

TOWARDS A NEW THEORY OF FINANCIAL
INTERMEDIATION

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Abstract

This thesis includes three interconnected essays which, building on the work by Hart and Zingales (2011), lay down the foundations for a new theory of financial intermediation. The first essay explains the Hart and Zingales (HZ) framework and shows that their results are not general. In the HZ model, there is a lack of simultaneous double coincidence of wants, and future income is not pledgeable. This implies that agents need money to trade. However, holding money entails an opportunity cost that leads to a waste of resources. Because of this inefficiency, pecuniary externalities have welfare consequences that private price-taking agents fail to internalize. I find that HZ's result, whereby the market produces inefficiently high levels of liquidity, cannot be generalized, because the conflict between private and social incentives to create money depends on agents' preferences. In the second essay I construct a framework that explains the transactions, precautionary and speculative demand for money. Again, the welfare analysis indicates that, depending on individuals' preferences, the market may produce inefficiently high or low levels of liquidity. The results also evidence that the speculative demand for money exists only when households are risk averse in their wealth. In that case, private and social incentives to hold money are stronger, but the market produces insufficient means of payment relative to the social optimum. The third essay introduces active financial institutions, and examines the role played by moral hazard in the provision of and demand for liquidity. Limited liability and the non-contractibility of bank investment policy induce highly levered financial institutions to invest in an inefficient gambling asset. I find that, when the probability that banks gamble is non-zero, the primary goal of public intervention is to address the moral hazard problem by restricting the creation of liquidity. Several policies to address this inefficiency are discussed and analyzed.

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Chapter 1

Introduction

Motivation

Recent financial crises have exposed the existence of market failures that can disrupt the process of financial intermediation and cause severe strain to the real economy. Yet, mainstream macroeconomic theory is not helpful in explaining this type of phenomena and, much less, the possible policy responses to them.

Prior to the 2007-2009 global financial crisis, macroeconomists and policy makers relied mostly on new Keynesian Dynamic Stochastic General Equilibrium models. The basis for this type of models is the Arrow-Debreu framework and its variant in finance, the Modigliani-Miller theorem. The latter rest on the assumption that markets are perfect and complete. Since there are no transactions costs, and all information about the economy is perfectly shared by everyone, Arrow-Debreu contingent-commodity markets achieve an ideal allocation¹. Consequently, there is no demand for money (or liquidity), and financial intermediation is unnecessary. Risk management is also pointless, as reallocating state-contingent assets in a complete market does not affect the market portfolio. Furthermore, the capital structure of individuals and institutions is irrelevant, because they can issue claims against the full value of new investments or future income. In the macroeconomic context, the predictions of this theory imply that agents' level of indebtedness and portfolio structure do not alter the final economic outcome; however, the evidence put forward by financial crises suggests otherwise.

The main motivation for this thesis is to offer a building block for a new theory of financial intermediation. The thesis includes three interconnected essays that build on the work by Hart and Zingales

¹This result is based on the two efficiency theorems of welfare economics; first, in the absence of external effects, any competitive allocation is Pareto-efficient; second, any Pareto-efficient allocation is competitive, provided that appropriate lump-sum transfers are made.

(2011), hereafter HZ. By incorporating elements of general equilibrium and principal-agent theory applied to corporate finance and banking, I develop a framework which is capable of addressing the kind of macroeconomic questions that normally arise during times of financial distress.

Outline

In the first essay (Chapter 2) I explain the Hart and Zingales (HZ) framework and show that their results are not general. This is an essential part of the thesis, as it provides the building blocks for the subsequent chapters.

The HZ framework builds on the classic Arrow-Debreu, or Walrasian, general equilibrium model. HZ first deviate from the AD framework by assuming that there is a lack of simultaneous double coincidence of wants. This implies that agents require a medium of exchange to trade, because rarely do two people, whose disposable possessions mutually suit each other's wants, meet and engage in barter (see Jevons, 1893 and Ostroy and Starr, 1990). Moreover, at any given point in time, an individual is either selling or buying (he cannot do both at the same time).

The other essential feature of the HZ model is that future income is not pledgeable. This assumption entails an important departure from the Arrow-Debreu Modigliani-Miller paradigm. Since non-pledgeability reduces the amount of wealth that can credibly be committed to settle transactions, agents build financial claims on a smaller resource base relative to the Walrasian world. Moreover, they demand stores of value and are willing to pay a premium for liquidity services. Consequently, money naturally emerges as the medium of exchange and (passive) financial institutions as the providers of this liquidity.

In the HZ framework, financial intermediaries issue notes (money) that certify the amount of pledgeable wealth agents have at their disposal. Notes are valuable as means of payment, because the issuing bank is independent from the parties involved in any trade transaction. Further, in order to obtain money, agents must deposit wealth in a bank. The resources deposited with banks are stored and cannot be invested in alternative (more profitable) ventures. Therefore, the creation of liquidity entails an opportunity cost.

The time structure assumed by HZ can be interpreted as that of a one-period model, where one market opens early and the other late. The authors assume that agents know whether they first buy or sell. The agents who buy first are liquidity-constrained, because they must acquire *costly* means of payment in order to consume. By contrast, the agents who sell first can use the proceeds to buy goods later in the period; hence, they do not need to deposit their wealth in unproductive bank-storage.

The costly creation of money implies that the HZ market economy does not reach its first-best outcome. In addition, when the economy is not perfect, prices do not fully accomplish their Walrasian duty of signaling scarcity through budget constraints. Instead, the price system also plays a role as driver of the market imperfection. Hence, pecuniary externalities have significant welfare consequences. What is more, since the market is perfectly competitive, agents are atomistic and fail to acknowledge the general equilibrium effects of their actions. Therefore, even if individuals are fully rational, they ignore the impact that their decisions have on prices and, thus, on the choice sets of other agents.

To sum up, the market economy of the HZ model generates two types of externalities. The non-pledgeability of future income induces agents to invest part of their wealth in unproductive storage. This entails a waste of resources that makes money creation costly, and prevents agents from exploiting all gains to trade. Furthermore, the costly provision of liquidity generates a pecuniary externality that has significant welfare consequences.

HZ find that, by spending more money than its socially desirable, price-taking private agents bid-up prices to a level that reduces their own welfare. In the second part of Chapter 2, I show that this is not a general result. The intuition is that, the costs generated by the lack of future income pledgeability are not always offset by the welfare effects of the pecuniary externality. Further, the relative size of the welfare losses generated by each of these inefficiencies, and how they relate to level of liquidity, hinges upon households' preferences. The level of income households have at their disposal depends on what they can pledge as means of payment. Hence, the non-pledgeability of future income has a direct effect on the demand for goods with high income elasticity. On the other hand, the pecuniary externality has a larger and indirect impact on the demand for goods with high price elasticity.

HZ assume that households are endowed with a quasi-linear utility function, which has very distinctive properties. For goods over which the utility function is concave, the demand function only depends on prices. By contrast, if the good enters the utility function linearly, its demand function depends on both, prices and income. HZ also suppose that the marginal utility of consumption is constant, while the marginal dis-utility of work is increasing. In this case, pecuniary externalities offset the welfare effects generated by future income non-pledgeability. As a result, private agents have *stronger* incentives to create liquidity than the planner. However, when I change slightly households' utility function - by assuming that preferences are still quasi-linear, but the marginal utility of consumption is decreasing, while the marginal dis-utility of labor is constant - the welfare analysis yields ambiguous results. That is, relative to the social optimum, private agents create insufficient means of payment in some cases, and excessive amounts in other cases.

It is essential that, in the first stages of the thesis, I provide an understanding of the sensitivity of HZ results to households' preferences. This is due to the fact that in the subsequent chapter, I will need to assume a different utility function than the one used by HZ, in order to motivate agents to hold money for speculative (or diversification) purposes.

In the HZ framework there are no uninsurable risks. Therefore, their model can only explain the transactional demand for money. Yet, as it was first stipulated by Keynes (1936), agents hold money not only because it is a medium of exchange (transactions motive), but also because it hedges against unexpected needs (precautionary motive), and it serves to diversify away the risk in investment portfolios (speculative motive).

The purpose of the second essay (Chapter 3) is to construct a framework that explains these three motives, and to assess whether the market produces an inefficient level of liquidity in that context. To this end, I extend the HZ model in three ways. First, I assume that the timing of transactions in the different commodity markets is random. This form of idiosyncratic risk induces agents to hold money for precautionary motives: they hedge against the uninsurable possibility of encountering a buying opportunity and being unable to make the purchase. Second, I introduce aggregate risk by assuming that real projects can go bankrupt. This implies that agents hold larger quantities of money because it is a safer asset. Finally, since a static model of aggregate risk and bankruptcy is not persuasive, I also adapt the HZ model to a simple dynamic setting: a two-period model.

The results evidence that the speculative motive only exists when households are risk averse in their wealth. This is consistent with the speculative money demand theory put forward by Tobin (1956, 1958). Tobin addressed some of the criticisms to Keynes' ideas about the speculative money demand. Keynes' analysis indicated that individuals hold money for speculative reasons only when the expected return on bonds is equal or less than on money. By contrast, Tobin argued that individuals care about the relative expected returns on assets as much as they do about their relative risk. Therefore, his theory predicted that agents generally hold diversified portfolios. The latter is, clearly, a more realistic description of reality.

The welfare analysis of Chapter 3 implies that there is a conflict between private and social incentives to create liquidity. As in Chapter 2, the latter stems from the fact that price-taking agents do not internalize the pecuniary externality generated by the costly creation of liquidity. The results also show that, when households hold money for transactions and precautionary motives, depending on their preferences, the market may provide inefficiently high or low levels of liquidity. Again, the reason is that, the specification

of households preferences determines whether the welfare costs emanating from the lack of future income pledgeability and its ensuing pecuniary externality, offset or reinforce each other.

When the risk of bankruptcy is introduced and the speculative motive for holding money is non-trivial, the market generates two types of inefficiencies relative to the Walrasian optimum. Because agents need to acquire costly means of payment to trade, the ensuing waste of resources generates a distortion in the form of missed trade opportunities. Second, unlike the Walrasian economy, agents' decision to hold money trades-off the profitability of risky assets and the liquidity and safety of money. Therefore, the market economy is unable to allocate risks efficiently.

On the other hand, if agents are risk-averse in wealth, the possibility of bankruptcy strengthens private and social incentives to hold liquidity. The reason is that, holding money insures against the possibility of becoming liquidity-constrained (idiosyncratic risk), but also against wealth losses during an economic downturn (aggregate risk). Notwithstanding, the market produces an inefficiently low level of liquidity relative to what is socially optimal. This is due to the fact that, the welfare effects generated by the pecuniary externality do not offset the social costs imposed by the lack of future income pledgeability.

According to some, Tobin's attempt to improve upon Keynes' rationale for the speculative demand for money was only partly successful. Their argument is that the role of speculative money holdings is not clear if there are risk-less assets that earn interest. In that case, money, as envisaged by Tobin, would be crowded out by ultra-liquid interest-earning securities. In reality, banks do pay interest in deposits. However, they also have strong incentives to take individuals' money and not repaying it, especially in the absence of oversight. Therefore, because of agency problems in the banking system, risk-less assets which pay higher returns than money do not really exist.

The purpose of the third essay (Chapter 4) is to develop a model that can explain the behavior of banks in a more realistic way and to examine whether, in the presence of a moral hazard problem, private and social incentives to create liquidity change. I, therefore, relax the assumption that banks are passive and safe. Instead, financial institutions are conceived as the investment project of a group of *outside* shareholders called bankers. Bankers invest their own wealth as equity and are granted a banking charter. Further, they make strategic decisions about their capital structure and investment portfolio: they choose how many deposits (debt) to take, and whether to invest in a safe or an inefficient gambling asset. Bankers are risk-neutral and enjoy limited liability. Finally, bank investment policy is assumed to be non-verifiable.

In this setup, a moral hazard problem arises for two reasons. First, bankers' payoffs have a lower zero-bound while the upside gain is unbounded. Second, deposit contracts cannot specify a banker's payoffs contingent on his investment decision. Therefore, when bankers gamble, they get high private returns if the gamble pays off; but if the gamble fails, the costs are borne by depositors.

I show that, when a financial institution is more levered, bankers' incentives to gamble are stronger (bank risk-shifting). This is due to the fact that, when the bank is highly capitalized, risk-taking behavior is constrained by the prospect of shareholder losses. Furthermore, when a bank engages in risk-taking behavior, it can afford to promise high deposit rates, as these are only paid in some states of the world.

From a social perspective, bank risk-taking is never optimal. This is so, even though a gambling bank can reduce the cost of money creation by promising higher deposit rates. The intuition is that, even when agents are risk neutral in wealth, the transactional role of money makes them risk-averse with respect to changes in the value of money. If banks fail, the value of money drops to zero, and trade collapses altogether. On the other hand, if banks effectively pay the higher promised rate, the value of money increases, but there is no additional benefit to it.

Consequently, regardless of how households' preferences are specified, if the probability that banks gamble is non-zero, the central planner would never intervene to promote the creation of liquidity; if he did, then banks would have stronger incentives to gamble.

In the last part of Chapter 4, I analyze the effects of different types of regulation to address the moral hazard problem. Capital requirements (or debt limits) eliminate risk-taking behavior, but create an inefficient shortage of liquidity. Liquidity requirements only mitigate the moral hazard problem by weakening bankers risk-taking incentives. However, this measure prevents money from losing its value when banks fail, thus restoring trade in the bad states of nature.

Market structure regulations, such as entry barriers, are also explored. Because of the agency problem, pecuniary externalities have important welfare consequences. In a competitive industry, bankers have extremely strong incentives to compete for deposits and promise higher rates. This is due to the fact that competition erodes profits, thus making the gamble more desirable. Since banks do not internalize the costs that gambling imposes on depositors, policies that limit competition in the financial industry mitigate the moral hazard problem.

Finally, the possibility of regulating banks' ownership structure is considered. A monopolist bank, mutually owned by all households in the economy, will solve the moral hazard. Further, this kind of

bank will succeed in internalizing the pecuniary externalities generated by the costly creation of liquidity. The intuition underlying this result is that, a monopolist mutual bank is tantamount to a central planner.

Remarks about Modeling Choices

Several of the assumptions I will make throughout the thesis may seem arbitrary and unrealistic. Therefore, below I explain why this set of assumptions is necessary to formulate a simple, and yet valid, theory of financial intermediation.

Finite Horizon Economy. Following HZ, the model is initially developed within a one-period setup, but it is then extended to a two-period framework. This is not a rich enough structure to address macroeconomic questions concerned with short-run and long-run dynamics. However, the framework could easily be adapted for that purpose by embedding it in an over-lapping generations model.

On the other hand, a finite-horizon monetary model must necessarily answer the question of whether, and why, money has positive value. This is a challenge that has had monetary economists puzzled for years. This puzzle was formalized by Hahn (1965) into what has become known as the *Hahn Paradox*. The paradox refers to the unexplainable fact whereby agents hold positive quantities of money, even though it is not rational for them to do so. In the last period of their lives, individuals would not want to hold money, because it makes no intrinsic contribution to their utility or technology. Thus, by backwards induction, they should not hold any money in the periods before. In this thesis I solve the Hahn paradox in the same way as HZ do. Money is valuable as a medium of exchange, because it is backed by resources from which agents do derive utility. In the final period, agents can go to a bank and exchange their money holdings for the resources that are backing it up. In other words, I use the “commodity money” approach to solving the Hahn paradox.

Commodity versus Fiat Money. In the literature there are two approaches for introducing money into a general equilibrium model and solving the Hahn paradox. One assumes a form of commodity money, and the other fiat money. Although the model developed in this thesis uses the commodity money approach, it does not restore to the “money in the utility function” assumption. The latter gives money a positive value by assumption and, therefore, does not provide an explanation for the Hahn Paradox. By contrast, the HZ framework is a finite horizon model with no fiat money (although fiat money could be included), where the best good (or asset) that can be used as money is valuable and has a low opportunity cost. The first of these properties ensures that the good can be credibly pledged in payment (as collateral). The second property implies that this setup can be used to explain any investment decision that trades-off

profitability and liquidity. This is not too unrealistic. After all, before the introduction of fiat money, the main commodity that was used as money, gold, was not particularly useful *per se*.

Walrasian Setup. The Walrasian general equilibrium setup is the most natural environment to construct a financial intermediation model. This framework is suitable to address macroeconomic questions, and it can easily be scaled to more complex environments. Moreover, with this kind of models one can analyze the general equilibrium (or price) effects of individuals' decisions. These pecuniary externalities are trivial if markets are perfect, but have significant welfare consequences otherwise. The reason is that, when markets are not perfect, the price mechanism becomes a driver of the friction. This implies, as a corollary, that perfect competition may not be ideal, since price-taking agents do not internalize the general equilibrium effects of their actions.

The non-pledgeability of Future Income and the HZ Framework. Assuming that all income is not pledgeable is not unrealistic. In real life, agents may be able to pledge only a share of their future labor income for institutional reasons, verifiability problems or incentive considerations. The non-pledgeability of future income differentiates the HZ framework from an otherwise canonical Walrasian model, as it induces agents to demand liquidity (in the form of money) and to be willing to pay a premium for it (in the form of an opportunity cost). This friction makes money and financial intermediaries essential, because trade would not take place without them. Further, the non-pledgeability of future income makes (some) agents liquidity-constrained, thus giving rise to a pecuniary externality that has important welfare effects. Hence, the value of HZ work is showing that, even in the absence of frictions such as agency problems or asymmetric information in the financial industry, the private provision of liquidity is inefficient and calls for public intervention.

Welfare Analysis. In the presence of market frictions, the relevant policy analysis must not examine whether a planner can evade the market's imperfections and achieve an unconstrained first-best outcome. Instead, public intervention should be evaluated on the grounds of whether a planner, while being subject to the same constraints as the agents in the decentralized market, can engineer a Pareto improvement. That is, policy measures should be set so as to achieve a *constrained Pareto efficient* outcome (see Greenwald and Stiglitz, 1986; and Geanakoplos and Polemarchakis, 1986). I use the less restrictive notion of *ex-ante* constrained Pareto inefficiency, as opposed to the ex-post criterion. The reasons for doing this is that, I assume individuals do not misperceive the probabilities of uncertain events ² and, even if they did (such that a planner would want to focus on results rather than expectations), the planner's decision would still have to be made and implemented ex-ante.

²See Hammond (1981) for a thorough discussion.

Chapter 2

Transactions Demand for Money and the Inefficient Creation of Liquidity

2.1 Introduction

This chapter starts by explaining the model developed by Hart and Zingales (2011), hereafter HZ. In order to micro-found the transactions demand for money, HZ develop a simple general equilibrium framework, where there is a lack of double coincidence of wants and future income is not pledgeable. While the authors find that, relative to what is socially optimal, the market produces an excessive amount of liquidity, in the second part of the chapter I demonstrate that this result cannot be generalized. The market may also produce insufficient means of payment, because the divergence between private and social incentives to create liquidity hinges upon households' preferences. In showing this, I provide a general understanding of the HZ model, which is essential for the subsequent chapters of the thesis.

This framework highlights the importance of introducing money into macroeconomic models by evidencing that, even in the absence of frictions such as agency problems or asymmetric information in the banking industry, or incomplete markets, the private sector is unable to efficiently provide liquidity services. What is more, the market equilibrium is constrained-inefficient. Holding money entails an opportunity cost, which leads to a waste of resources. Therefore, the lack of future income pledgeability (direct externality) prevents the market from reaching its Walrasian or first-best benchmark. Moreover, because of the distortions generated by this externality, pecuniary externalities have significant welfare consequences. However, since price-taking agents ignore the general equilibrium effects of their actions, the economy cannot achieve the second-best outcome either. In other words, private and social incentives to create liquidity generally diverge.

The implications of the welfare analysis (whether the market produces too much or too little liquidity) is sensitive to households' preferences because, the level of income households have at their disposal depends on what they can pledge as means of payment; therefore, the non-pledgeability of future income (direct externality) has a direct effect on the demand for goods with high income elasticity. By contrast, the consumption of goods with high price-elasticity is affected indirectly via the price mechanism or pecuniary externality.

The basis for the HZ framework is the classic Walrasian (or Arrow-Debreu) general equilibrium model. HZ first deviate from the Walrasian framework by assuming a lack of simultaneous double coincidence of wants. This implies that agents require a medium of exchange to trade, because two people, whose disposable possessions mutually suit each other's wants, do not often meet and engage in barter. Moreover, at any given point in time, an individual is either selling or buying (he cannot do both at the same time).

In the HZ model, the economy is inhabited by two types of consumers: the P -type, who grow potatoes, and the W -type, who make whiskey. Just as each agent is single-minded about production, each type is also assumed to consume only one good: the commodity he does not produce. In other words, households are not self-sufficient. Therefore, the P -type consume the whiskey the W -type sell, and vice versa.¹ The simultaneous double coincidence of wants implies that, for instance, rarely does a potato grower buy whiskey from the customer who purchases his potatoes. What is more, even if such an encounter did happen, P would not be able to simultaneously sell potatoes and buy whiskey from W .

To capture this, HZ assume a finite horizon economy, where one market opens early and the other late. In addition, agents are supposed to know whether they first buy or sell. If, say, the P -type buys first and sells second, he would earn labor income late and consume early. By contrast, the W -type would earn income early and spend late.

HZ also assume that households are endowed with gold. Unlike potatoes and whiskey, gold can serve as a store of value and for productive investments. Accordingly, the economy offers two investment alternatives for gold: agents can place it in storage or invest it in profitable projects, which are illiquid.

The other essential feature of the HZ model is that future income is not pledgeable. Because of this friction, the amount of wealth that can credibly be committed to settle transactions is smaller than in a Walrasian setup. This induces agents to demand stores of value and to be willing to pay a premium

¹A more general version of this framework would allow agents to consume both, the good the other household-type produces and the good they produce themselves. The current setting, nonetheless, is sufficient to motivate trade.

for liquidity services. Consequently, money naturally emerges as the medium of exchange and (passive) financial institutions as the providers of this liquidity.

Stored gold is the only source of wealth that can be used as means of payment. Consumers cannot credibly commit to make payments out of future labor income, because they cannot be forced to work on later dates. Similarly, future investment returns are non-verifiable (they can be diverted or hidden) and are, therefore, not acceptable in payment. The problem with stored gold is that it can easily be stolen and burdensome to carry. Thus, agents *strictly* prefer to trade with notes that provide proof of their holdings of stored-gold. But an independent party must certify the existence of the gold backing-up those notes. To illustrate, assume otherwise: let buyers write notes on themselves. Each of these notes may or may not accurately represent the gold held by its issuer. Since sellers cannot trust or verify this, such notes would become worthless as a medium of exchange. The independent issuer of gold-backed notes is, in fact, a financial intermediary (or bank). Hence, banks guarantee the safety of the payments system: by issuing gold-backed notes that credibly serve as a medium of exchange, intermediaries insure that the debts of the payers (buyers) are settled with the payees (sellers).

Gold deposits give households access to liquidity, which they need in order to buy goods. Note, however, that each monetary unit has an opportunity cost equivalent to the forgone return on the alternative investment. Thus, the creation of means of payments entails an investment decision that trades-off profitability and liquidity.

The creation of liquidity is, therefore, necessary and costly. For this reason, competitive markets almost never lead to *Walrasian* (or Pareto efficient) outcomes. When the economy is not perfect, prices do not fully accomplish their Walrasian duty of signaling scarcity through budget constraints. They also play a role as drivers of the market imperfection. Hence, in the HZ setup the market economy generates pecuniary externalities that have significant welfare consequences.

In sum, the non-pledgeability of future income induces agents to invest part of their wealth in the banks' unproductive storage. This entails a waste of resources that makes money creation costly, and prevents agents from exploiting all gains to trade. Furthermore, the costly provision of liquidity generates a pecuniary externality, whereby price-taking agents ignore the effects of their spending decisions on their own budget and the budget sets of others. In particular, HZ find that by spending more money than its socially desirable, agents bid-up prices to a level that is detrimental to their own welfare. As a consequence the market produces an inefficiently high level of liquidity.

HZ assume that households are endowed with a quasi-linear utility function, where the marginal utility of consumption is constant and the marginal disutility of work increasing. These preferences have very distinctive properties. For goods over which the utility function is concave, the demand function only depends on prices. On the other hand, if the good enters linearly in the utility, its demand function depends on both, prices and income. Recall that the level of income households have at their disposal depends on what they can pledge as means of payment. Therefore, the fact that future income is not pledgeable has a direct effect on the consumption of the goods that enter households' utility *linearly*. For the rest of the goods, this market imperfection only affects their level of consumption via the pecuniary externality.

This explains why, by changing slightly the assumption about households' preferences, HZ results are overturned in some cases. In the second part of the chapter I assume that preferences are still quasi-linear, but the marginal utility of consumption is decreasing, while the marginal dis-utility of labor is constant. With this assumption, the welfare effects generated by the lack of future income pledgeability are not always offset by the losses ensuing from the pecuniary externality. Thus, it is possible for private agents to have *weaker* incentives to create liquidity than the planner.

The rest of the chapter proceeds as follows. Section 2.2 provides a summary of the related literature. Section 2.3 explains the HZ framework and its results. Section 2.3 proves the lack of generality of these results. Finally, Section 2.4 concludes.

2.2 Related Literature

The literature on banking focuses largely on the role played by financial intermediaries in reducing or exacerbating the costs of transactions, informational asymmetries or agency problems. While still important, these frictions are ignored for now. The focus of this chapter is on the inefficiencies inherent in the process of liquidity creation, when the latter only plays a transactional role. Hence, this chapter is more related to the literature on money than banking.

The work by Baumol (1952) and Tobin (1956) motivated a micro-founded approach to the transactions demand for money, which has had microeconomists submerged in a quest to resolve the Hahn Paradox². Ostroy and Starr (1990) provide an excellent survey of attempts in this direction, which mainly seek to build models that deviate from the Arrow-Debreu economy in a way that gives money a positive value.

²The Hahn paradox is the unexplainable fact whereby economic agents hold positive quantities of money, even though it is not rational to do so. In the last period of their lives, individuals do not want to hold money because it makes no intrinsic contribution to their utility or technology. Hence, by back-wards induction, they should not hold money in any of the periods before. See Hahn (1965).

I solve the Hahn paradox following the approach taken by HZ. Money is valuable as a medium of exchange, because it is backed-up by resources from which agents do derive utility. In the final period, agents can go to a bank and exchange their money holdings for the resources that back it up. Note that, even though HZ use the “commodity money” approach to introduce money into a general equilibrium model and solve the Hahn paradox, they do not restore to the “money in the utility function” assumption.

Tobin also published a series of capital account models which assumed away the hypothesis employed by Keynes, whereby all assets are perfect substitutes. This allowed him to envisage a general framework with more than two assets and an explicit role for financial intermediaries. Moreover, Tobin argued that when agents hold money, be it for transactional, precautionary or speculative purposes, they bear an opportunity cost: the forgone return on alternative investments. This view is consistent with the costly creation of liquidity phenomenon of the HZ model. Therefore, as far as monetary theory is concerned, this chapter is closer to Tobin’s theory.

In the HZ model, money is used as means of payment because of the liquidity shortage created by the non-pledgeability of future income. Hence, this chapter is also related to the finance literature on liquidity, which offers various conceptual definitions. Diamond (1986), Jones and Ostroy (1984), and Morris and Shin (2003) suggest that liquidity refers to the thinness of the market for some good (an agent has an illiquid good if he cannot sell it quickly for a fair price). Holmstrom and Tirole (1998) define liquidity as the availability of instruments that can be used to transfer wealth across periods (an economy is more liquid than another if it has more markets). Fostel and Geanakoplos (2008) define “financial liquidity” as agents’ ability to borrow against the present value of future income or collateral, and “physical liquidity” as the flexibility to move goods across different projects (with storage facilities representing the most liquid asset); similarly, for Shubik (1999) or Kiyotaki and Moore (2002), liquidity refers to a substance, like gold, which is accepted as a means of payment. The model in this chapter develops a concept of liquidity which is consistent with the last of these streams of the literature.

From a methodological standpoint, the idea that competitive equilibria in economies with financial frictions are constrained inefficient was first developed by Kehoe and Levine (1993). They show that private contracts fail to internalize their effect on equilibrium prices and, thus, on financial constraints. Other contributions that use constrained efficiency analysis to study the role played by pecuniary externalities in economies with financial frictions include Caballero and Krishnamurthy (2001,2003), Allen and Gale (2004), and Lorenzoni (2008). More recently, Korinek (2011) has used this kind of analysis to explain fire-sales phenomena.

Relative to the banking literature, the model is closer to the work by Gorton and Pennacchi (1990). The authors suggest that the uniqueness of demand deposits lies in their role as medium of exchange. They assume an exchange economy model inhabited by risk-neutral agents. Consequently, the role of banks as providers of means of payment simply redistributes welfare. By contrast, the HZ model assumes a constant returns to scale production function, which albeit primitive, is able to capture the effects of liquidity provision in the levels of income and welfare.

2.3 Baseline Framework: The Hart and Zingales Model

This section explains the HZ model³. HZ consider a finite horizon economy where time extends over three dates $d = \{0, 1, 2\}$. There are two types of households in equal numbers. The P -type produce potatoes and consume whiskey, and the W -type do the opposite. The authors assume constant returns to scale. Hence, one unit of labor by P yields one potato, and one unit of labor by W produces a unit of whiskey. Each household type is also assumed to be endowed with an amount e_g of gold only on date 0, where $e_g \geq 1$.

On date 0, nature determines the order in which trade will take place. If the whiskey market opens first and the potato market second, the P -type is an early consumer who earns labor income late, while the W -type is a late consumer who earns labor income early. Further, since potatoes and whiskey are perishable, they must be consumed when they are produced.

Gold, on the other hand, is durable. It can serve as a store of value and for productive investments. Accordingly, the economy offers two investment alternatives: households can deposit gold in storage facilities, or invest it in real projects. Real projects require financing on $d = 0$ and are illiquid. If a project is financed, households cannot withdraw gold from it before its completion date, on $d = 2$. Moreover, the project generates a gross return $\bar{R} > 1$, which is payable in gold. $\bar{R}$ is assumed to be certain.

HZ assume that consumers are endowed with quasi-linear preferences. Further, they suppose that the marginal dis-utility of labor is increasing, while the marginal utility of consumption (gold and purchased goods), is constant. Formally,

$$U^h = g^h + c_b^h - \frac{1}{2} (l^h)^2$$

³Relative to the original work, I have introduced some differences of notation. Instead of having potato and whiskey producers, HZ visualize an economy populated by builders and doctors, where each type provides services in his capacity as builder (doctor) and contracts services from the type endowed with the other set of skills. Similarly, HZ consider wheat, not gold, to be the investment good.

Because of its properties, I label this kind of preferences “*leisure-concave*”. In the equation above, h is the household-type index, so $h = \{P, W\}$. l^h is the effort exerted by h to produce the good he sells, and g^h is his total consumption of gold. On the other hand, b is the index of purchased goods: $b = w$ if households want whiskey, and $b = p$ if they buy potatoes. Thus, c_b^h represents the amount of good b consumed by household h . By assumption, $b = w$ when $h = P$ (P -type households consume whiskey), and $b = p$ when $h = W$ (W households consume potatoes).

Finally, as it is customary in the general equilibrium literature, each consumer-type is treated as a continuum of identical atomistic agents who take prices as given. Thus, prices and traded quantities are determined competitively.

2.3.1 Walrasian Optimum

Initially, the benchmark corresponding to a frictionless economy is established. In an ideal world, households would be able to finance current consumption with their labor income (the proceeds from selling the goods they produce, be it early or late) as well as with the future returns on their investments. This is the assumption made in classic Walrasian or Arrow-Debreu theory. Thus, in a Walrasian world, it is optimal for households to invest their complete endowment of gold in the real project.

The behavior of potato producers is, therefore, described by the following optimization problem

$$\max_{g^P, c_w^P, l^P} U^P = g^P + c_w^P - \frac{1}{2} (l^P)^2$$

s.t.

$$g^P + P_w c_w^P = P_p l^P + \bar{R}e_g$$

where c_w^P is the amount of whiskey consumed by P , l^P the amount of time he spends growing potatoes, and g^P his total consumption of gold. Finally, P_w and P_p represent the prices of whiskey and potatoes respectively.

For P , gold and whiskey are perfect substitutes. Thus the first order condition with respect to c_w^P is

$$c_w^P = \begin{cases} 0 & \text{if } P_w > 1 \\ \left(\frac{P_p}{P_w}\right)^2 + \frac{\bar{R}e_g}{P_w} & \text{if } P_w < 1 \\ 0 \leq c_w^P \leq \left(\frac{P_p}{P_w}\right)^2 + \frac{\bar{R}e_g}{P_w} & \text{if } P_w = 1 \end{cases} \quad (2.1)$$

If $P_w > 1$, P -type households prefer gold to whiskey, if $P_w = 1$ they are indifferent, and, if $P_w < 1$ they prefer whiskey to gold. The marginal utility of gold for the potato growers is 1 if $P_w > 1$ and $\frac{1}{P_w}$ if $P_w < 1$. These corner solutions affect P 's labor supply decision. If $P_w \geq 1$, he chooses to work P_p , because that level of labor maximizes $P_p l^P - \frac{1}{2} (l^P)^2$. Similarly, when $P_w < 1$, P households work $\left(\frac{P_p}{P_w}\right)$, which is the marginal return of labor for that case.

Therefore, the solution to P 's labor supply is given by

$$l^P = \begin{cases} P_p & \text{if } P_w \geq 1 \\ \frac{P_p}{P_w} & \text{if } P_w < 1 \end{cases} \quad (2.2)$$

In the Walrasian world, households are symmetric in all aspects but the good they want and the good they sell. Hence, the optimization problem and first order conditions for W households are the mirror of P 's.

W 's potato consumption decision is given by

$$c_p^W = \begin{cases} 0 & \text{if } P_p > 1 \\ \left(\frac{P_w}{P_p}\right)^2 + \frac{\bar{R}e_g}{P_p} & \text{if } P_p < 1 \\ 0 \leq c_p^W \leq \left(\frac{P_w}{P_p}\right)^2 + \frac{\bar{R}e_g}{P_p} & \text{if } P_p = 1 \end{cases} \quad (2.3)$$

and his labor supply schedule is

$$l^W = \begin{cases} P_w & \text{if } P_p \geq 1 \\ \frac{P_w}{P_p} & \text{if } P_p < 1 \end{cases} \quad (2.4)$$

Further, the whiskey and potato markets must clear in equilibrium. Therefore,

$$c_w^P = l^W \quad \text{and} \quad c_p^W = l^P$$

Markets do not clear if either $P_w > 1$ or $P_p > 1$. In the first case, demand for whiskey is zero while the supply for that good is positive. The same applies for the potato market if $P_p > 1$. On the other hand, no equilibrium satisfies $P_w < 1$ and $P_p < 1$. These two conditions imply that demand for gold is zero, while its supply is positive and equal to $2\bar{R}e_g$. The same rationale indicates that, if $P_w < 1$ and $P_p = 1$ (or $P_w = 1$ and $P_p < 1$), then the whiskey (or potato) market does not clear.

Hence, there is a unique Walrasian equilibrium satisfying

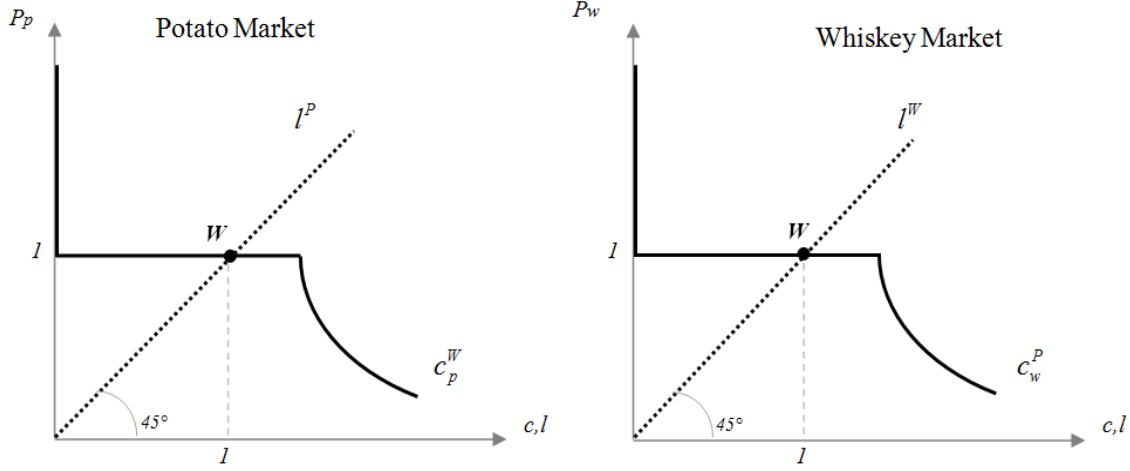
$$l^P = l^W = P_p = P_w = 1 \quad (2.5)$$

Figure 2.1 below provides a graphic illustration. The straight lines represent demand curves and the dotted lines supply curves. The W point on each panel represents the Walrasian equilibrium of that market.

In the potatoes market, the demand curve reflects the preferences of potato buyers: the W -type. This curve is, therefore, given by equation (2.3). The kink on the demand curve is the point where W households would use their entire income, labor income as well as future investment returns, to buy potatoes at a price $P_p = 1$. On the other hand, the supply of potatoes is determined by the labor choice of the P -type. The latter is given by equation (2.2). Since $P_w = 1$ in equilibrium, the potato supply curve is represented by a 45 degree line that passes through the origin. The equilibrium in the whiskey market is completely symmetric.

Prices and trade levels are equal to unity in both markets. This implies that households pledge only their labor income to buy goods. Since gold resources are not used to purchase goods, the Walrasian equilibrium lies to the left of the kink on the demand curve.

Figure 2.1: Walrasian Equilibrium



2.3.2 Market Equilibrium

In the HZ model there is lack of simultaneous double coincidence of wants. This implies that it is unlikely for a household-type, P say, to buy whiskey from the same customer who purchases his potatoes. Moreover, even if that implausible encounter took place, P would not be able to buy whiskey at the same time as he is selling potatoes. Therefore, agents need a medium of exchange to trade.

HZ also suppose that future income is not pledgeable. The rationale for this assumption is that, gold invested in real projects is locked-in until $d = 2$, and its future returns are non-verifiable. Hence, this source of income cannot be pledged to pay for current consumption. Similarly, future labor earnings can be diverted or hidden: agents cannot credibly commit to make payments out of future labor income, because they cannot be forced to work on later dates. Consequently, only gold kept in storage is an acceptable means of payments.

The problem with stored gold is that it can easily be stolen and burdensome to carry. Thus, agents strictly prefer to trade with notes that provide proof of their stored gold. Furthermore, an independent party must certify the existence of that gold. To illustrate why this is the case, assume otherwise: let buyers write notes on themselves in order to use them as means of payment. Each of these notes may or may not accurately represent the gold held by its issuer. Since sellers cannot trust or verify this, such notes would become worthless as a medium of exchange.

The independent issuer of gold-backed notes is a financial intermediary (or bank). On date 0, intermediaries provide liquidity to households by issuing notes that trade at par: each note is backed by one

unit of gold, and each note represents a claim to a single unit of gold on date 2. By issuing notes that credibly serve as a medium of exchange, banks guarantee the safety of the payments system: they insure that the debts of the payers (buyers) are settled with the payees (sellers).

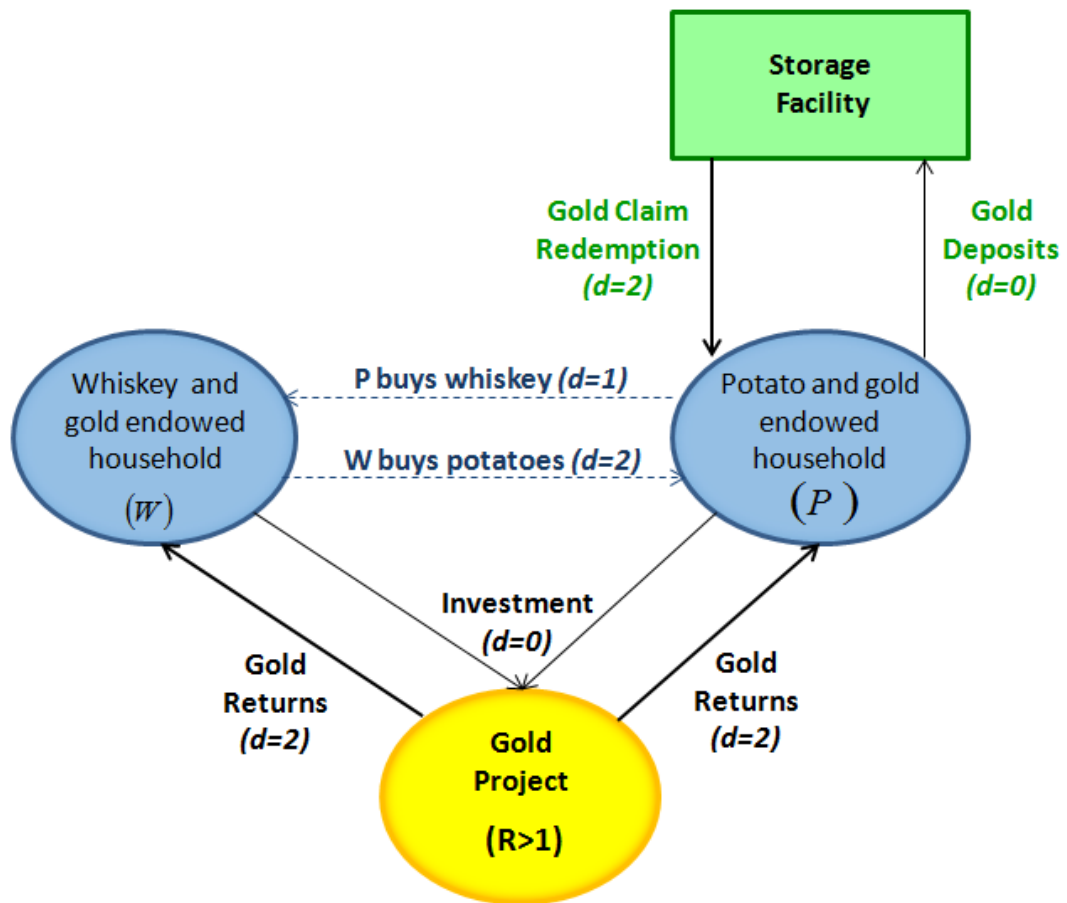
Gold deposits give households access to liquidity, which they need in order to buy goods. However, each unit of money has an opportunity cost $\bar{R}$ of forgone gold returns. Thus, the creation of means of payments entails an investment decision that trades-off profitability and liquidity.

On date 0, each household-type learns whether he will first buy or sell, and makes an investment decision accordingly. On dates 1 and 2 households trade in the order previously determined. Without loss of generality, assume nature chooses the P -type consumers to be buyers on $d = 1$ and the W -type on $d = 2$ (the reverse case is completely symmetric).

