

# WIMP matter power spectra and small scale power generation

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## ABSTRACT

Dark Matter (DM) is generally assumed to be massive, cold and collisionless from the structure formation point of view. Although this translates into a scale-free power-law matter power spectrum, a more correct statement is that DM indeed experiences collisional damping, but on a scale which is supposed to be too small to be relevant for structure formation. The aim of this paper is to present a Cold (although “collisional”) Dark Matter candidate whose matter power spectrum is damped, discuss its consequences on structure formation and see whether it is distinguishable from standard candidates. To achieve this purpose, we first calculate the collisional damping and free-streaming scales of conventional (namely neutralinos) and non conventional DM candidates (say light particles heavier than  $\sim 1$  MeV but lighter than  $O(10)$  GeV). The latter are supposed to annihilate to get the correct relic density and can be considered as Cold Dark Matter (CDM) particles in the sense that they become non relativistic before their thermal decoupling epoch. Unlike neutralinos, however, the linear matter power spectrum of light DM candidates can be damped on scales of  $\sim 10^3 M_\odot$  (due to their interactions). Since these scales are of cosmological interest for structure formation, we then perform a series of numerical simulations to obtain the corresponding non linear matter power spectra  $P(k)_{nl}$  at the present epoch. We show that because of small scale regeneration, they all resemble each other at low redshifts, *i.e.* become very similar to a typical CDM matter power spectrum on all but the smallest scales. Therefore, even if lensing measurements (at redshift below unity) were to yield a  $P(k)_{nl}$  consistent with CDM models, this would not constitute a sufficiently robust evidence in favour of the neutralino to rule out alternative DM candidates, even if their linear matter power spectrum is damped on relatively large scales.

## 1 INTRODUCTION

Despite decades of improvement in observations and experimental results, the nature of DM is still under investigation. It is often claimed that the accuracy of the measurement of the Cosmic Microwave Background spectrum up to the second acoustic peak provides very strong evidence in favour of CDM scenarios. However other DM models, *e.g.* Warm Dark Matter (WDM) (Schaeffer & Silk 1988), or scenarios in which DM has non-negligible interactions with photons (IDM) (Boehm et al. 2002c), also predict an identical CMB spectrum to that measured so far.

In contrast with CDM, however, WDM particles do have a cut-off in the linear matter power spectrum ( $P(k)_l$ ), at a scale fixed by the warmon mass (the linear matter power spectrum of IDM also has a cut-off but it is much less drastic than in WDM and fixed by the ratio of the elastic scattering cross section of DM with photons or neutrinos to its mass). Because of this cut-off, WDM particles lighter than  $\sim 1$  keV (Narayanan et al. 2000) or 10 keV (Yoshida et al. 2003), which experience damping on scales smaller than  $\sim 10^9 M_\odot$  or  $\approx 10^4 M_\odot$  respectively, would be

excluded by the WMAP detection of reionization at large redshift (Kogut et al. 2003) should it be confirmed (the major caveat being the efficiency of reionization sources see *e.g.* Knebe et al. (2003)). On the other hand, it might be difficult to discriminate among typical CDM, WDM or any kind of particles whose linear matter power spectrum is damped at cosmological scales by using non-linear matter power spectra only, because of the small-scale regeneration mechanism which tends to make the WDM and CDM  $P(k)_{nl}$  very similar, at least at redshifts  $z \leq 1$ . Thus, the determination of the matter power spectrum by lensing measurements (Mellier et al. 2002) would probably fail to constrain the nature of DM, although there might exist other properties (like the structural parameters of DM haloes) on which lensing observations may set tighter constraints.

Although a warmon with mass just above  $\sim 10$  keV is still a viable possibility, CDM remains the most popular scenario, mostly because:

- it is generally considered as the most simple and natural solution. Indeed, it does not rely on any particle physics

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parameter, in contrast with the WDM scenario where results depend on the warm mass,

- the most popular CDM candidate – namely neutralinos (Fayet 1977) – is still allowed by cosmology and particle physics while WDM candidates proposed so far, *e.g.* gravitinos or sterile neutrinos, appear much more constrained and very close to be excluded unless one invokes peculiar mechanisms (Baltz & Murayama 2003; Hansen et al. 2002).

Nevertheless, it is worth to keep in mind that neutralinos (or any other well-known CDM particle candidate) have not been discovered so far, so there is room for other possibilities.

Coming back to structure formation, DM particles are assumed to be fairly massive in order to avoid large free-streaming effects that would wash out the DM primordial fluctuations (Davis et al. 1981). Furthermore, to prevent the Silk damping mechanism (Silk 1967; Silk 1968) from operating, one requires that they should not have any electromagnetic interactions. This has actually led some authors (Gunn et al. 1978; Peebles 1982) to propose, in the early eighties, that DM could be made of Weakly Interacting Massive particles (WIMPs), a statement which is now commonly accepted. Because of this “weakly interacting” property, one generally neglects DM interactions in numerical simulations. As a result Dark Matter, and WIMPs in particular, are said to be collisionless regarding structure formation, their linear matter power spectra being generally taken as a “true” scale-invariant spectrum with no cut-off.

Obviously, such a statement is a bit too drastic. If they are produced thermally, the DM particles must have non-negligible interactions (weak or not, provided they are not electromagnetic) which might have consequences on structure formation. Those interactions turn out to be also very useful as they are at the basis of direct and indirect experimental detection of DM candidates (CRESST collaboration 2001; Barrau 2001). So it is worth to try to assess their detailed impact on structure formation, instead of neglecting them *a priori*.

DM elastic scattering cross sections turn out to be important for structure formation because they fix the epoch at which Dark Matter thermally decouples, *i.e.* they specify the epoch at which the collisional damping stops and the free-streaming starts. It is therefore of crucial importance to determine the strongest interactions a WIMP can have and what are their effects on structure formation. This eventually provides a definition of the particles really permitted by structure formation models, thereby extending the notion of WIMPs to all particles having interactions that are not necessarily weak but small enough so as to preserve fluctuations on scales where we know they exist (Boehm et al. 2001). As a result, this opens new possibilities for Cold DM candidates, bearing in mind that their  $P(k)_l$  would then differ from a “true” scale invariant spectrum.

The aim of this paper is to answer the question as to whether or not structure formation can distinguish standard CDM candidates like neutralinos from other more exotic alternatives. To this purpose, we first review the different damping mechanisms that can affect DM linear matter power spectra, with emphasis on the “mixed” damping mechanism introduced by Boehm et al. (2001). We then

compute the damping scales of neutralino primordial fluctuations and compare them with those of lighter DM particles (having a mass  $O(10 \text{ GeV}) > m_{dm} > O(\text{GeV})$ ). The latter indeed turn out to be viable Dark Matter candidates provided their dominant annihilation cross section are S-wave suppressed so as to satisfy both the relic density criteria and the gamma ray/radio constraints. This condition can be achieved for example by introducing a “new” kind of interaction (not based on the exchange of Standard Model gauge bosons) that would have eluded experimental searches up to now (Boehm & Fayet 2003).

The light DM  $P(k)_l$  being potentially damped on scales relevant to structure formation, we perform four WDM simulations<sup>1</sup> with different scale cut-offs and one CDM numerical simulation for the sake of comparison. We present the evolution of the non-linear matter power spectra with redshift for all these models in section 4 where we perform a detailed comparison of our WDM models with the typical standard CDM spectrum.

In light of these numerical results, we conclude that even if lensing observations favoured a standard CDM  $P(k)_{nl}$ , they would not exclude the existence of a cut-off in the linear matter power spectrum, unless they could probe the distribution of matter up to very small scales with great accuracy. Therefore, we emphasize that it is really difficult to rule out “exotic” DM models on the basis of the non linear matter power spectra alone.

## 2 DAMPING LENGTHS

The objects we know (*e.g.* galaxies, clusters of galaxies) are thought to form by gravitational collapse of primordial density fluctuations. However, in order to grow significantly, these fluctuations need to survive well-known damping mechanisms that would wash them out. If the dominant species that constitutes these fluctuations was a light neutrino or any relativistic species, then the relevant damping mechanism would be free-streaming. This free-streaming would generate a cut-off in the linear matter power spectrum at very large scales and eventually turn out to be in contradiction with small-scale observations since all galaxies would have to be created out of pancake and/or filament fragmentation. A cut-off at large scale is also expected if the main component of these matter density fluctuations is baryonic matter, because of their very large interactions with photons; this is known as Silk damping. Since both these models appear in contradiction with observations, there is a need for another kind of matter, called DM, that would be massive enough to avoid significant free-streaming and without any “standard” electromagnetic interactions.

In this paper, we argue that — quite similarly to WDM

<sup>1</sup> Light DM  $P(k)_l$  are in fact expected to be oscillating. Also one expects some power at very small scales (albeit much less than in CDM scenarios). Therefore, they may appear intermediate between standard CDM and WDM linear matter power spectra. However, we use a WDM  $P(k)_l$  to perform our simulations so as to make the point regarding the small-scales regeneration mechanism.

scenarios where the free-streaming length can be as large as  $10^6 - 10^9 M_\odot$  — there exist realistic scenarios where the collisional damping lengths of CDM can be significant (at least up to  $10^3 M_\odot$ ). These DM particles would be Cold in the sense that they annihilate and decouple after they become non-relativistic, but on the other hand they would have a linear matter power spectrum very different from what is generally expected. We also show that another damping effect, called mixed damping, can become particularly important in the case of light DM particles and is responsible for this peculiar behaviour.

The latter can be understood as an intermediate effect between collisional damping and free-streaming. It takes place when DM is thermally coupled to a species  $i$  that is already free-streaming. This means that it basically starts when species  $i$  thermally decouples (time  $t_{dec(i)}$ ) and lasts until DM stops being in thermal equilibrium with  $i$  (which occurs at time  $t_{dec(dm-i)}$ ). The condition for the mixed damping to take place is therefore written as:  $t_{dec(dm-i)} > t_{dec(i)}$ .

