



# Department of Economics Discussion Paper Series

## Contagion in Derivatives Markets

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Number 839

First Published: November, 2017

Revised: January, 2018

# Contagion in Derivatives Markets<sup>†</sup>

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January 5, 2018

## Abstract

A major credit shock can induce large intra-day variation margin payments between counterparties in derivatives markets, which may force some participants to default on their payments. These payment shortfalls become amplified as they cascade through the network of exposures. Using detailed DTCC data we model the full network of exposures, the shock-induced payments, the initial margin collected, and liquidity buffers for about 900 firms operating in the U.S. credit default swaps market. We estimate the total amount of contagion, the marginal contribution of each firm to contagion, and the number of defaulting firms for credit shocks of different magnitudes. A novel feature of the model is that it allows for a range of possible responses to balance sheet stress, including delayed or partial payments. These ‘soft default’ options distinguish our approach from conventional network models, which typically assume that full default is triggered whenever the default boundary is breached.

**Keywords:** Financial networks, contagion, stress testing, credit default swaps.

**JEL Classification Numbers:** D85, G23, L1

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<sup>†</sup>For constructive comments on earlier drafts the authors thank Nina Boyarchenko, Pierre Collin-Dufresne, Rama Cont, Gregory Feldberg, Samim Ghamami, Paul Glasserman, Jonathan Glicoes, Benjamin Munyan, Matthew Pritsker, Emil Siriwardane, Stathis Tompaidis, and participants at the NBER Summer Institute Risks of Financial Institutions, London Quantitative Finance Seminar, Bank of England One Bank Seminar, and the Conference on Banks, Systemic Risk, Measurement and Mitigation in Rome Italy.

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# 1 Introduction

The recent financial crisis highlighted the potential risks posed by derivatives markets. As the crisis unfolded, large sellers of protection such as American International Group, Inc. (AIG) became liable for payments on credit default swaps (CDS) contracts they had previously sold to banks and dealers as protection against credit defaults. Until the crisis, these protection sellers received a steady stream of payments from protection buyers and they rarely had to pay out claims. When the crisis hit, the sudden calls for payments put great pressure on the sellers, some of which had a thin capital base. In particular, AIG had to be rescued by the U.S. Federal Reserve in order to meet its margin obligations on CDS contracts to dealers, who in turn were threatened if the payments were not made.

This incident casts a spotlight on the potential risks posed by the CDS market to the stability of the financial system. Prior to the crisis this market grew extremely rapidly. From its inception in the 1990s to 2007 the total notional value of CDS contracts rose to about \$60 trillion, though it has since declined to about \$35 trillion. Under a shock to credit markets the sellers of CDS protection become liable for payments to the buyers. AIG got into trouble because it had sold protection on large pools of subprime mortgages to a variety of banks and broker-dealers. During the crisis the value of these mortgages deteriorated sharply, which triggered margin calls that AIG was unable to meet (Financial Crisis Inquiry Commission (2011)).

The AIG episode prompted many changes in derivatives markets, however payment risks remain. In this paper we analyze the propagation of losses through network spillover effects of losses and consequent payment reductions, which we refer to as contagion. The horizon over which contagion occurs critically differentiates our focus. Payment expectations in many derivatives markets are typically met once a day, but in some cases may be subject to much shorter compliance periods. These stringent expectations preclude firms from engaging in financing short-term liquidity, which could require more time. Consequently, the market must be capable of meeting the immediacy of payment obligations with access to liquid

reserves.

Since the crisis, various reforms have been introduced to mitigate destabilizing risks in the financial system. First, requirements for posting initial margin as security against nonpayment have been strengthened. This has occurred both in the scope of firms that are required to post initial margin and in the segregation of initial margin accounts, so that one counterparty's losses cannot be covered by another counterparty's funds. These developments have facilitated a movement towards exchanging variation margin on a daily basis. Variation margin payments, or bilateral exchanges in contractual value, were more infrequent prior to the financial crisis and were not the subject of regulation.

A second reform prompted by the crisis is to incentivize firms to trade contracts through central counterparties (CCPs). Central clearing is encouraged through higher capital and margin requirements for non-centrally cleared contracts.<sup>1</sup> An advantage of central clearing is that it fosters contractual standardization and shortens intermediation chains, which can help to reduce network contagion (Cont and Minca (2016)). A disadvantage is that it concentrates risk at a single point of failure and imposes on the CCP's counterparties much shorter variation margin compliance windows than existed before the crisis.

In this paper we model the propagation of losses in derivatives markets through network spillover effects, taking account of the short time frame within which payments must be made, and also the key role played by the CCP. We focus on the CDS market. We estimate the total amount of contagion, and the contribution of individual firms to contagion, under a range of assumptions about their response to balance sheet stress. The model builds on the framework of Eisenberg and Noe (2001), but it has several novel elements that are specific to OTC derivatives markets. In particular the model incorporates two safety valves that mitigate network spillover effects. One is the posting of initial margin as a security deposit against potential default. The second is the maintenance of in-house cash reserves and dedicated lines of credit to manage daily fluctuations in the demand for variation margin

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<sup>1</sup>However these incentives may not be sufficient to induce firms to clear in all cases; for an analysis of this issue see Ghamami and Glasserman (2016).

payments. Initial margins and cash reserves vary substantially among firms depending on the risk characteristics of their portfolios, which we observe in the data. The model therefore incorporates heterogeneity in firms' balance sheets as well as their positions in the network.

A second contribution of this paper is to allow for a range of possible strategies to cope with short-term liquidity stress, including delayed or partial payment and payment with illiquid collateral. We show how to accommodate a range of such responses in the model and then explore their implications for contagion using detailed exposure data provided to the Office of Financial Research by the Depository Trust & Clearing Corporation (DTCC). The data provide a detailed picture of the network of counterparty exposures in the United States CDS market at particular dates, including exposures between banks, dealers, hedge funds, asset managers, and insurance companies. We also use the data to estimate the initial margin posted by each counterparty and the liquidity buffers that they maintain to manage daily fluctuations in margin calls.

The final contribution of this work is to estimate the total payment shortfalls that would result from a severe but not implausible market shock, namely, the Federal Reserve's 2015 Comprehensive Capital Analysis and Review (CCAR) trading book shock. This shock was designed to test the robustness of the financial system under severe conditions, while embedding co-movements in the value of credit instruments that we are not in a position to estimate ourselves. The shock implies a sudden decrease in the value of corporate and sovereign debt on which CDS contracts are written, and thus results in large demands for variation margin payments between counterparties to these contracts.

The plan of the paper is as follows. In the next section we provide an overview of the literature on network contagion, including recent papers on CDS markets. In section 3 we introduce the network model and demonstrate the existence of a payments equilibrium under very general conditions on the firms' response functions. In section 4 we discuss specific examples of such response functions, including nonpayment, partial payment, payments with illiquid collateral, and the circumstances under which their counterparties might accept such

arrangements in lieu of declaring default. In sections 5-6 we describe the DTCC data in detail and show how it can be used to estimate the initial margins posted by counterparties, the amount of cash reserves firms need to manage their own accounts, and the variation margin payments induced by a shock.

In section 7 we bring these elements of the model together and estimate the total amount of contagion that would be produced by the CCAR shock. We also conduct a sensitivity analysis to assess how the amount of contagion is affected by the size of the shock, the size of the liquidity buffers, and the strategies that firms use to manage balance-sheet stress. We find that even when liquidity buffers are large enough to handle daily fluctuations in variation margin payments with 99.7% probability, and the initial margins are large enough to cover payment delinquencies with 99.5% probability (based on historical DTCC data), the amount of default contagion under the CCAR shock could be very substantial. In particular, the percentage of total payments that are in default is on the order of 5-12%, and about 15% of all market participants would be in technical default. We also estimate the marginal contribution to contagion of individual firms by asking how much total contagion would be reduced if their payments could be fully guaranteed by some outside entity. We find that the CCP contributes considerably less at the margin than do several large non-member firms that are peripheral to the network but have very large imbalances in their buy and sell positions.

## **2 Related Literature**

The financial crisis of 2007-09 has sparked a rapidly growing literature, both theoretical and empirical, on financial networks and systemic risk. A central theme of this literature is the relationship between network structure and the risk of contagion. Network connections can have a positive effect by diversifying the risk exposures of individual market participants, but they can also have a negative effect by creating channels through which shocks can

spread. The tension between these two forces has been explored in a variety of papers: see among others Allen and Gale (2000); Freixas et al. (2000); Gai and Kapadia (2010); Gai et al. (2011); Haldane and May (2011); Blume et al. (2011); Cont et al. (2013); Elliott et al. (2014); Acemoglu et al. (2015).

A key methodology for analyzing how networks propagate contagion is based on Eisenberg and Noe (2001), who show how payment shortfalls that originate at some nodes can cascade through the network causing an ever-widening series of defaults. There is now a substantial literature that builds on their approach (as we do here); see in particular Upper and Worms (2004), Elsinger et al. (2009), Rogers and Veraart (2013), Elliott et al. (2014) and Acemoglu et al. (2015). For general surveys of the literature see Bisias et al. (2012) and Glasserman and Young (2016).

There is also a literature that focuses specifically on the network structure of CDS markets. The potential for contagion in these markets was first highlighted by Cont (2010), who emphasized the importance of adequate liquid reserves to cope with large and sudden demand for variation margin. This paper also analyzed the extent to which a CCP can mitigate contagion, a topic that was subsequently treated by Duffie and Zhu (2011), Cont and Kokholm (2014) and Cont and Minca (2016). Various authors have studied the structure of CDS exposures and the potential for contagion among European banks; see in particular Brunnermeier et al. (2013), Peltonen et al. (2014), Vuillemeys and Peltonen (2015), and Clerc et al. (2014). Cont and Minca (2016) analyze the combined network of exposures in the CDS and interest rate swap markets together, and argue that central clearing in both markets can significantly reduce the probability and magnitude of illiquidity spirals.

