


Experimental Quantum Advantage in the Odd-Cycle Game

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We report the first experimental demonstration of the odd-cycle game. We entangle two atoms separated by ~ 2 m, and the players use them to win the odd-cycle game with a probability $\sim 26\sigma$ above that allowed by the best classical strategy. The experiment implements the optimal quantum strategy, is free of loopholes, and achieves 97.8(3)% of the theoretical limit to the quantum winning probability. We perform the associated Bell test and measure a nonlocal content of 0.54(2)—the largest value for physically separate devices, free of the detection loophole, ever observed.

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Quantum advantage refers to the use of quantum systems to perform a given task more efficiently than classical physics allows. But what does it take to convince anyone that quantum advantage is real? We suggest answering this question by identifying a problem (i) that can be expressed in everyday terms, and (ii) whose optimal classical solution is evident without a formal mathematical argument, and proposing an experiment (iii) that is feasible, (iv) that is a faithful implementation of the problem, and (v) whose result will immediately convince sceptics.

The quantum advantage for random circuit sampling [1] and Gaussian boson sampling [2] is impressive, but these problems involve complex probability distributions, and their classical solutions have not yet been proven optimal [3,4]. Even the violation of a Bell inequality [5] may not suffice for our purpose, since the classical limit of the inequality may not be evident without formal mathematical proof. One candidate could be the three-player Greenberger-Horne-Zeilinger (GHZ) game [6,7]. However, the state-of-the-art fidelity of GHZ states between three separated systems [8] does not yet permit quantum advantage at this game. We can give an entangled photon to each player [9], but then we have to contend with the detection loophole [10] and add some postprocessing. As we shall see, the odd-cycle game [11], however, arguably satisfies both (i) and (ii). In this Letter, we present a realization of the optimal quantum strategy to win this game. Our implementation satisfies (iii)–(v), as the game is played

by independent, physically separated players, and is free of loopholes (no extra assumptions are needed).

Bipartite nonlocal games [12] involve two cooperating players, Alice and Bob, interacting with a referee (Fig. 1). The players can choose a strategy involving quantum entanglement to increase their winning chances compared to a local, classical strategy. Nonlocal games differ from bipartite Bell tests in the way the local measurement settings are chosen. In bipartite Bell tests, the parties are assumed to have free will and choose their inputs independently. In nonlocal games, the rules include that a referee chooses all inputs (queries) for all players according to a predefined joint distribution and that the players return their outputs without consulting each other. The referee then evaluates the players' responses to the inputs using a winning condition.

This definition of nonlocal games precludes some of the loopholes that typically bedevil bipartite Bell tests. For instance, the freedom-of-choice assumption becomes obsolete because the referee chooses the settings instead of the players. Moreover, as no communication is allowed

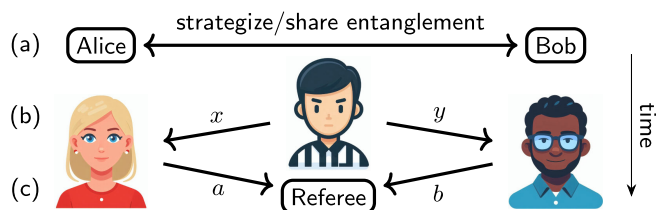


FIG. 1. One round of a nonlocal game consists of three steps. (a) The players strategize and/or share entanglement. (b) A referee provides inputs x and y to the players. (c) The players independently output a and b , and the referee evaluates the winning condition for that round. Many rounds are repeated to estimate the winning probability for each strategy.

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between players and all inputs originate from the referee, it is neither necessary nor possible to impose spacelike separation between the input of one player and the output of another player, unlike in loophole-free Bell tests [13–17].