In the following, we shall focus on  $i = \nu$ , since DM–neutrino interactions are likely to satisfy the previous relationship, *i.e.*

$$t_{dec(dm-\nu)} > t_{dec(\nu)}$$

or, equivalently,

$$T_{dec(dm-\nu)} < T_{dec(\nu)},$$

where  $T_{dec(dm-\nu)}$  is the temperature at which the  $dm-\nu$  interactions stop having an effect on the DM fluid and  $T_{dec(\nu)}$  is the neutrino temperature at their decoupling (say in a standard scenario:  $T_{dec(\nu)} = T_{dec(\nu-e)} \sim 1 \text{ MeV}$ ).

The mixed damping effect has never been considered before because its existence has only been recently pointed out by Boehm et al. (2001) (and its physical relevance by Boehm & Fayet (2003)) but also because DM candidates are generally thought to have thermally decoupled before or at the onset of 1 MeV (*i.e.*  $T_{dm-\nu} > T_{dec(\nu-e)}$ ) so that the condition for this damping to occur was thought to never be satisfied.

Let us now suppose that we are in a scenario where  $T_{dec(dm-\nu)} < T_{dec(\nu)}$ . The damping scale of DM primordial fluctuations then originates from i) the collisional damping effect due to the coupling of DM particles with species that are still collisional (this actually includes the neutrino contribution for  $T \gtrsim T_{dec(\nu)}$ ), ii) the mixed damping effect, originating from interaction with neutrinos and acquired during the period  $[t_{dec(\nu)}, t_{dec(dm-\nu)}]$ , and finally iii) the DM free-streaming for DM temperatures below  $T_{dec(dm)}$  (in fact  $T_{dec(dm-\nu)}$  if  $dm - \nu$  are the strongest DM interactions). The associated damping scales will be denoted  $l_{cd}$ ,  $l_{md}$ ,  $l_{fs}$  respectively. Their expressions are given (according to Boehm et al. (2001)Boehm et al. (2002b)) by<sup>2</sup>:

$$l_{cd}^2 \propto \int^{t_{dec(dm)}} \frac{\rho_{dm} v_{dm}^2}{\rho \Gamma_{dm} a^2} dt + \int^{t_X} \frac{\rho_\nu c^2}{\rho \Gamma_\nu a^2} dt + \sum_{i \supset e, \gamma \dots} \int^{t_{dec(dm-i)}} \frac{\rho_i v_i^2}{\rho \Gamma_i a^2} dt \quad (1)$$

$$l_{md}^2 \propto \int_{t_{dec(\nu)}}^{t_{dec(dm-\nu)}} \frac{\rho_\nu c^2}{\rho H a^2} dt \sim \left(\frac{ct}{a}\right)^2 \Big|_{t_{dec(dm-\nu)}} \quad (2)$$

$$l_{fs} \propto \int_{t_{dec(dm)}}^{t_0} \frac{v}{a} dt \sim \text{Max} \left(\frac{ct}{a}\right) \Big|_{t_{dec(dm)}-t_0} \quad (3)$$

with  $i$  denoting all the species that can transmit their collisional damping to DM fluctuations,  $\Gamma_{dm}$  the total DM interaction rate,  $\Gamma_i = \sum_j \Gamma_{i-j}$  the sum of the partial interaction rates of species  $i$  ( $j$  being the species in thermal equilibrium with  $i$ ),  $H$  the Hubble constant and  $a = a(t)$  the scale-factor at a time  $t$ . Here  $t_X$  denotes either  $t_{dec(dm-\nu)}$  if  $t_{dec(dm-\nu)} < t_{dec(\nu)}$  (in which case there is no mixed damping since DM thermally decouples from neutrinos before they enter their free-streaming regime) or  $t_{dec(\nu)}$  if  $t_{dec(dm-\nu)} > t_{dec(\nu)}$  (since in this case the integration of  $1/\Gamma_\nu$  for  $t > t_{dec(\nu)}$  does not have any meaning). We emphasize that eq.(1) and eq.(2) supposes implicitly that  $t_{dec(dm-i)} < t_{dec(i)}$  for  $i \neq \nu$ , or in other words, that the mixed damping species is only due to neutrinos.

We also draw attention to the fact that we wrote  $l_{md}$  and  $l_{cd}$  as two separate contributions when in fact, the mixed damping (eq.2) is part of the collisional damping (eq.1). Thus, even if  $t_{dec(dm-\nu)} > t_{dec(\nu)}$ , we should write only one contribution, namely:

$$l_{cd(dm-\nu)}^2 \propto \int^{t_{dec(\nu)}} \frac{\rho_\nu c^2}{\rho \Gamma_\nu a^2} dt + \int_{t_{dec(\nu)}}^{t_{dec(dm-\nu)}} \frac{\rho_\nu c^2}{\rho H a^2} dt$$

with

$$l_{cd}^2 \lesssim \int^{t_{dec(dm)}} \frac{\rho_{dm} v_{dm}^2}{\rho \Gamma_{dm} a^2} dt + \sum_{x \supset \nu, \gamma \dots} l_{cd(dm-x)}^2,$$

but we will avoid this notation in what follows (and come back to eq.1 and eq.2) to underline the importance of the mixed damping regime.

The reason why neutrinos are expected to be the only species of interest for the collisional and mixed damping can be easily understood. Since the transport coefficients that enter eq.1 are proportional to  $\frac{\rho_i v_i^2}{\Gamma_i}$ , only photons and neutrinos are expected to give a large contribution to  $l_{cd}$  (and therefore to  $l_{md}$ ). Their velocity is indeed maximal and they do not experience any annihilations that could reduce their energy density. To contribute to the collisional damping, however, the DM–photon interactions should be such that  $t_{dec(dm-\gamma)} \sim t_{rec}$  (with  $t_{rec}$  the recombination epoch). Such a condition actually requires very large (and therefore unlikely) values of the  $dm-\gamma$  elastic scattering cross section so neutrinos should eventually turn out to be the only species of interest as far as the estimate of  $l_{cd}$  and  $l_{md}$  is concerned (since  $t_{dec(dm-\nu)} \sim t_{dec(\nu)}$  is expected to rely on interactions of weak intensity).

In what follows, we shall therefore make the reasonable assumption that  $t_{dec(dm-\gamma)} \ll t_{rec}$ , and neglect the photon contribution to the collisional damping effect.

<sup>2</sup> The normalization factors are not shown in these formulae but we do take them into account in our calculations.

Let us now compute the terms associated with neutrinos in  $l_{cd}$  and  $l_{md}$ . The interaction rate  $\Gamma_\nu = \sum_j \Gamma_{\nu-j}$  can be decomposed as  $\Gamma_\nu = \Gamma_{\nu-e} + \Gamma_{\nu-dm} + \sum_{j \neq e} \Gamma_{\nu-j}$ . We shall neglect the terms  $\sum_{j \neq e} \Gamma_{\nu-j}$ , since the  $\Gamma_{\nu-j}$  are seen to be always smaller than  $\Gamma_{\nu-e}$ , and make the assumption  $\Gamma_{\nu-dm} < \Gamma_{\nu-e}$ . The neutrino decoupling is then given by  $\Gamma_{\nu-e} \sim H$ , as expected in a standard scenario. The non conventional case  $\Gamma_{\nu-dm} > \Gamma_{\nu-e}$  will be discussed later for completeness.

- Let us first study the case where  $t_{dec(dm-\nu)} \leq t_{dec(\nu)} \equiv t_{dec(\nu-e)}$ .

This is a standard situation without mixed damping effects so we only need to discuss the neutrino contribution to  $l_{cd}$ . Despite the term  $\rho_\nu c^2$  which appears very promising, the maximum collisional damping that standard neutrinos can really transmit to DM fluctuations is  $l_{cd}^2(dm-\nu) \lesssim \int^{t_{dec(\nu-e)}} \frac{\rho_\nu c^2}{\rho \Gamma_{\nu-e} a^2} dt \sim [O(100 \text{ pc})]^2$ , (the maximum damping length being reached when  $t_{dec(dm-\nu)} \simeq t_{dec(\nu-e)}$ ). Thus, if a DM candidate decouples from neutrinos at  $\sim 1$  MeV, the cut-off in the linear matter power spectrum due to DM-neutrino collisions should be around  $O(0.1 M_\odot)$ , which is a remarkable property!

This is actually much larger than the scale computed by Misner (1967) (say  $10^{-4} - 10^{-3} M_\odot$ ) relative to the damping of electronic primordial fluctuations<sup>3</sup>. But one can check that his computation, now rewritten in terms of the Weinberg formulation (Weinberg 1971), indeed corresponds to  $l_{Misner} \propto \int^{t_{dec(\nu-e)}} \frac{\rho_\nu c^2}{\rho \Gamma_{\nu-e} a^2} dt$ , so that  $l_{Misner} \equiv l_{cd} \simeq 100 \text{ pc}$ .

This damping has to be compared with the free-streaming motion acquired by the DM during the period  $[t_{dec(dm)}, t_0]$  ( $t_0$  denoting the present epoch and  $t_{dec(dm)} \equiv t_{dec(dm-\nu)} \sim t_{dec(\nu-e)}$  if  $dm - \nu$  is indeed the last DM interaction). For annihilating particles heavier than a few MeV, which is a reasonable assumption in the case of a usual WIMP candidate<sup>4</sup> decoupling at  $t_{dec(\nu-e)}$ , we find a free-streaming mass of

$$M_{fs} \lesssim \left( \frac{m_{dm}}{\text{MeV}} \frac{10^{10}}{10^4} \right)^{-3/2} 10^9 M_\odot \sim \left( \frac{m_{dm}}{\text{MeV}} \right)^{-3/2} M_\odot,$$

so  $M_{cd} \sim 0.1 \left( \frac{m_{dm}}{\text{MeV}} \right)^{3/2} M_{fs}$  for WIMPs decoupling as late as  $t_{dec(\nu-e)}$  Boehm et al. (2001).

- Let us now consider the case  $t_{dec(dm-\nu)} > t_{dec(\nu)} \equiv t_{dec(\nu-e)}$ .

Since the mixed damping becomes relevant, the DM primordial fluctuations are expected to be washed out by neutrino free-streaming motion (instead of neutrino collisions). Free-streaming being extremely efficient, one expects a significant damping effect for a temperature  $T_{dec(dm-\nu)}$  just

<sup>3</sup> The existence of DM was actually not considered at that epoch.

