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A Note on Simple MSV Solution Methods for

Rational Expectations Models of Monetary Policy

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Abstract

We analyse the derivation of optimal monetary policy under discretion and commitment when lagged expectations appear in the Phillips curve, making use of the comparatively simple MSV approach which does not require transformation of the model into state-space form.

Key Words: Monetary Policy, Rational Expectations, Solution Methods, Minimal State Variable, Undetermined Coefficients. JEL: C61, E52, E58

Introduction

The monetary policy literature increasingly uses models with many lags and leads of endogenous variables together with expectations formed at different dates. These features make the models harder and more cumbersome to solve for optimal policy and are the focus of what follows. A large literature (for recent contributions and surveys see Binder and Pesaran, 1995, Christiano, 2002,

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Klein, 2000, Sims, 2002, and Zadrozny, 1998) presents solution methods for given systems of equations with these features but there have been fewer contributions concerning the derivation of the policy optimality conditions which form part of such systems. Recent examples of the latter include Soderlind (1999), Gerali and Lippi (2003) and Steinsson (2001) which work with the state space form of the model. Dennis (2003) shows that it may be time consuming or impossible to translate more complex models into the state space form required for these optimisation algorithms and presents alternatives using a representation much closer to the structural form of the model.

This note pursues a similar agenda, working directly with the model, but differs by using solely the Undetermined Coefficients-Minimal State Variable (UC-MSV) approach of McCallum (1983, 1999, 2003) demonstrated for standard models by McCallum and Nelson (2000). Its contribution is to extend the relatively simple methods used in the latter paper to deal with the presence of past dated expectations in the Phillips curve. The mathematics is unsophisticated compared with state space methods but offers two benefits. Firstly it may be applicable when the latter is infeasible or unwieldy (nor does it require transformation to the canonical form used by Dennis, 2003). Secondly the numerical part of the solution amounts to the solution of a set of simultaneous equations which implies that i) the required numerical methods are very accessible (available on standard spreadsheet packages as well as more powerful software), and ii) the solution is easy to use 'in reverse' whereby model or objective function parameters may be derived from given output criteria such as matching data characteristics or minimising loss under delegation. This avoids repeated 'forward' applications of alternative methods via grid searches (for examples see Jensen, 2002, and Soderlind, Soderstrom and Vredin, 2002).

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We are interested in solving for optimal monetary policy under discretion and commitment when the Phillips curve may contain many lags and expectational leads (at different dates) of the variables of interest (typically inflation and the output gap or marginal cost plus cost push shocks).[2] We solve such a model in the appendix[3] but for clarity of exposition the main part of the paper considers the much simpler form (1) which is just sufficient to raise the necessary issues. Notation is π for inflation, y for the output gap and χ for cost push shocks which are assumed to be iid. Also for simplicity we assume the standard quadratic loss function (2) where β is the discount factor. The absence of an interest rate smoothing term means that we may disregard the IS relationship and think of the policy maker choosing output/inflation directly to minimise (2) subject to (1).

$$0 \; = \; \alpha_0 \pi_t + \alpha_1 E_t[\pi_{t+1}] + \alpha_2 \pi_{t-1} + \alpha_3 E_{t-1}[\pi_t] + \alpha_4 E_{t-2}[\pi_{t-1}] + \gamma y_t + \varepsilon_t \tag{1}$$

$$L \; = \; \Sigma_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda y_t^2] \tag{2}$$

1. Discretion

With the Phillips curve (1) as the constraint, the key issue to be addressed arises from the lagged expectation, $E_{t-2}[\pi_{t-1}]$, the presence of which means that we cannot assume an appropriate undetermined coefficients (UC) reduced form for inflation and substitute out the expectational terms. This is possible with expectations of time t variables but not of t-1 or earlier variables. For a given

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set of assumed state variables for π_t, substituting out $E_{t-2}[\pi_{t-1}]$ will introduce an extra lag of each state variable which is inconsistent with the original assumed form. For example if we assumed the reduced form $\pi_t = A_0\varepsilon_t + A_1\pi_{t-1}$, substituting out $E_{t-2}[\pi_{t-1}]$ will introduce π_{t-2} and adding π_{t-2} will introduce π_{t-3} and so on. Instead of substituting out the expectational terms we make lagged expectations explicit state variables. Defining $z_t = E_t[\pi_{t+1}]$ we substitute z_t into (1) to give (3) and thus conjecture the UC-MSV forms (4).