Their work differs from the present paper in the methodologies used to study contagion, and in the focus on the European rather than the U.S. market.<sup>2</sup> More recently, Ali et al. (2016) examine the network structure of the CDS market in the United Kingdom. These

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<sup>2</sup>There has been less research on the CDS market in the United States, mainly due to limited data access. To get around this difficulty, Markose et al. (2012) use imputation techniques to estimate CDS exposures from publicly reported balance sheet data (see also Markose (2012) and the BIS Macroeconomic Assessment Group on Derivatives (2013)).

authors argue, as do Glasserman and Young (2016), that the systemic importance of market participants is not fully captured by conventional measures of centrality. The size and structure of financial firms' balance sheets, in addition to their position in the network, is crucial to understanding how much risk they pose to the system as a whole.

### 3 Network Contagion Model

We take as given the network of CDS contracts that exist between counterparties at a given point in time and study the impact of a sudden shock to credit markets. Such a shock triggers large variation margin (VM) payments from the sellers to the buyers of CDS protection. Suppose, for example, that a firm sold \$100 million in protection against the default of corporation C over the next five years. The shock decrease the value of C's debt and correspondingly increases the implied probability that C will default within the contract period. This increases the value of the CDS contract to the buyer, and increases the liability of the seller to the buyer. This change in value must be paid by the seller to the buyer as variation margin; moreover the payment is typically due within one day. The upshot is that shocks to the credit markets impose expectations of rapid cash payments between participants in the CDS market.

Let  $\bar{p}_{ij}$  denote the VM payment obligation from firm  $i$  to firm  $j$  that results from a given shock. Let  $\bar{p}_i = \sum_{j \neq i} \bar{p}_{ij}$  denote the total payment obligation of  $i$  to all other firms. In what follows we shall restrict attention to those firms  $i$  such that  $\bar{p}_i > 0$ . The others do not transmit payment shortfalls; instead they act as shock absorbers. In the present context these firms are buyers of protection (not sellers) and under a shock they will have no VM obligations.

The *relative liability* of firm  $i$  to firm  $j$  is

$$a_{ij} = \bar{p}_{ij} / \bar{p}_i. \tag{1}$$



Note that for each  $i$ ,  $\sum_{j \neq i} a_{ij} \leq 1$ ; moreover  $\sum_{j \neq i} a_{ij} < 1$  if firms  $i$  owes payments to one or more firms with no obligations (which are not indexed). It follows that the matrix  $A = (a_{ij})$  is row substochastic.

Consider a node  $i$ , and let  $c_{ki}$  denote the amount of initial margin (IM) it collects from counterparty  $k$ . The purpose of the IM is to cover the deficiency in VM payments. In particular if counterparty  $k$  fails to pay VM to  $i$  in a timely manner, the position will be closed out or novated to another firm, and the IM will be applied to any losses that are incurred between the time the payment was due and the time it takes to novate or close out the position.

In addition, each firm  $i$  maintains cash reserves and short-term lines of credit that allow it to manage daily fluctuations in VM payments and receipts. Denote  $i$ 's *liquidity buffer* by  $b_i$ . In section 5.3 we shall show how to estimate these buffers from the DTCC data.

Given a shock, let  $p_{ki} \leq \bar{p}_{ki}$  denote the actual payment made by  $k$  to  $i$ . If  $p_{ki} < \bar{p}_{ki}$  the difference will be made up out of the IM sitting in  $k$ 's account at firm  $i$  provided that  $\bar{p}_{ki} - p_{ki} \leq c_{ki}$ . If  $\bar{p}_{ki} - p_{ki} > c_{ki}$ , however, then the difference  $\bar{p}_{ki} - (p_{ki} + c_{ki})$  must be borne by  $i$ . We define the *stress* at  $i$ ,  $s_i$ , to be the amount by which  $i$ 's payment obligations exceed it's incoming payments (buttressed by the counterparties' initial margins) plus  $i$ 's liquidity buffer, that is,

$$s_i = s_i(p) = \bar{p}_i - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) - b_i \quad (2)$$

Note that even when all of  $i$ 's counterparties pay in full, that is  $p_{ki} = \bar{p}_{ki}$  for all  $k$ , stress will still be positive if  $i$ 's payments obligations exceed its incoming payments by more than its liquidity buffer  $b_i$ .

The final element of the model is to specify how balance sheet stress translates into actual payments. We argue that in practice firms will respond to stress in diverse ways, depending on their access to credit, their relationship with counterparties, and their risk management practices. We do not have enough information about individual firms to model

these responses explicitly. Instead we shall consider a range of possible responses, and then show how they can be used to bound the total amount of network contagion holding other parameters fixed.

Given a level of stress  $s_i$  at firm  $i$ , let  $s_{ij}(p) = a_{ij}s_i(p)$ , and let  $p_{ij} = f_{ij}(s_{ij}, \bar{p}_{ij}) \in [0, \bar{p}_{ij}]$  be the *expected payment* that  $i$  makes to counterparty  $j$ . We call  $f_{ij}$  a *stress response function*. We shall impose two regularity conditions: (i)  $f_{ij}$  is monotone nonincreasing in  $s_{ij}$ , that is higher levels of stress lead to lower (or at least not higher) payments; (ii)  $f_{ij}$  is *upper-semicontinuous* in  $s_{ij}$ , that is, for every convergent sequence  $\{s_{ij}^k\} \rightarrow s_{ij}^*$  and for every  $\bar{p}_{ij}$ ,

$$\limsup_{s_{ij}^k \rightarrow s_{ij}^*} f_{ij}(s_{ij}^k, \bar{p}_{ij}) \leq f_{ij}(s_{ij}^*, \bar{p}_{ij}). \quad (3)$$

In the next section we shall describe specific examples of such stress response functions, and how they result from strategic decisions of the counterparties. Given such functions  $f_{ij}$  for every  $i$  and  $j$ , we are now in a position to define the notion of payments equilibrium. For every payment vector  $p = (p_{ij})_{1 \leq i, j \leq n}$  such that  $0 \leq p_{ij} \leq \bar{p}_{ij}$ , define the function

$$\forall i \neq j, \quad \Phi(p)_{ij} = f_{ij}(s_{ij}(p), \bar{p}_{ij}) \quad (4)$$

It is straightforward to show that  $s_{ij}(p)$  is continuous in  $p$ , hence  $\Phi(p)$  is upper-semicontinuous in  $p$ . (The composition of a continuous function and an upper-semicontinuous function is upper-semicontinuous.) The function  $\Phi(p)$  is monotone and order-preserving on the bounded lattice  $L = \Pi_{1 \leq i, j \leq n} [0, \bar{p}_{ij}] \in \mathbb{R}^{n^2}$ , where the (partial) order on  $L$  is defined by  $p \leq p'$  if  $p_{ij} \leq p'_{ij}$  for all  $i, j$ .

Consider the recursively defined sequence

$$p^1 = \Phi(\bar{p}), p^2 = \Phi(p^1), p^3 = \Phi(p^2), \dots \quad (5)$$

**Proposition 1.** *The sequence in (5) converges to the greatest fixed point of  $\Phi$ .*

*Proof.* Since  $\bar{p}$  is the maximal element in  $L$ ,  $p^1 = \Phi(\bar{p}) \leq \bar{p}$ . Since  $\Phi$  is order-preserving  $p^2 = \Phi(p^1) \leq \Phi(\bar{p}) = p^1$ . Proceeding inductively we see that the sequence  $\{p^k\}$  in (5) is monotone non-increasing. It is also bounded below by the zero-vector. By the monotone convergence theorem,  $\{p^k\}$  converges to its greatest lower bound.

We claim that  $p^*$  is a fixed point of  $\Phi$ . Since  $\Phi$  is order-preserving and  $p^* \leq p^k$  for all  $k$ ,  $\Phi(p^*) \leq \lim_{k \rightarrow \infty} \Phi(p^k)$ . Since  $\Phi$  is upper-semicontinuous,  $\Phi(p^*) = \Phi(\lim_{k \rightarrow \infty} p^k) \geq \lim_{k \rightarrow \infty} \Phi(p^k)$ . It follows that  $\Phi(p^*) = \lim_{k \rightarrow \infty} \Phi(p^k)$ . By construction  $p^{k+1} = \Phi(p^k)$  for all  $k$ , hence  $\Phi(p^*) = \lim_{k \rightarrow \infty} \Phi(p^k) = \lim_{k \rightarrow \infty} p^{k+1} = p^*$ , so  $p^*$  is fixed point of  $\Phi$ .