<sup>4</sup> The damping scale of annihilating WIMPs, having a mass above a few MeV, is  $l_{fs} \simeq O(200) \text{ kpc} \left( \frac{m_{dm}}{\text{MeV}} \right)^{-1/2} \left( \frac{a_{dec(dm)}}{10^{-4}} \right)^{1/2}$ , or equivalently  $M_{fs} \simeq 10^9 M_\odot \left( \frac{m_{dm}}{\text{MeV}} \right)^{-3/2} \left( \frac{a_{dec(dm)}}{10^{-4}} \right)^{3/2}$ ,  $a_{dec(dm)}$  being the scale-factor when the DM thermally decouples.

slightly below  $T_{dec(\nu-e)}$  (assuming standard neutrinos *i.e.*  $T_{dec(\nu)} = T_{dec(\nu-e)}$ ). But can  $T_{dec(dm-\nu)}$  be lower than  $T_{dec(\nu)} \equiv T_{dec(\nu-e)}$  for ordinary WIMPs?

What determines whether the mixed damping regime exists or not is basically the strength of the  $dm - \nu$  interactions. The stronger they are, the lowest  $T_{dec(dm-\nu)}$  is, and the more likely the condition  $T_{dec(dm-\nu)} < T_{dec(\nu)}$  is going to be fulfilled. Accordingly, we now proceed to establish the relationship between  $T_{dec(dm-\nu)}$  and the interaction rate  $\Gamma_{dm-\nu}$ .

## 2.1 Relationship between $T_{dec(dm)}$ and $\Gamma_{dm-\nu}$

The temperature can be written as  $T_{dec(dm-\nu)} \simeq T_0/a_{dec(dm-\nu)}$  where  $a_{dec(dm-\nu)} = (\tilde{\Gamma}_{dec(dm-\nu)}/\tilde{H}_r)$  is the scale-factor at  $t_{dec(dm-\nu)}$ ,  $\tilde{\Gamma}_{dec(dm-\nu)} = \Gamma_{dec(dm-\nu)} a_{dec(dm-\nu)}^3$  is the effective (comoving) interaction rate at the same epoch and  $\tilde{H}_r \sim 2 \cdot 10^{-20} \text{ s}^{-1}$  (in the radiation dominated era, which is the relevant epoch for our purpose).

The relationship between the interaction rate  $\Gamma_{(dm-\nu)}$  and the elastic scattering cross section  $\sigma_{(dm-\nu)}$  between DM and neutrinos can be obtained by writing the Euler equations for the DM and neutrino fluids (in the Newtonian gauge) (Boehm et al. 2002c):

$$\begin{aligned} \dot{v}_{dm} &= k\Phi - H v_{dm} - S^{-1} a \dot{\mu} (v_{dm} - v_\nu) \\ \dot{v}_\nu &= k\Phi + \frac{1}{4} k \delta_\nu - \frac{1}{6} k \pi_\nu - a \dot{\kappa} (v_\nu - v_b) \\ &\quad - a \dot{\mu} (v_\nu - v_{dm}), \end{aligned}$$

with

$$\dot{\mu} = \sigma_{(dm-\nu)} c n_{dm}, \quad \dot{\kappa} = \sigma_{(\nu-e)} c n_e, \quad S^{-1} \propto \frac{\rho_\nu}{\rho_{dm}}.$$

Here  $H$  is the Hubble parameter,  $\Phi$  the Bardeen potential,  $\delta_\nu$  the neutrino density perturbations,  $\pi_\nu$  the anisotropic stress tensor and  $v_{dm}$ ,  $v_\nu$  the velocity divergence of DM and neutrinos respectively. We then get:

$$\begin{aligned} \tilde{\Gamma}_{dec(dm-\nu)} &= S^{-1} \dot{\mu} a^3 |_{t_{dec(dm-\nu)}} \\ &= \sigma_{(dm-\nu)} c \tilde{n}_\nu T_{dec(dm-\nu)} / m_{dm} \end{aligned} \quad (4)$$

which implies, if we parameterize the cross section as

$$\sigma_{(dm-\nu)} c = b_{el(dm-\nu)} T^n$$

( $n \geq 0$ ) and use the definition  $T_{dec(dm-\nu)} \propto a_{dec(dm-\nu)}^{-1}$ :

$$\tilde{\Gamma}_{dec(dm-\nu)} = b_{el(dm-\nu)} \tilde{n}_\nu T_{dec(dm-\nu)}^{n+1} / m_{dm},$$

$$T_{dec(dm-\nu)} = \left( \frac{T_0 \tilde{H}_r m_{dm}}{b_{el(dm-\nu)} \tilde{n}_\nu} \right)^{1/(n+2)} \quad (5)$$

$$a_{dec(dm-\nu)} = \left( \frac{b_{el(dm-\nu)} \tilde{n}_\nu T_0^{n+1}}{\tilde{H}_r m_{dm}} \right)^{1/(n+2)} \quad (6)$$

$$\begin{aligned} \sigma_{(dm-\nu)} c &= b_{el(dm-\nu)} T_{dec(dm-\nu)}^n \\ &= \left( \frac{b_{el(dm-\nu)}^{\frac{2}{n+2}} T_0 \tilde{H}_r m_{dm}}{\tilde{n}_\nu} \right)^{\frac{n}{n+2}} \end{aligned} \quad (7)$$

Here  $\tilde{n}_\nu = n_\nu a^3$  is the comoving number density and

$T/m_{dm}$  a factor that reflects the efficiency of the  $dm-\nu$  collisions in transmitting the neutrino momentum to the DM. In other words, our effective  $\Gamma_{(dm-\nu)}$  corresponds to a standard interaction rate (usually defined as the product of  $\sigma\nu$  times a number density) times the transferred momentum and  $\tilde{\Gamma}_{dec(dm-\nu)}$  is the same interaction rate, but comoving and estimated at  $t_{dec(dm-\nu)}$ .

As expected, one can check from eq.5 that the largest the cross section (and incidentally  $b_{el(dm-\nu)}$ ) is, the lowest  $T_{dec(dm-\nu)}$  is. For  $n = 0$ ,  $a_{dec(dm-\nu)} \sim 2.3 \cdot 10^6 \sqrt{\sigma_{(dm-\nu)} c} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-1/2}$  (assuming that  $\sigma_{(dm-\nu)}$  indeed yields a decoupling in the radiation dominated era).

For  $n = 2$  (the most likely value), we find:

$$T_{dec(dm-\nu)} \sim 2.5 \cdot 10^{-3} b_{el}^{-1/4} \left(\frac{m_{dm}}{\text{MeV}}\right)^{1/4} \text{cm}^{-1}, \quad (8)$$

$$a_{dec(dm-\nu)} \sim 5 \cdot 10^3 b_{el}^{1/4} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-1/4},$$

$$\sigma_{(dm-\nu)} c \sim 6 \cdot 10^{-6} b_{el}^{1/2} \left(\frac{m_{dm}}{\text{MeV}}\right)^{1/2} \text{cm}^3 \text{s}^{-1} \quad (9)$$

(with  $b_{el} \equiv (b_{el(dm-\nu)}/\text{cm}^5 \text{s}^{-1})$ )

## 2.2 Conditions for mixed damping to exist

By using the definition 4, we actually introduce a ‘‘non standard’’ notation where, in fact,  $\tilde{\Gamma}_{dec(\nu-dm)} \neq \tilde{\Gamma}_{dec(dm-\nu)}$ . The relationship between these two interaction rates can be written as:  $\tilde{\Gamma}_{dec(\nu-dm)} = \frac{\rho_{dm}}{\rho_\nu} \tilde{\Gamma}_{dec(dm-\nu)}$  with  $\rho_{dm}$  and  $\rho_\nu$  being the energy densities of DM and neutrinos respectively. Roughly, one can write  $\tilde{\Gamma}_{(\nu-dm)} = \sigma_{dm-\nu} c \tilde{n}_{dm}$  and  $\tilde{\Gamma}_{(dm-\nu)} = \sigma_{dm-\nu} c \tilde{n}_\nu \frac{T}{m_{dm}}$ .

If one requires neutrinos to decouple at  $\sim 1$  MeV (which corresponds to the standard scenario) and DM to be still influenced by neutrinos, then the condition for the mixed damping effect to take place should in fact be rewritten (still assuming monotonic behaviour) as:

$$\Gamma_{\nu-dm}|_{t_{dec(\nu-e)}} < \Gamma_{\nu-e}|_{t_{dec(\nu-e)}} \quad (10)$$

$$\Gamma_{\nu-e}|_{t_{dec(\nu-e)}} \lesssim \Gamma_{dm-\nu}|_{t_{dec(\nu-e)}}, \quad (11)$$

which means, with our notation:

$$\sigma_{dm-\nu} < \sigma_{\nu-e} \left(\frac{n_e}{n_{dm}}\right) \Big|_{t_{dec(\nu-e)}}$$

$$\sigma_{dm-\nu} > \sigma_{\nu-e} \left(\frac{n_e}{n_\nu} \frac{m_{dm}}{T}\right) \Big|_{t_{dec(\nu-e)}}$$

or (if one assumes that DM has finished annihilating before (or at the onset of)  $t_{dec(\nu-e)}$ ):

$$\sigma_{dm-\nu} < A \sigma_{\nu-e} x_d^{-3/2} e^{x_d} \quad (12)$$

$$\sigma_{dm-\nu} \gtrsim \sigma_{\nu-e} \left(\frac{m_{dm}}{\text{MeV}} \frac{g_e}{g_\nu}\right). \quad (13)$$

with  $A \sim \left(\frac{g_e}{g_{dm}}\right) \frac{\zeta(3)}{\pi^2} (2\pi)^{3/2}$  ( $\zeta(3)$  the Riemann zeta function of 3) and  $x_d = m_{dm}/T_{fo} \in [12-20]$  for  $m_{dm} \in [1-10^3]$  MeV ( $T_{fo}$  being the freeze-out temperature).

