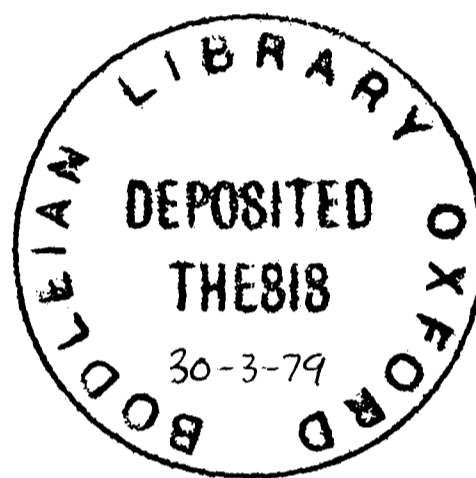


SIMULTANEOUS IDENTIFICATION AND CONTROL
OF
DISCRETE TIME SINGLE INPUT SINGLE OUTPUT SYSTEMS

by
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ABSTRACT

"Simultaneous identification and control of discrete time single input single output systems"

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This thesis is concerned with suboptimal adaptive control of discrete linear stochastic processes whose parameters are unknown. The suboptimal adaptive controllers considered are (i) Open Loop Feedback Optimal (OLFO) controller, (ii) self-tuning controller, and (iii) optimal k step ahead controller. Two more controllers, certainty about parameter (CAP) controller and no learning (NOL) controller, that provide bounds on the performance of these adaptive controllers are also considered. Performance of these controllers have been evaluated for a first order process through monte-carlo simulations.

Simulation of OLFO controller together with the bounding controllers for the first order process when there is only one unknown parameter revealed that OLFO controller is unsuitable to control unstable processes and would be an unwise choice even for controlling stable processes. Self-tuning and OK controllers have been simulated for the first order process with all the parameters unknown. Three cases for the unknown parameters have been considered. They are: (i) constant unknown parameters (ii) slowly time-varying unknown parameters and (iii) rapidly time-varying unknown parameters. Simulation results showed that in certain regions

of the unknown parameter space the cost produced by self tuning controller and OK controller are very similar, in certain regions the OK controller produces lesser cost than the self-tuning controller and in certain other regions both controllers perform very badly. But self-tuning controller always took only half as much computing time as OK controller.

A necessary condition for convergence of OK controller to a linear constant parameter controller having the same functional form as CAP controller is found out using the ideas of uniform complete observability. For a first order process under OK controller the only occasion the condition would be violated is when there is 'turn-off'.

Finally, it is shown that using the combined state/parameter estimator in the place of extended Kalman filter the computational requirement of OK controller can be reduced. For the first order process, OK controller with the combined estimator took only sixty percent as much computing time as the OK controller with extended Kalman filter without any appreciable deterioration in the performance.

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CHAPTER ONE

INTRODUCTION

1.1 Introduction

Optimal control of linear gaussian stochastic systems with quadratic performance criterion has been a solved problem ever since the early sixties, when the fundamental works of Kalman on linear filtering and of Bellman on decision theory came about. The optimal controller is known (1) to consist of an optimal linear (Kalman) filter cascaded to an optimal control law derived using Bellman's principle of optimality. But in many practical situations even if the dynamics of the system could be assumed linear and stochastic, the various parameters involved in the dynamics are seldom known. Examples of such situations in aerospace systems are given in (4,5), process control are given in (2), economics are given in (3). There are essentially two approaches to the control of such systems:

i) Identify the unknown parameters that describe the dynamics using an appropriate off-line identification algorithm and then use optimal stochastic control to control the identified system. Thus, in this approach identification and control take place separately. The disadvantage of this approach is that the identification procedure often involves time consuming experiments and computations. Further this approach would be unsuitable if the unknown dynamics change in time (6).

ii) Simultaneously identify and control the unknown dynamic system. As identification is carried out on-line this approach is suitable even when the unknown dynamics change in time and consequently there is growing interest towards this approach. The controller in this

approach performs two functions: identification of the unknown dynamics and controlling of the unknown system. Such controllers are commonly known as stochastic adaptive* controllers(7). Unfortunately optimal adaptive controller , except for very simple systems, requires enormous computer memory (8) and hence is not practically implementable. This has motivated the need for a suboptimal adaptive controller that is a good trade off between optimality and easy implementability. The structure of a suboptimal adaptive controller, as will be seen in Chapter 2, is a suboptimal estimator cascaded to a suboptimal control law. Suboptimal adaptive controllers in the literature are all designed on the basis of this structure.

*Here after 'stochastic adaptive' will be referred to simply as 'adaptive'.

1.2 Brief survey of suboptimal adaptive controllers in literature

This section presents only a brief survey of the work that has been done in the field of suboptimal adaptive controllers. A detailed survey can be found in (7) which has over 80 references.

Jenkins and Roy (10), Saridis and Lobbia (11) obtained a suboptimal adaptive controller using the well known certainty-equivalence (12, 13) principle. These certainty-equivalent controllers consider the estimates of unknown parameters to be their true values and are referred to as naive feedback controllers by Bertsekas (12). Deshpande et al. (14), Athans and Wilner (16), Athans et al. (15) all used the Multiple Model Adaptive Control (MMAC) method in which the unknown parameter process is assumed to belong to one of a number of known parameter linear models. For each linear model the optimal control can be evaluated since all parameters for that model are known. The probability of the process belonging to any particular model is then evaluated using a certain algorithm. The adaptive control is calculated as the weighted average of the various model controls with the probabilities generated acting as weights.

Using certainty-equivalence principle Åström and Wittenmark (6) derived a self-tuning regulator which required only the knowledge of certain combinations of the unknown parameters. Many practical applications of this regulator have been reported in the literature (50, 51, 52). Later Clarke and Gawthrop (17) extended the work of Åström and Wittenmark and derived a self-tuning controller for a more general performance criterion.

Barshalom and Sivan (20), Aoki (23), Spang (24), Tse and Athans (18), Ku and Athans (19) all used the Open Loop Feedback Optimal (OLFO) policy of Dreyfus (25) and obtained a suboptimal adaptive control law. It is

shown in Chapter 4 that OLFO controller is computationally very demanding and unsuitable for processes that are unstable.

Jacobs and Hughes (26) proposed a 'neutral' control algorithm for sub-optimal adaptive control. Their algorithm yields a control law which, in general, is characterised by extremely complex equations and consequently can only be used for very simple systems. Hughes (68), Potter and Jacobs (27), Bohlin (29), Åström and Wittenmark (28) used a one step ahead (OSA) control where the control law minimises only the next output. This control is a special case of the optimal k step ahead (OK) control (30) where the control law is designed to minimise k step ahead output.

Tse and Barshalom (31, 22) derived an actively adaptive dual controller which performed much better than a certainty-equivalent controller. But calculation of the control law in their algorithm involves numerical minimisation at every step and this makes their controller far from 'easily implementable'. Sternby (32) derived a two step control in which the control law minimises the next two outputs; a logical extension of one step ahead control. But as in Tse and Barshalom's (31) controller it also suffers from computational complexity.

1.3 Adaptive Control problem considered in this thesis

As mentioned in section 1.1 adaptive control problems are characterised by situations where it is necessary to simultaneously learn about the dynamic characteristic of a process and to control its behaviour. It is well known (7, 34) that such situations are mathematically characterised by equations which are both non-linear and stochastic and consequently defy theoretical analysis. In the absence of a satisfactory theoretical approach, knowledge regarding the performance of any adaptive controller can best be built up from systematic simulation studies. It is believed that simulation study on a simple but non-trivial mathematical model can lead to conclusions of some general validity. This thesis is therefore primarily concerned with the adaptive control of single input single output stochastic processes satisfying a first order difference equation:

$$y(i) + a(i-1)y(i-1) = b(i-1)u(i-1) + \xi(i) + c(i-1)\xi(i-1) \quad 1.1$$

where u is the input or control, y is the output, ξ is an uncorrelated stationary zero mean normal random sequence (white noise) having a known variance σ_ξ , i is the discrete time index and a , b , c are the parameters that describe the dynamics of the process and their true values are unknown. Further a , b , c could be either constants or time-varying. The purpose of control is to regulate the output y to a desired value of zero, and the performance is measured by a quadratic cost function

$$I_N = \sum_{j=i_0}^{i_f-1} \{ y^2(j) + qu^2(j) \} + y^2(i_f) \quad 1.2$$

where q is a positive weighting factor on cost of control, i_0 and i_f are the starting and finishing times respectively and N is the number of control stages from i_0 to i_f .

The suboptimal adaptive controllers investigated are OK controller, the self-tuning controller of Clarke and Gawthrop (17) and OLFO controller. Two more controllers that provide bounds (35) on the cost achieved by these adaptive controllers are also considered. These are the certainty about parameter (CAP) controller which is given perfect information about a , b , c and the no learning (NOL) controller which makes no attempt to learn about the parameters a , b , c . CAP controller would provide a lower bound and NOL controller would provide an upper bound on the cost achieved by the adaptive controllers.

1.4 A note on noise model

In equation 1.1 the noise affecting the output $y(i)$ at any time is $e(i)$ where

$$e(i) = \xi(i) + c(i-1)\xi(i-1). \quad 1.3a$$

If in a particular problem the coloured noise $e(i)$ is physically meaningful and stationary, then using representation theorem (2) $e(i)$ can always be represented by

$$e(i) = \xi(i) + c\xi(i-1) ; |c| \leq 1.$$

ξ s and c then do not carry any physical meaning; they are mathematical fictions used to generate the physically meaningful $e(i)$.

If in a particular problem ξ s and c of equation 1.3a are really meaningful and not mere mathematical fictions used to generate $e(i)$, then it is possible for the parameter c to be constant or time-varying and have a value inside or outside the unit circle. If c is constant and has a value greater than one then the noise $e_1(i)$,

$$e_1(i) = \xi(i) + c\xi(i-1) ; |c| > 1 , \quad 1.3b$$

can also be represented[†], invoking representation theorem (2), as

$$e_1(i) = \xi_1(i) + \frac{1}{c}\xi_1(i-1). \quad 1.4a$$

ξ_1 is an uncorrelated zero mean stationary normal random sequence (white noise) with a variance σ_{ξ_1} given by

$$\sigma_{\xi_1} = c^2 \sigma_{\xi} . \quad 1.4b$$

If the actual value of c is unknown then the actual value of σ_{ξ_1} will also be unknown.

[†] Valid only if the noise ξ is stationary.

A noise $e_1(i)$ produced by a constant $|c| > 1$ should be represented by equation 1.3b if an adaptive controller design requires knowledge of the variance of the white noise. If on the other hand an adaptive controller design does not require knowledge of this variance then $e_1(i)$ can be represented either by equation 1.3b or 1.4a. As will be seen in subsequent chapters OK and OLFO controller require this knowledge whereas self-tuning controller does not require it.

If the parameter c is time-varying and greater than one then the noise $e_2(i)$,

$$e_2(i) = \xi(i) + c(i-1)\xi(i-1) \quad |c(i)| > 1, \quad 1.5$$

is non-stationary and representation theorem can not be invoked. So there is no alternative representation for $e_2(i)$.

When the parameter c is constant, the noise model of equation 1.3a is non-minimum phase if $|c| > 1$ and minimum phase if $|c| \leq 1$.

1.5 Contribution of the thesis

Much of the work in this thesis is concerned with the evaluation of the performances of the three adaptive controllers - viz. Open Loop Feedback Optimal Controller, Self-Tuning Controller, OK Controller - through simulations. A preliminary simulation study pointed out the need to divide the unknown parameter space into regions in order to obtain meaningful results. A contribution of this thesis is to present a basis for such division of the unknown parameter space into various regions. Three factors form the basis for the division:

- i) Open loop stability of the process to be controlled,
- ii) apriori knowledge about the sign of the control gain parameter,
- iii) non-minimum phaseness of the noise model.

Simulation study carried out on a regional basis showed that Open Loop Feedback Optimal controller is unsuitable to control unstable processes and would be an unwise choice to control stable processes. The simulation study also showed that there are regions in the parameter space where both self-tuning and OK controller produce similar costs and regions where the self-tuning controller produces more cost than the OK controller and regions where both these controllers perform very badly. But the computing time and memory requirement of the self-tuning controller is always found to be less than that of the OK controller.

Suboptimal estimator used in OK controller in the above simulation study was extended Kalman filter (38) although later it was found that a combined state/parameter would be computationally more economical.

An important requirement for any adaptive controller is convergence. Using the idea of uniform complete observability a necessary condition for the convergence of OK controller to a linear constant parameter controller has been obtained and, for a first order process, occasions when this condition would be violated are found out. Further, conditions for closed loop stability are presented assuming the OK controller has converged to a linear constant parameter controller.

Finally it is shown how, by using a combined state/parameter estimator (36,56) in the place of an extended Kalman filter, the computational requirement of an adaptive controller can be considerably reduced with out any deterioration in the performance. Although the combined estimator has been used in the place of extended Kalman filter in (36, 56, 57), it is believed that this thesis is the first to report the use of this combined estimator in an adaptive control problem.

1.6 Organisation of the thesis:

Chapter 2 of this thesis contains a description of controller structure, presents various suboptimal adaptive controllers and bounding controllers. The controller description is based on a state space representation of the controlled process, hence the input output equation 1.1 is transformed to a state-space form in section 2.1. This chapter also brings out the interrelations between the various adaptive controllers presented.

Chapter 3 describes the computer program used to simulate the adaptive and bounding controllers and also presents details of the division of parameter space into regions.

Open Loop Feedback Optimal Control law is derived in Chapter 4 for a special case of equations 1.1 and 1.2 (only parameter b is unknown but constant, and $q=0$) and, using the program described in Chapter 3, simulated along with the two bounding controllers. The simulation results, presented in graphic form, are discussed in section 4.4 and the conclusions are stated in section 4.5.

Chapter 5 presents the simulation results of self-tuning controller, OK controller and the two bounding controllers for the first order process described in section 1.3 when the three unknown parameters are (i) constants, (ii) slowly time-varying, (iii) rapidly time-varying. Simulation results are presented in section 5.2 and conclusions about relative merits of the adaptive controllers are stated in section 5.3.

Chapter 6 considers the convergence of OK controller using an extended Kalman filter as suboptimal estimator. A necessary condition for convergence to a linear constant parameter controller having same

functional form as CAP controller has been obtained for a general n^{th} order single input single output process through a theorem proved in section 6.3. In section 6.4 the occasions when this condition would be violated are examined for a first order controlled process. Assuming the OK controller has converged to a constant parameter linear controller, the stability conditions for the closed loop system are presented in section 6.6.

Chapter 7 considers the combined estimator of (36, 56) for simultaneous state and parameter estimation. The computational advantages of the combined estimator over the extended Kalman filter are discussed and the governing equations for combined estimator are given in section 7.2. Simulation results of OK controller with (i) the combined estimator, and (ii) the extended Kalman filter are presented in section 7.3 for the first order process of equation 1.1 when the unknown parameters are constants.

Finally the main conclusions of this thesis are summarised in Chapter 8.

CHAPTER TWO

CONTROLLERS

2.1 Introduction

This chapter describes the structure of controllers, presents equations for various suboptimal adaptive and bounding controllers and summarises their interrelations.

A controller design based on an input output representation of the controlled process does not take the transient process dynamics into account. Since in many problems the transient behaviour of the controlled process is important, the description of controllers in this Chapter is based on a state-space representation of the controlled process. The input output form of equation 1.1 is therefore transformed to a state-space form in section 2.2 whereby all unknown parameters become unknown states.

Section 2.3 discusses optimal controller structure which provides a basis for the design of suboptimal adaptive controllers. Suboptimal adaptive controllers designed on this basis consists of a suboptimal estimator cascaded to a suboptimal control law. In section 2.4 equations for a commonly used suboptimal estimator, extended Kalman filter (38), are presented for the controlled process described in section 2.2. Five suboptimal control laws are described in sections 2.5, 2.6, 2.8, 2.9. They are: Open Loop Feedback Optimal (OLFO) control (25, 18, 19), Neutral control (26), Optimal k step ahead (OK) control, Certainty-equivalent k step ahead control and Self-tuning control (17). The two bounding controllers, viz. CAP controller and NOL controller, are described in section 2.7. Finally, a summary of the interrelations between the various controllers is presented in section 2.10.

2.2 Controlled process in state-space form:

Controlled process has been characterised in chapter 1 by the following equation:

$$y(i) + a(i-1)y(i-1) = b(i-1)u(i-1) + \xi(i) + c(i-1)\xi(i-1). \quad 1.1$$

A state-space representation of the above equation requires three state variables to represent the three unknown parameters a , b , c and a fourth state variable to represent the dynamics of the controlled process. It is convenient to define

$$x_1(i) \equiv -c(i),$$

$$x_2(i) \equiv b(i),$$

$$x_3(i) \equiv c(i) - a(i),$$

2.1a

$$x_4(i) \equiv y(i) - \xi(i).$$

2.1b

Substitution of equation 2.1 into equation 1.1 gives a first order difference equation for the state x_4 ,

$$x_4(i+1) = x_1(i)x_4(i) + x_2(i)u(i) + x_3(i)y(i). \quad 2.2a$$

If the unknown parameters are time-varying and stochastic then they are represented by first order difference equations for the other three states

$$x_j(i+1) = g_j x_j(i) + \alpha_j + \xi_j(i) \quad j = 1, 2, 3 \quad 2.2b$$

where g_j , α_j are assumed to be known coefficients ($|g_j| < 1$) and ξ_j are uncorrelated zero mean normal random sequences having variance σ_j . Equation 2.2b specifies stochastic variables having mean value

$$\bar{x}_j = \frac{\alpha_j}{(1-g_j)} \quad 2.2c$$

and variance

$$\text{var}(x_j) = \frac{\sigma_j}{(1-g_j^2)} \cdot \quad 2.2d$$

If the unknown parameters are constants

$$g_j = 1.0$$

$$\alpha_j = 0.0$$

$$\sigma_j = 0.0$$

and equation 2.2b becomes

$$x_j(i+1) = x_j(i) \quad 2.2e$$

2.3 Controller Structure

The starting point for discussion of a controller design is the cost function I_N ,

$$I_N = \sum_{j=i_0}^{i_f-1} \{ y^2(j) + q u^2(j) \} + y^2(i_f). \quad 1.2$$

At any time i the cost to go from i to i_f is random because of the stochastic variables ξ in equations 2.1 and 2.2, so the control $u(i)$ must be designed to minimise the expected value $J_n(i)$ (n is number of stages from i to i_f) of the cost from i to i_f conditional on the information available at time i

$$J_n(i) = E \left[\sum_{j=i}^{i_f-1} \{ y^2(j) + q u^2(j) \} + y^2(i_f) \mid i \right]. \quad 2.3$$

If there were no uncertainty about the parameters a , b , c of equation 1.1 there would be no need to consider the state variables of equation 2.1a. Equation 2.2a would be linear in a single state variable x_4 and could be regarded together with equation 2.1b as an innovation representation of the input output equation 1.1. The optimal control law would be given by standard LQG results (1); it would be the certainty-equivalent CAP control law

$$U_n^{\text{CAP}}(i) = -k_n [x_1 E(x_4(i) \mid i) + x_3 y(i)] \quad 2.4$$

where k_n is the certainty-equivalent feedback gain specified by a Riccati equation and $E(x_4(i) \mid i)$ is the mean of the conditional probability distribution $p(x_4(i) \mid i)$ generated by a Kalman filter. The optimal controller separates, as shown in fig. 2.1, into two operations of estimation and control found in all stochastic controllers (12, 37).

In the presence of uncertainty about some or all of the coefficients a , b , c some or all of the state variables of equation 2.1a must be used and equation 2.2a may become linear, bilinear or non-linear. If the only uncertain parameter were a , the only additional state variable needed would be x_3 and linearity would be preserved because the term $x_3(i)y(i)$ in equation 2.2a can be regarded as linear in x_3 with a time-varying coefficient $y(i)$ whose current value is known at current time i . If there were also uncertainty about b the state variable x_2 would be needed and equation 2.2a would not be linear but bilinear because the term $x_2(i)u(i)$ is bilinear (27). In the presence of uncertainty about c the state variable x_1 is needed and equation 2.2a is then non-linear because $x_1(i)x_4(i)$ is non-linear. When equation 2.2a is either bilinear or non-linear the optimal control is characterised by insoluble equations. When 2.2a is non-linear, but not when it is bilinear, estimation of states is a non-linear filtering problem which also has no practical optimal solution.

The consequence of uncertainty about a , b and c is thus that one or both of the two operations shown in fig. 2.1, estimation and control, cannot be performed optimally. Practical adaptive controllers for the controlled process of equation 1.1 are therefore suboptimal. Fig. 2.1 provides a basis for designing suboptimal adaptive controllers in the form of a suboptimal control law cascaded with a suboptimal estimator. In this chapter attention is focussed on suboptimal control laws and suboptimal estimation where necessary is performed by an extended Kalman filter (37, 38). One possible disadvantage of extended Kalman filter is that it would become computationally extravagant for higher order systems having many more than four state variables. In chapter 7 an alternative suboptimal estimation scheme is described.

2.4 Extended Kalman filter (37, 38)

The extended Kalman filter is derived by linearising the dynamic equation 2.2a about the current estimate and assuming all random variables to be normally distributed. It generates estimates $\hat{\underline{x}}(i)$ and $\underline{\Sigma}(i)$ of the mean and the covariance matrix of the conditional probability distribution $p(\underline{x}(i)|i)$. For the system of equations 2.1b and 2.2 the extended Kalman filter equations are

$$\hat{\underline{x}}(i+1) = \hat{\underline{x}}(i+1|i) + \underline{\Sigma}(i+1)\underline{C}^T\sigma_{\xi}^{-1}\{y(i+1) - \hat{x}_4(i+1|i)\}$$

$$\hat{\underline{x}}(i+1|i) = \begin{bmatrix} \varepsilon_1 \\ \\ \varepsilon_2 \\ \\ \varepsilon_3 \\ \\ 0 & u(i) & y(i) & \hat{x}_1(i) \end{bmatrix} \hat{\underline{x}}(i) + \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \sigma_{14}(i) \end{bmatrix}$$

$$\underline{\Sigma}(i+1) = \underline{\Sigma}_0 - \underline{\Sigma}_0\underline{C}^T(\sigma_{\xi} + \underline{C}\underline{\Sigma}_0\underline{C}^T)^{-1} \underline{C}\underline{\Sigma}_0$$

$$\underline{C} = [0 \ 0 \ 0 \ 1]$$

$$\underline{\Sigma}_0 = \underline{A}(i)\underline{\Sigma}(i)\underline{A}^T(i) + \underline{\Sigma}_{\xi}$$

$$\underline{A}(i) = \begin{bmatrix} \varepsilon_1 \\ \\ \varepsilon_2 \\ \\ \varepsilon_3 \\ \\ \hat{x}_4(i) & u(i) & y(i) & \hat{x}_1(i) \end{bmatrix}; \quad \underline{\Sigma}_{\xi} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ 0 \end{bmatrix}$$

2.5 Approximation to future cost: OLFO control and neutral control

Suboptimal control laws can be derived by considering the insoluble equation which characterises optimal control. The equation is derived by dynamic programming (12). It characterises optimal control by defining the optimal cost function

$$f_n(i) \equiv \min_{u(i), \dots, u(i_f-1)} \{J_n(i)\} \quad 2.6$$

and takes the form of a functional recurrence equation (12, 47)

$$f_n(i) = \min_{u(i)} \{y^2(i) + qu^2(i) + E[f_{n-1}(i+1) | i]\} \quad 2.7$$

embodying the principle of optimality. The optimal $u_n^{\text{OPT}}(i)$ is the value of $u(i)$ which minimises the right hand side of equation 2.7 and cannot generally be found in any closed analytic form because the last term $E[f_{n-1}(i+1) | i]$ cannot generally be evaluated in closed form. In LQG problems, as when equation 2.2a is linear, $f_n(i)$ is quadratic in the conditional mean, the last term can be evaluated and equation 2.7 can be solved. Suboptimal controls, for problems where $E[f_{n-1}(i+1) | i]$ cannot be evaluated, can be derived by replacing $E[f_{n-1}(i+1) | i]$ in equation 2.7 by approximate expressions. The significance of $E[f_{n-1}(i+1) | i]$ is that it represents the optimal cost to be expected from all future stages in the summation of equation 2.3; different approximate expressions for future costs lead to different suboptimal control laws.

One approximation is to replace $f_{n-1}(i+1)$ in equation 2.7 by the optimal cost that could be achieved over future stages if there were to be no future observations of the output y so that future control would be open loop control. The resulting suboptimal control law is open loop feedback optimal control (25). Another approximation is to replace $f_{n-1}(i+1)$ in equation 2.7 by the optimal cost that could be achieved

over future stages if there were to be no uncertainty about any of the states at any future time (26). The resulting suboptimal control is neutral (13) in the sense that it would exclude any probing (39) action. These two suboptimal controls, OLFO control u^{OLFO} and neutral control u^{NUT} , are characterised by equations which are too complex to implement for the general class of process of equations 1.1 and 1.2. The OLFO control is nevertheless simulated for a special case (no uncertainty about a , c and $q = 0$) in chapter 4. The neutral control can be expressed in closed form (26) for the restricted class of problems where cost of control is excluded from the cost function of equation 1.2 ($q = 0$); it is then identical with the optimal k step ahead (OK) control derived below.

2.6 OK control law

The OK control law is derived by replacing $f_{n-1}(i+1)$ in equation 2.7 by $y^2(i+k)$ where the integer k represents time delay in the controlled process. The resulting control law is similar to the 'minimum variance' control (2) in that it only minimises the k step ahead output $y(i+k)$. With this approximation equation 2.7 becomes

$$f_n(i) = \underset{u(i)}{\text{Min}} \{y^2(i) + qu^2(i) + E[y^2(i+k)|i]\}. \quad 2.8$$

For the process described by equation 1.1 the value of k is one ($k=1$). Substituting equations 2.1b and 2.2a into equation 2.8 gives

$$f_n(i) = \underset{u(i)}{\text{Min}} \{y^2(i) + qu^2(i) + E[x_1(i)x_4(i) + x_2(i)u(i) + x_3(i)y(i) + \xi(i+1)]^2 | i\}. \quad 2.9$$

The $u(i)$ that minimises the right hand side of the above equation is the OK control and is given by

$$u^{OK}(i) = - \frac{E[x_1 x_2 x_4 | i] + y E[x_2 x_3 | i]}{E[x_2^2 | i] + q}, \quad 2.10$$

where for notational simplicity $x_1(i)$ etc. are written as x_1 .

The conditional expectation in equation 2.10 could be evaluated if the conditional probability distribution $p(\underline{x} | i)$ were normal. The extended Kalman filter equations 2.5 provide estimates of the mean and covariance matrix of $p(\underline{x} | i)$, based on the assumption that it is normal. These estimates are used, together with standard results (40) about moments of normal distributions, to give an explicit expression for the OK control law

$$u^{OK}(i) = - \frac{[\hat{x}_1 \hat{x}_2 \hat{x}_4 + \hat{x}_1 \sigma_{24} + \hat{x}_2 \sigma_{14} + \hat{x}_4 \sigma_{12} + (\hat{x}_2 \hat{x}_3 + \sigma_{23})y]}{\hat{x}_2^2 + \sigma_{22} + q}. \quad 2.11$$

2.7 CAP control law and NOL control law:

CAP control law and NOL control law are both k step ahead control laws.

The CAP control law, given perfect information about the parameters

x_1, x_2, x_3 , minimises the right hand side of equation 2.8 or 2.9.

It provides a lower bound on the cost of all adaptive controls designed to minimise the k step ahead output. This lower bounding CAP control law is given by

$$u_1^{\text{CAP}}(i) = -\left[\frac{x_1 x_2 \hat{x}_4 + x_2 x_3 y}{x_2^2 + q}\right] \quad 2.12$$

where \hat{x}_4 , the conditional mean of the dynamic state x_4 , is generated by a Kalman filter having the following equations.

$$\hat{x}_4(i) = \hat{x}_4(i|i-1) + \sigma_{44}(i)\sigma_\xi^{-1}[y(i) - \hat{x}_4(i|i-1)], \quad 2.13a$$

$$\hat{x}_4(i|i-1) = x_1 \hat{x}_4(i-1) + x_2 u(i-1) + x_3 y(i-1),$$

$$\sigma_{44}(i) = \sigma_\xi x_1^2 \sigma_{44}(i-1) \{\sigma_\xi + x_1^2 \sigma_{44}(i-1)\}^{-1}. \quad 2.13b$$

The upper bound on the cost is provided by a control law identical to equation 2.12, 2.13 except that the actual values x_1, x_2, x_3 of the states representing a, b, c are replaced by a priori estimates $\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)$. This upper bounding NOL control law is given by

$$u_1^{\text{NOL}}(i) = -\left[\frac{\hat{x}_1(0)\hat{x}_2(0)\hat{x}_4(i) + \hat{x}_2(0)\hat{x}_3(0)y(i)}{\hat{x}_2^2(0) + q}\right]. \quad 2.14$$

This control law has no mechanism for using measurements to update the estimates of the parameters and is therefore described as 'no learning'. The NOL control provides an upper bound by indicating the worst that can happen as a result of uncertainty about a, b, c.

2.8 Certainty-equivalent k step ahead control law:

An alternate way to derive suboptimal control law is to use certainty-equivalence (12, 13). The procedure is to consider an equivalent deterministic problem in which all random variables are replaced by their mean values and to use the optimal feedback control law for this problem as a suboptimal control law in the controller of fig. 2.1. The arguments of the certainty-equivalent control law are the current conditional means of the random variables. The certainty-equivalent k step ahead control law to minimise the k step ahead output for the process of equation 2.1b, 2.2 (where k=1) is

$$u_1^{CE}(i) = -\frac{[\hat{x}_1 \hat{x}_2 \hat{x}_4 + \hat{x}_2 \hat{x}_3 y]}{\hat{x}_2^2 + q} \quad 2.15$$

where notation is simplified as in equation 2.11. The certainty equivalent k step ahead control of equation 2.15 could also be obtained from the optimal k step ahead (OK) control of equation 2.11 by neglecting all the covariance terms. The difference between the two controls can be described as 'caution' (39), and the poor performance of certainty-equivalent control laws in some adaptive control problems (9) has been attributed (9,39) to lack of caution.

2.9 Self-tuning controller

The self-tuning controller uses a control law which is a certainty-equivalent version of the asymptotic CAP control law u_1^{CAP} . The CAP control law of equation 2.12 in terms of a, b, c is

$$u_1^{\text{CAP}}(i) = - \frac{[-c\hat{x}_4(i) + (c-a)y(i)]}{(b + \lambda)}; \quad \lambda \equiv \frac{q}{b} \quad 2.16$$

and the estimate \hat{x}_4 evolves according to the Kalman filter equations 2.13a, 2.13b. Asymptotic versions of the Kalman filter equations 2.13, when $i \rightarrow \infty$, depend on whether $|c| \gtrless 1$ as follows:

If $|c| \leq 1$, equation 2.13b becomes $\sigma_{44}(i) \rightarrow 0$ and equation 2.13a in terms of a, b, c becomes

$$\hat{x}_4(i) = -c\hat{x}_4(i-1) + bu(i-1) + (c-a)y(i-1); \quad 2.17$$

and substituting u from equation 2.16, $\hat{x}_4(i) = -\lambda u(i-1)$. 2.18

If $|c| > 1$ equation 2.13b becomes $\sigma_{44}(i) \rightarrow (1 - \frac{1}{c^2})\sigma_\xi$

and equation 2.13a becomes

$$\hat{x}_4(i) = -c\hat{x}_4(i-1) + bu(i-1) + (c-a)y(i-1) + (1 - \frac{1}{c^2})\{y(i) - (-c\hat{x}_4(i-1) + bu(i-1) + (c-a)y(i-1))\}; \quad 2.19$$

and substituting $u(i)$ from equation 2.16

$$\hat{x}_4(i) = -\frac{\lambda}{c^2} u(i-1) + (1 - \frac{1}{c^2})y(i). \quad 2.20$$

Substituting these asymptotic expressions for \hat{x}_4 into the CAP control law of equation 2.16 gives

$$u_1^{\text{CAP}}(i) = - \frac{[c\lambda u_1^{\text{CAP}}(i-1) + (c-a)y(i)]}{(b + \lambda)}; \quad \text{when } i \rightarrow \infty \text{ and } |c| \leq 1. \quad 2.21$$

$$u_1^{\text{CAP}}(i) = - \frac{[\frac{\lambda}{c} u_1^{\text{CAP}}(i-1) + (\frac{1}{c} - a)y(i)]}{(b + \lambda)} ; \text{ when } i \rightarrow \infty \text{ and } |c| > 1. \quad 2.22$$

This asymptotic control law (equations 2.21, 2.22) can be shown to set the optimal (least squares) prediction of a generalised output ϕ to zero (17) where

$$\phi(i+1) = y(i+1) + \lambda u(i) \quad 2.23$$

as follows. Substituting equations 2.1b and 2.2a into 2.23 and replacing $\hat{x}_4(i)$ by its asymptotic value from equations 2.18, 2.20, the prediction $\hat{\phi}(i+1|i)$ asymptotically becomes

$$\hat{\phi}(i+1|i) = c\lambda u(i-1) + bu(i) + (c-a)y(i) + \lambda u(i) \quad \text{if } |c| \leq 1 \quad 2.24$$

$$\hat{\phi}(i+1|i) = \frac{\lambda}{c} u(i-1) + \frac{1}{c}y(i) + bu(i) - ay(i) + \lambda u(i) \quad \text{if } |c| > 1. \quad 2.25$$

Clearly the control law of equation 2.21, 2.22 sets this prediction to zero.

The certainty-equivalent control law used in the self tuning controller is based on the CAP control law of equation 2.21, 2.22;

$$u^{\text{ST}}(i) = - \frac{[\hat{c}\lambda u^{\text{ST}}(i-1) + \widehat{(c-a)}y(i)]}{\widehat{(b + \lambda)}} \quad \text{when } |c| \leq 1 \quad 2.26$$

$$u^{\text{ST}}(i) = - \frac{(\frac{\lambda}{c})u^{\text{ST}}(i-1) + \widehat{(\frac{1}{c} - a)}y(i)}{\widehat{(b + \lambda)}} \quad \text{when } |c| > 1. \quad 2.27$$

An estimator to generate $\hat{c}\lambda$, $\widehat{(c-a)}$, $\widehat{(b + \lambda)}$ or $\frac{\lambda}{c}$, $\widehat{(\frac{1}{c} - a)}$, $\widehat{(b + \lambda)}$ is derived by assuming equations 2.24, 2.25 are satisfied. This assumption would not be true unless the estimated values had been identical with the true values for such a long time that the resulting control system had settled to its steady state. The assumption is a form of linearisation. From (17) the generalised output ϕ can be written in terms of $\hat{\phi}$ as

$$\phi(i+1) = \hat{\phi}(i+1|i) + \epsilon(i+1) \quad 2.28$$

where $\epsilon(i+1)$ is the prediction error. Substituting for $\hat{\phi}$ from equations 2.24, 2.25,

$$\phi(i+1) = \underline{z}^T(i)\underline{\theta} + \epsilon(i+1) \quad 2.29$$

$$\underline{z}(i) = \begin{bmatrix} u(i-1) \\ u(i) \\ y(i) \end{bmatrix} ; \quad \underline{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} c\lambda \\ (b+\lambda) \\ (c-a) \end{bmatrix} \quad \text{if } |c| \leq 1 ;$$

$$\underline{\theta} = \begin{bmatrix} \frac{\lambda}{c} \\ (b+\lambda) \\ (\frac{1}{c} - a) \end{bmatrix} \quad \text{if } |c| > 1 .$$

The estimates of $\underline{\theta}$ can be obtained by Kalman filter equations,

$$\hat{\underline{\theta}}(i+1) = \hat{\underline{\theta}}(i+1|i) + \underline{P}(i+1)\underline{z}(i)\{\phi(i+1) - \underline{z}^T(i)\hat{\underline{\theta}}(i+1|i)\}. \quad 2.30a$$

When the unknown parameters are constants,

$$\left. \begin{aligned} \hat{\underline{\theta}}(i+1|i) &= \hat{\underline{\theta}}(i) ; \\ \underline{P}(i+1) &= \beta^{-1} [\underline{P}(i) - (\beta + \underline{z}^T(i)\underline{P}(i)\underline{z}(i))^{-1} \underline{P}(i)\underline{z}(i)\underline{z}^T(i)\underline{P}(i)] ; \end{aligned} \right\} \quad 2.30b$$

where $\underline{P}(i)$ is a matrix proportional to the covariance of the estimated parameters and β is an exponential forgetting factor ($0 < \beta < 1$) introduced to prevent the norm of $\underline{P}(i)$ approaching zero more rapidly than $\hat{\phi}(i+1|i)$ (17).

When the unknown parameters are time-varying either of two modifications could be used;

- i) Assume the controller parameters $\underline{\theta}$ are constants and use the same estimation equations 2.30a, 2.30b but choose an appropriate forgetting factor to account for the time-varyingness of the actual controller parameters.
- ii) Model the controller parameters $\underline{\theta}$ as time-varying parameters. This involves translation of process parameter dynamics of equation 2.2b into controller parameter dynamics.

When $|c(i)| \leq 1$ the controller parameter dynamics is

$$\begin{bmatrix} \theta_1(i+1) \\ \theta_2(i+1) \\ \theta_3(i+1) \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \cdot \begin{bmatrix} \theta_1(i) \\ \theta_2(i) \\ \theta_3(i) \end{bmatrix} + \begin{bmatrix} -\lambda\alpha_1 \\ \alpha_2 + \lambda - \lambda\varepsilon_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} -\lambda\varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} ;$$

and the estimation equation 2.30b becomes

$$\hat{\underline{\theta}}(i+1|i) = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \hat{\underline{\theta}}(i) + \begin{bmatrix} -\lambda\alpha_1 \\ \alpha_2 + \lambda - \lambda\varepsilon_2 \\ \alpha_3 \end{bmatrix},$$

$$\underline{P}(i+1) = \underline{P}(i+1|i) - (1 + \underline{z}^T(i)\underline{P}(i+1|i)\underline{z}(i))^{-1} \underline{P}(i+1|i)\underline{z}(i)\underline{z}^T(i)\underline{P}(i+1|i), \quad 2.30c$$

$$\underline{P}(i+1|i) = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \underline{P}(i) \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} + \begin{bmatrix} \lambda^2\sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}.$$

Here there is no need to introduce a forgetting factor since the norm of $\underline{P}(i)$ would not approach zero.

When $|c(i)| > 1$, linear translation of process parameter dynamics into controller parameter dynamics is impossible; so in this case modification (i) is the only choice.

From equations 2.26, 2.27 it is clear that the estimate $(\hat{b} + \lambda)$ (or $\hat{\theta}_2$) must be prevented from becoming zero. This is achieved by initially fixing θ_2 at a nominal value and estimating only the two parameters θ_1 and θ_3 (17, 48). It is shown in (67, 48) that as long as the initially fixed value $\hat{\theta}_2(0)$ is not less than 0.5 of true θ_2 the self-tuning controller would converge, otherwise (i.e. $\frac{\hat{\theta}_2(0)}{\theta_2} < 0.5$) the control would become unbounded and the estimates would never converge.

2.10 Summary

Relationship between control laws are clarified by summarising them here. The following control laws have been defined.

i) certainty-equivalent CAP control law (section 2.3)

$$u_n^{CAP}(i) = -k_n [x_1 E[x_4(i)|i] + x_3 y(i)], \quad 2.4$$

ii) OLFO control law (section 2.5)

assume no future measurement and solve equation 2.7,

iii) neutral control (section 2.5),

assume perfect future information and solve equation 2.7,

iv) OK control law (section 2.6),

$$u^{OK}(i) = - \frac{[\hat{x}_1 \hat{x}_2 \hat{x}_4 + \hat{x}_1 \sigma_{24} + \hat{x}_2 \sigma_{14} + \hat{x}_4 \sigma_{12} + (\hat{x}_2 \hat{x}_3 + \sigma_{23})y]}{\hat{x}_2^2 + \sigma_{22} + q}, \quad 2.11,$$

v) certainty-equivalent k step ahead control law (section 2.8),

$$u_1^{CE}(i) = - \frac{[\hat{x}_1 \hat{x}_2 \hat{x}_4 + \hat{x}_2 \hat{x}_3 y]}{\hat{x}_2^2 + q}, \quad 2.15$$

vi) self-tuning control law (section 2.9),

$$u^{ST}(i) = - \frac{[\hat{c} u^{ST}(i-1) + (\hat{c}-a)y(i)]}{(b+\lambda)} \quad \text{if } |c| \leq 1, \quad 2.26$$

$$u^{ST}(i) = - \frac{[\frac{\lambda}{c} u^{ST}(i-1) + (\frac{1}{c} - a)y(i)]}{(b+\lambda)} \quad \text{if } |c| > 1, \quad 2.27$$

vii) k step ahead CAP control law (section 2.7),

$$u_1^{CAP}(i) = - \frac{[x_1 x_2 \hat{x}_4 + x_2 x_3 y]}{x_2^2 + q}, \quad 2.12$$

viii) no learning control law (section 2.7),

$$u_1^{\text{NOL}}(i) = - \frac{[\hat{x}_1(0)\hat{x}_2(0)\hat{x}_4(i) + \hat{x}_2(0)\hat{x}_3(0)y(i)]}{\hat{x}_2^2(0) + q}. \quad 2.14$$

The OK control law of equation 2.11 is more complicated than the various certainty-equivalent control laws because it takes into account the uncertainties in the parameter estimates, i.e. it is cautious. The self-tuning control law is simple partly because it is a certainty-equivalent control law which neglects the uncertainties in the estimates and partly because it neglects the non-linearities by assuming steady state; i.e. assuming equations 2.24, 2.25 are satisfied. Further it regulates only ϕ (17). In general there is no guarantee that a control designed to regulate ϕ would perform well when judged by the original cost function 1.2, except when $q = 0$ in which case $\phi = y$. Then the self-tuning controller regulates the output y and the control law of equation 2.26, 2.27

$$u^{\text{ST}}(i) = - \frac{\widehat{(c-a)}}{\widehat{b}} y(i) \quad \text{if } |c| \leq 1$$

$$u^{\text{ST}}(i) = - \frac{\widehat{(\frac{1}{c} - a)}}{\widehat{b}} y(i) \quad \text{if } |c| > 1$$

is the same as the self-tuning regulator control law (6). Other simplifications result when cost of control is zero ($q=0$):

i) the certainty-equivalent CAP control law is the k step ahead CAP control law

$$u_n^{\text{CAP}}(i) = u_1^{\text{CAP}}(i) = \frac{-[x_1 \hat{x}_4 + x_3 y]}{x_2}.$$

ii) the OK control law of equation 2.11 is the neutral control law of (26).

