ABSTRACT

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The Hypothetical Method in Plato's Middle Dialogues

The purpose of this dissertation is to offer an interpretation of Plato's hypothetical method in his middle dialogues. The hypothetical method is given in three accounts, one in the *Meno*, the *Phaedo* and the *Republic*.

These three accounts of the method, besides their affinities, seem to present some differences. The first main problem is to see whether we can speak of one single method, or of three different hypothetical method. Plato, in the *Meno* (86e) says that his method is similar to one which the geometers use and gives as an elucidation of it, an obscure geometrical problem to which I offer a new solution. The second main problem of this dissertation is to examine whether there is any relation (and if there is, of what kind) between Plato's accounts of the hypothetical method and the various methodologies in Greek geometry at that time.

I show in this dissertation that we have one method, in a broad sense, that employs hypotheses and proceeds in two ways: firstly, a way upwards (or backwards) towards the premisses of the argument or towards prior questions and, secondly, a way downwards from the premisses to the desired conclusion. The upward way is a heuristic process, whilst the downward one is deductive. Although we have essentially one method in all three dialogues, it is somewhat differentiated from one dialogue to the next (and in particular between the *Phaedo* and the *Republic*). In the three accounts of the hypothetical method, I see three stages of an evolutionary process similar to a corresponding one which took place in the evolution of the method of indirect proof in geometry. More precisely, I argue that the three accounts of Plato's method reflect three corresponding stages in the evolution of the reductive method of Hippocrates of Chios (*apagoge*) to the geometrical method of analysis and synthesis.

I argue furthermore, as regards the relation between Plato's philosophy and mathematics, that the axiomatization of geometry had an impact on Plato's conception of knowledge and upon his conception of dialectic. Moreover, I try to show that we have good reasons to suppose that Plato proposed a programme of reducing the principles of mathematics into the fewest possible and his contribution to this programme was decisive.

There is another problem regarding Plato's hypothetical method. In his middle dialogues, Plato clearly speaks about a new philosophical method of great importance and he gives extensive theoretical accounts of it. The strange thing is that it seems (and here almost all scholars agree) that nowhere does he apply his method (with the exception of a small-scale application of it in the *Meno*). However, this does not seem very likely. In chapters V and VI, I shall argue that we have extensive applications of the method in both the *Phaedo* and the *Republic*.

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στό Γρηγόρη Βλαστό

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P R E F A C E

The text of the Platonic corpus used for citation is that of the Oxford edition, edited by J. Burnet. Citations from the Aristotelian corpus are from the Oxford edition (ed. W D Ross).

Bibliographical references are made by citing the name of the author, the year of the edition and the page number.

Because of technical problems, it was not possible to have all Greek citations in Greek chararters. Where the grammatical form of the word is not relevant, Greek words are transliterated. All transliterated words are underlined. Greek phrases, words that occur in citations by other authors and words whose grammatical form is relevant to my discussion are in Greek letters.

Throughout this thesis, I use the following abbreviations;

a) Platonic dialogues

Crito	Cri	Euthydemus	Euthd
Charmides	Charm	Phaedo	Phd
Laches	Lach	Symposium	Symp
Lysis	Lys	Phaedrus	Phdr
Euthyphro	Euthp	Republic	Rep
Protagoras	Prot	Parmenides	Parm
Gorgias	Gorg	Theaetetus	Theaet
Meno	Men	Sophist	Soph
Hippias Maior	H Ma	Politicus	Pol
Hippias Minor	H Mi	Philebus	Phil
Cratylus	Crat	Timaeus	Tim

b) Aristotelian works

Analytica Priora	An Pr
Analytica Posteriora	An Post
Topica	Top
Physica	Phys
Metaphysica	Met
Ethica Nicomachea	E.N.
Ethica Eudemia	E.E.
Categoriae	Cat

c) Other Works

Diogenes Laertius, Vitae Philosophorum (ed. H S Long), D.L.

Proclus, In Primum Euclidis Elementorum Librum Commentarii (ed. G Friedlein), In Eucl

Proclus, In Rempublicam (ed. Kroll), In Remp

H Diels and W Kranz, Die Fragmente Der Vorsokratiker (6 ed., Berlin 1951-2), D.K.

H G Liddell, R Scott and H S Jones, A Greek-English Lexicon (Oxford, 1940) LSJ

d) Journals

Classical Quarterly	C Q
Philosophical Review	P R
Classical Review	C R
Archiv für Geschichte der Philosophie	Ar. Ges. Phil.
American Journal of Philology	A J P
Review of Metaphysics	Rev Met
Oxford University Press	OUP
Cambridge University Press	CUP

C O N T E N T S

INTRODUCTION

The purpose of this dissertation is to study Plato's hypothetical method in the middle dialogues. Plato's early dialogues have mainly an 'aporetic' character and the relevant method there is the Socratic elenchus. This method is used to disprove and not to prove something.

The hypothetical method - characteristic of the middle dialogues - is a 'positive' method that is used to confirm or prove something. It is given in three accounts, one in the Meno (86dff), one in the Phaedo (100aff) and one in the Republic (510ff). There is also a short passage in the Cratylus (436c-d) that refers to this method.

These three accounts of the method, besides their affinities, seem to present some differences. The first main problem is to see whether we can speak of one single method, or of three different hypothetical methods. Plato, in the Meno (86e), says that his method is similar to one which the geometers use and gives as an elucidation of it, an obscure geometrical problem to which I offer a new solution. The second main problem of this dissertation is to examine whether there is any relation (and if there is, of what kind) between Plato's accounts of the hypothetical method and the various methodologies in Greek geometry at that time. To this end, I devote chapter 2 of this thesis to the examination of the Greek geometrical method of analysis and synthesis and the method of apagoge (reduction).

I show in this dissertation that we have one method, in a broad sense, that employs hypotheses and proceeds in two ways: firstly, a way upwards (or backwards) towards the premisses of the argument or towards prior questions and, secondly, a way downwards from the premisses (or principles) to the desired conclusion. The upward way is a heuristic process, whilst the downward one is deductive. Although we have essentially one method in all three dialogues, it is somewhat differentiated from one dialogue to the next (and in particular between the Phaedo and the Republic). In the three accounts of the hypothetical method, I see three stages of an evolutionary process similar to a corresponding one which took place in the evolution of the method of indirect proof in geometry. More precisely, I argue that the three accounts of Plato's method reflect three corresponding stages in the evolution of the reductive method of Hippocrates of Chios (apagoge) to the geometrical method of analysis and synthesis.

I argue furthermore, as regards the relation between Plato's philosophy and mathematics, that the axiomatization of geometry had an impact on Plato's conception of knowledge and upon his conception of dialectic. Moreover, I try to show that we have good reasons to suppose that Plato proposed a programme of reducing the principles of mathematics into the fewest possible and his contribution to this programme was decisive.

There is another problem regarding Plato's hypothetical method. In his early dialogues, Plato applies continuously the Socratic elenchus although he does not give any theoretical account of it. In his late dialogues, he introduces the method of division and gives descriptions and extensive applications of it. In his middle dialogues, Plato clearly speaks about a new philosophical method of great importance and he gives extensive theoretical accounts of it. The strange thing is that it seems (and here almost all scholars agree) that nowhere does he apply his method (with the exception of a small-scale application of it in the *Meno*). However, this does not seem very likely. In chapters V and VI, I shall argue that we have extensive applications of the method in both the *Phaedo* and the *Republic*.

CHAPTER I

CHRONOLOGY - THE WORD HYPOTHESIS

1. Chronology

There is no objection to the authenticity of the *Phaedo* and the *Republic*. Objections to the genuineness of the *Meno* were raised in the 19th century by Ast and Scharschmidt [1]. Nowadays, all scholars regard the *Meno* as an authentic dialogue. Its authenticity is confirmed by some Aristotelian references,two of them indirect (*Polit*.1260a14-19, cf.*Meno* 71e-73c; and *An.Pr.* 69a20-9, cf. *Meno* 87b-d) and two direct (*An.Pr.* 67a21 [2] and *An.Post.* 71a27-9 [3]).There is also, above all, a reference to the *Meno* by Plato himself (*Phaedo* 72e) and a possible reference in the *Republic* (506c) to *Meno*97a-b.

It is generally accepted that we can divide the platonic dialogues into three groups: a) early (or Socratic) dialogues, b) middle dialogues and c) late ones . The division line between the first two groups of dialogues is taken to be Plato's first visit to Italy and Sicily [4] that probably took place in 388-7B.C. when Plato was forty years old [5].

It is very probable that the foundation of the Academy followed Plato's return to Athens after his first visit to Sicily. This is stated by the late biographers (eg *D.L.* 3.7). How long afterwards, they do not say, but most scholars assume that it was not that long [6].

As regards the chronological order of platonic dialogues, four types of aid have been invoked: a) literary criticism, b) philosophical considerations, c) stylometric and linguistics, d) external evidence and cross-references.

Scholars unanimously put the Phaedo and the Republic among the middle dialogues [7]. They also agree that the Meno precedes the Phaedo, and the latter the Republic. The problem that now arises is whether the Meno is an early or middle dialogue, that is, whether it was written before of after Plato's first visit to Sicily [8]. Cicero (De.Rep. 1,16) says that Plato during his first visit to Italy and Sicily, became intimate with the Pythagorean statesman and mathematician Archytas. Moreover, Cicero says (De.Rep. 10) that Plato's theory of the immortality of the soul and his interest in mathematics have their origin in the pythagorean philosophy. If Cicero's testimony is correct, then the Meno should have been written soon after Plato's return from Sicily. Indeed the Meno is the most 'mathematical' of Plato's dialogues. There are three very relevant mathematical passages (73e-76a, 82a-85b, 86e-87a), the immortality of the soul is stated, the distinction between knowledge and true belief is drawn, and pythagorean ideas of reminiscence are introduced. Moreover, Plato, in the Meno, tries for the first time to give a positive answer to the problem of knowledge and introduces a new method in his philosophy, besides the Socratic elenchus. Our view is that the Meno is in fact a transitional dialogue and with it we have the beginning of a new philosophical orientation of Plato, that characterizes his middle period. Hence one is inclined to accept Bluck's view that the Meno was written soon after Plato's return from Sicily, probably c.386-5 B.C. [9].

The prominent view regarding the Symposium is that it was written later than the Phaedo and after 385 B.C. probably a few years later [10]. If we accept the above view, we have to date the *Phaedo* somewhere between 386 and 383 B.C.. Scholars agree that the *Symposium* precedes the *Republic*. The *Republic* is a very long dialogue that Plato probably had been writing for years. Books VI and VII (or V, VI and VII) were probably written later than Books II, III, IV, VIII and IX,and the same view is maintained about Book X [11]. The prevailing view regarding the *Republic* is that its composition occupied the greater part of the decade 380-370 and it was finished c.375-70 [12].

The problems are more difficult as regards the *Eu thydemus* and the *Cratylus*. Earlier scholars used to put the *Cratylus* before the *Phaedo* and *Symposium*. However, since the fifties, the argument from apparent affinities in content (mainly Plato's preoccupation with language) with the so called 'critical group' (*Parmenides*, *Theaetetus*, *Sophist*, *Politicus*) has won much more favour, though still without unanimity [13]. On the grounds of the passage *Crat* 436c8-d7, that describes the dialectician's use of the hypothetical method, I think that we have to put the *Cratylus* among the middle dialogues and somewhere between the *Meno* and the *Phaedrus* [14].

I also think that the *Euthydemus* may be later than is commmonly supposed. We find in it a possible reference to the theory of Forms (300d-301a) and at 290b-c, a conception of the distinction between mathematics and dialectic very similar to that of the *Republic*, Books VI and VII.

I am inclined to believe that, as regard the dialogues written between the first two visits to Sicily, the following order holds - *Meno*, *Phaedo*, *Symposium*, *Republic*, *Phaedrus*, *Parmenides*, *Theaetetus*. The *Euthydemus* and the *Cratylus* are placed somewhere between the Meno and the Phaedrus [15].

As far as my interpretation of Plato's hypothetical method is concerned, the following minimum hypothesis is enough:

a) The Meno precedes the Phaedo and the latter the Republic;

b) The Phaedo was written after Plato's return from Sicily;

c) The middle books of the Republic (VI and VII) were written in the late 370s.

d) There is at least an interval of 10 years between the Meno and the Republic VI and VII.

This minimum hypothesis meets the criteria adopted by most scholars.

2. The Word hypothesis

A. In Plato

The words ὑπόθεσις, ὑποτίθημι have their origin in the verb τίθημι. This verb is very common in the platonic dialectic and it means mainly (in reference to mental action), 'to hold', 'to regard as' (Rep, 331a,376e, Soph, 246e), 'to assume' (Rep, 340a, 430a, Phd 79a, 100a, Theaet, 191c), 'to classify' (Soph, 235a, 267c, Rep, 475d), 'to judge' (Pol, 257b), 'to define' (or better, to put a definition) (Rep, 334e, 340b, 361b). It is often used to characterise a proposition as a starting point of a discussion or an argument (Phd, 79a).

The meanings of the verb ὑποτίθημι are closely related to those of τίθημι. According to LSJ, the verb ὑποτίθημι has the following meanings: a) 'to place under', 'to put under'; b) 'to place under as a foundation', 'to lay down', 'to lay down as a principle or rule', 'to presuppose'; c) 'to propose to oneself for discussion of argument', 'to propose (to do)'.

The noun ὑπόθεσις takes its meanings entirely from the verb ὑποτίθημι. Its appearance in dialectical contexts is earlier than Plato, probably dating from the late 5th century.

In the Hippocratic treatise of the late 5th century *On the Ancient Medicine* [16], we have an early occurence of the words *hypothesis* and *hypotithemi*. This Treatise is written in the form of a dialectical debate against various kinds of opponents, both 'sophists' and 'doctors' [17]. The opening sentence runs as follows: "All who, on attempting to speak or to write on medicine, have assumed (*hypothemenoi*) for themselves a postulate (*hypothesin*) as a basis for their discussion .... who narrow down the causal principle (*archen tes aities*) of diseases and of death among men, and make it the same in all cases, postulating (*hypothemenoi*) one thing or two...." The author contrasts medicine with speculative cosmology (I, 20-7) and concludes that the former has no need of hypotheses, although in the latter - because of its obscure and problematic subject - we are obliged to use hypotheses (cf II, 25-6; XIII, 1-19). Here the meaning of the word 'hypothesis' is 'proposition assumed for the sake of an argument' [18]. The 'hypothesis' is introduced here to prove something else. The hypothesis is unprovable and untested (although not self-evident). Hence, it has rather the meaning of 'principle' (cf the *archai* of the cosmologists) or better, that of 'postulate'.

Another significant passage concerning the use of the word 'hypothesis' in the 5th century's dialectic is found in Thucydides (E, 85-111) in the dialogue between the leaders of the Athenians and the Melians. In 90, the leaders of Melos say that they are obliged to speak within the framework of a hypothesis: "ὑμεῖς οὕτω παρὰ τὸ δίκαιον τὸ ξυμφέρον λέγειν ὑπέθεσθε ".

In platonic dialectic, we find the word 'hypothesis' in many senses. Platonic Definitions defined a hypothesis as an 'undemonstrated principle' (arche anapodeiktos). But in Plato's dialogues, the usual meaning of the word hypothesis is 'starting point for the discussion' [19].

Sometimes, the 'hypothesis' is posited after an argument in favour of its truth (Prot 316b). The 'hypothesis' is always posited provisionally and not dogmatically. We ought always to bear in mind that the hypothesis might be false (Rep 388e, 437a, Phd 107b, Crat 428d, Theaet 165d). So a 'hypothesis' is not necessarily an absolute principle. This which 'is posited' (or 'hypothesised') is not something known; it belongs rather to the area of belief. There are, in its use, all the degrees of confidence, from the simple acceptance (Phil 13b) to the strong confidence (Rep 437a, Phd 92d) [20]. When hypothesis is posited under the strong agreement of the two interlocutors, it has the meaning of an homologema (Men 87d3).

The word 'hypothesis' also has the meaning of a proposed theory for elenchus (Theaet 183b, where the hypothesis is the theory of flux), or a subject proposed for discussion (Parm 127d) or generally the subject matter of a discussion (Laws 812a).

The 'hypothesis' in Plato is either a proposition to be proved (or mainly disproved) (Charm 165a, 171d, 172c, Euthp 11c, Phd 94b, Theaet 165d) or a proposition adopted to prove (or disprove) something else (Charm 160d, Men 87d, Phd 92d, Prot 339d, Rep 437a) [21].

A 'hypothesis' can be an existential proposition (Parm 135e, Soph 237a, Phd 100b), or a definition (Charm 163a, 172c, Euthp

9d). But usually a 'hypothesis' is neither a definition nor an existential proposition (<u>Charm</u> 160d, <u>Phdr</u> 236b, <u>Rep</u> 437a, <u>H Ma</u> 286b, <u>Prot</u> 339d, 361b, <u>Tim</u> 61d).

When a hypothesis is considered arbitrary or disputable, Plato uses the verb 'συγχωρεῖν' in order to ask the cons‿ent of the interlocutor (<u>Charm</u> 172c, 162e, <u>Phil</u> 13b, <u>Phdr</u> 236b, <u>Phd</u> 91d, 100b, <u>Theaet</u> 183b-c).

The terms that Plato uses to denote the consequences, which results from the hypothesis, are the verb 'συμβαίνειν' (<u>Men</u> 87a, <u>Theaet</u> 165c10, 164b, <u>Charm</u> 164c9, <u>Gorg</u> 480e, <u>Parm</u> 142c), and the noun 'τὰ συμβαίνοντα' (<u>Gorg</u> 495b, 496e, 479c, 508b, <u>Meno</u> 87b, <u>Rep</u> 437a, <u>Parm</u> 136a1, b8, 142b3). But sometimes, he uses also the nouns 'τὰ ἑπόμενα' (<u>Crat</u> 436d) [22], 'τὰ ἑξῆς' (<u>Phd</u> 100c) [23], 'τὰ ὁρμηθέντα' (<u>Phd</u> 101d-e).

If the two interlocutors agree on an assertion, this is an 'ὁμολόγημα' ('ὁμολογία'). The <u>homologema</u> does not usually have the provisional character that the hypothesis has, nor is it put under elenchus [24]. However, this is not a general rule. Sometimes we find it in close relationship to the word 'hypothesis' (see <u>Rep</u> 437a, <u>Theaet</u> 165c). So on occasion, an <u>homologema</u> is only a provisional agreement.

B. <u>In Mathematics</u>

As we can see from the dialectical treatises of the Hippocratic corpus (<u>On ancient Medicine</u>, <u>On the Art</u>, <u>On Breaths</u>) and the above mentioned passage of Thucydides, the word hypothesis was already in use in the dialectic of the late 5th century. We do not know if the word hypothesis was used in the 5th century's mathematics. In the oldest mathematical text known (the four quadratures of lunes by Hippocrates of Chios - reported by Eudemus) [25] originating in the last third of the 5th century, the word 'hypothesis' does not occur. But there is an occurence of the word arche which in post-Euclidean mathematics, is often synonymous with the word hypothesis [26]. However we cannot know if Hippocrates himself used this word. Plato in the Meno (86e), referring probably to Hippocrates' method (cf chapter III, section 4) says "making use of a hypothesis like the geometers". From the above mentioned passage we might infer that, at that time the word hypothesis was already part of the mathematical vocabulary.

We can distinguish two different meanings of the word 'hypothesis' in mathematics: a) an *ad hoc* selected proposition (presupposition) that is posited at the beginning of a proof, and b) a strictly determined principle.

In Men 86e, the word hypothesis has the first meaning. Hippocrates' arche also has the same meaning (see chapter III, section 4). We find such propositions in Euclid and later mathematics under the name 'lemma'. According to Proclus (In Eucl 211), when we use a lemma for solving a problem, we asssume the lemma as known, we solve our problem on the basis of it, and after that we prove the lemma. So the lemma is a kind of hypothesis for the solution of our problem. Before the proof of the lemma, our original problem has been solved on the basis of a hypothesis.

The second meaning of the word hypothesis is related to the organisation of Geometry into an axiomatic system. So the first meaning of hypothesis may be earlier than the second.

Plato, speaking in the Republic (510b-511d) about the principles of mathematics [27] calls them 'hypotheses' although mathematicians view them as archai (this is probably suggested at

511b5 and c7) or as self-evident principles (510d1). We might then suppose that in Plato's time, mathematicians called their first principles archai and after Plato's attack against them in the Republic, the word hypothesis took also the meaning of 'first principle' in mathematics. So in Proclus the word hypothesis is synonymous with the word arche [28].

In the language of mathematics, the term hypothesis seems to characterize all the non-demonstrable principles, either axioms (or postulates) or definitions [29]. Aristarchus of Samos begins his book On the sizes and distances of the Sun and the Moon by stating six propositions (principles) without labelling them. But three lines later, referring to the first principle, he labels it 'hypothesis' [30]. Aristarchus' principles are axioms or postulates in the Euclidean sense. But very often, the word hypothesis is used to describe definitions.

Plato, in the famous passage of the Republic (510c-d), gives three examples of the mathematicians' principles (odd and even, three kinds of angles, various geometrical figures). All these mathematical notions are introduced in Euclid's Elements with definitions (definitions VII, 6,7 for the odd and the even, definitions I,10,11,12 for the three kinds of angles and defintions I,15,18,19,20,21,22 for the geometrical figures). Proclus uses the word hypothesis in the same meaning (In Eucl 178) "the first principles of Geometry are divided into three groups: hypotheses, postulates and axioms .... hypotheses or what are called definitions, we have considered in the foregoing" (cf also 76.6). On p.76, Proclus distinguishes three kinds of principles according to Aristotle. After giving an account of hypothesis according to Aristotle (p76.13-5, cf An Post 76a30-77a4) he ofers an example of a 'hypothesis' which is clearly the case of a definition [31]. Later on (p77.20), Proclus adds that "often, however, they are all (ie. first principles) called hypotheses". Archimedes (On conoids and spheroids ed, Heib, 249.13, 253.1) uses also the verb hypotithesthai to introduce definitions.

The terminology concerning first principles in Greek geometry is very loose, especially in Archimedes. We can hardly find strictly determined terms concerning the various kinds of principles. So Archimedes, in his work De· Sphera et Cylinder, begins by stating two different kinds of principles with the labels axioms and postulates (lambanomena). But in the 2nd, 4th, 5th and 6th axioms, as well as the 1st postulate, are definitions. After stating his two groups of principles, Archimedes continues "τούτων δὲ ὑποκειμένων", that is to say, "with these hypotheses...."

In conclusion then we can say that in Greek mathematics, we cannot find a fixed terminology. The word hypothesis usually characterises all kinds of geometrical principles. It is still very probable that before the axiomization of geometry, the word hypothesis was used to describe propositions that were useful for proofs. After it (and probably after Plato's criticism of mathematics in the Republic) the word 'hypothesis' took the meaning of 'first principle', henceforth becoming more or less synonymous with the word arche.

NOTES TO CHAPTER I

[1]. See Bluck, 1961, 108; Hoerber, 1960, 78.

[2]. "ὁμοίως ὁ ἐν Μένωνι λόγος ὅτι ἡ μάθησις ἀνάμνησις"

[3]. "εἰ δέ μὴ τὸ ἐν Μένωνι ἀπόρημα συμβήσεται"

[4]. Cf Crombie, 1962,11; Guthrie, 1975, 53; Field, 1948, 74; Ross, 1951,10; Robin, 1971,29; Natorp, 1903/1929, 15; Bluck, 1961, 118; Stenzel, 1974, 99; Sharples, 1985,2; Bostock, 1986,4. C Kahn (1981, 306-10) divides the early dialogues into two groups and holds that the dialogues belonging to the second group (La, Char, Lys, Euthp, Prot, Euthd plus the transitional Meno) were written between 386 and 380, after Plato's return from Sicily.

[5]. See 7th Letter 324a6. Morrison,(1958 and 1964) claims that the Meno and the Phaedo were written before Plato's first journey to Sicily, but the Republic afterwards.

[6]. The foundation of the Academy is usually posited in 387-6 B.C. Ryle is an exception (1966, 282). However, it is probable that, although the Academy was founded soon after Plato's return, it was a few years until it took its final form as an institution.

[7]. Cf for example, Ross, 1951,10 (cf also the five tables reported by Ross on p2); A. E. Taylor, 1937,11; Guthrie, 1975, 53; Crombie, 1962, 11; Hackforth, 1955,7; Field, 1948, 29-30; G. Ryle, 1966, ch VII; Bluck, 1955, 144; Bury, 1932, pLXVII; Friedlander, 1969, 448; Bostock, 1986,2. Even Shorey, who castigates this sort of inquiry, admits that we can distinguish three groups of dialogues (see Shorey, 1933, 58; 1903, 78).

[8]. Bluck (1961, 118), J. Thomas (1980,22), Raven (1965, 68), Thompson (1933, lvi), Guthrie (1975, 53), Robin (1971, 29), R.

Robinson (1953, 121-2), Lassere, (1964, 15), Cambiano (1971, 142), Maccioni (1978,10), Dodds (1959,43) view the Meno as a middle dialogue. On the other hand, D. Ross (1951, 10), Field (1948, 74), Crombie (1962,11) view it as an earlier dialogue.

[9]. Its date has been assigned as 384 by Thompson (1933, lvi), 380 by Lassere (1964,15) and Kahn (1981,307), 387-6 by Raven (1965,71) and between 389 and 386 by Guthrie (1975, 236). Wilamowitz (see Hoerber, 1960,78) and Stenzel (1974, 102) consider the Meno as the programme of the Academy.

[10]. Cf for example, Bluck, 1961, 119; Field, 1948, 74; Bury, 1932, LXVII; Robin, 1971,19; Guthrie, 1975, 365. Symp 193a has often been taken as an allusion to the dioikismos of Mantinea in 385, or at any rate as indicating that Plato must have had that event in mind. See contra, H. Mattingly, 1958; Morrison, 1964, 43sq. K J Dover (1965) argued convincingly in favour of the classical view, and posits Symposium between 385 and 378.

[11].Nettleship, 1901, 162; Grube, 1935,21.

[12]. Cf Hackforth, 1952,7 n1; Guthrie, 1975, 437; Field, 1948, 71; Cambiano, 1967,144. It is generally accepted that the Republic precedes the Parmenides and the the Theaetetus, and the latter was written between 369-7B.C. The prevalent view about Phaedrus' position is between the Republic and the Parmenides.

[13]. Allen (1954) views it as contemporary with Theaetetus. Runciman (1962, 2) places it before the Phaedrus. On the other hand, Friedlander (1969,453) regards it as early gialogue.

[14]. Cf also Rose, 1964.

[15]. G.E.L.Owen (1965) suggested that the Timaeus and Critias belong to the middle period. While they "undoubtedly follow the Republic and possibly follow the Phaedrus, they precede the

'critical group' which begins with the <u>Parmenides</u> and <u>Theaetetus</u>" (p318). See contra, H. Cherniss, 1965.

[16]. On the date, see G.E.R. Lloyd, 1963, 108-26.

[17]. Cf G.E.R. Lloyd, 1979, 95

[18]. See in C.C.W. Taylor, 1967,196; Cohen, 1969, 137-40; Leszl, 1981, 320-1.

[19]. See <u>Gorg</u> 454c, <u>Phd</u> 92c, <u>Theaet</u> 165d1,183b, <u>Rep</u> 550c, <u>Tim</u> 61d, <u>Charm</u> 171d, <u>Prot</u>361b, <u>Soph</u> 237a 244c, <u>Parm</u> 136a-b, <u>H Ma</u> 302e. Sometimes, the noun 'hypothesis' indicates the starting-point for practical action (see <u>Laws</u> 743c, 812a).

[20]. Cf Robinson, 1953,93-4. Robinson says that "positing is only that kind of believing in which deliberately and consciously we adopt a proposition with the knowledge that after all it may be false" and below that, "what is posited is always provisional and tentative".

[21]. For this double use of the word Hypothesis, see Robinson, 1953, 110-3. A. Szabó (1967) is certainly wrong in saying that "the Greek ὑπόθεσις in dialectic and mathematics is a starting point which is impossible to prove; and it does not need to be proved, because the partners in the debate agree upon it" (p3). Szabó clearly confuses the meaning of the word hypothesis in dialectic with this of <u>homologema</u>.

[22]. The usual meaning of the word <u>hepomenon</u> is simply 'following' and not logical consequense (cf <u>Charm</u> 154a, <u>Phil</u> 66a, <u>Rep</u> 399c 422b, <u>Men</u> 75b, <u>Tim</u> 54d, <u>Laws</u> 690a 744d).

[23]. Cf also <u>Gorg</u> 454c. But usually the word 'ἑξῆς' is an adverb with the meanings of 'one after another', 'in order', 'next to', (cf <u>La</u> 182b, <u>Euthp</u> 288e, <u>Rep</u> 484b, 528a,b,d, <u>Tim</u> 20b, <u>Pol</u> 259d, <u>Prot</u> 314e).

[24]. For the two axioms of equality that we find in Theaet 155a, Plato uses the word homologema. For J. Klein (1965, 63) homologia means an agreement on one thesis acceptable by the interlocutors or of common acceptance, and the necessity of the agreement on technical terms. "...homologia governs the method of synthetic mathematics as well as of any other apodeitic discipline". According to Cornford (1932/1965, 66), the homologema shows the consistency of an argument or the agreement of the interlocutors. For the meanings of the words homologein, synchorein in Plato, see Lyons, 1963,105.

[25]. See Simplicius, In Phys A2, ed Diels, 60-8.

[26]. See Proclus, In Eucl (Friedlein), 26-7.

[27]. Plato describes geometry as a science already organized into an axiomatic system (see chapter VI, section 1).

[28]. See p26-7: "For, in mathematics later propositions are always dependent on their predecessors, and some are counted as starting-points (archai), others as deductions from the primary hypotheses"; p58 "...which we say is ungeometrical because it arises from hypotheses quite different from the principles of geometry".

[29]. Cf Szabó, 1978, 323-6; Cornford, 1932/1965, 63-5.

[30]. Cf also Archimedes, Sand-Reckoner, (Heiberg), ii p218.7-11. It was an usual phenomenon with ancient mathematicians to state at the beginning of their works, all their 'first principles' (axioms or definitions indifferently) without so labelling them (cf the pre-Euclidean mathematical work of Autolycus of Pitane, On the Moving Sphere; Archimedes, On Spirals).

[31]. "That a circle is a figure of such and such sort we do not

know by a common notion in advance of being taught, but....." (p76.14-16).

CHAPTER II

THE GREEK GEOMETRICAL METHOD OF ANALYSIS

Because my interpretation of Plato's hypothetical method depends heavily on the geometrical methods of <u>apagoge</u> and analysis, I propose here to examine these two geometrical methods of indirect proof. The interpretation of the geometrical method of analysis and synthesis presents many problems that need clarification. Moreover no interpreter has worked systematically on the method of <u>apagoge</u>. I shall devote this chapter then, to a discussion of the above methods.

1. <u>The term analysis</u>

The words ἀνάλυσις, ἀναλύειν originate from the verb λύειν and the preposition ἀνὰ. According to <u>LSJ</u>, λύειν means 'loose', 'loosen', 'unfasten', 'untie', 'set free', 'release', 'undo', 'do away with'. When it is used in compound words, the preposition ἀνὰ means 'up to', 'upwards', 'up', 'back', 'backwards' or sometimes has the sense of 'strengthening', 'improvement' or repetition.

However the verb λύειν denotes actions (loose, untie, set free etc) that have not any reference to determinations of place such as up or down. It is very difficult for a verb to mean 'λύω ἄνω' or 'λύω κάτω' (that is, loose up or down).

In <u>LSJ</u>, we find that the main and primary meanings of ἀναλύειν are 'unloose', 'untie', 'undo', 'get rid of', 'set free', 'do away'. Therefore we see that the two verbs (λύειν, ἀναλύειν) are nearly identical in their meanings.

We find the word ἀναλύειν (mainly in the form of 'ἀλλύειν') seven times in Homer, once in Hesiod, once in Pindar, once in Sophocles, once in Euripides and once in Xenophon [1]. The word is absent from the vocabulary of Aeschylus, Herodotus, Thucydides,Aristophanes, Lysias, Isaeus, Isocrates and Plato[2]. The word does not appear in the index (10th volume) of the <u>Oeuvres completes d' Hippocrate</u> (E.Littré, Paris, 1861), or in Diels' Index of Presocratics. It seems then that while the word ἀναλύειν was in use in Homer, it dropped out of use later (it is worthwhile to note here that the noun ἀνάλυσις occurs only once -in Sophocles- , and the adjective ἀναλυτικὸς is completely absent from all the ancient texts until Aristotle).

The words ἀναλύειν, ἀνάλυσις, ἀναλυτικὸς, occur again in Aristotle. However it seems that it was not Aristotle who reintroduced the word into Greek vocabulary. There are some passages in Aristotle (eg. <u>E.N.</u>1112b20-4, <u>Soph.El.</u>175a28) in which it seems that the term 'analysis' was already used by mathematicians. The mathematical origin of the above Aristotelian terms was recognised long ago[3]. It is very probable that the word analysis was in use among the mathematicians of Aristotle's time, and Aristotle took it from them. Mathematicians reintroduced the word analysis into Greek vocabulary (after it had been disused for a long time) to state a demonstrative process that goes in an opposite way to the usual, that is to say, backwards or upwards. In this way, the preposition ἀνὰ has now got meaning in the composite word ἀνάλυσις (ἀνάλυσις = solution backwards or upwards).

A.Szabó [4] claims that the meaning of the words ἀναλύειν, ἀνάλυσις is not what it is usally taken to be, which is 'taking apart' ('analysing' in our twentieth-century sense), but it is rather like 'untying' or 'unloosing'. The role of our modern 'analysing' he reserves for διαιρεῖν, διαίρεσις [5]. "...as the antithetical concepts are: διαιρεῖν and συνθεῖναι exactly for our modern 'analyse' and 'synthesize'. In any case, Greek σύνθεσις and modern 'synthesis' seem to be identical .... but I do not think that in classical Greek the words ἀναλύειν, ἀνάλυσις could ever be synonyms for διαιρεῖν, διαίρεσις respectively" (1974, p.119).

If Szabó had said only that the original meaning of the word ἀναλύειν was 'untying', he would be right. But from Aristotle onwards we also find this word in the sense of 'taking apart' [6]. In An.Pr. 49a19, 51a18, 51a32, 47a4, 50a30, 50b3, 50b30, 51a2, etc, the words ἀνάλυσις, ἀναλύειν are synonymous with ἀναγωγὴ, ἀνάγειν (cf. also Soph.El. 168a19). In N.E. 1112b18-25 -a passage in which the word analysis is used in a mathematical context - ἀνάλυσις (b24) is almost synonymous with εὕρεσις (b19). Therefore we can see that the words ἀνάλυσις, ἀναλύειν have a lot of other meanings besides this of 'untying'. But even Szabo's claim that διαίρεσις and σύνθεσις are antithetical concepts and also that διαίρεσις and ἀνάλυσις have no common meanings, is wrong. Aristotle uses the word διαίρεσις at De Caelo 299b6-11 in the same context in which he uses the word ἀνάλυσις in De Gen.Cor. 329a23. Moreover Proclus (In Eucl., ed.Friedlein, 57) speaks of geometrical analysis (as opposed to synthesis) as a transition from complex matters into more simple ones. But more characteristic is another passage (p.382.1-2) in which Proclus uses the word ἀνάλυσις as exactly synonymous to διαίρεσις ("χρὴ τοίνυν εἰδέναι ὅτι πᾶν σχῆμα εὐθύγραμμον εἰς τρίγωνα ἀναλύεται").

Therefore we can distinguish three different meanings in the word analysis as showing a demonstrative process: a) as a movement backwards (in this sense it is almost synonymous to the word ἀπαγωγὴ), b) as a movement upwards (synonymous to ἀναγωγὴ), and c) as resolution (or breaking up) of a complex matter to its simple elements.

The problem that now arises is which of these three characteristics is the original one in the mathematical use of the word analysis. The word analysis in Greek mathematics is strictly related to a demonstrative method. Its main characteristic is that it accepts the conclusion (or the problem to be solved) as given and draws consequences from it. In this sense it is a backwards movement and it is similar to the reductive method of Hippocrates of Chios. But at the same time, in pre-Euclidean mathematics (as I shall try to show later), the method of analysis was used to function as a movement from specific problems to more general ones, or as a tool for the reduction of mathematical propositions to their principles. In this sense, the method of analysis is a movement upwards.

However, in early Greek mathematics, both proof and mathematical figure are called διαγράμματα (see Arist. Met. 998a25; An.Pr.,46a8; Ross on Met.998a25; Plato Phaedo 73b1; Crat. 436d2; Xenophon Mem.IV,7). Moreover, as we shall see later, the analysis of a figure is an essential characteristic of the method of geometrical analysis. Thus we can suppose that the meaning of ἀναλύειν διάγραμμα is probably transferred from the proofs to the figures and it means the breaking up of a figure into its parts[7].

2. <u>Definitions of the geometrical method of analysis and synthesis</u>

Two definitions of Greek geometrical analysis have been preserved: a) the definitions of Pappus (<u>Collectio</u>,VII, Praef.1-3; ed. Hultsch pp.634.3-636.30) and b) the interpolated definitions of analysis and synthesis in Euclid's <u>Elements</u> XIII (together with five alternative proofs of the propositions XIII 1-5, by means of analysis and synthesis).

A) <u>PAPPUS</u>

634.10 Analysis is the way from what is sought [ζητουμένου] -as if
it were admitted [ὁμολογουμένου]- through its concomitants
[ἀκολούθων; the usual translation reads: consequences] in
order to reach something admitted in synthesis. For in analysis
we suppose [ὑποθέμενοι] wthat is sought to be already done
and we inquire from what it results [συμβαίνει], and again
what is the antecedent of the latter, until we on our backward
634.15 way light upon something already known or having the position of
a beginning [τῶν ἤδη γνωριζομένων ἢ τάξιν ἀρχῆς ἐχόντων]. And
we call such a method analysis, as being a solution backwards
[ἀνάπαλιν λύσιν]. In synthesis, on the other hand, we
suppose what was reached last in analysis to be already done,
634.20 and arranging in their natural order as consequents the former
antecedents [καὶ ἑπόμενα τὰ ἐκεῖ προηγούμενα κατὰ φύσιν
τάξαντες] and linking them one with another, we in the end
arrive at the construction of the thing sought. And this we
call synthesis. Now analysis is of two kinds. One seeks the
truth [ζητητικὸν τοῦ αληθοῦς], being called theoretical. The
634.25 other serves to carry out what was desired to do [ποριστικὸν
τοῦ προταθέντος], and this is called problematical. In the
theoretical kind we suppose the thing sought as being and as
636.1 being true, and then we pass through its concomitants
[consequences] in order, as though they were true and existent
by hypothesis [καθ' ὑπόθεσιν], to something admitted; then,
if that which is admitted is true, the thing sought is
true, too, and the proof will be the reverse of the analysis
636.5 [καὶ ἡ ἀπόδειξις ἀντίστροφος τῇ ἀναλύσει]. But if we come
upon something admittedly false [ψεύδει ὁμολογουμένῳ], the
thing sought will be false, too. In the problematical kind
we suppose the desired thing to be known, and then we pass
through its concomitants [consequences] in order, as though
they were true, up to something admitted. If the thing
636.10 admitted is possible or can be done [δυνατὸν ἢ καὶ
ποριστὸν], that is, if it is what the mathematicians call
given [δοθὲν], the desired thing will also be possible. The

proof will again be the reverse of analysis. But if we come upon something admittedly impossible [ἀδυνάτῳ ὁμολογουμένῳ], the problem will also be impossible.

(the translation is taken from Hintikka and Remes 1974,pp.8-10)

B) <u>Euclid's scholiast</u>

Τί ἐστιν ἀνάλυσις καὶ σύνθεσις

ἀνάλυσίς μὲν οὖν ἐστιν λῆψις τοῦ ζητουμένου ὡς ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπί τι ἀληθὲς ὁμολογούμενον

Analysis is an assumption of what is sought as if it were admitted <and the passage> through its consequences [concomitants] to something admitted <to be> true.

σύνθεσις δὲ λῆψις τοῦ ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπὶ τὴν τοῦ ζητουμένου κατάληξιν ἤτοι κατάληψιν.

Synthesis is an assumption of what is admitted <and the passage> through its consequences [concomitants] to the finishing or attainment of what is sought.

(the translation is taken from T.Heath 1926,vol.I, p.138)[8].

The problem which most of the critics of the Greek geometrical analysis have addressed, is the problem of convertibility between analysis and synthesis. The classical view is supported by Hankel, Cantor, Zeuthen, Heath, Robinson and Cherniss [9].

According to this view, analysis "was a method of discovering, a method of discovering either proofs of geometrical propositions or the solutions of geometrical problems. It was followed by a synthesis, and the latter constituted in the first place a check on the analysis, to make sure that there had been no error; but secondly, provided that there had been no error, it constituted the actual proof or solution for the sake of which the analysis was undertaken.......For this method to work, the implications must be reciprocal. Not merely must (1) imply (2), but (2) must imply (1). The chain 1-2-3-4-5 must give an unbroken series of necessitations whichever way you proceed along it. In other words, the propositions concerned must be convertible" (Robinson, 1936/1969, pp.1-2). Therefore, according to the classical interpretation, the last step in the analysis becomes the first in the synthesis, which repeats the steps of the analysis in reverse order until the original assumption is reached and so proved. Both, analysis and synthesis, are deductive. According to this interpretation, reductio ad absurdum is a special case of analysis. In this case one may conclude, without synthesis, that the original assumption is false or the solution impossible.

According to the second interpretation [10], the method of analysis is not one of deduction. The procedure is not to see what follows from the original assumption but to see from what the original assumption follows, and, having discovered this, to proceed backwards until a proposition p is reached independently known to be true. Then, by synthesis, the original assumption is deduced through p and so proved. It is only synthesis which is deductive, and not analysis. Cornford renders "διὰ τῶν ἑξῆς ἀκολούθων" of Pappus' passage, not as "through its successive consequences" (Heath, 1926, p.138) but as "through the succession of sequent steps" (Cornford, 1932/1965, p.71) [11].

Pappus' passage has given rise to this controversy. His account seems to be self-contradictory as far as the direction of analysis is concerned [12]. In Pappus' passage we have four accounts of analysis: (a)634.10-12, (b)634.13-17, (c)636.1-4 (theoretical analysis), (d)636.6-9 (problematical analysis). Accounts (a), (c), and (d) agree with each other and with the interpolated definitions in Euclid XIII. These accounts seem to give support to the classical view of analysis. That is, they seem to describe analysis as a downward movement of deduction from an initial assumption. The classical view is also supported by passages 636.5-6 and 636.10-12 from which we may suppose that Pappus regards reductio ad absurdum as a case of analysis.

However, account (b) seems to give support to Cornford's view, because it describes analysis as an upward movement to prior assumptions from which the initial assumption follows.

N.Gulley tries to escape from this puzzling situation, accepting that "Pappus is repeating two different formulations of the method, one describing it as an upward movement to prior assumptions from which the initial assumption follows; the other as a downward movement of deduction from an initial assumption" (1958, p.13).

Mahoney (1968-9, 13-9) claims that either Pappus contradicts himself or passage 634.13-22 is an interpolation. Since it is unlikely that Pappus contradicts himself, the above passage must be spurious.

However the problem of Pappus' self-contradiction may be due to the accepted translation of "τὰ ἀκόλουθα" as "logical consequences". Hintikka and Remes showed that 'ἀκόλουθον' does not always mean logical consequence [13]. They supply foundation for their claim by refering to other passages in Pappus' Collection in which the word ἀκόλουθον clearly does not have the meaning of logical consequence [14]. They also observe that in the passage in which Pappus describes synthesis - an apparently deductive procedure - he does not use the word ἀκόλουθον [15].

Therefore if we accept that the word ἀκόλουθον is used by Pappus in a loose sense, then the problem of reversibility remains open (if we rely only upon Pappus' description). Pappus' account can easily describe both a deductive and a non deductive analysis.

3. Aristotle and ancient commentators on analysis

Let us now see how Aristotle and the ancient commentators describe geometrical analysis.

In several passages, Aristotle indicates clearly the method recognised as geometrical analysis. First of all, the famous passage in Ethics (E.N. 1112b15ff) in which the process of deliberation in the sphere of actions is compared with analysis in Geometry. As Hintikka and Remes observe, this passage presents many similarities with Pappus' description, both in vocabulary and structure [16]. In both passages we have a hypothetical starting point (analysis as an opposite movement to synthesis) and the last step in analysis is the first in synthesis. Moreover, both passages speak about the case in which analysis ends in an impossibility. For Aristotle, analysis is an upward movement (ἕως ἂν ἔλθωσιν ἐπὶ τὸ πρῶτον αἴτιον, ὃ ἐν τῇ εὑρέσει ἔσχατόν ἐστιν) and the analytical procedure is an heuristic one (cf.Pappus, 634.5). It seems -from 1112b15-20 - that Aristotle has a propositional interpretation in mind. The meaning of the word 'διάγραμμα' in the passage is not very clear (proof or figure). But in practising the method of geometrical analysis (especially problematical analysis) one relies very much on the analysis of the figure.

In Met 1051a21-33, Aristotle emphasizes the role of auxiliary constructions in Geometry. However it is not certain whether in this passage Aristotle refers to geometrical analysis [17].

In Physics 200a16ff, Aristotle observes that "a straight line, being what it is, it is necessary that the angles of a triangle should be equal to two right angles, but the converse is not necessarily true" (cf also An.Post. 94a28-35). Therefore, Aristotle recognizes that convertibility was not always possible in mathematics.

In <u>Soph.El.</u> 175a26-29, he says that, in Geometry, synthesis may fail even after a successful analysis, that is, that an analysis might not be converted in a synthesis [18].

Nevertheless there is a passage in Aristotle (<u>An Post</u> 78a6ff) that shows perhaps a deductive analysis. Aristotle says that mathematical propositions are more commonly convertible than dialectical ones, and this is what makes analysis in mathematics easier [19]. However, the passage says neither that (in mathematics) we always have convertibility nor that analysis is possible only when we employ convertible propositions, but that it is easier. Therefore, the passage does not give us any strong evidence that analysis is always deductive [20]. The commentators of Aristotle explain analysis here as a geometrical method of discovering the premiss from which a proposition assumed to be true can be demonstrated (Themistius <u>In An.Post.</u> I,26.22-4; Philoponus, <u>In An.Post.</u> I,162.16-28) and so they give an interpretation of the passage which implies that analysis need not be deductive. But however we interpret this passage of Aristotle, it is almost certain that in Aristotle's time there was a recognized form of geometrical analysis which worked with non-reciprocal relationships.

Gulley showed (1958, pp.4-12) that for the ancient commentators, analysis had the following characteristics: a)It can be applied to many fields by γραμματικοὶ, φυσιολόγοι, φιλόσοφοι καὶ γεωμέτραι [21]. b) It is a process of resolution of a whole to its parts, or an ascent to what is prior, to principles or causes, from which the assumed proposition can be demonstrated. It is contrasted with the downward movement of synthesis or <u>apodeixis</u>. c) As a logical method, it is classified as one of the four divisions of dialectic (ἀνάλυσις, ἀπόδειξις, διαίρεσις, ὁρισμὸς), a classification for which the Greek commentators find the basis in Plato [22].

Now geometrical analysis is presented in the same terminology: a) It is an upward movement towards the principles (Proclus, 255.12ff; Philop., In An.Post. I, 162.25ff; Alexan., In An.Pr. I 7.15-8) [23]. The distinction between mathematical and dialectical analysis is not one of direction, but one of simplicity and complexity in the field of inquiry (Themist., In An.Post. I, 26.24-33). b) Similarly with Aristotle and Pappus, all Greek commentators emphasize the heuristic capacity of the analytical method. The method helps us to find proofs, lemmas, or principles [24].

It seems then that in Aristotle and in Greek commentators we have the recognition of a method of analysis in Geometry, described as an upward movement, and according to which the relation between premiss and conclusion is not necessarily reversible. Moreover, as Gulley observes (1958, 9), "what seems to be assumed in Aristotle, is that geometrical propositions can be ordered in a hierarchy, and it is principally in its use in furnishing such an ordered science in Geometry that the importance of a method of non-deductive analysis would appear to lie; its task would be that of systematising geometrical knowledge and co-ordinating results by leading back to first principles - to axioms or definitions or something already demonstrated".

4. Three examples of the geometrical method of analysis and synthesis

Let us now examine geometrical analysis as we find it not in descriptions or definitions but in geometrical demonstrations. I shall give three examples of demonstrations with analysis and synthesis. The first is the second of the five interpolated proofs in Euclid XIII, and the other two examples are from Pappus' Collectio, one of theoretical and one of problematical analysis.

I Euclid, Elements XIII,2 (interpolated)

1. ENUNCIATION

A) That which is given (δεδομένον). Let (Aa) a straight line CAD be such that $CD^2 = 5DA^2$

and (Ab) let 2DA=AB (see fig. 1)

B) The thing sought (ζητούμενον)

AB is divided in extreme and mean ratio at C ($AC^2 = CB.AB$), and AC>CB.

D A C B

fig.1

2. ANALYSIS

For since

1. BA=2AD therefore BA.AC = 2AD.AC, and $CB.AB=AC^2$ (ζητούμενον)
2. $BA.AC + CB.AB = 2AD.AC + AC^2$

   but since $BA.AC + CB.AB = AB^2$

3. $AB^2 = 2AD.AC + AC^2$, (and since AB=2AD)
4. $4AD^2 = 2AD.AC + AC^2$
5. $5AD^2 = 2AD.AC+AC^2+AD^2$ (and hence by Eucl.II,4)
6. $5AD^2 = (AC+AD)^2 = CD^2$

3. SYNTHESIS

For since

1. $CD^2 = 5AD^2$, and $CD^2=AC^2+AD^2+2AC.AD$
2. $5AD^2 = AC^2+AD^2+2AC.AD$
3. $4AD^2 = AC^2+2AC.AD$, and since AB=2AD or $AB^2=4AD^2$
4. $AB^2 = AC^2+2AC.AD$, and since $AB^2=AB.AC+AB.BC$

5. $AB.AC+AB.BC = AC^2+2AC.AD$

and since $AB=2AD$ or $AB.AC=2AD.AC$

6. $AB.CB=AC^2$

II Pappus Collectio, Book IV, prop.4 (theorem)

1. ENUNCIATION

A) That which is given (δεδομένα)

Let ABG be a circle with centre E. Let BG be a diameter, and AD a tangent (that meets the circle at A); it meets the extended BG at D. Join D and F (any point on the circumference). Draw (the diameter) AEH. Join H and F; join H and J (where FD meets the circumference). The diameter BG meets HF at K, and HJ at L (see fig.2).

B) The thing sought (ζητούμενον)

That EK=EL

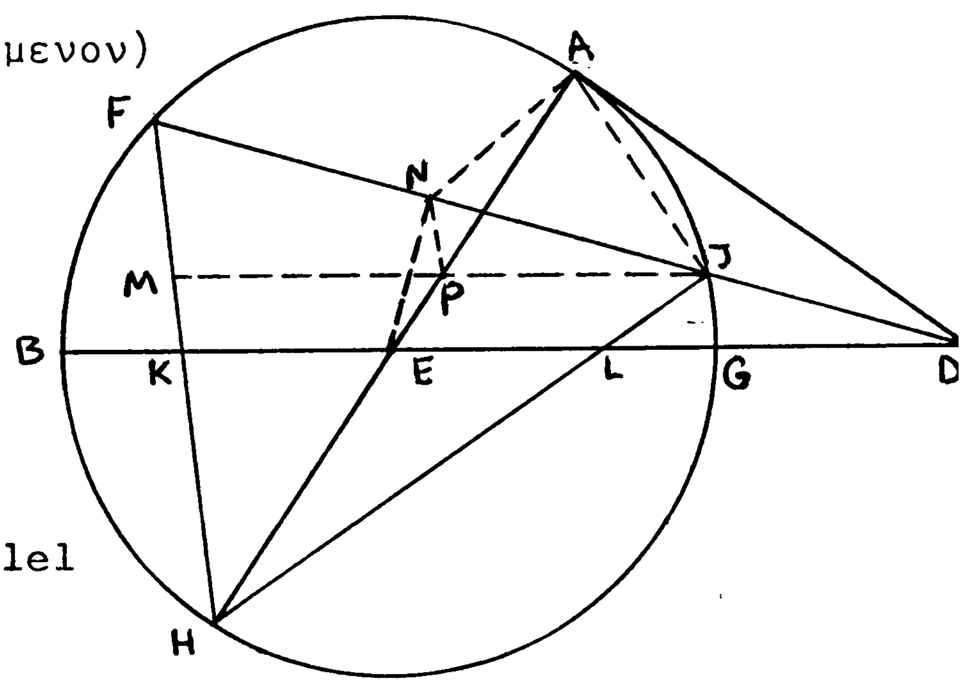


figure 2

2. ANALYSIS

A) Analysis proper[25]

Let it be so (γεγονέτω)

1. Let a straight line JM parallel to BG be described through J. first auxiliary construction; the zetoumenon assumption is not used

2. MP=PJ (by construction and by the zetoumenon assumption KE=EL)

3. Let a perpendicular from E to FJ be described.

2nd auxiliary construction; the zetoumenon assumption is not used

4. FN=NJ (Eucl. III,3)

5. Let N and P joined

3rd auxiliary construction; the zetoumenon assumption is not used

6. NP and FM are parallels (Eucl.,VI,2)

7. Let J and A be joined

4th auxiliary construction; the zetoumenon assumption is not used

8. The angle JNP=NFM=JAP (Eucl. I,29; III,21)

9. The points J,A,N,P are on a circle (converse of Eucl.III,21)
(the introduction of the circle JANP is based on the zetoumenon)

10. The angle ANJ=APJ=AEL (Eucl.I,29; III,21)

11. The points A,N,E,D are on a circle

B) Resolution

It is really so (ἔστιν δὲ)

1. The angles DAE and END are both right angles (and so the points A,N,E,D do lie on a circle)

3. SYNTHESIS

A) Construction (κατασκευὴ)

In the few theoretical analyses there are in the Collectio the construction seems to be missing. The same constructions which are introduced in the analysis proper are used in the synthesis.

B) Proof (ἀπόδειξις)

1. Because the angles DAE and END are both right, the points A,N,E,D are on a circle.

2. Consequently the angle AND=AED

3. But because ED and PJ are parallels, the angle AED=APJ

4. Hence the points A,N,P,J are on a circle

5. Hence the angle JAP=JNP

6. But the angle JAP=JFM

7. Therefore the angle JFM=JNP

8. Hence FM and NP are parallels

9. FN=NJ

10. Hence MP=PJ

11. Because MJ and KL are parallels, MP/KE=PJ/EL or MP/PJ=KE/EL. Therefore, KE=EL.

III Pappus, <u>Collectio</u>, VII, prop.108 (problem)

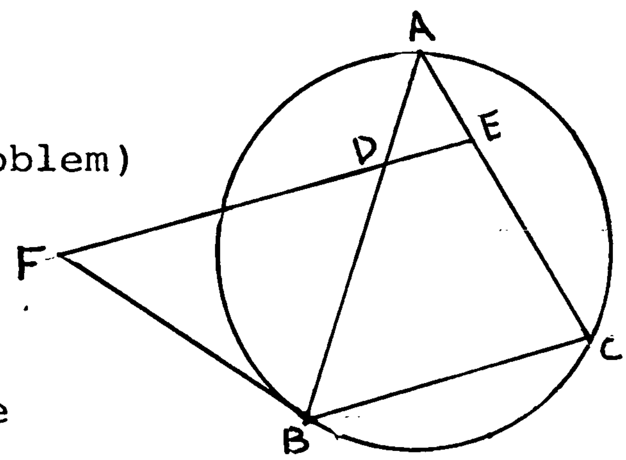


fig.3

1. ENUNCIATION

A) That which is given (δεδομένα)

Given two points D,E within a given circle

B) The thing sought (ζητούμενον)

To draw lines from D and E meeting the circumference at A, so that if these lines are produced, to meet the circumference at B and C, CB will be parallel to DE (see figure 3).

2. ANALYSIS

A) Analysis proper

Let it be so (γεγονέτω)

1. Draw BF tangent to the circle at B (auxiliary construction)
2. Draw the line EDF (auxiliary construction)
3. DE and BC are parallels (zetoumenon assumption).
4. The angle FBA=BCA=DEA .
5. B,F,A,E lie on a circle
6. Hence rectangle BD.DA=rec FD.DE; but BD.DA is given

B) Resolution

1. BD.DA is given
2. Hence FD.DE is given
3. But DE is given; hence FD is given.
4. D is given; therefore F is given.

5. FB has been drawn tangent from a given point to a given circle; therefore FB is given in position.

6. But the circle is given; so point B is given.

7. Point D is also given; hence BD is given.

8. Since the circle is given, point A is given.

9. But both D and E are given; hence DA and EA are given.

3. SYNTHESIS

A) Construction (κατασκευὴ)

1. Let ABC be the circle and D,E the two given points.
2. Let any line ADB be drawn.
3. Make rectangle ED.DF=rec AD.DB
4. From F draw FB tangent to the circle.
5. Draw lines BDA and AEC.

B) Proof (ἀπόδειξις)

1. AD.DB=ED.DF
2. Hence A,F,B,E are on a circle.
3. Hence angle FBA=AED
4. But because angle FBA=BCE, then angle AED=BCE.
5. Therefore BC is parallel to DE.

5. Discussion of the method

The first of the three examples seems to justify the classical interpretation of the method of analysis and synthesis in which both the two processes are deductive. Indeed, Robinson's interpretation -and Mahoney's as regards the theoretical analysis- is based on this example [26]. In this example we have indeed a deductive process in both directions, where the last step in the first process is the first in the second. Both movements go downwards.

If we observe all the descriptions of geometrical analysis, we shall see that in analysis we begin from the theorem to be proved or the problem to be solved (say B), and we arrive at a proposition C, that is true independently of our hypothesis (B). Then synthesis starts from C and arrives at B.

However this is only half of the truth as far as the method of analysis is concerned. Every enunciation of a theorem or a problem has two parts; the given and the thing sought. Analysis starts from both, the given and the thing sought (cf examples II and III), and during its process we utilize other known propositions (axioms or theorems - let them be K). So in the analytical process, the ultimate premiss is the conjunction $K \cap A \cap B$ (A=the given, B= the required). The proposition C at which the analysis finishes its process must be independently true. This means, independent only of B. Hence C is either a) an axiom (that is an element Ki of the set K), or b) a theorem (that is Kj), or c) a proposition that can be proved from $K \cap A$ (in this case we have $K \cap A \rightarrow C$).

In example I, analysis does not start from the conjunction $K \cap A \cap B$ and does not finish in a proposition C. The enunciation of the theorem has the form: if Aa and Ab, then B (the given has two parts). The 'analysis' begins from Ab and B and concludes to Aa.

So we have: <u>analysis</u>: if Ab and B then Aa

<u>synthesis</u>: if Aa and Ab then B

But this obviously is not a case of geometrical analysis and synthesis. We do not prove a theorem by analysis and synthesis but <u>two converse</u> propositions only by synthesis (deductively) [27].

Converse propositions are often found in Euclid's Elements (ie. I,5; I,6), sometimes under the same enunciation (ie. III,14).

Therefore it is wrong to view the five proofs interpolated in Elements XIII as examples of analysis and synthesis, because there is no analysis [28]. Even the two definitions at the beginning of the interpolated proofs do not accord with them (cf. "ἐπί τι ἀληθὲς" and not "ἐπί τι δεδομένον"). If we regard them as cases of analysis it is impossible to find where their heuristic value lies.

