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NOT SO DEMANDING: PREFERENCE STRUCTURE, FIRM BEHAVIOR, AND WELFARE

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NOT SO DEMANDING: PREFERENCE STRUCTURE, FIRM BEHAVIOR, AND WELFARE*

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Abstract

We introduce two new tools for relating preferences and demand to firm behavior and economic performance. The "Demand Manifold" links the elasticity and convexity of an arbitrary demand function; the "Utility Manifold" links the elasticity and concavity of an arbitrary utility function. Along the way we present some new families of demand functions; show how the structure of demand and preferences determine the responses of monopoly firms and monopolistically competitive industries to exogenous shocks; characterize the efficiency of a monopolistically competitive equilibrium; and present a quantitative framework for predicting the welfare effects of exogenous shocks.

Keywords: Heterogeneous Firms; Quantifying Gains from Trade; Super- and Sub-Convexity; Supermodularity.

JEL Classification: F23, F15, F12

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1 Introduction

Assumptions about the structure of preferences and demand matter enormously for comparative statics in trade, industrial organization, and many other applied fields. Consider just a few examples of classic questions, and the answers to them, which have attracted recent attention:

1. Competition Effects: Does globalization reduce firms' markups? Yes, if and only if the elasticity of demand falls with sales.[1]

2. Pass-Through: Do firms pass on more than 100% of cost increases? Yes, if and only if the demand function is more than log-convex.[2]

3. Selection Effects: Do more productive firms select into FDI rather than exports? Yes, if and only if the elasticity and convexity of demand sum to more than three.[3]

4. Price Discrimination: Does it raise welfare? Yes, if demand convexity falls as price rises.[4]

5. Welfare: Is monopolistic competition efficient? Yes, if and only if preferences are CES.[5]

In each case, the answer to an important real-world question hinges on a feature of preferences or demand which seems at best arbitrary and in some cases esoteric. All bar specialists may have difficulty remembering these results, far less explicating them and relating them to each other.

There is an apparent paradox here. These applied questions are all supply-side puzzles: they concern the behavior of firms or the performance of industries. Why then should the answers to them hinge on the shape of demand functions, and in all but the last case on

[1]See Krugman (1979) and Zhelobodko, Kokovin, Parenti, and Thisse (2012).

[2]See Bulow and Pfleiderer (1983), Fabinger and Weyl (2012), and Weyl and Fabinger (2013).

[3]See Helpman, Melitz, and Yeaple (2004) and Mrázová and Neary (2011). The result holds when exports incur iceberg trade costs. See Section 2.4 below.

[4]See Schmalensee (1981) and Aguirre, Cowan, and Vickers (2010).

[5]See Dixit and Stiglitz (1977) and Dhingra and Morrow (2011).

their second or even third derivatives? The paradox is only an apparent one. In perfectly competitive models, shifts in supply curves lead to movements along the demand curve, and so their effects hinge on the slope or elasticity of demand. When firms are monopolists or monopolistic competitors, as in this paper, they do not have a supply function as such; instead, exogenous supply-side shocks or differences between firms lead to more subtle differences in behavior, whose implications depend on the curvature as well as the slope of the demand function.

Different authors and even different sub-fields have adopted a variety of approaches to these issues. Weyl and Fabinger (2013) show that many results can be understood by taking the degree of pass-through of costs to prices as a unifying principle. Macroeconomists frequently work with the "superelasticity" of demand, due to Kimball (1995), to model more realistic patterns of price adjustment than allowed by CES preferences. In our previous work (Mrázová and Neary (2011)), we showed that, since monopoly firms adjust along their marginal revenue curve rather than the demand curve, the elasticity of marginal revenue itself pins down some results. Each of these approaches focuses on a single demand measure which is a sufficient statistic for particular results. This paper complements these by showing how the different measures are related and by providing a new perspective on how assumptions about the functional form of demand determine conclusions about comparative statics.

The key idea we explore is the value of taking a "firm's eye view" of demand functions. To understand a monopoly firm's responses to infinitesimal shocks it is enough to focus on the local properties of the demand function it faces, since these determine its choice of output: the slope of demand determines the firm's level of marginal revenue, which it wishes to equate to marginal cost, while the curvature of demand determines the slope of marginal revenue, which must be decreasing if the second-order condition for profit maximization is to be met. Measuring slope and curvature in unit-free ways leads us to focus on the elasticity and convexity of demand, and our major innovation is to show that for a given demand function these two parameters are related to each other. We call the implied relationship the

"Demand Manifold", and show that it is a sufficient statistic linking the functional form of demand to comparative statics properties. It thus allows us to illustrate existing results and develop many new ones in a simple and compact way; and it accommodates a wide range of demand behavior, including some new demand functions which provide a parsimonious way of nesting better-known ones.

A "firm's-eye view" is partial-equilibrium by construction, of course. Nevertheless, it can provide the basis for understanding general-equilibrium behavior. To demonstrate this, we show how our approach allows us to characterize the responses of outputs, prices and firm numbers in the canonical model of international trade under monopolistic competition due to Krugman (1979). We are also able to throw light on the welfare effects of globalization in this model, by introducing a second illustrative device, the "Utility Manifold". Analogously to the demand manifold, this links the elasticity and the convexity of an arbitrary utility function, and is a key determinant of the efficiency properties of an imperfectly competitive equilibrium and the welfare effects of exogenous shocks. This is of interest both in itself, and in the light of Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), who provide a rare exception to the rule that the functional form of demand matters for comparative statics results. Extending earlier work by Arkolakis, Costinot, and Rodríguez-Clare (2012), they show that the gains from trade in a wide class of monopolistically competitive models are affected little by departing from CES assumptions. Our results do not contradict theirs, but they suggest some notes of caution when we wish to quantify the gains from trade in models that allow for a wide range of demand behavior.

The plan of the paper follows this route map. Section 2 introduces our new perspective on demand, showing how the elasticity and convexity of demand condition comparative statics results. Section 3 shows how the demand manifold can be located in the space of elasticity and convexity, and explores how a wide range of demand functions, both old and new, can be represented by their manifold in a parsimonious way. Section 4 illustrates the usefulness of our approach by applying it to a canonical general-equilibrium model of international trade under monopolistic competition, and characterizing the implications of assumptions about functional form for the quantitative effects of exogenous shocks. Section 5 concludes, while the Appendix gives proofs of all propositions and notes some more technical extensions.

2 Demand Functions and Comparative Statics

2.1 A Firm's-Eye View of Demand

Almost by definition, a perfectly competitive firm takes the price it faces as given. Our starting point is the fact that a monopolistic or monopolistically firm takes the demand function it faces as given. Observing economists will often wish to solve for the full general equilibrium of the economy, or to consider the implications of alternative assumptions about the structure of preferences (homotheticity, separability, etc.). By contrast, the firm takes all these as given and is concerned only with maximizing profits subject to the demand function it perceives. For the most part we write this demand function in inverse form, $p = p(x)$, with the only restrictions that consumers' willingness to pay is continuous, twice differentiable, and strictly decreasing in sales: $p'(x) < 0$. It is sometimes convenient to switch to the corresponding direct demand function, $x = x(p)$, with $x'(p) < 0$, the inverse of $p(x)$.

Because we want to highlight the implications of alternative assumptions about demand, we assume throughout that marginal cost is constant.[6] Maximizing profits therefore requires that marginal revenue should equal marginal cost and should be decreasing with output. These conditions can be expressed in terms of the slope and curvature of demand, measured by two unit-free parameters, the elasticity ε and convexity ρ of the demand function:

$$\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0 \quad \text{and} \quad \rho(x) \equiv -\frac{xp''(x)}{p'(x)} \tag{1}$$

[6]Zhelobodko, Kokovin, Parenti, and Thisse (2012) show that variable marginal costs make little difference to the properties of models with homogeneous firms. In models of heterogeneous firms it is standard to assume that marginal costs are constant.

These are not unique measures of slope and curvature, and our results could alternatively be presented in terms of other parameters, such as the convexity of the direct demand function, or the Kimball (1995) superelasticity of demand. (See Section A in the Appendix for more details and references.) If the first-order condition holds for a zero marginal cost, it implies that the elasticity cannot be less than one:

$$p + xp' = c \geq 0 \quad \Rightarrow \quad \varepsilon \geq 1 \tag{2}$$

As for the second-order condition, if marginal revenue decreases with output, then our measure of convexity must be less then two:

$$2p' + xp'' < 0 \quad \Rightarrow \quad \rho < 2 \tag{3}$$

These restrictions can be visualized in terms of an admissible region in $\{\varepsilon, \rho\}$ space, as shown by the shaded region in Figure 1.[7]

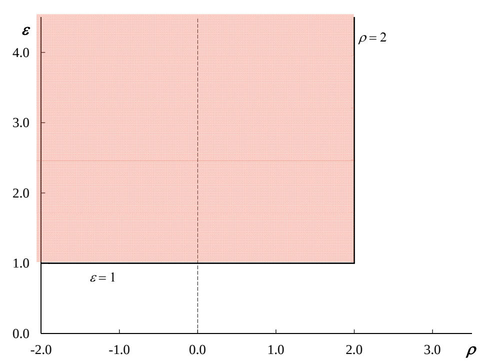


Figure 1: The Admissible Region

[7]The admissible region is $\{\varepsilon, \rho\} \in \{1 \leq \varepsilon \leq \infty, -\infty \leq \rho < 2\}$. We focus on the case where $\varepsilon \leq 4.5$ and $\rho \geq -2.0$, since this is where most interesting issues arise. Note that the admissible region is larger in oligopolistic markets, since both boundary conditions are less stringent than (2) and (3). See the Appendix, Section B, for details.

2.2 Superconvexity

In general, both ε and ρ vary with sales. The only exception to this is the case of CES preferences or iso-elastic demands:[8]

$$p(x) = \beta x^{-1/\sigma} \quad \Rightarrow \quad \varepsilon = \sigma, \quad \rho = \rho^{CES} \equiv \frac{\sigma+1}{\sigma} > 1 \tag{4}$$

Clearly this case is very special: both elasticity and convexity are determined by a single parameter. The curve labeled "SC" in Figure 2 illustrates the implied relationship between ε and ρ for all members of the CES family: $\varepsilon = \frac{1}{\rho-1}$, or $\rho = \frac{\varepsilon+1}{\varepsilon}$. Every point on this curve corresponds to a different demand function: firms always operate at that point irrespective of the values of exogenous variables. In this respect too the CES is very special, as we will see. The Cobb-Douglas special case corresponds to the point $\varepsilon = 1, \rho = 2$, and so has the dubious distinction of being just on the boundary of both the first- and second-order conditions.

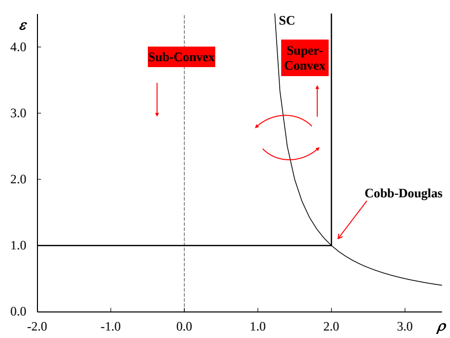


Figure 2: The Super- and Sub-Convex Regions

The CES case is important in itself but also because it is an important boundary for comparative statics results. Following Mrázová and Neary (2011), we can define a local property of any point on an arbitrary demand function as follows:[9]

[8]That CES demands are sufficient for constant elasticity is obvious. That they are necessary follows from setting $-\frac{p(x)}{xp'(x)}$ equal to a constant σ and integrating.

[9]As noted in Mrázová and Neary (2011), superconvexity of the inverse demand function $p(x)$ is equivalent to superconvexity of the corresponding direct demand function $x(p)$.

Definition 1. A demand function $p(x)$ is superconvex at a point (p_0, x_0) if and only if $\log p(x)$ is convex in $\log x$ at (p_0, x_0).

Direct calculation shows that superconvexity can be expressed in terms of ε and ρ as follows:

$$\frac{d^2 \log p}{d(\log x)^2} = \frac{1}{\varepsilon}\left(\rho - \frac{\varepsilon+1}{\varepsilon}\right) = \frac{1}{\varepsilon}\left(\rho - \rho^{CES}\right) \geq 0 \tag{5}$$

Recalling (4), this implies that superconvexity of a demand function at an arbitrary point is equivalent to the function being more convex at that point than a CES demand function with the same elasticity. Hence the CES or "SC" curve in Figure 2 divides the admissible region in two: points above the curve are strictly superconvex, points below are strictly subconvex, while all CES demand functions are both weakly superconvex and weakly subconvex.

The first comparative static property which superconvexity illuminates is the relationship between demand elasticity and sales. From (5), the elasticity of demand increases in sales (or, equivalently, decreases in price), $\varepsilon_x \geq 0$, if and only if the demand function $p(x)$ is superconvex:

$$\varepsilon_x = \frac{\varepsilon}{x}\left(\rho - \frac{\varepsilon+1}{\varepsilon}\right) \tag{6}$$

So, ε is independent of sales only along the CES or SC locus, it increases with sales in the superconvex region to the right, and decreases with sales in the subconvex region to the left. Many authors, including Marshall (1920), Dixit and Stiglitz (1977), and Krugman (1979), have argued that subconvexity is more plausible on intuitive grounds, though empirical evidence can be found both for and against it.[10] These results are not new, though it does not seem to have been noted before that, in Figure 2, they imply something like the comparative-statics analogue of a phase diagram: the arrows indicate the direction of movement as sales rise.

Superconvexity also matters for *relative pass-through*: the effects of cost changes on firms'

[10]The empirical evidence is reviewed in Zhelobodko, Kokovin, Parenti, and Thisse (2012). Marshall devoted a whole chapter of his *Principles* to arguments in favor of subconvexity, which he called the "Law of Elasticity." It is sometimes called "Marshall's Second Law of Demand."

proportional profit margins.[11] From the first-order condition, the relative margin $\frac{p-c}{p}$ equals $-\frac{xp'}{p}$, which is just the inverse of the elasticity ε. Hence an increase in marginal cost c, which other things equal must lower sales, is associated with a *higher* elasticity; this in turn implies a *lower* proportional profit margin if and only if demands are subconvex:

$$\frac{d\log p}{d\log c} = \frac{\varepsilon+1}{\varepsilon}\frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{d\log p}{d\log c} - 1 = -\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(2-\rho)} \gtreqless 0 \tag{7}$$

However, for *absolute pass-through*, a different criterion applies, to which we turn next.

