

# Positive Trend Inflation and Determinacy in a Medium-Sized New Keynesian Model\*

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## Abstract

This paper studies the challenge that increasing the inflation target poses to equilibrium determinacy in a medium-sized New Keynesian model without indexation fitted to the Great Moderation era. For moderate targets of the inflation rate, such as 2 or 4 percent, the probability of determinacy is near one conditional on the monetary policy rule of the estimated model. However, this probability drops significantly conditional on model-free estimates of the monetary policy rule based on real-time data. The difference is driven by the larger response of the federal funds rate to the output gap associated with the latter estimates.

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# 1 Introduction

In response to the Great Recession, there has been a call to rethink macroeconomic policy. An element of this proposal is the possibility of increasing the inflation target from 2 to 4 percent; see [Blanchard, Dell’Ariccia and Mauro \(2010\)](#), [Krugman \(2013\)](#), and [Ball \(2014\)](#). The rationale for a higher inflation target is that it would strengthen the capacity of the Federal Reserve to reduce interest rates when economic conditions deteriorate.

Increasing the inflation target, and thereby the level of trend inflation, raises questions about its costs and challenges. This paper focuses on the particular challenge that increasing the inflation target poses to equilibrium determinacy, a key indicator of the underlying ability of central banks to anchor inflation expectations and avoid self-fulfilling economic fluctuations.<sup>1,2</sup> Previous research by [Hornstein and Wolman \(2005\)](#), [Kiley \(2007\)](#), and [Ascari and Ropele \(2009\)](#) shows that the Taylor principle, which says that central banks should respond more than one for one to inflation, is not enough to guarantee determinacy when trend inflation is positive. More recently, [Coibion and Gorodnichenko \(2011\)](#) provide empirical support for this theoretical finding in the context of a calibrated small-sized New Keynesian model of the U.S. economy. [Hirose, Kurozumi and Van Zandweghe \(2017\)](#) present additional empirical evidence on the failure of the Taylor principle and its implications for the U.S. economy using an estimated small-sized New Keynesian model.

This paper contributes to this literature by examining the relation between positive trend inflation and determinacy in the U.S. through the lens of an estimated medium-sized New Keynesian model, which constitutes the backbone of several DSGE models used for monetary policy analysis. This class of models includes crucial features to understand business cycle dynamics and the effects of monetary policy—such as capital accumulation, investment adjustment costs, and capital

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<sup>1</sup>Equilibrium determinacy refers to the existence of a locally unique solution in a linear rational expectations model. Henceforward, we make reference to this equilibrium concept as determinacy. Self-fulfilling fluctuations (also called sunspot shocks) arise when there is indeterminacy, that is, the existence of multiple solutions in a linear rational expectations model. It is important to note that this paper abstracts from considering the type of indeterminacy associated with the zero lower bound constraint on interest rates. For an analysis of this relevant issue, see [Benhabib, Schmitt-Grohé and Uribe \(2001\)](#), [Mertens and Ravn \(2014\)](#), and [Aruoba, Cuba-Borda and Schorfheide \(2017\)](#).

<sup>2</sup>See [Ascari, Phaneuf and Sims \(2015\)](#) and [Blanco \(2017\)](#) for a comprehensive welfare based analysis of the benefits and costs associated with a higher inflation target.

utilization (e.g., [Christiano, Eichenbaum and Evans \(2005\)](#), [Smets and Wouters \(2007\)](#), [Justiniano and Primiceri \(2008\)](#), and [Altig et al. \(2011\)](#)). To this end, we begin by estimating an off-the-shelf version of that class of models by using U.S. data at quarterly frequency for the period 1984Q1–2008Q2, a sample characterized by low inflation and stable economic conditions. Thus, the model provides an empirically credible framework suited to quantitatively investigate the extent to which an increase in trend inflation to 4 percent could lead to self-fulfilling fluctuations in the U.S. economy.

We offer three contributions to the literature. First, we quantify the extent to which the model-implied probability of determinacy decreases with trend inflation in an estimated medium-sized model. Conditional on our estimated model—and policy rule—an increase in trend inflation to 4 percent would be unlikely to lead the U.S. economy to experience indeterminacy. The probability of determinacy is near one for levels of trend inflation as high as 4 percent—the value suggested by several of the recent proposals to increase the inflation target.

Second, we revisit the relation between the systematic component of monetary policy, trend inflation and determinacy. This is important because the reaction of monetary policy to economic conditions is the main factor underlying the equilibrium properties of New Keynesian models. Despite some quantitative differences, our analysis qualitatively confirms the main lessons of a large literature examining the existence of equilibrium in small-sized New Keynesian models.<sup>3</sup> These lessons can be summarized as follows: i) the response of the federal funds rate to inflation that is necessary to achieve determinacy is increasing in the level of trend inflation; ii) responding to the output gap is destabilizing, i.e., it leads to indeterminacy; iii) responding to output growth is stabilizing, i.e., it leads to determinacy; iv) monetary policy inertia is stabilizing; and v) when central banks respond to expected inflation, both weak responses as well as strong responses to expected inflation are destabilizing unless there is strong monetary policy inertia. Our simulations show that responding to the output gap is particularly dangerous for determinacy in our estimated model.

Third, we study the model-implied probability of determinacy at each FOMC meeting since the

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<sup>3</sup>For a recent survey of this literature, see [Ascari and Sbordone \(2014\)](#).

1970s given the levels of trend inflation and the systematic component of monetary policy that were likely to have prevailed at the time of those meetings, through a series of counterfactuals based on [Coibion and Gorodnichenko \(2011\)](#). [Coibion and Gorodnichenko \(2011\)](#) compute real-time estimates of the Taylor rule, allowing for time-varying coefficients and a time-dependent trend inflation, which then they feed to a calibrated small-sized New Keynesian model. We complement their analysis by feeding their estimates into our estimated medium-sized New Keynesian model.

The resulting time-varying probability of determinacy provides additional evidence to support the view put forward by [Clarida, Galí and Gertler \(2000\)](#) and [Lubik and Schorfheide \(2004\)](#) that the U.S. economy was more likely under indeterminacy during the 1970s and under determinacy in the post-Volcker era. In addition, our simulations show that changes in monetary policy as well as changes in the level of trend inflation play important roles in delivering this outcome—coherently with the results in small-sized models in [Coibion and Gorodnichenko \(2011\)](#) and more recently in [Hirose, Kurozumi and Van Zandweghe \(2017\)](#). In particular, high trend inflation kept the U.S. economy subject to self-fulfilling fluctuations until 1983, despite the switch in policy happening two years earlier, while low trend inflation in the 1990s was a major factor behind the high likelihood of the U.S. economy being in a determinate equilibrium.

The paper is organized as follows. Section [2](#) presents the model and its estimation. Section [3](#) examines the relation among determinacy, monetary policy and positive trend inflation in our estimated model. Section [4](#) revisits the analysis of Section [3](#) when using real-time measures of the systematic component of monetary policy. Section [5](#) concludes.

## 2 Model and Estimation

### 2.1 Model

The model is taken off the shelf from the New Keynesian literature ([Yun \(1996\)](#), [Christiano, Eichenbaum and Evans \(2005\)](#) and [Smets and Wouters \(2007\)](#)) except that we do not allow for indexation. This assumption is crucial for our analysis, and there are two reasons for our choice. First, there is no strong evidence to support the presence of price or wage indexation; see [Coibion](#)

and Gorodnichenko (2011) and Christiano, Eichenbaum and Trabandt (2016). Second, this is a standard assumption in the literature studying the macroeconomic effects of positive trend inflation. Because the model is otherwise standard, it suffices for our purposes to briefly describe its main features. The economy evolves in discrete time  $t$ , and it is populated by firms, households, financial intermediaries, and a government.

### 2.1.1 Firms

**Final-Good Firms** A final good  $Y_t^d$  is produced by final-good firms, using a continuum of intermediate goods  $Y_{it}$ , and it is sold to households in a competitive market at a price  $P_t$ . The production function used to produce final goods features constant returns to scale, and it is of the Dixit-Stiglitz form with elasticity of substitution between inputs equal to  $\eta_p$ . The final-good firms purchase inputs  $Y_{it}$  in a monopolistically competitive market at price  $P_{it}$ , and they maximize profits subject to a standard CES production function; the optimal demand for each intermediate good is increasing in the quantity of final goods and decreasing with respect to its price, that is:

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta_p} Y_t^d.$$

**Intermediate-Good Firms** A continuum of intermediate-good firms indexed by  $i$  supplies intermediate goods to the final-good firms. When setting prices, intermediate-good firms are subject to a nominal friction in the style of Calvo. In particular, at any given time, a fraction  $\nu_p$  of them are not able to change prices. Each intermediate-good firm faces the demand function  $Y_{it}$  described above. The production function of intermediate-good producers is Cobb-Douglas with a fixed cost of production  $\Phi$ :

$$Y_{it} = A_t (K_{it}^d)^\alpha (L_{it}^d)^{1-\alpha} - \Phi Z_t.$$

The fixed cost is scaled by a composite index of technological progress  $Z_t = A_t^{\frac{1}{1-\alpha}} \mu_t^{\frac{\alpha}{1-\alpha}}$ , which is determined by the weighted product of a unit root stochastic process for neutral technological change  $A_t$  and a unit root stochastic process for investment-specific technological change  $\mu_t$ .

Intermediate-good firms face perfectly competitive factor markets. Taking factor prices as given,

each firm selects the amount of labor  $L_{it}^d$  and capital  $K_{it}^d$  that minimizes the cost of producing output  $Y_{it}$ . Firms are subject to a working capital constraint; they must take a loan from financial intermediaries at a borrowing cost equal to  $R_t$  to pay workers in advance of production.

Before concluding the description of the intermediate-good firms, it is worth noticing that while analytically the expression for the non-linear first order condition (FOC) characterizing the price setting behavior of a firm under the case of no indexation is nearly identical to the one under the case of full indexation, there are significant economic differences in the price setting behavior of firms between these cases. To see this, consider the problem that each intermediate good producer faces when setting the price of its good. For our purposes, it is useful to assume that the price set by those firms that are not able to change prices equals  $P_{it} = P_{it-1}\Pi^{\chi_p}$ , where  $\Pi$  is the steady state level of gross inflation and  $\chi_p$  is a parameter that can take values in the set  $\{0, 1\}$ . Clearly, the presence of the parameter  $\chi_p$  allows us to consider our benchmark case of no indexation (i.e.,  $\chi_p = 0$ ) as well as the case of full indexation to steady state inflation (i.e.,  $\chi_p = 1$ ).

In this environment, the intermediate good producer  $i$  sets its price  $P_{it}$  in order to maximize the present value of current and future profits

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \nu_p^s \frac{\Xi_{t+s}}{\Xi_t} \left[ \left( \frac{P_{it}\Pi^{s\chi_p}}{P_{t+s}} - \text{MC}_{t+s} \right) \right] Y_{it+s},$$

where  $\beta$  is the subjective discount factor of the households,  $\Xi_{t+s}$  is marginal value of a dollar to the households (treated as exogenous by the firm),  $P_{it}\Pi^{s\chi_p}$  is the price that would prevail in period  $t + s$ ,  $\text{MC}_{t+s}$  is the real marginal cost of producing a unit of the intermediate good, and  $Y_{it+s} = \left( \frac{P_{it}\Pi^{s\chi_p}}{P_{t+s}} \right)^{-\eta_p} Y_{t+s}^d$  is the demand that the producer  $i$  would face in period  $t + s$ . Taking the FOC with respect to  $P_{it}$ , assuming symmetry  $P_{it} = P_t^*$  for all  $i$ , and writing the resulting FOC recursively we obtain

$$\begin{aligned} g_{1t} &= \frac{\eta_p - 1}{\eta_p} g_{2t} \\ g_{1t} &= \Xi_t \text{MC}_t Y_t^d + \beta \nu_p \mathbb{E}_t \left( \frac{\Pi^{\chi_p}}{\Pi_{t+1}} \right)^{-\eta_p} g_{1t+1} \\ g_{2t} &= \Xi_t \Pi_t^* Y_t^d + \beta \nu_p \mathbb{E}_t \left( \frac{\Pi^{\chi_p}}{\Pi_{t+1}} \right)^{(1-\eta_p)} \frac{\Pi_t^*}{\Pi_{t+1}^*} g_{2t+1}, \end{aligned}$$

where  $\Pi_t = P_t/P_{t-1}$  and  $\Pi_t^* = P_t^*/P_t$ .

