

Stephen Priest *Remarks on Logic* (2012 Draft)

REMARKS ON LOGIC

by

Stephen Priest

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ANALYTIC A POSTERIORI

An analytic proposition is true if and only if a definition is true. For example, the analytic proposition

'A vixen is a female fox'

is true if and only if the definition

'"Vixen" means "female fox" in English'

is true. It follows that an analytic proposition is logically equivalent to a definition, but what is it for a definition to be true ?

A definition is true if and only if some proposition correctly identifying the meaning of a linguistic expression is true. A proposition correctly identifies the meaning of a word if and only if it identifies that word's sense or that word's reference or both. A word has a sense if and only if it has a verbal definition and a word has a referent if and only if there is something it may be used to mention.

It follows that a definition is true if and only if a proposition stating the sense of a word is true or a proposition stating the reference of a word is true, or both. For example,

'"Vixen" means "female fox" in English'

is true if and only if either

'"Vixen" has the same sense as "female fox" in English'

is true, or

'"Vixen" has the same reference as "female fox" in English'

or both, are true.

'*a posteriori*' is an epistemological predicate which means 'knowable only through experience'. It follows that *p* is a *posteriori* if experience is necessary for deciding *p*'s truth value. '*a posteriori*' contrasts with '*a priori*' which is an epistemological predicate that means 'decidable intellectually' or 'knowable independently of experience'. It follows that *p* is a *priori* if experience is not necessary for deciding *p*'s truth value.

To say that *p* is a *priori* is not to say that no experience is necessary for knowing *p*'s meaning or understanding *p*. Rather, if *p* is a *priori*, once *p* is understood *p*'s truth value may be decided without further experience.

My argument that analytic propositions are a *posteriori* and not a *priori* is:

P1. The truth of any definition is known a *posteriori*.

P2. Any analytic proposition is logically equivalent to a definition.

[P3. There is nothing more to knowing the truth value of an analytic proposition than knowing the truth value of the definition that is logically equivalent to it.]

P4. Any proposition logically equivalent to an a *posteriori* proposition is an a *posteriori* proposition.

therefore

C. Any analytic proposition is a *posteriori*.

Although not a logical impossibility, it is an impossibility that a definition should be known a *priori*: that one who did not know the meaning of a word should by the operation of pure thought discover that word's definition. It is not contradictory to suppose that someone might introspect meanings or guess correctly the contents of a dictionary instead of opening it. However, it is empirically impossible so it never happens.

Coming to know the definition of a word in a language depends upon listening to users of the language or looking up the word in a dictionary. Listening and reading are empirical activities, so knowledge of definitions is a *posteriori* knowledge.

An analytic proposition is logically equivalent to a definition because if a definition is true then an analytic proposition is true and if a definition is false then an analytic proposition is false. To see this consider

'A vixen is a female fox'.

If 'vixen' did not mean 'female fox' this statement would be false (unless 'vixen' denoted some entity contingently identical with the referent of 'female fox' but then the proposition would not be analytic). Conversely, from the fact that 'vixen' means 'female fox' in English it follows that 'A vixen is a female fox' is true. It is because the sense and reference of 'vixen' is the sense and reference of 'female fox' that 'A vixen is a female fox' is true.

Nothing except the truth of a definition is necessary and sufficient for the truth of an

analytic proposition except what is necessary and sufficient for the truth of that definition.

The analytic proposition

'A vixen is a female fox'

does not say anything that

'"Vixen" has the sense and reference of "female fox"'

does not say and

'"Vixen" has the sense and reference of "female fox"'

does not say anything that

'A vixen is a female fox'

does not say.

Although 'A vixen is a female fox' has the grammatical form of an identity statement it asserts no ontological identity. It asserts that 'vixen' and 'female fox' have one sense, not two, and one referent, not two.

It follows that what someone knows when they know an analytic proposition is exactly what they know when they know the definition that is logically equivalent to that analytical proposition. The content of their knowledge is identical.

It follows that if someone knows a definition *a posteriori* then they know an analytic proposition *a posteriori*. All definitions are *a posteriori* so all analytic propositions are *a posteriori*.

But: analytic truths are deduced from definitions *a priori*.

But: definitions are stipulative and so *a priori*.

Is the proposition expressed by an analytic sentence necessary? This depends on the relation between:

p's meaning is sufficient for p's truth

and

p is necessarily true

If p essentially means p then p could not have had any meaning other than the meaning p has. If it is a contingent fact about p that p means that p then p could have had a different meaning and p's meaning might not have been sufficient for p's truth. Then no analytic truth is necessary because meanings could have been otherwise. If anything is contingent, what means what is contingent. I suggest this definition of synonymy:

'a and b are synonymous' iff (def.): a means b

and this definition of 'means':

'a means b' iff (def.): The meaning of a is identical to the meaning of b.

However, this is a contingent identity statement because it means either: The referent of a is identical with the referent of b, or the sense of a is identical with the sense of b, or both. 'a' might not have had 'b's reference because, if there are any contingent facts, then it is a contingent fact that some human beings use 'a' to refer to

what they use 'b' to refer to. 'a' might not have had 'b's sense because, if there are any contingent facts, it is a contingent fact that human beings define 'a' just as they define 'b'. It follows that 'a' might not have had 'b's sense and reference. If the putative necessity of analytic truths consists in their being expressions of synonymy then that necessity is an illusion: Analytic truths are all contingent.

I suggested above that a proposition is analytic if and only if its meaning is sufficient for its truth. This is not enough to make an analytic truth a necessary truth. The logical status of analytic truths is in some ways analogous to Cartesian truths: 'I exist' and 'I am thinking now' are arguably true just on condition they are formulated but they could have been false. 'A bachelor is an unmarried man' is true because 'bachelor' means 'unmarried man' but it is not necessarily true, because 'bachelor' might not have meant 'unmarried man'.

Part of the illusion that analytic truths are necessary truths has been created by a confusion between two modalities: the necessity of the inference from the meaning of a sentence to its truth value with the necessity of its having that truth value.

(A) S means that p therefore p

does not entail and is not entailed by

(N) Necessarily p

I maintain that where S is analytic (A) is true, but (N) false. p is analytic if and only if p 's meaning is sufficient for p 's truth. ' S means that p ' entails ' p is true'. Although this entailment is

necessary, it does not follow that *p* is a necessary truth. It is fallacious to infer the necessity of a conclusion from the necessity of an inference to that conclusion.

Definitions of analytic:

(1) *p* is analytic if and only if *p*'s predicate is contained in *p*'s subject.

(ie, if explicating the subject requires mentioning the concept(s) expresses by the predicate.

(2) *p* is analytic if and only if *p* is true by definition.

(3) *p* is analytic if and only if *p* is true in virtue of the meanings of *p*'s constituent terms.

Analytic propositions are to be distinguished sharply from metaphysical identities. These are not true by definition and are necessary (*de re*).

(Creation is the realm of contingency. God and the soul are the realm of necessity.)

2. Quine's Attack on Empiricism

In his seminal paper, *Two Dogmas of Empiricism*, Quine claims to be an empiricist but attacks the analytic/synthetic distinction. (5) He says 'this is a distinction which I reject' (p. 43). Here I argue that Quine's attack on the analytic/synthetic distinction is self-refuting.

Quine rejects a 'fundamental cleavage between truths which are *analytic*, or grounded in meanings

independently of matter of fact, and truth[s] which are *synthetic*, or grounded in fact' (p. 20).

This use of 'fact' in drawing the analytic/synthetic distinction is a mistake made not only by Quine but, *inter alia*, by: Ayer, Hume and Leibniz. There are exactly as many facts as truths, because it is true that p if and only if it is a fact that p . Analytic propositions are truths, so analytic propositions are factual or express facts. For example, it is a fact that $2+2=4$, so if ' $2+2=4$ ' is analytic then there is at least one analytic fact. So this use of 'fact' fails to distinguish analytic propositions from others in the *prima facie* way Quine and his opponents intend. Quine is using 'fact' here as synonymous with 'non-linguistic fact'. For example, Quine says it is not 'factual' that $2+4=4$, and that no bachelors are unmarried (p. 34). I take it that for 'is not factual' we are to read 'implies nothing about non-linguistic reality'. If 'fact' does mean here 'fact that is not about language' then the usage, although stipulative, is imprecise because there are facts about language.

Quine's 'grounded in' in '*analytic*, or grounded in meanings' is a misleading spatial metaphor, because if a is grounded in b then b is necessary for a . But meaning, and a proposition meaning what it does, are necessary for any proposition being a truth. So being 'grounded in meanings' does not distinguish analytic propositions from propositions which are not analytic. So this use of 'grounded' does not draw the analytic/synthetic distinction in the *prima facie* way Quine seeks.

Quine says of the expression 'true in all possible worlds': 'Picturesqueness aside, *this is to say that* the truths of reason are those that could not possibly be false' (p. 20) (my italics).

Quine's use of 'this is to say that' is self-refuting. Suppose:

'this'

(1) 'p is true in all possible worlds'

'is to say that'

(2) 'p could not possibly be false'

then in saying (1), (2) is being said, and in saying (2), (1) is being said. Sentences (1) and (2) express the same proposition and so are synonymous. The conjunction of (1) and (2):

(3) 'p is true in all possible worlds and p could not be false'

is analytic, because Quine is right in his view that:

'to say that "bachelor" and "unmarried man" are cognitively synonymous is to say no more or less than that the statement:

(3) All and only bachelors are unmarried men.

is analytic." (p. 28)

Crucially, Quine's 'to say that [...] is to say no more or less than that [...]' presupposes synonymy and therefore analyticity. If 'to say this is to say that' then the conjunction of this and that is analytic. Quine relies on synonymy and analyticity in his attack on synonymy and analyticity and so relies on the notion he rejects. Quine argues against what he presupposes, so if his view is true

it is false, so his view is false (or not true if contradictions are neither true nor false).

Quine's 'this is a distinction which I reject' (p. 43) makes it clear that Quine holds there is no analytic/synthetic distinction. He is not just criticising it as unclear. It is not clear that we should abandon a distinction just because its component concepts resist definition. It is hard to define 'red' or 'blue' phenomenologically but we do not therefore give up the phenomenological distinction between red things and blue things.

Quine makes a mistake about modality when he writes 'could not possibly be false'. What does 'could not possibly be false' add to 'could not be false'? If p is necessary then p could not be false. It is not necessary for p 's being necessary that it not be possible for p not to be possible to be false. Quine needs an argument for $\Box p \rightarrow \Box \Box p$.

When Quine says:

'Just what it means to affirm synonymy, just what the interconnections may be which are necessary and sufficient in order that the two linguistic forms be properly describable as synonymous, is far from clear' (p. 25)

synonymy is presupposed again: 'Just what it means to affirm synonymy' has the form '[...] is just what it means to affirm synonymy', and '[...] is what it means to [...]' is an attribution of synonymy. '[...] is what it means to [...]' is true if and only if the claim that '[...] is [...]' is analytic. Again, Quine's attack on analytic truth presupposes analytic truth.

It is false that being self contradictory depends on the analytic synthetic distinction, even though Quine thinks it true. (p. 20) A contradiction has the logical form: 'both p and not

p' so a contradiction is the conjunction of a proposition and its negation. Concerns about analyticity and synonymy are groundless here because a contradiction is the conjunction of one proposition and its negation. Just one proposition is affirmed and denied so there is no question of whether something can be synonymous with something else. The truth that any proposition is synonymous with itself does not show that the conjunction of a proposition with its negation depends on the analytic/synthetic distinction. There are indeed piecemeal problems of synonymy in the formalising of ordinary language, for example in deciding whether two sentences express the same proposition but these problems are not integral to what a contradiction is and do not show that contradictions depend on the analytic/synthetic distinction.

Quine says:

'a statement is analytic when it is true by virtue of meanings and independently of fact'. (p. 21)

'by virtue of' is unclear here. As I argued above, ' p 's meaning is necessary for p 's truth' does not distinguish an analytic truth from any other truth so I suggest this definition of 'analytic' as superior to Quine's:

'analytic' (def.): p is analytic if and only if p 's meaning is sufficient for p 's truth.

So, if p is analytic, that p means that p is sufficient for p 's truth or, at the level of sentences: if S expresses a proposition that is analytic, that S means that p is sufficient for S 's truth. My view is the opposite of Quine's. If p is analytic, then p 's meaning is sufficient for p 's

truth so, if p is analytic, p 's truth is necessary for p 's meaning. If p is analytic, p 's meaning is 'grounded' in p 's truth. Quine says 'Actually, we do know enough about the intended significance of 'analytic' to know that analytic statements are supposed to be true'. (32) This is right, but precisely because the meaning of an analytic proposition is sufficient for its truth.

Quine says 'of which a general term is true' (p. 21) but this makes no sense because terms cannot be true or false. I suggest: 'to which a general term is correctly applied'. Quine says '"Bachelor" has less than ten letters' (p. 27). This is a confusion of a mass adjective with a count adjective. I suggest: '"Bachelor" has fewer than ten letters'.

Quine thinks understanding 'necessary' presupposes understanding 'analytic':

'"necessarily" [...] Does the adverb really make sense? To suppose that it does is to suppose that we have already made satisfactory sense of "analytic".' (p. 29)

He deploys no argument for this, so suppose being necessary entails being analytic and we have not understood necessity unless we have understood this entailment.

I suggest 'necessary' does not depend on 'analytic'. To say that p is necessary is to say that p could not have had a different truth value from the truth value p has, so:

(NT) p is necessarily true if and only if [p and] not possibly not p

This claim does not entail that p is analytic.
[Even if it were true that all and only necessary

truths are analytic, this would be a fact requiring substantial philosophical argument.?] Nor does anything analytic, nor the negation of anything analytic, follow from 'p is necessarily false':

(NF) *p* is necessarily false if and only if [not *p* and] not possibly *p*

There is a genuine problem about the ontology of modality: What have we said about a portion of what is (linguistic or non-linguistic) when we say:

'This not only *is*, it could not not be'? On a literal view of the universe there is only what there is, so there seems no room for possibilities or impossibilities. One philosophical motive for construing all necessary truths as analytic is to avoid a problem of this sort. The strategy fails, however, because 'could not be otherwise' is as mysterious a property of the truth-value of a proposition as of a portion of non-linguistic reality.

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Quine uses the expression '*analytic for*' (p. 31). I suggest a tacit concession to the contingency of analytic truths here; as though *p* could be analytic for you but not for me, where this does not imply one of us is wrong. We just define words differently.

Quine says: 'Statements may be said simply to be cognitively synonymous when their biconditional [...] is analytic' (p. 31). What is synonymy?

If there is synonymy of sense then the verbal definition of one word is (numerically identical with) the verbal definition of another word. If someone orders: 'Tell me the meaning of *Wasser*' and then orders 'Tell me the meaning of *agua*' then, although we are explaining the meaning of two words, we only have to give one answer and explain that that does both jobs. To tell whether two or more expressions are synonymous: Count how many definitions we need. If the answer is 'One' the expressions are synonymous. If the answer is 'More than one' the expressions are not synonymous. 'Definition' is not used question-beggingly here.

Quine considers the claim 'nothing is red and green all over' (p. 31). This might be an *a posteriori* necessity. No matter how we changed language it is hard to see how something could be red and green all over, yet, that something is red and that something is green are *a posteriori* facts. On the other hand, if *p* is *a priori* if and only if *p* can be known with only enough experience needed

to understand (the concepts in) p , then 'Nothing is red and green all over' is *a priori*.

Quine claims that the whole of science, or the whole of a language, is the unit of truth and falsity. But a distinction is needed between 'What is confirmed or refuted?' and 'What do we need recourse to to confirm or refute something?' The individual sentence is what is confirmed or refuted. That is what we do. Parts of the rest of language or science are invoked, have to exist, for that confirmation or refutation to be possible. It would be something extremely peculiar for the whole of science to be confirmed or refuted. It makes little sense to speak of the whole of a language being confirmed or refuted because a language is not a set of consistent claims or a theory. Rather, it is a repository of meaning possibilities in which claim and counter claim can be made.

Quine's draws a relativist conclusion from his attack on the on the analytic/synthetic distinction: It is inappropriate to hold some propositions true come what may but others open to revision of truth value, or at least, such revision or lack of it is culturally and conceptually specific.

Quine thinks a consequence of the vacuousness of the analytic/synthetic distinction is that phenomenalism is false and relativism is true. Because there is an analytic/synthetic distinction Quine has proven neither conclusion.

Phenomenalism is false, not for the reason Quine gives, but because no claim about physical objects *qua* physical objects is logically equivalent to a claim about sense contents *qua* sense contents. 'Exists independently of being perceived', for example, resists translation and so reduction.

Relativism is false because self-refuting. 'Everything is true' entails that 'Relativism is false' is true, so if relativism is true it is false. Therefore relativism is false or, if contradictions are neither true nor false, relativism is neither true nor false so not true. If there is truth, some things are true and some things are false. Giving up logical laws produces contradictions and so falsity or nonsense. Refusing to give up a proposition as true does not make it a necessary truth, or even a truth.

ANALYTIC - SYNTHETIC DISTINCTION

Definitions of 'analytic':

- A.1. Any analytic proposition is true, but not all true propositions are analytic.
- A.2. The meaning of an analytic proposition is sufficient for its truth.
- A.3. The truth of an analytic proposition is necessary for its meaning.
- A.4. An analytic proposition is true by definition.
- A.5. An analytic proposition is true in virtue of the meaning of its constituent terms.
- A.6. The negation of an analytic proposition is a contradiction.

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A.7. In an analytic proposition the predicate is 'contained' in the subject.

A.8. Analytic propositions provide no information about non-linguistic reality.

Definitions of 'synthetic':

S.1. Any synthetic proposition is either true or false
but not all propositions are synthetic.

S.2. The meaning of a synthetic proposition is insufficient for its truth.

S.3. The truth of a synthetic proposition is not necessary for its meaning.

S.4. A synthetic proposition is true or false but not by definition.

S.5. A synthetic proposition is true or false but not in virtue of the meaning of its constituent terms.

S.6. The negation of a synthetic proposition is internally consistent.

S.7. In a synthetic proposition the predicate is not 'contained' in the subject.

S.8. Synthetic propositions may provide information about non-linguistic reality.

A Note on the Logic of Empiricism

I define 'empiricism' as the thesis that there can be no *a priori* metaphysical propositions and all knowledge depends on empirical knowledge.

Empiricists typically hold that all and only necessary truths are analytic and all and only contingent propositions are *a posteriori*. Mill is not typical because he thinks *a priori* claims are covertly empirical and contingent.

To be clearer on what empiricism, and why it is inconsistent with rationalism, I draw three distinctions in philosophical logic: analytic/synthetic, *a priori* /*a posteriori*, and necessary/contingent. These six terms are used to classify propositions in six ways. A proposition is what is expressed by an indicative sentence or what is true or false if a sentence is true or false.

'*p*' stands for any proposition, '*q*' stands for any proposition distinct from *p*, '*r*' stands for any proposition distinct from *p* and *q*.

Definitions of 'analytic':

(1) *p* is analytic if and only if *p* is true by definition.

(2) *p* is analytic if and only if *p* is true 'in virtue of' its meaning

(3) *p* is analytic if and only if *p* is true irrespective of how the world is.

Definitions of 'synthetic':

(1) *p* is synthetic if and only if *p*, if true, is not true by definition.

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(2) *p* is synthetic if and only if *p*, if true, is not true 'in virtue of' its meaning.

(3) *p* is synthetic if and only if, if *p* is true, *p* is true because of how the world is.

Definitions of '*a priori*':

(1) *p* is *a priori* if and only if *p*'s truth value may be decided independently of empirical observation.

(2) *p* is *a priori* if and only if *p*'s truth value may be decided purely intellectually.

(3) *p* is *a priori* if and only if *p*'s truth value is decidable with no more experience than that required to understand *p*.

Definitions of '*a posteriori*':

(1) *p* is *a posteriori* if and only if *p*'s truth value may be decided only through empirical observation.

(2) *p* is *a posteriori* if and only if *p*'s truth value may not be decided purely intellectually (but requires experience).

(3) *p* is *a posteriori* if and only if *p*'s truth value is decidable only with more experience than that required to understand *p*.

Definition of 'necessary':

(1) *p* is necessary if and only if, if *p* is true *p* could not have been false.

Definition of 'contingent':

(1) p is contingent if and only if, if p is true p could have been false.

If empiricism entails there are no *a priori* metaphysical propositions, what is their logical and epistemological status? Empiricists either allocate reason a practical application in making the objects of experience intelligible or construe logic as a purely formal system of tautologies. For example, Hobbes uses and extends geometrical method to model the physical world, construing human beings and their politics as wholly physical. This gives geometrical reasoning an empirical application that partly anticipates Kant's construal of the sentences of geometry as synthetic *a priori*.

Locke thinks *a priori* propositions (such as 'Lead is a metal') are 'trifling'. He means p is trifling if and only if perceiving the truth of p is a necessary condition for understanding p , so p is trifling if and only if p is self evident. Hume thinks 'relations of ideas' are always and everywhere distinct from 'matters of fact', so the conceptual relations expressed by analytic truths cannot provide information about non-linguistic reality. Russell thinks 'Mathematics [...] consists of tautologies.' (69)

Mill's view that logic is empirical and contingent is a position that most philosophers of logic and mathematics would find quite extraordinary. The sentences of logic and mathematics which we count as necessary truths Mill thinks could in principle admit of inductive falsification. It is a logical consequence of Mill's view that, perhaps one day, we will find a circle that is a square (rather than just find

something that looks circular from one angle but square from another), or meet someone who is taller than himself (without his changing height) or discover that the proposition expressed by the sentence ' $1+1=2$ ' is false. Mill admits that it is overwhelmingly psychologically compelling that putatively necessary truths are true and could not be false, especially simple ones. His explanation for this is also empirical. So far, we have encountered only many instances of their confirmation and no instance of their refutation. This, however, is consistent with their being false. This reduction of deduction to induction is a confining of the rational to the empirical.

Empiricist construals of the *a priori* have in common the ruling out of *a priori* metaphysical knowledge. This is inconsistent with the rationalist thesis that transcendent truth may be discovered by thinking.

Is logic empirical? Empiricist logic: a contradictory concept. If anything is empirical it is not logic. If anything is logic it is not empirical. There is no empirical refutation of logic. There is no empirical origin of logic. There is no empirical acquisition of logic. Suppose someone set about denying these claims. Someone might read Zeno and say 'Something can both at a place and not be at that place after all'. 'I saw something moving. This, if anything is, is an empirical refutation of logic'.

