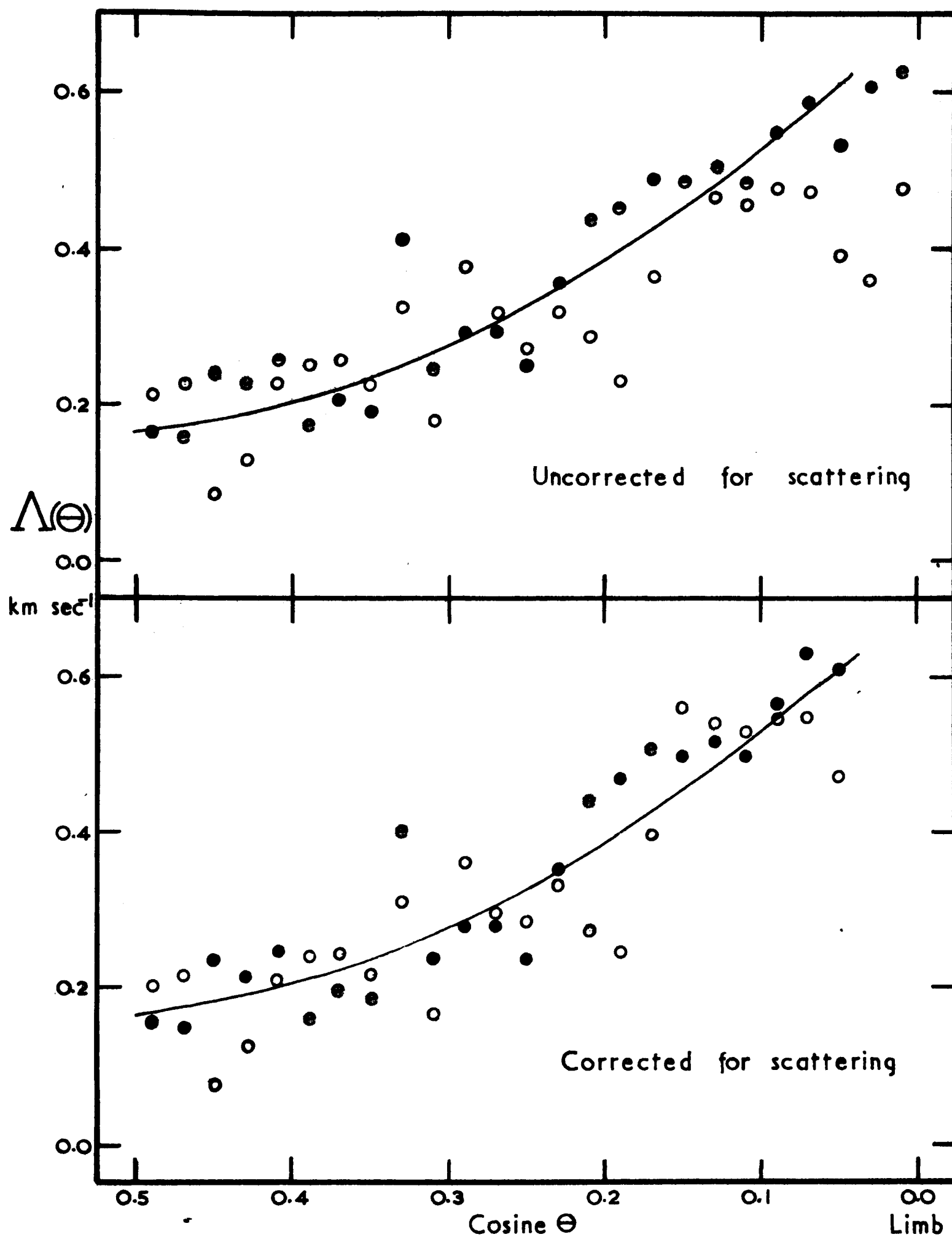


FIGURE 3.1 LIMB EFFECT AT THE EAST AND WEST LIMBS
 East • West ○
 The continuous curve is the final limb effect

Table 3.Final Limb Effect

<u>ρ/ρ_0</u>	<u>Δ (km/sec)</u>	
	L. A. Higgs	M. G. Adam
0.998	0.584	0.474
0.995	0.524	0.442
0.990	0.461	0.407
0.985	0.417	0.381
0.980	0.383	0.358
0.975	0.355	0.339
0.970	0.331	0.321
0.950	0.264	0.263
0.925	0.214	0.206
0.900	0.185	0.159
0.875	0.168	0.118
—	—	—
0.000	0.000	0.000

Included in this table are the corresponding values obtained by M. G. Adam for the solar equatorial region (1). The new determination appears to give larger shifts than those observed by her, but the difference between the two is entirely accounted for by differences in the standard solar positions used in the two determinations (see Table 1, Chapter 4). The uncertainty in these standard positions precludes the measurement of any general difference < 0.07 km/sec. Thus a latitude effect, if such exists, must be smaller than this figure. An analysis of the velocity shifts obtained from individual lines indicated small systematic differences between the shifts of the three

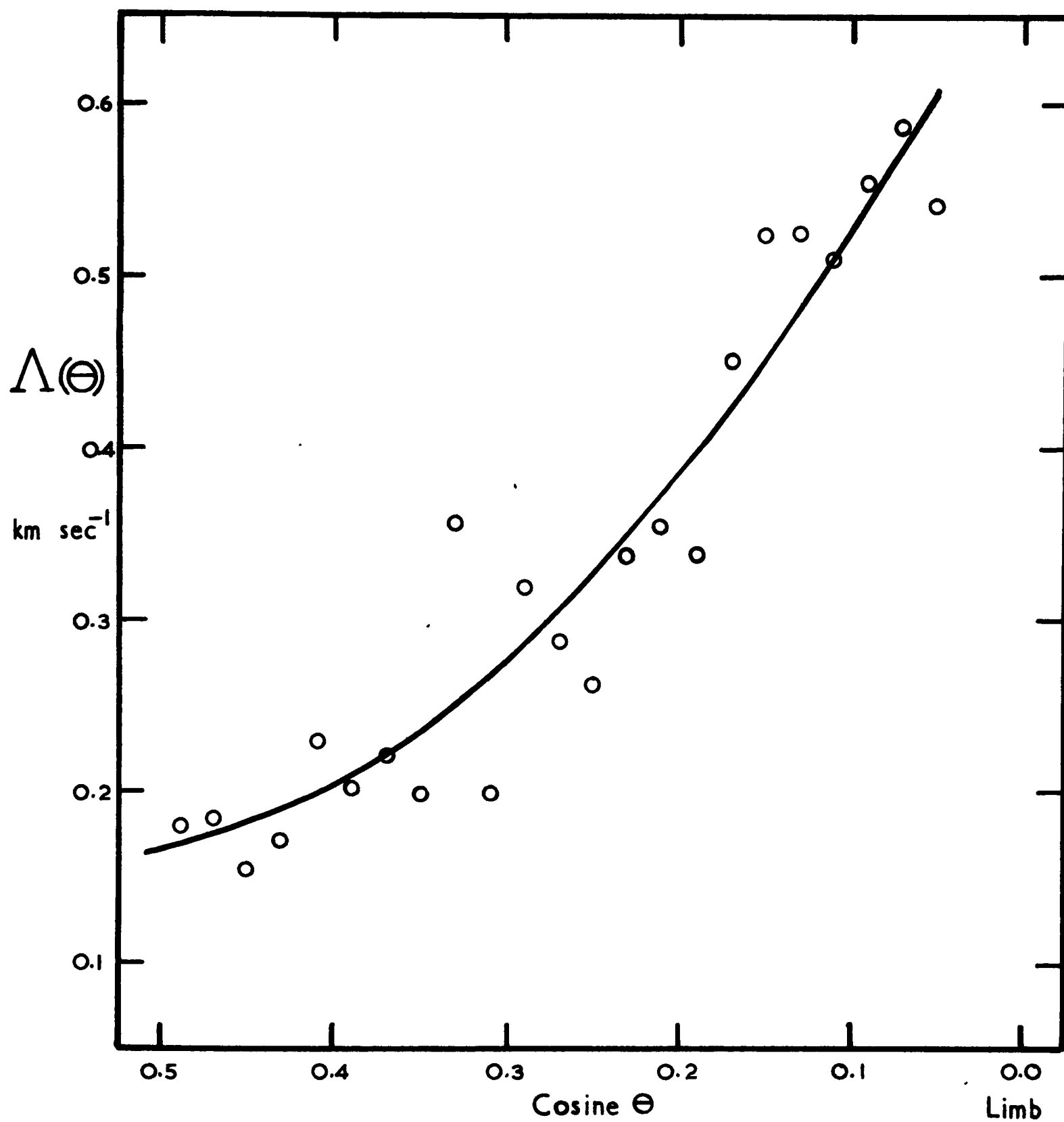


FIGURE 3.2 MEAN LIMB EFFECT

Averaged over both limbs o
 Least square fit —

lines, but these could also be accounted for by slight errors in the standard positions of the three lines. The question of the total red shift at the limb, especially the possibility of a super-gravity shift, cannot be answered until the red shifts of these lines at the centre of the disk are known (Part II).

P a r t I I

**DETERMINATION OF ABSOLUTE WAVELENGTHS AND CENTRE
RED SHIFTS**

A. Method.

The determination of red shifts requires the absolute wavelengths of both the solar lines and the corresponding terrestrial lines, formed in physical conditions as near those existing in the solar photosphere as possible, e.g. vacuum arc. At Oxford, the method of "circular channels" has been developed as a powerful interferometric technique for wavelength determinations, the theory and applications of which have been presented in the literature (4,5,6,7) and briefly discussed in Chapter 2.

Essentially the method consists of using a spectrograph in conjunction with a Fabry-Perot interferometer, the spectrograph serving only as a monochromator for the interferometer. If the light being analysed has a continuous spectrum with absorption lines, the observed interferometric pattern consists of a set of circular "heterochromatic" fringes, due to the continuous spectrum, which are crossed by absorption arcs forming part of another set of concentric circles intersecting the first. In addition, the interference pattern of the cadmium red line (λ 6438.4696) is superimposed on this pattern using a beam splitting cube. Overlapping is prevented by a system of diaphragms. For the measurement of arc wavelengths, in which one is concerned

only with a discrete line spectrum, the "heterochromatic" fringes do not exist and emission arcs now replace the absorption arcs.

The interference patterns are measured on a specially adapted Hilger photo-measuring micrometer with a rotating plate stage. From detailed computations, described in Chapter 2, using the measured arc positions, the wavelengths of the absorption or emission lines are compared with the wavelength of the cadmium line. Although complicated in its early stages, the method of circular channels has now become a standard procedure, and with the development of computer reduction of plate measurements, is now a fairly rapid method of wavelength determination.

B. Procedure.

The smaller Oxford telescope, giving an 18 cm solar image, was used for the determination of absolute wavelengths at the centre of the disk. The sun was imaged on the spectrograph slit, the slit height being 1.8 cm, and filters were used to equate the intensities of the solar and cadmium light beams passing through the interferometer. Two different spacers were used in the interferometer giving plate separations of 2.5 and 5 mm. Fused silica plates were

used which had been recoated at the beginning of the wavelength programme to give 7% transmission and 90% reflection in the red region. Seventy-four solar plates were taken, using Ilford panchromatic R40 plates, at exposures varying from one to three minutes, depending on the spacer used. Of these, twenty-one plates taken on four different days were measured and reduced, using the computer programme discussed in Chapter 2. Phase-change corrections and camera focal length corrections were adopted from earlier measurements (8). Both of these were small owing to the small difference between the wavelengths of the lines in question and the standard cadmium wavelength. For this reason, corrections for the dispersion of air in the interferometer were not necessary.

For the arc wavelengths, a vacuum arc run at about 9 cm Hg pressure replaced the Coudé beam from the telescope. Mild steel pole-pieces were used and a 10 mm arc was maintained at a current averaging 8 amperes. The usual method of guiding the arc and focusing it on the slit was employed (5), only the central portion of the arc actually being used in order to eliminate any pole effects. Owing to the low intensity of the arc lines, exposures of up to two hours were required and the cadmium intensity was correspondingly reduced using filters. Fifteen plates were taken,

using Ilford HP3 plates, and twelve of these were measured and reduced. The corrections for phase change, etc. were the same as for the solar plates. The calculated absolute wavelengths were reduced to absolute vacuum wavelengths using the empirical pressure depression of spectral terms given by Babcock (9). The largest pressure correction for the three iron lines amounted to only 0.9 mÅ.

C. Solar and Arc Wavelengths.

The calculated solar wavelengths obtained from the centre plates were corrected for the Doppler shifts due to the motion of the observer using the computer programme for radial velocity corrections. Since some plates were of better quality than others, it was decided to weight each of the calculated wavelengths on each plate according to the inverse square of its calculated mean error for that plate. A similar method was used to determine the average arc wavelengths. There were no serious differences in the wavelengths determined considering the two spacers separately. For the solar wavelengths, the 5 mm spacer gave wavelengths 0.6 mÅ longer than the 2.5 mm spacer, while for the arc wavelengths, the 2.5 mm spacer values were 1.0 mÅ longer. The final absolute wavelengths are given in Table 4, along with their standard deviations and the number of plates used

in each determination. For comparison purposes, the I.A.U. wavelengths are given for the solar lines and the Allegheny Observatory wavelengths (10) of the arc lines are given.

Table 4.Absolute WavelengthsSolar Spectrum Wavelengths

<u>El.</u>	<u>I.A.U.</u>	<u>Observed</u>	<u>No. Plates</u>
O ₂	6295.960 A	5.9603 ± 0.0002	10
Fe	6297.799	7.7989 0.0004	21
O ₂	6299.228	9.2283 0.0002	21
Fe	6301.508	1.5080 0.0003	21
O ₂	(6302.000)	1.9998 0.0001	21
Fe	6302.499	2.5000 0.0003	21
O ₂	6302.764	2.7640 0.0001	10

Iron Arc Wavelengths

<u>Alleg. Obs.</u>	<u>Observed</u>	<u>No. Plates</u>
6297.790 A	7.7906 ± 0.0006	10
6301.501	1.4990 0.0003	12
6302.493	2.4920 0.0004	12

D. Integrated-Disk Wavelengths.

Since it is known that local velocity fields may greatly affect wavelength determinations on the Sun, the question arises whether the twenty-one solar plates were enough to

eliminate these effects. As a check, it was decided to take some wavelength plates using integrated sunlight from all parts of the disk. An approximate calculation showed that, for the lines in question, the increase in line width expected from solar rotation is small, approximately an increase of 8%, thus the accuracy of circular channel measurements would not be affected. Ideal conditions for integrated-light wavelength measurements would be attained if every part of the slit and every part of the spectrograph collimator received truly integrated light. In practice it was found impossible to fulfil both requirements. Two optical arrangements were used, both of which required a parallel beam of solar light in the vicinity of the spectrograph slit. For this purpose, the Cassegrain mirror of the telescope was removed, and light passed down to the spectrograph using plane mirrors only.

In the first arrangement, the parallel beam was allowed to fall directly on the slit, thus satisfying the first condition above. To fulfil the second requirement, however, a very long slit is required. The slit height actually used, however, introduced vignetting at the collimator in such a manner that the upper and lower regions of the collimator received primarily light from the west and east limbs of the Sun respectively. To counteract this, an

image rotator was placed before the slit and the circular channel plates were taken in "mirror pairs". For the second plate of each pair, the image used for the first was inverted about the optical axis. In this way, wavelength shifts due to variations in the transmission properties of the grating across its surface were expected to be averaged out. The plates taken with this arrangement required exposure times of about 5 minutes.

A second arrangement using a lens train was also used. The optical arrangement is shown in Fig. 3.3. The parallel beam from the Coude mirror fell on a doublet lens, A, of 9.5 cm focal length, which produced an image of the Sun at B. A field lens, C, of 52.6 cm focal length, imaged the lens, A, on the spectrograph collimator, D. With this arrangement, the slit received integrated light, but the slit height again introduced vignetting at the collimator. The exposure times with this arrangement tended to be very long (about 15 minutes), and hence only a few plates were taken.

It was decided that the first arrangement was more reliable, owing to the presence of the image rotator, and two pairs of plates so obtained were measured. The average wavelength values determined for each of these mirror pairs agreed well with each other, although there

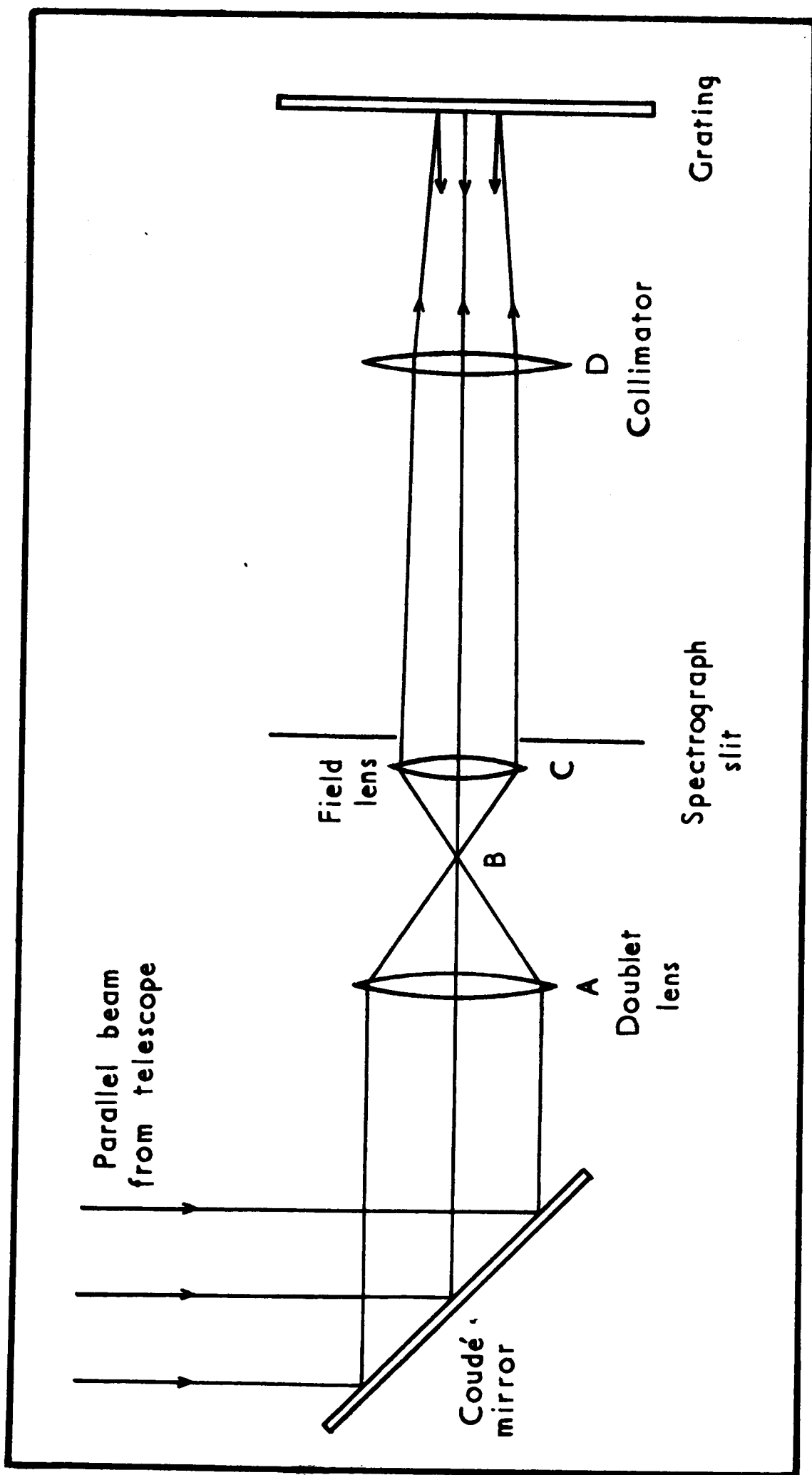


FIGURE 3.3 OPTICAL TRAIN FOR INTEGRATED DISK OBSERVATIONS
(not to scale)

were differences (~ 4 mÅ) between the wavelengths determined from the four plates individually.

Since wavelengths observed at the outer edge of the Sun are usually slightly redder than those at the centre, one might expect integrated light wavelengths to be slightly longer than those determined at the centre of the disk. A calculation of this difference was made using the limb effect curve and limb darkening from Part I. It was found that the wavelength difference was very critically dependent on the limb effect at $\rho/\rho_0 = 0.85$, which lies outside the range of the present observations. Using the 1948 curve (2), it was estimated to be in the range of 0.9 to 1.2 mÅ. In fact, the agreement between the mean wavelengths from the four integrated-disk plates (corrected for the observer's motion) and the absolute wavelengths determined at the centre of the disk is remarkable. Table 5 gives this comparison. This close agreement would seem to indicate that the absolute solar wavelengths found at the centre of the disk are free from effects due to local velocity fields.

Table 5. **Comparison of Solar Wavelengths.**

<u>Element</u>	<u>Centre</u>	<u>Integrated Disk</u>
O ₂	6295.9603 ± 0.0002 A	5.9605 ± 0.0002 A
Fe	6297.7989 0.0004	7.7980 0.0011
O ₂	6299.2283 0.0002	7.2288 0.0004
Fe	6301.5080 0.0003	1.5086 0.0010
O ₂	6301.9998 0.0001	2.0002 0.0003
Fe	6302.5000 0.0003	2.4999 0.0007
O ₂	6302.7640 0.0001	2.7636 0.0003

E. Red Shifts.

From the above absolute solar and arc wavelengths, the red shifts of the three lines may be determined. Since there is an appreciable gas pressure in the photosphere, the absolute vacuum wavelengths have been slightly corrected for the determination of red shifts. The photospheric pressure was estimated to be 0.05 atm. from the mean depth of formation of the lines derived by Minnaert (11), and the photospheric model given by H. H. Plaskett (12).

These are presented in Table 6. The observed shifts are slightly less than those expected according to the empirical shift vs. line strength relation given by Nichols and Clube (13).

Table 6.Centre Red Shifts.

<u>Line</u>	<u>Red Shift</u>
6297.8 A	8.1 ± 0.7 mÅ
6301.5	8.6 0.4
6302.5	7.6 0.5

The results of Part I may now be extended to give the red shifts of the three lines across the disk (with the reservations to be indicated in the next chapter). A final limb-effect curve on an absolute scale is given in Fig. 3.4, in which the 1959 curve and the new determination are presented with their regions of experimental error. The scale on the left of the figure gives the limb effect in km/sec, while that on the right gives the absolute red shift in km/sec. Also shown is the red shift required by the Principle of Equivalence. From this diagram, it is evident that, allowing for experimental error, the shift observed at the extreme limb exceeds the gravity shift, thus apparently ruling out the radial current interpretation of limb effect.

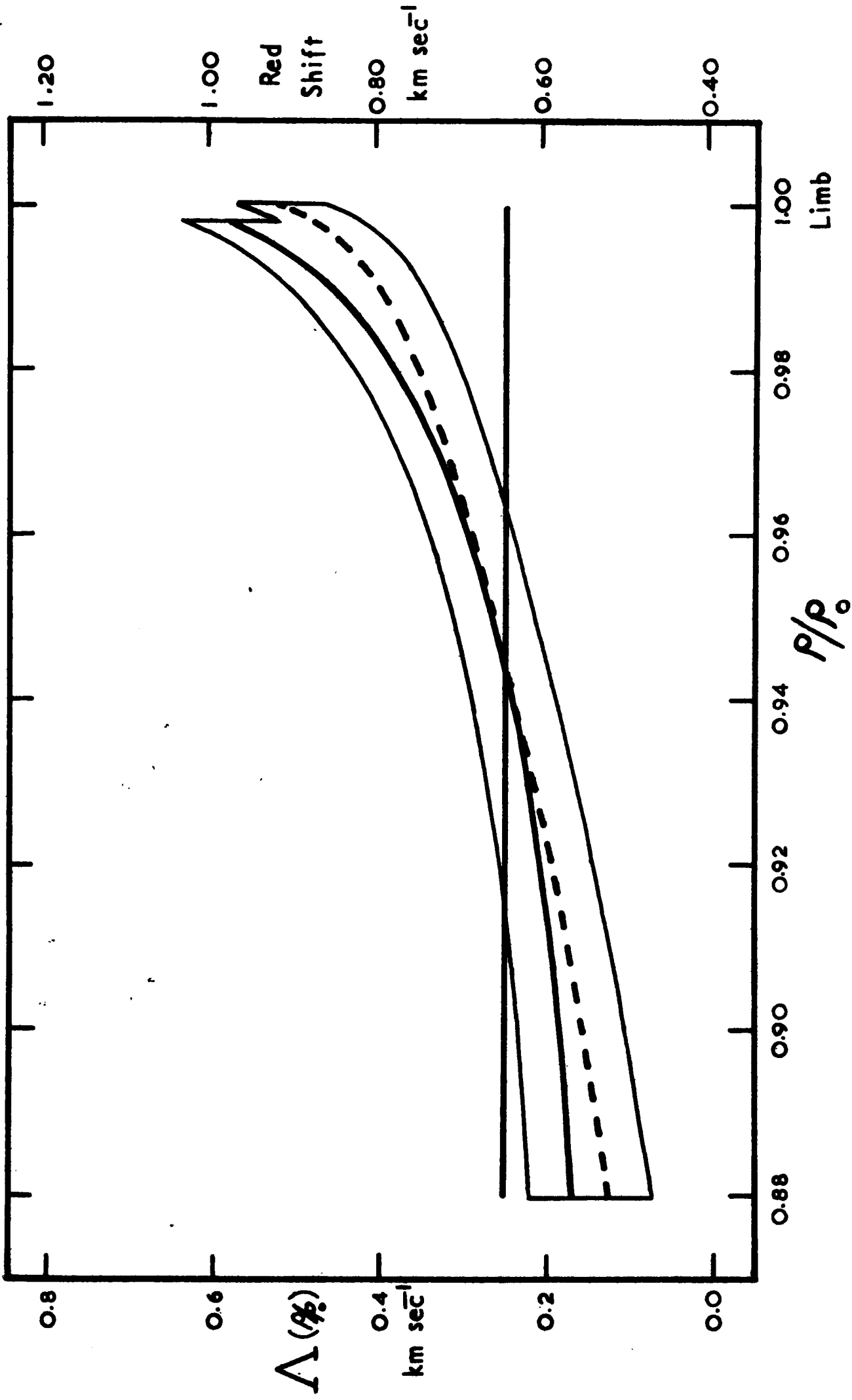


FIGURE 3.4 OBSERVED RED SHIFT

M.G. Adam 1959 ---

L.A. Higgs 1960 —

The uncertainty in the curves is indicated
The horizontal line gives the gravitational shift

References.

1. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.
2. A. B. Hart, M.N. 116, 38, 1956; Comm. Univ. Obs., Oxford, No. 54.
3. M. G. Adam, M.N. 108, 446, 1948; Comm. Univ. Obs., Oxford, No. 26.
4. P. J. Treanor, S. J., M.N. 109, 37, 1949; Comm. Univ. Obs., Oxford, No. 28.
5. M. G. Adam, M.N. 112, 149, 1952; Comm. Univ. Obs., Oxford, No. 36.
6. M. G. Adam, M.N. 115, 367, 1955; Comm. Univ. Obs., Oxford, No. 49.
7. M. G. Adam, M.N. 115, 385, 1955; Comm. Univ. Obs., Oxford, No. 50.
8. M. G. Adam and S. Nichols, M.N. 118, 51, 1958; Comm. Univ. Obs., Oxford, No. 65.
9. H. D. Babcock, Ap. J. 67, 240, 1928.
10. K. Burns and F. M. Walters, Publ. Alleg. Obs. VI, 159, 1929.
11. M. Minnaert, B.A.N. 10, 399, 1948.
12. H. H. Plaskett, "Vistas in Astronomy", Vol. I, p. 640, edited by A. Beer, Pergamon Press, 1955.
13. S. Nichols and S. V. M. Clube, M.N. 118, 73, 1958; Comm. Univ. Obs., Oxford, No. 67.

Chapter 4

OBSERVATIONS OF LINE PROFILES

P a r t I

INSTRUMENTAL EFFECTS ON THE GRATING WAVELENGTHS

In Part I of Chapter 3, it was stated that standard micrometer positions for a centre spectrum were adopted for the seven lines used, and in Part II, the determination of the absolute wavelengths of these lines was presented. Using the absolute oxygen wavelengths and micrometer positions, it was possible to calculate the micrometer positions we should expect from the measured absolute wavelengths of the iron lines. It was found that the calculated micrometer positions of the three solar lines were, on the average, 0.005 mm to the red side of the adopted standard positions. This corresponds to a wavelength difference of 4 mÅ. The calculated and standard micrometer positions are compared in Table 1, in which the standard positions used by M. G. Adam (1) are also included.

Table 1.Micrometer Positions.

<u>λ</u>	<u>Element</u>	<u>Standard Positions</u>		<u>Calculated Positions</u>
		M. G. Adam	L. A. Higgs	
6296.0 Å	O ₂	0.000 mm	0.000 mm	0.0000 mm
6297.8	Fe	2.571	2.570	2.5748
6299.2	O ₂	4.576	4.576	4.5764
6301.5	Fe	7.764	7.761	7.7683
6302.0	O ₂	8.456	8.457	8.4569
6302.5	Fe	9.154	9.153	9.1572
6302.8	O ₂	9.527	9.527	9.5268

On account of the small portion of the spectrograph slit (corresponding to 600 km on the surface of the Sun) used in the determination of the standard micrometer positions, one would expect some random fluctuations due to local velocity fields. Results obtained by A. B. Hart (2) indicate that r.m.s. velocity fluctuations of about 0.15 km/sec can be expected at a given point on the solar disc. Since ten plates were used in the determination of the standard positions, one might expect fluctuations in the mean positions of about ± 0.001 mm. Since the discrepancy in question was five times this value, it seemed unlikely that it was due to unusually strong velocity fields at the centre of the disk when the ten spectra were taken.

On the other hand, since the slit height used in the absolute wavelength measurements corresponded to 140,000 km on the surface of the Sun, one would expect all local velocity fluctuations to have been eliminated. The close agreement between the wavelengths determined at the centre of the disk and those obtained using integrated light tended to confirm this. It thus seemed difficult to accept the possibility that the absolute wavelengths were in error.

Since an asymmetric apparatus function of a spectrograph can produce appreciable wavelength shifts of spectral

lines (3), the possibility of such an effect in the grating spectra was considered. Using the Oxford Moll-type microphotometer, sixteen tracings of typical spectra were taken, in the forward and reverse directions, both at the centre of the disk and at several points near the limb. The dimensions of the microphotometer slit were chosen so as to correspond to the dimensions of the eyepiece diaphragm used in the micrometer measures. The tracings of a typical centre spectrum showed that the oxygen lines were approximately symmetric while the solar lines were definitely asymmetric, having pronounced violet flanks. Since this asymmetry is of such a magnitude as to be scarcely detectable in a diagram of the actual line profile, the following method of presentation will be used. The profile of a line is represented by a curve giving the mid-point of the profile at each value of the incident intensity, I . This is illustrated by the curve AB in Fig. 4.1 (a). In this manner, the observed profile of the strongest iron line (λ 6301.5) at the centre is presented in Fig. 4.1 (b).

To determine an approximate apparatus function for the 30,000 line/inch Bausch and Lomb grating, it was decided to use the terrestrial oxygen lines as essentially mono-

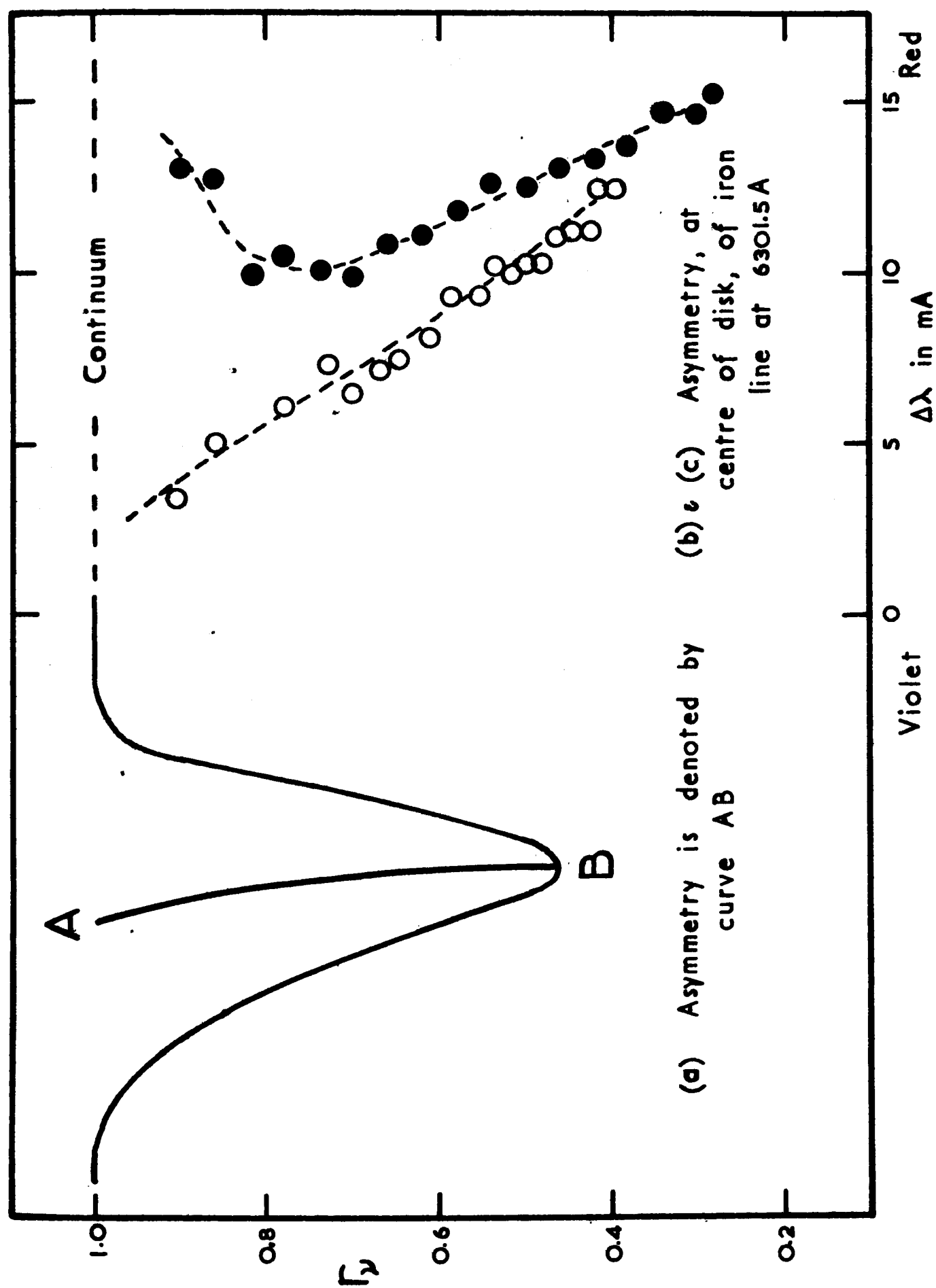


FIGURE 4.1 EFFECT OF GRATING APPARATUS FUNCTION ON PROFILES

Observed asymmetry O O (Bausch & Lomb grating)
 Computed true asymmetry ● ●

chromatic absorptions. The apparatus function is then given by the observed oxygen profiles. Allen (4), who has used atmospheric oxygen lines for similar purposes, gives theoretical values for the absorption coefficient in the line which leads to a line width of about 25 mÅ. This indicates the degree of approximation involved. Only the limb spectra were used for this purpose because the centre spectra were too over-exposed to show any possible asymmetry in the oxygen lines. All profiles were reduced to true intensity profiles using a characteristic curve for R50 plates, obtained in another investigation. From the profiles of the oxygen lines λ 6296.0 and λ 6299.2 an approximate apparatus function was obtained. It is shown in Fig. 4.2. Having a half-width of 69 mÅ, it is symmetric to a width of about 100 mÅ, after which it develops a violet wing. The uncertainty in the determination of the wings is about 1 per cent of the central intensity, whereas the asymmetry in the wings is three times this figure. Thus a true asymmetry is indicated.

Using this apparatus function, the observed centre profile was reduced to a true profile using the Burger-van Cittert iteration procedure given in Chapter 2. The

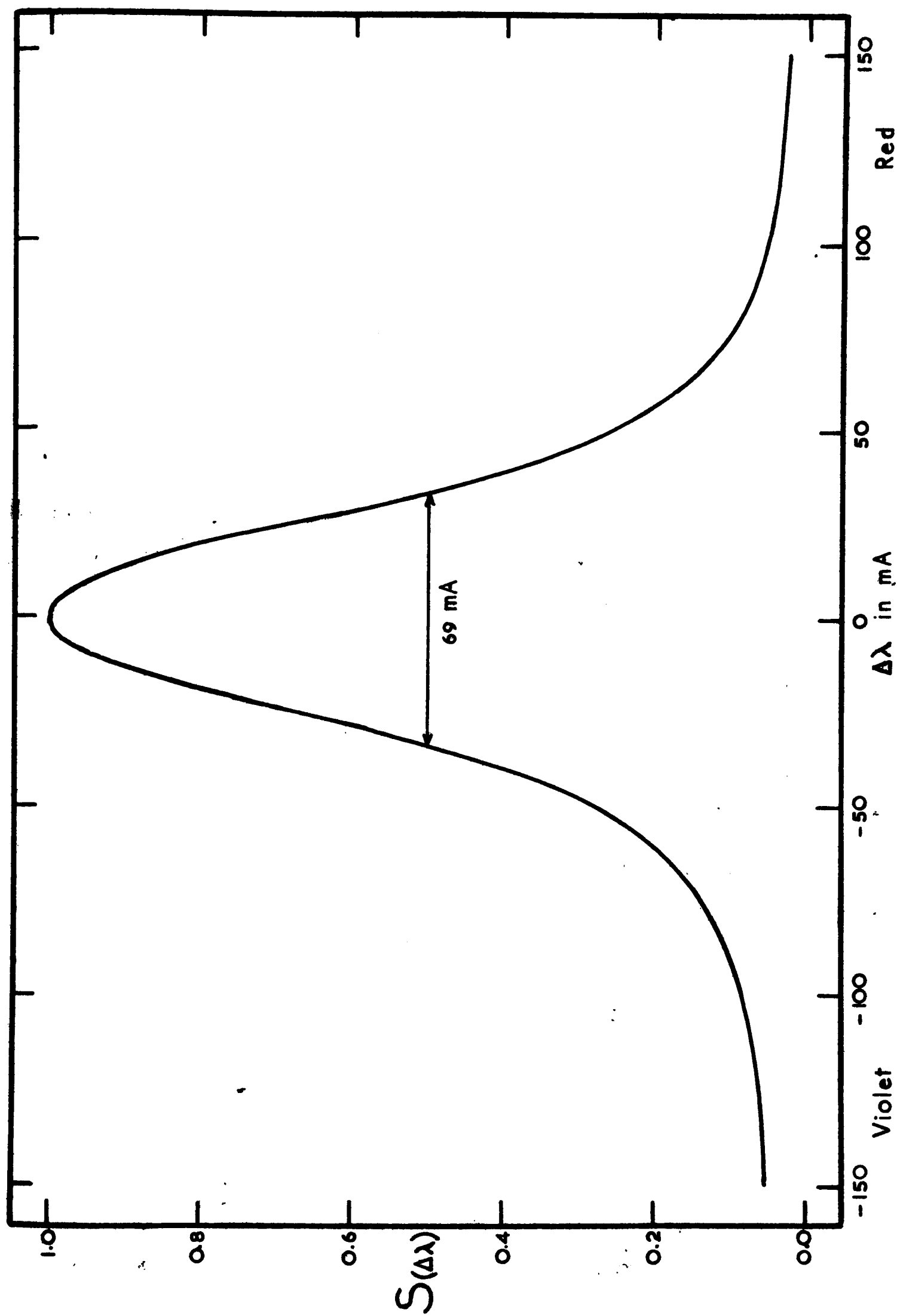


FIGURE 4.2 APPARATUS FUNCTION FOR BAUSCH & LOMB GRATING
First order at 6300 Å

Uncorrected for width of oxygen lines (~30mA) used for its determination

asymmetry of the true profile thus obtained is shown in Fig. 4.1 (c). From this it is seen that a large part of the observed asymmetry arises in the apparatus function and it was shown that, within the experimental error of the apparatus function determination, one could assume the true profile to be very nearly symmetric. In addition, it was estimated that the above apparatus function would shift the part of the solar profiles used in micrometer settings 3-4 mμ to the violet from their true wavelengths. This explains the apparent discrepancy between the grating and interferometric wavelengths, and indicates the precautions that must be taken in inferring wavelengths from grating spectra. Since this instrumental shift of grating wavelengths is independent of the point of observation on the solar disk, it has no effect on micrometer limb-effect determinations.

P a r t I I

LINE PROFILE AND ASYMMETRY DETERMINATION

A. Introduction and Procedure.

As indicated in Part I, the great difficulty in interpreting micrometer measurements of wavelengths lies in the undetermined relation between the position one accepts as the micrometer "centre" of the line and the true line profile. If one makes the simple assumption that all solar line profiles are of similar form, independent of the point of observation on the solar disk, such difficulties cannot affect any limb-effect determinations such as that discussed in Part I, Chapter 3. This is, however, a questionable assumption when one considers the complicated structure of the photosphere (see Part III, Chapter 1) and with the high dispersions now available, one should consider the possible variations of line profile as a function of disk position. A preliminary study had indicated that a detailed investigation of line profiles was worthwhile (see Part III, Appendix C), and therefore a detailed investigation of the true profiles, at the centre and near the limb, of the two iron lines, λ 6297.8 and λ 6301.5, was undertaken.

