

# On Truncated Generalized Gibbs Ensembles in the Ising Field Theory

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**Abstract.** We discuss the implementation of two different truncated Generalized Gibbs Ensembles (GGE) describing the stationary state after a mass quench process in the Ising Field Theory. One truncated GGE is based on the semi-local charges of the model, the other on regularized versions of its ultra-local charges. We test the efficiency of the two different ensembles by comparing their predictions for the stationary state values of the single-particle Green's function  $G(x) = \langle \psi^\dagger(x)\psi(0) \rangle$  of the complex fermion field  $\psi(x)$ . We find that both truncated GGEs are able to recover  $G(x)$ , but for a given number of charges the semi-local version performs better.

## 1. Introduction

It is by now well established that after a quantum quench integrable lattice models relax locally to generalized Gibbs ensembles (GGE) [1–6], *cf.* Ref. [7] for a recent review. The situation for integrable quantum field theories (QFT) is generally more complicated [2, 8–11]. A key issue is that, in quantum field theories, expectation values of ultra-local conservation laws typically suffer from ultraviolet divergences and in order to define a GGE density matrix an appropriate regularization procedure is needed. Here we follow the terminology of Ref. [12] and call conservation laws ultra-local, if they can be written as integrals of operators that act only at a point

$$Q_n = \int dx J_n(x), \quad n \in \mathbb{N}. \quad (1)$$

A related issue is that these ultra-local conservation laws are generally not complete, in the sense that the GGE based on them may be unable to describe the stationary state after an arbitrary global quantum quench. In Ref. [10] we have shown that in integrable QFTs one can construct a different set of conserved charges that fulfil a weaker form of locality. The densities  $\mathcal{I}(x, x + \alpha)$  of these *semi-local charges*  $\ddagger I(\alpha)$  act non-trivially only on a given finite interval  $[x, x + \alpha]$

$$I(\alpha) = \int dx \mathcal{I}(x, x + \alpha), \quad \alpha \in \mathbb{R}^+. \quad (2)$$

In contrast to the ultra-local conservation laws, the  $I_\alpha$  typically do not suffer from ultraviolet divergences. In the integrable QFTs considered in [10] the completeness of the set  $\{I_\alpha\}$  derives from the fact that in the infinite volume the charges are in one-to-one correspondence with the “mode occupation numbers”  $n(k)$  of the elementary excitations that exhibit purely elastic scattering. The expectation values of the ultra-local charges  $Q_n$  encode only the moments of the mode distribution  $n(k)$  and therefore their knowledge may be not sufficient to reconstruct  $n(k)$ . In this case the GGE built from the  $Q_n$  would not be complete. In integrable QFTs that support bound states (corresponding to string solutions of the Bethe Ansatz equations) the situation is more complicated and has been recently analyzed in Ref. [11].

In order to construct GGEs in practice, the concept of *truncated GGEs* introduced in Ref. [5] has proven very useful. It is based on approximating the full GGE by considering only certain subsets of conserved charges. By choosing an appropriate sequence of such subsets, the full GGE can then be defined by a limiting procedure. The main purpose of this note is to show how to implement a truncation procedure for GGEs built from semi-local charges. To do this we focus on the simplest possible case of a mass quench in the Ising field theory. We complement these considerations by a construction of truncated GGEs formed with regularized ultra-local charges.

## 2. Ising QFT and its conserved charges

The Hamiltonian of the Ising field theory is

$$H(m) = \int dx \left( \frac{iv}{2} [R(x)\partial_x R(x) - L(x)\partial_x L(x)] + imR(x)L(x) \right), \quad (3)$$

$\ddagger$  In Ref. [10] we called the conservation laws (2) “quasi-local”. In order to avoid confusion with the use of this term for lattice models, *e.g.* see [13], we will refer to them as semi-local in the following.

where  $R(x)$  and  $L(x)$  are real right and left-moving fermion fields. The model is diagonalised by the Bogoliubov transformation

$$R(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \sqrt{\frac{\omega(k) + vk}{2\omega(k)}} [e^{i\frac{\pi}{4}} Z(k)e^{-ixk} + \text{h.c.}] , \quad (4)$$

$$L(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \sqrt{\frac{\omega(k) - vk}{2\omega(k)}} [e^{-i\frac{\pi}{4}} Z(k)e^{-ixk} + \text{h.c.}] , \quad (5)$$

where  $\{Z(k), Z^\dagger(q)\} = 2\pi\delta(k - q)$ . The Hamiltonian takes the form

$$H(m) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \underbrace{\sqrt{m^2 + v^2k^2}}_{\omega(k)} Z^\dagger(k)Z(k) . \quad (6)$$

