

# International Trade in General Oligopolistic Equilibrium

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## Abstract

This paper presents a new model of oligopoly in general equilibrium and explores its implications for positive and normative aspects of international trade. Assuming “continuum–Pollak” preferences, the model allows for consistent aggregation over a continuum of sectors, in each of which a small number of home and foreign firms engage in Cournot competition. I show how competitive advantage interacts with comparative advantage to determine resource allocation and, specializing to continuum–quadratic preferences, I explore the model’s implications for the gains from trade, for the distribution of income between wages and profits, and for production and trade patterns in a two-country world.

## 1. Introduction

International markets are typically characterized by firms that are relatively large in the markets in which they compete. What are the implications of this undeniable fact for trade patterns, the gains from trade and the effects of trade policy on income distribution? These are some of the questions with which this paper is concerned.

Of course, these questions are not new. Since the late 1970s, they have been extensively addressed in the literature on the “new trade theory,” which has contributed enormously to our understanding of international markets. However, this literature really consists of two distinct strands that have relatively little in common with each other. On the one hand, general-equilibrium models of monopolistic competition have been applied to mostly positive questions of trade and location; on the other hand, partial-equilibrium models of oligopoly have been applied to mostly normative questions of “strategic” policy choice.<sup>1</sup> While both approaches have proved extremely fruitful, they suffer from some limitations. Models of monopolistic competition allow for increasing returns to scale and product differentiation in general equilibrium. However, since they assume that firms are atomistic and do not engage in strategic behavior, they represent little advance in descriptive realism over models of perfect competition. In particular, they sit uneasily with the growing evidence that a tiny

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proportion of firms account for the bulk of international trade, at least between developed countries;<sup>2</sup> and with recent theoretical and empirical work that suggests that idiosyncratic shocks to individual large firms do not cancel out and can instead generate aggregate fluctuations.<sup>3</sup> The assumption of instantaneous free entry and exit is also inconsistent with recent evidence that, at least in the short run, the bulk of adjustment to shocks occurs at the intensive margin within firms, rather than at the extensive margin of entry and exit.<sup>4</sup> Oligopoly models by contrast put large firms at center stage and allow for a wide range of sophisticated strategic interactions between them. However, since they typically ignore general-equilibrium interactions between markets, and especially between goods and factor markets, they cannot deal with many of the classic questions of international trade theory.<sup>5</sup>

This paper aims to advance the unfinished part of the new trade theory revolution, by developing a framework that should also have applications in other fields: a tractable but consistent model of oligopoly in general equilibrium. Previous attempts at this goal have encountered one of a number of related problems.<sup>6</sup> The essence of any oligopoly model is that firms have significant power in their own market and that they exploit this market power strategically. However, many attempts to model this formally have assumed that firms that are large in their own market are also large players in the economy as a whole, which opens a Pandora's Box of technical difficulties. For example, if firms are large in the economy, they influence the factor prices they face, in which case we would expect them to exercise this monopsony market power strategically. Moreover, if firms are large in the economy, they also influence national income, so they should take this too into account in their behavior. The resulting income effects often imply reaction functions that are extremely badly behaved, so that existence of equilibrium cannot be guaranteed even in the simplest models.<sup>7</sup> A more subtle difficulty is that, since firm owners influence the prices of their own outputs, they prefer lower rather than higher prices, the more they consume these goods. As a result, profit maximization leads to different outcomes depending on the tastes of profit recipients. Hence the apparent paradox that the properties of the model become sensitive to the choice of numéraire.<sup>8</sup>

Earlier writers have circumvented these difficulties either by ignoring them, or by explicitly modeling the simultaneous exercise of monopoly and monopsony power, or by assuming that firms maximize utility or shareholder wealth rather than profits.<sup>9</sup> None of these approaches has met with wide approval. In this paper I adopt a different approach. I assume that the economy consists of a continuum of sectors, each with a small number of firms. Factors are intersectorally mobile, so factor prices are determined at the economy-wide level. This makes it possible to model firms as having market power in their own industry but not in the economy as a whole. They behave strategically against their local rivals, but take income, prices in other sectors and factor prices as given. Profits are earned in equilibrium, but they are distributed to consumers in a lump-sum fashion. Hence the difficulties faced by other models of oligopoly in general equilibrium disappear.

Three technical building blocks are required to implement this approach, that views firms as "small in the large but large in the small." First, we need a specification of preferences and demand that is tractable at the sectoral level but also allows consistent aggregation over different sectors and agents. The Appendix draws on Pollak (1971) to characterize a general class of demand systems which meets these requirements, while Section 2 outlines the special case of continuum-quadratic preferences that we use to make explicit calculations of trade and welfare, and then shows how to measure welfare changes given these preferences. Second, we need to understand the implications of oligopolistic competition between firms located in different countries, which

differ in their cost structures. Even in partial equilibrium this requires considering the effects of market integration on production patterns. Section 3 extends the theory of oligopoly in open economies to consider these issues. Third, we need to link goods and factor markets in a consistent way. A natural framework in which to do this is the Ricardian continuum model of Samuelson (1964), in which each one of a continuum of sectors is assumed to have different costs at home and abroad. Whereas previous writers have explored this model under competitive assumptions, section 4 shows how it can form the basis for a model of general oligopolistic equilibrium.<sup>11</sup>

The remainder of the paper explores the model's implications for production and trade. Section 5 solves the model in autarky, while section 6 looks at free trade in a symmetric two-country world, and shows how trade and market structure affect welfare, income distribution and trade volumes. Section 7 considers small changes to a free-trade equilibrium in which countries need not be symmetric and shows how changes in competitive advantage interact with differences in comparative advantage to affect resource allocation and trade. Section 8 concludes, while the Appendix gives technical details and proofs of all results.

## 2. Preferences and Demand

### *Additive Separability*

The first desirable requirement of preferences is that they should allow a clear distinction between economy-wide and sector-specific determinants of demand. Following Dixit and Stiglitz (1977), a natural way to do this is to assume that preferences are additively separable. Thus the utility of a typical household (with household superscripts omitted for convenience) is defined as an additive function of the consumption levels of a continuum of goods defined on the unit interval:

$$U[\{x(z)\}] = \int_0^1 u[x(z), z] dz, \quad \frac{\partial u}{\partial x(z)} > 0, \quad \frac{\partial^2 u}{\partial x(z)^2} < 0. \quad (1)$$

This is to be maximized subject to the budget constraint:

$$\int_0^1 p(z)x(z)dz \leq I \quad (2)$$

where  $I$  is household income. Additive separability of the utility function has the key implication that the inverse demand for each good depends only on its own quantity and on the marginal utility of income  $\lambda$ :

$$p(z) = \lambda^{-1} \frac{\partial u[x(z), z]}{\partial x(z)}. \quad (3)$$

Following Browning et al. (1985), we call these “Frisch” demand functions, and their simple form makes it possible to model consistent oligopoly behavior in general equilibrium.<sup>12</sup> We assume that in each sector  $z$  there is a small number of firms producing a homogeneous good. They compete against their local rivals and take account of the endogenous determination of the price  $p(z)$ . The firms take the marginal utility of income as given, whereas it is endogenous in the economy as a whole. Moreover, the marginal utility of income serves as a “sufficient statistic” for the rest of the economy

in each sector. From the continuum assumption, individual firms are infinitesimally small in the overall economy and so it is fully rational for them to treat  $\lambda$  as fixed.

The distinction between (3) with  $\lambda$  parametric and with  $\lambda$  endogenously determined corresponds to the distinction between “perceived” and “actual” demand functions in the general-equilibrium formalization of Chamberlin (1933) by Negishi (1961). This approach to modeling demands is formally identical to the one used in monopolistically competitive models since Dixit and Stiglitz (1977) and Krugman (1979). However, the interpretation is very different. In monopolistic competition each good is produced by a single firm and all firms together constitute a single industry with the number of firms endogenous; whereas here there is a continuum of industries, with more than one firm in each industry and barriers to entry that prevent the emergence of either new firms or new industries.

### *Continuum-Pollak and Continuum-Quadratic Preferences*

Additive separability is a desirable prerequisite for a tractable model of oligopoly in general equilibrium, but it is not sufficient, at least in a multi-country context where we cannot assume the existence of a representative world consumer. We want to be able to aggregate demands consistently over individuals and countries. This requires that preferences be quasi-homothetic, so the expenditure function takes the Gorman (1961) polar form:

$$e[\{p(z)\}, u] = f[\{p(z)\}] + u g[\{p(z)\}] \quad (4)$$

where the functions  $f[\{p(z)\}]$  and  $g[\{p(z)\}]$  are linearly homogeneous in prices and so can be interpreted as price indexes:  $f$  is the “base-level” price index, corresponding to an arbitrary base indifference curve indexed by  $u = 0$ , and  $g$  is the “marginal” price index.

The two restrictions, additive separability (1) and quasi-homotheticity (4), are mutually independent. Fortunately, the set of utility functions that meets both restrictions has been characterized by Pollak (1971), assuming a discrete number of goods. His main result is restated for the case of a continuum of goods in the Appendix. Many of the qualitative results presented below hold for this “continuum–Pollak” specification of preferences. However, to obtain explicit solutions and, in particular, to carry out a global comparison between autarky and free trade, we specialize to the case of continuum–quadratic preferences:

$$U[\{x(z)\}] = \int_0^1 u[x(z)] dz \quad \text{where} \quad u[x(z)] = ax(z) - \frac{1}{2}bx(z)^2. \quad (5)$$

This implies that the inverse and direct demand functions for each good are linear conditional on  $\lambda$ :

$$p(z) = \frac{1}{\lambda} [a - bx(z)] \quad \text{and} \quad x(z) = \frac{1}{b} [a - \lambda p(z)]. \quad (6)$$

Solving for  $\lambda$  in this case gives:<sup>13</sup>

$$\lambda[\{p(z)\}, I] = \frac{a\mu_1^p - bI}{\mu_2^p}. \quad (7)$$

The effects of prices on  $\lambda$  are summarized by two functions,  $\mu_1^p$  and  $\mu_2^p$ , which are the first and second moments of the distribution of prices:

$$\mu_1^p \equiv \int_0^1 p(z) dz \quad \text{and} \quad \mu_2^p \equiv \int_0^1 p(z)^2 dz. \quad (8)$$

Hence, a rise in income, a rise in the uncentered variance of prices, or a fall in the mean of prices, all reduce  $\lambda$  and so shift the demand function for each good outwards.

