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## **OxCarre Research Paper 136**

# **Limit Pricing and the (In)Effectiveness of the Carbon Tax**

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# Limit Pricing and the (In)Effectiveness of the Carbon Tax<sup>\*</sup>

by

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### Short abstract

Demand for oil is very price inelastic. Facing such demand, an extractive cartel induces the highest price that does not destroy its demand, unlike the conventional Hotelling analysis: the cartel tolerates ordinary substitutes to its oil but deters high-potential ones. Limit-pricing equilibria of non-renewable-resource markets sharply differ from usual Hotelling outcomes. Resource taxes have no effect on current extraction; extraction may only be reduced by supporting its ordinary substitutes. The carbon tax applies to oil and also penalizes its ordinary (carbon) substitutes, inducing the cartel to increase current oil production. The carbon tax further affects ultimately-abandoned oil reserves ambiguously.

*JEL classification:* Q30; L12; H21; Q42

*Keywords:* Carbon tax; Limit pricing; Non-renewable resource; Monopoly; Demand inelasticity; Substitutes subsidies.

## Long abstract

This paper argues that the effectiveness of a carbon tax is very limited when limit pricing arises on the oil market; a possibility that all existing studies on the design of the optimal carbon tax assumed away.

Demand for energy, for fossil fuels like oil in particular, is notoriously very price inelastic, even in the long run. Facing such demand, an extractive cartel increases its profits with higher prices, as long as those prices do not destroy the demand for its resource. The academic literature inspired by Hotelling has extensively examined how market power on non-renewable resource markets is limited by the intertemporal constraint that reserves are exhaustible. In contrast, allowing low demand-elasticity levels highlights the static constraint that substitution possibilities may destroy profits.

The demand for the cartel's oil features kinks, each corresponding to the entry price of one competing substitute. Some substitutes may be tolerated by an oil-extracting cartel (e.g. some non-cartel oil and other fuels, including existing biofuels, solar and wind sources of energy...). However, when a substitution possibility has the potential to drastically deteriorate its market share, the cartel maximizes its profits by inducing the "limit price" that deters its entry. Limit-pricing equilibria of non-renewable resource markets sharply differ from the conventional Hotelling outcome; for instance, taxes on the cartel's resource become neutral on its current extraction regardless of their dynamics.

Environmental policies may still reduce current extraction quantities when limit pricing occurs. For that, policies must support the production of existing substitutes, i.e. those not deterred by the cartel's pricing. On the contrary, a carbon tax increases the cartel's current oil extraction: while its direct application to the oil (carbon) resource is neutral, its application to oil's (carbon) existing substitutes induces the cartel to take over the left market shares. The carbon tax further affects abandoned oil reserves, yet ambiguously: it shortens the oil extraction period, which may not compensate higher extraction levels during this period.

*JEL classification:* Q30; L12; H21; Q42

*Keywords:* Carbon tax; Limit pricing; Non-renewable resource; Monopoly; Demand inelasticity; Substitutes subsidies.

## I. Introduction

There are three basic facts about the market for oil and its energy substitutes. First, the demand for energy is very price inelastic; in particular, the long-run price elasticity of the oil demand is commonly estimated to be substantially lower than one.<sup>1</sup> Second, exploitable oil reserves are highly concentrated; the OPEC cartel controls most of them.<sup>2</sup> Third, besides substitutes currently competing with conventional oil (e.g. other fuels, biofuels, renewable energies from alternative sources), very abundant resource bases may replace in large part the cartel's resource: e.g. future-generation biofuels and energy-production technologies, huge unconventional oil resources...

Under standard cost conditions, a monopoly facing a relatively inelastic demand may increase its profits by charging higher prices (reducing quantities supplied). Yet, there is a limit up to which this monopoly will do so: high enough prices warrant the large-scale profitability of substitutes that would destroy its demand. When substitution possibilities amount to a perfect substitute producible without limit under constant returns ("back-stop technology" as coined by Nordhaus, 1973), the monopoly maximizes its profits by inducing the limit price which deters substitution: below, higher prices increase profits; above, profits vanish. Unlike the ordinary case where demand elasticity continuously rises with price, the entry of drastic substitution possibilities rules out standard monopoly pricing; hence the difference.

This limit-pricing theory carries over to the case of an extractive monopoly that exploits a finite stock of resource over time: as long as there is some resource to be

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<sup>1</sup>Krichene's (2005) estimate of the long-run price elasticity of the demand for crude oil is (absolute value) 0.26 for 1974-2004; a level that almost coincides with the 0.25 elasticity used in Hamilton (2009b). According to Hamilton (2009a, pp. 217-218), since crude oil only represents about half the retail cost of final oil-based products like gasoline, the demand elasticity of the former is typically much lower than that of the latter (e.g. Hausman and Newey, 1995; Kilian and Murphy, 2014; references in Krichene, 2005, and in Hamilton, 2009a). See Hamilton (2009b, p. 192) on why the price elasticity of the crude oil demand should be expected to be even smaller now than over the last decades.

<sup>2</sup>According to the US Energy Information Administration (EIA), 73 percent of proved oil reserves (i.e. recoverable at existing conditions) were controlled by OPEC members in 2013. Also according to the EIA, "OPEC member countries produce about 40 percent of the world's crude oil. [...] OPEC's oil exports represent about 60 percent of the total petroleum traded internationally. Because of this market share, OPEC's actions can, and do, influence international oil prices." (Available at <http://www.eia.gov/finance/markets/supply-opeccfm>).

exploited, the monopoly's intertemporal profits are maximum when the instantaneous profit-maximizing limit price is induced at each date, unlike most dynamic problems. The monopolistically-supplied resource may be OPEC's oil, and its backstop substitute may be some virtually-unlimited energy like fusion power (Nordhaus, 1973, p. 532), like high-potential future-generation biofuels, or like enormous deposits of unconventional oil that would become exploitable and economic at some break-even price.<sup>3</sup> Under stationary conditions, limit pricing means a constant extraction path, together with a constant price path, until the resource is entirely depleted. The possibility that limit pricing arises in non-renewable-resource markets with low demand elasticity has been anticipated by Salant (1977, p. 8) and Hoel (1978, p. 31), but has remained unexplored.<sup>4</sup>

There are two basic limitations to the exercise of market power by an oil cartel like the OPEC. The academic literature inspired by Hotelling (1931) has extensively examined the intertemporal constraint that stocks to be exploited are exhaustible (Stiglitz, 1976),<sup>5</sup> a constraint sometimes considered "irrelevant" (Adelman, 1990, p. 1). In contrast, the instantaneous constraint that high prices may trigger the entry of some oil substitutes has not received much consideration, but by business analysts (e.g. Stephen Schork).<sup>6</sup>

Besides its empirical foundations, the appeal of the limit-pricing theory for the oil market further relies on its explanatory advantages.<sup>7</sup> Most importantly, unlike conventional

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<sup>3</sup>For example, see the western Siberian shale oil deposits in the EIA's (2013) report on existing resources, and the resentful Russian viewpoint about the recent oil price fall in the Wall Street Journal (October 15, 2014, p. 16).

<sup>4</sup>See also Dasgupta and Heal (1979, p. 343) and Newbery (1981, p. 625). Those works especially dealt with the curious limit-pricing phase that may follow Stiglitz' (1976) non-renewable-resource monopoly pricing stage. In that context, see also the investigation on the effect of backstop subsidies by van der Ploeg and Withagen (2012, p. 353).

<sup>5</sup>In one case, Stiglitz' extractive monopoly behaves like a competitive sector; as Pindyck (1987) put it, "potential monopoly in extractive resource markets can be limited by the depletable of reserves". Stiglitz' paradox may be resolved by allowing limit pricing as in this paper.

<sup>6</sup>The influential energy industry analyst reported to CNBC on August 16, 2010: "OPEC is more concerned about long-term market share than they are about short-term price gains. (...). I speak with OPEC regularly, and [raising the entry barrier for alternative fuels] is consistently their main concern (...). The cheaper you make OPEC oil, the harder you make it to bring alternative fuels to bring on." (<http://www.theatlantic.com/business/archive/2010/08/why-opec-doesnt-mind-low-oil-prices/61557/>).

<sup>7</sup>Limit pricing easily accounts for the relative long-run stationarity of oil prices and quantities (see Gaudet, 2007; Livernois, 2009). Despite identical stationary conditions, the constant-price and constant-quantity outcome of limit pricing sharply contrasts with the conventional Hotelling-type interior equilibrium where marginal revenue rises at the profit-discounting rate (e.g. Stiglitz, 1976).

Hotellian monopoly models (e.g. Stiglitz, 1976), limit-pricing equilibria studied here are compatible with less-than-one demand-elasticity estimates.<sup>8</sup> Also, the theory fits well with the US Energy Information Administration’s evidence that OPEC’s spare production capacities have been typically utilized to stabilize oil prices,<sup>9</sup> though often “through trial and error” (Adelman, 2004, p. 20). Last but not least, the relevance of these equilibria can also be substantiated on the ground of various accounts by OPEC-related personalities and commentators.<sup>10</sup> 40 years ago already, Jamshid Amuzegar revealed that “The first of [OPEC’s] principles is that the price of oil should be equivalent to the cost of alternative sources of energy.”<sup>11</sup> More recently, at a time when oil prices were around US\$130, OPEC Secretary General Abdullah al-Badri recognized that “[OPEC was] not happy with prices at this level because there will be destruction as far as demand is concerned”.<sup>12</sup> Interestingly, this is much in line with current expectations about OPEC supply: either OPEC will cut production to counter the oil price fall, or it is allowing it so as to protect its market share in front of more-and-more cost-efficient alternatives.<sup>13</sup>

Limit pricing on non-renewable-resource markets has recently gained more attention (e.g. van der Ploeg and Withagen, 2012, p. 353). A closely related line of research was initiated by Gerlagh and Liski (2011, 2014) and followed by Jaakkola (2012). They provide the dynamic counterparts in resource markets of strategic entry-prevention equilibria in the spirit of Bain (1949), Dixit (1979), or Milgrom and Roberts (1982).<sup>14</sup> Oil-exporting

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<sup>8</sup>In Hotelling models, as Stiglitz (1976) put it, the restriction avoids that “one can obtain larger profits by reducing [the quantity]”; as is well known, a monopoly never operates in regions of the demand curve where the price elasticity is less than unity. For the extractive monopoly’s problem to be well behaved in absence of backstop substitute, Stiglitz (1976) and many others assumed away so low elasticity levels. This restriction is often embedded in the form of the monopoly’s gross revenue function; for instance, Lewis, Matthews and Burness (1979) assumed it to be decreasing with price everywhere.

<sup>9</sup>See the OPEC-supply section of the EIA’s “What drives crude oil prices?” analysis at <http://eia.gov/finance/markets/supply-opeccfm>. Appendix D reports an EIA’s data chart.

<sup>10</sup>Cairns and Calfucura (2012) concluded from their analysis of OPEC behavior, that Saudi Arabia’s (and OPEC’s) dominant strategy is to “restrain the price to conserve its market in the long-run.”

<sup>11</sup>In this interview (Time Magazine, October 14, 1974, p. 36.), made famous by Dasgupta and Heal (1979), the Iran’s Minister of the Interior and the Shah’s right-hand oil expert, was explaining that OPEC’s strategy is to have the oil price following the industrialized countries’ inflation.

<sup>12</sup>He later identified US\$100 as a more “comfortable” price (available at <http://www.reuters.com/article/2012/05/03/us-opeccsupply-idUSBRE8420UY20120503>).

<sup>13</sup>See for instance the Washington Post, October 14, 2014, at [http://www.washingtonpost.com/business/economy/oil-prices-plunge-as-production-rises-fueling-concern-in-opecc2014/10/14/9bfd877c-53c9-11e4-892e-602188e70e9c\\_story.html](http://www.washingtonpost.com/business/economy/oil-prices-plunge-as-production-rises-fueling-concern-in-opecc2014/10/14/9bfd877c-53c9-11e4-892e-602188e70e9c_story.html).

<sup>14</sup>See an illuminating literature review in Tirole (1988, p. 306).

countries interact with oil-consuming nations which may costly switch to alternative sources of energy; exporters maintain low enough prices for such investment strategy to remain dominated. In contrast, limit pricing arises in our analysis absent strategic interactions; no coordinated demand side is required.<sup>15</sup>

This paper examines the effects of taxes – like the carbon tax – on a non-renewable resource – like oil – when limit pricing arises from the low elasticity of the resource demand. The taxation of non-renewable resources is revisited in that context. Much research efforts currently revolve around the design of the optimal carbon tax: see the influential works by Chakravorty, Moreaux and Tidball (2008), Metcalf (2008), Sinn (2008), Golosov, Hassler, Krusell and Tsyvinski (2014), van der Ploeg and Withagen (2014), among many others. It is hoped that both the taxation of carbon resources like oil and the support to non-carbon substitutes are effective instruments to curb carbon emissions that are responsible for global warming. Moreover, relatively high tax rates are already applied to oil products in most countries. From existing governmental commitments and in light of current national and international policy discussions on climate change mitigation, it is to be anticipated that tax rates on carbon energies may further increase and that a more favorable fiscal treatment will be given to their non-carbon substitutes.

Yet, there exists no study of taxation-induced changes in non-renewable-resource quantities that consider the possibility that supply obeys a limit-pricing logic, whether in the literature on non-renewable-resource taxation (e.g. Gaudet and Lasserre, 2013) or in the literature about market power on resource markets. Studies on the effect of taxes on resource monopolies are typically based on Stiglitz’s (1976) Hotelling-type analysis; e.g. Bergstrom, Cross and Porter (1981) or Karp and Livernois (1992). As we will see, exclusively relying on this conventional treatment leads to wrong conclusions about the effects of large-scale environmental policies. Thus our analysis is not only interesting for the methodology of economic applications to the oil market, but it is also critical for the design of public policies against a climate-change problem labelled “the ultimate commons problem of the twenty-first century” (Stavins, 2011).

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<sup>15</sup>The limit price considered in the present paper may nevertheless be interpreted as the price level, taken as exogenous here, that would trigger the *development* of alternative energies; a limit price that is endogenous in Gerlagh-Liski strategic equilibrium.