It is obvious from (6) that the mode occupation numbers

$$N(k) = Z^\dagger(k)Z(k) \quad (7)$$

are conserved quantities. The Ising field theory has a set of conserved ultra-local charges of the form

$$Q_n^\pm = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \epsilon_n^\pm(k) N(k), \quad (8)$$

$$\epsilon_n^+(k) = \omega(k)k^{2n}, \quad \epsilon_n^-(k) = vk^{2n+1}. \quad (9)$$

It follows from the expressions for  $\epsilon_n^\pm(k)$  that expectation values of  $Q_n^\pm$  for sufficiently large  $n$  will generically suffer from ultra-violet divergences. Moreover, the functions  $k^m$  do not form a basis on the *infinite* interval  $k \in (-\infty, +\infty)$ , which poses the question whether the set  $\{Q_n^\pm\}$  is complete for constructing GGEs.

Applying the general construction of Ref. [10] to the Ising field theory results in two continuous families of semi-local conserved charges

$$I^\pm(\alpha) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \epsilon^\pm(k, \alpha) N(k), \quad (10)$$

$$\epsilon^+(k, \alpha) = \omega(k) \cos(k\alpha), \quad \epsilon^-(k, \alpha) = \sin(k\alpha) . \quad (11)$$

These charges are clearly in one-to-one correspondence with the mode occupation operators  $N(k)$ . It has been brought to our attention that the fact that (10) form a set of conserved charges for the Ising field theory has been previously noted in Ref. [14].

### 3. Mass quench

In the following we focus on a particular quantum quench, the one in which we prepare the system in the ground state  $|\Psi_0\rangle$  of  $H(m_0)$  and at time  $t = 0$  suddenly quench the mass parameter from  $m_0$  to  $m$ . The time evolution following this quench is induced by  $H(m)$ . This quench protocol has been previously studied in detail both on the lattice and in the continuum [4–6, 15]. In order to work out the occupation number densities we put our system into a large, finite box of length  $L$ . In this case, the finite volume Bogoliubov fermion creation/annihilation operators fulfil

$$\{Z_L^\dagger(k_n), Z_L(k_m)\} = L\delta_{n,m}, \quad k_n = \frac{2\pi n}{L}. \quad (12)$$

Denoting the ground state of  $H(m_0)$  in the finite volume by  $|\Psi_0\rangle_L$ , the mode occupation number densities are [15]

$$\frac{1}{L} \frac{\langle \Psi_0 | Z_L^\dagger(k_n) Z_L(k_n) | \Psi_0 \rangle_L}{L \langle \Psi_0 | \Psi_0 \rangle_L} = \frac{|K(k_n)|^2}{1 + |K(k_n)|^2} \equiv n(k_n), \quad (13)$$

where

$$K(k) = \tan \left[ \frac{1}{2} \arctan(vk/m) - \frac{1}{2} \arctan(vk/m_0) \right]. \quad (14)$$

We note that the mode occupation number densities are even functions of momentum,  $n(k) = n(-k)$ . In order to construct GGEs we require the expectation values on the initial state  $|\Psi_0\rangle_L$  of the conserved charges. For the ultra-local conserved charges, the expectation values suffer from ultraviolet divergences, which can be cured by regularizing the theory in terms of a sharp momentum cutoff  $\Lambda$ . This means neglecting, in any physical quantity, all contributions coming from modes higher than  $\Lambda$ . As we are dealing with a free theory, in which the modes are decoupled, this regularization does not spoil the integrability of the original theory. The ‘‘initial data’’ is then given by

$$\begin{aligned} i^\pm(\alpha) &= \lim_{L \rightarrow \infty} \frac{1}{L} \frac{\langle \Psi_0 | I^\pm(\alpha) | \Psi_0 \rangle_L}{L \langle \Psi_0 | \Psi_0 \rangle_L} = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \epsilon^\pm(k, \alpha) n(k), \\ q_{n,\Lambda}^\pm &= \lim_{L \rightarrow \infty} \frac{1}{L} \frac{\langle \Psi_0 | Q_{n,\Lambda}^\pm | \Psi_0 \rangle_L}{L \langle \Psi_0 | \Psi_0 \rangle_L} = \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \epsilon_n^\pm(k) n(k). \end{aligned} \quad (15)$$