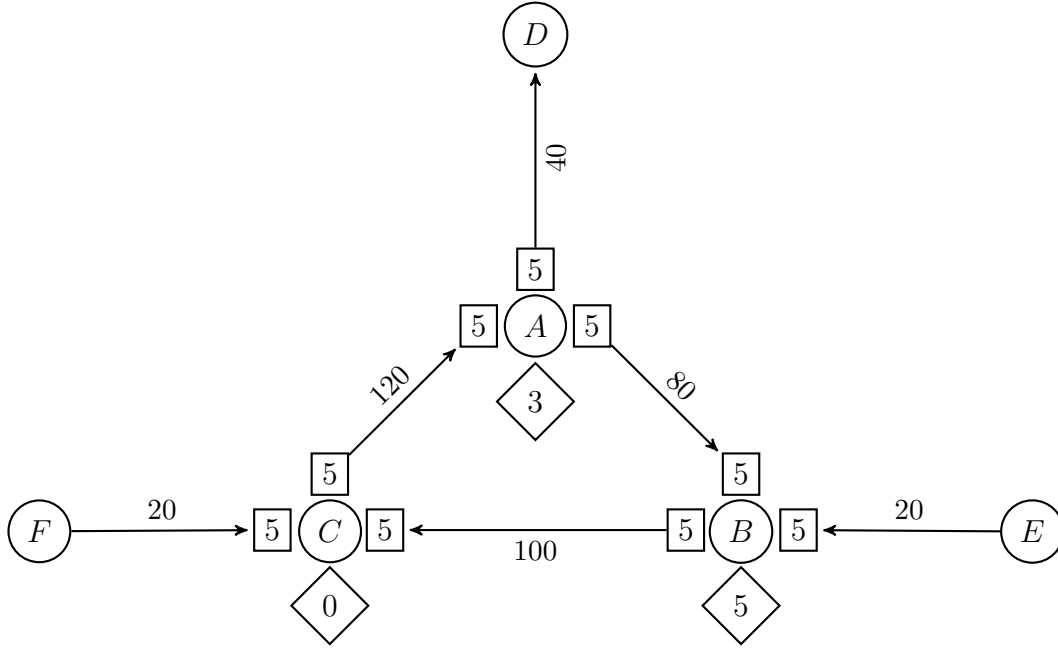
By Tarski's Theorem (Tarski (1955)),  $\Phi$  has a greatest fixed point  $p^+ \in L$ . Clearly  $p^+ \leq \bar{p}$ , hence  $p^+ = \Phi(p^+) \leq \Phi(\bar{p}) = p^1$ . Applying  $\Phi$  repeatedly, we conclude that  $p^+ \leq p^k$  for all  $k$ , hence  $p^+ \leq p^* = \lim_{k \rightarrow \infty} p^k$ . By assumption  $p^+$  is the maximal fixed point and we have just shown that  $p^*$  is also a fixed point, hence  $p^+ = p^*$ .  $\square$

A number of fixed-point results in the literature are particular instances of this proposition; see among others Rogers and Veraart (2013) and Elliott et al. (2014).

To illustrate the recursive computation of payments equilibrium, consider the three-firm example shown in Figure 1. Each firm collects IM from each of its counterparties, and these amounts are held in separate boxes. In addition each firm has a liquidity buffer that is held in a diamond. The amounts owed are shown next to the corresponding arrows. For example, firm A is owed 120 by C, and owes 80 to B and 40 to D. In what follows we shall assume that when a firm defaults it pays its counterparties the amount it owes them minus the pro rata stress, that is,  $f_{ij}(s_{ij}, \bar{p}_{ij}) = \bar{p}_{ij} - s_{ij}$  for all  $i$  and  $j$ .

Now suppose that firms E and F default completely. Then B can seize the 5 units of IM posted by E, but not the IM posted by its other two counterparties (A and C). In the first iteration of the algorithm, assume that B pays whatever it can to C: 5 in IM it collected from E, plus 5 in B's liquidity diamond, plus the anticipated 80 in payments from A. Thus in the first round we find that  $p_{BC}^1 = 90$ . This is less than B owes C, hence C can seize the 5 in IM it collected from B plus the 5 in IM it collected from F (which is in default) plus

**Figure 1:** Example Payment Network

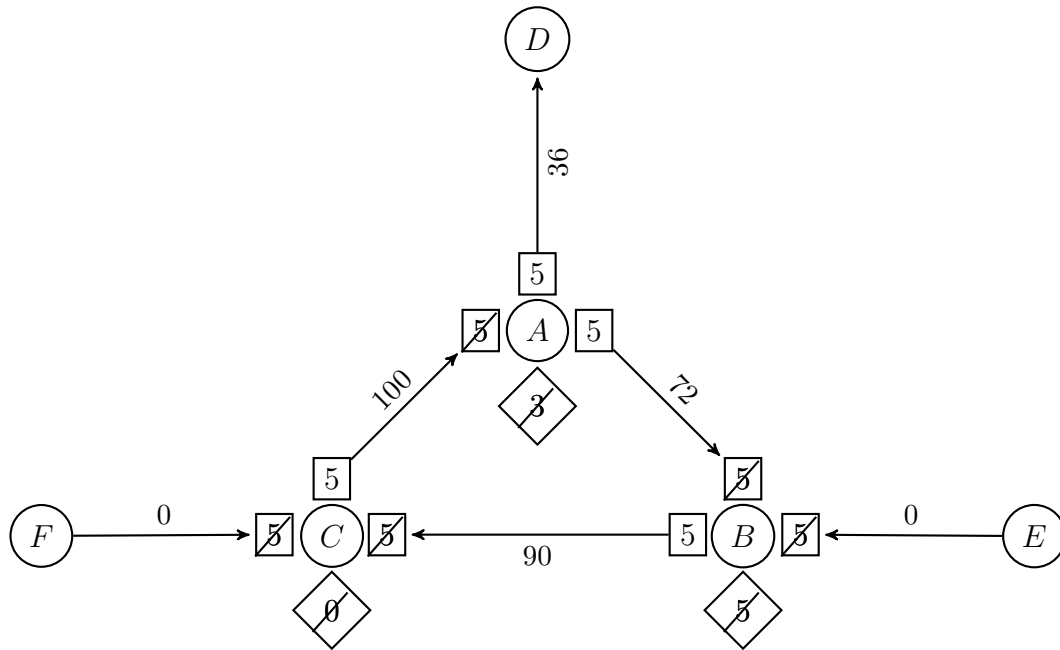


*Note:* Arc labels, diamonds, and squares denote variation margin payments, liquidity buffers, and initial margin stocks, respectively. A firm makes reduced payments when its counterparty(ies) are not forthcoming in their payment obligations. A firm may seize initial margin from such counterparty(ies) and use those proceeds, in addition to its own liquidity buffer, to make payments against its obligations.

*Source:* Authors' creation.

the 90 in anticipated payments from B, and pay 100 to A:  $p_{CA}^1 = 100$ . This is less than C owes A, hence A seizes the 5 in IM it collected from C, adding 3 from its liquidity diamond, and pays 108 to D and B in the proportions of one to two. Thus at the end of round 1 of the algorithm we have the (hypothetical) payments shown in Figure 2. Note, however that the ingoing and outgoing payments, supplemented by the IM and liquidity buffers, are not in balance. Therefore we need to apply the function  $\Phi$  again to determine the next round of payments. This process converges to the payments equilibrium shown in Figure 3.

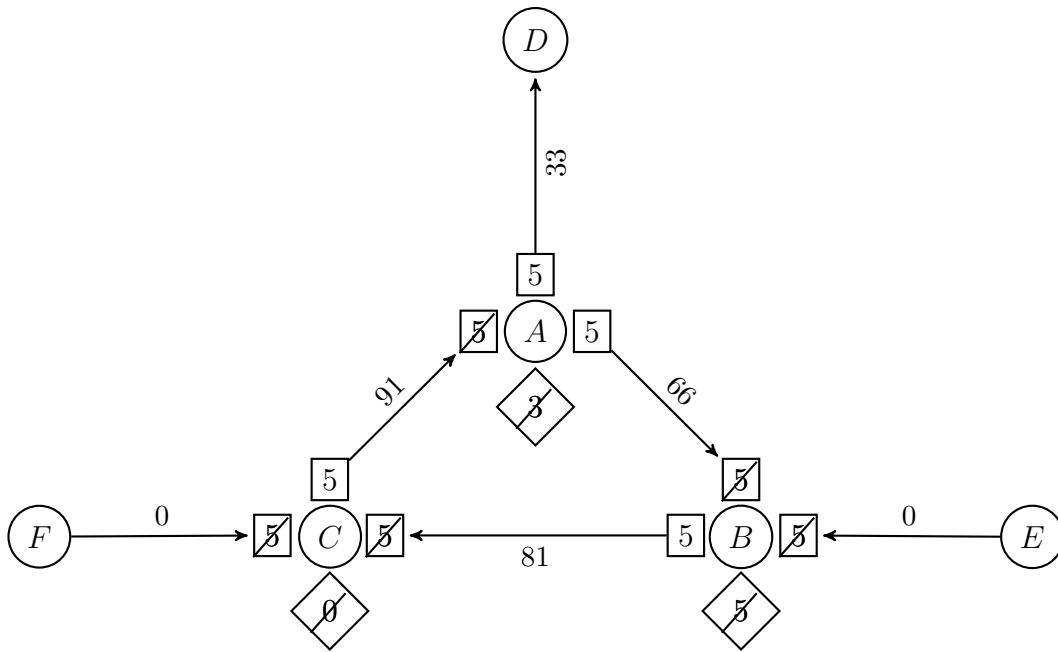
**Figure 2:** Example Payment Network at the End of Round 1



*Note:* Firms E and F default, which results in reduced payments (relative to obligations) from B to C, C to A, and A to B. B and C both seize their failing counterparties' IM stocks and resort to their own liquidity buffers. The resulting payment shortfall generates a corresponding response from A. Exhausted stocks and buffers are indicated with a stricken line.

*Source:* Authors' creation.

**Figure 3:** Example Network Equilibrium



*Note:* In equilibrium every firm's incoming payments, plus IM seizures and liquidity buffer, equal its outgoing payments.

*Source:* Authors' creation.