Using the 18 cm. image of the smaller Oxford telescope and its associated spectrograph, the Babcock grating giving a dispersion of 3.2 mm/A in the fourth order, spectra of the 6300 Å region, at both the limb and centre,

were obtained in June and July, 1960. Ilford HP3 and panchromatic R40 plates were used, the former being developed in Kodak Press Contrast developer and the latter in Parkhurst two solution developer. These plates were used in preference to the slower, finer grain plates more suitable for profile determinations because very poor observing weather limited the exposure times. With a slit width of 0.047 mm, exposure times of 5 to 15 sec were required.

On each plate, calibration exposures of the centre spectrum were taken, with and without a pre-calibrated seven step wedge, along with a limb spectrum, all with identical exposure times. For the centre spectrum without the wedge, a filter was used to reduce the density to that of the limb spectrum. For the latter, the solar image was guided so that the spectrograph slit was radial, crossing the limb approximately on the solar equator. Eleven plates were taken and of these, six were chosen for line profile determinations, three from each limb. These are listed in Table 2.

Table 2. High Dispersion Plates.

<u>Plate No.</u>	<u>Date U.T. 1960</u>		<u>Limb</u>
	<u>Limb Exp.</u>	<u>Centre Exp.</u>	
H4 (HP3)	June 28.6332	June 28.6273	West
H7 (HP3)	30.3620	30.3551	East
H12 (R40)	July 6.3872	July 6.3907	West
H14 (R40)	7.3477	7.4450	East
H16 (R40)	7.4574	12.3530	East
H18 (R40)	12.5579	14.5416	West

For each limb spectrum, the limb was estimated visually and marked, along with points approximately at $\rho/\rho_0 = 0.998$, 0.994, 0.989, and 0.983. Microphotometer tracings perpendicular to the dispersion then allowed the true limb to be determined, using the point of inflection in the limb darkening curve to indicate the position of the limb. From this, the true ρ/ρ_0 's of the marked positions on each plate were determined with an estimated uncertainty of ± 0.0008 . In order to obtain the calibration for each plate, three or four microphotometer tracings were made across the calibration spectra, also perpendicular to the dispersion.

At each marked ρ/ρ_0 , microphotometer tracings of the limb spectrum were made using a microphotometer slit height corresponding to $\Delta\rho/\rho_0 = 0.002$. Tracings were made of the two oxygen lines, λ 6296.0 and λ 6302.0, in addition to the two iron lines, λ 6297.8 and λ 6301.5 in such a

manner that profiles of one iron line and one oxygen line were present on each tracing. For most values of the tracings were taken in both the forward and reverse directions.

Tracings of the same lines at the centre were taken from the wedge-calibration spectrum on each plate, the wedge step being chosen to match the limb spectrum in photographic density. In all, tracings of 123 line profiles were obtained, the majority of these being of the two lines, Fe λ 6297.8 and O₂ λ 6296.0. Owing to the considerable grain of the plates, each profile was smoothed by drawing a smooth curve through the microphotometer profile. Fig. 4.10 (a) shows such a profile for the iron line, λ 6297.8, although not a typical one as far as asymmetry is concerned.

B. Apparatus Function.

In order to obtain true profiles of the spectral lines, the apparatus function of the 9 meter spectrograph and the associated 15,000 line/inch Babcock grating was required. As in Part I, it was decided to use the atmospheric oxygen lines to determine the apparatus function, but in a more exact manner than was done for the Bausch and Lomb grating. Two of the best spectral plates were chosen, one of the east limb (H14) and one of the west limb (H12), and all available tracing profiles of the oxygen line λ 6296.0 were measured. This gave 9 profiles from plate H12 and 8 from plate H14, at various values of ρ/ρ_0 from centre to extreme limb. Using the calibration tracings for these two plates and the computer programme for line profile reduction, true intensity profiles, uncorrected for the apparatus function, were determined for this line. From the calculated equivalent widths for each profile, a mean equivalent width was found for each plate. Knowing the equivalent width, the true line profile may be determined from an appropriate theory of line formation.

The theory of line formation given by Allen (4) and Van de Hulst (5) was adapted for this purpose. For an isothermal atmosphere, Allen has shown that the absorption

coefficient, integrated with respect to path length, assuming no Doppler broadening, is given by:

$$\alpha(x) = \frac{S_0 h_1 \sec z}{2\pi b_0} \ln \left(1 + \frac{b_0^2}{x^2} \right) \quad (4.1)$$

where $x = \nu - \nu_0$, z = zenith distance, h_1 is the atmospheric scale height, $b_0 = \frac{\gamma_0}{4\pi}$ where γ_0 is the local (at telescope level) damping constant, and S_0 is related to the product of the local air density and the transition probability for absorption. The effect of Doppler broadening can be included analytically, as shown by Van de Hulst, by calculating the Fourier transform of $\alpha(x)$ and adding a term due to thermal broadening. Thus the Fourier transform of the final integrated absorption coefficient is given by:

$$\mathcal{F}\{\alpha(x)\} = S_0 h_1 \sec z \frac{(1 - e^{-b_0 t})}{b_0 t} e^{-\frac{\delta^2 t^2}{4}} \quad (4.2)$$

where $\delta = \sqrt{\frac{2RT}{\mu}} \frac{\nu_0}{c}$ is the Doppler term.

If we now approximate $\alpha(x)$ by a Voigt function, $V(x)$ the Fourier transform of $V(x)$ is given by:

$$\mathcal{F}\{V(x)\} = e^{-\beta_0 - \beta_1 t - \frac{\beta_2^2 t^2}{4}} \quad (4.3)$$

Thus we must have

$$\beta_0 + \beta_1 t + \frac{\beta_2 t^2}{4} \approx -\ln \left\{ S_0 h_1 \sec z \frac{1 - e^{-b_0 t}}{b_0 t} e^{-\frac{\delta^2 t^2}{4}} \right\} \quad (4.4)$$

Expanding the logarithm by

$$-\ln \frac{1 - e^{-u}}{u} = \frac{u}{2} - \frac{u^2}{24} - O(u^4) \quad (4.5)$$

we find that

$$\beta_1 = \frac{b_0}{2} \quad (4.6)$$

$$\beta_2 = -\frac{b_0^2}{6} + \delta^2 \quad (4.7)$$

Thus we see that for an isothermal atmosphere, the effective damping constant, $4\pi\beta_1$, is $\frac{1}{2}$ the local damping constant, $4\pi b_0$.

From considerations based upon the asymptotic form of the curve of growth for the atmospheric oxygen lines, Van de Hulst concluded that, in a non-homogeneous atmosphere, relation (4.6) is only applicable for lines arising from states with excitation potentials of about 100 cm^{-1} . Since the $\lambda 6296.0$ line, ($P_2(6)$ of the α band) has an excitation potential of 81 cm^{-1} , this approximation was considered valid.

In order to calculate a value of γ_{eff} , the observational data given by Van de Hulst (6) was used. An effective damping constant of $7.89 \times 10^9 \text{ sec}^{-1}$ was adopted for lines observed at sea level at normal temperature and pressure,

as suggested by Van de Hulst. This value was corrected for a ground temperature of 291°K and an altitude of 64 meters, giving a final value of $8.09 \times 10^9 \text{ sec}^{-1}$. The "effective" temperature of the atmosphere was taken to be

$$T_{\text{eff}} = \frac{T_0}{1.20} = 242^{\circ}\text{K} \quad (4.8)$$

as given by Van de Hulst. If we now adopt the integrated absorption coefficient for an "equivalent" isothermal atmosphere, with the above effective damping constant and effective temperature, we find the Voigt parameters to be:

$$\beta_1 = 0.0215 \text{ cm}^{-1}$$

$$\beta_2 = 0.00399 \text{ cm}^{-1}$$

and $\beta_1/\beta_2 = 5.39$. Thus it seemed justified to consider the absorption coefficient to have a pure damping form given by:

$$\alpha(\nu) = \frac{\text{Constant}}{\pi \{(\nu - \nu_0)^2 + 0.0215^2\}} \quad (4.9a)$$

or

$$\alpha(\Delta\lambda) = \frac{K}{1 + 0.0138(\Delta\lambda)^2} \quad (4.9b)$$

Since the absorption in the oxygen lines is pure absorption, the true line profile is given by

$$I_\nu = r_\nu = e^{-\alpha_\nu} \quad (4.10)$$

Using this, equivalent widths were calculated for various values of K , giving a curve of growth for the line. This is shown in Fig. 4.3, along with later observational results. The true central intensities of the oxygen lines on the two plates were then found from the observed equivalent widths. As a test of the accuracy of the determinations, the ratio of the logarithms of the derived central intensities was compared with the ratio of the $\sec z$'s calculated from the observational data. The former was found to be 0.89 ± 0.06 , whereas the theory requires a value of 0.85.

The observed equivalent widths then enabled true profiles of the λ 6296.0 line to be calculated for each plate. Using these theoretical oxygen profiles as "apparatus functions", all the observed profiles were then reduced to "true" profiles using the computer programme for profile reduction (Chapter 2). The "true" profiles thus obtained are just the instrumental slit profiles. These were averaged together to give a mean apparatus function.

An independent determination of the apparatus function of the Babcock grating was carried out by G. Gompertz (7). Using a cadmium emission source (λ 6438.4696), the

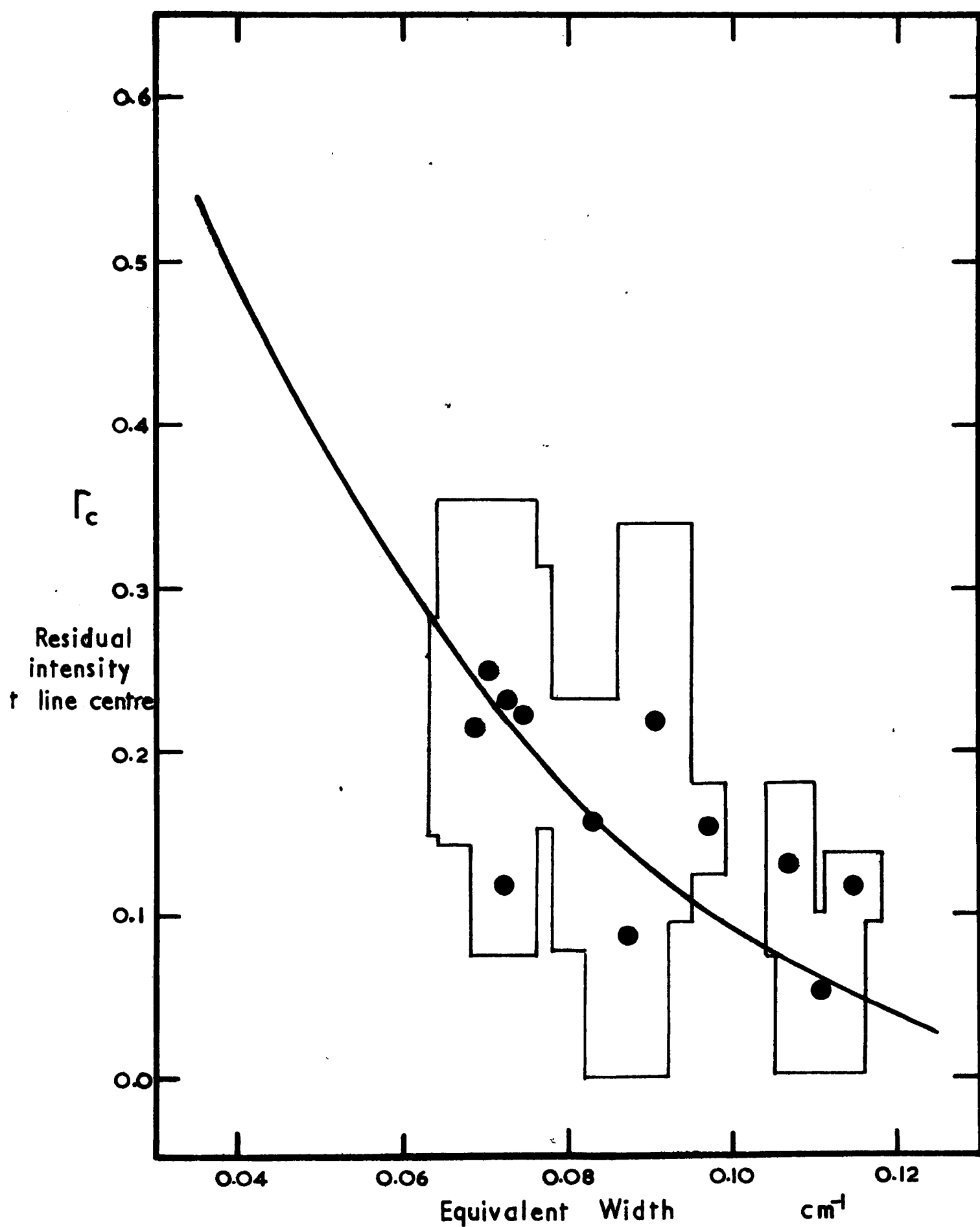


FIGURE 4.3 THEORETICAL CURVE OF GROWTH FOR ATMOSPHERIC OXYGEN LINES

Observed values for lines at 6296.0Å and 6302.0Å are given by ● and the uncertainty in these is indicated

apparatus function was derived from the observed emission line profile. To simplify comparison between the two determinations, the same spectrograph slit width was used in both. In Fig. 4.4 the apparatus function derived from the oxygen absorption lines is compared with that found by G. Gompertz from the cadmium emission line. Her determination has been corrected approximately for the line width of the cadmium line, assuming it has a Gaussian profile with a half width of 10 mÅ. It is seen from the figure that the two determinations agree well to a width of 100 mÅ, but thereafter disagree, especially in the red wing. This has been attributed to a blend in the oxygen line profile with the wing of a solar vanadium line situated 600 mÅ to the red.

Since there existed such a discrepancy between the two determinations in the wings, it was decided to use a composite apparatus function for the reduction of the solar line profiles. That is, for the wings, G. Gompertz's values were used and for the core, the values obtained from the oxygen profiles were adopted. The final apparatus function has a half width of 31.5 mÅ, and is slightly asymmetric, having a stronger violet flank than red flank.

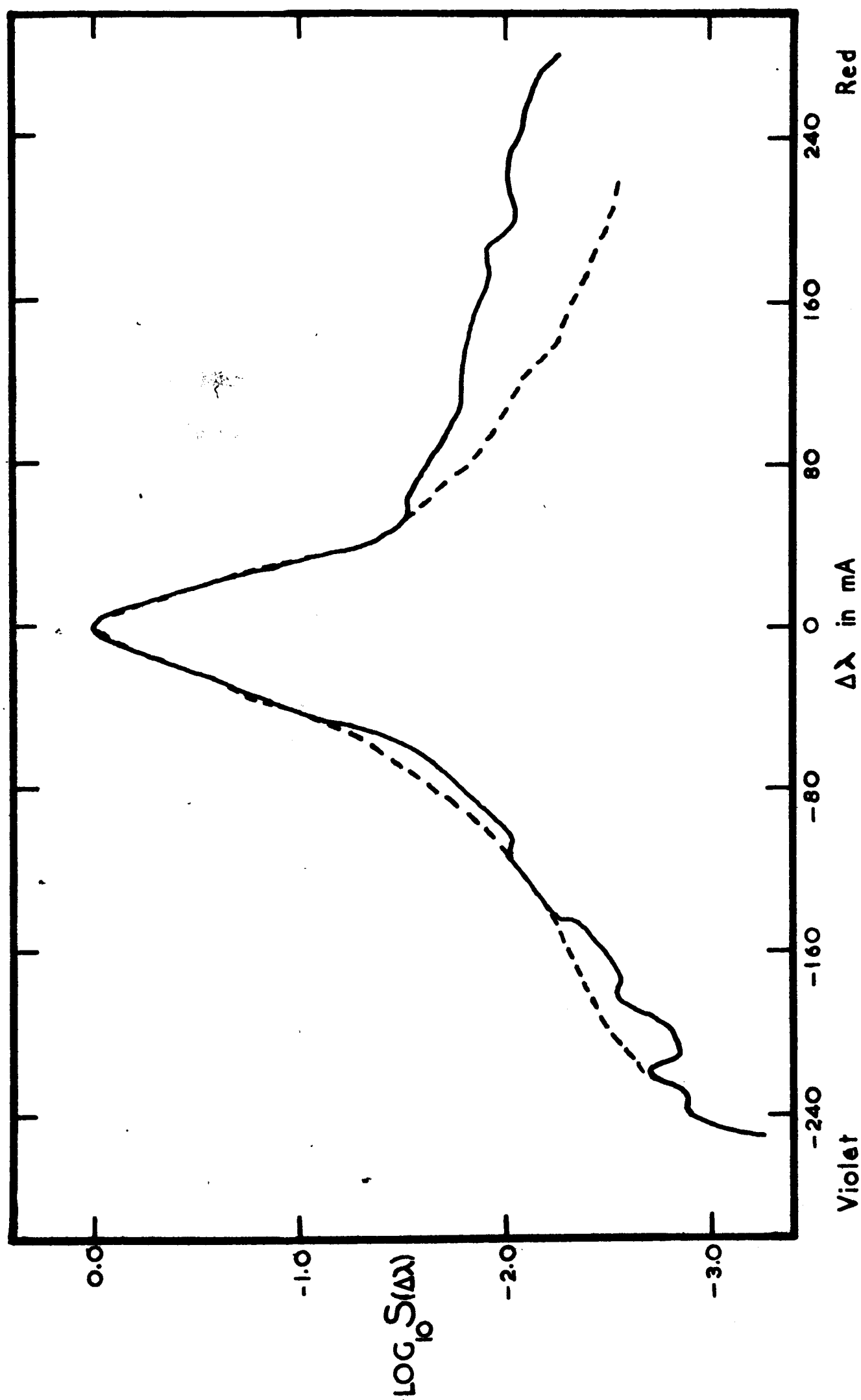


FIGURE 4.4 DERIVED APPARATUS FUNCTIONS FOR BABCOCK GRATING

L. Higgs — from oxygen absorption lines
 G. Comperitz - - - - from cadmium 6438 Å emission line

The consistency of this choice of apparatus function with the observed profiles of the oxygen lines not used in the determination of the apparatus function was tested by computing the true profiles for all the observed oxygen lines. There were found to be no systematic changes of line profile with ρ/ρ_0 for either of the two oxygen lines, and the mean computed asymmetries of these two lines, averaged for all the east-limb plates and for all the west-limb plates, are shown in Fig. 4.5. The red wing of the λ 6296.0 line is again obvious, as is a violet wing in the λ 6302.0 line. The latter was ascribed to a blend with the iron line at 6301.5 Å. Aside from these slight blends, the profiles are very nearly symmetric. Table 3 gives the mean equivalent widths, half widths and central intensities obtained for these lines from each plate. The errors given are standard deviations of the means. These central intensities and equivalent widths were then compared with the theoretical curve of growth and are indicated in Fig. 4.3. Since the accuracy of the determination of central intensities was so low, the agreement was considered satisfactory. It thus appeared that the adopted apparatus function and observed profiles were entirely consistent with the theoretical line profiles and the

assumption that the oxygen lines are symmetric.

Table 3. Oxygen Line Profiles.

<u>Line</u>	<u>Plate</u>	<u>Equivalent Width</u>	<u>Half Width</u>	<u>Central Intensity</u>
6296.0 A	H4	45.5 ± 1.4 mA	36.2 ± 1.7 mA	0.117 ± 0.021
	H7	43.8 2.3	32.4 2.6	0.051 0.049
	H12	38.5 0.9	31.6 1.2	0.153 0.027
	H14	42.3 1.2	36.3 2.3	0.127 0.053
	H16	32.8 1.7	29.0 1.6	0.156 0.078
	H18	34.6 2.0	27.2 1.7	0.083 0.086
6302.0 A	H4	29.6 0.3	33.7 1.2	0.222 0.008
	H7	28.5 1.5	26.5 1.4	0.118 0.041
	H12	29.2 2.5	29.1 5.8	0.232 0.081
	H14	35.8 1.8	33.1 6.2	0.217 0.122
	H16	27.2 2.0	27.0 1.6	0.216 0.067
	H18	27.8 2.5	27.4 2.0	0.248 0.107

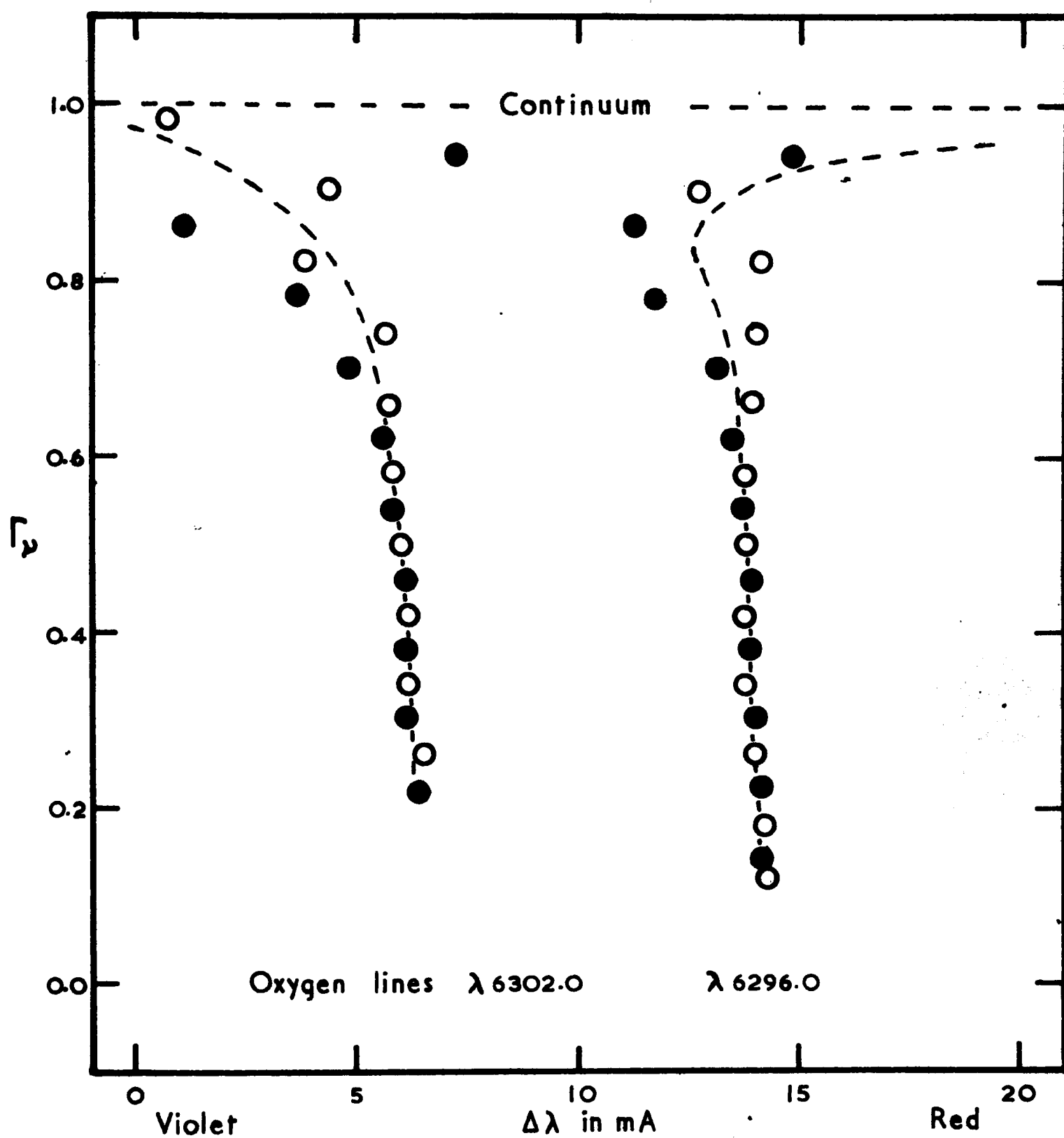


FIGURE 4.5 ASYMMETRY OF OXYGEN LINE PROFILES

- West-limb plates
- East-limb plates
- Mean asymmetry

C. Observed Asymmetries.

Using the apparatus function given above, all the observed profiles of the two iron lines were reduced to true profiles. For each limb, a mean asymmetry as a function of ρ/ρ_0 was then determined, from which it was found that there existed considerable centre-limb variation in asymmetry for both lines (see Appendix B for the centre-limb variation of other features of the line profiles). Owing to the large scatter in the observed points (which will be discussed in the next section) smooth curves were drawn through the observed asymmetry results at each value of ρ/ρ_0 . Since the accuracy of any asymmetry determination for residual intensities > 0.95 is very low, the results have been terminated at this point.

a) Centre Asymmetry.

It was found that the mean asymmetry observed at the centre for the two iron lines was the same for both east and west-limb plates, within the accuracy of the determinations. This is shown in Fig. 4.6 (a) where the average asymmetry of the line $\lambda 6297.8$ is given as determined for each group of plates. The results for all the plates were therefore averaged together and the mean observed

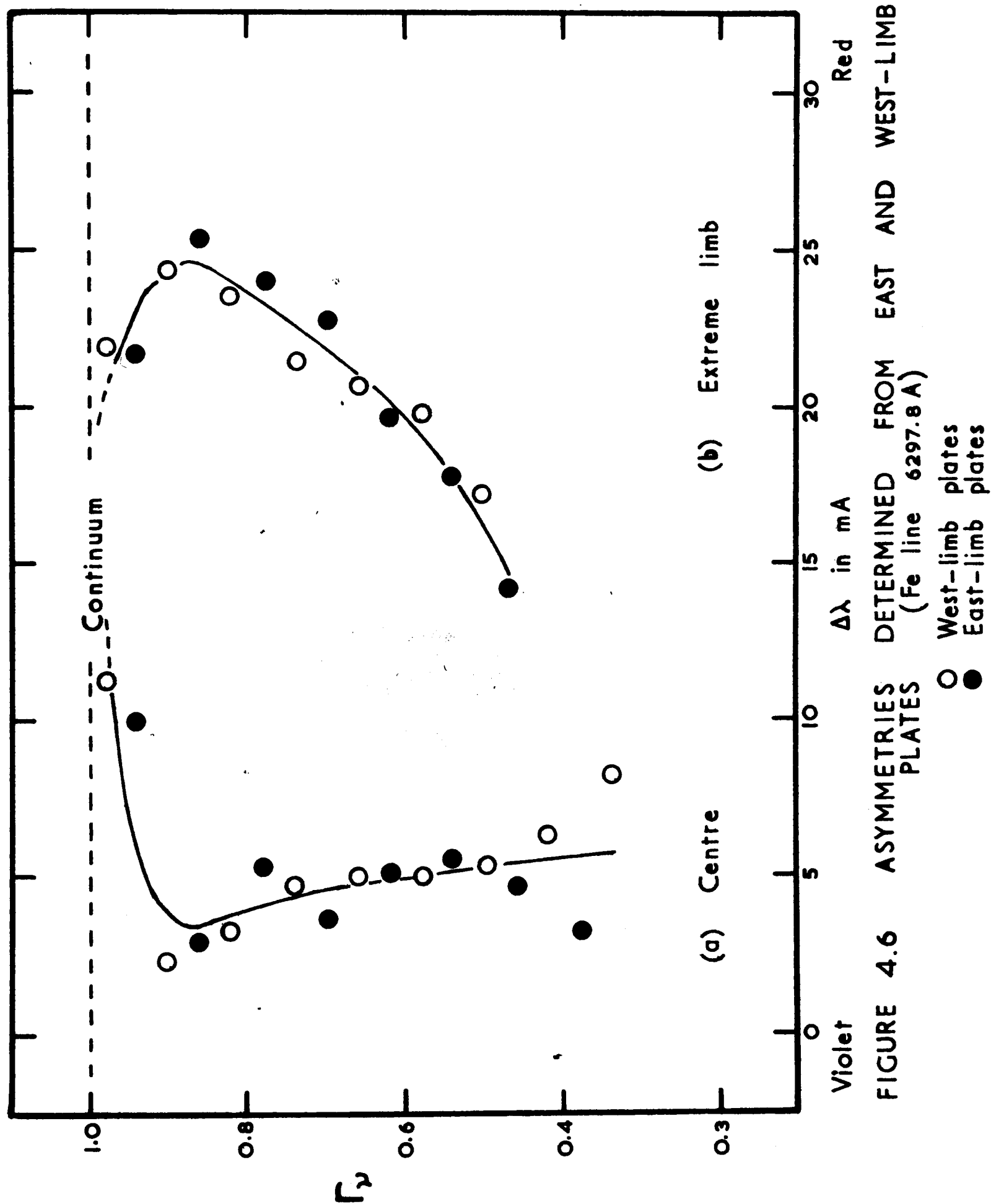


FIGURE 4.6 ASYMMETRIES DETERMINED FROM EAST AND WEST-LIMB PLATES (Fe line 6297.8 Å)

○ West-limb plates
● East-limb plates

asymmetries for both lines are given in Fig. 4.7. For comparison purposes, the results obtained for the same two lines in the preliminary low-dispersion survey are also given. The latter were obtained from the observed profiles (discussed in Appendix C) by computer reduction, using the apparatus function for the Bausch and Lomb grating found in Part I of this chapter. These results must be given much lower weight than those obtained with the Babcock grating since they apply to single determinations only. It is seen that both investigations yield slight violet flanks at the centre, the stronger iron line having the stronger asymmetry. In addition, all determinations indicate a deviation towards a red asymmetry for $r_v > 0.9$. At a residual intensity of 0.9, the weaker line has a violet asymmetry (with respect to the peak of the line) of about 1.5 mÅ. For the stronger line, this has a magnitude of about 3.5 mÅ.

b) Limb Asymmetry.

No marked difference was observed between the mean asymmetries at the two limbs, as is shown for the asymmetries observed at the extreme limbs in Fig. 4.6 (b). This indicated that scattering effects were negligible,

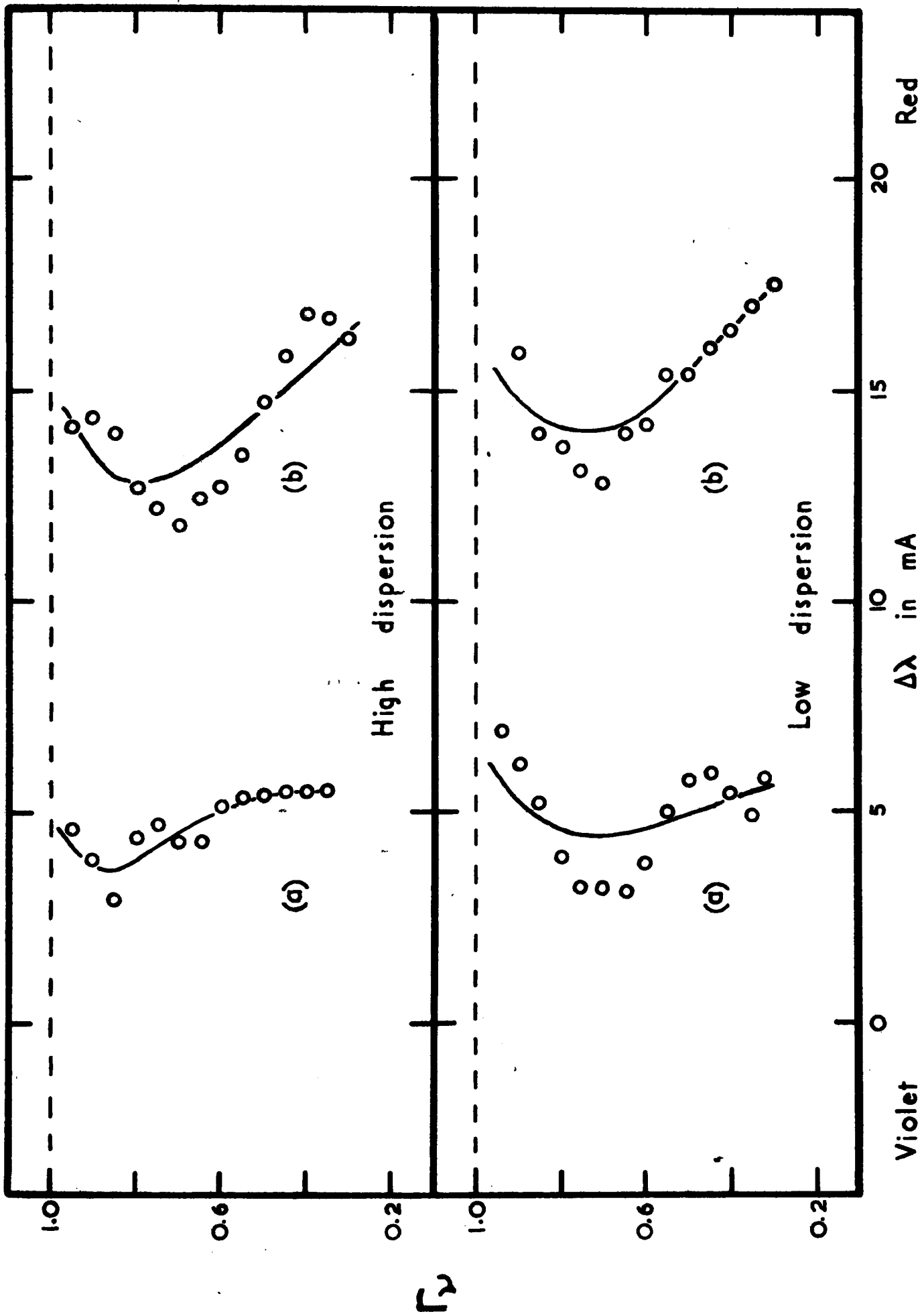


FIGURE 4.7 ASYMMETRY OF IRON LINES OBSERVED AT CENTRE
 (a) Fe 6297.8 Å
 (b) Fe 6301.5 Å

as was verified by a detailed investigation (see Appendix A). The asymmetry values obtained from the east and west limbs were then averaged together and mean results were obtained. These are given for the line λ 6297.8 in the top half of Fig. 4.8, in the lower half of which are given the corresponding results obtained from the preliminary survey. In both cases, the occurrence of a red flank at the limb is conspicuous. The line appears to be fairly symmetric until very near the limb ($\rho/\rho_0 \sim 0.998$) after which it develops the red asymmetry. This asymmetry at the limb does not appear to be constant, but varies greatly from plate to plate. Of the six extreme-limb profiles measured for this line, five showed this asymmetry to some extent, and two to an extreme degree. These two extreme cases will be discussed in more detail in a later section. It may thus be misleading to assume that the asymmetries given in Fig. 4.8 are constant; they may only represent some approximate time average.

The results obtained for the stronger iron line are given in the top half of Fig. 4.9. Again the preliminary results are given in comparison. The character of the asymmetry at the limb is slightly different from that

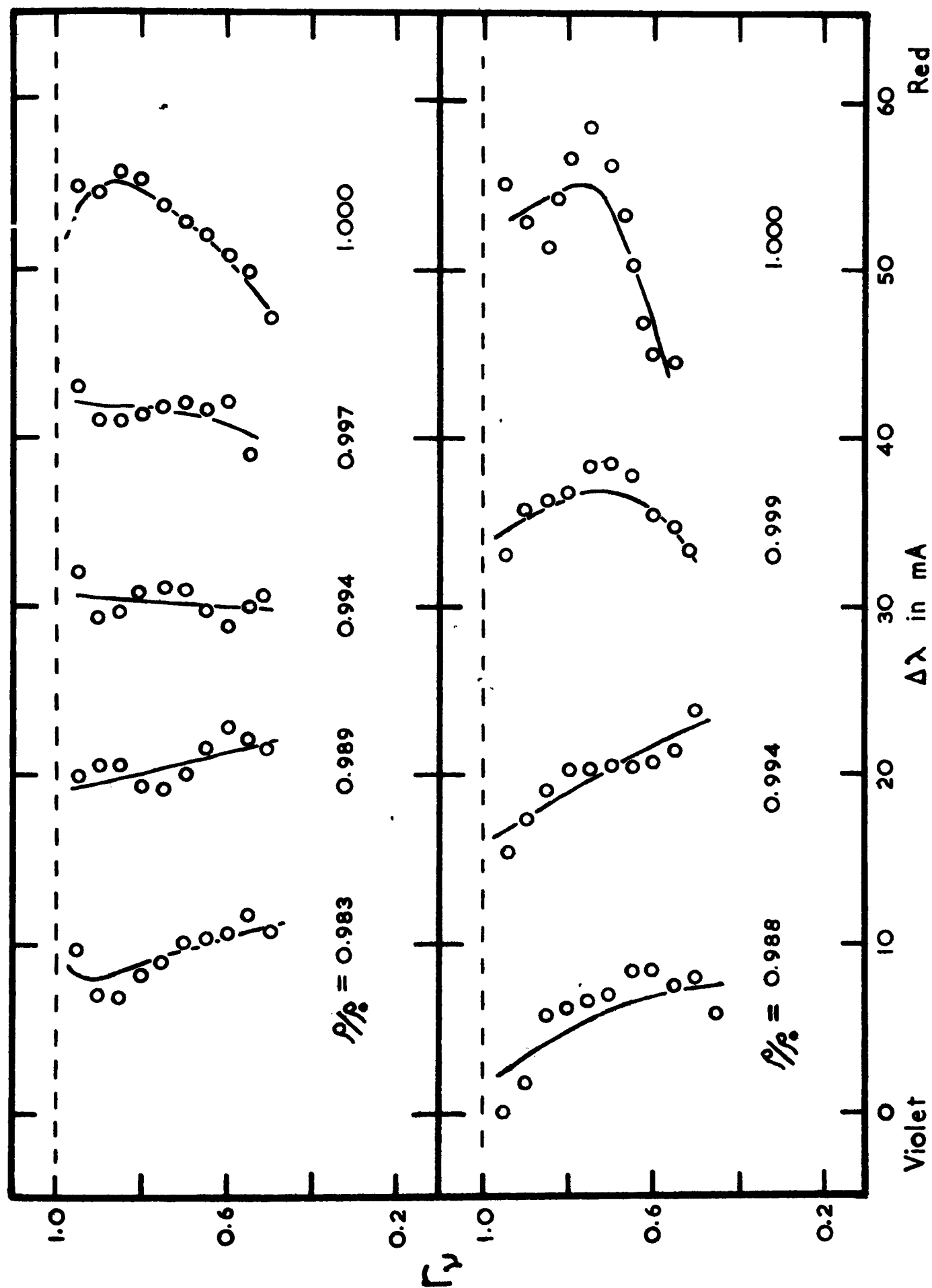


FIGURE 4.8 ASYMMETRY OF IRON LINE $\lambda 6297.8$ OBSERVED NEAR THE LIMB

Top: high dispersion
Bottom: low dispersion

observed for the weaker iron line, but there is still a strong red flank. There is, however, a slight violet wing for larger residual intensities which was not present in the weaker line. Also, the red asymmetry appears to set in at a lower value of ρ/ρ_0 (about $\rho/\rho_0 = 0.98$) than for the weaker line. From Fig. 4.9, we may conclude that, at a residual intensity of 0.85, the weaker line has a red asymmetry (with respect to the peak) of 7.5 mÅ, while for the stronger line this is slightly less, about 6.5 mÅ.

D. Accuracy of Asymmetry Curves.

A pronounced feature of the individual asymmetry determinations is the occurrence of large amplitude "ripples" in the derived asymmetry curves. These did not appear to arise wholly from the numerical reduction since asymmetry curves obtained from direct and reverse tracings of the same profile showed similar "ripples". On the other hand, one would not expect such a feature to be characteristic of the true solar line profiles. Further investigations indicated that this effect was due to the grain of the photographic plates. Although smooth curves were drawn through the microphotometer

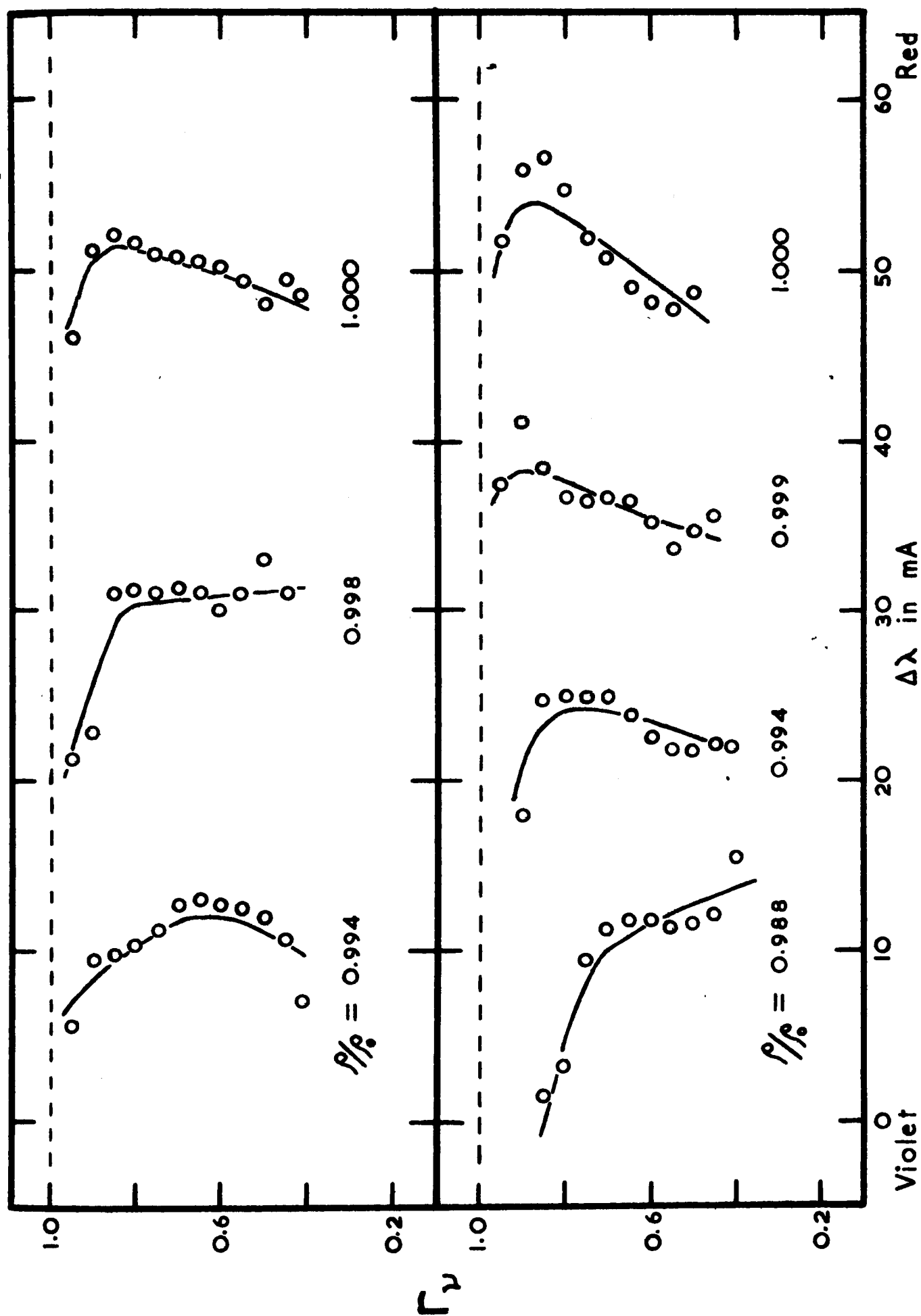


FIGURE 4.9 ASYMMETRY OF IRON LINE $\lambda 6301.5$ OBSERVED NEAR THE LIMB

Top: high dispersion
Bottom: low dispersion

profiles, slight irregularities in the hand-drawn curves remained, correlated with the original grain "noise" pattern. In the iteration process, these irregularities were magnified, giving true profiles that were quite irregular in form. Thus the computed asymmetry curves were distorted accordingly. Since only two or three iterations were required in the profile reduction, increasing the preset iteration error limit would not improve matters.

