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Sovereign contingent liabilities:
a perspective on default and debt crises

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Abstracts

Chapters 2-3: A global games approach to sovereign debt crises

The first chapters present a model that investigates the risks involved when a fiscal authority attempts to roll-over a stock of debt and there is the potential for coordination failure by investors. A continuum of investors, after receiving signals about the authority's willingness to repay, decides whether to roll-over the stock of debt. If an insufficient proportion of investors participates, the authority defaults. With one fiscal authority, private information results in a deterministic outcome. When a public signal is available, the model behaves in a similar manner to a sunspot model. In line with much of the global games literature, improving public information has an ambiguous effect on welfare. Finally, the model is extended to include a second fiscal authority, which captures a similar sunspot result and illustrates the potential for externalities in fiscal policy. Lower debt in the less indebted authority can push a more indebted authority into crisis. Lower debt makes the healthier authority relatively more attractive, which causes the investors to treat the heavily indebted authority more conservatively. In certain circumstances, this is sufficient to cause a coordination failure.

Chapter 4: A debt game with correlated information

This chapter models of debt roll-over where a continuum of investors receives correlated signals on whether a debtor is solvent or insolvent. The investors face a collective action problem: a sufficient proportion of investors must agree to participate in the debt roll-over for it to be a success. If an insufficient proportion of investors participates in the deal, the debtor will default. The game has a unique switching strategy, which results in global uncertainty being preserved. The *ex ante* distribution of play (conditional on the true solvency of the debtor) follows a Vasicek credit distribution. The *ex ante* probability of a debt crisis is affected by the exogenous model parameters. Of particular interest is the observation that increasing private noise unambiguously reduces the probability of a debt crisis. Unsurprisingly, increasing the fiscal space or return on debt also decreases the probability of a crisis.

Chapter 5: Bailouts and politics

The final chapter examines the political-economic equilibrium in a two-period model with overlapping generations and a financial sector, which is inspired by the model in Tabellini (1989). The public policy is chosen under majority rule by the agents currently alive. It demonstrates that the bailout policy adopted in the second period has important effects on the bank's financing decisions in the first period. By adopting a riskier financing regime (*i.e.* higher leverage) in the first period, the older generation can extract consumption from the younger generation in the second period. Sovereign backstops of the financial sector are state-contingent: they can appear costless for long periods of time but eventually result in a socialization of private-sector debt. It is this mechanism that makes implementing capital requirements costly to investors yet beneficial to the younger generation. The model also highlights two important issues: (i) bank capital is endogenous and (ii) proposed resolution mechanisms must be politically credible. It suggests that a major benefit of increasing and narrowing equity-capital requirements or increasing liquidity ratios is that they are implemented *ex ante* and therefore available either to absorb losses in the event of a crisis or to reduce the possibility of large drops in asset values. Finally, this chapter also provides a structure by which to interpret the stylized facts of Calomiris et al. (2014): that more populist political institutions are associated with more fragile financial systems.

Chapter I

Introduction

Why we should put ourselves out of our way to do anything for posterity,
for what has posterity ever done for us?ⁱ

From the fall of the Bourbons to the collapse of the Soviet Union, debt crises have shaped the course of history. To shed some light on how ruinous they can be, let's look at a little-discussed chapter in the history of my country: Canada.

Three decades before the country would adopt its distinctive red-and-white Maple Leaf flag, a sovereign debt crisis sparked the series of events that resulted in an independent nation, Newfoundland, abandoning democracy before being subsumed by Canada.

Newfoundland, the first self-governing colony in the British Empire, existed as an entirely separate country from Canada until 1949. From 1855, Newfoundland had its own government, set its own laws, and exhibited all the trappings of independence. The country even minted its own currency until a banking crisis in the late 19th century resulted in Newfoundland adopting the Canadian dollar as legal tender.

The roots for the crisis were sown well beforehand. After the First World War, when cod prices were high, Newfoundland embarked on a public expenditure binge. Despite its booming fishing-based economy, the government ran enormous deficits that doubled its debt in only 12 years. With the onset of the Great Depression in 1929, the price of cod plummeted, which left the government in a dire fiscal situation. By 1932, interest payments on government debt accounted for more than half of revenues; banks soon began refusing to lend money

ⁱ Attributed to Sir Boyle Roche, Member of Parliament for Tralee, County Kerry in the Irish House of Commons. See Geoghegan (1999), p. 110.

to Newfoundland. Having lost control of its currency, the country was unable to devalue or print money as a response to the crisis. Newfoundland, desperate for cash, even offered to sell significant parts of the country to Canada.

Amid the economic chaos, scandals about government corruption began to emerge. Civil unrest spread and parliament was ransacked by an angry mob looking for the Prime Minister. He managed to save himself (despite being unable to do likewise for his country's finances) and escaped from the throng, rather ingloriously, down a side alley. His respite was not long lived, however, and the government fell when he was arrested on charges of corruption soon afterwards. The election of 1932, the final election held by the Dominion of Newfoundland, returned a legislature that would eventually vote itself out of existence. On 16 February 1934, the legislature decreed an end to responsible government and transferred its power to an unelected British civil servant in London, effectively transforming Newfoundland back into a colonial dictatorship after nearly 80 years of democracy. 15 years later, fulfilling a condition for debt relief, a national referendum decided that Newfoundland would become a province of Canada.

Newfoundland's collapse was extraordinary. The fact that democracy was completely and voluntarily subordinated to creditors is, in many ways, unique in the history of sovereign debt crises. The repercussions faced by Newfoundland are nevertheless instructive. Although, by today's sensibilities, sacrificing sovereignty in order to avoid default would be extreme, creditors to a bailout still demand a pound of flesh in terms of policy reforms to ensure repayment – as we have seen recently in Greece's case. Other elements of Newfoundland's story are more familiar: it ran up large deficits during boom times, borrowing significant amounts of money from foreign banks (denominated in a foreign currency), and requested foreign involvement to prevent default. History has demonstrated that large buildups of debt are risky, especially if borrowing in a currency one doesn't control, as they can expose the economy to crises of confidence.

This thesis aims to shed some light on the problem of sovereign debt crises and how they arise. It was written during the European sovereign debt crisis of 2010-2013 and, it will be evident, was greatly influenced by it. (See Table 1.1 and accompanying Figure 1.1 for a timeline of these events.)

Chapter 2 will introduce a workhorse model of sovereign illiquidity that will be expanded in Chapters 3 and 4. A crucial element of this model is that a country's option to default, even if unused, can have profound effects on its ability to borrow. The fact that default is possible alters the behaviour of market participants today. Lenders' payoffs are a function of coordination; knowing what other lenders think is as important as whether the sovereign is willing to pay. If a public signal suggests that other investors won't lend, then the solvency of the sovereign is immaterial. The conflagration is, in some sense, an inescapable vicious cycle.

Chapter 5 introduces a different model that describes why sovereigns might assume the debts of the financial sector and how this, in turn, influences the financial sector's financing decision. There are situations in which it might be optimal for the sovereign to backstop the financial sector; however, this acts to redistribute resources between generations. Chapters 2, 3, and 4 can be read as an oblique commentary on the situation in Southern Europe while Chapter 5 is more relevant to the crisis in Ireland.

Finally, a note on style and structure: I have tried to keep footnotes to a minimum, choosing to use endnotes instead. Footnotes use Roman numerals and are used for short asides. Endnotes use Arabic/Indian numerals and are intended either for lengthier discussions or citations. Some mathematical details have been included in the Appendices; these are cited where relevant.

The 18th-century diplomat Talleyrand once wrote of the serially defaulting Bourbons that they had learned nothing and forgotten nothing. Perhaps we might be tempted to look at the situation in Europe and conclude, to adjust his description only slightly, that we have forgotten nothing – and yet still learned nothing either. I hope this thesis is a small step toward further understanding the most recent European crisis.

Date	Explanation	Graph note
15 September 2008	Lehman Brothers files for bankruptcy. ¹	Lehman Bros' bankruptcy
18 September 2008	ECB coordinates actions with other CBs to provide liquidity in USD markets (especially overnight). ²	
15 October 2008	ECB expands collateral list and offers supplementary 3- and 6-mo LTROs with fixed rates and full allotment. ³	
7 May 2009	Covered Bond Purchase Programme announced (€60 B). Also, announcement dates for 12-mo LTROs set. ⁴	CBPP.1
23 June 2009	First 12-mo LTRO announcement made (allotment and settlement took place on the 24 and 25 respectively).	
19 October 2009	The Greek government announces that their reported deficit (3.7% of GDP) needs to be revised upward to 12.7%. ⁵	Greek figures revised
2 May 2010	Eurozone countries and IMF announce €110 B bailout package for Greece. ⁶	Greek bailout
9 May 2010	EU announces the creation of a SPV that is authorized to borrow up to €440 B. ⁷	EFSSF
10 May 2010	Securities Market Programme announced with 'measures to address the severe tensions' in financial markets. Specifically, they announce their intention to intervene in the secondary market for public debt. ⁸	SMP
30 June 2010	CBPP.1 completed. ⁹	
28 November 2010	EU and IMF bail out Ireland (€67.5 B). ¹⁰	Irish bailout
16 May 2011	Eurozone approves €78 B bailout for Portugal. ¹¹	Portuguese bailout
21 July 2011	EU and IMF agree to cut the rate on the Irish bailout (from 6% to 3.5-4%) and increase the duration. ¹²	Irish renegotiation
21 July 2011	EU and IMF agree to an additional €109 bailout of Greece. ¹³	Greek bailout 2
9 September 2011	Jürgen Stark resigns from post as ECB Chief Economist. ¹⁴	
6 October 2011	Covered Bond Purchase Programme 2 announced (€40 B). ¹⁵	CBPP.2
27 October 2011	Eurozone and IMF agree with banks to accept a 50% write-off of Greek debt (€100 B). ¹⁶	Greek haircut (50%)
8 December 2011	Two 36-mo LTROs announced with fixed rates and full allotments; reserve requirements cut from 2% to 1% starting in 2012. ¹⁷	36-mo LTROs
9 March 2012	The largest sovereign debt restructuring in history; Greek debt reduced by €100 B. ¹⁸	Greek default
26 July 2012	Mario Draghi announces that he will 'do whatever it takes to preserve the euro'. ¹⁹	
2 August 2012	ECB announces intention to undertake operations in the secondary market specifically targeting the short end of the yield curve. ²⁰	OMT
6 September 2012	ECB announces Technical Framework for OMT; SMP is terminated. ²¹	
27 September 2012	The ESM comes into force after Germany completes the ratification process. ²²	ESM

Table 1.1: A rough timeline of the European debt crisis relating to Figure 1.1.

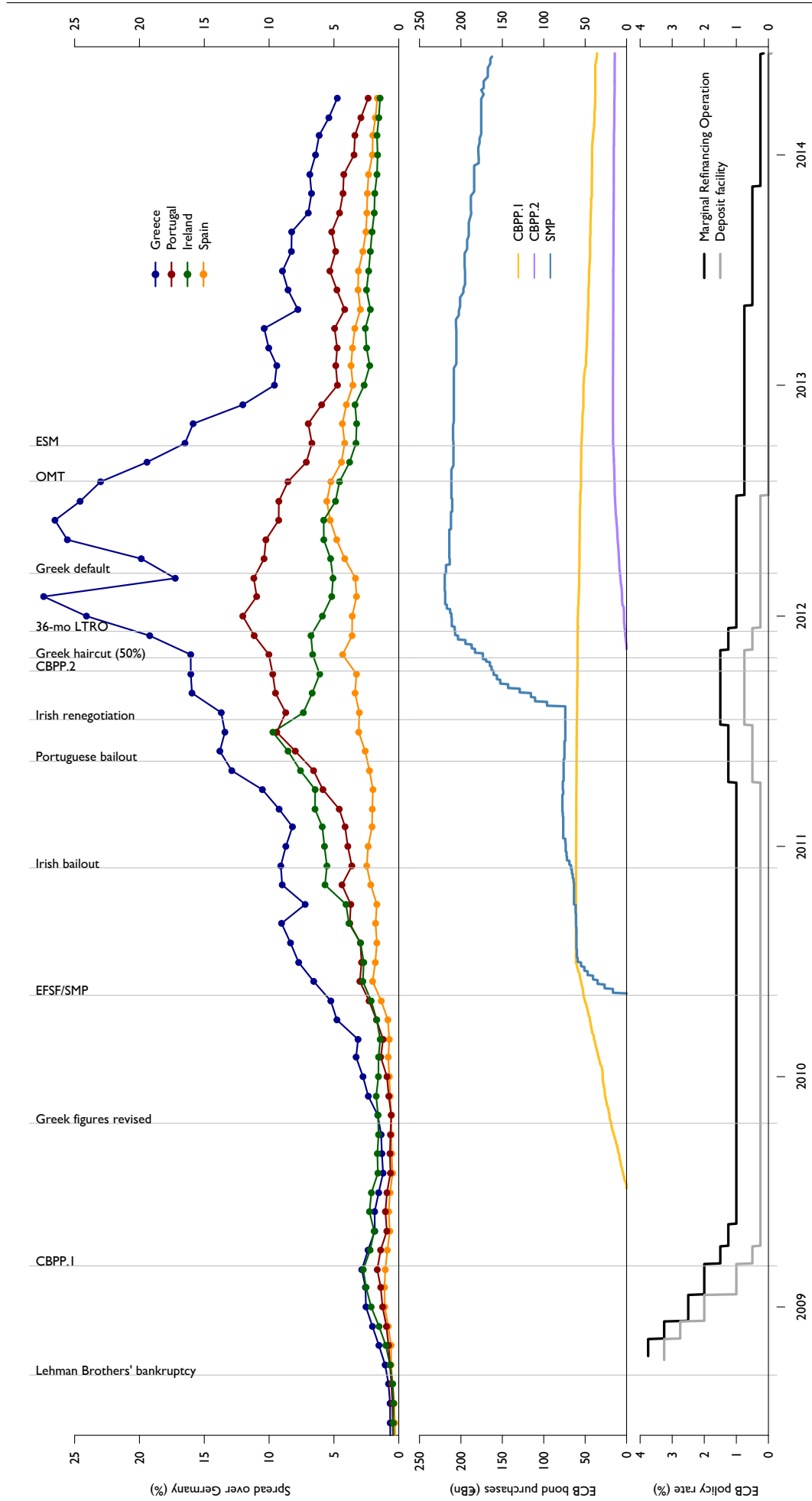


Figure 1.1: Source: ECB, Long-term interest rate for convergence assessment purposes (14 May 2014), ECB Monetary policy portfolio. SMP data compiled from ECB Press Releases.

Chapter 2

A global games approach to debt crises

It's not the speed that kills you; it's the sudden stop.ⁱ

Financial institutions hold large amounts of sovereign debt. This creates the potential for disruptions to the financial sector when a sovereign defaults. The fear, however, can result in investors being unwilling to roll-over the existing stock of sovereign debt. Such a fiscal disruption could itself cause the default the investors sought to avoid.

In Europe, such a concern persists; banks have large amounts of direct exposure to troubled sovereign debt and recapitalization could be required in the event of another sovereign default. Consequently, all levels of the European government have mobilized to arrest the crisis. Perhaps most controversially, the ECB has entered uncharted constitutional territory by announcing Outright Monetary Transactions (OMT). Although this intervention has almost certainly ended the most acute phase of the crisis (see Figure 1.1) these actions have resulted in a strong reaction. The German Federal Court has stated that the ECB is almost certainly acting beyond its mandate and has requested the European Court of Justice to rule on the constitutionality of the programme.¹ Even if we abstract from the specifics of the crisis in Europe, we might wonder under what conditions would the other Eurozone nations be willing to bail out an insolvent sovereign. Would there ever be the incentive to bail out a country, even if it were insolvent?

The formalization of last-resort lending as defined by Bagehot (1873) is useful in framing sovereign defaults and crises (although it is not directly analogous). The objective promulgated by Bagehot was that a policymaker should be willing to lend (on collateral) to a financial

ⁱThis is a banker's adage that was widely popularized by Dornbusch (1997).

institution that is illiquid but solvent. Conversely, the policymaker should allow for insolvent institutions to fail. In practice, it is very difficult to apply this rule to financial institutions since it is almost impossible to determine whether an illiquid institution is solvent. This reasoning becomes more complex when applied to sovereign institutions, an issue discussed by Krugman (1989), because a sovereign is neither strictly solvent nor insolvent; if it was *known* that a sovereign was willing to pay ('solvent') then investors would have no qualms about rolling-over the debt. Liquidity problems can arise specifically because there is uncertainty surrounding whether the sovereign will pay; that is, the country might not be able to access cash on a current basis and, as a result, not be able to meet its immediate debt obligations, even if it is willing to pay them. Such a system shares many similarities with the celebrated bank-run model by Diamond and Dybvig (1983). Just like bank runs, sovereigns can face sudden collapses in confidence that result in them being incapable of raising funds, which in turn can lead to a suspension of payments or default. Institutions lending to sovereigns face a certain degree of strategic complementarity in their actions: no single institution is capable of rolling-over the sovereign's debt and, if an insufficient number agree to participate in the deal, the sovereign might choose to default. This risk is illustrated in Figure 2.7, which shows the combined short-term debt and deficit of crisis countries as a proportion of GDP.

This strategic complementarity in lenders' actions leads debt markets to exhibit important coordination problems. A concrete example of such a coordination failure happened in 1989 when the Soviet Union attempted to roll-over its national debt by creating a consortium of banks. Of the 300 financial institutions that they approached, only five agreed to participate and the deal collapsed. The ensuing economic crisis acted in conjunction with other factors to bring down the Soviet Union (see Gaidar (2007) pp. 150-154 for an exposition of this specific instance of sovereign illiquidity and subsequent default).

Unlike illiquidity, which is a function of market dynamics, sovereign 'insolvency' is fundamentally a political decision and a function of how much pain politicians and political institutions are willing to sustain. One notable example of a leader with a very high pain threshold is Romania's Communist dictator Ceaușescu, discussed in Reinhart and Rogoff (2009). Ceaușescu ordered the export of much of the country's production in order to pay foreign creditors; food rationing was introduced and blackouts were commonplace. Most sovereigns would be unwilling to sustain such pain and would default before such drastic steps were necessary.

The specific points to be addressed in relation to this model are the interlinkages between governments' sovereign debts through the financial sector: particularly, what externalities exist between nations that share a common pool of external investors. This idea is explored in this thesis using a game theoretic model of debt roll-over; this analysis spans Chapters 2, 3, and 4. Here, Chapter 2 covers the simplest case: when there is only one sovereign authority trying to roll-over debt. Chapter 3 extends the model by considering the case where there are two countries that borrow from a single financial sector. Chapter 4 concludes the analysis of the model by considering the case when signals between financial institutions are correlated.

This chapter begins with a brief overview of the existing literature on global games. Section 2.2 introduces the game, which is similar to that discussed in Morris and Shin (2004). Section 2.3 provides a solution for the unique strategy relating to the perfect information game. Section 2.4 introduces public information and recovers a sunspot equilibrium for the game, which implies, somewhat paradoxically, that increasing the precision of public information does not necessarily increase welfare.

2.1 Relevant literature

The literature on global games largely derives from the formative paper by Carlsson and Van Damme (1993), which introduced the notion that global uncertainty implies that agents in two-agent, two-action game will play the risk-dominant action under any realization of payoffs; this has inspired a workable framework to model a wide variety of scenarios. The major benefit is that the implied unique equilibrium allows for a comparative statics analysis where, under perfect information, with multiple equilibria, such an analysis is not valid. The notion of deriving a unique equilibrium through perturbation dates back to, at least, Harsanyi (1973) and has been shown to hold for a large class of games (Frankel et al., 2003).

Since then, many papers have made significant advances. Fukao and Daigaku (1994) generalizes on the framework by investigating speculative-attack and network-externality models. Morris and Shin (1998) applies the framework to a currency-attack model and derives a unique equilibrium, which highlights the role of speculators' beliefs about others' behaviour. Morris and Shin (2002) introduces the notion of public information and demonstrates a well-known but counterintuitive result in the global games literature: that public information

can be detrimental to policy makers' goals. A similar result is demonstrated using an island economy in Amato et al. (2002), where better public information is 'double-edged' and drives the equilibrium away from fundamentals. Allen et al. (2006) presents a model with short-lived traders who play a 'beauty contest' game² and results in traders being overly influenced by public information and underweighting private signals. Rochet and Vives (2004) looks at the notion of 'lender of last resort' and how coordination failure in such a model can be reinterpreted to motivate Bagehot's taxonomy (see above). Myatt and Wallace (2008) breaks away from the strict public/private dichotomy of information adopted by previous papers and demonstrates, using an island economy model, that it is neither full clarity nor full suppression of information that is optimal; instead there is some ideal level of obfuscation. Myatt and Wallace (2012) further generalizes by modelling endogenous information acquisition in a 'beauty contest' coordination game; agents can observe multiple (costly) signals and they demonstrate that strategic complementarity generates an incentive for agents to listen to correlated sources. Angeletos and Werning (2006) also examines the effect of endogenous public information and demonstrates that the precision of endogenous public information is increasing with the precision of private information. Hellwig (2002) contributes by demonstrating that the use of the private information limit to select an equilibrium may not always be valid in coordination games; the equilibrium derived from the private information limit does not take into account the effects of public information (in the limit). Morris and Shin (2004) applies the global games approach to creditors attempting to roll-over the debt of a distressed borrower and relates the result to anomalies in the Merton (1974) model for pricing corporate debt.ⁱⁱ

The global games approach has not yet been applied directly to sovereign debt crises. With respect to coordination failure, the sovereign debt literature typically focuses on issues surrounding debt restructuring *after* a default and how this effects the incentive for the sovereign to repay (Eaton and Gersovitz, 1981; Grossman and Van Huyck, 1988). This literature has focused on bondholder committees (such as the Paris and London Clubs: see Eichengreen (2003)), reputation (Bulow and Rogoff, 1989) and contractual innovation (such as relaxing unanimous consent clauses: see Eichengreen and Kletzer (2003)).

ⁱⁱFor an overview of many of the techniques used in the global games literature (including in this chapter) a useful resource is Myatt et al. (2002). Morris and Shin (2003) provides an overview of many of the applied papers in this sub-field.

Although currency crises and sovereign debt crises are often interlinked (for example, the Mexican tequila crisis of 1995), there has been little work on the coordination required to roll-over sovereign debts and how shocks to the financial sector can act to spread this form of instability. In modelling the Mexican crisis, Cole and Kehoe (1996) used a sunspot model in order to capture the nature of financial firms refusing to roll-over the debts of a sovereign. This method of modelling is somewhat unsatisfactory since it leaves the crisis mechanism open as some *extrinsic* random perturbation. In a similar vein to the global games techniques used to model currency crises (Hellwig et al., 2005), the global games method can be used to model the coordination issue in deciding to roll-over a nation's debts and how, with multiple sovereigns, it can lead to a knock-on effect.

2.2 The one authority model

Consider a government that can finance itself only through taxes, net transfers or changes in its debt obligations. This could be for a variety of reasons, such as membership in a currency union, but for our immediate purposes let's assume that government is institutionally incapable of raising revenues through seigniorage. Further, assume that the growth in the government's debt must be bounded to grow asymptotically slower than the interest rate. It then follows that the present value of the government's outstanding debt obligations are exactly equal to the present value of the future surpluses. If the government's debt obligations increase, so must the future government surpluses. The immediate implication is that for a higher debt, governments must run higher surpluses in the future (*ceteris paribus*). But, of course, forcing governments to run surpluses can be painful; large debts necessarily imply that either future taxes must be higher or future primary spending must be lower. The 'cost' associated with adjustment is the cost that policymakers face (not necessarily acting benevolently); this cost is a function of how much pressure politicians can sustain from interest groups who are adversely impacted by the future and present budgetary changes.

Suppose that these politicians need not necessarily seek adjustment through taxes and expenditure: that they always have the option of defaulting on their debt but that there are costs associated with default. These costs are also perceived by the politician and are a function of how well creditors can pressure governments during the restructuring period, future capital market shutouts, sanctions from other nations, and many other variables. For our purposes,

however, suppose that these costs are increasing but bounded in debt or (even more simply) that they are constant regardless the level of debt.

As noted above, countries can sustain large levels of debt indefinitely; however, there is a theoretical maximum to the amount of debt that a country can sustain, which can be derived from basic theory. If we know the minimum amount of government expenditure required for the economy to function and the maximum amount of revenue that the state can tax (given by the maximum of the Laffer curve), these values determine the maximum possible surplus. Thus the theoretical upper bound on the value of debt is simply the present value of the stream of these very large surpluses, but it would be silly to believe that such a level could ever be achieved! Political pressures and economic pain would make default a viable alternative long before such an inflexible level of debt is achieved; we should instead concern ourselves with a *feasible* maximum level of debt.

If the national debt is increased, politicians must decide how and when to change future surpluses in order to keep the government's inter-temporal budget in balance. As debt increases, so does the cost of adjustment and, ultimately, the politician will perceive default to be less painful than continually raising taxes and cutting expenditure.

It follows that there is some *tolerable* maximum level of debt (B^*) above which the pain of future high taxes and low expenditure outweighs the pain of default. A country is defined as *solvent* if its debt is below this threshold where it is still willing to honour its debt.³

There are two levels of debt that are of interest: (i) the *tolerable maximum*, which we will denote as B^* which is a function of political pain and (ii) the *feasible maximum*, which is the maximum amount of debt that a country can actually realize. This thesis will demonstrate that the feasible and tolerable maximums are not necessarily the same: investors will refuse to lend to a sovereign long before it is insolvent.

2.2.1 The lenders

There is a financial sector composed of a continuum of investors who each choose whether to invest in government debt or in the safe asset. These lenders are aware of the existence of the solvency constraint that government faces and have only a fixed amount that they can

invest. If they knew the exact solvency cutoff, no investor would be willing to lend beyond this point. In reality, however, no one investor knows that true level. Each investor observes private information about the level.

There is an additional level of complexity: in order for the debt issuance to be a success, a sufficient proportion of investors must agree to join the conglomerate that rolls-over the government's debt. That is, at least l^* proportion of investors must purchase the government's debt in order for the issuance to be successful. Suppose that this proportion is an affine function of the debt level; by doubling the debt level, the country will need twice as many investors to contribute to the pot, which implies it needs twice the proportion of investors to roll-over the debt. Analytically, the coordination threshold is related to some normalized level of debt (say l_0, B_0):

$$l^* \equiv \frac{l_0}{B_0} B = \zeta B \quad (2.1)$$

An equivalent way to conceive of this constraint is that each investor has some fixed and infinitesimal ϵ of funds that they choose to invest in the safe asset or in government debt. The aggregation of resources from investors investing in the government must be enough to roll-over the government's debt. We will refer to this constraint as the *liquidity constraint* since it denotes a necessary level of coordination on the part of investors in order to successfully roll-over the debt. We will denote the proportion of investors that purchase the government's debt as l . Throughout this model, we will assume that:

$$\zeta B^* < 1 \quad (2.2)$$

2.2.2 Necessary conditions for government payment

We have outlined two necessary (and together sufficient) conditions for governments to pay their debts. First, the government must be solvent (willing to pay). Second, a sufficient number of investors must purchase the debt; this is the liquidity constraint. Formally, we can define these conditions as:

$$\begin{aligned} \text{Government is solvent} &\iff B \leq B^* \\ \text{Market is liquid} &\iff l \geq l^* \end{aligned}$$

When either of these constraints does not hold, the game ends and the government defaults. If both of these constraints do hold, then the government pays the fixed return r on its debt.

The condition for payment can be written as:

$$((l \geq l^*) \wedge (B \leq B^*)) \iff \text{Government pays debt} \quad (2.3)$$

2.2.3 Payoffs

Investors behave in a risk-neutral manner; in order to decide whether to roll-over government debt or to invest in the safe asset, investors must have an idea of the payoffs and probabilities of each outcome. When the government repays, investors holding government debt receive a return of r ; if the government is not able to roll-over its debts, it defaults and the investors lose a fixed amount β . In the event that the investor invests in the safe asset, they are guaranteed their money back but receive no return. In matrix form, the payoffs are:

	Pays	Defaults
Roll-over	r	$-\beta$
Safe asset	0	0

2.2.4 Formal specification of the game

Definition 2.1. Let Γ be a tuple (N, S, Θ, u, p) where:

- N is the set of investors and the government.
- Θ is the set of types: for investors, they are typified by the signal that they receive; the government is typified by its solvency condition.
- $u(\cdot)$ is the payoff function as defined in Section 2.2.3.
- p is the distribution of signals received by the investors conditional on B^* .

Corollary 2.0.1. *The tuple Γ is a Bayesian game.*

Proof. This follows from the definition of a Bayesian game in Appendix D.2. ■

Throughout this chapter (and subsequent chapters) the following parametric assumption will greatly simplify the analysis.

Assumption 2.1. (No lotteries) Assume that the cost of default is high compared to the return on repayment:

$$\beta > r \quad (2.4)$$

Theorem 2.1. *Under normally distributed private information, the game Γ has a unique, symmetric switching strategy κ^* . Depending on the parameters, this strategy is either finite or degenerate:*

$$\kappa^* = \begin{cases} \sigma \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) & \text{if } \frac{r}{r + \beta} \geq l^* \\ \infty & \text{otherwise} \end{cases} \quad (2.5)$$

where $\Phi(\cdot)$ is the standard normal CDF.

Note that a symmetric switching strategy is one in which (i) all agents adopt the same strategy and (ii) they will choose to purchase the debt if their private signal of the government's willingness to pay is above this threshold ($x_i > \kappa + B$). This is defined more rigorously in the subsequent section. For proof of Theorem 2.1 see Propositions 2.3 and 2.4.

Corollary 2.1.1. *The limit of the symmetric switching strategy of game Γ as the noise goes to zero simplifies to: all investors roll-over the debt when the government is solvent but choose the safe asset when the government is insolvent.*

The subsequent sections will demonstrate these results.

2.2.5 Perfect information equilibria

Like most models of debt markets, this one suffers from a multiplicity of equilibria; in fact, the model we have described has infinite equilibria under perfect information.

Proposition 2.2. *Under perfect information, the game Γ has infinite equilibria.*

Proof. There always exists a degenerate equilibrium where no one rolls-over the government's debt; this prevails regardless of whether the government is willing to pay or not. Even if the government is willing to pay, since no single investor is able to knock the government into a liquid position, there is no incentive for any investor to invest in the government debt; all investors keep their investments in the safe asset and receive a payoff of zero.

There are, however, infinite equilibria in which investors purchase government debt when the government is willing to pay ($B < B^*$); every investor chooses to roll-over the government's debt ($l = 1 > l^* \in (0, 1)$) and receives a payoff of r on some interval where government is willing to pay. To demonstrate that there are infinite equilibria, it serves to demonstrate formally one class of strategies:

$$\begin{aligned}
\text{Degenerate equilibrium strategy} &\equiv \{\text{Always Safe asset}\} \\
\text{Non-degenerate equilibrium strategy} &\equiv \left\{ \begin{array}{l} \text{Roll-over if } B \leq B^* - B_{crit}; \\ \text{Safe asset if } B > B^* - B_{crit} \end{array} \right\}
\end{aligned}$$

where $B_{crit} \geq 0$ defines some distance from the solvency constraint, at which all investors decide not to invest in the debt. From a Nash Equilibrium perspective, there no way to select a single B_{crit} over any other; they are all valid equilibria. Clearly, we might like to think that investors would be willing to roll-over government debt right up until the solvency constraint; although this is *an* equilibrium, it is only one of infinite equilibria. ■

Remark. Adding uncertainty will act as an equilibrium selection device; when the variance of the information goes to zero, it will distill a specific and non-trivial threshold level of debt (B_{crit}) above which all investors choose the safe asset and below which all investors coordinate in rolling-over the debt. This is achieved by solving the model under imperfect information and taking the limit of the variance of information to zero.

2.3 Private information

In reality, the level of debt that makes a government unwilling to meet its debt obligations is unobserved. Regardless, many private investors will have an idea about this level and will make their decisions to loan money to a government accordingly. Suppose that we have a continuum of investors, each receiving a private signal x_i about the threshold level B^* , where:

$$x_i = B^* + \epsilon_i \quad \text{where, } \epsilon_i \sim N(0, \sigma^2) \quad (2.6)$$

Suppose that investors adopt a cutoff strategy in which if their signal of the threshold is sufficiently larger than the level of debt, they will purchase the government's debt.

$$\text{Purchase debt} \iff x - B \geq \kappa$$

$$\text{Safe asset} \iff x - B < \kappa$$

Given that investors are informed on the strategies of the others, *ex post*, the liquidity constraint will be satisfied when:

$$l^* \leq 1 - \Phi\left(\frac{B + \kappa - B^*}{\sigma}\right) \quad (2.7)$$

where the RHS is the proportion of investors that received a private signal larger than $B + \kappa$. Note that $\Phi(\cdot)$ denotes the standard normal CDF; by a standard manipulation of the CDF, we calculate how many standard deviations the point is from the mean in order to determine what percent of signals are above this point. This equation demonstrates that achieving the liquidity constraint is directly a function of B^* ; in fact, we can solve for the threshold level at which this constraint is achieved:

$$B^* \geq B + \kappa + \sigma\Phi^{-1}(l^*) \equiv B_l \quad (2.8)$$

This equation demonstrates the minimum signal required to achieve the liquidity constraint; this is a function of how many investors receive a signal above their cutoff. As the cutoff (κ) increases, so does the minimum signal required to achieve l^* . Further, the minimum signal required increases as more coordination is required (an increase in l^*).

Note that B_l is composed of entirely known variables (we assume that investors know κ); if the true value of B^* is below *either* B or B_l then the government will not pay its debts. In the former case, this is because the government is unwilling to pay; in the latter case it is because the government is incapable of achieving sufficient coordination amongst the investors. It follows that there are two possible scenarios: (1) the solvency constraint is binding and $B > B_l$ or (2) the liquidity constraint is binding and $B_l > B$. Examining these inequalities, we see that the following must hold.

$$\text{Solvency binding} \iff \kappa < \sigma\Phi^{-1}(1 - l^*)$$

$$\text{Liquidity binding} \iff \kappa > \sigma\Phi^{-1}(1 - l^*)$$

I will examine these two possibilities separately and demonstrate that the former implies the existence of a unique and finite cutoff, while the latter necessarily leads to the degenerate outcome where all investors choose to invest in the safe asset regardless of their private signal.

2.3.1 Solvency binding: a unique, finite cutoff equilibrium

Proposition 2.3. *The game Γ , under private information, has a unique cutoff equilibrium when the solvency condition is binding. Formally:*

$$\frac{r}{r + \beta} \geq l^* \implies \kappa^* = -\sigma\Phi^{-1}\left(\frac{r}{r + \beta}\right) \quad (2.9)$$

Proof. When the solvency constraint is binding, the government pays whenever the debt B is less than the threshold B^* . That is, under these conditions, governments default because they are unwilling to pay (not because they are incapable of coordinating sufficient investors). Investors must then forecast, given their private signal, whether to invest or not. Investors will choose to roll-over the government's debt as long as the expected value of investing in debt is greater than investing in the safe asset. Formally, this is:

$$r \left(1 - \Phi \left(\frac{B-x}{\sigma} \right) \right) - \beta \Phi \left(\frac{B-x}{\sigma} \right) \geq 0 \quad (2.10)$$

Rearranging, we see that this is equivalent to:

$$x \geq B - \sigma \Phi^{-1} \left(\frac{r}{r+\beta} \right) \quad (2.11)$$

This defines the cutoff:

$$\kappa = -\sigma \Phi^{-1} \left(\frac{r}{r+\beta} \right) \quad (2.12)$$

■

For low returns on debt r and high default costs β , investors will need higher signals in order to be enticed into purchasing government debt; the standard deviation of the signal simply amplifies the effect. Interestingly, this cutoff is independent of the liquidity threshold l^* ; this is because no one need worry about the sovereign being illiquid since, under these parameters, it will already have defaulted.

This, however, will be an equilibrium only when the solvency constraint is binding; therefore, we must check that this is the case. That is:

$$-\Phi^{-1} \left(\frac{r}{r+\beta} \right) \leq \Phi^{-1} (1-l^*) \quad (2.13)$$

In order for the solvency constraint to be binding, it must be the case that:

$$\frac{r}{r+\beta} \geq l^* \quad (2.14)$$

When this inequality does not hold, the liquidity-constraint is necessarily tighter and the equilibrium strategy in Equation 2.12 is not valid. This yields a parsimonious condition:

whether the cutoff equilibrium is valid is a function of the exogenous parameters r, β , and l^* , respectively the return on debt, the cost of default, and the liquidity threshold. In order for this equilibrium to be valid, the liquidity threshold must be small relative to the benefits of lending. The condition can be reordered as a debt limit by substituting in the original definition of the liquidity constraint (see Equation 2.1):

$$\frac{r}{r + \beta} \zeta > B \quad (2.15)$$

Alternatively, the constraint can be viewed as setting a minimum on the return offered on the debt so as to prevent a liquidity crisis. Inverting the condition yields:

$$r > \frac{\beta l^*}{1 - l^*} \geq 0 \quad (2.16)$$

This demonstrates that the return must be significantly higher than that offered by the safe asset and must increase (to prevent a crisis) if either the default cost and liquidity threshold increases.

If Constraint (2.14) is not satisfied, debt is not rolled-over under the cutoff strategy in (2.12). It is noteworthy that σ plays *no role* in whether this equilibrium remains valid. In fact, taking the limit $\sigma \rightarrow 0$, the equilibrium still exists. The next subsections will demonstrate that this is the only finite symmetric cutoff equilibrium and, unless Constraint (2.14) holds, the system will necessarily slip into the degenerate equilibrium. This solution method generates an equilibrium value for B_{crit} , which was indeterminate under the perfect information scenario; *i. e.:*

$$B_{crit} = \inf \left\{ B^*, \frac{r}{r + \beta} \zeta \right\} \quad (2.17)$$

The next subsection demonstrates that this value defines the maximum level of debt sustainable under any given parameterization.

2.3.2 Liquidity binding: the degenerate equilibrium

Proposition 2.4. *When the liquidity condition is binding, the game Γ under private information has a unique degenerate equilibrium:*

$$\frac{r}{r + \beta} < l^* \implies \kappa \rightarrow \infty \quad (2.18)$$

Proof. When the liquidity constraint is binding, this is equivalent to $\kappa > \sigma\Phi^{-1}(1-l^*)$. It follows that the investor with signal x will choose to invest in the debt when the expected value of rolling-over is greater than investing in the safe asset. Remembering that the threshold signal at which the government does not pay is $B_l = B + \kappa + \sigma\Phi^{-1}(l^*)$, this inequality is given by:

$$r\left(1 - \Phi\left(\frac{B_l - x}{\sigma}\right)\right) - \beta\Phi\left(\frac{B_l - x}{\sigma}\right) > 0 \quad (2.19)$$

This rearranges to:

$$B + \kappa - \sigma\left[\Phi^{-1}\left(\frac{r}{r+\beta}\right) + \Phi^{-1}(1-l^*)\right] < x \quad (2.20)$$

Since the liquidity constraint is binding, $\Phi^{-1}\left(\frac{r}{r+\beta}\right) + \Phi^{-1}(1-l^*) < 0$. It follows that, when every other investor is playing strategy κ , investor i will prefer to choose a cutoff that is strictly more conservative than those chosen by the other players. Since every investor concludes this, it follows through iterated dominance of strategies that the cutoff goes to infinity. ■

When the liquidity constraint binds, all private signals are ignored by investors since the payoff is a function of others' actions, which are determined. That is, the only equilibrium outcome is one in which the investors always choose the safe asset and the government is incapable of rolling-over its debts. It follows that the only scenario under which the degenerate equilibrium is not the sole outcome is when the solvency constraint is binding, as in the previous section. Propositions 2.3 and 2.4 prove Theorem 2.1.

2.3.3 The maximum level of debt

In this model, the *tolerable maximum* (B^*) is defined by political willingness to repay the debt, which is not in general equal to the point at which a debt crisis occurs ($\frac{r}{r+\beta}\zeta$). This latter point can be referred to as the *crisis maximum*. The *feasible maximum* level of debt will be simply the smallest of these two values; depending on the parameterizations, it could be either. Formally:

$$B_{max} \equiv \inf\left\{B^*, \zeta\frac{r}{r+\beta}\right\} \quad (2.21)$$

If the maximum is defined by B^* , the country will outright default when this threshold is passed. If the maximum is defined by $\zeta\frac{r}{r+\beta}$, any debt beyond this level pushes the country into a liquidity crisis, in which they are unable to mobilize enough investors. This demonstrates the two outcomes that tend to typify government debt problems: either the government defaults ($B_{max} = B^*$) or creditors lose faith in the government ($B_{max} = \zeta\frac{r}{r+\beta}$). *Ex ante*,

investors do not know which of these two will bind; however, they do know that any level of debt beyond $\zeta \frac{r}{r+\beta}$ necessarily leads to crisis.

See Figure 2.1 for a diagram of the behaviour of the model. In fact, as debt increases, this game behaves much as Hemingway described going broke: “Gradually and then suddenly”.ⁱⁱⁱ For $\sigma > 0$, the proportion of investors holding debt decreases slowly as debt increases but, once $\frac{r}{r+\beta}\zeta$ is passed, everything falls apart at once.

2.3.4 The difference between default and crisis

This model has three possible outcomes, which are analogous to a successful roll-over, default, and crisis. The outcome when government is capable of rolling-over its debts results in the investors getting paid.

The two cases in which government debt does not pay, default and crisis, are analogous to their real-world equivalents. A sovereign crisis occurs when the degenerate equilibrium occurs due to the liquidity requirement being too high. This might occur even when a government is willing to pay; to this effect, the game is not unlike a Keynesian beauty contest, in which the opinion of others is very important to a given player’s actions. Investors know that there will not be a sufficient number of investors to roll-over the debts; as a result, all the investors choose the safe asset. Although an investor’s own signal might be that the government is willing to pay, because they believe that other investors will not be sufficiently enticed to invest, they choose to invest in the safe asset. Under the crisis scenario, no investors purchase government debt and so none is left holding defaulted bonds; this unrealistic outcome could be rectified by including a maturity structure to the debt.

The sovereign default case occurs when bonds do not pay because the government is simply unwilling to repay its debts. Many investors will purchase government debt because they receive signals that are higher than many of the other agents’; this illustrates another common mental model applied to many asset classes and is similar to the winner’s curse. In the event that the authority does not pay, the debt is held by the investors that value it most. Unfortunately for them, they have over-valued the debt and are stuck with a defaulted bond.

ⁱⁱⁱThis is from *The Sun Also Rises*.

Whether we observe crisis or default is a function of the debt maximum, which is itself determined by the parameters of the model. Under fixed parameters, a country is either a ‘crisis country’ or a ‘defaulting country’; perhaps an analogy can be drawn between this and the notion that some countries *graduate* from sovereign crises (Reinhart et al., 2003). Fundamentally, however, the quality of private information does *not* determine whether a country is one or the other and individual investors are incapable of knowing in which of the two cases they are. We will see that this is not the case when public information is introduced in Section 2.4.

2.3.5 Summary of the private-only information game

By setting up this simple and parsimonious one-shot model, in which symmetric agents receive perturbed signals on the government’s willingness to pay, we can distinguish between two important outcomes observed in sovereign debt markets: sovereign debt crises and government defaults. Further, the quality of private information does *not* determine which of these two states prevails.

The quality of private information has no bearing on the maximum level of debt that a government can sustain; it does, however, affect the proportion of investors that hold the debt. From a welfare perspective, better private information helps investors to act more appropriately: with better information, they are less likely to be holding debt when it does not pay and more likely to be holding it when it does.

The crisis scenario recovers an odd result in which the value of debt that causes a debt crisis is independent of the precision of the private information. The threshold value of debt is a function of when the system shifts from having the solvency-constraint bind to having the liquidity-constraint bind. This switch is determined only by the rate of return on debt, the pain of default, and the liquidity requirement (a function of the debt). This result is non-trivial and cannot be derived from the perfect information game by inspection, yet is derived by all players and commonly known under private-information structure. One way to conceive of the breakdown is that the beauty contest aspect of the sovereign debt begins to prevail. The valuation of the debt, in the opinion of any given investor, becomes a function solely of the actions and beliefs of the other investors. Holding everyone else’s strategies constant, each investor will want to behave more conservatively than their peers; applying

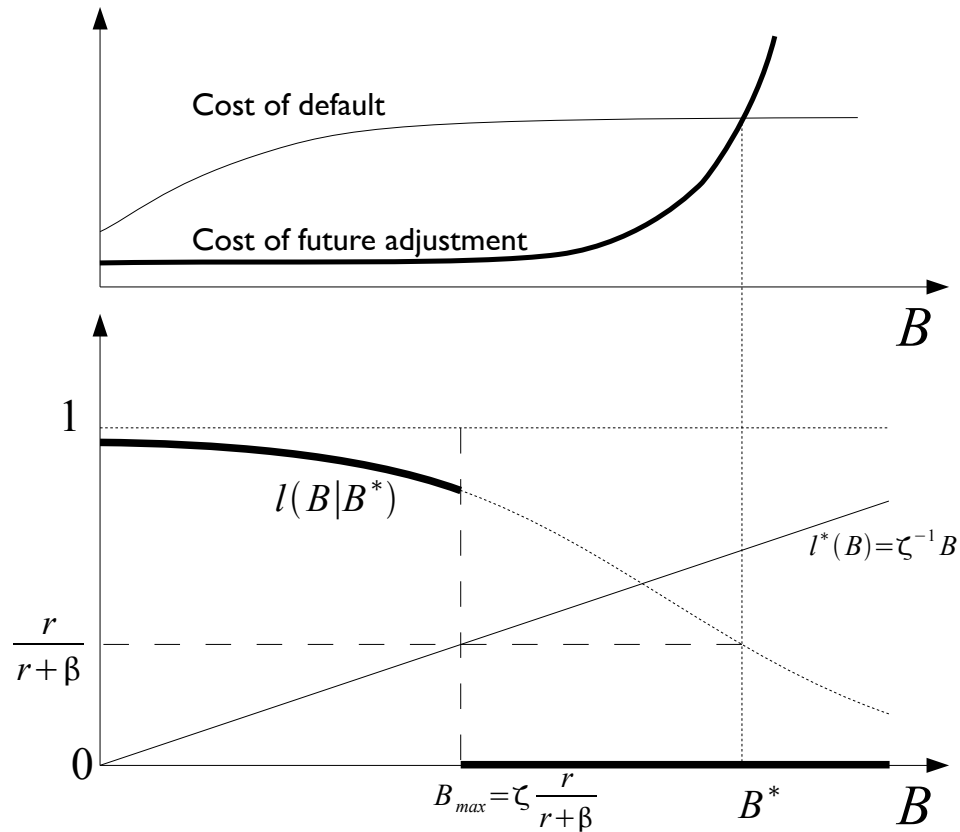


Figure 2.1: This diagram illustrates the private information game outcome as a function of the level of debt. The top panel shows the point at which the government will decide to default as a function of the size of the debt (B^*); this maps down onto the bottom panel. The thick black line is the proportion of investors who purchase the government debt conditional on B^* ; at debt level B , there will be $l(B|B^*)$ proportion of investors buying government bonds. Note that this line is discontinuous: it jumps to zero once the debt passes B_{max} ; this figure illustrates the outcome when a crisis occurs below B^* . The upward-sloping line out of the origin maps out the liquidity constraint as a function of the level of debt; for higher debt, more coordination is required. Note that the maximum level of debt is shown where $\frac{r}{r+\beta}$ intersects with the liquidity-threshold line. If we were to rotate the liquidity-threshold line down, this would push B_{max} out closer to B^* . At some point, the intersection between $\frac{r}{r+\beta}$ and the market-liquidity line would move beyond B^* , which would yield the default outcome (as opposed to the crisis outcome) for $B^* < B < \frac{B_0}{l_0} \frac{r}{r+\beta}$. As an aside, if we were to continue drawing the smooth $l(B, B^*, l^*)$ curve (instead of the discontinuous jump to zero), it would be equal to $\frac{r}{r+\beta}$ when $B = B^*$; this is shown by the downward-sloping dashed line.

this logic iteratively, the investors enter into a race to be the most conservative – leaving no one willing to roll-over the debt *even if* the signal suggests that the fiscal authority is overwhelmingly willing to repay the debts.

The proportion of investors holding debt (as a function of the level of debt) is:

$$l(B | B^*, r, \beta, B_0, l_0) = \begin{cases} 1 - \Phi\left(\frac{B-B^*}{\sigma} - \Phi^{-1}\left(\frac{r}{r+\beta}\right)\right) & \text{if } B \leq \zeta \frac{r}{r+\beta} \\ 0 & \text{if } B > \zeta \frac{r}{r+\beta} \end{cases} \quad (2.22)$$

When the solvency constraint binds, it follows that $\frac{r}{r+\beta}$ proportion of investors purchase debt when $B = B^*$.

The next section solves the game with public information, which can impact the overall outcome and shift the system between crisis and stability. Although the effect of improving public information is not unambiguous, as the variance goes to zero, we will be able to drive the debt maximum up to B^* from the lower crisis level by improving the quality of the public information signal.

2.4 Private and public information

Now consider the case in which the investors receive a private signal and observe a separate public signal. Both signals are normally perturbed values of the true value B^* with some specified variance. That is, the private signal for investor i is defined by:

$$x_i = B^* + \epsilon_i \quad \text{where, } \epsilon_i \sim N(0, \sigma^2) \quad (2.23)$$

The public signal is defined as:

$$y = B^* + \eta \quad \text{where, } \eta \sim N(0, \gamma^2) \quad (2.24)$$

Here, the investors must make two decisions: they must choose how to weight each of the signals and also what their cutoff should be. Denote the weight accorded to the public signal as α and the weight to the private component as $(1-\alpha)$. As long as $\alpha \in [0, 1]$, the forecast will be unbiased. If investors choose to minimize the variance of their forecast, they will adopt

an inverse variance rule where:

$$\alpha^* = \frac{\sigma^2}{\gamma^2 + \sigma^2} \quad (2.25)$$

If this variance-minimizing approach is used, their best guess of the true solvency cutoff is:

$$\hat{B}^* = \frac{\sigma^2 y + \gamma^2 x}{\gamma^2 + \sigma^2} \quad (2.26)$$

In general, however, we will denote their guess as:

$$\bar{B}^* = \alpha y + (1 - \alpha)x_i \quad (2.27)$$

Each investor will need to decide when to invest in sovereign debt or in the safe asset depending on their best guess of the government's willingness to pay \bar{B}^* and the announced level of government debt B . This statistic is an unbiased estimator of B^* with a variance of $\alpha^2 \gamma^2 + (1 - \alpha)^2 \sigma^2$. Suppose that investors adopt a cutoff strategy in which they play *roll-over* if the level of debt B is sufficiently below the best guess \bar{B}^* ; whenever the level of debt is above this level, they will choose to invest in the safe asset. The strategy κ is defined:

$$\begin{aligned} \text{Roll-over} &\iff \bar{B}^* - B \geq \kappa \\ \text{Safe asset} &\iff \bar{B}^* - B < \kappa \end{aligned}$$

Firstly, it is necessary to augment our definition of Γ in order to incorporate the public signal.

Definition 2.2. Let Γ' be the same as Γ (Definition 3.1) except augmented with an additional, unbiased public signal that is normally distributed with standard deviation γ .

Theorem 2.5. *The game Γ' has a unique switching strategy $\mathbb{S}_i^* = \{\kappa^*, \alpha^*\}$*

$$\kappa^* = \begin{cases} \frac{\gamma\sigma}{\sqrt{\sigma^2 + \gamma^2}} \Phi^{-1}\left(\frac{\beta}{r + \beta}\right) & \text{if } \frac{r}{r + \beta} \geq l^* \\ \infty & \text{otherwise} \end{cases} \quad (2.28)$$

and

$$\alpha^* = \frac{\sigma^2}{\gamma^2 + \sigma^2} \quad (2.29)$$

To show this result, firstly the relationship between the choice of cutoff κ and the investors' choice of weight α must be determined; then it is necessary to show that, given the cutoff, there is a dominant choice for α , which yields the weight shown in Equation (2.29).

From the information setup in Equations (2.23) and (2.24) it follows that the *ex post* distribution of signals will be normally distributed with mean $\alpha y + (1 - \alpha)B^*$ and variance $(1 - \alpha)^2\sigma^2$. Thus, the proportion of investors choosing to invest is given by:

$$l(\cdot|y, B^*) = 1 - \Phi\left(\frac{B + \kappa - \alpha y - (1 - \alpha)^2 B^*}{(1 - \alpha)^2 \sigma^2}\right) \quad (2.30)$$

As before, we can equate this with the liquidity threshold l^* and solve for the conditions under which the liquidity constraint or the solvency constraint will bind:

$$B_l \equiv \frac{B + \kappa - \alpha y}{(1 - \alpha)} - (1 - \alpha)\sigma^2\Phi^{-1}(1 - l^*) \quad (2.31)$$

It is worth noting that this is a function of the public signal y ; if the public signal is low, then the true value of B^* will need to be higher in order for sufficient investors to have signals large enough to be willing to buy debt. As in the private information case, there are two situations that could arise: (1) the solvency constraint is binding and $B \geq B_l$ or (2) the liquidity constraint is binding and $B_l > B$. Examining these inequalities, we see that the following must hold:

$$\begin{aligned} \text{Solvency binding} &\iff \kappa \leq \alpha(y - B) + (1 - \alpha)^2\sigma^2\Phi^{-1}(1 - l^*) \\ \text{Liquidity binding} &\iff \kappa < \alpha(y - B) + (1 - \alpha)^2\sigma^2\Phi^{-1}(1 - l^*) \end{aligned}$$

Like in the private-information case, these cases can be examined separately. Investors, with the variables in their information sets, are able to discern which of the two constraints is binding. It is interesting to note that there is some *ex ante* probability that either of these states could hold. When y is revealed, however, there is no ambiguity; this property is discussed below.

2.4.1 Choice of forecasting weight α

Investors face a problem of weighting two unbiased signals. As long as the weights they adopt sum to 1, it follows that their forecast will be an unbiased estimate of B^* . The investors' payoff, however, does not rely solely on guessing the 'correct' value of B^* , but the variance of their forecast will have a significant impact on their expected payoff. The variance of an investor's forecast \bar{B}^* can be denoted as V :

$$V = \alpha^2\gamma^2 + (1 - \alpha)^2\sigma^2 \quad (2.32)$$

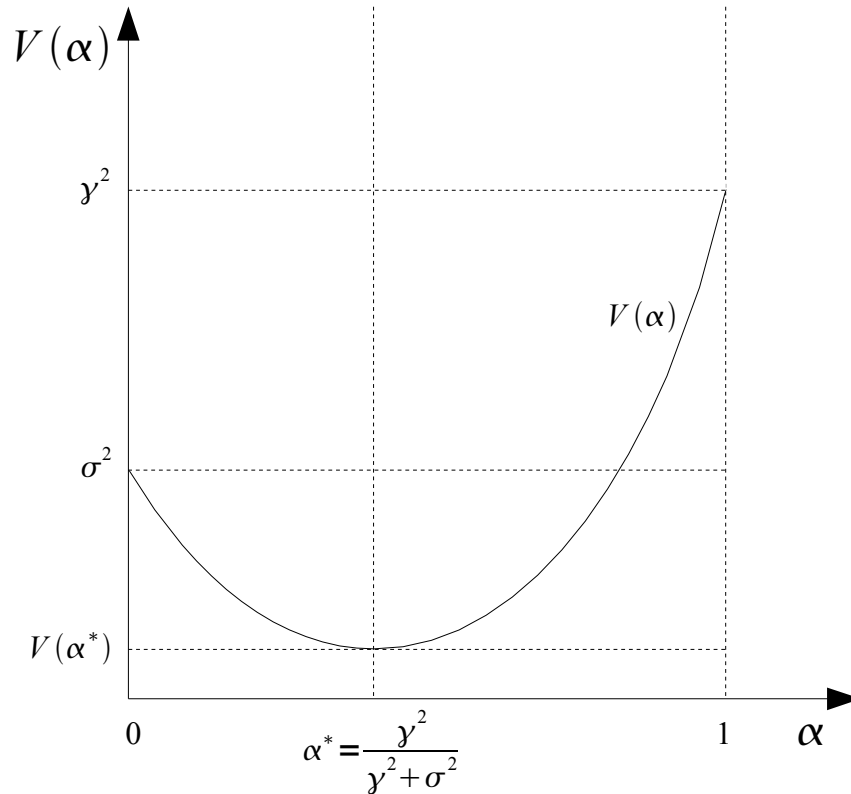


Figure 2.2: This figure shows the quadratic relationship between the weight placed on public information (α) and the variance of the forecast (V). The variance achieves a minimum at the inverse-variance rule α^* . When all weight is put on the public signal ($\alpha = 1$), the variance is simply the variance of the public signal (γ^2). Similarly, when all weight is put on the private signal ($\alpha = 0$), the variance of the forecast is the variance of the private signal (σ^2). This figure assumes that $\gamma > \sigma$, which need not be the case.

Immediately it is clear that there is a quadratic relationship between the variance of the forecast and the choice of α . This relationship is graphed in Figure 2.2. The minimum of this function is well-known: it is the inverse-variance weighting scheme familiar in much of Bayesian analysis. This variance-minimizing weighting is given in Equation (2.25).

The strategy adopted by investor i can be expressed as two terms:

$$\mathbb{S}_i = \{\kappa, \alpha\} \quad (2.33)$$

These terms are the cutoff (κ) and the forecasting weight (α). The cutoff κ denotes the level of signal above which the investor will choose to roll-over the sovereign's debt. For any choice

of κ , the dominant strategy is to select the forecast with minimum variance; this corresponds to the inverse variance rule associated with Bayesian updating. To show this, suppose there is an equilibrium in which all agents play the strategy $\{\kappa', \alpha'\}$; it follows that, unless $\alpha' = \alpha^*$ from Equation (2.25), there is a profitable deviation for anyone with a forecast that is greater than the cutoff level. Any forecast with $\alpha \neq \alpha^*$ has a strictly higher variance, which implies that, if the agent chooses to invest, they will have a higher probability of making a mistake and, hence, a lower expected payoff. The quadratic relationship between the variance of the weight term α is given by Equation (2.32) and illustrated in Figure 2.2. As a result, the strategy $\{\kappa', \alpha^*\}$ will strictly dominate $\{\kappa', \alpha\}$ for some non-empty subset of investors (specifically for those who have forecasts above the cutoff level). This argument is illustrated in Figure 2.3 and follows from the ‘no lotteries’ assumption 2.1, which guarantees that the expected payoff is decreasing in the variance of the forecast, while the signal that makes the investor indifferent is decreasing in the variance.

Ex ante, the expectation of the forecast is unbiased regardless the choice of α :

$$E[\bar{B}^*] = E[\hat{B}^*] = B^* \quad (2.34)$$

where \bar{B}^* is a forecast using any $\alpha \in [0, 1]$ while \hat{B}^* is the forecast using α^* . Naturally, it follows:

$$E\left[\frac{B - \bar{B}^*}{V}\right] < E\left[\frac{B - \hat{B}^*}{\gamma^2 \sigma^2 / (\gamma^2 + \sigma^2)}\right] \quad (2.35)$$

Note that V is defined in Equation (2.32). The values on both sides of the inequalities in square brackets are *ex ante* normally distributed with standard deviation 1. Then, it follows (because $\Phi(\cdot)$ is a monotonic, positive-valued function) that, as long as $\kappa > 0$:^{iv}

$$E\left[\Phi\left(\frac{B - \bar{B}^*}{V}\right)\right] < E\left[\Phi\left(\frac{B - \hat{B}^*}{\gamma^2 \sigma^2 / (\gamma^2 + \sigma^2)}\right)\right] \quad (2.36)$$

Under the strategy $\mathbb{S} = \{\kappa', \alpha'\}$, the expected payoff to agent i is:

$$E(\mathbb{S}; \bar{B}_i^*) = \begin{cases} 0 & \text{if } \bar{B}_i^* - B < \kappa' \\ r - (r + \beta)\Phi\left(\frac{B - \bar{B}_i^*}{V}\right) & \text{otherwise} \end{cases} \quad (2.37)$$

^{iv}This follows from the risk neutrality of the investors and the ‘no lotteries’ assumption 2.1.

