DISTRIBUTIONAL EFFECTS IN HOUSEHOLD MODELS: SEPARATE SPHERES AND INCOME POOLING

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Distributional effects in household models: separate spheres and income pooling.*

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Abstract

We derive distributional effects for a non-cooperative alternative to the unitary model of household behaviour. We consider the Nash equilibria of a voluntary contributions to public goods game. Our main result is that, in general, the two partners either choose to contribute to different public goods or they contribute to at most one common good. The former case corresponds to the separate spheres case of Lundberg and Pollak (1993). The second outcome yields (local) income pooling. A household will be in different regimes depending on the distribution of income within the household. Any bargaining model with this non-cooperative case as a breakdown point will inherit the local income pooling. We conclude that targeting benefits such as child benefits to one household member may not always have an effect on outcomes.

JEL codes: D10, C71, C72

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1 Introduction

Using policy to channel resources towards certain types of individuals within households, thereby exogenously altering the intra-household distribution of income, is an instrument widely used by governments, typically to further the welfare of children. Such transfers are usually put in the hands of mothers, on the basis of the belief that additional resources to mothers, over and above the level of income they generate by choice, leads to additional resources going to children. Economic evidence on these phenomena is relatively scarce. Two reasons can be invoked to justify this. On the one hand, scarcity of data has hampered progress on this issue\(^1\), and on the other, suitable conceptual tools have been developed only relatively recently. In the standard ‘unitary’ approach to household behavior, for instance, income is pooled at the household level and the identity of the recipient is irrelevant. Thus the issue of ‘targeting’ benefits to one household member can only be analysed outside the unitary framework.

Non unitary models can be classified into two broad categories, depending on whether they assume cooperation (hence Pareto efficiency) or non cooperation. Blundell, Chiappori and Meghir (2005) analyse the ‘targeting’ issue in a cooperative context. In the non-cooperative framework, two main avenues have been explored. One relies on non-cooperative Nash equilibrium and private provision of public goods. An alternative approach, introduced by Lundberg and Pollak (1993) (LP) relies on a ‘separate spheres’ approach, whereby in the absence of cooperation each individual within the household specializes into specific tasks (for instance, those that are ‘traditionally’ assigned to their gender). While intuitively appealing, the notion of ‘separate spheres’ has not been given a sound theoretical underpinning.

The main goal of the present contribution is to extend existing results, and to clarify the links between the ‘Nash equilibrium’ and ‘separate spheres’ approaches. In doing this we provide a framework which contains all current suggestions as special cases. We consider a model in which agents decide on the provision of several public goods; in this context, we analyse the Nash equilibrium with voluntary contributions. Our most important result is that in general there is at most one public good to which both agents contribute. Hence all public commodities, but possibly one, are exclusively provided by one agent only. We show that whether the two partners contribute to disjoint sets of public goods or to sets that have one good in common depends solely on preferences and the allocation of income within the household. Finally we show that if preferences and the intra-household distribution of income are such that both contribute to a common good, then an extension of the local income pooling result of Warr (1983) and Bergstrom et al (1986) holds. Specifically, in this case household demands for all goods are independent of individual incomes and only depend on aggregate household resources. The alternative case is that the sets of public goods to which each person contributes are disjoint; in this case the allocation of income matters. It is the latter that we interpret to be the ‘sepa-

\(^1\)The most convincing evidence is from Lundberg, Pollak and Wales (1997).
rate spheres’ model of Lundberg and Pollak (1993) which can thus be seen as a sub-case of the general, non-cooperative approach. Individual specialization, in this context, need not be assumed initially; rather, it emerges endogenously as an equilibrium feature. Finally, the definition of the individual ‘spheres’ is endogenous; we show how it is determined by individual preferences and the within household distribution of income.