We will use the previous equations only for particles

heavier than  $m_{dm} \gtrsim \text{MeV}$  (so that they stop annihilating before primordial nucleosynthesis). The mixed damping regime is expected to take place for DM particles with an elastic scattering cross section

$$\sigma_{(dm-\nu)} \gtrsim 2 \cdot 10^{-44} \left(\frac{m_{dm}}{\text{MeV}}\right) \text{cm}^2,$$

or

$$b_{el(dm-\nu)} \gtrsim 10^{-56} \left(\frac{m_{dm}}{\text{MeV}}\right) \text{cm}^5 \text{s}^{-1}$$

(bearing in mind that  $\sigma_{(dm-\nu)}$  cannot be too large so as to satisfy eq.12).

The elastic scattering cross section of heavy DM particles (with  $m_{dm} \gg m_e$ ) with neutrinos through the exchange of heavy particles  $X$  (with a mass  $m_X \gtrsim m_W$ ) is however likely to be such that  $\sigma_{dm-\nu} \ll \sigma_{\nu-e}$ . Since this is in contradiction with eq.13, one expects the mixed damping effect to be of interest only in non standard circumstances like resonant effects, light DM particles etc.

## 2.3 Expressions of the damping lengths

We can now express the different damping lengths in terms of the DM cross sections. Note that we assume  $n = 2$ ,  $a_{nr} \leq a_{dec(dm-\nu)} \leq a_{eq}$ ,  $\tilde{\Gamma}_{\nu-dm} < \tilde{\Gamma}_{\nu-e}$  and we make the assumption that DM annihilates.

$$l_{cd(dm-\nu)} \sim l_{\nu-e} \left(\frac{a_{dec(dm-\nu)}}{a_{dec(\nu-e)}}\right) \left(\frac{\tilde{\Gamma}_{dm-\nu}}{\tilde{\Gamma}_{(\nu-e)}}\right) \Big|_{t_{dec(dm-\nu)}}^{1/2}$$

$$\approx l_{\nu-e} \left(\frac{a_{dec(dm-\nu)}}{a_{dec(\nu-e)}}\right)^{5/2} \Big|_{a_{dec(dm-\nu)} < a_{dec(\nu-e)}}$$

$$\lesssim 100 \text{ pc} (10^{53} b_{el})^{5/8} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-5/8}$$

$$l_{md} \sim l_{\nu-e} \left(\frac{a_{dec(dm-\nu)}}{a_{dec(\nu-e)}}\right) \text{ if } a_{dec(dm-\nu)} \gtrsim a_{dec(\nu-e)}$$

$$\sim 100 \text{ pc} (1.4 \cdot 10^{54} b_{el})^{1/4} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-1/4}$$

$$l_{fs(dm)} \approx 200 \text{ pc} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-1/2} \left(\frac{a_{dec(dm)}}{a_{dec(\nu-e)}}\right)^{1/2}$$

$$l_{sd(dm)} \lesssim 200 \text{ pc} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-1/2} \left(\frac{a_{dec(dm)}}{a_{dec(\nu-e)}}\right)^{1/2}$$

As a comparison, in a non-standard scenario where  $T_{dec(\nu)} \equiv T_{dec(\nu-dm)} \lesssim 1$  MeV and  $T_{dec(dm-\nu)} < T_{dec(\nu-dm)}$ , the mixed damping would start at  $T_{dec(\nu-dm)}$  instead of  $T_{dec(\nu-e)}$ . This is important as most of the values of  $\sigma_{(dm-\nu)}$  one could be tempted to consider may yield the scenario  $T_{dec(\nu-dm)} < T_{dec(\nu-e)}$ . Indeed, only a small range of values of  $\sigma_{(dm-\nu)}$  can satisfy the conditions eq.10 and eq.11 associated with the mixed damping regime due to DM collisions with ‘‘standard’’ neutrinos. Note also that the case  $T_{dec(\nu-dm)} < T_{dec(\nu-e)}$  might have a significant impact on nucleosynthesis, so one has to treat it carefully.

### 3 WIMP DAMPING MASS

To get quantitative estimates of the damping lengths, we need to specify the DM characteristics. In particular, we need to determine when DM decouples from neutrinos and electrons (*i.e.*  $t_{dec(dm-\nu)}$  and  $t_{dec(dm-e)}$ ).

Similarly to  $T_{dec(dm-\nu)}$ , the decoupling temperature of DM from electrons, if  $m_{dm} > T_{dec(dm-e)} > m_e$ , is given by:  $T_{dec(dm-e)} = \left( \frac{T_0 \tilde{H}_r m_{dm}}{b_{el(dm-e)} n_e} \right)^{1/(n+2)} \text{ cm}^{-1}$  (where  $\sigma_{dm-e}$  is expected to be temperature dependent with  $n = 2$ ). However,  $T_{dec(dm-e)} < m_e$  is unlikely as this would require very large values of  $\sigma_{dm-e}$  to compensate the fact that electrons become non relativistic and drastically annihilate below their mass threshold.

More details are given in the next section but, generically, the elastic scattering cross section of fermionic or bosonic DM candidates with neutrinos or electrons can be written as:

$$\begin{aligned} \sigma_{dm-\nu} &\propto \frac{T^2}{(m_{dm}^2 - m_{S\nu}^2)^2} \\ \sigma_{dm-e} &\propto \frac{T^2}{(m_{dm}^2 - m_{Se}^2)^2} \end{aligned}$$

where the DM couplings to neutrinos or electrons are ‘‘hidden’’ in the proportional factor and with  $m_{S\nu}$ ,  $m_{Se}$  the masses of the exchanged particles (they are bosonic if DM is a fermion and fermionic if DM is a scalar). Note that  $m_{Se}$  (if charged) is necessarily larger than a few 100 GeV as no charged particle has been detected in past accelerator experiments, while  $m_{S\nu}$  can be below 100 GeV, provided one makes sure that these extra neutral particles are consistent with existing limits (they should be for example heavier than  $\sim 45$  GeV if significantly coupled to the  $Z$  to avoid an anomaly in the  $Z$  decay width).

When  $m_{S\nu}$  and  $m_{Se} > m_{dm}$ , we find typically for a fermionic or bosonic DM:

$$\begin{aligned} b_{el(dm-\nu)} &\propto 10^{-54} \left( \frac{m_{S\nu}}{100 \text{ GeV}} \right)^{-4} \text{ cm}^5 \text{ s}^{-1} \\ b_{el(dm-e)} &\propto 10^{-54} \left( \frac{m_{Se}}{100 \text{ GeV}} \right)^{-4} \text{ cm}^3 \text{ s}^{-1}, \end{aligned}$$

leading to

$$T_{dec(dm-\nu)} \gtrsim \text{MeV} \left( \frac{m_{S\nu}}{100 \text{ GeV}} \right) \left( \frac{m_{dm}}{\text{MeV}} \right)^{1/4}.$$

$T_{dec(dm-\nu)}$  and  $T_{dec(dm-e)}$  should actually differ by a factor proportional to a certain power of the couplings but we do not expect  $T_{dec(dm-e)}$  to be  $\ll 1$  MeV. Also these temperatures are expected to be slightly different depending on whether DM is bosonic or fermionic.

There is an exception to these formulae, however, in the case of light DM particles (with a mass below 100 MeV) which are required to get an annihilation cross-section dominated by a term in  $v^2$  to satisfy gamma ray fluxes (Boehm et al. 2002a); there may be other ways out but this is the simplest one. This implies, as explained in the subsection 3.2, that  $b_{el(dm-\nu,e)}$  depends rather on  $m_{dm}$  instead of  $m_{S\nu,e}$ .

In case of a mass degeneracy  $m_{S\nu}$  or  $m_{Se} \sim m_{dm}$ , these

decoupling temperatures become smaller, potentially making the mixed damping relevant. However, even if there exists a mass degeneracy large enough so that one would *a priori* expect  $T_{dec(dm-e)}$  to fall below 1 MeV, DM is likely to decouple from electrons at  $T \gtrsim m_e$  (due to the drastic change in the electron number density after  $e^-e^+$  annihilations).

The fact that  $b_{el(dm-e)}$  and  $b_{el(dm-\nu)}$  appear very close indicates that the DM thermal decoupling may be given by  $dm-\nu$  interactions instead of  $dm-e$ . It would be wrong, or at least dangerous, to focus only on  $dm-e$  interactions as the  $dm-\nu$  interactions determine whether the mixed damping regime exists or not.

Before computing the damping lengths of a typical fermionic candidate (say neutralinos) and a scalar candidate, we just mention that, in a situation where there is no mixed damping, the largest damping appears to be the free-streaming. The latter is indeed expected to erase all primordial fluctuations with a mass below:

$$M_{fs} \propto M_\odot c \left( \frac{m_{dm}}{\text{MeV}} \right)^{-15/8} \left( \frac{m_S}{100 \text{ GeV}} \right)^{-3/2}$$

where  $c$  denotes the couplings (at a certain power).

#### 3.1 Neutralinos

The expected mass for neutralino DM lies between  $[O(\text{GeV}), O(\text{TeV})]$  (Boehm et al. 2000). The lower limit set by the LEP experiment however is  $\sim 37$  GeV (ALEPH collaboration 2001) but this has been obtained by assuming a gaugino mass unification. Also within the Minimal Supersymmetric Standard Model (MSSM), the upper limit set by relic density calculations based on stau co-annihilations is about  $O(400)$  GeV (Ellis et al. 2000). Thus, one may find interesting to look at the damping mass of primordial fluctuations associated with neutralinos in the mass range  $[O(40) \text{ GeV}, O(400) \text{ GeV}]$ . (Looking at lighter neutralinos would actually be very interesting too, but if they are lighter than 10 GeV, one needs to make sure that their annihilation cross-section is suppressed by the square of the DM velocity to satisfy radio fluxes Boehm et al. (2002a), a condition which can perhaps be achieved by introducing, for example, a new light gauge boson that is not included in the Standard Supersymmetric Model.)

Let us therefore focus on neutralinos heavier than 40 GeV. The damping induced by neutrinos is basically given by the interactions  $\chi_0 \nu \rightarrow \chi_0 \nu$  through the exchange of sneutrinos. The latter has been searched in LEP experiments, notably from single photon events, and a limit of  $\sim 84$  GeV has now been set on its mass (under the assumptions of gaugino and sfermion mass unification as well as no sfermion mixing (ALEPH collaboration 2002)).