ARITHMETICAL TRUTH

Arithmetic is not analytic because not 'true by definition' or 'true in virtue of meaning'.

12 is essentially constituted by $5+7$.

12 is not (identical to) $5+7$, because 12 and $5+7$ do not have all and only the same properties.

$5+7 = 12$ is synthetic because ' $5+7$ ' expresses a combination of two numbers and because it conveys the information that that combination (sum) constitutes 12.

$5+7=12$ is a priori because it is decidable purely intellectually.

$5+7=12$ is a posteriori because someone could tell me this fact, or I could read it off a computer print out. (Here 'a posteriori' does not mean 'knowable only through (sense) experience' but 'knowable through sense experience'. So $5+7=12$ is both a priori and a posteriori.

$5+7=12$ is a necessary truth because 12 could not have been 12 without being constituted by $5+7$ and 5 and 7 could not have been 5 and 7 unless their sum constituted 12.

12 has an infinite number of essential constituents.

$7+5$ and 12 differ in sense but not in number

The sense and reference of a number.

The sense and reference of a numeral.

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POSSIBILITY AND NECESSITY

Definitions of 'Necessary Truth':

P is a necessary truth if and only if P is true and could not be false.

P is a necessary truth if and only if the negation of P is a contradiction.

P is a necessary truth if and only if P is true in all possible worlds.

Definitions of 'Necessarily False':

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P is necessarily false if and only if P is false and could not be true.

P is necessarily false if and only if the negation of P is a tautology.

P is necessarily false if and only if P is false in all possible worlds.

Logical Possibility:

A state of affairs is logically possible if and only if it has a consistent description.

A state of affairs is logically impossible if and only if it has an inconsistent description.

Self-Evidence:

P is self-evident if and only if knowing P's truth value is a necessary condition for understanding P.

P is self-evident if and only if understanding P is a sufficient condition for knowing P's truth value.

P is self-evidently true if and only if knowing that P is true is a necessary condition for understanding P.

P is self-evidently true if and only if understanding P is a sufficient condition for knowing that P is true.

P is self-evidently false if and only if knowing that P is false is a necessary condition for understanding P.

P is self-evidently false if and only if understanding P is a sufficient condition for knowing that P is false.

The idea that what is is divided into two mutually exclusive and collectively exhaustive portions is a pervasive assumption not only of different kinds of philosophy, physics and theology but also that historically fluctuating body of belief and life-style known as 'common sense'. The idea is that part of what is not only is but could not not be and could not be other than how it is while the remaining part of what is, although it is and is how it is, could have not been and could have been other than how it is.

For example, the philosophical thesis that some propositions could not have the truth values they do have or that there are certain objects which could not lack certain of the properties they possess are symptomatic of this bifurcation. So too is the thesis of physics that certain events are necessitated by natural laws and the theological view that there exists a being who could not not exist. Finally, the pre-theoretical view that some things are inevitable or fated and the pseudo-scientific view that history has an inexorable destination all depend too on the dualism between what has to be and what could not be.

With few exceptions, philosophers have traditionally maintained that the distinction between necessity and contingency admits of no empirical explanation - which is nearly as good as admitting that it has no explanation. My aim is to provide an explanation of necessity which meets the following requirements:

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(i) it is recognised that necessity exists - explaining is not to mean explaining away in this context.

(ii) the explanation is to be empiricist- it explains what is not observable in terms of what is observable

(iii) the explanation is shown to be more likely to be true than false, and more likely to be true than competitors.

PROPOSITIONS, TRUTH, CONSISTENCY

P.1. A proposition is what is expressed by an indicative sentence.

P.2. A proposition is either true or false but not both.

P.3. The letter 'P' stands for any proposition.
'Q'
stands for any proposition distinct from P,
'R' for
any proposition distinct from both P and Q,
etc.

P.4. If P is true then P is not false.

P.5. If P is false then P is not true.

C.1. If the truth of P does not preclude the truth of Q
and if the truth of Q does not preclude the truth
of P then P and Q are consistent.

C.2. If the truth of P precludes the truth of Q or
if
the truth of Q precludes the truth of P, or
both,
then P and Q are inconsistent.

N.1. If the truth of P is a necessary condition
for the
truth of Q then if not P then not Q (ie if P
is not
true then Q is not true).

S.1. If the truth of P is a sufficient condition
for the
truth of Q then if P then Q (ie if P is true
then Q
is true).

NS.1. If the truth of P is both necessary and
sufficient
for the truth of Q then Q if and only if P.

NS.2. If P is a necessary condition for Q then Q is
a
sufficient condition for P.

NS.3. If P is a sufficient condition for Q then Q
is a
necessary condition for P.

RELATIONS

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A relation, 'R', is transitive if and only if, if a is related by R to b, and if b is related by R to c, then a is related by R to c.

A relation, 'R', is intransitive if and only if, if a is related by R to b, and if b is related by R to c, then a is not related by R to c.

A relation, 'R', is symmetric if and only if, if a is related by R to b, then b is related by R to a.

A relation, 'R', is asymmetric if and only if, if a is related by R to b, then b is not related by R to a.

A relation, 'R', is reflexive if and only if, a is related by R to a.

A relation, 'R', is irreflexive if and only if, a is not related by R to a.

A relation, 'R', is an equivalence relation if and only if R is transitive, symmetric and reflexive.

TYPES AND TOKENS, IDENTITY

Types and Tokens

a and b are of the same type if and only if a is like b in at least one respect.

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a and b are of only the same type if and only if a is like b in every respect.

a and b are of the same type if and only if a and b are qualitatively similar.

a and b are of only the same type if and only if a and b are qualitatively identical.

a and b are the same token if and only if a is b.

a and b are the same token if and only if a is the same one as b.

a and b are the same token if and only if a is numerically identical to b.

Identity

a is numerically identical to b if and only if a is b.

a is numerically identical to b if and only if a is the same one as b.

a is qualitatively similar to b if and only if a is like b in at least one respect.

a is qualitatively identical to b if and only if a is like b in every respect.

a is qualitatively similar to b if and only p if a and b are of the same sort (kind, type).

Numerical Identity as an Equivalence Relation

The relation 'is numerically identical to' is transitive, symmetrical and reflexive, therefore it is an equivalence relation.

Transitivity of Numerical Identity

If a is numerically identical to b and b is numerically identical to c , then a is numerically identical with c .

Symmetry of Numerical Identity

If a is numerically identical to b then b is numerically identical to a .

Reflexivity of Numerical Identity

a is numerically identical to a .

VALIDITY AND SOUNDNESS

Argument

An argument is a set of propositions such that some (the premises) are putative evidence for another (the conclusion).

Any proposition in an argument is either a premise or the conclusion.

If p is a premise then p is putative evidence for the conclusion.

If p is a conclusion then p is putatively proved by the premises.

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Any argument is either a deductive argument or an inductive argument but not both.

It makes no sense to talk of arguments as true or false.

Propositions are either true or false.

Any deductive argument is either valid or invalid but not both.

If an argument is valid then the truth of the premises is inconsistent with the falsity of the conclusion.

If an argument is invalid then the truth of the premises is consistent with the falsity of the conclusion.

If an argument is valid then if the premises are true then the conclusion is true.

If an argument is invalid then even if the premises are true, the conclusion could still be false.

Any deductive argument is either sound or unsound but not both.

If an argument is sound then it is valid and its premises are true.

If an argument is unsound then either it is invalid, or at least one premise is false, or both.

If an argument is sound then it is valid.

If an argument is valid then it is either sound or unsound.

All the propositions in a sound argument are true.

DEFINITION, SENSE AND REFERENCE

Definition

Every definition is either verbal or ostensive.

A word has a verbal definition if the meaning of that word may be explained in other words.

A word has an ostensive definition if the referent of the word may be indicated as its meaning.

The defining characteristics (or essential properties) of a thing are the features necessary for it to be the sort of thing it is (or for it to belong to a certain type or class).

The defining characteristics (or essential properties) of x are singularly necessary but jointly sufficient for x 's being F .

There is a family resemblance between the members of a class when no single property is common to all and only the members of that class, yet any two members of the class have at least one property in common.

Sense and Reference

A term has a sense if and only if it has a verbal definition.

A term has a reference if and only if it denotes something.

Two or more terms may have the same sense but different referents.

Two or more terms may have the same referent but different senses.

Having a sense is neither necessary nor sufficient for having a referent.

Having a referent is neither necessary nor sufficient for having a sense.

Use and Mention

When a word is mentioned, it is placed between single quotation marks; ' '. When a word is used the single quotation marks are omitted.

DOES PEGASUS EXIST?

Refute Quine 'On What There Is'

EPISTEMOLOGICAL EXTREMISM

Two putative epistemologies are both mutually inconsistent and false: extreme scepticism and extreme relativism. They are defined as follows:

(1) Extreme scepticism is true if and only if every proposition is false.

(2) Extreme relativism is true if and only if every proposition is true.

Any putative scepticism or relativism according to which some propositions are true and some false is not thereby different from a non-sceptical or non-relativist epistemology. Such local scepticism and local relativism are only distinguishable from each

other and from non-sceptical and non-relativist epistemologies through what is held true and what is held false.

The putatively relativist view that propositions held true are 'true for' whoever holds them is not distinctively relativist because it entails and is entailed by the ordinary belief that different people believe different propositions. If 'P is true for x' does not mean 'x believes P' it means nothing. The relativist view that any proposition held true is thereby true is false because with some exceptions (such as 'I hold at least one belief') beliefs are not sufficient for their own truth, even though every belief is sufficient for its own truth or falsity, every believed truth is sufficient for its own truth and every believed falsehood is sufficient for its own falsehood.

The putatively sceptical view that any belief could be false is true, so long as it is not construed as entailing that true beliefs may be false, but it is not usefully termed 'sceptical'. If 'could' is a logical 'could' here it amounts to the fact that it is not incoherent to suppose that no belief is immune to error. This scepticism is so weak that even someone who was confident that everything believed is true could consistently subscribe to it.

Extreme scepticism and extreme relativism are mutually inconsistent because if it is true that every proposition is false then it is false that every proposition is true and if it is true that every proposition is true then it is false that every proposition is false. It follows that at most one of scepticism and relativism is true. In fact, they are both false.

Scepticism is false because it is directly self refuting. If it is true it is false, so it is false. This is because if scepticism is true then

every proposition is false but, if that is true, then the proposition that every proposition is false is false. Here I assume that, if a proposition is true then false then it is false because contradictory, not nonsensical because contradictory.

Relativism is false because it is refutable on extremely weak assumptions. For example, for every proposition there exists the negation of that proposition and at least one of a proposition and its negation is false. It follows that there exists at least one falsehood, so relativism is false.

If a proposition is true then its negation is false. If relativism is true then its negation is false, but if the negation of relativism is false then relativism is false because if its negation is false there exists at least one false proposition.

Nevertheless, there exists an important asymmetry between scepticism and relativism. Scepticism is directly self-refuting but relativism is refutable on weak assumptions. The sceptical claim:

(1*) Every proposition is false.

is shown to be directly self-refuting as soon as its truth is asserted:

(1**) The proposition that every proposition is false is true.

(1*) and (1**) are mutually inconsistent and so (1*) is an exception to itself.

On the other hand, the relativist claim

(2*) Every proposition is true.

is not directly self-refuting in a parallel way because it is not inconsistent with the assertion of its truth:

(2**) It is true that every proposition is true.

(2*) is not an exception to itself.

In order to refute scepticism we do not have to make the assumption that there are true propositions, we simply apply the theory to itself. In order to refute relativism we do have to make the assumption that there are false propositions. However weak, this assumption is one which will seem to the relativist to beg the question.

ESSENTIALISM

Kripke:

Distinguish:

(1) Belief in modality *de dicto*

(2) Belief in modality *de re*

Essentialism is (2) (NN)

'the problem of essentialism':

'x' eg 'a lecturn'

'What are its essential properties?'

ie.

(1) 'What properties [...] are such that this object has to have them if it exists at all?'

and are

(2) '[...] such that if an object did not have it [them], it would not be this object?' (IN 86)

Kripke considers a particular object (a token) not a sort of object (a type, a universal).

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(1) Properties: F , G , such that only if a is F , G ... may a exist.

(2) If x does not have F , G ... then $-(x = a)$.

(or, if a is not F , G ... then $-(\exists x)$.)

(Russell's $E!$)

$\neg \exists x(Fx) > \neg (\exists a)(x=a)$

$\neg E!x(Fx) > \neg (E!a)(x=a)$

2 kinds of essentialism:

(1) Being F is a necessary condition for being G .
(type essentialism).

(2) Being F is a necessary condition for being x .
(token essentialism).

Kripke is a token essentialist.

(1) 'Being made of wood, and not of ice, might be an essential property of this lecturn' (IN 86-7)
or (a weaker claim)

(2) 'It is not made of ice' (IN 87)

(1) and (2) are putative examples of essential properties of the lecturn.

(1) The lecturn is made of wood.

(2) Could the lecturn have been made of ice?

(2) is an essentialist question.

Kripke's answer is No. Then it would not have been *that* lecturn. This *particular* one is made of wood. Kripke accepts that a lecturn could have been made of ice, frozen Thames water say, and could have been placed where this lecturn is. Kripke says 'one would have made of course, a different object.'

(IN 87)

2 kinds of difference:

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(1) numerical difference
(2) qualitative difference
logically depend on two kinds of identity:

(1) numerical identity
(2) qualitative identity

eg:

(1) If a is numerically identical with b then a is the same one as b .

ie. a is b .

(2) If a is qualitatively identical with b then a is (of) the same the same sort as b . ie. a the same kind of thing as b .

Note these logical relations:

(1) If a is numerically identical with b then a is qualitatively identical with b .

(2) If a is qualitatively identical with b it does not follow that a is numerically identical with b .

Kripke thinks

(1) The lecturn made of ice is qualitatively different from the lecturn made of wood.

therefore

(2) The lecturn made of ice is numerically different from the lecturn made of wood.

Is this valid?

Is it true that from: $\neg(a = \text{the same } F \text{ as } b)$ it follows that $\neg(a = b)$?

Suppose a is made of ice.

Suppose b is not made of ice.

then a is F , b is not F ,

then $\neg(a=b)$

so a contradiction is obtained: ' a is F and a is not F '

or $\text{Ex}(Fa \text{ and } \neg Fa)$ which is a contradiction.

'It would not have been this very lecturn.' (ID 87)

Why not?

(1) This lecturn is not made of ice. $\text{Ex } \neg(Fa)$

(2) There is a lecturn made of ice. $\text{Ex}(Fa)$

(3) The two lecturns are not (numerically) identical. $\neg(a=b)$

Kripke thinks a qualitative difference is logically sufficient for a numerical difference: $(x) (y)$

$(Fx \text{ . } \neg Fy) > \neg(x=y)$

Distinguish:

a may be F , G ...

a may not be both F and not F .

'this very object' is not made of ice.

This very object could not have been made of ice or it would not have been this very object.

Therefore: not being made of ice is an essential property of this object.

Contingent properties:

Kripke's lecturn:

'It could have been in another room from the room it is in fact in' (IN87)

'even at this very time'

'It could not have been made from the very beginning from water frozen into ice' (IN 87)

Are origin and composition essential?

Are spatio-temporal location and shape contingent?

* NN: Saul Kripke *Naming and Necessity* (Blackwell)

** IN: Saul Kripke 'Identity and Necessity' in
Stephen Schwartz (ed.) *Naming, Necessity and
Natural Kinds* (Cornell)

'Self-
Presentation'

Frege says

'Everyone is presented to himself in a special and
primitive way, in which he is presented to no-one
else' (12) (1)

What is this way ?

How is it known that Frege's claim is true ?

Is it true ?

Why does this mode of presentation exist ?

'this primitive way in which he is presented to
himself' (13)

Unless otherwise stated references are to
'Thoughts' in Gottlob Frege *Logical Investigations*
edited with a preface by P.T. Geach, translated by
P.T. Geach and R.H. Stoothoff (Oxford, Basil
Blackwell, 1977). The translators have 'thoughts'
for 'der Gedanke' even though 'der Gedanke' is
singular. The Quintons' translation 'The Thought'
is literal.

The use of indexicals rests on metaphysical
presuppositions. There are indexical beings as
well as indexical expressions.

Frege's Circle

In 'The Thought: A Logical Inquiry' Frege attacks the correspondence theory of truth in this way:

'Can it not be laid down that truth exists when there is correspondence in a certain respect? But in which? For what would we then have to do to decide whether something were true? We should have to inquire whether it were true that an idea and a reality, perhaps, corresponded in the laid down respect. And then we should be confronted by a question of the same kind and the game could begin again. So the attempt to explain truth as correspondence collapses.'

and concludes that it is probably impossible to define 'truth':

'And every other attempt to define truth collapses too. For in a definition certain characteristics would have to be stated. And in application to any particular case the question would always arise whether it were true that the characteristics were present. So one goes around in a circle. Consequently, it is probable that the content of the word 'true' is unique and undefinable.' (1)

I suggest that Frege has assimilated two kinds of question which are distinct. The cluster: 'What is truth?', 'What is the meaning of the word true?', 'What is the definition of 'truth'?' needs to be sharply distinguished from the cluster: 'How can p be known (to be true)?', 'How may p be verified (or falsified)?'. Questions of the first sort belong to semantics and metaphysics, questions of the second sort to epistemology.

Suppose by Frege's 'truth exists when there is correspondence in a certain respect' entails:

(1) 'p' is true if and only if p

and suppose we understand by Frege's 'We should have to inquire whether it were true that an idea and a reality, perhaps, corresponded [...]' as entailing:

(2) $\neg \Diamond Kx ('p' \text{ is true if and only if } p) \rightarrow \neg ('p' \text{ is true if and only if } p)$

It is false that the truth of (2) is a necessary condition for the truth of (1) (or, equivalently, it is false that (1) entails (2)). No contradiction is entailed by the supposition that (1) is true but (2) false. In general, it is false that $\Diamond Kpx$ is a necessary condition for the truth of p .

Frege's Circle is generated by an extreme anti-realist intuition that stands in need of argument: p cannot be true unless it could be discovered what the truth of p consists in. This is a much stronger claim than the thesis that p cannot be true unless p could in principle be shown to be true, but is easily shown to follow from it by substituting (1) for p in $\neg \Diamond Kxp \rightarrow \neg p$.

Although Frege is right that a regress is generated by the view he is considering, it is generated anyway by $\neg \Diamond Kxp \rightarrow \neg p$ simply by substituting Kxp for 'p'. The regress entails that knowledge is impossible, a sceptical and counter intuitive conclusion.

'The Thought: A Logical Inquiry' in Simon Blackburn and Keith Simmins (eds.) *Truth* (Oxford, 2003) p. 87

DEDUCTION AND INDUCTION

Any argument is either deductive or inductive but not both.

Deduction

In a deductive argument, putatively, if the premises are true then the conclusion is true.

In a valid deductive argument, if the premises are true then the conclusion is true.

In a valid deductive argument the truth of the premises is inconsistent with the falsity of the conclusion.

In an invalid deductive argument, it is false that if the premises are true then the conclusion is true.

In an invalid deductive argument, the truth of the premises is consistent with the falsity of the conclusion.

Any deductive argument is either valid or invalid but not both.

Induction

In an inductive argument, putatively, if the premises are true then the conclusion is probably true.

In a strong inductive argument, if the premises are true then the conclusion is probably true.

In a weak inductive argument, it is false that if the premises are true then the conclusion is probably true.

Any inductive argument is either strong or weak but not both.

In any inductive argument the truth of the premises is consistent with the falsity of the conclusion.

EXISTING

WHAT DOES 'EXISTS' MEAN?

'Exists' only has one meaning. It is not true that there is mathematical existence, physical existence, political existence, mental existence and so on. It is true that there are numbers, physical objects, governments, minds and so on. To say that these exist in different sense is to confuse two questions: 'What is it?' and 'Is it?'. The answer to 'Is it?' is 'Yes' or 'No', or 'It exists' or 'It does not exist'. The answer to 'What is it?' is 'It has mathematical properties' or 'It is something with the capacity to think'. Radically different kinds of thing exist in exactly the same sense of 'exist'. If you say 'Numbers exist' and 'Trees exist' you are using 'exist' with exactly the same meaning both times. The fact that numbers are not much like trees does not show that this is false. Each thing's being what it is is not what its being consists in.

Existing is a property, a property that nothing lacks, so 'exists' is a predicate. Existing is a type to which everything belongs. Saying that something exists does add to the concept of that thing. A concept user's concept of a thing is enriched if they learn that the thing exists. If a

concept of something is an ability to imaginatively discriminate it from what it is not, then this ability is enhanced by the news that the thing exists. Fictional things all exist, even though they are fictional. If they did not exist it could come as news to someone that they do not exist, that they are fictional

If we imagine something we imagine it as though it were existing. Fictional beings all exist because 'exists' does not mean 'Is a publically available four dimensional object'. Story books and people's imaginations would be very different if fictional objects did not exist. Fictional objects have that which nothing lacks. For this reason, it is hard to imagine something that does not exist. 'Exists' is a first level predicate that refers to existence. Is non-existence a property?

Is '...does not exist' only predicated of what does exist if existing is a property? What lacks the property of existing? 'Does not exist' is true of nothing, if it were true of something, it could not be true. 'Exists' is true of everything. There are only the things that exist. Only some of these are actual. There is no such thing as things that do not exist. They exist as possibilities. Only some of the things that exist are actual. 'Exists' means the same whether predicated of types or individuals. For example, 'Socrates does not exist' is false. 'Socrates is dead' is true. Socrates is not 'actual' is true. 'Socrates is not our contemporary' is true. 'Does not exist' is a predicate that is false of everything, a fundamental first order predicate. 'Does not exist' is false of everything that exists. Everything that exists is everything, so 'Does not exist' is false of everything. There is only what there is, so there is nothing to predicate 'Does not exist' of, so 'Does not exist' is not true of anything.

Existing is not a category. If something exists, does it have (other) properties. Although 'exists' is true of everything, it is not necessarily true because the things that exist might not have existed. The necessity of the entailment from 'This exists' to 'This exists' does not show that 'This exists' is necessary. Being is the source of beings but God is the source of being. Things exist contingently but God exists necessarily.