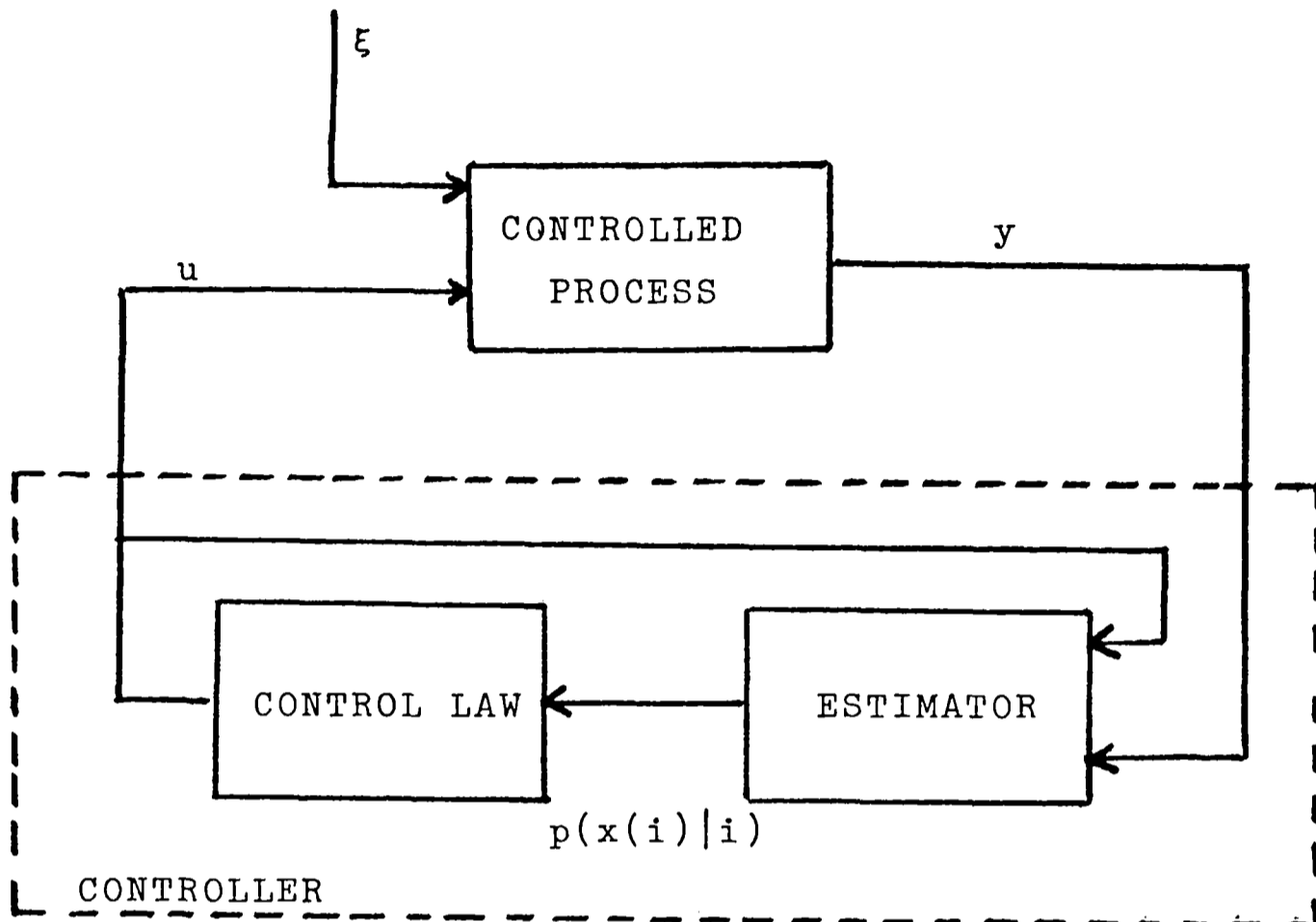


Fig.2.1 Separation of estimation and control in stochastic controller.

CHAPTER THREE

SIMULATION PROCEDURE

3.1 Introduction

Chapter 2 contained the description of various suboptimal adaptive controllers for a linear stochastic process with unknown parameters. It was pointed out in Chapter 1 that a good way to find the relative merits and demerits of these suboptimal adaptive controllers both in terms of cost achieved and computation required is by systematically simulating them along with the process to be controlled on a digital computer. A preliminary simulation study (43) indicated that to obtain meaningful results first the process parameter space should be divided into regions and then the various controllers should be simulated in each of these regions.

This chapter contains the description of the simulation program used throughout this thesis and the details of division of the parameter space into regions.

Section 3.2 describes the simulation program. The simulation program is written in FORTRAN language and it simulates the first order process described by equations 2.1, 2.2 under the various suboptimal controllers discussed in Chapter 2. This section also contains the details of the input data to be supplied to the program.

Section 3.3 contains the description of the division of the parameter space into regions. Altogether there are eight regions. The division is based on such factors as open loop stability of the controlled process, sign of the control gain parameter, and non-minimum phaseness of the noise model.

3.2 Description of simulation program

The simulation program is written with flexibility and reduced storage requirement in mind. This is achieved by arranging it in the form of a main program calling a number of subroutines. The program can simulate any or all of the different controllers discussed in the previous chapter. The main program reads in all the input data, realises the initial values of states and parameters and calls the controller subroutines in sequence. Each controller subroutine simulates the first order process under a particular controller. Data is transmitted between the main program and subroutines through COMMON areas; thus reducing the overall storage requirement. A flow diagram of the simulation program is given in fig. 3.1.

The input data to be supplied to the main program is as follows:

- 1) Initial estimates of the state and parameters; $\hat{\underline{x}}(0)$
- 2) Diagonal elements of the initial covariance matrix $\underline{\Sigma}(0)$; the off-diagonalelements of the initial covariance matrix are set to zero. If certain parameters are known initially the corresponding diagonal entry of $\underline{\Sigma}(0)$ is set to zero.
- 3) Values of the dynamics g_j of the unknown coefficients of equation 2.2b. For constant parameter case $g_j = 1.0$; $\alpha_j = 0$; $\sigma_j = 0$. For time varying parameter case the values α_j and σ_j are determined according to equations (cf. equations 2.2c, 2.2d).
 $\alpha_j = (1-g_j) \cdot \hat{x}_j(0)$, $\sigma_j = (1-g_j^2) \cdot \text{var}(x_j(0))$ which make the steady state mean and variance of x_j same as the mean and variance of its initial estimate.
- 4) The names of the controllers to be simulated.

- 5) Variance of the white process noise; σ_{ξ}
- 6) The weighting on cost of control; q
- 7) The exponential forgetting factor; β
- 8) Total number of stages N to be simulated.

A complete simulation or experiment consists of M repeated trials with the random numbers $\underline{x}(0)$, the random noise sequence $\xi(i)$ taking different values in each trial. Each trial involves simulating once an N stage process under a set of controllers. In each trial the random numbers $\underline{x}(0)$, the random noise sequence $\xi(i)$ are the same for all the controllers. The frequency of repetition of the random number generator was large enough so that repetition of random numbers did not occur. The expected cost J_N of equation 2.3 is estimated by averaging the M costs I_N of equation 1.2 generated by M different trials. The number of trials M was chosen on the basis that:

$$\text{Number of Trials (M)} \times \text{Number of Stages (N)} = 10,000.$$

This M gave, for 15% confidence interval (i.e. I_N (average) \pm 15%), a confidence level (66) of 90% when $N > 100$ and a confidence level of more than 95% when $N \leq 100$.

In order to be absolutely certain that the main program and the various subroutines are error free, quantities such as the estimates, control, cost, output etc. generated by the program were checked with a hand calculation of these quantities for a short trial of five stages ($N=5$). The simulation program takes about 50 sec. on a CDC 7600 computer for a complete simulation of OK controller, self-tuning controller, CAP controller and No learning controller[†]

[†] The covariance updating equation in the extended Kalman filter and in the least square estimator of ST controller could pose serious numerical problems resulting from rounding off errors. But this did not occur in any of the simulations reported in the thesis because the word length of the computer used was large enough (64 bit) for the number of stages N considered. (Maximum number of stages = 1000). However with many more stages than 1000 the numerical difficulty would show up and one way of solving this problem is to use a square root algorithm(48) which updates a square root of the covariance.

3.3 Division of parameter space into regions

This section presents the details of the division of the parameter space into meaningful regions. The motivation for such a division came from a preliminary simulation study (43) which was done in order to gain some idea about the performances of OK controller and self-tuning controller.

The division is based on the following points:

- i) Whether the process to be controlled is openloop stable or not;
- ii) Whether the sign of the control gain parameter is known a priori or not;
- iii) Whether the noise model is non-minimum phase or not.

For first order constant parameter linear processes of the form described by equation 1.1, the above three conditions become conditions on $|a|$, sign of b , $|c|$ respectively and give rise to eight regions in the parameter space a, b, c . These regions are:

Region 1	$ a < 1, c < 1$	}	and sign of b known.
Region 2	$ a > 1, c < 1$		
Region 3	$ a < 1, c > 1$		
Region 4	$ a > 1, c > 1$		
Region 5	$ a < 1, c < 1$	}	and sign of b unknown.
Region 6	$ a > 1, c < 1$		
Region 7	$ a < 1, c > 1$		
Region 8	$ a > 1, c > 1$		

Fig. 3.2 shows all the eight regions. In later chapters various sub-optimal adaptive controllers are simulated systematically in each of these above regions. Even when the parameters are time-varying and stochastic, still the simulations are performed on the basis of these regions.

3.4 Summary

The computer program used in later chapters for simulating the first order process with various suboptimal controllers has been described. A systematic procedure for conducting the simulation study is also given. According to this procedure first the parameter space is to be divided into regions and then the simulations are to be carried out in each of the regions. For a first order linear process with constant parameters these regions have been shown in fig. 3.2.

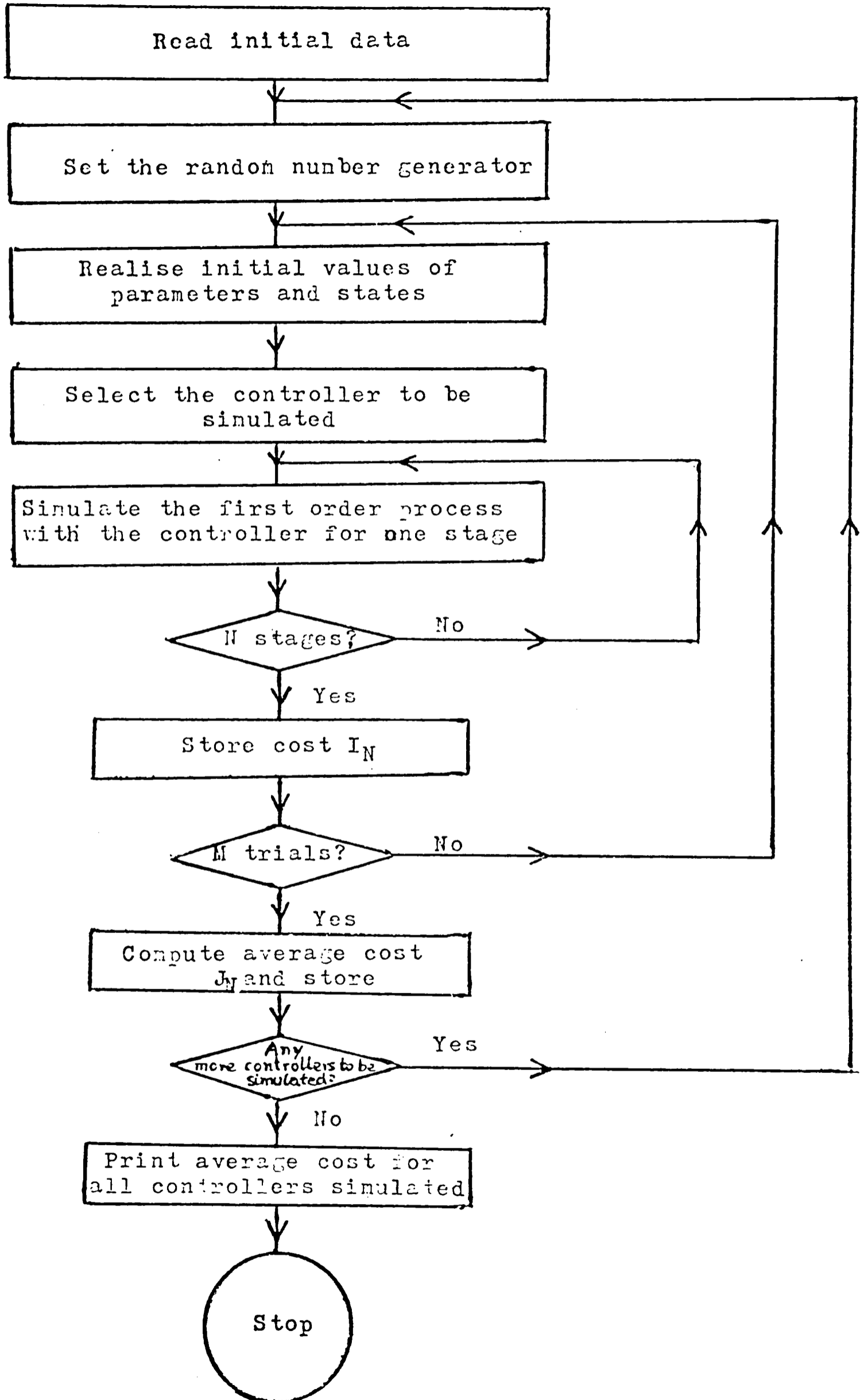


Fig.3.1 Flow chart of the simulation program.

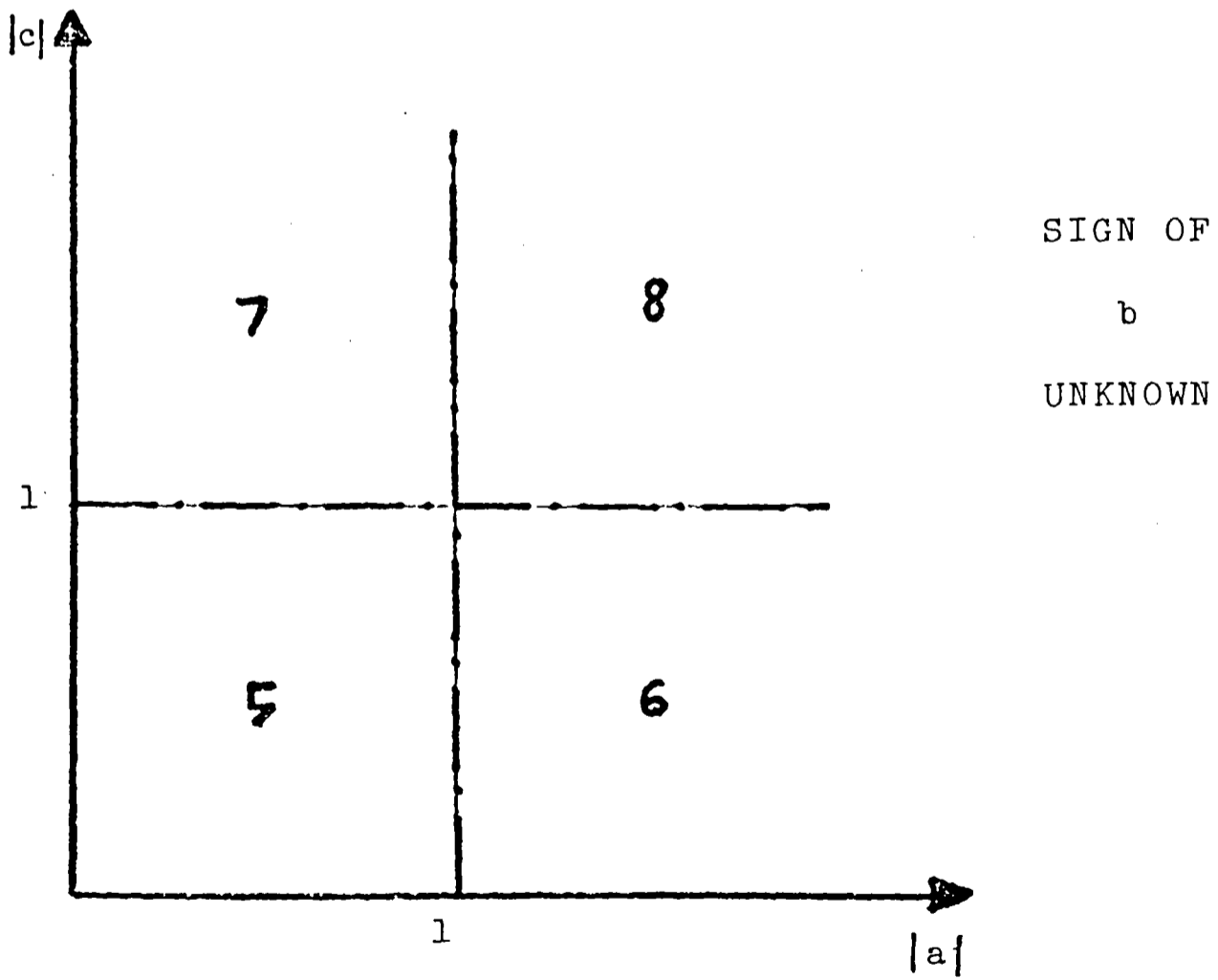
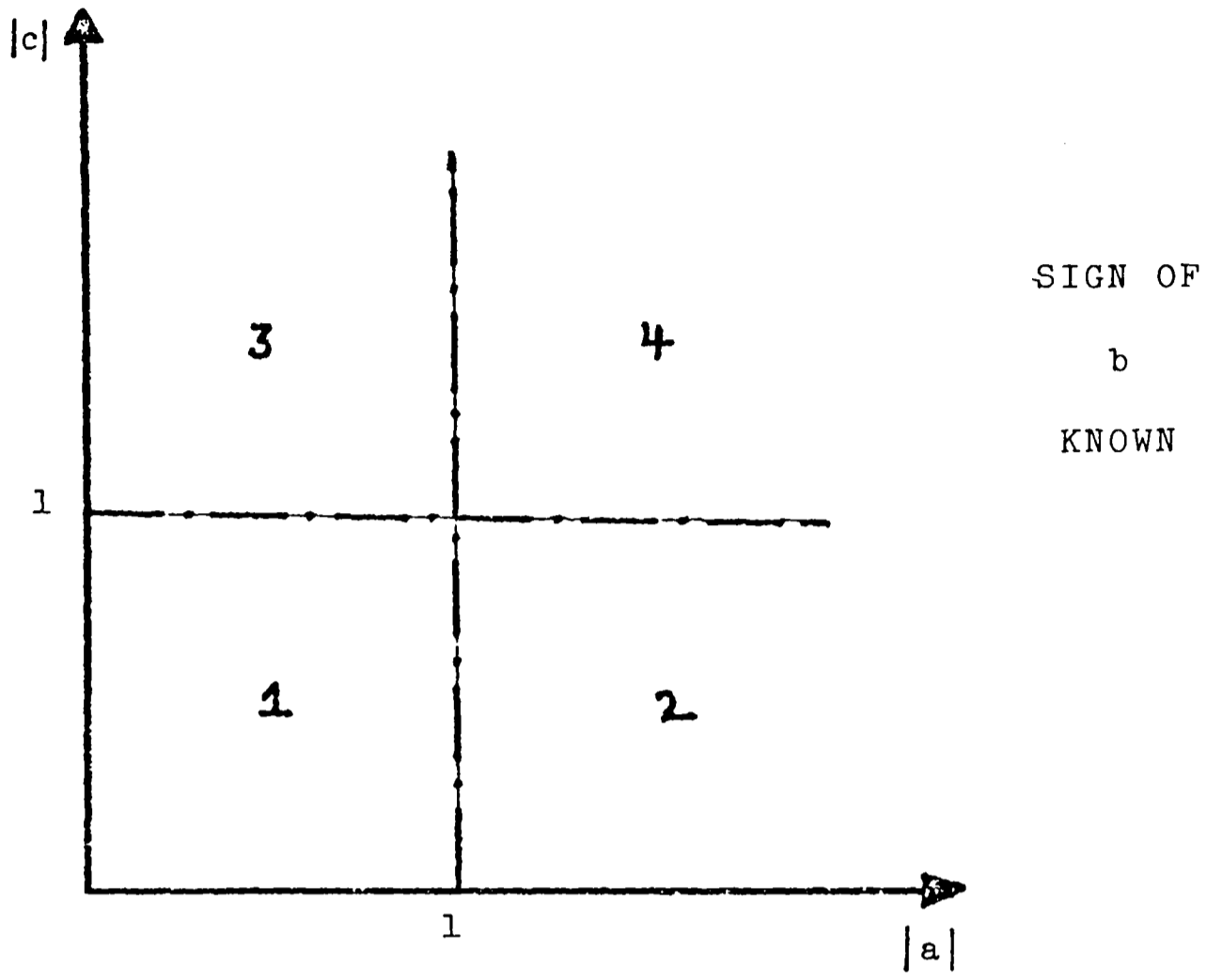


Fig.3.2 Regions in the parameter space a, b, c .

CHAPTER FOUR

PERFORMANCE OF OPENLOOP FEEDBACK OPTIMAL CONTROLLER

4.1 Introduction

Open Loop Feedback Optimal (OLFO) control policy is one of the oldest of all the suboptimal adaptive control policies. It was first proposed by Dreyfus (25) as early as in 1964. Dreyfus considered a three stage decision problem for which the OLFO policy gave a performance very close to what could be achieved under optimal policy. Though several people (18, 19, 20, 23, 24) have since considered this as a suboptimal control policy to control nonlinear stochastic processes, Tse and Athans (18) are believed to be the first to produce non-trivial simulation results. They used OLFO controller to suboptimally control a linear stochastic plant whose control gain parameters were all unknown. In terms of equation 1.1 or 2.2a the problem they considered is equivalent to a case where b or x_2 is the only unknown parameter. Later Ku and Athans (19, 49) extended Tse's work to cover problems where all the parameters were unknown and compared through simulations the performance of OLFO controller with that of a certainty-equivalent controller. Ku's work showed that the cost achieved under OLFO controller would be greater than that under certainty-equivalent controller for unstable controlled processes and very close to that under certainty-equivalent controller for stable controlled processes. The results of this chapter show that the cost achieved under OLFO controller is far greater than that under even NOL controller for unstable controlled processes and very close to that under NOL controller for stable controlled processes. Further OLFO controller required enormous computation. Thus the conclusion of this chapter is that OLFO controller is unsuitable to control unstable processes and because of its enormous computational requirement is

unattractive even for controlling stable processes.

OLFO control law is derived for a simple problem where the controlled process is characterised by equation 1.1 or 2.2 with b or x_2 the only unknown parameter and performance criterion governed by equation 1.2 with $q=0$. Since b or x_2 is the only unknown parameter, the controlled process is only bilinear and so estimates of uncertain states can be generated using a Kalman filter (8). The controller is then simulated along with the two bounding controllers, viz. CAP and NOL controller, in all the four regions 1, 2, 3, 4 of the parameter space shown in fig. 3.2. The simple case for equation 1.1, 1.2 is chosen as it illustrates the worthiness of OLFO controller.

The OLFO control law even for this simple problem is extremely complex and involves solving of Riccati equations. Furthermore its performance is very bad, worse than even the NOL controller, when the parameters of the controlled process are in regions 2 or 4. When the parameters are in regions 1 or 3 the performance of OLFO and NOL controllers are indistinguishable and close to that of the CAP controller.

4.2 OLFO control law

OLFO control policy is already described in section 2.5 of chapter 2.

In this section the resulting control law is derived for the first order process of equation 1.1 with b the only unknown constant parameter. The state space equations then become ,

$$x_2(i+1) = x_2(i),$$

$$x_4(i+1) = x_1 x_4(i) + x_2(i)u(i) + x_3 y(i), \quad 4.1a$$

$$y(i+1) = x_4(i+1) + \xi(i+1). \quad 4.1b$$

Further no cost of control is included in the performance criterion of equation 1.2. Thus equation 1.2 becomes

$$I_N = \sum_{i=i_0}^{i_f} y^2(i). \quad 4.1c$$

The procedure for deriving OLFO control law is as follows: At time i assume that no observations will be made in future. Under this assumption generate an open loop optimal control sequence $\{u^0(j|i)\}_{j=i}^{i_f-1}$ that minimises the expected value of the cost $J_n(i)$ from i to i_f based on all the information available at i . Implement only the first control $\{u^0(i|i)\}$ of this sequence. This control produces an output $y(i+1)$.

Now generate another sequence of open loop optimal controls $\{u^0(j|i+1)\}_{j=i+1}^{i_f-1}$ based on all the information available at $i+1$. As before implement only the first control $\{u^0(i+1|i+1)\}$ and continue the procedure. The controls $\{u^0(i|i)\}_{i=1}^{i_f-1}$ are the open loop feedback optimal controls $u^{OLFO}(i)$.

At time i the expected cost from i to i_f is $J_n(i)$ (c.f. equation 2.3);

$$J_n(i) = E\left\{ \sum_{j=i}^{i_f} y^2(j) \mid i \right\} \quad 4.2a$$

Substituting for y from equation 4.1b and taking the expectation,

$$J_n(i) = \sum_{j=i}^{i_f} \{ \hat{x}_4^2(j|i) + \sigma_{44}(j|i) \} + n\sigma_\xi \quad 4.2b$$

Finding the open loop optimal control sequence $\{ u^0(j|i) \}_{j=i}^{i_f-1}$ is a deterministic optimal control problem since the dynamics of $\hat{x}_4(j|i)$, $\sigma_{44}(j|i)$ are governed by the deterministic equations given below.

$$\left. \begin{aligned} \hat{x}_4(i+1|i) &= x_1 \hat{x}_4(i|i) + \hat{x}_2(i|i)u(i) + x_3 y(i) \\ \hat{x}_2(i+1|i) &= \hat{x}_2(i|i) \\ \sigma_{44}(i+1|i) &= x_1^2 \sigma_{44}(i|i) + u^2(i) \sigma_{22}(i|i) + 2x_1 u(i) \sigma_{24}(i|i) \\ \sigma_{24}(i+1|i) &= x_1 \sigma_{24}(i|i) + u(i) \sigma_{22}(i|i) \\ \sigma_{22}(i+1|i) &= \sigma_{22}(i|i) \end{aligned} \right\} \quad 4.3a$$

$$\left. \begin{aligned} \hat{x}_4(j+1|i) &= (x_1 + x_3) \hat{x}_4(j|i) + \hat{x}_2(j|i)u(j) \\ \hat{x}_2(j+1|i) &= \hat{x}_2(j|i) \\ \sigma_{44}(j+1|i) &= (x_1 + x_3)^2 \sigma_{44}(j|i) + u^2(j) \sigma_{22}(j|i) + \\ &\quad 2u(j)(x_1 + x_3) \sigma_{24}(j|i) + x_3^2 \sigma_\xi \\ \sigma_{24}(j+1|i) &= (x_1 + x_3) \sigma_{24}(j|i) + u(j) \sigma_{22}(j|i) \\ \sigma_{22}(j+1|i) &= \sigma_{22}(j|i) \end{aligned} \right\} \quad \begin{array}{l} \text{when} \\ i < j < i_f \end{array} \quad 4.3b$$

The initial conditions for these equations at $j=i$ are $\hat{x}_4(i|i)$, $\hat{x}_2(i|i)$, $\sigma_{44}(i|i)$, $\sigma_{24}(i|i)$, $\sigma_{22}(i|i)$. The two sets of equations 4.3a, 4.3b can be combined to one set of equations by a change of variables. A trajectory with initial conditions $\hat{x}_4(i|i)$, $\hat{x}_2(i|i)$, $\sigma_{44}(i|i)$, $\sigma_{24}(i|i)$, $\sigma_{22}(i|i)$ and following the dynamics of 4.3a, 4.3b will be identical with one starting from $\underline{z}(i|i)$ and having the following dynamics for $i \leq j < i_f$.

$$\begin{aligned}
 z_1(j+1|i) &= (x_1+x_3)z_1(j|i) + z_2(j|i)u(j) \\
 z_2(j+1|i) &= z_2(j|i) \\
 z_3(j+1|i) &= (x_1+x_3)^2 z_3(j|i) + u^2(j)z_5(j|i) + 2(x_1+x_3)u(j)z_4(j|i) + \\
 &\quad x_3^2 \sigma_\xi \\
 z_4(j+1|i) &= (x_1+x_3)z_4(j|i) + u(j)z_5(j|i) \\
 z_5(j+1|i) &= z_5(j|i)
 \end{aligned}
 \tag{4.4a}$$

$$\underline{z}(i|i) = \begin{bmatrix} z_1(i|i) \\ z_2(i|i) \\ z_3(i|i) \\ z_4(i|i) \\ z_5(i|i) \end{bmatrix} = \begin{bmatrix} \frac{x_1 \hat{x}_4(i|i) + x_3 y(i)}{(x_1+x_3)} \\ \hat{x}_2(i|i) \\ \frac{x_1^2}{(x_1+x_3)^2} \sigma_{44}(i|i) - \frac{x_3^2}{(x_1+x_3)^2} \sigma_\xi \\ \frac{x_1 \sigma_{24}(i|i)}{(x_1+x_3)^2} \\ \sigma_{22}(i|i) \end{bmatrix}
 \tag{4.4b}$$

Further the open loop optimal control sequence that minimise $J_n(i)$ of equation 4.2b would be identical with the control sequence that minimise $J'_n(i)$,

$$J'_n(i) = \sum_{j=i}^{i_f} \{z_1^2(j|i) + z_3(j|i)\}
 \tag{4.5}$$

and can be obtained via dynamic programming. The details of derivation of the control sequence is in Appendix 1. The resulting open loop feedback optimal control law $u^{\text{OLFO}}(i)$ is given below:

$$\begin{aligned}
 u^{\text{OLFO}}(i) &= -K^{-1}(i|i) \{ [V_{11}(i+1|i) \hat{x}_2(i|i) + V_{12}(i+1|i) \sigma_{22}(i|i)] \\
 &\quad [x_1 \hat{x}_4(i|i) + x_3 y(i)] + [V_{12}(i+1|i) \hat{x}_2(i|i) + V_{22}(i+1|i) \sigma_{22}(i|i) + \\
 &\quad B(i+1|i)] [x_1 \sigma_{24}(i|i)] \}
 \end{aligned}
 \tag{4.6}$$

where

$$K(i|i) = \begin{bmatrix} \hat{x}_2(i|i) \\ \sigma_{22}(i|i) \end{bmatrix}^T \begin{bmatrix} V_{11}(i+1|i) & V_{12}(i+1|i) \\ V_{12}(i+1|i) & V_{22}(i+1|i) \end{bmatrix} \begin{bmatrix} \hat{x}_2(i|i) \\ \sigma_{22}(i|i) \end{bmatrix} + B(i+1|i).$$

V_{11} , V_{12} , V_{22} , B are solved backwards in time according to the Riccati equations A11, A12, A13, A14 in Appendix 1. The estimates $\hat{x}_4(i|i)$, $\hat{x}_2(i|i)$, $\sigma_{44}(i|i)$, $\sigma_{24}(i|i)$, $\sigma_{22}(i|i)$ are generated by a Kalman filter as the controlled process is bilinear (8, 27).

The OLFO control law, like the OK control law of equation 2.11 is cautious in that the control law is a function of the covariance of the unknown parameters. But the OLFO control law is more complicated and involves solving of time consuming Riccati equations on-line.

4.3 CAP and NOL control laws

The two bounding control laws $u_1^{\text{CAP}}(i)$ and $u_1^{\text{NOL}}(i)$ for the process of equations 4.1a, 4.1b can be obtained by setting $q = 0$ in equations 2.12, 2.14 and further replacing $\hat{x}_1(0)$ and $\hat{x}_3(0)$ by the true values of x_1 and x_3 in equation 2.14.

4.4 Simulation results

The program described in chapter 3 is used to simulate the process described by equations 4.1 when controlled by (i) OLFO controller (ii) CAP controller (iii) NOL controller in each of the four regions 1, 2, 3, 4 of fig. 3.2. Table 4.1 gives the data used to specify these regions

Region	x_1	$\hat{x}_2(0)$	x_3	$\sqrt{\sigma_{22}(0)}$
1	0.25	4.0	0.25	0.5
2	0.5	4.0	4.0	0.5
3	4.0	4.0	-4.5	0.5
4	2.0	4.0	2.0	0.5

Table 4.1 Data specifying regions of fig. 3.2

Data common to all the trials are the following:

$$\hat{x}_4(0) = 10.0, \sigma_{44}(0) = 1.0, \sigma_{\xi} = 1.0.$$

The value of $\hat{x}_2(0)$ and $\sqrt{\sigma_{22}(0)}$ are so chosen to ensure negligible possibility of any uncertainty about the sign of x_2 .

Figs. 4.1, 4.2, 4.3, 4.4 show the simulation results. They are drawn to logarithmic scales with average cost of I_N as ordinate and number of stages N as abscissa.

Figs. 4.1 and 4.3 show that in regions 1 and 3 the cost achieved by OLFO and NOL controllers are indistinguishable and close to the lower bound achieved under CAP controller. The closeness of NOL cost to CAP cost indicates that for the simple process of equations 4.1a, 4.1b almost any controller would perform well when the process parameters occupy these regions.

In regions 2 and 4 (figs. 4.2, 4.4) both OLFO controller and NOL controller performed very badly. Furthermore, the cost achieved by OLFO controller far exceeded (more than 10 times) the cost achieved by NOL controller. This indicates that OLFO controller is totally unsuitable when the parameters of the process to be controlled are in regions 2 or 4.

Regions 2 and 4 characterise unstable controlled processes. The bad performance of OLFO controller in these regions may be because OLFO controller, since it assumes no future observations, would be an overly cautious controller (22) and consequently would fail to apply sufficient control. When the parameters of the controlled process are in regions 1 or 3 this overly cautious controller will, nonetheless, bring the output close to zero as the controlled process is stable.

The computing time taken by the three controllers are of the order CAP:NOL:OLFO equals 1:1:12.

The reason for Ku not observing such bad OLFO controller performance as in figs. 4.2 and 4.4 may be (i) length of a run N, for his unstable process (pole at 1.2) was short (N=30) and (ii) his average cost was based on too few trials (M=20).

4.5 Conclusions

OLFO control law has been derived for a simple process having only one unknown parameter. Its performance is then evaluated through computer simulations. The simulation study leads to the following conclusions:

- i) The OLFO control policy yields a control law which is cautious;
- ii) The resulting control law (equation 4.6) is extremely complex and involves solving of time consuming Riccati equations on-line.
- iii) The performance of OLFO controller is never better than and sometimes (for unstable processes) worse than that of NOL controller.
- iv) The above conclusion together with the computing time it takes makes OLFO controller unsuitable for unstable controlled processes and an unwise choice even for stable controlled processes.

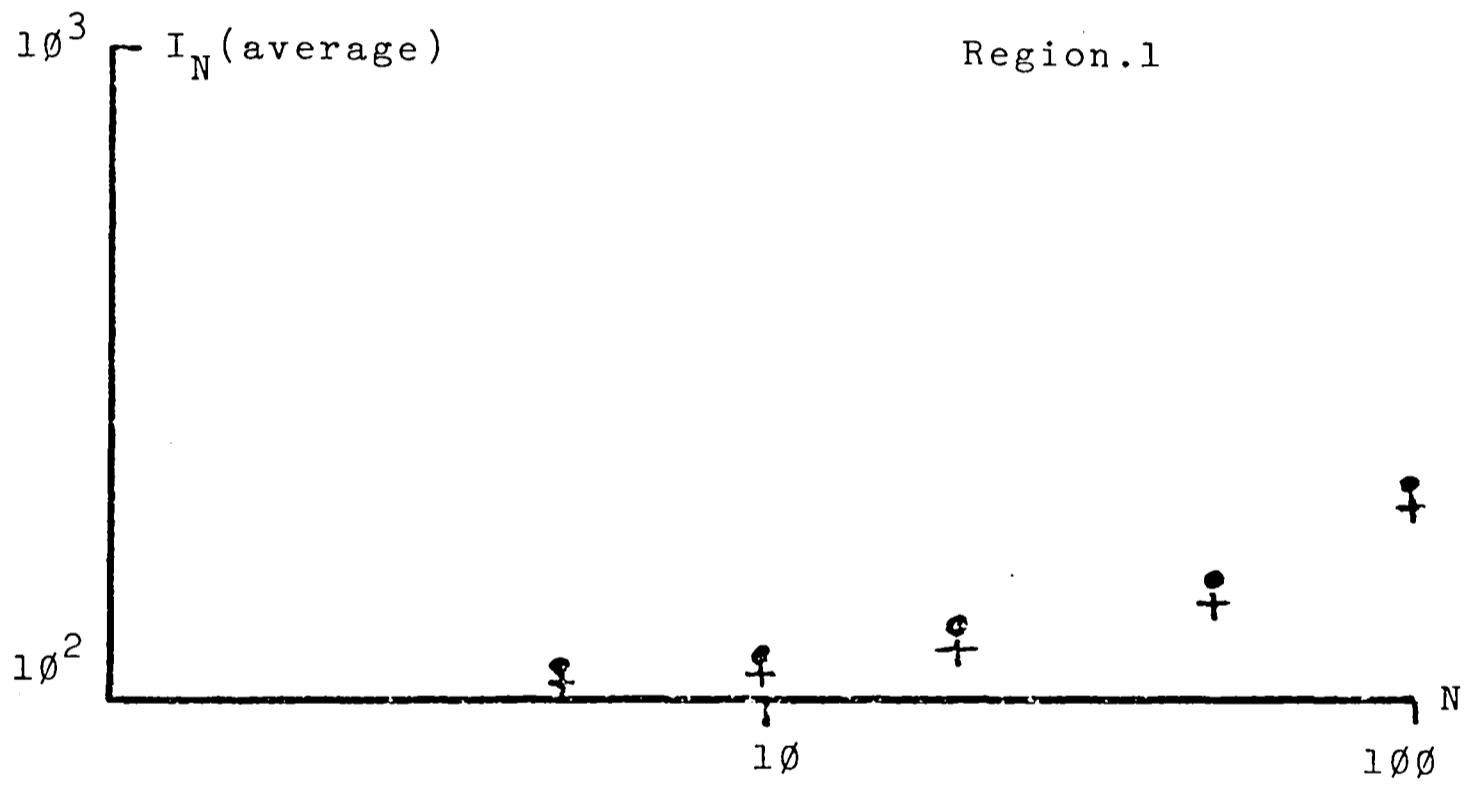


Fig.4.1 +CAP

• OLFO & NOL (indistinguishable)

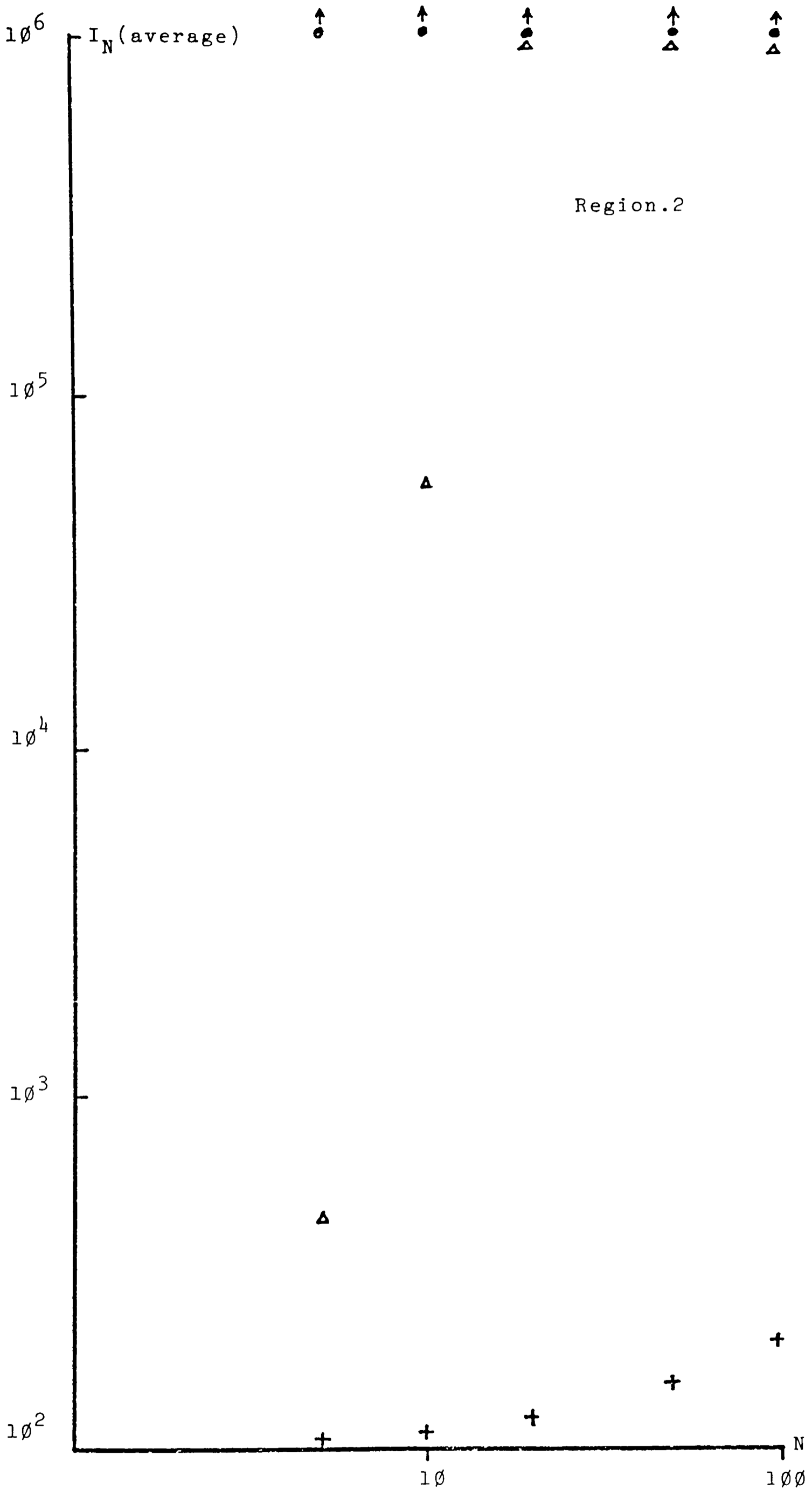


Fig.4.2 +CAP, •OLFO, Δ NOL.

