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On Price Taking Behavior in a Nonrenewable Resource Cartel-Fringe Game

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On price taking behavior in a nonrenewable resource cartel-fringe game*

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Abstract

We consider a nonrenewable resource game with one cartel and a set of fringe members. We show that (i) the outcomes of the closed-loop and the open-loop nonrenewable resource game with the fringe members as price takers (the cartel-fringe game à la Salant 1976) coincide and (ii) when the number of fringe firms becomes arbitrarily large, the equilibrium outcome of the closed-loop Nash game does not coincide with the equilibrium outcome of the closed-loop cartel-fringe game. Thus, the outcome of the cartel-fringe open-loop equilibrium can be supported as an outcome of a subgame perfect equilibrium. However the interpretation of the cartel-fringe model, where from the outset the fringe is assumed to be price-taker, as a limit case of an asymmetric oligopoly with the agents playing Nash-Cournot, does not extend to the case where firms can use closed-loop strategies.

Key words: cartel-fringe, dominant firm versus fringe, price taking, nonrenewable resources, dynamic games, open-loop versus closed-loop strategies.

JEL Classification: D43, Q30, L13.

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1 Introduction

Salant (1976) was the first to consider a model of the oil market in a situation where supply comes from a coherent cartel and a large group of fringe members. In particular he studied the case of zero extraction costs and a continuum of fringe members. The model was later generalized by Ulph and Folie (1980), again with a continuum of fringe members, but for positive constant marginal extraction costs, possibly differing between the cartel and the fringe. The cartel takes as given the production path of the fringe and chooses a price path, whereas the fringe firms are price takers and determine their production paths given the price path. The cartel and the fringe simultaneously choose their respective strategy. Because each firm's strategy is in the form of a path it is an open-loop game. We call this game the open-loop cartel-versus-fringe game. An important contribution of Salant (1976) is to provide a microfoundation of this model by showing that it is a limiting case of an asymmetric oligopoly model where fringe firms don't act as price takers but as Nash players. More precisely, consider the asymmetric oligopoly game with one dominant firm (e.g., with a low cost of extraction and/or larger reserves) and a finite number of fringe firms who compete à la Cournot in the natural resource market. Salant (1976) shows that when the number of fringe firms becomes arbitrarily large the equilibrium outcome of this open-loop Nash game coincides with the equilibrium outcome of the open-loop cartel-versus-fringe game¹.

Open-loop strategies are acceptable in environments where firms can commit over the whole time horizon to a production path or a price path, for instance under the assumption of a perfect futures' market. However, this may not be an acceptable way to model firms' strategies in environments where they have information about stocks at future dates and have the flexibility to change their course of actions during the game: the equilibrium obtained with open-loop strategies may not be subgame perfect. In the latter case, we consider the set of closed-loop strategies where a firm chooses states (i.e., stocks) dependent strategies².

In this paper we provide a formulation of a closed-loop cartel-fringe equilibrium and

¹In the sequel we will use the convention that in the cartel-versus-fringe game the fringe members are price takers. The expression Nash-Cournot is reserved for the case where the firms act as Nash-Cournot players, taking competitors' supply as given. Nevertheless, the non-cartel players are called fringe firms.

²Reinganum and Stokey (1985) consider an intermediate case where the period of commitment to an extraction is positive and finite.

determine its relationship with (i) the open-loop cartel-fringe equilibrium (Salant 1976) and (ii) an asymmetric closed-loop Nash equilibrium.

Our first objective is to provide a definition of the closed-loop equilibrium and to show that, for this definition, the open-loop and closed-loop cartel-fringe equilibrium do coincide. This is an important robustness property of the open-loop cartel-fringe equilibrium derived in Salant (1976). The difficulty in providing a plausible formulation of a closed-loop cartel-fringe model lies in reconciling the intrinsic *myopic* behavior of a fringe firm assumed through price taking and the rather sophisticated (or farsighted) behavior assumed by the use of closed-loop strategies. We propose the following scenario for the closed-loop model: each fringe firm takes the price path as given and determines its extraction strategy which is allowed to depend on its own stock only; the cartel takes each fringe firm's strategy as given and determines a pricing strategy (or alternatively a production strategy) that depends on its own stock and all fringe's stocks. The outcome of this simultaneous move game is an equilibrium if the market of the resource is in equilibrium at each moment. Our methodology is related to the work done by Groot et al. (1992, 2003) who studied the case of the cartel being a Stackelberg leader and the fringe being a price taker. The cartel-fringe model with Stackelberg leadership was first introduced by Gilbert (1978). It is well-known that in this model the open-loop Stackelberg equilibrium concept suffers from time inconsistency for plausible parameter values, and is therefore not a feedback equilibrium (see Newbery (1981) and Ulph (1982)). But open-loop and closed-loop equilibrium outcomes do coincide for at least some parameter values. In this paper we consider the case where the cartel and fringe firms *simultaneously* choose their respective strategies.

The second objective concerns the Nash game. We show that the open-loop equilibrium of this game does not coincide with the closed-loop equilibrium. More specifically, we consider a model where each firm exploits a private exhaustible resource and where one firm (called the cartel) has a cost advantage over the other firms. All firms compete à la Cournot in the resource market. The cartel chooses a strategy that specifies the extraction rate at each moment as a function of the state, described by the vector of stocks of all firms, at that moment. While the cartel takes the strategy of all other firms as given, its extraction rate depends on its own stock as well as on all other firms' stocks. When weighing the impact of an extra unit of extraction at a given moment it takes into account three effects (i) the additional revenue, (ii) the reduction of its

available stock and (iii) the impact of this change in its own stock on the extraction of its competitors. This latter effect, that we refer to as the feedback effect, is absent when firms use open-loop strategies. Also all other firms use a closed-loop strategy. The fact that the equilibrium outcome of the open-loop Nash game cannot be supported as the outcome of an equilibrium of the closed-loop game is due to the presence of the feedback effect. This result is closely related to (unpublished) work by Polasky. Polasky (1990) shows in a discrete time model with a finite number of players that the open-loop equilibrium is not subgame perfect if the exhaustion dates of firms differ. He then considers a duopoly model with linear demand and equal and constant marginal extraction costs. He also postulates an exogenous instant of time T , after which the extracted commodity is worthless. He then claims that if the per period profit function is quadratic in extraction and depends only on current extraction (and not on existing stocks) and if no firm exhausts before T , open-loop and feedback equilibria coincide. But then he proves that in the duopoly model with equal initial stocks and equal constant marginal extraction costs and in the absence of an exogenous T , the open-loop and the feedback equilibrium do not coincide because one firm can and will manipulate its own exhaustion time in a profitable way. The present paper uses a continuous time formulation of a nonrenewable resource oligopoly and allows for asymmetries between firms (in terms of costs, stocks and number of firms in each category).

In deriving our conclusion we exploit the analysis in Benckroun, Halsema and Withagen (2009) which provides a full characterization of the open-loop Nash equilibrium as well as of the open-loop cartel-fringe model, for all possible constant marginal extraction costs. Benckroun et al. (2009) is closely related to Lewis and Schmalensee (1980) and Loury (1986) who have studied the case of a finite number of oligopolists. The former authors were mainly interested in the order of exploitation and their analysis mainly concerns the case of two players. Loury studies the case of equal costs. All these papers focus on the case where firms use open-loop strategies.

This analysis allows to determine if there is a relationship between the closed-loop cartel-fringe equilibrium and the closed-loop Nash equilibrium in an asymmetric oligopoly. More precisely, we show that, contrary to the case where firms use open-loop strategies, the limit case of an asymmetric closed-loop Nash equilibrium where we let the number of the fringe firms go to infinity³ does not correspond to the outcome of

³while keeping the aggregate resource stock unchanged

a closed-loop cartel-fringe equilibrium where the fringe firms are assumed to be price takers. The feedback effect does not vanish as the market power of each fringe firm is diluted by the increase in the total number of fringe firms.

This paper adds to the literature on the limit points of monopolistic competition and the relationship between the limit of equilibria of finite-player games and equilibria of the nonatomic limit game (e.g., Roberts (1980), Novshek and Sonnenschein (1980), Mas-Colell (1982) and Mas-Colell et al. (1995) Chapters 12 and 16). Fudenberg and Levine (1988), perhaps the paper to which our analysis is the most related, extends that literature and considers a finitely repeated game. It considers a closed-loop and an open-loop model. In the closed-loop model the history of the play is common knowledge at the beginning of each stage. In an open-loop model players cannot observe the play of their opponents. They consider the limit of nonatomic games and show that "if the nonatomic game has unique open- and closed-loop equilibria, all open and closed equilibria approaching the limit must be near each other." Thus our result that the open- and closed-loop equilibria do not coincide even in the limit where the number of firms tends to infinity is in sharp contrast with the conclusion of Fudenberg and Levine (1988). In our paper we consider a dynamic game where the resource stocks are depleted over time. The history of the game is summarized in the information about the resource stocks and closed-loop strategies prescribe a production rate at each moment according to the level of stocks of resource. The contrast of our result with the conclusion of Fudenberg and Levine (1988) can be explained by the fact that (i) while we vary the number of firms that exploit the high cost mine and allow it to become arbitrarily large, the overall supply over time of all these firms is fixed and (ii) in our framework the time horizon is endogenous. In the limit case where the number of fringe firms tends to infinity, the dominant firm may reallocate the extraction of its own stock over time, slowing down its extraction in the early stages of the game and increasing its sales as the fringe's stock is depleted.