Here we have defined

$$Q_{n,\Lambda}^\pm = \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \epsilon_n^\pm(k) N(k). \quad (16)$$

We note that, unlike the original ultra-local charges  $Q_n^\pm$ , their regularized versions are no longer ultra-local but have instead densities that act non-trivially on the entire system. Moreover, in the presence of the cut-off, the functions  $\epsilon_n^\pm(k)$  form a basis on the finite interval  $[-\Lambda, \Lambda]$ , and therefore for the cutoff theory we expect that the charges  $Q_{n,\Lambda}^\pm$  are also able to construct a GGE. It is then interesting to compare the results obtained from the GGE’s constructed from the two sets of conserved charges, namely the semi-local ones and the regularized version of the ultra-local charges.

As  $n(k)$  is an even function the expectation values of  $I^-(\alpha)$  and  $Q_{n,\Lambda}^-$  are identically zero, and these charges do not play a role in the respective GGEs, *cf.* Ref. [5]. From here on we focus on the following Green’s function (other Green’s functions can be analysed in the same way)

$$G(x, t) = \lim_{L \rightarrow \infty} \frac{L \langle \Psi_0(t) | \psi^\dagger(x) \psi(0) | \Psi_0(t) \rangle_L}{L \langle \Psi_0 | \Psi_0 \rangle_L}, \quad \psi(x) = \frac{1}{\sqrt{2}} (R(x) + iL(x)). \quad (17)$$

At late times,  $G(x > 0, t)$  approaches stationary values

$$\mathfrak{g}(x) = \lim_{t \rightarrow \infty} G(x, t) = \int_0^\infty \frac{dk}{2\pi} \frac{m \cos(kx)}{\omega(k)} (2n(k) - 1), \quad x > 0. \quad (18)$$

The question we are interested is how well  $\mathfrak{g}(x)$  can be described by the two different kinds of truncated GGEs based on the sets  $\{I_\Lambda(\alpha)\}$  and  $\{Q_{n,\Lambda}\}$  respectively.

## 3.1. Truncated GGE built from semi-local charges

Following the rationale of Ref. [5], we now wish to construct a truncated GGE by retaining only a finite subset of the conserved charges  $I^+(\alpha)$

$$\rho_{\text{sl},M} = \frac{1}{\mathcal{Z}_{\text{sl},M}} \exp \left( - \sum_{m=1}^M \lambda^+ (\alpha_m) I^+(\alpha_m) \right), \quad (19)$$

where the Lagrange multipliers are fixed by the conditions

$$\lim_{L \rightarrow \infty} \frac{1}{L} \frac{\langle \Psi_0 | I^+(\alpha_m) | \Psi_0 \rangle_L}{\langle \Psi_0 | \Psi_0 \rangle_L} = \lim_{L \rightarrow \infty} \frac{1}{L} \text{tr} (\rho_{\text{sl},M} I^+(\alpha_m)), \quad m = 1, 2, \dots, M. \quad (20)$$

Eqns (20) do not require regularization except for  $\alpha_m = 0$ , which corresponds to the energy density and which is ultraviolet divergent for the mass quench considered. It is important to take the energy density into account, as the semi-local conservation laws with  $\alpha \neq 0$  cannot distinguish between eigenstates characterized by a density  $n(k)$  and eigenstates with density  $n(k) + c/\omega(k)$  because

$$\int_{-\infty}^{\infty} \frac{dk}{2\pi} \epsilon^+(k, \alpha) \frac{c}{\omega(k)} = 0, \quad \alpha \neq 0. \quad (21)$$

Given that the energy density is ultraviolet divergent for the mass quench, one could simply replace it by the density itself. This would remove the ambiguity (21). However, in order to solve the initial conditions (20), it is in practice very helpful to introduce a cutoff. We therefore proceed by introducing semi-local charges with a cutoff  $\Lambda$

