Defacing Euclid:
Reading and annotating the *Elements of Geometry* in early modern Britain

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Euclid's *Elements* was culturally pervasive in early modern Britain, but the details of its visibility and use have not been studied closely. This essay will consider the variety of Euclidean texts visible to British readers in this period, the sites where the text was engaged with, and the types of study and use readers appear to have engaged in based on the marks they left in surviving copies.

The early modern world did not possess a single, stable version of Euclid's *Elements*: far from it. Across Europe, the *Elements* was printed nearly once a year on average from the *editio princeps* of 1482 to the end of the seventeenth century (indeed, much the same rate of reprinting would continue well into the nineteenth century).[1]

Editors, booksellers and printers enacted a range of different agendas in the approaches they took to the *Elements*, and they produced as a result a diverse range of texts. The story is in many ways similar to the wider story of the changing patterns of text production and consumption – particularly in universities – in the first two centuries or so of print: but there are some distinctive features too. The intentions of editors and printers can in many cases be read from the gross features of the editions themselves, or from indications on the title page about the motivations for the new imprint. Some editors revealed their agendas at length in prefaces, and in some cases we have correspondence or other evidence bearing on the matter.

A substantial fraction of the nearly two hundred imprints of the Euclid's *Elements* from 1482 to 1700 were imported into Britain and Ireland during that same period. But it would be a mistake to suppose that new versions of the text were simply imported as they were produced and older texts discarded, so that readers had before them a changing, but momentarily uniform, view of the text. The evidence of book lists and library catalogues, curricula and the provenance of individual

[1] Pietro Riccardi, *Saggio di una bibliografia Euclidea* (Bologna, 1887–1893); Charles Thomas-Stanford, *Early Editions of Euclid's Elements* (London, 1926); Max Steck, *Bibliographia Euclideana* (Hildesheim, 1981); online database of Euclidean editions (forthcoming) available at www.readingeuclid.org.

copies tells a very different story. Institutions and individuals collected multiple copies of the *Elements*, and every site in which Euclidean texts were read afforded access to multiple, sometimes numerous, different versions of the text. Readers had the opportunity to compare and collate, to use one version of the text to question others: and the evidence shows that some of them did just that.

Thus we also should not imagine that readers or students engaged passively with the Euclidean text, 'learning' its contents by rote or silently accepting its authority. On the contrary, a typical early modern copy of Euclid's *Elements* contains dozens or hundreds of small manuscript emendations and (sometimes) supplementary remarks, bearing witness to various kinds of engagement with the text including suspicion, disagreement, and aggressive re-writing and re-working. Thus, marks in copies of the *Elements* contain valuable information about the engagement of individuals in different locations with the text in its various forms.

We can thus begin to build up a picture of the "natural history" of the *Elements* in early modern Britain: necessary groundwork to understanding the impact of the text in a period when the cultural profile of mathematics, and its importance to intellectual culture more generally, were changing rapidly.

1. The copious text: the Elements and textual scholarship from Ratdolt to Grynäus

The text of the *Elements* existed in a large number of more or less distinct Latin versions by the later fifteenth century, of which by far the most important for printed editions was that of Campanus of Novara (d. 1296),[2] printed at Venice in 1482.[3] A brief epitome of Euclidean geometry attributed to Boethius was also widely visible, printed in collections of his *Opera* from

[2] Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (2nd ed., Cambridge, 1925), vol. 1, pp. 93–113; Marshall Clagett, 'The Medieval Latin Translations From the Arabic of the Elements of Euclid, With Special Emphasis on the Versions of Adelard of Bath', *Isis* 44 (1953), pp. 16–42; John Murdoch, 'Euclid: Transmission of the Elements', in *Complete Dictionary of Scientific Biography*, vol. 4 (2008), pp. 437–59.

[3] Erhard Ratdolt, *Preclarissimus liber elementorum Euclidis perspicacissimi* (Venice, 1482).

1491 onwards:[4] but the complex medieval transmission of these two texts was effectively invisible to readers of early modern print. By the turn of the sixteenth century the humanistic reinvention of disciplines was under way, and one of its outcomes was a humanistic vision of Euclid as instigator of a broad and deep tradition of geometrical learning. Bartolomeo Zamberti (1473–1539) produced a new Latin translation of the *Elements*, and from 1505 several editors prepared editions in which this and the version of Campanus were collated together, with supplementary commentary and other material. The result was a massive text in which for a typical theorem there was given two versions of the enunciation, two versions of the proof, editorial commentary and possibly other matter such as alternative proofs from the manuscript traditions. Editions of this general type were produced at Venice by Zamberti and at Paris by Jacques Lefèvre d'Etaples.[5] In these editions the demonstrations functioned almost as commentary in relation to the definitions as the primary text, reflecting a widely held view that the received demonstrations were the work not of Euclid but of the fourth-century redactor Theon of Alexandria, and also reflecting the existence of a number of substantially different versions of the demonstrations and the practice of teachers in reworking them for different audiences.[6]

In 1533 Simon Grynäus put the Greek text of Euclid into print for the first time, at Basel.[7] Based on manuscripts from Paris and Venice, this edition made possible new kinds of engagement with the text, and it may well be thought to have inaugurated the theme of 'mathematical classicism' that ran through – particularly – later English engagements with the ancient texts on geometry and number theory. Grynäus included in his edition a text of Proclus's commentary on the first book of the *Elements*. Material from the commentary had been in circulation for some time, for instance in the *De expetendis et fugiendis rebus* of Giorgio Valla (1501) which translated large sections from various Greek Platonists,[8] but this publication of the Greek text would become a

[4] Boethius, *Opera* (Venice, 1491–92, reprinted 1497–99), containing 'liber primus Geometricum Euclidis a Boetio in Latinum translatae'.

[5] Bartolomeo Zamberti, *Euclidis Megarensis ... elementorum libros. xiij.* (Venice, 1505); Jacques Lefèvre d'Étaples, *Euclidis Megarensis Geometricorum elementorum libri XV* (Paris, 1516).

[6] Richard Oosterhoff, *Making Mathematical Culture: Books, universities, and the craft of knowledge In Renaissance Paris* (Oxford, forthcoming).

[7] Simon Grynäus, *Ευκλείδου Στοιχειῶν βιβλ. ιε* (Basel, 1533).

