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The historical role of energy in UK inflation and productivity with implications for price inflation[☆]

Jennifer L. Castle[a,1], David F. Hendry[a,*,1], Andrew B. Martinez[b,1]

[a] Climate Econometrics and Nuffield College, University of Oxford, UK
[b] US Department of the Treasury, United States of America

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ABSTRACT

We model UK price and wage inflation, productivity and unemployment over a century and a half of data, selecting dynamics, relevant variables, non-linearities and location and trend shifts using indicator saturation estimation. The four congruent econometric equations highlight complex interacting empirical relations. The production function reveals a major role for energy inputs additional to capital and labour, and although the price inflation equation shows a small direct impact of energy prices, the substantial rise in oil and gas prices seen by mid-2022 contribute half of the increase in price inflation. We find empirical evidence for non-linear adjustments of real wages to inflation: a wage-price spiral kicks in when inflation exceeds about 6%–8% p.a. We also find an additional non-linear reaction to unemployment, consistent with involuntary unemployment. A reduction in energy availability simultaneously reduces output and exacerbates inflation.

1. Introduction

The recent and relatively sudden increases in inflation rates in many countries, especially in energy and food, have posed serious financial problems for lower-income families with high expenditure shares on those items. The price increases in energy (oil and natural gas) and food have been large, stimulated by a mix of recovery from COVID-19, supply chain issues and the Russian invasion of Ukraine, exacerbated by Brexit in the case of the United Kingdom. At their heights since the start of 2022, natural gas prices more than tripled, and crude oil prices more than doubled, as did those of corn and wheat.

High rates of inflation are not new to the UK, although their low rates for the last 30 years may have lulled memories. A major advantage of long-run consistent time series is that they include many wars, oil (and other) crises and unanticipated major events like pandemics, so can provide evidence on the role of energy in inflation. There are growing risks that inflation has become embedded and persistent in the UK economy. Thus, it is of crucial importance to understand how the cost of energy feeds through into inflation and its second round effects. The main drawback of our data being annual is that the current situation was not foreseen in 2021 and so could not have been forecast by our models. We circumvent that last problem by calculating projections of inflation based on our empirical models but using recent data observations.

Fig. 1 reports the time series over 1860–2021 for logs of UK nominal wages (w) and prices (p) and their rates of inflation (calculated as $\Delta \log(X) = \Delta x$ for a level X). There have been huge increases in nominal annual wages and prices (700 fold and 100 fold respectively since 1860, compared to productivity rising 7-fold overall) with annual price inflation rates reaching more than 20% on three occasions. Real wages ($w - p$) have risen at varying rates, closely tracking productivity (measured by GDP per worker per year, $y - l$) and the two clearly cointegrate, yielding the basis for an economic theory model of real wages. Note the flatlining of both real wages and productivity from 2008 onwards, an issue we return to in Section 3. However, over this century, GDP divided by population (i.e., real income per capita) has risen by 17% (employment growth has exceeded population growth) so UK policies to successfully limit greenhouse gas emissions have not limited economic growth (see Castle and Hendry, 2022).

Fig. 2(a) shows that total UK energy use (E_t) rose almost 5-fold from 1860 to 1975, but has fallen since 2005, and now is back to 1955 levels after COVID lockdowns. The mix of sources has altered substantially from 100% coal in 1860 remaining dominant till the 1950s, but

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* Corresponding author.
E-mail addresses: jennifer.castle@magd.ox.ac.uk (J.L. Castle), david.hendry@nuffield.ox.ac.uk (D.F. Hendry), Andrew.Martinez@treasury.gov (A.B. Martinez).

[1] All three authors equally participated in the research and writing of this paper.

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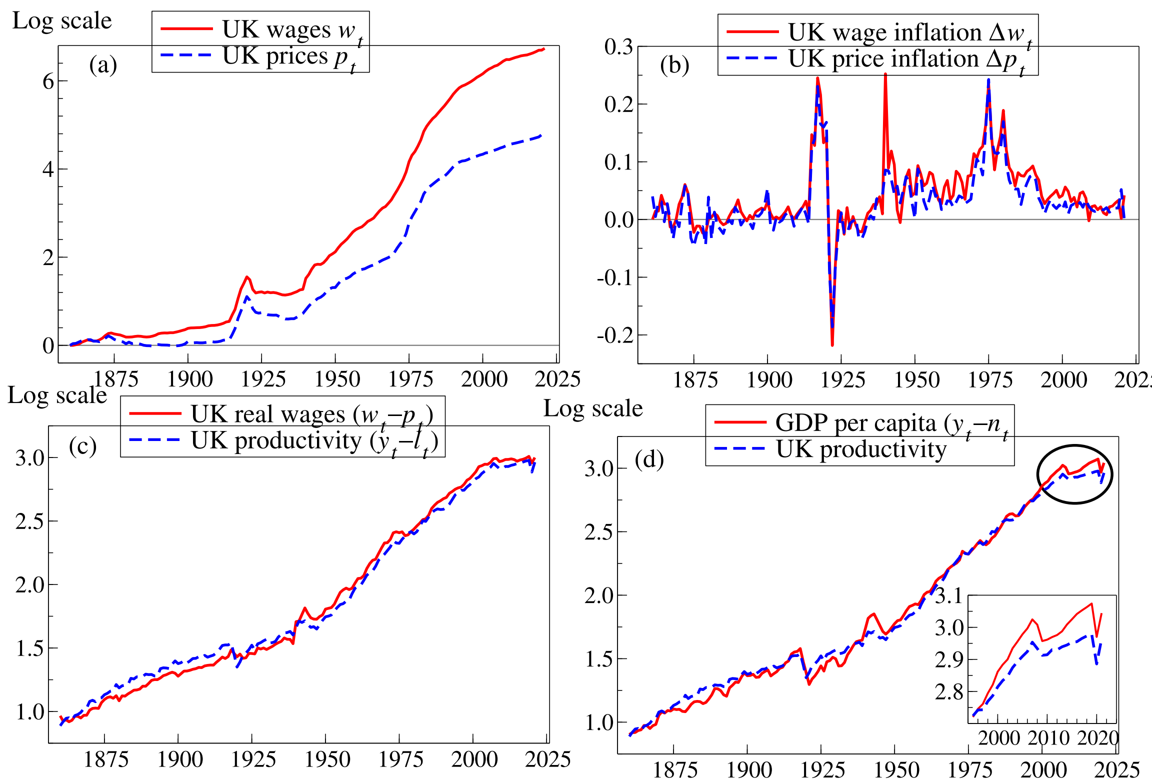

Fig. 1. (a) Logs of UK wages and prices, (b) their rates of inflation, (c) real wages and productivity in logs; (d) GDP divided by population, all 1860–2021.

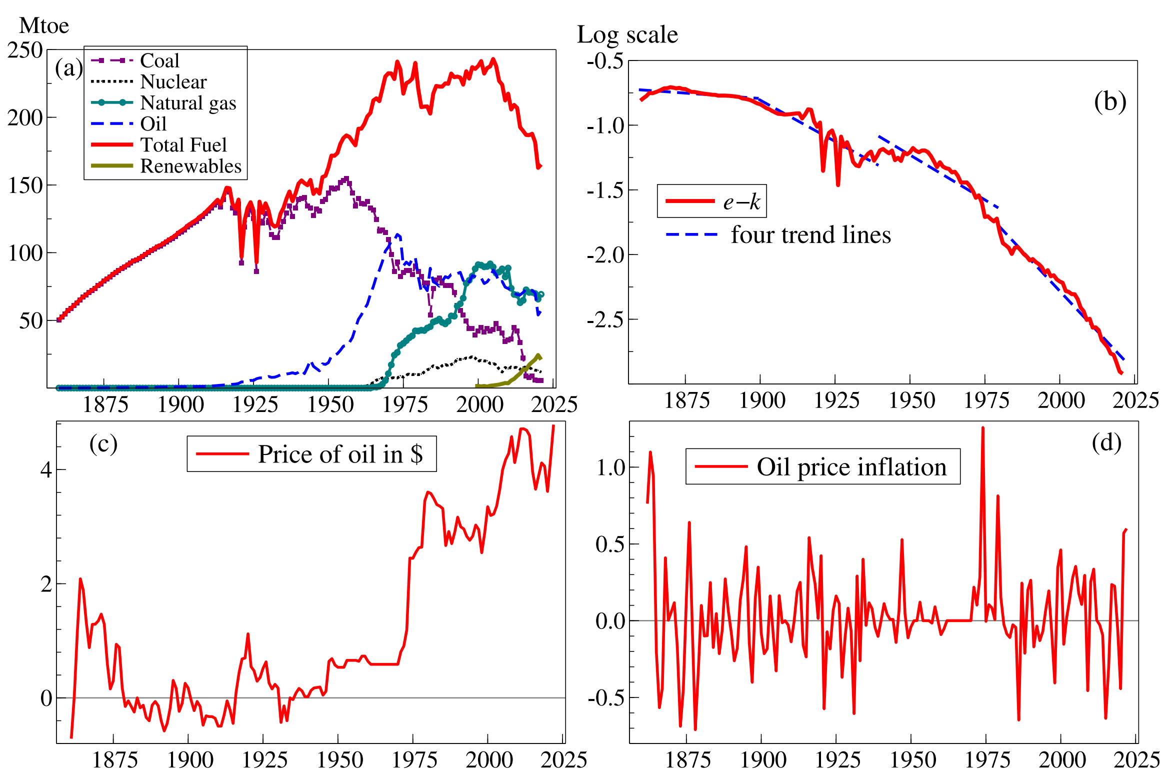

Fig. 2. (a) UK total energy use, E_t, calculated as the sum of coal, oil, natural gas, nuclear and other non-GHG all in Mtoe; (b) log energy per unit capital, $e-k$, with four sub-period trends; (c) logs of oil prices in \$; and (d) their rates of change (so $1.0 = 100\%$), all 1860–2021.

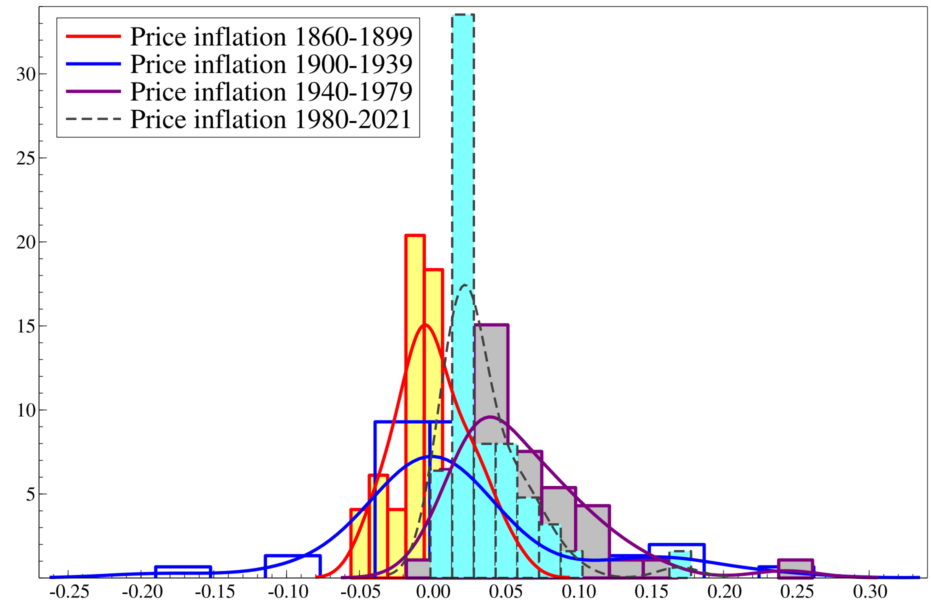

Fig. 3. Distributional shifts in UK price inflation by 40-year subsamples.

essentially none by the end of the period, now replaced by roughly equal amounts of oil, natural gas (both declining) and non-fossil (rising rapidly) all measured in millions of tons of oil equivalent (Mtoe).[2] Energy per unit of capital ($e - k$) shown in Fig. 2(b) has fallen greatly from the changing mix of fuels and from efficiency improvements, relatively slowly till the mid-1950s then more rapidly since at about 2% p.a. The impacts on nominal oil prices (P_o) of the 1970s oil crises, 2010 speculation and COVID-19 are clearly visible in Panels (c) and (d). In addition, UK natural gas prices have risen more than 200% since 2019 and have increased even faster during 2022. The resulting large rises in UK electricity prices despite extensive renewable supply are partly because wholesale electricity auctions reflect marginal prices, but also the UK mothballed its last gas storage facility in 2017.

One of the most obvious data features is the non-constancy of change, so even log differences are non-stationary from distributional shifts as seen in Fig. 3. Thus, the modelling approach undertaken must be able to handle all forms of change, outlined in Appendix A (also see Castle and Hendry, 2019). The next four sections build empirical econometric models of real wages in Section 2, highlighting the relevance of a wage-price spiral and a non-linear unemployment effect; unemployment in Section 3 where a profits proxy explains most of the variation in unemployment; productivity in Section 4 where the crucial role for energy is shown; and price inflation in Section 5 where many domestic and global factors are found to drive price inflation. Using the models developed, Section 6 combines the price and wage inflation models to make projections of price inflation based on energy price rises seen during 2022. Section 7 concludes. Appendix A sketches our econometric tools for modelling non-stationary time series and Appendix B records details of the data series used along with their sources.

2. An empirical model of UK real wages

Real wage models tend to fall into two categories, both of which rely on the underlying theory that real wages are determined by the marginal product of labour, but the first sees inflation expectations accorded a key role in feed-forward mechanisms of the New Keynesian Phillips Curve (NKPC), see, e.g. Galí and Gertler (1999) and Galí et al. (2001), and the second focuses on feed-back mechanisms through dynamic models as in Castle and Hendry (2009).

Fig. 4(a) records real wages for the UK since 1860. Panel (b) plots the cointegrating relation between real wages and output per worker, which proxies the marginal revenue product of labour with a cointegrating weight of 1. This is also the labour share in national income, and while it is not integrated given the evidence of cointegration, it is clearly not stationary with location shifts, notably at the end of WorldWar II (WWII) and at the beginning of the Thatcher era. Panel (c) plots the unemployment rate, which in a traditional Phillips curve relationship is assumed to be a driver of real wage growth. However, the data is persistent and non-stationary with shifting means attributable to exogenous shocks such as wars and policy, but it is not integrated. The annual change in the unemployment rate, recorded in Panel (d), also shows a changing variance so even the difference is non-stationary.

Eyeballing the data on changes in real wages in Fig. 4(a) it is hard to discern changes in growth rates, but Step Indicator Saturation (SIS; see Appendix A) can be applied (here at 0.1%) to check on location shifts unconditionally, recorded in the solid black line in Fig. 5(a). This reveals a doubling of the growth rate of real wages post WWII from 0.8% to 1.7%. Applying SIS to the growth rate in productivity in Panel (c) highlights the upwards shift from 1.2% to 1.7%, but at a different time and by a different magnitude to the shift in real wages. As the location shifts do not co-break and the large outliers do not align it suggests a much more complex empirical model of real wages. Panel (b) records many step shifts in price inflation, and Panel (d) shows those in unemployment: the location shifts in $U_{r,t}$ and Δp_t also do not match. In the general model of real wages below, we include both the level and the change in the unemployment rate. The change allows for possible dynamic labour supply effects, i.e. if unemployment is growing the pool of potential labour supply is increasing, lowering wages, and we find the level of inflation enters non-linearly. Price inflation is included as a catch-up mechanism if wages have been eroded due to less than complete adjustment to past inflation, playing an important non-linear role in the form of a wage-price spiral.

[2] Long-run estimates of UK energy consumption have also been compiled by Humphrey and Stanislaw (1979), Warde (2007) and Awaworyi Churchill et al. (2023). Fig. 16 in Appendix B records these alternative estimates which are based on similar sources, and as Fig. 17 shows, are broadly comparable with our measure when it is expressed in tons of coal equivalent, tce.

