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Observations on naming and labelling practices in angle work with teachers: how might small language choices support mathematical activity?

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There is a growing interest in how word choice in mathematics classroom interactions can influence the meanings and perspectives that students develop. In this paper we contribute to this by offering examples to highlight the variation in how mathematical objects and relationships can be described and the implications these differences may have on the learning of some students. These examples are drawn from a discussion between a group of researchers and teachers working together on a geometric task as part of a project where they are collaborating to develop resources to support language-responsive teaching. We focus in particular on the naming and labelling of angles and the relationships between angles, and the different roles these names and labels may have for the fluency, precision and meaning in mathematical communication.

Keywords: Noticing, naming and labelling, fluency and precision.

Introduction

Language plays a central role in teaching and learning mathematics. It enables students to express and explore both abstract concepts and the semantic relationships between them, as well as being constituent to practices such as arguing, reasoning, and justifying. The attendant importance of language to pedagogy is reflected in a growing body of international research exploring the potential for language-responsive approaches to teaching mathematics (Erath et al., 2021). Recent research, in particular, has highlighted the importance of the choice of specific words, phrases or representations to communicate about meaning but also to support the development of that meaning (Götze & Baiker, 2023).

In this paper and in our work with teachers, we emphasise smaller units of mathematical talk and language and the relationship between these and the mathematical meanings and concepts they relate to. This builds on a growing body of research focusing on the intertwining of these smaller units with mathematical practices (Götze & Baiker, 2023; Planas et al., 2024; Ingram, 2021). These smaller units include individual words, phrases or sentences that focus on a mathematical object, relationship or action and may or may not be specific to the mathematics register.

One example where this level of granularity has been helpful concerns the connected actions of naming and labelling. With specific regard to naming, much of the existing research has been on teachers' noticing of naming in their own talk (e.g. Adler, 2021; Planas et al., 2024), with contrasts made between mathematically correct and precise names and more ambiguous names or pronouns such as 'it' or 'that'. In classroom interactions, mathematical correctness and precision may be in tension with developing a shared meaning and the efficiency and fluency of communication (Adler, 1998; Planas et al., 2024).

The account below draws on a small section of one episode from a design-based collaborative research project focused on language-responsive mathematics teaching to illuminate the importance of small words and short phrases and sentences in mathematical communication, and the value of paying attention to their various possible uses and meanings, and the consequences these may have on learners. While we offer data from the project in this paper, these are to illustrate the arguments we are making which are tentative and evolving as we work alongside teachers and students. Our aim is not to present findings, but rather to provoke a conversation around these small units and their role in mathematics teaching and learning.

The professional development (methods)

The authors are part of a research team conducting a wider design-based research project (Cobb et al., 2003) which involves working alongside a group of ten secondary mathematics teachers in a series of professional development (PD) sessions. A central part of each session is a discussion built around mathematically rich tasks that some of the teachers already use in their classrooms. These discussions are intended to foreground topic-specific language demands and discourse practices, in line with the recommendations of Erath et al. (2021), with separate sessions focusing on linear equations, probability and angle properties of parallel lines.

Participation in the project is intended to promote teacher awareness of the use and import of language, with the goal of supporting pedagogy. It is hoped that drawing attention to challenges and variation relating to language use, within and across topic-specific domains, will support teachers in recognising, working with and, where appropriate, facilitating flexibility in learners' own mathematical thinking and use of language. This awareness can arise from the activities and discussions led by the research team, through their discussions with each other about their experiences using the tasks, as well as reflection on what their students say in their lessons that follow the PD. In this way the PD is based on the Discipline of Noticing (Mason, 2002).

The PD workshop from which the data for the discussion below is taken focused on identifying the linguistic demands and opportunities for student reasoning in a task involving a sequence of (connected) geometrical diagrams focused on using properties of parallel lines to identify a missing angle. The interaction we focus on involves the teachers and researchers/teacher educators reporting back to each other how they identified the missing angle, with each teacher focusing on a different image. All participants had access to all the diagrams, but the discussion was deliberately managed so that the diagrams could not be annotated. This forced the participants to communicate their reasoning verbally.