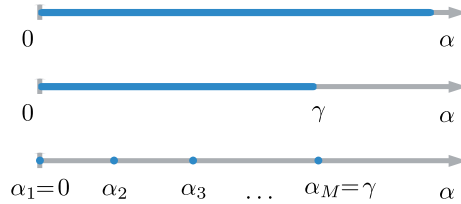
$$I_{\Lambda}^{\pm}(\alpha) = \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \epsilon^{\pm}(k, \alpha) N(k), \quad (22)$$

and then solve the initial conditions

$$\lim_{L \rightarrow \infty} \frac{1}{L} \frac{\langle \Psi_0 | I_{\Lambda}^+(\alpha_m) | \Psi_0 \rangle_L}{\langle \Psi_0 | \Psi_0 \rangle_L} = \lim_{L \rightarrow \infty} \frac{1}{L} \text{tr} (\rho_{\text{sl},M} I_{\Lambda}^+(\alpha_m)), \quad m = 1, 2, \dots, M. \quad (23)$$

This set of coupled nonlinear equations can be efficiently solved numerically by increasing the number of charges  $M$  by one at a time. The set  $\{\alpha_k | k = 1, \dots, M\}$  that defines the truncated GGE is conveniently chosen as shown in Fig. 1

$$\alpha_j = (j-1) \frac{\gamma}{M-1}, \quad j = 1, \dots, M. \quad (24)$$



**Figure 1.** The truncation scheme of the semi-local GGE is implemented firstly imposing a fixed cut-off in  $\alpha$ , denoted by  $\gamma$ , and then discretising the interval  $[0, \gamma]$  in  $M$  equidistant points.

Once the Lagrange multipliers  $\lambda + (\alpha_m)$  have been computed we can evaluate the expectation value

$$\begin{aligned} \mathfrak{g}_{\text{sl},M}(x) &= \text{tr}\left(\rho_{\text{sl},M}\psi^\dagger(x)\psi(0)\right) = \int_0^\infty \frac{dk}{2\pi} \frac{m \cos(kx)}{\omega(k)} (2n_{\text{sl},M}(k) - 1), \quad x > 0, \\ n_{\text{sl},M}(k) &= \left[1 + \exp\left(-\sum_{m=1}^M \lambda^+(\alpha_m)\epsilon^+(k, \alpha_m)\right)\right]^{-1}. \end{aligned} \quad (25)$$

### 3.2. Truncated GGE built from (regularized) ultra-local charges

One can repeat the construction of the previous section but this time using the regularized ultra-local charges (16). The density matrix is now

$$\rho_{\text{ul},M} = \frac{1}{\mathcal{Z}_{\text{ul},M}} \exp\left(-\sum_{m=1}^M \lambda_m^+ Q_{m,\Lambda}^+\right), \quad (26)$$

and the Lagrange multipliers  $\lambda_m^+$  are fixed by the conditions

$$\lim_{L \rightarrow \infty} \frac{1}{L} \frac{\langle \Psi_0 | Q_{m,\Lambda}^+ | \Psi_0 \rangle_L}{\langle \Psi_0 | \Psi_0 \rangle_L} = \lim_{L \rightarrow \infty} \frac{1}{L} \text{tr}\left(\rho_{\text{ul},M} Q_{m,\Lambda}^+\right), \quad m = 1, 2, \dots, M. \quad (27)$$

A direct numerical solution of (27) is difficult, because the expectation values of  $Q_{m,\Lambda}^+$  increase rapidly with the index  $m$  and there is a lack of continuity in the values of the Lagrange multipliers  $\lambda_m^+$  with respect to the total number of charges, i.e. the solutions  $\{\lambda_m^+\}$  for the ensembles with  $M$  and  $M + 1$  charges will differ significantly. In order to obtain a stable procedure we find it convenient to work with particular linear combinations of the charges  $Q_{n,\Lambda}^+$

$$\tilde{Q}_m^+ = 2 \int_0^\Lambda \frac{dk}{2\pi} \omega(k) T_{2(m-1)}(k/\Lambda) N(k), \quad (28)$$

where  $T_m(x)$  are Chebyshev polynomials. In practice we re-express (26) and (27) in terms of these linear combinations and solve for the corresponding Lagrange multipliers. From these we can then determine the  $\lambda_m^+$ . The Green's function takes the same form as in (25) but now with the ultra-local density matrix

$$\begin{aligned} \mathfrak{g}_{\text{ul},M}(x) &= \text{tr}\left(\rho_{\text{ul},M}\psi^\dagger(x)\psi(0)\right) = \int_0^\Lambda \frac{dk}{2\pi} \frac{m \cos(kx)}{\omega(k)} (2n_{\text{ul},M}(k) - 1), \quad x > 0, \\ n_{\text{ul},M}(k) &= \left[1 + \exp\left(-\sum_{m=1}^M \lambda_m^+ \epsilon_m^+(k)\right)\right]^{-1}. \end{aligned} \quad (29)$$