[8] Oosterhoff, *Making Mathematical Culture*.

crucial touchstone for broadly neoplatonist ideas about mathematics as well as for discussion of the structure of the Euclidean proofs, and of geometrical proof in general.[9]

Many of these editions can be shown to have been imported into Britain soon after their printing, or in some cases to have arrived in Britain later in the sixteenth century. Grynäus' text of Proclus was edited from a manuscript in Corpus Christi College, Oxford, and the edition as a whole was dedicated to Cuthbert Tunstall, diplomat, bishop, friend of More and Erasmus and author in 1522 of a work on arithmetic. Several copies of Grynäus's edition were transmitted to Oxford as a result of these connections, and other British scholars, including More, were also the recipients of presentation copies.[10] And it seems to have been usual for at least the larger university colleges to acquire a range of different versions of the Euclidean text during the sixteenth century: a typical college acquired at least one of the humanist Latin versions as well as Grynäus' Greek edition; less interest seems to have been taken in the very earliest printed editions, which perhaps did little for teaching purposes that a manuscript did not. Often these acquisitions were by donation: the books were bought by individual fellows of the college and donated or bequeathed for the use of students, sometimes for the use of specific parts of the student body.[11]

Furthermore, many of these copies bear marks made by early readers, allowing us to say in some detail how they were used. Practically all bear marks of ownership (inscriptions, bookplates or stamps): typically revealing little about mathematical reading in particular, although sometimes

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[9] See the marginalia cited below as well as for instance Federigo Commandino, *Euclidis Elementorum libri XV* (Pesaro, 1572), sig. **1v, where the vocabulary is explicitly drawn from Proclus; also Eckhard Kessler, 'Clavius entre Proclus et Descartes', in *Les jésuites à la Renaissance: Système éducatif et production du savoir*, ed. Luce Giard (Paris, 1995), pp. 285–308; and Paolo Mancosu, 'Aristotelian Logic and Euclidean Mathematics: Seventeenth-Century Developments of the Quaestio de certitudine mathematicarum', *Studies in History and Philosophy of Science* 23 (1992), pp. 241–265.

[10] Grynäus, *Ευκλείδου Στοιχειῶν βιβλ. ιε*, a2r–a5v. Bodleian Library, University of Oxford, Byw. C 3.3: copy presented to Thomas More.

[11] These remarks are based mainly on a perusal of the copies surviving in Queen's, All Souls, Balliol, New, Corpus Christi, Magdalen, Merton, Worcester, Jesus, Exeter and Brasenose Colleges in Oxford and Trinity College in Cambridge.

recording the ownership history of a copy in useful ways.[12] Setting these aside, I have seen evidence for several different kinds of engagement with the Euclidean text.

First, some readers relentlessly improved the Greek in respect of its spelling and grammar. Some copies bear dozens or even hundreds of tiny changes: accents, omicron for omega (or vice-versa) and also some quite compulsive policing of the distinction between proof and construction where it appears in headings or in the verbal formula which closes a proof (ὅπερ ἔδει ποιῆσαι/ ὅπερ ἔδει δεῖξαι).[13] These readers seem to have believed (in many cases quite probably rightly) that they knew better than the printer or editor, whether it was a question of the proper spelling of a Greek word or the proper deployment of a particle, or the distinction between proof and construction.

By perhaps the middle of the sixteenth century this impulse to improve the Euclidean text also began to extend to the Latin. Readers corrected grammatical errors and mere infelicities; increasingly they made spelling changes reflecting their views about proper Latinity: medial ch was emended to h (thus michi became mihi); lost spellings with æ were restored from plain e or the variety of sometimes ambiguous forms of decorated e deployed by some printers (thus hec or hęc became hæc).[14]

The impulse to correct a printed text was a widespread one in the Renaisance, and its appearance here does not necessarily require any special explanation.[15] In the case of privately owned books, it

[12] Roger E. Stoddard, *Marks in Books, illustrated and explained* (Cambridge, MA, 1985); H.J. Jackson, *Marginalia: readers writing in books* (New Haven, 2001), p. 19.

[13] For example, Merton College, University of Oxford, 40.J.15, a copy of Grynäus, *Ευκλείδου Στοιχειῶν βιβλ. ιε*, marked probably by Henry Savile; Balliol College, University of Oxford, St Cross 0625 d 06, a copy of the same edition apparently marked at Balliol.

[14] For example, Merton College, University of Oxford, 40.J.17, a copy of Zamberti, *Euclidis Megarensis ... elementorum libros. xiij* donated by Richard Rawlyns (1460–1536) for the use of the college's students ('studentibus Collegij in geometria et perspectiua'), with various corrections of this kind to Latin spelling and grammtical endings.

[15] Anthony T. Grafton, *The Culture of Correction in Renaissance Europe* (London, 2011); also Steven N. Zwicker, 'The constitution of opinion and the pacification of reading' in Sharpe and Zwicker, *Reading, society, and politics*, pp. 295–316 at p. 298: 'What we have come to appreciate from recent work on the various modes of manuscript and print publication in the Renaissance is how active were the processes of early modern reading, how intimate the relations between writing and reading, how implicated in the dynamic of production was consumption.'

may have been associated with a wish to be seen to be reading Euclid: the visual display of one's proficiency as a reader. There are few better ways to demonstrate to others that you have turned over every page of a text than to make trivial improvements to every page; nor are there many better ways to display your command of Greek than to make systematic improvements to a printed text in that language. Systematic improvement of the text is to be found today almost exclusively in older collections such as those of universities and colleges, reflecting at least in part the fact that newer libraries and private collections often valorised 'clean' books when deciding what to purchase and what to retain.[16] That, nevertheless, detailed correction may have had a particular importance in some institutional situations is suggested by another kind of mark we find in these early copies.

Some readers of these early Euclidean editions used them as a source for copying out sections of the text. Thus for example at Corpus Christ College, Oxford a copy of Lefèvre d'Etaples' edition of 1516 was donated by the president John Claymond for the use of the students, his inscription specifically suggesting that students would use it to 'copy out' Euclidean material.[17] Direct evidence for the copying of text sections is very rare indeed, but quite common is a subtler type of mark related to the copying of diagrams. It was not uncommon for a reader to lay a sheet of paper on top of the open book and copy or trace a diagram onto it. If a sharp pen or pencil was used, the printed page beneath would receive clear indentations matching the shape of the diagram, and in many copies these are still clearly visible, particularly under raking light. If compasses were used to draw circles or arcs, the point would leave a hole in the printed page, and these too can often be clearly seen today. Finally, if the copy was made in ink and the book was closed on the loose sheet soon after it was made, the facing page would sometimes receive an reversed ink-transfer of the diagram, and these too can sometimes be clearly identified today for what they are.[18]

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[16] Jackson, *Marginalia*, p. 235; Karen Sanchez-Eppler, 'Marks of Possession: Methods for an Impossible Subject', *Transactions and Proceedings of the Modern Language Association of America* 126 (2011), pp. 151–59 at pp. 151–3.