Is 'Exists' Equivocal?

'Exists' only has one meaning. It is not true that there is mathematical existence, physical existence, political existence, mental existence and so on. It is true that there are numbers, physical objects, governments, minds and so on. To say that these exist in different senses is to confuse two questions: 'What is it?' and 'Is it?'. The answer to 'Is it?' is 'Yes' or 'No', or 'It exists' or 'It does not exist'. The answer to 'What is it?' is 'It has mathematical properties' or 'It is something with the capacity to think'. Radically different kinds of thing exist in exactly the same sense of 'exist'. If you say 'Numbers exist' and 'Trees exist' you are using 'exist' with exactly the same meaning both times. The fact that numbers are not much like trees does not show that this is false. Each thing's being what it is is not what its being consists in.

Is Existing a Property?

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thing. A concept users concept of a thing is enriched if they learn that the thing exists. If a concept of something is an ability to imaginatively discriminate it from what it is not, then this ability is enhanced by the news that the thing exists.

Is 'Exists' a Predicate?

What is a predicate? A predicate is what remains of an indicative sentence when its grammatical subject is deleted. By this criterion, the deletion of 'x' from 'x exists' leaves 'exists' as a predicate. 'exists' appears a strange predicate because it is not an empirical predicate.

Do Fictional Objects Exist?

Fictional things all exists, even though they are fictional. If they did not exist it could come as news to someone that they do not exist, that they are fictional. If we imagine something we imagine it as though it were existing. This is a necessary condition for imagining something rather than nothing, in the sense of imagining having a content. Fictional beings all exist because 'exists' does not mean 'Is a publically available four dimensional object'. Story books and people's imaginations would be very different if fictional objects did not exist. Fictional objects have that which nothing lacks. For this reason, it is hard to imagine something that does not exist.

'Exists' is a first level predicate that refers to things that exist.

So, is not-existing a property? Is '..does not exist' only predicated of what does exist if existing is a property? What lacks the property of

existing? 'Does not exist' is true of nothing, if it were true of something, it could not be true.

Does Everything Exist?

'Exists' is true of everything. There are only the things that exist. Only some of these are actual. There is no such thing as things that do not exist. They exist as possibilities.

Is Existing Being Actual?

Only some of the things that exist are actual. 'Exists' means the same whether predicated of types or individuals. For example, 'Socrates does not exist' is false. 'Socrates is dead' is true. 'Socrates is not actual' is true. 'Socrates is not our contemporary' is true. 'Does not exist' is a predicate that is false of everything, a fundamental first order predicate. 'Does not exist' is false of everything that exists. Everything that exists is everything, so 'Does not exist' is false of everything. There is only what there is, so there is nothing to predicate 'Does not exist' of, so 'Does not exist' is not true of anything. Existing is not a category. If something exists, does it have (other) properties. Although 'exists' is true of everything, it is not necessarily true because the things that exist might not have existed.

Is Existence a Property?

No.

'Existence is essentially a property of a propositional function. It means that the propositional function is true in at least one

instance.' Bertrand Russell *Logic and Knowledge*
(Unwin, London, 1971) p. 232

'[...] to be is to be the value of a variable' W.V.O.
Quine 'Designation and Existence' *The Journal of*
Philosophy XXXVI 1939, pp. 707-8

'Socrates is', '(Ex) (Socrates = x)'.

'actuality' and the 'there-is' uses of 'is'.

What is the meaning of the existential quantifier?

Rewrite in terms of existing and existence:

Existence and nothing are opposites. Existence
lacks nothing. Nothing lacks existence.

If something is then there is at least one of it
and if something is not then there is none of it.
However, the existence of the things that are is
not countable in any ordinary sense. The existence
of what is is an absolute one (or 'unity' in the
true sense). An absolute one does not admit even
the logical possibility of being any other number.

Existence entails time without duration, or
presence. Existence is not abstract but concrete.

The question 'What is existence?' is not the
question 'What exists?'. The answer to 'What
exists?' is a list of all and only the types of
things there are. If some list is right, the
question 'What is existence?' has not been
answered. To answer 'What is existence?' is to show
the difference between being on the right list and
off it. Suppose God, physical objects, numbers,
acts of political aggression, minds and
mathematical points exist. What is thereby true of

them? What do these have in common entailing existence?

Being is not being something. Being is not being physical, perceptible, mathematical or political. Being something is not what being consists in. Being consists in being rather than not being not being something rather than something else.

Not being is not being something or not being anything. Not being is not existing.

Nevertheless, if something is identical with something that exists then it exists, even though its being identical with something is not what its being consists in.

Being is that which nothing lacks. Being is the property that nothing lacks. This is true both in the sense that it is not the case that there is anything that lacks being and in the sense that nothing, in the sense of nothing at all, does lack being. It is then contradictory to say 'There is nothing', unless this means 'It is not the case that there is anything'. It is also contradictory to say 'There are non-existent objects' or 'There are things that do not exist'. These entail 'There exist objects that do not exist' which is contradictory because it entails that 'There are objects' is both true and false.

Being is not becoming, even if everything that there is is in a state of becoming, because becoming is not what being rather than not being consists in. Becoming is the transition between being and not being and between not being and being. The transition is hard to think because we think of things as being. It can be experienced.

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Being does not change. Although the things that are change the being of the things that are does not change. Being cannot begin to be or cease to be, even though the things that are begin to be and cease to be. Being is immortal. Being makes truth possible. Because the things that are are, it is true that the things that are are. There is truth and falsity because of what is and what is not, but what is could not be unless it had that which nothing lacks. Change, beginning and ending, truth and falsity, are properties of the beings that are, not the being of the beings that are.

Being is pure, not empirical. Being is like invisibility. An invisible object could be detected by the sense of sight if someone were seen to collide with it. It could be seen *that* it exists but it could not be seen. You can see, hear, taste, smell or feel *that* something is but you cannot directly sense its being. Being is an invisible property. You point out that empirical things exist by pointing out those things, but this is only to point out that they exist, not to point to their existence.

The being of what is whatever is may be found within. You are at the centre of being. You are the interiority of being.

There is nothing outside of being, not even nothing, because being is an inside without an outside. (Rather as the surface of a physical object is an outside without an inside.)

Being is the being of God. In creation, God creates being and lends being to make the things that are be.

Superficially or empirically, we know what being is. If something differed from itself it could not exist. The use of the existential quantifier enshrines formally the doctrine that 'exists' is not a predicate but 'exists' is a

predicate nothing lacks. 'Existence is the fundamental first-level property. 'Exists' is the fundamental first-level predicate.

Being something depends up being. If something that exists has a property then that property exists. To talk of predicates is to talk of existence: the existence of properties. In 'something is' and 'something is physical' 'is' has the same meaning. To talk of numerical identity is to talk of existence. To say 'a is b' is to say that one thing exists here and can be truly named 'a' or truly named 'b'.

'Being' in the sense that designates being is not a count noun. 'Being' which admits of prefacing by 'a', or of a plural, is a count noun.

Being is a type to which everything belongs.

P is logically possible if and only if God could bring it about that p.

All possibilities exist, in the mind of God. The totality of possibilities is the mind of God. God actualises some possibilities.

Why is there anything?

The Being of God.

Being necessarily is. It is contradictory to say being is not. The necessity of being is the necessity of God's being. Beings exist contingently. Being exists necessarily. There could have been pure being, but only self-sustaining. Pure being would have to be God. The properties of pure being are the properties of God.

Being is timeless. Beings are temporal.

Being is not physical.

Being is one.

Being is necessary for beings.

Being is indivisible.

Is this pantheism? No: God is not the beings that are, because the being of the beings that are is not the beings that are.

The necessity of the entailment from 'This exists' to 'This exists' does not show that 'This exists' is necessary.

Being is the source of beings but God is the source of being. Things exist contingently but God exists necessarily.

Being and nothing are opposites. Being lacks nothing. Nothing lacks being. If something is then there is at least one of it and if something is not then there is none of it. However, the being of the things that are is not countable in any ordinary sense. The being of what is is an absolute one (or 'unity' in the true sense). An absolute one does not admit even the logical possibility of being any other number.

Being entails time without duration, or presence. Being is not abstract but concrete.

The question 'What is being?' is not the question 'What exists?'. The answer to 'What exists?' is a list of all and only the types of things there are. If some list is right, the question 'What is being?' has not been answered. To answer 'What is being?' is to show the difference between being on the right list and off it. Suppose God, physical objects, numbers, acts of political aggression, minds and mathematical points exist.

What is thereby true of them? What do these have in common entailing existence.

Being is not being something. Being is not being physical, perceptible, mathematical or political. Being something is not what being consists in. Being consists in being rather than not being not being something rather than something else. Not being is not being something or not being anything. Not being is not existing. Nevertheless, if something is identical with something that exists then it exists even though its being identical with something is not what its being consists in.

Being is that which nothing lacks. Being is the property that nothing lacks. This is true both in the sense that it is not the case that there is anything that lacks being and in the sense that nothing, in the sense of nothing at all, does lack being. It is then contradictory to say 'There is nothing', unless this means 'It is not the case that there is anything'. It is also contradictory to say 'There are non-existent objects' or 'There are things that do not exist'. These entail 'There exist objects that do not exist' which is contradictory because it entails that 'There are objects' is both true and false.

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A Short History of Modality

'Water is H₂O'

Meaningless

Unknown

Necessarily False

Contingently false

Contingently true

Necessarily true a priori

Necessarily true a posteriori

Water is not H₂O. Water is made out of H₂O. Nothing is what it is made out of.

But: destroy the statue but lump of bronze survives.

Destroy the H₂O the water does not survive.

Existence of H₂O is nec and suff for the existence of water.

IS SELF-IDENTITY A PROPERTY?

Does

(x) (x = x)

ascribe any property to x ? If it does then a famous argument against the possibility of there being any contingent identity statement would seem to work. If it does not then that argument is not valid. The argument in question is

- (1) $(x) (y) [(x = y) \supset (Fx \supset Fy)]$
- (2) $(x) (x = x)$
- (3) $(x) (y) (x = y) \supset [(x = x) \supset (x = y)]$
- (4) $(x) (y) [(x = y) \supset (x = y)]$

The crucial inference is line (3) from lines (1) and (2). If (3) can be validly derived from (1) this is because (2) is a substitution instance of 'F' in 'Fx' in line (1). This in turn, presupposes that self-identity can be a property.

If $(x) (x = x)$ does ascribe a property to x then the conjunction of lines (1) to (3) is a logical truth or tautology. If $(x) (x = x)$ does not ascribe a property to x then the claim is false (4), the conclusion that all identity statements are necessary is an economical formulation of (3).

If it is allowed that being identical with x is a property of x then being a property of x must also be a property of y , if it is true that if x is y then every property of y is a property of x and every property of x is a property of y . $(x) (x = x)$ can be no exception if a property.

If groups are found for successfully denying that $(x) (x = x)$ ascribes a property to x then $(x) (x = x)$ can not be allowed as a substitution instance of 'F' in 'Fx' in line (1). If x is self-identical with x and if x is y does it follow that y is self-identical with x ? Only if self-identity

is a property. If this is denied then the argument is circular.

There are grounds for denying that self-identity is a property. If anything that is true of x is a property of x , then by this criterion self-identity is a property because self-identity is (necessarily) true of everything. It might be the case, though, that not everything that is true of x is a property of x . I suggest that if being self-identical is a property then it is a very peculiar one indeed - it is a property that everything has. Further, in any description of x , which consisted of an enumeration of the essential and inessential properties of x (i.e. those forming part of the concept of x and those contingently true of x) self-identity would not be mentioned. This is because being self-identical is neither part of the concept of x nor is it contingently true of x , it is necessarily true of x but not in virtue of the meaning of x .

So, although $(x) (x = x)$ is of course true of x , as it adds nothing to the concept of x and is not contingently true of x I hold that self identity is not a property of x .

If this is accepted then the self-identity of x cannot yield in conjunction with (1), line (3). This is because 'F' predicates of x only what is part of the concept of x or contingently true of x . I hold that 'F' is not part of the concept of x nor contingently true of x , but is still a 'property of x ' is self-contradictory.

If it is objected that self-identity is a 'logical property' of x then I can allow this. If we want to use 'property' in this sense that can be

stipulated. But, and this is the essential point, self-identity is not a property of x in particular or x uniquely. Self-identity is a property of everything and anything whatsoever; including the purported object y . It at once follows from this that purported object y . It at once follows from this that $(x) (x = x)$ can not do the work of showing x and y to be necessarily identical if identical. The argument is either circular or false. This is because whatever 'it' is it is self-identical.

If $x = y$ then the object known under the names ' x ' and ' y ' is self identical.

If x is not y then x is self identical and y is self identical.

The notion of self identity is an a priori logical truth and not a property in any ?????? sense. It ????????

There is a difficulty about the coherent interpretation of the first line of the argument which is the indiscernibility of identicals or the expression of the reflexivity of identity:

$$(\forall x) (y) [(x = y) \supset (Fx = Fy)]$$

The ambiguity is between whether one object is being discussed here or two. Of course the theorem purports to state the logical conditions for numerical identity. But for this we cannot read 'for any two objects x and y ...' because the commitment to two objects and not one is already then made. Suppose we read instead 'for any two purported objects x and y ...' so as to deliberately leave open the commitment to one

object or two. Then instead of reading 'if x is y' we can read 'if x and y name one and the same object ...' which avoids the incoherence of saying two objects are one by holding that one and the same object is picked out under two different descriptions. Up to this point the ambiguities can be ironed out. How should we read though what is materially implied by the line so far? The following is incoherent; 'every property of x is a property of y and every property of y is a property of x' even if x is y. This is because this way of speaking still implies that two different objects are picked out by 'x' and 'y', and if this is so then the claim is self-contradictory. It is logically impossible for two numerically distinct objects to have any one property numerically in common. The claim can be made coherent and perhaps true by formulating it: 'every predicate in description x must apply to the same object as every predicate in description y and vice versa'.

It may be that we need a sortal concept to individuate this object under a description other than x and y. Then we can say

$x = y$

Whether or not we do need that once the reflexivity claim can be interpreted coherently we can consider the role of self-identity in it.

Suppose

$(\forall x) \quad (x=x)$

does ascribe a property to x and so the substitution for

Fx

in

$$(\forall x) (\forall y) [(x = y) \supset (Fx = Fy)]$$

is legitimate. Is it then coherent to maintain that $(x = x)$ can be a property of y ? There is a strong reason for supposing that it is not. This theorem expresses the self identity of x and it is not coherent to hold that something can be self-identical with something else. These two sentences do not express the same proposition, or, the first does express a proposition but the second does not:

1. y is identical with y .
2. y is self identical with x

y can only be self identical with y and x can only be self identical with y but y is not a coherent formulation of this to write ' x is self-identical with y '. The word 'self' here expresses the reflexivity of identity, but the expression of this reflexivity is quite out of place at this point in the argument. The point is that the object that is known under the names ' x ' and ' y ' is self-identical whatever it is, but something cannot be self-identical with something else. Self-identity is not in this sense a relation. There is no difficulty about formulising y is identical with x :

$$y = x$$

but what of y is self-identical with x ? The argument against contingent identity statements contains no such formula: precisely because such a formula could not be well formed. It is precisely the repetition of the variable ' x ' in the expression $(x) (x = x)$ which forces the interpretation 'self identical with ...' This means

that the step from $(Fx = Fy)$ is illegitimate if Fx expresses the self-identity of x .

The question 'how can something be self identical with itself?' is not appropriate to this issue. The ambiguity is between objects and concepts. The object that is individuated by the concepts 'x' and 'y' is self identical, and x is y . To say that y is self identical with x is to beg the question of nec and cont. (or is it?). What does $x = x$ mean except that x is not y ? This is ambiguous

1. The object known as 'x' is not knowable under the concept 'y'
2. Something (x) is not itself.

1. is feasible (logically possible) i.e. that certain descriptions e.g. an object are inapplicable (contingently). This is ???????
2. is logically impossible

If 'x is y' is true, it could have been false, if this means a certain description might not have been true of an object. It could not have been false if it means that some object might not have been self identical. But would not a certain description's inapplicability have entailed its non self identity? No. Whatever descriptions an object is individuated under it remains itself.

If y is built into the concept of the object which is x then x could not have been y . For a condition for being itself i.e. both x and y , is obviously that it be both x and y . This is logically necessary for it to be the object it is. Could it have been a different object. This very same object could not have been other than the object it is. Simply because it would not then have been that

every same object. What is logically possible (if the notion of possibility is to have any application whatsoever is that there should have been an object identical to x in every respect except one: it was not y , or, at least was not known under the description ' y '. (actuality is not possibility - only a part of it) - digression on possibility; the *de re* modalist mistakes (assimilates) causal or historical actuality for possibility).

There are reasons for thinking that $(x) (x = x)$ is not a legitimate substitution for Fx in ... (1) ... Kant on existence. (2 issues) -Fy

Kripke and Empiricism

Empiricist logic and epistemology must in important respects be wrong if Kripke's arguments in his influential *Naming and Necessity* are sound. Kripke argues that some propositions are both necessary and *a posteriori* and some propositions are both contingent and *a priori*: claims inconsistent with the views that all and only analytic truths are necessary, all and only necessary truths are *a priori* and all and only contingent truths are *a posteriori*. (32)

Is Kripke right? Kripke points out that 'analytic', 'necessary', and 'certain' are 'categories of truth' (p. 34) but '*a priori*' is 'a concept of epistemology' (p. 34). ('Certain' clearly also has an epistemological use that is not truth entailing, as in: 'He was certain that p , but not p '.) Kripke says '*a priori* truths are those which can be known independently of any experience' (p. 34?).

Kripke says it is a mistake to substitute 'must' for 'can' here so *p* can be known *a priori* and known 'on the basis of experience' or believed 'on the basis of a *posteriori* evidence' but need not be. '*p* is *a priori*' and '*p* is *a posteriori*' are not mutually consistent. '*p* is *a posteriori*' means '*p* is knowable only through experience' so experience is necessary for knowing *p*. But experience is not necessary for knowing *p* if *p* is *a priori* so '*p* is both *a priori* and *a posteriori*' entails a contradiction: Experience is necessary for knowing *p* and experience is not necessary for knowing *p*.

In drawing the *a priori/a posteriori* distinction, it is not clear what counts as experience and what counts as thinking. It is not clear where experience stops and thinking begins. Thinking is arguably a kind of experiencing. If you doubt this ask: How do I know that I think? Experience entails thinking if it entails judging and is not mere reception of data. The *a priori/a posteriori* distinction is only as clear as the distinction between thought and experience. The rationalism/empiricism distinction is also only as clear as the distinction between thought and experience. Suppose I remember that *p*. Do I thereby know *p a priori*? Suppose I vividly imagine that *p*, and evidence entailing *p*, and *p*. Do I thereby know *p a priori*? (There is no belief here and belief is often thought necessary for knowing.)

Are there contingent *a priori* propositions? Kripke provides an example from Wittgenstein: 'Stick *S* is one metre long at *t* 0' (p. 56) (and other cases of fixing the reference of a term). Kripke also suggests 'Aristotle had *F* or *G*...' (p. 62-3). Kripke also suggests: 'Neptune exists' (p. 79 SEE KRIPKE'S OWN FOOTNOTE) Are these propositions really contingent *a priori*?

I suggest the proposition about the stick is contingent *a posteriori*. We decided to call *that* length one metre, but could have decided otherwise. Indeed, had that length been very slightly longer or shorter we would very likely still have called it 'one metre'. I suggest 'Aristotle had *F* or *G*...' is contingent *a posteriori* or necessary *a posteriori*, depending on whether *F* or *G* are properties that Aristotle could have lacked. We find out about Aristotle's properties by reading or listening so *a posteriori*. (Reading seems an acute example of a fusion of thought and experience which blurs the *a priori/a posteriori* distinction.)

I suggest, if Neptune is a planet then 'Neptune exists' is contingent *a posteriori*. It was a scientific discovery that that planet which is Neptune exists so 'Neptune exists' is *a posteriori*. Neptune might not have existed so 'Neptune exists' is contingent.

Nevertheless, Kripke is right in his view that there exist contingent *a priori* propositions.

I suggest there exist at least the following classes of contingent *a priori* propositions: existential propositions, first person singular psychological ascriptions, inferences to *p* where *p* is contingent, and many propositions of philosophy.

'There is something rather than nothing' is contingent if there might have been nothing. It is *a priori* because no inquiry outside of pure thought is required to verify it, as thinking is itself something rather than nothing. If it is necessary that there is something rather than nothing then it is necessary *a priori*. However, the necessity of the inference from anything's existing to there being something rather than nothing does not show that it is necessary that there is something rather than nothing.

'I exist' is contingent *a priori*. It is contingent because I might not have existed. It is *a priori* because I do not have to inspect anything outside my own thinking to verify it. It is mysterious what something's being me consists in, but this mystery is not dispelled by any *a posteriori* procedure.

'Thinking exists' is contingent *a priori*. It is contingent because there might not have been any thinking. It is *a priori* because that there is thinking may be established just by thinking.

'I am thinking' is contingent *a priori*. It is contingent because I might not have been thinking. It is *a priori* because I can establish that I am thinking just by thinking.

'I am thinking that *p*' is contingent *a priori*. It is contingent because I might not have been thinking that *p*. It is *a priori* because I can establish that I am thinking that *p* just by thinking that *p*.

'I am in mood $\emptyset$ ' is contingent *a priori*. It is contingent because I might not have been in mood $\emptyset$. It is *a priori* because I can establish that I am in mood $\emptyset$ just by being in mood $\emptyset$.

Inferences to *p* where *p* is contingent. Suppose someone 'works out intellectually' that *p* by deriving *p* soundly from premises he already knows. He then knows *p a priori*. Suppose *p* is true but *p* could have been false. If so, then *p* is contingent. It follows that *p* is contingent *a priori*.

Many propositions of philosophy (for example of metaphysics) are decided intellectually, not by experience, and are contingent. In these cases the observable universe does not decide between *p*, and not *p*. If knowable, the following are *a priori*: 'There was a first event', 'God exists', 'The mind is the brain', 'People have free will', 'Memory is the criterion for personal identity'. They might

not be contingent, for example if it is necessary that God exists, or if, necessarily, there was a first thing that happened.