## 4 Stress Response Functions

We now consider the form of the stress response functions in more detail. Recall that when firm  $i$  is under stress ( $s_i > 0$ ), it is unable to meet its payment obligations in full, even after seizing all of the the IM posted by the counterparties who failed to pay and after exhausting its own liquidity buffer, including its access to short-term lines of credit. Thus, even if  $i$  were to offer partial payment to its counterparties, it would still be in technical default and the contract will be annulled. Why then should  $i$  offer to pay anything? The answer is that under some circumstances  $i$ 's counterparty might prefer to offer  $i$  some forbearance, i.e. to accept partial payment and leave the contract in place for the time being, and firm  $i$  would also prefer this outcome to being declared in default.

Consider a counterparty  $k$  to whom  $i$  owes  $\bar{p}_{ik}$ . When  $i$ 's total shortfall is  $s_i > 0$ , the pro rata shortfall to  $k$  is  $s_{ik} = a_{ik}s_i$ . Assume that  $k$  collected  $c_{ik}$  in IM to protect against  $i$ 's default. Then  $k$  is presented with the following options:

- (a) *Foreclosure*. Annul the contract, seize the IM (but not more than the amount owed), and attempt to recover the remaining amount from  $i$  at a future date. (In the short run  $i$  pays nothing.) Assuming that the recovery fraction is  $r \in [0, 1]$ , this option has value

$$c_{ik} \wedge \bar{p}_{ik} + r(\bar{p}_{ik} - c_{ik})_+. \quad (6)$$

- (b) *Temporary Forbearance*. Accept  $i$ 's offer of partial payment  $\bar{p}_{ik} - s_{ik}$ , leave the contract and IM in place for now, and attempt to recover the remaining balance later.

Option (b) creates the potential for moral hazard: in the future firm  $i$ , and perhaps other counterparties of  $k$ , may be tempted to behave recklessly in anticipation of  $k$ 's forgiving nature. Let the expected future cost to  $k$  of the induced moral hazard be  $h_{ik}$ . Then the temporary forbearance option has expected value

$$\bar{p}_{ik} - s_{ik} + r s_{ik} - h_{ik}. \quad (7)$$

Forbearance is (weakly) preferred to foreclosure if, and only if, (7)  $\geq$  (6), which holds if and only if

$$s_{ik} \leq \bar{p}_{ik} - c_{ik} - h_{ik}/(1 - r). \quad (8)$$

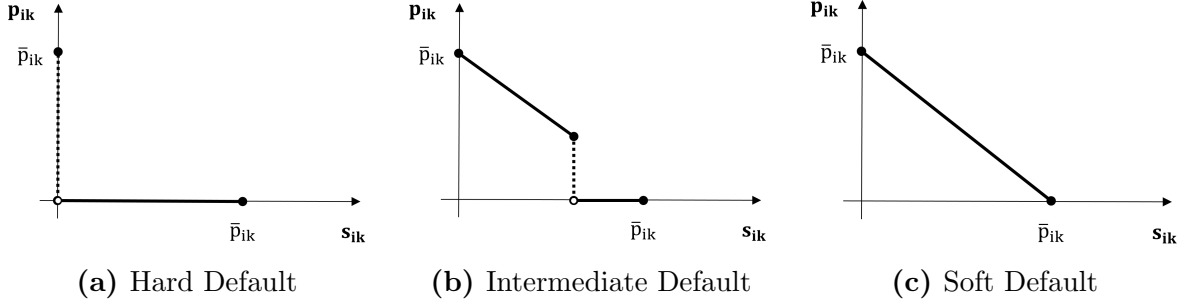
Expression (8) shows that forbearance becomes less attractive the more weight firm  $k$  gives to moral hazard, the higher the expected rate of recovery, and the more IM it collects from  $i$ . If the right-hand side of (8) is negative then  $k$  does not grant forbearance to  $i$  for any value of  $s_{ik}$ . In this case  $i$  defaults completely on its obligations, and the stress response function takes the form shown in Figure 4 (left panel). We shall call this the *hard default* option.

At the opposite extreme, if  $c_{ik} = h_{ik} = 0$ , then  $k$  grants forbearance no matter how large the shortfall. In this case the response function takes the form shown in Figure 4 (right panel). We shall call this the *soft default* option. The intermediate case is that  $i$  pays what it can and  $k$  grants forbearance provided that the shortfall is not too large (Figure 4, middle panel). Note that in all three cases the response function is upper-semicontinuous, hence Proposition 1 holds.

In the empirical application to follow, we shall not attempt to model the response functions for particular firms. Rather we shall view hard and soft default as placing bounds on the range of plausible responses, and estimate the amount of contagion produced by each.



**Figure 4: Stress Response Functions**



*Note:* Three cases are illustrated: (a) is the situation where  $k$  declares  $i$  to be in default and  $i$  pays nothing; (c) describes the situation where  $k$  grants forbearance to  $i$  for any value of  $s_{ik}$ ; (b) is an intermediate situation in which reduced payments are accepted if  $s_{ik}$  is not too large; otherwise  $k$  declares  $i$  to be in default and  $i$  pays nothing.

*Source:* OFR analysis.

## 5 Estimating the Model Parameters

In this section we show how to estimate the key elements of the network contagion model – variation margin, initial margin, and liquidity buffers – from the DTCC data. The data reports the positions on all standardized and confirmed CDS involving U.S. entities since 2010. Positions represent the extant swap transactions with comparable risk characteristics between each pair of counterparties. They include detailed information about the underlying reference entities, the notional amount bought and sold, the inception and termination dates, and other terms of the contract. We also use data from Markit to estimate single-name credit spreads for all reference entities in the positions we observe.

### 5.1 Variation Margin

Variation margin (VM) payments are cash transfers made by a firm to its counterparties to account for changes in the value of the CDS contracts. These variation margin payments are made on a daily basis. From the protection seller's perspective a CDS derives positive value from premia received until the contract terminates or the underlying reference entity defaults (whichever comes first); in the latter case the seller's contract value is reduced by

expected protection payment. The sources of value are switched from the standpoint of the protection purchaser: at contract inception, the present value of premia paid is balanced by the expected value of default payments. The value of the contract varies with market credit spreads through their concurrent impact on the present value of premia receipts and expected value of default payments.

Each CDS contract's value is established between counterparties  $i$  and  $j$  at time  $t$ , on a set of reference entity characteristics  $r$  and a notional amount of protection  $N$ . Through the use of a hazard rate curve, built with credit spreads over different contractual maturities, and the current credit spread at  $t$  we are able to estimate the net present value of the contract. The change in value of this contract between successive periods is the variation margin  $VM_{ij}(z, r, t, t + 1)$ . The sum of changes across all contracts between  $i$  and  $j$  is the bilateral variation margin

$$VM_{ij}(t, t + 1) = \sum_r VM_{ij}(N, r, t, t + 1). \quad (9)$$

We estimate the weekly change in contract values, and the induced VM payments, for each pair of firms in our data set over the period January 1, 2010 to October 21, 2016. The terms of the firms' CDS contracts come from DTCC while data on credit spreads and discount rates come from Markit and Bloomberg respectively. Contracts on indices or portfolios of reference entities are handled by disaggregating them into their single-name equivalents. The details of these calculations are described in the Appendix.

## 5.2 Initial Margin

The role of initial margin (IM) is to cover potential deficiencies in VM payments by a firm's counterparties, including the cost of closing out or transforming the position in case of default. Initial margins collected from counterparties are held in segregated accounts and can only be used to cover losses induced by a given counterparty's failure to pay. A portion

of the IM is typically held in cash or cash equivalents, and the remainder is held in assets that can be liquidated on short notice but not necessarily at full value.

Not all counterparties are required to post IM. For example broker-dealers only need to post IM in contracts with other broker-dealers and the CCP. Other market participants, such as hedge funds and asset managers, need to post IM with broker-dealers, commercial banks, and the CCP, but not with each other. Table 1 shows the matrix of IM posting requirements as of October 6, 2014 (the date of the shock we shall study).<sup>3</sup>

**Table 1:** Initial Margin Payment Matrix

	Participant:	Receiver				
		CCP	Dealers	Commercial Bank	HF/Asset Manager	Other
Payer	CCP	-	-	-	-	-
	Dealers	✓	(-)	(-)	-	-
	Commercial Bank	✓	✓	(-)	-	-
	HF/Asset Manager	✓	✓	✓	-	-
	Other	✓	✓	✓	-	-

*Note:* This table shows the direction of initial margin exchange (indicated by a checkmark) between CDS counterparties at the time of the CCAR stress studied in this paper. Certain market participants, whose flows are parenthetically indicated, were required to commence exchange of initial margin under new BCBS and IOSCO guidelines introduced in 2016.

*Source:* OFR analysis.

To determine the amount of IM posted (where IM is required) we adopt a conventional portfolio-at-risk measure, namely a 99.5 percent VaR with a 10-day margin period of risk (BCBS and IOSCO (2015)). For each pair of firms  $i$  and  $j$ , the DTCC data tells us the portfolio of CDS contracts for which  $i$  and  $j$  were the counterparties on the date of the shock. Using Markit data we can infer the price changes, and hence the VM that would have been exchanged between  $i$  and  $j$ , if they had held this same portfolio over the prior 1,000 days. We then find the amount  $c_{ij}$  such that on all but five days out of 1,000 the net amount of VM that  $i$  owed  $j$  was less than  $c_{ij}$ .<sup>4</sup>

<sup>3</sup>Starting in September 2016, new BIS-IOSCO regulations require broker/dealers and commercial banks to post IM with other broker-dealers and commercial banks but they do not need to post with asset managers, hedge funds, or insurance companies.