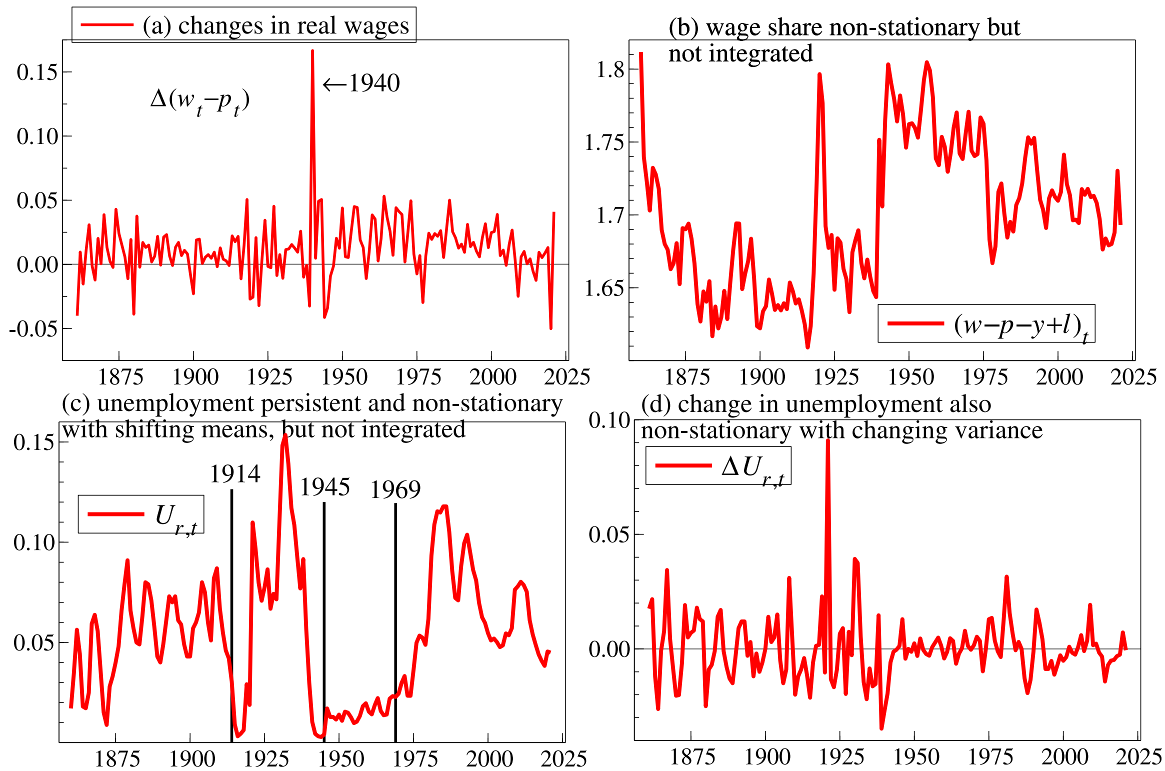

Fig. 4. (a) Changes in real wages; (b) wage share; (c) unemployment rate; and (d) changes in the unemployment rate, all 1860–2021.

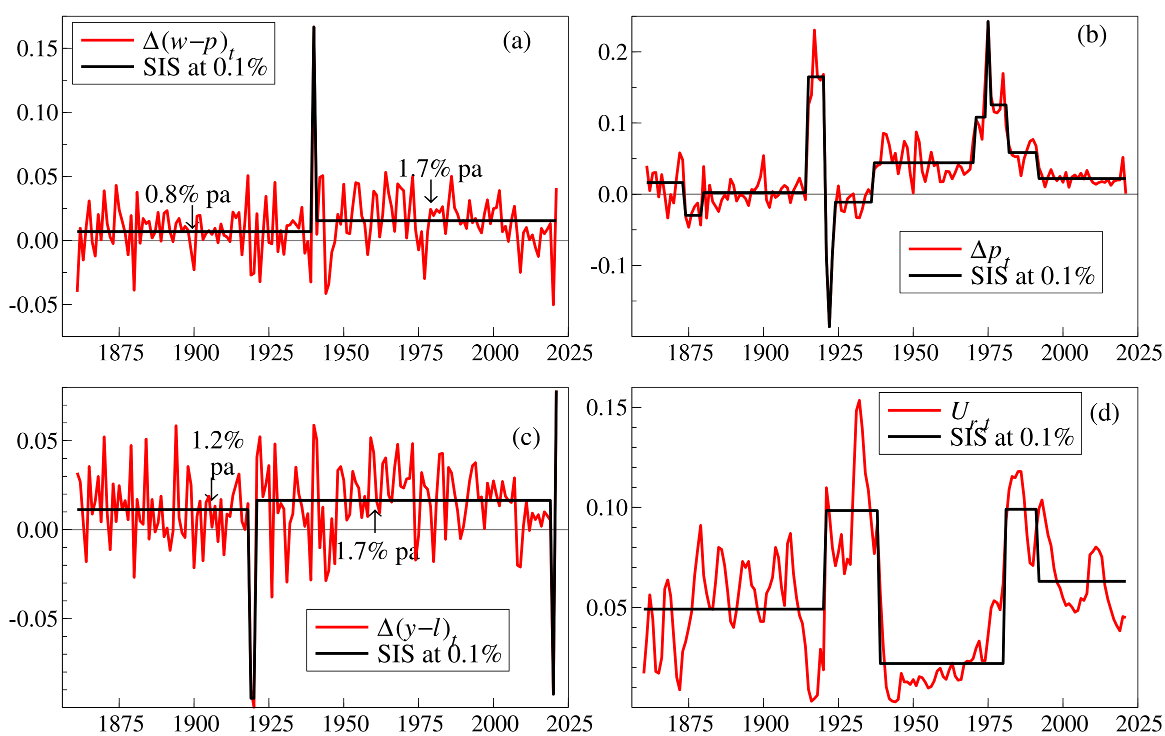

Fig. 5. Step indicator saturation (SIS) and regime shifts: selecting at 0.1% (a) change in real wages; (b) price inflation; (c) productivity; (d) unemployment rate.

2.1. Modelling non-linearities in real wage determination

We begin by specifying a general unrestricted model of the change in real wages, $\Delta(w-p)_t$, to include an intercept, two lags of the dependent variable $\Delta(w-p)_{t-i}$ and the labour share of income, $(w-p-y+l)_{t-i}$ for $i = 1, 2$, as well as the contemporaneous values and two lags of labour productivity changes, $\Delta(y-l)_{t-j}$, the unemployment rate, $U_{r,t-j}$, and price inflation, Δp_{t-j}, for $j = 0, 1, 2$. We apply Impulse Indicator Saturation (IIS; see Appendix A) and SIS to detect outliers and location shifts retaining regressors and selecting indicators at 0.1%; then selecting regressors at 1%.

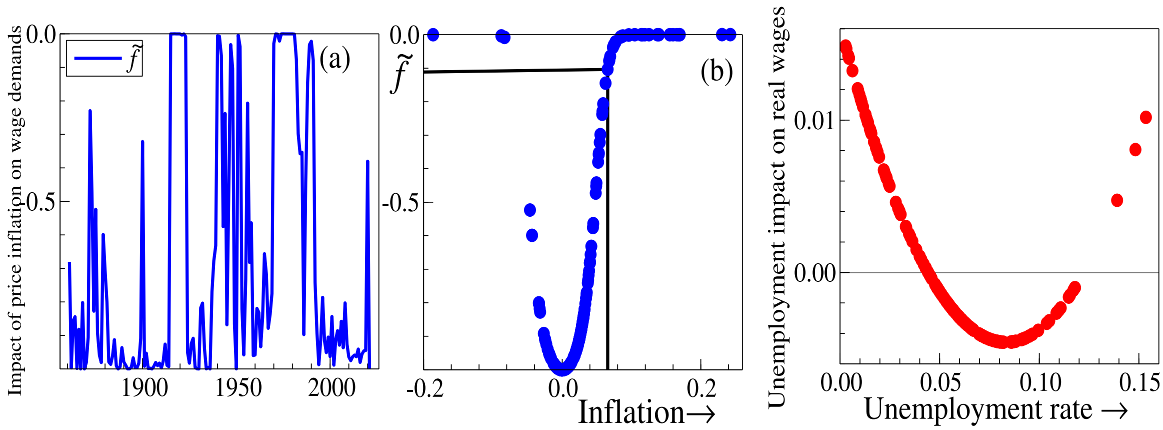

Fig. 6. (a) Regime switches from logistic smooth transition regression model; (b) cross-plot of regime switches against inflation showing wage-price spiral and (c) cross plot of total unemployment impact on real wages against the unemployment rate, showing involuntary unemployment.

We also include polynomials of price inflation and the unemployment rate to capture non-linearities. As a logistic smooth transition function has a Taylor approximation (see Luukkonen et al., 1988), the retained polynomials of price inflation pointed towards the following functional form for a wage-price spiral:

$$f(z) = \left(1 + \exp\left\{-z_t\right\}\right)^{-1}; \qquad z_t = \gamma\left(\frac{x-c}{\sigma_x}\right), \tag{1}$$

where the transition variable $x = 100\,(\Delta p)^2$ (squared annual inflation as a percentage). γ determines the rapidity of transition and c determines the transition point, both of which are determined by grid search.[3] The resulting non-linear function is:

$$\widetilde{f} = \frac{1}{0.88}\left(\left[1 + \exp\left(-10\left(100\,(\Delta p)^2 - 0.2\right)\right)\right]^{-1} - 1\right) \tag{2}$$

where the scaling adjusts the function to be bounded between $[-1, 0]$. (2) delivers a mapping close to the formulation in Castle and Hendry (2014) but improves the estimation precision of the non-linear effect $\widetilde{f}_t \times \Delta p_t$ in the real-wage model (the standard error falls from 0.12 to 0.07).

Fig. 6(a) records the regime switches over time. The non-linear mapping (2) shown in Fig. 6(b) is V-shaped as a function of price inflation. The $\widetilde{f}$ values are calculated from observed data using the estimated coefficients and show that the non-linearity induces a wage-price spiral once price inflation exceeds 6%–8% p.a. There is little reaction of real wages below this threshold, but workers become more attentive as price inflation rises above the threshold and act to prevent further erosion of their real wages. This is consistent with the model of inattentive producers in Reis (2006) and has implications for the UK as CPI inflation exceeded 10% in 2022.

It is also important to include the second non-linear term, $(U_{r,t} - 0.05)^2$, discussed in Castle and Hendry (2014). As the unemployment rate is intrinsically positive, but enters the model with a negative coefficient, the combined term is positive until 5%, then has an increasingly negative impact until the unemployment rate exceeds approximately 8%, but then increases: see Fig. 6(c). The long-run effects from the equilibrium correction feedback would be almost six times larger. Such an effect could initially reflect a loss of worker's bargaining power, but then represent movements along the marginal product curve, raising real wages of those still employed both from more capital per worker and the unemployed being the less productive workers. Importantly, the high real wages are not causing high unemployment, but result from the unemployment being involuntary

The final selected model is reported in Eq. (3) using the automatic model selection algorithm *Autometrics* (Doornik, 2009). It was obtained by starting from the general unrestricted model and first selecting indicators retaining regressors, then selecting the significant regressors (see Hendry and Doornik, 2014). Estimation was over the period 1862–2015 where 1_{xxxx} is an indicator function taking the value 1 for the observation at date $xxxx$ and 0 otherwise, and S_{xxxx} is a step indicator taking the value 1 till $xxxx$ and 0 after. $\hat{\mu}$ is the sample mean of $(w-p-y+l)$. Four selected consecutive impulse indicators during WWII are combined as their coefficients were equal and opposite signed: $\text{IWWII} = (1_{1942} + 1_{1943} - 1_{1944} - 1_{1945})$.[4] Six years of data are retained for forecasting from 2016–2021, which includes the COVID-19 pandemic shock.[5]

$$\begin{aligned}
\widehat{\Delta(w-p)_t} = \;& \underset{(0.044)}{0.380}\,\Delta(y-l)_t + \underset{(0.047)}{0.142}\,\Delta(y-l)_{t-1} - \underset{(0.030)}{0.145}\,\Delta^2 p_t \\
& - \underset{(0.035)}{0.175}\,(U_{r,t} - 0.05) + \underset{(0.691)}{3.00}\,(U_{r,t} - 0.05)^2 - \underset{(0.055)}{0.224}\,\Delta_2 U_{r,t} \\
& + \underset{(0.073)}{0.422}\,(\widetilde{f}_t \times \Delta p_t) - \underset{(0.012)}{0.130}\,S_{1939} + \underset{(0.017)}{0.182}\,S_{1940} - \underset{(0.012)}{0.073}\,S_{1941} \\
& - \underset{(0.012)}{0.045}\,I_{1916} - \underset{(0.011)}{0.049}\,I_{1977} + \underset{(0.006)}{0.028}\,IWWII \\
& - \underset{(0.030)}{0.171}\,(w - p - y + l - \hat{\mu})_{t-2} + \underset{(0.002)}{0.017}\,S_{2012} \\
& \hat{\sigma} = 1.1\% \quad \mathsf{R}^2 = 0.79 \quad \mathsf{F}_{\mathsf{ar}}(2, 137) = 0.25 \quad \mathsf{F}_{\mathsf{arch}}(1, 152) = 0.03 \\
& \chi^2_{nd}(2) = 0.62 \quad \mathsf{F}_{\mathsf{Het}}(19, 130) = 2.5^{**} \quad \mathsf{F}_{\mathsf{reset}}(2, 137) = 2.34
\end{aligned} \tag{3}$$

The model is reasonably well-specified other than failing the test for heteroskedasticity at 1%. Fig. 7 records the graphical statistics for the model. The forecast Chow test is $\mathsf{F}_{\mathsf{Chow}}(6, 139) = 0.52$ and the t-test for a zero forecast innovation mean is $t(5) = 1.1$. Both tests show the model performs well over the forecast horizon. This is a truly *ex ante* forecast exercise as the model was developed prior to the forecast period, and the data has since been updated so allows a test of the model over the period 2016–2021, albeit conditioning on known contemporaneous regressors. Fig. 8 computes the 1-step ahead forecasts both in differences and then cumulating to levels. Both show remarkable constancy and pick up the fall in real wages in 2020 due

[3] A grid consisting of $\gamma \in [0.1, 20]$ and $c \in [0.1, 1]$ was searched over.

[4] The test of reduction from the 4 impulse indicators to a combined indicator is accepted $\mathsf{F}(3, 136) = 2.04$ (p-value $= 0.11$).

[5] Lower case denotes logs, $\Delta^2 = (x_t - x_{t-1}) - (x_{t-1} - x_{t-2})$, and $\Delta_2 = (x_t - x_{t-2})$. Coefficient standard errors are shown in parentheses, $\hat{\sigma}$ is the residual standard deviation, F_{ar} tests for residual autocorrelation (see Godfrey, 1978), $\mathsf{F}_{\mathsf{arch}}$ tests for autoregressive conditional heteroscedasticity (see Engle, 1982), $\mathsf{F}_{\mathsf{het}}$ tests for residual heteroskedasticity (see White, 1980), $\chi^2_{\mathsf{nd}}(2)$ tests for non-Normality (see Doornik and Hansen, 2008), $\mathsf{F}_{\mathsf{reset}}$ tests non-linearity (see Ramsey, 1969) and $\mathsf{F}_{\mathsf{chow}}$ tests for parameter constancy (see Chow, 1960). One star indicates test significance at 5%, two at 1%.