## 4. Results

In the following we present results for two quench processes (i) a *small quench*  $m_0 = m/2 \rightarrow m$ ; (ii) a *large quench*  $m_0 = 5m \rightarrow m$ . Here large/small is measured by the density of excitations  $n(k)$  after the quench, cf. Refs [4]. In order to better distinguish between the different ensembles we focus on the dimensionless mode-distribution dependent part of the Green's functions

$$g(x) = 2v \int_0^\infty \frac{dk}{2\pi} \frac{\cos(kx)}{\omega(k)} n(k). \quad (30)$$

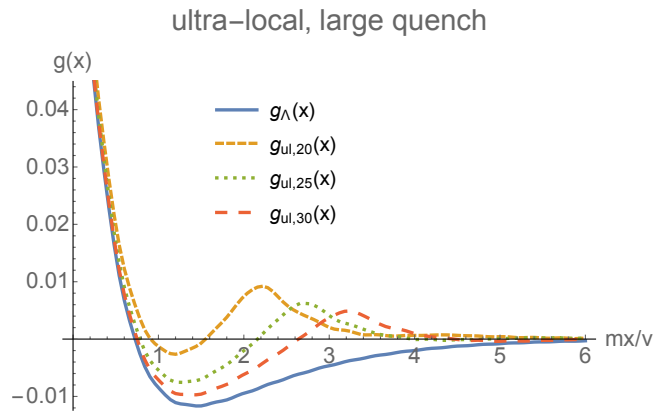
The cutoff  $\Lambda$  entering our construction of the ensembles is chosen such that in the spatial range considered in our plots the function

$$g_\Lambda(x) = 2v \int_0^\Lambda \frac{dk}{2\pi} \frac{\cos(kx)}{\omega(k)} n(k) \quad (31)$$

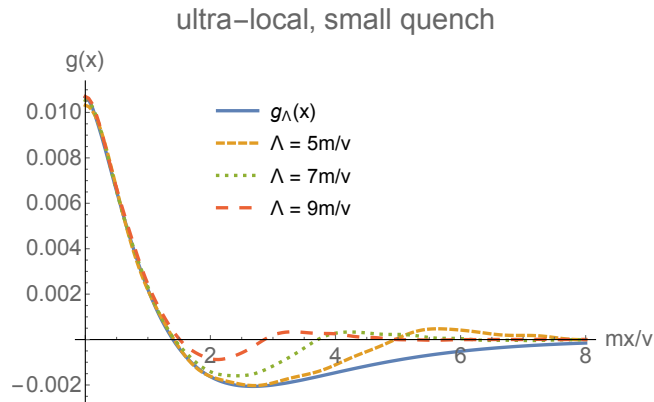
reproduces the full answer  $g(x)$  up to errors that are negligible on the scale of our plots. The values we use are  $\Lambda = 5m/v$  for the small quench and  $\Lambda = 20m/v$  for the large quench.

#### 4.1. Truncated ultra-local GGE

Our truncated ultra-local GGEs are characterized by the number  $M$  of conserved charges and the momentum cutoff  $\Lambda$ . We first analyze the dependence on  $M$  for a fixed value of  $\Lambda$ .



**Figure 2.** Truncated ultra-local GGE predictions  $g_{ul,M}(x)$  for increasing numbers  $M$  of ultra-local charges. Here the cutoff is fixed at  $\Lambda = 5m/v$ . We see that the range in  $mx/v$  over which the truncated GGE provides an accurate description of the stationary state result  $g_\Lambda(x)$  grows as  $M$  is increased.



**Figure 3.** Dependence of the truncated ultra-local GGE predictions  $g_{ul,M}(x)$  with  $M = 15$  charges on the cutoff  $\Lambda$ .