<sup>4</sup>In the case of the CCP, we estimate the IM that would be required to meet a 10-day 99.5 percent VaR

### 5.3 Liquidity Buffers

The IM collected by firm  $i$  from its counterparties is dedicated to covering shortfalls in payments to  $i$  from its counterparties; it cannot be accessed to meet  $i$ 's obligations to others. To cover its own obligations the firm maintains a liquid buffer  $b_i$ , which includes cash or cash-equivalents and short-term lines of credit. These buffers are not part of the DTCC data, nor are they available from public data sources.<sup>5</sup> Instead we shall estimate their magnitude by considering how much cash a prudently managed firm would need to manage its *net* VM obligations. These numbers can be estimated from the weekly inflows and outflows of VM at the firm level, which are derived from DTCC data as described above.

Fix a firm  $i$  and let  $X_i(t)$  denote the total net VM payment that  $i$  owes to all of its counterparties on a given day  $t$ . If  $X_i(t) > 0$  then  $i$  owes more than it is owed; the reverse holds if  $X_i(t) < 0$ . Note that in general,  $E[X_i(t)] = 0$ . Let  $N_i(t)$  be the gross notional value of  $i$ 's CDS contracts at time  $t$  and let  $\tilde{X}_i(t) = X_i(t)/N_i(t)$ . The *volatility* of  $i$ 's CDS portfolio over a given period  $[0, T]$  is

$$\sigma_i = \sqrt{(1/T) \sum_{t=1}^T (\tilde{X}_i(t))^2} \quad (10)$$

We estimate the volatility of each firm's CDS portfolio over the entire period for which we have data (January 1, 2010 to October 21, 2016). Although the composition of each firm's portfolio may have changed over the period, we are interested in measuring the overall volatility of its positions, which reflects its general risk management policy and degree of hedging. We find that there is substantial heterogeneity among different types of firms, as

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bilaterally for each of its counterparties, and then scale up the estimates by a common factor so that the total IM collected corresponds to the CCP's total reported IM at the end of 2014 (ICE (2016)).

<sup>5</sup>For some market participants, such as bank holding companies, liquidity coverage ratios are reported at the level of the parent company. However the liquidity buffers available for their CDS operations cannot be inferred from the reported aggregate numbers. Furthermore, the IM posted by counterparties counts toward the LCR, but in fact IM is siloed by counterparty and is not available to meet general liquidity demands. An exception is the CCP, which does report the size of the buffer that it can draw upon when the IM is depleted. This buffer consists mainly of the guarantee fund, which contained assets worth \$1.6 billion at the time of the study (ICE (2016)).

shown in Table 2. In particular, the members of the CCP have an average portfolio volatility that is an order of magnitude smaller than the portfolio volatility of non-member commercial banks. Moreover the latter are less than half as volatile as the holdings of hedge funds, asset managers, and insurance companies.

**Table 2:** Average portfolio volatility by type of firm

Firm Type	Average $\sigma$
Members	0.0009
<i>Non-members:</i>	
Commercial Banks	0.0106
Insurance & Pension Cos.	0.0224
Asset Managers & Hedge Funds	0.0235

*Note:* Historical volatilities are computed using weekly data from DTCC. For each firm, CDS portfolio volatility reflects changes in prices and bilateral exposures.

*Source:* Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

To estimate the size of  $i$ 's liquidity buffer, we again employ a portfolio-at-risk measure. Specifically let  $F_i(\tilde{X})$  be the CDF of  $\tilde{X}_i$  over the period of observation, and let  $\theta$  be a VaR level such as  $\theta = 0.99$  or  $\theta = 0.997$ .<sup>6</sup> Let  $\tilde{x}_{i,\theta}$  be the value such that  $F_i(\tilde{x}_{i,\theta}) = \theta$ . At a given date  $t^* \in [0, T]$ , we estimate  $i$ 's liquidity buffer to be  $b_{i,\theta} = \tilde{x}_{i,\theta} N_i(t^*)$ .

Although the portfolio volatility of CCP members is on average much smaller than that of non-members, the size of their portfolios is so much larger that many of them require higher absolute amounts to manage their VM payments at a given level of risk (see Table 3). These numbers are substantial: each of the top five firms would (on average) need to maintain liquidity of a quarter to half a billion dollars just to manage their CDS operations.<sup>7</sup>

<sup>6</sup> $\theta = 0.99$  is equivalent to the third largest net negative change in VM observes in our data set and  $\theta = 0.997$  is equivalent to largest net negative change in VM observes in our data set.

<sup>7</sup>The average liquid reserves among U.S. broker-dealers as of late 2014 was about \$1 billion for all of their operations. Thus our estimated liquidity buffers for CDS operations represent a substantial fraction of total reserves (SIFMA (2015)).

**Table 3:** Average Liquidity Buffers for the Top Five Members and Top Five Non-members, ordered by amount of net VM owed (millions of dollars).

	$\theta = 0.99$	$\theta = 0.997$
Top 5 Members	204	455
Top 5 Non-members	189	348
CCP	1,800	1,800

*Source:* Authors’ calculations using data provided by Depository Trust & Clearing Corporation and Markit.

## 6 VM Payments Induced by the CCAR Shock

We apply the Federal Reserve’s 2015 CCAR global market shock, which prescribes a sudden widening of credit spreads for corporate, state, municipal, and sovereign debt according to their rating class. The shock is applied on all outstanding positions as of October 6, 2014. The change in credit spreads alters the value of the premium and payment legs of CDS contracts that reference various classes of debt. These changes in CDS contract values induce variation margin (VM) payment obligations between counterparties. The methodology for estimating the VM payments is described in detail in the Appendix; here we provide an overview of the results.

The CCAR market shock prescribes a scenario in which there is a increase in credit spreads, as shown in Table 4. As an example, a AAA Advanced Economy proportional shock of 130% on a market spread of 100 basis points results in a new market spread of 230 basis points. These proportional shocks can be used to estimate market spreads and shifts in hazard rate curves over successive periods, from which we can compute the expected variation margin payments (see Appendix A.3 for details).

Figure 5 shows the net VM payment obligations between the CCP, members of the CCP, and other non-member firms on the CCAR shock date.<sup>8</sup> In addition, there are many non-member firms that have positions directly with members, as well as positions with the CCP

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<sup>8</sup>For each pair of firms payments are bilaterally netted, e.g. if A owes B 100 and B owes A 90, then the net payment is 10 from A to B.

**Table 4:** The Impact of the 2015 CCAR Severely Adverse Global Market Trading Shock.

<b>Corporate Credit</b>							
<i>Advanced Economies</i>							
	AAA	AA	A	BBB	BB	B	<B or Not Rated
Spread Widening (%)	130.0	133.0	110.2	201.7	269.0	265.1	265.1
<i>Emerging Markets</i>							
	AAA	AA	A	BBB	BB	B	<B or Not Rated
Spread Widening (%)	191.6	217.2	242.8	277.5	401.9	436.4	465.8

<b>State &amp; Municipal Credit</b>							
	AAA	AA	A	BBB	BB	B	<B or Not Rated
Spread Widening (bps)	12	17	37	158	236	315	393

<b>Sovereign &amp; Supra Credit</b>							
See FRB worksheet: CCAR-2015-Severely-Adverse-Market-Shocks-data.xlsx							

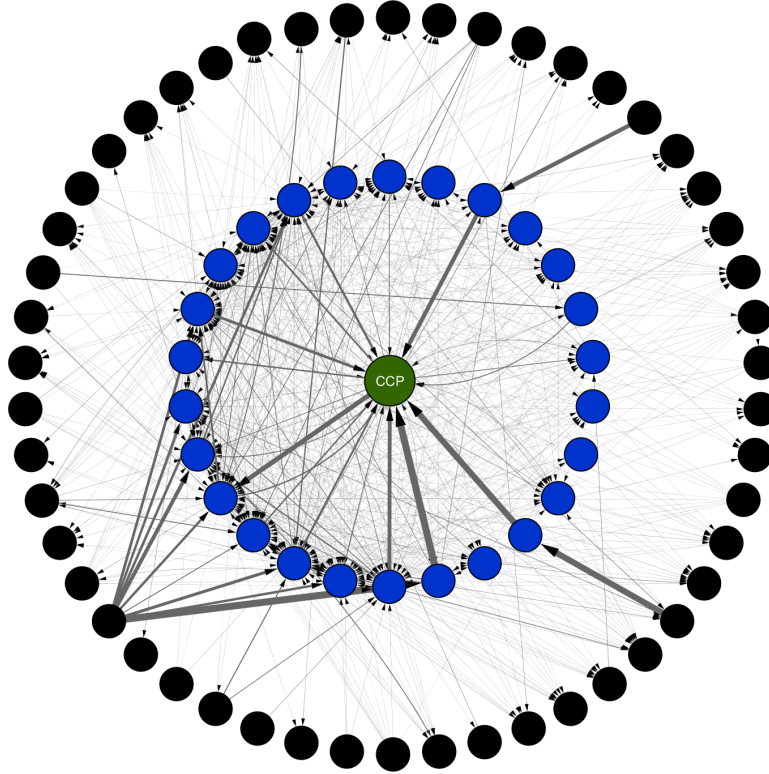
Source: Federal Reserve Board (FRB (2016))

that are guaranteed by members. There are over 900 such firms, including a wide variety of hedge funds, asset managers, and insurance companies.