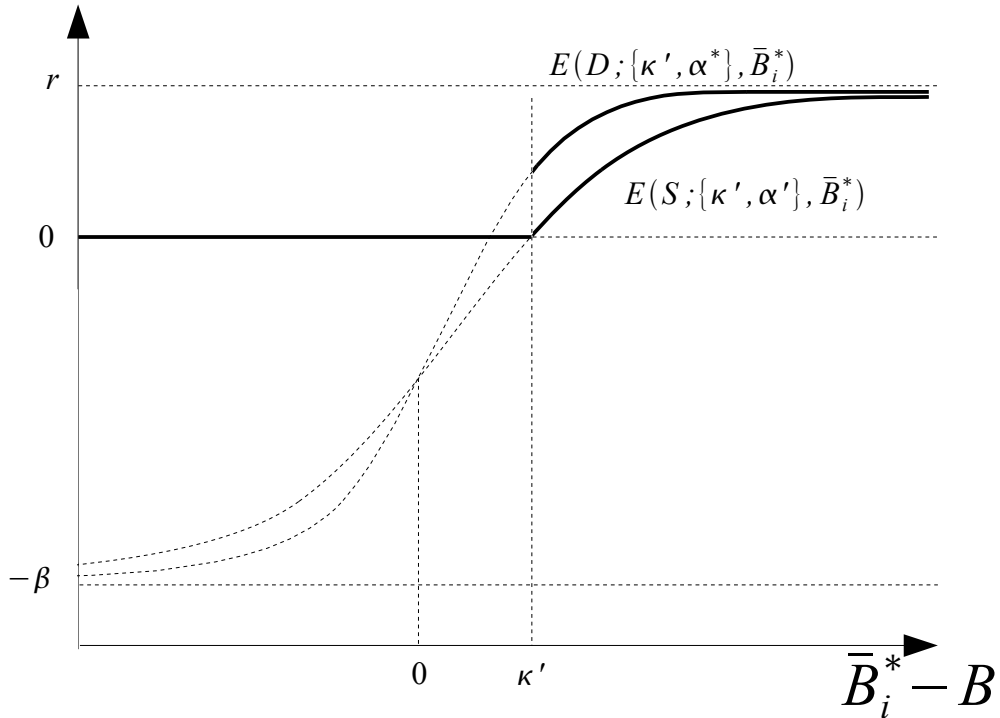


Figure 2.3: This figure demonstrates graphically how the choice of α^* dominates any other choice of α . The expected payoff of strategy $S = \{\kappa', \alpha'\}$ is shown. In this diagram, the cutoff κ' is at the point where the expected value of playing S as a function of \bar{B}^* is zero; this need not be the case. The crucial point is that there exists a deviation from S , denoted by D , in which the agents keep the same cutoff but increase the precision of their forecast by adopting α^* as their weight. There is a subset of agents (who have signals above the cutoff) who will strictly increase their expected payoff by reducing the variance of their forecasts.

This strategy will be dominated for some investor i who under strategy \mathbb{S} would invest; the dominant strategy will be denoted by $\mathbb{D} = \{\kappa', \alpha^*\}$:

$$E(\mathbb{D}; \bar{B}_i^*) = \begin{cases} 0 & \text{if } \bar{B}_i^* - B < \kappa' \\ r - (r + \beta)\Phi\left(\frac{B - \bar{B}_i^*}{V}\right) & \text{otherwise} \end{cases} \quad (2.38)$$

By Equation (2.36), \mathbb{S} is strictly dominated by \mathbb{D} for some subset of investors as long as the ‘no lotteries’ holds (Assumption 2.1). Thus it follows that any equilibrium strategy must have:

$$\alpha^* = \frac{\gamma^2}{\gamma^2 + \sigma^2} \quad (2.39)$$

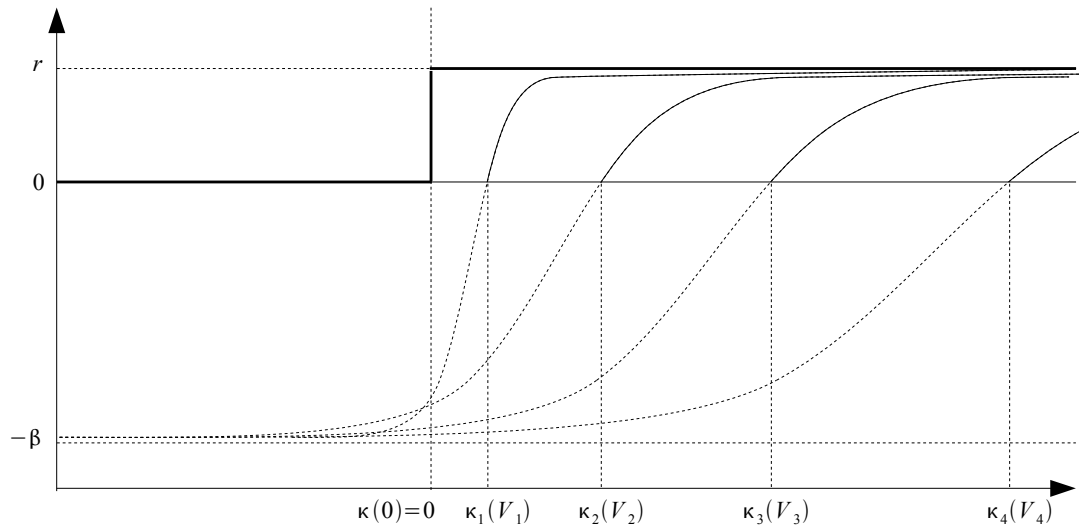


Figure 2.4: This figure illustrates the relationship between the variance (V) of the estimator \bar{B}^* and the expected value of investing. Further, the point κ , where the investor is indifferent between investing and not, is clearly illustrated. As the variance of the estimator approaches zero, the payoff profile for the investor is zero if the government is unwilling and r if the government is willing (this is denoted by the thick black line). For positive variance, the cutoff will be above zero and increasing in the variance of the estimator. Further, the expected value will be higher when the variance is lower. This figure illustrates this process for several variances, where $0 < V_1 < V_2 < V_3 < V_4$.

2.4.2 Solvency binding: a cutoff equilibrium

Given that the solvency constraint is binding, an investor will purchase the debt as long as the expected value is greater than zero (their second-best option). This yields the inequality:

$$r \left(1 - \Phi \left(\frac{B - \bar{B}^*}{V} \right) \right) - \beta \Phi \left(\frac{B - \bar{B}^*}{V} \right) \geq 0 \quad (2.40)$$

where V , the variance of the estimator \bar{B}^* , is given by Equation (2.32), which is illustrated in Figure 2.2. Rearranging Equation (2.40), the investor will choose to invest as long as:

$$\bar{B}^* \geq B + V \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \quad (2.41)$$

This implies that the cutoff κ , as defined vis-à-vis \bar{B}^* , is:

$$\kappa = V \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \quad (2.42)$$

As shown in Section 2.4.1 and Figure 2.4, the dominant choice of α^* is given by the inverse variance rule. This in turn implies that the forecast of B^* is given by \hat{B}^* (as defined in Equation 2.26). The equilibrium cutoff strategy is then given by:

$$\kappa = -\frac{\gamma\sigma}{\sqrt{\sigma^2 + \gamma^2}}\Phi^{-1}\left(\frac{r}{r + \beta}\right) \quad (2.43)$$

Substituting in the formula for the optimal forecast \hat{B}^* , we can rewrite the strategy in terms of the private signal alone; the investor will choose to invest as long as the private signal is sufficiently large:

$$x_i \geq \left(1 + \frac{\sigma^2}{\gamma^2}\right)B - \sqrt{\sigma^2 + \gamma^2}\frac{\sigma}{\gamma}\Phi^{-1}\left(\frac{r}{r + \beta}\right) - \frac{\sigma^2}{\gamma^2}y \quad (2.44)$$

This defines an equivalent cutoff strategy based solely on the private signal. Formally, we see that an investor will invest as long as:

$$x_i - B \geq \frac{\sigma^2}{\gamma^2}B - \sqrt{\sigma^2 + \gamma^2}\frac{\sigma}{\gamma}\Phi^{-1}\left(\frac{r}{r + \beta}\right) - \frac{\sigma^2}{\gamma^2}y \equiv \kappa_x \quad (2.45)$$

Under this formulation, it is clear that the higher the debt (B), the higher the private signal must be in order for an investor to be willing to invest in government debt (*ceteris paribus*). According to other parameters, this private-information formulation behaves similarly to the cutoff already described.

Now that we have specified a cutoff strategy for investors to play, we need to check that it is still valid given that the solvency constraint binds. We can arrange it such that we constrain the value of the public signal; that is, if the public signal is sufficiently high, the solvency constraint will bind. If, however, the public signal is below some publicly known threshold, the liquidity constraint will bind. The solvency constraint binds when:

$$B - \frac{\gamma}{\sigma}\left[\gamma\Phi^{-1}(1 - l^*) + \sqrt{\sigma^2 + \gamma^2}\Phi^{-1}\left(\frac{r}{r + \beta}\right)\right] < y \quad (2.46)$$

It should be noted that *ex ante* players will not know in what state the game is but when their signals are revealed every investor will know with certainty whether the solvency constraint is binding.

2.4.3 Liquidity binding: the degenerate equilibrium

Appealing to both the previous section and to Section 2.4.1, we will work forward assuming $\alpha = \alpha^*$ from Equation (2.29).⁴ When the liquidity constraint is binding, an investor will be willing to invest in government debt as long as:

$$r \left(1 - \Phi \left(\frac{\sqrt{\sigma^2 + \gamma^2}(B_l - \hat{B}^*)}{\gamma\sigma} \right) \right) - \beta \Phi \left(\frac{\sqrt{\sigma^2 + \gamma^2}(B_l - \hat{B}^*)}{\gamma\sigma} \right) > 0 \quad (2.47)$$

We also know that when $\hat{B}^* = B + \kappa$ the investor will be indifferent between the safe asset and government debt; using this equality, we can solve for the value of κ :

$$\kappa = \frac{\gamma^3}{\sigma\sqrt{\sigma^2 + \gamma^2}} \Phi^{-1} \left(\frac{r}{r + \beta} \right) + \frac{\gamma^2}{\sigma} \Phi^{-1}(1 - l^*) + (y - B) \quad (2.48)$$

This equation will be simplified; we must now check to see that this cutoff is consistent with the liquidity constraint binding. In order to have this constraint bind, we must have:

$$\kappa > \frac{\sigma\gamma^2}{\sigma^2 + \gamma^2} \Phi^{-1}(1 - l^*) + \frac{\sigma^2}{\sigma^2 + \gamma^2} (y - B) \quad (2.49)$$

Substituting in the κ for which we solved, if this is to hold then we must *also* have:

$$y > B - \frac{\gamma}{\sigma} \left[\gamma \Phi^{-1}(1 - l^*) + \sqrt{\sigma^2 + \gamma^2} \Phi^{-1} \left(\frac{r}{r + \beta} \right) \right] \quad (2.50)$$

This is the condition for having the solvency constraint bind from Equation 2.46; *i.e.* we have found a contradiction. There is no cutoff equilibrium when the liquidity constraint binds. It follows that we have the degenerate equilibrium; investors cannot coordinate their behaviour and, like in the private-information case, everything falls apart through iterated dominance.

2.4.4 A meaningful interpretation of sunspot equilibria

If the timing of the game is such that B is selected before the revelation of the public information, there will always be some finite probability that the system is pushed into a crisis due to a bad public signal. This is almost identical to the Cole and Kehoe (1996) model in which investors coordinate their behaviour based on a sunspot *extrinsic* to any economic interpretation; in this model, the information is more than a sunspot – it is based on a public signal of the government's willingness to pay and the coordination problem faced by investors.

It is straightforward to calculate the *ex ante* probability of being pushed into a crisis; it is simply the probability that the public signal is sufficiently bad to push us into the parameter space where the solvency constraint no longer binds. Formally, this is:

$$\Pr(\text{Coordination failure}) = \Phi\left(\frac{B - B^*}{\gamma} - \frac{1}{\sigma}\left[\gamma\Phi^{-1}(1 - \zeta B) + \sqrt{\sigma^2 + \gamma^2}\Phi^{-1}\left(\frac{r}{r + \beta}\right)\right]\right) \quad (2.51)$$

As the variance of public information goes to zero, the probability of such a crisis also converges on zero as long as $B < B^*$. If $B > B^*$, the probability of a crisis converges to 1 as the quality of public information improves.

2.4.5 The maximum level of debt

Alternatively, we could assume that the public information is revealed and then back out the maximum government debt that is possible given this signal. As in the private-information model, the maximum level of debt that a country can sustain is not necessarily set by the government's willingness to pay B^* but is also a function of when coordination can break down. Analytically this can be quite difficult to define but conceptually it is no more difficult than the private-information case.

$$B_{max} \equiv \inf \left\{ \sup_B \left\{ B < y + \frac{\gamma}{\sigma} \left[\gamma\Phi^{-1}(1 - \zeta B) + \sqrt{\sigma^2 + \gamma^2}\Phi^{-1}\left(\frac{r}{r + \beta}\right) \right] \mid B < B^* \right\}, B^* \mid y \right\} \quad (2.52)$$

This basically says, given the public signal, that either the maximum level of debt is the government's willingness to pay B^* or that it is the maximal sustainable amount before coordination breaks down, whichever of these two is smaller.

2.5 Discussion

This model is significantly different from the private-information model; adding public information can create coordination problems but whether reducing the variance of public information is better than the private-only case is ambiguous. The crucial difference is that, under private information, it is known with certainty if the game results in crisis or not; under public information, there is some known, *ex ante* probability that the crisis could occur.

In the private-only information model, under the ‘sovereign crisis’ scenario, the maximum debt limit is equal to $\frac{r}{r+\beta}\zeta$; under the public-information model (under the same parameters), this corresponds to a situation in which $\gamma = \infty$. That is, the private-information model is nested within the public-information model (when the public signal is uninformative): below the debt threshold, the probability of a crisis is zero; above the threshold, the probability jumps to 1. As we dial down the public variance, we realize the sunspot situation in which, for any level of debt, it is possible to have a crisis. For large levels of variance, it is ambiguous whether this is an improvement since there is now positive probability of a crisis occurring on the interval $B \in (0, B_{max})$; however, the probability of having a crisis on $B \in (B_{max}, B^*)$ is less than 1. As we decrease the public variance even further ($\gamma \rightarrow 0$), the effect becomes unambiguous since the probability of a crisis goes to zero. In the limit, the maximum level of debt is no longer $\frac{r}{r+\beta}\zeta$ but has increased to B^* . That is, if we have low-variance public information available we can not only decrease the probability of a crisis but also, in effect, *increase* the effective debt limit of a country. This is illustrated in Figure 2.5.

Alternatively, since the effect of changing the variance of public information has an ambiguous effect on the probability of a crisis, we can solve for the probability-maximizing variance. This can be loosely motivated by noticing (below the private-information debt maximum) that the probability of a crisis is zero when $\gamma = 0$ and $\gamma = \infty$; we might conjecture that there is some finite γ that is strictly positive and that maximizes the probability of a crisis. It is messy but straightforward to demonstrate this argument and solve for such a value; this value is a global maximum so any variance above or below it will imply a lower probability of a crisis. The direct implication is the marginal effect of reducing public-information variance on the probability of a crisis is positive when the variance is above this maximum and negative when below. Consequently, this implies that we cannot arbitrarily increase the probability of a crisis by changing the variance of public information; the probability is bounded and for low levels of debt might be infinitesimal. Figure 2.6 plots this probability-maximizing variance as a function of debt using the same parameters as Figure 2.5, with the maximum probability of a crisis. It might be the case that a low-variance public signal is preferable to private-only information; meanwhile, private-only information is preferred to a high-variance public signal.

Under the second scenario, in which a government’s willingness to pay is the maximum level of debt, when we introduce public information, we introduce a positive probability of a crisis

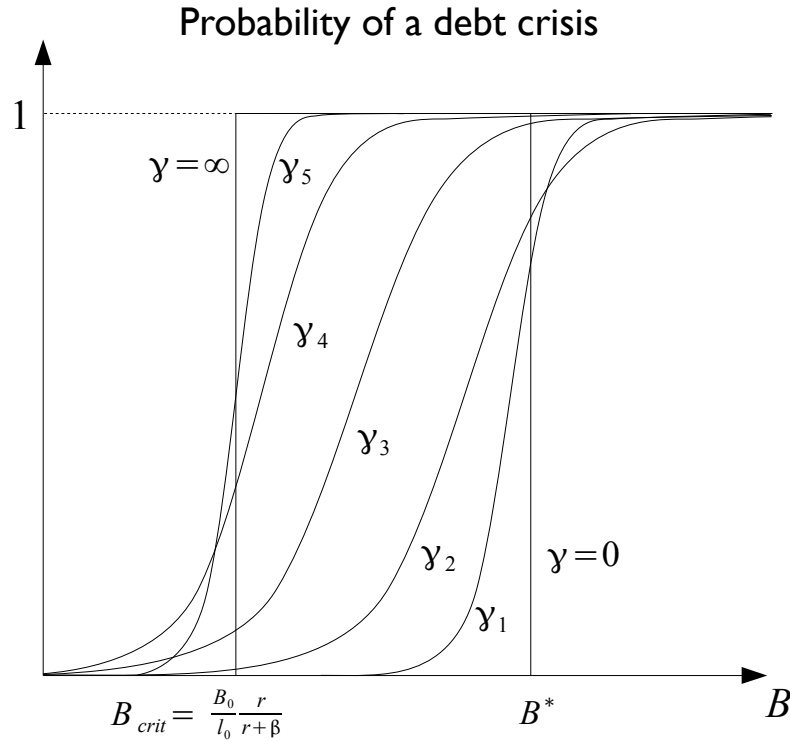


Figure 2.5: Under private-only information, which corresponds to $\gamma = \infty$, the probability of a debt crisis below B_{crit} is zero; for any debt above this value, the probability of a crisis is exactly 1. To demonstrate the effect of increasing the precision of public information, it serves to run through a few cases in which $\infty > \gamma_5 > \gamma_4 > \gamma_3 > \gamma_2 > \gamma_1 > 0$. If public information is not particularly informative (γ_5), for any level of debt below B_{crit} there is some positive probability of a crisis; above B_{crit} the probability of a crisis is strictly less than 1. If we continue to reduce the variance (γ_4), the probability of a crisis below B_{crit} will increase, while decreasing above the B_{crit} . Further reducing the variance to γ_3 and γ_2 ultimately begins to reduce the probability of a crisis below B_{crit} and reduces the probability of a crisis above B_{crit} . At some point, however, when the variance of public information is low enough, γ_1 , the probability of a debt crisis occurring above B^* will begin to increase towards 1; it should be clear that, as the variance of public information goes to zero, the outcome converges on a probability of zero below B^* and a probability of one above B^* . Clearly, increasing the precision of public information is not necessarily a good thing; better public information can serve to derail the coordination of investors attempting to roll-over the debt.

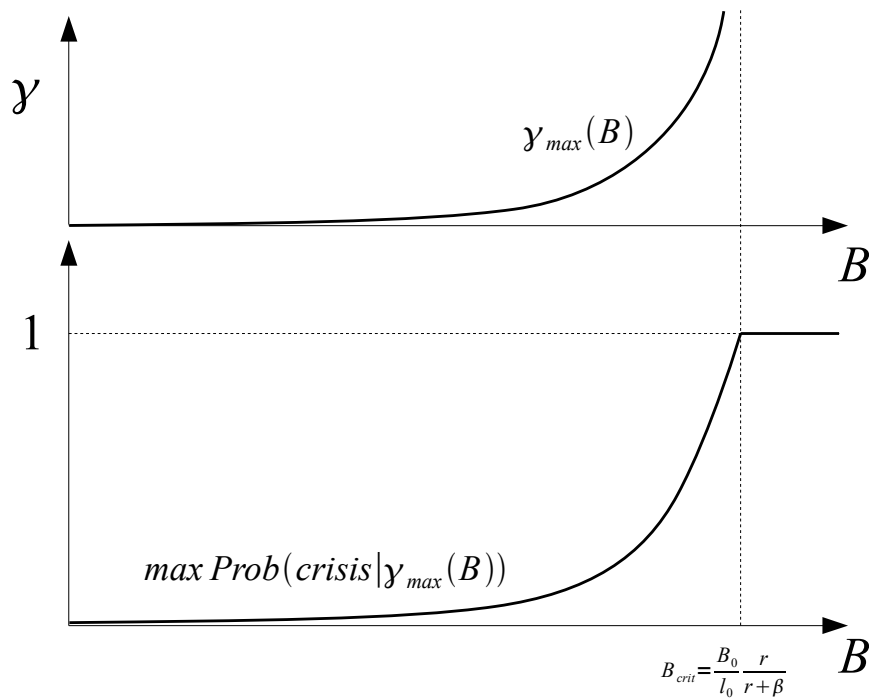


Figure 2.6: In the public-information setting, the effect of increasing the variance of public information on the probability of a crisis is ambiguous. Despite this, at each level of debt, there will be some crisis-probability-maximizing level of variance; we can conclude this loosely by recognizing in Figure 2.5 that the probability of a crisis on the interval below the private-only crisis threshold (*i.e.* B_{crit}) is zero for both $\gamma = 0$ and $\gamma = \infty$ but higher for any finite and strictly positive variance. We can solve for the value of public-information variance that maximizes the probability of a crisis. Here we plot in the top graph the standard deviation that maximizes the probability of default. It follows that any standard deviation on public information larger *or* smaller than the plotted line will necessarily lead to a lower crisis probability. As debt approaches the private-information threshold, the standard deviation that maximizes the probability goes to infinity since, above this threshold, the probability of a crisis will always be less than 1 for any finite variance. Note that this also demonstrates that the probability of a crisis for a given level of debt must be bounded. That is, for low levels of debt, we cannot arbitrarily increase the probability of a crisis by manipulating the variance of public information; in some cases, even the maximum probability will be vanishingly small. This is demonstrated in the lower graph, which plots the maximum possible probability of a debt crisis; the probability of such a crisis is negligible for low levels of debt. As debt approaches the feasible maximum at B_{crit} , the probability of a crisis quickly goes to 1.

whenever the debt is below this threshold. When $\gamma = 0$ or when $\gamma = \infty$, the probability of a crisis on the interval below the threshold is 0; in the former case, this is because we achieve the perfect information equilibrium. In the latter, it is because the signal is uninformative and thus ignored. Whenever $\gamma \in (0, \infty)$, there is some positive probability that a spurious crisis is caused by a bad draw of the public signal. There is, of course, a corresponding benefit if we do not want investors to be holding government obligations that will not pay, since under a private-information ‘sovereign default’ parameterization, many investors may be caught purchasing debt above the government’s willingness to pay. With public information, this outcome becomes less likely since, when debt is issued above the cutoff B^* , it is more likely that no investors purchase it. See Figure 2.6 for an illustration of this case.

One implication of the model is that the risk of a debt crisis could be related not only to the perceived solvency of the government but also the coordination required to roll-over and issue new debt. This is relevant when considering the European sovereign debt crisis. Figure 2.7 shows the maturing debt and fiscal deficit as a percent of GDP of the crisis countries. This is a crude measure of the coordination required in any given year required for the government to successfully borrow. It is evident that many crisis countries needed to raise the equivalent of 25-35% of their GDP *every year* during the crisis. Not only could this make us question their solvency but also whether investors could remain sufficiently coordinated.

The model also suggests that public statements and declarations can further exacerbate these coordination problems. In Greece’s case, a public declaration from the Minister of Finance regarding how economical the previous government had been with accurate budgetary information was sufficient to push the country into crisis. His statement publicly informed investors that Greece was much closer to their solvency threshold than many investors had originally believed. Not only would the investors revise their own priors but they were informed that everyone else’s signals were adversely affected as well.

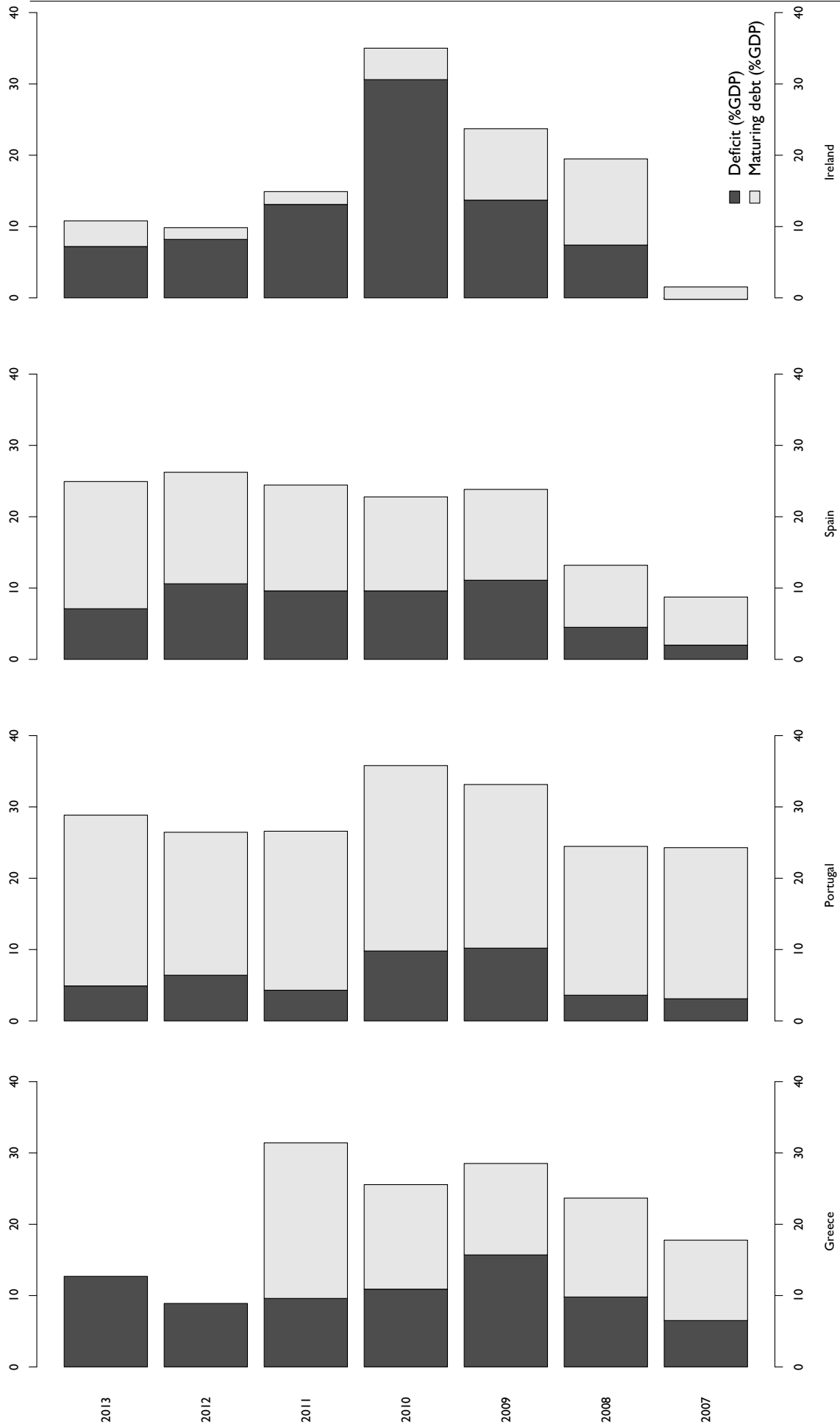


Figure 2.7: Note that there is no debt maturity data for Greece in 2012 or 2013.
 Source: Eurostat; Annex A, Chapter 2 of the Irish Report on the Account of the Public Services 2012.

Chapter 3

A debt game with multiple fiscal authorities

In the *Financial Times* on March 8, 2011, Martin Wolf in arguing why the eurozone will survive the current sovereign crisis noted:

It is important to remember, that Greece, Ireland and Portugal amount to only 6 per cent of the eurozone GDP. Even Spain is only 11 per cent. Moreover, overall eurozone public debt is only 84 per cent of GDP, while its fiscal deficit is 6 per cent. Both numbers are better than those of the US.¹

In the months after this was written, the European sovereign debt crisis only degenerated: Portugal had to be bailed out; the Irish bailout was renegotiated; Greece was bailed out for a second time and subsequently defaulted. America, however, proceeded through the crisis without significant borrowing problems – in spite of their politicians’ best attempts. Figure 3.1 shows the evolution of the debt in both the EA17 and the United States and it bears out Wolf’s point: the US saw a much larger relative increase in their borrowing yet there was no crisis.

One obvious distinction between the United States and the Eurozone is the level of fiscal integration and the dispersion of debt. The public debt of United States is largely an obligation of the federal government, which was famously created by Alexander Hamilton after the American War of Independence. The Eurozone, however, is composed of 17 separate and autonomous fiscal authorities. Wolf ignores this point in his analysis; however, whether it is important is certainly debatable. I wish to show that the existence of multiple fiscal authorities can increase the instability of the system since it increases the potential coordination problems. There could be social gains if fiscal authorities merged; this holds especially in the case that one authority is closer to the threshold debt level than the other.

There is a second major reason why America did not have a sovereign debt crisis: the US treasury is able to borrow in US dollars. Countries that are able to borrow in their own currency face little or no risk of a sovereign liquidity crisis. This is because the Central Bank is able to step in and credibly fulfill the government's liquidity needs.² Investors are aware of this and, consequently, countries such as the United States, Canada, and the United Kingdom faced little risk of a debt run. De Grauwe (2011) and De Grauwe and Ji (2012) argue that this was the main reason for the sovereign illiquidity observed in Europe during recent crisis. De Grauwe (2013) argues that the ECB ought to fulfill the role of sovereign lender of last resort in the Eurozone. This chapter, however, assumes that the Central Bank is incapable of intervening to stop the run on the sovereign.

There are many other explanations for why the Eurozone and not other economies was hit by a debt crisis. For instance, the Eurozone had many internal imbalances and the ECB allowed nominal GDP to drop dramatically. Acknowledging these other factors, the Eurozone has an additional problem that sovereign debt is distributed unevenly throughout the union.

This chapter models this partial cause of the crisis: the distribution of debt across a currency union is very important. This goes beyond the idea that countries can share 'fiscal capacity'; the strategic interaction of investors lending across multiple countries can have non-trivial implications. In short, a low-debt country can push a high-debt country into crisis by adopting austerity measures. In this two-country case, the high-debt country's crisis threshold becomes a function of the *other* country's behaviour. The fiscal externalities propagate by means of the strategic decisions made by investors.

Figure 3.3 demonstrates that the European debt crisis was highly asymmetric across the currency union. The major problems were concentrated in the pejoratively acronymed 'PIIGS': Portugal, Ireland, Italy, Greece, and Spain. Figure 3.2 shows how the debt in the EA17 became even more unevenly distributed over the crisis. This model might help provide some additional insight into why the European sovereign debt structure was (and remains) so much more fragile than America's.

This chapter extends the model from Chapter 2 to include multiple authorities. The structure and mechanics of the game are very similar. Section 3.1 extends the model to include two

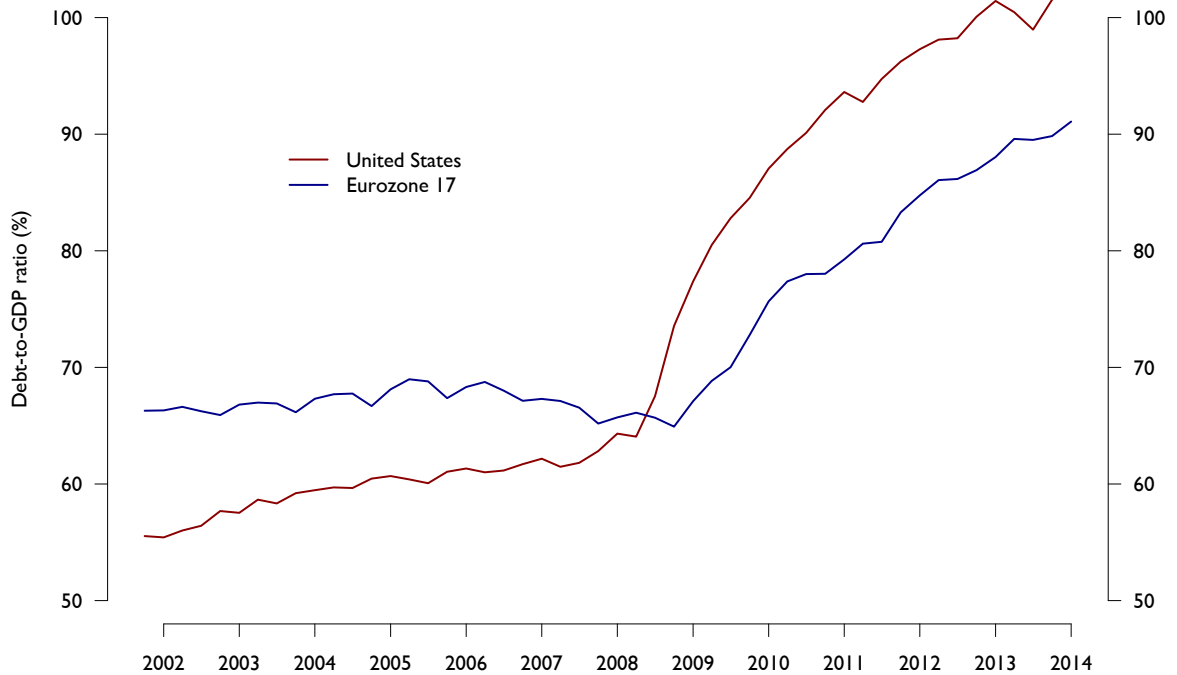


Figure 3.1: Source: FRED, Federal Reserve Economic Data, from the Federal Reserve Bank of St. Louis; Rolling 4-quarter Debt and GDP statistics, Eurostat. Author's calculations.

fiscal authorities and defines the game. Section 3.2 solves the equilibrium when the investors have access to private information. This results in a class of cutoff equilibria that are discussed in Section 3.4. Section 3.3 examines the game when investors have access to a public signal. Like in the one-country model (see Chapter 2), this results in a sunspot outcome. In closing, I discuss the implications that merging fiscal authorities could have on the probability of a debt crisis.

3.1 Extending the game to two fiscal authorities

This section will generalize the model to include two authorities and specify how the solvency and liquidity constraints fit into this game.

3.1.1 The solvency constraint: B^*

Assume we have two equally-sized countries (I for Ireland and G for Germany) that have the same institutions.ⁱ Like in the one-authority case, the countries will only repay their debts if they are both solvent and liquid. To be solvent, the country's debt must be below some

ⁱAppendix C.1 explains how the model can be extended to examine countries of different sizes.

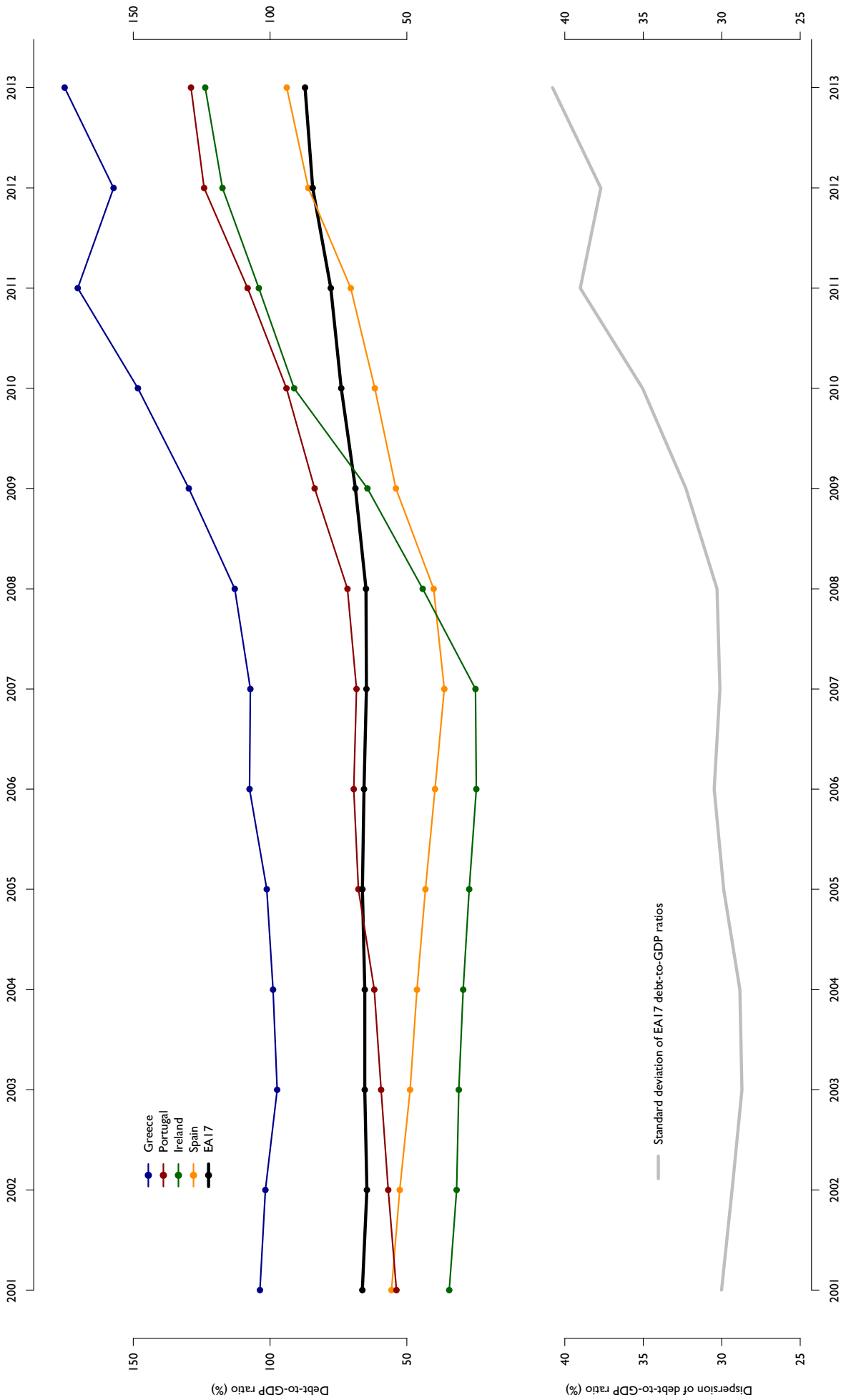


Figure 3.2: The top panel shows the evolution of the debt-to-GDP ratio of several crisis countries over the course of the crisis. This gives some idea of how the distribution of sovereign debt through the EA17 became more skewed over the course of the crisis. The bottom panel shows the standard deviation of the debt-to-GDP ratios of the EA17 members. It is evident that the standard deviation increases considerably over the crisis. By this metric, the dispersion of debt-to-GDP ratios increased by 35% over the course of the crisis.
 Source: Eurostat. Author's calculations.

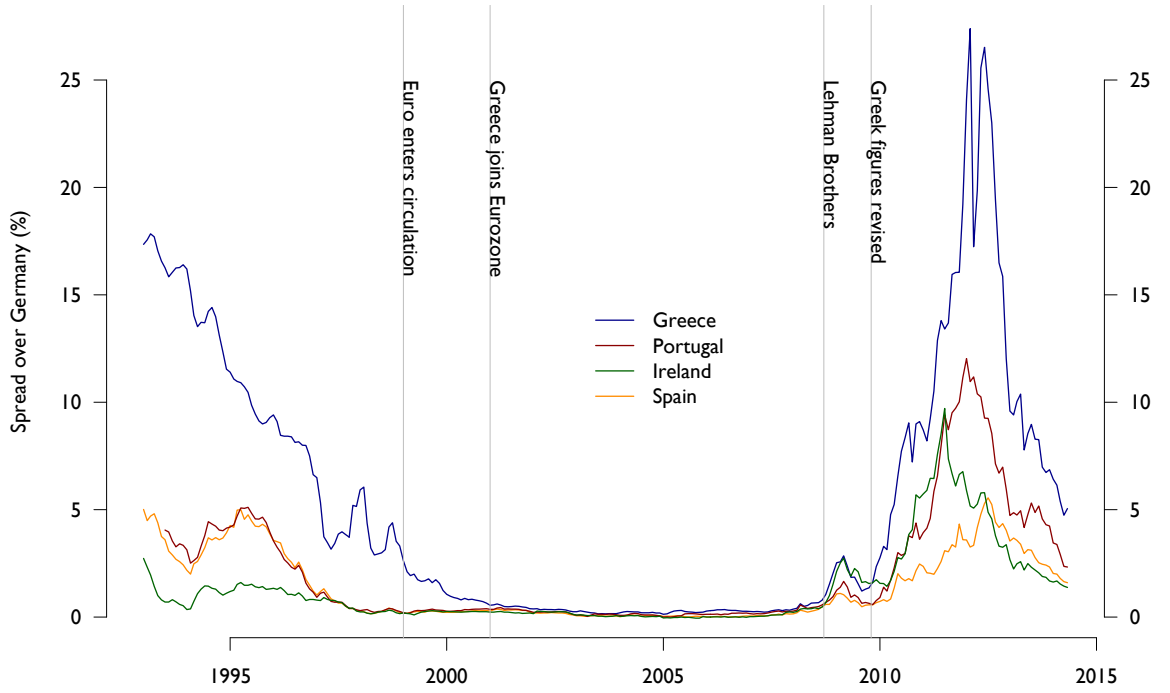


Figure 3.3: Source: Eurostat Monthly long-term interest rates for convergence purposes, 1993-2014. Author's calculations.

threshold level ($B_G \leq B^*$). For the country to be liquid, a sufficient proportion of investors must be willing to roll over their debt ($l_G \geq l_G^*$). Country G 's conditions for repayment are:ⁱⁱ

$$((l_G \geq l_G^*) \wedge (B_G \leq B^*)) \iff G \text{ pays debt} \quad (3.1)$$

where B_G denotes country G 's level of debt and l_G denotes the proportion of investors who hold country G debt.

For country I , the conditions are similar:

$$((l_I \geq l_I^*) \wedge (B_I \leq B^*)) \iff I \text{ pays debt} \quad (3.2)$$

Since both countries have the same type of institutions, they both have the same threshold level of debt B^* . Without loss of generality, assume that I is more indebted than G : $B_I \geq B_G$.

3.1.2 Investor actions

The investors' behaviours are restricted to three possible actions. Investors have two units of investment with which they can perform one of the following three actions:

ⁱⁱThe ' \wedge ' operator is the logical conjunction (*i.e.* AND gate): it outputs true if both operands are true.

- *Safe asset*: Invest both units in the safe asset, earning a guaranteed payoff of 0
- *G-only*: Invest one unit in authority G and the second unit in the Safe asset:
 - If G pays, the portfolio earns a return of r
 - If G does not pay, the investor goes bankrupt and loses β
- *Both*: Invest one unit in authority G and the second unit in authority I :
 - If both authorities pay, the portfolio earns a return of \bar{r}
 - If either authority defaults, the investor goes bankrupt and loses β

Like in the one-authority case, it serves to make the following simplifying assumption.

Assumption 3.1. (No lotteries) The cost of default is high compared to the return on debt and the return on the high-debt country is higher than the low-debt country, which in turn pays a higher rate than the safe asset:

$$\beta > \bar{r} > r > 0 \tag{3.3}$$

The payoffs of each investment bundle contingent on payment are shown in the matrix below:

	Both pay	Only G pays	Neither pay
Safe asset	0	0	0
G-only	r	r	$-\beta$
Both	\bar{r}	$-\beta$	$-\beta$

Where r is the return received on the bundle when only G is rolled-over, while \bar{r} is a (weighted) average of G and I 's interest rates; the important point is that $\bar{r} > r$.

The above is sufficient to define a game:

Proposition 3.1. *The above defines the game Γ , a tuple (N, S, Θ, u, p) where:*

- N is the set of investors (indexed on \mathbb{R}) and the governments I and G .
- The actions for the investors are Both, G-only, and Safe asset; the governments play deterministically, repaying if solvent and liquid, defaulting otherwise.
- Θ is the set of types: investors are typified by the signal they receive; the governments are typified by their solvency condition.

- $u(\cdot)$ is the payoff function as defined in the above table.
- p is the distribution of signals received by the investors conditional on B^* .

Corollary 3.1.1. *The game Γ has infinite equilibria under perfect information.*

The proof omitted since it is identical to the one-authority case. The subsequent sections will solve the model for private (see Section 3.2) and private and public information (see Section 3.3).

3.2 Private information

Investors each receive a private signal x_i that is a normally distributed signal of B^* . Just like in the one-authority case:

$$x_i = B^* + \epsilon_i, \quad \text{where } \epsilon_i \sim \mathcal{N}(0, \sigma^2) \quad (3.4)$$

I will solve the switching strategy:

$$\mathbb{S}_i = \begin{cases} x_i < \kappa_G + B_G & \implies \text{Safe asset} \\ \kappa_G + B_G < x_i < \kappa_I + B_I & \implies \text{G-only} \\ \kappa_I + B_I < x_i & \implies \text{Both} \end{cases} \quad (3.5)$$

where κ_G and κ_I are the cutoffs for the private signal that define the relevant intervals for the strategy. That is, if the private signal (x_i) falls on the appropriate interval, as defined in Equation (3.5), the agent takes the specified action. It follows that, exactly like in the one-authority model, either the liquidity constraint is binding or the solvency constraint is. There are four classes of equilibria depending on whether either authority has liquidity bind first or solvency bind first:ⁱⁱⁱ

1. G has solvency binding; I has solvency binding: (S,S)
2. G has liquidity binding; I has solvency binding: (L,S)
3. G has solvency binding; I has liquidity binding: (S,L)

ⁱⁱⁱAs a reminder, under a cutoff strategy, either the solvency *or* liquidity constraint breaking will result in the authority not paying. Any agent will be able to infer which constraint will break first, to which I refer as ‘bind’. If one constraint is binding, then the other must be slack.

4. G has liquidity binding; I has liquidity binding: (L,L)

The first of these cases (S,S) yields a cutoff strategy where *Safe asset*, *G-only*, and *Both* are all played. The latter three yield equilibria where either only one or two of the three possible actions are played. I will demonstrate that altering the parameters can be interpreted as the fiscal authority migrating between safe and crisis states. There will be several important barriers and bifurcation points where the stable equilibria become untenable. I will first solve for the most interesting case (S,S) before quickly demonstrating that the latter three cases are degenerate and discussing the overall implications. The main conclusion is that a switching strategy exists.

Theorem 3.2. *The game Γ , when augmented with private information as defined in Equation (3.4), has the following switching strategies:*

	$\kappa_I < \sigma\Phi^{-1}(1 - l_I^*)$	$\kappa_I > \sigma\Phi^{-1}(1 - l_I^*)$
$B_G > C_1$	$\Delta \leq \Delta^* \implies \begin{cases} \kappa_I = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right) \\ \kappa_G = \infty \end{cases}$	$\kappa_I = \infty$
	$\Delta > \Delta^* \implies \begin{cases} \kappa_I : \frac{\beta+r}{\beta+\bar{r}}\Phi\left(\frac{\Delta+\kappa_I}{\sigma}\right) = \Phi\left(\frac{\kappa_I}{\sigma}\right) \\ \kappa_G = -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right) \end{cases}$	$\kappa_G = -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right)$
$B_G < C_1$	$\kappa_I = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right)$	$\kappa_I = \infty$
	$\kappa_G = \infty$	$\kappa_G = \infty$

where $\Delta = B_I - B_G$. The critical point Δ^* is defined by Equation (3.18) and C_1 is the lower root of $l_G(B_G) - l_G^* = 0$.

The proof of this theorem is established over the subsequent sections and follows from several intermediate results. Note that this is *not* a unique switching strategy; there is a single other cutoff that behaves qualitatively in the same manner. In the interest of space, I have omitted defining this additional equilibrium. Section 3.2.4 interprets the equilibrium in Theorem 3.2 by mapping it onto the parameter space (B_I, B_G) .

3.2.1 Solvency and liquidity binding: defining the constraints

If ‘solvency binds’ for country k , when country k defaults it is because the government is insolvent ($B^* < B_k$). If ‘liquidity binds’, when a country defaults it is because the government is illiquid ($l_k < l_k^*$). Given some set of parameters and cutoff strategies, the investors

can discern which of the two constraints is binding. In order to find all the equilibria of this game, we need to check all four possible cases: (S,S), (L,S), (S,L), and (L,L).

Since investors are playing a cutoff strategy, the proportion of investors purchasing *Both* is the proportion of investors who receive a signal above the cutoff κ_I :

$$l_I(B^*) = 1 - \Phi\left(\frac{B_I + \kappa_I - B^*}{\sigma}\right) \quad (3.6)$$

I define a new variable B_I^l , which is the threshold level B^* , at which sufficient investors roll-over country I in order to just meet the liquidity constraint conditional on their cutoff strategy:

$$1 - \Phi\left(\frac{B_I + \kappa_I - B^*}{\sigma}\right) = l_I^* \iff B_I^l = B_I + \kappa_I - \sigma\Phi^{-1}(1 - l_I^*) \quad (3.7)$$

This then implies the following lemma.

Lemma 3.3. *Solvency binds for I if and only if $B_I^l < B_I$, which occurs if and only if:*

$$\kappa_I < \sigma\Phi^{-1}(1 - l_I^*) \quad (3.8)$$

Otherwise, liquidity binds.

For G , the proportion of investors that play *G-only* is given by:

$$l_G(B^*) = \Phi\left(\frac{B_I + \kappa_I - B^*}{\sigma}\right) - \Phi\left(\frac{B_G + \kappa_G - B^*}{\sigma}\right) \quad (3.9)$$

Given that κ_I and κ_G are finite, $l_G(B_G) = l_G^*$ will have two, one, or no solutions. Let's examine the case where solutions exist (*i.e.* l_G^* is sufficiently small or $B_I + \kappa_I - B_G - \kappa_G$ is sufficiently large); then we will have solutions C_1 and C_2 where, without loss of generality, $C_1 \leq C_2$. It is easy to demonstrate that $l_G(B_G)$ is symmetric around the point $B_I + \kappa_I + B_G + \kappa_G$ and that this implies:

$$C_1 + C_2 = B_I + B_G + \kappa_I + \kappa_G \quad (3.10)$$

Reordering, yields the condition:

$$B_G - C_1 = -(B_I - C_2 + \kappa_I + \kappa_G) \quad (3.11)$$

This then implies the following lemma.

Lemma 3.4. *Solvency binds for G if and only if $B_G > C_1$; otherwise, liquidity binds.*

Solvency binding for G , with the ‘no lotteries’ assumption, implies that $B_I < C_2$. The above lemmas are summarized in the following table:

	$\kappa_I < \sigma\Phi^{-1}(1 - l_I^*)$	$\kappa_I > \sigma\Phi^{-1}(1 - l_I^*)$
$B_G > C_1$	(S,S)	(S,L)
$B_G < C_1$	(L,S)	(L,L)

3.2.2 Separating equilibria: Scenario I: (S,S)

Lemma 3.5. *Under the parameterization (S,S) there are two possible separating equilibria. If countries are sufficiently similar ($\Delta^* \leq B_I - B_G$), investors treat the two countries as equivalent and choose to invest in either Both or Safe asset according to:*

$$\kappa_I = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r} + \beta}\right) \quad (3.12)$$

If the countries are sufficiently different ($\Delta^ > B_I - B_G$), the investors treat the two countries dissimilarly. They adopt the cutoff strategy:*

$$\begin{aligned} \kappa_I &: \frac{\beta+r}{\beta+\bar{r}}\Phi\left(\frac{\Delta+\kappa_I}{\sigma}\right) = \Phi\left(\frac{\kappa_I}{\sigma}\right) \\ \kappa_G &= -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right) \end{aligned} \quad (3.13)$$

where $\Delta = B_I - B_G$.

When the solvency constraint is binding for both authorities, there is a cutoff strategy in which, if the signal is sufficiently high, then the investor will play *Both*; if the signal is sufficiently low, the investor will play *Safe asset*; and if the signal falls between these intervals, the investor will play *G-only*. In order for this equilibrium to hold, the authorities must be sufficiently different; if their debts are too similar, *Both* will dominate *G-only* and investors will play a cutoff between *Safe asset* and *Both*.

Proof. Given signal x_i , the expected value of playing *G-only* is given by:

$$r\left(1 - \Phi\left(\frac{B_G - x_i}{\sigma}\right)\right) - \beta\Phi\left(\frac{B_G - x_i}{\sigma}\right) \quad (3.14)$$

The expected value of playing *Both* is given by:

$$\bar{r} \left(1 - \Phi \left(\frac{B_I - x_i}{\sigma} \right) \right) - \beta \Phi \left(\frac{B_I - x_i}{\sigma} \right) \quad (3.15)$$

Both of these must be compared to one another and to *Safe asset*. Equating the payoff of *G-only* to zero and substituting in $x_i = B_G + \kappa_G$, we get:

$$\kappa_G = -\sigma \Phi^{-1} \left(\frac{r}{r + \beta} \right) \quad (3.16)$$

It is possible that the signal, which equates *Both* with zero, is lower than $B_G + \kappa_G$. This would imply that only *Both* is played on this interval (and consequently *G-only* is never played). Solving for this signal yields:

$$\kappa_I = -\sigma \Phi^{-1} \left(\frac{\bar{r}}{\bar{r} + \beta} \right) \quad (3.17)$$

If investors are ever to play *G-only* it must be that $B_G + \kappa_G < B_I + \kappa_I$, which, using the variable $\Delta = B_I - B_G$, implies:

$$\Delta > \kappa_G - \kappa_I = \sigma \left[\Phi^{-1} \left(\frac{\bar{r}}{\bar{r} + \beta} \right) - \Phi^{-1} \left(\frac{r}{r + \beta} \right) \right] \equiv \Delta^* \quad (3.18)$$

This specific value (Δ^*) will be of particular significance to our analysis. This threshold difference (Δ^*) is a function that is increasing in both \bar{r} and the variance of private information. It is decreasing in r and is unaffected by the variance of private information. This condition implies that if the two authorities have similar levels of debt then *Both* will dominate *G-only*. It follows, when $\Delta \leq \Delta^*$, the resulting cutoffs would be $\kappa_G = \infty$ and $\kappa_I = -\sigma \Phi^{-1} \left(\frac{\bar{r}}{\bar{r} + \beta} \right)$ since *G-only* is never played. This is illustrated in the lower panel of Figure 3.4.

If, however, $B_I - B_G > \Delta^*$, we have a case where both *G-only* and *Both* are played (see the upper panel of Figure 3.4). As solved above, the cutoff for *G* will be:

$$\kappa_G = -\sigma \Phi^{-1} \left(\frac{r}{r + \beta} \right) \quad (3.19)$$

The cutoff for *I* is given from where the signal $x_i = B_I + \kappa_I$ equates the expected payoff from *Both* with *G-only*. This yields an implicit function for κ_I :

$$\kappa_I : \frac{\beta + r}{\beta + \bar{r}} \Phi \left(\frac{\Delta + \kappa_I}{\sigma} \right) = \Phi \left(\frac{\kappa_I}{\sigma} \right) \quad (3.20)$$

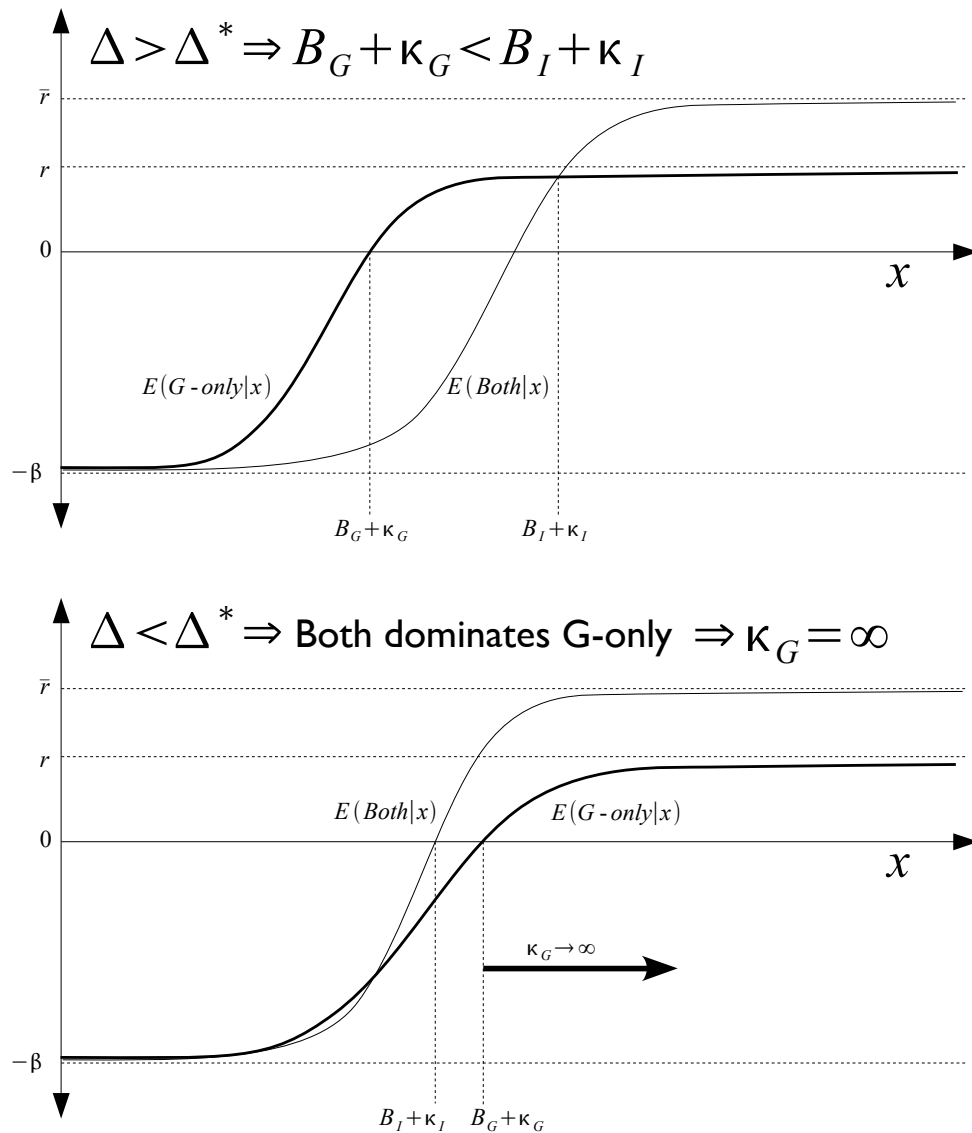


Figure 3.4: Both graphs illustrate the expected value of actions as a function of the private signal received. In the top panel, the debts are sufficiently different. As a result, a stable cutoff equilibrium can be achieved in which all three actions are taken. The lower panel illustrates what occurs when the debts are too similar and *Both* dominates *G-only*; the result is that *G-only* is never played and $\kappa_G \rightarrow \infty$.

By the implicit function theorem, it is straightforward to demonstrate that κ_I is increasing in Δ . Further, to verify, substitute $\Delta = \Delta^*$ into the implicit function that $\kappa_I(\Delta^*) = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right)$. This yields a continuous (albeit only piecewise differentiable) function, $\kappa_I(\Delta)$:

$$\kappa_I(\Delta) = \begin{cases} -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right), & \Delta \leq \Delta^* \\ \frac{\beta+r}{\beta+\bar{r}}\Phi\left(\frac{\Delta+\kappa_I}{\sigma}\right) = \Phi\left(\frac{\kappa_I}{\sigma}\right), & \Delta > \Delta^* \end{cases} \quad (3.21)$$

The limit of this function, as $\Delta \rightarrow \infty$, is $\sigma\Phi^{-1}\left(\frac{\beta+r}{\beta+\bar{r}}\right)$. The corresponding function for $\kappa_G(\Delta)$ is:

$$\kappa_G(\Delta) = \begin{cases} \infty, & \Delta \leq \Delta^* \\ -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right), & \Delta > \Delta^* \end{cases} \quad (3.22)$$

Both of these cutoff strategies are plotted as the thick black lines in Figure 3.5. Finally, it follows, given our cutoff functions and the ‘no lotteries’ condition, $B_I < C_2$. From the construction of C_2 :

$$C_2 - B_I = B_G - C_1 + \kappa_I + \kappa_G \quad (3.23)$$

Since $B_G - C_1 > 0$ by construction, we can show that $\inf\{\kappa_I + \kappa_G\} > 0$:

$$\inf\{\kappa_I + \kappa_G\} = -\sigma \left[\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right) + \Phi^{-1}\left(\frac{r}{r+\beta}\right) \right] \quad (3.24)$$

Since $\beta > \bar{r} > r$, it follows that:

$$\Phi^{-1}\left(\frac{r}{r+\beta}\right) < \Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right) < 0 \implies \inf\{\kappa_I + \kappa_G\} > 0 \implies B_I < C_2 \quad (3.25)$$

■

Thus I have solved for the cutoff strategies adopted by the financial institutions when solvency is the binding condition for both G and I . As long as the debts held by the two institutions are sufficiently different, we get a cutoff strategy that employs all three possible actions: *Safe asset*, *G-only*, and *Both*. If the debts are too similar, the additional payoff from playing *Both* will tend to dominate playing *G-only*, since $\bar{r} > r$. In that case, investors will treat both countries as one, choosing to invest in both or neither.