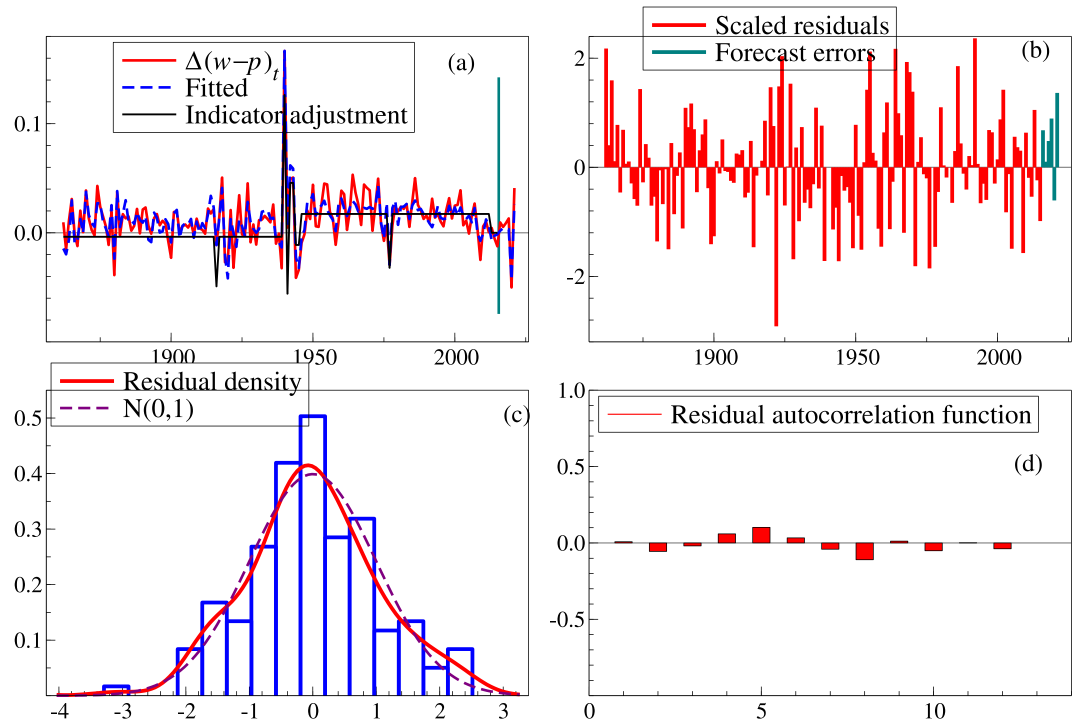

Fig. 7. Real-wage model (3) graphical statistics including (a) fitted and actual values along with the indicator adjustment path; (b) residuals and forecast errors; (c) residual density; and (d) residual autocorrelation function.

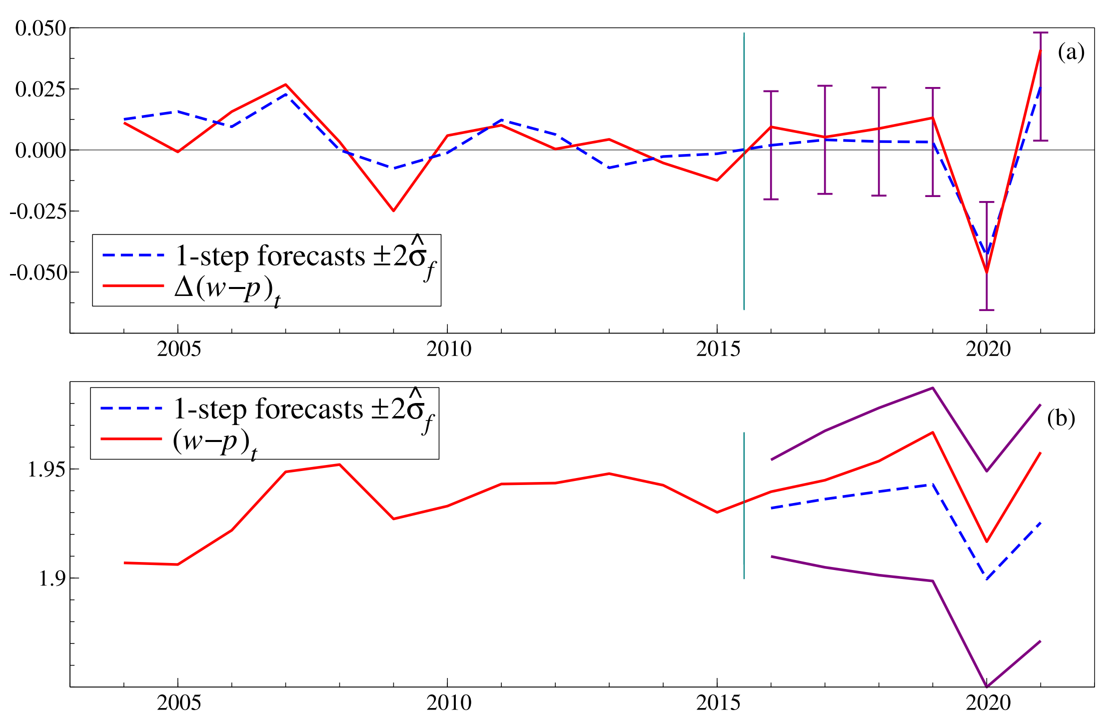

Fig. 8. 'Forecasts' for real wages in (a) differences and (b) levels over the pandemic.

to the pandemic. The results show that stable models can be developed despite highly non-constant data.

2.2. Testing super exogeneity

To justify a single equation model of real-wage growth conditioned on non-stationary contemporaneous variables requires super exogeneity to be satisfied. Super exogeneity is a concept of invariance which requires that the parameters in the conditional and marginal distributions (both in levels and in changes) are unrelated: see Engle and Hendry (1993). Without super exogeneity holding, shifts in the distributions of its conditioning variables will make the parameters of (3) non-constant. Indicator saturation estimators (ISEs) can be used to test for the super exogeneity of the conditioning variables (see Hendry

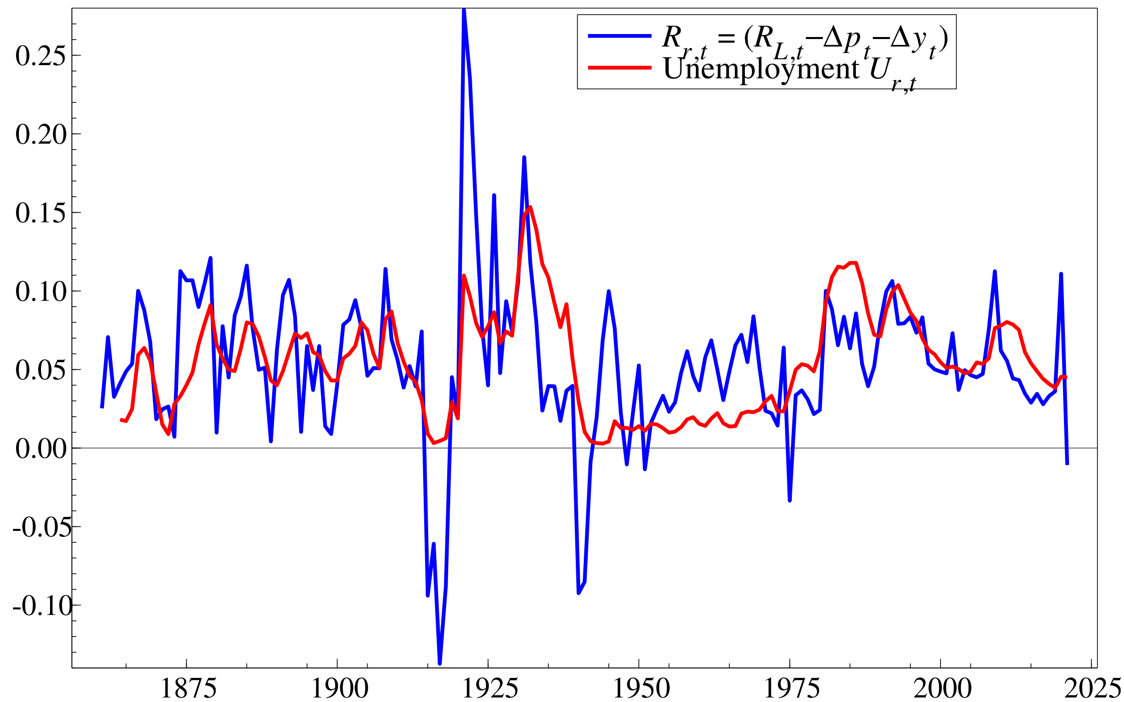

Fig. 9. Profits proxy and the unemployment rate, 1860–2021.

and Santos (2010) and Castle et al., 2017). Under the null hypothesis of super exogeneity, the parameters in the conditional model (3) are invariant to shifts in the marginal models of the included regressors, so any indicators or step shifts that are found in the marginal models should not enter the conditional model.

To test this hypothesis we use a VAR(2) in $(y-l)$, Δp and U_r, retaining all regressors and selecting outliers and shifts using IIS+SIS at $\alpha = 0.1\%$, switching off diagnostic tests for the selection as the marginal models are not congruent. Saturation found 5 impulse and 7 step indicators, noting that at such tight significant levels the probability of retaining an indicator or step that was not relevant is negligible. The retained indicators for the three marginal models were then included in (3), excluding two indicators that overlap with the retained indicators in the conditional model, and tested for their significance. The resulting super exogeneity test of $\mathsf{F}_{\mathrm{SE,IIS+SIS}}(10, 129) = 1.03$ (p-value $= 0.42$) is insignificant. Hence, we can conclude that it is valid to condition on the contemporaneous regressors in (3) and we can proceed to interpret the empirical results.

2.3. Interpretation

The short-run impact of changes in productivity on changes in real wages is ≈0.5, a rather rapid incorporation of productivity increases into real wage increases, but symmetric, so also reflects a dampening of real wage growth due to the productivity slowdown since 2008. There is a strong equilibrium correction of −0.17 from the labour share of income reflecting a long-run feedback to real unit labour costs of roughly 20% p.a, or a half life of 4 years. The coefficient of $(\widetilde{f}_t \times \Delta p_t)$ is highly significant, reflecting the importance of the non-linearity, but the effect is < 1 so is dampened avoiding an additional unit root.

The non-linear unemployment effect can be re-written as $\approx -0.5U_{r,t}$ $(1-6.0U_{r,t})$, which is increasingly negative until the unemployment rate exceeds ≈8.3%, but is then decreasingly negative. This non-linearity was not detected in the Castle and Hendry (2009) paper, highlighting the importance of the model selection method with general non-linearities revealing its role in an explanation of real wages.

Fig. 7(a), records the fitted and actual values of $\Delta(w-p)_t$ with indicators affecting real wages, as IIS and SIS were essential to identifying a stable model of real wage growth. There is a key role for the step indicators in explaining the higher growth rate of real wages post WWII (1.7% p.a., versus 0.8% p.a. pre-1945), even though $\Delta(y-l)$ is included and displays a similar pattern, suggesting the spike in $\Delta(w-p)$ in 1940 induced a permanent location shift which is not explained by the variables in the model. One possible explanation could be the increase in female labour force participation after WWII following a rapid up-skilling of the labour force during the war; see e.g. Bernstein and Martinez (2021). The step shift in 2012 suggests this increase has been reversed over the last decade; as steps are defined taking the value 1 prior to their date, real-wage growth has experienced a level shift down by 1.7%p.a. since 2012. The economic variables in the model do not explain this step shift which poses serious policy questions, but is fundamental for the forecast performance of the model over 2016–2021.

The equation standard error of 1.1% compares to an unconditional standard deviation of 2.3% for real wage growth over the same period, although the equation standard error also reflects the steps and impulses (excluding the steps and impulses $\hat{\sigma} = 1.86\%$). The model is remarkably constant over the period of the 'Great Recession' and the 'flat lining' of real wages. Given such constancy over a period of structural change, we can derive insights into possible effects on real wages of the current economic pressures in the UK. Rapidly rising price inflation is a key area of concern, but low productivity derived from a recession is likely to dampen this positive effect on real wage growth. The current tight labour market driving low unemployment suggests the second non-linearity is unlikely to impact real wages.

3. An empirical model of UK unemployment

Unemployment plays a key role in determining real wages, both as the change in unemployment and as a non-linear relationship with real wages, possibly capturing movements along the marginal product curve. As such, a model of the determinants of unemployment is useful for a well-specified marginal model to test, for example, super exogeneity, as well as an understanding of the system more generally.

Following from Hendry (2001), Castle et al. (2016) formulated and estimated a model of unemployment estimated up to 2014, and here we extend their analysis to 2021, testing parameter constancy over the extended sample from 2018.

The empirical model from Hendry (2001) assumes the unemployment rate, $U_{r,t}$, is the outcome of supply and demand for labour, aggregated across all prospective workers, with labour demand derived from the demand for goods and services. This implies a highly complex data generating process (DGP) which is approximated by assuming employment increases if hiring is profitable, and falls if not. As there is no good annual data over the last century and a half for profits we use a proxy. Changes in revenues are linked to changes in GDP, Δy_t, reflecting the demand side, and the close link between $(w-p)_t$ and $(y-l)_t$ (seen above) suggests labour costs and revenues are equilibrated. On the supply side, capital costs depend on real borrowing costs, $(R_L - \Delta p)_t$, where R_L is the nominal long-term interest rate. Combining, we approximate profits by the difference between the proxies for costs and for revenues (measured negatively for graphical convenience):

$$R_{r,t} = (R_L - \Delta p - \Delta y)_t. \tag{4}$$

Fig. 9 records this measure of the profits proxy along with the unemployment rate adjusted to match means. While there are some deviations between the two series they tend to move closely together suggesting a 'cointegrating' relation between the unemployment rate and the profits proxy: see Castle et al. (2021) for a monthly model of the unemployment rate including non-linear transformations and ISEs.

3.1. Modelling unemployment by the profits proxy

Here we formulate a dynamic model of $U_{r,t}$ by commencing with a model in levels with two lags of both $U_{r,t}$ and $R_{r,t}$. Non-linear functions are not included in the initial specification as the index test of non-linearity provides no evidence of non-linear functional form at the 1% significance level ($\chi^2_{nl}(12) = 22.7^*$, see Castle and Hendry, 2010). The model in levels with saturation over 1863–2015 yields:[6]

$$\begin{aligned}\widehat{U}_{r,t} = {} & \underset{(0.07)}{1.26}\, U_{r,t-1} - \underset{(0.06)}{0.36}\, U_{r,t-2} + \underset{(0.002)}{0.006} + \underset{(0.02)}{0.15}\, R_{r,t} - \underset{(0.02)}{0.08}\, R_{r,t-1} \\ & - \underset{(0.007)}{0.052}\, \Delta 1_{1922} + \underset{(0.008)}{0.036}\, 1_{1930} - \underset{(0.008)}{0.035}\, 1_{1939} \\ & \widehat{\sigma}_\epsilon = 0.83\% \;\; \mathsf{R}^{*2} = 0.94 \;\; \mathsf{F}_{\mathsf{ar}}(2,143) = 1.94 \;\; \chi^2_{\mathsf{nd}}(2) = 9.36^{**} \\ & \mathsf{F}_{\mathsf{Chow}}(6,145) = 0.30 \\ & \mathsf{F}_{\mathsf{arch}}(1,151) = 0.11 \;\; \mathsf{F}_{\mathsf{Het}}(10,140) = 1.13 \;\; \mathsf{F}_{\mathsf{Reset}}(2,143) = 2.49\end{aligned} \tag{5}$$

All the pairs of impulse and step saturation indicators cancelled as differences so were essentially equivalent to including $\Delta 1_{1922}$, 1_{1930} and 1_{1939}. $U_{r,t}$ is non-stationary from distributional shifts, but not from a unit root: $\mathsf{t}_{\mathsf{unitroot}} = -4.22^{**}$ (see Ericsson and MacKinnon, 2002). The model's graphical statistics are recorded in Fig. 10.