Like all members of the Gorman polar form family, quadratic preferences as in (5) imply that all income–consumption curves are linear (though not necessarily through the origin) so tastes are homothetic at the margin. Hence they allow for consistent aggregation over individuals, or, in a trade context, countries, with different incomes, provided the parameter  $b$  is the same for all.<sup>14</sup> In particular, in a two-country world, if the foreign country's preferences are represented by (5), with  $a^*$  instead of  $a$ , and if free trade prevails so prices are the same in both countries, then world demands are:

$$p(z) = a' - b' \bar{x}(z) \quad \text{and} \quad \bar{x}(z) \equiv x(z) + x^*(z) = \frac{1}{b} [\bar{a} - \bar{\lambda} p(z)]. \quad (9)$$

Here  $\bar{a} \equiv a + a^*$  is the world direct demand intercept, and  $\bar{\lambda} \equiv \lambda + \lambda^*$  is the world marginal utility of income. World demands depend on total world income  $\bar{I} \equiv I + I^*$ , where  $I^*$  is foreign income, but not on its distribution between countries or between wages and profits. Finally, the parameters in the world inverse demand function, taken as given by firms but endogenous in general equilibrium, are:  $a' \equiv \frac{\bar{a}}{\bar{\lambda}}$  and  $b' \equiv \frac{b}{\bar{\lambda}}$ .

### Measuring Welfare Change

How to compare welfare between two equilibria,  $A$  and  $B$ , which differ by a finite amount? The problem is more complicated than in many trade models, for a number of reasons. First, unlike cases where the elasticity of trade is constant, as in Arkolakis et al. (2012), we cannot integrate underneath a given import demand function. Second, unlike models where preferences are homothetic, we have to face up to the fact that the quantitative magnitude of welfare change depends on the reference prices used; in particular, it differs depending on whether we adopt an equivalent-variation approach, using *ex ante* prices as reference, or a compensating-variation approach, using *ex post* prices as reference. Finally, unlike partial-equilibrium models, we have to take account of the fact that income is endogenous. In what follows, we first review how to deal with these problems in general, and then outline a convenient short-cut that can be used in general equilibrium when preferences are quadratic.

We begin with the expenditure function, which is monotonically increasing in utility:

$$u^B > u^A \iff e(p, u^B) > e(p, u^A) \text{ for any } p. \quad (10)$$

In particular, if we choose as reference prices those corresponding to the initial equilibrium  $A$  (which, in a trade context, we can think of as the autarky equilibrium), we can define our measure of welfare change as follows:

$$\Delta e^{AB} \equiv e(p^A, u^B) - e(p^A, u^A). \quad (11)$$

This is related to, though not the same as, the equivalent variation, the amount that an individual with income  $I^A$  facing prices  $p^A$  would be willing to pay in order

to avoid a change such that the new price vector is  $p^B$  and her income is  $I^B$ :  $EV_{AB} = e(p^A, u^B) - e(p^B, u^B)$ . To see how the two are related, subtract income in the new equilibrium,  $I^B = e(p^B, u^B)$ , from both sides of (11):

$$u^B > u^A \iff \Delta e^{AB} > 0 \iff EV_{AB} > I^A - I^B. \quad (12)$$

This gives what Dixit and Weller (1979) call “the basic test for utility increase in going from  $A$  to  $B$ : the gain in consumer’s surplus should exceed any loss in lump-sum income.”

All this holds for any specification of preferences. When preferences exhibit the Gorman polar form,  $e(p, u) = f(p) + ug(p)$ , the expression for welfare change simplifies greatly:

$$\Delta e^{AB} = (u^B - u^A)g(p^A). \quad (13)$$

In particular, with continuum-quadratic preferences, the marginal price index  $g(p)$  is the uncentered standard variation of prices  $(\mu_2^p)^{1/2}$ . (See the Appendix.) It follows from (13) that a quantitative assessment of welfare change requires only that we evaluate the difference  $u^B - u^A$ , a short-cut that is not available in general for preferences other than the Gorman polar form, as (11) makes clear.

The final step in operationalizing this approach is to relate the levels of utility,  $u^A$  and  $u^B$ , to the properties of the equilibrium. The standard way of doing this is to relate utility to quantities consumed in order to calculate from (11) a money metric measure of welfare change:  $e[p^A, u(q^B)] - e[p^A, u(q^A)]$ . However, with quadratic preferences, it is more convenient to use what Mrázová and Neary (2014) call the “Frisch indirect utility function,” utility as a function of prices and the marginal utility of income. To derive this, substitute from the direct Frisch demand functions in (6) into the direct utility function (5):

$$\begin{aligned} V^F[\lambda, \{p(z)\}] &= \int_0^1 x(z) \left[ a - \frac{1}{2}bx(z) \right] dz \\ &= \frac{1}{2b} \int_0^1 [a - \lambda p(z)][a + \lambda p(z)] dz = \frac{1}{2b} \int_0^1 [a^2 - \lambda^2 p(z)^2] dz \\ &= \frac{1}{2b} (a^2 - \lambda^2 \mu_2^p), \quad \mu_2^p \equiv \int_0^1 p(z)^2 dz. \end{aligned} \quad (14)$$

Ignoring the constant term, this implies that utility equals minus the squared marginal utility of income times the second moment of prices:

$$\tilde{U} = -\lambda^2 \mu_2^p \text{ where: } \tilde{U} \equiv 2bV^F - a^2. \quad (15)$$

This is the most convenient way of evaluating consumer welfare in many applications. In practice, as we will see below, the values of nominal variables, including  $\lambda$ , are indeterminate in general equilibrium. Hence we can choose  $\lambda$  as numéraire, so welfare is just minus the second moment of prices.

### 3. Specialization Patterns in an International Oligopoly

Turning from demand to supply, consider the determination of equilibrium in a single international oligopolistic sector, with foreign variables denoted by an asterisk. It is

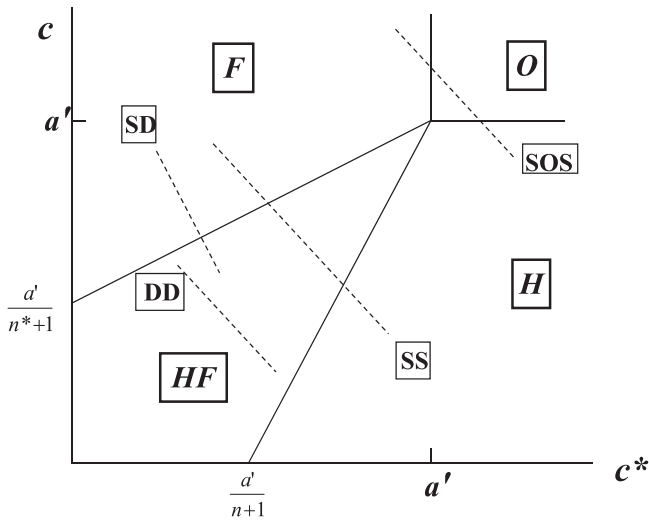


Figure 1. Illustrative Equilibrium Configurations for Given  $n$  and  $n^*$

convenient to suppress the sector index  $z$  in this subsection only. Firms are Cournot competitors, choosing their outputs on the assumption that their rivals (both home and foreign) will keep theirs fixed.<sup>15</sup> In addition, there are barriers facing new firms, so oligopoly rents are not eroded by entry. Of course, some incumbent firms may choose to produce zero output in equilibrium, effectively dropping out of the market if they cannot make positive profits.

Assume that international markets are fully integrated and there are no transport costs or other barriers to international trade, so the same price prevails at home and abroad. For simplicity we also assume that there are no fixed costs of production. Firms face a given world inverse demand function: (9) in the case of quadratic preferences, (A6) in the Appendix in the general case of continuum–Pollak preferences. They take costs and the marginal utility of income as given, but exercise market power in their own sector. We assume a given number  $n$  of home firms, all of which have the same marginal cost  $c$ , so all home firms have the same equilibrium output, denoted by  $y$ . Similarly, there is a given number  $n^*$  of foreign firms, all with the same marginal cost  $c^*$  and the same equilibrium output  $y^*$ . Market clearing implies that total sales to both home and foreign consumers equal the sum of total production by home and foreign firms:  $\bar{x} = \bar{y} \equiv ny + n^*y^*$ .

We wish to understand how specialization patterns depend on home and foreign marginal costs. We do this by identifying different specialization regions in  $\{c, c^*\}$  space, as shown in Figure 1.<sup>16</sup> The profits of a typical home firm depend on its own and its foreign rivals' marginal costs, and on the number of firms of each type:  $\pi(c, c^*; n, n^*)$ . This allows us to define two threshold levels of the home marginal cost  $c$ , for given numbers of home and foreign firms  $n$  and  $n^*$ . The first of these is where home profits are zero when there are no active foreign firms:  $\pi(c, c^*; n, 0) = 0$ . This defines a threshold  $c$  above which home production is unprofitable even in the absence of foreign competition (since  $\pi_{c^*} = 0$  when  $n^* = 0$ ). For demand functions with no choke price this threshold is infinite: in the absence of fixed costs, home firms can produce profitably at any cost short of infinity. The second threshold is where home profits are zero when foreign firms are active:  $\pi(c, c^*; n, n^*) = 0$ . This defines a locus that



gives the threshold home marginal cost as a function of the foreign marginal cost, when both types of firms are active. The slope of this equals  $\frac{dc}{dc^*} = -\frac{\pi_{c^*}}{\pi_c}$ . Since profits are always decreasing in a firm's own costs, this locus is upward-sloping provided the cross-effect of rivals' costs is positive,  $\pi_{c^*} > 0$ . It is possible to find examples where this cross-effect is negative (see the appendix to Neary, 2002b), but they can reasonably be ruled out as implausible. Similarly, in most cases we expect the effect of own costs to dominate that of rivals' costs, so  $\pi_{c^*} < -\pi_c$ , implying that the zero-profit locus is less steeply-sloped than the 45° line:  $\frac{dc}{dc^*} < 0$ .

Combining these conditions with similar restrictions on the foreign firms allows us to illustrate the possible equilibria in  $\{c, c^*\}$  space, for given  $n$  and  $n^*$ . The solid lines indicate the boundaries of the regions in which the market is served by firms from both countries (denoted “HF”), from one (denoted “H” and “F”) or from none (denoted “O”). (The dashed lines will be considered in the next section.) Region O can only exist if demand has a choke price: it would not arise with CES preferences for example. Region HF is of particular interest: it is a “cone of diversification”, in which high-cost producers in one country co-exist with lower-cost producers in the other. The reason for this is the persistence of barriers to entry. In the competitive limit with completely free entry by atomistic firms, as both  $n$  and  $n^*$  tend towards infinity, the cone collapses to the 45° line and the model becomes identical to that of Dornbusch et al. (1977).

In the case of quadratic preferences, we can go further and characterize fully the loci in Figure 1. To do this, we need explicit expressions for outputs and prices: calculating these in partial equilibrium is straightforward and the results are given in Table 1. Inspecting the first row of this table, we see that home firms will produce positive output only in one of two circumstances: either  $c$  must be below  $a'$  when foreign firms are not active, or  $c$  must be below  $\frac{a' + n^*c^*}{n^* + 1}$  when they are active.