[17] Corpus Christi College, University of Oxford, Rare Books Collection, Δ.10.1, inscription on title page: 'in usum discipulorum + ut inde exscriberent theoremata euclidis'.

[18] For example: Magdalen College, University of Oxford, Old Library q.11.17 (copy of Lefèvre d'Étaples, *Euclidis Megarensis Geometricorum eleme[n]torum libri XV*) has indentations indicating the copying of a number of diagrams in Book 1; Trinity College, University of Cambridge, NQ.7.82 (copy of Thomas Rudd, *Euclides elements of geometry* (London, 1651)), with prick-marks indicating copying using a compass (very likely by Isaac Newton) throughout pp. 1–

Thus it is possible to say with confidence that some readers – presumably the students for whom the copies were donated or purchased – copied out selected parts of the Euclidean text, particularly the diagrams. A limitation of this type of evidence is of course that no precise date can be assigned to the copying; and there is no good reason to suppose that books were only used for this purpose when they were relatively new. The generously sized diagrams of the early sixteenth-century imprints may well have remained useful for this purpose well into the next century, while newer, smaller imprints were used in this way less frequently (though they were sometimes so used; see below). These observations suggest the conjecture that institutionally-owned copies, or more generally copies that were intended to be taught from, were deliberately and systematically corrected by teachers, with the intention of making the text more suitable for selective copying by students.

Another type of engagement with the text was to collate one edition of the *Elements* with another edition or with a manuscript; many readers left clear traces of this activity in the form of more substantial textual emendations – at the phrase rather than the character level – or the replacement or insertion of substantial passages of text, occasionally with notes as to their sources.[19] These readers, again, seem have assumed that the editors of printed versions were not to be implicitly trusted, and in some cases at least that the local manuscript resources were likely to be superior to those employed by the editors (in some cases that is a judgement with which modern scholarship would be likely to concur).

It is quite possible that collation of this kind arose, like more detailed textual improvement, from a concern to present students with a better version of the *Elements* from which to copy. It also seems possible that a more general teaching agenda gave rise to this kind of work, particularly where the alternative readings copied in provided alternative strategies of proof or significantly changed the content or the implications of definitions.

40; Trinity College, University of Cambridge, S.10.72 (copy of Oronce Fine, *In sex priores libros geometricorum elementorum Euclidis Megarensis demonstrationes* (Paris, 1536)), p. 17, a probable ink transfer of a diagram on a loose leaf.

[19] For example, Bodleian Library, University of Oxford, Auct. K 3.8 (copy of Grynäus, *Ευκλείδου Στοιχειῶν βιβλ. ιε*), collated throughout with texts of Commandino and Montaureus, with reference also to at least two manuscripts.

It is characteristic of mathematical annotation quite generally that it is selective in its attention to a text;[20] this is certainly the case for the evidence of copying and of textual collation I have seen in these early editions of the *Elements*. Textual improvers generally began at the start of the text and either continued to the end or gave up at some intermediate point: after a few pages, after book 1 or book 2, or (quite often) after book 6. Those who copied parts of the text gave their attention to small selections of propositions, and there were a few favourite propositions that recur again and again in this evidence: most notably 1.5 (the Pons Asinorum), 1.6 and 1.47 (Pythagoras' theorem). The same is true of textual collation, which did not typically extend over the entire book but was typically applied to a selection of books (often books 1–6) and to a selection of propositions within those books. Definitions were given particular attention, notably those in books 1 and 5. In general it seems quite clear that books 1–6 were studied rather more than books 7–15 (most editions from this period included books 14 and 15, sometimes with a note explaining their later genesis than the remainder of the text); that in the second half of the *Elements* books 11–13 (the stereometric books) were studied more than the material on arithmetic, number theory and commensurability in books 7–10; and that book 1 and to a lesser degree book 2 were studied most of all.

2. Euclid in the vernacular

The massive humanist editions, and the Greek text of Grynäus, formed the basis for vernacular versions of the *Elements* from the mid-sixteenth century onwards: Italian (Venice, 1543), German (Augsburg, 1555), French (Paris, 1564), Spanish (Seville, 1567), English (London, 1570). These versions imitated the large size and textual copiousness of their models. The English Euclid produced at London in 1570 by Henry Billingsley followed this model. Based on the Greek of Grynäus and a selection of Latin editions including those of Zamberti (1505), Gracilis (1558) and Foix (1566), it incorporated comments and remarks from a variety of printed sources ('manifolde additions, Scholies, Annotations and Inventions ... gathered out of the most famous and chiefe Mathematiciens, both of old time and in our age') as well as new material from mathematician

[20] Benjamin Wardhaugh, '"The Admonitions of a good-natured Reader": How Georgians read mathematics', *Historia mathematica* (forthcoming).

John Dee. It filled five hundred folios.[21] As well, and like a number of other editions, it incorporated a large fold-out chart as part of its prefatory matter setting out in synoptic form the parts of mathematics. Such synoptic views of mathematics – textual or diagrammatic – went back to the humanist impulse of Lefèvre in particular,[22] but by this date they also owed something to the more elaborate and methodologically refined deployment of such charts by Ramus and his followers (and it is perhaps by this route as much as by any other that Ramist ideas about mathematics and its study entered the English-speaking world). It is notable that when such charts were attached to the *Elements* they divided up mathematics or geometry as a body of knowledge about methods and objects; there was very rarely, in all the dozens of editions of the text, any attempt to display synoptically the logical relationship between theorems.[23] Billingsley's edition also incoroprated the celebrated preface of mathematician John Dee which set out a vision of the nature of mathematics; Dee drew on a number of sources including the commentary of Proclus and other Platonist writings, but he also had a very clear personal agenda concerning the deployment of mathematics to solve practical problems in areas ranging from music to astrology.[24]

It is frankly somewhat unclear who puchased or used these first vernacular versions of the *Elements*. Editors – in prefaces or title pages – often gestured towards artisans who might value geometrical knowledge but be unable to read the classical languages,[25] yet these editions partook of the physical gigantism of their Greek and Latin predecessors and were often expensive folios, unlikely to be within the realistic purchasing power of the generality of artisans. I have seen none with marks that suggest use in the workshop (by contrast with later generations of practical mathematical books, which do in some cases clearly possess the dirt and the wounds that might be

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[21] Henry Billingsley, *The elements of geometrie of the most auncient philosopher Euclide of Megara* (London, 1570); Diana M. Simpkins, 'Early Editions of Euclid in England', *Annals of Science* 22 (1966), pp. 225–249.
and secondary.