After the original computations had been made, however, a new numerical method for the solution of the integral equation became available. This was the Fourier transform method discussed in Chapter 2, which is a non-iterative process and has the advantage that some degree of smoothness is forced upon the derived true profile. All the extreme limb profiles of the line λ 6297.8 were re-reduced using this method, along with some representative profiles at other values of ρ/ρ_0 . This indicated that the distortions in the asymmetry curves, especially at the centre and at intermediate values of ρ/ρ_0 , were spurious. For an individual asymmetry determination, it was concluded that the Burger-van Cittert method of iteration can give rise to "ripples" of amplitude 3-4 mÅ in the asymmetry curves. These new calculations showed, however, that

the red asymmetry discovered at the limb was much greater than any distortions due to grain, since the new mean asymmetry curves for the east and west extreme limb profiles agreed almost exactly with the previous determinations.

The smooth curves in Figs. 4.7, 4.8 and 4.9 have been adopted following these considerations. The uncertainty in the smooth curves in the Babcock determination is estimated to be ± 1 mA for residual intensities < 0.8 and increasingly larger for residual intensities > 0.8 , owing to the larger uncertainties in the determination of the wings of the profile. The uncertainty in the smooth curves for the preliminary observations is ± 2 mA. Thus we see that the violet asymmetry indicated at the centre is only slightly significant, while the red asymmetry at the limb is considerably larger than the uncertainty introduced by the numerical reduction.

At the extreme limb, other causes of error can enter into the asymmetry determinations. The spectral plates are here underexposed and grain fluctuations give a signal-to-noise ratio as small as 7:1, as compared with 40:1 for the centre exposures. It is therefore possible to introduce a completely fictitious asymmetry in the process of drawing a smooth profile by hand. The fact

that one limb profile out of the six measured for λ 6297.8, and one out of the six measured for λ 6301.5 displayed asymmetry in the opposite sense to the mean, and that these two "anomalous" profiles were taken from the two tracings with the lowest signal-to-noise ratio, tended to indicate that errors of this sort existed. It is hoped that this inaccuracy has been reduced by averaging together as many profiles as possible at each ρ/ρ_0 . It is, however, impossible to give any accurate estimate of the uncertainty in the limb profiles due to this cause.

E. Examples of Extreme Asymmetry.

Although most plates indicated some red asymmetry of the line λ 6297.8 at the limb, two plates, H7 and H18, gave asymmetries of exceptionally large magnitude. The first of these was taken on June 30, and referred to a point at the east limb with solar coordinates, $B = -2.89^\circ$, $L = 280.5^\circ$. The microphotometer profile of the line λ 6297.8 at this position is shown in Fig. 4.10 (a). The red asymmetry can clearly be seen. This was the most asymmetric profile observed, and Fig. 4.10 (b) gives a true intensity profile for the line, after correction for the apparatus function. The stronger iron line, λ 6301.5, was also found to be highly asymmetric

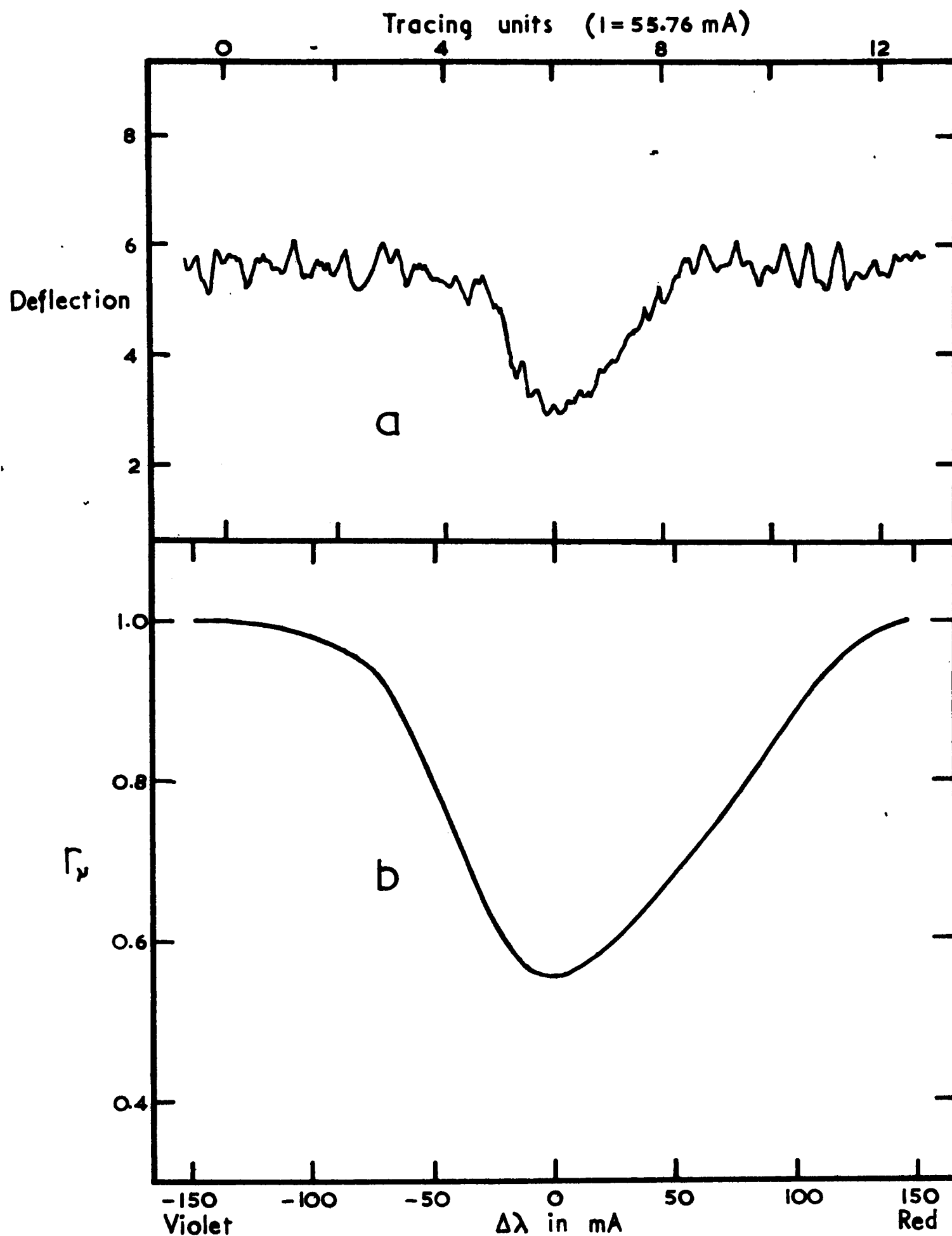


FIGURE 4.10 EXTREMELY ASYMMETRIC IRON LINE PROFILE ($\lambda 6297.8$)

- a Profile recorded by microphotometer
- b Derived true intensity profile

at this point as is shown in Fig. 4.11, where the asymmetries of both lines are given.

The second strongest asymmetry was observed at the west limb on a plate taken on July 12, referring to the point $B = -2.38$ and $L = 295.1$. The asymmetry found for the weaker iron line is shown in Fig. 4.11 (c). In this case, the stronger ($\lambda 6301.5$) line showed not a red asymmetry but a violet asymmetry, yet the line at $6302.5 \text{ \AA}$ showed a strong red asymmetry. This was one of the cases of a poor signal-to-noise ratio mentioned above, and smaller weight should be attached to these determinations than to that of the $\lambda 6297.8$ iron line. It is quite possible that the asymmetry observed for the $\lambda 6301.5$ line is unreal.

The close agreement between the solar coordinates of the points of observation for these two plates indicated the possibility of a localized disturbance on the Sun. Examination of solar activity maps showed no special activity at these points of observation, but did show the presence of a large filamentary prominence roughly on the same meridian. It extended from $B = 8^\circ$ to 40° and its existence could be traced back to June 9 and ahead to August 10. It thus appeared to be one of the dominant features during the period of observation, and it may be that the associated chromospheric disturbance extended

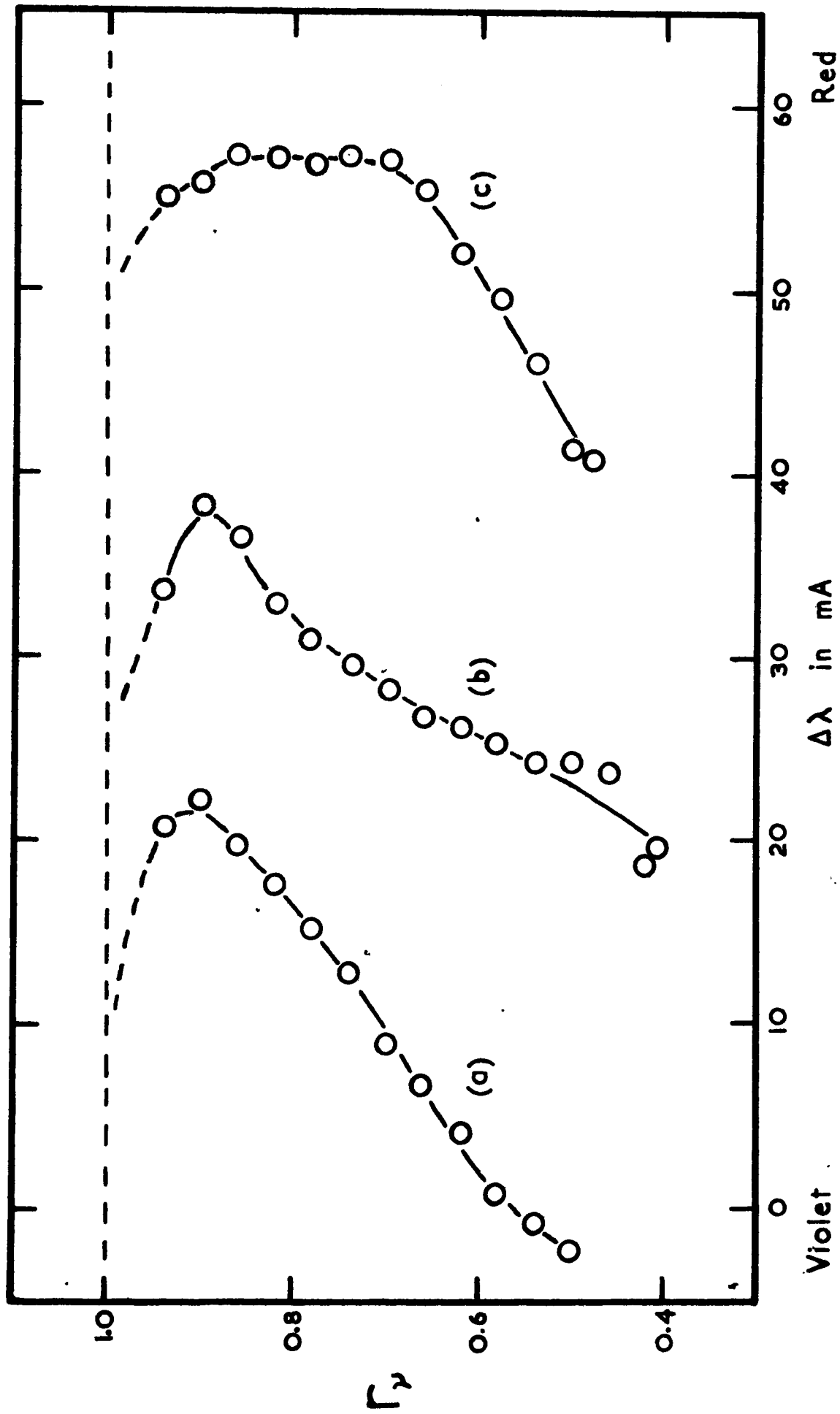


FIGURE 4.11 EXTREME OBSERVED ASYMMETRIES

(a)	Fe 6297.8 Å	June 30 , 1960
(b)	Fe 6301.5 Å	June 30 1960
(c)	Fe 6297.8 Å	July 12 1960

southward to the point of observation. On the other hand, a centre spectrum of the same solar region showed no unusual asymmetry in the iron lines. None of the other limb spectra were found to correspond to points on the Sun in the near vicinity of any large prominence or area of plage activity.

References

1. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.
2. A. B. Hart, M.N. 116, 38, 1956; Comm. Univ. Obs., Oxford, No. 54.
3. H. H. Plaskett, M.N. 112, 99, 1952; Comm. Univ. Obs., Oxford, No. 33.
4. C. W. Allen, Ap. J. 85, 156, 1937.
5. H. C. van de Hulst, Annal. d'Ap. 8, 1, 1945.
6. H. C. van de Hulst, Annal. d'Ap. 8, 12, 1945.
7. G. Gompertz, Univ. Obs., Oxford, private communication.

References

1. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.
2. A. B. Hart, M.N. 116, 38, 1956; Comm. Univ. Obs., Oxford, No. 54.
3. H. H. Plaskett, M.N. 112, 99, 1952; Comm. Univ. Obs., Oxford, No. 33.
4. C. W. Allen, Ap. J. 85, 156, 1937.
5. H. C. van de Hulst, Annal. d'Ap. 8, 1, 1945.
6. H. C. van de Hulst, Annal. d'Ap. 8, 12, 1945.
7. G. Gompertz, Univ. Obs., Oxford, private communication.

Chapter 5

DISCUSSION OF OBSERVATIONS

P a r t I

RELEVANT OBSERVATIONAL DATA

A. New Observations of Red Shifts.

In order to discuss the red shift observations presented in the last two chapters, especially to compare them with theoretical predictions, the fundamental results must be noted. These may be summarized as follows. For the three medium-strength iron lines (6300 Å) investigated, it was found that:

1) At the centre of the disk, the mean red shift was 0.39 km/sec (0.25 km/sec less than the gravitational red shift).

2) The red shift increased toward the limb (limb effect), exceeding the gravity shift for points nearer the limb than $\cos \theta = 0.30$. At the limb, the mean red shift was approximately 1 km/sec.

B. New Observations of Asymmetry.

In addition to the wavelength data, the observed asymmetry of the line profiles is of great importance in any interpretation of the observations. The fundamental asymmetry observations for the two iron lines at 6297.8 Å and 6301.5 Å may be summarized as follows:

1) At the centre, both lines displayed slight violet asymmetries, amounting to 2-3.5 mÅ at a residual intensity

of ~ 0.8 . There were some indications that this persisted up to $\cos \theta = 0.17$.

2) Beyond $\cos \theta = 0.15$, both lines developed strong red asymmetries. The magnitude of the red asymmetry fluctuated, at times being as large as 25 mÅ at a residual intensity of ~ 0.8 , and there was some evidence that the occurrence of a large asymmetry was associated with the presence nearby of a large prominence.

C. Existing Granulation Data.

Owing to the importance of granulation in any discussion of photospheric phenomena, a brief summary of present observational data concerning granulation will be given. The best photographs of granulation obtained to date have been taken using balloon-borne telescopes (1,2) and many of the values quoted below refer to such observations.

1) Granular Size: Mean value of $1''$ or 700 km (2)

2) Granular Area: Schwarzschild (2) finds that the total granular area exceeds that of the background, and states that the latter appears to consist of dark "lines" separating the granules.

3) Granular Contrast (J_g/J_b in the notation of Chapter 1): Three recent values are 1.046 (2), 1.066 (3) and 1.073 (4).

4) Contrast Near the Limb: An analysis of the

Schwarzschild granulation photographs (5) shows that the 1" granules are visible to $\cos \theta = 0.2$, after which they are replaced by a pattern of 5" granules visible to $\cos \theta = 0.05$ ($< 2''$ from the limb).

5) Macroturbulent Velocities: These are derived from Doppler shifts of line profiles and refer to elements considerably larger than the 1" granules. Detailed investigations using the McMath-Hulbert vacuum spectrograph (6,7) yield a mean macroturbulent velocity of ~ 0.3 km/sec (for 5.5" elements) which varies little from the centre to $\cos \theta = 0.35$. This confirmed an earlier value of ~ 0.35 km/sec (8,9). De Jager (10) concludes that these values are consistent with mass-motion velocities of ~ 1.4 km/sec for the 1" granules (assuming the Doppler shifts in line profiles to be due to the statistical contribution of unresolved elements).

6) Correlation Between Intensity and Velocity: All observations yield a small, negative correlation (9,11,3), indicating that, in general, the hotter elements are rising.

P a r t I I

COMPARISON WITH EXISTING THEORIES.

A. The Path-Length Interpretation.

Finlay-Freundlich and Forbes (12) have suggested that the red shift is due to a path-length interaction (photon-photon) between the observed photons and the photospheric radiation field. They predict that the red shift, $\delta\lambda$, is given by:

$$\delta\lambda = X + Y f(\theta, R/\ell_0) \quad (5.1)$$

where X is a residual line-shift term and Y is an interaction term. The function $f(\theta, R/\ell_0)$ is the path length of the photon in a layer of thickness ℓ_0 (in units of ℓ_0), and is given by:

$$f(\theta, R/\ell_0) = -\frac{R}{\ell_0} \cos \theta + \sqrt{1 + 2\frac{R}{\ell_0} + \left(\frac{R}{\ell_0} \cos \theta\right)^2} \quad (5.2)$$

This then gives a limb effect, $\Lambda(\theta)$, of:

$$\Lambda(\theta) = Y \{ f(\theta, R/\ell_0) - 1 \} \quad (5.3)$$

From an analysis of earlier limb-effect determinations, they found that the best fit was given by $Y = 2.04 \text{ mÅ}$ (corresponding to 0.097 km/sec at 6300 Å) and $R/\ell_0 \approx 100$. In order to see whether such an empirical relationship was displayed by the new limb-effect data, the new values were

plotted against $f(\theta, 100) - 1$. This is shown in Fig. 5.1, the uncertainty in the observations near the limb being indicated (the limb is at an abscissa of 13.2). The predicted linear relation is also shown and it is obvious that the observed curve does not follow the predicted law near the limb. These predictions were, however, based on the 1948 Oxford limb effect which has been shown to give values of Δ slightly lower than more recent observations (13). An analysis of the new data was therefore made to see if it satisfied a relation such as (5.3), and it was found that the observations were given almost exactly by relation (5.3) with $Y = 0.184$ km/sec and $R/\ell_0 = 11$ ($\ell_0 = 63,000$ km). It must be stressed, however, that such an agreement gives no support to a path-length theory in preference to other theories, since almost any form of limb-effect curve can be fitted by a proper choice of parameters. Moreover, physical explanation of the interaction is lacking (photon-photon interaction cross-sections are many orders of magnitude too small), and no explanation is given for the residual shifts (X term in equation (5.1)) of 4 mÅ at the centre. Such a theory, based essentially on the fitting of an observed curve to an equation with two adjustable parameters and lacking in any physical foundation, cannot be seriously considered unless new substantiation

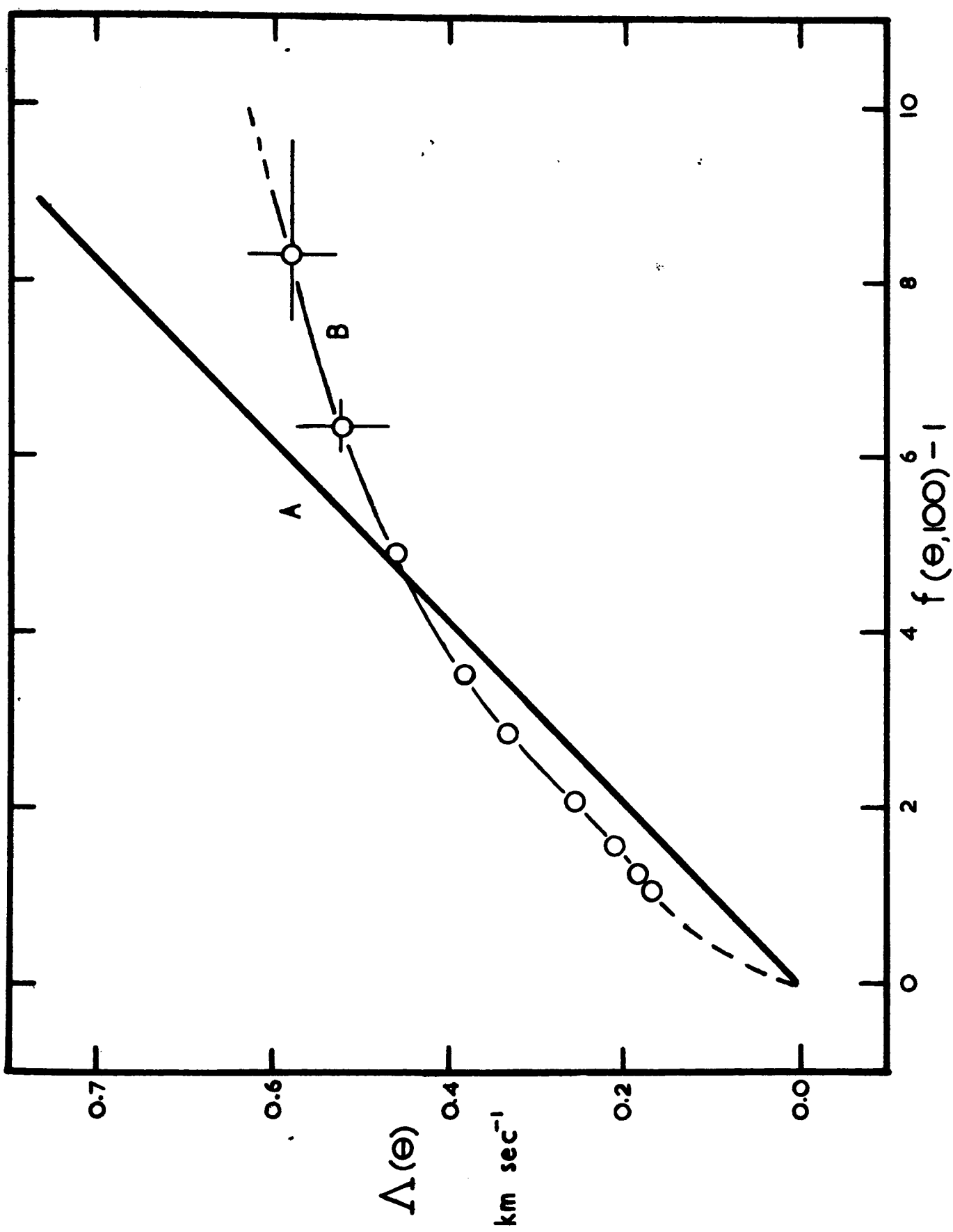


FIGURE 5.1 THE PATH-LENGTH INTERPRETATION COMPARED WITH OBSERVATION
 A Linear relation predicted by Finlay-Freundlich and Forbes
 B Observed limb effect from Figure 3.2

were to be found.

B. Schröter's Two Stream Theory.

It can easily be shown that the red-shift theory arising from the two-stream photospheric model proposed by Schröter (14) is not supported by the new observations. According to Schröter's interpretation, the spectral lines are asymmetric at the centre of the disk, and this asymmetry decreases toward the limb. Wavelength shifts then arise corresponding to the asymmetry at the intensity level in the line profile that is used for micrometer settings. This will be referred to as the "mean" asymmetry of the profile.

From detailed computations, Schröter found that an iron line of 150 mÅ equivalent width should have, at the centre, a "mean" violet asymmetry of 0.42 km/sec (expressing the asymmetry as an equivalent Doppler velocity). Moreover, this asymmetry should be even larger for weaker lines. On the other hand, the interferometric determination of centre wavelengths gives a mean red shift that is only 0.25 km/sec less than the gravity shift, about half the difference predicted by Schröter. The discrepancy of 5 mÅ between the observed red shift and the predicted red shift cannot be due to any error in the wavelength deter-

minations.

Even if it is assumed that the asymmetries at the centre that were computed by Schröter are in error, the new limb-effect curve is not of the form predicted from the two stream theory. The predicted relation between red shift and $\cos \theta$ is:

$$\delta \lambda \text{ (km/sec)} = 0.636 - \bar{v} \beta_{\theta} \cos \theta \quad (5.4)$$

where β_{θ} is related to the granular contrast (Fig. 1.6) and $\bar{v}$ refers to the "mean" asymmetry at the centre of the disk. If we take the observed value of 0.25 km/sec for $\bar{v}$, equation (5.4) gives the red-shift curve in Fig. 5.2. This is compared with the observed red shift curve and it can be seen that the two curves differ greatly, especially near the limb. Also shown are the 1959 observations of M. G. Adam (13). Obviously, no theory based on the radial current hypothesis can predict a super-gravity shift at the limb such as has been observed.

It might again be noted that, as in the case of the path-length theory, it is possible to find parameters such that equation (5.4) gives a limb effect of the proper form (a $\bar{v}$ of 0.57 km/sec fits the observed limb-effect values very well). This would again, however, mean that the centre red shifts were in error by 7 mÅ and would require

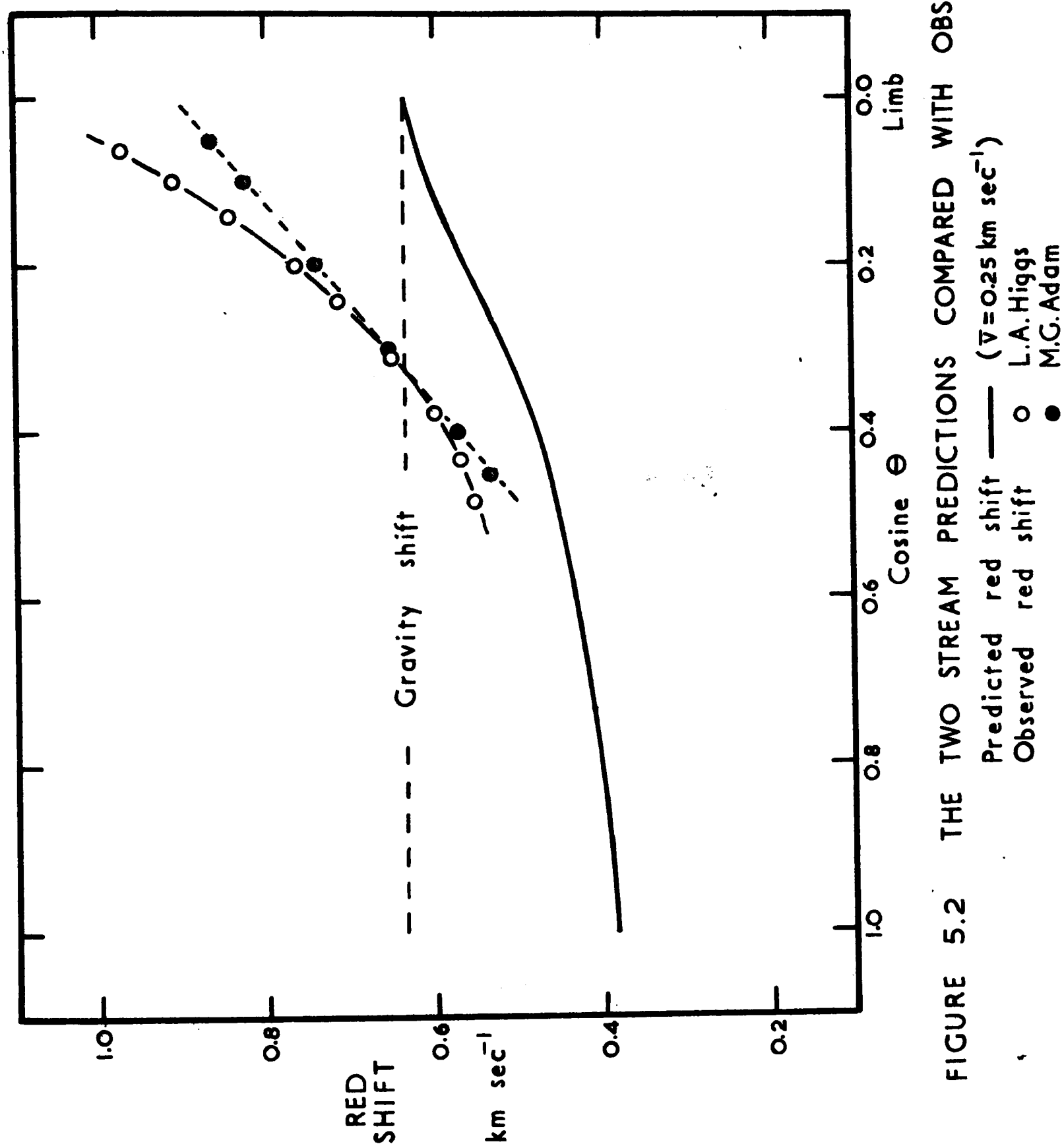


FIGURE 5.2 THE TWO STREAM PREDICTIONS COMPARED WITH OBSERVATION

Predicted red shift — ($\bar{v} = 0.25 \text{ km sec}^{-1}$)
 Observed red shift ○ L.A. Higgs ● M.G. Adam

asymmetries at the centre about three times larger than those observed.

In summary, unless some large source of error exists in the interferometric determination of centre wavelengths, Schröter's theory cannot possibly account for the observed red shift. On the other hand, the fundamental physical data that entered into this theory requires revision and it is possible that radial convection currents contribute to the deviations from the gravity shift in the central portion of the disk. This will be discussed in more detail in the next part of this chapter. As stated above, however, for regions near the limb some new agency is required for the explanation of the red shifts and especially the newly discovered red asymmetry. The latter feature will also be discussed in the next section.

P a r t I I I

POSSIBLE RED SHIFT MECHANISMS

A. Effects of Granulation on Centre Line Profiles.

Since granulation appears to be associated with convective motions in the photosphere, these motions may be expected to influence line profiles. Although the two stream theory developed by Schröter has been shown to be in conflict with the recent observations, it is still possible that part of the red shifts at the centre are due to convective motions. Certainly Schröter's model was based on granulation data that has since been revised (see Chapter 1 and Part I, Chapter 5). In order to estimate the effects of such revisions on the predictions of the two stream model, an approach similar to that of M. G. Adam (15) was used. It was assumed that Schröter's model photosphere could be replaced by rising elements of temperature T_1 , velocity v_1 , and surface area a_1 along with sinking elements of temperature T_2 , etc. From equation (1.9), the observed velocity is then:

$$\bar{V} = \frac{a_1 J_1 v_1 - a_2 J_2 v_2}{a_1 J_1 + a_2 J_2} \quad (5.4)$$

The ratio of the two velocities is obtained from (cf. equation (1.13)):

$$\frac{v_1}{v_2} = \frac{a_2}{a_1} \frac{T_1}{T_2} \quad (5.5)$$

If Schröter's model is assumed to be characterized by $J_1/J_2 = 1.45$, $T_1 - T_2 = 620^\circ$, $T_1/T_2 = 1.10$, $a_1 = a_2$, and $\bar{v} = 0.42$ km/sec (for a line with an equivalent width of 150 mÅ), then the equivalent velocities are $v_1 = 1.90$ km/sec and $v_2 = 1.73$ km/sec. Using this result, the effect of varying any of the parameters was estimated by assuming (from equation (1.14)) that the resulting v_1^* is given by:

$$\frac{v_1^*}{v_1} = \frac{v_g^*}{v_g} = \frac{\sqrt{d^* \Delta T_g^*}}{\sqrt{d \Delta T_g}} = \frac{\sqrt{d^* (T_1 - T_2)^*}}{\sqrt{d (T_1 - T_2)}} \quad (5.6)$$

and the corresponding v_2 is given by:

$$v_2^* = \left(\frac{a_1}{a_2}\right)^* \left(\frac{T_2}{T_1}\right)^* v_1^* \quad (5.7)$$

It was found that if the contrast were changed to 1.10, then the resulting $\bar{v}$ would be 0.06 km/sec. If, in addition, the size of the granules were increased from 280 km to 700 km and the granular area were double that of the background, the $\bar{v}$ would be 0.11 km/sec. Moreover, the equivalent velocities, v_1 and v_2 , would be ~ 1.5 km/sec, not inconsistent with granulation observations. It is evident from these approximate calculations that the observed velocity is primarily dependent upon the adopted contrast value. Since the observed contrast seems to be even less

than the value of 1.10 used here, it is very unlikely that a medium strength line could appear to be shifted any more than 0.20 km/sec by convective motions, compared with the observed value of 0.25 km/sec. On the other hand, the observed centre asymmetries of 0.09 - 0.14 km/sec are of the same order of magnitude as the $\bar{v}$'s computed above, so that it seems that they may be due to Doppler effects as assumed by Schröter.

It thus appears that about half of the deviation from the gravity shift for the present observations, may be due to granulation effects. The low granulation contrast now in favour, and the small microturbulent and macroturbulent velocities (~ 2 km/sec) that have been observed do not allow more than this to be attributed to convective motions. The problem of the centre red shifts, therefore, is no nearer solution and some new effect will be required in order to explain the observations. Moreover, if convective motions are unable to account completely for the centre red shifts, they are even less able to account for limb effect. Obviously a new theoretical treatment of the problem is required, but, unfortunately, at the moment no mechanism is known that can account for the observed deviations from the gravity shift and their variation across the disk.

B. Super-gravity Shift and Asymmetry at the Limb.

The observations presented in Chapter 4 definitely indicate that, at the extreme limb, the iron lines at 6300 Å have, on the average, a large red asymmetry. Since it has been observed, on different occasions, in spectra obtained with two completely different optical systems, it must be considered a true solar effect. Such an asymmetry must affect a micrometer determination of limb effect and it is therefore probable that the super-gravity shift observed at the limb is just a manifestation of this asymmetry. Confirmation of this would require a determination at the limb of the absolute wavelength shifts at various levels in the line profiles, but at present such an analysis has not been completed (preliminary results indicate that the cores of the lines are shifted by approximately the gravity shift, the super-gravity shift arising in the wings). Such a relation between asymmetry and wavelength shifts is supported by the fact that the observed asymmetry of about 8 mÅ is comparable with the observed super-gravity shift of 6-10 mÅ. On the other hand, the red asymmetry appears to set in at $\cos \theta \approx 0.15$, whereas a super-gravity shift certainly exists at $\cos \theta = 0.25$. In any case, a physical process was sought that could give rise to a red asymmetry in the lines at the extreme limb.

Brief consideration shows that none of the more common mechanisms that bring about shifts of the atomic energy levels, e.g. Zeeman effect, Stark effect, isotope shifts, etc. can produce asymmetries of up to 25 mÅ (assuming known physical conditions in the photosphere). Even less plausible is the interpretation that the asymmetry is due to Doppler shifts. Only two possible causes of the asymmetry could consequently be found. The first is that a fraction of the photons emitted by the photosphere may lose energy by Compton scattering, thus introducing some red asymmetry. Secondly, the red asymmetry may be a chromospheric effect; the observed line profile may be affected by chromospheric emission, absorption or scattered light.

Compton Scattering.

In order to estimate the effects of Compton scattering on lines observed at the extreme limb, the probability that a photon would be scattered while passing tangentially through the solar atmosphere was computed. At 6300 Å, the Compton scattering cross-section is very nearly the classical Thomson cross-section:

$$\sigma_0 = \frac{8\pi}{3} \frac{e^4}{m^2 c^4} = 6.65 \times 10^{-25} \text{ cm}^2 \quad (5.8)$$

and the probability that a photon will be scattered in passing through the atmosphere is given by $P = \int N_e s_0 dx$, where the integral is along the line of sight. In order to find values of P for various paths through the photosphere, a model photosphere similar to that of H. H. Plaskett (18) ($\log N_e$ varying from 13.48 to 10.00) was adopted, and as an extreme case, it was assumed that all the photons originated at an optical depth of $\tau = 3$. It was thus found that at the extreme limb ($\cos \theta = 0.000$), $P = 0.02$, and at the centre of the disk, $P = 0.0002$.

From the Compton scattering equation for a stationary electron:

$$\frac{\Delta \nu}{\nu} = - (1 - \cos \theta) \frac{h \nu}{m c^2} \quad (5.9)$$

the expected wavelength shift at the limb is 24.3 mÅ.

Allowance for the distribution of electron velocities, however, introduces a violet shift of the order of

$(1 - \cos \theta) \frac{kT}{m c^2}$ as well as a thermal broadening of the line formed by the scattering. Detailed computations by Dirac

(17) indicate that at 6300 Å, the "thermal" line shift and the Compton line shift are almost equal and hence no

asymmetry could be produced by the scattering. From

these considerations, three conclusions concerning the

effects of Compton scattering on line profiles may be drawn:

- 1) Even at the extreme limb, only about one photon in fifty observed photon will have undergone any scattering.
- 2) The energy lost by the photon in such a scattering will on the average be nearly zero.
- 3) Even if a wavelength shift were introduced, the thermal velocities of the photospheric electrons spread the scattered photons into a broad line 20 Å wide, so that no observable asymmetry could be introduced.

Chromospheric Effects.

The effects of chromospheric superposition (a chromospheric emission or absorption line superimposed on a photospheric line either due to formation in the line of sight or by scattering) is very important in the region near the limb. If the chromospheric line differs in wavelength from the photospheric line, then an asymmetry could appear in the observed profile. Such a difference in wavelength has been indicated (19,20), the chromospheric lines being shifted slightly to the violet. Moreover, a chromospheric origin of the red asymmetry is supported by its apparent connection with chromospheric activity that was discussed in Chapter 4.

On this interpretation, it was found that the two observed

line profiles with the largest asymmetry (see Fig. 4.10) could be represented as the sum of a symmetric absorption line plus a weak symmetric emission (or absorption) line, displaced 70 mÅ to the violet (or red) side of the main line. The observed equivalent width of the 6297.8 Å line on plate H7, being smaller than average, indicated that the weak line was an emission line. It is interesting to note that Goldberg et al. (18) found that the profile of $H\alpha$ in a flare-associated prominence could be accounted for by a weak chromospheric emission line displaced 1.5 Å to the violet (which they attributed to deuterium α). Using the observed fact that the asymmetry of the H7 profile could be represented as a difference of residual intensity of 0.18 between the two flanks, at a residual intensity of 0.93 in the violet flank, the variation of the asymmetry with $\cos \theta$ was investigated assuming it due to superposition.

H. H. Plaskett (16) has shown that an observed line profile (neglecting scattering) is given by:

$$I_{\nu}(\theta) = I_{\nu}^*(\theta) e^{-\tau(\theta)} + \int_0^{\tau(\theta)} B_{\nu} e^{-t(\theta)} dt(\theta) \quad (5.10)$$

where I_{ν}^* is the true photospheric line profile, τ is the optical depth of the chromosphere in the line of sight,

and B_ν is the chromospheric source function in the spectral line. In order to estimate the magnitude and $\cos \theta$ dependence of this form of superposition, a very simplified model of the lower chromosphere was adopted, an isothermal layer of 1000 km depth. The optical depth, τ , in the line of sight was assumed to give by $k f(\theta, R/\ell_0)$, k being a constant and $f(\theta, R/\ell_0)$ being given by equation (5.2) with $R/\ell_0 = 696$. Since the layer is isothermal, B_ν is independent of τ and equation (5.10) may be written as:

$$I_\nu(\theta) = I_\nu^*(\theta) + \{B_\nu - I_\nu^*(\theta)\} \{1 - e^{-k f(\theta)}\} \quad (5.11)$$

It can be seen that if $B_\nu > I_\nu^*$ there is effectively an emission effect, while if $B_\nu < I_\nu^*$, the chromosphere contributes to the line absorption. By dividing through by the intensity in the continuum, $I_c(\theta)$, this may be written in terms of residual intensities:

$$\Delta r_\nu = r_\nu - r_\nu^* = \left\{ \frac{B_\nu}{I_c(\theta)} - r_\nu^* \right\} \{1 - e^{-k f(\theta)}\} \quad (5.12)$$

Taking the value of $I_c(0)$ to be $4.08 \times 10^{-5} \text{ erg.cm}^{-2}$ and B_ν as:

$$B_\nu = \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT_e}} \quad (5.13)$$

it was found that $T_{\odot}$ must be less than 4620° for the observed profile to be an absorption effect. Consequently, the superposition effect (fitting the observed data at $\cos \theta = 0.024$) was computed for two models, one with $T_{\odot} = 4500^{\circ}$ (absorption) and one with $T_{\odot} = 5500^{\circ}$ (emission). The absorption model so found was not very satisfactory as it led to a rather large chromospheric optical depth at the limb. In Fig. 5.3, the results of these computations are shown, $\Delta r_{\odot}$ being plotted against $\cos \theta$ for each model (curves A and B). From this it can be seen that chromospheric emission becomes important at $\cos \theta \approx 0.15$ (for the case considered), which is in the same region that the red asymmetry appeared.