The total elastic scattering cross section is given by:

$$\sigma_{\chi-\nu} \approx \frac{c_l^4 T^2}{16\pi (m_\chi^2 - m_\nu^2)^2}, \quad (14)$$

with  $c_l$  the coupling between sneutrino, neutrino and neutralino. There are two cases for which it is interesting to compute the damping scale, namely:

- a heavy or light neutralino much lighter than sneutrinos, ( $m_{\tilde{\nu}} > m_\chi$ ),
- a heavy or light neutralino degenerate with sneutrinos ( $m_{\tilde{\nu}} \simeq m_\chi$ ) so as to benefit from a resonance that would enhance the cross section and therefore the damping effect.

Let us first consider a case where there is no mass degeneracy. One gets

$$b_{el(dm-\nu)} \approx c_l^4 10^{-54} \left( \frac{m_{\tilde{\nu}}}{100 \text{ GeV}} \right)^{-4} \text{ cm}^5 \text{ s}^{-1},$$

which appears to be much smaller than the value required to have mixed damping. So free-streaming is the most relevant effect, erasing up to:

$$M_{fs} \approx c_l^{3/2} M_\odot \left( \frac{m_{dm}}{\text{MeV}} \right)^{-15/8} \left( \frac{m_{\tilde{\nu}}}{100 \text{ GeV}} \right)^{-3/2}$$

*i.e.*

$$M_{fs} \approx 8 \cdot 10^{-10} - 10^{-11} \left( \frac{c_l}{0.3} \right)^{3/2} \left( \frac{m_{\tilde{\nu}}}{100 \text{ GeV}} \right)^{-3/2} M_\odot$$

for  $m_{\tilde{\nu}} > m_\chi$  and  $m_\chi = 40 - 400 \text{ GeV}$ . This is to be compared with

$$M_{cd} \approx 5 \cdot 10^{-18} - 4 \cdot 10^{-16} M_\odot \left( \frac{c_l}{0.3} \right)^{15/2} \left( \frac{m_{\tilde{\nu}}}{100 \text{ GeV}} \right)^{-15/2}$$

for  $m_\chi = 40 - 400 \text{ GeV}$ . All these values are actually far from the calculation done by (Hofmann et al. 2001).

In case of a mass degeneracy, on the other hand, the  $\chi_0 - \nu$  elastic cross section,

$$\sigma_{\chi-\nu} \propto \frac{c_l^4 T^2}{(-m_{\tilde{\nu}}^2 + m_\chi^2 + 2Tm_\chi)^2},$$

can become temperature independent provided  $m_\chi^2 - m_{\tilde{\nu}}^2 < 2Tm_\chi$ . Let us define  $m_{\tilde{\nu}}$  as  $m_{\tilde{\nu}} = m_\chi(1 + \delta)$ . So

- for  $T > (2\delta + \delta^2)m_\chi/2$ , the cross section is temperature independent:

$$\sigma_{\chi-\nu} \propto \frac{c_l^4}{4 m_\chi^2}, \quad (15)$$

- while for  $T < (2\delta + \delta^2)m_\chi/2$ , the cross section is temperature dependent:

$$\sigma_{\chi-\nu} \propto \frac{c_l^4 T^2}{m_{\tilde{\nu}}^4}. \quad (16)$$

For example, one expects  $\sigma_{\chi-\nu}$  to be constant at temperatures above 10.5 GeV, if one assumes  $m_\chi \sim 100 \text{ GeV}$  and  $\delta = 0.1$ .

Thus, unless the degeneracy appears to be adequate so that DM can decouple at  $T \lesssim 1 \text{ MeV}$ , the damping generated by the  $dm-\nu$  collisions should eventually be computed in two steps. First using formula 15 up to  $T_c = (2\delta + \delta^2)m_\chi/2$  and then using eq.16 up to  $T_{dec(dm-\nu)}$  (where  $T_{dec(dm-\nu)}$  will be estimated by use of eq.16). One nevertheless expects the damping to be dominated by the late times so using eq.16 only should finally give a good estimate.

One can compute the neutralino thermal decoupling temperature in ordinary situations from eq.14, using eq.8 and eq.9. One finds

$$T_{dec(dm-\nu)} \sim 5.3 \text{ MeV} \left( \frac{c_l}{0.3} \right)^{-1} \left( \frac{m_{\tilde{\nu}}}{100 \text{ GeV}} \right) \left( \frac{m_{dm}}{\text{MeV}} \right)^{1/4},$$

for  $T < (2\delta + \delta^2)m_\chi/2$  (*i.e.* 75 MeV for  $m_{dm} = 40 \text{ GeV}$  and 133 MeV for  $m_{dm} = 400 \text{ GeV}$ ). A similar temperature is expected for  $T_{dec(dm-e)}$  as the electrons are relativistic above  $m_e$  and behave like neutrinos. On the other hand, if the degeneracy was about  $10^{-3}$  for  $m_{dm} = 100 \text{ GeV}$  (*i.e.*  $\delta = 10^{-5}$ ,  $m_{\tilde{\nu}} = 100.001$ ), then only one step would be necessary to compute the thermal decoupling epoch as the cross section remains constant up to  $T \lesssim \text{MeV}$ . As a result,  $T_{dec(dm-\nu)}$  could be less than  $\sim 1 \text{ MeV}$  for  $m_\chi \sim 100 \text{ GeV}$ , which would further considerably increase the damping mass (at least up to  $1 M_\odot$ ), although such a degeneracy cannot be taken seriously.

### 3.2 Light DM

Although the most popular DM candidate certainly appears to be a ‘‘heavy’’ neutralino, the possibility of having light DM candidates may still be a viable solution. In particular, if not coupled to the  $Z$ , light scalar particles (with  $O(\text{MeV}) \lesssim m_{dm} \lesssim O(10 \text{ GeV})$ ) may be solution to the DM issue, provided they are coupled to a light gauge boson  $U$  (with a mass  $m_U \gtrsim m_{dm}$ ) or to heavy fermionic particles  $F$  (although in this case, the gamma ray, radio flux as well as  $g-2$  constraints might impose that DM be made of non self-conjugate particles and require to introduce a set of new particles in order to kill the too large  $F$  contribution to the muon and electron  $g-2$ ).

When coupled to fermionic particles  $F$ , light scalars are not expected to experience a large damping effect because of the mass difference between DM and the  $F$  particles (supposed to be very heavy from past accelerator experiments). There is an exception, however, if DM mainly annihilates through a very light particle  $F^0$  (with a mass  $m_{F^0} \sim m_{dm}$ ) which is not significantly coupled to the  $Z$  (and which would have escaped accelerator limits).

On the other hand, if DM is coupled to a new and light gauge boson  $U$ , one expects damping effects to be significantly enhanced. The elastic scattering cross section  $\sigma_{dm-\nu}$  (associated with the exchanged of a  $U$  boson through a t-channel) is given by:

$$b_{el(dm-\nu)} = 10^{-33} C_U^2 f_{U_l}^2 \left( \frac{m_U}{\text{MeV}} \right)^{-4} \text{ cm}^5 \text{ s}^{-1}.$$

In Boehm & Fayet (2003), it is shown that – to satisfy both relic density and  $g-2$  constraints – the product of the couplings  $C_U$  and  $f_{U_l}$  (which correspond to the  $U$  boson couplings to DM and to ordinary particles respectively) must satisfy the relationship  $C_U f_{U_l} = (3-12) 10^{-8} \left( \frac{m_{dm}}{\text{MeV}} \right)^{-1} \left( \frac{m_U}{\text{MeV}} \right)^2$  so

$$b_{el(dm-\nu)} \sim [9-144] 10^{-49} \left( \frac{m_{dm}}{\text{MeV}} \right)^{-2} \text{ cm}^5 \text{ s}^{-1}$$

(where we assume ‘‘universal’’  $f_{U_l}$ , *i.e.* same value for  $U-e-\bar{e}$  and  $U-\mu-\bar{\mu}$  for instance). This implies a decoupling scale-factor:

$$a_{dec(dm-\nu)} \sim (5-10) 10^{-9} \left( \frac{m_{dm}}{\text{MeV}} \right)^{-3/4}$$

which is independent of  $m_U$  and maximal for small values of the DM mass. As an example, the decoupling epoch for  $m_{dm} = 10 \text{ MeV}$  corresponds to  $a_{dec(dm-\nu)} \sim (0.8-1.5) 10^{-9}$

which implies a mixed damping mass of about

$$M_{md} \sim 21 - 172 M_{\odot} \text{ (for 10 MeV)}.$$

This damping mass obviously gets smaller for larger DM mass and it is worth to mention that the cross section for  $m_{dm} = 10$  MeV is close to the upper limit that arises from the condition of free-streaming neutrinos, namely  $b_{el(dm-\nu)} \lesssim 1.86 \cdot 10^{-56} \left(\frac{g_e}{g_{dm}}\right)^2 \left(\frac{m_{dm}}{\text{MeV}}\right)^{-1} x_d^{-3} e^{2x_d} \text{ cm}^5 \text{ s}^{-1}$  (so  $m_{dm} = 10$  MeV is close to the upper limit for  $b_{el(dm-\nu)} \sim 144 \cdot 10^{-49} \left(\frac{m_{dm}}{\text{MeV}}\right)^{-2} \text{ cm}^5 \text{ s}^{-1}$ ). Note that one would naively expect a bigger effect in the case of light DM due to the smallness of  $m_{dm}$  but the latter is actually compensated by the smallness of the product  $C_U f_{U_i}$ .

With the expression  $a_{dec(dm-\nu)}$  obtained above (valid only if DM particles are coupled to a  $U$  boson), one can readily see that the mixed damping regime is at work only for a DM mass in the range  $m_{dm} \in [\sim 5, (500 - 700)]$  MeV (heavier DM would decouple before 1 MeV, lighter particles would give rise to the alternative scenario where the neutrino decoupling occurs at  $T_{dec(\nu)} < \text{MeV}$ ).