We then solve for the long-run 'cointegrating' relation and transform the dynamic model to differences including the lagged long-run relationship. The resulting selected model over 1863–2015 yields:

$$\begin{aligned}\Delta\widehat{U}_{r,t} = {} & \underset{(0.05)}{0.36}\, \Delta U_{r,t-1} + \underset{(0.016)}{0.15}\, \Delta R_{r,t} - \underset{(0.022)}{0.10}\, E_{Ur,t-1} \\ & - \underset{(0.007)}{0.052}\, \Delta 1_{1922} + \underset{(0.008)}{0.036}\, 1_{1930} - \underset{(0.008)}{0.035}\, 1_{1939} \\ & \widehat{\sigma}_\epsilon = 0.83\% \;\; \mathsf{R}^{*2} = 0.67 \;\; \mathsf{F}_{\mathsf{ar}}(2,145) = 1.94 \;\; \chi^2_{\mathsf{nd}}(2) = 9.38^{**} \\ & \mathsf{F}_{\mathsf{arch}}(1,151) = 0.10 \;\; \mathsf{F}_{\mathsf{Het}}(8,142) = 0.88 \;\; \mathsf{F}_{\mathsf{Reset}}(2,145) = 0.83\end{aligned} \tag{6}$$

where the long-run relation is given by:

$$E_{Ur} = U_r - 0.054 - 0.72 R_r. \tag{7}$$

We test the constancy of the model by computing the 1-step *ex post* forecasts for 2016–2021 which delivers a Chow test of $\mathsf{F}_{\mathsf{Chow}}(6,147) = 0.30$, demonstrating remarkable stability of the simple model over the COVID-19 pandemic.

Fig. 11 records the graphical statistics. Although there is one diagnostic failure, the simple model can explain a large amount of the variation in the unemployment rate over the past century and a half.

3.2. Interpretation

The underlying economic theory is found to be empirically consistent over a century and a half; when the real long-term interest rate, $R_L - \Delta p$, equals Δy, then $R_r = 0$, and equilibrium U_r is about 5%, close to the historical average. Since the financial crisis, quantitative easing has lowered $R_L - \Delta p$, offsetting a large fall in Δy, so $R_{r,t}$ only rose briefly, which goes some way to understanding why U_r rose less than anticipated over the Great Recession. There is a rise in unemployment during 2008, which nevertheless was much smaller than expected, given a fall of more than 6% in real GDP. The earlier in-sample period saw many key changes, including two world wars, unemployment benefits, and vast industrial changes, yet only 1 differenced and 2 impulse indicators are needed with just one explanatory variable. Castle et al. (2016) compared (6) to more-conventional models (e.g.) using Δy and showed the latter were far poorer. Despite the success of the model, *ex ante* forecasts would require forecasts of the profits proxy, so we next investigate a system model of unemployment and the profits proxy.

3.3. System model of unemployment and profits proxy

The two variable system in levels for $U_{r,t}$ and $R_{r,t}$ over 1863–2017, with 4 observations held for forecasts up to 2021, applying IIS and SIS at 0.1% to select outliers and steps while holding the regressors fixed and then selecting the regressors at 1%, yields:

$$\begin{aligned}\widehat{U}_{r,t} = {} & \underset{(0.087)}{1.27}\, U_{r,t-1} - \underset{(0.079)}{0.37}\, U_{r,t-2} - \underset{(0.024)}{0.005}\, R_{r,t-1} + \underset{(0.002)}{0.005} \\ & - \underset{(0.010)}{0.005}\, 1_{1874} - \underset{(0.010)}{0.019}\, 1_{1915} - \underset{(0.010)}{0.002}\, 1_{1917} + \underset{(0.010)}{0.009}\, 1_{1926} \\ & - \underset{(0.010)}{0.036}\, 1_{1939} - \underset{(0.010)}{0.013}\, 1_{1940} + \underset{(0.010)}{0.010}\, 1_{1975} \\ & - \underset{(0.010)}{0.092}\, S_{1920} + \underset{(0.016)}{0.132}\, S_{1921} - \underset{(0.012)}{0.040}\, S_{1922} \\ & - \underset{(0.008)}{0.036}\, S_{1929} + \underset{(0.007)}{0.036}\, S_{1931} \\ & \chi^2_{\mathsf{nd}}(2) = 14^{**} \;\; \mathsf{F}_{\mathsf{ar}}(2,140) = 1.86 \;\; \mathsf{F}_{\mathsf{arch}}(1,152) = 0.01 \\ & \mathsf{F}_{\mathsf{Het}}(8,139) = 1.56\end{aligned} \tag{8}$$

$$\begin{aligned}\widehat{R}_{r,t} = {} & \underset{(0.067)}{0.514}\, R_{r,t-1} + \underset{(0.243)}{0.112}\, U_{r,t-1} - \underset{(0.220)}{0.123}\, U_{r,t-2} - \underset{(0.005)}{0.001} \\ & + \underset{(0.028)}{0.078}\, 1_{1874} - \underset{(0.028)}{0.159}\, 1_{1915} - \underset{(0.028)}{0.134}\, 1_{1917} + \underset{(0.030)}{0.124}\, 1_{1926} \\ & - \underset{(0.028)}{0.005}\, 1_{1939} - \underset{(0.028)}{0.133}\, 1_{1940} - \underset{(0.027)}{0.091}\, 1_{1975} \\ & - \underset{(0.027)}{0.241}\, S_{1920} + \underset{(0.043)}{0.188}\, S_{1921} + \underset{(0.035)}{0.065}\, S_{1922} \\ & - \underset{(0.023)}{0.082}\, S_{1929} + \underset{(0.020)}{0.073}\, S_{1931} \\ & \chi^2_{\mathsf{nd}}(2) = 2.83 \;\; \mathsf{F}_{\mathsf{ar}}(2,140) = 1.89 \;\; \mathsf{F}_{\mathsf{arch}}(1,152) = 1.66 \\ & \mathsf{F}_{\mathsf{Het}}(8,139) = 1.84\end{aligned} \tag{9}$$

The models are well-specified and satisfy the single-equation diagnostic tests, apart from normality for the unemployment rate equation, as well as the system statistics reported in Table 1. System residuals and root mean square forecast errors (RMSFEs) are reported in Table 2 where both the 1-step ahead and 2-step ahead forecast errors for $U_{r,t}$

[6] R^{*2} is the coefficient of determination excluding the impulse indicators.

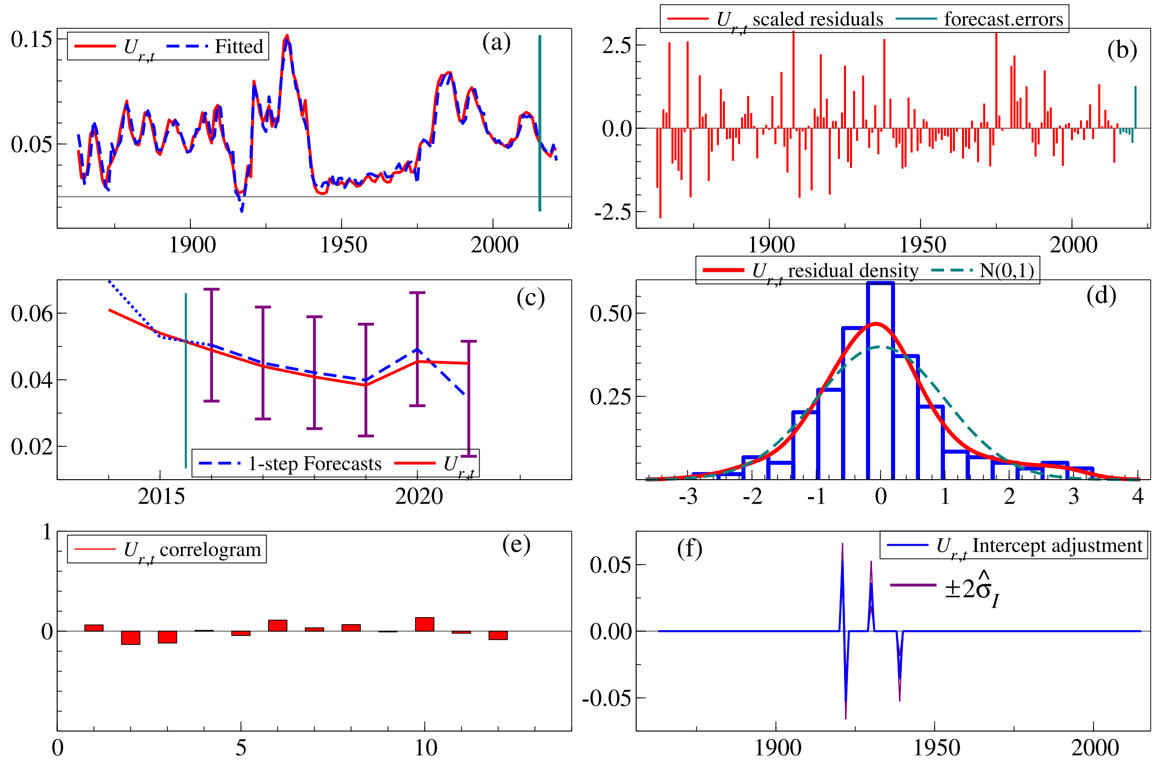

Fig. 10. Graphical statistics of the dynamic unemployment model in levels (5), including (a) fitted and actual values; (b) scaled residuals; (c) 1-step ahead forecasts; (d) residual density; (e) residual correlogram; and (f) the implied intercept adjustment from the retained indicators with $\pm 2\hat{\sigma}_I$.

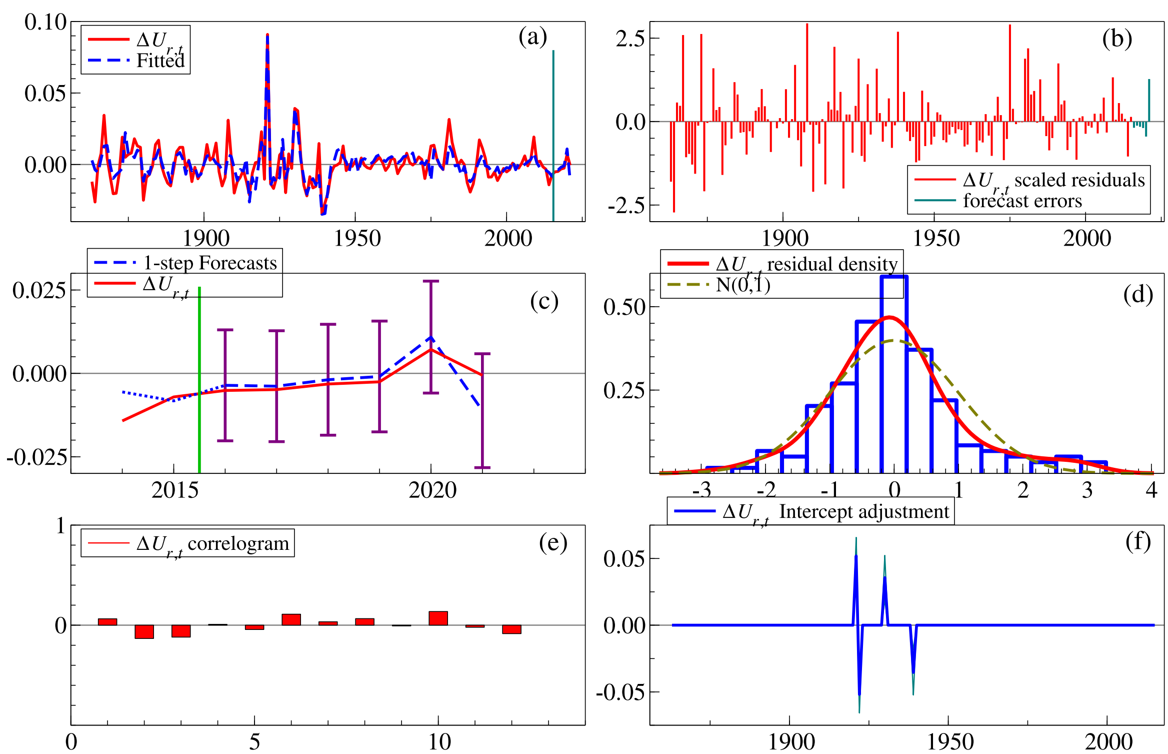

Fig. 11. Graphical statistics of the dynamic unemployment model in differences (6), including (a) fitted and actual values; (b) scaled residuals; (c) 1-step ahead forecasts; (d) residual density; (e) residual correlogram; and (f) the implied intercept adjustment from the retained indicators.

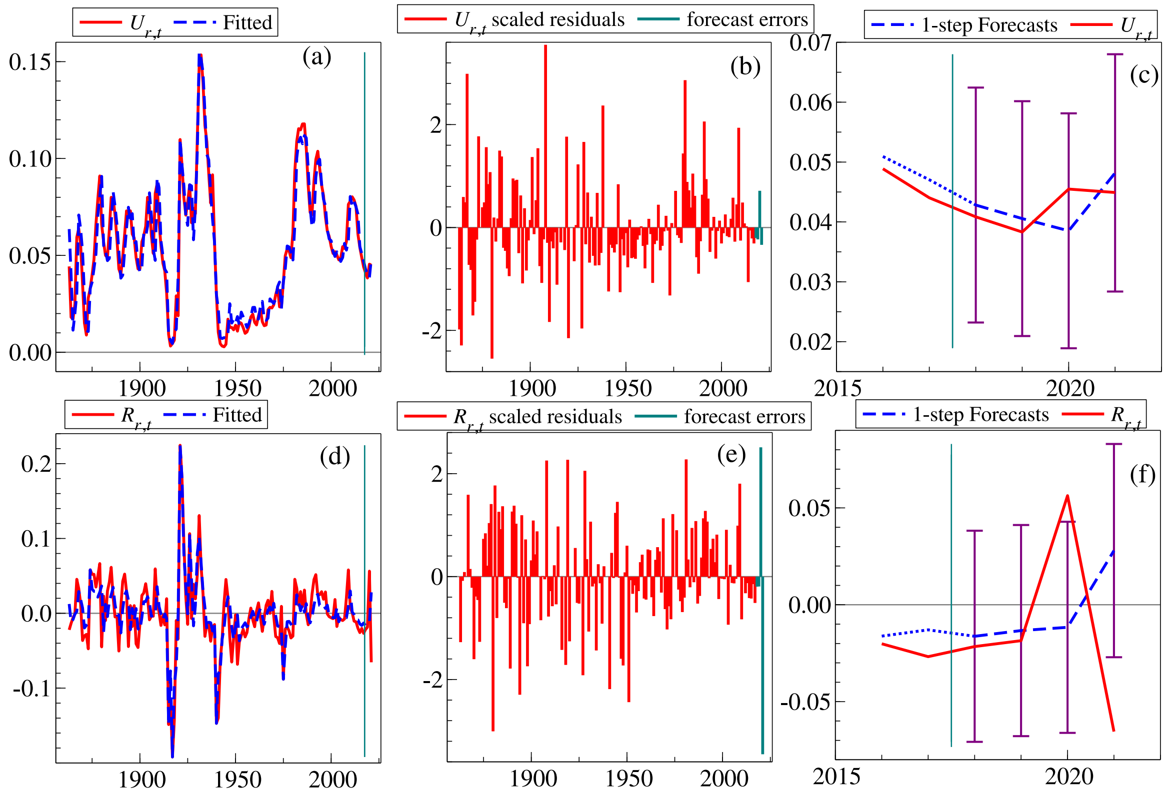

Fig. 12. Graphical statistics of the two variable system of U_r and R_r, including (a) fitted and actual values for U_r; (b) scaled residuals for U_r; (c) 1-step ahead forecasts for U_r; (d) fitted and actual values for R_r; (e) scaled residuals for R_r; and (f) 1-step ahead forecasts for R_r.

Table 1
System statistics for (8) and (9).

Statistic	Outcome
$\mathsf{F}_{\mathsf{VAR}}(8, 268)$	1.98
$\mathsf{F}_{\mathsf{VRESET}}(8, 268)$	2.26
$\chi^2_{\mathsf{VND}}(4)$	15.8**
$\mathsf{F}_{\mathsf{VHET}}(27, 391)$	1.75
$\mathsf{Corr}(\hat{e}_{U_r}, \hat{e}_{R_r})$	0.61

Table 2
System residual $\hat{\sigma}$s and RMFSEs for 1 and 2-steps ahead forecasts over 2018–2021.