Let f^h denote the amount of gold deposited by household h in the bank. In the HZ setup, $f^P > 0$ and $f^W = 0$ in equilibrium. Because W works early and consumes late, he can use his labor income in order to buy potatoes. By contrast, P consumes early and works late. Hence, he needs to deposit a positive amount gold in the bank; otherwise, he would not have access to any means of payment when his opportunity to buy whiskey arises. It is important to note that, the ordering of the transactions does not insure that W will choose not to deposit gold with the bank. $f^W = 0$ because, at the level of prices where the potato and whiskey markets clear, W 's utility is *monotonically decreasing* in gold deposits. Thus, it is optimal for him not to make any gold deposits. This is formally shown below.

Figure 2.2 below describes the structure of the economy for this particular case. On date 0, the P -type deposits f^P units of gold in the bank and receives the same quantity of notes. On $d = 1$, he uses these f^P notes to pay for whiskey. The amount of money spent by P on date 1 corresponds to W 's labor income. W uses the latter to buy potatoes on date 2. Therefore, after the second trading session is closed, P ends up holding f^P notes, which he redeems for gold at the bank.

Figure 2.2: The Hart and Zingales (2011) Economy



Behavior of Household P

Since households are assumed to be endowed with *leisure-concave* quasi-linear preferences, P maximizes

$$U^P = g^P + c_w^P - \frac{1}{2} (l^P)^2$$

On date 0, P invests $(e_g - f^P)$ units of gold in the project, and deposits the rest in the bank. Hence, he receives f^P notes and uses them to buy c_w^P units of whiskey on $d = 1$. Since whiskey sells at a price P_w

$$c_w^P = \frac{f^P}{P_w}$$

On date 2, P produces and sells potatoes at a price P_p . At the end of the trading session, he is left with $P_p l^P$ notes, which he exchanges for gold at par. In addition, on $d = 2$ the gold project pays-off. Thus, the overall consumption of gold by P is given by

$$g^P = \bar{R} (e_g - f^P) + P_p l^P$$

In sum, P deposits f^P units of gold in the bank and exerts a level of effort l^P to maximize his utility (U^P) . That is,

$$\max_{f^P, l^P} U^P = \bar{R} (e_g - f^P) + P_p l^P + \frac{f^P}{P_w} - \frac{1}{2} (l^P)^2 \quad (2.6)$$

An additional unit of liquidity allows P to derive utility from the consumption of $1/P_w$ units of whiskey. However, it also induces him to forgo $\bar{R}$ units of gold. Hence, if $P_w > 1/\bar{R}$ the P household prefers gold to potatoes, if $P_w = 1/\bar{R}$ he is indifferent, and if $P_w < 1/\bar{R}$ he prefers whiskey to gold.

$$f^P = \begin{cases} 0 & \text{if } P_w > \frac{1}{\bar{R}} \\ 0 < f^P < e_g & \text{if } P_w = \frac{1}{\bar{R}} \\ e_g & \text{if } P_w < \frac{1}{\bar{R}} \end{cases} \quad (2.7)$$

On the other hand, P 's labor supply choice is given by

$$l^P = P_p \quad (2.8)$$

That is, P chooses to exert a level of effort l^P to maximize $P_p l^P - \frac{1}{2} (l^P)^2$. The first of these terms is the contribution of labor income to P 's consumption of gold, and the second term is the dis-utility generated by work.

Behavior of Household W

W -type households maximize

$$U^W = g^W + c_p^W - \frac{1}{2} (l^W)^2$$

On date 1, W sells l^W units of whiskey at a price P_w . His labor income is, therefore, equal to $P_w l^W$ notes, which he uses to buy potatoes on date 2. Since potatoes sell at a price P_p

$$c_p^W = \frac{P_w l^W + f^W}{P_p}$$

where f^W is the amount of gold that W deposits at the bank on $d = 0$ in exchange for money. W spends his money holdings buying potatoes on $d = 2$. Therefore, his total consumption of gold is given by the return of the project on his initial gold investment $(e_g - f^W)$.

$$g^W = \bar{R} (e_g - f^W)$$

To sum up, W chooses to deposit f^W units of gold in the bank and to exert a level of effort l^W , in order to maximize his utility (U^W) . Formally,

$$\max_{f^W, l^W} U^W = \bar{R} (e_g - f^W) + \frac{P_w l^W + f^W}{P_p} - \frac{1}{2} (l^W)^2 \quad (2.9)$$

W 's decision to deposit gold at the bank is given by

$$f^W = \begin{cases} 0 & \text{if } P_p > \frac{1}{\bar{R}} \\ 0 < f^W < e_g & \text{if } P_p = \frac{1}{\bar{R}} \\ e_g & \text{if } P_p < \frac{1}{\bar{R}} \end{cases} \quad (2.10)$$

This first order condition has an analogous interpretation to equation (2.7). On the other hand, W 's labor supply choice is described by

$$l^W = \frac{P_w}{P_p} \quad (2.11)$$

That is, for whiskey producers the marginal return of work is (P_w/P_p) . The intuition is that, for every unit of effort exerted, W earns P_w notes on date 1 and, on date 2, he can use each of these notes to buy $(1/P_p)$ potatoes.

Market Clearing Conditions

In general equilibrium, the labor income earned by whiskey producers equals the amount of money spent by P households in the whiskey market:

$$P_w l^W = f^P$$

Hence, W 's demand for potatoes is given by

$$c_p^W = \frac{P_w l^W + f^W}{P_p} \iff c_p^W = \frac{f^P + f^W}{P_p}$$

On the other hand, the potatoes supply curve, equation (2.8), is

$$l^P = P_p$$

Therefore, market clearing in the potato market requires

$$c_p^W = l^P \implies \frac{f^P + f^W}{P_p} = P_p \quad (2.12)$$

Similarly, while P households' demand for whiskey is given by

$$c_w^P = \frac{f^P}{P_w}$$

the supply of whiskey, equation (2.11), is

$$l^W = \frac{P_w}{P_p}$$

Consequently, market clearing in the whiskey market entails

$$c_w^P = l^W \implies \frac{f^P}{P_w} = \frac{P_w}{P_p} \quad (2.13)$$

Market Equilibrium

Following HZ, the equilibrium is solved for under the conjecture that $f^P > 0$ and $f^W = 0$. The underlying motivation is that households' utility is strictly decreasing in gold deposits (f^h). P buys first, so he needs to access means of payments on $d = 0$. By contrast, W is a late consumer, so he can use his labor income to buy the good he wants without bearing the opportunity cost of making gold deposits. Put differently, P is *liquidity constrained* and W is not.

To guarantee an interior solution to P 's optimization problem, HZ also conjecture that $e_g > f^P$. This implies that, since $e_g > f^P > 0$, first order condition (2.7) reduces to

$$P_w = \frac{1}{\bar{R}} \quad (2.14)$$

Combining the potato and whiskey market clearing conditions, equations (2.12) and (2.13), yields

$$P_p = (f^P)^{(1/2)} \quad (2.15)$$

$$P_w = (f^P)^{(3/4)} \quad (2.16)$$

Equation (2.16) can be used to substitute out P_w from (2.14). This gives the equilibrium level of liquidity in the economy, which is denoted by f^* :

$$(f^*)^{-(3/4)} = \bar{R} \quad (2.17)$$

$$\Longleftrightarrow f^* = \bar{R}^{-4/3}$$

This equation implies that the demand for means of payment is decreasing in the opportunity cost of money holdings ($\bar{R}$). It then follows that the equilibrium levels of trade in the whiskey and potato markets are, respectively,

$$l^W = (f^*)^{(1/4)}$$

$$l^P = (f^*)^{(1/2)}$$

Table 2.1 below summarizes the competitive market outcome.

Table 2.1: Competitive Market Equilibrium in the HZ economy

Deposit Market	
Liquidity Holdings in the Household Sector	
$f^* = \bar{R}^{-4/3}$	
Potato Market	
Trade/Labor	Prices
$c_p^W = l^P = (f^*)^{1/2}$	$P_p = (f^*)^{1/2}$
Whiskey Market	
Trade/Labor	Prices
$c_w^P = l^W = (f^*)^{1/4}$	$P_w = (f^*)^{3/4}$

Since $\bar{R} > 1$, it must be the case that $f^* < 1$. Further, $e_g \geq 1$ by assumption; therefore, the equilibrium level of liquidity satisfies the conjecture whereby $e_g > f^P$. Finally, the following relation between equilibrium prices and trade obtains.

$$1 > P_p > P_w = \frac{1}{\bar{R}}$$

$$l^P < l^W < 1$$

The first inequality implies that $P_p > \frac{1}{\bar{R}}$. According to condition (2.10), this confirms that the W -type household is at a corner where $f^W = 0$.

Recall that in the Walrasian equilibrium the price vector is $[P_p, P_w] = [1, 1]$, and a unit of each good is traded in each commodity market: $l^P = l^W = 1$. Hence, relative to the Walrasian optimum the market generates less trade, and prices are lower. The proposition below summarizes.

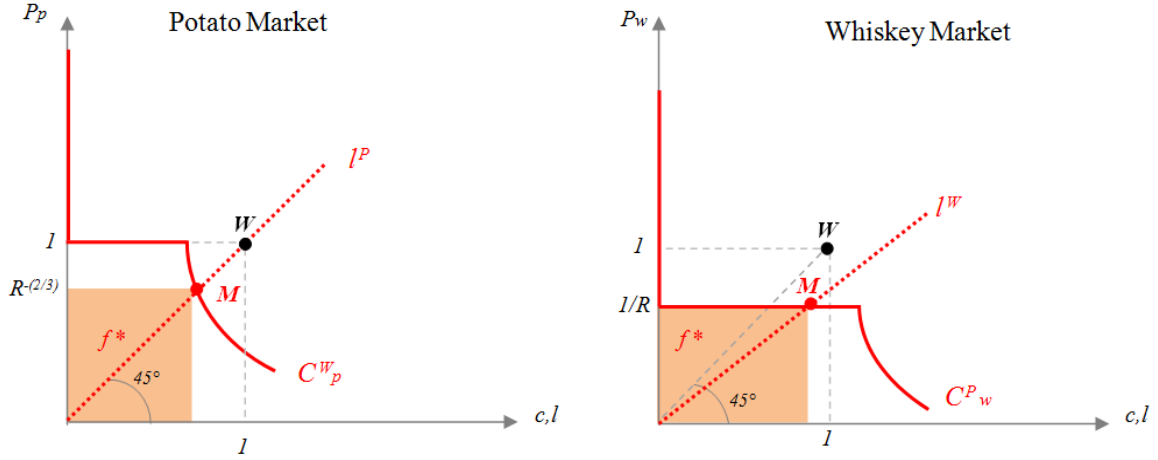
Proposition 2.3.1 *If $\bar{R} > 1$ and $e_g \geq 1$, the market equilibrium is characterized by less trade than the Walrasian equilibrium.*

In words, when future income is not pledgeable and $\bar{R} > 1$, an inefficiency in the form of missed trading opportunities emerges. Since high-yield investments and future labor income cannot be pledged to finance current consumption, there is an opportunity cost to money creation ($\bar{R}$). The latter entails a waste of resources that prevents agents from exploiting all gains to trade. Consequently, the economy cannot achieve the Walrasian outcome, whereby enough liquidity would be available to induce a level of trade of one unit per market. Note that in the Walrasian economy there would be more liquidity but less money than in the market economy. In other words, money is the consequence of the liquidity shortage brought about by the non-pledgeability of future income.

Figure 2.3 (below) provides a graphic illustration of the market equilibrium in the potato and whiskey markets. These are depicted by the left-hand side and right-hand side graphs respectively. In each commodity market, the equilibrium is denoted by point M and, as before, W represents the Walrasian optimum.

In the potato market (left-hand side graph), the supply curve is given by the labor choice of household P , equation (2.8). The latter is represented by a 45 degree line that passes through the origin. The demand for potatoes is determined by the expenditure decision of household W . W is not liquidity

Figure 2.3: Market Equilibrium



constrained, so he can use his date 1 labor income to buy potatoes on date 2. In equilibrium $P_p = (f^*)^{(1/2)} < 1$. Thus, W spends all his labor income in potatoes, and the equilibrium level of trade in the potato market lies to the right of the kink on the demand curve.

The right-hand side graph of Figure 2.3 depicts the equilibrium in the whiskey market. Equation (2.11) implies that whiskey makers supply labor in proportion to the relative price of whiskey (P_w/P_p). Since $P_p < 1$ in equilibrium, the supply curve of whiskey passes through the origin, but the latter is flatter than a 45 degree line. The demand for whiskey is given by the spending decision of P (first order condition 2.7), who is liquidity constrained. In equilibrium an interior solution obtains, so $P_w = 1/\bar{R}$. Therefore, the equilibrium in the whiskey market (point M), is located to the left of the kink on the whiskey demand curve. This is due to the fact that, P does not deposit all his gold with the bank ($e_g > f^*$).

The shaded area to the left and bottom of point M on both graphs represents the amount of liquidity spent by each household, or the monetary value of each transaction: f^* .

2.3.3 Social Optimum

If an economy is not perfect, the relevant policy analysis should not examine whether a planner can avoid the market imperfections and achieve an unconstrained first-best outcome. Instead, a reasonable yardstick to evaluate public intervention is that of *constrained Pareto efficiency*. This type of analysis establishes whether, while being subject to the same constraints as the agents in the decentralized market, the planner can engineer a Pareto improvement (see Greenwald and Stiglitz, 1986; and Geanakoplos and

Polemarchakis, 1986).

Therefore, in the HZ economy the planner must be constrained to take as given the non-pledgeability of future income and its ensuing costs. On the one hand, these costs are related to the fact that money creation entails a waste of resources. On the other hand, the planner must acknowledge that, when the economy is not perfect, prices do not fully accomplish their Walrasian duty of signaling scarcity through the budget constraints. Instead, prices also play a role as drivers of the market friction (the non-pledgeability of future income). This implies that there are pecuniary externalities with non-negligible welfare consequences. What is more, since the market is perfectly competitive, agents are atomistic price-takers and fail to acknowledge the general equilibrium effects of their actions. That is, even if they are fully rational, households ignore the impact of their decisions on prices and, thus, on the choice sets of other agents. Consequently, the planner must also take into account the effect of money holdings on prices.

When solving the constrained efficiency problem, the planner maximizes households' expected utility. Ex-ante (on date 0), whiskey and potato producers are equally likely to be buyers on date 1. Because of the symmetry of the model, U^P from the previous section represents the payoff of the liquidity-constrained type (the buyer on date 1) and U^W the payoff to the non-liquidity constrained (the buyer on date 2). Therefore, the expected utility of each household-type is given by

$$\frac{1}{2}U^P + \frac{1}{2}U^W = \frac{1}{2} \left[g^P + c_w^P - \frac{1}{2} (l^P)^2 \right] + \frac{1}{2} \left[g^W + c_p^W - \frac{1}{2} (l^W)^2 \right]$$

For simplicity, I use instead $U^P + U^W$, which is a linear transformation of the the social welfare function above and is, therefore, equivalent. Let S denote the objective function of the planner. Then,

$$S = U^P + U^W = \left[\bar{R}(e_g - f) + P_p l^P + \frac{f}{P_w} - \frac{1}{2} (l^P)^2 \right] + \left[\bar{R}e_g + \frac{P_w l^W}{P_p} - \frac{1}{2} (l^W)^2 \right]$$

To identify the minimal conditions whereby the planner can improve upon the competitive market outcome, assume he has a regulatory tool that determines the amount of liquidity held by the household sector: f_s .⁴ Moreover, unlike agents in the decentralized market, the planner internalizes the effect of liquidity holdings on prices. Hence, the centralized problem is given by

⁴Think, for instance, that the planner sets the maximum amount of resources that can be invested in the banking sector (or in the real project). If, relative to the market equilibrium, the planner wants agents to hold less liquidity, he sets a maximum level of bank deposits. If, on the contrary, the planner wants households to hold more money, he limits real investments ($e_g - f$).

$$\begin{aligned} \max_{f_s} S &= \bar{R}(e_g - f_s) + P_p l^P + \frac{f_s}{P_w} - \frac{1}{2} (l^P)^2 + \bar{R}e_g + \frac{P_w l^W}{P_p} - \frac{1}{2} (l^W)^2 \\ s.t. \quad P_p &= f_s^{1/2} ; \quad P_w = f_s^{3/4} \end{aligned}$$

Imposing the planner's constraints, and using the results of Table (2.1) that relate trade and liquidity, the problem is simplified as follows

$$\max_{f_s} S = f_s^{1/4} + \frac{1}{2} [f_s + f_s^{1/2}] + \bar{R}(e_g - f_s) + \bar{R}e_g \quad (2.18)$$

The first order condition yields

$$f_s \ni \frac{1}{2} + \frac{1}{4} [f_s^{-(1/2)} + f_s^{-(3/4)}] = \bar{R} \quad (2.19)$$

f_s is, therefore, the social optimum level of liquidity.

Comparing the left-hand-side of equations (2.17) and (2.19), yields the following relation between the market and the social optimum levels of liquidity

$$f^* > f_s \quad \text{for} \quad \bar{R} > 1 \quad \text{and} \quad e_g \geq 1$$

Proposition 2.3.2 *If $\bar{R} > 1$ and $e_g > 1$, the market equilibrium generates an excessive amount of liquidity and trade relative to the social optimum.*

Proof If $\bar{R} > 1$ and $e_g > 1$, the level of liquidity produced by the market is consistent with the interior solution to P 's optimization problem with respect to f^P . In other words, the level of liquidity created by the market is given by equation (2.17). The right-hand side of both, (2.17) and (2.19), is equal to $\bar{R}$; the left-hand side (LHS) of (2.17) is decreasing in f^* ; and the LHS of (2.19) is decreasing in f_s . Hence, if the LHS of both equations is evaluated at any given level of f , $f^* > f_s$ if the LHS of (2.17) is larger than the LHS of (2.19). That is,

$$\begin{aligned} f^{-(3/4)} &> \frac{1}{2} + \frac{1}{4} [f^{-(1/2)} + f^{-(3/4)}] \\ \iff \frac{1}{4} (f^{-(3/4)} - f^{-(1/2)}) &> 0 \quad \text{iff} \quad f < 1 \end{aligned}$$

Equation (2.17) implies that $f < 1$ because $\bar{R} > 1$. Thus, $f^* > f_s$ ⁵.

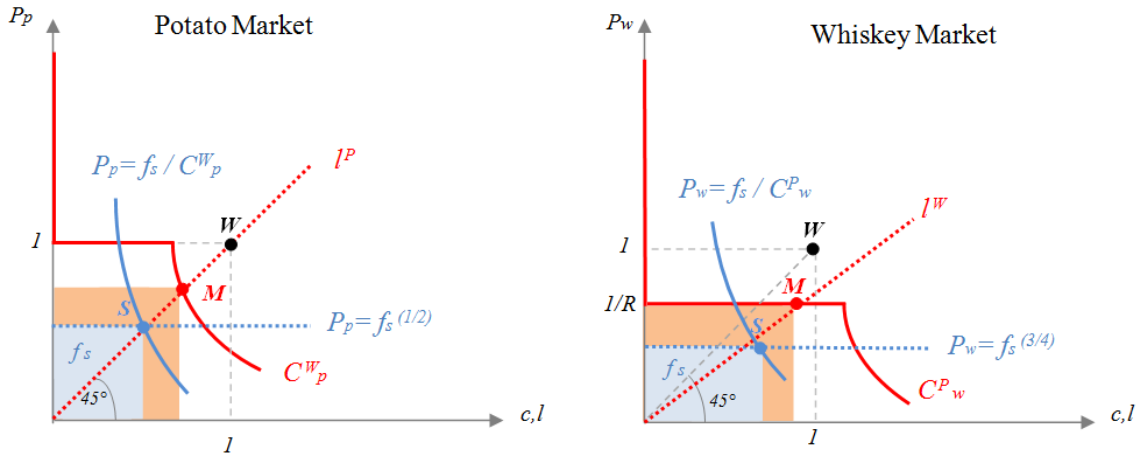
Trade in the whiskey and potato markets is respectively given by $l^W = f^{(1/4)}$ and $l^P = f^{(1/2)}$. These functions are monotonically increasing in f . Since $f^* > f_s$, it follows that the equilibrium level of trade in the competitive market economy is higher than what is socially optimal.

QED

Figure 2.4 (below) provides a graphic comparison between the competitive market equilibrium and the social optimum. The market equilibrium is illustrated as in Figure 2.3, and the planner's choice of liquidity is superimposed. The latter is represented by the blue area and point S . Note that, as stated by Lancaster and Lipsey (1956) in their *general theory of the second best*, other than satisfying agents' budget constraints, the choices made by the planner rarely coincide with households' first order conditions.

Households' budget constraints are represented by the blue downward-sloping curves, while the planner's constraints (the equations relating liquidity and prices) correspond to the horizontal blue dotted lines.

Figure 2.4: Social Optimum



Private sector agents have *stronger* incentives to create liquidity than the social planner. The conflict between private and social incentives stems from the fact that price-taking households do not internalize the general equilibrium effects of their actions. The incentives of the planner do not particularly diverge

⁵Note that if $\bar{R} = 1$, the inequality above would become an equality, and social and private solutions would coincide: $f^* = f_s = 1$.

from those of the non liquidity-constrained type, as W households do not deposit gold in the bank, so their investment decision does not entail a waste of resources. By contrast, liquidity-constrained agents must exchange gold for money in order to finance their consumption. In doing so, they only acknowledge the partial equilibrium effects of their actions. That is, when judging how much money to hold in pursuit of their private benefit, P households fail to realize that higher levels of liquidity bid-up the price of whiskey and potatoes. What is more, they ignore that prices rise to a level that is detrimental to their own welfare. This result deserves further explanation.

At the margin, an additional unity of money raises the price of potatoes by $(1/2)f^{-1/2}$ and the price of whiskey by $(3/4)f^{-1/4}$. Therefore, the indirect effect of P 's money holdings on his own utility is

$$\frac{\partial U^P}{\partial P_w} \cdot \frac{\partial P_w}{\partial f} + \frac{\partial U^P}{\partial P_p} \cdot \frac{\partial P_p}{\partial f} = -f^{-(3/4)} + \frac{1}{2} < 0$$

and the effect on the welfare of W is

$$\frac{\partial U^W}{\partial P_w} \cdot \frac{\partial P_w}{\partial f} + \frac{\partial U^W}{\partial P_p} \cdot \frac{\partial P_p}{\partial f} = \frac{1}{4}f^{-(1/2)} > 0$$

Even though lower levels of liquidity reduce the prices of both, whiskey and potatoes, this is beneficial only to P . The reason is that the relative price of potatoes is decreasing in f .

$$\frac{P_p}{P_w} = f^{-1/4} \implies \frac{\partial P_p/P_w}{\partial f} = -\frac{1}{4}f^{-5/4} < 0$$

That is, had the liquidity-constrained agent chosen to hold less money, the value of the goods he sells (potatoes) would have appreciated relative to the value of the good he buys (whiskey).

Since $0 < f < 1$ in equilibrium, the negative welfare effect of the pecuniary externality on the liquidity-constrained consumer is higher than its positive impact on the non-liquidity constrained. This is due to the fact that additional money holdings impose an opportunity cost only on the liquidity-constrained type. Similarly, by giving up an additional unit of consumption (via a reduction in f), the liquidity-constrained type (P) earns more leisure time than W . To illustrate, let MRS^P and MRS^W denote the marginal rates of substitution between labor and consumption of the purchased good for P and W , respectively. In equilibrium

$$\begin{aligned}
MRS^P &= -\frac{\partial U^P / \partial l^P}{\partial U^P / \partial c_w^P} = \frac{P_p}{1} = f^{1/2} \\
MRS^W &= -\frac{\partial U^W / \partial l^W}{\partial U^W / \partial c_P^W} = \frac{P_w}{P_p} = f^{1/4}
\end{aligned}$$

$$\implies MRS^P > MRS^W \quad \forall \quad 0 < f < 1$$

That is, a reduction in f induces P to cutback the number of hours worked by a larger proportion than W .

This result evidences that pecuniary externalities can create significant welfare losses, unlike in the Walrasian setup, where the welfare effects caused by price changes are negligible: if prices increase, the welfare gains of the sellers are netted out by the costs to the buyers⁶. By contrast, in the competitive market economy of the HZ model, pecuniary externalities matter, as equilibrium prices do not reflect the opportunity costs faced by each household-type; that is, the fact that, by choosing a higher level of consumption, the liquidity-constrained agent forgoes more gold wealth and leisure than the non liquidity-constrained. For this reason, if the planner chooses a lower level of liquidity relative to what private agents would choose under the market equilibrium, the ensuing welfare gains offset the losses. This is indeed socially desirable because, ex-ante, P and W are equally likely to become the liquidity-constrained type.

Another way of showing that the competitive price mechanism fails to fulfill its Walrasian duty is by demonstrating that, at the social optimum level of liquidity, private agents would want to hold more money. This is shown by means of the envelope theorem,

$$\left. \frac{\partial U^P}{\partial f} \right|_{f=f_s} = (f_s)^{-(3/4)} - \bar{R} > 0$$

The right-hand side inequality above follows from the following facts: at the market equilibrium, $(f^*)^{-3/4} = \bar{R}$; the social optimum level of liquidity is smaller than the market's choice; and both, f^* and f_s , are smaller than unity.

To sum up, HZ expose two types of externalities. In the Walrasian economy all sources of income can be pledged to finance current consumption. As a consequence, households do not invest gold in

⁶Similarly, if $\bar{R} = 1$ private and social solutions about the optimal level of liquidity would not differ. In this case, the opportunity cost of creating liquidity would be zero. Thus, making gold deposits with the banking sector would not lead to a waste of resources, and the Walrasian optimum would be achieved.

unproductive storage. When future income is not pledgeable, however, the market economy leads to an inefficient outcome whereby agents trade less than in the Walrasian case. This is due to the fact that, liquidity-constrained agents waste resources in the creation of means of payment. The costly creation of liquidity generates yet another inefficiency in the form of a pecuniary externality. In a competitive setup, pecuniary externalities arise because atomistic agents take prices as given and fail to internalize the general equilibrium effects of their actions. In particular, liquidity-constrained agents do not realize that, by spending more money than is socially desirable, prices rise to a level that reduces their own welfare.

2.4 Lack of Generality of HZ Results

HZ assume that households are endowed with a quasi-linear utility function, where preferences over the purchased good and gold are linear, while preferences over leisure are concave⁷.

Quasi-linear preferences have very distinctive properties. For goods over which the utility function is concave, the demand function is derived from the first order conditions, and the quantity of goods demanded only depends on prices. By contrast, if the good enters linearly in the utility function, its demand curve is determined by the budget constraint, and the quantity of goods demanded depends on both, prices and income. In the HZ setup, the level of income households have at their disposal is determined by the resources that can be pledged as means of payment. Therefore, the non-pledgeability of future income has a direct effect on the consumption of goods that enter linearly in the utility function. For the rest of the goods, this friction only affects their demand indirectly, through the price mechanism (via the pecuniary externality).

Thus, two legitimate questions arise: are HZ results robust to changes in the assumption about households' preferences? If they are not, how sensitive are they to this assumption? To answer these questions, I use the simplest possible deviation from the set of preferences assumed by HZ: suppose that households are still endowed with quasi-linear preferences, but the marginal utility of consumption is now decreasing, while the marginal dis-utility of leisure is constant. Furthermore, let agents have separable logarithmic preferences over the consumption of gold and the purchased good. This specification is equivalent to a symmetric Cobb-Douglas utility function, which is one of the simplest forms of concave preferences.

The utility function of household-type h is, therefore, given by

⁷Such an assumption is consistent with the pay-off functions normally used in principal-agent theory, but macroeconomic models that use quasi-linear preferences often have leisure as the good that enters linearly.

$$U^h = \ln(g^h) + \ln(c_b^h) - l^h \quad (2.20)$$

where the b subscript denotes the good purchased by household h ⁸. Because of its properties, I label this type of preferences *consumption-concave*.

2.4.1 The Walrasian Optimum with Consumption-Concave Preferences

Households follow the same behavior as in section 2.3.1. The only difference is that their objective function is now consistent with (2.20). Recall that in a frictionless economy, consumers invest all their gold in the real project. Since agents are symmetric, it suffices to show the optimization problem of only one household-type, say the P -type. Potato producers maximize

$$\max_{g^P, c_w^P, l^P} U^P = \ln(g^P) + \ln(c_w^P) - l^P$$

s.t.

$$g^P + P_w c_w^P = P_p l^P + \bar{R}e_g$$

The solution to this optimization problem is summarized by the first order conditions below.

$$c_w^P = \frac{P_p}{P_w} \quad (2.21)$$

$$g^P = P_p \quad (2.22)$$

Using these equations to substitute out $P_w c_w^P$ and P_p from the budget constraint, yields P 's labor supply schedule

$$l^P = 2 - \frac{\bar{R}e_g}{P_p} \quad (2.23)$$

By symmetry, the first order conditions for W correspond to the mirror of P 's. Therefore,

⁸Recall that, $b = w$ if $h = P$ (P households buy whiskey), and $b = p$ if $h = W$ (W households buy potatoes).

$$c_p^W = \frac{P_w}{P_p} \quad (2.24)$$

$$g^W = P_w \quad (2.25)$$

$$l^W = 2 - \frac{\bar{R}e_g}{P_w} \quad (2.26)$$

In general equilibrium the potato and whiskey markets clear. Thus

$$l^P = c_p^W$$

$$l^W = c_w^P$$

After imposing the conditions above and some algebraic manipulation, the Walrasian equilibrium obtains. Table 2.2. (below) summarizes.

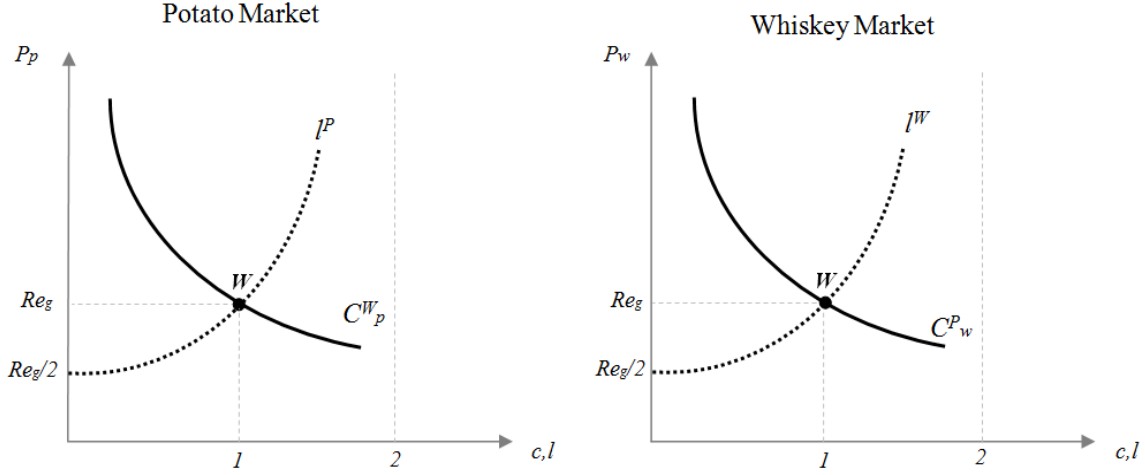
Table 2.2: Walrasian Optimum with Consumer-Concave Preferences

Potato Market	
Trade/Labor	Prices
$c_p^W = l^P = 1$	$P_p = \bar{R}e_g$
Whiskey Market	
Trade/Labor	Prices
$c_w^P = l^W = 1$	$P_w = \bar{R}e_g$

Figure 2.5 (below) provides a graphic illustration of this equilibrium. In each graph, point W represents the Walrasian equilibrium. In the potato market (left-hand side graph), the demand curve is given by the first order condition of W households with respect to the consumption of potatoes (equation 2.24). The latter is downward sloping and convex in the price-quantity plane. The supply curve corresponds to equation (2.23), which is the labor supplied by P households. The latter is upward sloping and horizontally asymptotic in 2.

The equilibrium in the whiskey market is entirely symmetric (left-hand side graph). The demand curve is derived from P 's first order condition with respect to c_p^W (equation 2.21), and the supply curve is represented by W 's labor choice (equation 2.26).

Figure 2.5: Walrasian Equilibrium



Note that, as with the previous set of preferences, the level of trade in the Walrasian economy is equal to one unit in each market. However, prices are now proportional to households initial wealth (e_g) and the return on the gold project ($\bar{R}$), so they are larger than 1.

2.4.2 Market Equilibrium with Consumption-Concave Preferences

As in section 2.3.2, the market economy is characterized by a lack of simultaneous double coincidence of wants, and the fact that future income is not pledgeable. This implies that agents need a medium of exchange to trade, and that the latter must take the form of gold-backed notes issued by an independent financial intermediary. The gold backing-up these notes corresponds to households gold deposits with the banking system, which are made on date 0. When households deposit gold with the bank, they receive, in exchange, an equal amount of notes. In addition, each note represents a claim to a single unit of gold on date 2.

Recall that, on date 0, each household-type learns whether he will first buy or sell and makes an investment decision accordingly. As in the previous section, assume nature determines that P -households consume early and the W -type late. This is with out loss of generality as the alternative case is entirely symmetric.

Behavior of Household P

On date 0, household P invests $(e_g - f^P)$ units of gold in the project, and deposits f^P in the bank. P , therefore, receives f^P notes and uses these to buy c_w^P units of whiskey on date 1. On date 2, he sells potatoes, and uses the proceeds to claim gold from the bank. He also collects the returns from the gold project. Thus, the optimization problem of household P is given by

$$\begin{aligned} \max_{f^P, l^P, c_w^P} U^P &= \ln(g^P) + \ln(c_w^P) - l^P \\ \text{s.t.} & \\ f^P &= P_w c_w^P \\ e_g &\geq f^P \geq 0 \end{aligned}$$

Where $g^P = \bar{R}(e_g - f^P) + P_p l^P$. The first order conditions to this problem are given by

$$\bar{R}f^P = P_p \quad (2.27)$$

$$g^P = P_p \quad (2.28)$$

$$c_w^P = \frac{P_p}{\bar{R}P_w} \quad (2.29)$$

These equilibrium conditions are used to substitute out f^P from the second budget constraint, which gives P 's labor supply schedule

$$l^P = 1 - \frac{\bar{R}(e_g - f^P)}{P_p} = 2 - \frac{\bar{R}e_g}{P_p} \quad (2.30)$$

Behavior of Household W

To finance his consumption of potatoes on date 2, W uses the proceeds from the whiskey he sells on date 1, as well as the notes he obtains from depositing f^W units of gold on date 0. Since potatoes sell for a price P_p , his consumption of whiskey is given by

$$c_p^W = \frac{P_w l^W + f^W}{P_p}$$

On the other hand, W 's consumption of gold corresponds to the payoffs of the gold project on his initial investment $(e_g - f^W)$. That is,

$$g^W = \bar{R}(e_g - f^W)$$

In sum, W optimizes

$$\begin{aligned} \max_{f^W, l^W} U^W &= \ln(g^W) + \ln(c_p^W) - l^W \\ \text{s.t.} \\ P_w l^W + f^W &= P_p c_p^W \\ e_g &\geq f^W \geq 0 \end{aligned}$$

Where $g^W = \bar{R}(e_g - f^W)$. The first order conditions with respect to c_p^W gives

$$c_p^W = \frac{P_w}{P_p} \quad (2.31)$$

On the other hand, W 's demand for money is described by

$$f^W = \begin{cases} 0 & \text{if } P_w \geq e_g \\ e_g - P_w & \text{if } P_w < e_g \end{cases} \quad (2.32)$$

Combining equation (2.31) with the first budget constraint gives

$$f^W = P_w(1 - l^W)$$

Therefore, W 's labor supply choice is given by

$$l^W = \begin{cases} 1 & \text{if } P_w \geq e_g \\ 1 - \frac{f^W}{P_w} = 2 - \frac{e_g}{P_w} & \text{if } P_w < e_g \end{cases} \quad (2.33)$$

Market Clearing Conditions

In the potato market, demand is determined by W 's first order condition (2.31), whereas the supply corresponds to P 's labor choice; that is, equation (2.30). Therefore, market clearing in the potato market requires

$$c_p^W = l^P \implies \frac{P_w}{P_p} = 2 - \frac{\bar{R}e_g}{P_p} \quad (2.34)$$

Similarly, while P 's demand for whiskey is given by first order condition (2.29), the supply is determined by W 's labor choice according to equation (2.33). Consequently, market clearing in the whiskey market entails

$$\frac{P_p}{\bar{R}P_w} = \begin{cases} 1 & \text{if } P_w \geq e_g \\ 2 - \frac{e_g}{P_w} & \text{if } P_w < e_g \end{cases} \quad (2.35)$$

Market Equilibrium

Assume first that $P_w \geq e_g$. From W 's first order conditions it must be the case that $f^W = 0$ and $l^W = 1$. Evaluating the market clearing conditions at these values gives

$$P_w = \frac{\bar{R}e_g}{2\bar{R} - 1}$$

But, since $\bar{R} > 1$, the equation above implies that $P_w < e_g$, which gives a contradiction.

By contrast, if the equilibrium is solved for under the conjecture that $P_w < e_g$, no contradiction arises. The intuition underlying this result is that, at the price level where the potato and whiskey markets clear, W 's objective function is concave in f^W . In other words, W values consumption more than in the case of quasi-linear leisure-concave preferences. Therefore, although he can use his labor income to buy potatoes, it is optimal for W to deposit some gold with the bank.

Since the two households exchange gold for money on date 0, they both bear the ensuing opportunity cost. The structure of the economy for this case is depicted by Figure (2.6), and the equilibrium trade levels and prices are displayed in Table 2.3 (below).

Figure 2.6: Nominal Flows of the Economy

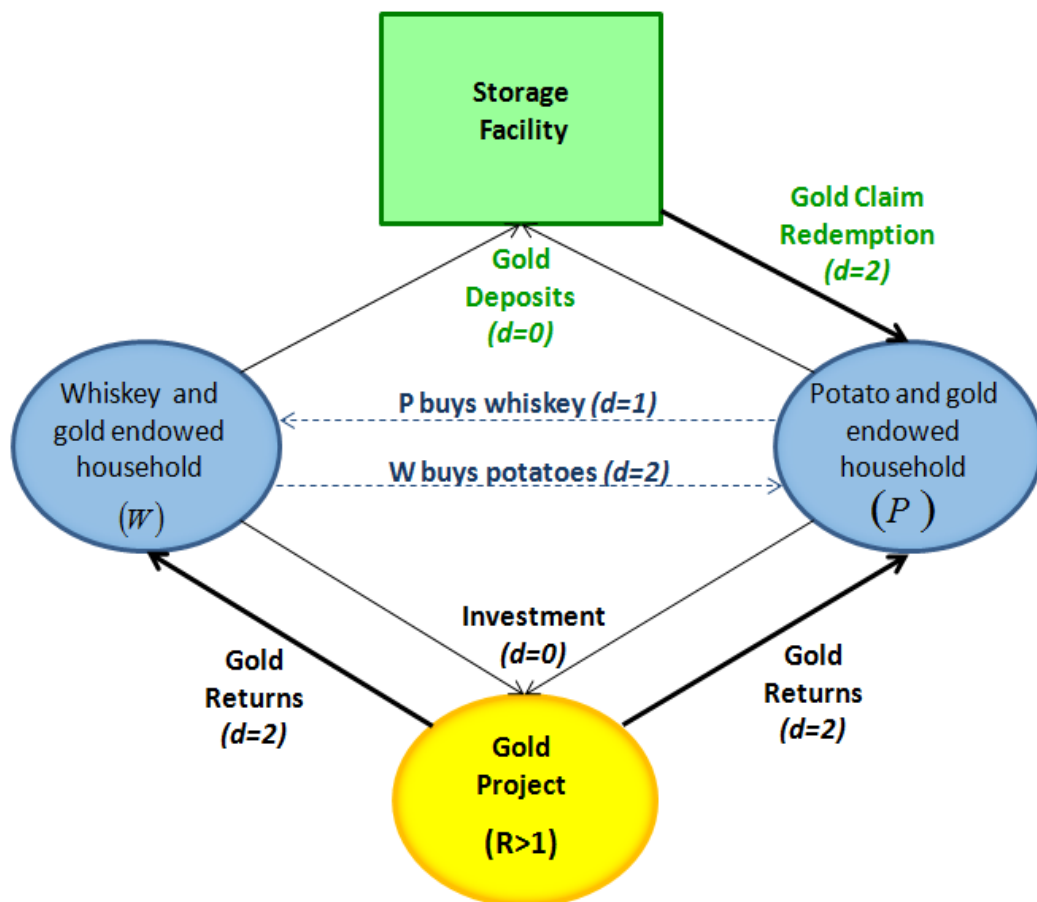


Table 2.3: Competitive Market Equilibrium with Consumption-Concave Preferences

Deposit Market	
Liquidity Holdings in the Household Sector	
P Household	W Household
$f^{P*} = \left(\frac{2\bar{R}+1}{4\bar{R}-1}\right) e_g$	$f^{W*} = \left(\frac{\bar{R}-1}{4\bar{R}-1}\right) e_g$
Potato Market	
Trade/Labor	Prices
$c_p^W = l^P = \frac{f^P + f^W}{\bar{R}(e_g - f^P) + f^P + f^W}$	$P_p = \bar{R}(e_g - f^P) + f^P + f^W$
Whiskey Market	
Trade/Labor	Prices
$c_w^P = l^W = \frac{f^P}{f^P + f^W}$	$P_w = f^P + f^W$

In addition, Figure 2.7 (below) provides a graphic illustration of the market equilibrium. The left-hand-side graph represents the outcome for the potato market and the right-hand-side graph for the whiskey market. In each graph the competitive market equilibrium is denoted by point M , and the Walrasian optimum by W . In addition, demand curves are represented by straight lines and the supply curves by dashed lines.

The demand curve in the potato market is given by equation (2.31). The latter is downward sloping and convex in the quantity-price plane. The supply of potatoes is determined by P 's first order condition with respect to labor (equation 2.30). Therefore, the supply curve is upward-sloping, convex and has an horizontal asymptote in $l^P = 2$.

On the other hand, the whiskey demand curve corresponds to P 's first order condition with respect to c_w^P , which is given by equation (2.29). The latter is upward sloping and convex. The whiskey supply schedule corresponds to W 's labor choice, equation (2.33). Thus, the whiskey supply curve is convex and upward-sloping for $P_w < e_g$, and perfectly inelastic and equal to one elsewhere.