Before proceeding, consider the effects of an increase in  $n$ , the number of home firms, on the outputs of foreign firms. From the expressions for foreign output in the second row of Table 1, foreign output falls. In addition, the locus separating the HF and H regions in Figure 1 shifts to the left, as foreign firms exit marginal sectors. In

Table 1. Equilibrium Outputs, Prices, Demands and Net Imports ( $m$ ) in Different Oligopoly Regimes with Quadratic Preferences

Regime:	H [ $n > 0, n^* = 0$ ]	HF [ $n > 0, n^* > 0$ ]	F [ $n = 0, n^* > 0$ ]
$y$	$\frac{a' - c}{b'(n+1)}$	$\frac{a' - (n^* + 1)c + n^*c^*}{b'(n + n^* + 1)}$	0
$y^*$	0	$\frac{a' - (n+1)c^* + nc}{b'(n + n^* + 1)}$	$\frac{a' - c^*}{b'(n^* + 1)}$
$\bar{x} = \bar{y} = ny + n^*y^*$	$n \frac{a' - c}{b'(n+1)}$	$\frac{\bar{n}a' - nc - n^*c^*}{b'(n + n^* + 1)}$	$n^* \frac{a' - c^*}{b'(n^* + 1)}$
$p = a' - b'\bar{x}$	$\frac{a' + nc}{n+1}$	$\frac{a' + nc + n^*c^*}{n + n^* + 1}$	$\frac{a' + n^*c^*}{n^* + 1}$
$x = \frac{1}{b}(a - \lambda p)$	$\frac{(n+1)a - \lambda(a' + nc)}{b(n+1)}$	$\frac{(n + n^* + 1)a - \lambda(a' + nc + n^*c^*)}{b(n + n^* + 1)}$	$\frac{(n^* + 1)a - \lambda(a' + n^*c^*)}{b(n^* + 1)}$
$m = x - ny$	$-\frac{(n+1)a^* - \lambda^*(a' + nc)}{b(n+1)}$	$\frac{\left(n^* + \frac{\lambda^*}{\lambda}\right)(a + n\bar{\lambda}c) - \left(n + \frac{\lambda}{\bar{\lambda}}\right)(a^* + n^*\bar{\lambda}c^*)}{b(n + n^* + 1)}$	$x$



line with the Ricardian nature of the model, we can say that foreign production contracts at both the intensive and extensive margins. This perspective will prove useful in understanding the model's properties.

#### 4. Linking Factor and Goods Markets

To embed the sectoral structure from the last section in general equilibrium, we need to specify how costs are determined. As explained in the introduction, we assume a Ricardian cost structure. Each sector requires an exogenously fixed labor input per unit output, denoted  $\alpha(z)$  and  $\alpha^*(z)$  in the home and foreign countries respectively. Hence the unit costs in sector  $z$  are:

$$c(z) = w\alpha(z), \quad c^*(z) = w^*\alpha^*(z). \quad (16)$$

Here  $w$  and  $w^*$  denote the wages in each country, which are common across sectors, and determined by the condition that labor demand and supply are equal.

What restrictions do we need to impose on the  $\alpha(z)$  and  $\alpha^*(z)$  functions? As in all applications of the continuum model, we make the mild technical restriction that they are continuous in  $z$ . In addition, we suppose (without loss of generality) that goods are ordered such that the home country is more efficient at producing goods with low values of  $z$ . In the diagrams below it is implicitly assumed that  $\alpha(z)$  and  $\alpha^*(z)$  are respectively increasing and decreasing in  $z$ , but this is far stronger than needed. Dornbusch et al. (1977) assume that the ratio  $\alpha(z)/\alpha^*(z)$  is increasing in  $z$ , but this is not sufficient to ensure that the model is well behaved. The precise condition we need is:

**ASSUMPTION 1.** *When both home and foreign firms operate,  $y(z)$  and  $y^*(z)$  are respectively decreasing and increasing in  $z$ .*

Lemma A1 in the Appendix relates this to underlying parameters, and Lemma A2 shows that it reduces to the Dornbusch–Fischer–Samuelson assumption in the competitive limit.

The assumptions we have made ensure that, for given technology and wage rates, there is a functional relationship between the home and foreign unit costs in each sector. Moreover, Assumption 1 ensures that in Figure 1 the locus representing this relationship cuts each boundary of the  $HF$  region at most once; otherwise it can take any form. The dashed lines in Figure 1 illustrate some possible equilibrium configurations. For example, the line labeled  $SS$  illustrates an equilibrium in which both countries are partly specialized. In this case, there are two threshold sectors,  $\tilde{z}$  and  $\tilde{z}^*$ , with  $0 < \tilde{z}^* < \tilde{z} < 1$ . All sectors for which  $z$  is less than  $\tilde{z}$  are competitive in the home country; all sectors for which  $z$  is greater than  $\tilde{z}^*$  are competitive in the foreign country; and home and foreign firms co-exist in sectors between  $\tilde{z}^*$  and  $\tilde{z}$ .

Other equilibrium configurations are also illustrated in Figure 1. The line labeled  $DD$  denotes one in which both countries are fully diversified, producing all goods; that labeled  $SD$  denotes one in which the home country is partly specialized and the foreign country fully diversified; and that labeled  $SOS$  denotes one in which both countries are partly specialized but some goods are not produced in either. (The latter case is a curiozum and will not be considered further.) Hence it is possible for either  $z$  to equal one and/or  $z^*$  to equal zero. Which of these cases obtains is determined endogenously as part of the full general equilibrium of the model, to which we now turn. We begin by considering the determination of equilibrium in autarky.

## 5. General Oligopolistic Equilibrium: Autarky<sup>17</sup>

To close the model of a single isolated economy we need only one further condition: the home labor market must clear:

$$L = \int_0^1 \alpha(z) n y(z) dz. \quad (17)$$

This equates the supply of labor  $L$ , assumed exogenous, to the aggregate demand for labor, which is the sum of labor demand from all sectors. The model is now easily solved. We can eliminate firm output  $y(z)$  from (17) using the expression in the first row of Table 1, and then use the first equation in (16) to eliminate  $c(z)$ . Making these substitutions gives:

$$L = \frac{n}{b(n+1)} \int_0^1 \alpha(z) [a - \lambda w \alpha(z)] dz. \quad (18)$$

This equation has only a single unknown, the product  $\lambda w$ , which is the consumer's real wage at the margin. Changes in the units in which nominal magnitudes are measured lead to equal and opposite changes in the values of  $\lambda$  and  $w$ , but no change in  $\lambda w$ .

Evaluating the integral in (18), we can solve for the equilibrium marginal real wage in autarky:

$$w_a \equiv (\lambda w)_a = \left( a\mu_1 - \frac{n+1}{n} bL \right) \frac{1}{\mu_2} \quad (19)$$

where  $\mu_1$  and  $\mu_2$  denote the first and second moments of the home technology distribution:

$$\mu_1 \equiv \int_0^1 \alpha(z) dz \quad \text{and} \quad \mu_2 \equiv \int_0^1 \alpha(z)^2 dz. \quad (20)$$

From (19), the wage in autarky is increasing in  $n$  and  $\mu_1$  and decreasing in  $L$  and  $\mu_2$ .

To calculate welfare in autarky, recall from equation (15) that it depends inversely on the second uncentered moment of the price distribution. The latter can be calculated explicitly by using the Cournot equilibrium price formula from Table 1, and by evaluating  $\mu_2^p$  from (8) to obtain:

$$\tilde{U}_a = -(\lambda^2 \mu_2^p)_a = -\frac{1}{(n+1)^2} (a^2 + 2an\mu_1 w_a + n^2 \mu_2 w_a^2). \quad (21)$$

So a higher wage raises the second moment of prices and hence lowers welfare. Substituting from (19) for  $w_a$ , this can be expressed in terms of underlying parameters as follows (details are given in the Appendix):

$$\tilde{U}_a = -\frac{a^2}{(n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2} \quad (22)$$

where  $\sigma^2$  is the variance of the home technology distribution:

$$\sigma^2 \equiv \int_0^1 [\alpha(z) - \mu_1]^2 dz = \mu_2 - \mu_1^2. \quad (23)$$

Equation (22) shows that autarky welfare is increasing in  $n$ . This is a familiar competition effect, but it has to be qualified in general equilibrium: the competition effect is stronger the greater the variance of costs across sectors,  $\sigma^2$ , and it is zero if all sectors have identical costs ( $\sigma^2 = 0$ ), the case called the “featureless economy” in Neary (2003c). Increased competition in all sectors raises the aggregate demand for labor, but the general-equilibrium constraint of full employment means that output can only increase if labor is reallocated from less to more efficient sectors. When all sectors are identical, this channel is blocked off, and so the welfare costs of imperfect competition vanish: a result first pointed out by Lerner (1934).

Equation (22) also implies that a mean-preserving spread in the distribution of costs raises autarky welfare:  $\bar{U}_a$  is increasing in  $\sigma^2$  for given  $\mu_1$ . This reflects two conflicting effects. On the one hand, from (5), consumers dislike heterogeneous consumption levels, and hence from (15) they dislike heterogeneous prices, so a rise in  $\sigma^2$  tends to reduce welfare at given wages. On the other hand, more heterogeneous technology across sectors implies from (19) a fall in the wage, which reduces the second moment of prices from (21), and hence tends to raise welfare. It can be checked that the second effect dominates, so welfare is increasing in  $\sigma^2$ .<sup>18</sup>

## 6. Free Trade with Symmetry and Full Diversification

Consider next a free-trade equilibrium in which both countries are fully diversified. Assume also that the countries are symmetric in the sense that they are the same size:  $L = L^*$ ; have the same tastes:  $a = a^* = \frac{1}{2}\bar{a}$ ; the same industrial structure:  $n = n^*$ ; and the same technology moments:  $\mu_1 = \mu_1^*$  and  $\mu_2 = \mu_2^*$  (where  $\mu_1^*$  and  $\mu_2^*$  are defined analogously to the home moments in (20)). In equilibrium they therefore have the same marginal utility of income:  $\lambda = \lambda^* = \frac{1}{2}\bar{\lambda}$ ; and the same wage:  $w = w^*$ .