2.3 Super-Pass-Through

The criterion for absolute pass-through from cost to price has been known since Bulow and Pfleiderer (1983). Differentiating the first-order condition $p+xp' = c$, we see that an increase in cost must raise price provided only that the second-order condition holds, which implies a different expression for the effect of an increase in marginal cost on the absolute profit margin:

$$\frac{dp}{dc} = \frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{dp}{dc} - 1 = \frac{\rho-1}{2-\rho} \gtreqless 0 \tag{8}$$

Hence we have what we call "Super-Pass-Through", a more-than-100% pass-through of costs to prices, if and only if ρ is greater than one. Figure 3 shows how the admissible region in $\{\varepsilon, \rho\}$ space is divided into sub-regions corresponding to super- and sub-pass-through. The boundary between the sub-regions corresponds to a log-convex direct demand function, which is less convex than the CES.[12] So it is immediately obvious that superconvexity implies super-pass-through, but the converse does not hold.

We have already seen in the introduction that some comparative statics results hinge on whether pass-through increases with sales, or, equivalently, decreases with price, as high-

[11] This is the sense in which the term "pass-through" is used in international macroeconomics. See for example Gopinath and Itskhoki (2010).

[12] Setting $\rho = 1$ implies a second-order ordinary differential equation $xp''(x) + p'(x) = 0$. Integrating this yields $p(x) = c_1 + c_2 \log x$, where c_1 and c_2 are constants of integration, which is equivalent to a log-convex direct demand function, $\log x(p) = \gamma + \zeta p$.

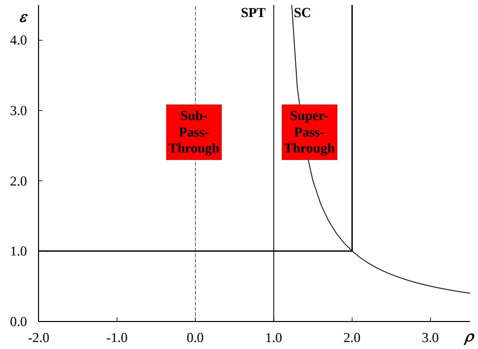


Figure 3: The Super- and Sub-Pass-Through Regions

lighted by Weyl and Fabinger (2013). A necessary and sufficient condition for this is the following:

Lemma 1. *Pass-through increases with sales if and only if* $\rho(1+\rho-\chi) < 0$*, where* $\chi \equiv -\frac{xp'''}{p''}$.

Proof. Differentiating the pass-through expression in (8) yields:

$$\frac{d^2p}{dxdc} = \frac{\rho_x}{(2-\rho)^2}\frac{dx}{dc} = \frac{1}{p'}\frac{\rho_x}{(2-\rho)^3} \tag{9}$$

so pass-through is increasing in sales if and only if convexity, ρ, is decreasing in sales: $\rho_x < 0$. Differentiating ρ in turn and collecting terms yields:

$$dp/dc\rho_x = \frac{\rho}{x}(1+\rho-\chi) \tag{10}$$

□

By analogy with Kimball (1990), we can call χ the "Coefficient of Relative Prudence of Demand," or simply "prudence."

Lemma 1 combined with our diagram in $\{\varepsilon, \rho\}$ space provides a convenient way of checking how pass-through varies with price for a given demand function. For most demand functions, both elasticity and convexity vary with sales; and we have already seen that the variation of elasticity with sales depends on whether demand is sub- or super-convex. From this it is easy to infer how convexity must vary with sales using the fact that:

$$\frac{d\varepsilon}{d\rho} = \varepsilon_x \frac{dx}{d\rho} = \frac{\varepsilon_x}{\rho_x} \tag{11}$$

Recalling from Lemma 1 that the direction of change of pass-through depends only on the sign of ρ_x, we can conclude that pass-through increases with sales if and only if $\frac{d\varepsilon}{d\rho}$ and ε_x have opposite signs. Since from (6) ε_x is positive if and only if the demand function is superconvex, we can conclude the following:

Lemma 2. *Pass-through increases with sales if and only if either convexity and elasticity move together in the superconvex region, or convexity and elasticity move in opposite directions in the subconvex region.*

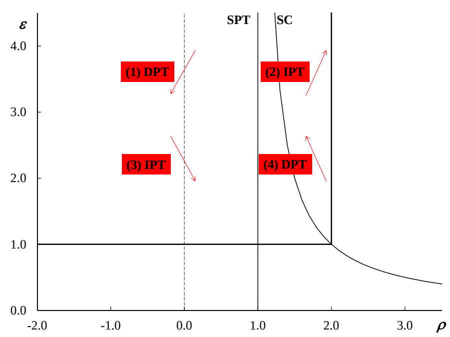


Figure 4: Increasing and Decreasing Pass-Through

This result is illustrated in Figure 4, where cases (2) and (3) represent the two configurations that are consistent with increasing pass-through, while cases (1) and (4) imply decreasing pass-through.

2.4 Supermodularity

The third criterion for comparative statics responses that we can locate in our diagram arises in models with heterogeneous firms. Consider a choice between two ways of serving a market, one of which incurs lower variable costs but higher fixed costs than the other.[13] Let $\pi(c, t)$ denote the maximum operating profits which a firm with marginal production costs c can earn facing a marginal cost of accessing the market equal to t. Mrázová and Neary (2011) show that a sufficient (and, with additional restrictions, necessary) condition for more efficient firms (i.e., firms with lower c) to select into the activity with lower relative marginal cost (i.e., lower t) is that the firm's *ex post* profit function is *supermodular* in c and t. This criterion is very general, but in an important class of models it takes a relatively simple form. This class is where the profit function is twice differentiable, and depends on c and t multiplicatively:

$$\pi(c,t) \equiv \max_x \tilde{\pi}(x,c,t) \qquad \tilde{\pi}(x,c,t) = [p(x) - tc]\, x \tag{12}$$

This specification encompasses a number of important models. Interpreting t as an iceberg transport cost, it represents the canonical model of horizontal foreign direct investment (FDI) as in Helpman, Melitz, and Yeaple (2004): firms have to choose between proximity to foreign consumers - FDI incurs zero access costs - or "concentration", exporting from their home plant; the latter saves on fixed costs but requires that they produce tx units in order to sell x in the foreign market. Alternatively, interpreting t as the wage that must be paid to c workers to produce a unit of output, it represents the canonical model of *vertical* FDI, as in Antràs and Helpman (2004): firms in the high-income "North" wishing to serve their home market face a choice between producing at home and paying a high wage, or building a new plant in the low-wage "South." Finally, interpreting t as the premium over the marginal cost c which a firm will incur if it fails to invest in superior technology, equation (12) represents a model of choice of technique as in Bustos (2011).

When the profit function is twice differentiable in c and t, supermodularity is equivalent

[13] Mrázová and Neary (2011) call the resulting selection effects "Second-Order," contrasting them with "First-Order Selection Effects" which arise from the decision whether to serve a market or not. First-order selection effects depend only on the first derivative of the profit function with respect to marginal cost, and are much more robust than second-order selection effects.

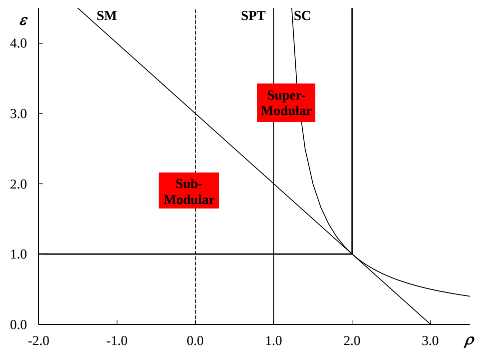


Figure 5: The Super- and Sub-Modularity Regions

to a positive value of π_{ct} over the relevant range. Moreover, when the profit function is given by (12), Mrázová and Neary (2011) show that π_{ct} is positive if and only if the elasticity of marginal revenue with respect to sales is less than one.[14] When this condition holds, a 10% reduction in the marginal cost of serving a market raises sales by more than 10%, so more productive firms have a bigger incentive to engage in FDI than in exports. The elasticity of marginal revenue depends on turn on the elasticity and convexity of demand:

$$\pi_{ct} = \frac{\varepsilon + \rho - 3}{2 - \rho} x \tag{13}$$

It follows that supermodularity holds in models described by (12) if and only if $\varepsilon + \rho > 3$. This criterion defines a third locus in $\{\varepsilon, \rho\}$ space, as shown in Figure 5. Once again it divides the admissible region into two sub-regions, one where either the elasticity or convexity or both are high, so supermodularity prevails, and the other where the profit function is submodular. The locus lies everywhere below the superconvex locus, and is tangential to it at the Cobb-Douglas point. Hence, supermodularity always holds with CES demands, the case assumed in Helpman, Melitz, and Yeaple (2004), Antràs and Helpman (2004) and Bustos (2011), among many others. However, when demands are subconvex and firms are large (operating

[14] By the envelope theorem, $\pi_c = \tilde{\pi}_c = -tx$. Hence, $\pi_{ct} = -x - t\frac{dx}{dt} = -x - \frac{tc}{2p' + xp''} = -x + \frac{\varepsilon - 1}{2 - \rho} x$. Writing revenue as $R(x) = xp(x)$, so marginal revenue is $R_x = p + xp'$, the elasticity of marginal revenue (in absolute value) is seen to be: $-\frac{xR_{xx}}{R_x} = \frac{2-\rho}{\varepsilon - 1}$. Combining these results gives (13).

at a point on their demand curve with relatively low elasticity), submodularity prevails, and so the standard comparative statics results are reversed.

2.5 Summary

Figure 6 summarizes the implications of this section. The three loci, corresponding to constant elasticity (SC), convexity equal to one (SPT), and elasticity of marginal revenue equal to one (SM), place bounds on the combinations of elasticity and convexity consistent with particular comparative statics outcomes. Of eight logically possible sub-regions within the admissible region, three can be ruled out because superconvexity implies both super-pass-through and supermodularity. From the figure it is clear that knowing the values of the elasticity and convexity of demand which a firm faces is sufficient to predict its responses to a very wide range of exogenous shocks.

Region	SC	SPT	SM
1			
2			✓
3		✓	
4		✓	✓
5	✓	✓	✓

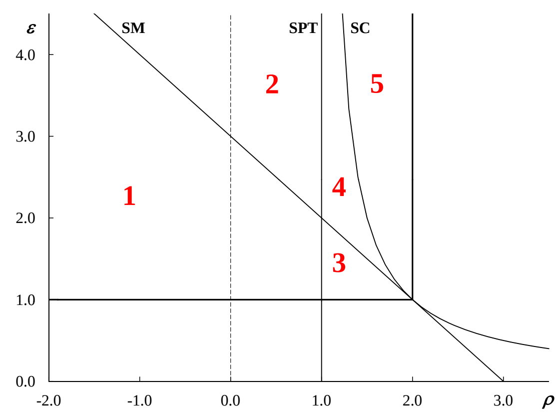


Figure 6: Regions of Comparative Statics

3 The Demand Manifold

3.1 Introduction

So far, we have shown how a very wide range of comparative statics responses can be signed just by knowing the values of ε and ρ which a firm faces. Next we want to see how different assumptions about the form of demand determine these responses. Formally, we seek to characterize the set of ε and ρ consistent with any given demand function $p = p_0(x)$:

$$\left\{\varepsilon, \rho : \varepsilon = -\frac{p_0(x)}{xp_0'(x)},\ \rho = -\frac{xp_0''(x)}{p_0'(x)},\ p = p_0(x)\right\} \tag{14}$$

We have already seen that this set and hence the comparative statics responses implied by particular demand functions are pinned down in two special cases: with CES demands the firm is always at a particular point in $\{\varepsilon, \rho\}$ space, while with linear demands it must lie along the $\rho = 0$ locus. Can anything be said more generally? The answer is "yes", because, for most demand functions, at least one of $\varepsilon = \varepsilon(x)$ and $\rho = \rho(x)$ can be inverted to solve for x, and the resulting expression substituted into the other to give a relationship between ε and ρ:

$$\varepsilon = \bar{\varepsilon}(\rho) \equiv \varepsilon\left[X(\rho)\right] \quad \text{or} \quad \rho = \bar{\rho}(\varepsilon) \equiv \rho\left[X(\varepsilon)\right] \tag{15}$$

We write this in two alternative ways, since either one or the other may be more convenient depending on the context. The relationship between ε and ρ defined implicitly by (14) is not in general a function, since it need not be globally single-valued; but neither is it a correspondence, since it is locally single-valued. So we call it the "Demand Manifold" corresponding to the demand function $p_0(x)$.[15]

The first advantage of working with the demand manifold rather than the demand function itself is that it can be located in $\{\varepsilon, \rho\}$ space, which immediately reveals the implications

[15] We are grateful to Kevin Roberts for pointing out that it is indeed a manifold. More formally, it is a one-dimensional smooth manifold, or a smooth plane curve in the Euclidean plane R^2. Locally it is defined by an equation $f(\varepsilon, \rho) = 0$, where $f : R^2 \to R$ is a smooth function, and the partial derivatives $\partial f/\partial\varepsilon$ and $\partial f/\partial\rho$ are never both zero.

of assumptions made about demand for comparative statics. A second advantage, departing from the "firm's-eye-view" that we have adopted so far, is that the manifold is often independent of exogenous parameters even though the demand function itself is not. Expressing this in the language of Chamberlin (1933), exogenous shocks typically shift the perceived demand curve, but they need not shift the corresponding demand manifold. When this property of "manifold invariance" holds, exogenous shocks lead only to movements along the manifold, not to shifts in it. As a result, it is particularly easy to make comparative statics predictions. Clearly, the manifold cannot in most cases be invariant to changes in all parameters: even in the CES case, the point manifold is not independent of the value of σ.[16] However, for many demand functions, including some of the most widely-used, the manifold turns out to be invariant with respect to some of their parameters, so it provides a parsimonious summary of their implications for comparative statics. In the remainder of this section, we show that manifold invariance provides a fruitful organizing principle for a wide range of demand functions.