Since $\bar{R} > 1$, the following relation between equilibrium prices and trade can be established

$$\bar{R}e_g > P_p > P_w$$

$$l^P < l^W < 1$$

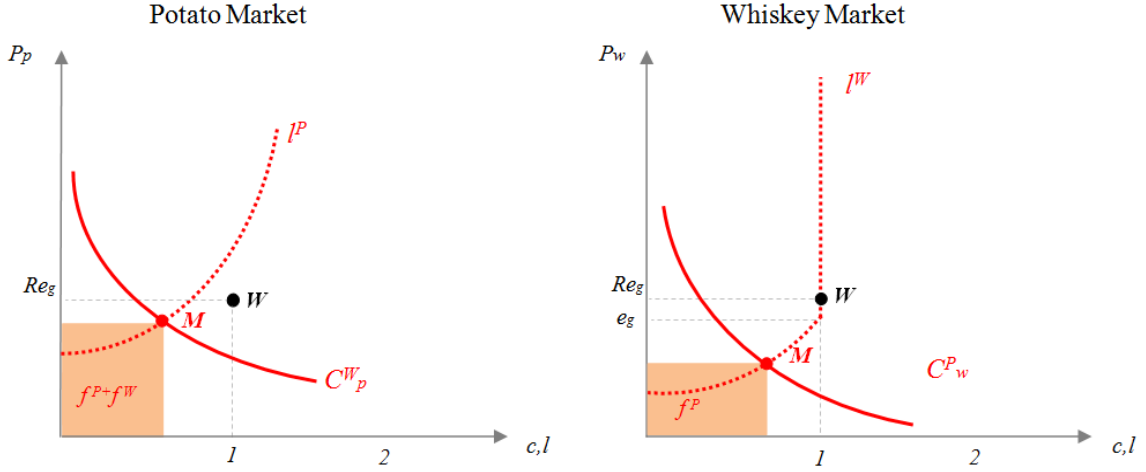
In the Walrasian equilibrium a unit of each good was traded in each market, and the prices of the two goods were equal to $\bar{R}e_g$. Therefore, with the set of preferences assumed in this section, the market also generates less trade and lower prices relative to the Walrasian optimum. Again, this is due to the fact that, the non-pledgeability of future income induces the market to engage in costly liquidity provision. This, in turn, generates an inefficiency in the form of missed trade opportunities.

Furthermore, note that

$$f^P > f^W > 0$$

This implies that, in contrast to the HZ model, the two households are liquidity constrained.

Figure 2.7: Market Equilibrium with Consumption Concave Preferences



2.4.3 Social Optimum

The costly creation of means of payment entails a waste of resources. This inefficiency prevents price-taking behavior from achieving the Walrasian optimum, which implies that pecuniary externalities have significant welfare effects. Therefore, as in section 2.3.3, I use the *constrained efficiency* approach to assess whether a planner can engineer a Pareto improvement relative to the market equilibrium.

Assume the centralized planner is endowed with a regulatory tool that determines the amount of liquidity that either household P , household W or both, can hold. Moreover, let f_s^h denote the quantity of money that the planner would want household h to spend. Unlike the agents in the decentralized market, the planner internalizes the effect of money holdings on prices. Therefore, his optimization problem is given by

$$\max_{f_s^P, f_s^W} S = U^P + U^W = \ln(g^P) + \ln(c_w^P) - l^P + \ln(g^W) + \ln(c_P^W) - l^W$$

s.t.

$$P_w = f_s^P + f_s^W$$

$$P_p = \bar{R}(e_g - f_s^P) + f_s^P + f_s^W$$

After imposing the constraints of the planner to substitute out P_p and P_w and some algebraic manipulation, the optimization problem is simplified as follows

$$\max_{f_s^P, f_s^W} S = \ln(f_s^P) + \ln(e_g - f_s^W) - \frac{f_s^P + f_s^W}{\bar{R}(e_g - f_s^P) + f_s^P + f_s^W} - \frac{f_s^P}{f_s^P + f_s^W}$$

The first order condition with respect to the amount of money the planner would like household P to hold is

$$\frac{\partial S}{\partial f_s^P} = \frac{1}{f_s^P} - \frac{\bar{R}(e_g + f_s^W)}{[\bar{R}(e_g - f_s^P) + f_s^P + f_s^W]^2} - \frac{f_s^W}{(f_s^P + f_s^W)^2}$$

Would the planner induce P to hold an amount of money which is different from his choice under the market equilibrium? The equation above does not have a simple closed form solution. Thus, to answer this question, I examine how small deviations from the market equilibrium change social welfare. I do this by means of the envelope theorem,

$$\left. \frac{\partial S}{\partial f_s^P} \right|_{f_s^P = f^{P*}, f_s^W = f^{W*}} = \frac{4\bar{R} - 1}{9\bar{R}^2 (2\bar{R} + 1)^2 e_g} [14\bar{R}^3 - 36\bar{R}^2 + 21\bar{R} + 1]$$

Since $\bar{R} > 1$

$$\left. \frac{\partial S}{\partial f_s^P} \right|_{f_s^P = f^{P*}, f_s^W = f^{W*}} = \begin{cases} \geq 0 & \text{if } \bar{R} \geq \frac{1}{14} (3 + 11\sqrt{15}) \approx 1.62 \\ < 0 & \text{if } \bar{R} < \frac{1}{14} (3 + 11\sqrt{15}) \approx 1.62 \end{cases}$$

This result implies that, whether P engages in excessive (or insufficient) liquidity creation depends on the parameters of the model. If the return on real investments is high ($\bar{R} > 1.62$), the planner would induce household P to hold more liquidity relative to the market equilibrium. By contrast, if $\bar{R}$ is below 1.62, P has stronger incentives to hold money than the planner, and public intervention would limit the creation of liquidity. Note that only the latter case is consistent with the results of the welfare analysis in HZ.

On the other hand, the planner's first order condition with respect to W 's liquidity holdings is given by

$$\frac{\partial S}{\partial f_s^W} = \frac{f_s^P}{(f_s^P + f_s^W)^2} - \frac{\bar{R}(e_g - f_s^P)}{[\bar{R}(e_g - f_s^P) + f_s^P + f_s^W]^2} - \frac{1}{f_s^P + f_s^W}$$

Again, to assess whether household W 's incentives to create liquidity differ from those of the planner, I use the envelope theorem.

$$\left. \frac{\partial S}{\partial f_s^W} \right|_{f_s^P=f^{P*}, f_s^W=f^{W*}} = -\frac{4\bar{R}-1}{9\bar{R}^2(2\bar{R}+1)^2} e_g [4\bar{R}^3 + 18\bar{R}^2 - 21\bar{R} - 1] < 0 \quad (2.36)$$

The inequality above results from the fact that $\bar{R} > 1$. Therefore, it is always the case that W holds an excessive amount of liquidity relative to the social optimum.

To sum up, in order to improve welfare, the planner can either induce W to hold less money; provide P with incentives to hold less (more) money if $\bar{R} < 1.62$ ($\bar{R} > 1.62$); or do both. These results deserve detailed explanation and comparison with the original HZ model.

First, unlike the case where households preferences are leisure-concave, here both households hold liquidity. Hence, the two agents are liquidity-constrained, and the P -type need not face the highest opportunity cost of holding money. Put differently, while in the HZ model the welfare losses borne by P (the only liquidity-constrained agent) are larger than W 's, here the marginal welfare losses brought about by the pecuniary externality may be just as important for W as they are for P .

Second, in the original HZ model, higher money holdings simply bid prices up via a demand effect. In the present case, however, additional money holdings by W increase the price of whiskey and potatoes proportionally. By contrast, additional money holdings by P increase the price of whiskey proportionally, and reduce the price of potatoes in proportion to $(\bar{R} - 1)$. These price effects result from the fact that, changes to households' money demand choice alter their labor decision in a way that shifts the supply curves. I explain this by means of a comparative statics exercise.

Consider first the case where P holds more liquidity. Figure 2.8 (below) provides a graphic illustration. The initial equilibrium is given by the red lines, while the green curves show how supply and demand curves shift as f^P increases. Developments in the potato market are depicted in the right-hand side graph, and the left-hand side graph explains changes to the whiskey market equilibrium.

If potato producers hold more money, they face a larger wealth shortfall in their final stock of wealth. This provides P with incentives to work more, hence shifting the supply of potatoes outwards. On the other hand, as f^P increases, the level of spending in the whiskey market rises, and the ensuing expansion of the demand curve leads to an increase in the equilibrium price of whiskey. Finally, trade in both markets is higher with higher levels of f^P .

Figure 2.8: Comparative Statics, f^P Increases

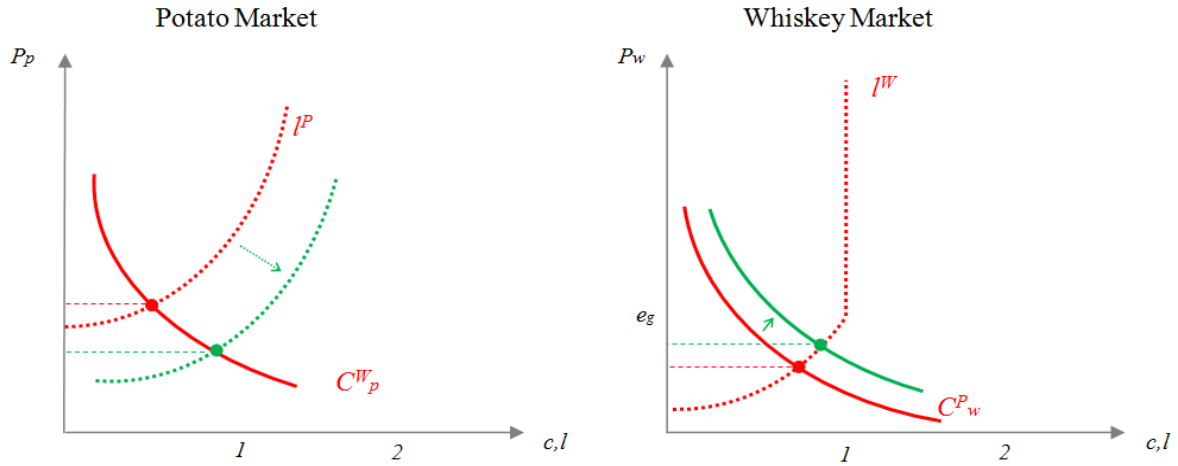
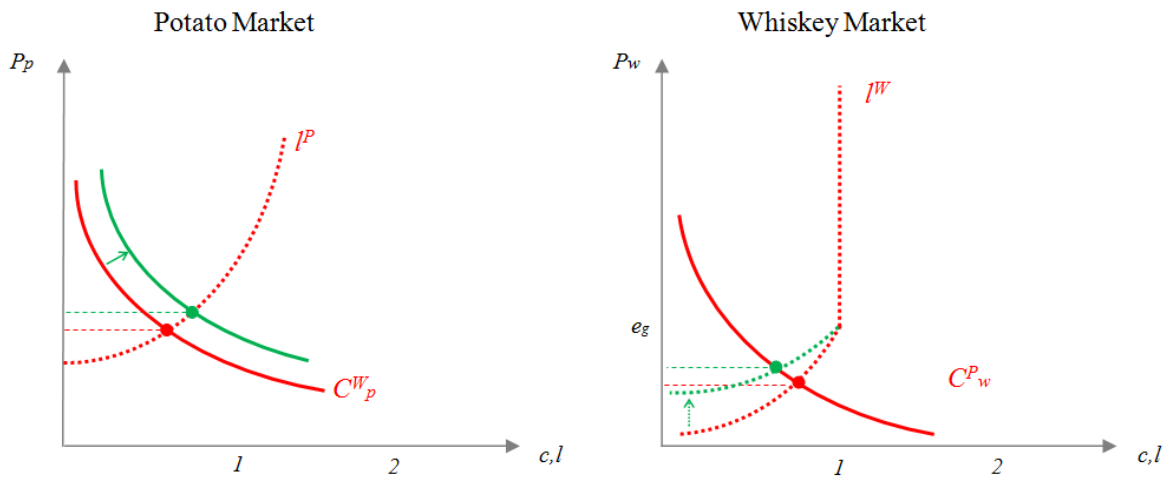


Figure 2.9 (below) describes what happens in equilibrium if f^W increases. If the W -type chooses to hold more money, whiskey makers choose to work less. Since W does not need to hold positive money balances to trade, when he holds less money he is substituting labor income with money holdings. Consequently, the supply of whiskey contracts, thereby raising the equilibrium price of this good. In addition, as f^W increases, spending in the potatoes market rises. The resulting expansion of the demand curve increases the equilibrium price of potatoes as well. Note that trade in the potato market rises, while the volume of whiskey transactions decreases.

Figure 2.9: Comparative Statics, f^W Increases



This exercise demonstrates that, with consumption-concave preferences, the non-pledgeability of future income and, thus, agents decision to hold money, has a direct effect on agents labor supply choice.

As in the previous section, when private agents make decisions about how much money to hold, they do not internalize the price effects described above. The indirect effect of W 's money holdings on P 's utility and his own are, respectively,

$$\frac{\partial U^P}{\partial P_w} \cdot \frac{\partial P_w}{\partial f^W} + \frac{\partial U^P}{\partial P_p} \cdot \frac{\partial P_p}{\partial f^W} = \frac{1}{P_p} - \frac{1}{P_w} - \frac{-\bar{R}(e_g - f^P)}{P_p^2} = \frac{-\bar{R}(e_g - f^P)}{P_p} \left[\frac{1}{P_p} + \frac{1}{P_w} \right] < 0$$

$$\frac{\partial U^W}{\partial P_w} \cdot \frac{\partial P_w}{\partial f^W} + \frac{\partial U^W}{\partial P_p} \cdot \frac{\partial P_p}{\partial f^W} = \frac{1}{P_w} - \frac{1}{P_p} - \frac{f^W}{P_w^2} = \frac{\bar{R}(\bar{R} - 1)e_g^2}{P_p P_w^2 (4\bar{R} - 1)} > 0$$

That is, as W chooses to hold more money, the prices of both, potatoes and whiskey increase, but the relative price of potatoes falls⁹. Therefore, the pecuniary externality has a negative effect on P and a positive effect on W . Moreover, the net effect is negative and equal to

$$-\frac{f^W}{P_w^2} - \frac{\bar{R}(e_g - f^P)}{P_p^2} < 0$$

Hence, if the planner intervenes to regulate f^W , he would induce household W to hold less money. Such an intervention would be socially desirable because, ex-ante, the probability of becoming liquidity constrained is the same for P and W .

Consider now the general equilibrium effects of the money demand choice by P on his own welfare

$$\frac{\partial U^P}{\partial P_w} \cdot \frac{\partial P_w}{\partial f^P} + \frac{\partial U^P}{\partial P_p} \cdot \frac{\partial P_p}{\partial f^P} = -\frac{1}{P_w} - \frac{(\bar{R} - 1)P_w}{P_p^2} < 0$$

and the effect on the welfare of W :

$$\frac{\partial U^W}{\partial P_w} \cdot \frac{\partial P_w}{\partial f^P} + \frac{\partial U^W}{\partial P_p} \cdot \frac{\partial P_p}{\partial f^P} = \frac{f^P}{P_w^2} + \frac{\bar{R} - 1}{P_p} > 0$$

Recall that, as per explained by the comparative statics analysis on Figure (2.8), if f^P increases, the price of whiskey rises and the price of potatoes falls. Since the relative price of the good sold by P decreases, the pecuniary externality has an adverse effect on his welfare and a positive impact on the well-being of W .

⁹ $\frac{\partial(P_p/P_w)}{\partial f^W} = \frac{-\bar{R}(e_g - f^P)}{P_p^2} < 0$.

The net effect of the pecuniary externality is given by

$$\sum_{h=P,W} \frac{\partial U^h}{\partial P_w} \cdot \frac{\partial P_h}{\partial f^P} + \frac{\partial U^h}{\partial P_p} \cdot \frac{\partial P_p}{\partial f^P} = \frac{\bar{R}(\bar{R}-1)(e_g - f^P)}{P_p^2} - \frac{f^w}{P_w^2}$$

which has an ambiguous sign. This implies that, for some parameter values, the positive effects caused by the pecuniary externality due to higher liquidity holdings by P , may increase households' welfare ex-ante.

I now assess the net welfare effects of the pecuniary externality by considering small deviations from the market equilibrium. Using the envelope theorem, it follows that

$$\sum_{h=P,W} \frac{\partial U^h}{\partial P_w} \cdot \frac{\partial P_h}{\partial f^P} + \frac{\partial U^h}{\partial P_p} \cdot \frac{\partial P_p}{\partial f^P} \bigg|_{f_s^P=f^{P*}, f_s^W=f^{W*}} = \frac{\bar{R}^2 e_g^3}{(4\bar{R}-1)^3} (14\bar{R}^3 - 36\bar{R}^2 + 21\bar{R} + 1)$$

The expression above is positive if $\bar{R} > 1.62$ and negative if $\bar{R} < 1.62$. Therefore, the planner would induce household P to hold more (less) liquidity relative to the market equilibrium if $\bar{R}$ is above (below) the 1.62 threshold. Note that, if $\bar{R} > 1.62$ and the planner is considering changes to f^P as the policy instrument, the implications of the welfare analysis are not consistent with the results put forward by HZ. That is, there are cases, where the market may produce inefficiently low levels of liquidity, such that the planner would want to put a cap on productive investments.

Intuitively, when $\bar{R}$ is high, the potato supply curve expansion generated by higher levels of f^P is also large (see Figure 2.8). Therefore, the welfare gains to W , in terms of acquired trade opportunities, will offset the costs borne by household P . Again, since households face an equal probability of becoming the late consumer, from an ex-ante perspective, this measure is socially desirable.

Judging which type of intervention (changes to f^P or f^W) is optimal, is beyond the scope of this thesis. Such a choice would not only depend on the parameters of the model, but also on the tools available to the planner. The means to implement the social optimum are particularly important if the planner is seeking to induce P to hold more money and W to hold less. Assume, for instance, that the tools available to the planner are taxes. In such a world, the planner could restrict the creation of liquidity by levying a tax on gold deposits, and stimulate it by taxing real investment returns. Since agents are heterogeneous in their money demand choices, and the conflict between social and private incentives may differ across households, several questions arise. What tax rate should the planner impose on the investment activities undertaken by each household-type? Would it be better to only induce one

household-type to change his monetary holdings? If so, which household-type should be targeted? This is an issue I will explore in future research.

For illustrative purposes, Figure (2.10) provides a graphic comparison of the competitive and social equilibria, when the planner induces household P to hold more liquidity. Figure (2.11), on the other hand, depicts a situation where the planner chooses to restrict the creation of liquidity. In both cases, the market equilibrium is illustrated as in Figure 2.7, and the social planner's choice of liquidity is depicted by the blue area and point S . The choice made by the planner is not consistent with agents' first order conditions, but households' budget constraints still hold. These are represented by the blue downward-sloping curves. The planner's constraints, which determine the relation between liquidity and prices, are depicted by the horizontal blue dotted lines.

Figure 2.10: Social Optimum: the Planner Restricts Real Investments

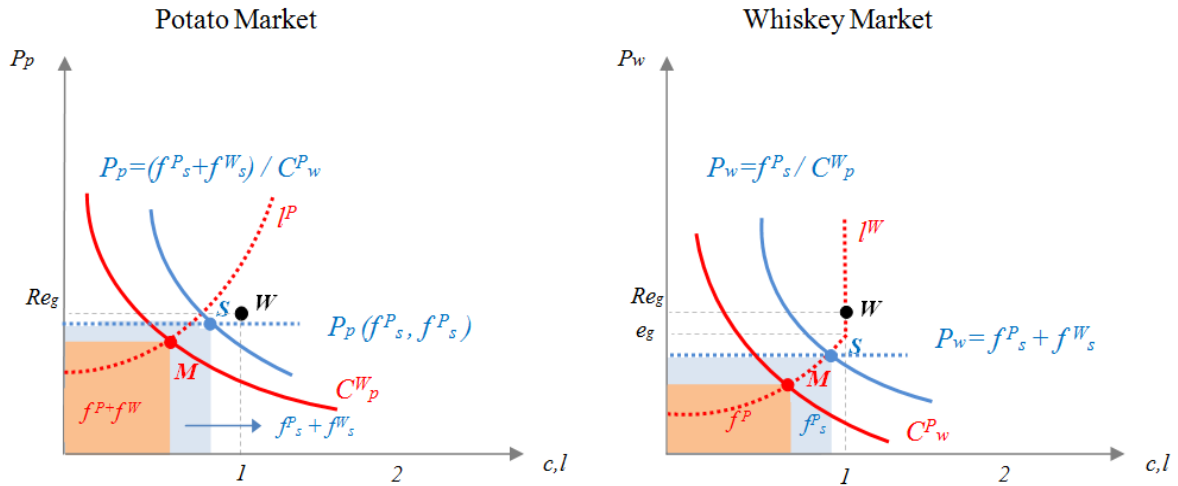
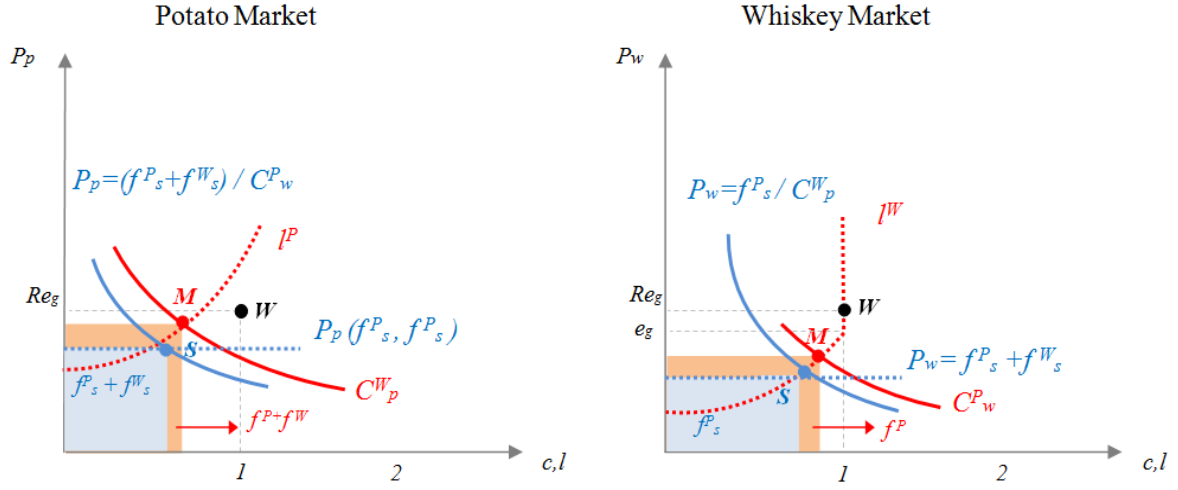


Figure 2.11: Social Optimum: the Planner Restricts Liquidity Creation



The welfare analysis in this section evidences that the results put forward by HZ are not general. What is more, their results are sensitive to the assumption about households' preferences and the parameters of the model.

If one assumes that households are endowed with quasi-linear consumption-concave preferences, the welfare costs emanating from the non-pledgeability of future income and the pecuniary externality may reinforce each other. In particular, when $\bar{R}$ is high, the inefficiency created by the non-pledgeability of future income is significant, so the social planner may choose to move away from the market equilibrium and towards the Walrasian optimum, by inducing the liquidity constrained-type to hold more money. This contrasts with the results obtained under the set of preferences assumed by HZ, where the welfare losses generated by the pecuniary externality always offset the costs imposed by the non-pledgeability of future income. Therefore, in their model, it is always the case that the planner moves away from the Walrasian optimum.

2.5 Concluding Remarks

This chapter has explained the main result of the HZ model at more length and proved that it cannot be generalized. HZ show that even in the absence of asymmetric information or an agency problem, the private provision of liquidity is inefficient. Future income non-pledgeability makes money creation costly, which prevents agents from exploiting all gains to trade. Furthermore, this distortion generates a welfare-reducing pecuniary externality. This is due to the fact that in the presence of market failures, prices become drivers of these frictions, and price-taking agents do not internalize this.

I showed that the way these two inefficiencies interact is sensitive to households' preferences. Private and social incentives to create liquidity differ because price-taking agents fail to internalize the pecuniary externality emanating from the lack of future income pledgeability. With quasi-linear consumption-concave preferences, the supply of labor is sensitive to income and prices, whereas the demand for gold and goods only depends on prices. This implies that the non-pledgeability of future income has a direct impact on the supply of goods and labor, but it only affects the consumption of commodities through the pecuniary externality. This is the exact opposite of what happens if one assumes quasi-linear leisure-concave preferences, as HZ do. In that case, the demand for goods and gold is sensitive to income and prices; hence the non-pledgeability of future income has a direct impact on demand schedules. On the other hand, the supply of labor and goods depends only on prices; thus, supply schedules only respond to the lack of future income pledgeability through the price mechanism.

This finding implies that the presence and severity of liquidity imbalances can be formulated as an empirical issue.¹⁰ The ultimate goal of such an exercise would be to understand the effects of (all) the externalities generated by the lack of future income pledgeability, and to guide policy intervention accordingly.

I have kept the policy analysis simple: restricting (or stimulating) the creation of liquidity can be achieved by simply imposing a maximum limit on gold deposits (or gold investments). However, as discussed in the last section of this chapter, the possibility of introducing taxes could be examined. For instance, HZ explore the effects of introducing government money. Publicly issued money would only be held by private agents, if it can be used to pay taxes. HZ assume that the government can impose sales taxes, and that agents can use government money to pay for these. The authors find that government money can crowd-out the demand for private liquidity, which in their setup is excessive. However, they also find that the ensuing tax charges generate a dead-weight loss that offsets the gains from such intervention. This happens even in the case of a Pigouvian tax system.

An alternative form of intervention would be to allow banks to lend fiat money to households. This requires a more sophisticated financial system than the one modeled in this chapter, but I hope to explore this option in future work.

An additional concern is that, the time structure assumed by HZ is not consistent (nor rich enough) to allow for an interpretation of their model as a short-run macroeconomic framework. Hence, adapting

¹⁰The term "imbalance" makes reference to the fact that, relative to what is socially desirable, private agents may have weaker (or stronger) incentives to create liquidity.

the model to a dynamic setting may yield results that are more appropriate to guide empirical work and policy analysis. This possibility is explored in the next chapter.

Finally, the HZ model only explains the transactional role of money. This is due to the fact the framework assumes away the existence of uninsurable risks. Yet, as it was first postulated by Keynes (1936), in addition to the transactions motive, individuals also hold money for precautionary and speculative reasons. The purpose of the next chapter is to address this issue.

Chapter 3

Precautionary Money Demand, the Speculative Motive and Bankruptcy

3.1 Introduction

Keynes postulated that agents hold money because it is a medium of exchange (transactions motive), hedges against an unexpected need (precautionary motive) or serves as a store of wealth (speculative motive). The purpose of this chapter is to develop a framework that explains these three motives, and to assess whether the market produces an inefficient level of money in that context. To this end, I construct a model that builds on the work by Hart and Zingales (2011) -HZ-, which was explained in Chapter 2.

In order to capture the three components of money demand, I extend the HZ framework in three ways. First, I assume that the timing of transactions in the different commodity markets is random. This form of idiosyncratic risk induces agents to hold money for precautionary motives: they hedge against the uninsurable possibility of encountering a buying opportunity and being unable to make the purchase. Consequently, and unlike the HZ model, agents are identical ex-ante, as all household-types need to hold positive amounts of money before trading starts. Second, I introduce aggregate risk by assuming that real projects can go bankrupt. This implies that agents hold larger quantities of money because it is a safer asset. Finally, since a static model of aggregate risk and bankruptcy is not persuasive, I also adapt the HZ model to a simple dynamic setting: a two-period model. An additional advantage of changing the time structure into a dynamic setup is to overcome an unrealistic characteristic of the HZ framework. HZ implicitly assume that the amount of time it takes households to consume non-durable goods, is just as long as the time elapsed between the initiation and completion dates of real investment projects. In reality, however, investment decisions are riskier and made over a longer term horizon than day-to-day

consumption choices.

In Chapter 2, I concluded that the welfare analysis of the HZ model is sensitive to households' preferences. For some parameter values, HZ results were overturned when households' utility function, originally a quasi-linear leisure-concave function, was replaced with quasi-linear consumption-concave preferences. For this reason, in this chapter I continue using these two sets of preferences. It is important to note that, in the presence of aggregate risk, the consumption-concave utility function implies that households are risk-averse in wealth, whereas with leisure-concave preferences agents are risk-averse in wealth.

The results of the welfare analysis evidence that there is a conflict between private and social incentives to create liquidity. As in Chapter 2, the latter stems from the fact that price-taking agents do not internalize the pecuniary externality generated by the costly creation of liquidity. Furthermore, depending on households' preferences, the market can produce inefficiently high or low levels of liquidity. The rationale for this ambiguity is that, the level of income households have at their disposal depends on what they can pledge as means of payment; hence, the non-pledgeability of future income has a direct effect on the demand for goods with high income elasticity. Because of its ensuing pecuniary externality, this friction also has an indirect impact on the demand for goods with high price elasticity. Similarly, whether the welfare effects emanating from these two inefficiencies offset or reinforce each other, hinges upon the specific characteristics of agents' utility function.

When households are assumed to have quasi-linear leisure-concave preferences, the market produces an excessive amount of liquidity relative to what is socially desirable. This result is consistent with HZ findings. By contrast, when households have consumption-concave preferences, private agents create insufficient amounts of means of payment relative to the social optimum. This result is robust to all feasible parameter values, which implies that, in the model developed in this chapter, HZ findings are more strongly overturned relative to the model in Chapter 2.

I show that the speculative motive for holding money only exists when households are risk averse in wealth. Moreover, when agents hold money for speculative (or diversification) purposes, the market generates two types of inefficiencies relative to the Walrasian optimum. Because agents need to acquire costly means of payment to trade, there is a waste of resources which prevents agents from exploiting all gains to trade. Second, unlike the Walrasian economy, agents' decision to hold money trades-off profitability with liquidity and safety. Therefore, the market economy is unable to allocate risks efficiently.

Moreover, if individuals are risk-averse in their wealth, private and social incentives to hold liquidity are stronger in the presence of aggregate risk. This is due to the fact that, in addition to lubricating trade and offering insurance against the possibility of becoming liquidity-constrained, money also insures against the risk of a wealth rundown during an economic downturn. However, the incentives of the private sector to create liquidity are too weak from a social standpoint. As in the certainty case, this result is driven by the fact that the welfare losses generated by the lack of future income pledgeability and its ensuing pecuniary externality reinforce each other.

In this chapter I assume a finite horizon economy where time extends over three dates ($d = \{0, 1, 2\}$) and two trading periods $t = \{1, 2\}$, where a large number of transactions take place between any two consecutive dates.

There are two types of households in equal numbers. The P -type produce potatoes, and the W -type whiskey. Assume, constant returns to scale. Thus, during a trading period, one unit of labor by P yields one potato, and one unit of labor by W produces a unit of whiskey. Just as each agent is single-minded about production, each type is also assumed to consume the good he does not produce. Further, whiskey and potatoes are non-durable, hence agents can only consume these goods in the period in which they are produced.

Each household is also endowed with an amount e_g of gold only on date 0, where $e_g \geq 1$. Unlike the other two goods, gold is durable. Therefore, it can serve as a store of value and for productive investments. Accordingly, the economy offers two investment alternatives: households can deposit gold in storage facilities, or invest it in a real project. The latter is illiquid and irreversible. If financed on $d = 0$, the project is completed on $d = 2$, and investors cannot withdraw gold during that period. The project generates a gross gold return of $\bar{R} > 1$, which is, initially, assumed to be certain.

As in Chapter 2, the lack of simultaneous double coincidence of wants implies that agents need a medium of exchange to trade. Moreover, future income is not pledgeable, so only gold deposits can be pledged as a means of payment. Agents *strictly* prefer to pay with notes that provide proof of their gold deposits, because gold can easily be stolen and burdensome to carry. Moreover, for any of these notes to be accepted in payment, an independent party, or financial intermediary, is required to be its issuer¹.

Following Banerjee and Maskin (1996), the different goods markets are assumed to be well organized, but geographically dispersed and decentralized. That is, for each good there is a known location where that good can be bought and sold, but there is no central clearinghouse.

¹See Chapter 2 for a thorough explanation of all the claims made in this paragraph.

Since each household has dual roles as producer and consumer, it is convenient to think of it as having two identities, or members. For instance, if the household is a potato-producer, one incarnation stays at home in the potato region and sells potatoes in his own shop. Hence, there will be many small shops in each region: one for each trader. By contrast, the other member goes out to buy the good the household wants. Continuing with the example of the P -household, the buyer goes to a shop in the whiskey region and transacts directly with a whiskey-seller there.

Just as there is no central clearinghouse for the whole economy, neither are there clearinghouses within the individual regions. The geographic dispersion of markets implies that a buyer can only go to one region during a trading session. Once in a region a buyer can visit whichever shop he wishes. This ensures that the shops within the region are competitive.

Assume there is a large number of contingencies (and an even larger number of combinations of these contingencies) that could delay the production process of any given good or change the traveling time of buyers. Hence, within a trading period, agents cannot possibly know the exact point in time when they will make a sale or a purchase. This implies that, households do not know whether they will have earned any income when an opportunity to buy emerges. Furthermore, the risk of being liquidity-constrained is not insurable². That is, insurance markets are assumed to be incomplete.

This is the kind of scenario that Keynes envisaged when he developed the concept of precautionary demand for money. According to him, there are situations where it is not possible to specify the universe of possible outcomes for a given process. Thus, when facing this kind of uncertainty, agents cannot protect themselves through the usual appeal to insurance; instead, they resort to the possession of money.

The incomplete markets setting entails a stark difference between the model developed in this chapter and the HZ framework. Because the timing of transactions is uncertain, and insurance markets are incomplete, agents are identical ex-ante: in order to insure themselves against the risk of being liquidity-constrained, both household-types hold positive amounts of money before trading starts, and this is independent from their preferences. By contrast, HZ assume that, before markets open, agents learn whether they first buy or sell. Thus, early consumers are liquidity-constrained and are forced to acquire costly means of payment, while late consumers, depending on their preferences, may, or may not, choose to hold money.

²The term “liquidity-constrained” is used to describe the situation where an agent encounters an opportunity to buy, but he has not earned any labor income yet.

Finally, to capture the speculative demand for money, the last part of the chapter introduces the possibility of default, and assumes that the latter represents a source of aggregate risk. The simplest way to do this is by assuming that the real project can go bankrupt. Technically, the returns of the gold project are now supposed to be stochastic such that, in the bad states of nature, the project fails, and investors lose everything. Hence, agents may choose to hold money simply to hedge potential losses from the real project.

The rest of the chapter proceeds as follows. Section 3.2 provides a summary of the related literature. In Section 3.3 the model with only idiosyncratic risks is developed. Further, it examines whether the market provides liquidity inefficiently under two different assumptions about households' preferences. Section 3.4 undertakes the same analysis for the case where real investments can go bankrupt. Finally, Section 3.5 concludes.

3.2 Related Literature

In addition to the references listed in Chapter 2, this chapter is also related to the Keynes-Tobin debate on the determinants of money demand and the literature of incomplete markets and bankruptcy.

As mentioned earlier, Keynes (1936) was the first to ask ‘why do individuals hold money?’. He postulated the transactions, precautionary and speculative motives, as the underlying drivers of money demand. According to him, only the speculative component is sensitive to interest rates, because agents choose to hold money when they expect bonds to yield lower returns. Tobin(1956, 1958), on the other hand, showed that all three motives are sensitive to interest rates, because holding money entails an opportunity cost: the returns that could have been earned on other assets. In the HZ model, the lack of future income pledgeability implies that liquidity holdings have an opportunity cost in terms of forgone investment returns. Therefore, the framework constructed in this chapter is closer to Tobin’s work.

On the other hand, Keynes’ analysis of the speculative demand for money indicated that individuals hold only money when the expected return on bonds is less than on money. Only when the expected returns on money and bonds are equal, do agents hold both. Thus, according to Keynes, practically no one holds a diversified portfolio as a store of wealth, which is an extremely unrealistic result. By contrast, Tobin (1956) argued that individuals care about the relative expected returns on assets as much as they care about their relative risk. Hence, since individuals are risk-averse, they generally hold diversified portfolios. In the model developed here, money has a lower return relative to real investments, and the speculative money demand only exists when households are risk-averse. Consequently, my results are

consistent with Tobin's exposition of the speculative motive.

In the incomplete markets literature, the seminal pieces are Bryant (1980) and Diamond and Dybvig (1983). Similar contributions include Ramakrishnan and Thakor (1984) and Allen and Gale (1997). Like the framework developed in this chapter, these models treat banks as providers of risk-sharing benefits, when liquidity needs arise (liquidity insurance). The difference between the two approaches is that, while in the model I develop the random timing of transactions precludes the existence of complete markets to provide insurance against the risk of being liquidity-constrained, in the Bryant and Diamond and Dybvig models markets are incomplete because agents ignore the proportion of consumers that effectively become liquidity-constrained.

This chapter also builds on the literature of constrained inefficiency under incomplete markets. The seminal papers are those by Hart (1975) and Stiglitz (1982). Hart (1975) argues that if the set of assets is incomplete, then for a generic subset of endowments, the resulting equilibrium allocations are Pareto suboptimal. More recent contributions include Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003) and Lorenzoni (2008). Gromb and Vayanos (2002) analyze financially constrained arbitrageurs. They show that private agents generally fail to engage in the socially efficient amount of arbitrage between two risky assets, because they do not internalize the pecuniary externalities involved in fire sales events. Caballero and Krishnamurthy (2003) and Lorenzoni (2008), build a model where, entrepreneurs raise finance for a risky investment project and face the risk of a future binding financial constraint. In both papers, entrepreneurs engage in excessive investment, because of pecuniary externalities that arise from the potentially binding constraints in subsequent periods.

Key pieces in the literature of bankruptcy include the papers by Stiglitz and Weiss (1981), and Hart and Moore (1994). However, these models are partial equilibrium. Consequently, they fail to capture the pecuniary externalities that arise in a general equilibrium framework with financial frictions. The first paper to include bankruptcy in a general equilibrium model with complete markets is Shubik (1972). More recent papers include Kocherlakota (1996) and Alvarez and Jermann (2000). These papers build on the literature of dynamic consistency and introduce individual rationality constraints as endogenous debt limits. Thus, by construction, debtors never default in these models.

The framework developed here is, therefore, closer to the general equilibrium model of incomplete markets developed by Dubey, Geanakoplos and Shubik (2005). In their model, default can arise in equilibrium, and its effects on the real economy are tractable. However, the authors consider bankruptcy to be a continuous variable. They compute the probability of default by assuming exogenous non-

pecuniary penalties that represent the bankruptcy code. While their model explains the drivers of default, this chapter only focuses on the effects of bankruptcy on the real economy and liquidity demand.

3.3 A Model of Transactions and Precautionary Demand for Money

Assume a finite horizon economy with three dates ($d = \{0, 1, 2\}$) and two periods: $t = \{1, 2\}$. The time elapsed between two consecutive dates corresponds to a trading round, where a large number of transactions randomly take place.

There are two types of households in equal numbers. During each trading session, the P -type produce potatoes and the W -type whiskey. Assume constant returns to scale. Hence, one unit of labor by P yields one potato, and one unit of labor by W produces a unit of whiskey. Each agent is single-minded about production as well as consumption. Therefore, each household-type is assumed to consume the good which he does not produce. Put differently, the P -type consume whiskey, and the W -type potatoes. These two goods are non-durable, so agents can only consume them in the period in which they are produced.

Each household is also endowed with an amount e_g of gold only on date 0. Assume $e_g \geq 1$. Gold is durable. Thus, it can serve as a store of value and for productive investments. Households can deposit gold in storage facilities, or invest it in a profitable and illiquid project. If financed on $d = 0$, the project is completed on $d = 2$, and investors cannot withdraw gold during that period. The project generates a gross gold return of $\bar{R} > 1$, which is assumed to be certain for the time being.

Assume first that households are endowed with leisure-concave quasi-linear preferences: the marginal utility of gold consumption and the purchased good is constant, while the marginal dis-utility of labor is increasing. Let U^h denote the utility function of household h . This is given by

$$U^h = g^h + \sum_{t=1}^2 \left[c_{b,t}^h - \frac{1}{2} (l_t^h)^2 \right]$$

h is the index indicating the household type: $h = \{P, W\}$. g^h is the cumulative consumption of gold by household h , l_t^h the amount of labor he puts into producing the good he sells in period t , and $c_{b,t}^P$ the amount of good b he consumes in period t . Note that $b = w$ when $h = P$, since P households consume whiskey. Similarly, $b = p$ when $h = W$.

The different goods markets are geographically dispersed and decentralized. For each good there is a known location where that good can be traded, and trade is bilateral. As discussed in the introduction, in this context it is conceptually helpful to think of a household as being populated by two members: a buyer and a seller. In order to buy whiskey, the buyer from the P -household must go to the whiskey region and purchase directly from a whiskey-seller there, and vice versa. Note that, each household executes at most two transactions per period, one in each capacity.

An infinite number of contingencies could setback the production process of any good or delay the traveling time of buyers. Thus, agents cannot possibly know the exact point in time when they will make a sale or a purchase. This implies that households ignore whether they will have earned any income when an opportunity to buy emerges. Since it is not possible to specify the entirety of outcomes involved in the process of producing, buying or selling goods, the risk of becoming liquidity-constrained is not insurable (markets are incomplete). Consequently, on date 0 the two household-types resort to the possession of money to hedge against this type of idiosyncratic risk. Moreover, since agents are symmetric in all aspects except for the good they want and the good they sell, they choose to hold the same amount of liquidity.

Finally, there are many small shops in each region, one for each household. These shops do not rely on a clearinghouse to operate. Therefore, when a buyer arrives to the region where the good he wants is sold, he can visit the shop of his choosing. This ensures that the shops within each region are perfectly competitive, so prices and traded quantities are determined accordingly.

3.3.1 Walrasian Optimum

The first-best benchmark is identified by solving the model for the Walrasian version of this economy. Classic Walrasian theory assumes that households can use all sources of current and future income to finance consumption. In such an ideal world, households would invest their entire endowment of gold in the real project, as they would be able to finance their current consumption with the proceeds from their future labor and investment income.

Moreover, since households are symmetric, to derive the equilibrium conditions that describe the behavior of the household sector, it suffices to solve the optimization problem for only one household-type, the P -type say. The optimization problem for potato producers is given by

$$\max_{c_{w,t}^P, l_t^P, g^P} U^P = g^P + \sum_{t=1}^2 \left[c_{w,t}^P - \frac{1}{2} (l_t^P)^2 \right]$$

s.t.

$$\begin{aligned} g_1^P + P_{w,1}c_{w,1}^P &\leq P_{p,1}l_1^P + \delta \bar{R}e_g \\ g_2^P + P_{w,2}c_{w,2}^P &\leq P_{p,2}l_2^P + (1 - \delta) \bar{R}e_g \\ g^P &= g_1^P + g_2^P \end{aligned}$$

g_t^h is the amount of gold consumed by h in period t . $\delta \in [0, 1]$ represents the fraction of final gold wealth that households pledge to pay for consumption in period 1. The remainder is used to finance period-2 consumption. Since agents are consumption smoothers, $\delta = 1/2$.