Statistic	U_r	R_r
$\hat{\sigma}$	0.0097	0.0271
RMFSE_1	0.0041	0.0579
RMFSE_2	0.0042	0.0437

are smaller than the in-sample residuals. The 1-step ahead forecast errors for $U_{r,t}$ are very similar to those for the single equation model despite not including the contemporaneous profits proxy. The forecasts for R_r are poor over 2020 and 2021 due to the COVID-19 pandemic. Fig. 12 records the model fit and residuals along with 1-step ahead forecast errors. Indicators and steps are needed mostly for the Great Depression and World Wars. The highly significant step indicators in (8) are not needed in (5) nor are most of the impulse indicators, effectively demonstrating the super exogeneity of $R_{r,t}$ in (5). Overall, the results suggest that the relationship between the unemployment rate and the profits proxy has been remarkably stable over the past century and a half despite many shocks including wars, recessions, technology and policy changes, but COVID has highlighted the difficulty of forecasting the profits proxy when there is an unanticipated large fall in Δy_t from lockdowns. Conversely, the UK job retention scheme (furlough) mitigated the impact of lockdowns on recorded unemployment (see Castle et al., 2021).

4. Productivity

Rates of technical progress have varied greatly over time, inducing changing trends in the relationships between output and capital. The real-wage data showed its close relation to productivity, so we model the 'production function' as relating $(y-l)$ to $(k-l)$ with changing trends and location shifts.[7]

One part of the recent UK 'productivity puzzle' is that real GDP per capita has risen although productivity has not. This is due to a large rise in employment relative to population leading to a ratio that is at its highest recorded in peacetime. This also leads to the very different trends since 1995 in output per worker and per person highlighted in the box in Fig. 1(d). As the proportion of the population employed rises, many of the jobs left for the new workers are lower productivity, but by raising total output, aggregate income rises.

4.1. Including energy in the productivity model

Given the concern about the rise in the prices of oil and gas since 2021, and the possibility of severe supply cuts, we reconsider the role of energy in determining the UK 'production function' developed in Hendry (2001, 2022). Both studies modelled the relation between output per employee per annum and the capital labour ratio, and found

[7] Previous models used $(k-n)$ where n is the log of the measured 'working population', namely $N = L + U$. This corrects for the potential artefact that while Y responds very rapidly to increases in U, K cannot, as can be seen in the 1920s, but proved problematic in more recent data.

Table 3
Correlations are between $\Delta(y-l)\&\Delta(k-l)$, $\Delta(y-l)\&\Delta(e-k)$, and $\Delta(k-l)\&\Delta(e-k)$.

Date	Statistic	$\Delta(y-l)$	$\Delta(k-l)$	$\Delta(e-k)$
1861–1945	Mean	0.9%	0.9%	−0.5%
	SD	2.7%	2.6%	7.9%
	Correlation	0.08	0.22	−0.42
1946–2021	Mean	1.7%	2.2%	−2.2 %
	SD	2.2%	1.6%	3.2%
	Correlation	0.23	0.39	−0.12
1861–2021	Mean	1.3%	1.5%	−1.3%
	SD	2.5%	2.2%	6.2%
	Correlation	0.17	0.22	−0.38

several trend shifts. Here we augment the dynamic model in the second paper with a measure of total energy use, E_t, calculated as the sum of coal, oil, natural gas, renewables (wind, solar and hydroelectric) and nuclear all measured in millions of tons of oil equivalent (Mtoe), shown in Fig. 2(a).

Net trends in the relationship between $(y-l)$ and $(k-l)$ depend on the extent and rapidity with which technical progress is embodied in the capital stock both for labour productivity and energy efficiency, or is 'disembodied' as in organisational improvements. Fig. 2(b) shows the dramatic drop in energy relative to capital, with four sub-period (quarter sample) trends. The large swings in the inter-war period are 'captured' as transient events by impulse indicators, after which the trend indicators for the first two sub-periods were essentially the same so were combined with a coefficient of −0.005 compared to −0.023 for the remainder of the sample. Overall the reduction in energy relative to capital is more than 80%, much of which is due to the changing energy mix: see Kaufmann (1992).

The means and standard deviations (SDs) of the growth rates $\Delta(y-l)$, $\Delta(k-l)$ and $\Delta(e-k)$ have changed greatly over time, as Table 3 records for a mid-period split, as have their correlations, although the mean growth rates and SDs of $\Delta(y-l)$ and $\Delta(k-l)$ have been similar within each period.

To model the production relation, we formulate a general model over 1863–2021 of y_t on l_t, k_t, e_t and their first lagged values, together with the trend shift, step shift and outlier indicators from Hendry (2022). These are selected using trend-indicator saturation (TIS, which allows for a potential trend shift at every point in time: see Castle et al., 2019). τ_{xxxx} denotes a segmented trend that commences with a negative value at the beginning of the sample and increases to zero at date xxxx so trends are not carried forward. The homogeneity restriction to a model of $(y-l)_t$ on $(k-l)_t$ and $(e-k)_t$, their lags and indicators yields $\mathsf{F}(3,140)=1.95$ so that reduction was imposed. Transforming to an equation of $(y-l)_t$ on a constant, t, $(k-l)_t$, $(e-k)_t$ and lags, plus all retained indicators, selecting at 1% results in the model for 1863–2021 in (10).

$$\begin{aligned}\widehat{(y-l)}_t = \; & \underset{(0.04)}{0.47}\,(y-l)_{t-1} + \underset{(0.025)}{0.225}\,(k-l)_t + \underset{(0.019)}{0.147}\,(e-k)_t \\ & + \underset{(0.23)}{1.11} + \underset{(0.0013)}{0.0090}\,t + \underset{(0.0007)}{0.0032}\,\tau_{1889} - \underset{(0.002)}{0.009}\,\tau_{1910} \\ & + \underset{(0.0015)}{0.005}\,\tau_{1921} + \underset{(0.002)}{0.014}\,\tau_{1939} - \underset{(0.002)}{0.018}\,\tau_{1946} \\ & + \underset{(0.002)}{0.0073}\,\tau_{2006} - \underset{(0.003)}{0.0073}\,\tau_{2010} - \underset{(0.009)}{0.037}\,S_{1893} \\ & + \underset{(0.014)}{0.084}\,S_{1918} - \underset{(0.012)}{0.080}\,S_{1939} - \underset{(0.015)}{0.093}\,1_{1920} \\ & - \underset{(0.015)}{0.095}\,1_{2020} \\ & \hat{\sigma} = 1.34\% \;\; \mathsf{R}^2 = 0.9996 \;\; \mathsf{F}_{\mathsf{ar}}(2,140) = 1.48 \\ & \chi^2_{\mathsf{nd}}(2) = 0.04 \;\; \mathsf{t}_{\mathsf{ur}} = -13.2^{**} \\ & \mathsf{F}_{\mathsf{arch}}(1,157) = 0.01 \;\; \mathsf{F}_{\mathsf{Het}}(25,131) = 0.69 \\ & \mathsf{F}_{\mathsf{Reset}}(2,140) = 0.46\end{aligned} \tag{10}$$

Fig. 13 records the graphical statistics of the model. Although the simple correlation between $(y-l)$ and $(e-k)$ is −0.97, the partial correlation in (10) is +0.53. The 1926 general strike and longer miners' strike led to a temporary 36% fall in coal output followed by a 41% increase in the next year, and this supply shock produced a 4% fall in GDP followed by an 8% rise, roughly matching the effect derived from the coefficient of $(e-k)$ in (10). Unlike the very sharp GDP falls of around 10% in 1919 and 1920 induced by massive reductions in the government deficit from about 40% of GDP to near zero, no indicator was needed for the 1926 drop, so the change in energy supply captured the fall. To test the invariance of the coefficient of $(e-k)_t$ in (10), we modelled it as a function of $(y-l)_{t-1}$, $(k-l)_{t-1}$ and $(e-k)_{t-1}$ (retained) selecting SIS+TIS at $\alpha=0.01\%$, then applied IIS at 0.01% and finally selected over all retained variables at 0.1%. The indicators retained were τ_{1920}, τ_{1922}, τ_{1925}, τ_{1948}, 1_{1921}, 1_{1926}, 1_{1927}, and testing their significance in (10) yielded $\mathsf{F}(6,136)=1.60$ (1_{1921} was redundant), so super exogeneity is not rejected.

The derived long-run coefficient of l is 0.58, similar to the aggregate labour share of about 2/3:

$$\begin{aligned}\widetilde{(y-l)}_{LR,t} = \; & \underset{(0.047)}{0.42}\,(k-l)_t + \underset{(0.037)}{0.28}\,(e-k)_t + \underset{(0.003)}{0.017}\,t + \underset{(0.001)}{0.006}\,\tau_{1889} \\ & - \underset{(0.003)}{0.017}\,\tau_{1910} + \underset{(0.003)}{0.009}\,\tau_{1921} - \underset{(0.004)}{0.026}\,\tau_{1939} - \underset{(0.004)}{0.034}\,\tau_{1946} \\ & + \underset{(0.004)}{0.014}\,\tau_{2006} - \underset{(0.006)}{0.014}\,\tau_{2010} \\ & - \underset{(0.018)}{0.069}\,S_{1893} + \underset{(0.025)}{0.157}\,S_{1918} - \underset{(0.024)}{0.15}\,S_{1939} + \underset{(0.38)}{2.07}\end{aligned} \tag{11}$$

Fig. 13(f) records the equilibrium-correction values $q_{(\mathrm{y-l}),t}=(y-l)_t-\widetilde{(y-l)}_{LR,t}$.

Expressed as a production function, where A_t collects all the deterministic functions:

$$\widetilde{Y}_{LR,t} = A_t L_t^{0.58} K_t^{0.14} E_t^{0.28} \tag{12}$$

The role of capital may seem too low to neoclassical economists but increases in energy use were as crucial to the industrial revolution as the machinery to utilise it, and $(k-l)$ and $(e-k)$ are very highly negatively correlated (see Cleveland et al., 1984). Transforming to an equilibrium-correction form yields:

$$\begin{aligned}\widehat{\Delta(y-l)}_t = \; & \underset{(0.0013)}{0.012} + \underset{(0.05)}{0.28}\,\Delta(k-l)_t + \underset{(0.02)}{0.16}\,\Delta(e-k)_t - \underset{(0.05)}{0.58}\,q_{(\mathrm{y-l}),t-1} \\ & - \underset{(0.013)}{0.10}\,1_{1919} - \underset{(0.014)}{0.086}\,1_{1920} + \underset{(0.014)}{0.042}\,1_{1940} - \underset{(0.013)}{0.10}\,1_{2020} \\ & \hat{\sigma} = 1.34\% \;\; \mathsf{R}^2 = 0.72 \;\; \mathsf{F}_{\mathsf{ar}}(2,149) = 2.01 \;\; \chi^2_{\mathsf{nd}}(2) = 0.10 \\ & \mathsf{F}_{\mathsf{arch}}(1,157) = 1.46 \;\; \mathsf{F}_{\mathsf{Het}}(6,148) = 0.27 \;\; \mathsf{F}_{\mathsf{Reset}}(2,149) = 0.35\end{aligned} \tag{13}$$

The fitted and actual values and residual graphs of the equilibrium-correction productivity model (13) are shown in Fig. 14. The model is well-specified, passing all the diagnostic tests. The slowdown in productivity is mostly driven by the trend shift in 2010 in levels, with a corresponding intercept shift in the differenced model. The intercept adjustment in Fig. 14(e) shows few indicators are needed beyond the trend shifts in (11) to model the change in output per worker.

5. Price inflation

Hendry (2001, 2015) derived a model for UK inflation that included excess demand for output, money, and national debt; unemployment, exchange rate, unit labour costs, interest rates, wages, world and energy prices. The selected model found significant roles for excess demand for goods and services, world and energy prices, M4 growth and short and long-term interest rates, and an equilibrium correction markup

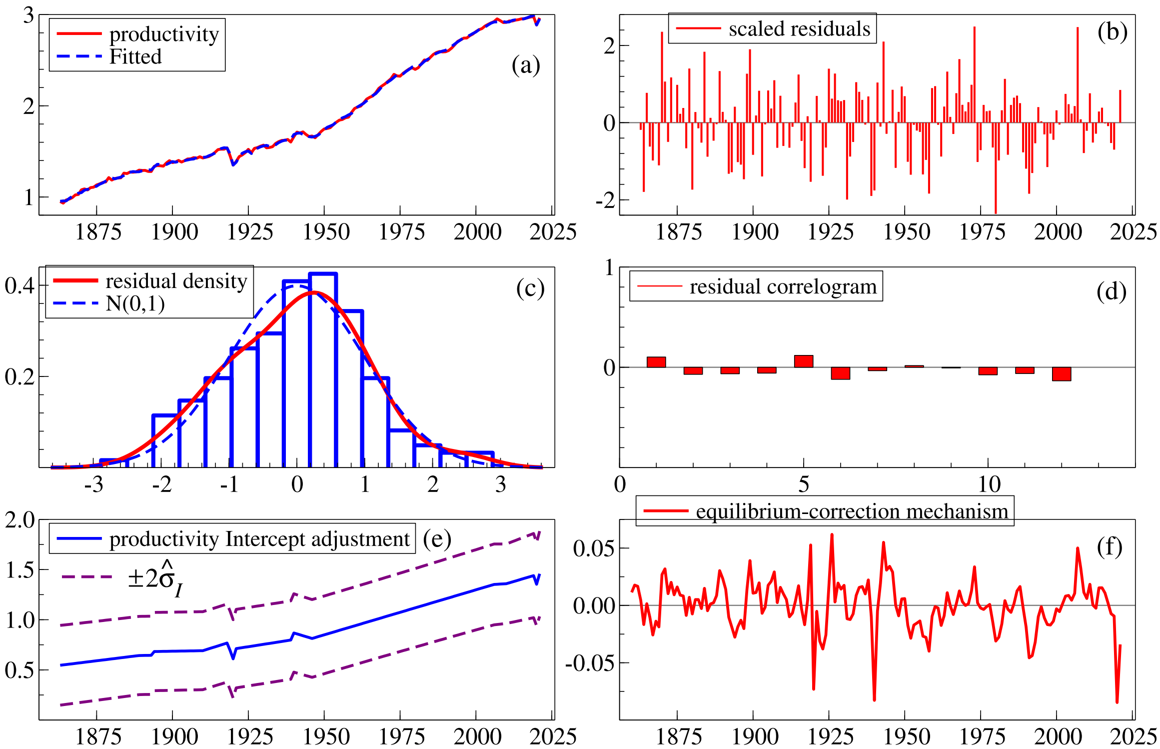

Fig. 13. Graphical model statistics for (10) including (a) model fit; (b) residuals; (c) residual density; (d) residual autocorrelation function; (e) implied indicator adjustment with $\pm 2\hat{\sigma}_I$; and (f) the long-run cointegrating relation after solving the dynamic model for the long-run solution.

