

What expertise can tell about mathematical learning and cognition?

Francesco Sella^{1,2} & Roi Cohen Kadosh¹

¹*Department of Experimental Psychology, University of Oxford, UK*

²*Department of Psychology, University of Sheffield, UK*

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The acquisition of new knowledge and skills is one of the most powerful and impressive of human capacities. During the first years of life, children display an overwhelming increase in their perceptual, motor, linguistic, and social skills, which keep evolving through the existence. One of the central aims of the educational system is to facilitate and maximise the acquisition of new abilities in an optimal balance between the requirements of modern society and individual ambitions. In this vein, the interaction between learning and instruction represents a crucial process to understand how our neurocognitive system works and to study the limits of our cognitive capacities as human beings. Performance variability has been widely investigated with the aim to identify the cognitive, emotional and biological factors that characterise people who underachieve. In the context of mathematical and numerical cognition, much research has been dedicated to the study of individuals with low math achievement (e.g., dyscalculia) whereas less attention has been placed on the examination of individuals who gain expertise in math. The current paper aims to provide the reader with a brief overview of the cognitive and neural characteristics of math expertise. We hope that this would increase the awareness for such neglected line of research in the field of mathematical cognition and education, and would generate a new line of studies to provide better mechanistic understanding and foster math expertise.

Math experts constitute a fascinating object of study because they are arithmetically competent, as required to pass regular school examinations, and also master advanced mathematical knowledge, which heavily depends on logic and formal reasoning. In this vein, math experts fundamentally differ from exceptional calculators. While the former frequently possess excellent reasoning capacities with good calculation skills the latter display extraordinary calculation skills, which are probably the consequence of protracted obsessive drill, but not necessarily acquire advanced mathematical competences (Fehr, Weber, Willmes, & Herrmann, 2010; Pesenti, 2005; see Butterworth, 2006, for the characteristics of

calculation expertise). The investigation of mathematical expertise can shape numerical and mathematical cognition theories by providing insights about the modifications that the neurocognitive system undergoes when mastering mathematical knowledge. Here, we argue that a deep understanding of mathematical expertise can be achieved by combining both quantitative and qualitative methods within a developmental perspective.

Several studies have highlighted the contribution of domain-general and domain-specific cognitive processes in explaining individual differences in mathematical learning (Passolunghi & Lanfranchi, 2012; Schneider et al., 2016; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014). Another branch of research has emphasised the contribution of emotional, motivational and social factors, such as math anxiety, attitudes toward math, socio-economic status and home numeracy (Benavides-Varela et al., 2016; Dowker, Sarkar, & Looi, 2016; Gunderson, Ramirez, Levine, & Beilock, 2011; Skwarchuk, Sowinski, & Lefevre, 2014; Suárez-Pellicioni, Macarena, Núñez-Peña, María Isabel, & Colomé, 2015; Thompson, Napoli, & Purpura, 2017). Mathematical expertise has been linked to above-average intelligence, memory capacities, education, motivation, and practice (Butterworth, 2006; Pesenti, 2005).

Experts usually undergo an extended period of deliberate practice that leads to the gradual acquisition of new cognitive mechanisms, which can explain the higher performance and at the same time, reduce the influence of other factors (Ericsson & Charness, 1994; Ericsson & Ward, 2014). Conceivably, after such prolonged and intensive training, profound changes could be expected in one's numerical representation, strategy use, and engagement of domain-general cognitive functions. Such extended and intense training characterises math expertise, rather than mathematical giftedness. Although it is likely that mathematical giftedness would lead to higher motivation and engagement with mathematical material, and that those who are mathematically gifted would be more likely to be engaged in prestige

mathematical programmes, prolonged and intensive training is a critical factor to reach expertise. In other words, we defined expertise “as consistently superior performance on a specified set of representative tasks for a domain” (Ericsson & Lehmann, 1996, p. 277), rather than stemmed purely from an innate ability (see Ericsson, 2014, for further discussion).

From the beginning of their education, individuals with math disabilities struggle in building basic numerical knowledge (e.g., arithmetic facts, symbolic magnitude representation, and simple arithmetic procedures; Geary, Hoard, & Hamson, 1999; Landerl, Bevan, & Butterworth, 2004) lagging behind their typically developing peers (see Figure 1). Similarly, the difference in math achievement between math proficient and average-skilled individuals might be marked by the understanding of fractions and divisions, which represents the base for more complex mathematical concepts (Siegler et al., 2012). At this stage, students with low or average capacities may decide to avoid additional math courses and direct their interest toward other scholastic subjects. In this case, the exclusion of further math education drastically shrinks the range of choices in future scholastic and career plans, such as choosing an occupation that does not require any particular mathematical competence. Conversely, proficient students keep improving their mathematical knowledge and have a broader range of possibilities for their long-term career plans. In this vein, the level of math competence modulates the probability of becoming an expert in mathematics, whereby above average achievers have a higher probability compared to individuals with inferior math skills. Nevertheless, good math competence does not imply that someone will become an expert in mathematics. A series of structure activities whose aim is to improve knowledge and performance (i.e., deliberate practice; Ericsson, 2016; Ericsson et al., 1993; for the role of deliberate practice in math learning see Lehtinen, Hannula-Sormunen, McMullen, & Gruber, 2017) is required to reach the status of expert. A proficient student is more likely to undertake extra exercises, autonomously explore the subject, and attend