Although one can interpret the LP separate spheres as stated in the last paragraph, it is not clear that this is the interpretation that Lundberg and Pollak (1993) have. They emphasise ‘traditional gender roles’ whereas our model implies that when the two partners contribute to different public goods, the actual sets depend on the within household allocation of income and tastes which have no social gender specific analogue. The two interpretations have radically different empirical implications. First, separate spheres (disjoint contributions) in our model is a local phenomenon; for different allocations of income within the household we might or might not have separate spheres. When we do not have separate spheres then we have local income pooling. Conversely, for LP, separate spheres holds for all allocations of income within the household and we would never observe local income pooling. Second, the LP interpretation implies that we should see all wives and all husbands acting in the same way in respect to contributions to public goods (since the definitions of the spheres is societal) whereas our interpretation would have that although husbands and wives will may contribute to disjoint sets of public goods, the mix of these will vary across households.

Two additional remarks can be made. First, our result has a wider bearing than intra-household allocation, as the Nash equilibrium is often used to represent situations involving large number of agents and of goods, for instance the provision of public goods in society. Second, the scope of our conclusions is not limited to non-cooperative models. Several existing contributions consider cooperative models based on bargaining, with individual outside options modeled as stemming from non-cooperative solutions. Then the local income pooling result implies local income pooling in the bargaining outcomes, at least whenever the underlying non-cooperative outcomes exhibits this feature.

2 Nash equilibrium with voluntary contributions to the public goods

2.1 Framework

We consider a two person ($J = A, B$ with $A$ being a ‘she’ and $B$ being a ‘he’ ) household which faces fixed prices and allocates a given income between different goods. Agent $J$ has income $Y^J$, and $Y = Y^A + Y^B$ denotes the household’s total income. We assume in all that follows that goods are either public or private\(^2\) and that each person has a representable preference ordering over the within

\(^2\)We could allow that goods have the possibility of having both a private and a public nature; this complicates the notation without adding anything of substance.
household allocation of goods. Denote person $J$’s $n$-vector of their private good by $q^J$ and let the $m$-vector of public goods be denoted $Q$. Let $q = q^A + q^B$ be the vector of household consumption of the private good. Prices of private (resp. public) goods are denoted $p = (p_1, ..., p_n)$ (resp. $P = (P_1, ..., P_m)$)

The household budget constraint is:

$$P'Q + p'q = Y \quad (1)$$

Preferences are assumed to be egoistic, in the sense that each person’s utility function is defined over public goods and the individual’s private consumption, $v^J(q^J, Q)$.\(^3\)

2.2 Definition

In the Nash equilibrium with voluntary contributions to the public goods, each individual chooses how to allocate their income between the private goods and the amounts they contribute to the public goods, given the level of contributions of the other individual to the public goods. The household’s expenditure on a public good is the sum of the individual contributions to that public good.\(^4\)

A solution in this problem is a vector of contributions to the public goods $(g^A, g^B)$ such that each individual’s belief are confirmed in equilibrium, that is such that $(q^J, g^J)$, $J = A, B$ are solutions of the programs:

$$\begin{cases}
\text{Max } q^J, g^J & U^J(q^J, g^A + g^B) \\
pq^J + P^J g^J \leq Y^J \\
g^J_i \geq 0, i = 1, ..., m
\end{cases} \quad (2)$$

This program can be rewritten equivalently in terms of private goods and of public goods for the household. For $A$:

$$\begin{cases}
\text{Max } q^A, Q & U^A(q^A, Q) \\
pq^A + P'Q \leq Y^A + P'g^B \\
Q \geq g^B
\end{cases} \quad (3)$$

One can write a similar program for $B$, with $B$ choosing the quantity of private goods he consumes and the quantity of public goods the household consumes in equilibrium. Under standard properties (continuous differentiability, strict quasi concavity), an equilibrium always exists in this game.

2.3 Properties of the Nash equilibrium

We now study the features of Nash equilibria in this context. We say that member $A$ contributes to public good $j$ if $g^A_j > 0$. We then have the following result:

\(^3\)The analysis below can easily be extended to the case in which there is caring. We shall return to this in the conclusion.