As mentioned above, while the recursive expressions for the cases of  $\chi_p = 0$  and  $\chi_p = 1$  are similar, the price setting behavior is significantly different. In particular, the first order approximation of the FOC with  $\chi_p = 1$  and with hats denoting log-deviations relative to the steady state implies that

$$\hat{\Pi}_t^* = (1 - \beta\nu_p)\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\nu_p)^s \hat{M}C_{t+s} + \beta\nu_p \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\nu_p)^s \hat{\Pi}_{t+s+1}.$$

In contrast, the first order approximation of the FOC with  $\chi_p = 0$  implies that

$$\begin{aligned} \hat{\Pi}_t^* &= (1 - \beta\nu_p \Pi^{\eta_p}) \mathbb{E}_t \sum_{s=0}^{\infty} \gamma_{p,1}^s \hat{M}C_{t+s} + \mathbb{E}_t \sum_{s=0}^{\infty} ((1 - \beta\nu_p \Pi^{\eta_p}) \gamma_{p,1}^s - (1 - \beta\nu_p \Pi^{\eta_p-1}) \gamma_{p,2}^s) (\hat{\Xi}_{t+s} + \hat{Y}_{t+s}^d) \\ &+ \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\nu_p \Pi^{\eta_p} \eta_p \gamma_{p,1}^s - \beta\nu_p \Pi^{\eta_p-1} (\eta_p - 1) \gamma_{p,2}^s) \mathbb{E}_t \sum_{s=0}^{\infty} \hat{\Pi}_{t+s+1}, \end{aligned}$$

where  $\gamma_{p,1} = \beta\nu_p \Pi^{\eta_p}$  and  $\gamma_{p,2} = \beta\nu_p \Pi^{\eta_p-1}$ .<sup>4</sup> Two clear differences emerge in the case of no indexation relative to the case of full indexation. First, the optimal price set by the firm not only depends on inflation and the marginal cost, but also on output and the marginal value of a dollar to the households. Second, the level of trend inflation directly affects the price setting behaviour of the firm. In general, it can be verified numerically that as the level of trend inflation increases the firm puts more weight on distant future values of marginal costs, inflation, output, and the marginal value of a dollar to the households. Hence, the firm becomes more forward looking. See [Coibion and Gorodnichenko \(2011\)](#) for an insightful discussion of these and other issues in the context of a small-sized New Keynesian model.

### 2.1.2 Households

A continuum of infinitely lived households indexed by  $j$  populates the economy. Each household supplies labor  $L_{jt}$  to the production sector through a representative labor aggregator that combines labor in the same proportion as firms would do. The optimal demand for each type of labor is

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<sup>4</sup>This approximation requires assuming that  $\gamma_{p,1} < 1$  and  $\gamma_{p,2} < 1$ .

proportional to the aggregate labor demand  $L_t^d$ :

$$L_{jt} = \left( \frac{W_{j,t}}{W_t} \right)^{-\eta_w} L_t^d,$$

where  $\eta_w$  denotes the elasticity of substitution between labor types. Households supply labor in a monopolistically competitive market at wage  $W_{jt}$ . When setting wages, they are subject to a nominal friction in the style of Calvo. In particular, at any given time, a fraction  $\nu_w$  of them are not able to change wages. In addition to supplying labor  $L_{jt}$ , households consume  $C_{jt}$ , hold real balances  $q_{jt} = Q_{jt}/P_t$  (where  $Q_{jt}$  stands for nominal balances), and accumulate capital. Households obtain utility from consumption and from holding real balances, and disutility from supplying labor. Households' resources consist of capital income net of capital utilization costs  $R_{jt}^k K_{jt-1}$ , labor income  $W_{jt}L_{jt}$ , firms' profits  $F_{jt}/P_t$ , net lump-sum transfers from the government  $T_{jt}/P_t$ , revenues from interest  $R_t$  earned on deposits  $d_{jt} = (M_{jt-1}/P_t - q_{jt})$  where  $M_{jt-1}$  denotes the stock of money that households hold at the beginning of period  $t$ , and net cash flows obtained by participating in a market for state contingent securities  $A_{jt-1}$ .<sup>5</sup> Households' expenditures consist of consumption, investment in physical capital  $X_{jt}$ , and state contingent securities  $\bar{A}_{jt}$ . In sum, the households' budget constraint boils down to

$$C_{jt} + X_{j,t} + \bar{A}_{jt} + \frac{M_{jt}}{P_t} = R_t d_{jt} + q_{jt} + (r_t U_{jt} - \mu_t^{-1} a(U_{jt})) K_{jt-1} + W_{jt} L_{jt} + \frac{F_{jt}}{P_t} - \frac{T_{jt}}{P_t} + A_{jt-1}.$$

Each household maximizes its utility functional subject to the budget constraint described above. Households' present discounted utility is separable in consumption, labor and real balances, and it is of the form:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left[ \log(C_{jt+s} - b C_{jt+s-1}) - \psi_L d_{L,t+s} \frac{L_{jt+s}^{1+\tau}}{1+\tau} + \psi_q \frac{q_{jt+s}^{1-\sigma_q}}{1-\sigma_q} \right].$$

The stream of utility is discounted by the subjective discount factor. There is habit in consumption

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<sup>5</sup>The net return on capital  $R_{jt}^k = r_t^k U_{jt} - \mu_t^{-1} a(U_{jt})$ , where  $U_{jt}$  denotes capital utilization, consists of two parts: the rate of return of renting capital  $r_t^k U_{jt}$  and capital utilization costs  $\mu_t^{-1} a(U_{jt})$  per unit of physical capital. Capital utilization costs are convex as determined by  $a(U_{jt}) = \gamma_1 (U_{jt} - 1) + \frac{\gamma_2}{2} (U_{jt} - 1)^2$  (with  $\gamma_1 > 0$  and  $\gamma_2 > 0$ ).



governed by the parameter  $b$ . Labor disutility is a function of the inverse of the Frisch-elasticity of labor denoted by  $\tau$ . The parameter  $\psi_L$  is a scale parameter that normalizes hours worked at the steady state. The parameters  $\psi_q$  and  $\sigma_q$  do not affect the equilibrium conditions of the model. Utility flows are subject to preference shocks  $d_t$ , and the disutility for labor is subject to shocks  $d_{L,t}$ , which affect the supply of labor.

The amount of physical capital available for production evolves according to the following law of motion:

$$K_{jt} = (1 - \delta)K_{jt-1} + \mu_t \left( 1 - S\left(\frac{m_{It}X_{jt}}{X_{jt-1}}\right) \right) X_{jt},$$

where  $\delta$  is the depreciation rate,  $m_{It}$  is an exogenous stochastic process for the marginal productivity of investment and  $S(x)$  is an increasing and convex investment adjustment cost function whose specific functional form is  $S(x) = 0.5\kappa(x - \Lambda_z)^2$ , where  $\Lambda_z$  denotes the growth rate of the economy at the steady state.

To conclude the description of the households, it is worth to emphasize that—as it was the case with the intermediate-good firms optimally setting prices—the level of trend inflation directly affects the wage setting behaviour of the households. To see this, consider the problem of a household that optimally sets her wage in period  $t$ . Accordingly, the household chooses the wage  $W_{jt}$  to maximize the present value of her earnings net of the disutility costs associated with labor, that is,

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \nu_w^s \left( -d_{t+s} \psi_L d_{L,t+s} \frac{L_{jt+s}^{1+\tau}}{1+\tau} + \Xi_{jt+s} (\Pi \tilde{Z})^{s\chi_w} \left( \prod_{\ell=1}^s \frac{1}{\Pi_{t+\ell-1}} \right) W_{jt} L_{jt+s} \right),$$

where  $L_{jt+s} = (\Pi \tilde{Z})^{-s\eta_w \chi_w} (\prod_{\ell=1}^s \Pi_{t+\ell})^{\eta_w} W_{jt}^{-\eta_w} W_{t+s}^{\eta_w} L_{t+s}^d$ ,  $\tilde{Z}$  is the steady state level of composite technological progress, and  $\chi_w$  is a parameter that can take values in the set  $\{0, 1\}$ . The parameter  $\chi_w$  allows us to consider our benchmark case of no wage indexation (i.e.,  $\chi_w = 0$ ) as well as the case of full wage indexation to steady state inflation and technological progress (i.e.,  $\chi_w = 1$ ). Up to a first order approximation, assuming symmetry  $W_{jt} = W_t^*$  for all  $j$ , and with hats denoting

log-deviations relative to the steady state, the optimal wage setting behaviour when  $\chi_w = 0$  is

$$\begin{aligned}\hat{W}_t^* &= \frac{\eta_w(1+\tau)}{1+\eta_w\tau} \mathbb{E}_t \sum_{s=0}^{\infty} \left( (1 - \beta\nu_w \Pi^{\eta_w(1+\tau)}) \gamma_{w,1}^s - \frac{(1 - \beta\nu_w \Pi^{\eta_w-1})}{\eta_w(1+\tau)} \gamma_{w,2}^s \right) \left( \tau \eta_w \hat{W}_{t+s} + \tau \hat{L}_{t+s}^d - \hat{\Xi}_{t+s} \right) \\ &+ \frac{\eta_w(1+\tau)}{1+\eta_w\tau} \mathbb{E}_t \sum_{s=0}^{\infty} \left( (1 - \beta\nu_w \Pi^{\eta_w(1+\tau)}) \gamma_{w,1}^s - \frac{(1 - \beta\nu_w \Pi^{\eta_w-1})}{\eta_w(1+\tau)} \gamma_{w,2}^s \right) \left( \eta_w \hat{W}_{t+s} + \hat{L}_{t+s}^d \right) \\ &+ \frac{\eta_w(1+\tau)}{1+\eta_w\tau} \mathbb{E}_t \sum_{s=0}^{\infty} \left( \beta\nu_w \Pi^{\eta_w(1+\tau)} \gamma_{w,1}^s - \frac{\beta\nu_w \Pi^{\eta_w-1}(\eta_w - 1)}{\eta_w(1+\tau)} \gamma_{w,2}^s \right) \hat{\Pi}_{t+s+1},\end{aligned}$$

where  $\gamma_{w,1} = \beta\nu_w \Pi^{\eta_w(1+\tau)}$  and  $\gamma_{w,2} = \beta\nu_w \Pi^{\eta_w-1}$ .<sup>6</sup> The expression for the case in which non-optimizing households index their wages to steady state inflation and technological progress is identical to the one just described except for the fact that  $\Pi$  must be replaced by  $\tilde{Z}^{-1}$ . Consequently, it is only in the case of no indexation that the level of trend inflation directly affects the wage set by households. In general, it can be verified numerically that as the level of trend inflation increases households put more weight on distant future values of wages, labor supplied, inflation, and the marginal value of a dollar.

### 2.1.3 Financial Intermediaries

Financial intermediaries receive funds from households by an amount equal to  $d_{jt} = M_{jt-1} - q_{jt}P_t$ , and then they lend these funds to intermediate-good firms through intra-period loans, which are in turn used to pay for labor services  $L_t^d$ . The interest rate charged on these loans equals  $R_t$ .

### 2.1.4 Government

In the model, the Ricardian equivalence holds, and the government issues risk-free bonds  $B_t$ , collects lump-sum taxes  $T_t$ , and purchases final goods by an exogenous amount  $G_t$ . In addition, the government sets the nominal interest rate  $R_t$  according to a mixed Taylor rule specification as in [Coibion and Gorodnichenko \(2011\)](#):

$$\log(R_t) = c + \rho_{R_1} \log(R_{t-1}) + \rho_{R_2} \log(R_{t-2}) + (1 - \rho_{R_1} - \rho_{R_2}) \left( \psi_\pi \mathbb{E}_t \hat{\Pi}_{t+1} + \psi_y (\hat{y}_t - \hat{y}_t^{fp}) + \psi_{gy} g y_t \right) + \varepsilon_{R,t},$$

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<sup>6</sup>The approximation requires assuming that  $\gamma_{w,1} < 1$  and  $\gamma_{w,2} < 1$ .

where  $\log(R_t)$  denotes the log nominal interest rate,  $c$  is a constant equal to  $\log(R_{ss})(1 - \rho_{R_1} - \rho_{R_2})$ , and  $R_{ss}$  denotes the steady-state nominal interest rate.  $\hat{\Pi}_t$  denotes the inflation rate in deviations from the inflation target,  $\hat{y}_t$  denotes log deviations of output relative to its steady state,  $\hat{y}_t^{fp}$  denotes log deviations of output relative to its steady state in the flexible price economy, and  $gy_t = \hat{y}_t - \hat{y}_{t-1}$  denotes the growth rate of output. The flexible price economy is modeled by removing the nominal frictions.<sup>7</sup>

The nominal interest rate responds to expected future inflation, to the current output gap, and to the current output growth. The magnitudes of these responses are denoted by  $\psi_\pi$ ,  $\psi_y$ , and  $\psi_{gy}$ , respectively. In addition, monetary policy evolves gradually; the degree of inertia is characterized by the coefficients  $\rho_{R_1}$  and  $\rho_{R_2}$ . Finally,  $\varepsilon_{R,t}$  denotes unexpected monetary policy shocks. Our choice of this Taylor rule specification is based on two grounds. First, we want to compare our results with those obtained by [Coibion and Gorodnichenko \(2011\)](#). Second, there are studies, [Coibion and Gorodnichenko \(2012\)](#) and [Ascari, Castelnovo and Rossi \(2011\)](#), showing that this specification is the best fitting among a number of alternatives for the post-World War II U.S. data.

### 2.1.5 Market Clearing and Exogenous Stochastic Processes

The aggregate market-clearing condition implies that production equals aggregate demand scaled by price dispersion  $v_{pt}$ ,  $Y_t = v_{pt}Y_t^d$ . In addition, labor and capital markets clear, i.e.,  $\int L_{j,t}^d dj = L_t^d$  and  $K_t^d = \int K_{i,t}^d di = U_t K_{t-1}$ , where  $U_t$  denotes aggregate capital utilization and  $K_{t-1}$  denotes the aggregate level of physical capital available at period  $t$ .

To conclude, Table 1 specifies the stochastic processes for the exogenous variables in log deviations from the steady state; that is, for each variable  $x$ , we describe the law of motion of  $\hat{x}_t = \log(x_t) - \log(x_{ss})$ . In addition, we let  $\tilde{x}$  denote  $\tilde{x}_t = \frac{x_t}{x_{t-1}}$ .