We present the model and the open-loop cartel-fringe equilibrium in the next section. In section 3, we treat the case of a closed-loop cartel fringe game. In section 4, we investigate the relationship between the cartel-fringe game and the oligopoly game.

2 Model and the open-loop cartel-fringe game

There are two types of mines c and f , distinguished by their marginal extraction costs. There is one c -type mine, owned by a firm called the cartel⁴, and there are n mines of the f -type. The owner of an f -mine is called a fringe member. Marginal extraction costs are constant: k^c and k^f . The cartel's initial stock is S_0^c . Fringe firm i ($i = 1, 2, \dots, n$) is endowed with an initial stock S_{0i}^f . Demand for the resource is stationary and linear with a choke price \bar{p} : $p(t) = \bar{p} - d(t)$, where $p(t)$ is the price at time t , $d(t)$ is the quantity demanded at time t and $\bar{p} > \max\{k^c, k^f\}$. We work in continuous time, which starts at time 0. Extraction rates at time $t \geq 0$ are denoted by $q^c(t) \geq 0$ and $q_i^f(t) \geq 0$. Define $q^f(t) = \sum_{i=1}^n q_i^f(t)$ and $S_0^f = \sum_{i=1}^n S_{0i}^f$ as aggregate supply and aggregate initial stocks of the fringe firms. In an equilibrium at each moment $t \geq 0$ the price of the resource is given by $p(t) = \bar{p} - q^c(t) - q^f(t)$. For the time being all fringe firms are assumed identical with regard to their stocks: $S_{0i}^f = S_0^f/n$. Any feasible extraction path for a firm is subject to the condition that its total extraction over time equals its initial stock. This is called the resource constraint. It reads

$$\int_0^\infty q^c(s)ds = S_0^c$$

for the cartel and

$$\int_0^\infty q_i^f(s)ds = S_{0i}^f$$

for fringe member i , respectively. The resource constraints are formulated as an equality because in any equilibrium all resource stocks will get exhausted in view of the assumption that $\bar{p} > \max\{k^c, k^f\}$. We also define $S(t) = (S^c(t), S^f(t)) = (S^c(t), S_1^f(t), S_2^f(t), \dots, S_n^f(t))$, the vector of resource stocks at time t .

The open-loop cartel-fringe game is specified in Salant (1976) and unfolds as follows. There is a coherent cartel and a number of fringe firms each possessing a stock of the nonrenewable resource. Each fringe firm takes the price path as given and chooses a path of extraction, whereas the cartel takes the extraction path of the fringe as given and determines a price path, and thereby its supply. Since fringe firms are price takers,

⁴The dominant firm terminology can also be found in the literature. The cartel-fringe terminology (that we use in this paper) and the dominant firm versus fringe terminology are interchangeable.

their actual number is not relevant, only their combined stock matters. All firms choose their respective strategies simultaneously. The outcome of this game is an equilibrium if market equilibrium holds at every moment. We denote the open-loop equilibrium of cartel-fringe game by OL-CFE.

Let S , C and F denote intervals of time with simultaneous supply, sole supply by the cartel and sole supply by the fringe. We have the following proposition.

Proposition 1

The OL-CFE is characterized as follows:

i. Suppose

$$\frac{1}{2}(\bar{p} + k^c) < k^f$$

Then the equilibrium sequence is $C \rightarrow S \rightarrow F$, with the F phase collapsing if S_0^c is 'small'.

ii. Suppose

$$\frac{1}{2}(\bar{p} + k^c) = k^f$$

Then the equilibrium sequence is $S \rightarrow F$.

iii. Suppose

$$\frac{1}{2}(\bar{p} + k^c) > k^f$$

Let $\sigma_{CFE} \equiv \frac{k^f - k^c}{\bar{p} + k^c - 2k^f}$. The OL-CFE depends on the initial stocks as displayed below.

Stocks	$S_0^c/S_0^f < \sigma_{CFE}$	$S_0^c/S_0^f = \sigma_{CFE}$	$S_0^c/S_0^f > \sigma_{CFE}$
OL-CFE	$S \rightarrow F$	S	$S \rightarrow C$

This proposition generalizes the OL-CFE derived in Salant (1976) to the case where extraction costs of the fringe and the cartel may be positive and different. Its proof is omitted here, but can be found in Benckroun et al. (2009).

3 The closed-loop cartel-fringe game

While the open-loop formulation of the cartel-fringe model is widely used and analyzed in the literature, there exists, to our knowledge, no analysis of a closed-loop formulation of the cartel-fringe game. This paper is a first attempt to specify a closed-loop formulation. The difficulty lies in reconciling the intrinsic *myopic* behavior of a fringe firm assumed

through price taking and the rather sophisticated (or farsighted) behavior assumed by the use of closed-loop strategies.

We propose the following scenario: each fringe firm takes the price path as given and determines its extraction strategy which is allowed to depend on its own stock only; the cartel takes the closed-loop *representation* of the fringe's production path as given and determines a pricing strategy (or alternatively a production strategy) that depends on its own stock and all fringe firms' stocks. The outcome of this simultaneous move is an equilibrium if the market of the resource is in equilibrium at each moment. We denote the closed-loop equilibrium of cartel-fringe game by CL-CFE. Formally

Definition: Closed-loop Cartel-Fringe Equilibrium (CL-CFE)

A vector $(\pi, \phi^c, \phi^f, \phi_1^f, \dots, \phi_n^f)$ with a price path $\pi = \pi(t)$ and closed-loop extraction rules $\phi^c = \phi^c(t, S(t))$, $\phi^f(t, S(t))$ and $\phi_i^f = \phi_i^f(t, S_i^f(t))$ ($i = 1, 2, \dots, n$) is a closed-loop Cartel-Fringe Equilibrium (CL-CFE) if

i. the resource constraint is satisfied for all firms, where $q^c(t) = \phi^c(t, S(t))$ and $q_i^f(t) = \phi_i^f(t, S_i^f(t))$ ($i = 1, 2, \dots, n$)

ii. given ϕ^f

$$\begin{aligned} & \int_0^\infty e^{-rs} [\text{Max} \{ \bar{p} - \phi^f(s, S(s)) - \phi^c(t, S(t)), 0 \} - k^c] \phi^c(t, S(t)) ds \\ & \geq \int_0^\infty e^{-rs} [\text{Max} \{ \bar{p} - \phi^f(s, S(s)) - \hat{\phi}^c(t, S(t)), 0 \} - k^c] \hat{\phi}^c(t, S(t)) ds \end{aligned}$$

for all feasible strategies $\hat{\phi}^c$.

iii. for all $i = 1, 2, \dots, n$, given π

$$\int_0^\infty e^{-rs} [\pi(s) - k^f] \phi_i^f(s, S_i^f(s)) ds \geq \int_0^\infty e^{-rs} [\pi(s) - k^f] \hat{\phi}_i^f(s, S_i^f(s)) ds$$

for all feasible strategies $\hat{\phi}_i^f$.

iv. $\phi^f(t, S(t)) = \sum_{i=1}^n \phi_i^f(t, S_i^f(t))$

v. for all $t \geq 0$: $\pi(t) = \text{Max} \{ \bar{p} - \phi^f(t, S^f(t)) - \phi^c(t, S(t)), 0 \}$

The function $\phi^f(t, S^f)$ corresponds to the aggregate extraction of the fringe written in a closed-loop form. It is not a strategy per se, it arises from the individual optimal

choice of each fringe firm of a production path, and gives the *behavior* of the fringe as a function of the vector of stocks. The cartel takes the fringe's behavior, $\phi^f(t, S^c, S^f)$, as given and determines its pricing (or production) strategy which is allowed to depend on its own stock and the fringe's stock. Condition v states that, for any $t \geq 0$, given a vector of stocks, the realization of $\bar{p} - \phi^f(t, S^f(t)) - \phi^c(t, S(t))$ yields the price $\pi(t)$ taken as given in the fringe's problem stated in iii.

The assumption about the fringe firms' behavior is important and is a modelling choice. One could employ alternative assumptions regarding the fringe firm's degree of sophistication. For instance a fringe firm could be allowed to consider the price rule as given but not the price path; in which case the fringe firm can still influence the price path through its influence on its own stock. This latter behavior of the fringe firm did not appeal to us because it assumes that a fringe firm, while determining its best response to a strategy of the cartel, is aware of the impact of its own stock on the market price but is not aware of the impact of its own quantity sold on the same market price. This implication appears rather contradictory and hence undesirable. Thus, and in keeping with the typically assumed *myopic* behavior of a fringe firm, we retain the assumption that each fringe firm takes the price path as given and that it may condition its extraction rate on its own stock only. The reason is that given a price path, the only payoff relevant information for a fringe firm is its own available stock.