In order to estimate the $\cos \theta$ variation of any superposition effect due to scattering of chromospheric light inside the limb, the scattering function derived in Appendix A was used. Assuming that the chromospheric emission line had a uniform intensity over a region extending outward from the limb for a distance of ~ 1000 km, the scattered intensity was found for various values of $\cos \theta$ (choosing the intensity of the line in the chromosphere to fit the above observations). This scattering superposition effect is indicated by curve C in Fig. 5.3. Clearly this effect is also important for $\cos \theta < 0.15$.

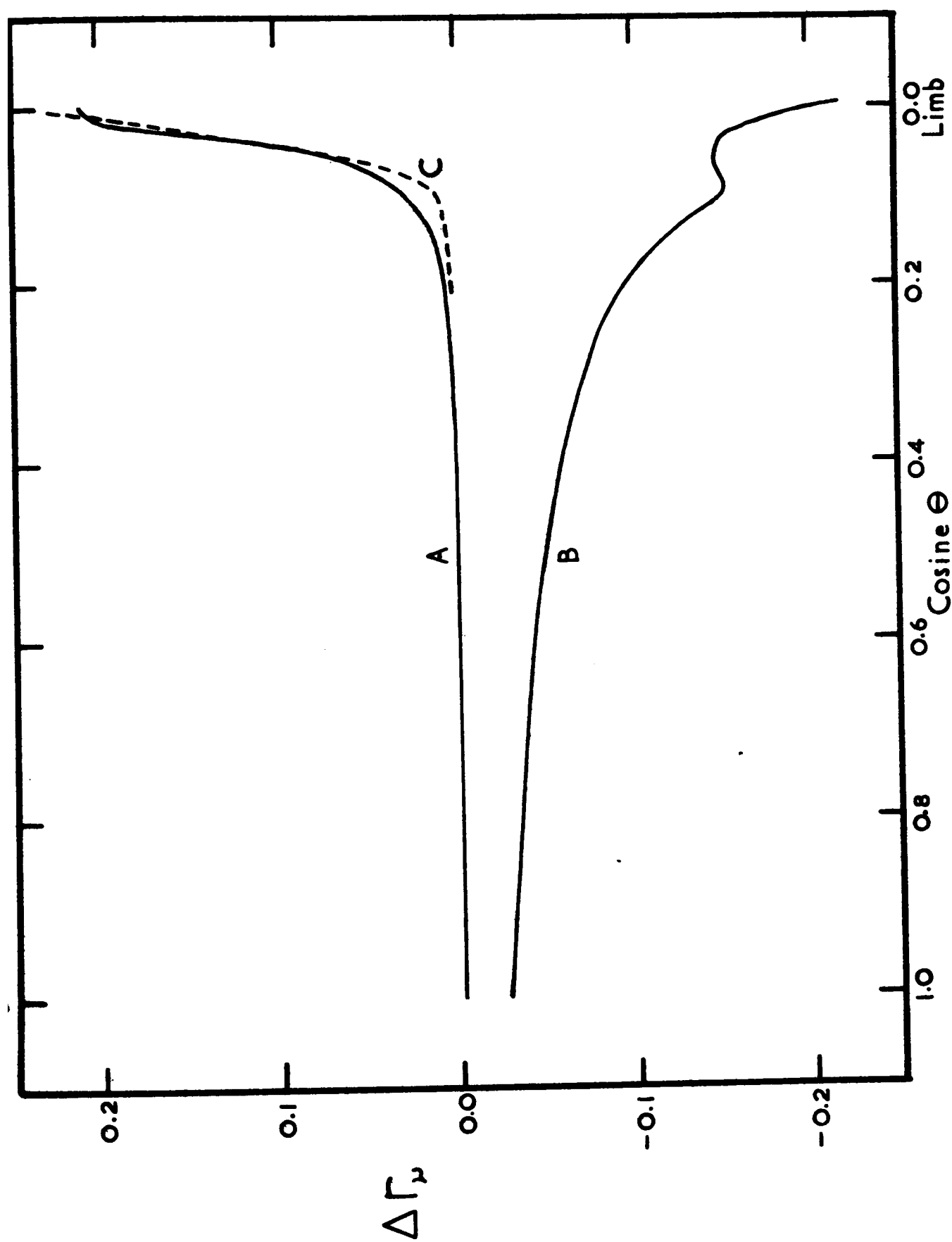


FIGURE 5.3 THE EFFECTS OF CHROMOSPHERIC SUPERPOSITION

- A Chromospheric emission $T = 5500^\circ\text{K}$
- B Chromospheric absorption $T = 4500^\circ\text{K}$
- C Scattering of chromospheric light

If one therefore assumes that the chromospheric lines are shifted to the violet side of the corresponding photospheric lines, it seems that the red asymmetry at the limb (and possibly the super-gravity shift) is due to chromospheric superposition. This, however, only transfers the problem to that of why chromospheric wavelengths are shifted to the violet.

P a r t I V

CONCLUSIONS.

From the foregoing discussion of the limb-effect and red-shift measurements, one fact emerges: there is no theory at present that satisfactorily accounts for the red shifts at the centre and limb effect. It now appears that the temperature inhomogeneities in the photosphere are insufficient to attribute the centre red shifts to Doppler effects. Moreover, a shift at the limb exceeding the gravity shift has been confirmed. Although this has been tentatively attributed to chromospheric effects, such an explanation is very lacking in any physical foundation as yet. Clearly a new theoretical investigation of mechanisms that could create deviations from the gravity shift, both at the centre and the limb, is required. Many observations have been made to date of accurate wavelengths at the centre of the disk, but very few accurate limb effect curves have been provided. The latter are, however, of utmost importance for a new theoretical treatment; especially valuable would be a limb-effect curve for a line having a centre red-shift exceeding the gravity shift. Finally, no longer will wavelength determinations alone be sufficient for a detailed investigation of solar wavelength shifts, but profiles are also needed (especially near the limb). The great complexity of the problem of line

formation in the photosphere would indicate that it will be some time before a satisfactory explanation of wavelength observations will be found, and until then, the investigation of deviations from the gravity shift will remain one of the most intriguing, although exacting, lines of solar research.

References.

1. D. E. Blackwell, D. W. Dewhurst, and A. Dollfus, M.N. 119, 98, 1959.
2. M. Schwarzschild, Ap. J. 130, 345, 1959.
3. R. Servajean, Annal. d'Ap. 24, 1, 1961.
4. J. Gaustad and M. Schwarzschild, M.N. 121, 260, 1960.
5. F. N. Edmonds Jr., Ap. J. 131, 57, 1960.
6. L. Goldberg and A. K. Pierce, "Handbuch der Physik", Bd. LII, p. 67, Springer, Berlin, 1959.
7. L. Goldberg, O. C. Mohler, W. Unno, and J. Brown, Ap. J. 132, 184, 1960.
8. R. S. Richardson and M. Schwarzschild, Ap. J. 111, 351, 1950.
9. H. H. Plaskett, M.N. 114, 251, 1954; Comm. Univ. Obs., Oxford, No. 44.
10. C. de Jager, "Handbuch der Physik", Bd. LII, p. 84, Springer, Berlin, 1959.
11. F. E. Stuart and J. H. Rush, Ap. J. 120, 245, 1954.
12. E. Finlay-Freundlich and E. G. Forbes, Annal. d'Ap. 19, 183, 1956.
13. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.
14. E. H. Schröter, Zs. f. Ap. 41, 141, 1957.
15. M. G. Adam, M.N. 108, 446, 1948; Comm. Univ. Obs., Oxford, No. 26.
16. H. H. Plaskett, M.N. 116, 419, 1956; Comm. Univ. Obs., Oxford, No. 58.
17. P.A.M. Dirac, M.N. 85, 825, 1925.

18. H. H. Plaskett, "Vistas in Astronomy", Vol. I, p. 640, edited by A. Beer, Pergamon Press, 1955.
19. D. H. Menzel, Publ. Lick Obs., 17, Part 1, 1931.
20. A. D. Thackeray, M.N. 97, 674, 1937.

Appendix A

EFFECTS OF SCATTERED LIGHT

A. Method.

In order to determine the effect of scattering on line profiles observed near the limb, the scattering function of the atmosphere plus optical system must be known for the time of observation. In order to determine this, several plates were taken on July 14, 1960, of light scattered beyond the Sun's limb. Ilford HP3 plates were used and were developed in Parkhurst two solution developer. Owing to the low intensities involved, exposures of 1 - 2 minutes were required. On each plate, two calibration exposures of the centre spectrum were taken, one with and one without the pre-calibrated wedge. For these, the light intensity was reduced by a factor of twenty using a combination of filters. For the scattered light spectrum, a rotating sector covered one half of the spectrograph slit. When in rotation, this reduced the exposure in the lower half to 3% of that in the upper half. The solar image was guided so that the limb fell on the lower half of the slit, allowing full exposure to the light beyond the limb, yet giving a limb spectrum of comparable density. The fourth exposure on each plate was another of the limb spectrum, but with the intensity reduced by an accurately calibrated filter. This removed the necessity of an accurate calibration of the rotating sector.

Three of these plates were chosen for photometric measurement. For each plate, two microphotometer tracings were made across the four exposures, perpendicular to the dispersion, at wavelengths near those of the spectral lines being studied.

B. Scattering Function.

The scattering function was determined following closely the method of M. G. Adam (1), differing only in that intensity measurements were to be used in place of wavelength measurements. Characteristic curves were computed from the calibration spectra for the three scattered-light plates, and observed intensity curves for the region $0.94 < \rho/\rho_0 < 1.13$, were obtained for each plate. The true position of the limb was estimated from the point of inflection in the intensity curve. All three curves agreed with one another, thus indicating that the scattering function was fairly constant throughout the period of observation. A mean curve was taken, and was fitted to the true limb darkening curve, used in Part I, Chapter 3, by assuming that the scattering decreased the continuum intensity at $\rho/\rho_0 = 0.94$ by 3.6%. It was assumed that the scattering process could be satisfactorily represented by the one dimensional

integral equation:

$$\overline{I}(x_0) = \frac{\int_{-1}^{+1} I(x) S(x-x_0) dx}{\int_{-1}^{+1} S(x-x_0) dx}$$

where $\rho/\rho_0 = |x|$, $I(x)$ is the true intensity in the continuum (6300Å), $\overline{I}(x_0)$ is the observed intensity at x_0 and $S(x-x_0)$ is the scattering function. Using the above limb darkening curve, the scattering function was derived from the observed intensity at points beyond the limb. For this determination, it was assumed that there was negligible scattering by $S(x-x_0)$ beyond $|x-x_0| = 1$. The light scattered at great distances allowed a very good determination of the wings of $S(x-x_0)$ but the core was much less precisely defined. It was found to be Gaussian in form to a width of 0.008, having a half-width of 0.0046 (4.4"). Thereafter, it has appreciable flanks and very extended low-intensity wings. Long range scattering is consequently large, 5.2% of the light being scattered into the far wings ($|x-x_0| > 0.122$). As a check on the consistency of this determination, the derived function was used to determine an "observed" intensity distribution inside the limb. This was compared with the observed distribution obtained from the limb spectrum taken with the calibrated filter. The observed and calculated curves

agreed in form, but there appeared to be a displacement of 0.002 in ρ/ρ_0 between them. This was attributed to errors in the determination of the position of the true limb, and not to any great inaccuracies in the determination of the scattering function.

C. Application to Line Profiles.

At the extreme limb, where both intensity and wavelengths are rapidly varying, scattering can be expected to introduce asymmetry into the observed line profiles. The asymmetry so introduced should be different for the east and west limbs. In order to determine the magnitude of any such asymmetry, an "observed" λ 6297.8 profile (corrected for the apparatus function) was calculated for $\rho/\rho_0 = 0.999$ at both the east and west limbs, using the scattering function derived above. It was assumed that the true line profile was symmetric for all values of ρ/ρ_0 , and the general form of the profiles was taken from Table 1, Appendix B. The limb darkening used in Part I, Chapter 3 was adopted, and the wavelength variation across the disk was found from a computed mean Earth's motion and solar rotation correction, plus a limb effect. For the west limb, an extreme limb effect (that obtained

in Part I, Chapter 3) was used, while for the east limb, a limb effect giving the gravity shift at the limb was used. These were adopted since they give, for each limb, the extreme value of any asymmetry introduced by scattering. The resulting asymmetries are shown in Fig. 1. The central line gives the centre of the profile assuming no scattering, and the two neighbouring curves give the profile, as a result of scattering, at the two limbs. It can be seen that a slight asymmetry is introduced but that it should have no resultant effect when the profiles from both limbs are averaged together (excluding the slight violet wing which is common to both limbs). It was therefore decided that no corrections, due to scattering, were necessary for the observed asymmetry curves.

References.

1

1. M. G. Adam, M.N. 119, 460, 1959; Comm. Univ. Obs., Oxford, No. 71.

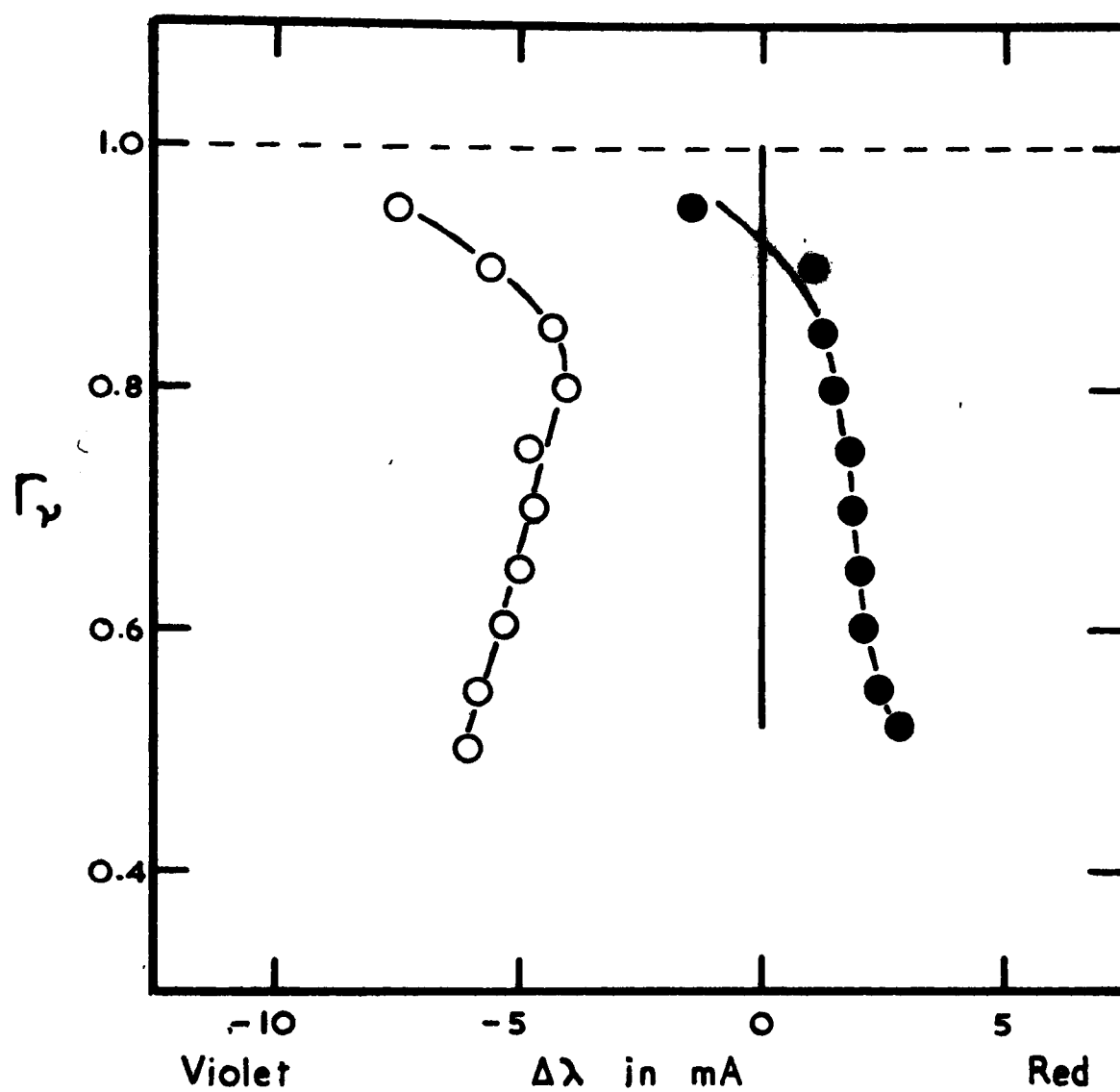


FIGURE 1 ASYMMETRY INTRODUCED BY SCATTERING

(Fe 6297.8 Å at $\rho/\rho_0 = 0.999$)

Unscattered line —
 East limb ●
 West limb ○

Appendix B

CENTRE-LIMB VARIATION OF LINE PROFILES

In addition to the asymmetry of the observed line profiles, the reduction procedure yielded a large amount of information on the general properties of the profiles of the two iron lines, λ 6297.8 and 6301.5. In Table 1, the mean observed equivalent widths, half-widths and central intensities of these two lines are given as a function of ρ/ρ_0 , after correction for the apparatus function. These were obtained by averaging together the results obtained at the two limbs. The errors that are quoted are the standard deviations of the means.

Table 1. Centre-Limb Variation of Iron Line Profiles.

<u>Line</u>	<u>Position (ρ/ρ_0)</u>	<u>Equivalent Width</u>	<u>Half-Width</u>	<u>Central Intensity</u>
6297.8 A	1.000	65.2 $\pm$ 2.4 mA	120.6 $\pm$ 4.0 mA	0.493 $\pm$ 0.017
	0.997	65.6 1.9	134.1 2.0	0.541 0.008
	Limb 0.994	67.9 1.6	135.6 2.1	0.516 0.008
	0.989	66.3 0.9	127.2 2.5	0.515 0.011
	0.983	70.5 2.6	131.5 3.9	0.490 0.006
	Centre 0.000	68.5 1.6	96.4 2.4	0.348 0.011
6301.5 A	1.000	95.3 3.4	141.8 7.7	0.412 0.045
	Limb 0.997	88.4 1.3	138.0 5.8	0.424 0.005
	0.994	97.3 2.0	144.2 8.6	0.404 0.012
	Centre 0.000	122.1 4.4	133.7 5.4	0.269 0.013

From these values, it is seen that the two lines have quite different centre-limb variations. The weaker line

λ 6297.8, has a fairly constant equivalent width but is much broader at the limb than at the centre, while the stronger line, although having a constant half-width, decreases in equivalent width towards the limb. It is interesting to note that the observed (at $\cos \theta = 1.00$ and 0.15) centre-limb variation of equivalent widths given by the table is quite different from that predicted by Schröter (1) from his two stream model (for $\cos \theta = 0.3$). This may indicate that after an increase in equivalent width towards the limb, the lines weaken for very small values of $\cos \theta$. No other observations so near the limb have been made, so that no conclusions on this point may be drawn.

The equivalent widths found for the two lines at the centre agree well with the values of 69.3 mÅ and 122.2 mÅ given by Minnaert et al. (2). The corresponding central intensities are, however, somewhat smaller than the values of 0.50 ± 0.02 and 0.30 ± 0.02 given by Allen (3) for the two lines. (No correction to the central intensities has been made for the effects of scattered light, but a brief investigation showed that such corrections amount to less than 2.5% of the continuum intensity for $\rho/\rho_0 = 0.999$).

References.

1. E. H. Schröter, Zs. f. Ap. 41, 141, 1957.
2. M. Minnaert, "Preliminary Photometric Catalogue of Fraunhofer Lines", Utrecht, 1960.
3. C. W. Allen, Mem. Comm. Solar Obs. 1, No. 5, 1934.

Appendix C

THE SOLAR RED SHIFT

A paper appearing in "The Monthly Notices of the Royal Astronomical Society", Volume 121, No. 5, p. 421, 1960.

MONTHLY NOTICES
OF THE
ROYAL ASTRONOMICAL SOCIETY
Vol. 121 No. 5

THE SOLAR RED-SHIFT

L. A. Higgs

(Communicated by the Director, University Observatory, Oxford)

(Received 1960 July 27)

Summary

The centre-to-limb increase in wave-length of three medium strength iron lines in the solar spectrum has been determined, using micrometer measurements of grating spectra. The observations were made at solar latitudes of approximately $\pm 55^\circ$, and at radial distances from the centre between 85 and 100 per cent of the radius of the disk. The limb-effect curve obtained, after corrections for scattering of light, is in good agreement with the curve obtained recently at Oxford (5) for solar equatorial regions.

The absolute wave-lengths in the solar spectrum of the lines used in this investigation and the vacuum arc wave-lengths of the three iron lines were determined using the method of circular channels. The centre red-shifts of the three lines, together with the limb-effect curve, give a red-shift at the extreme limb substantially exceeding the shift required by the principle of equivalence.

It is shown that the line profiles of the solar lines in the grating spectra are affected by the apparatus function of the spectrograph. Allowing for this, the solar lines at the centre appeared to be symmetric but an unexplained asymmetry at the extreme limb was discovered. This indicates that wave-length measurements alone are insufficient for a determination of the nature of limb effect.

Introduction

It is an established fact that the wave-lengths of the Fraunhofer lines in the solar spectrum are dependent upon the point of observation on the solar disk. In general, the wave-length of a line increases as the point of observation moves toward the limb. This difference of wave-length between the limb and the centre of the disk has received the general name "limb effect". In addition to this effect, most Fraunhofer lines have longer wave-lengths than do the corresponding transitions observed in a terrestrial laboratory. This effect will be referred to as the "red-shift" of the spectral lines. Thus the red-shift of a line at a given disk position is given by the red-shift of the line at the centre plus the limb effect.

The principle of equivalence predicts a wave-length shift of all solar lines with respect to corresponding terrestrial lines observed under similar physical conditions by an amount given by

$$\frac{\Delta\lambda}{\lambda} = 2.12 \times 10^{-6}.$$

This shift, referred to as the "gravity shift", is, however, independent of the point of observation on the Sun. Thus other physical processes must be invoked to describe the observed variations in the red-shifts at the centre of the disk and for the very existence of limb effect.

The early history of red-shift and limb-effect determinations may be found in the literature (14, 12, 7, 20, 13, 11). In 1948, M. G. Adam (1) published the results of an investigation of the wave-lengths of several solar lines at various positions along a polar disk radius. It was found that the limb effect reached an appreciable magnitude only along the last 10 per cent of a solar radius, and indicated the necessity of more detailed observations near the limb. Recently, in 1959, such a redetermination of solar limb effect was published by M. G. Adam (5). This gave the results of a study of the centre-limb variation of the wave-lengths of three medium strength iron lines in solar equatorial regions. The wave-length shifts were determined from micrometer measurements of spectra obtained using the 32 cm image of the larger Oxford telescope and its associated grating spectrograph. These indicated that the red-shift observed at the limb for these lines much exceeded the gravity-shift. Recent observations by Schröter (19) of limb effect agree well with the Oxford measurements in the common range of observation.

Because of the great theoretical importance of the shift at the limb, a further determination of the limb-effect curve for the same three solar lines as used in (5) has been carried out, and forms the subject of this paper. Although no great differences exist between the 1948 and 1959 limb-effect curves, referring respectively to polar and equatorial regions, the present determination was made at intermediate solar latitudes ($\sim \pm 55^\circ$) with a view to the detection of a latitude effect if such exists.

Part I presents the micrometer determination of the centre-limb variation of the wave-lengths of the three solar lines following the procedure adopted by M.G. Adam for the 1959 determination. In Part II, the determination of the absolute wave-lengths at the centre of the disk of the lines used in the investigation is given. Part III deals with an apparent discrepancy arising from the measurements in Parts I and II and presents some preliminary measurements of the profiles of the three solar lines at the centre and limb.

PART I

Limb effect from micrometer measurement

I.1. Method.—The same method of determining the centre-limb change in wave-length was adopted as was used in (5). The method is a differential one in which the positions of the solar lines on a spectral plate are compared with those of a set of standard lines, in this case terrestrial oxygen lines, whose wave-lengths are assumed constant. Since observations were confined to the part of the disk near the limb and the centre of the disk, such a method requires two essentially different sets of measurements. First, a set of standard positions of the solar lines at the centre of the disk with respect to the reference lines is required, and secondly, a set of measurements of the positions of the solar lines at various disk positions with respect to those of the standard lines is needed. Then the difference between the two gives the limb effect at various disk positions, when corrections have been made for the effects of the rotation of the Sun and motion of the observer. These shifts can then be converted to a limb effect expressed either in wave-length units or, as is customary, in equivalent Doppler velocities. For such a procedure to be valid, the dispersion used in the two sets of measurements must be identical, otherwise corrections must be applied, all measurements being reduced to one standard dispersion.

We shall use the following terminology for disk positions. A given radial position will be specified by θ , where θ is the angle between the line of sight and a true solar radius at the point of observation. Alternatively, a position is specified by ρ/ρ_0 where ρ_0 is the radius of the solar disk, and ρ is the radius to the point of observation. To a high degree of approximation, these are related by

$$\sin \theta = \rho/\rho_0.$$

1.2. *Solar spectra*.—The spectral plates used in the determination were some of a series obtained in 1957 (5), others of which were used in the 1959 limb-effect study. They were taken using the 35 m solar telescope with its associated spectrograph, the Bausch and Lomb grating giving a dispersion of 1.4 mm/A in the first order. The plates used were all Ilford panchromatic type R50 and were developed in Parkhurst two-solution developer.

Ten plates taken at the centre of the disk were used for the determination of standard positions and six plates of the region near the limb were used. For the limb plates, the solar image was guided so that the spectrograph slit was radial, the limb falling upon the slit. These plates were divided equally between the east and west limbs. The sixteen plates used are listed in Table I.

TABLE I
Details of measured plates

Centre plates	Limb plates		
Date U.T. 1957	Date U.T. 1957	Limb	Solar latitude
May 21.3812	May 24.3929	West	-50.0 to -59.0
22.6342			
27.4411	24.3979	West	-48.7 to -57.8
28.4674			
31.4592	27.3944	East	48.1 to 60.0
June 12.4850			
13.5754	June 16.4084	West	-50.6 to -63.7
13.6131			
14.4505	16.4141	East	51.0 to 61.8
14.6085			
	16.4233	East	48.9 to 58.5

The spectral lines are the same as those used by M. G. Adam and are listed in Table II, with their I.A.U. wave-lengths. There is no I.A.U. wave-length for the third oxygen line and the value given is an estimated one (5). Table II also gives their Rowland intensities, and for the solar lines, the excitation potentials of the lower levels and the transitions involved. The oxygen lines are associated with the α band of atmospheric oxygen.

TABLE II
Measured lines

I.A.U. wave-length	Element	Rowland intensity	Excitation potential	Transition
6295.960 A	Atm. O ₂	3		
6297.799	Fe	5	2.21 volts	$a^5P-y^5D^0$
6299.228	Atm. O ₂	3		
6301.508	Fe	7	3.64	$z^5P^0-e^5D$
(6302.000)	Atm. O ₂	2		
6302.499	Fe	5	3.67	$z^5P^0-e^5D$
6302.764	Atm. O ₂	2		

I.3. Centre measurements.—The spectra were measured on a Hilger photo-measuring micrometer fitted with a transverse micrometer motion. A diaphragm in the focal plane of the eyepiece limited the field of view to a strip of spectrum 0.145 mm wide. This was positioned parallel to the plate dispersion so that only a small section of each spectral line was visible. The plates were measured in the forward and reverse directions at the slit height corresponding to the centre of the disk and average line positions were found. These were referred to the first oxygen line after corrections for the observer's motion had been applied to the solar lines, the dispersions for the three lines being 33.99, 33.96 and 33.95 km/sec/mm. The average positions of the seven lines were then found from the ten plates. These were adopted as the standard centre positions and the oxygen positions were used to indicate the standard dispersion. These standard centre positions are listed in Table VI, Part III, where they are compared with the standard positions used by M. G. Adam (5).

I.4. Limb measurements.—Each limb plate was measured at about forty slit heights, each corresponding to a different radial position (different ρ/ρ_0). The positions of measurement were so chosen that they fell at approximately equal intervals in $\cos \theta$. In order to compute the value of ρ/ρ_0 for each "spectrum" measured, the limb plates were marked perpendicular to the dispersion at one millimetre intervals from the inner edge of the spectrum. The positions of each of the measured "spectra" were referred to these marks, and the position of the limb was determined visually with respect to them at both ends of the plate. The image radius having been determined for the times of observation, a ρ/ρ_0 was calculated for each set of measurements.

The measurements were made in the same manner as for the centre plates and mean positions of the seven lines were determined at each value of ρ/ρ_0 . Using the observed oxygen positions, these were corrected to the standard dispersion, assuming a linear correction factor. The difference between the corrected iron line positions and the standard iron line positions then gave the limb effect plus relative velocity corrections for the corresponding value of ρ/ρ_0 . The shifts so found for the three lines were then converted to equivalent Doppler velocities and averaged to give a mean shift for that value of ρ/ρ_0 .

I.5. Accuracy of measurement.—It was found that the velocity shifts determined as a function of ρ/ρ_0 from a single plate exhibited large fluctuations with amplitudes up to 0.2 km/sec. From an analysis of the micrometer settings in the centre spectra, it was concluded that the uncertainty in an individual reading of the position of an oxygen line was ± 0.05 km/sec, and for an iron line ± 0.07 km/sec. Similarly, an analysis of the limb measurements yielded an uncertainty in an iron line setting of ± 0.06 km/sec. From these figures, it is found that the uncertainty in the mean shift of all three solar lines at a given disk position due to measurement alone is ± 0.05 km/sec. Since this uncertainty is much smaller than the observed fluctuations, the latter are attributed to the effects of local velocity fields on the Sun (15).

From an analysis of micrometer measurements of the limb position with respect to the millimetre marks, it was concluded that the value of ρ/ρ_0 are uncertain by about ± 0.0003 . At the limb, this corresponds to ± 0.02 in $\cos \theta$.

I.6. Relative velocity corrections.—The corrections to be applied to the mean shifts obtained for each value of ρ/ρ_0 due to the motion of the observer and solar rotation were calculated using the Oxford Computing Laboratory "Mercury"

computer. Two programmes were written for this purpose. The first provides the coordinates of the centre of the disk and various other solar parameters at the time of exposure. The input for this consists of the times of exposure and certain data directly as tabulated in the Nautical Almanac. The second programme uses the output of the first plus the observational ρ/ρ_0 values as input data and calculates for each observed position: $\cos \theta$, solar latitude, correction for observer's motion, correction factor for solar rotation. The calculations of the corrections follow the procedure given by H. H. Plaskett (18) and, following M. G. Adam (5), the law of solar rotation is taken to be

$$R_B = R_0 \cos B(1 - 0.229 \sin^2 B)$$

where R_0 is the solar equatorial velocity. In the calculation of the solar rotation corrections, R_0 was regarded as an undetermined parameter, to be supplied from the observations themselves.

The mean shifts at each disk position were then corrected for the observer's motion, giving at each point a residual shift

$$\Delta(\theta) = R_0 \cdot f(\theta) + \Lambda(\theta)$$

where $R_0 \cdot f(\theta)$ is the correction for solar rotation, $f(\theta)$ being the calculated factor, and $\Lambda(\theta)$ is the limb effect. This gives essentially 249 equations in R_0 and the $\Lambda(\theta)$. The 1948 limb-effect curve showed, however, that $\Lambda(\theta)$ varies only slowly with θ except very near the limb. Thus the observations were split into thirteen groups, each covering a small range in $\cos \theta$ over which $\Lambda(\theta)$ could be considered approximately constant. For each group of observations, R_0 and $\overline{\Lambda(\theta)}$ were then determined by a least squares solution. As in the 1959 determination, the values of R_0 were fairly constant for the inner groups but decreased rapidly as the limb was approached. This was attributed to the effect of scattered light (5). From the four inner groups of observations, little affected by scattering, a mean value of 1.922 km/sec was determined for R_0 . Using this, the residual shifts were corrected for solar rotation, leaving values of $\Lambda(\theta)$.

I.7. Scattering corrections.—Since the positions of the observed points near the limb were uncertain by ± 0.02 in $\cos \theta$, and scattering effects depend primarily on the limb being observed (east or west), the shifts obtained from the three east limb plates were averaged together in intervals of 0.02 in $\cos \theta$, and similarly for the west limb. These results are shown in the top half of Fig. 1. The large scatter in the observed mean shifts is attributed to local velocity-field effects (15), and it is seen that the west limb values are lower near the limb than those for the east limb. This is as would be expected, since scattering effects are far greater for the west limb where solar rotation and true limb effect combine to give a rapid variation of wave-length near the limb.

The scattering effect was assumed to be represented mathematically by the simplified expression

$$\overline{u(x_0)} = \frac{\int_{-1}^{+1} I(x)u(x)S(x-x_0)dx}{\int_{-1}^{+1} I(x)S(x-x_0)dx}$$

where $u(x)$ is the true velocity shift (in the line of sight) at $\rho/\rho_0 = |x|$, $I(x)$ is the intensity at x , $S(x-x_0)$ is the scattering function and $\overline{u(x)}$ is the observed velocity shift. The limb darkening curve, $I(x)$, used by M. G. Adam was chosen, and, in the absence of any other "scattered light" exposures of the sky spectrum, the scattering function used by her was adopted (5).

To compute corrections for scattering, mean observed velocity-shift curves were obtained for the two limbs. Each was then smoothed and a true velocity shift curve was approximated by iteration, using the above expression for the scattering. From this, a scattering correction factor as a function of $\cos \theta$ was determined for each limb. Using this, the two observed limb-effect plots were then corrected, and the final values for the two limbs are given in the lower half of Fig. 1. Due to limitations in the iteration process, reliable scattering corrections could not be found for points nearer the limb than $\cos \theta = 0.05$, and hence the limb-effect results terminate at this point.

I.8. Final limb effect.—The east and west limb results were then averaged together to give a final limb-effect curve. This is given in Fig. 2. It is evident that local velocity-field effects still persist in the final curve despite the averaging of six plates. A smooth curve (a parabolic formula in $\cos \theta$ fitted by least squares) through the observed points gives the values tabulated in Table III as a function of ρ/ρ_0 . Included in this table are the corresponding values obtained by M. G. Adam for the solar equatorial region (5). The new determination appears

TABLE III

Final limb effect

ρ/ρ_0	Δ (km/sec)	
	L. A. Higgs	M. G. Adam
0.998	0.584	0.474
0.995	0.524	0.442
0.990	0.461	0.407
0.985	0.417	0.381
0.980	0.383	0.358
0.975	0.355	0.339
0.970	0.331	0.321
0.950	0.264	0.263
0.925	0.214	0.206
0.900	0.185	0.159
0.875	0.168	0.118
0.000	0.000	0.000

to give larger shifts than those observed by her, but the difference between the two is entirely accounted for by differences in the standard solar positions used in the two determinations (see Table VI, Part III). The uncertainty in these standard positions precludes the measurement of any general difference < 0.07 km/sec. Thus the latitude effect, if such exists, is of too small a magnitude to be detectable by these measurements. An analysis of the velocity shifts obtained from individual lines indicated small systematic differences between the shifts of the three lines, but these could also be accounted for by slight errors in the standard positions of the three lines. The question of the total red-shift at the limb, especially the possibility of a super-gravity shift,

cannot be answered until the red-shifts of these lines at the centre of the disk are known (Part II).

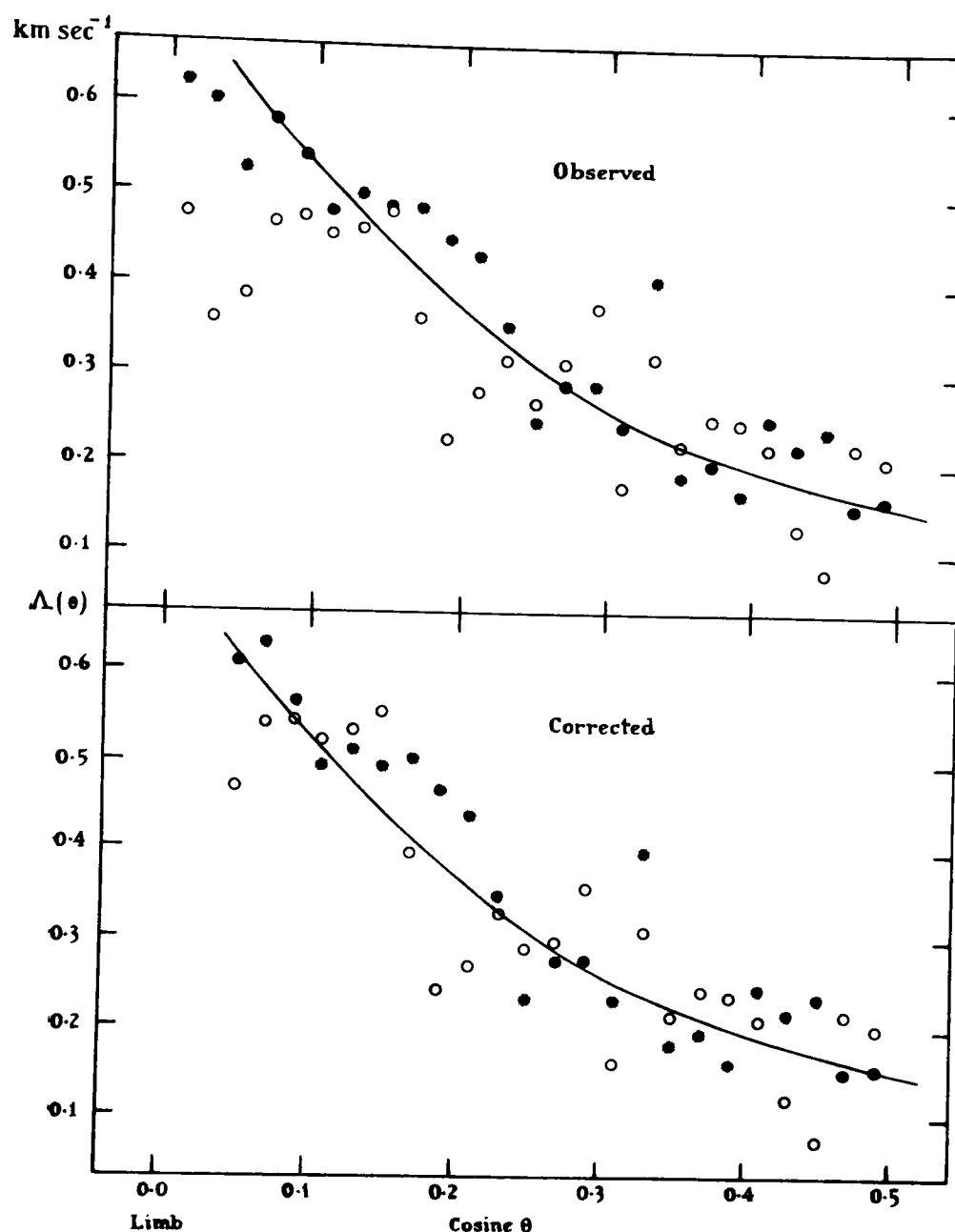


FIG. 1.—Mean observed limb effect before and after corrections for scattering.

Open circles: west limb.

Filled circles: east limb.

Continuous curve: final limb effect.

PART II

Determination of absolute wave-lengths and red-shifts

II.1. *Method.*—The determination of red-shifts requires the absolute wave-lengths of both the solar lines and the corresponding terrestrial lines, formed in physical conditions as near as those existing in the solar photosphere as possible, e.g. vacuum arc. At Oxford, the method of “circular channels” has been developed as a powerful interferometric technique for wave-length determinations, the theory and applications of which have been presented in the literature (21, 2, 3, 4).

Essentially the method consists of using a spectrograph in conjunction with a Fabry-Perot interferometer, the spectrograph serving only as a monochromator for the interferometer. If the light being analysed has a continuous spectrum with absorption lines, the observed interferometric pattern consists of a set of circular "heterochromatic" fringes, due to the continuous spectrum, which are crossed by absorption arcs forming part of another set of concentric circles

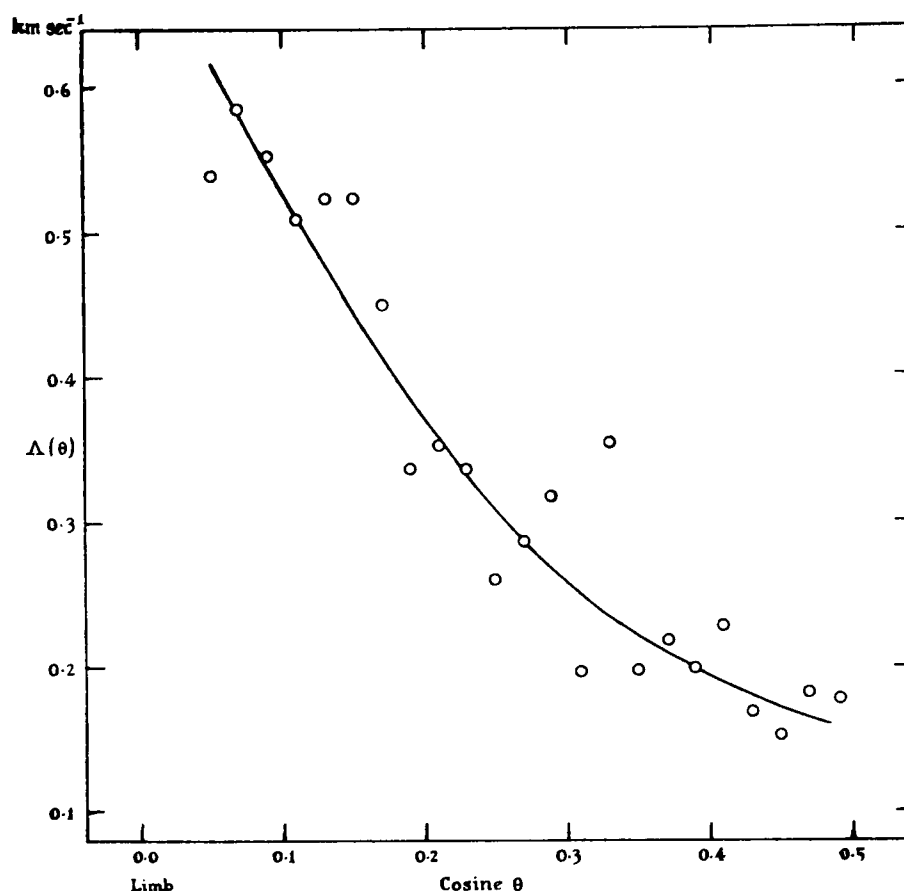


FIG. 2.—Final limb effect.

