Effects of tidal torques on 1I/2017 U1 (‘Oumuamua)

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Abstract

As observations of ‘Oumuamua were collected well into the outbound component of its hyperbolic orbit, it is not obvious what effects Sol had on its rotational dynamics. Therefore, we simulate ‘Oumuamua as a prolate spheroid and triaxial ellipsoid of uniform mass density and show that the experimentally observed angular velocities remain largely unchanged during ‘Oumuamua’s flyby, supporting previous work suggesting that, in the absence of a collision during its interstellar journey, the asteroid was tumbling in the same manner as when it left its original solar system.

Keywords: ‘Oumuamua, Rotational dynamics, Celestial mechanics

1. Introduction

The recent discovery of interstellar object 1I/2017 U1 (‘Oumuamua) [1, 2, 3, 4, 5] has attracted substantial interest due to its tumbling motion as well as its spin rate. In particular, the rotational motion of ‘Oumuamua has inspired speculation as to its origins [6, 7], as well as implications for understanding early planet formation [8], and the structure of nearby solar systems [9, 10]. The tumbling of ‘Oumuamua was expected to dampen on very large time-scales due to its anelastic properties [11, 12, 13] and this time-scale is of the order of 10^{10}-10^{12} years, with the lower and upper bounds determined by supposing the composition of the interstellar object is ice or rock typical of C-type asteroids respectively [14]. The hyperbolic (and hence non-recurrent) nature of ‘Oumuamua’s orbit suggests a careful analysis of its rotational behavior is needed in order to justify claims about its origins. Given that discovery of the interstellar asteroid and data regarding its

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rotational dynamics were collected after the perihelion of its flyby past Sol, the question of whether gravitational torques had a significant effect on ‘Oumuamua’s tumbling still remains.

Most studies of oblate bodies focus on perturbations from spherical geometries or regular ellipsoids [15, 16, 17], although dynamical models for more general rigid bodies exist [18, 19]. These simple modeling assumptions were used to analyze the lightcurve data for Oumuamua [20]. The effects of gravitational torques have previously been shown to exert a non-trivial effect on the spin state of an asteroid [21]. Originally, this theory was applied to prolate two-axial objects in such situations as the creation of asteroids and hyperbolic planetary flybys, but has since been extended to include more general ellipsoidal geometries [22].

In the present work, however, we show by simulation that the experimentally observed rotational velocities and dimensions give ‘Oumuamua a significantly large angular momentum which is unaffected by the exerted gravitational torque, thereby demonstrating that the asteroid broadly retains its tumbling dynamics during its hyperbolic flyby with Sol. Coupled with work suggesting that the rotations of ‘Oumuamua will be dampened on a time-scale of 10^{10}-10^{12} years [14], these results suggest that, barring the occurrence of a collision during its journey, the interstellar asteroid will continue to tumble in the same manner as when it left its parent solar system for a very long time.

2. Equations of orbital and rotational motion for the ‘Oumuamua-Sol system

We simulate a two-body system featuring only ‘Oumuamua and Sol, which have masses given by m_O and m_S respectively, and where ‘Oumuamua is allowed to tumble in 3D. Furthermore, it is assumed that ‘Oumuamua has a uniform mass distribution. Vectors in the system can be described in terms of either the fixed heliocentric frame whereby Sol is at the origin (denoted by the superscript F) or the orthogonal rotating frame whose basis vectors align with the principal axes of ‘Oumuamua for all time (denoted by the superscript R). This latter basis is described by unit vectors $\boldsymbol{e}'_1$, $\boldsymbol{e}'_2$, and $\boldsymbol{e}'_3$, which correspond to the semi-major axes of the asteroid with lengths given by a, b, and c respectively.