In every period, the demand for whiskey by P is described by the equation below

$$c_{w,t}^P = \begin{cases} 0 & \text{if } P_{w,t} > 1 \\ \left(\frac{P_{p,t}}{P_{w,t}} \right)^2 + \frac{\bar{R}e_g}{2P_{w,t}} & \text{if } P_{w,t} < 1 \\ c_{w,t}^P \ni 0 \leq c_{w,t}^P \leq \left(\frac{P_{p,t}}{P_{w,t}} \right)^2 + \frac{\bar{R}e_g}{2P_{w,t}} & \text{if } P_{w,t} = 1 \end{cases} \quad \forall \quad t \quad (3.1)$$

In words, since whiskey and gold are perfect substitutes from P 's perspective, he prefers gold to whiskey if $P_{w,t} > 1$, he is indifferent if $P_{w,t} = 1$, and he prefers whiskey to gold if $P_{w,t} < 1$.

This implies that the labor supply of P depends on his choice of whiskey consumption. If $P_{w,t} \geq 1$, P 's marginal return to labor is $P_{p,t}$, which is the level that maximizes $P_{p,t}l_t^P - \frac{1}{2}(l_t^P)^2$. Similarly, when $P_{w,t} < 1$, the marginal return of producing potatoes is $\left(\frac{P_{p,t}}{P_{w,t}} \right)$, which is the level of labor that maximizes $\left[\frac{P_{p,t}}{P_{w,t}}l_t^P - \frac{1}{2}(l_t^P)^2 \right]$.

Therefore, the labor supply by P (the potato supply curve) is given by

$$l_t^P = \begin{cases} P_{p,t} & \text{if } P_{w,t} \geq 1 \\ \frac{P_{p,t}}{P_{w,t}} & \text{if } P_{w,t} < 1 \end{cases} \quad \forall \quad t \quad (3.2)$$

The optimization problem of the W household is symmetric. Hence, his set of first order conditions mirror the choices of P . W 's demand for potatoes is described by

$$c_{p,t}^W = \begin{cases} 0 & \text{if } P_{p,t} > 1 \\ \left(\frac{P_{w,t}}{P_{p,t}}\right)^2 + \frac{\bar{R}e_g}{2P_{p,t}} & \text{if } P_{p,t} < 1 \\ c_{p,t}^W \ni 0 \leq c_{p,t}^W \leq \left(\frac{P_{w,t}}{P_{p,t}}\right)^2 + \frac{\bar{R}e_g}{2P_{p,t}} & \text{if } P_{p,t} = 1 \end{cases} \quad \forall \quad t \quad (3.3)$$

and the supply of whiskey by

$$l_t^W = \begin{cases} P_{w,t} & \text{if } P_{p,t} \geq 1 \\ \frac{P_{w,t}}{P_{p,t}} & \text{if } P_{p,t} < 1 \end{cases} \quad \forall \quad t \quad (3.4)$$

In equilibrium, the whiskey and potato markets clear in every period. Thus, the following conditions must also be satisfied

$$c_{w,t}^P = l_t^W \quad \text{and} \quad c_{p,t}^W = l_t^P \quad \forall \quad t$$

Markets do not clear if, for any $t \in \{1, 2\}$, either $P_{w,t} > 1$ or $P_{p,t} > 1$. In the first case, demand for whiskey is zero, while its supply is positive. The same applies for the potato market if $P_{p,t} > 1$. On the other hand, there is no equilibrium where $P_{w,t} < 1$ and $P_{p,t} < 1$. These conditions imply that demand for gold is zero, but the latter is in positive net supply and equal to $2\bar{R}e_g$. Similarly, if $P_{w,t} < 1$ and $P_{p,t} = 1$ (or $P_{w,t} = 1$ and $P_{p,t} < 1$), the whiskey (potato) market does not clear.

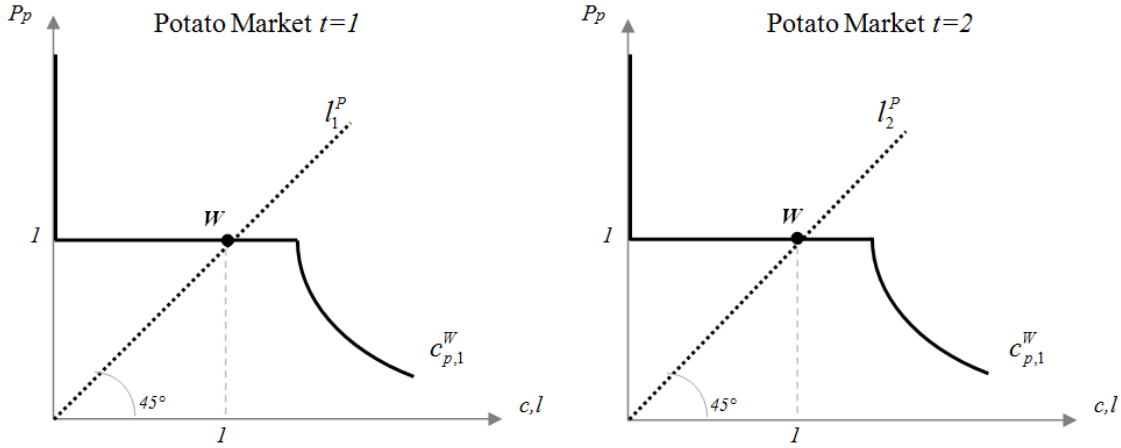
Therefore, there is a unique Walrasian equilibrium where

$$l_t^P = l_t^W = P_{p,t} = P_{w,t} = 1 \quad \forall \quad t \quad (3.5)$$

Figure 3.1 (below) provides a graphic illustration. The Walrasian equilibrium is the same in the potato and whiskey markets. Therefore, it suffices to show the first-best outcome for only one good, potatoes say. The Walrasian optimum is represented by point W . The left-hand side graph corresponds to the equilibrium in the first period and the right-hand side graph to the second period.

In each period, the demand curve for potatoes is given by W 's first order condition, equation (3.3). The kink on the demand curve corresponds to the point where W would use all his income, present or future labor income as well as future investment returns, to buy potatoes at a price $P_{p,t} = 1$. On the other hand, the supply of potatoes is determined by the labor choice of P . The latter is given by equation (3.2). Since $P_{w,t} = 1$ in equilibrium, the potato supply curve is represented by a 45 degree line that passes through the origin. The equilibrium in the whiskey market in both periods is completely symmetric.

Figure 3.1: Walrasian Equilibrium with Leisure-Concave Preferences



3.3.2 Market Equilibrium

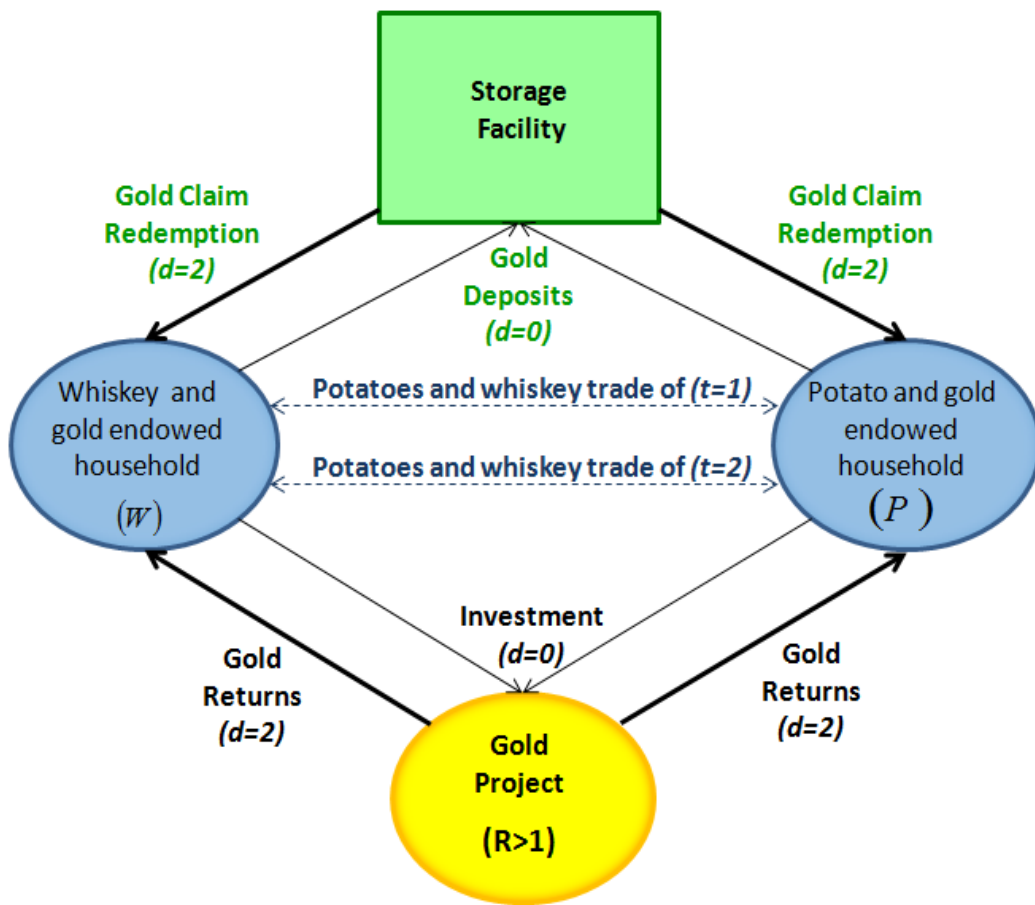
In the market economy, the lack of simultaneous double coincidence of wants implies that agents need a medium of exchange to trade. Furthermore, the non-pledgeability of future income induces the market to create costly means of payment. Gold deposited in storage is the only source of pledgeable income available to consumers. This is due to the fact that future gold returns are non-verifiable, and future labor income can be hidden or diverted. In addition, settling payments with gold is hazardous, so agents prefer to make purchases with notes that certify the existence of the gold they have kept in storage. Banks naturally emerge as the providers of liquidity in this system, since gold-backed notes are only accepted in payment if an independent party issues them.

These notes are a financial claim with the following characteristics: a unit of gold deposited on date 0 buys one note; and each note is worth a unit of gold on date 2. This implies that making gold deposits is not profitable, while the real investment yields a return of $\bar{R} > 1$. Hence, accessing means of payment

has an opportunity cost $\bar{R}$.

The trade and liquidity creation mechanisms are the same as in the HZ framework. The difference is that, since agents cannot know the timing in which buying and selling transactions will take place, they are identical from an ex-ante point of view. That is, both household-types demand liquidity on date 0, in order to insure themselves against the risk of being liquidity-constrained. In fact, as in the Walrasian case, P and W households are now identical in all aspects but their labor skills and the goods they want. This implies that the outcomes in the whiskey and potato markets are symmetric. Figure 3.2 (below) provides an illustration of the structure of the economy³.

Figure 3.2: Nominal Flows of the Economy



Households' Behavior

Since consumers are symmetric, to understand the behavior of the household sector, it suffices to solve the optimization problem for only one consumer-type. Take the potato producer.

³At a first glance, this figure may seem the same as Figure 2.6. Yet note, that unlike the model developed in Chapter 2, both agents claim gold back from the bank on $d=2$. In addition, while in the Chapter 2 model each good is traded only once and markets open sequentially, in this chapter the two goods are traded in each of the trading sessions.

On date 0, P deposits f_1^P units of gold in the bank in exchange for the same amount of notes. During the first round of trade, he spends this money buying whiskey from household W . At the end of this trading session, P finds himself with $P_{p,1}l_1^P$ notes, which correspond to the proceeds from his potato sales. He then chooses f_2^P of these notes to buy whiskey in the second trading round. At the end of this session, P redeems the notes he is left with for an equal amount of gold. He also receives the returns from the gold he invested in the project, which are equal to $\bar{R}(e_g - f_1^P)$.

P 's optimization problem is, therefore, given by

$$\max_{l_t^P, c_{w,t}^P, f_t^P} U^P = g^P + \sum_{t=1}^2 \left[(c_{w,t}^P) - \frac{1}{2} (l_t^P)^2 \right] \quad (3.6)$$

s.t.

$$\begin{aligned} P_{w,1}c_{w,1}^P &= f_1^P & (\lambda_1^P) \\ P_{w,2}c_{w,2}^P &= f_2^P & (\lambda_2^P) \\ P_{p,1}l_1^P &\geq f_2^P & (\mu^P) \\ e_g \geq f_t^P &\geq 0 & \forall t \end{aligned}$$

where $g^P = \bar{R}(e_g - f_1^P) + (P_{p,1}l_1^P - f_2^P) + P_{p,2}l_2^P$.

λ_t^P is the Lagrange multiplier of the period- t budget constraint. μ^h represents the multiplier of the complementary slackness (CS) condition. The latter specifies whether, during the second round of trade, P spends in full his first-period income. $c_{w,t}^P$ denotes the quantity of whiskey purchased by household P in period t , and l_t^P the amount of time he works during that period. $P_{p,t}$ is the price of potatoes in period t , and $P_{w,t}$ the price of whiskey. Finally, f_t^P denotes the amount of notes spent by household P in period t , and g^P his terminal stock of gold.

In equilibrium the CS condition binds, because accessing means of payment is costly. Consequently, holding liquidity and not spending it is inefficient. When the second trading session starts, P anticipates that, due to his symmetry with W , by the end of the period he will have earned as much money as he has spent. Therefore, it is optimal for him not to hoard liquidity in $t = 2$. That is,

$$P_{p,1}l_1^P = f_2^P$$

This implies that households' labor supply choice is given by

$$l_1^P = \frac{P_{p,1}}{P_{w,2}} \quad (3.7)$$

$$l_2^P = P_{p,2} \quad (3.8)$$

In the first period, the marginal return of labor for the potato producer is $P_{p,1}/P_{w,2}$. In $t = 1$, every unit of effort gives P an income of $P_{p,1}$, and he derives utility from the amount of goods this money buys in the second round of trade ($1/P_{w,2}$). By contrast, in $t = 2$ the amount of time P works yields a unitary income of $P_{p,2}$. However, he can only spend that money claiming an equivalent amount of gold from the bank. For this reason, the marginal return of labor equals $P_{p,2}$.

Note that the consumption of gold and the purchased good (whiskey) are perfect substitutes. Furthermore, an additional unit of money allows P to buy $1/P_{w,1}$ units of whiskey, but it also generates a cost of $\bar{R}$ units of forgone gold returns. Hence, P 's choice of money holdings in the first period is given by

$$f^P = \begin{cases} 0 & \text{if } P_w > \frac{1}{\bar{R}} \\ 0 < f^P < e_g & \text{if } P_w = \frac{1}{\bar{R}} \\ e_g & \text{if } P_w < \frac{1}{\bar{R}} \end{cases} \quad (3.9)$$

If $P_w > 1/\bar{R}$, P prefers gold to potatoes, so he does not deposit gold with the bank; if $P_w = 1/\bar{R}$, he is indifferent and deposits a positive amount of gold; finally, if $P_w < 1/\bar{R}$, he prefers whiskey to gold, so he monetizes all his gold. Although P also uses money to buy whiskey in the second period, the price of whiskey in $t = 2$ is not relevant for his investment decision on $d = 0$. The reason is that, the money P spends in the second round of trade corresponds to his first-period labor income.

By symmetry, the mirror of these equilibrium conditions are satisfied when the optimization problem for household W is solved.

In general equilibrium, the potato and whiskey markets must clear in all periods. Thus

$$c_{p,t}^W = l_t^P \quad \forall t$$

$$c_{w,t}^P = l_t^W \quad \forall t$$

Combining these conditions and households' budget constraints with the fact that households' CS conditions bind, yields the following result

$$f_1^P = f_1^W = f_2^P = f_2^W = f \quad (3.10)$$

That is, the demand for liquidity is the same across households and constant across time. Since households' labor and consumption choices are identical, this implies that the prices of potatoes and whiskey are the same in every period.

$$P_{p,t} = P_{w,t} \quad \forall t \quad (3.11)$$

Consequently, there are three possible equilibria.

- *Case 1:* $P_{p,1} = P_{w,1} < \frac{1}{\bar{R}}$
- *Case 2:* $P_{p,1} = P_{w,1} > \frac{1}{\bar{R}}$
- *Case 3:* $P_{p,1} = P_{w,1} = \frac{1}{\bar{R}}$

In Case 1, households would choose to deposit all their gold with the bank. Thus, market clearing in commodity markets would require

$$P_{p,1} = P_{w,1} = \sqrt{e_g}$$

But $e_g \geq 1$ by assumption, which yields a contradiction: if $P_{p,1} = P_{w,1} = \sqrt{e_g} < \frac{1}{\bar{R}}$, then $\bar{R}^2 e_g < 1$, which cannot be true since $e_g \geq 1$ and $\bar{R} > 1$. This implies that agents could not possibly optimize at this price level.

In Case 2, households' demand for potatoes and whiskey would be zero in the first period. However, equation (3.7) implies that the supply of both goods would be positive in $t = 1$. In other words, commodity markets would not clear, which rules out this case as a possible equilibrium.

The only possible equilibrium is characterized by an interior solution to households optimization problem, where both, P and W , deposit a portion of their gold in the bank. In this case,

$$P_{p,1} = P_{w,1} = \frac{1}{\bar{R}} \quad (3.12)$$

Combining equations (3.7), (3.8) and (3.12) with the market clearing conditions, gives the equilibrium level of liquidity (f^{**}).

$$f^{**3/4} = \bar{R} \quad (3.13)$$

Table 3.1 (below) summarizes the levels of trade and prices obtained in equilibrium.

Table 3.1: Market Equilibrium in the Baseline Model

Liquidity Holdings	
Per Capita	Aggregate
$f^{**} = \bar{R}^{-4/3}$	$2f^{**} = 2\bar{R}^{-4/3}$
First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = f^{**1/4}$	$P_{p,1} = P_{w,1} = f^{**3/4}$
Second Period	
Trade/Labor	Prices
$l_2^P = l_2^W = f^{**1/2}$	$P_{p,2} = P_{w,2} = f^{**1/2}$

Proposition 3.3.1 *If $\bar{R} > 1$ and $e_g \geq 1$, the market equilibrium generates less trade than the Walrasian optimum.*

Proof If $\bar{R} > 1$ and $e_g \geq 1$, the market equilibrium is characterized by the interior solution to households' money demand problem. That is, the outcome in the gold-deposit, whiskey and potato markets, corresponds to the equilibrium set out in Table 3.1. These results evidence that trade in the market economy is increasing in f^{**} . In addition, f^{**} is less than 1, if $\bar{R} > 1$, which is true by assumption. Consequently, trade and prices are smaller than unity. By contrast, all traded quantities in the Walrasian optimum are equal to 1.

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Figure 3.3 (below) provides a graphic illustration of the competitive market equilibrium in the potatoes market, which is, by symmetry, representative of all the goods markets. The graphs also compare the competitive market outcome with the Walrasian optimum. The former is denoted by point M , and the latter by W .

In the potato market, the supply curves are given by the labor choice of household P (dashed lines), and the demand curves by the consumption choice of W (straight lines). The left-hand side graph depicts the equilibrium for the potato market in the first period, and the right-hand side graph for the second period.

As explained earlier, the marginal return to P 's labor in the first period is given by $P_{p,1}/P_{w,2}$. This implies that the supply curve is a straight line that passes through the origin. Further, since $P_{w,2} < 1$ in equilibrium, the potato-supply schedule is flatter than a 45 degree line. On the other hand, for every note that W spends in potatoes on $t = 1$, he sacrifices the return on the alternative investment ($\bar{R}$). Since gold and potatoes are perfect substitutes for W , the potato demand schedule in $t = 1$ is given by

$$c_{p,1}^W = \begin{cases} 0 & \text{if } P_{p,1} > 1/\bar{R} \\ \frac{e_g}{P_{p,1}} & \text{if } P_{p,1} < 1/\bar{R} \\ 0 < c_p^W < \frac{e_g}{P_{p,1}} & \text{if } P_{p,1} = 1/\bar{R} \end{cases} \quad (3.14)$$

The market equilibrium is characterized by the interior solution, where $P_{p,1} = 1/\bar{R}$. Thus, W deposits only a fraction of his gold to buy whiskey. This implies that the equilibrium level of trade in the potato market lies to the left of the kink on the demand curve. The latter corresponds to the point where W would have spent his entire gold endowment in the consumption of whiskey at a price $1/\bar{R}$.

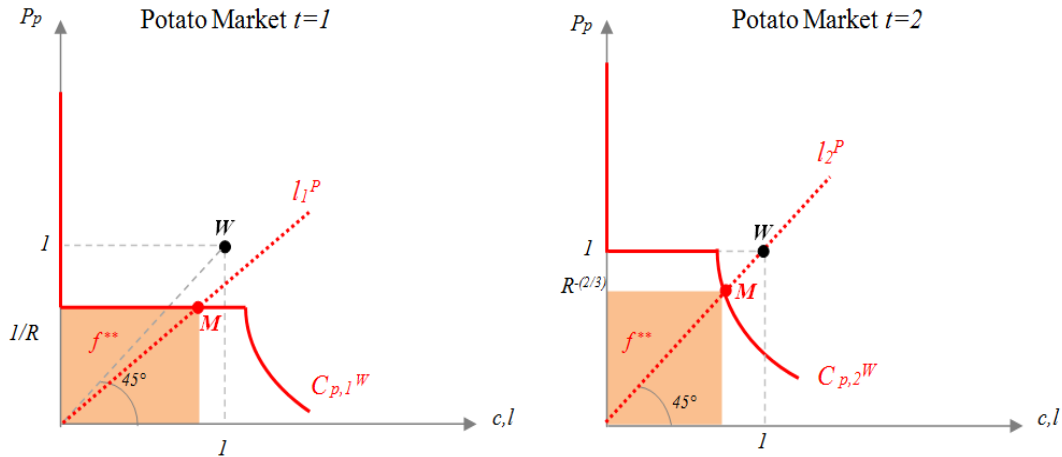
Recall that, in the second period, the marginal return of labor for P is given by $P_{p,2}$. Consequently, the supply curve is a 45 degree line that passes through the origin. On the demand side, the use of money itself does not entail any cost. This is due to the fact that in $t = 2$, the amount of money W can spend is equal to his first-period-labor income. Since, gold and potatoes are perfect substitutes for W , the potato demand schedule in the second period is given by

$$c_{p,2}^W = \begin{cases} 0 & \text{if } P_{p,2} > 1 \\ \frac{P_{w,1}l_1^W}{P_{p,2}} = \frac{f^*}{P_{p,2}} & \text{if } P_{p,2} < 1 \\ 0 < c_p^W < \frac{f^{**}}{P_{p,2}} & \text{if } P_{p,2} = 1 \end{cases} \quad (3.15)$$

If $P_{p,2} > 1$, W prefers gold to potatoes, if $P_{p,2} = 1$ he is indifferent, and if $P_{p,2} < 1$ he prefers potatoes to gold. In equilibrium, all prices are less than 1. Therefore, W spends his entire first-period income buying potatoes in the second trading session. This implies that the equilibrium level of trade in the potato market lies to the right of the kink on the demand curve.

Finally, the shaded area to the left and bottom of M in both graphs represents the monetary value of each transaction: f^{**} .

Figure 3.3: Market Equilibrium for a Representative Goods Market



In each period, equilibrium prices and trade levels are the same across markets. This is due to the fact that, before each trading session starts, both households hold money to hedge against the risk of being liquidity-constrained. Therefore, the two household-types follow the same behavior. This contrasts with

the HZ model, where agents know whether or not they will be liquidity-constrained. Hence, in their setup, each household-type follows a unique behavior, and the equilibrium in each market is different.

However, in the model developed in this chapter, prices and traded quantities do change over time. What is more, the market outcome is more inefficient in the second period: fewer goods are traded at higher prices. The intuition underlying this result is that, households' propensity to work/sell is lower in $t = 2$. As Figure 3.3 shows, the supply of goods is more inelastic in the second period, because the marginal return to labor is higher in $t = 1$ than in $t = 2$. This is due to the fact that, individuals can spend their first-period income buying goods in the second trading session, but they can only use their second-period income to claim gold from the bank, which is a venture that does not create value.

3.3.3 Social Optimum

The market economy is characterized by the facts that future income is not pledgeable, and agents do not have access to complete insurance markets. This implies that, in order to trade and insure themselves against the risk of being liquidity-constrained, agents must deposit gold in unproductive storage on $d = 0$. But doing so entails a waste of resources. Consequently, prices do not fully accomplish their Walrasian duty of signaling scarcity through the budget constraints. Instead, they also play a role as drivers of the aforementioned market imperfections. In other words, pecuniary externalities have non-trivial welfare effects, which agents fail to acknowledge, because they follow price-taking behavior⁴.

When markets are not perfect, the best benchmark to evaluate public intervention is that of *constrained Pareto efficiency*⁵. This type of analysis establishes whether, while being subject to the same constraints as the agents in the decentralized market, the planner can engineer a Pareto improvement.

In this setup, the planner must be constrained to take as given the non-pledgeability of future income and market incompleteness. Moreover, unlike the agents in the decentralized market, the planner internalizes the pecuniary externality. Thus, he also takes into account the effect of liquidity holdings on prices.

To identify the minimal conditions whereby the planner can improve upon the competitive market outcome, assume he has a regulatory tool that determines the amount of liquidity held by each household. Denote the latter by f_s .⁶

⁴See Greenwald and Stiglitz (1986).

⁵See Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986).

⁶Think, for instance, that the planner sets the maximum amount of resources that can be invested in the banking sector (or in the real project). If, relative to the market equilibrium, the planner wants agents to hold less liquidity, he sets a maximum level of bank deposits. If, on the contrary, the planner wants households to hold more money, he limits real investments ($e_g - f$).

In addition, when solving the constrained efficiency problem, the planner maximizes households' expected utility. Since households are symmetric, and the equilibrium in the potato and whiskey markets is identical, the planner's objective function is given by the utility function of any of the two household-types. Take the potato producers. The planner, therefore, optimizes

$$\begin{aligned}\max_{f_s} U^P &= g^P + \sum_{t=1}^2 \left[c_{w,t}^P - \frac{1}{2} (l_t^P)^2 \right] \\ &\equiv \max_{f_s} [\bar{R}(e_g - f_s) + f_s] + \sum_{t=1}^2 \left[c_{w,t}^P - \frac{1}{2} (l_t^P)^2 \right]\end{aligned}$$

s.t.

$$P_{w,1} = P_{p,1} = f_s^{3/4}$$

$$P_{w,2} = P_{p,2} = f_s^{1/2}$$

After imposing the planner's constraints, and using the results in Table 3.1 that relate trade and liquidity, the problem is simplified as follows

$$\max_{f_s} U^P = \left(f_s^{1/4} - \frac{1}{2} f_s^{1/2} \right) + \left(f_s^{1/2} - \frac{1}{2} f_s \right) + [\bar{R}e_g + (1 - \bar{R}) f_s] \quad (3.16)$$

The first order condition with respect to f_s yields

$$f_s \ni \frac{1}{2} + \frac{1}{4} \left[(f_s)^{-(3/4)} + (f_s)^{-(1/2)} \right] = \bar{R} \quad (3.17)$$

The level of f_s that solves the equation above corresponds to the social optimum level of liquidity.

Proposition 3.3.2 *If $\bar{R} > 1$ and $e_g \geq 1$, the market equilibrium leads to an excessive amount of liquidity and trade relative to the social optimum.*

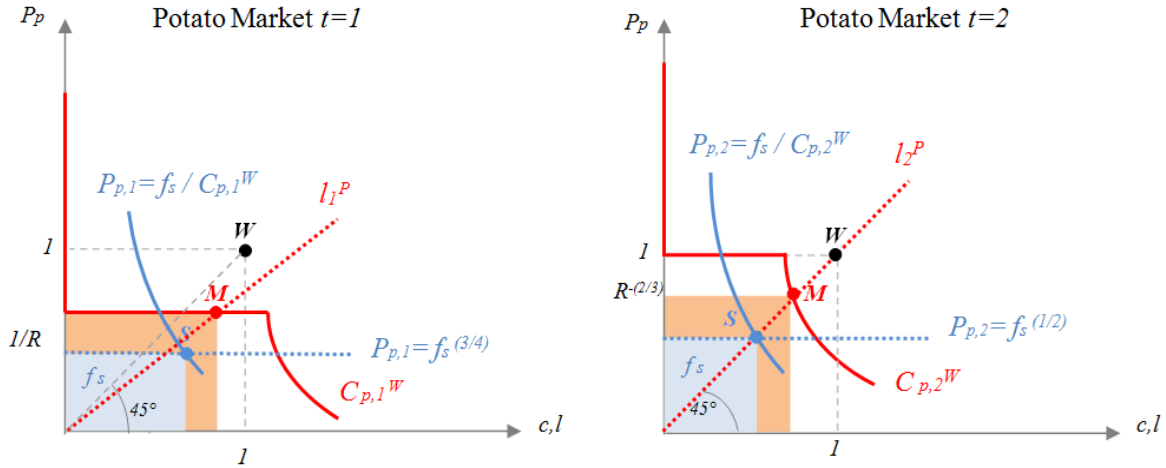
Proof Equation (3.13) is the same as equation (2.17) in Chapter 2, and (3.17) is the same as (2.19). Therefore, by proposition 2.3.2 $f^{**} > f_s$. That is, the market produces too much liquidity relative to the social optimum.

The results in Table (3.1) show that trade in the first period is given by $l_1^P = l_1^W = f^{1/4}$, and in the second period by $l_2^P = l_2^W = f^{1/2}$. Hence, the amount of trade in the market economy increases with the level of liquidity. Since $f^{**} > f_s$, it follows that the equilibrium level of trade in the competitive market economy is higher than the social optimum.

QED

Figure 3.4 (below) compares the competitive market equilibrium and the social optimum. The latter is denoted by point S. The market equilibrium is illustrated as in Figure 3.3, and the social planner's choice of liquidity, which is represented by the blue area, is superimposed over the market one. As it is customary in this kind of analysis, except for satisfying agents' budget constraints, the planner's choices do not coincide with the decisions private agents make in pursuit of their private benefit. Consumers' budget constraints under the social optimum are represented by the blue downward-sloping curves. The planner's own constraints, on the other hand, correspond to the horizontal blue dotted lines.

Figure 3.4: Social Optimum for a Representative Goods Market



This analysis exposes two types of externalities. First, because insurance markets are incomplete and future income is not pledgeable, the market generates sub-optimally low levels of liquidity and trade with respect to the Walrasian optimum. However, relative to the social optimum, the market produces too much liquidity and generates too much trade. Private and social incentives to create liquidity diverge, because costly money creation generates a pecuniary externality, which price-taking agents fail to internalize. More precisely, private agents fail to acknowledge that, by spending more money than it is socially desirable, prices rise to a level that is detrimental to their own well-being.

This is due to the fact that, when commodity prices increase, households' welfare losses - in their capacity as buyers - more than offset the gains they receive when acting as sellers.

Furthermore, if the planner intervenes to restrict the creation of liquidity, commodity prices in the first period fall relative to the second period. This entails a transfer of resources (or trade) from the second to the first period. Hence, households' welfare improves, because supply curves are more elastic in $t = 1$. In other words, in the absence of public intervention, the losses in terms of missed trade opportunities are larger.

3.3.4 Consumption-Concave Preferences

Thus far, the welfare results predicted by the model developed in this chapter coincide with those of the HZ framework. But recall from Chapter 2 that HZ's findings are sensitive to the assumption about households' preferences. Therefore, this section solves the model under a slightly different assumption about agents' utility function.

Assume that households are still endowed with quasi-linear preferences. However, the marginal marginal dis-utility of labor is now assumed to be constant and the marginal utility of consumption decreasing. Let U^h denote the utility function of household h .

$$U^h = \ln(g^h) + \sum_{t=1}^2 [\ln(c_{b,t}^h) - l_t^h]$$

where $h = \{P, W\}$. g^h is the cumulative consumption of gold by household h , l_t^h the amount of time he works in period t , and $c_{b,t}^P$ the amount of good b he consumes in period t . Recall that $b = w$ when $h = P$ (P households consume whiskey), and $b = p$ when $h = W$ (W households consume potatoes).

Walrasian Optimum

In the Walrasian economy, households would be able to finance consumption with all sources of current and future income. Therefore, they would invest all their gold in the real project.

Since agents are symmetric, it is sufficient to solve the optimization problem for only one type, say the P -type. The optimization problem of potato producers is given by

$$\max_{g^P, c_{w,t}^P, l_t^P} U^P = \ln(g^P) + \sum_{t=1}^2 [\ln(c_{w,t}^P) - l_t^P]$$

s, t

$$\begin{aligned} g_1^P + P_{w,1}c_{w,1}^P &= P_{p,1}l_1^P + \delta \bar{R}e_g \\ g_2^P + P_{w,2}c_{w,2}^P &= P_{p,2}l_2^P + (1 - \delta) \bar{R}e_g \\ g^P &= g_1^P + g_2^P \end{aligned}$$

g_t^P is the amount of gold consumed by P in period t , and $\delta \in [0, 1]$ represents the fraction of final gold wealth he pledges to pay for consumption in period 1 (he uses the remainder to finance his period-2 consumption). Since agents are consumption smoothers, $\delta = 1/2$.

The first order conditions are given by

$$c_{w,t}^P = \frac{P_{p,t}}{P_{w,t}} \quad \forall t \quad (3.18)$$

$$g_t^P = P_{p,t} \quad \forall t \quad (3.19)$$

Further, P 's labor supply schedule is obtained by combining the two equations above and the budget constraints.

$$l_t^P = 2 - \frac{\bar{R}e_g}{2P_{p,t}} \quad \forall t \quad (3.20)$$

By symmetry, the mirror of these equilibrium conditions describe the behavior of household W . Moreover, in equilibrium the whiskey and potato markets clear. Hence, the equalities below must hold in every period.

$$l_t^P = c_{p,t}^W \quad \forall t$$

$$l_t^W = c_{w,t}^P \quad \forall t$$

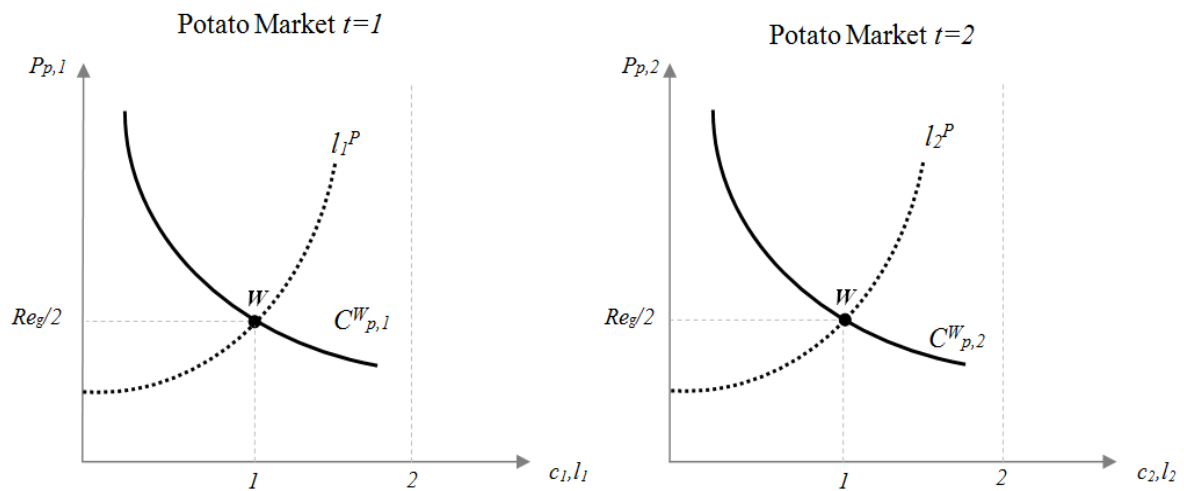
Combining households' first order conditions and budget constraints with the market clearing conditions, solves the model for the Walrasian economy. Table 3.2 (below) summarizes.

Table 3.2: Walrasian Optimum with Consumption-Concave Preferences

First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = 1$	$P_{p,1} = P_{w,1} = \frac{\bar{R}e_g}{2}$
Second Period	
Trade/Labor	Prices
$l_2^P = l_2^W = 1$	$P_{p,2} = P_{w,2} = \frac{\bar{R}e_g}{2}$

Figure 3.5 (below) provides a graphic illustration of the Walrasian optimum for a representative market: the potato market. The demand curves are represented by straight lines and correspond to W 's first order conditions with respect to potato consumption (the mirror of P 's first order condition 3.18). On the other hand, the supply curve is given by P 's labor supply choice, equation (3.20).

Figure 3.5: Walrasian Optimum for the Potato Market with Consumption-Concave Preferences



Market Economy

As in section 3.3.2, the market economy is characterized by a lack of simultaneous double coincidence of wants, and the fact that future income is not pledgeable. Consequently, agents require a medium of exchange to trade, and only gold-backed money is useful as means of payment. Only financial intermediaries can issue this money, as banks alone, in their capacity as independent parties to any trade transaction, are able to certify the existence of the gold backing-up each note.

Further, markets are incomplete. Therefore, households cannot insure themselves against the randomness in the timing of transactions. Instead, both household-types choose to hold positive amounts of money on date 0. This implies that agents are symmetric in all aspects but their consumption bundle. What is more, because of this symmetry, the behavior of either P or W is representative of the household sector. Hence, I only look at the optimization problem of household P . The optimization problem of potato growers is given by

$$\max_{l_t^P, c_{w,t}^P, f_t^P} U^P = \ln(g^P) + \sum_{t=1}^2 [\ln(c_{w,t}^P) - l_t^P]$$

s.t.

$$\begin{aligned} P_{w,1}c_{w,1}^P &= f_1^P & (\lambda_1^P) \\ P_{w,2}c_{w,2}^P &= f_2^P & (\lambda_2^P) \\ P_{p,1}l_1^P &\geq f_2^P & (\mu^P) \\ e_g &\geq f_t^P \geq 0 & \forall t \end{aligned}$$

where $g^P = \bar{R}(e_g^P - f_1^P) + (P_{p,1}l_1^P - f_2^P) + P_{p,2}l_2^P$

In equilibrium, the CS condition binds ($\mu^P > 0$). This is due to the fact that, accessing means of payment is costly. Thus, holding liquidity and not spending it is inefficient. Further, P anticipates that, because of his symmetry with W , he will have earned as much money as he has spent during the second trading session. Therefore, it is not optimal for him to hoard liquidity in $t = 2$.

Since households are symmetric and the CS condition binds, it must be the case that

$$f_1^P = f_1^W = f_2^P = f_2^W = f \quad (3.21)$$

That is, the demand for liquidity is the same across households and constant across time. Consequently, P 's first order conditions can be specified as follows

$$g^P = \bar{R}f \quad (3.22)$$

$$f = P_{p,1} \quad (3.23)$$

$$g^P = P_{p,2} \quad (3.24)$$

By combining these equations with each period's budget constraint, the first order conditions with respect to whiskey consumption obtain.

$$c_{w,1} = \frac{P_{p,1}}{P_{w,1}} \quad (3.25)$$

$$c_{w,2} = \frac{P_{p,2}}{\bar{R}P_{w,2}} \quad (3.26)$$

For household P , the marginal utility of labor and, hence, potatoes is constant and equal to -1 in every period. Therefore, the two equations above capture the conventional result, whereby the marginal rate of substitution between two goods is equal to their price ratio. Note, however, that the price of whiskey in period 2 is multiplied by $\bar{R}$. The intuition underlying this result is that, in order to buy the good they want, agents need money. Since money has an opportunity cost $\bar{R}$, which is accrued in $t = 2$, this cost is factored into the value of the good agents purchase during the second trading session.

The labor supply schedules are obtained by combining first order conditions (3.22) through (3.25) and budget constraint (3.21)

$$l_1^P = (\bar{R} + 1) - \left[\frac{\bar{R}e_g - (\bar{R} - 1)f}{P_{p,1}} \right] = \bar{R} \left(2 - \frac{e_g}{P_{p,1}} \right) \quad (3.27)$$

$$l_2^P = 1 - \frac{\bar{R}(e_g - f)}{P_{p,2}} = 2 - \frac{\bar{R}e_g}{P_{p,2}} \quad (3.28)$$

By symmetry, the mirror of equilibrium conditions (3.23) through (3.29) are satisfied when the optimization problem for household W is solved.

Let f^{**} denote the per-capita equilibrium level of liquidity. The latter can now be obtained from first order condition (3.23).

$$\begin{aligned} \bar{R}f^{**} = g^P &\iff \bar{R}f = \bar{R}(e_g - f^{**}) + f^{**} \\ \implies f^{**} &= \frac{\bar{R}e_g}{2\bar{R} - 1} \end{aligned} \quad (3.29)$$

Moreover, in general equilibrium the potato and whiskey markets clear in every period. Thus,

$$c_{p,t}^W = l_t^P \quad \forall t$$

$$c_{w,t}^P = l_t^W \quad \forall t$$

Since households' labor and consumption choices are identical, the prices of potatoes and whiskey are the same in every period. Table 3.3 displays the equilibrium solution for the gold-deposit and commodity markets.

Proposition 3.3.3 *If $\bar{R} > 1$ the market generates less trade than the Walrasian optimum only in the second period.*

Proof Due to the symmetry across household-types, in each period the equilibrium prices of whiskey and potatoes are the same. Thus, equation (3.26) implies that one unit of each good is traded in $t = 1$. By contrast, from equation (3.27), it can be concluded that $1/\bar{R}$ units of each commodity are traded in $t = 2$. Recall that, in the Walrasian equilibrium, the level of trade in each market is equal to 1 for $t = \{1, 2\}$. This implies that trade is efficient in the first period. On the other hand, if $\bar{R} > 1$, relative to the Walrasian optimum, the market generates less trade in the second period.

Table 3.3: Market Equilibrium with Consumption-Concave Preferences

Liquidity Holdings	
Per Capita	Aggregate
$f^{**} = \frac{\bar{R}e_g}{2\bar{R}-1}$	$2f^{**} = \frac{2\bar{R}e_g}{2\bar{R}-1}$
First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = 1$	$P_{p,1} = P_{w,1} = f^{**}$
Second Period	
Trade/Labor	Prices
$l_2^P = l_2^W = \frac{1}{\bar{R}}$	$P_{p,2} = P_{w,2} = \bar{R}(e_g - f^{**}) + f^{**}$

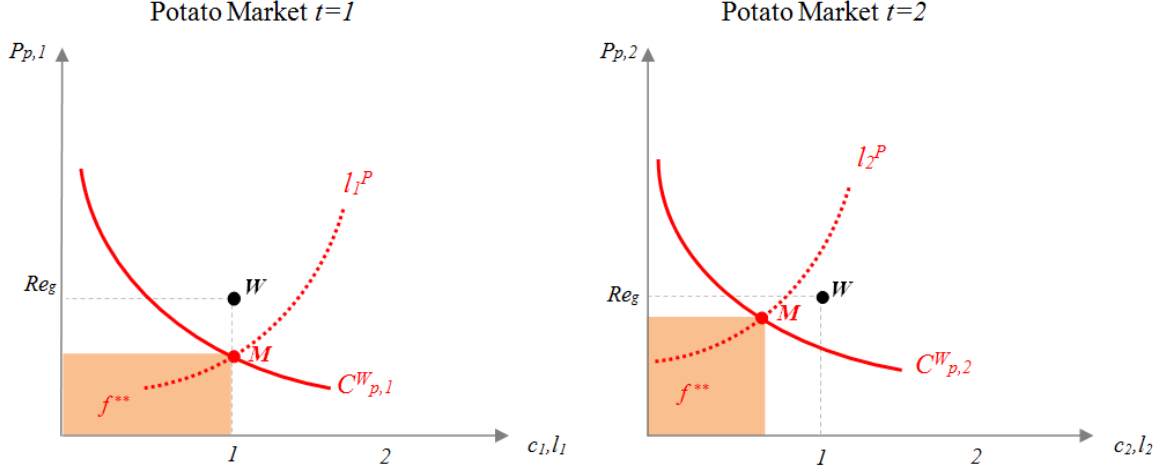
QED

Trade is more inefficient in the second period, because households' propensity to spend and work is lower in $t = 2$ than in $t = 1$. Agents need money to buy the good they want. However, money has an opportunity cost, which is accrued during the second period. Thus, when purchasing a good in $t = 2$, households incorporate the cost of using money to the price of the good they buy. Therefore, trade is lower in $t = 2$ than in $t = 1$. On the other hand, income earned during $t = 2$ can only be used to claim gold back from the bank, which does not create value. By contrast, first-period labor income can be used to purchase goods during the second trading session. Consequently, agents are more reluctant to work/sell in the second period. This supply-side effect is larger than the demand-side effect, which is why prices in $t = 2$ are higher relative to $t = 1$.