We argue that with a price taking fringe, there exists a CL-CFE that yields the same outcome as the OL-CFE outcome, for any composition of the initial stocks. The proof consists of three steps. First we build a closed-loop representation of each fringe firm's production path under the open-loop cartel-fringe equilibrium (Lemma 1 below). Then we show that for the cartel, the closed-loop representation of its open-loop equilibrium price is a best response to the fringe firms' closed-loop strategy (built in the first step) (Lemma 3 below). We complete the proof by noting that for each fringe firm, the closed-loop representation of its open-loop equilibrium strategy (built in the first step), is a best response to the open-loop cartel-fringe (OL-CFE) price path.

We only present the details of the proof for the case where the sequence of the OL-CFE is $S \rightarrow C$, i.e., when $\frac{1}{2}(\bar{p} + k^c) > k^f$ and for S^c and S^f such that $S^c/S^f > \sigma_{CFE}$. A similar treatment and the same conclusion regarding the existence of a CL-CFE that yields the same outcome as the OL-CFE, hold when the OL-CFE sequence is $S \rightarrow F$.

To write closed-loop representations of the open-loop equilibrium paths it will be

useful to define the following function

$$h(z) = \ln\left(\frac{1}{z}\right) + z - 1.$$

with domain⁵ $(0, 1]$. It can easily be checked that the function h is strictly decreasing over $(0, 1]$ with $\lim_{z \rightarrow 0} h(z) = \infty$ and $\lim_{z \rightarrow 1} h(z) = 0$. Therefore, for any $A \geq 0$ there exists a unique solution in $(0, 1]$ to $h(z) = A$.

For any $S^f \geq 0$, let x be the unique solution in $(0, 1]$ to

$$h(x) = \frac{rS^f}{\bar{p} + k^c - 2k^f} \quad (1)$$

and for any $S^c, S^f \geq 0$, let y be the unique solution in $(0, 1]$ to

$$h(y) = \frac{2rS^c + rS^f}{\bar{p} - k^c}. \quad (2)$$

In the sequel we will omit the time argument when there is no danger of confusion.

Lemma 1

For any $(S^c, S^f) > 0$ such that the OL-CFE sequence is $S \rightarrow C$, the OL-CFE coincides with the outcome of the following closed-loop strategies:

$$\phi^f(S^f) = q^f(x) = (\bar{p} + k^c - 2k^f)(1 - x) \quad (3)$$

and

$$\phi^c(S^c, S^f) = q^c(x, y) = \frac{1}{2}(\bar{p} + k^c - 2k^f)(x - 1) - \frac{1}{2}(\bar{p} - k^c)(y - 1) \quad (4)$$

where x and y are respectively the unique solutions in $(0, 1]$ to (1) and (2).

Proof: see Appendix A.

Note that when $S^f = 0$ we have $x = 1$ and when $S^f = S^c = 0$ we have $y = 1$. Therefore, the closed-loop strategies given in (3) and (4) also represent the open-loop extraction paths during the last phase C , where the cartel is the sole supplier, with

$$\phi^f(0) = q^f(1) = 0$$

and

$$\phi^c(S^c, 0) = q^c(1, y) = \frac{1}{2}(\bar{p} - k^c)(1 - y).$$

⁵The reason why we focus on this domain is transparent in Lemma 1 and its proof, see e.g. (3).

We also remark that the strategies are feedback strategies (they do not depend on time explicitly); this is due to the fact that the problem of each firm is autonomous.

The closed-loop representation of the production paths allows to get the cartel's discounted sum of profits in a closed-loop form.

Lemma 2

For any $(S^c, S^f) > 0$ such that the OL-CFE sequence is $S \rightarrow C$, a closed-loop representation of the cartel's discounted sum of profits at an initial date t with stocks (S^f, S^c) is

$$\begin{aligned} \Pi^c(t, x, y) = & \frac{e^{-rt}}{4r} \{ 4(k^f - k^c)^2(1 - x) + 4(k^f - k^c)(\bar{p} + k^c - 2k^f)x \ln\left(\frac{1}{x}\right) \\ & + (\bar{p} + k^c - 2k^f)^2(x - x^2) + (\bar{p} - k^c)^2(y^2 + x - 2y) \} \end{aligned} \quad (5)$$

where x and y are respectively the unique solutions in $(0, 1]$ to (1) and (2).

Proof: see Appendix B.

We are now able to state the following.

Lemma 3

For any $(S^c, S^f) > 0$ such that the OL-CFE sequence is $S \rightarrow C$, the cartel's closed-loop strategy (4) (representation of the cartel's open-loop equilibrium extraction) is a best response to the fringe's closed-loop behaviour (3) (representation of the fringe's open-loop equilibrium extraction).

Proof: see Appendix C.

Given the price path of the OL-CFE and using the symmetry among the fringe firms, it is straightforward to show that the following strategy

$$\phi_i^f(S_i^f) = q_i^f(x) = \frac{1}{n} (\bar{p} + k^c - 2k^f) (1 - x_i) \quad (6)$$

where for any $S^f \geq 0$, x_i is the unique solution in $(0, 1]$ to

$$h(x_i) = \frac{nrS_i^f}{\bar{p} + k^c - 2k^f} \quad (7)$$

is a closed-loop representation of the best response of the fringe firm to the OL-CFE price path.

The resource market clearing condition is obviously satisfied since it is satisfied under the OL-CFE and the closed-loop strategies replicate the output path and therefore the price path of that equilibrium⁶.

Proposition 2

For any $(S^c, S^f) > 0$ such that the OL-CFE sequence is $S \rightarrow C$, there exists a CL-CFE that yields the same outcome as the OL-CFE.

Remark: The same treatment and result hold for the case where the OL-CFE's sequence is $S \rightarrow F$. Given the similarity (in approach and length) of the proof with the case presented in Proposition 2, it is omitted.

An interesting property of the OL-CFE is its relationship to an asymmetric oligopoly game where a finite number of fringe firms are not assumed to be price takers and are rather assumed to have market power. Salant (1976) shows that, in the limit case where the number of fringe firms tends to infinity, the outcome of the oligopoly game, where firms use open-loop production paths, converges to the outcome of the OL-CFE. In the rest of the paper we investigate the relationship between the CL-CFE and the equilibrium of an oligopoly game where firms use closed-loop strategies.

4 The cartel-fringe game and the oligopoly game

We start by a formal definition of an Open-loop Nash Equilibrium and report its relationship to the OL-CFE. We then give the definition of a Closed-loop Nash Equilibrium and analyze the relationship between the outcome of a closed-loop oligopoly game and a closed-loop cartel fringe game.

4.1 The open-loop Nash equilibrium

Definition: Open-loop Nash Equilibrium (OLNE)

A vector $q(\cdot) \equiv (q^c(\cdot), q_1^f(\cdot), \dots, q_n^f(\cdot))$ with $q(t) \geq 0$ for all $t \geq 0$ is an open-loop Nash equilibrium if

- i. all resource constraints are satisfied

⁶For $\phi^c(S^c, S^f)$ and $\phi^f(S^f)$ and given initial stocks, the realizations of $p(t; \phi^c(S^c, S^f), \phi^f(S^f))$ yields the OL-CFE price path.

ii.

$$\begin{aligned} & \int_0^{\infty} e^{-rs} [\text{Max} \{ \bar{p} - q^c(s) - q^f(s), 0 \} - k^c] q^c(s) ds \\ & \geq \int_0^{\infty} e^{-rs} [\text{Max} \{ \bar{p} - \hat{q}^c(s) - q^f(s), 0 \} - k^c] \hat{q}^c(s) ds \end{aligned}$$

for all feasible \hat{q}^c .

iii. for all $i = 1, 2, \dots, n$

$$\begin{aligned} & \int_0^{\infty} e^{-rs} [\text{Max} \{ \bar{p} - q^c(s) - q^f(s), 0 \} - k^f] q_i^f(s) ds \\ & \geq \int_0^{\infty} e^{-rs} [\text{Max} \{ \bar{p} - q^c(s) - \sum_{j \neq i} q_j^f(s) - \hat{q}_i^f(s), 0 \} - k^f] \hat{q}_i^f(s) ds \end{aligned}$$

for all feasible \hat{q}_i^f .

Benchekroun et al. (2009) characterize the OLNE of this game⁷ and have established the following proposition.

Proposition 3

i. *Suppose*

$$\frac{1}{2}(\bar{p} + k^c) < k^f$$

For a given S_0^f , there exists $\tilde{S}_0^c > 0$ such that the OLNE sequence reads $C \rightarrow S \rightarrow F$ if $S_0^c > \tilde{S}_0^c$ and $S \rightarrow F$ if $S_0^c \leq \tilde{S}_0^c$.

ii. *Suppose*

$$\frac{1}{2}(\bar{p} + k^c) = k^f$$

Then the OLNE yields $S \rightarrow F$

iii. *Suppose*

$$\frac{1}{2}(\bar{p} + k^c) > k^f$$

⁷They allow for an arbitrary number of firms that have the c -type mines. For our present purpose this is less relevant.

Let $\sigma \equiv \frac{\bar{p} + nk^f - (n+1)k^c}{n(\bar{p} + k^c - 2k^f)}$. The OLNE sequence depends on the initial stocks as displayed below

Stocks	$S_0^c/S_0^f < \sigma$	$S_0^c/S_0^f = \sigma$	$S_0^c/S_0^f > \sigma$
OLNE	$S \rightarrow F$	S	$S \rightarrow C$

We provide elements of the derivation of the OLNE in Appendix D.