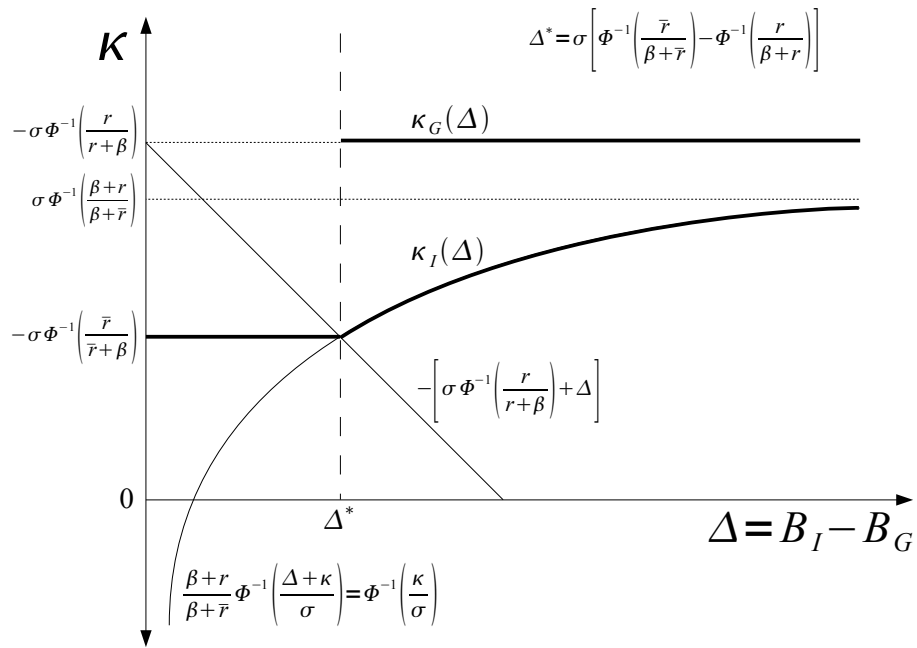


Figure 3.5: This figure plots both $\kappa_G(\Delta)$ and $\kappa_I(\Delta)$ as bold lines. It also demonstrates that Δ^* is the point that shifts the system from one where *Both* dominates *G-only* (as demonstrated by the point where the downward sloping line intersects $\kappa_I(\Delta)$) and how $\kappa_I(\Delta)$ is continuous in Δ .

3.2.3 'Degenerate' equilibrium classes 2-4: (L,S), (S,L), (L,L)

The remaining scenarios only ever imply that one or two of the three possible actions are ever played. The strategies will be of the form:

$$(L, S) \implies \mathbb{S}_i = \begin{cases} x < \kappa_I + B_I & \text{Safe asset} \\ x > \kappa_I + B_I & \text{Both} \end{cases} \quad (3.26)$$

$$(S, L) \implies \mathbb{S}_i = \begin{cases} x < \kappa_G + B_G & \text{Safe asset} \\ x > \kappa_G + B_G & \text{G-only} \end{cases} \quad (3.27)$$

$$(L, L) \implies \mathbb{S}_i = \{\text{Safe asset}\} \quad (3.28)$$

3.2.3.1 (L,S): Liquidity binds for G , Solvency binds for I

Through iterated domination κ_G necessarily increases without limit, which leaves investors playing only *Both*. Firstly, it is worth noting that in order to have liquidity binding for G we need to have:

$$\kappa_G > \sigma\Phi(1 - l_G^*) \quad (3.29)$$

This implies that in order to migrate out of (S,S) we need to have:

$$\frac{r}{r + \beta} < l_G^* \quad (3.30)$$

Under (L,S), the expected value for playing *G-only* is:

$$r - (r + \beta)\Phi\left(\frac{C_1 - x}{\sigma}\right) \quad (3.31)$$

Equating this with the payoff from playing *Safe asset* and substituting in $x_i = B_G + \kappa_G$, the cutoff as a function of C_1 is:

$$\kappa_G(C_1) = C_1 - B_G - \sigma\Phi^{-1}\left(\frac{r}{r + \beta}\right) \quad (3.32)$$

Inverting yields a function for C_1 dependent on the cutoff κ_G :

$$C_1^\kappa(\kappa_G) = \kappa_G + B_G + \sigma\Phi^{-1}\left(\frac{r}{r + \beta}\right) \quad (3.33)$$

Further, by inspection, $C_1^\kappa(\kappa_G)$ is a positively sloped function. As discussed above, however, C_1 is also implicitly defined as the lower solution to the function:

$$C_1^l(\kappa_G): \quad l_G^* = \Phi\left(\frac{B_I + \kappa_I - C_1}{\sigma}\right) - \Phi\left(\frac{B_G + \kappa_G - C_1}{\sigma}\right) \quad (3.34)$$

Any equilibrium must satisfy both these functions: a necessary condition is the existence of a fixed point, *i. e.* a solution for $C_1^\kappa(\kappa_G^*) = C_1^l(\kappa_G^*)$. I will show that there is no fixed point and, consequently, no equilibrium.

The function $C_1^l(\kappa_G)$ is bounded from below by an analytically more tractable function:

$$C_1^l(\kappa_G) > C^*(\kappa_G) = B_G + \kappa_G - \sigma\Phi^{-1}(1 - l_G^*) \quad (3.35)$$

The proof that the cutoff $\kappa_G \rightarrow \infty$ consists of demonstrating that $C_1^l(\kappa_G) > C^*(\kappa_G) > C_1^\kappa(\kappa_G)$, which follows as long as $l_G^* > \frac{\bar{r}}{\bar{r} + \beta}$, the condition for (L,S) from Equation (3.30). It follows that in the (L,S) equilibrium *G-only* is never played and the cutoff $\kappa_I = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r} + \beta}\right)$ is adopted by investors. This reasoning is illustrated in Figure 3.6. It follows that under (L,S) the action *G-only* is never taken. The only equilibrium strategy involves investing in either *Both* or *Safe asset*.

3.2.3.2 (S,L): Solvency binds for *G*, Liquidity binds for *I*

Under the scenario in which liquidity is binding for *I*, there *is* an equilibrium where *Both* and *G-only* are played; however, this special case necessarily occurs within the parameter space that can also sustain the (S,S) equilibrium (discussed above). If the game migrates out of the region that sustains (S,S), any (S,L) equilibrium results in investors never playing *Both*.³ Firstly, it serves to determine the point at which solvency breaks down for authority *I* in the (S,S) scenario. Since the function $\kappa_I(\Delta)$ has a minimum equal to $-\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r} + \beta}\right)$, a *necessary* condition to migrate from (S,S) to (S,L) is:

$$-\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r} + \beta}\right) > \sigma\Phi^{-1}(1 - l_I^*) \iff \frac{\bar{r}}{\bar{r} + \beta} < l_I^* \quad (3.36)$$

Equating the expected values of playing *G-only* and *Both* yields:

$$\bar{r} - (\bar{r} + \beta)\Phi\left(\frac{B_I^l - x_i}{\sigma}\right) = r - (r + \beta)\Phi\left(\frac{B_G - x_i}{\sigma}\right) \quad (3.37)$$

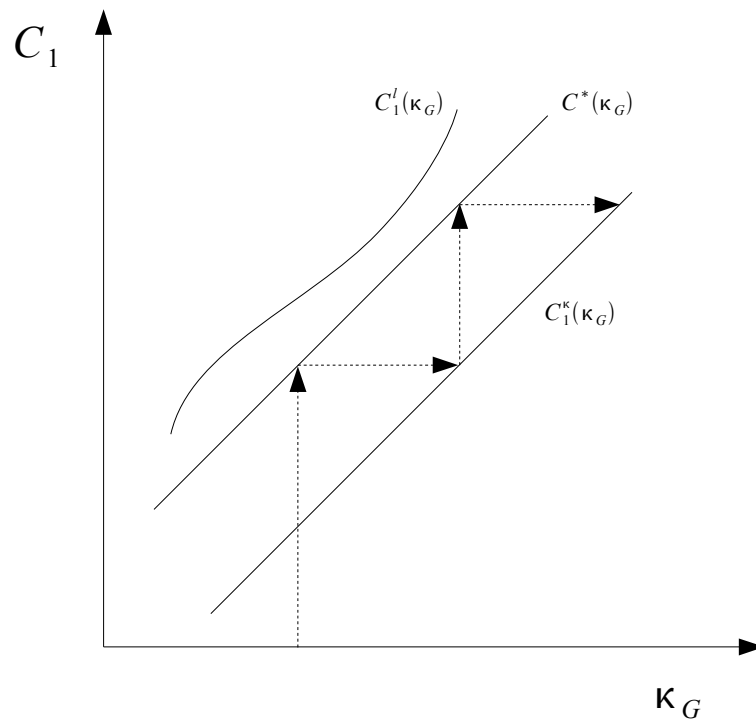


Figure 3.6: This diagram demonstrates the reasoning behind the proof that in the (L,S) scenario $\kappa_G \rightarrow \infty$. There is no point that satisfies the system and, through iterated domination of strategies, *G-only* is never played. Instead, investors play a cutoff strategy for *Both*.

where $B_I^l = B_I + \kappa_I - \sigma\Phi^{-1}(1 - l_I^*)$. We find the κ_I that solves this equation by substituting in $x_i = B_I + \kappa_I$. Simplifying yields:

$$\kappa_I = -(B_I - B_G) + \sigma\Phi^{-1}\left(\frac{(\bar{r} + \beta)(1 - l_I^*)}{r + \beta}\right) \quad (3.38)$$

We also know that $B_G + \kappa_G < B_I + \kappa_I$, for consistency; if it does not, then *Both* must dominate *G-Only*. In order to sustain this equilibrium, it follows that:

$$l_I^* < \frac{\bar{r}}{\bar{r} + \beta} \quad (3.39)$$

That is, this separating equilibrium can only exist within a subset of the parameter space that allows for the (S,S) equilibrium; outside of this subset, we have the equilibrium where only *G-only* is played according to the cutoff strategy described above, where $\kappa_G = -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right)$.

Under (S,L) there does exist a specific set of circumstances in which a similar equilibrium to the one laid out under (S,S) can be achieved. The parameter space, however, in which this exists is a subset of the parameter space that sustains the (S,S) equilibrium. Thus, if debt levels migrate out of the parameter space (S,S) into (S,L), it follows that the only equilibrium is a cutoff to discriminate between *Safe asset* and *G-only*; *Both* is never played.

3.2.3.3 (L,L): Liquidity binds for *G*, Liquidity binds for *I*

In this scenario, either the expected value of playing *G-Only* or *Both* must be greater than playing *Safe asset*. By the logic of iterated dominance discussed in the one-authority case (Section 2.3), both cutoffs explode, which implies that only *Safe asset* is played.

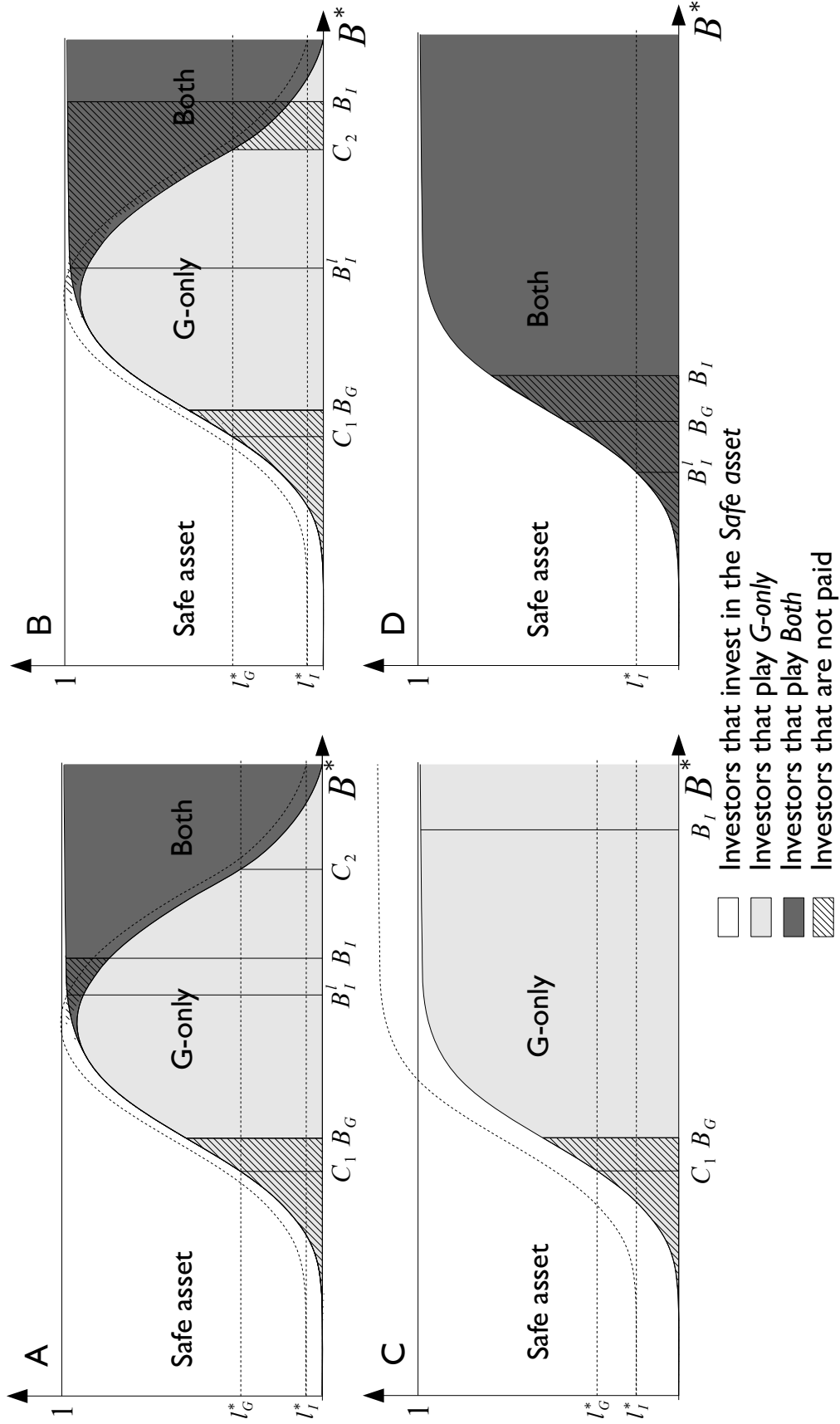


Figure 3.7: **Panel A:** This figure illustrates the proportions of investors playing each strategy as a function of the true underlying value of B^* when investors adopt the cutoff strategy from (S,S). **Panel B:** This panel illustrates an impossible outcome in (S,S) – before such a scenario could occur the bottom solvency constraint would break and investors would adopt the strategy discussed in (L,S) and shown in panel D. **Panel C:** An illustration of the the proportion of investors that play *G-only* as a function of B^* under (S,L). **Panel D:** The proportion of investors that play *Both* as a function of B^* under (L,S).

In summary, the following cutoff strategies are played:

	$\kappa_I < \sigma\Phi^{-1}(1 - l_I^*)$		$\kappa_I > \sigma\Phi^{-1}(1 - l_I^*)$
$B_G > C_1$	$\Delta \leq \Delta^* \implies$	$\left\{ \begin{array}{l} \kappa_I = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right) \\ \kappa_G = \infty \end{array} \right.$	$\kappa_I = \infty$
	$\Delta > \Delta^* \implies$	$\left\{ \begin{array}{l} \kappa_I : \frac{\beta+r}{\beta+\bar{r}}\Phi\left(\frac{\Delta+\kappa_I}{\sigma}\right) = \Phi\left(\frac{\kappa_I}{\sigma}\right) \\ \kappa_G = -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right) \end{array} \right.$	$\kappa_G = -\sigma\Phi^{-1}\left(\frac{r}{r+\beta}\right)$
$B_G < C_1$	$\kappa_I = -\sigma\Phi^{-1}\left(\frac{\bar{r}}{\bar{r}+\beta}\right)$		$\kappa_I = \infty$
	$\kappa_G = \infty$		$\kappa_G = \infty$

3.2.4 Mapping the strategies onto the parameter space

The above section solved for a version of the debt model from Chapter 2 with two fiscal authorities borrowing money from a single financial sector. The investors received private signals about the solvency of the governments and decided whether to roll over their debts or not. Theorem 3.2 showed that there exists four classes of cutoff strategies depending on the model parameters.

Figure 3.7 demonstrates the proportion of investors who take specific actions. Panel A shows the (S,S) equilibrium, where investors are willing to play all 3 actions conditional on their information. As the true solvency level (B^*) increases, the distribution of the signals rises. For very low levels of solvency, almost all investors will choose to invest in the safe asset. As the solvency increases, there will be a significant proportion of investors who lend to G ; at even higher levels, investors will receive sufficiently optimistic signals to invest in both countries. Panel B illustrates an impossible outcome: country G cannot be pushed into default as a result of better information on B^* . Although we never assumed this result directly, it follows from the above analysis. Panels C and D show degenerate cases where either G -only or $Both$ are played. In order to further interpret the solutions from Theorem 3.2, I will make the following assumption.

Assumption 3.2. The liquidity threshold (l_k^*) for country k is linearly related to their debt (B_k):

$$l_k^* = \zeta_k B_k \tag{3.40}$$

An important question is whether rolling over debt under two authorities is any different from the single-authority case in Chapter 2. In order to compare the two, I will map the outcomes of the game in the (B_I, B_G) -space. This is shown in the top panel of Figure 3.8.

In order to construct this figure, note that $B_G < B_I$ so only consider the area underneath the 45° line. We also know that if the two countries are sufficiently similar ($B_I - B_G < \Delta^*$), investors adopt a cutoff strategy that only plays *Both*; this can be mapped onto the space by drawing another 45° line shifted down by Δ^* . Next, it is important to map the critical points for both authorities: for G the cutoff occurs when their debt becomes too large ($B_G = \zeta_G \frac{r}{r+\beta}$). I 's critical point is more complicated since the cutoff strategy adopted by investors is a function of the difference between the two countries ($\kappa_I(\Delta)$). This function is increasing but bounded; further, the critical point is when the following condition is broken:

$$\kappa_I(\Delta) < \sigma \Phi^{-1}(1 - l_I^*) \quad (3.41)$$

This implies a very interesting result: as G decreases its debt, this in turn increases the cutoff adopted by investors towards I . Since the LHS of this condition is affected by B_G , authority G can affect whether I is pushed into crisis. Put into more plain language: there are cases in which austerity measures by G could push I into crisis by inducing investors to behave more conservatively towards the high-debt country. This implies that G 's debt policy can have externalities for the high debt country I .

We can map this crisis line as an upward-sloping and concave-downward line rising from an asymptote at $\zeta_I \frac{\bar{r}-r}{\bar{r}+\beta}$. This I -crisis line meets the $B_G = B_I - \Delta^*$ line at $\zeta_I \frac{\bar{r}}{\bar{r}+\beta}$. To the right of the I -crisis line and below the G -crisis line, the G -only cutoff strategy is adopted from (S,L). It is worth noting that, in this region, only authority G is able to roll-over its debts; as a result, the total amount of combined debt in this region is limited to simply the maximum that G can sustain, which is $\zeta_G \frac{r}{r+\beta}$.

Above the G -crisis line and to the left of the I -crisis line we have the outcome outlined in (L,S), where *Both* is played using a cutoff strategy. Above the G -crisis line and to the right of the I -crisis line, only *Safe asset* is ever played. The maximum possible debt achievable by both countries combined is in the top-right corner of the triangle where $B_G + B_I = 2\zeta_I \frac{\bar{r}}{\bar{r}+\beta}$.

The G -crisis line is not a function of σ , which implies improving private information does not change the crisis point. These crisis lines have a very similar interpretation as the crisis point in the one-authority model. These delineate the (B_I, B_G) space into regions in which the ‘beauty contest’ elements of the game dominate. Just like in the one-authority case, when the beauty contest prevails for one action, players want to be ever more conservative in their approach. The net result is that such an action is never taken in equilibrium.

In order to compare the outcome of two authorities with only one, we can contrast the above outcome to a single authority that has debts $B_O = B_G + B_I$ and a solvency constraint of $B_O^* = 2B^*$. Under this scenario, although the maximum combined level of debt is the same, many more combinations of debt are made possible. For instance, the region to the right of the I -crisis line and below $B_G = 2\zeta_I \frac{\bar{r}}{\bar{r} + \beta} - B_I$ is now in the realm of possibility. The intuition is straightforward: when there were two authorities, G may have had extra debt capacity but was unable to ‘share’ it with I . In the region denoted by ‘B’ in Figure 3.8, both G and I could be made better off in the two-authority model if they agreed to unite and maintain a separating equilibrium. I would move part of its debts onto G ’s books and G could receive some compensation in return.

Although this analysis does not take into account any moral hazard or fairness issues that might arise, it is interesting that G and I could both be made better off by amalgamating their debts into one authority. The region denoted by ‘A’ maps out further debt bundles that are now feasible. When there were two authorities, no debt bundle in region ‘A’ could be rolled-over; with only one authority, bundles in ‘A’ are entirely feasible.

3.3 Private and public information

Consider now the game Γ from Proposition 3.1 augmented with two signals available to the investors: one public and one private. The investors will use these two signals to forecast optimally the true underlying value of debt threshold B^* . Using the same notation as the one-authority case from Chapter 2, the private signal, denoted by x_i , has variance σ^2 ; the

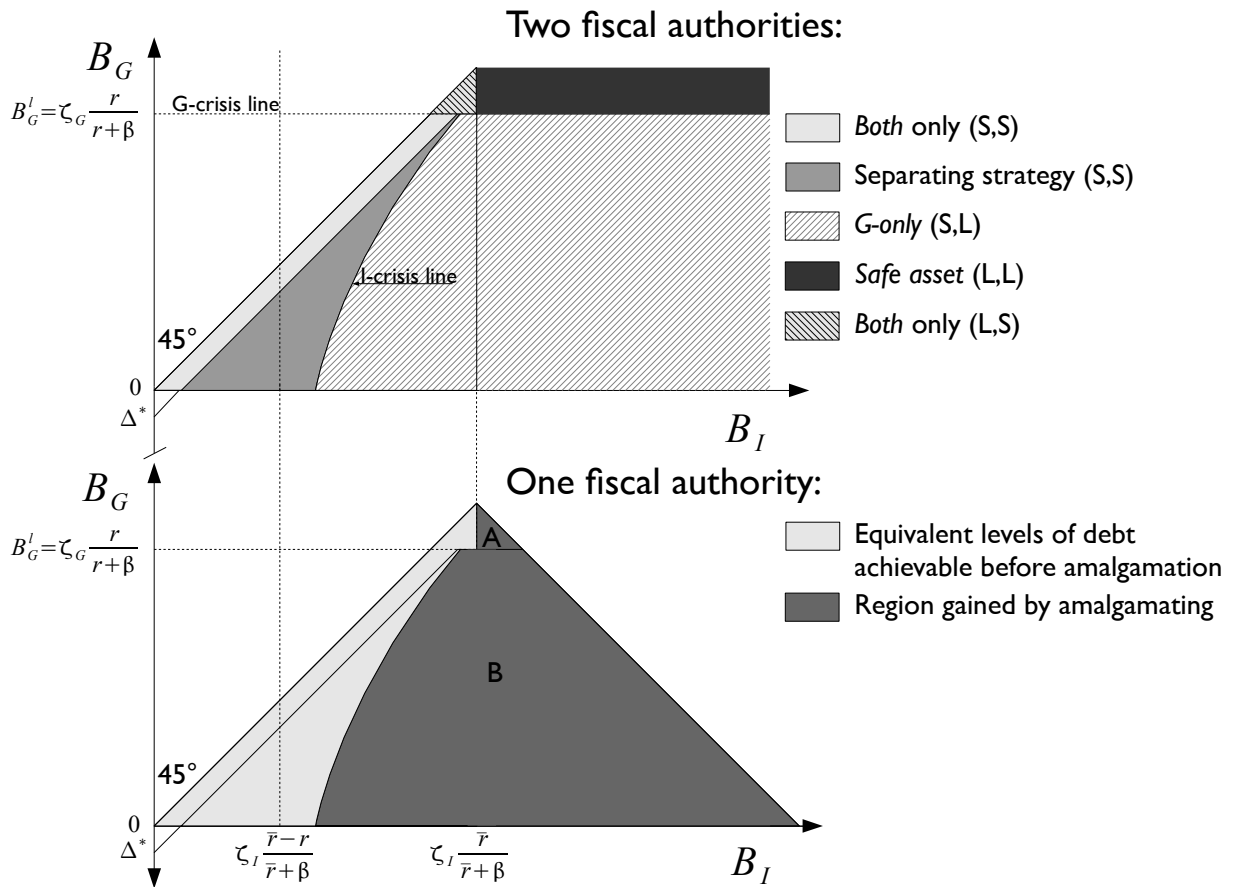


Figure 3.8: Mapping the strategies from the two-authority case onto the (B_I, B_G) space and comparing the outcome to a one-authority model.

public signal, denoted by y , has variance γ^2 . The best forecast available to an individual is:^{iv}

$$\hat{B}^* = \frac{\sigma^2 y + \gamma^2 x}{\gamma^2 + \sigma^2} \quad (3.42)$$

Like in the previous section, our solution will be a cutoff equilibrium strategy of the form:

$$\mathbb{S}_i = \begin{cases} \hat{B}^* < \kappa_G + B_G & \implies \text{Safe asset} \\ \kappa_G + B_G < \hat{B}^* < \kappa_I + B_I & \implies \text{G-only} \\ \kappa_I + B_I < \hat{B}^* & \implies \text{Both} \end{cases} \quad (3.43)$$

Theorem 3.6. *The game Γ from Proposition 3.1, when augmented with normally distributed public and private signals, yields three classes of non-degenerate cutoff equilibria.*

- *In the first instance, the two countries are too similar to distinguish ($\Delta < \Delta^*$). The outcome collapses to the one-authority case (see Theorem 2.5).*
- *If the countries are sufficiently different ($\Delta > \Delta^*$), there are two possible classes of equilibria:*
 - *In the first – denoted (S,L) – a set of cutoff strategies is adopted such that there are three critical values of the public signal (y):*

$$y_{min}^{(S,L)} < y_{mid}^{(S,L)} < y_{max} \quad (3.44)$$

For signal draws below $y_{min}^{(S,L)}$, both countries face a crisis. For draws $y_{min}^{(S,L)} < y < y_{mid}^{(S,L)}$, only G is able to roll-over their debts. For draws $y_{mid}^{(S,L)} < y < y_{max}$, both countries can roll-over their debt but investors adopt separate cutoff strategies for each. Finally, for signals above y_{max} , investors treat the two countries as one. See the upper-left panel of Figure 3.9 for an illustration.

- *In the second – denoted (L,S) – a set of cutoff strategies is adopted such that there are three critical values of y :*

$$y_{min}^{(L,S)} < y_{mid}^{(L,S)} < y_{max} \quad (3.45)$$

For signal draws below $y_{min}^{(L,S)}$, both countries face a crisis. For draws $y_{min}^{(L,S)} < y <$

^{iv}Section 2.4.1 shows that this weighting is a dominant action; the same reasoning applies here.

$y_{mid}^{(L,S)}$, both countries can roll-over their debts (they are treated as one country). For draws $y_{mid}^{(L,S)} < y < y_{max}$, both countries can roll-over their debt but investors adopt separate cutoff strategies for each. Finally, for signals above y_{max} , investors treat the two countries as one. See the lower-left panel of Figure 3.9 for an illustration.

In all cases, the investors weight their signals using the inverse-variance rule.

An informal way to conceive of this equilibrium is to consider the model when only private information is available (see Section 3.2). Under the private-only information, there were four classes of equilibria and, given a specific set of parameters, only one of these classes would prevail. When the model switched from a good equilibrium to a bad one, this was interpreted as a debt crisis. This section similarly has four classes of equilibria; however, they are not fully determined by the model's parameters. Conditional on the parameters, the public signal will determine which of the four classes prevails: the public signal shifts the equilibrium state from good to bad. Like in the one-authority case (see Section 2.4), this is interpreted as a sunspot. The probability of a crisis is the likelihood that the public signal is drawn from a specific interval.

In the remainder of this section, I will solve for the cutoff strategy stated in Theorem 3.6. Like in Section 3.2, this will require examining four possible classes of equilibria: (S,S), (S,L), (L,S), and (SS). The borders between these classes will determine the intervals on which the public signal causes a crisis.

3.3.1 Solvency and liquidity binding: defining the constraints

Since the public signal is common knowledge, the distribution of forecasts (\hat{B}^*) is:

$$\hat{B}^* \sim N\left(\frac{\sigma^2 y + \gamma^2 B^*}{\gamma^2 + \sigma^2}, \frac{\gamma^4 \sigma^2}{(\gamma^2 + \sigma^2)^2}\right) \quad (3.46)$$

Given this distribution and the cutoffs, it is possible to calculate the proportion of investors taking each action. The proportion of investors playing *Both* is:

$$l_I(B^*) = 1 - \Phi\left(\frac{(\gamma^2 + \sigma^2)}{\gamma^2 \sigma} \left(B_I + \kappa_I - \frac{\sigma^2 y + \gamma^2 B^*}{\gamma^2 + \sigma^2}\right)\right) \quad (3.47)$$

Solving for the cutoff between the solvency and liquidity constraints yields:

$$\begin{aligned}
 \text{Solvency binds for } I &\iff \kappa_I < \frac{\gamma^2 \sigma}{\sigma^2 + \gamma^2} \Phi^{-1}(1 - l_I^*) + \frac{\sigma^2}{\gamma^2 + \sigma^2} (y - B_I) \\
 \text{Liquidity binds for } I &\iff \kappa_I > \frac{\gamma^2 \sigma}{\sigma^2 + \gamma^2} \Phi^{-1}(1 - l_I^*) + \frac{\sigma^2}{\gamma^2 + \sigma^2} (y - B_I)
 \end{aligned} \tag{3.48}$$

Inverting these conditions demonstrates that investors do not *ex ante* know which constraint is binding; only once y is revealed is it known whether liquidity or solvency is binding for I . Since the *ex ante* distribution for y is known to the investors (normal with mean B^* and standard deviation γ), the investors can calculate the probability of either case prevailing. Given B^* and κ_I , the probability of either condition binding is:

$$\begin{aligned}
 \text{Pr(Solvency binds for } I) &= 1 - \Phi\left(\frac{\gamma^2 + \sigma^2}{\gamma \sigma^2} \kappa_I + \frac{\gamma}{\sigma} \Phi^{-1}(l_I^*) + \frac{B_I - B^*}{\gamma}\right) \\
 \text{Pr(Liquidity binds for } I) &= \Phi\left(\frac{\gamma^2 + \sigma^2}{\gamma \sigma^2} \kappa_I + \frac{\gamma}{\sigma} \Phi^{-1}(l_I^*) + \frac{B_I - B^*}{\gamma}\right)
 \end{aligned} \tag{3.49}$$

Turning our attention to the the investors playing *G-only*, the proportion that chooses to invest in country G is given by:

$$l_G(B^*) = \Phi\left(\frac{(\gamma^2 + \sigma^2)}{\gamma^2 \sigma} (B_I + \kappa_I) - \frac{\sigma}{\gamma^2} y - \frac{B^*}{\sigma}\right) - \Phi\left(\frac{(\gamma^2 + \sigma^2)}{\gamma^2 \sigma} (B_G + \kappa_G) - \frac{\sigma}{\gamma^2} y - \frac{B^*}{\sigma}\right) \tag{3.50}$$

The proportion of investors playing each action is greatly affected by the public signal. For instance, if the signal y is very large then everyone can infer that it is unlikely for many investors to play *G-only*. The public signal y directly shifts the distribution of signals left and right depending on how large or small it is. For very small public signals, not enough people will be able to roll-over the debt and the equilibrium will unravel.

3.3.1.1 (S,S): Solvency binding for both countries

Suppose that solvency binds for both countries. The expected value of playing *G-only* is:

$$r - (r + \beta) \Phi\left(\frac{\sqrt{\gamma^2 + \sigma^2}}{\gamma \sigma} (B_G - \hat{B}^*)\right) \tag{3.51}$$

Similarly, the expected value for playing *Both* is:

$$\bar{r} - (\bar{r} + \beta) \Phi\left(\frac{\sqrt{\gamma^2 + \sigma^2}}{\gamma \sigma} (B_I - \hat{B}^*)\right) \tag{3.52}$$

The solving for the cutoff for *G-only* against *Safe asset* yields:

$$\kappa_G = -\frac{\gamma\sigma}{\sqrt{\gamma^2 + \sigma^2}} \Phi^{-1}\left(\frac{r}{r + \beta}\right) \quad (3.53)$$

When comparing *G-only* to *Both*, the debts of each authority must be sufficiently different from one another. In a manner similar to the private-only information case, if the two debts are too similar, *Both* will tend to dominate *G-only*; as such, we solve for how different the two debts must be:

$$\Delta^* \equiv \frac{\gamma\sigma}{\sqrt{\gamma^2 + \sigma^2}} \left[\Phi^{-1}\left(\frac{\bar{r}}{\beta + \bar{r}}\right) - \Phi^{-1}\left(\frac{r}{\beta + r}\right) \right] \quad (3.54)$$

As long as the debt levels are sufficiently different, the cutoff for *I* can be solved using the implicit function:

$$\kappa_I: \frac{\beta + r}{\beta + \bar{r}} \Phi\left(\frac{\sqrt{\gamma^2 + \sigma^2}}{\gamma\sigma}(\Delta + \kappa_I)\right) = \Phi\left(\frac{\sqrt{\gamma^2 + \sigma^2}}{\gamma\sigma}\kappa_I\right) \quad (3.55)$$

This function behaves in a manner very similar to the private-information setting. Since the cutoff for *G* exists analytically, it is straightforward to write it directly in terms of the x_i and y signals. The investor will prefer to play *G-only* over *Safe asset* as long as:

$$y > \frac{\gamma^2 + \sigma^2}{\sigma^2} (B_G + \kappa_G) - \frac{\gamma^2}{\sigma^2} x \quad (3.56)$$

or

$$y > \frac{\gamma^2 + \sigma^2}{\sigma^2} B_G - \frac{\gamma}{\sigma} \sqrt{\gamma^2 + \sigma^2} \Phi^{-1}\left(\frac{r}{r + \beta}\right) - \frac{\gamma^2}{\sigma^2} x \quad (3.57)$$

Under any given parameterization, the cutoff for playing *Both* can be determined ($\kappa_I(\Delta)$); investors in general will prefer to play *Both* over *G-only* when:

$$y > \frac{\gamma^2 + \sigma^2}{\sigma^2} (B_I + \kappa_I) - \frac{\gamma^2}{\sigma^2} x \quad (3.58)$$

Finally, *Both* will be preferred over *Safe asset* as long as:

$$y > \frac{\gamma^2 + \sigma^2}{\sigma^2} B_I - \frac{\gamma}{\sigma} \sqrt{\gamma^2 + \sigma^2} \Phi^{-1}\left(\frac{\bar{r}}{\bar{r} + \beta}\right) - \frac{\gamma^2}{\sigma^2} x \quad (3.59)$$

In both of these situations, the indifference curves in the (x_i, y) -space are negatively sloped parallel lines with slope $-\frac{\gamma^2}{\sigma^2}$. This strategy, however, is not valid for *all* values of y . Since a change in y has the effect of shifting the $l_G(B^*)$ and $l_I(B^*)$ functions left and right, the

public signal can knock the system out of this equilibrium. As the public signal becomes more pessimistic (decreasing y), inevitably one of the liquidity constraints will bind over the solvency constraint and move the system into an equilibrium similar to those discussed in (S,L) and (L,S) in the private-only information game (see Section 3.2). These constraint switches (either for G or I) are functions of, among other things, how large l_G^* and l_I^* are; the higher the demand for liquidity is, the easier it is for the equilibrium to fall apart. Depending on the parameters, as the public signal becomes more pessimistic (y decreases) the system moves from the (S,S)-like equilibrium discussed above to one resembling either (S,L) or (L,S) and finally, for very low public signals, to an equilibrium resembling (L,L) where no one is able to roll-over their debts.

Additionally, there is a somewhat counterintuitive outcome when the public signal is very high: the action *Both* will dominate *G-only*. This is because, as the public signal increases, the proportion function $l_G(B^*)$ continually shifts leftward until $C_2 < B_I$. At this point, the cutoff strategy for which we solved is no longer valid: *G-only* now pays off only on the interval (B_G, C_2) (see Figure 3.7, panel B). As a result, investors necessarily behave more conservatively towards *G-only* by increasing their cutoff for only investing in the low-debt country (κ_G) and decreasing their cutoff for investing in both (κ_I). The proportion of investors playing *G-only* necessarily shrinks, which again reduces the payoff from playing *G-only*; inevitably, this vicious cycle destroys any cutoff equilibrium where *G-only* is played. Alternatively, *Both* comes to dominate and a cutoff strategy is played between it and *Safe asset*. The threshold for the public signal to trigger this outcome cannot be solved analytically; however, given parameters, it can easily be solved numerically. For our purposes, we will refer to this threshold public signal value as y_{max} : any public signal above this threshold implies investors treat both the high-debt and low-debt country equally.

Thus, there are generally two classes of parameterizations in this game: (i) where, as the public signal decreases, I fails first or (ii) where *G-only* is dominated by *Both* and both authorities fail together. I will refer to the former case as (S,L) and the latter as (L,S).

3.3.1.2 (S,L): I fails first

Under this class of equilibrium, as the public signal decreases, the liquidity constraint binds on I before binding on G . Given that I fails first, the public signal that causes I to fail is:

$$y < \frac{\gamma^2 + \sigma^2}{\sigma^2} \kappa_I + \frac{1}{\sigma\gamma^2} \Phi^{-1}(l_I^*) \equiv y_{mid}^{(S,L)} \quad (3.60)$$

Here we can see the effect of increasing the debt can have on the probability of a crisis occurring. Assume that increasing debt increases the liquidity threshold (l_I^*): since the probability of a debt crisis is the probability of drawing y less than this threshold, increasing debt directly increases the threshold *and* increases the cutoff κ_I . In contrast to the one authority case, in which there was only a direct effect of increasing the debt on the probability of default, here there are *two* forces that increase the probability of a crisis. Generally, under this class of parameterizations, the probability of a debt crisis is higher under two authorities than under one since the option of a more secure authority's debt leads to investors treating I more conservatively than they otherwise would. The threshold signal where G is pushed into a crisis is then:

$$y < \frac{\gamma\sqrt{\gamma^2 + \sigma^2}}{\sigma} \Phi^{-1}\left(\frac{\beta}{r + \beta}\right) + \frac{1}{\sigma\gamma^2} \Phi^{-1}(l_G^*) \equiv y_{min}^{(S,L)} \quad (3.61)$$

To summarize the outcomes, given $\Delta > \Delta^*$:

(S,L)	κ_I	κ_G
$y > y_{max}$	$-\frac{\gamma\sigma}{\sqrt{\gamma^2 + \sigma^2}} \Phi^{-1}\left(\frac{\bar{r}}{\bar{r} + \beta}\right)$	∞
$y_{max} > y > y_{mid}^{(S,L)}$	$\frac{\beta+r}{\beta+\bar{r}} \Phi\left(\frac{\sqrt{\gamma^2 + \sigma^2}}{\gamma\sigma} (\Delta + \kappa_I)\right) = \Phi\left(\frac{\sqrt{\gamma^2 + \sigma^2}}{\gamma\sigma} \kappa_I\right)$	$-\frac{\gamma\sigma}{\sqrt{\gamma^2 + \sigma^2}} \Phi^{-1}\left(\frac{r}{r + \beta}\right)$
$y_{mid}^{(S,L)} > y > y_{min}^{(S,L)}$	∞	$-\frac{\gamma\sigma}{\sqrt{\gamma^2 + \sigma^2}} \Phi^{-1}\left(\frac{r}{r + \beta}\right)$
$y_{min}^{(S,L)} > y$	∞	∞

This strategy is plotted in Figure 3.9 on the (x_i, y) space. The *ex ante* probability of a debt crisis for authority I is the probability that $y < y_{mid}^{(S,L)}$, while the probability of a debt crisis for authority G is the probability that $y < y_{min}^{(S,L)}$. The probability that I fails and G does not is the probability that $y_{min}^{(S,L)} < y < y_{mid}^{(S,L)}$. Like in the one-authority model, we have recovered a sunspot result that can shift the system into crisis, in which no investors are willing to roll-over debt.

In comparison to the one-authority model, under the (S,L) scenario, the probability that the whole debt of both authorities is not rolled-over is necessarily higher. That is, if both authorities were to amalgamate, the total probability of a debt crisis would be reduced. Authority I would clearly be better off since the probability would unambiguously drop. Authority G , on the other hand, would be worse off unless adequately compensated by I ; the probability of a debt crisis is *higher* under the one-authority model than the probability of G alone having a debt crisis. See the upper-left panel of Figure 3.9 for an illustration of this equilibrium.

3.3.1.3 (L,S): Both countries fail together

There is another set of parameters under which both countries will fail together. To find this, consider the case where G 's liquidity constraint binds for a y higher than for I . Like in the previous section, there will be four intervals on which the public signal can fall. Each interval implies a different strategy set is adopted by the investors. Like in the (S,L) case, if $y > y_{max}$ then *Both* will come to dominate *G-only*. As y decreases below this value, eventually the liquidity constraint will bind for G and the separating equilibrium will break down. This occurs when y is such that C_1 drops below B_G . The analytical solution to this value does not exist but it can be solved numerically for any set of parameters; denote this threshold value of public information as $y_{mid}^{(L,S)}$. When the public signal is immediately below this value, a cutoff strategy employing only *Both* and *Safe asset* is used. There is yet another lower-threshold value of public information, which leads to the degenerate equilibrium where all investors play *Safe asset*. This value is given by:

$$y < \frac{\gamma\sqrt{\gamma^2 + \sigma^2}}{\sigma} \Phi^{-1}\left(\frac{\beta}{\bar{r} + \beta}\right) + \frac{1}{\sigma\gamma^2} \Phi^{-1}(l_I^*) \equiv y_{min}^{(L,S)} \quad (3.62)$$

It is worth noting that this is the same sunspot cutoff for which we solved in the one-authority model (see Section 2.4). It follows that under this parameterization the sunspot equilibrium behaves almost identically to that in the one-authority model: the total probability of a crisis is the same. The only substantive difference is that for some moderate values of public information investors adopt a separating strategy in which all three actions are employed. See the lower-left panel of Figure 3.9 for an illustration of this equilibrium.

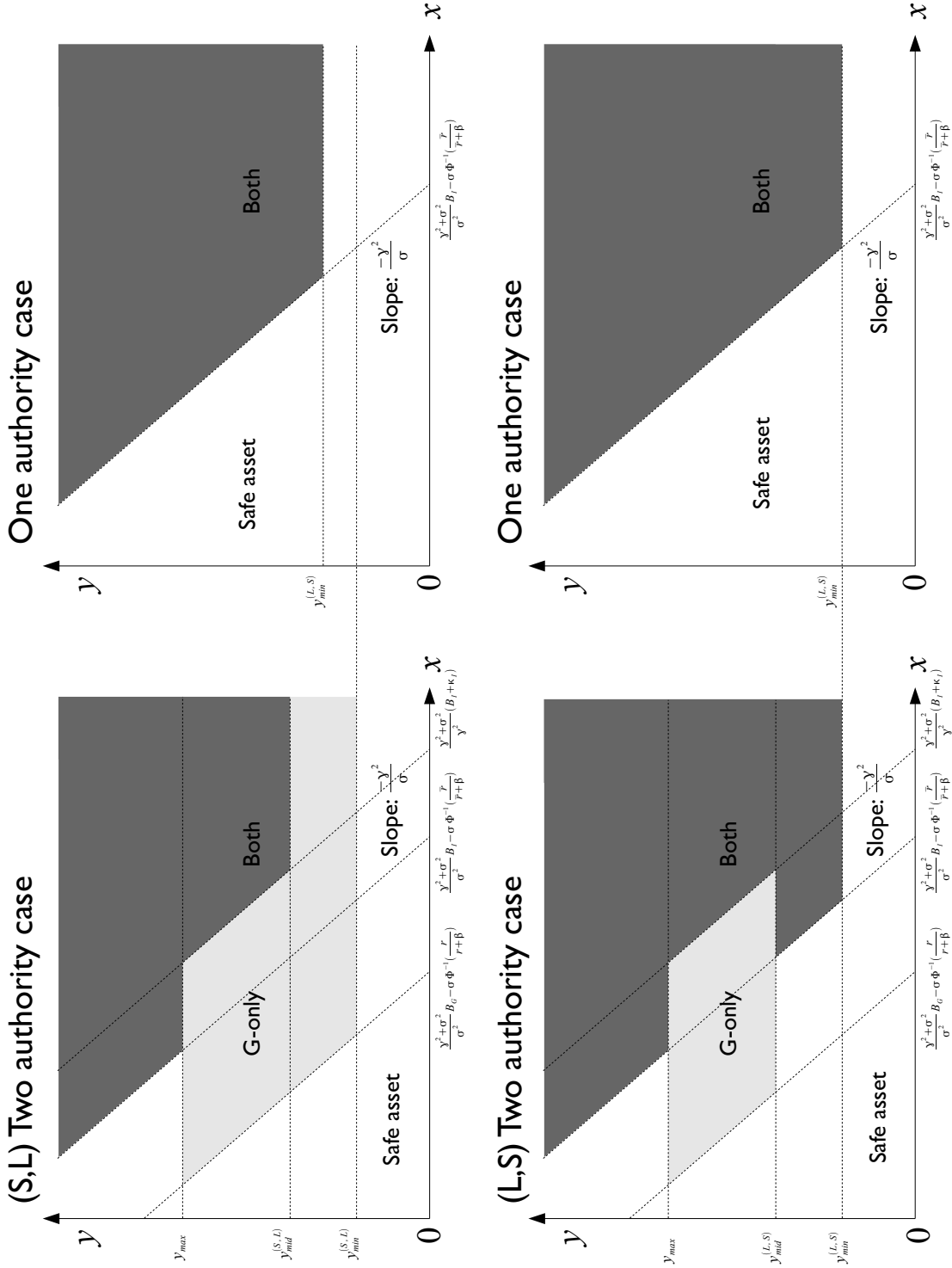


Figure 3.9: These graphs map out the strategy sets under (S,L) and (L,S) as a function of the public (y) and private (x_i) signals. The top left panel maps out the (S,L) case and compares it to the one authority case (top right); the comparison demonstrates that the probability of a crisis resulting in default by country G is higher under the one authority case than under the two authority model. The bottom left graph maps the strategy adopted under the (L,S) case. The probability of G being forced into a crisis is the same under the one and two authority scenarios.

3.3.2 Summary of the two-authority model with public information

Adding public information to the two-authority model recovers an equilibrium similar in structure to both a sunspot model and the one-authority outcome. Generally, there are three possible parameterizations that lead to three separate strategies.

The first strategy is when $\Delta < \Delta^*$; under this scenario, the action *Both* comes to dominate *G-only* and the strategies adopted by the investors are indistinguishable from those in the one-authority case. In the second parameterization, referred to as (S,L), all three actions are employed. Under this scenario, there are two crisis sunspots: the first pushes *I* into crisis, while the second pushes both *I* and *G* into crisis. The third possible strategy set adopted, referred to as (L,S), has only one crisis point where both *I* and *G* enter crisis together.

Just like in the private-only information case, in this one-shot model authority *I* cannot directly drag down *G* through the financial sector. This is because investors are not naïve in forming their strategies and can foresee that such an outcome is untenable. Further, since *I* is incapable of exerting such externalities on *G*, there is no immediate incentive for *G* to bail out *I*. Of course, this reasoning assumes that *G* considers only the intra-period outcome; if the game would be repeated the result could be very different.

3.3.3 A multi-stage game: an incentive for bailouts?

The previous section demonstrated that there is no specific incentive for *G* to bail out *I*; however, this lack of *intra-temporal* incentive does not imply that there does not exist an *inter-temporal* incentive. It is possible to imagine the scenario in which the game is extended to two periods. The first period is composed of the two-authority public-information game. The second stage, if *I* is unwilling to pay in the first stage, becomes the one-authority public-information game in which the investors who purchased *I* are removed from the game. This has the net effect of increasing the liquidity requirement in the second period for *G* and increases the sunspot probability of default. In order for this to hold directly, assume that investors (in the first stage) act myopically and discount the second stage completely. In this manner, we can treat the first stage exactly like that of the 2-authority public information game.

In the event that I defaults due to a low draw of $B^* < B_I$, $l_I(B^*)$ investors will ‘die’ in the process. Before the next stage of the game begins, G has the opportunity to ‘bail out’ I , by assuming a portion of their debts (*i.e.* $B_I - B^*$) or let them fail.

If G lets I fail, the one authority game is played with a reduced number of investors; however, if G chooses to bail out I , the one-authority game is played with the two authorities having merged into one. Although we have not strictly defined the payoffs in this proto-game, clearly the choice that G faces is between two outcomes: an increased probability of a crisis and no cost of debt-assumption versus no increase in the probability of a crisis but the cost of assuming $B_I - B^*$. Depending on how much G values reducing the probability of a crisis, it is perfectly plausible that they would decide a small bailout of $B_I - B^*$ would be cheaper in expectation than an increase in the sunspot probability of a crisis due to $l_I(B^*)$ investors dying. Here we have a very strange result: due to the knock-on effect of an increase in the sunspot probability, a low-debt nation could be willing to bail out an insolvent nation. This, however, does not take into account any moral hazard or reputational considerations that would be important in a dynamic setting.

3.4 Discussion

Chapter 2 introduced a model in which one fiscal authority attempts to roll-over its debt with a continuum of investors. Under perfect information, there is a multiplicity of equilibria. Introducing private information on the sovereign’s willingness to pay refines the equilibrium to a unique cutoff strategy, which is a function of the country’s level of debt. The one-authority private-information model qualitatively replicates both the ‘beauty contest’ aspects of sovereign debt and the non-linearities in how investors respond to levels of debt.

Investors know that there is strategic complementarity in their actions when purchasing sovereign debt; although the payoff when debt pays (r) is itself not a function of coordination amongst the investors, whether or not the government pays depends on the proportion of investors choosing to roll-over the debt. Holding everything else constant, as debt increases so does the coordination required to roll-over the debt. Under low levels of debt, investors adopt a cutoff strategy in which they are willing to invest in the authority’s debt given their private signal is sufficiently high. Assuming that the coordination required to roll-over debt

increases in the quantity of debt, there exists a critical level of debt at which the coordination becomes too onerous. Investors realize that, under such a heavy debt burden, the outcome is determined by whether enough investors choose to roll-over the debt. Regardless of their private signal, investors conclude that everyone else will rush for the exits; optimally, they choose not to roll-over the debts and the government is forced to default.

Section 2.4 further generalized the one-authority model by introducing public information. With this addition, the equilibrium appears very similar to a sunspot model, which captures how this system can stochastically jump between stable and crisis equilibria. In fact, this result demonstrates an awkward implication: two identical countries may have drastically different macroeconomic outcomes. If a random public signal is drawn below some threshold value, a crisis occurs; if it is drawn from above the threshold, the stable outcome prevails. Although many of the results of this extension are intuitive – such as the probability of a crisis increasing with the level of debt – better public information is not necessarily socially beneficial. For low levels of debt we have the counter-intuitive result that reducing the variance of public information can increase the probability of a crisis. For high levels of debt, however, better public information unambiguously results in a lower probability of a crisis.

This chapter extends the game to include two fiscal authorities attempting to roll-over debts with a continuum of investors. Under perfect information, we get a similar multiplicity of equilibria that existed under the one-authority perfect-information case. By adding private information, we solved for four classes of equilibria, which are determined by the parameterization of the model. If the difference between the debts of the two countries is small, investors will play a single cutoff, effectively treating the two as a single country. If the authorities are sufficiently different, investors use separate cutoffs for each country. For pessimistic signals, an investor will choose to invest solely in the *safe asset*. For moderate signals, they will choose to invest in the low-debt country. Only if they receive a high signal will they invest in both countries. This outcome is analogous to a ‘separating’ equilibrium because the two countries are treated differently as a result of their fundamentals.

Perhaps the most interesting implication of this model is that, under the separating equilibrium, the high-debt country’s crisis point is a function of the low-debt country’s level of debt. The comparative statics imply that if the low-debt further reduces its debt, the investors will

behave more conservatively towards the high-debt country. In certain circumstances, this is sufficient to push the high-debt country into crisis. There can be somewhat counter-intuitive externalities from austerity. The two-authority model also demonstrated that merging two fiscal authorities could increase the maximum level of debt. Bundles of debt that cannot be rolled-over under the two-authorities can be rolled-over if the two authorities merged. This effect is not solely a function sharing ‘debt capacity’ but is a result of removing the externality that the lesser-debt authority imposed on the high-debt authority.

Finally, we looked at the case in which there were two fiscal authorities, and investors received public and private information regarding the authorities’ willingness to pay. Just like in the one-authority model, we recovered a sort of sunspot equilibrium, in which the public signal knocks the system in and out of various possible equilibria. Under this extension, there are three classes of parameterizations; two of them imply that the lesser-debt country would be indifferent about creating a single fiscal authority. In the third parameterization, however, the lesser-debt authority explicitly would not want to merge with the higher-debt authority since it necessarily implies a higher sunspot probability of a crisis. Although this entails that there is no intra-temporal incentive for the lesser-debt country to bail out the high-debt authority, an obvious extension of the model would be to create a two-stage game where the failure of the high-debt country could impact the low-debt country by increasing the coordination threshold in the subsequent period. Just like in the one-authority model, an increase in the precision of public information does not necessarily yield a lower probability of a crisis.

Despite this model’s highly simplified view of reality, it does shed some light on issues regarding sovereign crises and the stability of debt with regards to multiple authorities compared to those relating to one consolidated authority. At the risk of overreaching, the model might provide some insight into why, among other reasons, the Eurozone sovereign debt structure is perceived as more fragile than American sovereign debt.

Chapter 4

A debt game with correlated information

This chapter presents a model of debt roll-over in which a continuum of investors receives correlated signals on whether a debtor is solvent or insolvent. The investors face a collective-action problem, whereby a sufficient proportion of investors must agree to participate in the debt roll-over for it to be a success; if an insufficient proportion of investors participates in the deal, the debtor will default. One way of conceiving of correlated signals is by decomposing the noise component of signals into public and private noise. Higher public noise will result in more correlation between the signals received by the investors. The formal structure of this model is the same as those presented in Chapters 2 and 3.

Myatt and Wallace (2008), among others, provide a cogent interpretation of correlated signals: a sender imperfectly observes a parameter and sends a signal to a receiver, who only imperfectly observes the message. If two such receivers observe a signal from the same sender, their observed signal should be correlated (differing only by the private noise introduced by their own imperfect observation technology). In the case of sovereign debt, the sender might be a Credit Rating Agency (CRA), which attempts to measure the level of debt at which the sovereign will default. Investors, however, only imperfectly interpret the information from the CRA. Individual investors know how they each interpret the signal from the CRA, but are unsure whether other agents' interpretations are the same.

Mathematically, using normally distributed noise, the CRA observes some signal y of the fundamental parameter θ such that:

$$y|\theta \sim N(\theta, \nu^2) \tag{4.1}$$

A signal Q_i is then communicated imperfectly to the investor i (the receiver). The receiver i 's information-processing technology introduces some additional normally distributed noise into the signal (orthogonal to the sender's noise):

$$x_i | (\theta, y) \sim N(y, \zeta_i^2) \quad (4.2)$$

Combining the sender's and receiver's noise, the final signal to the investor of the true parameter θ will have variance:

$$\sigma^2 = \nu^2 + \zeta_i^2 \quad (4.3)$$

Further, the signals observed by different investors are not independent. The correlation between signals generating from the CRA, assuming an equally precise receiver technology across agents ($\zeta_i = \zeta_j$), will be:

$$\rho = \frac{\nu^2}{\nu^2 + \zeta^2} \quad (4.4)$$

Thus, correlated signals between agents can be interpreted as opaque but informative signals from a common origin.

I will use this informational setup to examine a model in which individual agents receive correlated signals about a sovereign's willingness to pay. In theory, there exists some level of debt at which a sovereign would be better off simply to default. In reality, however, it is very difficult to determine where exactly this level is. Partly to get around this information problem, CRAs have entered the market to provide signals to investors about such opaque metrics. The issue I wish to examine is how such opaque signals can themselves create coordination failures.

This chapter will present a highly stylized model of investors who choose whether to roll-over a sovereign's debt or not. I will set up a game between a continuum of investors, who receive correlated signals regarding the sovereign's willingness to pay, and the sovereign, who mechanically defaults if its debt is above some specified level. I will prove the existence of a cutoff strategy used by the agents and demonstrate that this will result in the possibility of 'debt crises'. Further, I will solve for the distribution of play, which is an isomorphism of the Vasicek credit risk model that is widely used in the risk management industry.

Section 4.2 demonstrates the unique switching strategy, which results in global uncertainty being preserved; Section 4.3 shows the comparative statics of the equilibrium strategy. Section 4.4 solves for the *ex ante* distribution of play (conditional on the true solvency of the debtor) and shows that the investors' actions will be distributed according to a Vasicek credit distribution. The *ex ante* probability of a debt crisis is affected by the exogenous model parameters. Of particular interest is the fact that increasing private noise unambiguously reduces the probability of a debt crisis. Unsurprisingly, increasing the fiscal space or return on debt also decreases the probability of a crisis. Appendix B.1 contains several numerical simulations of the model.

4.1 Actions, payoffs, and information

The agents consist of a continuum of investors, who decide whether to invest in sovereign debt or not, and the sovereign, who decides whether to honour its debts or not. Two conditions must be satisfied in order for the sovereign to honour its debts: it must be *willing* to pay (solvent) and the investors must be sufficiently *coordinated* to roll-over the debt (liquid).

Formally, being *solvent* implies that the sovereign debt is below some threshold level B^* ; I will refer to this level of debt as the sovereign's 'debt capacity'. The difference between its debt capacity and its actual level of debt ($B^* - B = \Delta^*$) is its 'fiscal space'. If the stock of debt is above the debt capacity (B^*), then sovereign would do better to default than to honour its contractual obligations. Thus, in order to honour its obligations, it is necessary that the sovereign has positive fiscal space ($\Delta^* > 0$).¹

To be *liquid*, a sufficient proportion of investors (l^*) must agree to participate in the debt issuance. Note that l (without the asterisk) denotes the proportion of investors that chooses to roll-over the sovereign debt while l^* (with the asterisk) denotes the liquidity threshold.

$$\text{Solvent} \iff \Delta^* \geq 0$$

$$\text{Liquid} \iff l \geq l^*$$

The sovereign pays only when both of these conditions are satisfied. Formally, that is:

$$((l \geq l^*) \wedge (\Delta \geq \Delta^*)) \iff \text{Government pays} \quad (4.5)$$

When the sovereign pays, its bonds pay return r . When the sovereign defaults (whether through coordination failure or unwillingness to pay) the bondholders lose β .

	Pays	Defaults
Sovereign debt	r	$-\beta$
Safe asset	0	0

The investors observe the signal Q_i of the sovereign's fiscal space:

$$Q_i = B^* - B + \sigma X_i \quad (4.6)$$

$$= \Delta^* + \sigma X_i \quad (4.7)$$

where B^* is the unobserved capacity level of debt while B is the actual (observed) level of debt. The noise parameter X_i is standard normal and is scaled by the standard deviation σ . The noise is equicorrelated² across players; thus we can decompose the noise into public and private components:

$$X_i = \sqrt{\rho}y + \sqrt{1-\rho}z_i \quad (4.8)$$

The public component y is present in all agents' signals. The private component z_i is idiosyncratic to agent i . Both y and z_i are independent standard normal random variables. It follows that:

$$E[Q_i] = \Delta^* \quad (4.9)$$

$$\text{Var}[Q_i] = \sigma^2 = \nu^2 + \zeta^2 \quad (4.10)$$

$$\text{Corr}(Q_i, Q_j) = \rho = \frac{\nu^2}{\nu^2 + \zeta^2} \quad \forall i \neq j \quad (4.11)$$

The parameter ρ is the correlation between observed signals. As discussed in the introduction, ρ can be interpreted as the proportion of variance introduced by a CRA into a signal of the fiscal space Δ^* . The lower ρ , the more precise is the public noise relative to private. That is, the Credit Rating Agency imperfectly observes the government's fiscal space (Δ^*), which introduces some error (with variance $\sigma^2\rho$). The CRA sends this information to the individual investors, who each imperfectly observe the signal, which introduces more, idiosyncratic noise with variance $\sigma^2(1-\rho)$. The total variance of the signal observed by the investor is σ^2 but, due to the shared nature of the CRA noise, the investors' signals will be correlated.

Using the terminology introduced in Morris and Shin (2003), I conjecture a pure ‘switching’ strategy adopted by investors that is conditional on their signal. If an investor receives a ‘good’ signal, they will purchase the debt. If they receive a ‘bad’ signal, they will opt for the safe asset. In order to specify this strategy, we must solve for the cutoff that makes them indifferent between purchasing debt and the safe asset.

$$\begin{aligned} \text{Purchase debt} &\iff Q_i \geq \kappa^* \\ \text{Safe asset} &\iff Q_i < \kappa^* \end{aligned}$$

In this setup, if the signals to investors were purely private ($\rho = 0$) or purely public ($\rho = 1$), the investors would be able to infer whether the liquidity or solvency condition was tighter. As a result, the outcome would be determined by the tighter constraint, which was demonstrated in Chapter 2. With correlated signals, however, the investors will not be able to discern which constraint is tighter: solving for the equilibrium will require examining both the solvency and liquidity conditions in turn.