Figure 3.6 provides an illustration of the market equilibrium in the potatoes market and compares it with the Walrasian optimum. The demand curves are represented by straight lines. These correspond to W 's first order conditions with respect to the consumption of potatoes, which mirror equations (3.26) and (3.27). The supply curves are given by P 's labor supply schedule and are represented by dashed

lines. For the first and second periods, the supply curves are obtained from equations (3.28) and (3.29), respectively. In addition, the market equilibrium is denoted with point M , and the Walrasian optimum with point W . Finally, the shaded area corresponds to the monetary value of each transaction.

Figure 3.6: Market Equilibrium: Potato Market with Consumption-Concave Preferences



Social Optimum

As in Section 3.3.3, the central planner maximizes the utility of the representative household and, unlike decentralized agents, he takes into account the effect of liquidity on prices. Assume the planner has the means to fix the amount of liquidity held by each household, and denote this quantity by f_s . The centralized problem is, therefore, given by

$$\begin{aligned} \max_{f_s} U^P &= \ln(g^P) + \sum_{t=1}^2 [\ln(c_{w,t}^P) - l_t^P] \\ \text{s.t.} \\ P_{p,1} = P_{w,1} &= f_s \\ P_{p,2} = P_{w,2} &= \bar{R}(e_g - f_s) + f_s \\ e_g &\geq f_s \geq 0 \end{aligned}$$

After imposing the planner's constraints and households' budget constraints, the problems is simplified as follows

$$\max_{f_s} U^P = \ln(f_s) - \frac{f_s}{\bar{R}(e_g - f_s) + f_s}$$

The first order condition to this problem yields the social optimum level of liquidity. That is,

$$f_s \ni \bar{R}^2 e_g^2 - \bar{R} e_g f_s (2\bar{R} - 1) + (\bar{R} - 1)^2 f_s^2 = 0 \quad (3.30)$$

The proposition below demonstrates that the planner would always intervene to stimulate the creation of liquidity.

Proposition 3.3.4 *If $\bar{R} > 1$, the market creates an inefficiently low a level of liquidity relative to the social optimum.*

Proof Solving for f_s from equation (3.31) gives the following quadratic polynomial

$$f_s = \frac{\bar{R} e_g (2\bar{R} - 1)}{2(\bar{R} - 1)^2} \pm \frac{\bar{R} e_g}{2(\bar{R} - 1)^2} \left[(2\bar{R} - 1)^2 - 4(\bar{R} - 1)^2 \right]^{(1/2)} \quad (3.31)$$

This implies that the planner can choose one of two possible values for f_s . The first term on the right hand side of equation (3.32) is larger than f^{**} . Therefore, the social optimum level of liquidity obtained with the positive root of (3.32) is greater than the market solution (i.e. $f_s > f^{**}$). But since $\bar{R}(2\bar{R} - 1)/2(\bar{R} - 1)^2 > 1$, then the solution would violate the resource constraint whereby $e_g \geq f_s$. Hence, the solution with the positive root is not feasible.

With the negative root, $f_s > f^{**}$ if

$$2\bar{R}^2 - 1 - (4\bar{R} - 3)^{(1/2)} (2\bar{R} - 1) > 0$$

After some algebraic manipulation, the inequality above reduces to

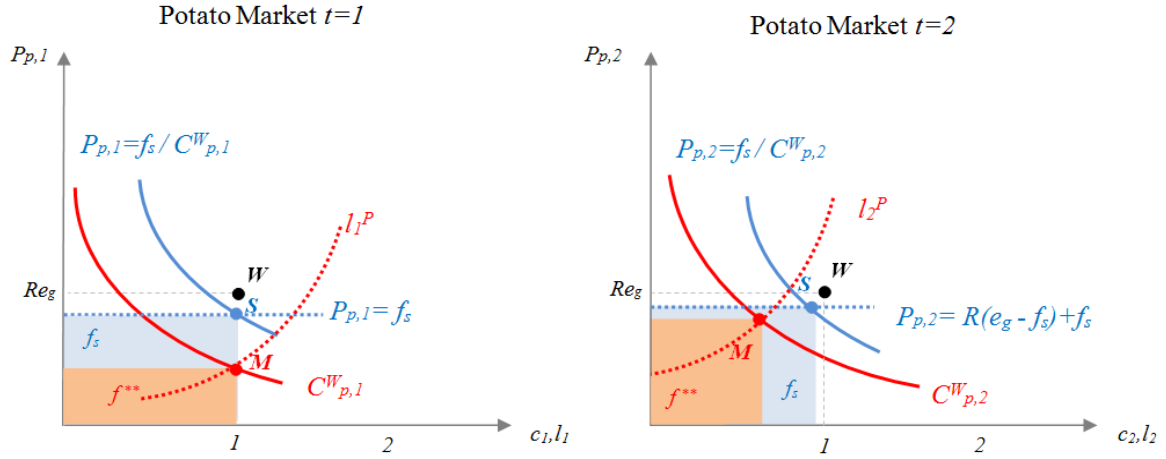
$$(\bar{R} - 1) > 0$$

Thus, the market generates too low a level of liquidity relative to the social optimum.

QED

Figure 3.7 (below) provides a graphic comparison between the competitive market equilibrium and the social optimum. The market equilibrium is illustrated as in Figure 3.6, and the social planner's choice of liquidity is depicted by the blue area and point S . The choice made by the planner is not consistent with agents' first order conditions, but households' budget constraints still hold. These are represented by the blue downward-sloping curves. The planner's constraints, which determine the relation between liquidity and prices, are depicted by the horizontal blue dotted lines.

Figure 3.7: Social Optimum: Potato Market with Consumption-Concave Preferences



With consumption-concave preferences, private sector agents have *weaker* incentives to create liquidity than the social planner. This result is the exact opposite to the case where quasi-linear leisure-concave preferences are assumed. Nonetheless, the underlying cause for the divergence between private and social incentives to create liquidity is the same. The lack of future income pledgeability implies that individuals need costly means of payments to trade, which entails a waste of resources. Because of this inefficiency, households' welfare is also affected through the price mechanism (pecuniary externality), as decentralized agents follow price-taking behavior and fail to internalize this pecuniary externality.

For the set of preferences assumed in this section, agents trade efficiently in the first period. Hence the pecuniary externality has a negligible welfare effect in $t = 1$. In the second period, however, households trade fewer goods than in the Walrasian economy. What is more, by pursuing their own benefit, private agents fail to realize that, if they had held more money, prices in the second period would have fallen to a level that would have improved their own welfare. This is explained below.

If households hold more money, the supply curves of the different goods expand by more than the demand curves. This is due to the fact that, while an additional unit of money affects the purchasing

power of buyers on a one-to-one basis, the supply curve expands by a proportion $\bar{R} > 1$. The shift in the supply curve is explained by the fact that, when agents deposit gold in the bank in exchange for money, they forgo $\bar{R}$ units of gold. Hence, in their role as sellers, households try to make-up for this wealth shortfall. To sum up, as money holdings increase, second-period prices fall and trade increases⁷.

This implies that, in the absence of public intervention, the pecuniary externality imposes larger losses on households in their capacity as consumers relative to the gains they receive as sellers. Intuitively, in the second period agents incorporate the (accrued) opportunity cost of using money into their private valuation of the good they buy. While this cost affects households' marginal rates of substitution, it is not reflected in the competitive equilibrium price vector.

Furthermore, by increasing the level of liquidity in the economy, the planner reduces the price of goods in the second period relative to the first period. This implies that trade increases in $t = 2$, which is when the effects of the direct inefficiency, in the form of missed trade opportunities, are more severe.

3.4 Aggregate Risk, Bankruptcy and the Speculative Demand for Money

In this section I introduce aggregate risk into the model by assuming that real projects can go bankrupt. This extension is necessary to explain the speculative demand for money. In addition, the possibility of default is essential to develop a comprehensive theory of financial intermediation because, in a world without bankruptcy, most kinds of financial intermediation would be unnecessary. If the risk of default was negligible, all agents would be able to borrow or lend at the same risk-free interest rate, and there would be a single capital market where borrowers and lenders would meet directly.

Accordingly, assume that the returns of the gold project are stochastic. This source of aggregate uncertainty is resolved during the interim date ($d = 1$). At that point in time, households learn whether the real project will yield a high or a low return. Let $s = \{H, L\}$, denote the set of possible states, and $s^* = \{1, H, L\}$ the set of all state-periods. H and L represent, respectively, the high and low states. If $s = L$, projects fail and investors lose everything. α is the likelihood of a high state realization. That is, real projects go bankrupt with probability $(1 - \alpha)$.

⁷Note that I focus only on the effects of this measure in $t = 2$, because the pecuniary externalities in the first period have trivial effects.

$$R = \begin{cases} R^H > 1 & \text{with probability } \alpha \\ R^L = 0 & \text{with probability } (1 - \alpha) \end{cases}$$

$\bar{R}$ still denotes the expected gross return of the project, which is assumed to be larger than 1. Hence,

$$\bar{R} = \alpha R^H > 1$$

Finally, suppose that agents cannot buy insurance to protect themselves against the risk of the project failing. This implies that households' final stock of wealth is uncertain. Hence, if agents have quasi-linear leisure-concave preferences, they are risk-neutral in their wealth. By contrast, when households are assumed to have quasi-linear consumption-concave preferences, they are risk-averse in wealth. As mentioned earlier, whether households are risk-neutral or risk-averse is essential to the debate on the speculative demand for money. If agents only care about expected returns (they are risk-neutral), then, as predicted by Keynes, only in very special circumstances would they hold a diversified portfolio. In the model developed here, money is strictly less profitable than real projects. Therefore, the speculative demand for money does not exist when preferences are leisure-concave. This is demonstrated below.

The rest of this section proceeds as follows. I first assume that households are endowed with quasi-linear leisure-concave preferences. For this case, the Walrasian optimum benchmark is established first, then the market equilibrium is solved for and, finally, the latter is compared with the social optimum. This exercise is then undertaken for the case where households are endowed with a quasi-linear consumption-concave utility function.

3.4.1 The Case of Leisure-Concave Preferences

Walrasian Optimum

Assume first a Walrasian economy. This implies that households can finance their consumption with all sources of current and future income. Therefore, households invest all their gold in the real project. Since agents are symmetric, it suffices to solve the optimization problem for only one consumer-type, P say.

The optimization problem of potato producers is given by

$$U^P = g_1^P + c_{w,1}^P - \frac{1}{2} (l_1^P)^2 + \alpha \left[g_H^P + c_{w,H}^P - \frac{1}{2} (l_H^P)^2 \right] + (1 - \alpha) \left[g_L^P + c_{w,L}^P - \frac{1}{2} (l_L^P)^2 \right]$$

s.t.

$$\begin{aligned} g_1^P + P_{w,1} c_{w,1}^P &\leq P_{p,1} l_1^P + \delta \bar{R}e_g \\ g_s^P + P_{w,s} c_{w,s}^P &\leq P_{p,s} l_s^P + (1 - \delta) \bar{R}e_g \end{aligned}$$

Where the subscript s denotes the state realized in period 2. Since P smooths consumption across time, $\delta = 1/2$.

For any period or state s^* , the solution to P 's labor supply is given by

$$l_{s^*}^P = \begin{cases} P_{p,s^*} & \text{if } P_{w,s^*} \geq 1 \\ \frac{P_{p,s^*}}{P_{w,s^*}} & \text{if } P_{w,s^*} < 1 \end{cases} \quad \forall \quad s^* = \{1, H, L\}$$

On the other hand, P 's whiskey demand schedule is described by

$$c_{w,s^*}^P = \begin{cases} 0 & \text{if } P_{w,s^*} > 1 \\ \left(\frac{P_{p,s^*}}{P_{w,s^*}} \right)^2 + \frac{\bar{R}e_g}{2P_{w,t}} & \text{if } P_{w,s^*} < 1 \quad \forall \quad s^* = \{1, H, L\} \\ c_{w,s^*}^P \ni 0 < c_{w,s^*}^P < \left(\frac{P_{p,s^*}}{P_{w,s^*}} \right)^2 + \frac{\bar{R}e_g}{2P_{w,s^*}} & \text{if } P_{w,s^*} = 1 \end{cases}$$

This is the same solution as in the aggregate certainty case; thus, the Walrasian equilibrium is given by equation (3.5). That is,

$$l_t^P = l_t^W = P_{p,t} = P_{w,t} = 1 \quad \forall \quad t$$

Competitive Market Equilibrium

Households are symmetric. Hence, the behavior of any consumer-type, say P , is representative of the household sector. When gold consumption is uncertain, the optimization problem of potato producers in the market economy is given by

$$\max_{f_{s*}^P, c_{w,s*}^P, l_{s*}^P} U^P = c_{w,1}^P - \frac{1}{2} (l_1^P)^2 + \alpha \left[g_H^P + c_{w,H}^P - \frac{1}{2} (l_H^P)^2 \right] + (1 - \alpha) \left[g_L^P + c_{w,L}^P - \frac{1}{2} (l_L^P)^2 \right]$$

s.t.

$$\begin{aligned} P_{w,1} c_{w,1}^P &= f_1^P & (\lambda_1^P) \\ P_{w,H} c_{w,H}^P &= f_H^P & (\lambda_H^P) \\ P_{w,L} c_{w,L}^P &= f_L^P & (\lambda_L^P) \\ P_{p,1} l_1^P &\geq f_H^P & (\mu_H^P) \\ P_{p,1} l_1^P &\geq f_L^P & (\mu_L^P) \end{aligned}$$

where

$$\begin{aligned} g_H^P &= R^H (e_g^P - f_1^P) + (P_{p,1} l_1^P - f_H^P) + P_{p,H} l_H^P \\ g_L^P &= (P_{p,1} l_1^P - f_L^P) + P_{p,L} l_L^P \end{aligned}$$

In equilibrium the CS conditions bind in both states. Since accessing means of payment is costly, households are better-off by using the cash they have at hand, than by holding on to it. If a good state is realized, agents anticipate to earn as much money as they spend in $t = 2$, because they are symmetric. Hence, it is optimal for them not to hoard liquidity. In the bad state, money turns out to be costless. Thus, agents can only benefit from spending it.

This implies that P 's money holdings are constant across time and states. By symmetry, this equilibrium condition also holds for household W . Therefore, it must be the case that

$$f_{s*}^P = f_{s*}^W = f \quad \forall \quad s^* = \{1, H, L\}$$

Because households are symmetric in all aspects but the good they want and the good they sell, the equation above implies that trade and prices are the same in the high and low states of nature. In other words, households' choices in the second-period are not state-contingent. Therefore, the market equilibrium deployed in Table 3.1, which corresponds to the aggregate certainty case, obtains.

Introducing bankruptcy into the model is irrelevant if agents are endowed with quasi-linear leisure-concave preferences, because they are risk-neutral in wealth. This implies that households are not concerned with the risks involved in the profitable gold venture. They only care about the expected returns on the project, and these are the same as in the certainty case.

Social Optimum

The social planner chooses a level of liquidity f_s , which maximizes the utility of the representative household. Take the P -type. The centralized problem must take into account the budget constraints of potato growers. Thus, the social objective function is given by

$$\begin{aligned} \max_{f_s} U^P &= \bar{R}(e_g^P - f_s) + \frac{f_s}{P_{w,1}} + P_{p,1}l_1^P - \frac{1}{2}(l_1^P)^2 \\ &+ \alpha \left[\frac{f_s^P}{P_{w,H}} + P_{p,H}l_H^P - \frac{1}{2}(l_H^P)^2 - f_s \right] \\ &+ (1 - \alpha) \left[\frac{f_s^P}{P_{w,L}} + P_{p,L}l_L^P - \frac{1}{2}(l_L^P)^2 - f_s \right] \end{aligned}$$

Moreover, the planner internalizes the effects of liquidity on prices. Therefore, he is constrained to take as given the relation between liquidity and prices, as specified below.

$$\begin{aligned} P_{w,1} &= P_{p,1} = f_s^{3/4} \\ P_{w,H} &= P_{p,H} = f_s^{1/2} \\ P_{w,L} &= P_{p,L} = f_s^{1/2} \end{aligned}$$

Imposing the planner's constraints simplifies the problem as follows.

$$\max_{f_s} U^P = \left(f_s^{1/4} - \frac{1}{2}f_s^{1/2} \right) + \left(f_s^{1/2} - \frac{1}{2}f_s \right) + [\bar{R}e_g + (1 - \bar{R})f_s]$$

This is the same problem that the planner faces in absence of aggregate uncertainty. Hence, since the market equilibrium also coincides with the outcome of the certainty case, the welfare analysis and results of the non-bankruptcy setting apply here. That is, the market produces inefficiently high levels of liquidity and trade relative to the social optimum.

3.4.2 The Case of Consumption-Concave Preferences

Walrasian Optimum

If households are endowed with quasi-linear consumption-concave preferences, they are risk-averse with respect to gold wealth. In this case, the first-best benchmark may not be characterized by an outcome where households invest all their gold in the project. This is due to the fact, gold storage is a vehicle whereby agents can diversify away aggregate risk.

When all sources of income are pledgeable, the optimization problem of the representative household, the P -type, is given by

$$\begin{aligned} \max_{g_{s*}^P, c_{w,s*}^P, l_{s*}^P, \phi} U^P &= \ln(c_{w,1}^P) - l_1^P + \alpha [\ln(g_1^P + g_H^P) + \ln(c_{w,H}^P) - l_H^P] \\ &+ (1 - \alpha) [\ln(g_1^P + g_L^P) + \ln(c_{w,L}^P) - l_L^P] \end{aligned}$$

s.t.

$$\begin{aligned} g_1^P + P_{w,1}c_{w,1}^P &= P_{p,1}l_1^P + \delta [(1 - \phi) \bar{R} + \phi] e_g & (\lambda_1^P) \\ g_H^P + P_{w,H}c_{w,H}^P &= P_{p,H}l_H^P + [(1 - \phi) R^H + \phi] e_g - g_1^P & (\lambda_H^P) \\ g_L^P + P_{w,L}c_{w,L}^P &= P_{p,L}l_L^P + \phi e_g - g_1^P & (\lambda_L^P) \end{aligned}$$

where

$$\begin{aligned}
g_1^P &= \delta [(1 - \phi) \bar{R} + \phi] e_g \\
g_H^P &= [(1 - \phi) R^H + \phi] e_g - g_1^P \\
g_L^P &= \phi e_g - g_1^P
\end{aligned}$$

δ is the proportion of expected terminal gold wealth households consume in the first period, and ϕ denotes the fraction of gold agents choose to keep in storage. Since households are consumption smoothers, then $\delta = 1/2$. The other first order conditions to this problem are given by

$$c_{w,s*}^P = \frac{P_{p,s*}}{P_{w,s*}} \quad \forall \quad s^* = \{1, H, L\} \quad (3.32)$$

$$\left[\frac{\alpha}{g_1^P + g_H^P} + \frac{1 - \alpha}{g_1^P + g_L^P} \right]^{-1} = P_{p,1} \quad (3.33)$$

$$g_1^P + g_s^P = P_{p,s} \quad \forall \quad s = \{H, L\} \quad (3.34)$$

$$\frac{g_1^P + g_H^P}{g_1^P + g_L^P} = \frac{\alpha (R^H - 1)}{1 - \alpha} \quad (3.35)$$

By combining equations (3.34) through (3.36), the equilibrium level of ϕ can be obtained.

$$\phi = \frac{(1 - \alpha) \bar{R}}{\bar{R} - \alpha} \quad (3.36)$$

ϕ is decreasing in α . Thus, if the real project involves a higher risk (lower α), the Walrasian optimum is characterized by an investment portfolio where more gold is invested in unproductive-safe storage.

On the other hand, P 's labor supply schedules are obtained by combining the first order conditions above and the budget constraints.

$$l_1^P = \bar{R} + 1 - \frac{\bar{R}e_g}{P_{p,1}} \quad (3.37)$$

$$l_H^P = 2 - \frac{\bar{R}e_g}{P_{p,H}} \quad (3.38)$$

$$l_L^P = 2 - \frac{(1-\alpha)\bar{R}e_g}{(\bar{R}-\alpha)P_{p,L}} \quad (3.39)$$

By symmetry, the mirror of equations (3.33)-(3.40) describe the behavior of household W . Moreover, in equilibrium the whiskey and potato markets clear. Hence,

$$l_{s*}^P = c_{p,s*}^W \quad \forall s^*$$

$$l_{s*}^W = c_{w,s*}^P \quad \forall s^*$$

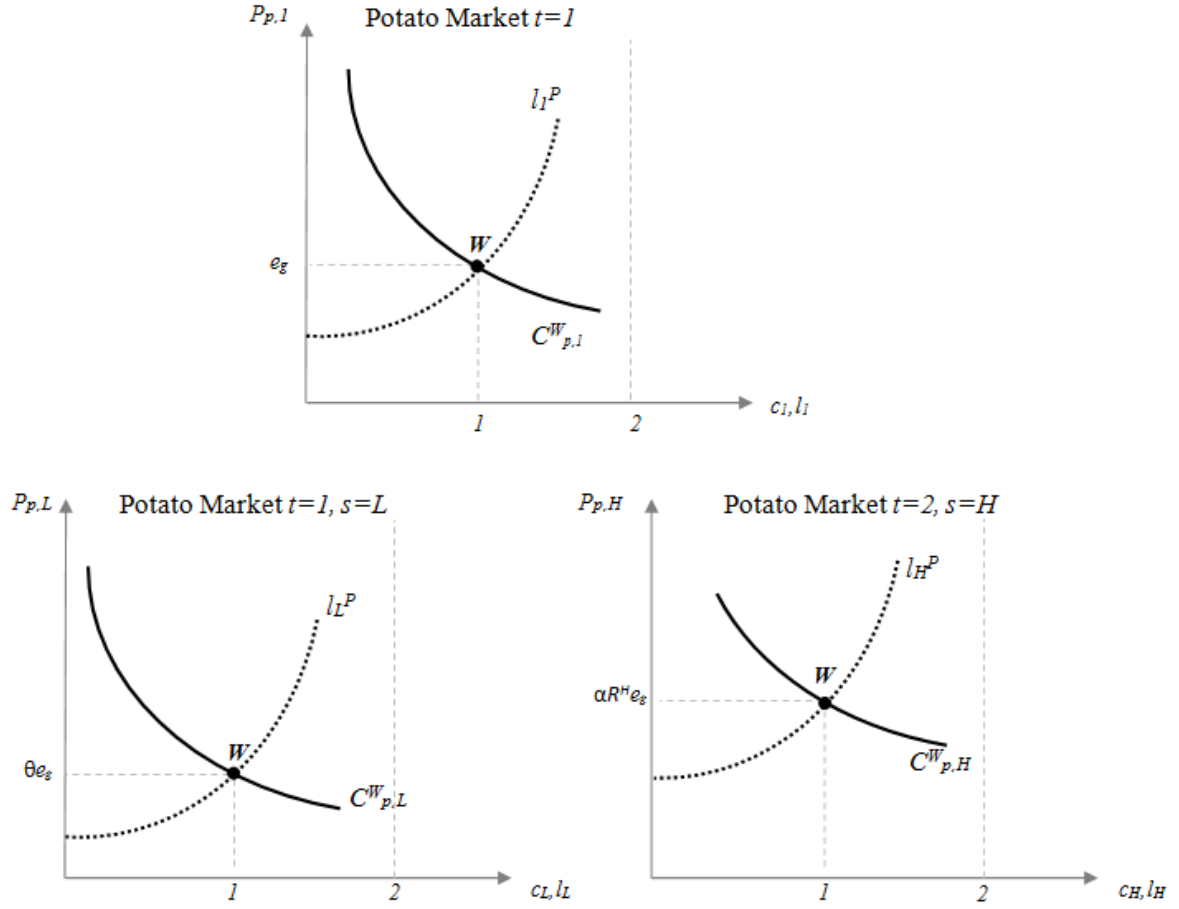
Since, the market outcome in the whiskey and potatoes market is symmetric, equation (3.33) implies that, in equilibrium, one unit of each good is traded in every state-period. The complete Walrasian equilibrium is summarized in Table 3.4 (below). Further, Figure 3.8 provides a graphical illustration of the Walrasian optimum for a representative market: the potato market. In each graph, point W represents the equilibrium. The first period outcome is described by the top graph, the second period-low state by the bottom-left graph, and the second period-high state by the bottom-right graph. The demand curves are represented by straight lines and correspond to W 's first order conditions with respect to potato consumption (the mirror of P 's first order condition 3.33). The supply curves, equations (3.38) through (3.40), are depicted by dashed lines.

Prices are higher in the good state than in the bad state, but trade is always equal to 1. The reason is that, while households' propensity to work is decreasing in gold wealth, in the Walrasian economy their purchasing capacity rises with the final stock of gold wealth. To illustrate, assume a bad state is realized. This reduces households' purchasing power, which leads to a contraction of the demand curve. Due to the poor performance of the project, agents also choose to work more in order to compensate for their wealth shortfall. This expands the supply curve to ensure that trade is kept at one unit in each market, but prices end up being lower than if a good state had realized.

Table 3.4: Walrasian Optimum with Consumption-Concave Preferences

Gold Investments	
Storage	Real Projects
$\phi = \frac{(1-\alpha)\bar{R}}{\bar{R}-\alpha}$	$1 - \phi = \frac{\alpha(\bar{R}-1)}{\bar{R}-\alpha}$
First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = 1$	$P_{p,1} = P_{w,1} = e_g$
Second Period-High State	
Trade/Labor	Prices
$l_H^P = l_H^W = 1$	$P_{p,H} = P_{w,H} = \bar{R}e_g$
Second Period-Low State	
Trade/Labor	Prices
$l_L^P = l_L^W = 1$	$P_{p,L} = P_{w,L} = \frac{(1-\alpha)\bar{R}e_g}{\bar{R}-\alpha}$

Figure 3.8: Walrasian Equilibrium with Consumption-Concave Preferences and Bankruptcy



Market Economy

In the market economy, future income is not pledgeable. Consequently, agents need costly means of payment, and the latter is provided by banks. Further, markets are incomplete. That is, households cannot buy insurance against idiosyncratic or aggregate risks.

In particular, since agents cannot insure against the risk of being liquidity-constrained, the two household-types choose to hold money on date 0. What is more, by symmetry, the liquidity demand schedules of potato and whiskey producers are the same. Therefore, to understand the behavior of the household sector, it suffices to solve the optimization problem for only one household-type, P say.

The optimization problem of the potato producer is given by

$$\max_{g_{s*}^P, c_{w,s*}^P, l_{s*}^P} U^P = \ln(c_{w,1}^P) - l_1^P + \alpha [\ln(g_H^P) + \ln(c_{w,H}^P) - l_H^P] + (1 - \alpha) [\ln(g_L^P) + \ln(c_{w,L}^P) - l_L^P]$$

$s.t$

$$P_{w,1}c_{w,1}^P = f_1^P \quad (\lambda_1^P)$$

$$P_{w,H}c_{w,H}^P = f_H^P \quad (\lambda_H^P)$$

$$P_{w,L}c_{w,L}^P = f_L^P \quad (\lambda_L^P)$$

$$P_{p,1}l_1^P \geq f_H^P \quad (\mu_H^P)$$

$$P_{p,1}l_1^P \geq f_L^P \quad (\mu_L^P)$$

where

$$\begin{aligned} g_H^P &= R^H (e_g - f_1^P) + (P_{p,1}l_1^P - f_H^P) + P_{p,H}l_H^P \\ g_L^P &= (P_{p,1}l_1^P - f_L^P) + P_{p,L}l_L^P \end{aligned}$$

In equilibrium the CS conditions bind in both states. In expectation, liquidity is costly. Thus, households are better-off by using the cash they have at hand, than by holding on to it. If a good state is realized, agents anticipate that they will earn as much money as they spend in $t = 2$. Hence, it is optimal for them not to hoard liquidity. In the bad state, money turns out to be costless. Thus, agents can only benefit from spending it. To sum up $\mu_s^P > 0 \quad \forall s$.

Since households are symmetric and the CS conditions bind, then it must be the case that

$$f_{s*}^P = f_{s*}^W = f \quad \forall \quad s^* \tag{3.40}$$

In words, households' demand for liquidity is constant across time and states. Using this equilibrium condition, the rest of the first order conditions can be expressed as follows

$$\bar{R}f = g_H^P \quad (3.41)$$

$$c_{w,1}^P = \frac{P_{p,1}}{P_{w,1}} \quad (3.42)$$

$$c_{w,H}^P = \frac{P_{p,H}}{R^H P_{w,H}} \quad (3.43)$$

$$c_{w,L}^P = \frac{P_{p,L}}{P_{w,L}} \quad (3.44)$$

$$f = P_{p,1} \quad (3.45)$$

$$g_s^P = P_{p,s} \quad \forall \quad s \quad (3.46)$$

Combining these equilibrium conditions with the budget constraints gives the labor supply schedule for every state-period.

$$l_1^P = \alpha R^H \left(1 + \frac{1}{\alpha}\right) - \frac{R^H e_g}{P_{p,1}} \quad (3.47)$$

$$l_H^P = \left(1 + \frac{1}{\alpha}\right) - \frac{R^H e_g}{P_{p,H}} \quad (3.48)$$

$$l_1^P = 1 \quad (3.49)$$

The mirror of equations (3.42) - (3.50) hold for household W . Let f^T denote the per-capita equilibrium level of liquidity. The latter can be solved for from equation (3.42).

$$\begin{aligned} \bar{R}f^T &= g_H^P \\ \implies f^T &= \frac{\bar{R}e_g}{\bar{R}(1 + \alpha) - \alpha} \end{aligned} \quad (3.50)$$

Note that, as the risk of an economic downturn increases (lower α), households deposit more of their gold endowment in the banking system. This implies that agents hold money for three reasons: to pay for goods (transactions motive); to hedge against the possibility of encountering a buying opportunity and not being able to make the purchase (precautionary motive); and to diversify away the risk of losing their wealth in the project (speculative motive). Does the market invest the right proportion of resources in the safe asset? The answer is no. The proposition below elaborates.

Proposition 3.4.1 *If $\bar{R} > 1$, the market invests too many resources in the risky project relative to the Walrasian optimum.*

Proof Relative to the Walrasian equilibrium, the market allocates more resources to the risky project if $\phi e_g > f^T$. Plugging-in the equilibrium values for ϕ and f^T in the latter inequality yields

$$\frac{1 - \alpha}{\bar{R} - \alpha} < \frac{1}{\bar{R}(1 + \alpha) - \alpha}$$

$$\implies \bar{R} > 1$$

which is true by assumption.

QED

Table 3.5 (below) displays the complete market equilibrium. Prices in the first period and in the bad state are equal to f^T and are, therefore, decreasing in α . When the risk of bankruptcy in the real sector is high (lower α), households deposit more gold in the banking system. Since more money chases goods, prices increase.

By the same rationale, the demand for goods in the good states of nature is also inversely proportional to α . Notwithstanding, $P_{p,H}$ and $P_{w,H}$ are increasing in the probability of a good state realization. The intuition is that, lower levels of α induce agents to hold more money. Hence, if $s = H$, households work more to compensate for the wealth shortfall brought about by their increased demand for liquidity. For every additional unit of gold deposited in the bank, the supply of goods expands by $R^H > 1$. By contrast, the demand expands one for one with f^T . In words, the supply-side effect dominates, thereby explaining the positive relation between α and prices in the good state.

If a bad state is realized, the labor supply is perfectly inelastic and equal to 1. This implies that trade is efficient. Intuitively, the cost of money is effectively zero when $s = L$; therefore, agents try to exploit all gains to trade. By contrast, trade is inefficient in the good states of nature. When making a purchase in the second period, agents must include the cost of using money into the price of the good they buy. When $s = H$, the cost of accessing means of payment is $R^H > 1$, which deters trade. This is summarized in the proposition below.

Proposition 3.4.2 *If $R^H > 1$, the market generates less trade than the Walrasian optimum only in the good states of nature.*

Table 3.5: Market Equilibrium with Bankruptcy Risk

Liquidity Holdings	
Per Capita	Aggregate
$f^T = \frac{\bar{R}e_g}{R(1+\alpha)-\alpha}$	$\hat{f}^T = 2f^T = \frac{2\bar{R}e_g}{R(1+\alpha)-\alpha}$
First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = 1$	$P_{p,1} = P_{w,1} = f^T$
Second Period, High State	
Trade/Labor	Prices
$l_H^P = l_H^W = \frac{1}{R^H}$	$P_{p,H} = P_{w,H} = \bar{R}(e_g - f^T) + f^T$
Second Period, Low State	
Trade/Labor	Prices
$l_L^P = l_L^W = 1$	$P_{p,L} = P_{w,L} = f^T$

Proof In equilibrium, the prices of whiskey and potatoes are equal in every state-period. Thus, first order conditions (3.43) through (3.45) imply that, while one unit of each good is traded in the first period and the bad states of nature, $1/R^H$ units of potatoes and whiskey are traded if a good state is realized in $t = 2$. Recall that, in the Walrasian equilibrium, one unit of each good is traded in every state-period. It then follows that, if $R^H > 1$, the market generates less trade than the Walrasian optimum only in the good states of nature.

QED

Propositions 3.4.1 and 3.4.2 highlight two types of inefficiencies relative to the first-best benchmark. On the one hand, since future income is not pledgeable, agents must invest resources in unproductive storage. This generates an inefficiency in the form of missed trade opportunities, when money is in effect

costly ($s = H$). Second, unlike the Walrasian economy, agents' choice of gold deposits trades-off the profitability of the real project with the liquidity and safety of money holdings. Thus, in the presence of uninsurable aggregate risks, the market is unable to efficiently allocate resources across the different asset classes.

Figure 3.9: Market Equilibrium with Consumption-Concave Preferences and Bankruptcy

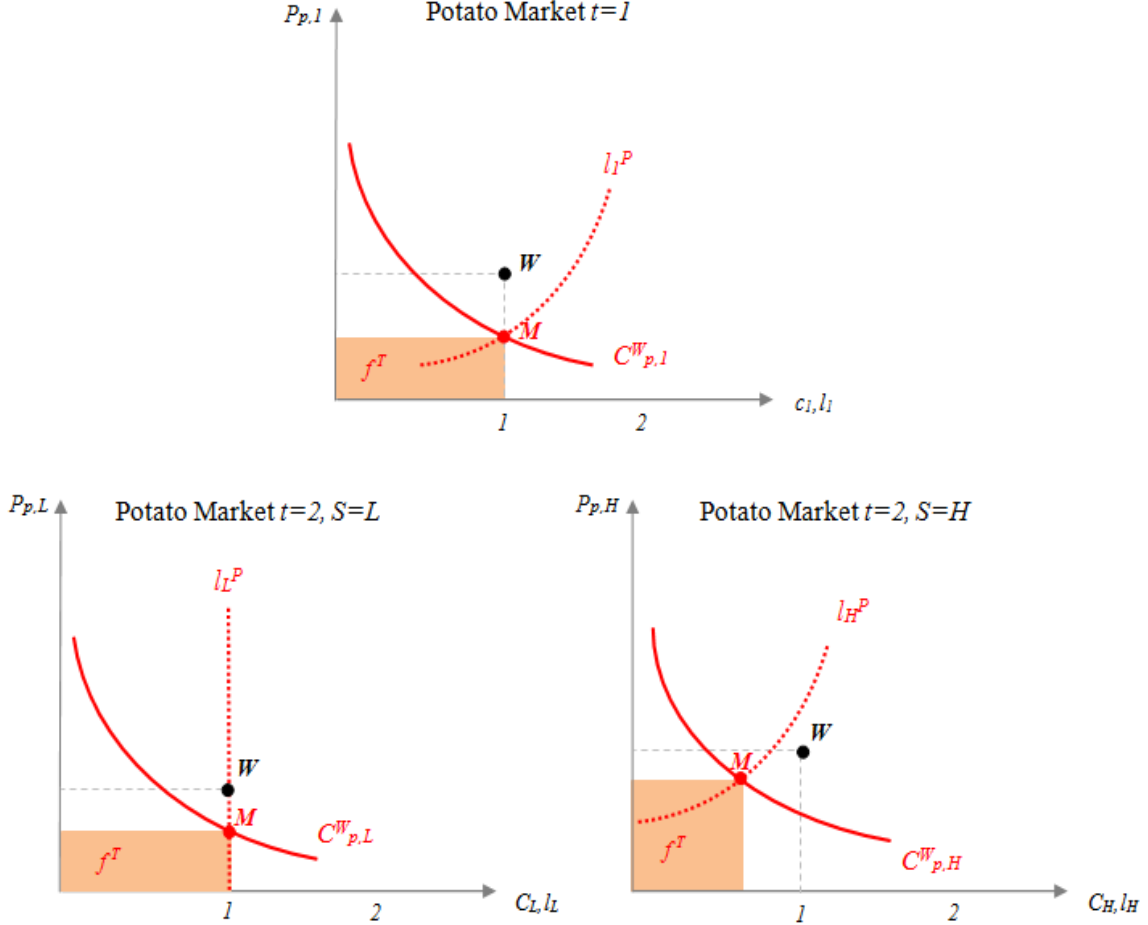


Figure 3.9 (above) provides the graphic illustration of the market equilibrium for a representative goods market and compares it with the Walrasian optimum. In each graph the market equilibrium is denoted with point M , and the Walrasian optimum with point W . The first period outcome is described by the top graph, the second period-low state by the bottom-left graph, and the second period-high state by the bottom-right graph. The demand curves are represented by straight lines. These are derived from the mirror of first order conditions (3.43) through (3.45) for household W . The supply curves, on the other hand, correspond to equations (3.48) through (3.50). These are represented by the dashed lines. Finally, the shaded area represents to the monetary value of each transaction.

Social Optimum

Assume the central planner has a regulatory tool that allows him to determine the per-capita level of liquidity (f_s). He maximizes the utility of the representative household, P , and is constrained to take into account the non-pledgeability of future income, the fact that insurance markets are incomplete, and the effect of liquidity on prices (which decentralized agents ignore because they are price-takers). His optimization problem is, therefore, given by

$$\max_{f_s} U^P = \ln(c_{w,1}^P) - l_1^P + \alpha [\ln(g_H^P) + \ln(c_{w,H}^P) - l_H^P] + (1 - \alpha) [\ln(g_L^P) + \ln(c_{w,L}^P) - l_L^P]$$

s.t.

$$\begin{aligned} P_{p,1} &= P_{w,1} = f_s \\ P_{p,H} &= P_{w,H} = R^H (e_g - f_s) + f_s \\ P_{p,L} &= P_{w,L} = f_s \\ e_g &\geq f_s \geq 0 \end{aligned}$$

The planner also takes into consideration households' budget sets. Thus, after imposing the market clearing conditions and the planner's constraints, the problem is simplified as shown below.

$$\max_{f_s} U^P = \ln f_s - \frac{\alpha R^H}{R^H (e_g - f_s) + f_s} \quad (3.51)$$

The first order condition of this problem is given by

$$f_s \ni (R^H e_g)^2 - R^H e_g f_s [2(R^H - 1) + \alpha] + [(R^H - 1) f_s]^2 = 0 \quad (3.52)$$

where f_s is the social optimum level of liquidity per-capita.

Proposition 3.4.3 *If $R^H > \alpha R^H > 1$, the market produces too low a level of liquidity relative to the social optimum.*

Proof Equation (3.53) implies that the solution to f_s is given by the quadratic polynomial below.

$$f_s = \frac{R^H e_g}{R^H - 1} + \frac{\alpha R^H e_g}{2(R^H - 1)^2} \pm \frac{R^H e_g}{2(R^H - 1)^2} [4\alpha(R^H - 1) + \alpha^2]^{(1/2)} \quad (3.53)$$

Consequently, there are two possible solutions. Since $R^H > 1$, the first term on the right hand side of this equation is larger than the market solution f^T . The latter is given by equation (3.51). This implies that the level of f_s that solves the polynomial with the positive root is larger than f^T . But since this solution would violate the resource constraint whereby $e_g \geq f_s$, it is not feasible.