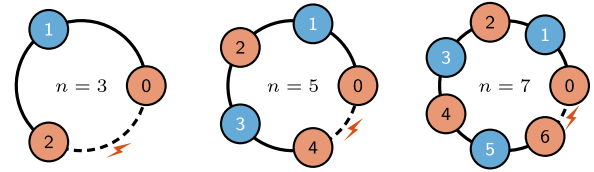
A nonlocal game exhibits quantum advantage if, when players share entanglement, they achieve a winning probability ω_q that is greater than the probability ω_c of winning using the best classical strategy. For the simplest nonlocal game, the Clauser-Horne-Shimony-Holt (CHSH) game [5], the quantum strategy wins on average $\omega_q = \cos^2(\pi/8) \approx 85\%$ of rounds compared to the best classical strategy with $\omega_c = 75\%$. Without knowledge of quantum physics, an observer would be drawn to conclude that the players communicate telepathically in order to win the game more frequently than mathematically possible. For this reason, “pseudo-telepathy” is attributed to games for which $\omega_q = 1$, such as the magic square game [12,18–20]. However, it can be proven that the quantum advantage of any bipartite nonlocal game with an output alphabet of size 2, such as the CHSH game and the odd-cycle game, is restricted to $\omega_q < 1$ [21].

Nonlocal games have so far only been demonstrated in a photonic system [22]; earlier photonic experiments explored nonlocality and related concepts [23–30]. In those cases, imperfect detection efficiencies made it necessary to ignore no-click outcomes while invoking the fair-sampling assumption to acknowledge the possibility of hidden variables biasing the distribution of photon loss events. This constitutes the detection loophole, which introduces margin for the players to cheat in nonlocal games by claiming photon loss depending on the inputs.

The detection efficiency requirements needed to avoid this loophole are challenging to overcome in photonic systems [14]. However, atomic systems can be measured with near-perfect efficiency, and have been used to perform detection-loophole-free tests of contextuality, and to violate Bell inequalities [16,31–34]. Here, we implement the odd-cycle game for the first time and show experimentally that the quantum strategy outperforms the best classical strategy. We use remotely entangled trapped atomic ions, which are among the leading platforms for quantum networking in terms of entanglement fidelity and rate [35]. This allows us to argue convincingly that the players are physically separated by a macroscopic distance and that each of them controls and acts on a different physical system in a totally independent way. Furthermore, as trapped ions can be measured with high efficiency, we do not invoke the fair-sampling assumption to justify the quantum advantage.

To illustrate the problem underlying the odd-cycle game, imagine setting a round table with an odd number of seats n and alternating red and blue plates. Because there is an odd number of seats, but only two colors, there is always one offending adjacent pair of plates with the same color [Fig. 2(a)]. In the odd-cycle game, a referee individually asks two isolated players, Alice and Bob, to specify the

(a) Classical strategy



(b) Quantum strategy

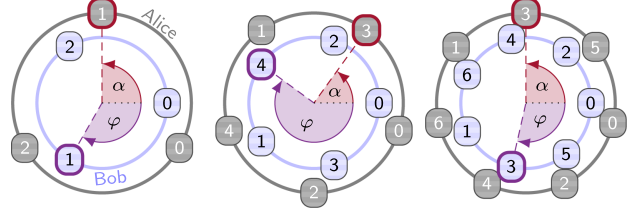


FIG. 2. Two cooperating players, Alice and Bob, try to convince a referee that the vertices of an odd cycle can be two colored. The best classical strategy (a) is for the players to agree on one particular coloring before the game (examples shown for $n \in \{3, 5, 7\}$). This strategy wins all queries except the one where adjacent vertices inevitably have the same color (see dashed edges). In the quantum strategy (b), Alice and Bob each measure their half of an entangled Bell state. The optimal qubit rotation angles [Eq. (1)] for any given query (vertex to specify the color of) are shown, with example inputs highlighted. Alice’s angles (outer circle) are shifted by an angle $\pi/(2n)$ with respect to Bob’s (inner circle). Using the correlated nature of the measurement outcomes, the players can win the game more often than using the optimal classical strategy.

color for a given seat, and arranges that they are given either the same seat, or adjacent seats, with Alice’s seat x always to the “left” of Bob’s seat y , such that $y \equiv x + 1 \pmod{n}$. For example, if $n = 3$, the set of inputs is $\{00, 01, 11, 12, 22, 20\}$. In each round, the referee selects one of the $2n$ possible queries with uniform probability. Each of Alice and Bob returns a bit indicating the color of the plate to the referee. The players win a round of the game if and only if their responses (a, b) satisfy the following winning condition: when given the same seat, their response should be the same, and when given adjacent seats, their responses should be different. Since they are not allowed to communicate during the game, their best classical strategy is to agree on a particular coloring before the game, such as $a = x \pmod{n}$ and $b = y \pmod{n}$, as illustrated in Fig. 2(a) for $n \in \{3, 5, 7\}$. The players win all queries except the one where, in their chosen arrangement, adjacent seats inevitably have been assigned the same color. Therefore, using the optimal classical strategy, the winning probability is $\omega_c = 1 - 1/(2n)$.