Mahoney, interpreting theoretical analysis in the light of these propositions, is led to a wrong distinction between theoretical and problematical analysis: "it is really this form of analysis [the theoretical one] that demands consideration of logical reversibility" (1968/9, p.329); and on p.330: "problematical analysis seeks answers not proofs; hence the problem of reversibility is of secondary importance".

Let us now examine the other two examples. First of all let us make a preliminary note. The method of analysis is a helpful method for solving difficult problems (or proving difficult theorems). Its application to simple problems (or theorems) has no meaning. The method is useless. We can understand its heuristic value only if we apply it in difficult cases in which the use of auxiliary constructions is a necessity.

The first thing we have to emphasize is the crucial role that the auxiliary constructions play in analysis [29]. In fact theorems that do not need auxiliary constructions, do not need analysis either [30]. Through the auxiliary constructions we introduce new mathematical entities in the proof and hence new mathematical relations in the theorem (or problem). The first task of analysis is the discovering of these auxiliary constructions, and after that the discovery of the proof.

We can see from the second example above mentioned, that analysis begins from both the given (A) and the required (B) (and perhaps other known propositions K) and arrives at a proposition C ($K \cap A \cap B \longrightarrow C$) that is known independently of the required. On the other hand synthesis begins from K and A, takes the proposition C into account, and arrives at B ($K \cap A \cap C \longrightarrow B$). Thus the proposition C is one of the main premisses from which we can deduce the required B. But C is not given. Hence, the role of analysis is to find C [31], and finding this, to find the proof (cf. Geminus' definition of analysis as "ἀποδείξεως εὕρεσις"). However in order to find C, we have first to proceed to some auxiliary constructions (now analysis seems to be an analysis of a figure; cf. Aristotle, <u>Met.</u> 1051a21ff).

The heuristic secret of the method, is to use the theorem to be proved (or the problem to be solved), to find a proof for it. When we have to prove a theorem (or to solve a problem) there is - besides the given figure and the given mathematical relations- a great number of possible auxiliary constructions and an even greater number of geometrical relations among these geometrical entities. It is very difficult -and sometimes impossible- to choose the suitable auxiliary constructions (and relations). With the aid of analysis (using the required) we eliminate the number of possible choices and we have something to guide us. Nevertheless the method of analysis is not a mechanical procedure. It is not a decision procedure with the help of which we can proceed mechanically. Although we have exploited the structure of the theorem to be proved, a lot of choices still remain.

In analysis there is a fact of discovery. This is closely related to the experience, the intuition and the mental capacity of the geometer [32]. The classical interpretation describes analysis as a method in which we draw conclusions (deductively) from the theorem to be proved until we arrive at something known. However this description is misleading. Although it does not view analysis as mechanical procedure, it does not pay attention to the role that intuition, experience, and mental ability of the gepmeter play in analysis. The problem is not whether we draw conclusions or not, but how we choose the appropriate chains of conclusions. This is because in every step in analysis we have a lot of choices in both auxiliary constructions and consequences that can be drawn from the original assumpion. Thus, before one can find a successful analysis one may have a lot of unsuccessful ones. Moreover when one works by analysis one is searching to establish one's required proposition. Hence the question "how can I establish it" goes together with the question "what conclusions can I draw from it". Therefore in analysis we have two procedures bound together: a) drawing conclusions and b) looking for premisses.

For example, consider the problem: Given two points A,B and a straight line. To find, on the line, a point M such that AM + BM is a minimum (see figure 4).
(The problem is solved by finding a point A1 symmetrical to A, and drawing the line A1B)

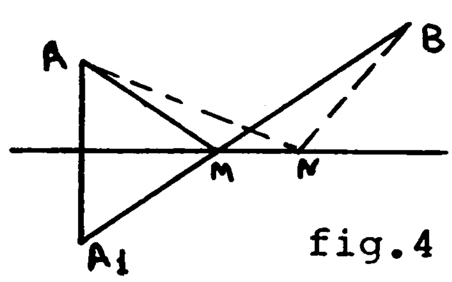


fig.4

<u>Analysis</u>: Let M be the required point such that AM+BM = min. Let N

be another point on the line. Then AM+MB < AN+NB. But now, how can one compare these two broken lines? For, in order to be able to compare two broken lines with the same ends, one must include the other (Eucl. I,21). This is impossible in this figure. So, one must transform it (the figure) suitably by an (or some) auxiliary construction. However, if one find the image A1 of A then ....

All these series of thoughts in a geometer's mind are obviously a search for premisses and auxiliary constructions rather than a deductive series of conclusions from the assumption to be proved. However, when the proof (or the solution) is over and we write it, we formulate it in a different way as a series of constructions and logical consequenses.

We may now say that the main problem of an interpretation of the method of analysis and synthesis is not so much whether analysis is deductive or not, but the process of thought of the geometer when he practices analysis. The interpretation that requires analysis always to be deductive assumes that the geometer draws conclusions (deductively) from his initial asssumption (the problem to be solved). The other interpretation requires analysis to be non-deductive and assumes that the geometer looks for premisses. However the above example of analysis shows that, independently of whether or not analysis is deductive, the geometer may search for auxiliary constructions and premisses for his proof without deducing conclusions from his initial problem. The search for premisses does not exclude the deductiveness of the analysis. The geometer always expects that if he succeeds in an analysis he will be able to proceed to synthesis (although this last sometimes might not happen). However he is mainly interested in finding a solution for his problem and not in carrying out deductively all the consequences of his initial problem.

Carpus (reported by Proclus) says that the method of analysis is mainly applied to problems rather than to theorems [33]. We must now see if there is any essential difference between theoretical and problematical analysis.

In Pappus' definition of analysis and synthesis we do not see any crucial methodological difference between theoretical and problematical analysis. But if we look carefully at the two examples mentioned above, we shall notice a lot of differences. We have emphasized the role of the new mathematical entities that are introduced by the auxiliary constructions. In a theoretical analysis (cf example II) we have two kinds of mathematical entities: a) the given ones; and b) the new ones that are introduced in the process of analysis, through the auxiliary constructions. In theoretical analysis the figure is given and the zetoumenon introduces a new mathematical relation, but not new entities. Thus, the auxiliary constructions are based <u>on the given figure</u> The heuristic role of theoretical analysis is based only on the fact that we exploit the new geometrical relation (the zetoumenon).

Now, in problematical analysis (cf example III) we have three kinds of mathematical entities: a)the given ones, b)the entities that the zetoumenon introduces, and c) those that are introduced in the process of analysis. In problematical analysis the given figure is different from the one that analysis begins its process with [34]. In problematical analysis the auxiliary constructions are based almost exclusively <u>on the zetoumenon</u>. The heuristic role of problematical analysis is far greater than that of theoretical analysis, because now we exploit a)the mathematical entities found in the zetoumenon, and b)a great number of geometrical relations in which the geometrical entities introduced by the zetoumenon are involved. Hence in many problems it is impossible to proceed to the solution without analysis (cf the examples of the note 34), while in theorems, it is always possible to proceed to their proof without analysis [35].

We said that in problematical analysis the auxiliary constructions are based on the zetoumenon. Therefore two different types of justification are needed: a)justification of the analysis, b) justification of the auxiliary constructions. We can see (examples II and III) that analysis and synthesis each have two parts. But while in problematical analysis the resolution is the bigger part, in theoretical analysis it is rudimentary (if not absent) [36]. In problematical analysis the resolution serves to establish the independence of whatever constructions are necessary, from the zetoumenon; ie. to establish that all the geometrical entities needed in the argument were 'given' (resolution is opposite to construction).

The rudimentary role of resolution in theoretical analysis, shows only that the last proposition in the analytical process (C), is independent from the zetoumenon (cf example II) [37].

Lakatos and Szabó claim that analysis is a procedure of testing conjectures [38]. This interpretation, does not take into account the role of auxiliary constructions and the double character of analysis (drawing conclusions and searching for premisses). It is surely wrong and it cannot explain fully the heuristic usefulness of the method.

When we propound a problem to solve, it is not known that the problem is solvable (cf. Pappus, Coll. III, praef. 1, pp.30-1). We begin from our conjecture and we draw conclusions (and we carry out auxiliary constructions). There are three possibilities: a) we arrive at something impossible, b) we arrive at something constructable, or c) we arrive at something that does not help us to solve or to reject the problem. In the third case we shall try again through other auxiliary constructions and other series of conclusions. So in the general procedure of 'testing' our conjecture (the problem to be solved), we have other 'subconjectures' (finding the suitable auxiliary constructions) and 'tests'. But all these 'subconjectures' are not done at random. We always have in mind the problem to be solved and our auxiliary constructions are chosen in such a way as to relate the required to the given (or other known propositions). This procedure of conjecture and test is at the same time an analysis of figure and a search for premisses. The required guides us. The same holds good when we have to prove a theorem.

In the accounts of Pappus and Aristotle(E.N.,III2b), we find that, instead of something known, the end-point of an analysis may be an impossibility. According to the classical interpretation, we find here evidence that analysis proceeds downwards. The outcome of the 'analysis proper' would be a reductio ad absurdum argument [39]. Although such a posibility may exist, to infer that this fact shows that analysis is always deductive is not valid. We can easily have a non deductive analysis which ends in an impossibility without having a reductio ad absurdum proof [40].

Reductio ad absurdum is an indirect proof. Proclus (p. 255) classifies analysis and reductio ad absurdum among the proofs that proceed to the starting points, but while analysis is affirmative, reductio is destructive. Reductio ad absurdum starts not from the proposition to be proved (say A), but from its contradictory (-A), and in leading us to an impossibility, proves A [41]. Although it presents many similarities with analysis, its logic and applicability are different. Reductio is applicable in theorems and not in problems, while analysis is applicable mainly in problems. Reductio is the most effective method of proving the converse (say B) of a theorem (say A), by admitting -B and arriving at an impossibility, exploiting A [42].

Reductio ad absurdum is not then a case of analysis and this was recognised by Proclus. Analysis is always followed by synthesis, reductio never. Reductio leads to negative proofs , never to affirmative ones. According to the classical interpretation (that takes analysis to be deductive), analysis is the real proof and synthesis is reduced to a mere confirmatory device. But in all ancient tradition, synthesis is considered as an integral part of the method, perhaps the most important. Only synthesis is called also ἀπόδειξις (cf. Pappus' definition). Analysis discovers the premisses but only synthesis demonstrates. For Greek mathematicians, as for Aristotle, there is a clear differentiation between the premisses of a demonstration and its conclusion. Premisses are prior and better known than the conclusion. So the difference in direction between analysis and synthesis is an important one. Analysis (going upwards or backwards) is heuristic. Synthesis, on the other hand (going downwards) is demonstrative.

6. Ἀπαγωγή (reduction) - the early analysis

Many historians of mathematics seem to assume that the early analysis was apagoge (reduction) [43]. Zeuthen [44] has suggested that the first part of the late analytical proof system, which we have called 'analysis proper' was called apagoge. Apagoge is earlier than analysis. Hippocrates of Chios (second half of 5th century) was the first who systematically employed it. Because apagoge is very relevant for our explanation of Plato's hypothetical method, let us examine in greater detail Hippocrates of Chios and the method he used to practise in geometry.

Hippocrates of Chios is famous, as a mathematician, for three achievements: a) for squaring the lunes [45], b) for the reduction of the problem of doubling the cube to the finding of two mean proportionals [46], and c) for being the first - according to Eudemus - to compile Elements [47].

However, apart from these chievements, the name of Hippocrates is related to a geometrical method called apagoge. Historians of mathematics have not paid sufficient attention to this point.

Proclus says the following regarding apagoge:

> Reduction is a transition from a problem or theorem to another which, if known or constructed, will make the original problem or theorem evident. For example, to solve the problem of doubling the cube, geometers shifted their inquiry to another on which this depends, namely, the finding of two mean proportionals; and henceforth they devoted their efforts to discovering how to find two means in continuous proportion between two given straight lines. It is reported that the first to effect reduction of difficult (perplexing) constructions (ἀπορουμένων διαγράμματων) was Hippocrates of Chios who also squared the lunes [48].

I shall offer some remarks on Proclus' account of apagoge: i) Apagoge is a method that can be applied both to theorems and problems. However, it seems that it was used mainly in problems, at least by Hippocrates (...the first to effect reduction of difficult constructions...). The above passage may also imply that apagoge is more useful in difficult than in simple cases.

ii) Proclus says that Hippocrates "effected reduction of difficult constructions" (plural). That means that apagoge was a method that Hippocrates often practised.

iii) Proclus does not say that Hippocrates was the first to practise (or to invent) apagoge, but he was the first to effect reduction of difficult constructions. Heath [49] claims that we cannot "attribute to Hippocrates the discovery of the method of geometrical reduction" only on the strength of Proclus' passage. However, even if Hippocrates did not discover apagoge, he was the first who practised it systematically.

I shall now examine the method that Hippocrates used is his famous proofs of the quaring of lunes, as Eudemus reports them.

Scholarly opinion is unanimous regarding the genuineness of the following passage that is of great importance for the method that Hippocrates used to practise:

> He (Hippocrates) started with, and laid down as the first of the theorems useful for his purpose (ἀρχὴν μὲν οὖν ἐποιήσατο καὶ πρῶτον ἔθετο τῶν πρὸς αὐτοὺς χρησίμων), the proposition that similar segments of circles have the same ratio to one another as the squares on their bases have. And this he proved by first showing that the squares on the diameters have the same ratio as the circles [50].

The stages in Hippocrates' proofs are the following: 1) He enuciates the problem (say B), 2) he takes as the starting point (ἀρχὴν μὲν οὖν ἐποιήσατο) a proposition L, that is, "similar segments of circles have the same ratio as the squares on their bases", 3) he proved L (ἐδείκνυεν) by means of another proposition M, that is, "the squares on the diameters have the same ratio as the circles", 4) he proved M (δεῖξαι), and 5) he solved the four cases of the squaring of the lunes utilizing L.

In other words, Hippocrates first proposes the problem to be solved (B), then he finds a proposition L that lets him solve the problem (or reduces B to L). L constitutes the principle (arche) for the construction of the original problem. Thirdly, he reduces L to another proposition M, and he proves L via M (M is now the principle for the proof of L). Fourthly, he proves M (presumably deriving it from some known propositions), and finally he solves his original problem by means of L.

It is easy to see that Hippocrates' demonstrative procedure is different from the usual one known from Euclid. In fact, as regards the first four stages, Euclid would proceed in the reverse order, that is 4-3-2-1. It seems then that Hippocrates applied his usual method of apagoge in the case of the squaring of the lunes.

As regards the four proofs reported by Eudemus -whether we accept Diels' or Heath's version of the original text - there is the problem of what is due to Eudemus and what to Hippocrates himself. The text of the proofs is very elliptical and contains many gaps. In the second problem, the actual squaring of the lune is omitted ("presumably as being obvious" Simplicius remarks). It is certain that Eudemus does not quote Hippocrates word for word.

The four proofs have a direct and not an indirect form. However, many times, proofs that are discovered indirectly (through analysis or apagoge) are presented in their exposition in a direct form. If Hippocrates used apagoge in any of the four proofs, this should have happened in the third proof which is the most difficult. Looking at this proof (the text here is very problematic) we can observe that there is in it a reduction of the original problem to a problem of neusis ('inclination', or 'verging') (64.18); moreover, there is a passage (genuine according to Diels, spurious according to Heath) which clearly reminds us of the indirect method of apagoge [51]; furthermore, from Eudemus' report of this squaring, we have the feeling that although the problem had been solved indirectly, only the direct part of the demonstration was reported.

Hippocrates' arche is not a principle in the Euclidean sense. It is an ordinary geometrical proposition which Hippocrates - according to Eudemus - had proved. Euclid would never use this word for such a proposition. Hippocrates' arche (it is not certain, however, if Hippocrates himself used this word or its use is due to Eudemus) is a proposition which helps him to solve his problem; it is a relative starting point that itself needs proof. In this sense it is the same as a lemma. Proclus (In Eucl 211) defines a lemma as follows:

> The term lemma is often used to designate any proposition invoked for the purpose of establishing another, as when we assert that a proof can be made from such-and-such a lemma; but the specific meaning of lemma in geometry is 'a proposition requiring confirmation'. Whenever for a construction or a demonstration we assume something that has not been demonstrated but needs to be proved, in such a case, considering that the assumed proposition, though doubtful, is worthy of inquiry on its own account, we call it a lemma. It differs from a postulate and an axiom in being a matter for demonstration, whereas they are invoked in their own right without demonstration to establish other propositions.

From Proclus' statement we can see that when we use a lemma for solving a problem, we assume the lemma to be known, we solve our problem on the basis of it, and after that we prove the lemma. So the lemma is kind of hypothesis (or relative principle) for the solution of our problem. We find exactly the same procedure in Euclid. Indeed, when Euclid uses a lemma for the proof of a theorem (eg XI 23, XII 2,4) he proves first the theorem on the basis of the lemma and after that he proves the lemma. Therefore before the proof of the lemma, the theorem has been proved only on the basis of a hypothesis. So in theorem XI 23, Euclid uses the verb 'ὑπόκειται' when he utilizes the - not yet demonstrated - lemma. This is exactly what Hippocrates did when he proved that "similar segments of circles have the same ratio to one another as the squares on their bases" through the proposition (lemma, arche or hypothesis) "the squares on the diameters have the same ratio as the circles".

"Apagoge is a transition from a problem or theorem to another, which, if known or constructed, will make the original problem or theorem evident". It is now plain that the above definition of Proclus fits exactly the method of Hippocrates. Apagoge is a transition from a problem (or theorem) to another which is probably easier to solve. The original problem is solved hypothetically on the assumption of this second problem (or theorem). We have a real solution of our original problem only after the solution of the second one.

The method of Hippocrates is an indirect one. If we have to solve a difficult problem and we cannot foresee from which propositions (or constructions) the solution must start, we can proceed indirectly (backwards) and reduce our problem (B) to another one (L). This last constitutes the _arche_ (or lemma or hypothesis) for the solution of B. We prove B (hypothetically) on the basis of L and after that we try to prove L in the same way. Thus, this method is mainly one of discovering lemmas and proofs. Its advantages are that a) it utilizes the structure of the very proposition that we have to prove, and b) it does not presuppose a complete system of anterior propositions.

We can easily see that the method of reduction of a problem or theorem to another is analogous to the later method of geometrical analysis. Because of that some historians of mathematics thought that Hippocrates used (or discovered) the method of analysis [52]. However we have no evidence to attribute analysis to Hippocrates simply because he used to practise a kind of indirect proof.

Nevertheless although Hippocrates did not practise (or discover) the method of analysis, his analytical-reductive method was the base for the subsequent development of analysis. Both _apagoge_ and analysis accept the problem to be solved (or the theorem to be proved) as a starting point and -according to Proclus' terminology- they proceed to the principles and they are 'affirmative' (that is, they prove and not disprove something). In the same way as analysis, _apagoge_ is used mainly in difficult problems. Finally, both are methods for discovering proofs and lemmas. In fact the method of analysis is nothing else other than an evolution and elaboration of _apagoge_. _Apagoge_ is very primitive. Its only secret is that it takes the required as conjecture and, drawing conclusions from it, works in a different way from ordinary deductive proof. _Apagoge_ has no place in an elaborate post- euclidean geometrical system. _Apagoge_,starting from the problem to be solved (say B), reduces it to another problem (or theorem -say L). The problem B is now solved hypothetically on the basis of L. We can solve L directly or indirectly. If we proceed to solve L via another reduction, we may reduce L to another geometrical proposition M (L is solved again hypothetically on the basis of M). If we continue the same procedure, reducing M to another proposition N and so on, we may arrive at a proposition C that is already known independently. Now the original problem is solved.

The similarity between the method of analysis and synthesis and the method of successive reductions is striking. There is however a difference in their structure. In the method of analysis and synthesis the way up (analysis) and the way down (sythesis) are separated. In apagoge, both ways are involved in every step of the method. This means that every reduction (every movement backwards) is immediately followed by a deductive movement downwards, and only after that we proceed to a new movement backwards. We can show the two different procedures as follows:

A. Method of analysis and synthesis

1.Analysis:B(plus A and K) $\dashrightarrow$ L $\dashrightarrow$ M $\dashrightarrow$ N $\dashrightarrow$ ...... $\dashrightarrow$ C

2.Synthesis:C(plus A and K) $\longrightarrow$ ...... $\longrightarrow$ N $\longrightarrow$ M $\longrightarrow$ L $\longrightarrow$ B

B. Apagoge

B(plus A and K) $\rightleftarrows$ L $\rightleftarrows$ M $\rightleftarrows$ N $\rightleftarrows$ ...... $\rightleftarrows$ C [53]

We can easily see now that if we separate the two procedures (up and down) in the method of successive reductions, we have exactly the method of analysis and synthesis.

We do not know whether Hippocates' reductions were always deductive or not. However it seems that his main iterest was the proof of his problem B through his principle (arche) L and not the manner in which we arrive at L from B. We may see this in the case of his reduction of the Delian problem. Hippocrates reduced this problem to that of finding two mean proportionals between a number and its double. That is, he reduced the proposition $x^3=2a^3$ (1) to the proposition $a/x=x/y=y/2a$ (2), and hypothetically solved (1) through (2).

Let us try to see what kind of analysis Hippocrates may have used and how he arrived at proposition (2) from (1). If we go from (2) to (1) the proof is very simple:

$$a/x=x/y=y/2a \quad \left[\begin{array}{l} x^2=ay \\ \\ y^2=2ax \end{array}\right. \quad \left[\begin{array}{l} y=x^2/a \\ \\ x^4/a^2=2ax \end{array}\right. \quad x^3=2a^3$$

If, on the contrary, we go from (1) to (2), then the proof is difficult because it presupposes the introduction of another unknown variable (i.e. y). We have: $x^3=2a^3$, we posit $y=x^2/a$ and then

$$\left[\begin{array}{l} y=x^2/a \\ \\ x^3=2a^3 \end{array}\right. \quad \left[\begin{array}{l} x^2=ay \\ \\ x^3=2a^3 \end{array}\right. \quad \left[\begin{array}{l} x^2=ay \\ \\ axy=2a^3 \end{array}\right. \quad \left[\begin{array}{l} a/x=x/y \\ \\ xy=2a^2 \end{array}\right. \quad a/x=x/y=y/2a$$

It is very difficult to imagine that Hippocrates reduced proposition (1) to (2) using the second procedure. It is more probable that proposition (2) was found by him intuitively and after that, Hippocrates tried the first procedure. This must be done in the following way: In the epoch of Hippocrates, the problem of the duplication of square ($x^2=2a^2$) and its reduction to the finding of a mean proportional between a number and its double ($a/x=x/2a$) was well known. It seems then that by analogy, Hippocrates thought that the problem of duplication of cube could be reduced to the finding of <u>two</u> mean proportionals.

In Archimedes [54] we have an interesting case in which the relation between apagoge and analysis appears clearly. Archimedes propounds the problem "to cut a given sphere, so that the segments of the sphere shall have, one towards the other, a given ratio", and he proceeds to solve it analytically. In his solution we have the following steps: a) analysis of the main problem (say A) which is reduced to a particular case (say B1) of a general problem (say B); b) analysis of the general problem B which ends with a diorismos; c) synthesis of the problem B according to the three cases that have resulted from the diorismos; d) proof that in the particular case B1, to which the original problem has been reduced, there is always a real solution; e) synthesis of the original problem.

It is generally accepted that analysis is strictly related to diorismos (condition of solvability of a problem) [55]. Indeed analysis may well show the necessity for a diorismos by indicating, in the final stage of its reasoning, that the problem in question admits a solution only under certain conditions. Diorismos is practised in the second part of analysis (resolution), and it can establish only necessary and not sufficient conditions of solvability, in advance of a synthesis (cf. also Hintikka-Remes, 1974, pp.58-9). So Archimedes' analyses (cf. the above example) frequently end with a diorismos. According to Heath (1926,I, p.142) this was common in the analyses of problems. In the Euclidean systematic form of exposition, diorismos discovered by analysis, is incorporated into enuntiation of the proposition (cf. Eucl. VI, 28).

Although it is possible that cases of diorismoi were known by the Pythagoreans (cf. Heath, 1921,I, p.230) -that is, before the method of analysis- its systematic use and formulation is possible only with the help of the method of analysis.

7. <u>The role of indirect proof in the axiomatization of geometry</u>

We said that <u>apagoge</u> was the forerunner of analysis. <u>Apagoge</u> is connected with the geometry of the 5th century and especially with the name of Hippocrates. In this age, geometry was a set of different and independent theorems and problems, and different deductive chains. Geometrical propositions were still being presented with proofs which were hardly standardized, since Eudemus asserts that the order and method of arrangement of the <u>Elements</u> were work of Leodamas, Leon, Theaetetus, Archytas and others- all mathematicians of the beginning of the 4th century (see in Proclus, p.66).

In such a situation, an indirect method works more easily than a direct one. It is easier (in difficult cases) to consider our problem as solved and then to seek a solution by working back from the original assumption. In analysis (and <u>apagoge</u>), taking as an assumption the proposition to be proved, it is possible to begin without the need of other premisses. So the indirect proof does not presuppose a complete system of anterior propositions. Here lies the revolutionary role of <u>apagoge</u> and analysis in mathematics before axiomatization. At this time the <u>apagoge</u> of Hippocrates -and the early analysis later on- functioned as the tool that related different theorems, problems and deductive chains. The procedure of analysis leads us to the discovery of other propositions (or hidden lemmas). Analysis treats these new propositions as related, sets them in order and creates a hierarchy among mathematical propositions, giving order to the disorder.

We may suppose that in pre-axiomatic mathematics, analysis has the following features:

a)It leads to an interrelation of the various mathematical propositions and proofs. Going backwards, it often reduces geometrical propositions to more basic theorems or more simple constructions (cf. the reduction of the squaring of lunes to the general theorem "circles have the same ratio as the squares on their diameters", by Hippocrates of Chios). Often geometers have tried to show that a particular problem (or theorem) is a special case of a more general one. The analytical method is useful in such reductions (cf. the above example of Archimedes).

b) It leads to the discovery, and proof of new theorems and problems. Apagoge and analysis lead to the discovery of hidden lemmas (cf. Proclus, p.211.11-3).

c) Analysis leads to the discovery of new fields of investigation in mathematics. The Delian problem leads to the problem of finding two mean proportionals between two numbers and from there to the conic sections. The problems of the trisection of an angle and the squaring of lunes lead to the problems of inclinations.

d) Analysis is the best method of discovering hidden assumptions and so asking for evidence for things that appeared to be self-evident. It is not a chance occurence that at the time when Hippocrates used his reductive method, another eminent mathematician, his compatriot Oenopides was working on problems of elementary constructions (Proclus, pp.285,333), but was using purely strict methods for the first time [56]. We may also say that analysis (as well as reductio ad absurdum), in asking for reasons for things that appear evident, serves as a significant tool for the antiempirical trend in mathematics [57].

e) Analysis leads -as we said- to diorismos.

f) Analysis plays a great role in the reduction of geometry to its first principles, and hence its transformation into an axiomatic system [58]. It is very probable that because of this role which the method of analysis played in the axiomatization of Geometry, it was viewed, even from its beginning as an upward movement of reduction into first principles. It is this function that Aristotle and the Greek commentators stress in their descriptions of the method.

In the first years of the 4th century, a new generation of mathematicians appear (Leodamas, Archytas, Theaetetus, Neoclides, Leon, Eudoxus). These mathematicians changed the structure of the geometrical research. According to Eudemus, geometry was formulated into an axiomatic system by this generation of geometers. Before the time of the axiomatization of geometry, analysis had been a revolutionary method. Its primary task was not so much to prove propositions but to discover new truths and hidden assumpions. Its second task was to corroborate these lemmas and to organise them hierarchically into an axiomatic system [59]. After the axiomatization, analysis changed its character. As Lakatos says (1978, p.100): "if one suggested a geometrical conjecture, the question was whether it follows from Euclid's postulates and axioms and not whether it is true". The only role of analysis was then to solve problems (to find proofs rather than lemmas). Its heuristic capacity lay only there [60].

After the axiomatization the early analysis had a two-way evolution: a) It was transformed (or rather, it gave rise) to an investigation of converse propositions in mathematics [61], and b) mainly it became a technique of solving problems. In fact the Treasury of Analysis was -according to Pappus- a corpus of books "for the use of those who, after going through the usual elements, wish to obtain power to solve problems set to them involving curves, and for this purpose only it is useful" [62]. These books contain problems of every kind and techniques for solving them.

8. Plato, Leodamas, and the method of analysis

The method of analysis is associated with the names of Plato and Leodamas of Thasos. There are three passages that refer to the birth of the method. a) Diogenes Laertius (III,24) says that Plato "explained [εἰσηγήσατο] to Leodamas of Thasos the method of inquiry by analysis"; b)Proclus (p.211) says that "...the best being the method of analysis, which traces the desired result back to an acknowledged principle. Plato, it is said, taught this method to Leodamas, who also is reported to have made many discoveries in geometry by means of it"; c) The earlier author of the Index Herculanensis (Col. Y.,14ff; ed. Meckler p.17) speaks of the rise of analysis in Plato's time without ascribing it to Plato himself.

Some of the critics [63], on the basis of the first two passages, seem to attribute the invention of the method to Plato. However Diogenes indicates that Favorinus was the source of his statement (besides, Diogenes does not constitute a reliable source). Proclus clearly had some reservations about saying that Plato transmitted the method to Leodamas, for he defended himself with the phrase "it is said". Moreover the earlier Index

Herculanensis does not ascribe the method to Plato. Although Heath (1926, I,36) accepts that the source of Proclus' statement is Eudemus, he claims that the method of analysis is more ancient and probably goes back to Hippocrates; he thinks that Proclus had in mind the philosophical method described in *Republic* VII, which -according to Heath- "does not refer to mathematical analysis at all" (1921, I,291).

Cherniss (1950-1, 417-8) observes that although Aristotle refers to geometrical analysis several times, he does not suggest that its formulation was the work of Plato.

I think that is very rash to ascribe the invention of the method to Plato. Plato was good at mathematics but he was not a mathematician. He was influenced by mathematics but there is no evidence that he conducted mathematical research. Moreover there is no reason why Plato should have taught the method only to Leodamas. The only thing we can say about the relation of Plato's name with the method of analysis, is that Proclus and other later authors thought that they could find applications of the geometrical method of analysis in Plato's dialectic [64]. If we want now to pay any attention to the story that Plato taught Leodamas the method of analysis, the only conjecture we might make is that Plato, knowing the method of Hippocrates (who flourished in the last half of the 5th century in Athens) [65], turned the attention of Leodamas to the method of *apagoge*; after that, Leodamas developed this method to the more sophisticated method of analysis and synthesis.

However neither can we attribute the method of analysis to Hippocrates. Although Hippocrates' *apagoge* is probably the forerunner of analysis, it is not the same as it. The method is analysis and synthesis, and we have consciously two opposite processes that are related by the 'resolution'. Hippocrates' apagoge was not such an elaborated method.

Consequently I think that it is more plausible to ascribe the method of analysis to Leodamas "older than Neoclides, the teacher of Leon, and Leon was himself a little older than Eudoxus" (Proclus, p.66). Leodamas was a contemporary of Plato and older than Theaetetus (if we accept that the order of the names Eudemus gives us is chronological) who was born c.415 [66].

Therefore, if we suppose that Leodamas was born between 430 and 425 B.C., we may suppose that he was at his prime between 395 and 380 B.C. Again if we accept that the method of analysis presupposes diorismoi, and that Leon, "a little older than Eudoxus", flourished between 377 and 370 (see ch. VII, section 2), we may suppose that analysis was in use before 377. Therefore, although we have not strong evidence, I think that the existing evidence points towards ascribing the invention of the method of analysis and synthesis to Leodamas in the period between 395 and 377 B.C.

We may summarize here some of our results. Both apagoge and the method of analysis and synthesis are indirect methods of proof. They are useful in difficult cases (and mainly in problems) because they utilize the structure of the proposition to be proved. They proceed into two directions: a way upwards (or backwards) from the proposition to be proved to its premisses; and a way downwards which is demonstrative and contains the actual proof. The way up (analysis or reduction) of the two methods is mostly, but not always, deductive. Practising the way up, we have two processes of thought: a) searching for premisses, and b) drawing conclusions. A deductive analysis is not incompatible with the fact that a geometer tries to find lemmas and premisses for his proof. The relevant thing on the way up is to find premisses for the proof rather than to draw systematically and deductively all the conclusions from the original problem to the premiss from which the proof (way down) will start.

Analysis is (and so it was thought by Aristotle and the ancient commentators) a method that leads up to principles, and because of that, its contribution to the axiomatization of geometry was relevant. The geometrical method of analysis and synthesis is closely related (and probably gave rise) to <u>diorismos</u>. It seems that Leodamas of Thasos was the first to practise analysis, probably somewhere between 395 and 377 B.c.

The geometrical method of <u>apagoge</u> is the early form of the method of analysis and synthesis and is connected with the name of Hippocrates of Chios. It consists in the reduction of one geometrical proposition to another and its main difference from the method of analysis and synthesis is that the two ways (up and down) are not separated, but they are both involved in every step of the method. That is, every movement backwards (or upwards) is immediately followed by a deductive movement downwards and only after that do we proceed to the new movement upwards.

NOTES TO CHAPTER II

[1] Cf. H.Ebelyn, Lexicon Homericon; M. Hofinger, Lexicon Hesiodorum; I.Rumpel, Lexicon Pindaricum; F.Ellendt, Lexicon Sophocleum; C.D. Beckio, Index Graecitatis Euripidae; F.G. Sturzius, Lexicon Xenophonteum.

[2] Cf. G.Dindorfius, Index Aeshyleum; J.E. Powell, A Lexicon to Herodotus; E.A. Betant, Lexicon Thucydideum; O.J. Todd, Index Aristophaneus; D H. Holmes, Index Lysiacus; M.Denomme, Index Isaeus; S.Preuss, Index Isocrateus; L.Brandwood, A Word Index to Plato.

[3] Blancanus: Aristotelis loca mathematica, p.35ff, Bologna 1615, see in B. Einarson 1936; cf. also Hintikka and Remes 1974, p.85; N. Gulley 1958, p.7.

[4] 1974, pp.118-9.

[5] Cf. "the so-called elements are the last things into which the bodies can be taken apart [διαιρεῖται]" (Arist. Met. 1014a31); and "element means a simpler part into which a compound can be analysed [taken apart - διαιρεῖται]" (Proclus, In Eucl., ed. Friedlein, p.72). The translations of Proclus' passages reported here are taken from G.Morrow's translation.

[6] Cf. Arist. De Gen.Cor., 329a23 "τῶν στοιχείων ὄντων στερεῶν μέχρι ἐπιπέδων ποιεῖται τὴν ἀνάλυσιν". The word ἀνάλυσις here is synonymous with διάλυσις in De Gen.Cor., 315b32; cf. also Meteor. 339b2, and De Caelo 270a24.

[7] Einarson (1936, p39) claims exatly the opposite.

However the analysis of figures does not presuppose the analytic method. And since the method of analysis and the corresponding term occur in mathematics in the first third of the

4th century (cf section 8), although the analysis of figures is something many years older, I think that the term analysis is transfered from the proofs to the figures.

[8] Following Heath (1926, I, p.138) I prefer the definition of synthesis reported by the Mss B and V because it is the most intelligible. Ms P, followed by Heiberg and Stamatis, gives the following definition of synthesis: "σύνθεσις δὲ λῆψις τοῦ ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπί τι ἀληθὲς ὁμολογούμενον". The interpolation, being in all the 'Theonine'. Mss, took place before Theon of Alexandria (4th century A.D.) (cf Heath 1926, I ch.5; III, p.442).

[9] For Hankel, Cantor, Zeuthen and Heath see in Heath 1926, I,p.139; Robinson 1936/1969; Cherniss 1950-1, pp.414-9. Cf also K.Sayre 1969, pp.22-4.

[10] Cornford 1932/1965. For a similar view see Scolnicov 1973, ch.1.

[11] Cornford (1936/1969, 72) is mistaken in saying that "you cannot follow the same series of steps first one way, then the opposite way, and arrive at logical consequences in both directions". Working with convertible propositions is is always possible to arrive at logical consequences in both direcions; cf Robinson (1936/1969, p.8; cf also Lakatos 1978, p.81)

[12] Cf. Gulley 1958, p.1; Mahoney 1968-9, p.323.

[13] 1974, pp.13-9; "τὸ ἀκόλουθον in Pappus' description of analysis and synthesis does not mean a logical consequence, but it is a more vague term for whatever 'corresponds to' or better 'goes together with' the desired conclusion in the premisses from which it can be deduced, perhaps in the sense of enabling one to deduce the conclusion from them. Hence our translation 'concomitant'

instead of the usual 'consequence'" (p.14).

[14] Cf. Pappus -in Hultsch's ed.- 30.9-11, 84.7, 252.10, 352.3, 864.24, 90.4, 34.14-5.

[15] Cf also Hintikka and Remes 1974, pp.81-2,90-1, for the use of the word ἀκόλουθον in Proclus, Speusippus, Stoics and Albinus.

[16] "The vocabulary of the two passages already witnesses to this similarity. Cases in point are τέλος at 1112b15 and at Hultsch 634.22; ἀρχὴ at 1112b28 and at 634.17; σκοποῦσιν at 1112b16 and ἐπισκοποῦσιν at 1112b17 and σκοπούμεθα at 634.14; εὑρέσει at 1112b19 and εὑρετικὴν at 634.6; ζητεῖν, ζήτησις at 1112b20 and 22 and ζητητικὴν at 634.24; ἐντύχωσιν at 1112b24-5 and ἐντύχωμεν at 636.6-7; πορισθῆναι at 1112b26 and ποριστὸν at 636.11 and ποριστικὸν at 636.25" (1974, p.86).

[17] Cf Heath 1949, p.216.

[18] Cf. also Eutocius <u>In Archimedes</u>, ed. Heiberg, III, p.160.10ff: "therefore D.AC is either greater than BE^2.EA or equal to it or less than it; if it is greater,there will be no synthesis,as has been shown in the analysis". On the other hand it is always possible to have a synthesis (proof or solution) without an analysis being possible. Of course we have no evidence for such cases because in such problems (or theorems) the method of analysis is not applied.

[19] It is worth mentioning that Aristotle speaks as if the application of the method of geometrical analysis to dialectic were a usual phenomenon in his time.

[20] Cf. Ross, 1949, p.549; Gulley 1958, p.10.

[21] Alex. <u>In An.Pr.</u>, I,7.12ff (Wallies); Ammonius, <u>In An.Pr.</u> I,5.10ff (Wallies); Philoponus <u>In An.Pr.</u> I,5.16ff (Wallies);

Eustratius *In An.Post.* II,3.10ff (Hayduck); David *In Porph.Is.*, 9.11ff (Busse).

[22] Cf. Ammon. *In Porph.Is.* 34.20ff (Busse); *In An.Pr.*I, 5.22ff; Philop. *In An.Pr.*I, 5.18ff; *In An.Post.*II, 334.24ff (Wallies); Eustr. *In An.Post.* II, 3.13ff; David *In Porph.Is.* 88.6ff; Albinus *Didaskalikos* V,4 and 5 (Louis); Proclus *In Eucl.* 62.12ff. Albinus distinguishes three kinds of philosophical analysis in Plato. The second is very similar to the descriptions of geometrical analysis by the Greek commentators.

[23] Proclus (p.255.10ff) says that all mathematical proofs are either from principles or to principles, and the latter he divides into those θετικαὶ τῶν ἀρχῶν (analysis as opposed to synthesis), and those ἀναιρετικαὶ (reductio ad absurdum).

[24] Cf.Proclus, p.211, as a method of discovering lemmas; Geminus' definition of analysis (reported by Ammonius, *In An.Pr.I* 5.27-8) as "ἀποδείξεως εὕρεσις"; and Philoponus' account of geometrical analysis (*In An.Post.* 162.22ff) as a movement leading to principles; similarly in Proclus, p.246. Lemma is a geometrical proposition invoked for the purpose of establishing another (see p.47)

[25]. Both analysis and synthesis are composed of two parts. Although the names of the two parts of synthesis are κατασκευὴ, ἀπόδειξις we have no standard terminology for the two parts of the analysis. Hankel uses the names 'transformation' and 'solution'. Following Hintikka and Remes (1974, pp.24-5) I shall use the terms 'analysis proper' and 'resolution'.

[26] The example on which Robinson's and Mahoney's interpretations are based, is the first of the five interpolated proofs and not the second. But all the five proofs have exactly

the same structure.

[27] Regarding the conversion of geometrical propositions, see Proclus, pp.69,252,409. He distinguishes four kinds of conversion: simple conversion, "when conclusion and hypothesis can be interchanged as wholes", and three kinds of partial conversion "when the whole with a part, and a part with the whole are interchangable, and when only a part with a part" (p.69). Proclus observes that not all geometrical prositions are convertible (p.253.18). In our example we have conversion of the second kind, while in the fourth proof interpolated in Euclid XIII, we have simple conversion.

Speaking in a very general sense we can say that even the direct proof of two converse propositions is a special case of analysis and synthesis. But as the above passage shows, this was not viewed in this way by the Greek mathematicians.

[28] It is surprising that none of the critics has observed this fact. Even Hintikka and Remes regard them as cases of analysis.

[29] It is surprising that none of the critics, except Hintikka and Remes, pay attention to the role of the auxiliary constructions in analysis. On the other hand Hintikka and Remes base nearly all their interpretation there.

[30] Proclus (p.246) says that: "the geometers have made composite propositions for the purpose of analysis". This may imply that difficult geometrical propositions are treated mainly via the method of analysis and synthesis.

[31] Cf. Proclus, p.211, where analysis is presented as a method of finding lemmas; cf.also Lakatos 1978, p.72.

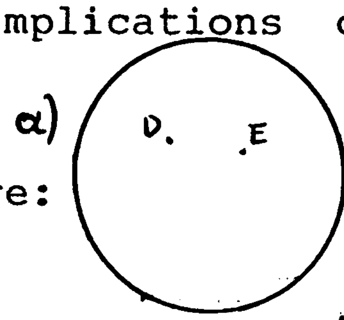
[32] Cf. Proclus p.211: "the best aid in the discovery of lemmas is a mental aptitude for it. For we can see many persons who are

keen at finding solutions but do so without method ..... Nevertheless there are certain methods that have been handed down, the best being the method of analysis".

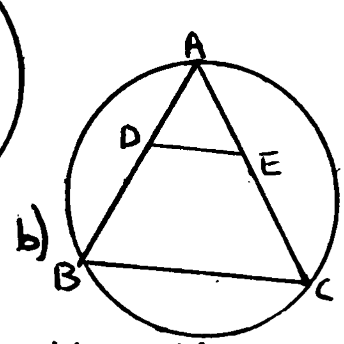
[33] "And for problems one common procedure, the method of analysis, has been discovered, and by following it we can reach a solution; for thus it is that even the most obscure problems are pursued. But the handling of theorems is a difficult matter, and no one to this day, he declares, has been able to teach a uniform way of approaching them" (p. 242). Carpus lived in the 1st or 2nd century A.D. (see Heath 1926, I,p.21). We know that Hippocrates of Chios had applied his reductive method mainly in problems (see section 6). Similarly the two analyses that Eutocius, in his commentary on Archimedes, attributes to Menaechmus, are analyses of problems (see Archimedes ed. Heiberg, vol.III, pp.92ff). Moreover it seems that for Aristotle the paradigmatic kind of analysis was the problematic one (see Hintikka and Remes, 1974, pp.86-7).

Through problems we construct new mathematical entities and through theorems we prove new mathematical relations. According to Pappus (Collectio, iii, Praef.,I; see I Thomas, 1939-41, II,567) "the term problem means an inquiry in which it is proposed to do or to construct something and the term theorem, an inquiry in which the consequences and necesssary implications of certain hypotheses are investigated.

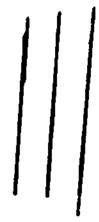
[34] Cf.example III: a) the given figure:

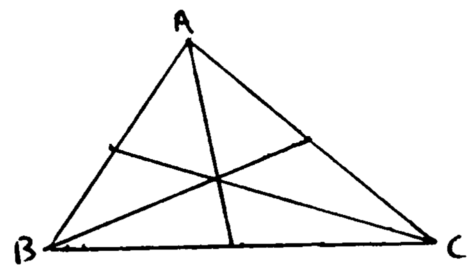


b) the figure from which analysis begins:



cf. also the problem: to construct a triangle from its three medians.

a)the given figure: 

b)the figure from which analysis begins: 

[35] Problematical analysis can be more easily described as an analysis of a figure than can theoretical analysis.

[36] We can also see that the 'analysis proper' is opposite to 'apodeixis' (proof) and now we can understand why Pappus -in his description of theoretical and problematical analysis- contrasts analysis to apodeixis.

[37] Hintikka and Remes (and also all other critics) do not observe these crucial differences between theoretical and problematical analysis. They think that "methodologically and epistemologically the two kinds of analysis are on a par" (1974,46-7). They claim that the resolution is most of the time rundimetnary, and that in theoretical analysis its role is the same as in problematical analysis (p.60). They base their claim mainly on the fact that there are in Pappus' Collection three theorems where "the resolution is also essentially like that of a problematical analysis. As in problematical analysis, there is also a separate construction in synthesis" (p.59). However these three 'theorems' are surface-loci theorems. In loci theorems, although we do not have to show (prove) the existence of new mathematical entities, we have to construct them. So the analysis in a locus-theorem introduces (as in problems) new mathematical entities, and the auxiliary constructions are often based on the zetoumenon.

In fact Pappus (Hultsch p.652; see also in Heath 1926, I,p.11) says that loci are neither theorems nor problems but porisms (about porisms -not in the sense of corollary- see Proclus pp.212,30). Porisms are geometrical propositions between theorems and problems. Therefore it is wrong to base the similarity between theoretical and problematical analysis on the assumption that loci are theorems.

[38] Cf. Lakatos 1978, pp.72-3: "Rule of analysis and synthesis: Draw conclusions from your conjecture, one after the other, assuming that it is true. If you reach a false conclusion, then your conjecture was false. If you reach an indubitably true conclusion, your conjecture may have been true. In this case reverse the process, work backwards and try to deduce your original conjecture via the inverse route from the indubitable truth to the dubitable conjecture. If you succeed, you have proved your conjecture". And in p.95-6: "But if we start from a proposition P and draw consequences from it rather than try to look for premisses from which it follows, then what we objectively do is test rather prove. Thus in analysis we test conjectures". Cf. also Szabó (1974, p.123): "I think analysis is rather merely a testing of conjecture".

[39] Cf. Robinson 1936/1969, pp.2-3; Heath 1926, I,p.140.

[40] Hintikka and Remes argued convincingly at this point: "the reference to an impossible outcome might then be interprєted in the light of the plain but sadly negleted fact that even a search for antecedents from which a given theorem could be proved can run into a dead end. In principle such a contingency need not even result in a reductive proof, as a closer logical examination shows. That is the only legitimate conclusion might be a non sequitur instead of a reductio proof." (1974, p.77; cf. also pp.78-80)

[41] For definitions of reductio ad absurdum, see Proclus

pp.254,255; Aristotle, An.Pr. 40b25, 41a24.

[42] Cf. Proclus p.295: "This is the method [reductio] ordinarily used for proving the converse of a theorem although in problems, at least, our geometer admits also leading constructions".

[43] See Heath 1921, I,p.291; Sarton 1952, p.433.

[44] See in Hintikka and Remes, 1974,p.85.

[45] Simplicius In Phys, 55-9 (Diels).

[46] Pseudo-Eratosthenes to king Ptolemy, in Eutocius, Comm. on Archimedes sphere and cylinder, ii (Archimedes, ed. Heiberg, III,88).

[47] Proclus In Eucl 66.7-9 (Friedlein).

[48] Proclus In Eucl 213 (Friedlein). The translation is adapted from Morrow 1970.

Heath (1926, I,135), I Thomas (1939-41, I,253) and Morrow translate "τῶν ἀπορουμένων διαγράμματων" as "of difficult constructions". This translation is poor - a better translation would be "of perplexing demonstrations" - but I think that Proclus' meaning comes out in either translation.

In geometry, an "ἀπορούμενον διάγραμμα" is the case of a proposed problem (or theorem) in which the geometer has reached an impasseor is at a loss as regards its solution (or proof). Such a geometrical problem (or theorem) is either a difficult one (but solvable) or one impossible to solve. In our case, Proclus says that Hippocrates did effect his reductions; therefore, this is not the case of impossibility but of difficulty. Hence, I think that we can safely translate 'ἀπορουμένων' as 'difficult'.

'Διάγραμμα' refers both to problems and theorems so its translation as 'construction' (that refers only to problems) is not literally accurate. However, as far as we know, Hippocrates dealt mainly with problems. So I think that it is not wrong - in the case of Hippocrates - to translate 'διάγραμμα' as 'construction'.

[49] 1926, I,135.

[50] Simplicius In Phys 61.5-9. Translation from Heath, 1921, I,187. Simpicius says that quotes Eudemus word for word.

[51] "ἐὰν οὖν δείξω τῇ ἀπὸ τοῦ κέντρου ἐπὶ τὸ Κ ἴσην τὴν ἀπὸ τοῦ κέντρου ἐπὶ τὸ Β, δῆλον ὅτι τὸ γραφόμενον τμῆμα κύκλου διὰ τοῦ ΕΚΗ ἥξει καὶ διὰ τοῦ Β, καὶ περιλήψεται κύκλου τμῆμα τὸ τραπέζιον".

[52] See Sarton, 1952/1970, 433; Gow, 1884,170, where he refers also to Bretschneider and Hankel.

[53] A= the given; B= the required; K= other known propositions or axioms.

I use the symbol ⟹ to describe the way up (analysis) accepting that it is not always deductive and that a geometer searches mainly for premisses when he analyzes.

[54] De sphaera et cyl., ii,prop.4; see in Thomas 1939-41, II,126-57; and in Heath 1921, II, 43-5.

[55] Cf. Heath 1926, I,142; Hintikka and Remes 1974,58; Lakatos 1978, 75,95; Mahoney 1968-9,328; Hankel (see in Hintikka and Remes, 58); Cherniss 1950-1, 419; Lassere 1964, 107.

Cherniss refers to the Index Herculanensis (Col.Y., 14ff; Meckler,p.17) in which the enunciation of diorismos is connected with the rise of analysis. For diorismos see Proclus 202,208 and the interpolated definition in Pappus (Hultsch, 30.14-6) "diorismos is a preliminary distinction [προδιαστολὴ] of when and how and in what cases the problem is possible". Proclus (66-7) attributes the discovery of diorismos to Leon who was younger than

Leodamas (the discoverer of analysis).

[56] "It is clear that the geometrical reputation of Oenopides could not have rested on the mere solution of such simple problems as these. Nor of course could he have been the first to draw perpendicular in practice; the point may be that he was the first to solve the problem by means of ruler and compasses only.......Similarly Oenopides may have been the first to give the theoretical rather than the practical construction for the problem I,23 which we find in Euclid" (Heath 1921, I,175).

[57] Szabó (1978) showed the relation between indirect proof and antiempiricism in mathematics.

[58] It is not an accident that the only *Elements* of geometry originating in the 5th century are ascribed to Hippocrates. It is difficult to believe that Hippocrates' *Elements* had an axiomatic form similar to that of Euclid. But it is certain that they were a collection of hierarchically ordered mathematical propositions. I shall discuss the problems of axiomatization of geometry and of the first *Elements* in chapter VII, sections 1,2.

[59] Cf. Lakatos 1978,99: "Analysis is only revolutionary when it engineers a breakthrough from a low-level naive conjecture to a research programme"; and in p.199 "In my view the most exciting analyses of Greek geomery were pre-euclidean and their role was to generate Euclid's axiomatic system".

[60] See Lakatos 1978,99: "once the mathematical machinery is established, analysis, 'working backwards' may still be applied as a heuristic tool in puzzle-solving, but it becomes clear that its role is only psychological. It helps the imagination to produce valid proofs or explanations in terms of a given research programme"

[61] The first geometer known to investigate converse propositions systematically is Menaechmus (see Proclus, 254), who also was practising the method of analysis (see Eutocius, in *Archimedes* ed. Heiberg, III,92). Therefore we can say that the method of analysis and synthesis gives rise to an inquiry into convertibility. It seems (from Proclus, 253-4) that Menaechmus had observed that geometrical conversion is something usual in theorems and not in problems, and also that he had demonstrated the falsity of many conversions.

[62] Pappus *Collectio*, vii praef. 1; in Thomas 1939-41,

[63] Mugler 1948, 283-7; Sarton 1952, 433.

[64] Cf Plato *Rep.*510-11; *Phaedrus*245c-e; *Phaedo* 101d-e. Cf also Arist. *E.N.* 1095a32-b1 (εὖ γὰρ καὶ Πλάτων ἠπόρει τοῦτο καὶ ἐζήτει πότερον ἀπὸ τῶν ἀρχῶν ἢ ἐπὶ τὰς ἀρχάς ἑστιν ἡ ὁδός), and Proclus, p.255 (πᾶσαι αἱ μαθηματικαὶ πίστεις ἢ ἀπὸ τῶν ἀρχῶν εἰσιν ἢ ἐπὶ τὰς ἀρχάς). Cherniss (1950-1, 419) says that "Aristotle's reference might seem to have to do with geometrical analysis".

[65] See Philoponus *In Phys* (Vitelli) 31.3-9; Heath, 1921, I,22,183. Cf Aristotle *E.E.* 1247a17-25.

[66] Regarding the historical problem of the chronologies of Theaetetus, Leon and Eudoxus, see chapter VII, section 2.

It is almost certain, and generally accepted that the first part of Proclus' 'Summary of Geometers' is taken from (or at least is based on) Eudemus' *History of Geometry* (see Heath 1921, 118-120; Van der Waerden 1954, 91). Proclus, after his reference to the geometers of the Academy, says: "All those who have written histories bring to this point their account of this science. Not long after these men came Euclid...." (68.5-8). Eudemus (Aristotle's pupil) was a little older than Euclid, exactly like 'those who have written histories' in Proclus' passage; moreover Eudemus is the only known person who had ever written a history of geometry before Euclid's time. Therefore, I think that we can safely identify Eudemus with 'those who have written histories'.

CHAPTER III

THE METHOD OF HYPOTHESIS IN THE 'MENO'

1. The hypothesis

The dialogue opens with Meno's question of whether virtue is teachable (διδακτὸν) or not (70a)[1]. Socrates, however, wants them first to examine the question of what virtue itself is, before answering the question about the teachability of virtue (71a). This is because we cannot know a property of something, without first knowing what it is (71b4). After three unsuccesful attempts at giving a satisfactory definition of virtue, Meno comes up with a sophistic paradox concerning the impossibility of any inquiry. Socrates answers by introducing the theory of recollection (81a-85b), and, at 86c, asks Meno again to give a new definition of virtue. Meno refuses to do so, and suggests that they examine the first queston of the dialogue, that is to say, whether virtue is teachable. But as we do not know what virtue is, Socrates suggests that they employ the hypothetical method as it is used by geometers (86e) [2].

Then a mathematical example follows as an explanation of the method:

> I mean by hypothesis the way in which the geometers often inquire into things. When they are asked, for example, about an area, whether it is possible for this area to be inscribed as a triangle in this circle here. One of them may say: I do not know yet whether this is so, but I think I have a sort of hypothesis which is useful in this case and is the following: if this given area is ...[such and such] ... one thing seems to me to follow, and another thing if it is not such. Therefore, using a hypothesis I shall be able to tell you about the inscription of the area in the circle, whether or not this is possible (86e-87a)

After that, the application of the hypothetical method to the problem of whether virtue is teachable follows:

> Let us do the same about virtue. Since we don't know what it is or what it resembles, let us use a hypothesis in investigating whether it is teachable or not. We shall say: 'What thing among those connected with the soul would virtue be, to make it teachable or not teachable?' Well, in the first place, if it is anything else but knowledge, is there a possibility of anyone teaching it - or, in the language we used just now, reminding someone of it? We needn't worry about which name we are to give to the process, but simply ask: will it be teachable? Isn't it plain to everyone that a man is not taught anything except knowledge? (87b3-c2) (translation adapted from Guthrie)

Consequently, "if virtue is knowledge, it is obvious that it is teachable" (87c). So the problem is to search to detemine whether virtue is knowledge or not. But Socrates, before examining this, speaks about another hypothesis:

> Well then, do we assert that virtue is good? And this hypothesis stands, that it is good? (87d2-3)

Based on this second hypothesis (virtue is good), Socrates comes to the conclusion that virtue is knowledge (87d-89c). Therefore:

> SOCRATES. Since then goodness does not come by nature, is it got by learning?
> MENO. I don't see how we can escape the conclusion. Indeed it is obvious on our assumptions that, if virtue is knowledge, it is teachable (εἴπερ ἐπιστήμη ἐστὶν ἀρετή, ὅτι διδακτόν ἐστιν) (89b9-c4) (Guthrie).

Then Socrates has doubts regarding the prior proof and Meno says: "But what has occurred to you to make you turn against it and suspect that virtue may not be knowledge?" . And then a long empirical argument follows about the fact that there are no teachers of virtue.

There are two major problems with the interpretation of the above passage: a) what is the main hypothesis and what is the structure of the hypothetical method? and b) what is the geometrical method that Plato refers to? This chapter is devoted to those two questions.

Let us first consider the geometrical example: the problem (say r) is that of inscribing a certain rectilinear area in a given circle (86e6-7). The geometer says that he cannot answer whether the problem is solvable or not. But he can answer "from a hypothesis" (say s). If the condition that the hypothesis states is fulfilled, then the problem is solvable; if not, the problem is not solvable. In other words: if the area has a certain property then it can be inscribed in the circle, if not, then it cannot be. So we see that the geometer finds a proposition s (hypothesis) that constitutes a condition of solvability of his problem r (that is, $s \rightarrow r$ and $-s \rightarrow -r$).

In this way the problem is solved only hypothetically (on the basis of s). In order to have a real solution to it (or proof that it is unsolvable) we must know whether s is true or false. That is, we have to solve the problem that the proposition s constitutes (or to prove it impossible).

Let us now examine closer the hypothetical method: the first question which arises is which is the (main) hypothesis of the argument? Robinson (1953, 117) observes that there are three propositions that may be regarded as the main 'hypothesis': i) 'Virtue is knowledge' (87b-c), ii) 'virtue is good' (87d1-2) and iii) 'if virtue is knowledge, it will be teachable' (89d3-4).

But it is easy to see from 89c2-4, that 'virtue is good' cannot be the main hypothesis [3]. Similarly, if we see passage 89c2-4 in relation to 89d1-2 (... and suspect that virtue may not be knowledge), it seems that the proposition 'if virtue is knowledge then it is teachable' cannot be the hypothesis. Consequently, the hypothesis must be 'virtue is knowledge'.

(Moreover, if we accept one of the above ii) or iii) propositions as the hypothesis, we can hardly find any similarity between the hypothetical argument and the geometric example.) [4]

The basic evidence in favour of the proposition 'virtue is knowledge' as the hypothesis, is passage 89c2-4. The common reading is: "and plainly, Socrates, on our hypothesis that virtue is knowledge, it must be teachable" (W R M Lamb, LOEB Classical Library, 1924). Cherniss (1947, p140, n.38), notices that "the position of ὅτι before the 'it is teachable' shows that the εἴπερ clause states the hypothesis [5]. I think that Cherniss' reading of 89c2-4 is supported by passage 87c5-6 (εἰ δέ γ' ἐστὶν ἐπιστήμη τις ἡ ἀρετή, δῆλον ὅτι διδακτὸν ἂν εἴη). Here the word δῆλον before the ὅτι shows that the two propositions are separate and distinct and we can read it as follows: "if virtue is a sort of knowledge, it is clear that it will be teachable".