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<sup>7</sup>Because of the working capital constraint of the intermediate-good firms, when computing the solution to the flexible price economy the standard practice of removing the monetary policy equation does not apply. This is because the value of the nominal interest rate must be pinned-down in order to determine the real marginal cost of intermediate-good producers. Consequently, we determine the value of the nominal interest and thereby inflation by working with the Taylor rule specification described above modified in order that the nominal interest rate reacts to current inflation instead of reacting to expected inflation; otherwise, the solution to the model would be indeterminate. In any case, the results of the paper are almost identical when removing the working capital constraint from the model.

Variable	Stochastic Process
Preference shock	$\hat{d}_t = \rho_d \hat{d}_{t-1} + \sigma_d \varepsilon_{d,t}$
Labor preference shock	$\hat{d}_{L,t} = \rho_{d_L} \hat{d}_{L,t-1} + \sigma_{d_L} \varepsilon_{d_L,t}$
Neutral technological progress	$\hat{\bar{a}}_t = (1 - \rho_{\bar{A}}) \log(\Lambda_{\bar{A}}) + \rho_{\bar{A}} \hat{\bar{a}}_{t-1} + \sigma_A \varepsilon_{\bar{A},t}$
Marginal productivity of investment	$\hat{n}_{I,t} = \rho_{m_I} \hat{n}_{I,t-1} + \sigma_{m_I} \varepsilon_{m_I,t}$
Investment-specific technological progress	$\hat{\mu}_t = (1 - \rho_{\bar{\mu}}) \log(\Lambda_{\bar{\mu}}) + \rho_{\bar{\mu}} \hat{\mu}_{t-1} + \sigma_{\bar{\mu}} \varepsilon_{\bar{\mu},t}$
Government spending	$\hat{g}_t = \rho_G \hat{g}_{t-1} + \sigma_G \varepsilon_{G,t}$

Table 1: Exogenous stochastic processes

## 2.2 Estimation

When studying the equilibrium determinacy of New Keynesian models in the absence of indexation, it is common to work with small-sized calibrated models.<sup>8</sup> The appeal of such an approach is that, in these cases, determinacy can be characterized either analytically (e.g., [Ascari and Ropele \(2009\)](#)) or numerically by a small number of parameters (e.g., [Coibion and Gorodnichenko \(2011\)](#)). Unfortunately, the addition of capital and other typical features of New Keynesian models used for policy analysis—such as capital adjustment costs, capital utilization, sticky wages, and the systematic component of monetary policy—implies that determinacy can no longer be characterized analytically and that it becomes a complicated function of a larger number of structural parameters.

The challenge to characterize the determinacy region of medium-sized New Keynesian models without indexation implies that the standard approach for the estimation of New Keynesian models allowing for indeterminacy, see e.g. [Lubik and Schorfheide \(2004\)](#) and [Justiniano and Primiceri \(2008\)](#), becomes non-trivial. This is because there is no unique threshold around which to center the prior for the parameters characterizing indeterminacy: in our model the degree of indeterminacy is greater than one for most of the parameter space and one cannot simply use a parameter such as the response of the federal funds rate to expected inflation to numerically find the region that separates determinacy from indeterminacy as done in [Justiniano and Primiceri \(2008\)](#).<sup>9</sup>

<sup>8</sup>An exception is [Carlstrom and Fuerst \(2005\)](#), who work with a medium-sized calibrated New Keynesian model.

<sup>9</sup>In previous research, we have estimated our model centering the prior around the orthogonality solution of [Lubik and Schorfheide \(2003\)](#), but unfortunately the convergence properties of the estimated parameters were not

Motivated by the complexity of estimating this type of models under indeterminacy, we take a pragmatic approach. Specifically, we estimate the model described in Section 2.1 conditional on determinacy using quarterly U.S. data for the period 1984Q1–2008Q2 and use the estimation results to discipline our analysis of determinacy. The variables included in the estimation are real GDP growth, real consumption growth, real investment growth, hours worked, the nominal interest rate, GDP deflator inflation, and real wage inflation. Because our estimation procedure closely follows the literature, we relegate details to Appendix A.1.

Parameters	Posterior				Prior		
	Mode	Mean	5 Percent	95 Percent	Distribution	Mean	Std
$\psi_\pi$	2.33	2.42	1.96	2.93	Gamma	1.70	0.30
$\psi_y$	0.02	0.03	0.00	0.08	Gamma	0.13	0.10
$\psi_{gy}$	0.52	0.52	0.29	0.76	Gamma	0.13	0.10
$\rho_{R1}$	1.30	1.28	1.17	1.39	Normal	1.00	0.20
$\rho_{R2}$	-0.44	-0.43	-0.52	-0.32	Normal	0.00	0.20
$\kappa$	3.94	4.12	2.88	5.55	Gamma	3.00	0.75
$b$	0.82	0.82	0.71	0.90	Normal	0.15	0.05
$(\eta_p - 1)^{-1}$	0.23	0.23	0.16	0.29	Normal	0.15	0.05
$(\eta_w - 1)^{-1}$	0.16	0.17	0.11	0.25	Gamma	2.00	0.75
$\tau$	1.07	1.37	0.70	2.22	Gamma	0.25	0.10
$\nu_p$	0.81	0.81	0.77	0.84	Beta	0.50	0.20
$\nu_w$	0.52	0.51	0.36	0.65	Beta	0.50	0.20

Table 2: Estimated Structural Parameters

While we estimate most of the model parameters, some of them are calibrated. The steady-state level of inflation is set equal to 1.0061, which is equal to 2.5 percent in annualized terms (the average in our sample). The subjective discount factor  $\beta$  is set equal to 0.9979 to match the average real interest rate in our sample. The capital share of the economy  $\alpha$  and the depreciation rate of capital  $\delta$  are set equal to 0.225 and 0.025, respectively, based on [Schmitt-Grohe and Uribe \(2012\)](#). The steady-state level of government purchases over GDP,  $\eta_G$ , is set equal to 0.2, which is the average in our sample. The growth rate of the economy at the steady-state  $\Lambda_Z$  is set

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satisfactory. We think that this could be related to the degree of indeterminacy of the model. We leave the further pursuit of this venue—as well as the exploration of the techniques proposed by [Farmer, Khramov and Nicolò \(2015\)](#) and [Bianchi and Nicolò \(2017\)](#)—for future research.

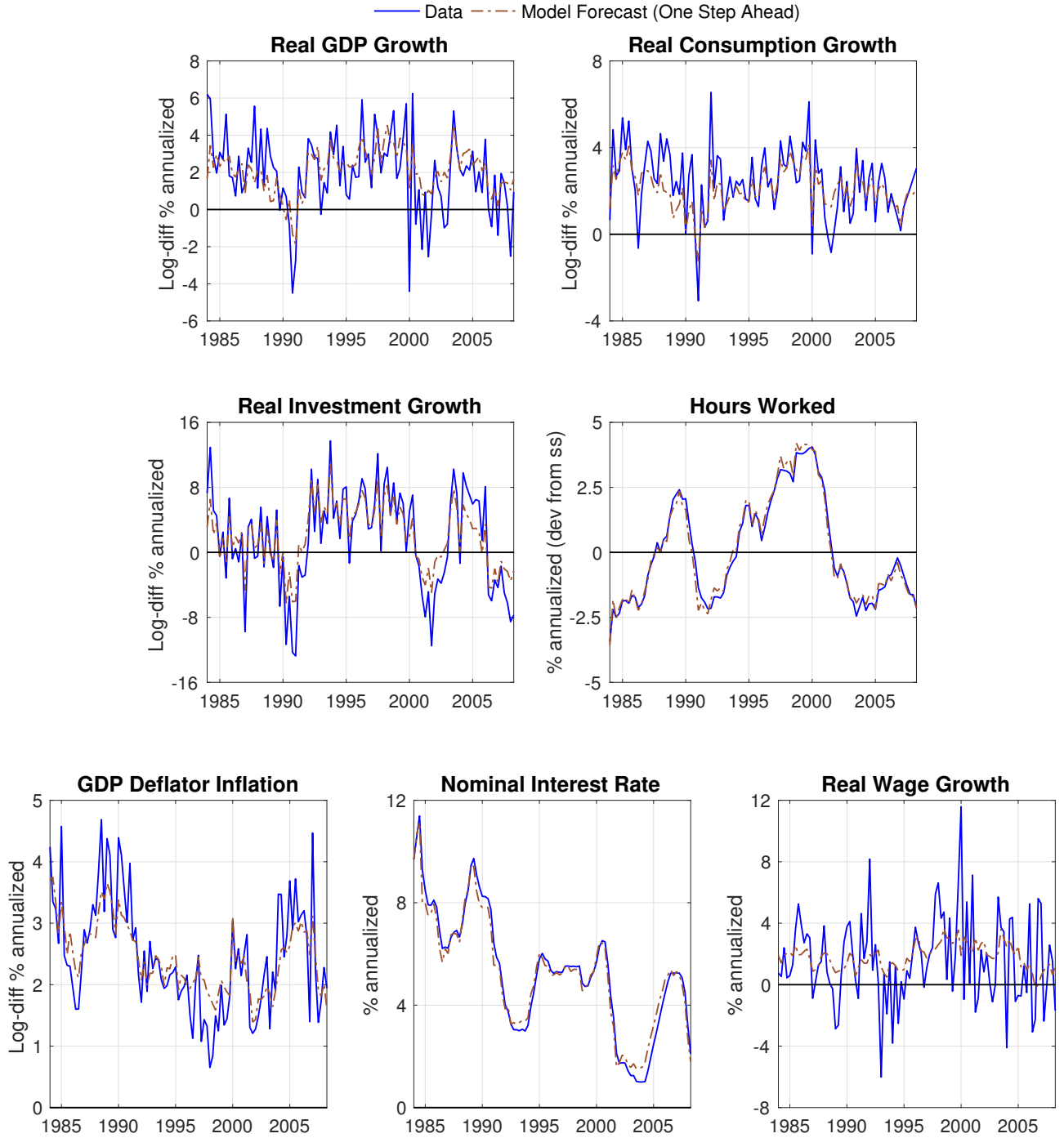


Figure 1: One-Step-Ahead Forecasts

equal to 1.0048, which matches the average growth rate of the economy in our sample. Similarly, the steady-state growth rate of the investment-specific shock  $\Lambda_\mu$  is set equal to 1.0060, which is calibrated to match the average relative price of investment in terms of consumption in our sample using [Justiniano, Primiceri and Tambalotti's \(2011\)](#) data. The inverse of the elasticity of substitution of capital utilization with respect to the rental rate of capital  $\sigma_a$  is not well identified,

and it is set equal to 1.1739 following [Smets and Wouters \(2007\)](#). We set the autocorrelation of the unit root investment-specific technological progress  $\rho_{\bar{\mu}}$  equal to zero given that the autocorrelation of the marginal efficiency of investment,  $\rho_{m_I}$ , already captures autocorrelation in investment shocks. The volatility of the unit root process for investment-specific technological change  $\sigma_{\bar{\mu}}$  does not have good convergence properties; hence, we set it equal to the posterior mode that would be obtained when  $\sigma_{\bar{\mu}}$  is treated as one of the estimated parameters.

Table 2 shows the posterior mode, the posterior mean and the 90 posterior percent credible sets of the estimated structural parameters of the model. Our estimates are in line with the literature; therefore, for ease of exposition, we discuss them in Appendix A.2.<sup>10</sup>

Figure 1 plots the data and the posterior mean of the model implied one-step-ahead forecasts. In particular, starting at the initial values of 1984Q1, we compute  $\mathbb{E}(\mathbf{x}_{(t+1)} | \mathcal{Y}_t)$  for  $t = 1984Q1, \dots, 2008Q2$ , where  $\mathbf{x}_t$  denotes each of the observables used in the estimation and where  $\mathcal{Y}_t$  denotes the agents' information set at time  $t$ . Overall, our model is able to track the evolution of key macroeconomic variables.

### 3 Determinacy and Positive Trend Inflation

In this section, we examine the relation between determinacy and positive trend inflation. First, we use our estimated medium-sized model to quantify the extent to which the ability of central banks to induce determinacy—a key indicator of the underlying ability of central banks to anchor inflation expectations and avoid self-fulling economic fluctuations—is undermined at high values of trend inflation. Second, we analyze how the systematic component of monetary policy affects determinacy in the presence of positive trend inflation.