To implement the optimal quantum strategy [11] (see Supplemental Material [36] for an alternative proof of optimality), Alice and Bob share a Bell state $|\Psi^+\rangle = (|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)/\sqrt{2}$ prior to each round of the game. The states $|0\rangle$ and $|1\rangle$ are eigenstates of the Pauli operator \hat{Z} . In the following, we will refer to vertices in an odd cycle

instead of seats on a round table. Once the players have generated entanglement, the referee distributes the queried vertices x and y to Alice and Bob, who perform a qubit rotation $\hat{R}(\theta) = \cos(\theta/2)\hat{I} - i\sin(\theta/2)\hat{Y}$ with rotation angles given by

$$\alpha_x = \varphi_x - \frac{\pi}{2n}, \quad (1a)$$

$$\beta_y = -\varphi_y, \quad (1b)$$

respectively, where $\varphi_k = \pi k(n-1)/n$ [Fig. 2(b)]. Finally, they obtain their responses a and b by measuring their qubits in the \hat{Z} basis. Alice inverts her measurement result to obtain her output bit a , and Bob uses his measurement result directly as output bit b . This process implements a qubit-qubit strategy [40] with a winning probability of $\omega_q = \cos^2[\pi/(4n)]$.

In our experiment [Fig. 3(a)], each player controls their own apparatus for trapping and manipulating a $^{88}\text{Sr}^+$ trapped-ion qubit [35]. The systems are separated by ~ 2 m and controlled independently. The referee implements a state machine with synchronized digital communication with the control systems of the two players. The trapped ions are interfaced to optical fibers using free-space collection optics. To generate remote entanglement in preparation for a round of the odd-cycle game, both ions are simultaneously excited by a 10-ps laser pulse. Hereafter, the polarization of each of the spontaneously emitted 422-nm single photons is entangled with the emitter's spin state [Fig. 3(b)]. A photonic Bell state measurement at a central heralding station swaps entanglement from the ion-photon pairs onto the remote ions. One of four two-photon coincidence events $\gamma \in \{0, 1, 2, 3\}$,

each corresponding to a different photon detector click pattern, heralds the creation of

$$|\Psi^+(\vartheta_\gamma)\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + e^{i\vartheta_\gamma}|1\rangle_A|0\rangle_B). \quad (2)$$

To produce the same state $|\Psi^+\rangle$ independent of γ , Bob applies the phase gate $\exp(i\hat{Z}\vartheta_\gamma/2)$ to his qubit. Generating entanglement prior to the start of a game takes, on average, ~ 30 ms; generation attempts are performed at a 1-MHz rate for 250 μs with interleaved periods of 770 μs consisting of Doppler cooling and electromagnetically induced transparency cooling of the axial motion. Compared to [35], additional ground state cooling and shorter attempt periods increased Bell state fidelities to ~ 0.97 . The subsequent interaction with the referee lasts ~ 600 μs , which includes communication of the inputs (< 1 μs), qubit manipulation (~ 20 μs), qubit readout (550 μs), communication of the outputs (< 1 μs), and delays (~ 30 μs).

The players implement the input-dependent rotation using a sequence of two laser pulses on the 674-nm qubit transition $|0\rangle \leftrightarrow |\tilde{1}\rangle$ [Fig. 3(b)] followed by state-dependent fluorescence readout. First, a $\pi/2$ pulse around the \hat{X} axis effectively swaps the \hat{Z} axis with the \hat{Y} axis. Then, a $\pi/2$ pulse with a relative phase of $\pi - \theta$ applies the target rotation $\theta \in \{\alpha_x, \beta_y\}$ around the \hat{Z} axis while swapping the \hat{Z} and \hat{Y} axes back. This way, the variable angle θ is encoded in a digital phase instead of a physical pulse duration, for which independent calibrations would be necessary. Both players transmit their outputs electronically in real time to the referee to conclude one round of the game.