P Friedländer cites passage 87b2-4 to support the view that the hypothesis is 'virtue is knowledge' [6]. but this becomes more clear if we compare the above passage with 87b7 ("if it is other than knowedge") and 87c5 ("if virtue is some kind of knowledge").

Recently, H Zyskind and R Sternfeld [7] have given a different answer to the question 'what is the hypothesis'. They suppose that the hypothesis is 'knowledge alone is teachable' (87c2-3) and they read the passage 89c2-4 as follows: "and it is plain, Socrates, on the hypothesis, since virtue is knowledge, that it is teachable". The argument runs as follows: "'Knowledge alone is teachable' is a universal equivalence expressible logically as: 'For any X, x is knowledge, if and only if x is teachable' ...... From this the single conditional can be detached: 'For any x, if x is knowledge, then x is teachable'. And by universal instantiation with virtue as the object instantiated, one derives: 'If virtue is knowledge, then virtue is teachable'" (p132, n.12). So we have the following hypothetical syllogism:

if virtue is knowledge, then virtue is teachable
virtue is knowledge
----
virtue is teachable

Zyskind and Sternfeld's interpretation seems to me untenable for four reasons: a) They translate the εἴπερ as 'since' and not as 'if', although some lines later (89d3-5: τὸ μὲν γὰρ διδακτὸν αὐτὸ εἶναι, εἴπερ ἐπιστήμη ἐστίν, οὐκ ἀνατίθεμαι μὴ οὐ καλῶς λέγεσθαι) clearly the εἴπερ means 'if'. b) The statement that 'only knowledge is teachable' (87c2-3) is introduced as something 'plain to all' and not as a hypothesis. c) If we accept that the hypothesis is 'knowledge alone is teachable' then it is impossible to find the corresponding hypothesis of the geometrical problem [8]. d) Plato seems to believe that he introduces a new method. But accepting the above interpretation we can hardly find something new in his methodology.

I think then that linguistic evidence leads us to accept the proposition 'virtue is knowledge' as the (main) hypothesis [9].

2. The argument by hypothesis

Apart fom the linguistic argument in favour of 'virtue is knowledge' as the basic hypothesis, there are other (and probably stronger) arguments based on the meaning of the passage. But first of all let us look at the internal structure of the argument.

Socrates says that he is going to inquire into the problem of the teachability of virtue in the same way as the geometer did with his problem (87b2-3). "Since we don't know what it is or its quality, let us investigate it, whether it is teachable of not, through a hypothesis" (87b3-4). Our problem is whether or not virtue is teachable. Socrates asks "which of the things connected with the soul must virtue be, to make it teachable or not teachable?" (87b6-7). So Socrates begins his inquiry in an indirect manner. He starts from his problem and asks about a premiss from which his original problem follows. "Well, in the first place, if it is of another kind than knowledge, is it then teachable or not..... Or is it at least plain to everyone that a man is not taught anything except knowledge?" (87b6-c3). It seems then that Plato thinks that the terms 'knowledge' and 'teachable' are co-extensive.

"If then, virtue is some sort of knowledge, clearly it could be taught" (87c5-6). Hence, from our original proposition 'virtue is teachable' (p), we reach the proposition 'virtue is knowledge' (q). Because of the co-extensiveness of the notions 'knowledge' and 'teachable' it is clear that $q \longleftrightarrow p$ (cf 98d10-12). But Plato insists not so much on the above formula of the equivalence between p and q, as on the conjunction of the two implications $q \longrightarrow p$ and $-q \longrightarrow -p$ (cf 87a6-7, 87b1-2, 87b4, 87b6 and 87c8-9). So he is interested in seeing in proposition q the limiting condition of the truth or falsity of p, and so in being able to decide (according to the truth or falsity of q) whether p is true or false [10].

"So that (question) is easily settled .... The next thing apparently, is to ask whether virtue is knowledge or other than knowledge" (87c8-12). It is very clear that Plato thinks that the argument by hypothesis has two stages. The first one is now finished. At this stage of the argument, we find the parallel with the mathematical passage and Plato's clear statement that he is applying the hypothetical method (87b2-3).

After we have found the limiting condition q (or hypothesis - 'virtue is knowledge') for virtue to be teachable (p), the next step is to see whether the hypothesis is really true or false.

"Well now, do we assert that virtue is good? and this hypothesis stands that it is good? (87d2-3). Beginning the second stage of the hypothetical argument, Socrates states a new hypothesis: 'virtue is good'. It is not clear whether we can translate "ἀγαθὸν αὐτό φαμεν εἶναι τὴν ἀρετήν" as "virtue itself is good" (Stock, 1904, loc. cit.), "virtue is good in itself" or simply "virtue itself is good" (Bluck, 1961, p327). I find the third reading the most suitable. So we do not have to take 'virtue' and 'good' as coextensive terms (but even if we accept the second of the above readings this does not affect the argument).

We must also notice that the hypothesis 'virtue is good' is introduced as something indisputable, under the complete agreement of the two interlocutors ("and this hypothesis stands" - 87d3). Therefore 'virtue is good' plays the role of a principle for the subsequent proof.

The argument may be outlined as follows:

1. Virtue is good (87d2)
2. Whatever is good is useful (87e2)

∴ 3. Virtue is useful (87e3)

4. If something is useful, it is knowledge (89a1-2)

∴ 5. Virtue is knowledge (89a3)

First of all we can notice that the second stage of the hypothetical argument constitutes a direct proof of the proposition 'virtue is knowledge'.

Secondly, there is a problem of whether Plato considers the notions of good and knowledge as co-extensive. From 87e2, (every good is useful) it seems that between the terms 'good' and 'useful', there is an inclusion relationship (good $\subset$ useful). The same seems to be happening with 'useful' and 'knowledge' (89a1, φρόνησις ἂν εἴη τὸ ὠφέλιμον), that is, useful $\subset$ knowedge) [11]. Hence, 'good' $\subset$ 'knowledge' (cf 87d4-8) and 'virtue' $\subset$ 'knowledge' (89a3-4: "virtue is knowledge whether in whole or in part") [12]. Hence, it seems that 'good' and 'knowledge' are not co-extensive notions, or at least that Plato is only interested in inclusion relationships (or one way implications) and this is enough for his proof that virtue is knowledge.

So, if we put t for 'virtue is good' and q for 'virtue is knowledge', we have $t \rightarrow q$ but not $q \rightarrow t$ (or $-t \rightarrow -q$). Therefore t is not a limiting condition of q. 'Virtue is good' is a proposition posited as true (like a principle) at the beginning of the proof of 'virtue is knowledge' and not a condition of the truth or falsity of it [13].

Let us now examine the argument more closely. Socrates goes on with the proof through hypothesis as geometers would (86e). The 'hypothesis' is a proposition on whose truth or falsity depends the truth or falsity of the problem (87b). So the hypothesis is a proposition to which our problem has been reduced. The geometers reach this proposition (hypothesis) analysing their problem (that is, in an indirect way). They consider their first problem as true and starting from that, they arrive, through analysis, at another proposition. This proposition can be known or unknown. If the final proposition which analysis reaches (that is, the one to which the original problem has been reduced) is not known, we must prove it (in an indirect or direct way) so that the first problem can also be proved.

Geometers use this method when their problem is difficult to solve and they cannot see the point from which the proof must start. Facing these difficulties, they proceed in a reverse way. They start from the problem to be solved and reduce it to a proposition which is already known or to another problem which is easier to solve.

Socrates does the same. He starts from the proposition 'virtue is teachable' and analysing (or drawing conclusions from it), he reaches the proposition 'virtue is knowledge' (86b-c). In this way, and because the terms 'teachable' and 'knowledge' are co-extensive, the truth or falsity of our original proposition 'virtue is teachable' depends on the truth or falsity of the proposition 'virtue is knowledge'. This latter proposition is the 'hypothesis' through which we can prove our original proposition.

So far the similarity with the geometrical problem is obvious. The hypothesis (virtue is knowledge) is the 'limiting condition' for the truth or falsity of the proposition 'virtue is knowledge'. [14]

Here ends the first step of the argument. The term 'virtue' is unknown and we cannot draw any direct conclusion from the proposition 'virtue is knowledge'. In fact, the reduction of the proposition 'virtue is teachable' to 'virtue is knowledge' was due only to an analysis of the term 'teachable'.

Socrates tells us that he uses the hypothetical method because we do not know what virtue is (87b3). However, when a mathematician faces a problem, all the concepts are known (defined, or, speaking in Plato's terminology, one has some true beliefs about them) [15]. Therefore, the hypothetical method does not overcome the difficulty of the non-definition of virtue.

If we now want the argument by hypothesis to proceed further, we must find a premiss (or a second hypothesis) from which to deduce the (first) hypothesis 'virtue is knowledge'. If this second 'hypothesis' needs proof, we have to find a third hypothesis from which we shall be able to deduce the second one and so on until we find a proposition (the last hypothesis) which is known and whose truth is acceptable to both interlocutors. This 'last hypothesis' must be a statement regarding virtue, otherwise we shall never be able to connect the term 'virtue' with the term 'teachable' (or with any other attribute) because we know nothing about virtue. Therefore, this 'last hypothesis', although not being a definition of virtue, must be a statement referring to the essence of virtue or a 'partial' definition of it.

Socrates proceeds, indeed in the above mentioned way. He cannot draw any direct conclusions from 'virtue is knowledge' but he finds a premiss (second hypothesis) that is, 'virtue is good', from which he is able to deduce the first hypothesis (virtue is knowledge). The proposition 'virtue is good' is a premiss (or a principle) for the proof of 'virtue is knowledge'. Plato calls it an 'hypothesis' [16] but it is in fact a homologema because it is posited under the complete agreement of the two interlocutors [17]. When The proposition 'virtue is teachable' is proved once true and once false, Socrates and Meno do not challenge the validity of this hypothesis [18].

'Virtue is good' is not the only premiss (or hypothesis) from which the proposition 'virtue is knowledge' has been inferred.

But Plato names only this one as hypothesis. This happens because of the special role of this proposition, as the hypothesis put in order to overcome the obstacle of the unknown character of virtue. The other 'hypotheses' (that is, 'all that is good is useful' and 'if something is useful, it is knowledge') are usual premisses that are introduced under the agreement of the two interlocutors - something very common in platonic dialectic [19].

As we saw earlier, while 'virtue is good' (t) entails 'virtue is knowledge' (q), the converse is not true. Therefore the relation between t and q is not biconditional ($t \rightarrow q$ but not $q \rightarrow t$). Therefore, t is only a sufficient condition for q and it cannot be considered as a limiting condition. (As 'virtue is knowledge' (q) is a limiting condition for 'virtue is teachable' (p)). [20]

The argument by hypothesis in the Meno consists mainly in two reductions. Firstly, Socrates reduces the proposition 'virtue is teachable' (the original problem) to the proposition 'virtue is knowledge' (the first hypothesis). Secondly, the proposition 'virtue is knowledge' is reduced to the proposition 'virtue is good'. In both cases, after the reduction there is a proof of the original proposition, on the basis of the 'hypothesis' to which this last was reduced (ie $q \rightarrow p$, $t \rightarrow q$). The first proof (that $q \rightarrow p$) is simple and has only one step (87c5-6), the second proof (that $t \rightarrow q$ - 87d2-89a3) is a long one with many steps and premisses and it employs also an empirical argument to obtain a premiss.

The two reductions are also different. The first reduction is deductive and we obtain the 'hypothesis' q ('virtue is knowledge') by analysing and drawing conclusions from our original problem p ('virtue is teachable'). Therefore, p implies q and the relation between p and q is that of equivalence ($p \leftrightarrow q$).

In the second reduction the 'hypothesis' (t) is not obtained by drawing conclusions from q. We obtain the 'hypothesis' t rather by an insight of what is prior, of what could be a premiss for the proof of q. In this case, the finding of t is a search for premisses, a grasp of something prior which would be sufficient for the proof of q. What is relevant is not to draw conclusions from the proposition to be proved (q) but to search and find the appropriate premiss (t) from which (and together with other premisses) we could deduce q. The second reduction is not deductive and the argument by hypothesis does not employ convertible propositions.

The reduction of one proposition to another may be either deductive or non-deductive. The two steps of the argument also show two different processes of thought. In the first, we start from our problem and we draw conclusions from it, arriving at another proposition which, if we reverse the process, entails the original. In the second, we start again from our problem, and we do not draw conclusions, but search to find a premiss for the solution of our problem (that is we ask: which proposition do I have to hypothesize in order to deduce my original one?). In the second process, the proof from the hypothesis to the original proposition may have many deductive steps.

It seems then that convertibility of propositions is not an essential feature of the hypothetical method [21]. Philosophy, unlike mathematics [22] very often employs non-convertible propositions and a method that uses only equivalences is almost useless. As Bluck (1961, 50) remarks, the whole hypothetical method in the Meno is a combination of the method of geometers and that of Socrates. It seems that the hypothetical method in the Meno is almost the same as the method in the Phaedo (100a, 101d-e) [23]. In this way, the proposition 'virtue is knowledge' is a hypothesis for the proof of 'virtue is teachable' and a conclusion from a 'higher' hypothesis, namely 'virtue is good'.

In the above interpretation of the method, I have tacitly assumed that the application of the hypothetical method to the problem of the teachability of virtue stops at 89a [24]. This is also the common view. But after the 'proof' that 'virtue is teachable', Socrates reverses himself and disproves it in a long empirical argument (89d - 97a). The argument runs as follows: 'virtue is teachable', $\rightarrow$ 'there are teachers and students of virtue' (see 89d6-8, 98e1-2); but since 'there are no teachers and students of virtue', 'virtue is not teachable'. [25]

The argument has almost the same form as the hypothetical argument with the terms 'virtue', 'teachable' and 'knowledge', with the difference that here we diprove our original proposiition (that is: $q \rightarrow p$ and $-q \rightarrow -p$, but $-q$, so $-p$). So we might infer that Plato proceeds to a second application of the hypothetical method [26] and the hypothesis is "there are teachers and students of virtue'.

3. ANALYSIS AND DIORISMOS

I shall now pass to the second problem concerning the hypothetical method in the Meno, that is, to see which method of the geometers Plato refers to (86e). On the evidence (in Proclus and Diogenes Laertius, see p.56) that Plato was the first to teach the analytic method to Leodamas of Thasos, many commentators maintain that the hypothetical method in the Meno coincides with the geometrical method of analysis and synthesis [27].

The method of analysis and synthesis consists in assuming the theorem to be proved (hypothesizing it) and reducing it to another and so on until one comes to a proposition which one knows to be true independently of the original problem. After that, starting from this known proposition, one is able to deduce one's theorem.

As we have already said (p.56-7) it is very unlikely that Plato himself discovered the geometrical method of analysis and synthesis. Moreover, it is not certain that at the time when the Meno was written (c. 386), the method of analysis was in practice among the mathematicians and known to Plato.

The question which now arises is whether the method mentioned in the Meno is that of analysis and synthesis or not. The hypothetical method in the Meno is definitely analytic, in the sense that it starts from the problem to be solved and proceeds backwards in order to find a solution for it. But there are some differences:

1. If the method were that of analysis, the 'hypothesis' should be 'virtue is teachable' (or in the case of the mathematical problem, 'assume a triangle inscribed in a circle'). This means that what is accepted as being true is the problem itself and not the proposition to which the problem is reduced. [28]

2. In the method of analysis and synthesis, the analysis starts from the problem to be solved and, through subsequent intermediate steps, arrives at a proposition known independently from the given one. Only, when the analysis ends do we start the synthesis (proof), beginning from the last proposition found in the course of the analysis and proceeding in the opposite way until we reach

(deduce) our original problem. The two ways of the method are clearly separated (see p.50).

On the other hand, in the Meno, the process is different. We start again indirectly from our proposition to be proved p and we arrive at a proposition q through which we are able to deduce p. After that, we do not continue to proceed backwards (analytically) but we prove p via q. Only then do we try to confirm our hypothesis q and proceed again backwards finding a new 'hypothesis' t from which we are able to deduce q. Then we proceed again downwards and we prove q via t. Here our proof is finished. We see then that in the Meno, the way from our problem to its premisses and vice versa are not separated like in the method of analysis and synthesis. Every step backwards (or upwards) is followed by a demonstrative step downwards.

Therefore I can now conclude that the hypothetical method in the Meno is not the geometrical method of analysis.

Almost all critics who do not accept that the hypothetical method in the Meno coincides with analysis, assert that the 'hypothesis' is simply a case of diorismos[29].

If the hypothesis is a diorismos, this does not necessarily mean that the method cannot be analysis. On the contrary, in geometry the method of analysis very often ends with a diorismos [30].

According to Proclus [31] "Leon, a student of Neoclides [who was younger than Leodamas, the discoverer of the method of analysis] ..... discovered the diorismos, that is when a problem is solvable and when it is not".

The hypothesis in the Meno (that is, that 'virtue is knowledge', as well as the 'hypothesis' of the mathematical passage) is very similar to a diorismos. The hypothesis, as the diorismos is a condition of solvability of the given problem. However, I have some objections to such an interpretation.

a) As we have already said (p.52-3) although cases of diorismoi may have been known to the Pythagoreans, the systematization and the formal recognition of diorismos must be subsequent to the method of analysis. Given that, and the fact that Leon (pupil of Plato - see chapter VII, section 2) flourished after 380 BC, it is probable that the theory of diorismos is subsequent to the composition of the Meno (c. 368).

b) As we saw, Plato proposes a method of proof which employs hypotheses. Therefore, we have a hypothetical method. But diorismos does not constitute a method. Diorismos is simply a proposition that asserts a condition of the solvability of a problem.

c) A diorismos is not a proposition that appears in the deductive chain of the solution of a problem. It is outside of it. But in our example, from the acceptance of the proposition 'virtue is knowledge', we can conclude that 'virtue is teachable'.

d) If we had a diorismos in the mathematical passage, it would have consisted of the following sentence: "the given area must be not larger than that of the equilateral triangle which is inscribed in the circle" (see p.109).

e) Applied to virtue: if the 'hypothesis' were a diorismos, a further proof could not be necessary. A diorismos concerns the final limiting conditions for the solvability of the problem and does not constitute a new problem which needs solution.

f) Pappus Collectio VII, 2 (bracketed by Hultsch) gives a definition of *diorismos*. "*Diorismos* is a preliminary distinction [προδιαστολή] of when, how and in how many uses the problem is possible. Therefore a *diorismos* must give us an advanced specification (prelimiary distinction)" [32]. But in our 'hypothesis' we have no such preliminary distinction.

4. Apagoge (reduction)

In Aristotle's *Prior Analytics* (69a20-36) there is a passage that probably refers to our dialogue. In this passage, Aristotle talks about a demonstrative process which he calls *apagoge* [33]:

> XXV. We have Reduction (1) when it is obvious that the first term applies to the middle, but that the middle applies to the last term is not obvious, yet nevertheless is more probable or not less probable than the conclusion; or (2) if there are not many intermediate terms between the last and the middle; for in all such cases the effect is to bring us nearer to knowledge. (1) E.g., let A stand for 'that which is teachable', B for 'knowledge' and C for 'justice'. Then that knowledge can be taught is evident; but whether virtue is knowledge is not clear. Then if BC is not less probable or is more probable than AC, we have reduction; for we are nearer to knowledge for having introduced an additional term, whereas before we had no knowledge that AC is true.
>
> (2) Or again we have reduction if there are not many intermediate terms between B and C; for in this case too we are brought nearer to knowledge. E.g., suppose that D is 'to square', E 'rectilinear figure' and F 'circle'. Assuming that between E and F there is only one intermediate term - that the circle becomes equal to a rectilinear figure by means of lunes - we should approximate to knowledge. When, however, BC is not more probable than AC, or there are several intermediate terms, I do not use the expression 'reduction'; nor when the proposition BC is immediate; for such a stalemate implied knowledge. (Tredennik, LOEB Classical Library, 1938)

Aristotle's allusion to the *Meno* is characteristic. If in his example we substitute in C the word virtue for the word justice, we will have exactly the example of the *Meno* (a little further on, in the same passage, Aristotle, refering to C, says instead of justice, virtue; this reinforces our hypothesis). As Bluck suggests, the Meno is the only place where we know that the question of whether virtue is teachable was made to depend on, or was reduced to, the question of whether it is knowledge.

Aristotle distinguishes two cases in which we have apagoge. a) "When it is obvious that the first term applies to the middle, but that the middle applies to the last term is not obvious, yet nevertheless is more probable, or not less probable, than the conclusion". b) "If there are not many intermediate terms between the last and the middle".

Indeed, if we posit, as Aristotle says, A for teachable, B for knowledge, C for virtue (that is to say, exactly the same example as in the Meno), then it is obvious that AB (that is to say, 'knowledge is teachable'), whereas BC ('virtue is knowledge') is uncertain. Moreover, BC is equally or more probable than AC - 'virtue is teachable' - (since the truth or the falsity of AC depends on the truth or falsity of BC).

The type of argument by apagoge is, it might be said, semi-demonstrative, semi-dialectical since it has a major premiss (in our case AB: 'Knowledge is teachable') which is known, and a minor premiss which for the moment is only assumed (BC: 'virtue is knowledge'). The unknown character of the minor premiss is the essential feature of this kind of syllogism according to Aristotle (see 69a35-6). Now the minor premiss is getting "more known" (ἐγγὺς ἂν εἴη τοῦ εἰδέναι, 69a34; cf 69a28) - and the argument more probable - if we introduce a middle term (good) which links the subject 'virtue' to the predicate 'knowledge'.

It is worth comparing the example of the Meno with the second example that Aristotle gives us.

A teachable	D to square
B knowledge	E rectilinear
- - - - - - -	- - - - - - - - -
M good	N lunes
------------	--------------
C virtue	F circle

AB knowledge is teachable	DE the rectilinear is squared
BC virtue is knowledge	EF the circle becomes rectilinear
- - - - - - - - - - - -	- - - - - - - - - - - - - - - - - - - -
MC virtue is good	NF the circle is sum of lunes(?)
--------------------	------------------------------
AC virtue is teachable	DF the circle is squared

If we compare the first example of Aristotle with the hypothetical argument presented by Plato in the <u>Meno</u>, we can find many similarities (although Aristotle gives a syllogistic form to the argument): a) our problem is the 'proof' of the proposition 'virtue is teachable'; b) the proposition 'only knowledge is teachable' is something evident (cf. <u>An Pr</u> 69a25-6: ἡ μὲν οὖν ἐπιστήμη ὅτι διδακτὸν φανερόν; <u>Meno</u> 87c2: παντὶ δῆλον); c) in both cases the truth or falsity of the original problem depends on the truth or falsity of the proposition 'virtue is knowledge'; d) this last proposition remains uncertain but it becomes "more known" by the introduction of a middle term ('good' in the case of the <u>Meno</u>).

The second example clearly refers to the mathematician of the 5th century Hippocrates of Chios. Hippocrates is the only Greek mathematician whose name is connected with lunes. In <u>Soph. El.</u>, 175b15, Aristotle explicitly refers to Hippocrates in relation to the quadrature of the circle and the lunes. Moreover, we know already that Hippocrates systematically practised the geometrical method of <u>apagoge</u> in the last half of the 5th century in Athens (see p.45,57).

It seems then that we have evidence that the hypothetical method in the <u>Meno</u> is the <u>apagoge</u> of Hippocrates.

We have already seen (p.45-6, 50) that the apagoge (as practiced by Hippocrates) starts from the problem to be solved (P) and reduces it to another (Q). The proposition Q is a principle (arche for Hippocrates) for the solution of the original problem P [34]. On the truth of this last proposition, depends the solution of the original problem. Hippocrates solves P on the assumption of its arche Q (that is, hypothetically) and after that he asks about the validity of Q. If Q needs confirmation, then he finds another proposition T from which he will be able to deduce Q. Now the T is an arche for the proof of Q, and he deduces Q using T. If T is a proposition known to be true, the proof finishes here. If not, we proceed in the same way until we arrive at something which is independently known.

It is easy to see that Hippocrates' procedure is the same as that of the Meno. Here, similarly, our original problem p (virtue is teachable) is reduced to the proposition (hypothesis) q (virtue is knowledge), and p is proved hypothetically on the assumption of q. Only after that, is the question of the validity of the hypothesis q raised (87c11-12). Because q needs confirmation, we find another hypothesis t (virtue is good) on the basis of which we are able to deduce q. After that we deduce q utilising t and our proof is finished because t is taken to be known.

The hypothetical method in the Meno also fits very well with Proclus' definition of apagoge as "a transition from a problem or theorem to another which, if known or constructed, will make the original problem or theorem evident" (In Eucl, 213).

Moreover, in the Meno, as in the apagoge of Hippocrates (see p.51), the reduction of one proposition to another is not always a deductive process of drawing consequences but also a search for premisses.

I think that I can now conclude that the hypothetical method in the *Meno* is *apagoge* [35] because: 1. The process of the hypothetical method in the *Meno* is similar to the *apagoge* as it is presented through the definitions of Aristotle and Proclus. 2. The hypothetical method has the same structure as the method of *apagoge* that Hippocrates of Chios used to practise (in the Aristotelian definition of the *apagoge*, there is a clear allusion to Hippocrates). 3. There is an expicit allusion in Aristotle that the hypothetical method in the *Meno* is *apagoge*.

NOTES TO CHAPTER III

[1] The word διδακτὸν can be translated both as 'being taught' and 'teachable'. Most of the interpreters prefer the second reading and I too find it more suitable. Socrates is inquiring into the conceptual characteristics of virtue (cf Grimm, 1962, pp9-17) and hence the question whether virtue is διδακτὸν or not means whether 'virtue is able to be taught'. On the other hand, the question 'whether virtue is taught or not' is one whose answer depends only upon empirical facts, that is whether there are teachers of virtue of not. So something that is teachable may not be taught (see Rep 528b-c, where although solid geometry is teachable, there are, as it happens, no teachers of it. cf Sternfeld and Zyskind, 1978, p68; Gould, 1955, p134). But it is not certain if Plato had distinguished this double meaning of the word.

[2] Although the point has been disputed (eg Crombie, 1963, p563) it seems certain to me that Plato himself believes he is introducing a new method. The manner in which Socrates asks Meno's permission to introduce his method, the reference to the geometers, and the explanation of the method through a geometrical problem strongly support the above position (cf also Raven, 1965, p61-2).

[3] But this is not to deny that 'virtue is good' is not also a hypothesis. As far as I know, the only interpreter who accepts that 'virtue is good' is the main hypothesis of the argument is Sesonske (1968,93).

[4] In the first edition of his book, Robinson took the main hypothesis to be 'if virtue is knowledge, it is teachable'; but, after the criticism of Cherniss (1947,140) and Friedländer (1945,

255) in their reviews of his book, he was forced to revise his interpretation. So in his second edition, Robinson accepts the view that the hypothesis is 'virtue is knowledge'.

Crombie (1963,553) accepts (like Robinson in his first edition) as the hypothesis the proposition 'if virtue is knowledge, it is teachable'. However he does not give any convincing argument for this choice.

[5] cf also Bluck,1961,339.

[6] Friedländer (1945,255) says that the passage "'ὑποθέμενοι αὐτὸ σκοπῶμεν εἴτε διδακτὸν εἴτε οὐ διδακτόν ἐστιν' (87b4) does not mean 'let us ... inquire on a hypothesis whether it is teachable ot not teachable' but 'hypothesizing it let us inquire whether...' The very act of hypothesizing, then, precedes the inquiry whether virtue is teachable, and the object of hypothesizing is it, namely, virtue or the nature of virtue. This, moreover, and only this, renders the analogy with the mathematical example precise: the geometer makes a hypothesis about the nature of his figure ('this figure is such and such') and then he draws his conclusion ('whether it is possible or not... ')". So according to Friedländer's reading, we have evidence that the hypothesis is 'virtue is knowledge'. But Friedländer's reading has also other implications. According to it, 'virtue is knowledge' is a hypothetical answer to the question what virtue is. Robinson (1953) accepts Friedländer's view and says that Socrates and Meno "hypothesize a certain account of the essence of virtue" (p53) and that 'virtue is knowledge' is "a definition or partial definition of virtue" (p202). Grimm (1962, 19-23), Thompson (1937,63), Crombie (1963,537), Stahl (1971, 185-7), Guthrie (1975, pp259-60) and Beddu-Addo (1984,2-3) are of the same

opinion.

Bluck claims that αὐτὸ is not the object of ὑποθέμενοι; "αὐτὸ is far more likely to be the object of σκοπῶμεν, as the αυτο at 86e3 above is the object of σκοπεῖσθαι. If so, υποθεμενοι is used absolutely, as ὑποθέμενος was at a6 above" (cf also Sharples, 1985, p161)

K Buchnam (see Grimm, 1962,21-2) refers to 97b10-c1 (καὶ τοῦτό ἐστιν ὃ νυνδὴ παρελείπομεν ἐν τῇ περὶ τῆς ἀρετῆς ..σκέψει ὁποῖον τι εἴη λέγοντες ὅτι φρόνησις μόνον ἡγεῖται τοῦ ὀρθῶς πράττειν) as an indication that the inquiry concerning virtue as knowledge is an investigation of ὁποῖον. I think that this view is also supported by 87b6-7 where Plato uses the word ποῖον, asking for the consequence of teachability.

It seems to me that Bluck's reading gives a better parallel to the geometrical example. "Socrates hypothesizes not the given figure or its nature, but that the relation of an area to a line in a cirlce is determinative of the area's inscribability in the circle. Similarly, in the case of virtue, the hypothesis states that the presence or absence of knowledge is determinative of virtue's teachabilty or non-teachability" Zyskind and Sternfeld (1976, 131, n9).

So I think that 'virtue is knowledge' does not refer to the essence of virtue and hence it is not a (partial) definition of virtue. 'Virtue is knowledge' may refer to the ὁποῖον question about virtue (cf also Thompson (1937, 153) and Sternfeld and Zyskind (1978,56)).

[7] Zyskind and Sternfeld (1976, 130-134).

[8] According to Zyskind and Sternfeld, the hypothesis is a universal equivalence relation expressible as 'For any x, x is

knowledge if and only if x is teachable'. Correspondingly, in the geometrical example, we must have: 'For any x, x is extended (in the circle) along one of its sides and falls short by another x equal to the extended one, if and only if it is inscribable in the circle'. But a) it is very unreasonable for a geometer to produce such statements, because he is working on concrete problems, and b) the above proposition is evidently false. It is only true if and only if x is a plane parallelogram (see chapter IV section 1).

[9] At this point, most of the interpreters agree. See Bluck (1961,86), A E Taylor (1937, 139), Klein (1965,210), Thompson (1937,153), Rose (1970,3). J Thomas (1980, 171), Grimm (1962,20), Stahl (1971, 186), Cambiano (1971,150), Matthews (1972,28), C C W Taylor (unpublished paper, 4-6), Cahn (1981, 318) and Scolnicov (1973,54).

[10] In the geometrical passage (let r be the original problem about inscribability and s the proposition - condition - to which the problem is reduced) we have only the following formula of the equivilence between r and s: $s \rightarrow r$ and $-s \rightarrow -r$. Expressing the problem in this way, we can say that the hypothesis is a double one (cf Thompson 1937, 153). C C W Taylor (unpubl. paper, 11) thinks that we have two parallel arguments rather than a single argument. "In each case, he [Plato] starts from a question of 'r of -r?' whose answer is unknown. His procedure is then to look for sufficient conditions for either alternative. However, later on, when Plato hypothesizes a second hypothesis (virtue is good) the pattern of the argument is different.

[11] cf also Bluck (1961), 335.

[12] For a thorough examination of the argument, see Thomas 1980, 173-8. Thompson (1937) takes 89d3-4 to mean "... we conclude

virtue or some part (meros) of virtue to be knowledge" (p161). But according to the argument, it is impossible for some part of virtue to fall outside of knowledge.

[13] The argument presents defects, the most outstanding of which is a decisive shift from episteme to phronesis (88c). I have overlooked this point and I have translated both episteme and phronesis as knowledge, because my aim was only to examine the structure of the argument. For a critisism of the argumment cf Klein (1965, 211-6), Sternfeld and Zyskind (1978, 58-61), Teloh (1981, 47-50) and Heijbeor (1955, 106).

[14] Cf Bluck (1961, 76,87), Cambiano (1967, 135-6) and Maccioni (1978,25).

[15] See 82b-c where Socrates and the slave-boy agree on some properties of the square.

[16] As we saw (p. 9-10, 12-4) the meaning of the word hypothesis is very loose. it contains the meanings of the words; homologema, axiom, definition and simple hypothesis.

[17] Friedländer (1969, 285) says that 'virtue is good' is "something obvious because of the close relation of the two terms". And Guthrie (1975,259) claims that 'virtue is good' is a tautology because "arete is the noun corresponding to the adjective agathon". This explains the easy acceptance of the proposition by Meno not simply as a mere hypothesis but as something stronger. However, as concerns the argument, even if we accept that 'virtue' and 'good' have identical meaning, the new introduced term (good) helps Plato to draw a consequence, namely the utility of virtue. So goodness unlike virtue is treated as known.

[18] The hypothesis 'virtue is good' seems to be quite

similar to the hikanon of the Phaedo (101e1). Cf also Taylor (unpubl. paper, 9).

[19] Rose (1970, 5-8) insists that a number of 'hypotheses' are employed in the hypothetical argument as if this were the basic feature of the method.

[20] Bluck (1961,88) thinks that as 'virtue is knowledge' is the limiting condition of 'virtue is teachable', so 'virtue is good' is the limiting condition of 'virtue is knowledge'. See similarly, Cambiano (1971,150). Stahl (1970) thinks that the method employs only convertible propositions.

[21] Stahl (1971) claims that we can find in Plato's hypothetical method in the Meno the beginnings of the propositional 'stoic logic'. "In Plato the method of hypothesis is already full formed as a kind of argument theory" (p182). He explains the method through a series of modus ponens and modus tollens arguments and claims that "clearly we have the recognized modus ponens argument form of propositional logic before us" (p188). However, Plato's hypothetical method is borrowed from geometry. And in mathematics, every argument has (or can have) the form of modus ponens. The distance between this and the statement that Plato consciously uses modus ponens arguments and propositional logic (as the forerunner of the Stoics) is great.

[22] But even in mathematics, analysis or reduction of a problem to another is not always deductive (see p.27,39).

[23] Cf A E Taylor (1937, 139), Cherniss (1947, 140), Robinson (1953, 118), Rose (1970,2), C C W Taylor (unpubl. paper, 8-9), Bluck (1961, 89), Cambiano (1971, 150) and Bedu-Addo (1984,10).

[24] Cf Robinson (1953, 117): "This is probably the end of the hypothetical procedure, for after page 89 neither the word

'hypothesis' nor any methodological remark occurs in the dialogue".

[25] C C W Taylor interpreting the Meno in the light of the Phaedo says regarding the empirical argument: "On my suggestion this is to be interpreted in the terms of the Phaedo as a refutation of the hypothesis by the demonstration of an inconsistency in its consequences" (p18). This does not seem right to me because: a) the empricial argument does not show inconsistencies in the consequences of the hypothesis (virtue is knowledge) but proves as false the original problem of the teachability of virtue. The whole argument has nothing to do with the hypothesis. The hypothesis is never exploited during the course of the argument. b) According to the Phaedo, firstly, we state a hypothesis capable of solving our problem, secondly, we test it through its consequences and, thirdly, we reduce it to a higher one. But here (if we accept that 'virtue is good' is a higher hypothesis) according to Taylor, firstly, we reduce our hypothesis to the higher one and after that we test the (first) hypothesis.

[26] cf Zyskind and Sternfeld (1978,67), Stahl (1971, 189-91). Kahn (1981, p318-9) thinks that there is an application of the hypothetical method in the final argument of Protagoras. Scolnicov (1973, 58-56) thinks that the application of the method starts with Meno's paradox. However his argument does not seem to me convincing.

[27] For the hypothetical method in the Meno as the geometrical method of analysis, see Natorp (1903/1929, 39), Klein (1965,207), Farquarson (1923, 21-6), Heijboer (1955, 106), Gulley (1958,7), Gulley (1962, 12.15), Stahl (1971, 193-7) and C Taylor (unpubl. paper, 9-19).

[28] cf Robinson (1953, 121) and Cambiano (1971,137).

[29] About the hypothesis as a case of *diorismos*, see Heath (1921, I,303), Frajese (1963, 112-3), Mahoney (1968-9, 324), A E Taylor (1937, 211-9), Cambiano (1967, 138), Maccioni (1978, 164), Sterfeld and Zyskind (1978, 54) Cherniss (1951, 419) and Bedu-Addo (1984,6).

[30] See p.52. Gulley (1958) accepts that the hypothetical method is the analysis and the 'hypothesis' is a *diorismos*.

[31] Proclus, *In Eucl.*, 66-7.

[32] cf Bluck (1961, 81, n2). In Euclid, the *diorismos* is stated in the enuciation of the problem.

[33] The word *apagoge* must have been inroduced very early in the language of Greek mathematics. Cf the expressions "εἰς ἄτοπον ἀπαγωγή"; "ἀπαγωγὴ εἰς αδύνατον" (*reductio ad absurdum*). We know, furthermore, that Zeno of Elea was using the method of *reductio ad absurdum*.

[34] The *arche* of Hippocrates and the 'hypothesis' of Plato in the *Meno* are not real principles in the ordinary sense of the axiomatized mathematics. They are usually ordinary propositions that need proof. They are only relative principles (cf Cambiano, 1971,130). In this sense, they are similar to lemmas, that is propositions used to establish others and which themselves need confirmation (see p.47-8).

[35] Bluck (1961, 82) accepts that the hypothetical method in the *Meno* is the *apagoge* but only because of the evidence of Aristotle. He cannot give a clear answer because he finds difficulties with the interpretation of the second 'hypothesis', 'virtue is good'.

Heijboer, Cambiano and Scolinov accept that the hypothetical method in the *Meno* is the method of Hippocrates. But Heijboer claims that Hippocrates was using the method of analysis. Cambiano gives a good interpretation of the method of Hippocrates, recognizes its analytical-reductive character, but does not correlate it with the *apagoge*. He says that "the most exact term for showing the hypothetical process described by Plato, is this of *anagoge*, reduction of a problem to another". But *anagoge* is the dictionary translation of the word reduction. The same Latin word *reductio* is used for the rendering of two Greek words with different senses - *anagoge* (to lead up) and *apagoge* (to lead away). Scolnicov accepts that the method is *apagoge*. However he draws no distinction between *apagoge* and analysis.

CHAPTER IV

PLATO'S MENO: 86E-87A

1. Discussion of the literature

> λέγω δὲ τὸ ἐξ ὑποθέσεως ὧδε, ὥσπερ οἱ γεωμέτραι πολλάκις σκοποῦνται, ἐπειδάν τις ἔρηται αὐτούς, οἷον περὶ χωρίου, εἰ οἷόν τε ἐς τόνδε τὸν κύκλον τόδε τὸ χωρίον τρίγωνον ἐνταθῆναι, εἴποι ἄν τις ὅτι "Οὔπω οἶδα εἰ ἔστιν τοῦτο τοιοῦτον, ἀλλ ὥσπερ μέν τινα ὑπόθεσιν προὔργου οἶμαι ἔχειν πρὸς τὸ πρᾶγμα τοιάνδε· εἰ μέν ἐστιν τοῦτο τὸ χωρίον τοιοῦτον οἷον παρὰ τὴν δοθεῖσαν αὐτοῦ γραμμὴν παρατείναντα ἐλλείπειν τοιούτῳ χωρίῳ οἷον ἂν αὐτὸ τὸ παρατεταμένον ᾖ, ἄλλο τι συμβαίνειν μοι δοκεῖ, καὶ ἄλλο αὖ, εἰ ἀδύνατόν ἐστιν ταῦτα παθεῖν. ὑποθέμενος οὖν ἐθέλω εἰπεῖν σοι τὸ συμβαῖνον περὶ τῆς ἐντάσεως αὐτοῦ εἰς τὸν κύκλον, εἴτε ἀδύνατον εἴτε μή".

This mathematical passage presents many difficulties with respect to its interpretation, geometrical reconstruction, and its comparison with Plato's application of the hypothetical method later on in the dialogue.

Numerous interpreters have given different solutions. In 1832 Patze counted twenty two theories [1]. With Blass, in 1861, the number had risen to thirty [2] and Gueroult (1933/1969, 129) in 1933 counted more than a hundred; since then many more have appeared.

The more interesting explanations are those of H Butcher (which J Cook Wilson improved fifteen years later), of A Benecke, of L Farquharson, of A Heijboer, and of K Gaiser [3].

Sir T Heath, independently of Cook Wilson, gives the same solution in his book <u>A History of Greek Mathematics</u> (1921, I,298-303). R Sternfeld and H Zyskind (1977, 206-11) give essentially the same solution as Heijboer, but for the special case in which the side of the rectangle which is a chord of the circle, is equal to the side of the equilateral triangle inscribed in the circle. E. Σταμάτης (1951, 218-27) has given an interesting solution which however is too advanced to be correct. It is based on the theorems of Euclid's Elements VI 27,28, and of Apollonius' Conics 1,15, and it is an undoubted fact that when the Meno was written the two latter theorems at least were not known [4].

I begin my examination of the problem by stating some conditions that a correct reconstruction of it must fulfil: 1) the solution of the problem must be in harmony with a grammatically correct interpretation of the passage; 2) the problem must be solved with the mathematics of that time; 3) the geometrical problem must be related to the problems which occupied the geometers of that time; 4) the method of solving the problem must be the same as the method which Plato applies for the proof of the proposition 'virtue is teachable'.

First some remarks on the passage before I give my translation and interpretation.

A. Certain scholars [5] interpret "τόδε τὸ χωρίον τρίγωνον" as "this triangle here". but as Heijboer rightly observes (1955, 90), the accusative 'τρίγωνον' is taken predicatively and functions as an adjective. Moreover such an interpretation would make the geometrical problem and the 'hypothesis' superfluous (cf Bluck, 1961, 443; Thompson, 1937, 149).

B. Benecke [6] takes chorion to mean the square ABCD of side two feet which had already been drawn by Socrates (82c sq) in the problem of the duplication of the square. The circle is that which

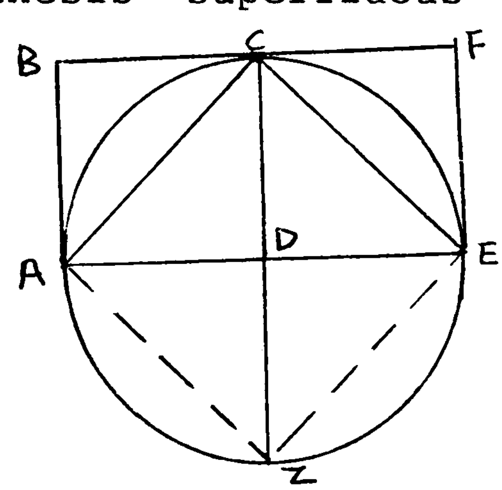


fig. 1

is described around the double square ACEZ (fig.1), and the required triangle is the ACE. However, in this case, the problem does not have a general character [7], while "οἷον περὶ χωρίου" shows that the 'given area' is not a particular figure as Benecke argues. Moreover, in this way, the problem always has a solution and the "ὑποθεμενος οὖν ἐθέλω εἰπεῖν σοι τὸ συμβαῖνον περὶ τῆς ἐντάσεως αὐτοῦ εἰς τὸν κύκλον εἴτε ἀδύνατον εἴτε μή" has no meaning. The 'hypothesis' of Plato is superfluous.

C. Butcher (1888, 219-25) [8] takes chorion to mean a 'given rectangle'. Now if this 'given rectangle' fulfils the condition (hypothesis), it can be inscribed into the given circle. However, as Butcher himself observes, according to his solution, the condition is sufficient but not necessary, because it is possible for a rectangle to be inscribed as a triangle without this condition being fulfilled. Cook Wilson (1903, 222-9) and Heath (1921, I,300) tried to improve Butcher's solution in this direction.

They claim that "οἷον παρὰ τὴν δοθεῖσαν αὐτοῦ γραμμὴν" does not refer to the original area (X) which is arbitrary but to another (see fig.2), a parallelogram, of equal area to the first, exactly as in Euclid VI 28, the applied figure may be any kind of parallelogram (cf Archimedes De sph. et cyl., I,16, and Pappus, Collectio vi ed. Hultch, 544.8-10, who use the word chorion in the sense of a parallelogram). It seems then that when geometers say chorion, they take it from the point of view of its magnitude and not its shape, and they consider that it can always be transformed into a parallelogram [9].

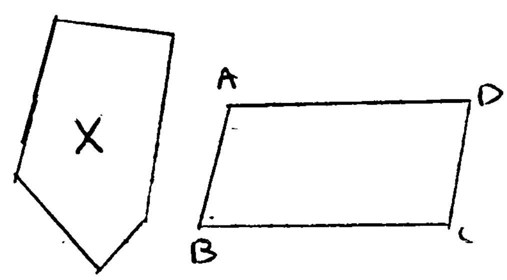


fig.2

D. Many scholars (Benecke, Tannery, Butcher, Cook Wilson, Heath) translate "τὴν δοθεῖσαν αὐτοῦ γραμμὴν" as "the diameter of the circle"; but in the text the <u>gramme</u> is clearly related to <u>chorion</u> ("εἰ μέν ἐστιν τοῦτο τὸ χωρίον τοιοῦτον, οἷον παρὰ τὴν δοθεῖσαν αὐτοῦ γραμμὴν παρατείναντα"). A circle's <u>gramme</u> is usually its circumference [10] and not its diameter (cf Eucl. I, def.15: "κύκλος ἐστὶ σχῆμα ὑπο μιᾶς γραμμῆς περιεχόμενον"). Moreover since the circle is given, the word δοθεῖσαν is superfluous if the <u>gramme</u> denotes the diameter. Surely "παρὰ τὴν αὐτοῦ γραμμὴν should have been enough.

E. Butcher, Cook-Wilson and Heath take the "τοιούτῳ ... οἷον" to mean "similar". However, if it means similar, then, the word αὐτό (that is 'itself') is superfluous. "Of a figure similar to another figure we might vaguely say that it is like the other, but we could not possibly say that it is <u>like the other figure itself</u>" [11]. The word αὐτὸ suggests rather equality than similarity.

Cook Wilson's and Heath's solution has two more disadvantages (see fig.3: BC = diameter, rect. ABCD = triangle BDH). Firstly, to find a corner point D on the circumference, such that the rectangle ABCD be equal in magnitude to the given area X, presupposes, as Heath observes (1921,I,300-1)

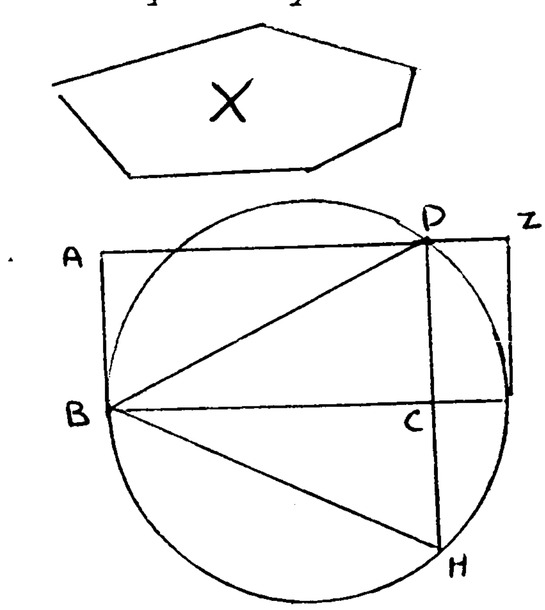


fig.3

(1921, I,300-1), the use of conic sections; therefore, the problem is unsolvable using the mathematics of the time of the <u>Meno</u>. The problem must be comprehensible to Meno and not unsolvable. Moreover, the condition laid down should be such as to enable the geometer to ascertain whether the problem has a solution (ὑποθέμενος ἐθέλω εἰπεῖν σοι .....) [12]. If we accept Heath's interpretation, then, the geometer is not able to tell us about the possibility of the inscription.

Secondly, the above solution gives only isosceles triangles although Plato speaks generally about 'triangle'. The terminology describing various kinds of triangles was already established at Plato's time (see Euthp 12d, Tim 53c-55c). Therefore, if Plato had meant an isosceles triangle he would have said it.

F. According to Proclus (In Eucl 419-20) and to Euclid (I 44 and VI 28,29) the word 'ἐλλείπειν' is used when an area ABCD applied to a straight line AE, does not cover it entirely, but leaves a segment BE (fig.4). From Proclus' passage, it seems that the Pythagoreans were using the term in this sense. In our text it is plausible to suppose that the ἐλλείπειν has this technical sense, especially because it is used together with the other technical term 'παρατείνειν'. However in Heijboer's solution (fig.5 - AD=DZ and rect. ABCD = tr.IAB) the rectangle ABCD has been extended lengthwise along the base AB, while it falls short in height. Moreover 'ἐλλείπειν' is not in relation to a given straight line, as it should have been.

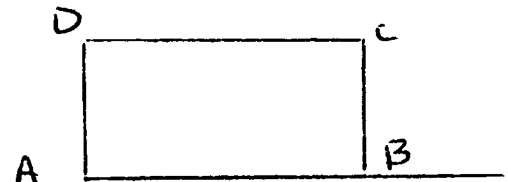


fig 4

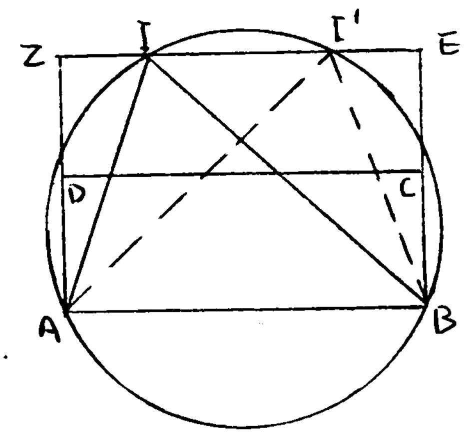


fig.5

G. Farquharson (1925, 21-6) proceeds analytically. Let AEC be a triangle equal in magnitude to the given area, and inscribed in the given circle (fig.6). Bisect any one of the sides - say AE at B and join B to the vertex C. Through A and E draw parallels to

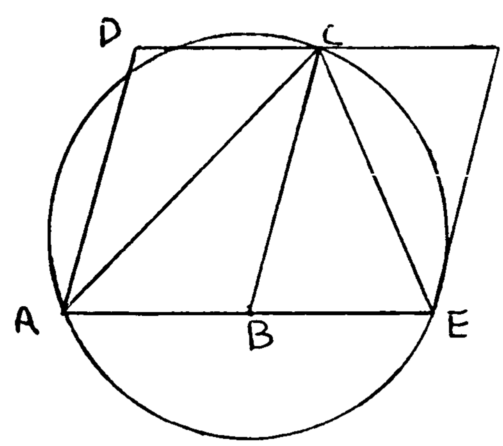


fig. 6

BC and through C a line DCF parallel to AE and meeting the parallels AD and EF at D and F. Each of the parallelograms ABCD and BEFC is equal in area to the triangle AEC (The sides AC and CE of the triangle are diagonals of the two parallelograms).

Again, we can solve the problem by finding a chord (say AE) of the circle such that a parallelogram equal to the given figure, described on the half chord, and placed so that one of its diagonals (say AC) is also a chord of the circle.

However Farquharson does not explain how the geometer will find the appropriate chord in any given case, or how he can determine whether or not the problem has a solution. Moreover, in Farquharson's solution, the word 'δοθεῖσαν' does not have the usual mathematical meaning of 'given'. Here although the gramme refers to chorion (or to a parallelogram equal), it is not something given but it is the required (cf Bluck, 1961,458).

Bluck thinks that we can overcome the difficulties of Farquharson's interpretation if we take 'its given line' to mean 'the line (chord) given for it', "assuming that we are given not only a particular χωρίον and a particular circle, but a particular chord in the circle as well; and the question is whether the χωρίον can be inscribed in that circle as a triangle with the given chord as base" (1961,459). However there is nothing in the text suggesting that we might have a given chord.

H. Gaiser (1964, 275-81) tries to improve the solutions of Farquharson and Heijboer. He thinks that Plato is interested rather in finding the diorismos of the problem than in finding the solution of it. The possibility of the inscription of a parallelogram triangularly in a circle depends on whether the given area (parallelogram) is not greater than the equilateral triangle inscribed in the circle. The procedure is the following:

We find a chord (AH) of the circle (fig.7) which is equal to the side of the equilateral triangle inscribed in the circle; We transform our original parallelogram ABCD (utilizing the theorem of <u>gnomon</u>, Eucl. I,43) to another (AEFG) whose side AE is half of the side (AH) of the equilateral triangle inscribed in the circle. If the side GF cuts or touches the circle, the inscription is possible and our solution is the triangle IAH. If the side GF falls out of the circle, then, the inscription is impossible (or in other words our area is too large to be inscribed in the circle).

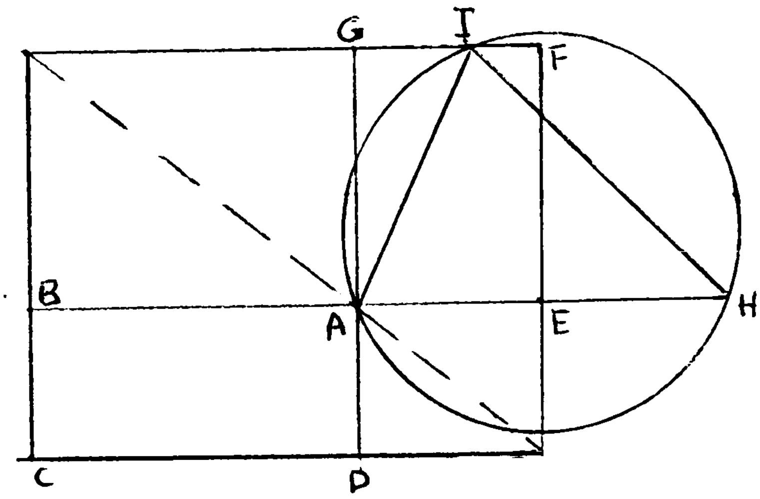


fig. 7

Gaiser's solution (and all others that utilize the side of the equilateral triangle inscribed in the circle) covers all cases and gives a real <u>diorismos</u> for the inscription. Every other solution cannot cover all cases because the equilateral triangle is the greatest of all inscribed triangles in a given circle. However, we do not know if this fact was known when the <u>Meno</u> was written. Gaiser himself (who proves it via the theorem Eucl. VI,27) is dubious about it. Moreover, there is no reference to an equilateral triangle in the text.

Gaiser suggests (281) that we can follow his solution without applying the parallelogram to the specific chord AH (side of the inscribed equilateral triangle) but to any chord of the circle. In this way we avoid the above objections, but our solution does not cover all the cases. However (in both versions of Gaiser's solution) the 'given line' is a chord of the circle and not a side of the parallelogram (given area).

2. <u>The proposed solutions</u>

Let me first give a translation of the passage taking into account the above.

> I mean by hypothesis, the way in which the geometers many times inquire into things. When they are asked, for example, about an area, whether it is possible for this area to be inscribed as a triangle in this circle here. One of them may say "I do not know yet whether this is so, but I think that I have a sort of hypothesis which is useful in this case, and it is the following: if this [rectilinear] area is such that when it is applied [in the circle, as a parallelogram] along its given line [ie one of its sides] it falls short by another area [parallelogram] equal to the applied one, then it seems to me that one thing follows, but if it is impossible to be so, then something different follows. Therefore, using a hypothesis, I shall be able to tell you about the inscription of the area in the circle, whether or not this is possible".

In the solution I propose, I shall try, in addition to other points, to follow the method that Plato applies in the problem of the teachability of virtue. Or in other words, the method I shall follow will be the geometrical method of <u>apagoge</u> that Hippocrates of Chios used to practise.

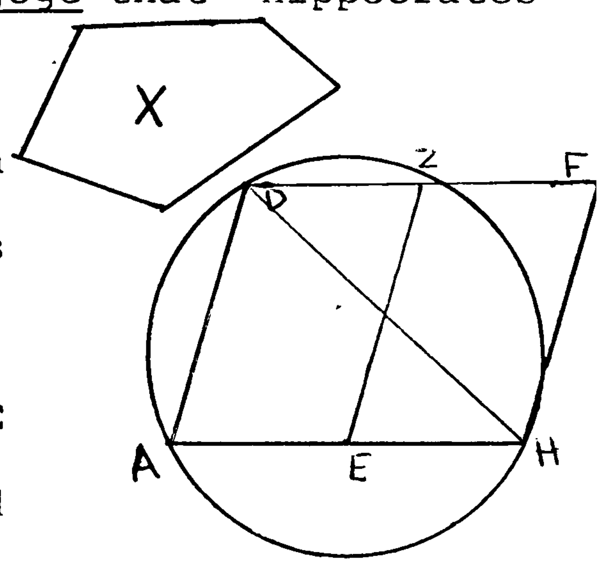


fig. 8

Let X be a rectilinear figure and O a circle. Let the given area X be inscribed as the triangle ADH in the circle O (figure 8). Let E be the midpoint of AH and through E and H draw parallels EZ and HF to AD, and through D draw the straight line DZF parallel to AH. Now two equal parallelograms AEZD and EHFZ have been formed. Each of these parallelograms is equal in area to the triangle DAH. Then the parallelogram AEZD is extended along its given line (side) AH, and it falls short by an area (parallelogram) EHFZ equal to extended AEZD. Therefore, if it is possible for our area X to be transformed to such a parallelogram AEZD, that if, when it applied (in the circle) along its given line AH, it falls short by a parallelogram EHFZ equal to AEZD, then the problem has a solution.

We have followed the method of apagoge. We have reduced our original problem P (the inscription of a given area X into a given circle O as a triangle) to another problem Q (the transformation of our area X into a parallelogram AEZD, such that if it is extended along its side AH where A, D, and H all lie on the circle, it falls short by a parallelogram EHFZ equal to it). If Q is solvable, then P is solvable.

Our condition (hypothesis) is exactly the same as that of Plato. But from "using a hypothesis I shall be able to tell you .... whether or not this is possible", it is clear that the geometer always has the possibility of finding out whether this condition is really valid of not. So the solution cannot stop here. We must see whether the given area X can be transformed into such a parallelogram AEZD which fulfills the condition. So we shall try to solve this second problem Q (Hypothesis), exactly as Plato does at 87d-89c, where he proves the proposition (hypothesis) 'virtue is knowledge'.

<u>I The first Solution</u>

Let O be the given circle and X the rectilinear area. We can always regard the area X as being a parallelogram (or as being transformable into a parallelogram ABCD - Eucl. I,45). We put the parallelogram ABCD on the circle such that vertex A is on the circumference. Draw the radius OA and with this as diameter draw a circle which meets the side BC at the point E. Draw AE which meets the circumference at H. Through D draw DZ parallel to AE which meets BC at Z. The parallelogram AEZD is equal in area to ABCD.

Moreover, the angle OEA is a right angle, since OA is the diameter so OE is perpendicular to the chord AH, and bisects AH. Thus AE=EH, and parallelogram AEZD = parallelogram EHFZ.