[22] Oosterhoff, *Making Mathematical Culture*.

[23] For a modern take see Ian Mueller, *Philosophy of mathematics and deductive structure in Euclid's Elements* (Cambridge, MA and London, 1981); the one early modern attempt known to me is Petrus Ramus, *Scholae mathematicae* (Frankfurt, 1559).

[24] John Dee, 'To the unfained lovers of truthe', in Billingsley, *Elements of geometrie*, at a.iij^{r}–A.iij^{r}.

[25] Billingsley, *Elements of geometrie*, [hand]ii^{v}: 'many good wittes both of gentlemen and of others of all degrees'.

expected from direct contact with, say, carpentry).[26]

On the other hand, in university situations, in which teaching was carried out in Latin, it is not obvious that students or teachers would have had much use for a vernacular crib (and I have never seen vernacular translation written into the margins of Latin or Greek editions). Although a number of university colleges seem to have acquired a copy of the English Euclid,[27] they do not seem to have made much use of it, to judge from the marks (or rather, the frequent lack of them) on surviving copies.

3. Teaching editions: Clavius and others

Some editors from the 1530s turned away from the large, bulky and costly editions of the *Elements* that had been produced so far, and found ways to render the text much smaller. Some printed the enunciations only: most famously Petrus Ramus at Paris (1545).[28] Such a move was justified by the history of the text: the enunciations were generally believed to be the only part of the Greek text actually composed by Euclid; furthermore, the proofs in the Latin text of Campanus were of that translator's composition. Such editions can also be seen as meeting the practical need for a Euclidean synopsis – more adequate than the Boethian synopsis – that could be actually purchased by the generality of students and feasibly carried about and taken to lectures. Again, they emphasise the charater of the text as a collection of assertions, not a collection of proofs, and they efface any sense that the *Elements* is a texbook of logic or an exemplar of demonstrative methods. They also perhaps emphasise a sense that the proofs were interchangeable, malleable, to be supplied by the individual teacher or learner to suit the particular needs of the situation. They thus

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[26] Benjamin Wardhaugh, 'Consuming Mathematics: John Ward's *Young Mathematician's Guide* (1707) and Its Owners', *Journal for Eighteenth-Century Studies* 38 (2015), pp. 65–82 at p. 72.

[27] The following hold copies which appear to have been acquired at an early date: the universities of Glasgow, Aberdeen; in Oxford, Balliol, New, Queen's, St John's, Christ Church, Corpus Christi and Worcester colleges; in Cambridge, Trinity College.

[28] Petrus Ramus, *Euclidis elementorum libri quindecim* (Paris, 1549); also Conrad Dasypodius, *Εὐκλείδου τῶν πέντε καὶ δέκα Στοιχειῶν, ἐκ τῶν τοῦ Θέωνος συνουσιῶν τὸ πρῶτον*; *Εὐκλείδου τῶν πέντε καὶ δέκα Στοιχειῶν ἐκ τῶν τοῦ Θέωνος συνουσιῶν τὸ δεύτερον* and *Propositiones reliquorum librorum geometriæ Euclidis* (Strasbourg, 1564), a three-part version aimed at school use in which books 3–15 are given as enunciations only.

prompt a very different kind of engagement with the *Elements* than the textually copious editions that had preceded them. Ramus's own view of the Euclidean text, as Robert Goulding has shown, saw it as malleable, needing to be cured of its diseases, to be altered and updated in response to the needs of mathematical practice:

> Ingrediamur igitur in ipsa Euclidis elementa, inque viscera ipsa penitus subeamus: sanguinem, carnem, ossa retexamus: intimas propositi ad curandum morbi caussas perscrutemur.[29]

This went well beyond the traditionally selective approach to the Euclidean text, and it was controversial. In Britain, mathematician and classical scholar Henry Savile sharply attacked the Ramist view of the *Elements*, insisting that the text – proofs and all – was a beautiful, even a perfect body of classical knowledge, for which revision was inappropriate:

> In pulcherrimo Geometriæ corpore duo sunt nævi, duæ labes, nec, quod sciam, plures[.][30]

I believe that this was the more representative British view at this time. Evidence that Ramus's edition of the *Elements* was taken up in Britain is very scant; Ramist editions with sixteenth-century British provenance are conspicuous by their absence from nearly all collections. And the evidence of ownership and annotations in the more copious versions of the text from this period does not suggest that readers were (yet) either discarding the received proofs in favour of their own versions, nor engaging in wholesale rewriting or restructuring of the text to meet their own purposes. Such things would change in the following century.

The decisive events in the printing of Euclid's *Elements* later in the sixteenth century were the production of two new versions of the Latin text. The first, printed at Pesaro in 1572, was a fresh translation from the Greek of Grynäus (and collated with at least one manuscript), by Federigo

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[29] Petrus Ramus, *Scholæ mathematicæ* (Basel, 1569), p. 91; see Robert Goulding, *Defending Hypatia: Ramus, Savile, and the Renaissance Rediscovery of Mathematical History* (Dordrecht, 2010).

[30] Henry Savile, *Prælectiones tresdecim in principium Elementorum Euclidis* (Oxford, 1621), p. 140.

Commandino.[31] Tacitly ackowledging that not all of those educated in Latin culture were also able to work with the text in Greek, Commandino stayed faithfully close to the Greek sources he knew; but he also translated, for the first time, numerous Greek scholia. This new project inaugurated a wave of new Latin Euclids intended to serve various agendas.