4.2 The closed-loop Nash equilibrium

A closed-loop strategy for a firm is a decision rule that gives the extraction rate at t as a function of t and the vector of stocks at time t , $S(t) = (S^c(t), S_1^f(t), S_2^f(t), \dots, S_n^f(t))$. The definition reads as follows⁸.

Definition: Closed-loop Nash Equilibrium (CLNE)

A vector of closed-loop strategies $\phi \equiv (\phi^c, \phi_1^f, \dots, \phi_n^f) = (\phi^c, \phi^f)$ is a closed-loop Nash equilibrium if

- i. the resource constraint is satisfied for all firms, where $q^c(t) = \phi^c(t, S(t))$ and $q_i^f(t) = \phi_i^f(t, S(t))$ ($i = 1, 2, \dots, n$)
- ii.

$$\begin{aligned} & \int_0^\infty e^{-rs} [\text{Max}\{\bar{p} - \sum_{i=1}^n \phi_i^f(s, S(s)) - \phi^c(t, S(t)), 0\} - k^c] \phi^c(t, S(t)) ds \\ & \geq \int_0^\infty e^{-rs} [\text{Max}\{\bar{p} - \sum_{i=1}^n \phi_i^f(s, S(s)) - \hat{\phi}^c(t, S(t)), 0\} - k^c] \hat{\phi}^c(t, S(t)) ds \end{aligned}$$

for all feasible strategies $\hat{\phi}^c$.

- iii. for all $i = 1, 2, \dots, n$

$$\begin{aligned} & \int_0^\infty e^{-rs} [\text{Max}\{\bar{p} - \sum_{j=1}^n \phi_j^f(s, S(s)) - \phi^c(t, S(t)), 0\} - k^f] \phi_i^f(s, S(s)) ds \\ & \geq \int_0^\infty e^{-rs} [\text{Max}\{\bar{p} - \sum_{j \neq i}^n \phi_j^f(s, S(s)) - \hat{\phi}_i^f(s, S(s)) - \phi^c(t, S(t)), 0\} - k^f] \hat{\phi}_i^f(s, S(s)) ds \end{aligned}$$

⁸For both the OLNE and CLNE we give an ad-hoc definition for this resource game. For a more formal treatment we refer to Dockner et al. (2000) or Başar and Olsder (1995).

for all feasible strategies $\hat{\phi}_i^f$.

In this section we determine whether the OLNE can coincide with the CLNE.

The case $S \rightarrow F$

Proposition 1 provides conditions for the OLNE to contain the sequence $S \rightarrow F$. We seek to determine if there exists a CLNE, that is therefore subgame-perfect, that replicates the exploitation path of the OLNE, given a vector of initial stocks. Fix some $\tau \geq 0$. The cartel takes the strategy of the fringe $\phi^f(t, S)$ as given and chooses a closed-loop strategy $\phi^c(S, t)$ that maximizes its discounted sum of profits

$$\int_{\tau}^{\infty} e^{-rs} \left(\text{Max} \left\{ \bar{p} - q^c(s) - \sum_{i=1}^n \phi_i^f(s, S(s)), 0 \right\} - k^c \right) q^c(s) ds \quad (8)$$

subject to

$$\int_{\tau}^{\infty} q^c(s) ds \leq S^c(\tau) \quad (9)$$

and

$$\int_{\tau}^{\infty} \phi_i^f(s, S(s)) ds \leq S_i^f(\tau), \quad i = 1, 2, \dots, n \quad (10)$$

for all non-negative couples (S, t) , with $q^c(s) = \phi^c(s, S(s))$.

The Hamiltonian associated with the cartel's problem is given by

$$H^c(q^c, S, \mu_c^c, \mu_{fi}^c, t) = e^{-rt} \left(\text{Max} \left\{ \bar{p} - q^c - \sum_{i=1}^n \phi_i^f(t, S), 0 \right\} - k^c \right) q^c - \mu_c^c q^c - \sum_{i=1}^n \mu_{fi}^c \phi_i^f(t, S)$$

where μ_c^c is the costate variable associated with S^c and μ_{fi}^c is the costate variable associated with S_i^f . Applying the maximum principle and assuming we are in an S -phase gives the following set of necessary conditions for an interior solution at time t :

$$e^{-rt} \left(\bar{p} - 2q^c(t) - \sum_{i=1}^n \phi_i^f(t, S(t)) - k^c \right) - \mu_c^c(t) = 0 \quad (11)$$

$$\dot{\mu}_c^c(t) = -\frac{\partial H^c}{\partial S^c} = \sum_{i=1}^n (e^{-rt} q^c(t) + \mu_{fi}^c(t)) \frac{\partial \phi_i^f(t, S(t))}{\partial S^c} \quad (12)$$

$$\dot{\mu}_{fi}^c(t) = -\frac{\partial H^c}{\partial S_i^f} = \sum_{i=1}^n (e^{-rt} q^c(t) + \mu_{fi}^c(t)) \frac{\partial \phi_i^f(t, S(t))}{\partial S_i^f} \quad (13)$$

Appendix D provides a further characterization of the OLNE in this case, based on Benckroun et al. (2009). It is shown that along the phase of simultaneous supply, taken to be from time 0 till time t_1 , the production paths of the fringe and the cartel along the OLNE are given by

$$(n+2)q^c(t) = \bar{p} + n(k^f + \lambda^f e^{rt}) - (n+1)(k^c + \lambda^c e^{rt}) \quad (14)$$

$$\frac{2+n}{n}q^f(t) = \bar{p} + (k^c + \lambda^c e^{rt}) - 2(k^f + \lambda^f e^{rt}) \quad (15)$$

where λ^c and λ^f are the constant own shadow prices of the resource stocks of the cartel and the fringe members respectively. Hence, in view of (11) and (14), for a CLNE to result in the extraction path of the OLNE, we must have $\mu_c^c(s) = \lambda^c$, for all instants $s \geq t$ for all $t \geq 0$. From necessary condition (5) it follows that then

$$\sum_{i=1}^n (e^{-rt}q^c(t) + \mu_{fi}^c(t)) \frac{\partial \phi_i^f(t, S(t))}{\partial S^c} = 0$$

Given the symmetry of fringe firms we must have either $e^{-rt}q^c(t) + \mu_{fi}^c(t) = 0$ where q^c is the OLNE path of the cartel, and therefore $\mu_{fi}^c(t) = -e^{-rt}q^c(t)$, or $\partial \phi_i^f(S(t), t) / \partial S^c = 0$. The first possibility is in contradiction with the necessary conditions since it implies from (6) that $\dot{\mu}_{fi}^c(t) = 0$, but $e^{-rs}q^c(t)$ is not constant along the OLNE. Thus

$$\frac{\partial \phi_i^f(t, S(t))}{\partial S^c} = 0 \text{ for all } t \geq 0, \text{ for every fringe firm } i.$$

Given the symmetry of fringe firms we have

$$\frac{\partial \phi_i^f(t, S(t))}{\partial S^c} = \frac{1}{n} \frac{\partial \phi^f(t, S(t))}{\partial S^c} = \frac{1}{n} \frac{\partial q^f(t)}{\partial S^c}$$

which gives from (8)

$$\frac{2+n}{n} \frac{\partial \phi^f(t, S(t))}{\partial S^c} = \frac{\partial (\lambda^c - 2\lambda^f) e^{rt}}{\partial S^c}$$

Note that $\partial \phi^f(t, S(t)) / \partial S^c$ represents the impact of changing the cartel's stock at time t on the extraction of the fringe at time t . However, this change is anticipated at time 0.

Along the OLNE we also have

$$\lambda^f = e^{-rT}(\bar{p} - k^f) \text{ and } \lambda^c = \frac{n}{n+1}\lambda^f + \frac{e^{-rt_1}(\bar{p} + nk^f - (n+1)k^c)}{n+1}$$

As explained in appendix D the first of these equations states that the market price at the instant T of exhaustion of the resource equals the choke price; the second equation follows from the requirement that the price path is continuous. The two equations yield

$$\lambda^c - 2\lambda^f = \left(\frac{n}{n+1} - 2 \right) e^{-rT} (\bar{p} - k^f) + \frac{1}{n+1} e^{-rt_1} (\bar{p} + nk^f - (n+1)k^c) \quad (16)$$

The time of transition t_1 and the final time T satisfy (see appendix D):

$$(2+n)rS_0^c = (\bar{p} + nk^f - (n+1)k^c) (rt_1 - 1 + e^{-rt_1}) \quad (17)$$

$$r \left(S_0^f + \frac{n}{n+1} S_0^c \right) = \frac{n}{n+1} (\bar{p} - k^f) (rT - 1 + e^{-rT}) \quad (18)$$

At any time $t \leq t_1$ the time of transition t_1 and the final time T satisfy:

$$(2+n)rS^c(t) = (\bar{p} + nk^f - (n+1)k^c) (r(t_1 - t) - 1 + e^{-r(t_1-t)}) \quad (19)$$

$$r \left(S^f(t) + \frac{n}{n+1} S^c(t) \right) = \frac{n}{n+1} (\bar{p} - k^f) (r(T - t) - 1 + e^{-r(T-t)}) \quad (20)$$

If the OLNE is followed from time 0 to time t , that is if $S^c(t)$ and $S^f(t)$ correspond to the stocks at time t under the OLNE, then the solution (t_1, T) should also solve the system above (this is from the time consistency of the OLNE).