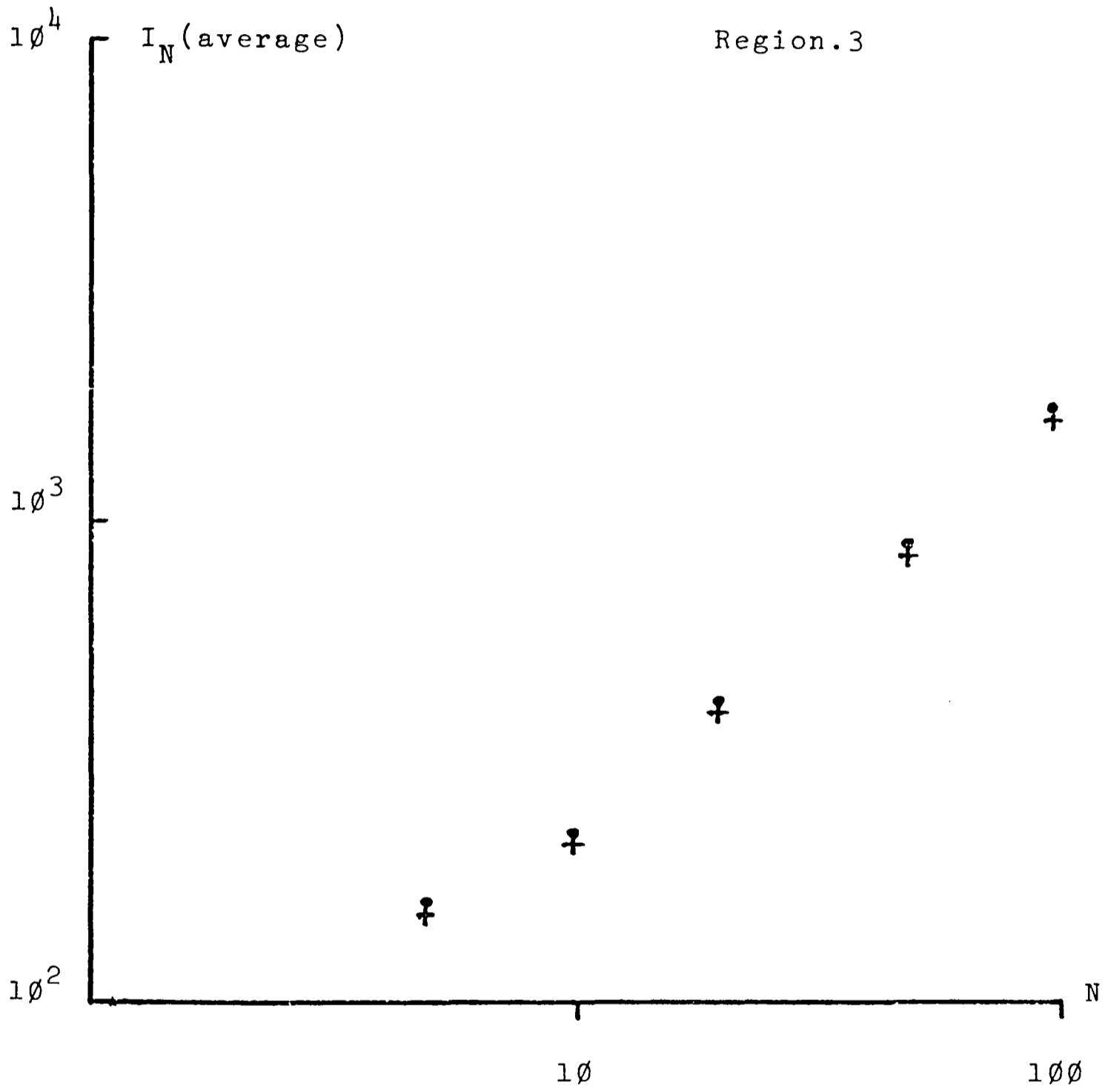


Fig.4.3 + CAP

• OLFO & NOL (indistinguishable)

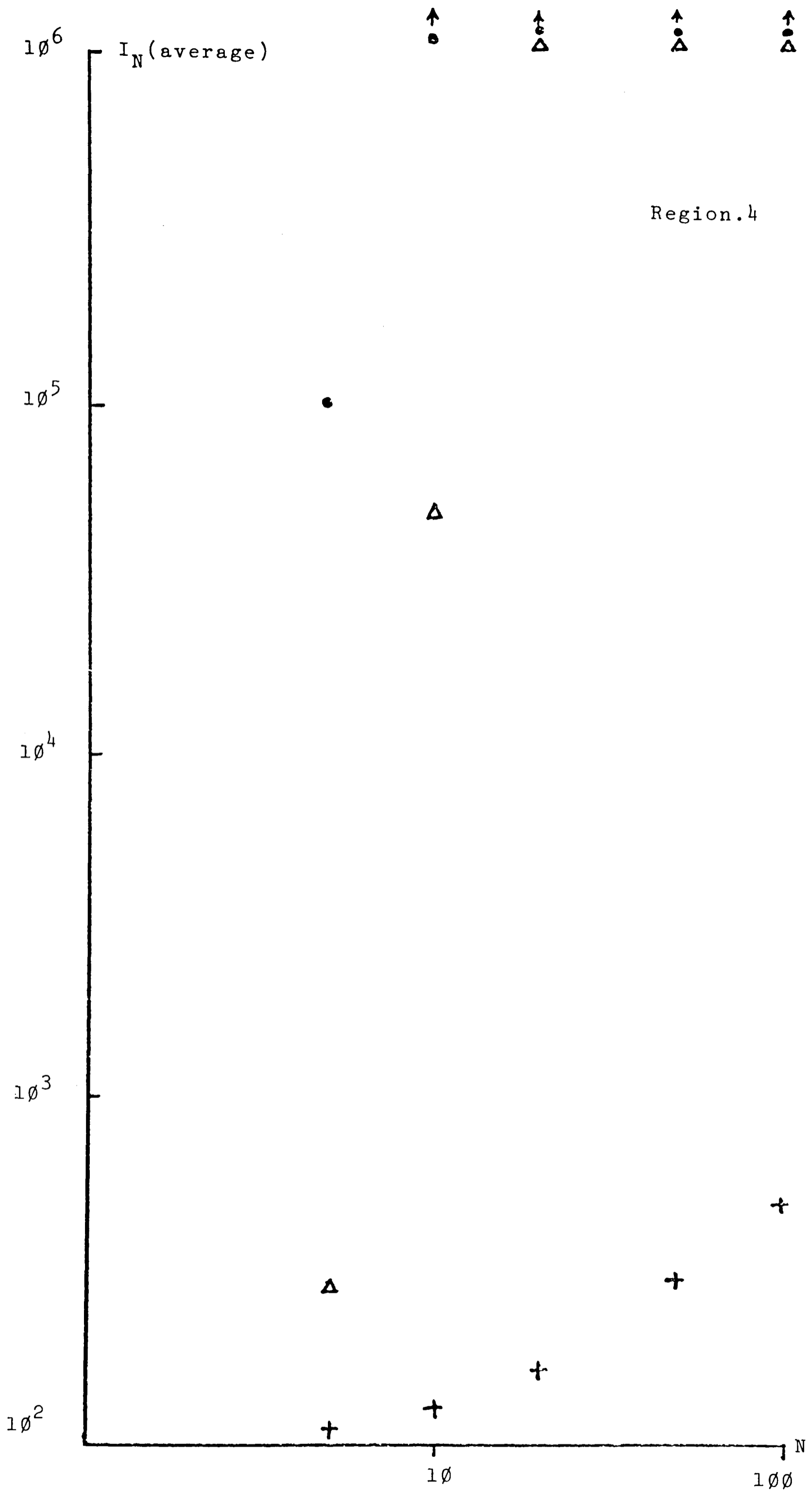


Fig.4.4 +CAP, •OLFO, ΔNOL

CHAPTER FIVE

PERFORMANCE OF OK CONTROLLER AND SELF-TUNING CONTROLLER

5.1 Introduction

This chapter contains the simulation results for OK controller, self-tuning controller, CAP controller and NOL controller. The process to be controlled is described by the equation

$$x_4(i+1) = x_1(i)x_4(i) + x_2(i)u(i) + x_3(i)y(i) \quad 2.2a$$

$$y(i) = x_4(i) + \xi(i) \quad 2.1b$$

with the parameters x_1 , x_2 , x_3 unknown. The parameters could further be constants or slowly time-varying or rapidly time-varying.

The simulation results showed that OK controller and self-tuning controller produces similar costs when

- i) unknown parameters of the controlled process are constants and occupy regions 1, 2, 3, 4, or
 - ii) unknown parameters are time-varying but occupy only region 1; and the OK controller produces less cost than self-tuning controller when
 - iii) unknown parameters are constants and occupy regions 5, 7, or
 - iv) unknown parameters are slowly time-varying and occupy regions 2, 3, 4, 5, 7, or
 - v) unknown parameters are rapidly time-varying and occupy regions 2, 3, 5, 7;
- and both controllers perform badly when

vi) unknown parameters occupy regions 6, 8, or

vii) unknown parameters are rapidly time-varying and occupy region 4.

5.2 Simulation Results

This section presents results of 192 simulations, consisting of eight different values of N (no. of stages), for each of three conditions of uncertainty in each of eight regions (fig. 3.2) in the unknown parameter space. Table 5.1 gives data used to specify each of the eight regions.

Region	$\hat{x}_1(0)$	$\hat{x}_2(0)$	$\hat{x}_3(0)$	$\sqrt{\sigma_{11}(0)}$	$\sqrt{\sigma_{22}(0)}$	$\sqrt{\sigma_{33}(0)}$
1	0.1	4.0	0.1	0.2	0.5	0.2
2	0.0	4.0	4.0	0.2	0.5	1.0
3	4.0	4.0	-4.0	0.2	0.5	0.2
4	4.0	4.0	4.0	1.0	0.5	1.0
5	0.1	0.2	0.1	0.2	0.5	0.2
6	0.0	0.2	4.0	0.2	0.5	1.0
7	4.0	0.2	-4.0	0.2	0.5	0.2
8	4.0	0.2	4.0	1.0	0.5	1.0

Table 5.1 Data specifying the eight regions of Fig. 3.2

The three conditions of uncertainty specify whether the unknown parameters are constants or slowly time-varying or rapidly time-varying and are obtained by assigning different values to g_j of equation 2.2b. Table 5.2 gives the g_j s specifying the three conditions of uncertainty.

(i) constant	$g_j = 1.0$
(ii) slowly time-varying	$g_j = 0.8$
(iii) rapidly time-varying	$g_j = 0.6$

Table 5.2 Data specifying conditons of uncertainty.

Data common to all simulations was as follows:

$$\hat{x}_4(0) = 10.0, \quad \sigma_{44}(0) = 1.0, \quad \sigma_{\xi} = 1.0, \quad q = 0.01$$

For self-tuning (ST) controller forgetting factor β was chosen as 0.99 for constant parameter case (condition (i) of Table 5.2). For time-varying parameter cases, when $|c(i)| \neq 1$, as described in section 2.9 two modifications to the basic self-tuning algorithm are possible. ST controller was simulated using both modifications and the forgetting factor used for modification (i) was $\beta = g$. Modification (i) produced larger costs than modification (ii) except for region 1 where both modifications produced very similar costs. Hence, where both modifications are possible, simulation results shown for ST controller are using modification (ii).

Figs. 5.1 to 5.24 show the simulation results. Figs. 5.1 to 5.8 are results when the unknown parameters are constants, figs. 5.9 to 5.16 are results when the unknown parameters are slowly time-varying and figs. 5.17 to 5.24 are results when the unknown parameters are rapidly time-varying. They are drawn to logarithmic scales and have the following features.

- 1) Figs. 5.1, 5.9, 5.17 show that in region 1 the costs achieved by the two adaptive controllers under all the three conditions of uncertainty are indistinguishable and very close to the lower bound achieved by the CAP controller. The cost achieved by the no learning controller under all the three conditions of uncertainty is close to that of the adaptive controllers. This indicates that almost any controller will perform well when the controlled process is in region 1 and the desired value of y is zero. Since the performance of the no learning controller is close to that

of the adaptive (learning) controllers, adaptation is only marginally beneficial in this region.

- 2) No learning controller performed very badly outside regions 1 and 5. It produced costs so large that they are not shown in Figs. 5.2-5.4, 5.6-5.8, 5.10-5.12, 5.14-5.16, 5.18-5.20, 5.22-5.24. This indicates that a more sophisticated controller like adaptive controller is need when the process is outside regions 1 and 5.
- 3) In regions 2 and 3 both adaptive controllers performed equally well and produced costs close to the lower bound of the CAP controller when the uncertain parameters were constants (Figs. 5.2, 5.3). But when the parameters were time-varying (conditions (ii) & (iii) of Table 5.2) ST controller produced higher costs (Figs. 5.10, 5.11, 5.18, 5.19) than OK controller. This is because the self-tuning control law is derived assuming the controller parameter estimates have all converged to their true values which is only possible when the parameters are constants.
- 4) Fig. 5.4 shows that in region 4 under uncertainty condition (i), both ST and OK controller perform equally well. But, as shown by fig. 5.12, under conditon (ii) the ST controller performance is totally unsatisfactory and the OK controller produces costs much greater (about 10 times) than would be produced by the CAP controller. Under condition (iii) both the adaptive controllers performed very badly. This is shown in fig. 5.20.
- 5) The scatter of the OK points for condition (ii) and the very bad performance for condition (iii) suggest that the average cost of equation 2.3 may not be well defined for processes with time-varying coefficients in region 4 under OK controller. This could

be resulting from an unstable closed loop system. The question then arises whether the optimal adaptive controller would result in a stable closed loop system? If not, the solution to the functional recurrence equation 2.7 might be non-existent for these processes. Finding an answer to this question is a subject for future research.

- 6) The performance of ST controller in regions 5, 6, 7 and 8 was so bad for all the three conditions of uncertainty that it could not be shown in figs. 5.5-5.8, 5.13-5.16, 5.21-5.24. The reason for the bad performance is as follows: In these regions the sign of the control gain parameter x_2 (or b) and hence the self-tuning controller parameter θ_2 (equation 2.29) is not known a priori. So it is inevitable that the fixed value of $\hat{\theta}_2(0)$ will violate the convergence and stability requirement given in section 2.9 for certain runs, thus producing a very bad performance.
- 7) OK controller performed well in regions 5 and 7 for all the three conditions of uncertainty. Its performance in regions 5 and 7 is similar to that in regions 1 and 3; thus implying that prior knowledge about the sign of x_2 (or b) is not an important factor as long as $|x_1 + x_3|$ (or $|a|$) < 1 .
- 8) In regions 6, 8 even the CAP controller performed very badly for $q = 0.01$. This was because the CAP controller was producing an unstable closed loop system for several runs. The control weight q , was then changed to zero and the results for this $q (=0)$ are shown in figs. 5.6, 5.8, 5.14, 5.16, 5.22, 5.24. ST controller and OK controller still performed very badly and produced huge costs. The reason for the bad performance of ST controller is the same as given in (6) above. The bad performance of the OK controller is due to stochastic instabilities which have been reported

elsewhere (68, 69) and described as 'escapes'. In (68, 69) it is further shown for a simple example that even optimal adaptive controller produces 'escapes'. So it may be that adaptive control is impossible when the process occupies these regions.

- 9) The logarithmic scales of figs. 5.1 to 5.24 are such that convergence (as N increases) of non-CAP costs towards CAP costs indicates convergence of non-CAP controllers towards CAP controller. With constant unknown parameters OK controller and ST controller both have this convergence except for the special cases discussed in (6) and (8) above. Convergence of OK controller is never slower than, sometimes (in region 4) slightly faster than the convergence of ST controller.
- 10) The performance of CAP controller indicate that the average cost achieved by it depends only on $|x_1|$ and not on x_2 or x_3 . For the case $q = 0$ it can be shown using standard analysis (37) that for large N the average cost J_N achieved by the CAP controller is:

$$\begin{aligned}
 J_N &\rightarrow N \cdot \sigma_\xi && \text{if } |x_1| \leq 1 \\
 J_N &\rightarrow x_1^2 N \sigma_\xi && \text{if } |x_1| > 1.
 \end{aligned}$$

- 11) Finally the ST controller took only half as much computing time as the OK controller for any trial. This is because i) the ST controller requires estimates only of the controller parameters where as the OK controller requires estimates of the process parameters and of the states, and ii) the ST control law is simpler than the OK control law.

The computing time taken by CAP, ST and OK controllers are of the order 1:3:6.

Fig. 5.25 shows distribution of costs when $N=100$, process parameters constants and in region 2. Distribution of costs for other regions also had similar shape; an initial hump and a long tail. Similar cost distributions are also reported in (8, 9).

5.3 Conclusions

The simulation results show that the performance of OK controller is never worse than and sometimes appreciably better than that of ST controller. The reasons for this could be that the OK controller is based on a more accurate approximation to the underlying non-linear process equations (linearisation in the extended Kalman filter of section 2.4 rather than linearisation by assuming convergence as in section 2.9), and that OK controller is cautious where as the ST controller is certainty-equivalent (or naive) (12).

It has not been possible to obtain theoretical proofs for convergence of OK controller to CAP controller. However some theoretical proofs are obtained in chapter 6 showing convergence of OK controller to a linear constant parameter controller.

The advantages ST controller has over OK controller are 1) lesser computing requirement and 2) simpler control law. For a process of order n and delay k the ST controller estimates only $2n + k - 1$ controller parameters (48) where as the OK controller estimates $3n$ process parameters and $n+k-1$ states[†]. As the order of the process becomes large, computational requirement of OK controller would become excessive and the difference in the requirement between OK and ST controller would become significant. Computational requirement of OK controller may be reduced to some extent by replacing the extended Kalman filter by a more economical suboptimal estimator. This has been investigated in Chapter 7.

[†] See Appendix 3

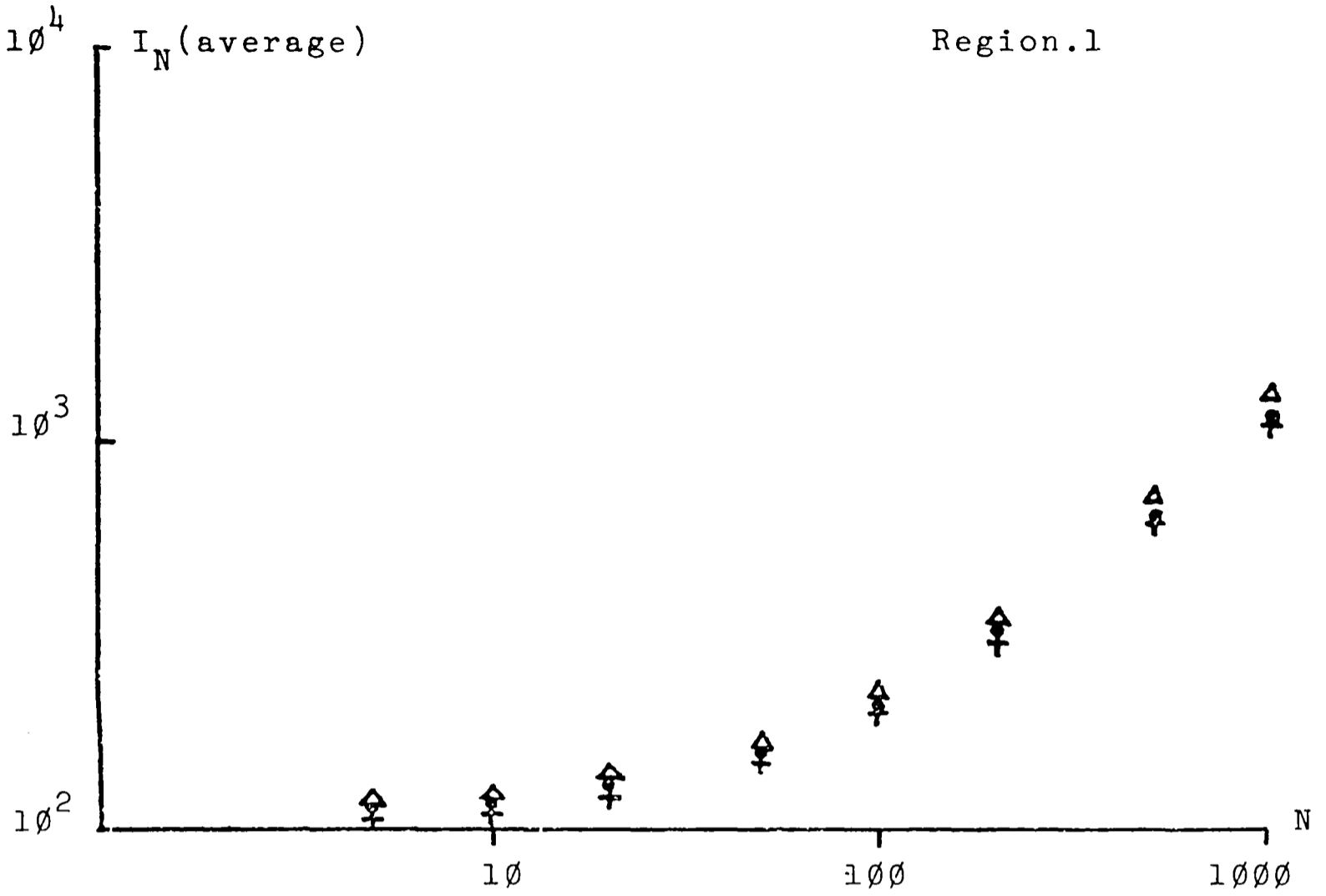


Fig.5.1 +CAP, •ST & OK (indistinguishable), Δ NOL.

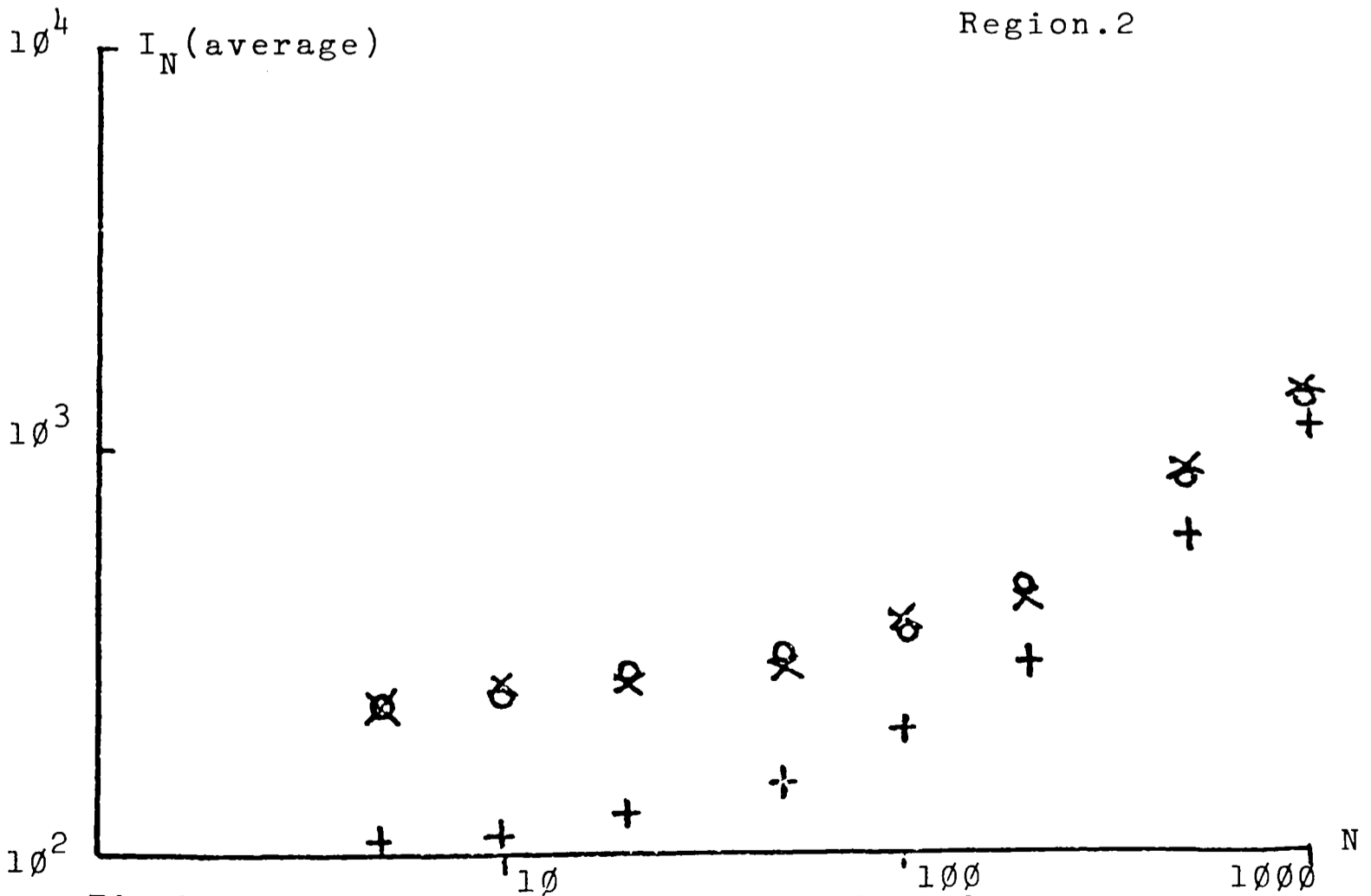


Fig.5.2 +CAP, oST, xOK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters constants.

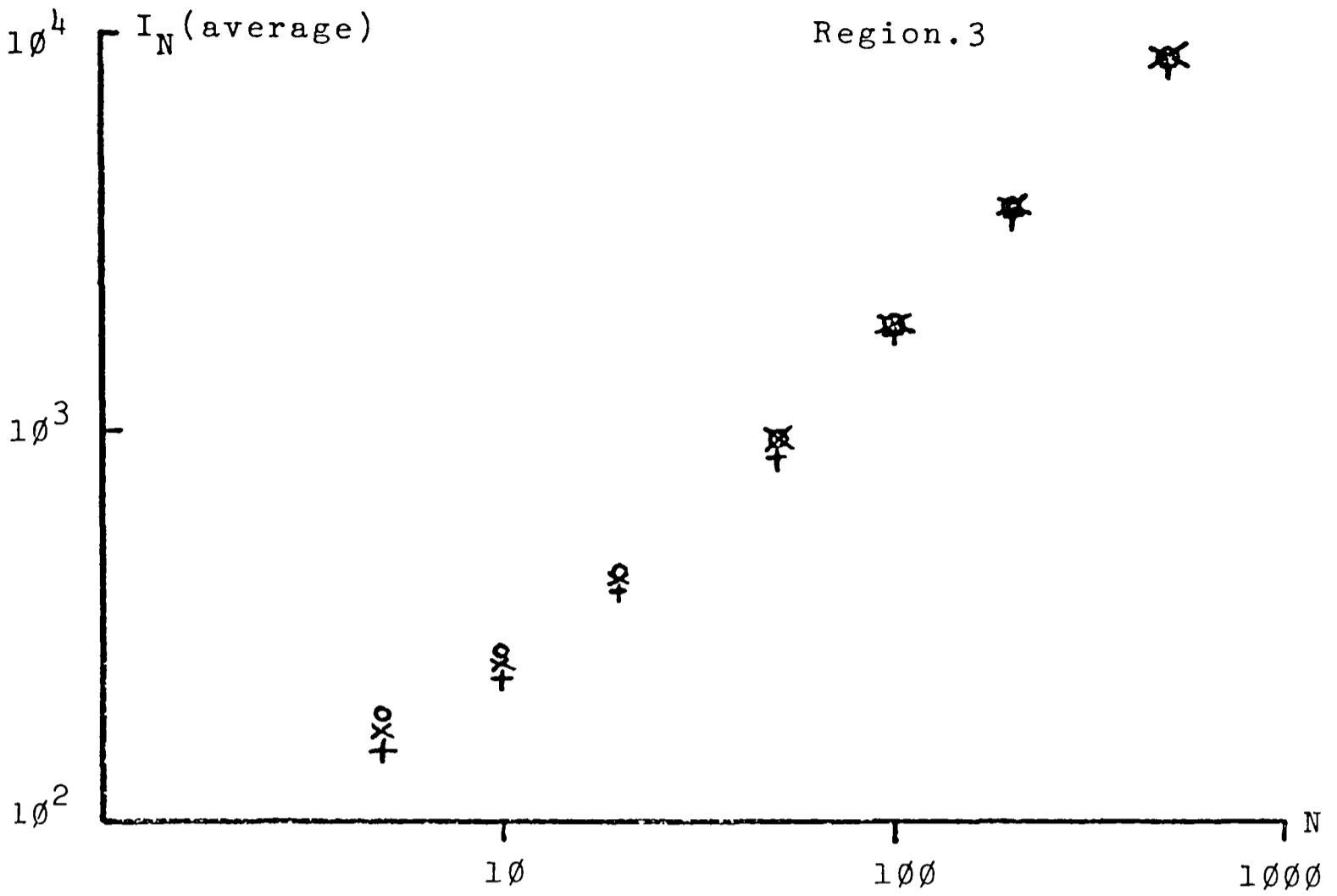


Fig.5.3 + CAP, o ST, x OK; NOL performed very badly.

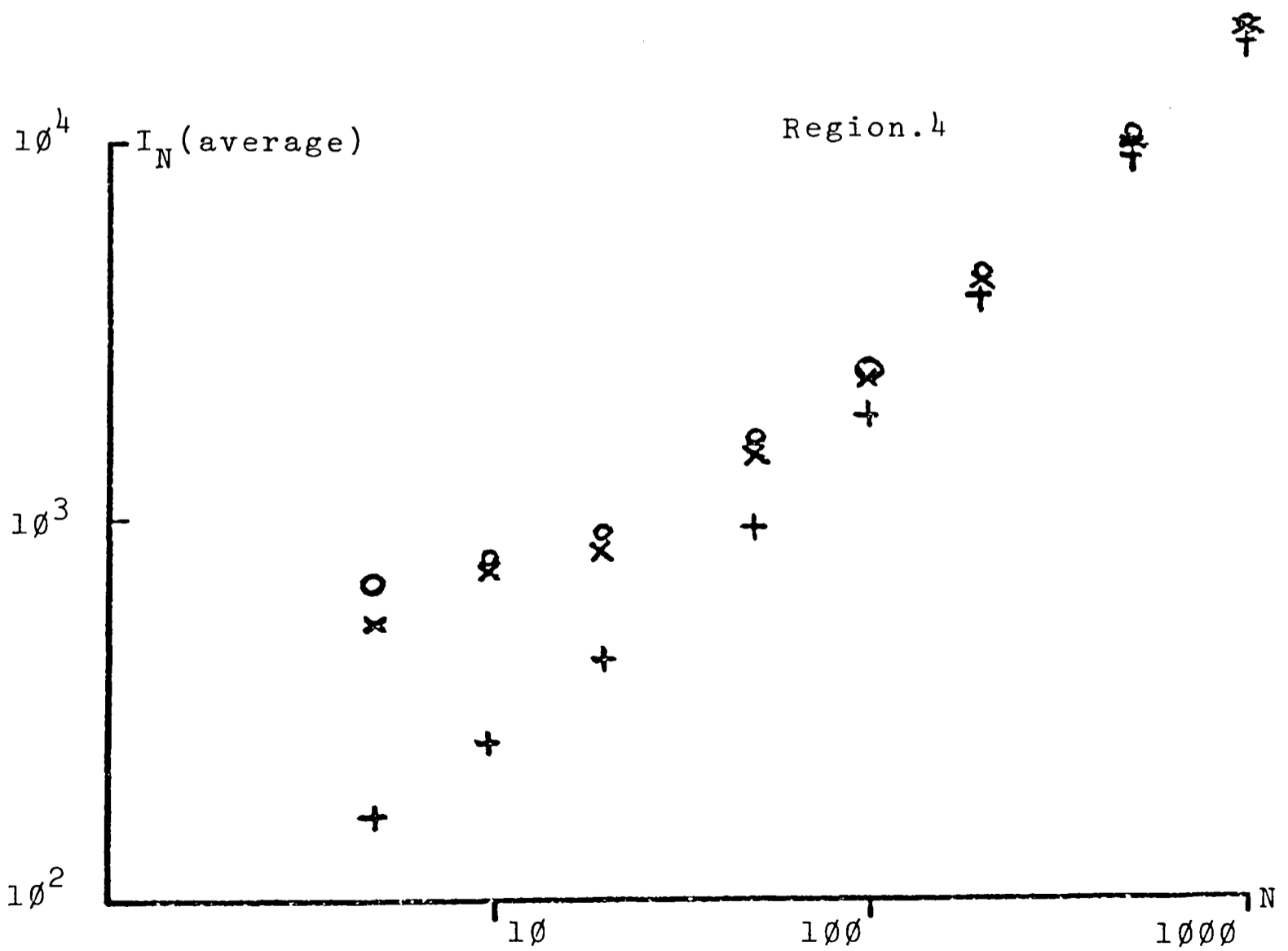


Fig.5.4 + CAP, o ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters constants.

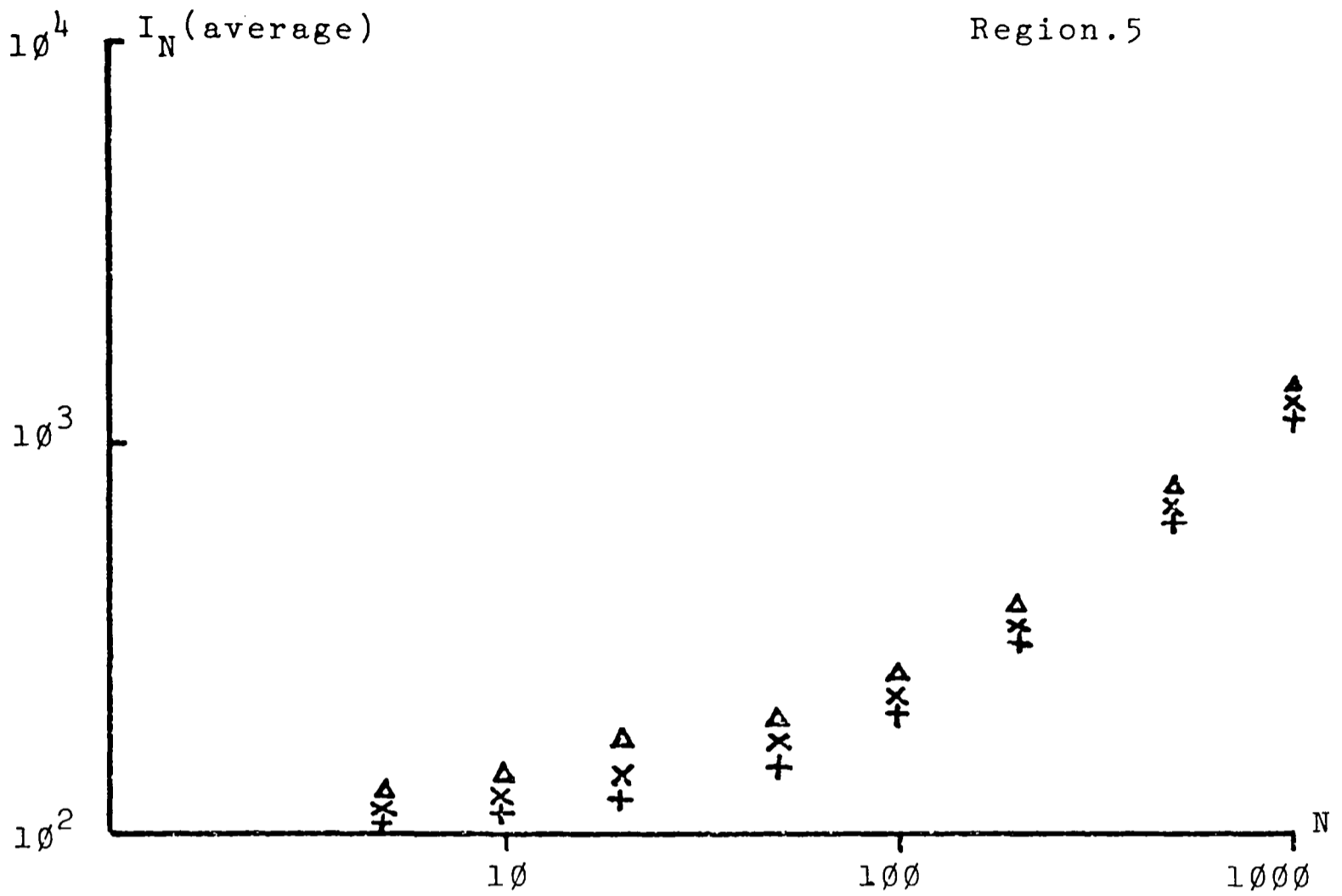


Fig.5.5 + CAP, x OK, Δ NOL; ST performed very badly.

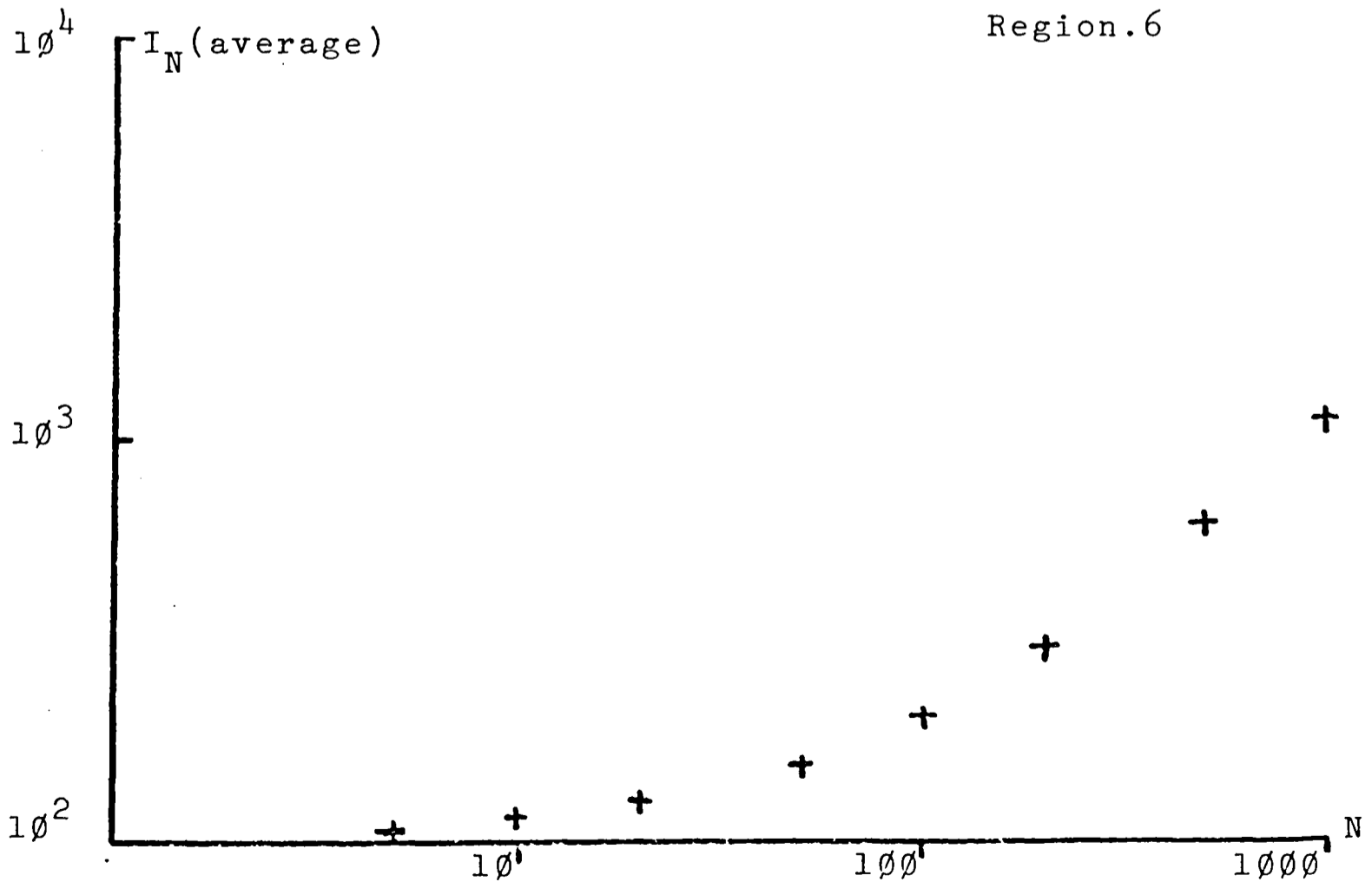


Fig.5.6 + CAP; ST, OK, NOL performed very badly.

Average cost Vs Number of stages; unknown parameters constants.

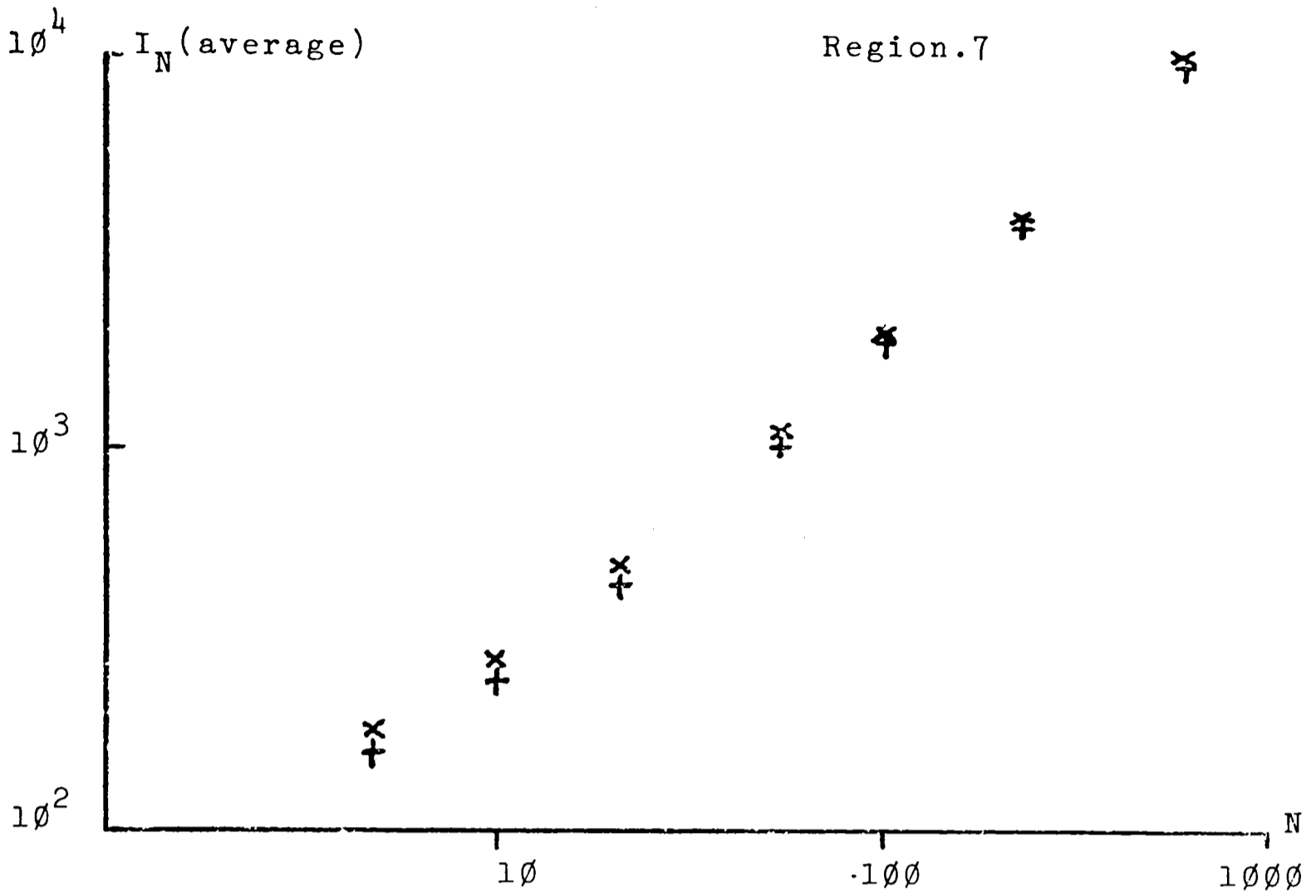


Fig.5.7 + CAP, x OK; ST, NOL performed very badly.