4.1.1 An alternative interpretation: the intelligibility and accuracy of signals

A different interpretation of this information structure is that of ‘intelligibility’ and ‘accuracy’. The public noise can be seen as the accuracy of the government’s financial statements. The intelligibility of these statements is the extent to which different investors view the same piece of information and draw differing conclusions.

Using the mathematical framework above, the financial disclosures of a government are more *accurate* if the variance of the public component of the noise (ν^2) is smaller. For a small public variance, the *average* signal will be close to the true value of the fiscal space. The government’s disclosures are more *intelligible* if the variance of the private component (ζ^2) is smaller: there is strong agreement amongst investors as to what this implies for the fiscal space. The subsequent sections will demonstrate that, if the state wishes to reduce the probability of a crisis, they can do this either by increasing their fiscal capacity (through some form of austerity or political reforms) or by making their financial disclosures less intelligible. Highly intelligible signals will *increase* the probability of a debt crisis.

This result provides one explanation for the well-known obscurity of government finances: short-sighted policymakers may opt to decrease financial transparency in order to reduce the

odds of a crisis. Of course, obfuscating government financial data is not a sustainable long-run strategy: eventually the truth will out. Acknowledging this temptation might provide support for the creation and maintenance of independent agencies that report on the fiscal state of the government.

4.1.2 Formal specification of the game

Using the ‘types’ formalism from Harsanyi (1967), this is a supermodular Bayesian game where the set of players (\mathcal{I}) is the continuum of investors and the government. The set of actions \mathcal{S}_i for each investor i is either to *Purchase debt* or the *Safe asset*; the government chooses either to pay or default (deterministically). A pure strategy for player i is an injective mapping $s_i : \Theta_i \rightarrow \mathcal{S}_i$, which determines an action for each possible type of player i .

The investors’ types ($\theta_i \in \Theta_i$) are defined by the signal they receive ($Q_i \in \mathbb{R}$); the government’s type is defined by its debt capacity $\Delta^* \in \mathbb{R}$. The payoff function is defined according to the strategies and types of the players:

$$u_i(s_i, s_{-i}, \theta_i, \theta_{-i}) = \begin{cases} r & \text{if } \Delta^* \geq 0 \text{ \& } l \geq l^* \\ -\beta & \text{otherwise} \end{cases} \quad (4.12)$$

where l is the proportion of investors that plays *Purchase debt*.

Finally, the player types are distributed according to an equicorrelated normal distribution with mean Δ^* , variance σ^2 and correlation ρ . The players have common knowledge of the game and all have improper uniform priors.

If Δ^* were common knowledge, there would be one equilibrium for $\Delta^* < 0$: all investors choose *Safe asset* and the government would default. For $\Delta^* \geq 0$, there are infinite pure strategy Nash equilibria. Obviously, the degenerate strategy, where all investors choose *Safe asset*, remains an equilibrium. It is also possible to construct any number of arbitrary switching strategies. When Δ^* is not common knowledge, we have a global games setup.

In the following section, I will show that there is a unique symmetric switching equilibrium. Using the methods established by Milgrom and Roberts (1990), this implies that the result is

globally stable and dominance solvable. Further, using those by Bernheim (1984) and Pearce (1984), the solution will be rationalizable.

Proposition 4.1. *The above game constitutes a Bayesian game (as in Definition D.2) denoted by the tuple:*

$$\Gamma = (N, S, \Theta, p, u) \quad (4.13)$$

- *The set of agents (N) are indexed by the real line (\mathbb{R}).*
- *The agents can take the actions $s_i \in \{\text{Purchase debt, Safe asset}\}$.*
- *The agents' types correspond to the signal they have received $\Theta = \mathbb{R}$.*
- *The payoff function u_i is given by Equation (4.12).*
- *The types are distributed normally according to (4.6).*

Proof. Follows directly from Definition D.2. ■

Proposition 4.2. *The game Γ is supermodular.*

Proof. The definition for supermodular games is given in Definition D.4. Firstly, every finite set is compact. Secondly, u_i exhibits increasing differences since, if another agent changes to investing in the sovereign, the payoff to i of rolling-over can never decrease but, in some circumstances, will increase. Lastly, the utility u_i is upper semi-continuous in l , which fulfills the final condition. ■

Remark. Applying Topkis's Theorem (see Theorem D.6) implies that the best-response functions must be increasing in the other players' actions.

The analysis is greatly simplified by making the following assumptions.

Assumption 4.1. (No lotteries) Assume that the return on government debt is small relative to the loss of a default: $r < \beta$.

Assumption 4.2. Agents have improper uniform priors regarding the sovereign's fiscal space.

4.2 The equilibrium strategy

The sovereign will be willing to honour its debts when they have positive fiscal space ($\Delta^* \geq 0$). A single investor, however, only observes Q_i , a noisy signal of the fiscal space (Δ^*). Abstracting away from the liquidity condition, the investor would be willing to purchase the sovereign debt as long as the expected return was higher than the safe option. Investors will rationally conclude that (given their information and improper uniform priors) that their belief of the fiscal space (Δ^*) is normally distributed with mean Q_i and standard deviation σ . This pins down their subjective probability that the sovereign is solvent – specifically, it is the probability that $\Delta^* \geq 0$. For any signal Q_i observed, there is an infinite set of private and public components (z_i and y respectively) that could result in Q_i . There is no great difference between the two components: for a given signal Q_i , a larger (more positive) error (σX_i) inevitably implies a lower true value of the fiscal space Δ^* . Conversely, a smaller (more negative) error implies a larger value of the fiscal space Δ^* . Given the observed signal Q_i , there are pairs of public and private error components (y, z_i) that imply the sovereign is solvent and others that imply it is insolvent.

As an illustration, consider the case in which the signal Q_i just happened exactly to equal the true value fiscal space Δ^* . In this instance, y and z_i must exactly cancel out:

$$\begin{aligned} \Delta^* &= Q_i \\ \implies X_i &= 0 \\ \implies z_i &= -\sqrt{\frac{\rho}{1-\rho}}y = -\frac{\nu}{\zeta}y \end{aligned} \tag{4.14}$$

The relation in Equation (4.14) is mapped in Figure (4.1) as the grey line. To solve for the values of the private and public components that will imply that the sovereign is insolvent, $\Delta^* < 0$. Begin with the boundary condition that $\Delta^* = 0$:

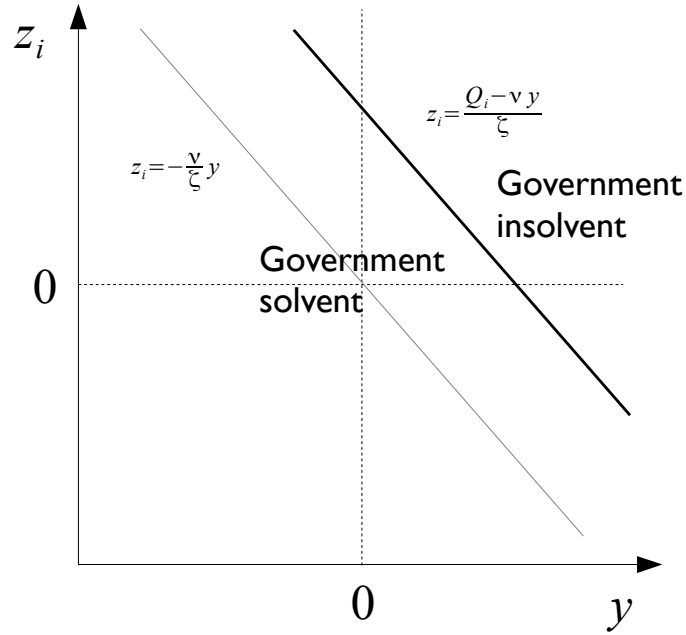


Figure 4.1: This figure illustrates where in the (y, z_i) -space the sovereign is willing and unwilling to pay its debts. The light grey line maps out where the signal $Q_i = 0$ while the black line maps out the boundary between where sovereign is willing to pay and where it is unwilling.

$$Q_i = \Delta^* + \sigma X_i \quad (4.15)$$

$$\implies Q_i = \sigma X_i \quad (4.16)$$

$$\implies X_i = \frac{Q_i}{\sigma} \quad (4.17)$$

$$\implies z_i = -\sqrt{\frac{\rho}{1-\rho}} y + \frac{Q_i}{\sigma\sqrt{1-\rho}} \quad (4.18)$$

$$= \frac{Q_i - \nu y}{\zeta} \quad (4.19)$$

The line given by (4.19) is the boundary that separates the domains over which sovereign is willing and unwilling to honour its debt. This is illustrated as the black line in Figure (4.1): for a given signal Q_i , above-right of this line represents the sub-domain where the sovereign is insolvent, to the below-left is where the sovereign is solvent.

Proposition 4.3. *The probability that the sovereign is solvent conditional on Q_i is:*

$$\text{Prob}(\Delta^* \geq 0 | Q_i) = \Phi\left(\frac{Q_i}{\sigma}\right) \quad (4.20)$$

Proof. Instead of the straightforward approach, I will use the limit from (4.19) and integrate over z_i and y in order to prove the result. The integral is:

$$\text{Prob}((\Delta^* \geq 0 | Q_i)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{\rho}{1-\rho}} y + \frac{Q_i}{\sigma \sqrt{1-\rho}} \phi(z) \phi(y) dz dy \quad (4.21)$$

$$= \int_{-\infty}^{\infty} \phi(y) \Phi \left(\sqrt{\frac{\rho}{1-\rho}} y + \frac{Q_i}{\sigma \sqrt{1-\rho}} \right) dy \quad (4.22)$$

$$= \Phi \left(\frac{Q_i}{\sigma} \right) \quad (4.23)$$

which is the same as (4.20).³ ■

4.2.1 The liquidity condition

The liquidity condition is somewhat more complicated than the solvency condition given in Equation (4.19). Firstly, consider the *ex post* distribution of signals, which will be normally distributed with mean $\Delta^* + \nu y$ and standard deviation ζ . From i 's perspective, the other investors follow a cutoff strategy (κ_{-i}); it follows that the proportion choosing to purchase the sovereign debt is given by:

$$l(\cdot) = \Phi \left(\frac{\Delta^* + \nu y - \kappa_{-i}}{\zeta} \right) \quad (4.24)$$

If the public component y was observable, the investor would be able to infer whether the solvency condition or the liquidity condition was tighter. That is, the investor would know with certainty that the sovereign would become insolvent before becoming illiquid or *vice versa*. Because the public component y is unknown, they don't have such a luxury. Instead, there are bundles of (z_i, y) that mean solvency is the tighter constraint and others that mean liquidity is tighter. Equating the proportion from (4.24) to the liquidity threshold l^* and solving for Δ^* yields:

$$\Delta^* = \kappa_{-i} - \nu y + \zeta \Phi^{-1}(l^*) \equiv \Delta_l \quad (4.25)$$

As the public component of the signal decreases, this pulls down the average signal received by agents. A lower public component necessarily implies that a higher Δ^* would be required for the sovereign to attain the same proportion of investors choosing to roll-over. Conversely, as the public component (y) gets larger, the fiscal space (Δ^*) required to meet the liquidity constraint (Δ_l) declines linearly.

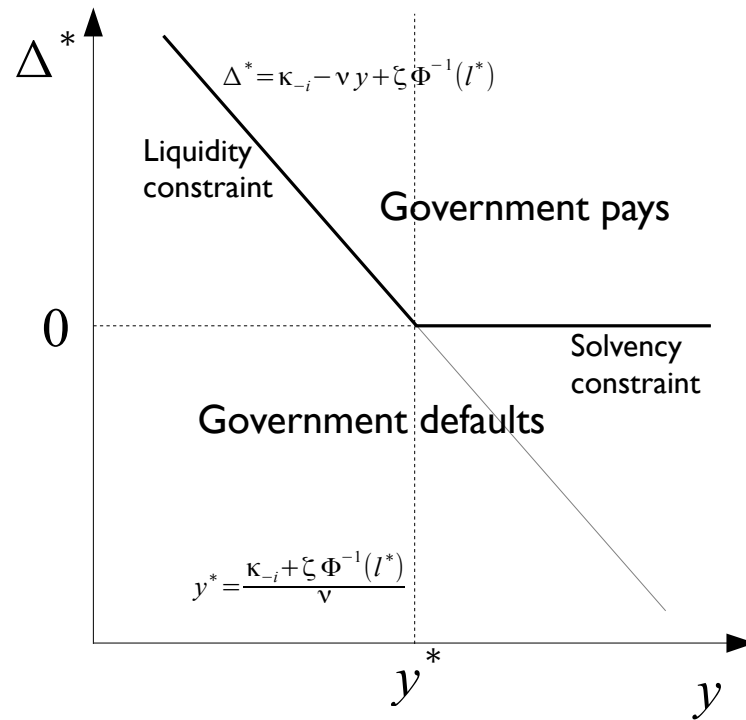


Figure 4.2: This shows under what conditions the government will default or pay as a function of the public-noise component (y) and the fiscal space (Δ^*). The government pays only when both the willingness and coordination constraints are met. The willingness constraint is shown by the horizontal line at $\Delta^* = 0$, while the coordination constraint is shown by the downward-sloping line. The boundary shown by the thick black line is the threshold between payment and default. When (y, Δ^*) are above-right of this black line, the government will pay (both the willingness and coordination constraints are met) while to the below-left, it will default.

Eventually Δ_l drops below 0 and the tighter constraint switches from liquidity to solvency; we can map (4.25) into the (y, Δ^*) -space to show this (see Figure 4.2). The value of the public component where this shift occurs is:

$$y^* = \frac{\kappa_{-i} + \zeta \Phi^{-1}(l^*)}{\nu} \quad (4.26)$$

Figure 4.2 shows that when both the liquidity and solvency requirements are met (above-right of the black line), the sovereign pays. When either of the conditions is not met, the sovereign defaults on the debt (below-left of the black line). As $\rho \rightarrow 1$ (a perfectly public signal), the liquidity condition converges to the solvency condition. In contrast, the limit $\rho \rightarrow 0$ does not make sense in this situation. When information becomes completely private, the value for y^* will explode to either ∞ or $-\infty$ depending on whether the liquidity condition or the solvency condition is tighter. In Figure 4.2, this limit would be illustrated as the downward-sloping liquidity condition becoming flatter. In the limit, the liquidity condition will be a flat line either above or below the solvency condition.

We can now use this information to update the (y, z_i) -space that we began sketching in Figure 4.1. As long as $y > y^*$, the solvency condition will be tighter (just like in the previous figure). If, however, $y < y^*$, the liquidity condition will be tighter. In Figure 4.2 we saw that, as the public component decreases, the fiscal space Δ^* necessary to ensure roll-over increased; in Figure 4.3, we consider the case in which the signal Q_i is fixed and hence, as the public error component y decreases, Δ^* necessarily increases (if we hold the private error component z_i constant). These two effects exactly offset one another. We can demonstrate this mathematically by solving for boundary of the liquidity condition in the (y, z_i) domain:

$$Q_i = \Delta_l + \nu y + \zeta z_i \quad (4.27)$$

$$\implies z_i^* = \frac{Q_i - \kappa_{-i}}{\zeta} + \Phi^{-1}(1 - l^*) \quad (4.28)$$

This is independent of the public component. Figure 4.3 presents the sub-domains over which the sovereign pays and defaults. Integrating the bivariate standard normal distribution over these sub-domains yields the probability that the sovereign pays (in equilibrium $\kappa_{-i} = \kappa_i = Q_i$):

$$z_i^* = \Phi^{-1}(1 - l^*) \quad (4.29)$$

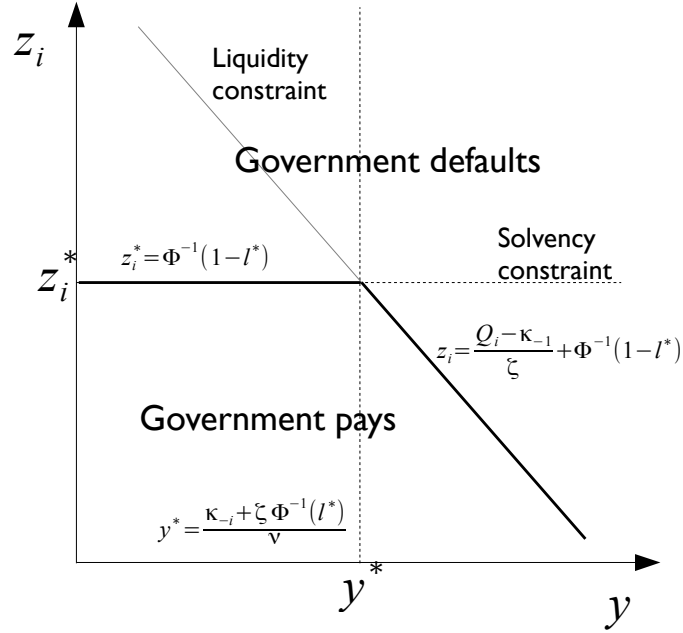


Figure 4.3: This demonstrates when the government defaults and pays as a function of the private and public components of the signal. The downward-sloping line is the coordination constraint while the horizontal line at z_i^* is the willingness constraint. The threshold condition for payment is shown by the thick black line. Below-left of this line, the government is both coordinated and solvent; above-right, the government will default.

4.2.2 Existence and uniqueness of the cutoff strategy

In order to determine the expected utility of investing in the sovereign debt it will be useful to know the probability of repayment conditional on the agent's signal.

Proposition 4.4. *In the game Γ , agent i 's subjective probability that the sovereign repays, conditional on their signal Q_i , is:*

$$I(Q_i, \kappa_{-i}; \sigma, \rho, l^*) = \int_{-\infty}^{z_i^*} \phi(z_i) \Phi\left(\frac{Q_i - \zeta z_i}{\nu}\right) dz_i \quad (4.30)$$

Proof. In order to calculate the probability of payment or default (both necessary in determining the expected value of the investors' actions) we need only integrate a bivariate standard normal over the relevant domains. Firstly we will integrate across the public component y from $-\infty$ to the solvency condition, which is a function of the private component, and then across the private component z_i from $-\infty$ to the liquidity condition (z_i^*). For agent i , this will be a function of the cutoff strategy adopted by the other agents (κ_{-i}).

$$I(Q_i, \kappa_{-i}; \sigma, \rho, l^*) = \int_{-\infty}^{z_i^*} \int_{-\infty}^b \phi(y) \phi(z_i) dy dz_i \quad (4.31)$$

where:

$$z_i^* = \frac{Q_i - \kappa_{-i}}{\zeta} + \Phi^{-1}(1 - l^*) \quad (4.32)$$

$$b = \frac{Q_i - \zeta z_i}{\nu} \quad (4.33)$$

This can be simplified to the result in (4.30). ■

The integral from Equation (4.30) cannot be solved analytically (in terms of elementary functions) for finite limits – it is the CDF of a bivariate normal distribution.⁴ This integral is the probability that the sovereign pays its debt. We can use it, along with a fixed-point argument, to solve for the equilibrium cutoff strategy.

Theorem 4.5. *In game Γ , agent i 's best response function is implicitly defined by:*

$$I(\kappa_i, \kappa_{-i}) = \frac{\beta}{r + \beta} \quad (4.34)$$

where $I(\kappa_i, \kappa_{-i})$ is defined according to (4.30).

Proof. The expected utility to agent i , who has received signal Q_i , for investing in the sovereign debt is:

$$ER(Q_i, \kappa_{-i}) = r I(Q_i, \kappa_{-i}) - \beta [1 - I(Q_i, \kappa_{-i})] \quad (4.35)$$

It follows from (4.35) that the cutoff κ_i is defined as the level of the signal Q_i that makes investors indifferent between their two available actions. Thus, κ_i solves the following equation:

$$ER(\kappa_i, \kappa_{-i}) = r I(\kappa_i, \kappa_{-i}) - \beta [1 - I(\kappa_i, \kappa_{-i})] = 0 \quad (4.36)$$

Reordering proves Theorem 4.5. ■

In equilibrium, the symmetric outcome will be when the strategy adopted by investor i is equal to the switching strategy adopted by all the other agents. Formally, this is:

$$\kappa_i = \kappa_{-i} = \kappa^* \quad (4.37)$$

Theorem 4.6. *There exists a finite, positive, unique symmetric cutoff strategy for game Γ , which is defined by:*

$$I(\kappa^*) = \frac{\beta}{r + \beta} \quad (4.38)$$

This is as long as $l^ \leq r/(r + \beta)$. If $l^* > r/(r + \beta)$, the only solution is degenerate, where $\kappa^* \rightarrow \infty$.*

Before proving this theorem, it is useful to establish two intermediate results with Lemmas 4.7 and 4.8.

Lemma 4.7. *The best response function for agent i exhibits strategic complementarities.*

Proof. In order to demonstrate strategic complementarity, it is sufficient to show that the best-response function is increasing in the actions of the other players by using the implicit differentiation to get $d\kappa_i/d\kappa_{-i}$. The derivatives of $I(\cdot)$ with respect to the relevant arguments are:⁵

$$\frac{dI}{d\kappa_{-i}} = -\frac{1}{\nu} \phi \left(\frac{\kappa_i - \kappa_{-i}}{\nu} + \Phi^{-1}(1 - l^*) \right) \Phi \left(\frac{(\nu - \zeta)\kappa_i + \zeta\kappa_{-i}}{\nu^2} - \frac{\zeta}{\nu} \Phi^{-1}(1 - l^*) \right) < 0 \quad (4.39)$$

and:

$$\begin{aligned} \frac{dI}{d\kappa_i} &= -\frac{dI}{d\kappa_{-i}} \\ &+ \frac{\nu}{\sqrt{2\pi}(\nu^2 + \zeta^2)} \exp \left[\left(\frac{1}{\nu^2} - \frac{\zeta^2}{\zeta^2 + \nu^2} \right) \kappa_i \right] \Phi \left(\frac{\sqrt{\nu^2 + \zeta^2}}{\nu} z_i^* - \frac{\zeta\kappa_i}{\sqrt{\nu^2 + \zeta^2}} \right) \end{aligned} \quad (4.40)$$

$$> 0 \quad (4.41)$$

Further, using implicit differentiation, the slope of the best-response curve will be given by:

$$\frac{d\kappa_i}{d\kappa_{-i}} = -\frac{dI/d\kappa_{-i}}{dI/d\kappa_i} \quad (4.42)$$

Using the results from (4.39), (4.41), and (4.42), it follows immediately that this is greater than zero. ■

Lemma 4.8. *The slope of the best response function is bounded:*

$$0 < \frac{d\kappa_i}{d\kappa_{-i}} < 1 \quad \forall \kappa_{-i} \in \mathbb{R} \quad (4.43)$$

and the slope of the best-response function is unambiguously increasing in κ_{-i} .

Proof. First, the slope of the best-response function (see the proof to Lemma 4.7) takes the form:

$$\frac{d\kappa_i}{d\kappa_{-i}} = \frac{A}{A+B} \quad (4.44)$$

where $A, B > 0$, $B \rightarrow \infty$ as $\kappa_{-i} \rightarrow -\infty$ and $B \rightarrow 0$ as $\kappa_{-i} \rightarrow \infty$. In the limit, as the cutoff adopted by other players becomes increasingly optimistic ($\kappa_{-i} \rightarrow -\infty$), their behaviour has little effect on the optimal choice of cutoff for individual i . In effect, if all other players almost certainly choose to roll-over the debt, individual i will base the decision only on whether the government appears to be solvent. As such, the limiting strategy $\kappa_{-i} \rightarrow -\infty$ is:

$$\lim_{\kappa_{-i} \rightarrow -\infty} \kappa_i = \sqrt{\nu^2 + \zeta^2} \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \quad (4.45)$$

As the strategy played by others becomes very conservative ($\kappa_{-i} \rightarrow \infty$), the strategy adopted by agent i also grows, in line with the strategy adopted by others. The slope of the best-response function goes to 1 and the cutoff adopted by agent i grows similarly, with the strategy adopted by others. In the limit, the best-response function converges to a slant asymptote:

$$\kappa_i = \kappa_{-i} + \underbrace{\zeta \left[\Phi^{-1} \left(\frac{\beta}{r + \beta} \right) - \Phi^{-1} (1 - l^*) \right]}_{<0} \quad (4.46)$$

By assuming $\frac{\beta}{r+\beta} < 1 - l^*$, the constant term in the slant asymptote is negative and the slope of the best response curve is bounded between $0 < d\kappa_i/d\kappa_{-i} < 1$ and monotonically increasing. ■

Proof. (Theorem 4.6) The logic of this proof is shown in Figure 4.4. It follows from Lemmas 4.8 and 4.7 that a unique fixed point must exist. This is equivalent to stating that there exists a unique switching equilibrium. This game is also supermodular since we are able to demonstrate that the slope of the best-response function is increasing in the cutoff adopted by the other investors.

From the above analysis, we are searching for a single value of $\kappa^* = \kappa_i = \kappa_{-i}$ that will equate the expected value of rolling-over government debt to zero (the return on the safe asset). From Equation (4.34), the specific value κ^* solves the equation:

$$I(\kappa^*) = \frac{\beta}{r + \beta} \quad (4.47)$$

Equation (4.47) implicitly defines the equilibrium cutoff strategy adopted by the investors. In order to solve for κ^* , it is useful to examine how $I(\kappa)$ behaves more generally.

By the fundamental theorem of calculus, we can easily demonstrate that $I(\kappa)$ is strictly monotonically increasing in κ . Further, since it is differentiable, it is also continuous:

$$\frac{\partial I}{\partial \kappa} = \frac{\exp\left(-\frac{\kappa^2}{2(\nu^2 + \zeta^2)}\right)}{\sqrt{2\pi(\nu^2 + \zeta^2)}} \Phi\left(\frac{\sqrt{\nu^2 + \zeta^2}}{\nu} \left(\Phi^{-1}(1 - l^*) - \frac{\zeta \kappa}{\nu^2 + \zeta^2}\right)\right) > 0 \quad (4.48)$$

See Appendix C.2.1 for the derivation of Equation (4.48).

The function $I(\kappa)$ is bounded from above and below as $\kappa \rightarrow \infty$ and $\kappa \rightarrow -\infty$ respectively. The results are remarkably simple:

$$\lim_{\kappa \rightarrow \infty} I(\kappa) = 1 - l^* \quad (4.49)$$

$$\lim_{\kappa \rightarrow -\infty} I(\kappa) = 0 \quad (4.50)$$

From Section 4.2.2, the continuity and monotonicity of $I(\kappa)$ implies the existence of a unique equilibrium as long as:

$$\frac{r}{r + \beta} \geq l^* \quad (4.51)$$

This is a remarkably simple condition to guarantee the existence of a finite cutoff equilibrium. The condition is a function solely of exogenous variables that are known to the players of the game. If the condition does not hold, the coordination required for roll-over, relative to the cost and benefits of lending to the sovereign, is too great: in that case, *only* the degenerate outcome is a possible solution. If the condition does hold, there exists a unique cutoff κ^* ; by appropriately selecting r and β (both exogenous), any value positive of κ^* can be sustained. See Figure 4.5 for further illustration. ■

Although there is no analytical solution for κ^* , I have demonstrated both the existence and uniqueness of a positive, real equilibrium strategy. Further, κ^* can be approximated arbitrarily precisely using numerical methods (see Appendix B.1 for numerical simulations). Finally, see Appendix C.2.2 for special cases where the closed-form solution exists.

Using general results from the literature on supermodular games, it will follow that this strategy can be achieved with iterated deletion of strictly dominated strategies and is rationalizable. Further, the equilibrium has desirable (convergent) best-response dynamics. The stable nature of this equilibrium (equivalent to the process of iterated deletion of strictly dominated strategies) is illustrated in Figure 4.4.

4.3 Comparative statics of the cutoff κ^*

Now that I have demonstrated that a unique equilibrium exists in this game, this section examines the comparative statics. Specifically, I will demonstrate how the optimal strategies change, conditional on different exogenous parameters. Most of the results demonstrated here will be intuitive and will be useful in illustrating the aggregate behaviour of the model. Generally in this section, I will assume that the existence and uniqueness conditions are satisfied.

The equilibrium κ^* is implicitly defined by Equation (4.47) and is a function of the return on government debt (r), the loss from default (β), the coordination required for roll over (l^*), and the standard deviations of sender and receiver noise (ν and ζ respectively).ⁱ The first two variables (r, β) are on the RHS of Equation (4.47) and do not appear on the LHS. Similarly, the remainder of the exogenous variables, they only occur on the LHS of (4.47). I will consider each in turn and demonstrate how they affect the equilibrium cutoff κ^* .

A very useful result in the following comparative statics is the following lemma.

Lemma 4.9. *An increase in the equilibrium cutoff strategy ceteris paribus results in an increase in the probability of repayment:*

$$\partial I / \partial \kappa > 0 \tag{4.52}$$

Proof. This follows from the proof of Theorem 4.6; see Equation (4.48). ■

Proposition 4.10. *Increasing the cost of default (β) results in a higher equilibrium cutoff; increasing the return on government debt (r) results in a lower equilibrium cutoff.*

ⁱThe last two variables can be trivially transformed to the standard deviation of the signal noise (σ), and the correlation between the signals (ρ).

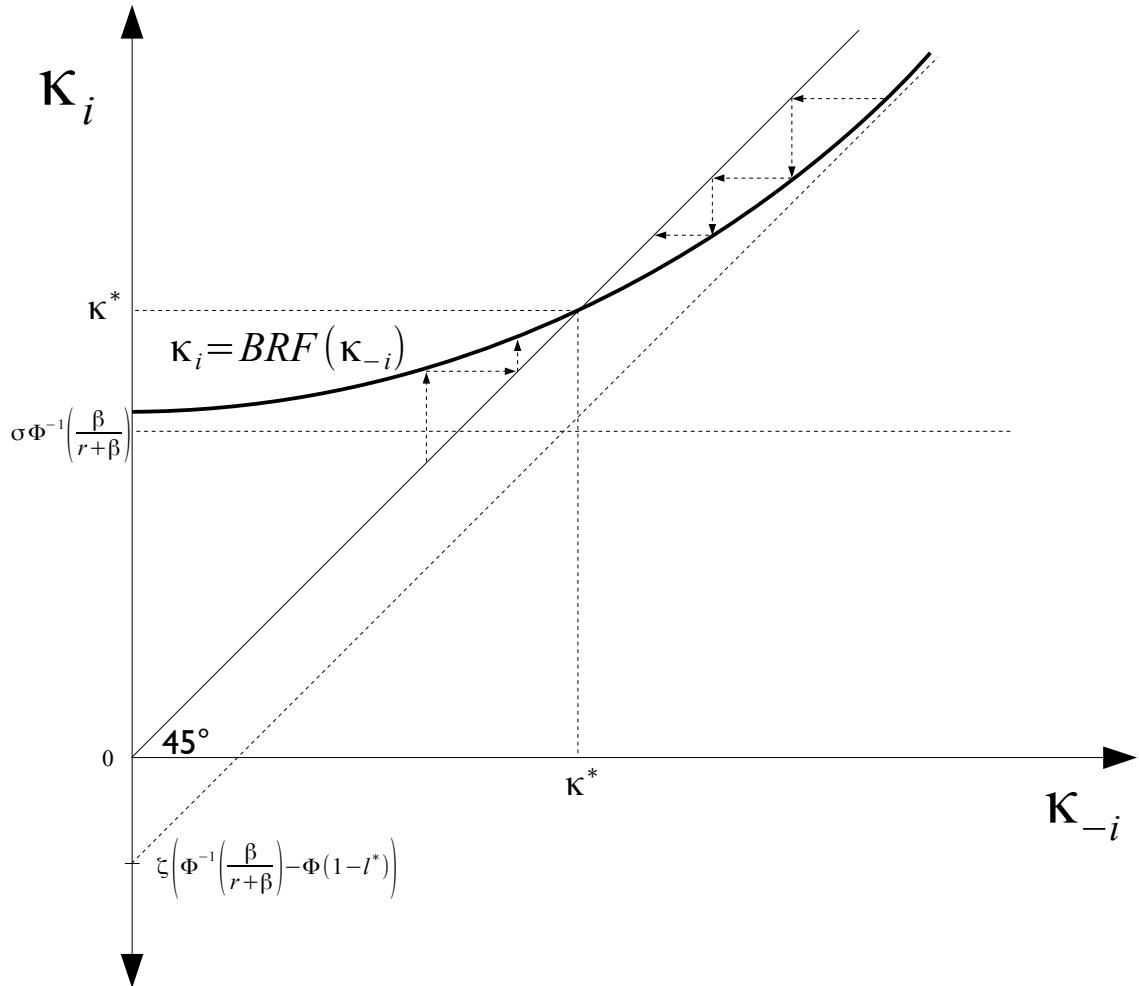


Figure 4.4: This figure demonstrates the best-response choice of κ_i as a function of κ_{-i} (denoted by the thick black line). The best response function has a single fixed point where $\kappa_i = \kappa_{-i}$ and the ‘dynamics’ of this equilibrium are stable. That is, we can use iterated deletion of strictly dominated strategies to show that κ^* is the unique cutoff equilibrium in this game. Begin by choosing any $\kappa_{-i} \neq \kappa^*$ as a candidate equilibrium strategy. Immediately, every agent under this proposed strategy will want to deviate to a cutoff that is closer to κ^* ; any candidate strategy more extreme can be excluded as being strictly dominated. This process continues until we end up at the unique equilibrium strategy κ^* .

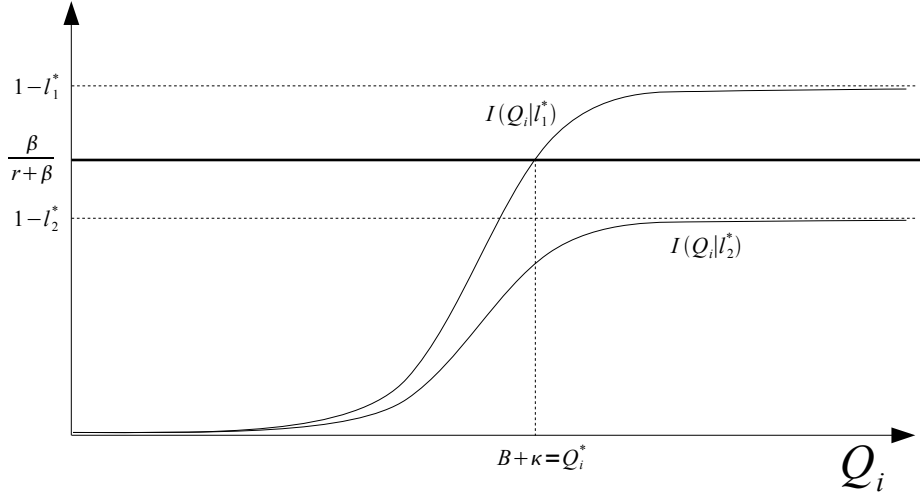


Figure 4.5: This graph demonstrates that the condition from Equation 4.51 is both necessary and sufficient to provide a unique cutoff equilibrium. Assuming that $l_1^* < r/(r + \beta) < l_2^*$, we can see that one case (l_1^*) results in a cutoff equilibrium (shown as $B + \kappa = Q_i^*$). If the agent receives a signal above Q_i^* , they will invest in sovereign debt; if not, they will choose the money market. The second case (l_2^*) results in the degenerate outcome in which agents invest only in the safe asset.

Proof. By direct computation and using Lemma 4.9, it follows:

$$\frac{\partial \kappa^*}{\partial r} = \frac{-\beta/(r + \beta)^2}{\partial I(\kappa)/\partial \kappa} < 0 \quad (4.53)$$

$$\frac{\partial \kappa^*}{\partial \beta} = \frac{r/(r + \beta^2)}{\partial I(\kappa)/\partial \kappa} > 0 \quad (4.54)$$

■

Proposition 4.11. *Increasing the coordination requirement (i.e. the liquidity requirement) results in a higher equilibrium cutoff.*

Proof. The coordination term l^* enters the $I(\kappa)$ only through the upper limit on the integral. Therefore, we can solve for the derivative of $I(\kappa)$ with respect to l^* using an extension to the fundamental theorem of calculus.

$$\begin{aligned} \frac{\partial I}{\partial l^*} &= \frac{\partial}{\partial l^*} \int_{-\infty}^{\Phi^{-1}(1-l^*)} \phi(x) \Phi\left(\frac{\kappa - \zeta x}{\nu}\right) dx \\ &= \phi(\Phi^{-1}(1-l^*)) \Phi\left(\frac{\kappa - \zeta \Phi^{-1}(1-l^*)}{\nu}\right) \left(\frac{\partial}{\partial l^*} \Phi^{-1}(1-l^*)\right) \\ &= -\phi(\Phi^{-1}(l^*)) \Phi\left(\frac{\kappa + \zeta \Phi^{-1}(l^*)}{\nu}\right) \left(\frac{1}{\phi(\Phi^{-1}(l^*))}\right) \\ &= -\Phi\left(\frac{\kappa + \zeta \Phi^{-1}(l^*)}{\nu}\right) < 0 \end{aligned} \quad (4.55)$$

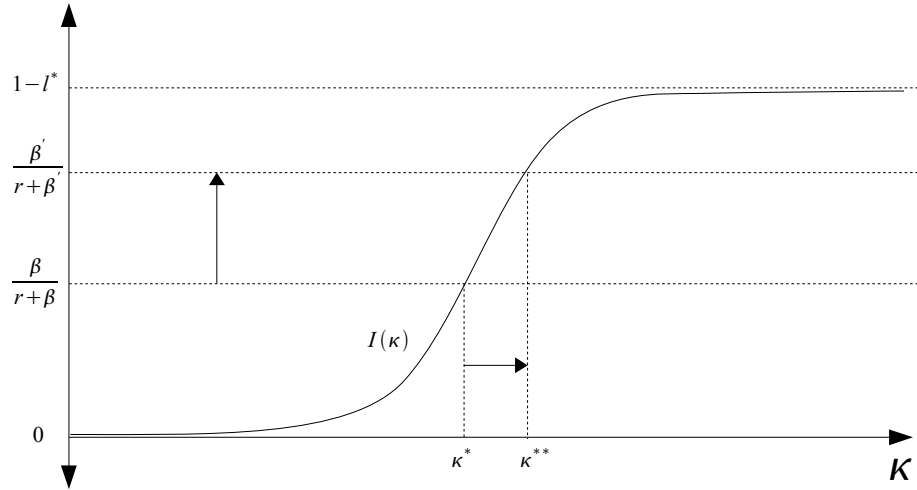


Figure 4.6: By increasing β , the RHS of Equation (4.47) increases, while the LHS remains unchanged. In the diagram, this has the effect of shifting up the $\beta/(r + \beta)$ line, while the $I(\kappa)$ line remains unchanged. As long as Condition (4.51) holds, there will be a new equilibrium κ . In this case, the equilibrium strategy shifts from κ^* to κ^{**} as the cost of default increases from β to β' .

This implies, then, that there is a positive relationship between the equilibrium cutoff (κ^*) and the coordination variable (l^*).

$$\frac{d\kappa^*}{dl^*} = \frac{\Phi\left(\frac{\kappa + \zeta\Phi^{-1}(l^*)}{\nu}\right)}{dI(\kappa)/d\kappa} > 0 \quad (4.56)$$

That is, as the liquidity requirement increases, the behaviour of the investors becomes more conservative. Formally:

$$\frac{d\kappa^*}{dl^*} = \frac{\Phi\left(\frac{\kappa + \zeta\Phi^{-1}(l^*)}{\nu}\right)}{dI(\kappa)/d\kappa} > 0 \quad (4.57)$$

■

Remark. Graphically, by increasing the liquidity condition l^* , the probability of payment (denoted by $I(\kappa)$) is effectively reduced (the $I(\kappa)$ line is squashed down), which results in more conservative equilibrium strategies. This is illustrated in Figure 4.7.

Proposition 4.12. *The equilibrium cutoff is proportional to the standard deviation of the signal noise: $\kappa^* = \sigma\kappa'$.*

Proof. This fact can be demonstrated by solving for the derivative of κ with respect to σ ,

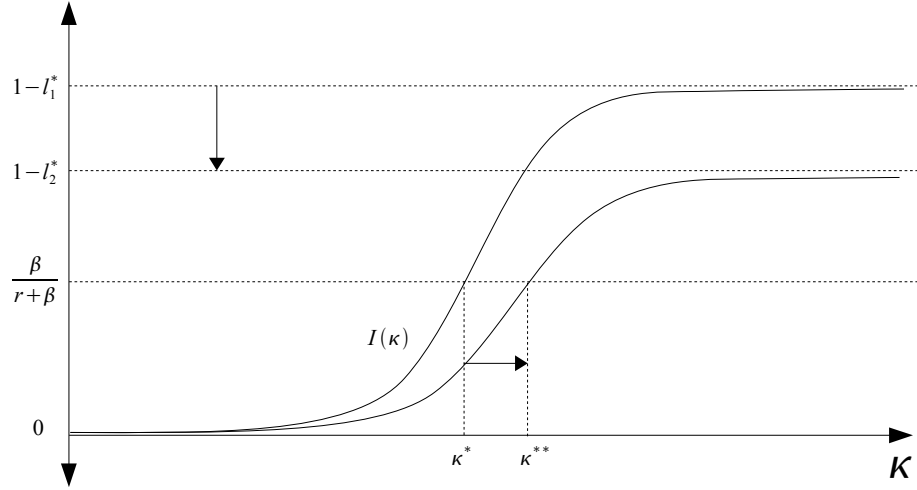


Figure 4.7: By increasing l_1^* to l_2^* , the RHS of Equation (4.47) is unchanged, while the LHS is decreased. In the diagram, this has the effect of squashing the $I(\kappa)$ downward. As long as Condition (4.51) holds, there will be a new equilibrium κ . In this case, the equilibrium strategy shifts from κ^* to κ^{**} as the minimum coordination increases.

which yields:

$$\frac{1}{\sigma\sqrt{\rho}} \left[\frac{\partial \kappa}{\partial \sigma} - \frac{\kappa}{\sigma} \right] \int_{-\infty}^{\Phi^{-1}(1-l^*)} \phi(x) \phi \left(\frac{\kappa}{\sigma\sqrt{\rho}} - \sqrt{\frac{1-\rho}{\rho}} x \right) dx = 0 \quad (4.58)$$

This greatly simplifies to the first-order linear differential equation:

$$\kappa^* = \sigma \kappa' \quad (4.59)$$

The initial condition can be given by the case where $\sigma = 0$, which implies $\kappa = 0$. Thus, the solution to the ordinary differential equation is:

$$\kappa^* = \sigma \hat{\kappa}(r, \beta, l^*, \rho) \quad (4.60)$$

where $\hat{\kappa}$ is constant with respect to σ ; the standard deviation of total noise is simply a scaling parameter. The constant $\hat{\kappa}$ is a function made up of the remaining exogenous variables (excluding σ). Hence, it is clear that the equilibrium cutoff varies proportionately with the signal noise. ■

Proposition 4.13. *Increasing the correlation between the signals results in a higher equilibrium cutoff being adopted.*

Proof. Since $\partial I/\partial \kappa$ is positive, we can show that $\partial I/\partial \rho$ is negative, which will imply a positive relationship between the correlation and the equilibrium cutoff. That is, as the proportion of the noise in the signal becomes more ‘public’ (while holding the total noise constant) investors will behave more conservatively. With some algebra, it follows:

$$\begin{aligned} \frac{\partial I}{\partial \rho} &= - \frac{\exp \left[-\frac{\sigma^2 \Phi^{-1}(1-l^*)^2 - 2\kappa \Phi^{-1}(1-l^*) \sqrt{1-\rho+\kappa^2}}{2\sigma^2 \rho} \right]}{4\pi \sqrt{\rho(1-\rho)}} \\ &< 0 \\ &\implies \frac{\partial \kappa^*}{\partial \rho} > 0 \quad \forall \rho \in (0, 1) \end{aligned} \tag{4.61}$$

We can conclude that the relationship between the correlation between signals and the cutoff is positive. ■

Corollary 4.13.1. *For low levels of correlation, increasing the correlation has little effect on the equilibrium cutoff. Conversely, for high levels of correlation, there is a very strong effect on the equilibrium cutoff.*

Proof. Using L’Hôpital’s Rule:

$$\lim_{\rho \rightarrow 0^+} \frac{\partial \kappa^*}{\partial \rho} = 0 \tag{4.62}$$

$$\lim_{\rho \rightarrow 1^-} \frac{\partial \kappa^*}{\partial \rho} = \infty \tag{4.63}$$

Figure 4.8 illustrates the relationship between the equilibrium strategy κ^* and correlation of signals ρ . ■

An alternative interpretation of correlated signals is that noise is introduced in two stages: one error term directly from the CRA (the public component) and the varying ‘interpretations’ of the signals (the private component).

The total variance of the signal is simply the sum of the public and private components:

$$\sigma^2 = \nu^2 + \zeta_i^2 \tag{4.64}$$

The correlation is the proportion of the total variation for which the public component accounts:

$$\rho = \frac{\nu^2}{\nu^2 + \zeta_i^2} \tag{4.65}$$

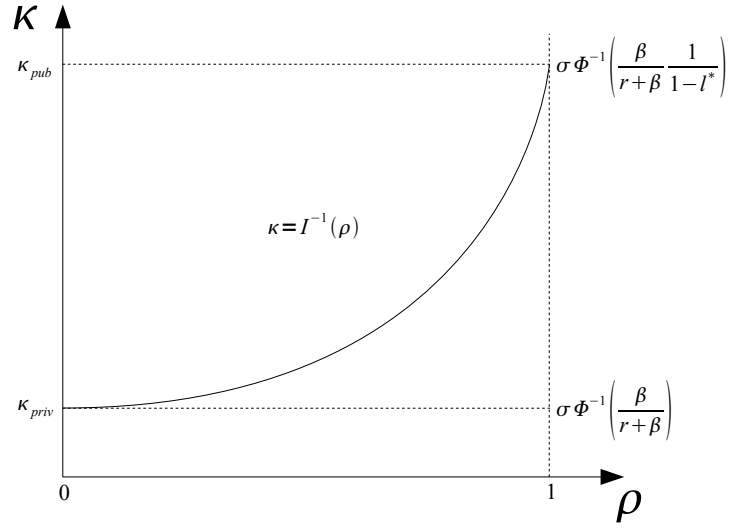


Figure 4.8: This figure illustrates the relationship between the equilibrium value of κ and $\rho \in [0, 1]$. As was demonstrated in Proposition 4.13, the marginal effect of a change in correlation of information at $\rho = 0$ is zero; at $\rho = 0$, the equilibrium cutoff is κ_{priv} . As the correlation increases, so does the marginal effect of ρ on κ . This increases the equilibrium cutoff value as the correlation increases. As $\rho \rightarrow 1$, the marginal effect explodes to infinity and the equilibrium cutoff converges to κ_{pub} .

The $I(\kappa)$ function can be written as:

$$I(\kappa) = \int_{-\infty}^{\Phi^{-1}(1-l^*)} \phi(x) \Phi\left(\frac{\kappa - \zeta x}{\nu}\right) dx \quad (4.66)$$

Proposition 4.14. *The equilibrium cutoff is unambiguously increasing in the standard deviation of public noise.*

Proof. This follows from Propositions 4.12 and 4.13. By increasing the standard deviation of the public-noise component (holding private noise constant), the correlation between signals and the total volatility of signals will increase. Both of these effects lead to an increase in the equilibrium cutoff κ^* , so there is an unambiguously positive relationship between the public noise volatility (ν) and the equilibrium strategy (κ^*).

$$\begin{aligned} \frac{\partial I}{\partial \nu} &= 2\nu \frac{\partial I}{\partial \sigma} + \frac{2\nu\zeta^2}{(\nu^2 + \zeta^2)^2} \frac{\partial I}{\partial \rho} \\ &< 0 \\ &\implies \frac{\partial \kappa^*}{\partial \nu} > 0 \quad \forall \nu \in \mathbb{R}_+ \end{aligned} \quad (4.67)$$

■

As the volatility of public noise goes to infinity, the total noise also explodes and the signals between agents become perfectly correlated; this results in the equilibrium strategy also exploding to infinity ($\kappa^* \rightarrow \infty$). Conversely, as the public noise volatility drops to zero, the only remaining volatility is due to the private noise; similarly, the correlation between signals drops to zero. In this case, the game converges on the private-only information game, in which $\kappa^* = \kappa_{\text{priv}}$ as in Equation (C.6).

The volatility of the private noise, however, is not as simple. As private-noise volatility increases, it acts to increase the total volatility of the signals but also to reduce the correlation between signals. These two changes have opposing effects on the equilibrium cutoff. For small volatilities of private information, the correlation effect will dominate: increasing private-information volatility will *reduce* the equilibrium cutoff. This effect, however, is soon dominated by the total variance effect and, beyond a certain point, increasing the private-information variance will inevitably *increase* the cutoff strategy adopted.

Proposition 4.15. *The effect of an increase in the private noise on the equilibrium cutoff is ambiguous. If the standard deviation is below a cutoff ζ^* , the effect is negative; if the standard deviation is above the cutoff, the effect is unambiguously positive.*

Proof. (Sketch) In order to see this formally, consider the marginal effect of private noise on the optimal cutoff as the private noise gets very small. In this instance, the marginal effect will be *negative*; that is, going from zero private noise to a small amount of private noise will result in a less conservative cutoff. Obviously, this result is somewhat counterintuitive.

$$\begin{aligned}
 \lim_{\zeta \rightarrow 0^+} \frac{\partial I}{\partial \zeta} &= \lim_{\zeta \rightarrow 0^+} \left(\frac{\zeta}{\sqrt{\zeta^2 + \nu^2}} \frac{\partial I}{\partial \sigma} - \frac{2\nu^2 \zeta}{(\nu^2 + \zeta^2)^2} \frac{\partial I}{\partial \rho} \right) \\
 &= \lim_{\zeta \rightarrow 0^+} \underbrace{\frac{\zeta}{\sqrt{\zeta^2 + \nu^2}} \frac{\partial I}{\partial \sigma}}_{=0} - \lim_{\zeta \rightarrow 0^+} \frac{2\nu^2 \zeta}{(\nu^2 + \zeta^2)^2} \frac{\partial I}{\partial \rho} \\
 &= \frac{1}{2\pi \nu} \exp \left[-\frac{\Phi^{-1}(1 - l^*)^2}{2} - \frac{\kappa^2}{2\nu^2} \right] \\
 &> 0 \\
 &\implies \lim_{\zeta \rightarrow 0^+} \frac{\partial \kappa^*}{\partial \zeta} < 0
 \end{aligned} \tag{4.68}$$

Conversely, as the private noise gets large, the correlation between signals is weak. Increasing the private noise increases the total variance and results in a more conservative (larger) cutoff. As the private noise ζ becomes very large, the total noise explodes to infinity and

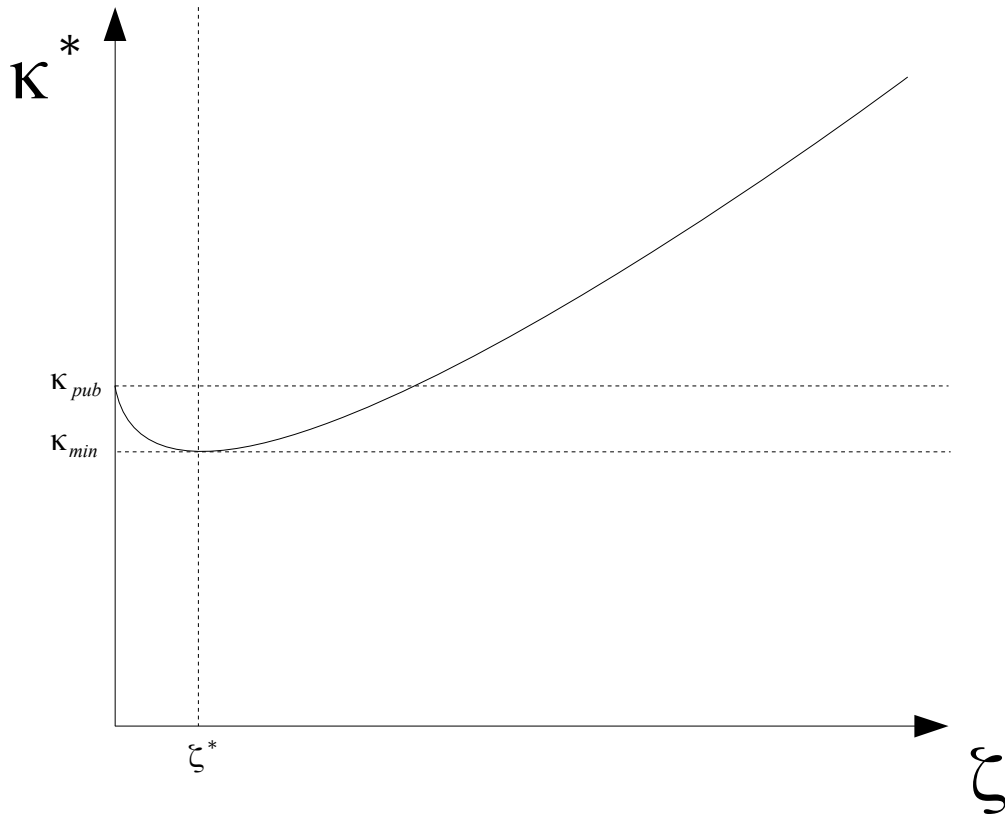


Figure 4.9: The equilibrium cutoff is, for low levels of ζ , decreasing in the standard deviation of private noise. The cutoff will achieve a global minimum at some positive level of private noise (denoted as ζ^* in the figure) before beginning to increase. As ζ gets large, the cutoff will continue to increase.

the correlation drops toward zero. In the limit, the game converges to the private-only information and the cutoff explodes to infinity as the total signal noise increases without bound. There exists some intermediate level of private noise (ζ^*) at which the effect of an increase switches from having a negative effect on the cutoff to a positive effect. ■

Any increase in private noise will have two effects: (i) an increase in the total noise of the signal (σ) and (ii) a reduction in the correlation between signals (ρ). For small amounts of private noise (low ζ), a decrease in correlation between signals has a larger effect on the cutoff than an increase in the total noise. As such, for small ζ , increasing private noise will result in investors adopting more optimistic cutoffs (smaller κ^*). As the private noise increases, however, at some point the effect of decreasing the correlation between signals (ρ) is outweighed by the increase in the total noise (σ). Thus, for larger ζ an increase in the private noise will result in more conservative behaviour (larger κ^*). The intuition behind this result is that more correlation increases the probability of a coordination failure, *ceteris paribus*. With high correlation, it is likely that, if a bad signal is transmitted, everyone has

received the bad signal and a run will occur. With low correlation, it is likely that much of the noise cancels out and the average signal is close to the fundamental value. In this manner, increasing private noise can have a salutary effect on the behaviour of the investors. In fact, there will be a level of private noise that minimizes the cutoff they adopt.

A similar result was demonstrated in Morris and Shin (2002), a notable paper that examines the benefits and costs of central-bank transparency. They modelled a continuum of agents who received two signals (one private and one public) and who were attempting to co-ordinate their behaviour in a ‘beauty contest’ game. Due to the nature of the game, the agents would overweigh the public signal. Because the public information has disproportionate influence on the agents’ decisions, improving the precision of public information did not always increase the welfare of the agents when good private information was already available. Morris and Shin (2002) summarized the result:

Increased precision of public information is beneficial only when the private information of the agents is not very precise. If the agents have access to very precise information ..., then any increase in the precision of the public information will be harmful. Thus, as a rule of thumb, when the private sector agents are already very well informed, the official sector would be well advised not to make public any more information, unless they could be confident that they can provide public information of very great precision.

The result presented here is somewhat different. In Morris and Shin (2002), the conclusion implies the possibility of keeping public signals noisy: in their game, increasing the precision of *private* information unambiguously increased welfare.

In this chapter, there are only private signals that correlate with one another. Reducing the public source of noise results in agents unambiguously behaving more optimistically. Reducing the receiver noise, however, has more nuanced effects. There are cases in which reducing private noise will result in investors behaving more conservatively. I will demonstrate below that neither of these results clearly translate over to an effect on the probability of crisis. In fact, an increase in the standard deviation of the private component, despite its ambiguous effect on κ^* , will unambiguously decrease the probability of a debt crisis. Conversely, increasing the standard deviation of the public component will have ambiguous effects on the probability of crisis.

4.3.1 Summary table of comparative statics of κ

The following table summarizes the results from the above sections. It gives the domain over which the exogenous variable is defined, the description of the variable, the marginal effect that the variable has on the equilibrium strategy, the sign of the effect and the limits of the strategy.

Variable (x)	Description	Marginal effect ($d\kappa/dx$)	Sign	$\lim_{x \rightarrow \sup(x)} \kappa$	$\lim_{x \rightarrow \inf(x)} \kappa$
$\beta \in \{\mathbb{R}_+ \beta > r\}$	Cost of default to investor	$\frac{r}{(r+\beta)^2} / \frac{dI(\kappa)}{d\kappa}$	Positive	$\kappa = I^{-1}(1/2)$	∞
$r \in \{\mathbb{R}_+ \beta > r\}$	Return to investor from sovereign debt	$-\frac{\beta}{(r+\beta)^2} / \frac{dI(\kappa)}{d\kappa}$	Negative	∞	$\kappa = I^{-1}(1/2)$
$l^* \in (0, 1)$	Coordination required for payment	$\Phi\left(\frac{\kappa}{\sigma\sqrt{\rho}} + \sqrt{\frac{1-\rho}{\rho}}\Phi^{-1}(l^*)\right) / \frac{dI(\kappa)}{d\kappa}$	Positive	$\sigma\Phi^{-1}\left(\frac{\beta}{r+\beta}\right)$	∞
$\nu \in \mathbb{R}_+$	Standard deviation of public noise	$-\left(2\nu \frac{\partial I}{\partial \sigma} + \frac{2\nu\zeta^2}{(\nu^2+\zeta^2)^2} \frac{\partial I}{\partial \rho}\right) / \frac{dI(\kappa)}{d\kappa}$	Positive	∞	κ_{priv}
$\zeta \in \mathbb{R}_+$	Standard deviation of private noise	$-\left(2\zeta \frac{\partial I}{\partial \sigma} - \frac{2\nu^2\zeta}{(\nu^2+\zeta^2)^2} \frac{\partial I}{\partial \rho}\right) / \frac{dI(\kappa)}{d\kappa}$	Neg/Pos	∞	κ_{pub}
$\sigma \in \mathbb{R}_+$	Total standard deviation of signal	κ/σ	Positive	∞	0
$\rho \in (0, 1)$	Correlation between signals	$\frac{\exp[A]}{4\pi\sqrt{\rho(1-\rho)}} / \frac{dI(\kappa)}{d\kappa}$ ii	Positive	$\sigma\Phi^{-1}\left(\frac{\beta}{r+\beta}\right)$	$\sigma\Phi^{-1}\left(\frac{\beta}{r+\beta} \frac{1}{1-l^*}\right)$

ii $A = -\frac{\sigma^2\Phi^{-1}(1-l^*)^2 - 2\kappa\Phi^{-1}(1-l^*)\sqrt{1-\rho+\kappa^2}}{2\sigma^2\rho}$

4.3.2 The *ex post* global uncertainty of Δ^*

With correlated information, once the game is complete, not all uncertainty is eliminated. If some external institution (say, a central bank) were to observe the outcome of the game, they would still not be able to infer the true value of Δ^* . In other words, with correlated signals, we preserve global uncertainty.

Proposition 4.16. *Global uncertainty is preserved in the game Γ .*

Proof. The sovereign's fiscal space (Δ^*) cannot be inferred because the public component of the correlated signal has biased the mean of the signal distribution away from Δ^* . Consider the two possible cases from Theorem 4.6: when no cutoff equilibrium exists (*i.e.* Condition 4.51 does not hold) and when it does. When the cutoff equilibrium does not exist then no investor chooses to invest in sovereign debt. As a result, no information on Δ^* can be garnered by an external observer who only sees the investors' actions. If, however, a cutoff equilibrium does exist, then the external observer will see the proportion of investors that chose to roll-over the sovereign debt. Given this information (and knowledge of all the parameters of the model, except Δ^*), the observer will be able to discern only a noisy signal; let's call this value $\hat{\Delta}^*$, which is a linear combination of the actual solvency condition value (Δ^*) and the public noise component of the signal (y), which is not observable. Formally:

$$\hat{\Delta}^* = \Delta^* + \sigma\sqrt{\rho}y \quad (4.69)$$

■

Remark. An external observer would be able to observe only an unbiased, normally distributed *signal* of the true solvency parameter (Δ^*) with variance $\rho\sigma^2$. This result can be used to examine the case of what would happen if there was some lender of last resort in a non-trivial setting: the third-party observer would infer from the signal that (given the outcome of the game) there is a some probability that the sovereign is illiquid but solvent. In the model setup, with Assumption 4.2, this would be the probability that $\Delta^* > 0$:

$$\Pr(\Delta^* > 0|l) = \Phi\left(\frac{\hat{\Delta}^*}{\sigma\sqrt{\rho}}\right) = \Phi\left(\frac{\hat{\Delta}^*}{\nu}\right) \quad (4.70)$$

If this framework were to be used to analyze the outcome of potential sovereign bailouts, this would obviously have effects on the decisions made by the investors; examining this scenario in full is beyond the scope of this chapter.