\(^4\)Or equivalently, a function of the sum of the contributions. A more general assumption would be to allow for the household’s consumption of public goods to be a function of the individual contributions. This would make it possible to capture for instance semi-publicness for some goods. We do not allow for this.
Proposition 1 (Types of Equilibrium). Let $m^A$ be the number of public goods to which $A$ contributes, and $m^B$ the number of public goods to which $B$ contributes. In general, $m^A + m^B \leq m + 1$; that is, there is at most one public good to which both contribute. If all public goods are bought, either $m^A + m^B = m$ or $m^A + m^B = m + 1$.

Proof. Take any Nash equilibrium. The result is obviously true if $m^B = 0$. Therefore, assume that $m^B \geq 1$ and that, with no loss of generality, member $A$ contributes to public commodities 1 to $m^A$ while $B$ contributes to commodities $m - m^B + 1$ to $m$. Define $\rho^A = Y^A + P^A g^B$, and let $(q^A_1, q^A_2, \ldots, q^A_m)$ denote the solution to program (3). Then $(q^A_1, q^A_2, \ldots, q^A_m)$ is uniquely defined as a function of $(p, P, \rho^A)$; similarly, $(q^B_1, q^B_2, \ldots, q^B_m)$ is uniquely defined as a function of $(p, P, \rho^B)$, where $\rho^B = Y^B + P^B g^A$.

Now, assume that $m^A + m^B \leq m + 2$. Then $\rho^A$ and $\rho^B$ must satisfy:

$$Q^A_{m^A - 1}(p, P, \rho^A) = Q^B_{m^B - 1}(p, P, \rho^B)$$

$$Q^A_m(p, P, \rho^A) = Q^B_m(p, P, \rho^B)$$

as well as the budget constraint:

$$\rho^A + \rho^B = y - p' (q^A_1(p, P, \rho^A) + q^B_1(p, P, \rho^B))$$

These three, algebraically independent equations in two unknowns are generically incompatible. Specifically, let $(\bar{\rho}^A, \bar{\rho}^B)$ denote a solution to equations (4) and (5); assume that the determinant

$$D = \begin{vmatrix} \frac{\partial Q^B_{m^B - 1}}{\partial \rho^A} & \frac{\partial Q^A_{m^A - 1}}{\partial \rho^A} \\ \frac{\partial Q^A_m}{\partial \rho^A} & \frac{\partial Q^B_m}{\partial \rho^A} \end{vmatrix}$$

is non zero at $(p, P, \bar{\rho}^A, \bar{\rho}^B)$. Then the solution $(\bar{\rho}^A, \bar{\rho}^B)$ is locally unique. Moreover, $(\bar{\rho}^A, \bar{\rho}^B)$ fails to satisfy condition (6) except for one specific value $\bar{y}$ of $y$, namely:

$$\bar{y} = \bar{\rho}^A + \bar{\rho}^B + p' (q^A_1(p, P, \bar{\rho}^A) + q^B_1(p, P, \bar{\rho}^B))$$

We conclude that a solution fails to exist for almost all $y$. ■

Note that the result is only ‘general’ (or ‘generic’), in the sense that it is ‘almost always’ satisfied. Still, it is possible, for arbitrary preferences, that it is violated at specific points (but then typically these points are locally unique). Also, one can find preferences such that the result is violated upon an open set. This is the case, for instance, when public goods are separable and (sub) preferences over the public goods are identical across individuals, a case studied in Lechene and Preston (2005); in that case, the determinant $D$ is identically null. Note, however, that such preferences are not robust to local perturbations.
A precise statement of these ‘genericity’ conditions would require some heavy mathematical apparatus (transversality theory) that is outside the scope of this paper; instead, a detailed example is provided below.

The next proposition states explicitly an implication for the case $m^A + m^B = m + 1$.

**Proposition 2 (Local Income Pooling)** When there is one public good to which both household members contribute ($m^A + m^B = m + 1$), redistributions of income between household members which do not exceed the amount of each individual’s contributions to the jointly contributed public good have no effect on households expenditures.