3.2 Demand Functions Separable in a Parameter

From now on we make explicit that any demand function depends on some exogenous parameters, which we denote in general by a vector ϕ: $p = p(x, \phi)$, so the manifold can be written in full as either $\bar{\varepsilon}(\rho, \phi) = \varepsilon \left[X(\rho, \phi), \phi\right]$ or $\bar{\rho}(\varepsilon, \phi) \equiv \rho \left[X(\varepsilon, \phi), \phi\right]$.[17] Our first result is that manifold invariance holds when the demand function is multiplicatively separable in ϕ:

Proposition 1. *The Demand Manifold is invariant to shocks in a parameter ϕ if either the direct or inverse demand function is multiplicatively separable in ϕ:*

$$\text{(a) } x\left(p, \phi\right) = \zeta\left(\phi\right) \tilde{x}\left(p\right); \quad \text{or} \quad \text{(b) } p\left(x, \phi\right) = \beta(\phi)\tilde{p}(x) \tag{16}$$

[16]The manifold corresponding to the linear demand function is a relatively rare example which is invariant with respect to all demand parameters.

[17]Formally, manifold invariance requires that either $\bar{\varepsilon}_\phi = \varepsilon_x X_\varepsilon + \varepsilon_\phi = -\varepsilon_x \frac{\rho_\phi}{\rho_x} + \varepsilon_\phi = 0$ or $\bar{\rho}_\phi = \rho_x X_\rho + \rho_\phi = -\rho_x \frac{\varepsilon_\phi}{\varepsilon_x} + \rho_\phi = 0$.

The proof is in the Appendix, and relies on the simple property that with separability of this kind the values of both the elasticity and convexity are independent of ϕ.

This result has two important corollaries. First, when preferences are additively separable, the inverse demand function for any good equals the marginal utility of that good times the inverse of the marginal utility of income, which we denote by λ. The latter is a sufficient statistic for all economy-wide variables which affect the demand in an individual market, such as aggregate income or the price index. Setting $\beta(\phi)$ in equation (16) (b) equal to λ^{-1} implies the following:

Corollary 1. *With additive preferences, $U = \int_{\omega\in\Omega} u[x(\omega), \omega]\,\mathrm{d}\omega$, the Demand Manifold for any good ω is invariant to changes in aggregate variables: $p[x(\omega), \omega, \lambda] = \lambda^{-1}\tilde{p}[x(\omega), \omega]$, $\tilde{p}[x(\omega), \omega] = u'[x(\omega), \omega]$.*

Given the pervasiveness of additive separability in theoretical models of monopolistic competition, this is an important result, which implies that in many models the manifold is invariant to economy-wide shocks.

A second corollary of Proposition 1 comes by noting that, setting $\zeta(\phi)$ in (16) (a) equal to market size s, yields the following:

Corollary 2. *The Demand Manifold is invariant to neutral changes in market size: $x(p, s) = s\tilde{x}(p)$.*

This corollary is particularly useful since it does not depend on the functional form of the demand function. An example which illustrates this is the logistic direct demand function, equivalent to a logit inverse demand function (see Cowan (2012)):

$$x(p, s) = \left(1 + e^{p-a}\right)^{-1} s \quad \Leftrightarrow \quad p(x, s) = a - \log\frac{x}{s - x} \tag{17}$$

Here x/s is the share of the market served: $x \in [0, s]$; and a is the price which induces a 50% market share: $p = a$ implies $x = \frac{s}{2}$. The elasticity equals $\varepsilon = p\frac{s-x}{s}$, while the convexity equals $\rho = \frac{s-2x}{s-x}$, which must be less than one. Eliminating x and p yields a closed-form expression for the manifold:

$$\bar{\varepsilon}(\rho) = \frac{a - \log(1-\rho)}{2-\rho} \tag{18}$$

which is invariant with respect to market size s though not with respect to a. Figure 7 illustrates this for values of a equal to 2 and 5.[18]

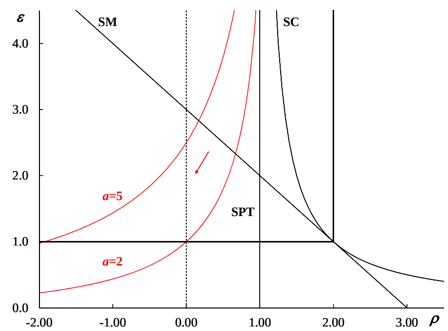


Figure 7: The Demand Manifold for the Logistic Demand Function

The logistic is just one example of a whole family of demand functions, many of which can be derived from log-concave distribution functions: Bergstrom and Bagnoli (2005) give a comprehensive review of these. The power of the approach introduced in the last section is that we can immediately state the properties of all these functions: they imply sub-pass-through and, *a fortiori*, subconvexity, while they are typically supermodular for low values of output and submodular for high values. Other things equal, an increase in market size must raise the output of a monopoly firm, implying an adjustment as shown by the arrow in the figure.

3.3 Bipower Inverse Demand Functions

A different approach to exploring the relationship between demand functions and the corresponding demand manifolds is to ask which demand functions correspond to particular

[18]The value of ρ determines market share and the level of price relative to a: $x = \frac{1-\rho}{2-\rho}s$ and $p = a - \log(1-\rho)$. In particular, when the function switches from convex to concave (i.e., ρ is zero), the elasticity equals $\frac{a}{2}$, market share is 50%, and $p = a$.

forms of the manifold itself. The first result of this kind characterizes the demand functions which are consistent with a linear manifold:

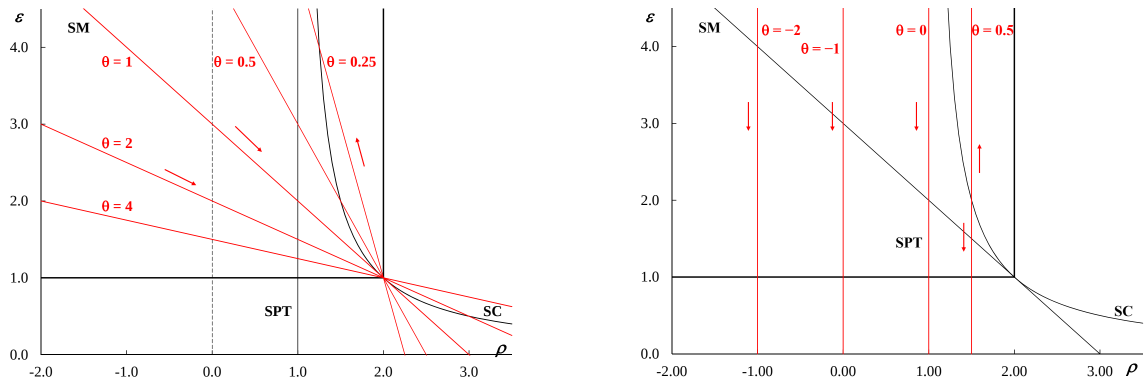
Proposition 2. *The Demand Manifold is linear in ε and ρ if and only if the inverse demand function takes a bipower form:*

$$p\left(x\right)=\alpha x^{-\eta}+\beta x^{-\theta}\quad\Leftrightarrow\quad\bar{\rho}\left(\varepsilon\right)=\eta+\theta+1-\eta\theta\varepsilon \tag{19}$$

Sufficiency follows by differentiating $p(x)$ and calculating the manifold directly. Necessity follows by setting $\rho(x)=a+b\varepsilon(x)$, where a and b are constants, and solving the resulting Euler-Cauchy differential equation. Details are in the Appendix, Section D.1. Clearly, this manifold is invariant with respect to the parameters α and β, so changes in exogenous variables such as income or market size which only affect α or β do not shift the manifold. Putting this differently, we need four parameters to characterize the demand function, but only two to characterize the manifold and therefore to predict the comparative statics responses reviewed in Section 2.

The demand system (19) nests a wide range of demands. One nested system is the "inverse PIGL" ("price-independent generalized linear") system, which is dual to the direct PIGL system of Muellbauer (1975), to be considered in the next sub-section. Setting η in (19) equal to one expresses expenditure $p(x)x$ as a "translated-CES" function of sales: $p(x)=\frac{1}{x}(\alpha+\beta x^{1-\theta})$. This system implies that the elasticity of marginal revenue defined in footnote 14 is constant and equal to θ: $\eta=1$ implies from (19) that $\varphi=\frac{2-\rho}{\varepsilon-1}=\theta$. The limiting case as $\theta\to 1$ is the inverse "PIGLOG" ("price-independent generalized logarithmic") or inverse translog, $p(x)=\frac{1}{x}(\alpha'+\beta'\log x)$.[19] This implies that the elasticity of marginal revenue is unity, and so, as noted in Mrázová and Neary (2011), it coincides with the supermodularity locus: $\eta=\theta=1$ implies from (19) that $\bar{\rho}\left(\varepsilon\right)=3-\varepsilon$. Panel(a) of Figure 8 shows the demand manifolds for some members of this family.

[19]To show this, rewrite the constants as $\alpha=\alpha'-\frac{\beta'}{1-\theta}$ and $\beta=\frac{\beta'}{1-\theta}$, and apply l'Hôpital's Rule.



(a) The Inverse PIGL Family

(b) The Bulow-Pfleiderer Family

Figure 8: Demand Manifolds for Bipower Inverse Demand Functions

A second and even more important special case of (19) comes from setting $\eta = 0$, giving the demand function $p(x) = \alpha + \beta x^{-\theta}$.[20] This is the iso-convex or "constant pass-through" family of Bulow and Pfleiderer (1983): from (19) with $\eta = 0$, convexity ρ equals a constant $\theta + 1$, so from (8) $\frac{1}{1-\theta}$ measures the degree of absolute pass-through for this system. Pass-through can be more than 100%, as in the CES case ($\alpha = 0$, $\theta = \frac{1}{\sigma} > 0$); equal to 100%, as in the log-linear direct demand function ($\theta \to 0$, so $p(x) = \alpha' + \beta' \log x$, implying that $\log x(p) = \gamma + \zeta p$); or less than 100%, as in the case of linear demand ($\theta = -1$ so pass-through is 50%). This family has many other attractive properties.[21] It is necessary and sufficient for marginal revenue to be affine in price. It can be given a discrete choice interpretation: it equals the cumulative demand that would be generated by a population of consumers if their preferences followed a Generalized Pareto Distribution.[22] Finally, as shown by Weyl and Fabinger (2013) and empirically implemented by Atkin and Donaldson (2012), it allows the division of surplus between consumers and producer to be calculated without knowledge of quantities. Panel(b) of Figure 8 shows the demand manifolds for some members of this family.

[20] Because η and θ enter symmetrically into (19), it is arbitrary which we set equal to zero. For concreteness and without loss of generality we assume $\beta \neq 0$ and $\theta \neq 0$ throughout.

[21] See the Appendix Section D.2 for more details.

[22] See Bulow and Klemperer (2012).

3.4 Bipower Direct Demand Functions

A second characterization result linking a family of demand functions to a particular functional form for the manifold is where the direct demand functions have the same form as the inverse demand functions in Proposition 2:

Proposition 3. *The Demand Manifold is such that ρ is linear in the inverse and squared inverse of ε if and only if the direct demand function takes a bipower form:*

$$x(p) = \gamma p^{-\nu} + \zeta p^{-\sigma} \quad \Leftrightarrow \quad \bar{\rho}(\varepsilon) = \frac{\nu + \sigma + 1}{\varepsilon} - \frac{\nu\sigma}{\varepsilon^2} \tag{20}$$

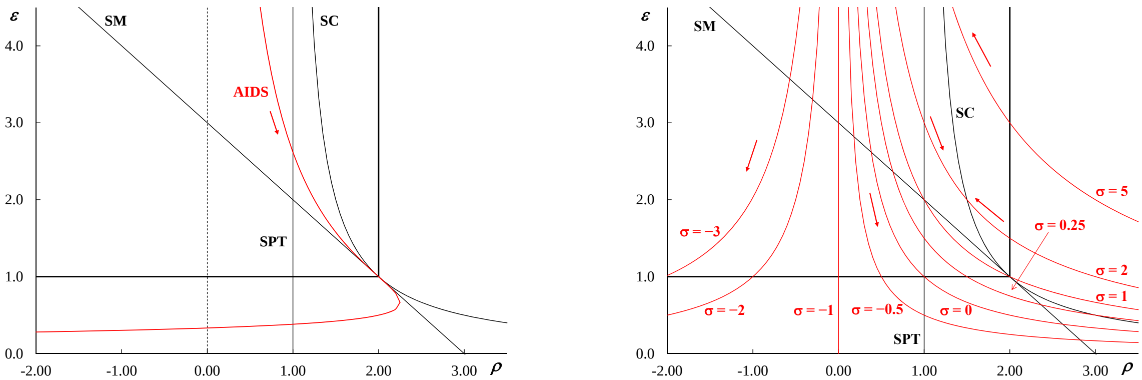
Formally, this proposition follows immediately from Proposition 2 by exploiting the duality between direct and inverse demand functions: see the Appendix, Section E.1, for details. Substantively, it nests some of the most widely-used demand functions in applied economics.

Two special cases of (20), dual to the special cases of (19) already considered, are of particular interest. The first, where $\nu = 1$, is the direct PIGL system of Muellbauer (1975): $x(p) = \frac{1}{p}\left[\gamma + \zeta p^{1-\sigma}\right]$, which implies that expenditure $px(p)$ is a translated-CES function of price. From (20), the manifold is given by: $\bar{\rho}(\varepsilon) = \frac{(\sigma+2)\varepsilon - \sigma}{\varepsilon^2}$. The PIGLOG is the limiting case as σ approaches 1 so $\bar{\rho}(\varepsilon) = \frac{3\varepsilon - 1}{\varepsilon^2}$.[23] This is consistent with both the AIDS model of Deaton and Muellbauer (1980), which is not in general homothetic, and with the homothetic translog of Feenstra (2003). In both cases, expenditure $px(p)$ is affine and decreasing in $\log p$: $x(p) = \frac{1}{p}(\gamma' + \zeta' \log p)$, $\zeta' < 0$.

Panel (a) of Figure 9 illustrates the properties of the PIGLOG/AIDS/translog manifold without the need for any calculations.[24] This demand function is the only example we have found which is both subconcave and supermodular throughout the admissible region. As for

[23] To see this, take the limit as in footnote 19.