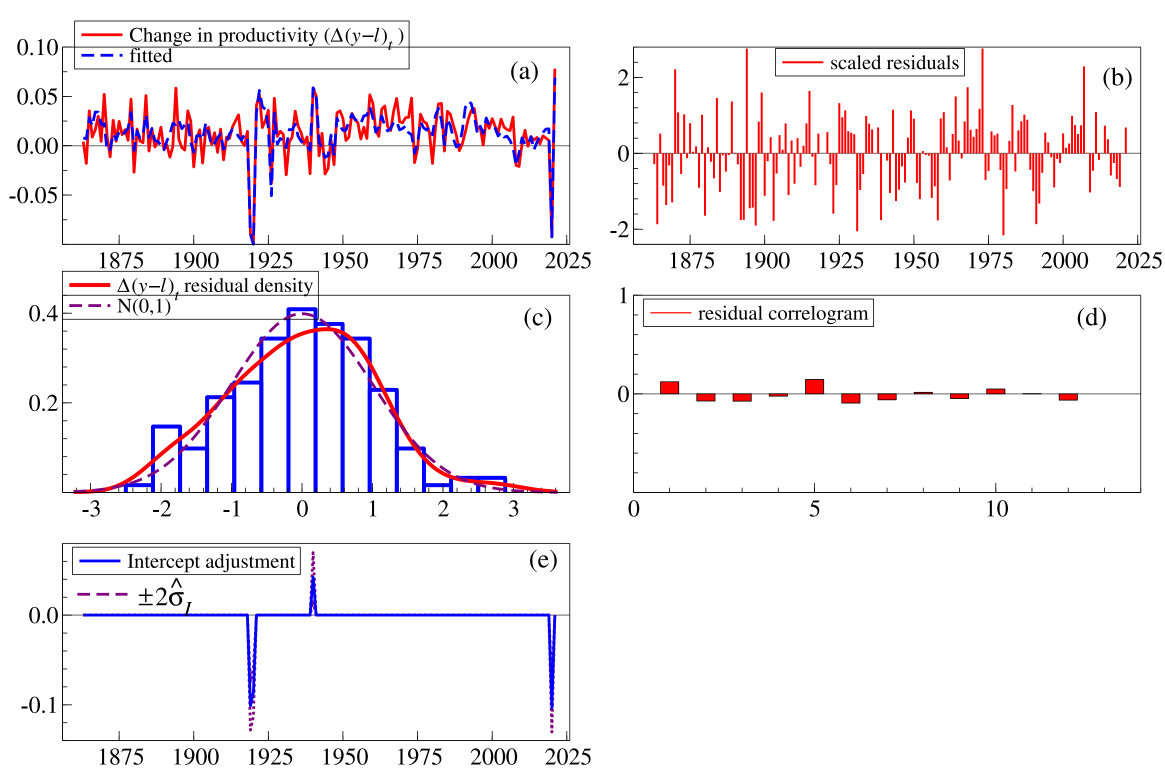

Fig. 14. (a) actual and fitted values of $\Delta(y-l)$; (b) scaled residuals; (c) residual density; (d) residual correlogram; and (e) intercept adjustment with $\pm 2\hat{\sigma}_I$.

of prices over unit labour costs. There was little inertia via lagged inflation, but a small direct impact of wages via unit labour costs. Neither unemployment nor inflation expectations were found to be relevant but many impulse and step indicators were needed, explaining events outside of the economic variables included.

Here we build a model of price inflation as measured by the implicit GDP deflator updated to 2021, although we commence the sample in 1965 given the many structural changes and non-constancies in the earlier period. The model selection methodology described in the Appendix A is applied and the resulting model for 1965–2021 is:[8]

$$\begin{aligned}\widehat{\Delta p}_t \quad = \quad & \underset{(0.04)}{0.29}\ \Delta p_{t-1} + \underset{(0.03)}{0.11}\ \Delta m_{t-1} + \underset{(0.10)}{0.16}\ \Delta R_{s,t-1} + \underset{(0.003)}{0.005}\ \Delta_2 p_{o,t} \\ & + \underset{(0.02)}{0.07}\ \Delta p_{w,t} - \underset{(0.09)}{0.43}\ \left(R_s - R_l - \mu_R\right)_{t-1} + \underset{(0.05)}{0.46}\ \Delta ulc_t \\ & + \underset{(0.005)}{0.03}\ I_{agg} - \underset{(0.002)}{0.005}\ \text{ChinaEffect}_t \\ & \hat{\sigma} = 0.89\%\ \ \mathsf{R}^2 = 0.98\ \ \mathsf{F}_{\mathsf{ar}}(2,46) = 0.12\ \ \chi^2_{\mathsf{nd}}(2) = 0.20 \\ & \mathsf{F}_{\mathsf{arch}}(1,55) = 1.01\ \ \mathsf{F}_{\mathsf{Het}}(17,39) = 0.99\ \ \mathsf{F}_{\mathsf{Reset}}(2,46) = 0.61\end{aligned} \tag{14}$$

The supplementary data file details the data and sources but for convenience we summarise the relevant regressors here. Δm is the growth rate of broad money, R_s the short-term interest rate and $R_s - R_l - \mu_R$ is the difference between the short and long interest rates corrected for a zero mean over the full sample such that $\mu_R = \overline{R_s - R_l}$. $\Delta_2 p_{o,t} = p_{o,t} - p_{o,t-2}$ is the smoothed growth rate over two years of a commodity price index linked to oil post 1997, measured in £, Δulc is a measure of the change in unit labour costs, and $\Delta p_{w,t}$ is a measure of world inflation based on a trade-weighted world price index measured in £. I_{agg} is an aggregated index of retained indicator variables based on Hendry (2001): $I_{agg} = 1_{1970} + 1_{1971} - 1.5 \times 1_{1973} + 1.5 \times 1_{1975} + 1_{1980}$. Finally, ChinaEffect is a step shift taking the value 0 to 1993 and 1 from 1994 onwards to represent the downward pressure from Chinese price competition.

Fig. 15 records the model fit in panel (a), scaled residuals in panel (b), residual density in panel (c) and residual autocorrelation function in panel (d). The model passes all diagnostics and is well-specified with an equation standard error of 0.9% despite inflation ranging from upwards of 24% to 0%. There is some inflation inertia, along with effects from broad money growth and interest rates, both the change in the short-run and the short-long spread. Energy prices, which are proxied by commodity prices, do not play a significant role but given the large changes in energy prices in 2022 can still have a sizable impact on inflation and so are retained. There is a sizable coefficient on unit labour costs which, linking back to Section 2.1, shows the important role for a wage price spiral effect in the price inflation equation. We do not find a role for excess demand for goods and services or the mark-up of prices over unit labour costs unlike the earlier studies. Given the congruent specification we next investigate the implications for current inflation under alternative scenarios for energy prices.

6. Wage-price spirals and inflation projections

Having developed models for price and wage inflation, unemployment and productivity we can undertake scenario analysis to investigate what the impact of the energy price rises seen in 2022 will be on UK price inflation. A full system model of wages, prices, productivity and unemployment is left for future research, but combining the single equation models is informative.

Table 4
Estimates for the parameters in the solved out inflation model.

Parameter	λ	ρ	γ	ω	ζ	η	κ	$\hat{\alpha}_{\tilde{f}=0}$	$\hat{\alpha}_{\tilde{f}=-0.5}$
Estimate	0.46	0.29	0.42	−0.18	−0.15	−0.72	0.52	5.25	3.48

We summarise the four empirical equations as:

1. Real wage equation (3). Let $(y+p-w-l) = \pi$ denote the markup of nominal output over the wage bill and write $\Delta(w-p)_t = \cdots$ as $\Delta w_t = \Delta p_t + \cdots$:

$$\Delta w_t = \left(1 - \zeta + \gamma \widetilde{f}_t\right) \Delta p_t + \zeta \Delta p_{t-1} + \kappa \Delta (y-l)_t + \omega U_{r,t} + D_{\Delta w} \tag{15}$$

where we use the fact that $\Delta^2 p_t = \Delta p_t - \Delta p_{t-1}$ and $D_{\Delta w}$ summarises the other drivers in the real wage model. We assume $\Delta_2 U_{r,t} \approx 0$ and we close off the non-linearity of unemployment in the wage equation for the purposes of our projections. The non-linearity kicks in when unemployment exceeds 8% and we make a simplifying assumption that unemployment will remain below the threshold level over the coming years.

2. Long-run unemployment equation (7). From the profits proxy, $R_r = R_L - \Delta p - \Delta y$:

$$U_{r,t} = \eta \Delta p_t + D_{Ur} \tag{16}$$

where D_{Ur} collects the other drivers including $R_{L,t}$ and Δy_t.

3. Productivity equation (13):

$$\Delta (y-l)_t = D_{yl} \tag{17}$$

where D_{yl} includes $\Delta(k-l)_t$ and $\Delta(e-k)_t$.

4. Price equation (14). Using the definition of the change in unit labour costs as $\Delta ulc_t = (\Delta w_t + \Delta l_t - \Delta y_t)$:

$$\Delta p_t = \lambda \Delta w_t + \rho \Delta p_{t-1} - \lambda \Delta (y-l)_t + D_{\Delta p} \tag{18}$$

where $D_{\Delta p}$ includes the exogenous drivers Δm_{t-1}, $\Delta p_{w,t}$, $R_{s,t-1}$, $R_{l,t-1}$, $\Delta_2 p_{o,t}$.

Solving out for price inflation results in:

$$\Delta p_t \approx \hat{\alpha}_{\tilde{f}_j} \left[\lambda (\kappa - 1) D_{yl} + \lambda \omega D_{Ur} + \lambda D_{\Delta w} + D_{\Delta p}\right] \tag{19}$$

where $\hat{\alpha}_{\tilde{f}_j} = 1/\left[1 - \rho - \lambda\left(1 + \gamma \widetilde{f}_t + \omega\eta\right)\right]$ and we approximate Δy_t by $\Delta(y-l)_t$ which is a simplifying assumption but over the short-term the forecasts should not be too different.

Table 4 reports the estimates for the parameters in the solved out inflation model assuming two cases; the first is the limit when workers demand 100% inflation compensation ($\widetilde{f}_t = 0$) and the second is when workers demand 50% of the observed price inflation as compensation in wages ($\widetilde{f}_t = -0.5$).

Substituting back in the other drivers for wage and price inflation and solving for the long-run solution so that the other regressors enter contemporaneously, since:

$$\Delta(y-l)_t = 0.44\Delta(k-l) + 0.28\Delta(e-k) + 0.005,$$

for $\widetilde{f} = 0$ this results in:

$$\begin{aligned}\Delta p_t \approx\ & 0.58\Delta m_{t-1} + 0.03\Delta_2 p_{o,t} + 0.37\Delta p_{w,t} - 1.42 R_{s,t-1} - 0.84 R_{s,t-2} \\ & - 0.31 R_{l,t} + 2.26 R_{l,t-1} + 0.41\pi_{t-2} - 0.37\Delta(k-l)_t - 0.24\Delta(e-k)_t.\end{aligned} \tag{20}$$

Thus, increases in the growth rate of M4, energy prices and world prices, and in the markup and long-term interest rates all raise inflation, whereas increases in capital and energy, short-term interest rates, and unemployment reduce it. Also, a reduction in energy availability of

[8] Similar results are obtained if we estimate using instrumental variables for Δulc_t as it implicitly uses Δw_t. The instruments included the lagged wage share, lagged change in unit labour costs, lagged output gap, the change in the working population and the change in the unemployment rate.

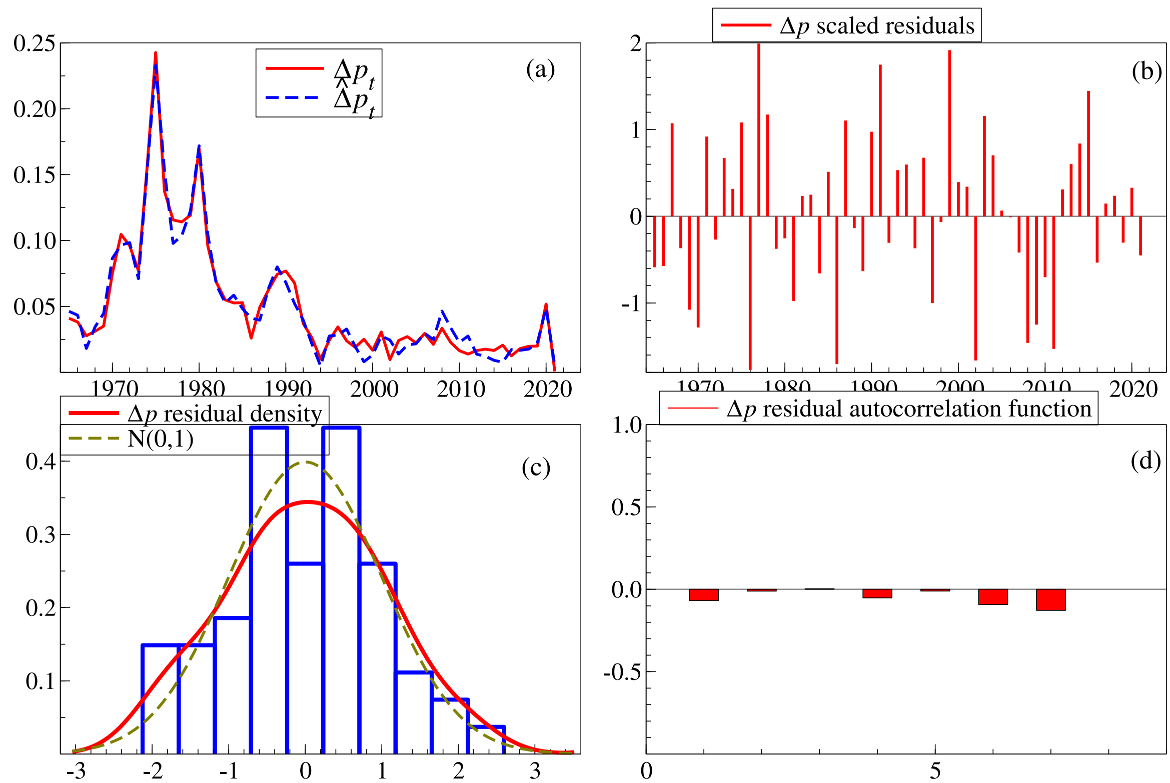

Fig. 15. Graphical statistics for the price inflation model in (14).

Table 5
Baseline assumptions for inflation projections for 2022 (a central scenario and a 'bad' scenario), 2023 and 2024.

Variable	Assumptions			
	Made in June 2022		Made in March 2023	
	2022	2022 (bad)	2023	2024
Δm_{t-1}	4%	8%	3.1%	3.2%
$\Delta_2 p_{o,t}$	150%	300%	−34%	−34%
$\Delta p_{w,t}$	6%	10%	4%	4%
$R_{s,t-1}$	0.1%	0.1%	3.5%	5.0%
$R_{s,t-2}$	0.1%	0.1%	0.1%	3.5%
$R_{l,t}$	3.0%	4.0%	4.0%	3%
$R_{l,t-1}$	0.5%	0.5%	3.0%	4%
π_{t-2}	5%	−5%	4%	4%
$\Delta(k-l)_t$	1%	1%	1%	1.0%
$\Delta(e-k)_t$	−10%	−10%	−15%	−15%

Table 6
Nowcasts and forecasts of UK inflation.

	Annual inflation (%)			
	Full wage pass-through	50% wage pass-through	80% wage pass-through	Bank of England
2022	12.5	8.3	10.4	3.4
2022 (bad)	15.7	10.4	13.1	3.4
2023	7.7	5.1	6.4	7.7
2024	5.3	3.5	4.4	3.3

10% (say) would simultaneously reduce output by 2.8% and exacerbate inflation by 2.4%.

Table 5 records the baseline assumptions for the conditional projections of inflation for (20), where $\Delta_2 p_{o,t}$ uses an equally-weighted average of a 50% increase for oil and 250% increase for natural gas, resulting in a 150% increase in commodity prices for the central 2022 scenario, whereas the 'bad scenario' has a 300% increase. The assumptions for 2022 were based on judgement and were made in June 2022 whereas the assumptions for 2023 and 2024 used higher frequency data when available and obtained projections using Cardt, a statistical forecasting device that is designed to be robust to structural breaks, see Doornik et al. (2020).

Table 6 reports the conditional projections of inflation given the assumptions in Table 5. Average annualised inflation over 2022 was 9%. In 2022 commodity prices rose by 170%, close to our projected scenario of 150%, and we use this along with other realised values of world inflation, broad money growth and the long and short term interest rates to infer a pass-through from price inflation to wage inflation of 80% ($\tilde{f} = -0.2$).[9] Given this reverse-engineered estimate of the wage-price spiral effect for 2022 we obtain projections of UK inflation for 2023 of 6.4% and 4.4% for 2024.

Table 7 shows the breakdown of components driving inflation under the assumptions given. Despite a small and insignificant direct effect of commodity prices on inflation in (14), the impact of the energy price increase contributes a third of the projected contribution to price inflation in 2022. Given the contributions to inflation from the model, short term interest rates would need to rise to 7.2% to offset the direct contributions of energy price rises in the central scenario, *ceteris paribus*, assuming 80% pass through of price inflation to wage inflation. Fortunately the 'bad scenario' did not materialise but as during the 1970s oil crises, interest rates would need to rise towards 10% in the second

[9] Data on capital per worker and energy per unit of capital for 2022 is not available so we use our projections.