additional advanced courses at school whenever possible, or even hire a private math teacher. All these conditions increase the chances to pursue further math learning at university. Well-structured deliberate practice can be realised only with great effort, constant motivation and sufficient resources (Ericsson et al., 1993). Just a few individuals even among a group of proficient students may decide to become a math expert. Therefore, a crucial research question concerns the cognitive and non-cognitive factors that lead a student to actively improve his or her (most likely) already good math skills (but not exceptional as in math-gifted individuals). What are these promoting factors? Can these factors be detected early in the development or do they appear only at later stages in education? More broadly, extensive research should focus on the constellation of factors that drive students to approach the study of mathematics (e.g., self-efficacy, motivation, interest; Moore et al., 2014). Fostering these approaching factors in the general population has the potential to increase the number of students taking up a career in STEM disciplines, thereby fulfilling the need of graduates capable of contributing to the scientific and technological advances of nations (European Commission, 2011; Kuenzi, 2008).

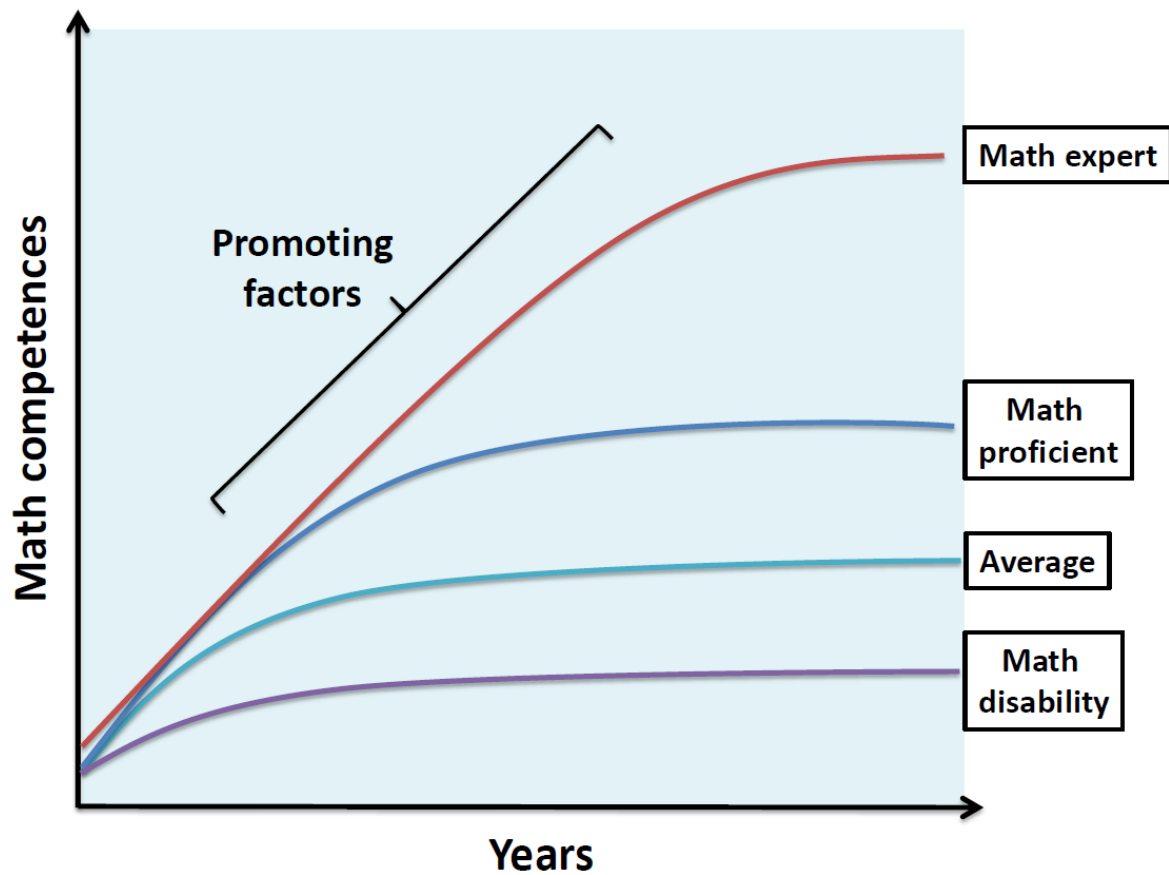


Figure 1. Hypothetical developmental trajectories of math competences as a function of years of education for individuals considered as experts, proficient-, average-, and low- achievers. In the early years of school, children with math disability struggle to build basic numerical competences and lag behind their typically developing individuals. Math proficient individuals distinguish themselves from the average achievers by mastering more advanced mathematical concepts such as divisions, fractions and basic algebra. Future math expert and skilled math individuals may initially show a similar superior math achievement compared to the general population. However, only some individuals may be characterised by those promoting factors (e.g., positive attitude toward math, deliberate practice) that lead to becoming math experts.

Math expertise: A cognitive profile

Years of extensive math training lead experts to the acquisition of new and advance mathematical concepts, but also to a qualitative change in their numerical representations and in the way they frame mathematical problems (see Table 1). For example, professional mathematicians are more accurate in estimating the results of numerical operations and also show a more extensive repertoire of arithmetic strategies compared to accountants and non-

math students (Dowker, 1992; Dowker, Flood, Griffiths, Harriss, & Hook, 1996). The implementation of different strategies in mathematicians has been also found in the case of fractions comparison (Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013) and for the solution of algebraic problems (Star & Newton, 2009). Mathematicians also possess a more efficient processing of symbolic numerical information as indexed by their ability to quickly compare two-digit numbers, superior numerosity estimation skills (Castronovo & Göbel, 2012) and more accurate spatial mapping of numbers, even though the latter ability appears to be mediated by visuospatial skills (Sella, Sader, Lolliot, & Cohen Kadosh, 2016; see also Wei, Yuan, Chen, & Zhou, 2012). Mathematicians' representation of symbolic numbers may lack a robust spatial connotation, which was instead