Theoretical stance

The members of the research team have experience of different theoretical frames, including ethnomethodology, sociocultural theory and systemic functional linguistics (SFL), and we consider this diversity to have enriched our engagement with the data. What is often central to each of these perspectives is the focus on communication in interaction, and an understanding of learning as developing shared meanings and ways of communicating.

From a SFL perspective, in mathematics education research the focus is on how linguistic communication represents ideas and interpersonal relationships (Halliday, 1994). From an ethnomethodological perspective, both what is said and how it is said make a difference to how people act in interaction (Heritage, 1984). From a sociocultural perspective, communication carries with it the social and cultural meanings that have been established over time (Werstsch, 1988). Beyond mathematics education, there is also a wealth of research on social interaction that has demonstrated how word choice can impact behaviour (e.g., Heritage & Robinson, 2011) and the perspectives we take.

For the purposes of this paper, language is language use, that is, the use of linguistic forms, such as words, phrases and sentences, in articulation with other modes of communication, such as visual diagrams. Moreover, we view language use as indicative of choice (words, phrases, gestures or images), though not necessarily conscious choice, and that the meanings which may be construed depend on the interactional context. Within mathematics education research, word choice is an under-researched aspect of linguistic communication. Language use at the level of choosing single words, phrases or relatively short sentences has important functions in mathematical communication, including the function of naming concepts, processes or practices in particular ways, each of which encodes meanings.

Following Mason (2010), we treat learning mathematics as involving students perceiving similarities, relationships and properties and discerning differences or features in a new way. Learning about angle properties of parallel lines involves recognising the similarities in the relative positions of the lines and angles that result in angles having the same size, which we might name as alternate or corresponding angles. Distinctions need to be made between lines that have the properties of being parallel and those that do not, so that the properties of parallel lines can be used either dynamically by transforming one angle into another through rotation or translation, or statically by deducing sequentially the relationships between different angles.

Results and discussion

In this paper, we draw on two aspects of the interaction in the PD session to highlight the variation in language used and the implications this has for ambiguity in communication that is often indicative of fluency in communication and the tensions between mathematical clarity and precision and the communication of reasoning. We show how word choice conveys different meanings and perspectives (i.e. where attention is being focused or directed), as well as how the sequencing of these words can affect the communication of reasoning.

In Table 1, we have collected the different names used to refer to angles or relationships between angles during the discussion by the participants around the sequence of tasks. The names have been partly organised in rows to reflect some immediate connections, but in general the structure of the table does not necessarily indicate related use or adjacent cases.

This table demonstrates a substantial variation in how angles may be named. Some of the names for an angle are simple labels (e.g. A, angle A), whilst some reflect properties of the angle such as its size (e.g., the seventy degree angle). Other names locate the angle in relation to another angle (e.g. the alternate angle) whilst some follow an agreed convention, relating the angle to other geometric elements (e.g. BDC[2]). In switching between these, students have to navigate this variation, often in contexts where the name and the relationship or property being named is a relatively new idea.

A single angle	**A collection of two or more angles**	**Relationships between angles**
the seventy degree angle	the angles	angles in a triangle add up to one hundred and eighty degrees
the alternate angle	the alternate angles	are alternate
A	lots of 40° angles	equal
it		equivalent
angle A		angles at a point equal three hundred and sixty
angle forty degrees		vertically opposite angles are equal
a plane angle		opposite angles
co-interior angle	co-interior angles	co-interior angles
third angle		the corresponding angles
the one hundred and twenty		angles in a triangle
BDC		angles around a point

Table 1: Ways that angles or relationships between angles were named

The names used to indicate relationships also vary. Some of these reflect permanent or generalised relationships or rules (e.g., angles on a straight line add to one hundred and eighty degrees), while others are local to the specific angles being talked about (e.g., the alternate angles). Sometimes, the clarity of a label can shift over time; for instance, as one of our participating teachers noted, naming angles with their size can be effective until more than one angle of the same size becomes relevant in an argument, as is often the case when teaching angles in parallel lines.