[24] For completeness, we give the calculations here. Subconcavity follows since $\rho - \frac{\varepsilon+1}{\varepsilon} = -\frac{(\varepsilon-1)^2}{\varepsilon^2} \leq 0$. Supermodularity requires $\varepsilon + \rho - 3 = \frac{(\varepsilon-1)^3}{\varepsilon^2} \geq 0$, which holds if and only if $\varepsilon \geq 1$. Super-pass-through requires $\rho \geq 1$, which implies $\varepsilon \leq \frac{1}{2}(3 \pm \sqrt{5})$, i.e., $\varepsilon \in (0.38, 2.62)$. Finally, with $\varepsilon_x < 0$ always, decreasing pass-through holds if and only if $E_\rho < 0$, i.e., if and only if the curve is above its maximum value of ρ at $[\varepsilon, \rho] = [0.667, 2.25]$.



(a) The Translog Demand Function

(b) The Pollak Family

Figure 9: Demand Manifolds for Bipower Direct Demand Functions

pass-through, it exhibits sub-pass-through for high elasticities (i.e., low outputs) but super-pass-through otherwise, and pass-through is always decreasing in the admissible region. So, it provides a counter-example to Fabinger and Weyl (2012), who say: " ... it seems natural to assume that demand is globally either [log-concave or log-linear or log-convex]".[25]

A second important special case of (20) comes from setting $\nu = 0$. This gives the family of demand functions due to Pollak (1971): $x(p) = \gamma + \zeta p^{-\sigma}$. This family includes many widely-used demand functions, including the CES, quadratic, Stone-Geary (or "LES" for "Linear Expenditure System") and "CARA" ("constant absolute risk aversion"). Pollak showed that these are the only demand functions that are consistent with both additive separability and quasi-homotheticity (so the expenditure function exhibits the "Gorman Polar Form"). Just as (20) is dual to (19), so the Pollak family of direct demand functions is dual to the Bulow-Pfleiderer family of inverse demand functions. An implication of this is that, corresponding to the property of Bulow-Pfleiderer demands that marginal revenue is linear in price, Pollak demands exhibit the property that the marginal *loss* in revenue from a small increase in price is linear in sales.[26] This implies that the coefficient of absolute risk aversion for these demands

[25] Though they are careful to qualify this statement: "[Assumption 1] is satisfied by nearly all common statistical distributions and demand functions. Whether this calls into question typical demand functions and distributions, or bolsters Assumption 1, is left for the reader to decide."

[26] Recall from footnote 21 that Bulow-Pfleiderer demands $p(x) = \alpha + \beta x^{-\theta}$ satisfy the property: $p + xp' =$

is hyperbolic in sales, which is why, in the theory of choice under uncertainty, they are known as "HARA" ("hyperbolic absolute risk aversion") demands following Merton (1971).[27] Not surprisingly, the demand manifold is also a rectangular hyperbola: when $\nu = 0$, the left-hand side of (20) becomes $\bar{\rho}(\varepsilon) = \frac{\sigma+1}{\varepsilon}$.

Panel (b) of Figure 9 illustrates some members of the Pollak family. The CARA demand function is the limiting case when $\sigma \to 0$: the direct demand function becomes $x = \gamma' + \zeta' \log p$, $\zeta' < 0$, which implies that the inverse demand function is log-linear: $\log p = \alpha + \beta x$, $\beta < 0$.[28] The CARA manifold is $\bar{\rho}(\varepsilon) = \frac{1}{\varepsilon}$, which is a rectangular hyperbola through the point $\{1.0, 1.0\}$. Except in the CARA case (when the elasticity equals $-\frac{\zeta'}{x}$), the elasticity of demand for all members of the Pollak family is $\sigma\zeta\frac{p^{-\sigma}}{x} = \sigma\frac{x-\gamma}{x}$. This must be positive, which requires that σ, ζ and $x - \gamma$ must have the same sign. Hence the CARA function is the dividing line between two sub-groups of demand functions and their corresponding manifolds, with σ either negative or positive. For negative values of σ, γ is an upper bound to consumption: the best-known example of this class is the linear demand function, corresponding to $\sigma = -1$. By contrast, for strictly positive values of σ, γ is a "subsistence" level of consumption and there is no upper bound. All members of the family with positive σ are translated-CES functions, and as the arrows indicate, they asymptote towards the corresponding "untranslated-CES" function as sales rise without bound; for example, the LES demand function, with σ equal to one, asymptotes towards the Cobb-Douglas.[29]

$\theta\alpha+(1-\theta)p$. Switching variables, we can conclude that Pollak demands $x(p) = \gamma+\zeta p^{-\sigma}$ satisfy the property: $x + px' = \sigma\gamma + (1 - \sigma)x$.

[27] The Arrow-Pratt coefficient of absolute risk aversion is $A(x) \equiv -\frac{u''(x)}{u'(x)}$. With additive separability this becomes $A(x) = -\frac{p'(x)}{p(x)} = -\frac{1}{px'(p)}$. Using the result from footnote 26, this implies: $A(x) = \frac{1}{\sigma(x-\gamma)}$, which is hyperbolic in x.

[28] As noted by Pollak, this demand function was first proposed by Chipman (1965), who showed that it is implied by an additive exponential utility function. Later independent developments include Bertoletti (2006) and Behrens and Murata (2007). Differentiating the Arrow-Pratt coefficient of absolute risk aversion defined in footnote 27 gives $\frac{\partial A(x)}{\partial x} = -\frac{u'u'''-(u'')^2}{(u'')^2} = -\frac{pp''-(p')^2}{(p')^2} = 1-\varepsilon\rho$, so absolute risk aversion is constant if and only if $\varepsilon = \frac{1}{\rho}$.

[29] Note how these differ from the Bulow-Pfleiderer case in Figure 8, especially in the super-pass-through region. With Bulow-Pfleiderer demands, firms diverge from the CES benchmark along the SC locus as sales increase, whereas with Pollak demands they converge towards it; and both these statements hold whether demands are super- or subconvex.

Finally, the demand functions are strictly superconvex if and only if both σ and γ are strictly positive.[30]

3.5 Demand Functions that are Not Manifold-Invariant

In the rest of the paper we concentrate on the demand functions introduced here which have invariant manifolds. Appendix Section F presents two examples of demand functions which have non-invariant manifolds, and which nest some important cases, such as the "Adjustable Pass-Through" (Apt) demand function of Fabinger and Weyl (2012). These have manifolds with the same number of parameters as the demand function, four or more, which implies a clear trade-off. On the one hand, they are more flexible and so have greater potential to match any desired relationship between ε and ρ. On the other hand, they are more difficult to work with theoretically or to estimate empirically.

4 Monopolistic Competition in General Equilibrium

4.1 The Positive Effects of Globalization

To illustrate the power of the approach we have developed in previous sections, we turn in the remainder of the paper to apply it to a canonical model of international trade, a one-sector, one-factor, multi-country, general-equilibrium model of monopolistic competition. To highlight the new features of our approach, we focus on the case considered by Krugman (1979), where countries are symmetric, trade is unrestricted, and firms are homogeneous. For the present we assume only that preferences are symmetric, and that the elasticity of demand depends only on consumption levels, which in symmetric equilibrium means on the amount consumed of a typical variety, denoted by x. We do not need to make explicit our assumptions about preferences until we consider welfare in Section 4.2.

[30]The criterion for superconvexity, $\rho \geq \frac{\varepsilon+1}{\varepsilon}$, becomes $\frac{\sigma\gamma}{x\varepsilon} \geq 0$ for Pollak demands. This holds strictly if and only if σ and γ have the same sign and are non-zero; and this cannot be true when σ is strictly negative, since as we have seen this implies that $x - \gamma$ is also strictly negative, so γ is strictly positive.

Symmetric demands and homogeneous firms imply that we can dispense with firm subscripts from the outset. Industry equilibrium requires that firms maximize profits by setting marginal revenue MR equal to marginal cost MC, and that profits are driven to zero by free entry (so average revenue AR equals average cost AC):

$$\text{Profit Maximization (MR=MC):} \quad p = \frac{\varepsilon(x)}{\varepsilon(x) - 1} c \tag{21}$$

$$\text{Free Entry (AR=AC):} \quad p = \frac{f}{y} + c \tag{22}$$

The model is completed by market-clearing conditions for the goods and labor markets:

$$\text{Goods-Market Equilibrium (GME):} \quad y = kLx \tag{23}$$

$$\text{Labor-Market Equilibrium (LME):} \quad L = n(f + cy) \tag{24}$$

Here L is the number of worker/consumers in each country, each of whom supplies one unit of labor and consumes an amount x of every variety; k is the number of identical countries; and n is the number of identical firms in each, all with total output y, so $N = kn$ is the total number of firms in the world. Since all firms are single-product by assumption, N is also the total number of varieties available to all consumers.

Equations (21) to (24) comprise a system of four equations in four endogenous variables, p, x, y and n, with the wage rate set equal to one by choice of numéraire. To solve for the effects of "globalization", an increase in the number of countries k, we totally differentiate the equations, using "hats" to denote logarithmic derivatives, so $\hat{x} \equiv d \log x$, $x \neq 0$:

$$\text{MR=MC:} \quad \hat{p} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon(\varepsilon - 1)} \hat{x} \tag{25}$$

$$\text{AR=AC:} \quad \hat{p} = -(1 - \omega)\hat{y} \tag{26}$$

$$\text{GME:} \quad \hat{y} = \hat{k} + \hat{x} \tag{27}$$

$$\text{LME:} \quad 0 = \hat{n} + \omega \hat{y} \tag{28}$$

Consider first the MR=MC equilibrium condition, equation (25). Clearly p and x move together if and only if $\varepsilon + 1 - \varepsilon\rho > 0$, i.e., if and only if demand is subconvex. This reflects the property noted in Section 2.2: higher sales are associated with a higher mark-up if and only if they imply a lower elasticity of demand. As for the free-entry condition, equation (26), it shows that the fall in price required to maintain zero profits following an increase in firm output is greater the smaller is $\omega \equiv \frac{cy}{f+cy}$, the share of variable in total costs, which is an inverse measure of returns to scale. This looks like a new parameter but in equilibrium it is not. It equals the ratio of marginal cost to price, $\frac{c}{p}$, which because of profit maximization equals the ratio of marginal revenue to price $\frac{p+xp'}{p}$, which in turn is a monotonically increasing transformation of the elasticity of demand ε: $\omega = \frac{c}{p} = \frac{p+xp'}{p} = \frac{\varepsilon-1}{\varepsilon}$. Similarly, equation (28) shows that the fall in the number of firms required to maintain full employment following an increase in firm output is greater the larger is ω. It follows by inspection that all four equations depend only on two parameters, which implies:

Lemma 3. *The local comparative statics properties of the symmetric monopolistic competition model with respect to a globalization shock depend only on ε and ρ.*

Solving for the effects of globalization on outputs, prices and the number of firms in each country gives:

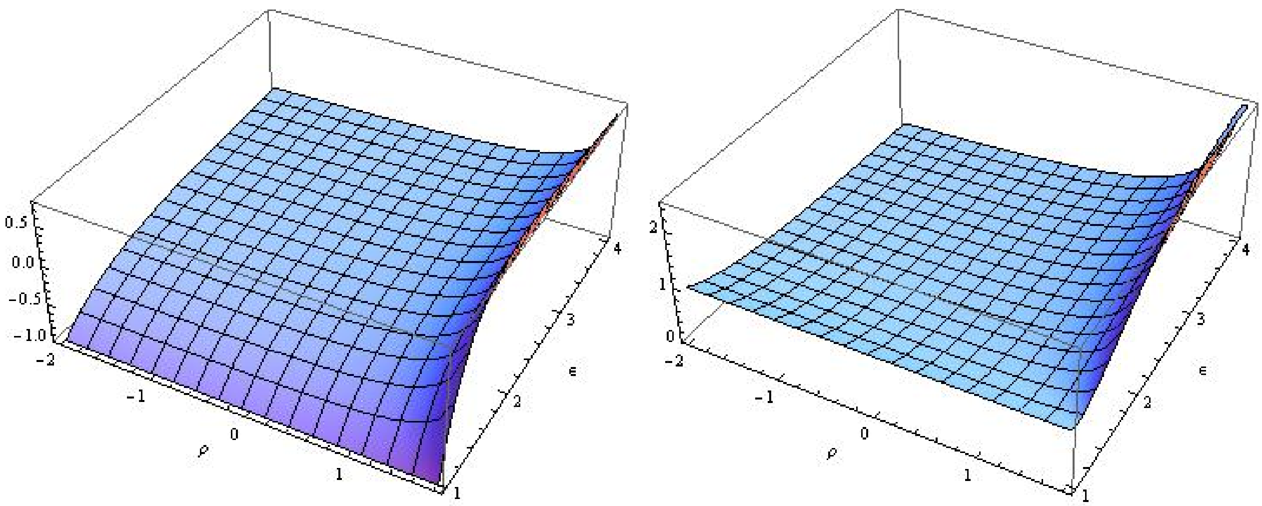
$$\hat{y} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon\,(2-\rho)}\hat{k}, \quad \hat{p} = -\frac{1}{\varepsilon}\hat{y}, \quad \hat{n} = -\frac{\varepsilon - 1}{\varepsilon}\hat{y} \tag{29}$$

(Details of the solution are given in the Appendix, Section G.) The signs of these depend solely on whether demands are sub- or superconvex, i.e., whether $\varepsilon + 1 - \varepsilon\rho$ is positive or negative. With subconvexity we get what Krugman (1979) called "sensible" results: globalization prompts a shift from the extensive to the intensive margin, with fewer but larger firms in each country, as firms move down their average cost curves and prices of all varieties fall. With superconvexity, as noted by Zhelobodko, Kokovin, Parenti, and Thisse (2012), all these results are reversed. (See also Neary (2009).) The CES case, where

$\varepsilon + 1 - \varepsilon\rho = 0$, is the boundary one, with firm outputs, prices, and the number of firms per country unchanged. The only effects which hold irrespective of the form of demand are that consumption per head of each variety falls and the total number of varieties produced in the world and consumed in each country rises:

$$\hat{x} = -\frac{1}{2-\rho}\frac{\varepsilon-1}{\varepsilon}\hat{k} \ < \ 0, \qquad \hat{N} = \frac{(\varepsilon-1)^2 + (2-\rho)\,\varepsilon}{\varepsilon^2\,(2-\rho)}\hat{k} \ > \ 0 \tag{30}$$

In qualitative terms these results are not new. The new feature that our approach highlights is that their quantitative magnitudes depend only on two parameters, ε and ρ, the same ones on which we have focused throughout.