In the case of the negative root, the planner chooses a level of liquidity higher than the market if

$$\frac{2R^H - (2 - \alpha) - \alpha^{1/2} [4(R^H - 1) + \alpha^2]^{1/2}}{2(R^H - 1)^2} > \frac{1}{R^H(1 + \alpha) - 1}$$

After some algebraic manipulation the inequality above reduces to

$$\alpha(4 + \alpha)(R^H)^2 + 2R^H(1 - \alpha^2) + \alpha > 0$$

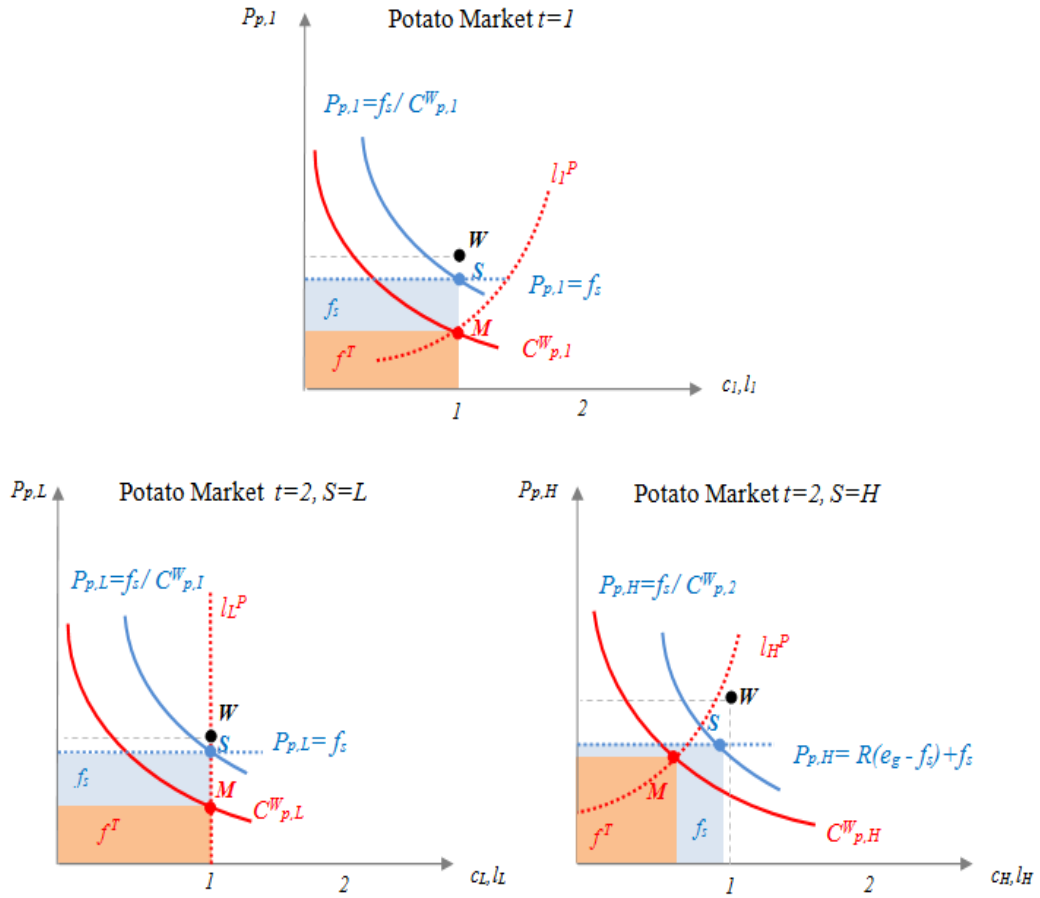
This always holds if $0 \leq \alpha \leq 1$ and $R^H > 1$, which is true since $R^H > \alpha R^H > 1$. Therefore, $f_s > f^T$.

QED

When the possibility of bankruptcy is introduced and agents are risk-averse in their wealth, private and social incentives to hold liquidity become stronger. The reason is that, in addition to providing liquidity services and insuring against the risk of being liquidity-constrained, money also diversifies the risk of wealth write-downs during an economic downturn. Nonetheless, the market produces an inefficiently low level of liquidity relative to what is socially desirable. Since future income is not pledgeable and insurance markets are incomplete, the price mechanism becomes a driver of these frictions. Moreover, private agents do not internalize these pecuniary externalities, because they follow price-taking behavior. Due to their risk-aversion, agents value consumption greatly. This implies that, the inefficiencies generated by the market frictions and its ensuing pecuniary externalities do not offset each other. In other words, households do not realize that, had they chosen to hold more money, the prices of commodities in the good states of nature would have fallen to a level that would have improved their own welfare (recall that trade is inefficient only in $s = H$).

Figure 3.10 (below) provides a graphic comparison between the competitive market equilibrium and the social optimum. The market equilibrium is illustrated as in Figure 3.9, and the social planner's choice of liquidity is depicted by the blue area and point S . The choice made by the planner is not consistent with agents' first order conditions, but households' budget constraints still hold. These are represented by the blue downward-sloping curves. The planner's constraints, which determine the relation between liquidity and prices, are depicted by the horizontal blue dotted lines.

Figure 3.10: Social Optimum with Consumption-Concave Preferences and Bankruptcy



3.5 Concluding Remarks

This chapter showed that, in an economy where agents are subject to idiosyncratic and aggregate risks, the non-pledgeability of future income and markets' incompleteness motivates households to hold money for reasons other than to trade. Individuals also demand liquidity for precautionary motives, as money insures against the risk of being liquidity-constrained. On the other hand, since the expected return on money is lower than on the alternative investment, risk-neutral agents do not hold money for diversification purposes, but risk-averse households do.

Moreover, when the speculative demand for money is non-negligible an additional distortion emerges relative to the first-best benchmark. Since, in contrast to the frictionless economy, agents' decision to hold money trades-off profitability and liquidity, the market is unable to allocate risks efficiently.

I also showed that, private and social incentives to create liquidity diverge, because price-taking agents do not internalize the pecuniary externality generated by the costly creation of liquidity. I demonstrated that the direction of this inefficiency is contingent on households' preferences. That is, depending on the assumption one makes about households' utility function, the market may produce inefficiently high or low levels of liquidity.

The model developed in this chapter is subject to the same criticism as Tobin's theory of money demand. Tobin's critics argue that, by assuming that money is a safe asset with zero net return, its role as a diversifying asset is not well defined: if there are risk-less assets that earn interest, such as debit cards linked to savings accounts, money should be crowded-out by these ultra-liquid securities. In reality, banks do pay interest on deposits. However, they also have strong incentives to take individuals' money and not repaying it.

Thus, an important extension to the model developed in this chapter is the introduction of an active financial sector. In reality, banks make strategic decisions about their capital structure and loan portfolio. What is more, some of these decisions are neither contractible nor efficient, which can generate moral hazard problems. In the presence of such an agency problem, banks may offer to pay a positive return on gold deposits in order to attract more resources. While positive deposit rates would reduce the opportunity cost of money holdings, banks may engage in gambling activities and default on depositors. Hence, the liquidity provided by the banking sector may no longer be regarded as a safe asset. In such a setup, one would be able to assess the importance of money as a diversifying asset. This is the topic of the next chapter.

Chapter 4

Liquidity and Bank Risk-Shifting

4.1 Introduction

The purpose of this chapter is to explain the conditions that can lead to a conflict between bankers' incentives and depositors' interests. To this end, I construct a model that introduces moral hazard into the framework developed in Chapter 3. Extending the model in this way alters the tension between private and social incentives to create liquidity. In a world where financial institutions can use depositors' wealth to make profits, bankers may be able to promise higher deposit rates. This would reduce the opportunity cost of holding money and mitigate the inefficiencies therein. The only problem with this scenario is that bankers may also undertake inefficient investments and default on depositors. Like costly liquidity, a deposit default event is detrimental to social welfare. Therefore, this chapter examines what friction should a social planner be mostly concerned with. Accordingly, possible regulatory measures are formulated and discussed.

To construct a model of moral hazard in the banking industry, I deviate from the previous two chapters by relaxing the assumption that financial intermediaries are passive and safe. But this raises an essential question: where does the authority to make decisions about banks' assets rest? In an ideal world, control rights would be irrelevant, since every issue could be specified in an enforceable contract that would cover all future contingencies (nothing would be left to decide outside of the contract). In the presence of market frictions, however, comprehensive contracts are not feasible, and the allocation of control rights matters: only those with authority can make decisions about what is not specified in the contract. In this chapter, bank behavior is modeled using the concept of "residual control rights over nonhuman assets" proposed by Hart and Moore (1990). The latter explains the existence of outside-ownership corporate structures and allows for agency problems to arise in equilibrium.

Following Hart and Moore (1990), financial institutions are now conceived as the investment project of a group of *outside* shareholders called bankers. Bankers are risk-neutral and enjoy limited liability. They invest their own wealth as equity of the bank and are granted a charter. The banks first decide how much debt to take (capital structure choice), and then choose whether to invest in a safe or an inefficient gambling asset. Further, it is assumed that banks investment policy is non-verifiable.

In this setup a moral hazard problem arises. Limited liability implies that bankers' payoffs have a lower zero-bound, while their upside gain is unbounded. Moreover, the non-verifiability of bank investment policy means that deposit contracts cannot specify a banker's payoff contingent on his investment decision. As a consequence, when bankers engage in risk-taking behavior, they walk-out with high private returns if the gamble pays-off, while depositors bear the losses if the gamble fails.

The results show that moral hazard is more severe when financial institutions are poorly capitalized. This provides support for the invalidity of the Modigliani-Miller (1958) irrelevance theorem. In a perfect world, such that all the assumptions of the Modigliani-Miller framework hold, bank capital structure would be irrelevant. This is due to the fact that financial institutions would be able to pledge their entire income stream to depositors. Moreover, higher returns on banks' portfolios (and thus deposit rates), would *efficiently* induce households to deposit more of their wealth in the financial system. By contrast, in an economy where bankers' actions cannot be verified, gambling banks can afford to promise higher deposit rates, as these are only paid in some states of nature. Furthermore, claims on the full value of the bank cannot be issued: risk-taking incentives are constrained by the prospect of shareholder losses; thus, bankers must hold a sufficiently large stake in order to behave properly.

This implies that high levels of liquidity may not be socially desirable: as more money is created, leverage in the financial system increases, and bankers' incentives to be prudent fall. However, a gambling bank can also reduce the opportunity cost of holding money, because unlike the prudent bank, it promises higher deposit rates. Consequently, the appropriate policy response must weigh the welfare costs generated by each of these inefficiencies.

I show that, when the probability that banks gamble is non-zero, social welfare improves if the planner prioritizes addressing the moral hazard problem over reducing the opportunity cost of money. This is due to the fact that, regardless of how preferences are specified, agents are risk-averse with respect to changes in the value of money because of its transactional role. Intuitively, if banks fail, the value of money drops to zero and trade collapses altogether. If, on the contrary, banks pay the high (promised) deposit rate, the value of money increases, but households cannot derive additional benefits from it.

Thus, the central planner would never intervene to promote the creation of liquidity; if he did, he would provide banks with stronger incentives to gamble¹. To sum up, financial regulation must focus on aligning bankers' incentives with depositors' interests through policies that limit the creation of inside money.

This chapter concludes by proposing a set of regulatory measures to address the moral hazard problem. Although I do not, at this stage, put forward the possible policies (or policy combinations) that generate a Pareto-improvement, the advantages and limitations of each tool are discussed. Capital requirements (or debt limits) eliminate risk-taking behavior, but create an inefficient shortage of liquidity. Liquidity requirements only mitigate the moral hazard problem. However, this measure prevents money from losing its value when banks fail, thus restoring trade in the bad states of nature. Market structure regulations, such as entry barriers, are also explored. The motivation is that, pecuniary externalities in the deposit market have important welfare consequences because of the agency problem. In a competitive industry, bankers have extremely strong incentives to compete for deposits and promise higher rates. This is due to the fact that competition erodes profits and makes the gamble more desirable. Thus, policies that limit competition in the financial industry alleviate the moral hazard problem. Finally, the possibility of regulating banks' ownership structure is considered. The control-based notion, upon which this framework is constructed, can also explain the existence of cooperatives. When a bank is mutually owned, its assets are controlled by its members (owner-depositors), who make decisions democratically. A monopolist mutual bank solves the moral hazard problem. What is more, it internalizes the pecuniary externalities generated by the costly creation of liquidity, because its structure is tantamount to a central planner.

The rest of the chapter proceeds as follows. Section 4.2 summarizes the relevant literature for this chapter. Section 4.3 develops the baseline model. In addition, the competitive equilibrium is computed, and the conditions leading to bank risk-shifting are determined. Subsequently, in Section 4.4 the objectives of financial regulation are determined by analyzing the effects of moral hazard and costly money creation on households' welfare. Alternative instruments of prudential regulation are also discussed. Finally, section 4.5 concludes.

It should be noted that, for ease of exposition, I develop the model under the assumption that households are endowed with quasi-linear leisure-concave preferences. However, in the Appendix of the chapter the model is solved for the case of quasi-linear consumption-concave preferences. The results are qual-

¹Note that this result contrasts with the conclusions of the previous two chapters where, contingent on households' preferences, the planner could either stimulate or restrict the creation of liquidity.

itatively the same. Both specifications predict bank risk-shifting phenomena, and that moral hazard imposes larger welfare losses on households than the costly creation of liquidity. Thus, under both specifications, the findings suggest that public intervention should primarily focus on addressing the agency problem.

4.2 Related Literature

In addition to the references cited in chapters 2 and 3, this chapter is related to the literature of agency problems applied to banking. As mentioned earlier, the model uses the notion of residual control rights over nonhuman assets developed by Hart and Moore (1990)². This concept allows for the definitions of two ownership structures: outside ownership and cooperatives. The former implies that banks' assets are controlled by outside owners who maximize profits. By contrast, under the cooperative structure, economic agents form a mutually owned bank and make democratic decisions on a one-member, one-vote basis.

I model banks in a way that is closer to the standard static model of moral hazard. In particular, the results imply that bank risk-taking is coupled with high leverage (bank risk-shifting). The seminal paper of this line of work is Jensen and Meckling (1976). These authors were the first to introduce risk-shifting as an agency conflict between equity and debt holders of a levered firm.

This chapter also relates to the empirical literature on banking crises. Even several years prior to the 2007-2009 financial meltdown, microeconomic evidence put forward by Cole et. al. (1992) and Boyd and Gertler (1993), concluded that problems in the banking sector were mainly due to risk-taking behavior. Similarly, Kane (1989) and Cole et al. (1995) document the problem of “gambling on resurrection”. The latter refers to cases where, banks choose a risky portfolio that pays out high profits if the gamble succeeds, but leaves depositors (or their insurers) with the losses if the gamble fails. Caprio, et. al. (1997) and Fischer et.al. (1997) also find that moral hazard plays an important role in bank failures.

Some observers have suggested that deposit insurance is the cause of the moral hazard problem in the financial industry. However, whether a formal system of deposit insurance is in place is of limited relevance. This is due to the fact that, in the event of a financial crisis, there will almost always be a bailout. Moreover, the fact that there have been financial crises in countries with and without formal deposit-insurance schemes, suggests that eliminating this system does not solve the problem. The model developed in this chapter is consistent with the latter view. Like Furlong and Keeley (1989), Innes(1990),

²Similar work includes Grossman and Hart (1986) and Hart (1995).

Matutes and Vives (1996) and Hellman et. al. (2000), this chapter argues that moral hazard arises because bankers' payoffs are protected by limited liability.

In the model developed here, the non-verifiability of bank investment policy is also important to explain moral hazard. Seminal papers on this line of work include Diamond (1984) and Hart and Moore (1994). In the first piece, the revenues of borrowers cannot be observed and the cost of auditing is assumed to be infinite. Hart and Moore (1994), on the other hand, suggest that human capital is inalienable. Thus, revenues may be hidden or diverted, which implies that borrowers' may even default strategically.

Finally, this chapter contributes to the ongoing discussion on financial regulation. My analysis of capital requirements is related to Rochet (1992), who shows that this kind of measures reduce banks' incentives to gamble. In a more comprehensive model, Dewatripont and Tirole (1994) argue that capital requirements fail to recognize all the relevant sources of risk. This has been a major issue after the 2007-2012 crises. Recent contributions that have emerged in response to these events include Tresselt and Verdier (2010), Korinek (2011), and Osorio (2011). These are general equilibrium models where, in addition to capital adequacy, alternative measures to mitigate the adverse feedback effects of bank failure are discussed. This chapter shows that liquidity requirements, which are now being considered as part of the Basel III accord, deliver such an outcome.

On a similar vein, Hellman et. al. (2000) claim that the greater emphasis on capital requirements under the first Basel Accord, is not optimal. The authors show that capital requirements alone do not reach a constrained Pareto efficient outcome and propose the use of barriers to entry as a complimentary tool. Similar contributions include Bhattacharya (1982) and Smith (1984), who suggest that deposit rate controls can reduce financial instability, and Caprio and Summers (1996), who emphasize the importance of charter value for prudential regulation.

The term "charter value" refers to the fact that bankers need a charter to run a bank. When a bank defaults, its charter is either revoked or transferred to a new holding. Therefore, the threat of losing the charter may act as a disciplinary device against risk-taking. Seminal papers in the "charter value" literature include Suarez (1994), Matutes and Vives (1996), and Hellman et.al. (2000). These authors develop dynamic moral hazard models, where the costs of bankruptcy (the loss of future payoffs and reputational damage) may induce banks to behave prudently. The higher the present value of future profits, the lower the incentive to adopt risky decisions. Banking charters are also mentioned in earlier work by Marcus (1984) and Keeley (1990), who develop simpler one-period models. In these papers, the charter is defined as shareholders' claim, which is contingent on bank solvency. As in the dynamic

models, the authors show that the value of the charter is increasing in market power: profits are eroded by market competition, thus raising the relative value of risky investment strategies. In these models, however, the charter value is taken as given. Gorton and Winton (1995) endogenize the charter value, by relating it to banks' private valuation of borrowers' information. When analyzing the possibility of introducing barriers to entry, I adapt the model to the Monti-Klein framework of imperfect competition. Like the aforementioned papers, the results suggest that barriers to entry induce banks to engage in non-price competition, which creates charter value and reduces bank risk-taking incentives. Further, the model is general equilibrium. Hence, the charter value is endogenous and corresponds to the bankers' value function.

4.3 A Model with Active Banks

Consider a finite horizon economy in which time extends over three dates $d = \{0, 1, 2\}$. The time elapsed between two consecutive dates constitutes a trading period. Trading sessions are denoted with subscripts $t = \{1, 2\}$. That is, the first round of trade takes place between dates 0 and 1 ($t = 1$), and the second between dates 1 and 2 ($t = 2$).

There are two types of households in equal numbers. The P -type grow potatoes (p), and the W -type produce whiskey (w). The real sector is endowed with a constant returns to scale technology. One unit of labor by P yields one potato, and a unit of labor by W produces a unit of whiskey. Since potatoes and whiskey are perishable, they must be consumed in the period in which they are produced. Further, assume that, just as agents are single minded about production, they only purchase the good they cannot produce. That is, the P -type buy whiskey and the W -type potatoes.

Each household type is also endowed with an amount e_g of gold only on date 0. Let $e_g \geq 1$. Unlike potatoes and whiskey, gold is durable. Hence, gold can serve as a store of value and for productive investments. As in the previous chapters, households can deposit their gold with financial intermediaries, or invest it in real projects. Real projects are illiquid: if financed on $d = 0$, investors cannot withdraw their gold from the project until date 2, which is when the project is completed.

There is a lack of simultaneous double coincidence of wants, and future income is not non-pledgeable. Hence, agents need a medium of exchange to trade. Only notes that certify the existence of gold deposited with intermediaries can be used as means of payments³. In addition, there is uncertainty about the timing

³Chapter 2 develops this argument thoroughly.

of buying and selling transactions within each trading session. Therefore, both P and W hold positive amounts of this money on date 0⁴.

Financial institutions issue notes on a one-to-one basis. On date 0 households deposit gold in the bank in return for notes. Each note is backed by a unit of gold, and each note is a gold claim redeemable on $d = 2$ at an agreed deposit rate. Denote this rate with R^D . The gold-deposit market clears when households and banks reach an agreement on the future value of gold-backed notes (R^D).

The economy is subject to a single source of aggregate risk, which is captured by the fact that gold projects have stochastic returns. Moreover, even though there may be various investment opportunities with different expected returns, these are assumed to be positively correlated. Aggregate uncertainty is resolved during the interim date ($d = 1$). At that point in time agents learn whether real investments yield high or low returns. Let $s = \{H, L\}$, denote the set of possible states and $s^* = \{1, H, L\}$ the set of all state-periods. H and L represent, respectively, the second period's high and low states. If $s = L$, projects fail, and investors lose everything. α is the likelihood of a high state realization; hence, real projects go bankrupt with probability $(1 - \alpha)$.

Let $\bar{R}$ denote the expected return of the project that households invest in and R the realized return. The latter has the following properties

$$R = \begin{cases} R^H > 1 & \text{with probability } \alpha \\ 0 & \text{with probability } (1 - \alpha) \end{cases}$$

Assume $\alpha R^H = \bar{R} > 1$. This implies that accessing means of payments is costly because, an additional unit of money entails an (expected) opportunity cost $\bar{R}$.

In this chapter, a bank is conceived as the investment project of a group of *outside* shareholders called bankers. Bankers are risk-neutral and enjoy limited liability. On date 0, they put an amount E of gold as equity and are granted a banking charter. Subsequently, intermediaries can choose whether to invest their assets in a real project, which is stochastically inefficient, or to leave it in storage. Further, banks' investment policy is non-verifiable.

The investment opportunity that the bank has access to has an expected return $\bar{\rho} < 1$. ρ , the actual return of the project, has the following characteristics

⁴See Chapter 3 for a comprehensive explanation.

$$\rho = \begin{cases} \rho^H > 1 & \text{with probability } \alpha \\ 0 & \text{with probability } (1 - \alpha) \end{cases}$$

Note that $\alpha\rho^H = \bar{\rho} < 1$, and $\rho^H < R^H$.

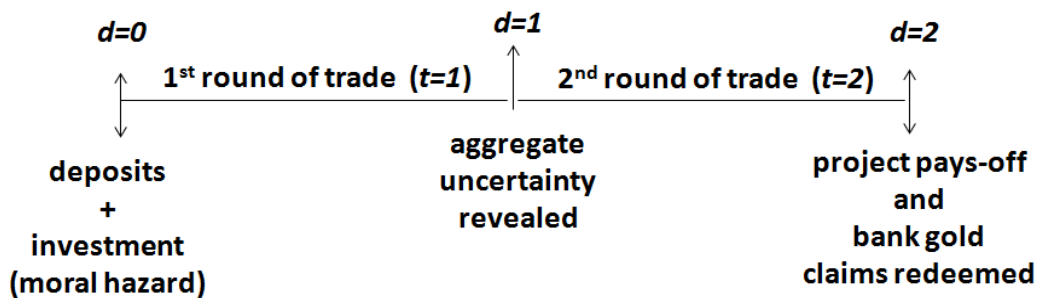
The assumptions made about the investment opportunity of the bank deserve further comment. The assumption that $\bar{\rho} < 1$ reflects the fact that, as argued by Hart and Moore (1994) and Diamond and Rajan (2000, 2001), bankers have less expertise than entrepreneurs in managing real investments. Another argument for this assumption, which describes more accurately the financial industry of today, is that banks transform cash-flows without creating value through derivatives and other financial markets.

More importantly, even though the gambling asset has higher risk and lower expected returns than the safe asset, the banker may still find it attractive. Because of limited liability, banker's payoffs have a lower zero-bound, while their upside gain is unbounded. In addition, bank investment policy is non-verifiable. This implies that deposit contracts can only specify the promised nominal deposit return, because it is not feasible to write contracts that establish a relation between banker's payoffs and the bank's investment policy. For these reasons, when bankers invest in the gambling asset, they get high private returns if the gamble pays off, but if the venture fails, the costs are borne by depositors.

From a social point of view, the planner is willing to give a charter to a bank that may engage in gambling activities, only because financial intermediaries are necessary to provide liquidity services to individuals.

The time structure of the model is depicted in Figure 4.1 (below).

Figure 4.1: Time Structure

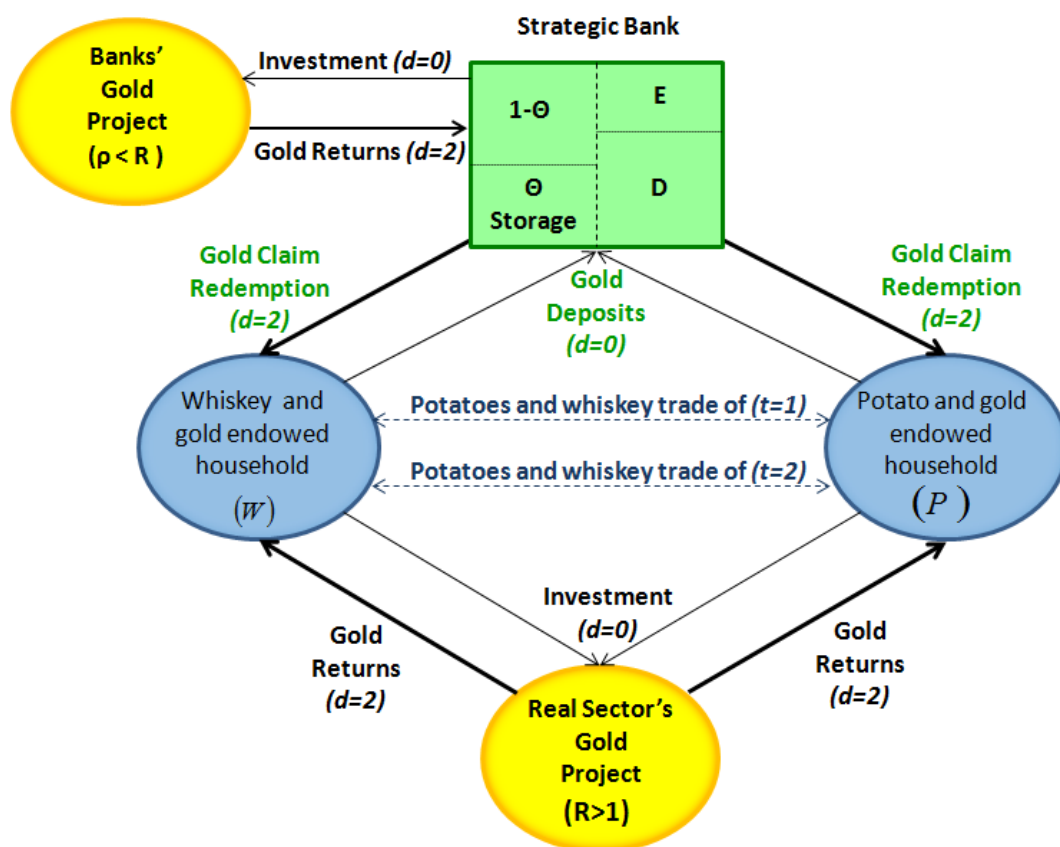


As it is customary in the general equilibrium literature, each consumer-type is treated as a continuum of identical atomistic agents who take prices as given. Thus, prices and traded quantities are determined competitively. Similarly, the financial industry is comprised by a large number of homogeneous small banks. Hence, households and intermediaries take deposit rates as given. Moreover, since bankers are risk-neutral, perfect competition drives their profits to the break-even point.

To sum up, households' resources are channeled towards the real sector in two competing ways: direct financing (by households) and indirect financing (by banks). The first alternative entails a trade-off between profitability and liquidity, because each unit of gold deposited in the banking system has an expected opportunity cost $\bar{R} > 1$. On the other hand, when projects are financed indirectly, banks necessarily engage in risk-taking behavior. In this case, the expected costs of liquidity creation decrease, but households are at risk of losing their deposits.

Figure 4.2 (below) describes the structure of the economy.

Figure 4.2: Nominal Flows of the Economy



4.3.1 Household Sector

Assume that households are endowed with quasi-linear preferences, where the marginal utility of consumption is constant and equal to 1, while the marginal disutility of work is increasing ⁵. P -type households buy whiskey and work in order to grow potatoes, while the W -type purchase potatoes and work in order to make whiskey. $P_{w,s*}$ and $P_{p,s*}$ denote, respectively, the prices of whiskey and potatoes in state-period s^* .

Within each trading session, households cannot possibly know the timing of selling and buying transactions. Hence, it is possible for agents to encounter a buying opportunity before earning any labor income. In that particular case, an individual is said to be “liquidity-constrained”. Suppose this kind of idiosyncratic risk is not insurable. Agents then restore to holding money on $d = 0$ in order to protect themselves against the risk of being liquidity-constrained⁶.

Similarly, households cannot buy insurance to protect themselves against aggregate risk. This type of uncertainty is revealed during the interim date ($d = 1$). Thus, the first trading period is characterized by a single state, whereas s possible states can be realized before the second round of trade starts. This implies that, in the second period, households’ decisions are state-dependent.

Since bank investment policy is non-verifiable, households only learn the value of their claims on the last date, $d = 2$. This is when banks’ deposit repayments are due. At that point in time, households go to the bank to redeem their notes at the prevailing deposit rate. The latter diverges from the contracted rate if banks have previously engaged in risk taking behavior, and a bad state is realized. In that case, the actual gross return on deposits drops to zero and households lose their gold savings.

This implies that, as part of their decision-making process, households must form expectations about the probability that banks invest prudently. This probability is denoted with $\tilde{\theta}$.

Households are symmetric in all aspects but the goods they buy and sell. Therefore, the behavior of the household sector can be described by analyzing the conduct of just one consumer-type, the potato producers say.

On the first date, P deposits f_1^P units of gold in the bank in exchange for the same amount of notes. During the first round of trade, he uses these notes to buy whiskey. By the end of $t = 1$, P receives $P_{p,1}l_t^P$ notes from his sales of potatoes, and chooses f_2^P of these to buy whiskey in the second period.

⁵In the Appendix the model is solved for under the assumption that households have quasi-linear preferences and the marginal disutility of labor is constant, while the marginal utility of consumption is decreasing.

⁶See Chapter 3 for a through analysis of this claim.

Subsequently, P redeems the notes he is left with for gold at the prevailing deposit rate, and receives the returns on the gold project.

Household P maximizes

$$\begin{aligned}
\max_{l_{s*}^P, c_{w,s*}^P, f_{s*}^P} U^P &= (c_{w,1}^P) - \frac{1}{2} (l_1^P)^2 \\
&+ \alpha \left[g_H^P + (c_{w,H}^P) - \frac{1}{2} (l_H^P)^2 \right] \\
&+ (1 - \alpha) \left[g_L^P + (c_{w,L}^P) - \frac{1}{2} (l_L^P)^2 \right]
\end{aligned} \tag{4.1}$$

s.t.

$$\begin{aligned}
P_{w,1} c_{w,1}^P &= f_1^P & (\lambda_1^P) \\
P_{w,H} c_{w,H}^P &= f_H^P & (\lambda_H^P) \\
P_{w,L} c_{w,L}^P &= f_L^P & (\lambda_L^P) \\
P_{p,H} l_H^P &\geq f_H^P & (\mu_H^P) \\
P_{p,L} l_L^P &\geq f_L^P & (\mu_L^P)
\end{aligned} \tag{4.2}$$

where

$$\begin{aligned}
g_H^P &= R^H (e_g - f_1^P) + R^D (P_{p,1} l_1^P + P_{p,H} l_H^P - f_H^P) \\
g_L^P &= \tilde{\theta} R^D (P_{p,1} l_1^P + P_{p,L} l_L^P - f_L^P)
\end{aligned}$$

λ_{s*}^P is the Lagrange multiplier of P 's budget constraint in state-period s^* , and μ_s^P represents the multiplier of the complementary slackness (CS) condition in state s . $c_{w,s*}^W$ is the quantity of whiskey P buys and consumes in state-period s^* , l_{s*}^P denotes the labor he supplies, and f_{s*}^P represents the amount of notes he spends. Finally, e_g is P 's initial endowment of gold, while g^P denotes his terminal stock of gold. The first order conditions with respect to P 's labor decision are given by

$$l_1^P = P_{p,1} \left(\frac{\alpha}{P_{w,H}} + \frac{1-\alpha}{P_{w,L}} \right) \quad (4.3)$$

$$l_H^P = P_{p,H} R^D \quad (4.4)$$

$$l_L^P = P_{p,L} \tilde{\theta} R^D \quad (4.5)$$

In the first period, the marginal return of labor for the potato producer is $P_{p,1} \left(\frac{\alpha}{P_{w,H}} + \frac{1-\alpha}{P_{w,L}} \right)$. In $t = 1$, every unit of effort gives P an income of $P_{p,1}$. Therefore, he derives utility from the amount of goods this money is expected to buy in the second round of trade; that is, $\left(\frac{\alpha}{P_{w,H}} + \frac{1-\alpha}{P_{w,L}} \right)$. In $t = 2$ the amount of time P works yields a unitary income $P_{p,H} R^D$ if the good state is realized, and $P_{p,H} \tilde{\theta} R^D$ if the bad state is realized. This is due to the fact that, in the second period, P can only use the money he earns to claim gold back from the bank at the effective deposit rate. The latter is equal to the contracted rate if a good state is realized. By contrast, in the bad states of nature, households expect to earn $\tilde{\theta} R^D$ on their deposits.

From P 's perspective, gold and whiskey are perfect substitutes. Further, an additional unit of money allows P to buy $1/P_{w,1}$ units of whiskey in $t = 1$, but it also generates an opportunity cost of $\bar{R}$ units gold. Therefore, P 's choice of money holdings is determined as follows.

$$f_1^P = \begin{cases} 0 & \text{if } P_w > \frac{1}{\bar{R}} \\ 0 < f_1^P < e_g & \text{if } P_w = \frac{1}{\bar{R}} \\ e_g & \text{if } P_w < \frac{1}{\bar{R}} \end{cases} \quad (4.6)$$

If $P_{w,1} > 1/\bar{R}$, P prefers gold to potatoes, so he does not deposit gold with the bank. If $P_{w,1} = 1/\bar{R}$ he is indifferent and deposits a positive amount of gold. Finally, if $P_{w,1} < 1/\bar{R}$, P prefers whiskey to gold, so he deposits all his gold in the bank. P also uses money to buy whiskey in the second period; however, the price of whiskey in $t = 2$ is not relevant for his choice of money holdings on $d = 0$. The reason is that, P uses his first-period labor income to buy goods during the second round of trade.

By symmetry, the mirror of these equilibrium conditions are satisfied when the optimization problem of household W is solved. Moreover, in general equilibrium, the potato and whiskey markets must clear in all state-periods. Thus,

$$c_{p,s*}^W = l_{s*}^P \quad \forall s^*$$

$$c_{w,t}^P = l_{s*}^W \quad \forall s^*$$

Combining the above equations with households' first order conditions and budget constraints yields the following equilibrium conditions

$$f_1^P = \frac{1}{\bar{R}^2} \left(\frac{\alpha}{P_{w,H}} + \frac{1-\alpha}{P_{w,L}} \right) \quad (4.7)$$

$$f_H^P = P_{p,H}^2 R^D \quad (4.8)$$

$$f_L^P = P_{p,H}^2 \tilde{\theta} R^D \quad (4.9)$$

Households' preferences are monotonically increasing, and money is costly but does not have any intrinsic value. Consequently, for any state realization, agents choose to spend all of their first-period income in the second round of trade. In other words, the complimentary slackness conditions bind.

$$\mu_H^P > 0 \implies f_H^P = P_{p,1} l_1^P = f_1^W$$

$$\mu_L^P > 0 \implies f_L^P = P_{p,1} l_1^P = f_1^W$$

The second equality in each of the equations above is derived from W 's budget constraints. In addition, by symmetry

$$\mu_H^W > 0 \implies f_H^W = P_{w,1} l_1^W = f_1^P$$

$$\mu_L^W > 0 \implies f_L^W = P_{w,1} l_1^W = f_1^P$$

The last four equations suggest that P and W demand the same amount of money in every state-period. That is,

$$f_{s*}^P = f_{s*}^W = f \quad \forall s^* = \{1, H, L\}$$

In sum, households' labor, consumption and investment choices are identical. Therefore, the prices of potatoes and whiskey are the same in every period and state.

$$P_{p,s*} = P_{w,s*} \quad \forall s^* \quad (4.10)$$

This implies that there are three possible equilibria.

- *Case 1:* $P_{p,1} = P_{w,1} < \frac{1}{\bar{R}}$
- *Case 2:* $P_{p,1} = P_{w,1} > \frac{1}{\bar{R}}$
- *Case 3:* $P_{p,1} = P_{w,1} = \frac{1}{\bar{R}}$

Assume the conditions for Case 1 hold. If prices in the first period are below $\frac{1}{\bar{R}}$, households would choose to deposit all their gold with the bank. Further, the market clearing conditions in the first period would require that

$$P_{p,1} = P_{w,1} = \sqrt{e_g}$$

But $e_g \geq 1$ by assumption, which yields a contradiction. $P_{p,1} = P_{w,1} = \sqrt{e_g} < \frac{1}{\bar{R}}$ implies $\bar{R}^2 e_g < 1$. Since this last inequality is not true, agents could not possibly optimize at this price level.

In Case 2, households' demand for potatoes and whiskey would be zero in $t = 1$, while the supply for these goods would be positive (see equation 4.3 above). This implies that commodity markets would not clear, thereby ruling out this case as an equilibrium.

The only possible equilibrium is characterized by an interior solution to households optimization problem, whereby both, P and W , deposit a portion of their gold in the bank. That is,

$$P_{p,1} = P_{w,1} = \frac{1}{\bar{R}} \quad (4.11)$$

Combining the equation above with the other first order conditions as well as market clearing conditions gives

$$f^* = \bar{R}^{-4/3} (R^D)^{1/3} \left[\alpha + (1 - \alpha) \tilde{\theta}^{1/2} \right]^{2/3}$$

f^* denotes the equilibrium level of liquidity per-capita in every state-period. As expected, households demand for liquidity is positively related to the contracted deposit rate (R^D), the conjectured probability that banks will invest prudently ($\tilde{\theta}$), and the likelihood that a high state is realized (α). On the other hand, the demand for money falls when the opportunity cost of liquidity creation ($\bar{R}$) increases.

Since households are symmetric, the aggregate amount of money in circulation in every state-period, which is denoted by D , is simply

$$D = 2f^* = 2\bar{R}^{-4/3} (R^D)^{1/3} \left[\alpha + (1 - \alpha) \tilde{\theta}^{1/2} \right]^{2/3} \quad (4.12)$$

The inverse demand function of deposits is useful to make a diagrammatic illustration of the demand for liquidity. The latter is obtained by solving for R^D from equation (4.12). This curve is denoted by **DD**.

$$R^D = \frac{2D^3 \bar{R}^4}{\left[\alpha + (1 - \alpha) \tilde{\theta}^{1/2} \right]^2} \quad (\text{DD})$$

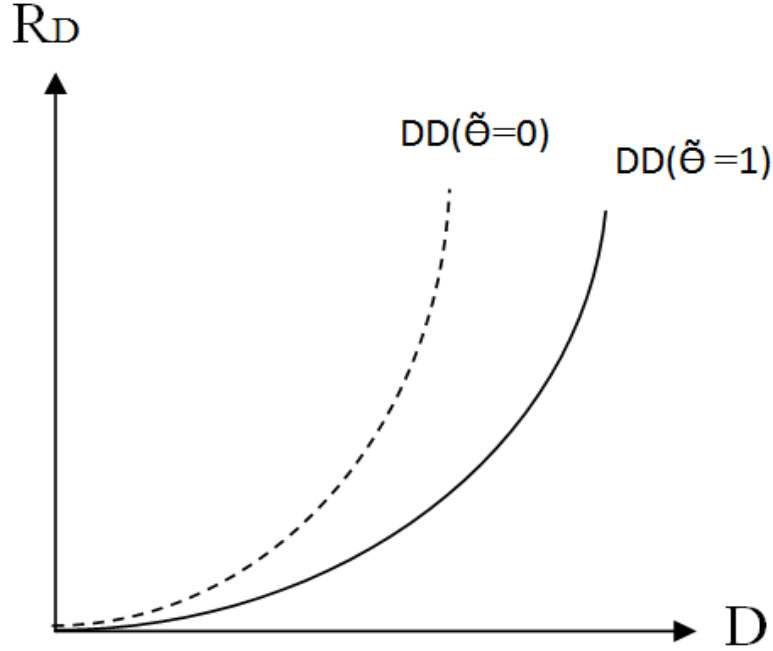
R^D is increasing and convex in D (see Figure 4.3 below). As households become more pessimistic regarding the likelihood that banks will behave prudently, the demand for liquidity contracts (shifts to the left). Figure 4.3 also illustrates the two extreme cases. The straight line is the demand curve when banks are expected to invest in the safe asset with probability 1. By contrast, the dashed line depicts the money demand schedule when depositors expect intermediaries to gamble 100% of the time.

For a given outcome in the gold deposit market, Table 4.1 (below) summarizes the equilibrium trade and price levels in the whiskey and potato markets.

Trade and price levels are decreasing in $\bar{R}$. When real projects become more profitable ($\bar{R}$ rises), the opportunity cost of liquidity creation increases. This induces households to demand less money, which reduces commodity prices and trade levels.

In the second period, trade is increasing in the contracted deposit rate, as the latter can potentially reduce the opportunity cost of liquidity. Prices in $t = 2$, however, are decreasing in R^D . Higher deposit rates increase the value of gold claims, thereby providing households with incentives to work more in the second period. This induces the supply curve to pivot clockwise, thereby reducing the equilibrium price level.

Figure 4.3: Deposit Demand Schedule



In the good state, prices and trade are increasing in $\tilde{\theta}$. When intermediaries are more likely to invest prudently (higher $\tilde{\theta}$), the actual return on deposits is more likely to equal the contracted one. This reduces the *effective* opportunity cost of money, thus stimulating spending and trade. This is also true in the bad state. However, since depositors can lose their gold savings when $s = L$, $\tilde{\theta}$ has an additional impact on prices and trade. Further, while the additional effect of $\tilde{\theta}$ on prices is negative, its effect on trade is positive. This is explained below.

Note that the equilibrium in the commodity markets depends on deposit market outcome. Hence, different combinations of R^D and $\tilde{\theta}$ yield different market equilibria. To illustrate the importance of moral hazard, Figure 4.4 (below) provides a graphical illustration of the market outcome when $\tilde{\theta}$ is small (when depositors allocate a high probability to the event that banks will gamble). Since the equilibrium is symmetric across goods markets, the figure focuses on the outcome for just one commodity: potatoes.

In the potato market, the supply curve is given by the labor choice of household P , and the demand by the consumption choice of W . In each graph the equilibrium is denoted with point M . The first-period outcome is described by the top graph, the second period-low state by the bottom-left graph, and the second period-high state by the bottom-right graph. The demand curves are represented by straight lines and the supply curves by dashed lines.