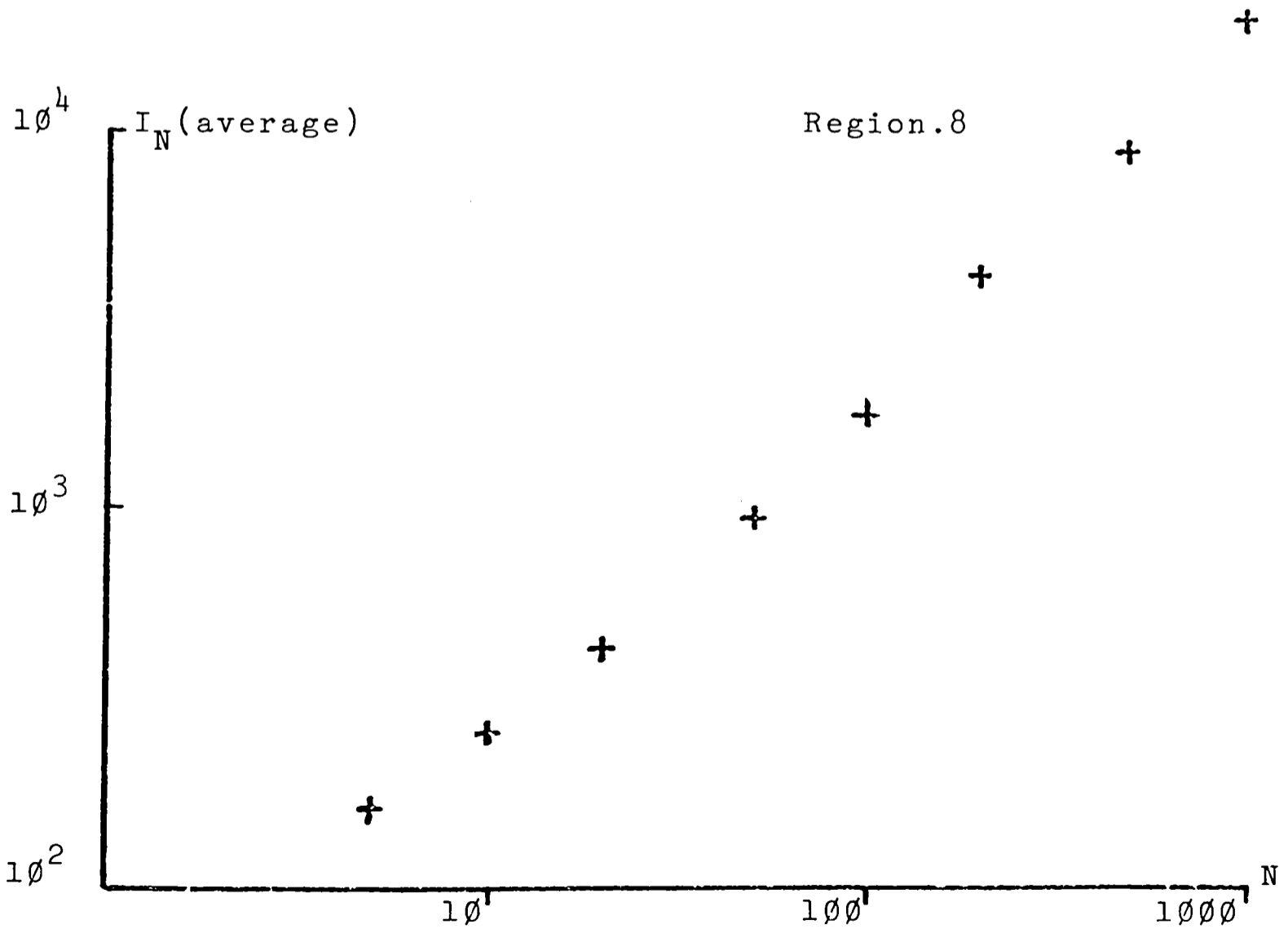


Fig.5.8 + CAP; ST, OK, NOL performed very badly.

Average cost Vs Number of stages; unknown parameters constants.

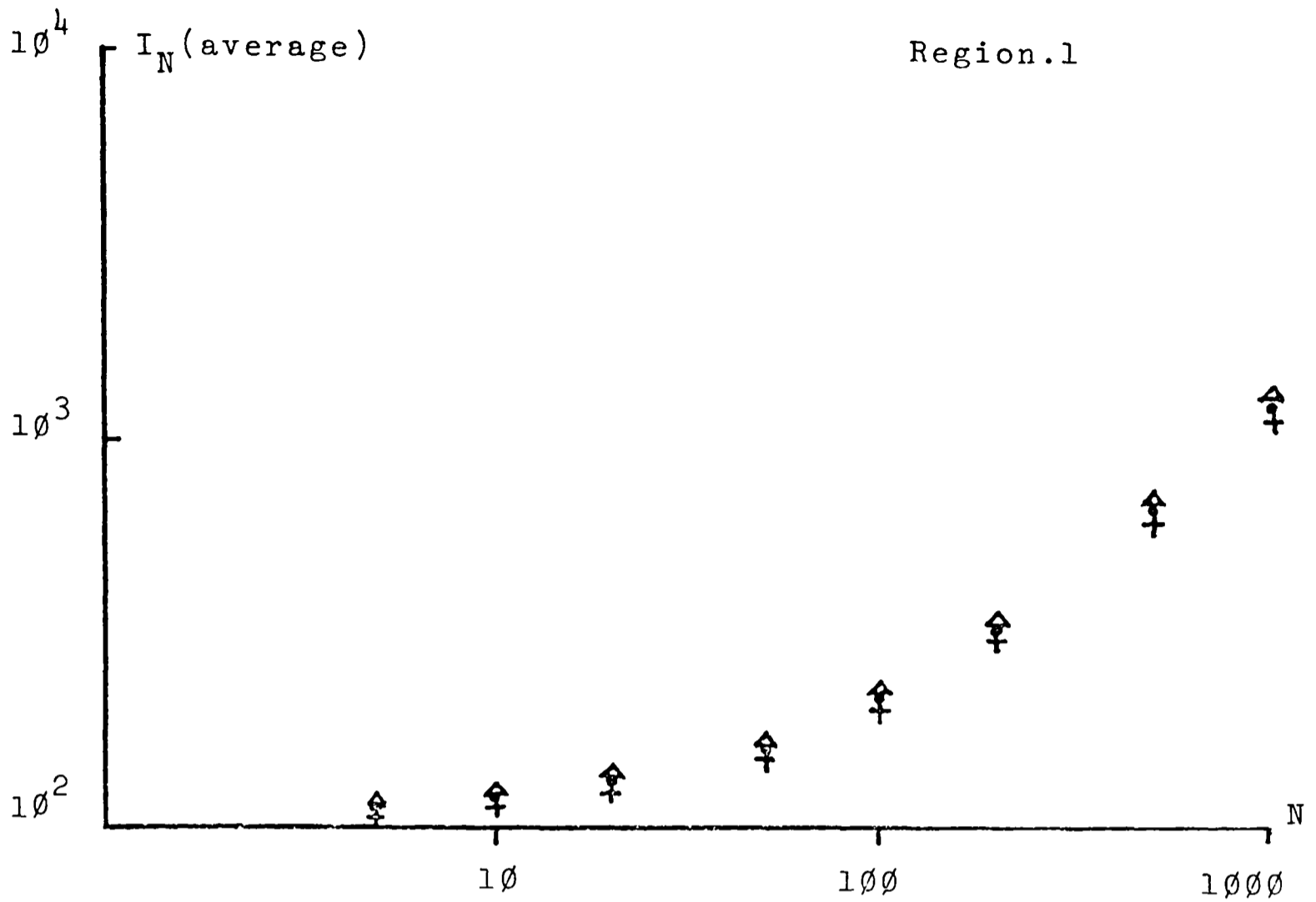


Fig.5.9 + CAP, • ST & OK (indistinguishable), Δ NOL.

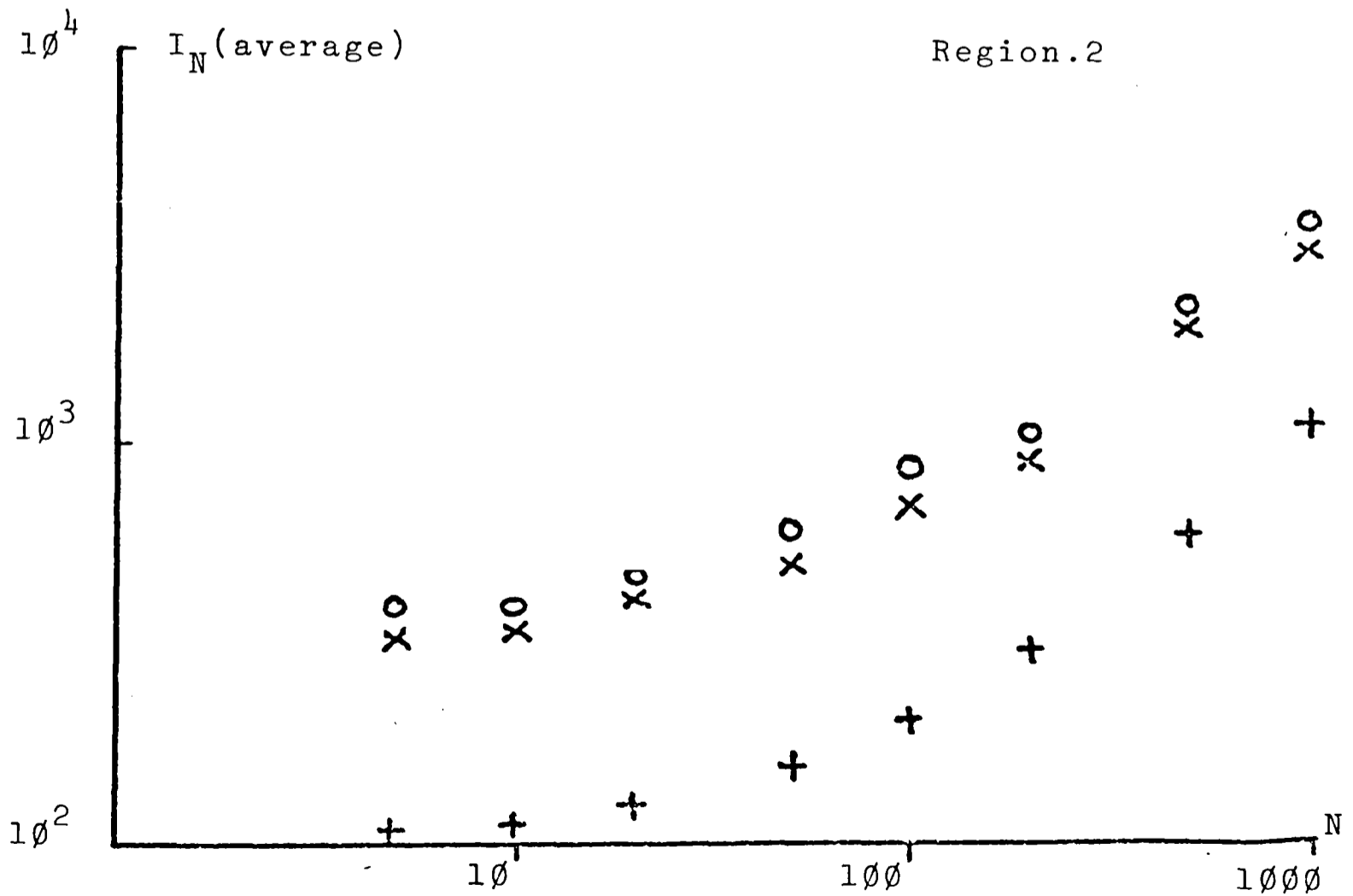


Fig.5.10 + CAP, o ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters slowly time-varying.

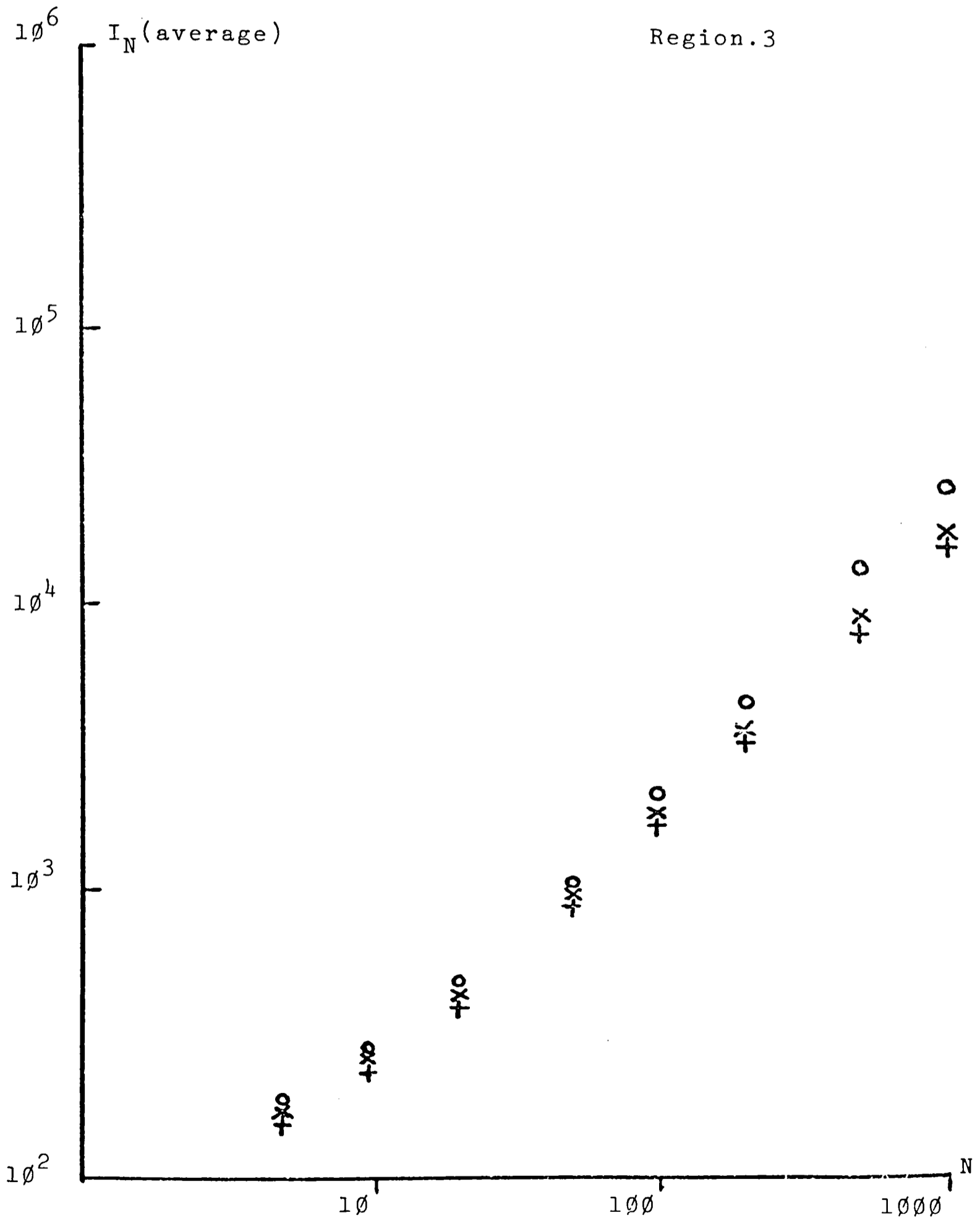


Fig.5.11 + CAP, o ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters slowly time-varying.

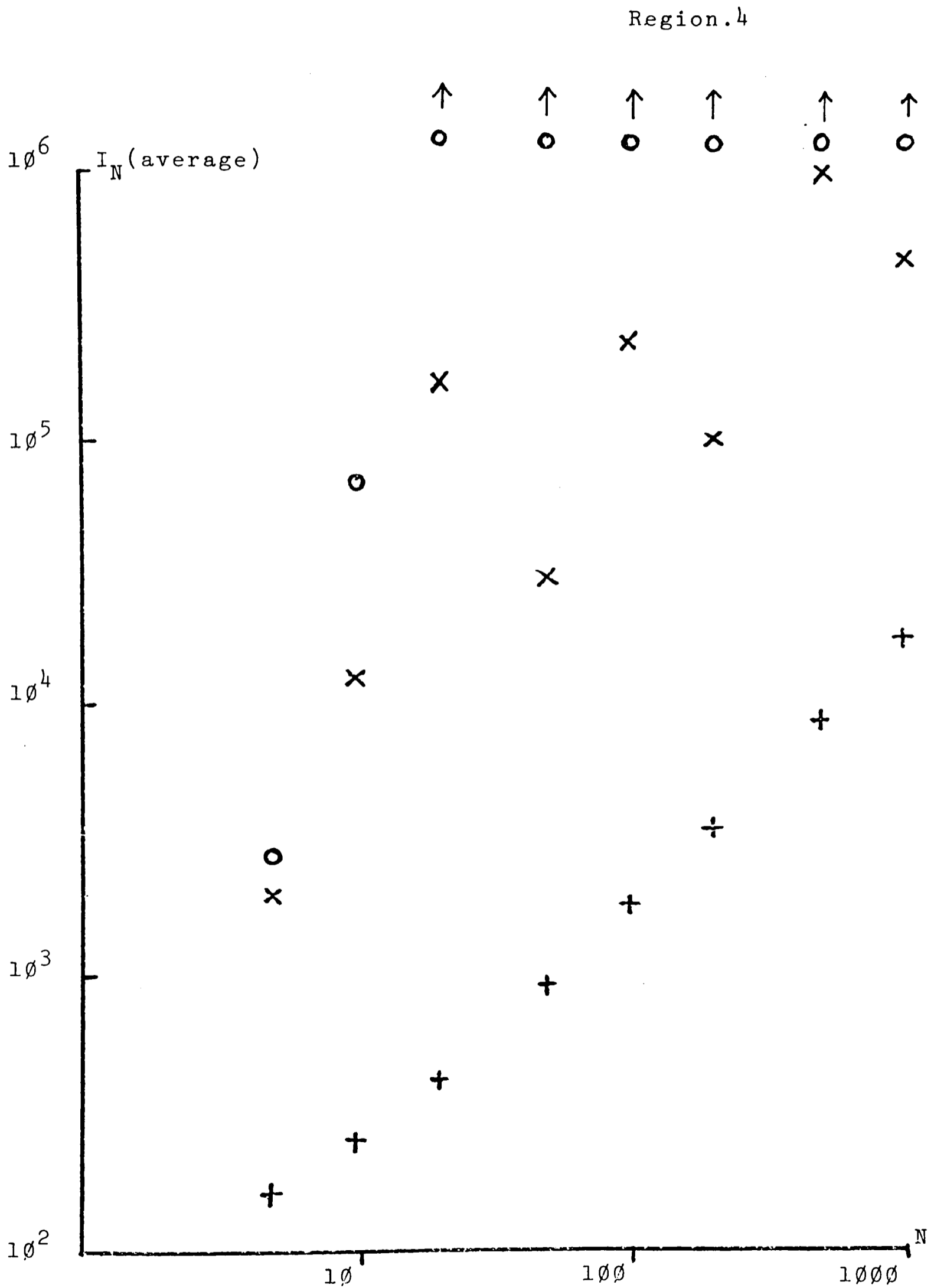


Fig.5.12 + CAP, o ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters slowly time-varying.

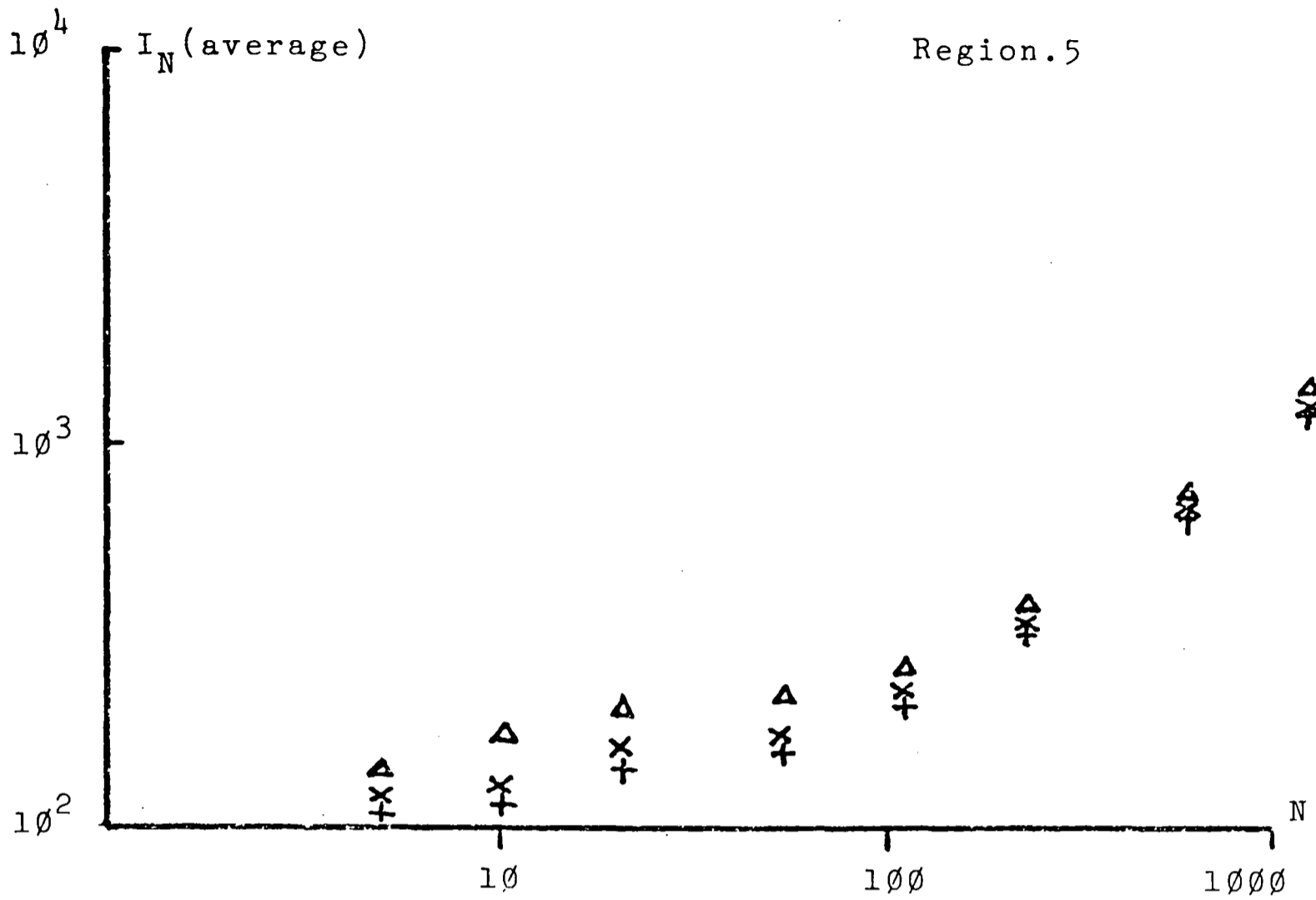


Fig.5.13 + CAP, x OK, Δ NOL; ST performed very badly.

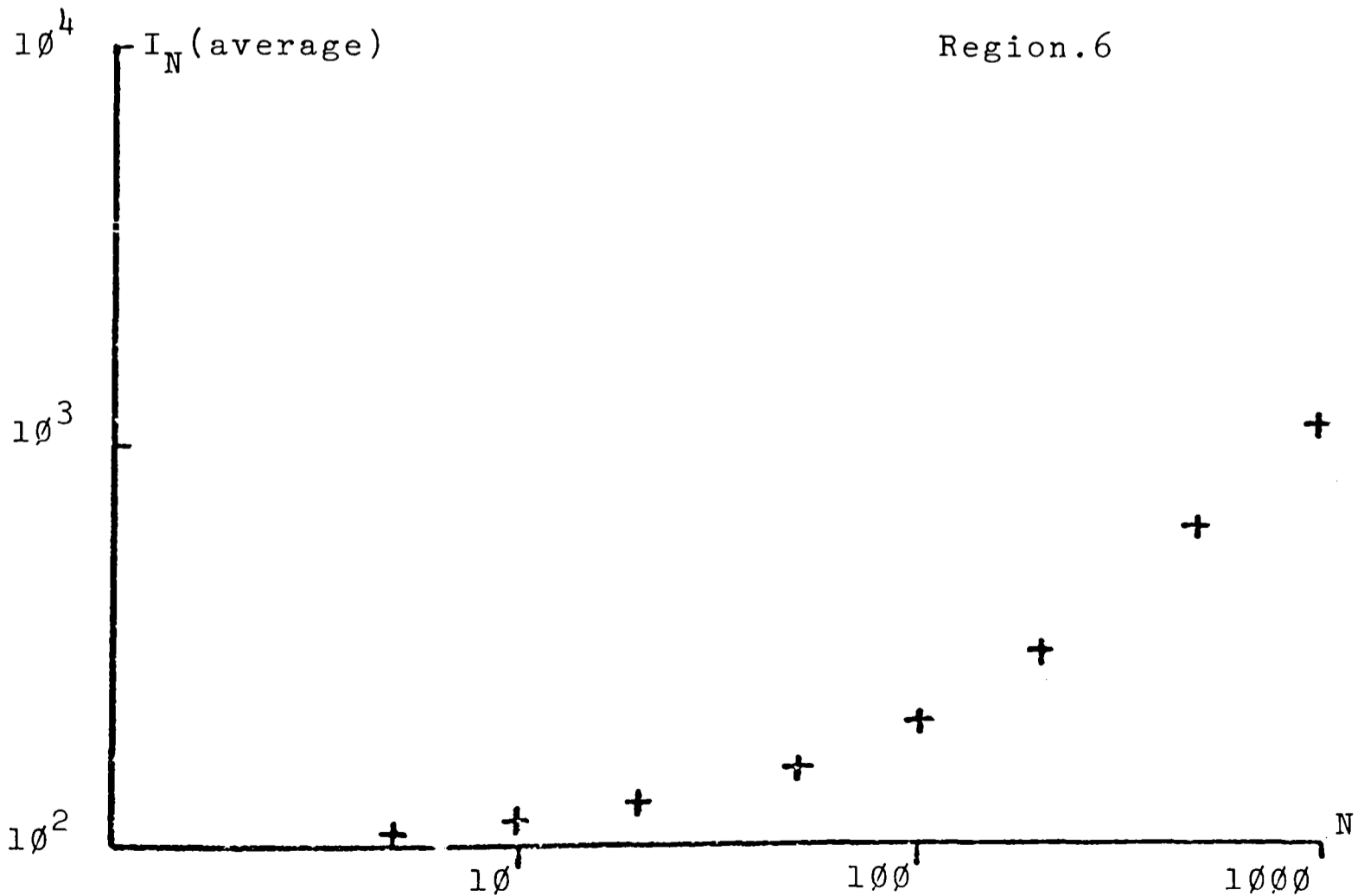


Fig.5.14 + CAP; ST, OK, NOL performed very badly.

Average cost Vs Number of stages; unknown parameters slowly time-varying.

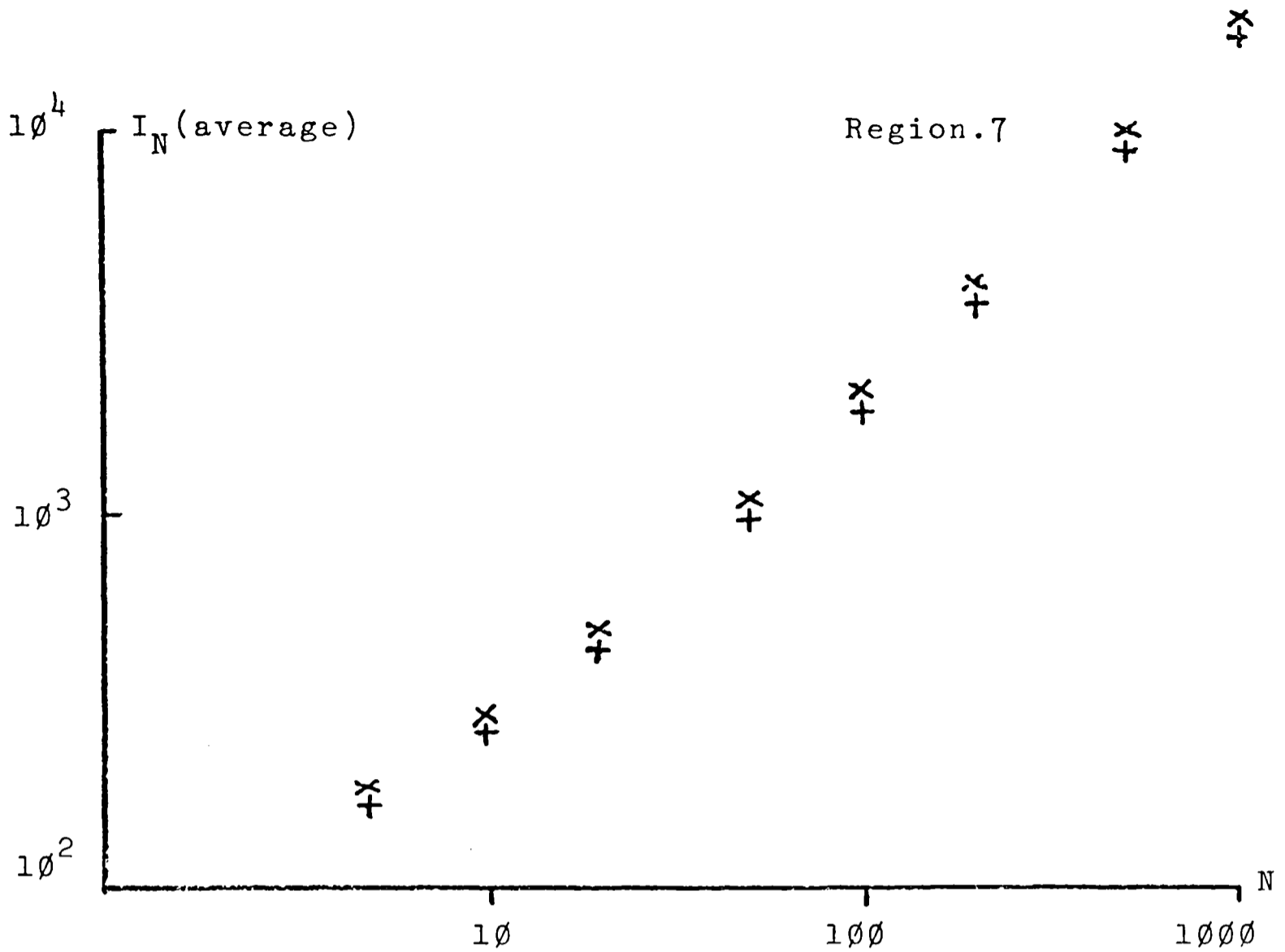


Fig.5.15 +CAP, xOK; ST, NOL performed very badly.

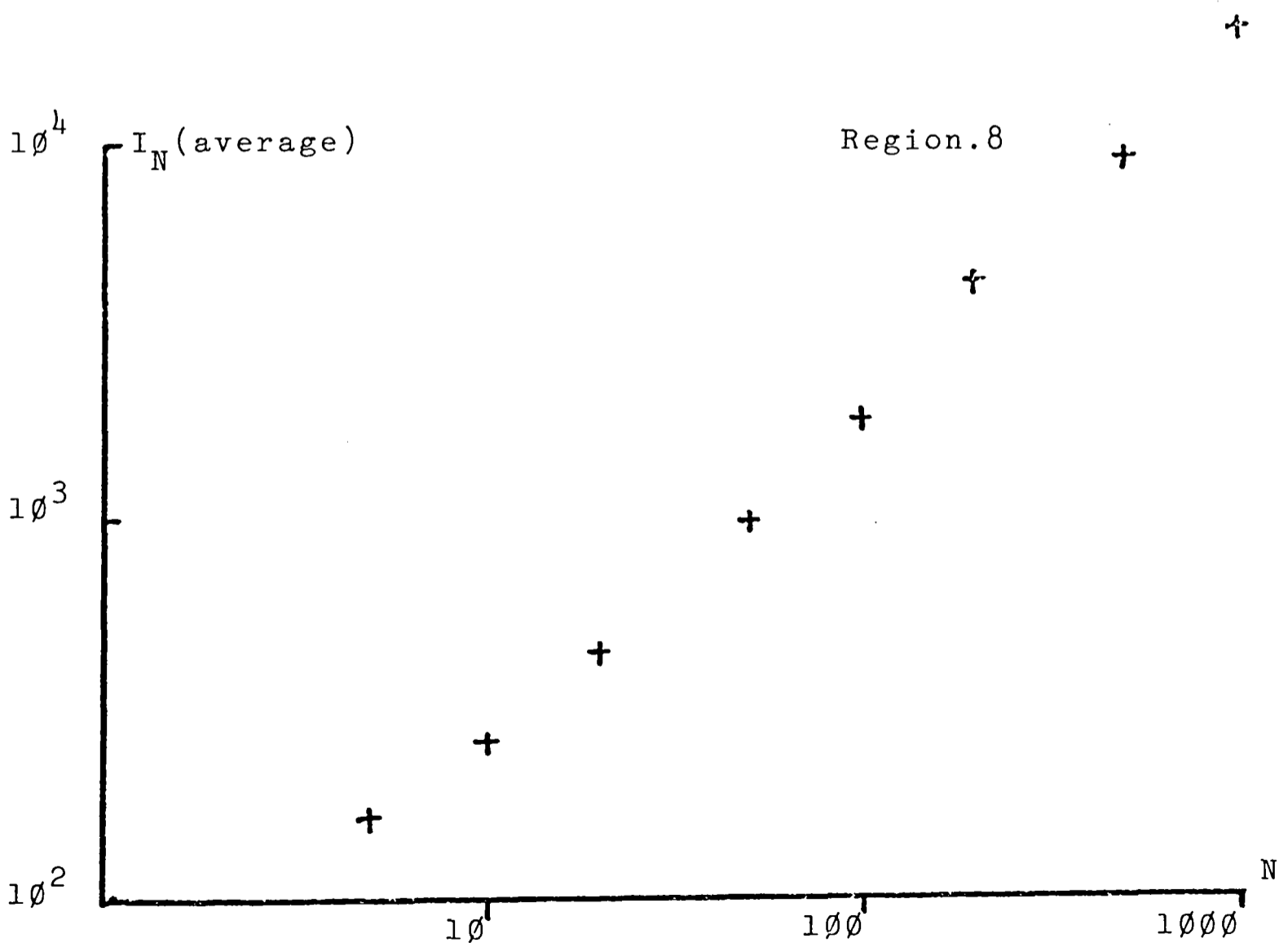


Fig.5.16 +CAP; ST, OK, NOL performed very badly.

Average cost Vs Number of stages; unknown parameters slowly time-varying.

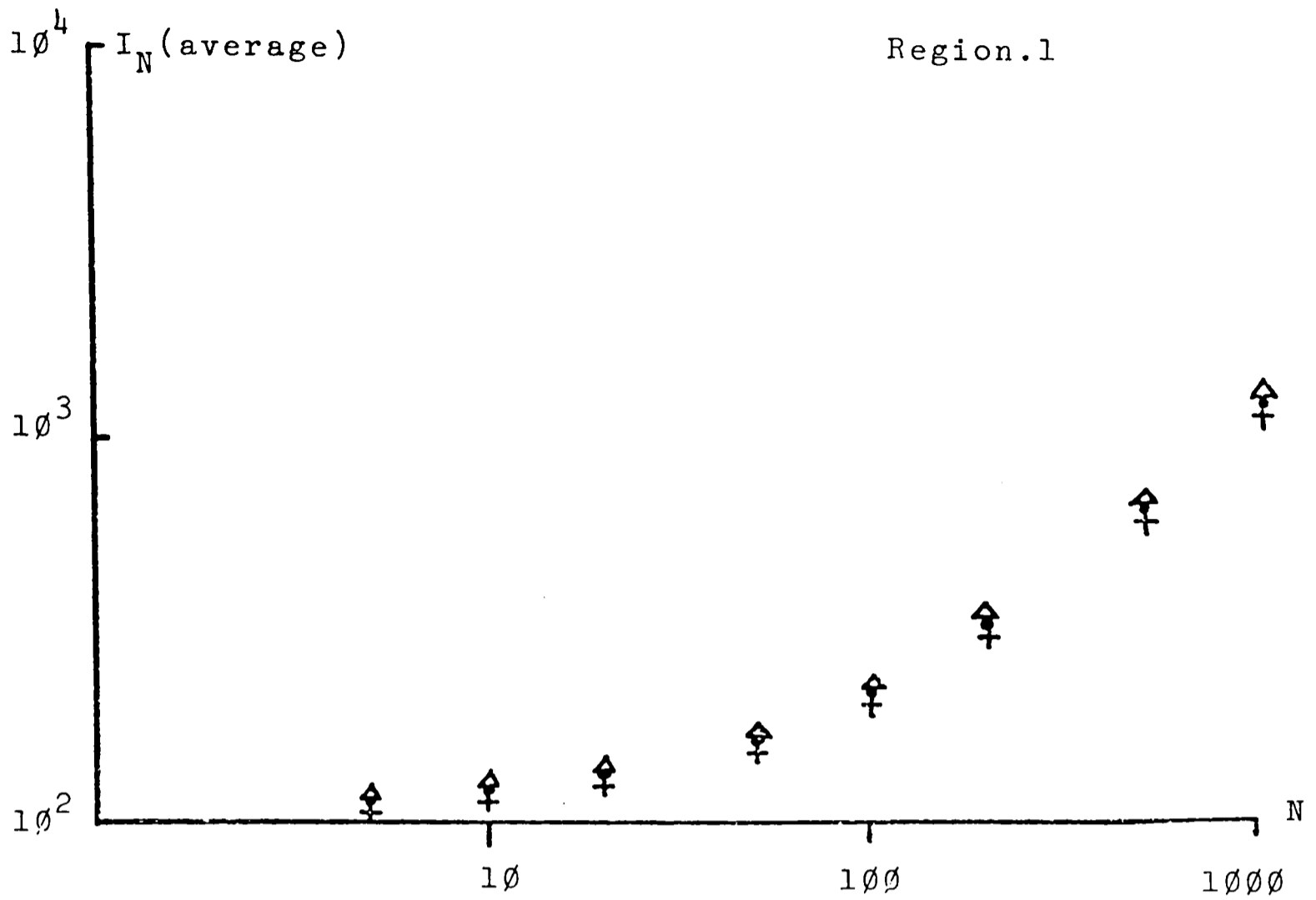


Fig.5.17 + CAP, • ST & OK (indistinguishable), Δ NOL.

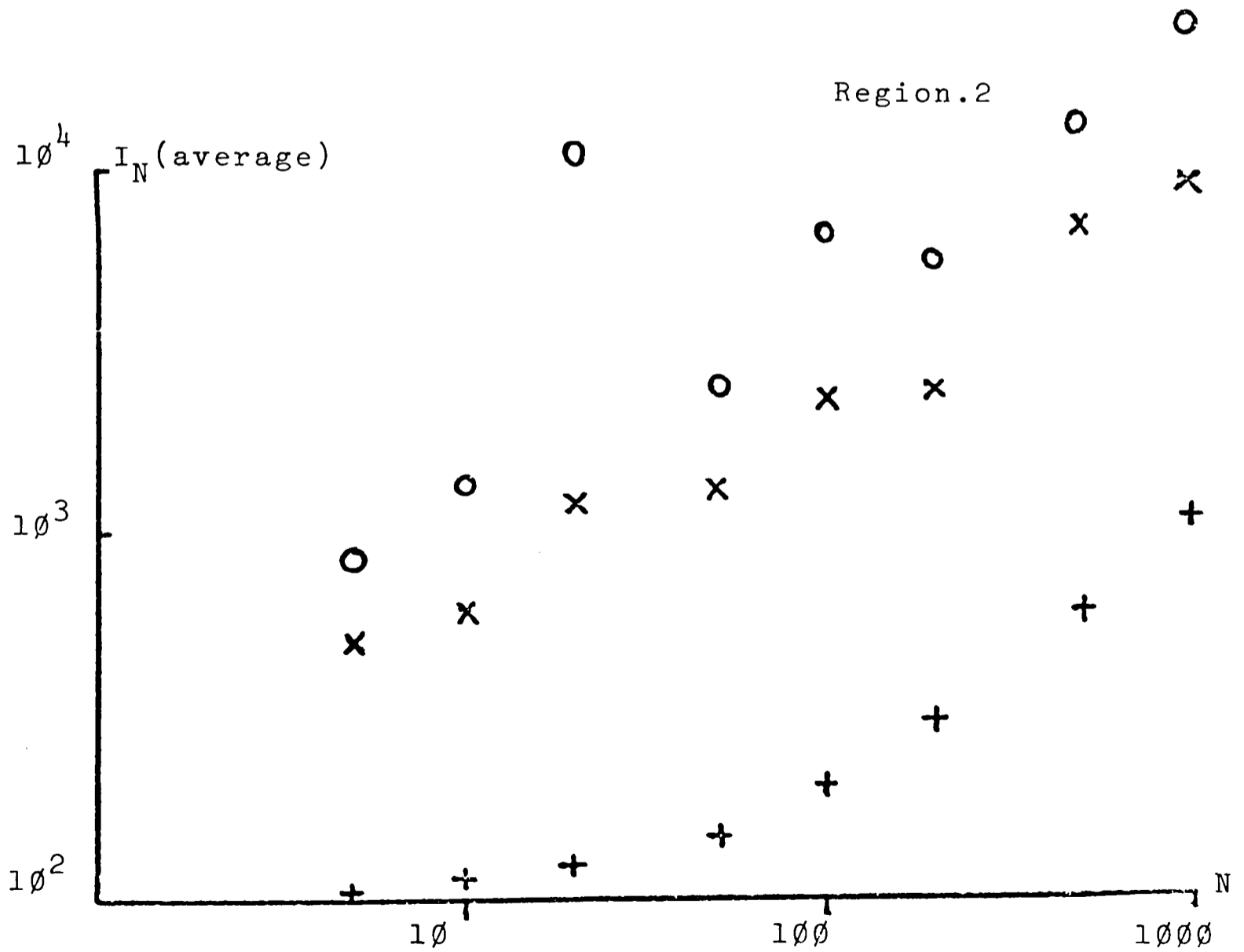


Fig.5.18 + CAP, ○ ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters rapidly time-varying.

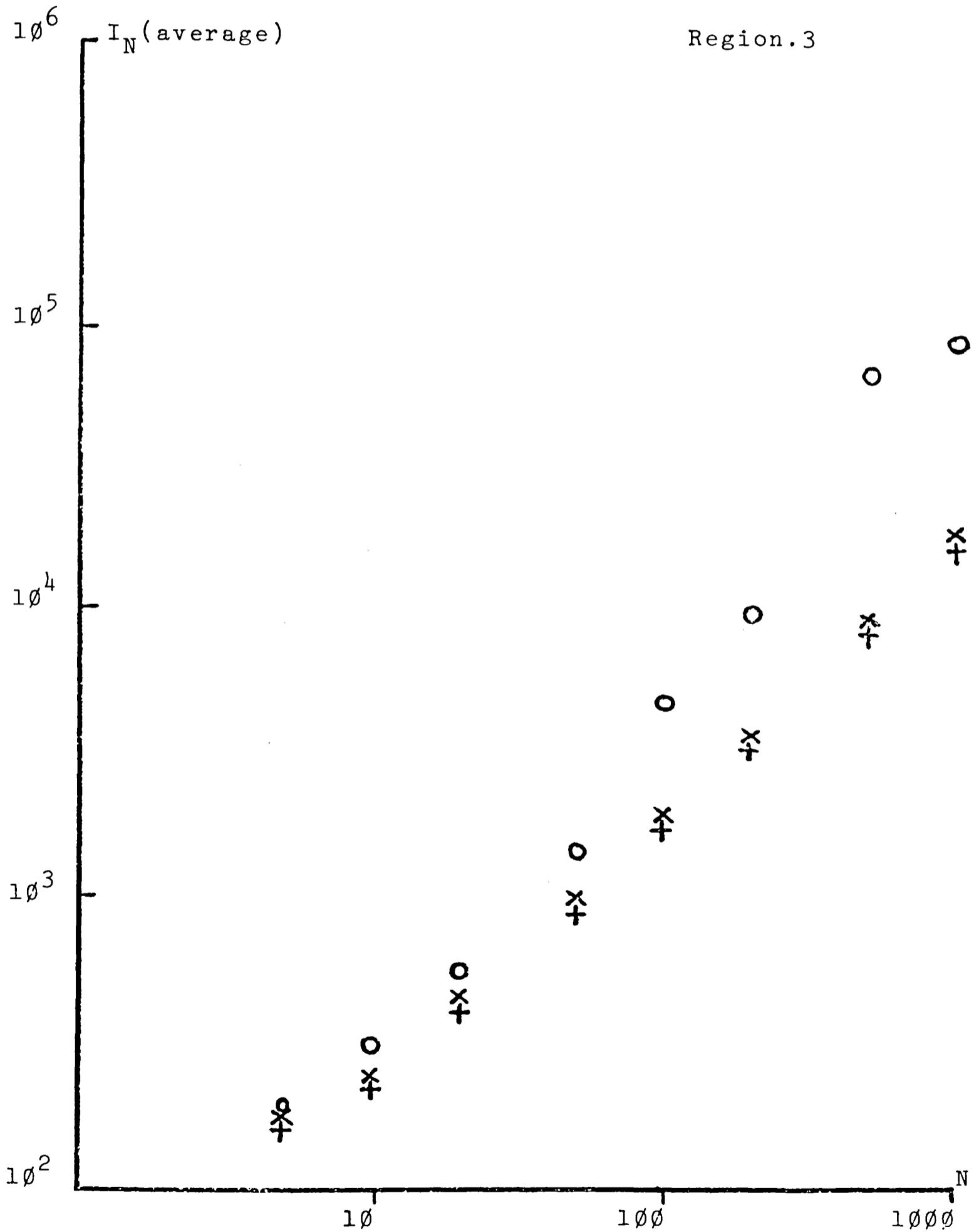


Fig. 5.19 + CAP, o ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters rapidly time-varying.