If these are not knowable, but their negations are knowable, then their negations are contingent *a priori*. At most one of *p* and not *p* can be true and at most one can be false. If *p* is contingent *a priori* *p* is true, because if *p* is *a priori* *p* is true and if *p* is contingent *a priori* *p* is *a priori*.

Are contingent *a priori* propositions synthetic? If so Kripke is a Kantian, even though Kant thinks all synthetic *a priori* propositions are necessary. Contingent *a priori* propositions are not analytic[?]. (But perhaps analytic truths are not necessary, or not all necessary, so perhaps analytic truths are contingent. Are they *a priori*?) Every permutation is possible.

If *p* is *a priori* it is possible to know *p*, but possible for whom? Possible for you? For me? 'I am thinking that *p*' is knowable *a priori* by me but that I am thinking that *p* is not knowable *a priori* by you. If it is knowable by you, it is only knowable *a posteriori* by you. If *p* is knowable *a priori* by anyone then *p* is *a priori*. For God? The modality of 'can' is obscure.

Kripke argues that there are *a posteriori* necessities. There is no incoherence in '*p* is necessary *a posteriori*' because the conjunction of '*p* is necessary' and '*p* is *a posteriori*' does not entail a contradiction. He maintains a kind of essentialism which entails the necessity of origin, constitution and identity. Is he right? This is partly a matter of intuitions about which conceptual revisions we would make in counterfactual situations. If we made the strange discovery that motor cars grew on trees and were not made in factories, I suggest we would not give up the view that there are motor cars. We would

seek an explanation of our gross mistake about their origin. If we found out that everything with the appearance and behaviour of a cat had the internal make-up of a demon, I suggest we would not conclude that there are no cats but that cats are demons, rather than, say, animals. Kripke's lectern could have been made of ice. If we insist it would not then be that very lectern that is because that very lectern is not made of ice but wood. However, if anything differs from Kripke's lectern in any respect no matter how minor it is not Kripke's lectern, by Leibniz's Law. If Kripke's lectern is *F* and something is not *F* then it is not Kripke's lectern. However, Kripke's lectern might have had all the properties it has except those which depend upon being made of wood are replaced by those which depend upon being made of ice.

I suggest there is a history of modality. For example, the history of the modal status of 'Water is H₂O' has been, I conjecture, as follows: first; senseless, then consecutively; unknown, contingently false *a posteriori*, contingently true *a posteriori*, necessarily true *a priori*, necessarily true *a posteriori*. I see no reason to suppose the history of its modal status has stopped.

LOGICAL AXIOMS

It is the mute point whether Aristotle's 'laws of thought' merit that title or whether they are 'laws of being' or indeed any kind of law at all, whether on a legal or a scientific metaphor. They can though be given a *de re* and a *de dicto* construal. *De re* they are

- i. A is A (everything is what it is)

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ii. Not both A and not A (nothing is both A and not A whatever A is)

iii. Either A or not A. (everything is either A or not A whatever A is).

De dicto they are:

i. $p = p$ If p is true p is true.

ii. $\neg(p \cdot \neg p)$ It is not the case that p is both true and false.

iii. $p \vee \neg p$ Either p is true or p is false. All for any constant sense of 'p'.

i. Is the law of identity ii. is the law of non-contradiction.

iii. is the law of excluded middle.

Aristotle make two important claims about them; they are true and they are indubitable. Here 'true' means true in a way not amenable to logical or empirical demonstration [Indubitable' means: it isn't possible to disbelieve them (if you understand them)]. In just this sense they are axiomatic; they cannot themselves be logically derived from more primitive propositions - all propositions presuppose their truth.

Is this true? Aristotle formulates the LNC (law of non contradiction) thus:

LNC 'For the same thing to hold good and not to hold good simultaneously of the same thing and in the same respect is impossible. (Given any further specifications which might be added against the dialectical difficulties).

Metaphysics Gamma p7 trans Kirwan).

He says that this is the 'firmest of all principles' (7) implying I think that the laws of identity and excluded middle depend on the LNC but not vice versa. He doesn't say of others that they are the firmest of all. We can look at the logical relations between them later () but for the moment Aristotle's words can be taken in another sense. Immediately after the claim that this is the 'firmest principle' he says 'For it is impossible for anyone to believe that the same thing is and is not'. (7). The for here implies that LNC's being the firmest principle depends on its indubitability. Now, I think it isn't a good idea to make an axiom of logic rest on a psychological criterion to the effect that 'if p is indubitable then p is axiomatic'. This is because it does not follow from any or all persons inability to doubt that p that p is true. This is because from no proposition making a claim about a mental state can a logical axiom be derived. This is just because no proposition can yield an axiom as a logical consequence.

Independently of justifications of purported axioms it is worth debating the truth of the psychological claims. Is it true that it is impossible to believe both p and not p, or what amounts to the same thing, to doubt that if p then p? I think this claim must be false. The first reason is, that if it were impossible to believe that both p and not p then it would not be possible for us to ever discover that we had inconsistent beliefs but we do, so it is possible. Aristotle seems to be assuming a particular concept of belief which excludes A's believing both p and not p? What sort of concept might this be? Well, on two accounts of what a belief is beliefs can't really be inconsistent. On the first of these to

believe p is to have the conscious thought that p is true. Thus, to believe that p and that not p would be to simultaneously have two thoughts; the thought that p and the thought that not p . Depending on how we count thoughts it might not be possible to have two thoughts simultaneously anyway; I mean not even two consistent thoughts like the thought that p and the thought that q . This might just be psychologically impossible, but somehow I doubt it. What could this impossibility amount to though over and above the fact that in fact p cannot be both true and not true.

On the second construal of 'belief' it is a necessary condition of A 's believing that p that A believe that they believe that p . Here there isn't so much a problem about believing p and that not p but about believing that one believes that both p and not. In other words, if one is conscious of inconsistency in one's beliefs one eradicates it: one abandons one or both beliefs. This only fits Aristotle's case if it is made a condition of A 's believing that p that A believe that A believes that p .

There are though two compelling reasons for not making this a condition of belief. Firstly, it generates an infinite regress; A cannot believe that A believes that p without believing that A believes that p without ... Secondly it seems manifestly false that we believe we have every belief that we have. There are, I think a very large number of beliefs that each of us has that we are completely unaware of. We believe them but we are completely unaware of them. For example, suppose we ask a person who has not interest in philosophy and no knowledge of it whether there are any physical objects. Yes, of course there are,

here are some, he replies. Had he always believed this yes: of course he had but he had never had such a ridiculous thought that there weren't or there were.

There is something wrong in common to both these doctrines about beliefs. They make beliefs into introspectible particulars whereas beliefs are dispositions to allocate the truth value 'true' to propositions on good evidence. (.....). If we accept the dispositional account we are not committed to the view that p is believed just on condition that p is thought or mentally entertained, nor even just on condition that p has been thought at all.

On this account we can allow for what is the case: persons hold mutually inconsistent beliefs. One and the same person may have a tendency or propensity to allocate the truth value 'true' to p and the truth value 'false' to p. They do not know that they are doing this, they do not believe that they are doing this and if they are rational they would stop doing one or the other or both if they came to notice they are doing this. I would not want to say that a person just believes that p so long as he thinks, says, writes that p and that he believes that not p just so long as he thinks, writes says that not p. So long as a person has the disposition, so long he has the belief. (I must emphasise that this dispositional analysis of belief is not consistent with Ryle's dispositional account in The Concept of Mind). A person may then have two dispositions; to allocate mutually exclusive truth values to a proposition. Of course the dispositions are not exercised simultaneously. Some dispositions are either numerically distinct ones simultaneously, or one disposition understood

under different descriptions. In this case though the attempt to allocate the two exclusive truth values simultaneously would be likely to reveal the inconsistency. The important point against Aristotle is that there is something that is thus revealed - the belief that both p and not p ; or as he puts it the belief 'that the same thing is and is not' (7).

Although Aristotle is wrong to say that it is impossible to believe that p and not p a related remark seems correct. This is (8) 'it is not necessary that the things one says one should also believe'. A's assertion that p is not sufficient for A's beliefs that p for A may be insincere in the assertion that p . There are several logical relations worth pointing out here. A may believe that p but never assert that p . A may assert that p but never believe that p . P may be true but A neither believe nor assert that p . P may be false but A may both believe and assert that p . All these are possible permutations.

Aristotle ascribes to some (perhaps that followers of Heracleitus) that they 'assert that it is possible for the same thing to be and not be, and (assert that it is possible) to believe so' (8). Whether or not some persons subscribed to both these views they are logically independent. It may or may not be the case that it is possible for the same thing to be and not to be - and Aristotle has persuasive arguments against this (as perhaps Hegel has for it). Quite independently of this it may or may not be possible to believe that the same thing is and is not. I have argued that it is possible to believe this' it is a case of being inconsistent in ones beliefs; of holding both p and not p .

I won't examine the objections brought against LNC by Hegel and others until chapter (). Here I'll just deal with Aristotle's defence of the principle in *Metaphysics Gamma*. He has three main defences. One is that no logical justification is either appropriate or possible. A second is that to deny the LNC is to subscribe to it - I shall call this the 'self-refutation argument' and the first the 'no argument argument'. The third 'is' that the LNC holds is a necessary condition for meaning (one version) for truth or falsity (another version). I shall call this the 'semantic argument'.

The no-argument arguments conclusion is: 'some, owing to lack of training actually ask that it (LNC) be demonstrated: for it is lack of training not to recognise of which things demonstration ought to be thought, and of which not' (p8). There are really two defences telescoped together here. One is that unless certain propositions are treated as axioms an infinite regress is generated. p is derived from ' p 'p8 from p ' etc etc. So the desired ultimate justification is never reached. The other is that suppose a justification for logic is possible: that some propositions will have to be taken as axioms, then there isn't a better one than LNC. There just isn't any claim which is more self-evident and indubitable than this one and all other conceivable propositions presuppose its truth and there is not conceivable proposition from which it may be derived. What this means is that one way or another there cannot be a deductive proof of LNC. There are no premises more primitive than LNC from which LNC could be derived as a logical consequence.

Is this true? One possible candidate for an equally if not more axiomatic principle is I the

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law of identity that A is A, if p then p. Is this more primitive than LNC? Certainly to deny I sit to deny LNC. In other words to say A is not A, in a sense which violates I is to say that A is both A and not A, and anyone who said this would have denied LNC. Of course they have not thereby refuted it. Suppose a person were to deny LNC they might do this by denying I. For example a person might assert that both p and not p, ie A is both A and not A, in other words something both is and is not itself. It would seem from this that I and LNC are mutually dependent; that it isn't possible to deny one and yet maintain the other, at least not consistently.

By choice of axioms though either LNC or I can be regarded as primitive because:

$p = p$ entails

$\neg(p \cdot \neg p)$

and

$\neg(p \cdot \neg p)$

entails

$p=p$

In other words

$(p=p) = \neg(p \cdot \neg p) .$

I and LNC are materially equivalent. We could then by stipulation take the I as primitive and logically derive LNC from it. In that sense LNC would no longer be axiomatic. There is though no

intrinsic reason for doing this rather than taking LNC as primitive, because I does depend on it. One criterion for choice of axioms here might be simplicity. I and LNC are both compound propositions but I use only one logical constant and expresses simple self identity. If this is accepted then the quest for a logical justification shifts to I. A regress is still present.

If it is true that no justification is possible for the axioms of logic, how do we know they are true? We do believe them yet, if no evidence can be produced for their truth how is our belief well founded? Aristotle says they are just not the sort of propositions for which justification should be sought. We need thought to be able to sort these sorts of proposition from those that do need justification. What are the hallmarks by which we recognise axioms? We can't say 'they seem not to rest on more primitive claims' because they might. It might just be that we do not know what they are. In that case we could not distinguish cases where there were more primitive axioms, but where we were ignorant of them, from cases - as Aristotle claims here - where there are no more primitive axioms to discover. There are perhaps three candidates for this criterion. One subjective or psychological, one intrinsic to the proposition, one empirical.

The psychological criterion for distinguishing axioms from non axioms with hidden premises is that in the case of axioms we cannot conceive of what it would be like for them to rest on more primitive axioms. We don't know what they would be like.

This is not a very good criterion because although it is no doubt true in the case of genuine logical axioms that it is inconceivable what these

purported premises would be like, this might also be true of propositions which did rest on more primitive premises. Our powers of imagination might not be up to thinking those either. If we make a distinction between a contingent 'can' and a necessary 'can' here then there is a danger of begging the question. One reason why it might be logically impossible to imagine such a premise is that such a premise logically could not exist; and a logically impossible state of affairs ? is unimaginable. This begs the question. If we leave the 'can' as a contingent 'can' the claim is too weak. It becomes a psychological fact about us that we cannot conceive of certain premises' natures; a subset of these premises might be more primitive than some purported axioms.

The intrinsic criterion is self-evidence. It might be thought that this criterion could not be sharply distinguished from a second sort of psychological or subjective criterion. After all, if p is self evident, p must be self evident to some actual or possible mind. Self evidence is a relation between understand and understood. This criterion says p is axiomatic - rests on no premises more primitive than p - if p is self evident. p is self evident if it isn't possible to understand p without believing/seeing that p is true. (If being self evident entails being true we have to write 'seeing that' or 'knowing that' here. If it does not then we write 'believing'). There are two related drawbacks to this criterion. Firstly, it isn't clear whether it is a necessary or a contingent fact that (given it is a fact for some p) that p can only be understood if seen to be true. This depends on the reason why this is true. If they are psychological the chances are this fact is contingent. If they are logical the fact will

probably be necessary. Whatever the outcome of this issue though a second ambiguity remains. If it is clear that p's being self evident is neither a logically necessary nor sufficient condition for p's being axiomatic even though the connection is very close. Axiomatic means 'primitive', 'not derivable from more primitive premises'. 'Self evident means 'known to be true if understood'. Even if it were true that all and only axioms were self evident it would not logically follow from this that 'axiomatic' and 'self evident' were synonyms. If it is a truth then that all and only axioms are self evident then this is a contingent truth even though a generalisation without exceptions. As a criterion for recognising axioms it is open to the classic problem of induction. p's being self evident will provide no logical guarantee of p's being axiomatic, only very strong inductive evidence.

But perhaps the connection between self-evidence and being axiomatic might be so strong as to make it a logical truth that all and only axioms are self-evident even if 'axiomatic' and 'self-evident' are not synonyms. One way of strengthening the connections is to say certain propositions are self evident because tautologous or analytic. In other words they depend upon for their truth. This of course leaves I as axiomatic and I is a tautology or is analytic and does not itself (it seems) depend on any other premise for its truth. If we shelve the problem of I as axiom for a moment. We can see that self-evidence not only makes redundant logical dependence on more primitive axioms. If p is self-evident because tautologous or analytic then it isn't necessary that p depend on more primitive axioms. Of course p's being tautologous does not in itself logically exclude p's not being

axiomatic. p may be a complex tautology derivable from simpler premises. This stronger condition - logical exclusion - would be required for [p 's being tautologous to logically guarantee p 's being axiomatic. If p is axiomatic then p is tautologous. If p is tautologous it doesn't logically follow that p is axiomatic.

A distinction between 'analytic' and 'tautologous' is needed here. p may be a very complex tautology and so not self-evident. In other words, in the case of complex tautologies it is possible to understand them without seeing they are true. Indeed, it is frequently the case that propositions can only be exhibited as contradictory, contingent or tautologous after a lengthy decision procedure. There is though a subset of tautologies which are self-evident. It is these we call 'analytic'. A second criterion is that p is analytic if p is tautologous in virtue of its semantics. We can combine these to define 'analysis'; p is analytic if and only if p is self evident and tautologous in virtue of its semantics.

If p is analytic is p axiomatic? Not necessarily. Axioms may be logically complex although the three logical axioms are not. They may be so logically complex as to not be self evident and further, they may be tautologous in virtue of purely logical and not semantic features. So being analytic is neither a necessary nor a sufficient condition for being axiomatic.

The weaker claim does hold though. If p is analytic that is tautologous and self-evident, then p can be treated as axiomatic. It does not depend on any other proposition for its truth. This claim is much too weak for distinguishing axioms from

non-axioms in the foundations of logic though because the number of proposition which may be treated as axioms by stipulation is very large. The most we can say is that the axioms of logic do not need more primitive premises on this criterion, and this is just what Aristotle says. We cannot show (a priori) that there are no such axioms.

The final criterion is the empirical one. Here we have to construe logical axioms as very large exceptionless empirical generalisations. On this view, where the axioms are read de re A is A means something like 'everything is itself', not (A and not A) means something like 'nothing both possesses and lacks a certain property'. The criterion suggested is that p may be taken as axiomatic if p is not open to empirical falsification. Not if it is true as it well might be that the three logical axioms are not empirically falsifiable then this might well be a consequence of the fact they are tautologies. There is a strong objection to making the opposite claim: the claim that 'tautology' is just a shorthand term for 'true proposition not refutable empirically'. This is that there are two sorts of exceptionless empirical generalisation. One sort which could not (necessarily) be falsified empirically (even if de re we could read these as 'empirical necessities') the other which not been falsified but where this is contingent fact about them. But the objection to distinction is that it depends on the notion of necessity and contingency which itself depends on the axioms. That p is not refuted by empirical evidence is very weak inductive evidence that p is tautologous. It is even weaker evidence that p is axiomatic.

The least unsatisfactory of the three criteria is the intrinsic one: p may be axiomatic if self

evident. Propositions which are not self evident may be treated as axioms if tautologous but it is true that the three axioms of logic are self-evident. What we need is an elucidation of those features of those three axioms in virtue of which they cannot be treated as not axiomatic. The very weak claim that they may be axiomatic although tentatively established is one true in virtue of properties they shall with a very wide class of propositions. It is not as though we could just chose to make these ones axiomatic by stipulation: we have no other option, it seems. It is important to note a feature of the 'no argument argument' here. It may in the end be true - Aristotle may be right - that no argument deductive or inductive - can be given yielding the logical axioms as conclusions. Separate issue though is why this should be so - why no argument is possible or appropriate for axioms. It is no good just saying 'this is part of the concept of an axiom'. Of course it is but we want to know what the criteria are for 'axiom' and whether they are fulfilled by the 'axioms of logic', or the propositions that go by that name.

It may be that we do not have strict logical criteria for distinguishing logical axioms from axioms in general. It may just be that unless there were these axioms logic would not be possible. Suppose that there were a justification for them. It still might not be the case that the justification provided the grounds for holding them as axiomatic. Indeed this couldn't be so. If they could be justified they would not be axiomatic and so fall into the class of propositions to be distinguished from axioms.

There is one further way of approaching the no-argument argument but detailed consideration of this will have to wait until the chapter on Hegel (). This is the approach which denies that there are logical foundations at all. One aspect of a logical system may be invoked to justify as true some other aspect of the system but there is no point of reference outside the system by which any claim within the system may be ultimately substantiated. It maybe that that the relationship of the purported 'axioms of logic' one to another are best understood on the 'circle' model and not on the 'foundations' model. If so we are not forced to take any single proposition as axiomatic. Nor, by not doing so, are we forced into an infinite regress.

The Self Refutation Argument

The conclusion of the self-refutation argument is that if a person denies the LNC then they implicitly subscribe to it. By making an assertion they claim that p, that is that p is true. To do they is to deny that p is (also) not true. To deny the law of LNC is to assert that it isn't not true.

Aristotle (Metaphysics Gamma 9-10) tends to telescope this argument or assimilate it to the semantic argument. These are in fact two arguments whose merits can be appraised separately.

How could the opponent of LNC reply to the self refutation charge? He could try this: he could say that his denial of the LNC was indeed true and false and it was precisely because of this is was a refutation of LNC. He has produced, he claims, something that is both true and false. This will not be straightforwardly, or only a denial of LNC

but it will be an 'exception' to LNC. The exception can be referred to using a statement in accordance with the LNC to deny the LNC. For example: 'It is the case that both p and $\neg p$ so LNC is false' or has at least one exception to it. Thus although Aristotle is right that to deny LNC is to implicitly subscribe to it the denial of LNC that proposition itself might not be the grounds for denying the LNC. The denier of LNC thus gives an example of LNC in giving an example of an exception. He does not have to confine himself to the exception in denying LNC, nor to the denial.

Whether this is a plausible tactic for the 'R' sceptic depends on whether there are exceptions to LNC. We shall consider some candidates under ().

The Semantic Argument

Asking the opponent of LNC to 'stage something either to be or not to be ... begs what was originally at issue' (9) so he should be asked 'at least to signify something both to himself and to someone else' (9). Here the assumption is that coherence is a condition of sense. If the opponent says something that is both meaningful to himself and can be used to communicate ie is meaningful to others, then that person will have admitted the truth of LNC.

Is this true? To decide this imagine the opposite. A person contradicts himself - both p and not p . Does this make sense either to him or to others? A case could be made out that it did. This would have two aspects. Firstly, the claim must make sense to have the virtue of being false. If it did not express a genuine proposition then the denial

on LNC would not even be false: it would be nonsensical. We can if we wish allow this, but the price of a contradictions being nonsensical is that it cannot also be false. Secondly, it could be argued that the denial of LNC must make sense in order for certain facts to be known about it: that it is a denial, that it is a contradiction, if it is denied that both p and $\neg p$ has sense this is I suspect because the claim is false; logically false. 'Signifying' something here is being tacitly read to mean 'saying something is the case' is asserting that p . And, of course anything of the form 'both p and not p ' cannot be used to assert that p . It at most follows from this that contradictions are useless, uninformative and cannot be used to make assertions or true claims. It doesn't follow that they are senseless.

Suppose p has sense and suppose not p has sense. We must suppose this for both p and not p to be a contradiction. Indeed, p must have the same sense in each occurrence or else both p and not p would not be a contradiction. How then has the combination of p and not p in the compound proposition deprives p of its sense? Well, p and not p is the conjunction of a proposition and its negation in the same compound proposition. This proposition is exhausted by p and the negation of p . One clear effect of this is to deprive the compound proposition of the truth value 'true'. It could be argued that it has deprived it of the value 'false' also, but for the moment we should work with the assumption that contradictions are falsehoods. So far we have not touched the semantics of ' p '. All we have said is that p has the same sense in both ' p ' and 'not p '. This thought, I think, is to show that there are semantic preconditions for contradictions.