Table 4.1: Commodity Prices and Trade Levels given R^D

First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = \frac{(R^D)^{1/3} [\alpha + (1-\alpha)\tilde{\theta}^{1/2}]^{(2/3)}}{\bar{R}^{1/3}}$	$P_{p,1} = P_{w,1} = \frac{1}{\bar{R}}$
Second Period - High State	
Trade/Labor	Prices
$l_H^P = l_H^W = \frac{(R^D)^{2/3} [\alpha + (1-\alpha)\tilde{\theta}^{1/2}]^{1/3}}{\bar{R}^{2/3}}$	$P_{p,H} = P_{w,H} = \frac{[\alpha + (1-\alpha)\tilde{\theta}^{1/2}]^{(1/3)}}{(R^D)^{1/3} \bar{R}^{2/3}}$
Second Period - Low State	
Trade/Labor	Prices
$l_L^P = l_L^W = \frac{\tilde{\theta}^{1/2} (R^D)^{2/3} [\alpha + (1-\alpha)\tilde{\theta}^{1/2}]^{2/3}}{\bar{R}^{1/3}}$	$P_{p,L} = P_{w,L} = \frac{[\alpha + (1-\alpha)\tilde{\theta}^{1/2}]^{1/3}}{\tilde{\theta}^{1/2} (R^D)^{1/3} \bar{R}^{2/3}}$

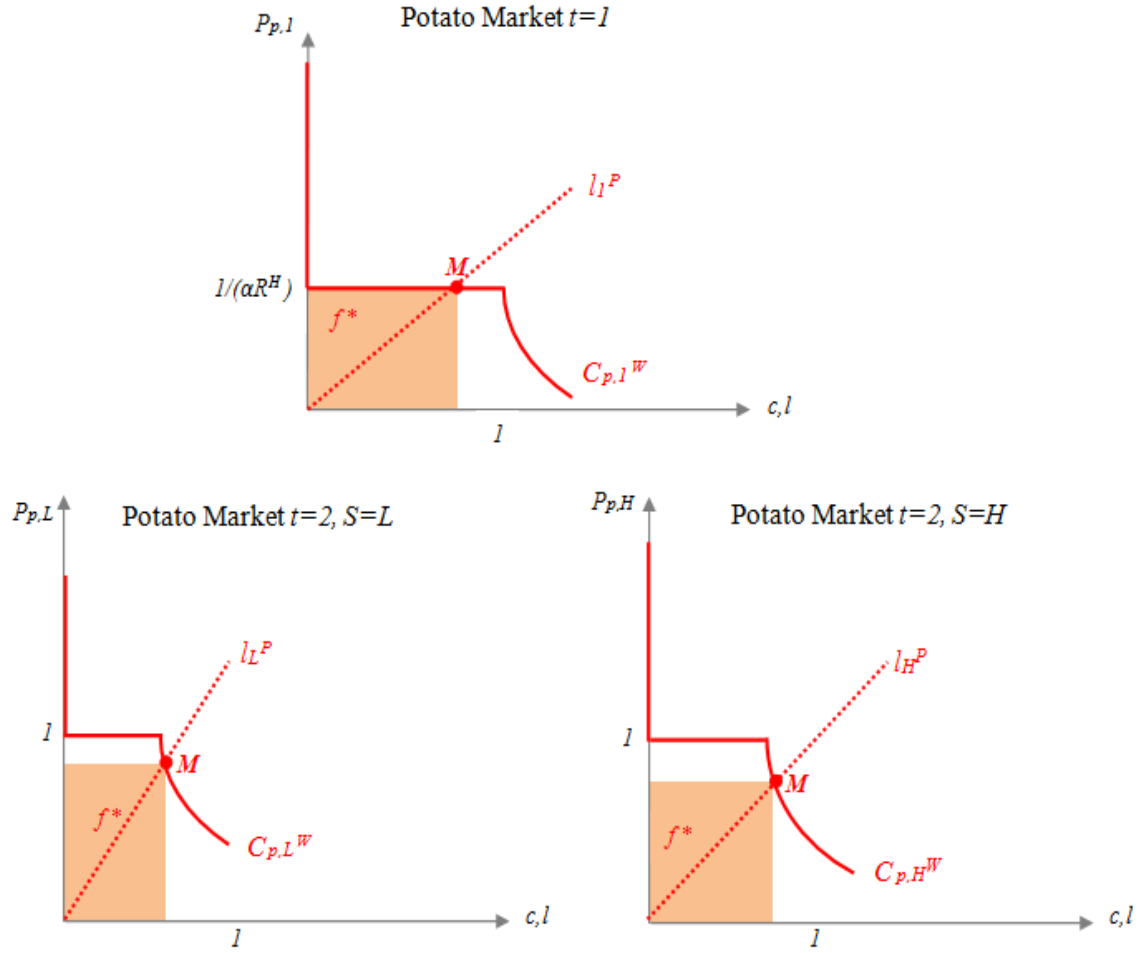
The marginal return of P 's labor in the first period is given by equation (4.3). This implies that the supply curve is a straight line that passes through the origin. On the other hand, the equilibrium is given by the interior solution to households deposit decision. This implies that household W deposits only a fraction of his gold with the bank. Consequently, the equilibrium level of trade in the potato market in $t = 1$ lies to the left of the kink on the demand curve. The latter corresponds to the point where household W would have spent his entire gold endowment in the consumption of whiskey at a price $1/\bar{R}$.

In the second period, the marginal return of labor for P is proportional to $P_{p,H}$ if a good state is realized, and proportional to $P_{p,L}$ otherwise. Therefore, in both states the supply curve passes through the origin. On the demand side, the use of money itself does not entail any cost. This is due to the fact that, in $t = 2$, the amount of money W can spend is given by his first-period-labor income. In both cases, W would choose to spend his entire first-period income buying potatoes. This implies that the equilibrium level of trade in the potato market lies to the right of the kink on the demand curve.

However, if bad state is realized, lower values of $\tilde{\theta}$ imply that the value of money as a medium of exchange decreases. This is due to the fact that, the probability of bank-issued notes being effectively

backed by gold is smaller. Hence, with lower values of $\tilde{\theta}$, the labor supply is more inelastic (see equation 4.5) and the expected value of the resources households can spend drops (the kink on the demand curve gets closer to origin across the horizontal axis). For this reason, the level of trade in the bad state is lower than in the high state, while prices are higher in the bad state than in the good state.

Figure 4.4: Market Equilibrium with Active Banks



Note that, in the extreme case where θ tends to zero,

$$\lim_{\tilde{\theta} \rightarrow 0} l_L^P = \lim_{\tilde{\theta} \rightarrow 0} l_L^W = \infty$$

$$\lim_{\tilde{\theta} \rightarrow 0} P_{p,L} = \lim_{\tilde{\theta} \rightarrow 0} P_{w,L} = 0$$

In words, when the probability that banks undertake risky investments is high ($\tilde{\theta} \rightarrow 0$) and a bad state is realized, commodities are no longer traded and prices rise to infinitely high levels. The intuition is that, if depositors are certain that banks have engaged in risk-taking behavior, when a bad state is realized, money becomes worthless because it cannot be converted back into gold. Therefore, agents wishing to spend money in exchange for goods cannot find a counter-party willing to take their notes.

This is similar to a fire-sale phenomenon: bad news directly lower expectations about the fundamental value of money. Since the shock is likely to render banks insolvent, the value of gold-claims collapses to 0. Further, potato and whiskey markets shut-down, as the money market dries-up. This is due to the fact that, since future income is not pledgeable, transactions in commodity markets cannot take place if money is not backed by gold.

4.3.2 Financial Sector

The banking sector is comprised by a continuum of identical atomistic price-taking financial institutions. Each bank is conceived as the investment project of a group of *outside* shareholders called bankers. These outside investors put down an amount of gold E , which constitutes the bank's equity. E is exogenous and is public knowledge. Banks liabilities are made up of promised payments to depositors (principal plus interest), while the value of assets corresponds to the gross returns on the bank's loan portfolio. Finally, bankers are risk-neutral, enjoy limited liability and are initially granted a banking charter.

Financial intermediaries take gold-deposits from the household sector, thereby making a capital structure decision. Subsequently, they choose an investment strategy which is non-verifiable. This implies that feasible deposit contracts cannot specify payoffs contingent on the investment policy implemented by the banker. For this reason, households and bankers cannot contract on anything but the nominal return on gold deposits. The latter is determined competitively. Furthermore, although households learn which state of nature is realized on $d = 1$, the actual deposit return is only revealed to them on $d = 2$. This is when banks repay (or default) on their obligations.

Let Π^I denote the *expected* payoff to the banker from implementing investment strategy I . $I = \{S, R\}$, where S denotes the safe investment strategy whereby gold is kept in storage, and R indicates the risky strategy. When $I = R$ banks invest in the inefficient gambling asset with expected return $\bar{\rho} < 1$. Because of the convexities introduced by limited liability and bankers risk-neutrality, intermediaries choose either S or R . Strategy S dominates whenever $\Pi^S \geq \Pi^R$. Otherwise, banks engage in risk-taking behavior.

Banks may also randomize between S and R . In that case, bankers are indifferent between investing prudently or deviating towards the risky option, but they choose to play “safe” a fraction θ of the time.

Due to limited liability, bankers’ downside losses are limited, while their upside gain is unbounded. Moreover, the non-verifiability of bank investment policy implies that, when bankers invest in the gambling asset, they get high private returns if the gamble pays off. If this venture fails, however, costs are borne by depositors. Consequently, a moral hazard problem arises in this setup.

For simplicity of exposition, this section first explains banks’ pure strategy decision-making process. The representative bank takes D deposits from the household sector. Then, bankers implement the investment policy that gives them the highest payoff. Finally, bankers only enter the financial industry, if they expect to at least break-even (constraint 4.14 below).

$$\max_{D, I=\{S, R\}} \Pi^I = \max \{ \Pi^S, \Pi^R \} \quad (4.13)$$

$s.t.$

$$\Pi^I \geq E \quad (4.14)$$

where

$$\begin{aligned} \Pi^S &= \max \{ (E + D) - R^D D, 0 \} \\ \Pi^R &= \alpha [\max \{ \rho^H (E + D) - R^D D, 0 \}] + (1 - \alpha) [\max \{ -R^D D, 0 \}] \\ \implies \Pi^R &= \max \{ \bar{\rho} (E + D) - \alpha R^D D, 0 \} \end{aligned}$$

Due to bankers’ risk-neutrality, perfect competition drives their expected profits to the break-even point. Hence, constraint (4.14) is binding.

$$\Pi^I = \bar{E}$$

Thus, bankers invest safely if the following two conditions are satisfied.

- $\Pi^S = E$
- $\Pi^S \geq \Pi^R$

The first of these conditions implies that, when banks play safe, $R^D = 1$.

$$\begin{aligned}\Pi^S &= \max \{(E + D) - R^D D, 0\} = E \\ \implies R^D &= 1\end{aligned}\tag{4.15}$$

By contrast, when banks choose to engage in risk-taking behavior, the conditions below must be satisfied.

- $\Pi^R = E$
- $\Pi^R > \Pi^S$

The combination of these two conditions implies that $R^D > 1$. That is, when a deposit contract specifies a nominal return larger than 1, banks cannot credibly commit to invest safely.

If banks gamble, the supply curve is derived by solving for R^D from the break-even condition.

$$\begin{aligned}\Pi^R &= \alpha [\max \{\rho^H (E + D) - R^D D, 0\}] + (1 - \alpha) [\max \{-R^D D, 0\}] = E \\ \implies R^D &= \rho^H - \left(\frac{1 - \bar{\rho}}{\alpha} \right) \frac{E}{D}\end{aligned}\tag{4.16}$$

Note that R^D is increasing and concave in D .

The next section will show that households' welfare decreases when banks engage in risk-taking behavior. In the meantime, assume that the safe investment policy is in the best interest of depositors. Therefore, banks incentive compatibility constraint is given by

$$\begin{aligned}\Pi^S &\geq \Pi^R \\ \implies E + D - R^D D &\geq \alpha [\rho^H (E + D) - R^D D] \\ \implies \frac{D}{E} &\leq \frac{1 - \alpha \rho^H}{\alpha (\rho^H - 1)}\end{aligned}$$

The term on the right hand side of the last inequality is the maximum debt-to-equity ratio consistent with the incentive compatibility constraint. Denote this ratio by d^* . Thus,

$$d^* = \frac{1 - \alpha\rho^H}{\alpha(\rho^H - 1)} \quad (4.17)$$

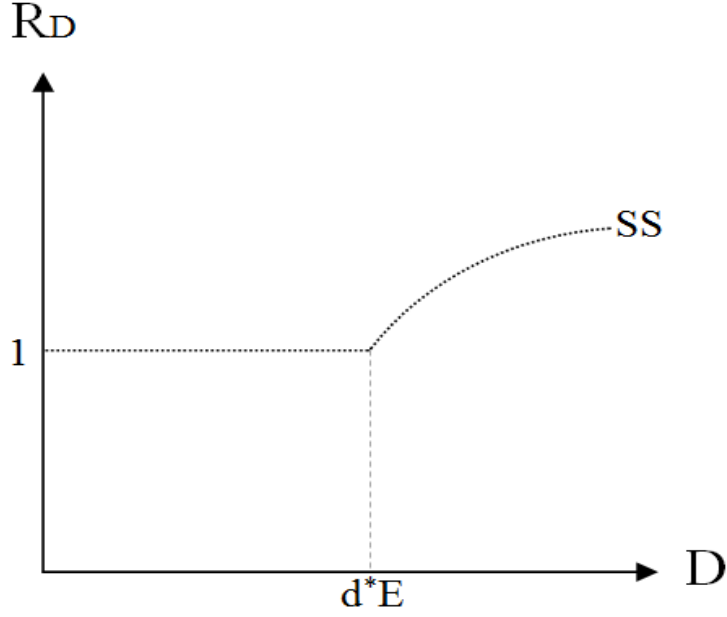
This critical leverage ratio is decreasing in both α and ρ^H . When the economy is less likely to experience an economic downturn (high α), less capital is needed to induce banks to invest safely. On the other hand, lower returns on the gambling asset (smaller ρ^H) make the risky policy more inefficient. This exacerbates the moral hazard problem, thereby requiring banks to lever less in order to discourage risk-taking.

If banks lever up beyond d^* , they will have incentives to gamble. By contrast, when bank capital is sufficiently large, risk-taking incentives are constrained by the prospect of shareholder losses. Accordingly, the incentive compatibility condition **(IC)** can be expressed in terms of the level of bank capital

$$E \geq \frac{D}{d^*} \quad \textbf{(IC)}$$

This implies that, for a given level of bank capital E , the supply of deposits is consistent with equation (4.15) when $D \leq d^*E$. Beyond this level, the supply of deposits is given by equation (4.16). Intuitively, when bankers intend to take on the risky project, they can afford to promise higher deposit rates, because they will repay depositors only in some states of the world. Figure 4.5 (below) provides a graphical illustration. The graph corresponds to the supply curve of deposits, which is denoted by SS.

Figure 4.5: Deposit Supply Curve



4.3.3 Competitive Equilibrium

In equilibrium, households' conjecture about the likelihood that banks invest safely must be consistent with the strategy implemented by the banker ($\tilde{\theta} = \theta$). In addition, the gold deposit market must clear. Since banks can undertake either a safe or a risky investment strategy, there are 3 types of equilibria.

Type I: Safe Equilibrium. First, let households conjecture that banks will make safe investments. Hence $\tilde{\theta} = 1$ and households' deposit-demand schedule is given by

$$R^D = D^3 \bar{R}^4 \quad (4.18)$$

According to the incentive compatibility condition, banks effectively invest in the safe technology ($\theta = 1$) if

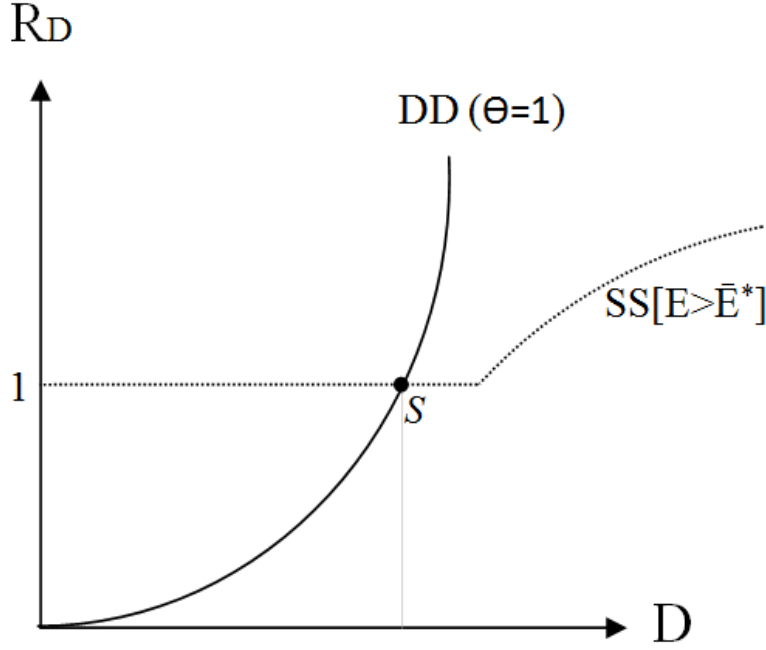
$$E \geq \frac{D}{d^*}$$

Recall that banks can only credibly commit to the risk-free strategy if $R^D = 1$. In addition, market clearing implies that the deposit demand schedule, equation (4.18), can be used to substitute out D from the inequality above. Using these two conditions transforms the incentive compatibility constraint as follows.

$$E \geq \frac{2}{\bar{R}^{4/3} d^*} \quad (4.19)$$

Let $\bar{E}^* = \frac{2}{\bar{R}^{4/3} d^*}$. When $E \geq \bar{E}^*$ financial intermediaries invest in the safe asset, and households correctly anticipate this ($\tilde{\theta} = \theta = 1$). Moreover, deposit contracts trade at par ($R^D = 1$), and the equilibrium amount of liquidity in circulation per period is $D = 2\bar{R}^{-4/3}$. This result is depicted in Figure 4.6. The equilibrium is denoted by point S .

Figure 4.6: Deposit Market Equilibrium when Banks are Highly Capitalized



Type II: Risk-Shifting Equilibrium. Take now the case where depositors expect banks to engage in risk-taking behavior ($\tilde{\theta} = 0$). Households' deposit-demand schedule is given by

$$R^D = \frac{D^3 \bar{R}^4}{\alpha^2} \quad (4.20)$$

Financial intermediaries would deviate and invest safely if, given $\tilde{\theta} = 0$,

$$E \geq \frac{D}{d^*}$$

Using equation (4.20) to substitute out D from the inequality above, and imposing the safe-equilibrium condition whereby $R^D = 1$, yields

$$E \geq \frac{2\alpha^{2/3}}{\bar{R}^{4/3} d^*}$$

Consequently, when depositors expect banks to gamble, bankers will effectively engage in risky investments if

$$E < \frac{2\alpha^{2/3}}{\bar{R}^{4/3} d^*}$$

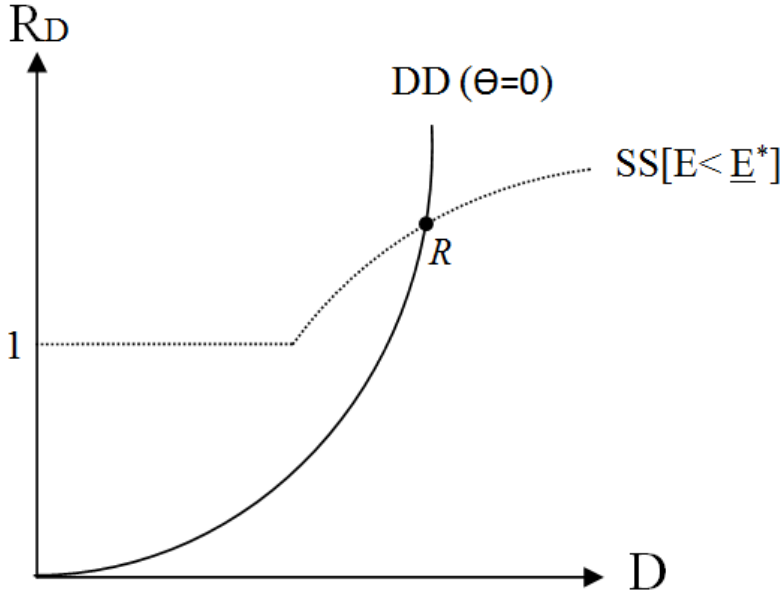
Let

$$\underline{E}^* = \frac{2\alpha^{2/3}}{\bar{R}^{4/3} d^*}$$

Note that $\underline{E}^* = \alpha^{2/3} \bar{E}^*$. Therefore, $\underline{E}^* < \bar{E}^*$.

In words, if the banking sector is poorly capitalized ($E < \underline{E}^*$), financial intermediaries engage in risk-taking behavior. Households correctly anticipate this, so $\tilde{\theta} = \theta = 0$. Further, since banks' leverage ratio is higher than d^* , the deposit rate is larger than unity. Bankers can afford to promise high deposit rates when they gamble, because they repay depositors only when a high state is realized. This type of equilibrium is denoted by R and is depicted in Figure 4.7 (below).

Figure 4.7: Deposit Market Equilibrium when Banks are Poorly Capitalized



Type III: Mixed Strategy Equilibrium. What happens in the cases where bankers are endowed with a level of capital $E \in (\underline{E}^*, \bar{E}^*)$? The deposit market outcome is characterized by a mixed strategy. There-

fore, bankers must be indifferent between the safe and risky strategies. Again, since bank shareholders are risk-neutral, perfect competition drives bankers expected profits to the break-even point. Thus, the mixed strategy equilibrium must satisfy

$$\Pi^R = \Pi^S = E$$

The second equality requires that $R^D = 1$. In addition, since $\Pi^R = E$, the equilibrium must lie on the kink of the **(SS)** curve. Intuitively, if $R^D > 1$ the risk-less strategy is always dominated, and gambling is *strictly* optimal. Hence, when bankers randomize it must be the case that $R^D = 1$. Moreover, since the deposit rate allows banks to afford the prudent strategy ($R^D = 1$), they take just the right amount of deposits to be indifferent between the safe and the risky strategies. The level of bank debt that satisfies this conditions is

$$\begin{aligned} D &= Ed^* \\ \implies D &= E \left(\frac{1 - \alpha \rho^H}{\alpha (\rho^H - 1)} \right) \end{aligned}$$

On the other hand, equation (4.12) implies that when $R^D = 1$, households demand for deposits is given by

$$D = \frac{[\alpha + (1 - \alpha) \tilde{\theta}^{1/2}]^{2/3}}{\bar{R}^{4/3}}$$

Consequently, in equilibrium banks must undertake a safe policy with probability $\tilde{\theta} = \theta$, such that

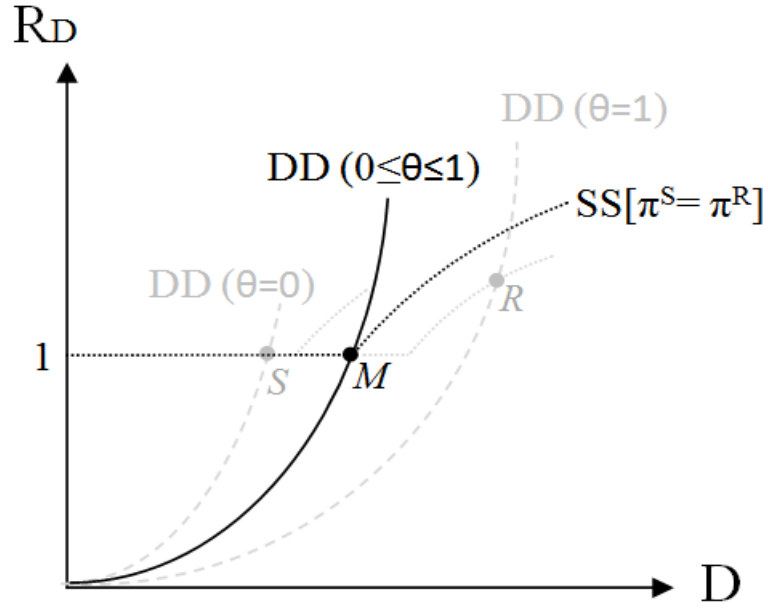
$$E \left(\frac{1 - \alpha \rho^H}{\alpha (\rho^H - 1)} \right) = \frac{[\alpha + (1 - \alpha) \theta^{1/2}]^{2/3}}{\bar{R}^{4/3}} \quad (4.21)$$

Solving for θ from equation (4.21) gives

$$\theta = \left[\left(\frac{Ed^*}{2} \right)^{3/2} \bar{R}^2 - \alpha \right]^2 \left(\frac{1}{1 - \alpha} \right)^2 \quad (4.22)$$

This type of equilibrium is depicted in Figure 4.8 (below) and represented by point M . For comparison purposes, I have superimposed this market outcome on the safe and risky equilibria.

Figure 4.8: Deposit Market Equilibrium when Banks are Fairly Capitalized



As expected, the probability that bankers behave prudently is increasing in the level of bank capital. Moreover, note that

$$\theta = \begin{cases} 1 & \text{if } E = \bar{E}^* \\ 0 & \text{if } E = \underline{E}^* \end{cases}$$

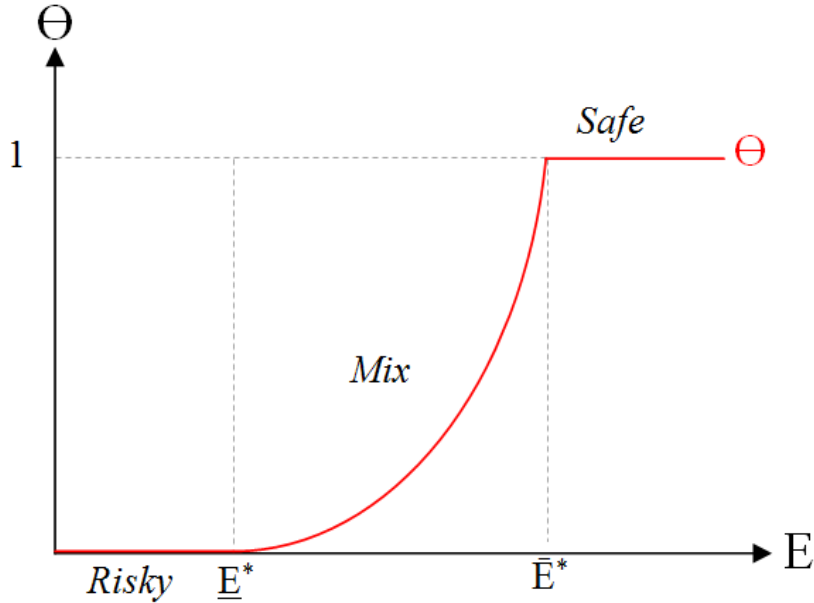
These results are summarized in the next proposition.

Proposition 4.3.1 $\exists \underline{E}^*$ and $\bar{E}^*$ such that

- If $E > \bar{E}^*$ banks invest in the safe asset, and deposit contracts trade at par ($R^D = 1$).
- If $E < \underline{E}^*$ banks invest in the inefficient gambling asset, and deposit contracts trade at a discount on $d = 0$ ($R^D > 1$).
- If $\underline{E}^* \leq E \leq \bar{E}^*$ banks and households randomize. Bankers invest in the safe asset a fraction $0 < \theta < 1$ of the time, and deposit contracts trade at par.

The model produces an insightful inverse relationship between bank capital and risk-taking incentives. In the finance literature this phenomenon is known as risk-shifting. Here, the intuition underlying this result is that, due to the prospect of shareholder losses, bankers are less likely to deviate towards risky investments when the level of bank capital is higher. Figure 4.9 summarizes this results.

Figure 4.9: Bank Capital and Investment Policy



Note that both, $\underline{E}^*$ and $\bar{E}^*$ are decreasing in $\bar{R}$. When $\bar{R}$ is high, the soaring opportunity cost of money induces households to demand less liquidity. This forces bankers to lever less, thereby weakening their incentives to gamble. Consequently, the minimum level of capital required to discourage risk-taking behavior decreases. As explained previously, d^* is decreasing in α and ρ^H . Hence, these parameters have a positive relation with $\underline{E}^*$ and $\bar{E}^*$. When returns on the gambling asset are low (small ρ^H) or the economy is more likely to experience an economic downturn (low α), the gambling asset is more

inefficient. This exacerbates the moral hazard problem. Consequently, higher levels of bank capital are necessary to prevent banks from gambling.

A lower level of α also increases the gap between $\underline{E}^*$ and $\bar{E}^*$. Thus, when real projects are more likely to fail, the prospect of shareholder losses becomes imminent. In other words, difficult macroeconomic conditions, on their own, tend to align bankers' incentives with depositors' interests. In this case, lower levels of bank capital are required to prevent banks from implementing a pure-gambling strategy.

4.4 Remarks on Financial Regulation

Hereafter, regulation refers to any type of intervention that addresses the moral hazard problem. However, one must first determine whether regulation is indeed needed. In addition, one must assess whether the planner would prioritize addressing the moral hazard problem over reducing the opportunity cost of money.

4.4.1 Is Regulation Needed?

Type I: Safe Equilibrium. The safe equilibrium corresponds to the market outcome of the framework developed in Chapter 3, which abstracts from the moral hazard problem. In that case, the market creates an aggregate amount of liquidity $2f^{**}$, where

$$2f^{**} = 2\bar{R}^{-4/3} \quad (4.23)$$

This equation is equivalent to the level of deposits obtained from equation (4.12) when $R^D = 1$ and $\tilde{\theta} = \theta = 1$ (that is, when banks are highly capitalized and, therefore, play safe). Recall from Chapter 3 that, in this context, public intervention is evaluated on a *constrained efficiency* basis: the planner maximizes households' welfare while taking as given the non-pledgeability of future income and insurance markets' incompleteness. Moreover, the planner internalizes the pecuniary externalities ensuing from these market imperfections. This is due to the fact that, private agents follow price-taking behavior and fail to internalize the general equilibrium effects of their actions. Therefore, the planner also takes into account the effect of liquidity holdings on prices.

Assume the planner has a regulatory tool that determines the amount of liquidity held by each household. Denote the latter by f_s . The planner, therefore, optimizes

$$\max_{f_{s*}^P, c_{w,s*}^P, l_{s*}^P} U^P = c_{w,1}^P - \frac{1}{2} (l_1^P)^2 + \alpha \left[g_H^P + c_{w,H}^P - \frac{1}{2} (l_H^P)^2 \right] (1 - \alpha) \left[g_L^P + c_{w,L}^P - \frac{1}{2} (l_L^P)^2 \right]$$

s.t.

$$\begin{aligned} P_{w,1} = P_{p,1} &= f_s^{3/4} \\ P_{w,s} = P_{p,s} &= f_s^{1/2} \quad \forall \quad \{H, L\} \end{aligned}$$

Using the results set out in Table (4.1), imposing the planner's constraints and setting $R^D = 1$ and $\tilde{\theta} = \theta = 1$, simplifies the problem as follows

$$\max_{f_s} U^P = \left(f_s^{1/4} - \frac{1}{2} f_s^{1/2} \right) + \left(f_s^{1/2} - \frac{1}{2} f_s \right) + [\bar{R}e_g + (1 - \bar{R}) f_s]$$

The first order condition with respect to f_s yields

$$f_s \ni \frac{1}{2} + \frac{1}{4} \left[(f_s)^{-(3/4)} + (f_s)^{-(1/2)} \right] = \bar{R} \quad (4.24)$$

From proposition 3.3.2 in Chapter 3, equations (4.23) and (4.24) imply that the planner would intervene to restrict the creation of liquidity.

Type II: Risk-Shifting Equilibrium. When banks engage in risk-taking behavior, they are able to afford high deposit rates ($R^D > 1$). This can make depositors better-off by reducing the opportunity cost of accessing means of payment. Recall that, due to the lack of future income pledgeability and insurance markets' incompleteness, money creation is costly and detrimental to households' well-being⁷. Hence, regulatory measures that address the moral hazard problem are desirable if the social costs imposed by bank risk-shifting are larger than the gains brought about by the reduction to the opportunity costs of liquidity.

By proposition 4.3.1, if $E < \underline{E}^*$, banks invest in the gambling asset with probability 1 and promise depositors a nominal return $R^D = \rho^H - \left(\frac{1 - \alpha \rho^H}{\alpha} \right) \frac{E}{D} > 1$. At this rate, households make a positive amount of gold-deposits, say D^* , which satisfies

⁷See chapters 2 and 3 for a formal proof.

$$D^* = \bar{R}^{-4/3} \alpha^{2/3} [R^D]^{1/3} = \bar{R}^{-4/3} \alpha^{2/3} \left[\rho^H - \left(\frac{1 - \alpha \rho^H}{\alpha} \right) \frac{E}{D^*} \right]^{1/3}$$

Discouraging bank risk-taking is the planner's main objective if, for a given level of D^* , households would have been better-off had bankers invested safely and promised to pay $R^D = 1$.

The equilibrium in the whiskey and potato market is symmetric (see Table 4.1), and the liquidity demand function is the same for both household types. This implies that the indirect utility function of P and W is identical. Because of this symmetry, the social welfare function ($\mathcal{W}$) can simply be established as the utility function of household P .

$$\begin{aligned} \mathcal{W} = & (c_{w,1}^P) - \frac{1}{2} (l_1^P)^2 + \alpha \left[g_H^P + (c_{w,H}^P) - \frac{1}{2} (l_H^P)^2 \right] \\ & + (1 - \alpha) \left[g_L^P + (c_{w,L}^P) - \frac{1}{2} (l_L^P)^2 \right] \end{aligned} \quad (4.25)$$

Let D_i^* denote the per-capita demand for liquidity under the type II equilibrium. Given D_i^* , $\mathcal{W}^R$ denotes households' welfare when banks engage in risk-taking behavior ($\theta = 0$). Using the equilibrium conditions set out in Table 4.1, the following expression obtains

$$\mathcal{W}^R = \alpha^{1/2} (R^D D_i^*)^{1/4} + \frac{\alpha (R^D D_i^*)^{1/2}}{2} + \frac{\alpha}{2} (R^D D_i^*) + \bar{R} (e_g - D_i^*)$$

Now assume that, somehow, intermediaries are forced to invest in the safe asset ($\theta = 1$). This implies that bankers can only afford to pay depositors $R^D = 1$. Let $\mathcal{W}^S$ denote the social welfare when banks behave prudently given D_i^* . Using equation (4.25) and the equilibrium conditions in Table 4.1 gives

$$\mathcal{W}^S = [D_i^*]^{1/4} + \frac{1}{2} \left[(D_i^*)^{1/2} + D_i^* \right] + \bar{R} (e_g - D_i^*)$$

Let $\Delta \mathcal{W} = \mathcal{W}^S - \mathcal{W}^R$. Bank risk-shifting imposes a negative externality on the household sector if $\Delta \mathcal{W} > 0$.

$$\Delta \mathcal{W} = (D_i^*)^{1/4} \left[1 - \alpha^{1/2} (R^D)^{1/4} \right] + \frac{(D_i^*)^{1/2}}{2} \left[1 - \alpha (R^D)^{1/2} \right] + \frac{D_i^*}{2} [1 - \alpha R^D]$$

Since $R^D > 1$ when banks gamble, then it must be the case that

$$\Delta\mathcal{W} > (D_i^*)^{1/4} \left[1 - (\alpha R^D)^{1/2} \right] + \frac{(D_i^*)^{1/2} + D_i^*}{2} \left[1 - \alpha R^D \right]$$

After some algebraic manipulation to the right hand side of the inequality above, the following expression obtains

$$\Delta\mathcal{W} > (D_i^*)^{1/4} \left[1 - (\alpha R^D)^{1/2} \right] + \left[1 + \frac{(D_i^*)^{1/2} + D_i^*}{2} \right] \left[1 - (\alpha R^D)^{1/2} \right] > 0$$

The second inequality follows because $1 > (\alpha R^D)^{1/2}$. Intuitively, banks can never afford to pay a deposit rate higher than ρ^H , thus $R^D < \rho^H$. Further, since the gambling asset is inefficient $\alpha\rho^H < 1$, then $\alpha R^D < 1$ which is equivalent to $1 > (\alpha R^D)^{1/2}$.

Type III Mixed Strategy Equilibrium. In the mixed strategy equilibrium, bankers cannot afford to promise returns above $R^D = 1$. In this case there are no social gains from bank risk-shifting. Therefore regulatory measures to address the moral hazard problem are strictly necessary.

In sum, when the probability of financial institutions engaging in risk-taking behavior is non-zero, bankers impose a negative externality on depositors. Consequently, financial regulatory measures aimed at addressing the moral hazard problem are welfare improving.

The remainder of this section explores different forms of financial regulation to protect the banking system from moral hazard. Traditionally, this kind of measures have consisted of a mixture of monitoring individual transactions (ensuring, for instance, that adequate collateral is put up), regulations concerning self-dealing, capital requirements, and entry restrictions. In some countries, restrictions have been placed on lending in particular areas ⁸.

I start by examining capital requirements, which is the more conventional and widely accepted policy tool. Asset regulations and entry rules have also been proposed as alternative (or complimentary) measures to preserve the solvency of the banking system. These are, therefore, analyzed next. Finally, I explore the possibility of regulating banks' ownership structure. For instance, Hansmann (1996) shows that mutual banks have superior monitoring skills than for profit companies; thus, the former do a better job at controlling the moral hazard problem.

⁸The intent of these restrictions was only partially to enhance the safety and soundness of the banking system; these restrictions were also intended to direct credit towards - what were viewed at the time as - more productive investments.

Further research is necessary to determine which policy (or policy mix), is better. I nevertheless offer this analysis as a stepping stone to help understand the advantages and limitations of each regulatory tool. My results are based on comparative statics exercises. This has the advantage of providing insights into the fundamentals that determine the effectiveness of each measure.

4.4.2 Capital Requirements or Leverage Ratios

Capital requirements are widely accepted as a regulatory policy tool. Minimum capital adequacy ratios force bankers to have more of their own wealth at risk. This induces them to internalize the inefficiency generated by their gambling. Clearly, once banks have enough of their own capital at stake, they become motivated to invest prudently.

In this setup, minimum capital requirements are equivalent to leverage limits, because bank capital is exogenous. Therefore, the central planner is assumed to set a maximum debt-to-equity ratio. By inspection of the three types of equilibrium, the central bank would want to set the maximum leverage ratio at d^* .

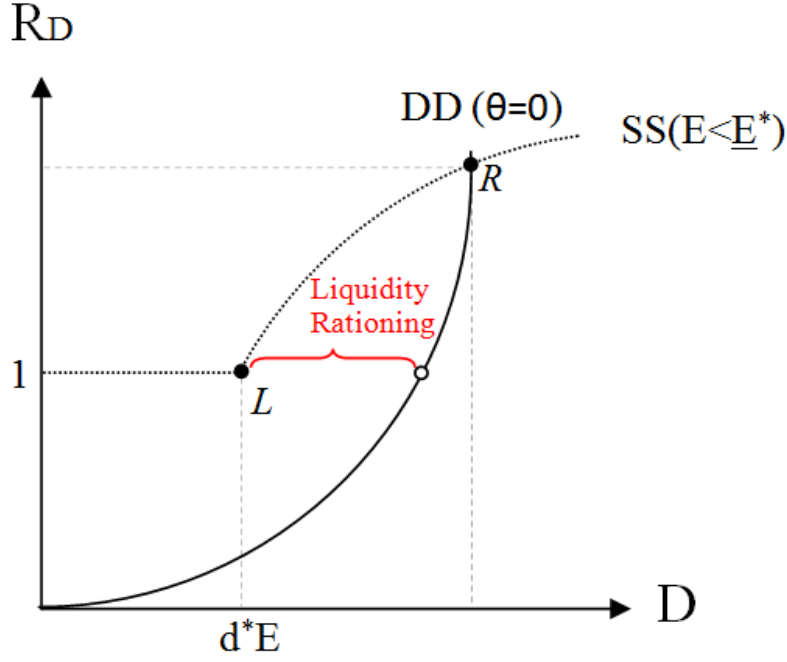
$$d^* = \frac{1 - \alpha\rho^H}{\alpha(\rho^H - 1)}$$

For a given level of bank capital E , this ratio determines the maximum level of debt that the financial institution can take and still satisfy the incentive compatibility constraint.

The shortcoming of this type of regulation is that it induces banks to ration their provision of liquidity. Figure 4.10 (below) illustrates. Note also, that the lower the level of bank capital, the higher the degree of liquidity rationing.

This result can be interpreted along the lines of the argument put forward by Gorton and Winton (1995). The authors reject the use of only capital requirements to address the moral hazard problem in the financial industry. They argue that, if banks are required to hold a minimum level of capital ex-ante, they may simply choose not open a business. This would reduce households welfare significantly, because financial institutions would refrain from producing means of payment.

Figure 4.10: Effect of Leverage Ratios when Banks are Poorly Capitalized



4.4.3 Liquidity Requirements

A liquid asset can be quickly sold or pledged as collateral at its true value. Moreover, liquid assets are, by definition, less profitable than risky ones. In this setup, stored gold is the liquid asset. The motivation for imposing a minimum liquidity requirement is to force banks to build a buffer of stored gold against bad shocks. In other words, this type of asset restriction allows banks to engage in better risk-management practices by diversifying their portfolio risk. Further, liquidity requirements can also help limit the feedback (or general equilibrium) effects of bank failure (see Kashyap et.al. 2010 and Osorio 2011).

Accordingly, let the government impose a minimum liquidity requirement, whereby banks are forced to invest a fraction L of their assets in the safe technology. Since the level of bank capital and the deposit return are public knowledge, the banks' optimization problem becomes

$$\max_{D, I=\{S, R\}} \Pi^I = \max \{\Pi^S, \Pi^R\} \quad (4.26)$$

s.t.

$$\Pi^I \geq E \quad (4.27)$$

where

$$\Pi^S = \max \{ (E + D) - R^D D, 0 \}$$

$$\begin{aligned} \Pi^R &= \alpha \left[\max \{ \rho^H (1 - L) (E + D) + L (E + D) - R^D D, 0 \} \right] \\ &+ (1 - \alpha) \left[\max \{ L (E + D) - R^D D, 0 \} \right] \end{aligned}$$

$$\Rightarrow \Pi^R = \max \{ \bar{\rho} (1 - L) (E + D) + \alpha L (E + D) - \alpha R^D D, 0 \}$$

Note that liquidity requirements are binding only in the case where banks choose to invest in the risky asset. Due to bankers' risk-neutrality, perfect competition drives banks' expected profits to the break-even point. Hence, constraint (4.27) is binding.

$$\Pi^I = \bar{E}$$

Bankers invest safely if

$$\Pi^S = E \quad \text{and} \quad \Pi^S \geq \Pi^R$$

This implies that $R^D = 1$, when banks invest prudently. By contrast, when banks choose to engage in risk-taking behavior, it must be the case that

$$\Pi^R = E \quad \text{and} \quad \Pi^R > \Pi^S$$

The combination of these two conditions implies that $R^D > 1$. Moreover, in this case the supply curve is derived by solving for R^D from the break-even condition

$$\begin{aligned} \Pi^R &= \max \{ \bar{\rho} (1 - L) (E + D) + L (E + D) - \alpha R^D D, 0 \} = E \\ \Rightarrow R^D &= \rho^H (1 - L) + L - \left(\frac{1 - \alpha \rho^H (1 - L) - \alpha L}{\alpha} \right) \frac{E}{D} \end{aligned} \tag{4.28}$$

Note that R^D is decreasing in L . In words, bankers promise lower deposit returns when they invest a larger fraction of their portfolio in the risk-free asset. This is due to the fact that liquidity requirements

force banks to pay back depositors in all states of nature. Consequently, bankers cannot afford to offer deposit rates as high as in the absence of regulation.

The incentive compatibility condition indicates that bankers refrain from gambling if

$$\Pi^S \geq \Pi^R$$

$$\implies E + D - R^D D \geq \bar{\rho}(1 - L)(E + D) + \alpha L(E + D) - \alpha R^D D$$

Given that banks must break even in equilibrium, the safe strategy implies that $R^D = 1$. Hence, the inequality above can be re-expressed as follows:

$$\frac{D}{E} \leq \frac{1 - \alpha\rho^H + L\alpha(\rho^H - 1)}{\alpha(\rho^H - 1)(1 - L)}$$

Denote the term on the right hand side of the last inequality by d^L . Hence

$$d^L = \frac{1 - \alpha\rho^H + L\alpha(\rho^H - 1)}{\alpha(\rho^H - 1)(1 - L)} \quad (4.29)$$

Compare the critical leverage ratio with and without liquidity requirements. These correspond to d^L and d^* (from equation 4.17) respectively. Since d^L is increasing in L , then

$$d^L > d^*$$

In the presence of minimum liquidity requirements, the level of leverage above which financial institutions engage in risk-shifting is higher. Intuitively, liquidity requirements are a market-based device to insure deposits.