(a) Changes in Prices (b) Changes in the Number of Varieties

Figure 10: The Effects of Globalization

Figure 10 gives the quantitative magnitudes of changes in the two variables that matter most for welfare, prices and the number of varieties. (In the next section we will see how exactly they affect welfare.) In each panel, the vertical axis measures the proportional change in either p or N relative to k as a function of the elasticity and convexity of demand. These three-dimensional surfaces are independent of the functional form of demand, so we can combine them with the results on demand manifolds in the previous section to compute the implications of different assumptions about demand for the quantitative magnitude of the effects of globalization. We know already from equations (29) and (30) that prices fall if and only if demand is subconvex and that product variety always rises. The figures show in addition that lower values of the elasticity of demand are usually associated with greater falls in prices and larger increases in variety;[31] while more convex demand is always associated with greater increases (in absolute value) in both prices and the number of varieties.

To summarize this sub-section, Lemma 3 implies that the demand manifold is a sufficient statistic for the positive effects of globalization in the Krugman (1979) model, just as it is for the comparative statics results discussed in Section 2. However, to make quantitative predictions about how welfare is affected we need to know how consumers trade off the changes in prices and product variety illustrated in Figure 10. We turn to this next.

4.2 Welfare: Additive Separability and the Utility Manifold

To quantify the welfare effects of globalization, we follow Dixit and Stiglitz (1977) and assume that preferences are additively separable. Hence the overall utility function, denoted by U, is a monotonically increasing function of an integral of sub-utility functions, denoted by u, each defined over the consumption of a single variety: $U = f\left[\int_0^N u\{x(\omega)\}\,\mathrm{d}\omega\right]$, $f' > 0$. Marginal utility must be positive and decreasing in the consumption of each variety: $u'(x) > 0$ and $u''(x) < 0$. With symmetric preferences and no trade costs the overall utility function becomes: $U = f\,[Nu(x)]$. So, welfare depends on the extensive margin of consumption N times the utility of the intensive margin x.

Using the budget constraint to eliminate x, we write the change in utility in terms of its income equivalent $\hat{Y}$, which is independent of the function f (see the Appendix, Section H, for details):

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{N} - \hat{p} \tag{31}$$

[31] Though these properties are reversed if demands are highly convex: $\hat{p}/\hat{k}$ is increasing in ε if and only if $\rho < 1 + \frac{2}{\varepsilon}$, and $\hat{N}/\hat{k}$ is decreasing in ε if and only if $\rho < \frac{2}{\varepsilon}$.

Here $\xi(x) \equiv \frac{xu'(x)}{u(x)}$ is the elasticity of utility with respect to consumption: clearly, it must lie between zero and one if preferences exhibit a taste for variety. (See also Vives (1999).) Moreover, since welfare rises more slowly with N the higher is ξ, it is an inverse measure of preference for variety. This parameter plays the key role of trading off changes in prices and number of varieties from a consumer's perspective. To understand this role, we relate $\xi(x)$ to the elasticity of demand, $\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)}$, which is an inverse measure of the concavity of the sub-utility function: $\varepsilon(x) = -\frac{u'(x)}{xu''(x)}$.[32] Given these two parameters that are unit-free measures of the slope and curvature of u, we can proceed in a similar manner to Sections 2 and 3: we first show how different configurations of ξ and ε determine the properties of the model, and in particular the efficiency of equilibrium; we then solve for the relationship between the two parameters, which we call the "utility manifold", that is implied by specific assumptions about the form of u.

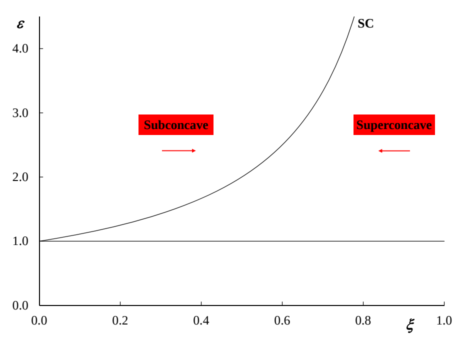


Figure 11: The Sub- and Superconcave Regions in $\{\varepsilon, \xi\}$ Space

Figure 11 illustrates part of the admissible region in $\{\varepsilon, \xi\}$ space. As in previous sections, the CES case is an important threshold. Corresponding to the CES inverse demand function from (4), $p(x) = \beta x^{-1/\sigma}$, is the sub-utility function:

$$u(x) = \frac{\sigma}{\sigma - 1}\beta x^{\frac{\sigma-1}{\sigma}} \quad \Rightarrow \quad \xi(x) = \xi^{CES} \equiv \frac{\sigma - 1}{\sigma}, \;\; \varepsilon(x) = \varepsilon^{CES} \equiv \sigma \tag{32}$$

[32] A utility function $u_1(x)$ is more concave than a different utility function $u_2(x)$ at a common point if $\frac{xu_1''}{u_1'} < \frac{xu_2''}{u_2'}$. Switching both from negative to positive and inverting, we can conclude that $u_1(x)$ is more concave than $u_2(x)$ if $\varepsilon_1(x) < \varepsilon_2(x)$.

Both ε and ξ depend only on σ, so we can solve for the locus of $\{\varepsilon, \xi\}$ combinations, i.e., the locus of utility point-manifolds, consistent with CES preferences: $\varepsilon = \frac{1}{1-\xi}$, or $\xi = \frac{\varepsilon - 1}{\varepsilon}$. This locus is labeled SC in Figure 11. By analogy with our treatment of demand in Section 2.2, it defines a threshold degree of concavity of the sub-utility function, and we call a utility or sub-utility function "superconcave" if it is more concave than that threshold:

Definition 2. A utility function $u(x)$ is superconcave at a point (u_0, x_0) if and only if $\log u(x)$ is concave in $\log x$ at (u_0, x_0).

Analogously to Section 2.2, a utility function is superconcave at a point if and only if it is more concave than a CES function with the same elasticity of utility at that point:

$$\frac{d^2 \log u}{d(\log x)^2} = \xi \left(\frac{\varepsilon - 1}{\varepsilon} - \xi \right) = \xi \left(\frac{1}{\varepsilon^{CES}} - \frac{1}{\varepsilon} \right) \leq 0 \tag{33}$$

Since ε is an inverse measure of concavity, the region below and to the right of the SC locus in Figure 11 corresponds to superconcavity of utility. In addition, superconcavity of utility is equivalent to the elasticity of utility decreasing in consumption:

$$\xi_x = \frac{\xi}{x} \left(\frac{\varepsilon - 1}{\varepsilon} - \xi \right) \leq 0 \tag{34}$$

It follows that the elasticity of utility is associated with higher levels of consumption as indicated by the arrows in Figure 11: rising in the subconcave region and falling in the superconcave region, in a similar manner to how the elasticity of demand varies with consumption in Figure 2.[33]

[33] Vives (1999), pages 170-1, argues that the latter case, preference for variety that is increasing in consumption, or superconcavity of utility, is intuitively plausible, though, as he notes, this is not the view of Dixit and Stiglitz (1977).

4.3 Globalization, Efficiency and Welfare

To relate superconcavity of utility to the efficiency of equilibrium, we return to the expression for welfare change in (31). The change in the extensive margin can be decomposed into two components, one exogenous and international, the other endogenous and intranational: $\hat{N} = \hat{k} + \hat{n}$. We can eliminate the latter using the full-employment and zero-profit conditions, equations (26) and (28): $\hat{n} = -\omega\hat{y} = \frac{\omega}{1-\omega}\hat{p}$. This allows us to decompose the change in real income into a *direct* and an *indirect* effect of globalization:

$$\hat{Y} = \frac{1-\xi}{\xi}\hat{k} + \frac{\omega - \xi}{\xi(1-\omega)}\hat{p} \tag{35}$$

The direct effect given by the first term is positive, so we focus on the indirect effect, which works through the induced change in prices. A non-zero coefficient of $\hat{p}$ implies that there is scope for increasing real income even in the absence of globalization, in other words, that the initial equilibrium is *inefficient*. Hence the condition for efficiency in this model is that ω and ξ should be equal. To see why, note that the social optimum requires that ξ should equal one in a competitive economy: with no fixed costs (so $f = 0$ and $\omega = 1$), it is optimal to produce as many varieties as needed to indulge consumers' taste for variety. By contrast, when fixed costs are strictly positive ($f > 0$ so $\omega < 1$), efficiency requires that ξ be strictly less than one. Recalling that ω is an inverse measure of returns to scale, the intuition for this is that more strongly increasing returns to scale mandate substitution away from the extensive towards the intensive margin. The social optimum occurs when the gain to consumers from an additional variety, of which ξ is an inverse measure, exactly matches the cost to society of setting up an additional firm, of which ω is an inverse measure. Absent efficiency, we can say that varieties are *under-supplied* if and only if ξ is less than ω. Recalling Definition 2 and the fact that ω equals $\frac{\varepsilon-1}{\varepsilon}$ in equilibrium, we can conclude:

Corollary 3. *Relative to the efficient benchmark where $\xi = \omega$, varieties are under-supplied if and only if utility is subconcave, i.e., $\xi \leq \omega$.*

The intuition underlying this is that subconcavity of utility, $\xi \leq \frac{\varepsilon-1}{\varepsilon}$, which is a partial-equilibrium property of preferences, turns out in general equilibrium to imply that varieties are under-supplied: $\xi \leq \omega$. Note that there is not a unique efficient equilibrium in general: any equilibrium which lies along the SC locus in Figure 11 is efficient.

Quantifying the gains from globalization in this model is straightforward when the initial equilibrium is efficient, but this will occur in only two cases. The first is when preferences are CES. In that case, as we have already seen, both ω and ξ equal $\frac{\sigma-1}{\sigma}$. Quantitatively, the gains from globalization, $\hat{Y}/\hat{k}$, reduce to $\frac{1}{\sigma-1}$, exactly the expression found for the gains from trade in a range of CES-based models by Arkolakis, Costinot, and Rodríguez-Clare (2012). Qualitatively, this result is familiar from Dixit and Stiglitz (1977): only CES preferences ensure that the gains from greater variety are exactly matched by the losses from reducing the scale of production of existing varieties. A second case in which efficiency occurs is when it is brought about by government intervention: for given k, optimal competition policy chooses the welfare-maximizing level of n (which in turn determines p and x). This case requires strong assumptions: benevolent and cooperative governments, plus an institutional framework for anti-trust policy which does not impose any further inefficiencies of its own. We abstract from policy issues from now on, and focus on the implications of non-CES preferences.

Absent efficiency, we can assess the sign and magnitude of the gains from globalization with reference to the CES benchmark, which is also the efficient or "first-best" benchmark. Consider first the direct gain in (35), given by the coefficient of $\hat{k}$. It exceeds the CES benchmark $\frac{1}{\varepsilon-1}$ if and only if $\xi \leq \frac{\varepsilon-1}{\varepsilon}$, which gives our first result:

Proposition 4. *The direct gain from globalization exceeds the CES benchmark if and only if utility is subconcave.*

This makes sense: consumers always gain directly from more varieties, and gain by more when varieties are initially under-supplied.

Consider next the indirect effect, which will be positive if the exogenous shock brings the world economy closer to efficiency. Recalling from (29) that prices rise if and only if demand is superconvex, we can sign this as follows:

Proposition 5. *The indirect gain from globalization is positive if and only if either: (a) demand is superconvex and utility is subconcave; or (b) demand is subconvex and utility is superconcave.*

In case (a), prices rise, but varieties are under-supplied, so the loss at the intensive margin is offset by the gain to consumers of increased variety. This reasoning is reversed in case (b): now varieties are over-supplied, but efficiency is increased as consumers gain from the fall in prices.

Improving efficiency is sufficient for a globalization shock to raise welfare but it is not necessary. To assess the full effect, we have to combine the indirect or induced-efficiency effect with the direct effect. First we obtain two alternative sufficient conditions for the overall gains to be positive from the two equations for the change in real income, (31) and (35):

Proposition 6. *Gains from globalization are guaranteed if either: (a) demand is subconvex; or (b) utility is subconcave.*

Putting this differently, losses from globalization are possible only if demand is superconvex *and* utility is superconcave. The intuition for this proposition combines that from previous results. In case (a), if demand is *subconvex*, i.e., if $\frac{\varepsilon+1}{\varepsilon}$ is greater than ρ, then prices fall. As a result, from (31), consumers reap a double dividend: welfare rises at both the intensive and extensive margins. In this case, the elasticity of utility is irrelevant for the sign of welfare change. In case (b), if utility is *subconcave*, i.e., if $\frac{\varepsilon-1}{\varepsilon}$ is greater than ξ, then varieties are under-supplied in the initial equilibrium. Even if prices rise, consumers value variety sufficiently that the gain at the extensive margin from the increase in the number of varieties offsets the losses at the intensive margin due to higher prices.

Second, we can calculate a necessary and sufficient condition for gains from globalization. Substitute for $\omega = \frac{\varepsilon-1}{\varepsilon}$ and $\hat{p}$ into (35) to obtain an explicit expression for the gain in welfare:

$$\hat{Y} = \frac{1}{\xi\varepsilon}\left[1 - \left(\xi - \frac{\varepsilon-1}{\varepsilon}\right)\frac{\varepsilon-1}{2-\rho}\right]\hat{k} \tag{36}$$

Now there are three sufficient statistics for the change in welfare, only one of which has an unambiguous effect. The gains from globalization are always decreasing in ξ: unsurprisingly, consumers gain more from a proliferation of countries and hence of products, the greater their taste for variety. By contrast, the gains from globalization depend ambiguously on both ε and ρ. Of course, the values of the three key parameters do not in general vary independently of each other, which suggests how we should proceed. Equation (36) gives the change in real income as a function of ξ, ε, and ρ only. These in turn are related to each other and to ϕ, the vector of non-invariant parameters, via the demand manifold, $\varepsilon = \bar{\varepsilon}(\rho, \phi)$, and the utility manifold, $\xi = \bar{\xi}(\varepsilon, \phi)$. To get an explicit solution for the gains from globalization we thus have to solve three equations in $4 + m$ unknowns, where m is the dimension of ϕ. This can be visualized in three dimensions when m equals one, so we turn next to consider how ξ varies in the two families of demand functions with only a single non-invariant parameter discussed in Section 3.