Although the countries are symmetric, they are not necessarily identical. As we will see, a key role is played by the difference between their technology distributions. To parameterize this difference, we first define the “uncentered” covariance  $\gamma$  of the two technology distributions as:

$$\gamma \equiv \int_0^1 \alpha(z)\alpha^*(z)dz \quad (24)$$

while we define the “centered” covariance  $\omega$  as:

$$\omega \equiv \int_0^1 [\alpha(z) - \mu_1][\alpha^*(z) - \mu_1^*]dz = \gamma - \mu_1\mu_1^*. \quad (25)$$

Using the standard property that  $\mu_2 + \mu_2^* \geq 2\gamma$ , so with symmetry  $\mu_2 \geq \gamma$ , we can now define  $\delta$ :

$$\delta \equiv \mu_2 - \gamma = \sigma^2 - \omega \quad (26)$$

as a measure of the technological dissimilarity between the two countries, or simply as a measure of comparative advantage. Only when  $\delta$  attains its minimum value of zero, so comparative advantage is zero, are the two countries identical.

The labor-market equilibrium condition is identical to that in autarky, equation (17), except that the expression for output now comes from the central column of Table 1. This differs in two respects from the autarky case. First, home firms now face

competition from foreign firms in all markets. Second, the size of the market has increased; this is reflected in the fact that the slope of the perceived inverse demand function,  $b'$ , has fallen: it now equals  $\frac{b}{\lambda} = \frac{b}{2\lambda}$  instead of  $\frac{b}{\lambda}$ . Making these substitutions into (17), integrating and solving as in the previous section, we can derive the wage in both countries:

$$w_f \equiv (\lambda w)_f = \left( a\mu_1 - \frac{2n+1}{2n} bL \right) \frac{1}{\mu_2 + n\delta}. \quad (27)$$

Comparing this with the autarky wage (19), there are three sources of difference, which we can identify with a market size effect, a competition effect and a comparative advantage effect. The market size effect, represented by the term  $-\frac{1}{2n}$  in (27) in place of  $-\frac{1}{n}$  in (19), tends to raise the wage: doubling the number of consumers raises the demand for labor and so the equilibrium wage in both countries. By contrast, the competition effect, represented by the term  $-(2n+1)$  in (27) in place of  $-(n+1)$  in (19), tends to reduce the wage, as firms face more competitors in their home market and so scale down their sales there. Taken together, the market size effect dominates the competition effect: the term  $-\frac{2n+1}{2n}$  in  $w_f$  exceeds the corresponding term  $-\frac{n+1}{n}$  in  $w_a$ , so the opening up of a new foreign market more than compensates for additional competition at home, with labor demand and hence the wage tending to rise. However, this can be offset by the third effect, the comparative advantage effect, represented by the term  $\delta$  in the denominator of (27): the higher is  $\delta$ , the more free-trade output tends to be higher in sectors with relatively lower labor requirements (because low-cost home firms compete against high-cost foreign rivals) and conversely, so depressing the aggregate demand for labor and tending to reduce the wage. Overall, therefore, the change in the wage between autarky and free trade is indeterminate.

This comparison of wages is instructive in suggesting the change in incentives for labor usage as a result of moving to free trade. However, it has no direct implications for utility. Although  $w_f$  in (27) is the wage evaluated by the domestic marginal utility of income  $\lambda$  (not the world marginal utility of income  $\bar{\lambda}$ ), it is still not directly comparable with  $w_a$  since these only measure real wages at the margin. We turn therefore to consider the gains from trade themselves.

### Gains from Trade

As in the previous section, we measure aggregate welfare using the second moment of the price distribution. Evaluating this (using the Cournot equilibrium price formula from the central column of Table 1, with  $w = w^*$ ) gives:

$$\tilde{U}_f = -(\lambda^2 \mu_2^p)_f = -\frac{1}{(2n+1)^2} \left[ a^2 + 4an\mu_1 w_f + 2n^2(2\mu_2 - \delta)w_f^2 \right]. \quad (28)$$

There are gains from trade if and only if this expression is greater than the corresponding expression in autarky, given by (21). Comparing the two, there are three sources of difference. The first (reflected in differences in the denominators) is a direct competition effect: with more firms in all markets, prices tend to be bid down, reducing their variability and so raising welfare. The second difference (corresponding to the coefficient of the third term in brackets) reflects a direct comparative advantage or technological dissimilarity effect: the greater is  $\delta$ , the more high-cost home firms tend to face low-cost foreign firms and vice versa, so tending to reduce price variability across

sectors and raise welfare. Finally, the third difference arises from the difference in wages. If free-trade wages are exactly twice those in autarky, reflecting the doubling of the market size, then this effect does not arise. However, a free-trade wage that is more than twice that in autarky tends to raise prices relative to autarky and so works against gains from trade. Of course, we have seen in the previous subsection that the difference in wages depends on the same factors, market size, competition and comparative advantage, as the direct effects. Hence we need further analysis to determine the overall gains from trade.

To proceed, we first restate (28) in terms of underlying parameters (details in the Appendix):

$$\tilde{U}_f = -\frac{a^2}{(2n+1)^2} \frac{2\sigma^2 - \delta}{2\mu_2 - \delta} - \left( a\mu_1 \frac{2\mu_2}{2\mu_2 - \delta} - bL \right)^2 \frac{2\mu_2 - \delta}{2(\mu_2 + n\delta)^2}. \quad (29)$$

To prove that the gains from trade are always non-negative, we need to show that this cannot be less than the corresponding expression in autarky (22). We first consider two special cases. The more extreme is the featureless world, where all sectors are identical at home and abroad. Formally,  $\sigma^2 = \delta = 0$  and  $\mu_2 = \mu_1^2$ . In this case (22) and (29) are equal:  $\tilde{U}_a = \tilde{U}_f = -(a - bL/\mu_1)^2$ , and so there are no gains from trade. Summarizing:

LEMMA 1. *In the featureless world where  $\sigma^2 = \delta = 0$ , welfare in autarky and in free trade are identical, and both are independent of the number of firms.*

This extends our earlier formalization of Lerner's insight: when the "degree of monopoly" is the same in all sectors, neither free trade nor competition policy has any scope for raising welfare.

The second special case we consider is where the two countries are still identical, so  $\delta = 0$ , but sectors are heterogeneous:  $\sigma^2 > 0$ , implying that  $\mu_2 = \gamma > \mu_1^2$ .<sup>19</sup> In Figure 1, the cost locus in this case coincides with a segment of the 45° line. Equation (29) now reduces to:

$$\tilde{U}_f = -\frac{a^2}{(2n+1)^2} \frac{\sigma^2}{\mu_2} - \frac{(a\mu_1 - bL)^2}{\mu_2}. \quad (30)$$

The second term is identical to the corresponding term in the expression for autarky welfare (22), but the first term is unambiguously larger because the number of firms has risen, and the difference is increasing in  $\sigma^2$ . So:

LEMMA 2. *When the two countries are identical,  $\delta = 0$ , but there is some heterogeneity across sectors,  $\sigma^2 > 0$ , there are unambiguous gains from trade because of the competition effect and the extent of gains is increasing in  $\sigma^2$ .*

It is easy to show that the dispersion in productivities across sectors  $\sigma^2$  determines the dispersion of markups. So Lemmas 1 and 2 provide a micro-foundation in our model for the importance of markup dispersion in making gains from trade possible, as emphasized by Hsieh and Klenow (2007), Epifani and Gancia (2011) and Edmond et al. (2015).

Finally, we need to show that the gains from trade are increasing in the degree of comparative advantage  $\delta$ . Since welfare in autarky is independent of  $\delta$ , we need only differentiate expression (28) for welfare in free trade:

$$\frac{\partial \tilde{U}_f}{\partial \delta} = \frac{2n}{(2n+1)^2} \left[ nw_f^2 - 2\{a\mu_1 + n(2\mu_2 - \delta)w_f\} \frac{\partial w_f}{\partial \delta} \right] > 0. \quad (31)$$

This shows that an increase in comparative advantage, reflecting greater technological dissimilarity between countries, has two effects. First, it raises free-trade welfare at given wages: as we already saw in discussing (29), an increase in comparative advantage reduces the variability of prices across sectors and so raises welfare. Second, it raises welfare by reducing the wage: as we already saw in discussing (27), an increase in comparative advantage skews the pattern of output and hence the demand for labor towards more efficient, and hence less labor-intensive sectors. Formally,  $\frac{\partial w_f}{\partial \delta} = -\frac{nw_f}{\mu_2 + n\delta} < 0$ . Hence the total effect is unambiguous:

LEMMA 3. *For given  $\sigma^2$ , the gains from trade are strictly increasing in the degree of comparative advantage  $\delta$ .*

Combining these three lemmas, we can conclude:

PROPOSITION 1. *The gains from trade are always positive, strictly so provided there is some heterogeneity in technology between sectors ( $\sigma^2 > 0$ ), and increasingly so the greater is  $\sigma^2$  and the greater the degree of comparative advantage  $\delta$ .*

Lemma 3, which shows that international differences in technology increase the gains from trade, is not too surprising, though it should be stressed that all the analysis applies without complete specialization in production, which is the source of gains from trade in the competitive Ricardian model. Here more efficient sectors expand and less efficient ones contract but do not cease production altogether. Lemma 2 is the most novel part of the result, since it shows that the pro-competitive effects of trade can raise welfare even when countries are identical, both *ex ante* and *ex post*. By contrast, in trade models based on the Dixit–Stiglitz model, countries differ *ex post* since they produce different varieties of the monopolistically competitive good in free trade. Lemmas 1 and 2 also highlight the importance of taking a general-equilibrium perspective: the competition effect of opening up to trade is only effective if there is scope for allocation of labor between sectors.

### *Trade and Income Distribution*

A different classic issue that arises in comparing autarky and free trade is the change in the functional distribution of income. Under competitive assumptions, the Ricardian model is ill-suited to address this question, since all national income accrues to labor. With oligopoly, however, profit recipients must also be taken into account. Of course, given the model's assumptions, the distribution of income between wages and profits has no welfare significance, but it is still of interest to explore how it is affected by the move from autarky to free trade. (Just as the classic result on the gains from trade in competitive models does not eliminate the interest of results such as the Stolper–Samuelson theorem on the distribution of income in multi-factor models.) The question is also of particular interest, since it has been shown in partial-equilibrium oligopoly models that profits must generally fall as a result of moving to free trade: see Anderson et al. (1989). As we will see, this need not hold in general equilibrium.

Consider first the case where the two countries are identical, so there is no comparative advantage basis for trade:  $\delta = 0$ . We have seen that in this case the market size effect dominates the competition effect, so the wage (relative to the marginal utility of income) is higher in free trade than in autarky. It can also be shown that profits are

lower (see Appendix for proof). Hence, it follows that wages rise not only absolutely but also as a share of national income. Letting  $\theta_a$  denote the share of wages in nominal GDP in autarky and  $\theta_f$  the corresponding share in free trade, we have:

**PROPOSITION 2.** *With no comparative advantage effect, so  $\delta = 0$ , the share of wages in national income is higher in free trade than in autarky:  $\theta_f > \theta_a$ .*

So, if the two countries are identical, moving to free trade implies a shift in the distribution of national income towards wages at the expense of profits. Essentially, this is the same result as Anderson et al. (1989), extended to a continuum of sectors, which can differ within countries (since the result holds for any value of  $\sigma^2$ ) but not between them.