found in professionals who regularly use math for their job and doctoral students from humanities (Cipora et al., 2015; for similar results see Hoffmann, Mussolin, Martin, & Schiltz, 2014). Overall, mathematicians display a flexible and accurate representation of numerical information that is coupled with the efficient use of optimal cognitive strategies. Their flexibility might be rooted in better executive functions and lower cognitive load during math-related tasks, similar to what was found in other fields such as language (Bialystok & Feng, 2009; Tzelgov & Cohen Kadosh, 2009).

Math expertise: A neural profile

At the neural level, both structural and functional differences have been found in the brains of math experts as usually compared to controls matched for salient characteristics, such as intelligence and academic status, but without advanced mathematical expertise. Not surprisingly, years spent as a mathematician were associated with an increased grey matter density in the parietal cortex (Aydin et al., 2007), which constitutes the primary brain area responsible for processing of numerical and visuospatial information (Cohen Kadosh, Lammertyn, & Izard, 2008; Dehaene, Piazza, Pinel, & Cohen, 2003; Klingberg, Forssberg, &

Westerberg, 2002) (see Table 1). Significant differences also emerge in the frontoparietal network, which plays a crucial role in the development of arithmetical and mathematical competencies (Evans et al., 2015; Kucian et al., 2013; Rotzer et al., 2008; Tsang, Dougherty, Deutsch, Wandell, & Ben-Shachar, 2009). For instance, those who are involved in maths training programme during their adolescence compared to control group who do not take part in such programme show larger surface area and thinner cortex in the frontal and parietal regions compared to age- and IQ-matched controls (Navas-Sanchez et al., 2014, 2016). Although, one needs to take into account that these studies do not indicate that those adolescents will eventually become a math expert.

At the functional level, mathematical expertise was linked to a reduced extent of activation, especially in the frontal areas of the brain, when individuals were solving algebraic expressions (Jeon & Friederici, 2016). This reduced activation, which has been associated with neural efficiency, marks the presence of more efficient and automatized processing (Chein & Schneider, 2012; Shiffrin & Schneider, 1977), and has been shown to be causally related to mathematical performance (Snowball et al., 2013). Accordingly, math-gifted adolescents show a rapid reduction of neural activity compared to peers when learning how to solve number reasoning tasks (Zhang, Gan, & Wang, 2015). More broadly, age-related changes in mental arithmetic have been related to increased functional specialization of the left inferior parietal cortex and a reduction in activation of the hippocampus and dorsal basal ganglia, which characterised younger individuals who still heavily relied on memory and attentional resources when solving arithmetic problems (Rivera, Reiss, Eckert, & Menon, 2005). Similarly, math-gifted adolescents show greater neural efficiency in right-lateral frontoparietal regions compared to peers. A widespread pattern of brain activation characterises children with math learning disability when solving arithmetic problems, even though numerical training can efficiently reduce the initial widespread pattern to the extent to

resemble the brain activity of typically developing individuals (Iuculano et al., 2015).