Open circles: mean of east and west limbs.

Continuous curve: smooth limb effect.

intersecting the first. In addition, the interference pattern of the cadmium red line ($\lambda 6438.4696$) is superimposed on this pattern using a beam splitting cube. Overlapping is prevented by a system of diaphragms. For the measurement of arc wave-lengths, in which one is concerned only with a discrete line spectrum, the "heterochromatic" fringes do not exist and emission arcs now replace the absorption arcs.

The interference patterns are measured on a specially adapted Hilger photo-measuring micrometer with a rotating plate stage. From detailed calculations, described in earlier papers, using the measured arc positions, the wave-lengths of the absorption or emission lines are compared with the wave-length of the cadmium line. Although complicated in its early stages, the method of circular channels has now become a standard procedure, and with the development of computer reduction of plate measurements is now a fairly rapid method of wave-length determination.

II.2. *Reduction of measurements.*—The formerly tedious reduction procedure has been superseded by automatic reduction using the “Mercury” electronic computer of the Oxford Computing Laboratory. A programme for this purpose has been written and is suitable for most conceivable wave-length determinations.

The input data consist of general plate information and the plate measurements. The former include: the number of lines that have been measured; chromatic phase changes, camera focal length corrections and approximate wave-lengths for each line; and special instruction parameters to indicate the specific reduction procedure to be followed. The output consists among other things of a tabulation of the fractional orders of interference of all the absorption or emission lines and the absolute wave-lengths of all the lines. In addition, the mean error due to the scatter of the measurements is tabulated for each value. The total reduction time for a typical plate is reduced by this procedure to approximately twenty minutes, almost all of which is taken in preparing the input data on punched tape.

II.3. *Procedure.*—The smaller Oxford telescope, giving an 18 cm solar image, was used for the determination of absolute wave-lengths at the centre of the disk. The Sun was imaged on the spectrograph slit, the slit height being 1.8 cm, and filters were used to equate the intensities of the solar and cadmium light beams passing through the interferometer. Two different spacers were used in the interferometer giving plate separations of 2.5 and 5 mm. Fused silica plates were used and had been recoated at the beginning of the wave-length programme to give 7 per cent transmission and 90 per cent reflection in the red region. Seventy-four solar plates were taken, using Ilford panchromatic R40 plates, at exposures varying from one to three minutes, depending on the spacer used. Of these, twenty-one plates taken on four different days were measured and reduced. Phase-change corrections and camera focal length corrections were adopted from earlier measurements (6). Both of these were small due to the small differences between the wave-lengths of the lines in question and the standard cadmium wave-length. For this reason, corrections for the dispersion of air in the interferometer were not necessary.

For the arc wave-lengths, a vacuum arc run at about 9 cm Hg pressure replaced the coudé beam from the telescope. Mild steel pole-pieces were used and a 10 mm arc was maintained at a current averaging 8 amperes. The usual method of guiding the arc and focusing it on the slit was employed (2), only the central portion of the arc actually being used in order to eliminate any pole effects. Due to the low intensity of the arc lines, exposures of up to two hours were required and the cadmium intensity was correspondingly reduced using filters. Fifteen plates were taken, using Ilford HP3 plates, and twelve of these were measured and reduced. The corrections for phase change, etc., were the same as for the solar plates. The calculated absolute wave-lengths were reduced to absolute vacuum wave-lengths using the empirical pressure depression of spectral terms given by Babcock (9). The largest pressure correction for the three iron lines amounted to only 0.9 mÅ.

II.4. *Solar and arc wave-lengths.*—The calculated solar wave-lengths obtained from the centre plates were corrected for the Doppler shifts due to the motion of the observer using the computer programme developed for Part I. Since some plates were of better quality than others, it was decided to weight each of the calculated wave-lengths on each plate according to the inverse square

of its calculated mean error for that plate. A similar method was used to determine the average arc-wave-lengths. There were no serious differences in the wave-lengths determined considering the two spacers separately. For the solar wave-lengths, the 5 mm spacer gave wave-lengths 0.6 mÅ longer than the 2.5 mm spacer, while for the arc wave-lengths, the 2.5 mm spacer values were 1.0 mÅ longer. The final absolute wave-lengths are given in Table IV, along with their standard deviations and the number of plates used in each determination. For comparison purposes, the I.A.U. wave-lengths are given for the solar lines and the Allegheny Observatory wave-lengths (10) of the arc lines are given.

II.5. *Integrated disk wave-lengths*.—Since it is known that local velocity fields may greatly affect wave-length determination on the Sun, the question arises whether the twenty-one solar plates were enough to eliminate these effects. As a check, wave-length plates were taken using integrated sunlight from all parts of the disk. Ideal conditions for integrated-light wave-length measurements would be attained if every part of the slit received truly integrated light and if every part of the spectrograph collimator received integrated light. The first condition was satisfied by removing the Cassegrain mirror of the telescope and allowing a parallel beam of light to fall directly on the spectrograph slit. The second condition, on the other hand, could not be satisfied. To counteract this, an image rotator was placed before the slit and the circular channel plates were taken in “mirror pairs”. For the second plate of each pair, the image used for the first was inverted about the optical axis. In this way, wave-length shifts due to variations in the transmission properties of the grating across its surface were expected to be averaged out. Two pairs of plates were measured.

TABLE IV

Absolute wave-lengths and red-shifts

Solar spectrum wave-lengths

El.	I.A.U.	Observed	No. Plates
O ₂	6295.960 Å	5.9603 ± 0.0002	10
Fe	6297.799	7.7989 ± 0.0004	21
O ₂	6299.228	9.2283 ± 0.0002	21
Fe	6301.508	1.5080 ± 0.0003	21
O ₂	(6302.000)	1.9998 ± 0.0001	21
Fe	6302.499	2.5000 ± 0.0003	21
O ₂	6302.764	2.7640 ± 0.0001	10

Iron arc wave-lengths

Alleg. Obs.	Observed	No. Plates	Red-shift
6297.790 Å	7.7918 ± 0.0006	10	7.1 ± 0.7 mÅ
6301.501	1.5008 ± 0.0003	12	7.2 ± 0.4
6302.493	2.4938 ± 0.0004	12	6.2 ± 0.5

The average wave-length values determined for each of these mirror pairs agreed well with each other, although there were differences (~ 4 mÅ) between the wave-lengths determined from the four plates individually. Since limb effect is known to concern only light coming from the extreme outer portion of the solar disk, one would expect the integrated disk wave-lengths to agree substantially with those determined for the centre of the disk. In fact, the agreement between

the mean wave-lengths from the four integrated disk plates (corrected for the observer's motion) and the absolute wave-lengths determined at the centre of the disk is remarkable. Table V gives this comparison. This close agreement would seem to indicate that the absolute solar wave-lengths found at the centre of the disk are free from effects due to local velocity fields.

II.6. *Red-shifts*.—From the above absolute solar and arc wave-lengths, the red-shifts of the three lines may be determined. These are presented in the last column of Table IV. These observed shifts are found to be slightly greater than those expected according to the empirical shift *vs.* line strength relation given by Nichols and Clube (16).

TABLE V
Comparison of solar wave-lengths

Element	Centre	Integrated disk
O ₂	6295.9603 ± 0.0002 Å	5.9605 ± 0.0002
Fe	6297.7989 ± 0.0004	7.7980 ± 0.0011
O ₂	6299.2283 ± 0.0002	7.2288 ± 0.0004
Fe	6301.5080 ± 0.0003	1.5086 ± 0.0010
O ₂	6301.9998 ± 0.0001	2.0002 ± 0.0003
Fe	6302.5000 ± 0.0003	2.4999 ± 0.0007
O ₂	6302.7640 ± 0.0001	2.7636 ± 0.0003

The results of Part I may now be extended to give the red-shift of the three lines across the disk (within the reservations to be indicated in Part III). A final limb-effect curve on an absolute scale is given in Fig. 3, in which the

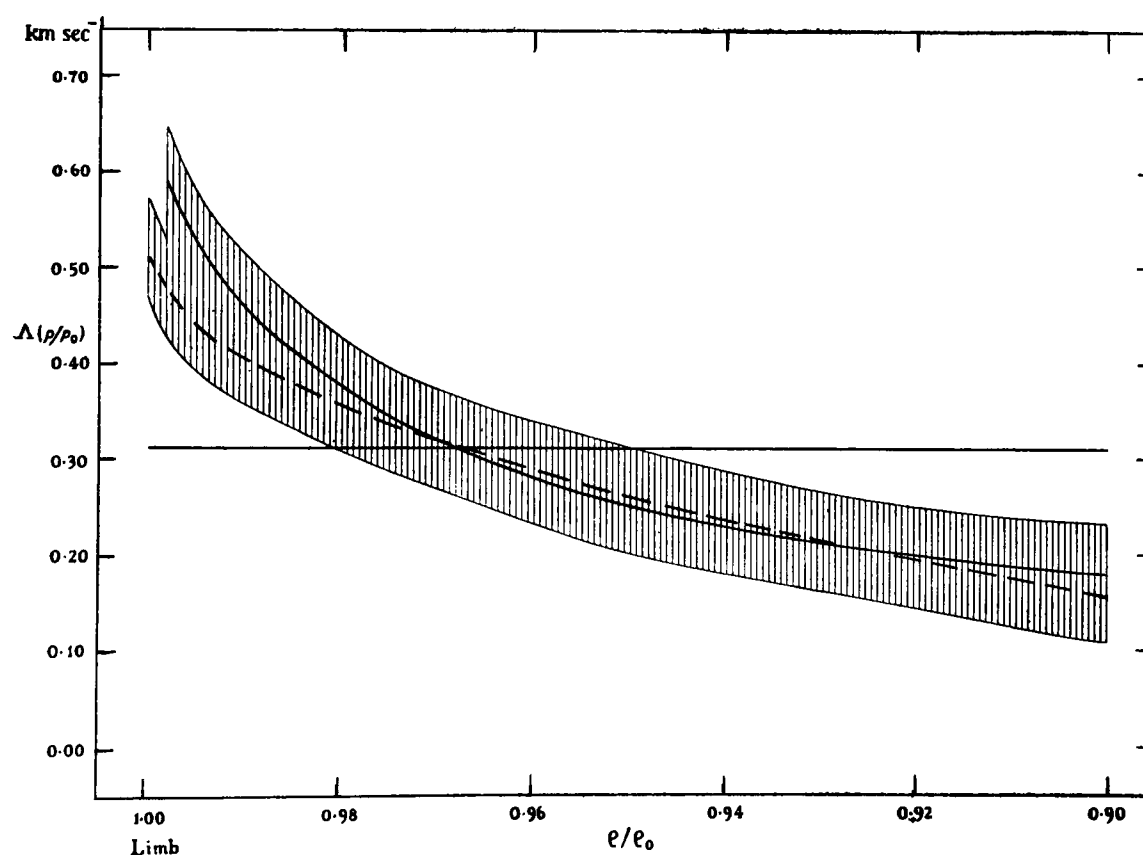


FIG. 3.—Limb effect and the gravity shift. The uncertainty in the limb-effect determinations is indicated by the shaded band.

Continuous curve: 1960 limb effect.
Dashed curve: 1959 limb effect.
Horizontal line: gravity shift.

1959 curve and the new determination are presented with their regions of experimental error. Also shown is the red-shift required by the principle of equivalence. From this diagram, it is evident that, allowing for experimental error, the shift observed at the extreme limb exceeds the gravity-shift, thus apparently ruling out the radial current interpretation of limb effect.

PART III

Line profiles

III.1. *Centre wave-lengths*.—In Part I of this investigation, standard micrometer positions for a centre spectrum were adopted for the seven lines used, and in Part II, the determination of the absolute wave-lengths of these lines was presented. Using the absolute oxygen wave-lengths and micrometer positions, it was possible to calculate the micrometer positions corresponding to the absolute wave-lengths of the iron lines. It was found that the calculated micrometer positions of the three solar lines were, on the average, 0.005 mm to the red side of the adopted standard positions. This corresponds to a wave-length difference of 4 mÅ. The calculated and standard micrometer positions are compared in Table VI, in which the standard positions used by M. G. Adam (5) are also included.

TABLE VI

Micrometer positions

	El.	Standard positions		Calculated positions
		M. G. Adam	L. A. Higgs	
6296.0 Å	O ₂	0.000 mm	0.000 mm	0.0000 mm
6297.8	Fe	2.571	2.570	2.5748
6299.2	O ₂	4.576	4.576	4.5764
6301.5	Fe	7.764	7.761	7.7683
6302.2	O ₂	8.456	8.457	8.4569
6302.5	Fe	9.154	9.153	9.1572
6302.8	O ₂	9.527	9.527	9.5268

On account of the small portion of the spectrograph slit (corresponding to 600 km on the surface of the Sun) used in the determination of the standard micrometer positions, one would expect some random fluctuations due to local velocity fields. Results obtained by A. B. Hart (15) indicate that r.m.s. velocity fluctuations of about 0.15 km/sec can be expected at a given point on the solar disk. Since ten plates were used in the determination of the standard positions, one might expect fluctuations in the mean positions of about ± 0.001 mm. Since the discrepancy in question was five times this value, it seemed unlikely that it was due to unusually strong velocity fields at the centre of the disk when the ten spectra were taken.

On the other hand, since the slit height used in the absolute wave-length measurements corresponded to 140 000 km on the surface of the Sun, one would expect all local velocity fluctuations to have been eliminated. The close agreement between the wave-lengths determined at the centre of the disk and those obtained using integrated light from the whole disk tended to confirm this. It thus seemed difficult to accept the possibility that the absolute wave-lengths were in error.

Since an asymmetric apparatus function of a spectograph can produce wave-length shifts of spectral lines (17), the possibility of such an effect in the grating spectra was considered. Using the Oxford Moll-type microphotometer, sixteen tracings of typical spectra were taken, in the forward and reverse directions, both at the centre of the disk and at several points near the limb. The dimensions of the microphotometer slit were chosen so as to correspond to the dimensions of the eyepiece diaphragm used in the micrometer measures. The tracings of a typical centre spectrum showed that the oxygen lines were approximately symmetric while the solar lines were definitely asymmetric, having pronounced violet flanks. Since this asymmetry is of such a magnitude as to be scarcely detectable in a diagram of the actual line profile, the following method of presentation will be used. The profile of a line is represented by a curve giving the mid-point of the profile at each value of the incident intensity, I . This is illustrated by the curve AB in Fig. 4(a). In this manner, the profile at the centre of the strongest iron line is presented in Fig. 4(b).

To determine an approximate apparatus function for the 30 000 line/inch Bausch and Lomb grating, it was decided to use the terrestrial oxygen lines as essentially monochromatic absorptions. The apparatus function is then given by the observed oxygen profiles. Allen (8), who has used atmospheric oxygen lines for similar purposes, gives theoretical widths for these lines of about 18 mÅ, which indicates the degree of approximation involved. Only the limb tracings were used for this purpose because the centre spectra were too over-exposed to show any possible asymmetry in the oxygen lines. All profiles were reduced to true intensity profiles using a characteristic curve for R50 plates, obtained in another investigation. From profiles of the oxygen lines $\lambda 6296.0$ and $\lambda 6299.2$ an approximate apparatus function was obtained. Having a half-width of 69 mÅ, it is symmetric to a width of about 150 mÅ, after which it develops a violet wing. The uncertainty in the determination of the wings is about 1 per cent of the central intensity, whereas the asymmetry in the wings is three times this figure. Thus a true asymmetry is indicated.

Using this apparatus function, it was shown that the observed asymmetry of the solar lines at the centre could be accounted for by purely instrumental effects, the true solar profiles being symmetric. In addition, it was estimated that the above apparatus function would shift the part of the solar lines used in micrometer settings 3–4 mÅ to the violet from their true wave-lengths. This explains the apparent discrepancy between the grating and interferometric wave-lengths. Since this instrumental shift of grating wave-lengths is independent of the point of observation on the solar disk, it has no effect on micrometer limb-effect determinations.

III.2. *Limb profiles.*—To investigate the profiles of the solar lines near the limb, tracings of six limb spectra were measured, each at different values of ρ/ρ_0 . The east-limb plates were used for these in order to reduce the effects of scattered light. Fig. 4(c) to (f) shows the observed “profile” of the strongest iron line ($\lambda 6301.5$) as a function of ρ/ρ_0 . There seems to be a systematic change in profile as one approaches the limb, with the asymmetry reversing in the core of the line, indicating effects other than instrumental slit pattern. Although these curves all refer to the strongest iron line, the other two iron lines show similar behaviour.

Several causes of the difference between centre and limb profiles were considered. The most plausible of these was that the scattering of light might create very asymmetric line profiles at the limb where both the intensities and wave-lengths of the lines are varying rapidly. To test this, an "observed" profile was computed for $\rho/\rho_0 = 0.997$, assuming the true line profile to be symmetric. In this calculation, the same scattering function was used as in Part I, the apparatus function was taken to be that above, and the wave-lengths were calculated using the limb-effect curve in Part I. It was found that scattering could not account for the observed asymmetry of the lines at the limb, its only effect being a slight shift of the whole line to the violet. In a similar manner, it was shown that the finite height of the microphotometer slit could not account for the observed profiles.

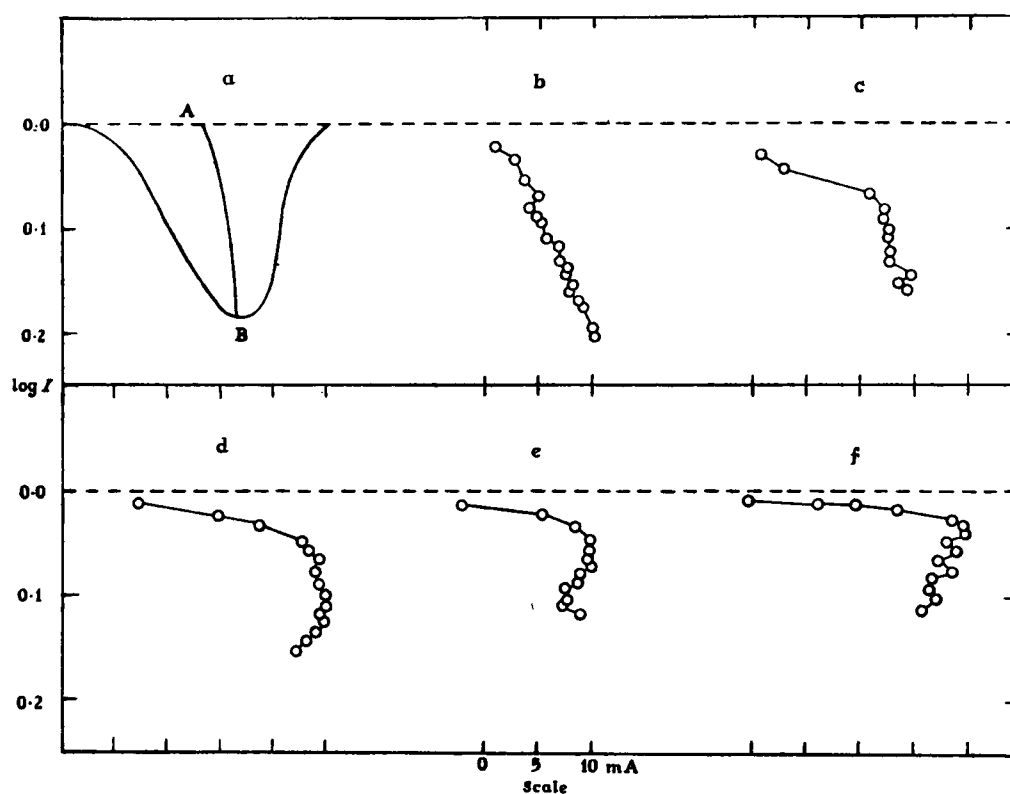


FIG. 4.—Measured asymmetry of the 6301.5 Å iron line.

- (a) Explanatory profile.
- (b) Centre.
- (c) $\rho/\rho_0 = 0.988$.
- (d) $= 0.994$.
- (e) $= 0.999$.
- (f) $= 1.000$.

It thus appeared possible that the solar lines at the extreme limb are formed with asymmetric profiles. The unexplained asymmetry is seen to set in at a value of ρ/ρ_0 of about 0.98 (about 20" from the limb). Such an asymmetry has never been previously observed*; on the other hand, observations so near the limb are few. Assuming that such true asymmetry existed, an attempt was made to correct the observed profiles for the apparatus function and thus determine the amount of the true asymmetry. Unfortunately, the asymmetries concerned were too small and the apparatus function insufficiently well determined for any

* Although asymmetry of infra-red oxygen lines has been detected at the centre by H. H. Voigt, *Zs. f. Ap.*, 40, 365, 1956.

iteration procedure to yield reliable results. It did appear, however, as if the solar lines at the limb have very pronounced red flanks.

The asymmetry observed in limb lines, if real, completely changes our views as to the nature of limb effect. A wave-length shift to the red will in all probability be produced by the strong red flanks which are indicated for the limb lines, but the interest is now changed from wave-length measurement, an unsuitable means of measuring line asymmetry, to a photometric survey of the nature and origin of the asymmetry observed. To this end, observations are at present in progress with a Babcock grating, the apparatus function of which is being accurately determined, giving greater dispersion and resolving power than the Bausch and Lomb grating so far employed.

Acknowledgments

I wish to express my thanks to Dr M. G. Adam for her guidance throughout this investigation, for many helpful discussions on all aspects of the subject, and for her assistance in making many of the observations. I should also like to thank Dr Hindmarsh for silvering the interferometer plates, the staff of the Oxford Computing Laboratory for their assistance in the reduction of numerical data, and the Rhodes Trustees for the scholarship which has made this work possible.

University Observatory,
Oxford:
1960 July 15.

References

- (1) M. G. Adam, *M.N.*, **108**, 446, 1948; *Comm. Univ. Obs. Oxford*, No. 26.
- (2) M. G. Adam, *M.N.*, **112**, 149, 1952; *Comm. Univ. Obs. Oxford*, No. 36.
- (3) M. G. Adam, *M.N.*, **115**, 367, 1955; *Comm. Univ. Obs. Oxford*, No. 49.
- (4) M. G. Adam, *M.N.*, **115**, 385, 1955; *Comm. Univ. Obs. Oxford*, No. 50.
- (5) M. G. Adam, *M.N.*, **119**, 460, 1959; *Comm. Univ. Obs. Oxford*, No. 71.
- (6) M. G. Adam and S. Nichols, *M.N.*, **118**, 51, 1958; *Comm. Univ. Obs. Oxford*, No. 65.
- (7) W. S. Adams, *Ap. J.*, **31**, 30, 1910.
- (8) C. W. Allen, *Ap. J.*, **85**, 156, 1937.
- (9) H. D. Babcock, *Ap. J.*, **65**, 147, 1927.
- (10) K. Burns and F. M. Walters, *Publ. Allegheny Obs.*, **6**, 159, 1929.
- (11) J. Evershed, *M.N.*, **91**, 260, 1931.
- (12) J. Evershed and T. Royds, *Kodaikanal Obs. Bull.*, No. 49, 1916.
- (13) E. F. Freundlich, A. v. Brunn and H. Brück, *Zs. f. Ap.*, **1**, 43, 1930.
- (14) J. Halm, *Astronomische Nachrichten*, **173**, 273, 1907.
- (15) A. B. Hart, *M.N.*, **116**, 38, 1956; *Comm. Univ. Obs. Oxford*, No. 54.
- (16) S. Nichols and S. V. M. Clube, *M.N.*, **118**, 73, 1958; *Comm. Univ. Obs. Oxford*, No. 67.
- (17) H. H. Plaskett, *M.N.*, **112**, 99, 1952; *Comm. Univ. Obs. Oxford*, No. 33.
- (18) H. H. Plaskett, *M.N.*, **112**, 117, 1952; *Comm. Univ. Obs. Oxford*, No. 34.
- (19) E. H. Schröter, *Mitt. Astrophys. Obs. Potsdam*, Nr. 83, 1959.
- (20) C. E. St John, *Ap. J.*, **67**, 195, 1928.
- (21) P. J. Treanor, S. J., *M.N.*, **109**, 37, 1949; *Comm. Univ. Obs. Oxford*, No. 28.

Errata (Appendix C).

Page 430, Table IV: The observed arc wavelengths and red shift values should be as in Tables 4 and 6 of Chapter 3 (thesis).

Page 431, line 8: "greater" should be "less".

Page 431, Fig. 3: The horizontal line should be at an ordinate of 0.25 km/sec.

Page 435, References: Reference (9) should be H.D. Babcock, Ap. J., 67, 240, 1928.

Appendix D

AUTOCODE PROGRAMMES FOR THE "MERCURY" COMPUTER

PART I. SOLAR COORDINATES AND RADIAL VELOCITY CORRECTIONS

A. Introduction.

Several programmes were written for the derivation of various solar coordinates and radial velocity corrections to solar wavelengths for given points and times of observation. These consist of programmes OBS 1, 1A, 1B, 3, 3C-1 and 3C-2. Of these, the most general one, OBS 3C-1, will be given in detail, followed by short descriptions of the other five.

B. Programme OBS 3C-1.B. 1. Description.

This programme is used to compute, from the ϕ and ρ_0 of the observed point on the disk at a known U.T., the values of $\cos \gamma_1$, $\cos \gamma_2$, $\cos \gamma_3$, $\sin B$ and the radial velocity corrections $C_1 + C_2$ and f (see Chapter 2 for explanations of the symbols). In addition, the following solar coordinates and parameters are computed for the time of observation: α_0 , δ_0 , H_0 , X , Y , Z , $\dot{X}$, $\dot{Y}$, $\dot{Z}$, B_0 , L_0 , P and S . (If ϕ depends only on H_0 , it

is also computed).

B.2. Input Data.

The input data, aside from the U.T., ϕ , and ρ/ρ_0 values, consist of short tables copied from the Astronomical Ephemeris. A separate data tape is required for each date of observation, and this is compiled as follows: an integer, $n-1$, where n is the number of observations on the given day ($n \leq 30$), followed by the U.T.'s corresponding to the n observations; a constant, A , which is defined by $\phi = H_0 + A$ (degrees) if ϕ depends on H_0 , otherwise $A = 99$; the equation of time, B_0 , P , S , and L_0 for the three consecutive days (0 hrs U.T.) centred on the date of observation; the apparent right ascension and declination for the five consecutive days centred on the date of observation; the values of X , Y , and Z (arbitrary equinox) for the nine days centred on the date of observation; an integer, m , where $m-12$ is the number of disk positions for which radial velocity corrections are required at the first time of observation, followed by the $(m-12)$ values of ρ/ρ_0 and ϕ (if ϕ is derivable

from H_0 , i.e., $A \neq 99$, then the ϕ 's are not given here), with ϕ in degrees; the m for the second time of observation followed by the $(m-12)$ pairs of coordinates ρ/ρ_0 and ϕ ; the m for the third time of observation, etc. As many data tapes (different dates of observation) may be entered, one after the other, as desired.

Quantities that enter as three columns (U.T., E. of T., S, R.A., and Dec.) must have two "SPACE" instructions between the columns, and a number must be given for each column, if only a zero. All numbers are given exactly as in the Astronomical Ephemeris except that an hour column must be added to the E. of T. values and a degree column to the S values.* Any $\rho/\rho_0 > 1$ is treated as $\rho/\rho_0 \equiv 1$ in the computation of radial velocity corrections.

The required layout of the data is illustrated by a sample data tape given on the next two pages. It is for two spectral plates taken at 9 hrs 22 min 39 sec U.T. and 9 hrs 17 min 33 sec U.T. on July 6, 1960. The first plate was of the centre of the disk, and the second was of the west limb.

* The three L_0 values and five R.A. values entered from the Astronomical Ephemeris must not contain a discontinuity of 2π radians. That is, 10° , 350° , 330° must be entered as 370° , 350° , 330° .

TYPICAL INPUT DATA

OBS 96 - 1
217

1	Number of observations less one				
9 22 39	U.T.'s for observations				
9 17 33					
99	Constant related to ϕ , indicates ϕ 's are to be individually specified				
00 -04 -23.66					
00 -04 -33.90	Equation of time				
00 -04 -43.77					
337					
348	H_0				
358					
-0.87					
-0.41	P				
0.04					
00 15 45.36					
00 15 45.37	S				
00 15 45.39					
307.43					
294.19	L_0				
280.96					
6 52 13.94	22 53	56.0			
6 56 21.09	22 48	35.8			
7 00 27.89	22 42	51.7	Right ascension on left, declination on right		
7 04 34.32	22 36	44.0			
7 08 40.38	22 30	12.8			
-0.1784063	0.9183297	0.3982433			
-0.1950343	0.9154810	0.3970085			
-0.2116058	0.9123753	0.3956622			
-0.2281164	0.9090136	0.3942048	$X, Y, \text{ and } Z \text{ (equinox 1960.0)}$		
-0.2445614	0.9053974	0.3926368			
-0.2609369	0.9015279	0.3909587			
-0.2772386	0.8974062	0.3891712			
-0.2934622	0.8930337	0.3872748			
-0.3096040	0.8884113	0.3852698			
13	Number of ρ/ρ 's plus 12				
0.0000 270	ρ/ρ and ϕ				

17.

Number of p/p_0 's plus 12

0.99948	270
0.9966	270
0.9939	270
0.9885	270
0.9826	270

p/p_0 and ϕ

B.3. Output.

The output for a given date of observation consists of two sections. In the first, general solar coordinates and parameters are given for the various times of observation, and in the second, certain parameters referring to the points of observation are given. The first section consists of two columns, broken into twelve sections. These give, in order, the fraction of day, H_0 , B_0 , P , S , L_0 , ϕ , α_0 , δ_0 , X , Y , and Z . (If ϕ is not derivable from H_0 , the ϕ values are meaningless). For angular quantities, the first column gives the value in degrees and the second gives the value in radians. For X , Y , and Z , the second column gives the derivatives (A.U./solar day) of these quantities at the times of observation. The values of each of these quantities for the different times of observation appear one below the other, in the order in which the U.T.'s of the observations were given on the data tape.

The second part of the output consists of seven columns, giving ρ/ρ_0 , $\cos \theta$ or $-\cos \gamma_3$, the correction to radial velocities for the Earth's motion (rotation and

orbital motion) in km/sec, $\sin B$ at the point of observation, the correction factor, f , for solar rotation ($-f.R_0$ is the true correction in km/sec), and the other two direction cosines at the point of observation, $\cos \gamma_1$ and $\cos \gamma_2$. For each time of observation, the ρ/ρ_0 's appear below one another in the order in which they were entered in the data.

As an example, the output for the data given for July 6, 1960 is presented on the next two pages.

046-

B.4. Programme and Operating Instructions.

The programme is given after section C along with explanatory notes. The operating instructions are as follows:

1. Enter programme tape in reader.
2. Set hand-switch 4, others clear (unless ? is required).
3. ITB
4. On reading $\rightarrow$, remove programme tape.
5. Enter data tape in reader.
6. Tap hand-switch 9.
7. For extra data tapes, same as instructions 5 and 6.

TYPICAL OUTPUT

OBS 30 - 1

221

F	00.39072917 00.38718750	Fraction of day corresponding to the two times of observation	
H	318.25333118 316.97847652	5.55456845 5.53231804	H_0 in degrees (left) and radians (right)
B	3.52026321 3.51990511	0.06144018 0.06143393	B_0 in degrees and radians
P	-00.23298157 -00.23457926	-00.00406630 -00.00409418	P in degrees and radians
S	00.26260462 00.26260460	00.00458332 00.00458331	S in degrees and radians
L	289.01946163 289.06632328	5.04434119 5.04515908	L_0 in degrees and radians
PHI	417.25333118 415.97847652	7.28244440 7.26019398	ϕ in degrees and radians (in this case these figures are meaningless)
RA	105.51759219 105.51395535	1.84162939 1.84156593	Right ascension in degrees and radians
EC	22.67522972 22.67558902	00.39575742 00.39576368	Declination in degrees and radians
(	-00.25096828 -00.25091026	-00.01638343 -00.01638368	X and $\dot{X}$
.	00.90391556 00.90392916	-00.00384193 -00.00384104	Y and $\dot{Y}$
!	00.39199419 00.39200009	-00.00166611 -00.00166572	Z and $\dot{Z}$

R/R ₀	COS ₃	E.M.	SIN B	F	COS ₁	COS ₂
0.00000	1.00000	0.20139	0.0614	-0.00000	0.00000	0.00000
0.99948	0.0322	0.33821	0.0063	0.99758	0.9976	-0.0612
0.99660	0.0824	0.33783	0.0094	0.99469	0.9948	-0.0606
0.99390	0.1103	0.33747	0.0111	0.99199	0.9921	-0.0602
0.98850	0.1512	0.33676	0.0136	0.98659	0.9867	-0.0593
0.98260	0.1857	0.33597	0.0157	0.98068	0.9809	-0.0585

Direction cosines and radial velocity corrections
(the quantity under F is not strictly a correction).
The values for the two times of observation are
separated by the space.

C. Other Similar Programmes.

C.1. Programme OBS 1.

This is essentially Chapter 2 of OBS 3C-1. For fifty pre-determined values of ρ/ρ_0 , it computes $\cos \theta$, C_1+C_2 , $\sin B$, and f , assuming that all the ρ/ρ_0 's refer to the same ϕ . The ρ/ρ_0 's are: 1.000 to 0.980 in steps of 0.002, and 0.940 to 0.840 in steps of 0.005. The input data consist of the following numbers, referring to the time of observation: H_0 , δ_0 , X , Y , Z , $\dot{X}$, $\dot{Y}$, $\dot{Z}$, B_0 , P , ϕ , and S , entered one after the other. All angles must be in radians, X , Y , and Z in A.U. and their derivatives in A.U. /solar day. A 999 on the data tape will stop the computation on an "END" instruction, but is usually unnecessary, and as many data tapes may be entered as required. The operating instructions are as for OBS 3C-1. The output consists of the first five columns of the OBS 3C-1 output.

C.2. Programme OBS 1A.

This is the same as OBS 1 except that the ρ/ρ_0 's are entered as data. The data tape then consists of the 12 numbers as for OBS 1 plus the following: an integer n ,

($n \leq 60$) where $n-12$ is the number of ρ/ρ_0 's to be specified, followed by the $(n-12)\rho/\rho_0$'s. The output is the same as for OBS 1.

C.3. Programme OBS 1B.

This programme gives a mean total radial-velocity correction, along a solar radius, over a number of times of observation. The data consist of the twelve numbers required for OBS 1, for a given time of observation, repeated for each time of observation. As many sets of twelve numbers (times of observation) may be entered as desired (< 512). These are followed by the number 999 and the value to be adopted for R_0 in km/sec.

For each set of twelve numbers (time of observation), the computations of OBS 1 are carried out for fifty values of ρ/ρ_0 , going from 1.00 to 0.00 in steps of 0.02. The first part of the output then consists of the OBS 1 output for each time of observation. From the computed C_1+C_2 and f values for each time of observation, a mean velocity shift curve is then computed for all the times of observation and this curve ($f.R_0 - C_1 - C_2$) is tabulated for the fifty values of ρ/ρ_0 . Only one data tape can be used without re-entering the programme tape.

C.4. Programme OBS 3.

This is essentially Chapters 1 and 0 of OBS 3C-1. For given times of observation, it computes various general solar coordinates and parameters for physical observations. The input is the same as the first part of that for OBS 3C-1, i.e., everything up to the first m value, and the output is correspondingly the same as the first part of the OBS 3C-1 output. Up to 73 times of observation may be entered and as many data tapes as required may follow one another.

C.5. Programme OBS 3C-2.

This is exactly the same as OBS 3C-1 except that the values of $\cos \gamma_1$ and $\cos \gamma_2$ are not included in the output.

TITLE

RADIAL VELOCITY CORRECTIONS FOR SOLAR SPECTRA - HIGGS

CHAPTER 1

A→25

B→25

C→80

D→10

E→75

F→75

V→75

X→26

E→10

U→*30

Y→30

1) I=0(1)4

P)=2)

3) READ(E0)

READ(E1)

READ(E2)

JUMP12

2) A1=15E0

P)=4)

JUMP3

4) B1=E0

REPEAT

I=0(1)4

E1=A1

REPEAT

NEWLINE

NEWLINE

620,27

620,18

620,1

620,0

NEWLINE

M)=5)

K=0(1)T

JUMP10

5) PRINT(E8) 3,8

SPACE

PRINT(E8/180) 3,8

NEWLINE

REPEAT

NEWLINE

I=0(1)4

E1=B1

REPEAT

NEWLINE

Repeat for each of five days, centred on
the day of observation

Read right ascension (α .)

Convert to hours

Convert to degrees

Read declination (δ .) and convert to degrees

Copy right ascensions

Print RA

Repeat for each time of observation

Interpolate right ascension

Print right ascension (degrees)

Print right ascension (radians)

Copy declinations


```

620,27 .
620,4
620,5
620,3
620,0
NEW LINE
M)=6)
K=0(1)T
JUMP10
6) PRINT(E8) 3,8
SPACE
PRINT(E8/180) 3,8
UK=E8/180
NEW LINE
REPEAT
X7(220)U0,0
NEW LINE
I=0(1)26
READ(X1)
REPEAT
S=0(1)2
NEW LINE
JUMP20,S>0
620,27 .
620,24
620,0
AN=250
BN=280
NEW LINE
JUMP22
20) JUMP2 3, S>1
620,27 .
620,25
620,0
AN=310
BN=340
NEW LINE
JUMP22
23) 620,27 .
620,26
620,0
AN=370
BN=400
NEW LINE
22) K=0(1)T
I=0(1)4
J=S+3I+6

```

Print DEC

Repeat for each time of observation
Interpolate declination
Print declination (δ_0) in degrees

Print δ_0 in radians

Store δ_0 for each time of observation

Read the 9x3 matrix of X, Y, and Z, row by row (Row gives the day, the three columns are X, Y, and Z)
Repeat the following for X, Y, and Z (i.e. for each column of matrix)

Print X

Print Y

Print Z

Repeat for each time of observation

Select a column from five central rows (column depends upon the value of S)

```

EI=XJ
REPEAT
M)=9)
JUMP 10
9) PRINT (E8) 3,8
UK=E8
SPACE
I=0(1)4
J=0(1)4
Q=S+3I+3J
DJ=XQ
REPEAT
N)=7)
JUMP 11
7) EI=D7
REPEAT
M)=8)
JUMP 10
8) PRINT (E8) 3,8
YK=E8
NEWLINE
REPEAT
X7(A7) U0,0
X7(B7) Y0,0
REPEAT
NEWLINE
NEWLINE
NEWLINE
ACROSS 1/2
10) E5=E3-E1
E6=E3-2E2+E1
E7=E4-2E3+2E1-E0
E8=E2+VK E5/2+VKV KE6/2+VKV KV KE7/12-V KE7/12
JUMP(M)
11) D5=D3-D1
D6=D4-2D3+2D1-D0
D7=6 D5/H-D6/H
D7=D7/12
JUMP(N)
12) E0=E0+E1/60+E2/3600
JUMP(P)
PSA
CLOSE

```

Select values of X, Y, or Z (depending on S)
on the five days centred on the day of
observation
Interpolate X, Y, or Z for the time of observ.
Print X, Y, or Z

Repeat for five days centred on day of observ.

Select values of X, Y, or Z (depending on S)
on the five days centred on the day in question

Compute derivative of X, Y, or Z for the day
in question (0 hrs U.T.)

Interpolate $\dot{X}$, $\dot{Y}$, or $\dot{Z}$ at time of observation
Print $\dot{X}$, $\dot{Y}$, or $\dot{Z}$

Store X, Y, and Z, $\dot{X}$, $\dot{Y}$, and $\dot{Z}$ for each time
of observation

Interpolate to third differences

Comput derivative

Convert hours, minutes and seconds to
hours, or degrees, minutes and seconds to
degrees

CHAPTE R2

A→6 0
B→2 5
C→2 5
D→6 0
E→2 0
F→2 0
G→2 0
H→2 0
U→2 0
V→2 0
W→2 0
X→2 0
Y→2 0
Z→2 0
E→2 0
1) X6 (1 0) E0, 0
X6 (2 2 0) F0, 0
X6 (2 5 0) G0, 0
X6 (3 1 0) H0, 0
X6 (3 7 0) U0, 0
X6 (2 8 0) V0, 0
X6 (3 4 0) W0, 0
X6 (4 0 0) X0, 0
X6 (4 0) Y0, 0
X6 (7 0) Z0, 0
X6 (1 0 0) E0, 0
X6 (1 6 0) C6, 0
P=0(1) 3
SPACE
REPEAT
620, 27 .
620, 18
620, 0
620, 23
620, 27 .
620, 18
620, 0
620, 16
P=0(1) 4
SPACE
REPEAT
620, 27 .
620, 3
620, 15
620, 19
620, 0
*620, 19

Read H. , for each time of observation
Read δ. (from store)

Read X
Read Y
Read Z
Read X
Read Y
Read Z
Read B.
Read P
Read S
Read φ

Print R/R0

Print 0083

```

P=0(1) 5
SPACE
REPEAT
620,27 .
620,5~
620,0
620,28
620*27 .
620,13
620,0
620,28
P=0(1) 4
SPACE
REPEAT
620,27 .
620,19
620,9
620,14
620,0
SPACE
620,27 .
620,2~
620,0
P=0(1) 5
SPACE
REPEAT
620,27 .
620,6~
620,0
P=0(1) *6
SPACE
REPEAT
620,27 .
620,3~
620,15
620,19
620,0
620,1
P=0(1) 4
SPACE
REPEAT
620,27 .
620,3~
620,15
620,19
620,0
620,2
NEWLINE

```

Print E.M.