**Proof.** Assume, without loss of generality, that both agents contribute to commodity 1, while $A$ is an exclusive contributor for commodities 2, ..., $p$ and $B$ is an exclusive contributor for commodities $p + 1$, ..., $m$ (where $2 \leq p \leq m$). Then the $(2n + m)$ vector $(q^A, q^B, Q)$ satisfies $n + p - 1$ first order conditions for $A$, $n + m - p$ first order conditions for $B$ plus the global budget constraint. None of these conditions depend on individual incomes (all but the last do not depend on incomes at all and the last only depends on aggregate income). Hence the set of solutions does not depend on individual incomes.

Proposition 2 generalizes the remarkable result first obtained by Warr (1983) in the case of one public good, then also by Kemp (1984), and Bergstrom, Blume and Varian (1986) for several public goods.\(^5\) It shows, in particular, that a given household may or may not pool income; in fact, pooling behavior obtains endogenously as the outcome of non-cooperation for certain ranges of the relevant parameters. A heterogeneous sample of households may therefore contain “pooling” and “non-pooling” households, a fact which has important implications for empirical work.

Proposition 1 is more original. It states that in a non-cooperative setting, individuals specialize in funding public goods, so that all public goods but maybe one, are exclusively funded by one individual. Proposition 1 can thus be interpreted as a ‘separate spheres’ result: in practice, each publicly consumed commodity (but maybe one) belongs exclusively to the ‘sphere of influence’ of one of the household members.

Two remarks are however in order. First, in contrast with Lundberg and Pollak’s approach, respective spheres are *endogenously* determined. Specifically, for any equilibrium vector $(q^A, q^B, Q)$, for any public good $j$ which is consumed, we have the following characterization (assuming that both people buy the first private good):

- either $\frac{\partial U^A}{\partial q^A_j} < \frac{p_j}{p_1}$, then $\frac{\partial U^B}{\partial q^B_j} = \frac{p_j}{p_1}$ and $B$ is an exclusive contributor (so that commodity $j$ belongs to $B$’s sphere),

\(^5\)Kemp’s method of proof relies on counting equations and unknowns, but assumes that all agents contribute to all public goods, which is impossible in general. Bergstrom, Blume and Varian use a revealed preference argument. Lechene and Preston (2005) give a proof in the same spirit as the proof provided here, but at an interior equilibrium, that is under restrictions on preferences so that all agents contribute to all public goods.
In other words, for all public goods but possibly one, the marginal willingness to pay (out of private consumption) of one of the partners is smaller than the marginal cost.

The second remark is that the definition of the ‘spheres’ is not fixed; it depends on individual incomes. For instance, when a member’s income is low enough, this member will in general contribute to no public good. This implies that any change in income distribution that affect the members’ respective incomes may change the definition of the spheres. A precise illustration is given below in a specific example.

Before turning to the example, we discuss briefly the possible implications of this analysis if agents use a bargaining models that takes the non-cooperative outcome as a breakdown point. In this case the bargaining outcomes inherit some of the features of the non-cooperative outcomes. In particular, the same segments of local pooling will hold for all goods.

2.4 An example

We now study an example with one private good and two public goods (denoted $G$ and $H$ for simplicity). Individual preferences are Cobb-Douglas:

$$u^A(q^A, G, H) = q^A G^a H^\alpha$$
$$u^B(q^B, G, H) = q^B G^b H^\beta$$

We denote by $A$’s income by $\rho$ and $B$’s by $(1 - \rho)$. We are particularly interested in analyzing changes in demand resulting from variations in the income share $\rho$ (keeping total income constant at unity). We assume, as a normalization, that

$$\frac{a}{\alpha} > \frac{b}{\beta}$$

so that $A$ cares (relatively) more for commodity $G$ and $B$ cares (relatively) more for commodity $H$.