### *Structure and Main Results*

We start with a very simple limit-pricing setup: a finite stock of homogenous resource (oil, say) is depleted by a monopoly that faces a relatively price-inelastic demand, and substitution opportunities are summarized by a backstop technology; other aspects are progressively incorporated to the analysis.

In that first setting, we introduce a specific tax applied to the extracted flow of resource and we examine its effect in the spirit of Gaudet and Lasserre (2013). Unlike Hotelling models where only constant-present-value taxes are neutral (Dasgupta, Heal and Stiglitz, 1981), we show that resource taxes have in general no effect on current extraction. The goal of reducing the consumption flow of the resource cannot be achieved in the short term by directly penalizing extraction. Additionally, subsidizing the backstop at any date induces more extraction; unlike the “green paradox” (Sinn, 2008), the effect is contemporary rather than the result of an intertemporal substitution. This is the object of Section 2.

A backstop technology represents the possibility that the oil resource be completely replaced in the long run, by a virtually-infinite resource base capable of meeting all demand requirements. Following Nordhaus’ example, nuclear fusion would provide such energy abundance that oil would no longer be economically scarce. In contrast, currently-exploited substitutes to OPEC’s oil only offer limited substitution possibilities: the production of existing energy goods usually exhibits decreasing returns to scale because it relies on some scarce primary factors.<sup>16</sup> On these grounds, Section 3 introduces ordinary substitutes to the monopoly’s resource that have imperfectly-elastic supplies, unlike the

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<sup>16</sup>For non renewables (carbon fuels, uranium), scarcity arises from the finiteness of total exploration prospects and/or from the fact that low-cost reserves specifically are limited. Similarly for standard bio-fuels, as well as for solar and wind energy production, scarcity arises from land limitations. For instance, at the microeconomic level of a wind turbine, returns to scale should be increasing because the turbine involves a fixed set-up cost and almost-constant marginal costs of maintenance; at the macroeconomic level however, the unit cost of wind energy output must be increasing both because of land supply limitations and because the marginal land is of worse quality as far as wind exploitation is concerned. See for instance Chakravorty, Magné and Moreaux (2008) and Heal (2009) on land requirements and large-scale substitution of fuel products. Land availability is considered an issue as soon as further use of land causes rents to rise. The same is true for hydropower exploitation: for example in Switzerland, the 25 projects of new hydroelectric power plants will exhibit an expected average unit cost that is twice as large as that of the existing plants (Swiss Federal Office of the Energy, 2013, p. 7).

backstop. Each substitute is characterized by its entry price and has a rising marginal cost function.

Current substitution possibilities leave a (residual) demand for the cartel's resource, whose curve progressively reflects the multiplicity of substitutes, with kinks and increasing demand elasticity at those kinks. On the one hand, the backstop technology has the potential to destroy the entire resource demand. Profit maximization thus requires that it be deterred as in Section 2. On the other hand, ordinary substitutes are not sufficient threats to the resource market share to warrant deterrence. Extraction profits may increase with higher prices despite the fact that ordinary substitutes become economic, unlike the backstop. Limit pricing is compatible with ordinary substitutes being produced. In that context, resource taxes remain neutral and backstop subsidies retain their positive effect on extraction. In contrast, subsidies to (taxes on) ordinary substitutes increase (reduce) their production and so induce a reduction (rise) in the extraction flow by the same amount.

The above results are also obtained in Appendix A, where a stationary and much simplified version of the model is presented.

The energy-market model of Section 3 allows to examine the carbon tax. Not only oil, but some of its energy substitutes contain carbon. The carbon tax is formally equivalent to several taxes, each being applied to one carbon-containing good, to the extent of its carbon content. According to the above results, the fact that the carbon tax is directly applied to the cartel's oil has no effect on the equilibrium resource quantity. Currently-produced carbon substitutes to the resource are ordinary substitutes in our analysis. The fact that the carbon tax also penalizes these substitutes reduces their production, yet this reduction is compensated by an increase in the cartel's supply.

In Section 4, we consider a Ricardian resource that is incompletely depleted: extraction may become uneconomic before exploitable reserves are exhausted. Throughout the limit-pricing exploitation period, taxation policies retain their effects on current extraction, but may further affect ultimately extracted quantities. The carbon tax increases resource extraction, but shortens the resource exploitation period; its effect on the ultimately extracted quantity turns out to be ambiguous.

Finally in Section 5, with further details in the Appendix, we discuss limit-pricing equilibria in less parsimonious models integrating various aspects of the oil market. First and foremost, the models of Sections 3 and 4 are isomorphic to one with a competitive fringe supplying the same resource as the cartel (e.g. Salant, 1976), once an ordinary substitute is interpreted as the fringe's production; using recent elasticity estimates, we assure the empirical relevance of limit pricing in that case. We also elaborate on exploration and reserve development (e.g. Gaudet and Lasserre, 1988), as well as on the multiplicity of demand segments (e.g. Hoel, 1984).

## II. A Simple Limit-Pricing Model and the Effects of Taxation Policies

This section presents a limit-pricing model of a homogenous non-renewable resource market, where substitution possibilities are solely represented by a backstop substitute. We study the effects of taxes on the resource and of subsidies to the substitute.

### A. Static Limit Pricing

Consider first a single date  $t$ . At this date, a monopoly produces some energy resource flow  $q$  at a constant marginal cost  $c_t > 0$ .

The total energy demand is given by the function  $\overline{D}_t(p)$  of its price  $p$ ; it is continuously differentiable and strictly decreasing. We assume that the price elasticity of the energy demand is lower than unity all along the demand curve:  $\xi_{\overline{D}_t}(p) \equiv -\overline{D}'_t(p)p/\overline{D}_t(p) < 1$ .

There is a backstop technology by which a competitive sector can produce a perfect substitute to the resource at a constant positive marginal cost  $p_t^b > c_t$ . The demand notion that is relevant to the monopoly is the residual demand it faces.<sup>17</sup> Let us denote it with  $D_t(p) \leq \overline{D}_t(p)$ . When  $p < p_t^b$ , the production of the substitute is not profitable and thus the residual demand for the resource amounts to the entire energy demand  $D_t(p) = \overline{D}_t(p)$ . When  $p \geq p_t^b$ , the substitute becomes profitable, so that the market establishes the resource price to  $p = p_t^b$ . Were the resource price strictly higher than  $p_t^b$ , the resource demand would be destroyed:  $D_t(p) = 0$ . For notational simplicity

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<sup>17</sup>The presence of a competitive fringe producing an identical resource amounts to interpreting the residual demand for each price as being net of the fringe's supply for that price (e.g. Salant, 1976). More on that further below, in Section 5.

and without any consequence on our message, we assume as is standard that if  $p = p_t^b$  consumers give priority to the resource: at this price, the monopoly may serve the entire demand  $D_t(p_t^b) = \overline{D}_t(p_t^b)$ , assumed to be strictly positive.

To sum up, we make the following assumption.

**Assumption 1 (Low price elasticity of the resource demand)**

*For all prices  $p < p_t^b$ , the residual demand  $D_t(p) = \overline{D}_t(p)$  for the monopoly's resource is strictly positive and exhibits a low elasticity*

$$\xi_{D_t}(p) \equiv -D'_t(p)p/D_t(p) < 1; \quad (1)$$

*at price  $p = p_t^b$ , the monopoly may serve any demand portion  $q \in [0, \overline{D}_t(p_t^b)]$ ; for prices  $p > p_t^b$ , its demand vanishes.*

Figure 1 illustrates the residual demand schedule and its kink at price  $p = p_t^b$ .

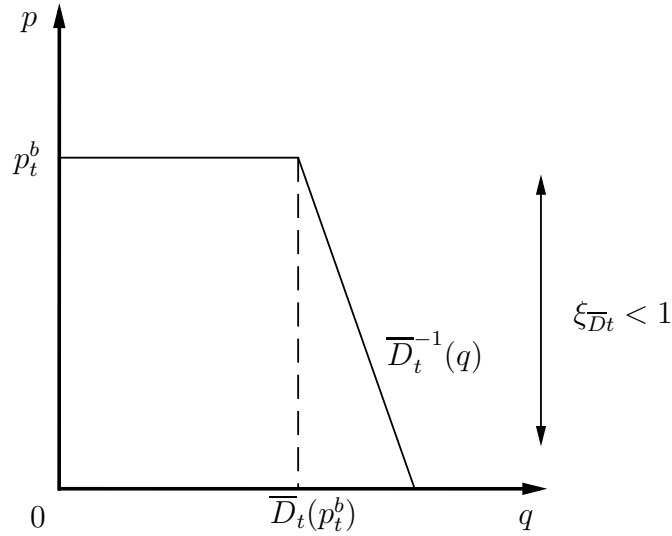


Figure 1: Residual demand for the resource with a backstop technology

Which production level maximizes the monopoly's profits in that context? If the monopoly supplies an amount  $q$  that is lower than the threshold quantity  $\overline{D}_t(p_t^b) > 0$ , it meets the demand at the resource market price is  $p = p_t^b$ ; the monopoly's spot profit is  $(p_t^b - c_t)q$ , strictly increasing in  $q$ . With a higher supply  $q > \overline{D}_t(p_t^b)$ , the monopoly

depresses the price below  $p_t^b$ ; its spot profit as function of the resource quantity becomes  $(\overline{D}_t^{-1}(q) - c_t)q$ , which is strictly decreasing in  $q$  because demand is sufficiently inelastic.<sup>18</sup> Indeed, marginal profit may be written  $p(1 - 1/\xi_{\overline{D}_t}(p)) - c_t$ , where  $\xi_{\overline{D}_t}(p) < 1$  implies the term into parentheses to be negative. To sum up, the instantaneous profit is

$$\pi_t(q) = \begin{cases} (p_t^b - c_t)q, & \text{increasing, for } q \leq \overline{D}_t(p_t^b) \\ (\overline{D}_t^{-1}(q) - c_t)q, & \text{decreasing, for } q > \overline{D}_t(p_t^b) \end{cases}, \quad (2)$$

which is maximized by the supply level  $q_t = \overline{D}_t(p_t^b)$  that induces the limit price  $p_t = p_t^b$ , the maximum price that deters the entry of the backstop.

The limit-pricing optimum differs from the usual optimum of a static monopoly. Convention has it that the demand function is differentiable everywhere, so that its elasticity to price evolves continuously along its curve. As is well known in that context, a conventional monopoly always deviates from less-than-unity elasticity demand sections because it enjoys higher prices (e.g. Tirole, 1988, p. 66). In contrast, when the entry of a substitute causes a kink to the demand as per Assumption 1, the elasticity at this kink jumps from a low level to a very high level which reflects that the demand is destroyed. This entry threat maintains the profit-maximizing monopoly supply on the lower-than-unity elasticity section of its demand; at the limit-pricing monopoly solution, higher elasticity levels are not observed.

### B. Intertemporal Limit Pricing of Extraction

Consider now that the resource is non-renewable; it is available in a finite quantity  $Q_0 > 0$ , that is to be extracted over the continuum of dates  $t \in [0, +\infty)$ .

In that case, the monopoly's problem becomes intertemporal. Assuming a discount rate  $r > 0$ , the stream of discounted profits amounts to

$$\int_0^T \pi_t(q_t) e^{-rt} dt, \quad (3)$$

where the function  $\pi_t(q_t)$  is given by the function (2) and where the terminal date  $T \geq 0$  is endogenous. The monopoly chooses the extraction path  $(q_t)_{t \geq 0}$  in such a way as to

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<sup>18</sup>The less-than-one demand elasticity as per Assumption 1 is sufficient, not necessary. It can easily be shown that the extraction profit  $(\overline{D}_t^{-1}(q) - c_t)q$  is strictly decreasing under elasticity levels  $\xi_{\overline{D}_t} < p_t^b/(p_t^b - c_t)$ ; a threshold greater than one in general, but with zero extraction costs.

maximize (3) under the exhaustibility constraint

$$\dot{Q}_t = -q_t, \text{ with } Q_T \geq 0, \quad (4)$$

where  $Q_t$  denotes the remaining stock at date  $t$ , and  $Q_0 > 0$  is given.

In such dynamic problems, the relevant instantaneous objective is the Hamiltonian function. The Hamiltonian at some date  $t \geq 0$  does not only consist of the present-value static profit objective  $\pi_t(q_t)e^{-rt}$ ; it is corrected by a linear term that reflects the opportunity cost of extracting the scarce resource. For the problem of maximizing (3) under (4), the Hamiltonian writes

$$\mathcal{H}(q_t, Q_t, \lambda_t, t) \equiv \pi_t(q_t)e^{-rt} - \lambda_t q_t, \quad (5)$$

where  $\lambda_t \geq 0$  is the multiplier associated with constraint (4).  $\lambda_t$  must be interpreted as the discounted scarcity value of the resource. By the Maximum Principle, it is constant over time at the producer's optimum:  $\lambda_t = \lambda$ .<sup>19</sup>

A necessary condition for the optimal choice of extraction  $q_t$  is that the Hamiltonian (5) is maximized at all dates of the extraction period. Since  $\lambda q$  is linear in  $q$ , as well as  $\pi_t(q)$  in (2) at the left of its maximum, it follows that the Hamiltonian is maximum for the same supply level  $q_t^m = \overline{D}_t(p_t^b)$  as the instantaneous revenue  $\pi_t(q)$  in (2), as long as the discounted marginal revenue  $(p_t^b - c_t)e^{-rt}$  remains greater than the scarcity value  $\lambda$  (See Figure 2).

In the stationary version of the model,  $p^b$  and  $c$  are constant with  $p^b > c$ , so that the discounted marginal revenue  $(p^b - c)e^{-rt}$  is strictly decreasing because of discounting. In the non-stationary model used here, it need not be so. For simplicity, we make the following assumption that excludes supply interruptions during the resource exploitation phase;<sup>20</sup> an assumption that will be maintained until Section 4.

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<sup>19</sup>The time independence of  $\lambda$  along the optimal producer path is standard in models of Hotellian resources. It arises from the fact that the Hamiltonian does not depend on  $Q_t$  because the resource is homogenous. In Section 4, we will examine the case of heterogenous resources.