Aristotle is wrong then to think that coherence is a proposition for sense. Rather sense is precondition for incoherence. Of course very radical incoherence; say grammatical incoherence will destroy sense but no conjunctions of propositions unless the simple propositions conjoined in a contradiction taken singly made sense contradictions would not be possible. This is true I think on any true account of what sense is. Suppose truth conditional semantics is true. Then it is a condition for p's making sense that it be knowable what it would be for p to be true or for p to be false. Here there are not (or don't seem to be) any conceivable empirical falsity conditions. There are no empirical objects of the imagination in this case???? This doesn't mean that we don't know what conditions have to be fulfilled for p's being self-contradictory. They are semantic and logical conditions, not empirical ones. P's conjunction with the negation of p is both necessary and sufficient for a contradiction, this is conceivable and knowable, therefore contradictions have sense.

Suppose it is objected to this that being false, or being logically false it has no truth conditions. There isn't any possible state of affairs it corresponds to which would verify it and therefore it is senseless. The reply to this is that absence of truth conditions is a problem then for giving sense to any false proposition and not just to contradictions in particular. It not being clear how it not being the case that p can be instantiated anymore than it not possibly being that case that p but if we say this is some state of affairs call it 'q' we have difficulty in pointing to some features of 'q' in virtue of which it provides the falsity conditions for p. Yet, despite

these considerations we do not deny sense to false propositions, only truth.

There is a class of false propositions which do not have falsity conditions. If I say 'x is not the case' and x is the case then x's obtaining falsifies the denial that x is the case. In general denials which are untrue have falsity conditions, the negations of assertions do not; at least not in any clear cut empirical sense. This is because there is no such thing as a non-existent state of affairs. Of course it is true that an infinite number of states of affairs do not obtain but this is not to be understood as the actual obtaining of some very peculiar subset of states of affairs in general; the non-existent ones.

If contradictions have sense then it isn't true that 'if he does offer this, there will be demonstration, for there will already be something definite' (9) because 'something definite' is 'p' the sense of which remains constant between 'p' and 'not p'. Aristotle's next claim 'eliminating statement be submits to statement' (9) is ambiguous but, I think, true on either construal. This could mean, the denier of LNC is forced to subscribe to it at least in the minimal sense that he makes a denial. Although this is true it isn't part of the semantic argument it is part of the self-refutation argument and belongs with it. On another construal a person who asserts 'both p and not p' does not succeed in producing something that is both true and false but something that is false. It would be necessary to show that something could be both true and false to refute LNC but asserting both p and not p does not do this according to Aristotle: it asserts something false. If this is false then what logical form could be open to someone who y

thought there were propositions which were both true and false? They may wish to express this by using exactly the forms 'both p and not p'. It may be that this is inexpressible because there cannot be a proposition both true and false. If this is possible then a case needs to be made out for saying that both p and not p is false, and doesn't lack both true values or possess both truth values.

p	.	- p
T	F	F T
T	F	T F
F	F	F T
F	F	T F

If how p and not p is exhibited as a contradiction. For any combination of truth values of p it comes out false. We could say; this is just how the symbols . and - are used; it is part of their meaning. Constants lacking these functions or having different ones would not be them. There would remain the question of justifying that use though and, chances are, it rests on LNC.

One way of extending the possible truth values of p and p belong 'p is true if not p is false' and p is false if not p is true' is to say that p may be neither true or false, or in other words, both p and not p are false. This is not a violation of LNC. To do that we would have to say p is true and not p is true. This is rather a violation of LEM (law of excluded middle). This says every proposition is either true or false. It is part of the concept of a proposition to be capable of being true or false, or that which is expressed by a sentence capable of being true or false but LEM isn't just an analysis of the concept of a proposition. It at least means 'every proposition

is true or false but not both'. Here we are considering the case where p and not p are both false. This might be thought to be impossible by LNC because if not p is false then p is true and if p is false then not p is true. Might there not be exceptions to this though. Suppose we say of which neither p nor p is true; a state of affairs to which neither a proposition nor its negation is applicable; one to which the application of the concepts of the proposition results in nonsense. Such an example would be difficult to find because if it doesn't make sense to say some x is F then that x lacks the property F and this fact is reportable by p where p is ' x lacks F ' or by ' $\text{not } q$ ' where q is ' x is F '. If it doesn't make sense to say F is F though, it could be argued that it doesn't make sense to deny it either. Making sense of the denial, it might be argued, involves making sense of the affirmation. If the affirmation makes no sense neither can the denial.

Suppose a sentence cannot not make sense just in virtue of being a contradiction. Aristotle has a refinement of the Semantic Argument which is that a predicate must be applied consistently to be meaningful. In particular - F cannot be applicable exactly where F is applicable, and it isn't possible for everything to be F is F is to have sense: 'the name signifies to be or not to be this particular thing, so that it could not be that everything was so and so and not so and so' (9).

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$(\exists x)$

$(\forall x)$

$\rightarrow$

$\leftrightarrow$

$\supset$

•

$\diamond$

Abschattungen: see aspects. [SUGGESTED]

anomie. 85 Breakdown of the conventions of everyday life. Weakening of a society's collective self-image or social laws.

Anomic terror is the psychological state of the individual stripped of the social mores which legitimate their death. *Durkheim argues that suicide rates increase during periods of anomie. The raising of philosophical questions arguably calls into question established world views and partly deconditions the individual. If so, philosophy is partly conducive to anomie.

SP

Emile Durkheim *The Rules of Sociological Method* tr. W.D. Halls, ed. Stephen Lukes (London, 1982)

Peter L. Berger and Thomas Luckmann *The Social Construction of Reality* (Harmondsworth, 1967)

aspects. 160 Ways of appearing. What appears in ways of appearing.

In Wittgenstein's philosophy, what is seen in *'seeing as'. *Wittgenstein distinguishes the 'continuous seeing' of an aspect from the 'dawning' of an aspect, suggests that the concept of an aspect is like the concept of a (re)presentation (*Vorstellung*) and says 'aspect-blindness' is like the lack of a 'musical ear'. According to Wittgenstein, seeing aspects is 'subject to the will' but does not entail the existence of any 'private object'. In a change of aspect, paradoxically, there seems to be a new perception yet what is presented remains unchanged.

In German phenomenology, the phenomenological profiles or adumbrations known as *Abschattungen* through which spatial items such as shapes and colours are given directly in perception. *Husserl thinks physical objects are presented through *Abschattungen* but non-spatial items, notably mental processes, are not.

SP

Ludwig Wittgenstein *Philosophical Investigations*
tr. G. E. M. Anscombe (Oxford, 1953) esp. pp. 194-6, 206-8, 210, 213-14, 536

Edmund Husserl *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*
First Book tr. F. Kersten (The Hague, 1983) esp. §41, §44, §97

coherence. 160 p and q are coherent if and only if the possible *truth of p does not preclude the possible truth of q and the possible truth of q does not preclude the possible truth of p . It

follows that the concept of coherence presupposes the concept of truth so truth cannot be explained in terms of coherence without circularity.

Nevertheless, coherence is necessary for truth because if $\{p, q\}$ form an incoherent set then at least one of p and q is false. Coherence also provides a test for truth because if it can be shown that $\{p, q\}$ form an incoherent set, and it is known that one of p, q is true then it follows that the other is false and if it can be known which is true then it can be known which is false. However, coherence is not sufficient for truth because the coherence of $\{p, q\}$ is consistent with the falsity of both p and q .

conceivability. 160 Admissibility by the mind. Thinkability. If imagination entails the generation of mental imagery then everything imaginable is conceivable but not everything conceivable is imaginable. Logical possibility is necessary for conceivability but logical possibility does not consist in conceivability. A putative state of affairs is logically impossible if and only if its description entails a contradiction. A putative state of affairs is conceivable if and only if it can be thought.

conceptual analysis. 360 The attempt to solve philosophical problems by defining words or being clear about how concepts are used. In practice, conceptual analysis involves logical deduction See *implicature.

context. 160 The parts of a text immediately preceding or succeeding a part of a text that are either necessary for its meaning or sufficient for its meaning or both. Derivatively, whatever is spatially or temporally juxtaposed to something

that contributes to that thing being what it is. Notoriously, quotation is sometimes criticised for being taken out of context but that is what quotation is.

formal and material mode. 85 A distinction between two ways of using language. In the formal mode, the expressions of a language refer only to linguistic items: concepts and the logical relations between them. In the material mode the expressions of a language refer to non-linguistic items: objects, events and the relations between them. The distinction is analogous to the *use/mention distinction and due to *Carnap.

Rudolf Carnap *The Logical Syntax of Language* (1934)

free logics. 160 Logical systems in which existential commitment is not assumed. So, (Ex) (Fx) does not necessarily follow from Fa . Logic in which names and predicates need not refer. Logic whose sentences are not made true or false by what does or does not exist.

future contingents. 85 Either: a prediction neither true nor false at the time it is made, or: a future state of affairs that might or might not obtain. On the first definition, a future contingent is a proposition about the future, or the content of a future tense indicative sentence, on the second the possible truth condition for such a proposition. See Sea-Battle. [BUT PROPOSITIONS ARE TRUE OR FALSE]

fuzzy logic. 160 Logical system which allows degrees of truth. For example, where '1' denotes truth and '0' falsity, p might be true to degree

0.7 and so false to degree 0.3. In general, if p is true to degree n then p is false to degree $1 - n$.

L. A. Zadeh 'Fuzzy Sets' in *Information and Control* (1965)

Idealism, British. 360 Movement in nineteenth and twentieth century British philosophy according to which ultimate reality is mental or spiritual, rather than physical. *Bradley, *Green, and *Bosanquet, think matter is not real. Physical objects and the subjective points of view of conscious individuals stand in a system of *internal relations called 'the absolute' or 'absolute mind', so British idealism is mostly a kind of *absolute idealism. *McTaggart, although denying the existence of God and the reality of time and matter, holds that we are essentially immortal souls.

Newton might be an early exponent of absolute idealism

More recently, non absolute or individualistic versions of idealism have been defended in Britain by John Foster and Howard Robinson. They argue that it is incoherent to maintain that ultimate reality is physical. Timothy *Sprigge argues for a kind of absolute idealism. Although late twentieth century British philosophy was dominated by the materialist and positivist ideas common to English language philosophy, it is not impossible that idealism will have a new role once non-reductivist explanations of consciousness come to be taken more seriously. It is now clear, if it was not clear then, that twentieth century materialism, naturalism and positivistic science wholly failed to provide any kind of explanation of the existence and reality of one's own subjective mind.

SP

John Foster *The Case for Idealism* (London, 19)

Theories of the Mind (London, 1991)

'Hegel and Newton: Can Science Explain the Scientist?' in Michael Petry (ed.) *Hegel and Newton* (The Hague, 19)

Howard Robinson *Matter and Sense* (Cambridge, 19)

Howard Robinson (ed.) *Objections to Physicalism* (Oxford, 1993)

Timothy Sprigge *The Vindication of Absolute Idealism* (Edinburgh,)

Idealism, German. 720 Movement in late eighteenth and early nineteenth century German philosophy according to which ultimate reality is mental or spiritual, or at least not physical. *Kant, *Hegel, *Fichte and *Schelling are usually classed as the German idealists, and *Leibniz sometimes included.

Arguably, Kant is not any kind of idealist. He called his critical philosophy 'transcendental idealism' but realised this was a mistake when readers and critics understood this to be a kind of idealism. In the second (B) edition of the *Critique of Pure Reason* he expressly included a new chapter called 'The Refutation of Idealism' (B).

'Idealism' in 'transcendental idealism' is in fact correctly understood as 'anti-realism'. Kant thinks there are no metaphysical propositions, so transcendental idealism is metaphysical anti-realism. Kant makes many claims consistent with idealism but they are misunderstood as entailing idealism. Famously, he insists we have knowledge only of appearances, not of things in themselves. This means we can only know things as they appear to us, not as they could not appear to us. There is nothing to suggest that appearances are mental ('A

motor car appeared around the street corner'). 'We can only know possible objects of experience' means: We can only know things as they would appear to us if we were experiencing them. This does not entail that those things are mind dependent, only that our knowledge has a perspectival nature. Kant makes it clear that the notion of a noumenon is a 'limiting' concept, and 'existence' is a category, so whatever interpretation we give to 'noumena' it had better not imply that there are any. Kant thinks we can only know objects if they are spatio-temporal, Euclidian, Newtonian, countable, could interact causally and are substances. If this epistemological conservatism carries any ontological implication it is materialism, not idealism.

There is a huge secondary literature on Kant, most of it, from Hegel to contemporary British, German and American commentators, mistakenly construing Kant as some sort of idealist.

SP

Henry Allison *Kant's Transcendental Idealism* (19)

Graham Bird *Kant's Theory of Knowledge* (London, 1961)

Theories of the Mind (1991)

(ed.) *Hegel's Critique of Kant* (Oxford, 1987)

implicature. 85 Meaning that is supplementary or contrary to that logically entailed by a sentence. For example in: 'What did you think of his argument?' 'Impeccable scholarship', or in: 'Oh, you are awful.' The discovery of conversational and conventional implicature by Paul Grice in the 1960's cast doubt on the solution or dissolution of

philosophical problems by *linguistic philosophy because it suggests meaning cannot be wholly and appropriately displayed by *conceptual analysis. If you ask someone 'Could you direct me to the library?' and they smuggly answer only 'Yes' it is appropriate to ask them if they have heard of conversational implicature.

SP

Paul Grice

langue and parole. 85 A distinction drawn by the Swiss founder of linguistics, Ferdinand de Saussure (1857-1913) in his lecture series *Cours de linguistique générale* (1916), between language as a system of formal rules and language in its everyday use.

SP

Ferdinand de Saussure *Course in General Linguistics* (London, 19)
esp. pp.

light of nature. 85 In Cartesian philosophy, reason. The faculty by which knowledge, especially *a priori* knowledge is discerned. For example, Descartes says

Rene Descartes *Meditations*

logical laws.

Propositions true on logical grounds alone. Logical truths. For example, the laws of non-contradiction,

identity, excluded middle, and double negation. In propositional calculus the law of non-contradiction is:

$\neg(p \ \& \ \neg p)$,

'It is not the case that both p and not p' ,

in predicate calculus:

$(\forall x) \neg(Fx \ \& \ \neg Fx)$

'For any x , it is not the case that x is F and x is not F' .

In propositional calculus the law of identity is:

$(p \rightarrow p)$,

'If p then p' ,

in predicate calculus:

$(\forall x) (Fx \rightarrow Fx)$,

'For any x , if x is F then x is F' ,

in predicate calculus with identity:

$(\forall x) (x = x)$,

'For any x , x is x' ,

in modal predicate calculus with identity:

$\Box(\forall x) (x = x)$,

'Necessarily, for any x , x is x' .

In propositional calculus the law of excluded middle is:

$$p \vee \neg p,$$

'Either p or not p ' ,

in predicate calculus:

$$(\forall x) (Fx \vee \neg Fx) ,$$

'For any x , either x is F or x is not F ' .

In propositional calculus the laws of double negation are:

$$\neg\neg p \rightarrow p,$$

'If not not p then p ' ,

$$\text{and } p \rightarrow \neg\neg p,$$

'If p then not not p ' ,

and in predicate calculus:

$$(\forall x) (\neg\neg Fx \rightarrow Fx)$$

'For any x , if x is not not F then x is F '

$$\text{and } (\forall x) (Fx \rightarrow \neg\neg Fx) ,$$

'For any x , if x is F then x is not not F '

Aristotle does not distinguish sharply between logical laws, laws of thought and laws of being, so

the consistent, the *conceivable and what could exist coincide, and the inconsistent, the inconceivable and what could not exist coincide. Aristotle's informal statements of the law of non-contradiction include:

'For the same thing to hold good and not to hold good simultaneously of the same thing and in the same respect is impossible' (Metaphysics ¹ 1005b):

$$(\forall x) \neg (Fx \cdot \neg Fx)$$

or arguably:

$$(\forall x) \neg \Diamond (Fx \cdot \neg Fx),$$

and

'Nor [...] is it possible that there should be anything in the middle of a contradiction' (1011b):

$$\neg \Diamond (p \cdot \neg p)$$

His statement of the law of excluded middle is

'but it is necessary either to assert or deny any one thing of one thing' (1011b),

$$(\forall x) (Fx \vee \neg Fx)$$

or arguably;

$$\bullet (\forall x) (Fx \vee \neg Fx)$$

Aristotle says it shows a lack of education to demand a proof of logical laws. He does however bring a self-refutation argument against their putative denial by his pre-Socratic predecessors:

Protagoras, who thinks that every claim is true but there is no truth over and above belief by or appearance to persons, and Heraclitus, who thinks that everything is changing in every respect so there is no truth. Aristotle points out that saying anything meaningful or true, for example making Protagorean or Heraclitian claims, presupposes logical laws.

*Mill maintains that logical laws are not a *priori* or necessary, but empirical generalisations confirmed by all experience but, so far, refuted by none. He thinks all deduction is really induction.

*Quine has suggested revision of the law of excluded middle to simplify quantum mechanics.

*Plantinga has commented that this is rather like revising a law of arithmetic to simplify the doctrine of the Holy Trinity.

It is widely taken as axiomatic that if the description of a putative phenomenon entails a violation of a logical law then that phenomenon cannot exist. However, if we are persuaded, for example, that Zeno has found contradictions in the concept of motion (for example: If x moves then x is at a place at a time and x is not at that place at that time) we do not thereby conclude that nothing moves; "Foolish, foolish us! We thought things moved. But no. That philosopher Zeno has shown that the concept of motion entails a contradiction. Clearly we should give up this widespread, perceptually compelling but incoherent assumption! Motion is *logically impossible*." Rather, we retain the view that things move and look for a consistent theory of motion. The implications for philosophy, science and theology are wide. Perhaps time travel is not logically impossible, it is just that we so far lack a consistent theory of it. Arguably, something is

possible if and only if there is at least one consistent description of it.

Perhaps nothing is logically impossible, because contradictions do not pick out any putative states of affairs. If not, they do not pick out any impossible putative states of affairs.

"Ah yes, 'Both $(p \rightarrow p)$ ', it is the putative state of affairs picked out by *that* sentence that could not come about!"

But what state of affairs could not come about?

E. J. Lemmon *Beginning Logic* (London, 1967)

Aristotle's Metaphysics Books   
translated with notes by
Christopher Kirwan (Oxford, 1971) Book Gamma.

John Stuart Mill *A System of Logic* 2 vols. (London, 1879)

Alvin Plantinga *The Nature of Necessity* (Oxford, 1974)

W. V. O. Quine 'Two Dogmas of Empiricism' in his *From a Logical Point of View* (Harvard, 1953) pp. 20-46

logic of discovery. 160 *Deduction in the testing of scientific theories. For example, the exhibiting of logical relations between the sentences of a theory (such as equivalence, derivability, consistency, inconsistency) or between a theory and established theories; the logical inferring of predictions from a theory. *Popper argues against the view that scientific theories are tested

inductively but for their deductive and empirical falsification.

Karl R. Popper *The Logic of Scientific Discovery* (London, 1980) Ch. 1 Sect. 3 pp. 32-4.

paralogisms 85 In Kant's critical philosophy, a fallacious argument. In the *Critique of Pure Reason* Kant identifies three paralogisms in the first edition and a fourth in the second edition. They are invalid inferences to conclusions expressing the simplicity of the soul, the personality of the soul, the immortality of the soul and the existence of the external world. In each case, the mistake is to try to derive a tenet of transcendental realism from transcendental idealism.

Immanuel Kant's Critique of Pure Reason trans. Norman Kemp Smith (London, 19)

projectivism 85 The thesis that some apparent properties of the external world really belong to the mind that perceives it. For example, the world appears coloured, auditory, olfactory, gustatory, actions seem good or evil and objects ugly or beautiful but according to projectivism these characteristics are at least partly due to our mental constitution.

quantification 85 The application of quantifiers (for example: 'all', 'some', 'a few' 'more than half'). In predicate logic, the prefacing of a sentence by either the universal quantifier ($\forall x$) or the existential quantifier ($\exists x$). Quantification turns a sentence with free variables into a sentence with bound variables, for example: Fx into $(\forall x) Fx$.

The propriety of the universal and existential quantifiers is not beyond philosophical question. The use of the universal quantifier assimilates 'all' and 'every' which are arguably distinct concepts. The use of the existential quantifier enshrines formally the doctrine that 'exists' is not a predicate, but 'exists' might be some kind of predicate. Quantification theory is predicate logic.

SP

Martin Davies *Meaning and Quantification* (London, 19)

Frege

realism, modal. See semantics, possible worlds
[SUGGESTED]

reductionism, psychological, in personal identity
160

sea-battle argument 160 Argument criticised by Aristotle in *de Interpretatione* Book IX. Either there will be a sea-battle tomorrow or there will not. These possibilities are mutually exclusive and jointly exhaustive. It seems, therefore, to be a necessary truth that there will be a sea-battle tomorrow or there will be no sea-battle. The future is inevitable now, it is just that we are now ignorant of which future event will happen. In criticism, Aristotle argues that from the necessity of the disjunction $(p \vee \neg p)$ it does not follow that the disjunct p is necessary nor that the disjunct $\neg p$ is necessary. Therefore the argument does not

validly establish that future events are inevitable.

SP

Aristotle *de Interpretatione* (Oxford) Book 9

semantics, possible worlds 160 The meanings of *modal logic. Possible worlds are the referents of the sentences of modal logic. For example, $[p]$ is true if and only if p is true in all possible worlds, $\langle p \rangle$ is true if and only if p is true in some possible world, $\neg \langle p \rangle$ is true if and only if p is true at no possible world. According to modal realism, all possible worlds exist. The difference between the present world and other possible worlds is not that this one exists and the others do not, but that it is actual. Actuality is indexed to a world because, for example, this world being actual consists in our occupying it. It is debatable whether there is a possible world in which nothing exists. It seems possible that there should have been nothing, yet existence would seem necessary to anything's being a world.

SP

Saul Kripke 'Semantical Considerations on Modal Logic' *Acta Philosophica Fennica* 16 (1963) 83-94 reprinted in Leonard Linsky (ed.) *Reference and Modality* (Oxford, 1971) pp. 63-87)

David Lewis *On the Plurality of Worlds* (Oxford, 1986)

transcendence. 160 Existence beyond. For example, God, numbers and universals are sometimes argued to

exist beyond space, time, the physical world or experience (in some non-spatial sense of 'beyond'). In theology the transcendence of God is contrasted with his immanence or pervasion of the world. In Kant's critical philosophy, 'transcendent' or metaphysical knowledge of non-spatio-temporal reality that cannot be subsumed under the *categories is impossible. 'Transcendental' knowledge, *a priori knowledge of how knowledge is possible, is however possible.