#### 3.1 Probability of Determinacy

We begin by quantifying the probability of determinacy implied by our model by drawing from the posterior distribution of the estimated parameters and assessing whether there is determinacy at

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<sup>10</sup> Table A.3.1 in Appendix A.3 presents the estimated parameters for the exogenous stochastic processes.

different levels of trend inflation. Specifically, the probability of determinacy at a given level of trend inflation  $\bar{\Pi}$  is computed as follows:

$$\Pr(\text{Determinacy} \mid \bar{\Pi}) = \frac{\sum_{i=1}^n \mathbf{1}_d(\theta(i), \bar{\Pi})}{n},$$

where  $\theta(i)$  denotes the  $i$ -th draw of the posterior distribution of the vector of estimated parameters  $\theta$ ,  $n$  denotes the number of posterior draws used in the analysis, and  $\mathbf{1}_d(\theta(i), \bar{\Pi})$  equals one when the solution to the first order log-linear approximation of our model evaluated at  $(\theta(i), \bar{\Pi})$  is determinate and zero otherwise.<sup>11</sup> In order to make clear how the probability of determinacy is computed, it is useful to start by noting that  $n$  is a function of the length of the Markov Chain used in the estimation procedure. Such length equals 2,400,000 after excluding a 20 percent burn-out period. Out of these draws we keep one every 1,000 and as a result we end up with 2,400 effective draws from the posterior distribution of the estimated parameters, hence  $\{\theta(i)\}_{i=1}^n$  where  $n=2,400$ . Given a value of  $\bar{\Pi}$ , for each of the  $n$  effective posterior draws it is possible to solve the first order log-linear approximation of our model and determine whether the implied solution is determinate. More specifically, we solve the model evaluated at  $(\theta(i), \bar{\Pi})$  using Chris Sims's Gensys solution method and we set  $\mathbf{1}_d(\theta(i), \bar{\Pi}) = 1$  when the solution to the linear rational expectation model is unique and we set  $\mathbf{1}_d(\theta(i), \bar{\Pi}) = 0$  otherwise.<sup>12</sup>

Clearly, since the model is estimated assuming determinacy, when  $\bar{\Pi}$  equals the calibrated value used in the estimation (1.0061, i.e. a level of trend inflation of about 2.5 percent annualized) each of the  $n$  draws will imply that the model is determinate so that  $\Pr(\text{Determinacy} \mid \bar{\Pi}) = 1$ . The same occurs for lower values of trend inflation. However, as  $\bar{\Pi}$  is set to values higher than the one used in the estimation there are posterior draws for which the model is not determinate implying  $\Pr(\text{Determinacy} \mid \bar{\Pi}) < 1$ . This is possible because we do not re-estimate the model for each value of  $\bar{\Pi}$ ; instead, we compute the probability of determinacy using the posterior draws associated with  $\bar{\Pi} = 1.0061$ , the average value of trend inflation in our estimation sample.<sup>13</sup>

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<sup>11</sup>The vector  $\theta$  consists of the parameters listed in Tables 2 and A.3.1. Note that the function  $\mathbf{1}_d(\theta(i), \bar{\Pi})$  also implicitly depends of the remaining calibrated parameters which are fixed throughout our analysis.

<sup>12</sup>The model is log-linearized in Mathematica and then it is exported to MATLAB in order to use Gensys. Equivalent results can be obtained using Dynare.

<sup>13</sup>Below we will discuss the implications of this approach. Nevertheless, for now it suffices to point out that



Figure 2 shows the probability of determinacy associated with levels of trend inflation between 0 and 10 percent.<sup>14</sup> The probability of determinacy equals one when trend inflation equals zero. As the level of trend inflation increases, the probability of determinacy remains near one for values of trend inflation as high as 3 percent, which is a value higher than the average inflation rate realized over the period 1984Q1–2008Q2. As trend inflation increases above 3 percent, the probability of determinacy begins to decline. However, the probability of determinacy is greater or equal than 0.9 for values of trend inflation as high as 5 percent, which is 1 percentage point higher than the 4 percent level associated with the majority of the proposals to raise the inflation target. For values of trend inflation above 5 percent, the probability of determinacy declines sharply with trend inflation: it drops to about 0.6 for the value of trend inflation equal to the average inflation in the U.S. during the period 1975–1981 (i.e., 6.6 percent) and is about 0.1 when trend inflation equals 10 percent.

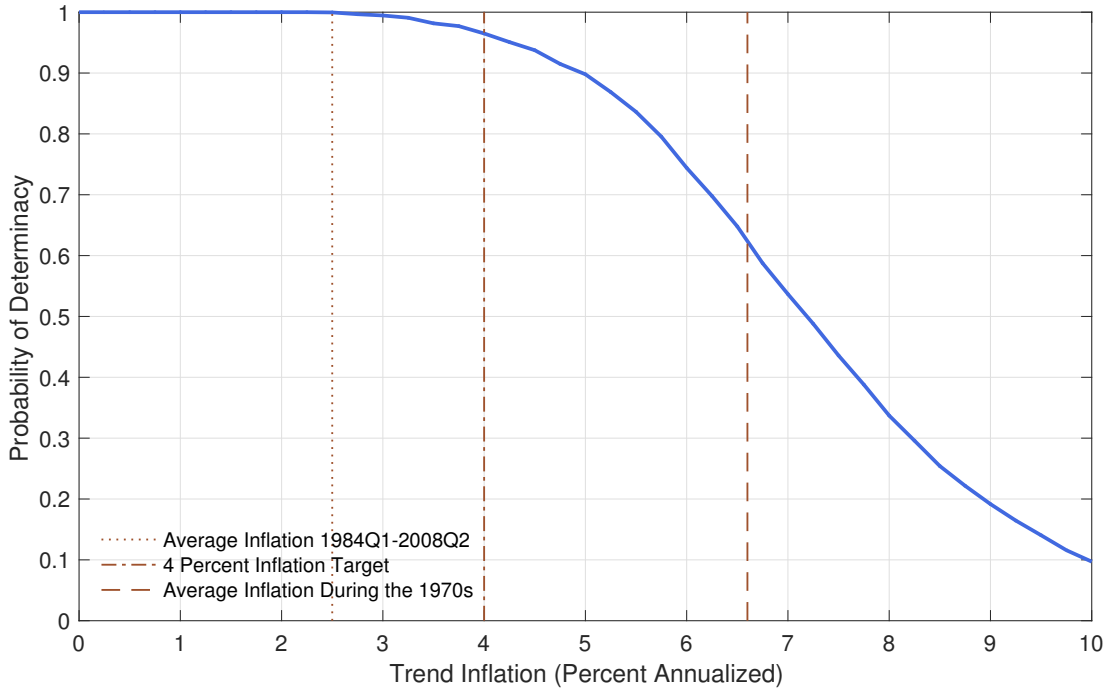


Figure 2: Probability of Determinacy and Trend Inflation

our approach complements the analysis of [Coibion and Gorodnichenko \(2011\)](#), who condition their analysis in a calibrated model.

<sup>14</sup>The choice of 10 percent as an upper bound is motivated by [Gagnon \(2009\)](#), who finds evidence of strong state price dependence for levels of trend inflation higher than 10 percent.

Two results stand out. First and foremost, Figure 2 documents that *ceteris paribus* trend inflation negatively affects determinacy in a medium-sized model. To the best of our knowledge, we are the first to show that this result, usually obtained in small-sized New Keynesian models, holds in a medium-sized model embedding the key features of models used in central banks for policy analysis and forecasting. Second, Figure 2 suggests that conditional on our estimated policy rule for the Great Moderation period, an increase in trend inflation to 4 percent would be unlikely to lead the U.S. economy to experience indeterminacy.

At this juncture, it is important to highlight that the results shown in Figure 2 depend on our estimated monetary policy rule for the Great Moderation—a period of active monetary policy (see e.g., [Lubik and Schorfheide \(2004\)](#))—and also to re-state that the model has been estimated assuming determinacy. As a consequence, as trend inflation increases determinacy will be more likely in our approach than in an alternative approach in which periods of passive monetary policy (such the seventies) and indeterminacy are taken into account. To assess the extent to which the results are affected by our approach of focusing on a period of active monetary policy and determinacy, we re-compute the probability of determinacy by considering simulations in which the posterior draws associated with the monetary policy rule of our model are replaced by the parameters of the monetary policy rule estimated by [Coibion and Gorodnichenko \(2011\)](#) for the pre-Volcker era, a period often associated with passive or accommodative monetary policy and indeterminacy.

Figure 3 reports the results from these simulations. The solid line replicates the probability of determinacy implied by our estimated model shown in Figure 2. The dashed line marked with circles shows the probability of determinacy implied by our estimated model when all the parameters of the monetary policy rule are set to [Coibion and Gorodnichenko's \(2011\)](#) point estimates for the pre-1979 period. Such point estimates are 1.04, 0.52/4, and 0 for the response to expected inflation, the output gap, and output growth, respectively, and 1.34 and -0.44 for the coefficients describing the degree of monetary policy inertia. The probability of determinacy implied by the pre-1979 monetary policy rule is equal to zero for all the levels of trend inflation under analysis. This is consistent with the view, put forward by [Clarida, Galí and Gertler \(2000\)](#),

that passive monetary policy led the U.S. economy to indeterminacy during the pre-1979 period.

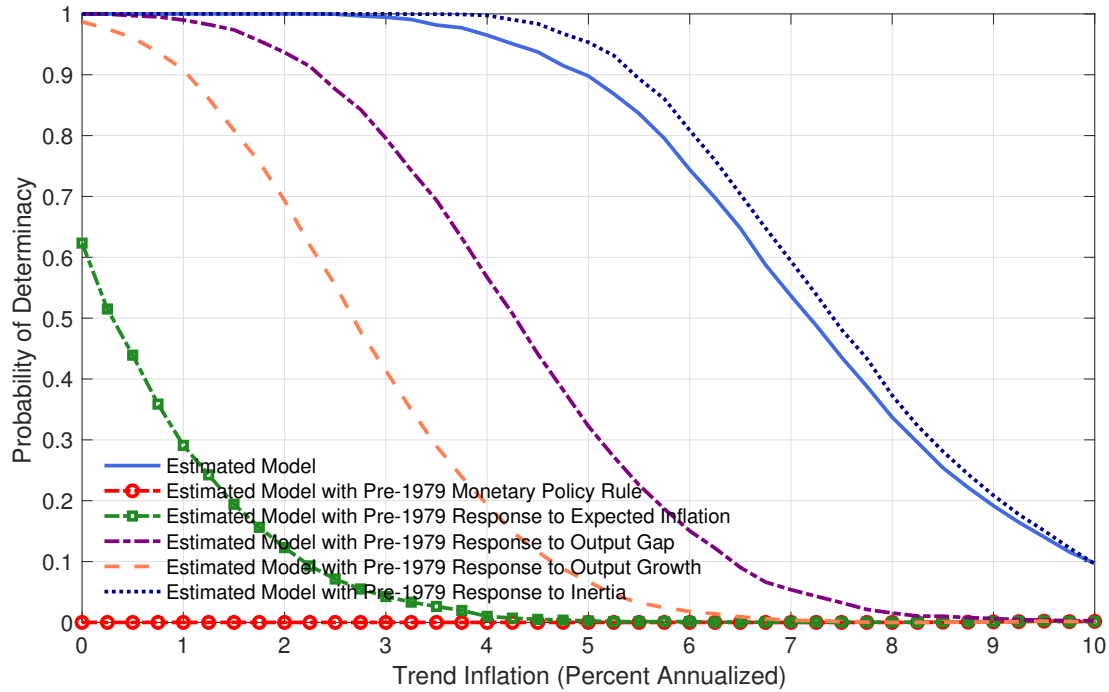


Figure 3: Probability of Determinacy and Trend Inflation

To understand the stark difference between the probability of determinacy implied by our estimated policy rule and that implied by the pre-1979 monetary policy rule, it is insightful to consider the systematic components of monetary policy one at a time. Accordingly, the dashed line marked with squares shows the probability of determinacy implied by our estimated model when we change the response to expected inflation in our estimated rule by setting it equal to the pre-1979 monetary policy rule. The probability of determinacy is slightly above 0.6 when trend inflation equals zero, and it exponentially decreases toward zero as trend inflation increases. In fact, the probability of determinacy at a level of trend inflation equal to 4 percent is almost zero. Note that the response to expected inflation estimated by [Coibion and Gorodnichenko \(2011\)](#) for the pre-1979 monetary policy rule (1.04) is substantially lower than our posterior mean estimate of 2.42 and lies below the 90 percent credible set equal to [1.96, 2.93].

The dashed-dotted line shows the probability of determinacy implied by our estimated model when the response to the output gap is set equal to the pre-1979 monetary policy rule. As was the

case when switching the response to expected inflation, a switch in the response to the output gap also implies a lower probability of determinacy at all the levels of trend inflation under analysis. In this case, the pre-1979 response to the output gap ( $0.52/4$ ) is greater than the 90 percent credible set  $[0.00, 0.08]$  resulting from the posterior estimates of our model. The dashed line shows the probability of determinacy implied by our estimated model when the response to output growth is set equal to the pre-1979 monetary policy rule. As was the case above, this simulation implies a lower probability of determinacy for all the levels of trend inflation under analysis. The pre-1979 response to the output growth (0) is smaller than the 90 percent credible set  $[0.29, 0.76]$  associated with our Great Moderation estimates. Finally, the dotted line shows the probability of determinacy implied by our estimated model when the parameter describing the monetary policy inertia is set equal to the pre-1979 monetary policy rule. In contrast to the previous cases, the probability of determinacy is higher than the probability implied by our estimated model. This is because the pre-1979 overall policy inertia ( $0.90 = 1.34 - 0.44$ ) is slightly higher than the posterior mean associated with our Great Moderation estimates, 0.85.

In sum, we find that in a medium-sized New Keynesian model fitting the behavior of standard macroeconomic variables for the Great Moderation, an increase in trend inflation to 4 percent does not create a significant risk of self-fulfilling fluctuations for the U.S. economy. However, conditional on [Coibion and Gorodnichenko's \(2011\)](#) estimated rule for the pre-1979 period, the U.S. economy would have experienced indeterminacy for any level of positive trend inflation. Thus, consistent with previous studies, our results suggest that monetary policy in the 1970s could have been the cause of self-fulfilling fluctuations and thus the main source of the Great Inflation. It follows that the role of trend inflation crucially depends on a particular policy in place. In particular, our simulations show that a higher response to expected inflation, a lower response to the output gap, a higher response to output growth and a higher inertia in the monetary policy rule diminish the probability of determinacy for any given level of trend inflation. This calls for a deeper investigation of the relationship among monetary policy, trend inflation, and determinacy. This is what we turn to next.