Two key features of the network are: (i) non-members tend to owe members rather than each other; (ii) the largest non-members contribute substantially more stress than the largest members because they are large net sellers of CDS protection. These points are highlighted in Table 5, which shows the average VM owed by the top five members versus the average VM owed by the top five non-members (ordered by net amount of VM owed). The market structure described here affects the results described in Section 7 in several critical ways. First, liquidity buffers (and additionally for the CCP its default fund) will play critical roles in resolving stress in the network. The exhaustion of such resources for members and the CCP leads to payments contagion. Second, the initial channel for stress are payments due from non-members to members, which in some cases are several times larger than their liquidity buffers.

<sup>8</sup>The CCP actually has 30 members but the exposures of several of them are aggregated in the data at the bank holding company level, which leads to 26 observable members.

**Figure 5:** Variation Margin Payment Network for CCP Members and a subsample of Non-members, based on the 2015 CCAR Shock.



*Note:* The network diagram plots the central counterparty clearing house (CCP) (in green), CCP members (in blue), and a sample of CCP non-members (in black). The width and direction of each arrow indicate the relative size of the net VM payment owed bilaterally between counterparties.

*Source:* Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

**Table 5:** Variation Margin Payments for CCP Member and Non-member Firms (millions of dollars) under the CCAR shock.

Firms	Avg Variation Margin Owed By	Avg Variation Margin Owed To	Avg Net Variation Margin Owed	Avg Liquidity Buffer ( $\theta = 0.997$ )
Top 5 Members	1,679	1,102	577	455
Top 5 Non-members	1,946	53	1,893	348
CCP	8,747	8,747	-	1,800

*Source:* Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.



## 7 Empirical Results

We apply this framework to evaluate the potential amount of contagion in the CDS market that would be produced by a shock such as CCAR.

### 7.1 Payment Deficiencies and Firms in Default

Define the total *impact* of the shock to be the total deficiency in VM payments summed over all directed edges in the network. If  $p_{ij}$  is the payment from  $i$  to  $j$  at the greatest fixed point then the *payment deficiency* from  $i$  to  $j$  is  $d_{ij} = \bar{p}_{ij} - p_{ij}$ , and the total deficiency is

$$D = \sum_{1 \leq i, j \leq n} d_{ij}. \quad (11)$$

We shall estimate the total payment deficiency under a range of shocks that are calibrated to the 2015 CCAR shock as the benchmark case. Given a scaling factor  $\alpha > 0$ , we multiply all VM payments under the CCAR shock by the factor  $\alpha$ , and compute the resulting payment deficiency under two scenarios: hard default and soft default. As we argued in section 4, these yield plausible upper and lower bounds on the level of stress transmission by individual firms, and hence on the total amount of contagion. The results are summarized in Table 6 for three sizes of shock:  $\alpha = 0.75$ , 1, and 1.25. The three shock variations scale up the non-member imbalances highlighted in Table 5 and illustrate the nonlinear relationship between shock scale  $\alpha$  and the absolute payment deficiency. Specifically, initial margin stocks and liquidity buffers (the only resources under hard default) are insufficient to cover payment imbalances under stress. This inadequacy results in a convex and increasing level of stress propagation as a function of the scale of the shocks.

Focusing on the benchmark case ( $\alpha = 1$ ) we see that the difference between hard and soft default is not particularly extreme: the percentage of firms in default is nearly the same in the two cases (14.7 percent and 16.8 percent), whereas the payment deficiency increases by a factor of 2.5 (from 4.6 percent to 11.2 percent). When we look at  $\alpha = 1.25$ , we see that

**Table 6:** Payment Deficiency and Firms in Default Under Six Scenarios ( $\theta = 0.997$ )

	Absolute Payment Deficiency(\$b)	Relative Payment Deficiency(%)	Firms in Default(%)	Members in Default(%)	CCP Default
$\alpha = 0.75$					
Soft Default	6.9	3.7	12.0	19.2	
Hard Default	19.3	10.3	13.5	50.0	
$\alpha = 1$					
Soft Default	11.1	4.6	14.7	34.6	
Hard Default	27.1	11.2	16.8	57.7	
$\alpha = 1.25$					
Soft Default	16.5	5.4	17.4	46.2	
Hard Default	69.6	12.0	19.9	73.1	✓

*Source:* Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

the CCP fails in the hard default scenario but does not fail in the soft default scenario.

Which of these two scenarios is more likely? We would argue that most firms would not be inclined to grant forbearance given the precedent that would be established and the ensuing moral hazard. This suggests that the hard default option is closer to reality. We should also note that a number of our assumptions about the model parameters may under-estimate the amount of contagion even in this case. In particular, initial margin is typically not all held in cash, and in crisis situations it can only be transformed into cash at a discount. Hence the cash value of the initial margins are probably less than our estimates. Second, liquidity buffers consist in part of access to short-term lines of credit, which could dry up in crisis.

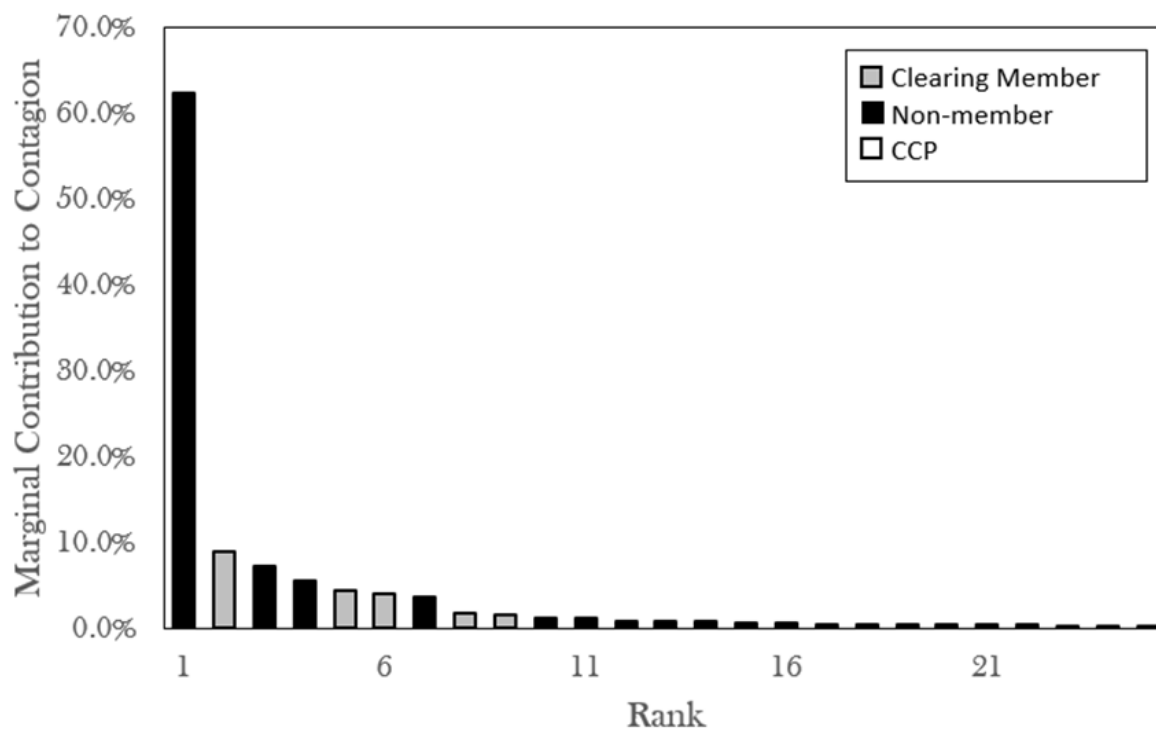
## 7.2 Marginal Contribution of Individual Firms to Contagion

The model can also be used to assess the amount that individual firms contribute to contagion at the margin. Given a shock size  $\alpha$  and a default scenario (hard or soft), we compute the total payments deficiency,  $D = \sum_{1 \leq i, j \leq n} d_{ij}$ , over all firms. Now choose a particular firm  $i$  and suppose (hypothetically) that its payment obligations to all counterparties could be guaranteed, say by giving  $i$  access to a special return fund. The resulting payments deficiency

$D_i$  will be at most  $D$ , and the relative difference  $(D - D_i)/D$  is  $i$ 's *marginal contribution to contagion* under the given scenario ( $\alpha$ , hard/soft), expressed as a percentage reduction in total payment deficiency.