<sup>20</sup>The analysis can easily accommodate supply interruptions, as when the limit price  $p_t^b$  falls short of  $c_t$  for some dates of the exploitation period. This would not modify the analysis in any insightful manner. Section 4 considers the possibility that limit-pricing extraction becomes uneconomic after some date.

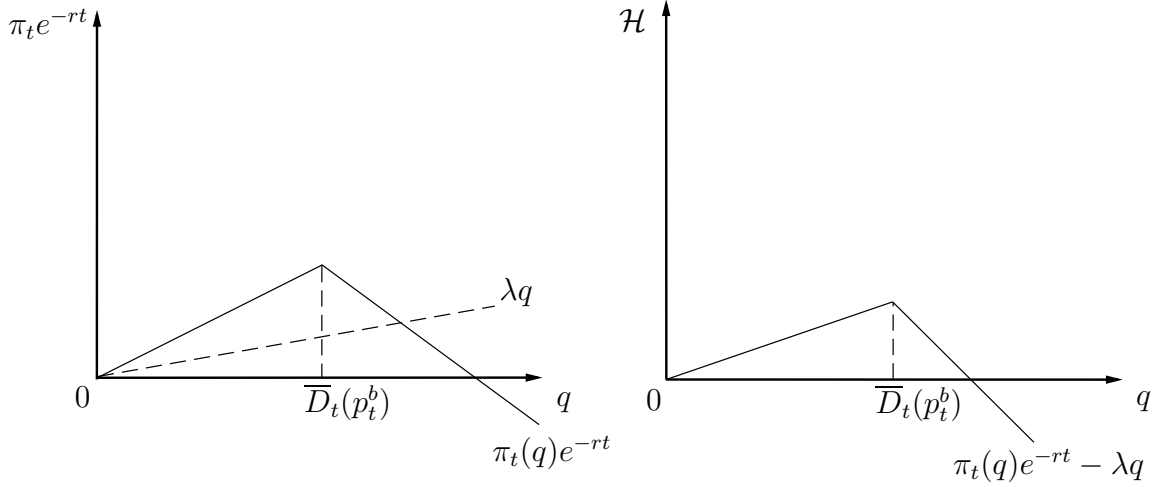


Figure 2: Instantaneous profit and Hamiltonian value

**Assumption 2 (Complete and uninterrupted extraction)**

*For all dates  $t \geq 0$ , the limit-pricing marginal revenue is strictly positive and strictly decreasing in present-value terms.*

Absent any policy, the limit-pricing marginal revenue is  $p_t^b - c_t$ . By Assumption 2, for all  $t \geq 0$ ,  $p_t^b > c_t$  and  $(p_t^b - c_t)e^{-rt}$  is strictly decreasing, as in the standard stationary treatment.

Assume, as a statement to be contradicted, that  $\lambda$  is nil. Since the present-value marginal revenue  $(p_t^b - c_t)e^{-rt}$  is always strictly positive by Assumption 2, extraction must be  $q_t^m = \overline{D}_t(p_t^b) > 0$  at all dates  $t \geq 0$ . Clearly, this would violate the exhaustibility constraint (4) in finite time.

Therefore we must conclude that the opportunity cost of extraction  $\lambda$  is strictly positive, so that the dynamic constraint (4) is relevant and the resource is economically scarce. Despite that, the rest of the resolution formally shows that the monopoly's optimum consists in the limit-pricing quantity  $q_t^m = \overline{D}_t(p_t^b)$  from date 0, until the resource is exhausted; the reader may also directly refer to the heuristic verification that follows Proposition 1.

Now let us contradict that  $p_0^b - c_0 < \lambda$ . Were this true, by Assumption 2,  $(p_t^b - c_t)e^{-rt}$  would fall short of  $\lambda$  for all  $t \geq 0$ ; thus no extraction at all would be optimal. Since  $p_t^b > c_t$

for all  $t \geq 0$ , this would be trivially dominated by some strictly positive extraction.

It follows that the marginal extraction profit  $(p_t^b - c_t)e^{-rt}$  is greater than or equal to the opportunity cost  $\lambda$ , from date 0, until the terminal date  $T^m$ . At date  $T^m$ ,

$$(p_{T^m}^b - c_{T^m})e^{-rT^m} = \lambda > 0, \quad (6)$$

i.e. extraction stops when the marginal extraction benefit meets the extraction opportunity cost. Since  $\lambda > 0$ ,  $T^m$  must also be the exhaustion date:  $Q_{T^m} = 0$ . Since extraction is  $q_t^m = \overline{D}_t(p_t^b) > 0$  all along the exploitation period  $[0, T^m]$ , the exhaustion date  $T^m$  is characterized by

$$\int_0^{T^m} \overline{D}_t(p_t^b) dt = Q_0. \quad (7)$$

Combining conditions (6) and (7) determines  $\lambda$ .

In the stationary model, the limit-pricing quantity  $q^m = \overline{D}(p^b)$  is constant, that induces the limit price  $p^m = p^b$ . The terminal date  $T^m$  is given by  $T^m = Q_0 / \overline{D}(p^b)$ , which implies  $\lambda = (p^b - c)e^{-rT^m}$ .

When Assumptions 1 and 2 are verified, the general properties of the limit-pricing equilibrium in absence of taxation policies are summarized in the following proposition.

**Proposition 1 (Limit-pricing equilibrium)**

1. *The monopoly supplies  $q_t^m = \overline{D}_t(p_t^b) > 0$ , and so induces the limit price  $p_t^m = p_t^b$  that deters the backstop-substitute production, at each date  $t$  of the extraction period  $[0, T^m]$ ;*
2. *The limit-pricing equilibrium leads to the complete exhaustion of the resource at the date  $T^m$  such that  $\int_0^{T^m} \overline{D}_t(p_t^b) dt = Q_0$ .*

It can easily be verified that deviations from this extraction path would decrease the sum of the monopoly's discounted profits. Two types of deviations are possible. First, consider reallocations of an infinitesimal quantity  $\Delta > 0$  of resource from any date  $t$  to any date  $t' \neq t$  such that  $t, t' < T^m$ . Reducing extraction by  $\Delta$  at date  $t$  decreases present-value profits by  $(p_t^b - c_t)\Delta e^{-rt}$  while increasing extraction at date  $t'$  decreases profits as well, since profits are decreasing for quantities exceeding the limit-pricing extraction  $q_t^m$ .



Second, consider reallocations of an infinitesimal quantity  $\Delta > 0$  of resource from any date  $t \leq T^m$  to any date  $t' > T^m$ . Again, reducing extraction by  $\Delta$  at date  $t$  decreases present-value profits by  $(p_t^b - c_t)\Delta e^{-rt}$ . On the other hand, increasing extraction at date  $t'$ , from zero, by  $\Delta$ , increases present-value profits by  $(p_{t'}^b - c_{t'})\Delta e^{-rt'}$ . However by Assumption 2,  $(p_{t'}^b - c_{t'})e^{-rt'} < (p_t^b - c_t)e^{-rt}$ , so that the overall effect on the discounted stream of profits remains negative.

This section shows a very peculiar characteristic of limit-pricing equilibria of non-renewable-resource markets. Unlike most dynamic problems, Proposition 1 implies that the maximization of the intertemporal profit objective (3) is compatible with pursuing the maximization of instantaneous profits (2) at each date of the exploitation period. This is so despite the fact that the dynamic exhaustibility constraint (4) is active. Thus if we observe that a resource monopoly maximizes instantaneous profits, we should not conclude that such rule is not (privately) optimal.

In other words, the limit-pricing theory reconciles two apparently incompatible analyses of oil supply. On the one hand, the conventional treatment adopted by most resource economists requires that the dynamic dimension added by oil's exhaustibility be taken into account. On the other hand, Adelman (1990) and many energy analysts offer an interpretation of OPEC's behavior where the exhaustibility constraint plays no relevant role.

Besides, there are two noticeable differences between the limit-pricing equilibrium arising here and conventional Hotelling equilibria. First, throughout the extraction period, the equilibrium present-value marginal revenue  $(p_t^b - c_t)e^{-rt}$  of the monopoly may be time varying – it is decreasing in the stationary model –, unlike Hotelling analysis where it is always constant, equal to the scarcity value  $\lambda$ ; this is so despite the assumption that the resource is homogenous in both cases. Second, the stylized fact that the oil demand has a less-than-unity price elasticity at equilibrium is observed. This is incompatible with conventional treatments of monopoly power on resource markets, either because low-elasticity levels are assumed away (e.g. Stiglitz), or because the discontinuity of demand elasticity resulting from large-scale drastic substitution possibilities is not taken into account.

### C. Taxes on the Non-Renewable Resource

Let  $\theta_t$  be a specific resource tax (or subsidy if negative) applied to the producer resource price  $p_t$  at each date  $t \geq 0$  to determine the consumer resource price  $p_t + \theta_t$ .<sup>21</sup>

The consumer price at which the substitute becomes profitable is  $p_t^b$ , regardless of the tax. Therefore the resource supply that induces this limit consumer price remains the one given by the demand relation:  $q_t = \overline{D}_t(p_t^b)$ . With a lower supply, the substitute is profitable: the market establishes the (tax-inclusive) consumer resource price at level  $p_t^b$  and thus the resource producer price at level  $p_t^b - \theta_t$ . With a greater supply  $q_t \geq \overline{D}_t(p_t^b)$ , only the resource may be produced so that the (tax-inclusive) consumer price is given by the inverse demand  $\overline{D}_t^{-1}(q_t)$ : the price accruing to the producer becomes  $\overline{D}_t^{-1}(q_t) - \theta_t$ .

It turns out that the problem of the previous section is only modified to the extent that the instantaneous profit becomes

$$\pi_t(q) = \begin{cases} (p_t^b - \theta_t - c_t)q, & \text{increasing, for } q \leq \overline{D}_t(p_t^b) \\ \left(\overline{D}_t^{-1}(q) - \theta_t - c_t\right)q, & \text{decreasing, for } q > \overline{D}_t(p_t^b) \end{cases} \quad (8)$$

The modification amounts to integrating the tax  $\theta_t$  to the marginal cost  $c_t$ .

Let Assumption 2 apply in this context, where the cost  $c_t$  in the absence of policies is replaced here by  $c_t + \theta_t$ . The assumption implies focusing on taxes that leave extraction attractive along the exploitation period. First, the property that the limit-pricing marginal revenue  $p_t^b - c_t - \theta_t$  remains positive for all  $t \geq 0$  excludes so high taxes that would leave no extraction profits at all. Second, the property that  $p_t^b - c_t - \theta_t$  is decreasing in present value excludes taxes (subsidies) that are falling (rising) too rapidly. The two conditions rule out the possibility that depletion be interrupted during the exploitation phase.<sup>22</sup>

Once Assumption 2 is adjusted that way, the analysis of the previous subsection follows through, unchanged, and the same limit-pricing equilibrium described in Proposition 1

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<sup>21</sup>This is a consumer tax. As for instance in Bergstrom et al. (1981), its effect is formally equivalent to that of a tax falling on the producer.

<sup>22</sup>In the stationary model, the assumption holds in particular for all constant taxes (and subsidies)  $\theta < p^b - c$ , as discounting implies their present value  $(p^b - c - \theta)e^{-rt}$  to decrease. It also holds for all rising taxes (falling subsidies), as well as for those taxes (subsidies) that are not too decreasing (increasing) over time. For example let a tax  $\theta_t$  have an initial level  $\theta > 0$  and be rising at a negative rate  $\alpha < 0$ :  $\theta_t = \theta e^{\alpha t}$ . It can easily be shown that Assumption 2 applies as long as  $\alpha > 1 - (p^b - c)/\theta$ , with  $p^b - c > \theta$ . In the time-dependent model where  $p_t^b - c_t$  is decreasing, the set of admissible taxes is broader.

is realized. Indeed the quantity that the monopoly needs to supply so as to deter the backstop production remains, at each date  $0 \leq t \leq T^m$  of the exploitation period,  $q_t^m = \overline{D}_t(p_t^b)$ , regardless of whether the resource is taxed or not; in the limit-pricing equilibrium, the path of resource taxes have no effect on the monopolist's extraction. Meanwhile, its revenues are reduced by the tax burden  $\theta_t \overline{D}_t(p_t^b)$  at each extraction date.

The following proposition summarizes the effect of resource taxes that are compatible with Assumption 2.

**Proposition 2 (Effect of resource taxes)**

*Resource taxes leave resource extraction unchanged.*

Extreme resource taxes eliminated by Assumption 2 might cause resource supply interruptions during the exploitation phase. First, too high taxes  $\theta_t \geq p_t^b - c_t$  for some  $t \leq T^m$  would expropriate the entire profit at the monopolist's optimum; the monopolist in that case would be better-off with no extraction. Second, taxes that are falling so rapidly that discounted marginal revenue is greater at distant dates  $t > T^m$  than during the exploitation period would lead the monopolist to completely shift extraction away from the exploitation phase.

There also exist some neutral resource taxes in standard Hotelling models. Dasgupta et al. (1981) showed that specific resource taxes rising at the rate at which profits are discounted leave the extraction of a competitive sector unchanged; they do not modify the intertemporal no-arbitrage condition that prevails in any Hotelling competitive equilibrium. As noticed by Karp and Livernois (1992), this also applies under monopoly.<sup>23</sup> Also, under competition as well as in a monopoly, extreme taxes that eat the entire Hotelling rent do not warrant any extraction.

Although reminiscent of Dasgupta et al.'s (1981) and Karp and Livernois' (1992) neutrality result, the result of Proposition 2 is much stronger. The novelty lies in the

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<sup>23</sup>In Hotelling equilibria, whether under competition or monopoly, there exists a family of optimal resource tax/subsidy paths. This family is indexed by a tax component  $Ke^{rt}$ , where  $K$  is some scalar. As Karp and Livernois (1992, p. 23) put it: "If the amount  $Ke^{rt}$  is added to [the optimal unit tax], the monopolist will still want to extract at the efficient rate, provided that the dynamics rationality constraint is satisfied (...)."

fact that resource taxation neutrality in limit-pricing equilibria does not require taxes to obey any particular dynamics.