### 3.2 Monetary Policy, Determinacy and Trend Inflation

A large number of studies have explored the relation between equilibrium determinacy and positive trend inflation in small-sized New Keynesian models, as recently reviewed by [Ascari and Sbordone \(2014\)](#). The main lessons of this line of research that can be summarized as follows: i) the response of the federal funds rate to inflation (current or expected) that is necessary to achieve determinacy is increasing in the level of trend inflation; ii) responding to the output gap is destabilizing, i.e., it leads to indeterminacy; iii) responding to output growth helps determinacy; iv) monetary policy inertia is also stabilizing; and v) when central banks respond to expected inflation, weak responses as well as strong responses to expected inflation are destabilizing unless there is strong monetary policy inertia.

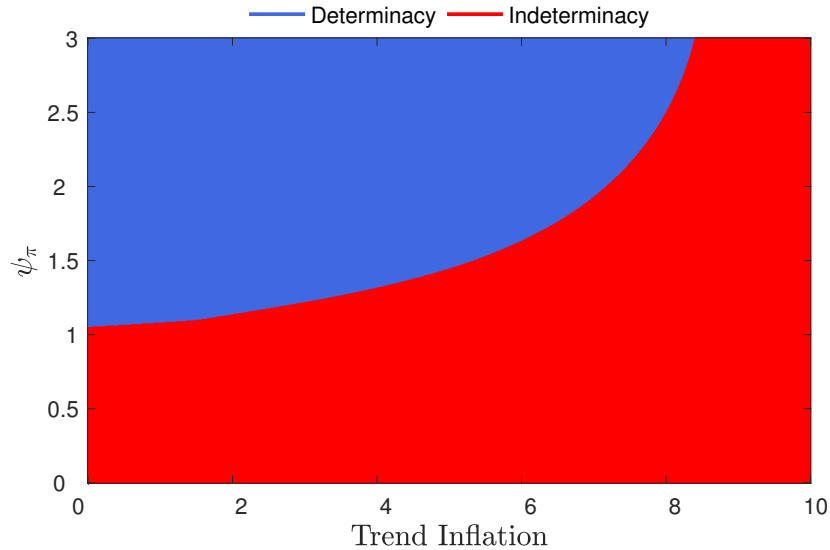


Figure 4: Response to Inflation, Trend Inflation and Determinacy

In line with the previous literature for small-sized models, we investigate the determinacy regions of the parameter space to scrutinize these findings in our estimated medium-sized model. Figures 4 and 5 plot the determinacy and indeterminacy regions when varying the policy parameters or the level of trend inflation.<sup>15</sup> In particular, Figure 4 characterizes the equilibrium of the model as a function of trend inflation and the response of the federal funds rate to expected

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<sup>15</sup>Determinacy areas are depicted in blue and indeterminacy areas are depicted in red.

inflation ( $\psi_\pi$ ), with the remaining parameters fixed at their posterior mean and calibrated values, respectively. Henceforward, unless stated otherwise, when considering the effects of certain parameters on determinacy, the remaining parameters are fixed in this manner. Figure 5 shows how the systematic components of monetary policy affect determinacy at different values of trend inflation: In Panel (a), trend inflation equals 2.5 percent annualized (the average during the Great Moderation era); in Panel (b), trend inflation equals 4 percent annualized (in line with the recent suggested increase in the inflation target). More specifically, the upper left subplot in Panel (a) describes determinacy as a function of the response of the federal funds rate to expected inflation ( $\psi_\pi$ ) and to the output gap ( $\psi_y$ ). The upper right subplot repeats the analysis focusing on the response of the federal funds rate to expected inflation  $\psi_\pi$  and to output growth  $\psi_{gy}$ . The lower left subplot describes determinacy as a function of  $\psi_y$  and  $\psi_{gy}$ . The lower right panel shows determinacy when varying the response to expected inflation and the degree of monetary policy inertia  $\psi_\rho \equiv \psi_{\rho_{R1}} + \psi_{\rho_{R2}}$ . Panel (b) replicates Panel (a) when trend inflation is set to 4 percent annualized. Note that, as expected, the determinacy regions shrink with trend inflation in all the panels. Based on these figures, we now revisit the role of the systematic components of monetary policy in our estimated model.

As in small-sized models, Figure 4 shows that the response of the federal funds rate to inflation that is necessary to achieve determinacy is increasing in the level of trend inflation. This effect seems, however, to be less pronounced than the one reported in the small-sized calibrated New Keynesian model of [Coibion and Gorodnichenko \(2011\)](#). Conditional on a level of trend inflation equal to 4 percent the calibrated model of [Coibion and Gorodnichenko \(2011\)](#) implies that the federal funds rate response to inflation must be greater than 4 in order to induce determinacy. In contrast, in our estimated model determinacy arises even when the the federal funds rate response to inflation is slightly greater than 1 which is broadly in line with the small-sized New Keynesian model of [Hirose, Kurozumi and Van Zandweghe \(2017\)](#) estimated for the Great Moderation.

Next, we consider the destabilizing effect of responding to the output gap. The upper left subplots in Panels (a) and (b) of Figure 5 show that the determinacy region substantially shrinks as the response to the output gap increases. When the policy rule does not respond to the

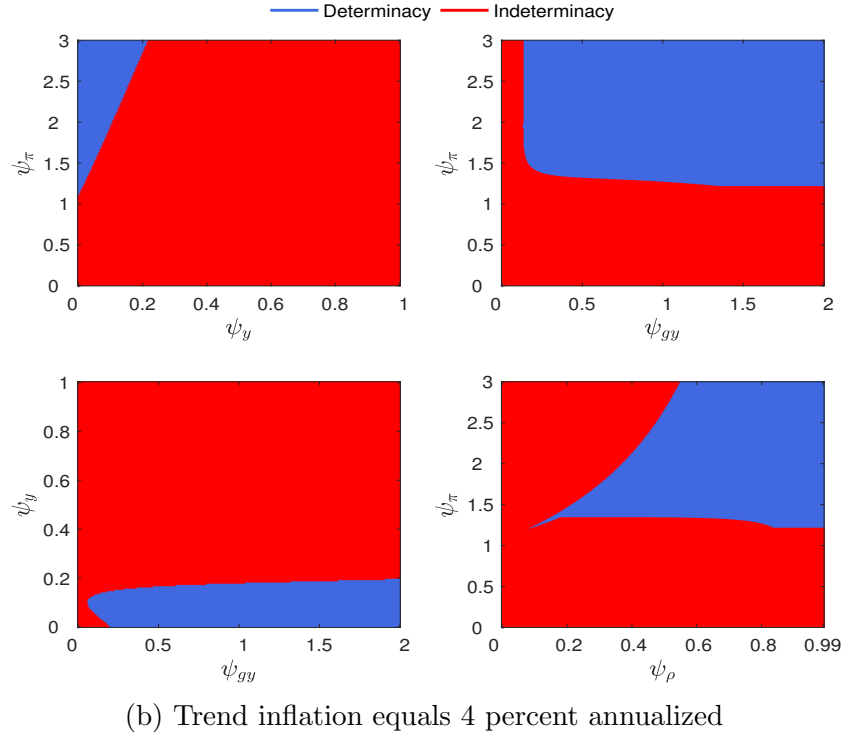
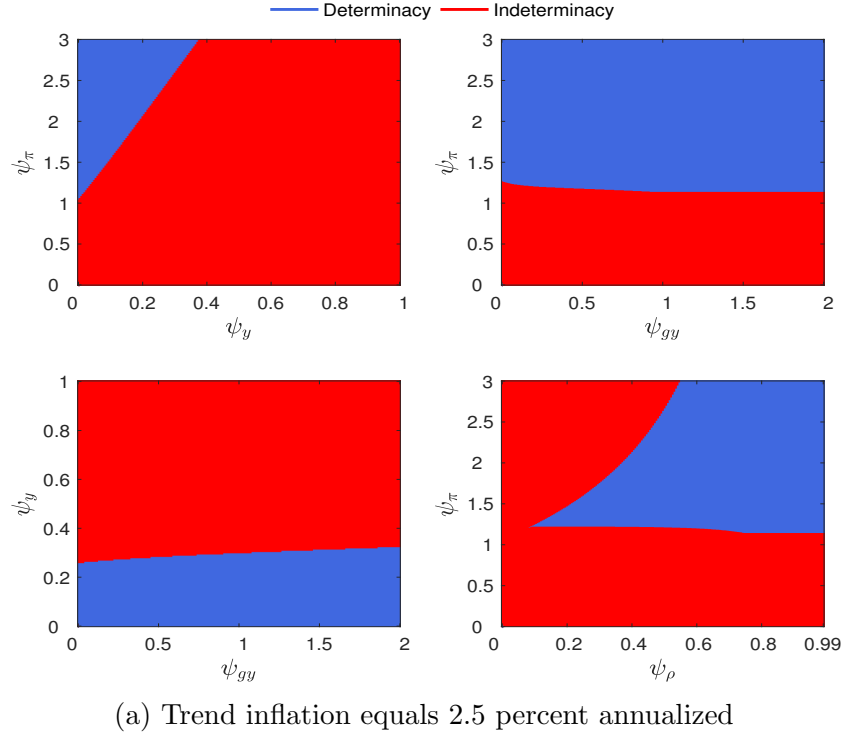


Figure 5: Panel (a) shows how the systematic components of monetary policy affect determinacy when trend inflation equals the average level observed during the Great Moderation (about 2.5 percent annualized). Panel (b) shows how the systematic components of monetary policy affect determinacy when trend inflation equals 4 percent, as recently considered by [Blanchard, Dell’Ariccia and Mauro \(2010\)](#), [Krugman \(2013\)](#), and [Ball \(2014\)](#).

output gap, a level of  $\psi_\pi$  that satisfies the Taylor principle ( $\psi_\pi > 1$ ) is sufficient to guarantee determinacy. However, very small positive values of  $\psi_y$  require a very strong increase in  $\psi_\pi$  to guarantee determinacy, even when trend inflation is as low as 2 or 4 percent. This contrasts with some findings for small-sized New Keynesian models, in which small but positive responses to the output gap lead to lower minimum responses to inflation to achieve determinacy, as in the case with zero trend inflation (see [Coibion and Gorodnichenko \(2011\)](#), p. 349).

While large responses to the output gap lead to indeterminacy, responding to output growth is stabilizing. The upper right subplot of Panel (b) in Figure 5 shows that for typical responses to inflation such as those near 1.5, the model becomes determinate as the response to output growth increases. A similar conclusion arises by examining the lower left subplots in Panels (a) and (b) of Figure 5: as the response to output growth increases, equilibrium determinacy is possible even with somewhat larger responses to the output gap. However, this effect is relatively small: responding to output growth has very limited power to counterbalance the destabilizing effects of responding to the output gap.

We conclude by revising the role of monetary policy inertia. The lower right subplots in Panels (a) and (b) of Figure 5 show that for low values of monetary policy inertia our forward looking Taylor rule leads to indeterminacy, in line with [Carlstrom and Fuerst \(2005\)](#). As the degree of monetary policy inertia increases, there is a wide range of responses of the federal funds rate to expected inflation that are consistent with determinacy. In addition, given that our monetary policy rule reacts to expected inflation, in the absence of substantial monetary policy inertia, weak as well as strong responses to expected inflation can lead to indeterminacy, consistent with [King \(2000\)](#) and [Carlstrom and Fuerst \(2005\)](#).

In sum, the main lessons on the relation among systematic monetary policy, determinacy, and positive trend inflation derived from small-sized New Keynesian models qualitatively hold on a medium-sized estimated New Keynesian model that includes the typical features relevant for monetary policy analysis. There are some quantitative differences though, most notably that the destabilizing effect of responding to the output gap is very strong even for quite low levels of trend inflation.



## 4 Monetary Policy and Trend Inflation Since the 1970s

### 4.1 Probability of Determinacy and Monetary Policy in Real Time

The previous section revolves around assessing the probability of determinacy conditional on the monetary policy parameters estimated for the Great Moderation period. A common concern of such an approach is that the estimated parameters for the monetary policy rule may not represent the true response of policymakers to economic conditions at the time of each FOMC meeting. For instance, [Orphanides \(2002\)](#) and [Orphanides and Williams \(2006\)](#) highlight the importance of using real-time measures of expected inflation, output growth, and the output gap to obtain an accurate characterization of the systematic component of monetary policy.

We address such concerns by computing the model-implied probability of determinacy at each FOMC meeting since the 1970s given the systematic component of monetary policy and the levels of trend inflation that were likely to have prevailed at the time of those meetings.