However, allowing for comparative advantage introduces a countervailing tendency. We have already seen that the wage in free trade is lower the greater the extent of comparative advantage:  $\frac{\partial w_f}{\partial \delta} < 0$ . It turns out that profits in free trade need not be increasing in  $\delta$ . However, it is the case that the wage share must be lower. As shown in the Appendix:

**PROPOSITION 3.** *The greater the degree of comparative advantage, the lower the share of wages in free-trade national income:  $\frac{\partial \theta_f}{\partial \delta} < 0$ .*

In effect, barriers to the entry of new firms ensure that incumbent firms appropriate the benefits of specialization according to comparative advantage. Moreover, this effect can outweigh the market size effect, which as we have seen tends to raise wages and reduce profits. It is straightforward to construct examples where this happens, so that moving from autarky to free trade lowers the share of wages in national income, despite the fact that labor is the only productive factor in the economy.

### *The Volume of Trade*

Next, we want to consider how the volume of trade is affected by the degree of competition. The level of net imports in a typical sector,  $m(z)$ , equals home demand less home production,  $x(z) - ny(z)$ . Using the results from Table 1, specialized to the symmetric fully diversified case, this equals:

$$m(z) = \frac{1}{2b} n w_f [\alpha(z) - \alpha^*(z)]. \quad (32)$$

Thus net imports are positive if and only if home firms are less productive than foreign. In the symmetric case, trade patterns are determined solely by comparative advantage. Equation (32) also shows that, for given relative labor efficiencies, the volume of trade increases in proportion to the number of firms and to the wage rate. Totally differentiating (32):

$$\hat{m}(z) = \hat{n} + \hat{w}_f \quad (33)$$

where “hats” denote proportional changes. We have already seen that the wage rate may fall as the world economy becomes more competitive. However, it cannot fall sufficiently to lead to a contraction of trade. Substituting for the change in  $w_f$  from (27) into (33) yields:



$$\frac{\hat{m}(z)}{\hat{n}} = \frac{\mu_2 w_f + \frac{bL}{n}}{2a\mu_1 - \frac{2n+1}{n}bL} > 0. \quad (34)$$

So lower wages may dampen but not reverse the direct trade-expanding effect of higher  $n$ . Hence we can conclude:<sup>20</sup>

**PROPOSITION 4.** *An increase in the number of firms raises the volume of imports in all sectors.*

Of course, since real income rises as the economy becomes more competitive, this result is not surprising. Of greater interest is whether trade rises faster than consumption. Totally differentiating the expression for  $x(z)$  from Table 1, the proportional change in consumption equals:

$$\hat{x}(z) = \frac{1}{2n+1} \hat{n} - \frac{[\alpha(z) + \alpha^*(z)]w_f}{4a - [\alpha(z) + \alpha^*(z)]w_f} \hat{w}_f. \quad (35)$$

Combining (33) and (35), the effect of an increase in the number of firms in all world markets on the share of imports in home consumption is:

$$\hat{m}(z) - \hat{x}(z) = \frac{2n}{2n+1} \hat{n} + \frac{4a}{4a - [\alpha(z) + \alpha^*(z)]w_f} \hat{w}_f. \quad (36)$$

So the effect of an increase in competition on the import share in partial equilibrium (i.e. at constant wages) is unambiguously positive, but this could be offset if technology is sufficiently dissimilar that wages fall. Substituting for  $\hat{w}_f$  gives an ambiguous result:

$$\begin{aligned} \frac{\hat{m}(z) - \hat{x}(z)}{\hat{n}} &\propto \frac{2n+1}{n} [2a + n\{\alpha(z) + \alpha^*(z)\}w_f] bL \\ &\quad + 2an[2\mu_2 - \delta - \{\alpha(z) + \alpha^*(z)\}\mu_1]w_f. \end{aligned} \quad (37)$$

Only the second term can be negative, so this implies a sufficient condition for the import share to rise:

**PROPOSITION 5.** *A sufficient condition for an increase in the number of firms to raise the share of imports in consumption in sector  $z$  is that the sector is not extremal in its technology, i.e. that  $\alpha(z) + \alpha^*(z) \leq \frac{2\mu_2 - \delta}{\mu_1}$ .*

The sufficient condition in this proposition could be violated in some sectors, but it must hold on average in all sectors. So we can conclude:

**PROPOSITION 6.** *An increase in the number of firms raises the average share (in absolute value) of net imports in consumption across all sectors.*

These results show clearly that oligopoly tends to reduce trade volumes. An obvious implication is the light this may throw on the “mystery of the missing trade” documented by Trefler (1995): real-world trade volumes are much less than the Heckscher–Ohlin model suggests they should be. Davis and Weinstein (2001) go some way to solving the mystery, while remaining in a competitive Heckscher–Ohlin framework. Propositions 5 and 6 suggest a different explanation for low import shares, and point towards testable hypotheses linking concentration levels and technology to trade volumes.

## 7. Changes in International Competitiveness

An alternative approach to examining the model's properties is to look at small perturbations around an arbitrary free-trade equilibrium. In this case we do not need to assume quadratic preferences, so the results hold for all members of the continuum–Pollak family of preferences set out in the Appendix. In this section I first illustrate the determination of equilibrium and then consider the effects of a particularly interesting exogenous shock: an increase in the number of firms in all home sectors. This can be interpreted as a more stringent anti-trust policy, so our model permits the first formal analysis of the general-equilibrium effects of such policies in open economies.<sup>21</sup> Alternatively, it can be interpreted as an improvement in the “competitive advantage” of the home country. The importance of competitive advantage as a determinant of firm and national performance is much discussed in business schools: see, for example, Porter (1990). However, it has not as yet been formalized in general equilibrium as here.<sup>22</sup>

In the general asymmetric two-country model, there are four equilibrium conditions: a labor-market clearing condition and a condition for the threshold sector in each. Consider first the market for labor in the home country. In equilibrium the total labor supply  $L$  must equal aggregate labor demand, which in turn equals the sum of labor demand from those sectors labeled  $z \in [0, \bar{z}^*]$  in which home firms face no foreign competition ( $n^* = 0$ ), and from those with  $z \in [\bar{z}^*, \bar{z}]$  in which both home and foreign firms operate ( $n^* > 0$ ):<sup>23</sup>

$$L = L^D(w_+, w_+^*; n) = \int_0^{\bar{z}^*} n\alpha(z)y(z)|_{n^*=0}dz + \int_{\bar{z}^*}^{\bar{z}} n\alpha(z)y(z)|_{n^*>0}dz. \quad (38)$$

Aggregate labor demand is summarized by the function  $L^D$ , where the signs below the arguments indicate the responsiveness of labor demand to changes in its determinants. These signs are justified formally in the Appendix and can be explained intuitively as follows. Note first that labor demand is unaffected by small changes in the threshold sectors  $\bar{z}$  and  $\bar{z}^*$ . Changes in either of these thresholds implies entry or exit of extra firms (home firms in the case of  $\bar{z}$ , foreign in the case of  $\bar{z}^*$ ) which are just at the margin of profitability and hence whose effect on aggregate labor demand can be ignored.<sup>24</sup>

Consider next the effects of an increase in the home wage  $w$ . At the intensive margin, this raises production costs for all active home firms and hence lowers their demand for labor. At the extensive margin, if home firms in some sector are just on the threshold of profitability (i.e. if  $\bar{z}$  is less than one), they will no longer be able to compete. Hence the margin of home specialization changes: the threshold home sector  $\bar{z}$  falls and for this reason too home demand for labor falls, though for small changes this effect is negligible. The net effect is a fall in the total demand for labor at home. Similar arguments show that home demand for labor is increasing in the foreign wage  $w^*$ .

Analogous arguments apply to the foreign country, where the labor-market equilibrium condition is:

$$L^* = L^{*D}(w_+, w_+^*, n) = \int_{\bar{z}}^1 n^*\alpha^*(z)y^*(z)|_{n=0}dz + \int_{\bar{z}^*}^{\bar{z}} n^*\alpha^*(z)y^*(z)|_{n>0}dz. \quad (39)$$

Now the responsiveness of labor demand to home and foreign wages is reversed, but the two continue to exert opposing effects. Like home labor demand, that in foreign is unaffected by small changes in the threshold sectors  $\bar{z}$  and  $\bar{z}^*$ . Hence the two labor-market-equilibrium conditions, (38) and (39), can be solved for home and foreign wages as an independent sub-system.

We are now ready to consider the comparative statics properties of the model. At initial wages, an increase in the number of home firms in all sectors reduces output per firm but not by enough to offset the rise in the number of firms (see the Appendix for details). Hence home demand for labor increases. Similar but opposite effects in the foreign country reduce labor demand there. The presumptive outcome is that the equilibrium wage rises at home and falls abroad. The Appendix derives the exact conditions for both a relative and absolute rise in home wages, and proves the following:

**PROPOSITION 7.** *A sufficient condition for an increase in  $n$  to raise the home country's relative wage,  $w/w^*$ , is that the own-effects of wages on labor demand dominate the cross-effects (as they must if the initial equilibrium is symmetric).*

**PROPOSITION 8.** *A sufficient condition for an increase in  $n$  to raise the home wage  $w$  and lower the foreign wage  $w^*$  is that the own-effects of wages and of  $n$  on labor demand dominate the cross-effects (as they must if the initial equilibrium is symmetric).*

Finally, consider the effects of an increase in  $n$  on specialization patterns. From the expressions for output, the threshold sectors in each country,  $\tilde{z}$  and  $\tilde{z}^*$  are defined by the following equations:

$$y(\tilde{z}) \geq 0 \iff \bar{a} - (n^* + 1)w\alpha(\tilde{z}) + n^*w^*\alpha^*(\tilde{z}) \geq 0, \quad \tilde{z} \leq 1 \quad (40)$$

$$y^*(\tilde{z}^*) \geq 0 \iff \bar{a} - (n + 1)w^*\alpha^*(\tilde{z}^*) + nw\alpha(\tilde{z}^*) \geq 0, \quad \tilde{z}^* \geq 0. \quad (41)$$

Each pair of inequalities in (40) and (41) is complementary slack. So, in (40) for example, if  $y(\tilde{z})$  is strictly positive, then  $\tilde{z}$  equals one: this is the case where home firms are profitable in all sectors, so the home country is fully diversified in equilibrium. By contrast, if  $\tilde{z}$  is strictly less than one, then  $y(\tilde{z})$  is zero: this is the case where home firms in sectors with  $z \geq \tilde{z}$  are unprofitable, so the home country is partially specialized in equilibrium. We consider this case (so  $\tilde{z} < 1$  and  $y(\tilde{z}) = 0$ ) and totally differentiate equation (40) to obtain:

$$\frac{d\tilde{z}}{dn} = \frac{\partial \tilde{z}}{\partial w} \frac{dw}{dn} + \frac{\partial \tilde{z}}{\partial w^*} \frac{dw^*}{dn}. \quad (42)$$

Since wage changes affect the threshold sector in an unambiguous manner, we can again state a sufficient condition:

**PROPOSITION 9.** *A sufficient condition for an increase in  $n$  to lower the home threshold sector is that the home wage rises and the foreign wage falls.*

This result is not expressed in terms of primitive parameters, but from Proposition 8 we can see that it will always hold if the initial equilibrium is symmetric. It has a striking implication: an improvement in the home country's competitive advantage raises output in all sectors at initial wages. However, the induced wage changes make marginal home sectors uncompetitive. Hence improved competitive advantage leads the home country to specialize more in accordance with comparative advantage, exiting some sectors as home wages rise.