From (16) we have

$$\frac{\partial (\lambda^c - 2\lambda^f)}{\partial S^c} = -r \left(\frac{n}{n+1} - 2 \right) e^{-rT} \frac{\partial T}{\partial S^c} (\bar{p} - k^f) - r e^{-rt_1} \frac{\partial t_1}{\partial S^c} \frac{(\bar{p} + nk^f - (n+1)k^c)}{n+1} \quad (21)$$

We derive $\partial T / \partial S^c$ and $\partial t_1 / \partial S^c$ from (19) and (20)

$$(2+n)r = (1 - e^{-r(t_1-t)}) r (\bar{p} + nk^f - (n+1)k^c) \frac{\partial t_1}{\partial S^c} \quad (22)$$

$$r = (1 - e^{-r(T-t)}) r (\bar{p} - k^f) \frac{\partial T}{\partial S^c} \quad (23)$$

Now we substitute them into (21) to obtain

$$\frac{\partial (\lambda^c - 2\lambda^f) e^{rt}}{\partial S^c(t)} = r \left(\frac{n+2}{n+1} \right) \left(\frac{1}{e^{r(T-t)} - 1} - \frac{1}{e^{r(t_1-t)} - 1} \right) \neq 0$$

since $T > t_1$. Hence, for any equilibrium that reads $C \rightarrow S \rightarrow F$ or $S \rightarrow F$ a necessary condition for the CLNE to yield the OLNE extraction path is not met. Note that the result holds true even in the limit case where $n = \infty$ since for $n \rightarrow \infty$ we have

$$\frac{\partial (\lambda^c - 2\lambda^f)}{\partial S^c} = r \left(\frac{1}{e^{r(T-t)} - 1} - \frac{1}{e^{r(t_1-t)} - 1} \right) \neq 0 \quad (24)$$

The argument also goes through for any cost constellation that yields this equilibrium sequence. We have thus shown the following.

Proposition 4a

Suppose the OLNE yields the sequence $S \rightarrow F$. Then the OLNE extraction path cannot be obtained as the extraction path of a CLNE. This is true even when $n \rightarrow \infty$.

Remark: The fact that the time horizon is endogenous and differs between firms when their respective stock endowments differ is essential in establishing (24) and proving Proposition 4a. This contrasts with Fudenberg and Levine (1988) which shows, in the case of a repeated game where the time horizon is fixed and where firms' supply is unconstrained, that all open and closed equilibria approaching the limit must be near each other when the nonatomic game has unique open- and closed-loop equilibria.

For a vector of strategies to qualify as a non-degenerate CLNE it must specify extraction rates for all possible values of the initial stocks. Since there always exists a range of initial stocks such that the OLNE yields the sequence $S \rightarrow F$ we conclude from proposition 2 that there exists no CLNE that will replicate the OLNE for *all* values of the vector of stocks. A less restrictive condition is to require that the OLNE outcome be replicated by a non-degenerate CLNE *only for a subset* (of positive measure) *of initial stocks*. Proposition 4a does not rule out the possibility that for initial values of the vector of stocks such that the OLNE sequence is $S \rightarrow C$, an OLNE outcome may be replicated by a non-degenerate CLNE. We therefore turn to

The case $S \rightarrow C$

We know from Proposition 1 that if $n = 1$ and $k^f < \frac{1}{2}(\bar{p} + k^f)$ the equilibrium reads $S \rightarrow C$ if the initial resource stock of the fringe is not too large. We seek to determine whether there exists a feedback Nash equilibrium, that is therefore subgame-perfect, that replicates such an OLNE, given a vector of initial stocks. Along the phase

of simultaneous supply equations (7) and (8) hold, where, in the case at hand

$$\lambda^c = e^{-rT}(\bar{p} - k^c) \text{ and } \frac{1}{2}(\bar{p} + k^c + e^{rt_1}\lambda^c) = k^f + e^{rt_1}\lambda^f$$

and where the transition date t_1 and the exhaustion date T are respectively given by

$$\frac{2+n}{n}rS_0^f = (\bar{p} + k^c - 2k^f)(rt_1 - 1 + e^{-rt_1})$$

and

$$r\left(S_0^f + \frac{1}{2}S_0^f\right) = (\bar{p} - k^c)(rT - 1 + e^{-rT}).$$

It readily follows that $q^f(t)$ is independent of S^c . Contrary to the previous case we will henceforth concentrate on the fringe. The problem is that we cannot repeat the steps taken in the previous case, since we have to be clear about what to mean by a marginal change in the stock of one of the fringe members, keeping the other stocks fixed. This poses a difficulty because it has been assumed that all fringe members are equal, and the OLNE has been derived under that assumption. However, it is not difficult to conceptualize what will happen if one fringe member is given an addition to its reserve. All other fringe members will exhaust their resource before this fringe member under consideration does, as is formally demonstrated in Appendix E. Hence it is left with the cartel as sole competitor. We are therefore done if we can show that the OLNE and the CLNE do not coincide for the case of a single cartel and a single fringe member. Due to symmetry this is straightforward since we can repeat the steps taken in the previous case, *ceteris paribus*, and obtain the same negative result. For the sake of completeness the proof is given in detail in appendix F.

Proposition 4b

Suppose the OLNE yields the sequence $S \rightarrow C$, i.e.,

$$\frac{1}{2}(\bar{p} + k^c) > k^f \text{ and } \frac{S_0^c}{S_0^f} > \sigma$$

Then the OLNE extraction path cannot be the outcome of a CLNE extraction path. This is true even when $n \rightarrow \infty$.

Proposition 2 along with Proposition 4a and 4b allow us to draw an important conclusion regarding the microfoundation of the cartel-fringe model.

Proposition 5

The CL-CFE does not coincide with the outcome of the limit case of the asymmetric oligopoly CLNE where the number of fringe firms tends to infinity.

This is in sharp contrast with Salant (1976) where price taking behaviour of the fringe is justified as the limit case of an asymmetric oligopoly where the number of fringe firms is arbitrarily large⁹. The difference is due to the presence of the additional level of interaction in the game with closed-loop strategies. In the case of a closed-loop Nash game, when deriving its best response to the competitors' strategies, each firm (large and small) can still impact the extraction rates of its competitors (even though it takes their strategies as given). This additional layer of interaction in a CLNE makes the OLNE and the CLNE differ and does not vanish as the market power of fringe firms goes to zero. When firms can use closed-loop strategies, the outcome of the game where the fringe is assumed from the outset to be price taker is not useful to predict the outcome of the limit case where the market power of the fringe firms becomes arbitrarily small.

5 Conclusions

We have considered the exploitation of nonrenewable resources under imperfect competition with asymmetric firms. For the cartel-fringe model, we specified and solved a closed-loop game, with a price taking fringe. We have shown that the outcomes of the closed-loop and the open-loop cartel-fringe game (à la Salant 1976) coincide. However, we have also shown that, in contrast with Salant (1976) the interpretation of the cartel-fringe model, where the fringe is assumed from the outset to be price taker, as a limit case of an asymmetric oligopoly where the number of fringe firms tends to infinity, does not extend to the case where firms can use closed-loop strategies. Indeed, when the number of fringe firms becomes arbitrarily large, the equilibrium outcome of the closed-loop Nash game does not coincide with the equilibrium outcome of the closed-loop cartel-fringe game. In our model we focused on the polar cases of open-loop strategies where firms can commit to a production for the whole time horizon and closed-loop strategies

⁹The micro-foundation of price taking behavior as yielding an outcome that corresponds to the limit of equilibria of nonatomic games when the number of firms tends to infinity is quite well established in static games (for more details see e.g., Roberts (1980), Novshek and Sonnenschein (1980), Mas-Colell (1982) and Mas-Colell et al. (1995) Chapters 12 and 16).

where firms cannot commit to a production plan for any period of time. However we expect our conclusions to hold even in the intermediate case where the length of the period of commitment to a production plan is positive and finite à la Reinganum and Stokey (1985). In particular, we expect that the equilibrium outcomes of two dynamic games that differ only with respect to the length of the period of commitment to be in general distinct even in the limit case where the number of firms becomes arbitrarily large. While noncooperative dynamic game theory has been quite successfully in different areas of economic theory (besides environmental and resource economics, e.g., research and development, investment and capacity building, advertising, growth under imperfect property rights¹⁰) the choice of the length of the period of commitment is seldom discussed. It is generally chosen by the modeller at the outset and is sometimes made with the objective of facilitating the tractability of the model at hand. This choice can have important implications on the outcome of the game even in limit case where the number of players tends to infinity.