#### *D. Subsidies to the Backstop Substitute*

Alternatively, let  $\gamma_t^b$  be a specific subsidy to the backstop substitute, applied to the backstop's producer price, which is also its marginal cost  $p_t^b$ . Thus, the problem in absence of taxation is only modified to the extent that the price of the backstop substitute  $p_t^b$  should be replaced by the consumer net-of-subsidy price  $p_t^b - \gamma_t^b$ . Unlike a resource tax, a backstop subsidy  $\gamma_t^b$  always affects the limit-pricing equilibrium.

When the substitute consumer price is reduced to  $p_t^b - \gamma_t^b$ , the resource supply that deters its production rises to  $\overline{D}_t(p_t^b - \gamma_t^b) > \overline{D}_t(p_t^b)$ ; indeed, more resource needs to be supplied to deter a cheaper backstop. Also, low resource quantities  $q_t < \overline{D}_t(p_t^b - \gamma_t^b)$  that warrant the production of the substitute reduce the resource price to  $p_t^b - \gamma_t^b$ , so that the marginal extraction profit of the monopolist becomes  $p_t^b - \gamma_t^b - c_t$ .

We only consider subsidies that do not violate Assumption 2, so that a continuous resource supply is warranted throughout the exploitation period. This avoids extreme subsidies that would cause resource supply interruptions. First,  $p_t^b - \gamma_t^b - c_t > 0$ , for all  $t \geq 0$ : the condition assumes away subsidies that would destroy extraction profits because the substitute would be available to consumers for a price  $p_t^b - \gamma_t^b$  lower than the resource extraction cost  $c_t$ . Second,  $p_t^b - \gamma_t^b - c_t$  remains decreasing in present value for all  $t \geq 0$ : the condition rules out backstop subsidies that are so decreasing over time that they would make extraction less attractive during the exploitation phase than at more distant dates.

With Assumption 2, the instantaneous extraction profit with backstop subsidies becomes

$$\pi_t(q) = \begin{cases} (p_t^b - \gamma_t^b - c_t)q, & \text{increasing, for } q \leq \overline{D}_t(p_t^b - \gamma_t^b) \\ \left(\overline{D}_t^{-1}(q) - c_t\right)q, & \text{decreasing, for } q > \overline{D}_t(p_t^b - \gamma_t^b) \end{cases}, \quad (9)$$

and the same dynamic analysis applies as before. It follows that at each date of the resource exploitation phase, the monopoly chooses the limit-pricing supply  $q_t^m = \overline{D}_t(p_t^b - \gamma_t^b)$  that deters the backstop production.

Figure 3 illustrates how backstop subsidies shift the demand kink along the demand

curve and modify the limit-pricing equilibrium. The following proposition summarizes the effect of subsidies to a backstop substitute in the context of this section.

**Proposition 3 (Effect of subsidies to the backstop substitute)**

*Subsidies to the backstop substitute increase the resource current extraction.*

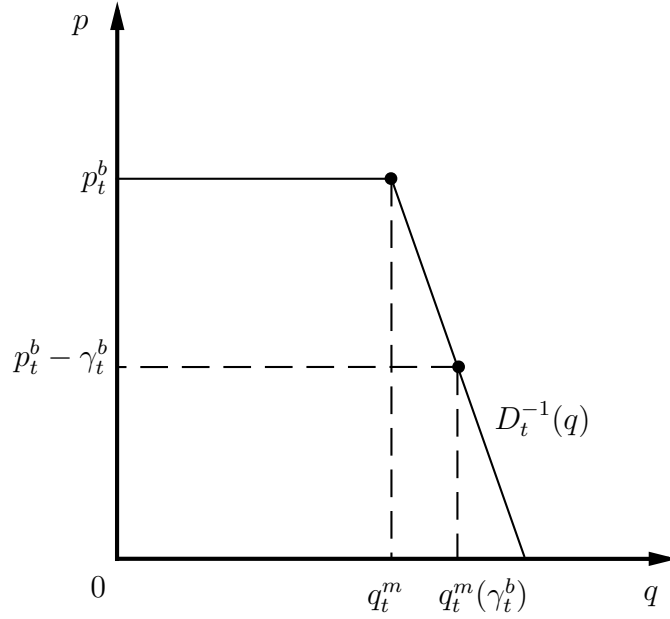


Figure 3: Limit-pricing equilibrium and the effect of a backstop subsidy

If taxation policies aim at reducing current oil extraction, the model of this section yields a rather pessimistic message. Leave aside extreme policies that would cause supply disruptions: not only are resource taxes strongly neutral, but subsidizing the backstop substitute induces the monopoly to increase its supply.

### III. Ordinary Substitutes

A backstop technology is a standard and meaningful modeling device. It represents the possibility that the resource be completely replaced, as a result of a virtually-infinite resource base. Whether in conventional Hotelling-type equilibria or in the limit-pricing equilibrium of Section 2, such backstop technology is never used before the exhaustion

date, after which it becomes the exclusive source of energy.

In contrast, empirical evidence shows that ordinary substitutes to oil are currently traded and consumed on energy markets, such as other regular fuels, biofuels, and alternative energies. Yet, each substitute remains far from meeting a large fraction of the energy demand.

In this section, we do away with the assumption that there is a single (backstop) substitute and allow for the possibility that some ordinary substitutes may be used along the resource extraction phase. Limit pricing to deter the backstop substitute is not incompatible with ordinary substitutes being produced during the resource exploitation phase.

#### *A. The Model*

The elasticity of the residual demand is often interpreted as the extent of substitution opportunities (e.g. Lewis et al., 1979). Marshall (1920) argued that, ordinarily, demand curves should be expected to have the property that the price elasticity is increasing with price. In this section, there are several substitutes whose entries sequentially kink the resource demand and increase its elasticity.

The backstop substitute retains the same role as in Section 2; for prices greater than its entry price  $p > p_t^b$ , it offers an unlimited substitution opportunity that will induce the resource monopoly to deter its production.

We further consider ordinary substitutes. Like the backstop, ordinary substitutes are assumed to be perfect ones and are produced competitively.<sup>24</sup> Yet they only offer relatively limited substitution possibilities because their production exhibits decreasing returns to scale. In fact, ordinary substitutes offer so low substitution possibilities, that the resource monopoly does not find optimal to deter them. In brief, we define them in the following way, that will be given more precise grounds shortly below.

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<sup>24</sup>Similarly one may consider substitutability to be partial because some ordinary substitutes to oil only replace the resource for some uses (Hoel, 1984); the case of various uses with use-specific imperfect substitutes will be discussed in Section 5.

**Definition 1 (Ordinary substitute)**

*With an ordinary substitute, Assumption 1 remains satisfied at all dates  $t \geq 0$ .*

As already argued in the Introduction, the supply of existing energy goods is subject to limitations that typically arise because of the scarcity of some factors.<sup>25</sup> Whether this scarcity is static (e.g. land, as in the case of biofuels, and wind and solar energies) or dynamic (e.g. finite exploitable reserves, as in the case of non-renewable fuels), higher instantaneous prices always warrant a higher instantaneous supply, yet at some greater marginal costs.<sup>26</sup> Thus for simplicity, we assume that the production of substitutes is static and the only good that we explicitly treat as non renewable is the resource supplied by the monopoly.

We consider for brevity a single ordinary substitute. As will be clear, the analysis immediately accommodates more than one such substitutes. The ordinary substitute is produced for all prices strictly greater than  $p_t^o > 0$ ; we further assume

$$c_t < p_t^o < p_t^b, \text{ for all } t \geq 0, \quad (10)$$

so as to exclude the uninteresting case where the ordinary substitute is deterred at the same time as the backstop.<sup>27</sup> Thus the ordinary substitute may be produced along the resource exploitation phase. We now examine the three sections of the residual resource demand curve, as is represented in Figure 4.

i) For all prices  $p \leq p_t^o$ , no substitute is competing with the resource at all. Hence, the residual demand that the monopoly is facing is the entire demand  $D_t(p) = \overline{D}_t(p)$ . Such range of prices is induced by sufficiently high monopoly extraction

$$q \geq \overline{D}_t(p_t^o) \quad (11)$$

over which

$$\pi_t(q) = \left( \overline{D}^{-1}(q) - c_t \right) q \text{ is decreasing} \quad (12)$$

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<sup>25</sup>See especially Footnote 16.

<sup>26</sup>In the case of a non-renewable competitively-produced substitute, supply is still characterized by the equalization of price with marginal costs, once marginal costs are adjusted to comprise the opportunity cost of extraction. See Sweeney (1993, pp. 775-776) for the interpretation of the instantaneous supply of a non-renewable resource.

<sup>27</sup>In principle, there may be substitutes, backstop or ordinary, with entry prices exceeding the equilibrium limit price, that are not produced over the limit-pricing extraction phase.

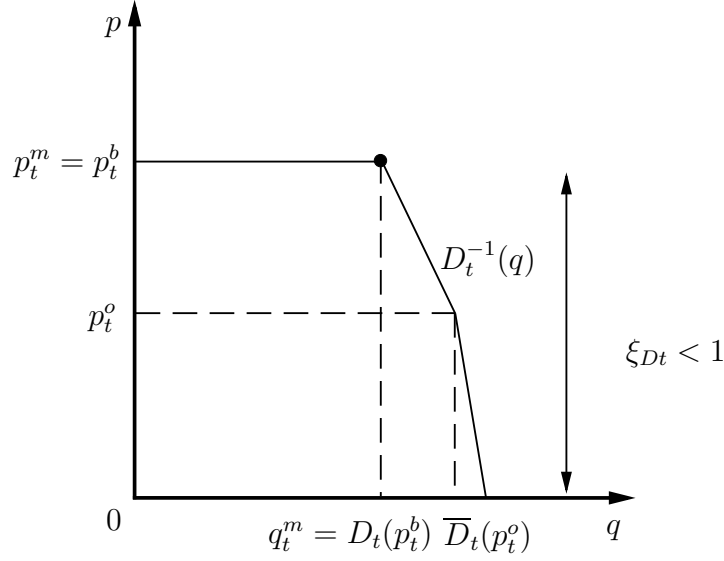


Figure 4: Residual demand and limit-pricing equilibrium with backstop and ordinary substitutes

by Assumption 1.

*ii)* For prices  $p_t^o < p \leq p_t^b$ , only the ordinary substitute is competing with the resource, as the resource price exceeds the entry price  $p_t^o$ , which is its marginal cost at the origin:  $p_t^o \equiv C_t^{o'}(0) > 0$ . Unlike the backstop, the ordinary substitute is unable to meet a large fraction of the resource demand without exhibiting substantial cost increase. Thus the marginal cost  $C_t^{o'}(x)$  of producing a quantity  $x$  of ordinary substitute is differentiable, strictly rising and the ordinary-substitute supply function  $S_t^o(p) \equiv C_t^{o'-1}(p)$  is continuous, with  $S_t^o(p) > 0$  if and only if  $p > p_t^o$ .

Yet the price elasticity of the ordinary substitute's supply  $\xi_{S^o_t}(x) = C_t^{o'}(x) / (C_t^{o''}(x)x)$  is low in the sense that the elasticity  $\xi_{D_t}(q)$  of the residual demand  $D_t(p) = \overline{D}_t(p) - S_t^o(p)$  satisfies the inequality

$$\xi_{D_t}(q) = \frac{e}{q} \xi_{\overline{D}_t}(e) + \frac{x}{q} \xi_{S^o_t}(x) < 1, \quad (13)$$

where  $e = q + x$  is the total energy supply. This way, Assumption 1 is verified, as per Definition 1.

The range of prices  $p_t^o < p \leq p_t^b$  over which only the ordinary substitute is produced



is induced by the monopoly's intermediate supplies

$$D_t(p_t^b) \leq q < \overline{D}_t(p_t^o), \quad (14)$$

with

$$D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b), \text{ assumed strictly positive.}^{28} \quad (15)$$

Over this range, it follows from (13) that

$$\pi_t(q) = (D_t^{-1}(q) - c_t) q \text{ is decreasing.} \quad (16)$$

*iii)* For all prices  $p > p_t^b$ , the backstop has the capacity of meeting the entire demand while remaining more attractive than both the ordinary substitute and the resource.<sup>29</sup>

Since the backstop is supplied competitively, any monopoly's supply as low as

$$q < D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b) \quad (17)$$

induces the resource price  $p = p_t^b$ , under which

$$\pi_t(q) = (p_t^b - c_t) q, \text{ is increasing.} \quad (18)$$

To sum up, the instantaneous profit with an ordinary substitute writes

$$\pi_t(q) = \begin{cases} (p_t^b - c_t) q, & \text{increasing, for } q < D_t(p_t^b) \\ (D_t^{-1}(q) - c_t) q, & \text{decreasing, for } D_t(p_t^b) \leq q < \overline{D}_t(p_t^o) \\ (\overline{D}_t^{-1}(q) - c_t) q, & \text{decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases}, \quad (19)$$

and is thus maximized by the supply level

$$q_t^m = D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b). \quad (20)$$

Thus, once  $q_t^m$  of Section 2 is adjusted to become (20), the dynamic analysis of Section 2 applies as before under Assumption 2. The following proposition summarizes the properties of the limit-pricing equilibrium in the context of this section.

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<sup>28</sup>The assumption that  $D_t(p_t^b) > 0$  despite the ordinary substitute is the counterpart of  $\overline{D}_t(p_t^b) > 0$  in Section 2. This way, Assumption 1 is satisfied, which eliminates the uninteresting case where the backstop supply and the residual resource demand do not intersect at all.

<sup>29</sup>Instead of the backstop, limit pricing may seek to deter a substitute produced under decreasing – but slowly – returns. Consider a substitute with a sufficiently high, although not infinite, supply elasticity; beyond its entry price, it may cause the residual demand to be sufficiently elastic for the monopoly's profit to be increasing. The analysis easily accommodates that case, for no additional interesting insight.