4.4 Globalization and Welfare with Bipower Preferences

The first example we consider is that of bipower demands, given by the demand function (19) in Section 3.3. Integrating that function yields the corresponding utility function, which also takes a bipower form:[34]

$$u(x) = \frac{1}{1-\eta}\alpha x^{1-\eta} + \frac{1}{1-\theta}\beta x^{1-\theta} \tag{37}$$

[34] It is natural to set the constant of integration to zero, which implies that $u(0) = 0$. A non-zero value of $u(0)$ could be interpreted to imply that consumers gain or lose from the introduction of new varieties even if they do not consume them, though Dixit and Stiglitz (1979) argue to the contrary. We return to this issue in the next sub-section.

Proceeding as in Proposition 2, we can derive a closed-form expression for the utility manifold in this case:[35]

Proposition 7. *If and only if the utility function is as in* (37)*, the utility manifold is:*

$$\bar{\xi}(\varepsilon) = \frac{(1-\eta)(1-\theta)\varepsilon}{(1-\eta-\theta)\varepsilon+1} \tag{38}$$

Direct calculations now yield:

Proposition 8. *With bipower utility as in* (37)*, demand is subconvex if and only if* $(\eta\varepsilon - 1)(\theta\varepsilon - 1) \geq 0$*; utility is superconcave if and only if* $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{(1-\eta-\theta)\varepsilon+1} \geq 0$

Recalling the sufficient conditions for gains from globalization given in Proposition 6, this implies:

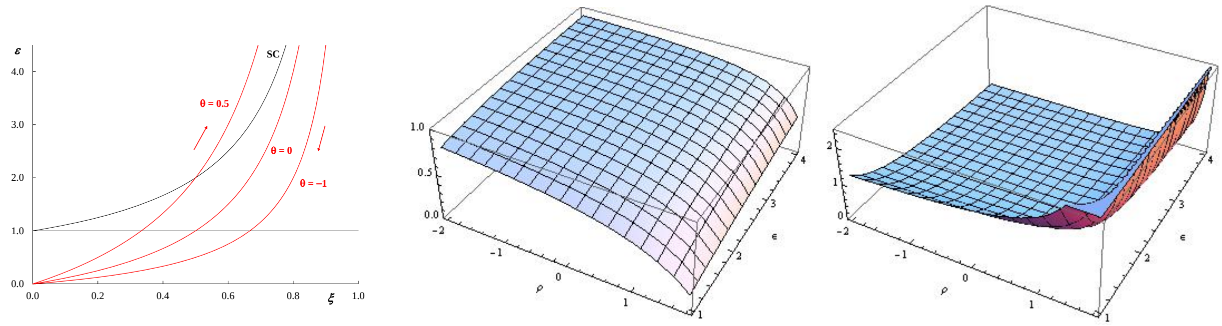
Corollary 4. *With bipower utility as in* (37)*, gains from globalization are guaranteed if* $(1-\eta-\theta)\varepsilon + 1 > 0$.

If this condition holds, Proposition 8 implies that the only possible combinations allowed by the utility function (37) are *either* subconvex demand and superconcave utility *or* superconvex demand and subconcave utility; either prices fall, or consumers don't lose too much when they rise because varieties are initially under-supplied. It follows from Proposition 6 that welfare must increase as the world economy expands.

In the special case of Bulow-Pfleiderer demands, when η is zero and so $\theta = \rho - 1$, the sufficient condition from Corollary 4 holds, and we can characterize the possible outcomes as follows:

Corollary 5. *With Bulow-Pfleiderer demands, so utility is given by* (37) *with* $\eta = 0$*, demand is subconvex and utility is superconcave if* $\theta > 0$ *and* $\alpha > 0$*; otherwise demand is superconvex and utility is superconcave. In both cases, globalization must raise welfare.*

[35]Proofs of all results in this sub-section are in the Appendix, Section I.



(a) Utility Manifold (b) Elasticity of Utility (c) Change in Real Income

Figure 12: Globalization and Welfare: Bulow-Pfleiderer Demands

Panel (a) of Figure 12 illustrates, showing three utility manifolds from the Bulow-Pfleiderer family. Recalling from (30) that consumption of each variety falls, globalization always moves the equilibrium in the opposite direction to the arrows. To the right of the SC locus, utility is superconcave and so, from Proposition 5, demand is subconvex. Globalization leads to a rightward movement, and so always moves the economy closer to efficiency, though the elasticity of utility changes in a paradoxical way. From Corollary 3, varieties are initially over-supplied, as the "efficiency gap" $\xi - \omega$ is positive. Globalization reduces this gap, though at the same time it *raises* ξ: consumers' preference for variety falls. A similar outcome follows, *mutatis mutandis*, to the left of the SC locus. Here too globalization raises efficiency, though by moving the efficiency gap, initially negative, closer to zero, while at the same time lowering ξ. In both cases, the global economy converges asymptotically towards efficiency, though in different ways depending on whether θ is greater or less than zero. From Figure 8 in Section 3.3, if θ is greater than zero, so demands are strictly log-convex, the equilibrium converges towards a CES equilibrium with positive profit margins. By contrast, if θ is equal to or less than zero, so demands are log-concave, the equilibrium converges towards a quasi-perfectly-competitive outcome in which price equals marginal cost and the elasticity of utility equals one. Products are still differentiated, but consumers no longer care about variety.

The remaining panels of Figure 12 illustrate how the initial value of ξ and the change in welfare vary with ε and ρ. As panel (b) shows, the elasticity of utility is always between zero and one, is increasing in the elasticity of demand, and decreasing in convexity: there is a greater taste for variety at high ρ. Recalling from equation (31) that ξ is an inverse measure of the weight that consumers attach to the variety changes shown in Figure 10, we find the implications for the gains from globalization shown in Panel (c). The gains are always positive, are decreasing in ε and increasing in ρ.

4.5 Globalization and Welfare with Pollak Preferences

The second example we consider is the Pollak/HARA demand function from Section 3.4. It turns out that the welfare implications of this specification depend crucially on how we normalize the sub-utility function. To highlight the contrast with the Bulow-Pfleiderer case in the previous sub-section, we focus in the text on the case considered by Pollak (1971) and Dixit and Stiglitz (1977). (In the Appendix we consider an alternative specification, due to Pettengill (1979), which yields different results.) This gives the following sub-utility function:

$$u(x) = \frac{\sigma}{\sigma - 1}\beta(x-\gamma)^{\frac{\sigma-1}{\sigma}} \tag{39}$$

Solving for the elasticity of utility, the utility manifold is:

$$\bar{\xi}(\varepsilon) = \frac{\sigma - 1}{\varepsilon} \tag{40}$$

Combining this with the demand manifold from Section 3.4, we can express the elasticity of utility as a function of ε and ρ only: $\xi = \frac{\varepsilon\rho - 2}{\varepsilon}$. Since only values of ξ between zero and one are admissible, we restrict attention to the range $\frac{2}{\varepsilon} < \rho < 1 + \frac{2}{\varepsilon}$.[36] It follows that the configurations of demand and utility are the opposite to those in the Bulow-Pfleiderer case:[37]

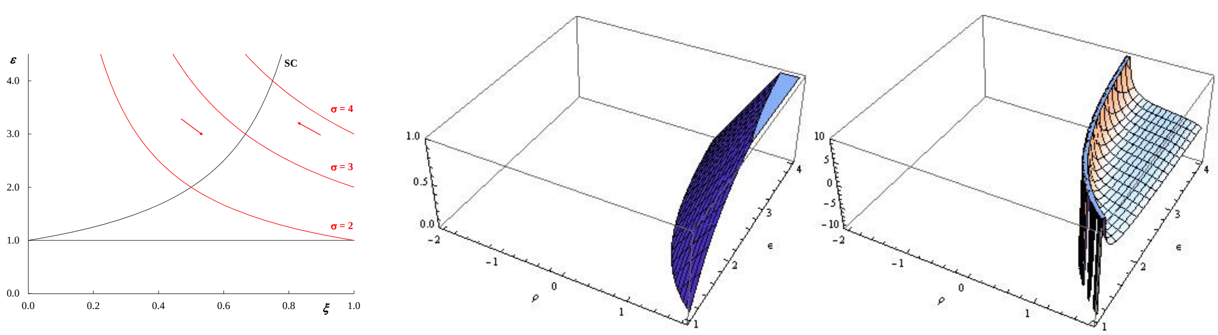
Proposition 9. *With Pollak preferences as in* (58) *and* $0 < \xi < 1$*, demand is superconvex*

[36] See Pettengill (1979) and Dixit and Stiglitz (1979). The restricted range excludes the Linear Expenditure System ($\rho = \frac{2}{\varepsilon}$) but includes demand functions that are both more and less convex than the CES ($\rho = \frac{\varepsilon+1}{\varepsilon}$).

[37] This extends a result of Vives (1999), p. 371, who shows that with HARA preferences utility is subconcave when demand is subconvex.

and utility is superconcave if and only if $\gamma > 0$.

Panel (a) of Figure 13 illustrates. To the left of the SC locus, utility is subconcave and so, from Proposition 9, demand is subconvex. Hence globalization lowers prices and raises welfare, even though it *reduces* efficiency: the "efficiency gap" is initially negative, and is reduced further as the elasticity of utility falls: varieties are increasingly under-supplied. Efficiency also falls to the right of the locus, and now this indirect effect can dominate the direct effect, leading to a net loss in welfare, as prices rise and varieties are increasingly over-supplied. In both cases, the global economy converges asymptotically towards an inefficient equilibrium: in the former case profit margins are driven to zero and varieties are under-supplied, whereas in the latter case either the elasticity of demand or of utility converges to unity.



(a) Utility Manifold (b) Elasticity of Utility (c) Change in Real Income

Figure 13: Globalization and Welfare: Pollak Preferences

The remaining panels of Figure 13 illustrate how the elasticity of utility and the gains from globalization vary with ε and ρ in this case. The contrast with the Bulow-Pfleiderer case in Figure 12 could hardly be more striking. Panel (b) shows that the elasticity of utility is now increasing in *both* ε and ρ: consumers have a lower taste for variety at high ρ, which we know from Figure 10(a) is when prices increase most. As a result, welfare can fall with globalization. As panel (c) shows, the gains from globalization are decreasing in both ε and ρ, and are *negative* for sufficiently convex demand: as shown in the Appendix, the exact condition for this is $\rho > \frac{\varepsilon^2+2\varepsilon-1}{\varepsilon^2}$. This provides, to our knowledge, the first concrete example of the "folk theorem" that welfare can fall if preferences are (in our terminology) sufficiently superconvex. Perhaps equally striking is that welfare rises by more for lower values of ε and ρ: estimates based on CES preferences grossly *underestimate* the gains from globalization in much of the subconvex region, just as they fail to predict losses from globalization in the superconvex region.

5 Conclusion

In this paper we have presented a new way of relating the structure of demand and utility functions to the positive and normative properties of monopolistic and monopolistically competitive markets. By adopting a "firms'-eye view" of demand, we have shown how the elasticity and convexity of demand determine many comparative statics responses. In turn, we have shown how the relationship between these two parameters, which we call the "demand manifold," provides a parsimonious representation of an arbitrary demand function, and a sufficient statistic for many comparative statics results. The manifold is particularly useful when it is unaffected by changes in exogenous variables, a property which we call "manifold invariance." We have introduced some new families of demand systems which exhibit manifold invariance, and have shown that they nest many of the most widely used functions in applied theory. For example, our "bipower direct" family provides a natural way of nesting translog, CES and linear demand functions.[38]

To illustrate the usefulness of our approach, we have shown how it allows a parsimonious way of understanding how monopolistically competitive economies adjust to external shocks, as well as a framework for quantifying the effects of globalization. The demand manifold turns out to be a sufficient statistic for the positive implications of globalization in general equilibrium. As for the normative implications, we have shown that the same approach can be applied to an arbitrary utility function. The relationship between the elasticity

[38] Alternative ways of nesting translog and CES demands, though with considerably more complicated demand manifolds, appear in Novy (2013) and in Pollak, Sickles, and Wales (1984).

and concavity of such a function, which we call the "utility manifold," plays a similar role to the demand manifold. We have shown how to compute the gains from trade for an arbitrary utility function, and have illustrated how sensitive are estimates of these gains to alternative specifications of preferences. Our approach suggest that the CES case is an unreliable reference point for calibrating the effects of exogenous shocks. It has been known since Dixit and Stiglitz (1977) that CES preferences ensure efficiency, but the implications of this observation for calibration have not been brought out. When the initial equilibrium is not efficient, any shock has both a direct effect and an indirect effect whose implications hinge on whether it brings the economy closer to or further away from efficiency. As a result, while positive gains are guaranteed with CES preferences, losses from trade are possible with demands that are more convex than CES, and gains from trade can be greater than in the CES case when demands are less convex. Quantifying the gains from trade *assuming* CES preferences is going to miss some important effects.

Many extensions of our approach naturally suggest themselves. There are many other topics where functional form plays a key role in determining the implications of a given set of assumptions: applications to choice under uncertainty and to oligopoly immediately come to mind. As for our application to the gains from globalization in monopolistic competition, the framework we have presented can straightforwardly be extended to allow for trade costs and heterogeneous firms.[39] Finally, the families of demand functions we have introduced provide a natural setting for estimating relatively flexible functional forms, and direct attention towards the parameters which matter for comparative statics predictions.

[39] Models combining trade costs and heterogeneous firms with general non-CES preferences have been considered by Zhelobodko, Kokovin, Parenti, and Thisse (2012), Bertoletti and Epifani (2012), and Dhingra and Morrow (2011).