## 8. Conclusion

This paper has developed a tractable but consistent model of oligopoly in general equilibrium; and used it to take a small step towards completing the "new trade theory" agenda of integrating international trade with industrial organization. The step is

a small one because the functional forms assumed are special, and because many simplifications are made in specifying agents' behavior and the workings of goods and factor markets. Nevertheless, it is hopefully a step in the right direction. The model allows for consistent aggregation over a continuum of sectors, each of which is characterized by Cournot competition between home and foreign firms. The model makes explicit the links between goods and factor markets, and so is able to give a coherent yet tractable analysis of the effects of a variety of exogenous shocks.

The key idea in the paper is that oligopolistic firms should be modeled as large in their own markets but small in the economy as a whole. This perspective avoids at a stroke all the problems (of non-existence, ambiguity of profit maximization, sensitivity of the model's properties to the choice of numéraire, etc.), which have concerned writers such as Gabszewicz and Vial (1972), Roberts and Sonnenschein (1977), and Eaton et al. (2013), who have tackled the problem of oligopoly in general equilibrium. Hopefully it thus opens up a rich vein of research, combining the insights of modern theories of industrial organization with those of applied general equilibrium theory.

The paper's central idea could be operationalized in a great variety of ways. Here I have chosen to work with quadratic preferences on the demand side and a Cournot–Ricardo (or Brander–Samuelson) specification of goods and factor markets. While the individual building blocks are familiar, the full model exhibits many novel properties and throws light on a number of substantive issues. In particular, the model shows that trade between economies that are identical both *ex ante* and *ex post* is welfare-improving because it enhances competition, although (contrary to partial-equilibrium intuition) the competition effect can only raise welfare to the extent that sectors are heterogeneous within each country; that moving to free trade may tilt the distribution of income towards profits at the expense of wages, the more so the greater the countries differ, as the gains from specializing according to comparative advantage benefit profit-earners disproportionately; that barriers to entry reduce trade volumes both absolutely and relative to total consumption, suggesting a plausible (and testable) explanation for Treffer's "mystery of the missing trade"; and that a rise in one country's competitive advantage is likely to raise its relative wage and lead it to specialize more in the direction of comparative advantage.

There are many obvious ways in which the approach adopted here could be extended. I have already explored in a simplified version of the model the implications for the effects of trade on income inequality of having more than one factor of production and of allowing firms to engage in entry-preventing behavior (see Neary, 2002a). The model also makes it possible to explore how trade liberalization and other shocks can affect market structure itself, for example by changing the incentives for cross-border mergers and acquisitions (see Neary, 2007). Overall I hope the model points towards a richer theory of imperfect competition in open economies than is possible in models where entry is never difficult and firm behavior is never strategic, whether under perfectly or monopolistically competitive assumptions.

## Appendix

### *Continuum–Pollak Preferences*

The result of Pollak (1971), which characterizes the set of utility functions that satisfies both additive separability (1) and quasi-homotheticity (4), can be restated for the case of a continuum of goods as follows:<sup>25</sup>

PROPOSITION 10 (Pollak, 1971). *A utility function satisfies both additive separability and quasi-homotheticity if and only if it takes the form:*

$$U[\{x(z)\}] = \int_0^1 \alpha(z) [\chi\{x(z) - \beta(z)\}]^\theta dz \quad (\text{A1})$$

subject to the parameter restrictions:

$$\chi\{x(z) - \beta(z)\} > 0 \quad (1 - \theta)\chi > 0 \quad \theta(1 - \theta)\alpha(z) \geq 0. \quad (\text{A2})$$

Here  $\{\alpha(z)\}$ ,  $\{\beta(z)\}$  and  $\theta$  are constants that can be positive or negative, while  $\chi$  is a scalar indicator variable that equals either +1 or -1. The restrictions in (A2) are needed to ensure that the expression in square brackets is positive and that marginal utility is positive and diminishing.<sup>26</sup> These restrictions in turn define four and only four cases, two corresponding to ranges of  $\theta$  and two to limiting special cases:<sup>27</sup>

- (1) *Translated CES*:  $-\infty < \theta < 1$  and  $\theta \neq 1$ , implying  $\chi = 1$  and so  $x(z) > \beta(z)$ . In this case the indifference curves exhibit a constant elasticity of substitution (CES), though defined not with respect to the origin but to a “translated” origin whose coordinates are  $\{\beta(z)\}$ , sometimes interpreted as a “minimum subsistence level” (though some or all of the  $\{\beta(z)\}$  can be negative).
- (2) *Generalized Quadratic*:  $1 < \theta < \infty$ , implying  $\chi = -1$  and so  $x(z) < \beta(z)$ . In this case utility rises as the indifference curves converge towards the point  $\{\beta(z)\}$ , rather than away from it as in case 1, so  $\{\beta(z)\}$  should now be interpreted as a “bliss point”.
- (3) *Stone–Geary*:  $\ln U = \int_0^1 \alpha(z) \ln \{x(z) - \beta(z)\} dz$ . This translated Cobb–Douglas case is the limit of (A1) as  $\theta \rightarrow 0$ , with  $x(z) > \beta(z)$  and  $\chi = 1$ .
- (4) *Additive Exponential*:  $U = -\int_0^1 \alpha(z) e^{-\beta(z)x(z)}$ ,  $\beta(z) > 0$ . This case is the limit of (A1) as  $\theta \rightarrow -\infty$ , with  $x(z) > \beta(z)$  and  $\chi = 1$ .

Many well-known demand systems are subsumed within this system. For example, the special case of 1 with  $\beta(z) = 0, \forall z$ , gives the familiar homothetic CES; the special case of 3 with  $\beta(z) = 0, \forall z$ , gives the Cobb–Douglas; and the special case of 2 with  $\theta = 2$  gives the quadratic as in (5) (with minor rewriting of the constants).

Henceforward we concentrate on cases 1 and 2.<sup>28</sup> Maximizing utility (A1) subject to the budget constraint (2) leads to the implied direct demand functions, which can be written in two alternative ways as follows:

$$x(z) = \beta(z) + \chi \left[ \lambda \frac{p(z)}{A(z)} \right]^{\frac{1}{\theta-1}} = \beta(z) + \left[ \frac{p(z)}{A(z)g[\{p(z)\}]} \right]^{\frac{1}{\theta-1}} \frac{I - f[\{p(z)\}]}{g[\{p(z)\}]} \quad (\text{A3})$$

where  $A(z) \equiv \chi\theta\alpha(z) > 0$ , and  $f[\{p(z)\}]$  and  $g[\{p(z)\}]$  are linear and CES price indices respectively:

$$\begin{aligned} f[\{p(z)\}] &\equiv \int_0^1 \beta(z)p(z)dz \\ g[\{p(z)\}] &\equiv \left[ \int_0^1 p(z)^{\frac{\theta}{\theta-1}} A(z)^{\theta-1} dz \right]^{\frac{\theta-1}{\theta}} = \left[ \int_0^1 p(z) \left\{ \frac{p(z)}{A(z)} \right\}^{\frac{1}{\theta-1}} dz \right]^{\frac{\theta-1}{\theta}}. \end{aligned} \quad (\text{A4})$$

The demand functions in (A3) are respectively Frisch and Marshallian from a consumer theory perspective, or perceived and actual from an industrial organization perspective. Within each sector, firms take the marginal utility of income as given, and face a perceived or Frisch demand function that takes either a translated isoelastic form (for  $x(z) > \beta(z)$  and  $\chi = 1$ ) or a “generalized linear” form (for  $x(z) < \beta(z)$  and  $\chi = -1$ ). From the economy-wide (and the observing economist’s) perspective, the marginal utility of income depends in turn on all prices and on income:

$$\lambda = \frac{[\chi \{I - f[\{p(z)\}]\}]^{\theta-1}}{(g[\{p(z)\}])^\theta}. \quad (\text{A5})$$

Hence the actual or Marshallian demand function for each good depends on its own price and on “supernumerary income”  $I - f[\{p(z)\}]$  (negative for  $\chi = -1$ ), both deflated by the marginal price index  $g[\{p(z)\}]$ . Finally, for welfare analysis we can substitute the demand functions into (A1) to get the indirect utility function, which implies the Gorman polar form for the expenditure function as in (4).

With a single representative consumer in each country, the world inverse Frisch demand curve faced by firms in the industry is:

$$p(z) = \chi \theta \alpha'(z) [\chi \{\bar{x}(z) - \bar{\beta}(z)\}]^{\theta-1} \text{ where : } \alpha'(z) \equiv \frac{\alpha(z)}{\bar{\lambda}}. \quad (\text{A6})$$

Here  $\bar{x}(z) = x(z) + x^*(z)$  is world demand, and  $\bar{\beta}(z)$  and  $\bar{\lambda}$  are appropriate aggregates of the individual country terms.<sup>29</sup> The term  $\bar{\lambda}$  can be interpreted as the world marginal utility of income, which is taken as given by firms but determined endogenously in general equilibrium as already noted.