$$0 \;=\; \alpha_0\pi_t + \alpha_1 z_t + \alpha_2\pi_{t-1} + \alpha_3 z_{t-1} + \alpha_4 z_{t-2} + \gamma y_t + \varepsilon_t \tag{3}$$

$$\pi_t = A_0\varepsilon_t + A_1\pi_{t-1} + A_2 z_{t-1} + A_3 z_{t-2} \quad y_t = B_0\varepsilon_t + B_1\pi_{t-1} + B_2 z_{t-1} + B_3 z_{t-2} \quad z_t = C_1\pi_t + C_2 z_{t-1} \tag{4}$$

The UC-MSV form for z_t in (4) follows directly from that for π_t, and for this simple case it is easy to solve directly for the C coefficients in terms of the A ones, but this is not the case in larger models so for illustration we add the definition of z_t and its reduced form in (4) to the system with all the A, B and C coefficients treated as undetermined. Hence we form the Lagrangean (5) from which the first order condition for inflation is given by (6), with expectations evaluated at time t when inflation is determined. Following McCallum and Nelson (2000), the underlying principle reflected in (6) is that the discretionary policy maker cannot directly influence expectations through explicit commitment but they may nevertheless influence their own future choices by the choice of current variables that will become lagged state variables in the future. This is formalised by the policy maker treating the coefficients in (3) as constants when optimising. The first order condition for the output gap is simply $0 = \lambda y_t + \gamma\varphi_t$ which allows us to substitute the Lagrangean multiplier out of (6) to give (7). If (1) contained output gap dynamics this step would not be possible and we would add a

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UC-MSV form for the multiplier to (3) and retain the inflation and output gap first order conditions as separate equations.

$$L_0 = E_0\Sigma_{t=0}^{\infty}\beta^t[\pi_t^2+\lambda y_t^2+\varphi_t(\alpha_0\pi_t+\alpha_1 z_t+\alpha_2\pi_{t-1}+\alpha_3 z_{t-1}+\alpha_4 z_{t-2}+\gamma y_t+\varepsilon_t)] \qquad (5)$$

$$0 = \pi_t+(\alpha_0+\alpha_1 C_1)\varphi_t+\beta(\alpha_0 A_1+\alpha_2+\alpha_3 C_1)E_t[\varphi_{t+1}]+\beta^2(\alpha_2 A_1+\alpha_4 C_1)E_t[\varphi_{t+2}] \qquad (6)$$

$$\tfrac{\gamma}{\lambda}\pi_t=(\alpha_0+\alpha_1 C_1)y_t+\beta(\alpha_0 A_1+\alpha_2+\alpha_3 C_1)E_t[y_{t+1}]+\beta^2(\alpha_2 A_1+\alpha_4 C_1)E_t[y_{t+2}] \qquad (7)$$

The solution comprises values for the A, B, C coefficients in (4) which are obtained by substituting (4) into (3) and (7) for variables dated t or later, and into the definition of z_t for variables dated t+1 or later, in this case just π_{t+1}. This results in three equations comprising terms in the state variables identified in (4). Since those equations must be satisfied for any values of the state variables, their coefficients must equal zero which gives us ten simultaneous equations in the ten UC coefficients in (4) and thus the solution to the model. Given numerical values for (4) it is easy to derive impulse responses to the cost push shock and predicted moments. In turn, the explicit simultaneous equations structure of the solution makes it easy to solve the model backwards by seeking values for the Phillips curve or loss function coefficients that give rise to particular properties of the solution subject to the simultaneous equations being satisfied.

2. Commitment

The issue we address is the presence of pairs of terms such as π_t and $E_{t-1}[\pi_t]$ in he Phillips curve. We illustrate the approach to this most simply by setting $\alpha_4=0$ since that term adds little to the discussion.