4.4 Calculating the *ex ante* distribution of play

Despite not being able to derive an analytical expression for the equilibrium cutoff when $\rho \in (0, 1)$, it is possible to derive the *ex ante* probability distribution of what proportion of investors will choose to invest in the risky sovereign debt.

I will use a conditional independence framework, similar to a portfolio loss distribution with a single systematic factor, which was popularized by Vasicek (1991) and later Vasicek (2002). The resulting distributions are highly skewed and leptokurtic. A normal distribution would not be a good approximation of the uncertainty of play in this game.

From here, we will assume the perspective of the sovereign, who has knowledge of all variables (including the willingness limit B^* or fiscal space Δ^*). The sovereign, however, will not know what signals have been sent to the population.

Theorem 4.17. *The proportion of agents who will decide to invest in the sovereign debt is ex ante distributed according to a Vasicek single factor loan portfolio value, with CDF:*

$$F(l) = \Phi \left(\frac{\zeta \Phi^{-1}(l) - \Delta^* + \kappa^*}{\nu} \right) \quad (4.71)$$

The probability of a debt crisis is:

$$p_c = \Phi \left(\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu} \right) \quad (4.72)$$

Proof. First, define the variable L_i , where $L_i = 1$ if the agent i successfully rolls-over and $L_i = 0$ if not. If the public noise component y was observable, the conditional probability of not purchasing the sovereign debt would be given by:

$$p(y) = \text{Prob}(L_i = 0|y) \quad (4.73)$$

Conditional on the public noise component y , agents' actions are independently and identically distributed, random variables with finite variances. Thus, we can appeal to the Law of Large Numbers: as $n \rightarrow \infty$, their sample average ($L = \sum_i L_i/n$) should converge on the

expectation. Reordering yields:⁶

$$\text{Prob}(L \leq x) = \text{Prob}(p(y) \leq x) = \text{Prob}(y \geq p^{-1}(x)) = \Phi(-p^{-1}(x)) \quad (4.74)$$

Exploiting this manipulation yields the CDF of l , the percent of investors who choose to roll-over the sovereign debt conditional on the exogenous variables:

$$F(l) = \Phi\left(\sqrt{\frac{1-\rho}{\rho}}\Phi^{-1}(l) - \frac{\Delta^* - \kappa^*}{\sigma\sqrt{\rho}}\right) \quad (4.75)$$

$$= \Phi\left(\frac{\zeta\Phi^{-1}(l) - \Delta^* + \kappa^*}{\nu}\right) \quad (4.76)$$

In order to determine the *ex ante* probability of a liquidity crisis, denoted p_c , substitute in the threshold level of coordination l^* :

$$p_c \equiv \Pr(l < l^*) = \Phi\left(\sqrt{\frac{1-\rho}{\rho}}\Phi^{-1}(l^*) - \frac{\Delta^* - \kappa^*}{\sigma\sqrt{\rho}}\right) \quad (4.77)$$

$$= \Phi\left(\frac{\zeta\Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu}\right) \quad (4.78)$$

■

Remark. Before the game is played, the percentage of investors who will choose to roll-over the debt is a random variable. This random variable (l) is distributed according to a Vasicek distribution; the CDF of this distribution is given by Equation (4.71) and its PDF is illustrated in Figure 4.10. In order to derive the probability of crisis, we need to calculate the probability that not enough investors will choose to roll-over the debt. That is, the probability that l is drawn below l^* , or $\Pr(l < l^*)$, which is the value given in Equation (4.72).

Here we have a measure for the probability that a government faces a liquidity crisis conditional on its fiscal space (Δ^*). From the sovereign's perspective, the fiscal space might be known but it cannot credibly be communicated to the investors. In this instance, the sovereign might be willing to pay its debts but it still faces a positive probability of a liquidity crisis (denoted by p_c). Note that the probability of a crisis is unambiguously decreasing in the fiscal space, which implies that if the sovereign were to reduce its debts, the probability of a debt crisis would also decrease. This would suggest that sovereigns might want to reduce their debt burdens slowly over time in response to an unanticipated debt shock.

4.4.1 Comparative statics of the probability of a debt crisis p_c

Proposition 4.18. *The probability of a debt crisis is increasing in the cost of default and decreasing in the return on debt.*

Proof. The cost of default (β) and the return on the debt (r) affect the probability of crisis only through the choice of cutoff (κ^*). Unsurprisingly, the probability of a crisis (conditional on the fiscal space Δ^*) increases in the cost of default to investors and decreases in the return on debt.

$$\frac{\partial p_c}{\partial \beta} = \frac{1}{\nu} \phi \left(\underbrace{\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu}}_{>0} \right) \underbrace{\frac{\partial \kappa^*}{\partial \beta}}_{>0} > 0 \quad (4.79)$$

$$\frac{\partial p_c}{\partial r} = \frac{1}{\nu} \phi \left(\underbrace{\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu}}_{>0} \right) \underbrace{\frac{\partial \kappa^*}{\partial r}}_{<0} < 0 \quad (4.80)$$

■

Remark. Here we see a potentially stabilizing force in sovereign debt markets. If interest rates on the debt rise (*ceteris paribus*), the *ex ante* probability of a debt crisis decreases. This is because more investors who receive borderline signals will opt to roll-over the debt than under a lower interest rate. Consequently, a higher interest rate results in more stability in demand for the government's debt. Obviously, there are certain circumstances not captured in this model where this might not be the case. For instance, if the high interest rates began to have an effect on the solvency of the country. In this model, that would be a situation in which changing the interest rate r would have effects on the fiscal space Δ^* . By increasing the rate of return on debt (r), if this made the debt less sustainable (a decrease in Δ^*), the probability of a crisis could easily increase. This model, perhaps unrealistically, assumes that the fiscal space and interest rate are unrelated.

Proposition 4.19. *Increasing the coordination required to roll-over the debt increases the probability of a crisis.*

Proof. By direct computation:

$$\frac{\partial p_c}{\partial l^*} = \phi \left(\underbrace{\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu}}_{>0} \right) \left[\underbrace{\frac{\zeta}{\nu} \frac{\partial}{\partial l^*} \Phi^{-1}(l^*)}_{>0} + \underbrace{\frac{1}{\nu} \frac{\partial \kappa^*}{\partial l^*}}_{>0} \right] > 0 \quad (4.81)$$

■

Remark. In order to roll-over the sovereign debt, a larger number of investors must receive adequate signals. Increasing the proportion of investors who must receive appropriate signals increases the chance that the signals sent will not be sufficiently large to roll-over the debt. Note also that as the coordination requirement goes to zero ($l^* \rightarrow 0$) the probability of a crisis goes to zero. This model assumes that the proportion of investors needed for rolling-over the debt is unrelated to the fiscal space. It might be plausible that a smaller fiscal space (*i.e.* higher debt) would require more investors to participate in the roll-over and this might further increase the probability of crisis.

Proposition 4.20. *The effect of increasing the standard deviation of total noise can either increase or decrease the probability of a crisis depending on the sign of the fiscal space. Better information when the government is solvent results in a lower probability of a debt crisis. Similarly, better information when the government is insolvent results in a higher probability of a debt crisis.*

Proof. In order to determine the effect of the increasing the standard deviation of total noise (σ), we can use the result from the proof of Proposition 4.12, which showed that κ^* was proportional to σ . We can use this result to simplify our calculations greatly:

$$p_c = \Phi \left(\sqrt{\frac{1-\rho}{\rho}} \Phi^{-1}(l^*) - \frac{\Delta^*}{\sigma\sqrt{\rho}} + \frac{\hat{\kappa}(r, \beta, l^*, \rho)}{\sqrt{\rho}} \right) \quad (4.82)$$

$$\frac{\partial p_c}{\partial \sigma} = \underbrace{\left[\frac{-1}{2\sigma^2\sqrt{\rho}} \right] \phi \left(\sqrt{\frac{1-\rho}{\rho}} \Phi^{-1}(l^*) - \frac{\Delta^* - \kappa^*}{\sigma\sqrt{\rho}} \right)}_{<0} \Delta^* \quad (4.83)$$

It follows that the sign of the effect is a function of whether the fiscal space is positive or negative – that is, whether the government is solvent or insolvent. ■

Proposition 4.21. *The effect of correlation on the probability of a crisis is ambiguous and depends on the government's solvency. For a solvent government, increasing the correlation will unambiguously increase the probability of a debt crisis.*

Corollary 4.21.1. *For an insolvent government, the effect of a change in correlation is ambiguous. There is a level of fiscal space above which the effect of increasing the correlation is always positive. Below this threshold, the effect is ambiguous: for low levels of correlation, increasing correlation will decrease the probability of a debt crisis. As the correlation approaches 1, the probability of a debt crisis inevitably increases. The threshold value of fiscal*

space is:

$$\Delta^* = \sigma \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \quad (4.84)$$

For numerical simulations of the result, please see Appendix B.1, specifically Figure B.3.

It is useful to establish some preliminaries about the probability of a debt crisis when the correlation is varied. Under perfectly private information ($\rho \rightarrow 0$), the probability of default is given by:

$$\lim_{\rho \rightarrow 0^+} p_c = \begin{cases} 1 & \text{if } \Delta^* < \sigma \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \\ 0.5 & \text{if } \Delta^* = \sigma \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \\ 0 & \text{if } \Delta^* > \sigma \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \end{cases} \quad (4.85)$$

An interesting result here is that, when Condition 4.51 holds, the critical value of Δ^* for which the probability switches is negative.

Remark. Under strictly private information, there are circumstances in which an insolvent sovereign faces zero probability of a crisis. The converse, however, does not hold: under private information, we should not observe a crisis when the sovereign is solvent. By observing a crisis under private information, we can infer that the sovereign is insolvent.

Under perfectly public information ($\rho \rightarrow 1$), the probability of default is given by:

$$\lim_{\rho \rightarrow 1^-} p_c = \Phi \left(\Phi^{-1}\left(\frac{\beta}{r+\beta} \frac{1}{1-l^*}\right) - \frac{\Delta^*}{\sigma} \right) \quad (4.86)$$

$$< 1 \quad (4.87)$$

Thus, the probability of default under purely public information is always strictly less than 1, regardless of the parameters of the model. This is because there is always some chance that the public signal is so extreme that it convinces a sufficient proportion of investors to roll-over the sovereign's debt. Of course, the probability of a crisis could be very large or small depending on the parameters of the model. The marginal effect of the correlation on the probability of crisis is ambiguous. For instance, if the limit as $\rho \rightarrow 0$ is 1, then the probability of a crisis could decrease as ρ increases, reach a minimum, and then increase toward the limit as $\rho \rightarrow 1$. See the numerical simulations in Appendix B.1 for an illustration of the effects, specifically Figures B.2 and B.3.

Proposition 4.22. *The effect of public noise on the probability of a crisis is ambiguous and depends on the government's solvency. For a solvent government, increasing the public noise will unambiguously increase the probability of a debt crisis.*

Corollary 4.22.1. *For an insolvent government, the effect of a change in public noise is ambiguous. There is a level of fiscal space for above which the effect of increasing the public noise is always positive. Below this threshold, the effect is ambiguous: for low levels of public noise, increasing public noise will decrease the probability of a debt crisis. As the public noise gets large, the probability of a debt crisis inevitably increases. The threshold value of fiscal space is:*

$$\Delta^* = \sigma \left[\Phi^{-1}(l^*) + \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \right] \quad (4.88)$$

See Appendix B.1 for numerical simulations of this result, specifically Figure B.7.

A change in the standard deviation of public noise can be decomposed into simultaneous changes in the correlation between signals (ρ) and the total standard deviation of the signal (σ). As the standard deviation of the public noise increases, it increases both the correlation and standard deviation of the total noise. Increasing the correlation has an ambiguous effect on the probability of a crisis (see Proposition 4.21), while the effect of an increase in standard deviation of total noise depends on the fiscal space (see Proposition 4.20). The probability of default is given by:

$$p_c \equiv \Pr(l < l^*) = \Phi \left(\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu} \right) \quad (4.89)$$

Consider what happens to this probability when $\nu \rightarrow 0$; *i.e.* the cutoff goes to the private outcome:

$$\lim_{\nu \rightarrow 0^+} p_c = \begin{cases} 1 & \text{if } \Delta^* < \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \right] \\ 0.5 & \text{if } \Delta^* = \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \right] \\ 0 & \text{if } \Delta^* > \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \right] \end{cases} \quad (4.90)$$

Conditional on the fiscal space, the probability of a crisis will converge to 1, 0.5, or 0 as the public noise becomes small. For low levels of fiscal space, the crisis will occur almost certainly while, for large amounts of fiscal space, the crisis is highly unlikely. The threshold value of fiscal space is a function of the private noise (ζ), the liquidity constraint (l^*), and the costs

and benefits of holding the debt (β and r). In general, when a non-degenerate equilibrium exists, the threshold value will be negative. In fact, this result is almost identical to what we found in Proposition 4.21.

As the standard deviation of public noise becomes very large, the probability of a crisis converges to a constant, which is a function of the liquidity constraint and the costs and benefits of default:

$$\lim_{\nu \rightarrow \infty} p_c = \frac{\beta}{r + \beta} \frac{1}{1 - l^*} \quad (4.91)$$

As the standard deviation of the public noise increases, it must converge toward the limit as $\nu \rightarrow \infty$; the derivative can give us the sign of marginal effect of changing the variable. Unsurprisingly, for low levels of fiscal space, the probability of crisis will be decreasing in the standard deviation of public information. When there are high levels of fiscal space, increasing the standard deviation of public noise results in a higher probability of a crisis. The threshold value of fiscal space is the same as for the limits in Equation (4.90). It follows, fairly uninformatively: that the marginal effect of a change in the standard deviation of public information is zero when the standard deviation is near zero:

$$\lim_{\nu \rightarrow 0^+} \frac{\partial p_c}{\partial \nu} = 0 \quad (4.92)$$

On its own, this result is not very interesting; however, it is possible to determine whether it approaches zero from below or above by using the signum function. Consider the following:

$$\text{sgn}\left(\frac{\partial p_c}{\partial \nu}\right) = \text{sgn}\left(\frac{\Delta^* - \kappa^* - \zeta \Phi^{-1}(l^*)}{\nu} + \frac{\partial \kappa}{\partial \nu}\right) \quad (4.93)$$

Taking the limit as the standard deviation of public information goes to zero yields:

$$\lim_{\nu \rightarrow 0^+} \text{sgn}\left(\frac{\partial p_c}{\partial \nu}\right) = \begin{cases} -1 & \text{if } \Delta^* < \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \\ 0 & \text{if } \Delta^* = \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \\ 1 & \text{if } \Delta^* > \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r+\beta}\right) \right] \end{cases} \quad (4.94)$$

which gives us the result that, for low levels of the standard deviation, the probability will *at first* be increasing or decreasing conditional on the fiscal space. In some circumstances, the

probability might overshoot the limit as $\nu \rightarrow \infty$ and the convergence will not be monotonic. Please see the numerical simulations for illustrations of this effect in Appendix B.1. In general, the effect of changing the standard deviation of the public noise has an ambiguous effect of the probability of a crisis: the other exogenous variables play an important role in determining the sign of the effect. Unlike the standard deviation of public noise, increasing the standard deviation of private noise has unambiguous effects on the probability of a crisis.

Proposition 4.23. *The probability of a crisis is unambiguously decreasing in the standard deviation of private information.*

Proof. First note that the probability of a crisis as ζ gets large decreases to zero:

$$\lim_{\zeta \rightarrow \infty} p_c = \lim_{\zeta \rightarrow \infty} \Phi \left(\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu} \right) \quad (4.95)$$

$$= \lim_{\zeta \rightarrow \infty} \Phi \left(\frac{\zeta}{\nu} \underbrace{\left[\Phi^{-1}(l^*) + \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \right]}_{<0} - \frac{\Delta^*}{\nu} \right) \quad (4.96)$$

$$= 0 \quad (4.97)$$

To demonstrate that the probability monotonically goes to zero as ζ increases, consider the derivative of the probability with respect to the standard deviation of private noise:

$$\frac{\partial p_c}{\partial \zeta} = \frac{1}{\nu} \underbrace{\phi \left(\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu} \right)}_{>0} \left[\Phi^{-1}(l^*) + \frac{\partial \kappa}{\partial \zeta} \right] \quad (4.98)$$

The sign of the derivative is decided by the elements in the square brackets. Under the assumptions of the model, this will always be negative. To determine the sign of the derivative as ζ gets large, we can use the $\text{sgn}(\cdot)$ operator:

$$\text{sgn} \left(\frac{\partial p_c}{\partial \zeta} \right) = \text{sgn} \left(\frac{1}{\nu} \phi \left(\frac{\zeta \Phi^{-1}(l^*) - \Delta^* + \kappa^*}{\nu} \right) \left[\Phi^{-1}(l^*) + \frac{\partial \kappa}{\partial \zeta} \right] \right) \quad (4.99)$$

$$= \text{sgn} \left(\Phi^{-1}(l^*) + \frac{\partial \kappa}{\partial \zeta} \right) \quad (4.100)$$

The derivative of κ with respect to ζ is increasing in ζ . If the sign of (4.100) is negative as $\zeta \rightarrow \infty$, the probability of crisis is unambiguously decreasing in ζ .

$$\lim_{\zeta \rightarrow \infty} \frac{\partial \kappa}{\partial \zeta} = \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \quad (4.101)$$

This, in turn, implies that:

$$\text{sgn}\left(\frac{\partial p_c}{\partial \zeta}\right) = \text{sgn}\left(\Phi^{-1}(l^*) + \frac{\partial \kappa}{\partial \zeta}\right) = -1 \quad (4.102)$$

It follows that the probability of a crisis is monotonically decreasing in the standard deviation of private noise. ■

Remark. This is an unusual result, especially regarding the ambiguous effect that ζ has on the equilibrium cutoff: the effect of an increase in ζ on the probability of a crisis is unambiguous. Increasing the standard deviation of the private noise introduced in the signal will reduce the probability of a crisis. This is because the spread of signals increases and, consequently, the proportion of investors who receive higher signals increases; thus, the probability of a sufficient proportion of investors receiving an adequate signal increases.

As the standard deviation of the private noise becomes large, the signals between become largely uncorrelated and the standard deviation of each signal also becomes very large. This provides a somewhat counterintuitive and problematic result: worse private understanding of noisy public signals, in general, reduces the probability of a crisis. This holds true whether the sovereign is solvent or insolvent. In the terminology I used in the introduction, reducing the intelligibility of signals (or increasing the disagreement between investors holding the public noise error constant) will *reduce* the probability of a crisis. More intelligible signals can greatly increase the probability of a crisis.

4.4.2 Additional properties of the *ex ante* distribution of play

The previous sections covered the derivation of the Vasicek distribution of play and the comparative statics with respect to the probability of crisis. This section presents several properties of the distribution. The behaviour of this distribution is well known and many of its properties are demonstrated in Vasicek (2002).

Proposition 4.24. *The proportion of agents who will decide to invest in the sovereign debt is ex ante distributed according to a Vasicek single factor loan portfolio value, with PDF:*

$$f(l) = \frac{\zeta}{\nu} e^{-\frac{1}{2}} \left[\left(\frac{\zeta \Phi^{-1}(l) - \Delta^* + \kappa^*}{\nu} \right)^2 - \Phi^{-1}(l)^2 \right] \quad (4.103)$$

Or, using different notation:

$$f(l) = \sqrt{\frac{1-\rho}{\rho}} e^{-\frac{1}{2} \left[\left(\sqrt{\frac{1-\rho}{\rho}} \Phi^{-1}(l) - \frac{\Delta^* - \kappa^*}{\sigma \sqrt{\rho}} \right)^2 - \Phi^{-1}(l)^2 \right]} \quad (4.104)$$

Proof. This can be shown by differentiating the CDF from Theorem 4.17 directly. The PDF is illustrated in Figure 4.10. ■

Remark. This distribution could equally have been written using only two parameters (p, ρ) . The parameter ρ denotes the correlation between the signals sent, while p is the *unconditional* probability of an investor choosing the safe asset:

$$p = \Phi \left(\frac{\Delta^* - \kappa^*}{\sigma} \right) \quad (4.105)$$

The PDF would then be written as:

$$f(l) = \sqrt{\frac{1-\rho}{\rho}} \exp \left(\frac{1}{2} \left[\Phi^{-1}(l)^2 - \left(\frac{\sqrt{1-\rho} \Phi^{-1}(l) - \Phi^{-1}(p)}{\sqrt{\rho}} \right)^2 \right] \right) \quad (4.106)$$

Proposition 4.25. *The mean value of l is equal to the unconditional probability of an investor choosing the safe asset:*

$$l_{Mean} = \Phi \left(\frac{\Delta^* - \kappa^*}{\sigma} \right) \quad (4.107)$$

Proposition 4.26. *The density of the distribution from Theorem 4.17 is unimodal when $\rho < 1/2$, with the mode occurring at:*

$$l_{Mode} = 1 - \Phi \left(\frac{\sqrt{1-\rho}}{1-2\rho} \frac{\Delta^* - \kappa^*}{\sigma} \right) \quad (4.108)$$

The distribution is monotone for $\rho = 1/2$ and has a U-shape when the correlation is above $1/2$.

Remark. In order to see the intuition behind this last result, consider the case when $\rho = 1$; in this situation, either everyone will choose to roll-over the debt ($l = 1$) or no one will ($l = 0$).

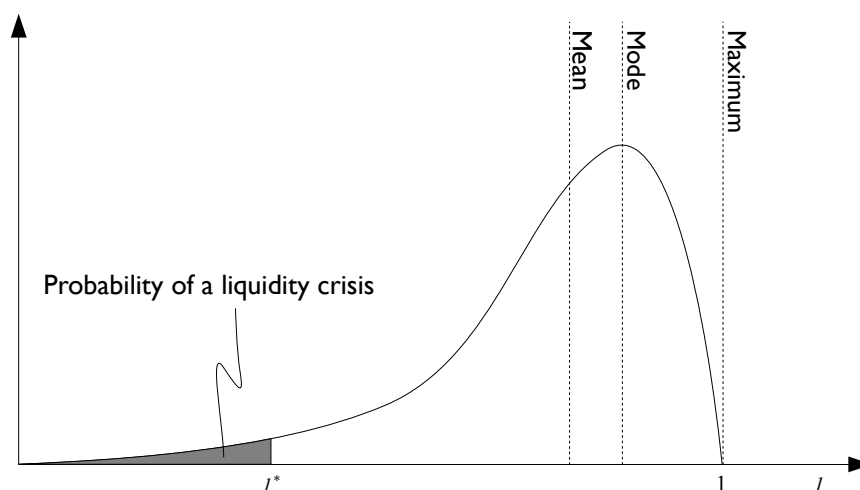


Figure 4.10: This figure demonstrates a stylized example of what the PDF of the *ex ante* distribution of play could look like when $0 < \rho < 1/2$. The formula for the PDF is given in Equations (4.103) and (4.104).

Variable (x)	Description	Effect on probability of crisis (dp_c/dx)
$\beta \in \mathbb{R}_+$	Cost of default to investor	Positive
$r \in \mathbb{R}_+$	Return to investor	Negative
$l^* \in (0, 1)$	Coordination required for payment	Positive
$\sigma \in \mathbb{R}_+$	Standard deviation of signal	Negative if solvent, positive if insolvent
$\rho \in (0, 1)$	Correlation between signals	Ambiguous
$\zeta \in \mathbb{R}_+$	SD of the private noise component	Negative
$\nu \in \mathbb{R}_+$	SD of the public noise component	Ambiguous
$\Delta^* \in \mathbb{R}$	Fiscal space	Negative

4.5 Discussion

There are two possible equilibrium strategies that can occur in this model depending on the parameterization: (1) a unique symmetric and finite cutoff equilibrium will exist or (2) the equilibrium strategy is the degenerate outcome. The latter case is somewhat trivial since it results in all players choosing to invest only in the safe asset regardless of their signal. The intuition behind this result is that the liquidity condition is too high relative to the cost and benefits of holding sovereign debt; the investors can infer this outcome directly from

the exogenous parameters of the model. When the condition in Equation (4.51) is satisfied, however, there exists a symmetric unique cutoff equilibrium strategy, which is the much more interesting case.

Before the game is played, anyone with knowledge of the sovereign's fiscal space can calculate the probability that the government will successfully roll-over the debt. The exogenous parameters of the model affect this probability. Increasing the cost of default to investors has a positive effect on the probability of a crisis, while increasing the return on the debt has a negative effect. Similarly, increasing the coordination requirement amongst the investors unambiguously increases the probability of a debt crisis. Increasing the standard deviation of total noise will increase the probability of the crisis if the government is insolvent but decrease the probability of a crisis if it is solvent. The correlation between signals has an ambiguous effect on the probability of a crisis. As the correlation gets small, the probability converges to either 0, 0.5 (a knife-edge case), or 1 but not necessarily as a function of whether the government is solvent. It is entirely possible for an insolvent government to have zero probability of crisis as the correlation goes to zero. The converse, however, is not true: solvent governments will always have zero probability of crisis (in the limit) as the correlation gets small. As the correlation increases, the probability of a crisis will converge to some single value independent of the government's fiscal space. The fiscal space also plays a very important and unambiguous role in determining the probability of a crisis: the higher the fiscal space, the lower the probability of a crisis.

The comparative statics of the probability of default become somewhat more interesting when we view the model through the lens of public and private noise components. The standard deviation of public noise component has an ambiguous effect on the probability of a crisis and, unsurprisingly, has an effect similar to that of the correlation between signals. This is interesting since, using the interpretation that the public noise is introduced by a CRA, better information from a CRA does not necessarily reduce the probability of a crisis. Conversely, it does not necessarily *increase* the probability of a crisis either. How the clarity of information from a CRA affects the probability of a crisis depends crucially on the other model parameters.

The standard deviation of private information, however, has an unambiguous effect on the probability of a crisis: increasing the standard deviation of the private component decreases

the probability of a crisis. This raises some interesting possibilities. The standard deviation of the public component can be thought of as a measure of the accuracy of the mean signal sent. The standard deviation of the private component, however, can be thought of as the agreement between investors on the fiscal space. In the introduction of this chapter, I used the terms ‘accuracy’ to denote the former and ‘intelligibility’ for the latter. By reducing the intelligibility of information on the fiscal space, a government can decrease the probability of a debt crisis. In some ways, this is a substitute for affecting the fiscal space. That is, if a government wishes to decrease the probability of a debt crisis, it has several levers: it may increase the return on the debt, increase its fiscal space (through political reform, tax increases, or expenditure cuts), or reduce the intelligibility of information.ⁱⁱⁱ Decreasing the intelligibility of information is a *substitute* for austerity. This thesis does *not* recommend exercising this lever over the others. It does, however, raise the interesting possibility that governments face an incentive to obfuscate information regarding their fiscal states. In fact, they do not necessarily want the information to be inaccurate *per se* but would want it to cause disagreement between investors. This result might provide some insight into why clear information on the finances of states has been so hard to find.^{iv} Moreover, adopting a strategy of obscuring information, although tempting in the short run, is likely the road to ruin. Providing confusing and abstruse information on a sovereign’s finances might confuse investors for a period of time but eventually the (quite literally) day of reckoning will arrive: the sovereign’s accounts will need to balance and this alone can send a very public signal to investors.

Perhaps a more fundamental question would be why government officials yield to the temptation and pursue the possibly self-defeating strategy of providing unintelligible information. Obviously, one possible and intuitive explanation is that the policymakers are acting myopically. This explanation is intuitive; policymakers choose the present benefit of reducing the probability of a crisis by increasing the probability of a debt crisis in the future. The government might wish to commit to a future path of highly intelligible information but, in each time period, the government wishes to reduce the intelligibility in order to reduce the probability of default in that period.

ⁱⁱⁱThis assumes that the government does not control the cost of default or the coordination amongst investors. The former is hard for the government to signal credibly and the latter is probably a function of the stock of debt, which the government cannot change without defaulting.

^{iv}See Reinhart and Rogoff (2009) for a lengthy discussion on the lack of transparency in sovereign financial statistics.

This explanation, in turn, suggests a solution: committing, through institutional design, to provide intelligible and accurate information on the government's fiscal situation. Institutions, such as the Congressional Budget Office (US), the Office for Budget Responsibility (UK), and the Office for National Statistics (UK) help investors and citizens to understand the government's finances. Although intelligible information might, in the short run, lead to the increased possibility of a debt crisis, the long-term benefits are almost certainly much larger. This chapter, however, does not fully deal with the potential tradeoffs between unintelligible signals today and future signals. This important question is left for future research.

Chapter 5

Bailouts and politics

5.1 Introduction

Principum munus est resistere levitati multitudinis.ⁱ
Inscription outside the Canadian Senate Speaker's chambers.

In *The Politics of Intergenerational Redistribution*, Tabellini (1989) uses a phrase to describe the incentives surrounding the repayment of public debt: issuing debt ‘creates facts’. The debt creates a constituency in favour of its repayment. Once debt has been issued, repudiation will have significant redistributive consequences. The average tax burden on the younger generation (the ‘children’) is a function of the average bond holdings of the older generation (the ‘parents’). However, since a child’s tax burden is not a function of specifically his parents’ bond holdings, children who have parents with larger than average bond holdings would prefer to pay slightly higher taxes than default on their parents’ bonds. A coalition of the parents and the children of the wealthy supports the repayment of the public debt. The net result, however, is a transfer of wealth from children to parents. This is in contrast to an unfunded social security system in which the coalition is between the old and the children of the poor. Although social security and public debt are identical from an accounting or neo-classical perspective, they have very different distributional impacts.

To use Tabellini’s language, institutions that are too-big-to-fail (TBTF), implicit guarantees on bank liabilities, and deposit insurance are ‘facts’. Their existence changes the incentives and decisions today, potentially at the expense of future generations. The promise of a

ⁱCicero, *Pro Milone* 22, which translates literally as “It is the duty of the nobles to oppose the fickleness of the multitude.” The Canadian Senate Speaker suggests that it be interpreted as: “It is the duty of senators to observe the common good and to resist opinions that fluctuate excessively, no matter who holds them.”¹

bailout (or a promise to let a bank fail) is credible only if it is politically stable when it must be executed. Although bailouts might harm the children, given the behaviour of their parents, the children might still prefer to bail out financial institutions than let them fail. If, somehow, the children could commit to not bailing out the banks, the parents would behave differently in the first period and the bailouts might be unnecessary – but how does framing the problem of TBTF as a political decision involving intergenerational transfers help to advance our understanding of financial regulation?

Firstly, it creates a negative result: if there are significant inter-generational externalities, this suggests that the legal innovation of ‘bail-in’ is not sufficient to deal with TBTF. In short, bail-in is a mechanism to resolve insolvent institutions by wiping out shareholders and converting creditors into equity-holders. The theory hinges on the (obviously correct) observation that the *status quo* resolution mechanisms are inadequate: there are significant negative externalities that result from the failure of systemically important institutions and the resolution itself can be very socially costly, as Lehman Brothers demonstrated. If this were the sole cause for bailouts, we might be justified in claiming that bail-in could be the solution. This reasoning, however, compares the imperfect *status quo* to the implementation of bail-in laws as intended. Although this comparison might be useful to understand the problem, weighing the imperfect reality against a perfect abstraction is not a fair comparison.

Equally, using this intergenerational model somewhat undermines proposals to use conditional contingent bonds (CoCos) to finance banks. Proponents of this approach argue that using CoCos as a form of financing pre-commits the bond-holders to face equity-like losses in a crisis (Goodhart, 2010). As does the literature on bail-in, this assumes that the contracts will be executed as intended. If they are not – through regulatory forbearance or some other mechanism – this proposal can hardly be considered an effective commitment device.

Consider an example from the recent financial crisis to illustrate this point. Under Basel II, subordinated debt was classified as Tier 2 capital, which was intended to be loss absorbing (albeit of a lesser class than Tier 1 capital). In the event of insolvency, subordinated debt was supposed to act as supplementary capital in a manner similar to bail-in: creditors who held subordinated debt of a failed institution should lose money.

Whether it should be necessary for banks to finance their operations by issuing subordinated debt has long been debated in regulatory circles.² A subordinated debt requirement, it is argued, can help to resolve some of the principal-agent and moral-hazard problems that are present in financial institutions: it imposes market discipline and provides useful information to both the banks and the regulators. Because subordinated debt would lose money under restructuring, the price should reflect the risk of the financial institution failing. Higher rates on subordinated debt signal that the bankers are undertaking riskier operations and must pay commensurately higher financing costs. Banks that adopt riskier strategies will inevitably find it harder to finance their operations. Similarly, the regulator is also able to observe which banks are perceived as having the greatest risk of failure. The subordinated debt acts as a proverbial canary in the coal mine. This reasoning rests on the assumption that losses will be imposed on subordinated debt-holders should the bank fail. In retrospect, however, it is clear that this assumption is flawed: in reality, the resolution authorities were unwilling to impose losses on subordinated debt.

For instance, the authors of the 2012 OECD Economic Survey of Spain noted (emphasis added, see p. 69):

Bank resolution has so far involved wiping out holders of common equity. **Debt, including subordinate debt, has generally been fully bailed out, including hybrid instruments with subordinated clauses which count as Tier 1 capital, such as preferential shares.** The authorities have avoided forcing creditors to take losses in order to prevent contagion risk and because of consumer protection concerns, as many banks have sold preferential shares and subordinate debt to their retail customers, in some cases perhaps without providing transparent information about the risks. However about half of the subordinate debt issued by Spanish banks has been subscribed by institutional investors where consumer protection concerns do not apply.

If regulators and policymakers were unwilling to implement the law as intended on subordinated debt, why would they be willing to implement bail-in? Understanding why regulators and policy-makers were unwilling to implement the law on subordinated debt could shed light on how to structure future reforms.

One explanation for bailouts is that the distributional consequences of imposing losses on creditors is clearly unpalatable. We can write clever laws and design independent institutions to force costs on creditors but, if our political representatives are subsequently unwilling to im-

pose these costs, the reforms are little more than window dressing. One way forward is to begin by taking these distributional consequences and political incentives into consideration. As Charles Calomiris has argued: “There’s no way to get politics out of the banking system, because banks require the state to give them their charters, to create and enforce the rules.”³

The main point of this model is *not* that we ought to adopt an Old Testament view when dealing with financial crises and ‘hose the bankers’; morality plays often make for bad economics. The point is that there is a fundamental difference between regulation that affects the financial sector *ex ante* and *ex post*. Time and time again, when dealing with a crisis, policymakers ignore rules that were written before the event. Discretion – or regulatory forbearance – can sometimes be beneficial but it precludes the possibility of commitment. The dichotomy between *ex ante* and *ex post* intervention should be present in our minds when designing and debating regulation. This chapter adds an additional argument in favour of raising and narrowing bank capital: using equity to finance bank operations is a form of pre-commitment to absorbing losses, which preserves the ability of regulators and policy-makers to implement other discretionary policies.⁴

From casual empiricism, bank bailouts have tended to focus on bailing out the creditors while allowing the equity-holders to lose money. Why is this the case? The bank creditors (including depositors) typically make up a significant proportion of the population; they are a large constituency that has a concentrated interest in receiving the face value of their investments. Self-interested politicians might, in a bid to placate this constituency, decide to backstop the bank’s obligations.

Further, it is equally important to consider the indirect beneficiaries from a riskier financing regime. Like the incidence of taxation, regarding which it might not matter who actually pays the tax, borrowers might benefit from bailout policies due to easier credit conditions. One potential effect of an implicit state guarantee on the financial sector is that many people who otherwise would not be able to borrow would find it easier to access credit. The electoral coalition that supports lax equity-capital controls might not be solely the owners of the bank.⁵

Table 5.1, below, shows the recovery rates of various state interventions in the financial sector (as a percent of GDP). In the United States, for instance, the IMF estimates that the US

government has more than recouped its support of the financial sector. Most other countries, however, have not been so fortunate: the average recovery rate has been only 58%. Excluding the United States, the average recovery rate has been only, roughly, 25%. Although these instances might have been intended as ‘costless interventions’ to stop a liquidity crisis, they have undeniably converted large amounts of private-sector debt into public-sector liability.

This normative analysis raises a separate question: why doesn’t the government also bail out the bank equity-holders? One justification could be that bank equity is traded on open markets and that there is therefore a reasonable (albeit imperfect) measure of the residual value of the bank’s assets. When a bank declares that it is insolvent, the market value of the equity will drop to (near) zero; it would then be difficult for a government to justify giving the book value of the bank’s equity to the investors when the market value is roughly zero. That is, it would be public information that the equity-holders had already absorbed some of the losses. However, this reasoning is problematic since the possibility of a bailout would increase the market value of the equity above zero, creating a self-fulfilling prophecy. This results in an underdetermined problem in which the value of the equity would be affected by the bailout policy and the bailout policy would be justified by the market value of the equity. Almost any valuation could justify any bailout policy and *vice versa*. This first reason, therefore, is not entirely compelling.

A different justification is that bank equity is only a small portion of the bank’s liabilities; consequently, the constituency that would benefit from an intervention is rather small. At the margin, imposing losses on the equity-holders reduces the cost of the bailout to the creditors, which benefits the taxpayers (or marginal voters). In short, there is actually a demand for bailouts when a bad state of the world occurs; a government’s ability, however, to extract them is limited by the political institutions and preferences of other voters.

This chapter aims partially to address these questions by presenting an OLG model in which there is an incentive to bail out failed institutions, despite the existence of a costless resolution technology. Using commitment to solve the problem – through bail-in, CoCos, or an independent resolution authority – simply disregards the problem. This is not to say that commitment mechanisms are unimportant but that there are important factors in financial regulation orthogonal to commitment that can help mitigate TBTF.

Country	Impact on gross public debt	Recovery to date	Impact on gross public debt, post-recovery
Belgium	7.5	3.2	4.3
Cyprus	10.9	0.0	10.9
Germany	12.5	1.9	10.5
Greece	30.9	6.8	24.1
Ireland	40.1	6.9	33.2
Netherlands	18.7	14.2	4.5
Slovenia	12.0	0.0	12.0
Spain	7.7	3.1	4.6
United Kingdom	10.3	2.1	8.3
United States	4.5	4.8	-0.3
Average	7.4	4.3	3.0

Table 5.1: This table shows that most countries have not recouped the cost of bailing out their financial sectors. This calculation does not take into account what may have happened had these interventions not taken place. Examining the recovery rates shows that the US did remarkably well; they have recovered more than 100% of the cost of the bailouts. Europe, however, has not been as successful.

Source: Table reproduced from IMF World Economic and Financial Surveys – Fiscal Monitor April 2014. Public Expenditure Reform: Making Difficult Choices, Table 1.6 (page 9). Units are percent of 2013 GDP.

A note on terminology: I will use the following terms interchangeably.

- Bust and recession;
- Debt-holder, bond-holder, and depositor;
- Bank capital, bank equity, and equity-capital.

The chapter will be structured as follows. Firstly, Section 5.2 introduces the relevant literature before explaining how it relates to the model. Section 5.3, describes the mechanics of the model, before solving for the economic equilibrium in Section 5.4. Assuming the parents have uniformly distributed income yields an analytically tractable version the model as found in Section 5.5, which can be used to explain a normative distinction between bailouts and deposit insurance. The distribution of the parents' income has no direct effect on the economic outcome; however, it changes the political incentives faced by the children. Section 5.6 considers the case when the children have heterogeneous income and demonstrates that the demand for a bailout comes from the rich even if they are unwilling to transfer their own money to their own parents. Finally, Section 5.7 concludes with a discussion of the results.

5.2 Relevant literature

This section contains references to relevant areas of the economics literature that relate to this chapter. Firstly, I discuss the difference between the ‘institutional’ and ‘classical’ views on bank financing decisions. Then, I show how these perspectives relate to the literature on regulatory capture and public choice. I conclude with a short discussion on how this chapter fits into the literature, inasmuch as it gives an alternative view on the function of bank bailouts and the inadequacy of many equity-capital regulatory minimums.

Perhaps the most important paper relating to the choice of debt and equity financing is Modigliani and Miller (1958) – hereafter MM – the well-known invariance proposition states that a firm’s value is independent of its capital structure. Moreover, the celebrated result shows that a firm’s weighted average cost of capital (WACC) is, under certain conditions, independent of its financing regime. This is not to say that ‘nothing matters’ in the selection of the debt-equity ratio; the result is more an exposition of *what* does not matter and, by extension, what factors can affect the WACC. See Miller (1988) for a discussion of this misunderstanding.

Regarding the microeconomics of banking, or ‘institutional’ view, much of the literature examines why MM does not hold for banks.ⁱⁱ If MM is a reasonable approximation of the banking sector, raising capital requirements would not be costly to either the bank or society. Miles et al. (2013), which estimates the cost of raising bank equity-capital requirements in the UK, notes that many of the private costs associated with raising equity-capital requirements involve the banking sector paying more in taxes due to the preferential tax treatment of debt. They conclude that these private costs can be largely ignored from a social welfare perspective, since the taxes paid by the bank are offset by the social benefit of government revenue, which is presumably used to provide public goods.⁶ Consequently, we might want to focus on why raising bank equity-capital requirements could be socially costly.

There are three commonly cited factors that suggest banks are different from ‘normal’ firms and imply that raising equity-capital requirements could be socially costly: (i) short-term debt might be an important tool used to discipline bank managers, (ii) banks’ specific eco-

ⁱⁱFor a comprehensive treatment of this subject, see Freixas and Rochet (1997).

conomic function is to provide highly liquid assets ('informationally insensitive financial assets'), and (iii) increasing capital requirements can worsen market externalities.

Point (i) relates to the principal-agent problem between the bank's shareholders and managers. The owners of the bank have a costly monitoring technology; however, the lenders of short-term debt effectively monitor the actions of the bank's managers. By forcing the managers to borrow from the short-term debt market, the owners know that the manager's behaviour will be policed. This idea is associated largely with the work of Jensen and Meckling (1979) and Myers (1977). This idea is also discussed in French et al. (2010)⁷ as one reason why increasing bank equity-capital requirements could be socially costly.ⁱⁱⁱ

Point (ii) relates to informationally insensitive financial instruments, as discussed in Dang et al. (2013); this notion was also popularized by Gorton and Ordoñez (2012).⁸ The general argument is that privately produced short-term securitized debt (liquid assets, such as demand deposits) is only efficient if agents do not need to acquire new information continually about the quality of the underlying assets or counter-parties. This is because acquiring new information is privately costly but socially beneficial. In a credit boom, banks and firms will borrow in such a way so that lenders do not seek out new information on the quality of their deposits; over time, all qualities of collateral become close substitutes. In such a state, consumption and output rise but so does the fragility of the economy. At some point, there is a small shock that causes the lenders to update their information, which results in a credit crunch. A socially optimal outcome requires balancing the cost of fragility against the cost of information acquisition.

Both of these arguments imply that short-term debt funding is a necessary, although potentially imperfect, tool that allows banks to be run more efficiently. This perspective, however, has been challenged by, among others, Admati and Hellwig (2013). Admati and Hellwig contends that points (i) and (ii) are mutually exclusive. This is because point (i) assumes that short-term debt-holders are constantly updating their priors on the solvency of financial institutions. This, argues Admati and Hellwig, contradicts the view that such liquid assets are 'informationally insensitive'. Admati and Hellwig also contends that these arguments are either incorrect or of little consequence to the equity-capital regulation debate. The ubiqui-

ⁱⁱⁱSee Rey and Stiglitz (1993) and Calomiris et al. (1991).

tous use of short-term debt for funding causes a major conflict between the bank owners and taxpayers, which dominates any internal principal-agent considerations. Regulators should focus more on reducing the moral-hazard problem than on allowing for bank owners to increase leverage. Similar arguments have been made in many papers and books.⁹

Point (iii), that capital adequacy requirements (especially when combined with mark-to-market regulations) can create self-amplifying spirals, has been of major interest when analysing the global financial crisis of 2007-08. Shin (2010) argues that there can be upward-sloping demand-reaction functions as a result of forcing financial institutions both to mark-to-market and to abide by a binding capital-adequacy ratio.¹⁰ Similar arguments are made in Adrian and Shin (2010). This literature has resulted in proposals to implement counter-cyclical capital-adequacy buffers. For instance, the Basel III framework includes a “discretionary counter-cyclical buffer” that allows national prudential regulators to add an additional 2.5 percentage points to capital buffers.¹¹ This literature does not imply that higher capital requirements are socially harmful *per se*, but suggests that the timing of an increase in capital ratios can have social welfare implications.

The traditional justifications for bank equity-capital minimums are well known and literally hundreds of papers have been written on the topic. The ‘classical’ view sees the primary function of bank capital as being able to absorb losses to assets, thereby insuring the bank creditors and aligning the incentives of the bank’s owners, which reduces moral hazard. This latter point could be described colloquially as ensuring that the owners have ‘skin in the game’.

The moral hazard associated with bank liabilities is often attributed to either the existence of deposit insurance schemes or (more recently) the concept of being ‘Too Big to Fail’. The former programme insures bank deposits (short-term liabilities) and has only relatively recently been instituted outside of the United States. The common economic justification for deposit insurance is Diamond and Dybvig (1983).¹² Solvent financial institutions are capable of facing socially harmful bank runs that destroy real wealth. Deposit insurance is intended to be a credible commitment to eliminate the incentive for depositors to run.

Until relatively recently, deposit insurance was almost exclusively an American policy. For much of their history, the Americans have had a notoriously fragile banking system partly

due to various idiosyncratic legal institutions (especially the prohibitions on branch banking). Despite implementing *national* deposit insurance only in 1933 through the Federal Deposit Insurance Corporation, the American states had previously instituted many deposit-protection schemes throughout the 19th century. Calomiris and White (1994) and Golembe (1960) discuss the history and evolution of deposit insurance in the US. For a survey on the use of deposit insurance schemes beyond the United States, see Demirgüç-Kunt and Kane (2002) and Beck et al. (2000).

Although deposit protection schemes are now *de rigueur* in the developed world, some cross-country evidence suggests that they might increase the probability of banking crises (Demirgüç-Kunt and Detragiache, 2002). Whether deposit insurance is socially beneficial in practice remains an unsettled question among economists (Demirgüç-Kunt and Huizinga, 2004). Despite this debate, there is no major disagreement that government-funded deposit insurance creates significant moral-hazard problems.

One solution to this moral-hazard problem has been to regulate minimum equity-capital requirements. This has two salutary effects: firstly, it reduces the probability and severity of an insolvency and, secondly, it reduces the incentive to invest in risky financial assets since the bank owners face more potential downsides to imprudent investments.

Kashyap et al. (2008) describes how this classical view justifies the existence of bank capital regulation according to four premises: (i) the need to protect society from losses in the financial sector, (ii) the need to align the incentives of the bank shareholders (i.e. having ‘skin in the game’), (iii) equity-capital requirements should be linked to the risk of bank assets, and (iv) banks must face market discipline if they are to continue operating. Dewatripont and Tirole (1994) also justifies the existence of government intervention because the underlying assets in the financial sector are high risk and the claims are held by dispersed and unsophisticated agents. They note that, since the widespread implementation of deposit insurance, the regulatory debate has necessarily shifted to equity-capital adequacy and control rights.^{iv}

There is a conflict between the institutional and classical views on equity-capital requirements for banks. The institutional view implies that high minimum requirements are socially costly

^{iv}See Dewatripont and Tirole (1994), pp. 37 & 112.

since they interfere with the bank's ability to balance internal incentives, while the classical view sees equity-capital as a necessary check on pervasive moral hazard in the financial sector.

The literature on regulatory capture, which stems from the seminal paper by Stigler (1971), is also relevant to this chapter. Broadly speaking, the danger of regulatory capture is that an industry might actively seek out regulation in order to construct barriers to entry. In other cases, an industry might usurp control of an existing regulatory body and use it to its own advantage. These ideas are often associated with agency theory and, what have been titled, the *Chicago School* and *Virginia Schools*. The former is associated with the work of Stigler, Becker, and Peltzman,^v while the latter is associated with Buchanan, Tullock, and Tollison.^{vi} Both of these schools focus on how interest groups affect the regulatory decision-making process, especially regarding cases in which there are no apparent market failures. The agency-theoretic approach, however, came in response to these two groups and is associated with the work of Laffont and Tirole (Laffont and Tirole, 1991); it combines the *Chicago* and *Virginia Schools* with the possibility of institutional design and organizational responses to genuine market failures.

A common thread through all the literature on regulatory capture is that a regulated industry (or interest group) can use public policy for its own benefit, typically by reducing competition. This can be successful because the interest group has concentrated interests and the costs are distributed across the rest of society.^{vii} Information asymmetries and the need for complex regulations, both prevalent in the financial sector, give incumbent firms more power to use the regulatory system to their advantage (Laffont, 1999). The relationship between regulatory capture and capital adequacy regulation is addressed in Hardy (2006). Hardy argues that, if incumbent banks successfully capture the regulator, they will *not* necessarily conspire to lower capital ratios. In fact, they will lobby for *higher* capital ratios to limit marginal banks, since equity financing is more costly to such institutions. There is, however, reason to remain sceptical of this conclusion: Hardy does not consider the subsidies to the financial sector through formal or implicit state-backed insurance to bank creditors. Such policies, along with the tax code, create incentives for banks to choose debt financing over equity.

^vSee Becker (1976), Peltzman (1976), Peltzman (1993), and Peltzman et al. (1989).

^{vi}See Buchanan et al. (1980) for a collection of relevant papers on rent-seeking.

^{vii}This is similar to the argument made by Olson (1971), which also describes the phenomenon of concentrated benefits and dispersed costs.

Figure 5.1 plots the book-equity to book-asset ratio (a crude metric of bank capital) that shows a secular decline in bank capital over the 19th and 20th centuries in the United States and Canada. Despite not being plotted here, the United Kingdom's financial sector shows a similar secular downward trend in book-equity ratios. Although this is not *prima facie* evidence against Hardy's model, it should give us pause: perhaps the banks (and their creditors) would instead conspire to reduce capital requirements in order to increase the contingent liability on the state.

This process was partly modelled by Akerlof and Romer (1993), which examines the moral hazard caused by implicit government insurance in the financial sector. In this model, which is used to explain the excessive losses during the S&L crisis, the owners of financial institutions exploit their lenders as a result of limited liability; the owners can then intentionally drive the institution into insolvency to capture private gains at the public's expense.¹³ Akerlof and Romer (1993) begins with the assumption that the government agrees to an inefficient contract to backstop the financial sector. This chapter seeks to explain *why* this might be the case. One interpretation is that policy 'mistakes' are not isolated incidents but endemic and that the financial sector will exploit them.

This chapter outlines a model in which elements of these three literatures combine to demonstrate that the equilibrium-regulated capital requirements will be insufficient to cover large shocks to the financial sector. Instead of the regulation being driven solely by the industry (as in the regulatory capture models), the voters willingly choose to have capital requirements that are insufficient to absorb future financial shocks. This is because they are able to pass some of the cost of the shock on to future generations; in doing so, they extract consumption from future generations and redistribute resources across generations. Some of the future voters will support backstopping the financial sector.

5.3 Model outline

This section discusses the main mechanics of the model. Firstly, Section 5.3.1 describes the preferences of both the parents and the children. Then 5.3.2 describes the mechanics of the bank. The model timeline is as follows.

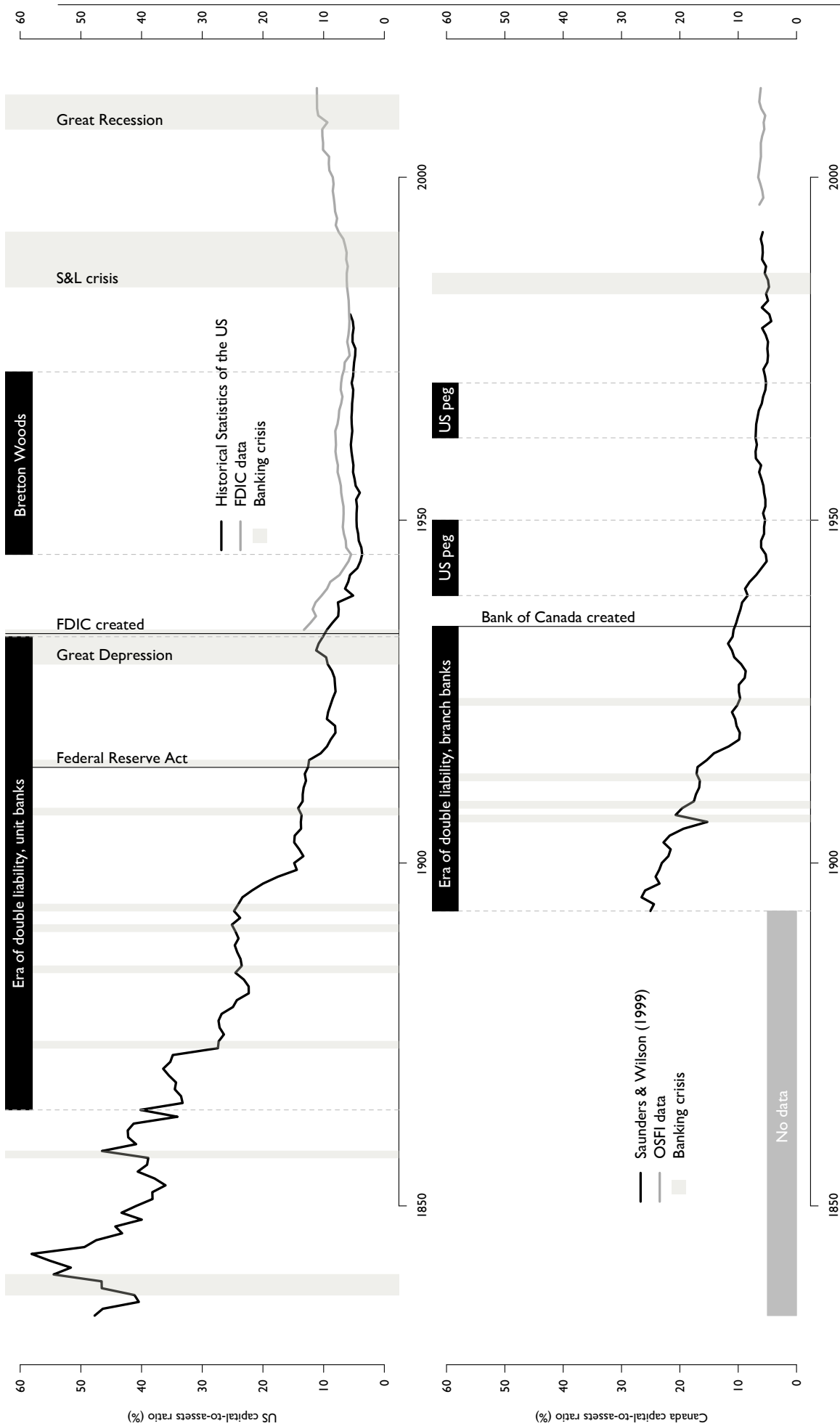


Figure 5.1: The banking crises are defined according to Reinhart and Rogoff (2009).¹⁴

Sources: *Historical Statistics of the United States*¹⁵, *FDIC Statistics on Banking Database*, *Office of the Superintendent of Financial Institutions (OSFI)*, *Summary Income Statements and BCAR Capital Adequacy Components*, and *Saunders and Wilson (1999)*.

1. Period 1:

- a) Parents choose their consumption and portfolio allocation: (c_1^i, e^i, d^i) .
- b) The competitive banks choose their balance sheet: (E_1, D_1) .
- c) These will jointly determine the interest rate offered on bank bonds: (R^d) .

2. Period 2:

- a) The state of the economy is revealed: $z = \{0, 1\}$ (boom or recession).
- b) This determines the banks' balance sheet: $(E_2, D_2, \tilde{R}^e, \tilde{R}^d)$.
- c) If recession ($z = 1$), the political choice of bailout policy is (τ, \bar{L}) .
- d) If boom ($z = 0$), the all contracts are honoured.
- e) Parents choose consumption: (c_2^i) .
- f) Children choose consumption: (c_K^i) .

Note that the children's consumption is denoted with a subscript K , which stands for 'kids'. The political-economic equilibrium, as defined by Tabellini (1989), must satisfy three points as follows.

- 1. Economic equilibrium: taking government policy as given, the economic choices of the agents are optimal and markets clear.
- 2. Political equilibrium: the policy chosen by the government is preferred by a majority of voters.
- 3. Rationality: the agents are temporally consistent inasmuch as their expectation of the future (conditional on the information they have) is correct.

Further, I will assume that recessions are rare: $0 < \alpha < 1/2$. Lastly, I will relax the Political Equilibrium assumption regarding majority voting mechanisms and examine possible super-majority rules. Section 5.5 will examine the case when parents have different incomes and demonstrates that this affects the electoral incentives of the children. Section 5.6 will examine the case when children have different incomes (and parents are all the same) and demonstrates that this can also have similar incentive effects.

5.3.1 Consumer preferences

There are two generations: parents and children. The parents are born in the first period and live for two periods; the children are born in the second period and only live one period. The game ends after the second period. Both generations have other-regarding preferences. That is, they receive some utility by observing their relatives' consumption. This utility, however, is small compared to that related to their own consumption. There is a single consumption good and no money.

In the first period of the game, only the parents are alive. They receive an endowment that they can either consume or save for retirement. The banks simultaneously choose how to finance their operations in order to maximize the expected return on equity. The banks make investments (risky loans to be repaid with gross interest R) and finance this by issuing bonds (debt that promises to pay a gross return R^d) and equity (a claim on the bank's residual assets after the debt has been repaid). Unlike banks, parents cannot invest directly in risky financial assets; they can, however, purchase bank bonds and bank equity.

In the second period of the game, the state of the economy is revealed: it is either a boom (probability $1 - \alpha$) or a recession (probability α). In a boom, all the loans made by the banks are repaid; in a recession, p proportion of the loans defaults. If a bank's assets are insufficient to pay the debtors, it is insolvent. In this case, the equity-holders are wiped out and the bond-holders *should* receive the residual value of the bank's balance sheet. Instead of executing the contracts, however, a political decision is made about whether a lump-sum tax should be levied on the children in order to compensate the bond-holders who lost money in the crisis.

The parent's objective function is quasi-linear:

$$W_i^P = c_1^i + E[u(c_2^i) + \delta V(c_K^i)] \quad (5.1)$$

where $u(c^i)$, the Bernoulli function, is the utility of the i^{th} parent's consumption in the second period. This is an unconventional utility function, however, it provides a tractable functional form that allows for the separation of risk and time-preferences. That is, the parents' portfolio selection problem is independent of their consumption problem. This same effect could be achieved using Epstein-Zin preferences, however, the voting behaviour in subsequent sections

would not be as tractable. A quasi-linear form allows for a clear exposition of the main results. The parents receive utility from observing their children's utility from consumption ($\delta V(c_K^i)$), where $\delta \in (0, 1)$ is a weighting parameter. $E(\cdot)$ is the expectation operator conditional on information available in the first period; the uncertainty in Period 2 is surrounding the parent's investment income. For simplicity, there is no discounting. The parent has the first-period constraints:

$$c_1^i + e^i + d^i \leq w^i \quad (5.2)$$

$$e^i, d^i \geq 0 \quad (5.3)$$

This budget constraint states that consumption and savings ($c_1^i + e^i + d^i$) in the first period must be less than or equal to income (w^i). Individuals have two options when saving: they can invest in a bank by purchasing either equity (e^i) or debt (d^i). They can issue neither their own bank bonds ($d^i \geq 0$) nor short stock ($e^i \geq 0$); agents cannot borrow, but this is not an issue since there will be no demand for borrowing in equilibrium. The parents' first-period income (w^i) is drawn from a distribution $F(\cdot)$, bounded on $[\underline{w}, \bar{w}]$. In the second period, the parent's budget constraint is:

$$c_2^i \leq \tilde{R}^e e^i + \tilde{R}^d d^i \quad (5.4)$$

This constraint states that the parent's second period income consists only of their *ex post* gross returns from equity ($\tilde{R}^e e^i$) and bank debt ($\tilde{R}^d d^i$); these will be defined more rigorously later.^{viii} In this model, the parents receive no endowment income in the second period so must allocate their endowment from the first period in order to maximize their expected utility. In the second period, income can be consumed (c_2^i). As a result of the stochastic defaults on bank assets, the gross returns on debt (\tilde{R}^d) and equity (\tilde{R}^e) are random variables; their distribution will be a function of how the defaults are distributed.

The children of the i^{th} parent has utility:

$$W_i^K = V(c_K^i) + \gamma u(c_2^i) \quad (5.5)$$

Where $V(c_K^i)$ is their direct utility of consumption and $\gamma \in (0, 1)$ is the weight of their parents' consumption utility in the second period. The children are born in the second period, live

^{viii}Note that the *ex post* return on debt is different from the *promised* return on debt, which is denoted R^d (with no tilde). In general $\tilde{R}^d \leq R^d$.

for a single period, and have a budget constraint:

$$c_K^i \leq w_K^i - z\tau \quad (5.6)$$

This states that the children's income net of taxes can be consumed (c_K^i) or gifted to their parents. They receive gross income (w_K^i) that is drawn from distribution $G(\cdot)$ and pays a lump-sum tax in a given state of the economy; z is a dummy variable, which is equal to 1 if there is a recession and 0 if not. The tax goes to 'recapitalizing' a failed bank (*i.e.* bailing out the creditors). The game ends after the second period, so all agents will optimally consume everything in the last period.