In the previous examples, however, we assumed that the number density of DM particles was equal to the number density of anti DM particles. If one relaxes this assumption (assuming non self-conjugate DM), then  $C_U$  can take much larger values (but likely to be  $\in [10^{-3}, O(1)]$ ). As a result, the decoupling scale-factor which is proportional to  $b_{el(dm-\nu)}^{1/4} \propto (C_U f_{U_i})^{1/2}$  can become at most  $\sim \sqrt{10^3}$  larger than our previous estimate, so that the damping mass that one expects in such situations could possibly reach  $\sim (\sqrt{10^3})^3 \sim 3 \cdot 10^4$  times the mass  $M_{md}$  estimated without the number density asymmetry. But a change in  $C_U$  is, in fact, particularly relevant for DM particles with  $m_{dm} \gtrsim O(10)$  MeV, for example, as the upper limit on the cross section (set by the condition of free-streaming neutrinos) may remain possible to satisfy.

Such particles, however, would not have any residual annihilations so they would not produce gamma rays but on the other hand, they would have an oscillating linear matter power spectrum largely damped below  $O(10^3 M_{\odot})$ , which finally provides a signature.

One can also consider larger values of  $f_{U_i}$  by relaxing the assumption of universal couplings. Indeed, the values mentioned before for  $C_U f_{U_i}$  actually supposed that the coupling  $dm - \nu - \bar{\nu}$  is of the same order of magnitude as the couplings  $dm - \mu - \bar{\mu}$  or  $dm - e - \bar{e}$  (constrained through the  $U$  contribution to the muon and electron anomalous magnetic moments). Writing  $f_{U_i}^e = x f_{U_i}^e$  (with  $x \geq 1$ ) ( $f_{U_i}^e$  being the relevant coupling for low DM mass), one finds that the damping mass is increased by a factor  $x^{3/2}$  and  $a(t)$  by a factor  $x^{1/2}$  (still provided one makes sure that the condition of free-streaming neutrinos is satisfied).

Such situations may finally allow for DM particles heavier than the limit 500 – 700 MeV to experience a non-negligible mixed damping (while they were not supposed to with smaller values of the couplings). Indeed, an increase in the couplings simultaneously allows for eq.13 to be satisfied and for  $M_{md}$  to be enhanced.

Note that the mixed damping mass is not expected to

become larger in case of a degeneracy between  $m_U$  and  $m_{dm}$ , because the interaction at the origin of the damping effect proceeds through a t-channel. On the other hand, such a degeneracy would affect the decay modes of the  $U$  boson which would then decay into ordinary particles only (*i.e.* into  $e^+e^-$  and  $\nu\bar{\nu}$  instead of  $dm dm^*$ , although one has to make sure this is not excluded by nuclear experiments).

The previous examples provide a theoretical framework to investigate the effect of a “collisional” cut-off in the linear matter power spectrum at scales  $\sim 10^3 M_{\odot}$ . This is actually too small compare to our resolution but the question of whether a cut-off in the linear matter power spectrum is still present in the  $P(k)_{nl}$  can be partially answered by investigating the effect of a cut-off at  $10^6 - 10^9 M_{\odot}$ .

The main feature, say oscillations, in the light DM linear matter power spectra should be somewhat similar to that obtained in Boehm et al. (2002c) (although the damping mechanisms are different). However, to extend our point to other class of DM candidates, we decide to perform simulations of collisionless WDM and see whether a cut-off in the linear matter power spectrum at large scale ( $10^6 - 10^9 M_{\odot}$ ) also appear in the  $P(k)_{nl}$  and at which redshift. If such a cut-off appears difficult to detect, then non conventional candidates as proposed in this paper will be very difficult to exclude from the measurement of the non linear matter power spectrum only.

#### 4 REGENERATING SMALL SCALE POWER

Estimations of the matter power spectrum on scales  $l \gtrsim 200 h^{-1}$  kpc are now routinely derived from weak and strong lensing techniques at low redshift, or indirectly from the correlation of spectra of the Ly- $\alpha$  forest at  $z \sim 3$ . Fair agreement of the observations with the slope of the non-linear power spectrum expected in the CDM paradigm is commonly regarded as a confirmation of the assumed “cold” nature of dark matter. This is a biased conclusion, as the  $n_{nl} = -1.4$  slope of  $P(k)_{nl}$  at these small scales ( $n = d \ln P / d \ln k$ ) and at this low redshift range is known to behave as an “attractor” to the evolution of  $n$ , for a variety of DM models (Soccimarro & Frieman 1996).

Using numerical simulations of structure formation with initial power spectrum restricted to two separated scale ranges, Bagla & Padmanabhan (1997) demonstrate that the non-linear growth of the small scale modes is mostly driven by the cascade of power due to the coupling with larger scale modes. As a result, DM models with very different  $P(k)_l$  on small scales but similar large scale power are expected to yield small-scale  $P(k)_{nl}$  of the same shape. Focusing on measurements of the Ly- $\alpha$  forest, Zaldarriaga et al. (2003) make the same point at a somewhat more observational degree, and clearly show how little information is obtained on the initial shape of  $P(k)_l$  at  $k \gtrsim 10 h \text{Mpc}^{-1}$  by inverting the flux power spectrum, as long as one starts with a sufficiently large prior in  $n_l$ .

Physically, the IDM and WDM models considered here are as well motivated as CDM (but for a little more complexity), and they are *a priori* as probable as the latter. It is therefore necessary to verify if, when and at which scale

their specific matter power spectrum becomes sufficiently close to that of CDM so that current observations are unable to separate between the two.

To assess this, we simulate the gravitational formation of structure using  $128^3$  collisionless particles in a small,  $5 h^{-1}$  Mpc side comoving box, with the cosmological parameters measured by *WMAP* (Spergel et al. 2003), starting from  $z=100$ . The mean inter-particle distance is  $39 h^{-1}$  kpc and our force resolution is one tenth this value. We evolve the particle distribution with the public version of *GADGET* (Springel et al. 2001). We consider a CDM and four WDM models with warmon masses 0.6, 1.1, 2 and 3.5 keV: the associated free-streaming lengths correspond to Lagrangian masses  $10^9$ ,  $10^8$ ,  $10^7$  and  $10^6 h^{-1} M_\odot$  respectively. The CDM and 0.6 keV WDM models bracket the series of models discussed in this work (including IDM), while the other 3 samples are intermediate cases. We keep the same phases for all the modes in the five simulations. In the following discussion we will not tackle the issue of cosmic variance. We normalize the modes  $\delta_k$  inside the box so that they correspond to  $\sigma_8 = 0.9$ . In doing so, we neglect the effect of primordial fluctuations with comoving scales greater than the box size: the power effectively realized in the box is 3 percent of the total cosmological power.

To estimate the dispersion expected if we were to include all the modes up to the scale of cosmic homogeneity, we have performed two series of similar simulations which we call test simulations. Checking this effect is necessary because of the combination of our limited dynamic range in scale and of our choice of a small simulation box.

The first series of test simulations consists of a CDM and a 0.6 keV WDM model with the same phases as above in a flat cosmology with the same  $\sigma_8$  and  $t_0$  but with different  $\Omega_0$ :  $\Omega_0=0.15$  and  $0.45$ , corresponding to a  $\delta_{>5h^{-1} \text{ Mpc}} = \pm\sigma_5/3$  background and  $\Omega_0=0.03$  and  $0.57$ , corresponding to a  $\delta_{>5h^{-1} \text{ Mpc}} = \pm\sigma_5$  background. In these simulations,  $h$  varies accordingly to ensure the same age  $t_0$  as the cosmological model preferred by the *WMAP* results. Recall that we take  $\sigma_8 = 0.9$  and we define  $\sigma_5$  as the RMS density fluctuations in spheres of radius  $5 h^{-1}$  Mpc. This series of simulation checks the effect of large scale bias in a peak-background split approach.

While we find the regeneration of small scale power for our 0.6 keV WDM model to be very similar in the  $\Omega_0=0.45$  and  $\Omega_0=0.57$  simulations to the result of the reference mean density simulation, the low density simulations (especially  $\Omega_0=0.07$ ) show that small scale regeneration occurs significantly earlier when one evolves in an underdense,  $5 h^{-1}$  Mpc region of the universe compared to a mean or high density region. This is expected, because for the same absolute normalisation of the power spectrum, the growth of modes occurs earlier in low density flat cosmologies. As a result, the cascade of power from the larger modes that we describe in detail below also occur earlier in low density cosmologies and the 0.6 keV WDM power spectrum catches up earlier on with the CDM power spectrum. However, this phenomenon will only strengthen our conclusions.

The second series is the same CDM and 0.6 keV WDM model as in our main series, changing only the box size from 5 to  $20 h^{-1}$  Mpc (and the mean inter-particle separation to

$156 h^{-1}$  kpc). This second series verifies that enough large-scale power is already realized inside the  $5 h^{-1}$  Mpc box for the same cascade of power to occur as would happen in a larger box. We have found that the neglected larger scale modes have little impact on our conclusions: the dispersion due to fluctuations in matter density is smaller than the differences we discuss in the power spectra of different DM models, and the regeneration of small-scale power is the same in the 5 and  $20 h^{-1}$  Mpc boxes at the overlapping wavelengths. For completeness sake we point out that we did find reduced small-scale regeneration in a  $1 h^{-1}$  Mpc box simulation because of lack of large scale power in this case. This allows us to conclude that  $5 h^{-1}$  Mpc is larger than, but close to the minimum box size necessary for our conclusions to be meaningful.

Figure 1 gives the matter power spectra  $P(k)_{\text{nl}}$  of our five simulations, from  $z=30$  to  $z=0$ . The spectra are computed up to  $k_{\text{Ny}} = 2\pi 64/5 \sim 80 h\text{Mpc}^{-1}$  using a deconvolved TSC scheme. To facilitate the comparison, they have been divided by the normalized linear growth factor  $D_{z=z_{\text{init}} \rightarrow z=z_{\text{plot}}}/D_{z=z_{\text{init}} \rightarrow z=0}$ .