The first order conditions give:

$$a \frac{q^A}{G} \leq 1, \alpha \frac{q^A}{H} \leq 1 \text{ for } A,$$
$$b \frac{q^B}{G} \leq 1, \beta \frac{q^B}{H} \leq 1 \text{ for } B.$$


with an equality when the agents contributes to the commodity under consider-
ation. Now assume, first, that $A$ is contributing to $G$ and that $B$ is contributing to $H$. Then

$$\frac{a q^A}{G} = \frac{\beta q^B}{H} = 1$$

If, moreover, $A$ also contributes to $H$, then $a q^A_H = 1$, hence $\frac{H}{G} = \frac{a}{\beta}$. Similarly, if $B$ also contributes to $G$, then $b \frac{q^B}{G} = 1$, hence $\frac{H}{G} = \frac{b}{\beta}$. It follows that $A$ and $B$ cannot simultaneously contribute to both public goods unless $\frac{a}{\alpha} = \frac{b}{\beta}$. This is exactly the meaning of the ‘in general’ qualification in the statement of Proposition 1: such a condition is ‘almost never’ satisfied, and when it is the situation is ‘knife-edge’ and not robust to infinitesimal perturbations (here, infinitesimal changes in the parameters).

The exact solutions as a function of $\rho$ are given by:

- if

$$\rho \leq \frac{b}{a + ab + a\beta + b}$$

then $A$ contributes to no public good, $B$ contributes to both public goods and

$$q^A = \rho, \ q^B = \frac{(1 - \rho)}{1 + b + \beta}, \ G = \frac{b(1 - \rho)}{1 + b + \beta}, \ H = \frac{\beta(1 - \rho)}{1 + b + \beta}$$

- if

$$\frac{b}{a + ab + a\beta + b} < \rho \leq \frac{b(a + 1)}{a + ab + a\beta + b}$$

then $A$ contributes to $G$, $B$ contributes to $G$ and $H$, and

$$q^A = \frac{b}{a + ab + a\beta + b}, \ q^B = \frac{a}{a + ab + a\beta + b}, \ G = \frac{ab}{a + ab + a\beta + b}, \ H = \frac{a\beta}{a + ab + a\beta + b}$$

Note that, in that case, demand does not depend on $\rho$, as stated in Proposition 2, since both agents contribute to $G$.

- if

$$\frac{b(a + 1)}{a + ab + a\beta + b} < \rho \leq \frac{\beta(a + 1)}{a + a\beta + a\beta + \beta}$$

then $A$ contributes to $G$, $B$ contributes to $H$, and

$$q^A = \frac{\rho}{a + 1}, \ q^B = \frac{(1 - \rho)}{\beta + 1}, \ G = \frac{a\rho}{a + 1}, \ H = \frac{\beta(1 - \rho)}{\beta + 1}$$

This is the ‘pure separate sphere’ case, in which each public good is funded by one agent.
• if 
\[ \frac{\beta (a + 1)}{a + \alpha \beta + a \beta + \beta} < \rho \leq \frac{\beta (a + a + 1)}{a + \alpha \beta + a \beta + \beta} \]
then \( A \) contributes to \( G \) and \( H \), \( B \) contributes to \( H \), and
\[
q^A = \frac{\beta}{a + \alpha \beta + a \beta + \beta}, \quad q^B = \frac{\alpha}{a + \alpha \beta + a \beta + \beta}, \quad G = \frac{\alpha \beta}{a + \alpha \beta + a \beta + \beta}, \quad H = \frac{\alpha \beta}{a + \alpha \beta + a \beta + \beta}
\]
and again demand does not depend on \( \rho \).

• finally, if 
\[ \frac{\beta (a + a + 1)}{a + \alpha \beta + a \beta + \beta} < \rho \]
then \( A \) contributes to \( G \) and \( H \), \( B \) contributes to no public good, and
\[
q^A = \frac{\rho}{1 + a + \alpha}, \quad q^B = (1 - \rho), \quad G = \frac{a \rho}{1 + a + \alpha}, \quad H = \frac{\alpha \rho}{1 + a + \alpha}
\]

It is simple to show that the outcomes are inefficient for all values of \( \rho \in (0, 1) \). At the endpoints the outcomes are efficient since then one or other person is a dictator. If we take a bargaining model which has the non-cooperative outcomes as a breakdown point then the household demands for both public goods will be higher than in the non-cooperative case (with equality at the endpoints). Importantly, the bargaining outcomes will then have intervals of income pooling over the same values of \( \rho \) as the non-cooperative case.