5.3.2 The bank

There is a competitive bank industry in which each bank seeks to maximize the expected return to its owners. It's worth noting that, if we consider the case in which there are no bailouts in the second period, the bank's leverage is *irrelevant* to the economic outcome of the model. Debt and equity are only names for various real claims on the bank's assets; their price will change such that the equilibrium intertemporal allocation is independent of the institution's financial structure.^{ix} Later sections will show that the political effect of increasing leverage is to create a constituency in favour of state support of the financial sector; the economic effect is to facilitate a transfer of resources between and across generations.

The bank invests its funds in some real technology and finances this through debt and equity contracts.¹⁶ It promises to pay debt-holders R^d and simultaneously holds equity, which can absorb losses. Think of the bank as a farm that receives corn from the parents in exchange for either a debt or an equity contract. The bank then plants the corn, which returns either a bountiful harvest (return R with probability $1 - \alpha$) or meagre harvest (return $(1 - p)R$ with probability α). The expected return on the entire portfolio of assets is $(1 - \alpha p)R$. Note, however, that this is not in general equal to the *ex ante* expectation of the return on the bank's liabilities; if agents expect a bailout in a recession, the expected value of the bailout will increase the value of the claims on the bank. Before examining the bank's behaviour in more detail, it is important to clarify several terms.

^{ix}This can be seen simply as an application of the Modigliani-Miller theorem. For a demonstration of this result, see Appendix C.3.3.

Definition 5.1. A bank is **well capitalized** when it has sufficient capital to absorb losses in a recession. It is **undercapitalized** when creditors (without government intervention) would face losses in a recession.

Even if a bank is undercapitalized, creditors may face no losses due to government bailouts. An equivalent phrasing is that a bank is undercapitalized if, in the recession, it is insolvent.

Definition 5.2. A bank's debt is **safe** if, given government policy, the bank will repay the full face value of the bond in the second period. Conversely, if creditors will face losses in a recession, the bank is **unsafe**.

Being well capitalized implies that a bank is safe; the converse is not true. For instance, if a full bailout is guaranteed, the bank is safe but not necessarily well capitalized.

The returns on equity and debt depend on how the bank is financed. If the bank has sufficient capital, the debt is riskless. If, however, the bank is undercapitalized, the debtors could face losses in a recession (although not necessarily). Much of the return on bank equity is driven by the spread between bank borrowing and lending costs: $R - R^d$. This wedge is driven by two interacting aspects of the model: (i) there is some chance that assets do not fully repay and (ii) the losses are first absorbed by the equity-holders. In equilibrium, equity-holders insure bond-holders against losses and, consequently, receive a higher expected return as compensation.

5.3.2.1 The bank's balance sheet

In the first period, the bank must finance its loans to consumers through some combination of debt and equity. Note that MM does not hold with this model as debt and equity are treated differently under insolvency: at the margin, increasing equity will not necessarily decrease the risk on bonds and can change the total value of the bank. The bank's balance sheet is given by the identity:

$$A_1 = E_1 + D_1 \tag{5.7}$$

The LHS is the assets held by the bank, where A_1 is the total quantity of assets in period 1. The RHS is the liabilities, where D_1 is the amount of debt it issues and E_1 is the amount of

capital. “Leverage” is defined as the ratio of assets to bank capital:^x

$$L_1 = (E_1 + D_1)/E_1 = A_1/E_1 \quad (5.8)$$

The value of the assets in the second period is:

$$A_2 = \begin{cases} (1-p)RA_1 & \text{with probability } \alpha \text{ (bust)} \\ RA_1 & \text{with probability } 1-\alpha \text{ (boom)} \end{cases} \quad (5.9)$$

In a recession, the bank loses pRA_1 in assets and must also repay depositors $R^d D_1$. Either the bank has sufficient capital to absorb this shock ($(1-p)R(E_1 + D_1) \geq R^d D_1$) or it does not. As mentioned above in Definition 5.1, I will refer to these two outcomes as a bank being either ‘well capitalized’ or ‘undercapitalized’ respectively. This capitalization condition can be rearranged to show that, as long as the bank has a sufficiently low leverage, it is well capitalized:

$$L_1 \equiv \frac{E_1 + D_1}{E_1} \leq \frac{R^d}{R^d - (1-p)R} \equiv L_{max} \quad (5.10)$$

Alternatively, this states that, as long as the gross return on assets in the worst-case scenario equals the promised return on bonds ($(1-p)R \geq R^d$), there will be sufficient assets remaining in the bad state regardless of how the bank is financed. The bank could finance its operations entirely using debt (infinite leverage) and still remain solvent as long as it promised a sufficiently low return. If, however, the bank promises a sufficiently high return on bonds ($(1-p)R < R^d$), it must have sufficient risk-absorbing capital in order to offer risk-free bonds. I will focus on the latter case, which is of more practical interest.

5.3.2.2 The return on equity

The equity-holders receive any residual value from the assets after the debt-holders have been paid but, as a result of limited liability, cannot lose more than their initial investment of E_1 .

^xIn reality, leverage can be much more complicated than this simple metric, especially if the bank is able to buy and sell even simple derivatives.

The *ex post* return on equity is a function of the bank's leverage and the state of the economy:

$$\tilde{R}^e = \begin{cases} & \text{Boom} & \text{Bust} \\ \text{If } L_1 \geq \bar{L} & R + (R - R^d) \frac{D_1}{E_1} & 0 \\ \text{If } L_1 < \bar{L} & R + (R - R^d) \frac{D_1}{E_1} & (1 - p)R + [(1 - p)R - R^d] \frac{D_1}{E_1} \end{cases} \quad (5.11)$$

This function can be written far more parsimoniously as:

$$\tilde{R}^e = \max\left(0, (1 - zp)R + [(1 - zp)R - R^d] \frac{D_1}{E_1}\right) \quad (5.12)$$

where z is a dummy variable for bust. The spread that the bank can charge ($R - R^d$) is not independent of the leverage of the institution. As the bank leverage increases, the risk to debt-holders also increases; it is only reasonable to conclude that they must be compensated for this additional risk, and eventually the spread must fall as leverage increases. At some point, the leverage is so high that there will be no remaining assets to cover the debt obligations in a bust. This is why, in a bust, either the return on equity is either zero – the equity-holders are wiped out – or there are some remaining assets.

5.3.2.3 The return on debt

The debt-holder's *ex post* gross return (ignoring any bailouts) is summarized in the following table.

$$\tilde{R}^d \equiv \frac{D_2}{D_1} = \begin{cases} & \text{Boom} & \text{Bust} \\ \text{If } L_1 > \bar{L} & R^d & (1 - p)R\left(1 + \frac{E_1}{D_1}\right) \\ \text{If } L_1 \leq \bar{L} & R^d & R^d \end{cases} \quad (5.13)$$

which can be rewritten as:

$$\tilde{R}^d = (1 - z)R^d + z \min\left(R^d, (1 - p)R\left(1 + \frac{E_1}{D_1}\right)\right) \quad (5.14)$$

where z is a dummy variable that denotes a bust. The debt-holders are promised a fixed return of R^d ; however, if the bank is undercapitalized and a recession occurs, they receive the residual value of the balance sheet plus any bailout (which is omitted from this function). If the bank is well capitalized, the bond is a safe asset that repays regardless the state of the economy. The bond-holders' losses are related to how the bank was financed in the first period: less equity implies larger losses. Ultimately, a bank that is entirely financed by debt imposes all the losses on debt-holders if it does not receive a bailout. In such a scenario, the bond-holders are equity-holders in all but name.

As stated above, when the bank is well capitalized the return on bonds is riskless: the bond-holder receives the promised gross return R^d with certainty. If, however, the bank is undercapitalized, the bond-holder receives the promised return when there is a boom but only the residual value of the assets (and possibly a bailout) in a bust, which might be less than the promised gross return R^d . The bailout of creditors occurs in the second period and will be contingent on the funding scheme adopted by the bank; the bailout policy is inferred by the parents in the first period.

5.3.2.4 The insolvency procedure

Under insolvency, the bank's debt is repaid sequentially; the last bond issued will be the first bond defaulted on. The distribution of bonds to the creditors is random. As such, if creditors happened to be holding debt on which losses must be imposed, they would lose a fixed proportion of their bonds (due to the law of large numbers) but the marginal value of purchasing an additional bond would be zero. In equilibrium, no creditor would purchase a bond that would be defaulted on since buying the marginal unit of debt is stochastically dominated by purchasing bank equity.

Using this procedure implies that the banks cannot, in equilibrium, issue debt beyond the leverage limit: no parent would agree to buy the marginal bond. Losses are proportional to debt holdings and the marginal value of a bond beyond the leverage limit is zero. Thus, the safe leverage limit condition will be the solution to the bank's optimization problem.

Proposition 5.1. *The bank, in equilibrium, will choose its debt and equity such that the bank's leverage is equal to the safe leverage limit.*

Proof. The expected return on bank equity is given by:

$$E(R^e) = (1 - \alpha)(R - R^d)\frac{D + E}{E} + R^d + (1 - p)\alpha R\frac{D + E}{E} - R^d\frac{D}{E} \quad (5.15)$$

Taking the partial derivative with respect to D :

$$\frac{\partial E(R^e)}{\partial D} = (1 - \alpha)(R - R^d)\frac{1}{E} + (1 - p)\alpha R\frac{1}{E} - R^d\frac{1}{E} \quad (5.16)$$

This is greater than zero as long as the expected return on bank assets is greater than the cost of financing:

$$(1 - \alpha p)R > R^d \quad (5.17)$$

The bank, therefore, will want to increase its leverage *up to* the leverage limit. The bank, however, cannot issue debt beyond this limit because no parent would purchase it. The marginal debt issuance pays R^d in the boom state and 0 in the bust; this is stochastically dominated by purchasing equity, which pays R^e in the boom and 0 in the bust. Therefore we conclude that the bank will increase their leverage up to the limit and would like to increase it beyond the limit; however, no parent would finance this expansion in equilibrium. ■

Perhaps, however, it might be more realistic if every creditor is paid out up to a maximum amount and loses anything above the threshold. Under that procedure, the rich would face disproportionately large losses, while the poor would completely recover their investments. One major issue with this alternative procedure is that the same bonds, if held by different people, would be worth different amounts. This disparate impact would result in a very strong arbitrage opportunity.

In fact, a similar incentive is generated by deposit insurance, in which amounts under the insurance threshold are fully backed while amounts above the threshold are at risk. In the United States, some financial companies have even designed products to take advantage of the differential treatment of deposits by creating a product called “Certificate of Deposit Account Registry Service” (CDARS). If a client wishes to deposit an amount larger than the FDIC-guaranteed minimum with a financial institution, they can purchase a CDARS. Their deposit is then broken up and, using a network of swap contracts, distributed through thousands of FDIC-insured institutions. The nominal exposure at each individual institution is below the insurance threshold, so the CDARS is able to convert a large uninsured sum

into a large insured sum. See Kane (2014) for further discussion of these practices. Using a proportional-loss rule eliminates such arbitrage opportunities.¹⁷

An implication from this model is that deposit insurance and bank bailouts are qualitatively different. It might be tempting to conclude that bailouts and deposit insurance are similar since they both result in transfers to creditors. This model demonstrates that the electoral coalition that supports a bailout is different from one that supports deposit insurance.

5.4 Economic equilibrium

This section considers the model when parents' income is heterogeneously distributed. In order to solve the economic equilibrium, I will first begin in Subsection 5.4.1 with the consumption decision in the second period of the model. Then, in Subsection 5.4.2, I will focus on the first-period consumption and portfolio-allocation decisions made by the parents. This requires solving for their behaviour conditional on the bailout policy adopted in the second period (for which is solved for in Section 5.5). Lastly, in Section 5.4.3, I will solve for the economic equilibrium. Appendix C.3.1 contains the full mathematical derivations of the solutions discussed in this section.

Before solving for the equilibrium behaviour of the model, it is important to make some simplifying assumptions.

Assumption 5.1. The parents' Bernoulli function is $u(\cdot) = \ln(\cdot)$.

Assumption 5.2. Parents and children cannot make direct transfers to one another.

This assumption will be relaxed in the discussion of the model. It serves to demonstrate that there are cases in which children will not want to transfer resources to their parents but will support a bailout. Similarly, there will be children who are against a bailout but would still like to transfer resources to their parents.

5.4.1 The Period 2 choices

Given the positive (but decreasing) marginal utility of consumption, both the parent and the child will simply consume all of their income in the second period.

Proposition 5.2. *In the second period, both parents and children will simply consume their endowment net of taxes, making no transfers to relatives. Formally:*

$$\begin{aligned} c_2^{i*} &= \tilde{R}^e e^i + \tilde{R}^d d^i \\ c_K^{i*} &= w_K - z\tau \end{aligned} \tag{5.18}$$

where \tilde{R}^e and \tilde{R}^d are the ex post realizations of the return on equity and debt respectively.

Note that the children's variables are denoted with subscript K (for 'k'ids).

5.4.2 The Period I choices

The only difference between parents in the first period is their endowment income; they all have no second-period income so will want to save at least some of their endowment. This section will examine the parents' decision when the bank offers an equity contract such that in a bust the equity returns zero (gross) and the debt is just honoured. That is, the leverage of the bank is L_{max} , the right-hand side of Equation (5.10).^{xi}

5.4.2.1 Behaviour under a safe bank

The parents will want to smooth their consumption by saving some of their endowment. They have two ways of doing this: investing in bank debt or investing in bank equity. An interior solution to the parents' first-period maximization must result in them being indifferent at the margin between their portfolio allocation of equity and debt, and consumption today. Given the quasi-linear utility function, an interior solution will hold only for parents with sufficiently high income. Without loss of generality, suppose that the bank's equity contract pays:^{xii}

$$\tilde{R}^e = \begin{cases} R^e & \text{if boom} \\ 0 & \text{if bust} \end{cases} \tag{5.19}$$

Given the bank is safe, the debt contract will always pay R^d independent of the state of the world. The optimal amount of consumption in the first period to a parent is:

$$c_1^*(w^i) = \max(0, w^i - 1) \tag{5.20}$$

^{xi}Considering this case alone will be sufficient because the equilibrium solution to any other level of leverage can be constructed by re-labelling a portion of the equity as debt. Regardless, I will show that this will be the equilibrium outcome given the insolvency procedure and the incentives of the bank managers.

^{xiii}Here, I am using R^e as a placeholder; from the previous section, $R^e \equiv R + (R - R^d) \frac{D_1}{E_1}$.

This result might appear unusual to those familiar with traditional OLG models; it follows from the assumption that parents have quasi-linear utility. The quasi-linear utility allows the separation of the consumption and portfolio problems, which is helpful in retaining tractability when solving for the political equilibrium. The solution in Equation (5.20) states that if parents have sufficient income ($w^i \geq 1$), they will consume some amount in the first period after setting aside 1 unit to be invested for the second period. If, however, they don't have sufficient income ($w^i < 1$), they will save everything for the second period. This is because the marginal utility of consumption in the second period is high for low levels of consumption. So all parents will save; the rich will save more than the poor but the very rich ($w^i \geq 1$) will have a marginal propensity to save of zero.

This might seem like an unrealistic outcome: the poorest parents consume nothing in the first period. Yet this need not be our interpretation: it could be the case that parents have other sources of consumption and we are only considering consumption over and above a minimum threshold, which could be modelled using a Stone-Geary utility framework. This would alter our interpretation: the poorest parents would be consuming the minimum amount of consumption in the first period and saving all excess income for their retirement. Rich parents, conversely, can afford to consume above the minimum threshold in Period 1. The equilibrium amount saved as bank debt is:

$$d^*(w^i) = \min\left(\frac{\alpha R^e}{R^e - R^d} w^i, \frac{\alpha R^e}{R^e - R^d}\right) \quad (5.21)$$

All parents choose to invest a fixed proportion of their savings in debt regardless of their income level. That is, bank debt is the same fixed proportion of each parent's portfolio. This follows from the assumption of a CRRA Bernoulli function in the utility, of which log-utility is a special case. Again, note that poor parents ($w^i < 1$) are savings constrained and split their first-period income between equity and debt; they consume nothing. This unrealistic implication can be relaxed by departing from quasi-linear utility. This functional form, however, greatly simplifies the analysis since it implies the aggregate savings are inelastic. Introducing a more complicated utility function would allow for the aggregate savings to be elastic but would not greatly change the analysis. The proportion allocated to debt is increasing in the return on debt (R^d) and decreasing in the return on equity (R^e). The amount invested in

equity, as a function of income, is:

$$e^*(w^i) = \min\left(\frac{(1-\alpha)R^e - R^d}{R^e - R^d}w^i, \frac{(1-\alpha)R^e - R^d}{R^e - R^d}\right) \quad (5.22)$$

Again, as a result of CRRA preferences, parents allocate a fixed proportion of their savings to bank equity. The amount allocated to equity is increasing in the return on equity (R^e) and decreasing in the return on debt (R^d). A sufficient and necessary condition for the parent to hold bank equity is that the expected return on equity must be greater than that of bank debt. A keen observer might note:

$$e^*(w^i) + d^*(w^i) = \min(w^i, 1) \quad (5.23)$$

This confirms the result that, as income increases, parents will weakly increase their savings. An implication of the quasi-linear, time-separable preferences is that consumption in the first period is a luxury good; conversely, saving is a necessity. In short, poor parents are savings constrained while rich parents allocate any marginal income to consumption. The breakdown of consumption and savings as a function of income (*i.e.* $w^i = e^*(w^i) + d^*(w^i) + c_1^*(w^i)$) is shown in Figure 5.2.

Proposition 5.3. *The parents' first-period behaviour, as a function of income, can be characterized by the functions:*

$$c_1^*(w^i) = (5.20)$$

$$d^*(w^i) = (5.21) \quad (5.24)$$

$$e^*(w^i) = (5.22)$$

Proof. See Appendix C.3.1. ■

One might wonder about what would happen if (i) the bank equity retained some value in the bust state or (ii) the bank's debts were not safe. For (i), the bank would choose to issue more debt in order to increase the expected return on bank equity. For (ii), the issue can be discounted in a somewhat more subtle manner: at the margin, parents will never choose to buy debt from a bank beyond the safe level of leverage. This is because the additional debt will be defaulted on in the bust. Consequently, purchasing equity would dominate purchasing debt beyond the safe level. It follows that the demand for bank debt (in equilibrium) must be zero beyond the point at which deposits are safe.

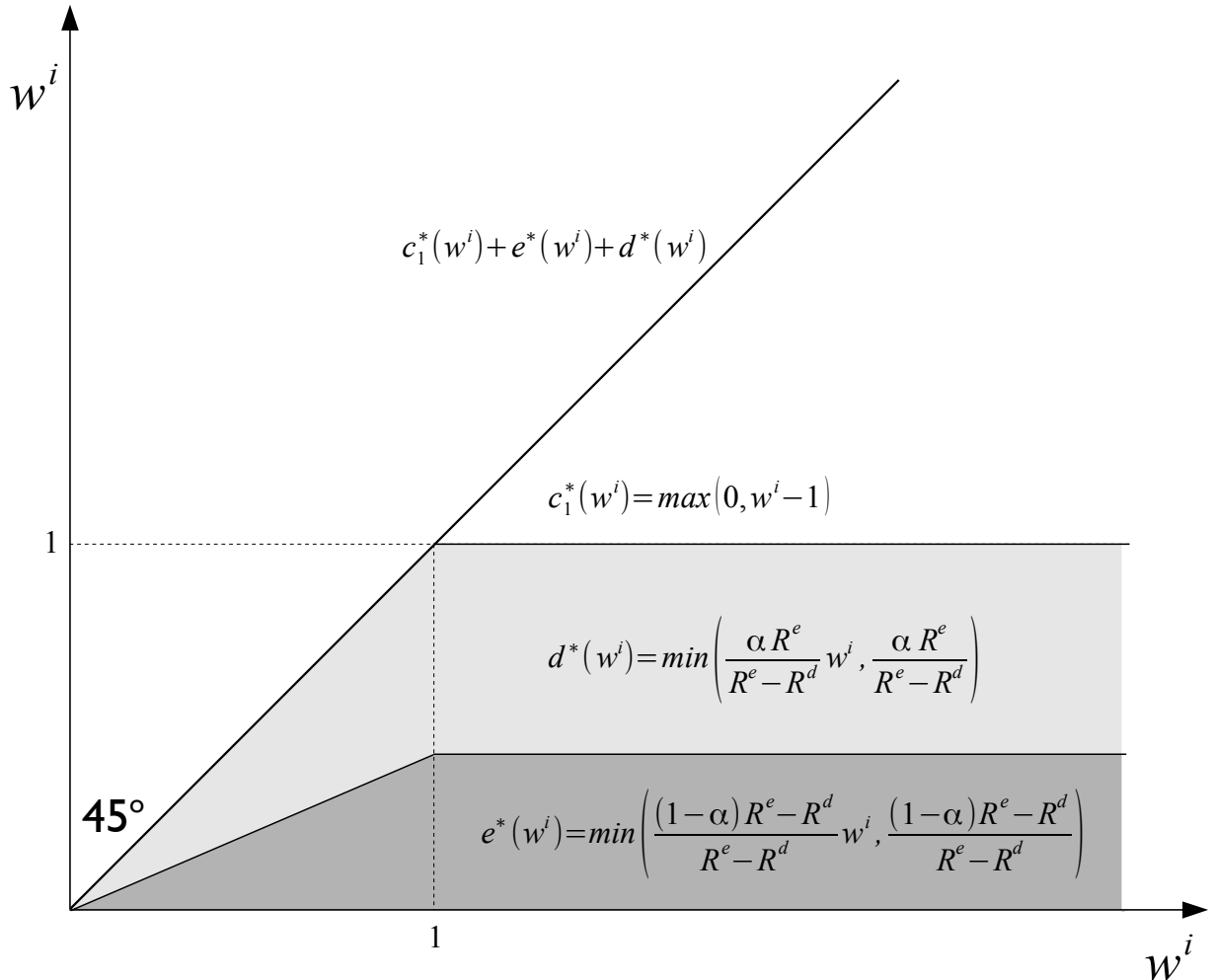


Figure 5.2: This figure shows the allocation of the parent's first-period income as a function of their income ($w^i = e^*(w^i) + d^*(w^i) + c_1^*(w^i)$). It shows that, for low levels of income ($w^i < 1$), all income is saved; the savings are allocated to both debt and equity in fixed proportions. Above the threshold ($w^i > 1$) the marginal utility of consumption is strictly greater than that of saving and so all marginal increases in income go to consumption. In short, the parent with sufficient income will always allocate up to 1 unit of income to saving (regardless of the prices) and consume the rest. This sort of extreme behaviour is a direct consequence of assuming time-separable, quasi-linear log-preferences. That is, the Bernoulli function exhibits CRRA.

5.4.2.2 The supply of leverage

Given the utility function uses a special case of CRRA risk-preferences, parents purchase the financial assets in fixed proportions regardless of their income. This implies that the distribution of income for the parents is broadly irrelevant to the economic equilibrium of the model; however, it will be relevant to the political interpretation. Aggregating the demand for each asset is straightforward: it is the proportion invested in the asset multiplied by the average savings. Let E^* and D^* be the aggregate demands for equity and debt respectively. The average amount that a parent saves is ($0 < \underline{w} < 1 < \bar{w}$):

$$E^* + D^* = S \equiv \int_{\underline{w}}^{\bar{w}} e^*(w^i) + d^*(w^i) dF(w^i) = \int_{\underline{w}}^1 w^i dF(w^i) + [F(\bar{w}) - F(1)] \quad (5.25)$$

where S denotes the average amount saved by the parents. If we assume income is uniformly distributed on $[\underline{w}, \bar{w}]$, the amount saved is:

$$E^* + D^* = \frac{1 + 2(\bar{w} - 1) - \underline{w}^2}{2(\bar{w} - \underline{w})} \equiv S \quad (5.26)$$

Every parent, regardless of income, chooses to hold debt and equity in the same proportions. The aggregate leverage $((E^* + D^*)/E^*)$, however, is solely a function of the individual parent's preference, as if there was a representative agent. The average savings in the numerator is simply cancelled by the aggregate savings in the denominator:

$$\frac{E^* + D^*}{E^*} = \frac{e^*(w^i) + d^*(w^i)}{e^*(w^i)} = \frac{1}{e^*(w^i)} \quad \forall w^i \in [\underline{w}, \bar{w}] \quad (5.27)$$

From Section 5.3.2, regarding the mechanics of the bank, the return on equity conditional on a boom is:

$$R^e \equiv R + (R - R^d) \frac{D_1}{E_1} = (R - R^d) L_1 + R^d \quad (5.28)$$

Since parents demand assets in fixed proportions, the supply of leverage to the bank can be expressed as a function of the gross return on debt and model parameters.

Lemma 5.4. *The aggregate leverage supplied by the parents to a bank offering safe debt is:*

$$L_S(R^d; \alpha, R) = \frac{R - (1 - \alpha)R^d}{(1 - \alpha)(R - R^d)} \quad (5.29)$$

Proof. The first part of the proposition can be demonstrated by substituting Equations (5.21)

and (5.22) into Equation (5.27) and replacing the placeholder R^e with (5.28). The last step requires solving for the internally consistent supply of leverage. ■

The supply of leverage by the parents to the bank is increasing in the gross return on debt (R^d) and decreasing in the exogenous return on the bank's assets (R). This should be rather intuitive: holding everything else equal, a larger return to the bank's assets increases the return on equity; thus the parents would prefer to alter their portfolio in that direction. The amount of leverage parents offer is increasing in the risk of a bust (α). This is because we are assuming that the bank is offering safe deposits; increasing the probability of a bust then only decreases the expected return on equity. Given the insolvency procedure (discussed in Section 5.3.2.4), issuing an additional unit of debt beyond the safe level implies that the specific bond will not pay in a bust. Consequently, the equilibrium leverage is pinned down by safe level of leverage, which is determined by the political process.

5.4.3 Economic equilibrium

The previous sections determined the consumption decisions and both the supply of leverage from the parents and the demand for leverage from the banks. In the first period, the parents' consumption and savings decisions are defined according to Proposition 5.3. The market for bank debt must also clear; the supply of leverage from the parents must equal the demand for leverage from the banks. Respectively, these are:

$$L_S = \frac{R - (1 - \alpha)R^d}{(1 - \alpha)(R - R^d)} \quad (5.30)$$

and

$$L_D = \bar{L}(\tau) \quad (5.31)$$

The level of leverage at which the bank becomes unsafe (\bar{L}) is a function of the political equilibrium, which is discussed in Section 5.5. The contracted return on debt (R^d) equates these two conditions so that the market for bank debt clears. The equilibrium return on debt as a function of the safe leverage level can be determined by inverting Equation (5.30) and substituting in Equation (5.31):

$$R^{d*}(\bar{L}(\tau)) = \frac{(1 - (1 - \alpha)\bar{L}(\tau))R}{(1 - \alpha)(1 - \bar{L}(\tau))} \quad (5.32)$$

This equation demonstrates that, in order to sustain higher levels of leverage, the return paid on debt must rise. For higher leverage, the amount of bank equity is lower but the debt remains safe because of the anticipated bailout. In equilibrium, the agents must be indifferent between holding debt and equity so, if the amount of safe debt increases, *ceteris paribus*, the return on equity will also increase.

In the second period, the state of the economy is revealed. If it is a boom, then all the agents fully consume their endowments. In a bust, however, the political equilibrium will determine the safe level of leverage and bail out any debt below this level. Once the contracts and bailouts have been resolved, the agents all consume their income.

5.4.3.1 The interest rate as a function of debt

In equilibrium, the total amount invested in bank equity will be:

$$E_1^* = e^* S = \frac{(1 - \alpha)(R - R^d)}{R - (1 - \alpha)R^d} S \quad (5.33)$$

The total invested in bank debt is:

$$D_1^* = d^* S = \frac{\alpha R}{R - (1 - \alpha)R^d} S \quad (5.34)$$

where S is the average savings per person in the economy, as defined in Equation (5.25). The equilibrium interest rate is a function of the debt:

$$R^{d*}(D) = \frac{(D - \alpha S)R}{(1 - \alpha)D} = \frac{(d - \alpha)R}{(1 - \alpha)d} \quad (5.35)$$

Note that α , R , and S are exogenous parameters. This equation must hold in equilibrium and is central in the discussion on the political equilibrium (Section 5.5).

The return on debt (R^d) is the price that equilibrates the market for saving. The market clearing condition is:

$$E_1 + D_1 = S \quad (5.36)$$

This follows from the parents' quasi-linear utility; their aggregate savings are completely inelastic. The promise of a bailout will have an effect only on the allocation within the

agents' portfolios. This might be somewhat unrealistic since future bailouts have no effect on parents' decision regarding *how much* to save. This abstraction, however, greatly increases the tractability of the model. If there will be a bailout in the second period, the parents will simply relabel their claims on the bank's assets in order to take advantage of this.

5.4.3.2 Illustration: commitment to no bailouts

Suppose the government can commit to *not* bailing out the banks. In this case, the safe level of leverage is:

$$\bar{L}_{FM} = \frac{R^d}{R^d - (1-p)R} \quad (5.37)$$

The subscript *FM* denotes that this is the outcome that would occur in the 'free market'. The supply of leverage remains unchanged as:

$$L_S = \frac{R - (1-\alpha)R^d}{(1-\alpha)(R - R^d)} \quad (5.38)$$

Using these two equations yields the equilibrium allocation as a function of the exogenous parameters: the probability of a recession (α), the effect of a recession on the bank's balance sheet (p), and the return on investment conditional on a boom (R). Equating these two equations yields the equilibrium safe-interest rate (R^{d*}) and leverage (L^*):

$$\begin{aligned} R_{FM}^{d*} &= \frac{(1-p)R}{1 - (1-\alpha)p} \\ L_{FM}^* &= \frac{1}{(1-\alpha)p} \end{aligned} \quad (5.39)$$

This solution is illustrated in Figure 5.3. The equilibrium interest rate is increasing in the return on the bank's assets. The leverage, however, is independent of the return on the bank's balance sheet and is solely a function of the probability and magnitude of a bust (α and p respectively). The return on equity in the boom state is:

$$R_{FM}^{e*} = \frac{R}{1-\alpha} \quad (5.40)$$

The equilibrium aggregate levels of debt and equity are:

$$\begin{aligned} D_{FM}^* &= (1 - (1-\alpha)p) S \\ E_{FM}^* &= (1-\alpha)p S \end{aligned} \quad (5.41)$$

Where S is the aggregate level of savings from Equation (5.25). The parents' state-contingent consumption is:

$$c_2^i = \begin{cases} \min(Rw^i, R) & \text{if boom} \\ \min((1-p)Rw^i, (1-p)R) & \text{if bust} \end{cases} \quad (5.42)$$

The consumption in either state of the economy is not a function of the probability of a bust, only of the amount of assets remaining in the bank. This should not be entirely surprising since, when there is no bailout and the parents have CRRA utility, in equilibrium they each hold a small version of the bank's total balance sheet. In the bust state, the average parent would receive $(1-p)RS$. The richest parents ($w^i > 1$) would receive $(1-p)R$ and the poorest parent would get $(1-p)R\underline{w}$.

5.4.4 Investment and bailouts

Suppose that the parents expect the future generation to contribute a lump sum tax of τ to a bailout in the bust state.^{xiii} Given that the investment decision must be consistent with the expected bailout, the proportion of savings that is allocated to bank equity and debt is, for $\tau \in [0, RS]$:

$$\begin{aligned} E^*(\tau) &= (1-\alpha) \left(pS - \frac{\tau}{R} \right) = E_{FM}^* - (1-\alpha) \frac{\tau}{R} \\ D^*(\tau) &= S - (1-\alpha) \left(pS - \frac{\tau}{R} \right) = D_{FM}^* + (1-\alpha) \frac{\tau}{R} \end{aligned} \quad (5.43)$$

This shows that the proportion of savings allocated to bank equity-capital is negatively affected by a bank bailout. This is because, knowing a bailout is forthcoming, the parents will be willing to raise their portfolio allocation of debt; the banks respond by financing their operations with more debt, which increases their expected return on equity at the margin. In order to equilibrate the market for debt and equity, the promised return on debt must also increase as the bailout τ increases. The promised bailout simply causes the parents to relabel their savings (which were equity, under the free market outcome) as debt. This follows from the fact that the total amount of savings is completely inelastic. If the government commits to not bailing out the financial sector ($\tau = 0$), it implies the result from Equation (5.41). If, however, the government backstops the maximum loss ($\tau = pRS$), the banks will have no equity. The return on debt is:

$$R^d(\tau) = \frac{(1-p)SR + \tau}{S - (1-\alpha) \left(pS - \frac{\tau}{R} \right)} \quad (5.44)$$

^{xiii}Since the mass of children is one, the average tax and total tax are τ as well.

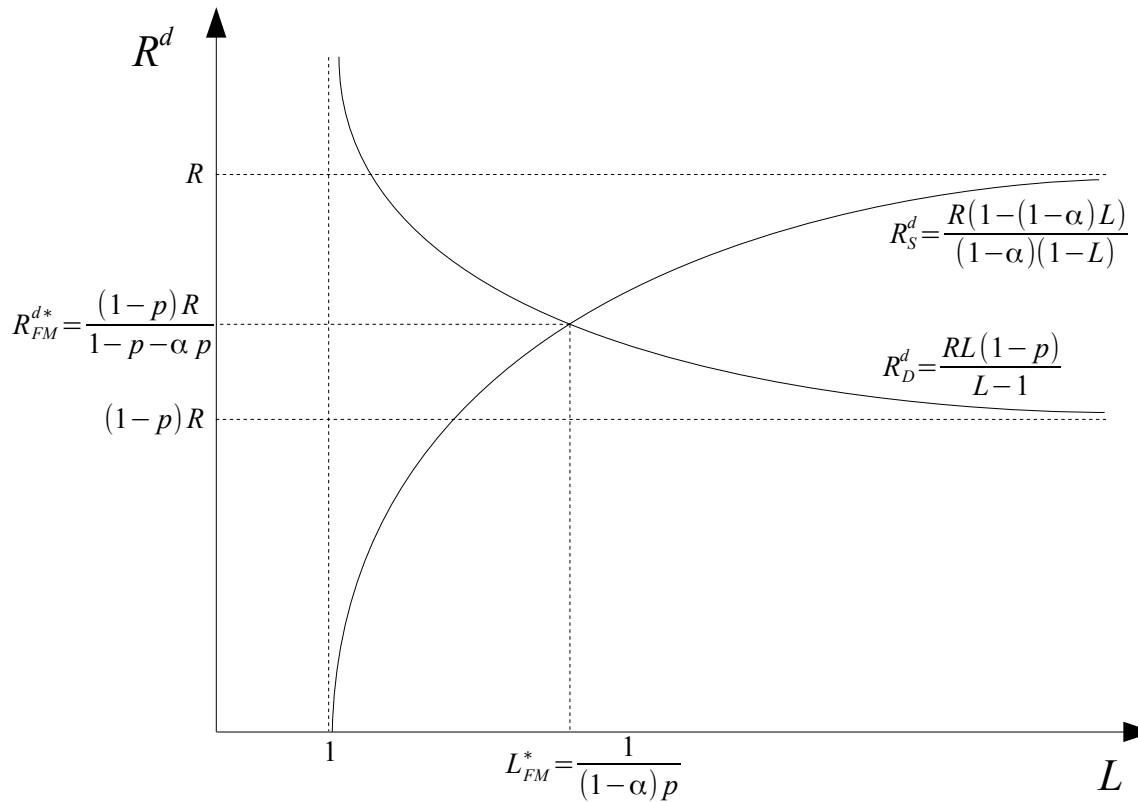


Figure 5.3: This figure demonstrates the equilibrium leverage and interest rate when the government does *not* intervene in the second period. The downward-sloping curve (R_D^d) is the maximum amount of leverage that the bank can have while still offering riskless deposits. For lower levels of interest the bank can sustain more leverage without adding any risk. The parents, however, are willing to hold debt and equity in only fixed proportions and so there is a market supply of leverage (denoted R_S^d). The parents' willingness to hold more debt in their portfolio is positively related to the interest paid on debt. This results in the upward-sloping leverage supply curve. Note that both curves have a vertical asymptote at $L = 1$; this should be intuitive as leverage is undefined below this level. Further, both functions have horizontal asymptotes that, with the continuity of the functions, guarantee a solution. The equilibrium outcome is denoted by the return R_{FM}^{d*} and the leverage L_{FM}^* .

The numerator of this equation shows the bank's remaining assets plus the bailout; the denominator is the notional liability from the first period; the ratio is the gross return. When the government promises a full bailout ($\tau = pRS$), the return on debt is the same as the underlying assets in the boom state (R). The return promised on debt is unambiguously increasing in the size of the expected bailout.

$$\frac{dR^d}{d\tau} = \frac{\alpha SR^2}{((1-p+\alpha p)RS + (1-\alpha)\tau)^2} > 0 \quad (5.45)$$

In contrast, the return on equity is:

$$\begin{aligned} R^e(\tau) &= (R - R^d(\tau)) \frac{D^*(\tau)}{E^*(\tau)} + R \\ &= \frac{R}{1-\alpha} \end{aligned} \quad (5.46)$$

This implies a counterintuitive result: the return on equity is *independent* of the bailout policy adopted by the government. Consider the decision made by a well-capitalized bank if the government announces bailout policy τ . If the well-capitalized bank chooses *not* to increase leverage, its return on equity would decrease as the market price of debt decreases (*i.e.* as the yield on debt increases). This is because the other banks increase their leverage, which depresses the price of safe debt. In order to keep its return on equity high, the well-capitalized bank will follow suit and increase its leverage. From an individual bank's perspective, increasing leverage increases that bank's return on equity. This, however, is similar to the prisoner's dilemma: each individual bank has the same incentive, so the aggregate amount of debt traded in the market increases, which depresses the price of debt. No individual bank has any effect on the price; their aggregate actions, however, do. The increased return on debt exactly cancels the effect of the increased leverage. In equilibrium, the return on equity will be independent of the bailout policy. This implies that the effect of the bailout occurs through changes in the price of debt and the balance sheet of the bank. In this model, the bailout policy does not directly affect the return on equity; the risk premium over bank debt, however, is reduced. This yields the following result.

Proposition 5.5. *When the government adopts the bailout policy $\tau \in [0, pRS]$ in Period 2, the bank's balance sheet in equilibrium will be:*

$$\begin{aligned} E^*(\tau) &= (1 - \alpha) \left(pS - \frac{\tau}{R} \right) \\ D^*(\tau) &= S - (1 - \alpha) \left(pS - \frac{\tau}{R} \right) \end{aligned} \quad (5.47)$$

The promised return on debt is:

$$R^d(\tau) = \frac{(1 - p)SR + \tau}{S - (1 - \alpha) \left(pS - \frac{\tau}{R} \right)} \quad (5.48)$$

This is honoured even in the bust state. Finally, the return on equity is independent of the bailout policy:

$$R^e(\tau) = \frac{R}{1 - \alpha} \quad (5.49)$$

5.5 Political equilibrium: heterogeneous depositors

This section considers what happens when a bust occurs and the bank has insufficient capital to absorb the losses. The government must decide whether to bailout the banks and, if so, by how much. The policy (τ) is selected using a political process; a politically viable policy is such that at least 50% of the voting population supports the policy. The subsequent sections will demonstrate that all the voters have single-peaked preferences over the bailout policy τ so the results from Black et al. (1958) and Downs (1957) can be adopted: the public policy will be determined by the median voter.

In the second period, the representative bank's balance sheet $(E_1(\tau), D_1(\tau))$ and individual debt contracts $(d^i(\tau), R^d(\tau))$ are treated as exogenous since they are determined in the first period. They must, however, be consistent with the political equilibrium. Losses are imposed proportionately on all creditors (ie. *pari passu*).^{xiv} Although a bailout solution is not necessarily guaranteed, I will show that under certain parameterizations the politically viable set is non-empty. For certain parameterizations, the younger generation will be willing to bailout the old. Further, this is *not* because they simply wish to transfer more consumption to their own parents.

^{xiv}Note that although the losses are proportionate to the holding of debt, the losses are imposed entirely on the marginal bond held by the parents. This is described in further detail in Section 5.3.2.4.

In short, the political process will determine what values of $\tau \in [0, pRS]$ are politically viable; the bailout policy will then determine the equilibrium financing of the bank.

5.5.1 The parents' voting preferences

From above, we can re-write the parents' objective function in the second period as:

$$W_i^P = c_1^i + u(c_2^i) + \delta V(c_K) \quad (5.50)$$

An application of the Envelope Theorem gives us the effect of increasing the bailout. An increase in the bailout has two direct effects: firstly, it directly increases the parents' consumption in the second period (in proportion to their debt holding); secondly, it decreases their child's consumption. The marginal effect of a change in bailout policy is:

$$\frac{\partial W_i^P}{\partial \tau} = \frac{d^i}{D_1} u' \left([(1-p)RS + \tau] \frac{d^i}{D_1} \right) - \delta V'(w_k - \tau) \quad (5.51)$$

Increasing the bailout increases the tax on the child, which reduces the parents' welfare by the child's weighted marginal utility of income, but it also increases the parents' consumption by a factor of how many deposits they hold relative to the average. That is, if a parent has five times the average deposit, increasing the bailout tax by 1 unit results in 5 units being transferred to the parent while reducing the child's consumption by only 1. Assuming that parents have log-utility, this effect cancels with the diminishing marginal utility:

$$\begin{aligned} \frac{\partial W_i^P}{\partial \tau} &= \frac{d^i}{D_1} \left([(1-p)RS + \tau] \frac{d^i}{D_1} \right)^{-1} - \delta V'(w_k - \tau) \\ &= [(1-p)RS + \tau]^{-1} - \delta V'(w_k - \tau) \end{aligned} \quad (5.52)$$

Thus the parents' preference over the bailout policy (under log-preferences) is independent of their own holdings of bank debt. The differences in opinion across parents is a function of how the bailout policy affects their children, which is discussed in Section 5.6. The parents' optimal bailout policy is implicitly defined by:

$$\frac{1}{\delta} \frac{d^i}{D_1} u' \left([(1-p)RS + \tau_P^i] \frac{d^i}{D_1} \right) = V'(w_k - \tau_P^i) \quad (5.53)$$

This is graphed in Figure 5.4. Using log-utility, this simplifies to:

$$[(1-p)RS + \tau_P^i]^{-1} = \delta V'(w_k - \tau_P^i) \quad (5.54)$$

That is, the parents' preferences over the bailout are single-peaked around τ_P^i . If we further assume that the child has log-preferences over consumption, the optimal bailout for the parent is:

$$\tau_P^i = \frac{w_k - \delta(1-p)RS}{(1+\delta)} \quad (5.55)$$

The less weight a parent puts on their child's consumption (the smaller δ), the larger the desired bailout. This is because taxing the child does not affect the parent as much. Contrast a bailout with a direct transfer from the child; the marginal effect of a direct transfer t is:

$$\frac{\partial W_i^P}{\partial t} = u' \left((1-p)RS \frac{d^i}{D_1} + t \right) - \delta V'(w_k - t) \quad (5.56)$$

This implies that, if a parent has more-than-average deposits, they will prefer a bailout to a direct transfer since it will be less costly to their child. Conversely, parents with less-than-average deposits will prefer direct transfers from their children over bailouts. It is worth highlighting that if the parents do not care about the children ($\delta = 0$) then their preference will be to have a full bailout; this is a corner solution, in which $\tau_P^i = pRS$ (assuming the children have sufficient income).

5.5.2 The children's voting preferences

The children's preferences are similar to the parents'. Applying the Envelope Theorem, the marginal effect on a child's wellbeing of a change in the bailout is:

$$\frac{\partial W_i^k}{\partial \tau} = \gamma \frac{d^i}{D_1} u' \left([(1-p)RS + \tau] \frac{d^i}{D_1} \right) - V'(w_k - \tau) \quad (5.57)$$

Assuming that the parents have log-preferences yields a similar result to the one above, the child's optimal choice of bailout is independent of the parent's income. Under log-preferences, the marginal effect is:

$$\begin{aligned} \frac{\partial W_i^k}{\partial \tau} &= \gamma \frac{d^i}{D_1} \left([(1-p)RS + \tau] \frac{d^i}{D_1} \right)^{-1} - V'(w_k - \tau) \\ &= \gamma [(1-p)RS + \tau]^{-1} - V'(w_k - \tau) \end{aligned} \quad (5.58)$$

The optimal choice of bailout is defined implicitly as:

$$\gamma \frac{d^i}{D_1} u' \left([(1-p)RS + \tau_k^i] \frac{d^i}{D_1} \right) = V'(w_k - \tau_k^i) \quad (5.59)$$

Like above, if we consider the case in which the child has log-preferences, the child's preferences over the bailout policy are single-peaked around τ_k^i , where:

$$\tau_k^i = \frac{\gamma w_k - (1-p)RS}{(1+\gamma)} \quad (5.60)$$

As long as the children's income is sufficiently large, it is possible that they will support a non-zero bailout. This is because, at high levels of income, their marginal utility of consumption is low.

Contrast the children's preferences over a bailout with their willingness to transfer resources directly to their parents. Note that increasing the bailout by 1 unit resulted in the parents' consumption increasing by d^i/D_1 units. When children transfers money directly to their parents, 1 unit only increases the parent's consumption by 1 unit. For children with parents who have more than the average level of deposits ($d^i > D_1$), a bailout is strictly preferred to direct transfers; this is because other children will be subsidizing the transfer. If the parents have less-than-average deposits ($d^i < D_1$), the children would simply prefer to give their parents money over a bailout. The marginal effect of transferring t consumption directly to the parents is:

$$\frac{\partial W_i^k}{\partial t} = \gamma u' \left((1-p)RS \frac{d^i}{D_1} + t \right) - V'(w_k - t) \quad (5.61)$$

The children of the rich (*i.e.* with higher-than-average deposits) could conceivably support a bailout policy even though they do not want to transfer resources to their parents. If, however, the children of the rich want a bailout, the children of the poor would rather directly transfer their parents resources than bail them out. Despite this, all children will have the *same* preferences over a bailout; Section 5.6 relaxes this implication and considers the case in which children have different levels of income. The children's choice of bailout is illustrated in Figure 5.4. If the children did not care about the parents ($\gamma = 0$), they would want a transfer *from* the parents. That is, the bailout would be a corner solution at $\tau_k^i = 0$.

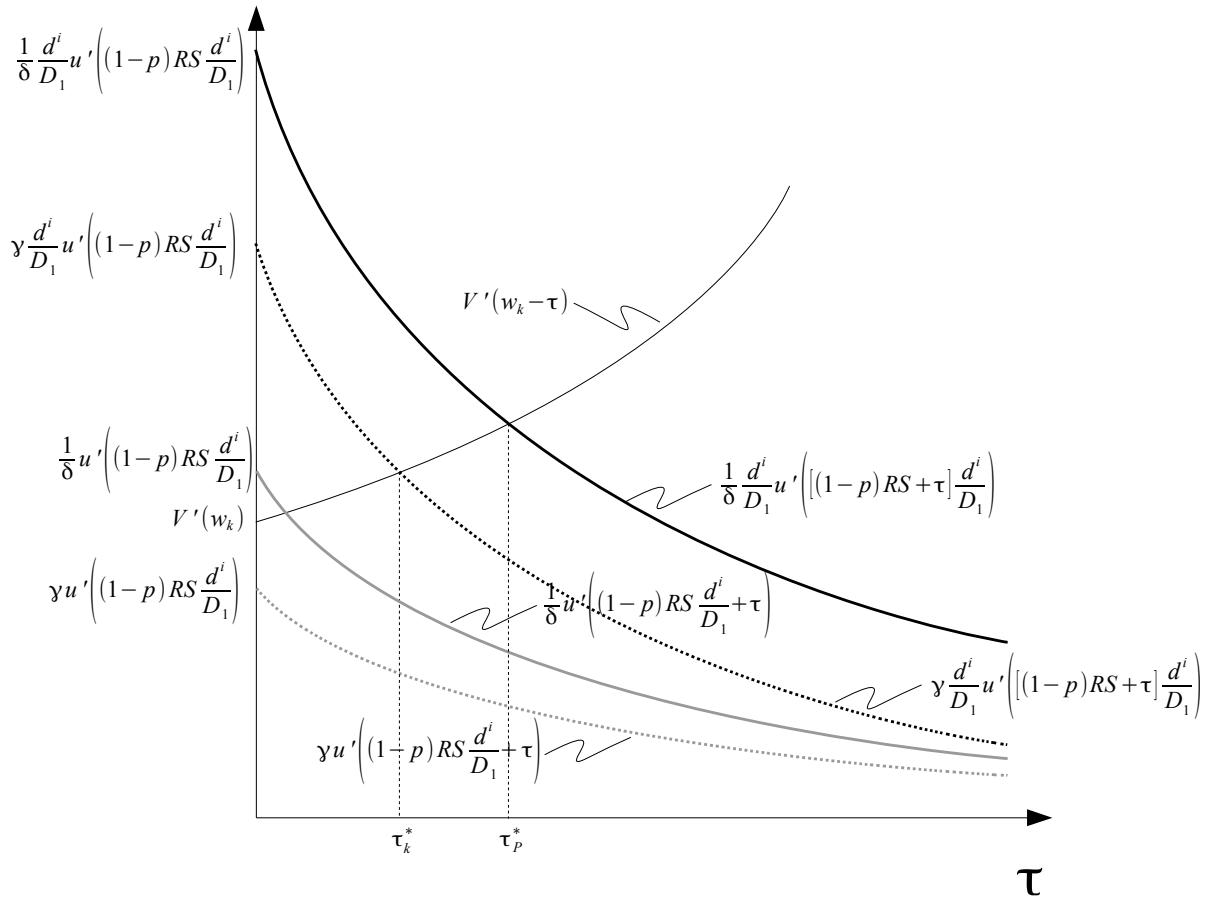


Figure 5.4: This figure graphs the preferences for a child-parent pair, where the parent has more than the average amount of deposits. The downward-sloping, dashed lines relate to the child's preferences. The grey line denotes the positive effect on the child's utility of transferring resources directly to the parent. In this diagram, the child would like to receive a transfer of resources from the parent, since this line intersects the $V'(w_k - \tau)$ line on the negative part of the domain. The downward-sloping, dashed, black line shows the positive utility to the child associated with a bailout (the LHS of Equation (5.59)). The intersection of this curve with the upward sloping marginal utility of income ($V'(w_k - \tau)$) yields the child's optimal choice of the bailout. Since the parent has higher-than-average deposits, the marginal benefit of a bailout is always higher than that of a direct transfer.

The figure also graphs the parent's preferences. The marginal effects on the parent are denoted by the downward-sloping continuous lines. The black line shows the marginal effect of a bailout; where it intersects the marginal utility of income for the child yields the optimal bailout policy for the parent (denoted τ_p^*). The grey line represents the marginal utility from a direct transfer from the child. Although the parent would like to receive some money from their child (given this line intersects with the $V'(w_k - \tau)$ curve), parents would prefer a bailout to the direct transfer.

5.5.3 The median voter

Following from Black et al. (1958) and Downs (1957), we can consider the policy preferences of the median voter. This is because all the agents have single-peaked preferences over the bailout. If the bailout is the only option (and direct transfers are not allowed), the median voter is either a child or a parent. Since all parents have the same preferences, as do all children, this is a knife-edge case. There is an equal number of children and parents so there are two possible median voters. For the sake of completeness, I will consider both cases.

Proposition 5.6. *Under log preferences for both the parent and the child, the parent's preferences over the bailout policy τ are single-peaked. The parents achieve their maximum at:*

$$\tau_P^i = \frac{w_k - \delta(1-p)RS}{(1+\delta)} \quad (5.62)$$

The children achieve theirs at:

$$\tau_k^i = \frac{\gamma w_k - (1-p)RS}{(1+\gamma)} \quad (5.63)$$

5.5.3.1 Median voter: parent

If a parent casts the decisive vote, the bailout policy will be:

$$\tau_P^i = \frac{w_k - \delta(1-p)RS}{(1+\delta)} \quad (5.64)$$

Conditional on this being in the appropriate domain ($\tau \in [0, pRS]$), this determines the economic equilibrium. The bank's equity will be:

$$E^* = \frac{1-\alpha}{1+\delta} \left((p + (1-2p)\delta)S - \frac{w_k}{R} \right) \quad (5.65)$$

This is decreasing (on the relevant domain) in both the child's income and the parent's weight on the child's consumption. The less the parents care about their children, the less capital they will have to absorb losses. The return on debt is:

$$R^d = \frac{R((1-p)RS + w_k)}{(1 - (1-\alpha)p + \alpha\delta)RS + (1-\alpha)w_k} \quad (5.66)$$

This confirms the result: if the parents care less about the children, the promised return on debt will be higher.

5.5.3.2 Median voter: child

If the child casts the decisive vote, the bailout policy will be:

$$\tau_k^i = \frac{\gamma w_k - (1-p)RS}{(1+\gamma)} \quad (5.67)$$

This implies that, in equilibrium, the amount of equity is:

$$E^* = \frac{1-\alpha}{1+\gamma} \left((1+p\gamma)S - \gamma \frac{w_k}{R} \right) \quad (5.68)$$

Again, this shows that, if the children care less about the parents, the bailout will be smaller and the amount of equity larger (when the bailout is greater than zero). The return on debt is:

$$R^d = \frac{\gamma R((1-p)RS + w_k)}{(\gamma[1 - (1-\alpha)p] + \alpha)RS + \gamma(1-\alpha)w_k} \quad (5.69)$$

This shows that, when the bailout is non-zero, the promised return is higher than the free market rate.

5.5.4 Deposit insurance and bailouts

The preceding section discussed the political equilibrium in which the older generation is able to secure a bailout from the younger generation. This held independent of whether a child or a parent was the marginal voter. In both cases there existed parameterizations by which a non-zero bailout was offered. The children of the rich supported this proposal even though they might be unwilling to transfer resources to their own parents. The children of the poor, however, would prefer to transfer resources to their parents directly. The net result was a redistribution both across and between generations. The children of the poor inordinately subsidized the rich depositors.

This raises a normative difference between bailout policies and deposit insurance. Both policies involve the government assuming the liabilities of a private entity, which creates a moral-hazard problem. It is not clear that either policy is economically very different. The policies, however, do differ in whom they benefit and how they are structured; these normative concerns are important. The bailout policy in this model was stable because the rich benefitted disproportionately. Under deposit insurance, there is a cap on the amount paid out to the depositors, which would affect the electoral coalition.

Consider modifying the above game so that depositors would receive insurance only on some amount below \bar{d} of deposits. The government would allow any deposits above this level to default. The policy problem would be to decide how high \bar{d} should be. If the deposit insurance level is sufficiently high, it is indistinguishable from a bailout. If, however, it is set somewhere below the maximum deposit, then all depositors (and their children) below the threshold have no incentive to vote in favour of an increase. The same electorate that supports the full bailout of the banks would not, at the margin, vote to increase the deposit insurance.

This is because the incentives under deposit insurance and bailouts are very different. If you are a depositor below \bar{d} then you face *no* losses in a bust. Any increase in the threshold is a transfer from your child to some other depositor, about whom you do not care. It is similar for your child: their income is reduced and redistributed to a stranger. Once you are already covered under deposit insurance, there is *no incentive* at the margin to lobby to increase the limit. It follows that the electoral coalition that supports a bailout of the banks, when losses are imposed proportionately, is not the same as the electoral coalition that supports deposit insurance.

5.5.5 Summary

This section solved for the situation in which parents have heterogeneous income and their children are all identical. The difference in income between the parents has no direct effect on the economic equilibrium of the game: the aggregate level of savings was independent of the parents' income distribution. It did, however, determine the *political* outcome, which in turn affected the financing decision of the bank. This was because, in equilibrium, parents with higher income invested more heavily in bank debt. This implied that, in a bailout, they would receive a larger transfer. From the children's perspective, at the margin, assuming log-utility, the effect of the size of the parents' deposits exactly cancels the richer parents' decreased marginal utility of consumption. Although richer parents have a lower marginal utility of consumption, because the bailout is related to their holdings of bank debt, their children will support a bailout even though the children would not be willing to transfer their own consumption to their own parents. The next section considers the case when the children have different levels of income.

5.6 Political equilibrium: heterogeneous children

This section will consider an alternative setup to the model. Suppose that the parents are all identical and have income $w^i > 1$. They will all have 1 unit of consumption invested in bank debt and equity as described in Section 5.4; it follows for the remainder of this section that $S = 1$. Suppose further that the children have different incomes that are drawn from the distribution $G(\cdot)$, bounded on $[\underline{w}, \bar{w}]$. Given the variation in the younger generation's endowments, the political equilibrium will be different.

The preferences defined in the previous section will not substantially change. Consider the biologically related parent-child pair denoted by the index i . The parent's ideal choice of bailout is defined implicitly by:

$$\frac{1}{\delta} \frac{d^i}{D_1} u' \left([(1-p)R + \tau_P^i] \frac{d^i}{D_1} \right) = V'(w_k^i - \tau_P^i) \quad (5.70)$$

The child, however, has preferences:

$$\gamma \frac{d^i}{D_1} u' \left([(1-p)R + \tau_k^i] \frac{d^i}{D_1} \right) = V'(w_k^i - \tau_k^i) \quad (5.71)$$

Note that each biologically related parent-child pair has different preferences over the bailout, although all are functions of the child's income. In general, the parents will always prefer a larger bailout than their biological children. As the children's income increases, they will prefer a larger bailout; this is illustrated in Figure 5.5.

There also exists a biologically unrelated parent-child pair with the same peak in their preferences. A parent who has a child with low income w_p^i and a child with higher income w_k^i have the same peak at τ^* ; assuming log-preferences, these two incomes are linearly related:

$$w_k^i = \frac{(1 + \gamma)w_p^i + (1 - \delta\gamma)(1 - p)R}{(1 + \delta)\gamma} \quad (5.72)$$

where w_k^i is the level of income the child must have in order to have the same preference as the parent, whose child has income w_p^i . Consider the policy that is set by the median voter; the government's policy is defined by the preferences of a decisive pair of voters, a parent

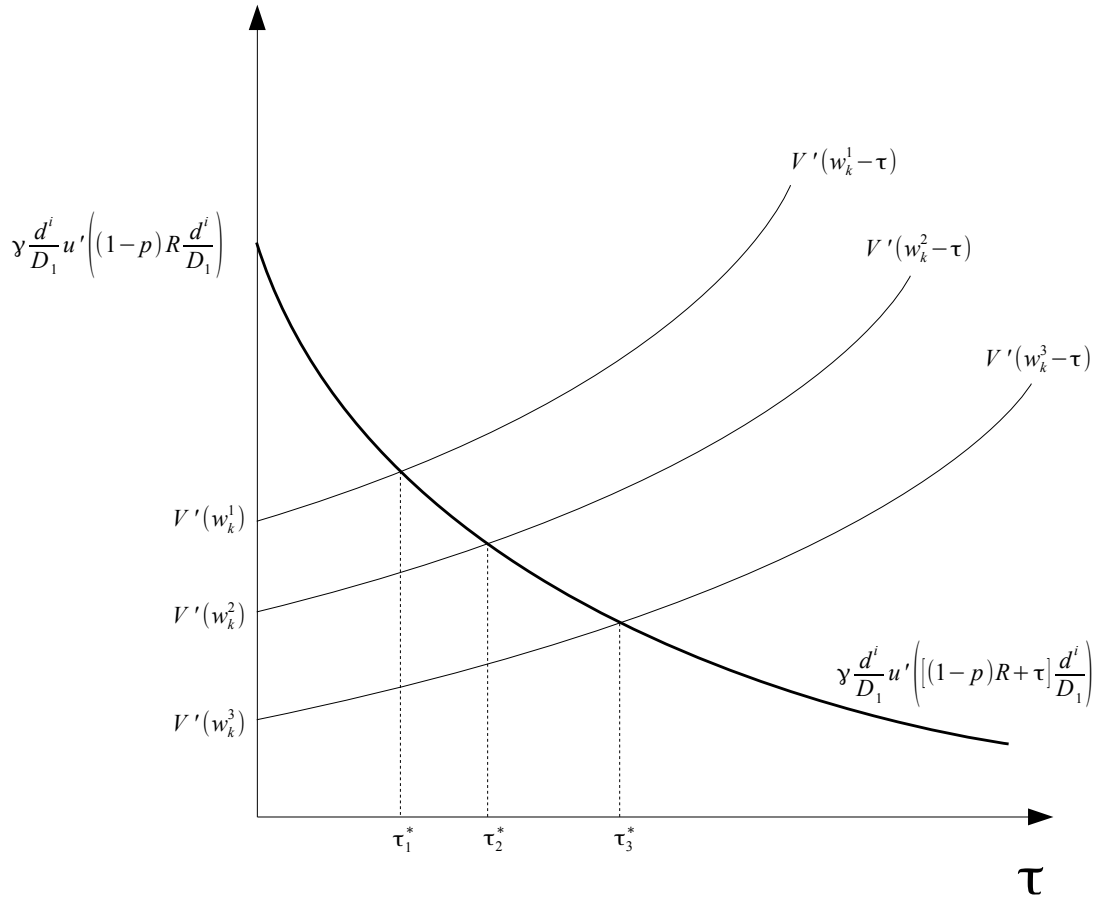


Figure 5.5: This figure illustrates how increasing children's income changes their optimal choice of bailout. In this illustration, $w_k^3 > w_k^2 > w_k^1$; the child with the highest income also has the lowest marginal utility of income and so prefers the largest bailout. The distribution of the children's income implies a distribution of preferences for τ^* .

and child (not necessarily biologically related) such that:

$$1 - \frac{1}{2} [G(w_P^i) + G(w_k^i)] = 0.5 \quad (5.73)$$

Using the linear relationship between w_k^i and w_P^i , the decisive parent voter (w_P^{med}) is implicitly defined as the solution to:

$$1 - \frac{1}{2} \left[G(w_P^{med}) + G \left(\frac{(1 + \gamma)w_P^{med} + (1 - \delta\gamma)(1 - p)R}{(1 + \delta)\gamma} \right) \right] = 0.5 \quad (5.74)$$

As an illustration, consider the case in which the children's income is uniformly distributed on $[\underline{w}, \bar{w}]$, and the decisive pair is defined by:

$$1 - \frac{1}{2(\bar{w} - \underline{w})} \left[w_P^{med} + \frac{(1 + \gamma)w_P^{med} + (1 - \delta\gamma)(1 - p)RS}{(1 + \delta)\gamma} - 2\underline{w} \right] = 0.5 \quad (5.75)$$

Policy	w_P^{med}	w_k^{med}	τ^{med}	E_1	D_1	L_1	R^d
Bailout	1.78	7.22	0.82	0.036	0.964	27.7	1.9917
No bailout	–	–	–	0.405	0.595	2.469	1.8487

Table 5.2: This numerical solution corresponds to the parameters: $\alpha = 0.1$; $\gamma = 0.3$; $\delta = 0.5$; $\bar{w} = 8$; $\underline{w} = 1$; $p = 0.45$; $R = 2$. Consider a scenario in which, over a period of time during which a risky asset can double in value, there is a 10% chance that the asset could lose 45% of its value. Given the preference parameters, in a bust state, $\sim 88\%$ of the older generation will vote in favour of a bailout while only $\sim 11\%$ of younger generation will. The bailout is a transfer that is $\sim 18\%$ of the average child's income the the older generation. As a result of the bailout, the bank's leverage is $\sim 30\times$ instead of the safe leverage of $\sim 2.5\times$. The return on equity remains unchanged; however, the return on debt is much higher with the bailout. Nearly all the returns of the underlying assets are paid out as interest on debt.