Immanuel Kant's Critique of Pure Reason trans.
Norman Kemp Smith (London, 19)

transcendental idealism [SUGGESTED]
transcendental realism [SUGGESTED]
transcendentals [SUGGESTED]

use and mention 85 A distinction between talk about the world by means of a word and talk about that word. For example, in 'Numbers are abstract objects' the word 'number' is used, but in '"Number" has six letters' the word 'number' is mentioned. To mark the distinction, it is customary to place a word between quotation marks if it is mentioned. As a test for the distinction, translate the sentence embedding the word into another language. If the word is mentioned it is appropriate to leave it untranslated, if used, inappropriate. It is important to observe the distinction to avoid confusion between ascribing properties to language and ascribing properties to non-linguistic reality.

[look at Edward Craig's *Encyclopaedia* to avoid duplication]
[look at Oxford libraries and internet for refs & biographies]

The Knowledge of God (Revelation)

Time Travel and Possible Worlds

Time travel consists in making another possible world actual. The past is a possible world. The past + you is another possible world. Have you visited the past? Say the people in the past that you visit are numerically identical with the people in the (ordinary hiastorucal past) that you have not visited (yet). They preserve transworld identity despite the fact that you are in one world and not the other of ther two they are in. Use modal logic to illustrate the possibility of time travel. Modal realism is the ontology.

'"Exists" and the Existential Quantifier'

Heaven is sky-like. ie all around you, not just above you. The earth beneath your feet stops you seeing that the sky is all around you.

Is Anything Impossible?

What is a contradiction? The term has many senses: the conjunction of a proposition and its negation, that is any proposition of the form 'both p and not p' ($p \cdot \neg p$). The claim that some entity both possesses and lacks some property; that is that it both possesses and lacks that property in exactly the same respect and at exactly the same time; (or to put it more precisely - the respect in which x possesses some property F is precisely the respect in which the same x lacks that same property F, simultaneously). These are two formal senses of 'contradiction'. But there exist informal senses too. Hegelians hold that the ultimately correct or

complete metaphysical description of reality must contain contradictions, or at least propositions which appear in their surface grammar to be self-contradictory. On this view; which perhaps finds its ancient precursors in Heraclitus and Plotinus, an ontology of process and dialectical interaction of subject and object is held to defy ready classification in a formal framework. Similarly, Marxists frequently claim historical and social transformations are generated by ??? "contradictions". On both the Hegelian and the Marxist views contradictions do not just obtain in purely formal, linguistic or even intellectual reality. Contradictions exist in metaphysical and social reality as it is in itself and it is because of this that reality can be accurately characterised as contradictory by use ??? of language.

Not only are there different sorts of contradictions or different sense of 'contradiction'. There is wide disagreement about why there are contradictions; not in the empirical sense which could be answered by an inquiry into human speech acts but in the a priori of 'what makes contradiction possible?' What are the necessary and sufficient conditions of a proposition being self-contradictory that it have the logical form 'both p and not p'. This is ??? it begs the question. Anyone who had the relevant concept of contradiction would already understand the sense of 'both p and not p', anyone lacking that concept of ?? would not find the ??? explanatory. Nor in the sense is having the logical form 'both p and not p' a necessary condition for a proposition being self-contradictory. There are complex and compound propositions whose logical form is correspondingly more complex than 'both p and not p'. In a

different sense though, having this logical form is both necessary and sufficient. Any compound contradiction does reduce to the logical form 'both p and no p'. Anything that does ????? thus reduces is not self contradictory anything that can be thus reduced to self-contradictory. Anything that is of this logical form is self contradictory.

The two questions: what is a contradiction? and How are contradictions possible? are closely related, but they are not the same question. The second question will have answers of the form 'There would not be contradictions unless ... there were for example; minds, languages, societies, brains, physical objects; some combination of these, none of these. The answer to the first question will take the form: 'A contradiction is ...' for example, a proposition conjoined with and its negation, two thoughts that cannot be believed simultaneously, an objects both possessing and lacking a property simultaneously, something's being other than what it is, that is, not being itself, a tension or set of incompatible tendencies in social or psychological reality, for example: a divided mind as in akrasia, that is ???/ a contradiction within the person; or a clash of interests between persons: that is ??? a contradiction between persons. We should not talk too early at calling the divided mind or the divided society a 'contradiction'. It may well be that such dilemmas in the person and conflicts within society and between societies are best characterised as 'contradictions'. For example suppose a person desires that p and that not p. One is tempted to say 'Yes, but surely not in the same respect' but that is the problem: they desire that p and they desire that not p in exactly the same respect ?????????? Two issues have to be

carefully distinguished here: the motives from which a person (A) desires and does not desire that p and on the other hand the sense in which they desire' that p. and not p. They may desire that p and that not p in exactly the same respect yet from different' indeed from opposite motives. Of course in characterising such a dilemma we are not committed to the view that both p and not p, nor is the subject of akrasia; A neither has to believe that both p and not p nor need it be true that both p and not p. A simple desires that p and not p. To say that A desires both p and not p is, in one sense at least misleading. A would not be satisfied if per impossible both p and not p obtained in the same respect: the status of affairs A desires is not desired as logically impossible to A; ie A does not desire a state of affairs that logically could not obtain as one that logically could not obtain. That would be senseless, or perhaps insanity. (Perhaps he is insane, we can return to that). He desires p and he desires not p but he doesn't desire both p and not p. To desire that p and to desire that not p is not formally incoherent. It isn't incoherent in the sense that to believe that p and that not p, or to sincerely assert that p and not p is formally inconsistent. And, if we wish we can stipulate that all and only formally inconsistent claims are contradictions. This seems a less than useful stipulation though. Notice that to desire that p is to desire that p be true, or to desire that p obtain. Similarly to desire that not p is to desire that p be false, or to desire that p not obtain. It is at least true that these desires are 'irrational' on these criteria. Both desires cannot be fulfilled simultaneously (or perhaps they can). There are at least plenty of senses in which the desire that p and the desire that not p cannot be fulfilled

simultaneously: all those are cases where p and not p have exactly the same sense.

A parallel case can be presented for regarding conflicts of interests between groups of persons as contradiction. It does not have to be the case that both p and not p nor does it have to be believed or even supposed to be coherent that p and not p , in order for sense to be given to 'social contradiction' or 'historical contradiction'. Suppose there are two interest groups 1 and 2. Suppose further that it is in 1's interest that p , I know that it is in 1's interest that p and that, consequently, I will attempt to make p true, ie, make p obtain if it is possible that I make p obtain. Suppose now that not p is in 2's interest, 2 knows that not p is in its interest and that consequently 2 will (attempt to) prevent p 's being true or prevent p 's obtaining if possible. We can make a distinction between a proposition and its truth conditions (or a proposition and its falsity conditions). I will attempt to make the truth conditions of p obtain. 2 will try to prevent the truth conditions of p from obtaining.

Is this a contradiction? Formally the answer is 'no'. The conflict is not correctly characterised by 'it is the case that both p and not p '. But, as with the case of the individual's dilemma, 1 desires that p be true, 2 desires that p be false. This is not the inconsistency of 1's holding that both p and not p , nor even of 1's holding that p and 2's holding that not p . But, in 1's desire and 2's desire both being fulfilled would require that both p and not p . In order for the conflict to be resolved, to the satisfaction of both parties a logically impossible state of affairs would have to obtain. And that is itself logically impossible.

If it is necessarily not possible that p then necessarily it is . It is not logically possible that p . Because of this there is a close analogy between one aspect of the logic of the conflict resolution and one feature of formal contradictions. It is logically impossible that a contradiction be true. A logically impossible state of affairs is frequently defined precisely as one which it would be self contradictory to describe. Now, if this true, then contradictions not only do not have truth/falsity conditions but it is in principle impossible that they should have. So, there is no possible state of affairs which would falsify or confirm a contradiction and there is no possible state of affairs which would consist in the resolution of the conflict.

This last point might be doubted. Conflicts are resolved, some purported formal contradictions turn out on analysis to be paradoxes. Both these claims are true but are not objections to the unresolability of a conflict or the purported truth of a contradiction in the relevant way. This is the way in which the truth of p excludes the truth of not p and vice the truth of not p excludes the truth of p , or where p 's being the case is incompatible with p 's being the case. No conflict could be resolved, under these conditions, by both p and not p being true. Of course there may be compromises, as there are, where p and not p are understood in different senses of p .

This sort of argument, I think, is a prima facie case for calling certain sorts of conflict 'contradictions' . In chapter we shall see whether Hegelian and Marxist social and historical theory produces, or detects contradictions in this sense or in some still looser sense.

I have taken it for granted so far that contradictions do exist in language. It maybe that they would not unless they also existed in thought, but even someone who thinks that (and there are arguments for and against it) is committed to the view they exist in language or, more precisely, are linguistic items at least. Even someone who thought there would be contradictions even if there were no language, no language users and no conscious beings could concede that, given that there is language, there are contradictions. His account of why there are contradictions might not give much weight to linguistic features like truth or falsity - he might think that there were contradictions in language which reported paradoxical states of affairs; states of affairs which seem to us to be logically impossible. Or, more weakly the evidence for p is as strong as the evidence that not p and there is no way, in principle, of deciding whether p or not p is the case. In this situation to be verificationist we could say that the claim p and that the claim not p are each senseless. There is no way in principle of verifying them therefore they are devoid of sense. Even a weaker form of verificationism, say a truth functional semantics might be used to argue this. On this view, to understand p is to know what it would be for p to be true, to understand not p is to know what it would be for p to be false. The next step is to say that if it is in principle impossible to verify or falsify a proposition then it is in principle impossible to conceive what it would be for that proposition to be true or false, and so to know its truth value either. These moves seem to me mistaken. I won't raise general objections to verificationism here but not an objection that is decisive against both

the specific verificationist and truth conditional accounts of the sense of p and not p here. It is not true that the only reason why it might be in principle impossible to know the truth value of a proposition is that that proposition has no truth conditions. Of course on many respectable accounts - including these two - if p has no truth/falsity conditions then p is not a proposition at all. This is just part of the concept of a proposition. What is possible is that p or not p should be true and that it still remains logically impossible that the correct truth value be known. It could be guessed. But then there would be no way of confirming or falsifying the guess, even if it was guessed correctly. Suppose for example p denotes A's mental state at t_1 , A dies at t_2 and absolutely no evidence that either p or not p remains. Here, p is either true or false, say here it is true, but it is in principle impossible that p 's truth value should ever be known. (K=JTB).

It still might be the case that contradictions are senseless. It just is not the case that the reason why they are senseless is that there is no way in principle of deciding whether p is true or whether not p is true. This is because propositions may have truth conditions even if in principle we cannot know what they are and, at least on one account, if they have truth conditions they have sense. This is an objection to the verificationist account but not the truth conditional semanticists account. p has sense because we know what it would be for p to be true or false. Even if we cannot in principle determine p 's truth value we can conceive it, that is conceive of what it would be for p to have it.

Does it follow that contradictions have sense? They are false but don't have falsity/conditions. I think they do have sense. This is because p can have sense and not p can have sense; the contradiction p and not p cannot express a true proposition but expressing a true proposition is not the same as having sense. If p can be thought to be true then p 's truth conditions can be imagined. If not p can be thought to be true then not p 's truth conditions can be imagined. It follows that p has sense and also that not has sense. I think it follows from this that ' p and not p ' has sense, although the quite separate point that it cannot be used to say something true is also true.

It is also logically possible that a proposition should have a truth value, we know it has a truth value, we don't know what that truth value is and we can't conceive of what it would be for the proposition to be true (or false). to show this it is necessary to assume realism (which may or may not be true). That is it is true that every proposition is neither true or false independently of our knowledge of or beliefs about its truth value. Secondly; we can know p has a truth value. Suppose a reliable machine asserts that p (ie p is true). Suppose next the machine tells us p is a genuine proposition ie, it has a truth value. Suppose, the machine being reliable we believe it. Suppose further, assuming realism, that p is true. Next - the machine explodes. I've said it was a reliable machine so say, someone demolishes it. Suppose p is a claim about the past that the sole remaining evidence for was stored in this machine and that the evidence was destroyed with the machine. Then: we know that p is a genuine proposition; we have no idea of p 's truth value;

we don't know what it would be for p to be true or false (we don't know say in 3 possible strength - what the sense of p is (say p is in a foreign language) ii what event p purports to denote iii we do know ii but not whether the event took place). Despite this; p is true, p has sense, p is a genuine proposition.

I think this sort of argument leaves open the question of whether p and not p or p or not p best characterise a situation. It also leaves room for the possibility that there may be state of affairs which appear to us to be logically impossible which are not. There is an unthought way in which the contradiction can be shown to be a paradox. I leave open also at this stage the question of whether there might be contradictions that are informative; or even contradictions that have non-linguistic falsity conditions. In chapter ... we shall look at some examples where the truth conditions of not p seem to be fulfilled and the truth conditions of not p seem to be fulfilled. If it at least seems to us that this is the case then the state of affairs might be described as a contradiction. It is another question again whether such a purportedly contradictory state of affairs could actually obtain. A new sense perhaps who would have to be given to 'could'.

(Note that the question of whether a contradiction can be informative is not the same as the question of whether it can be informative that p is self-contradictory. It is easy to see that someone who did not know that some (complex) proposition was self-contradictory would come by some new information if this was pointed out to him. It is less easy to see how the contradiction thus exposed gave him any new information).

What is the notion of logical impossibility that should be invoked here? The formal criterion begs the question. If we say that S is a logically impossible state of affairs because p is self-contradictory and p characterises S then we invoke the concept of a contradiction to explain the concept of impossibility. But we may want the concept of impossibility to explain what a contradiction is. We might for example want to say (what seems to be true) that a contradiction is necessarily false, that is, that it is impossible that it be true, impossible that it denote any state of affairs.

Is there then a concept of impossibility - logical impossibility which does not include the idea of a contradiction? One weak candidate is this: S is logically impossible if and only if no conscious being can conceive of what it would be for S to obtain. There are at least two thing wrong with this, may be three. Firstly, there might be a ??? failure of the powers of the imagination of all conscious beings. From that alone it would not follow that S was impossible. It might obtain but be unimaginable. Secondly, the formulation makes use of 'cannot'. If this is just a practical 'cannot' the formulation is too weak and falls to the first objection. If it is a logical "cannot" then the formulation is circular, in one of two ways. It either invokes the notion of a contradiction or the notion of conceivability in the sense in which they need to be explained. Thirdly, and finally, it is not at all clear which has the priority here ie, which concept is to be taken as logically primitive; the concept of a contradiction or the concept of inconceivability. The problem is: is 'p and not p' logically

impossible because we cannot conceive of what would have to be the case for p and not p to obtain? Or, on the other hand, is it inconceivable that both p and not p just because not p and not p is self contradictory? The present formulation requires the truth of the first formulation, not the second, but arguments can be presented for each conclusion.

There are then different levels which there might be contradictions, either in the same or in different senses. They may exist in language: in the sense that a person has contradicted himself if he has said 'both p and not p ' - although it is not clear what he has done in both asserting and denying p . They may exist in thought in the sense that a person may think incoherently p and not p . They exist in a weaker or derivative sense in psychological dilemmas and human conflicts. Might there be contradictions about physical objects too?

De Re and De Dicto

There is a debate about essentialism in logic. Essentialism is the view that there are some properties of an object, call it x in the absence of which either x would not exist, or else would not exist as x . The position needs careful formulation. Suppose we say x 's essential properties are just those in absence of which x would not be x . Immediately this must be all of x 's properties. If x here means 'this particular actual x ', or indeed any particular actual x then x 's essential properties must be all of x 's properties. Any object which lacks any property possessed by x is not x . Not this x . Essentialism in this extreme formulation becomes an uninteresting doctrine because it applies to absolutely every property of everything. It is a

spelling court of the law of identity ($x = x$) in terms of properties of x .

Suppose then we refine this sort of essentialism and we say: those properties of x are essential to x in the absence of which x would not be an object of the sort we call ' x '. It follows from this that if any object lacks some essential property similar to those of x then that object is not of the same sort as x . It isn't an x . Suppose for example that it is an essential property of a physical object to have size. Suppose further that some object O has no size, then it follows that O is not a physical object.

This is better than saying that O could not exist as a physical object unless O had size, because having size is a necessary condition of anything being a physical object quite irrespective of whether it exists or not. Of course this is not to deny that 'a physical object with no size exists' describes a logically impossible state of affairs, but this is precisely because the sentence contains a contradiction. It claims that something has no size and is a physical object, which is (partly) to claim that something both has and lacks size, or that both p and not p . Any compound proposition which on analysis reduced to a contradiction or which contains a contradiction is itself a contradiction.

There are objections to essentialism which purport to undermine the de re de dicto distinction. But essentialism can be weakened to take account of these objections to preserve the de re de dicto distinction. Suppose it is true that entities do not have essential properties in themselves, quite independently of the criteria for their

individualisation, or the defining descriptions under which they are referred to. In other words suppose the Ayer/Quine account of necessity is true, that it is a consequence of conventionalism truth that there are necessities. So for example, on this view, size is not a property a physical object has essentially and intrinsically. It is just that if any x does not have size then we do not count that x as a physical object. Suppose this is true.

This seems to me to leave the de re/de dicto distinction completely unaffected. It is still possible to ascribe properties to objects essentially and it is still possible to generate necessary propositions. Consider:

(1) This physical object necessarily has size.

(2) It is a necessary truth that this physical object had size.

(1) ascribe an essential property to an object. (2) above ascribes necessary truth to a proposition: 'this physical object has size'. This is sufficient for there to be a de re/de dicto distinction. There is a quite different question which leaves the distinction unaffected. This is the question of whether there could be de re necessities without de dicto necessities or whether there could be de dicto necessities without de re necessities or - a third possibility - whether there is some sort of mutual logical dependence between the two. Any of these could be argued for. Even if this turned out to be true: any de re claim can be explained in de dicto terms but not all de dicto terms can be explained in de re terms. In other words if de re is

'reducible' to de dicto'. It still would not follow that the distinction was not well founded - that there did not exist two categories of different logical types. Consider this analogy: Supposed phenomenalism is true: that any claim about a po can be translated into a claim or set of claims about actual or hypothetical sense experiences. It wouldn't follow from this that there was not a quite well founded distinction between physical objects and sense experiences. The two sorts of thing are just not of the same logical type.

There are then de re necessities but there are different possible accounts of how de re necessities are possible. On one view objects would possess essential properties irrespective of human convention. On another they would not. There is a problem about what objects are; about whether linguistic idealism is true, not about logical categories.

That there are de re necessities is very important for what a contradiction is. We have assumed so far then there are contradictions in language: de dicto contradictions. Now I think there are also de re contradictions. A de re contradiction is produced by denying of some object x that it has one or more of its essential properties. For example:

(3) This physical object has no size.

is a de re contradiction. It is necessarily false but the false is made about a physical object. Although contradictory this claim is not self contradictory. It is not necessarily false in virtue of its semantics but in virtue of the

essential properties of a physical object. Of course this is not to deny that

(4) It is necessarily false that this physical object has no size.

is both de dicto and necessarily true. It predicates necessary falsehood of the proposition 'this physical object has no size'.

We should look now at the separate question of which has the wider scope: de re or de dicto contradictions. A case could be made out that de dicto contradictions are a subset of de dicto contradictions. On this view, necessarily lacking a property can be predicated of different sorts of entity. For example, the negative property of not having size is necessarily lacked by an physical object. So, it might be said, propositions are just a special case. Those which are contradictions necessarily lack the property of being true. The predicates remain unaffected through translations between de re and de dicto contexts: O is necessary lacks whether O is a proposition or an object. Just as de dicto a proposition necessarily is F where F is necessarily has the property of being true and this po necessarily has F' where F'' is the property of having size.

To think otherwise is to prejudice the concept of a property. A property of x is anything that is true of x regardless of the content of any true sentence describing x and regardless of what x is. For example x might be a physical object or apposition. X might be a physical or linguistic item. On some views of what a

proposition is a proposition might not be a linguistic item anyway: they are at least what are expressed by linguistic items such as sentences capable of being true or false.

What would be the consequence of accepting two views about essentialism for the doctrine of de re contradiction? The two views I have in mind are; i) the relaxed essentialism whether if x lacked any property which it does in fact possess then x would not be x , and ii) the Ayer/Quine view that which properties are essential to x depends on the chosen criteria for the individuation of x . The consequence is that; firstly - all empirical truths are necessary truths (de re); secondly; all empirical falsehoods are in fact de re contradictions, that is necessary falsehoods de re.

Why should we balk at this consequence? We happily accept that p and not p is a contradiction. The consequence of essentialism is that where p is falsified by a non-linguistics entis??? possessing or lacking a certain property then the proposition used to ascribe that property is necessarily false. Of course this is quite compatible with its being a contingent fact that x possesses the properties it does have or indeed, that x exists at all. If an object does exist, and if it is of the sort x it will have certain essential properties, indeed, every object that exist has the properties it does have and not a different set. Given this then i) x exists ii) x has certain specifiable properties including these are de re contradictions:

(5) x does not exist

(6) x lacks property F

If false they are necessarily false.

INDUCTION

Each of us is directly acquainted only with our sense data, universals, and perhaps ourselves. It follows that knowledge of anything else – the past, the future, other people, indeed anything outside our immediate private experience – must be by other means. Most of our knowledge depends upon induction, on inductive inference from facts about our immediate experience to other facts. It follows that the problem of the justification of induction is not only a problem in the philosophy of science; it is central to the possibility of nearly all our knowledge.

If we are asked why we believe the sun will rise tomorrow morning, we will reply, 'Because it always has risen.' If we are asked why we believe that, we reply, 'Because certain laws of motion hold.' If we are asked why we think those will continue to hold, we can only reply, 'Because they always have done.' In each case our reasoning is inductive. We argue that because something has happened, it will continue to happen. The problem is, as we saw above (pp. 157–9), that because x has happened n times it does not logically follow that x will happen again. Once this is realized it is hard to see how the occurrence of any finite set of events, no matter how large, is evidence that a further event of that sort will occur.