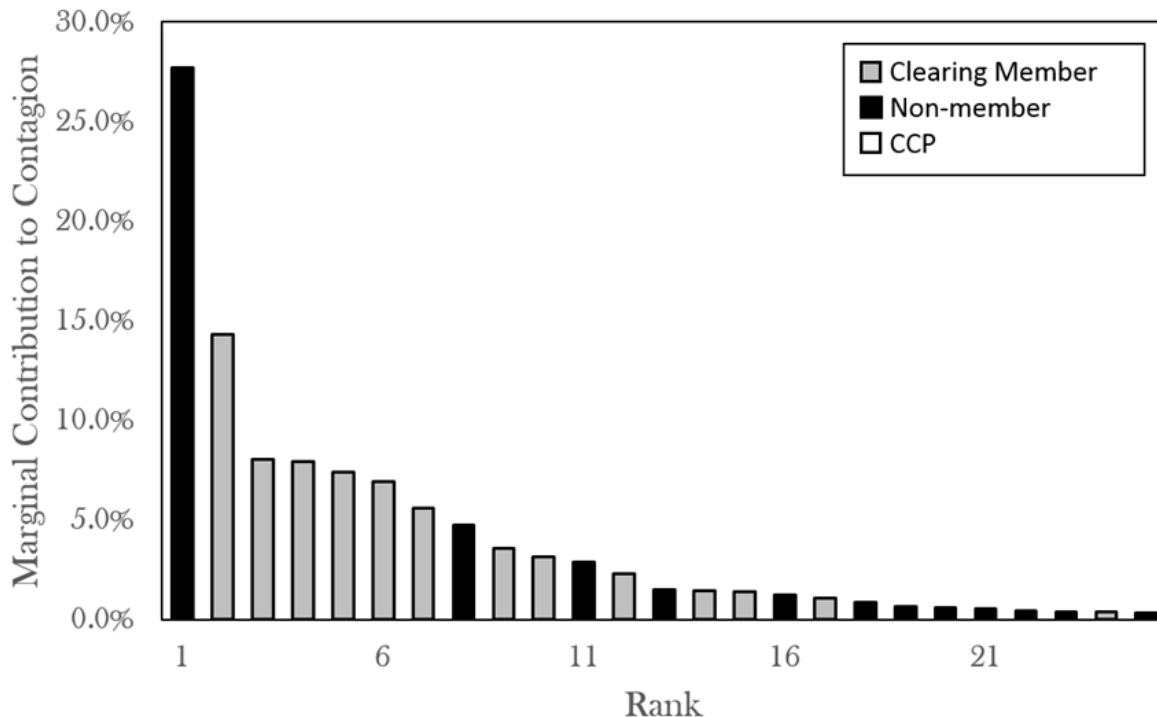
Figure 6 and 7 show the marginal contribution of the largest contributors to contagion under hard and soft default (and  $\alpha = 1$ ). In both cases, the largest single contributor to contagion is a non-member firm, which is a large net seller of protection. Although the total payment deficiency is much larger under hard default, the CCP does not default in either case.

**Figure 6:** Distribution of Firms' Marginal Contributions to Contagion under Soft Default



*Source:* Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

**Figure 7:** Distribution of Firms' Marginal Contributions to Contagion under Hard Default



*Source:* Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit.

## 8 Conclusion

In this paper, we have analyzed the network of counterparty exposures in the CDS market. In contrast to much of the prior work on stress-testing, contagion, and fire sales, we track the potential effects of a shock using actual financial exposures and the Federal Reserve's supervisory stress scenarios as of a particular date. We estimate the impact of the shock on the value of market participants' portfolios, and by implication the variation margin (VM) owed between the contracting parties. A significant feature of this market is that demands for VM must be met over very short time horizons. A failure to meet these demands leads to payment shortfalls that become amplified as they cascade through the network.

We have examined the potential contribution to network contagion of the 30 members of the major CCP in this market (ICE Clear Credit), and also the potential contribution

of the major non-members. We found that network exposures significantly increase the amount of contagion. Furthermore there are many members (and some non-members) that contribute substantially more to contagion than does the CCP, in spite of the fact that they are peripheral in the network. Our analysis suggests that more attention should be paid to firms that are very large and have highly unbalanced CDS positions, whose failure can trigger large systemic losses as happened with AIG in 2008. It also highlights the key role of liquidity buffers in coping with large and sudden demands for variation margin that result from a credit shock.

Our study is limited to the analysis of a specific part of the derivatives market, and does not encompass the full range of shocks to which firms may be exposed. In particular, we have not included exposures to interest rate swaps, which form a substantially larger market (in notional terms) than the CDS market, but which are not part of our data set. Under a severe credit shock, firms may be subjected to simultaneous payment demands over multiple lines of business, increasing the stress on their resources and possibly leading to even higher rates of default than we have estimated here.

## References

- Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. *The American Economic Review*, 105(2):564–608.
- Ali, R., Vause, N., and Zikes, F. (2016). Systemic risk in derivatives markets: a pilot study using CDS data. Bank of England Working Paper 38.
- Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108(1):1–33.
- BCBS and IOSCO (2015). Margin requirements for non-centrally-cleared derivatives. Technical report, BIS and OICU-IOSCO, Basil, Switzerland.
- Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012). A survey of systemic risk analytics. *Annual Review of Financial Economics*, 4(1):255–296.
- Blume, L., Easley, D., Kleinberg, J., Kleinberg, R., and Tardos, E. (2011). Network formation in the presence of contagious risk. In *Proceedings of the 12th ACM Conference on Electronic Commerce*, pages 1–10. ACM.
- Brunnermeier, M., Clerc, L., and Scheicher, M. (2013). Assessing contagion risks in the CDS market. Banque de France Financial Stability Review 17.
- Clerc, L., Gabrieli, S., Kern, S., and El Omari, Y. (2014). Monitoring the European CDS market through networks: Implications for contagion risks. European Securities and Markets Authority Working Paper 1.
- Cont, R. (2010). Credit default swaps and financial stability. Banque de France Financial Stability Review 14.
- Cont, R. and Kokholm, T. (2014). Central clearing of OTC derivatives: bilateral vs multi-lateral netting. *Statistics & Risk Modeling*, 31(1):3–22.

- Cont, R. and Minca, A. (2016). Credit default swaps and systemic risk. *Annals of Operations Research*, 247(2):523–547.
- Cont, R., Moussa, A., and Santos, E. B. (2013). Network structure and systemic risk in banking systems. *Handbook of Systemic Risk*, pages 327–368.
- Duffie, D. and Zhu, H. (2011). Does a central clearing counterparty reduce counterparty risk? *Review of Asset Pricing Studies*, 1(1):74–95.
- Eisenberg, L. and Noe, T. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.
- Elliott, M., Golub, B., and Jackson, M. (2014). Financial networks and contagion. *The American Economic Review*, 104(10):3115–53.
- Elsinger, H. et al. (2009). *Financial networks, cross holdings, and limited liability*. Oesterreichische Nationalbank Austria.
- Financial Crisis Inquiry Commission (2011). *The financial crisis inquiry report: Final report of the national commission on the causes of the financial and economic crisis in the United States*. Public Affairs.
- FRB (2016). Stress tests and capital planning: Comprehensive capital analysis and review. Technical report, Board of Governors of the Federal Reserve System.
- Freixas, X., Parigi, B. M., and Rochet, J.-C. (2000). Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, 32(3):611–638.
- Gai, P., Haldane, A., and Kapadia, S. (2011). Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453–470.
- Gai, P. and Kapadia, S. (2010). Contagion in financial networks. Bank of England Working Paper 383.

- Ghamami, S. and Glasserman, P. (2016). Does OTC derivatives reform incentivize central clearing? Office of Financial Research Working Paper 16-07.
- Glasserman, P. and Young, H. P. (2016). Contagion in financial networks. *Journal of Economic Literature*, 54(3):779–831.
- Haldane, A. and May, R. (2011). Systemic risk in banking ecosystems. *Nature*, 469(7330):351–355.
- ICE (2016). ICE Clear Credit LLC: Financial statements: Years ended December 31, 2015 and 2014. Technical report, Intercontinental Exchange.
- Luo, L. S. (2005). Bootstrapping default probability curves. *Journal of Credit Risk*, 1(4):169–179.
- Macroeconomic Assessment Group on Derivatives (2013). Macroeconomic impact assessment of OTC derivatives regulatory reforms. Technical report, Bank for International Settlements.
- Markose, S. (2012). *Systemic risk from global financial derivatives: A network analysis of contagion and its mitigation with super-spreader tax*. Number 12-282. International Monetary Fund.
- Markose, S., Giansante, S., and Shaghghi, A. R. (2012). Too interconnected to fail: financial network of US CDS market: Topological fragility and systemic risk. *Journal of Economic Behavior & Organization*, 83(3):627–646.
- Peltonen, T. A., Scheicher, M., and Vuillemeys, G. (2014). The network structure of the CDS market and its determinants. *Journal of Financial Stability*, 13:118–133.
- Rogers, L. C. and Veraart, L. A. (2013). Failure and rescue in an interbank network. *Management Science*, 59(4):882–898.



- SIFMA (2015). Financial and operational combined uniform single (focus) report filings: 2015 Q4. Technical report, SIFMA.
- Siriwardane, E. (2015). Concentrated capital losses and the pricing of corporate credit risk: Data appendix. Office of Financial Research Working Paper 14-10.
- Tarski, A. (1955). A lattice-theoretical fixpoint theorem and its applications. *Pacific Journal of Mathematics*, 5(2):285–309.
- Upper, C. and Worms, A. (2004). Estimating bilateral exposures in the German interbank market: Is there a danger of contagion? *European Economic Review*, 48(4):827–849.
- Vuillemeys, G. and Peltonen, T. A. (2015). Disentangling the bond–CDS nexus: A stress test model of the CDS market. *Economic Modelling*, 49:32–45.

## Appendix – Evaluating CDS Portfolios

Here we describe the methodology for estimating the mark-to-market value of each counterparty’s exposures at a given date for both single-name and index positions. We describe a bootstrap procedure to generate a schedule of hazard rates consistent with the market for all traded credit curves. We then describe the process of index disaggregation to single-name equivalents. Finally, we describe how we arrive at expressions for variation margin payments under stress.

### A.1 Bootstrapping Credit Curves

We calibrate hazard rate schedules associated with each of the reference entities in the contracts we observe. The CDS market quotes credit spreads through a range of standard terms: 1-year, 3-year, 5-year, 7-year, and 10-year, and sometimes longer maturities. Each additional term generates a hazard rate over a corresponding increment: the 3-year term generates a 1-3 year increment, the 5-year term generates a 3-5 year increment, and so on through 10 years. The bootstrapping technique we employ here generates a piecewise constant schedule of hazard rates. A CDS contract struck at inception at the market spread for a standard maturity, and valued using the schedule of hazard rates through that maturity, will have equal default and premium legs. Bootstrapping permits us to value any position whose remaining maturity at the time of stress may not correspond to market-quoted maturities.