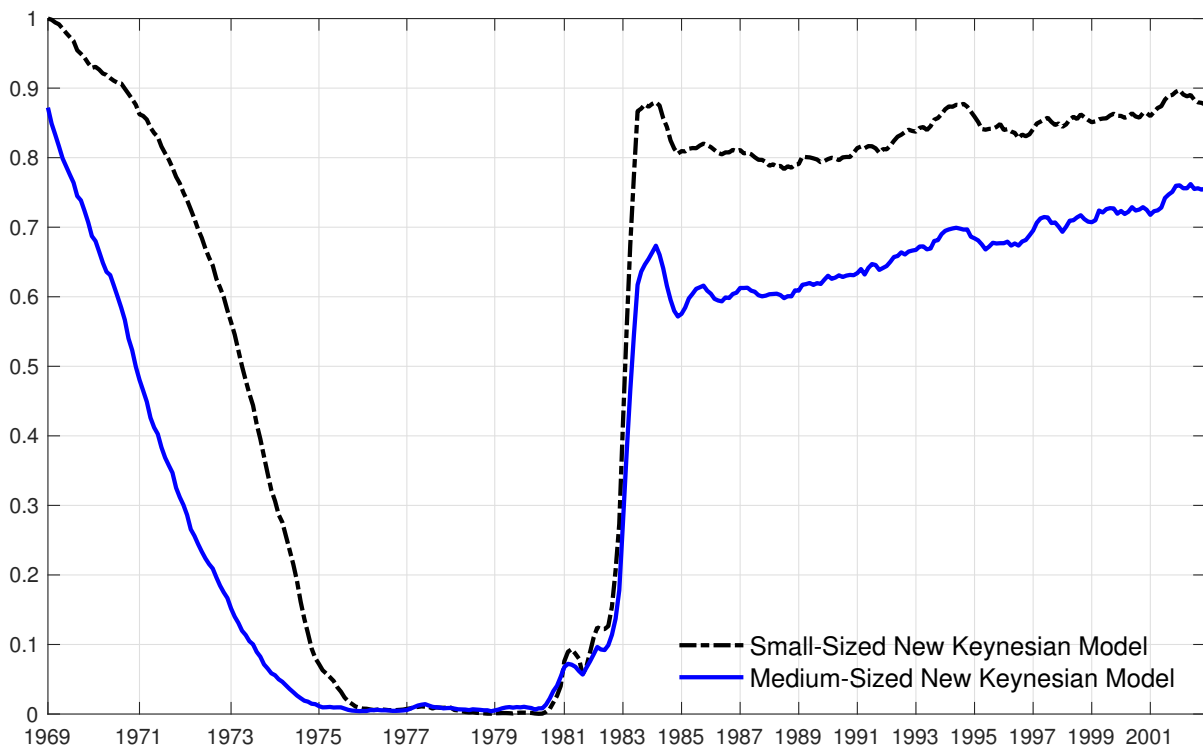


Figure 6: Probability of Determinacy Implied by New Keynesian Models

We begin by computing the probability of determinacy at the time of each FOMC meeting using Coibion and Gorodnichenko’s (2011) real-time and time-varying estimates for the systematic component of monetary policy and their smooth estimates for trend inflation. We do this by drawing from the distribution of time-varying monetary policy parameters and trend inflation; essentially, we replicate the time-varying inflation case shown in Figure 4 of Coibion and Gorodnichenko (2011) using our estimated medium-sized New Keynesian model. Accordingly, at the time of each FOMC meeting the probability of determinacy is computed based on 1,000 draws obtained from the distribution of the systematic component of monetary policy and trend inflation estimated by Coibion and Gorodnichenko (2011). The remaining structural parameters are drawn from the posterior distribution implied by our estimation. The solid line in Figure 6 shows the probability of determinacy implied by our medium-sized model and the dashed-dotted line reproduces the probability reported by Coibion and Gorodnichenko (2011) when using their small-sized model.

Overall, the probability of determinacy implied by the medium-sized New Keynesian model broadly tracks that of the small-sized New Keynesian model. Hence, the small- and medium-sized models support the view, put forward by Clarida, Galí and Gertler (2000) and Lubik and Schorfheide (2004), that the U.S. economy was more likely under indeterminacy during the 1970s and under determinacy during the post-Volcker era. Even so, the medium-sized model attributes a significantly lower probability of determinacy to the early 1970s, and it also attributes a lower probability of determinacy during the post-Volcker period. The probabilities of determinacy reported in Figure 6 encompass the estimation uncertainty associated with the level of trend inflation and with the monetary policy parameters. As a consequence, the simulation is not informative about the underlying sources affecting the probability of determinacy. Next, we will disentangle these effects by computing conditional probabilities of determinacy: the probability of determinacy conditional on trend inflation and the probability of determinacy conditional on monetary policy.

## 4.2 Identifying the Role of Trend Inflation and of Monetary Policy

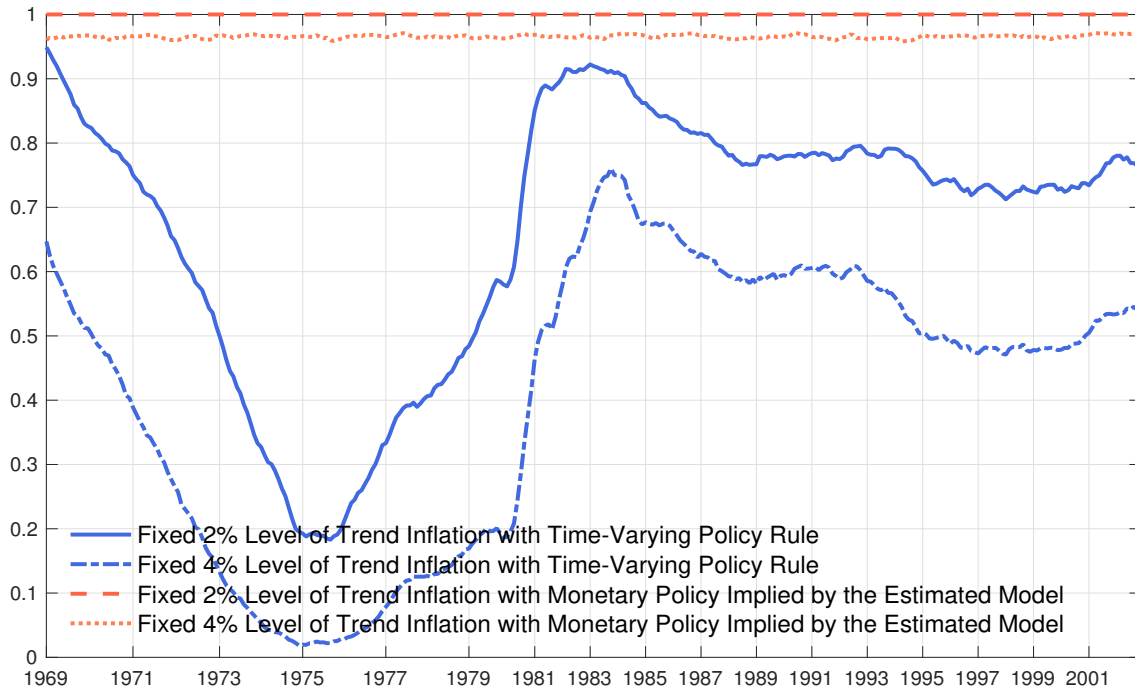
To identify the role of monetary policy in shaping Figure 6, we compute the probability of determinacy conditional on trend inflation by fixing the level of trend inflation at either 2 or 4 percent and by drawing from the distribution of time-varying monetary policy parameters estimated by Coibion and Gorodnichenko (2011). As it was the case in Figure 6, the remaining structural parameters are drawn from the distribution implied by our estimated model. The solid and the dashed-dotted lines in Panel (a) of Figure 7 exhibit big swings in the implied probability of determinacy in line with Figure 6. This implies that the systematic component of monetary policy is important: even if trend inflation had been equal to 2 percent, the probability of determinacy would have been lower than 0.2 around the mid-1970s. Moreover, a constant level of 4 percent trend inflation would lead to a similar narrative than the one emerging from Figure 6: indeterminacy was very likely in the U.S. economy for the entire 1970s, up to 1982. The relevance of monetary policy becomes even clearer if we compute the probability of determinacy at the same levels of trend inflation by drawing from the posterior distribution of the monetary policy rule estimated in Section 2. The dashed line equals 1 and dotted line hovers steadily around 0.9, suggesting that neither the 2 percent nor the 4 percent trend inflation level implies a large probability of indeterminacy.

To identify the role of trend inflation in shaping Figure 6, we compute the probability of determinacy conditional on monetary policy by fixing the monetary policy parameters and by drawing from the distribution of the level of trend inflation estimated by Coibion and Gorodnichenko (2011).<sup>16</sup> When conditioning on monetary policy, we consider three cases for the parameters associated with the monetary policy rule: (i) the estimates for the post-1982 period computed in Section 2; (ii) Coibion and Gorodnichenko's (2011) pre-1979 estimates; and (iii) Coibion and Gorodnichenko's (2011) post-1982 estimates.<sup>17</sup> The solid line in Panel (b) of Figure 7 shows the probability of determinacy implied by our model when fixing the policy parameters at Coibion and Gorodnichenko's (2011) pre-1979 estimates. The probability of determinacy equals zero at the time

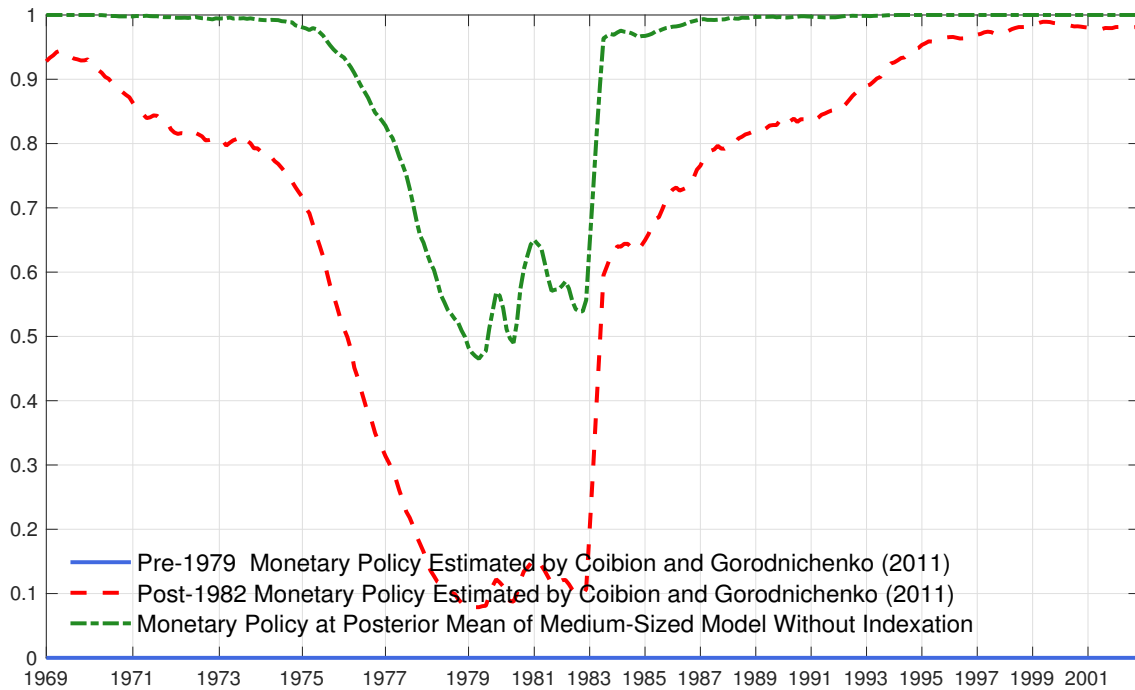
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<sup>16</sup>Again, the remaining structural parameters are drawn from the distribution implied by our estimated model.

<sup>17</sup>Such point estimates are 2.2, 0.43/4, and 1.56 for the response to expected inflation, the output gap, and output growth, respectively; and 1.05 and -0.13 for the coefficients describing the degree of monetary policy inertia.



(a) The Role of Monetary Policy Given Trend Inflation



(b) The Role of Trend Inflation Given Monetary Policy

Figure 7: Interaction Between Monetary Policy and Trend Inflation

of each FOMC meeting since 1969. The dashed line shows the probability of determinacy implied by our model when fixing the policy parameters at [Coibion and Gorodnichenko's \(2011\)](#) post-1982 estimates. This probability is larger than 0.5 at the time of most FOMC meetings except for those

meetings that occurred between 1976 and 1983 when estimated trend inflation was above 6 percent. This indicates that conditional on passive monetary policy such as during the pre-1979 period, even moderate levels of trend inflation would cause indeterminacy. In contrast, conditional on active monetary policy such as during the post-1979 period trend inflation would cause indeterminacy to be more likely than determinacy only if it was above 6 percent.

As was the case with Panel (a), the relevance of monetary policy becomes even clearer when we use the estimates of Section 2. The dashed-dotted line shows the probability of determinacy conditional on the posterior mean of the monetary policy rule. The probability has always been near one at the time of every FOMC meeting since 1969 except for the period the period 1976–1983 when trend inflation is estimated to be higher than 6 percent (annualized) on average. Hence, active monetary policy is a key driving force of determinacy as long as trend inflation is not too high.

## 4.3 Implications

### 4.3.1 The Role of Monetary Policy When Trend Inflation Equals 4 Percent

The results in Sections 4.1 and 4.2 show the importance of the systematic component of monetary policy and suggest some caution in interpreting the results presented in Section 3.1. When one considers estimates of the monetary policy rule that take into account the information available to policymakers at the time of each FOMC meeting, the probability of determinacy at a level of trend inflation equal to 4 percent is remarkably lower than the probability reported in Section 3.1. Although the latter is above 0.9, the former is only a tad above 0.5 at the end of Coibion and Gorodnichenko’s (2011) sample and slightly below 0.5 in the second part of the 1990s.