This can be inverted to yield the income of the decisive parent's child:

$$w_P^{med} = \frac{\gamma(1+\delta)[\bar{w} + \underline{w}] - (1-p)(1-\gamma\delta)R}{2\gamma + \gamma\delta + 1} \quad (5.76)$$

Substituting this into the parent's preference yields the political equilibrium:

$$\tau^{med} = \frac{\gamma[\bar{w} + \underline{w}] - (1-p)(1+\gamma\delta)R}{2\gamma + \gamma\delta + 1} \quad (5.77)$$

This shows that the equilibrium bailout is increasing in the average income $((\bar{w} + \underline{w})/2)$ and decreasing in the residual value of the bank's assets $((1-p)R)$. This solution is illustrated in Figure 5.6. For a numerical example of this equilibrium with a uniform distribution, see Table 5.2.

5.6.1 Elitism and populism

The above section considered the policy adopted as a result of the median voter theorem. This was valid since all the agents had single-peaked preferences and so the median voter's preference is the dominant policy under pairwise majoritarian voting. Given this restriction on preferences, pairwise majoritarian voting is a non-dictatorial aggregation rule that satisfies independence of irrelevant alternatives (Black et al., 1958). The political-equilibrium bailout is simply the one that is preferred by the median agent(s).

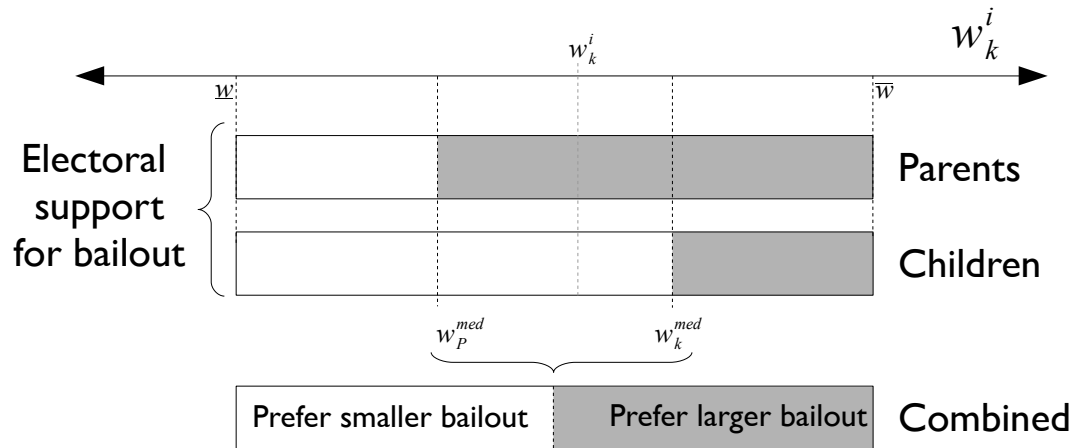


Figure 5.6: This figure illustrates how the child's income maps to the support for a bailout τ^{med} (as defined in Equation 5.77). The top horizontal axis plots the child's income, which indexes all the agents (both parents and children). Any biologically related pair is identified by the child's income. To illustrate this, take an arbitrary w_k^i and draw a line straight down: this picks out a parent and child who are related. In general, however, these two individuals have very different views on the bailout. The arbitrary w_k^i chosen in the figure shows a parent who supports a larger bailout and a child who would prefer a smaller one. There are, however, parents and children who have the same preferences; they just aren't related. The parent whose child earns w_p^{med} has the same preferences over the bailout as the child who earns w_k^{med} . The bailout policy is set by the median-voter pair.

The choice of the median agents is not arbitrary. They are important because their preferences can never be defeated by an alternative under majority voting (*i.e.* they are always a Condorcet winner). I will interpret this outcome as the 'populist' outcome; this is because the bank's contracts are overruled by a simple majority of the population. To determine the effect of anti-populist political institutions on bailouts, consider an asymmetric super-majority rule. Instead of taking the median voter as determinant of policy, suppose that $\theta \in [0.5, 1]$ proportion of the agents must support the bailout. The policy will be determined by the θ -quantile voter.

Assumption 5.3. A non-populist government targets policy to the θ -quantile voter.

True super-majority rules create the possibility of blocking coalitions, cycling and other complex strategic interactions, which are beyond the scope of this chapter. As a short cut, however, I am going to assume that the political system targets the preferences of the θ -quantile voter instead of the median voter. From a theoretical point of view, this is a crude assumption. The preferences of the θ -quantile voter will not in general survive pairwise voting. Regardless, if the government targets the θ -quantile voter, θ proportion of the voters would

desire a bailout larger than the one adopted. The analogue is that, using the median-voter rule, only 50% of the agents want a bailout that is larger than the one adopted.

Like in the median-voter setting, the optimal policy $\tau^*(\theta) \in [0, pRS]$ will be determined by a pair of voters. The threshold voters are decisive inasmuch as they act like a median voter under a majority-rule system and the political process targets their preferences. Consider, then, the income distribution of the children, denoted with CDF $G(\cdot)$. The θ -quantile voters are parent and child, defined implicitly, such that:

$$1 - \frac{1}{2} [G(w_P^\theta) + G(w_k^\theta)] = \theta \quad (5.78)$$

where w_P^θ is the income of the decisive parent's child and w_k^θ is the income of the decisive child. Using the linear relationship between w_k^θ and w_P^θ , the decisive parent voter (w_P^d) is implicitly defined as the solution to:

$$1 - \frac{1}{2} \left[G(w_P^d) + G\left(\frac{(1+\gamma)w_P^d + (1-\delta\gamma)(1-p)R}{(1+\delta)\gamma}\right) \right] = \theta \quad (5.79)$$

Using the implicit function theorem, it is possible to demonstrate that the marginal effect of reducing populism (*i.e.* raising θ) is:

$$\frac{\partial w_P^d}{\partial \theta} = -\frac{1}{2} \left[g(w_P^d) + \frac{1+\gamma}{\gamma(1+\delta)} g\left(\frac{(1+\gamma)w_P^d + (1-\delta\gamma)(1-p)RS}{(1+\delta)\gamma}\right) \right] < 0 \quad (5.80)$$

where $g(\cdot)$ is the PDF of $G(\cdot)$. That is, as we raise the necessary proportion for the winning coalition, the decisive voter will necessarily come from a family with lower income. The bailout policy is ultimately a function of the institutional parameter θ : $\tau^*(w_P^d(\theta))$.

The institutional parameter determines who is decisive in the political process and the decisive pair determines the bailout policy. Decreasing the populism of the institution makes the decisive pair more conservative (*i.e.* the pair prefers a smaller bailout). In this manner, the institutional parameter θ ultimately dictates the capital structure and fragility of the banks.

Suppose that the children have income that is uniformly distributed on $[\underline{w}, \bar{w}]$. The decisive

θ	τ^θ	w_P^θ	w_k^θ	E_1	D_1	L_1	R^d	$2\tau^\theta/(\bar{w} + \underline{w})$
50%	0.82	1.78	7.22	0.036	0.964	27.7	1.9917	18.2%
60%	0.58	1.42	6.18	0.144	0.856	6.94	1.9626	12.8%
70%	0.34	1.06	5.14	0.252	0.748	3.97	1.9251	7.5%
NB	0	–	–	0.405	0.595	2.469	1.8487	–

Table 5.3: This numerical solution uses the same parameters as Table 5.2: $\alpha = 0.1$; $\gamma = 0.3$; $\delta = 0.5$; $\bar{w} = 8$; $\underline{w} = 1$; $p = 0.45$; $R = 2$. The parameters imply a scenario where, over a period of time where a risky asset can double in value, there is a 10% chance that the asset could lose 45% of its value. The table shows that, as the institutional parameter θ increases, the bailout decreases. When $\theta = 50\%$, the outcome is the same as in Table 5.2. However, when θ is raised to 60% and 70%, the banks finance their operations using more equity and the size of the bailout drops.

pair, identified by w_P^d , is defined by:

$$1 - \frac{1}{2(\bar{w} - \underline{w})} \left[w_P^d + \frac{(1 + \gamma)w_P^d + (1 - \delta\gamma)(1 - p)R}{(1 + \delta)\gamma} - 2\underline{w} \right] = \theta \quad (5.81)$$

This can be inverted to yield the income of the decisive parent's child:

$$w_P^\theta(\theta) = \frac{\gamma(1 + \delta) [2(1 - \theta)(\bar{w} - \underline{w}) + 2\underline{w}] - (1 - p)(1 - \gamma\delta)R}{2\gamma + \gamma\delta + 1} \quad (5.82)$$

Table 5.3 shows a numerical solution to this equilibrium and demonstrates how it changes as the institutional parameter θ varies. As θ increases, the size of the bailout drops, the bank's equity increases, and the leverage decreases.

The extent of the bailout is a function of how populist the political institutions are. The model implies that more populist institutions will result in larger bailouts. Consequently, banks will operate in a more risky manner under populist institutions and more conservatively under elitist ones.

5.7 Discussion

The model presented in this chapter shows that a bank's capital structure 'creates facts'. By choosing an unsafe level of leverage, a financial institution can 'bake in' the incentive for a future government to bail out its creditors. The bailout is forthcoming despite the existence

of a costless resolution mechanism; it occurs as a result of the distributional consequences of insolvency. When a bank fails, there are simply not enough assets to repay the promises to creditors. Although the law dictates that the creditors should face the cost of failure, the political process can be used to overturn these contracts. A bailout involves imposing a lump-sum tax on the younger generation and redistributing it to the creditors, their parents. If it is known that the political process will interfere during the insolvency, the financial institutions will alter their behaviour in anticipation. In this respect, bank bailouts create a self-fulfilling prophecy: expectations of bailouts cause banks to finance their operations in a risky manner, which causes future bailouts.

Some support for a bailout must come from the younger generation. Many children will vote in favour of this intergenerational redistribution even though they do not favour transferring their own money to their own parents. This is because, at the margin, some parents benefit more from the bailout than its cost to their child in taxes. As an illustration of this externality, if a child's parent holds five times the average debt, then an additional dollar taken from the child results in the parent receiving five dollars, the remainder being taken from children who are (possibly) less enthusiastic towards the bailout policy. If all parents suffered equally from the failure it would be equivalent to children giving their own parents their own money.

The result is closely associated to the asymmetry of the voting process in the first period: the children do not get a vote before they are born. This suggests that the problem might be more subtle than just one of time-inconsistency. If the regulators in the first period are responsible to the voters, they will have no incentive to rein in the banks. This is because the voters do not seek to avoid a bailout. The political process will neither apply the necessary preventative constraints on the financial sector nor impose losses on future creditors.

Further analysis of the model implied (i) that institutional independence could help to mitigate the political temptation to bailout the financial sector and that this results in banks choosing a safer financing regime, (ii) that increasing capital requirements is costly to investors in the bank, because it reduces the implicit subsidy and (iii) that there is a normative distinction to be made between bailouts and deposit insurance. In this section, I will focus on the implications the model has for bank regulation and whether it could inform a view on the recent financial crisis; specifically, the cases of Ireland and Canada.

5.7.1 Implications for bank regulation

One of the awkward implications of the model presented in this chapter is that the parents have no incentive to finance the banks using a sufficient amount of capital. Their incentive is to use as much debt as the younger generation is willing to bail out. Banks will increase their leverage as long as they are able to issue safe bonds. The parents do not want to implement binding capital adequacy requirements since this would reduce the state-contingent transfer from the younger generation. Although, one hopes, public policy is not made with such cynical intentions, this suggests that there are political costs to increasing capital requirements: increased equity financing reduces the implicit subsidy to the banks' creditors. This is why capital requirements are 'costly' and MM does not hold.

The Basel capital adequacy framework proved insufficient in the most recent financial crisis. For several European countries, the socialization of privately accumulated debts lay at the heart of the sovereign debt crisis. Allowing banks to hold capital against risk-weighted assets provided many opportunities to increase their leverage and incentivized banks to hold sovereign debt, which was given zero risk weighting. This latter policy was particularly problematic as it created a feedback mechanism between the sovereigns and the financial sector. The most acute period of this crisis was ended only when the ECB intervened by backstopping the sovereigns. This model partly explains why the Basel framework was insufficient: there is little incentive to raise capital requirements *ex ante* or to fulfill contracts *ex post*.

In this simple model, if it is possible to force banks to finance their operations with more equity, the problem of TBTF can be mitigated. Increasing and narrowing capital requirements would be a possible solution to TBTF. This is hardly a novel conclusion and it is congruent with the arguments made by, among others, Admati and Hellwig (2014). More capital would be available to absorb losses, which protects taxpayers. The model, however, suggests that achieving this goal could be very difficult: raising capital requirements does not necessarily benefit anyone alive today since the costs of bailouts are likely borne by future generations. The difficult problem is that populist political institutions might be unwilling to enforce sufficiently large capital requirements and elitist ones could be subject to industry capture. It is unclear which of these extremes is preferable.

The model also suggests scepticism over bail-in and subordinated debt proposals. This is because they are unlikely to be implemented as intended in a crisis. Future electoral coalitions will choose to provide bailouts to creditors before imposing losses on them; this is no secret. Conversely, the model suggests that policy interventions that affect the financial sector before a crisis will have a higher potential of reducing TBTF. These proposals might include ‘limited purpose banking’ (Chamley and Kotlikoff, 2009; Kotlikoff, 2010) and narrow banking (Cochrane, 2014; De Grauwe, 2009). Fully exploring these options is beyond the scope of this chapter.

5.7.2 The effect of political institutions: elitism or populism

The model presented in this chapter showed that the banks’ financing decision was affected by the political process. By altering the threshold level of support required to pass a bailout measure, it follows that a higher threshold results in a lower bailout. This, in turn, causes the banks to finance their operations in a safer manner. It suggests that the moral hazard and time-inconsistency problems in the financial sector can be aggravated by populist institutions. Such a conclusion confirms the historical work in Calomiris et al. (2014), which concludes that anti-populist democratic institutions are associated with financial stability. The subsequent sections discuss the claims made in Calomiris et al. (2014) but contend they need not be adopted with enthusiasm: there remains room for healthy scepticism.

5.7.2.1 A historical perspective on elitist institutions

To illustrate the historical argument, consider the banking sectors in Canada and the United States. In the Great Depression alone, over 9,000 American banks failed. By contrast, from 1867 – the founding of Canada – to today, there have been approximately 11 bank failures in Canada (Ben-Ishai, 2009). This is for several historical and institutional reasons. Perhaps most importantly, branch banking was common in Canada over this period of time. In the United States, branch banking was effectively outlawed by the ironically named ‘free banking acts’ imposed by the states. The American system of ‘unit banking’ (banks with no branches) largely persisted until the 1980s.

From today’s perspective, the unit banking system appears almost designed to increase the fragility of a financial system. A unit bank could not (or was severely limited in its ability to) open branches in different locations. Consequently, it was difficult to diversify any

idiosyncratic regional risk. This resulted in many towns having only a single monopolistic local bank. This policy was supported by state-level electoral coalitions of agrarian populists and, unsurprisingly, small bankers. Similar political movements existed in Canada and they sought to impose comparable policy interventions but were stymied by the relatively anti-populist nature of the Canadian political system. To implement these policies in Canada would have required a national electoral coalition.

The Social Credit movement, for instance, was particularly active in Western Canadian provincial politics for much of the 20th century.^{xv} In 1937, a Social Credit government in Alberta tried to implement their heterodox monetary theories. The (Federal) Prime Minister King intervened, asking the Albertan Premier Aberhart to seek a reference from the Supreme Court before passing the sweeping economic reforms. The constitution clearly gave power to regulate the banks to the Federal government. Aberhart, however, refused so King (through the Albertan Lieutenant-Governor) invalidated the legislation by denying it Royal Assent. Aberhart was not easily defeated and he tried to pass similar legislation only months later. Eventually the dispute between King and Aberhart was settled by the Supreme Court, who sided with the Federal government and reasserted the Federal control over monetary and banking policy (Mallory, 1948). This implied that small, provincially-based coalitions could not co-opt the regulation and administration of the financial sector; any attempt to do so would need to be successful on a national level. In the United States, in contrast, state-level electoral coalitions were sufficient to pass sweeping financial reforms and interventions.

Another enlightening Canadian episode is the 1910 Farmer's Bank insolvency; see Carr et al. (1994) for further exposition of this bank failure and its aftermath. The insolvency was caused by gross mismanagement: the bank had invested heavily in silver-mining speculations that did not pan out. In December 1910, the bank suspended convertibility and depositors lost everything. In response, the creditors *and* equity-holders organized a campaign for a Federal government bailout. This was the era of double-liability so the equity-holders had a strong incentive to support the creditors in this endeavour. On a wave of populist zeal, the Conservative Minister of Finance shepherded a bill through the House of Commons that intended to compensate the depositors and creditors of the Farmer's Bank. The bill, however, never became law as the unelected Senate refused to pass the bailout.

^{xv}See Macpherson (2013).

These historical anecdotes are congruent with the model. The anti-populist structure of the Canadian Federal government greatly raised the co-ordination required for any electoral coalition to re-write the banking laws or pass bailouts. Even though there were several attempts, it became clear that the elitist Federal institutions acted as a form of commitment to enforcing bank contracts, regardless of the distributional consequences.

This echoes the ‘conservative’ Central Banker conclusion from Rogoff (1985). In some instances, it might be necessary to select an official (or institution) that has very *different* preferences from society in order to achieve a desirable outcome. Here, an anti-populist institution is a commitment device that acts to prevent popular bailouts; future generations benefit as a result of the institution *limiting* their choices.

This perspective suggests that a necessary condition for a stable financial sector is the strong independence of the financial regulator and bank-resolution authority. Recent moves to house the macro-prudential regulatory authority inside the independent central bank might be helpful reforms along this line.¹⁸ There is, however, the corresponding danger that housing these tools in the monetary authority could increase the political temptation to reduce its independence. Further, an independent resolution authority is moot if the government intervenes before it has a chance to resolve a troubled institution.

5.7.2.2 Tempering the enthusiasm for elitist institutions

The historical contrast between Canada and the United States has been popularized in Calomiris et al. (2014), Bordo et al. (1994) and Bordo and Redish (1987). They conclude that the Canadian banking system has been stable compared to other developed countries because of its relatively anti-populist constitution, the prevalence of branch banking, and the unusual nature of Canadian bank charters. Calomiris et al. (2014) concludes that it is the anti-populist nature of the Canadian Federal government that has prevented the populist-capture of the Canadian financial industry.

Despite these institutional characteristics, it does not follow that the Canadian government will retain its anti-populist nature. Recently, the Canadian Senate, whose anti-democratic structure Calomiris et al. (2014) credits, has fallen into disrepute. There are credible political movements to elect or abolish the upper chamber.¹⁹

In the most recent crisis, for instance, it is not clear that the government did not provide an implicit guarantee to the banks. In fact, there was a significant likelihood of a bailout had something gone awry. This was made evident when Ed Clark, the CEO of Toronto Dominion Bank (TD) responded to a question about whether it was necessary for Canadian banks to raise additional capital during the 2008-09 financial crisis. He was remarkably candid in his spoken remarks and revealed his expectation that bank debt was backed by the full faith and credit of the Government of Canada (emphasis added):²⁰

Canada will emerge, as long as we don't do anything stupid, as the only country in the world where the banks didn't need the government[']s help. [The banks] needed the government in a sense to be a positive[*sic*], and [the government is] being helpful in lots of little ways, but we're not talking the kind of nationalization that's gone on around the world. [...]

[W]hat happened in our preferred share issue, is that the average consumer sat there and said, well, this is 6.25%, but it's actually 8% pretax, effectively government guaranteed, maybe not explicitly, but what are the chances that TD Bank is going to not be bailed out if it did something stupid? And so where else do I get 8% government investments right now?

Fortunately, Canadian bank creditors did not need to exercise their option on the Canadian government. However, the belief that such state-contingent arrangements exist is clearly problematic. It somewhat undermines the claim that the banking industry 'didn't need government help'; the help came through implicit insurance. Had the government of Canada needed to resort to extreme measures (recapitalization or nationalization) the country would certainly not be considered a model to copy. Such survivorship and state-contingent considerations are important when examining the historical analysis and options for regulatory reform. Although a country might appear to be the model of stability, the expectation of government bailouts can create significant risk of sovereign fiscal disruption. It is worth recalling that, before the financial crisis of 2008, the last major Irish banking failure occurred in 1885 (Ó Gráda, 2012).

From 2000-08, the Irish government ran modest fiscal surpluses; few, if any, thought Irish fiscal policy was recklessly loose. The bailouts to the financial sector, however, effectively bankrupted the Irish state. The state interventions in the financial sector dwarfed the surpluses from the boom years (see Figure 5.7). Although the financial sector was a significant part of the Irish economy, it was not obvious that, prior to the crisis, they were in such a dangerous position. Even during the crisis, Merrill Lynch, consultants to the Irish Treas-

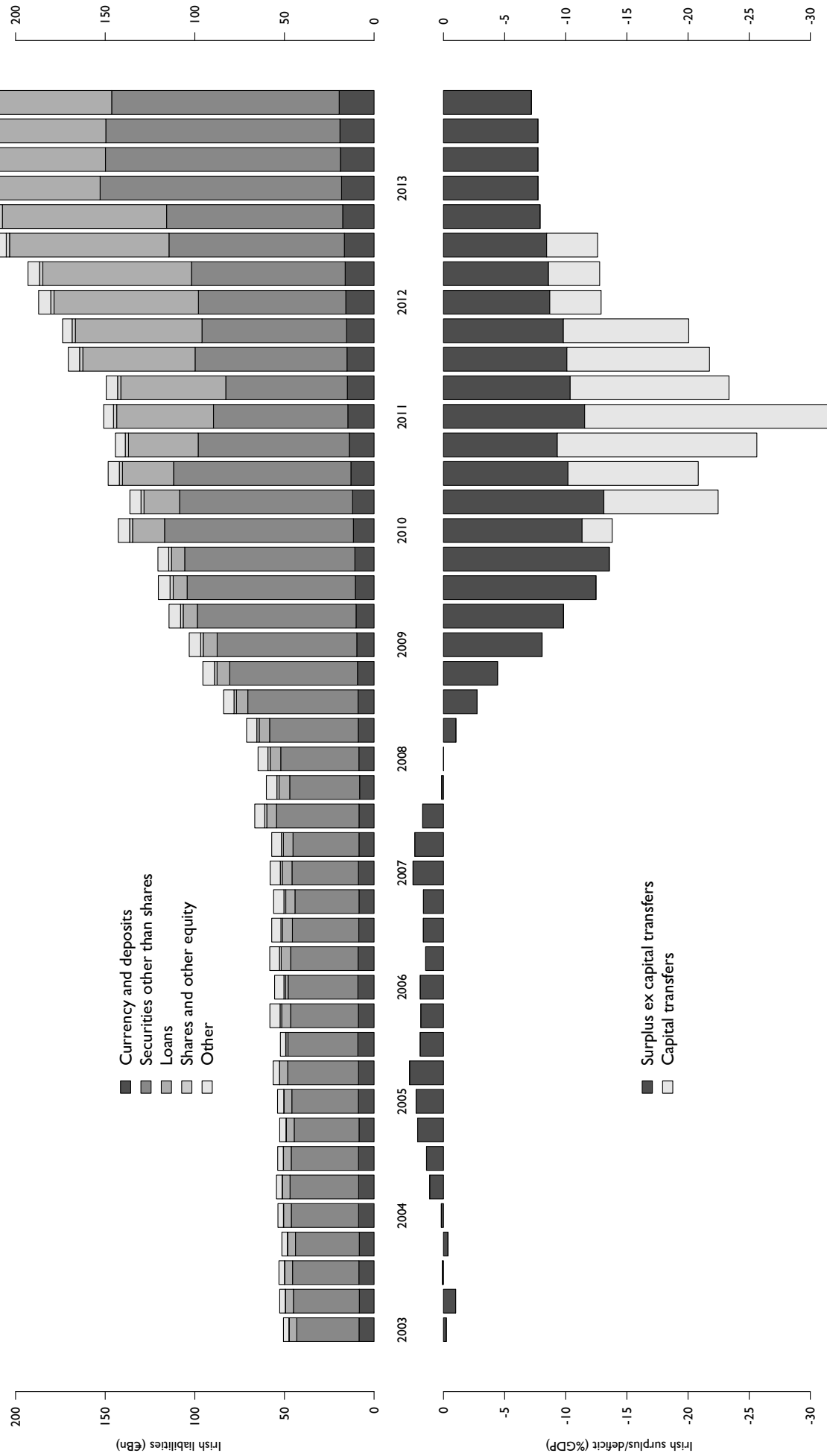


Figure 5.7: The top panel shows the Irish government's liabilities, which rose considerably during the crisis. The reason for the discrepancy between the quarterly government debt and the liabilities is that the former is measured at nominal value while the latter are marked to market. The lower panel shows the Irish government's quarterly deficit, when measured as a 4-quarter moving average. Source: *Quarterly Financial Accounts for Ireland: Quarter 1, 2002 - Quarter 4, 2013*. Author's calculations.

sury, thought that the banking industry was well capitalized only hours before the sovereign backstop was announced (memo dated September 28, 2008):²¹

It is important to stress that at present, liquidity concerns aside, all of the Irish banks are profitable and well capitalised.

The external consultants were adamant that resolution not be pursued:

Whilst we set out the various strategic options within this memo, we have also fully considered, and utterly discounted, one additional outcome – allowing an Irish bank to fail and go into liquidation without any government intervention. Whilst this option would initially have no financial impact on the government, the resulting shock to the wider Irish banking system could, in our view, be very damaging. [...] Therefore in order to minimize the impact of any bank failure on the rest of the broadly sound domestic financial institutions, we strongly advocate a more controlled interventionist approach.

The final option that the consultants proposed, which was ultimately pursued by the Irish government, was to backstop the Irish financial industry completely. The consultants estimated that the recapitalization operation would be €6.5-€16.4 billion.²² The actual total outlay was of €62.8 billion (~ 40% of GDP).²³ For a discussion on the fiscal consequences of this decision, see Whelan (2013).

When considering the stability of the Canadian banking system, it is important to remember what happened in Ireland. A country that appeared to have relatively conservative fiscal policy was made, for all intents and purposes, insolvent by backstopping its banking system. Further, the Irish banking system had a relatively good track record of stability before the crisis. In order to determine how to reform financial regulation, we must consider both successes and failures. Although the results regarding elitist institutions seem plausible (and are congruent with the model presented in this chapter) more work needs to be done on this topic.

Chapter 6

Conclusion

This thesis has presented two different types of model in order to give some insight into the problems associated with sovereign debt, default and contingent liabilities on the state. Chapter 2 presented a simple global games model that, under private information, yielded a deterministic outcome. This simple model served as the foundation for Chapters 3 and 4, which introduced the possibility of multiple authorities and correlated information respectively. Chapter 5 departed from the basic model but considered the question of why private debts are socialized.

The major result from Chapter 2 followed from the introduction of public information. This implied that the equilibrium strategy would behave like a sunspot equilibrium. This conclusion is both informative and troubling: two countries with the same macroeconomic characteristics could face very different outcomes. The switching occurred because the agents were able to discern when the ‘beauty contest’ aspects of the game would dominate. Knowing that the other agents faced the same incentive to run caused a self-fulfilling prophecy for an outcome.

Chapter 3 generalized the model by considering the incentives under multiple authorities. The major conclusion from this chapter is that countries that borrow from a common financial sector, can impose externalities on one another. Specifically, it is possible for a low-debt country to push a high-debt country into crisis by adopting austerity measures. Merging the two fiscal authorities could, under fairly general circumstances, increase the amount of debt that they could roll-over. This was more than just a function of sharing debt-capacity; it was also partly driven by the elimination of fiscal externalities. When public signals are

introduced to the game, it exhibits sunspot behaviour similar to that in the outcome modelled in Chapter 2. This showed that there is never an intra-temporal incentive to merge the authorities since the lower-debt authority would see an increase in the probability of a crisis. It is, however, possible under certain parameterizations for the two countries to enter into a Coasean arrangement: there are situations in which the high-debt country could compensate the low-debt country enough to incentivize a merger.

The model from Chapter 2 was further generalized in Chapter 4 with the introduction of correlated signals. This game had a unique symmetric cutoff strategy, which resulted in global uncertainty being preserved. The major result of this chapter showed that the *ex ante* distribution of play converged to the Vasicek credit distribution. Further, it implied that there existed a perverse incentive for countries to make their financial information less intelligible to investors.

Lastly, Chapter 5 addressed the question of sovereign bailouts of the financial sector. This chapter deviates from the previous three in its modelling strategy; however, the topic is closely related. It uses a simple overlapping-generations model to demonstrate that there exists a political incentive to bail out failed financial institutions. Consequently, financial institutions respond by increasing their leverage above the safe threshold in order to take advantage of the implicit insurance of their bonds. When such an equilibrium exists, the resulting bailout transfers resources from the younger generation to the older. Somewhat paradoxically, children who would not support giving their own parents their own money will support a bailout (as long as their parents are owed a sufficiently large sum). Further, the model implied that there is an important normative distinction between bailouts and deposit insurance, although on the surface they both appear to socialize private losses. The important distinction relates to the incentive for various subsets of the population to support the policy. Under a bailout, all holders of the debt benefit from a bailout proportionately. Under deposit insurance, if investors have deposits under the threshold, they have no incentive to support a more generous deposit insurance scheme. Finally, the major conclusion came from the examination of different policy rules: the more elitist the political institution, the smaller the bailout will be. This is because a larger political coalition is required to sustain a bailout – a conclusion that is congruent with the historical evidence.

Appendices

Appendix A

Variable lists

A.1 Chapter 2

	Explanation	Formula
l^*	The threshold proportion of coordination for liquidity.	$l^* \in [0, 1]$
l	The realized proportion of coordination for liquidity.	$l \in [0, 1]$
B^*	The threshold level of debt for solvency.	$B^* \in \mathbb{R}_+$
B	The actual level of government debt.	$B \in \mathbb{R}_+$
B_l	The level of debt at which a liquidity crisis is triggered.	$B_l \in \mathbb{R}_+$
ζ	The linear relation between debt and liquidity.	$l^* = \zeta B$
r	The return on debt conditional on repayment.	$r \in \mathbb{R}_+$
β	The loss associated with default.	$\beta \in \mathbb{R}_+$
x_i	The normally distributed private signal for agent i .	$x_i = B^* + \epsilon_i$
y	The normally distributed public signal.	$y = B^* + \eta$
α	The forecast weight put on the public signal.	$\alpha \in [0, 1]$
\bar{B}^*	The agent's belief of the expected value of B^* .	$\bar{B}^* = \alpha y + (1 - \alpha)x_i$
σ	The standard deviation of the private signal.	$\sigma \in \mathbb{R}_+$
γ	The standard deviation of the public signal.	$\gamma \in \mathbb{R}_+$
κ_i	The cutoff strategy adopted by agent i .	
κ^*	The symmetric equilibrium cutoff strategy.	
$\Phi(\cdot)$	The standard normal CDF.	$[2\pi]^{-1/2} \int_{-\infty}^x e^{-t^2/2} dt$
$\phi(\cdot)$	The standard normal PDF.	$[2\pi]^{-1/2} e^{-x^2/2}$

A.2 Chapter 3

	Explanation	Formula
l_k^*	The threshold coordination for country k .	$l_k^* \in [0, 1]$
l_k	The realized coordination for country k .	$l_k \in [0, 1]$
B_k^*	The threshold level of debt for solvency for country k .	$B_k^* \in \mathbb{R}_+$
B_k	The actual level of government debt for country k .	$B_k \in \mathbb{R}_+$
B_l	The level of debt at which a liquidity crisis is triggered.	$B_l \in \mathbb{R}_+$
Δ	The difference in the debt level between the two countries.	$\Delta = B_I - B_G$
Δ^*	A threshold difference that causes the equilibrium to bifurcate.	
\bar{r}	The return on debt conditional on both countries paying.	$\bar{r} \in \mathbb{R}_+$
r	The return on debt conditional on G paying.	$r \in [0, \bar{r}]$
β	The loss associated with default.	$\beta \in \mathbb{R}_+$
x_i	The normally distributed private signal for agent i .	$x_i = B^* + \epsilon_i$
y	The normally distributed public signal.	$y = B^* + \eta$
α	The weight put on the public signal.	$\alpha \in [0, 1]$
\bar{B}^*	The agent's belief of the expected value of B^* .	$\bar{B}^* = \alpha y + (1 - \alpha)x_i$
σ	The standard deviation of the private signal.	$\sigma \in \mathbb{R}_+$
γ	The standard deviation of the public signal.	$\gamma \in \mathbb{R}_+$
p_c	The <i>ex ante</i> probability of a crisis conditional on Δ^* .	$p_c = \Pr(l < l^*)$
κ_k^i	The cutoff strategy adopted by agent i for country k .	
κ_k^*	The symmetric equilibrium cutoff strategy for country k .	
$\Phi(\cdot)$	The standard normal CDF.	$[2\pi]^{-1/2} \int_{-\infty}^x e^{-t^2/2} dt$
$\phi(\cdot)$	The standard normal PDF.	$[2\pi]^{-1/2} e^{-x^2/2}$

A.3 Chapter 4

	Explanation	Formula
l^*	The threshold proportion of coordination for liquidity.	$l^* \in [0, 1]$
l	The realized proportion of coordination for liquidity.	$l \in [0, 1]$
B^*	The threshold level of debt for solvency.	$B^* \in \mathbb{R}_+$
B	The actual level of government debt.	$B \in \mathbb{R}_+$
Δ^*	The fiscal space.	$\Delta^* = B^* - B$
B_l	The level of debt at which a liquidity crisis is triggered.	$B_l \in \mathbb{R}_+$
r	The return on debt conditional on repayment.	$r \in \mathbb{R}_+$
β	The loss associated with default.	$\beta \in \mathbb{R}_+$
Q_i	The normally distributed (correlated) signal for agent i .	$Q_i = \Delta^* + \sigma X_i$
X_i	The normally distributed (correlated) noise.	$X_i = \sqrt{\rho} y + \sqrt{1 - \rho} z_i$
ρ	The correlation between agents' signals.	$\rho = \nu^2 / (\nu^2 + \zeta^2)$
σ	The standard deviation of the agents' signals.	$\sigma^2 = \nu^2 + \zeta^2$
ν	The standard deviation of the private component of the noise.	$\nu \in \mathbb{R}_+$
ζ	The standard deviation of the public component of the noise.	$\zeta \in \mathbb{R}_+$
z_i	The normally distributed private component of the signal.	$z_i \sim N(0, 1)$
y	The normally distributed public component of the signal.	$y \sim N(0, 1)$
κ_i	The cutoff strategy adopted by agent i .	
κ^*	The symmetric equilibrium cutoff strategy.	
$\Phi(\cdot)$	The standard normal CDF.	$[2\pi]^{-1/2} \int_{-\infty}^x e^{-t^2/2} dt$
$\phi(\cdot)$	The standard normal PDF.	$[2\pi]^{-1/2} e^{-x^2/2}$

A.4 Chapter 5

Explanation	Formula
c_1^i The i^{th} parent's consumption in the first period.	
c_2^i The i^{th} parent's consumption in the second period.	
e^i The i^{th} parent's choice of equity in the first period.	
d^i The i^{th} parent's choice of bonds/deposits in the first period.	
w^i The i^{th} parent's income in Period 1.	$w^i \sim F$
w_k^i The i^{th} child's income in Period 2.	$w_k^i \sim G$
c_K^i The i^{th} child's consumption in the second period.	
L_1 The representative bank's leverage.	$L_1 = \frac{E_1 + D_1}{E_1}$
E_1 The representative bank's capital in the first period.	$\int e^i(w_1^i) dG(w_1^i)$
D_1 The representative bank's debt in the first period.	$\int d^i(w_1^i) dG(w_1^i)$
A_1 The representative bank's assets in the first period.	$\int b^i(w_1^i) dG(w_1^i)$
R The return on the bank's assets in the boom state.	
R^d The promised return on the bank's bonds.	
R^e The return on the bank's equity.	$R^e = R + (R - R^d) \frac{D_1}{E_1}$
α The probability of a recession in the second period.	
p The proportion of bank loans that default.	
τ The second-period bailout policy.	$\tau \in [0, pRS]$
θ The decisive supermajority voter.	$\theta \in [0.5, 1]$

Appendix B

Numerical examples

B.1 Chapter 4

B.1.1 Using correlated information terminology

Consider a country that has a debt of €480 B and a fiscal capacity of €500 B; in this case, the nation's fiscal space would be €20 B ($\Delta^* = 20$). Suppose also that the standard deviation of the public signals was €50 B ($\sigma = 50$). Suppose further that the bonds pay a 7% return ($r = 0.07$) and that, if it defaults, the government will repudiate 75% its debt ($\beta = 0.75$) and 5% of investors will be required to roll-over the debt ($l^* = 0.05$). Consider what happens when we allow the correlation between signals (ρ) to vary between zero and one.

Firstly, we can check the Condition (4.51) holds, implying that there is a non-degenerate equilibrium:

$$0.95 = 1 - l^* > \frac{\beta}{r + \beta} = 0.9146 \quad (\text{B.1})$$

For zero correlation between signals ($\rho = 0$), the cutoff selected by investors is:

$$\kappa_{priv} = \sigma \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \approx 68.4928 \quad (\text{B.2})$$

When there is zero correlation between the signals, we know that the distribution of the signals will be centred around $\Delta = 20$ with standard deviation of 50. Any investor who receives a signal over 68.4928 will choose to roll-over the debt – this amounts to about 16.6% of investors and so there will be no liquidity crisis. This occurs with certainty, so under perfectly private information the probability of a crisis is zero.

As the correlation between signals increases, so does the optimal cutoff; when the correlation approaches 1, the cutoff monotonically approaches the public information cutoff:

$$\kappa_{pub} = \sigma \Phi^{-1} \left(\frac{\beta}{r + \beta} \frac{1}{1 - l^*} \right) \approx 89.1905 \quad (\text{B.3})$$

When information is perfectly public, every investor receives the same information. They will all choose to roll-over the debt as long as the signal they receive is at least as large as the cutoff. In this case, there is only a probability that the signal will be sufficiently large. The probability of a signal above the cutoff being sent is only 8.3%; when $\rho = 1$ the probability of a crisis is 91.7%.

For values of $\rho \in (0, 1)$, we can see that the probability of a crisis is monotonically increasing as the correlation increases; this is illustrated in Figure B.1. This numerical example illustrates one of the results of this model: the information structure can have enormous effects on the probability of a debt crisis. Information that is more highly correlated has two major effects on the outcome. Firstly, it reduces the diversification of signals – in the extreme either all agents will choose to roll-over the debt or none will. Secondly, it causes agents to behave more conservatively; the cutoff required for them to roll-over the debt is higher when correlation between signals is higher. This result demonstrates that the probability of default for a sovereign is not necessarily simply a function of fundamentals but also of market coordination and information structure. In this example, the debt crisis is purely due to the information structure and the inability of the investors to coordinate in rolling-over the debt. The government is fundamentally solvent yet the probability of crisis can range from 0% to over 90% depending solely on the correlation structure of the information.

Increasing the fiscal space of the sovereign (Δ^*) unambiguously reduces the probability of a crisis regardless of the level of correlation. This is illustrated in Figure B.2, where the probability of a crisis is calculated (as a function of the correlation) for multiple levels of fiscal space. The effect of correlation on the probability of crisis, however, is dependent on the fiscal space.

$$\Delta_{crit}^* = \sigma \left[\Phi^{-1}(l^*) + \Phi^{-1} \left(\frac{\beta}{r + \beta} \right) \right] \approx -13.7499 \quad (\text{B.4})$$

Figure B.3 demonstrates that, if $\Delta^* \geq \Delta_{crit}^*$, the probability of crisis will be unambiguously

increasing in the correlation. If, however, $\Delta^* < \Delta_{crit}^*$, increasing the correlation above zero will result in a reduction in the probability of crisis. As the correlation gets larger, it will eventually begin to cause an increase in the probability of crisis. For changes in correlation to have these effects, the government must be insolvent.

B.1.2 Using public and private noise terminology

Consider a country that has a debt of €480 B and a fiscal capacity of €500 B; in this case, the nation's fiscal space would be €20 B ($\Delta^* = 20$). Suppose also that the standard deviation of noise introduced into public announcements is €30 B ($\nu = 30$). Suppose further that the bonds pay a 7% return ($r = 0.07$), that the government will repay only 25% of the debt in a crisis ($\beta = 75\%$) and that 5% of investors are required to roll-over the debt ($l^* = 0.05$). Consider what happens as the standard deviation of private noise (ζ) goes from zero to infinity.

Suppose that the private noise is zero ($\zeta = 0$) so that the public cutoff is adopted:

$$\kappa_{pub} = \nu \Phi^{-1} \left(\frac{\beta}{r + \beta} \frac{1}{1 - l^*} \right) \approx 53.514 \quad (\text{B.5})$$

The equilibrium cutoff achieves a global minimum of $\kappa^* \approx 53.39$ when $\zeta \approx 2.4$. For low levels of private noise ($\zeta < 2.4$), the equilibrium cutoff is decreasing as the noise increases. Once the noise is above $\zeta = 2.4$, the cutoff unambiguously increases as ζ increases. This is shown in Figures B.4 and B.5.

Computing the probability of a crisis when $\zeta = 0$ is straightforward:

$$p_c(\zeta = 0) = \Phi \left(\frac{\Delta^* - \kappa_{pub}}{\nu} \right) \approx 86.8\% \quad (\text{B.6})$$

As the standard deviation of the private noise component increases (ζ), the probability of a crisis unambiguously decreases from approximately 86.8% to 0%. This is because as ζ gets large, the correlation between signals goes to zero. The equilibrium cutoff scales with the standard deviation of the total noise in the signal so, as the correlation between signals approaches zero, the probability of a crisis approaches zero. The signals, in the limit, approach perfectly private signals.

Contrast this with changing the standard deviation of the public noise; as the public noise gets small, the probability of a crisis will converge to either 0, 0.5, or 1 depending on other parameters in the model. Specifically, there is a threshold value of Δ^* that determines how the probability converged. For low levels of fiscal space (below the threshold value) the probability will converge to 1. For higher levels, it will converge to 0. The threshold value converges to 0.5. This threshold value is given by:

$$\Delta^* = \zeta \left[\Phi^{-1}(l^*) + \Phi^{-1}\left(\frac{\beta}{r + \beta}\right) \right] \approx -8.249914 \dots \quad (\text{B.7})$$

As the standard deviation of the public noise gets large, the probability converges to a finite value:

$$\lim_{\nu \rightarrow \infty} = \frac{\beta}{r + \beta} \frac{1}{1 - l^*} \approx 0.96277 \dots \quad (\text{B.8})$$

Figure B.7 demonstrates the probability of a crisis as a function of ν for different levels of Δ^* , both above and below the threshold level of fiscal space.

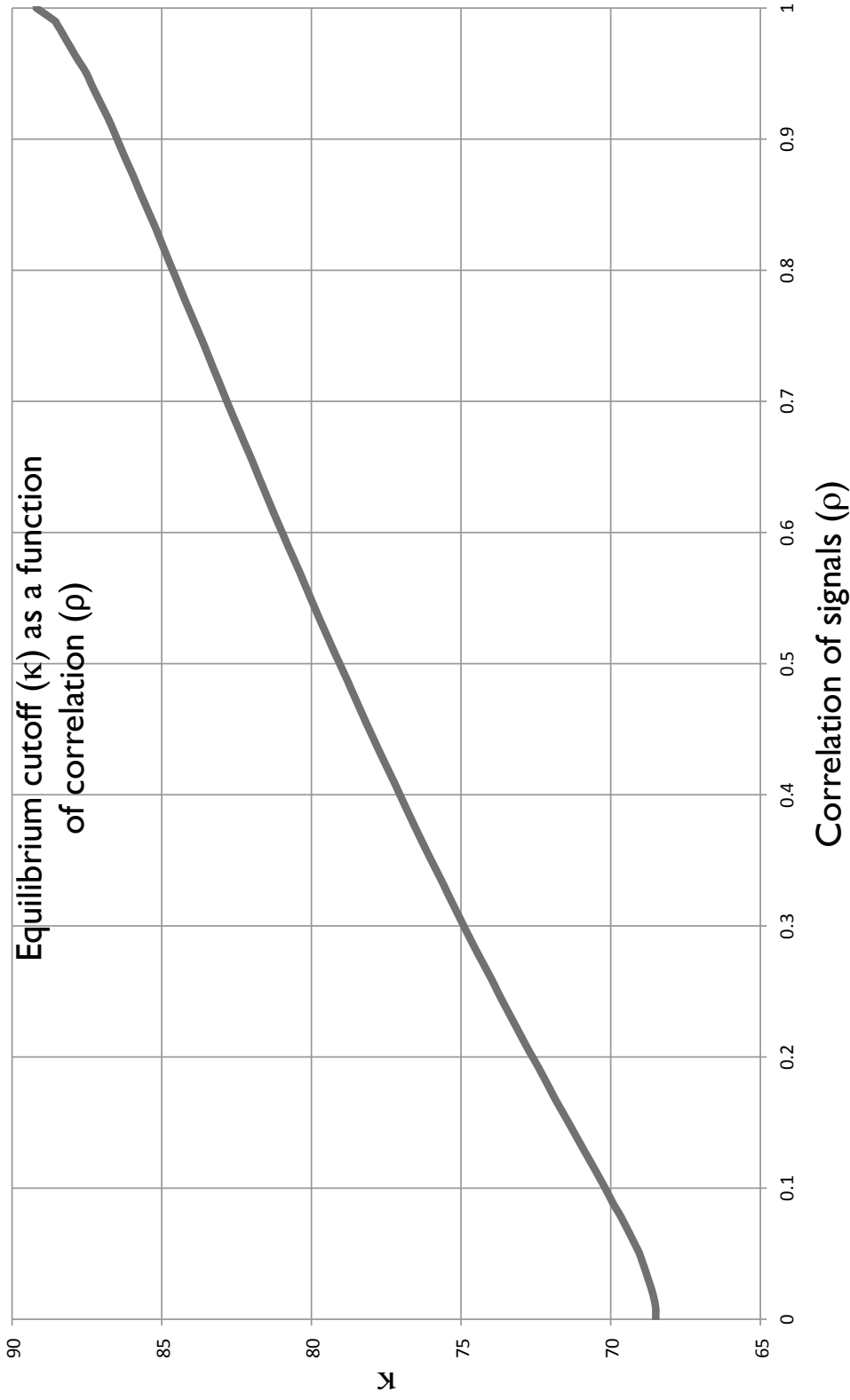


Figure B.1: This graph demonstrates the equilibrium choice of κ^* as a function of the correlation between signals (ρ , on the horizontal axis). This shows that as correlation goes from zero to one, the equilibrium cutoff rises monotonically from the private information case ($\kappa_{priv} \approx 68.5$) to the public information case ($\kappa_{pub} \approx 89.2$). In this numerical example: $\beta = 0.75$; $r = 0.07$; $\sigma = 50$; $l^* = 0.05$. The correlation (ρ) is allowed to vary between zero and 1.

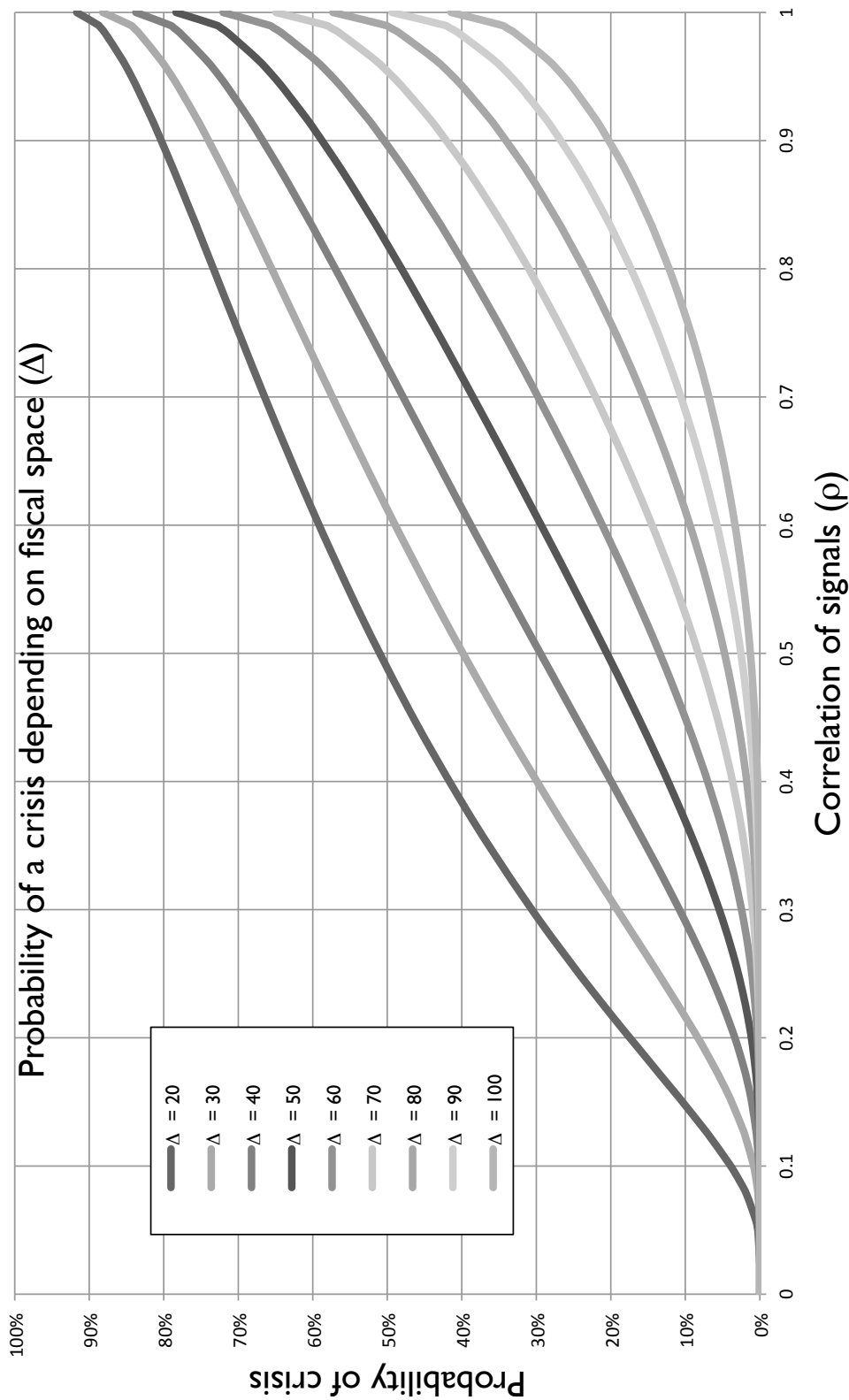


Figure B.2: This graph shows the probability of a crisis as a function of the correlation (ρ) for different levels of fiscal space (Δ^*). It shows that, as the correlation of signals increases, so does the probability of a crisis – even though the sovereign’s fundamentals (its fiscal space, the cost of default, the return on debt, etc.) remain unchanged. This increase in the probability of crisis is caused by two factors. Firstly, with correlated signals, an important degree of diversification is lost. Secondly, investors require more optimistic signals of the sovereign’s solvency in order to roll-over the debt when correlation is higher. The figure also demonstrates that, for every level of correlation, more fiscal space reduces the probability of a debt crisis. In this numerical example: $\beta = 0.75$, $r = 0.07$, $\sigma = 50$, $l^* = 0.05$. The correlation (ρ) is allowed to vary between zero and 1; the different lines show the probability of crisis for different levels of fiscal space, specifically $\Delta^* = \{20, 30, 40, 50, 60, 70, 80, 90, 100\}$

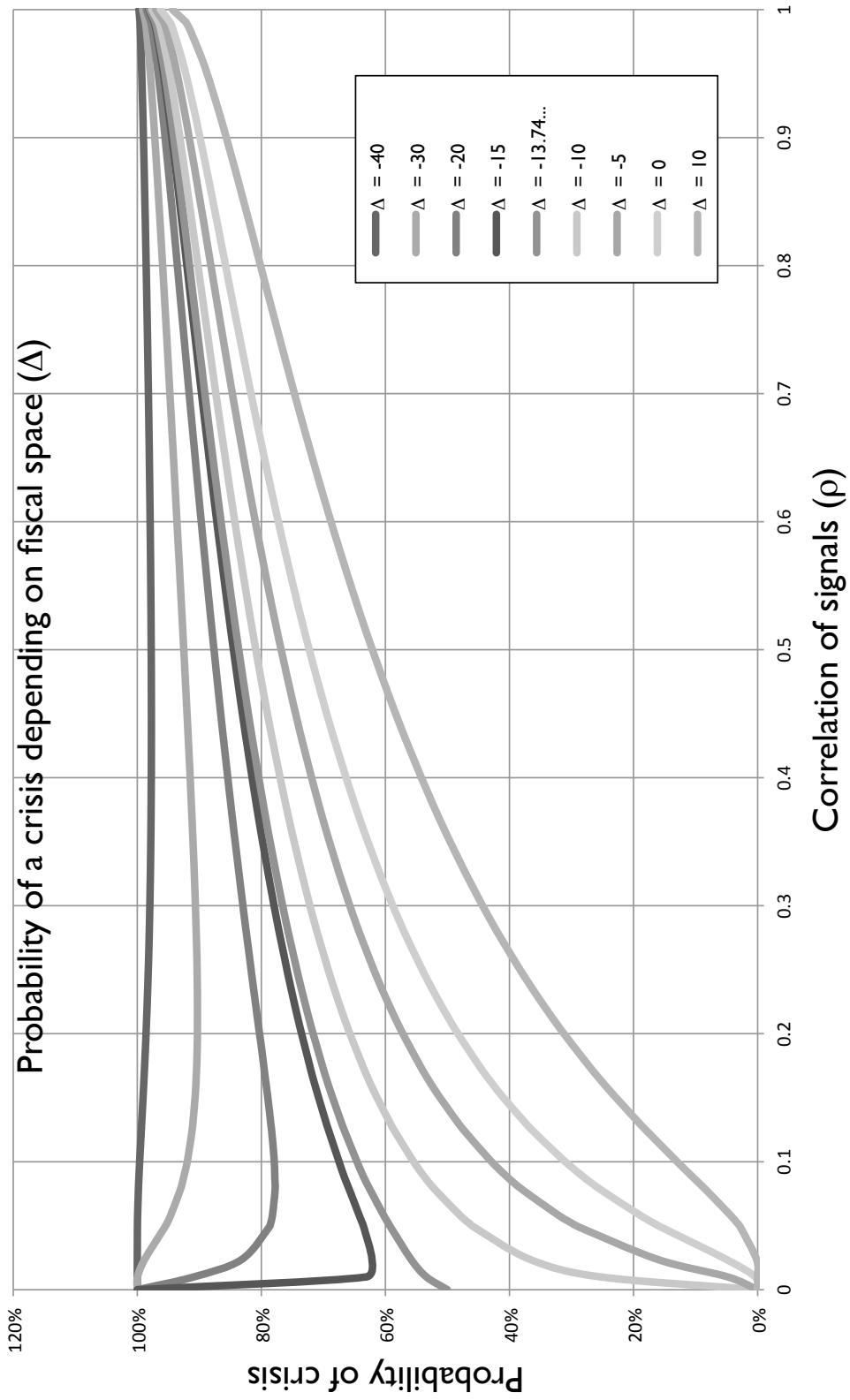


Figure B.3: This graph demonstrates that the effect of correlation on the probability of crisis is a function of the fiscal space Δ^* . When the fiscal space is large (and above some critical threshold -13.7499), increasing the correlation unambiguously increases the probability of default. For fiscal space below this level, the probability of crisis will at first decrease and then increase as correlation gets larger.

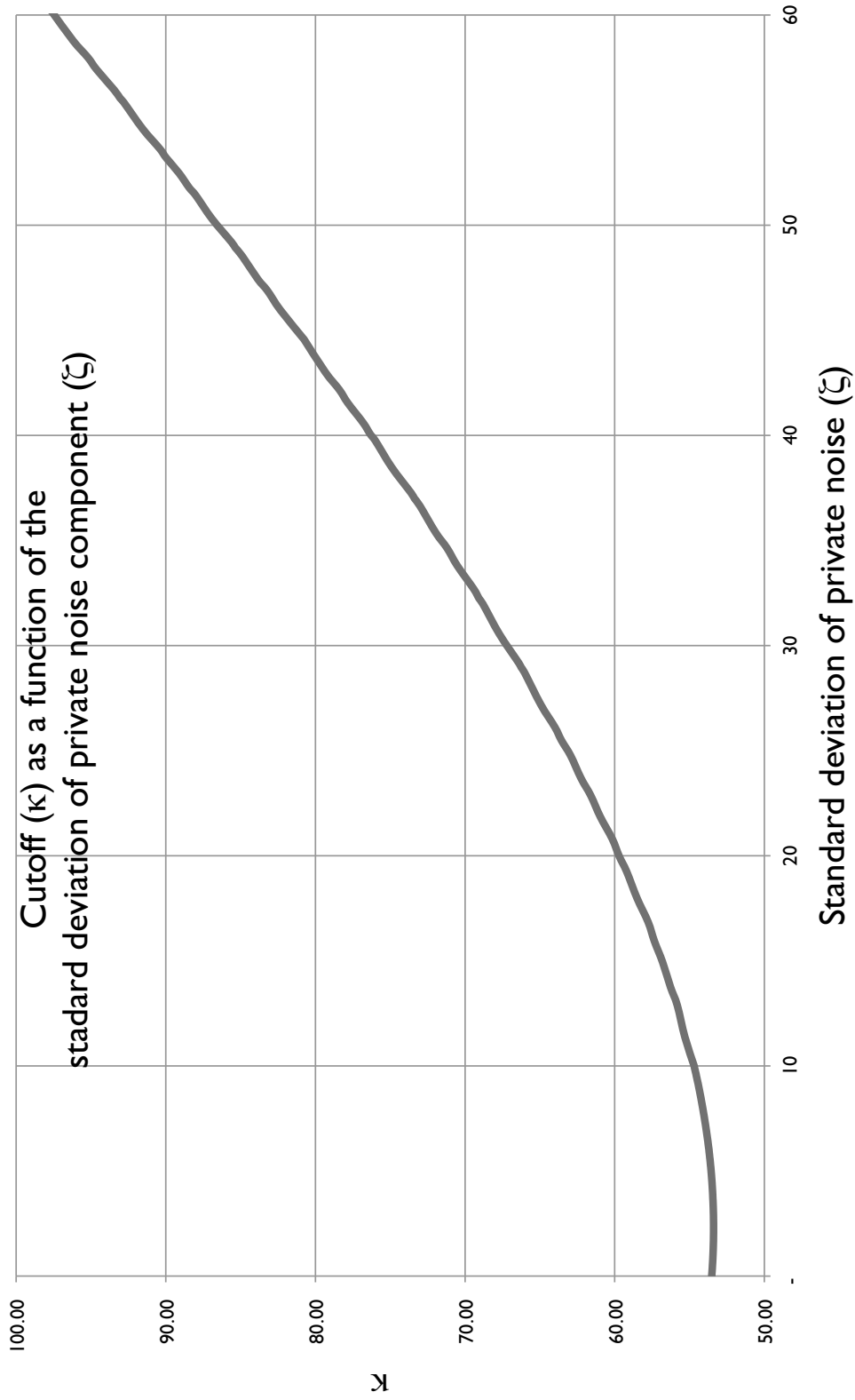


Figure B.4: This graph demonstrates the relationship between the standard deviation of the private noise (ζ) and the equilibrium cutoff selected (κ^*). In this numerical example: $\nu = 30$; $r = 0.07$; $\beta = 75\%$; and $l^* = 0.05$. Despite appearances, the optimal cutoff is not monotonically increasing in ζ but achieves a global minimum of $\kappa \approx 53.39$ when $\zeta \approx 2.4$. This is more clearly shown in Figure B.5.