Commandino's edition seems to have been imported into Britain in only quite modest numbers, but his Latin text was used as a basis by Henry Briggs in his 1620 Latin–Greek edition,[32] which certainly was widely disseminated in Britain. Far more frequently purchased in Britain was the hugely important new edition of 1574, specifically aimed at university teaching, producd by the Jesuit mathematician Christoph Clavius.[33] A two-volume octavo, and one of the first significant editions of the text in that smaller format, it was reprinted half a dozen times, and the evidence is very clear that it was adopted as a teaching text in universities across Europe.

The sheer quantity of text it contained was enormous: Clavius compiled material from Campanus, Zamberti and Commandino and claimed he had included as a result a total of 1234 distinct propositions (as against fewer then 500 in most predecessors).[34] He also gave a wealth of extra material from various sources: commentary and geometrical explanation of the kind to be found in vernacular editions such as Billingsley's, but that few previous Latin editors had thought necessary.

Subsequent editions appeared at Rome in 1589 and 1603, at Cologne in 1591, 1607 and 1627, and at Frankfurt in 1607 and 1654. There is hardly an ancient university college in Britain that does not possess at least one version of the edition, with many colleges acquiring multiple copies. At Oxford and Cambridge the edition was used well into the eighteenth century.[35] Its longevity is a testament

[31] Federigo Commandino, *Euclidis Elementorum libri XV* (Pesaro, 1572); subsequently reprinted in 1619.

[32] Henry Briggs, *Elementorum Euclidis libri VI priores* (London, 1620).

[33] Christoph Clavius, *Euclidis elementorvm libri XV* (Rome, 1574).

[34] Christoph Clavius, *Euclidis elementorum libri XV* (Frankfurt 1607), f*6^{r}.

[35] Trinity College, University of Cambridge, T.38.13–14: copy of Clavius, *Euclidis elementorum libri XV* (Frankfurt, 1607, bearing the signature of Richard Bentley on the title page and detailed marks of attention throughout the indices of propositions, problems and theorems, vol. 1, a*1^{r}–7^{r}, c*5^{r}–d1^{v}. Queen's College, University of Oxford, Upper Library: 40a.A.14–15: copy of Clavius, *Euclidis elementorum libri XV* (Frankfurt, 1654), donated in 1669 by Clement Ellis (1630–1700) 'Taberdarijs Coll: Reg: & eorum successoribus' (vol. 1, second free front endpaper; vol. 2, third free

to its success in meeting the needs of readers and indeed in shaping the types of use the *Elements* received and thus the types of expectations readers had of a printed edition.

Two new types of reading, though not absent from earlier editions, come to real prominence here, and dominate the marks in, particularly, surviving copies of the Clavius editions. First, some readers saw the text as a series of factual assertions, noticed that many of them were wrong in detail, and corrected them. Geometrical labels might be adjusted to bring them into line with the accompanying printed diagram (or vice versa); other small errors such as "aequalis" for "inaequalis" or a misplaced "non" might be emended. Incorrect cross-references might be corrected.[36] A typical reader would select certain theorems for attention – as before, overwhelmingly in books 1–6, and most frequently in books 1 and 2 – and add as many as a dozen small corrections on each printed page.

This is one of the most characteristic ways of reading a mathematical text: in the seventeenth and eighteenth centuries correction of this kind becomes overwhelmingly dominant in mathematical annotations.[37] It corresponds to the traditional – and frequently correct – image of mathematical reading done with pen in hand, the reader always ready to use mathematical skills to correct mathematical errors. Some readers allowed their correction to be directed in part by a printed list of errata, but very few limited their corrections to those thus sanctioned. More were moved by the authorial apologies that accompanied printed errata lists, which frequently pointed to the special difficulty of correcting mathematical print (Billingsley's was typical: 'no one man though he be neuer so diligent and circumspecte can espie all thinges'), and took these as an invitation to undertake further autonomous correction and improvement.[38]

That this kind of emendation – as distinct from the correction of spelling and grammar – became

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front endpaper). Queen's College, University of Oxford, Upper Library, 40a.A.18–19: another copy of Clavius, *Euclidis elementorum libri XV* (Frankfurt, 1654) donated by Stephen Green (c. 1685–) 'In Usum Taberdariorum' (front pastedown) and bearing detailed marks of attention throughout the list of contents, a*1^{r}–4^{v}.

[36] For example, Queen's College, University of Oxford, Upper Library: 40a.A.14–15: copy of Clavius, *Euclidis elementorum libri XV* (Frankfurt, 1654), with corrections concentrated in Book 6.

[37] Wardhaugh, 'Consuming Mathematics' and idem, 'The admonitions', *passim*.

[38] Billingsley, *Elements of geometrie*, fol. 463^{v}.

so common with the advent of the smaller-format teaching editions of Euclid suggests the conjecture that it was particularly relevant to teaching or to learning, and there are certainly numerous cases from the early seventeenth century onwards where annotation of this kind seems to have been performed by student rather than teacher: the corrections are faulty or accompanied by rough notes on other university subjects including basic arithmetic; or occasionally an ownership inscription makes the status of the annotator clear.[39] There is room to suspect that schools in particular positively encouraged students to read with pen in hand and to correct the text as they worked through it, and perhaps that universities also fostered this culture of (distinctively mathematical) reading.[40]

Second, many readers went further than mere correction, and re-worked the printed text more thoroughly: selecting, numbering, cross-referencing or indexing. They explicitly selected parts of the text for attention, marking up the contents list or the index. They added new cross-references or references to other works and they inserted explanatory remarks and examples or modified the printed diagrams by the addition of numbers, turning them into specific numerical examples of the general propositions they – as printed – illustrated. In extreme cases they copied elaborate explanatory schemes from other works, notably Proclus's commentary.[41]

This type of annotation is particularly revealing about the agendas of the annotators, and certain copies of the Clavius editions in particular bear marks which clearly show which propositions were

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[39] For example, New College, University of Oxford, BT3.207.19, a copy of John Leeke and George Serle, *Euclid's Elements of geometry* (London, 1661), has incorrect emendations as well as emendations subsequently reversed in the same hand. Trinity College, University of Cambridge, T.46.16, a copy of Gulielmus Ingenierius, *Euclidis elementorum libri XV* (Pesaro, 1619) bears corrections in a number of hands as well as marginal doodling and evidence for copying of diagrams.

[40] Wardhaugh, 'The admonitions'; idem, 'Rehearsing in the margins: mathematical print and mathematical learning in the early modern period', in Alice Jenkins (ed.), *Palgrave Handbook of Mathematics and Literature* (forthcoming).