These results are summarized in figure 1, in which the horizontal axis represents the values of \( a \) and the vertical axis the expenditures on the two public goods. We take values of
\[
\{a, \alpha, b, \beta\} = \{5/3, 8/9, 15/32, 1/2\}
\]
which gives ‘join’ points at \( \rho = \{1/8, 1/3, 1/2, 2/3\} \). In interval I person \( A \) does not contribute to either public good. As income is transferred to her she spends it on her private good and \( B \) cuts back expenditures on both public goods and his private good. Thus we see that expenditure on good \( G \) falls even though person \( A \) cares relatively more for this good. As an example, transferring income from father to mother will not necessarily lead to higher expenditures on children even if the mother cares more for the children. At the value \( \rho = 1/8 \) person \( A \) starts to contribute to good \( G \) and we enter an interval of income pooling (II). As even more income is transferred to \( A \) we reach a point \( \rho = 1/3 \) in this case) at which \( B \) stops contributing to good \( G \). This gives the pure separate spheres interval III. Intervals IV and V are obvious counterparts to II and I respectively. This figure shows many of the important features of our model:

• Local income pooling will hold for some values of the distribution of income but not for others.
The household demands for public goods are not necessarily monotone in the distribution of income, even if one partner cares relatively more for one good than the other.

The join points for the different regimes are the same across all goods (we do not show the private good expenditures but this property holds for them as well; see the conditions above). This is potentially important for empirical work; without this property, the chances of successfully devising powerful tests for the patterns displayed here would be remote. On the other hand, if preferences vary across women (different $a$ and $\alpha$ for different households) and across men (different $b$ and $\beta$ for different households) then the join points themselves will be heterogeneous which will have to be taken into account.

3 Conclusion

We have considered a non-cooperative model of household allocation to different goods. We have shown that if preferences are egoistic then the voluntary contributions game gives two distinct regimes for household behaviour. In the first regime there is one public good to which both partners contribute and we...
have local income pooling. In the second regime, the two partners contribute to distinct sets of public goods (separate spheres) and a local re-distribution of income will lead to a change in household demands. One important corollary of the latter is that a reallocation to $A$ may lead to a decrease in the household demand for the public good that $A$ values most. This analysis also has implications for bargaining models if the household uses the non-cooperative outcomes suggested here as a breakdown point. In that case the bargaining outcomes will inherit the local income pooling from the non-cooperative model. Moreover, the analysis also implies the strong restriction that the only distribution factor$^6$ is relative income.

Allowing for caring (so that $A$’s preferences are represented by a weighted sum of her felicity function and his felicity function and similarly for $B$) leaves the analysis unchanged, except that we add flat (local income pooling) segments to the demands for all goods at extreme values of the household distribution of income (that is, $\rho$ close to zero or unity). This follows since at such values the high income and caring person will be effectively transferring resources to the low income partner and any local re-distribution is simply undone. These regions of income pooling are analogous to those that arise in the Rotten Kid Theorem which also relies on one person having most of the resources and caring for the one with low resources. Once again, any bargaining model will inherit these flats at extreme values of the within household income distribution.

The positive and policy implications of our analysis are quite sharp: even if households do not have a common utility function (the unitary model) they may exhibit local income pooling for some values of the within household distribution of income. When they do not pool income locally it must be that they are contributing to separate spheres. The exact importance of the local income pooling intervals and the separate spheres intervals will depend on preferences. For some values of preferences we may have that the household will almost never respond to changes in the within household allocation of income and that targeting income will have little effect. We end by emphasising that we stress the ‘may’ in the preceding sentences. We do not make the general claim that we believe redistribution or targeting does not matter in general - alternative theories to the ones we have analysed here (e.g., cooperative approaches) will give more or less income pooling and potential for targeting. Whether or not households pool income (locally or globally) is, in the end, an empirical issue.

References


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$^6$A distribution factor is a household variable that enters the household demand functions but does not influence the preferences of the two partners.