Figure 8 reveals the two main reasons behind this difference: the different values of the response to the output gap and the higher uncertainty in Coibion and Gorodnichenko’s (2011) estimates relative to ours. The figure shows the histograms implied by Coibion and Gorodnichenko’s (2011) time-varying parameter draws for the post-1982 period (green bars) relative to the histograms obtained from our estimation (blue bars). Specifically, we compute a histogram for each of the parameters characterizing the systematic component of monetary policy: the response of the federal

funds rate to expected inflation  $\psi_\pi$ , the response of the federal funds rate to the output gap  $\psi_y$ , the response of the federal funds rate to output growth  $\psi_{gy}$ , and the degree of monetary policy inertia  $\rho \equiv \rho_1 + \rho_2$ .

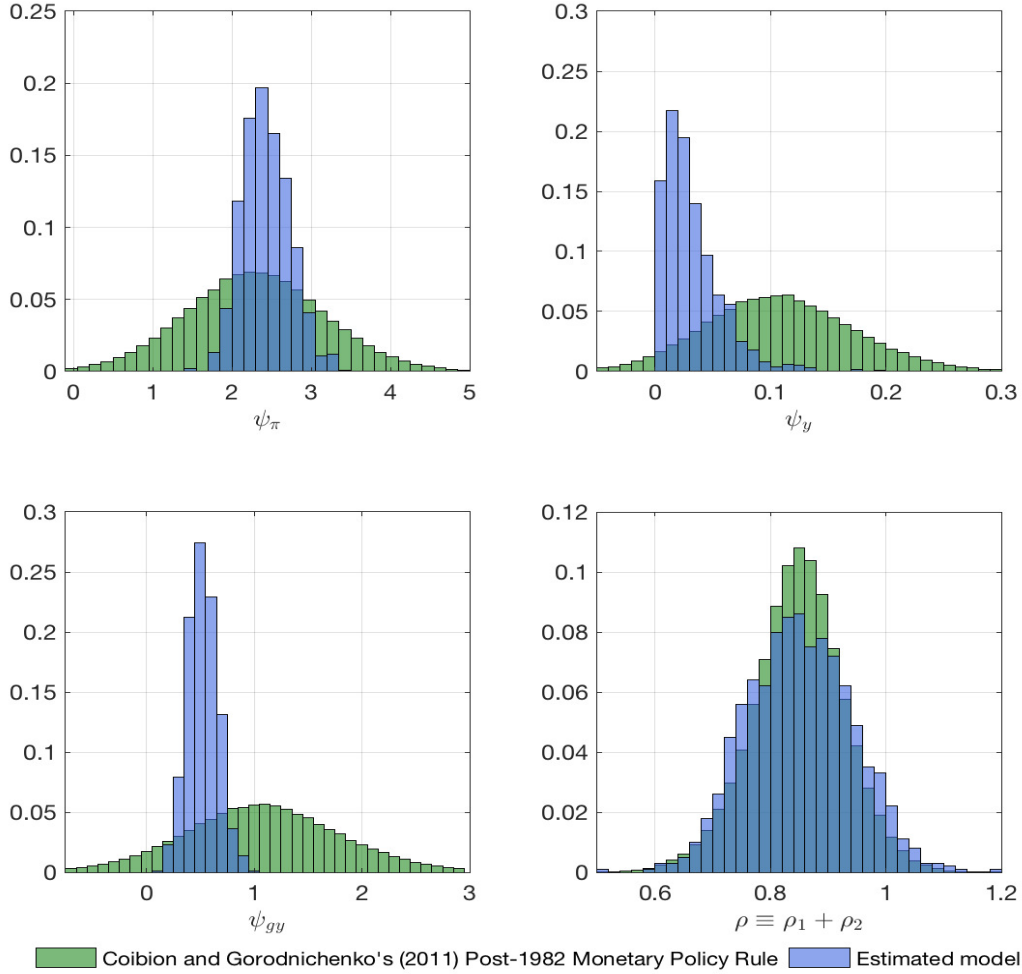


Figure 8: Histograms of the Draws of Monetary Policy Coefficients

It is insightful to begin by examining the histograms for  $\psi_\pi$  and  $\psi_y$ . The histogram for  $\psi_\pi$  associated with our posterior estimates is concentrated around 2.4, and it assigns low probability to values below 2. The histogram for  $\psi_\pi$  associated with Coibion and Gorodnichenko's (2011) estimates is concentrated around a similar value, i.e. 2.3, but it features a significantly larger variance. In particular, it assigns nearly 40 percent probability to values of  $\psi_\pi$  that are below 2. The histogram for  $\psi_y$  associated with our posterior estimates is quite concentrated near zero and assigns a negligible probability to values above 0.1. In contrast, the histogram for  $\psi_y$  associated

with Coibion and Gorodnichenko's (2011) estimates is almost symmetrically distributed around 0.1, and it exhibits a larger variance than the one implied by our estimates. About half of the parameter draws for  $\psi_y$  are above 0.1.<sup>18</sup>

The interaction of these histograms with the results in Section 3.2 is key to providing a rationale for our findings. As shown in Section 3.2, the medium-sized model is quite sensitive to the relation between trend inflation and  $\phi_y$ . In particular, Figure 5b shows that when the level of trend inflation equals 4 percent and  $\phi_y$  is larger than 0.1,  $\phi_\pi$  must be roughly larger than 2 to achieve determinacy. Now consider the event  $A = \{(\psi_\pi, \psi_y) \in \mathbb{R}_+^2 : \psi_\pi \leq 2 \text{ and } \psi_y > 0.1\}$ . The probability attributed to event  $A$  by the histograms associated with Coibion and Gorodnichenko's (2011) estimates is about 20 percent, while the probability attributed by the histograms associated with our posterior estimates is almost zero.<sup>19</sup> Hence, the parameters implied by our posterior estimates are more likely to induce determinacy.

In addition, while the responses of the federal funds rate to output growth implied by our posterior estimates are concentrated around 0.5 and assign almost zero probability to responses to output growth smaller than 0.1, Coibion and Gorodnichenko's (2011) time-varying parameter draws for the responses of the federal funds rate to output growth during the post-1982 period assign a near 10 percent probability to a response to output growth smaller than 0.1. As shown in the lower left subplot of Figure 5b, a positive response to output growth is a force toward determinacy conditional on positive responses to the output gap.

Altogether, the analysis above provides a rationale for the different probabilities of determinacy implied by our estimated model relative to those obtained when using real-time measures of monetary policy. In brief, relative to our posterior estimates, the distribution of the parameters estimated by Coibion and Gorodnichenko (2011) assigns a higher probability mass to strong responses to the output gap and to small responses to expected inflation and output growth. The different distribution of the draws for  $\phi_\pi$  and especially for  $\phi_y$  explains most of the difference

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<sup>18</sup>We divide by 4 the value of  $\phi_y$  in Coibion and Gorodnichenko's (2011) estimates because they estimate the Taylor rules using annualized rates, while the Taylor rule in the model is written in terms of quarterly rates. See footnote 20, p. 357 in Coibion and Gorodnichenko (2011).

<sup>19</sup>To facilitate the comparison with Coibion and Gorodnichenko (2011) we have ignored the correlation between  $\psi_\pi$  and  $\psi_y$  when computing these probabilities.

between the dotted and the dashed-dotted lines depicted in Figure 7a.

Given the emphasis on the role of the federal funds rate response to the output gap as a key source of the risk of indeterminacy, it is reasonable to ask what would happen to the probability of determinacy reported in Figure 2 if the data on the output gap used in Coibion and Gorodnichenko (2011) were exploited when estimating the model described in Section 2.1.<sup>20</sup> To answer this question we re-estimate the model using the same approach than in Section 2.2 except that we add Coibion and Gorodnichenko’s (2011) real-time output gap (demeaned) as an additional observable.<sup>21</sup> A summary of the estimation results is presented in Appendix A.4.

We highlight two lessons from such an exercise. First, even though the estimated response to the output gap is in line with the estimates reported in Table 2, there are several differences in the remaining parameters. The most consequential is the frequency of optimal wage setting which decreases significantly. More specifically, based on the posterior mean estimate for  $\nu_w$  obtained when using the output gap as an additional observable, households optimally set wages once every 13 to 14 months, which is significantly above the 6 to 7 months frequency associated with the benchmark posterior mean estimate reported in Table 2. A lower frequency of optimal wage setting implies that households become more forward-looking which tends to make indeterminacy more likely in New-Keynesian models without indexation. Altogether, this suggests that the probability of determinacy shown in Figure 2 would be substantially lower if it were computed using the posterior distribution of the parameters obtained from estimating the model with the expanded set of observables. Figure 9 confirms this intuition. Notwithstanding, it is worth noting that when using Coibion and Gorodnichenko’s (2011) real-time output gap as an additional observable the model fit of inflation deteriorates significantly—as shown in Figure A.4.1—and hence the results shown in Figure 9 should be interpreted with caution.

Second, the fact that the estimated response of the federal funds rate to the output gap obtained when using Coibion and Gorodnichenko’s (2011) real-time output gap as an additional observable is similar to the one reported in Table 2 suggests that the model-free approach of Coibion and Gorodnichenko (2011) is most likely the key driver behind the higher values for  $\psi_y$  reported in

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<sup>20</sup>We thank an anonymous referee for raising this question.

<sup>21</sup>We expanded this series up to 2008Q2 using data from the Philadelphia Fed Real-Time Data Center.



Figure 8.

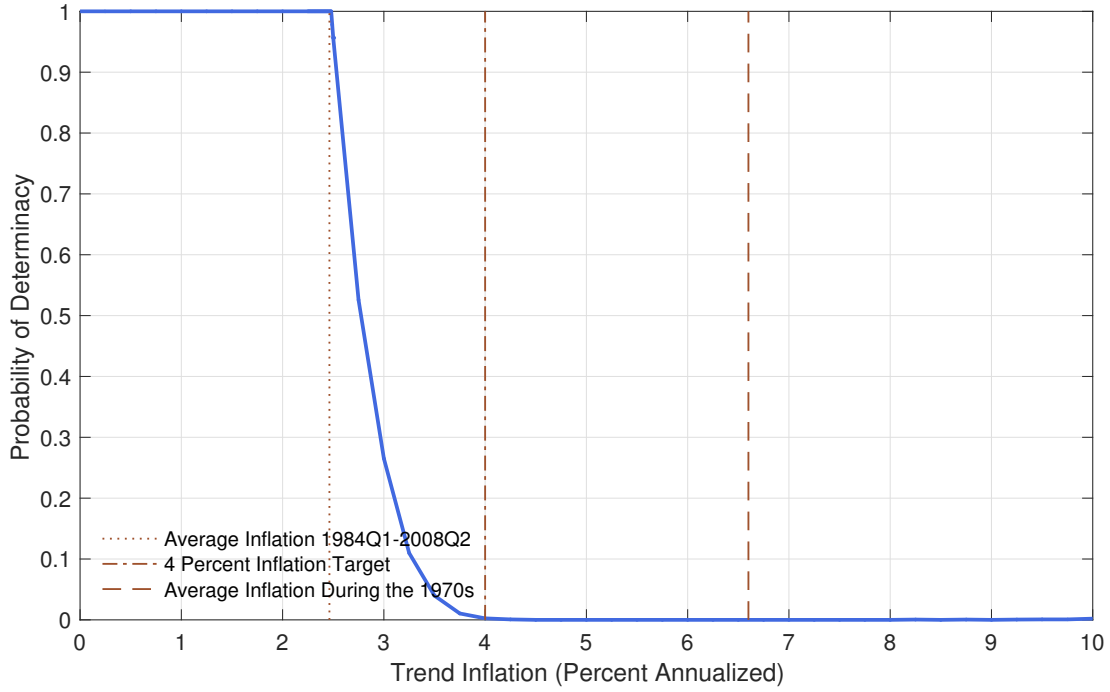


Figure 9: Probability of Determinacy and Trend Inflation

#### 4.3.2 The Role of Trend Inflation

The previous section supports the view that the systematic component of monetary policy is important to induce determinacy as put forward by [Clarida, Galí and Gertler \(2000\)](#) and [Lubik and Schorfheide \(2004\)](#). In this subsection, we show that trend inflation also plays a role supporting the view of [Coibion and Gorodnichenko \(2011\)](#).

To this end, we compare the probability of determinacy conditional on [Coibion and Gorodnichenko's \(2011\)](#) time-varying monetary policy when trend inflation is fixed at 4 percent (shown by the dashed-dotted line in [Figure 7a](#)) with the probability of determinacy conditional on [Coibion and Gorodnichenko's \(2011\)](#) time-varying monetary policy when trend inflation is allowed to vary (shown by the solid line in [Figure 6](#)). While the pattern of these conditional probabilities is similar, there are two notable differences during the period from 1977 until 1983 and during the 1990s. These differences provide useful insights on the role of trend inflation for determinacy.