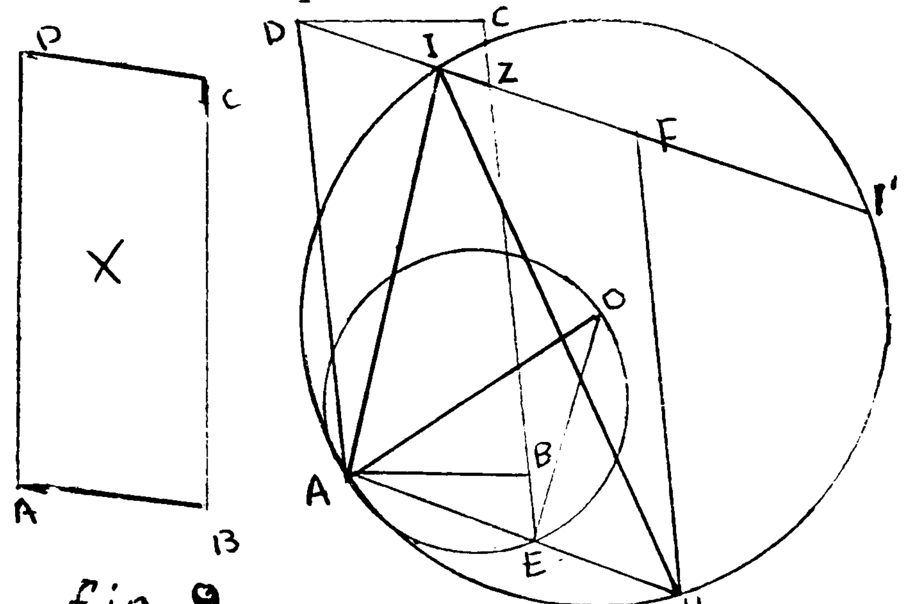


fig. 9

Rectangle ABCD is transformed into the parallelogram AEZD, equal in area, "such that when it is applied [in the circle] along its given side [AH], it falls short by another parallelogram [EHFZ] equal to the applied one [AEZD]". So the required triangle is IAH, equal to DAH.

II The Second Solution

Let (figure 10) X (= prallelogram ABCD) be the rectilinear area and O the circle. Place the ABCD in the circle having vertex A on the circumference.

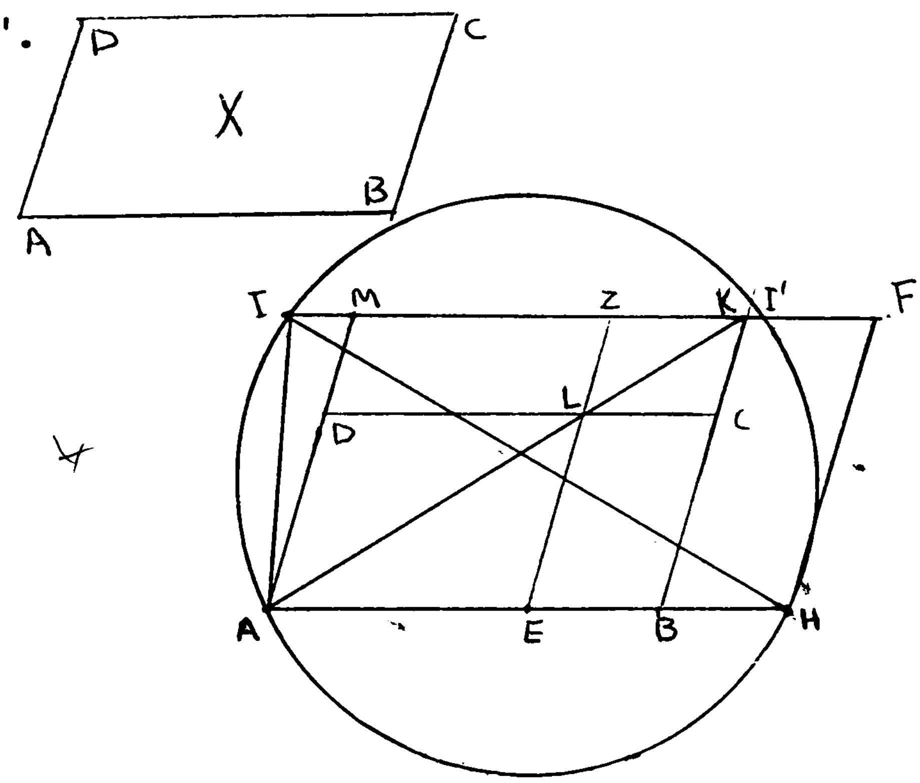


fig. 10

The side AB meets the circumference at H. Let E be the midpoint of the chord AH. Draw EL parallel to AD which meets CD at L. Draw AL which intersects BC at K. Through K draw a parallel to AH which meets AD at M and EL at Z. The parallelogram AEZM is equal in area to the parallelogram ABCD because parallelogram DLZM is equal to parallelogram EBCL (Eucl.1,43).

Because AE=EH, the parallelogram ABCD is transformed to a parallelogram AEZM of equal area which is "such that when it is applied [in the circle] along its given line [AH], it falls short on this line by another rectilinear are [parallelogram EHFZ] equal to the extended one [AEZM]". The straight line MF intersects the circumference at points I and I'. The triangles IAH and I'AH are the two solutions of the problem, symmetrical with respect to the diameter EO. The theorem (Eucl.1,43) on which this solution is based is a fundamental theorem of the early Pythagorean mathematics which resulted from the study of 'gnomon'.

The same construction can be achieved in a different way, applying theorem VI,12 of Euclid (construction of the fourth proportional) in combination with V,18 (a/b=c/d=a+b/a=c+d/c). See figure 11 where the parallelogram ABCD (the given area) is transformed to the parallelogram AEZM and after that to the triangle IAH.

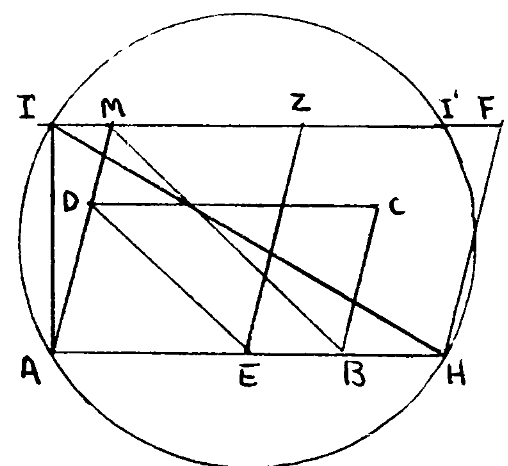


fig. 11

III The Third Solution

Let (figure 12) ABCD be our rectilinear area (in the form of a parallelogram) and O the circle. Extend the side AB to E such that AB=BE. With centre a random point A' on the circumference, draw a circle having radius AE that meets circle O at the points E' and Z' (A'E'=AE).

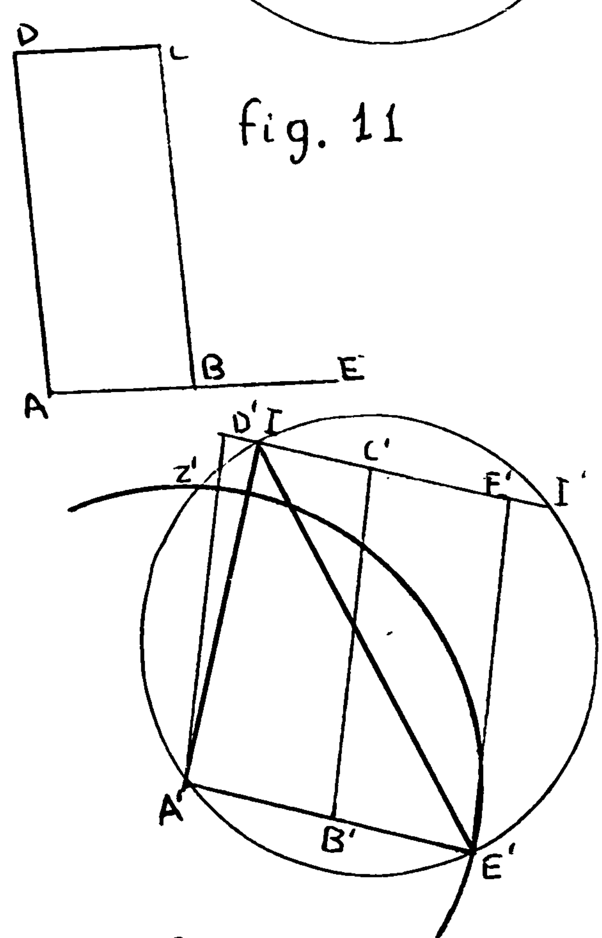


fig. 12

B' is the midpoint of A'E'. On the line A'B' construct a parallelogram A'B'C'D' congruent to ABCD (A'B'=AB). This parallelogram is extended along its side A'E' and falls short by another parallelogram (B'E'F'C') equal to it. The line D'C' meets the circle at the points I and I'. Triangle IA'E' is the required triangle.

The proposed solutions here have a lot of advantages in comparison to the preceding ones. 1) The 'area' is taken simply as a random rectilinear area (or parallelogram). 2) "Its given line" is considered as a side of the parallelogram. 3) The toioyto hoion is considered as equality and not as similarity. 4) elleipein and parateinanta refers to the same side of the parallelogram. 5) The interpretation of elleipein fits with the interpretation which Proclus gives and with the way Euclid uses it. 6) The triangle is a random one and not necessarily an isosceles or equilateral one. 7) The problem is resolved using only a rule and compass, and mathematics known at the time at which the dialogue was written. 8) The proposed solutions follow the method which Plato uses to solve the problem of the teachability of virtue.

The first solution seems to be more general than the other two in the sense that it is totally independent of the shape and the specific position of the parallelogram ABCD in the circle. It suffices that only one vertex of ABCD lies on the circumference.

The possibilities, that the second solution covers, depend on the chord AH on which the side AB of the parallelogram ABCD is placed. The possibilities that the third solution cover depend on the specific shape of the parallelogram ABCD. Another possible objection that applies to my first and second solutions (as well as to the solution of Gaiser) is that the phrase "when to extend it along one of its sides it falls short ..." refers to side AE of the transformed parallelogram AEZM although it probably should have referred to side AB of the original parallelogram ABCD. The third solution avoids this objection. Another merit of the third solution is its great simplicity.

NOTES TO CHAPTER IV

[1] See Heijboer,1955,89.

[2] See Heath, 1921,I,298.

[3] Butcher, 1888, 219-25; Cook-Wilson, 1903, 222-9; Benecke, 1867 (see Bluck, 1961, 447 sq); Farquharson, 1925,21-6; Heijboer, 1955, 89-122; Gaiser, 1964, 264-282.

[4] For a presentation of the various solutions to the geometrical problem, see Maccioni, 1978, 173-188, and Sharples, 1985, 158-60. For a presentation and critical discussion of them see Bluck, 1961, Appendix.

[5] P Kucharski, 1938 and Apelt,1922 (see Bluck, 1961,443 n.1)

Frajese (1963,112) claims that Meno 86e-87a, refers not to one, but to two different geometrical problems. The first is the inscription of a triangle (having given area) in a given circle. Plato proposes this problem to be solved and together with it he points out the method that one has to follow in order to solve it. The method is the one used in problems of elliptical applications of areas. Frajese justifies his claim only by referring to the many unsuccessful attempts by numerous scholars to give a satisfactory interpretation. However, the most obvious thing in the passage is that the question is about one geometrical problem.

[6] Others who accept Benecke's solution are: Gow, 1884,175; J Thomas, 1980,168; P Tannery and M Cantor (see Thompson, 1937,232).

M Timpanaro-Cardini (1951, 402-9), following Frajese, accepts that there are two geometrical problems and not one; but she objects to his view that the questioner of the geometer wants to learn whether or not the problem has a solution and not only the possible method of its solution. She thinks therefore that the first problem (inscription of a triangle in a given circle) is solvable and she accepts Benecke's solution.

[7] Brumbaugh (1954, 33-5) accepts Benecke's solution, but he thinks that Socrates has in mind a more general theorem, the converse of Eucl. III,35. However, he presents to Meno the simplest version of this theorem (when the two chords AE and CZ are perpendicular diameters), because Meno could not understand something more difficult.

Similarly, Benecke supports his very simple reconstruction of the problem, saying that Meno could not understand something more complicated. However this claim does not hold good. Meno is presented in the dialogue as a well educated young man, pupil of Gorgias. Meno seems to enjoy the mathematical discussion between Socrates and the slave-boy, and he seems well acquainted with the problem of incommensurability between a side and the diagonal of a square (see also Klein, 1965, 64-6,99,206).

Gueroult (1935/1969, 130-7) tries to generalize Benecke's solution, thinking that the problem is analogous to Eucl. VI,28,29. However he fails to give a general solution according to Eucl. VI,28. His only conclusion is that the side of the square, which we must to inscribe triangularly in the circle, has to be less than or equal to the radius of the circle, in order to have solution. But this is mistaken because we can inscribe in a circle triangles having area greater than that of a square with side the radius of the circle.

[8] Stock (1904,39-42) and Thompson (1937,148) accept Butcher's solution.

[9] As Heijboer (1955,99) points out "the word χωρίον, unlike

the word σχῆμα ... suggests the idea of area; any area may be thought as a product, and the natural geometrical picture of a product is a rectangle".

[10] Although Thompson (1937,150) accepts Butcher's solution, he admits that the gramme must refer to the area and not to the circle. Gaiser (1964,274) argues that the phrase "its given line" could not possibly refer to the diameter and suggests (280) either reading αὖ τού<του> or else interpreting αὐτοῦ not as 'of it' or 'for it', but as the adverb 'here'.

[11] Heijboer, 1955,120; cf also Bluck, 1961,450. See contra Klein, 1965, 208-9. The word hoion occurs four times in the passage 86e5-87a5, each time with a different meaning.

[12] Cf Bluck, 1961,449. Others who accept that the problem has no solution are: Frajese, 1963,112, and Stock, 1904,22.

[13] This solution is very similar to that of Gaiser in the case in which he aplies the parallelogram to a random chord of the circle and not to one which is equal to the side of the equilateral triangle inscribed in the circle. I discovered Gaiser's paper long time after I had formulated this solution. However, I think that this solution is the less satisfactory of the three I propose.

C H A P T E R V

PLATO'S HYPOTHETICAL METHOD IN THE "PHAEDO"

1. The Strongest 'logos'

Plato's account of the hypothetical method is given in two parts: 100a and 101d-e. The two parts are separated by an interval in which Plato introduces and discusses his 'hypothesis' that the Forms are 'causes' of characteristics in particular things. Moreover, we can divide passage 101d-e into two parts, namely 101d1-5 and 101d5-e1, because they present different steps in the course of the method.

The main problem in passages 100a and 101d1-5 is the metaphor of 'accord' and 'disaccord' that occurs twice, once in each of them. I am going to examine these two passages together in this section.

100a: "But anyhow, this was how I proceeded [ὥρμησα]: hypothesing on each occasion the theory [or proposition - λόγος] I judge strongest, I put down as true whatever things seem to me to accord with it, both about a reason and about everything else [Hackforth: whether the question is about causes or anything else]; and whatever do not, I put down as not true."

101d1-5: "But if anyone hung on to the hypothesis itself, you would dismiss him, and you wouldn't answer till you should examine its consequences [ὁρμηθέντα], to see if, in your view, they are in accord or discord with each other [1]."

The indisputable fact is that in passages 100a and 101d-e, Plato presents his methodological remarks as relevant to the philosophical investigation. Whether those remarks constitute a single and complete method of inquiry or not, is another problem that I intend to confront later (section 3).

The first problem that arises, is what is meant at 100a4 by "hypothesising in each occasion the logos I judge strongest"? The word logos has been rendered by commentators in various ways ('proposition', 'definition', 'judgement', 'account', 'theory' etc) [2].

It seems to me that the rendering of logos as definition is too restrictive. For - as Gallop remarks (1975, 178-9) - "the meaning of the 'theory I judge strongest' must be gathered, partly at least, from Socrates' illustrations at 100b-101c". The examples of the logos or 'hypothesis' that Plato gives in the above passage preclude logos at 100a4 from meaning only definition. I find it more plausible then to take logos to mean more generally 'proposition' or 'statement', following the more accepted way of interpretation [3]. This does not mean that the logos or hypothesis might not sometimes be a definition, or a more complex proposition including a definition.

At 100a6, Plato says that his method is the same "whether the question is about causes or anything else". That means that the method is not designed to apply only to the search for 'causes', but that it is a more general method that can be applied in any field of philosophical inquiry [4].

After the exposition of the method at 100a3-7, Plato proceeds to give a better elucidation of what he had in mind ("but I'd like to explain my meaning more clearly, because I do not imagine you understand it as yet"; 100a7-8). At 100b1, he says that what he is going to tell us "is not something new"; and in 100b4-7, we see that what is not new is the theory of Forms.

We might suppose that what Plato is going to explain (100a7-8) is what "is not new" (100b1) namely the theory of Forms (100b4-7). In this way, the "strongest logos" (100a4) may refer to the theory of Forms and not generally to the method. We might then conclude that the method is not a general one, but only limited to the search for the 'causes'. However, this contradicts Plato's statement at 100a6 ("whether the question is about causes or anything else").

We have then to suppose that, when Plato says "I'd like to explain my meaning more clearly", he means that he is going to explain his theoretical account of the method (and mainly the choice of the 'strongest hypothesis') better by applying it to the problem of finding the 'causes' of coming to be and destruction of 'things' (99d, 95e9-96a2, 96a9-10). Under this interpretation, although the method is a general one, the "it is not something new" refers to the theory of Forms which is the 'strongest hypothesis' in the elucidation of the method that Plato gives at 100b [5].

The elucidation that Plato offers consists of (i) "hypothesizing that a beautiful itself by itself, is something and so are a good, and a large and all the rest" (b5-7), and (ii) that the particular things are good, large, etc because they participate in the corresponding Forms (100c4-6, 100d-e). We must notice that at 100b5, Plato uses the word hypothemenos exactly as at 100a3 where he introduces the method.

Moreover, he uses the verb synchorein (100b8) so asking the consent of his interlocutor to introduce the hypothesis (see p10). It seems certain then that Plato views the existence of the Forms as a hypothesis. The problem now is whether Plato's hypothesis is only the existence of the Forms or more generally the whole theory of 'Forms', (existence of Forms, characteristics of Forms, participation of particulars in Forms).

It is true that when Plato introduces the theory of participation of particular things in the Forms, he does not use the word *hypothemenos*. Moreover, "τὰ ἑξῆς" at 100c3 (cf "ἐκ τούτων" at 100b8) might indicate that the theory of participation is a consequence of the existence of Forms. Based on this evidence some commentators argue that the hypothesis is only the existence of the Forms, and not the theory of participation. The theory of participation is a consequence of the existence of the Forms [6].

But let us look at the problem more closely. At 100b3, Plato says: "I am going to set about displaying [*epideixasthai*] to you the kind of reason [cause] I have been dealing with", and at 100b7-9: "if you grant me that and agree that those things exist I hope that from them I shall display [*epideixein*] to you the reason [cause] and find out [*aneuresein*] that soul is immortal".

From the second passage we may infer that there are three steps in Plato's reasoning: a) the acceptance of the hypothesis (Forms exist); b) the explanation of how Forms are 'causes' and c) the immortality of the soul.

We can easily see that Plato (102sq) proves the immortality of the soul (c) based on the whole theory of 'Forms-causes ((a)+(b)). The problem is whether or not the theory of participation, (b), is a consequence of the existence of Forms, (a).

However, it seems that this is not the case. We cannot find an inference from (a) to (b) in 100b-101c and *ta exes* (and not *symbainonta*) at 100c3 may imply consistency and not deducibility. We have also to observe that (in the above passage 100b7-9), in the transition form (a) to (b), Plato uses the verb *epideixein* (display) while in the transition from (b) to (c) (or from (a)+(b) to (c)), he uses the verb aneuresein (find out). The use of the verb epideixein may indicate that Plato does not regard the transition from (a) to (b) as a logical inference. Moreover, at 100c7, after Plato introduced the theory of participation, he uses the verb synchorein asking the consent of the interlocutor: Plato could never use this verb if (b) was an inference from (a).

I think we have to suppose that the hypothesis Plato states is the conjunction of (a) and (b). This is confirmed by passage 102a10-b2 ("As I recall, when these points had been granted [synechorethe] him, and it was agreed [homologeito] that each of the forms was something, and that the other things, partaking in them took the name of the forms themselves") [7]. Moreover, the word-similarity between passages 100d8-e3 and 101d1-2 (cf 100d8-e3: ἀσφαλέστατον, ἐχόμενος, ἀσφαλὲς; 101d1-2: "ἐχόμενος ἐκείνου τοῦ ἀσφαλοῦς τῆς ὑποθέσεως") indicates that the hypothesis Plato has in mind at 101d is rather the theory of 'Forms-Causes' [8].

Let us now see if - assuming that the hypothesis is the conjunction of (a) and (b) - passage 100b-101c gives an explanation of the method described at 100a. Plato says at 100b that beginning from the existence of the Forms [9] - something already assumed at 65d-66a, 72e-76c and 78c-80b - he is going to display the 'cause', or in other words, to formulate a further point of the main hypothesis about Forms; and on the basis of this hypothesis he is going to find out (or to prove) that the soul is immortal. I think that in passage 100b, we mainly find the explanation of 100a. After that Plato proceeds to the demonstration of the soul's immortality according to the procedure stated at 100a. At 100c3-7, he formulates his hypothesis about participation and 'causation'; at 100c9-e3, he shows that his hypothesis is 'safe' (cf 100a4: <u>erromenestaton</u>); at 100e8-101c9, he compares his hypothesis with that of the natural philosophers (already rejected at 96-97); and at 102b-106c, he states his principle of the 'cleverer causes' and proves the immortality of the soul.

According to Bluck (1957, 21-3), one of the main defects of the propositional interpretation of the method - that takes the two passages 100a and 101d to be concerned with one and the same method - lies in that fact that it cannot explain why Plato should intersperse his account of the method with an illustrative example [10]. Nevertheless, there is no difficulty in explaining the dispersion of Plato's account of the method. Plato's main problem at 95e-99e is the search of the 'cause' of generation, existence and destruction (the proof of the soul's immortality depends on it). The passage at 100b illustrates 100a, and passage 100c-101c formulates the hypothesis about 'causes' and shows that it is 'safe' (or strongest). Now Plato is finished with the general problem of causation. He makes some other remarks about his philosophical method and he proceeds to the proof of the soul's immortality.

At 100a, Plato says that the method consists of hypothesizing the theory (proposition) judged strongest and putting down as true whatever things seem to accord with it, and as not true whatever do not. Two main problems now arise: a) how do we judge the strongest hypothesis? and b) what is the meaning of the word 'accord' and hence what is the procedure described at 100a?

I have already accepted i) that the method has to do with propositional reasoning and that Plato intends to prove the soul's immortality applying his methodology (cf100b) and ii) that passsage 100c-e may be regarded as illustrating the 'theory judged strongest'.

As regards now the first question, we can easily see from 95e-96a that the immortality of the soul depends on the problem of 'causation'. At 100b, Socrates says that he is going first to exhibit the 'causes' and after that to give a proof of the soul's immortality. Later he uses the hypothesis of 'Forms-Causes' to prove the immortality of the soul. Therefore, the hypothesis is posited in order to prove a proposition [11]. However, it seems that Plato regards that there are many such propositions from which we must choose the strongest (100a4). The problem is, how can we judge which is the strongest hypothesis? We may plausibly regard passage 100c-e as an answer to this question. The proposition which is hypothesised at 100c is the theory of participation. At 100c9-101e3, Plato tries to show - through the example of 'beautiful' (and similarly through 'large' and 'small' at 100e5-6) that his hypothesis is safe, that is to say, that his hypothesis about 'causation' can be applied safely to everything [12], and it is the strongest because it is the least refutable [13].

We see then that here, Plato's hypothesis is a general account (or principle) about causation. Plato states it and after that corroborates it via a series of examples. This seems to be the Socratic epagoge (induction): the generalisation through some examples and the assertion of a general account or principle (cf Aristotle Topics I,12). Plato applies it sometimes in the Phaedo, ie at 70d7-71b10 where he first states the principle of opposites and later on he gives a series of examples, of which the above principle is a generalisation.

Therefore, in the case in which our hypothesis is a general theory, it must also conform to the particular cases of it. Of course the conformity of the hypothesis to the particular cases does not constitute a proof of the hypothesis. The hypothesis so chosen remains tentative [14].

Let us now examine the second question, concerning the procedure described at 100a. Some think that passage 100b-101c may best be regarded as illustrating not only 'the theory judged strongest' but also the putting down as true whatever things seem to accord with it, and as not, whatever do not. In this case the hypothesis is always a general theory and something is in accord with the hypothesis if it is an application of it. Something disaccords with it, when it is an application of another hypothesis conflicting with the first hypothesis [15]. So "x is beautiful because it participates in the Form of Beauty" accords with the hypothesis and hence it is true, while "x is beautiful because of its colour or shape" disaccords with the hypothesis and hence is false. If we accept the above interpretation of passage 100a-101c, then we must also accept that the hypothesis is always a theory of explanation of particular facts. So the method has not wider application. But passages 100a6 and 100b show exactly the opposite. Moreover, according to the above interpretation passage 100c-101c cannot be regarded as an explanation of 100a. This is because the theory of Physicists (or the various instances of this hypothesis) is not rejected because it disaccords with the hypothesis (Forms-Causes), but because it leads to contradictions (cf 101b1-2: "and it is surely monstrous that anyone should be large by something small").

Therefore, I think that passage 100c-101c has only to do with the choice of the 'strongest hypothesis'. This does not mean that I reject the ossibility that passage 100a might also describe a procedure according to which applications of the - already accepted - strongest hypothesis are regarded as true, while instances of another hypothesis - conflicting with the strongest one - are regarded as false.

2. The Metaphor of accord

Let us now see how we can explain the meaning of the word 'accord' at 100a. As Robinson (1953, 126) remarks, the two most natural conjectures as to Plato's meaning of the terms symphonein and its contradictory are the following:

i) consistent with - inconsistent with
ii) deducible from, entailed by - not deducible from, not entailed by

In the first case, all propositions which are not inconsistent with the hypothesis must be true. This seems a rash and unwarranted thing to say. We are not justified in adopting as true every proposition that is not contradicted by our hypothesis. It is unlikely that this is what Plato means. Moreover, Plato deduces the immortality of the soul from the 'Forms-causes' hypothesis. He does not simply try to show that they are consistent with each other.

In the second case, we have to accept that every proposition that does not follow from our hypothesis is false. This result is equally (if not more) unwarranted [16]

Let us now apply the two above interpretations to the passage 101d1-5 ("until you had considered the [logical] consequences [horme thenta] of the hypothesis to see if they accord or disaccord with each other") [17].

According to the second interpretation (deducibility - non-deducibility), the meaning of the passage is that after drawing the logical consequences of the hypothesis we check whether these consequences are deducible or not from each other. However, although such a procedure may not be meaningless, it has no methodological value. We may then suppose that Plato means that we must check not whether the consequences of the hypothesis are deducible from each other, but whether or not they are <u>really</u> consequences of the hypothesis. But, again, it seems very implausible that Plato puts such an emphasis on the minor activity of checking one's logical calculations. Nowhere in the middle dialogues does Plato confront the possibility of wrong calculations from true hypotheses. The most important task in dialectic are examining and giving account of the hypotheses (cf <u>Crat.</u>, 436c8-a7). Moreover, such an interpretation does not allow us to find at 94a-b (where the 'hypothesis' that soul is harmony is refuted) an application of 101d1-5, as it seems to be.

If we apply the first interpretation (consistency - inconsistency) at 101d, another problem arises: how can the consequences of the hypothesis be inconsistent with each other? That seems to be paradoxical. But there are two ways of escaping from this paradox. i) The hypothesis may be a complex proposition and one of its parts is latently inconsistent with another. ii) The hypothesis may have "conflicting consequences when combined with some of our permanent beliefs" [18]. This happens at 94a-b, where the hypothesis that soul is harmony is refuted (it contradicts the standing belief that souls are not equally virtuous) [19]. It seems then that the rendering of the words symphonein - diaphonein as 'consistent with' - 'inconsistent with' offers a plausible interpretation of passage 101d1-5.

Ross (1951, 27) and A E Taylor (1937, 201) claim that 'accord' means logical consequence and 'disaccord' (or not accord) inconsistency. But this reading, although it gives a plausible interpretation of 100a, creates problems with the interpretation of 101d (if we accept that hormethenta at 101d4 means logical consequences - as Taylor and Ross deem). In the above case, we must translate passage 101d4-5 as follows: "you would refuse to answer until you had considered the (logical) consequences of the hypothesis to see if they follow from each other (accord) or if they are inconsistent (disaccord). (The word allelois, 101d5, refers both to 'accord' and 'disaccord'). However, as we have said, the rendering of 'accord' as 'follow from each other' seems implausible. Moreover, if we interpret 'accord' as 'follow from the hypothesis', then, in the case in which our hypothesis is a self-contradictory one or some of the consequences contradict with some of our standing beliefs, then it is possible both that the consequences are really consequences of the hypothesis and that they are inconsistent.

Hackforth (1955, 139-40), following almost the same interpretation as Ross and A E Taylor, takes passage 100a to mean that "any propostion, arrived at by what the inquirer deems a valid process of deduction, is accepted, and the contradictory of any such proposition is rejected". Hackforth believes that passage 101d does not describe something additional to 100a but gives only the detail of the process described. Nevertheless, his account of 101d does not conform well to his interpretation of 100a [20]. Hackforth takes the hormethenta to mean intermediate propositions (G, F, E, D, C, B) between the hypothesis (H) and the proposition to be proved (A); "that is why they are not called τὰ συμβάντα; there is only one συμβὰν, for there is only one proposition conceived as an ultimate result" (139-40). If all the intermediate propositions are valid inferences from the hypothesis, then they will be 'in accord' with each other (and they are considered as true). But if one of them (say F) is challenged and the challenge cannot be rebutted, then F and G are 'at variance'. "The one is not a valid inference from the other" (p140). However, Hackforth's interpretation of 'disaccord' at 101d5 is different from his interpretation of 'disaccord' at 100a; where something does not agree with the hypothesis when it is the contradictory of what is a valid inference from H. Moreover, according to Hackforth's interpretation, passage 101d3-5 gives only a check of one's logical calculations. But this is unlikely (see p128)

Sayre (1969, 12ff) advances two different interpretations. He argues that Plato remains deliberately vague. Thus we cannot answer the question of which of those interpretations Plato himself has in mind in writing the <u>Phaedo</u> 100a-101d. "I think the proper response to this question is to refuse its implication that only one interpretation is acceptable" (39).

According to his first interpretation, the propositions entailed by the hypothesis are accepted as true and the negations of those entailed are rejected as false. Passage 101d1-5 describes merely a test of the accuracy of one's deduction.

According to the second interpretation, "the relationship of agreement between two propositions is merely one of consistency and Socrates' immediate audience is relied upon to understand, as in the context of geometrical analysis with which they surely were familiar, that a proposition which is consistent with another entails that other proposition" (p 33). Sayre finds that both interpretations have many disadvantages (pp37-8) [21]. He says that the advantages of the first interpretation are the disadvantages of the second (p37). And it seems that he considers that the one is not incompatible with the other (p39)

However, there is a serious disadvantage with both interpretations. Sayre accepts (without giving any plausible justification) that the method Plato describes at 100a-101d refers only to convertible propositions (equivalences). In his second interpretation, he takes for granted (or as a fact), without any argumentation, that Greek geometrical analysis employs only convertible propositions, so having always a deductive character. Such a thing is very improbable (if not wrong) as we saw in chapter II (p39-40). Moreover, in the course of the proof of the soul's immortality (which Sayre considers as an application of the method), we certainly do not always have convertible propositions.

It seems then that we cannot have any plausible interpretation of the passage, if we interpret the word <u>symphonein</u> in terms of consistency or deducibility. The only way is to interpret passage 100a in terms of deducibility - inconsistency and passage 101d1-5 in terms of consistency - inconsistency. But as Robinson (1953, 130) remarks, "it seems very unlikely that in two passages so closely connected in time and significance Plato would mean different things by the metaphor" [22].

I think that in order to give a plausible interpretation of passages 100a and 101d, we have to look at the meaning of the words <u>symphonein</u>, <u>diaphonein</u>, and <u>hormethenta</u> in the Platonic corpus and to try to answer the question of why Plato uses the word _symphonein_ at 100a (_hormethenta_ at 101d) and not the word _symbainein_ (_symbainonta_ at 101d) which is the technical term that Plato uses for logical consequences.

Sayre (1969, 17) finds three meanings of the term 'accord' (_symphonein_). "Two assertions might agree in what they say by saying exactly the same thing. The formalization of this relationship would be one of logical equivalence. Two assertions, again, might agree in that one says only what the other says, although not all the other says. This suggest the relationship of entailment. And yet again, two assertions might agree merely in that neither disagrees with the other. The relationship here is consistency".

Robinson (1953, 129), examining the meaning of the words 'accord' and 'disaccord' elsewhere in the Platonic corpus, concludes that: "there is no passage in which they certainly indicate deducibility or the absence thereof; but there are several in which they certainly indicate consistency or inconsistency". But the terms 'accord' and 'disaccord' (generally speaking) are not contradictories but contraries. Two proposition may accord with each other, or disaccord, or neither accord nor disaccord (eg 'the bird sings' and 'children thrive on milk') [23]. In this last case, the two propositions, although they are consistent with each other, they do not accord. Hence the term 'consistent' is too broad to describe exactly the relationship of accord. On the other hand, the term _diaphonein_ (disaccord) has always had the meaning of inconsistency or conflict (cf _Gorg_, 482b6, 482b8, _Crat_, 394c7, _Pol_, 292b4 and _Laws_, 689a7, 691a5, 859a1, 860a5).

The accord between two propositions presupposes a kind of relationship between them (not necessarily deducibility). We can have such a relationship when we speak within a framework of a coherent system (deductive or not) or within a restrictive universe of discourse, the limits of which - in Plato's dialectic - are understandable from the context [24].

However, when we speak within the framework of such a system or restrictive universe, we can consider that two propositions (or two things) are either in accord or in disaccord with each other, so excluding the case of neither being in accord nor in disaccord. In such a case, we can consider the terms 'accord' and 'disaccord' as contradictories and not contraries and then 'disaccord' is synonymous with the absence of accord (not accord). In fact, Plato uses the words διαφωνεῖν, οὐ συμφωνεῖν, ἀσύμφωνον as synonymous [25]. At 92c - a passage very similar to 101d5 - Plato uses the words συνῳδῶ, οὐ συνῳδὸς to indicate consistency and inconsistency. The word συνᾴδειν is very similar in meaning to the word συμφωνεῖν. Both are methaphors from music (cf Rep, 402d9, Gorg, 482b8, and Prot, 333a). When in music we say that two sounds are consonant or dissonant, we always take for granted that they refer to the same melody. There is not therefore a third possibility. Similarly in 92c, the words συνῳδῶ, οὐ συνῳδὸς have the meaning of consistency - inconsistency, because they function in the framework of an argument.

I think that we can now draw some conclusions. a) When Plato uses the words 'accord' and 'not accord' (at 100a), and the words 'accord' and 'disaccord' at 101d, we have to take the terms 'not accord' and 'disaccord' as synonymous. b) The term 'disaccord' always shows inconsistency or conflict. c) The terms 'accord' and 'disaccord' are taken as contradictories and not as contraries.

d) Plato's method works within a restrictive universe of discourse, the limits of which are understandable from the context. e) However, we cannot render the term 'accord' as consistency in general. We can safely say that 'accord' means 'consistency' only when we first accept that we speak (or work) within a certain context (or framework) [26]. So the meaning of 'accord' is near to coherency. However , the word 'accord' may have an even more narrow sense, that of 'entailed by' (for example, in mathematics). Therefore, I think that we cannot express the meaning of the word 'accord' in terms of deducibility or consistency, but we have to fix - in each occasion - its meaning according to the context.

In Cratylus there is a passage (436c8-d7) which is very relevant to the interpretation of the hypothetical method in the Phaedo, that corroborates our above conclusions [27]. "If the giver (of the names) made a mistake in the first place and then distorted the rest to meet it, compelled them to accord (symphonein) with him, it would not be at all surprising, just as in diagrams sometimes, when a slight and inconspicuous mistake is made in the first place, all the huge mass of consequences (hepomena) agree (homologein) with each other. It is about the beginning (arches) of every matter that every man must make the big discussion and his big inquiry, to see whether it is rightly laid down or not (hypokeitai); and only when that has been adequately examined should he see whether the rest appear to follow from it" (Robinson, 1953,147).

This passage is primarily concerned with names and mainly with the connexion between the first names and the composite ones. However, the language and the meaning of the passage are similar to Phaedo 100a-101d. It seems natural to take symphonein (at 436d1) to mean 'be consistent with' rather than 'follow from'. But at 436d2-4, Plato clarifies his statement through a geometrical example. In this case, accord of the hypothesis (or hypotheses) with its consequences (hepomena) cannot mean anything else but deducibility. Plato's statement about the accord between the hypothesis and its results, is a general one that can be applied to every matter (d4-5). So in grammar, the first names may accord with the composite ones, in mathematics the principles accord with the theorems and in dialectic the hypotheses accord with their results. It seems again that the term 'accord' takes its exact meaning from the context in which it works. Moreover, the passage shows that when we work within a certain framework, the use of hypothesis implies a certain orderly sequence in the system. We cannot speak simply about consistency. I think that Plato (at Crat 436c-d) is deliberately vague in his terminology, because his account is applied to 'every matter' and not only to mathematics.

Probably the same happens with the Phaedo passage (100a). Although Plato has mainly in mind a deductive process (for the proof of a proposition; eg soul is immortal) he acknowledges that dialectic is not like mathematics. A usual procedure in dialectic is to establish premisses via induction (epagoge; see for instance: Meno, 87e-88c and Phaedo, 70e-71c) or to formulate a general theory of explanation (Phaedo 100b-100e). This procedure of establishing premisses, general laws or theories, is a vital part of dialectic. And although induction is not regarded by Plato as a method of proof, all premisses obtained in such a way (together with others) must be part of a coherent system of propositions (propositions relevant to the case of discussion). We can use instances of these general propositions in our proofs (cf <u>Phaedo</u>, 71d) or we can estimate the truth or falsity of instances of theories on the basis of our theory, already accepted as true (strongest) (100c-101c).

Things in dialectic are more complicated than in mathematics. Therefore, I think that Plato consciously uses the term <u>symphonein</u> at 100a (and at <u>Crat</u>, 436d) in order to cover all these possibilities. But this implies that the term Plato uses is not precise and we have to understand from the context its exact meaning.

Returning back to the method described at 100a, if we accept (as I think) that the main goal of positing a hypothesis is to prove another proposition (immortality of the soul, in this case), then the meaning of the word 'accord' at 100a is nearer to that of 'deduce'. But the term 'accord' indicates also the relation between the 'Forms-causes' hypothesis and its applications. In this last case, we can say that the rejected alternative (explanations via physical terms) are 'not in accord' with the hypothesis.

We can explain in a similar way Plato's use of the word <u>hormethenta</u> (and not <u>symbainonta</u>) at 101d4. It seems that <u>hormethenta</u> are not only the logical consequences in a narrow sense of propositional calculus [28], but also instances or application of the hypothesis (if the hypothesis is a general theory or a general statement regarding particular things) [29]. The consequences of the hypothesis may be not only the final proposition in an argument which is shown to be true or false, but also the intermediate steps. That is, all the consequences of the hypothesis [30]. We can also suppose that Plato's phrase "the consequences of the hypothesis" may very well include the hypothesis itself [31].

Plato seems to mean at 101d1-5 that if someone objects to the hypothesis [32], then we have to examine the consequences of it, to see if any contradiction arises among them. If contradictions arise, we must abandon the hypothesis; if they do not, the hypothesis holds well. It seems then (and this is the opinion of the great majority of commentators) that passage 101d1-5 constitutes a test of the hypothesis. The test consists of seeing whether or not the consequences of the hypothesis are consistent with each other. So here, the terms 'accord' and 'disaccord' mean 'consistent' and 'inconsistent'; but when we speak about consistency here, we do not mean consistency in general but only between propositions that are consequences of some initial principles (the hypothesis and some other premisses) [33].

The problem that now arises is how could consequences springing from a single hypothesis be 'in disaccord' with each other? We have already said that this may happen in two ways: a) when the hypothesis is a complex proposition and one of its parts is latently inconsistent with another. This inconsistency can be revealed through the consequences of the hypothesis. b) When the hypothesis (or some of its consequences) contradicts some of our permanent beliefs. In this case, the contradiction may be revealed among the consequences of the hypothesis when one (or some) of them is (are) combined with one of those beliefs that contradict the hypothesis.

We have said that Plato introduces his hypothesis within a set of known or accepted beliefs or propositions (say, P) and that these propositions are all regarded as true. We prove the proposition in question by taking as premisses the hypothesis and some of the propositions of the set P. The test of the hypothesis (described at 101d1-5) has meaning only if all the propositions of the set P are true. If one of them is false, we can have conflicting consequences from the hypothesis, although the hypothesis is true [34]. But it seems that Plato does not consider this case in the Phaedo.

Passages 100a and 100b, according to our interpretation, tell us that we can use a hypothesis in order to prove a proposition and passage 101d1-5, that we can test the falsity of the hypothesis through elenchus [35] (we cannot test the truth; even if the hypothesis resists the test, it is not regarded as true). But both of these procedures are not Plato's innovation and they do not constitute any new method [36]. We saw in the first chapter the various meanings of the word 'hypothesis' in Plato's dialogues. We also saw that the act of hypothesising a proposition is something common in Platonic (and pre-Platonic) dialectic, occuring in many dialogues, early, middle and late. The hypothesis is posited with the aim either a) to be proved (or rather to be disproved) or b) to prove (or disprove) something else. In the first case, the hypothesis is the proposition to be tested by elenchus and it is usually posited by the interlocutor of Socrates (cf Charm, 165a, 171d, 172 and Euthp, 11c). In the second case, the hypothesis is a proposition assumed by the two interlocutors, during the elenchtic procedure (in the Socratic dialectic of the early dialogues) as a premiss, the aim of which is testing (or disproving) the original proposition posited under elenchus (cf Charm, 160d, 169d, and Prot, 339d). So positing hypotheses is not something unfamiliar to Socratic _elenchus_. On the contrary, it is an inseparable part of the _elenchtic_ procedure.

In the first category of the above cases, we have exactly the same procedure as in the _Phaedo_ 101d1-5 [37]. The second case is similar to the procedure described at _Phaedo_ 100a with only one difference: whilst in early Socratic dialectic, we try to disprove a propostion, in the _Phaedo_, Plato posits the hypothesis in order to prove something. So Plato uses the hypothesis in a positive way and not in a negative way. But even that is not an innovation. Doctors [38] or natural philosophers posited hypotheses (principles) at the beginning of their systems from which they could derive other propositions or explain facts [39].

So the two passages, 100a and 101d1-5, do not really give a new method. Both procedures (and mainly that of 101d1-5) are found in the Socratic dialectic (the procedure of 100a is found also at _Meno_, 87d-89a). That may explain the apparent paradox (see Robinson, 1953, 139) of how Simmias understands (at 92c-e and 85c-d) the hypothetical method (that of 100a and 101d) before Socrates explains it to Cebes in 100a.

3. _Hypothesizing a higher hypothesis_

In 101d5-e1, Plato moves to a new part of the expostion of his method:

"and when you have to give an account [διδόναι λόγον] of the hypothesis itself, you would give it in the same way [ὡσαύτως], once again hypothesizing another hypothesis, whichever should seem best of those above [τῶν ἄνωθεν βελτίστη], till you come to something adequate [ἱκανὸν]".

The first problem that arises is what is meant here by 'giving an account' of the hypothesis? In Plato's dialectic, 'giving an account' of something is strictly connected with having knowledge of something [40]. We can give an account either of an object (a Form) or of a proposition.

The most natural way of interpreting the 'giving an account' of an object (a Form) (Phaedo, 76b4-9, 78d1-3) is to take it to mean 'giving a definition of it' [41]. However, this definition must be a correct definition of the Form. And we can show the correctness of this definition only by testing it by elenchus (or by defending it by rational argument) [42]. So, even in the case of the 'giving an account of a Form', we can have logical argumentation. As Gallop remarks (1975, 133), "giving an account in this sense and giving one in the definitional sense, are, of course, related, the former being required for the latter". Giving an account of a proposition meant usually giving and argument or a proof for it [43]. It is plausible then to suppose that 'giving an account of the hypothesis', at 101d6, means giving an argument or a proof of it [44]. The hypothesis at 100b, (theory of Forms) is posited in order to prove the immortality of the soul. Plato now, in his methodological remarks at 101d5-e1, says that the account we shall give of the hypothesis must be given "in the same way" by positing a 'higher' hypothesis. That means that we give an account of the hypothesis by deducing it from another 'higher' hypothesis [45]. This interpretation is reinforced by passage 95d7-e1 (see n. 43) from which we can infer that 'giving an account' of the soul's immortality means 'giving a proof' of it [46].

According to Robinson (1953, 136) we give an account of the hypothesis only when someone objects to it. The text however, does not give evidence in support of Robinson's claim. Although the check of the hypothesis (d3-5) is conneted with the objection of the interlocutor, it is not clear whether 'giving and account' of the hypothesis is part of what one has to do when 'someone grabs hold on it', or a necessity which confronts one on other occasions.

Dorter (1982, 132) claims that one has to 'give an account' of the hypothesis either a) when its consequences are inconsistent, or b) when "although consistent, but have failed to persuade because the premisses have been challenged". His second case is similar to Robinson's view, which I referred to above. The first case seems to me unlikely, because if the test by elenchus (d3-5) shows that the consequences are inconsistent, then we do not proceed to a higher hypothesis but we give up our original hypothesis and postulate another one, which is able to solve our original problem without leading to inconsistencies (cf the rejection of the 'hypothesis' of the physicists because it leads to contradictions).

The initial hypothesis must be derived from another hypothesis. The new hypothesis is to be the one that 'seems the best of those above' (τῶν ἄνωθεν βελτίστη). That is to say, Plato applies the process described at 100a to his initial hypothesis. But what does the 'above' mean here? At 101d5-e1, the hypothesis (H) is considered as a proposition which needs confirmation. To this end, we find another hypothesis H1 that is a premiss (probably the main one) for the proof of H. After that we find another hypothesis H2 which is a premiss for the proof of H1 and so forth until we arrive at something 'adequate' (say I). In this way, the procedure described at 101d5-e1 is a movement upwards from our problem (proposition to be proved) to its premisses. Therefore, every 'higher' hypothesis is a logically prior proposition.

According to what has been said so far, the 'higher' hypothesis is posited in order to confirm the original one. Therefore the 'higher' hypothesis is a logically prior hypothesis in the context of our argumentation. 'Above' then means simply 'that comes before' and the process described at 101d5-e1 is both a movements upwards (or backwards) to premisses. I find this interpretation the most plausible one [47].

However, some other interpretations have been proposed. Many readers think that 'higher' here means 'more general' [48]. So Archer-Hind (1883, 140) holds that 'the above' means more comprehensive hypotheses, further removed from the particulars and that 101d5-e1 describes a process "from particulars to species, from species to genus, from genus to a more comprehensive genus, and so ascend step by step until we arrive at one which will satisfy our needs". I find this interpretation unlikely because it is too restrictive and does not accord with Plato's practice of the method in the dialogue [49]. The rendering of 'above' as what 'comes before' is more vague and covers the relation of species-genus as well.

Bluck (1955, 15-17, 169-70) proposes a similar interpretation. 'Above' means again 'more comprehensive', 'more universal' and 'more remote from particulars', but the relation between a hypothesis and its 'higher' is not necessarily that of species-genus (1957, 26). According to him, the ascent to 'higher' hypothesis (which is the same as the dialectician's ascent in the <u>Republic</u>), shows a teleological hierarchy of Forms somehow culminating in the Good. His hypotheses are not propositions but 'notions' or 'mental images' and the passage 101d-e has nothing to do with logic, propositional reasoning or deductive arguments. But Bluck's views do not find explicit support from the text. His rendering of the word 'hypothesis' as 'notion' is very unusual and unjustifiable. How 'notions' have 'results' and 'give account' is a mystery. Moreover, we have to give up any idea that Plato deals with methodology or logic at 100a-101e [50].

Recently, Dorter (1982, 132-7) has expressed another view about the meaning of 'higher hypothesis'. According to him, we proceed to a higher hypothesis only when our original hypothesis is challenged. The 'higher' hypothesis does not entail the original one, but "would therefore be an attempt at a better formulation [hypothesis], sufficient (101e1) to overcome the previous objection" (p132). Dorter finds three hypotheses (in section 95-102): "that of the physical scientists which explained to some extent becoming but not essential existence; the simple-minded hypothesis of Forms, which explained to some extent essential existence but not becoming; and the more sophisticated hypothesis of forms which unifies the virtues of these two nearly antithetical hypotheses" (p137).

Dorter's interpretation seem to me untenable for the following reasons: a) if the 'higher' hypothesis is a re-formulation of the original hypothesis, then it is inexplicable how the 'higher' hypothesis can 'give an account' of the original one; b) in which sense can we accept that the re-formulation of a theory is 'higher' than the original formulation? c) why does Plato re-formulate his 'simple-minded' hypothesis although it is not challenged and no inconsistencies arise from it? d) I cannot understand how we can consider the theory of Forms-causes as a reformulation of the physical scientists' hypothesis.

Plato says that the new hypothesis must be the "best of those above" (101d7). That means that here, exactly as when we choose the original hypothesis (100a3-4: "hypothesizing on each occasion the theory I judge strongest"), there are probably many 'higher' hypotheses from which we may think we can infer our original one. From all these hypotheses we must choose the best one (or strongest -100a4). The new hypothesis (higher) is 'best' in the sense of being the most likely to lead to the required conclusion and such that no contradictions arise from it. It may be best as well in the sense that it is less questionable and probably more easily implied from a 'higher' one. The 'higher hypothesis' is posited 'in the same way' (<u>hosaytos</u>) as the original one. The word <u>hosaytos</u> shows us also that the higher hypothesis is tested by "seeing whether its consequences are in accord or disaccord with each other" (d4-5) exactly as the original one.

Passages 100a and 101d9-e1 seem to describe two opposite movements of the method. In 100a, we posit a hypothesis (H) and we draw its consequences (in order to prove our original proposition). We have therefore a deductive downward way. Passage 101d5-e1 describes an opposite movement which consists in an upward way from our hypothesis (H) to a 'higher' hypothesis (H1), from which we can deduce H. After that we find another higher hypothesis H2 from which we can deduce the H1 and so on until we arrive at something adequate. The word <u>hosaytos</u> (101d6) shows that every time we want to give an account of a hypothesis, we follow the same procedure described at 100a. That is, we find a higher hypothesis and we deduce our initial hypothesis from it

[51]. Therefore, the process described at 101d5-e1 is nothing other than the same procedure described at 100a (and exemplified at 100b) but repeated many times until the arrival at something 'adequate'.

The problem is how do we find a 'higher' hypothesis. There is no evidence in the text that there is a deductive process from the original problem to the hypothesis, or from a hypothesis to another 'higher'. On the contrary, it seems that the choice of a new hypothesis, made every time, consists in an intuitive grasp of what may be a premiss for the proof of the original problem of a 'lower' hypothesis [52]. That this is so appears to be the case from 100a3-4 ("hypothesizing on each occasion the theory I judge strongest"). So the downward movement from a hypothesis to its lower one ($Hn \rightarrow Gn \rightarrow \ldots\ldots \rightarrow Cn \rightarrow Bn \rightarrow Hn-1$) is a process of proof and the upwards movement, from a hypothesis to its 'higher' (Hn-1 Hn) is a search for premisses. In the course of the upwards movement the 'higher' hypotheses are obtained by an intuitive grasp of what is prior and by inductive reasoning [53]. The 'intuition' does not give knowledge. It helps us only to find a premiss, a new hypothesis that must be given a logos and can never pretend to be knowledge without being given a logos.

4. "Until you come to something adequate"

"Until you came to something adequate [ἱκανὸν]" (e1). Our problem is now to see what "something adequate" means. In 107b, exactly after the proof of the soul's immortality, we find a very relevant passage: "...so the initial hypotheses, even if they are acceptable to you people, should still be examined more clearly: if you analyse [διέλητε] them adequately [ἱκανῶς], you will, I believe, follow the argument to the furthest point to which man can follow it up; and if you get that clear [κἂν τοῦτο αὐτὸ σαφὲς γένηται], you will seek nothing further" [54].

The 'initial hypothesis' mentioned in the above passage, must consist in, or at least include, the theory of Forms. The theory of Forms is the hypothesis upon which the last argument relied and the main principle (or hypothesis) to which the argument in the Phaedo again and again returns as its main support.

Why does Plato speak about hypotheses (plural) and not about hypothesis (singular)? We can give several explanations: a) we have already said (section 1 of this chapter) that in every argument, we use usually more than one premiss, although we name only one hypothesis. It is possible then that the 'initial hypotheses' are all premisses that have led the last argument to its end (Rose, 1966, 472 n.29); b) The 'initial hypotheses' are the hypotheses used in all arguments of the dialogue, and not only in the last one; c) the 'initial hypotheses' are all the simple propositions about Forms that constitute the Theory of Forms (ie "Forms exist", "sensible particulars partake to Forms", "Forms have such and such qualities" etc, cf 102b1-2); d) "'hypotheses' (plural) could refer to the Theory alone, the positing of each Form being thought as a separate hypothesis" [55].

Let us now see whether we can identify the 'something adequate' (101e1) with the Theory of Forms or not. The theory of Forms is never 'proved' in the dialogue (through a higher hypothesis). The interlocutors have repeatedly declared their unalterable conviction of its validity (eg 77a). So we can say that the theory of Forms is the ultimate (speaking in terms of the upwards path - 101d5-e1) hypothesis or the first principle upon which the proof of the immortality of the soul relies. In this sense, we can say that the theory of Forms is 'something adequate'. However, there are reasons in favour of the non-identification between theory of Forms and 'something adequate'. Plato says that the theory of Forms is a hypothesis (cf 76d7-e7; 92d6-7) "but the argument about recollection and learning has come from a hypothesis worthy of acceptance"; 100b5-6 "hypothesizing that a beautiful, itself by itself, is something, and so are a good and a large and all the rest")

At 100d8-e1, Plato calls the 'hypothesis' of the theory of Forms safe, exactly as the hypothesis to be tested and to be deduced from a 'higher' one is called at 101d2. Moreover, the wording of passage 100d8-e1 is similar to that of 101d1-3 (see p123). We can plausibly assume then that the theory of Forms is the 'safe hypothesis' of 101d2. But the 'something adequate' (101e1) seems to be different from the 'safe hypothesis'. Therefore, it seems that the theory of Forms is not identical to the 'something adequate'.

Passage 107b reminds us of passage 101d5-e1. In the latter the 'safe' hypothesis is reduced to a 'higher' one and after that, through a continuous regress from one hypothesis to another, we arrive at something 'adequate'. At 107b, Socrates recommends that Simmias and Cebes analyse further the initial hypotheses (which are, or at least include, the theory of Forms). "... if you analyse them adequately [ἱκανῶς], you will follow the argument to the furthest point to which one can follow it up; and if you get that clear you will seek nothing further". If the 'safe' hypothesis at 101d is the theory of Forms - like the initial hypothesis at 107b5 - then we can plausibly say that the 'something adequate' (101e1) is the furthest point at which if we arrive, we shall seek nothing further (cf also, 'adequate' at 101e1 and 'adequately' at 107b7; moreover, we can plausibly say that the adequate analysis of 107b, is the analysis that leads to the destination of the adequate of 101e1; both the process of 'hypothesizing a higher hypothesis' and the analysis of the 'initial hypotheses' show an upward - or backward - path leading to principles).

Many commentators claim that 'adequate' means 'something adequate to satisfy an objector' [56]. According to Robinson (1953, 137), adequate "is not 'adequate to satisfy yourself'; for you were already satisfied with the first hypothesis". But the _Phaedo_ as a dialogue concerns the philosopher (here Socrates). For the philosopher, it is more difficult to be satisfied, or persuaded, than any interlocutor (see 91a7-9: "my concern will not be, except perhaps incidentally, that what I say shall seem true to those present, but, rather that it shall, as far as possible, seem to myself") [57]. If the goal of the hypothetical method was to persuade the interlocutors, then it would be identical to eristic which exactly tries to persuade the audience (cf 91a3-5) [58]. But Socrates clearly contrasts his method with the practice of the _antilogikoi_ (91a-c and 101e). According to Robinson's interpretation the 'adequate' becomes something epistemologically inferior to the 'ἀξία ὑπόθεσις' (92d5-6), the 'ἐρρωμενεστάτην ὑπόθεσιν' (100a) or the 'ἀσφαλῆν ὑπόθεσιν' (100d, 101d). The words ἀξία, ἐρρωμενεστάτη, ἀσφαλῆς show that the hypothesis has an objective character of validity, while Robinson's 'adequate' has not.

Ross holds that 'adequate' means 'that satisfies both you and your opponent' [59]. If Ross, in saying 'you', means the Philosopher and if we accept that the philosopher is not satisfied by any conviction but only with the truth, then I could agree with Ross. The philosopher (Socrates or Plato) is interested in truth and not in convincing anybody (see 91b1-c5 "... that if what I say proves true, it is surely well to have been persuaded; ... but for you part, if you take my advice, you will care little for Socrates but much more for the truth: if I seem to you to say anything true, agree with it; but if not, resist it with every argument you can, taking care that in my zeal I do not deceive you and myself alike") [60]. Morevoer, Socrates' recommendation at 107b to Simmias and Cebes to re-examine the initial hypotheses even if they are acceptable to them, shows again that the goal of the hypothetical method is not to persuade the interlocutors, and therefore the <u>hikanon</u> is not 'something adequate' simply to satisfy someone.

Similarly 'something adequate' cannot mean 'some adequate hypothesis'. It seems that the 'adequate' is contrasted at 101d with the hypotheses which as such are insufficient and not adequate (Friedländer, 1943, 256). As Gallop rightly observes: " a mere hypothesis, it might seem, could not be 'adequate', if its sole merit were that no objections to it could be found" (1975, 190-1). Hypotheses need a further account or analysis to be supported (101d5-6, 107b5-6). On the contrary, the 'adequate' seems to be something self-sufficient [61]. It is something that is not derivable from anything else (higher) in the context in question and is not open to any challenge [62]. The 'adequate' is not something that none of the interlocutors objects to, but something to which no objections can be raised. It seems that the 'adequate' is not just an 'adequate hypothesis'.

Socrates, in his defence speech (64a-67c), claims that the body is a hindrance for those seeking the truth (65a-b) and that philosophers come closest to knowledge when they are rid as far as possible of the sense's influence (65c-e). There is no full knowledge in this life; the soul can obtain full knowledge only after death (66e-67a, 67b-c). There is however, a track (atrapos) (66b4) for philosphers to acquire truth in this life [63]. We can approach knowledge in this life when we separate the souls from the body as far as possible.

In a very relevant passage and similar to the one about hypotheses, Simmias (85c-d) makes some remarks: a) it is impossible or very difficult to obtain knowledge in this life; b) however, one - if he is not a feeble spirit - must test in every way what people say; c) "on these questions (ie immortality of the soul) one must achieve one of two things: either learn or find out how things are" [64]; d) "or, if that is impossible, then adopt the best and least refutable of human doctrines, embarking on it as a kind of raft..., unless one could travel more safely and with less risk, on a securer conveyance afforded by some divine doctrine".