Appendices

A Alternative Measures of Slope and Curvature

As our measure of demand slope, we work throughout with the price elasticity of demand, which can be expressed in terms of the derivatives of either the inverse or the direct demand functions $p(x)$ and $x(p)$: $\varepsilon \equiv -\frac{p}{xp'} = -\frac{px'}{x}$. Many authors have used the inverse of this elasticity, $e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon}$, under a variety of names: the elasticity of marginal utility: $e = -\frac{d \log u'(x)}{d \log x}$; the "relative love for variety" as in Zhelobodko, Kokovin, Parenti, and Thisse (2012); or (in monopoly equilibrium) the profit margin or Lerner Index of monopoly power: $e = \frac{p-c}{p}$. This has the advantage that its definition is symmetric with that of curvature ρ (and also with those of the elasticity of utility ξ and the "prudence" of demand χ to be discussed below). Offsetting advantages of using ε include its greater intuitive appeal, and the fact that it focuses attention on the region of parameter space where comparative statics results are ambiguous.

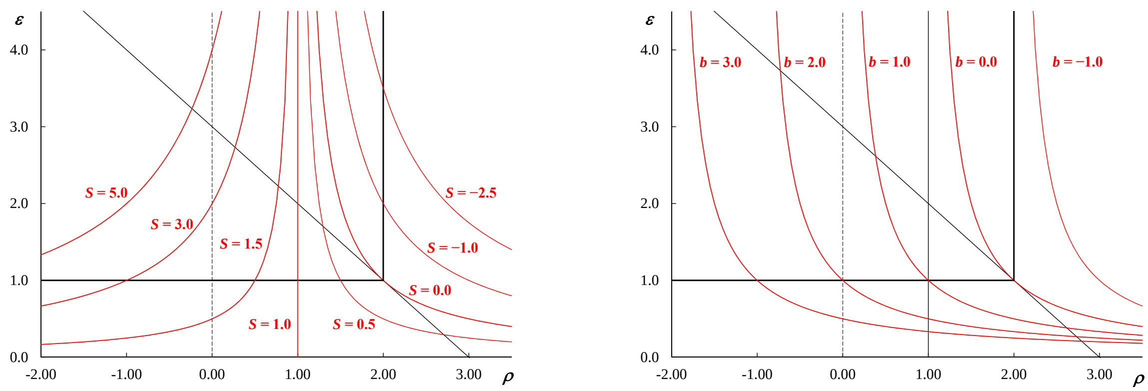
Turning to measures of curvature, the convexity of inverse demand which we use throughout equals the elasticity of the slope of inverse demand, $\rho \equiv -\frac{xp''}{p'} = -\frac{d \log p'(x)}{d \log x}$. Its importance for comparative statics results was highlighted by Seade (1980), and it is widely used in industrial organization, for example by Bulow, Geanakoplos, and Klemperer (1985) and Shapiro (1989). An alternative measure is the convexity of the direct demand function $x(p)$: $r(p) \equiv -\frac{px''(p)}{x'(p)}$. A convenient property is that e and r are dual to ε and ρ:

$$e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon} \qquad r \equiv -\frac{px''}{x'} = \frac{pp''}{(p')^2} = \varepsilon\rho \tag{41}$$

$$\varepsilon \equiv -\frac{p}{xp'} = \frac{1}{e} \qquad \rho \equiv -\frac{xp''}{p'} = \frac{xx''}{(x')^2} = er \tag{42}$$

We use these properties in the proof of Proposition 3 below.

Yet another measure of demand curvature, widely used in macroeconomics, is the superelasticity of Kimball (1995), defined as the elasticity with respect to price of the elasticity of demand, $S \equiv \frac{d\log\varepsilon}{d\log p}$. Positive values of S allow for asymmetric price setting in monopolistic competition. It is related to our measures as follows: $S = \frac{d\log\varepsilon}{d\log x}\frac{d\log x}{d\log p} = \left(\frac{x\varepsilon_x}{\varepsilon}\right)(-\varepsilon) = \varepsilon + 1 - \varepsilon\rho$ (using (6)), so it is positive if and only if demand is subconvex. Figure 14(a) illustrates loci of constant superelasticity, $\rho = \frac{\varepsilon+1-k}{\varepsilon}$. Formally, they correspond to the family of Pollak manifolds, $\bar{\rho}(\varepsilon) = \frac{\sigma+1}{\varepsilon}$, displaced rightwards to be symmetric around the log-linear ($\rho = 1$) rather than the linear ($\rho = 0$) demand manifold.



(a) Iso-Superelasticity Loci

(b) Klenow-Willis Demand Manifolds

Figure 14: Kimball Superelasticity

Kimball himself did not present a parametric family of demand functions, and closed-form expressions for the demand functions which correspond to the loci in Figure 14(a) are not available. Klenow and Willis (2006) introduce a parametric family which has the property that the superelasticity is a linear function of the elasticity: $S = b\varepsilon$. Substituting for S leads to the family of demand manifolds $\bar{\rho}(\varepsilon) = \frac{(1-b)\varepsilon+1}{\varepsilon}$, which are lateral displacements of the CES locus. Figure 14(b) illustrates some members of this family.

We note in footnotes some implications of these alternative measures. The choice between them is largely a matter of convenience. We express all our results in terms of ε and ρ, partly because this is standard in industrial organization, partly because (unlike e and r) the inverse demand functions are easily integrated to obtain the direct utility function, and partly because (unlike ε and S) they lead to simple restrictions on the shape of the demand manifold as shown in Proposition 2. However, our results could just as well be expressed in terms of e and r or of ε and S. Details of these alternative ways of presenting them are available on request.

B Oligopoly

We consider only monopoly and monopolistic competition in the text, but our approach can also be applied to oligopolistic markets. Consider for simplicity the case of Cournot competition with homogeneous goods. Now we need to distinguish market demand X from the sales of a typical firm i, x_i, with the elasticity and convexity of the demand function $p(X)$ defined in terms of the former: $\varepsilon \equiv -\frac{p}{Xp'}$ and $\rho \equiv -\frac{Xp''}{p'}$. The first- and second-order conditions are now $p + x_i p' = c_i \geq 0$ and $2p' + x_i p'' < 0$. Since these differ between firms, the restrictions they imply on the admissible region must be expressed in terms of the maximum and minimum market share: $\varepsilon \geq \max_i(\omega_i)$ and $\rho < 2\min_i(\frac{1}{\omega_i})$, where $\omega_i \equiv \frac{x_i}{X}$ is the market share of firm i. Relative to the monopoly case, the admissible region expands unambiguously, since $\max_i(\omega_i) \leq 1 \leq \min_i(\frac{1}{\omega_i})$, though its boundaries now depend on endogenous variables. Equally important in oligopoly, as we know from Bulow, Geanakoplos, and Klemperer (1985), is that many comparative statics results hinge on strategic substitutability: the marginal revenue of firm i is decreasing in the output of every other firm. This is equivalent to $p' + x_i p'' < 0$, $\forall i$, which in our notation implies a restriction on convexity that lies within the admissible region: $\rho < \min_i(\frac{1}{\omega_i}) \geq 1$. With n symmetric firms, the admissible region becomes $\{\varepsilon, \rho\} \in \{\varepsilon \geq \frac{1}{n}, \rho < 2n\}$, while the criterion for strategic substitutability is $\rho < n$.

C Proof of Proposition 1

If demands are separable in ϕ, both the elasticity and convexity are independent of ϕ. In the case of direct demands, $x(p,\phi) = \zeta(\phi)\,\tilde{x}(p)$ implies:

$$\varepsilon = -\frac{px_p(p,\phi)}{x(p,\phi)} = -\frac{p\tilde{x}'(p)}{\tilde{x}(p)} \quad \text{and} \quad \rho = \frac{x(p,\phi)x_{pp}(p,\phi)}{[x_p(p,\phi)]^2} = \frac{\tilde{x}(p)\tilde{x}''(p)}{[\tilde{x}'(p)]^2} \tag{43}$$

A special case of this is indirect additivity, where the indirect utility function can be written as: $\int v\,[p(\omega)/I]\,d\omega$. Roy's Identity implies that: $v'\,[p(\omega)/I] = -\lambda x(\omega)$, where λ is the marginal utility of income, from which the direct demand functions can be written in multiplicative form: $x(p,\phi) = -\lambda^{-1}(\phi)\tilde{x}(p/I)$.

Similar derivations hold for inverse demands. $p(x,\phi) = \beta(\phi)\tilde{p}(x)$ implies:

$$\varepsilon = -\frac{p(x,\phi)}{xp_x(x,\phi)} = -\frac{\tilde{p}(x)}{x\tilde{p}'(x)} \quad \text{and} \quad \rho = -\frac{xp_{xx}(x,\phi)}{p_x(x,\phi)} = -\frac{x\tilde{p}''(x)}{\tilde{p}'(x)} \tag{44}$$

Also we have a similar corollary. Direct additivity of the utility function implies: $\int u\,[x(\omega)]\,d\omega$. The first-order condition is: $u'\,[x(\omega)] = \lambda p(\omega)$, which implies that the indirect demand functions can be written in multiplicative form: $p(x,\phi) = \lambda^{-1}(\phi)\tilde{p}(x)$.

D Bipower Inverse Demands

D.1 Proof of Proposition 2

To prove sufficiency, we first define $A \equiv \alpha x^{-\eta}$ and $B \equiv \beta x^{-\theta}$, so the demand function can be written as $p(x) = A+B$. Calculating the first and second derivatives yields: $xp' = -\eta A - \theta B$ and $x^2p'' = \eta(\eta+1)A + \theta(\theta+1)B$. Adding x^2p'' to $\eta\theta p$ yields:

$$x^2p'' + \eta\theta p = (\eta+\theta+1)(\eta A + \theta B). \tag{45}$$

Using the expression for xp', this implies:

$$x^2p'' + (\eta + \theta + 1)xp' + \eta\theta p = 0. \tag{46}$$

Dividing by xp' gives the desired result: $\bar{\rho}\,(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon$.

To prove necessity, assume the manifold is linear, so $\rho(x) = a + b\varepsilon(x)$ where a and b are constants. Substituting for $\rho(x)$ and $\varepsilon(x)$ and collecting terms yields:

$$x^2p''(x) + axp'(x) - bp(x) = 0 \tag{47}$$

To solve this second-order Euler-Cauchy differential equation, we change variables as follows: $t = \log x$ and $p(x) = g(\log x) = g(t)$. Substituting for $p(x) = g(t)$, $p'(x) = \frac{1}{x}g'(t)$ and $p''(x) = \frac{1}{x^2}\left[g''(t) - g'(t)\right]$ into (47) gives a linear differential equation:

$$g''(t) + (a-1)g'(t) - bg(t) = 0 \tag{48}$$

Assuming a trial solution $g(t) = e^{\lambda t}$ gives the characteristic polynomial: $\lambda^2 + (a-1)\lambda - b = 0$, whose roots are $\lambda = \frac{1}{2}\left[-(a-1) \pm \sqrt{(a-1)^2 + 4b}\right]$. Only real roots make sense, so we assume $(a-1)^2 + 4b \geq 0$. If the inequality is strict, the roots are distinct and the general solution is given by $g(t) = \alpha e^{\lambda_1 t} + \beta e^{\lambda_2 t}$, where α and β are constants of integration. If $(a-1)^2 = -4b$, the roots are equal and the general solution is given by $g(t) = (\alpha + \beta t)e^{\lambda t}$. In both cases, the solution may be found by switching back from t and $g(t)$ to $\log x$ and $p(x)$, recalling that $e^{\lambda \log x} = x^\lambda$. Hence, in the first case, $p(x) = \alpha x^{\lambda_1} + \beta x^{\lambda_2}$, and in the second case, $p(x) = (\alpha + \beta \log x)x^\lambda$. The final step is to note that the sum of the roots is $\lambda_1 + \lambda_2 = 1 - a$ and their product is $\lambda_1\lambda_2 = b$, which implies the relationship between the coefficients of the manifold and those of the implied demand function stated in the proposition. This completes the proof.

D.2 Properties of Bipower Inverse and Bulow-Pfleiderer Demands

Subconvexity of demand is equivalent from (5) to $\frac{\varepsilon+1}{\varepsilon} \geq \rho$. Substituting for ρ from the bipower inverse demand manifold (19) implies that subconvexity holds for this family if and only if: $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{\varepsilon} \geq 0$. In the case of Bulow-Pfleiderer demands, this reduces to $\theta\varepsilon \leq 1$.

Except in the case of log-linear direct demand (when the elasticity equals $-\frac{p}{\beta'}$), the elasticity of demand for all members of the Bulow-Pfleiderer family is $\frac{1}{\theta\beta}\frac{p}{x^{-\theta}} = \frac{1}{\theta}\frac{p}{p-\alpha}$. This must be positive, which requires that θ, β and $p-\alpha$ must have the same sign. There are thus three possible parameter configurations: when θ is negative, *direct* demand is log-concave and subconvex, and $\alpha > p$; when θ is positive and α is negative, direct demand is log-convex and subconvex; finally, when both parameters are positive, demand is log-convex and superconvex, and $p > \alpha$.

Next, we wish to show that Bulow-Pfleiderer demands are necessary and sufficient for marginal revenue to be affine in price. Sufficiency is immediate: marginal revenue is $p+xp' = \theta\alpha+(1-\theta)p$. Necessity follows by solving the differential equation $p(x)+xp'(x) = a+bp(x)$, which yields $p(x) = \frac{a}{1-b} + c_1 x^{b-1}$, where c_1 is a constant of integration. As for relating these demands to the Generalized Pareto Distribution, this requires a suitable transformation of the invariant parameters. Setting $\alpha = \mu - \frac{\sigma}{\theta}$ and $\beta = \frac{\sigma}{\theta}k^{\theta}$ gives: $x(p) = k\left[1 + \frac{\theta(p-\mu)}{\sigma}\right]^{-\frac{1}{\theta}}$. The regular Pareto (the case where $\mu = \frac{\sigma}{\theta}$) is observationally equivalent to the CES demand function ($\alpha = 0$).

E Bipower Direct and Pollak Demands

E.1 Proof of Proposition 3

The proof follows immediately by noting that the direct demand function in Proposition 3, $x(p) = \gamma p^{-\nu} + \zeta p^{-\sigma}$, is the dual of the inverse demand function in Proposition 2, $p(x) = \alpha x^{-\eta}+\beta x^{-\theta}$. Hence Proposition 2 with appropriate relabeling implies that the direct demand function $x(p) = \gamma p^{-\nu} + \zeta p^{-\sigma}$ is necessary and sufficient for a linear *dual* manifold, that is to say, an equation linking the dual parameters r and e: $\bar{r}(e) = \nu + \sigma + 1 - \nu\sigma e$. Recalling from (41) that $e = \frac{1}{\varepsilon}$ and $r = \varepsilon\rho$ gives the desired result.