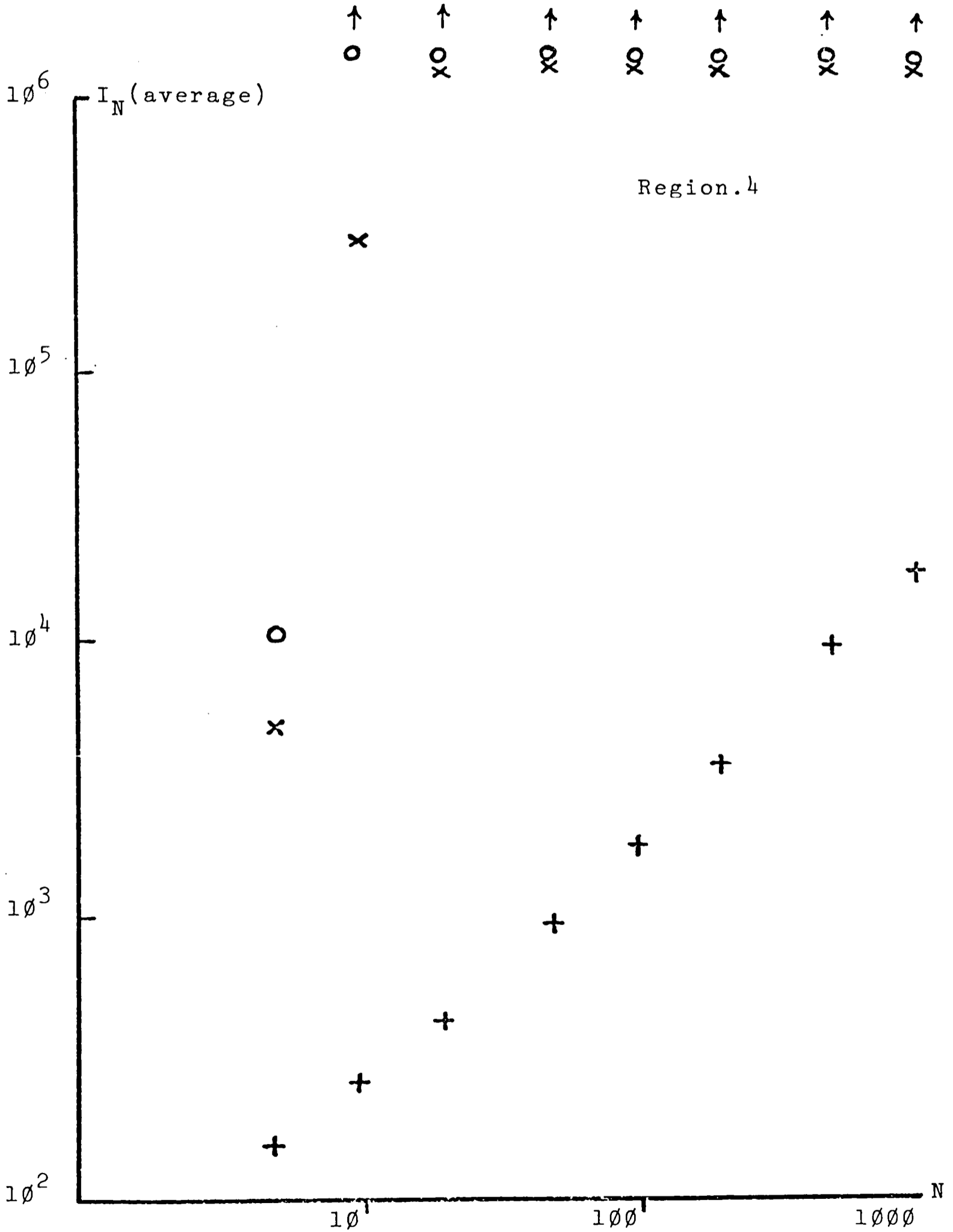


Fig.5.20 + CAP, o ST, x OK; NOL performed very badly.

Average cost Vs Number of stages; unknown parameters rapidly time-varying.

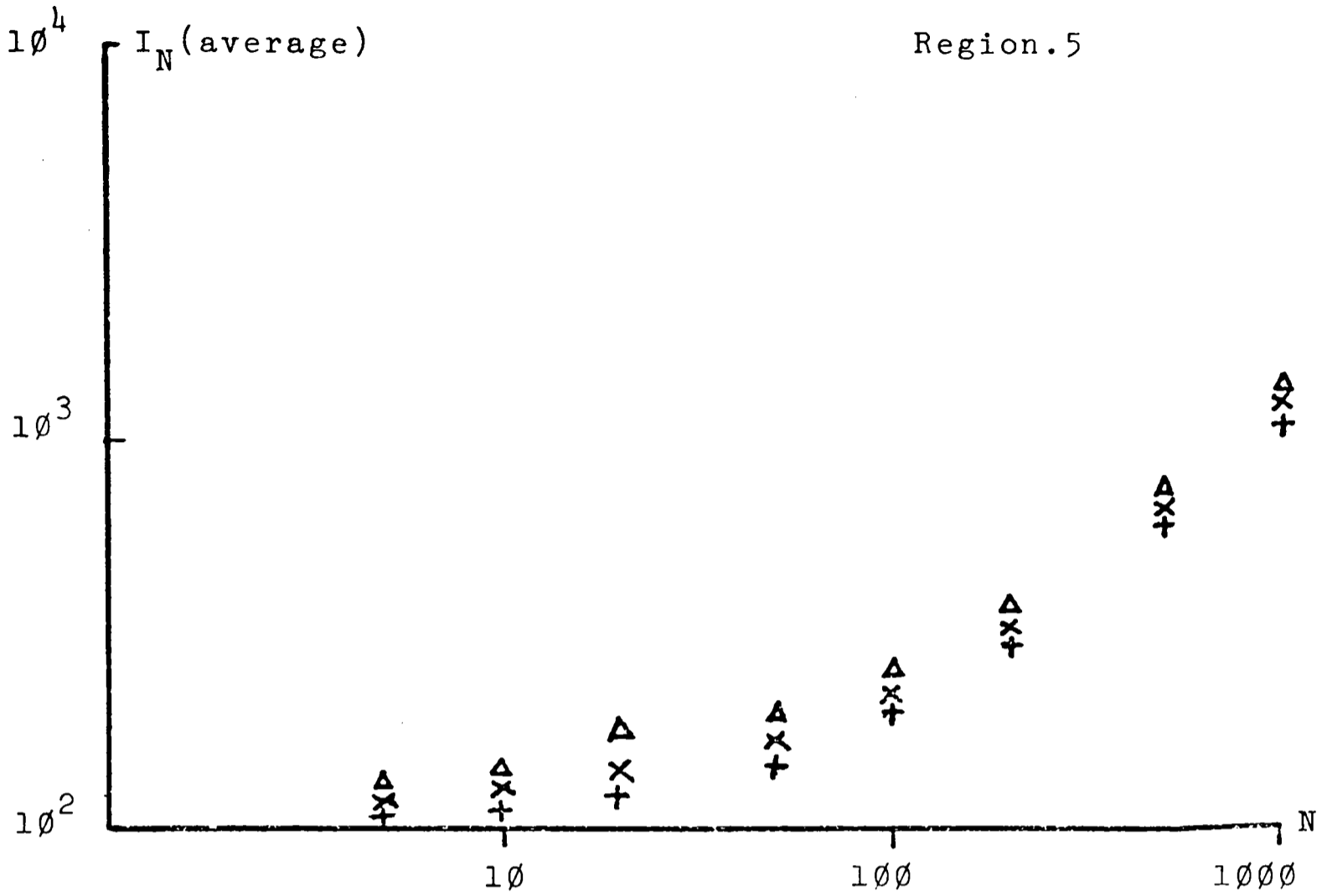


Fig.5.21 + CAP, x OK, Δ NOL; ST performed very badly.

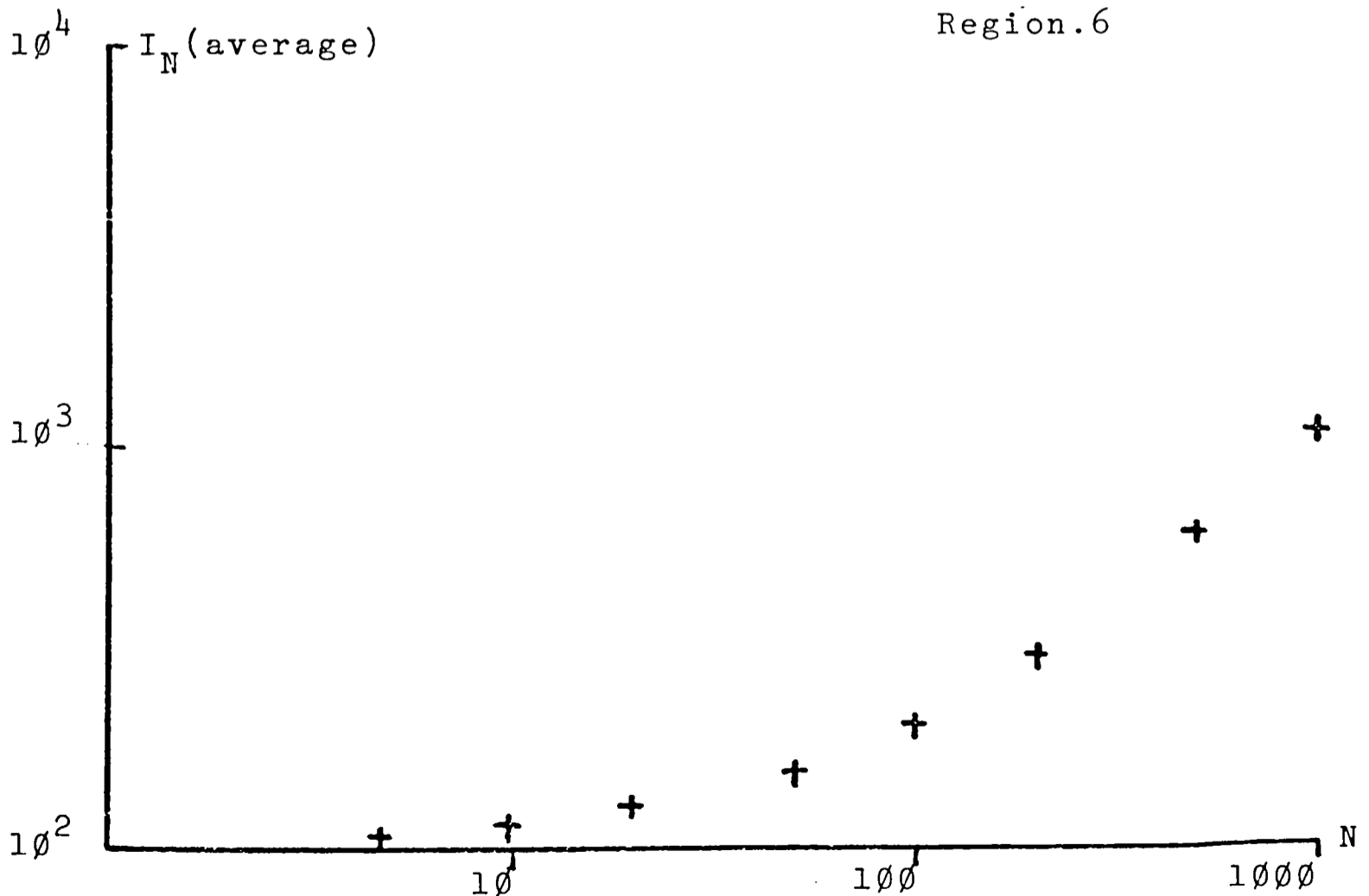


Fig.5.22 + CAP; ST, OK, NOL performed very badly.

Average cost Vs Number of stages; unknown parameters rapidly time-varying.

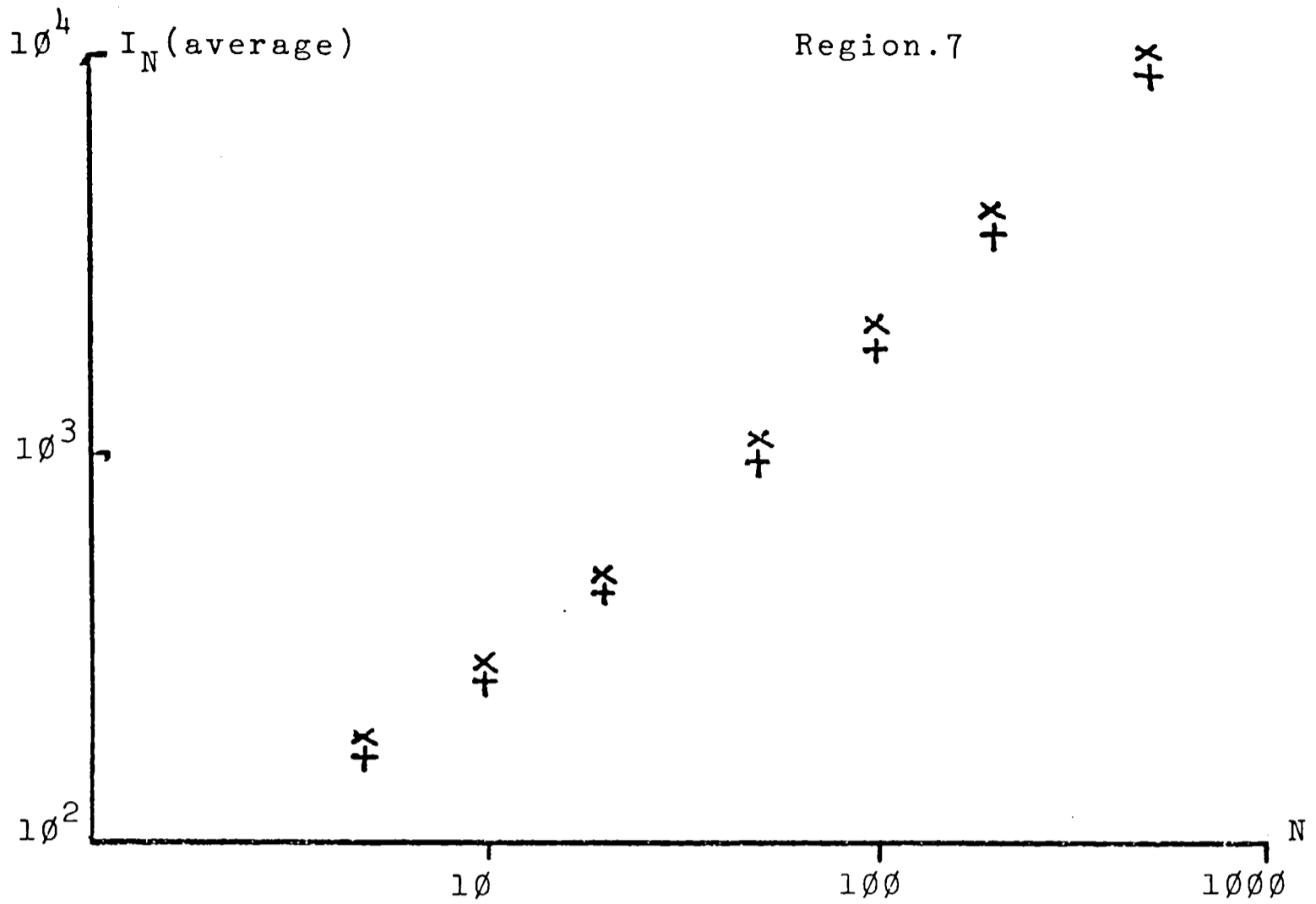


Fig.5.23 + CAP, x OK; ST, NOL performed very badly.

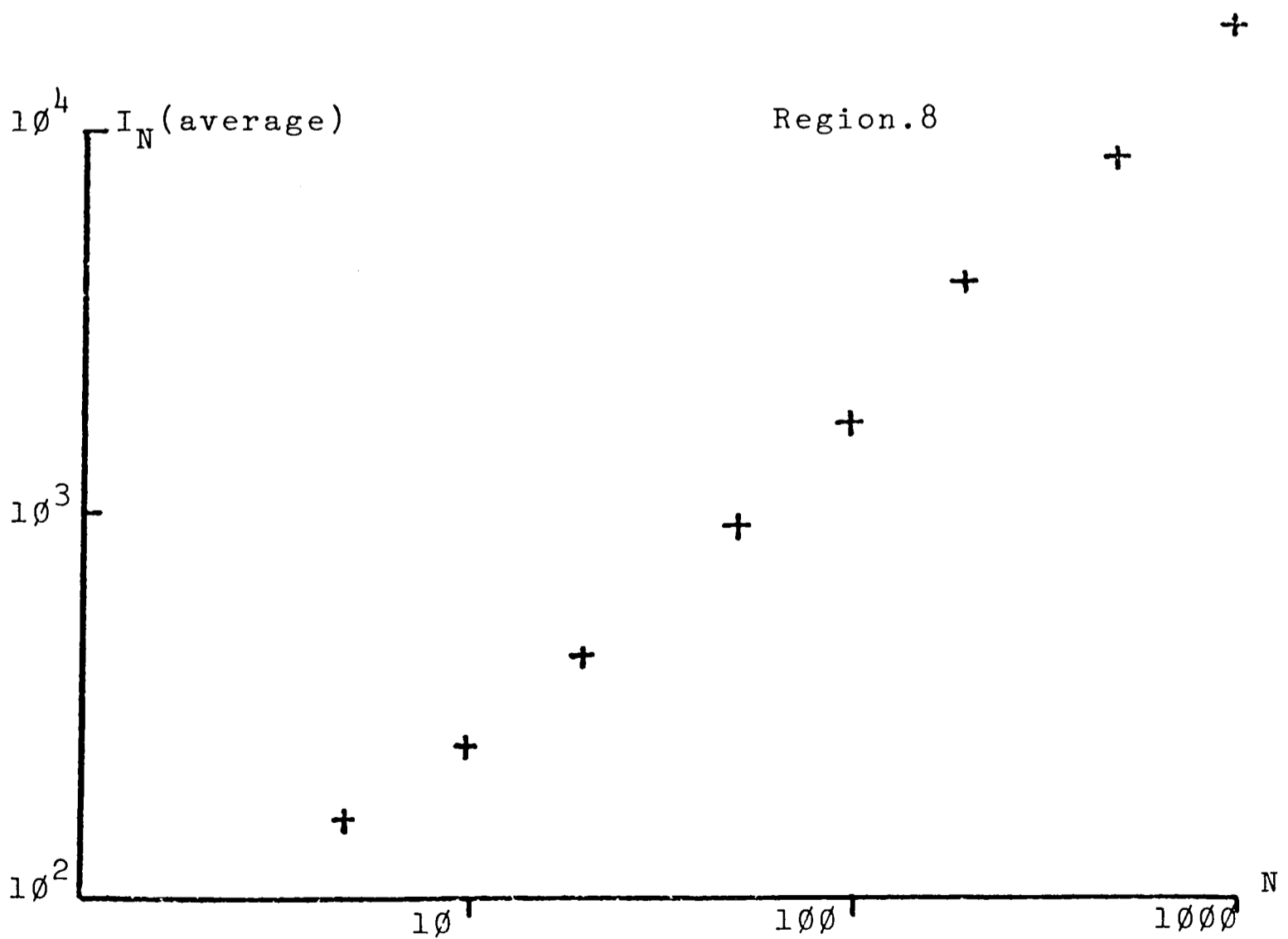


Fig.5.24 + CAP; ST, OK, NOL performed very badly.

Average cost Vs Number of stages; unknown parameters rapidly time-varying.

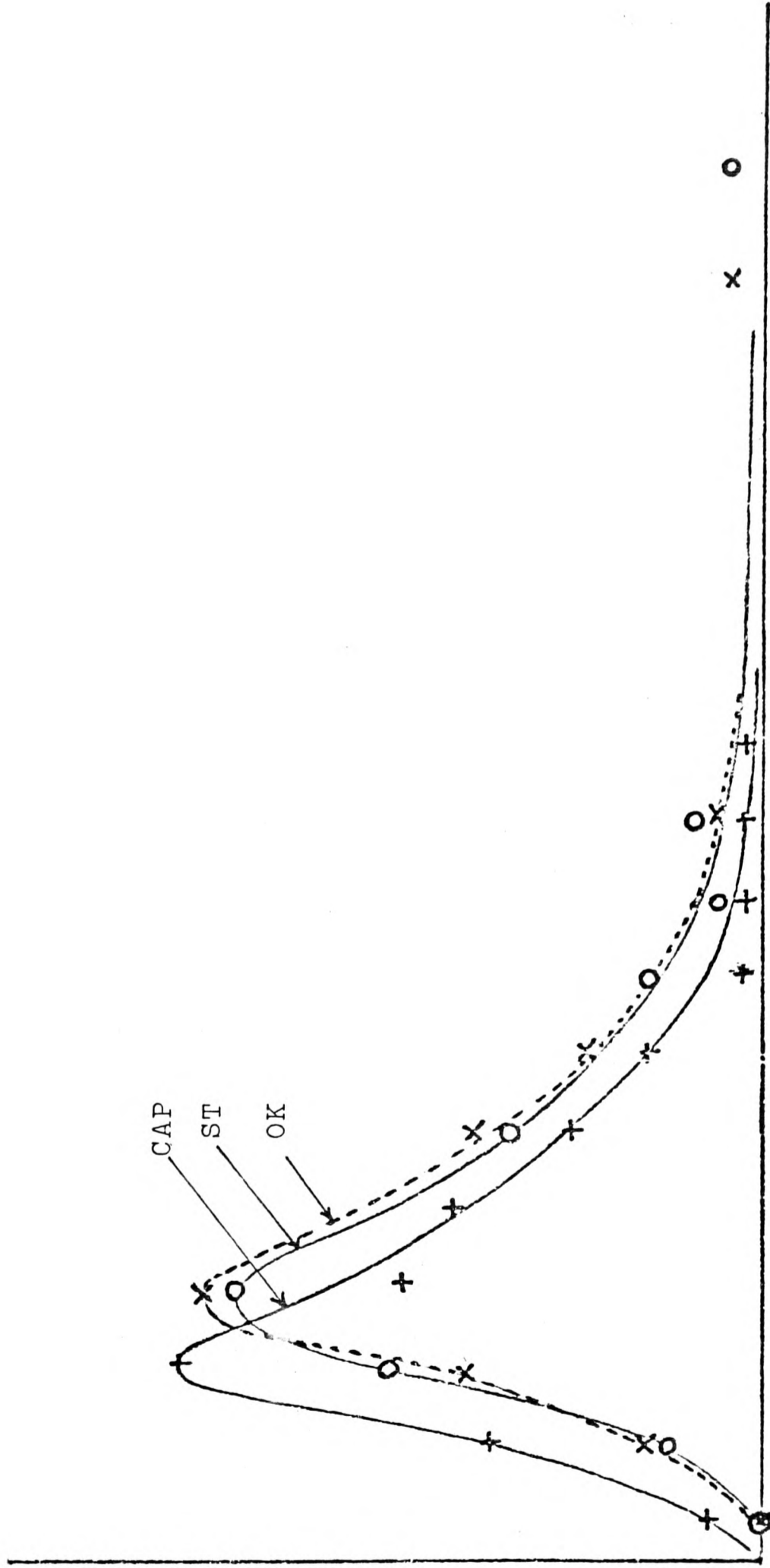


Fig. 5.25 Distribution of costs when $N=100$, unknown parameters constants and in region 2.

CHAPTER SIX

CONVERGENCE AND STABILITY OF OK CONTROLLER

6.1 Introduction

Simulation results of chapter 5 indicate OK controller converging to CAP controller when the unknown parameters of the controlled process are constants. A necessary condition for this convergence is that the non-linear OK control law should converge to a linear constant parameter control law having the same functional form as CAP control law. So perhaps the first step towards a theoretical analysis of convergence of the OK controller is to investigate when or whether the OK control law will converge to a linear constant parameter control law. This can be done by studying the convergence of the extended Kalman filter used in the OK controller because when the estimates of the unknown parameters of the extended Kalman filter converge to some constant values, the OK control law would converge to a linear constant parameter control law.

Ljung (41) studied the convergence of extended Kalman filters when used to generate estimates of states and parameters of linear processes. But his analysis assumes that the control input u is not generated by a feedback, and so cannot as such be applied to adaptive control problems. In (53, 55) Ljung et al. studied convergence of a class of adaptive controllers which used a linear control law (e.g. u^{ST}). Extension of the analysis in (41, 53, 55) to an adaptive control problem where there is a non-linear feedback control law (e.g. u^{OK}) leads to intractable mathematics.

However, a necessary condition for convergence of OK control law to a linear constant parameter control law is that the estimated covariances of the unknown parameters generated by the extended Kalman filter should converge to zero. A sufficient condition would be that these covariances should go to zero faster than the divergence of the estimates of the unknown parameters. In this chapter it is shown through a theorem that the necessary condition would be satisfied if certain matrix pair of the extended Kalman filter is uniformly completely observable. In order for the analysis to be general, an n^{th} order constant parameter single input single output (SISO) stochastic difference equation is used to represent the controlled process.

In section 6.2 extended Kalman filter equations are presented for the n^{th} order constant parameter controlled process. In section 6.3 the theorem for convergence to zero of the estimated covariances of the unknown parameters is proved assuming certain matrix pair of extended Kalman filter is uniformly completely observable (UCO). When the controlled process is first order this UCO condition reduces to a requirement that a determinant should always remain non-zero. The occasions when this determinant become zero and the reasons for that are discussed in section 6.4.

Time histories of several trials for a first order process under OK controller showed the estimated covariances of the unknown parameters always converging to zero and the estimates of the unknown parameters always converging to constant values. This suggests, at least for first order processes under OK controller, (i) that UCO requirement is not a stringent one but is always satisfied, (ii) that the convergence of the estimated covariances of the unknown parameters is always faster

than the divergence of the estimates of the unknown parameters.

A typical time history in region 4 is given in section 6.5.

In section 6.6 closed loop stability conditions are derived for a first order process under the OK controller when the estimates of the unknown parameters have all converged to constant values.

6.2 Extended Kalman filter for n^{th} order SISO process

The controlled process is described by the following n^{th} order SISO stochastic difference equation.

$$y(i) + a_1 y(i-1) + \dots + a_n y(i-n) = b_0 u(i-k) + \dots + b_{n-1} u(i-k-n+1) + \xi(i) + c_1 \xi(i-1) + \dots + c_n \xi(i-n) \quad 6.1$$

where y , u are the output and input respectively, ξ is an uncorrelated zero mean random sequence having variance σ_ξ ,

$a_1, \dots, a_n, b_0, \dots, b_{n-1}, c_1, \dots, c_n$ are the unknown constant parameters. k is the delay between input and output ($k \geq 1$). n is the order of the process.

The extended Kalman filter is designed on the basis of a state space representation of the controlled process. This requires state variables $[\underline{\alpha}]$, $[\underline{\beta}]$ and $[\underline{\gamma}]$ where

$$\underline{\alpha} \equiv [\alpha_1, \dots, \alpha_n]^T$$

$$\underline{\beta} \equiv [\beta_1, \dots, \beta_n]^T$$

$$\underline{\gamma} \equiv [\gamma_1, \dots, \gamma_n]^T$$

to represent the unknown parameters $a_1 \dots a_n, b_0 \dots b_{n-1}, c_1 \dots c_n$.

The dynamics of the process is represented by another state variable \underline{x} where

$$\underline{x} \equiv [x_1, \dots, x_{n+k-1}]^T.$$

The state variables $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$ are related to a s, b s and c s of equation 6.1 as follows:

$$\alpha_j = -c_j \quad j = 1, \dots, n$$

$$\beta_j \equiv b_{j-1} \quad j = 1, \dots, n$$

$$\gamma_j \equiv c_j - a_j \quad j = 1, \dots, n \quad 6.2a$$

The dynamic state \underline{x} is related to the single output y as

$$x_1(i) \equiv y(i) - \xi(i) \quad 6.2b$$

Substituting equations 6.2a, 6.2b into equation 6.1 gives the state space dynamics of the controlled process as

$$\begin{bmatrix} \underline{x}(i+1) \end{bmatrix} = \begin{bmatrix} \alpha_1 & & & \\ \vdots & & & \\ & & [I] & \\ & & & \alpha_n \\ 0 & & & & 0 \dots & & 0 \end{bmatrix} \begin{bmatrix} \underline{x}(i) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} u(i) + \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} y(i) \quad 6.3$$

The extended Kalman filter equations for the process of equations 6.3 are as follows:

$$\begin{bmatrix} \hat{\underline{\alpha}}(i+1) \\ \hat{\underline{\beta}}(i+1) \\ \hat{\underline{\gamma}}(i+1) \\ \hat{\underline{x}}(i+1) \end{bmatrix} = \begin{bmatrix} \hat{\underline{\alpha}}(i) \\ \hat{\underline{\beta}}(i) \\ \hat{\underline{\gamma}}(i) \\ \hat{\underline{x}}(i+1|i) \end{bmatrix} + \underline{\Sigma}(i+1) \underline{c}^T \underline{\sigma}_{\xi}^{-1} (y(i+1) - \hat{x}_1(i+1|i)) \quad 6.4a$$

$$\begin{bmatrix} \hat{\underline{x}}(i+1|i) \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_1(i) \\ \vdots \\ \hat{\alpha}_n(i) \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} [I]_{n+k-2, n+k-2} \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} \hat{\underline{x}}(i) \end{bmatrix} + \begin{bmatrix} \sigma_{\alpha_1 x_1} \\ \vdots \\ \sigma_{\alpha_n x_1} \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \hat{\beta}_1(i) \\ \vdots \\ \hat{\beta}_n(i) \end{bmatrix} u(i) + \begin{bmatrix} \hat{\gamma}_1(i) \\ \vdots \\ \hat{\gamma}_n(i) \\ \vdots \\ 0 \end{bmatrix} y(i) \quad 6.4b$$

$$\underline{\Sigma}(i+1) = \underline{\Sigma}_0 - \underline{\Sigma}_0 \underline{c}^T (\sigma_\xi + \underline{c} \underline{\Sigma}_0 \underline{c}^T)^{-1} \underline{c} \underline{\Sigma}_0 \quad 6.5a$$

$$[\underline{c}]_{1, 4n+k-1} = [[0]_{1, 3n} \quad , 1, [0]_{1, n+k-2}]$$

$$\underline{\Sigma}_0 = \underline{A}(i) \underline{\Sigma}(i) \underline{A}^T(i) \quad 6.5b$$

Since the unknown parameters are constant,

$$[A(i)]_{4n+k-1, 4n+k-1} = \begin{bmatrix} [I]_{3n, 3n} & & & [0]_{3n, n+k-1} \\ \hat{\underline{x}}_n(i) [I]_{n, n} & [0]_{k-1, n} & y(i) [I]_{n, n} & \\ [0]_{k-1, n} & u(i) [I]_{n, n} & [0]_{k-1, n} & \\ & & & \begin{bmatrix} \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_n \\ \vdots \\ 0 \end{bmatrix} \\ & & & \begin{bmatrix} [I]_{n+k-2, n+k-2} \\ \vdots \\ 0 \end{bmatrix} \end{bmatrix}$$

$$[\underline{\Sigma}(i)]_{4n+k-1, 4n+k-1} = \begin{bmatrix} [\Sigma_{pp}(i)]_{3n, 3n} & [\Sigma_{px}(i)]_{3n, n+k-1} \\ [\Sigma_{px}^t(i)]_{n+k-1, 3n} & [\Sigma_{xx}(i)]_{n+k-1, n+k-1} \end{bmatrix}$$

$\underline{\Sigma}(i) \equiv$ the estimated covariance matrix generated by the extended Kalman filter and

$\underline{\Sigma}_{pp} \equiv$ the estimated covariance matrix of the coefficients $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$

Σ_{xx} \equiv the estimated covariance matrix of the states \underline{x}

Σ_{px} \equiv the estimated covariance between the state \underline{x} and the coefficients $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$.

$[I]_{j,k}$ \equiv Identity matrix of j rows and k columns.

$[O]_{j,k}$ \equiv Null matrix of j rows and k columns.

6.3 Convergence of parameter covariance $\underline{\Sigma}_{pp}$

Equations 6.5 governing the estimated covariance $\underline{\Sigma}$ generated by the extended Kalman filter are similar to equations governing covariance matrices of Kalman filters for linear systems having time-varying coefficients. So the convergence properties of $\underline{\Sigma}_{pp}$ in equations 6.5 can be studied through standard observability conditions (38,42) of the matrices $\underline{A}(i)$ and \underline{C} . The observability conditions are expressed in terms of transition matrices $\underline{\Phi}$,

$$\underline{\Phi}(i,j) = \underline{A}(i-1) \cdot \underline{A}(j+1) \underline{A}(j) \quad i > j$$

$$\underline{\Phi}(i,i) = [I] .$$

The pair $\{ C, A \}$ is completely observable at time j if and only if the observability matrix

$$\underline{M}(j+N-1, j) = \sum_{i=j}^{j+N-1} \underline{\Phi}^T(i, j) \underline{C}^T \sigma_{\xi}^{-1} \underline{C} \underline{\Phi}(i, j)$$

is positive definite* (N is a +ve integer; here $N=4n+k-1$) and uniformly completely observable if and only if $M(j+N-1, j)$ is +ve definite for all j .

Theorem 1

If $\{ C, A \}$ is a uniformly completely observable pair then the parameter covariance $\underline{\Sigma}_{pp}$ of the extended Kalman filter will asymptotically converge to zero.

Proof

The covariance matrix $\underline{\Sigma}(j)$ is related to the observability matrix as (42)

* $M(j+N-1, j) > 0$ (+ve definite) only if rank $[\underline{C}^T \cdot \underline{\Phi}^T(j+1, j) \underline{C}^T : \underline{\Phi}^T(j+N-1, j) \underline{C}^T] = 4n+k-1$

$$\underline{\Sigma}(j) = \underline{\Phi}(j,1) [\underline{\Sigma}^{-1}(0) + \underline{M}(j,1)]^{-1} \underline{\Phi}^T(j,1) \quad 6.6$$

when $\underline{\Sigma}(0) > 0$. It follows then that

$$\underline{\Sigma}(j) \leq \underline{\Phi}(j,1) \underline{M}^{-1}(j,1) \underline{\Phi}^T(j,1).$$

The transition matrix $\underline{\Phi}(j,1)$ is of the form

$$\underline{\Phi}(j,1) = \begin{bmatrix} [\mathbf{I}]_{3n,3n} & [0]_{3n,n+k-1} \\ [\phi_{21}]_{n+k-1,3n} & [\phi_{22}]_{n+k-1,n+k-1} \end{bmatrix} \quad 6.7$$

and so

$$\underline{\Sigma}(j) \leq \begin{bmatrix} [\mathbf{I}] & [0] \\ [\phi_{21}] & [\phi_{22}] \end{bmatrix} \underline{M}^{-1}(j,1) \begin{bmatrix} [\mathbf{I}] & [\phi_{21}]^T \\ [0]^T & [\phi_{22}]^T \end{bmatrix} \quad 6.8$$

$$\text{Define } \underline{B}(j,1) = \begin{bmatrix} B_{11}(j,1) & B_{12}(j,1) \\ B_{21}(j,1) & B_{22}(j,1) \end{bmatrix} \equiv \underline{M}^{-1}(j,1)$$

Inequality 6.8 can be written as,

$$\left[\begin{array}{c|c} \Sigma_{pp}(j,1) & \Sigma_{px}(j,1) \\ \hline \Sigma_{px}^T(j,1) & \Sigma_{xx}(j,1) \end{array} \right]$$

\leq

$$\left[\begin{array}{c|c} B_{11}(j,1) & B_{11}(j,1)\phi_{21}^T(j,1) + B_{12}(j,1)\phi_{22}(j,1) \\ \hline \{ B_{11}(j,1)\phi_{21}^T(j,1) + B_{12}(j,1)\phi_{22}(j,1) \}^T & [\{ B_{11}(j,1)\phi_{21}^T(j,1) + B_{12}(j,1)\phi_{22}(j,1) \}^T \\ & \phi_{21}^T(j,1)] \\ & + \\ & [\{ \phi_{21}^T(j,1)B_{12}(j,1) + \phi_{22}^T(j,1)B_{22}(j,1) \} \\ & \phi_{22}^T(j,1)] \\ & \dots\dots\dots 6.9 \end{array} \right]$$

or

$$\Sigma_{pp}(j,1) \leq B_{11}(j,1)$$

$\dots\dots\dots 6.10$

But the pair $\{\underline{C}, \underline{A}\}$ is uniformly completely observable, so $\|M(j,1)\|$ increases monotonically without bound (38,42) as j increases.

$\lim_{j \rightarrow \infty} \|M(j,1)\| \rightarrow \infty$, where $\|\cdot\|$ is the spectral norm.

Since $\underline{B}(j,1) \equiv \underline{M}^{-1}(j,1)$,

$\lim_{j \rightarrow \infty} \|B(j,1)\| \rightarrow 0$; or the matrix $B(j,1)$ tends to a null matrix as

$j \rightarrow \infty$.

It follows then that,

$\lim_{j \rightarrow \infty} \|B_{11}(j,1)\| \rightarrow 0$.

But since from equation 6.10

$$\underline{\Sigma}_{pp}(j,1) \leq \underline{B}_{11}(j,1),$$

$\lim_{j \rightarrow \infty} \|\underline{\Sigma}_{pp}(j,1)\| \rightarrow 0$ or the matrix $\underline{\Sigma}_{pp}(j,1)$ tends to a null matrix

as $j \rightarrow \infty$.

This completes the proof of the theorem.

Corollary:

From equation 6.6 it follows that $\underline{\Sigma}(j)$ is at least positive semi definite;

or

$$\underline{\Sigma}(j) \geq \underline{0}.$$

So, the above theorem implies that

$$\lim_{j \rightarrow \infty} [\underline{\Sigma}_{px}(j,1)] \rightarrow [0]$$

6.11

Theorem 1 does not imply that the estimates of the unknown parameters

generated by the extended Kalman filter converge to constant values. However, if it is assumed that the parameter estimates do not diverge faster than the convergence of Σ_{pp} , then Theorem 1 does imply that the parameter estimates converge asymptotically to constant values.

6.4 Convergence condition of Σ_{pp} for a first order process

The requirement for the proof of convergence to zero of the estimated parameter covariance Σ_{pp} was that the matrix pair $\{\underline{C}, \underline{A}\}$ should be uniformly completely observable.

When the controlled process is of first order this requirement is that the matrix $\underline{M}_1(i+3, i)$,

$$\underline{M}_1(i+3, i) = \begin{bmatrix} \underline{C} \\ \underline{c} \underline{A}(i) \\ \underline{c} \underline{A}(i+1)\underline{A}(i) \\ \underline{c} \underline{A}(i+2)\underline{A}(i+1)\underline{A}(i) \end{bmatrix}^T \quad 6.12$$

should have a rank 4 for all i . Since the controlled process is first order it is perhaps more convenient to refer to equations 2.1 and 2.2 than putting $n=1$ in equation 6.1. Substituting \underline{C} and \underline{A} from equation 2.5 and performing simple algebraic manipulations the matrix $\underline{M}_1(i+3, i)$ can be expressed as

$$\underline{M}_1(i+3, i) = \begin{bmatrix} 0 & \hat{x}_4(i) & \hat{x}_4(i+1) & \hat{x}_4(i+2) \\ 0 & u(i) & u(i+1) & u(i+2) \\ 0 & y(i) & y(i+1) & y(i+2) \\ 1 & \hat{x}_1(i) & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \hat{x}_1(i+1) & \hat{x}_1(i+2) \cdot \hat{x}_1(i+1) \\ 0 & 0 & 1 & \hat{x}_1(i+2) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\underline{M}_1 would have a rank 4 for all i only if the determinant of

$$\begin{bmatrix} \hat{x}_4(i) & \hat{x}_4(i+1) & \hat{x}_4(i+2) \\ u(i) & u(i+1) & u(i+2) \\ y(i) & y(i+1) & y(i+2) \end{bmatrix} \neq 0 \quad \forall i \quad 6.13$$

There are two reasons why the requirement 6.13 might be violated.

i) One of the sequence \hat{x}_4, u, y might be identically zero.

The sequence y would never be identically zero. The best regulation of y can be achieved under certainty about parameter control u_1^{CAP} with no cost of control term ($q=0$ in equation 1.2), when y becomes identical with the random noise sequence ξ (2). Under OK controller with a non zero cost of control ($q \neq 0$), y and the estimate \hat{x}_4 would both be farther from zero. When there is no cost of control ($q=0$) the estimate \hat{x}_4 under OK controller can converge to zero; but not before the parameter estimates have converged to constant values since σ_{44} and $\hat{x}_4(i|i-1)$ (equation 2.5) can not go to zero, i.e. not before Σ_{pp} has converged to zero. Thus condition 6.13 would not be violated until there was no need for further convergence.

The sequence u can sometimes be zero over finite intervals of time. This is described as 'turn-off' (8, 69). The convergence condition 6.13 would then be violated. However it is shown in (8) for OK controller, that the likelihood of 'turn-off' diminishes as the number of unknown parameters in the controlled process increases.

ii) Two or more of the sequence \hat{x}_4, u, y might be directly correlated.

A simple, constant linear feedback control law of the form

$$u(i) = k_1 \hat{x}_4(i) + k_2 y(i) \quad 6.14$$

would cause the determinant in 6.13 to be zero. The OK control law has the form of equation 6.14 only when the estimates of all parameters have converged; then the convergence condition 6.13 would be violated but there would be no need for further convergence.

Thus as long as there is no 'turn-off' the convergence condition 6.13 would not be violated under OK controller. Since there are three unknown parameters even for first order processes, it is unlikely that there will be turn-off for OK controller when all the parameters are unknown.

6.5 Numerical Example

Extensive simulation of constant parameter first order process when controlled by OK controller showed Σ_{pp} always converging to zero and the parameter estimates always converging to constant values. A typical time history in region 4 is shown in figs. 6.1, 6.2, 6.3, 6.4. Details of the trial are in Table 6.1.

	x_1	x_2	x_3
True Value	1.193	2.815	5.073
Initial Estimate	0.0	4.0	4.0
Initial Variance	1.44	1.44	1.44
Converged Estimate	1.132	2.728	5.013

Table 6.1 Details of Simulation, $q=0.01$, $\sigma_\xi=1.0$,
Length of run = 200 stages.

The estimates of parameter covariances go **to** zero as expected and the estimates of parameters converge to constant values. The converged values are however biased.

From the converged estimates for x_1 , x_2 , x_3 the converged estimates for a, b, c can be evaluated;

$$\tilde{a} = -(\tilde{x}_1 + \tilde{x}_3) = -6.145$$

$$\tilde{b} = \tilde{x}_2 = 2.728$$

$$\tilde{c} = -\tilde{x}_1 = -1.132$$

where tilde represents converged values.