### *Continuum–Quadratic Preferences*

We have seen in the text how to derive the Frisch indirect utility function in the case of quadratic preferences. For completeness, we show here how this relates to the more familiar Marshallian indirect utility function and to the expenditure function. First, we use the expression in (7) to eliminate the marginal utility of income from the Frisch indirect utility function:

$$V[I, \{p(z)\}] = \frac{1}{2b} \left[ a^2 - \left( \frac{a\mu_1^p - bI}{\mu_2^p} \right)^2 \mu_2^p \right]. \quad (\text{A7})$$

A monotonically increasing transformation of utility allows us to rewrite this in the Gorman polar form:  $\tilde{V}[\{p(z)\}, I] = \frac{I - f(p)}{g(p)}$ :

$$\tilde{V}[\{p(z)\}, I] \equiv -\frac{1}{b} (a^2 - 2bV)^{1/2} = \frac{I - \frac{a}{b}\mu_1^p}{(\mu_2^p)^{1/2}}. \quad (\text{A8})$$

This shows that, with quadratic preferences, the two price indices  $f(p)$  and  $g(p)$  are simple transformations of the mean and the uncentered standard deviation of prices, respectively. Finally, we can invert the indirect utility function to obtain the expenditure function written in the Gorman polar form,  $e[\{p(z)\}, u] = f(p) + ug(p)$ :

$$e[\{p(z)\}, u] = \frac{a}{b} \mu_1^p + \tilde{u} (\mu_2^p)^{1/2} \quad \text{where : } \tilde{u} = -\frac{1}{b} [(a^2 - 2bu)]^{1/2}. \quad (\text{A9})$$

### *Proof of Proposition 1*

We give the steps in deriving the free-trade level of welfare (29). Deriving the autarky level of welfare proceeds along similar lines: details are in Neary (2003c). Rewrite the expression in brackets in (28) as a difference of squares and then substitute for  $w_f$  from (27):

$$\begin{aligned} -(2n+1)^2 \tilde{U}_f &= a^2 + 4an\mu_1 w_f + 2n^2(2\mu_2 - \delta)w_f^2 \\ &= a^2 + 2(2\mu_2 - \delta)nw_f \left[ nw_f + \frac{2a\mu_1}{2\mu_2 - \delta} \right] \\ &= a^2 + 2(2\mu_2 - \delta) \left[ \left( nw_f + \frac{a\mu_1}{2\mu_2 - \delta} \right)^2 - \left( \frac{a\mu_1}{2\mu_2 - \delta} \right)^2 \right] \\ &= a^2 \left( 1 - \frac{2\mu_1^2}{2\mu_2 - \delta} \right) + 2(2\mu_2 - \delta) \left( nw_f + \frac{a\mu_1}{2\mu_2 - \delta} \right)^2 \\ &= a^2 \frac{2\sigma^2 - \delta}{2\mu_2 - \delta} + 2(2\mu_2 - \delta) \left( \frac{an\mu_1 - \frac{2n+1}{2}bL}{\mu_2 + n\delta} + \frac{a\mu_1}{2\mu_2 - \delta} \right)^2 \\ &= a^2 \frac{2\sigma^2 - \delta}{2\mu_2 - \delta} + (2n+1)^2 \left( a\mu_1 \frac{2\mu_2}{2\mu_2 - \delta} - bL \right)^2 \frac{2\mu_2 - \delta}{2(\mu_2 + n\delta)^2}. \end{aligned} \quad (\text{A10})$$

This gives equation (29). The proofs of Lemmas 1, 2 and 3 follow by inspection.  $\square$

### *Proof of Proposition 2*

We first calculate aggregate profits in autarky. Aggregating across sectors:

$$\Pi_a = \int_0^1 n\pi(z)dz \quad \pi(z) = [p(z) - w_a\alpha(z)]y(z) = by(z)^2. \quad (\text{A11})$$

Substituting first for  $y(z)$  and then for  $w_a$  from (19) gives:

$$\Pi_a = \frac{n}{b(n+1)^2} (a^2 - 2a\mu_1 w_a + \mu_2 w_a^2) = \left[ \frac{na^2}{b(n+1)^2} \sigma^2 + \frac{bL^2}{n} \right] \frac{1}{\mu_2}. \quad (\text{A12})$$

Next, we calculate aggregate profits in free trade. A similar series of steps to that above yields:

$$\Pi_f = \frac{2n}{b(2n+1)^2} \left[ a^2 - 2a\mu_1 w_f + \{\mu_2 + 2n(n+1)\delta\}w_f^2 \right]. \quad (\text{A13})$$



Specializing to the case of no comparative advantage,  $\delta = 0$ , we have:

$$\Pi_f|_{\delta=0} = \frac{2n}{b(2n+1)^2} \left[ a^2 - 2a\mu_1 w_f + \mu_2 w_f^2 \right] \quad w_f|_{\delta=0} = \frac{a\mu_1 - \frac{2n+1}{2n} bL}{\mu_2} \quad (\text{A14})$$

which simplifies to:

$$\Pi_f|_{\delta=0} = \left[ \frac{2na^2}{b(2n+1)^2} \sigma^2 + \frac{bL^2}{2n} \right] \frac{1}{\mu_2}. \quad (\text{A15})$$

Comparing this with autarky profits  $\Pi_a$  in (A12) gives:

$$\Pi_f|_{\delta=0} - \Pi_a = - \left[ \frac{2n^2-1}{(2n+1)^2} \frac{na^2}{b(n+1)^2} \sigma^2 + \frac{bL^2}{2n} \right] \frac{1}{\mu_2} < 0. \quad (\text{A16})$$

Thus profits are lower in free trade when  $\delta = 0$ , and we have already seen that the wage is higher. It follows that the share of labor is higher, which proves the proposition.  $\square$

### *Proof of Proposition 3*

The logarithmic change in the share of wages is by construction related to the changes in the wage and the level of profits:

$$d\ln \theta_f = d\ln w_f - d\ln I_f = (1 - \theta_f) (d\ln w_f - d\ln \Pi_f). \quad (\text{A17})$$

Logarithmically differentiating the level of profits from (A13) yields:

$$\begin{aligned} Hd\ln \Pi_f = & -2[a\mu_1 - \{\mu_2 + 2(n+1)n\delta\}w_f]w_f d\ln w_f \\ & + 2(n+1)n\delta w_f^2 d\ln \delta \end{aligned} \quad (\text{A18})$$

where  $H \equiv a^2 - 2a\mu_1 w_f + \{\mu_2 + 2(n+1)n\delta\}w_f^2$ .

Substituting into (A17) yields:

$$\frac{H}{1 - \theta_f} d\ln \theta_f = \left[ a^2 - \{\mu_2 + 2(n+1)n\delta\}w_f^2 \right] d\ln w_f - 2(n+1)n\delta w_f^2 d\ln \delta. \quad (\text{A19})$$

So, an increase in the degree of comparative advantage has a direct effect on the wage share, which is negative, but the indirect effect via the wage itself is ambiguous. To eliminate the latter, logarithmically differentiate the wage from (27):

$$d\ln w_f = - \frac{n\delta}{\mu_2 + n\delta} d\ln \delta. \quad (\text{A20})$$

Finally, substituting into (A19) yields:

$$\frac{H}{1-\theta_f} \frac{d \ln \theta_f}{d \ln \delta} = - \left[ a^2 + (2n+1) \mu_2 w_f^2 \right] \frac{n \delta}{\mu_2 + n \delta} < 0. \quad (\text{A21})$$

Thus the negative direct effect dominates the indirect effect via the change in wages, which proves the proposition.  $\square$

### Comparative Statics of Free Trade

When both countries are partly specialized, so  $0 < \tilde{z}^* < \tilde{z} < 1$ , and equations (40) and (41) which define the threshold sectors hold with equality, the total differential of the system is as follows:

$$\begin{bmatrix} L_w & L_{w^*} & 0 & 0 \\ L_w^* & L_{w^*}^* & 0 & 0 \\ -(n^*+1)\alpha(\tilde{z}) & n^*\alpha^*(\tilde{z}) & -H & 0 \\ -n\alpha(\tilde{z}^*) & (n+1)\alpha^*(\tilde{z}^*) & 0 & -H^* \end{bmatrix} \begin{bmatrix} dw \\ dw^* \\ d\tilde{z} \\ d\tilde{z}^* \end{bmatrix} = \begin{bmatrix} -L_n \\ -L_n^* \\ 0 \\ -J \end{bmatrix} dw \quad (\text{A22})$$

where  $H$ ,  $H^*$  and  $J$  are defined in (A31) and (A33) below. Consider first the derivatives of the home labor-market equilibrium condition (38). Differentiating this with respect to home and foreign wages, and using the expressions in Table 1 to sign the individual terms, gives:

$$L_w = n \int_0^{\tilde{z}^*} \alpha(z) \frac{\partial y(z)}{\partial w} \Big|_{n^*=0} dz + n \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z) \frac{\partial y(z)}{\partial w} \Big|_{n^*>0} dz < 0 \quad (\text{A23})$$

$$L_{w^*} = n \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z) \frac{\partial y(z)}{\partial w^*} \Big|_{n^*>0} dz > 0. \quad (\text{A24})$$

So, home labor demand is decreasing in the home wage and increasing in the foreign wage. Next, differentiating with respect to the threshold sectors:

$$L_{\tilde{z}} = \alpha(\tilde{z})ny(\tilde{z}) = 0 \quad \text{and} \quad L_{\tilde{z}^*} = \alpha(\tilde{z}^*)\bar{x}(\tilde{z}^*) - \alpha^*(\tilde{z}^*)ny(\tilde{z}^*) = 0 \quad (\text{A25})$$

where the last result follows from (41) with  $y^*(\tilde{z}^*)=0$ . So, for small changes, home labor demand is independent of both home and foreign threshold sectors. Finally, differentiating with respect to the number of home firms:

$$L_n = n \int_0^{\tilde{z}^*} \alpha(z) \frac{\partial y(z)}{\partial n} \Big|_{n^*=0} dz + n \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z) \left[ y(z) + n \frac{\partial y(z)}{\partial n} \Big|_{n^*>0} \right] dz \quad (\text{A26})$$

$$= \frac{1}{n+1} \int_0^{\tilde{z}^*} \alpha(z)y(z) \Big|_{n^*=0} dz + \frac{n^*+1}{n+n^*+1} \int_{\tilde{z}^*}^{\tilde{z}} \alpha(z)y(z) \Big|_{n^*>0} dz > 0 \quad (\text{A27})$$

implying that home labor demand is increasing in the number of home firms in each sector.

The derivatives of the foreign labor demand schedule are derived similarly by totally differentiating (39). Foreign labor demand is decreasing in the foreign wage and increasing in the home wage. It is independent of both the home and foreign threshold sectors. Finally:

$$L_n^* = n^* \int_{\tilde{z}^*}^{\tilde{z}} \alpha^*(z) \frac{\partial y^*(z)}{\partial n} \Big|_{n>0} dz = - \frac{n^*}{n+n^*+1} \int_{\tilde{z}^*}^{\tilde{z}} \alpha^*(z) y^*(z) \Big|_{n>0} dz < 0. \quad (\text{A28})$$

So foreign labor demand is decreasing in the number of home firms in each sector. Solving the sub-system consisting of the first two equations in (A22) and evaluating at a stable equilibrium gives:<sup>30</sup>

$$\frac{dw}{dn} \propto -L_n L_{w^*}^* + L_n^* L_{w^*} \quad \text{and} \quad \frac{dw^*}{dn} \propto L_n L_w^* - L_n^* L_w. \quad (\text{A29})$$

Converting to proportional changes and subtracting gives:

$$\frac{\hat{w} - \hat{w}^*}{\hat{n}} \propto L_n^* (w L_w + w^* L_w^*) - L_n (w^* L_{w^*}^* + w L_w^*). \quad (\text{A30})$$

The expressions in brackets are respectively negative and positive provided the own-effects of wages dominate the cross-effects in home and foreign labor demand respectively, which proves Proposition 8. If we assume in addition that the positive own-effect of  $n$  on home labor demand dominates its negative cross-effect, so  $L_n + L_n^* > 0$ , then  $w$  must rise and  $w^*$  must fall. This proves Proposition 7.