The qualitative evolution of the power spectrum is similar to that found by Knebe et al. (2002) in their simulations of WDM models with warmon masses of 0.5 and 1 keV. At  $z=30$ , shortly after starting the simulations, the shape of the measured spectrum is in all cases close to the linear power spectrum of the initial conditions. By  $z=10$ , non-linear effects have already significantly altered the shape of all power spectra on the smallest scales  $k \gtrsim 10 h\text{Mpc}^{-1}$ . At  $z=7$ , WDM power spectra are already reasonably close to CDM, with the exception of the 0.6 keV simulation, where the power spectrum is still steeper and keeps the signature of the  $10^9 h^{-1} M_\odot$  cut-off. At  $z<2$  however, the power spectrum of the 0.6 keV WDM simulation is within a factor of 2.5 of that of the CDM down to the smallest scales probed. At  $z=1$ , the curves can be considered the same within 10 percent. These plots confirm that the late  $P(k)_{\text{nl}}$  is similar down to scales as small as  $80 h^{-1}$  kpc at  $z \lesssim 2$  for models with so different input power spectra such as CDM and the 0.6 keV WDM scenario. We find the slope of the evolved power spectrum to tilt from  $n \sim -1$  for  $k < 10 h\text{Mpc}^{-1}$  to  $n \sim -1.6$  for  $k > 10 h\text{Mpc}^{-1}$ . Furthermore, at a fixed comoving scale, the power spectrum of WDM models with the larger warmon mass (and smaller free-streaming length) catches up earlier with the CDM power spectrum than do models with larger free streaming lengths. The intuitive guess that at a fixed WDM model, power on large scales matches earlier CDM power than power on a smaller scale is confirmed. This is of course the signature of the exponential cut-off present in the linear power spectrum.

These monotonic behaviours are more clearly seen in Figure 2. There, we first divide our wavelength coverage ( $k \in [0.1 \text{ } 1.9] h\text{Mpc}^{-1}$ ) into 6 consecutive bins, corresponding to the 6 panels shown. In each bin we estimate the average of the power spectrum ( $P(k)_{\text{nl}}$ ) for the CDM simulation and the four WDM simulations at each redshift. We then divide the time evolution of the averaged power spectrum  $\langle P(k)_{\text{nl}}(z) \rangle$  of each WDM simulation by the time evolution of the averaged CDM power spectrum. We only discuss the upper left and lower right panels, the other 4 being intermediate cases shown here for reference. On the upper left

panel (bin associated to the largest scales), the ratio of the average power measured in the WDM models to the average power in the CDM model is within 10 percent of unity from  $z=30$  down to  $z=0$  for all warmons but the lightest, *i.e.* the 0.6 keV particle. In this last case, the signature of the initial exponential cut-off is already visible on large scales, with a factor 2 less power than CDM. This deficit gradually disappears over  $10 > z > 2$ . The lower right panel (bin associated to the smallest scales that we probe) clearly separates our four WDM models. The average power of the 3.5 keV WDM model is the first to catch up with CDM, at  $z \sim 12$ . The increase in the ratio of 2 and 1 keV WDM power to CDM power is abrupt before  $z \sim 10$ , slows down in  $10 > z > 2$  and becomes comparable to CDM at low redshift. The ratio of the 0.6 keV WDM to CDM power increases less steeply than the 1 and 2 keV power initially, seems to slow down at  $z < 1$ , but has not yet exactly matched the CDM power at  $z=0$ : there is a remaining 10 percent offset (but probably indistinguishable with current observations). However, in all these cases, and also including the four remaining panels, the “monotonic” feature of the process of small scale regeneration is preserved. Figure 2 contains the same information as Figure 1, it only has more sampling points in redshift. We can extrapolate that the power of the 0.6 keV model may not have matched the power of the CDM model at  $z=0$ , if one would look at scales smaller than  $\log_{10} k = 1.9$ . As we want to simply illustrate the late-time degeneracy in the power spectrum between a 0.6 keV WDM model and a CDM model at scales  $\sim 100 h^{-1}$  kpc, we leave this issue for further work.

Regarding the regeneration of small-scale power we find that observations (e.g. lensing) probing the matter distribution at  $z \lesssim 1$  at scales as small as  $100 h^{-1}$  kpc will be unable to detect the signature of the initial exponential cut-off of WDM models with warmon mass as small as 0.6 keV. For this warmon mass, the signature of a cut-off will only be clearly visible on lenses at  $z \lesssim 4$ , not even considering systematic errors nor cosmic variance. All WDM models with larger warmon mass will be increasingly harder to single out. Alternatively, one may need to probe scales smaller than  $100 h^{-1}$  kpc. The converse, and our main point, is that finding at the above scales a  $z=0$  power spectrum in agreement with that expected from the non-linear evolution of an initial CDM model cannot rule out physically motivated alternatives such as WDM or a light DM scenario presented in this paper.

We emphasize that this is true even if we take into account the possible effect of the modes with wavelength larger than our simulation box: while evolving in a  $1 \sigma$  overdense region (defined with respect to our neglected modes) does not change the regeneration of small scale modes compared to what happens in a mean density region, simulating a  $1 \sigma$  underdense region we find small scale regeneration to occur even earlier.

To conclude this section, we note that the mass function of collapsed objects is also a sensitive probe *at high*  $z$  of a possible exponential cut-off in the primordial power spectrum. It has the advantage over power spectrum measurements that large variations in the abundance of haloes at the epoch of the formation of the first stars or later at reionization can be rather tightly constrained even with current

observations, at a fixed physical prescription for the evolution of the baryonic component. The drawback resides in first, the current large freedom in physical models of reionization or of the formation of the first stars and second, in that the mass function is also very sensitive to a possible small-scale primordial non-gaussianity. While we plan to address these issues in more detail in the future, Figure 3 gives the  $z=0$  (left panel) and  $z=10$  (right panel) mass function of collapsed dark matter haloes in our simulations. Note the mass range probed ( $5 \times 10^5$  to  $10^{12} h^{-1} M_{\odot}$ ). At  $z=10$ , the deficit of haloes of all masses is evident in the 0.6 keV WDM model compared to CDM. It is also apparent in the simulated WDM models with higher warmon mass, although mainly at the low-mass end of the curves. The  $z=10$  abundance of the largest,  $10^{10} h^{-1} M_{\odot}$  mass haloes is similar in the CDM and the 1.1, 2, 3.5 keV WDM models. At  $z=0$  the abundances of haloes with  $M_{\text{tot}} \gtrsim 10^{11} h^{-1} M_{\odot}$  match in all 5 models. In the range  $10^{8.5} \lesssim M_{\text{tot}} \lesssim 10^{11} h^{-1} M_{\odot}$ , the mass function of the 0.6, 1.1 and 2 keV models shows a deficit of haloes compared to CDM, with the 0.6 keV model showing the lowest abundance. At  $M_{\text{tot}} \lesssim 10^{8.5} h^{-1} M_{\odot}$ , halo abundances tend to coincide again.

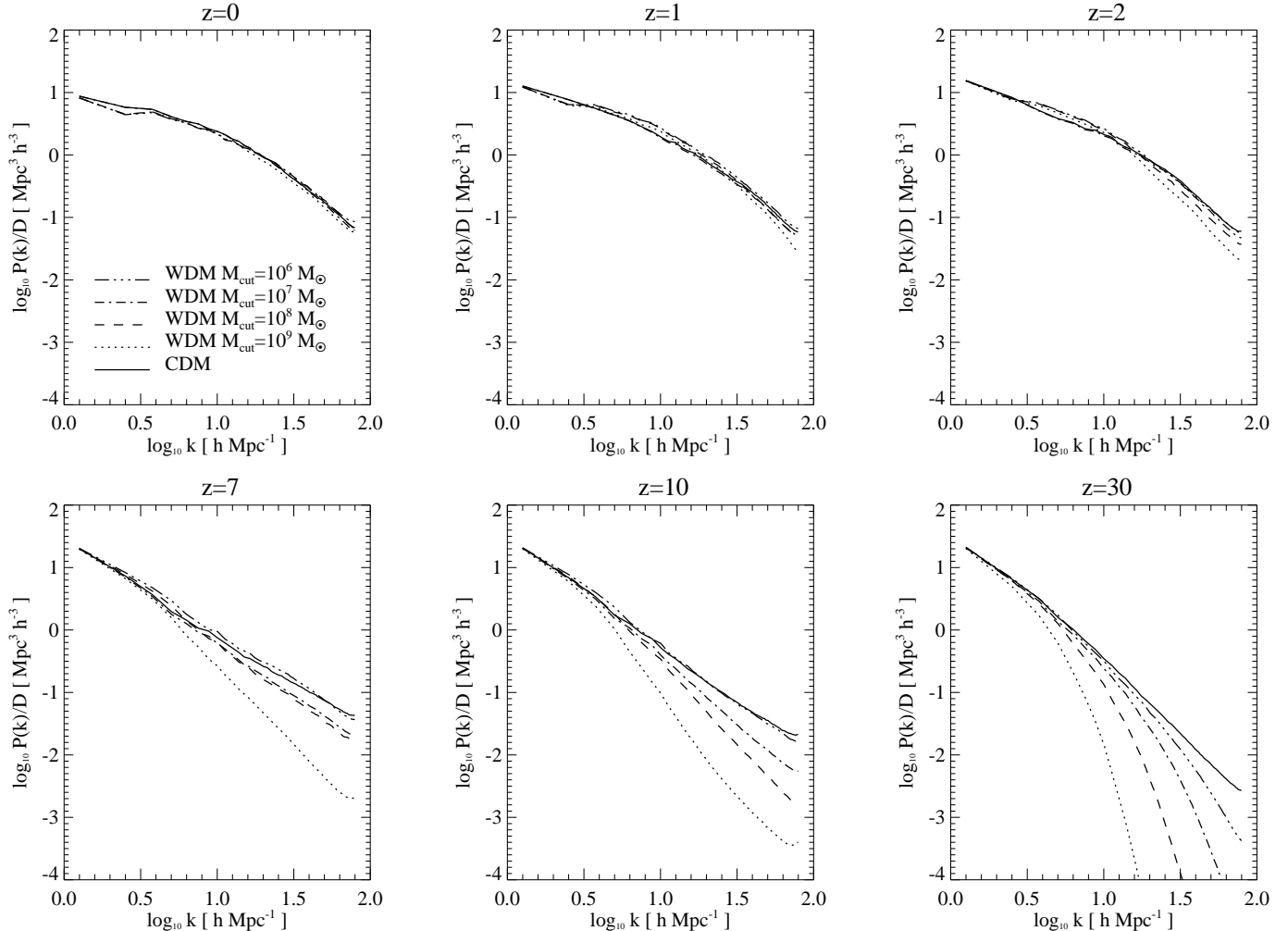
## 5 CONCLUSION

In this paper we compare two distinct DM candidates. One is the most popular candidate, say the lightest neutralino, while the other one is light DM particles.