To transform an arbitrary vector $\boldsymbol{k}$ between these basis descriptions, we use the equation:

$$\boldsymbol{k}^R = M^T \boldsymbol{k}^F, \tag{1}$$

where M is an orthogonal 3×3 matrix whose entries are given by:

$$M = \begin{bmatrix} (e'_{11})^F & (e'_{21})^F & (e'_{31})^F \\ (e'_{12})^F & (e'_{22})^F & (e'_{32})^F \\ (e'_{13})^F & (e'_{23})^F & (e'_{33})^F \end{bmatrix}, \tag{2}$$

with the unit vectors of the rotating basis $(\boldsymbol{e}'_i)^F = \left((e'_{i1})^F, (e'_{i2})^F, (e'_{i3})^F\right)$, for $i = \{1, 2, 3\}$, forming the columns.

However, M will depend on time in general and will evolve based on the angular velocities of the ellipsoid as it rotates. We denote these angular velocities in the fixed and rotational axes as $\boldsymbol{\omega}^F = \left(\omega_1^F, \omega_2^F, \omega_3^F\right)$ and $\boldsymbol{\omega}^R = \left(\omega_1^R, \omega_2^R, \omega_3^R\right)$ respectively, which determines the velocity at which 'Oumuamua rotates around the spanning unit vectors in the fixed and rotational bases. The vectors of the rotational basis evolve according to the relation [23]:

$$\frac{\mathrm{d}\,(\boldsymbol{e}'_i)^F}{\mathrm{d}t} = \boldsymbol{\omega}^F \times (\boldsymbol{e}'_i)^F. \tag{3}$$

For the case where the two gravitational bodies are separated by a length-scale which is much larger than the size of the individual bodies, the masses can be effectively treated as points (see [19] for a more in-depth discussion). Furthermore, we note that the mass ratio of 'Oumuamua and Sol m_O/m_S is of the order of 10^{-21}. As such, we can assume that Sol exhibits negligible orbital motion so that it remains fixed at the origin whilst only 'Oumuamua moves due to gravity. Therefore, we have a single equation describing the translation of 'Oumuamua's center of mass, denoted by the vector $\boldsymbol{r}_O^F = \left(x_O^F, y_O^F, z_O^F\right)$:

$$\frac{\mathrm{d}^2\boldsymbol{r}_O^F}{\mathrm{d}t^2} = \frac{Gm_S}{\left|\boldsymbol{r}_O^F\right|^3}\boldsymbol{r}_O^F, \tag{4}$$

where G is the universal gravitational constant.

To model the tumbling motion of 'Oumuamua we employ the Euler equations of rigid body motion [24]. To determine the torques in the rotating basis, one can either use a gravitational potential approach (e.g. as discussed in [25]) or a continuum mechanical approach (e.g. evaluating equation (42) in [19] using elliptical coordinates). Both methods are equivalent and give the result:

$$I_{11}\frac{\mathrm{d}\omega_1^R}{\mathrm{d}t}+\left(I_{33}-I_{22}\right)\omega_2^R\omega_3^R+\frac{3\left(I_{33}-I_{22}\right)Gm_S y_0^R z_0^R}{\left|\boldsymbol{r}_0^F\right|^5}=0, \tag{5}$$

$$I_{22}\frac{\mathrm{d}\omega_2^R}{\mathrm{d}t}+\left(I_{11}-I_{33}\right)\omega_1^R\omega_3^R+\frac{3\left(I_{11}-I_{33}\right)Gm_S x_0^R z_0^R}{\left|\boldsymbol{r}_0^F\right|^5}=0, \tag{6}$$

$$I_{33}\frac{\mathrm{d}\omega_3^R}{\mathrm{d}t}+\left(I_{22}-I_{11}\right)\omega_1^R\omega_2^R+\frac{3\left(I_{22}-I_{11}\right)Gm_S x_0^R y_0^R}{\left|\boldsymbol{r}_0^F\right|^5}=0, \tag{7}$$

where $I_{11}=m_O\left(b^2+c^2\right)/5$, $I_{22}=m_O\left(a^2+c^2\right)/5$, and $I_{33}=m_O\left(a^2+b^2\right)/5$ are the principal moments of inertia of ‘Oumuamua, $\left(x_O^R, y_O^R, z_O^R\right)$ is the position of ‘Oumuamua's center of mass as described by the rotating basis and where Sol is again taken to be at the origin. Note that vector norms are preserved under the orthogonal transformation between the fixed and rotating basis. Therefore, we use the center of mass positions described in the fixed frame for convenience.