**Proposition 4 (Limit-pricing equilibrium with an ordinary substitute)**

*In presence of an ordinary substitute,*

1. *The monopoly supplies  $q_t^m = D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b) > 0$  as per (20), and so induces the limit price  $p_t^m = p_t^b$  that deters the backstop substitute's production, at all dates  $t$  of the extraction period  $[0, T^m]$ ;*
2. *The limit-pricing equilibrium leads to the complete exhaustion of the resource at date  $T^m$  such that  $\int_0^{T^m} D_t(p_t^b) dt = Q_0$ ;*
3. *All along the extraction period  $[0, T^m]$ , the ordinary substitute is produced in quantity  $S_t^o(p_t^b) > 0$ .*

In the stationary model, the limit-pricing quantity  $q^m = D(p^b) = \overline{D}(p^b) - S^o(p^b) > 0$  is constant, so that the exhaustion date is  $T^m = Q_0 / (\overline{D}(p^b) - S^o(p^b))$ .

Absent taxation policies, the limit-pricing equilibrium at any date  $t$  of the exploitation phase is depicted in Figure 4. As far as taxation policies are concerned, the distinction between the deterred backstop and the on-use ordinary substitute, will turn out to be fundamental.

*B. Taxes on the Non-Renewable Resource*

The same way as in Section 2, a unit consumer tax  $\theta_t$  leaves unchanged the consumer price  $p_t^b$  at which the backstop substitute enters, and thus the limit extraction quantity  $D_t(p_t^b)$ , given by (20), that deters its entry. It also leaves the entry price  $p_t^o$  unchanged. Thus the tax only modifies the instantaneous revenue (19) to the extent that, for any extraction quantity  $q$ , the price accruing to the producer is the inverse demand  $D_t^{-1}(q)$  reduced by the tax  $\theta_t$ ; as if the cost  $c_t$  was augmented by the levy  $\theta_t$ .

When Assumption 2 is adjusted to the case of a resource tax, the instantaneous profit function becomes

$$\pi_t(q) = \begin{cases} (p_t^b - \theta_t - c_t) q, & \text{increasing, for } q < D_t(p_t^b) \\ (D_t^{-1}(q) - \theta_t - c_t) q, & \text{decreasing, for } D_t(p_t^b) \leq q < \overline{D}_t(p_t^o) \\ (\overline{D}_t^{-1}(q) - \theta_t - c_t) q, & \text{decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases} . \quad (21)$$

Thus to the extent that the tax does not violate Assumption 2 – it warrants no interruption of resource supply –, it will not affect the monopoly's limit-pricing path described in Proposition 4: the strong neutrality result of resource taxes and subsidies holds as per Proposition 2 in presence of an ordinary substitute.

### C. Subsidies to the Backstop Substitute

Subsidies to the backstop substitute also have the same effect as in Section 2, regardless of whether there is an ordinary substitute.

Consider a subsidy  $\gamma_t^b \geq 0$  to the backstop substitute. Its price is reduced to  $p_t^b - \gamma_t^b$ , which is also the resource price whenever the backstop is profitable. The extraction quantity that deters the entry of the backstop substitute is thus increased to

$$D_t(p_t^b - \gamma_t^b) = \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b), \quad (22)$$

instead of  $D_t(p_t^b)$  as in (20).

As long as backstop subsidies leave a strictly positive limit-pricing revenue to the monopoly, as by Assumption 2, its revenue is only modified in this respect. It rewrites

$$\pi_t(q) = \begin{cases} (p_t^b - \gamma_t^b - c_t) q, & \text{increasing, for } q < D_t(p_t^b - \gamma_t^b) \\ (D_t^{-1}(q) - c_t) q, & \text{decreasing, for } D_t(p_t^b - \gamma_t^b) \leq q < \overline{D}_t(p_t^o) \\ (\overline{D}_t^{-1}(q) - c_t) q, & \text{decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases}, \quad (23)$$

with the exact same consequence as in Section 2 for the effect of  $\gamma_t^b$ : the equilibrium limit-pricing extraction  $q_t^m$  is increased as per (22).

### D. Subsidies to Ordinary Substitutes

In the limit-pricing equilibrium of Proposition 4, the production of the backstop substitute is deterred by the monopoly. Currently-used substitutes must all be ordinary substitutes that satisfy Definition 1. As this section shows, in a limit-pricing context, the effect of subsidies to existing substitutes differs from the effect earlier identified of subsidies to the backstop.

With a subsidy  $\gamma_t^o \geq 0$  to the consumption of the ordinary substitute, the resource price at which its production is profitable becomes  $p_t^o - \gamma_t^o$ . Thus the extraction level below which the substitute enters is reduced to  $\overline{D}_t(p_t^o - \gamma_t^o)$  instead of  $\overline{D}_t(p_t^o)$  in (11).

For all resource prices  $p > p_t^o - \gamma_t^o$  – equivalently all extraction levels  $q < \overline{D}_t(p_t^o - \gamma_t^o) -$  that warrant the production of the ordinary substitute, its supply expressed as a function of the resource price is augmented to  $S_t^o(p + \gamma_t^o)$ . Accordingly, the residual demand for the resource is reduced by the same amount:  $D_t(p) = \overline{D}_t(p) - S_t^o(p + \gamma_t^o)$ .

Hence at the entry price  $p_t^b$  of the backstop substitute, the subsidy  $\gamma_t^o$  increases the ordinary substitute's production to  $S_t^o(p_t^b + \gamma_t^o)$  and reduces the residual demand faced by the monopoly by the same quantity; extraction to be supplied so as to deter the backstop's production is, instead of (20),

$$D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o), \quad (24)$$

lower than in absence of subsidy.

Definition 1 and Assumption 1 rule out the case where the ordinary substitute would satisfy the entire resource demand at some price below the backstop's price  $p_t^b$ . Thus by assumption, the residual resource demand at the limit price  $D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$  is strictly positive. This eliminates extreme subsidies  $\gamma_t^o$  that would make the ordinary substitute meet the entire energy demand, i.e. such that  $S_t^o(p_t^b + \gamma_t^o) > \overline{D}_t(p_t^b)$ , thus causing disruptions of resource supply.

Thus (19) rewrites

$$\pi_t(q) = \begin{cases} (p_t^b - c_t) q, & \text{increasing, for } q < \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o) \\ (D_t^{-1}(q) - c_t) q, & \text{decreasing, for } \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o) \leq q < \overline{D}_t(p_t^o - \gamma_t^o) , \\ (\overline{D}_t^{-1}(q) - c_t) q, & \text{decreasing, for } q \geq \overline{D}_t(p_t^o - \gamma_t^o) \end{cases} \quad (25)$$

where threshold quantities  $\overline{D}_t(p_t^o - \gamma_t^o)$  and  $\overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$  are reduced by the subsidy.

Thus the dynamic analysis of Section 2 follows through, and a limit-pricing equilibrium realizes, in which the monopoly supplies less, so as to induce the unchanged limit price  $p_t^b$ :  $q_t^m = \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$ , decreasing with  $\gamma_t^o$ . In Figure 5, the shift from the dark curve to the red curve depicts the reduction in the residual demand faced by the monopoly as a consequence of the subsidy to the ordinary substitute, and the resulting reduction in the limit-pricing resource quantity.

The message of the following proposition sharply contrasts with that of Proposition 3.

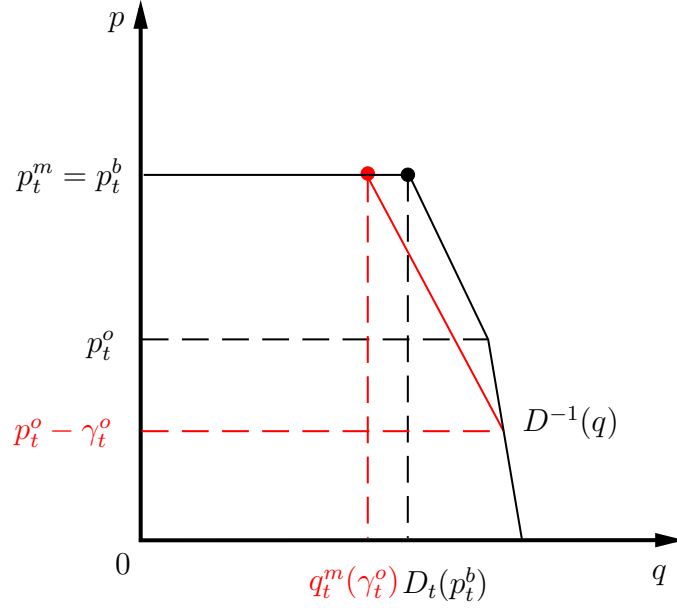


Figure 5: Limit-pricing equilibrium and the effect of a subsidy to the ordinary substitute

**Proposition 5 (Effect of subsidies to the ordinary substitute)**

*Subsidies to an ordinary substitute,*

1. *Increase the substitute current production;*
2. *Reduce the resource current extraction by the same quantity.*

Vive versa, taxes on an ordinary substitute reduce its current production and increase the resource demand by the same amount. Maximizing extraction profits requires serving the increased resource demand, as is illustrated by the shift from the red to the dark residual-resource-demand curve in Figure 5. Assume that in front of the reduction in the ordinary substitute's production, the monopoly was not adjusting its supply. Then, the backstop substitute would become profitable, and would meet the extra resource demand left by the ordinary substitute – the amount between the red and the dark kinks. Increasing its supply, so as to conquer the market share left by the ordinary substitute, at the backstop price  $p_t^b$  increases the monopoly's extraction profits until the backstop is completely excluded.

### *E. The Carbon Tax*

The carbon tax is applied to the carbon content of energy goods. Thus the carbon tax is formally equivalent to several taxes, each applied to a carbon-energy good, to an extent that reflects its unit carbon content.

In particular, the carbon tax comprises a tax on the oil resource of the monopoly as earlier examined. The result of Proposition 2 is valid in the context of this section, which indicates that such tax has no direct effect on the monopoly's resource supply. Indeed as earlier explained, it modifies neither the entry price of the backstop substitute nor the limit-pricing extraction.

Energy goods that are substitutes to the cartel's oil also contain carbon (e.g. other oil, gas, coal). Those carbon substitutes are currently produced and are ordinary substitutes in our analysis. Thus the carbon tax also acts as a tax on the ordinary substitute. The analysis of the previous subsection indicates that such tax (a negative subsidy) reduces the supply of the substitute and increases the monopoly resource supply by the same amount.

Since the carbon tax combines a resource tax with a tax on the ordinary substitute, its effect immediately results from Propositions 2 and 5, as summarized in the following corollary.<sup>30</sup> As the ordinary substitute's production is reduced by the carbon tax, the monopoly finds optimal to serve rather than abandon the left market share.

#### **Corollary 1 (Effect of the carbon tax)**

*The combination of a resource tax, with a tax on the ordinary substitute,*

- 1. Decreases the substitute current production;*
- 2. Increases the resource current extraction by the same amount.*

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<sup>30</sup>When units of the substitute contain more carbon than the resource, the carbon tax reduces carbon consumption, despite the fact that the increase in resource supply compensates the decrease in the substitute's production. Vice versa, when the substitute is less carbon intensive than the resource, the carbon tax increases carbon consumption.

#### IV. Resource Heterogeneity, Exploitation Duration and the Ultimately Extracted Quantity

In the simple limit-pricing model of Sections 2 and 3, resource reserves are completely depleted. In such context, our analysis showed that resource taxes like the carbon tax are very limited instruments to curb resource consumption and carbon emissions.

As a matter of fact, reserves of oil are highly heterogenous (see for instance the discussion in Hamilton, 2009a, 225-226). One standard way to take resource heterogeneity into account is to assume that marginal extraction costs increase as less reserves are to be extracted, as when the resource is Ricardian and its units are exploited in order of their respective costs. This approach has been recently used for instance by van der Ploeg and Withagen (2012, 2014) in works on the carbon tax.<sup>31</sup>

Stock effects notoriously introduce incentives to extract the resource less rapidly (Dasgupta and Heal, 1979). This section extends the limit-pricing model of Sections 2 and 3 to the Hotelling-Gordon cost representation just described. This extension makes the limit-pricing model comparable with the conventional non-renewable-resource monopoly model of Karp and Livernois (1992); it turns out that the limit-pricing outcome survives the introduction of stock effects.

Also with stock effects, the ultimately extracted quantity becomes endogenous because extraction can stop before the complete depletion of available reserves: the benefit derived from the last units to be extracted need not meet too high extraction costs. Thus in principle, more reserves may become economic or uneconomic as a result of a policy. This possibility, assumed away by classical papers on the taxation of resource monopolies (Bergstrom et al., 1981; Karp and Livernois, 1992),<sup>32</sup> is considered a fundamental aspect of climate policy.

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<sup>31</sup>The view that exploited reserves contribute to increasing extraction costs has been initiated by Hotelling (1931, p. 152), consolidated by Gordon (1967), and perfected, among others, by Weitzman (1976) and Salant et al. (1983).

<sup>32</sup>Karp and Livernois (1992) specifically considered that reserves are fully exploited, despite stock effects and taxation.

### A. The Model

Assume now that at each date  $t \geq 0$  the marginal extraction cost  $c_t$  is given by the decreasing function

$$c_t \equiv C_t(Q_t) > 0 \quad (26)$$

of remaining reserves  $Q_t \geq 0$ ; marginal cost increases as remaining reserves diminish. The function  $C_t$  is assumed differentiable everywhere.

To consider the possibility that extraction be incomplete, we do away in this section with Assumption 2 that the cost of extraction is always covered by extraction benefits. Absent any taxation policy, the discounted marginal limit-pricing revenue is  $(p_t^b - c_t)e^{-rt}$  with  $c_t = C_t(Q_t)$ , which may now be negative.