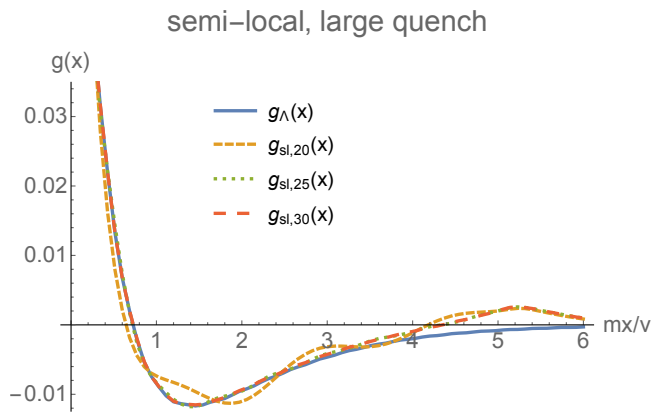
Fig. 2 shows the effect of increasing the number of charges  $M$  employed in the ultra-local GGE, while keeping the cutoff fixed. We see that increasing  $M$  extends the region in  $mx/v$ , over which the truncated GGE provides an accurate approximation  $g_{\text{UL},M}(x)$  of the stationary Green's function  $g_\Lambda(x)$ . We next investigate the cutoff dependence at a fixed number  $M$  of charges. We consider a small quench so that a small number of charges already provides good results. As shown in Fig. 3 increasing  $\Lambda$  at fixed  $M$  (and of course adjusting the Lagrange multipliers accordingly) gives a truncated GGE that provides a good description of  $g(x)$  over a *decreasing* range in  $mx/v$ .

We may summarize our results for truncated ultra-local GGEs as follows:

- (i) In order to obtain an accurate description of  $g(x)$  on a given range  $[0, x_0]$  we require a sufficiently large cutoff  $\Lambda$ .
- (ii) Given  $\Lambda$  one needs to increase the number of ultra-local charges  $M$  until convergence of  $g_{\text{ul},M}(x)$  on the interval  $[0, x_0]$  has been achieved. The larger  $\Lambda$ , the higher  $M$  needs to be.

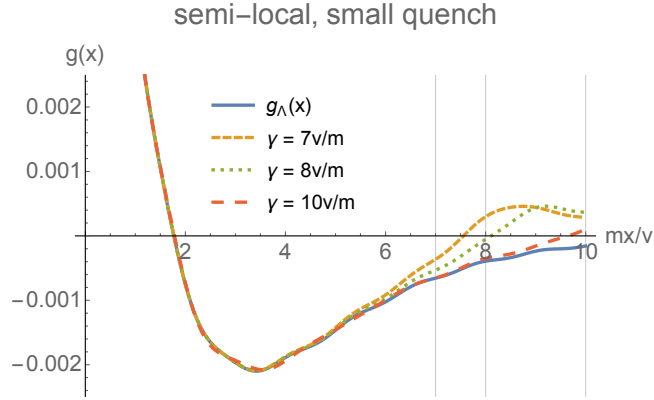
#### 4.2. Truncated semi-local GGE

Our truncated semi-local GGEs depend on three parameters: (i) the number of charges  $M$ ; (ii) the ‘‘spread’’  $\gamma$ ; (iii) the cutoff  $\Lambda$ . We find that, unlike in the ultra-local case, the cutoff dependence does not play an important role and we therefore exclude it from further discussion. To proceed we simply fix the cutoff  $\Lambda$  a sufficiently large value to ensure that the difference  $|g_\Lambda(x) - g(x)|$  is negligible on the interval  $[0, x_0]$  of interest. We first consider the  $M$ -dependence at a fixed value of  $\gamma$ , cf. Fig. 4.



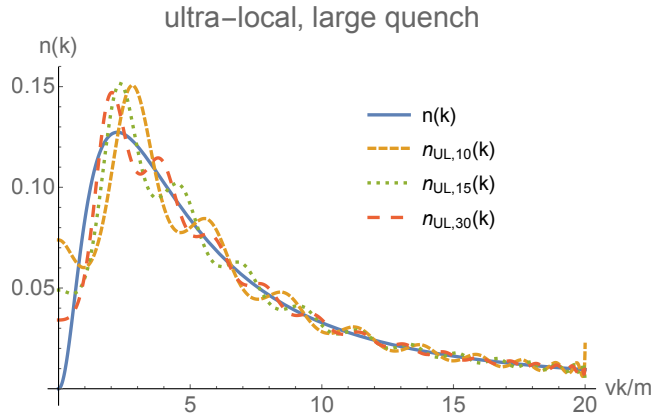
**Figure 4.**  $M$ -dependence of the truncated semi-local GGE prediction for the Green's function. The cutoff has been fixed at  $\Lambda = 20m/v$  and the spread is  $\gamma = 5v/m$ .