[41] See note 36 above for examples of marked indices. Jesus College, University of Oxford, Fellows' Library: K.6.5 Gall.: copy of Christoph Clavius, *Commentaria in Euclidis Elementa geometrica* (Mainz, 1612) with added remarks and many added cross-references. Trinity College, University of Cambridge, T.41.48: copy of Isaac Barrow, *Euclide's elements* (London, 1660), with numerical examples added to Book 10. New College, University of Oxford, BT3.207.12(1): copy of Briggs, *Εὐκλείδου Στοιχειῶν Βιβλία ἓγ* with proofs divided into sections according to the Proclean scheme at pp. 6–13.

studied and in what order. Copies used by students at both Oxford and Cambridge bear detailed marks of this kind, in which some theorems are marked for attention and others neglected, and indeed in which the lists of contents and indices of theorems are marked up – in one case with the selected theorems numbered – so as to make the intended course of study unambiguous. This is most naturally interpreted as evidence about what took place in college teaching. Evidently some users, very probably tutors or donors (or both) were lightly customizing printed copies of the *Elements* – restructuring the text, in effect – in order to direct attention to a particular course of study, almost invariably based on selections from the propositions in books 1–6.[42] (Looking ahead into the eighteenth century, textbooks constructed by seletion and rearrangement from the *Elements* would become an increasingly important part of the world of printed geometry.)[43]

This was a period in which university lecture theatres in Britain also saw an increase in Euclidean teaching. The setting up of the Savilian chairs at Oxford in 1619 was a key moment for this kind of attention to the Euclidean text in Britain, and it responded explicitly to a sense that such attention had been neglected hitherto. In the Savilian statutes the first duties of the professor of geometry were to interpret publicly the *Elements* of Euclid, the *Conics* of Apollonius and the works of Archimedes; by 1661 these lectures were being delivered twice a week at eight in the morning, with second-year students obliged to attend on pain of a fine (of sixpence per lecture missed).[44] The *Praelectiones ... in principium Elementorum Euclidis* of Henry Savile, published in 1621, and the lectures of Savilian Professor John Wallis, delivered in 1651–53 and surviving in manuscript, indicate the character of the lectures that were given.[45] Both of these lecturers covered three

[42] See note 36 above. The 'taberdars', for whom the Queen's College copies were given, were BA graduates who were in effect doing MA work and waiting for fellowships to open up. For more on them and their books see Edward Bernard, *Catalogi Manuscriptorum Angliæ* (Oxford, 1697), 2nd. pagination, p. 30; also the evidence in Will Poole, 'A Royalist Mathematical Practitioner in Interregnum Oxford: The Exploits of Richard Rawlinson (1616–1668)' (forthcoming), showing that manuscript epitomes of the *Elements* were an important resource in this particular context.

[43] Such as the anonymous *Euclidis elementa geometrica* (London, 1666).

[44] *Statuta universitatis Oxoniensis* (Oxford, 1857), p. 241; *Statuta Selecta è Corpore Statutorum Universitatis Oxon* ([Oxford], 1661), p. 7.

[45] Henry Savile, *Praelectiones ... in principium Elementorum Euclidis* (Oxford, 1621); Bodleian Library, University of Oxford, MS Don. d. 45). On lecturing see also Mordechai Feingold, *The mathematicians' apprenticeship : science, universities and society in England, 1560–1640* (Cambridge, 1984).

different approaches to Euclidean material: philological attention to the text as text; philosophical reflection on the nature and definition of mathematical entities; and the mathematical content of a selection – in Savile's case quite a small selection – of specific propositions and their proofs. These three types of attention correspond at least in part to the distinct types of textual annotation discussed above; they are differently reflected also in the evidence we have in a final university context where Euclid was visible: responsions and disputations, the public exercises at which candidates for degrees displayed their mastery (or not) of the curricular subjects. The best evidence for this is perhaps the repeated importation into Britain and later printing at Oxford of the *Quæstiones geometricæ* by Peter Ryff (first printed in Frankfurt in 1600), from which students could learn standard answers to various chestnuts. A familiar range of different approaches to the subject were in evidence here. Some of the questions were, so to speak, meta-questions ('how is geometry subdivided?'; 'what are the things which it is proposed to measure?'). Some dealt with geometrical definitions and the issues they raised ('how is a line defined?'; 'what do you call a rectangle?' 'what distinctions do straight and curved lines admit?). Others worked through selected geometrical propositions in detail, suggesting that students were being experted to memorize specific proofs.[46] Ryff's *Quæstiones* were explicitly concerned with the geometry of Ramus as well as of Euclid, suggesting that by this date a more aggressive approach to the text had become the norm in (at least some) British universities.

As well as at university, private study of the *Elements* was certainly a feature of anecdotes about mathematics in this period, although it is a moot point whether any reader ever made significant progress with the text wholly unaided.[47] Perhaps most famous is John Aubrey's story about Thomas Hobbes:

> He was (vide his life) 40 yeares old before he looked on geometry; which happened accidentally. Being in a gentleman's library in ..., Euclid's Elements lay open, and 'twas the

[46] Peter Ryff, *Quaestiones geometricae* (Frankfurt, 1600); subsequent editions at Frankfurt in 1602, 1621 and 1649, and at Oxford in 1665.

citation and pages.

[47] Emily Winterburn has explored the myth of mathematical self-teaching in a somewhat later period: Winterburn, Emily, 'Philomaths, Herschel, and the myth of the self-taught man', *Notes and Records of the Royal Society of London* 68 (2014), pp. 207–225.

> 47 El. libri I. He read the proposition. 'By G--,' sayd he, 'this is impossible!' So he reads the demonstration of it, which referred him back to such a proposition; which proposition he read. that referred him back to another, which he also read. Et sic deinceps, that at last he was demonstratively convinced of that trueth. This made him in love with geometry.[48]

Hobbes did in fact become sufficiently proficient at Euclidean-style proof to publish works in that style, although it is fair to say he never acquired the degree of mastery he wished to, that would have enabled his original contributions in the field to gain acceptance by others.[49] If the truly private reader of Euclid was an exceptional figure, a more usual route involved the guidance of a tutor. These contexts are often poorly documented, and direct evidence for Euclidean study in them is therefore elusive, but there seems little doubt that aristocratic youths who were being prepared for university would have been introduced to the *Elements* as one of the key mathematical texts of the university curricula. This is also perhaps a context in which the use of a vernacular Euclid is somewhat more rather than less likely.[50] The copies of Billingsley's Euclid that are marked are those which appear to have been in private hands, and they found readers who carried out textual corrections and did little more, hinting perhaps at the calmer and more private environment of the private rather than the institutional library.