The above two passages possess many similaritites. Both Simias and Socrates assume that although there is such a thing as full knowledge, it is impossible for this to be obtained in this life (Simmias is a little more optimistic here) and we probably have the same remark at 99c8-9 where Socrates 'was deprived' of the knowledge of the Good. Both Simmias and Socrates agree that there is a way (approach) to knowledge in this life. This way (track) consists in the separation as far as possible of the soul from the body (Socrates) or in adopting the best and least refutable doctrine (Simmias). Both the above characteristics are found in passage 99-101. Socrates finds refuge in logoi (in opposition to the physicists who use sense-experience) in order to find the truth of the things (99e5-6) and for this task, he hypothesizes the logos he judges strongest [65]. The upward path at 101d5-e1 (the regress to higher hypotheses) shows that we have a continuous approach to knowledge. In this approach, the 'adequate' is the last step. When we arrive there, we obtain probably 'certainty' and 'knowledge' which is sufficient for the thing we are investigating and there is no need for further seeking. However, this is only human knowledge. It cannot give perfect certainty (85d3-4). In every step upwards, we obtain more and more justified confidence. Even before the arrival at the last step of the upward path ('adequate'), we may have highly justified confidence about the truth of our initial proposition. So, although the theory of Forms is not itself 'something adequate' but needs further analysis, Socrates has no reservations about the validity of his final argument.

Some commentators claim that the 'adequate' of the Phaedo is like the unhypothesized principle of the Republic [66]. However, the text does not give support for such a view and Socrates' remark at 99c that he abandoned the search for the Good shows rather the opposite [67]. The method of hypothesis in the Phaedo is proposed in order to prove something sufficiently. Thus it seems that the method functions in a concrete context every time. Plato's method presents the practice of the philosopher (in order to find something - 101e3) as opposed to the practice of the antilogikoi. Hence, Plato probably "does not expect us to ascend to the αὐτὸ ἀγαθὸν every time we seek to establish a contested point" [68]. Moreover, in the Republic, the 'unhypothesized' is the unique principle for all knowledge. But in the Phaedo, Socrates does not say that the 'adequate' is such a unique principle. The text says "ἐπί τι ἱκανὸν", that is "to something adequate" and not "ἐπὶ τὸ ἱκανὸν". Therefore, it is plausible to suppose that there is not only one 'adequate' but many.

What is then the nature of the 'adequate'? The 'adequate' is the last step of the upward path, that is of the regress (or analytical process - 107b7) of one hypothesis to another. From this we can infer all the hypotheses in our subject matter. The 'adequate' cannot be reduced to any other (107b9). It is the starting point (principle) and it is adequate in the sense that it needs no justificiation itself, and that it renders the system of propositions derived from it logically coherent. In this sense, the 'something adequate' is very similar to the first principles of Geométry. Like those, the 'adequate' (or the 'adequates') is obtained via an analytical and reductive process and it gives reason for all the propositions of the subject matter. So speaking within the limits of the particular dialectical inquiry, the 'adequate' is something sufficient and self-evident exactly as mathematicians view their principles. Plato says, at Rep. 510c, that the mathematicians consider their principles not like hypotheses but like self-evident principles, which do not need any further justification. I think that Plato considers the 'adequate' (or the 'adequates') in a similar way to the mathematicians' consideration of their principles. In this sense, we can say that the 'adequate' is a first and absolute principle, but only for the subject matter in question [69]. It is not self-explanatory and it is self-sufficient only speaking within the context of the subject matter in question. No objection can be raised to it but it is not able to give absolute certainty and knowledge for the totality of all 'knowable' things as the Good in the Republic does. The above interpretation fits also very well with what Olympiodrorus says about the ἱκανὸν: "ἄμεινον δέ, τὸ ἀεὶ ὁμολογούμενον φάναι καὶ τοὺς αὐτοπίστους ὑποθέσεις τε καὶ ἀρχὰς" (ed Norvin, 225.13-15)

At 101e, Plato recommends that we should not "jumble things as the contradiction-mongers do, by discussing the starting point [ἀρχῆς] and its consequences at the same time, if that is, you wanted to discover any of the things that are".

Apart from the problems of interpretation that this passage creates [70], it seems that Plato gives advice to keep the two parts of the method, as described at 100a and 101d5-e1, separate, or, according to Robinson (1953, 141) to distinguish "at every instance between what you are assuming and what you are deducing" [71].

Plato's advice may enable one to "discover... the things that are". It seems th n that Plato considers his method as an heuristic one (method of discovery) rather than as a method of proof. The heuristic process of the method is the upward path of 'hypothesizing a higher hypothesis' (see n. 53). In hypothesizing a higher hypothesis, we try to discover premisses and (therefore) proofs for the lower one. Moreover, testing the 'higher' hypothesis, we eliminate disjunctive cases or we ensure the consistency of our system of propositions.

Robinson (1953, 135-6, 138, 140) considers the process of 'hypothesizing a higher hypothesis' as a positive test of the hypothesis; it is an extrinsic and not an essential part of the hypothetical method. On the other hand, the 'seeing whether the result accord' is a negative test of the hypothesis and an intrinsic and integral part of the hypothetical method. Robinson thinks that the hypothetical method in the Phaedo is only what is described at 100a [72], that is to say proving a proposition through a hypothesis. For Robinson, the part of the method described at 101d5-e1 aims only at convincing the interlocutor to agree with the hypothesis. So the upward path has nothing to do with epistemology.

Contrarily to Robinson, I think, that the 'hypothesizing a higher hypothesis' is an integral part of the method [73]. In the Phaedo's hypothetical method, there are two different parts (or stages) that Plato wants to keep distinct (101e2-3). These two parts are tightly connected with each other. The first (100a) concerns mainly the solution of a problem through a hypothesis [74]. The second (101d5-e1) concerns the truth of the hypothesis itself. Thus we have a real proof (and knowledge) of our proposition to be proved, only when the second procedure is exhausted (passage 100a presents only a hypothetical solution to our problem). Without the second part of the method, there is no knowledge and we cannot say that we have really solved our original problem.

I have already argued that the procedures described and 100a and 101d2-5 were not Plato's innovation, but something common in the dialectic of the 5th century B.C.. The new and most important thing is the justification of the hypotheses themselves by following the same process and not just the use of a hypothesis in order to prove something. This is what constitutes Plato's innovation in the dialectical inquiry. Only, the upward path is that which makes the hypothetical method a method of philosophical inquiry (101e3). Only the upward path is able to guarentee the truth of the hypotheses (premisses) and to lead us to knowledge (cf P126-7).

In the Meno, the justification of the hypothesis ("virtue is knowledge") is a very relevant part of the method described there. In the Republic, the upward path of the dialectician is the proper and the most relevant activity of the dialectical inquiry; it is the activity that gives truth and transforms the hypothesis into knowledge. In Cratylus 436d (see p135), Plato clearly says that the proper and most significant activity in any inquiry is the discussion of the starting-points and not their consequences.

I think then that the main goal of the hypothetical method is the justification of the hypotheses, and from this point of view, the process of 'hypothesizing a higher hypothesis' is equally if not more relevant than the part of the method described at 100a.

5. Analysis, Apagoge and the Hypothetical Method in the 'Phaedo'

In this section I shall argue against the theory that the hypothetical method in the Phaedo is modelled upon the pattern of the geometrical method of analysis and synthesis. After that I shall support the thesis that, as with the Meno, the method in the Phaedo is the geometrical method of apagoge adapted to the needs of philosophy.

First of all, I shall summarize some of our results regarding the structure of the method.

In the third section of this chapter, I argued that in the process of Plato's hypothetical method in the Phaedo, we can discern two opposite movements; a) a deductive downward movement from the hypothesis to its consequences (or from a 'higher' to a 'lower' hypothesis; b) a heuristic upward movement from our original problem to the hypothesis (or from a hypothesis to a 'higher' one). The two movements are not separated but interrelated. They both occur in every stage of the method between a hypothesis (Hn) and its 'higher' (Hn+1) [75]. The upward movement of the method is a regress towards what is logically prior. Its business is to discover (and clarify) the premisses that are necessary for the solution of our problem. Such premisses can be called in one sense 'causes'. In this upward way, a higher hypothesis is obtained not deductively from the lower one but rather by an intuitive insight that helps us to find a stronger foundation on which our proof will rely.

Some commentators have claimed that Plato's hypothetical method in the Phaedo is similar to the hypothetico-deductive method of modern experimental science [76]. The similarities are indeed striking, despite the fact that Plato's attitude is rather anti-empirical. There is not any experimental or external (ie out of the deductions of the hypothesis) test in Plato's method [77]. The test described at 101d1-5 refers to possible internal logical inconsistencies in the consequences of the hypothesis.

Moreover, even in the cases in which it seems that Plato states a general principle (or theory) by use of induction, we do not usually have truly empirical induction. The general principle is of a rather logical character and the examples (instances of the principle or theory) have mainly an explanatory function [78].

According to the classical conception of the hypothetico-deductive method, experience or experimental predictions of a theory confirm the scientific theory [79]. But according to Plato, we have confirmation of a theory (hypothesis) not so much 'by seeing if its results agree with each other' but mainly 'by deducing our hypothesis from a higher one' (see διδόναι λόγον - 101d6). We can find a similarity between Plato's upwards process and Hempel's model of scientific theory in terms of covering laws deduced from other laws further up a hierarchy of increasing generality [80]. But even this model does not correspond well to Plato's upward process because in the latter a 'higher' hypothesis is not necessarily a more general theory, whose 'lower' hypothesis is a specific case. Plato's hypotheses are many times premisses from which (conjunctively with other premisses) we can deduce our original proposition. For example, the immortality of the soul (in the Last argument) is not a specific case of the theory of Forms.

It has often been observed by commentators that Plato's hypothetical method has logical - mathematical characteristics and that we can trace in it some influence from the mathematics of Plato's time [81]. Many of them claim that Plato's hypothetical method in the Phaedo is similar to the Greek geometrical method of analysis [82].

I have already argued (see p38-9) that the method of Greek geometrical analysis is mainly a heuristic method of finding lemmas and proofs. The heuristic character of analysis consists not in its deductive or otherwise character, but in the fact that, taking the required as given, we can exploit more mathematical entities and relations in our proof and that we can utilize the structure of the same proposition we have to prove in the search for its proof.

The question of whether analysis is deductive or not is a secondary one. The geometer, practicing analysis tries to find premisses (lemmas) which will enable him to solve his problem. In this process, experience, intuition and the mental ability of the geometer are always the most helpful things. The manner in which we discover a proof and the manner in which we write it down after we have discovered it are very different.

We can easily see that the main characteristics of Plato's hypothetical method in the Phaedo are almost the same as those of the geometrical method of analysis and synthesis. In both methods, we have a demonstrative deductive downward movement and a heuristic upward one. In the way up, the intuitive ability of grasping what is prior plays a very relevant role. In both methods we start from the required (the problem to be solved) and we look for premisses (or lemmas) that will enable us to solve our problem [83]. The upward movement ends when we arrive at something independently known as true, which does not need further confirmation. There is no doubt that Plato was well aware of the current mathematical developments and it is very probable that he had thought about the application of the new and revolutionary method of geometrical analysis to dialectic. In the Meno, Plato says clearly that he borrows his method from the geometers and we have no reason to believe that the same does not happen in the Phaedo [84]. This is probably the reason why Simmias, Cebes and Echecrates (all Pythagoreans) found Socrates' account of the method 'perfectly clear' "even for someone of small intelligence" (102a4-5) [85].

Plato's recommendation (101e) about the distinction between a) the discussion about the arche, and b) the discussion about its consequences (as well as Aristotle's testimony - N.E. 1095a32 - that Plato attached great importance to keeping apart the arguments towards arche and from it) reminds us of the geometer's practice when he applies the method of analysis and synthesis (cf Cherniss, 1951, 413).

We might find some more evidence that Plato's hypothetical method in the Phaedo is modelled upon the method of analysis and synthesis in Aristotle and ancient commentators.

First of all, Plato's exposition of the method is very similar in wording and meaning to the Aristotelian passage in Ethics (EN, 1112b13ff) that refers to the geometrical method of analysis and synthesis (see page 27).

In both passages, we start by hypothesizing something (Phaedo 100a2-3 καὶ ὑποθέμενοι ἑκάστοτε λόγον; EN 111b15 ἀλλὰ θέμενοι τὸ τέλος). There are many propositions from which our initial hypothesis can be inferred (Phd 101d7 τῶν ἄνωθεν; EN 112b15 καὶ διὰ πλειόνων μὲν φαινομένου γίγνεσθαι) and we choose the best one among them (Ph101d7: ἥτις τῶν ἄνωθεν βελτίστη φαίνοιτο; EN112b17 διά τινος ῥᾷστα καὶ κάλλιστα ἐπισκοποῦσι). In both Plato and Aristotle, we have an upward movement that consists of the search for prior things (propositions) (Phd 101d6-7 ἄλλην αὖ ὑπόθεσιν ὑποθέμενος ἥτις τῶν ἄνωθεν; EN 112b15-6 ἀλλὰ θέμενοι τὸ τέλος τὸ πῶς καὶ διά τινων ἔσται σκοποῦσι) and for both Plato and

Aristotle, this method is a heuristic one (Phd 101e3 εἴπερ βούλοιό τι τῶν ὄντων εὑρεῖν; 107b9 ζητήσετε: EN 112β19 ἐν τῇ εὑρέσει, b20 ζητεῖν, b22,23 ζήτησις).

The upward heuristic process stops for Plato at the ἱκανὸν (101e1) and for Aristotle at the πρῶτον αἴτιον (b19) (cf also Phd 107b9 οὐδὲν ζητήσετε περαιτέρω; EN 1112b19-20 τὸ πρῶτον αἴτιον, ὃ ἐν τῇ εὑρέσει ἔσχατόν ἐστιν). We must note that the search in which Plato introduces his hypothetical method is a search for the 'causes' [86].

Secondly, Ammonius (In An Pr, Wallies, 6.6-8), David (In Porph Is, Busse, 104.1,ff) and Albinus (Didaskalikos, Herman, V 5) after giving their accounts of syllogistic analysis (Albinus after the account of his second kind of philosophical analysis) give an example of it in the proof of the proposition that the soul is immortal. This seems to be more evidence that Plato's hypothetical method in the Phaedo is the same as (or similar to) the method of analysis and synthesis. But the proof of the immortality of the soul that commentators give is not that of the Phaedo but that of the Phaedrus (245c-e) [87].

Thirdly, it seems that Albinus (Didaskalikos V 6) thought that Plato's hypothetical method in the Phaedo was a kind of philosophical analysis. His account of the third kind of philosophical analysis (hypothetical analysis) is clearly the same as the method in the Phaedo:

Ἡ δὲ ἐξ ὑποθέσεως ἀνάλυσίς ἐστι τοιαύτη· ὁ ζητῶν τι ὑποτίθεται αὐτὸ ἐκεῖνο, εἶτα τῷ ὑποτεθέντι σκοπεῖ τὶ ἀκολουθεῖ, καὶ μετὰ τοῦτο εἰ δέοι λόγον ἀποδιδόναι τῆς ὑποθέσεως, ἄλλην ὑποθέμενος ὑπόθεσιν, ζητεῖ εἰ τὸ πρότερον ὑποτεθὲν πάλιν ἐστὶν ἀκόλουθον ἄλλῃ ὑποθέσει, καὶ τοῦτο μέχρις οὗ ἄν ἐπί τινα ἀρχὴν ἀνυπόθετον ἔλθῃ ποιεῖ [88].

The above would appear to make it very plausible to suppose that Plato has formulated his hypothetical method in the Phaedo in the model of Greek geometrical analysis and synthesis. However there are some reasons, which make this interpretation impossible. Greek geometrical method of analysis and synthesis has two different and separate ways. Only when the analysis stops (arriving at some principles) does synthesis begin its path. On the contrary, in Plato's method of hypothesis, the two ways (analytic and synthetic or upward and downward) are not separate. Both processes are involved in each step of the method. We can present schematically the processes of the two methods as follows:

A. Method of analysis and synthesis;

analysis - $G \Longrightarrow F \Longrightarrow \cdots \Longrightarrow B \Longrightarrow A$

synthesis - $G \longleftarrow F \longleftarrow \cdots \longleftarrow B \longleftarrow A$

B. Plato's hypothetical method in the Phaedo;

$$\underset{}{G} \underset{1}{\rightleftarrows} F \underset{2}{\rightleftarrows} E \underset{3}{\rightleftarrows} \cdots \underset{5}{\rightleftarrows} B \underset{6}{\rightleftarrows} A$$

We can see clearly this difference if we compare Albinus' second kind of philosophical analysis (similar to the geometrical method of analysis and synthesis) [89] and his third (similar to Plato's method of hypothesis in the Phaedo).

Moreover, the proposition we hypothesize in the geometrical method of analysis is the proposition that we want to prove, that is proposition G. On the contrary, Plato calls hypotheses all the other propositions (F......A) whereby we prove G.

We can safely conclude then that Plato's hypothetical method in the Phaedo is not the geometrical method of analysis and synthesis.

In Chapter III (p89-91), I have argued that the hypothetical method in the Meno is similar to the geometrical method of apagoge that Hippocrates of Chios used to practise. I have already said (pp49-50) that apagoge was the forerunner of the method of analysis and synthesis.

Apagoge is the reduction of one problem (say G) to another (say F) that is probably easier to solve. In apagoge we start from the problem G, which we have to solve and we look for another problem F upon which G depends ($G \Longrightarrow F$). After finding F we deduce G from F ($G \longleftarrow F$) and so we have solved G hypothetically on the basis of F. So in practising apagoge, the hypothesis is not our original problem G (as in the method of analysis) but the problem F to which our original problem G is reduced.

G is now solved only hypothetically. For a real solution of it we must solve problem F. If we apply the same reductive method to F, we must find another proposition E ('higher' hypothesis) upon which F depends and to prove F through E (that is $F \longleftarrow E$) [90]. If we continue the same process to its end, we may arrive at a proposition A that is either a principle of geometry or a proposition known already. Now if we separate the two ways (up and down) and we proceed firstly from G to A and after from A to G, we have the method of analysis and synthesis. In this sense the method of successive reductions was historically the first stage of the method of analysis [91]. A schematic presentation of the method of successive reductions (apagoge) is the following:

$$G \underset{1}{\rightleftarrows} F \underset{2}{\rightleftarrows} E \rightleftarrows \cdots\cdots \underset{5}{\rightleftarrows} B \underset{6}{\rightleftarrows} A$$

Now we can see easily that Plato's hypothetical method in the Phaedo has exactly the same structure as the early geometrical apagoge analysis.

However, we must not suppose that Plato applies blindly a geometrical method to dialectic. He is conscious of the difference between dialectic and mathematics and his use of the terms 'accord' and 'disaccord' shows that he looks not only for deducibility but also for coherency. Moreover, the philosopher depends also on the possible objections of his interlocutor. Plato's method is not exclusively mathematical or anti-empirical. There is room also in the hypothetical method for inductive reasoning, formulation of general theories and application of them to particlular things. Finally, adapting the method of apagoge to dialectic, Plato adds the activity of seeing whether the consequences of the hypothesis "are in accord or disaccord with each other", so introducing a check of the hypothesis.

6. Does Plato apply his method in the dialogue?

i. We have already seen that Plato's method of hypothesis in the Phaedo, as described in 100a and 101d, consists of three parts: a) hypothesizing a proposition and positing as true whatever agrees with it and as false whatever disagrees with it; b) seeing whether any contradiction arises out of the hypothesis; c) hypothesizing a 'higher' hypothesis in order to give an account of the 'lower' one. Is the method practised in the dialogue? If so, then in which part of it and to what extent?

Almost all critics find that Plato illustates the first part of the method not only in passage 100b - 107a, but also in other parts of the dialogue [92]. Some commentators also find that we have an example of the second part of the method in the refutation of the Simmias' thesis that the soul is harmony [93].

As regards the third part of the method, the common view is that it is not undertaken in the Phaedo [94], although it has been repeatedly observed, and it is now widely accepted, that the various arguments in the Phaedo are not self-contained and independent of each other but they are mutually related and closely integrated [95].

On the contrary, I think that Plato applies his method - and especially the first and third parts - throughout the whole dialogue [96]. I will try to justify my view in two ways: a) by following the Phaedo's argument and showing that we can trace in its development the application of the method; b) by finding linguistic similarities between the hypothesis passage and other passages throughout the dialogue, or passages that contain allusions to the hypothetical method. This will be done in the course of the examination of the argument.

ii. The initial problem of the dialogue is the justification of Socrates' attitude in front of death (63e-64a). This attitude reflects the real philosophical way of life that Socrates experienced. True philosophy then becomes practice for dying (melete thanatou - 67d-e). The justification of Socrates' philosophical way of life consists of the analysis of the presuppositions on which true philosophy is based. To this end, Plato advances a theory about the soul, the theory of Forms, the theory of recollection and a new methodology.

After an introductory argument against suicide, Socrates formulates the original problem of the dialogue: "a man who has truly spent his life in philosophy feels confident when about to die, and he is hopeful that, when he has died, he will win great benefits in the other world"(63e9-64a2). Socrates says that he is going to give an account (logon apodounai - 63e9) in order to defend his thesis (cf the same expression at 101d where we have to give an account of the hypothesis).

Socrates' defence begins with two agreed premisses: a) there is such a thing as death; b) death is the separation of the soul from the body (64c2-8; cf Gorg. 524b). This definition of death involves a sharp distinction between soul and body and indirectly the soul's survival after death [97].

The argument of Socrates' defence may be summarised as follows. The philosopher seeks to attain wisdom (phronesis, 65a9, 65b9, 66a6, 66e3)(i). To this end, the body is a hindrance, because the cognitive activity belongs to the soul alone(65a-c, 66a)(ii). The philosopher dismisses the body and the bodily pleasures and is directed towards the soul (64e)(iii). In this sense the philosopher tries to release his soul, as far as possible, from its communion with the body (65a1-2,67a)(iv).

At 65d, Socrates introduces a third premiss: there are such things as 'the just itself', 'the beautiful itself' etc. These things are not to be apprehended by any of our senses but only by the soul [98].

As long as our soul is contaminated by the body, we shall never adequately gain knowledge and truth. We can gain them only when we die, since then the soul will be alone by itself apart from the body (66b, 66d-67a)(v).

But even in this life there is a 'track' that can lead the philosopher closest to knowledge(66b2-4). This track consists in "consorting with the body as little as possible, and not communing with it ... and not infecting ourselves with its nature, but remaining pure from it"(67a) [99].

So it is absurd for a man who spent his life in a way that brought him as near as possible to being dead, then to complain of death when it came(67d12-e2)(vi) [100].

The decisive step in the argument is the (v). Behind this we can find the assumptions that the soul a) survives death and b) retains its individuality and its cognitive character after death.

iii. Cebes objects to the argument exactly at this point. Our original problem has been solved on the basis of the hypothesis that the soul survives death. This hypothesis is now objected to by Cebes, so we have to give an account of it. The new problem we have to solve now - as it is stated by Cebes - is that "when the man has died, his soul exists, and that it possesses some power and wisdom(70b2-4). A provisional justification of the hypothesis (immortality of the soul) and of the philosopher's life might be found in the ancient doctrine of Paligenesis(70c-d). The ancient doctrine, however, is not well enough established and needs the support of an argument. We must note here that Socrates - via the doctrine of Paligenesis - seems to reformulate the question. It is rather the reincarnation than the immortality of the soul that will give reason for the philosopher's life (I shall come back to this point later).

The Antapodosis argument [101] can be divided into three parts: 70c4-71e2, 71e3-72a10, 72a11-72e2. The first part concerns itself with pre-natal death and existence; the other two, with post-mortem death and existence. Cebes' other requirement that the soul "possesses some power and wisdom"(70b3-4), is going to be supplied by the Recollection argument [102].

The first part of the Antapodosis argument is based on two principles and one premiss [103]. The principle of Opposites (70d9-e2) runs thus: "for all things subject to coming-to-be ..... everything comes to be in this way: opposites come to be only from opposites - in the case of all things that actually have an opposite". We can state it more formally as follows:

If f and g are opposite properties, then any object that acquires the property f at a time t possesses the property g during a period immediately prior to t; and conversely.

The principle of Existence is only hinted at in 70c8-d2 (where we are told that if living people come into being from the dead, then their souls existed in Hades before this event) and it seems that Plato does not distinguish it from the principle of Opposites. The two principles are considered by him to be one premiss. We can formulate this principle as follows:

If anything that exists at a time t acquires a property at t, then it existed during a period immediately prior to t.

The premiss of the argument is: Being alive and being dead are opposites.

From these three propositions we have:

If anything (that exists at t) acquires at t the property of being alive, then it both exists and possesses the property of being dead during a period immediately prior to t [104].

As we can easily see (cf 103a-b), the argument is concerned with things that have opposite properties and not with the opposite properties themselves [105]. Moreover, the principle of Opposites speaks about a single subject undergoing a change [106]. What is the object that undergoes the change from life to death? Plato's answers that it is the soul [107].

Therefore, the soul comes to be alive at t (or acquires the property of being alive, or of being incarnate), from being dead (or from having the property of being dead, or of being discarnate) [108] and exists during a period immediately prior to t.

Therefore, our souls exist in Hades (71e2).

The first part of the Antapodosis argument shows the pre-natal existence of the soul and is based on the first premiss (including the principle of Opposites and the principle of Existence) and on the premiss that 'being alive' and 'being dead' are opposite properties (premiss 2). The latter premiss is never challenged and holds well throughout the dialogue (it is probably considered as obvious). From the conclusion of the argument (that our souls exist in Hades) we can infer that the soul is something substantially different and independent from the body (cf 70c8-9 where soul's substantiality is assumed). Nevertheless the positing of the soul as a subject undergoing properties living and dead is not a conclusion but a basic premiss of the argument (premiss 3).

Two other arguments follow now of the necessity of the process of return to life. At 71a12-b10, Plato had stated that the principle of Opposites involves two processes of transition between a pair of opposites. But this does not entail the actuality of the process of 'coming-to-be again', and therefore reincarnation.

The first of the two arguments says that if there were no return to life, nature would be lame; it is not; therefore, there is return to life (71e4-72a10). Many commentators have not taken the argument seriously and I do not know if Plato himself has taken it so.

The second argument about the 'coming to life again' is introduced to justify admissions made earlier (72a11-12). These have rested upon the postulate that nature is not lame and therefore there is such a thing as coming to life again. The argument says that if the opposite process passing from the 'dead' [109] to the 'living' again did not take place, sooner or later everything would be permanently dead, because the 'stock' of souls [110] would be exhausted (72c6-d3).

The main assumption of the argument is that nature will never be permanently extinguished (premiss 4). This assumption is left unexplained and is also questionable. It is only a pious hope and it cannot constitute a real principle of the argument. Besides this assumption two other premisses are assumed, although Plato does not state them: a) there is not an unlimited stock (of souls) from which new lives could originate (premiss 5) [111]; b) there is not creation of life ex nihilo (premiss 6). These premisses are not challenged or questioned throughout the dialogue. Without them the argument has no force.

The soul's return to life was proved via premisses ('nature is not lame' and 'nature will never be extinguished') that are questionable and regard (apply to) the physical world. However, the right explanation of what happens in this world must depend on higher realities. Furthermore if we prove the immortality of the soul on the basis of those assumptions and without first an inquiry into its nature, the soul is treated as if it is a sensible, natural object.

We must also note that Plato distinguishes between only two cases: a) a linear process that ends up in the death of all living nature; and b) a cyclical process that never stops. He does not consider the case that the soul, although participating in the cyclical process, after some reincarnations could be extinguished (objection of Cebes; 87a-88b). But I will come back to this point later.

The whole argument proves the continuous reincarnations of the soul and so gives a reason for the doctrine of Paligenesis. The soul's survival after death is a result of this doctrine (cf 72a4-8,72d8-e1).

The soul is taken to be substantial (cf premiss 3) and independent from the body throughout the Antapodosis argument. In the first part of it (70d7-71e2) the soul is described simply as something that can acquire the properties of 'being alive' and 'being dead'. In the rest of the argument (71e4-72e1), the soul is described mainly as the principle of life and motion [112]. The soul's return back to life is that which does not allow nature to be permanently dead (72a11-b5). It is the soul that is responsible for the coming-to-be of living things (72c5-d3). Bringing life is an essential character of the soul. The soul must be continuously reincarnated in order to keep the nature alive. This essential feature of the soul is exploited in the last argument.

The soul is considered as an individual soul and Plato aims to prove <u>personal</u> immortality (cf. 70c6-8, 71e2, 72a8-9, 72d8-e2). Nevertheless the Antapodosis argument does not prove the individuality of the soul (perhaps it takes it as given) [113]. It is the Recollection argument that will do this. Although in Socrates' defence, the soul was presented as 'cognitive principle' (see eg 65b9,e7,65c2-5), this feature of the soul is absent here [114].

Many commentators claim that the principle of Opposites is an empirical Law of nature [115] and is connected with Heraclitus' views. It is also said that Plato himself recognises that the argument is not sound, later on in the Phaedo [116]. But the principle of Opposites is posited rather as a logical truth than as an empirical natural law (see anagaion, 70e4). Gallop [117] rightly remarks that "it takes no empirical study to discover that what comes to be larger must have been smaller, or what comes to be worse must have been better. Nor do these truths apply only to physical phenomena". That the principle 'applies' to the natural phenomena does not imply that it is a natural law.

We can easily see that Plato does not consider the argument unsound from the passages 77c-d and 103a-c where he refers back to it (or to its main premiss) without any hint against its soundness. That some of its premisses need further justification, is another matter. This is exactly one of the major features of the essence of the hypothetical method.

iv. The Recollection argument (72e3-76e7) runs as follows:

1) If anyone is to be reminded of a thing, he must have known that thing at some time previously (73c1-2).

2) There is such a thing as the equal itself (74a11-12).

3) We do know the equal itself (74b2-3).

4) We have been reminded of the equal itself (by seeing equal

things)(74c7-9).

5) We must have acquired the knowledge of the equal itself, before that time when we first saw equal things and saw that they fell short of Forms (74e9-75a3).

6) Not from perception (74a11-b12).

7) Not in this life (74c4-5).

8) Not at the time of birth (76d1-5).

9) Before birth (76b11-13).

The Recollection argument -as Plato himself notes at 76d7-e7 - is based essentially on the assumption of the existence of the Forms. It shows that the soul exists before birth and possesses wisdom, thus supplementing the Antapodosis argument. The soul is here presented as the 'cognitive principle' by which the quest for knowledge is pursued. That the soul possesses wisdom is a very relevant premiss for the justification of the philosopher's life [118] (note that in Socrates' defence, the soul is mainly presented as cognitive principle). The Recollection argument also shows the individuality of the soul stressing the point of the soul's memory [119]. We must also note that the Recollection argument exactly like the Antapodosis one, is based on the assumption that the soul and the body are substantially different things (cf the definition of death - 64c4-8).

At 77c6-d5 Socrates, answering Simmias' and Cebes' objection (77a9-c5) that the after-existence of the soul has not so far been proved, says that it is already proved if we combine the two arguments (Antapodosis and Recollection). So Plato considers the two arguments complementary. It is also worthwhile to note that according to Plato, the Antapodosis argument does not show only the <u>post mortem</u> existence of the soul. It had sought to establish that souls pass through an eternal cycle of alternating 'death' and 'life', so justifying the doctrine of Paligenesis. From 77d3-5 ("surely it must also exist after it has died, given that it has to be born again"), we see that Plato regards the soul as a principle of life, its essential character being to animate (bodies) endlessly.

We may summarize the argument so far as follows. Socrates' defence regarding the philosopher's hope depends on the premiss (hypothesis) that the soul 'exists after the death and possesses some power and wisdom'. The first part of the Antapodosis argument proves the <u>pre-natal</u> existence of the soul (depending mainly on the Principle of Opposites) and that "the soul, so far as it exists, alternates between life and death". The second part of the argument proves the <u>post-mortem</u> existence of the soul (depending mainly on the premiss that 'nature will never be extinguished'). The whole argument proves continuous reincarnations of the soul and therefore immortality. The Recollection argument proves the <u>pre-natal</u> existence of the soul and that the "soul possesses some power and wisdom"; in this way it is complementary to the Antapodosis argument. The required conclusion is entailed by the theory of recollection, which, in its turn depends on the existence of Forms. We can easily see then, that the whole argument proceeds from hypotheses to 'higher' ones so exemplifying the method described at 100a and 101d5-e1.

v. The Antapodosis argument was not convincing for Simmias and Cebes and they do not think that the <u>post-mortem</u> existence of the soul has been proved. The Affinity argument (78b4-81a10) is an argument by analogy based on a sharp distinction between two kinds of things, namely Forms and particulars. It runs as follows: The composite is dispersible, the changeable composite, the visible changeable. The incomposite is indispersible, the unchangeable incomposite, the invisible unchangeable (78c-79a). Particulars are visible and changeable. Forms are invisible and unchangeable (78c10-79a4). We are partly body and soul (79b1-2).

A) Body resembles the visible and therefore the changeable and the dispersible. Soul resembles the invisible and therefore the unchangeable and the indispersible (79b4-16).

B) Through the body we study the visible and the changeable. Through the soul we study the invisible and the unchangeable. Therefore, body resembles the changeable and soul resembles the unchangeable (79c2-e7).

C) By nature, the body serves and the soul rules. Similarly by nature, the mortal serves and the divine rules. Therefore, the body resembles the mortal and the soul the divine (80a).

Given that the body (corpse) although visible and dispersible, is not dispersed immediately after death but survives for a considerable time, the soul that is invisible must be "completely indispersible or something close to it" (80b9-10).

The Affinity argument tries to establish the soul's survival of death upon the soul' essential nature and not upon independent factors as the Antapodosis argument did. The argument relies on the distinction between Forms and particulars and on the substantial distinction between soul and body (it has been already taken for granted in the Antapodosis and Recollection arguments). The argument -being an argument by analogy- does not establish the indissolubility or the immortality of the soul [120].

Plato himself recognises that the argument is not rigorous (cf. <u>eikos</u>, 78c7; <u>homoioteron</u>, 79b4, b16, e1, e4; <u>homoion</u>, 80a3; <u>homoiotaton</u>, 80b3, b5) and does not establish the indissolubility of the soul (cf. "or something close to it", 80b10; and the comparison with the body's duration after death at 80c-d). It is rather a rhetorical argument which has to do with the persuasion of the "child in us" (77e). However it seems that Plato thinks that the argument does prove (or at least gives good ground for) the survival of the soul after death for a period of time.

The argument is not independent from what has been argued or proved so far, but a sequel to the preceding ones (see 78b1: "but let's go back to the point where we left off, if you've no objection"). The main concern of the argument (from Socrates' point of view) is to give grounds for the reincarnation of the soul. This is plain from passage 80d-84b. The immortality of the soul cannot alone give any justification for the philosopher's life. But reincarnation, if accepted, justified or proved, can constitute the basis on which a theory about the future of the individual soul will rest, so giving justification for the philosopher's life. Reincarnation thus obtains moral consequences [121].

How is it possible for Plato to think that from the Affinity argument (or from the soul's survival after death) he can draw conclusions about the reincarnation of the soul? The argument intended to prove reincarnation was the Antapodosis one. And if we accept, as many commentators did (see note 116), that the Antapodosis argument is not considered valid by Plato and so is dropped, then Plato's obstinacy about reincarnation after the Affinity argument is inexplicable. We may then suppose that the Affinity argument is not independent of the previous ones, and that the Antapodosis argument is not dropped but only one of its premisses is challenged. In this case, the Affinity argument is intended to give an account of this challenged premiss, or to give an alternative proof of reincarnation.

From the six premisses of the Antapodosis argument (see p167-9) the only one that it is possible to challenge, is the fourth (that the living nature will never be extinguished) [122]. This premiss is the most debatable and unjustifiable, and the one that proves directly the soul's return to life and therefore reincarnation. If this premiss is challenged, then the Antapodosis argument collapses. We have no reason to suppose that Simmias and Cebes view this premiss to be false. It is simply uncertain and debatable. So if we want to give a reason for the reincarnation and the philosopher's life, we have either to prove this premiss or to base the Antapodosis argument on other premisses. Both reincarnation and the proposition 'nature will never be extinguished' can be proved (given that the other five premisses of the Antapodosis argument are valid) on the basis of the combined force of the following two premisses:

(a) post-mortem existence of the soul

(b) as far as the soul exists, it alternates between life and death (let us call it the Antapodosis hypothesis) [123].

The second premiss is something that has already been proved in the first part of the Antapodosis argument (cf n.108). The Affinity argument 'proves' the first premiss, so giving support to the Antapodosis argument. However, the above two premisses are not enough to prove reincarnation (or the proposition "nature will never be extinguished").

We have already said that in the Antapodosis argument Plato distinguished between only two cases: i) a linear process that ends with the death of nature; ii) an endless cyclical process. He disregarded the case of a finite cyclical process and it seems that he had taken for granted that (c) every cyclical process is an endless one. Premiss (a) says only that the soul survives death for a period of time and not that it is immortal. Therefore, without premiss (c), we cannot prove reincarnation.

Cebes objects to premiss (c). There are not only two possible processes but also a third - the finite cyclical process. So given all the other premisses and even accepting the conclusion of the Affinity argument (that the soul survives death for some time), we cannot prove the endless character of the cyclical process and without it, reincarnation is not enough to guarantee the destiny of the soul after death. It is possible that after many reincarnations a soul may be exhausted and at last extinguished (87a-88b).

A theory concerning the _post mortem_ future of the soul can be based on reincarnation only if we first prove the essential imperishability of the soul.

Granted the soul's 'divinity' and affinity to Forms (Affinity argument) we cannot tackle the new problem relying on premisses about the uniformity of the nature. We must first inquire into a genuine theory of 'causation'. Cebes' objection calls for consideration of the _aitiai_ of becoming and perishing _in general_ (95e8-96a1; see _holos_ at 95e9). Only if we solve this problem can we inquire into the soul's immortality.

vi. Before Cebes' objection, Simmias had already raised another one (85e-86d). He proposes another conception of the soul. The soul has not substantiality but it is an 'attunement' (harmony) of the bodily constituents. So the soul can have all (or most) of the characteristics that the Affinity argument attributed to it without being immortal (or even being more durable than the body). Simmias' theory is not a mere counter-example of the Affinity argument as many commentators have supposed [124]. It is something more. It objects to the basic premiss of all three arguments so far, that the soul is a substantially different thing from the body. Because of that, Socrates proceeds to refute Simmias' theory (92e-94c) although Simmias himself had previously admitted (91e-92d) that the theory of Recollection and the theory that the soul is harmony contradict each other and had accepted that the theory of Recollection is preferable because it had "come from a hypothesis worthy of acceptance" (92d6-7).

As Gallop (1975, 157) remarks, it is plausible to see Simmias' withdrawal at 92c-d "as an application ot the 'hypothetical method' that Socrates will describe at 100a3-7". The Harmony theory condradicts (diaphonein) the Recollection one which is derived 'from a hypothesis worthy of acceptance' (92d6-7 , cf c1-2) ie the Theory of Forms. Therefore it must be considered false. Note also the similarity between synodo and synodos at 92b5-8 and symphonein and me symphonein at 100a5-8 both in language and meaning (see p133). We had earlier (85b10-d10) a reference to the method of hypothesis described at 100a5-8, again by Simmias (see p150). Simmias speaks in the same terms as Socrates in 99c-100a (cf especially 85c7-d2 with 99c8-d1). We can find similarity between Socrates' warning against misology (especially at 90b4-91c5) and his resort to logoi at 99e-100a (see Gallop, 1975, 154) and at 101e. The treatment of the arguments by antilogikos (see 90-bc, 101e) is exactly the opposite of that of the philosopher who uses the hypothetical method.

Socrates, refuting Simmias' theory, shows that the soul is not a mere epiphenomenon of the body and so establishes indirectly the substantiality of the soul. In Simmias' theory we can see an example of the second part of the hypothetical method. The 'harmony' theory is twice called 'hypothesis' (93c10: "but obviously anyone maintaining the hypothesis would say something of that sort"; 94b1: "..if the hypothesis that soul is attunement were correct") and is refuted because it leads to contradictions [125].

vii. After Cebes' objection, Socrates posits the new problem we have to solve if we want to give reason for the philosopher's life (95c-d), that is the immortality of the soul (95c1, c8, d1). It is the first time that Socrates himself says clearly that his problem is the immortality of the soul. Up till now, he has spoken about the doctrine of Paligenesis or reincarnation (70c-d, 81-83). He has also said that the soul exists in Hades (71e2), that the soul comes to life again (72a7-8, d8-9, 77d1-4; at 77d1-4 the soul's survival of death is a corollary of its comming to life again), that the soul existed before life and had wisdom (76c11-13, 77a) and that the soul is non-dispersible after death (77b, 80b).

Socrates develops the argument according to his method trying to secure all the presuppositions that can justify the philosopher's life and attitude in front of death and so depends upon the argument from 'higher' hypotheses.

At 95cff Socrates sets out to prove the immortality and imperishability of the soul [126]. But this problem is reduced to the more general one of finding the aitiai of the coming-to-be and the perishing of all things (95e-96a). Only if we solve the latter can we solve the former.

There are now two different theories that give an explanation of "the reasons for each thing, why each thing comes to be, why it perishes, and why it exists" (96a9-11). The first is that of the physical philosopher who relies on sense-experience, which leads to contradictions (96e-97b, 100e-101c) [127]. Socrates' explanation relies on the theory of Forms, and mainly on the participation of the particulars to Forms (100c-d).

At this point Socrates explains the hypothetical method [128] and says that on the basis of the theory of Forms he will give an explanation (theory) about 'causation' and after that he will prove the immortality of the soul (100b). Here we can see clearly the application of the third step of the method. The immortality of the soul depends on (the hypothesis of) Socrates' theory of 'causation' (cf. also 95e4- 96a4) and the latter on (the hypothesis of) the theory of Forms. We might trace in 100c-e an application of the activity of 'seeing whether the consequences of the hypothesis are in accord or disaccord with each other'. The consequences of Socrates' theory of Causation are in accord with each other having in this way a confirmation of the hypothesis.

viii. At 102a10-b3 Socrates and his interlocutors agree (see homologeito at 101b1) on the theory of Forms (existence of Forms and participation of the particulars in them). After that Socrates proceeds to expound his theory of causality [129]. In

102b3-d4 he shows how it is possible for a particular to have at the same time two opposite properties (when the opposite properties are not essential attributes of it; 102c1-4).

In 102d5-103a3 Socrates states the principle of Exclusion of Opposites: "F-ness and f-ness in us never admit their opposite". If the opposite Form (or form in us)'advances' then F-ness (or the f-ness in us) either 'perishes' or 'gets out of the way' [130]. The principle of Exclusion of Opposites concerns not things but their attributes (103b). So the apparent contradiction between it and the principle of Opposites at 70e-71a vanishes. Not only that, but the principle of Exclusion of Opposites gives reason for the principle of Opposites. The principle of Opposites says that 'x becomes large from being small' and thus shows how change (becoming and perishing) is possible. The principle of exclusion of Opposites says that 'when largeness in x (or the Largeness) advances, the smallness in x (or the Smallness) perishes or withdraws' and so 'x changes and becomes large from being small'. Thus the principle of Opposites is an inference from (or a case of) the principle of Exclusion of Opposites, and the latter gives an explanation of the possibility of becoming and perishing [131]. The principle of Opposites -basic premiss for the proof of reincarnation- is now dependent upon the theory of Forms. This is the reason why Plato here (103a-c) explicitly refers back to 70a-71a.

Socrates goes on now to formulate his principle of the 'subtle' aitiai that constitutes the main premiss of the Last argument: "it is not only the opposite that does not admit its opposite; there is also that which brings up an opposite into whatever it enters itself; and that thing, the very thing that brings it up, never admits the quality opposed to the one that's brought up" (105a1-5); or in other words: "if A necessarily entails F, then it cannot admit G" (F and G opposites) [132]. The above principle is directly dependent on the principle of Exclusion of Opposites.

The Last argument runs now as follows:

If A necessarily accompanies F then A cannot admit G (105a1-5).

The soul necessarily accompanies life (105d3-4).

Life is opposite to death (105d6-9).

∴ The soul cannot admit death (105e4-5).

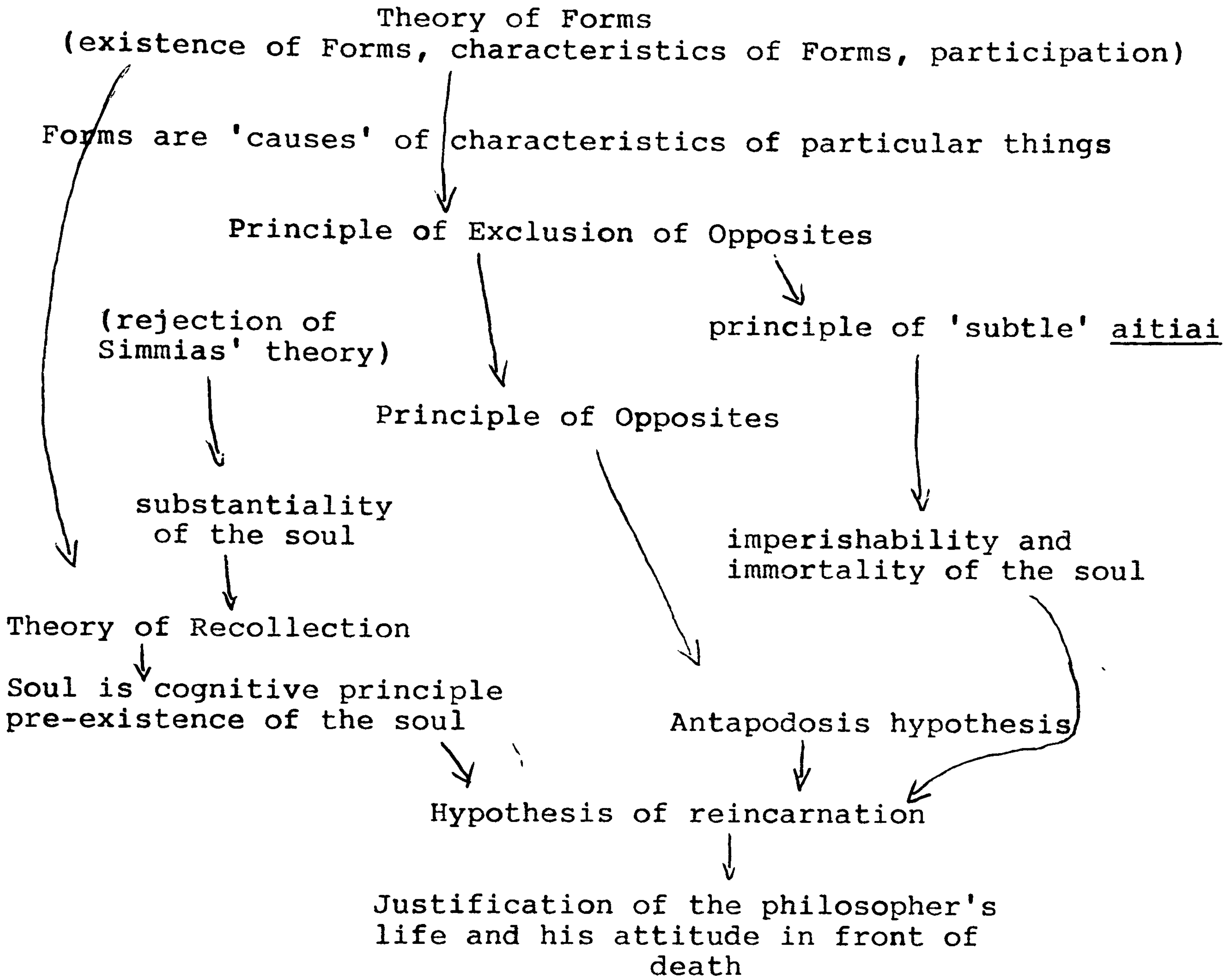
What will not admit death is immortal (105d13-e3).

∴ The soul is immortal (105e6-7).

Now another argument follows to show that the soul, being immortal, is imperishable [133].

Since the soul cannot admit death, it must either withdraw or perish at the approach of death (106b-c; cf. 102d5-103a2). But something perishes when it ceases to be what it was before, that is when it loses its essential attribute. So a lump of snow perishes at the approach of heat and becomes water (see 106a3-6). Now if the immortal is imperishable, then the soul is imperishable (106c9-d1). The essential character of the immortal (or of the soul) is 'being alive' (see 105d3-4). The immortal (soul) perishes when it ceases to live (and so becomes dead). The immortal (soul) never admits death (see 105e4-5). Therefore the immortal (soul) is imperishable (106e1-4) [134].

ix. We can see the exemplification of the hypothetical method in the _Phaedo_ in the following diagram.



The second part of the method was practised twice, in the rejections of the Simmias' theory about the soul and the physical philosopher's theory of 'causality' (and perhaps at 100c-e in the confirmation of Soctrates' theory of causation via the agreement of its consequences).

The way up and the way down of the method are not separated but occur in every stage between a hypothesis and a higher one, so following the model of <u>apagoge</u>.

Besides this reconstruction of the whole argument of the dialogue, the many terminological points that I have made show again that the hypothesis that Plato applies his method in the dialogue is not implausible.

NOTES TO CHAPTER V

[1] The translation is from Gallop (1975). Hereafter I shall use his translation.

[2] See Guthrie, 1975, 352. Bluck (1955, 13-4, 104-6) translates logos at 100a1,4 as Socratic definition. For Archer-Hind, 1883, 139: the hypothesis is the "notion or definition, λόγος under which the object to be explained falls".

[3] Cf A E Taylor (1937, 201); Hackforth (1955, ad loc.); Robinson (1953, 126); Sayre (1969, 5); Ross (1951, 27); Burnet (1911, ad loc); Hardie (1936, 66); Stahl (1960, 426-7) and Bostock (1986, 160-1).

[4] Cf Gulley, 1962, 41; Dorter, 1982, 127; Sayre, 1969, 6 n.6; Stahl, 1960, 427. The word hekastote at 100a4 might show again the generality of the meth

[5] Archer-Hind (1883, 136-7) takes the "it is not something new" to refer to the method of hypothesis and not to Forms (similarly Stahl, 1960, 429). This passage forces Burnet (1911, ad loc) and A E Taylor, 1937, p176, pp202-3) (who hold that the Phaedo presents the real Socrates) to accept that the theory of Forms is Socratic and not Platonic. Burnet say that "if Plato had been the real author of the 'Theory of ideas', and if, as is commonly believed, it was pronounced for the first time in the Phaedo, this sentence would be a pure mystification" (pp110-11). However, the 'not something new' can plausibly refer to the beginning of the dialogue (65d-66a) where Socrates (Plato) introduces for the first time the Forms. (see also Grube, 1935, Appendix 1). Bluck (1955, 161sq) thinks that at 100b1 we have a shifting from Socratic to Platonic doctrine.

[6] See Hackforth, 1955, 143: "It is the existence of Forms that

is assumed, but nothing beyond that. There is a relation between Forms and sensibles, a relation which involves causality, is not part of the hypothesis, but rather regarded as an immediate and inseparable consequence of their existence". Cf also Dorter, 1982, 128; Ross, 1951, 28; Plass, 1960, 104; Stahl, 1960, 429-30. On the other hand, for the argument that the hypothesis is the whole theory of Froms, see Bedu-Addo (1979, 129 n.15) and Hardie (1936, 67).

[7] In this passage, the words <u>homologeito</u> and <u>synechorethe</u> refer both to the existence of Forms and the theory of participation.

[8] Another equally plausible interpretation is to distinguish between the participation of the particulars to Forms from Socrates' theory of causation. In this way, the hypothesis is that Forms exist <u>and</u> that particulars participate to Forms; the theory of causation is an immediate inference of this hypothesis. This interpretation might find support from passages 95e-96a and 99c-d, which show that the present problem is the inquiry into the 'causes' and that the problem of the immortality of the soul depends on it.

[9] From the existence of something we cannot infer anything. but the existence of x is a necessary condition for every assertion about x. cf 74a12: "... the equal itself: are we to say that there is something or nothing?"

[10] Bluck (1955, p14 and Appendix 6) expresses the view that the hypotheses of 100b-101e are not propositions but 'provisional notions' of Platonic Forms and therefore the passage is not an account of propositional reasoning at all. On the other hand, the <u>logoi</u> at 99e-100a are Socratic definitions and the two passages

(100a and 101d) do not describe one and the same method (cf also Bluck (1957)). Bluck's views have been well critisized by Ackrill (1956), Cross (1956), Lemmon (1957) and Rosenmeyer (1956) in their reviews of his book. Bluck has considerably revised his opinions in his Plato's Meno (1961, pp101-4). I do not intend to critisize Bluck's interpretation here.

[11] At 100a, Plato does not say that the hypothesis is posited in order to prove a proposition (or to solve a problem). However, at 101d, it is clear that the 'higher' hypothesis is posited in order to confirm the 'lower' and Plato says that this must be done "in the same way", that is, as at 100a. It seems then that the hypothesis is posited in order to give an account of something.

[12] The 'safe' causes of 100c-e obtain their safety from the fact that they are uninformative. Cf Vlastos (1969/1973); C C W Taylor (1969) and Burge (1971).

[13] Cf 85c, where Simmias said that one should "adopt the best and least refutable logos. In passage 92d, Simmias admits that the theory of recollection was based on a "hypothesis worthy of acceptance". This means that one must have good grounds for positing a hypothesis.

[14] We might regard that the procedure of positing the 'strongest hypothesis' is a first test of the hypothesis (cf Bedo-Addo (1979, 115).

[15] Cf Bedu-Addo, 1979, 115-8. Bluck's interpretation of passage 100a is quite similar although he does not relate it to 100b-101c (1955/1976, p111). he takes the "ὑποθέμενος .... λόγον" to mean a Socratic (of the real Socrates) definition. So Socrates accepts as genuine instances (alethe) of the thing concerned whatever seems to conform (symphonein) to it and rejects

what did not. But such a procedure does not constitute any method. Moreover, nowhere in the Socratic (early Platonic) dialogues do we find such a procedure, and the rendering of the word 'true' at 100a5 as 'genuine instance' sounds very improbable. Cf also the critiques of Ackrill, Cross, Lemmon, Rosenmeyer (see n.10) and Gallop (1975, 180-1).

[16] Robinson (1953, 127) says that there is no other interpretation apart from the two above mentioned. "We have to chose between consistency and deducibility as the meaning of 'accord'. The better is consistency for more than one reason. The paradox to which it leads is much less than that to which deducibility leads". I cannot understand why there is not another interpretation (cf also Friedländer (1945, 255)).

Robinson although he accepts as more plausible the interpretation of consistency - inconsistency, he concludes (p128) that "Plato here does not say quite all that he means ... The whole idea in his mind is that which follows from the hypothesis is to be set down as true, and what whose contradictory follows from the hypoothesis is to be set down as false". Stahl (1960, 428) says that Plato confuses what does not follow from the hypothesis with what contradicts it.

[17] I take for the present, the word hormethenta to mean 'logical consequences' according to the classical interpretation of the passage. Cf Robinson, 1953, 129; A E Taylor, 1937, 201; Ross, 1951, 28; Hackforth, 1955, 139-40; Burnet, 1911, 114; Sayre, 1969, 29, 33; Murphy, 1936, 41; Dorter, 1982, 131; C C W Taylor, unpublished paper, p17; Bostock, 1986, 170.

I take passage 101d3-5 to be genuine and I am not going to discuss the various emendations of the text or the view that it is an interpolation (see Archer-Hind, 1883, 140; Jackson, <u>Journal of Philos.</u>, X, p148).

[18] Cf Robinson, 1953, 132-3; Bostock, 1986, 170-1.

[19] We must note again (see p82-3) that the consequences of a hypothesis are not consequences from only that proposition. In every argument there are many premisses, but only one is considered as the hypothesis - probably the most crucial for the argument. So Plato considers as the hypothesis, the main premiss which, in conjunction with a set of 'standing beliefs', gives the conclussion. Cf Rose, 1966, 471-2; Scolnicov, 1973, 258.

[20] See Bluck, 1957, 22-3; Ackrill (1958); Gallop, 1975, 180.

[21] The first interpretation of Sayre has already been critisized previously. In the second one (consistency - inconsistency), Sayre tries to avoid its undesirable conclusions, that we have already seen on page 127, by saying that Plato speaks in the context of Greek geometircal analysis. So although (at 100a), 'accord' means 'consistent' with, as a matter of fact, we have implications from the hypothesis. But then another problem arises: the meaning of the word 'accord' at 100a is different from that at 101d.

[22] It might be objected that at the same passage, Plato uses the verb ἔχεσθαι in two different ways in d1 and d3. In d1, ἐχόμενος means 'clinging to' (or 'holding firmly on to', or 'insisting') and in d3 ἔχοιτο has the sense of 'object to' (or 'take issue with' or challenge'). I do not think that the passage needs emendation (as Madvig's ἔφοιτο instead of ἔχοιτο - see Gallop (1975, 235); or Huby's (1959, 14 n.1) ὀχούμενος ἐπὶ instead of ἐχούμενος. As Gallop remarks "<u>echomenos</u> has occurred at 100d9 in precisely the sense required

at 101d1". The verb ἔχεσθαι changes its meaning according to the different contexts in which it occurs, and this is not unnatural in a colloquial passage (cf also Bluck (1955, 116). But we cannot accept the same for the word symphonein - a very crucial and central term in Plato's methodological account. This would imply a very astonishing carelessness on Plato's part.

[23] Cf Friedländer (1945, 255); Bostock (1986, 163).

[24] Cf Scolicov (1973, 81).

[25] See especially Gorg 482b7-c3 (Prot 333a, Gorg 457e, 461a, Laws 860c1, 861a10, 691a, Crat 413b.)

[26] For a smiliar view, see Scolicov (1973, 80-1).

[27] It is surprising that commentators have not paid attention to this passage in their interpretations of the hypothetical method in the Phaedo.

[28] Robinson (1953, 129), Sayre (1969, 33) and Hackforth (1955, 139-40) take ormethenta to mean logical consequence.

[29] Plass (1960, 110; cf 104-5) deems that the hormethenta "are in the broadest sense all relevant propositions brought up in the course of a discussion". He believes that the theory of participation is not a hypothesis but a consequence of the existence of Forms (I have already discussed this). Moreover, he regards the various explanations of particular things according to the natural philosophers as consequences (hormethenta) of the hypothesis. But in this way, passage 101d1-5 has no meaning, and does not constitute any test of the hypothesis, because we always have conflicting consequences, even if the hypothesis is true.

[30] I think that the passage of Crat 436d-e and especially the use of the word hepomena at 436d4 and d7, supports this opinion. Cf also Hackforth (1955, 139-40).

[31] Cf also Robinson (1953, 131)

[32] It seems strange that Plato advises us to test the hypothesis only when someone objects to it. Plato probably has in mind the usual practice of dialogue. The person who leads the discussion has good reasons to choose his hypothesis, and he is sure about the truth of it (the choice of the hypothesis constitutes then a first test of it). The hypothesis has so far passed the test and is regarded by him as confirmed (100e1, 101d2). If the interlocutors have no objection to the hypothesis, then we have agreement (homologia) and there is no need for further examination. However, we might suppose that even the leader of the discussion may object to the hypothesis.

[33] Exactly as in geometry where two theorems (when the one is not a premiss used for the proof of the other) which originated from the same set of axioms, are consistent with each other (within the framework of geometry). Cf Crat 436d2-4.