The positive message of our analysis is that in a dynamic cartel-fringe game, if the assumption of price taking behavior can be assumed at the outset, for example on the ground of realism, the outcome of the open-loop cartel-fringe equilibrium can be supported as the outcome of a subgame perfect equilibrium. This robustness is an important feature of the equilibrium of the open-loop cartel fringe model which may render this model more attractive to contribute to important questions currently being debated. For example, two related issues have recently received a lot of attention; namely, the order of depletion of different nonrenewable resources that differ by their polluting content (Chakravorty, Moreaux and Tidball (2008)) and the Green Paradox (Sinn (2008), Van der Ploeg and Withagen (2010), Gerlagh (2010), Hoel (2008), Grafton et al. (2010)). Basically the Green Paradox stresses that the neglect of the supply side of conventional fossil fuels may give rise to counterproductive policy recommendations. For example, subsidizing backstop technologies may lead to faster exhaustion of fossil fuels, rather than delaying the time of exhaustion. As for the order of use of different resources that differ in their pollution content, Chakravorty et al. (2008) show that the ordering of extraction need not be driven by whether a resource is clean or dirty and that pollution regulation may have the perverse effect of accelerating the use of the polluting resource.

¹⁰See for example Dockner et al. (2000).

The Green Paradox has thus far only been investigated for the polar cases of perfect competition and monopoly whereas Chakravorty et al. (2008) consider the problem of a social planner. Examining these questions in the more realistic case where the market structure of some resources (e.g., oil) is of the cartel-fringe type is definitely a relevant and promising line of future research where the methodology used in this paper can be instrumental.

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Appendix A

In this appendix we characterize the OL-CFE in the case where it reads $S \rightarrow C$ and provide a closed-loop representation of the cartel and the fringe's production paths and price path.

When the OL-CFE reads $S \rightarrow C$, we have:

For any $t \in [0, t_1]$

$$p(t) = k^f + \lambda^f e^{rt} \quad (25)$$

$$q^f(t) = (\bar{p} + k^c - 2k^f) - (2\lambda^f - \lambda^c) e^{rt} \quad (26)$$

$$q^c(t) = (k^f - k^c) + (\lambda^f - \lambda^c) e^{rt} \quad (27)$$

and for any $t \in [t_1, T]$

$$q^f(t) = 0$$

$$q^c(t) = \frac{1}{2} (\bar{p} - k^c - \lambda^c e^{rt}) \quad (28)$$

$$p(t) = \frac{1}{2} (\bar{p} + k^c + \lambda^c e^{rt}) \quad (29)$$

where the transition time t_1 and the terminal time T are given by

$$e^{rt_1} = \frac{\bar{p} + k^c - 2k^f}{2\lambda^f - \lambda^c} \quad (30)$$

$$e^{rT} = \frac{\bar{p} - k^c}{\lambda^c}. \quad (31)$$

and where the constants λ^c and λ^f are determined using the resource constraints

$$(\bar{p} + k^c - 2k^f) (t_1 - t) - (2\lambda^f - \lambda^c) \frac{(e^{rt_1} - e^{rt})}{r} = S^f(t) \quad (32)$$

$$(k^f - k^c) (t_1 - t) + (\lambda^f - \lambda^c) \frac{(e^{rt_1} - e^{rt})}{r} + \frac{1}{2} (\bar{p} - k^c) (T - t_1) - \frac{1}{2} \lambda^c \frac{(e^{rT} - e^{rt_1})}{r} = S^c(t) \quad (33)$$

The details of these derivations are direct generalizations of the OL-CFE derived in Salant (1976) to the case of positive and different extraction costs.

We now give a closed-loop representation of the OL-CFE in the case where it reads $S \rightarrow C$. Let

$$x \equiv \frac{(2\lambda^f - \lambda^c) e^{rt}}{\bar{p} + k^c - 2k^f} = e^{r(t-t_1)} \text{ and } y \equiv \frac{\lambda^c e^{rt}}{\bar{p} - k^c} = e^{r(t-T)} \quad (34)$$

We show that x and y can be determined as the unique solutions to respectively (1) and (2). Substitution of t_1 from (30) into (32) yields after algebraic manipulations

$$(\bar{p} + k^c - 2k^f) \left(\frac{1}{r} \ln \left(\frac{\bar{p} + k^c - 2k^f}{2\lambda^f - \lambda^c} \right) - t \right) - (2\lambda^f - \lambda^c) \frac{\left(\frac{\bar{p} + k^c - 2k^f}{2\lambda^f - \lambda^c} - e^{rt} \right)}{r} = S^f(t) \quad (35)$$

which can be simplified to

$$\ln \left(\frac{\bar{p} + k^c - 2k^f}{2\lambda^f - \lambda^c} e^{-rt} \right) - 1 + \frac{(2\lambda^f - \lambda^c)}{(\bar{p} + k^c - 2k^f)} e^{rt} = \frac{rS^f(t)}{(\bar{p} + k^c - 2k^f)} \quad (36)$$

or

$$\ln \left(\frac{1}{x} \right) + x = \frac{rS^f(t)}{\bar{p} + k^c - 2k^f} + 1 \quad (37)$$

Combining (32) and (33) gives after simplification

$$(\bar{p} - k^c) (T - t) - \lambda^c \frac{(e^{rT} - e^{rt})}{r} = 2S^c(t) + S^f(t)$$

Substituting T from (31) gives after manipulations

$$\ln \left(\frac{1}{y} \right) + y = r \frac{2S^c(t) + S^f(t)}{\bar{p} - k^c} + 1 \quad (38)$$

Thus x and y depend on S^f and (S^f, S^c) respectively and combined with (27) and (26) along with (34) give a closed loop representation of the open-loop paths (4) and (3).

For any $t \in [t_1, T]$ we have $S^f = 0$ and $x = 1$. It can easily be checked that substituting $x = 1$ into (3) and (4) yields the extraction path of the cartel when it is a sole supplier ($q^f = 0$ and (28) hold) ■

Appendix B

After substitution of (26) and (27) the cartel's profits are given by

$$\begin{aligned}\Pi^c &= \int_t^{t_1} e^{-rs} (k^f - k^c + \lambda^f e^{rs}) (k^f - k^c + (\lambda^f - \lambda^c) e^{rs}) ds \\ &\quad + \int_{t_1}^T e^{-rs} \frac{1}{2} (\bar{p} - k^c - \lambda^c e^{rs}) \frac{1}{2} (\bar{p} - k^c + \lambda^c e^{rs}) ds\end{aligned}$$

or

$$\begin{aligned}r\Pi^c &= (k^f - k^c)^2 (e^{-rt} - e^{-rt_1}) + (k^f - k^c) (2\lambda^f - \lambda^c) (rt_1 - rt) + \lambda^f (\lambda^f - \lambda^c) (e^{rt_1} - e^{rt}) \\ &\quad + \frac{1}{4} (\bar{p} - k^c)^2 (e^{-rt_1} - e^{-rT}) - \frac{1}{4} (\lambda^c)^2 (e^{rT} - e^{rt_1})\end{aligned}\quad (39)$$

We first write λ^f and λ^c as functions of x and y defined in (34) and get

$$\lambda^f e^{rt} = \frac{(\bar{p} + k^c - 2k^f)x + (\bar{p} - k^c)y}{2}$$

and

$$\lambda^c e^{rt} = (\bar{p} - k^c)y$$

We then determine t_1 and T as functions of x and y using (30), (31). Substituting λ^f , λ^c , t_1 and T as functions of x and y into (39) gives after algebraic manipulations (5) ■

Appendix C

To prove this claim we show that (5) satisfies the Hamilton Jacobi Bellman (HJB) equation of the cartel's problem. Since $x = x(S^f)$, from (31), and $y = y(S^f, S^c)$, from (32), we define

$$V^c(t, S^f, S^c) = \Pi^c(t, x, y)$$

We check now that V satisfies the HJB equation for all (S^f, S^c) (such that the equilibrium sequence is $S \rightarrow C$)

$$-\frac{\partial V^c}{\partial t} = \frac{\partial V^c}{\partial S^f} (-\phi^f) + \text{Max}_{q^c} \left\{ (\bar{p} - k^c - \phi^f - q^c) q^c e^{-rt} + \frac{\partial V^c}{\partial S^c} (-q^c) \right\} \quad (40)$$

with q^f given by (3). This is done in two steps: (i) we first check that q^c given by (4) solves the maximization problem; (ii) we show that when ϕ^f is given by (3) and q^c is given by (4) the function $V^c(t, S^f, S^c) = \Pi^c(t, x, y)$ satisfies the cartel's HJB equation.

(i) The first order condition associated with the maximization problem gives

$$(\bar{p} - k^c - \phi^f - 2q^c) e^{-rt} - \frac{\partial V^c}{\partial S^c} = 0$$

or

$$q^c = \frac{1}{2} \left(\bar{p} - k^c - \phi^f - \frac{\partial V^c}{\partial S^c} e^{rt} \right) \quad (41)$$

We now compute the derivative

$$\frac{\partial V^c}{\partial S^c} = \frac{\partial \Pi^c}{\partial x} \frac{\partial x}{\partial S^c} + \frac{\partial \Pi^c}{\partial y} \frac{\partial y}{\partial S^c}$$

We have $\frac{\partial x}{\partial S^c} = 0$ and $\frac{\partial y}{\partial S^c} = \frac{2r}{\bar{p} - k^c} \frac{y}{y-1}$ and

$$\frac{\partial \Pi^c}{\partial y} = \frac{e^{-rt}}{4r} (2(\bar{p} - k^c)^2 y - 2(\bar{p} - k^c)^2)$$

Hence

$$\begin{aligned} \frac{\partial V^c}{\partial S^c} &= \frac{e^{-rt}}{4r} 2(\bar{p} - k^c)^2 (y-1) \frac{2r}{\bar{p} - k^c} \frac{y}{y-1} \\ \frac{\partial V^c}{\partial S^c} &= e^{-rt} (\bar{p} - k^c) y = \lambda^c \end{aligned} \quad (42)$$

Substitution of $\frac{\partial V^c}{\partial S^c} e^{rt}$ and of ϕ^f from 3 gives

$$q^c = \frac{1}{2} (\bar{p} - k^c - (\bar{p} + k^c - 2k^f) (1-x) - (\bar{p} - k^c) y) \quad (43)$$

which (after simplification) is identical to (4).