[2] Note: This is what was said so the angle notation usually used in formal mathematics writing has not been added

We notice here that there are only subtle differences in the ways that some of these properties are used to describe angles and their properties or relationships. Such differences might be difficult to notice, particularly for students facing wider linguistic challenges. For example, *the alternate angle* refers to an angle with a specific relationship to the angle being previously talked about. *The alternate angles* refers to two angles that have the relationship that they are equal (in size) because they are alternate angles, and this relationship can be perceived as the result of a transformation (rotation around a point) or as a static rule for equal angles (so-called Z angles). Finally, *are alternate* focuses on the relationship as a property that these angles have.

Mason (2010, p. 23) offers that "each technical term in mathematics signals the fact that those who developed the term needed a label". However, we note here that although these needs may be consequent from a task, they originate with individuals; students will have different needs as they are cast in different roles as speakers and listeners within the classroom, as they develop their mathematical knowledge, and as they engage in different mathematical activities. The subtle semantic differences between names may fade when viewed through the lens of experience and with fluency, but each difference holds some potential to confuse or frustrate a learner and is thus worthy of attention. Even whilst working as experienced mathematicians, our participating teachers showed differences in how they engaged with the process of labelling. After a discussion of the final problem, one teacher ventured "I think the labelling definitely helped that one", but a second teacher reflected how the "labelling was helpful, but it took me some time to actually match the labelling to the picture."

Fluency and precision

We also offer here that fluency or precision is not an absolute, but rather inclusive of sensitivity to what is important and what is sufficient, both of which can be influenced by and inferred from context. This resonates with Grice's maxims of communication and Adler's dilemma of transparency (1998). When we interpret the situation as being one in which what is unsaid is understood, efficiency can be emphasised over precision (e.g., angles around a point as an abbreviation for angles around a point sum to three hundred and sixty degrees), and this assumption of a shared understanding of what is unsaid suggests a fluency in the ideas, rules or properties being communicated.

Precision has two roles in mathematics classroom interaction – precision in the use of technical terms, but also precision in terms of the shared attention on what is being talked about. These are not necessarily the same in the teaching and learning of mathematics, with mathematical precision sometimes obscuring or distracting from the focus (Adler, 1998). Precision in the shared attention can also be achieved through the use of meaning-related words and phrases and not solely through technical terms from a mathematics register.

Meaning is dependent upon the interactional context and the choice of words. Communication about relationships, in particular, needs students to pay attention not only to the relationship, but also to the objects between which the relationship exists. Describing an angle as "the seventy degree angle" or as "the alternate angle" emphasises different features of the angle – is it the size that is important or the relationship to another angle? We also note that naming by size draws attention to a local property of the problem whereas naming by relationship may allow for variation in the problem that maintains this relationship, drawing attention to a more global property of the angle. Where the focus is on relationships between angles, a lack of precision in the naming of the angles or even incorrect use of names or non-mathematical names may not impede communication and could also support the focus on the relationship, e.g. this angle here is alternate to that one there.

Adler (1998) stresses the need for teachers to make decisions about whether to emphasise mathematical precision or emphasise meaning by using words familiar to the students. What we stress here is that this is not a dichotomous choice, and this choice needs to be influenced by focusing on what we want students to notice and the distinctions we want students to make. Mathematical ideas, objects or relationships that are in focus can be foregrounded by using the technical term, while more informal language can support the backgrounding of other features.

Language in service of intersubjectivity

We now offer more detail of two incidents from the same PD session which we consider to be interesting.

The first instance arose when a teacher described verbally their method of solving a problem that involved a diagram that included two pairs of parallel lines and three angles labelled with their size and a fourth angle whose size needed to be found (Figure 1a). They began with the following step:

> "I added in three parallel lines that cut through the seventy degree angle, the hundred and fifty degree angle and the hundred degree angle, that were all parallel to the brown lines…"

There was some confusion in the group after the end of this description, with a second teacher sharing that "…because I'm not seeing it, it's hard for me to visualise what's happening" and indicating some confusion about the parallel lines. The first teacher then reframed the key step:

> "So I inserted three parallel lines that were all parallel to the browns, at the seventy, at the one hundred and fifty and at the hundred…"

Even though this contained no new information, this reformulation proved to be effective, and the second teacher said that they could now place the parallel lines. One reading of this situation is that in the new version of their statement, the first teacher shared the information in an order that was more aligned with the (unseen) visualisation of the second teacher. In this case, a small shift in language use would have had a meaningful consequence for the learner. The placement of the "three parallel lines" requires the listener to know parallel to what and where to draw these lines (through the three labelled angles).