E.2 Properties of Bipower Direct and Pollak Demands

Not surprisingly, these are dual to the properties of the bipower inverse and Bulow-Pfleiderer demands. Substituting for ρ from the bipower direct demand manifold (20) into (5) shows that subconvexity holds for this family if and only if: $(\varepsilon - \nu)(\varepsilon - \sigma) \geq 0$. In the case of Pollak demands (with $\nu = 0$), this reduces to $\varepsilon \geq \sigma$.

Except in the case of CARA or log-linear inverse demand, the elasticity of demand for all members of the Pollak family is $\sigma\zeta\frac{p^{-\sigma}}{x} = \sigma\frac{x-\gamma}{x}$. This must be positive, which requires that σ, ζ, and $x-\gamma$ must have the same sign. There are thus three possible parameter configurations: when σ is negative, *inverse* demand is log-concave and subconvex, and $\gamma > x$; when σ is positive and γ is negative, inverse demand is log-convex and subconvex; finally, when both parameters are positive, inverse demand is log-convex and superconvex, and $x > \gamma$.

F Demand Functions that are not Manifold-Invariant

In this section we introduce two new demand systems whose demand manifolds can be written in closed form, though they depend on all the parameters, and so are not manifold invariant. We consider in turn: the "Doubly-Translated CES" super-family, which nests both the Pollak and Bulow-Pfleiderer families; and the "Translated Bipower-Inverse" super-family, which nests both the "Apt" (Adjustable pass-through) system of Fabinger and Weyl (2012) and a new family which we call the inverse "iso-prudence" system.[40]

[40] A third super-family is the dual of the second, the "Translated Bipower-Direct" super-family. Reversing the roles of p and x in equation (51) below leads to a "dual" manifold giving the inverse elasticity e as a function of the direct convexity r with the same form as (53). Special cases of this include the dual of the Apt system and the direct "iso-prudence" system (i.e., the demand system necessary and sufficient for $-px'''/x''$ to be constant). It does not seem possible to express the manifold $\bar{\varepsilon}(\rho)$ in closed form for this family.

F.1 The "Doubly-Translated CES" Super-Family

We can nest the Pollak and Bulow-Pfleiderer families as follows: $p(x) = \alpha + \beta (x-\gamma)^{-\theta}$. The elasticity and convexity of this function are:

$$\varepsilon(x) = \frac{1}{\theta}\frac{p}{p-\alpha}\frac{x-\gamma}{x} \qquad \rho(x) = (\theta+1)\frac{x}{x-\gamma} \tag{49}$$

When γ is zero this reduces to the Bulow-Pfleiderer case. Assuming $\gamma \neq 0$, we have $\rho \neq \theta+1$, and so the expression for ρ in (49) can be solved for x: $x = \frac{\rho}{\rho-(\theta+1)}\gamma$. Substituting into the expression for ε yields:[41]

$$\bar{\varepsilon}(\rho) = \left[1 + \frac{\alpha}{\beta}\left\{\frac{\theta+1}{\rho-(\theta+1)}\gamma\right\}^{\theta}\right]\frac{\theta+1}{\theta}\frac{1}{\rho} \tag{50}$$

This is a closed-form expression for the manifold but it depends on all four parameters, except in special cases such as the Pollak family, when, with $\alpha = 0$, it reduces to $\bar{\varepsilon}(\rho) = \frac{\theta+1}{\theta}\frac{1}{\rho}$.

F.2 The "Translated Bipower-Inverse" Super-Family

This demand function adds an intercept α_0 to the bipower-inverse family of Section 3.3:

$$p(x) = \alpha_0 + \alpha x^{-\eta} + \beta x^{-\theta} \tag{51}$$

Differentiating gives the elasticity and convexity:

$$\varepsilon(x) = \frac{\alpha_0 x^{\eta} + \alpha + \beta x^{\eta-\theta}}{\eta\alpha + \theta\beta x^{\eta-\theta}} \qquad \rho(x) = \frac{\eta(\eta+1)\alpha + \theta(\theta+1)\beta x^{\eta-\theta}}{\eta\alpha + \theta\beta x^{\eta-\theta}} \tag{52}$$

Assuming as before that $\rho \neq \theta+1$, and also that $\eta \neq \theta$, we can invert $\rho(x)$ to solve for x: $x(\rho) = \left[\frac{\eta\alpha}{\theta\beta}\frac{(\eta+1)-\rho}{\rho-(\theta+1)}\right]^{\frac{1}{\eta-\theta}}$. Substituting into $\varepsilon(x)$ gives a closed-form expression for the

[41] Here and elsewhere, the parameters must be such that, when the exponent (here θ) is not an integer, the expression which is raised to the power of that exponent is positive.

manifold:

$$\bar{\varepsilon}(\rho) = \frac{1+\eta+\theta-\rho}{\eta\theta} - \left[\frac{\eta}{\beta}\left\{(\eta+1)-\rho\right\}\right]^{\frac{\eta}{\eta-\theta}} \left[\frac{\theta}{\alpha}\left\{\rho-(\theta+1)\right\}\right]^{-\frac{\theta}{\eta-\theta}} \frac{\alpha_0}{\eta\theta\left(\eta-\theta\right)} \tag{53}$$

In general, this depends on the same five parameters as the demand function (51). It is best understood by considering some special cases:

(1) Bipower Inverse: The cost in additional complexity of the "translation" parameter α_0 is apparent. Setting this equal to zero, the expression simplifies to give the bipower inverse manifold as in Proposition 2: $\bar{\rho}(\varepsilon) = 1+\eta+\theta-\eta\theta\varepsilon$.

(2) Apt Demands: Fabinger and Weyl (2012) show that the pass-through rate (in our notation, $\frac{dp}{dc} = \frac{1}{2-\rho}$) is quadratic in the square root of price if and only if the inverse demand function has the form of (51) with $\eta = 2\theta$. This reduces the number of parameters by one, so the demand manifold simplifies to: $\bar{\varepsilon}(\rho) = \frac{1+3\theta-\rho}{2\theta^2} - \frac{[(2\theta+1)-\rho]^2}{\rho-(\theta+1)} \frac{2\alpha}{\beta^2\theta^2}\alpha_0$.

(3) Iso-Prudence Demands: Setting $\eta = -1$ is sufficient to ensure that demand prudence, $\chi \equiv -\frac{xp'''}{p''}$, is constant, equal to $\theta+2$. It is also necessary. To see this, write $xp''' = -\chi p''$, where χ is a constant, and integrate three times, which yields $p(x) = c_0 + c_1 x + \frac{c_2}{(1-\chi)(2-\chi)} x^{2-\chi}$, where c_0, c_1 and c_2 are constants of integration. This is identical to (51) with $\eta = -1$ and $\theta = \chi - 2$. Note that iso-convexity implies iso-prudence, but the converse does not hold; just as CES implies iso-convexity, but the converse does not hold.

It should be apparent that the demand manifold in (53) is not particularly convenient to work with. However, if we are mainly interested in pass-through, then we do not need to work with the demand manifold at all, since the key conditions in (8) and (10) do not depend on the elasticity of demand (a point stressed by Weyl and Fabinger (2013)). In such cases, our approach can be applied to the *slope* rather than the *level* of demand. By relating the elasticity and convexity of this slope to each other, we can construct a "Demand Slope Manifold" corresponding to any given demand function, and the properties of this manifold are very informative about when pass-through is increasing or decreasing with sales. It can be shown that the demand slope manifold of the Apt and iso-prudent demand functions are particularly convenient in this respect.

G Calculating the Effects of Globalization

To solve for the results in (29), use (27) to eliminate $\hat{x}$ from (25) and then solve (25) and (26) for $\hat{p}$ and $\hat{y}$, with $\hat{n}$ determined residually by (28). The results in (30) are obtained by using $\hat{x} = \hat{y} - \hat{k}$ and $\hat{N} = \hat{k} + \hat{n}$.

H Change in Real Income: Details

With symmetric preferences and identical prices for all goods, the budget constraint becomes: $I = \int_0^N p(\omega)x(\omega)\,\mathrm{d}\omega = Npx$. So consumption of each good is: $x = \frac{I}{Np}$. Substituting into the direct utility function yields its indirect counterpart:

$$V(N,p,I) = f\left[Nu\left(\frac{I}{Np}\right)\right] \tag{54}$$

We can now define equivalent income $Y(N,p)$ as the income that preserves the initial level of utility U_0 following a shock:

$$V\left(N,p,\frac{I}{Y}\right) = U_0 \tag{55}$$

For small changes (so equivalent and compensating variations coincide), we logarithmically differentiate, with I fixed (since it equals exogenous labor income), to obtain: $\hat{N} - \xi(\hat{N} + \hat{p} + \hat{Y}) = 0$. Rearranging gives the change in real income in (31).

I Welfare with Bipower Inverse Demands

I.1 Proof of Proposition 7

The proof of Proposition 7 follows by adapting that of Proposition 2. Consider sufficiency first. Just as the bipower inverse demand function in (19) implies equation (46), so the bipower utility function in (37) leads with appropriate relabeling to:

$$x^2 u'' + \left[(\eta - 1) + (\theta - 1) + 1\right] x u' + (\eta - 1)(\theta - 1) u = 0. \tag{56}$$

Dividing by xu' and replacing $\frac{xu''}{u'}$ by $-\frac{1}{\varepsilon}$ and $\frac{u}{xu'}$ by $\frac{1}{\xi}$ yields equation (38) as required. The proof of necessity also proceeds as in Proposition 2, *mutatis mutandis*.

I.2 Proof of Proposition 8

We have already seen in Section D.2 that superconvexity of demand for the bipower inverse family is equivalent to $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{\varepsilon} \leq 0$. Similarly, superconcavity of utility is equivalent from (33) to $\xi \geq \frac{\varepsilon-1}{\varepsilon}$. Substituting for ξ from the bipower utility manifold (38) implies that superconcavity of utility holds for this family if and only if: $\frac{(\eta\varepsilon-1)(\theta\varepsilon-1)}{[(1-\eta-\theta)\varepsilon+1]\varepsilon} \geq 0$.

I.3 Welfare with Bulow-Pfleiderer Demands

In the special case of Bulow-Pfleiderer demands as in Corollary 5, the sufficient condition from Corollary 4 simplifies to $(1-\theta)\,\varepsilon + 1$. Since $\theta = \rho - 1$, this equals $(2-\rho)\,\varepsilon + 1$, which must be strictly positive from the firm's second-order condition. Hence the conditions for subconcavity of utility are the same as those for superconvexity of demand derived in Section D.2 above: $\theta > 0$ and $p > \alpha > 0$. Finally, the utility manifold (38) simplifies to: $\bar{\xi}(\varepsilon) = \frac{(1-\theta)\varepsilon}{(1-\theta)\varepsilon+1}$. Expressing this as a function of ε and ρ: $\xi(\varepsilon, \rho) = \frac{(2-\rho)\varepsilon}{(2-\rho)\varepsilon+1}$. Hence ξ always lies between zero and one.

J Welfare with Pollak Demands

J.1 Proof of Proposition 9

Substituting from the Pollak demand manifold $\bar{\rho}(\varepsilon) = \frac{\sigma+1}{\varepsilon}$ into the condition for superconvexity of demand, $\rho \geq \frac{\varepsilon+1}{\varepsilon}$, gives $\frac{\sigma-\varepsilon}{\varepsilon x} \geq 0$. Recalling that the elasticity ε equals $\sigma\frac{x-\gamma}{x}$, this reduces to $\frac{\sigma\gamma}{\varepsilon x} \geq 0$. Exactly the same condition arises when we substitute from the Pollak utility manifold $\bar{\xi}(\varepsilon) = \frac{\sigma-1}{\varepsilon}$ into the condition for superconcavity of utility, $\xi \geq \frac{\varepsilon-1}{\varepsilon}$. The restriction that $\xi > 0$ implies $\sigma > 1$, from which it follows that $\gamma > 0$ is necessary and sufficient for both superconvexity of demand and superconcavity of utility.

J.2 Gains from Globalization with Pollak Demands

Substituting for $\xi = \frac{\varepsilon\rho-2}{\varepsilon}$ into the general expression for welfare change in equation (36) gives in this case:

$$\hat{Y} = \frac{\varepsilon}{\varepsilon\rho - 2}\left[1 - \frac{(\varepsilon-1)^2}{\varepsilon^2(2-\rho)}\right]\hat{k} \tag{57}$$

This is negative when $\rho > \rho^Y \equiv \frac{\varepsilon^2+2\varepsilon-1}{\varepsilon^2}$. To confirm that this lies in the admissible range $\rho \in \left[\underline{\rho}, \overline{\rho}\right] \equiv \left[\frac{2}{\varepsilon}, 1+\frac{2}{\varepsilon}\right]$, note that $\rho^Y - \underline{\rho} = \frac{\varepsilon^2-1}{\varepsilon^2} > 0$ and $\overline{\rho} - \rho^Y = \frac{1}{\varepsilon^2} > 0$.

J.3 Alternative Normalizations of the Sub-Utility Function

In the text we follow Dixit and Stiglitz (1977) and do not include a constant of integration in the sub-utility function. As Dixit and Stiglitz (1979) point out, this need not imply that $u(0)$ is strictly positive: we can define $u(x) = \max\left\{0, \frac{\sigma}{\sigma-1}\beta(x-\gamma)^{\frac{\sigma-1}{\sigma}}\right\}$ which is discontinuous at $x = 0$, but in all respects is a valid utility index. An alternative approach, due to Pettengill (1979), is to choose the constant of integration itself to ensure that $u(0) = 0$:

$$u(x) = \frac{\beta}{\sigma-1}\left[(\sigma x + \gamma)^{\frac{\sigma-1}{\sigma}} - \gamma^{\frac{\sigma-1}{\sigma}}\right] \tag{58}$$

It can be checked that the elasticity and convexity of demand are unaffected by this renormalization. However, the elasticity of utility is very different. It now behaves more like the Bulow-Pfleiderer case, though it is not consistent with superconvex demands. All this confirms that the elasticity and convexity of demand are not sufficient statistics for the welfare effects of globalization, and that small changes in the parameterization of utility can have big implications for the quantitative effects of changes in the size of the world economy.

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