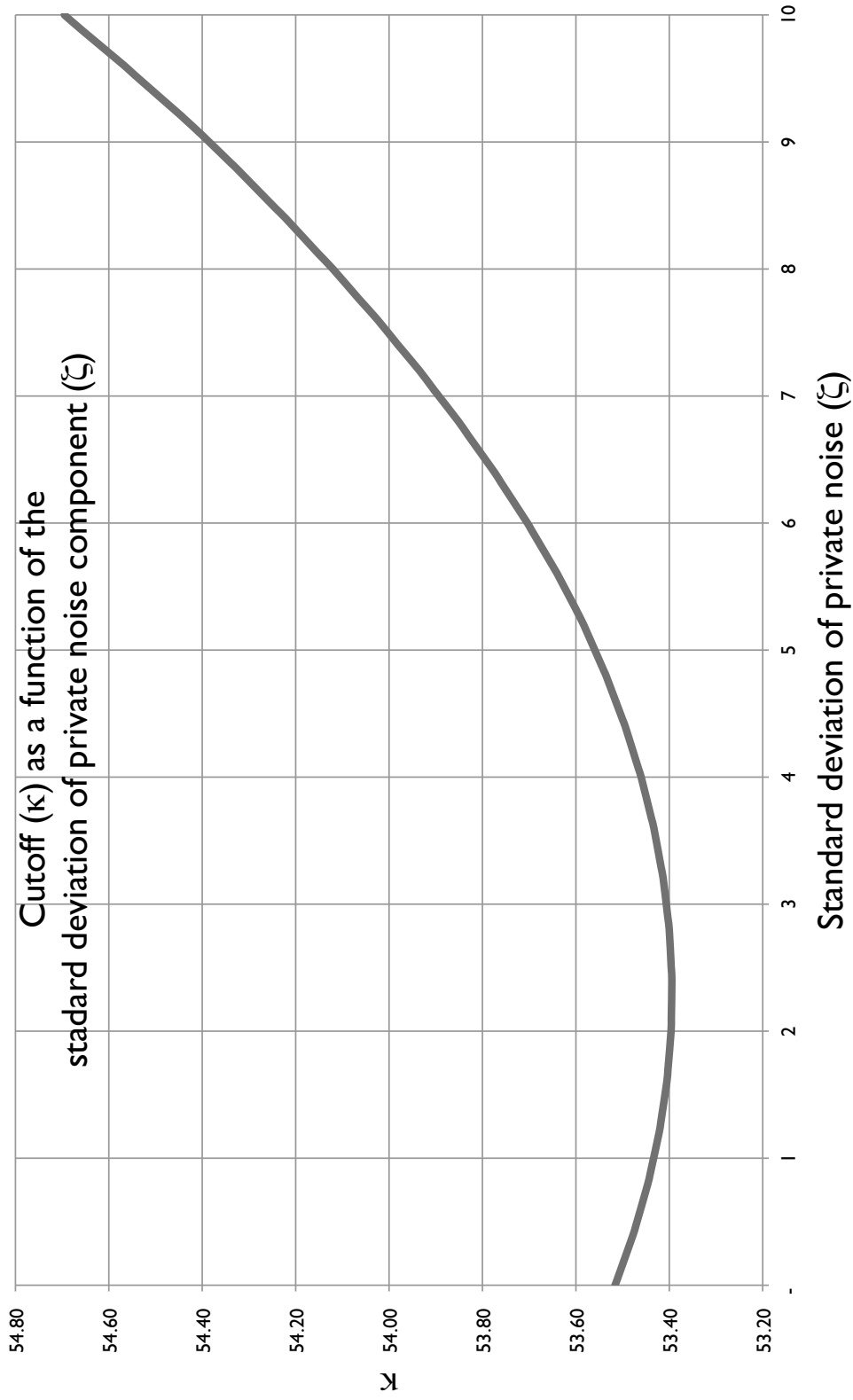


Figure B.5: This graph shows the same numerical example as Figure B.4 but zoomed in around the global minimum. It demonstrates that, for low values of ζ , the cutoff is decreased as ζ increases. The cutoff achieves a global minimum when $\zeta \approx 2.4$ and $\kappa \approx 53.39$.

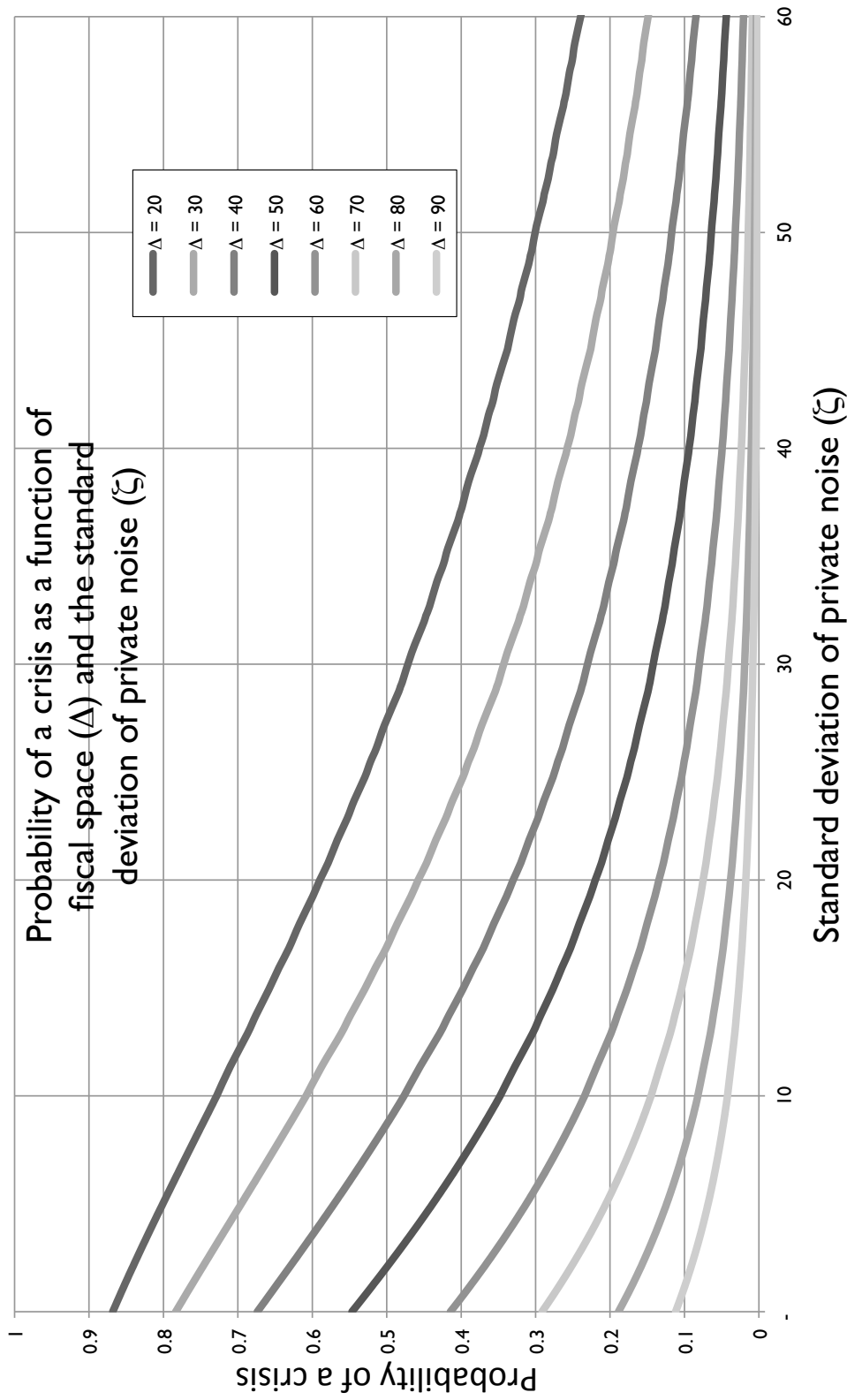


Figure B.6: This graph shows the probability of a crisis as a function of the standard deviation of private noise (ζ) for different levels of fiscal space (Δ^*). It shows that, as the standard deviation of the private-noise component increases, the probability of a crisis falls. The figure also demonstrates that, for every ζ , more fiscal space reduces the probability of a debt crisis. In this numerical example: $\beta = 0.75$; $r = 0.07$; $\nu = 30$; $l^* = 0.05$. The different lines show the probability of crisis for different levels of fiscal space, specifically $\Delta^* = \{20, 30, 40, 50, 60, 70, 80, 90\}$.

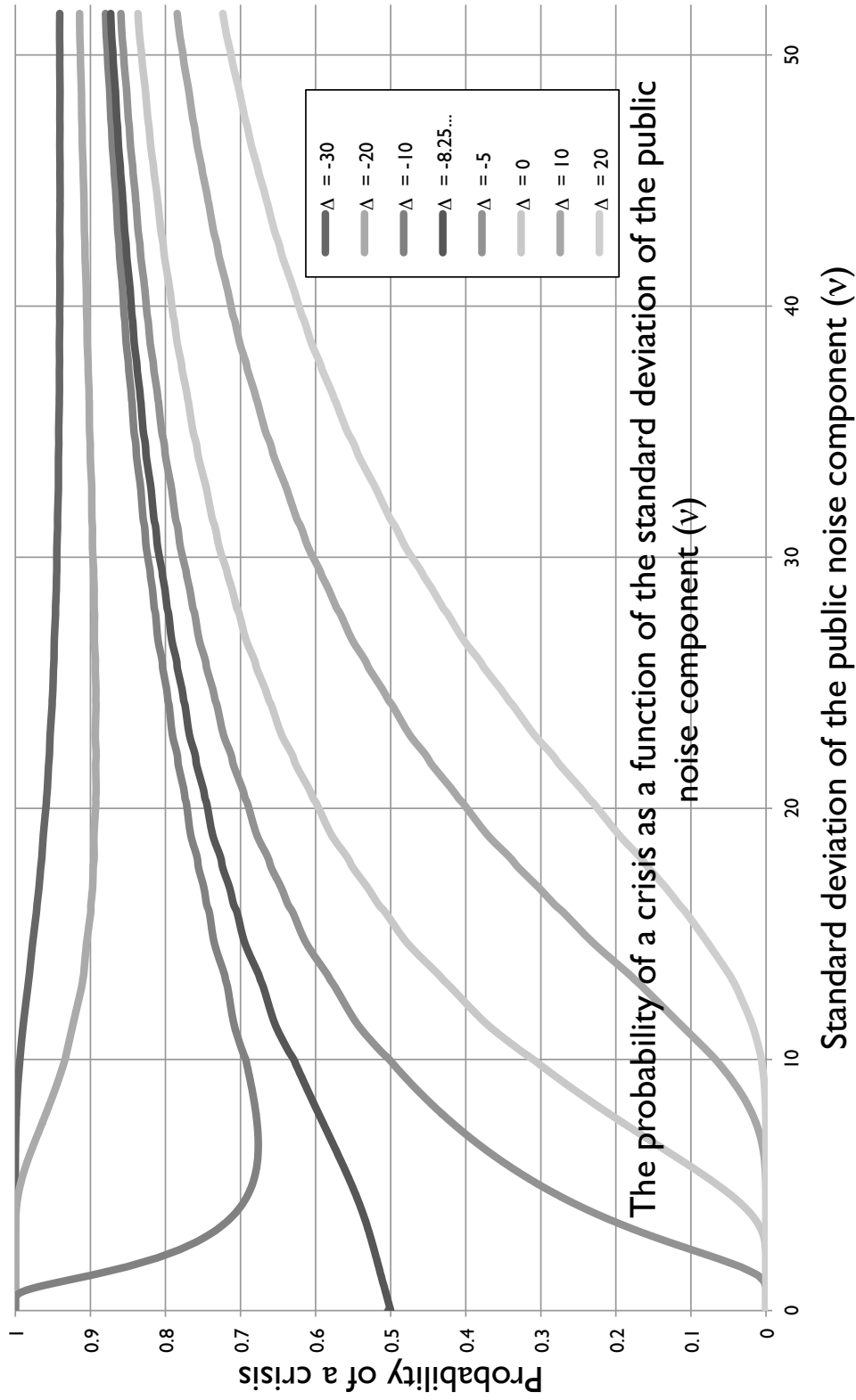


Figure B.7: This graph demonstrates the probability of a crisis as a function of ν and shows that the effect can be quite different depending on the value of Δ^* . There is a critical value of Δ^* that determines whether the probability of a crisis converges to 0, 0.5, or 1 as $\nu \rightarrow 0$. In this numerical example, the critical threshold is given by $\Delta^* \approx 8.2489\dots$. As ν gets large, regardless the value of Δ^* , the probability of crisis will converge to ~ 0.963 . In this numerical example: $\beta = 0.75$; $r = 0.07$; $\zeta = 30$; $l^* = 0.05$. The different lines show the probability of crisis for different levels of fiscal space, specifically $\Delta^* = \{-30, -20, -10, -8.25, -5, 0, 10, 20\}$

Appendix C

Mathematical details and extensions

C.1 Allowing the fiscal authorities to differ in size

In this appendix, I will demonstrate how the two-country game can be modified to allow for countries of different sizes.

C.1.1 The solvency constraint: B^*

Assume that one nation makes up α proportion of the total economy while the other nation controls the remaining $1 - \alpha$. Like in the one authority case, as debt increases, there is a point where the cost of fiscal adjustment outweighs the cost of default. Since both countries have the same institutional framework then they will have the same solvency constraint proportional to the size of their economy.

Using the same notation as in chapter 3, consider the two countries I and G .

Their threshold levels of debt can be related by $B^* = B_I^* = \frac{\alpha}{1-\alpha} B_G'^* = B_G^*$. With such a setup, it is common knowledge whose debt is more onerous (even without knowing the true value of B^*). As before, assume that country I has a more onerous level of debt. For notational convenience, the actual figures pertaining to country G can be denoted with a prime; once they have been projected into the I -space I will drop the prime. Figure C.1 demonstrates this process.

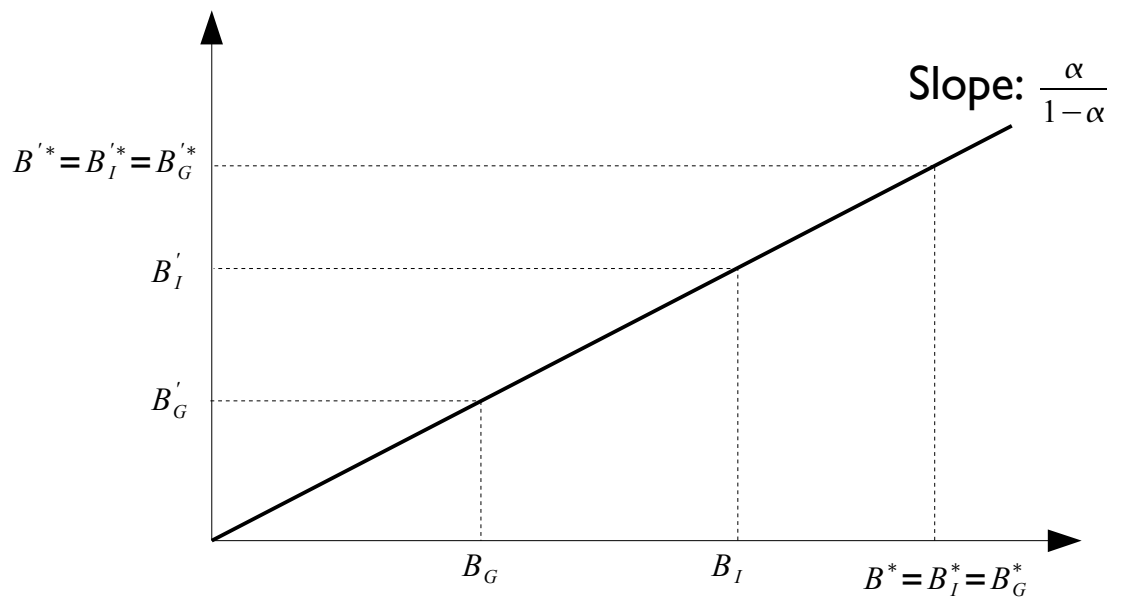


Figure C.1: This figure denotes the method through which we can directly compare the debts of two countries. Country G accounts for α of the combined economy, while country I accounts for the remainder. We project all the relevant variables onto one axis in order to complete our analysis. Without loss of generality, I denote the country with a less onerous debt as G and the country with a more onerous debt as I .

C.2 Chapter 4

C.2.1 Details of Theorem 4.6

$$\frac{\partial I}{\partial \kappa} = \frac{1}{\nu} \int_{-\infty}^{\Phi^{-1}(1-l^*)} \phi(x) \phi\left(\frac{\kappa - \zeta x}{\nu}\right) dx > 0 \quad (\text{C.1})$$

This derivative can be further simplified; consider only the two normal PDFs in the integral:

$$\begin{aligned} \phi(x) \phi\left(\frac{\kappa - \zeta x}{\nu}\right) &= \phi(x) \phi\left(\frac{\zeta x - \kappa}{\nu}\right) \\ &= \frac{1}{2\pi} \exp\left[-\frac{1}{2} \frac{\nu^2 + \zeta^2}{\nu^2} \left(x^2 - 2 \frac{\zeta}{\nu^2 + \zeta^2} \kappa x + \frac{\kappa^2}{\nu^2 + \zeta^2}\right)\right] \\ &= \frac{1}{2\pi} \exp\left[-\frac{1}{2} \frac{\nu^2 + \zeta^2}{\nu^2} \left(x - \frac{\zeta \kappa}{\nu^2 + \zeta^2}\right)^2 - \frac{\kappa^2}{2(\nu^2 + \zeta^2)}\right] \\ &= \frac{1}{2\pi} \exp\left(-\frac{\kappa^2}{2(\nu^2 + \zeta^2)}\right) \exp\left[-\frac{1}{2} \frac{\nu^2 + \zeta^2}{\nu^2} \left(x - \frac{\zeta}{\nu^2 + \zeta^2} \kappa\right)^2\right] \\ &= \underbrace{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\kappa^2}{2(\nu^2 + \zeta^2)}\right)}_{\text{Positive constant}} \phi\left(\frac{\sqrt{\nu^2 + \zeta^2}}{\nu} \left(x - \frac{\zeta \kappa}{\nu^2 + \zeta^2}\right)\right) \end{aligned} \quad (\text{C.2})$$

Thus, it follows that $I(\kappa)$ is strictly increasing in κ :

$$\begin{aligned} \frac{\partial I}{\partial \kappa} &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\kappa^2}{2(\nu^2 + \zeta^2)}\right) \int_{-\infty}^{\Phi^{-1}(1-l^*)} \phi\left(\frac{\sqrt{\nu^2 + \zeta^2}}{\nu} \left(x - \frac{\zeta \kappa}{\nu^2 + \zeta^2}\right)\right) dx \\ &= \frac{\exp\left(-\frac{\kappa^2}{2(\nu^2 + \zeta^2)}\right)}{\sqrt{2\pi(\nu^2 + \zeta^2)}} \Phi\left(\frac{\sqrt{\nu^2 + \zeta^2}}{\nu} \left(\Phi^{-1}(1-l^*) - \frac{\zeta \kappa}{\nu^2 + \zeta^2}\right)\right) > 0 \end{aligned} \quad (\text{C.3})$$

C.2.2 Possible closed-form solutions for κ^*

In the limit, we can obtain closed-form solutions when $\rho \rightarrow 0$ or $\rho \rightarrow 1$. For perfectly private information ($\rho = 0$), the solution becomes:

$$\Phi\left(\frac{\kappa_{\text{priv}}}{\sqrt{\nu^2 + \zeta^2}}\right) = \frac{\beta}{r + \beta} \quad (\text{C.4})$$

which gives:

$$\kappa_{\text{priv}} = \sqrt{\nu^2 + \zeta^2} \Phi^{-1}\left(\frac{\beta}{r + \beta}\right) \quad (\text{C.5})$$

$$= \sigma \Phi^{-1}\left(\frac{\beta}{r + \beta}\right) \quad (\text{C.6})$$

Under perfectly private information, we see that the cutoff is scaled by the standard deviation of the signal (σ). Further, the cutoff is decreasing in r (the benefits of holding debt) and

increasing in β (the cost of default). For perfectly public information, the solution becomes:

$$(1 - l^*)\Phi\left(\frac{\kappa_{\text{pub}}}{\sigma}\right) = \frac{\beta}{r + \beta} \quad (\text{C.7})$$

which gives:

$$\kappa_{\text{pub}} = \sigma\Phi^{-1}\left(\frac{\beta}{r + \beta} \frac{1}{1 - l^*}\right) \quad (\text{C.8})$$

This condition is remarkably similar to the private-information cutoff but includes a very interesting additional term: the coordination cutoff l^* . When full coordination is required (as l^* approaches 1), the cutoff explodes to ∞ . Similarly, when no coordination between investors is required (l^* approaches 0), the cutoff approaches the private information strategy. In general, the public information cutoff will be more conservative than the private-only information one. Formally:

$$\kappa_{\text{priv}} \leq \kappa_{\text{pub}} \quad (\text{C.9})$$

C.3 Chapter 5

C.3.1 Optimization problem with quasi-linear log-utility

C.3.1.1 Parents' economic decision in the first period

The parent's problem in the first period is:

$$\begin{aligned} & \underset{c_1^i, e^i, d^i, t_p}{\text{maximize}} && W_i^P = c_1^i + \alpha \ln(c_L^i) + (1 - \alpha) \ln(c_H^i) + \gamma V(c_k) \\ & \text{subject to:} && c_1^i + e^i + d^i \leq w_1^i - t_p \\ & && c_L^i \leq R^d d^i \\ & && c_H^i \leq R^d d^i + R^e e^i \\ & && c_k \leq w_k - t_k + t_p \\ & && c_1^i \geq 0, c_L^i \geq 0, c_H^i \geq 0, d^i \geq 0, e^i \geq 0, t_p \geq 0 \end{aligned} \quad (\text{C.10})$$

This results in the following KKT complementary slackness conditions.

$$\begin{aligned}
[c_1]: & \quad 1 - \lambda_1 = 0 \quad \text{if } c_1 > 0 \\
[c_L]: & \quad \frac{\alpha}{c_L} - \lambda_2 = 0 \quad \text{if } c_L > 0 \\
[c_H]: & \quad \frac{(1-\alpha)}{c_H} - \lambda_3 = 0 \quad \text{if } c_H > 0 \\
[c_k]: & \quad \gamma V'(c_k) - \lambda_4 = 0 \quad \text{if } c_k > 0 \\
[e^i]: & \quad -\lambda_1 + R^e \lambda_3 = 0 \quad \text{if } e^i > 0 \\
[d^i]: & \quad -\lambda_1 + R^d [\lambda_2 + \lambda_3] = 0 \quad \text{if } d^i > 0 \\
[t_p]: & \quad -\lambda_1 + \lambda_4 = 0 \quad \text{if } t_p > 0 \\
[\lambda_1]: & \quad c_1^i + e^i + d^i + t_p - t_k = w^i \quad \text{if } \lambda_1 > 0 \\
[\lambda_2]: & \quad R^d d^i = c_L \quad \text{if } \lambda_2 > 0 \\
[\lambda_3]: & \quad R^e e^i + R^d d^i = c_H \quad \text{if } \lambda_3 > 0 \\
[\lambda_4]: & \quad w_k - t_k + t_p = c_k \quad \text{if } \lambda_4 > 0
\end{aligned} \tag{C.11}$$

These give the necessary criteria for an optimal solution. In this problem, it is easy to check that the consumption of c_L and c_H must always be positive (as long as $w^i > 0$). For the following, I will assume that the parents do not make any transfers to the child. This then implies (assuming $w_k - t_k > 0$):

$$\begin{aligned}
t_p = 0 & \implies \lambda_1 \geq \lambda_4 \\
& \implies \begin{cases} 1 \geq \gamma V'(c_k) & \text{if } w^i > 1 \\ \frac{1}{w^i} \geq \gamma V'(c_k) & \text{if } w^i \leq 1 \end{cases} \tag{C.12}
\end{aligned}$$

If the parent does not send the child any transfer, this is because the marginal utility of consumption to the parent in the first period is higher than the marginal utility (to the parent) of giving more resources to the child. Given the quasi-linear preferences, the parent's marginal utility of consumption in the first period is always greater than or equal to 1. It follows that a sufficient condition for no transfers to the children is:

$$1 \geq \gamma V'(w_k - t_k) \tag{C.13}$$

C.3.1.2 Interior solution

Suppose we have a solution in which $t_p = 0$ and $c_1 > 0$; it then follows that $\lambda_1 = 1$. This then implies that the unique solution to the system (C.11) is:

$$\begin{aligned}
 c_1^* &= w^i - 1 \\
 c_L^* &= \frac{\alpha R^e R^d}{R^e - R^d} \\
 c_H^* &= (1 - \alpha) R^e \\
 d^* &= \frac{\alpha R^e}{R^e - R^d} \\
 e^* &= 1 - \frac{\alpha R^e}{R^e - R^d} \\
 t_p^* &= 0 \\
 \lambda_1^* &= 1 \\
 \lambda_2^* &= \frac{R^e - R^d}{R^e R^d} \\
 \lambda_3^* &= \frac{1}{R^e} \\
 \lambda_4^* &= \gamma V'(w_k - t_k)
 \end{aligned} \tag{C.14}$$

It is worth noting that $e^* + d^* = 1$. In order for this solution to be valid, the first-period income must be sufficiently large: $w^i > 1$. If this does not hold, there is a corner solution in which the agent consumes nothing in the first period.

C.3.1.3 Corner solution

If there is a corner solution, $c_1 = 0$. This then implies that λ_1 is not necessarily equal to 1. Solving the system (C.11) then yields the unique solution:

$$\begin{aligned}
c_1^* &= 0 \\
c_L^* &= \frac{\alpha R^e R^d}{R^e - R^d} w^i \\
c_H^* &= (1 - \alpha) R^e w^i \\
d^* &= \frac{\alpha R^e}{R^e - R^d} w^i \\
e^* &= 1 - \frac{\alpha R^e}{R^e - R^d} w^i \\
t_p^* &= 0 \\
\lambda_1^* &= \frac{1}{w^i} \\
\lambda_2^* &= \frac{R^e - R^d}{R^e R^d} \frac{1}{w^i} \\
\lambda_3^* &= \frac{1}{R^e} \frac{1}{w^i} \\
\lambda_4^* &= \gamma V'(w_k - t_k)
\end{aligned} \tag{C.15}$$

It is worth noting that $e^* + d^* = w^i$; the savings completely exhaust the first-period income. This solution is valid as long as $0 < w^i \leq 1$.

C.3.1.4 Parent's solution in the first period

We can use the above solutions to determine how the parent will behave, conditional on their income:

$$\begin{aligned}
c_1^*(w^i) &= \max(0, w^i - 1) \\
c_L^*(w^i) &= \min\left(\frac{\alpha R^e R^d}{R^e - R^d} w^i, \frac{\alpha R^e R^d}{R^e - R^d}\right) \\
c_H^*(w^i) &= \min((1 - \alpha) R^e w^i, (1 - \alpha) R^e) \\
d^*(w^i) &= \min\left(\frac{\alpha R^e}{R^e - R^d} w^i, \frac{\alpha R^e}{R^e - R^d}\right) \\
e^*(w^i) &= \min\left(\frac{(1 - \alpha) R^e - R^d}{R^e - R^d} w^i, \frac{(1 - \alpha) R^e - R^d}{R^e - R^d}\right)
\end{aligned} \tag{C.16}$$

As a result of assuming time-separable quasi-linear log-preferences, consumption in the first period is a luxury good; conversely, saving is a necessary good. For low levels of income ($w^i < 1$), agents will dedicate all their income to saving. Once income is sufficiently high ($w^i \geq 1$), the agent then allocates any marginal income to consumption. Further, parents will allocate their portfolios to debt and equity in fixed proportions, independent of income. Again, this follows from CES utility, a class of risk preferences of which log-utility is a special

case. In order for the parents to allocate any of a portfolio to bank equity, the risk-neutral expected return on equity $((1 - \alpha)R^e)$ must be greater than the return on safe debt (R^d) , which should be intuitive.

C.3.2 The child's economic decision

The child's problem in the second period is conditional on the parent's second-period income w_2^i :

$$\begin{aligned}
 & \underset{c_k, t_k}{\text{maximize}} && W_i^K = V(c_k) + \delta \ln(c_2) \\
 & \text{subject to:} && c_k \leq w_k - t_k - \tau \\
 & && c_2^i \leq w_2^i + t_k \\
 & && c_k \geq 0, t_k \geq 0
 \end{aligned} \tag{C.17}$$

This optimization has the following KKT complementary slackness conditions:

$$\begin{aligned}
 [c_k]: & \quad V'(c_k) - \lambda_1 = 0 \quad \text{if } c_k > 0 \\
 [c_2]: & \quad \frac{\delta}{c_2} - \lambda_2 = 0 \quad \text{if } c_2 > 0 \\
 [t_k]: & \quad -\lambda_1 + \lambda_2 = 0 \quad \text{if } t_k > 0 \\
 [\lambda_1]: & \quad w_k - t_k - \tau = c_k \quad \text{if } \lambda_1 > 0 \\
 [\lambda_2]: & \quad w_2^i + t_k = c_2 \quad \text{if } \lambda_2 > 0
 \end{aligned} \tag{C.18}$$

If the child is does not give the parent a transfer, it must be the case that:

$$V'(w_k - \tau) > \frac{\delta}{c_2} \tag{C.19}$$

For no children ever to want to transfer resources to their parents, the marginal utility of consumption for a child must be greater than the weighted marginal utility of the poorest parent in the bust state. A sufficient condition for no children to transfer resources to their parents directly is:

$$V'(w_k - \tau) \geq \delta \frac{R^e - R^d}{\alpha R^e \underline{w}} \tag{C.20}$$

If this is the case, the childred optimally consume all their income:

$$\begin{aligned}
 c_k^* &= w_k \\
 t_k^* &= 0
 \end{aligned} \tag{C.21}$$

C.3.3 The irrelevance of bank leverage

If we consider a more primitive form of the model presented in this chapter, it is easy to demonstrate that leverage is irrelevant. Instead of a bank choosing leverage, suppose that the parents simply have access to the bank's investment technology. That is, they can invest in a technology that pays R with probability $(1-\alpha)$ and $(1-p)R$ with probability α . Leverage has no meaning in this instance since the parent is exposed only to the underlying asset.

$$\begin{aligned}
 & \underset{c_1^i, s^i}{\text{maximize}} && W_i^P = u(c_1^i) + E[u(c_2^i)] \\
 & \text{subject to:} && c_1^i + s^i \leq w_1^i \\
 & && c_2^i \leq (1-zp)R \\
 & && s^i \geq 0, c_1^i \geq 0, c_2^i \geq 0
 \end{aligned} \tag{C.22}$$

The parent then decides how much to consume in the first period (c_1^i) and how much to save (s^i); the dummy variable $z = 1$ when there is a recession (with probability α) and 0 otherwise.

The solution to this optimization implies that poor parents ($0 < w^i < 1$) will save all their income, while the rich ($w^i \geq 1$) will save a threshold amount ($s^* = 1$) and consume the remainder. This is the same solution that was found in the model presented in this chapter; labelling different parts of the underlying asset 'debt' and 'equity' has no fundamental economic meaning unless they are treated differently in insolvency. The choice of leverage is underdetermined.

This is why, in the model presented in this chapter, I have assumed that the banks seek to maximize the return on equity (as opposed to economic profit). This provides the bankers with an incentive to increase the leverage of their institution (*ceteris paribus*). Given the compensation schemes of many major financial institutions, this doesn't seem like an absurd assumption.

The banker's assumed objective in conjunction with the liquidation procedure (see Section 5.3.2.4) and the possibility of a bailout implies that leverage will be economically meaningful in this model.

Appendix D

General theorems and definitions

Some of the following theorems and definitions are referenced in the text. These are as stated by Levin and Randel (2001).

D.1 Games

Definition D.1. A strategy vector σ^* is a **Nash Equilibrium** if for each player i and each strategy σ_i of player i :

$$U_i(\sigma^*) \geq U_i(\sigma_i, \sigma_{-i}^*) \quad (\text{D.1})$$

Definition D.2. A **Bayesian game** consists of the tuple (N, S, Θ, p, u) :

- N is a set of players.
- $S_i \in S$ is a set of actions for each player i .
- $\theta_i \in \Theta$ is a type for each player i .
- $u = (u_1, \dots, u_n)$, where $u_i : S \times \Theta \rightarrow \mathbb{R}$ is a payoff function for each i .
- $p(\theta_1, \dots, \theta_n)$ is a (joint) probability distribution over types.

Remark. Note that we can apply Definition D.2 more generally, allowing for each player who has a different signal to be considered a type.

Definition D.3. The strategy profile $s(\cdot)$ is a **Bayesian Nash Equilibrium** if, for all $i \in N$ and for all $\theta_i \in \Theta$, we have that:

$$s_i(\theta_i) \in \arg \max_{s'_i \in S_i} \sum_{\theta_{-i}} p(\theta_{-i} | \theta_i) u_i(s'_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) \quad (\text{D.2})$$

Alternatively in the non-finite case:

$$s_i(\theta_i) \in \arg \max_{s'_i \in S_i} \int u_i(s'_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) P(d\theta_{-i} | \theta_i) \quad (\text{D.3})$$

Definition D.4. The game $(S_1, \dots, S_n; u_1, \dots, u_n)$ is a **supermodular game** if for all i :

- S_i is a compact subset of \mathbb{R} .
- u_i is upper-semi-continuous in (s_i, s_{-i}) .
- u_i has increasing differences in (s_i, s_{-i}) .

Definition D.5. A **Harsanyi game with incomplete information** is a vector:

$$(N, (T_i)_{i \in N}, p, S, (s_t)_{t \in \chi_{i \in N} T_i})$$

where:

- N is a finite set of players.
- T_i is a finite set of types for player i , for each $i \in N$. The set of type vectors is denoted by $T = \chi_{i \in N} T_i$.
- $p \in \Delta(T)$ is a probability distribution over the set of type vectors that satisfies $p(t_i) := \sum_{t_{-i} \in T_{-1}} p(t_i, t_{-i}) > 0$ for every player $i \in N$ and every type $t_i \in T_i$.
- S is a set of states of nature, which will be called **state games**. Every state of nature $s \in S$ is a vector $s = (N, (A_i)_{i \in N}, (u_i)_{i \in N})$, where A_i is a nonempty set of actions of player i and $u_i : \chi_{i \in N} A_i \rightarrow \mathbb{R}$ is the payoff function of player i .

D.2 The theorem of maximum

For parametric constrained optimization problems, The Theorem of Maximum states (see Levin and Randel (2001)):

Theorem D.1. (The Theorem of Maximum) *Consider the class of parametric constrained optimization problems:*

$$\max_{x \in D(\theta)} f(x, \theta) \quad (\text{D.4})$$

This is defined over the set of parameters Θ . Suppose that (i) $D : \Theta \rightarrow X$ is continuous and compact-valued, and (ii) $f : X \times \Theta \rightarrow \mathcal{R}$ is a continuous function. Then:

1. $x^*(\theta)$ is non-empty for every θ .
2. x^* is upper semi-continuous.
3. V (the value function) is continuous.

D.3 Kuhn-Tucker

Definition D.6. Consider a point x that satisfies constraints $g_k(x, \theta) \leq b_k$ for all k . We say **constraint k binds at x** if $g_k(x, \theta) = b_k$ and is **slack** if $g_k(x, \theta) < b_k$. If $B(x)$ denotes the set of binding constraints at point x , then **constraint qualification holds at x** if the vectors in the set $\{Dd_k(x, \theta) | k \in B(x)\}$ are linearly independent.

We use this to state the KT theorem:

Theorem D.2. (Kuhn-Tucker) *Suppose that for the parameter θ , the following conditions hold: (i) $f(\cdot, \theta)$, $g_1(\cdot, \theta), \dots, g_K(\cdot, \theta)$ are continuously differentiable in x ; (ii) $D(\theta)$ is non-empty; (iii) x^* is a solution to the optimization problem; and (iv) constraint qualification holds at x^* . Then:*

1. *There exist non-negative numbers $\lambda_1, \dots, \lambda_K$ such that:*

$$Df(x^*, \theta) = \sum_{k=1}^K \lambda_k Dg_k(x^*, \theta) \quad (\text{D.5})$$

2. *For $k = 1, \dots, K$,*

$$\lambda_k (b_k - g_k(x^*, \theta)) = 0 \quad (\text{D.6})$$

Theorem D.3. (Kuhn-Tucker with non-negativity constraints) *Consider the constrained optimization problem with non-negativity constraints. Suppose that for parameter θ the following conditions hold: (i) $f(\cdot, \theta)$, $g_1(\cdot, \theta)$, ..., $g_k(\cdot, \theta)$ are continuously differentiable; (ii) $D(\theta)$ is non-empty; (iii) x^* is a solution to the optimization problem; (iv) constraint qualification holds at x^* for all the constraints (including any binding non-negativity constraints). Then:*

1. *There are numbers $\lambda_1, \dots, \lambda_I$ such that*

$$\frac{\partial f(x^*, \theta)}{\partial x_j} + \mu_j = \sum_{k=1}^I \lambda_k \frac{\partial g_k(x^*, \theta)}{\partial x_j} \quad \forall j = 1, \dots, n \quad (\text{D.7})$$

2. For $k = 1, \dots, I$:

$$\lambda_k(b_k - g_k(x^*, \theta)) = 0 \quad (\text{D.8})$$

3. For $j = 1, \dots, n$

$$\mu_j x_j^* = 0 \quad (\text{D.9})$$

Lemma D.4. *If we suppose that the conditions of the Kuhn-Tucker theorem are satisfied and that (i) $f(\cdot, \theta)$ is quasi-concave and (ii) $g_1(\cdot, \theta), \dots, g_K(\cdot, \theta)$ are quasi-convex, any point x^* that satisfies the Kuhn-Tucker conditions is a solution to the constrained optimization problem.*

D.4 The implicit function theorem

Consider a system of equations of the form:

$$h_k(x, \theta) = 0 \quad \forall k = 1, \dots, n \quad (\text{D.10})$$

where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $\theta = (\theta_1, \dots, \theta_s) \in \mathbb{R}^s$. We can think of x as the variables and θ as the parameters. Consider a solution \hat{x} of the system at $\hat{\theta}$. We say that the system can be locally solved at $(\hat{x}, \hat{\theta})$ if there exists some uniquely determined function $\eta: A \rightarrow B$ such that for all $\theta \in A$:

$$h_k(\eta(\theta), \theta) = 0 \quad \forall k = 1, \dots, n \quad (\text{D.11})$$

We call η the implicit solution of the system.

Theorem D.5. (The implicit function theorem) *Consider the system of equations and suppose that (i) $h_k(\cdot)$ is continuously differentiable at (x, θ) for all k ; (ii) \hat{x} is a solution at $\hat{\theta}$; and (iii) the matrix $D_x h(\hat{x}, \hat{\theta})$ is non-singular:*

$$\text{rank} \begin{pmatrix} D_x h_1(\hat{x}, \hat{\theta}) \\ \dots \\ D_x h_n(\hat{x}, \hat{\theta}) \end{pmatrix}_{n \times n} = n \quad (\text{D.12})$$

Then:

1. The system can be locally solved at $(\hat{x}, \hat{\theta})$.

2. The implicit function η is continuously differentiable and

$$D_{\theta}\eta(\hat{\theta}) = -[D_x h(\hat{x}, \hat{\theta})]^{-1} D_{\theta} h(\hat{x}, \hat{\theta}) \quad (\text{D.13})$$

D.5 Topkis's Monotonicity Theorem

Theorem D.6. (Topkis's Monotonicity Theorem) *If f is supermodular in (x, θ) then $x^*(\theta) = \arg \max_{x \in D}$ is nondecreasing.*

Notes

Chapter I. Introduction

1. *Lehman Files for Bankruptcy; Merrill Is Sold*, New York Times, September 15, 2008. <http://www.nytimes.com/2008/09/15/business/15lehman.html>
2. Bank of England News Release: Coordinated Central Bank Action to Improve US\$ Liquidity, <http://www.bankofengland.co.uk/archive/Documents/historicpubs/news/2008/054.pdf>
3. ECB Press Release, 15 October, 2008. <https://www.ecb.europa.eu/press/pr/date/2008/html/pr081015.en.html>
4. ECB Press Release, 7 May, 2009. https://www.ecb.europa.eu/press/pr/date/2009/html/pr090507_2.en.html
5. *Revised Greek figures cause outrage*, EU Observer, 20 October 2009. <http://euobserver.com/economic/28853>
6. EU, IMF agree \$147 billion bailout for Greece, 2 May 2010, Reuters. <http://www.reuters.com/article/2010/05/02/us-eurozone-idUSTRE6400PJ20100502>
7. See <http://www.efsf.europa.eu/about/index.htm>:

The European Financial Stability Facility (EFSF) was created by the euro area Member States following the decisions taken on 9 May 2010 within the framework of the Ecofin Council. The EFSF's mandate is to safeguard financial stability in Europe by providing financial assistance to euro area Member States within the framework of a macro-economic adjustment programme.

And http://www.efsf.europa.eu/attachments/20111019_efsf_framework_agreement_en.pdf.

8. ECB Press Release, 10 May 2010. <https://www.ecb.europa.eu/press/pr/date/2010/html/pr100510.en.html>
9. ECB Press Release, 30 June 2010. <https://www.ecb.europa.eu/press/pr/date/2010/html/pr100630.en.html>
10. See Irish National Treasury Management Agency website: <http://www.ntma.ie/business-areas/funding-and-debt-management/euimf-programme/>

The Government agreed, on 28 November 2010, to a three-year financial support programme for Ireland by the EU and IMF. External support amounted to €67.5 billion.

11. ECOFIN statement, 16 May 2011. https://www.consilium.europa.eu/uedocs/cms_data/docs/pressdata/en/ecofin/122011.pdf

12. *Ireland's bailout interest rate reduced by EU leaders*, BBC, 22 July 2011. <http://www.bbc.co.uk/news/world-europe-14245861>

European leaders in Brussels agreed on Thursday evening to lower the rate from around 6% to between 3.5% and 4% as part of a wider EU debt deal.

The length of time to pay back the loan has also been extended from seven and a half years to 15 years.

13. EU leaders agree €109bn Greek bail-out, FT, 21 July 2011. <http://www.ft.com/cms/s/0/952e0326-b3af-11e0-855b-00144feabdc0.html>

14. ECB Press Release, 9 September 2011. <https://www.ecb.europa.eu/press/pr/date/2011/html/pr110909.en.html>

15. ECB Press Release, 6 October 2011. https://www.ecb.europa.eu/press/pr/date/2011/html/pr111006_3.en.html

16. *Merkel: 50 pct haircut to cut Greek debt by 100 bln euros*, Reuters, 27 October 2011. <http://www.reuters.com/article/2011/10/27/eu-greece-programme-idUSL5E7LR02120111027>

Private owners of Greek bonds will accept a 50 percent writedown on their investment, enabling both a 100 billion euro cut in Greece's sovereign debts and allowing a new Greek programme of aid of 100 billion euros, German Chancellor Angela Merkel said on Thursday.

17. ECB Press Release, 8 December 2011. https://www.ecb.europa.eu/press/pr/date/2011/html/pr111208_1.en.html

18. *The wait is over*, The Economist, March 17 2011. <http://www.economist.com/node/21550271>

19. ECB Speeches: Verbatim remarks by Mario Draghi at the Global Investment Conference in London, 26 July 2012. <https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html>

At the event, Mario Draghi stated (punctuation in original):

When people talk about the fragility of the euro and the increasing fragility of the euro, and perhaps the crisis of the euro, very often non-euro area member states or leaders, underestimate the amount of political capital that is being invested in the euro.

And so we view this, and I do not think we are unbiased observers, we think the euro is irreversible. And it's not an empty word now, because I preceded saying exactly what actions have been made, are being made to make it irreversible.

But there is another message I want to tell you.

Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.

20. ECB Introductory statement to the press conference, 2 August 2012. <https://www.ecb.europa.eu/press/pressconf/2012/html/is120802.en.html>

21. ECB Press Release, 6 September 2012. https://www.ecb.europa.eu/press/pr/date/2012/html/pr120906_1.en.html
22. *Note Verbale*, Council of the European Union General Secretariat, 27 September 2012. http://www.efsf.europa.eu/attachments/esm_entry_into_force.pdf

Chapter 2. A global games approach to debt crises

1. Although the German Federal Court (Das Bundesverfassungsgericht) does not have jurisdiction to rule directly on the question of EU law it made the unprecedented move to refer the question of the legality of OMT directly to the European Court of Justice. The German court noted that the prohibition on monetary financing of sovereign debt (Art. 123) could not be circumvented using functionally equivalent measures such as using neutralizing interest rate spreads, selective bond purchases, and the wedding of ECB policy to the ESM or EFSF. As such, the reference states that OMT aims at circumventing the prohibition on monetary financing and is incompatible with German primary law. They challenge the ECJ to provide an alternative interpretation of the programme so that it can be ‘interpreted in conformity with primary law’.

For a critical discussion regarding the constitutionality of OMT that predates the German decision see Yowell (2014). The German court’s decision (in English) can be found here http://www.bverfg.de/entscheidungen/rs20140114_2bvr272813en.html

2. As originally described in Keynes (1936), a ‘beauty contest’ is a game where payoffs are a function of the average play. He noted:

“[P]rofessional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one’s judgment, are really the prettiest, nor even those which the average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.”

This type of reasoning has been formalized with Level-k Thinking or, as Camerer et al. (2004) call it, a “cognitive hierarchy” model.

3. The notion of a sovereign’s willingness to pay is explicitly discussed in Eaton and Gersovitz (1981). The paper examines the effect that exclusion from capital markets has on incentives to repay. The model implies that lenders will set a debt ceiling for the borrower, which is determined by the borrower’s willingness to repay and above which they will be unwilling to lend.

4. In referring to Section 2.4.1, simply replace B by B_l . Under this construction, the logical reasoning is exactly the same. One agent changing strategy will have no effect on the aggregate B_l – it simply demonstrates that any equilibrium that does not choose α appropriately is untenable.

Chapter 3. A debt game with multiple fiscal authorities

1. *Why the eurozone will survive*, The Financial Times, March 8, 2011. <http://www.ft.com/cms/s/0/5030759e-49bd-11e0-acf0-00144feab49a.html>

2. From Woodford (1995, 1999), this raises an issue regarding the ‘Fiscal Theory of the Price Level’, where an economy can be in one of two regimes. There exists a ‘Ricardian’ equilibrium where the Central Bank controls inflation; government bonds are not net wealth and the fiscal authority responds to monetary policy. If, however, a Central Bank can be forced to intervene whenever investors do not want to purchase government debt, this provides a mechanism for the Treasury to force the Central Bank to finance its debt. The economy could end up in the fiscal dominant outcome (*i.e.* a ‘non-Ricardian’ regime), where monetary policy reacts (passively) to monetize the government’s debt. Leith and Wren-Lewis (2006) examines the consequences of this problem in a monetary union and shows that the Central Bank can act to stabilize the debt of one of the member countries. There are equilibria where active monetary policy is maintained, sometimes as a result of fiscal transfers between the members of the union in order to maintain national solvency. There also exists a passive monetary outcome where the Central Bank ensures the solvency of the country running destabilizing fiscal policy, while the other country ensures the stability of its own debt stock. Fully exploring these outcomes is beyond the scope of this chapter.

3. It is this particular strategy that implies the strategy from theorem 3.2 is not unique. It follows that we do not have a unique equilibrium for every point in the parameter space for this model. It is straightforward but tedious to demonstrate that the alternative equilibrium is not qualitatively different in the conclusions; due to space considerations, this is omitted.

Chapter 4. A debt game with correlated information

1. There are several reasons why the sovereign would not be able to repay debts above some threshold level. For instance, it could be that the political system is incapable of increasing future surpluses sufficiently (and the monetary authority is committed to not monetizing the debt) or because the NPV of defaulting is simply less than the cost of honouring the debts. Tabellini (1989) provides a good discussion on how honouring sovereign debt is a function of intergenerational politics in that the older generation issues the debt to redistribute consumption in their favour at the expense of future generations. This process, however, cannot be continued without limit: under certain conditions, it would be optimal for the young to default on the debts to the older generation.

2. It is worth noting that it is only reasonable to consider the cases in which $0 < \rho < 1$. It does not make sense to consider the cases in which signals are negatively correlated because a necessary condition for the equicorrelated normal distribution to be well-defined is:

$$\frac{-1}{n-1} < \rho < 1 \quad (\text{D.14})$$

where n is the number of dimensions in the multivariate normal. In our case, $n \rightarrow \infty$ since we are assuming a continuum of investors. It follows from this that the correlation must be bounded: $0 < \rho < 1$. For $\rho = 0$ or $\rho = 1$, the signal is either perfectly private or perfectly public respectively.

3. The step from (4.22) to (4.23) is an application of Lemma 1 from Gupta and Pillai (1965):

For any real numbers α and β :

$$\int_{-\infty}^{\infty} \phi(x) \Phi(\alpha x + \beta) dx = \Phi\left(\frac{\beta}{\sqrt{1 + \alpha^2}}\right) \quad (\text{D.15})$$

4. Liouville proved that $\int e^{-x^2} dx$ cannot be expressed in elementary terms. For a discussion of this proof (and the more general result), see Conrad (2005). It follows from this result that the bivariate normal density function is integrable but that the functional form cannot be expressed analytically. There are many numerical methods available to approximate such functions quickly.

5. By Lagrange's theorem, it follows that:

$$\frac{d}{dx} \int_{\alpha(x)}^{\beta(x)} f(x, t) dt = f(x, \beta(x)) \frac{d\beta}{dx} - f(x, \alpha(x)) \frac{d\alpha}{dx} + \int_{\alpha(x)}^{\beta(x)} \frac{\partial f}{\partial x} dt \quad (\text{D.16})$$

6. For a more extensive discussion of this manipulation, see Vasicek (1991).

Chapter 5. Bailouts and politics

1. See <http://www.parl.gc.ca/About/Senate/WordsOfWisdom/PMERLM1-e.htm>

2. For some papers discussing this issue see the following. For a short primer see Haubrich (1998); for some empirical estimates on the whether it is an effective disciplining tool see Goyal (2005); for a regulatory proposal see Evanoff and Wall (2000).

3. *Charles Calomiris: There's no way to get politics out of the banking system* by Jonathan Derbyshire, Prospect Magazine, March 3, 2014.

4. See Dewatripont and Tirole (1994) for a lengthier discussion on how financial regulation can be decomposed into either *discretionary* or *non-discretionary* regulations; specifically, see Chapter 14 and pp. 220-221.

5. In the interest of tractability, the model presented in this chapter will focus on investors. This assumption, however, does not necessitate an interpretation that the demand for lax regulation comes solely from this class of individual. From a welfare perspective, a riskier financing regime would benefit the recipients of these loans. Borrowers from the bank, however, would also have little incentive to follow through on a promise to bailout bank creditors – their preferences are time-inconsistent. This normative perspective might explain why some politicians advocated for greatly reduced lending standards in the run-up to the crisis and then opposed bailouts to creditors.

6. Miles et al. (2013) concludes that the social and private costs of substantially raising bank equity-capital are small. If bank leverage were halved from 30× to 15×, they estimate:

...bank WACC rises by 18 bps (5.51%-5.33%); with no MM offset this rise would be 33bps (5.66%-5.33%). So the rise in WACC is about 55% of what it would be if there was no MM effect (18/33). Put another way, the MM offset is about 45% as large as it would be if MM held exactly.

They estimate that the socially optimal level of bank equity-capital ratio is roughly 16-20%. The negative welfare effects of incorrectly setting equity-capital ratios are asymmetric: setting them too high is less socially costly than setting them too low. They conclude:

...that even proportionally large increases in bank capital are likely to result in a small long-run impact on the borrowing costs faced by bank customers. Even if the amount of bank capital doubles our estimates suggest that the average cost of bank funding will increase by only around 10-40bps. (A doubling in capital would still mean that banks were financing more than 90% of their assets with debt). But substantially higher capital requirements could create very large benefits by reducing the probability of systemic banking crises.

7. See Chapter 5 of French et al. (2010), pp. 44-46:

Capital requirements are not free. The disciplining effect of short-term debt, for example, makes management more productive. Capital requirements that lean against short-term debt push banks toward other forms of financing that may allow managers to be more lax. [...]

It is difficult for the bank's stockholders or its board of directors to control this conflict directly because the managers have much more information about the bank's investment opportunities and the projects they select. Short-term debt can reduce these agency problems. If a bank has a significant amount of short-term debt in its capital structure, it must continually raise new funding to repay the current creditors. This forces the company and its managers to meet a continual market test, so managers have less opportunity to enrich themselves at the expense of the bank's owners.

It is worth noting that, despite this criticism, the recommendation from the chapter is that capital requirements should be increased and that institutions with more short-term debt should have higher capital requirements. They conclude that, due to externality of systemic risk caused by short-term debt, there is more short-term debt than is socially optimal.

8. Similar ideas have been argued by others. For instance, Bernanke (1983) argues that, in addition to the monetary impact of a collapsing banking sector (see Friedman and Schwartz (1963) for this argument), a collapse in credit helped propagate the Great Depression. A key element of Bernanke's story is that a shock can result in the value of collateral collapsing, which reduces the efficiency of credit allocation. This perspective helps to explain the length and severity of the Great Depression; he notes that there is little reason to believe that money would remain non-neutral for such a long period of time.

9. For example: Admati et al. (2012); Admati and Hellwig (2014); Admati et al. (2011, 2013); Admati and Hellwig (2013) make similar arguments. Kotlikoff (2010) and Chamley and Kotlikoff (2009) argue that banks should be regulated in order to have 100% equity-capital and a mutual fund structure (a system known as Limited Purpose Banking).

10. This is explained in Chapter 2 of the book; Shin uses a Value-at-Risk constraint (VaR), however, in his framework it is formally indistinguishable from a binding capital-adequacy ratio or a leverage limit.

11. For instance, see Section 1.IV in the *Basel III: A global regulatory framework for more resilient banks and banking systems - revised version June 2011*, which can be downloaded at <http://www.bis.org/publ/bcbs189.htm>

12. It is worth noting that there is some dispute as to whether, despite the title, Diamond and Dybvig (1983) actually justifies deposit insurance over suspension of convertibility or lender of last resort (Selgin, 1993). In the Diamond and Dybvig model, there is no functional difference between deposit insurance and lender of last resort. In fact, all of the banks in their model remain solvent so the deposit insurer (or lender of last resort) could intervene

without any fear of saving an insolvent bank. They do, however, argue that the lender of last resort is inferior to deposit insurance since doubt in the lender of last resort might trigger runs. Deposit insurance is a commitment to backstop the institutions:

If the lender of last resort is not required to bail out banks unconditionally, a bank run can occur in response to changes in depositor expectations about the bank's creditworthiness. A run can even occur in response to expectations about the general willingness of the lender of last resort to rescue failing banks, as illustrated by the unfortunate experience of the 1930s when the Federal Reserve misused its discretion and did not allow much discounting. In contrast, deposit insurance is a binding commitment which can be structured to retain punishment of the bank's owners, board of directors, and officers in the case of failure.

By introducing the possibility of insolvency (and thus moral hazard), any assessment of support programs would need to consider both type-I and type-II errors.

13. In the Akerlof and Romer (1993) model, as an analytical convenience, it is the government that lends directly to the financial institutions. This is to mimic the state backing of the creditors; the paper, however, does not address why the government would do this.

14. The relevant figure from Reinhart and Rogoff (2009) is Figure 10.1 and the data on banking crises can be downloaded from <http://www.reinhartandrogoff.com/>. Equally, it is possible to use the data by loading the *Ecdat* library in R, which can be found at <http://cran.r-project.org/web/packages/Ecdat/index.html>.

15. See Carter et al. (2006), Table Cj265-272 Commercial banks liabilities: 1834-1980, which can be found at <https://hsus.cambridge.org/HSUSWeb/toc/tableToc.do?id=Cj265-272>.

16. It is equally possible to set up this model such that the bank facilitates borrowing and lending between the parents. In order to see how this would work, parents would need to consume in both the first and second periods. Suppose that all parents have the same (positive) second-period income. Parents who have low period-1 income will want to borrow in order to smooth consumption; symmetrically, those who have high period-1 income would lend to them through a financial intermediary. A friction is introduced if, in a recession, p proportion of the loans are forgiven.

17. From Kane (2014), p. 22 (emphasis in original):

[A]ny bank can circumvent the FDIC's statutory limits on account coverage by writing bilateral swap contracts through the Promontory International Network. Although it operates as a swaps broker, Promontory is not itself federally regulated, even though (and perhaps because) its founders include a former Comptroller of the Currency and a former Governor of the Federal Reserve System. This firm offers a funds placement service that is equivalent to an OTC swap product. Placing funds across the network allows a single depositor to push its FDIC coverage from \$250,000 per depositor envisioned in the FDIC Act to as much as \$50 million. Promontory named this product the Certificate of Deposit Account Registry Service (CDARS). CDARS acts as a clearinghouse. It re-books balances in excess of FDIC limits (i.e., statutorily uninsured deposits) from one institution as below-the-limit increments in deposits that are *swapped* into other institutions. The injustice of the arbitrage is that Promontory collects fees for re-booking these deposits, while the FDIC incurs the costs of providing the overline coverage. It seems likely that during crisis years deposits supplied through CDARS helped to sustain the life of more than a few zombie banks.

The lessons inherent in the CDARS program generalize to the broader swaps market. Almost anything that carries an explicit or implicit government guarantee can be swapped in great volume and achieving high volume establishes the equivalent of a squatters right. This is because authorities are reluctant to roll back innovations once they have achieved widespread use. CDARS shows that *swaps* designed to arbitrage the safety net can easily go viral if authorities do not intervene. At this writing, approximately 3,000 financial institutions belonged to the CDARS deposit-swapping network. Like sponsors of the first retail money-market funds, the sponsors of CDARS extended the effective reach of the US safety net and dared authorities to do something about it.

18. The post-crisis reforms have included the United States creating the Financial Stability Committee (within the Federal Reserve) and the Bank of England taking responsibility for the regulation of the financial sector through the Prudential Regulatory Authority.

19. This could be the exception that proves the rule: the democratic reform movements have been set back considerably after a Supreme Court reference on the constitutionality of Senate reform. The Government of Canada posed a series of constitutional questions regarding reforming the Senate to the Supreme Court. The court decided that any significant alteration to the powers and privileges of the upper house would need to be made by constitutional convention. In the Canadian system, this would require the approval of 7 provinces (out of 10) that comprise 50% of the population.

See *Reference re Senate Reform*, November 2013; 2014 SCC 32; File No. 35203.

Available at: <https://scc-csc.lexum.com/scc-csc/scc-csc/en/13614/1/document.do>

20. TD Bank Financial Group, RBC Capital Markets Canadian Bank CEO Conference. January 8, 2009. <http://www.td.com/document/PDF/investor/td-investor-2009rbc-trans.pdf>

21. The memo (Document 3) has been made available by the Houses of Oireachtas at:

<http://www.oireachtas.ie/viewdoc.asp?fn=/documents/Committees30thDail/PAC/Reports/document1.htm>

22. Presentation to the Irish Department of Finance, 18 November 2008, Merrill Lynch.

23. Written Answers on Bank Recapitalization, 18 April 2012. Dáil Éireann Debate Vol. 761 No. 3, p.157. Available at: <http://debates.oireachtas.ie/dail/2012/04/18/00157.asp>

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