The second event concerned a diagram with four line segments in two separated parts (Figure 1b).

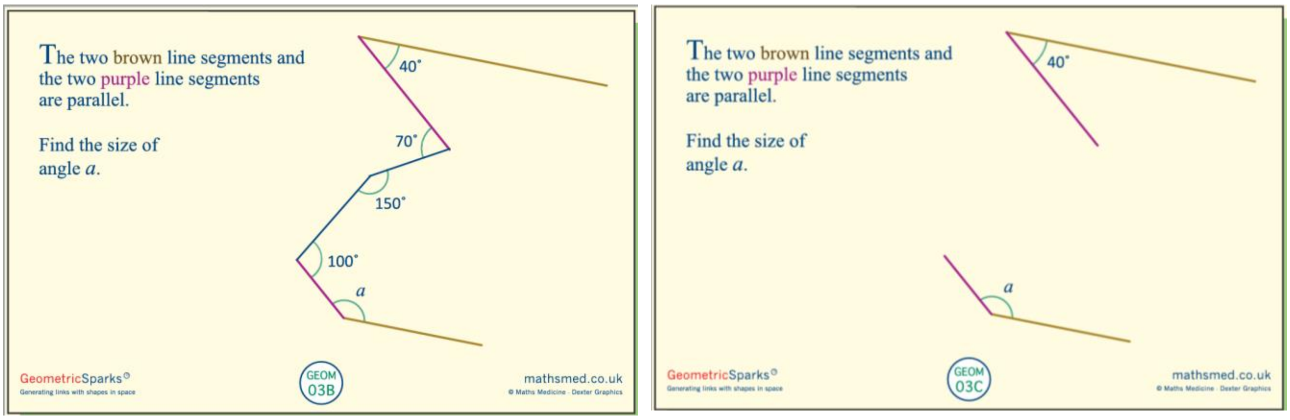
A solution to this problem was shared by one of the researchers:

> "I'm not extending or adding lines… I'm not thinking about the static geometry I'm thinking about dynamic geometry… if I move it, I translate angle a up or to the left or to the right… it remains angle a… I change the position, but I don't change the measure."

The researcher then offered the step of overlapping the brown line segments, so that the two angles labelled in the diagram were placed next to each other, and the two purple line segments would form one longer line segment.

In the fruitful discussion that followed, the same teacher who had expressed confusion in the first incident several minutes later indicated that sufficient intersubjectivity had been achieved. Moreover, it had prompted further mathematical thinking:

> "I really like that approach… to be truthful, that would not be my first option but no, I've just learned something because even while you were explaining I've just seen another way of doing that question…"



Figures 1a and 1b: Two angles tasks (reprinted with permission from Küchemann, 2023)

In a contrasting way to that described above, it is possible that this success may have been supported by a closer match between the researcher's verbal account and the teacher's subsequent internal visualisation. This invites the question of whether this alignment may hint at the difficulties students and teachers may encounter when following and making sense of a different perspective (e.g. making sense of a dynamic account when your own method was based on a static account), potentially supported by specific linguistic features.

Conclusion

Our work to date on this project has repeatedly pointed towards the importance of paying attention to small words and phrases within mathematical language, and being mindful of the multiple interpretations which can hide behind them. That is, there is a need to make sure we are naming what we are paying or drawing attention to. There is worth in becoming more aware of, and beginning to unpick, the complexity of what we are asking learners to do.

Angles, and the language associated with naming angles, can invoke diverse intuitions and understandings: of a static object, of a measure, of a vertex, and of a dynamic turn. Relationships between angles become even more complex, and incautious use of language risks consolidating or compounding students' difficulties. This concern is particularly salient in the cases of students with other linguistic disadvantages.

We do not propose any specific measures here to manage this concern or suggest what good practice should look like (or even that any unqualified set of standard practices could exist). Instead, we offer that by supporting the noticing skills of teachers, we are both developing their awareness of this issue and upholding their agency to make informed and deliberate decisions.

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