Further, households will no longer anticipate a scenario were they might lose their gold savings. This implies that trade in commodity markets will not collapse if a bad state of nature is realized. To illustrate better, assume that liquidity requirements insure a fraction V of deposits. Thus,

$$L \ni L(E + D) = VR^D D$$

Anticipating this, the optimization problem of the representative household, say the P type, becomes

$$\begin{aligned}
\max_{l_{s*}^P, c_{w,s*}^P, f_{s*}^P} U^P &= (c_{w,1}^P) - \frac{1}{2} (l_1^P)^2 \\
&+ \alpha \left[g_H^P + (c_{w,H}^P) - \frac{1}{2} (l_H^P)^2 \right] \\
&+ (1 - \alpha) \left[g_L^P + (c_{w,L}^P) - \frac{1}{2} (l_L^P)^2 \right]
\end{aligned} \tag{4.30}$$

where

$$\begin{aligned}
g_H^P &= R^H (e_g - f_1^P) + R^D (P_{p,1} l_1^P + P_{p,H} l_H^P - f_H^P) \\
g_L^P &= \tilde{\theta} R^D (P_{p,1} l_1^P + P_{p,L} l_L^P - f_L^P) + (1 - \tilde{\theta}) V R^D (P_{p,1} l_1^P + P_{p,L} l_L^P - f_L^P)
\end{aligned}$$

This objective function is subject to the set of constraints (4.2). Thus, households demand for liquidity is now given by

$$D = \frac{2 (R^D)^{1/3} \left[\alpha + (1 - \alpha) \left[\tilde{\theta} + (1 - \tilde{\theta}) V \right]^{1/2} \right]^{2/3}}{\bar{R}^{4/3}}$$

The equilibrium in commodity markets is characterized in terms of R^D , as well as the regulation parameters, L and V . Table 4.2 (below) summarizes the goods markets outcome.

$$\text{where } k = \alpha + (1 - \alpha) \left[\tilde{\theta} + (1 - \tilde{\theta}) V \right]^{1/2}.$$

When $\tilde{\theta} \approx 0$ and a bad state is realized, commodity prices do not soar to infinity and trade does not collapse, because $V > 0$ (liquidity requirements successfully insure a fraction V of gold deposits). In other words, if poorly capitalized banks invest in the risky asset and the gamble fails, money does not lose its value. Gold claims are still useful as a medium of exchange and, therefore, can still support trade. This result shows how liquidity requirements limit the feedback effects of bank failure to the real sector.

Table 4.2: Commodity Prices and Trade Levels given R^D , L and V

First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = \frac{(R^D)^{1/3} k^{2/3}}{\bar{R}^{1/3}}$	$P_{p,1} = P_{w,1} = \frac{1}{\bar{R}}$
Second Period - High State	
Trade/Labor	Prices
$l_H^P = l_H^W = \frac{(R^D)^{2/3} k^{1/3}}{\bar{R}^{2/3}}$	$P_{p,H} = P_{w,H} = \frac{k^{1/3}}{(R^D)^{1/3} \bar{R}^{2/3}}$
Second Period - Low State	
Trade/Labor	Prices
$l_L^P = l_L^W = \frac{[\tilde{\theta} + (1-\tilde{\theta})V]^{1/2} (R^D)^{2/3} k^{2/3}}{\bar{R}^{1/3}}$	$P_{p,L} = P_{w,L} = \frac{k^{1/3}}{[\tilde{\theta} + (1-\tilde{\theta})V]^{1/2} (R^D)^{1/3} \bar{R}^{2/3}}$

4.4.4 Chartering Policy

Several economists have shown that increased competition in deregulated financial markets worsen the moral hazard problem⁹. Thus, an alternative form of prudential regulation is to use barriers to entry to increase bank rents, or create charter value. This kind of measures can be thought of as tightening the requirements for the creation of new banks or branches. The value of the charter acts as a disciplinary device because, if it is high enough, bankers will choose not to gamble to avoid losing the rents therein. In the context of this model, bankers' profits determine the value of the charter.

Standard models of imperfect competition have long been applied to analyzing the banking industry¹⁰. I use the Monti-Klein model of monopolistic competition a la-Cournot. In this framework each bank is a (local) monopolist in the market for deposits. The results show that, by limiting the degree of competition in the financial industry, the charter value increases and, thus, risk-taking incentives are reduced.

⁹See Matutes and Vives (1996), Hellman et. al. 2000, and Boyd and DeNicolo (2005).

¹⁰See Frexias and Rochet (2008) for a literature review.

The Monopolistic Competition Model

Let s denote the fraction of households of each type that is served by one bank. Thus, $\frac{1}{s}$ represents the number of banks in the financial industry. If $s = 0$ banks are price-takers and the industry is perfectly competitive. Similarly, if $s = 1$ there is a single monopolist bank in the economy.

The total number of notes in circulation (D) is given by

$$D = sd_0 + (1 - s)\bar{d} \quad (4.31)$$

where d_0 is the quantity of notes issued by a representative bank, and $\bar{d}$ the average number of notes issued by the rest of the banks in the industry.

Assume banks engage in a Cournot-type of competition. Thus the market outcome is characterized by a Nash equilibrium. Moreover, since financial institutions are homogeneous, they implement the same strategy in equilibrium. Each bank takes as given the other banks' actions, but when making its capital structure and investment policy decisions, it acknowledges the impact of its own and its competitors' actions on the deposit rate. That is, each bank perceives the deposit market clearing condition to be as follows

$$\begin{aligned} D &= sd_0 + (1 - s)\bar{d} = K(R^D)^{(1/3)} \\ \Leftrightarrow R^D &= \left(\frac{D}{K}\right)^3 = \left(\frac{sd_0 + (1 - s)\bar{d}}{K}\right)^3 \end{aligned} \quad (4.32)$$

where $K = 2\bar{R}^{-(4/3)} \left[\alpha + (1 - \alpha)\tilde{\theta}^{(1/2)}\right]^{(2/3)}$

The optimization problem of the representative bank is, therefore, given by

$$\begin{aligned} \max_{d_0, I} \Pi^I &= \max \{\Pi^S, \Pi^R\} \\ &s.t. \\ \Pi^S &\geq E \\ \Pi^R &\geq E \\ R^D &= \left(\frac{sd_0 + (1 - s)\bar{d}}{K}\right)^3 \end{aligned}$$

where

$$\Pi^S = \max \{ (E + d_0) - R^D d_0, 0 \}$$

$$\Pi^R = \alpha [\max \{ \rho^H (E + d_0) - R^D d_0, 0 \}]$$

Prudent Behavior. First, examine the case where banks invest prudently. Each bank foresees the deposit market clearing condition to be

$$D_s = \frac{K}{(3s + 1)^{(1/3)}}$$

where D_s denotes the aggregate level of gold-deposits taken by the financial industry when banks choose to invest assets safely.

Hence, bank expected profits are given by

$$\Pi^S = E + \frac{3sK}{(3s + 1)^{(4/3)}}$$

As expected, if $s \rightarrow 0$, bank debt and profit levels tend to their perfect competition levels. Note that banks break-even condition only binds in the perfect competition case ($s = 0$). The expected pay-off from making prudent investments (Π^S) is increasing in s ¹¹. Hence, banks rents from investing prudently are higher when the financial industry is less competitive.

Risk-taking Behavior. If banks, on the other hand, engage in risk-taking behavior, their optimization problem is as follows

¹¹ $\frac{\partial \Pi^S}{\partial s} = 3K(3s + 1)^{(-4/3)}(1 - s) > 0$ for $s < 1$. The derivative reaches its maximum value of zero, when the financial industry is comprised by a single monopolist bank ($s = 1$).

$$\begin{aligned} \max_{d_0} \Pi^R &= \alpha [\max \{ \rho^H (E + d_0) - \alpha R^D d_0, 0 \}] \\ &s.t. \end{aligned}$$

$$\Pi^R \geq E \iff \frac{\alpha d_0 (\rho^H - R^D)}{1 - \alpha \rho^H} \geq E$$

$$R^D = \left(\frac{s d_0 + (1-s) \bar{d}}{K} \right)^3$$

Letting μ denote the multiplier of the break-even condition, the Lagrangian of this optimization problem can be written as

$$\mathcal{L} = \alpha \rho^H E + \alpha d_0 \left[\rho^H - \left(\frac{s d_0 + (1-s) \bar{d}}{K} \right)^3 \right] + \mu \left[\frac{\alpha d_0 (\rho^H - [s d_0 + (1-s) \bar{d}]^3 / K^3)}{1 - \alpha \rho^H} - E \right]$$

The first order condition is

$$\frac{\partial \mathcal{L}}{\partial d_0} = \alpha \rho^H - \frac{\alpha}{K^3} (3D^2 s d_0 + D^3) - \frac{\alpha \mu}{1 - \alpha \rho^H} \left(\frac{3D^2 s d_0 + D^3}{K^3} \right) = 0$$

The Nash equilibrium is symmetric. Therefore, in equilibrium $d_0 = \bar{d}$, which implies that $d_0 = D$. Evaluating the first order condition at $d_0 = D$ gives ¹²

$$\rho^H = \left(\frac{D}{K} \right)^3 \left[1 + \frac{\mu}{1 - \alpha \rho^H} \right] (3s + 1) \quad (4.33)$$

Case 1: $\mu > 0 \implies \Pi^R = E$

When the break-even condition binds ($\mu > 0$), the expected pay-off from gambling is

$$\Pi^R = \alpha \rho^H + \alpha \left[\rho^H - (D/K)^3 \right] D = E$$

D_μ is the equilibrium aggregate level of deposits in the financial industry, when banks engage in risky investments and $\mu > 0$. The latter satisfies

¹²That is, solving for $\left. \frac{\partial \mathcal{L}}{\partial d_0} \right|_{d_0=D} = 0$.

$$D_\mu \ni D_\mu [\rho^H K^3 - D_\mu^3] = EK^3 \left(\frac{1 - \alpha \rho^H}{\alpha} \right) \quad (4.34)$$

Since the right hand side of equation (4.34) is positive, so is the bracketed term on the left hand side.

$$(\rho^H K^3 - D_\mu^3) > 0 \implies D_\mu < K (\rho^H)^{1/3}$$

Let $\bar{D}_\mu = K (\rho^H)^{1/3}$. This represents the highest level of bank-debt that allows financial institutions to break-even, and make risky investments.

$$\text{Case 2: } \mu = 0 \implies \Pi^R > E$$

Let D_0 denote the equilibrium aggregate level of deposits, when banks engage in risky investments and $\mu = 0$. Using first order condition (4.33)

$$D_0 = K \left(\frac{\rho^H}{3s+1} \right)^{1/3} \quad (4.35)$$

Consequently, banks' expected pay-off from making risky investments is

$$\Pi^R = \alpha \rho^H E + 3\alpha s K \left(\frac{\rho^H}{3s+1} \right)^{(4/3)} > E$$

Lemma 4.4.1 *Given households conjecture about the likelihood that banks investing prudently $(\tilde{\theta})$, if banks make risky investments, there is a unique level of bank capital (E_0^R) such that*

$$\begin{aligned} \Pi^R &> E & \text{if } E < E_0^R \\ \Pi^R &= E & \text{if } E \geq E_0^R \end{aligned}$$

Proof If banks engage in risk-taking behavior, the break-even (complementary slackness) condition does not bind whenever

$$\begin{aligned} \Pi^R &= \alpha \rho^H E + 3\alpha s K \left(\frac{\rho^H}{3s+1} \right)^{(4/3)} > E \\ \implies \frac{3s\alpha K}{1 - \alpha \rho^H} \left(\frac{\rho^H}{3s+1} \right)^{(4/3)} &> E \end{aligned} \quad (4.36)$$

Denote with E_0^R the left-hand side of the inequality above. That is,

$$\frac{3s\alpha K}{1 - \alpha\rho^H} \left(\frac{\rho^H}{3s+1} \right)^{(4/3)} = E_0^R \quad (4.37)$$

Recall that

$$K = 2\bar{R}^{-(4/3)} \left[\alpha + (1 - \alpha) \tilde{\theta}^{(1/2)} \right]^{(2/3)}$$

For every $\tilde{\theta}$, there is a unique K . Therefore, equation (4.37) implies that, for every $\tilde{\theta}$, there is also a unique E_0^R . From, first order condition (4.33) and inequality (4.36), the desired result obtains

$$\begin{aligned} \mu = 0 &\implies \Pi^R > E && \text{if } E < E_0^R \\ \mu > 0 &\implies \Pi^R = E && \text{if } E \geq E_0^R \end{aligned}$$

QED

Note that E_0^R is increasing in s .¹³ In words, as the financial industry becomes less competitive (s increases), the critical level of bank capital below which gambling pays more than breaking-even, increases.

Moreover, since $\rho^H > 1$ and $\frac{1}{3s+1} < 1$, the following inequalities are satisfied

$$\begin{aligned} \bar{D}_\mu &> D_0 > D_s && \text{if } 1 \geq s > 0 \\ \bar{D}_\mu &= D_0 > D_s && \text{if } s = 0 \end{aligned}$$

This implies that there is risk-shifting. That is, banks undertake risky investments when they are more levered.

Proposition 4.4.2 *Given households' conjecture about the probability that banks invest safely $(\tilde{\theta})$, if $s > 0$ and*

- $\bar{\rho} \geq \alpha^{1/4}$, $\exists E_\mu^R < E_0^R$ such that

– Banks invest prudently when $E \geq E_\mu^R$

¹³ $\frac{\partial E_0^R}{\partial s} = \frac{3\alpha K(\rho^H)^{4/3}(3s+1)^{1/3}}{1-\alpha\rho^H} (1-s) \geq 0$.

– Banks make risky investments when $E < E_\mu^R$.

- $\bar{\rho} < \alpha^{1/4}$, banks invest prudently.

Proof Lemma 4.4.1 indicates that implementing a risky investment strategy leads to an outcome whereby the break-even condition may or may not bind. Therefore, the two cases need to be analyzed.

Assume first that $E < E_0^R$. If the bank undertakes a risky investment, by Lemma 4.4.1 $\Pi^R > E$. Using equation (4.36), the bank's expected payoff from gambling gives

$$\Pi^R = \rho^H E + 3\alpha s K \left(\frac{\rho^H}{3s+1} \right)^{(4/3)} > E$$

If, on the other hand, the bank invests safely, its payoff is

$$\Pi^S = E + \frac{3sK}{(3s+1)^{(4/3)}}$$

In equilibrium, the bank implements a safe investment policy whenever

$$\begin{aligned} \Pi^S &\geq \Pi^R \\ \implies E &\geq E_0^R - \frac{3sK}{1 - \alpha\rho^H} \left(\frac{1}{3s+1} \right)^{4/3} \\ \implies E &\geq \frac{3sK \left[\alpha (\rho^H)^{4/3} - 1 \right]}{(1 - \alpha\rho^H) (3s+1)^{4/3}} \end{aligned} \tag{4.38}$$

Let E_μ^R equal the right hand side of inequality (4.38)

$$E_\mu^R = \frac{3sK \left[\alpha (\rho^H)^{4/3} - 1 \right]}{(1 - \alpha\rho^H) (3s+1)^{4/3}} \tag{4.39}$$

If $\bar{\rho} \geq \alpha^{1/4} \implies \rho^H \geq \alpha^{-3/4}$, and the right hand side of inequality (4.38) is positive. Thus, banks invest prudently whenever their level of bank capital is at or above E_μ^R and engage in risk-taking behavior otherwise.

If $\bar{\rho} \geq \alpha^{1/4} \implies \rho^H < \alpha^{-3/4}$, (4.38) implies that $E_\mu^R < 0$. Since bank-capital is non-negative, banks invest prudently.

Suppose now that $E > E_0^R$. If the bank undertakes a risky investment, by Lemma 4.4.1 $\Pi^R = E$. If, on the contrary, the bank invests in the safe asset, its pay-off is equivalent to

$$\Pi^S = E + \frac{3sK}{(3s+1)^{(4/3)}}$$

Consequently, if $s > 0$

$$\Pi^S = E + \frac{3sK}{(3s+1)^{(4/3)}} > \Pi^R = E$$

That is, if banks can extract monopolistic rents ($s > 0$), they refrain from engaging in risk-taking behavior.

QED

This result is depicted in Figures 4.11 and 4.12 below.

Figure 4.11: Bank risk-taking behavior if $\bar{\rho} \leq \alpha^{1/4}$ and $s > 0$

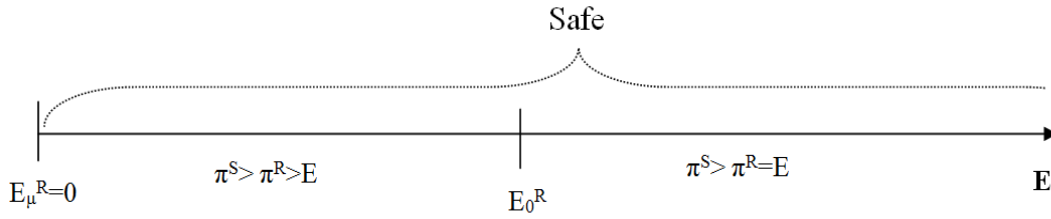
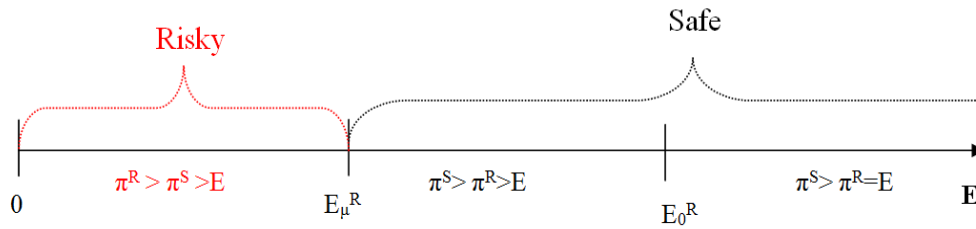


Figure 4.12: Bank risk-taking behavior if $\bar{\rho} > \alpha^{1/4}$ and $s > 0$



Is a non-competitive banking industry more beneficial to households? To answer this question one must compare banks behavior when competition is perfect ($s = 0$) and imperfect ($s > 0$). From proposition 4.4.2, it is straightforward to see that, when $\bar{\rho} \leq \alpha^{1/4}$ non-competitive banking always improves households welfare. Intuitively, if the gambling asset is very inefficient ($\bar{\rho} < \alpha^{1/4}$), the monopolistic rents banks can extract by gambling are lower than if they choose to invest safely. Therefore, banks always choose the risk-less strategy. What is more, as s increases, bankers' rents from playing safe increase, whereas the payoff from gambling remains constant and equal to E .

On the other hand, if ($\bar{\rho} \geq \alpha^{1/4}$) and the banks have a low level of capital $E < E_\mu^R$, the monopolistic rents from gambling are larger than the rents from implementing the safe strategy. Hence, in the latter case banks would undertake the risky strategy. However, there may still be cases where imperfect competition can improve households' welfare.

If, for a given $\tilde{\theta}$ (or K), the level of bank capital required to induce a bank to play safe (E_μ^R) under imperfect competition is lower than in the perfect competition case (E^*), then barriers to entry improve households welfare. That is, E^* and E_μ^R must satisfy

$$E^* > E_\mu^R$$

$$\text{where } E^* = \alpha K \left(\frac{\rho^H - 1}{1 - \alpha \rho^H} \right) \text{ and } E_\mu^R = 3sK \left[\frac{\alpha(\rho^H)^{4/3} - 1}{(1 - \alpha \rho^H)(3s + 1)^{4/3}} \right].$$

$$E^* > E_\mu^R \iff \alpha(3s + 1)^{4/3}(\rho^H - 1) - 3s[\alpha(\rho^H)^{4/3} - 1] > 0 \quad (4.40)$$

The inequality above holds for most parameter values, but it ceases to hold when $\alpha \rho^H = \bar{\rho} \rightarrow 1$ and/or $\alpha \rightarrow 0$. However, if the project is *almost* efficient ($\bar{\rho} \rightarrow 1$), the moral hazard problem nearly disappears. On the other hand, if the project is very risky ($\alpha \rightarrow 0$), banks will choose not to gamble in the perfectly competitive setup. In terms of the parameters of the model, $E^* \rightarrow 0$ so even poorly capitalized banks will invest prudently.

In words, although imperfect competition does not eliminate the moral hazard problem when ($\bar{\rho} \geq \alpha^{1/4}$), there is scope for welfare improving barriers to entry. This is reflected in the fact that, relative to the perfectly competitive setup, the number of cases where bank risk-shifting takes place is smaller if $s > 0$.

4.4.5 Ownership Structure

This section considers the possibility of regulating banks' ownership structure. When a firm is not organized as a conventional for-profit joint company, the alternatives include worker and consumer co-operatives, partnerships, and not for profit firms. In this chapter, I have used the theory of the firm developed by Hart and Moore (1990) to model banks behavior. Their framework relies in the concept of residual control rights over non-human assets, which besides outside ownership, explains the existence of cooperatives. Cooperatives reduce market distortions, such as those due to monopoly or externalities (see, for instance, Hart and Moore, 1996; and Renstrom and Yalcin, 2003).

In the banking industry cooperatives are known as mutual banks. Hansmann (1996) shows that mutual banks gained business at the expense of for profit rivals in the nineteenth century, because mutual banks were more able to control the moral hazard problem. Throughout the twentieth century increased regulation reduced the scope for moral hazard. In particular, widespread deposit insurance gave savers less reason to prefer mutual banks. As a result they became less common. This regulation was partly counter-productive because mutual banks had a significantly lower default rate during the US savings and loans crisis of the 1980s.

In this setup, mutual banks are owned by depositors. Since depositors are better-off under safe bank investment policies, mutual banks do not engage in risk-taking incentives.

The Mutual Bank Model

Assume there is a fixed number of mutual banks, $\frac{1}{s}$, where $0 < s < 1$. The limiting case where $s = 0$ can be interpreted as a situation where every household can set up his own bank. Suppose also that each bank offers the households in its constituency the following service: deposit an amount of gold up to f_0 and receive notes (or checks) equal to f_0 . Hence, f_0 is the bank's policy instrument. Further, f_0 is the same for all customers. Finally, assume the bank can commit to deliver the amount of liquidity it announces¹⁴.

Consider a single banks choice. The bank takes as given the fact that it serves a fraction s of the household population, and that the average choice of other banks is $\bar{f}$. Hence, the total value of notes in circulation, F , is given by

$$F = sf_0 + (1 - s)\bar{f} \tag{4.41}$$

¹⁴Note that if the market level of deposits, f^* , is smaller than f_0 the bank policy would be irrelevant.

The first term on the right hand side represents the contribution of this bank to the total money supply, and the second term the contribution of other banks.

The mutual bank chooses to maximize the utility of a representative member, by taking into account the effect of f_0 and $\bar{f}$ on prices. That is, mutual banks acknowledge that the demand for any good, say potatoes in period t , is given by

$$\frac{F}{P_{p,t}}$$

Hence, a mutual bank maximizes the utility of the representative household in its constituency and takes into account the effect of its deposit policy on prices. Since households are symmetric, the optimization problem of the representative financial institution is given by

$$\begin{aligned} \max_{f_0} U^P &= g^P + c_{w,1}^P - \frac{1}{2} (l_1^P)^2 + \alpha \left[c_{w,H}^P - \frac{1}{2} (l_H^P)^2 \right] \\ &+ (1 - \alpha) \left[c_{w,L}^P - \frac{1}{2} (l_L^P)^2 \right] \end{aligned}$$

As shown in Chapter 3, the equilibrium price and trade levels are the same in the good and bad states. Therefore, the state subscripts (H and L) are dropped and replaced with the period subscript 2.

$$\begin{aligned} c_{w,1}^P &= \frac{f_0}{P_1} \\ c_{w,2}^P &= \frac{sF}{P_2} \\ l_1^P &= \frac{P_1}{P_2} \\ l_2^P &= P_2 \\ P_{w,1} &= P_{p,1} = F^{3/4} \\ P_{w,2} &= P_{p,2} = F^{1/2} \end{aligned}$$

Substituting out prices and imposing the market clearing conditions, simplifies the mutual bank's problem as follows

$$\max_{f_0} U^P = \bar{R}(e_g - f_0) + \frac{f_0}{F^{3/4}} + F \left(\frac{3}{2} - s \right) + F^{1/2} \left(s - \frac{1}{2} \right) \quad (4.42)$$

The first order condition to this problem is given by

$$-\bar{R} + \frac{1}{F^{3/2}} \left(F^{3/4} - \frac{3}{4} s F^{-1/4} f_0 \right) + s \left(\frac{3}{2} - s \right) + \frac{1}{2} F^{-1/2} s \left(s - \frac{1}{2} \right) = 0$$

In equilibrium, f_0 is the same for all banks. That is, banks are identical, and the Nash equilibrium is symmetric. Hence $f_0 = \bar{f}$, which implies that $f_0 = F$. Evaluating the first order condition above at $f_0 = F$, gives the equilibrium level of deposits taken by each bank (F^*):

$$F^* \quad \ni \quad F^{*-3/4} \left(1 - \frac{3}{4} s \right) + s \left(\frac{3}{2} - s \right) + \frac{s}{2} F^{*-1/2} \left(s - \frac{1}{2} \right) = \bar{R} \quad (4.43)$$

Proposition 4.4.3 *Perfectly competitive mutual banks ($s = 0$) restore the safe market equilibrium, and a monopolistic mutual bank ($s = 1$) restores the social optimum level of liquidity.*

Proof When $s = 0$ equation (4.43) becomes

$$F^* = \bar{R}^{-4/3}$$

This equation coincides with the market equilibrium level of deposits when banks invest safely (see equation 4.23).

When $s = 1$ equation (4.43) becomes

$$\frac{1}{2} + \frac{1}{4} \left(F^{*-3/4} + F^{*-1/2} \right) = \bar{R}$$

This is the same solution a social planner would obtain if banks invest safely (see equation 4.24).

QED

In a mutual bank setup, there is no longer a conflict between the incentives of depositors and the owners of the bank. Hence, the level of liquidity generated by a depositor-mutually owned bank is the same as that provided by a safe financial institution. Chapters 2 and 3 develop the model in such a setup and show that, even in the absence of an agency problem, a competitive banking sector creates an inefficient level of liquidity. This is due to the fact that, price-taking depositors do not take into account the effects of their actions on commodity prices. What is more, the social optimum level of liquidity corresponds to the choice a planner would make by maximizing households' welfare and internalizing the effect of money holdings on whiskey and potato prices. The result above shows that only a monopolist

bank, mutually owned by all depositors in the economy, is able to internalize all these externalities. The reason is that, when there is no moral hazard, such a bank structure is tantamount to a central planner.

4.5 Concluding Remarks

This chapter introduced active financial institutions into the model developed in Chapter 3. Banks were conceived as the investment project of a group of *outside* shareholder-bankers. In this setup a moral hazard problem emerged, because bankers' payoffs were assumed to be protected by limited liability and bank investment policy to be non-verifiable. The results showed that more levered financial institutions have stronger incentives to engage in risk-taking behavior (bank risk-shifting). Furthermore, this agency problem proved to impose larger welfare losses on households than the costly creation of liquidity. Accordingly, various policy recommendations to solve moral hazard were considered.

Although the effects, advantages and limitations of each regulatory measure were discussed, I did not examine which policies, or policy combinations, could generate a Pareto-improvement. This is something I wish to explore in the future. Additional considerations for future research are discussed in the concluding section below.

Appendix: Market Equilibrium when Households have Consumer-Concave Preferences

When households are endowed with consumer-concave preferences, the optimization problem of the representative household, P , is given by

$$\begin{aligned} \max_{l_{s*}^P, c_{w,s*}^P, f_{s*}^P} U^P &= \ln(c_{w,1}^P) - \frac{1}{2} (l_1^P)^2 + \alpha \left[\ln(g_H^P) + \ln(c_{w,H}^P) - \frac{1}{2} (l_H^P)^2 \right] \\ &+ (1 - \alpha) \left[\ln(g_L^P) + \ln(c_{w,L}^P) - \frac{1}{2} (l_L^P)^2 \right] + S \end{aligned}$$

s.t.

$$\begin{aligned} P_{w,s*} c_{w,s*}^P &= f_{s*}^P & (\lambda_{s*}^P) \\ P_{p,s} l_s^P &\geq f_s^P & (\mu_s^P) \end{aligned}$$

where

$$\begin{aligned} g_H^P &= R^H (e_g - f_1^P) + R^D (P_{p,1} l_1^P + P_{p,H} l_H^P - f_H^P) \\ g_L^P &= \tilde{\theta} R^D (P_{p,1} l_1^P + P_{p,L} l_L^P - f_L^P) \end{aligned}$$

Further, $S > 0$. S is the amount of consumption households derive from a government insurance guarantee. This assumption is necessary to obtain an interior solution. Otherwise, households utility would be $-\infty$. In other words, since households are risk-averse, the risk of losing their entire wealth pushes them to a corner autarchic solution where they would refrain from depositing wealth in the bank and from trading.

After the Lagrange multipliers are substituted out and the budget constraints imposed, the first order conditions are given by

$$\begin{aligned}
P_{p,1} &= \left(\frac{\alpha}{f_H^P} + \frac{1-\alpha}{f_L^P} \right)^{-1} \\
g_H^P &= R^D P_{p,H} \\
g_L^P &= \tilde{\theta} R^D P_{p,L} \\
\alpha R^H f_1^P &= g_H^P \\
\mu_H^P &= \frac{\alpha}{f_H^P} - \frac{\alpha R^D}{g_H^P} \\
\mu_L^P &= \frac{1-\alpha}{f_L^P} - \frac{(1-\alpha) \tilde{\theta} R^D}{g_L^P}
\end{aligned}$$

In equilibrium the CS conditions bind in both states. In expectation, liquidity is costly. Thus, households are better-off by using the cash they have at hand, than by holding on to it. If a good state is realized, agents anticipate that, by symmetry, they will earn as much money as they spend in $t = 2$. Hence, it is optimal for them not to hoard liquidity. In the bad state, money turns out to be costless. Thus, agents can only benefit from spending it. This implies that $\mu_s^P > 0$ and

$$f_1^P = f_H^P = f_L^P \quad (4.44)$$

In words, households' demand for liquidity is constant across time and states. Therefore, the per-capita equilibrium level of liquidity (f^*) is given by

$$\alpha R^H f_1^P = g_H^P \quad \implies \quad f^* = \frac{\bar{R} e_g}{\bar{R}(1+\alpha) - \alpha R^D}$$

Using the remaining first order conditions, the equilibrium levels of trade and prices can be calculated. Table 4.3 (below) displays the results.

Provided that the government ensures a positive amount of deposits ($S > 0$), the market equilibrium has the same properties as in the case of leisure-concave preferences. In particular, note that in the bad states of nature, trade decreases with the probability that banks play risky ($\tilde{\theta}$), while prices rise.

Banks behavior is the same as in the model developed in the main text. However, the incentive compatibility condition changes, because the aggregate demand for deposits is now given by

$$D = \frac{2\bar{R} e_g}{\bar{R}(1+\alpha) - \alpha R^D}$$

Table 4.3: Market Equilibrium with Consumption-Concave Preferences

Deposit Market	
Liquidity Holdings in the Household Sector	
$f^* = \frac{\bar{R}e_g}{R(1+\alpha)-\alpha R^D}$	
First Period	
Trade/Labor	Prices
$l_1^P = l_1^W = 1$	$P_{p,1} = P_{w,1} = f^*$
Second Period, High State	
Trade/Labor	Prices
$l_H^P = l_H^W = \frac{1}{R}$	$P_{p,H} = P_{w,H} = R^H (e_g - f^*) + R^D f^*$
Second Period, Low States	
Trade/Labor	Prices
$l_L^P = l_L^W = \tilde{\theta}$	$P_{p,L} = P_{w,L} = \frac{f^*}{\tilde{\theta}}$

Using this equation and banks' incentive compatibility constraint, the proposition below follows.

Proposition A3.1. $\exists E^*$ such that

- If $E \geq E^*$, banks invest in the safe asset, and deposit contracts trade at par on $d = 0$.
- If $E < E^*$, banks invest in the inefficient gambling asset, and deposit contracts trade at a discount on $d = 0$.

Proof If depositors conjecture that banks will play safe, they effectively do so if

$$\Pi^S = E$$

$$\Pi^S \geq \Pi^R$$

This implies that $R^D = 1$ and

$$\bar{E} \geq \left[\frac{\alpha (\rho^H - 1)}{1 - \alpha \rho^H} \right] \left[\frac{2\bar{R}e_g}{\bar{R}(1 + \alpha) - \alpha} \right]$$

If, on the other hand, depositors conjecture that banks will engage in risk taking behavior, banks do so if

$$\Pi^R = E$$

$$\Pi^R > \Pi^S$$

The first equation implies that

$$R^D = R^H \left[1 + \alpha - \frac{2R^He_g}{D} \right]$$

Combining this equation with the demand for deposits and banks' break-even condition implies that $R^D > 1$. Moreover, D and R^D can be substituted out from inequality $\Pi^R > \Pi^S$, which gives

$$E < \left[\frac{\alpha (\rho^H - 1)}{1 - \alpha \rho^H} \right] \left[\frac{2\bar{R}e_g}{\bar{R}(1 + \alpha) - \alpha} \right]$$

letting

$$E^* = \left[\frac{\alpha (\rho^H - 1)}{1 - \alpha \rho^H} \right] \left[\frac{2\bar{R}e_g}{\bar{R}(1 + \alpha) - \alpha} \right]$$

concludes the proof of the proposition.

QED

As in the case of leisure-concave preferences, higher levels of liquidity raise bank leverage, and provides bank managers with incentives to gamble.

Remarks on Social Welfare Preferences

Chapters 2 and 3 showed that the results regarding the optimum level of liquidity were extremely sensitive to households preferences. In this setup, whenever the probability of banks engaging in risk-shifting is non-zero, this ambiguity is not an issue. This is due to the fact that, the welfare losses generated by the moral hazard problem outweigh the costs brought about by costly money creation. In other words, the planner always chooses to restrict the creation of means of payment in order to reduce banks' risk-taking incentives.

Note that the deposit insurance assumption was used as a device to obtain an interior equilibrium and to show its consistency with the case of leisure-concave preferences. Deposit insurance mitigates the feedback effects of bank risk-shifting, but they do not address the moral hazard problem. To illustrate, note that when assessing whether the costs generated by moral hazard outweigh the gains from a reduction in the cost of money (that is, by calculating $\Delta\mathcal{W} = \mathcal{W}^S - \mathcal{W}^R$ as in the main text), the deposit insurance term (S) cancels-out. Further, recall that whenever $S = 0$ and the probability that banks engage in risk-taking behavior is non-zero, households utility equals $-\infty$. Therefore, $\Delta\mathcal{W} = \infty$

In words, as long as the risk of bank risk-shifting is not immaterial, a central planner would always focus in policies that reduce banker's risk-taking incentives. This implies that all the results regarding the regulatory agenda proposed in the main text also apply in this case. Even the conclusions about bank ownership structure regulation are the same. The latter is sensitive to households preferences. Therefore, the case for a mutual bank when households have consumption-concave preferences is explored below.

Mutual Banks Behavior when Households have Consumer-Concave Preferences

The mutual bank maximizes the utility of the representative household in its constituency, taking into account the effect of its deposit policy on prices.

$$\begin{aligned}
 \max_{f_0} \quad U^P + U^W &= \ln(c_{w,1}^P) - l_1^P + \alpha [\ln(g_H^P) + \ln(c_{w,H}^P) - l_H^P] \\
 &+ (1 - \alpha) [\ln(g_L^P) + \ln(c_{w,L}^P) - l_L^P] \\
 &+ \ln(c_{p,1}^W) - l_1^W + \alpha [\ln(g_H^W) + \ln(c_{p,H}^W) - l_H^W] \\
 &+ (1 - \alpha) [\ln(g_L^W) + \ln(c_{p,L}^W) - l_L^W]
 \end{aligned}$$

$$\begin{aligned}
& s.t \\
c_{w,s^*}^P &= \frac{f_0}{P_{w,s^*}} \quad \forall \quad s^* \\
g_L^P &= F \\
g_H^P &= R^H (e_g - f_0) + F \\
P_{w,1} &= P_{p,1} = [s f_0 + (1-s) \bar{f}] = F \\
P_{w,L} &= P_{p,L} = [s f_0 + (1-s) \bar{f}] = F \\
P_{w,H} &= P_{p,H} = R^H (e_g - [s f_0 + (1-s) \bar{f}]) \\
&+ [s f_0 + (1-s) \bar{f}] = R^H (e_g - F) + F
\end{aligned}$$

If the banking sector is perfectly competitive ($s = 0$), the mutual bank's objective function becomes

$$\max_{f_0} U^P + U^W = \ln f_0 + \alpha \ln [R^H (e_g - f_0 + \bar{f})]$$

The first order condition of which is given by

$$f_0 = F = \frac{\bar{R} e_g}{\bar{R} (1 + \alpha) - \alpha} \quad (4.45)$$

This solution coincides with the market equilibrium in Chapter 3 (see equation 3.51).

On the hand, if the banking system is a perfect monopoly, the cooperative's objective function becomes

$$\max_{f_0} U^P + U^W = \ln f_0 - \frac{\alpha f_0}{R^H (e_g - f_0 + f_0)}$$

The first order condition is given by

$$f_0 \ni (R^H e_g)^2 - R^H e_g f_0 [2(R^H - 1) + \alpha] + [(R^H - 1) f_0]^2 = 0 \quad (4.46)$$

which coincides with the social optimum level of liquidity in Chapter 3 (see equation 3.53).

Chapter 5

Conclusion

This thesis developed a new framework of financial intermediation by building on the work by Hart and Zingales (2011). The first essay focused on the transactional function of money, and the role played by banks as liquidity providers. The results showed that, when agents cannot pledge their entire income stream to finance consumption, money is costly and prices become drivers of this distortion. Furthermore, the way these two externalities interact, and how they affect welfare, depends on households' preferences. This, in turn, determines whether the market produces an inefficiently low (or high) level of liquidity.

The second essay, examined the role played by financial institutions as providers of liquidity insurance. To this end, the model in Chapter 2 was adapted to a dynamic setting, and non-insurable idiosyncratic and aggregate risks were introduced. With these changes, the framework was able to explain the precautionary and speculative motives of money demand. In this context, the results regarding the over (or under) provision of liquidity, also proved to be sensitive to households' preferences.

The third and final essay introduced active financial institutions into the framework developed in Chapter 3. In this setup a moral hazard problem emerged because bankers' payoffs were assumed to be protected by limited liability and bank investment policy to be non-verifiable. This essay examined the role played by moral hazard on the supply and demand of liquidity in the economy. The results showed that, levered financial institutions have stronger incentives to engage in risk-taking behavior (risk-shifting). Further, the welfare losses emanating from bank risk-taking behavior proved to be larger than the costs generated by liquidity creation. Consequently, several regulatory measures to address the moral hazard problem were discussed.

The findings of this thesis could guide future empirical research on judging whether economies are experiencing an excessive (or limited) creation of liquidity. Further, the analysis can contribute to the

ongoing discussion on financial regulation.

Finally, the purpose of this thesis has been to lay down the foundations for a theory of financial intermediation that is capable of addressing the type questions that normally arise in times of financial distress. Yet, to get there more work needs to be done. Therefore, key topics on my research agenda going forward are discussed below.

Future Research

The model developed in this thesis is silent about the role played by the public sector as a provider of liquidity both, in normal times and during crises. Government money could be introduced into the model to mitigate the distortions generated by the costly creation of liquidity. However, as per explained by Cochrane (1998), the introduction of public liquidity would require the imposition of taxes, so that agents can pay them with the government's money. Hence, publicly supplied liquidity is only beneficial to the extent that the dead-weight losses generated by the tax charge are not too large. On the other hand, the government could be assumed to intervene only in the bad states of nature by injecting liquidity into the system. Here, another moral hazard problem could arise, because financial intermediaries may take too many risks in anticipation of a bailout.

A related issue is the analysis of how bailouts affect the government's budget and the real sector. As evidenced by the crises of 2007-2012, a significant part of the costs of a bailout can be borne by the government and the tax payers. Therefore, the origin and consequences of a "too-big-to-fail" problem are an important topic on my research agenda. When authorities confer a guarantee of survival to big banks for fear of causing severe troubles to the economy, they provide them with incentives to engage in risk-taking behavior. This increases the costs of a bailout to depositors (or the deposit insurance agency), which implies that chartering policies can backfire. Hence, strict closure rules could be a better tool to discipline banks.

In order to capture the "too-big-to-fail" problem, the time structure of the framework needs to be adapted, so that banks may be considered as ongoing concerns over many periods. A plausible approach would be to frame the optimization problem of the bank in a dynamic setting where its managers make a sequence of state-contingent investment and capital structure decisions. In addition, an indicator variable could be constructed, in order to specify whether the bank has gone bankrupt in a given state. For consistency purposes, the behavior of the household sector would also have to be specified in a longer time horizon. One possibility would be to adapt households' decision making process to an overlapping

generations framework.

By implementing these changes, the model should also be suitable for explaining other kinds of phenomena. These include moral hazard problems whereby banks engage in “short-termism” behavior or “gamble on resurrection”. Liquidity crisis episodes triggered by bank maturity miss-matches could also be modeled, and bank runs could take place. What is more, as shown by Diamond and Rajan (2000, 2001), the threat of a bank run could be considered as a market-based device to discipline banks. Further, with this dynamic structure, some parameters could be replaced with stochastic variables in order to model different types of shocks. This methodology would be a valid exercise to calibrate the parameters of the model to match actual data.

Another important extension to the model developed in this thesis, is to allow bank shareholders to choose the amount of resources they want put down as capital. One simple way of doing this is to assume that bank capital has an opportunity cost. This is a rather realistic assumption. If capital was costless, the pervasiveness of the moral hazard problem in the banking industry would not be an issue. Regulators would simply insure that banks hold sufficient capital to induce prudent investment, and banks would willingly comply.

But if the cost of capital is endogenously determined, there may be feed-back effects between this variable and regulatory policy interventions. This takes me to another point. Although the model developed in this thesis is general equilibrium, it still has an open end. Instead of assuming that bank capital is provided by outside investors, households could be allowed to buy shares of stock in financial institutions. In such a setting, one could examine whether bank capital is sufficiently large to support a prudent equilibrium. Since the return on equity would have to compensate shareholders for the lack of liquidity, bank capital may be scarce and lead to risk-shifting.

Finally, prudential regulation is meant to protect the banking system against moral hazard problems. As discussed in the last chapter, financial regulation can take many forms. Traditionally, it has consisted of a mixture of monitoring individual transactions, capital requirements, asset and entry restrictions. Since there are many potentially effective forms of intervention, future work should focus on determining the optimal form of prudential regulation. Following Hellman et.al. (2000) a Pareto frontier between any two policies could be built by fixing one of the tools at its critical level, (the level at which the bank is indifferent between gambling and investing prudently), and working out the values for the other tool that are consistent with prudent behavior. This kind of exercises could also be useful to underline the trade-offs between alternative regulatory measures.

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