In summary, the full system modeling the orbital and 3D rotational motion of ‘Oumuamua is given by equations (3)-(7). We note that in the regime whereby the separation distance is significantly larger than the size of ‘Oumuamua, the orbital motion of the asteroid decouples from its rotational motion. Therefore, one can solve for the positions of ‘Oumuamua's center of mass and substitute the result into the rotation equations (5)-(7) using the transformation matrix between the two coordinate systems M and applying relation (1). Furthermore, in reducing the problem to only consider the motion of ‘Oumuamua, we find that both the orbital and rotational motion of the asteroid is independent of its mass. This independence in its rotational motion follows from the definitions of the second moment of inertia I_{11}, I_{22}, and I_{33} in (5)-(7) which cancel with the mass from the torque contributions. Additionally, we note that for the case of a prolate spheroid (i.e. the two semi-major axes a and b are equal), (5) implies that ω_1^R will be constant for all time.

For the two-body system consisting of ‘Oumuamua and Sol, we must solve a system of fourteen equations: two equations to determine the orbital positions (4) (which, for simplicity, we will take to be in the x-y plane without loss of generality given that the motion of two bodies under gravity is purely planar in our regime), three equations to determine the three angular velocities in the rotational frame (5)-(7), and nine equations for the vector basis of the rotating frame (3).

Parameter/Initial Condition	Numerical value
Heliocentric gravitational constant	$Gm_S = 1.3271244 \times 10^{20}\mathrm{m}^3\mathrm{s}^{-2}$
Principal semi-axes of prolate spheroid	$(a, b, c) = (230\mathrm{m}, 35\mathrm{m}, 35\mathrm{m})$
Principal semi-axes of tri-axial ellipsoid	$(a, b, c) = (230\mathrm{m}, 35\mathrm{m}, 17.5\mathrm{m})$
Position of ‘Oumuamua	$\left(x_O^F, y_O^F\right) = \left(1.02136 \times 10^{12}\mathrm{m}, -1.32400 \times 10^{12}\mathrm{m}\right)$
Orbital velocity of ‘Oumuamua	$\left(\frac{\mathrm{d}x_O^F}{\mathrm{d}t}, \frac{\mathrm{d}y_O^F}{\mathrm{d}t}\right) = \left(-1.62069 \times 10^4\mathrm{m\ s}^{-1}, 2.42791 \times 10^4\mathrm{m\ s}^{-1}\right)$
Orientation of ‘Oumuamua	$\left(\boldsymbol{e}_1'\right)^F = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \quad \left(\boldsymbol{e}_2'\right)^F = (0, 1, 0) \quad \left(\boldsymbol{e}_3'\right)^F = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$
Angular velocity of ‘Oumuamua	$\left(\omega_1^R, \omega_2^R, \omega_3^R\right) = \left(-1.1 \times 10^{-4}\mathrm{rad\ s}^{-1}, 1.2 \times 10^{-4}\mathrm{rad\ s}^{-1}, 1.4 \times 10^{-4}\mathrm{rad\ s}^{-1}\right)$

Table 1: Numerical parameters and initial conditions used to simulate the observed hyperbolic orbit and rotation of ‘Oumuamua.

3. Numerical method and parameters used

We list the parameters and initial conditions used in the simulations in Table 1. In particular, we reference the heliocentric gravitational constant from the IAU 2009 system, as given in [26]. We used the eccentricity of ‘Oumuamua’s hyperbolic orbit and perihelion distance as reported in the NASA Jet Propulsion Laboratory’s Small-Body Database Search Engine (https://ssd.jpl.-nasa.gov/). The perihelion velocity was calculated assuming a two-body system with Sol. The length of its semi-axes was taken to be the values estimated in [4] whilst the experimentally observed angular velocities used were based off data presented in [3, 14].