6.5.1 Effect of biased estimates on the OK control law:

Once the parameter estimates have converged the extended Kalman filter of the OK controller becomes a Kalman filter and the OK control law has the same functional form as CAP control law. This means the asymptotic OK control law would have the same functional form as the asymptotic CAP control

law given by equations 2.21, 2.22 with the converged estimates of parameters replacing the true parameters. Asymptotic OK control law for the numerical example, where $|\tilde{c}| > 1$, is given by

$$u^{OK}(i) = -\left\{ \frac{(q/\tilde{b})}{\tilde{c}(\tilde{b}+q/\tilde{b})} u(i-1) + \frac{(1/\tilde{c} - \tilde{a})}{(\tilde{b} + q/\tilde{b})} y(i) \right\}$$

and the asymptotic CAP control law for the numerical example, where $|c| > 1$, is given by

$$u^{CAP}(i) = -\left\{ \frac{q/b}{c(b + q/b)} u(i-1) + \frac{(1/c - a)}{(b + q/b)} y(i) \right\} \quad 2.22$$

Substituting the values for $\tilde{a}, \tilde{b}, \tilde{c}$ the asymptotic OK control law becomes

$$u^{OK}(i) = 0.0012 u(i-1) - 1.9262 y(i).$$

Similarly substituting the values for a, b, c the asymptotic CAP control law becomes

$$u^{CAP}(i) = 0.0011 u(i-1) - 1.9257 y(i).$$

This indicates that although the parameter estimates could be biased the resulting OK control law can still asymptotically converge to CAP control law.

6.6 Stability of closed loop system when the parameter estimates have converged

As the parameter estimates converge, the OK control law tends to a linear constant parameter control law. Since the converged values of the estimates could in general be biased, it is possible for the resulting closed loop system to be unstable. In this section the stability conditions are derived for a first order process (equations 2.1, 2.2) when the OK controller has converged to a linear constant parameter controller.

Once the parameter estimates have converged, the extended Kalman filter becomes a Kalman filter generating the estimate of the dynamic state x_4 . The covariance equation of this Kalman filter is:

$$\sigma_{44}(i+1) = \frac{\sigma_{\xi} \tilde{x}_1^2 \sigma_{44}(i-1)}{\sigma_{\xi} + \tilde{x}_1^2 \sigma_{44}(i-1)} \quad 6.15$$

where tilde represents converged values of the parameter estimates. It can be easily shown that the steady stage value ($i \rightarrow \infty$) of this covariance, σ_{44}^{ss} , is zero if $|\tilde{x}_1| \leq 1$ and $(1 - \frac{1}{\tilde{x}_1^2})\sigma_{\xi}$ if $|\tilde{x}_1| > 1$.

Thus for very large i , the equations for the dynamic state and its estimate are

$$x_4(i+1) = x_1 x_4(i) + x_2 u^{OK}(i) + x_3 y(i) \quad 6.16$$

and

$$\hat{x}_4(i+1) = \tilde{x}_1 \hat{x}_4(i) + \tilde{x}_2 u^{OK}(i) + \tilde{x}_3 y(i) + \sigma_{44}^{ss} [y(i+1) - \{\tilde{x}_1 \hat{x}_4(i) + \tilde{x}_2 u^{OK}(i) + \tilde{x}_3 y(i)\}] \sigma_{\xi}^{-1} \quad 6.17$$

The OK control $u^{OK}(i)$ when the parameter estimates have all converged is:

$$u^{OK}(i) = - \frac{[\tilde{x}_1 \hat{x}_4(i) + \tilde{x}_3 y(i)] \tilde{x}_2}{\tilde{x}_2^2 + q} \quad 6.18$$

Substituting for $u^{OK}(i)$, equation 6.16 becomes,

$$x_4(i+1) = x_1 x_4(i) - \frac{x_2 \tilde{x}_2}{\tilde{x}_2^2 + q} [\tilde{x}_1 \hat{x}_4(i) + \tilde{x}_3 y(i)] + x_3 y(i). \quad 6.19$$

Equation 6.17, when $|\tilde{x}_1| \leq 1$ becomes

$$\hat{x}_4(i+1) = \tilde{x}_1 \hat{x}_4(i) - \frac{\tilde{x}_2^2}{(\tilde{x}_2^2 + q)} [\tilde{x}_1 \hat{x}_4(i) + \tilde{x}_3 y(i)] + \tilde{x}_3 y(i), \quad 6.20a$$

and when $|\tilde{x}_1| > 1$ becomes

$$\begin{aligned} \hat{x}_4(i+1) = \tilde{x}_1 \hat{x}_4(i) - \frac{\tilde{x}_2^2}{(\tilde{x}_2^2 + q)} [\tilde{x}_1 \hat{x}_4(i) + \tilde{x}_3 y(i)] + \tilde{x}_3 y(i) + \\ (1 - \frac{1}{\tilde{x}_1^2}) [y(i+1) - \{ \tilde{x}_1 \hat{x}_4(i) - \frac{\tilde{x}_2^2}{\tilde{x}_2^2 + q} (\tilde{x}_1 \hat{x}_4(i) + \tilde{x}_3 y(i)) + \tilde{x}_3 y(i) \}]. \end{aligned} \quad 6.20b$$

Closed loop dynamics, for very large i , are described by equations 6.19 and 6.20 and they are linear. The condition for stability of the closed loop system when $|\tilde{x}_1| \leq 1$ is that the two characteristic roots

$$\begin{aligned} \{ [\tilde{x}_2^2 (x_1 + x_3) + q(x_1 + x_3 + \tilde{x}_1) - x_2 \tilde{x}_2 \tilde{x}_3] \pm [\{ \tilde{x}_2^2 (x_1 + x_3) + \\ q(x_1 - \tilde{x}_1 + x_3) - x_2 \tilde{x}_2 \tilde{x}_3 \}^2 - 4 \tilde{x}_1 \tilde{x}_2 \tilde{x}_3 x_2 q]^{1/2} \} (2(\tilde{x}_2^2 + q))^{-1} \end{aligned} \quad 6.21a$$

must lie within the unit circle.

The general condition for stability when $|\tilde{x}_1| > 1$ is not simple. For the case when $q=0$ the condition is that the roots

$$\begin{aligned}
& \{ [\tilde{x}_2^2 \tilde{x}_1 (x_1 + x_3) - \tilde{x}_1 \tilde{x}_2 \tilde{x}_3 x_2 - x_2 \tilde{x}_2 (\tilde{x}_1^2 - 1)] \pm [\{ \tilde{x}_2^2 \tilde{x}_1 (x_1 + x_3) - \\
& \tilde{x}_1 \tilde{x}_2 \tilde{x}_3 x_2 + x_2 \tilde{x}_2 (\tilde{x}_1^2 - 1) \}^2 - 4 \tilde{x}_1 \tilde{x}_2 x_2 (\tilde{x}_2^2 (x_1 + x_3) (\tilde{x}_1^2 - 1) - \\
& x_2 \tilde{x}_2 \tilde{x}_3 (\tilde{x}_1^2 - 1))]^{\frac{1}{2}} \} (2 \tilde{x}_1 \tilde{x}_2^2)^{-1}
\end{aligned}
\tag{6.21b}$$

must be within the unit circle.

In (41) Ljung predicts the points of convergence of parameter estimates of extended Kalman filters used for non-adaptive control problems. If it were possible to predict the convergence points of extended Kalman filters used for adaptive control problems, then visualising \tilde{x}_1 , \tilde{x}_2 , \tilde{x}_3 as a point in the x_1 , x_2 , x_3 space and using equations 6.21a, 6.21b one could construct stability regions for a first order process under the OK controller.

6.7 Conclusions

Extended Kalman filter equations are presented for an n^{th} order SISO linear process with constant parameters whose values are unknown. It is shown through Theorem 1 that the estimated covariance of the unknown parameters generated by the extended Kalman filter (Σ_{pp}) asymptotically converges to zero when the matrix pair $\{C, A(i)\}$ is uniformly completely observable. Theorem 1 proves only the necessary condition for the convergence of the parameter estimates to constant values; ~~or~~ for the convergence of OK controller to a linear constant parameter controller. If it is assumed that the parameter estimates of the extended Kalman filter do not diverge faster than the convergence of Σ_{pp} , then theorem 1 proves the sufficient condition. For a first order SISO process the necessary condition becomes a requirement that

$$\det \begin{vmatrix} \hat{x}_4(i) & \hat{x}_4(i+1) & \hat{x}_4(i+2) \\ u(i) & u(i+1) & u(i+2) \\ y(i) & y(i+1) & y(i+2) \end{vmatrix} \neq 0 \quad \forall i.$$

Under the OK controller the only time this requirement would be violated before the parameters had converged is when there is a 'turn-off'. It has been conjectured (8) that such a violation of the necessary condition is an essential feature of 'turn-off'.

Several trials of first order process when controlled by OK controller showed the parameter estimates converging to constant values and their estimated covariances converging to zero. From this, it is conjectured that, at least for first order processes, the sufficient condition for the convergence of OK controller to a linear constant parameter controller is always satisfied.

Unfortunately it has not been able to predict the possible points of convergence of the parameter estimates. Extension of Ljung's (41, 53, 55) approach in which a non-linear differential equation describes the evolution of the parameter estimates, results in intractable mathematics because of the non-linear nature of the OK control law. However, if it were possible to predict the points of convergence of the parameter estimates, then for a first order process stability regions for closed loop system can be constructed using the conditions 6.21 derived in section 6.6.

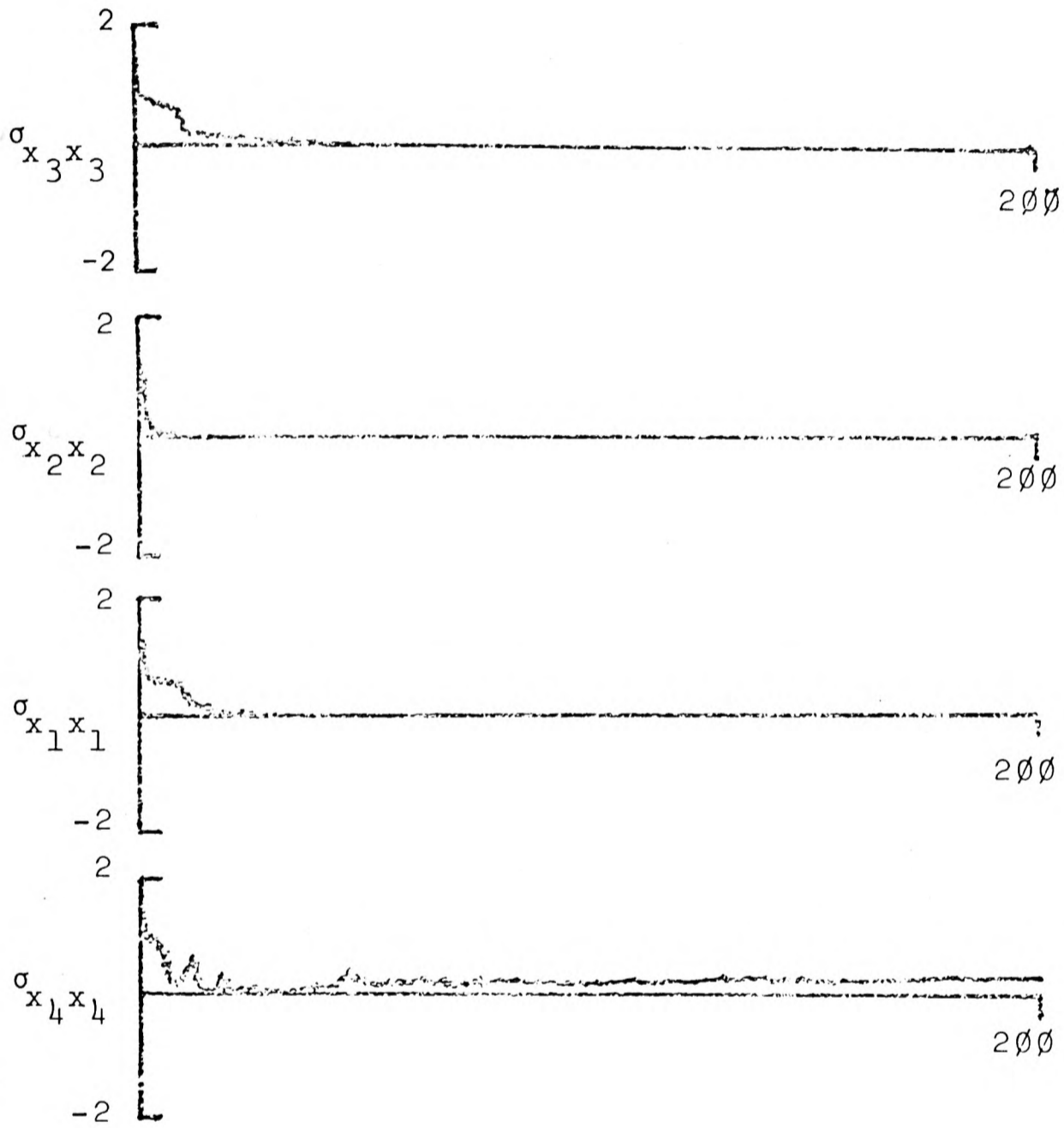


Fig.6.1 Extended Kalman filter generated estimates of unknown parameter covariances and state covariance.

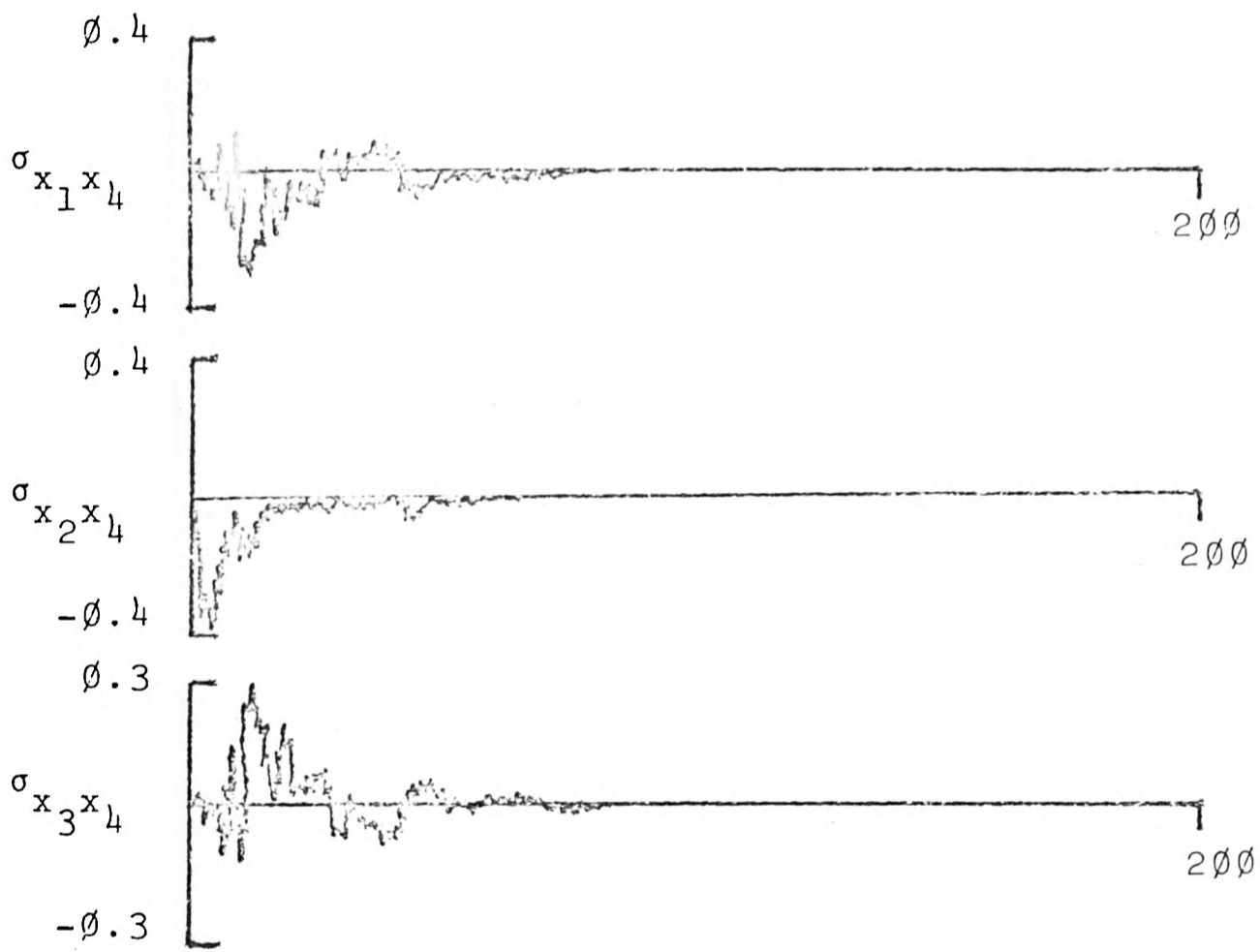


Fig.6.2 Extended Kalman filter generated estimates of covariances between unknown parameters and the state.

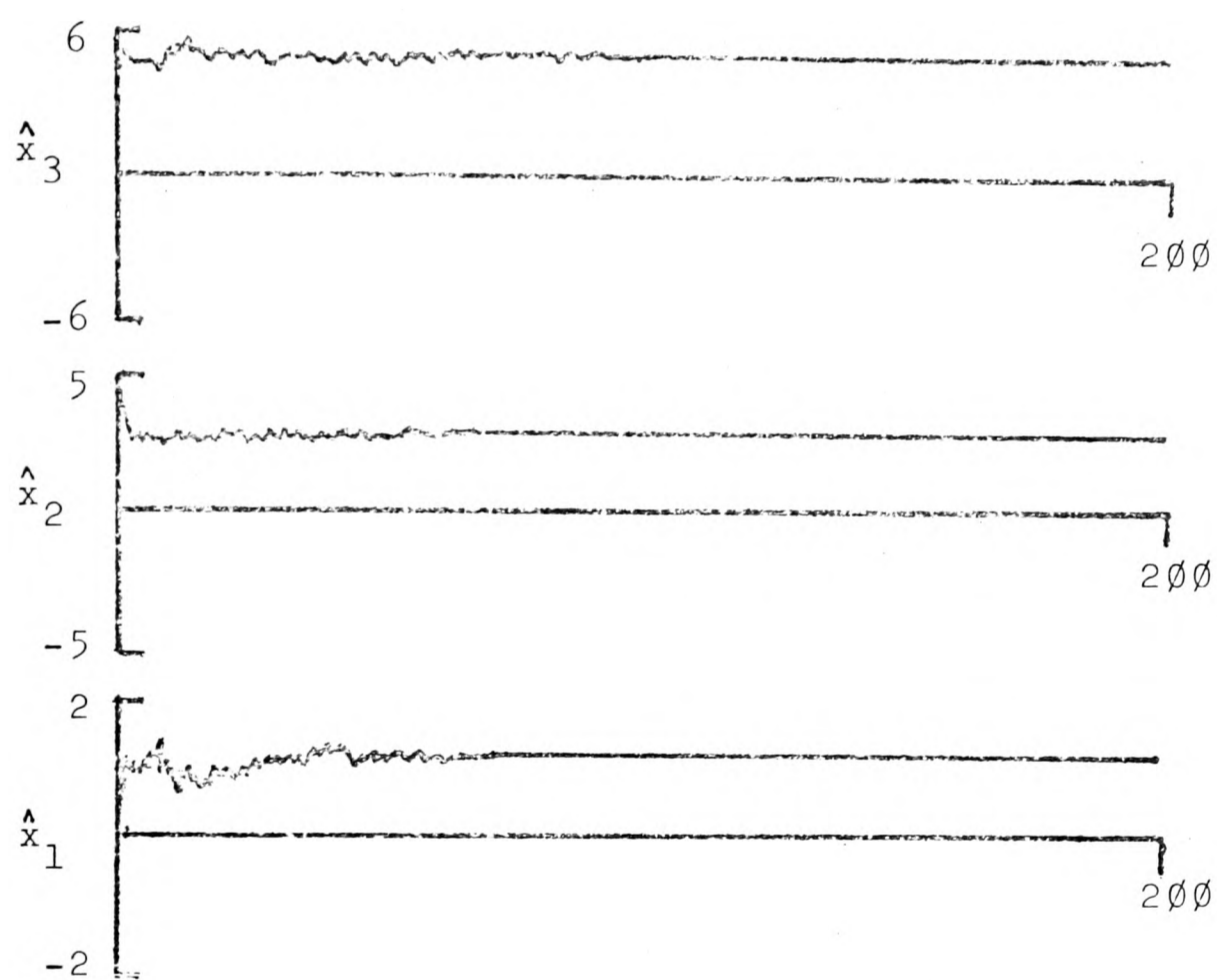


Fig.6.3 Extended Kalman filter generated estimates of unknown parameters.

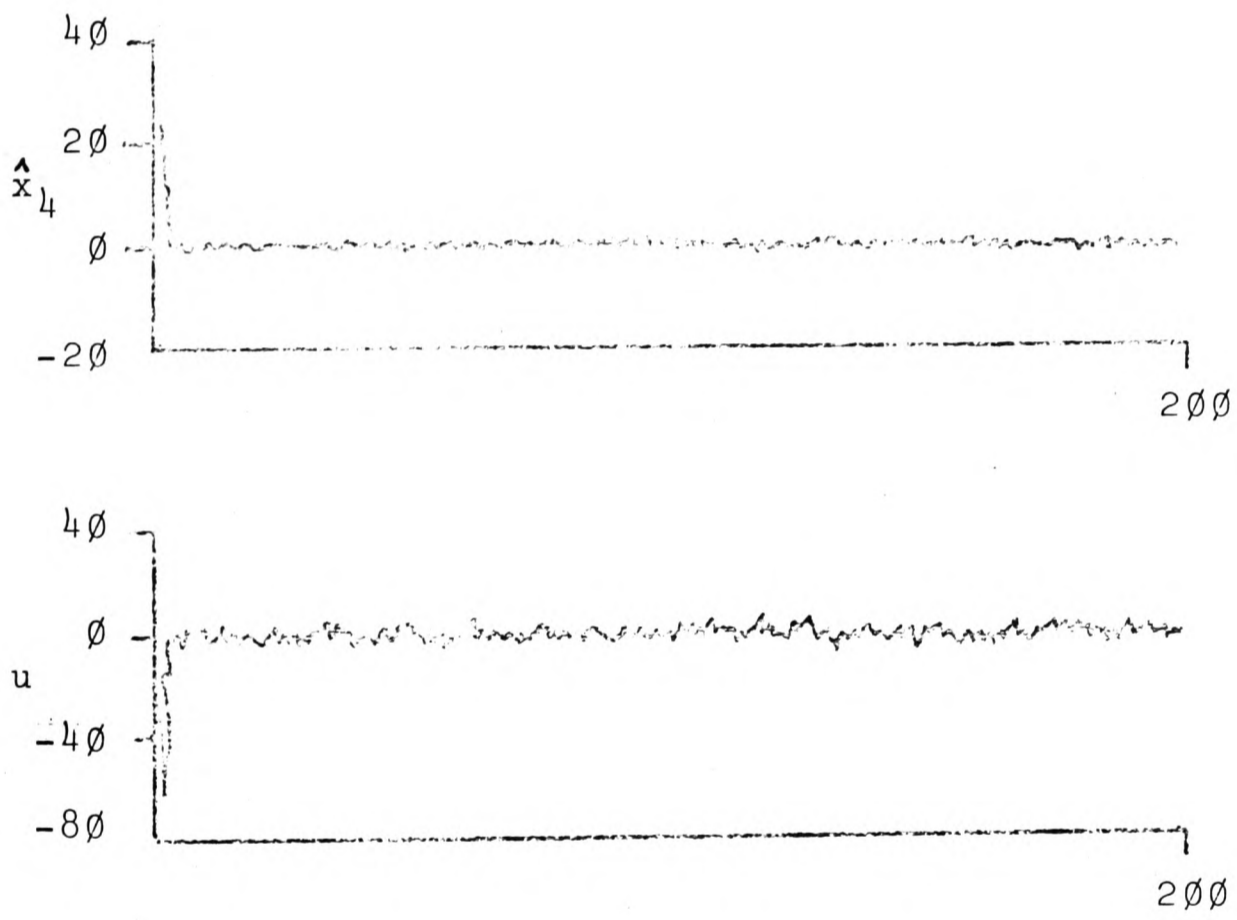


Fig.6.4 Control u and estimate of dynamic state x_4

CHAPTER SEVENAN ALTERNATIVE STATE AND PARAMETER ESTIMATOR7.1 Introduction

One of the conclusions of chapter 5 was that the OK controller when used for controlling higher order processes would suffer from excessive computational requirement . This excessive computational requirement comes mainly from the extended Kalman filter which generates the estimates of states and parameters. So, if extended Kalman filter could be replaced by a computationally economical state and parameter estimator then the overall computational requirement of the OK controller could be brought down considerably.

The suboptimal estimation scheme proposed in (36) and (56) for the combined estimation of states and parameters of an linear process is a computationally economical scheme. According to this scheme two linear estimators, one for estimating the states and the other for estimating the parameters, together generate the estimates of states and parameters of a linear process. If this combined estimator in addition to being computationally economical does not cause any deterioration in the performance of the OK controller, then it can justifiably replace the extended Kalman filter in the OK controller. In this chapter the performance of OK controller with combined estimator is compared with the performance of OK controller with extended Kalman filter. The controlled process is described by the first order equation 1.1 and the unknown parameters are assumed as constants.

In section 7.2 equations governing the combined estimator are presented for the first order process of equation 1.1. Section 7.3 contains simulation results when the first order process is controlled by

(i) OK controller with combined estimator, and (ii) OK controller with extended Kalman filter in all the eight regions of the parameter space shown in Fig. 3.2. The simulation results indicate that the combined estimator when used in the place of extended Kalman filter in the OK controller will not cause any significant deterioration in the overall performance, but at the same time will reduce the net computational requirement.

7.2 Combined Estimator

The combined estimator consists of two linear estimators: a parameter estimator generating the estimates of the unknown parameters and a state estimator generating the estimates of the states using the parameter estimates already generated. The structure of the combined estimator is shown in fig. 7.1.

7.2.1 Parameter Estimator

The parameter estimator considers the input output relationship of equation 1.1 and uses an algorithm of the type given in (54) for generating the estimates of the unknown parameters. The input output relationship of equation 1.1 is written below.

$$y(i+1) = \underline{z}^T(i)\underline{\theta} + \xi(i+1) \quad 7.1$$

where

$$\underline{z}(i) = \begin{bmatrix} y(i) \\ u(i) \\ \xi(i) \end{bmatrix} ; \quad \underline{\theta} = \begin{bmatrix} -a \\ b \\ c \end{bmatrix}$$

Since the parameters a, b, c are considered to be constants,

$$\underline{\theta}(i+1) = \underline{\theta}(i). \quad 7.2$$

For the system of equations 7.1, 7.2 the following estimation algorithm can be obtained by a formal application of the recursive least squares formula (61).

$$\hat{\underline{\theta}}(i+1) = \hat{\underline{\theta}}(i) + \underline{K}(i+1) [y(i+1) - \hat{\underline{z}}^T(i) \hat{\underline{\theta}}(i)] \quad 7.3$$

$$\underline{K}(i+1) = \underline{\Sigma}_{\theta}(i+1) \hat{\underline{z}}(i) \sigma_{\xi}^{-1} \quad 7.4a$$

where $\underline{\Sigma}_{\theta}$ is the estimated parameter covariance matrix and σ_{ξ} is the variance of the noise ξ .

$$\underline{\Sigma}_{\theta}(i+1) = \underline{\Sigma}_{\theta}(i) - \underline{\Sigma}_{\theta}(i)\hat{\underline{z}}(i) \left\{ \sigma_{\xi} + \hat{\underline{z}}^T(i) \underline{\Sigma}_{\theta}(i)\hat{\underline{z}}(i) \right\}^{-1} \hat{\underline{z}}^T(i)\underline{\Sigma}_{\theta}(i) \quad 7.4b$$

In equations 7.3, 7.4a, 7.4b $\hat{\underline{z}}(i)$ denotes a vector obtained from $\underline{z}(i)$ by replacing $\xi(i)$ by $\hat{\xi}(i)$ where from equation 7.1

$$\hat{\xi}(i) = y(i) - \hat{\underline{z}}^T(i-1)\hat{\underline{\theta}}(i) \quad 7.4c$$

Estimation of unknown parameters of a linear system using equations of the type 7.3, 7.4a, 7.4b, 7.4c is believed to be first proposed by Panuska (54). It is also sometimes known as extended least squares (ELS) method (53).

In this sub-section parameter estimator equations have been presented only for constant unknown parameter case. If the process parameters are time-varying then equations 7.3, 7.4b have to be modified. The modifications are that $\hat{\underline{\theta}}(i)$ and $\underline{\Sigma}_{\theta}(i)$ in equations 7.3, 7.4b should be replaced by their predicted values $\hat{\underline{\theta}}(i+1|i)$ and $\underline{\Sigma}_{\theta}(i+1|i)$. These can be easily obtained from the dynamics of the parameters.

7.2.2 State Estimator

The state estimator assumes the parameter estimates $\hat{\underline{\theta}}$ generated by the parameter estimator to be the true parameter values $\underline{\theta}$ and uses them in a linear Kalman filter to generate the estimate of the dynamic state \underline{x}_4 of equation 2.2a. Kalman filter equations generating the estimate $\hat{\underline{x}}_4$ are

$$\hat{\underline{x}}_4(i+1) = \hat{\underline{x}}_4(i+1|i) + \sigma_{44}(i+1) \sigma_{\xi}^{-1} \left\{ y(i+1) - \hat{\underline{x}}_4(i+1|i) \right\} \quad 7.5a$$

$$\hat{\underline{x}}_4(i+1|i) = \hat{\underline{x}}_1 \hat{\underline{x}}_4(i) + \hat{\underline{x}}_2 u(i) + \hat{\underline{x}}_3 y(i) \quad 7.5b$$

where $\hat{\underline{x}}_1, \hat{\underline{x}}_2, \hat{\underline{x}}_3$ are obtained from $\hat{\underline{\theta}}$ (cf. equation 2.1a) and

$$\hat{\underline{x}}_1 = -\hat{\underline{c}}$$

$$\hat{\underline{x}}_2 = \hat{\underline{b}}$$

$$\hat{\underline{x}}_3 = \hat{\underline{c}} - \hat{\underline{a}}.$$

The recursive equation for σ_{44} is

$$\sigma_{44}(i+1) = \frac{\hat{x}_1^2 \sigma_{44}(i) \sigma_\xi}{\hat{x}_1^2 \sigma_{44}(i) + \sigma_\xi} \quad 7.6$$

Since the state estimator assumes that all the parameters are known, estimates of covariances between state and parameters do not exist in the combined estimator. So in the OK control law one could put these terms (σ_{14}, σ_{24} in equation 2.11) as simply zeros. The resulting OK control law would be

$$u^{OK}(i) = \frac{-[\hat{x}_1 \hat{x}_2 \hat{x}_4 + \hat{x}_4 \sigma_{12} + (\hat{x}_2 \hat{x}_3 + \sigma_{23})y]}{\hat{x}_2^2 + \sigma_{22} + q} \quad 7.7$$

where q is the control weight (cf. equation 1.2). Alternatively one could approximate σ_{14} and σ_{24} as

$$\begin{aligned} \sigma_{14} &\sim \rho_1 \sqrt{\sigma_{\theta_3} \theta_3} \sigma_{44} \\ \sigma_{24} &\sim \rho_2 \sqrt{\sigma_{\theta_2} \theta_2} \sigma_{44} \end{aligned}$$

where ρ_1, ρ_2 are a priori chosen constants having a value between zero and one. In this chapter the simpler control law of equation 7.7 is used as the OK control law that goes with the combined estimator. As will be seen from the next section this control law does not produce any appreciable deterioration in the performance of the OK controller with combined estimator as compared with that of the OK controller with extended Kalman filter. The covariances σ_{12}, σ_{23} and σ_{22} in equation 7.7 are related to Σ_θ of equation 7.4b as

$$\sigma_{12} = -\sigma_{\theta_2} \theta_3 ,$$

$$\sigma_{22} = \sigma_{\theta_2 \theta_2}$$

$$\sigma_{23} = \sigma_{\theta_2 \theta_3} + \sigma_{\theta_1 \theta_2}$$

7.8

The extended Kalman filter equations for the first order process are given in section 2.4. The OK control law that goes with the extended Kalman filter is given by equation 2.11.

Whereas the extended Kalman filter for the first order process deals with a covariance matrix of size 4 x 4, the combined estimator only deals with covariance matrices of size 3 x 3 and 1 x 1. So one can expect a reduced computing time for the combined estimator.

7.3 Simulation Results and Conclusions

The first order process with constant unknown parameters is simulated under (i) OK controller with combined estimator, and (ii) OK controller with extended Kalman filter with the same random variables in all the eight regions of the parameter space described in chapter 3 . Data specifying each of these regions is the same as given in Table 5.1 of Chapter 5

Although no special precaution was taken to ensure that the state estimator of the combined estimator did not diverge due to poor parameter estimates, this was not a problem in any of the simulations. If this were a problem then some algorithmic modification of the estimated state covariance would be necessary. The simulation results are shown in Figs. 7.2 to 7.7. As in chapter 5 they are drawn to logarithmic scale with average cost as ordinate and number of stages N as abscissa.

Figs. 7.2, 7.3, 7.4, 7.6, 7.7 show that the cost achieved under both OK controllers are very close to each other when the unknown parameters occupy regions 1, 2, 3, 5 and 7.

When the unknown parameters occupy region 4 the performance of the OK controller with combined estimator, shown in fig. 7.5, is only marginally inferior to that of the OK controller with extended Kalman filter for short time (small N) processes. For long time processes, however, the performances of both the OK controllers are very close to each other.

When the unknown parameters are in regions 6, 8 both the OK controllers performed so badly that they are not shown. The reason for this is 'escape' (68,69,8); the same as that given in chapter 5 where similar bad performances have been reported.

On the computation side OK controller with combined estimator took only about sixty percent as much computing time as the OK controller with extended Kalman filter. Thus simulation results of this chapter lead to the conclusion that the combined estimator should be preferred to extended Kalman filter as the suboptimal estimator in OK controller. Although simulations have been carried out only for the constant parameter case, it is expected that similar conclusions regarding the combined estimator would result for time-varying cases as well.

Finally time history of a typical run in region 2 showing y , u , various estimates, costs, of OK controller with the two estimators is shown in figs. 7.8, 7.9, 7.10, 7.11.

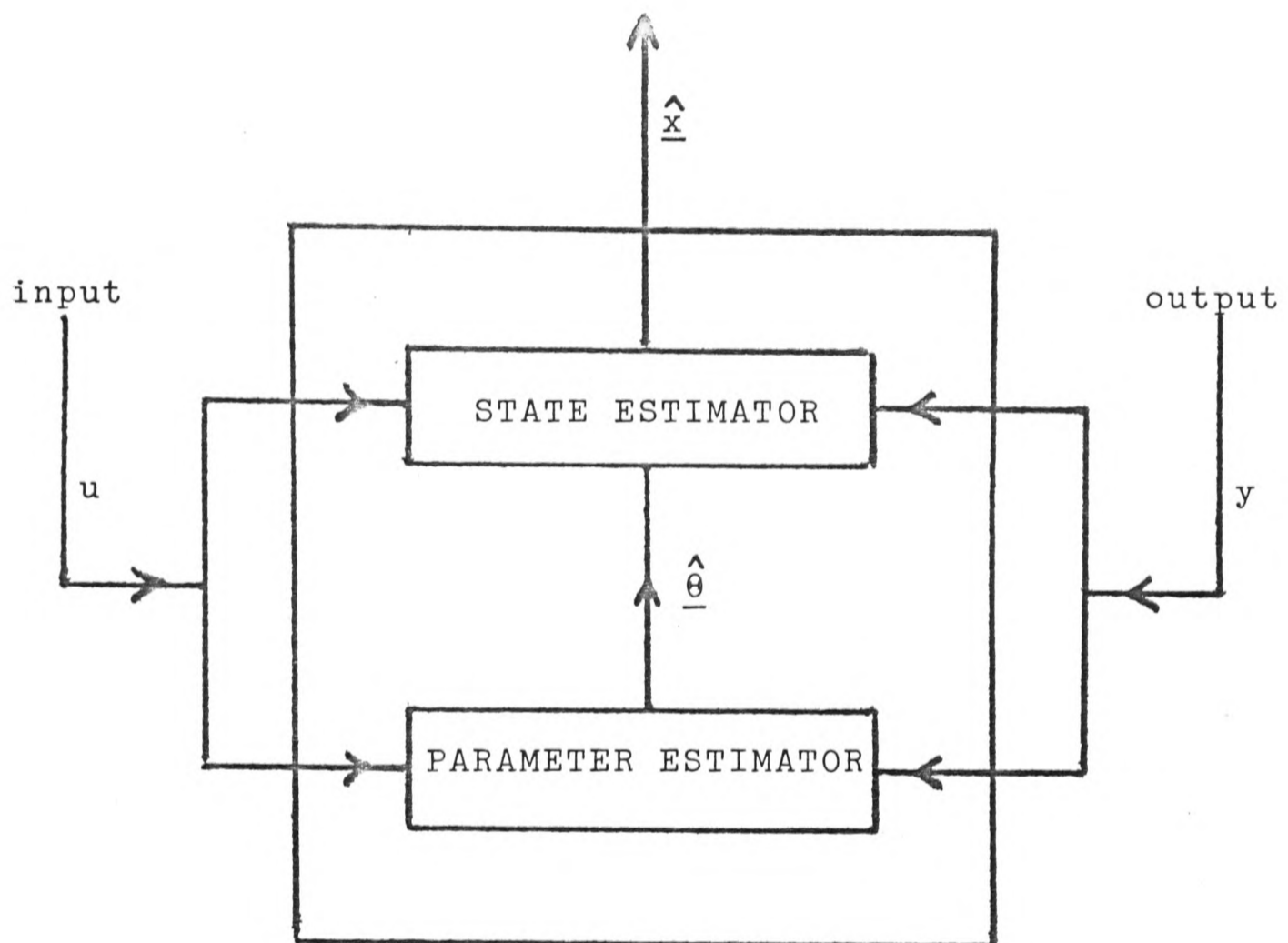


Fig.7.1 Structure of the combined state/parameter estimator.