Both these candidates are expected to have weak interactions. They both annihilate to get the proper relic density but, in the case of neutralinos, collisional damping effects are completely negligible while, in the case of light DM, the collisional damping (and more specifically the mixed damping) yields a cut-off in the linear matter power spectrum which can realistically reach  $10^3 M_{\odot}$ .

Because the existence of this cut-off seems a promising way to discriminate among these two extreme DM scenarios, we perform a series of numerical simulations to obtain the corresponding non-linear matter power spectra. To make the case even more obvious, in fact, we simulate scenarios with a more drastic cut-off than what is expected in our model. More realistic simulations of light DM scenarios will be published in a forthcoming paper but we do expect similar conclusions. This enables us to quantify the time evolution of  $P(k)_{nl}$  as a function of scale, and also to precisely assess to which extent the late time ( $z < 1$ ) small-scale power spectrum can be used as an efficient tools to distinguish between standard (collisionless) CDM and alternative scenarios.

Since we are limited by the size of our simulations, we simulate a cut-off at  $10^6$ ,  $10^7$ ,  $10^8$  and  $10^9 h^{-1} M_{\odot}$ . We show that the corresponding  $P(k)_{nl}$  is similar to the non-linear matter power spectrum of CDM particles for  $z < 1$ , so weak lensing measurements except on the smallest scales ( $100 \text{ kpc/h}$  or less for lenses located at  $z < 1$ ) are not able to discriminate between usual CDM particles and more exotic candidates. Note here that current measurements of the cosmic shear probe the shape of the matter power spectrum down to  $200 h^{-1}$  kpc, while planned upcoming surveys will



**Figure 1.** Snapshots of the dark matter power spectra measured in simulations of a series of DM models. CDM and 0.6, 1.1, 2 and 3.5 keV heavy warm dark matter corresponding to respectively  $10^9$ ,  $10^8$ ,  $10^7$  and  $10^6 h^{-1} M_{\odot}$  Lagrangian masses in the free-streaming length are shown. The power spectra have been divided by the linear growth factor to facilitate the comparison between redshifts. Note the exponential cut-off at  $z=30$  in the spectrum of WDM models and the similarity of the spectra of all five models over the simulated scales at  $z \leq 2$ .

go down to  $70 h^{-1} \text{kpc}$  and start to probe a critical region for a  $10^9 h^{-1} M_{\odot}$  cut-off.

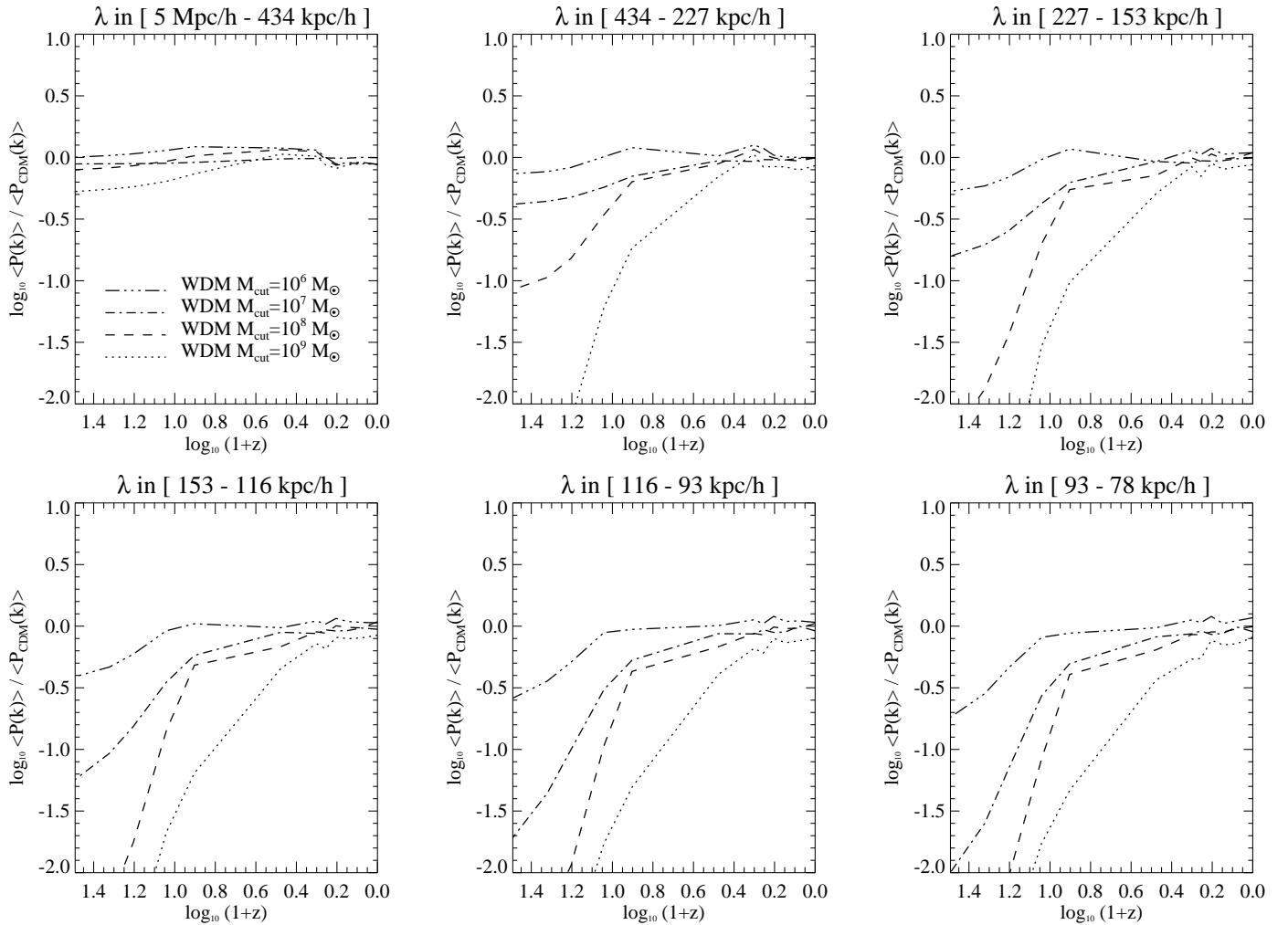
On the other hand, there exist other sensitive signatures of the nature of the dark matter, such as structural parameters, clustering of virialized dark matter haloes and the reionization epoch. However these are more indirect constraints and involve poorly understood interactions between baryons and DM, which considerably weakens their predictive/discriminative power.

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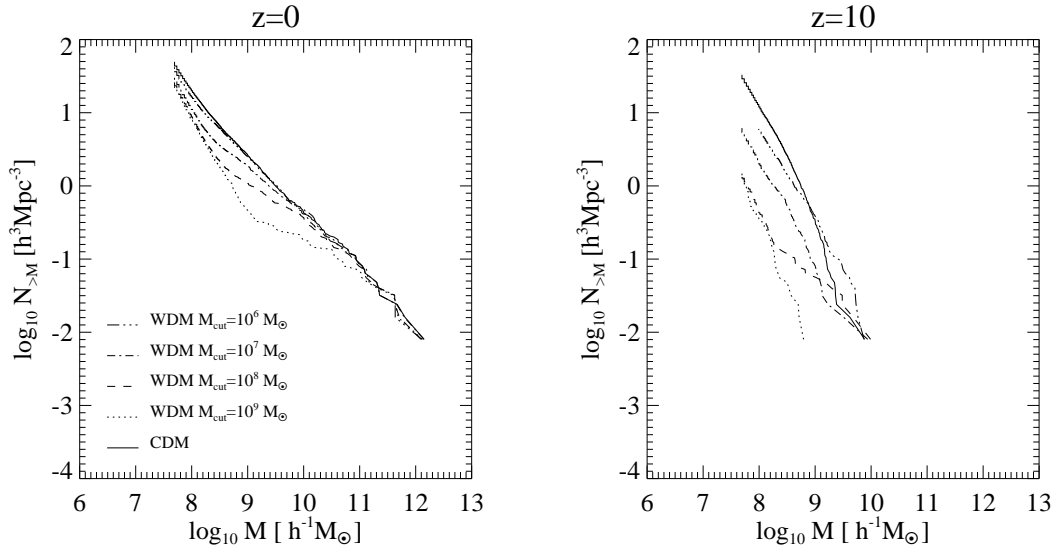
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**Figure 2.** Evolution with redshift of the ratio of (1) the mean of the power spectrum of each of the four WDM models (0.6, 1.1, 2 and 3.5 keV particles corresponding to respectively  $10^9$ ,  $10^8$ ,  $10^7$  and  $10^6 h^{-1} M_{\odot}$  Lagrangian masses) to (2) the mean of the power spectrum of CDM model, in six consecutive bins covering the whole wavelength range probed by the simulations. Note how the ratio converges to unity at late times for all WDM models in each of the 6 bins. The marginal exception is the evolution of the 0.6 keV WDM model which shows 10 percent less average power than CDM on the smallest scales bin at  $z=0$ . The power spectrum of this model may strongly depart from CDM on scales  $\log_{10}(k) \sim 2$  even at  $z=0$ .

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**Figure 3.** Early ( $z=10$ ) and late-time ( $z=0$ ) dark matter halo mass functions of the CDM model compared to those of the WDM models. Note the deficit by a factor of  $\sim 40$  of 0.6 keV WDM haloes at  $z=10$  compared to the abundance of CDM haloes.