It is relevant to this kind of relatively isolated reader – more so, perhaps, to those somewhat lower down the social scale – to know that the public lectures on geometry offered at Gresham College in London at least sometimes dealt with Euclidean matters. A comment recorded by John Aubrey states that

> Mr Rooke [i.e. Lawrence Rooke] sayd in a Lecture at Grisham Colledge, upon 23.6 that the last definition, viz the fifth) in the sicth booke is not found in some Auncient copies; &

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[48] John Aubrey, *'Brief Lives', chiefly of Contemporaries*, ed. Andrew Clarke (Oxford, 1898), p. 332. Broadly similar stories were told for instance about Newton: see D.T. Whiteside, *The Mathematical Papers of Isaac Newton* vol. 1 (Cambridge, 1967), p. 6, citing an account by Abraham de Moivre.

[49] Douglas Jesseph, *Squaring the circle : the war between Hobbes and Wallis* (Chicago and London, 1999).

[50] All Souls College, University of Oxford, Stack 2nd floor: 4:SR.59.c.23, a copy of Briggs, *Elementorum Euclidis libri VI priores*; Worcester College, University of Oxford, J.7.9, a copy of Billingsley, *Elements of geometrie*: each has inscriptions suggesting private ownership and neat annotations suggesting use in teaching.

conceives it hath [*text lost*] of some Scholiast.[51]

This provides a hint that the geometrical lecturing there took some interest in the history or the philology of the text, and therefore raises the suspicion that it may have been of a broadly similar character to what took place at the universities. But this is a world of which we know frustratingly little.

4. Symbols and algebra

The length of the text negated some of the benefits of the smaller format adopted by Clavius's edition, and the next phase of Euclidean publication, during the seventeenth century, saw the rapid adoption of smaller formats in a process whose culmination were a number of tiny duodecimo issues such as the 1644 Paris edition of Georges Fournier, later reissued at Cambridge.[52] Certain kinds of use – particularly marginal annotation – were not facilitated by this trend, but octavo and duodecimo Euclids certainly found buyers, and they found publishers willing to issue and reissue them throughout the seventeenth century. Brevity in these editions was achieved in various ways. By giving only one proof for each proposition, in some cases a new or truncated proof. By reducing commentary, sometimes to nil. By – most effectively – printing only a selection of the theorems. Typical of the smaller teaching editions, including those intended for use in schools, was the printing of just books 1–6 or sometimes 1–6 together with 11 and 12: that is, the geometrical (and stereometric) material, but not the arithmetic and theory of incommensurables.[53] The natural assumption is that this restriction reflected which sections of the text were in fact being studied in schools and universities, as indeed seems to be borne out by the evidence from annotations presented above.

By the eighteenth century it was well established that grammar schools throughout England taught Euclid as part of their core curriculum; that assumption was being established during the

[51] Worcester College, University of Oxford, MSS.5.4, blank leaf following a copy of Briggs, *Elementorum Euclidis libri VI priores*.

[52] Georges Fournier, *Euclidis sex primi Elementorum geometricorum libri* (Paris, 1643), issued as a duodecimo at Paris in 1644 and 1654, and at London in 1654 and 1665.

[53] Such as *Euclidis elementa geometrica* (London, 1666) (a duodecimo), reissued in 1678.

seventeenth century although the evidence is so far poorly studied. Certainly it is the case that some schools in this period were sites for Euclidean teaching.[54] And these contributed to a demand for editions physically smaller and easier to handle than that of Clavius, less stored with material of textual rather than geometrical interest, and more tailored to the parts of the text actually likely to be studied by beginners. This had its impact, too, on modes of Euclidean annotation.

At school, mathematics in general and geometry in particular was taught not so much as a body of knowledge as a set of skills. For elementary mathematics the assumption until the mid-eighteenth century was that the teacher possessed a copy of the text taught from, while the students possessed blank books into which they or more frequently the teacher copied sections for study. Beneath the teacher-copied section the student would fair-copy relevant exercises and show these to the teacher before being allowed to move on to the next section of material. Descriptions of school classrooms suggest strongly that what was learned was therefore conceived as a sort of performance, the teaching process consisting of demonstration performances by the teacher followed by private rehearsal by the student and culminating in a demonstration performance by the student.[55]

For Euclidean geometry, where it seems to have been more usual for students to possess their own printed books, all of this resulted in an almost compulsive use of the margins of the printed page as a space in which to rehearse, to rewrite a proof in one's own words, to work out an example, to make a copy of a printed diagram from elsewhere on the same page; to check comprehension by actually doing the mathematical operation of whatever kind. We find many marks of this kind in the editions of Euclid that were used by schoolchildren and university students.[56] And indeed, numerical worked examples also became a feature of printed editions of the text intended for

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[54] See Jonas Moore, *A New Systeme of the Mathematicks* (London, 1681), title page: the book includes the first six books of the *Elements* and was composed for the use of the Royal Mathematical School.

[55] Nerida Ellerton and M.A Clements, *Samuel Pepys, Isaac Newton, James Hodgson and the Beginnings of Secondary School Mathematics: A history of the Royal Mathematical School within Christ's Hospital, London 1673–1868* (Dordrecht, 2017); John Denniss, *Figuring It Out: Children's arithmetical manuscripts, 1680–1880* (Oxford, 2012).