Print SIN B

Print F

Print COS1

Print COS2

NEWLINE

 $I = 0(I) T$ $A_0 = E I$ $A_1 = F I$ $A_2 = G I$ $A_3 = H I$ $A_4 = U I$ $A_5 = V I$ $A_6 = W I$ $A_7 = X I$ $A_8 = Y I$ $A_9 = Z I$ $A_1 \times I = E I$

READ(A12)

 $J = X \text{INTPT}(A12)$ $2) K = 13(I) J$

READ(AK)

JUMP10, Cn=1

 $DK = C(I+6)$

JUMP11

10) READ(DK)

 $DK = E DK / 180$ $11) *B_0 = X \sin(DK)$ $?B_1 = X \sin(A_8)$ $?B_2 = X \cos(A_9 - DK)$ $B_3 = X \cos(DK)$ $B_4 = X \cos(A_1)$ $B_5 = X \cos(A_8)$ $?B_6 = 1/B_4$ $B_7 = X \sqrt{A_2 A_2 + A_3 A_3}$ $?B_8 = X \arctan(B_7, A_4)$ $?B_9 = X \arctan(A_2, A_3)$ $B_{18} = X \sin(A_9 - DK)$ $C_0 = AK$

PRINT(C0) 2, 5

 $C_1 = 1 - C_0 C_0$

JUMP7, C1 ≥ 0

 $C_1 = 0$ 7) $C_1 = X \sqrt{C_1}$

PRINT(C1) 1, 4

 $C_2 = X \arctan(C_1, C_0)$ $?B_{10} = X \sin(A_0 * - A_{11} B_0 C_0 B_6)$ $?B_{11} = X \sin(B_9 + A_{11} B_0 B_6 C_0)$ $?B_{12} = X \sin(B_8 + A_{11} B_3 C_0)$ $?B_{13} = X \sin(C_2 - A_{11} C_0)$ $?B_{14} = X \cos(A_1 + A_{11} B_3 C_0)$ $B_{15} = X \cos(B_9 + A_{11} B_0 B_6 C_0)$ $B_{16} = X \cos(B_8 + A_{11} B_3 C_0)$

Repeat for each time of observation

 H_0 δ_0 X Y Z X Y Z B_0 P S Read number of ρ/ρ_0 's + 12Repeat for each value of ρ/ρ_0 Read ρ/ρ_0 Copy ϕ if applicableRead ϕ if applicable

Convert to radians

 $\sin \phi$ $\sin B_0$ $\cos \phi$ $\cos \phi$ $\cos \delta_0$ $\cos B_0$ $\sec \delta_0$ $\arctan \left(\frac{Z}{\sqrt{X^2 + Y^2}} \right)$ $\arctan(Y/X)$ $\sin \phi$ Print ρ/ρ_0 $\cos \theta$ Print $\cos \theta$ ($-\cos \gamma_3$) $\sin H$ $\sin \alpha'$ $\sin \delta'$ $\sin \theta'$ $\cos \delta$ $\cos \alpha'$ $\cos \delta'$

```

B17=XCOS(C2-A11Co)      Cos 0*
C3=-0.2885B14B10-1732.34A5B15B16-1732.34A6B16B11-1732.34A7B12
PRINT(C3)2,5             Print Earth's motion correction (C1 + C2)
C4=B1B17+B5B13B2        Print sin B
PRINT(C4)1,4
C5=B18B5Co-0.229B18B5CoC4C4
PRINT*(C5)1,5           Print solar rotation correction (-C3/R0)
B20=1-C4C4
JUMP30,B20>0
B20=0
30)B21=X SQRT(B20)      Cos B
B19=B13B18/B21          Sin (L-L0)
B22=B19B5Co/B13
PRINT(B22)1,4           Print cos γ1
B23=1-B19B19
JUMP31,B23>0
B23=0
31)B23=X SQRT(B23)      Cos (L-L0)
B24=C4B5B23Co/B13-B21B*1Co/B13
PRINT(B24*)1,4          Print cos γ2
NEWLINE
REPEAT
NEWLINE
NEWLINE
REPEAT
ACROSS1/0
PSA
XSQRT
XARCTAN:
XSIN:
CLOSE

```

Return to beginning for new data

```

CHAPTER0
VARIABLES1
1)READ(T)               Read number of times of observation, on
                        given day, less one
O=T+1
NEWLINE
NEWLINE
620,27.
620,6.
620,0
NEWLINE
K=0(1)T
I=0(1)2
READ(E1)
REPEAT

```

Print F

Repeat for each time of observation

Read U.T.

```

P)=2)
JUMP I 2
2) VK=£0/24
PRINT (VK) * 3, 8
NEWLINE
FK=£0-0.08 3444444
REPEAT
READ (A)
C7=0
JUMP 20, A=99
JUMP 21
20) C7=1
21) H=1
I=0(I) 2
READ (£0)
READ (£1)
READ (£2)
P)=3)
JUMP I 2
3) E I=£0
REPEAT
I=0(I) 2
£I=E I
REPEAT
Q)=4)
K=0(I) T
JUMP I 3
4) E (K+3)=£5
C4=FK+E (K+3)+12
C(K+5)=15C4
REPEAT
NEWLINE
620, 27 .
620, 8 -
620, 0
NEWLINE
K=0(I) T
PRINT (C(K+5)) 3, 8
SPACE
PRINT (£C(K+5)/180) 3, 8
UK=£C(K+5)/180
NEWLINE
REPEAT
X7(10) U0, 0
NEWLINE
620, 27 .
620, 2 -
620, 0

```

Convert to hours

Fraction of day

Print fraction of day

Local mean time

Read number related to ϕ

Set $C_7 = 0$ if ϕ 's are derivable from H_0 ,
 $= 1$ otherwise

Read equation of time

Convert to hours

Repeat for each time of observation

Interpolate equation of time

 H_0 (hours) H_0 (degrees)

Print H

Repeat for each time of observation

Print hour angle in degrees

Print hour angle in radians

Store H_0 for each time of observation

Print B

```

NEWL INE
READ (E0)
READ (E1)
READ (E2)
Q=6)
K=0(1)T
JUMP I 3
6) PRINT (E5) 3, 8
SPACE
PRINT (EE5/180) 3, 8
UK=EE5/180
NEWL INE
REPEAT
*x7(40)U0,0
NEWL INE
620,27
620,16
620,0
NEWL INE
READ (E0)
READ (E1)
*READ (E2)
Q=25)
K=0(1)T
JUMP I 3
25) PRINT (E5) 3, 8
SPACE
PRINT (EE5/180) 3, 8
UK=EE5/180
NEWL INE
REPEAT
x7(70)U0,0
NEWL INE
620,27
620,19
620,0
NEWL INE
I=0(1)2
READ (E0)
READ (E1)
READ (E2)
P=5)
JUMP I 2
5) F I=E0
REPEAT
I=0(1)2
EI=FI
REPEAT

```

Read B_0

Repeat for each time of observation
Interpolate B_0 at time of observation
Print B_0 (degrees)

Print B_0 (radians)

Store B_0 for each time of observation

Print P

Read P

Repeat for each time of observation
Interpolate P at time of observation
Print P (degrees)

Print P (radians)

Store P for each time of observation

Print S

Read S

Convert to degrees

Q)=7)	
K=0(1)T	Repeat for each time of observation
JUMP 3	Interpolate S at each time of observation
7)PRINT(E5) 3,8	Print S (degrees)
SPACE	
PRINT(E5/180) 3,8	Print S (radians)
UK=E5/180	
NEW LINE	
REPEAT	
X7(100)U0,0	Store S for each time of observation
NEW LINE	
620,27,	
620,12	Print L
620,0	
NEW LINE	
READ(E0)	
READ(E1)	Read L ₀
READ(E2)	
Q)=9)	
K=0(1)T	Repeat for each time of observation
JUMP 3	Interpolate L ₀ at the time of observation
9)PR*INT(E5) 3,8	Print L ₀ (degrees)
SPACE	
PRINT(E5/180) 3,8	Print L ₀ (radians)
UK=E5/180	
NEW LINE	
REPEAT	
X7(130)U0,0	Store L ₀ for each time of observation
NEW LINE	
620,27,	
620,16	Print PHI
620,8	
620,9	
620,0	
NEW LINE	
K=0(1)T	Repeat for each time of observation
PRINT(C(K+5)+A) 3,8	Print ϕ , assuming it is derivable from H ₀
SPACE	
PRINT(C(K+5)E/180+EA/180) 3,8	Print ϕ (radians)
UK=C(K+5)E/180+EA/180	
NEW LINE	
REPEAT	
X7(160)U0,0	Store ϕ for each time of observation
AGROSSI/I	
12)E0=E0+E1/60+E2/3600	Convert hours, minutes and seconds to
JUMP(P)	hours, or degrees, minutes and seconds to
13)E3=E2-E0	degrees

$E_4 = E_2 - 2E_1 + E_0$
 $E_5 = E_1 + VK E_{3/2} + V KV K E_{4/2}$
JUMP(Q)
PSA
CLOSE
→
—

Interpolate to second differences

PART II. REDUCTION OF INTERFEROMETRIC (CIRCULAR CHANNEL) PLATES

A. Introduction.

In order to facilitate the reduction of large numbers of interferometric plates, programme OBS 4 was written. This computes accurate wavelengths, along with their standard deviations, of all the spectral lines on the circular channel plates from data obtained using a special measuring machine (see Chapter 2 for details).

B. Programme OBS 4.

B.1. Input Data.

The input data consist mainly of the measuring machine readings, and in designating these, the terminology set out in Chapter 2 will be followed. In addition, individual readings of arc positions will be specified as follows. The arcs (or interference rings) will be numbered outward from the centre of the plate, from 1 to N, starting from the first measured arc and where the Nth arc is the measured arc farthest from the centre. Thus a_3 refers to the reading for the third arc to the "a" side of the centre in the "a-c" arm of the cadmium cross.

An asterisk will denote the reading for the corresponding arc after the plate has been rotated 180° . Thus a_3^* refers to the reading of the above arc after the plate has been rotated (it matters not which of the two readings of each arc is referred to as the "rotated" reading). The arcs of lines perpendicular to the "a-c" arm will be designated by b_1 and d_1 (including the two pseudo-lines of cadmium). Finally (b_N) will refer to the angular reading obtained when setting on the b_N arc of a given line.

The input data consist of two parts, the first being basic data on the spectral lines whose wavelengths are required, and the second being the observed plate readings. The first part of the input is as follows: an integer s , where s is the number of lines of unknown λ on the "a" side of the "b-d" arm of the cadmium cross; an integer t , where t is the number of lines on the "c" side of the "b-d" arm; approximate wavelengths (in A units) for the $(s+t)$ lines, from the "a" side of the plate to the "c" side; phase change factors, k , for the $(s+t)$ lines in the above order; camera focal length factors, $F = (f_\lambda / f_{Cd})^2$ for the $(s+t)$ lines in the above order; an integer n , which is the first trial integral order of interference for the cadmium line; and finally

a control number. The latter is zero if the best of 100 trial integral orders of interference for the cadmium line (starting at n) is required, otherwise the trial integral order of interference is chosen for which the mean square of the difference between the trial orders of interference and the observed orders of interference (for the unknown lines) is less than the control number. Values must be given for both s and t , even if zero. The phase change factors are chosen so that the true etalon optical thickness for the unknown wavelength, λ , is $2t+k\lambda$, where $2t$ is the optical thickness for the cadmium wavelength.

The second part of the input data consists of the plate readings, which are given in two columns separated by two "SPACE"'s. The data start with the readings for the "a-c" arm of the cadmium cross, the first column giving the readings $a_N, \dots, a_1, c_1, \dots, c_N$. The second column gives the corresponding "rotated" readings, a_N^* etc. The number 999 terminates the "a-c" data. This is followed by the data for the "b-d" arm, the first column giving $b_N, \dots, d_N$, and the second b_N^* etc. Again 999 terminates the data. Following this data for the cadmium cross, the readings for each line of unknown λ and the cadmium pseudo-lines are given. These are

given in the following order: Cadmium "a" pseudo line, the unknown lines from the "a" side to the "c" side, and the Cadmium "c" pseudo-line. If all the unknown lines are confined to the "a" (or "c") side of the plate, only the Cadmium "a" (or "c") pseudo-line data are given. For each of these lines, the first column gives the readings $b_1, d_1, b_2, d_2, \dots, b_N, d_N$, and the second column gives the readings b_1^*, d_1^* etc. in the same order. These are followed by 999 and four numbers in two columns of two. The first column gives $\theta(b_N)$ and $\theta(d_N)$, and the second gives $\theta^*(b_N)$ and $\theta^*(d_N)$.

It must be noted that readings must be given for all the arcs from 1 to N (N may vary from line to line, but must be the same for the "a-c" and "b-d" arms of the cadmium cross). If certain arcs are not measurable, therefore, all the readings for those arcs must be entered as zeros.

Data for other plates may follow on the same data tape. If the first part of the input data is the same as for the preceding plate, the data for the next plate commence with the "a-c" readings. If, on the other hand, the first part of the input data is different from that for the preceding plate, the number 777 must be given

followed by the input data commencing with the integer s.

A sample input data tape is given on the next five pages, referring to the determination of the wavelengths of the seven spectral lines used in Chapter 3, using a 2.5 mm etalon spacer.

TYPICAL INPUT DATA

OBS 4
242

3 4

Number of unknown lines on side a and side c of interferometric plate

6295.96 0
6297.789
6299.228
6301.498
6302.000
6302.490
6302.764

Approximate wavelengths of the seven lines, a to c

0.001 0
0.001 0
0.001 0
0.001 0
0.001 0
0.001 0
0.001 0

Phase change corrections for the seven lines

0.99984 0
0.99984 0
0.99984 0
0.99984 0
0.99984 0
0.99984 0
0.99984 0

Camera focal length factors for the seven lines

8200

First trial integral order of interference for cadmium

0

Error limit (signifies that the best of 100 trial integral orders of interference is to be adopted)

46.367	83.751
47.851	82.253
49.495	80.613
51.332	78.790
53.446	76.662
56.047	74.062
74.076	56.030
76.674	53.447
78.792	51.328
80.624	49.497
82.261	47.862
83.753	46.372
999	

Data for a-c arm of cadmium cross

46.364	83.756
47.857	82.253
49.499	80.617
51.332	78.787
53.452	76.669
56.048	74.062
74.073	56.049
76.679	53.452
78.784	51.336
80.620	49.509
82.252	47.874
83.750	46.379
999	

Data for b-d arm of cadmium cross

47.856	82.264
47.848	82.249
46.363	83.755
46.368	83.744
44.982	85.141
44.984	85.130
43.688	86.442
43.692	86.430
999	

Data for cadmium A pseudo-line

34.7	134.6
226.7	46.7

52.391	77.730
52.394	77.717

Beginning of data for first unknown line

50.46 3	79.66 0
50.46 3	79.65 0
48.76 5	81.35 *9
48.76 8	81.35 2
47.23 3	82.89 2
47.23 7	82.88 5
45.81 5	84.31 1
45.82 3	84.30 5
999	

Data for first unknown line (terrestrial oxygen line in this case)

327.8	147.8
214.0	34.0

55.86 5	74.27 4
55.87 8	74.26 8
53.37 0	76.76 7
53.36 7	76.77 5
51.31 5	78.83 3
51.31 1 *	78.82 3
49.52 1	80.63 3
49.51 8	80.66 1 5
47.91 9	82.23 8
47.92 0	82.21 8
999	

Data for second unknown line (solar iron line)

339.4	159.2
202.4	22.5

55.65 2	74.48 0
55.64 3	74.47 1
53.18 3	76.94 3
53.18 9	76.93 4
51.15 0	78.97 7
51.15 7	78.97 0
49.38 2	80.75 0
49.39 0	80.73 9
47.78 4	82.35 2
47.79 1	82.34 0
999	

Data for third unknown line (terrestrial oxygen line)

351.1	171.0
190.7	10.8

74.376	55.743
74.387	55.768
76.877	53.260
76.842	53.299
78.926	51.215
78.905	51.218
80.715	49.438
80.684	49.456
82.299	47.838
82.283	47.858
999	

Data for the fourth unknown line (solar iron line)

188.7	8.8
33.1	173.2

75.263	54.834
75.280	54.848
77.578	52.541
77.581	52.549
79.529	50.597
79.529	50.596
81.240	48.889
81.238	48.894
82.788	47.345
82.784	47.351
999	

Data for the fifth unknown line (terrestrial oxygen line)

192.2	12.2
349.4	169.6

76.118	54.007
76.140	53.995
78.278	51.841
78.281	51.857
80.140	50.006
80.123	50.005
81.795	48.348
81.784	48.349
83.284	46.845
83.286	46.846
999	

Data for the sixth unknown line (solar iron line)

195.6	15.9
346.1	166.1

75.217	54.905
75.225	54.902
77.5237	52.583
77.549	52.587
79.496	50.634
79.490	50.643
81.208	48.921
81.208	48.930
82.757	47.371
82.756	47.378
999	

Data for the seventh unknown line
(terrestrial oxygen line)

198.2	18.1
343.6	163.6

80.621	49.494
80.625	49.498
82.263	47.844
82.260	47.861
83.756	46.368
83.750	46.375
85.143	44.982
85.134	44.991
999	

Data for the cadmium C pseudo-line

228.1	48.1
313.1	133.1

B.2. Computer Output.

Certain precautions have been written into the programme and it is possible that the computations will halt on an "END" instruction before all the output has been given. In such a case, a characteristic number terminates the output. These characteristic numbers and their interpretations are as follows. A 999 signifies that the number of arc readings given for the "a-c" and "b-d" arms were not equal. A 998 signifies either a machine fault in the computation of the fractional order of interference for the cadmium cross or extremely bad data (this should never occur). A 997 can only arise when the control number in the first part of the input is not zero, and it signifies that none of the 100 trial integral orders of interference gave a mean square difference between the computed and observed orders that was less than the control number. In such a case, the machine does not stop but reads data for the next plate.

Aside from these special cases, the output consists of the following numbers. After "DIFF", the value adopted for $4x_1$ is given, followed by the maximum absolute

values of $2S_a - 2S_c$ and $2S_b - 2S_d$. These are given in the same units as the arc readings in the input. The number after "DD" is the value of ΔD^{*2} for the cadmium cross (in the above units squared). The next group of quantities designated by "FO" are fractional orders of interference along with their standard deviations. The first of these, specified by "C" refers to the cadmium cross; the next, "CB" refers to only the "b-d" arm of the cross; "CA" refers to the Cadmium "a" pseudo-line; the next group of fractional orders, which are undesignated, refer to the unknown lines, from "a" to "c"; and finally "CC" refers to the Cadmium "c" pseudo-line. The number following "IO" gives the mean square difference between the computed (approximate) orders of interference and the observed orders of interference, and it is followed by the adopted integral order of interference for the cadmium line. (If the fractional orders of interference observed for the cadmium pattern are ~ 0.9999 or 0.0001 , this may be one less than the true value used in the computations.) Finally the accurate wavelengths of the unknown lines are given, from "a" to "c" (in A units) along with their standard deviations. Although given to five decimal places, the computed wavelengths are only accurate to

four places. On the next page, the output corresponding to the sample input is given as an example.

TYPICAL OUTPUT

OBS 4

250

CIRCULAR CHANNEL REDUCTION - HIGGS - 12 APRIL 61

01 : 271 :
02 037 :
03 035 :
00 061 :

DIFF -0.008500 0.0310 0.0210

ID 214.20554

FO C 0.51657, 0.00018

FO CB 0.51563 0.00023

FO CA 0.51877, 0.00093

FO 0.06533 0.00083

FO 0.61407, 0.00068

FO 0.69153 0.00076

FO 0.65446 0.00108

FO 0.98775 0.00113

FO 0.32884 0.00157

FO 0.96702 0.00083

FO CC 0.51709 0.00143

IO 0.0000058 8241.0

6295.96064 0.00082

6297.79173 0.00062

6299.22850 0.00061

6301.50016 0.00084

6301.99925 0.00090

6302.49250 0.00124

6302.76344 0.00076

Fractional orders of interference
for the seven unknown lines, followed
by their standard deviations

Accurate wavelengths for the seven
lines, followed by their standard
deviations

B.3. Programme and Operating Instructions.

The programme, along with explanatory notes is given on the next fourteen pages. The operating instructions are as follows:

1. Enter programme tape in reader.
2. Set hand-switch 4, others clear (unless ? is required).
3. ITB
4. On reading $\rightarrow$, remove programme tape.
5. Enter data tape in reader.
6. Tap hand-switch 9.
7. For extra data tapes, same as instructions 5 and 6.

TITLE
CIRCULAR CHANNEL REDUCTION - HIGGS - 12 APRIL 61

CHAPTER I

A→20

B→20

C→20

D→40

E→40

F→20

G→20

H→20

U→40

V→10

W→20

X→20

Y→20

Z→20

Z→100

I) $\epsilon_5 = 0$

J = 0(I)I

K = Q* - J - I

R = I - J

FP = E J K

ZP = F P

JUMP 20, 00.01 > Z P

FP = FP - 0.00000084868 F P P

FP = FP / H

GP = XFRPT (FP)

$\epsilon_5 = \epsilon_5 + G P$

20) REPEAT

$\epsilon_5 = \epsilon_5 / E$

DOWN 7 / 3

F = ϵ_5

620, 27

620, 6

620, 15

620, 0

SPACE

620, 27

620, 3

620, 2

620, 0

Repeat for each ring of cadmium pattern

$$28b.28d = D_n^2$$

Omit rings which were not measured

$$D_n^2$$

$$\epsilon_n$$

$$\bar{\epsilon}_n = \epsilon$$

Check that no special values of ϵ_n have entered into the derivation of ϵ , i.e. values such as 0.9999 and 0.0001

Print FO CB

```

PRINT(F) I, 5
G=0
J=0(I) I
JUMP I I, 0. 0I > Z J
FJ=XMOD(GJ-F)
G=G+FJFJ
II) REPEAT
G=G/E
E=E-I
G=G/E
G=X SQRT(G)
PRINT(G) I, 5
V4=G
JUMP 2I, I > F
F=F-I
2I) NEWL JNE
NEWL JNE
2) P=0
3) READ(E0)
JUMP 4, E0=999
READ(EI)
UP=XMOD(EI-E0)
P=P+I
JUMP 3
4) J=0(I) 3
READ(VJ)
REPEAT
5) E0=XMOD(V0-V2)
EI=XMOD(VI-V3)
JUMP 6, E0 > 180
JUMP 7
6) E0=360-E0
7) JUMP 8, EI > 180
JUMP 9
8) EI=360-EI
9) E0=E0/4+EI/4
E0=E0/180
E0=X COS(E0)
EI=U(P-I)/2+U(P-2)/2
E3=EIE0
X=E3
GOV P/2-I
I=XINTPT(G0)
DI=I
J=0(I) I
K=2J
ZJ=UK

```

Print ϵ

Repeat for each ring
Omit rings not measured
 $|\epsilon_n - \epsilon|$

Compute standard deviation of ϵ

Print standard deviation of ϵ

Retain the fractional part of ϵ (ϵ
may be a number such as 1.0001)

CADMIUM A PSEUDO-LINE

Read R, starting at the b arc farthest from the pattern centre

Read R' for the same arc after 180° rotation 2S₁ or 2S₂ depending on whether a b arc or d arc is measured

Read the four angles given for the extreme b and d arcs

Mean angle θ in degrees
 θ in radians

S₁ + S₂ for the largest measured ring (-2x)

Number of measured rings less one

Repeat for each ring

JUMP12, 00. 01 > UK
 GJ=UKU(K+1)-E3D
 GJ=GJ-00. 00000084868GJGJ

Omit rings not measured

$$2S_1, 2S_2 + 2x_1x_2 = D_n^2$$

JUMP13

12) GJV 0

Dn=Dn-1

13) REPEAT

E5=0

J=0(1) 1

JUMP22, 00. 01 > ZJ

GJ=XFRPT(GJ/H)

E5=E5+GJ

22) REPEAT

E=Dn+1

E5=E5/E

DOWN 7/3

NEWLINE

620, 27

620, 6

620, 15

620, 0

SPACE

620, 27

620, 3

620, 1

620, 0

PRINT(E5) 1, 5

G=0

J=0(1) 1

JUMP14, 00. 01 > ZJ

GJ=XMOD(E5-GJ)

G=G+GJGJ

14) REPEAT

G=G/E

G=G/Dn

G=XSQRT(G)

PRINT(G) 1, 5

V5=G

G=E5

JUMP23, 1 > G

G=G-1

23) NEWLINE

NEWLINE

ACROSS 1/2

PSA

XSQRT

XCOS

CLOSE

Set $D_n^{*2} = 0$ for rings not measured

Repeat for each ring

Omit rings not measured

ϵ_n

ϵ
 Check that no special values of ϵ_n have entered into the calculations

Print FO CA

Print ϵ

Repeat for each ring

Omit rings not measured

$|\epsilon - \epsilon_n|$

Compute standard deviation of ϵ

Print standard deviation of ϵ

Retain fractional part of ϵ

CHAPTER 2
VARIABLES

```

1) N = 0
L = R + 1
2) P = 0
3) JUMP 12, N > L
READ (E0)
JUMP 4, E0 = 999
READ (E1)
UP = XMOD(E1 - E0)
P = P + 1
JUMP 3
4) J = 0(I) 3
READ (VJ)
REPEAT
5) E0 = XMOD(V0 - V2)
E1 = XMOD(V1 - V3)
JUMP 6, E0 > 180
JUMP 7
6) E0 = 360 - E0
7) JUMP 8, E1 > 180
JUMP 9
8) E1 = 360 - E1
9) E0 = E0 / 4 + E1 / 4
E0 = E0 / 180
E0 = XCOS(E0)
E1 = U(P-1) / 2 + U(P-2) / 2
E3 = E1 E0
?YN = E3
K = S - 1
JUMP 10, N > K
JUMP 11
10) E3 = -E3
11) GOV P / 2 - 1
0) = 38
41) I = XINTPT(G0)
Dn = I
J = 0(I) I
K = 2J
ZJ = UK
JUMP 35, 0.01 > UK
GJ = UKU(K+1) - E3 D
GJ = GJ - 0.00000084868GJGJ
JUMP 36
35) GJ = 0
Dn = Dn - 1
36) REPEAT
JUMP(0)

```

Total number of wavelengths to be determined

LINES OF UNKNOWN WAVELENGTH

Read R, R' and compute $2S_1$, $2S_2$ etc. as above for the cadmium A pseudo-line

Change the sign of $2x$ if the line concerned has an $x > 0$ (i.e. it is on the c side of the centre)

Compute D_n^{*2} as above for the cadmium A pseudo-line

Control to allow some of above instructions to be used for the cadmium C pseudo-line

```

38) ?E4=CNANH/C
E5=0
J=0(1)1
JUMP5 0, 00 01 > ZJ
GJ=XFRPT(GJ/E4)
E5=E5+GJ
50) REPEAT
E=Dn+1
E5=E5/E
DOWN7/3
620, 27
620, 6
620, 15
620, 0
PRINT(E5)1;5
V=0
J=0(1)1
JUMP37, 00 01 > ZJ
GJ=XMOD(E5-GJ)
V=V+GJGJ
37) REPEAT
V=V/E
V=V/Dn
V=XSQRT(V)
PRINT(V)1;5
HN=V
?XN=E5
ANXINTPT(XN)
JUMP30, AN=0
XN=XN-1
30) NEWLINE
NEWLINE
N=N+1
JUMP2
12) P=0
JUMP20, T=0
13) READ(E0)
JUMP14, E0=999
READ(E1)
UP=XMOD(E1-E0)
P=P+1
JUMP13
14) J=0(1)3
READ(VJ)
REPEAT
15) E0=XMOD(V0-V2)
E1=XMOD(V1-V3)

```

$$\Delta D^{*2} = (\Delta D^{*2})_{Cd} \cdot \lambda / \lambda_{Cd} \cdot \text{focal length factor}$$

Compute ϵ_n for each ring

ϵ

Print FO

Print ϵ

Compute standard deviation of ϵ

Print standard deviation of ϵ

Store fractional part of ϵ

Repeat for next measured spectral line

Omit the following if there are no lines
on the c half of the plate

CADMIUM C PSEUDO-LINE

Computation as above for lines of unknown
wavelengths

```

JUMP16, E0 > 180
JUMP17
16) E0 = 360 - E0
17) JUMP18, E1 > 180
JUMP19
18) E1 = 360 - E1
19) E0 = E0 / 4 + E1 / 4
E0 = E0 / 180
E0 = X COS(E0)
E1 = U(P-1) / 2 + U(P-2) / 2
E3 = E1 E0
Z = E3
E3 = -E3
GO TO P / 2 - 1
0) = 39
JUMP41
39) E5 = 0
J = 0(1) 1
JUMP52, 00 01 > Z J
GJ = XFRPT(GJ/H)
E5 = E5 + GJ
52) REPEAT
E = Dn + 1
E5 = E5 / E
DOWN 7 / 3
620, 27
620, 6
620, 15
620, 0
SPACE
620, 27
620, 3
620, 3
620, 0
PRINT(E5) 1, 5
V = 0
J = 0(1) 1
JUMP40, 00 01 > Z J
GJ = XMOD(E5 - GJ)
V = V + GJ
40) REPEAT
V = V / E
V = V / Dn
V = X SQRT(V)
PRINT(V) 1, 5
V6 = V
Y = E5

```

Jump to previous routine to calculate D_n^{*2}

Compute $\bar{\epsilon}_n = \epsilon$

Print FO CC

Print ϵ

Compute standard deviation of ϵ

Print standard deviation of ϵ

```

JUMP 5 3, I > Y
Y = Y - I
53) NEWLINE
NEWLINE
DOWN 5 0 / 3
20) K = S - I
J = 0(1) K
E J = F + G Y J / X - F Y J / X
REPEAT
JUMP 21, T = 0
J = S(1) R
E J = F + Y Y J / Z - F Y J / Z
REPEAT
21) NEWLINE
ACROSS 1 / 3
PSA
XSQRT
XCOS
CLOSE

```

Retain fractional part of ϵ

Check that all cadmium ϵ 's are comparable,
e.g. no 0.9999's and 0.0001's
Repeat for each line with $x < 0$
Estimated ϵ_{cd} at the position of the line

Repeat for each line with $x > 0$
Estimated ϵ_{cd} at the position of the line

```

CHAPTER 3
VARIABLES
1) JUMP 17, B = 0
Q = 0(1) 100
E = 0
D = A + W + Q
J = 0(1) R
E J = CD 0 / A J
F J = XFRPT(E J)
F = XMOD(F J - X J + B J)
F J = F
F = 1 - F
JUMP 15, F > F J
E J = E J + 1
F J = F
15) E = E + F J F J
REPEAT
E = E / L
JUMP 2, B > E
REPEAT
2) JUMP 3, 100 > Q
U0 = 997
PRINT(U0) 3, 1
JUMP 6

```

Omit the following if the error limit = 0
Repeat for 100 trial integral orders

Trial order of interference for cadmium
Repeat for each line
Order of interference for wavelength of line
Trial ϵ for line
Difference between (observed ϵ - phase change
correction) and trial ϵ

Select best fit of ϵ and $1 + \epsilon$

Mean square of the difference
Stop when mean square is less than the
preset error limit

If no integral order for cadmium is found
that satisfies this requirement, print 997
and read new data

3) J=0(1) R
E J=XINTPT(E J)
REPEAT

620,27

620,9

620,15

620,0

PRINT(E)1,7

PRINT(D0-W)6,1

NEWLINE

4)K=S-1

J=0(1) K

W9=X SQRT(V4V4+V5V5)

W9=Y JV9/X

WJ=X SQRT(V4V4+V9V9)

REPEAT

JUMP5, T=0

J=S(1) R

W9=X SQRT(V4V4+V6V6)

W9=Y JV9/Z

WJ=X SQRT(V4V4+V9V9)

REPEAT

5)NEWLINE

J=0(1) R

EV D0C+E JC-WC

FVEJ+XJ-BJ

WJVE/F

PRINT(GJ)4,5

W7=X DIVIDE(WJ, D0+E J-W)

W8=X DIVIDE(HJ, EJ+XJ-BJ)

W9=X SQRT(V7V7+V8V8)

W9=G JV9

PRINT(V9)1,5

NEWLINE

REPEAT

NEWLINE

NEWLINE

JUMP6

7)J=0(1) I

JUMP10, 00 01 > ZJ

CHECK(E5, GJ, 00 3, 10)

JUMP9

10)REPEAT

UP

9)J=0(1) I

JUMP8, 00 01 > ZJ

Hn=XINTPT(GJ+00 5)

Repeat for each line
Adopted integral order at wavelength of line

Print IO

Print mean square of difference
Print cadmium integral order (this may be one less than the true value)

Repeat for lines with $x < 0$

Mean error (standard deviation) in cadmium fractional order at line position

Repeat for lines with $x > 0$

Mean error of cadmium fractional order of interference at line position

Repeat for each line
Cadmium order of interference multiplied by λ_{Cd}
Order of interference at line wavelength
Wavelength of line
Print wavelength of line

Compute mean error (standard deviation) of each wavelength
Print mean error of wavelength

Repeat for each ring of interference pattern
Omit rings not measured
Check that ϵ_n and ϵ agree within 0.3

If not, increase the low ϵ_n 's by 1 and compute a new ϵ

```

JUMP8, Hn > 0.9
GJ = G J + 1
8) REPEAT
E5 = 0
J = 0(1) I
E5 = E5 + G J
REPEAT
E5 = E5 / E
UP
12) J = 0(1) I
JUMP13, HJ = 666
GJ = XINTPT(HJ + 0.5)
JUMP13, GJ > 0.9
HJ = HJ + 1
13) REPEAT
G = 0
J = 0(1) I
JUMP35, HJ = 666
G = G + HJ
35) REPEAT
G = G / E
Gn = XFRPT(G)
UP
17) Q = 0(1) 100
ZQ = 0
E = 0
Do = A + W + Q
J = 0(1) R
EJ = CD0 / AJ
FJ = XFRPT(EJ)
F = XMOD(FJ - XJ + BJ)
*FJ = F
F = 1 - F
JUMP18, F > FJ
*FJ = F
18) ZQ = ZQ + FJFJ
REPEAT
REPEAT
K = XMIN(Z0, 0, 100)
620, 27
620, 9
620, 15
620, 0
PRINT(ZK/L) 1, 7
PRINT(A+K) 6, 1
NEWLINE

```

Repeat for each ring

As in routine 7), if some ε_n 's have special values, modify low values and compute a new ε

Repeat for 100 trial integral orders of interference for cadmium

Trial cadmium order of interference
Repeat for each line

As in routine 1), compute mean square difference between the trial ε 's and observed ε 's, averaged over all the lines

Choose the integral order giving the smallest mean square difference

Print IO

Print the minimum mean square difference
Print cadmium integral order of interference
(This value may be one less than actual value used in the calculations)

```

J=0(I)R
EJ=CA/AJ+WC/AJ+KC/AJ
F=XFRPT(EJ)
Bn=F
F=XMOD(F-XJ+BJ)
G=I-F
JUMP 9, G>F
JUMP 31, Bn>XJ
EJVE J-I
JUMP 19
31) EJ=EJ+I
?19) EJ=XINTPT(*EJ)
REPEAT
Do=A*W+K
JUMP 4
6) ACROSS 2/0
50) CHECK(G,F,0.2,54)
JUMP 55, G>F
G=G+I
JUMP 54
55) F=F+I
54) CHECK(F,Y,0.2,56)
Y=Y+I
56) CHECK(W,F,0.2,57)
W=W+I
57) UP
PSA
XSQRT
CLOSE

```

Repeat for each line
Approximate order of interference at line λ
Approximate ϵ for line

Integral order of interference adopted for each line
Adopted order of interference for cadmium

Check that all cadmium ϵ 's are comparable (cadmium cross, b-d arm of cross, pseudo-line a, and pseudo-line c)

CHAPTER 0

```

A→20
B→20
C→20
D→40
E→40
G→20
H→20
U→2
E→20
1) READ(S)
READ(T)
C=6438.4696
R=S+T-I

```

Read number of lines with $x < 0$ (on side a)
Read number of lines with $x > 0$ (on side c)
Standard cadmium wavelength

I=0(I) R	Read approximate wavelengths for each line
READ(AI)	
REPEAT	
I=0(I) R	Read phase change corrections for each line
READ(BI)	
REPEAT	
I=0(I) R	Read camera focal length factors for each line
READ(*CI)	
REPEAT	
READ(A)	Read approximate integral order of interference for cadmium, and the preset error limit
READ(B)	
NEWLINE	
2) Q=0	CADMIUM CROSS
NEWLINE	
3) READ(E0)	Read R, starting with the arc of the a-c arm having the most negative x
JUMP4, E0=999	
JUMP1, E0=777	
READ(E1)	Read R' for the same arc after 180° rotation
DQ=XMOD(E0-E1)	2S _a or 2S _b depending on arc
Q=Q+1	Count the total number of arcs
JUMP3	
4) P=0	
5) READ(E0)	As above, but for the arcs of the b-d arm of the cadmium cross
JUMP6, E0=999	
READ(E1)	
EP=XMOD(E0-E1)	2S _b or 2S _d depending on arc
P=P+1	Count total number of arcs
JUMP5	
6) U=Q-I	Total number of arcs in the a-c arm less one
UI=U0/2-0.45	Corresponding number of rings less one
I=XINTPT(UI)	
E=I+1	Number of rings
J=0(I) I	Repeat for each ring
K=Q-J-I	
JUMP**14, 0.01>DJ	Omit rings which were not measured
JUMP15	
14) E=E-I	
EJ=0	
JUMP16	
15) EJ=DJ-DK	2S _a -2S _b
16) REPEAT	
*D=0	
J=0(I) I	Average 2S _a -2S _b
D=D+EJ	
REPEAT	
D=D/E	

620, 27	
620, 4	
620, 9	
620, 6	Print DIFF
620, 6	
620, 0	
SPACE	
SPACE	
PRINT(D) I, 6	Print mean value of $2S_a - 2S_c = 4x$,
J=0(I) I	
EJ=XMOD(E J)	
REPEAT	
SPACE	
K=XMAX*(E 0, 0, I)	Select largest difference ($2S_a - 2S_c$)
PRINT(E K) I, 4	Print this value
U20 P-I	
CHECK(U2, U0, 0. 01, 7)	Check that the same number of arcs were
U0=999	measured in each arm of the cross. If not,
PRINT(U0) 3, I	print 999 and stop
END	
7) J=0(I) I	Repeat for each ring
K=Q-J-I	
EJ=E J-E K	$2S_b - 2S_d$
REPEAT	
J=0(I) I	
EJ=XMOD(E J)	
REPEAT	
K=XMAX*(E 0, 0, I)	Select largest difference ($2S_b - 2S_d$)
PRINT(E K) I, 4	Print this value
NEWLINE	
NEWLINE	
E=I+I	Number of rings in cadmium pattern
F=0	
G=0	
L=0	
M=0	
J=0(I) I	Repeat for each ring
K=Q-J-I	
P=I-J	
9) JUMP I 0, 0. 01 > DJ	Omit rings not measured
JUMP I I	
10) E=E-I	
GP=0	
JUMP I 2	
11) GP=DJDK/2+E J E K/2	D_n^2
GP=GP-0. 00000084868G RG P	D_n^{*2}
F=F+GP	

G=G+PGP	
L=L+PP	
M=M+P	
12) REPEAT	Least square solution (linear) for ΔD^{*2}
H=XDIVIDE(EG-MF,LE-MM)	and D_1^{*2}
E1=XDIVIDE(LF-MG,LE-MM)	$\frac{\Delta D^{*2}}{D_1^{*2}}$
F=XFRPT(E1/H)	
620,27	
620,4	Print DD
620,4	
620,0	
PRINT(H)4,5	Print ΔD^{*2}
NEWLINE	
NEWLINE	
620,27	
620,6	
620,15	
620,0	Print FO C
SPACE	
620,27	
620,3	
620,0	
SPACE	
PRINT(F)1,5	Print ϵ
W=F	
G=0	
J=0(1)1	Repeat for each ring
JUMP18,G,J=0	$D_n^{*2} / \Delta D^{*2}$
HJ=GJ/H	ϵ_n
HJ=XFRPT(HJ)	
G=G+HJ	
JUMP19	
18) HJ=666	
19) REPEAT	
G=G/E	$\bar{\epsilon}_n$
CHECK(G,F,0.0001,8)	Check that $\bar{\epsilon}_n = \epsilon$. If not, modify ϵ_n 's
DOWN12/3	assuming that some are slightly > 0 and
CHECK(G,F,0.0001,8)	others slightly < 1. If test fails, as
U0=998	it can only do in the case of very bad
PRINT(U0)3,1	data or a machine fault, print 998 and stop
END	
8) U=0	
J=0(1)1	Repeat for each ring
JUMP20,HJ=666	$ \epsilon_n - \bar{\epsilon}_n $
HJ=XMOD(HJ-G)	
U=U+HJHJ	
20) REPEAT	

U=U/E
F=E-I
U=U/F
U=X SQRT(U)
PRINT(U) I,5
NEWLINE
NEWLINE
ACROSS I/I
PSA
XSQRT
CLOSE

Compute standard deviation of ε

Print standard deviation of ε

PART III. SOLUTION OF THE FREDHOLM INTEGRAL EQUATION

A. Introduction.

In order to solve numerically the Fredholm integral equation arising in line profile studies due to the apparatus function (see Chapter 2), several programmes were written. Aside from the Fourier transform programme mentioned in Chapter 2, the most successful was the iteration programme, OBS 11 - E, based on the method of Burger and van Cittert. Two other programmes were also developed in this connection, however, one of which, OBS 11 - C, utilizes the Gauss-Seidel iterative method for the solution of the integral equation. Finally, a programme, OBS 11 - D, was written for the inverse operation. That is, knowing the true line profile and apparatus function, an "observed" profile is computed. In the following, the Burger-van Cittert iteration programme will be given in detail, and short instructions for the use of OBS 11 - C, and 11 - D will also be presented.