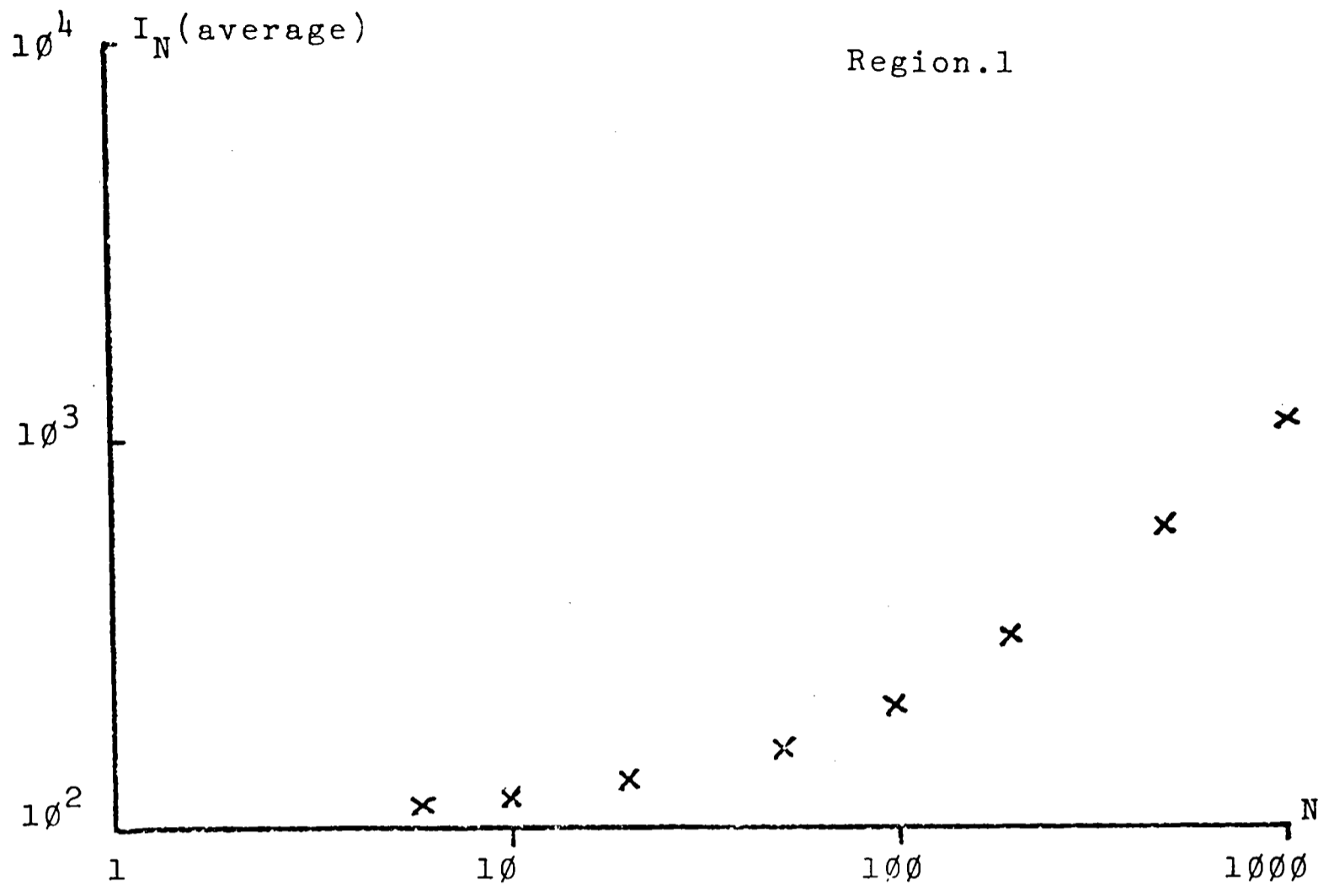


Fig.7.2 } OK controller with extended Kalman filter
 & OK controller with combined estimator (indistinguishable)

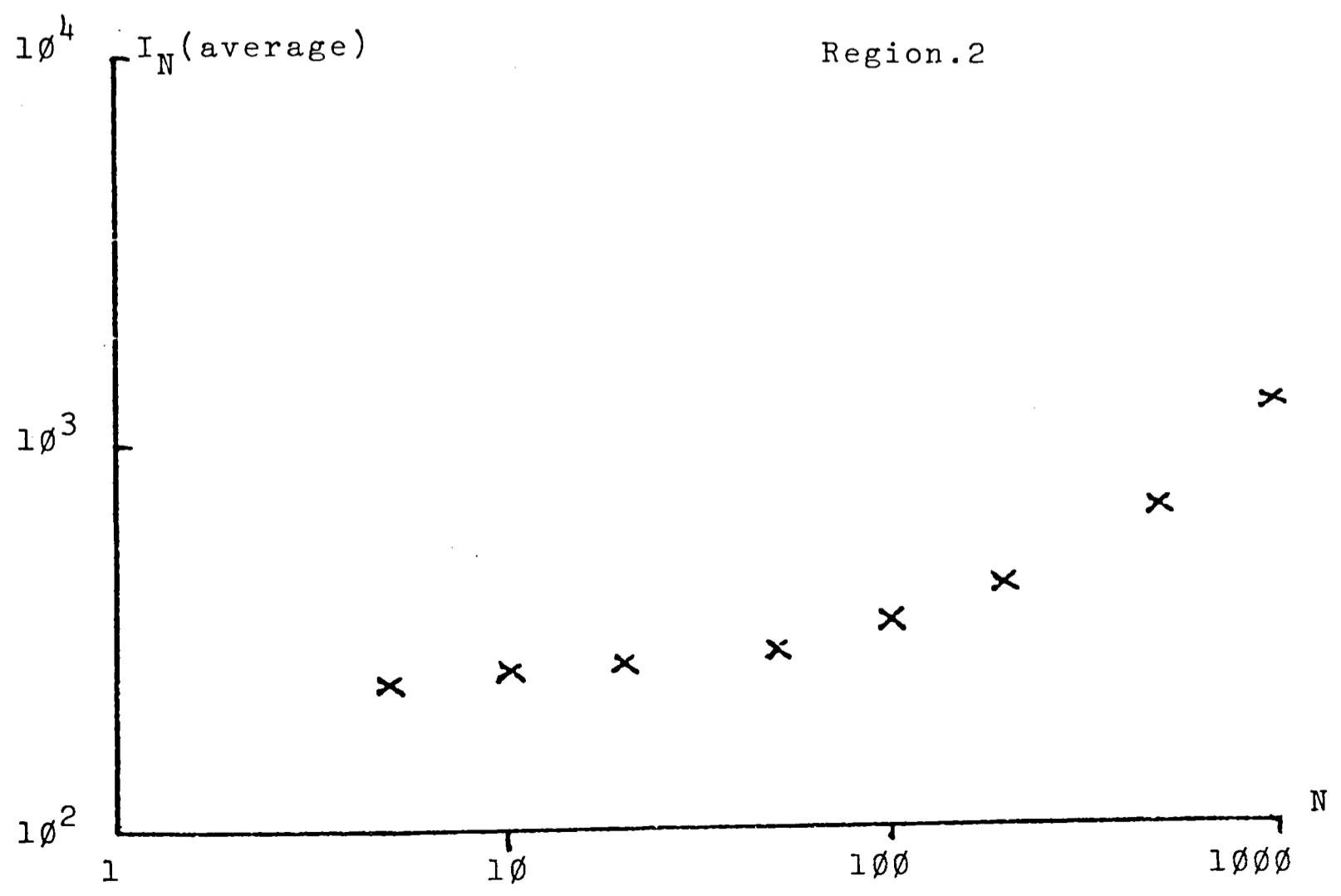


Fig.7.3 } OK controller with extended Kalman filter
 & OK controller with combined estimator (indistinguishable)

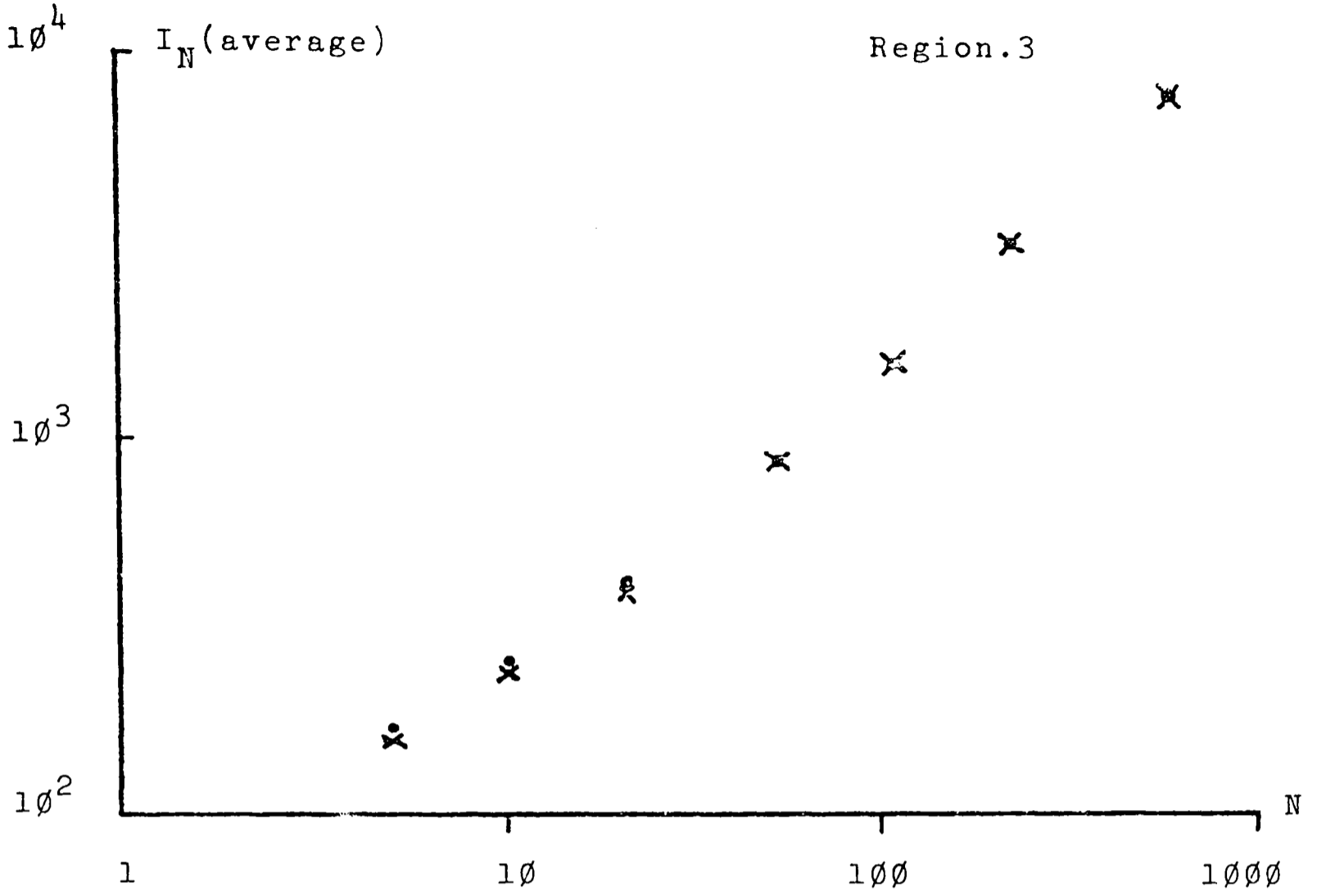


Fig.7.4 x OK controller with extended Kalman filter
 • OK controller with combined estimator.

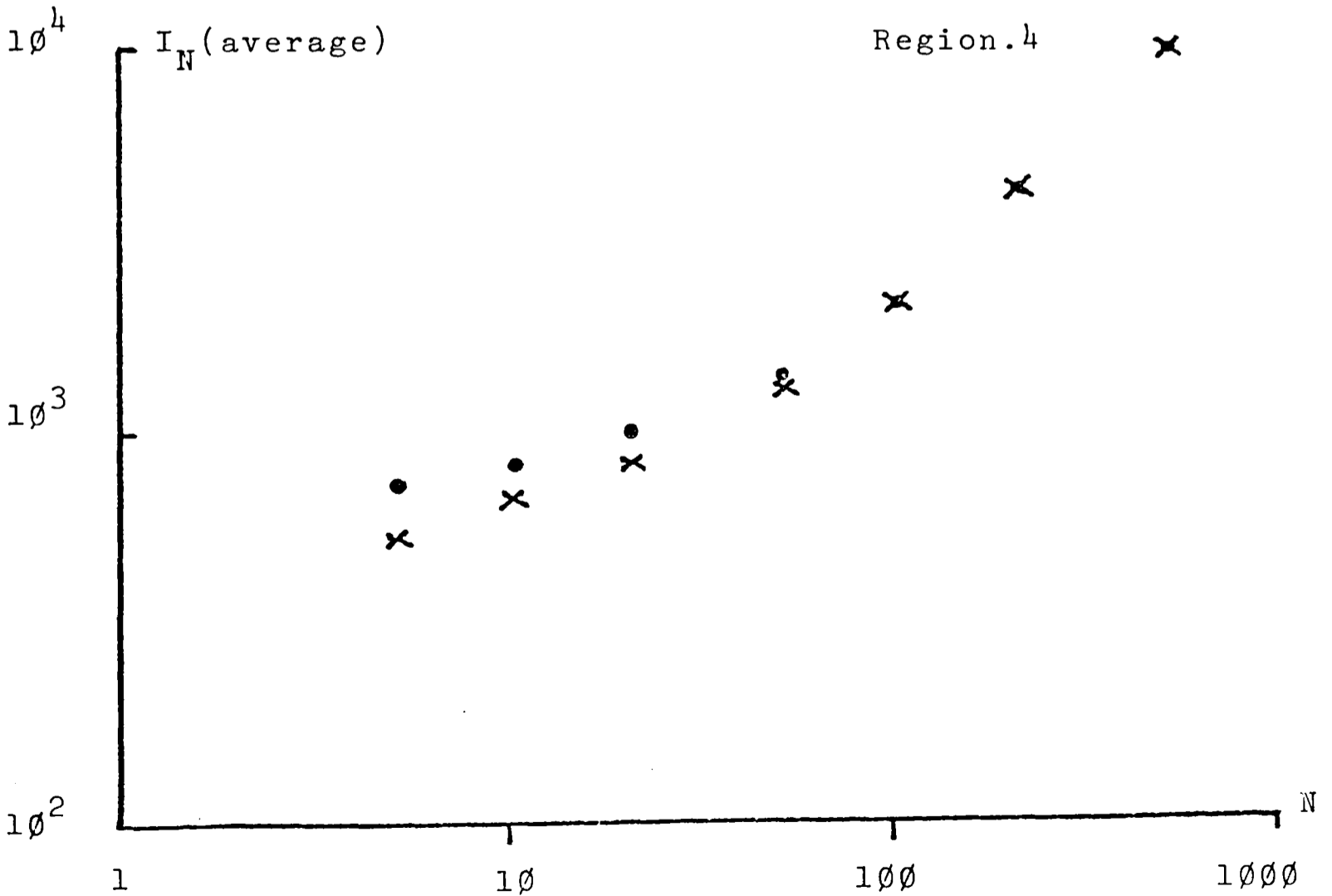


Fig.7.5 x OK controller with extended Kalman filter
 • OK controller with combined estimator.

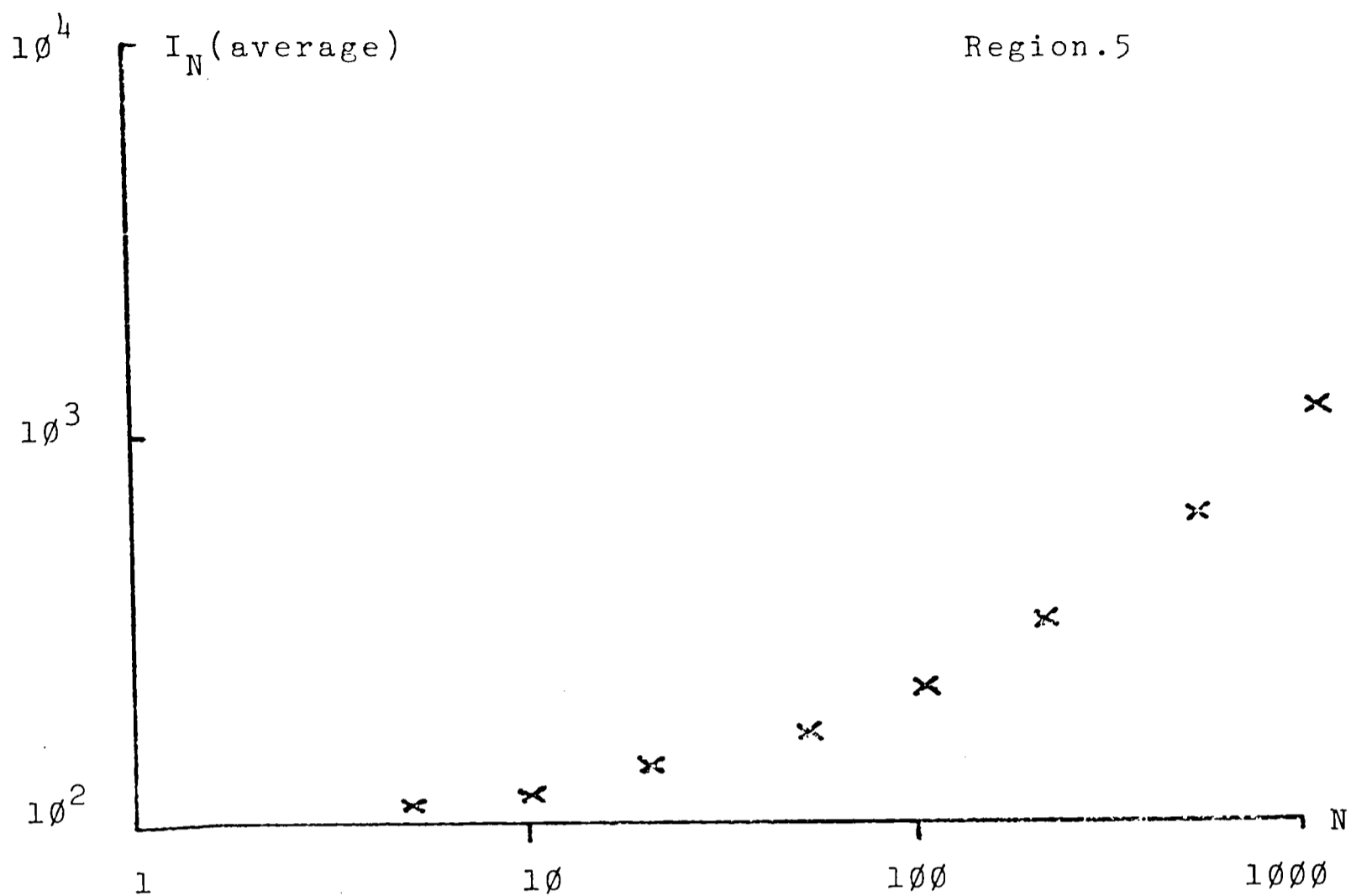


Fig.7.6 \times } OK controller with extended Kalman filter
 & OK controller with combined estimator (indistinguishable)

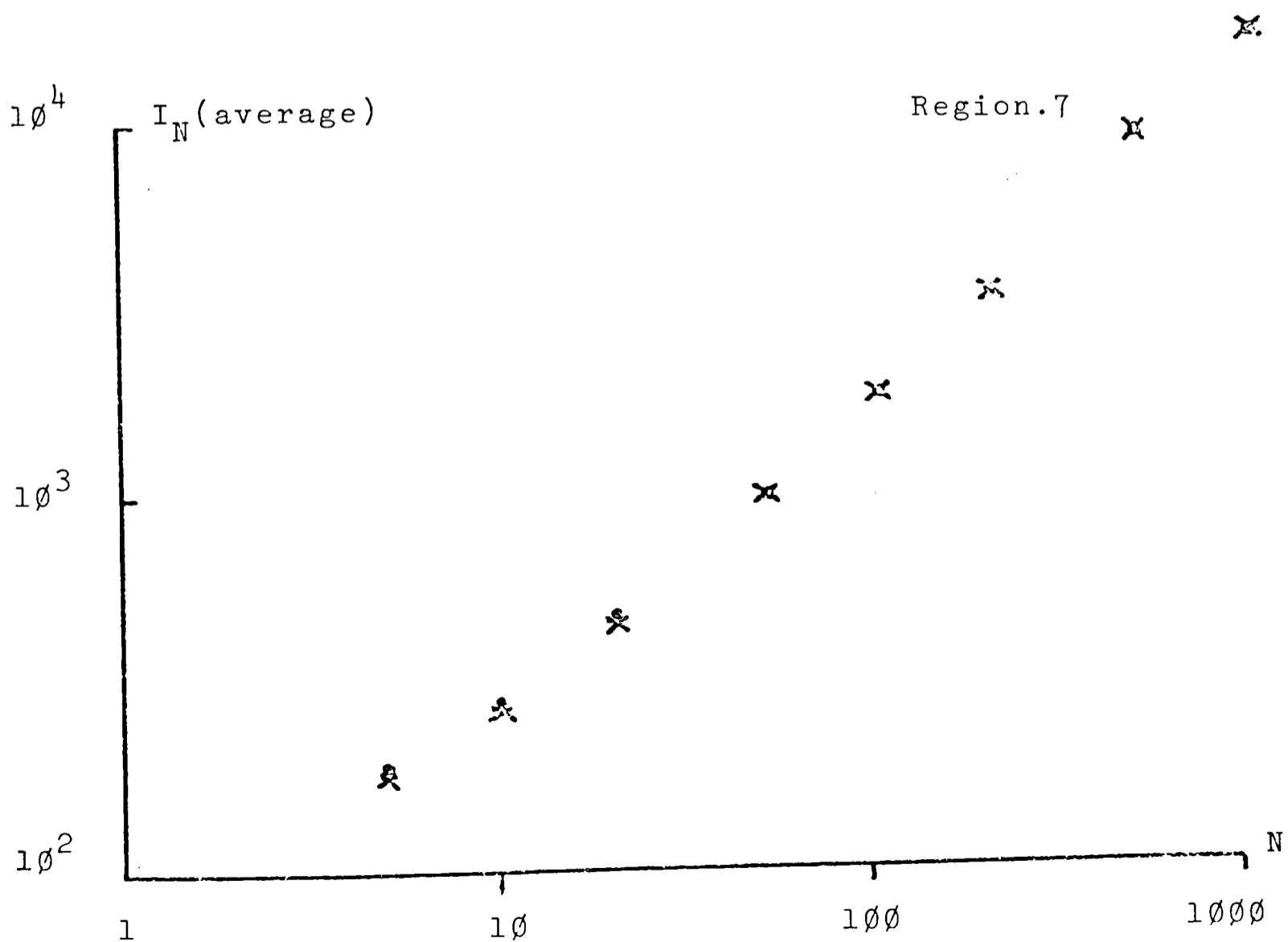


Fig.7.7 \times OK controller with extended Kalman filter
 • OK controller with combined estimator.

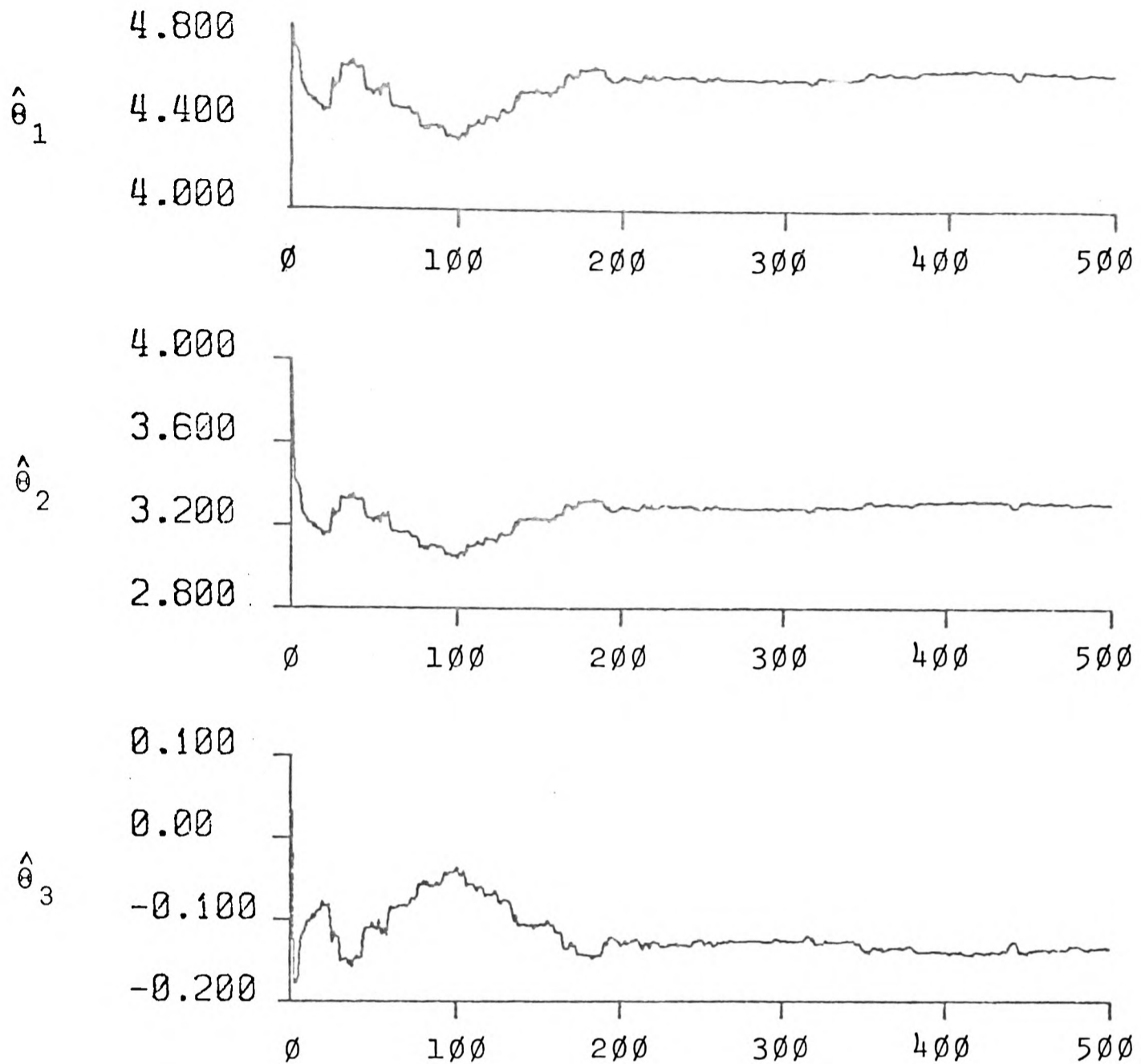


Fig.7.8 Parameter estimates $\hat{\underline{\theta}}$ from the combined estimator when used in the OK controller. True values are $\theta_1=4.025$, $\theta_2=2.860$, $\theta_3=-0.181$.

Time histories for the OK controller using (i) combined estimator, and (ii) extended Kalman filter.

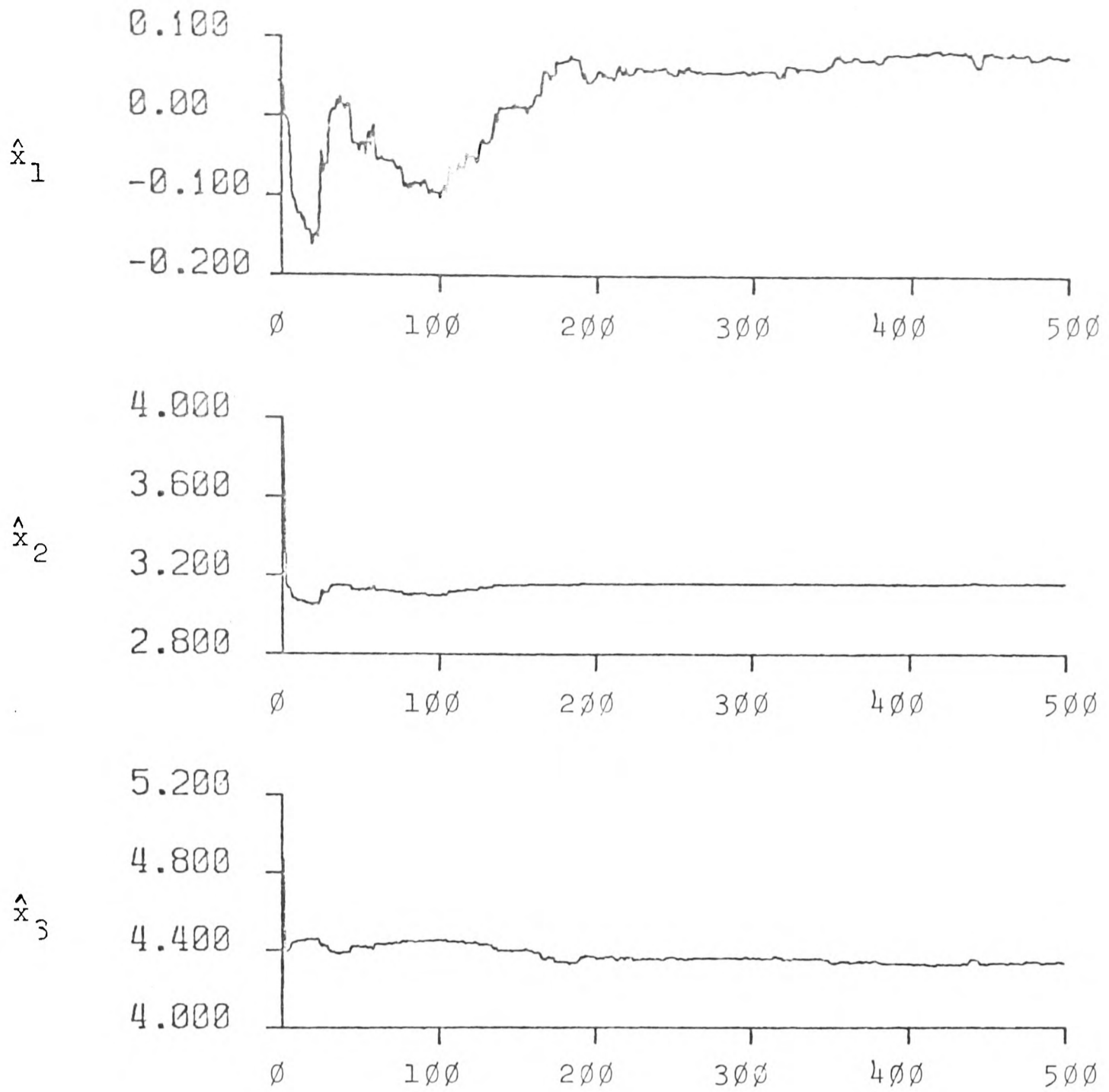


Fig.7.9 Parameter estimates $\hat{\underline{x}}$ from the extended Kalman filter used in the OK controller. True values are $x_1=0.181$, $x_2=2.860$, $x_3=3.844$.

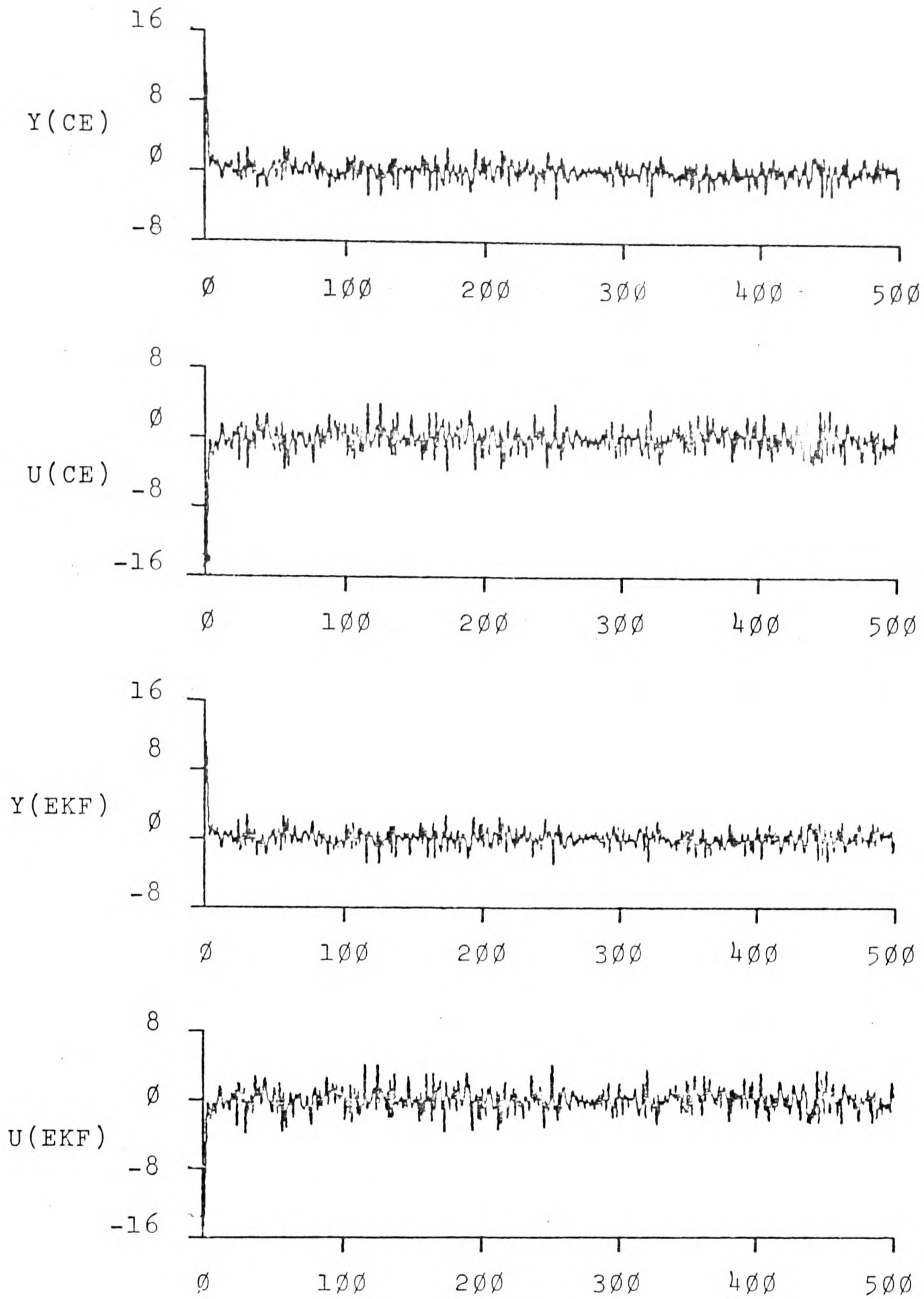


Fig.7.10 Controls and outputs for the OK controller with combined estimator(CE) and the OK controller with extended Kalman filter(EKF).

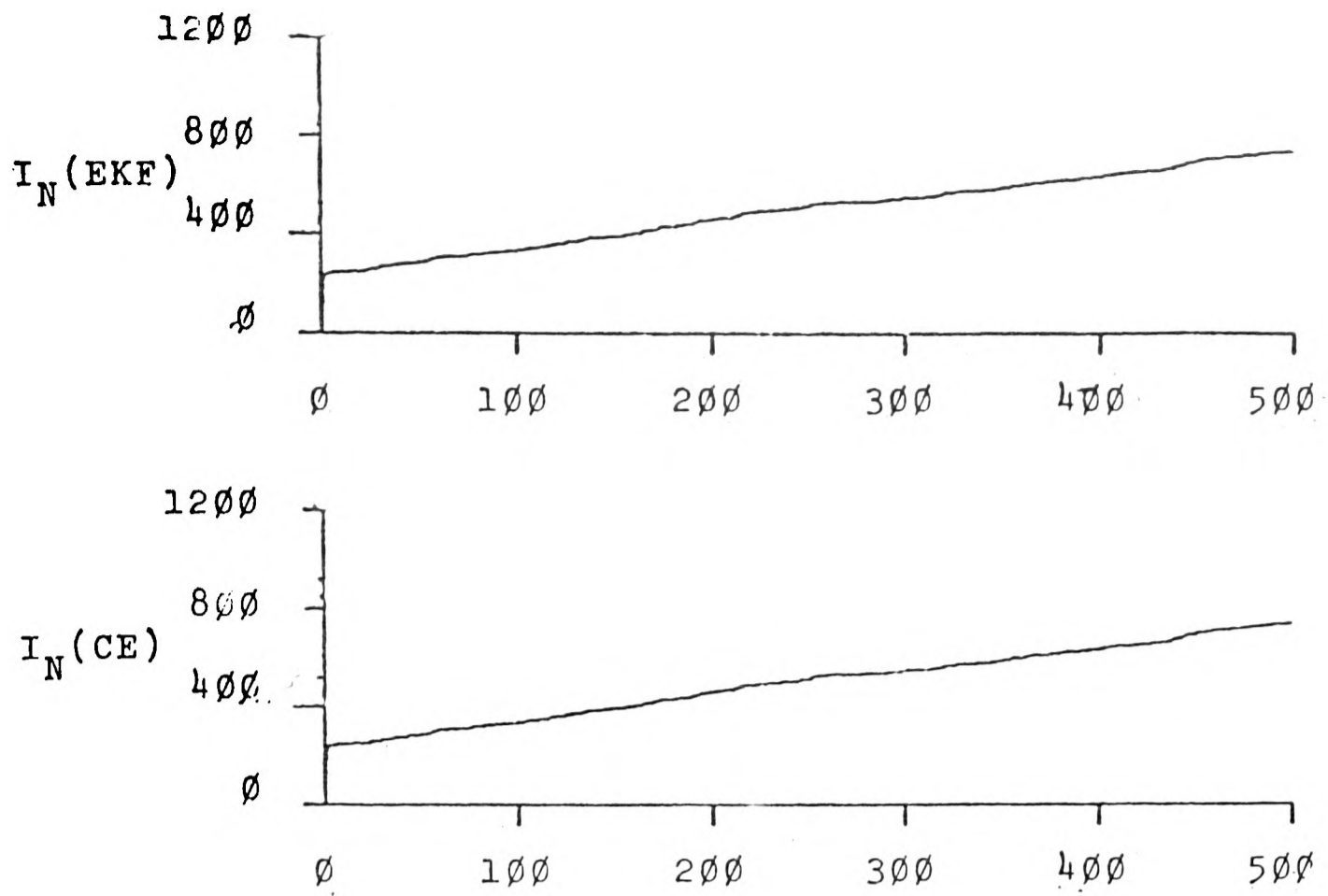


Fig.7.11 Cost (I_N) achieved by the OK controller with combined estimator(CE) and by the OK controller with extended Kalman filter(EKF).

CHAPTER EIGHT

CONCLUSIONS

8.1 Introduction

This thesis is concerned with the investigation of performances of the adaptive controllers - viz. OLFO controller, OK controller, ST controller, for a first order discrete-time single input single output stochastic process whose parameters are unknown using monte-carlo simulations. The following sections summarise the conclusions resulting from the investigation and also present some suggestions for future research.

8.2 Simulation study of OLFO controller

OLFO controller was simulated along with the two bounding (CAP and NOL) controllers for a simple case of equations 1.1 and 1.2. Even for this simple example OLFO controller took enormous computing time (12 times as much as that of lower bounding CAP controller) mainly because of its requirement to solve Riccati equations on-line. Further the cost achieved under OLFO controller was indistinguishably close to that of NOL controller for a stable controlled process and much higher than that of NOL controller for an unstable controlled process. This leads to the conclusion that OLFO controller is totally unsuitable for controlling unstable processes and would be an unwise choice for controlling stable processes.

8.3 Simulation study of OK and Self-tuning controller

8.3.1 Choice of Parameters

Adaptive problems were simulated in each of the eight regions of the unknown parameter space shown in fig 3.2. Data for the simulation, Table 5.1, were chosen to ensure that problems, generated at random as described in chapter 3, would remain within their specified regions. The discrete-time system simulated could be regarded as a sampled continuous-time system (for example under computer control) and it is of some interest to consider what is implied by the data of Table 5.1 about the relationship between

sampling rate and the response time of an underlying continuous-time process.

It is assumed that, as is usual with computer control, the continuous-time process is periodically sampled, at time intervals Δ , and that the sampled value is held constant (by a zero-order hold) over the sampling interval. There are two continuous-time processes which, when sampled in this way, would give rise to the discrete-time process of equation 1.1; they depend on the sign of the parameter 'a'. For $a < 0$, the first order discrete-time process of equation 1.1 could be generated by a first order continuous-time process of the form

$$Y(s) = \frac{K_1}{(1+sT_1)} U(s) + \frac{K_2(1+sT_2)}{(1+sT_1)} \Psi(s) \quad 8.1$$

where y, u, ψ are the continuous-time process output, control input and a zero mean gaussian stationary white noise disturbance respectively. 's' is the Laplace operator. This continuous-time process when sampled and held by a zero order hold gives the first order discrete-time process having

$$a = -e^{-\Delta/T_1} \quad 8.2a$$

$$b = K_1(1 - e^{-\Delta/T_1}) \quad 8.2b$$

$$c = -1 + \frac{T_1}{T_2}(1 - e^{-\Delta/T_1}) \quad 8.2c$$

$$\sigma_f^2 = \left(K_2 \frac{T_2}{T_1}\right)^2 \text{Var}(\Psi) \quad 8.2d$$

and a is always negative.

For $a > 0$, the first order discrete-time process could be generated by sampling, at a special frequency, an under damped ($\zeta < 1$) second order continuous-time process of the form

$$Y(s) = \frac{K_3 \omega_0^2}{s^2 + 2\zeta \omega_0 s + \omega_0^2} U(s) + \frac{K_4 (s^2 + \omega^2)}{(s^2 + 2\zeta \omega_0 s + \omega_0^2)} \Psi(s) \quad 8.3$$

The frequency ω_1 of oscillations of the process is related to ω_0 as $\omega_1 = \omega_0 \sqrt{1 - \zeta^2}$. This continuous-time process when sampled at

$$\Delta = \frac{k\pi}{\omega_1}, \quad k \text{ a positive odd integer,} \quad 8.4$$

and held by a zero order hold gives a first order discrete-time process having

$$a = e^{-\frac{k\pi\zeta}{(1-\zeta^2)^{1/2}}} \quad 8.5a$$

$$b = K_3 \left(1 + e^{-\frac{k\pi\zeta}{(1-\zeta^2)^{1/2}}} \right) \quad 8.5b$$

$$c = -1 + \frac{\omega^2}{\omega_0^2} \left(1 + e^{-\frac{k\pi\zeta}{(1-\zeta^2)^{1/2}}} \right) \quad 8.5c$$

$$\sigma_{\xi}^2 = (K_4)^2 \text{Var}(\Psi) \quad 8.5d$$

and a is always positive.

Table 8.1 gives the continuous-time parameter Δ or ζ for the discrete-time processes in the simulations specified by the data of Table 5.1. By 'likely values' in the table is meant those that lie between 'Low a ' and 'High a '. The table quotes 'Low a ' and 'High a ' as follows.

$$Av(a) = -(\hat{x}_1(0) + \hat{x}_3(0))$$

$$SD(a) = (\sigma_{11}(0) + \sigma_{33}(0))^{1/2}$$

$$\text{Low } a = Av(a) - SD(a)$$

$$\text{High } a = Av(a) + SD(a)$$

Region	Likely values of a		Continuous-time Parameter			
	Low a	High a	Δ (For a < 0)		\mathfrak{S} (For a > 0)	
			Low Δ	High Δ	Low \mathfrak{S}	High \mathfrak{S}
1,5 Av(a)=-0.2 SD(a)=0.28	-0.5 0	0 0.1	$0.7T_1$	∞	0.59	1
2,6 Av(a)=-4.0 SD(a)=1.02	-5	-3	$1.1 T_1 $	$1.61 T_1 $		
3,7 Av(a)=0.0 SD(a)=0.28	-0.3 0	0 0.3	$1.2T_1$	∞	0.36	1
4,8 Av(a)=-8 SD(a)=1.4	-9.4	-6.6	$1.89 T_1 $	$2.24 T_1 $		

Table 8.1 Continuous-time Parameter (Δ, \mathfrak{S}) corresponding to the likely values of a in the simulated discrete-time processes.

Whenever the likely values span over zero, as for regions 1,5,3,7, the likely values are grouped into two; (i) 'Low a' to $a=0$, and (ii) $a=0$ to 'High a'. The continuous-time parameter is obtained from equation 8.2a for $a < 0$ and 8.5a (assuming $k=1$) for $a > 0$. It can be seen that, for $a < 0$, the sampling interval Δ is mostly greater than the time constant T_1 of the continuous-time process which would therefore be sampled rather slowly. The simulations with $a < 0$ are thus mostly atypical of discrete-time control of first order continuous-time processes which would usually be sampled more rapidly, at a period of say $\Delta = 0.3 T_1$, giving rise to a value of $a = -0.74$. The simulations with $a > 0$ are also atypical because the condition of equation 8.4 also represent rather slow sampling and would not normally be achievable in practice.

8.3.2 Parameter variations

Consider the parameters of the continuous-time process to be time-varying and their dynamics are represented by a first order differential equation of the form

$$T_p \frac{dx}{dt} + x = k_p. \quad 8.6$$

When the continuous-time process is sampled at Δ intervals and held by a zero order hold the dynamics of the parameters in discrete-time would be of the form

$$x(i+1) = e^{-\Delta/T_p} x(i) + k_p (1 - e^{-\Delta/T_p}). \quad 8.7$$

The discrete-time model used to describe the parameter dynamics in the simulation study is (c.f. equation 2.2b)

$$x(i+1) = g x(i) + \alpha + \xi(i). \quad 8.7a$$

The disturbance is added in equation 8.7a to have continuing uncertainty about the parameter; it does not affect the speed of variation of the

parameter. Speed of variation of the parameter in equation 8.7a is governed by 'g' and comparing with 8.7

$$g = e^{-\Delta/T_p} \quad 8.7b$$

The value of 'g' used in the simulation study was $g=0.8$ and $g=0.6$. From equation 8.7b it can be seen that $g=0.8$, termed as 'slow' variation, corresponds to a situation when the parameter time constant T_p is 4.5 times the sampling interval Δ (or $T_p=4.5\Delta$) and $g=0.6$, termed as 'rapid' variation, corresponds to a situation when the parameter time constant T_p is twice the sampling interval ($T_p=2\Delta$).

8.3.3 Simulation results

Figs 5.1 - 5.24 which show the total cost against the number of stages N are perhaps not convenient to summarise the asymptotic behaviour of the various controllers. Asymptotic behaviour can be summarised by the incremental costs or rate of increase of average I_N with respect to N after many stages of operation. Small incremental costs represent good control and completely successful adaptation is indicated by incremental costs as small as those achieved by CAP controller.

Table 8.2 shows the incremental cost for 1000th stage (i.e. $y^2(1000) + q u^2(999)$) for CAP, OK, ST, NOL controllers for all the conditions of uncertainty in all the eight regions of the parameter space. The last column of the table contains the theoretically predicted incremental cost due to 'no control', obtained from equation A2.6 of Appendix 2, when the unknown parameters are constants and the process to be controlled is stable. The table together with figures 5.1 - 5.24 lead to the following conclusions.

(i) The performance of both OK and ST controller start to deteriorate as the parameters start varying with time. More the speed of parameter

	Conditions of uncer- tainty	CAP	OK	ST	NOL	No control
R e g i o n 1	g=1.0	1.005	1.005	1.005	1.009	1.022
	g=0.8	1.005	1.045	1.047	1.047	
	g=0.6	1.005	1.047	1.050	1.052	
R e g i o n 2	g=1.0	1.073	1.130	1.130	Huge	
	g=0.8	1.074	2.102	3.021	"	
	g=0.6	1.093	4.560	25.102	"	
R e g i o n 3	g=1.0	16.765	16.826	17.142	"	18.31
	g=0.8	16.807	17.472	24.487	"	
	g=0.6	16.847	17.486	93.360	"	
R e g i o n 4	g=1.0	20.160	20.332	22.786	"	
	g=0.8	20.301	562.44	Huge	"	
	g=0.6	20.34	Huge	"	"	

	Conditions of uncertainty	CAP	OK	ST	NOL	No control
R e g i o n 5	g=1.0	1.01	1.02	Huge	1.025	1.022
	g=0.8	1.021	1.076	"	1.085	
	g=0.6	1.025	1.087	"	1.093	
R e g i o n 6	g=1.0	1.00	Huge	"	Huge	
	g=0.8	1.00	"	"	"	
	g=0.6	1.00	"	"	"	
R e g i o n 7	g=1.0	16.671	17.65	"	"	18.31
	g=0.8	16.68	19.112	"	"	
	g=0.6	17.16	21.175	"	"	
R e g i o n 8	g=1.0	16.01	Huge	"	"	
	g=0.8	16.05	"	"	"	
	g=0.6	16.05	"	"	"	

Table 8.2 Incremental costs for 1000th stage.