We observe that, as long as the separation  $x$  is sufficiently much smaller than the spread  $\gamma$ , increasing  $M$  leads to a more accurate description of the stationary state Green's function. As soon as  $x \sim \gamma$  increasing  $M$  beyond a certain value ceases to improve the accuracy of the corresponding truncated GGEs. In Fig. 5 we show results for the  $\gamma$ -dependence of the truncated semi-local GGE for a fixed number of charges. We consider a small quench, for which  $M = 15$  already produces accurate results. We see that the effect of increasing  $\gamma$  is to increase the spatial interval, over which the truncated GGE provides a good approximation to the exact stationary Green's function.



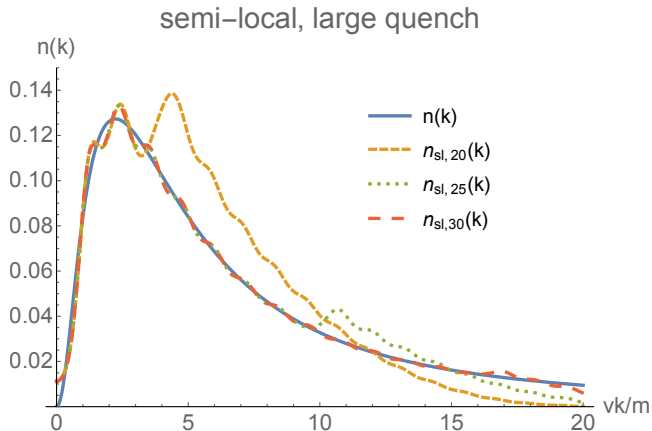
**Figure 5.**  $\gamma$ -dependence of the truncated semi-local GGE prediction for the Green's function  $g_{\text{sl},M}(x)$  for fixed number of conserved charges  $M = 15$ .

#### 4.3. Mode occupation number densities



**Figure 6.** Dependence of the mode occupation densities on the number of charges in truncated ultra-local GGE.

In the generalized micro-canonical ensemble [6, 16] the stationary state after a quantum quench to an integrable model is described by a simultaneous eigenstate of all conservation laws [6]. In general this state is characterized by its Bethe Ansatz root distribution functions. In our particular case of the Ising field theory this amounts to specifying the mode occupation number density  $n(k)$ . It is therefore an interesting question, how well our truncated GGEs reproduce the exact  $n(k)$  that describes the stationary state. In Figs. 6 and 7 we show results for large quench. We see that both our truncated GGE's give a reasonable account of  $n(k)$ . The quality of the the truncated GGE predictions for  $n(k)$  depends on the number of charges: increasing the number of ultra-local charges gives a better overall match in the interval  $[0, \Lambda]$  (see Fig. 6), while increasing number of semi-local charges extends the region of accuracy in  $k$  (see Fig. 7). We note that above we always considered spreads such that  $\gamma > \Lambda^{-1}$ . In this



**Figure 7.** Dependence of the mode occupation densities on the number of charges in the truncated semi-local GGE with spread  $\gamma = 5v/m$ .

regime the cutoff essentially regularizes the energy density without influencing other semi-local charges. We find that the mode occupation number densities have only a very weak  $\gamma$  dependence.

## 5. Conclusions

We have considered the implementation of truncated GGEs [5] for describing the stationary state after quantum quenches to integrable field theories, focussing on the simple case of a mass quench in the Ising field theory. Following Ref. [10] we have constructed two kinds of GGEs, built from ultra-local and semi-local conservation laws respectively. We have compared the predictions of the two ensembles for the two-point function  $G(x) = \langle \psi^\dagger(x)\psi(0) \rangle$  of Fermi fields in the stationary state reached at late times after the quench. We find that once appropriate regularization procedures have been put in place, both ensembles give a good account of  $G(x)$ , although, on balance, the ensemble built from semi-local charges performs slightly better. It would be interesting to generalize the results presented here to a fully interactive field theory such as a Lieb-Liniger model [18]. Interaction quenches in this model have been investigated by a number of groups [9]. A key difficulty compared to the simple case studied here is that correlations functions of interactive theories as the the Lieb-Liniger model cannot be calculated analytically, and one needs to resort to numerical methods such as ABACUS [19].

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