Equations (3)-(7) were solved under these parameters and conditions using NDSolve in Mathematica (with relative and absolute tolerances of 10^{-13}). Convergence checks were carried out using the Runge-Kutta scheme ‘ode45’ and the variable-order solver ‘ode113’ in Matlab (with relative and absolute tolerances of 10^{-12}). For local relative and absolute tolerances of 10^{-n}, simulating the ‘Oumuamua system to a final time of 10^ms incurs a global integration error of the order 10^{m-n}. Random parameter sweeps were carried out using the ‘default’ seed in Matlab and the uniform random number generator ‘rand’ for robustness, with 10^3 instances considered. In particular, we also simulated the orbital and rotational motion of ‘Oumuamua with 10^3 other parameter choices corresponding to the same magnitude of the initial angular velocity (uniformly sampling a spherical distribution to generate these), and with the initial inclination of the body chosen uniformly

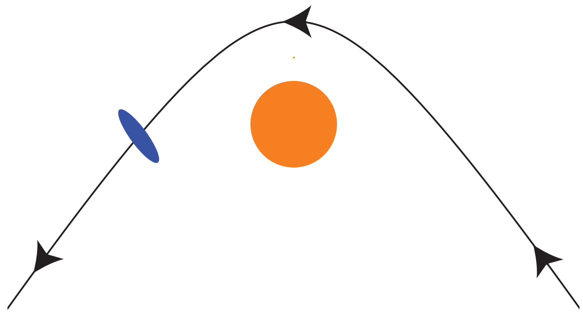

Figure 1: Schematic diagram of ‘Oumuamua (denoted by the blue ellipse, not to scale) as it completed its hyperbolic flyby from right to left past Sol (denoted by orange dot, not to scale) at periapsis distance 0.25534AU with simulated velocity of 87.429km s^{-1} in the fixed heliocentric frame where Sol is taken to be at the origin. Discovery of the interstellar body occurred on its outbound trajectory. (Color online)

between 0 and $\pi/2$ rad.

4. Tumbling in hyperbolic orbits for observed angular velocities

We now show that the impact of Sol's gravitational torque on ‘Oumuamua did not have a significant effect on the asteroid's rotational velocity and orientation due to its already large angular momentum. We model ‘Oumuamua as a rigid ellipsoid over a period of 10^8s or approximately 1157.4 days. We emphasize again that the mass of ‘Oumuamua does not affect its orbital and rotational motion, so results from our simulations still hold even though there is some disagreement in the literature about ‘Oumuamua's composition [27]. Initial angular velocities in the rotational frame are taken to give the body a rotational period of 2.15×10^{-4}rad s^{-1}, corresponding to an observed rotational period of 8.1hr [14, 6]. ‘Oumuamua's hyperbolic orbit (see Fig. 1) has eccentricity 1.1995 and the perihelion is reached at simulation time $t = 5 \times 10^7 s$. The orbital dynamics are taken to be planar, as Sol is the most dominant mass which ‘Oumuamua is influenced by while in the Solar System.

Due to its NPA rotation, ‘Oumuamua is assumed to rotate in three dimensions throughout its journey. There is uncertainty in the precise geometry of ‘Oumuamua, so we consider two possible geometries: a prolate spheroid (two-axes equal) and a tri-axial ellipsoid (three-axes distinct). We simulate the orbital and rotational motion of ‘Oumuamua using numerical parameters and initial conditions specified in Table 1 and plot the asteroid's angular velocities in the fixed heliocentric frame and quantify the non-uniform rotational motion by plotting the effective rotation rate ω_{eff} and dynamic moment of inertia I_D, both under the effect of solar tidal torques and the torque free regime, in Fig. 2. In particular, the latter two quantities are defined as [21]:

$$\omega_{\text{eff}} = \frac{\boldsymbol{\omega}^R \cdot \boldsymbol{I} \cdot \boldsymbol{\omega}^R}{\left|\boldsymbol{I} \cdot \boldsymbol{\omega}^{\boldsymbol{R}}\right|}, \tag{8}$$

$$I_D = \frac{\boldsymbol{\omega}^R \cdot \boldsymbol{I} \cdot \boldsymbol{I} \cdot \boldsymbol{\omega}^R}{\boldsymbol{\omega}^R \cdot \boldsymbol{I} \cdot \boldsymbol{\omega}^R}, \tag{9}$$

where $\boldsymbol{I} = \operatorname{diag}(I_{11}, I_{22}, I_{33})$.

Some comments are in order: The relative change of the effective rotation rate and the dynamic moment of inertia as a result of ‘Oumuamua's gravitational interaction with Sol is 1.7×10^{-10} and 1.6×10^{-7} of the initial effective rotation rate and 5.2×10^{-12} and 3.1×10^{-7} of the initial dynamic moment of inertia respectively for the simulated prolate spheroid and triaxial geometries respectively. Despite the maximal changes to these tumbling quantities occurring at the perihelion distance of the hyperbolic flyby, ‘Oumuamua settles to its original rotational behavior. As a result, the tumbling motion of ‘Oumuamua is largely unchanged before and after it passes through the perihelion in its hyperbolic flyby at simulation time $t = 5 \times 10^7$. This point is independent of whether the asteroid periodically rotates, as for the case of the prolate spheroid, or quasiperiodically precesses, when modeled as a triaxial ellipsoid. The components of ‘Oumuamua's angular velocities appear to be approximately the same on both its inbound and outbound trajectory through the Solar System.

To verify the robustness of these dynamics, we have simulated this system with 10^3 other parameter choices using the random seed discussed in Section 3. In all cases, the results were the same; namely, that the rotational behavior of the asteroid consisted of an NPA rotation that was unchanged by gravitational influences from Sol.

5. Conclusions

Our simulations show that, for the orbital data reported in [1], length dimensions given in [4], and rotational velocities found in [3, 14], the tidal torques applied by Sol did not significantly affect the rotational dynamics of ‘Oumuamua during its hyperbolic flyby. This result appears to be independent of whether ‘Oumuamua is modeled as a prolate spheroid or a triaxial ellipsoid