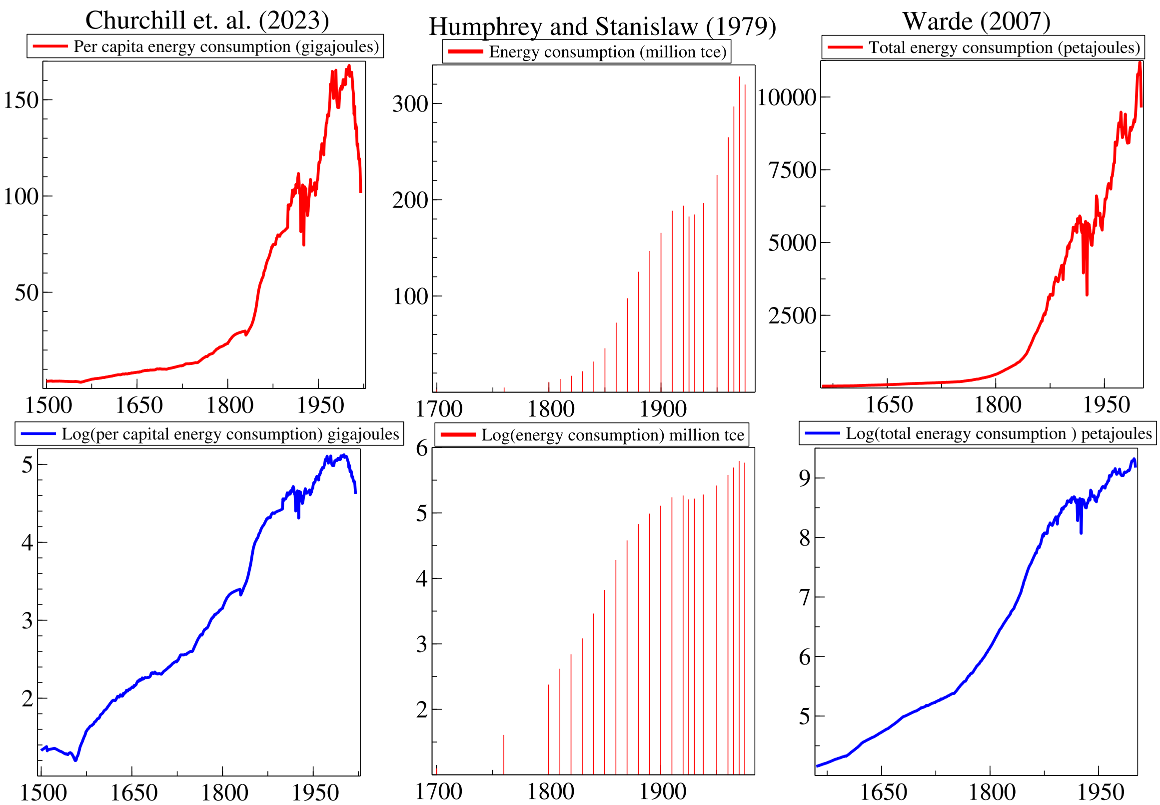

Fig. 16. Measures of energy consumption from Awaworyi Churchill et al. (2023), Humphrey and Stanislaw (1979) and Warde (2007).

Table 7
Contributions to Inflation. $\hat{p}$ denotes the projected inflation rate under alternative scenarios assuming 80% of price inflation is passed through to wage demands ($\widetilde{f}$ = −0.2). Contributions to R_s and R_l are summed over all periods.

	Δm_t	$\Delta_2 p_{o,t}$	$\Delta p_{w,t}$	R_s	R_l	π_t	$\Delta(k-l)_t$	$\Delta(e-k)_t$	$\hat{p}$
2022 ≈ % impact	1.9	3.3	1.8	−0.2	0.2	1.7	−0.3	2.0	10.4
2023 ≈ % impact	1.5	−0.7	1.2	−4.2	4.6	1.4	−0.3	3.0	6.4
2024 ≈ % impact	1.5	−0.7	1.2	−8.3	6.7	1.4	−0.3	3.0	4.4

scenario to dampen inflation down to the government set 2% target for inflation exceeding any levels seen since the 1970s (see Hendry, 2001).

7. Conclusions

The recent rise in UK price inflation was unanticipated, leading to a flurry of activity rethinking inflation models.[10] But high inflation rates are not new and history can shed light on the current inflationary climate. We use a long-run time-series dataset to model price inflation along with real wages, unemployment and productivity to gain insight into the current implications of inflationary pressures. The advantage of a long time-series of data is that there is a lot of variation which helps to identify explanatory factors. The disadvantage is that history is fraught with outliers, structural breaks, and distributional shifts (as is the future). The present is not at all like the past. In order to use the historical evidence we rely on econometric methods that can handle non-stationarities in the form of distributional shifts, resulting in congruent econometric models despite the vast change that the UK economy has experienced.

The paper highlights the importance of joint modelling of dynamics, location shifts, relevant variables and non-linearities. The automatic model selection approach implemented in our empirical models can handle many more variables than observations and this approach led to the detection of non-linearities which are fundamental in explaining the relationship between wages and prices. There is strong empirical evidence for non-linear adjustments of real wages to inflation where a wage-price spiral essentially adds a unit root to the wage-price process when inflation exceeds about 8%. Given that inflation was 8.8% in July 2022, this non-linearity is essential to understanding how inflation could potentially take off. We also found an additional non-linearity in unemployment which is consistent with involuntary unemployment.

Although the models are single equation, we use automatic tests for super-exogeneity to evaluate the modelling procedure. We show that the price, wage, productivity and unemployment equations can be combined along with the fundamental non-linearities to obtain projections for the contributions to current inflation. By imposing a 150% increase on energy prices (made up of 50% price rise in oil and 250% in natural gas in equal parts, a conservative estimate at the time of writing given the outturn of 170% increase) we show that inflation is projected to rise to 12.5% if workers demand full compensation for the price rises, close to the 11.1% annual inflation level observed in October 2022. Energy costs along with unit labour costs are fundamental to explaining past inflation episodes, and hence understanding current inflationary pressures.

[10] The Bank of England Monetary Policy Report in November 2021 included a central projection of inflation to rise to almost 5% in mid-2022, declining back to 2% over the two year horizon, but the widest confidence bands indicating a 90% confidence interval were at a maximum of 7% in mid-2022. The outturn was 8.8% in July 2022.

Appendix A. Econometric tools for modelling non-stationary time series

Given the manifest evidence of changing changes in all the variables associated with energy and economic outputs and their prices, *a priori* specification of a complete and correct model of the DGP is infeasible. Instead, to make the economic analysis empirically useful, model selection allowing for any number, magnitude, type, sign and timing of shifts is needed. Indicator saturation estimators (ISEs) offer a possible approach, available in software like *Autometrics PcGive* (see Doornik, 2009, and Doornik and Hendry, 2021), in EViews and as *gets* in R (see Pretis et al., 2018) based on a variant of machine learning for time series that uses block multi-path expanding and contracting searches. The main ISEs are impulse (IIS) for detecting outliers: see Hendry et al. (2008), analysed by Johansen and Nielsen (2009); step (SIS) for modelling location shifts (see Castle et al., 2015); trend (TIS) for trend shifts (applied in Walker et al., 2019 to health care management); multiplicative (MIS) for parameter changes (see Castle et al., 2020); and 'designed' (DIS) for modelling repeating shift patterns (e.g., the impacts of volcanic eruptions on temperatures as in Pretis et al., 2016); combinations of these, called super-saturation, are proposed in Ericsson and Reisman (2012). Impulse indicators are the first difference of step indicators which are the first difference of trend indicators, so SIS and TIS can capture outliers and the latter also steps, and while not parsimonious, that can be adjusted manually. Although stringent significance levels like $\alpha = 0.01\%$ are required to avoid excess numbers of irrelevant indicators being selected, these only apply to the indicators as all other regressors can be retained at that stage and only need selecting at more conventional significance levels like 1% later (see Hendry and Johansen, 2015).

The general approach to modelling non-stationary economic times series in Section 2–Section 5 commences with a very general model specification that allows for all possible explanatory variables, unknown functional forms of non-linearity, general dynamics, distributional shifts and outliers. Such generality necessarily implies more variables than observations at the outset, but it enables a congruent, well-specified model which nests the Data Generating Process (DGP) via the Theory of Reduction, see Hendry (1995, ch.9). Model selection using a tree search algorithm reduces the candidate set of regressors while allowing for complex correlations, ensuring that congruency is retained at every reduction stage, see Hendry and Doornik (2014).

General non-linear functions are used at the initial specification stage if agnostic on the specific functional form of possible non-linearities, and linearity in the parameters is maintained to ensure an efficient reduction although this is not required as reduction is done via maximum likelihood. Weierstrass's approximation theorem suggests using polynomials as the general non-linear functional form, and we use encompassing tests to identify specific functional forms against this general alternative. Section 2 demonstrates this approach by testing a general polynomial model against a logistic smooth transition wage-price spiral to obtain identification of the non-linearity inherent in the model.

Selection with more variables than observations inevitably means that the initial general model specification cannot be estimated. An iterative approach is needed with expanding as well as contracting searches to allow for correlations between variables that are not jointly included in each block search. Backtesting ensures that any reduced model encompasses the general model so there is no significant loss of information by eliminating regressors. Diagnostic checking also ensures the selected models are well-specified such that the model is a close approximation to the DGP. Finally, if a range of models is retained, denoted terminal models, then encompassing tests or information criteria can be used to select the final preferred model.

Having arrived at a model that is congruent with relevant explanatory variables and any breaks, outliers and non-linearities have been explicitly modelled, tests of exogeneity on contemporaneous regressors can be undertaken, see Engle and Hendry (1993). As a final stage, forecasts can be computed by extending the data set or having held back a subset of data to allow *ex ante* forecasts. Evaluating the forecasts does not validate the model as the forecast performance will depend on the out-of-sample data and need not indicate a poor model even if the forecasts are poor. However, poor forecasts could highlight *ex post* parameter non-constancy.

Despite searching over many candidate variables, Hendry and Johansen (2015) show that under the null of N possible candidate regressors that are all irrelevant, αN will be retained by chance even when $N > T$, where α is the significance level used to select the candidate regressors. Furthermore, if a theory model is retained without selection, and all other variables included in selection are orthogonalised with respect to the theory variables, then the resulting parameter estimates will be exactly the same as if the theory model was directly estimated. But as the data from Section 1 shows, any theory model that does not allow for change cannot be empirically relevant. Selection enables learning about non-constancies from data.

Throughout the modelling process we emphasise the joint nature of all modelling decisions. All aspects must be selected jointly for a coherent economic model, including all substantively relevant variables, their dynamics, outliers and location shifts, and non-linearities. Testing for each aspect individually and sequentially will not result in a well-specified model. For example, location shifts and non-linearities can be observationally equivalent and yet have very different economic interpretations and forecast implications, and not removing large outliers or shifts could hide the presence of other relevant variables or non-linearities.

Appendix B. Data definitions

Y_t	=	real GDP, £million, 1985 prices	[6], p.836, [9]a (1993), ONS code:YBHH at 2005 prices, [23].
P_t	=	implicit deflator of GDP, (1860=1)	[6], p.836, [9]a (1993), ONS code:ABML, [23].
K_t	=	total capital stock, £billions, 1985 prices	[6], [7], p.864, [8]b,c., [24].
$R_{l,t}$	=	bond rate, fraction	[8].
$R_{s,t}$	=	short-term interest rate, fraction	[8].
E_t	=	UK total energy use = $C + O + NG + NC + RN$	
O_t	=	Net oil usage, millions of tonnes	[21].
C_t	=	Coal volumes in millions of tonnes & MToe	[22].
NG_t	=	Natural gas volumes in MToe	[25].
NC_t	=	Nuclear energy use in TWh & MToe	[26].
Bio_t	=	Biomass in MToe	[27].
RN_t	=	Renewable energy: bio with wind+solar+hydroelectric in TWh & Mtoe	[26].
$P_{w,t}$	=	trade-weighted world price index	[1], [2], [10].
$P_{o,t}$	=	commodity/oil price index, £	[11].
U_t	=	unemployment	[7], [9]c (1993), ONS code: MGSC.
N_t	=	working population	[7], [9]c (1993), ONS code: MGSF.
$U_{r,t}$	=	U_t/N_t (unemployment rate, fraction)	
L_t	=	employment($= N_t - U_t$)	[4], [5].
W_t	=	nominal wage rates	[5], [12], [18].
M_t	=	Broad money M4	[6], p.148, [9], [20] ONS code: AUYN.

Additional definitions

x_t	=	$\ln(X)_t$ for any variable x_t
Δx_t	=	$\left(x_t - x_{t-1}\right)$
$\Delta^2 x_t$	=	$\Delta x_t - \Delta x_{t-1}$
t	=	deterministic trend commencing at 1 for the first observation in the sample
1_{xxxx}	=	1 for observation $xxxx$ and 0 otherwise
S_{xxxx}	=	1 for observations up to and including $xxxx$ and 0 otherwise
τ_{xxxx}	=	$-j, -j+1, -j+2, \dots, 0$ for observations up to $xxxx$ and 0 thereafter
IWWII	=	$1_{1942} + 1_{1943} - 1_{1944} - 1_{1945}$
I_{agg}	=	$1_{1970} + 1_{1971} - 1.5 \times 1_{1973} + 1.5 \times 1_{1975} + 1_{1980}$
ChinaEffect	=	0 for observations up to 1993 and 1 from 1994 onwards.

Data transformations

$w - p$	=	real wages
$y - l$	=	productivity (GDP per worker per year)
$k - l$	=	capital per worker per year
$e - k$	=	energy per unit of capital
ULC	=	$L_t W_t / P_t Y_t$ (unit labour costs)
π	=	$y + p - w - l$ (markup of nominal output over wage bill)
R_r	=	$R_l - \Delta p - \Delta y$ (profits proxy)

Sources:
[1] Friedman and Schwartz (1982);
[2] Attfield et al. (1995);
[3] Ericsson et al. (1998);
[4] Shadman-Mehta (1995) (who cites Sleeman (1981) and Thomas (1984) as sources);
[5] Phillips (1958);
[6] Mitchell (1988);
[7] Feinstein (1972) and Boyer and Hatton (2002);
[8] Bank of England and FRED;
[9] Bean (taken from (a) *Economic Trends Annual Supplements*, (b) *Annual Abstract of Statistics*, (c) *Department of Employment Gazette* and (d) *National Income and Expenditure*, as well as other sources cited here);
[10] Cameron and Muellbauer and IMF;
[11] UN Statistical Yearbook and Christopher Gilbert, spliced with oil prices since 1994;
[12] Office for National Statistics, Blue Book;
[13] Board of Trade (1860–1908);
[14] SS Stats;
[15] Annual Abstract of Statistics;

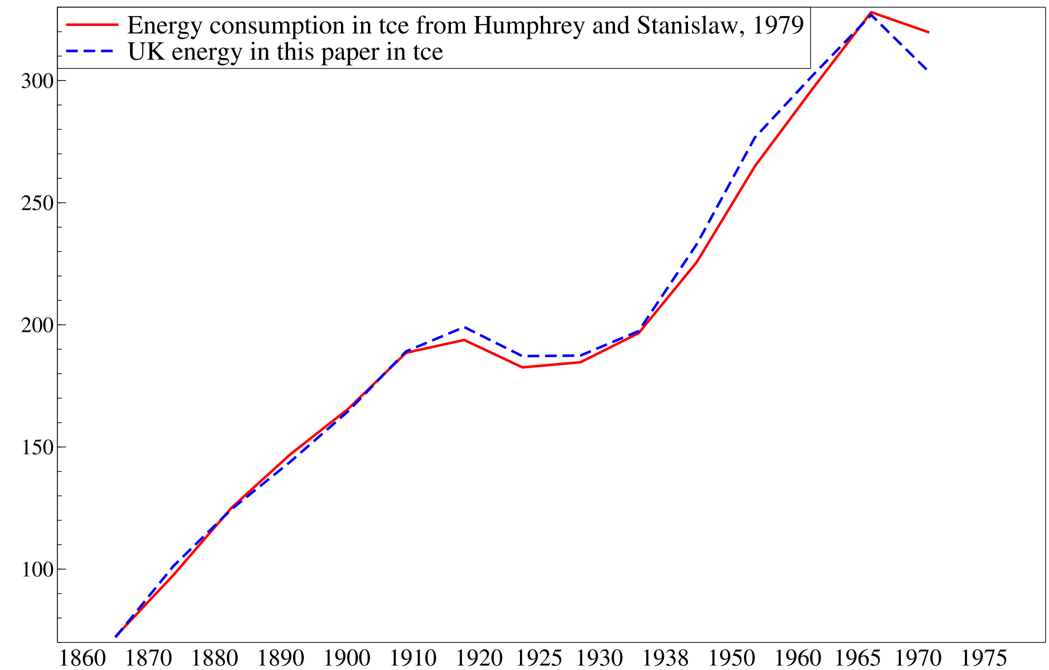

Fig. 17. Comparing our energy consumption with Humphrey and Stanislaw (1979) in tce.