We realize the odd-cycle game for $3 \leq n \leq 27$ and measure the experimental winning probability [Fig. 4(a)]. The number of vertices n is announced before the start of the game, and the players use it to calculate the measurement angles α_x and β_y [Eq. (1)] in every round. In total, 101000 rounds of the odd-cycle game were played. On average, the players perform at a $\langle \omega/\omega_q \rangle = 97.8(3)\%$ level with respect to the theoretical limit imposed by quantum mechanics. These values are significantly above the classical limits for $n \leq 19$, thus demonstrating a clear quantum advantage at this game. The observed reduction compared with the quantum limit is dominated by imperfections in the entangled state [Eq. (2)]. The measurement error is independent of the number of vertices n in the odd cycle and the particular inputs x and y because the qubit rotation angles [Eq. (1)] are encoded in a relative phase. Contrary to previous experimental demonstrations of nonlocal games using photonic systems [22], we employ shot-by-shot random switching of the inputs to the players. Moreover, as the players each have a quantum memory, they can operate in an “event-ready” manner. Thus, the first stage of the game, in which the players are allowed to “strategize” [Fig. 1(a)], is well separated temporally from the subsequent

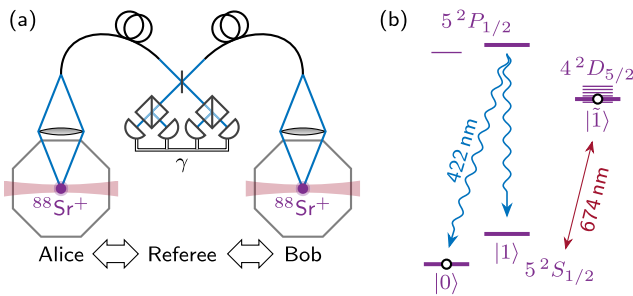


FIG. 3. (a) Alice and Bob each operate an ion trap loaded with a single $^{88}\text{Sr}^+$ ion. The ions are entangled photonically by entanglement swapping at a central heralding station. (b) Level diagram of $^{88}\text{Sr}^+$ (not to scale). Single photons for generating entanglement are spontaneously emitted on the 422-nm $S_{1/2} \leftrightarrow P_{1/2}$ transition. Upon a two-photon herald, the $S_{1/2}$ ground state qubits $\{|0\rangle = |m_J = -1/2\rangle, |1\rangle = |m_J = 1/2\rangle\}$ of the remote ions become entangled. A 674-nm π pulse then maps $|1\rangle$ to $|\tilde{1}\rangle = |D_{5/2}, m_J = -3/2\rangle$, and the query-dependent rotations are performed on the resulting optical qubit $\{|0\rangle, |\tilde{1}\rangle\}$.

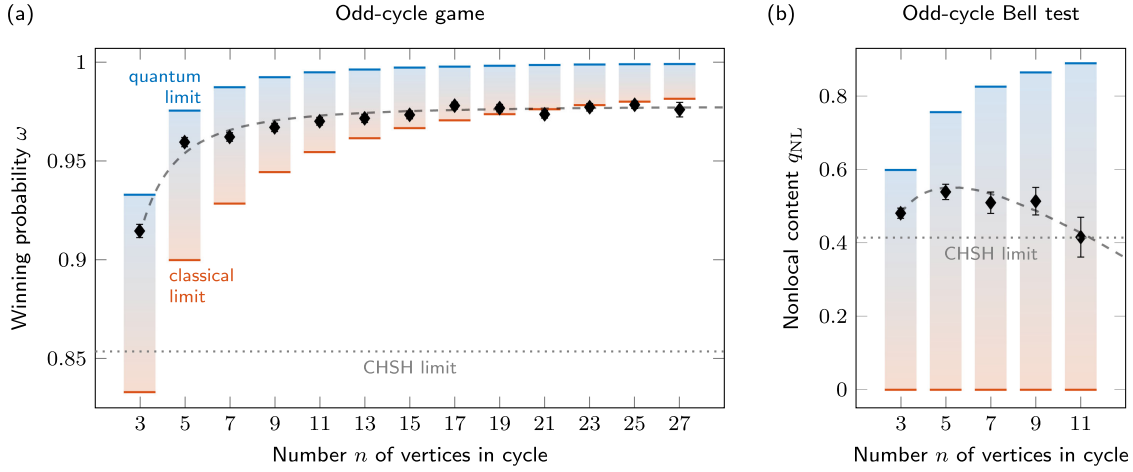


FIG. 4. Experimental results demonstrating quantum advantage and nonlocality in the odd-cycle game. (a) Alice and Bob use a preshared entangled state to increase their winning probability compared to the best classical approach. For $n = 3$, the winning probability is $\sim 26\sigma$ above the classical limit. Because of experimental imperfections, their quantum advantage is limited to $n \leq 19$. (b) The nonlocal content q_{NL} is measured in a separate Bell experiment, where Alice and Bob choose their inputs independently. Error bars represent the binomial standard error. Dashed curves show the theoretical prediction for performance at the 97.8% level to the quantum limit. Dotted lines indicate the quantum limit for the CHSH case for reference.