[34] Cf Meno 87d-89a, where the proposition "virtue is knowledge" contradicts the fact that "there are not teachers and students of virtue" although the hypothesis is true. The contradiction is due to the falsity of another premiss (If something is useful it is knowledge", 89a1-2) and not to the hypothesis.

[35] That the test of the hypothesis consists in the Socratic elenchus is shown from the rejection of the Simmias' hypothesis and that of the Physicists, where we arrive at contradictions among the consequences of the hypothesis by the use of elenchus. Cf also 85c (παντὶ τρόπῳ ἐλέγχειν ... δυσεξελεγκτότατον).

[36] See Stenzel, 1940, 7; Huby, 1959, 14; cf also Crombie, 1963, 540; Bluck, 1957, 24 about passage 100a.

[37] For other examples in the Platonic corpus, see Goodrich 1904, p9-10.

[38] Cf On the Ancient medicine, I, II, 25-6, XIII 1-19.

[39] There is a difference between Plato and Physicists. Plato's hypotheses work in an argumentative system while the Physicists' ones in an explanatory system. However, it seems that the hypothesis of Forms-Causes has both characteristics. It seems that doctors used their 'hypotheses' in both ways (see Lloyd, 1963).

[40] See Phd, 75b5-7: "If a man knows things, can he give an account [δοῦναι λόγον] or not? - Of course he can Socrates". At 95d6-e1, 'giving an account' and knowledge are very strictly connected. At 73a9-10, 'correct account' [ὀρθὸς λόγος] is also linked with knowledge (cf also 90c8-d7, Rep 534b, Theaet, 202c, Pol, 286a). Cross (1954, 442-9) pointed out, that in Plato's philosophy, logos is kept in the domain of language, and the strict connection between logos and knowledge shows that knowledge, for Plato, has a discursive or argumentative character. Cf Phd 65c, "so isn't it in reasoning [λογίζεσθαι] if anywhere at all, that any of the things that are become manifest to it [soul]?"; Phd 85c8-d, "... then adopt the best and least refutable of human doctrines [λόγον]"; Theaet 186d, "so knowledge is located not in our experience, but in our reasoning [συλλογισμῷ] about those things we mentioned.

[41] Cf also Theat 148d and Pol 285d8-9 where giving a logos of an object means giving a definition of it. For other interpretations of the 'giving an account of a Form', see: Hackforth, 1955, 72 and Gulley, 1962, 46-7.

[42] Cf Phaedo 78d1-3 ("is the being itself, whose being we

give an account of in asking and answering questions...") and Rep, 534b8-c2 where even as regards the nature of the Form of the Good the dialectician must distinguish it through rational arguments from all the other Forms and must submit it to every sort of elenchus.

[43] See Phd 95d, "anyone who neither knows nor can give a proof [account - λόγον διδόναι] that it's immortal should be afraid unless he has no sense" (Gallop is right here in rendering 'λόγον διδόναι' as giving a proof because the main problem in the dialogue is to give a proof of the immortality of the soul); Phd 63e9, where the expression 'διδόναι λόγον' is used for the (argumentative) defense of Soctrates' position that the philospher must not fear death. Cf also Gorg, 510a and Crat, 426a.

[44] Cf also Gallop, 1975, 190; Robinson, 1953, 136; Hackforth, 1955, 141; Bostock, 1986, 164

[45] The expression 'in the same way' surely refers to something described previously. The only recommendation mentioned in passage 101d is to check the consistency of the consequences of the hypothesis. Moreover, it is hardly acceptable that this check describes a positive procedure for 'giving and account' of the hypothesis. We may suppose then that Plato, saying "in the same way" refers not only to the check of the hypothesis (101d3-5) but also to 100a-b. This is also supported from the "hypothesizing another hypothesis" (101d6-7). Sayre (1969, 31) remarks that "this is one more reason for considering the remark about consistency at 101d as an extension of what was said at 100a.

[46] For a different view, see Bluck (1955) p169. According to him, 'giving a account' of the hypothesis means giving a "verbal explanation, a kind of definition".

[47] Is is also the most accepted interpretation: cf Robinson, 1953, 136-7; Hackforth, 1955, 141, Crombie, 1963, 543; Sayre, 1969, 32 and Gallop, 1975, 190

[48] Cf Archer-Hind, 1883, 140-1; Burnet, 1911, 114; Bluck, 1955, 15-17, 169-70; Raven, 1965, 99.

[49] Cf also Robinson's comments (1953, 137) on Archer-Hind

[50] For a full criticism of Bluck's views, see Ackrill (1956), Cross (1956), Lemmon (1957), Rosenmeyer (1956); cf also Sayre, 1969, 31-2

[51] Strictly speaking, we find the two opposite ways (up and down) in both passages 100a and 101d5-e1. In 100a, we find the way up in the act of "hypothesising on each occasion the <u>logos</u> I judge strongest" and in 101d5-e1, we find the way down in the deduction of the 'lower' hypothesis from the 'higher' one. However, in 100a, the emphasis is put on the way down and in 101d5-e1, on the way up. Within each step of the upward movement, say between Hn and Hn+I, there is an opposite movement (deduction) from Hn+I to Hn as described at 100a. We can schematicaly present the process in the following way;

A←B←C←...←G←H←B1←C1←...←G1←H1←B2←...←G2←H2←........←I

(I use the symbol ← to indicate deduction and the symbol ⇢ to indicate the ascent to higher hypotheses. 'A' stands for the original problem and 'I' for the 'adequate').

[52] Cf also Scolnicov, 1973, 83; Bluck, 1961, 103; Bedu-Addo, 1979, 123. My use of the word intuition here has always this specific meaning: it is not an intuition that gives direct knowledge or a blind grasp of the truth. It is rather a bright idea, a flash that helps us find a better judgement or leads us to a better basis or to a good proposal.

[53] See the manner in which the hypothesis of the Forms-causes is obtained at 100c. See also the same forthe law of opposites and the theory of recollection. Saying that the ascent from Hn-1 to Hn has an intuitive character while the descent from Hn to Hn-1 (Hn→Gn→...→Cn→Bn→Hn-1) is deductive, does not mean that between Hn and Hn-1, we always have one way implication (Hn →Hn-1). We may very well have equivalences (Hn↔Hn-1). What I want to emphasize is not so much the existence of a one or two way implication between a hypothesis and its 'higher', than the manner in which the philosopher proceeds in his finding of 'higher' hypotheses. In the course of the upward movement we do not draw conclusions from Hn-1 and so arrive at Hn, but we grasp intuitively the Hn as a premiss for the proof of Hn-1. The ascent to higher hypotheses is an heuristic process of finding premisses and proofs and not a demonstrative one (cf 101e3). When someone works in an heuristic way in order to find something, he functions psychologically differently from someone who draws logical consequences from given premisses. But of course, after one has discovered a proof, one can set it out deductively.

The above interpretation of the ascent to 'higher' hypotheses, although emphasizes the heuristic character of the ascent and the great role that intuition plays, does not exclude the possibility of a hypothesis to be found deductively from a 'lower' one. However, deductiveness is not the main characteristic of the ascent to 'higher' hypotheses, although a possible one.

[54] The reference of 'τοῦτο αὐτὸ' at 107b9 is uncertain. Two interpretations have been proposed: a) "if you make sure that you have followed up the argument as far as is humanly possible" (Burnet, 1911, p124) and b) "if the security of the hypotheses and

the validity of the deductions from them become plain" (Archer-Hind, 1883, p157). I find the first interpretation, that is also the most accepted (Hackforth, 1955, p161; Goodrich, 1904, p7; Shipton, 1979, p52, n.22; Bluck, 1955, p23, n.5), more likely. If we accept the second reading, it is difficult to explain Plato's recommentation for a further analysis.

[55] Gallop, 1975, p222. According to Bluck (1955, p23 n.5), the 'initial hypotheses' are "the most important of the notions of the 'causes' (ie Forms) concerned.

[56] Robinson, 1953, p137. See also A E Taylor, 1937, 201; Hackforth, 1955, 141; Dorter, 1982, 133; White, 1976, 77; Raven, 1965, 98; Crombie, 1963, 543; Matthews, 1972, 33; Plass, 1960, 113. Bluck (1961, 103) thinks that the 'adequate' is an homologema.

[57] Cf Gorg, 482b-c. Friedländer (1945, 256) says that: "when Socrates says 'you', he no doubt speaks of his own method; he means 'I' as well. The oddity would follow that Socrates is more easily satisfied than a somebody".

[58] Cf Tait, 1952, p113.

[59] 1951, p28. Somewhat similar is the view of Sayre (1969, 37) that the 'adequate' is one which is no longer open to challenge in the context in question.

[60] Cf 99e6 ("the truth of the things"), 65b9 ("when does the soul attain the truth"), 66a ("using his intellect alone, by itself and unsullied, he would undertake the hunt for each of the things that are, each alone by itself and unsullied..... and doesn't allow to gain truth and wisdom when in partnership with it.."). Cf also Gorg, 4872: "agreement between you and me will finally possess the goal of truth", Soph, 246d: "however, we are

not concerned with them so much as with our search for the truth" and Gorg, 482a.

[61] Cherniss (1947, 141) arguing against Robinson's interpretation that hikanon is 'something adequate to satisfy an objector' says: "ἐπειδὴ δὲ ἐκείνης αὐτῆς δέοι σε διδόναι λόγον certainly does not imply that you won't do so at all unless he does so object... Nor has Robinson any reason for saying that this ἱκανὸν is meant to be an hypothesis in the same sense as the hypothesis which lead up to it. The very wording τι ἱκανὸν in contrast to the ὑπόθεσιν ἥτις ... βελτίστη immediately preceding implies that Plato is thinking of it as something different, and the περὶ της ἀρχῆς immediately following suggest that he is thinking of it as an ἀρχὴ in a special sense."

[62] Although the 'adequate' is not derivable from anything else, we can plausibly suppose that the test by elenchus described at 101d4-5 may be applied to it. (cf Rep, 534b-c, where we are told that we can use elenchus even to the Good).

[63] I follow Shipton (1979, 35-4) as regards the meaning of atrapos. I do not intend to discuss the problem of the interpretation of the highly disputable passage, 66b. Nevertheless, that there is a way for philosophers to approach knnowledge in this life is clearly stated in 67a and 65e-66a.

[64] Cf 99c8-9: "but since I was deprived of this, proving unable either to find it for myself or to learn it from anyone else...". Cf also Archytas, D.K., 47, frag. B3: "Δεῖ γὰρ ἢ μαθόντα παρ' ἄλλῳ ἢ αὐτὸν ἐξευρόντα, ὧν ἀνεπιστάμων ἦσθα, ἐπιστάμονα γενέσθαι".

[65] I do not discuss the problem of what Socrates' second voyage consists in (if it is, a) the hypothetical method, b) the

search for formal causes, or c) the theory of Forms).

[66] See Friedländer, 1945, p256; Stahl, 1960, 435; Cherniss, 1947, p141 ("There is every reason to take the ἱκανὸν here as the equivalent of the ἀρχὴ ἀνυπόθετος of the Republic or rather to take the latter as a special case of the former"); Bedu-Addo, 1979, p124 n.24. Friedländer says that "there is hardly anyone who has not identified the ἱκανόν of the Phaedo with the ἀνυπόθετον of the Republic". But many commentators before Friedländer did not identify them (see Burnet, 1911, 114; Goodrich, 1904, 7; Stenzel, 1940, 10; Adam, 1902, 175; Archer-Hind, 1883, 141).

[67] Cf Archer-Hind, 1883, 141; Robinson, 1953, 143-4.

[68] Hackforth, 1955, 141. Cf also Tait, 1952, 114 ("To be truth, ἀλήθεια need not to be full truth... Truth will have been reached adequate to the subject of investigation as thus far comprehended .... [The adequate] has epistomological significance and ontological status even if it falls short of the full reality and intelligibility of the ἀρχή ἀνυπόθετος").

[69] For a similar interpretation of the 'adequate' (as something self-evident) see Bostock, 1986, 174-5. Cherniss (1947, 141-2, n.40) refers to Aristotle's Met 1005a19-b34 in order to support his view that the 'adequate' of the Phaedo is the same as the non-hypothetical principle of the Republic. However, I think that Aristotle's passage corroborates rather our interpretation than that of Cherniss. At this passage, Aristotle refers to the principles of sciences and philosophy. The various axioms of the sciences are said to be 'adequate' (hikanon) for carrying out the demonstrations in each special field (a25-6). Scientists who work within the limits of their field do not say

anything about their axioms as to whether or not they are true (a29-31). On the other hand, the law of non-contradiction ('which is the principle of all axioms - b33-4) is said to be non-hypothetical (_anypotheton_). It seems that Aristotle here has in mind Plato's epistemology; and we may suppose that the difference he finds between the axioms of the sciences and the law of non-contradiction is similar to that between the _hikanon_ of the _Phaedo_ and the _anypotheton_ of the _Republic_.

[70] The _arche_ (101e2) may either be a hypothesis (Robinson, 1953, p140; Burnet, 1911, p115; Crombie, 1963, p545; Archer-Hind, 1883, p141; Bostock, 1986, 167) or an 'adequate' (Cherniss, 1947, 141; Bedu-Addo, 1979, p123). We already know (see p47) that the word _arche_ may signify a relative starting-point for a proof. The 'discussing the _arche_' may mean either 'discussing _what_ hypothesis to postulate as an _arche_', or 'discussing how to prove this _arche_. (I owe this distinction to Prof Ackrill). I do not enter into the discussion of these problems. However, in _Crat_, 436d (see p135), it is quite clear that the _arche_ is not an unquestionable principle and the 'discussion of the _arche_ means 'discussion of its validity'.

[71] Aristotle (_EN_, 1095a32) says that Plato attached great importance to keeping apart the argument towards _arche_ and from it and he used to ask: "are we on the way from or to the first principles?" Bluck (1955, 171) thinks that the 'contradiction mongers' confuse propositions about the Forms with propositions about particulars named after them. Shipton (1979, 46) thinks that Plato refers mainly to scientists who confuse a cause with the necessary conditions of the cause (see 99a-b).

[72] The procedure described at 100a "is the proper aim of the

hypothetical method, namely, draw the consequences of the hypothesis and posit them as true" (1953, 134). Huby (1959) claims that in the 'hypotheses' passage of the Phaedo, Plato does not "announce a new theory of dialectic but an affirmation of certain quite elementary principles of psychological as much as of methodological importance" (p12); and that "it is probably a mistake to suppose that Plato at this stage at any rate had a clear concept of method in the abstract" (p14). Huby, like Robinson, does not consider the upward process described at 101d5-e1 as relevant, and she does not take it into account.

[73] See Tait, 1952, 113; Bedu-Addo, 1979, 122-3.

[74] Probably the activity of "seeing whether the results accord of discord with each other" (101d3-5) belongs to the first part. Passage 101d3-5 presents a test of the hypothesis and so it has something to do with its truth, for if we find inconsistency of some kind in the consequences of the hypothesis, we know not only that the hypothesis does not solve our problems, but also that it cannot be true. However, this is made by discussing the consequences of the hypothesis (cf 101e2-3).

[75] The method is completed by the activity of 'seeing whether any contradiction arises out of the hypothesis'. This activity also occurs in every step of the method.

[76] see Burnet (1911) on 101d3; Raven (1965, 96). Cf also Goodrich, 1904, 9-10.

[77] In Socratic method we may have mental experiments to find counter-examples, but we have no evidence that Plato considers this possibility in his method in the Phaedo

[78] cf 70d-71b, principle of Opposites; 100b9-e7 'safe causes'; 102d-103c principle of Exclusion of Opposites; 103d-105c 'subtle

causes'.

[79] see Carnap, 1950, C. G. Hempel, 1965; contra K Popper, 1959 and the 'non-rationalists' Kuhn, 1970 and P K Feyerabend, 1975. Bedu-Addo (1979, 130, n.33) thinks that Plato's method is similar to Popper's account of scientific progress depending on the notion of 'falsifiability'. However, Plato's hypothesis is not always proved false (by checking the consequences of it) and the 'higher' hypothesis is not a better reformulation of the 'lower' one (see p143)

[80] For instance, Newtonian mechanics can be conceived as a specific case (for velocities much less than C) of relativistic mechanics - the former being derivable from the latter. So Einstein's theory is a more general one that covers Newtonian one.

[81] see P.Natorp, 1903/1929, 130-1; J. Stenzel, 1940, 11, 38; R. Hackforth, 1955, 6; A.E. Taylor, 1937, 201-2; G. Cambiano, 1967, 140; Cambiano, 1971, 145; Raven, 1965, 96; Murphy, 1936, 41; Bluck, 1961, 85, 102; Scolnicov, 1973, 81-4 and chapter IX; Lemmon, 1957, 113.

[82] See Sayre, 1969, 22-4; Stahl, 1960; Cornford, 1932/1963, 87 n.3; Scolnicov, 1973, 81-4; C C W Talyor, unpubl. paper, 11-15

[83] Sayre (1963, 9-10, 25-7,33-9; and Stahl (1960, 421, 423, 426) (cf also Matthews p31-3) claim that Plato's method employs only convertible propositions exactly like the geometrical method of analysis and synthesis. But convertibility is not a necessary feature of the method of geometrical analysis. Dialectic, to an even greater extent, employs non-convertible propositions and Plato is conscious of it (cf <u>Pol.</u> 263b7-10, <u>Prot.</u> 350c, <u>Euthyp.</u>12c). Furthermore Plato's practice of the method (cf section 6) does not employ only convertible propositions (Sayre

recognises later at p37 that the Last argument does not involve convertible propositions).

[84] cf also Sayre p21: "it is entirely unlikely that Plato should have written on the subject of Philosophic methodology without considering, in the process, comparable methods which had already proven fruitful in contemporary mathematics".

[85] cf also Sayre(1969) pp22-3.

[86] We must note that in the passage of _Ethics_, Aristotle speaks of the process of deliberation in the sphere of action; and in the process of deliberation we have not propositional reasoning. However, Aristotle compares this process with geometrical analysis where we have propositional reasoning. There is another remarkable difference between the 'hypothesis' of Plato and Aristotle. Aristotle hypothesizes the proposition we have to prove (or the problem we have to solve). Plato does not call 'hypothesis' the _demonstranda_ but a proposition via which we shall prove our original proposition.

[87] cf Cherniss, 1951, 419.

[88] There is a possible difference between Albinus' account and Plato's description of the hypothetical method. Plato (at 100a) does not say that we hypothesize a _logos_ in order to prove (or to find) something, but simply that we hypothesize a _logos_ and we draw its conclusions. However, according to Plato's elucidation of the method at 100b, it is clear that the hypothesis is posited in order to prove a proposition (or to solve a problem). From the above account of Plato's hypothetical method in the _Phaedo_ we can observe the following: a) Albinus thinks that the two methodological accounts of Plato in the _Phaedo_ (100a and 101d) present one single method; b) he interprets the 'agree' of 100a as

'follow'; c) he drops the second part of the method ("seeing whether any contradiction arises out of the hypothesis"), probably regarding it as a non-essential characteristic of it; d) instead of 'adequate' he speaks about 'unhypothesized principle', but the word 'some' (tina) shows that this principle is not one unique and absolute like the 'Good' of the Republic.

[89] Albinus Didaskalikos V 5

Τὸ δὲ δεύτερον εἶδος τῆς ἀναλύσεως τοιοῦτον τί εστιν· ὑποτίθεσθαι δεῖ τὸ ζητούμενον, καὶ θεωρεῖν τίνα ἐστι πρότερα αὐτοῦ, καὶ ταῦτα ἀποδεικνύειν ἀπὸ τῶν ὑστέρων ἐπὶ τὰ πρότερα ἀνιόντα, ἕως ἂν ἔλθωμεν ἐπὶ τὸ πρῶτον καὶ ὁμολογούμενον, ἀπὸ τούτου δὲ ἀρξάμενοι ἐπὶ τὸ ζητούμενον κατελευσόμεθα συνθετικῷ τρόπῳ.

[90] Hippocrates of Chios had solved the problem of the squaring of the lunes, via three successive reductions (see p46).

[91] See p50. cf also T Heath 1921, I, 291: "Analysis" was "according to the ancient view nothing more than a series of successive reductions of a theorem or problem till it is finally reduced to a theorem or problem already known........... Not only did Hippocrates of Chios reduce the problem of duplicating the cube to that of finding two mean proportionals, but it is clear that the method of analysis in the sense of reduction must have been in use by the Pythagoreans".

[92] Cf e.g. R.Robinson, 1953, 142; D.Ross, 1951, 29); D.Gallop, 1975, 157.

[93] Cf R.Robinson, 1953, 142: "The second part....is realized in the deductions from the minor hypothesis that soul is harmony. That proposition leads the 'contradictions' that all souls are equally good (94a), and that soul never rules over body; and has

to be abandoned". Cf. also D.Gallop 1975, 157, 189. It is not certain whether the second part is a necessary one or depends only on the possible objection of the interlocutor. Moreover it seems that it is not a "distinct operation from the first, but rather a possibility for which we look out while engaged in the first" (Robinson, 1953, 142; see n.74 of this chapter).

[94] Cf eg R.Robinson, 1953, 142; D.Ross, 1951, 29; R.Hackforth, 1955, 140; D.Gallop, 1975, 190.

[95] Cf eg Archer-Hind, 1883, 8-9; A.E.Taylor, 1937, 177; D.Gallop, 1975, 103; Gomperz, 1905, 32.

[96] The only commentators who have a similar view are K.Dorter (1982, 131-7), S.Scolnicov (1973, chapter 4) and Stahl (1960, 421-6). The first finds an application of the third stage of the method in the passage 94a-107b (see my critique of him, p143). The second deems that all the dialogue is an exercise of the method (as going up to higher hypotheses). However his account about how the regress to higher hypotheses is practised in the Phaedo is not clear. The third thinks that the method involves only equivalent propositions and wherever he finds equivalences in the Phaedo, he argues that we have an application of the method.

[97] Cf Gorg.524b. It seems that this definition of death prejudges the question of the soul's survival (see especially 64c5-8). As Gallop rightly remarks (1975, pp.86-7) "'being dead' is taken to include 'the soul's being apart, alone by itself, separated from the body'. From this definition, conjoined with the admission that there is such a thing as death, it follows that the soul does exist apart from the body. If it did not, there would be no such thing as death, in the sense given to the word at

64c5-8". This definition captures the Homeric conception of psyche.

[98] The theory of Forms introduced here does not play any important role and is not necessary for the argument. Plato introduces and explains the theory of Forms in the Phaedo gradually (cf Grube, 1935, appendix 1). Note that the term eidos is introduced only at 102b1. The only thing we learn here about the Forms is that they exist and that they can be apprehended only by the soul.

[99] Cf p150 and note 63. Passage 66b2-4 is very similar in meaning to 99e-100a where Socrates, dismissing the empirical science, takes refuge in logoi, in order to find the truth of things. The conception of a (gradual) approach to knowledge, stated here, is realized in the ascent to 'higher' hypotheses at 101d.

[100] After that two short arguments follow (68c-69b) in order to show that only the philosopher is truly brave and temperate.

[101] Cf 71e8, 72a11, b8. Other labels used by the commentators for the same argument are 'Cyclical argument' and 'argument from opposites'.

[102] That the soul possesses 'some power' is also 'proved' by the Antapodosis argument since it states that the soul has an independent existence from the body, on both pre-natal and post-mortem forms. However, this power does not justify the hope of the philosopher.

[103] J.Barnes, 1978, pp401-5. I follow his formulation of the argument.

[104] I am not going to discuss the problem of the evaluation of

the argument, which depends mainly on the meaning we will ascribe to the term 'opposite', and on whether 'life' and 'death' can be characterized as opposite properties.

[105] "It was said then that one opposite thing comes to be from another opposite thing; what we're saying now is that the opposite itself could never come to be opposite to itself....Then, my friend, we were talking about things that have opposites" (103b2-7). Cf also 71a10, and Olympiodorus (In Platonis Phaedonem Scholia, ed.W.Norvin) p129.14-6.

[106] Cf 70e10-71a1 "And again, if it comes to be smaller, it will come to be smaller later from being larger before" (all verbs have the same subject). Cf also Strato (in Olympiodorus, ed. Norvin, p.221): "Opposites may come from opposites if a substratum persists but not if it is destroyed".

[107] Cf 70c5, 70c7-8, d1-2,7, 71e2, 72a7-8, 72d8-9, 73a1-2, 77d1-4. Many times in the Phaedo (eg 76e1-2, 115c3-5) self and soul are taken as synonymous; see also T.M.Robinson, 1970, 22-3, 32-3. It seems that Plato, taking the soul to be the object that underlies the change between death and life, begs the question (cf Gallop, 1975, 106, 109-10; R.Loriaux, 1969, 131). K.Dorter (1982, 36) very strangely takes the view that the underlying entity between life and death is not the soul, but both soul and body (not the composite), considered not as individuals but as corporeality and undifferentiated psychic-stuff correspondingly.

[108] The only way to work the argument is to take 'life' to mean 'union of soul and body' and 'death', 'separation of the soul from the body' (cf 64c2-5). In this way a soul has the property of 'being alive' when it is incarnated, and the property of 'being

dead' when it is discarnated. Thus, a 'dead' soul is not only a 'no londer-alive' soul, but also a 'not yet alive' one. Although Plato's language is ambiguous, I think that he may have been conscious of this difference and hence he regarded that the the first part of the Antapodosis argument (70d7-71e2) shows only the pre-natal existence of the soul (strictly speaking, the argument proves that the soul existed before birth and that as far as it exists, it alternates between life and death). In 70d7-71e2 Plato says that we 'come to be from the dead' and not that we 'come to be from the dead again'; only at 71e14 does he speak about anavioskesthai (cf also Barnes, 1978, 417 n.44; Hackforth, 1955, 64; Crombie 1962, 307-8). Otherwise I cannot find any reason for the introduction of the other two arguments (71e4-72a10, 72a11-72e1).

[109] In 71e3-72e1 the word 'dead' has the meaning of 'no longer alive'.

[110] I take 'the other things' to mean 'not yet alive souls', that is, sources or stock of souls waiting to come to life (cf Gallop 1975, 112; Bluck 1955, 19).

[111] Cf Rep. 611a where it is argued that the number of souls must be always the same.

[112] Cf also T.M.Robinson, 1970, 26; D.Gallop, 1975, 89; K.Dorter, 1982, 141.

[113] Cf P.Friedländer, 1969, 45.

[114] T.M.Robinson (1970, 26) says that Plato assumes the cognitive character of the soul at 71e2. But the only thing assumed there is the individuality and not the cognitive character of the soul.

[115] See Archer-Hind, 1883, 9-10; R.Bluck, 1955, 19-21; S.Scolnicov, 1973, 95; P.Friedländer, 1969, 44-5; A.E.Taylor, 1937, 185.

[116] See J.Burnet, 1911, 47; R.Bluck, 1955, 20-1; I.M.Crombie, 1962, 305; S.Scolnicov, 1973, 96, 98.

Scolnicov (chapter IV) holds that Plato applies his method throughout the dialogue. But his explanation of the application of the method is very obscure. He regards the Antapodosis argument as something unecessary that offers nothing. And this because "it was formulated in physical terms and referred to physical objects" (p.95). So "the line of the argument seems to have run to its end" (p.96). Therefore the first real argument of the dialogue is the Recollection one.

[117] 1975, 109; see also Barnes, 1978, 404-5.

[118] Plato never proves that cognitive soul and soul as principle of life are one and the same thing, but he takes it for granted (cf T.M.Robinson, 1970, 33).

[119] Or at least Plato thinks so. We may say that the memory of the general terms that the soul brings with it, is not an individual memory but a general form of memory the same for all souls.

[120] Not even an 'approximate indissolubility of the soul' as Hackforth (1955, 86) says; cf J.Ackrill, 1958, 109. The only conclusion we can infer from the argument is that it is very probable that the soul is more durable than the body.

[121] Cf Ostenfeld, 1982, 196: "It should be noticed that there is no <u>post mortem</u> judgement and punishment. Also, there is salvation for the good. Reincarnation has a moral purpose".

[122] The first premiss (principle of Opposites) is formulated rather as a logical law and is referred to by Plato at 103a-c without any hint against its validity. The second premiss ('life' and 'death' are opposites) is considered as something very obvious throughout the dialogue and is used as a premiss in the Last argument. The third premiss (the soul is substantially different from body) is a premiss of the Affinity argument as well (79b1-2). The fifth and sixth premisses are never explicitly stated by Plato in the Phaedo, and we have every reason to suppose that they are Plato's firm beliefs.

[123] It seems that the Antapodosis hypothesis implies that the soul is the principle of life, or that its essential characteristic (as far as it exists) is to bring life to bodies. Plato clearly uses this last premiss in his Last argument (105d3-4) and it is implicit at 72c-d and at 77d3-4. However, it is not clear from the text whether Plato considers it as a proposition about the soul's nature on which we must agree or as an inference from the Antapodosis argument. If we accept that bringing life is the essential characteristic of the soul such that if it loses it, it is no longer a soul, then the status of a soul that has escaped from the cycle of reincarnation is impossible. But it is exactly that that Socrates believes about the philosopher's soul after death (81a9, 114c). If we accept that the soul may escape from the cycle of reincarnation, then the Antapodosis and the Last argument lose their force.

Nevertheless I think that Socrates' remarks about the destiny of his soul do not play any role in the argument. Both passages 81a9 and 114c are in places of the dialogue where Plato does not unfold arguments but spells out a myth, and he does not intend us to take his words as established truth (cf the word eikos at 81d5, e2, 82a2, b5). Moreover Cebes' objection takes for granted that as far as the soul exists it alternates continually between life an death. He does not regard the case of a soul that although existing escapes the cycle of reincarnation.

[124] cf eg Bluck, 1955, 22; A E Taylor, 1937, 193; P Gallop, 1975, 197.

[125] See 94a8-10, 94c-e; note the linguistic similarity between enantia adein at 94c4 and diaphonein at 101d5. cf also p.133

[126] Compare logon didonai at 95d7 and didonai logon at 101d6.

[127] The physical explanations are rejected because they contradict the following principle: i) the same 'cause' cannot have opposite effects (100a6-8); ii) the same thing cannot have opposite 'causes' (97a-b); iii) a 'cause' cannot be the opposite of what it purports to explain (100a8-b2). (cf M S Cresswell, 1971, 246-7; E L Burge, 1971, 5; H Teloh, 1975, 19-20; C Stough, 1976, 10; Gallop, 1975, 174, 185-7). In the rejection of the explanations of the physical philosophers we can see another instance of the second part of the hypothetical method.

[128] Why does Plato explain his method here although he applies it in the whole dialogue? The hypothetical method is a conceptual procedure conducted purely within the realm of logoi. It would be premature for Plato to formulate the method before a clear distinction between physical and conceptual explanation has been drawn and the insufficiency of physical explanations pointed out.

[129] The mere proposition that 'y is f because it participates to F-ness' does not explain coming-to-be and perishing.

[130] I do not discuss the ontological status of the forms in us. I find plausible D.O'Brien's explanation (1967, 198-231, 1968, 95-106) about the distinction of the two cases. Smallness 'advances' and Largeness 'gets out of the way' in the case of comparison (with something larger). Smallness 'advances' and Largeness 'perishes' in the case of an actual change of size.

[131] Cf C.Stough, 1976, 8-9.

[132] I do not enter into the discussion of what are the 'subtle' aitiai (Forms, forms in us, or otherwise).

[133] Many commentators do not see any argument here; cf. eg A.E.Taylor, 1937, 206; J.Burnet, 1911, 123; R.Hackforth, 1955, 164.

[134] This reconstruction of the argument is essentially the same as that of Bluck (1955, 192-3). It is the most rigorous one and makes the Last argument sound and not 'lame' (cf 107a-b where Socrates Simmias and Cebes really believe that the Last argument has proved the immortality and imperishability of the soul). Another reconstruction of the argument, which depends mainly on 106d, is the following (Gallop, 1975, p215).

1) If the immortal, being everlasting, admits perishing, then there could hardly be anything that does not admit it (106d2-4).

2) There are things that are imperishable, viz. God and the Form of life, and anything else immortal may be (d5-7).

3) The immortal is imperishable (e1).

In this way the argument is dependent on the existence of the God and the Form of Life.

CHAPTER VI

MATHEMATICS, DIALECTIC AND THE FORM OF THE GOOD IN THE REPUBLIC

1. Mathematics is presented by Plato as an axiomatized science

Plato argues regarding the sciences of dianoia that: a) they are compelled to begin from hypotheses, not proceeding to a starting point but to an end (510b5-6); b) assuming knowledge of these hypotheses, the mathematicians do not expect to give any account of them either to themselves or to others, as if they were plain to all (510c6-d1); c) beginning from these hypotheses they go through the rest and end consistently with that which they set out to examine (510d1-3, cf 533c1-5) [1]; d) mathematics (or mathematical propositions) form a body constituted from first principles, conclusions and intermediate propositions woven together (συμπλέκεσθαι) (533c1-4).

The characteristics that Plato ascribes to mathematics give us a picture of it as a deductively organized science. Mathematics -according to Plato's description- is a science "where one could achieve not just bits of information but an understanding of the whole rationally organized system, and see a)what is basic b) what is derived and c)how each result depended on what had gone before" [2]. Therefore we have the picture of a science that, beginning from certain first principles (which are viewed as self-evident), constructs a consistent deductive system. All these characteristics are characteristics of a science already organized in an axiomatic system.

We must note that when Plato speaks about mathematics in contrast to dialectic (510-511, 533b-d), he principally has in mind geometry which was the most advanced branch of mathematics at that time. When Plato speaks about astronomy and harmonic (528e-531c), it is easy to see that these sciences were still at an empirical pre-theoretical level. Plato exhorts the astronomers to work in the same manner as geometers do ("προβλήμασιν ἄρα ἦν δ' ἐγὼ χρώμενοι ὥσπερ γεωμετρίαν οὕτω καὶ ἀστρονομίαν μέτιμεν" 530b6-7).

Therefore we may suppose that at the time at which the Republic was written, geometry had already been transformed, in a first stage, into an axiomatic system [3].

The geometer proceeds always from the hypotheses to the end. The procedure of geometry is only downwards; there is not ascent. The hypotheses function as limits and the geometer cannot "τῶν ὑποθέσεων ἀνωτέρω ἐκβαίνειν" (511a5-6)[4]. Nevertheless this description of geometry does not fit with what Plato says in the Meno that the geometers use the hypothetical method. In the Meno geometry proceeds both by ascent and by descent.

We can explain this change in Plato's description of geometry in the following way. In chapter II (pp.53-6), I argued that during the first decades of the fourth century geometers were using the geometrical method of analysis (or its forerunner apagoge) that involves both ascent and descent. The method of analysis was probably the strongest instrument that helped the transformation of geometry into an axiomatic system, functioning as an ascent to the first principles. After the axiomatization of geometry, the role of the geometrical analysis was not the ascent to the principles but only that of a heuristic method for solving problems. Therefore, it is very plausible to suppose that Plato's description of mathematics as having only descent shows that when the Republic was written, geometry had already been transformed into an axiomatic system.

2. The hypotheses of mathematics

Socrates says that with regard to the hypotheses of mathematics (510c-511a), the mathematicians a)hypothesize "the odd, the even, the figures, three kinds of angles and so on", b)treat them as if they knew them, making them hypotheses, c)refuse to give any account of them because they think that they are plain to everybody, d)are compelled to use hypotheses in their investigation and do not go to a beginning since they cannot get above the hypotheses.

First of all we must note that it is Plato who calls the principles of mathematics hypotheses. The mathematicians probably did not use this term for their principles. They accepted their principles unquestionably as true and as plain to all (see pp11-2). Only the philosopher recognizes that the principles of mathematics are unexamined assumptions whose truth has still to be established. Therefore Plato thinks that the mathematicians do not treat certain propositions as hypotheses when they ought to. The hypotheses of mathematics are not provisional or tentative, deliberately posited for the time being. From the mathematician's point of view the hypotheses are self-evident principles that do not need any confirmation [5].

Let us now see what kind of things the hypotheses of mathematics are. Plato says that the mathematicians hypothesize "the odd and the even, the figures, three kinds of angles and so on". These things are elementary notions of mathematics. However it seems somewhat surprising that Plato should cite a list of terms rather than a list of propositions to exemplify the hypotheses of mathematics [6]. We have therefore two possibilities: the hypotheses can be regarded either as single terms (concepts), or as propositions about these terms [7].

Professor Hare is probably the most prominent defender of the first option [8]. According to him the hypotheses of mathematics are not propositions but 'things' (Forms or concepts), and 'giving an account of them', means 'giving a definition of them'. Therefore Plato critisizes the mathematicians because they introduce entities like the 'odd' the 'even' and so on in their discussions without defining them. Furthermore their conclusions do not follow from propositions which are true by definition of the entities involved, but from propositions concerning these entities based only on intuition or empirical observation.

Professor Hare defends his thesis on four grounds: 1) the syntax of the proposition at 510c3-5 ("ὑποθέμενοι τό τε περιττὸν καὶ τὸ ἄρτιον...")where the 'odd' and the 'even' etc. are direct objects of the verb; 2) 'giving an account' means 'giving a definition'; 3) he refers to the mathematical proof of the Meno (duplication of the square) where the 'square' had not been defined; 4) he says that "definitions were never given by mathematicians in Plato's time" (p.27).

C.C.W.Taylor [9] has sufficiently shown the unsustainable character of Hare's thesis. Here I shall mainly follow his criticism. A) We have already seen in the first chapter that, for Plato, a hypothesis is an assumption of a proposition. Sometimes it has the form of an assumption of the existence of a thing but not the form of a postulated entity [10]. Moreover we have seen that in both the Meno and the Phaedo the hypothetical method is about propositions. B) We have already seen [11] that 'giving an account' does not only mean 'giving a definition' but also 'giving a proof'. This latter is the main meaning of 'giving an account' in Plato's hypothetical method in the above dialogues.

C)As Taylor showed (p. 200), the passage of the Meno does not constitute any evidence for Hare's thesis, firstly because Socrates was not a professional mathematician, secondly because the passage is not a piece of formal mathematical work, but an informal bit of instruction conducted not for any mathematical purpose, and thirdly because in a particular demonstration, a mathematician does not define his terms.

D) It is not true that the mathematicians of Plato's time did not give definitions. There is plenty of evidence for the opposite. In Euthyp. 12d (an early dialogue) we find a definition of the 'even'. Theaetetus (in Theaet. 147c-148b) begins his account of incommensurability by defining his technical terms. Aristotle says (Met. 987a19-21) that the Pythagoreans were the first to speak about the essence of objects and to define them [12].

The hypotheses cannot be axioms (common notions or postulates in the Euclidean terminology). Plato says that the mathematicians hypothesize the odd and the even, the figures, the three kinds of angles and so on. These notions are introduced in Euclid's Elements in definitions [13] and not in axioms involving them. It is quite probable that the first pre-euclidean Elements did not put forward axioms but only definitions. Axioms were too obvious to be put forward explicitly. For this reason Plato's examples of the mathematicians' hypotheses are the 'odd' and the 'even' and so on. We have evidence for this view from Aristotle: "Nothing, however, prevents some sciences from overlooking some of these - e.g. from not supposing that its kind is, if it is evident that it is ... and from not assuming what the attributes signify, if they are clear - just as in the case of the common items it does not assume what to take equals from equals signifies, because it is familiar" (Barnes) (An.Post. 76b15-22).

The problem now arising is that when Plato says that the mathematicians hypothesize "the odd and the even....", does he mean that they hypothesize the existence of these things, the definitions of these things or both?

The syntax of the passage gives support to the view that the hypotheses are propositions assuming the existence of these things. This view may find support from the Posterior Analytics (I, 2 and 10). According to the classical interpretation of An. Post. I,2, the hypotheses are assumptions of the existence of the basic terms of a science (e.g. points and lines in geometry) [14]. The existence of everything else (e.g. the various figures) has to be proved by construction and demonstration (see 76a35-6, 76b8-11, 92b15-16). However the account of hypotheses and definitions given in I,10 seems to be different from that of I,2. Furthermore Aristotle calls in some passages the basic starting points 'hypotheses' (An.Post.81b15, 76b36-7, An.Pr.24b1; see esp. Eth.Eud.1227b29 and 1222b23-41 where all the starting points of the theoretical sciences are called 'hypotheses) and in some other passages 'definitions'. It seems then that the term 'hypothesis' in Aristotle is not always restricted to existential propositions (as in An.Post.72a24-25), but is used as well to denote all the starting points of the sciences [15].

Existential propositions as such make little sense of Plato's description of the mathematicians' attitude. We cannot infer anything from existential propositions like 'the odd exists'. Moreover the mathematicians would hardly think that they knew the 'odd' and the 'even' merely by stating an existential proposition about them [16]. In Euclid's Elements no propositions are put forward which assume the existence of the various geometrical entities. There are only definitions about them. Only definitions and axioms can be starting points for proofs (cf. An.Post. 75b31). Furthermore as C.C.W.Taylor observes "sometimes it appears that Plato regarded as equivalent the expressions ὑποτίθεσθαί τι and ὑποτίθεσθαι τὶ εἶναι" [17]. In addition to this the term 'hypothesis' has not the specified meaning in Plato that it may have in Aristotle, but it is also used for definitions or other propositions [18]. Therefore it is very probable that the principles (hypotheses according to Plato) that the mathematicians used to put forward in their treatises were definitions [19].

On the other hand, while a definition can easily be a beginning of a proof, we cannot have a science without first assuming (or proving) the existence of the relevant things; so the proposition "the even is such and such" has no use if we do not know in the first place whether the 'even' exists or not. Stating their definitions, the mathematicians tacitly assume the existence of the defined entities. In this way, and from a philosopher's point of view, the definitions of the mathematicians contain the tacit assumption of the existence of the defined entities [20]. Moreover the 'definitions' of the mathematicians were sometimes complex propositions (or a set of propositions) including a proper definition plus a proposition asserting relations of the things defined. In this way, in 'definition' 17 (Book I) of his Elements, Euclid defines the diameter of a circle and after that -in the same proposition - he adds: "and such a straight line also bisects the circle"[21]. Moreover we find among Euclid's definitions some statements that can hardly be taken as definitions. For example, after the definitions of point and line, there is the proposition (posited as third definition) that "the extremities of a line are points" [22]. It is quite probable that these 'definitions' of Euclid are remnants of a pre-euclidean manner of the mathematicians of putting forward principles. Probably in pre-euclidean times, all sorts of principles were collectively posited at the beginning of the mathematical works without any distinction between them. We have such an example in Autolycus' of Pitane work <u>On a moving sphere</u> - a mathematical work earlier than Euclid's <u>Elements</u>. In this work, principles are exhibited collectively at the beginning of the work without any label indicative of their nature (cf Szabó,1978,225-6).

We can conclude therefore that the starting points of the mathematicians were definitions plus (probably) other propositions about the things defined [23]. From the philosopher's point of view these hypotheses include also the tacit assumption of the existence of these things.

Another thing that we must notice is that the hypotheses of mathematics are "the odd and the even, the figures, three kinds of angles", that is, things whose definitions are lower in the hierarchy of definitions and whose existence and relations are derivable from the existence, the definitions and the relational properties of things like 'number', 'point', 'line' etc. So in

Euclid, definitions of the most elementary geometrical objects like point, line, surface, plane surface, angle, boundary, figure (Book I, def.1,2,4,5,7,8,13,14), unit, number (Book VII, def. 1,2), are never invoked in proofs and are mathematically useless. On the other hand definitions of other objects (kinds of angles or of figures, odd and even etc) are frequently invoked and play an essential role in proofs[24]. This last point will be developed further later.

3.Διάνοια

There are two main problems connected with the second upper segment of Plato's Divided Line: a) the problem of the ontological status of the objects of dianoia; b)the problem of whether or not dianoia studies only mathematical concepts.

My interpretation of Plato's methodology does not depend on the answers to these two questions, and I am not going to examine them in depth. Nevertheless an answer to the above two problems will facilitate the overall understanding of Plato's dialectic.

With regard to the first question, the main problem is whether or not the objects of dianoia are Forms or the mathematical intermediates about which Aristotle speaks in Metaphysics[25]. The main advocate of the theory of mathematical intermediates in the British tradition is Adam [26]. However nowadays very few scholars accept that we can find intermediates in the Republic [27]. The fact is that nowhere in the Republic does Plato speak explicitly about mathematical intermediates. On the contrary there are at least two passages which show that Plato views the objects of dianoia as Forms: a)510d7-8 "the square itself.....the diameter itself"; b)511d2 "although they are intelligible and have a first principle" [28].

Therefore, the standard argument against the intermediates in the _Republic_ is still very strong. If Plato had consciously introduced a new ontological distinction of such great importance, he would have been more explicit and would have devoted more words to support and explain it.

Moreover, from Plato's description of the upper line (510b-511d) it seems that the distinction between _dianoia_ and _noesis_ is rather methodological than ontological.

Therefore I think that we can exclude the conscious introduction of mathematicals in the _Republic_ by Plato. It seems more safe to accept that the objects of _dianoia_ are Forms.

Nevertheless there is a passage (534a4-8) where Plato seems at least to recognize the problem of the attribution of different objects to the different states of mind [29]. Plato here, after saying that _doxa_ is concerned with _genesis_ and _noesis_ with _ousia_ he proceeds: "let us not discuss the proportion which holds between the objects of these states of mind and the twofold division of _doxaston_ and _noeton_, to avoid arguments many times as long as those we have had already". Here it is likely that Plato speaks about objects with regard to the four states of mind and recognizes that the relationships between objects is a matter more problematic than that of cognitive relationships. However, his refusal to discuss the problem can be interpreted in opposite ways. We cannot therefore have a clue from this passage as to whether Plato did ascribe different objects to the different states of mind [30].

The next question is whether _dianoia_ deals only with mathematics or whether it can be extended to others. Plato in the

Republic speaks about dianoia as dealing only with mathematics. There is no direct evidence about the extension of dianoia to moral issues. On the other hand, from the proportions of the Line, we cannot infer that dianoia deals with only a sub-class of the objects of the intelligible world.

It seems that Plato deals with the branches of mathematics known in the early fourth century B.C., because he had no other example of systematic and deductively organized knowledge. In principle what he says, is applicable to every deductive science [31].

Thus, nothing prevents us from supposing that Plato thought there could be ethical dianoia that used similar methods to mathematics, although there was not yet any recognisable branch of study, like geometry[32]. As in geometry, a drawn circle is an illustration of the Form of the Circle, so in ethics a just action may be an illustration of the Form of Justice. Geometrical and arithmetical equality is used by Plato in Gorgias 508a and in Laws 757b-c to express the difference between two kinds of justice. In Phil. 26 number and proportion are responsible for all good conditions including virtue, and in Prot.356d-357b the metreteke techne deals with the 'larger' the 'smaller' and the 'equal' not only with regard to numbers and magnitudes, but also with regard to pleasure and pain. Moreover the mathematical notions of order, proportion and harmony are central in Plato's moral and aesthetic doctrines [33].

4. Dialectic

In this section I shall discuss some points of dialectic and I shall come back in section 8 to reconsider the whole problem of dialectic. Plato does not say much about dialectic. When Glaucon asks Socrates what dialectic is and what the method is, he refuses to answer (532d-533a) saying that Glaucon would not be able to follow a full exposition.

However, Socrates makes a few remarks about dialectic in 510-511 and 532-534.

a) Starting from hypotheses, the soul [34] proceeds towards an ἀνυπόθετος ἀρχὴ, without making use of sensible images but using only Forms (510b).

b) In 511b3-c2, he explains dialectic in the following way: here the αὐτὸς ὁ λόγος (I take it to mean 'pure reason') treats the hypotheses not as starting points, but really as hypotheses (or, in other words, "as things to stand on and jump off from" [35]) until it arrives at the non-hypothetical, the starting point of everything; and then grasping it, it comes down to the end, ἐχόμενος τῶν ἐκείνης ἐχομένων ("clinging to the things that cling to that" [36]) without making use of anything sensible but starting and ending with Forms.

In the above passages, Plato speaks about dialectic in contrast with mathematics. Both mathematics and dialectic a)begin from hypotheses, b)include a 'downward path' from starting points, and c)deal with eternal and unchangable objects. Dialectic, unlike mathematics, a) includes an 'upward path' [37], b) arrives at a non-hypothetical principle, c) treats the hypotheses not as absolute self-evident truths but as mere hypotheses, and d)does not make use of sensible images. Mathematics assumes a certainty to which it is not entitled because its starting points are mere hypotheses (see 533c1-3). Dialectic alone achieves perfect knowledge because it arrives at the non-hypothetical principle [38].

The above passages give us an image of dialectic as a method of proving propositions. Dialectic gives an account of the hypotheses by deducing them from 'higher' ones. The non-hypothetical principle is the unique self-evident starting point of dialectic [39]. In this way, dialectic is a kind of super-science or meta-science which supplies the foundation for the principles of the sciences and organizes the whole field of the knowable in a single deductive system [40].

However, passages 532a5-b5 and 534b3-d1 give us a different picture of dialectic: "And do you call dialectical the man who takes the account of the essence of each thing? And if he cannot give an account to himself and others, then, in so far as he cannot, you will not call him intelligent of the matter?.....And the same with the good? If a man cannot by his account separate and distinguish the idea of the good from all else, and persevere through everything in the battle of refutation, eager to refute in reality and not in appearance, and to go through all these things without letting his argument be overthrown, you will not say that he knows the good itself or any other good?" [41].

The above passages suggest that dialectic is a kind of intellectual sight and its business is to see what each thing is. This will be done only if someone reaches the goal of the intelligible, that is, the Good itself. The dialectician is the man who grasps the λόγος of the essence of each thing. Dialectic here is concerned with "saying what a thing is" whereas proving is not mentioned. The meaning of 'giving an account' in 534b seems to be that of 'giving a definition'.

It seems then that the above passages give a different account of dialectic from those of Book 6. Here Plato seems to recommend that the philosopher must do something like conceptual analysis, clarifying concepts, distinguishing meanings and giving definitions [42].

Nowhere does Plato say clearly that the non-hypothetical principle is the Good. Similarly, the Form of the Good is never called an <u>arche</u>. However it is almost certain that the non-hypothetical principle is closely connected with the Good. The non-hypothetical first principle is situated at the highest point of the intelligible world in the simile of Line. The Form of the Good occupies the same position in the Sun analogy and in the Cave simile (517b-c). Passages 519c9-d1 and 526e1-4 say that the final goal of the education of guardians is the arrival at the Good. Similarly, in the account of dialectic at 532a5-b1 -a passage which clearly recalls the description of <u>noesis</u> in the Line - the last step of the dialectical ascent is not the non-hypothetical principle but the Good. We can plausibly assume then that the non-hypothetical principle is the Good [43]. The only difference, as Ross remarks (1951,42), is that "while the phrase 'the idea of good' points to a universal, the other phrase points to a proposition presumably one in which the idea of good is a term".

5. <u>ὁ μὲν γὰρ συνοπτικὸς διαλεκτικός - τὰς ὑποθέσεις ἀναιροῦσα</u>

The transition from the sciences of <u>dianoia</u> to dialectic is gradual. From the different sciences we pass to a stage of an overall comprehensive and synoptic overview of all sciences (531d). That is a comparative inquiry of the sciences and their principles ("συνακτέον εἰς σύνοψιν οἰκειότητός τε ἀλλήλων τῶν μαθημάτων καὶ τῆς τοῦ ὄντος φύσεως......καὶ μεγίστη γε, ἦν δ' ἐγώ, πεῖρα διαλεκτικῆς φύσεως καὶ μή· ὁ μὲν γὰρ συνοπτικὸς διαλεκτικὸς, ὁ δὲ μὴ οὔ" – 537c).

At this stage, the scientist (and future philosopher), grasping the essential features of each branch of study, finds relations between the separate hypotheses of the different sciences and begins to relate mathematical Forms to non-mathematical Forms within a more comprehensive analysis [44]. So the πάμπολυ ἔργον at531d5 refers to the bringing together the various mathematical sciences and the related mathematical Forms. Moreover in 537c1-3, Plato speaks about bringing together the 'μαθήματα' and the 'τοῦ ὄντος φύσεως' that is, mathematical plus other Forms. The hypotheses of the sciences seem to be similar to those of dialectic. This is clear from 533c7-8, where the procedure of dialectic as 'destroying' the hypotheses, is in contrast with the previous statement (533b6-c6) about the inability of the mathematicians to give an account of their hypotheses [45].

This stage probably belongs to <u>dianoia</u> (531d7: τοῦ προοιμίου; cf 537c1-3). But it is really a transitional stage between <u>dianoia</u> and <u>episteme</u>, which takes place only after the independent study of the various sciences (531c9-d4). This stage is necessary for the scientist who is going to become a philosopher. The future philosopher must be <u>synoptikos</u> 8537c7), that is, he must be able to see the affinities and communion between the various branches of knowledge. Only the man who has the capacity of seeing things together may become a philosopher (537c6). In this way, a scientist is potentially a philosopher and a philosopher is actually a scientist.

A passage that seems to give a bit more information about the dialectical method is 533c7-d1: "the dialectical method alone proceeds in this way, destroying the hypotheses, to the very beginning, in order to obtain confirmation".

A hypothesis may be regarded as destroyed in two senses: a) once its falsity is proved, and b) once its hypothetical character is removed.

Adam (1902; note on 533c) takes this passage to be a reference to the procedure of the early dialogues of setting up one theory after another, and showing its inadequacy (like the refutation of the definitions of justice in Book 1). But it hardly seems true that the hypotheses of mathematics are overthrown like the bad definitions of justice. A.E. Taylor [46] proposed a similar interpretation of 'destroying' as 'proving false'. He takes the hypothesis of the 'odd and the even' of 510c to mean that "every number is either odd or even"; but Plato thought this proposition false because there are also irrational numbers. Taylor gives similar interpretations to the other two 'hypotheses' namely the 'figures' and the 'three kinds of angles'. However, his account lacks any historical ground. As regards, for example, the first hypothesis, it is almost certain that Plato never thought of the irrationals as numbers [47].

It is impossible that all the hypotheses of mathematics are false propositions [48]. The mathematicians regard their hypotheses as self-evident principles (510d1). Mathematicians are 'dreaming about the being' because they have their hypotheses unexamined and can give no account of them (533b-c). Therefore dialectic 'destroys' the hypotheses by giving an account of them. In this way, the thing destroyed is not the hypothesis, but its hypothetical character [49].

There are two different interpretations of the phrase

'destroying the hypotheses' corresponding to the two main interpretations of the 'giving an account'.
'Giving an 'account' may mean 'giving
a definition' and 'destroying the hypotheses' means clarifying and defining concepts. In this way we do not prove the basic principles of mathematics but we achieve a better understanding of them [50].

This interpretation fits i) with the visual language that Plato uses in the Republic as regards the knowledge of the Forms and the Good and ii) with passages 532a9-b2, 533b1-2 and 534b3-5 where dialectic is concerned with 'saying what a thing is'. However passage 533c2-d1 is directly related to 533b6-c6 which refers to mathematics and its methodology. The task of dialectic here is to give an account of the hypotheses of mathematics and 'giving an account' here (c2) is more close to 'giving a proof'. Moreover, the above passage suggests also that the process of dialectic is a long path of articulated propositions starting from the hypotheses of mathematics and ending at the 'very beginning' (see c2-3: ἀρχὴ, τελευτὴ, μεταξὺ; cf also 511b). A process of clarifying concepts has not these characteristics. Moreover -as we show in section 2- the hypotheses of mathematics are neither concepts, nor definitions as the above interpretation requires.

I think therefore that it is more likely to take 'giving an account' to mean here 'giving a proof' and 'destroying the hypotheses' to mean 'proving them by means of higher hypotheses' similar to the procedure described in Phd 101d-e [51]. As we saw in chapter V (p 140), 'giving an account' even in the definitional sense requires rational argument. Plato's practice in the Republic confirms our opinion. Plato here finds (and defends) a definition of justice via a demonstrative process (see section

12).

Robinson [52] suggests that the hypotheses are destroyed only when we reach the non-hypothetical principle and obtain certainty and knowledge. Therefore, according to his interpretation, all propositions of mathematics (initial and derivative) plus all the propositions we obtain during the dialectical ascent (and before the arrival at the Good), should have been called 'hypotheses'. However Plato uses the term 'hypothesis' in the Republic (510c) in order to denote only the initial propositions from which the mathematicians begin their proofs. He does not consider as hypotheses derivative propositions (theorems). Furthermore "μόνη ταυτη πορεύεται τὰς ὑποθέσεις ἀναιροῦσα" suggest that "this 'destruction' occurs at each step of the upward path ...The tense of the participle, ἀναιροῦσα itself proves this, and to make doubly sure that this sense should not be overlooked Plato put the participle clause between πορευεται and ἐπ' αὐτὴν τὴν ἀρχὴν" (Cherniss, 1947,143).

Therefore, the hypotheses are destroyed not by losing their hypothetical character in general, but by losing their primitive hypothetical character. Every time we posit a 'higher' hypothesis in order to prove a 'lower' one, during our dialectical ascent, the latter loses its primitive hypothetical character and ceases to be a hypothesis. In this sense, and before our arrival at the Good (that gives knowledge and certainty), although derivative propositions dependent upon hypotheses have in a sense hypothetical character, according to Plato they do not deserve the name hypothesis.

6. <u>The defects of mathematics</u>

The two defects that Plato ascribes to mathematics are the use of sensible diagrams and the use of hypotheses.

Plato thinks that diagrams are a necessary feature of mathematics. The geometer needs an intuition of spatial figures in order to prove his theorems. Ross (1951,49) says that Plato is at fault in describing geometers as necessarily drawing diagrams; someone can easily make imaginative figures. However this is true only for very simple problems or theorems. It is impossible for difficult and complicated cases that need auxiliary constructions. Moreover, even imaginative figures are in a sense particulars.

It seems strange that Plato thinks that arithmetic employs sensible images as well, because we are used to the idea that arithmetic employs only symbols. However, and apart from the fact that symbols are in a sense images, the arithmetic of that time was a kind of geometrical arithmetic employing figures made by dots or pebbles. This is why mathematics spoke of triangular numbers, square numbers, and so on [53].

In every science, only when we investigate its primary concepts and the relations between them can we work without help from particulars. Dialectic proceeds without any help of sensible particulars because it employs only the primitive and simplest concepts (and relations between them) of the various sciences. It is impossible, even for the dialectician, to prove a complicated theorem (eg in conic sections) without a drawn figure. Therefore, dialectic cannot remedy this defect of mathematics. It is not remediable [54].

The second mark of mathematics is not that the mathematicians make use of hypotheses, but that they consider them self-evident (although they are not) and leave them unexamined (510c6-d1, 533b7-c5). Therefore the defect is not that the mathematicians use the hypothetical method, but that they fail to use it [55].

The main defect, that makes the sciences of dianoia inferior to dialectic, is their unexamined hypotheses (cf 533b-d). Mathematics cannot go above the hypotheses to arrive at the non-hypothetical principle (511a). Because of this, mathematics alone cannot give perfect knowledge, only hypothetical. Only dialectic proceeds to the non-hypothetical starting point and achieves perfect knowledge. Mathematics achieves the status of perfect knowledge only when the philosopher, after having arrived at the non-hypothetical principle reaches mathematics in his way down[56]. As far as mathematics continues to remain an independent axiomatic system with unexamined principles, it is not remediable.

Plato does not exhort the mathematicians to remedy their science from the above defects. The mathematician thinks his principles are self-evident and since he proves rightly his theorems, he is all right. The examination of the principles of mathematics is not the task of a mathematician, but the task of a philosopher [57]. The above 'defect' of mathematics is a defect only in comparison to dialectic. Plato does not criticize the mathematicians for it [58].