The CDS payment and premium legs are implicit functions of a hazard rate,  $\lambda$ , which enters the expression for the survival probability  $S(0, t, \lambda)$  through its definition as  $\exp\left(-\int_t^u \lambda ds\right)$ . Let  $Z(0, t_i)$  denote the risk-free discount factor through period  $i$ , which we compute from LIBOR rates from 1 through 6 months and swap rates from 9 months through 30 years. We assume CDS premia are paid on International Money Market (IMM) payment dates, consistent with market convention. Finally, we allow for the possibility that when a default occurs inter-period, CDS premia to the protection seller are pro-rated to the time of default.

For simplicity assume that  $\alpha = 0.5$  (i.e., that any default occurs at the inter-period halfway point). In subsequent notation,  $\Delta_t$  represents the daycount fraction for the time interval  $(t - 1, t)$  such that  $\sum_{t=1}^T \Delta_t = T$ . We use the ACT/365 convention standard in the CDS market.

We write the CDS payment and premium legs through maturity  $T$  as follows:

$$V_{prem}^{0 \rightarrow T}(\lambda, s_T) = s_T \sum_{t=1}^T Z(0, t) \Delta_t ((1 - \alpha)S(0, t) + \alpha S(0, t - 1)). \quad (\text{A1})$$

In any period, the payment leg derives its value from the incremental default probability over that time. Given the relationship between the default probability  $P(t, u, \lambda)$  and the survival probability  $S(t, u, \lambda)$ ,  $S(t, u, \lambda) = 1 - P(t, u, \lambda)$ , we can write the value of the payment leg as follows:

$$V_{pay}^{0 \rightarrow T}(\lambda) = (1 - R) \sum_{t=1}^T Z(0, t) (P(0, t) - P(0, t - 1)). \quad (\text{A2})$$

Let  $\lambda^*$  be the solution that sets the CDS payment and premia legs to fair value (equality) at inception, that is  $V_{contract}^{0 \rightarrow T}(\lambda^*, s_T) = V_{pay}^{0 \rightarrow T}(\lambda) - V_{prem}^{0 \rightarrow T}(\lambda, s_T) = 0$ .

Credit spreads are quoted for a sequence of maturities  $T_1, T_2, \dots, T_k$ . Quotes consistent across the term structure require that for each  $T_i \geq T$ , a vector of hazard rates  $\lambda^* = \{\lambda_{0, T_i}^*, \lambda_{T_i, T_{i+k}}^*\}$  exists such that  $V_{contract}^{0 \rightarrow T_{i+k}}(\lambda^*, s_{T_{i+k}}) = V_{contract}^{0 \rightarrow T_i}(\lambda_{0, T_i}^*, s_{T_{i+k}}) + V_{contract}^{T_i \rightarrow T_{i+k}}(\lambda_{T_i, T_{i+k}}^*, s_{T_{i+k}})$  for all  $k$ . We adopt a bootstrap procedure from Luo (2005) that ensures this by construction. The procedure generates hazard rates  $\{\lambda_{0, T_1}^*, \lambda_{T_1, T_2}^*, \dots, \lambda_{T_{k-1}, T_k}^*\}$  that correspond to quoted maturities of  $\{T_1, T_2, \dots, T_k\}$ . The default probability at any time  $t$  such that  $T_k \leq t \leq T_m$ , can be expressed as a function of bootstrapped hazard rates as follows:

$$P(0, t, \lambda^*) = P(0, T_k, \lambda_{0, T_k}^*) P(T_k, t, \lambda_{T_k, T_m}^*). \quad (\text{A3})$$

For notational convenience we will refer to  $P(0, t, \lambda^*)$  from here on as  $P(t)$ . We make the simplifying assumption that maturity dates fall upon IMM payment dates. We start from

the premise that  $\lambda_{\mathbf{0}, T_k}^*$  is known and is either a) the solution that equates (A1) and (A2), or b) is the bootstrapped solution vector described by the preceding recursive procedure. The parameters  $s_{T_m}$  are derived from market quotes. The conditional premia and payment legs are given as follows:

$$V_{prem}^{0 \rightarrow T_m}(\lambda_{T_k, T_m}; s_{T_m}, \lambda_{\mathbf{0}, T_k}^*) = s_{T_m} \left\{ C(\lambda_{\mathbf{0}, T_k}^*) + D(\lambda_{T_k, T_m}) - \sum_{t=T_k+1}^{T_m} Z(0, t) \Delta_t \left( P(t) - P(T_k) - \alpha \frac{P(t) - P(t-1)}{2} \right) \right\}, \quad (\text{A4})$$

where

$$C(\lambda_{\mathbf{0}, T_k}^*) = \sum_{\substack{(T_i, T_j) \in \\ \{(0, T_1) \dots \\ (T_{k-1}, T_k)\}}} \sum_{t=T_i}^{T_j} Z(0, t) \Delta_t \left[ (1 - P(t)) + \alpha \frac{(P(t) - P(t-1))}{2} \right],$$

$$D(\lambda_{T_k, T_m}) = \sum_{t=T_k}^{T_m} Z(0, t) \Delta_t (1 - P(T_k)).$$

Similarly,

$$V_{pay}^{0 \rightarrow T_m}(\lambda_{T_k, T_m}; \lambda_{\mathbf{0}, T_k}^*) = A(\lambda_{\mathbf{0}, T_k}^*) + \sum_{t=T_k+1}^{T_m} (1 - R) Z(0, t) (P(t) - P(t-1)), \quad (\text{A5})$$

where

$$A(\lambda_{\mathbf{0}, T_k}^*) = \sum_{\substack{(T_i, T_j) \in \\ \{(0, T_1) \dots \\ (T_{k-1}, T_k)\}}} \sum_{t=T_i}^{T_j} (1 - R) Z(0, t) (P(t) - P(t-1)).$$

Here  $\lambda_{T_k, T_m}^*$  is the value that sets Equations (A4) and (A5) to parity. Once this value

is determined, the hazard rate vector  $\lambda_{0,T_m}^*$  can be derived for subsequent stages of the bootstrap recursion. The resulting bootstrapped hazard rate schedule allows us to value a contract of any term by applying its contracted spread and the hazard rate schedule associated with its term.

## A.2 Portfolios of Single Name Equivalents

We disaggregate Markit credit indices to single-name constituents. For each position referencing a Markit credit index, we decompose the index using *Markit RED* data. This source provides the composition of the index at any point in time, taking into account index revisions and defaults. We employ the disaggregation technique described by Siriwardane (2015) in Section 2.

Each Markit credit index is described by its series and version. A series may have one or more versions. An index series factor,  $f_i$  is defined for every version  $i$  as  $f_i = 1 - \frac{D_{i-1}}{N}$ , where  $D_{i-1}$  is the number of defaults for an index series version  $i$  in  $1, 2, 3 \dots$ .  $D_0 = 0$ , so  $f_1 = 1$ . The weight of a constituent within a version must be computed on a given valuation date and is a function of the index composition on the date the position was established (trade date). In general, the index composition at the trade date may not be its composition at inception. The current weight  $w_i(u)$  for index version  $i$  of a constituent  $u$  whose inception index series weight is  $w_0(u)$  is given as

$$w_i(u) = \frac{w_0(u)}{f_i}. \quad (\text{A6})$$

As an example, an index with 43 original constituents at inception would have a per-constituent weight of  $w_1(u) = \frac{\frac{1}{43}}{1} = 0.0233$ . Version 2 of the index would have a per-constituent weight of  $w_2(u) = \frac{\frac{1}{43}}{0.953} = 0.0244$ . The per-constituent weight is scaled by the notional value of the index position to arrive at the effective single-name notional equivalent. We perform all calculations in this paper on a firm's single-name equivalent notional CDS

positions.

### A.3 Estimating Variation Margin

At the CCAR valuation date (October 6, 2014), we generate stressed portfolio values using the following approach. The change in value of exposures under stress follows from their valuation at baseline and revaluation after the market shock. For our purposes, we incorporate counterparty flows in the description of the net present value (NPV). For example, if  $x$  sells protection to  $y$ , then  $x$  is long the premium leg and short the payment leg. Stated alternatively,  $x$  writes the payment leg, while  $y$  writes the premium leg. Incorporating such flows and suppressing some earlier notation, we express  $V_{prem}^{0 \rightarrow T}(\lambda_m^*; s, \boldsymbol{\lambda}_{m-1}^*)$  as  $V_x^{prem}(\boldsymbol{\lambda}; s, T)$  and similarly the payment leg as  $V_y^{pay}(\boldsymbol{\lambda}, T_m)$ . The hazard rate environment that exists at valuation date  $t$  is given by  $\boldsymbol{\lambda}^t$ . Analogously, the environment at  $t$  under stress is  $\boldsymbol{\lambda}^{shock}$ .

The NPV of a swap of  $\$N$  notional on the as-of-date  $T$  is defined as follows:

$$NPV_{xy}(N, \boldsymbol{\lambda}^T, s, T) = N [V_x^{prem}(\boldsymbol{\lambda}^T; s, T) - V_y^{pay}(\boldsymbol{\lambda}^T, T)]. \quad (\text{A7})$$

Similarly, the NPV of the swap under stress is:

$$NPV_{xy}(N, \boldsymbol{\lambda}^{shock}, s, T) = N [V_x^{prem}(\boldsymbol{\lambda}^{shock}, s, T) - V_y^{pay}(\boldsymbol{\lambda}^{shock}, T)]. \quad (\text{A8})$$

The difference between these values determines the shock-induced variation margin payment due from  $x$  to  $y$  for each contract, from which we deduce the net payment due by summing over all contracts between the two parties.