Russell carefully distinguishes the logical problem, the fact that there is no logical inference from 'Some A s are F ' to 'All A s are F ', or 'The next A is an F ', from the psychological truth that the repetition of a type of event

creates the expectation that an event of that kind will occur again. In Russell's example, 'the man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken' (PP 63).

Numerous qualitatively similar repetitions cause expectations of further such repetitions, but the problem of induction is whether such repetitions provide evidence for further repetitions.

Two proposed solutions to the problem of induction turn out to presuppose induction, and so beg the question. Any appeal to the uniformity of nature is circular, because the principle that nature is uniform is the thesis that nature operates in accordance with exceptionless natural laws. Those may only be established inductively, if at all. Any argument that we know the future will resemble the past because past futures have resembled the past is equally inductive, because only induction warrants the belief that future futures will resemble the past.

Russell's solution is to accept that we cannot logically, that is deductively, derive 'All As are F' from 'Some As are F', but we do not, in inductive contexts, wish to preclude the possibility of some As not being F. The probability of all As being F, or the next A being F, is all we should seek. Russell's principle of induction is therefore as follows: the greater the number of cases of A being F, the greater the probability of all As being F, and the greater the number of cases of A being F the greater the probability of the next A being F. It is always more probable that the next A will be an F (other things being equal) than that all As are F, because although 'All As are F' makes 'The next A is F' true, 'The next A is F' does not make 'All As are F' true.

Russell's probabilistic solution to the problem of induction is not refuted if, for a particular A, 'A is not F' is true. Suppose we have seen only white swans, in a large and increasing population of swans. Suppose, however (as is the case), there is also a population of black swans. We were none the less rational in supposing all swans to be white even though that supposition was false. From the fact that an event is unlikely to occur it does not logically follow that it will not occur, and conversely if an event is likely to occur, it does not logically follow that it will occur.⁶

LOGICAL ATOMISM

Logical atomism is a theory of meaning designed to solve philosophical problems. Russell's version of the theory is strongly influenced by that of his pupil and colleague, Wittgenstein, and is presented in the eight lectures *The Philosophy of Logical Atomism*, and the short essay entitled 'Logical Atomism'.

Russell tells us that his work in the philosophy of mathematics first suggested to him the correctness of logical atomism. It is not too misleading to say that in the philosophy of mathematics Russell attempts to reduce mathematics to logic, while in the philosophy of language he attempts to reduce language to logic: that is, to reduce ordinary language to a logical language. His ambition is to produce a logically perfect language which will not generate philosophical problems, the thought being that the imprecise and misleading nature of natural language leads us into those problems.

The title 'logical atomism' is intended to capture two aspects of the doctrine it names. Atomism is the thesis that the universe (and in logical

atomism the language which names it) is intrinsically a plurality. There really are discrete elements constituting what exists, even if the set of such elements is infinite, and even if the elements are thoroughly interrelated. Atomism is thus opposed to the monism of, say, the neo-Hegelians, who think that the pluralistic nature of what exists is an illusion, or mere appearance.¹⁰ Russell calls the theory 'logical atomism' because the atomic constituents of what exists are logical rather than physical, or, for that matter, mental. A physical atom could in principle be divided, but a logical atom could not even in principle be analysed into constituents.

According to Russell, 'the world contains facts, which are what they are whatever we may choose to think about them' (LA 35). Logical atomism thus entails a kind of realism. Something either is or is not the case, regardless of whether we believe it to be the case. Propositions are either true or false, irrespective of whether they are believed to be true or false. A fact is whatever makes a proposition true or false, and a fact is a state of affairs that obtains quite independently of thought and language. No fact is any particular thing or person: 'a fact is the sort of thing that is expressed by a whole sentence, not by a single name like "Socrates" ' (LA 36), so that A has property F, or that A bears relation R to B, are facts. It does not make sense to speak of true or false facts: 'A fact cannot be either true or false' (LA 38). The bearers of truth values are propositions, and we may talk of beliefs, for example, as true or false because the propositions we believe are true or false. A proposition is treated by Russell as an indicative sentence - that is, a sentence used to assert what is or is not the case. Clearly, not all

sentences are propositions because not all sentences are indicative. Some are imperative, interrogative, or exclamatory. Another way of explaining the concept of a proposition is to say that it is a sentence which is always capable of being prefaced by the propositional 'that', so ' "that Socrates is alive" and "that two and two are four" ' (LA 39) are propositions in Russell's terms.

It follows that no facts are propositions and every proposition is an indicative sentence. Every proposition is a complex symbol, a symbol that has symbols for its parts. For example, a sentence containing more than one word is a complex symbol and its constituent words are the simple symbols which are its parts. It is essential to clarity, and to the avoidance of error, not to mistake the properties of words and sentences for properties of objects and states of affairs. Meaning is a relationship between language and non-linguistic reality: nouns mean objects, adjectival expressions name qualities, and whole indicative sentences mean facts. Although names name objects, propositions assert or deny facts and do so truly or falsely, in virtue of those facts.

Facts are the objects of analysis in logical atomism. Complex facts are analysed into atomic facts, but the only way of doing this is via the analysis of propositions: the analysis of molecular (or complex) propositions into atomic (or simple) propositions, and the analysis of atomic propositions into simple symbols.

Simple symbols are the ultimate constituents of propositions, and they denote the simple components of the facts which make the propositions true or false. 'Red', for example, is a simple symbol because it cannot be analysed into constituent

symbols, yet it may be a constituent of a symbol. Although 'red' can in one sense be defined (a description of what 'red' is can be given in terms of physics), it cannot be defined in the sense of 'analysed'. No verbal definition of 'red' could make a person unacquainted with the colour red know what red is. In this sense of 'red', ' "red" could not be understood except by seeing red things' (LA 49).

Atomic propositions are analysed into proper names and predicates. Proper names name particulars. A particular is a component of a fact which exists logically independent of other components and yet bears some relation to them. A predicate designates a quality, so 'red', for instance, is a predicate, but 'John' is a proper name. Russell says that 'to understand a name you must be acquainted with the particular of which it is the name' (LA 60).

Molecular propositions are truth functional. This means their truth or falsity depends upon the truth or falsity of their constituent propositions.

Propositional functions are sentences with at least one uninterpreted constituent, and they are expressed in a symbolism. 'P', 'Q', 'R', etc. are propositional variables, and '-', '·', 'v', '→', '≡' are logical constants. The logical constants may be defined by truth tables as follows:

P - P
T F
F T

'-P' (read 'not P') is true if and only if P is false.

P . Q
T T T
T F F

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F F T
F F F

'P . Q' (read 'P and Q') is true if and only if both P and Q are true.

P v Q
T T T
T T F
F T T
F F F

'P v Q' (read 'P or Q') is true if and only if either both P and Q are true or if only one of P and Q is true.

P → Q
T T T
T F F
F T T
F T F

'P → Q' (read 'if P then Q') is true if and only if either not P is true or Q is true or both.

P = Q
T T T
T F F
F F T
F T F

'P = Q' (read 'P if and only if Q') is true if and only if either both P and Q are true or both P and Q are false. The analysis of a propositional function may proceed in a purely logical, indeed algorithmic, manner. A complex propositional function is a tautology, a contradiction, or contingent, depending on the truth values of its constituent atomic propositional variables. A

proposition is atomic if, and only if, it has no constituent propositions.

The truth-functional analysis of propositions leaves entirely on one side questions about the semantic constituents of propositions. Indeed, propositional functions are undetermined in the sense that P , Q , etc. may stand for any (mutually distinct) propositions whatsoever. Russell says that 'One may call a propositional function necessary, when it is always true possible, when it is sometimes true impossible, when it is never true' (LA 87) but it does not make sense to ascribe these properties to propositions: 'Propositions are only true or false' (LA 88).

It is a source of philosophical error to confuse the properties of propositions with those of propositional functions, as it is to confuse properties of both of those with properties of facts. Being clear about grammar makes progress possible in philosophy. Indeed, 'practically all traditional metaphysics is filled with mistakes due to bad grammar' (LA 128-9).¹¹

MEANING AND TRUTH

Meaning

Russell subscribes to a referential theory of meaning; that is, he thinks the meaning of a word is the thing the word refers to: 'Words all have meaning, in the simple sense that they are symbols that stand for something other than themselves' (PM 47). Russell's referential theory is, however, more sophisticated than most and contains attempts to forestall philosophical objections.¹²

If the meaning of a word is held to be the object the word refers to, then certain difficulties arise. An object may be referred to by a number of

non-synonymous expressions: for example, 'the person with open eyes' and 'the person understanding this sentence' both refer to you but their meaning is not the same. Conversely, one description, say 'the head of department', may be used to refer to more than one person: a university professor at Cambridge and one at Edinburgh, for instance. If meaning were only reference, both these facts would be impossible. Again, if the meaning of

'The person writing this sentence is me'

were me, then if I ceased to exist the meaning of that sentence would cease to exist. Clearly, however, the meaningfulness of that sentence does not depend so closely on my existence.

If you assert that 'Scott is the author of *Waverley*' (LA 105), what you assert is true but not tautologically true, as it would be if meaning were only reference. It is contingently true because a definite human being, Scott, wrote a specific book, *Waverley*. It is not true by definition but in virtue of a certain fact. 'Scott' is a name and 'the author of *Waverley*' is a definite description. The definite description is true of the person named by the proper name, and that is what makes the whole proposition true. If one were to substitute a proper name for the definite description in this example the result would be a different proposition, and if that proper name referred to Scott the new proposition would be a tautology.

The distinction between proper names and definite descriptions explains how identity statements may be informative. Although 'George IV wished to know if Scott was the author of *Waverley*, it is not true

that George IV wished to know if Scott was Scott'
(LA 105) .

In *The Philosophy of Logical Atomism* Russell argues
that

the notion of meaning is always more or less
psychological' (LA 40) .

Nevertheless, as we have seen, names name
particulars, predicates denote qualities or
relations, and indicative sentences express facts;
thus there are three different kinds of meaning.
Giving the meaning of an expression involves
analysing that expression, and ideally the meanings
of a logically perfect language would be clear.
Ordinary language is riddled with ambiguities but
this is not necessarily a disadvantage for non-
philosophical work. Ambiguity gives ordinary
language a certain richness and allows for
originality in expression.

Russell's most influential contribution to the
theory of meaning is his 'theory of
descriptions'.¹³ A singular definite description, a
sentence of the form 'the so-and-so', may be true
or false, but if it is false because what it
putatively describes does not exist, then two
philosophical problems arise. If we say, 'the
present king of France is bald' is false, we seem
to be covertly asserting the existence of what does
not exist, and that is manifestly false. This is a
problem concerning existence. If we claim that
names name objects and propositions name facts,
then objects and facts are the meanings of names
and propositions. However, it then follows that if
the king of France does not exist, the meaning of
'the present king of France' does not exist and the

expression is meaningless. Similarly, if the king of France is not bald, then the fact that the king of France is not bald does not exist, so the meaning of 'the present king of France is bald' does not exist and that proposition is meaningless. This is a problem concerning meaning. The point of the theory of descriptions is to show in what way false existential claims are meaningful.

Russell's solution is to claim that 'a name has got to name something or it is not a name' (LA 100). If a putative name names nothing, it is a truncated description of the logical form 'x has such and such properties' (LA 100). The proposition 'the present king of France is bald' is in fact two propositions:

(1) 'There is a c such that c is now king of France'

(2) 'c is bald'.

Analysing the proposition this way avoids construing its denial, 'The present king of France is not bald', as entailing 'There is such a person as the king of France and that person is not bald' (LA 109), which is false. There are in fact two ways of denying the proposition: by denying the king of France exists and by denying the king of France is bald. So the denial of the proposition is not

(3) 'There is a c such that c is now king of France and c is not bald'

but

(4) 'Either there is not a c such that c is now king of France, or, if there is such a c, then c is not bald'. (LA 109)

If we wish to deny the proposition we have a choice of asserting either of the disjuncts of (4). In these ways we will not have said something meaningless and not have covertly asserted the existence of what does not exist.

Truth

Truth and falsity are properties of beliefs; but if I believe something, what I believe is a sentence; thus truth and falsity are derivatively properties of sentences. However, the truth or falsity of a sentence is logically independent of its being believed. Our beliefs may be true or false, and what is the case and what is not the case in no way depends on our beliefs.

A belief is true or false in virtue of the occurrence or non-occurrence of extra-linguistic states of affairs - 'facts' as Russell calls them in *The Philosophy of Logical Atomism*, or 'verifiers' as he calls them in *An Inquiry into Meaning and Truth*. For example, the truth of 'Caesar was assassinated' (IMT 215) consists in the occurrence of a particular event - the assassination of Caesar. This event is a fact which verifies the sentence.

Russell rejects the view that truth can be explained in terms of verifiability, perhaps for the very good reason that to 'verify' p means to demonstrate the truth of p, so any such explanation would be circular. Later, Russell subscribes to a correspondence theory of truth: p is true if, and only if, the fact expressed by p exists: 'The "verifier" is defined as that occurrence in virtue of which my assertion is true (or false)' (IMT 219).

A series of philosophical problems exists which concern what the truth of propositions about unobserved facts consists in: propositions about other minds, historical events, theoretical particles, for example. Russell, correctly I think, wishes to maintain that the truth of such propositions consists in the existence of these facts: 'on what may be called the realist view of truth, there are "facts", and there are sentences

related to these facts in ways which make the sentences true or false, quite independently of deciding the alternative' (IMT 232), but 'the difficulty is to define the relation which constitutes truth if this view is adopted' (IMT 234). I should say the truth of the proposition lies simply in the existence of the fact and the proposition, and there is no residual relation and so no residual problem.¹⁴

9 MATHEMATICS

Russell argues in the massive and influential three-volume work *Principia Mathematica* that mathematics is reducible to logic. Indeed he maintains that essentially there is no difference between mathematics and mathematical logic because any putative distinction between the two is arbitrary. As he puts it in *The Principles of Mathematics*, 'All mathematics is symbolic logic ... all mathematics is deduction by logical principles from mathematical principles' (PM 5), and in the *Introduction to Mathematical Philosophy*:

Logic has become more mathematical and mathematics has become more logical. The consequence is that it has now become wholly impossible to draw a line between the two; in fact the two are one. They differ as boy and man: logic is the youth of mathematics and mathematics is the manhood of logic. (IMP 194)

Russell wrote an enormous amount of outstanding quality on the philosophy of mathematics, but here I shall confine attention to one fundamental question: 'What is a number?' (IMP 11).

A number is not identical with the number of things there are that number of. For example, the number two is not identical with any two objects, the number three is not identical with any three objects, and so on. Rather, the number two is what is common to all pairs of objects, the number three

is what is common to all trios, and so forth. All pairs of objects form a class of objects, the class of pairs; all trios form a class, the class of trios. Russell maintains that a number is a class of such classes. The number two, for example, is the class of pairs, the number three the class of trios, etc., but further argument is needed to reach this conclusion on pain of using 'number' in a question-begging way.

To analyse the concept of number Russell uses the notions 'class', 'member' and 'similar'.

Classes may be defined extensionally or intensionally. To define a class extensionally is to enumerate the members of that class (a, b, c, ... n). To define a class intensionally is to mention a defining property of the members of the class, a property such that if an individual possesses that property then it logically follows that it is a member of that class, and such that if it lacks that property it logically follows that it is not a member of that class. Intension is logically prior to extension for Russell because extension can always be reduced to intension but intension cannot always be reduced to extension. For example, we could not in principle enumerate all the members of an infinite set, but we could state the defining characteristic of all members. The series of natural numbers (0, 1, 2, 3 ... n) is infinite, so any definition of 'natural number' in terms of classes must be by intension and not by extension. To define 'number' in terms of classes, it is necessary to know when two classes have the same number of members and so belong to the same class of classes. This is possible without knowing how many members those two classes contain. For example, if there were only monogamy the number of wives would be the number of husbands. Clearly, this is knowable a priori because the relation is

'one-one' (IMP 15). Russell distinguishes relations that are 'one-one', 'one-many', and 'many-one'. A relation is one-one if, and only if, 'if x has the relation in question to y , no other term x' has the same relation to y , and x does not have the same relation to any term y' other than y ' (IMP 15).

Clearly, a relation is one-many if, and only if, x has a relation to y and to y' but no term x' distinct from x has that relation to either y or y' ; and a relation is many-one if, and only if, both y and some term distinct from y , y' , has a relation to x but does not have that relation to x' , some term distinct from x . Notice the concept of number is nowhere employed in these definitions.

Russell uses the idea of a one-one relation to provide a criterion for the similarity of classes: 'two classes are said to be "similar" when there is a one-one relation which correlates the terms of the one class each with one term of the other class' (IMP 15-16). If a is related to b by relation R , then a is the 'domain' of R , and b is the 'converse domain' of R . The relation 'similar' between classes has three logical properties:

- (1) Reflexivity: 'Every class is similar to itself' (IMP 16).
- (2) Symmetry: 'If a class a is similar to a class b , then b is similar to a ' (IMP 16).
- (3) Transitivity: 'If a is similar to b , and b to c , then a is similar to c ' (IMP 16).

Russell defines 'number' in this way: 'A number is anything which is the number of a class' (IMP 19).

There is no circularity here because he uses 'class' and 'similar' to define 'the number of a class' without using 'number': 'The number of a class is the class of all those classes that are similar to it' (IMP 18). Thus the number two comes out as the class of all couples, the number three is the class of all classes of three objects, and

so on. Notice 'the class of all couples will be the number 2' (IMP 18). Numbers, then, are not Platonic, metaphysical or other-worldly, but classes of classes.¹⁵

Russell and Whitehead's attempt in *Principia Mathematica* to derive the order of mathematics from certain logical axioms was not entirely successful, as Gödel demonstrated.¹⁶ However, it is in large part because of their efforts that mathematical logic is the thriving subject it is today.¹⁷

10 CAUSATION

There is a critical and a constructive side to Russell's theory of causation. He presents a neo-Humean set of criticisms of our commonsensical concept of 'cause', and then replaces this with an account he regards as scientific.

Our belief that causes necessitate their effects can be expressed as follows: If A causes B, then if A happens B cannot fail to happen. This belief is deeply embedded in commonsense. It is nevertheless an illusion because there is no compulsion or inevitability in nature. As Hume saw, the belief that A necessitates B is only a psychological product of our witnessing events of type A followed always by events of type B within our experience. We expect B to follow A and mistake our subjective expectation for an objective compulsion or necessity. Hence, Russell says, the ideas of 'compulsion' and 'necessity' are 'purely anthropomorphic' (OP 121).

Observing that Bs follow As, we feel justified in inferring that any A will be followed by a B. However, we are equally entitled to infer the reverse: that any B was caused by an A:

When you get a letter you are justified in inferring that someone wrote it, but you do not

feel that your receiving it compelled the sender to write it' (OP 121).

Effects are no more necessitated by their causes than causes are necessitated by their effects. Our powers of prediction and retrodiction, and our expectations about past and future, are not to be taken for natural necessities. Thus 'science is concerned merely with what happens, not with what must happen' (OP 121). Russell's science generalizes probabilistically about Hume's constantly conjoined events. It dispenses with the idea of a force which, in Russell's view, 'is as mysterious as the influence of the stars in astrology' (OP 122).

Russell replaces the concept of causal necessity with the concept of a probabilistic law. If an event, A, occurs, then the probability of an event B occurring is greater or less depending on the number of previously conjoined events of sort A and sort B. Scientific generalizations are thus always probabilistic; the limiting deterministic case, where 'if A then B' has a probability of one, is rare and symptomatic of a young science: 'Genuine laws, in advanced sciences, are practically always quantitative laws of tendency' (OP 150).¹⁸

Cambridge Changes and the Problem of Induction

Induction Justified

Time does not make a difference

This a is a b, this a is a be etc therefore the next a will be a b.

The next inference is qualitatively identical with the previous one (except for time and other irrelevant features).

The next inference is therefore identical with an inference that worked was inductively 'correct'.

(There is still a problem about in what sense a correct inductive inference is an inference, and what its being correct consists in.)

'It is qualitatively identical with an inference that was justified'
is the justification of the next inductive inference.

(If something is qualitatively identical to something that is justified it is justified.)

The justification of the earlier inferences is that they worked. The conclusions came out true and the conclusions were believed because the premises were believed.

Of course induction falls short of deduction.

If there are new features which make the new inference fail this is not an objection to induction. The inference was justified because it was qualitatively identical with a justified inference.

GLOSSARY

aesthetics. The attempt to solve philosophical problems about art and the beauty of nature.

a fortiori. (Latin for: 'from the stronger') 'all the more evidently' or 'even more obviously', especially of the valid derivation of a conclusion of the form 'All *b*'s are *F*' or '*b* is *F*' from a premise of the form 'All *a*'s are *F*' where *b*'s are a subclass of *a*'s.

ambiguous. (1) having two and only two meanings (2) having at least two meanings. 'Ambiguous' is ambiguous, in sense (1), between meaning (1) and meaning (2).

analytic/synthetic distinction. Any analytic proposition is true, but not all true propositions are analytic. Definitions of 'analytic': (1) The meaning of an analytic proposition is sufficient for its truth. (2) The truth of an analytic proposition is necessary for its meaning. (3) An analytic proposition is true by definition. (4) An analytic proposition is true in virtue of the meaning of its constituent terms. (5) The negation of an analytic proposition entails a contradiction. (6) In an analytic proposition the predicate is 'contained' in the subject. Analytic propositions provide no information about non-linguistic reality.

Any synthetic proposition is either true or false but not all propositions are synthetic. Definitions of 'synthetic': (1) The meaning of a synthetic proposition is insufficient for its truth. (2) The truth of a synthetic proposition is not necessary for its meaning. (3) A synthetic proposition is true or false but not by definition. (4) A synthetic proposition is true or false but not in virtue of the meaning of its constituent terms. (5). The negation of a synthetic proposition is

internally consistent. (6) In a synthetic proposition the predicate is not 'contained' in the subject. (7) Synthetic propositions may provide information about non-linguistic reality. (8) synthetic propositions provide non-tautological information.

anti-realism. The thesis that if p is true then p is, at least in principle, provable.

a priori/a posteriori. A distinction between two ways of knowing that a proposition is true. Definitions of '*a priori*': (1) p is *a priori* if and only if p 's truth value may be decided independently of empirical observation. (3) p is *a priori* if and only if p 's truth value may be decided purely intellectually. (4) p is *a priori* if and only if p 's truth value is decidable with no more experience than that required to understand p .