In this context, at any date  $t$  when remaining reserves are  $Q_t$ , the instantaneous monopoly revenue (19) writes in a way similar to Section 3:

$$\pi_t(q, Q_t) = \begin{cases} (p_t^b - C_t(Q_t)) q, & \text{increasing or decreasing, for } q < D_t(p_t^b) \\ (D_t^{-1}(q) - C_t(Q_t)) q, & \text{decreasing, for } D_t(p_t^b) \leq q < \overline{D}_t(p_t^o) \\ (\overline{D}_t^{-1}(q) - C_t(Q_t)) q, & \text{decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases} \quad (27)$$

For large supplies that deter the backstop, the monopoly revenue remains decreasing by Assumption 1 and Definition 1. However,  $\pi_t(q, Q_t)$  may not always be increasing for low supplies  $q > D_t(p_t^b)$  that warrant the backstop production. It retains the same form as before, and exhibits the same limit-pricing maximum  $D_t(p_t^b) > 0$  only when the limit-pricing marginal revenue  $p_t^b - C_t(Q_t)$  is positive. Otherwise, extraction is not economic for the monopoly; zero extraction is optimal.

As previously, the monopoly seeks to maximize its intertemporal stream (3) of discounted profits  $\pi_t(q_t, Q_t)$  over the free extraction period  $[0, T]$  under the exhaustibility constraint (4). At any date  $t \geq 0$ , its relevant instantaneous objective for the optimal choice of extraction  $q_t$  is given by the Hamiltonian

$$\mathcal{H}(q_t, Q_t, \lambda_t, t) \equiv \pi_t(q_t, Q_t)e^{-rt} - \lambda_t q_t, \quad (28)$$

where  $\lambda_t \geq 0$  denotes the multiplier associated with (4).

As described in Section 2 (see Figure 2), the Hamiltonian admits the same maximum as the instantaneous revenue (27) whenever the discounted marginal profit  $(p_t^b - C_t(Q_t))e^{-rt}$



is greater than the extraction opportunity cost  $\lambda_t \geq 0$ . In that case,  $(p_t^b - C_t(Q_t)) e^{-rt} > 0$ , so that by (27) the optimal extraction is the limit-pricing supply  $q_t^m = D_t(p_t^b)$ .

In the spirit of Assumption 2, its following alternative assumes that limit-pricing marginal revenue decreases in present value; this is made for simplicity to eliminate supply disruptions along the exploitation period; phenomena of relatively minor economic interest. Unlike Assumption 2 however, the alternative Assumption 3 considers that extraction may become uneconomic.

### **Assumption 3 (Uninterrupted incomplete extraction)**

*The marginal limit-pricing revenue is strictly positive at date 0 for initial reserves  $Q_0 > 0$ ; for all dates  $t \geq 0$  and any given reserves  $Q_0 \geq Q \geq 0$ , it is continuously decreasing in present-value terms as long as it is positive.*

Thus with no taxation policies, the marginal limit-pricing revenue  $(p_t^b - C_t(Q_t)) e^{-rt}$  is positive at early dates, and decreases continuously with time for two reasons: for unchanged reserves by Assumption 3, and because diminishing reserves increase  $C_t$  by (26). Yet, unlike Sections 2 and 3, the value  $\lambda_t$  of the scarce resource underground is also decreasing in this context, to reflect that reserves exploited later are more costly: by the Maximum Principle,  $\dot{\lambda}_t = C'_t(Q_t)q_t e^{-rt} < 0$  at each date  $t$  when an amount  $q_t > 0$  is extracted. Appendix B shows that the marginal limit-pricing revenue  $(p_t^b - C_t(Q_t)) e^{-rt}$  always decreases more rapidly than the opportunity cost  $\lambda_t$ .

Appendix B further shows that the marginal revenue  $p_0^b - C_0(Q_0)$  initially exceeds  $\lambda_0$ . Thus the discounted marginal extraction revenue covers the scarcity value initially and until extraction stops at date  $T$ :

$$(p_t^b - C_t(Q_t)) e^{-rt} \geq \lambda_t, \quad \forall t \in [0, T]. \quad (29)$$

All along the exploitation phase  $[0, T]$ , the limit-pricing extraction  $q_t^m = D_t(p_t^b)$  is thus optimum, which induces the limit price  $p_t^m = p_t^b$ .

As far as the optimal terminal date  $T^m$  and abandoned reserves  $Q_{T^m}$  at that date are concerned, there are two possibilities, as detailed in Appendix B. Consider first that no reserves are abandoned, i.e.  $Q_{T^m} = 0$ . In that case, the limit-pricing extraction lasts

until reserves are exhausted:  $T^m$  is such that

$$\int_0^{T^m} D_t(p_t^b) dt = Q_0, \quad (30)$$

as in Section 3. Full exhaustion may only be optimal if the marginal limit-pricing revenue is not becoming negative before the exhaustion date  $T^m$  given by (30).

Otherwise, the terminal date is such that marginal revenue becomes nil:

$$(p_{T^m}^b - C_{T^m}(Q_{T^m})) e^{-rT^m} = 0, \quad (31)$$

with

$$Q_{T^m} = Q_0 - \int_0^{T^m} D_t(p_t^b) dt. \quad (32)$$

Appendix B shows that the system jointly determines the date  $T^m \geq 0$  when extraction stops, and abandoned reserves  $Q_{T^m} \geq 0$  at that date – equivalently the ultimately extracted quantity  $Q_0 - Q_{T^m} \leq Q_0$ .

We have the following proposition that summarizes the properties of the limit-pricing equilibrium in the context of this section.

**Proposition 6 (Limit-pricing equilibrium with incomplete extraction)**

*Under the assumptions of this section,*

1. *The monopoly supplies  $q_t^m = D_t(p_t^b) > 0$ , and so induces the limit price  $p_t^b$  that deters the backstop substitute's production, at all dates of the exploitation period  $[0, T^m]$ ;*
2. *Extraction is complete if there exists no date  $T > 0$  such that the marginal revenue  $p_T^b - C_T(Q_T)$  is nil with  $Q_T = Q_0 - \int_0^T D_t(p_t^b) dt > 0$ : in that case  $Q_{T^m} = 0$  and  $T^m$  is given by (30);*
3. *Otherwise, extraction is incomplete: the terminal date  $T^m$  and abandoned reserves  $Q_{T^m} > 0$  are determined by (31) and (32).*

In the sequel, we examine how taxation policies affect this equilibrium.

### B. Taxation Policies

Assume, at each date  $t \geq 0$ , a tax  $\theta_t > 0$  on the resource, a subsidy  $\gamma_t^b > 0$  to the backstop and a subsidy (tax) to the ordinary substitute  $\gamma_t^o > 0 (< 0)$ . In light of the analysis of Section 3, the monopoly's profit at date  $t \geq 0$ , with reserves  $Q_t \geq 0$ , writes in that context

$$\pi_t(q, Q_t) = \begin{cases} (p_t^b - \gamma_t^b - \theta_t - C_t(Q_t)) q, & \text{for } q < \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o) \\ (D_t^{-1}(q) - \theta_t - C_t(Q_t)) q, & \text{for } \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o) \leq q < \overline{D}_t(p_t^b - \gamma_t^b) , \\ (\overline{D}_t^{-1}(q) - \theta_t - C_t(Q_t)) q, & \text{for } q \geq \overline{D}_t(p_t^o - \gamma_t^o) \end{cases} \quad (33)$$

which has the same pattern as in (27). By Assumption 1 and Definition 1, extraction revenue (33) is decreasing for all quantities  $q < \overline{D}_t(p_t^o - \gamma_t^o)$  that do not warrant the backstop substitute's production. By Assumption 3, extraction revenue is increasing for all  $q < \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)$ , as long as the marginal revenue  $p_t^b - \gamma_t^b - \theta_t - C_t(Q_t)$  is strictly positive.<sup>33</sup>

Thus for policies that satisfy Assumption 3 and Definition 1, the same analysis as in absence of policies applies, so that the limit-pricing equilibrium realizes as follows. At each date  $t$  of the exploitation period  $[0, T^m]$ , resource extraction becomes

$$q_t^m = \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o), \quad (34)$$

that induces the limit-price  $p_t^m = p_t^b - \gamma_t^b$ . All along this period, it can easily be verified that the effects of  $\theta_t$ ,  $\gamma_t^b$  and  $\gamma_t^o$  on current extraction  $q_t^m$  remain those identified earlier in Propositions 2, 3 and 5.

When the resource is fully exhausted, the date at which exploitation ends is such that

$$\int_0^{T^m} (\overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)) dt = Q_0. \quad (35)$$

In that case, backstop subsidies anticipate the terminal date because they increase current extraction during the exploitation period. In contrast, subsidies to ordinary substitutes reduce current extraction, and so induce a longer depletion.

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<sup>33</sup>As for previous sections, Assumption 3 and Definition 1 amount to the following restrictions on the tax instruments under study. The resource tax and the backstop subsidy are not sufficiently high to make extraction uneconomic at early dates, and are not decreasing rapidly enough to make discounted marginal revenue increase. The subsidy to the ordinary substitute is not sufficiently high to destroy the (residual) resource demand.

When the marginal revenue  $(p_T^b - \gamma_T^b - \theta_T - C_T(Q_T)) e^{-rT}$  becomes negative for positive remaining reserves  $Q_T = Q_0 - \int_0^T q_t^m dt > 0$ , extraction stops at the terminal date  $T^m$  characterized as follows:

$$(p_{T^m}^b - \gamma_{T^m}^b - \theta_{T^m} - C_{T^m}(Q_{T^m})) e^{-rT^m} = 0 \quad (36)$$

with

$$Q_{T^m} = Q_0 - \int_0^{T^m} (\bar{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)) dt. \quad (37)$$

Marginal revenue in (36) is decreasing in the terminal date  $T^m$  and increasing in remaining reserves  $Q_{T^m}$  at that date. The remaining reserves in (37) are diminishing with the length of extraction  $T^m$ . Other things given, Appendix B shows that the two formulas systematically characterize the terminal date  $T^m$  and abandoned reserves  $Q_{T^m}$ , and can be used to examine the effects of any particular trajectory of tax instruments. It brings up the following general insights about the qualitative effects of policies with limit pricing.

There are two basic ways by which taxation policies may affect the marginal extraction revenue, thus the terminal date and abandoned reserves at that date. On the one hand, for unchanged remaining reserves  $Q_{T^m}$ , policies may deteriorate the marginal extraction revenue in (36) directly. On the other hand, policies that reduce (increase) current extraction  $q_t^m$  via (34), leave more (less) future reserves  $Q_{T^m}$  to be extracted as per (37), and so improve (deteriorate) the marginal revenue in (36) indirectly, because less reserves mean higher extraction costs.

For instance, since resource taxes do not affect current extraction (34) throughout the exploitation phase, they do not affect the reserves  $Q_t$  available for extraction at any date  $t$ . Thus they only anticipate the terminal date because they make extraction less profitable as per (36). It follows that resource taxes unambiguously reduce ultimately extracted reserves.

In contrast, for unchanged reserves, subsidies to ordinary substitutes do not affect directly the profitability of extraction in (36). Yet they reduce current extraction (34) all along the exploitation phase, so that, by (37), it takes longer to reach the cut-off level of remaining reserves that satisfy (36). Since extraction is less profitable over time, a later terminal date in (36) implies larger abandoned reserves.

Backstop subsidies induce extraction (34) to increase along the exploitation phase, and thus contribute to greater extraction costs in (36). Simultaneously in (36), but for unchanged reserves, they directly deteriorate extraction profitability. For these two reasons, backstop subsidies anticipate the terminal date. Yet they imply a lower extraction over a shorter period and thus have an ambiguous effect on ultimately abandoned reserves.

Hence the following results.

**Proposition 7 (Effect of policies with incomplete extraction)**

*When extraction is incomplete,*

1. *Resource taxes shorten the extraction period and reduce the ultimately extracted quantity;*
2. *Subsidies to the backstop substitute shorten the extraction period but have an ambiguous effect on the ultimately extracted quantity;*
3. *Subsidies to the ordinary substitute extend the extraction period, but reduce the ultimately extracted quantity;*

As a consequence, the carbon tax has the following effect.

**Corollary 2 (Effect of the carbon tax with incomplete extraction)**

*The combination of a resource tax, with a tax on the ordinary substitute,*

1. *Shortens the extraction period;*
2. *Affects the ultimately extracted quantity ambiguously.*

Indeed in light of Proposition 7, a resource tax and a tax (negative subsidy) on the ordinary substitute both contribute to shorten the extraction period. A resource tax alone thus reduces cumulative extraction; yet an ordinary-substitute tax tends to increase resource extraction at each date of the shorter extraction period.

## V. Discussion: Industrial Structure, Reserves' Production, Demand Segments...

This paper points at the empirical relevance of limit-pricing equilibria for the oil and energy market and shows that the effects of environmental taxation instruments in such context differ from conventional studies. In particular, taxes applied to flows of resources, when they warrant no supply disruption, are ineffective regardless of their time dynamics. As far as subsidies to oil substitutes are concerned, it is fundamental to make a distinction between two sorts of substitutes. On the one hand, limit pricing deters the entry of drastic substitution possibilities. Subsidies to a backstop substitute induce equilibrium extraction quantities to increase. On the other hand, currently used substitutes to oil – we called them ordinary – offer less drastic substitution possibilities that are compatible with limit pricing. Unlike the backstop, subsidies to any currently in-use substitutes do reduce current extraction quantities by an amount that depends on their respective elasticity of supply.

While we have restricted attention to a single ordinary substitute for simplicity, extension to several such substitutes is immediate. Since the effect of subsidies depends on the supply elasticity of the substitute, the objective of reducing carbon-resource extraction quantities in a cost-efficient manner may imply selecting non-carbon substitutes on the ground of their supply elasticity; an issue that is beyond the scope of the present work.

The simple model of Section 2 has focused on backstop substitution possibilities, and has assumed that the resource is entirely exhausted. Section 3 has completed the description of substitution possibilities, while Section 4 has considered incomplete resource exhaustion. Those extensions proved to neatly refine our results on the incidence of taxation policies. Yet, our results have been obtained in a relatively parsimonious model; one may question whether limit-pricing equilibria survive more complex setups. In the sequel, we discuss further aspects of the oil market.