(ii) We have

$$\frac{\partial V^c}{\partial t} = \frac{\partial \Pi^c}{\partial t} = -r\Pi^c$$

and

$$\frac{\partial V^c}{\partial S^c} = e^{-rt} (\bar{p} - k^c) y = \lambda^c \quad (44)$$

We now turn to $\frac{\partial V^c}{\partial S^f}$. We have

$$\frac{\partial V^c}{\partial S^f} = \frac{\partial \Pi^c}{\partial x} \frac{\partial x}{\partial S^f} + \frac{\partial \Pi^c}{\partial y} \frac{\partial y}{\partial S^f}$$

with

$$\frac{\partial x}{\partial S^f} = \frac{r}{\bar{p} + k^c - 2k^f} \frac{x}{x-1} \quad \text{and} \quad \frac{\partial y}{\partial S^f} = \frac{r}{\bar{p} - k^c} \frac{y}{y-1}$$

After simplification we have

$$\frac{\partial \Pi^c}{\partial x} = \frac{e^{-rt}}{4r} \left[4(k^f - k^c) (\bar{p} + k^c - 2k^f) \ln \left(\frac{1}{x} \right) + 2(\bar{p} + k^c - 2k^f)^2 (1-x) \right]$$

and thus

$$\frac{\partial \Pi^c}{\partial x} \frac{\partial x}{\partial S^f} = \frac{e^{-rt}}{4} \left[4 (k^f - k^c) \frac{x}{x-1} \ln \left(\frac{1}{x} \right) - 2 (\bar{p} + k^c - 2k^f) x \right]$$

We also have

$$\frac{\partial \Pi^c}{\partial y} = \frac{e^{-rt}}{4r} 2 (\bar{p} - k^c)^2 (y - 1)$$

and thus

$$\frac{\partial \Pi^c}{\partial y} \frac{\partial y}{\partial S^f} = \frac{e^{-rt}}{4} 2 (\bar{p} - k^c) y$$

We can now obtain $\frac{\partial V^c}{\partial S^f}$ as the sum of $\frac{\partial \Pi^c}{\partial y} \frac{\partial y}{\partial S^f}$ and $\frac{\partial \Pi^c}{\partial x} \frac{\partial x}{\partial S^f}$ which gives

$$\frac{\partial V^c}{\partial S^f} = (k^f - k^c) \frac{x}{x-1} e^{-rt} \ln \left(\frac{1}{x} \right) - \frac{1}{2} (\bar{p} + k^c - 2k^f) x e^{-rt} + \frac{1}{2} (\bar{p} - k^c) y e^{-rt}$$

The last step consists of checking that when substituting each term $\frac{\partial V^c}{\partial t}$, $\frac{\partial V^c}{\partial S^f}$, $\frac{\partial V^c}{\partial S^c} q^f$ and q^c into the HJB the equality holds for all $S^f, S^c \geq 0$. This step is skipped. It involves lengthy but straightforward algebraic simplifications only. More specifically it can be shown that each side of the HJB equation

$$-\frac{\partial V^c}{\partial t} = -q^f \frac{\partial V^c}{\partial S^f} - q^c \frac{\partial V^c}{\partial S^c} + (\bar{p} - k^c - q^f - q^c) q^c e^{-rt} \quad (45)$$

reduces to

$$\begin{aligned} & -x \left(\ln \frac{1}{x} \right) (k^c - k^f) (p + k^c - 2k^f) \\ & - \frac{1}{4} (p + k^c - 2k^f)^2 x^2 + \frac{1}{2} (p - k^c) (p + k^c - 2k^f) x + \\ & \frac{1}{4} (p - k^c)^2 y^2 - \frac{1}{2} (p - k^c)^2 y + (k^c - k^f)^2 \blacksquare \end{aligned}$$

Appendix D

Here we summarize the findings on the open-loop Nash equilibrium. There is one cartel and there are n fringe members. Each fringe firm i takes the strategy profile of its n competitors as given and maximizes its present value profits subject to the resource constraint. The corresponding Hamiltonian reads

$$H_i^f(q_i^f, q^c, q^f, \lambda_i^f, t) = e^{-rt} (\bar{p} - q^c - q^f - k^f) q_i^f + \lambda_i^f (-q_i^f)$$

where q^f and q^c denote the aggregate supply by the fringe and the supply by the cartel, respectively. For the cartel the Hamiltonian reads

$$H^c(q^c, \lambda^c, q^f, t) = e^{-rt} (\bar{p} - q^c - q^f - k^f) q^c + \lambda^c (-q^c)$$

Among the necessary conditions we have that the co-state variables are constant since stocks are absent from the Hamiltonians. In addition, the Hamiltonians are maximized with respect to the own supply of the agent. We use the symmetry among the fringe players, i.e. $q_i^f = q^f/n$ and $\lambda_i^f = \lambda^f$ for all i . Then we arrive at the following necessary conditions.

-along an F interval:

$$e^{-rt} (\bar{p} - q^f(t) - \frac{1}{n} q^f(t) - k^f) = \lambda^f$$

$$p(t) = \frac{1}{n+1} (\bar{p} + n (k^f + \lambda^f e^{rt})) \leq k^c + e^{rt} \lambda^c.$$

The first condition follows from the maximization of the Hamiltonian of player i . The second condition is necessary in order for the cartel not to supply.

-along a C interval:

$$e^{-rt} (\bar{p} - 2q^c(t) - k^c) = \lambda^c$$

$$p(t) = \frac{1}{2} (\bar{p} + k^c + \lambda^c e^{rt}) \leq k^f + e^{rt} \lambda^f$$

-along an S interval

$$(2+n)q^c(t) = \bar{p} + n (k^f + \lambda^f e^{rt}) - (n+1) (k^c + \lambda^c e^{rt})$$

$$\frac{n+2}{n} q^f(t) = \bar{p} + k^c + \lambda^c e^{rt} - 2 (k^f + \lambda^f e^{rt})$$

$$p(t) = \frac{1}{2+n} (\bar{p} + k^c + \lambda^c e^{rt} + n(k^f + \lambda^f e^{rt})).$$

Continuity of the price path at the different possible transitions gives:

- a transition at t from S to C or vice versa requires

$$\frac{1}{2}(\bar{p} + k^c + \lambda^c e^{rt}) = k^f + \lambda^f e^{rt}$$

- a transition at t from S to F or vice versa requires

$$\frac{1}{n+1}(\bar{p} + n(k^f + \lambda^f e^{rt})) = k^c + \lambda^c e^{rt}$$

- a transition at t from F to C or vice versa requires

$$\frac{1}{2}(\bar{p} + k^c + \lambda^c e^{rt}) = \frac{1}{n+1}(\bar{p} + n(k^f + \lambda^f e^{rt}))$$

We also have to take into account that at the moment of exhaustion T of all resource stocks, the price must have reached the choke level:

$$p(T) = \bar{p}$$

Consider the sequence $S \rightarrow C$, with C the final phase before exhaustion and where the transition takes place at instant of time t_1 and exhaustion at T . Then it is tedious but straightforward to derive (see Benchevkroun et al. (2009))

$$(2+n)rS_0^f = n(\bar{p} + k^c - 2k^f)(rt_1 - 1 + e^{-rt_1})$$

$$\begin{aligned} (2+n)rS_0^c &= -\frac{1}{2}n(\bar{p} + k^c - 2k^f)(rt_1 - 1 + e^{-rt_1}) + \\ &\quad (1 + \frac{1}{2}n)(\bar{p} - k^c)(rT - 1 + e^{-rT}) \end{aligned}$$

For the sequence $S \rightarrow F$ we have

$$\begin{aligned} (2+n)rS_0^f &= -\frac{n}{n+1}(\bar{p} + nk^f - (n^f + 1)k^c)(rt_1 - 1 + e^{-rt_1}) + \\ &\quad \frac{n(2+n)}{n+1}(\bar{p} - k^f)(rT - 1 + e^{-rT}) \\ (2+n)rS_0^c &= (\bar{p} + nk^f - (n+1)k^c)(rt_1 - 1 + e^{-rt_1}) \end{aligned}$$

Appendix E

In this appendix we modify the problem discussed in appendix D so as to allow for an additional fringe member with a larger stock than all other n fringe members. We will show that the stocks of all other fringe members will be depleted before the stock of this particular fringe member is. The variables referring to the larger fringe member are denoted by upper bars. Among the necessary conditions for an OLNE we have

$$\begin{aligned} e^{-rt} (p(t) - \bar{q}^f(t) - k^f) &\leq \bar{\lambda}^f \\ e^{-rt} (p(t) - \frac{1}{n} q^f(t) - k^f) &\leq \lambda^f \\ e^{-rt} (p(t) - q^c(t) - k^c) &\leq \lambda^c \end{aligned}$$

with equality holding if $\bar{q}^f(t), q^f(t)$ (aggregate supply of all other fringe members) and $q^c(t)$ are positive, respectively. Since the fringe members only differ with respect to the stocks, the shadow price of the larger stock is smaller than the shadow price of each smaller stock: $\bar{\lambda}^f < \lambda^f$. Let \bar{T} denote the time of exhaustion of stock of the larger fringe member. Then $p(t) \leq e^{rt} \bar{\lambda}^f + k^f$ for all $t \geq \bar{T}$. If $q^f(t) > 0$ for some $t \geq \bar{T}$ then $p(t) = e^{rt} \lambda^f + k^f + \frac{1}{n} q^f(t) > e^{rt} \bar{\lambda}^f + k^f \geq p(t)$, a contradiction.