[56] Trinity College, University of Cambridge, T.41.48, copy of Barrow, *Euclide's Elements* (London, 1660), with numerical examples and algebraic explanations particularly in Book 10. University College, London, Strong Room Euclid Octavo 1659 (2), copy of Barrow, *Euclidis Elementorum Libri xv* (London, 1659) with added numerical and algebraic examples.

students.[57]

A further development in the same direction followed. A final and most effective way to reduce the copiousness of the Euclidean text to a brevity suitable for study in the seventeenth-century school or university was to re-write the proofs once again, translating them into symbolic terms. Pierre Hérigone in 1639 was the first to carry this to book-length publication; enormously important for British readers was the version of Euclid 'breviter demonstrati' using symbols by the Cambridge Lucasian Professor Isaac Barrow.[58] In fewer than four hundred pages he gave all fifteen books with shortened proofs in a symbolic style that was possibly more comprehensible to mathematical readers of his day than the verbal exposition of nearly all previous versions of the *Elements*. The evidence of surviving copies, and the number of subsequent editions of Barrow's version (five in Latin and three in English) makes it clear that this was the most successful version of the *Elements* among British readers since that of Clavius, and those copies bear all the characteristic marks of study from this period: correction, selection and rearrangement, and copying, translation and marginal rehearsal of the material. Isaac Newton was one of the many readers who participated in this new world of algebraic study of the *Elements*: his annotated copies survive in the library of Trinity College Cambridge.[59]

The algebraized Euclid was, furthermore, particularly suited to the mathematical schools of the period, bridging as it did the emerging divide between Euclidean education for gentlemen and algebra for tradesmen.[60] This was the period when the specialist mathematical school was beginning to emerge: not just the prestigious national institutions such as the Royal Mathematical School at Christ's Hospital (founded in 1673), but also a multitude of smaller private establishments, many of which specialized in mathematics. Towards the end of the seventeenth century – specifically after the passage of the misnamed Toleration Act of 1689 – the Dissenting Academies began to be

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[57] For example the preamble to Book 5 in the Clavius editions.

[58] Pierre Hérigone, *Cursus mathematicus* (Paris, 1634), the text of Euclid reprinted in 1636 as *Les six premiers liures des Elements d'Euclide*; Isaac Barrow, *Euclidis Elementorum libri xv* (Cambridge, 1655).

[59] Notably Trinity College, University of Cambridge, NQ.16.201[1], copy of Barrow, *Euclidis Elementorum libri xv* (London, 1655), with algebraic explanations added in books 2, 5 and 10.

[60] See Shelley Costa, 'The "Ladies' Diary": gender, mathematics, and civil society in early-eighteenth-century England', *Osiris* 17 (2002), 49–73.

set up, to provide a route to higher intellectual training for those unwilling to matriculate at one of the Anglican universities. Mathematics was certainly taught in some of these (although it was systematically neglected in others),[61] and when it was there was an emphasis on material of a practical character. The context for reader's attention to Euclid's *Elements* was quite evidently very different from what it had been even a generation before, let alone a century, and the final edition to become important for English readers was that of Claude-François Milliet Dechales (1682), popular in France with six editions, and printed four times in English, receiving the rare compliment of two rival translations in 1685. Dechales wrote that

> The greatest part of those that learn Euclid's Elements, are very often dissatisfied therewith, because they know not the use of Propositions so inconsiderable in appearance; and yet so difficult: I thought it might be to good purpose, not only to make them as easie as possible, but also to add some Uses after each Proposition, to shew how they are applicable to Practice.[62]

Applicability to practice would dominate one strand of geometrical teaching in Britain through the new century, while the *Elements* was retained in more elite educational establishments as a means of focussing the attention and training the mind in sound reasoning, a bifurcation that came to be reflected in editions algebraic or non-algebraic, textually faithful to the sources or revised and rearranged.

5. Conclusion

During the period around 1690 Edward Bernard, Savilian Professor of Astronomy, worked with a number of his Oxford colleagues on a project to produce a new edition of Euclid's *Elements*.[63] His

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[61] Irene Parker, *Dissenting Academies in England, their rise and progress and their place among the educational systems of the country* (Cambridge, 1914); David A. Reid, *Science and pedagogy in the dissenting academies of Enlightenment Britain* (Ph.D. thesis, University of Wisconsin-Madison, 1999).

[62] Claude-François Milliet Dechales, *Les elemens d'Euclide, expliquez d'une maniere nouvelle & très facile* (Paris, 1682), translated in Reeve Williams, *The elements of Euclid, explained and demonstrated in a new and most easy method* (London, 1685), [A]4^{r}.

[63] Evidence includes the correspondence of Bernard with Thomas Smith (Oxford, Bodleian Library, MSS Smith 47 and 57), various heavily annotated printed copies of the *Elements* (Bodleian Auct. S 1.12–15, 8° B 16 Linc. and 8° C

ambitious project would have printed improved Greek and Latin texts (and possibly an Arabic text as well), together with textual and mathematical commentary including in at least some cases translations of the propositons into algebraic terms. Bernard's project would thus have brought together several different types of attention to the text, from detailed improvement of the Latin translation and collation of the Greek text with Oxford manuscripts, through the identifying of mathematical errors and logical lacunae in commentary, through to the translation of propositions into algebraic terms. It would have articulated in a strong way what has been called the mathematical classicism of this period of British mathematics (Oxford scholars in fact planned a whole series of classicising editions of ancient mathematical texts in this period, but only a few ever actually appeared in print). It would also have been a hybrid edition, attempting to do several different kinds of work and meet the needs of several diferent kinds of reader. Its complexity burgeoned excessively, and Bernard was advised by his friend Thomas Smith that

> these are not times to furnish out or encourage such an august edition; for men in this degenerate age wil not learne Greeke, or buy mathematicks att so great an expense of time and study and mony too.[64]

And indeed the project was abandoned at some point in the 1690s, although much of Bernard's textual work – though not the Arabic, nor the algebra – found its way into the somewhat simpler edition issued at Oxford by David Gregory in 1704.

The conditions for the publication of new versions of Euclid's *Elements* were changing, and it was difficult for editors to keep up. The pervasive presence of the Euclidean text and the demand for new editions of it had not changed, however. Changing types of textual attention would continue to be paid to it for two more centuries before it dropped from the university curriculum. Readers continued to emend the text grammatically and mathematically, to restructure it for their own

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134 Linc.), a similarly annotated 1625 copy of the Data (Auct. S 2.22) and the bound set of printed plates for Bernard's intended edition (MS. Bodl. 887). See Philip Beeley's chapter in Philip Beeley, Yelda Nasifoglu and Benjamin Wardhaugh (eds), *Reading Mathematics in the Early Modern World* (forthcoming).

[64] Oxford, Bodleian Library, MS Smith 57, pp. 429–30: letter of Thomas Smith to Edward Bernard, 13 November 1694.

purposes and to translate its content into different terms, rehearsing their mathematical and sometimes philological skills in its margins. Its pervasiveness therefore makes it a unique case study for the changing uses of texts and of print in this period, as well as for the changing cultural profile of mathematics.

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