individual information exchange with the referee [Figs. 1(b) and 1(c)].

In a second experiment, we perform the bipartite Bell test variant of the odd-cycle game by letting the players choose their inputs (x, y) independently. We use the Bell inequality associated with the winning probability of the odd-cycle game,

$$\omega = \frac{1}{2n} \sum_{j=0}^{n-1} P(0, 0|j, j) + P(1, 1|j, j) + P(0, 1|j, j+1) + P(1, 0|j, j+1) \leq \omega_c. \quad (3)$$

The nonsignaling correlations $P(a, b|x, y)$ can be decomposed in terms of local correlations $P_L(a, b|x, y)$ and non-local nonsignaling correlations $P_{NL}(a, b|x, y)$. Consider all possible decompositions of the form

$$P(a, b|x, y) = q_L P_L(a, b|x, y) + q_{NL} P_{NL}(a, b|x, y), \quad (4)$$

with $q_{NL} = 1 - q_L$ and $0 \leq q_L \leq 1$. The local content q_L^{\max} of $P(a, b|x, y)$ [41] is the maximum local weight over all decompositions of the form (4), i.e., $q_L^{\max} \doteq \max_{\{P_L, P_{NL}\}} q_L$. A correlation is local if and only if $q_{NL}^{\min} = 0$. The local content of a correlation attaining a winning probability ω given by (3) is upper bounded by $q_L^{\max} \leq (1 - \omega)/(1 - \omega_c)$ (see Supplemental Material [36]).

In total, 134 000 repetitions of the odd-cycle Bell test were performed. The experimentally measured nonlocal content [Fig. 4(b)] of up to $q_{NL} = 0.54(2)$ for $n = 5$ is, to the best of our knowledge, the highest detection-loophole-free value ever observed between physically separate devices (see [28] for experimental values obtained with

photons without closing the detection loophole, and [34] for the highest nonlocal content with the detection loophole closed, but using a single device). This value is $\sim 6\sigma$ above the theoretical value for the maximum quantum violation of the CHSH inequality $q_{NL} = \sqrt{2} - 1 \approx 0.414$.

The results from both the odd-cycle game and the associated Bell test were obtained using the outcomes from every round commencing with entanglement generation, so that the fair-sampling assumption was not required and the detection loophole fully closed. This eliminates any extra assumptions from the implementation of the odd-cycle game, rendering it loophole-free. For a loophole-free measurement of nonlocal content above the CHSH limit, the locality loophole is yet to be closed by separating Alice and Bob further to establish spacelike separation between their measurements [13–17].

The same resources that lead to quantum advantage in the odd-cycle game can be applied to practical scenarios in which collaborating agents cannot communicate, such as the rendezvous task [42–45]. More complex nonlocal games, such as the Kochen-Specker game and the magic square game [11,22], could be performed in our apparatus if the dimension of the entangled state is increased; by cotrapping a second ion [46], the players could distribute two Bell pairs at the start of each round and perform local operations to generate qutrit-qutrit entanglement, or four-qubit entanglement, to implement different quantum strategies. While the necessary experimental tools for these processes are available in our system [47], their fidelity would still need to be improved to demonstrate a quantum advantage at those games.

In summary, we have implemented the odd-cycle game, which is a nonlocal game where a qubit-qubit strategy

offers quantum advantage compared to the best classical strategy. Using remotely entangled trapped ions, we have realized a quantum advantage for odd cycles of up to $n = 19$ vertices. Since each round produces a valid set of outcomes, we are able to close the detection loophole in this demonstration. In addition, we have performed the Bell test associated with the odd-cycle game and measured the highest nonlocal content for physically separate systems where the detection loophole is closed.

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