(Note: For regions 6 and 8 the control weighting $q=0$)

variation, more is the deterioration in the performance.

(ii) In general OK controller produces lesser cost (both incremental and total) than ST controller for time-varying parameter processes. The difference between the costs due to ST and OK controller increases as the speed of parameter variation increases or 'g' of equation 2.2b decreases. Exceptions to this general conclusion are (i) regions 6 and 8 where both controllers performed badly even when the parameters were constants and (ii) region 4 with the parameters varying 'rapidly' ($g=0.6$) which also resulted in a bad performance for both controllers.

(iii) Table 8.3 shows the average cost/stage (Average I_N/N) for OK and ST controller when the unknown parameters are constants and the number of stages $N=5$. Average cost per stage for small N , such as $N=5$, is a good indicator of the transient performance of the controllers. As can be seen from the table the transient performance of ST controller is not as good as that of OK controller outside region 1. Since time-varying parameters can be considered to represent a continuing transient case it is not surprising that, outside region 1, OK controller produced lesser costs than ST controller for time-varying processes.

(iv) Both OK and ST controller seem to converge to the lower bounding CAP controller when the parameters are constants and there is no uncertainty regarding the sign of b . Convergence of OK controller is found to be never slower than and some times (region 4) slightly faster than that of ST controller. This can also be seen from figure 8.1 which shows excess incremental cost against number of stages for both these adaptive controllers in region 4. Excess incremental cost for an adaptive controller is the difference between the incremental cost for the adaptive controller and the incremental cost for CAP controller. This difference would go to

Region	OK	ST
1	22.65	22.93
2	46.16	47.05
3	33.83	36.81
4	109.52	136.34

Table 8.3 Average cost/stage when $N=5$ and unknown parameters constants.

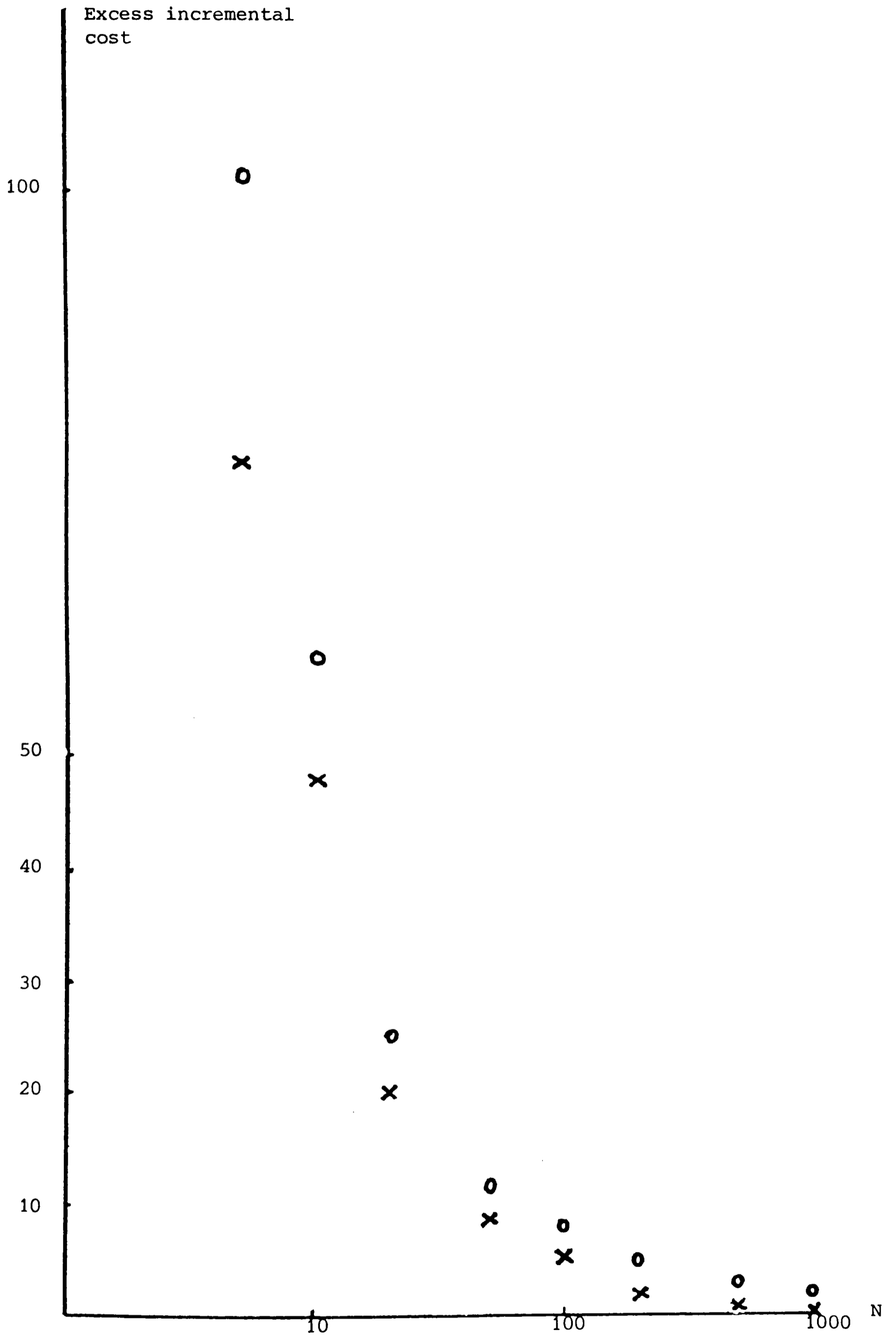


Fig.8.1 Excess incremental cost Vs Number of stages for OK controller (x) and ST controller (o). Unknown parameters are constants and are in Region.4.

zero as the adaptive controller converges to CAP controller.

(v) The computational requirement of ST controller is much less than that of OK controller. For the first order process ST controller took only half as much computing time as OK controller. Although use of the combined estimator in the place of extended Kalman filter reduced the computational requirement of OK controller, this reduced requirement is found to be still larger than that required by ST controller.

For constant unknown parameter processes with no uncertainty about the sign of b ST controller produced costs similar to those of OK controller. But when sign of b is not known (Regions 5 - 8) the ST controller implemented (i.e. $\hat{\theta}_2$ fixed) became unstable and produced huge costs. This bad performance was due to the violation of convergence and stability condition given in section 2.9. However this might have been avoided by fixing another parameter about which more is known instead of $\hat{\theta}_2$ in the Self-tuning algorithm and constraining the control magnitude. This has not been tried in the study reported here. Proper constraints on control magnitude may well make the ST controller perform as well in regions 5 and 7 as in regions 1 and 3. But in regions 6 and 8, since the process to be controlled is unstable and b is small, large inputs are needed to stabilise the process. So constraining the control magnitude in these regions could result in huge costs as observed by Sternby (32). Since OK controller also produced huge costs in these regions (6 and 8) it may be that adaptive control is impossible when the unknown parameters occupy regions 6 and 8.

It must be pointed out that in all the simulations the value of noise variance σ_ξ was assumed to be known exactly and only the parameters were considered unknown. This may be an idealised situation; in practice the noise variance is seldom known exactly. Since ST controller does not require knowledge of σ_ξ , its performance will not be sensitive to σ_ξ .

However the same cannot be said for OK controller. OK control law is cautious; it takes into account the uncertainties in the estimates of the parameters. Since σ_{ξ} affects these uncertainties, choosing a wrong value for σ_{ξ} would make all the estimates of the covariances of the parameters generated by the controller also wrong. How this 'erroneous' OK controller would perform has not been investigated in the thesis. Intuitively, too much undervaluing of σ_{ξ} ($\sigma_{\xi}(\text{controller}) \ll \sigma_{\xi}(\text{actual})$) should make the OK controller negligibly cautious resulting in a performance close to a certainty-equivalent controller and too much over valuing of σ_{ξ} ($\sigma_{\xi}(\text{controller}) \gg \sigma_{\xi}(\text{actual})$) should make the OK controller overly cautious and may result in a performance close to OLFO controller.

8.4 Convergence of OK controller

Convergence of OK controller using an extended Kalman filter was considered in chapter 6. Unfortunately it has not been possible to theoretically prove convergence of OK controller to the lower bounding CAP controller. However a necessary condition for convergence of OK controller to a linear constant parameter controller having the same functional form as CAP controller is found for an n^{th} order process. This condition is that the matrix pair $\{C, A(i)\}$ of the extended Kalman filter must be uniformly completely observable. For a first order process this condition is shown to reduce to a requirement that

$$\det \begin{vmatrix} \hat{x}_4(i) & \hat{x}_4(i+1) & \hat{x}_4(i+2) \\ u(i) & u(i+1) & u(i+2) \\ y(i) & y(i+1) & y(i+2) \end{vmatrix} \neq 0 \text{ for all } i.$$

The only occasion the above requirement would be violated before the OK

controller has converged is when there is 'turn off' which is characterised by the control staying at zero for finite lengths of time.

8.5 Combined Estimator

Simulation of OK controller with the combined estimator in the place of extended Kalman filter showed a reduction in the net computational requirement without any appreciable deterioration in the cost. This leads to the conclusion that the combined estimator should be preferred to extended Kalman filter as the suboptimal estimator in the OK controller.

8.6 Areas for further research

- (i) Since the combined estimator is found to be preferable to extended Kalman filter as the suboptimal estimator in OK controller, it may be worthwhile to study the convergence properties of OK controller with the combined estimator.
- (ii) To investigate the sensitivity of the performance of OK controller to the noise variance σ_{ξ} .
- (iii) Comparison of performance of these adaptive controllers for higher order ($n > 1$), higher time delay ($k > 1$) systems having a non-zero set point.

APPENDIX X.1

Derivation of open loop control sequence $u^o(j|i)$ via dynamic programming.

Problem Statement: Given the following dynamics for $\underline{z}(j|i)$

$$\begin{aligned} z_1(j+1|i) &= (x_1+x_3)z_1(j|i)+z_2(j|i)u(j), \\ z_2(j+1|i) &= z_2(j|i), \\ z_3(j+1|i) &= (x_1+x_3)^2 z_3(j|i)+u^2(j)z_5(j|i) \\ &\quad + 2(x_1+x_3)u(j)z_4(j|i) + x_3^2 \sigma_3^2, \\ z_4(j+1|i) &= (x_1+x_3)z_4(j|i)+u(j)z_5(j|i), \\ z_5(j+1|i) &= z_5(j|i), \end{aligned} \tag{4.4a}$$

find the control sequence $u(j|i)$ that minimises $J'_n(i)$,

$$J'_n(i) = \sum_{j=i}^{i_f} \{z_1^2(j|i)+z_3(j|i)\}. \tag{4.5}$$

Functional recurrence equation is given by (c f equation 2.7)

$$f_n(\underline{z}(j)) = \text{Min}_{u(j)} \{z_1^2(j)+z_3(j)+f_{n-1}(z(j+1))\}^\dagger. \tag{A1}$$

Assume $f_n(\underline{z}(j))$ has the following form.

$$\begin{aligned} f_n(\underline{z}(j)) &= \begin{bmatrix} z_1(j) \\ z_4(j) \end{bmatrix}^T \begin{bmatrix} V_{11}(j) & V_{12}(j) \\ V_{12}(j) & V_{22}(j) \end{bmatrix} \begin{bmatrix} z_1(j) \\ z_4(j) \end{bmatrix} \\ &\quad + B(j)z_3(j)+C(j) \end{aligned} \tag{A2}$$

† Time index ($|i$) is dropped for notational simplicity.

Functional recurrence equation A1, using the above form for $f_{n-1}(\underline{z}(j+1))$, becomes

$$f_n(\underline{z}(j)) = \text{Min}_{u(j)} \left\{ z_1^2(j) + z_3(j) + V_{11}(j+1) z_1^2(j+1) + 2V_{12}(j+1) z_1(j+1) \cdot z_4(j+1) + V_{22}(j+1) z_4^2(j+1) + B(j+1) z_3(j+1) + C(j+1) \right\} .$$

Substituting for $z_1(j+1)$, $z_3(j+1)$ and $z_4(j+1)$ from equation 4.4a $f_n(\underline{z}(j))$ becomes

$$f_n(\underline{z}(j)) = \text{Min}_{u(j)} \left\{ z_1^2(j) + z_3(j) + E + F u(j) + G u^2(j) \right\} , \quad A3$$

where

$$E \equiv (x_1 + x_3)^2 \left\{ V_{11}(j+1) z_1^2(j) + 2V_{12}(j+1) z_1(j) z_4(j) + V_{22}(j+1) z_4^2(j) + B(j+1) z_3(j) \right\} + B(j+1) x_3^2 \sigma_\xi^2 + C(j+1) , \quad A4$$

$$F \equiv 2(x_1 + x_3) \left\{ (V_{11}(j+1) z_2(j) + V_{12}(j+1) z_5(j)) z_1(j) + (V_{12}(j+1) z_2(j) + V_{22}(j+1) z_5(j) + B(j+1)) z_4(j) \right\} , \quad A5$$

$$G \equiv \left\{ V_{11}(j+1) z_1^2(j) + 2V_{12}(j+1) z_2(j) z_5(j) + V_{22}(j+1) z_5^2(j) + B(j+1) z_5(j) \right\} . \quad A6$$

The $u(j)$ that minimises right hand side of A3 is

$$u(j) = -\frac{F}{2G} . \quad A7$$

The $\{u(j)\}_{j=i}^{i_f-1}$ would be the open loop control sequence provided the assumed form A2 for $f_n(\underline{z}(j))$ is correct.

Substituting $u(j)$ into equation A3 gives

$$f_n(\underline{z}(j)) = z_1^2(j) + z_3(j) + E - \frac{F^2}{4G} .$$

Again substituting E, F, G from A4, A5, A6 in the above equation gives

$$\begin{aligned}
 f_n(z(j)) = & z_1^2(j) \left\{ 1 + (x_1 + x_3)^2 \left[z_5^2(j) (V_{11}(j+1)V_{22}(j+1) - V_{12}^2(j+1)) \right. \right. \\
 & + B(j+1) V_{11}(j+1) z_5(j) \left. \left. \left[z_2^2(j) V_{11}(j+1) + z_5^2(j) V_{22}(j+1) \right. \right. \right. \\
 & \left. \left. \left. + 2 z_2(j) z_5(j) V_{12}(j+1) + B(j+1) z_5(j) \right]^{-1} \right\} + \\
 & 2 z_1(j) z_4(j) \left\{ (x_1 + x_3)^2 \left[z_2(j) z_5(j) (V_{12}^2(j+1) - V_{11}(j+1)V_{22}(j+1)) \right. \right. \\
 & - B(j+1) V_{11}(j+1) z_2(j) \left. \left. \left[z_2^2(j) V_{11}(j+1) + z_5^2(j) V_{22}(j+1) \right. \right. \right. \\
 & \left. \left. \left. + 2 z_2(j) z_5(j) V_{12}(j+1) + B(j+1) z_5(j) \right]^{-1} \right\} + \\
 & z_4^2(j) \left\{ (x_1 + x_3)^2 \left[z_2^2(j) (V_{11}(j+1)V_{22}(j+1) - V_{12}^2(j+1)) \right. \right. \\
 & - B(j+1) (B(j+1) + 2 V_{12}(j+1) z_2(j) + V_{22}(j+1) z_5(j)) \left. \left. \left[z_2^2(j) V_{11}(j+1) \right. \right. \right. \\
 & \left. \left. \left. + z_5^2(j) V_{22}(j+1) + 2 z_2(j) z_5(j) V_{12}(j+1) + B(j+1) z_5(j) \right]^{-1} \right\} + \\
 & z_3(j) \left\{ 1 + (x_1 + x_3)^2 B(j+1) \right\} + \left\{ C(j+1) + B(j+1) x_3^2 \sigma_{\xi}^2 \right\} \quad A8
 \end{aligned}$$

$$\text{or} \quad f_n(z(j)) = \begin{bmatrix} z_1(j) \\ z_4(j) \end{bmatrix}^T \begin{bmatrix} V_{11}(j) & V_{12}(j) \\ V_{12}(j) & V_{22}(j) \end{bmatrix} \begin{bmatrix} z_1(j) \\ z_4(j) \end{bmatrix} +$$

$$B(j) z_3(j) + C(j)$$

Thus the assumed form A2 for $f_n(\underline{z}(j))$ is correct.

The open loop control sequence, after substituting for F and G in equation A7, is given by

$$u^o(j|i) = -K_1^{-1}(j|i) \left\{ (x_1 + x_3) \left[V_{11}(j+1|i) z_2(j|i) + V_{12}(j+1|i) z_5(j|i) \right] z_1(j|i) + \left(V_{12}(j+1|i) z_2(j|i) + V_{22}(j+1|i) z_5(j|i) + B(j+1|i) \right) z_4(j|i) \right\} \quad A9$$

$$\text{where } K_1(j|i) = z_2^2(j|i) V_{11}(j+1|i) + z_5^2(j|i) V_{22}(j+1|i) + 2 z_2(j|i) z_5(j|i) V_{12}(j+1|i) + B(j+1|i) z_5(j|i)$$

where the time index ($|i$) is re-introduced.

Open loop feedback optimal control, $u^{\text{OLFO}}(i)$, is the first control of the open loop loop optimal control sequence; i.e. $u^o(i|i)$.

$$u^{\text{OLFO}}(i) = -K^{-1}(i|i) \left\{ (x_1 \hat{x}_4(i|i) + x_3 y(i)) \left(V_{11}(i+1|i) \hat{x}_2(i|i) + V_{12}(i+1|i) \sigma_{22}(i|i) \right) + x_1 \sigma_{24}(i|i) \left(V_{12}(i+1|i) \hat{x}_2(i|i) + V_{22}(i+1|i) \sigma_{22}(i|i) + B(i+1|i) \right) \right\} \quad A10$$

where $z_1(i|i)$, $z_2(i|i)$, $z_4(i|i)$, $z_5(i|i)$ are substituted in terms of $\hat{x}_4(i|i)$, $\hat{x}_2(i|i)$, $\sigma_{24}(i|i)$, $\sigma_{22}(i|i)$ according to equation 4.4b of chapter 4.

$$K(i|i) = \begin{bmatrix} \hat{x}_2(i|i) \\ \sigma_{22}(i|i) \end{bmatrix}^T \begin{bmatrix} V_{11}(i+1|i) & V_{12}(i+1|i) \\ V_{12}(i+1|i) & V_{22}(i+1|i) \end{bmatrix} \begin{bmatrix} \hat{x}_2(i|i) \\ \sigma_{22}(i|i) \end{bmatrix} + B(i+1|i) \sigma_{22}(i|i)$$

V_{11} , V_{12} , V_{22} , B are solved backwards in time according to the following equations

$$V_{11}(j|i) = 1 + (x_1 + x_3)^2 \left[\sigma_{22}^2(j|i) \left(V_{11}(j+1|i) V_{22}(j+1|i) - V_{12}^2(j+1|i) \right) + B(j+1|i) V_{11}(j+1|i) \sigma_{22}(j|i) \right] \kappa^{-1}(j|i) \quad A11$$

$$V_{12}(j|i) = (x_1 + x_3)^2 \left[\hat{x}_2(j|i) \sigma_{22}(j|i) \left(V_{12}^2(j+1|i) - V_{11}(j+1|i) V_{22}(j+1|i) \right) - B(j+1|i) V_{11}(j+1|i) \hat{x}_2(j|i) \right] \kappa^{-1}(j|i) \quad A12$$

$$V_{22}(j|i) = (x_1 + x_3)^2 \left[\hat{x}_2^2(j|i) \left(V_{11}(j+1|i) V_{22}(j+1|i) - V_{12}^2(j+1|i) \right) - B(j+1|i) \{ B(j+1|i) + 2 V_{12}(j+1|i) \hat{x}_2(j|i) + V_{22}(j+1|i) \sigma_{22}(j|i) \} \right] \kappa^{-1}(j|i) \quad A13$$

$$B(j|i) = 1 + (x_1 + x_3)^2 B(j+1|i) \quad A14$$

$$V_{11}(i_f|i) = 1.0, \quad V_{12}(i_f|i) = 0.0, \quad V_{22}(i_f|i) = 0.0,$$

$$B(i_f|i) = 1.0.$$

APPENDIX.2

Derivation of 'no control' cost for constant unknown parameter process:

Process dynamics under 'no control' ($u=0$) is given by (c.f. equation 1.1)

$$y(i) + ay(i-1) = \xi(i) + c\xi(i-1). \quad A2.1$$

Using the backward shift operator z^{-1} , the process dynamics of A2.1 can be written as

$$y(i) = \frac{1 + cz^{-1}}{1 + az^{-1}} \xi(i). \quad A2.2$$

The cost under 'no control' will be finite only if equation A2.1 is stable or $|a| < 1$. When $|a| < 1$, using standard integration tables (37, Ch.6), the expected incremental cost for any given a, c can be expressed as

$$\frac{(1 + c^2 - 2ca) \sigma_{\xi}}{(1 - a^2)}, \quad A2.3$$

where σ_{ξ} is the variance of ξ . In terms of x_1, x_3 the expected incremental cost when $|x_1 + x_3| < 1$ is

$$\frac{(1 - x_1^2 - 2x_1x_3) \sigma_{\xi}}{1 - (x_1 + x_3)^2} \quad A2.3a$$

When the true values of x_1, x_3 are not known and are only specified by normal distributions, the expected incremental cost for no control would be the expected value of the expression A2.3a;

$$E \left[\frac{1 - x_1^2 - 2x_1x_3}{1 - (x_1 + x_3)^2} \right] \sigma_{\xi}. \quad A2.4$$

Expanding the denominator of A2.4 as a series, which is possible since $|x_1 + x_3|$ (or $|a|$) should be less than one for 'no control' to produce finite cost, the expression for expected incremental cost

becomes

$$E \left[(1 - x_1^2 - 2x_1x_3) \left\{ 1 + (x_1 + x_3)^2 + (x_1 + x_3)^4 + \dots \right\} \right] \sigma_{\xi}. \quad A2.5$$

Since the parameters x_1, x_3 are specified by normal distributions all their higher moments in A2.5 can be expressed in terms of their mean and variance. The expression for incremental cost when the series for $(1 - (x_1 + x_3)^2)^{-1}$ is approximated to two terms is derived below.

$$\begin{aligned} \left. \begin{array}{l} \text{Expected incremental} \\ \text{cost} \end{array} \right\} &\approx E \left[(1 - x_1^2 - 2x_1x_3) \left\{ (1 + (x_1 + x_3)^2) \right\} \right] \sigma_{\xi} \\ &\approx E \left[(1 - x_1^2 - 2x_1x_3) + (x_1^2 + x_3^2 + 2x_1x_3 - x_1^4 \right. \\ &\quad \left. - 5x_1^2x_3^2 - 4x_1^3x_3 - 2x_1x_3^3) \right] \sigma_{\xi}. \end{aligned}$$

Using standard formulas for higher moments of normal distributions (40) and noting that x_1 and x_3 are independent parameters, the above expectation can be written as

$$\begin{aligned} \left. \begin{array}{l} \text{Expected incremental} \\ \text{cost} \end{array} \right\} &\approx \left\{ (1 - m_1^2 - \sigma_{11} - 2m_1m_3) + [m_1^2 + \sigma_{11} + m_3^2 \right. \\ &\quad \left. + \sigma_{33} + 2m_1m_3 - (m_1^4 + 6m_1^2\sigma_{11} + 3\sigma_{11}^2) \right. \\ &\quad \left. - 5(m_1^2 + \sigma_{11})(m_3^2 + \sigma_{33}) - 4(m_1^3 + 3m_1\sigma_{11})(m_3) \right. \\ &\quad \left. - 2m_1(m_3^3 + 3m_3\sigma_{33}) \right\} \sigma_{\xi}, \quad A2.6 \end{aligned}$$

where $m_1, m_3, \sigma_{11}, \sigma_{33}$ are the mean and variance of the normal distributions that specify x_1 and x_3 .

Considering more terms in the series for $(1 - (x_1 + x_3)^2)^{-1}$ gives more accurate but more complicated expressions for the expected incremental cost

In the simulation study for regions 1 and 5, $m_1 = 0.10, \sigma_{11} = 0.04, m_3 = 0.10, \sigma_{33} = 0.04$. Substituting these values in equation A2.6 gives the expected incremental cost for 'no control' in these regions 1.022. Similarly in the simulation study for regions 3 and 7 $m_1 = 4.0, \sigma_{11} = 0.04, m_3 = -4.0, \sigma_{33} = 0.04$ and the expected incremental cost using equation A2.6 is 18.31 .

The relationship of the coefficients $\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_n, \gamma_1, \dots, \gamma_n$ to a s, b s, and c s of equation 6.1 is given by equation 6.2a of chapter 6.

The functional recurrence equation used for deriving OK control law is given by

$$f_n(i) = \text{Min}_{u(i)} \left\{ y^2(i) + q u^2(i) + E [y^2(i+k) | i] \right\} . \quad 2.8$$

To derive OK control law for a general k time delay case involves extremely tedious algebra. However it is possible to show through examples that to implement OK controller for an n^{th} order process having a delay k requires the generation of the estimates of all the $3n$ unknown parameters, $(n+k-1)$ states and their covariances.

Example 1. This example considers an n^{th} order process having a time delay $k=1$.

Functional recurrence equation 2.8 when $k=1$ becomes

$$f_n(i) = \text{Min}_{u(i)} \left\{ y^2(i) + q u^2(i) + E [y^2(i+1) | i] \right\} .$$

Substitution of equations 6.2b, 6.3 into the above gives

$$f_n(i) = \text{Min}_{u(i)} \left\{ y^2(i) + q u^2(i) + E \left[(\alpha_1 x_1(i) + \beta_1 u(i) + \gamma_1 y(i) + \xi(i+1))^2 | i \right] \right\} .$$

The $u(i)$ that minimises the right hand side of the above expression is $u^{ok}(i)$,

$$u^{ok}(i) = - \frac{E[\alpha_1 \beta_1 x_1 | i] + E[\beta_1 x_2 | i] + E[\gamma_1 \beta_1 | i] y(i)}{E[\beta_1^2 | i] + q} \quad A3.1$$

Assuming the conditional distributions to be normal $u^{ok}(i)$ becomes,

$$u^{ok}(i) = - \left\{ \hat{\alpha}_1 \hat{\beta}_1 \hat{x}_1 + \hat{\alpha}_1 \overline{\sigma_{\beta_1 x_1}} + \hat{\beta}_1 \overline{\sigma_{\alpha_1 x_1}} + \hat{x}_1 \overline{\sigma_{\alpha_1 \beta_1}} + \hat{\beta}_1 \hat{x}_2 + \overline{\sigma_{\beta_1 x_2}} + y(i) [\hat{\gamma}_1 \hat{\beta}_1 + \overline{\sigma_{\beta_1 \gamma_1}}] \right\} (\hat{\beta}_1^2 + \overline{\sigma_{\beta_1 \beta_1}} + q)^{-1}.$$

The control law requires only the estimates of the parameters $\alpha_1, \beta_1, \gamma_1$ and the states x_1, x_2 and their covariances. But to generate these estimates it is necessary to generate the estimates of all the other states, parameters and their covariances. So for an n^{th} order process having unit time delay the OK controller would have to generate the estimates of all the $3n$ unknown parameters, all the n states and their covariances.

Example 2. This example considers an n^{th} order process having a time delay $k=2$.

Functional recurrence equation 2.8 in this case becomes

$$f_n(i) = \min_{u(i)} \left\{ y^2(i) + q u^2(i) + E[y^2(i+2) | i] \right\}.$$

Substitution of equations 6.2b, 6.3 into the above equation, after

some algebraic manipulation, yields

$$f_n(i) = \text{Min}_{u(i)} \left\{ E \left[q u^2 + \beta_1^2 u^2 + 2 \beta_1 u \left\{ (\alpha_1 (\alpha_1 + \gamma_1) + \alpha_2) x_1 + (\alpha_1 + \gamma_1) x_2 + x_3 + (\gamma_1 (\gamma_1 + \alpha_1) + \gamma_2) y \right\} + \text{Non } u \text{ terms} \right] \middle| i \right\}$$

The $u(i)$ that minimises the right hand side of the above expression is $u^{\text{ok}}(i)$

$$u^{\text{ok}}(i) = - \left\{ E \left[(\alpha_1 (\alpha_1 + \gamma_1) + \alpha_2) \beta_1 x_1 \middle| i \right] + E \left[(\alpha_1 + \gamma_1) \beta_1 x_2 \middle| i \right] + E \left[\beta_1 x_3 \middle| i \right] + E \left[(\gamma_1 (\alpha_1 + \gamma_1) + \gamma_2) \beta_1 \middle| i \right] y(i) \right\} \cdot \left(E \left[\beta_1^2 \middle| i \right] + q \right)^{-1}$$

Assuming the conditional distributions to be normal $u^{\text{ok}}(i)$ becomes,

$$u^{\text{ok}}(i) = - \left\{ \begin{aligned} & [\hat{\alpha}_1^2 \hat{\beta}_1 \hat{x}_1 + 2 \hat{\alpha}_1 \hat{\beta}_1 \sigma_{\alpha_1 x_1} + 2 \hat{\alpha}_1 \hat{x}_1 \sigma_{\alpha_1 \beta_1} + \hat{\alpha}_1^2 \sigma_{\beta_1 x_1} \\ & + \hat{\beta}_1 \hat{x}_1 \sigma_{\alpha_1 \alpha_1} + 2 \sigma_{\alpha_1 \beta_1} \sigma_{\alpha_1 x_1} + \sigma_{\alpha_1 \alpha_1} \cdot \sigma_{\beta_1 x_1} + \hat{\alpha}_1 \hat{\beta}_1 \hat{\gamma}_1 \hat{x}_1 \\ & + \hat{\alpha}_1 \hat{\beta}_1 \sigma_{\gamma_1 x_1} + \hat{\alpha}_1 \hat{\gamma}_1 \sigma_{\beta_1 x_1} + \hat{\alpha}_1 \hat{x}_1 \sigma_{\beta_1 \gamma_1} + \hat{\beta}_1 \hat{\gamma}_1 \sigma_{\alpha_1 x_1} \\ & + \hat{\beta}_1 \hat{x}_1 \sigma_{\alpha_1 \gamma_1} + \hat{\gamma}_1 \hat{x}_1 \sigma_{\alpha_1 \beta_1} + \sigma_{\alpha_1 \beta_1} \cdot \sigma_{\gamma_1 x_1} + \sigma_{\alpha_1 \gamma_1} \cdot \sigma_{\beta_1 x_1} \\ & + \sigma_{\alpha_1 x_1} \cdot \sigma_{\beta_1 \gamma_1} + \hat{\alpha}_2 \hat{\beta}_1 \hat{x}_1 + \hat{\alpha}_2 \sigma_{\beta_1 x_1} + \hat{\beta}_1 \sigma_{\alpha_2 x_1} + \hat{x}_1 \sigma_{\alpha_2 \beta_1} \\ & + [\hat{\alpha}_1 \hat{\beta}_1 \hat{x}_2 + \hat{\alpha}_1 \sigma_{\beta_1 x_2} + \hat{\beta}_1 \sigma_{\alpha_1 x_2} + \hat{x}_2 \sigma_{\alpha_1 \beta_1} + \hat{\gamma}_1 \hat{\beta}_1 \hat{x}_2 \end{aligned} \right.$$

$$\begin{aligned}
& + \hat{\gamma}_1 \overline{\sigma_{\beta_1 x_2}} + \hat{\beta}_1 \overline{\sigma_{\gamma_1 x_2}} + \hat{x}_2 \overline{\sigma_{\gamma_1 \beta_1}} + \\
& \left[\hat{\beta}_1 \hat{x}_3 + \overline{\sigma_{\beta_1 x_3}} \right] + y(i) \left[\hat{\gamma}_1^2 \hat{\beta}_1 + 2 \hat{\gamma}_1 \overline{\sigma_{\gamma_1 \beta_1}} \right. \\
& + \hat{\beta}_1 \overline{\sigma_{\gamma_1 \gamma_1}} + \hat{\alpha}_1 \hat{\beta}_1 \hat{\gamma}_1 + \hat{\alpha}_1 \overline{\sigma_{\beta_1 \gamma_1}} + \hat{\beta}_1 \overline{\sigma_{\alpha_1 \gamma_1}} \\
& \left. + \hat{\gamma}_1 \overline{\sigma_{\alpha_1 \beta_1}} + \hat{\gamma}_2 \hat{\beta}_1 + \overline{\sigma_{\gamma_2 \beta_1}} \right] \cdot \\
& \left[\hat{\beta}_1^2 + \overline{\sigma_{\beta_1 \beta_1}} + q \right]^{-1}
\end{aligned}$$

The control law requires only the estimates of the parameters $\alpha_1, \alpha_2, \beta_1, \gamma_1, \gamma_2$ and states x_1, x_2, x_3 and their covariances. However to generate these estimates it is necessary to generate these estimates of all the other states, parameters and their covariances. Thus for an n^{th} order process having a time delay $k=2$ the OK controller would have to generate the estimates of all the $3n$ unknown parameters, all the $(n+1)$ states and their covariances.

Similarly for an n^{th} order process with k delay the OK controller would have to generate the estimates of $3n$ unknown parameters, $n+k-1$ states and their covariances.

BIBLIOGRAPHY

1. Athans, M., editor of "The linear quadratic gaussian problem", Special Issue IEEE Trans. AC, 16, 6. Dec. 1971.
2. Åström, K.J., Introduction to stochastic control theory, AP, 1970.
3. Chow, G.C., Analysis and control of dynamic economic systems, Wiley, 1975.
4. Elliot, J.R., "NASA's advanced control law program for the F-8 digital fly-by-wire aircraft", IEEE Trans. AC, 22, 5, pp 753-757, Oct. 1977.
5. Elliot, J.R., Organiser of "NASA's F-8 adaptive flight control program", Session WP6, IEEE Conf. on decision and control, Houston, Texas, 1975.
6. Åström, K.J. & Wittenmark, B., "On self-tuning regulators", Automatica, 9, 2, pp 185-199, 1973.
7. Wittenmark, B., "Stochastic adaptive control methods: a survey", IJC, 21, 5, pp 705-730, June, 1975.
8. Potter, R.V., "Some non-linear stochastic control problems", D.Phil. Dissertation, Dept. of Eng. Science, Univ. Oxford, 1977.
9. Patchel, J.W., "On the structure of stochastic control laws", D.Phil. Dissertation, Dept. of Eng. Science, Univ. Oxford, 1971.
10. Jenkins, K.W. & Roy, R.J., "A design procedure for discrete adaptive control system", Preprint JACC, pp 624-633, 1966.
11. Saridis, G.N. & Lobbia, R.N., "Parameter identification and control of linear discrete-time systems", IEEE Trans. AC, 17, 1, pp 52-60, Feb. 1972.
12. Bertsekas, D.P., Dynamic programming and stochastic control, AP, 1976.
13. Patchel, J.W. & Jacobs, O.L.R., "Separability, neutrality and certainty equivalence", IJC, 13, pp 337-342, 1971.
14. Deshpande, J.G., Upadhyay, T.N. & Lainiotis, .D.G, "Adaptive control of linear stochastic systems", Automatica, 9, 1, pp 107-115, Jan. 1973.
15. Athans, M. et al., "The stochastic control of the F-8C aircraft using a multiple model adaptive control (MMAC) method - Part I", IEEE Trans. AC, 22, 5, 768-780, Oct. 1977.
16. Athans, M. & Wilner, D., "A practical scheme for adaptive aircraft flight control systems", Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Centre, Edwards, Calif. pp 315-336, Apr. 1973.

17. Clarke, D.W. & Gawthrop, P.J., "Self-tuning controller", Proc. IEE, 122, 9, pp 929-934, Sept. 1975.
18. Tse, E. & Athans, M., "Adaptive stochastic control for a class of linear systems", IEEE Trans. AC, 17, 1, pp 38-51, Feb. 1972.
19. Ku, R., & Athans, M., "On the adaptive control of linear systems using the open-loop-feedback-optimal approach", IEEE Trans. AC, 18, 5, pp 489-493, Oct. 1973.
20. Bar-Shalom, Y. & Sivan R., "On the optimal control of discrete time linear systems with random parameters", IEEE Trans. AC, 14, 1, pp 3-8, Feb. 1969.
21. Bar-Shalom, Y. & Tse, E., "Dual effect, certainty equivalence and separation in stochastic control", IEEE Trans. AC, 19, 5, pp 494-500, Oct. 1974
22. Bar-Shalom, Y. & Tse, E., "Concepts and methods in stochastic control", in Control and Dynamic Systems: Advances in theory and applications, ed. C.T. Leondes, Vol. 12, 1975.
23. Aoki, M., Optimization of stochastic systems AP, 1967.
24. Spang, H.A., "Optimum control of an unknown linear plant using bayesian estimation error", IEEE Trans. AC, 10, 1, pp 80-83, Jan. 1965.
25. Dreyfus, S.E., "Some types of optimal control of stochastic systems", SIAM J. Control, 2, 1, pp 120-134, 1964.
26. Jacobs, O.L.R. & Hughes, D.J., "Simultaneous identification and control using a neutral control law", 6th IFAC Congress, Boston, 1975, Paper 30.2.
27. Jacobs, O.L.R. & Potter, R.V., "Optimal control of a stochastic bilinear system", paper at IMA conference on control, Leicester, 1976, also OUEL report no. 1156/76.
28. ^oAström, K.J. & Wittenmark, B., "Problems of identification and control", J. Math. Anal. and Apps., 34, pp 90-113, 1971.
29. Bohlin, T., "Optimal dual control of a simple process with unknown gain", Report TP 18.196, IBM Nordic Lab., Stockholm, 1969
30. Hughes D.J. & Jacobs, O.L.R., "Performance of LQG control systems using optimal k-step-ahead control laws", JACC Columbus, Ohio, Paper 22.4, 1973.
31. Tse, E. & Bar-Shalom, Y., "An actively adaptive control for linear systems with random parameters via dual control approach", IEEE Trans. AC, 18, 2, pp 109-117, Apr. 1973.

32. Sternby, J., "Topics in dual control", Doctoral dissertation, Dept. of Automatic Control, Lund Inst. Tech., Lund, Sweden, 1977.
33. Sternby, J. "A simple dual control problem with an analytical solution", IEEE Trans AC, 21, 6, pp 840-844, Dec. 1976.
34. Bellman, R., Adaptive control process: A guided tour, Princeton University Press, 1961.
35. Sawaragi, Y. & Yoshikawa, T., "Performance bounds for discrete time stochastic optimal control problems", IJC, 10, 5, pp 481-500, 1969.
36. Nelson, N.W. & Slear, E.B., "The simultaneous on-line estimation of parameters and states in linear systems", IEEE Trans. AC, 21, pp 94-98, 1976.
37. Jacobs, O.L.R. Introduction to control theory, Oxford University Press, 1974.
38. Jazwinski, A.H., Stochastic processes and filtering theory, AP, 1970.
39. Jacobs, O.L.R. & Patchel, J.W., "Caution and probing in stochastic control", IJC, 16, 1, pp 189-199, 1972.
40. Bar, W. & Dittrich, F., "Useful formulae for moment computation of normal random variables with non-zero means", IEEE Trans. AC, 16, 3, pp 263-265, Jun. 1971.
41. Ljung, L., "The extended Kalman filter as a parameter estimator for linear systems", Linköping University, Report LiTH ISY I 0154, 1977.
42. Sorenson, H.W., "Controllability and observability of linear stochastic time-discrete control systems" - in Advances in control systems ed. - C.T. Leondes, AP, Vol. 6, 1968.
43. Jacobs, O.L.R., Saratchandran, P. & Colville, J.R., "Comparison of two adaptive controllers", Elec. Letters, 12, 3, 1976.
44. Dreyfus, S.E., Dynamic programming and calculus of variations, AP, 1965.
45. Gawthrop, P.J. "Studies in identification and control", D.Phil. Dissertation, Univ. Oxford, 1976.
46. Åström, K.J., "Stochastic control theory and some of its industrial applications", IFAC Symposium on stochastic control, Budapest, September, 1974.
47. Jacobs, O., "On the functional recurrence equation of optimal discrete time stochastic control", IJC, 26, 5, pp 763-768, 1977.
48. Clarke, D.W., Cope, S.N. & Gawthrop, P.J., "Feasibility study of the application of microprocessors to self-tuning controllers", Dept. of Eng. Science, Univ. Oxford, Report No. 1137/75, 1975.

49. Ku, R., "Adaptive control of stochastic linear systems with unknown parameters", Electronics Systems Lab., MIT, Report ESL-R-477, May 1972.
50. Borisson, U. & Syding, R., "Self-tuning control of an ore crusher", Automatica, 12, 1, pp 1-7, 1976.
51. Sastry, V.A., Seborg, D.E. & Wood, R.K., "Self-tuning regulator applied to a binary distillation column", Automatica, 13, 4, pp 417-424, 1977.
52. Borisson, U. & Wittenmark, B., "Moisture control of a paper machine - an application of self-tuning regulator", Dept. of Aut. Control, Lund Inst. of Tech., Sweden, Report 7337, 1973.
53. Aström, K.J., Borisson, U., Ljung, L. & Wittenmark, B., "Theory and applications of self-tuning regulators", Automatica, 13, 5, pp 457-476, 1977.
54. Panuska, V., "An adaptive recursive least squares identification algorithm", Proc. 8th IEEE Symposium on Adaptive processes, 1969.
55. Ljung, L. & Wittenmark, B., "Analysis of a class of adaptive regulators", IFAC Symposium on stochastic control, Budapest, 1974.
56. Irwin, G.W. & Roberts, A.P., "The Luenberger canonical form in the state/parameter estimation of linear systems", IJC, 23, 6, pp 851-864, 1976.
57. Sud, S.K., Biswas, K.K. & Sinha, A.K., "Stochastic state variable model for paper machine head-box", Proc. IEE, 124, 12, pp 1249-1254, 1977.
58. Lee, R.C.K., Optimal estimation identification and control, MIT press, 1964.
59. Gelb, A., Applied optimal estimation, MIT press, 1974.
60. Kalman, R.E., "A new approach to linear filtering and prediction problems", Trans. ASME, Journal of Basic Eng., 82D, No. 1, pp 35-45, 1960.
61. Kalman, R.E., "New methods in Wiener filtering", Proc. 1st Symposium on Engineering applications of random function theory and probability, J.L. Bogdanoff and F. Kozin, Eds., Wiley, 1963.
62. Gawthrop, P.J., "Some interpretations of the self-tuning controller", Proc. IEE, 124, 10, pp 889-894, Oct. 1977.
63. Ljung, L., "Analysis of recursive stochastic algorithms", IEEE Trans. AC, 22, 4, pp 551-575, Aug. 1977.

64. Ljung, L. "Convergence of recursive stochastic algorithms", IFAC Symposium on stochastic control, Budapest, 1974.
65. Horowitz, I.M., Synthesis of feedback systems, AP, 1963.
66. McCollough, C. & Atta, L.V. Statistical Concepts, McGraw-Hill, 1963.
67. Ljung, L. & Wittenmark, B., "Asymptotic properties of self-tuning regulators", Dept. of Aut. Control, Lund Inst. of Tech., Sweden, Report 7404, 1974.
68. Hughes, D.J., "Studies in stochastic control theory", D. Phil. Dissertation, Dept. of Eng. Science, Univ. Oxford, 1974.
69. Hughes, D.J. & Jacobs, O.L.R., "Turn-Off, Escape and Probing in Nonlinear Stochastic Control", IFAC Symposium on stochastic control, Budapest, Sept. 1974.
70. Prasad, R.M., Sihna, A.K. & Mahalanabis, A.K., "Two stage identification of closed loop systems", IEEE Trans. AC, 22, 6, pp 987-988, Dec. 1977.