### A. *Competitive Fringe*

The industrial structure of the oil market differs from the frequently-used monopoly model. The OPEC cartel controls the majority of exploitable oil reserves; yet non-

OPEC reserves yield a substantial fraction of current oil production.<sup>34</sup> A more adequate representation of the market power exerted in the oil-production sector must take into account that a competitive fringe limits the power of the dominant cartelized extractor as in the model initiated by Salant (1976).<sup>35</sup>

The analysis of Sections 3 and 4 easily extends to that case. Indeed, although the fringe's oil production is identical to the cartel's production, it is analogous to the ordinary substitute introduced in Section 3 (competitively-supplied, perfect substitute to the cartel's resource). Thus it can be represented in the same manner. The residual demand that the monopoly is facing is that fraction of the total oil demand that exceeds the fringe's production. Because of reserve limitations, the elasticity of non-OPEC oil supply is notoriously very limited, and is even more so as non-OPEC producers have virtually no spare production capacities.<sup>36</sup>

As will be argued shortly, it is sensible to consider that non-OPEC oil supply satisfies Definition 1 of ordinary substitutes – equivalently, Assumption 1 holds in spite of the fringe –, by which OPEC's residual (net-of-fringe-supply) demand exhibits a lower-than-one price elasticity. Thus the limit-pricing analysis of Sections 3 and 4 carries over unchanged with the fringe and our results are relevant to the actual structure of the oil market.

Treating the fringe's oil supply as the ordinary substitute to the cartel's resource,  $x$  denotes the fringe's production and  $e = q + x$  the total oil supplied. The question is to know whether the inequality in formula (13) is verified, by which the cartel's residual-demand elasticity  $\xi_{Dt}$  is lower than one:

$$\xi_{Dt}(q) = \frac{\xi_{\overline{Dt}}(e)}{q/e} + \frac{x/e}{q/e} \xi_{S^ot}(x) < 1. \quad (13)$$

The formula gives this elasticity as a weighted sum of the elasticity of the total oil demand  $\xi_{\overline{Dt}}$  and the fringe's supply  $\xi_{S^ot}$ ;  $q/e$  and  $x/e$  are respectively the market shares of the

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<sup>34</sup>See Footnote 2 for more details.

<sup>35</sup>Issues about coordination within the OPEC cartel are out of the scope of this discussion for simplicity. See for instance Griffin (1985). In the most extreme conceivable case, the cartel would be completely ineffective. Saudi Arabia would make the price alone, thanks to very large spare capacities; the fringe would consist of all other producers, OPEC members or not.

<sup>36</sup>According to Hamilton, "In the absence of significant excess production capacity, the short-run price elasticity of oil supply is very low." (Hamilton, 2009b).

cartel and the fringe.

It is possible to verify that recent (long-run) elasticity estimates satisfy relation (13). Market shares are currently about  $q/e = 0.4$  and  $x/e = 0.6$ . For the price elasticity of the total oil demand, the value used in Hamilton (2009b) is 0.25, in line with Krichene's (2005) long-run estimate for the period 1974-2004. Hamilton (2009b, p. 192) argues that this elasticity should be expected to be even smaller. Taking this conservative value, basic algebra shows that (13) holds for any elasticity  $\xi_{S^o_t}$  of the fringe's supply such that

$$\xi_{S^o_t} \leq 0.25. \quad (38)$$

For instance, Golombek et al.'s (2013) estimates of the (long-run) non-OPEC oil supply elasticity are between 0.11 and 0.25, depending on their model's specification, which is compatible with (38).

As the above numbers indicate, it is sensible to consider that Assumption 1 holds, by which the residual demand that OPEC is facing exhibits a less-than-one elasticity. Yet testing Assumption 1 requires further empirical research. For instance, the specification of existing empirical models assumes away the possibility that OPEC is limit pricing (e.g. Golombek et al., 2013, p. 8).

### *B. Reserves' Production*

Section 4 assumes heterogenous reserves whose extraction cost rises as extraction goes. In that context, extraction may become uneconomic before reserves are completely depleted, so that in general taxation policies affect the exploitation duration, and the ultimately extracted quantity.

Another reason why policies may affect the ultimately exploited resource is that they discourage exploration and development efforts by which reserves become exploitable. In Appendix C, we borrow the approach of Gaudet and Lasserre (1988), also used for instance in Fischer and Laxminarayan (2005) or Daubanes and Lasserre (2014). In these models, the marginal cost of developing an amount of exploitable reserves is rising, as when resource units are developed in order of their respective development costs; reserves are established so as to equate the marginal development cost with the implicit value of



marginal reserves.

This extension does not modify qualitatively the limit-pricing outcome, nor the earlier-identified effects of policies on ultimately developed and exploited quantities.

### *C. Multiple Demand Segments with Various Degrees of Substitutability*

It is standard to rely on a unique decreasing function to describe the heterogeneity of the aggregate demand. Yet in reality, the oil demand is segmented. Segments mainly correspond to different uses of the resource (e.g. Hoel, 1984), and to different regions.

One particular resource use in one particular region can be represented by a particular demand function of a form similar to the demand of Section 3. Resource uses and regions may differ by their accessible possibilities of substitution, as well as by their regulation.

One can also consider substitutes to vary by their degree of substitutability with the resource. On the one hand, as imperfect substitutes only become profitable beyond a certain resource price, they introduce kinks to the oil demand as in Sections 3 and 4. On the other hand, imperfect substitutability amounts to a broader interpretation of the demand elasticity. On each segment, the sensitiveness of the resource demand at some resource price jointly reflects the elasticity of supply and the degree of substitutability of resource substitutes that are profitable at that price.

Limit pricing in that context intuitively arises from the entry threat of sufficiently substitutable alternative sources, on large enough demand segments. For instance, in the interview mentioned in Footnote 6, the energy industry analyst Stephen Schork later clarified OPEC's "main concern" (CNBC on August 16, 2010): the "shift of the sentiment in the US *especially* towards alternative fuels." [our italics].

### *D. Interpretation of the Limit Price*

The limit price may also be interpreted more broadly than the entry price of a (backstop) substitute that offers drastic substitution possibilities. In Gerlagh and Liski (2011), the backstop substitute needs to be developed: the falling limit price induced by the strategic oil producer is the price beyond which the costly development of the backstop substitute is irreversibly triggered, which destroys the oil demand after some lag.

Gerlagh-Liski equilibrium results from strategic interactions which are absent here. But their analysis may inspire a more general interpretation of the limit price as the price level beyond which a sufficiently drastic threat to the monopoly's profits would be carried out, whether it means the development of fuel reserves unexploitable up to that point, or research-and-development efforts aimed at mastering a new energy-generation technology.

## APPENDIX

### A Appendix to Sections 2 and 3: The Simple Stationary Case

This appendix reproduces the analysis of Section 2 under stationary conditions. In the spirit of Section 3, it also introduces an ordinary substitute under the simplifying assumption that its supply is perfectly inelastic.

Assume that the resource marginal extraction cost  $c$  and the backstop marginal production cost  $p^b$  are constant with  $p^b > c$ . The total energy demand  $\overline{D}(p)$  is stationary, and satisfies  $\xi_{\overline{D}}(p) < 1$ , for all  $p > 0$  as per Assumption 1.

At each date  $t$  when there is some resource left to be exploited, the monopoly's instantaneous profit writes

$$\pi(q) = \begin{cases} (p^b - c)q, & \text{increasing, for } q \leq \overline{D}(p^b) \\ (\overline{D}^{-1}(q) - c)q, & \text{decreasing, for } q > \overline{D}(p^b) \end{cases}, \quad (39)$$

and is maximized by the supply  $\overline{D}(p^b)$  which induces the limit price  $p^b$  that deters the backstop.

The intertemporal problem of maximizing the discounted stream of profits (3) under the exhaustibility constraint (4) implies the Hamiltonian function (5), where the scarcity value  $\lambda$  is constant. All along the extraction period  $[0, T]$ , the Hamiltonian is maximized by the same supply level

$$q^m = \overline{D}(p^b)$$

that maximizes the instantaneous profit.

Thus the maximized Hamiltonian

$$\mathcal{H}(q^m, Q_t, \lambda, t) \equiv (p^b - c)q^m e^{-rt} - \lambda q^m \quad (40)$$

is decreasing over time because profits are discounted at rate  $r > 0$ ; in the stationary case, Assumption 2 is superfluous. It can easily be verified that the maximized Hamiltonian is initially positive because  $p^b > c$  so that extraction is warranted. Also, one can verify that  $\lambda$  is strictly positive so that the exhaustibility constraint is not violated. Thus the resource is completely exhausted. At each date of the extraction period  $[0, T^m]$ , extraction is  $q^m$ , so that exhaustion occurs at the terminal date  $T^m = Q_0/q^m$ .

Since the duration of the exploitation period is free, the Hamiltonian must become nil at date  $T^m$ . This characterizes the scarcity value  $\lambda$  under limit pricing:  $\lambda = (p^b - c)e^{-r(Q_0/q^m)}$ , with  $q^m = \overline{D}(p^b)$ .

#### Effect of a Constant Resource Tax

Assume a constant tax on the resource  $\theta > 0$  that leaves positive extraction profits:  $\theta < p^b - c$ . The producer price of the resource is reduced by  $\theta$ , regardless of whether consumers are ready to pay  $\overline{D}^{-1}(q)$  or  $p^b$ , as when the backstop is profitable.

Thus the instantaneous monopoly's profit becomes

$$\pi(q) = \begin{cases} (p^b - \theta - c)q, & \text{increasing, for } q \leq \overline{D}(p^b) \\ (\overline{D}^{-1}(q) - \theta - c)q, & \text{decreasing, for } q > \overline{D}(p^b) \end{cases} . \quad (41)$$

The same analysis as in absence of tax follows through, with  $c + \theta$  instead of  $c$ . The limit-pricing equilibrium is not modified: it implies an unchanged extraction level  $q^m = \overline{D}(p^b)$  at each date preceding  $T^m = Q_0/q^m$ .

#### **Effect of a Constant Backstop Subsidy**

Assume a constant subsidy to the backstop  $\gamma^b > 0$ . The price at which the backstop is profitable becomes  $p^b - \gamma^b$  instead of  $p^b$ . Further assume that the backstop subsidy leaves positive extraction profits:  $p^b - \gamma^b > c$ . Then, the instantaneous profit of the monopoly writes

$$\pi(q) = \begin{cases} (p^b - \gamma^b - c)q, & \text{increasing, for } q \leq \overline{D}(p^b - \gamma^b) \\ (\overline{D}^{-1}(q) - c)q, & \text{decreasing, for } q > \overline{D}(p^b - \gamma^b) \end{cases} , \quad (42)$$

and the same analysis as in absence of policies applies with  $p^b - \gamma^b$  instead of  $p^b$ . The limit-pricing equilibrium is thus modified. All along the extraction period, the monopoly's extraction is  $q^m = \overline{D}(p^b - \gamma^b)$ , which is greater than  $\overline{D}(p^b)$  in absence of subsidies. The resource is exhausted earlier, at the terminal date  $T^m = Q_0/\overline{D}(p^b - \gamma^b)$ .

#### **Inelastically-Supplied Ordinary Substitute**

Assume that the demand the monopoly is facing is reduced by a constant amount  $S^o$ , exogenous, of a perfect substitute to the resource. Unlike the backstop, assume that this amount is limited so that it falls short of the monopoly's total demand:  $S^o < \overline{D}(p^b)$ . In that case, the limit-pricing extraction is modified as follows.

For any monopoly's supply  $q$  that deters the backstop, the resource price  $p$  is established in such a way that the market equilibrium  $q = \overline{D}(p) - S^o$  realizes. Therefore, the supply that induces the limit price  $p^b$  is reduced to  $\overline{D}(p^b) - S^o$  instead of  $\overline{D}(p^b)$ . Also, the inverse demand for the resource is reduced to  $\overline{D}^{-1}(q - S^o)$ .

Thus the monopoly's instantaneous profit becomes:

$$\pi(q) = \begin{cases} (p^b - c)q, & \text{increasing, for } q \leq \overline{D}(p^b) - S^o \\ (\overline{D}^{-1}(q - S^o) - c)q, & \text{decreasing, for } q > \overline{D}(p^b) - S^o \end{cases} , \quad (43)$$

which leads to the same dynamic analysis as before. The limit-pricing equilibrium realizes, with constant extraction  $q^m = \overline{D}(p^b) - S^o$  until the exhaustion date  $T^m = Q_0/(\overline{D}(p^b) - S^o)$ .

## **B Appendix to Section 4: Elements of Proofs**

The results of Section 4 are mostly shown in the main text. The main text also refers to the following elements.

#### **Limit-Pricing Marginal Revenue and Scarcity Value**

The limit-pricing marginal revenue, in present value terms, decreases more rapidly

than the multiplier  $\lambda_t$ ; this can be shown as follows.

At any date  $t$ , when remaining reserves are  $Q_t$  and extraction is  $q_t \geq 0$ , the derivative of the discounted marginal revenue  $(p_t^b - C_t(Q_t)) e^{-rt}$  with respect to time is

$$\frac{d((p_t^b - C_t(Q_t)) e^{-rt})}{dt} = \left[ \frac{d(p_t^b - C_t(Q_t))}{dt} - r(p_t^b - C_t(Q_t)) \right] e^{-rt} + C'_t(Q_t) q_t e^{-rt} \leq 0,$$

where the term between brackets is the increase in the discounted marginal revenue for given reserves. By Assumption 3, it is negative or zero. The second term  $C'_t(Q_t) q_t e^{-rt}$  corresponds to the decrease in the marginal revenue that arises because reserves diminish. It is strictly negative when extraction is non zero, and zero otherwise.

By the Maximum Principle, the latter term is also the time derivative of  $\lambda_t$ :

$$\dot{\lambda}_t = -\frac{\partial \mathcal{H}(q_t, Q_t, \lambda_t, t)}{\partial Q_t} = C'_t(Q_t) q_t e^{-rt} \leq 0.$$

It follows that

$$\frac{d((p_t^b - C_t(Q_t)) e^{-rt})}{dt} \leq \dot{\lambda}_t \leq 0.$$

### Extraction at Date 0

Consider, as a statement to be contradicted, that  $p_0^b - C_0(Q_0) \leq \lambda_0$ . Since the marginal revenue is decreasing more rapidly than  $\lambda_t \geq 0$ , then  $(p_t^b - C_t(Q_t)) e^{-rt} \leq \lambda_t$ , for all  $t \geq 0$ , where the equality may only hold as  $(p_t^b - C_t(Q_t)) e^{-rt} = \lambda_t = 0$ ; some extraction may be optimal in that case, but for no profit at all. Clearly, this is dominated by some extraction at initial dates since by Assumption 3,  $p_0^b - C_0(Q_0) > 0$ . Thus we must conclude that  $p_0^b - C_0(Q_0) > \lambda_0$ .