Plato says that the mathematicians are compelled to start from hypotheses (510b5, 511a4) and cannot go above them (511a5-6). This statement seems strange. What is it that compels the geometer not to examine his initial assumptions? The right answer seems to be the following: the geometer so long as he does geometry, is interested in proving theorems and solving problems. For this task, he needs some first principles (definitions, axioms) from which he will start and which he will use in his demonstrations. Mathematicians need definitions and axioms to use them in their proofs. Such definitions are those of 'the odd', 'the even', 'the square' and so on, but not those of 'the point', 'the number', 'the limit' etc (see p218-9). Therefore, examination of these latter things is not necessary for the pure mathematician and leaves him indifferent [59]. Their examination is the task of the philosopher.

Is there any necessary connexion between the two defects of mathematics? Plato says nothing about this and it seems to me rather unlikely. However, Robinson's suggestion [60] is not unsound. According to him, the mathematicians adopt this confident and dogmatic attitude to their assumptions because they seem to be directly given in sense experience. Thus the assumptions are <u>plain to all</u> in the sense of being embodied in the sensible diagrams which the mathematicians use. "In geometry", says Robinson, "the appeal to spatial intuition and the claim that one's postulates are certainties, go together".

7. <u>The Form of the Good</u>

For Plato the Good is the 'μέγιστον μάθημα' (504e-505a); problems connected with the Good are its nature, the knowledge of it, and its relation to the other Forms. It seems that Plato did not have a clear answer to these problems. At 506d-e Socrates refuses to speak clearly about the Good, saying that he does not know what it is and that he is able to talk only in imagery. What Plato says in the <u>Republic</u> about the Good is so brief and obscure that every attempt to give answers to the above questions will probably be conjectural. Moreover we cannot find any help from the other dialogues.

Plato's discussion of the Good occurs in 504ff. In 505a-b we are told that the Form of the Good has a privileged position among all other things. All our actions and pursuits are for the sake of the Good (505e) and its knowledge is of the greatest importance (505e-506b). Nevertheless people do not know it and identify it either with knowledge or with pleasure (505b-c). It seems then that people have a pre-intuition about the Good (see "ἀπομαντευομένη τὶ εἶναι" -505e), allowing them to recognize it as the most valuable thing although they have not knowledge of it.

The simile of the Sun (506d-509c) deals with the relation of the Good to the other Forms. We are told that the Good is the cause of every truth and knowledge, and makes not only other Forms but itself known (508e-509a). It "not only gives the objects of knowledge their being known but also their being and essence [reality] though the Good is not essence but still transcends essence in dignity and power" (509b).

How is the Good the cause of being, essence and knowledge of the Forms? One answer (given by Santas -1983) is that the Good is the 'cause' of the attributes of the Forms <u>qua</u> Forms or, in other words, of their <u>ideal</u> attributes; not of their <u>proper</u> attributes. Ideal attributes - Forms unlike particulars are unchangeable, ungenerated or indestructive - constitute the 'being and essence' of the Forms. Moreover it is precisely these attributes that make Forms perfectly knowable things, and the lack of these attributees that makes particulars unknowable (see 476ff). We may say then, that as particulars take their attributes by participation in Forms, so Forms take their ideal attributes by participation in the Good (and their proper attributes by self-predication). The Good then makes each Form to be the best object of its kind (p.239). The Good has only proper attributes, those being the ideal attributes of the other Forms. Accepting that 'participation' involves only 'formal causality', the Good is the formal cause [61] of the superlative goodness of kind of all the other Forms, their essence and knowability (p.240).

Nevertheless there are some things that Santas' interpretation does not explain. Firstly, if the only things that are able to participate in the Good are Forms, how is it possible to speak about 'good' men, horses, actions etc? Secondly, because the Good gives the Forms only their ideal atttributes, it cannot be the 'cause' of their actual knowledge. We cannot for example give a definition of Justice through the Good because a definition of Justice refers to its proper attributes and not to its ideal. In this way the Good is the 'cause' only of Forms' knowability and not of their actually being known as the Republic suggests. Thirdly, Santas' interpretation gives a picture of the Forms as being in relation with the Good but not to each other. But in this way we cannot have a long dialectical ascent and descent.

Many scholars think that the Good is the final cause of the Forms [62]. In this sense the good of everything is the final explanation of its existence. Hence the Forms are what they are in order that the whole might be good. It is almost certain that the Good is for Plato the teleological cause of particular actions (505e) and perhaps of the sensible World (see Phd 99c). However, it is difficult to see how the Good could be the final cause of eternal, unchangable and ungenerated things like the Forms [63].

Very few scholars think that the Good is an efficient cause of the forms as well [64]. The only piece of evidence about it is an indirect one through the analogy of the Sun (506e3-6: ἔκγονος, πατρός; 508b13; 509b2-5). But is is impossible to understand in which way the Good could be an efficient cause of the Forms (all of them being immutable, unchangeable and ungenerated things).

It is therefore difficult to see in what way the Good is the 'cause ' of the Forms. We can plausibly ascribe to it formal and perhaps final causality, specially regarding it from a man's point of view. However I do not think that we can regard it as an efficient and material cause.

Jaeger [65] identifies the Good with God. This is very unlikely because, as Ross remarks (1951,43), the Good is a nature or a universal although God is a being having a nature. The God is not goodness but a good being (cf Phd97b8-c6 where divine reason is distinguished from the good to which it looks in its goverment of the world).

Plato gives two different accounts of the knowledge of the Forms and the Good; a visual and a discursive one [66]. In the Republic there is a constant use of the imagery of vision, a conception of seeing the Forms with the mind's eye. Knowledge seems to be something non-verbal and non-articulated that every person has to do by himself. The visual account of knowledge is predominant when Plato speaks about the knowledge of the Good.

In the discursive account, the objects of knowledge (Forms) are interrelated and seem to be complex entities. The only passage in which Plato seems to give a discursive account of the knowledge of the Good is 534b-c. Here we are told that when one arrives at the highest point of the dialectical ascent, one has to separate and distinguish (or to define? -διορίσασθαι τῷ λόγῳ) by one's account the Form of the Good from everything else submitting it to a real elenchus. Plato does not say that we can arrive at the knowledge of the Good by applying the dialectical method. The non-hypothetical first principle is the highest point of the dialectical ascent. We cannot have something higher through which we shall give an account of it. Plato is conscious that a continuous regression of 'causes' or proofs leaves us always with an unexplained or unproved residuum; something that if granted, makes the rest intelligible, but for whose own being no reason can be assigned.

Similarly we cannot obtain knowledge of the Good via elenchus, because elenchus can show us only what a thing is not and not what it is [67]. Elenchus cannot establish propositions because we can never know if we have drawn and checked all the consequences. As we have said, the phrase 'non-hypothetical first principle' points to a proposition. We have the same feeling from passage 534b-c. Only propositions can be submitted to elenchus. If it is so, and according to our interpretation of the hypotheses of mathematics, the non-hypothetical first principle must be a proposition or a set of propositions about the Good [68] which asserts (a) that the Good exists and (b) that the Good is such and such and has such and such relations with the other Forms (similarly to the hypothesis of the Forms in the Phaedo).

Since we cannot acquire knowledge of the Good by using something 'higher' than it, and since elenchus cannot establish propositons, we must admit that the knowledge of the Good is of a different kind. It is a direct knowledge. The last step of the dialectical ascent -the knowledge of the Good- comes to the mind like a sudden flash of intuition or inspiration after a long training and apprenticeship in dialectic and after a thorough elenchus and examination of all the hypotheses [69]. Because of the direct and intuitive character of the knowledge of the Good, Plato speaks of it in terms of 'sight' and 'vision'.

The main advocate of the above interpretation is Robinson. However his account presents some difficulties. Nowhere does Plato say that knowledge is produced by a faculty that guarantees its own infallibility. Secondly we have no guarantee that after all this long dialectical path we shall finally get the intuition of the Good; and lastly, even if every dialectical ascent converges upwards on the one arche, there is no way to know that every future philosopher will fix his mental eye on the same thing and that this thing will be the Good.

Because of these difficulties, other interpretations have been proposed. The Good might be not a Form similar to the others and distinguished from them, but the totality of the world, or -regarding it from a functional point of view- the most general function or structure of all the particular functions in the world, or even the order and coherence that unifies and gives meaning to the totality of the things (Forms) [70].

In this way the knowledge of the Good is not an intuition of a premiss but an understanding of the subject matter or of the most general function of the whole system [71]. The direct knowledge of the Good produced by dialectic is the kind of insight derived from comprehending a whole system and seeing the particular functions and structures from the perspective of a complete understanding of the structure and function of the system.

Nevertheless even this interpretation of the Good and its knowledge does not exhaust Plato's conception of the Good. Plato many times speaks of the Good as being a Form among the others (although in a higher position) which is able to be known and probably defined (534b-c). Starting from the Good, (regarding it as the non-hypothetical principle) we must be able to give an account of the hypotheses (including those of mathematics). Moreover nowhere does Plato talk explicitly about the Good in functional terms. Furthermore, coherence and order cannot be identified with the Good although closely connected with it. Coherence by itself is not enough to explain a system or to give meaning to our lives [72].

Concluding this section, we may say that the Good is the formal and probably the final cause of the Forms. Moreover, the knowledge of the Good that the philosopher acquires is not obtained via the use of the hypothetical method. Elenchus helps us to distinguish the Good from everything else, but the full knowledge of it must be a direct one. However, it seems then that it is very difficult to form any clear idea of what was in Plato's mind when he spoke about the Good. Every interpretation seems to be conjectural. What he says of the Good is brief, obscure and metaphorical; and this, together with his silence in later dialogues, leads us to believe his statement of 506e that even he himself had no clear conception of it.

8. The upward and downward paths

Our problem is now to examine more closely what the dialectical method is and especially its upward path.

First of all we shall make two brief remarks. As we have already said, the dialectical path seems to be a long one and the way from the hypotheses of the sciences to the Good has many steps. The other point is that the central Books of the Republic give the impression that the Forms constitute a sort of pyramid with goodness at its apex, and the dialectical journey is an ascent of this pyramid [73]. If this view is correct then there is a hierarchy among the objects of knowledge and their relations; in this way Forms are interrelated and ordered among themselves, some being higher and others lower in the hierarchy. For example we can say that the odd and the even presuppose number as justice and courage presuppose virtue [74].

There are two ways to attempt to give an interpretation of the dialectical method. We may either try to explain it from the top, asking 'how does the Good give reason to the other Forms?' or from the bottom asking 'how can we proceed over the hypotheses of mathematics in order to give an account of them?'. Although some people treat these two questions as identical, they differ considerably. The first question regards the first steps of dialectic while the second, the last ones.

In the previous section we have concluded that Plato had no clear conception of the Good and its knowledge. The same is plausible with regard to the relation of the Good to the Forms high in the hierarchy. At 533a, Socrates refuses to give a clear account of the dialectical method saying that Glaucon is not able to follow him. Glaucon's question to Socrates about the dialectical method has to do probably with the last steps of the upward path because it is connected with the two previous references to dialectic by Socrates (532a1-b2, 532c), where the emphasis was on the arrival at the Good. Nevertheless in the few words that Socrates says about dialectic as a response to

Glaucon's question (533c)there is no reference to the Good.

However, although it is quite probable that the end of the dialectical ascent is obscure even to Plato himself, we have to suppose that he did have an idea of what the first steps of dialectic were like, or in other words of the way in which we can go over the hypotheses of mathematics. Otherwise we cannot explain how it is possible for Plato to speak about a method of which he knows nothing. It seems then that the right approach of an interpretation of the method is to give an explanation of the first steps of dialectic.

We have already seen that in Plato's application of the method in the Meno and in the Phaedo the hypotheses were not (or were not only) definitions. We have also seen, (in section 2) that the principles of the mathematicians were not only definitions and that the terms cited by Plato at 510c are derivative ones whose existence must be proved [75] and whose definitions are used in the mathematical proofs and are necessary to the mathematicians. In Euclid's Elements these terms are defined through other ones like point, line, number etc. We know that at the time of Aristotle, geometry had already reduced its priciples to fewer and 'higher' than those that Plato refers to. Moreover Aristotle (Met 1019a1-4) attributes to Plato the belief that what is prior in nature is that which can exist without other things, while they cannot exist without it. In this way the point must be prior to the line, the line to the plane etc. I think therefore that it is plausible to suppose that 'going over' (or 'giving an account of') the hypotheses of mathematics means reducing them to other fewer and 'higher' ones so that the former can be derived from the latter. This job is not for the mathematician. The pure mathematician is satisfied with his principles (odd, even, various figures etc) because he can work out all his demonstrations through these principles. The above task is a foundational-philosophical one, appropriate to the mathematician - philosopher.

In section 5 (p227), we gave an interpretation of 'destoying' the hypotheses in the sense of giving an argument or a proof of them via 'higher' hypothesis [76]. The ascent from the hypotheses of mathematics (even, odd, etc) to the higher ones (say number, point etc) is not only a processs of creating a hierarchy of definitions of some terms where every lower depends on higher ones [77]. In other words the definition of square depends on the rectangle, the definition of rectangle depends on the parallelogram, the latter on the quadrilateral and so on. As Proclus says [78] the geometer cannot discourse about the properties of triangles "unless he had previously shown the existence of triangles and their mode of construction" (Morrow). Therefore, together with the process of definitiions there is a parallel process of proving by construction the thing defined. Moreover there is in geometry a process of proving theorems. In this process theorems are proved on the basis of other more elementary (or 'higher', or 'prior') theorems. The same is valid for the problems of constructions which are solved on the basis oF simpler (prior) constructions and theorems (or axioms). Thus the construction of the parallelogram depends (among others) on the postulate of the parallels.

It is not possible to ascertain whether or not Plato was conscious of the differences between the above processes in geometry. However the fact is that in the Republic he thinks that he can find the definition of justice by using a deductive argument and defining it from 'higher' hypotheses (see section 12).

We have reason to believe (see section 3) that the sciences of <u>dianoia</u> are not exhausted in the various branches of mathematics. We may suppose that Plato thought that there are moral analogues that did not constitute readily recognisable branches of study, while the branches of mathematics did already exist in an abstract form which was the model of study that Plato was advocating. In all the sciences of <u>dianoia</u> there are common theorems like those of equality and concepts like order, harmony amd beauty [79].

It appears to me that Plato saw that the principles of mathematics were reducible to fewer and prior ones, and that all sciences of <u>dianoia</u> had in common their most basic theorems and concepts. We might suppose then that he thought that the sciences of <u>dianoia</u> should all be reduced to one science (σύνοψις ἐπιστημῶν), and this could not be fulfilled so long as their special principles (hypotheses) remained. After that he probably thought that we could continue the dialectical ascent until we arrive at the unique first principle which would be no longer a postulate, namely the Good. So the whole knowable could be organized in a system in the model of geometry.

The dialectical ascent to the higher hypotheses is not a deductive process [80] dealing only with convertible propositions. In equivalent propositions we cannot easily say that one of them gives an account of the other and that we can 'rise above' the hypotheses. Plato's ideal of knowledge as a pyramid having the Good at its apex is incompatible with a description of the dialectical ascent as a deductive process.

The upward path is rather a process of discovering premisses from which we can derive our initial hypothesis. As in geometrical analysis the geometer asks himself what conditions must be satisfied in order to solve his problem, so the dialectician asks himself what conditions must be satisfied if his assumption (hypothesis) is to be correct. In this process the experience and the mental ability of the dialectician, in order to have a correct insight and grasp of the appropriate conditions and premisses, is very relevant. The upward path is tentative; it is always possible that the dialectician might have started incorrectly. Then he can try again using other routes and premisses in order to establish his conclusion [81]. Sometimes the question of the correctness of a hypothesis might lead not just to one or some prior propositions but to an entirely new problem that must be solved [82]. It is very probable that in many steps of the dialectical ascent we may have deduction. However the dialectician is not interested in deducing premisses but in finding them. His interest is to find the conditions of his problem or the initial premisses that he will use in order to prove his hypothesis. In this way, a step upwards looks like a leap to, or a grasp of, the principles of the argument. Therefore, one step in our way up may include several steps in our way down where our process is demonstrative (we cannot have gaps in a demonstrative chain). Such an explanation of the dialectical ascent avoids the possible objection that the way down may be a reduplication of the way up.

According to the above interpretation the way down is a demonstrative process that establishes the truth and certainty of the whole system. It is true that nowhere does Plato clearly say that (he has not a precise logical vocabulary), but his description of the way down (511b7-c2) is very similar to his description of the process in mathematics (510d1-3). The difference between the way up and the way down seems to be the difference between the order of discovery and the order of exposition [83]. Going up to a higher hypothesis is discovering how to go down. We cannot be sure that we have gone upwards to a genuinely higher hypothesis until we are assured that we can go down. However, as we see from Aristotle's An.Post. and Euclid's Elements, the way down is not only a demonstrative process but also a way of presenting and expounding what has become intelligible, a rational organization and presentation of the subject that will facilitate its understanding.

It may be objected that since we know that the unique non-hypotheticl first prinnciple is the Good, there is no need for the way up. This objection is esily met. We may say that although we know that the Good is the supreme principle (we have probably a pre-intuition about it; see p.232), we do not know its nature (505b-c). So the way up is necessary in order to have a full understanding and knowledge of the Good.

A second objection may be that the dialectical ascent is a necessity only for the first dialectician who will realise the whole project of dialectic. After that the whole knowable will be organized and presented in due order. Therefore the systematized dialectic can be taught to the new dialecticians using only descent like mathematics; so there is no need for the way up any more. We might answer this objection by arguing that the non-hypothetical principle is not like an axiom (an unproved and assumed proposition). The dialectician must obtain knowledge of the Good and he can obtain this knowledge only by followiing the whole way up. The knowledge of the Good is a personal conquest.

The first dialectician who will reach the Good cannot teach it to others. Moreover the way up may culminate not only in the knowledge of the Good but in the understanding of the whole subject matter. The way up is also what makes the future philosopher have an overall view of the various sciences of dianoia, in other words to be συνοπτικὸς

This version of the deductive or argumentative interpretation [84] of the dialectical method we have defended so far, may find support from evidence external to the Republic. First of all, dialectic was evolved in the Academy into a kind of argumentative competition (see Aristotle's Topics) where someone has to defend or attack a thesis by argument [85]. Secondly, dialectic seems to be very similar to the hypothetical method in the Phaedo [86] which seems to have propositional and deductive character. The method of the Republic like that of the Phaedo is a method employing hypotheses. So we can speak about a hypothetical method in the Republic. Both methods are posited in order to establish something and not to refute it. Every hypothesis needs confirmation and this is done via a higher hypothesis (Rep 511b,533c7-8; Phd 101d). Both methods include a deductive way downwards, and a heuristic way upwards to higher hypotheses. In the method of the Phaedo we also find the activity of testing a hypothesis through elenchus. This activity seems at first sight to be absent in the Republic; but passage 534c, that speaks about elenchus that applies to the Good, suggests that even the test of the hypothesis might be present in the Republic. Moreover Plato uses almost the same terminology in describing the two methods [87].

However, the two methods are not identical. The main difference is that the method in the *Phaedo* does not claim to give infallible knowledge, because it does not arrive at a non-hypothetical first principle but only at something 'adequate'. The 'adequate' of the *Phaedo* is still a hypothesis although there is a general agreement about it or it is regarded as something 'plain to all'. Stahl (1960,445) finds another difference between the two methods. In the *Phaedo* the hypothesis is posited in order to prove something; but in the *Republic* the hypotheses are the problems to be proved. However the difference is only apparent. The hypotheses of the *Republic* are the principles from which the mathematicians prove their theorems and in this sense they are posited in order to prove something. Moreover at 533c8 Plato uses the term 'hypothesis' not only for the principles of mathematics but for propositions that clearly belong to dialectic.

9. Plato and the first principles of geometry

Another external piece of evidence that supports our interpretation, may be found in the evolution of the axiomatization of geometry and Plato's contribution to it. This is very relevant for my interpretation of Plato's method and therefore I deal with it at length in this section.

We said (p 219) that there are in Euclid two kinds of definitions: a) definitions of elementary geometric objects (point, line etc) that are never invoked in Euclid's proofs and that are mathematically useless, and b) definitions of other mathematical objects (kinds of angles, or of figures etc) that are frequently invoked and play an essential role in proofs. I will argue now that the second kind of definitions show an earlier stage of mathematical definitions that has a purely mathematical origin. The first kind of defintions represent a later stage, and it is formulated under the philosophical influence of Plato and the Academy.

In Rep. 510c2-5, Plato says that the mathematicians posit as principles [88] "the odd and the even, the various figures and the three kinds of angles" and starting from them, they prove all the others. Now, the definitions of all the above mathematical objects belong to the second class. Plato says clearly that all these are 'principles' (for Plato they are 'hypotheses') of mathematics. They have been formulated by the mathematicians. On the other hand, the task of the dialectician is to proceed beyond these 'hypotheses'. So while the mathematicians are responsible for the second kind of definitions the philosophers are responsible for the first. An 531e Plato says that there are among the mathematicians a few philosophers (he probabply has Theaetetus in mind). Hence, the first kind of definitions may be the task of the mathematicians-philosophers.

Let us now test this conjecture. In Euthp 12d, the even number (category two) is explained as that which is not scalene but isoceles, that is, according to the old geometrical 'pebble-arithmetic' pythagorean tradition. The definitions of the odd and the even in Euclid are different (Book VII, def. 6,7; "an even number is that which is divisible into two equal parts") [89]. Now in the Laws 895e, (the latest platonic dialogue, written many years after the death of Theaetetus) Plato defines an even number as "a number divisible into two equal parts" [90]

In Meno 76a, Plato gives a definition of 'figure' (category one) as "the limit (πέρας) of a solid" [91], and in Parm 145a, we have an explanation of the term limit ("what contains is a limit"). According to Aristotle, (<u>Met</u>. 992a20) Plato regarded the genus of point as being a 'geometrical fiction', calling a point, the beginning (or in other words a limit) of a line.

Now in Euclid, 'figure' is defined as "that which is contained by any boundary (ὅρος) or boundaries" (def.I.14), and 'boundary' is defined as "that which is an extremity (or limit -πέρας) of anything" (def. I,13). The similarities between the above definitions and Plato's one of figure and his explanation of 'limit' are striking. But the similarity becomes an identity, if we compare Plato's definition of figure with Euclid's def. XI,2: "a limit (πέρας) of a solid is surface".

I quote now some definitions from Euclid.

1) A <u>point</u> is that which has no part (def. I,1).

2) A <u>line</u> is breadthless length (def. I,2).

3) The extremities (or limits -πέρατα) of a line are points (def. I,3)

4) A <u>surface</u> is that which has length and breadth only (def. I.5).

5)The extremities (or limits-πέρατα) of a surface are lines (def. I,6).

6) A <u>solid</u> is that which has length, breadth and depth (def. XI,1).

7) An extremity (or limit -πέρας) of a solid is a surface (def XI,2).

We see that the definitions of a 'line', 'surface' and 'solid' are followed by statements (not really definitions) about their limits (def. 3,5,7). Definitions 3, 5, and 7 were probably the older definitions of point, line and surfce - as limits of line, surface and solid respectively. This is attested by Aristotle

(Met. 1060b14-16, Topics 141b5-10 and Categ 5a2-5). These definitions originated with Plato, as the definitions of 'figure' and 'point' (as beginnig of line) show [92]. These definitions were attacked sharply by Aristotle (cf the above passages) and probably by others. So they were replaced (in Euclid's Elements) by definitions 1,2,4. Nevertheless Euclid kept them as second definitions or as explanatory statements.

In Parm 137e, Plato gives a definition of the straight line (first category) as "that of which the middle covers the ends" (εὐθὺ γε οὗ ἄν τὸ μέσον τοῖν ἐσχάτοιν ἐπίπροσθεν ῇ). Aristotle quotes it in Topics (148b27) (οὗ τὸ μέσον ἐπιπρόσθει τοῖς ... πέρασιν). Proclus (In Eucl 109) also quotes this definition (ἧς τὰ μέσα τοῖς ἄκροις ἐπιπροσθεῖ) saying that it originated with Plato himself. Aristotle does not mention the name of Plato but states it in combination with the definition of the line as the extremity of a surface. From the way in which Aristotle speaks about the above definition of the straight line, we have the impression that is was a well known definition at that time [93].

I am going now to show the philosophical origin of the definitions of a unit (Eucl. VII,1) and of a point (Eucl. I,1). Plato gives a definition of a unit as that which has no parts (ἀμερὲς δήπου δεῖ παντελῶς τό γε ἀληθῶς ἓν κατὰ τὸν ὀρθὸν λόγον - Soph 245a8-9; cf Rep 526a). This definition is almost identical to Eucid's definition of a point (σημεῖόν ἐστιν οὗ μέρος οὐθέν) [94]. It is plausible to suppose that this Platonic definition of a unit was replaced later by the Euclidean one (VII,1: "a unit is that by virtue of which each of the things that exist is called one") when that definition of a point as 'an extremity' (or beginning) of a line was challenged. So the old definition of a unit became the definition of a point and a new definition of a unit was given (Eucl. VII,1). If we look at this later definition of a unit, we can see that it is formulated according to the Platonic philosophy. Everything that is called one, is called so by virtue of the unit (that is, the Form of Unit). The above two definitions of a unit are not the only ones [95]; but the common characteristic of all definitions of a unit is that a unit is not divisible [96]. The indivisibility of a unit however is a philosophical and not a mathematical conception [97].

If we pass now to Euclid's common notions (axioms) we may find that they also have their origin in dialectic and especially in the philosophy of Plato. We find in Euclid nine common notions. The first eight (the ninth is probably spurious [98]) are propositions about equality and inequality. In Parm 154b we find the fourth common notion of Euclid. In Euthd 293d4-5, and in Rep 437a we find two forms of the law of non-contradiction. We must however observe that Plato treats these propositions not as self-evident but as debatable cases. So in Charm 168c, he arrives at the absurd statement that something is double and half of itself (cf common notions 5,6), and later on (168e4-6) he states that this is impossssible (not generally but) in magnitudes and numbers.

In Theaet 162e he states three agreed statements (homologemata) as a basis for the discussion:

a) As long as something is equal to what it was, it does not come to be greater or smaller in size or number (than it was).

b) If something has nothing added to it or substracted from it, it is equal (to what it was).

c) Nothing is what it previously was not, without having come to be.

Plato does not say that the truth of the above propositions is something evident, but labels them as homologemata. All three are again propositions concerning equality and the way in which he states them, reminds us of the way in which Euclid puts forward his common notions. I think then that we may reasonably suppose that the mathematical axioms have their origin in dialectic [99].

It is probably not too much to say that Plato in the Republic posited a programme of reducing mathematics into its most elementary principles. We know that this programme was fulfilled by the mathematicians of the Academy (see Chapter VII, section 1) and it seems that even Plato himself contributed to the foundational problems of mathematics.

Although mathematics is inferior to philosophy, Plato regards a training in mathematics as an essential preliminary to dialectic. Mathematics constitutes a good training in abstract thought [100] and a good means for the elevation of the spirit to the world of the Forms (521d, 523a, 525c, 527b).

Plato is not interested in applied mathematics or in natural sciences. He wants his higher education disciplines to take us right away from the natural world. Thus he exhorts astronomers to deal not with the observed movements of the stars but with the ideal ones (529d).

At Plato's time only geometry, and probably arithmetic, had been formed (even in an incomplete way) into an axiomatic deductive system. Geometry is the model of the organization of the other sciences of dianoia. So Plato exhorts astronomers to organize their science on the model of geometry (530b). Dialectic can better proceed to its upward path if previously the various sciences of dianoia have been clearly organized as proper sciences on the model of geometry. Plato exhorts even geometers to free their science from empirical elements and to develop it in a more abstract theoretical level, blaming them because of their terminology and probably also because of their techniques (527a).

It seems plausible to suppose that Plato in the Republic is sketching a double programme of research which aims firstly to eradicate the above defects of the mathematical sciences and to develop new fields of research (like solid geometry) on the model of geometry; and secondly, to continue the axiomatization of mathematics reducing the principles of the sciences of dianoia to the most elementary (and prior) ones [101] in order to find the common principles and theorems of all the sciences (531d) and after that to proceed to the proper (or higher) dialectic. However this last is only a programme for the future, of which even Plato himself cannot say many things and whose realization he does not yet see.

10. Other Interpretations of dialectic

In spite of all evidence about our interpretation of the dialectical method in the Republc, there are some objections to it. Firstly,according to this view the non-hypothetical pinciple ought to be a proposition from which all the basic propositions of the sciences can be derived; but it is beyond the human power of mind to deduce everything from one principle [102]. Moreover we have already said that there are passages that suggest that the first principle (the Good) is not a proposition. It is difficult to see how we can deduce statements about Forms from statements about the Good. It is also difficult to see how such deductive procedures are to be connected with the apprehension of the Forms.

Secondly, it is difficult to see how we can obtain knowledge or give an account of the Good (534b-c). Because, if it were possible to give an account of the Good and to distinguish it from all others, then there would have to be specifications of the Good, as there are for the other Forms. But specifications of any thing are in terms of its relation to the Good. We may then conclude that there are no specifications of the Good. Thirdly, some passages suggest that the role of dialectic is rather an explanation or a clarification of concepts.

I have no answer to the first two questions. The only thing I can say, is that they refer not to the first steps of dialectic but to the last ones and to the Form of the Good; and it is almost certain that even Plato himself had not any clear answer about them. These objections therefore do not appear to argue against the above interpretation. As regards the third objection, we may say that in our interpretation there is room for making definitions and clarifying terms in so far as this is done in an argumentative way like that in which Plato defines 'justice' in Book 4.

Nevertheless the deductive model explains: a) the relevance of mathematics for dialectic and b) the reason why dialectic is 'higher' than mathematics for it gives an account of its hypotheses) [103]. Moreover we know that Plato was clearly impressed by mathematics more than any other science and his philosophical understanding was much in debt to what he knew from geometry. The deductive model gives justice to that as well.

Some scholars think that passages 534b8-c1 ("if a man cannot by his account separate and distinguish the Form of the Good from all else") and 537c (that the dialectician has the power of seeing things as a whole), suggest the method of division and generalization (accompanied by elenchus) which is an essential part of Plato's dialectic in his later dialogues [104]. Division might not be absent from Plato's conception of dialectic in the Republic. It is probably hinted at in Glaucon's statement in 532d8-e1, and more clearly stated in 454a [105]. However, it is rather unlikely that 534b-c suggests the method of division. Division cannot provide an account (definition) of the supreme Form, but only of a subordinate one. Robinson (1953,162-5) has shown sufficiently that this interpretation is very unlikely. The technical terms that Plato uses when discussing the method of division in the later dialogues do not occur in the Republic and especially in the passage of the Line. On the contrary, the terminology used in the Republic is similar to that of the Phaedo 100-101, where the method is clearly propositional. It is very doubtful that Plato could think of the Good as the summum genus and it is very difficult to undestand how the process of finding a genus in a group of species was in any sense 'destroying the hypotheses'. The whole description of dialectic in 510-511 and especially the 'downward path' is in terms of a propositional deductive method, similar to that of mathematics. Robinson suggests that "if the upward path were generalization, it would surely have to be empirical. Generalization picks the universal out of the particulars given to the sense" (1953,163); but Plato is very clearly against the senses and experience in the Republic. Moreover the above interpretation does not explain the relevance that Plato shows for mathematics.

Gosling (1973, ch VII) has proposed a functional interpretation of the dialectical method. According to him the thing required by dialectic is explication rather than demonstration. The relevant aim is clarity of the system and not deductiveness. From reflecting on the hypotheses of mathematics Plato is led to genuine _episteme_, an inquiry of how it is best that the world be structured as it is. "If one comes across a machine, then trying to understand it will consist in assigning a supposed function to the machine and its various parts..... An assumption about general function will create expectations about the function of parts and study of the actual capacities of parts will modify one's view of the general function..... The procedure could, of course, be described as making hypotheses, but the relationship between thesis and hypothesis is generally looser than entailment or presupposition" (116). The way up consists in going from specific explanations to more general ones and the way down follows the opposite direction.

The functional model seems to be implausible. There is nothing in the _Republic_ to support this view, and Gosling's references to the text do not help his interpretation. Plato talks not only of clarity but of truth and certainty. Gosling's functional interpretation does not explain the relevance that Plato assigns to mathematics and does not fit into the account of the hypothetical method (and its application to the argument) of the _Meno_ and the _Phaedo_. According to the functional model the hypotheses of mathematics ought to be explanations or "assumptions about function" (p.116); and the destruction of the hypotheses must be "modification of one's view of the general function". Gosling's interpretation is influenced by medicine and arts and crafts, and applies better to the particulars (world of _doxa_) than to the Forms. Forms are stable, unchanged, ungenerated and indestructible things. Their world is static like that of mathematics. Mathematical methods are more suitable for the understanding of the the world of Forms than the functional method which gives better results if applied to particulars.

11. Dialectic and the geometrical method of analysis and synthesis

We have said that the hypothetical method of the *Phaedo* seems to be very similar to the dialectical method of the *Republic*. However besides the fact that the dialectical method arrives at the non-hypothetical first principle, there is another relevant difference between the two methods. In the hypothetical method in the *Phaedo* every step upwards is followed by a step downwards, so that the two ways are not separated [106]. In the method of the *Republic* the two ways are clearly separated and only after we finish the way up we begin the way down. One follows first the whole way up until the arrival at the Good and after that one proceeds to the way down.

The forms of the two methods can be presented as follows:

A. Hypothetical method in the *Phaedo*

$$G \underset{1}{\rightleftarrows} F \underset{2}{\rightleftarrows} E \underset{3}{\rightleftarrows} \cdots \underset{5}{\rightleftarrows} B \underset{6}{\rightleftarrows} A$$

B. Dialectical method in the *Republic*

$$\text{way up } G \underset{1}{\Longrightarrow} F \underset{2}{\Longrightarrow} \cdots \underset{5}{\Longrightarrow} B \underset{6}{\Longrightarrow} A$$

$$\text{way down } G \underset{6}{\longleftarrow} F \underset{5}{\longleftarrow} \cdots \underset{2}{\longleftarrow} B \underset{1}{\longleftarrow} A$$

As we saw in p 161, this was the main difference between the hypothetical method in the *Phaedo* and the geometrical method of analysis and synthesis. The hypothetical method in the *Phaedo* has the same form as the geometrical method of *apagoge*, the latter being the model for the former. In chapter II (p49) we have argued that *apagoge* was the early stage of the method of analysis and synthesis. Because of that the hypothetical method in the *Phaedo* and that of the *Republic* (dialectic) presents so many similarities.

As in the method of analysis, so in the dialectical method we hypothesize the proposition to be proved (the principles of mathematics that will be proved by dialectic are called 'hypotheses'; in the method of the *Phaedo*, the problem to be solved is not called a hypothesis). The way up, similarly to analysis, is a search for premisses or of what is prior. So both the analysis and the dialectical ascent are described in terms of an upward process. They both also end their upward journey at a principle that is independently known. In both methods the two processes (up and down) are separated and only when the analysis (or the dialectical ascent) ends, the synthesis (or the dialectical descent) starts. Moreover both synthesis and dialectical descent are deductive.

Terms (ὑπόθεσις, ἀρχή, τελευτή, καταβαίνω, ἄνω, ζήτησις, εἶμι, ἄνειμι) that Plato uses in his description of dialectic occur in most of the accounts of the method of analysis and synthesis. More striking is the similarity of Plato's account of dialectic with Albinus' second kind of philosophical analysis (*Didaskalikos* V 5-6) and Aristotle's account of geometrical analysis in *E.N.* 1112b15ff (see p159-60) [107]. That Plato *did* use in dialectic the method of analysis and synthesis is shown by an argument in the Phaedrus (245c-e: a proof of the immortality of the soul) whose form is the same as that of the method of analysis and synthesis and which is the standard example of the ancient commentators when they speak of the application of the geometrical method of analysis and synthesis to philosophical arguments [108]. Moreover, as we said in chapter II (see p56-7), in antiquity Plato's name was strictly related to the geometrical method of anaalysis and synthesis [109].

Robinson (1953, 166) objects to the analysis theory of the upward path that geometrical analysis is always deductive although Plato's dialectical ascent is not. However, in our interpretation of the geometrical method of analysis we show that analysis is not necessarily deductive and that its main characteristic is the search for premisses and auxiliary constructions whereby we shall solve our problem.

Saying that the hypothetical method in the Republic (dialectic) follows the model of the geometrical method of analysis and synthesis does not mean that the two methods are identical or that Plato was not conscious of the differences between a mathematical proof and a philosophical argument. Although the general form is the same in both methods, Plato's dialectic is more complicated. We may use elenchus to check a hypothesis or induction to get a premiss, and sometimes the method is used to find definitions and clarify concepts.

12. The Dialectical Method and the Argument in the "Republic"

In the chapter on the Phaedo we have seen that we have good reasons to support the view that Plato does apply his hypothetical method throughout the argument of the dialogue. Our purpose now is to see whether or not we can trace an application of the dialectical method in the main argument of the Republic.

In section 11 (p255) we suggested that there are two main differences between the Republic's account of the dialectical method and that of the hypothetical method of the Phaedo: a) in the Republic the two processes of dialectic (ascent and descent) are clearly separated and b) the dialectical ascent arrives at the Form of Good - the first and unique Unhypothetical Principle.

Nevertheless at Republic 506d-e, Plato makes Socrates say that he does not know what the Good is, and that the only way to speak about it is via images. Furthermore at 532d-533a Socrates refuses to give a clear account of the dialectical method. It seems then that Plato considers the full application of the dialectical method and the arrival at the Good either as an ideal never fulfilled, except by the philosopher - king, or as a future philosophical programme, about which he cannot say anything now.

Given Plato's above remarks, it does not seem possible for us to find a full application of the dialectical method in the Republic, which includes the arrival at the Form of Good. However there is a passage in the fourth book (437a) that speaks about an introduced hypothesis, and whose terminology is very similar to that used at the Phaedo 100a and 101d-e.

Therefore it is probably worthwhile to have a look at the main argument of the Republic in order to see if we can find an application of the hypothetical method as described in the middle books of the Republic [110].

The first Book of the Republic is methodologically similar to the first or'Socratic' dialogues. The only method that we find there is the elenchus.

Book II opens with the statement of the central problem of the dialogue. It must be proved that in every way it is better to be just than to be unjust (357b1-2) [111].

Socrates and Glaucon agree that justice is desirable both for itself and for its consequences [112]. We see therefore that the conclusion of the argument is stated from the beginning and is firmly believed by the interlocutors. We are familiar with this kind of procedure from Euclidean geometry. There, the theorem to be proved is first stated, then the proof follows, and at the end we have the formula quod erat demonstrandum [113].

As Socrates has shown at the end of Book I (354b-c), and as the two brothers accept (368b4-6), the question of the superiority of justice presupposes an investigation into the essence of justice [114].

However the question "what is justice itself" is a very obscure one (368c8) that must be approached indirectly. At 368e Socrates says that justice is an attribute of both an individual and a city. But since a city is greater than an individual, "then perhaps justice may exist in greater proportions in the greater space, and be easier to discover. So if you are willing, we shall begin our inquiry as to its nature in cities, and after that let us continue our inquiry in the individual also, looking for the likeness of the greater in the form of the less" (368e-370a - Lindsay).

Socrates is assuming that justice in the individual is the same as justice in the city [115]. This is exactly the likeness between small and large letters (368d). Therefore the question of "what is justice in the individual" is reduced, via the above principle of likeness, to the the question "what is justice in the city". But such an inquiry presupposes an inquiry into the origin and nature of the city itself (369a5-10). In order to inquire into the origin of the city we must find why poeple come to live together in a city. This is, according to Plato, because individuals are not self-sufficient and they have many needs (369b5-8). This is the principle of the formation of the city.

So far the argument has been proceeding indirectly, not from some premisses to the conclusion (the proposition to be proved) but from the conclusion to the presuppositions of it. The procedure was not a demonstrative one but rather a tracing back to the premisses of the argument or presuppositions of the question. It was an inquiry into "what is prior", a backward (or upward) procedure which is not necessarily deductive. In this procedure the interlocutors are helped by their experience and intuition in their efforts to find out what is their problem.

Having found the principle of the formation of the city, Socrates proceeds downwards to find what justice is in the city. At 369c Socrates says that men, being in want of many things, come to live together in one place, doing different work and exchanging their products. To this common settlement we give the name of city. Therefore, according to Plato, the many needs lead to the division of labour and the formation of the first community [116]. A direct result of the principle of the division of labour is that better work is done if every man practices only one trade (370b4-5, 371c3-5). According to this last principle, the more needs we have, the more specialized men we need in order to satisfy them. Therefore we shall need merchants, shopkeepers, hired labourers, market place, coinage for the exchange etc (370d-371e).

This is the first or 'healthy' city (373b3), which is thought of wholly in material terms as a community devoted solely to production and consumption. After that we have the growth of the first city into a 'luxurious' one (372-4). The ideal (good) city develops not from the first city but from the luxurious one through a purging process that gets rid of what is unhealthy in it [117].

The process of the formation of the ideal city ends at 427c. At 427e6-8 Socrates says that "our city, as it has been rightly established, is perfectly good". Therefore it must have all the four virtues (wisdom, courage, temperance, justice). If therefore we manage to identify in the ideal city the three first virtues, then what remains must be justice (327e10- 328a6) [118].

At 433e12-13 it is agreed (ὁμολογοῖτο) that justice is "ἡ τοῦ οἰκείου τε καὶ ἑαυτοῦ ἕξις καὶ πρᾶξις", [119] and later on (434a-b; cf 435b) Plato makes clear that οἰκειοπραγία means mainly that every individual in the city functions within the limits of his own class and does the work which is appropriate to his class. In this way a just city is a well ordered city and the greatest injustice is the meddling and interchanging among the three classes (434b9-c5) [120].

Socrates has thus solved the first part of the problem posed in his simile of the small and large letters. Now we must look at the small letters to see whether they are the same as the large letters. If they are, then our search will have ended; if not, we shall have to go back to the city and check our findings there (434e).

The new problem now is to see if the small letters are the same as the large. This means that we must establish whether justice in the individual is the same as the accountof justice which we found in the city. This similarity presupposes that the city and the individual have the same structure (4355-c2). Only if we find the latter shall we be able to establish the similarity between justice in the city and justice in the individual. This means that we must try to find whether or not the human soul has three parts that correspond to the three classes in the city (435b4-7, 435e1-3). It is easy to see, through experience, that the three characteristics found in the three classes of the city - spirited character, love of learning, love of riches - exist in the human soul as well. The real problem is whether or not these three characters are three distinct elements of the soul (436a8-b3).

In order to solve this problem Plato resorts to a principle (hypothesis, 437a6) that many critics consider as a version of the law of non contradiction and which I shall call -following J. Annas [121]- the principle of conflict: "It is obvious that the same thing will not at one and the same time, in the same part of it, and in the same relation, do two opposite things or be in two opposite states; so that if we find such conjuction of opposites in these elements, we may be sure that they are not one, but several" (436b8-c1) (Lindsay).

At 436c-e Socrates confronts some possible objections to that principle and concludes: "In any case we do not want to be compelled to examine in detail all difficulties of this kind, and to waste our time in establishing their falsehood; and so we may proceed on the assumption (ὑποθέμενοι) that what we have said is right, agreeing (ὁμολογήσαντες) that if at any time we have to change our opinion all the consequences (συμβαίνοντα) of our assumption must be abandonned"(437a4-9). This passage is of great importance because it is very similar in both vocabulary and meaning to the description of the hypothetical method that Plato gives in the Phaedo (100a and 101d-e).

a) The hypothesis has the character of a premiss and is used to prove something that has already been stated from the beginning (here, the threefold character of the soul).

b) The hypothesis is posited provisionally.

c) If the hypothesis is proved false then all the consequences drawn from it must be abandonned [112].

The above passage gives us ground to believe that Plato consciously applies here his hypothetical method.

As regards justice in the individual, our main problem was to see if our account of the city applies to the soul. However, if it is to apply, the soul must have the same structure as the city. In order to prove this latter statement we resorted to the principle of conflict. Here the tracing back, in order to find the premisses of the argument, has finished. That constituted the 'upward path'. Now it is time for the proof or 'downward path'.

Plato, using the principle of conflict, proves the threefold character of soul. However this is only "fairly agreed" (ἐπιεικῶς ὡμολόγηται -441c5), for it depends on unproved premisses. Having found the three elements in the soul corresponding to those in the city, Plato then proceeds to the assignation of the virtues and at last he concludes that justice is the order and harmony among the three elements of the soul (443c-e). On the contrary, injustice is the destruction of the order and harmony of the soul (444d-e). From this immediately it follows that justice is more profitable than injustice because it is the health of the soul and therefore it is something good in itself (445a-b).

Now we have found what justice and injustice is, Socrates is ready to proceed to find the various kinds of injustice in the city and in the soul, and to examine whether or not justice is more desirable for man and makes him happier. But instead of that we have an interlude of three chapters.

At the beginning of Book 8 (543c7-544b2) Socrates and Glaucon explicitly refer back to the discussion of Book 4 (445c-e), before the 'digression' of Books 5-7. In Books 8-9 Plato tries to show that justice has also good consequences, in particular that it makes the agent happy.

In Books 8 and 9 Plato classifies the kinds of bad cities and the corresponding men, following the analogy between 'small and large letters'. The various kinds of badness consist of various disturbances of the right order (or harmony) between the 'elements' of the good city or the soul of the good individual. After that three direct arguments follow (576c-588a) that prove that the just is happier than the unjust.

Here the proof that the just man is happier than the unjust (and therefore that justice is more desirable than injustice) has finished; and with it the main argument of the Republic finished. Book 10 is a 'coda' that does not contribute to the main argument and deals with two themes: a)the banishment of poetry (595a-608c), and b) the rewards (or artificial consequences) of justice (608c-621d) [123].

We must note now that the argument of the Republic does not give any definite definition of justice and does not prove definitively the original thesis of the dialogue (i.e. that justice is more desirable than injustice). All the conclusions are tentative because they rely on unproved hypotheses. Moreover arguments by analogy give only approximate answers.

Plato recognises the approximative character of the answer that his argument gives. At 435d he says that "I do not think that we shall ever settle this whole question accurately by our present methods. The road which leads to that result is longer and more toilsome" (Lindsay). The 'longer way' here seems to concern the question whether the soul does or does not contain three elements, but in Book 6 (504b), where Plato refers back to this passage, it is clear that the 'longer way' refers to the 'μέγιστον μάθημα', that is the whole question about the soul, the virtues and their relation to the Form of Good [124].

For a final and definite answer we would need an investigation of the relation between 'just' and 'good'. A final definition of justice would have relied on the notion of good and the theory of Forms. However such an investigation was never undertaken. Books 5-7 show us, in an illustrative way, the right method for the arrival at the Good and knowledge, but they stop there. The dialectical method, as applied in the argument, does not arrive at the Unhypothetical Principle. The right argument about the question of the Republic is never fully stated and could not have been unless Plato had been willing to expound his doctrine of the Good. Therefore, the argument in the Republic gives only some steps of the dialectical programme and does not display dialectic in its final form.

We said that the hypothetical method in the Phaedo and that in the Republic (dialectic) differ in their structure in the sense that in the method of the Republic, the way up and the way downare separated, while in the Phaedo, the two ways occur in every step of the method. Let us now see if we can find this difference in the two applications of the method (or methods) in the Phaedo and the Republic.

In the Phaedo, the whole argument consisted of various particular arguments interrelated with each other. In every particular argument, a hypothesis was posited in order to prove the required conclusion and only after this proof, did we proceede to confirm the hypothesis, positing a 'higher' one. So, for example, on the basis of the principle of Opposites, we proved the Antapodosis hypothesis. After that, the principle of Opposites was proved on the basis of the principle of Exclusion of Opposites and later on, this last was proved on the basis of the theoy of Forms. Therefore, in every particular argument in the Phaedo, we had a stage of the method in which both ways (up and down) were present.

In the Republic, the two ways are clearly separated. Starting from the original problem, we have a continuous regress backwards to the presuppositions of the problem. Only when this process ends somewhere, do we take the opposite way (downwards) and we solve our initial problem. So, for example, the problem is to show that in every way it is better to be just than unjust. This question is reducėd to the problem of finding what justice is in the individual and this last to the question of what justice is in the city. The inquiry into justice in the city presupposes an inquiry into the origin and nature of the city and in order to find this we must first find the principle of the formation of the city. This last is that people have many needs and are not self-sufficient. Starting from that we change direction and proceeding downwards we find what justice in the city is. We see therefore that the dialectical method of the *Republic* is modelled on the pattern of the method of analysis and synthesis.

NOTES TO CHAPTER VI

[1]. The word 'ὁμολογουμένως' is ambiguous. It may mean either 'consistently' or 'on agreement' (between teacher and pupil or the interlocutors); see pp9-10 , and Cornford, 1932/1965, 66. The same ambiguity recurs in 533c: "how can such ὁμολογία ever become knowledge?". However here, as in Crat. 436d4, the word ὁμολογία refers to 'agreement' or 'consistency' between propositions. Moreover given that Plato speaks here about mathematics, I think that the rendering of 'ὁμολογουμένως' with 'consistently' gives the meaning that Plato had in mind better. About the passage of Cratylus see ch.5 section A.

[2]. Annas, 1981, 289; cf. Wedberg, 1955, 71.

[3]. I shall supply more evidence about the axiomatization of geometry in the next chapter.

[4]. 'Cannot' here does not mean that the hypotheses are insuperable obstacles, but that it is not the geometer's scope to go beyond the hypotheses in so far he works as a mathematician and not as a philosopher (see pp230-1 , and ch. VII, section 3).

[5]. Cf. Ross, 1951, 50; Cross and Woozley, 1964,242; Annas, 1981,287; Robinson, 1953,152.

[6]. We have seen in the first chapter that for Plato, a hypothesis is the supposing of a proposition. Sometimes it has the form of a postulated entity, but it is really an assumption of the existence of a thing. Moreover in both the Meno and the Phaedo we saw that the 'hypotheses' are propositions.

[7]. Cf. Crombie, 1963, 552; Annas, 1981, 277.

[8]. 1965, 21-30; cf. also Bluck, (1935, appendix 6, and 1957, 26-9) who regards the hypotheses as 'notions'. I have already

critisized Bluck's interpretation in ch. V, section 1.

[9]. 1967, 193-203; see also Solmsen, 1977,88.

[10]. Cf. C.C.W.Taylor, 1967,195-6; Stahl, 1960, 440-1.

[11]. Chapter V, section 1; cf.C.C.W.Taylor, 1967, 197.

[12]. See C.C.W.Taylor, 1967, 201; Heath, 1926, 117-20. Archytas investigated general definitions as well (see Aristotle _Met._ 1043a21).

[13]. Euclid's _Elements_ begin with three kinds of principles: definitions, common notions and postulates. It is not certain whether or not this threefold distinction of principles reflects the aristotelian one in _An.Post._ I,2 (definitions, axioms and hypotheses -or hypotheses plus postulates). Euclid's definitions are of general terms designating geometrical properties and objects (straight, curve, point, line, number, circle, centre, distance, angle etc). There are no definitions of: a)verbs describing mathematical activities (draw, extend etc), b) expressions of relative position (from...to, interior, between etc), c) expressions of extent (finite, ad infinitum etc), d) expressions of size comparison (equal, less etc) (see Mueller, 1981,39.

[14]. See Ross, 1951,51; Stahl, 1960, 441-3; Lee, 1935, 13-8; Heath, 1949,55; A.E.Taylor, 1937,291; Cornford, 1932/1965, 65. For different views see Robinson, 1953, 101; Barnes, 1975, 103-4; Leszl, 1981, 309-10; Landor, 1981; Gomez-Lobo, 1981, 32-3

[15]. Cf. Leszl, 1981, 229-30; C.C.W. Taylor, 1967, 18-9; Heath, 1949, 279-80; Landor, 1981, 309-11.

[16]. Cf Boyle, 1974, p.9.

[17]. 1967, 198; Taylor refers to and analyzes the passages _Parm._136a-c, _Tim._61d, _Tim._53c-d, _Epin._977a-d.

[18]. See p 9 ; cf. also C.C.W.Taylor, 1967, 199.

[19]. Cf. Adam, 1902, vol.II, 66; Bluck, 1961, 96; Boyle, 1974,9. In pp 11-13 we have seen that in the post-euclidean mathematics the term 'hypothesis' is used for all the kinds of principles and more specially for definitions.

[20]. Plato often insists on positing the existence of a thing at the beginning of an argument. See e.g.Phd 74a11-12, H.Ma 287c1-d2, Prot 330c. Aristotle also requires that one must first determine whether or not something exists before proceeding to the question, 'what it is' (An.Post 89b34, 89b38-90a3, 90a22-3, 92b4-7, 93a17-21); cf Ackrill,1981,364. On the contrary there are passages in An.Post (e.g. 72a23, 90b30-3) where Aristotle says that we must first know the 'what it is' and then to proceed to the demonstration of the 'being'. However we must note that when Aristotle says that, he speaks only about non-primitive items the existence of which has to be proved. For this problem in Aristotle see Ackrill, 1981, 364-9.

[21]. cf also definition 18.

[22]. Similarly def.1,6 asserts that "the extremities of a surface are lines". Cf also Book XI def.2 ("An extremity of a solid is a surface").

[23]. Cf. C.C.W.Taylor, 1967, 199.

[24]. See Mueller, 1981,40; Szabó, 1971, 221-2.

[25]. See e.g. 987b14-18, 1028b18-21, 1076a19-21, 1086a12; cf Proclus In Eucl. 3-4, Syrianus, Comm in Arist Gr VI 1, 82.20-26 (Kroll).

[26]. 1902, appendix 1 to Book 7; cf also Hardie, 1936,51-2.

[27]. See Wedberg, 1955, chs 4-5; Bretlinger, 1963; Burnyeat, 1984. For a criticism of Wedberg's views see Scolnicov, 1973, ch

VIII. Bretlinger's account for the existence of intermediates is based on three assumptions that are not obviously true: a) the 'faculties' (Book 5) are identical in meaning and function with the 'states of mind' (Book 6); b) the relation image/original implies always ontological distinctions; c) the passage 534a shows that Plato does accept ontological distinctions between the four states of mind.

Burnyeat's more sophisticated paper (which in a sense gives an improved version of Wedberg's arguments), does not bring any convincing evidence from the Republic about intermediates.

For an argumentation against the theory of intermediates in the Republic, see Ross, 1951, 59-67; Murphy, 1951, 167-8; Joseph, 1948, 49-52; Hackforth, 1942,3-4; Gosling, 1973, 109-111; Cross and Woozley, 1964,233-7. Cherniss (1942, 75-8) goes further and rejects as unreliable all Aristotle's testimony about platonic intermediates.

[28]. 525d6 ("...that it [arithmetic] strongly directs the soul upwards and compels it to discourse about the numbers themselves") seems to support as well that the objects of dianoia are Forms. However it is connected with 526a which some scholars think that may refer to mathematicals. In 597b Plato speaks about only one intelligible bed, i.e. the Form of Bed; cf also 7th Letter 342a-c and Symp 210a-211c where we cannot find anything like the 'intermediates' that Aristotle speaks about.

J. Annas (1975) claims that we cannot conclude anything about intermediates or the opposite from Plato's dialogues. Plato and Aristotle discuss different problems and have in mind different arguments.

[29]. Cf Crombie, 1963,93; Wedberg, 1955, 108; Burnyeat, 1984,6.

[30]. I cannot agree with Burnyeat (1984,8) that for Plato the main problem in the upper line is ontology and not methodology.

[31]. Cf A.E Taylor, 1937, 289. Aristotle in *Met* 1077a1-5, in his polemic against Plato and Platonists as regards the problem of 'mathematical intermediates', speaks about mathematicals in astronomy, harmonics, and optics. Proclus (*In Eucl* 73) includes among the 'sciences' not only branches of mathematics but natural sciences as well.

[32]. Cf Murphy, 1932,97; Raven, 1965, 158-9; Gould, 1955,178; Nettleship, 1901, 254-5; Crombie, 1962, 125-6; Crombie, 1963, 80-5; Gulley, 1962,60. Spinoza's *Ethics* is a good example of a mathematically organized moral theory.

[33]. See Crombie, 1962, 126; Gosling, 1973, 103.

[34]. Although Plato does not name dialectic here, but in 511b-c, it is clear from the context that he speaks about it.

[35]. See Crombie, 1963,509; Robinson, 1953, 156-7 who translate "ἐπιβιβάσεις τε καὶ ὁρμὰς" as "steps and sallies".

[36]. Robinson, 1953,149.

[37]. Rosenmeyer (1972) tries to show that in Plato the way from premisses to conclusions is not a way downwards but upwards (or at least horizontal). Similarly, the 'ascent' of dialectic is not really an ascent but a descent. However, his interpretation of the relevant passages (*Rep* 511, *Phd* 101d-e) is not convincing.

[38]. I think that Cherniss is right in saying (1947, 144): "Since ἀνυπόθετον is a coinage and not merely a negative of ὑπόθετος, a form which Plato does noe use, it probably has an etymological sense connected with the etymological turn given to ὑπόθεσις in 511b5-6, in which sense ὑποθέσεις is contrasted with ἀρχὰς. It ought not then to be thought of as

'unhypothesized', the negative of the action 'to hypothesize', but as 'not resting under something else to which it is a stepping stone'. It is ἀνυπόθετος in the sense that you cannot posit another hypothesis from which you can deduce it, and that is why it a true ἀρχὴ".

[39]. Sayre (1969,46) thinks that there is not only one non-hypothetical principle, but the "μέχρι τοῦ ἀνυποθέτου ἐπὶ τὴν τοῦ παντὸς ἀρχὴν (511b6-7) clearly suggests the opposite.

[40]. See 534e: "Now do you not think, I said, that we have placed dialectic like a coping stone on the top of our studies, and that there is no other study which deserves to be put above it" (Lindsay). Cf also Proclus <u>In Eucl</u>, (Friedlein), 75: "There is only one unhypothesized science, the others receiving their first principles from it..... For no science demonstrates its own first principles or presents a reason for them; rather each holds them as self-evident, that is as more evident than their consequences" (Morrow).

[41]. The translation is from Robinson, 1953,151. Cf 533b1-3; cf also 532a5-b2: "So too when anyone tries by dialectic through the discourse of reason unaided by any of the senses to attain to what each reality is and desists not until by sheer intelligence he apprehends the reality of good, then he stands at the real goal of the intelligible world" (Lindsay).

[42]. Cf Annas, 1981,287; Hackforth, 1942,7. These two different accounts of dialectic led Cornford (1932/1965, 80sq) to argue that we have two methods of dialectic, one mathematical and one ethical. Similarly, there are two non-hypothetical principles, the 'one' and the 'good'. Mathematical dialectic employs the geometrical method of analysis and synthesis while ethical

dialectic employs the socratic elenchus. However, this distinction does not find evidence in the text. Moreover his interpretation involves a narrowing of the significance of the Good which is, according to Plato, the 'greatest study'. For a more extensive criticism of Cornford's views see: Murphy, 1951, 188-9; Ross, 1951,54; Hackforth, 1942,5-6.

[43]. Among those who construe the non-hypothetical principle as the Good are Adam, 1902, ad loc.; Ross, 1951, 42,54; Robinson, 1953,160; Crombie, 1963, 550-1; Raven, 1965, 161-2; Cross and woozley, 1964, 251; Burnet, 1914, 230; N. White, 1976, 99,110; Murphy, 1951,147; Hare, 1963,35; Bluck,1961,283. See also Proclus, In Eucl (Friedlein), 31, and In Remp (Kroll), 283.