Nevertheless, math-gifted individuals showed enhanced activation in the parietal and frontal regions when performing reasoning and executive function tasks (Desco et al., 2011; see also O'Boyle et al., 2005).

The frontoparietal connectivity has been specifically related to math performance in children (Emerson & Cantlon, 2012) while a reduced white matter coherence in the fibres connecting the two hemispheres passing through the corpus callosum has been found in children with poor numerical skills (Cantlon et al., 2011). In line with these results, math-gifted adolescents show both greater intra-hemispheric frontoparietal connectivity and interhemispheric frontal connectivity compared to control peers (Prescott, Gavrilescu, Cunningham, O'Boyle, & Egan, 2010). The extensive experience with numerals shapes the ventral visual cortex as similarly happens in the case of letters when individuals learn to read (Dehaene et al., 2010). Advanced neuroimaging techniques have revealed the presence in the bilateral inferior temporal gyri of a visual number form area (Dehaene & Cohen, 1995), which specifically respond to numerals more than letters and other symbols (Cohen Kadosh, Cohen Kadosh, Kaas, Henik, & Goebel, 2007; Grotheer, Herrmann, & Kovacs, 2016; Shum et al., 2013). Mathematicians display enhanced activation of the visual number form area when presented with digits and mathematical formulas. Concurrently, mathematicians also displayed a reduced activation when presented with faces in the right fusiform gyrus, thereby suggesting expertise can modify the functional organisation of the ventral visual cortex (Amalric & Dehaene, 2016).

Finally, when asked to verify whether a complex mathematical statement is true, false, or makes no sense, mathematicians display the activation of a bilateral network including prefrontal, parietal, and inferior temporal areas, without activating language-related areas situated in the left hemisphere. This has led to the suggestion that advanced

mathematical reasoning might be coupled with a network responsible for the processing of numerical and spatial information rather than linguistic processing (Amalric & Dehaene, 2016, 2017).

	Characteristic	Study
Cognitive	Excellent computational estimation skills and use of arithmetic strategies	(Dowker, 1992; Dowker, Flood, Griffiths, Harriss, & Hook, 1996; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013)
	Rapid number comparison	(Castronovo & Göbel, 2012)
	Accurate spatial mapping of numbers	(Sella, Sader, Lollot, & Cohen Kadosh, 2016)
	Flexible spatial representation of numbers	(Cipora et al., 2015; Hoffmann, Mussolin, Martin, & Schiltz, 2014)
Neural	Increased grey matter in the parietal cortex	(Aydin et al., 2007)
	Larger surface area and thinner cortex in the frontal and parietal regions	(Navas-Sánchez et al., 2014, 2016)
	Neural efficiency in frontal areas	(Jeon & Friederici, 2016; Zhang, Gan, & Wang, 2015)
	Greater frontoparietal and interhemispheric connectivity	(Prescott, Gavrilescu, Cunnington, O’Boyle, & Egan, 2010)
	Enhanced activation in frontal and parietal areas during reasoning and executive function tasks	(Desco et al., 2011)

Table 1. Examples of the cognitive and neural characteristics associated with proficient mathematical individuals.

Summary

Despite these findings, it remains unclear whether the observed differences at the cognitive and neural level between math experts and non-experts are merely the by-product of the extensive mathematical experience or are essential requirements to become a math expert. Moreover, it is hard to tease apart math expertise from math giftedness. Individuals with precocious and extraordinary math skills are much more likely to engage in deliberate practice to further extend their mathematical competence. However, math-gifted individuals are a small minority of the population whereas a larger group of people have sufficient math skills to engage with further math education successfully. Only some of these individuals

decide to improve their math expertise further. What are the promoting factors moving these individuals toward a career involving further math education (as in the STEM fields)? What is the role of biological, cognitive and motivational factors in such a decision? What is the influence of external factors? Long-term quantitative and qualitative developmental studies are necessary to answer this question. Such longitudinal studies could provide valuable insights for maths education. For example, it could provide valuable suggestions for optimising and designing new cognitive training, and education programmes to strengthen cognitive components related to math expertise and continuously giving the opportunity to gain further math expertise in large part of the population, not only math-gifted individuals. Moreover, the implementation of advanced training can reveal how the brain and cognitive functions change when (possibly not-gifted) individuals complete an extensive practice. The combination of longitudinal and training studies can provide valuable information on the mastering of mathematical knowledge, which ultimately represents a powerful form of formal human reasoning.

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