[16] Office for National Statistics, Labour Market Trends;
[17] Crafts and Mills (1994);
[18] Feinstein (1990);
[19] Office for National Statistics, Labour Force Survey;
[20] Office for National Statistics, Economic Trends Annual Supplement.
[21] https://www.gov.uk/government/statistical-data-sets/crude-oil-and-petroleum-production-imports-and-exports Crude oil and petroleum: production, imports and exports 1890 to 2021, BEIS;
[22] Energy trends, BEIS and Carbon Brief;
[23] ONS;
[24] ONS Capital;
[25] Historical gas data: gas production and consumption and fuel input, BEIS
[26] Renewables (Solar [Large-scale Photovoltaic], Wind Onshore, Wind Offshore, Hydro) and Nuclear from Digest of UK Energy Statistics (DUKES), BEIS.
[27] bio

Hendry and Ericsson (1991), Hendry (2001), and Hendry (2015) provide detailed discussions about many of the macroeconomic time series.

Average weekly wages: a measure of full-time weekly earnings for all blue collar workers, where the coverage has been extended over time to include more occupations, and allows for factors such as changes in the composition of the manual labour force by age, sex, and skill, and the effect of variations in remuneration under piece rates and other systems of payments, but not adjusted for time lost through part-time work, short-time, unemployment etc. A reduction in standard hours worked that was offset by a rise in hourly wage rates would not be reflected in the index. From 1855–1880, the data are from Feinstein (1990), but not revised to increase coverage. The data also come from a number of sources on average wage rates for blue collar workers.

Nominal wage rates: hourly wage rates prior to 1946, then weekly wage rates afterwards, so the latter were standardised by dividing by normal hours. The trend rate of decline of hours is about 0.5% p.a. (based on a drop from 56 to 40 h per week between 1913 and 1990, with an additional increase in paid holidays), so unit labour costs were adjusted accordingly, and spliced to an average earnings index for the whole economy including bonuses [ONS: LNMM] from 1991 and rebased to 2000=1. Average earnings index was discontinued in 2010, and replaced with average weekly earnings. Exchange rate: annual £/$ rate till 1954, then an annual aggregate of quarterly data on the trade-weighted effective exchange rate, spliced to the £/$ rate in 1955. World prices: US prices till 1954, then a trade-weighted annual aggregate of quarterly data on the corresponding PPP values, from which the price data were derived and spliced to US prices in 1955.

There are undoubtedly important measurement errors in all these time series, but James (Duffy and Hendry, 2017) show that strong trends and large location shifts of the form prevalent in the data analysed here help offset potential biases in the long-run relation's estimated coefficients.

References

Attfield, C.L.F., Demery, D., Duck, N.W., 1995. Estimating the UK demand for money function: A test of two approaches. Mimeo, Economics Department, University of Bristol.

Awaworyi Churchill, S., Inekwe, J., Ivanovski, K., Smyth, R., 2023. Human capital and energy consumption: Six centuries of evidence from the united kingdom. Energy Econ. 117, 106465. http://dx.doi.org/10.1016/j.eneco.2022.106465.

Bernstein, D.H., Martinez, A.B., 2021. Jointly modeling male and female labor participation and unemployment. Econometrics 9, http://dx.doi.org/10.3390/econometrics9040046.

Boyer, G.R., Hatton, T.J., 2002. New estimates of British unemployment, 1870–1913. J. Econ. History 62, 643–675. http://dx.doi.org/10.1017/S0022050702001031.

Castle, J.L., Clements, M.P., Hendry, D.F., 2016. An overview of forecasting facing breaks. J. Bus. Cycle Res. 12, 3–23. http://dx.doi.org/10.1007/s41549-016-0005-2.

Castle, J.L., Doornik, J.A., Hendry, D.F., 2020. Multiplicative-Indicator Saturation. Working Paper, Nuffield College, Oxford University.

Castle, J.L., Doornik, J.A., Hendry, D.F., 2021. The value of robust statistical forecasts in the Covid-19 pandemic. Natl. Inst. Econ. Rev. 256, 19–43. http://dx.doi.org/10.1017/nie.2021.9.

Castle, J.L., Doornik, J.A., Hendry, D.F., Pretis, F., 2015. Detecting location shifts during model selection by step-indicator saturation. Econometrics 3 (2), 240–264. http://dx.doi.org/10.3390/econometrics3020240.

Castle, J.L., Doornik, J.A., Hendry, D.F., Pretis, F., 2019. Trend-Indicator Saturation. Working Paper, Nuffield College, Oxford University.

Castle, J.L., Hendry, D.F., 2009. The long-run determinants of UK wages, 1860–2004. J. Macroecon. 31, 5–28. http://dx.doi.org/10.1016/j.jmacro.2007.08.018.

Castle, J.L., Hendry, D.F., 2010. A low-dimension portmanteau test for non-linearity. J. Econometrics 158, 231–245. http://dx.doi.org/10.1016/j.jeconom.2010.01.006.

Castle, J.L., Hendry, D.F., 2014. Semi-automatic non-linear model selection. In: Haldrup, N., Meitz, M., Saikkonen, P. (Eds.), Essays in Nonlinear Time Series Econometrics. Oxford University Press, Oxford, pp. 163–197. http://dx.doi.org/10.1093/acprof:oso/9780199679959.003.0007.

Castle, J.L., Hendry, D.F., 2019. Modelling Our Changing World. Palgrave Mcmillan, London, https://link.springer.com/book/10.1007%2F978-3-030-21432-6.

Castle, J.L., Hendry, D.F., 2022. Econometrics for modelling climate change. In: Hamilton, J. (Ed.), Oxford Research Encyclopedia of Economics and Finance. Oxford University Press, Oxford, http://dx.doi.org/10.1093/acrefore/9780190625979.013.675.

Castle, J.L., Hendry, D.F., Martinez, A.B., 2017. Evaluating forecasts, narratives and policy using a test of invariance. Econometrics 5 (39), http://dx.doi.org/10.3390/econometrics5030039.

Castle, J.L., Shephard, N. (Eds.), 2009. The Methodology and Practice of Econometrics. Oxford University Press, Oxford, http://dx.doi.org/10.1093/acprof:oso/9780199679959.003.0007.

Chow, G.C., 1960. Tests of equality between sets of coefficients in two linear regressions. Econometrica 28, 591–605. http://dx.doi.org/10.2307/1910133.

Cleveland, C.J., Costanza, R., Hall, C.A.S., Kaufmann, R., 1984. Energy and the United States economy: A biophysical perspective. Science 225, 890–897, https://www.science.org/doi/10.1126/science.225.4665.890.

Crafts, N.F.R., Mills, T.C., 1994. Trends in real wages in Britain, 1750-1913. Explor. Econ. History 31, 176–194. http://dx.doi.org/10.1006/exeh.1994.1007.

Doornik, J.A., 2009. Autometrics. pp. 88–121, See Castle and Shephard (2009).

Doornik, J.A., Castle, J.L., Hendry, D.F., 2020. Card forecasts for M4. Int. J. Forecast. 36, 129–134. http://dx.doi.org/10.1016/j.ijforecast.2019.03.012.

Doornik, J.A., Hansen, H., 2008. An omnibus test for univariate and multivariate normality. Oxford Bull. Econ. Stat. 70, 927–939. http://dx.doi.org/10.1111/j.1468-0084.2008.00537.x.

Doornik, J.A., Hendry, D.F., 2021. Empirical Econometric Modelling using PcGive: Volume I, nineth ed. Timberlake Consultants Press, London.

Duffy, J.A., Hendry, D.F., 2017. The impact of near-integrated measurement errors on modelling long-run macroeconomic time series. Econometric Rev. 36, 568–587. http://dx.doi.org/10.1080/07474938.2017.1307177.

Engle, R.F., 1982. Autoregressive conditional heteroscedasticity, with estimates of the variance of United Kingdom inflation. Econometrica 50, 987–1007. http://dx.doi.org/10.2307/1912773.

Engle, R.F., Hendry, D.F., 1993. Testing super exogeneity and invariance in regression models. J. Econometrics 56, 119–139. http://dx.doi.org/10.1016/0304-4076(93)90103-C.

Ericsson, N.R., Hendry, D.F., Prestwich, K.M., 1998. The demand for broad money in the United Kingdom, 1878–1993. Scand. J. Econ. 100, 289–324, https://www.jstor.org/stable/3440779.

Ericsson, N.R., MacKinnon, J.G., 2002. Distributions of error correction tests for cointegration. Econom. J. 5, 285–318. http://dx.doi.org/10.1111/1368-423X.00085.

Ericsson, N.R., Reisman, E.L., 2012. Evaluating a global vector autoregression for forecasting. Int. Adv. Econ. Res. 18, 247–258. http://dx.doi.org/10.1007/s11294-012-9357-0.

Feinstein, C.H., 1972. National Income, Expenditure and Output of the United Kingdom, 1855–1965. Cambridge University Press, Cambridge.

Feinstein, C.H., 1990. New estimates of average earnings in the UK, 1880-1913. Econ. History Rev. 43, 595–632. http://dx.doi.org/10.2307/2596737.

Friedman, M., Schwartz, A.J., 1982. Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867–1975. University of Chicago Press, Chicago.

Galí, J., Gertler, M., 1999. Inflation dynamics: A structural econometric analysis. J. Monetary Econ. 44, 195–222. http://dx.doi.org/10.1016/S0304-3932(99)00023-9.

Galí, J., Gertler, M., Lopez-Salido, J.D., 2001. European inflation dynamics. Eur. Econ. Rev. 45, 1237–1270. http://dx.doi.org/10.1016/S0014-2921(00)00105-7.

Godfrey, L.G., 1978. Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. Econometrica 46, 1303–1313. http://dx.doi.org/10.2307/1913830.

Hendry, D.F., 1995. Dynamic Econometrics. Oxford University Press, Oxford, http://dx.doi.org/10.1093/0198283164.001.0001.

Hendry, D.F., 2001. Modelling UK inflation, 1875–1991. J. Appl. Econometrics 16, 255–275. http://dx.doi.org/10.1002/jae.615.

Hendry, D.F., 2015. Introductory Macro-Econometrics: A New Approach. Timberlake Consultants, London, http://www.timberlake.co.uk/macroeconometrics.html.

Hendry, D.F., 2022. Does an empirical economic relation have a life? A review essay. Hist. Political Econ. 163–179. http://dx.doi.org/10.1215/00182702-9699096.

Hendry, D.F., Doornik, J.A., 2014. Empirical Model Discovery and Theory Evaluation. Cambridge. MIT Press, Cambridge, Mass, http://dx.doi.org/10.7551/mitpress/9780262028356.001.0001.

Hendry, D.F., Ericsson, N.R., 1991. An econometric analysis of UK money demand in 'monetary trends in the United States and the United Kingdom' by Milton Friedman and Anna J. Schwartz. Am. Econ. Rev. 81, 8–38, https://www.jstor.org/stable/2006786.

Hendry, D.F., Johansen, S., 2015. Model discovery and Trygve Haavelmo's legacy. Econom. Theory 31, 93–114. http://dx.doi.org/10.1017/S0266466614000218.

Hendry, D.F., Johansen, S., Santos, C., 2008. Automatic selection of indicators in a fully saturated regression. Comput. Statist. 23, 317–335. http://dx.doi.org/10.1007/s00180-007-0054-z, Erratum, 337–339.

Hendry, D.F., Santos, C., 2010. An automatic test of super exogeneity. In: Watson, M.W., Bollerslev, T., Russell, J. (Eds.), Volatility and Time Series Econometrics. Oxford University Press, Oxford, pp. 164–193. http://dx.doi.org/10.1093/acprof:oso/9780199549498.003.0009.

Humphrey, W.S., Stanislaw, J., 1979. Economic growth and energy consumption in the UK, 1700–1975. Energy Policy 7 (1), 29–42. http://dx.doi.org/10.1016/0301-4215(79)90049-1.

Johansen, S., Nielsen, B., 2009. An analysis of the indicator saturation estimator as a robust regression estimator. pp. 1–36. http://dx.doi.org/10.1093/acprof:oso/9780199237197.003.0001, See Castle and Shephard (2009).

Kaufmann, R.K., 1992. A biophysical analysis of the energy/real GDP ratio: Implications for substitution and technical change. Ecol. Econom. 6, 35–56. http://dx.doi.org/10.1016/0921-8009(92)90037-S.

Luukkonen, R., Saikkonen, P., Teräsvirta, T., 1988. Testing linearity against smooth transition autoregressive models. Biometrika 75 (3), 491–499, http://www.jstor.com/stable/2336599.

Mitchell, B.R., 1988. British Historical Statistics. Cambridge University Press, Cambridge.

Phillips, A.W.H., 1958. The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957. Economica 25, 283–299, Reprinted as pp 243–260 in; 2000. Leeson, R. (Ed.), A.W.H. Phillips: Collected Works in Contemporary Perspective. Cambridge University Press, Cambridge.

Pretis, F., Reade, J.J., Sucarrat, G., 2018. Automated general-to-specific (GETS) regression modeling and indicator saturation for outliers and structural breaks. J. Stat. Softw. 86, 3, https://www.jstatsoft.org/article/view/v086i03.

Pretis, F., Schneider, L., Smerdon, J.E., Hendry, D.F., 2016. Detecting volcanic eruptions in temperature reconstructions by designed break-indicator saturation. J. Econ. Surv. 30, 403–429. http://dx.doi.org/10.1111/joes.12148.

Ramsey, J.B., 1969. Tests for specification errors in classical linear least squares regression analysis. J. R. Stat. Soc. B 31, 350–371, https://www.jstor.org/stable/2984219.

Reis, R., 2006. Inattentive producers. Rev. Econom. Stud. 73, 793–821. http://dx.doi.org/10.1111/j.1467-937X.2006.00396.x.

Shadman-Mehta, F., 1995. An Empirical Study of the Determinants of Real Wages and Employment: The Phillips Curve Revisited. (Unpublished thesis). Université Catholique de Louvain, Belgium.

Sleeman, A., 1981. The relation between unemployment and the rate of change of money wage rates in the United Kingdom. pp. 1851–1979, Paper presented to the Atlantic Economic Society, LSE, London.

Thomas, J.J., 1984. Wages and Prices in the United Kingdom, 1862-1913: A Re-Examination of the Phillips Curve. Presentation, ESRC Quantitative Economic History Study Group, Oxford.

Walker, A., Pretis, F., Powell-Smith, A., Goldacre, B., 2019. Variation in responsiveness to warranted behaviour change among NHS clinicians: A novel implementation of change-detection methods in longitudinal prescribing data. Br. Med. J. 367, l5205, https://www.bmj.com/content/367/bmj.l5205.

Warde, P., 2007. Energy Consumption in England and Wales, 1560–2004. ConsiglioNazionale Delle Ricerche. Istituto di Studi sulle Societé del Mediterraneo, Naples, https://histecon.fas.harvard.edu/energyhistory/data/Warde_Energy%20Consumption%20England.pdf.

White, H., 1980. A heteroskedastic-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica 48, 817–838. http://dx.doi.org/10.2307/1912934.