[44]. Cf Annas, 1981, 285-6; Gulley, 1962,59; Solmsen, 1977,88; Nettleship, 1901,270; Crombie, 1963,555. See also Proclus In Eucl, 7-8: "let us enumerate the single theorems that are common to them all [branches of mathematics], that is, the theorems generated by the single science that embraces alike all forms of mathematical knowledge; .... such are the theorems governing proportion .... likewise the theorems concerning ratios of all kinds...and the theorems about equality and inequality in their most general and universal aspects, not equality or inequality of figures, numbers or motions, but each of the two by itself as having a nature common to all its forms and capable of more simple apprehension. And certainly beauty and order are common to all branches of mathematics" (Morrow).

[45]. See also Murphy, 1951,177; Natorp, 1903/1929, 187; Guthrie, 1978, 525 n.2; Crombie, 1963, 555; Raven, 1965,161; Stenzel, 1974,285.

[46]. 1934; for a similar view, see Hardie, 1936,69.

[47]. Irrationals were always thought as magnitudes and therefore as objects of geometry, never of arithmetic. Only after Eudoxus' new definition (and theory) of proportion (which is in a period later than the Republic), that applies to both rationals and irrationals, could such a conception of irrationals be possible. For a critisism of Taylor see Ross, 1951, 56-7; Crosss and Woozley, 1964, 247-8.

[48]. The possibility is always open that some of the hypotheses of mathematics might be proved false.

[49]. Cf Robinson, 1953,161; Cross and Woozley, 1964,248; Annas, 1981,287; Hackforth, 1942,8; Gosling, 1973,108; Stahl, 1960,447; Rose, 1966, 471 n.1; Cambiano, 1971,194; Boyle, 1974,15 n.5; Ross, 1951,57.

[50]. See Crombie, 1963, 556-60; Murphy, 1951, 173-4; Gosling, 1973, 112-8. N White (1976, 97-8 and 112 n.43) proposed a slightly different view. For him, the geometers want theorems to be true statements about the sensible figures they draw in the sand. In this way, their definitions are about sensible things and not explicitly about Forms. Therefore dialectic 'destroys the hypotheses' by ceasing "to cast those axioms in the way in which the geometer ordinarily casts them and will instead adopt a different less misleading manner of putting them". However, Plato is clear that the geometers think about eternal unchangeable objects (510d6-8) and not about sensibles. Moreover, according to his interpretation, dialectic is not a long process as the word πορεύεται (533c8) suggests.

[51]. This is the most accepted interpretation. See eg Ross, 1951, 52-8; Robinson, 1953, 159-60; Cross and Woozley, 1964, 248-9; Stahl, 1960,447; Cherniss, 1942,143; Gulley, 1962, 55-8; A

E Taylor, 1937,293; Stenzel, 1974,282; Robin, 1971,60; Natorp, 1903/1929, 188; Burnet, 1914, 229-30; Wedberg, 1955, 44,60; Webberg, 1982, 70-1.

Crombie (1963,556) says that we cannot prove the axioms of mathematics. But as we saw in section 2, the hypotheses of mathematics that Plato mentions in 510c do not concern the genus of the subject and they can easily be derivative.

[52]. 1953,159; similarly, Hackforth, 1942,8.

[53]. See Heath, 1921, ch III.

[54]. Cf Hare, 1965,31. Gulley (1962,56) is wrong in saying that this defect is not an essential feature of mathematics which can be eradicated.

[55]. Cf Robinson, 1953,154-5; Cross and Woozley, 1964,244-5.

[56]. Cf Wedberg, 1982,71; Hare, 1965,30; N White, 1976,96; Cross and Woozley, 1964,242; Annas, 1981,279; Stahl, 1960,443. We must have in mind that the objects of mathematics are "intelligible with a principle" (511d2).

[57]. Cf Euthd 190b-c. For the close connexion of this passage to the theories of the central books of the Republic see Hawtrey, 1978.

[58]. However, there are 'defects' proper to mathematics and in this case Plato clearly critisizes the mathematicians and exhorts them to remedy these defects (see 527c, 530b, 531a).

[59]. Cf Aristotle Met 1005a11: "καὶ διὰ τοῦτο οὐ τοῦ γεωμέτρου Θεωρῆσαι τὶ τὸ ἐνάντιον ἢ τέλειον ἢ ὂν ἢ ἓν ἢ ταυτὸν ἢ ἕτερον, ἀλλ' ἢ ἐξ ὑποθέσεως"

[60]. 1953,155-6; cf Cross and Woozley, 1964,244-5.

[61]. Others that accept that the Good is only the formal cause of the Forms, are Ferguson, 1921-2, 134; Natorp, 1903/1929, 188;

and perhaps Ross, 1951, 140-2.

[62]. Among those who think of the Good as having final causality are Guthrie, 1978, 504-5; Stahl, 1960,448; Gosling, 1973, 26-8; Nettleship, 1901, 256-7; Irwin, 1977,223; Joseph, 1948,18; Bluck, 1961,95; Adam, 1902,172; Stenzel, 1940,42; N White, 1976,101; Teloh, 1981, 134-5

[63]. Cf Ross,1951,41; Santas, 1983, 258 n.15

[64]. See Stahl, 1960,448; Adam, 1902, 258 n.15.

[65]. Jaeger, 1944,vol I, 296-8; cf also Adam, 1902, 171

[66]. For these two different accounts of knowledge in Plato, see Teloh, 1981, ch 3; Annas, 1981, 282-4.

[67]. See Proclus In Remp 284-5; cf Hare, 1965, 33-4; Stahl, 1960, 448; Robinson, 1953, 176.

[68]. Cf N White, 1976,106; According to Hare (1965), Boyle (1974, 7-8), Hackforth (1942,7), Cornford (1932/1965, 85-6) and perhaps Robinson (1953, 176), the non-hypothetical principle is a definition of the Good.

[69]. See Symp 210e, Seventh Letter 341c; see also Friendländer, 1958,69; Robinson, 1953, 174; Crombie, 1963, 561; Cornford, 1932/1965, 93; Cross and Woozley, 1964, 252-3; Stenzel, 1940, 31,37; Boyle, 1974,13.

[70]. For various forms of this conception about the Good see: Gosling, 1973, 60-71, 116; Teloh, 1981,144; Cambiano, 1971, 196-7; Murphy, 1951, 184-5; Joseph, 1948, 18-23. In 427e, the well formed city is said to be a good city that contains all virtues. According to the early or platonic Aristotle (frag. 79, Rose) the Good is "πάντων ἀκριβέστατον μέτρον".

[71]. Gosling,1973,117.

[72]. Cf Joseph,1948,19; Murphy,1951,185-6.

[73]. See 511a5-c2, 532a7-b2, 533c7-d1.

[74]. Cf Crombie, 1963, 554-5. In Rep 443c-e, Plato defines justice in terms of order and harmony. In Euthp 12c we are told that justice is a 'higher' (more general) notion than holiness, and in Lach 192c-d, wisdom is 'higher' (more general) than courage. A notion that many times seems to have a high position is 'profitability' (Euthd 280c, Charm 171d, Men 87d-e).

[75]. See Arist. An Post 76a35-6, 92b15-6; Proclus In Eucl (Friedlein), 233; In Remp (Kroll), 292.

[76]. We must note one more time that in every argument there are usually many premisses. The 'hypothesis' is only one of them, perhaps the most important for the proof and the most debatable (see p82-3).

[77]. See Boyle, 1974, 11-3. According to him, there is a hierarchy of definitions where the definiens is the proximate (teleological?) cause of the existence of the definiendum; that is to say, if the quadrilateral is a 'plane figure contained by four straight lines', then "it is by reason of the plane figure itself that the quadrilateral itself exists" (p.12).

Bluck (1957) finds in the Republic a teleological hierarchy of 'notions' (hypotheses are for him 'notions'). Each 'higher' hypothesis is the proximate teleological cause of the 'lower' one. For example, the 'cause' of Refreshment is Health, the 'cause' of Health is Activity, the 'cause' of Activity is Virtue and the 'cause' of Virtue is Good. Bluck, in his later book on the Meno (1961), has changed considerably his views and proposed an interpretation which although it remains teleological, is quite near to the deductive one. According to him, the dialectician "having made his hypothesis must be able in some way to deduce his original hypothesis from it". However we cannot "deduce the existence of one Form from another although we might deduce its nature" (p.99).

I find it difficult to understand how is it possible to have teleological causality in mathematics or among the Forms. However we can allow that teleological reasoning involving the Good might complement the dialectical method of the Divided Line. Moreover both Boyle and Bluck seem to forget that constructions are proofs of existence of geometrical objects.

[78]. Proclus In Eucl, 233; see also Arist. An Post.

[79]. See Gorg, 508a and Arist. E.N., 1131b13ff about the arithmetical and geometrical equality in justice. Proclus (In Eucl, 8) says that beauty and order are common to all branches of mathematics.

[80]. Stahl (1960,445) thinks that the upward path is a deductive procedure.

[81]. In Rep 434e where Plato, after finding, what justice in the city is, on the assumption that justice in the city is the same as justice in the individual, proceeds to examine whether or not this assumption is true, and says that if it is not, we shall have to go back to the city and to check all our findings.

[82]. So in Book 2 of the Republic the question of the definition of justice leads to the problem of the formation of a city (see section 12).

[83]. Cf Annas, 1981, 291; Crombie,1963,558; Ross,1951,58.

[84]. The deductive interpretation is accepted by the majority of the scholars; see n.51.

[85]. Leszl (1981, 522-6), argues for the affinity between dialectic and mathematics even in Aristotle.

[86]. I do not know anybody who clearly denies the similarity of the two methods. About the close similarity of the two methods, see Robinson, 1953, 170-1; Cross and Woozley, 1964, 249-52; Bluck, 1961, 103; Ross, 1951, 57; Stahl, 1960, 445; Scolnicov, 1973, 281; Hackforth, 1942,6.

[87]. See Phd 101d1-e5: ὑποθέσεως, ὑπόθεσιν, ὑποθέμενος, ἐχόμενος, ἄνωθεν, ἀρχῆς, διαλεγόμενος, ὁρμηθέντα, λόγος, διδόναι λόγον . Rep 511a5-c2: ὑποθέσεων, ὑποθέσεις, ἐχόμενος, ἐχομένων, ἀνωτέρω, ἀρχὰς, διαλέγεσθαι, ὁρμὰς, λόγος. Rep 533c2 λόγον διδόναι.

[88]. They are principles for the mathematicians. Only for the philosopher are they hypotheses.

[89]. Knorr (1975,244), thinks that this definition is due to Theaetetus.

[90]. In Parm 137e, there is a definition of the 'round' (or the circle) as "that in which the furthest points in all directions are at the same distance from the middle [centre]". The 'sphere' is similarly defined as that which "is equal (equidistant) from the centre in all directions" (Tim 33b,34b). These definitions belong to category two, but the 'sphere' has a special interest for Plato's cosmology, and the 'round' is the opposite of the 'straight'. They are not probably Plato's contribution.

[91]. From the previous discussion in the dialogue, it seems that this definition originated with Plato himself. Meno, a well educated young man, is unable to give a definition of 'figure'.

[92]. According to Heath (1921, I,293), the euclidean "definition of a line as 'breadthless length' originated in the platonic school". It is also probable that the definition of 'solid' as "that which has length,breadth and depth" (Eucl. XI,1),

originated with Plato (see Rep 528a9-b3).

[93]. Cf Heath, 1926,I,166.

[94]. Cf Frajese, 1963,168-9.

[95]. See in Heath,1926, I,279, other definitions of a unit; eg Arist. Met 1089b35 (unit is "the indivisible in [the category of] quantity"); Thymaridas defined a monad as "limiting quantity".

[96]. Cf Rep 525d8-e4, Parm 137c-d, Phil 15b.

[97]. Cf Plato's Parm 137c-d, 149a. About the relation between the definition of unit and the Eleatic philosophy, see Szabó, 1978, 256-61.

[98]. Proclus (In Eucl, 239) refers to it as a principle not contained in the Elements.

[99]. Szabó (1978, 291-9; 1967, 7-8) argued convincingly that Euclid's seventh common notion responds to Zeno's paradox that "half the time is equal to its double", and does not allow in geometry the absurdities originated from it.

[100]. Cf Archytas D.K. 47.B1: "mathematicians seem to me to have excellent discernment and it is in no way strange that they should think correctly concerning the nature of particular things: For since they have passed an excellent judgement on the nature of the whole, they were bound to have an excellent view of separate things" (K Freeman).

[101]. See p241. When we speak here about axiomatization we have not to confuse the ancient (or Euclidean) axiomatization with the modern one. Modern axiomatization is not concerned with the factual truth or falsehood of the axioms, and it is formal not only in the sense that it is deductive, but of being indifferent to the particular meanings that may be assigned to the terms and symbols involved in the axiomatization. On the contrary, ancient

axiomatization has not these two characteristics. It is not only less formal than the modern but also its axioms must be true and its terms and symbols must have concrete meanings. Euclid's _Elements_ is not only geometry but also cosmology. Plato (or Euclid) could never accept the geometry of Rieman.

For different views about Greek axiomatization see Leszl, 1981, 273,280-4; Mueller, 1969, 289-309. For a comparison between modern and ancient axiomatic method see R Blanche, 1971, ch 1.

[102]. See Annas,1981,289; Gosling,1973,107; Ross,1951,55.

[103]. Cf also Annas,1981,288.

[104]. See Murphy,1951, 178-80; Raven, 1963, 173; Guthrie, 1978, 525; Adam, 1902, 172-5; Γεωργούλης, 1963, 495. Robinson (1953, 163) refers also to Zeller, Maier and Rodier. Irwin (1977, 223) thinks that the dialectical method is nothing other than the Socratic elenchus.

[105]. See Adam,1902, note on 534b; Cherniss, 1947, 142 n.42; Bluck, 1957, 29. See also 368d-e where Socrates distinguishes justice in the city and justice in the individual.

[106]. See ch V, p161.

[107]. For various accounts of the method of analysis and synthesis, see p28-9. For Albinus' second acount of philosophical analysis see p202n 89. This account is very similar to the various descriptions of the geometrical analysis.

[108]. See Ammonius, _In An Pr_ 6.6-8; David _In Porph Is_ 104.1ff; Albinus _Didaskalikos_ V,5.

[109]. Others who interpret the method of the Line by means of geometrical method of analysis and synthesis are: Cornford, 1932/1965; Scolnicov, 1973, 281-2; Hackforth, 1942,4. Bluck (1961,6-7) thinks that Plato's dialectical method fits _apagoge_ and

not analysis.

[110]. Speaking about the hypothetical method in the Republic, I mean the method of dialectic. The idea that Plato practises his hypothetical method throughout the main argument of the Republic was first elaborated by S. Scolnicov (1973; chapter V). Nevertheless his account is considerably different from mine. Scolnicov does not realize the second of the above mentioned differences between the method in the Phaedo and in the Republic which is reflected in the two applications of the method.

[111]. See also 354a. I do not intend to examine in detail, or to criticize and evaluate, Plato's arguments in the Republic. I am only interested in the form of its main argument.

[112]. The consequences that Socrates and the two brothers have in mind are not the rewards or reputations of justice that the vulgar have in mind when they say that justice is good only for the sake of its consequences (358a). Rewards and reputations are 'artificial' or 'incidental' consequences because they can go to the person who seems to be just but is not. The Gyges' story (359-360) confirms that the brothers exclude this kind of consequences. The examples of consequences that Glaucon gives (357b-c) show that by 'consequences of justice' Glaucon ought to mean things that inevitably result from being just. Therefore Glaucon asks Socrates to prove a very strong version of the thesis that justice is better than injustice. For this it must be shown that the just are happier than the unjust (367c5-d5, 368b6-7, 358c2-6, 361c3-d3). For the problem of the 'consequences of justice' see: J. Annas (1981), pp.65-70; Cross and Woozley (1964), pp66-8; I.M. Crombie (1962), pp.85-8; T. Irwin (1977), p.88.

[113]. Cf. Cross and Woozley (1964) p.62.

[114].Cf. Meno 71b, where the inquiry into the 'ὁποῖον' question presupposes an inquiry into the 'τὶ' question.

[115]. As the simile of the large and small letters shows, Plato takes for granted that justice in a city is the same as justice in a person. The examination of the large letters will facilitate that of the small letters. This procedure seems to be unwarrantably dogmatic. Nevertheless the close parallel that Plato draws at 435ff between the nature of a city and the nature of an individual makes the above assumpion more plausible.

[116]. This is the 'ἀναγκαιοτάτη πόλις'(369d6-11). Plato thinks that the division of labour is natural since it depends on our different capacities (370a8-b2).

[117]. See 399c. "The first city adds nothing [to the argument] except a context in which the Principle of Specialization is introduced in a plausible way" (J. Annas, 1981, p.79).

[118]. Here we have an argument by elimination which depends on some assumptions: a) the city, as they have planned it, is completely good and therefore it contains all virtues; b) there are only four virtues that fill out exactly the structure of the city; c)the four virtues are exactly the four named by Socrates (cf. W.K.C. Guthrie, 1975, p.471; Cross and Woozley, 1964, p.104; J. Annas, 1981, pp.110-11).

The method that Plato follows here is more effective in dealing with 'abstract quantity' and is probably borrowed from mathematics. We find a similar method in the two quadratures of lunes that Alexander attributes to Hippocrates of Chios (Simplicius in Phys., ed Diels, 56.1-57.24). Adam (1902, vol.I, p.255) is wrong in saying that the method here is akin to the method described in Meno 86e. Similarly Γεωργούλης (1962, p.396) is wrong claiming that this method is a special case of the method of exhaustion, discovered by Eudoxus.

[119]. Cf. 433d8-9 where justice is "the principle of each man doing his own business in the city" (Lindsay).

[120]. Justice is not identical to the principle of division of labour (or, one-man-one-job). However is is "a form of it" (433a) concerning not the non-confusion of jobs but that of classes. Cf. Cross and Woozley, 1964, pp.109-10; J. Annas, 1981, p.118; P. Friedlander, 1969, p.101; R.L. Nettleship, 1901, p.151.

[121]. J.Annas, 1981, p.137.

[122]. For the similarity of the above passage to Phaedo 100a and 101d-e, see H-P Stahl, 1960, pp.438-9; S. Scolnicov, 1973, pp.143-5.

[123]. S.Scolnicov (1973, pp.162-3) thinks that Book 10 does contribute to the main argument, and that the original question of the Republic concerns incidental consequences as well.

[124]. As regards the problem of the 'longer way', see W.K.C. Guthrie, 1975, p.473; Cross and Woozley, 1964, pp.112-4; I.M. Crombie, 1962, p.96; N. Murphy, 1951, pp.9-10.

CHAPTER VII

AXIOMATIZATION OF GEOMETRY AND PLATO'S CONCEPTION OF KNOWLEDGE

This chapter is mainly a concluding one in which I shall try to answer some more general questions regarding the relation between mathematics on the one hand and Plato's methodology and conception of knowledge on the other hand. The first section is devoted to the development of mathematics with special attention being paid to its axiomatization. In the second section I look at the dating of the first axiomatized Elements of geometry. In the third section, I shall try to show that Plato's conception of knowledge changes between the Meno and the Republic and this is due to the axiomatization of geometry. In the final section, I draw some conclusion regarding Plato's hypothetical method in the Meno, Phaedo and Republic.

1. Towards the axiomatization of geometry

For geometry, the fifth century B.C. was the 'problem-solving' period in which the three great problems appeared (squaring of a circle, duplication of a cube, trisection of an angle), and by the end of the century, mathematics was developed sufficiently in the sense that a great number of mathematical propositions had been discovered [1].

From Hippocrates' surviving proofs we may conclude that the material of Elem, Books I, III, VI and a great part of IV was known to him. The propositions of Books II and IV may have been known to Theodorus. According to Heath (1921, I,216) "we have ...

reason to suppose that there existed Elements of Arithmetic partly ... on the lines of Eucl. Book VII, while some propositions of Book VIII (eg 11,12) were also common property". According to V d Waerden (1954, 153) Book VIII is the work of Archytas. Solid geometry must have been developed to a considerable extent, for Anaxagoras and Democritus to outline a theory of perspective (Virtruvius De Architetura, VII praef.). Democritus found the formula for the the volumes of a pyramid and a cone (Archimedes, in the introduction of the Method), and the Pythagoreans knew at least three out of the five regular solids (Scholion No 1 to Eucl. XIII; in Heiberg's Euclid, vol.V, 654). Moreover the substance of Book XI 1-19,21 may have been known to Archytas (Heath, 1921, I,217) and propositions IX 21-34 seem to have their origin in the early Pythagoreans. Finally, the problem of the irrationals had been formulated. The incommensurability of the diagonal and the side of a square was proved by means of the Pythagorean theory of the 'odd' and the 'even', and Theodorus demonstrated the incommensurability of the sides of squares whose areas are 3,5,..., 17 times the area of a given square [2].

Greek mathematicians of the 5th century often employed heuristic or practical methods and were content with loose or informal proofs. However, informal procedures, such as the mechanical methods of Archytas, Hippias, and Archimedes, are the usual route by which new discoveries are made to advance a field. The 5th century solutions of the three great problems "involved very ingenious operations, apparently much more complex than those involved in, say, bisecting an angle. It is not surprising that the question of whether these solutions were satisfactory remained open;" (Mueller, 1969, 293). At that time, mathematics consisted mainly of a number of theorems and problems, sometimes with alternative proofs or solutions established by various mathematicians using different methods and hypotheses.

What was missing in the geometry of the beginning of the 4th century was its formulation into a unified system. At that time a new generation of geometers appeared (Archytas, Leodamas, Theaetetus, Neoclides, Leon, Eudoxus). According to Eudemus (in Proclus In Eucl, 66-7), the above mentioned new generation of mathematicians fulfilled this task. They paid more attention to rigorous methodology and foundation than to the solution of problems. Archytas (Arist. Met 1043a21-2) and Theaetetus (Plato Theaet 147d-e) tried to find general definitions, Leodamas discovered (?) and applied the method of analysis and synthesis, Leon discovered diorismoi and wrote the first Elements in an axiomatic form (see next section).

Plato described in a lively manner the difference between these two generations of mathematicians in the persons of Theodorus and Theaetetus (Theaet 143e-148d). Theodorus was interested in problems and their solution. He was not used to dialectical discussions (146b3). On the other hand Theaetetus had a great interest in philosophy, in foundational problems and general definitions in mathematics. These interests of the new generation of mathematicians were also related to the general intellectual atmosphere in Greece at the beginning of the 4th century. The epoch of the physicist-philosophers had definitely passed. There was a real interest in logical, ethical and political problems combined with a highly antiempirical trend. Mathematics overcame Protagoras' criticism (Aristotle Met 998a2-4) by passing from the sensible sphere to the intelligible one (see

Plato Rep 510d-e). In such a situation the need for definitions was urgent. The meaning of mathematical objects could be taken for granted no longer. It became clear also that these definitions were a different kind of proposition for which there was no issue of proof, for they precede all proof. So definitions constitute a separate set of propositions that is posited at the beginning of any mathematical treatise.

Another factor whose influence was decisive for the development of mathematical definitions and the axiomatization of geometry, was the discovery of incommensurability [3]. This discovery showed that there exist two kinds of mathematical objects: numbers and magnitudes; geometrical objects cannot be manipulated by means of numbers, and the problem of incommensurability, although unsolvable in arithmetic, it raises no difficulties in geometry (see Men 82b-85b). The impact of incommensurability was greatest in the Pythagorean school. Elsewhere, we hardly see any impact [4]. Nevertheless the 'geometrical pebble arithmetic' of the Pythagoreans was transformed into the 'metrical' geometry that we find in Book II of Euclid with the replacement of the pebbles by segments. This process - that involves a higher mathematical abstraction - was extended even to pure arithmetic [5].

The old Pythagorean theory of proportions was based on numbers (Eucl., VII, def.20). After the discovery of incommensurability, there was urgent need to modify this definition. Theaetetus, in his theory of irrationals, probably used a new definition based on the notion of anthyphairesis (see Knorr, 1975, 255-261), and later on, Eudoxus formulated the well known (Eucl. V, def. 5) definition of proportion which applies both to rationals and irrationals.

Thus incommensurability compelled mathematicians to work seriously upon definitions [6].

2. Elements

Eudemus (Proclus In Eucl, 66.7-9) says that the first person who wrote Elements was Hippocrates of Chios. We know nothing else about Hippocrates' Elements. The ascription of a book of Elements to Hippocrates, however fits very well with his reductive method. His method (apagoge) is very useful in reducing mathematical propositions to other ones, and so in creating a series of mathematical propositions in which the later depends on the earlier. It is therefore very probable that Hippocrates wrote a book containing mathematical propositions in order of dependence. However, it is very unlikely that Hippocrates' Elements was cast into an axiomatic form, that is, that Hippocrates put forward in his book a set of 'first principles' from which the theorems were derived. Hippocrates' principles (archai or hypotheses) were propositions which needed proof (lemmas) and not axioms or definitions which assisted proof. Moreover, it seems that the systematic theory of definitions in mathematics began only at the beginning of 4th century [7].

In chapter VI (section 1) we said that Plato, in the Republic, describes the mathematics of his time as being already transformed (probably imperfectly) into an axiomatic system. I now wish to examine the origin of Elements and find evidence for the dating.

Eudemus (Proclus In Eucl 66.19-23) reports that Leon "added many discoveries to those of his predecessors, so that Leon was able to compile a book of elements more carefully designed (ἐπιμελέστερον) to take account of the number of propositions that have been proved and of their utility. He also discovered diorismoi" (Morrow). That the name of Leon is related to diorismoi makes it more probable that he was the first to write Elements axiomatically organized. Diorismos (see p52) is closely related to the method of analysis and synthesis which played a great role in the axiomatization of geometry. After giving a long series of names (Leodamas, Archytas, Theaetetus, Neoclides, Leon, Eudoxus, Amylcas, Dinostratus, Menaechmus, Theudius and Atheneus), Eudemus adds: "these men lived together in the Academy, making their inquiries in common". Another early source, the Academ. Philos. Index Herculanensis (ed Meckler, 15-7), says that there was a great development in mathematics in the time of Plato under his supervision (ἀρχιτεκτονοῦντος καὶ προβλήματα διδόντος τοῦ Πλάτωνος, ζητούντων δὲ μετὰ σπουδῆς αὐτὰ τῶν μαθηματικῶν). "And so geometry advanced and analysis and diorismos arose". We see that the author of the Index Herculanensis connects the method of analysis and diorismos with Plato. This is further evidence that Leon - who discovered diorismos - was working in the Academy. His name is also related - in the Eudemus' passage - to the names of Leodamas and Eudoxus who have both been related to Academy.

The Academy was at that time a centre of mathematical research. We find there all the famous mathematicians of the 4th century [8]. The first theoretical account of arithmetic and music (Archytas), the theoretical research on definitions (Archytas, Theaetetus), the method of analysis and synthesis (Leodamas), the theory of incommensurability and stereometry (Theaetetus), the diorismoi (Leon), the first axiomatized Elements (Leon, Theudius [9]), the new theory of proportions and the method of exhaustion (Eudoxus), the investigation on convertible propositions and conic sections (Menaechmus), all these are achievements made by mathematicians related to Plato and the Academy [10].

If we accept that Plato in the Republic had in mind Leon's Elements, then we can conjecture the following: a) the first principles of his Elements were probably only definitions (510c); b) Leon's Elements contained a book (or books) on arithmetic (510c4 "the odd and the even"); c) it is quite probable that Leon's Elements did not contain stereometry (528b-c).

According to historians of mathematics, our only testimony about Leon is the above passage of Eudemus. There is however an unnoticed passage in Philostratus' Lives of Sophists (485) [11]. Here Philostratus speaks of a 'sophist' named Leon of Byzantium: "Leon of Byzantium was in his youth pupil of Plato, but when he reached man's estate he was called a sophist because his repartees were so convincing". Similarly Suidas (ad loc) speaks about Leon of Byzantium calling him "philosopher, peripatetic and sophist, student of Plato"; and Plutarch (Phoc XIV,4) refers to him as being, in his youth, a companion of Phocion in the Academy. According to Philostratus, Leon was not a real sophist but a philosopher (484.4-10). The most interesting thing is that Philostratus, immediately before mentioning Leon, calls the famous mathematician Eudoxus of Cnidus a sophist. It seems plausible then to suppose that Leon of Byzantium is one and the same person as Leon the mathematician. Philostratus says that Leon was in his youth, a pupil of Plato and that he was soon employed in rhetoric and politics in his native city Byzantium. We have then to suppose that Leon had reached his mathematical achievements early in his life. An upper limit for the time in which Leon wrote his Elements is the opening year of the Academy (Leon was a member of it), that is c.386. We must then put Leon's Elements after 386.

Let us now deal with some questions regarding the main dates of Leon's life. Eudemus says that Leodamas was a contemporary of Plato ("at this time [ie Plato's] lived Leodamas"). We may then suppose that Leodamas was born between 430 and 425. "Leodamas was older than Neoclides and Neoclides was a teacher of Leon". Therefore,if we accept a difference of 5-10 years between Leodamas and Neoclides, and a difference of 15-20 years between Neoclides and Leon (teacher - pupil), then we may posit the birth of Leon between 405 and 400. If we accept then that Leon wrote his Elements in an early stage of his life (something usual for mathematicians), we may posit Leon's Elements c.375 [12].

Eudemus says that Eudoxus was "a little younger than Leon". As regards the birth of Eudoxus we have three possible dates. A) Apollodorus says that Eudoxus flourished in 368-5 B.C., and taking the maturity of a man as the age of 40 years, many historians infer that he born in 408 B.C. [13]. B) It has been pointed out by F Gisinger that fr. 58 of the <u>γῆς περίοδος</u> would indicate that Eudoxus died after, rather than before Plato, that is, in or after 347. He therefore suggested the date of 400 B.C. [14]. C) More recent and more scholarly re-examinations of the evidence favour a date between 395 and 390 [15]. If "a little younger" indicates a difference of about five years and if we assume that Leon wrote his Elements in an age of c. thirty years old, then we must posit his Elements either a) c.383, b) c.375, c) between 370 and 365.

Philostratus gives two anecdotes from Leon's life. The first is when Philip of Macedon tried to take Byzantium (340 B.C.). The other took place "when this Leon came on an embassy to Athens, [and] the city had long been disturbed by factions and was being governed in defiance of established customs" (485.15-7). This happened somewhere between 357 (Byzantium's revolt against the Athenian league) and 355 (independence of Byzantium recognized by Athens). Leon is presented in this anecdote as a mature man, probably between 45 and 50 years old. In this case, his birth is posited somewhere between 407 and 397 B.C. This is better suited to the second and third possible dates of Eudoxus' birth (ie 400 or 395-390). We finally conclude therefore that the most plausible date of Leon's birth is 407-400 B.C., and the most plausible date for his Elements is 377-370 B.C.

If we accept that Plato wrote the middle Books of the Republic in the last years of the decade 380-370, then it seems that the axiomatization of geometry took place after the composition of the Phaedo and just before the completion of the Republic.

There is of course an element of conjecture in this dating, but the conclusion does fit in very well with what we have to say about Plato.

3. Plato's conception of knowledge in the "Meno" and the "Republic"

In Men 81c ff, Plato introduces the theory of recollection in order to answer Meno's paradox (80d-e) regarding the impossibility of inquiry. According to this theory, the soul is immortal, has been born many times, has 'seen' all things both here and in the other world, and has learned everything. So the soul can recall the knowledge of virtue or anything else it once possessed. In this way, seeking and learning are in fact nothing else but recollection (81c-d). One who does not know something, has in oneself true beliefs about it (85c). With the help of questions, the true beliefs are elicited; and if questions are put to one on many occasions and in different ways, at the end one will have knowledge (85c).

Recollection is a process in which we can distinguish three stages: a) the elimination of false beliefs, b) the eliciting of true beliefs, and c) the obtaining of knowledge (see the geometrical problem with the slave-boy - 82b-85c).

According to Plato, one's knowledge is recovered by oneself. This knowledge is not empirical; eliciting knowledge from ourselves does not involve sense-experience [16]. Socrates (76d-e) disregards the definition of colour based on empirical observation. Prof. Vlastos (1965) persuasively argued that even the example with the slave-boy and the use of diagrams does not involve empirical knowledge. The evidence from the dialogue shows that the knowledge that Plato had in mind was related to mathematical and ethical propositions.

Recollection includes (a) "all things" (81c6) or "virtue and other things" (81c8); we might therefore conclude that these 'things' are similar in kind or 'akin' to virtue. At 81d4-5, we are told that recollection includes (b) "all seeking and learning". Thus (b) excludes knowledge which demands no inquiry and restricts (a) to the things that the soul once knew.

At 98a Socrates says that knowledge is true belief "bound by working out the reason' (αἰτίας λογισμὸς). According to Vlastos (1965), the phrase "δήση αἰτίας λογισμῷ" implies rational thought in contrast to sense perception and is used "for knowledge reached and justified by formal inference and analysis in emphatic contrast to sensory cognition" (154). In this way "a statement becomes known when it is seen to follow logically from premisses sufficient for this purpose; to recollect it, then, would be to see that these premisses entail it" (155).

According to this interpretation, it seems then that recollection in the Meno is primarily about our acquisition of necessary truths and not of concepts (like in the Phaedo) [17]. This view is supported by the evidence from the text. The two arguments of the dialogue after the introduction of the theory of recollection deal with proofs of propositions and not with definitions or clarifications of concepts. However, although Plato's examples concern proofs of propositions, it seems that the theory is posited also in order to answer questions like "what is x?" (see 81c8, 81e2). In this case, to recollect probably means to gain an insight into a concept and to find intra-propositional logical relations between terms [18].

We said that to recollect something (speaking about the final stage of recollection) is to see that it is entailed by certain premisses. The premisses too can be recollected in the same manner. Is there any end to this procedure? If a proposition in order to be known must be entailed by some premisses, then we have either an infinite regress of proofs and premisses or circular demonstrations (see Arist. An Post 72b15ff). The same happens with defining concepts. Plato, in the Meno says nothing about the necessity of unproved first principles. For him something, in order to be known, must be "bound by working out the reason". But let us examine the problem more closely. At 81d plato says that "all nature is akin and the soul has learned everything, so that when a man has recalled only one thing.....there is no reason why he should not find out all the rest". There are three problems connected with the above statement. A) What does "all nature is akin" mean? B) Can we start from any one? C) How can we acquire knowledge of the one that we first come to know?

A) The word 'nature' cannot refer, as Bluck (1961,288) seems to suggest, to everything that exists, since that would imply recollection of particulars and kinship between particulars and eternal realities. Klein (1965,65) suggests that "by virtue of this assumption [kinship] everything, every bit the soul recollects can be understood as a 'part' of a 'whole' and can be traced back to common origin". Klein however does not tell us how an understanding of this part-whole relation or the common origin makes recollection possible.

"All nature is akin" may mean that all things that constitute nature are 'akin' to each other. That is, that they are the same kind of thing or akin ontologicaly. In this way if we find one we can find all others in the same way as this first one [19]. However the phrase "so that when a man has recalled one thing there is no reason why he should not find all the rest" suggests something stronger. It suggests not only that one can find all the rest in the same way as the first one, but also that the knowledge of this first thing will facilitate the knowledge of the rest. That means that things that constitute 'nature' are interrelated between them and they all form a coherent system; so the finding of one of them makes possible the finding of the rest [20].

We may also say that, things that constitute 'nature' are 'akin' to the soul (cf <u>Phd</u> 77-80) since the soul once learned all these things. Hence the objects of nature are ontologicaly similar to the soul and the concepts that the soul forms and the relations found among these concepts reflect the objects of nature and their relations [21].

B) Plato does not explain what or which must be the first one. Reading the phrase "ἓν μόνον ἀναμιμνησθέντα" it seems that whichever this first one is, if we have knowledge of it, then all the others can be known only by processes of thought. The word monon (only) supports our interpretation. Moreover, we have no evidence (in the Meno or in any other earlier dialogue) regarding the necessity of first principles from which all other knowledge must begin (as opposed to relative principles or homologemata peculiar only to an argument or discussion). Hence, it seems that the Meno allows us to start from anywhere and does not imply that one has to start from fixed first principles. We can pass from true beliefs about some things to their definitions and relations between them. The knowledge of the whole system of knowledge comes probably slowly through the understanding of its inner relations. The system of knowledge seems to be not like a pyramid with one (or some) principle at its apex. There is no real arche. The system of knowledge is rather like a network in which there is no concrete beginning or end. Everything may be the arche. Things that are in the centre of the network are more relevant and exhibit more relations than others [22]. The system is coherent, structured, but not hierarchically structured in the model of Euclidean geometry [23]. The conception that some propositions (or concepts) are 'higher' than others in the sense of the Republic seems to be absent in the Meno.

At 75c Socrates gives a definition of figure (or shape) as "the only thing which always accompanies colour". The concepts 'figure' and 'colour' are coextensive [24]. This definition has not the standard pattern of a definition by genus et differentia.

Meno does not attack Socrates' pattern of definition but only the fact that 'colour' is something unknown and hence we defined 'figure' ignotum per ignotius. Socrates (75c8, d6) accepts that if someone does not know what colour is, then we must find another definition (or another way of defining something). However, he thinks that there is nothing wrong with this pattern of definition [25]. In the above definition, 'figure' is not defined through 'higher' concepts, and the fact that Plato recognizes (in the Meno) such definitions as valid shows again that in his conception of knowledge, concepts are not hierarchically ordered like in Euclidean geometry. Moreover, the above definition of 'figure' allows us to define 'colour' in the same way; but in this case, the one definition would be the conversion of the other. We might then suppose that Plato, in the Meno, permits definitions where things are defined reciprocally. If we extend this to proofs, we might suppose that circular demonstrations are not excluded from Plato's system of knowledge in the Meno. However, we have no evidence regarding this last statement either in the Meno or in the early dialogues [26].

C) How can we obtain knowledge of the first thing? A possible answer could be: by some strong insight or intuition. However, it seems that this kind of knowledge is ruled out by Plato in the Meno. Knowledge is 'αἰτίας λογισμός' involving logical reasoning. 99e suggests that intuition might be useful only for obtaining true belief, although even this is not certain because Plato probably speaks ironically in this passage. The problem then remains. As Aristotle suggests (An Post 72b15ff; see also Barnes, 1976, 178-81), if we accept that our knowledge is only demonstrative and does not start from principles that are known independently and in a different way, then there are two possibilities: i) an infinite regress of proofs that results in the conclusion that knowledge is impossible, and ii) circular demonstration. It is certain here that Plato rejects the first alternative and we have no strong evidence regarding the second.

How is it then possible to start our process of knowledge? We might say that we can obtain knowledge of a thing A after a logical reasoning that begins from some true beliefs. In this case our 'knowledge' relies on true beliefs. But it seems that such 'knowledge' is not regarded by Plato as knowledge (cf 85c, where, after the proof, Socrates insists that the slave-boy has not yet got knowledge).

We probably arrive at perfect knowledge only when, after starting our inquiry from some true beliefs and after we proceed by logical reasoning to other propositions and so on, until we cover the whole field of knowledge, we return to our initial point of departure and give an account of it. Thus the system of knowledge is illuminated progressively and the complete knowledge and illumination comes only after we proceed rationally through all the inner relations of the system. We must observe that Plato says "if we <u>recall</u> one thing only" and not "if we know". We might then suppose that 'recall' refers not only to the last stage of recollection, but also to the 'eliciting of true beliefs'. In this way, the first thing we recall might be a true belief. However, it is quite probable that Plato, when he wrote the <u>Meno</u>, had neither a clear conception of these problems nor an answer to them. The first axiomatization of geometry is subsequent to the <u>Meno</u>, and when Plato wrote it he did not have the example of a hierarchically organized deductive science that starts from some first principles.

Plato's hypothetical method in the Meno is influenced by mathematics (86e) and especially by Hippocrates' method of apagoge (reduction). This method was at work in the geometry of the late 5th and the early 4th century. We have already said that the 5th century was the problem-solving period of geometry. At this time we have a great number of deductive chains each of which aims to solve a problem and has its own relevant principles. We saw (p47-8) that Hippocrates' principles were ordinary geometrical propositions that needed proof (lemmas), and not principles in the Euclidean sense. We also saw that apagoge reduces problems or theorems to other ones, and thereby leads to the unification of all these distinct deductive chains. However this is an incidental result of this method, which was mainly used as a technique of solving difficult problems. Apagoge does not necessarily lead to 'higher' or prior propositions. We cannot say that the problem of conic sections is 'higher' than that of finding two mean proportionals between two numbers or that this last is 'higher' than the Delian problem. Moreover, the word apagoge has not the meaning of 'leading upwards' but simply that of 'leading away'. Therefore, although apagoge led to the unification of various deductive chains in geometry, it was not this that led mathematics to its axiomatization.

Apagoge was active at the beginning of the 4th century, and it seems that Plato's conception of knowledge in the Meno reflects the situation of mathematics of that time where geometry was going to be organized in a unified deductive system, but where the necessity of undemonstrable first principles had not yet been yet realized.

In the *Republic* everything changes. Geometry is already axiomatized and Plato adopts the same hierarchical model. His system of knowledge is like a pyramid with the non-hypothetical first principle at its apex. The knowledge of this last is not demonstrative but of another kind. In this way he avoids infinite regress of proofs and circular demonstration. The metaphors of 'up' and 'down' suggest the hierarchical character of his model of knowledge. Even from the *Phaedo* (101d7-e3, 107b5) and the *Cratylus* (436d) Plato had seen the necessity of a hierarchy among the objects of knowledge and their relations [27]. However, he did not then have an answer either to the question of the principles of a system or to the way in which they could be made known. Correspondingly, as we saw Plato's hypothetical method in the *Republic* is different; it is now modelled on the pattern of the geometrical method of analysis and synthesis which was of great significance for the axiomatization of geometry.

There is a last problem we have to face before we finish this section. It has to do with the depreciation of mathematics by Plato in the *Republic*. In the *Meno* mathematics constitutes the model for all knowledge. Mathematical notions have an epistemic value equal to, if not higher than, moral ones. The definition of figure is given as a model of a definition of virtue; the theory of recollection is exhibited in practice via a geometrical problem, and the proof that 'virtue is teachable' follows a geometrical method. It seems then that in the *Meno*, Plato is convinced that mathematics constitutes knowledge and he demands the same for ethics.

In the *Phaedo* there is no evidence that mathematics is in a lower position than ethics, and the word *dianoia* (65e7, 66a2, 67c3) has not the cognitively inferior meaning that it has in the Republic.

In the Republic, mathematics is assumed to be inferior to dialectic. This is not because of the inferiority of mathematical concepts [28]. The inferiority of mathematics occurs mainly because it unquestionably uses hypotheses as starting points and cannot go beyond them (see chapter VI, section 7). Mathematicians are interested in solving their problems and posit as their principles propositions useful for this task. They cannot give reasons for their principles and, moreover, they are not interested in doing so. Between the composition of the Phaedo and the Republic geometry was organized into an axiomatic system. This means that geometry was transformed into a closed deductive system having its own first principles, using its own techniques and aiming at its own problems. Hence geometry was restricted to its own subject and was delimited from other sciences. In this way it lost its universal character no longer being able to supply the foundation for all knowledge [29].

Plato saw the inability of the axiomatized geometry to proceed beyond its principles, and also the tedency of other sciences to be organized like geometry. In this way all sciences are restricted to their own subjects, and only another 'higher' science (dialectic) is able to coordinate the various sciences and to organize the whole of knowledge in a system.

4. Plato's hypothetical method: its characteristics and its development in relation to geometrical methodologies

Plato's three accounts of the hypothetical method present so many similarities that we may speak of one single method. The main characteristics are the same in all three accounts of the method. However there are some differences that distinguish between these accounts.

Let us first enumerate the main characteristics common to all the accounts of the method. The main feature of Plato's hypothetical method is that it is a method of inquiry into causes that proceeds backwards in order to find premisses for the proof. The method is an indirect one that starts from the question to be answered. This last is posited from the beginning (Men86e5-87a, 87b2-4, Phd 100b, Rep511b5-6). The method is a positive one useful for proving something and not for disproof (Men 87c5-d3, Phd 101b, Rep 511b5-6); it has two directions: a) an upward (or backward), which is a search for premisses, for the discovery of a proof (Men 87c5-d3, Phd 101d, Rep 511b5-7), and b) a downward, which is demonstrative (Men 87d-92a, Phd 100a, Rep 511b7-c2). The hypothesis is posited in order to prove something and not in order to be proved (Men 87a, c5-6, d2-3, Phd 100a2-5, 100b, Rep 436b with 437a). The hypothesis is tentative, is posited provisionally and has need of confirmation. Confirmation occurs through another ('higher') hypothesis (Men 87d1-3, Phd 101d, Rep 533c7-8). If the hypothesis is proved to be false, then all its consequences are upset (Crat 436c-d, Phd 101d1-5, Rep 533c2-5, 437a).

Plato gives the name hypothesis to only one of the premisses that conjointly entail the conclusion, presumably that which in his eyes is the most important or the most questionable. The hypothesis is chosen on the basis of its ability to solve the original problem and because it seems to be the least refutable (Phd 100a4, 101d6-7).

There are some differences in the hypothetical method in the three dialogues. In the _Meno_ it is used mainly to solve a problem which is difficult to solve in a direct way. So we start indirectly, we analyse our original problem and we arrive at a proposition which can be proved either directly or indirectly, and upon which our original problem depends. The hypothetical method here is similar to the reductive method of Hippocrates of Chios. In the _Meno_ we do not have the conception that the hypothesis is 'higher' than the original problem and there is no specific emphasis in a continuous regress of hypotheses.

In the _Phaedo_, as in the _Meno_ the method is used in order to solve a problem. However there is here a great emphasis on the way in which we shall secure our hypothesis. This is done in the same way (that is indirectly) by positing a 'higher' hypothesis and so on until we arrive at something 'adequate'. So in the _Phaedo_ we find a clear conception of the two ways of the method. One upward and one downward. The method is again similar to the _apagoge_ of Hippocrates. The difference from the _Meno_ is that here we have an emphasis on the continuous reductions that lead us not just to something provable or accepted by the interlocutor, but to something 'adequate', which presumably means something that is either generally accepted as true or self-evident. The method of _apagoge_ in this form of successive reductions, is the initial stage of the more elaborated method of analysis and synthesis, the latter being an evolution of the former.

In the _Phaedo_, we also find another stage in the method which is absent from the _Meno_ and perhaps from the _Republic_. This stage consists in checking the hypothesis (through elenchus) "by seeing if its results agree with each other" (101d4-5).

In the Republic now, the primary task of the hypothetical method (Plato says dialectic) is not so much to solve a problem (in our case to give an account of the principles of the sciences) as to organize in a hierarchically structured system the whole system of knowledge and to arrive at the unique non-hypothetical first principle, namely the Good. We may submit this first principle to elenchus but we cannot offer a proof for it. Its knowledge is not demonstrative but of another kind. Probably we grasp it by an act of intuition. This principle gives the foundation for (and understanding of) the whole system of knowledge. The non-hypothetical principle of the Republic has not the tentative character of the hypotheses and is self-sufficient and self-explanatory. Only in the Republic does the method give perfect knowledge because of its arrival at the unique non-hypothetical principle.

The way up of in the method is again a search for premisses relying mainly on the experience and the ability of the philosopher to grasp what is prior. The way down is demonstrative and gives also a systematic and intelligible presentation of the subject. The two ways (up and down) reflect two aspects of the method, a personal and an objective.

In the Republic the two ways of the method are clearly separated from each other. One starts the way down only when one has finished the way up. The hypothetical method here is modelled upon the pattern of the geometrical method of analysis and synthesis.

According to our interpretation, the hypothetical method in the three dialogues is closely linked to the methods which the mathematicians of the time were following. It seems that there is a parallel between the evolution of the indirect methods of proof in geometry and the evolution of Plato's hypothetical method.

Apagoge, which is the model of the hypothetical method in the *Meno*, is the forerunner of the method of analysis and synthesis, the latter being the model for the dialectical method of the *Republic*. The intermediate stage that links the two methods is that which occurs if we proceed to successive applications of *apagoge* (successive reductions) aiming, not so much at arriving at a solvable problem, but at something evident that does not need proof. The hypothetical method in the *Phaedo* exhibits this stage.

However, we cannot suppose that Plato blindly and uncritically applies geometrical methods in philosophy. The above geometrical methods are adapted to dialectic. Elenchus of the hypotheses is introduced, and there is the possibility of proposing general theories or obtaining premisses through induction. Moreover, many times in dialectic, coherency is what we need and not deductiveness. Plato's hypothetical method is posited in order to meet all these needs of philosophy.

NOTES TO CHAPTER VII

[1]. The great names of this time are Anaxagoras, Oenopides of Chios, Democritus, Antiphon the sophist, Hippias of Elis, Hippocrates of Chios, Theodorus of Cyrene.

[2]. Theaet 147d: see also Lassere,1964,55-6; Knorr,1975, ch IV.

[3]. There is a problem regarding the dating of the discovery of incommensurability. Many dates have been proposed, from the last years of the 6th century to the last decade of the 5th century. All historians of mathematics agree that the discovery was the work of the Pythagoreans. Given that i) incommensurability was known to Hippocrates (see next note), ii) Democritus (born c.460) wrote a book "on irrational lines and solids", and iii) the early Pythagorean mathematics was too elementary and empirical, I am inclined to believe that the discovery took place somewhere between 450 and 420 B.C.

K Popper (1952/1965, 87-92) argues, without any evidence, that the discovery of incommensurability took place at the very end of the 5th century or at the beginning of the 4th century. According to him, Plato was the first who realized the disastrous results of this discovery to all atomistic theories and to all explanations through numbers. Plato encouraged geometry and geometrical methods and succeeded in turning almost all mathematics from arithmetic to geometry. Popper insists that Democritus did not know about incommensurability and translates the title of his book "περὶ ἀλόγων γραμμῶν ..." as "on the illogical lines". Popper seems to ignore the fact that geometry had flourished considerably in the late 5th century and that the Pythagoreans had begun to turn their arithmetic (or 'pebble arithmetic') into the 'geometrical algebra' (or 'metrical geometry') that we find in the second book of Euclid a long time before Plato.

[4]. It is almost certain that Hippocrates knew of the problem of incommensurability. We can see it from his reduction of the Delian problem and from his second, third and fourth quadratures of lunes (τὴν δὲ μίαν τὴν μείζω τῶν παραλλήλων τριπλασίαν ἑκάστης δυνάμει, see in I Thomas,I, 240). However his knowledge of irrationality had no effect on his geometry.

[5]. In the arithmetical books of Euclid, numbers are represented by segments.

[6]. For the problem of incommensurability and its effects on Greek mathematics see Knorr, 1975, chs I,II,VI,VIII,IX; Burkert, 1972, 455-65; V d Waerden, 1954, 154-7; Szabó, 1978, 199-216, 316-7; Heath, 1921,I, 154-7; Lloyd, 1979, 105-8; Lassere, 1964, 66-73; Neugebauer, 1957, 148-9; Mueller, 1969, 105-8.

[7]. It is unlikely that Hippocrates used the term 'Elements' for his book. The mathematical use of the word was well known to Aristotle (Topics 158b35ff, 163b23-25; An Post 84b21; Categ 14a39; Met 998a25-27, 1014a35-36). In Plato we find the word very often but in the sense in which letters are elements of a syllable (or a word). In the Tim it signifies the first physical elements. In Theaet 201d, it has the sense of the first constituents of every complex thing (see Morrow, 1970).

Proclus (In Eucl, 72-3) says that according to Menaechmus the term 'element' can be used in two ways: a) in the sense of a lemma, and b) in the sense of "the more primary members of an argument". According to the first meaning, every proposition used in a proof is an 'element'. According to the second, only axioms and postulates are 'elements'. The first meaning of the word, reminds us of Hippocrates' principles. Thus even if Hippocrates had used this word (in this sense), this says nothing about the existence of a set of first principles in Hippocrates' book.

[8]. The relation of Leodamas, Archytas, Theaetetus, Menaechmus, Leon, Eudoxus and Philippus of Medna (or Opus), to Plato and the Academy is testified also by other sources. We know these names were the leading ones in the mathematical research of the 4th century.

[9]. Historians of mathematics believe that Theudius' Elements was the mathematical text book in the Academy at Aristotle's time.

[10]. There is a great debate about whether this revolution in mathematics should be attributed to a certain extent to Plato himself. Some writers like Neugebauer (1957, 152), Mueller (1969) and Knorr (1982) think that the role of Plato and more generally of philosophy in mathematics has been overestimated. The certain thing however is that there was an interrelation between philosophy and mathematics, since most of the mathematicians of that time were also philosophers (Archytas, Theaetetus, leon, Eudoxus, Menaechmus, Philippus of Medna etc). This issue cannot be dealt with at length now. I believe however that Plato, although not a mathematician himself, was of great service to mathematics, especially (as I argued in Chapter VI, section 9) in connexion with the systematic treatment of first principles and its emancipation from the previous reliance on experience (see Plutarch Marc, 14.9-12) and probably proposing mathematical projects. (Aaccording to Sosigenes, Eudemus declared that Plato set the following problem to the astronomers; to find "what uniform and ordered movements must be assumed to account for the apparent movements of the planets"; see in Simplicius On de Caelo

488.20-31, Heiberg). Moreover both Plato's dialogues and the ancient tradition accord in giving us an image of Plato as one who was greatly interested in mathematics.

[11]. Philostratus was born c.170 A.D. and studied in Athens. The 'Lives' were written between 230 and 238 A.D. (see W Wright, Philostratus and Eunapius ed LOEB, introduction).

[12]. Of the same opinion is De Santillana, 1940,252.

[13]. See Heath, 1921,I, 322; Sarton, 1952/1970,441-2.

[14]. See in De Santillana, 1940,253. K von Fritz proposes also the same date (Philol., 1930, 478).

[15]. De Santillana, 1940, 261; P Merlan, 1960, Appendix, The life of Eudoxus, 98-104. Lassere (1964,86) posits Eudoxus' birth in 391.

[16]. Cf Moravcsik, 1971, 55,60; Gulley, 1962, 16-9; Gulley, 1954, 194; Cornford, 1952/1971, 120; Vlastos, 1965,153. Irwin (1977, 316 n.17) thinks that " some empirical contribution (like the perception of equals in the Phaedo) may still begin the recollection and influence the conclusion". In the Meno unlike the Phaedo, we have no evidence about the role of sense-experience in recollection. However, sensible things like diagrams may facilitate recollection. Regarding the theory of recollection in the Phaedo and the role of sense-experience, see Ackrill, 1973; Gulley, 1954; Gosling, 1965; Morgan, 1984; Gallop, 1975, 113-36.

Mugler (1948, ch 4) argued that the soul's previous knowledge is not empirical knowledge from the present life but empirical knowledge from previous lives. Not only is there no textual evidence for his claim, but also "any empirical explanation of a priori knowledge, if resting on the postulate of the pre-existence of the soul, involve[s] an infinite regress"

(Gulley, 1954, 196).

[17]. Cf Ackrill, 1973,177; Crombie, 1963, 138-9. Moravcsic (1971,60ff) thinks that Plato's theory of recollection in the Meno deals primarily with innate concepts in the human mind, rather than with propositions. Hare's (1965) interpretation relies on the distinction between the ability to use a term and the ability to offer a satisfactory definition of it, and restricts recollection to the domain of meaning.

[18]. See Vlastos, 1965, 157.

[19]. See Tinger, 1970,3; Sharples, 1985,49.

[20]. Cf Crombie, 1963, 140; Cornford, 1952/1971, 121; Moravcsic, 1971, 60; Gulley, 1962,9; Tinger, 1970,4.

[21]. See Crombie, 1963, 140: "The way our minds work corresponds to the way things are, and this is the kinship of the soul to the 'universal nature'".

[22]. For a modern and more sophisticated view of this conception of knowledge, see Quine, 1963, 42ff.

[23]. Many scholars interpreting the Meno in the light of the Republic (or of axiomatized Euclidean geometry), speak of a hierarchically and deductively organized system of knowledge with first principles at its top. See Vlastos, 1965, 155-6; Gulley, 1962, 14-5; Cornford, 1952/1971, 125. Moravcsic (1971, 60-1) objects to this model claiming that the objects of recollection are concepts that "form a 'field' of some sort, so that if one has brought one element to consciousness then this will bring with it the ability to bring to consciousness other members of the field as well". However he does not explain what he means by this 'field' and how the 'grasping' of the first thing will facilitate the finding of the others.

[24]. The word 'always' rules out the possibility of colour without figure (shape), but not the possibility of figure that does not accompany colour; but, in this case, the 'definition' of figure would not really be a definition because it would not include all things that fall under the name 'figure'. Others who think that the two terms are thought by Plato as coextensive are: Klein, 1965,59; Irwin, 1977, 136; Thomas, 1980, 98-9; Sharples, 1985, 131.

[25]. In fact, in the three definitions offered by Socrates, we have three different patterns. Socrates openly dismisses only the third definition (76d-e). Most scholars claim that Plato finds the type of the first definition unsatisfactory, or at least less satisfactory than the second definition of shape as the "limit of solid". For a different view see Nahknikian, 1971, 127. Hoerber (1960, 96-7) goes further and claims that the first definition is superior to the second, the "deepest philosophically" and the only one that reveals an insight into the cause.

[26]. In _Men_ 87d-89a, Socrates 'proves' that virtue is knowledge. The argument, roughly speaking, is the following: Virtue is good; every good is profitable; profitable involves right use and this last, _phronesis_ (which is taken to be synonymous to _episteme_ - knowledge). But at 73b-c and 74b, Socrates says that _phronesis_ or _sophia_ (these terms are almost synonymous) is a part of virtue. The argument, besides its other defects, seems to be circular. However it does not seem to me that Plato deliberately used or even noticed the circularity of the argument.

Aristotle (_An Post_ 72b15ff) refers to some people who hoped to save demonstrative knowledge by maintaining the validity of reciprocal demonstration. Cherniss (1944, 67-8, and 1951, 417 n.50) suggested that Aristotle does not refer here to Plato, Speusippus or Xenocrates (although this last tried to demonstrate a definition by the use of convertible terms - see An Post 91a35-b11), but to some followers of Xenocrates. Barnes (1976) suggested the mathematician and philosopher Menaechmus who worked on convertible propositions (Proclus In Eucl 183, 194-5) and used the term 'element' (In Eucl 72-73) not in the narrow sense of principle but for any proposition used to prove something; "in this sense many propositions can be called elements of one another when they can be established reciprocally (κατασκευάζεται....ἐξ ἀλλήλων)". Moreover Aristotle informs us that mathematicians tried to prove the 5th postulate by using circular proofs (An Post 64b39-65a9) and Proclus says that Apollonius tried to prove axioms (In Eucl 194-5). It seems then that there was a debate among mathematicians (and probably among philosophers) in post-platonic times regarding the validity of circular proof of mathematical principles. However we have no evidence for circular proofs in mathematics of the 5th or early 4th century.

[27]. Plato in the Phaedo and in the Cratylus does not refer to the whole of knowledge' as a hierarchically structured system (as in the Republic). He probably refers to restrictive 'fields' with their relative principles (p13-15). It is not certain whether such a principle (adequate) has to be thought of as a) something generally accepted to be true, or b) something self-evident.

[28]. Although it is possible that mathematical concepts like square, circle etc are lower in the hierarchy than others like beauty, justice or virtue.

[29]. Cf Cambiano, 1967, 142-3; 1971, 193-4.

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