We also express inflation and the multiplier by $\pi_t = a_0\varepsilon_0 + E_{t-1}[\pi_t]$ and $\varphi_t = c_0\varepsilon_0 + E_{t-1}[\varphi_t]$ where, following Sims (2002), the first terms are expectational errors. As a first step we may write the Lagrangean (8) for the commitment case, retaining for the time being all the information about the timing of expectations in the Phillips curve despite the presence of the time zero expectation at the front of the expression which reflects the policy rule being committed to at that time without the possibility of subsequent re-optimisation.

$$L_0 = E_0\Sigma_{t=0}^{\infty}\beta^t[\pi_t^2 + \lambda y_t^2 + \varphi_t(\alpha_0\pi_t + \alpha_1 E_t[\pi_{t+1}] + \alpha_2\pi_{t-1} + \alpha_3 E_{t-1}[\pi_t] + \gamma y_t + \varepsilon_t)] \qquad (8)$$

Firstly, as in McCallum and Nelson (2000), we could replace $E_t[\pi_{t+1}]$ with π_{t+1} since φ_t is uncorrelated with $a_0\varepsilon_{t+1}$ and hence if $\alpha_3 = 0$ the constraint in (8) would no longer contain expectations. However, if α_3 is non-zero this is not possible since $E_0[\varphi_t\pi_t]$ includes the product of c_0a_0 and the variance of the shock process whereas $E_0[\varphi_t E_{t-1}[\pi_t]]$ does not. To maintain the proper distinction between inflation and its prior expectation we model the policy maker as optimising separately over $E_{t-1}[\pi_t]$ and a_0. This is achieved by substituting the expression for inflation above into both (1) and (2) and forming the Lagrangean (9) from which the first order conditions for a_0 and $E_{t-1}[\pi_t]$ are given by (10), in which the c_i coefficients are from the Wold representation of the multiplier, $\varphi_t = \Sigma c_i\varepsilon_{t-i}$, and (11).

$$L_0 = E_0\Sigma_{t=0}^{\infty}\beta^t[(a_0\varepsilon_t + E_{t-1}[\pi_t])^2 + \lambda y_t^2 + \varphi_t[\begin{array}{c}(1+\alpha_0 a_0)\varepsilon_t + \alpha_2 a_0\varepsilon_{t-1} + \gamma y_t \\ +\alpha_1 E_t[\pi_{t+1}] + (\alpha_0+\alpha_3)E_{t-1}[\pi_t] + \alpha_2 E_{t-2}[\pi_{t-1}]\end{array}]] \qquad (9)$$

$$0 = a_0 + \alpha_0 c_0 + \beta\alpha_2 c_1 \qquad (10)$$

$$0 = \alpha_1 \varphi_{t-1} + \beta E_{t-1}[\pi_t] + \beta(\alpha_0 + \alpha_3)E_{t-1}[\varphi_t] + \beta^2 \alpha_2 E_{t-1}[\varphi_{t+1}] \tag{11}$$

Next we multiply (10) by $\beta\varepsilon_t$ and add the resulting expression to (11) to give (12).

$$0 = \alpha_1 \varphi_{t-1} + \beta\pi_t + \beta\alpha_0\varphi_t + \beta\alpha_3 E_{t-1}[\varphi_t] + \beta^2 \alpha_2 E_t[\varphi_{t+1}] \tag{12}$$

In turn we may use the first order condition for y_t, which is the same as under discretion, to substitute out φ_t from (11) so the resulting expression plus the Phillips curve that appears as the constraint in (9) give two equations that represent the solution. From these we identify ε_t, π_{t-1}, y_{t-1}, $E_{t-1}[\pi_t]$ and $E_{t-1}[y_t]$ as state variables so the MSV solution is given by expressing π_t and y_t as linear functions of these terms and substituting them into the solution equations.[4]

3. Conclusion

This paper has extended the comparatively simple MSV approach of McCallum and Nelson (2000) for deriving optimising monetary policy choices and the resulting dynamics of inflation and output to models of private sector behaviour which give rise to past dated expectations in the Phillips curve. Such expectations affect the nature of the solution under both commitment and discretion but may be dealt with without sacrificing the clarity and simplicity of this approach. The paper has also highlighted the potential benefits of being able to use this method to solve models in reverse given that the numerical part of the solution is restricted to a set of simultaneous equations.

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[4]An example of this solution approach under commitment is given in Appendix D of Mash (2003).

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