### Terminal Date and Ultimately Abandoned Reserves

Since the terminal date  $T$  when extraction stops is free, the Hamiltonian (28) – the relevant flow of extraction benefits – must be zero at that date. The standard transversality condition

$$(p_T^b - C_T(Q_T)) e^{-rT} = \lambda_T \quad (44)$$

must hold.

Also at the terminal date  $T$ , reserves left unexploited must be non negative by constraint (4):

$$Q_T \geq 0. \quad (45)$$

Therefore, another standard transversality condition must be satisfied, by which

$$\lambda_T Q_T = 0. \quad (46)$$

Hence two possibilities. Consider first that  $Q_{T^m} = 0$ . In that case, the limit-pricing extraction lasts until reserves are exhausted, so that  $T^m$  is characterized by (30).

Second, consider that  $Q_{T^m} > 0$  because the extraction of the last units is uneconomic. By (46), this can only be compatible with reserves having no more value at the terminal

date  $T^m$ :  $\lambda_{T^m} = 0$ . In this case, the terminal date  $T^m$  must satisfy

$$(p_T^b - C_T(Q_T)) e^{-rT} = 0, \quad (47)$$

with

$$Q_T = Q_0 - \int_0^T D_t(p_t^b) dt; \quad (48)$$

a system that will turn out to uniquely characterize the terminal date  $T^m$  and abandoned reserves  $Q_{T^m}$ : hence (31) and (32).

We analyze this system now. By Assumption 3, the marginal revenue in (47) is initially positive for low  $T$  when  $Q_T$  in (48) is close to  $Q_0$ . If  $T$  does not exist such that, together with  $Q_T$  in (48), it implies the marginal revenue in (47) to take a zero value, then extraction continues until  $Q_T = 0$ . In that case,  $Q_{T^m} = 0$  is solution as in the first possibility;  $T^m$  is given by (30), and the analysis is similar to that of Section 3 with complete exhaustion.

Thus the analysis of Section 4 is most interesting in the second possibility, when  $T$  exists such that  $Q_T > 0$  in (48) and  $T$  jointly satisfy (47). In this case, the solution is obviously unique since the marginal revenue on the left-hand side of (47) strictly decreases as  $T$  increases and reserves  $Q_T$  diminish. Precisely, it is decreasing in  $T$  for a given  $Q_T$ , and strictly decreasing when it is taken into account that an increase in  $T$  goes hand in hand with a decrease in  $Q_T$  as per (48).

Focus now on that unique interior solution when it exists. For that, it will be useful to consider  $T$  and  $Q_T$  as two variables that separately affect (47); the effect of  $T$  on  $Q_T$  being encompassed in (48). In (47), the discounted marginal revenue on the left-hand side is decreasing in  $T$  and increasing with  $Q_T$ . Thus the equation defines a positive relationship between  $T$  and  $Q_T$ , that we denote with the following function:

$$T = T_1(Q_T), \text{ increasing.} \quad (49)$$

According to (48), a greater  $Q_T$  is associated with a shorter extraction period that lasts until a lower  $T$ . This negative relationship is represented by the function

$$T = T_2(Q_T), \text{ decreasing.} \quad (50)$$

The intersection of the  $T_1$  and  $T_2$  relations defines either the unique interior solution  $(Q_{T^m}, T^m)$  given by (31) and (32) when they cross at the right of the  $Q_T = 0$  vertical axis ( $Q_{T^m} > 0$ ), or the complete-exhaustion solution  $Q_{T^m} = 0$  earlier mentioned otherwise. The graphical representation of Figure 6 will be useful shortly to identify how this solution modifies with parametric policy changes.

### Effects of Policies with Incomplete Extraction

The taxation policies under study in Section 4 are considered to satisfy Assumptions 1 and 3. In that context, the terminal date  $T^m$  and the ultimately abandoned reserves  $Q_{T^m}$  are characterized by (36) and (37), instead of (31) and (32). Under the same assumptions, the same analysis applies as in absence of policies: (36) and (37) can be represented with

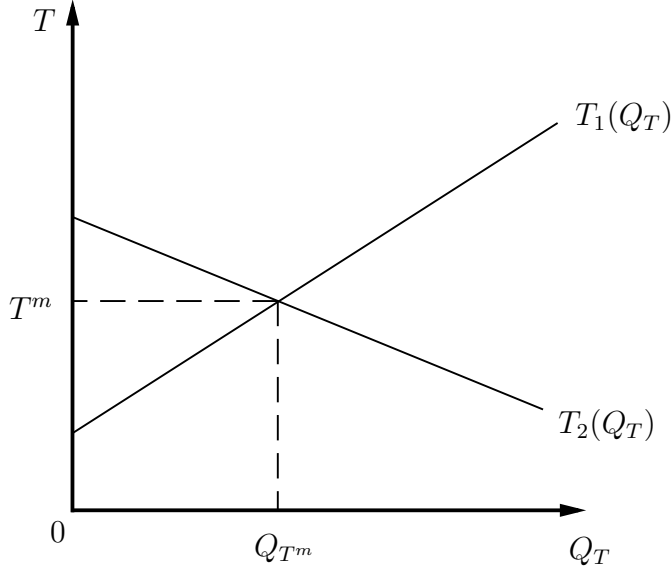


Figure 6: Graphical characterization of  $T^m$  and  $Q_{T^m}$

the  $T_1$  and  $T_2$  functions of (49) and (50), except that these functions now depend on policy parameters that enter (36) and (37).

We focus on the effects of taxation policies on the interior solution depicted in Figure 6. When the solution implies complete exhaustion, the analysis is the same as in Section 3 and is only concerned with the effects on current extraction levels over the exploitation period; effects on the length of this period are obvious. In what follows we derive the results presented in Section 4 by shifting the  $T_1$  and  $T_2$  curves of Figure 6 whose intersection characterizes  $Q_{T^m}$  and  $T^m$  in the limit-pricing equilibrium.

A tax  $\theta_t > 0$ ,  $\forall t \geq 0$ , only affects (36). For given reserves  $Q_T$ , it brings backward the date  $T$  when the (tax-inclusive) marginal revenue becomes zero. Thus a rise in the tax amounts to shifting down the  $T_1$  curve: it implies extraction until a lower  $T^m$ , and greater abandoned reserves  $Q_{T^m}$ .

A subsidy to the ordinary substitute  $\gamma_t^o > 0$ ,  $\forall t \geq 0$ , only affects (37). For given terminal reserves  $Q_T$ , it brings forward the terminal date  $T$  at which those reserves will be reached. Thus a rise in the subsidy amounts to shifting up the  $T_2$  curve: it implies extraction until a later  $T^m$ , and greater abandoned reserves  $Q_{T^m}$ . The opposite result is obviously obtained for a tax  $\gamma_t^o < 0$ ,  $\forall t \geq 0$ .

A subsidy to the backstop substitute  $\gamma_t^b > 0$ ,  $\forall t \geq 0$ , enters both (36) and (37). On the one hand, for given reserves  $Q_T$ , the subsidy brings backward the date  $T$  when the marginal revenue in (36) becomes zero; a subsidy rise amounts to shifting down the  $T_1$  curve. On the other hand, for given abandoned reserves  $Q_T$ , the subsidy reduces the date  $T$  when those reserves will be reached in (37); a subsidy rise amounts to shifting down the  $T_2$  curve. Those two changes to Figure 6 imply that subsidies to the backstop substitute

imply a shorter extraction period, i.e. until a lower  $T^m$ . Yet they have an ambiguous effect on abandoned reserves  $Q_{T^m}$  and thus on the ultimately extracted quantity  $Q_0 - Q_{T^m}$ .

These results are summarized in Proposition 7, which also yields Corollary 2.

## C Appendix to Section 5: Costly Exploration and Development Efforts

In the context of Section 4, consider that reserves  $Q_0 - Q_{T^m}$ , before being exploited, need to be produced by costly exploration and development efforts. Following Gaudet and Lasserre (1988), assume that the production of those reserves takes place at date 0 and is subject to decreasing returns to scale because, as exploration prospects are finite, it must be more and more difficult to produce new reserves. When reserves' production is costly, it cannot be optimum to produce more than what is to be exploited. Formally, the cost of producing  $Q_0 - Q_{T^m}$  is given by the increasing and strictly convex function  $E(Q_0 - Q_{T^m})$ . Let us further assume that  $E'(0) = 0$  so as to avoid the uninteresting situation where the development cost induces the monopoly to produce no reserves at all.

The objective (3) of the monopoly now incorporates the reserve-development cost function  $E$ . Thus the monopoly's problem is

$$\max_{(Q_0 - Q_T), (q_t)_{t \geq 0}} \int_0^T \pi_t(q_t, Q_t) e^{-rt} dt - E(Q_0 - Q_T), \quad (51)$$

subject to (4), where  $T$  is a free variable.

Despite this modification of the objective, the Hamiltonian associated with the above problem is the same as in Section 4, given by (28). The integration of reserves' production into the monopoly's problem affects neither the analysis of the limit-pricing exploitation phase, nor the transversality condition (44), but the transversality condition associated with the non-negativity constraint (45).

Specifically, condition (46) is modified as follows.  $Q_0$  may be entirely developed and completely exhausted as before and  $Q_T = 0$  if development and extraction cost conditions make it profitable. Such is compatible with the marginal reserve-production cost being lower than the implicit value of marginal reserves:  $E'(Q_0) \leq \lambda_T$ . Yet when reserves are not completely developed and extracted,  $Q_T$  is strictly positive, and the implicit value of marginal reserves  $\lambda_T$ , instead of being equalized to zero as in absence of reserve production cost, is equalized to the marginal cost  $E'(Q_0 - Q_T)$ . The transversality condition associated with the non-negativity constraint (45) becomes

$$Q_T (\lambda_T - E'(Q_0 - Q_T)) = 0. \quad (52)$$

When  $Q_T = 0$ , things go as in absence of reserve-development efforts; no adjustment to Section 4 is needed. When  $Q_T > 0$ , the condition tells that instead of a zero value as in Section 4,  $\lambda_T$  equals the positive marginal cost of reserve production:

$$\lambda_T = E'(Q_0 - Q_T).$$



Thus condition (44) yields, instead of (31),

$$(p_{T^m}^b - C_{T^m}(Q_{T^m})) e^{-rT^m} - E'(Q_0 - Q_{T^m}) = 0, \quad (53)$$

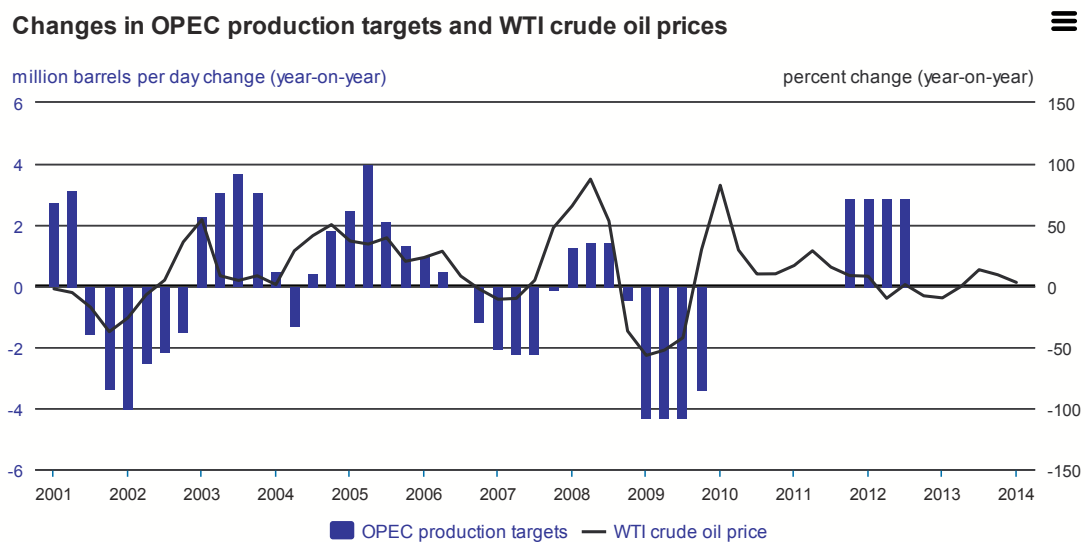
where  $Q_{T^m}$  is still given by (32).

In that case, (53) and (32) form the system that uniquely characterizes the terminal date  $T^m$  and abandoned reserves  $Q_{T^m}$ . Since the left-hand side of (53) is increasing with  $Q_{T^m}$  in the same manner as (31), the new system retains the same properties as in the analysis of Section 4. Also, the system (53)-(32) only differs from (31)-(32) by the marginal development cost term  $E'(Q_0 - Q_T)$ . Since this term is not directly affected by the taxation policies considered in this paper, the interested reader can easily verify that the policies' effects established in Section 4 carry over to the case of this appendix.

#### **D OPEC's Use of Available Production Capacities to Balance the Oil Market**

Figure 7 reports a chart from the OPEC-supply section of the EIA's analysis entitled "What drives crude oil prices?". Other data charts, in particular about OPEC's high spare production capacities, are available at <http://eia.gov/finance/markets/supply-opec.cfm>. According to the EIA's analysis, OPEC's use of its ability to respond to demand and price increases, unlike non OPEC, suggests that the cartel's supply pursues price management purposes.

OPEC production often acts to balance the oil market. Cuts in OPEC production targets tend to lead to price increases.



Source: U.S. Energy Information Administration, Thomson Reuters  
Updated: Quarterly | Last Updated: 3/31/2014

This chart shows changes in OPEC production targets compared to changes in oil prices. Reductions in OPEC production targets often lead to increases in oil prices.

Figure 7: Changes in OPEC production targets and oil prices

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