Consider next equations (40) and (41), which define the threshold sectors. Recalling Assumption 1 in the text (i.e. that  $y(z)$  is increasing and  $y^*(z)$  is decreasing in  $z$ ), it is easy to confirm:

LEMMA A1. *The conditions  $dy(\tilde{z})/dz < 0$  and  $dy^*(\tilde{z}^*)/dz > 0$ , are equivalent to  $H > 0$  and  $H^* > 0$  respectively, where  $H$  and  $H^*$  are defined as follows:*

$$H \equiv (n^* + 1)w\alpha'(\tilde{z}) - n^*w^*\alpha'^*(\tilde{z}) \quad \text{and} \quad H^* \equiv nw\alpha'(\tilde{z}^*) - (n+1)w^*\alpha'^*(\tilde{z}^*). \quad (\text{A31})$$

When  $y(\tilde{z})$  and  $y^*(\tilde{z}^*)$  are zero, we can substitute from (40) and (41) to obtain:

$$\begin{aligned} H > 0 &\iff \alpha^*(\tilde{z})\alpha'(\tilde{z}) > \alpha(\tilde{z})\alpha'^*(\tilde{z}) - \frac{\bar{a}}{n^*w^*} \\ &\text{and} \\ H^* > 0 &\iff \alpha^*(\tilde{z}^*)\alpha'(\tilde{z}^*) > \alpha(\tilde{z}^*)\alpha'^*(\tilde{z}^*) + \frac{\bar{a}}{nw}. \end{aligned} \quad (\text{A32})$$

Apart from the final terms, which are of order  $1/n^*$  and  $1/n$  respectively, these conditions are identical, except that they are evaluated at different points. Hence:

LEMMA A2. *In the competitive limit, as  $n$  and  $n^*$  approach infinity, Assumption 1 collapses to  $\alpha^*(z)\alpha'(z) > \alpha(z)\alpha'^*(z)$ ; i.e.  $\alpha(z)/\alpha^*(z)$  is increasing in  $z$ .*

It is clear from (A22) that Assumption 1 allows us to sign the partial effects of changes in wages on the threshold sectors:

LEMMA A3. Given Assumption 1, so  $H$  and  $H^*$  are positive, both  $\tilde{z}$  and  $\tilde{z}^*$  are decreasing in  $w$  and increasing in  $w^*$ .

Finally, while the home threshold sector in (40) does not depend directly on the number of home firms, (40) is related to it by the parameter  $J$ :

$$J \equiv w^* \alpha^*(\tilde{z}^*) - w \alpha(\tilde{z}) = \frac{\bar{a} - w \alpha(\tilde{z}^*)}{n+1} = y(\tilde{z}^*)|_{n=0} > 0. \quad (\text{A33})$$

Since  $J$  is positive it follows that, at given wages, an increase in the number of home firms raises the threshold foreign sector: i.e. it reduces the number of sectors that are competitive in the foreign country.

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## Notes

1. Helpman and Krugman (1985, 1989) give classic overviews of these two strands, respectively. Of course, the two strands overlap to some extent. For example, chapter 5 of Helpman and Krugman (1985) presents some models of oligopoly in general equilibrium, though without addressing the problems discussed below.
2. See, for example, Bernard et al. (2007), Mayer and Ottaviano (2008), and Freund and Pierola (2015).
3. See Gabaix (2011) and Di Giovanni and Levchenko (2012).
4. See, for example, Bernard et al. (2009) and Bricongne et al. (2012).
5. Naturally, this brief summary fails to do justice to an enormous literature. One conspicuous exception is the work of Brander (1981), subsequently extended by Brander and Krugman (1983), Weinstein (1992) and Yomogida (2008). Though confined to partial equilibrium, this shows that oligopolistic competition in segmented markets provides a distinct motive for “cross-hauling” or two-way trade. The model of the present paper can be viewed as a general-equilibrium generalization of Brander’s, though for simplicity only integrated markets are considered. Dixit and Grossman (1986) and Neary (1994) provide elements of a general-equilibrium foundation for open-economy oligopoly models, by endogenizing factor-market clearing and the government budget respectively.

6. For detailed references and further discussion, as well as a non-technical overview of the model presented here, see Neary (2003b,c).
7. See Roberts and Sonnenschein (1977).
8. See Gabszewicz and Vial (1972). The paradox is apparent rather than real, since what is at issue is not the units in which profits are measured, but the specification of profit-recipients' preferences.
9. Once again, this fails to do justice to a large literature. For studies of oligopoly in general equilibrium that de-emphasize the issues highlighted here, see for example Markusen (1984) and Ruffin (2003). Dierker and Grodal (1998) assume that firms maximize shareholders' real wealth. Eaton et al. (2013) review the difficulties of modeling oligopolistic markets in general equilibrium with a finite number of firms.
10. A similar approach is used in a different context by Hart (1982). More recently, Atkeson and Burstein (2008) independently develop a model of oligopoly in general equilibrium with continuum-CES preferences and apply it to study pricing-to-market. Their framework has in turn been applied by Edmond et al. (2015) to quantifying the misallocation of resources arising from differences in markups across sectors as in Hsieh and Klenow (2007) and Epifani and Gancia (2011). The importance for welfare of the structural underpinnings of differences in markups will become apparent in section 6 below.
11. See Dornbusch et al. (1977), Dornbusch et al. (1980), Wilson (1980), Matsuyama (2000) and Eaton and Kortum (2002) for extensions of the continuum model under perfect competition, and Romalis (2004) for an application to monopolistic competition.
12. More generally, inverse Frisch demand functions depend on the quantities of all goods and on the marginal utility of income, while direct Frisch demand functions depend on the prices of all goods and on the marginal utility of income. The special feature of additive separability is that the implied inverse Frisch demand function for each good depends only on the price of the good itself (as well as on  $\lambda$ ).
13. To do this, multiply either demand function in (6) by  $p(z)$ , integrate and use the budget constraint (2) to express in terms of total spending  $I$ .
14. They also rationalize my use of a single representative consumer to characterize demands in each country. Disaggregation within countries leads to essentially the same results, provided all consumers have Gorman polar form preferences, with the same  $b$  but possibly different values of  $a$ .
15. See Neary and Tharakan (2012) for an extension to Bertrand competition.
16. For an earlier use of this diagram in a very different context, see Collie (1991).
17. The closed-economy case is also discussed in Neary (2003c).
18. The partial derivative of  $\tilde{U}_a$  with respect to  $\sigma^2$  is  $\left[-\frac{a^2}{(n+1)^2}\mu_1^2 + (a\mu_1 - bL)^2\right]\frac{1}{\mu_2^2}$ . This simplifies to  $\frac{n(n+2)}{(n+1)^2}(a\mu_1 - \frac{n+1}{n+2}bL)(a\mu_1 - \frac{n+1}{n}bL)\frac{1}{\mu_2^2}$ , which must be positive when the wage as given in (19) is positive.
19. It is convenient to consider changes in  $\sigma^2$  and  $\delta$  separately, though for many distributions they are related. For example, if  $\alpha(z)$  and  $\alpha^*(z)$  are symmetric and linear in  $z$ , so  $\alpha(z) = \alpha_0 + \alpha z$  and  $\alpha^*(z) = (\alpha_0 + \alpha) - \alpha z$ , then:  $\delta = \frac{1}{6}\alpha^2 = 2\sigma^2$ .
20. Ruffin (2003) independently derives this result in a model with two oligopolistic sectors.
21. The relatively small literature on anti-trust policy in open economies is mostly cast in partial equilibrium. See, for example, Dixit (1984) and Horn and Levinsohn (2001). A notable exception is Francois and Horn (2007), who advocate an approach similar to that adopted here.
22. See Neary (2003a) for further references and for a non-technical exposition of the results presented here.
23. Since we consider only the free-trade equilibrium in this section, we dispense with the "f" subscript.
24. So, for example, using Leibniz's Rule,  $\frac{\partial L^D}{\partial \tilde{z}} = n\alpha(\tilde{z})y(\tilde{z}) = 0$  since  $y(\tilde{z}) = 0$ .
25. We write (A1) in its additively separable form rather than the more familiar translated CES form, since this implies a neater expression for the expenditure function below. Of course, since



utility is ordinal, we are free to write it as any monotonic transformation of an additive function. Our choice affects only those expressions that involve utility explicitly, such as the marginal utility of income.

26. From (A1),  $\frac{\partial U}{\partial x(z)} = \chi \theta \alpha(z) [\chi \{x(z) - \beta(z)\}]^{\theta-1}$  and  $\frac{\partial^2 U}{\partial x(z)^2} = -(1-\theta) \theta \alpha(z) [\chi \{x(z) - \beta(z)\}]^{\theta-2}$ .

27. This list exhausts all possible values of  $\theta$  except for two that are inadmissible:  $\theta = 1$  violates the requirement of strictly diminishing marginal utility,  $\frac{\partial^2 U}{\partial x(z)^2} < 0$ ; while  $\theta \rightarrow +\infty$  yields the Leontief (fixed-coefficients) utility function, which is not a member of the Gorman class.

28. The Stone–Geary case, which leads to the linear expenditure system of demand functions, is well known, while the additive exponential case has been extensively studied by Behrens and Murata (2007), who call it “CARA” (Constant Relative Risk Aversion) utility.

29. Thus:  $\bar{\beta}(z) = \beta(z) + \beta^*(z)$  and  $\bar{\lambda} = \left[ \lambda^{\frac{1}{\theta-1}} + (\lambda^*)^{\frac{1}{\theta-1}} \right]^{\theta-1}$ . The  $\alpha(z)$  and  $\theta$  parameters must be the same in both countries for consistent aggregation.

30. The stability condition is  $L_w L_{w^*}^* - L_{w^*} L_w^* > 0$ ; i.e. the product of the two own-effects on labor demand is greater than the product of the two cross-effects. In the space of  $\{w, w^*\}$ , with  $w$  on the vertical axis, where both labor-market equilibrium conditions are represented by upward-sloping loci, stability is equivalent to the foreign locus being more steeply sloped. See Neary (2003a).