Let's begin with the period 1977–1983. The probability of determinacy conditional on [Coibion](#)

and Gorodnichenko's (2011) time-varying monetary policy when trend inflation is fixed at 4 percent increases from below 0.2 in 1980 to about 0.5 in 1981. Because the level of trend inflation is fixed, this suggests that there is a structural change in the behavior of monetary policy around 1981—note that the dashed-dotted line in Figure 7a is above 0.5 in 1982 and at or above 0.7 in 1983. This is in contrast with the probability of determinacy conditional on Coibion and Gorodnichenko's (2011) time-varying monetary policy when trend inflation is allowed to vary, which is very close to zero between 1981 and 1983. That is, the probability of determinacy does not increase despite the change in the systematic component of monetary policy discussed above. Note that the probability of determinacy is above 0.5 only after the Volcker disinflation of 1979–1982 is accomplished.<sup>22</sup> This indicates that the high levels of trend inflation observed during the period from 1981 until 1983 prevented the change in the systematic component of monetary policy from increasing the probability of determinacy. This is in line with the narrative in Coibion and Gorodnichenko (2011).

Let's now turn to the role played by trend inflation during the 1990s. The probability of determinacy conditional on Coibion and Gorodnichenko's (2011) time-varying monetary policy when trend inflation is fixed at 4 percent had decreased on average since 1984 and it is consistently below 0.5 during the late 1990s, see the dashed-dotted line in Figure 7a. In contrast, the probability of determinacy conditional on Coibion and Gorodnichenko's (2011) time-varying monetary policy when trend inflation is allowed to vary has increased on average since 1987, reaching 0.7 during the early 1990s and remaining above that value for most of the remainder of the sample. The steady decline in the level of trend inflation during the 1990s explains this result. That is, the decline in trend inflation combined with active monetary policy tilted the U.S. economy toward a higher probability of determinacy. This effect is not present when we fix trend inflation at 4 percent, which explains the lower probability of determinacy.

## 5 Conclusion

We contribute to the debate on the costs and challenges associated with the recent proposal to increase the inflation target from 2 to 4 percent by studying the relation between trend inflation and

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<sup>22</sup>In 1983, the trend inflation estimate fell below 6 percent.

determinacy in an off-the-shelf estimated New Keynesian model without indexation. Specifically, we focus on the challenge that increasing the inflation target poses to equilibrium determinacy. Our main result suggests that such an increase in the inflation target does not imply a significant risk of self-fulfilling fluctuations for the U.S. economy. Importantly, this result is conditional on the estimated policy rule. When using real-time measures of the systematic component of monetary policy, the probability of determinacy drops significantly.

In our analysis, we abstract from price and wage indexation, and we assume a constant frequency of price adjustment. These are plausible assumptions for moderate levels of trend inflation; however, as trend inflation increases, for example, beyond 10 percent, firms are more likely to increase the frequency of price changes, see [Gagnon \(2009\)](#). This indicates that although models with state-dependent pricing could impact our conclusions, our findings provide a useful benchmark for the literature.

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# A Appendix

## A.1 Data

We estimate our model in Dynare 4.5.4. using Bayesian methods as described by [An and Schorfheide \(2007\)](#). We obtain 3 million draws from the posterior and discard the first 20 percent of them. The vector of observables contains data on inflation, growth rates of real GDP per capita, growth rates of real consumption per capita, growth rates of real investment per capita, growth rates of real wages, nominal interest rate, and deviations of hours worked from the steady-state.

The time series used to construct the vector of observables are retrieved from FRED, Federal Reserve Bank of St. Louis, and they are described as follows: (1) Real gross domestic product, billions of chained 2009 dollars, quarterly, seasonally adjusted annual rate; (2) Nominal gross domestic product, billions of dollars, quarterly, seasonally adjusted annual rate; (3) Personal consumption expenditures, nondurable goods, billions of dollars, quarterly, seasonally adjusted annual rate; (4) Personal consumption expenditures, services, billions of dollars, quarterly, seasonally adjusted annual rate; (5) Private residential fixed investment, billions of dollars, quarterly, seasonally adjusted annual rate; (6) Private nonresidential fixed investment, billions of dollars, quarterly, seasonally adjusted annual rate; (7) Effective federal funds rate, percent and annualized, quarterly, daily average aggregation, not seasonally adjusted; (8) Compensation per hour, nonfarm business sector, quarterly, seasonally adjusted, index 2009=100; (9) Civilian non-institutional population over 16, thousands of persons, quarterly; (10) GDP deflator =  $\frac{(2)}{(1)}$ ; (11) Real per capita GDP =  $\frac{(1)}{(9)} \frac{1E+9}{1E+3}$ ; (12) Real per capita consumption =  $\frac{(3)+(4)}{(10)(9)} \frac{1E+9}{1E+3}$ ; (13) Real per capita investment =  $\frac{(5)+(6)}{(10)(9)} \frac{1E+9}{1E+3}$ ; (14) Real wages =  $\frac{(8)}{(10)}$ ; (15) Federal funds rate =  $100 \left( \left( 1 + \frac{(7)}{100} \right)^{\frac{1}{4}} - 1 \right)$ ; (16) Average weekly hours, nonfarm business sector, quarterly, seasonally adjusted, index 2009=100; (17) Civilian employment over 16, thousands of persons, quarterly, average aggregate, seasonally adjusted; (18) Civilian non-institutional population over 16, thousands of persons; (19) Hours worked =  $100 \ln \left( \frac{\frac{(16)(17)1E+3}{100}}{(18)1E+3} \right)$ .

The growth rates of the GDP deflator, real per capita GDP, real per capita consumption, real per capita investment, and real wages are computed by transforming time series (10)-(14) to log

differences in percentages. The nominal interest rate corresponds to time series (15). Finally, the deviations of hours worked from the steady-state correspond to time series (19), demeaned.

## A.2 Structural Parameters

In this section, we begin by discussing the estimated parameters shown in Table 2.<sup>1</sup> For ease of exposition, we focus on the posterior mean. Consider the Taylor rule parameters. The response of the federal funds rate to one-period-ahead expected inflation  $\psi_\pi$  equals 2.42. The response of the federal funds rate to the output gap  $\psi_y$  and output growth  $\psi_{gy}$  is 0.03 and 0.52, respectively. The parameters describing the persistence of the Taylor rule  $\rho_{R1}$  and  $\rho_{R2}$  are 1.28 and  $-0.43$ , respectively. While our estimates for the response of the federal funds rate to inflation and the degree of monetary policy inertia are in line with Coibion and Gorodnichenko (2011), the response of the federal funds rate to the output gap and output growth is smaller than the findings of Coibion and Gorodnichenko (2011). Even so, our estimates are in line with Coibion and Gorodnichenko (2011) in that both imply a higher response to output growth than to the output gap.

Turning to the deep structural parameters, the investment adjustment cost parameter  $\kappa$  is equal to 4.12, which is between the posterior median reported by Justiniano and Primiceri (2008), 2.83, and the posterior mean reported by Smets and Wouters (2007), 6.23. The degree of habit persistence in our model is 0.82, which is slightly larger than the posterior mean reported by Smets and Wouters (2007), i.e., 0.68, and the posterior median reported by Justiniano and Primiceri (2008), 0.77. The markup for intermediate goods  $(\eta_p - 1)^{-1}$  is equal to 0.23, a tad larger than the value estimated by Justiniano and Primiceri (2008), 0.18. The markup for labor types  $(\eta_w - 1)^{-1}$  is equal to 0.17 as in Justiniano and Primiceri (2008).

The inverse of the Frisch elasticity of labor supply  $\tau$  is equal to 1.37, which is smaller than the posterior median reported by Justiniano and Primiceri (2008) and Smets and Wouters (2007) but close to that reported by Christiano, Eichenbaum and Trabandt (2016). The estimates for the parameters governing the frequency of price and wage adjustment  $\nu_p$  and  $\nu_w$  are 0.81 and 0.51, implying that firms adjust prices approximately once every 16 to 17 months, and wages

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<sup>1</sup>Overall, the structural and the exogenous parameters exhibit good mixing properties. The convergence diagnostics are available upon request from the authors.



are adjusted once every 6 to 7 months. Our estimates for  $\nu_p$  and  $\nu_w$  lie between the values estimated by [Justiniano and Primiceri \(2008\)](#) and the values estimated by [Smets and Wouters \(2007\)](#), respectively.

### A.3 Exogenous Parameters

Table [A.3.1](#) shows the estimated parameters of the exogenous processes.

Parameters	Posterior				Prior		
	Mode	Mean	5 Percent	95 Percent	Distribution	Mean	Std
$\rho_d$	0.46	0.48	0.23	0.73	Beta	0.50	0.20
$\rho_{dL}$	0.97	0.96	0.94	0.98	Beta	0.50	0.20
$\rho_A$	0.10	0.12	0.04	0.22	Beta	0.50	0.20
$\rho_{mI}$	0.71	0.70	0.60	0.79	Beta	0.50	0.20
$\rho_G$	0.96	0.96	0.95	0.98	Beta	0.50	0.20
$\sigma_d$	0.020	0.022	0.013	0.036	Inverse Gamma	0.001	2.00
$\sigma_{dL}$	0.019	0.025	0.016	0.040	Inverse Gamma	0.001	2.00
$\sigma_A$	0.005	0.005	0.005	0.006	Inverse Gamma	0.001	2.00
$\sigma_{mI}$	0.024	0.025	0.019	0.033	Inverse Gamma	0.001	2.00
$\sigma_R$	0.001	0.001	0.001	0.001	Inverse Gamma	0.001	2.00
$\sigma_G$	0.017	0.017	0.015	0.019	Inverse Gamma	0.001	2.00
$\sigma_{obs\pi}$	0.001	0.001	0.001	0.002	Inverse Gamma	0.001	2.00
$\sigma_{obs_w}$	0.007	0.007	0.006	0.008	Inverse Gamma	0.001	2.00

Table A.3.1: Estimated Parameters of the Exogenous Processes

### A.4 Estimating the Model Using Real-Time Output Gap Data

In this section we present a summary of the results obtained when using [Coibion and Gorodnichenko's \(2011\)](#) real-time output gap data as an additional observable in the estimation of the model described in Section [2](#). For easy of exposition, we focus on the deep structural parameters and abstract from discussing the parameters characterizing the law of motion of the shocks. Table [A.4.1](#) shows the results along with the posterior moments of Table [2](#) which we repeat here to facilitate a comparison. We refer to the latter as Posterior (Benchmark).

Figure [A.4.1](#) plots the posterior mean of the one-step-ahead forecasts for selected variables.

The fit of the model estimated by adding the [Coibion and Gorodnichenko's \(2011\)](#) real-time output gap as an additional observable is similar for most of the variables except for inflation which is poorly fit as shown by the upper left subplot.

Parameters	Posterior			Posterior (Benchmark)		
	Mean	5 Percent	95 Percent	Mean	5 Percent	95 Percent
$\psi_\pi$	3.25	2.73	3.83	2.42	1.96	2.93
$\psi_y$	0.04	0.02	0.07	0.03	0.00	0.08
$\psi_{gy}$	0.38	0.28	0.51	0.52	0.29	0.76
$\rho_{R1}$	1.23	1.10	1.36	1.28	1.17	1.39
$\rho_{R2}$	-0.41	-0.52	-0.29	-0.43	-0.52	-0.32
$\kappa$	4.05	2.88	5.39	4.12	2.88	5.55
$b$	0.71	0.63	0.79	0.82	0.71	0.90
$(\eta_p - 1)^{-1}$	0.20	0.14	0.27	0.23	0.16	0.29
$(\eta_w - 1)^{-1}$	0.24	0.18	0.30	0.17	0.11	0.25
$\tau$	1.67	1.10	2.35	1.37	0.70	2.22
$\nu_p$	0.84	0.80	0.87	0.81	0.77	0.84
$\nu_w$	0.78	0.72	0.83	0.51	0.36	0.65

Table A.4.1: Estimated Structural Parameters

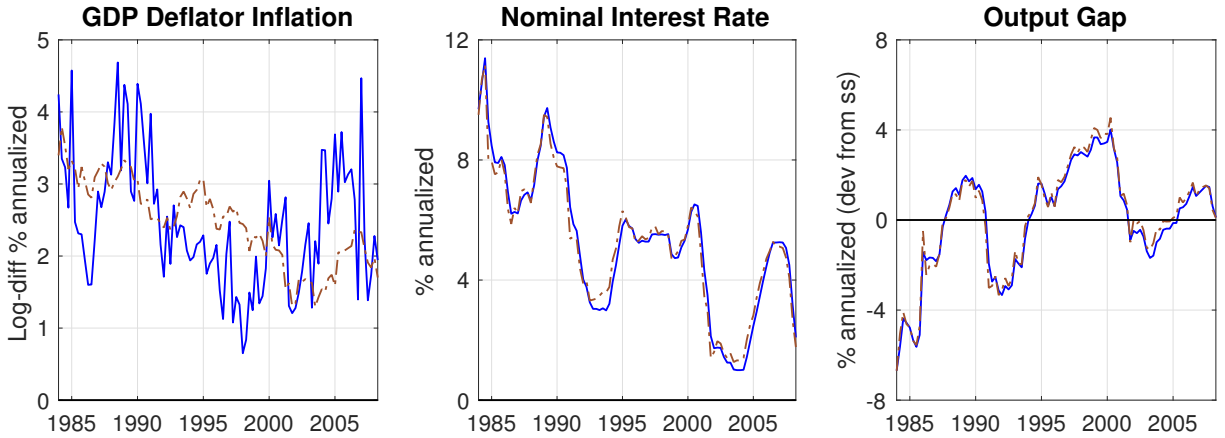


Figure A.4.1: One-Step-Ahead Forecasts of Selected Variables