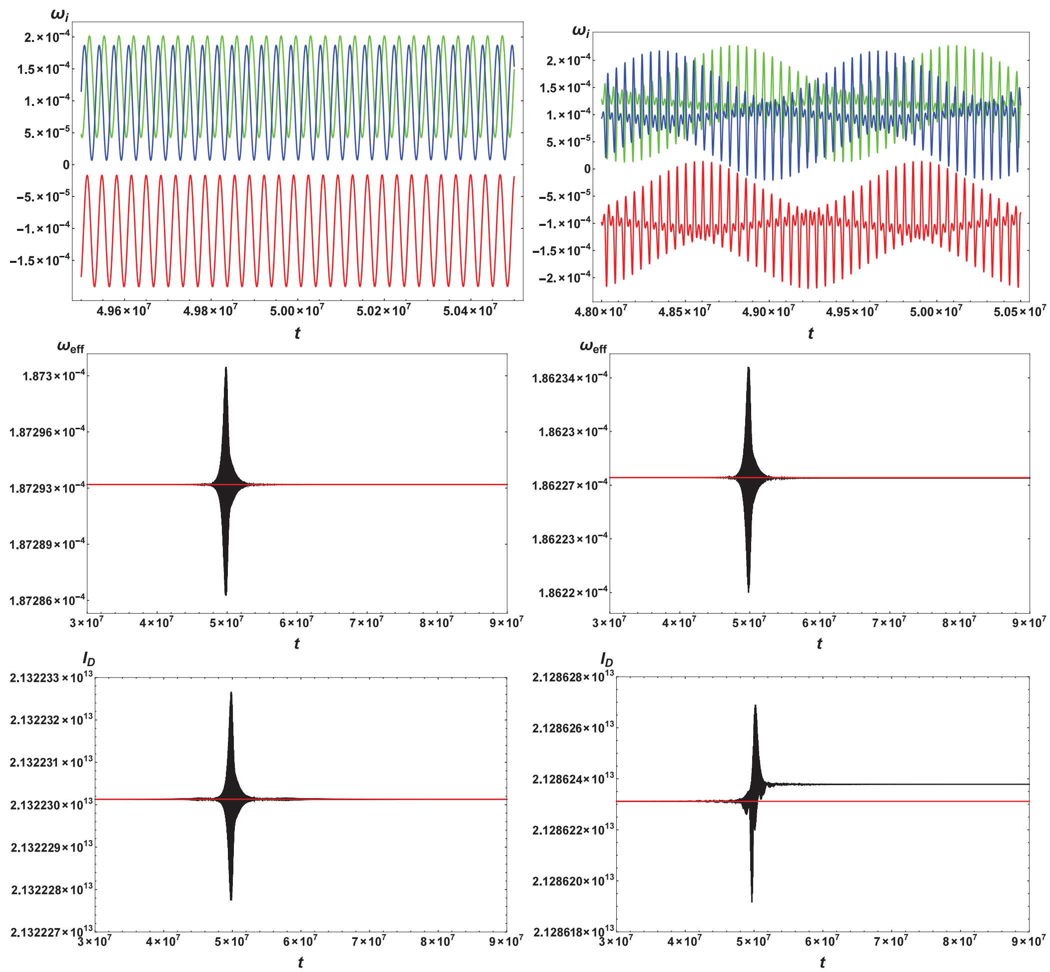

Figure 2: Simulation of 'Oumuamua's rotational dynamics at observed angular velocities during its hyperbolic orbit before and after the perihelion (which occurs at simulation time $t = 5 \times 10^7$s). 'Oumuamua is modeled as a prolate spheroid (left column) and tri-axial ellipsoid (right column). The first row shows the angular velocity components in the principal axes of the fixed heliocentric frame, with the first, second, and third components denoted by the red, green, and blue curves, respectively. Vertical units are in radians per second. The second row shows the effective rotation rate, as defined in (8) of 'Oumuamua under the effects of Sol's gravitational torque (black line) and in the torque free regime (red line), with vertical units given in radians per second. The third row shows 'Oumuamua's dynamic moment of inertia, defined in (9), under gravitational effects from Sol (black line) and in the torque-free regime (red line), with vertical units given in kilogram meter squared. Horizontal units in all panels are given in seconds. Numerical parameters and initial conditions are given in Table 1. The relative change between the initial and final effective rotation rates are 1.7×10^{-10} and 1.6×10^{-7} of the initial effective rotation rate respectively for the prolate spheroid and triaxial ellipsoid whilst the relative change of the dynamic moment of inertia is 5.2×10^{-12} and 3.1×10^{-7} of the initial dynamic moment of inertia for the two simulated geometries. Despite the tidal torque applied by Sol, the maximum of which occurs at the perihelion (i.e. $t = 5 \times 10^7$), 'Oumuamua retains its original NPA rotation in the outbound trajectory of its hyperbolic flyby with Sol for both prolate spheroid and triaxial ellipsoid geometries. (Color online)

and, furthermore, is independent of its mass. Therefore, despite disagreement as to the asteroid's composition, our final result still holds.

The angular momentum of ‘Oumuamua in all cases was significantly large so that changes to it brought about by applied solar torques did not result in an appreciable change. Indeed, the relative changes in ‘Oumuamua's effective rotation rate and dynamic moment of inertia during the flyby were less than 1.6×10^{-7} and 3.1×10^{-7} of the inbound quantities. Therefore, inferences regarding the interstellar asteroid's origins from observational data collected on its rotational dynamics appear to be well-founded. Given that the tumbling would take on the order of 10^{10}-10^{12} years to dampen, it is likely that ‘Oumuamua will continue to tumble as it did when it left its original solar system.

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