B. Programme OBS 11 - E.**B.1. Input Data.**

Since this programme is intended to be applied to line profile studies (although it is not restricted to such use), corresponding terminology will be adopted. The observed profile will be assumed to be tabulated from "violet" to "red", i.e., in the direction of increasing wavelength. If this is the case, the apparatus function must be tabulated from red to violet, opposite to the observed profile. Both of these functions must be tabulated at equal intervals, Δx , and the apparatus function need not be normalized. The observed profile should be of emission type, i.e., the tabulated values should approach zero in the wings (it is permissible for the wings to approach a non-zero value, A, if a large number of A's are tabulated at either end of the observed curve. The number of points so added should be comparable to the number of points in the apparatus function). The zero reference of the apparatus function is assumed to correspond to its maximum value.

The input commences with an integer specifying the

number of points at which the apparatus function is tabulated (≤ 180), followed by the values of the apparatus function at these points, tabulated from red to violet. Next is an integer giving the number of points at which the observed curve is tabulated (≤ 140). Following this is a number specifying the accuracy desired in the iteration, and should be comparable to the standard deviation of a single point of the observed curve. This controls the termination of the iteration as follows: If the n^{th} approximation to the true profile, when folded (broadened) by the apparatus function, differs from the observed profile by a mean difference $\bar{E}$, and a maximum difference E_{max} , the iteration ceases when either $\bar{E}$ is less than the above number or E_{max} is less than five times the above number. Finally, the observed profile is given, tabulated from violet to red, and is followed by another control number. The value of this latter number depends on what is to be taken as the first approximation to the true profile. If it is to be the observed profile, this number is zero; if it is to be the observed profile less a function of its second derivative (see Chapter 2 for details), this number must be the half width of the apparatus function

in units of $\Delta\lambda$. Several data tapes may be entered, one after the other.

A sample data tape is given on the next six pages. The apparatus function is that derived in Chapter 4 for the Babcock grating ($\Delta\lambda = 5 \text{ m\AA}$), and the observed profile is that of a typical iron line at 6300 Å.

TYPICAL INPUT DATA

OBS 11 - E
270

98

Number of tabulated values of apparatus
function

0.00000
0.00050
0.00100
0.00150
0.00200
0.00263
0.00264
0.00259
0.00260
0.00271
0.00292

Apparatus function, tabulated from red to
violet. This is the apparatus function
for the Babcock grating and spectrograph
as determined in Chapter 4. The interval,
 Δx , used here is 5 mÅ.

0.00318
0.00338
0.00354
0.00373
0.00396
0.00410
0.00396
0.00392
0.00415
0.00451

0.00507
0.00589
0.00673
0.00700
0.00750
0.00800
0.00870
0.00940
0.0100
0.0106

0.01113
0.0123
0.0141
0.0155
0.0179
0.0206
0.0229
0.0274
0.0333
0.0401

0.0553
0.0728
0.114
0.188
0.317
0.471
0.657
0.812
0.895
0.836

0.665
0.475
0.337
0.245
0.179
0.132
0.0979
0.0743
0.0593
0.0476

0.0380
0.0318
0.0273
0.0239
0.0205
0.0180
0.0152
0.0128
0.0110
0.00985

0.00890
0.00790
0.00710
0.00670
0.00647
0.00610
0.00573
0.00536
0.00500
0.00470

0.00448
0.00426
0.00403
0.00378

0.00357 .
0.00341
0.00325
0.00301
0.00273
0.00248

0.00229
0.00210
0.00195
0.00150
0.00100*
0.000*50
0.00000

134

0.0062

Number of tabulated values of observed
profile
Preset error limit, ϵ

0.0007 .
0.0009
0.0011
0.0014
0.0017 .
0.0020
0.0024
0.0028
0.0032
0.0036

Observed profile, tabulated from violet
to red. This is the profile of an iron
line at 6500 A and it is tabulated at
5 mA intervals.

0.0040
0.0045
0.0050
0.0055
0.0061
0.0067 .
0.0073

0.0075
0.0082
0.0089

0.0098
0.0107
0.0116
0.0140
0.0150
0.0158
0.0166
0.0173
0.0173
0.0179

0.0185
0.0190
0.0196
0.0202
0.0188
0.0199
0.0214
0.0232
0.0250
0.0276

0.0307
0.0349
0.0386
0.0424
0.0492
0.0533
0.0611
0.0748
0.0904
0.1079
0.1269

0.1488
0.1758
0.1996
0.2251
0.2484
0.2703
0.2914
0.3122
0.3317

0. 34 99
0. 36 70
0. 38 25
0. 39 84
0. 40 97
0. 41 81
0. 42 39
0. 42 82
0. 43 01
0. 42 96
0. 42 65
0. 42 17

0. 41 40
0. 40 61
0. 39 71
0. 38 63
0. 37 49
0. 36 07
0. 34 45
0. 32 59

0. 30 57
0. 28 24
0. 25 97
0. 23 35
0. 21 13
0. 19 14
0. 17 09
0. 15 21
0. 13 26
0. 11 28

0. 09 32
0. 07 92
0. 06 70
0. 05 90
0. 05 07
0. 04 45
0. 04 13
0. 03 84
0. 03 70
0. 03 83
0. 03 81

0. 03 89
0. 03 92
0. 03 95
0. 03 97

0.04 00
0.04 04
0.04 06
0.04 07
0.04 08
0.04 09
0.04 09
0.04 03
0.04 02
- - - -
0.04 02
0.04 01
0.04 00
0.03 99
0.04 23
0.04 18
0.04 11
0.04 00
- - - -
0.03 87
0.02 99
0.02 71
0.02 44
0.02 18
0.01 93
0.01 70
0.01 48
0.01 27
0.01 07
0.00 89
0.00 72
- - - -

6.32

Half-intensity width of the apparatus
function in units of 5 mÅ

B.2. Computer Output.

The output can take two forms, depending upon the way that the iteration terminates. If the iteration terminates due to an approximate solution fitting the observed profile, when folded, within the preset accuracy, the output is as follows. After "ITER." is given the number of the iteration at which a satisfactory solution was obtained. The number after "ERR." gives the E for this iteration, and the number after "M" gives the corresponding $E_{\max}$. The true profile thus obtained is then tabulated below "FINAL CURVE". The first column gives the abscissa (in units of Δx) corresponding to the points at which the observed curve was given (0 refers to the first point of the observed curve). The second column gives the corresponding values of the true profile (tabulated from violet to red).

If the iteration is stopped manually (see B.3) or stops because $\bar{E}_n > \bar{E}_{n-1}$ (which can only arise through rounding errors), then the output takes another form. After "BEST" is given $\bar{E}_{n-1}$ and $\bar{E}_n$, where n is the number of the last iteration completed. Then n is given in parentheses, followed by "M" and $E_{\max}$ for the last iteration. The approximate solution obtained from the last iteration is then tabulated as above.

The output resulting from the sample data tape given above is presented on the next three pages. The iteration in this case has halted since $E_{\max}$ is less than five times the preset accuracy. (The true profile derived here is rough because the second derivative of the observed curve was not at all smooth. It would thus have been better to set the last control number in the input equal to zero).

TYPICAL OUTPUT

OBS 11 - E
278

BURG. VAN CITT. ITERATION - HIGGS - OBS/11E

01 223
00 631

ITER. 1 : ERR. 1 0.0091 : M 0.0291 : Number of completed
iterations, mean and maximum
deviations

FINAL CURVE

0	0.0007
1	0.0016
2	0.0008
3	0.0013
4	0.0016
5	0.0018
6	0.0023
7	0.0028
8	0.0032
9	0.0035
10	0.0038
11	0.0044
12	0.0049
13	0.0053
14	0.0059
15	0.0074
16	0.0074
17	0.0072
18	0.0072
19	0.0087
20	0.0100
21	0.0085
22	0.0106
23	0.0148
24	0.0175
25	0.0155
26	0.0178
27	0.0179
28	0.0171
29	0.0171
30	0.0187
31	0.0183
32	0.0227
33	0.0208
34	0.0165

The computed true profile, tabulated
from violet to red at 5 mμ intervals

35	0. 01 51
36	0. 02 14
37	0. 02 21
38	0. 02 34
39	0. 02 42
40	0. 02 91
41	0. 03 54
42	0. 03 48
43	0. 04 14
44	0. 04 75
45	0. 04 44
46	0. 04 36
47	0. 06 35
48	0. 08 56
49	0. 1 01 5
50	0. 1 1 33
51	0. 1 4 12
52	0. 1 7 32
53	0. 2 0 44
54	0. 2 2 7 3
55	0. 2 5 5 0
56	0. 2 7 2 7
57	0. 2 9 3 7
58	0. 3 1 5 9
59	0. 3 3 6 4
60	0. 3 5 5 1
61	0. 3 6 8 8
62	0. 3 8 8 2
63	0. 4 0 8 2
64	0. 4 2 3 3
65	0. 4 2 5 9
66	0. 4 3 1 1
67	0. 4 3 5 6
68	0. 4 3 9 5
69	0. 4 3 7 6
70	0. 4 3 5 5
71	0. 4 2 8 0
72	0. 4 1 8 2
73	0. 4 0 9 4
74	0. 4 0 1 4
75	0. 3 9 2 1
76	0. 3 8 1 7
77	0. 3 6 9 9
78	0. 3 5 1 3
79	0. 3 3 5 1
80	0. 3 1 0 5
81	0. 2 9 0 2
82	0. 2 5 9 9

83	0. 2280
84	0. 2025
85	0. 1885
86	0. 17 01 :
87 ,	0. 1515
88	0. 1 360
89	0. 1 06 2
90	0. 0821 :
91	0. 06 39
92	0. 06 02
93	0. 05 32
94	0. 0451 :
95	0. 0367 ,
96	0. 0367 ,
97 ,	0. 0325
98	0. 0327 ,
99	0. 0367 ,
100	0. 0397 ,
101	0. 0380
102	0. 0403
103	0. 0394
104	0. 0395
105	0. 0399
106	0. 04 08
107 ,	0. 04 11
108	0. 04 07 ,
109	0. 04 07 ,
110	0. 04 20
111	0. 0415
112	0. 04 00
113	0. 0393
114	0. 04 04
115	0. 0410
116	0. 03 60
117 ,	0. 0390
118	0. 0444
119	0. 0469
120	0. 0391 :
121	0. 0519
122	0. 0446
123	0. 0285
124	0. 0171 :
125	0. 0259
126	0. 0213
127 ,	0. 01 88
128	0. 01 65
129	0. 01 45
130	0. 01 22
131	0. 0084
132	0. 0173
133	0. 0079

B.3. Programme and Operating Instructions.

The programme, along with explanatory notes, is given after section C. The operating instructions are as follows:

1. Enter programme tape in reader.
2. Hand-switches clear.
3. ITB
4. On reading $\rightarrow$, remove programme tape.
5. Enter data tape in reader.
6. Tap hand-switch 9.
7. The number of the iteration in progress is displayed (in binary) in register B2. After each iteration ($\sim$ 30 sec) there is a hoot.
8. To stop the iteration manually, after the hoot, set hand-switch 9. The computations will then stop on the completion of the next iteration, and the approximate solution will be printed out.
9. For extra data tapes, same as instructions 5 and 6.

C. Other Similar Programmes.

C.1. Programme OBS 11 - C.

This programme is also an iterative method (Gauss-Seidel) for the solution of the Fredholm equation. The latter is expressed as a set of linear equations which are solved by successive approximations. The input data is the same as for OBS 11 - E except that the last control number must be omitted, and the number of points in the observed curve may be ≤ 150 .

The output is identical with that of OBS 11 - E. In this case, the mean difference, $\bar{E}$, refers to the difference between successive approximations to the true profile. The iteration ceases when this is less than the preset error limit.

The operating instructions are the same as those for OBS 11 - E. The time^(seconds) required for each iteration is approximately $N/5$ where N is the number of points in the observed curve.

C.2. Programme OBS 11 - D.

This is a programme for computing the integral arising in the broadening problem, knowing the true

profile and apparatus function. That is, it computes the fold of the true curve with the apparatus function.

The input data consist of three parts. The first is an integer specifying the number of true profiles to be folded with the apparatus function concerned. This is then followed by the number of apparatus function points and the apparatus function, tabulated as in OBS 11 - E. Finally the number of points for the first true profile is given followed by the true profile, tabulated from violet to red at intervals, Δx . If more than one profile is to be folded with the given apparatus function, they are then tabulated, each preceded by an integer giving the number of its points, one after the other. Several such data tapes may be entered one after the other.

The output consists of the observed profile, tabulated under "FOLDED CURVE", along with the corresponding integral x values (similar to the OBS 11 - E output). The operating instructions are the same as 1 - 6 and 9 for OBS 11 - E.

TITLE

BURG. VAN CITT. ITERATION - HIGGS - OBS/11E

CHAPTER

A→179

B→139

C→139

F→15

1) READ(P)

P=P-1

I=0(1)P

READ(A1)

REPEAT

Read number of points in apparatus function

P=P+1

JUMP 3, P>179

I=1P(1)179

AI=0

REPEAT

Set remaining registers at zero

13)V=0

I=0(1)179

V=V+AI

REPEAT

I=0(1)179

AI=AI/V

REPEAT

Normalize the apparatus function

X7(400)A0,180

Store the apparatus function

UP

2) R=XINTPT(70,10-Q/2)

X6(0)BR,Q

X6(400)A0,180

M=R-1

JUMP 40, 0>M

I=0(1)M

BI=0

REPEAT

Read observed function from store and centre it in the 180 registers

40)L=R+Q

JUMP 25, L>139

I=L(I) I 39
BI=0
FE FE AT
25) X7(0) B0, I4 0
L=L-I

Store the observed function

K=XMAX(A0, 0, I7 9)

Find the peak of the apparatus function

26) READ(Z)
JUMP27, Z=0
28) Z=0.09016ZZ
ACROSS48/0

Read half-width of apparatus function; if zero, omit second difference routine; if non-zero, jump to second difference routine

27) Z=1000
P=0
Zn=5Hn
Yn=5A

Initial parameters for iteration, H_n is preset error limit (A is superfluous in this programme)

5) W=0
Un=0
P=P+1
002(P)
J=0(I) I 39
X=0

**Increase iteration number by one
Display present iteration number in register B2
Repeat for each pivotal point**

I=J(I) I 39
T=K+I-J
JUMP3, T>I7 9
X=X+BIAT
3) RE FE AT

Contributions scattered from points to the right of the point in question

Y=0
S=J-I
JUMP4, J=0

Contributions scattered from the points to the left of the point in question

I=0(I) S
T=K-J+I
JUMP4, 0>T
Y=Y+BIAT
4) RE FE AT

**Function obtained when trial solution is scattered by apparatus function
Read the observed function from the store**

I4) CJ=X+Y
FE FE AT
X6(0) A0, I4 0
J=0(I) I 39
U=XMOD(AJ-CJ)
JUMP17, M>J
JUMP17, J>L

Compare the scattered and observed functions

W=W+U
 JUMP17, Un>U
 Un=U
 17) REPEAT

 W=W/Q
 Wn=5W
 610(1)
 JUMP8, o>1
 HOOT
 JUMP8, W>Z
 Z=W
 JUMP20, Un>Wn
 JUMP18, Un>Zn
 JUMP7
 18) JUMP15, Un>Yn
 JUMP7
 20) JUMP6, W>Hn
 JUMP7
 6) JUMP15, W>A
 7) NEWLINE
 NEWLINE
 620, 27
 620, 9
 620, 20
 620, 5
 620, 18
 620, 28
 620, 0
 PRINT(P) 2, 0
 SPACE
 SPACE
 620, 27
 620, 5
 620, 18
 620, 18
 620, 28
 620, 0
 PRINT(W) 2, 4
 SPACE
 620, 27
 620, 13
 PRINT(Un) 2, 4
 NEWLINE
 NEWLINE
 JUMP16
 15) J=0(1)139
 BJ=BJ+AJ-CJ

Add deviations between the two functions

Find the maximum deviation, E_{max}

Mean deviation, $\bar{E}$

Read hand-switches; if hand-switch 9 is set, jump to 8); if not, hoet

If E has not decreased since last iteration, jump to 8)

ITERATION ANALYSIS. E_{max} greater than $5\bar{E}$:
 stop iteration if $\bar{E}$ is less than the
 preset error ϵ ; if not, continue.
 E_{max} less than $5\bar{E}$: stop iteration if E_{max}
 is less than 5ϵ ; if not, continue.

Print ITER.

Print index number of iteration just completed

Print ERR.

Print mean deviation, $\bar{E}$

Print maximum deviation, E_{max} , in parentheses

If iteration is to continue, form a new trial solution, $T(x)$

```

REPEAT
X6(400)A0,180
JUMP5
8)NEWLINE
NEWLINE
620,27
620,2
620,5
620,19
620,20
SPACE
PRINT(Z)2,4
PRINT(W)2,4
620,0
620,5
PRINT(P)2,0
620,0
620,6
SPACE
620,27
620,13
PRINT(Un)2,4
NEWLINE
NEWLINE

```

```

16) JUMP10,Cn=0
NEWLINE
620,27
620,6
620,9
620,14
620,1
620,12
SPACE
620,27
620,3
620,21
620,18
620,22
620,5
620,0
NEWLINE
NEWLINE

```

```

I=R(1)L
S=I-R
PRINT(S)2,0

```

Read apparatus function from store

The following applies for a stop (of iteration) due to non-convergence, or a manual stop by operator

Print BEST

Print previous mean deviation, $\bar{E}$
Print present mean deviation, $\bar{E}$

Print number of iteration last completed in parentheses

Print N

Print present maximum deviation, E_{max}

Jump to 10) if print out is to be suppressed (this is not applicable to this programme)

Print FINAL

Print CURVE

Repeat for each pivotal point

Print x

PRINT (BI)4,4
12)NEWLINE
REPEAT
NEWLINE

Print T(x)

10)ACROSS2/0

PSA
CLOSE

CHAPTER 0

A→179
B→139
C→139
F→15

1)DOWN I/I
Cn=I
O=5
READ(Q)
READ(Hn)
A=Hn

**Read the apparatus function, normalize it
and store it**

**Read number of pivotal points for observed curve
Read press t error limit,**

S=Q-I
I=0(I)S
READ(AI)
REPEAT

Read the observed function, O(x)

X7(0)A0,Q
ACROSS2/I

Store the observed function

2)JUMP I

48)J=3(I)I 36
F0=ZB(J-3)-2ZB(J-2)+ZB(J-1)
F1=ZB(J-2)-2ZB(J-1)+ZBJ
F2=ZB(J-1)-2ZBJ+ZB(J+1)
F3=ZBJ-2ZB(J+1)+ZB(J+2)
F4=ZB(J+1)-2ZB(J+2)+ZB(J+3)

Repeat for 134 observed points

**Compute second difference factor for
five points centred on point in
question**

F8=F0+F1+F2+F3+F4
F10=4F0+F1+F3+4F4
F14=0.48571F8-0.14285F10

**Least-square quadratic solution for
second difference factor at point in
question (Factor = A + Bx + Cx²)**

50) CJ=BJ-F I 4
REPEAT

Modify observed function to give first approximation

J=3(I) I 36
BJ=CJ
REPEAT

Replace observed function with first approximation

51) ACROSS27/I :

Return to main part of programme

PSA
CLOSE

PART IV. DETERMINATION OF CHARACTERISTIC CURVES

A. Introduction.

Programme OBS 11 - 111 was written as a sub-programme of the general reduction programme, OBS 11. It may, however, be used as a complete programme in itself for the derivation of photographic plate characteristic curves. Using least-square methods, an analytic form of the curve is derived from data obtained using a seven-step transmission wedge.

B. Programme OBS 11 - 111.

B.1. Input Data.

The input data consist of sets of numbers giving microphotometer readings for the seven wedge steps, corresponding readings for the glass plates of the wedge, and clear and black plate readings. Up to ten sets of readings (individual determinations) may be combined together for the determination of a characteristic curve. It will be assumed that the microphotometer deflection is given by a coordinate x . The input data then consist of the following: an integer giving the number of

sets of data (individual determinations) to be entered (≤ 10); an integer which is 1 if the characteristic curve is to be printed out in the form of a table, and is 0 if this print out is not required; an integer which gives the first register in the auxiliary store to be used for storage purposes (the programme requires $30n + 95$ auxiliary variables if n is the number of sets of input data) which is usually 0; seven pairs of numbers corresponding to the wedge steps, which will be described below, for the first determination, followed by similar sets of fourteen numbers for each of the other determinations; and finally, for each determination, pairs of numbers, the first of which gives the $\times$ reading for the clear plate, and the second of which gives the black plate or "zero" reading. These pairs are entered one after the other in the order that the sets of wedge readings were entered. The fourteen numbers in each set of wedge readings are entered in two columns. The first column gives the $\times$ readings for the seven wedge steps starting at the reading nearest the clear plate reading. The second column gives the corresponding readings for the glass plates alone, at the positions of the wedge steps. If a wedge reading is not considered

usable, it must be set equal to zero along with the corresponding glass plate reading. (In no other case should a wedge reading be exactly 0). As many data tapes may be entered, one after the other, as desired.

Since the clear plate reading may vary considerably across the microphotometer tracing and only one x_{cf} value may be entered for each set of data, the following procedure has been adopted. The x_{cf} entered in the data is chosen to refer to the wedge readings. The glass plate readings are then adjusted to correspond to this clear reading. If x_{cf} is the actual clear reading for the glass plate readings, the correction, δ , to be added to a glass plate reading, x , is given by:

$$\delta = \frac{(x_0 - x) (x_{cf} - x_{cf})}{(x_{cf} - x_0)}$$

A typical data tape is given on the next page. This is for an Ilford HP3 plate, the data referring to two wedge calibrations near 6300 Å.

TYPICAL INPUT DATA

OBS 11 - 111

293

2 1 0

Control numbers. The first indicates two sets of data, the second indicates a full print out is wanted, the third is the first auxiliary variable.

1.60 9.21
2.28 9.47
4.10 9.70
6.93 9.75
9.90 9.77
11.60 9.70
12.395 9.51

First set of data. On the left are the seven wedge readings, on the right the corresponding glass plate readings.

1.98 9.48
2.88 9.82
4.90 9.94
7.40 10.07
10.41 10.08
11.81 10.045
12.49 9.99

Second set of data

0.935 13.015
1.415 13.012

Clear and black reading for first set
Clear and black reading for second set

B.2. Computer Output.

The first part of the output consists of three columns. The first one, designated "ORD.", gives the order of the polynomial characteristic curve found for each set of data, and lastly, the order of the final adopted characteristic curve. The second column, "VAR.", gives the variance (see Chapter 2 for definition) of each of these determinations. The third column gives the percentage uncertainty in any intensity value determined from these polynomial fits. In all of these, the last row of numbers is the significant one, referring to the final curve.

If a print out of the characteristic curve was specified in the input, the above is followed by a set of numbers headed "COEFF." These are the coefficients in the polynomial finally adopted starting with a_0 . Finally, the adopted curve is given in tabular form with the natural logarithm of I/I_0 being given for values of y/y_0 . A $y/y_0 = 1$ refers to the "clear" plate. At the bottom of the table, two more numbers are given. The first gives the minimum y/y_0 in the input data, and the second the maximum y/y_0 , thus indicating the region for which the determination applies. The output corresponding to the sample data tape given above is shown on

the next page.

TYPICAL OUTPUT

OBS 11 - 111
296

PROG RAMME-111-OBS/11-H IGGS-7 MARCH 61

01 051
02 102
03 033
00 627

ORD.	VAR.		
2	0.00057	2.4	Results for first set of data
1	0.00234	5.0	Results for second set
4	0.00147	3.9	Final adopted curve

COEFF :

-1.34278
0.48349
-0.01275
-0.00343
0.00132

Coefficients of adopted polynomial

Y/Y ₀	LN(1/I ₀)
0.02	0.44573
0.10	-0.34767
0.18	-0.64394
0.26	-0.85336
0.34	-1.02843
0.42	-1.18815
0.50	-1.34278
0.58	-1.50003
0.66	-1.66783
0.74	-1.85694
0.82	-2.08631
0.90	-2.39959
0.98	-2.99609

Tabulated characteristic curve

0.0450 0.9513

Minimum and maximum y/y_0 's

B.3. Programme and Operating Instructions.

The programme, along with explanatory notes, is given on the next thirteen pages. The operating instructions are as follows:

1. Enter programme tape in reader.
2. Hand-switches clear.
3. ITB
4. On reading → , remove programme tape.
5. De-isolate Column 2, Drum 0.
6. Enter data tape in reader.
7. Tap hand-switch 9.
8. For extra data tapes, same as 6 and 7.
9. If programme stops shortly after beginning of computations, and hoots at a rapid rate, instruction 5 has been neglected. The computations will continue as soon as it is carried out.

TITLE
PROGRAMME - 111 - OBS / 11 - HIGGS - 7 MARCH 61

PROGRAMME - 111 :

CHAPTER 1

A=7 5
B=6
D=7 5
E=6
G=9
L=2 2
W=4
X=6
Y=9
Z=5
U=6

1) NEWLINE
NEWLINE
SPACE
SPACE
SPACE

620, 27
620, 15
620, 18
620, 4
620, 0
620, 28
I=0(1) 8

Print ORD.

SPACE
REPEAT

620, 27
620, 22
620, 1
620, 18
620, 0
620, 28
NEWLINE
NEWLINE

Print VAR.

5) P=1
4) N=7

Control parameters (P indicates which set
of data is being considered)

AV 0

22) D0 = 0.0735

D1 = 0.104

D2 = 0.156

D3 = 0.263

D4 = 0.430

D5 = 0.656

D6 = 1.000

JUMP7, A=3

*X6 (Cn+7 P-7) A0,7

I = 0(I)6

D1 = XLOG(D1)

JUMP2, A1 > 0.9999

JUMP2, 0.0001 > A1

A1 = XLOG(1/A1-1)

JUMP3

2) N = N-1

D1 = 0

A1 = 0

3) UI = A1

REPEAT

Q = 15**)

JUMP14

14) I = 0(I)6

D7 = 0

A7 = 0

J = 0(I)6

JUMP8, AJ = 0

JUMP9

8) AJ = A(J+1)

A(J+1) = 0

9) JUMP10, DJ = 0

JUMP11

10) DJ = D(J+1)

D(J+1) = 0

11) REPEAT

REPEAT

JUMP(Q)

15) K = 1(I)4

X7(AK) A0,N

X7(AK+N) D0,N

M = N

Transmission factors for the seven wedge steps

Read y/y_0 's for wedge steps from store

Ln (transmission factor)

Omit wedge y/y_0 's outside $0.0001 < y/y_0 < 0.9999$

Ln ($y_0/y - 1$)

If some y/y_0 's are omitted, decrease number of wedge steps accordingly and set the corresponding $\ln(t)$'s equal to zero

Ordering routine. This moves all non-zero $\ln(y_0/y - 1)$'s and $\ln(t)$'s to the lower-index registers, leaving all zeros in the higher-index registers

Repeat for four orders of polynomial
Store the $\ln(y_0/y - 1)$'s
Store the $\ln(t)$'s

Q=1
 PRESE RVE
 IDWN 1/2-111
 RE STORE
 O=K+2
 R=5K-5
 S=6K-5
 X6(Bn+6 0) ER, 0
 WK=E(S+1)
 RE FEAT

Find best polynomial (order K) fitting the observed data, using programme 523

Read polynomial coefficients and variance from store (W_K is the variance)

T=XMIN(W 0, 1, 4)
 S=6T-5
 R=5T-5

Choose the order of polynomial with minimum variance

JUMP 30, A=2
 JUMP 19, A=1
 JUMP 33
 30) NEWLINE
 JUMP 19

Control jumps

33) X6(Cn+7 B+7 P-7) X 0,7

Read the y/y_0 's for the glass plates from store

N=7

N 3
 JUMP 22

Control parameters

Restore original transmission factors

7) I=0(1) 6
 JUMP 12, XI=9
 JUMP 12, UI=0
 XI=XLOG(I/XI-1)
 JUMP 13
 12) DI=0
 UI=0
 XI=0
 N=N-1
 13) VI=0
 J=S(-1) R
 VI=EJ+V XI
 RE FEAT
 RE FEAT

Ln ($y_0/y - 1$) for glass plates, omitting those which were either not measured or which correspond to wedge steps that were omitted in the above computation of the first approximation to the characteristic curve

Compute ln (I/I_0) for glass plate readings using the approximate characteristic curve found from wedge readings alone

I=0(1) 6
 AI=UI
 JUMP 16, XI=0

Correct the original ln (t)'s for intensity variations across the wedge steps, using the

DI=XLOG(DI)
DI=DI+V1-V4
16) REPEAT

intensity incident on the fourth step as
a reference

Q=18)
JUMP14
18) AV1
JUMP15

Order the new $\ln(t)$'s and $\ln(y_0/y - 1)$'s

Reset control parameter and find new least-
square polynomial

Print the order of the best polynomial
Print corresponding variance

19) PRINT(T)4,0
PRINT(E(S+1))8,5
JUMP40,0****>E(S+1)
E(S+1)=X S Q R T(E(S+1))
E(S+1)=X E X P(E(S+1))
E(S+1)=X F R P T(E(S+1))
PRINT(100E(S+1))4,1
40) NEWLINE
JUMP31,A=2
JUMP34
31) ACROSS 31/3-111

Compute corresponding percentage
uncertainty in a determination of I
Print the percentage uncertainty

Control jump

34)V=0
I=2(I)7
VI=0
Bo=0.05+0.*I1
Bo=XLOG(I/Bo-1)
J=S(-1)R
VI=EJ+VIB0
REPEAT
JUMP20,P=1
JUMP21
20)GI=VI
21)VI=VI-GI
V=V+VI/6
REPEAT

Compute $\ln(I/I_0)$ for six values of y/y_0
using the adopted polynomial characteristic
curve

Find mean shift along $\ln(t)$ axis between the
characteristic curve just determined and that
found from the first set of data

*25) I=0(I)6
JUMP6,AI=0
DI=DI-V
6) REPEAT

Adjust the $\ln(t)$'s using the mean shift derived
above, placing all sets of observations on
a common basis

X7(CN+7B+7P-7)Do,7
X7(CN+7P-7)Ao,7
P=P+1
Q=XINTPT(B+0.5)
T=Q+1
JUMP4,T>P

Store the final $\ln(t)$'s and $\ln(y_0/y - 1)$'s
Increase P by one, indicating next
set of data

Return to 4) if all sets of data have been
considered

ACROSS33/3-111

PSA
XLOG
XXEXP
CLOSE

CHAPTE R2

A→9
B→9
C→9
D→9
E→9
W→100
X→100
Y→100
Z→100
V→5

1) R=Q
x6 (An) Xi , M
Y00 An+M
2) I=1 (1) N
W1=1
REPEAT
An An-N
3) x6 (An+N+M) Yi , N
9) I=1 (1) N
Z1=0
REPEAT
A0=0
B0=-1
C0=0
D0=1

J=0 (1) K
V1=0
V2=0
V3=0
I=1 (1) N
YI=YI-CJZI

The first part of this chapter is essentially the Ferranti library programme No. 523, which determines the least-square polynomial $(y = \sum a_i x^i)$ of order K best fitting N pairs of x and y values. These are stored in the auxiliary store, starting respectively at A₇ and A₇ + N

$V_4 = X_1 - A_J$
 $V_4 = V_4 Z_1 - B_{JW_1}$
 $W_1 = Z_1$
 $Z_1 = V_4$
 $V_4 = Z_1 Z_1$
 $V_1 = V_1 + V_4$
 $V_4 = V_4 X_1 -$
 $V_2 = V_2 + V_4$
 $V_4 = Y_1 Z_1 -$
 $V_3 = V_3 + V_4$
 REPEAT
 $A(J+1) = V_2 / V_1$
 $D(J+1) = V_1$
 $C(J+1) = V_3 / V_1$
 $B(J+1) = V_1 / D_J$
 $E(J+1) = V_3 C(J+1)$
 REPEAT
 $V_1 = 0$
 $V_3 = 0$
 $I = 1(1)N$
 $Y_1 = Y_1 - C(K+1) Z_1$
 $V_2 = Y_1 Y_1 -$
 $V_1 = V_1 + V_2$
 REPEAT
 $I(0) \times 7 (A_1 + N + M) Y_1, N$
 $V_2 = D_1 - K - 1$
 $JUMP_4, 0, I > V_2$
 $V_3 = V_1 / V_2$

 4) $I = 0(1) 100$
 $Z_1 = 0$
 REPEAT
 $Z_1 I = 1$
 $J = 1(1) K$
 $T = 10 J + 10$
 $S = J + 1$
 $I = 1(1) S$
 $T = T + 1$
 $O = T - 20$
 $L = T - 10$
 $ZT = Z(L-1) - A_{JZL} - B_{JZO}$
 REPEAT
 REPEAT
 $J = 0(1) K$
 $S = J + 1$
 $T = 10 J + 10$
 $I = 1(1) S$
 $T = T + 1$

$V_2 = C(J+1)ZT$
 $ZI = ZI + V_2$
 RE FE AT
 RE FE AT
 5) JUMP6, $Q > R$
 $S = I$
 $J = o(1)K$
 $S = S + J$
 $T = I \circ J + 1I$
 $I = o(1)J$
 $L = S + I$
 $Y_L = ZT$
 $T = T + 1$
 RE FE AT
 RE FE AT
 $S = K + 1$
 $I = 1(1)S$
 $Y(I+50) = DI$
 RE FE AT
 $x_7(Bn)Y_1, 60$
 6) $S = K + 1$
 $I = 1(1)S$
 $Z(I+10) = CI$
 $Z(I+20) = EI$
 RE FE AT
 $Z(K+2) = V_3$
 $Z(K+22) = V_1$
 $x_7(Bn+60)Z_1, 30$
 $Bm Bn+30$
 7) $R = R - 1$
 ANV AN+N
 ANV AN+N
 12) JUMP*2, $R > 0$
 UP

19) $x_6(Dn)V_0, 0$
 $x_6(Cn+14B)V_5, 1$

$A_2 = XEXP(V_5)$
 $A_2 = 1 + A_2$
 $A_2 = 1 / A_2$
 $\%E n = A_2$
 $A_3 = A_2 - 0.01$
 $A_3 = XLOG(1 / A_3 - 1)$
 $A_0 = V_5$
 $A_1 = A_3$
 $I = o(1)1$

Read polynomial coefficients from store
 Read minimum value of $\ln(y_0/y - 1)$ from store

Maximum value of y/y_0

```

C1=0
J=T(-1)0
C1=VJ+C1A1
REPEAT
REPEAT
C2=100C0-100C1
A2=A2-1
XD=C2A2
A2=XLOG(-A2)
E=C0-DA2

```

JUMP22, C=0

```

I=0(1)12
YI=0
A4=0.02+0.08I
PRINT(A4)2,2
JUMP20, A4>E7
A5=XLOG(1/A4-1)
J=T(-1)0
YI=YJ+Y1A5
REPEAT
JUMP21:
20) A4=XLOG(1-A4)
YI=DA4+E
21) PRINT(YI)4,5
NEWLINE
REPEAT

```

22) ACROSS54/ 3-111

```

PSA
XSQRT
XLOG
XEXP
CLOSE

```

Compute $\ln(I/I_0)$ corresponding to the maximum observed y/y_0 and to a y/y_0 that is 0.01 smaller

$$\left\{ \frac{d \ln(I/I_0)}{d y/y_0} \right\} \left\{ (y/y_0)_{\max} - 1 \right\} \\ \ln(I/I_0)_{\text{at max}} - D \ln \left\{ 1 - (y/y_0)_{\max} \right\}$$

Control jump concerning print out

Tabulated characteristic curve

Print value of y/y_0

Compute $\ln(I/I_0)$ from polynomial if y/y_0 is less than $(y/y_0)_{\max}$

Compute $\ln(I/I_0)$ from linear extrapolation otherwise
Print $\ln(I/I_0)$

CHAPTE R₃

VARIABLES₁

C→6

F→6

H→2

Y→12

1) Q=X INTPT(B+0.5)

T=7 Q

Reset parameters as in chapter 0

BW An+14B

CW An+14B+90

DW An+30B+90

J=I(1) Q

X6(Cn+7J-7)C0,7

X6(Cn+T+7J-7)F0,7

X6(Cn+2T+2J-2)H0,2

H2=H1-H0

I=0(1)6

JUMP40,C I=0

C I=H1/H2-C I/H2

40) JUMP41,F I=0

F I=H1/H2-F I/H2

JUMP42

41) F I=9

42) REPEAT

X7(Cn+7J-7)C0,7

X7(Cn+T+7J-7)F0,7

REPEAT

Read wedge readings from store

Read glass plate readings from store

Read clear plate and black plate (zero) readings
Y₀

Compute y/y₀ for wedge (C_I) and glass (F_I)
readings, omitting those not measured, the
latter being set equal to either zero (wedge)
or nine (glass).

Store the y/y₀'s for the wedge and glass
readings

As above

Checking routine to determine whether column
2, drum 0 is de-isolated. If not, numbers
read from drum are not the same as numbers
transferred to drum, and the machine hoots
until the store is de-isolated

2) BW An+14B

CW An+14B+90

DW An+30B+90

50) I=I(1)2

E0=1

X7(-2000)E0,1

X6(-2000)E1,1

CHECK(E0,E1,0.01,51)

HOOT

JUMP50

51) REPEAT

ACROSSI/1-111

33) N=7 Q
S=N
R=S-1
X6(CN) A0,N
X6(CN+N) D0,N

Read the final $\ln(t)$'s and $\ln(y_0/y - 1)$'s
from store

I=0(I) R
JUMP35, AI=0
JUMP*36
35) AI=10
36) REPEAT
K=XMIN(A0,0,R)
I=0(I) R
JUMP37, AI=10
JUMP38
37) AI=-10
38) REPEAT
L=XMAX(A0,0,R)
X7(CN+2N) AK,I
X7(CN+2N+1) A**XL,I

Choose the minimum and maximum values of
 $\ln(y_0/y - 1)$

Store the minimum and maximum values of
 $\ln(y_0/y - 1)$

I=0(I) R
JUMP24, AI=-10
JUMP25
24) N=N-1
DI=0
AI=0
25) REPEAT

Omit readings which have been omitted in
previous calculations

I=0(I) S
D(S+1)=0
A(S+1)=0
J=0(I) S
JUMP26, DJ=0
JUMP27
26) JUMP45, DJ=AJ
JUMP27
45) DJ=D(J+1)
D(J+1)=0
27) JUMP28, AJ=0
JUMP29
28) AJ=A(J+1)
A(J+1)=0
29) REPEAT
REPEAT*

Order all the values of $\ln(t)$ and
 $\ln(y_0/y - 1)$ so that all the zeros are in
the high-index registers

AV 2

Reset control parameter

ACROSS I 5 / I - I I I

Compute best polynomial fitting all the
observed data

31) I = 0(1) T

J = R + I

ZI = EJ

REPEAT

Copy the coefficients of the adopted poly-
nomial

O = T + 1

JUMP 32, C = 0

Order of the adopted polynomial plus one
Control jump concerning print out

NEWLINE

NEWLINE

SPACE

SPACE

SPACE

620, 27 .

620, 3

620, 15

620, 5

620, 6

620, 6

620, 0

620, 28

NEWLINE

NEWLINE

Print COEFF.

I = 0(1) T

PRINT(Z I) 3, 5

NEWLINE

REPEAT

Print the coefficients of the adopted
polynomial

NEWLINE

NEWLINE

SPACE

SPACE

620, 27 .

620, 25

620, 0

620, 23

620, 27 .

620, 25

620, 0

620, 16

I = 0(1) 4

SPACE

REPEAT

Print Y/Y_0

620, 27 .