Appendix F

Here we prove that the case $S \rightarrow C$ cannot be sustained as a closed-loop equilibrium. As was made clear in the main text as well as in Appendix E, we only have to consider the case of a single fringe member. Fix some $\tau \geq 0$. The cartel takes the closed-loop strategy of the fringe $\phi^f(S, t)$ as given and chooses a closed-loop strategy $\phi^c(S, t)$ that maximizes its discounted sum of profits

$$\int_{\tau}^{\infty} e^{-rs} (\bar{p} - q^c(s) - \phi^f(S(s), s) - k^c) q^c(s) ds$$

subject to

$$\int_{\tau}^{\infty} q^c(s) ds \leq S^c$$

and

$$\int_t^{\infty} \phi^f(S(s), s) ds \leq S^f$$

for all non-negative couples (S, t) , with $q^c(s) = \phi^c(S(s), s)$. The Hamiltonian for the cartel reads

$$H^c(q^c, S, \mu_c^c, \mu_f^c, t) = e^{-rt} (\bar{p} - q^c - \phi^f(S(t), t) - k^c) q^c - \mu_c^c q^c - \mu_f^c \phi^f(S, t)$$

where μ_c^c is the costate variable associated with S^c and μ_f^c is the costate variable associated with S^f . Applying the Maximum Principle gives the following set of necessary conditions for an interior solution at t (i.e. $q^f(t) > 0$ and $q^c(t) > 0$):

$$e^{-rt} (\bar{p} - 2q^c(t) - \phi^f(S(t), t) - k^c) - \mu_c^c(t) = 0$$

$$\dot{\mu}_c^c(t) = -\frac{\partial H^c}{\partial S^c} = (e^{-rt} q^c(t) + \mu_f^c(t)) \frac{\partial \phi^f(S(t), t)}{\partial S^c}$$

$$\dot{\mu}_f^c(t) = -\frac{\partial H^c}{\partial S^f} = (e^{-rt} q^c(t) + \mu_f^c(t)) \frac{\partial \phi^f(S(t), t)}{\partial S^f}$$

Next we consider the case where the OLNE consists of a final phase with $S \rightarrow C$. The Hamiltonian associated with the OLNE problem of firm j ($j = c, f$) reads

$$H^j(q^j, \lambda^j, t) = e^{-rt} (\bar{p} - q^j - q^i - k^j) q^j + \lambda^j (-q^j)$$

Among the necessary conditions we have that the co-state variable λ^j is constant. In addition the Hamiltonian is maximized. This implies that if at time t there is simultaneous supply we have

$$3q^c(t) = \bar{p} + k^f + \lambda^f e^{rt} - 2(k^c + \lambda^c e^{rt})$$

$$3q^f(t) = \bar{p} + (k^c + \lambda^c e^{rt}) - 2(k^f + \lambda^f e^{rt})$$

$$3p(t) = \bar{p} + k^c + \lambda^c e^{rt} + k^f + \lambda^f e^{rt}$$

Along the C interval we have

$$2q^c(t) = \bar{p} - k^c - \lambda^c e^{rt}$$

$$2p(t) = \bar{p} + k^c + \lambda^c e^{rt}$$

In addition, the equilibrium price is continuous at the time of transition t_1 . Moreover, at the final time T the price equals \bar{p} . Taking this into account we derive the stocks needed to have this equilibrium from some t in the S -phase on. At such an instant of time it holds that

$$3q^c(t) = \bar{p} + (k^f + \lambda^f e^{rt}) - 2(k^c + \lambda^c e^{rt})$$

$$\lambda^c = e^{-rT} (\bar{p} - k^f)$$

$$\frac{1}{3}(\bar{p} + k^c + \lambda^c e^{rt_1} + k^f + \lambda^f e^{rt_1}) = \frac{1}{2}(\bar{p} + k^f + \lambda^f e^{rt_1})$$

$$3rS^f(t) = (\bar{p} + k^c - 2k^f) (rt_1 - rt - 1 + e^{rt-rt_1})$$

$$3rS^c(t) = -\frac{1}{2}(\bar{p} + k^c - 2k^f) (rt_1 - rt - 1 + e^{rt-rt_1})$$

$$+ \frac{3}{2}(\bar{p} - k^c) (rT - rt - 1 + e^{rt-rT})$$

From here on the analysis proceeds along the same lines as in the other case treated in the main text. For completeness we write down the full argument. For a CLNE to result in the extraction path of the OLN \bar{E} , we must have $\mu_c^c(s) = \lambda^c$ for all instants $s \geq t$ for all $t \geq 0$. Therefore μ_c^c is constant. It follows that then

$$(e^{-rt}q^c(t) + \mu_f^c) \frac{\partial \phi^f(S(t), t)}{\partial S^c} = 0$$

This implies that either (i) $e^{-rt}q^c(t) + \mu_f^c(t) = 0$ where q^c is the OLN \bar{E} equilibrium path of the cartel and therefore $\mu_f^c(t) = -e^{-rt}q^c(t)$ or (ii)

$$\frac{\partial \phi^f(S(t), t)}{\partial S^c} = 0.$$

Condition (i) implies that $\dot{\mu}_f^c = 0$, but $e^{-rt}q^c(t)$ is not constant along the OLN \bar{E} . Hence, for a CLNE to result in the extraction path of the OLN \bar{E} , we must have

$$\frac{\partial \phi^f(S(t), t)}{\partial S^c} = 0$$

along the OLN \bar{E} where there is simultaneous supply. We next show that this condition is not met.

Our strategy is to assume that the open-loop equilibrium is subgame perfect. Consequently we represent extraction by the cartel as a function of time and the existing stocks. So, we first write

$$3 \frac{\partial \phi^c}{\partial S^f} = \frac{\partial (\lambda^f - 2\lambda^c) e^{rt}}{\partial S^f}$$

We have

$$\lambda^f - 2\lambda^c = -\frac{3}{2}e^{-rT}(\bar{p} - k^c) + \frac{1}{2}e^{-rt_1}(\bar{p} + k^c - 2k^c)$$

So

$$\frac{\partial (\lambda^f - 2\lambda^c)}{\partial S^f} = r \frac{3e^{-rT}}{2} \frac{\partial T}{\partial S^f} (\bar{p} - k^c) - r \frac{\partial t_1}{\partial S^f} \frac{e^{-rt_1}(\bar{p} + k^c - 2k^c)}{2}$$

The derivatives with respect to the stocks follow from the expressions derived above for these stocks.

$$3r = (\bar{p} + k^c - 2k^f) (r - re^{rt-rt_1}) \frac{\partial t_1}{\partial S^f}$$

$$0 = -\frac{1}{2}(\bar{p} + k^c - 2k^f) (r - re^{rt-rt_1}) \frac{\partial t_1}{\partial S^f} + \frac{3}{2}(\bar{p} - k^c) (r - re^{rt-rT}) \frac{\partial T}{\partial S^f}$$

Therefore

$$0 = -\frac{3}{2}r + \frac{3}{2}(\bar{p} - k^c) (r - re^{rt-rT}) \frac{\partial T}{\partial S^f}$$

Or

$$\frac{1}{1 - e^{rt-rT}} = (\bar{p} - k^c) \frac{\partial T}{\partial S^f}$$

and

$$\frac{3}{1 - e^{rt-rt_1}} = (\bar{p} + k^c - 2k^f) \frac{\partial t_1}{\partial S^f}$$

Substituting gives

$$\begin{aligned} \frac{\partial (\lambda^f - 2\lambda^c)}{\partial S^f} &= r \frac{3e^{-rT}}{2} \frac{1}{1 - e^{rt-rT}} - r \frac{3}{t_1 - e^{rt-rt_1}} \frac{e^{-rt_1}}{2} \\ &= \frac{3r}{2} \left(\frac{e^{-rT}}{1 - e^{rt-rT}} - \frac{e^{-rt_1}}{1 - e^{rt-rt_1}} \right) \\ &= \frac{3r}{2} \left(\frac{1}{e^{rT} - e^{rt}} - \frac{1}{e^{rt_1} - e^{rt}} \right) \end{aligned}$$

For any t we have that

$$f(X) = \frac{1}{e^{rX} - e^{rt}}$$

is strictly decreasing in X and therefore $\frac{\partial(\lambda^f - 2\lambda^c)}{\partial S^c} \neq 0$.