620, 12

620, 14

620, 0

Print $\ln(I/I_0)$

620,5
 620,27
 620,9
 620,0
 620,23
 620,27
 620,9
 620,0
 620,16
 620,6
 NEWLINE
 NEWLINE

32) X7(Dn)Z0,0
 F=XPARITY(T)
 JUMP52,F>0
 JUMP53
 52) ACROSSI 9/2-III
 53) JUMP54,C=0

I=0(I)I2
 YI=0
 Z5=0.02+0.08I
 PRINT(Z5)2,2
 Z5=XLOG(I/Z5-I)
 J=T(-I)0
 YI=ZJ+YIZ5
 REPEAT
 PRINT(YI)4,5
 NEWLINE
 REPEAT

54)UP

PSA
 XLOG
 CLOSE

CHAPTER 0

A→25
 X→75
 C→75

Store the coefficients of the polynomial
 Check whether the adopted polynomial is odd
 or even. If even, jump to chapter 2

Control jump concerning print out

Tabular presentation of characteristic curve

Print y/y_0

Compute $\ln(I/I_0)$ from polynomial

Print $\ln(I/I_0)$

1) READ(B)	Read number of sets of data
READ(C)	Read print-out control integer
READ(A1)	Read starting point in auxiliary store
CMV AN+14B+90	Storage positions in auxiliary store
DMV AN+30B+90	
Q=XINTPT(B+0.5)	Number of sets of data (individual determinations). T is number of wedge readings
T=7Q	
I=1(I)T	
READ(XI)	Read wedge reading
READ(CI)	Read corresponding glass plate reading
REPEAT	
S=2Q	
I=1(I)S	
READ(AI)	Read clear film and zero (black) readings
REPEAT	
X7(CN)XI,T	Store wedge readings
X7(CN+T)CI,T	Store glass plate readings
X7(CN+*2T)AI,S	Store clear film and zero readings
DOWN I/3-111	
NEWLINE	
X6(CN+*14B)A22,2	Read maximum and minimum observed
A22=XEXP(A22)	ln ($y_0/y - 1$)'s from store
A22=1+A22	
A22=1/A22	Compute maximum and minimum observed y/y_0 's
A23=XEXP(A23)	
A23=1+A23	
A23=1/A23	
PRINT(A23)1,4	Print minimum observed y/y_0
PRINT(A22)1,4	Print maximum observed y/y_0
JMP1	Return to beginning for next data tape
PSA	
XEXP	
CLOSE	
→	

PART V. REDUCTION OF LINE PROFILES

A. Introduction.

Programmes OBS 11 - E and OBS 11 - 111 were combined into a general programme, OBS 11, for the complete reduction of true line profiles and their asymmetry from microphotometer tracings, allowing for apparatus function effects and the photographic plate response. Although severely limited in the amount of data that can be considered for each profile, it is adaptable to a large number of reduction procedures. Because of its flexibility and speed it is very useful when large amounts of data must be analysed (although it is not as useful for single profile determinations owing to the length of the programme tape that must be entered).

B. Programme OBS 11.**B.1. Input Data.**

The basic data for this programme consist of measurements of a line profile on a microphotometer tracing, using X,Y coordinates superimposed on the tracing in a grid form. The X coordinate is in the direction of increasing wavelength and the Y coordinate measures the microphotometer deflection. In addition, data related to the apparatus function and characteristic curve must be given.

The data begins with the apparatus function. As for OBS 11 - E, if it consists of N tabulated values, $N (\leq 180)$ is given first, followed by the N values tabulated from red to violet at equal intervals, Δx . This tabulation interval, Δx , sets the interval to be used in all subsequent calculations. Only one apparatus function may be used (for many different profiles) without re-entering the programme tape (or a manual restart at first instruction of chapter O).

The characteristic curve data is then given. This is exactly the same as the input data for OBS 11 - 111, except that the integer specifying the first auxiliary

variable to be used is omitted. (Note that the coordinate now called Y was called x in the specifications for OBS 11 - 111).

The next part of the data consists of the five control parameters A, B, C, D, and E. The computations carried out in this programme make available four main sets of results (details may be found in Part V, Chapter 2). These are: the residual intensities at the measured points on the microphotometer tracing profile, the smooth profile through these values found by interpolation, the true profile derived by solving the integral equation, and finally the asymmetry of the true profile. The five control parameters then have the following effect, depending on their values.

A = the number of line profiles to be reduced using the characteristic curve data last entered (< 512)

If B = 2, the residual intensities of the observed points and the smooth interpolated profile are printed out

= 1, only the smooth interpolated profile is printed out

= 0, neither of these two sets of results are printed out

If $C = 2$, both the true line profile and
its asymmetry are printed out

$= 1$, only the true line profile is
printed out

$= 0$, only the asymmetry of the true
profile is printed out

If $D =$ non-zero integer, the true profile,
if being printed out, is printed out
at intervals of $D\Delta x$

$= 0$, the true profile is not printed out,
regardless of the value of C

$E =$ the preset iteration error limit (see
OBS 11 - E). The iteration termination
is controlled by the larger of E and the
fitting error arising in the interpolation
of the smooth observed line profile

The rest of the data refers to the line profile on
the microphotometer tracing. It begins with a number,
 $M (\leq 54)$, which is the number of observed points on
this profile. Following this are three numbers giving
the black plate, continuum, and clear plate Y readings
for the tracing at the position of the profile. Next
is a number, P , giving the relation between Δx and the
unit of the X coordinate on the tracing. If the latter
corresponds to 50 mA, and $\Delta x = 5$ mA, then $P = 10$.
Finally the Y and X values of the M observed points are
given in two columns, the Y 's on the left with the

corresponding X's on the right. These tabulated values must satisfy three restrictions: 1) the X's must be entered, increasing in value, from violet to red (opposite to the apparatus function), 2) the first X value entered must be 1 (thus corresponding to a grid line on the tracing), and 3) the largest X value given must be such that the integral part of $(PX_{\max} + X_{\max}) \leq 139$.

If more than one profile is to be reduced using the same characteristic curve data ($A > 1$), then the data for the next and successive profiles follows, commencing with the integer B. After the A sets of profile data have been entered, new characteristic curve data may be entered followed by more profile data.

The flexibility of the programme may be illustrated by a few examples. It should be noted that the true profile may be made equal to the observed profile by entering the apparatus function as 1.0, 1.0.

Apparatus function normal

A,B,C = 0,0,0	Gives asymmetry of true profile
= 0,1,0	Gives equivalent width of profile

Apparatus function as 1,1

A,B,C = 0,0,0	Gives asymmetry of observed profile
= 0,1,10	Prints observed profile at intervals of $10 \Delta x$

On the next five pages, a sample data tape is given. This is for the determination of the true profile of an iron line at 6300 A using the Babcock grating.

TYPICAL INPUT DATA

OBS 11

317

98

Number of points in apparatus function

00 00000
00 00050
00 00100
00 00150
00 00200
00 00263
00 00264
00 00259
00 00260
00 00271
00 00292

Apparatus function (as used in Chapter 4
for the Babcock grating) tabulated from
red to violet at 5 mA intervals ($=\Delta x$)

00 00318
00 00338
00 00354
00 00373
00 00396
00 00410
00 00396
00 00392
00 00415
00 00451

00 00507
00 00589
00 00673
00 00700
00 00750
00 00800
00 00870
00 00940
00 0100
00 0106

00 0*113
00 0123
00 0141
00 0155
00 0179
00 0206
00 0229
00 0274
00 0333
00 0401

00 0553
00 0728
00 114
00 188
00 317
00 471
00 657
00 812
00 895
00 836

00 665
00 475
00 337
00 245
00 179
00 132
00 0979
00 0743
00 0593
00 0476

00 0380
00 0318
00 0273
00 0239
00 0205
00 0180
00 0152
00 0128
00 0110
00 00985

00 00890
00 00790
00 00710
00 00670
00 00647
00 00610
00 00573
00 00536
00 00500
00 00470

00 00448
00 00426
00 00403
00 00378

0.00357 .
 0.00341
 0.00325
 0.00301
 0.00273
 0.00248

0.00229
 0.00210
 0.00195
 0.00150
 0.00100*
 0.00050
 0.00000

3 0

0 0
 0.275 1.*80
 0.77 1.91
 3.18 2.005
 10.15 2.10
 12.63 2.02
 13.005 1.875

Numbers specifying that there are three sets of data for the characteristic curve determination, and that a print out of the characteristic curve is not wanted

First set of data

0 0
 0.58 2.27 .
 1.06 2.39
 3.54 2.36
 10.375 2.41
 12.67 , 2.42
 0 0

Second set of data

0 0
 0 0
 0.59 1.82
 3.19 1.87
 10.22 1.98
 12.70 1.90
 13.005 1.72

Third set of data

0.198 13.082
 0.400 13.002
 -0.046 13.078

Clear and black plate readings

1 Number of profiles to be reduced

0 2 2 0.0015

Three control integers and iteration error limit

54 Number of points in observed line profile

0.01 8.305 12.525

Black plate, continuum and clear plate readings for the microphotometer tracing
 Dispersion of tracing (1 unit on tracing = 11.152 x 5 mA)

11.152

8.42 1.00
 8.43 1.20
 *8.515 1.80
 8.59 2.00
 8.67 2.20
 8.76 2.40
 8.87 2.60
 8.985 2.80
 9.09 3.00
 9.27 3.20
 9.35 3.30
 9.50 3.425

The 54 observed points. The Y (deflection) values are on the left and the X (wavelength) values are on the right

9.60	3.52
9.80	3.69
10.00	3.775
10.20	3.88
10.40	3.975
10.60	4.03
10.80	4.12
11.00	4.20
11.20	4.29
*11.40	4.39
11.60	4.505
11.80	4.70
11.925	4.90
12.00	5.00
12.03	5.20
12.03	5.30
12.005	5.40
11.91	5.60
11.80	5.785
11.60	5.97
11.40	6.10
11.20	6.20
11.00	6.31
10.80	6.41
10.60	6.525
10.40	6.67
10.20	6.765
10.00	6.865
9.80	6.92
9.60	7.01
9.315	7.20
9.105	7.40
8.98	7.60
8.88	7.80
8.78	8.00
8.69	8.20
8.62	8.40
8.56	8.60
8.465	9.00
8.405	9.40
8.40	9.80
8.40	10.00

B.2. Computer Output.

The first part of the output gives the characteristic curve determination and is the same as for OBS 11 - 111 (except that the minimum and maximum y/y_0 's entering into its determination are not printed).

The next part, if specified in the input, consists of the residual intensities at the observed points of the profile. Those values which arise from an extrapolation of the characteristic curve beyond the range of calibration observations are followed by asterisks.

The interpolation results are given next. A "999" at this point indicates a singular set of equations arising in the least square fitting. After "L.S. FIT", the root mean square fitting error (see programme for explanation) and the maximum fitting error arising in the interpolation procedure are given. If specified in the input, the interpolated observed curve is then given, headed "SMOOTH CURVE". The profile is given as an emission profile and the corresponding x values are given. The 0 of the x scale corresponds to the 0 of the X scale on the tracing. This convention will apply to all computed x values. Again if extrapolation has

entered into any of the intensity determinations, asterisks are printed. This will always arise at the two extrapolated ends of the profile.

The equivalent width of the profile (in units of Δx) is then given, after "EQUIV. WIDTH". Following this are the numbers indicating at what point the iteration terminated. These are the same as those given by OBS 11 - E. A number titled "MIN. PEAK" is then given which is the minimum value of the derived true profile, a useful indicator of "diffraction ripples" arising from the iteration. If the true profile was desired, it is then given under "FINAL CURVE". Again the x values and possible asterisks are given.

After "CENT. INT.", the central residual intensity of the true profile is given, along with the x value at which it occurs (in brackets). In addition, the half width (in units of Δx) is given after "HALF WIDTH". In this and the following parts of the output, the symbols +, *, and X refer to difficulties arising in the inverse interpolation procedures. The programme should be consulted for their interpretation.

Finally, if the asymmetry routine was used, the asymmetry of the true profile is given after "ASYMMETRY".

At each of several values of I (from 0 at the continuum in steps of 0.02 to the line peak) the $\bar{x}$ value of the line centre is given. If more than one peak occurs at that level, the $\bar{x}$ value is a weighted mean for all such peaks, and the number of secondary peaks (additional to the mean line peak) is indicated by the number of asterisks following the $\bar{x}$ value. If the asymmetry at a given level can not be found due to the fact that the profile never drops below that value at one end or the other, one of "POS.", "NEG.", or "NONE" is printed. These indicate, respectively, that the centre $\bar{x}$ value lies toward the red end of the profile, toward the violet end of the profile, or that no information about the line centre can be determined.

The output resulting from the sample data given previously is presented on the next three pages. The ripples in the asymmetry curve are mainly due to irregularities in the line profile which were enhanced by the iteration. This problem is discussed in Part II, Chapter 4.

TYPICAL OUTPUT

OBS 11

325

REDUCTION OF LINE PROFILES - HIGGS - 7 MARCH 61

01 051
02 102
03 033
04 048
05 069
06 153
07 324
00 152

ORD.

VAR.

Characteristic curve
results

4 0.00120 3.5

4 0.00000 0.0

4 0.00000 0.0

Solution for each set of
data

4 0.00030 1.7

Final adopted polynomial

L. S. FIT 0.0014 0.0038

Mean and maximum errors arising
in the least square fitting
for interpolation purposes

EQUIV. WIDTH 15.257

Equivalent width of line profile
(= 15.257 x 5 mA)

ITER. 3 ERR. 0.0011 M 0.0069

Number of iterations

MIN. PEAK -0.002

required for a satisfactory solution to the
integral equation, and the mean and maximum
deviations. The minimum intensity value in
the true profile

FINAL CURVE

0 -0.0018 *
2 -0.0008 *
4 0.0000 *
6 0.0016 *
8 0.0032 *
10 0.0054 *
12 0.0081 *
14 0.0055 *
16 0.0053 *

The computed true profile, tabulated from
violet to red at 10 mA intervals. The
asterisks indicate extrapolated values.
The zero x value corresponds to the zero
X value on the original microphotometer
tracing

18	0.0068	
20	0.0096	
22	0.0139	
24	0.0209	
26	0.0266	
28	0.0343	
30	0.0435	
32	0.0475	
34	0.0551	
36	0.0682	
38	0.0827	
40	0.0890	
42	0.1185	
44	0.1633	
46	0.2458	
48	0.3251	
50	0.4051	
52	0.4559	
54	0.4860	
56	0.5303	
58	0.5417	
60	0.5365	
62	0.5072	
64	0.4824	
66	0.4465	
68	0.3809	
70	0.3042	
72	0.2475	
74	0.2218	
76	0.1809	
78	0.0989	
80	0.0722	
82	0.0568	
84	0.0487	
86	0.0451	
88	0.0386	
90	0.0304	
92	0.0240	
94	0.0195	
96	0.0154	
98	0.0118	
100	0.0088	
102	0.0057	
104	0.0044	
106	0.0040	
108	0.0046	
110	0.0084	
112	0.0063	*
114	0.0041	*

I 16	0.0026	*
I 18	0.0013	*
I 20	0.0003	*
I 22	-0.0006	*

CENT. INT. 1 0.456 (57.916) Central (residual) intensity
 HALF WIDTH 24.379 of the true profile and x value of peak
 Half width (= 24.379 x 5 mA)

ASYMMETRY

0.02	58.687
0.04	58.364
0.06	58.228
0.08	58.524
0.10	58.099
0.12	59.711
0.14	59.903
0.16	60.151
0.18	60.170
0.20	60.113
0.22	59.762
0.24	59.191
0.26	58.928
0.28	58.833
0.30	58.762
0.32	58.708
0.34	58.667
0.36	58.673
0.38	58.641
0.40	58.664
0.42	58.702
0.44	58.741
0.46	58.931
0.48	59.002
0.50	58.666
0.52	58.309
0.54	58.086

Asymmetry of the true profile

Intensity values (1 - residual intensity)
 on left and centre $\bar{x}$ value on the right

B.3. Programme and Operating Instructions.

The programme, along with explanatory notes, is given on the following twenty-two pages. The operating instructions are as follows:

1. Enter programme tape in reader.
2. Hand-switches clear.
3. ITB
4. On reading $\rightarrow$, remove programme tape.
5. De-isolate Column 2, Drum 0.
6. Enter data tape in reader.
7. Tap hand-switch 9.
8. If programme stops shortly after beginning of computations and hoots at a rapid rate, instruction 5 has been neglected. The computations will continue as soon as it is carried out.
9. In the iteration routine, the number of the iteration in progress is displayed (in binary) in register B2. After each iteration ($\sim$ 30 sec) there is a hoot.
10. To stop the iteration manually, after the hoot set hand-switch 9. The computations will then stop on the completion of the next iteration, and the rest of the computations will continue.
11. For extra data tapes, same as instructions 6 and 7.

TITLE
REDUCTION OF LINE PROFILES - HIGGS - 7 MARCH 61

PROGRAMME - I I I

This sub-programme is identical with the programme OBS 11 - 111 (except for chapter 0). It will therefore be omitted in the following.

CHAPTER

A→7 5
B→6
D→7 5
E→6
G→9
L→22
W→4
X→6
V→9
Z→5
U→6

1) NEWLINE
NEWLINE
SPACE
SPACE
SPACE
620, 27
620, 15
620, 18
620, 4
620, 0
620, 28
I=0(1) 8
SPACE
REPEAT
620, 27
620, 22
620, 1
620, 18
620, 0
620, 28
NEWLINE
NEWLINE

5) P=1
4) N=7

620,5
 620,27 .
 620,9
 620,0
 620,23
 620,27 .
 620,9
 620,0
 620,16
 620,6
 NEWLINE
 NEWLINE

This concludes sub-programme 111

32) X7 (Dn) Z0,0
 F=X PARITY(T)
 JUMP52,F>0
 JUMP53
 52) ACROSS19/2-111
 53) JUMP54,C=0

I=0(1)12
 Y1=0
 Z5=0.02+0.08I
 PRINT(Z5)2,2
 Z5=XLOG(1/Z5-1)
 J=T(-1)0
 Y1=ZJ+Y1Z5
 FEFEAT
 PRINT(Y1)4,5
 NEWLINE
 FEFEAT

54)UP

PSA
 XLOG
 CLOSE

PROG RAMME -I

CHAPTE R1

A→57 .
B→57 .
Z→57 .
Y→3
E→16
U→1 39
X→1 39
**W→3

1)X6(0)A0,L
X6(0+L)B0,L
X6(0+**2L)Z0,L

**Read observed intensities, corresponding
x values and extrapolation indicators from
store**

P=0
M=N+2
W=0
Un=0

Repeat for each observed point

I=2(I)M
J=0(I)3
Q=I+J
EJ=B(Q-2)
YJ=A(Q-2)
FE FE AT
E15=Z1
E16=Z(1-I)

Choose four pairs of intensity and x values

**Indicates whether either of the inner pair of
values represent extrapolated points**

J=4(I)10
EJ=0
FE FE AT

Weights for the four observed points

Z=0
Y=0
W0=0.66
W1=1.34
W2=1.34
W3=0.66

J=O(1) 3
 Z=Z+LJ
 Y=Y+YJ
 REPEAT
 Z=Z/4
 Y=Y/4

Transform the coordinates of the four points to their centre of gravity

Coordinates of the centre of gravity

2) J=O(1) 3
 LJ=LJ-Z
 YJ=YJ-Y
 L4=L4+LJWJ*
 L5=L5+LJLJWJ
 L6=L6+LJLJLJWJ
 L7=L7+LJLJLJLJWJ
 L8=L8+YJWJ
 L9=L9+LJYJWJ
 LI0=LI0+LJLJYJWJ
 REPEAT

Transformed coordinates

Least square solution for second order polynomial ($y = Ax^2 + Bx + C$) through the four points

LI1=4L7L5-4L6L6+2L6L5L4-L7L4L4-L5L5L5
 LI2=4LI0L5-4L9L6-LI0L4L4+L6L4L8+L5L9L4-L5L5L8
 LI3=4L9L7-L7L4L8-4LI0L6+LI0L5L4+L5L6L8-L5L5L9
 LI4=L7L5L8-L7L4L9-L6L6L8+L6L9L5+LI0L6L4-LI0L5L5

LI0=XMOD(LI1)
 JMP30, 0, 000001 > LI0
 LI2=LI2/LI1
 LI3=LI3/LI1
 LI4=LI4/LI1

Coefficient determinant for the normal equations

Polynomial coefficients A, B, and C respectively

JMP3
 30) L0=999
 PRINT(L0) 3, 0
 NEWLINE
 NEWLINE
 UP

Print 999 if solution is singular and return to chapter 0

INTERPOLATION OF OBSERVED INTENSITY
 (at integral x values)

3) Yn=L2+Z
 X=P
 JMP5, X>Yn
 U=P-Z
 UP=L12UU+L13U+L14*+Y
 V=L12B(1-I)B(1-I)-2L12ZB(1-I)+ZZL12+L13B(1-I)-L13Z+L14-A(1-I)+Y
 V=XMOD(V)
 U=L12BIBI-2L12ZBI+ZZL12+L13BI-L13Z+L14-AI+Y
 U=XMOD(U)
 V=U/2+V/2
 JMP12, V>Un

Check that the integral x value in question lies between the two central observed points

Compute intensity at x (Up) and also find the mean difference (V) between the observed and computed values for the two central observed points. This is an estimate of the fitting error

```

JUMPI 3
12)  $U_n = V$ 
13)  $W = W + VV$ 
4) JUMP6,  $\varepsilon_{15} > 0$ 
JUMP6,  $\varepsilon_{16} > 0$ 
 $XP = 0$ 
JUMP7
6)  $XP = 1$ 
7)  $P = P + 1$ 
 $X = P$ 
JUMP5,  $X > Y_n$ 
JUMP3
5) REPEAT

 $Y = N + 1$ 
 $X = *X \text{ SQRT}(W/Y**)$ 
 $H_n = X*$ 
NEWLINE
NEWLINE
620, 27
620, 12
620, 0
620, 28
620, 27
620, 19
620, 0
620, 28
SPACE
620, 27
620, 6
620, 9
620, 20
620, 0
PRINT(X) 4, 4
PRINT( $U_n$ ) 3, 4
NEWLINE

```

```

 $Q = X \text{INTPT}(B(N+1) + G**)$ 
JUMP10,  $B_n = 0$ 

```

```

NEWLINE
NEWLINE

```

```

620, 27
620, 19
620, 13

```

Choose the maximum value of the fitting error
Sum the squares of the fitting errors for each x value

Set the extrapolation indicators at one for extrapolated points

Move to next integral value of x

Interpolate intensity at x either with the same polynomial as above, or with one calculated using a new set of four observed points

Find the root mean square of the fitting error and store this value

Print L.S. FIT

Print root mean square of fitting error
Print maximum fitting error

Largest x value for which an intensity was interpolated. Omit print out if instructed

Print SMOOTH CURVE

620,15
 620,15
 620,20
 620,8
 620,0
 SPACE
 620,27
 620,3
 620,21
 620,18
 620,22
 620,5
 620,0

NEWLINE
 NEWLINE
 P=0(1)Q
 PRINT(P)3,0
 PRINT(UP)4,4
 JUMP8,XP>0
 JUMP9
 8) SPACE
 620,3
 9)NEWLINE
 REPEAT
 NEWLINE

**10) Q=Q+1
 X7(0)U0,Q
 X7(0+140%)X0,Q
 ACROSS2/2-1

PSA
 XSQRT
 CLOSE

Print integral x value (units are Δx)
 Print interpolated intensity

Print an asterick after intensity if
 corresponding extrapolation indicator is
 set at one

Store interpolated intensities and
 extrapolation indicators

CHAPTER2

A→179
 B→139
 C→139

```

1) READ( P)
P=P-I
I=O(I) P
READ( A1 )
REPEAT

```

Most of this chapter is the same as
chapter 1, programme OBS 11 - E

```

P=P+I
JUMP 13, P>179
I=P(I) 179
A1=0
REPEAT

```

```

13) V=0
I=O(I) 179
V=V+A1
REPEAT
I=O(I) 179
A1=A1/V
REPEAT

```

```

X7( 400) A0, 180

```

```

UP

```

```

2) R=XINTPT( 70.10-Q/2 )
X6( 0) BR, Q
X6( 400) A0, 180
M=R-I
JUMP 40, *0>M
I=O(I) M
BI=0
REPEAT
40) L=R+Q
JUMP 41, L>139
I=L(I) 139
BI=0
REPEAT
41) X7( 0) B0, 140
L=L-I

```

```

W=0
I=O(I) 139
W=W+BI
REPEAT
NEWLINE
NEWLINE
620, 27 .
620, 5

```

Add intensity values to give equivalent
width (in units of Δx)

620, 17 ,
620, 21 :
620, 9
620, 22
620, 28
SPACE

Print EQUIV. WIDTH

620, 27 ,
620, 23
620, 9
620, 4
620, 20
620, 8-

620, 0
PRINT(W) 4, 3
NEW LINE
NEW LINE

Print the equivalent width

N=Q-1
K=XMAX(A0, 0, 17 9)

Z=1 000
P=0-----
Zn=5Hn
Yn=5A

5) W=0
Un=0-
P=P+1
002 (P)
J=0(1) 1 39
X=0-

I=J(1) 1 39
T=K+1-J
JUMP 3, T>17 9
X=X+BIAT
3) REPEAT

Y=0
S=J-1
JUMP 14, J=0

I=0(1) S
T=K-J+1
JUMP 4, 0>T
Y=Y+BIAT
4) REPEAT

```

14) CJ=X+Y
REPEAT
X6(0) A0, 140
J=0(1) 139
U=XMOD(AJ-CJ)
JUMP17, M>J
JUMP17, J>L
W=W+U
JUMP17, Un>U
Un=U
17) REPEAT

```

```

W=W/Q
Wn=5W
610(1)
JUMP8, o>1
HOOT
JUMP8, W>Z
Z=W
JUMP20, Un>Wn
JUMP18, Un>Zn
JUMP7
18) JUMP15, Un>Yn
JUMP7
20) JUMP6, U>Hn
JUMP7
6) JUMP15, W>A
7) NEWLINE
NEWLINE
620, 27
620, 9
620, 20
620, 5
620, 18
620, 28
620, 0
PRINT(P) 2, 0
SPACE
SPACE
620, 27
620, 5
620, 18
620, 18
620, 28
620, 0
PRINT(W) 2, 4
SPACE
620, 27

```

The termination criteria are the same as in OBS 11 - E except that the larger of the preset error limit, E, and the root mean square fitting error is taken as the allowed mean deviation

```

620, I 3
PRINT(Un) 2, 4
NEWL INE
NEWL INE
JUMP I 6
* I 5) J=0( I ) I 39
BJ=BJ+AJ-CJ
FE FE AT
X6( 400) A0, I 80
JUMP 5
8) NEWL INE
NEWL INE
620, 27 .
620, 2
620, 5
620, I 9
620, 20
SPACE
PRINT(Z) 2, 4
PRINT(W) 2, 4
620, 0
620, 5
PRINT( P) 2, 0
620, 0
620, 6
NEWL INE
NEWL INE
I 6) X6( 0+I 40) C0, Q
P=XMIN( B0, R, L)
NEWL INE
620, 27 .
620, I 3
620, 9
620, I 4
620, 28
SPACE
620, 27 .
620, I 6
620, 5
620, I
620, I I
620, 0
PRINT(BP) 3, 3
NEWL INE
NEWL INE
JUMP I 0, Cn=0

```

Read extrapolation indicators from store

Select the minimum intensity value in derived true curve

Print MIN. PEAK

Print the minimum intensity in the true profile

NEWLINE
620,27
620,6
620,9
620,14
620,1
620,12
SPACE
620,27
620,3
620,21
620,18
620,22
620,5
620,0
NEWLINE
NEWLINE

JUMP10, Vn=0
I=XINTPT(Vn+0.1)
S=XINTPT(L/Vn-R/Vn+0.01)

Omit print out if the tabulation interval = 0
Tabulation interval (units of Δx)

K=R+S
P=I
I=R(P)K
S=I-R

PRINT(S)2,0
PRINT(BI)4,4
JUMP11,CS>0
JUMP12

Print x value in units of Δx
Print true intensity values

11) SPACE
620,3
12)NEWLINE
REPEAT
NEWLINE

Print an asterisk if the corresponding
intensity has been derived from extrapolated
data

10)X7(0)BR,Q
ACROSSI/3-1

Store the computed true profile

PSA
CLOSE

CHAPTER 3

```

A→5 0
B→1 39
C→5 0
D→1 39
E→5 0
L→1 8
Y→3
W→3

1) X6(0) B0, Q      Read the true line profile from store
K=XMAX(B0, 0, N)     Find the maximum intensity value in the
                     profile

JUMP4 8, B(K+1) > B(K-1)
JUMP4 9              Ensure that two largest values are at K-1
48) K=K+1            and K

49) L0=K-2
L1=K-1               Choose four points near the peak to be fitted
L2=K                  by a least square polynomial
L3=K+1

Y0=B(K-2)
Y1=B(K-1)
Y2=BK
Y3=B(K+1)

T=19                Find the least square quadratic polynomial
DOWN 1 3/4-1        through the four points

19) Xn=-L1 3/L1 2    x value of peak (in transformed coordinates)
Yn=0.5L1 2XnXn+L1 3Xn+L1 4+Y    T(x) at the peak
W=1-Yn              Residual intensity at the peak (central intensity)

NEWLINE
NEWLINE
620*, 27 .
620, 3
620, 5
620, 14
620, 20
620, 28
SPACE
620, 27 .
620, 9
620, 14

```

Print CENT. INT.

620,20%
 620,28
 PRINT(W)4,3
 620,0
 620,5
 PRINT(X_n+Z)3,3
 620,0**
 620,%6
 NEWLINE
 NEWLINE

Print central intensity

Print x value of peak in parentheses

K=0
 A0=0.5Y_n

Half intensity level in the profile

620,27.
 620,8
 620,1
 620,12
 %620,6
 SPACE

Print HALF WIDTH

620,27.
 620,23
 620,9
 620,4
 620,20
 620,8
 620,0
 SPACE

JUMP41
 42)PRINT(V)4,3
 NEWLINE

Compute half width

Print half width (in units of Δx)

NEWLINE
 JUMP36,C_n=1
 NEWLINE
 NEWLINE

Omit asymmetry determination if instructed

620,27.
 620,1
 620,19
 620,25
 620,13
 620,13*

Print ASYMMETRY

620,5
 620,20
 620,18
 620,25
 620,0
 NEWLINE
 NEWLINE

K=I(1)50

AK=0.02K
JUMP36, AK>Yn

Specified intensity level, $I(x) = \text{Constant}$
Jump 36) if I exceeds the peak intensity

41) I=0(I)N
DI=0
REPEAT
P=0
X=X SIGN (AK-B0)
JUMP2, X>0
3) Y=X
X=X SIGN (AK-BN)
JUMP7, X>0
JUMP8

Check that the true profile, $T(x)$, is less
than I at both ends of the profile

2) I=0(I)N
Y=X
X=X SIGN (AK-BI)
JUMP4, Y≠X
JUMP5
4) DP=I
P=P+1
5) REPEAT
JUMP6, 0>X
JUMP38

Whenever $T(x) - I$ changes sign, place the
corresponding x value in D_p . P counts the
number of such points (less one) in the
profile

7) 620, 27
620, 14
620, 5
620, 7
620, 28
NEWLINE

If $T(x)$ is greater than I at the violet end
of the profile, print NEG.

JUMP37
6) 620, 27
620, 16
620, 15
620, 19
620, 28
NEWLINE

If $T(x)$ is greater than I at the red end of
the profile, print POS.

JUMP37
8) 620, 27
620, 14
620, 15
620, 14
620, 5
NEWLINE

If $T(x)$ is greater than I at both ends of the
profile, print NONE

JUMP37
38) R=XMAX(D0, 0, N)
JUMP37, R=0
R=XINTPT(0.5 R-0.40)
3=0(I)R

Number of points less one at which $T(x) = I$

R+1 = number of peaks in profile

P=2S
 T=P+1
 L=XINTPT(DP+0.10)
 M=XINTPT(DT+0.10)
 U=0

For each peak, L is the x value at the violet side and M that at the red side at the intensity level, I

JUMP9,L=1
 11)E0=L-2
 E1=L-1
 E2=L
 E3=L+1

Select x and y (T(x)) values for a least square determination of the exact x at which $T(x) = I$ on the violet side of the peak. The equation $y = Ax^2 + Bx + C$ is fitted to the four points about the position at which $T(x) = I$

Y0=B(L-2)
 Y1=B(L-1)
 Y2=EL
 Y3=B(L+1)

JUMP10
 9)L=L+1
 JUMP11
 10)T=12
 DOWN I *3/4-1
 JUMP21,U=1
 12)E18=X

If L = 1, use points 0,1,2,3 instead of -1,0,1,2

T is a control index

Down to least square sub-chapter

Exact x value at violet side of peak

JUMP14,M=N
 15)E0=M-2
 E1=M-1
 E2=M
 E3=M+1

Repeat the above for the red side of the peak

*Y0=B(M-2)
 Y1=B(M-1)
 Y2=BM
 Y3=B(M+1)
 JUMP16

14)M=M-1
 JUMP15
 16)T=17
 DOWN I 3/4-1
 JUMP21,U=1
 **

17)CS=X-E18
 ES=E18+0.50X-0.50E18
 21)REPEAT

Width of the peak (in units of Δx)
 Centre of the peak at intensity level I

U=0

```

V=0
S=0(I) R
U=U+CSE S
V=V+CS
REPEAT
  JUMP 42, K=0
  JUMP 39, 0.0000*I>V
  U=U/V
  JUMP 40
39) U=0
40) PRINT(AK) 2, 2
  PRINT(U) 3, 3
  JUMP 34, R>0
  JUMP 35
34) S=I(I) R
620, 0
620, 3
REPEAT
35) NEWLINE
37) REPEAT JUMP 42, K=0
NEWLINE

```

Repeat for each peak at the level in question

Jump 42) if half width is being determined

Weighted centre of profile at level I

Print intensity level I

Print $\bar{x}$ value of line centre

Print an asterisk after the $\bar{x}$ value of the line centre for each peak in addition to the main profile peak at the level I

36) UP

```

PSA
XSQRT
CLOSE

```

CHAPTER 4

```

A→50
B→139
C→50
D→139
E→50
F→18
Y→3
W→3

```

Least square sub-chapter. Similar to least square routine in chapter 1, programme - 1

```

I3) J=4(I) I0
LJ=0
REPEAT

```

Z=0

Y=0
W0=0.66
W1=1.34
W2=1.34
W3=0.66
J=0(1) 3
Z=Z+EJ
Y=Y+YJ
REPEAT
Z=Z/4
Y=Y/4

Weights for the four pints

Transformation to centre of gravity of the four points

22) J=0(1) 3
EJ=EJ-Z
YJ=YJ-Y
E4=E4+EJWJ*
E5=E5+EJEJWJ
E6=E6+EJEJEJWJ
E7=E7+EJEJEJEJWJ
E8=E8+YJWJ
E9=E9+EJYJWJ
E10=E10+EJEJYJWJ
REPEAT

Least square calculations for $y = Ax^2 + Bx + C$

E11=4E7E5-4E6E6+2E6E5E4-E7E4E4-E5E5E5
E12=4E10E5-4E9E6-E10E4E4+E6E4E8+E5E9E4-E5E5E8
E13=4E9E7-E7E4E8-4E10E6+E10E5E4+E5E6E8-E5E5E9
E14=E7E5E8-E7E4E9-E6E6E8+E6E9E5+E10E6E4-E10E5E5

Solution of normal equations

E10=XMOD(E11)
JUMP30,0.000001>E10
E12=E12/E11
E13=E13/E11
E14=E14/E11

Polynomial coefficients A, B, and C

JUMP2 3
30)6 20,0
620,26
U=1
ES=0
CS=0
UP
*2 3)E12=2E12
JUMP32,T=19

If solution is singular (coefficient determinant is zero), set U=1, print + sign and return to main chapter

2A
Refers to the determination of the peak of the profile

JUMP 33

32) UP

33) $E_{15} = X \text{ SQRT}(E_{13}E_{13} - 2E_{12}E_{14} + 2E_{12}AK - 2E_{12}Y)$

JUMP 39

100) 620, 31

U=1

ES=0

CS=0

UP

Rescue routine. If roots are complex, set
U=1, print erase symbol and return to main
chapter

39) $E_{16} = -E_{13}/E_{12} + E_{15}/E_{12}$ $E_{17} = -E_{13}/E_{12} - E_{15}/E_{12}$

Two real roots of the equation for
 $T(x) = I$

JUMP 24, $E_{17} > E_{16}$ $E_0 = E_{16}$ $E_{16} = E_{17}$ Set x_{17} equal to the larger root $E_{17} = E_0$ 24) CHECK($E_1, E_{16}, 2.0, 25$)

JUMP 27

25) CHECK($E_1, E_{17}, 2.0, 26$)

JUMP 29

27) CHECK($E_1, E_{17}, 2.0, 28$)

620, 0

620, 24

U=1

ES=0

CS=0

UP

28) $X = E_{17} + Z$

Select larger root

UP

29) $X = E_{16} + Z$

Select smaller root

UP

26) JUMP 28, $T = 17$

JUMP 29

If both roots are within 2 of the second
x value, select the smaller root for the
violet side of the peak and the larger one
for the red side.

PSA

CLOSE

CHAPTE R₀

A→25
 B→57
 C→75
 D→5
 E→5
 F→57
 G→57
 H→57
 Z→57
 X→75

1) DOWN 1/2-1
 2) READ(B)
 READ(C)
 AN 0
 CN 14B+90
 DN 30B+90
 Q=X IN TPT(B+0.5)
 T=7 Q

I=1(I) T
 READ(XI)
 READ(CI)
 REPEAT

S=2 Q

I=1(I) S
 READ(AI)
 REPEAT

X7(Cn) XI, T
 X7(Cn+T) CI, T
 X7(Cn+*2T) AI, S

DOWN 1/3-111

NEWLINE
 X6(Dn) D₀, 0
 X6(Cn+14B) E₀, 2
 Fn=E₀
 Gn=E₁

Read apparatus function
 Read data for characteristic curve
 determination (see chapter 0, programme
 OBS 11 - 111)

Compute characteristic curve

Read characteristic curve coefficients from store
 Read minimum and maximum $\ln(y_0/y - 1)$'s from
 store

8) READ (An)
 $An = An + 0.20$
 $H = 1$

Read number of profiles to be reduced

12) READ (Bn)
 READ (Cn)
 READ (Vn)
 READ (A)
 READ (N)
 READ (E₂)
 READ (E₃)
 READ (E₄)

Read three control integers indicating
 sequence of computations to be performed

Read preset error limit for iteration
 Read number of points in observed profile
 Read zero reading for microphotometer tracing
 Read continuum reading
 Read clear plate reading
 Read tracing dispersion (in terms of Δx)

READ (G)
 $En = \text{XEXP}(Fn)$
 $En = En + 1$
 $En = 1 / En$

Parity of adopted characteristic curve
 polynomial

$X_0 = \text{XPARITY}(T)$

y/y_0 for the continuum

$E_4 = E_2 - E_4$
 $E_3 = E_2 / E_4 - E_3 / E_4$
 JUMP 5, $E_3 > E_2$
 40) $E_3 = \text{XLOG}(1 / E_3 - 1)$

Compute $\ln(I/I_0)$ for the continuum (if the
 y/y_0 for the continuum is greater than the
 maximum y/y_0 entering into the characteristic
 curve determination, then use the extrapolation
 curve corresponding to the parity of the
 characteristic curve)

$V = 0$
 $J = T(-1)_0$
 $V = DJ + VE_3$
 READ AT
 $E_3 = V$
 JUMP 16
 15) JUMP 40, $0 > X_0$
 $E_3 = \text{XLOG}(1 - E_3)$
 $E_3 = DE_3 + E$

Read Y } for the observed points
 Read X }
 Compute y/y_0 for the observed points

Compute $\ln(I/I_0)$ for the observed points

16) $I = I(1)^N$
 $V = 0$
 READ (BI)
 READ (FI)
 $BI = E_2 / E_4 - BI / E_4$
 JUMP 7, $BI > E_2$
 41) $BI = \text{XLOG}(1 / BI - 1)$
 $J = T(-1)_0$
 $V = DJ + VBI$
 READ AT
 JUMP 9
 7) JUMP 41, $0 > X_0$

```

BI=XLOG(I-BI)
V=DBI+E
9) X=V-E3
X=XEXP(X)
JUMP3, F#>BI
JUMP3, BI>G1
Z(I+I)=0
BI=X
JUMP4
3) Z(I+I)=I
BI=X
4) JUMP10, B#≠2
PRINT(BI) 2,4
JUMP5, Z(I+I)>0
JUMP6
5) SPACE
620, 3
6) NEWLINE
10) REPEAT

```

```

K=XMAX(B0, I, N)
B=BK
I=I(I)N
JUMP20, I0>B
BI=BI/B
20) J=I+I
HJ=GFI
GJ=I-BI
REPEAT

```

```

NEWLINE
G0=0
G1=0
G(N+2)=0
G(N+3)=0
Z0=I
Z1=I
Z(N+2)=I
Z(N+3)=I
H0=-G
H1=0
H(N+2)=GFN+G
H(N+3)=GFN+2G

```

```

L=N+4
X7(0) D0, 0
X7(0) G0, L

```

Compute residual intensities for observed points

Set extrapolation indicator equal to one if y/y_0 of the observed point lies outside the range of observations used in the characteristic curve determination, otherwise set it at zero

Control jump concerning print out
Print the observed residual intensities

Print an asterisk if the residual intensity relies upon an extrapolation of data

Find the maximum observed residual intensity

If this value exceeds one, adjust all residual intensity values so that the maximum value is one

Store x values in units of Δx
Store the observed profile intensities (expressed as an emission profile)

Set values of intensity at both ends of the profile for extrapolation purposes

Extra x values for extrapolation

Store the characteristic curve coefficients, observed intensities, corresponding x values, and extrapolation indicators

X7 (O+L) H₀, L
X7 (O+2L) Z₀, L

DOWN I / I - I

I I) H = H + I
JUMP I 3, H > A n
X6 (o) D₀, 0
T = 0 - I
JUMP I 2
I 3) JUMP 2

PSA
X LOG
X EXP
CLOSE
→
-

Across to programme 1

Check whether all profiles have been
reduced, and store characteristic curve
coefficients

ACKNOWLEDGEMENTS

I wish to express my gratitude to Dr M. G. Adam for her continual guidance and inspiration throughout this investigation, and for her assistance in making many of the observations. In addition I wish to thank Professor H. H. Blaskett for the many helpful discussions we have had together on the limb-effect problem, especially for his suggestion of chromospheric effects as presented in Chapter 5. Finally I should like to thank the staff of the Oxford University Computing Laboratory for their assistance with the computational problems, especially Dr J. S. Rollett whose assistance has been invaluable, and the Rhodes Trustees for the scholarship that has made this research possible.

L. A. H.