Definitions of '*a posteriori*': (1) p is *a posteriori* if and only if p 's truth value may be decided only through empirical observation. (2) p is *a posteriori* if and only if p 's truth value may not be decided purely intellectually (but requires experience). (3) p is *a posteriori* if and only if p 's truth value is decidable only with more experience than that required to understand p .

argument. A set of propositions such that some (the premises) are putative evidence for another (the conclusion). Any argument is either a deductive argument or an inductive argument, but not both. It makes no sense to talk of arguments as true or false. Propositions are either true or false and so premises and conclusions are either true or false. Any deductive argument is either valid or invalid but not both, and either sound or unsound but not both.

conclusion. If p is the conclusion of an argument then p is putatively proven by the premises of that argument.

conjunction. and

consistent. If the truth of p does not preclude the truth of q and if the truth of q does not preclude the truth of p then p and q are consistent.

If the truth of p precludes the truth of q or if the truth of q precludes the truth of p , or both, then p and q are inconsistent.

contradiction. The conjunction of a proposition and its negation: ' p and not p '.

contradiction, law of (or, law of non-). The axiom that the conjunction of a proposition and its negation is false: $-(p \ \& \ -p)$

contradictories. p and q are contradictories if there are no conditions under which they have the same truth value: if p then not q and if q then not p . ' p if and only if q ' is always false and ' $\text{not } (p \text{ if and only if } q)$ ' is always true if p and q are contradictories.

contraries. p and q are contraries if there are no conditions under which p and q are both true, that is: if p then not q and if q then not p . If p and q are contraries, ' p and q ' is always false and ' $\text{not } (p \text{ and } q)$ ' is always true.

deduction. In a deductive argument, putatively, if the premises are true then the conclusion is true. In a valid deductive argument, if the premises are

true then the conclusion is true. In a valid deductive argument the truth of the premises is inconsistent with the falsity of the conclusion. In an invalid deductive argument, it is false that if the premises are true then the conclusion is true. In an invalid deductive argument, the truth of the premises is consistent with the falsity of the conclusion. Any deductive argument is either valid or invalid but not both. Any deductive argument is either sound or unsound but not both.

definition. Every definition is either verbal or ostensive. A word has a verbal definition if the meaning of that word may be explained in other words. A word has an ostensive definition if the referent of the word may be indicated as its meaning.

The defining characteristics (or essential properties) of a thing are the features necessary for it to be the sort of thing it is (or for it to belong to a certain type or class). The defining characteristics (or essential properties) of x are singularly necessary but jointly sufficient for x 's being F .

empiricism. The thesis that all concepts are acquired through experience and *a priori* metaphysical knowledge is impossible.

equivalence relation. A relation, ' R ', is an equivalence relation if and only if R is transitive, symmetric and reflexive.

epistemology. The attempt to answer philosophical questions about knowledge.

ethics. The attempt to answer philosophical questions about morality.

excluded middle, law of. The axiom that either a proposition or its negation, but not both, is true:
 $(p \vee \neg p)$

family resemblance. There is a family resemblance between the members of a class when no single property is common to all and only the members of that class, yet any two members of the class have at least one property in common.

functionalism. The thesis that what something is is essentially a cause or an effect or both. The claim that 'What is it?' is answered by answering 'What is it for?'. The claim that distinct items stand in different causal relations.

idealism. The thesis that if something exists it is a mind or logically depends on a mind.

identity. (1) the relation between a and b if a is b (2) the relation between a and b if a is exactly like b (3) the relation between a and b if a is like b . See numerical identity and qualitative identity.

identity, law of. The axiom that a proposition implies itself: $(p \rightarrow p)$

imperative. Sentence that expresses a command. Imperatives cannot be true or false, so do not express propositions. Nevertheless, quasi-entailments may hold between imperatives. For example, 'All a 's: become F !' quasi-entails 'This a : become F !'.

implication. p implies q if and only if, under all conditions, if p is true, then q is true.

indicative sentence. Sentence capable of being true or false. Sentence that expresses a proposition. See interrogative and imperative.

induction. In an inductive argument, putatively, if the premises are true then the conclusion is probably true. In a strong inductive argument, if the premises are true then the conclusion is probably true. In a weak inductive argument, it is false that if the premises are true then the conclusion is probably true. Any inductive argument is either strong or weak but not both. In any inductive argument the truth of the premises is consistent with the falsity of the conclusion.

interrogative. Sentence that expresses a question. Interrogatives cannot be true or false, so do not express propositions. Nevertheless, quasi-entailments may hold between interrogatives. For example: 'Should all a 's become F ?' quasi-entails 'Should this a become F ?'. .

logical behaviourism. The thesis that any sentence or set of sentences about minds may be translated without loss of meaning into a sentence or set of sentences about publicly observable behaviour. The thesis that a mind is a set of exercised or unexercised behavioural dispositions. The reduction of the inner and subjective to the outer and objective.

logical equivalence. p and q are logically equivalent if and only if p and q have the same truth value under all conditions: if p then q and if q then p .

logical truth. Proposition true on logical grounds alone. Proposition provable from no assumptions. See contradiction, law of; excluded middle, law of; identity, law of.

materialism. The thesis that if something exists it is physical, or logically depends upon something physical.

metaphysics. The attempt to answer philosophical questions about the ultimate nature of reality, or reality as a whole.

necessary and sufficient conditions. If the truth of p is a necessary condition for the truth of q then if not p then not q (ie if p is not true then q is not true). If the truth of p is a sufficient condition for the truth of q then if p then q (ie if p is true then q is true). If the truth of p is both necessary and sufficient for the truth of q then q if and only if p . If p is a necessary condition for q then q is a sufficient condition for p . If p is a sufficient condition for q then q is a necessary condition for p .

necessary truth. (1) p is a necessary truth if and only if p is true and could not be false. (2) p is a necessary truth if and only if the negation of p is a contradiction. (3) p is a necessary truth if and only if p is true in all possible worlds.

necessarily false. (1) p is necessarily false if and only if p is false and could not be true. (2) p is necessarily false if and only if the negation of p is a tautology. (3) p is necessarily false if and only if p is false in all possible worlds.

neutral monism. The thesis that mental and physical are two aspects of some underlying reality that is intrinsically neither mental nor physical.

numerical identity. a is numerically identical to b if and only if a is b .

a is numerically identical to b if and only if a is the same one as b . The relation 'is numerically identical to' is transitive, symmetrical and reflexive, therefore it is an equivalence relation. If a is numerically identical to b and b is numerically identical to c , then a is numerically identical with c . If a is numerically identical to b then b is numerically identical to a . a is numerically identical to a .

ontology. (1) The attempt to prove what exists and what does not exist. (2) The attempt to clarify or answer 'What is being?'.

phenomenology. The description of appearances, especially with the aim of showing how knowledge is possible.

phenomenalism. The thesis that any sentence or set of sentences about physical objects may be translated into a claim or set of claims about sense contents without loss of meaning.

Philosophy. The attempt to answer philosophical questions.

possible world. A way what is could be.

possibility. A state of affairs is logically possible if and only if it has a consistent description. A state of affairs is logically

impossible if and only if it has an inconsistent description.

predicate. That which remains of an indicative sentence if its subject is subtracted. An expression, or expression sense, used to ascribe a property.

premise. In an argument, a proposition that is putative evidence for the conclusion.

proposition. That which is expressed by an indicative sentence. The meaning of an indicative sentence. A proposition is either true or false but not both. The letter 'p' stands for any proposition. 'q' stands for any proposition distinct from p, 'r' for any proposition distinct from both p and q, etc. Any proposition in an argument is either a premise or the conclusion of that argument.

qualitative identity. a is qualitatively similar to b if and only if a is like b in at least one respect. a is qualitatively identical to b if and only if a is like b in every respect. a is qualitatively similar to b if and only if a and b are of the same sort (kind, type).

question. Linguistic expression of the desire to know, expressed in interrogative form.

philosophical question. Question with no available type of method for answering.

rationalism. The thesis that there are concepts that are not acquired empirically or the thesis that *a priori* metaphysical knowledge is possible, or both.

realism. The thesis that ' p is true' does not entail ' p is provable'. The thesis that what there is does not depend on the possibility of knowing or showing what there is.

reflexive. A relation, ' R ', is reflexive if and only if, a is related by R to a .
A relation, ' R ', is irreflexive if and only if, a is not related by R to a .

relation. A property of more than one thing or a property of one thing in more than one respect.

self-evident. p is self-evident if and only if knowing p 's truth value is a necessary condition for understanding p . p is self-evident if and only if understanding p is a sufficient condition for knowing p 's truth value. p is self-evidently true if and only if knowing that p is true is a necessary condition for understanding p . p is self-evidently true if and only if understanding p is a sufficient condition for knowing that p is true. p is self-evidently false if and only if knowing that p is false is a necessary condition for understanding p . p is self-evidently false if and only if understanding p is a sufficient condition for knowing that p is false.

sense and reference. A term has a sense if and only if it has a verbal definition. A term has a reference if and only if it denotes something. Two or more terms may have the same sense but different referents.

Two or more terms may have the same referent but different senses.

Having a sense is neither necessary nor sufficient for having a referent.

Having a referent is neither necessary nor sufficient for having a sense.

sound. If an argument is sound then it is valid and its premises are true, so its conclusion is true. If an argument is unsound then either it is invalid, or at least one premise is false, or both. If an argument is sound then it is valid. All the propositions in a sound argument are true.

subcontraries. If p and q are subcontraries then there are no conditions under which p and q are both false, that is: if not p then q and if not q then p . If p and q are subcontraries then ' p or q ' is always true and ' $\text{not } (p \text{ or } q)$ ' is always false.

subject. In logic and grammar: that which remains if the predicate is subtracted from an indicative sentence. In metaphysics and ontology: that which has experiences.

symmetric. A relation, ' R ', is symmetric if and only if, if a is related by R to b , then b is related by R to a . A relation, ' R ', is asymmetric if and only if, if a is related by R to b , then b is not related by R to a .

substance. (1) that which does not depend upon anything else for its existence. (2) that which bears properties but is not itself a property.

tautology. If p is a tautology then the semantics of p are sufficient for the truth of p . In propositional logic, a truth functional proposition that is true under all distributions of truth values to its atomic propositions. In a subject/predicate proposition that is a tautology,

overtly or covertly, a predicate is ascribed to the same subject twice.

theorem. Proposition provable from no assumptions.

transitive. A relation, ' R ', is transitive if and only if, if a is related by R to b , and if b is related by R to c , then a is related by R to c . A relation, ' R ', is intransitive if and only if, if a is related by R to b , and if b is related by R to c , then a is not related by R to c .

type/token. a and b are of the same type if and only if a is like b in at least one respect. a and b are of only the same type if and only if a is like b in every respect. a and b are of the same type if and only if a and b are qualitatively similar. a and b are of only the same type if and only if a and b are qualitatively identical.

a and b are the same token if and only if a is b . a and b are the same token if and only if a is the same one as b . a and b are the same token if and only if a is numerically identical to b .

truth. The relation between a proposition and its subject matter is that subject matter is as expressed by that proposition. If p is true then p is not false. If p is false then p is not true.

use and mention. In the sentence "The sky is blue." the word 'blue' is used not mentioned. In the sentence "'Blue' has four letters." the word 'blue' is mentioned, not used. When a word is mentioned, it is placed between single quotation marks; ' '. When a word is used the single quotation marks are omitted.

valid. If an argument is valid then the truth of the premises is inconsistent with the falsity of the conclusion. If an argument is invalid then the truth of the premises is consistent with the falsity of the conclusion. If an argument is valid then if the premises are true then the conclusion is true. If an argument is invalid then even if the premises are true, the conclusion could still be false. If an argument is valid then it is either sound or unsound. If an argument is invalid it is unsound.

Knowing the Philosophical

Answers

by

Contrary to popular belief, the answers to many philosophical questions are known.

For example, on a widely endorsed but not uncontroversial analysis of 'knowledge', if you justifiably believe there was no first event and I justifiably believe there was, then one of us knows whether there was a first event, because there either was or was not a first event. If you justifiably believe there is no God and I justifiably believe there is a God, then one of us knows whether there is a God. If I justifiably believe there is life after death and you justifiably believe there is no life after death then one of knows whether there is life after death, and so on for the answers to Is the mind the brain?, Is there free will?, Is everything physical?, Do numbers exist? and many others.

Stephen Priest *Remarks on Logic* (2012 Draft)

It is not a sound objection to philosophical knowledge that there is still well argued disagreement about the right answer to a philosophical question, because from:

(1) $\text{By } \neg p$

we cannot validly infer

(2) $\neg \text{Kxp}$

Nor is it a sound objection to philosophical knowledge that it is not known which of us knows the right answer to a philosophical question because neither:

(3) $\text{Ky } (\text{Kxp})$

nor

(4) KKxp

is required for

(5) Kxp

because (5) does not entail (3) and (4).

Nor is it required for

(6) $\text{Kxp} \vee \text{Kyp}$

that

(7) $(\exists z) [\text{Kz } (\text{Kxp}) \vee \text{Kz } (\text{Kyp})]$

nor that

(8) $(\exists z) [Kz (Kxp) \ \& \ Kz \neg (Kyp)] \vee [Kz \neg (Kxp) \ \& \ Kz (Kyp)]$

because (6) does not entail (7) and (8).

The answer to any philosophical question admitting of a bivalent answer might be known if good enough reasons for believing it can be adduced. The answers to philosophical questions are very close to us. It is the proofs of those answers that are remote. (1)

(1) The claim that some of us know the answers to some philosophical questions does not depend closely on the claim a knows that p if and only if a believes that p , a has reasons for believing that p and p is true.

Suppose knowledge is only having true information in the mind (or the brain), then if that information entails the answer to a philosophical question then someone knows that answer.

(2) I am grateful to Paolo Crivelli, Michael Inwood, Adrian Moore and Timothy Williamson for useful discussion.

$\neg(Kxp \rightarrow KKxp)$.

A DICTIONARY OF LOGIC

by

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A

'a'. The indefinite article in English. (1) At least one. (2) One and only one. (3) Something or other.

abstract noun. Noun used to denote what cannot be counted: 'butter', 'hydrogen', 'endurance'. Noun lacking a plural. See count noun, sortal.

accessibility. A possible world w is accessible from a possible world w' if and only if

actual. Anything is actual in a possible world if and only if it is in that world. Any world, w , is itself actual at w . To say that our world is the actual world is not to say that it exists but to say that we are in it. See modal logic, modal realism, possible worlds.

actualism. The doctrine that only the actual world exists. the doctrine that either only the actual world is real or the actual world is more real than other worlds.

ad absurdum. Latin for 'to absurdity'. See *reductio ad absurdum*.

adjective. Word used as a predicatively. Word used to ascribe a property.

adverb. Word used to denote the manner or time of an action or event.

affirmation. The claiming that a proposition is true. The expressing of a putative truth. The asserting of what is putatively the case.

affirmative. Appropriate for expressing a truth.

affirming the consequent, fallacy of. Wrongly taking as valid the invalid inference from $p \rightarrow q$ and q to p .

algorithm. Purely mechanical proof procedure or method of computation. Proof or computation that a machine could perform.

'all'. See quantifier.

ambiguity, lexical. Ambiguity caused by an ambiguous word.

Lexical ambiguity is not structural. See ambiguous, structural ambiguity.

ambiguity, structural. Ambiguity caused by grammatical construction. Structural ambiguity is not lexical. See ambiguous, lexical ambiguity.

ambiguous. (1) With two and only two meanings. (2) With at least two meanings.

ampersand. The name of a symbol for conjunction:
'&'.

analysis. The breaking into logical or semantic parts.

analytic. Definitions of 'analytic' and criteria for being analytic:

(1) Any analytic proposition is true, but not all true propositions are analytic. (2) The meaning of an analytic proposition is sufficient for its truth. (3) The truth of an analytic proposition is necessary for its meaning. (4) An analytic proposition is true by definition.

(5) An analytic proposition is true in virtue of the meaning of its constituent terms. (6) The negation of an analytic proposition is a contradiction. (7) In an analytic proposition the predicate is 'contained' in the subject. (8) Analytic propositions provide no information about non-linguistic reality.

analyticity. The property of being analytic.

and. ' p and q ' is the proposition that p is true and q is true. The meaning of 'and' ('&', ' . ' , '^') is defined in the propositional calculus by this truth table:

<u>p & q</u>		
	q	q
	T	F
p T	T	F
p F	F	F

So the whole proposition ' $p \ \& \ q$ ' is true when p is true and when q is true but otherwise false. That information may be displayed like this:

$p \ \& \ q$: TFFF

&-introduction (&I). The rule of the propositional calculus that legitimates the inference: $p, \ q \vdash p \ \& \ q$. Given p and given q , $p \ \& \ q$ follows as conclusion.

&-elimination (&-E). The rule of the propositional calculus which legitimises the inference: $p \ \& \ q \vdash p$. Given $p \ \& \ q$ as an assumption, p follows or q follows as conclusion.

antecedent. In a conditional proposition, the component proposition which precedes the implication sign: ' $\rightarrow$ '. For example, in ' $p \rightarrow q$ ', ' p ' is the antecedent. See consequent.

anti-realism. The thesis that the possibility of a proposition's being true or false does not exceed the possibility of demonstrating its truth or falsity. If p is true or false p is provable or refutable. ' p is true' entails p is provable. ' p is false' entails p is refutable. Anti-realism is inconsistent with realism.

antisymmetric. A relation R between any distinct objects x and y is antisymmetric if and only if x is related by R to y then y is not related by R to x . '**antisymmetric**' (def.): R is antisymmetric iff $(\forall x) (\forall y) \neg(x = y) \ \& \ (Rxy \rightarrow \neg Ryx)$.

asymmetric. A relation, ' R ', is asymmetric if and only if, if a is related by R to b , then b is not related by R to a . '**asymmetric**' (def.): R is asymmetric iff $(\forall x) (\forall y) (Rxy \rightarrow \neg Ryx)$.

'any'. See quantifier.

a posteriori. (1) *p* is *a posteriori* if and only if *p*'s truth value may be decided only through empirical observation. (2) *p* is *a posteriori* if and only if *p*'s truth value may not be decided purely intellectually (but requires experience). (3) *p* is *a posteriori* if and only if *p*'s truth value is decidable only with more experience than that required to understand *p*.

a priori. (1) *p* is *a priori* if and only if *p*'s truth value may be decided independently of empirical observation. (2) *p* is *a priori* if and only if *p*'s truth value may be decided purely intellectually. (3) *p* is *a priori* if and only if *p*'s truth value is decidable with no more experience than that required to understand *p*.

argument.

A set of propositions such that some (the premises) are putative evidence for another (the conclusion). It makes no sense to talk of arguments as true or false but valid or invalid, sound or unsound. Any argument is either a deductive argument or an inductive argument but not both.

arrow. The name of a sign for implication: ' $\rightarrow$ '.

arrow, double. The name of a sign for logical equivalence: ' $\leftrightarrow$ '.

assertible. Appropriate for advocating as true.

asserting. Advocating as true (whether sincerely or insincerely).

assertion. Proposition advocated as true (whether sincerely or insincerely). A token sentence expressing a proposition, especially as advocated as true..

attribute. Property. That which is reported by the predicate part of an indicative sentence or denoted by an adjective or adjectival expression.

attributive. (1) The grammatical place of an adjective when it qualifies a noun directly rather than predicatively. For example, in 'Genghis Khan was a tyrannous ruler' 'tyrannous' is used attributively. In 'The ruler Genghis Khan was tyrannous' 'tyrannous' is not used attributively but predicatively. (2) Use of expression which refers essentially.

arbitrary name. Letter symbolising an arbitrarily selected object in a universe of discourse: a , b , c ..., . No arbitrary name is a proper name. See arbitrarily selected object.

arbitrarily selected object. Any object selected from a domain or universe. If an arbitrarily selected object is F then it follows that all objects in the domain are F . In this respect the logical use of 'arbitrarily selected object' contrasts sharply with ordinary English. See arbitrary name.

argument. A set of propositions such that some, the premises, are putative evidence for another, the conclusion. It makes no sense to talk of arguments as true or false. Arguments are valid or invalid, sound or unsound. Any argument is either a deductive argument or an inductive argument but not

both. See conclusion, invalid, premise, sound, unsound, valid,

argument-frame (or argument-form). Pattern of argument. Logical form of argument. For example: $p \rightarrow q, p \vdash q$.

ascription. The asserting of possession. The ascription of a property to an object is the claiming that that object has that property.

assertion sign. The name of the sign ' $\vdash$ ' which means 'from...it may be proven that...' or 'from...it follows logically that...' or '...therefore...'. For example in: $p \ \& \ q \vdash p$. 'Assertion sign' is a misleading name because the assertion sign does not signify assertion.

It may be reversed to show provability in the other direction: ' $\dashv$ '. It may be placed back to back with a reverse assertion sign to show two-way provability: ' $\dashv\vdash$ '. The assertion sign is also called 'the provability sign' and 'turnstile'.

associative laws. Logical equivalences exhibiting properties of '&': $p \ \& \ (q \ \& \ r) \leftrightarrow (p \ \& \ q) \ \& \ r$, and ' $\vee$ ': $p \ \vee \ (q \ \vee \ r) \leftrightarrow (p \ \vee \ q) \ \vee \ r$.

assumption. Proposition taken for granted in an argument, rather than proven by it. Typically an assumption is used as a premise. Nevertheless 'assumption' does not mean 'premise'.

asymmetric. A relation, ' R ', is asymmetric if and only if, if a is related by R to b , then b is not related by R to a . 'asymmetric' (def.): R is asymmetric iff $(\forall x) (\forall y) (Rxy \rightarrow \neg Ryx)$

'at least n '. See quantifier.

'at least one'. One and possibly more than one. See quantifier.

'at least two'. Two and possibly more than two. See quantifier.

'at most one'. One and no more than one. See quantifier.

atomic proposition. Proposition with no proposition except itself as a part. ' p ' is atomic but ' $p \ \& \ q$ ' is not atomic. $\neg p$ is atomic.

atomic sentence. Sentence with no sentence except itself as a part.

Sentence expressing one and only one atomic proposition. See atomic proposition.

aut. Latin for 'or' in the exclusive sense: p or q but not both. This is not the sense of 'or' depicted in ordinary propositional calculus. (See vel.)

axiom. Proposition for which no proof is required. Axioms typically feature as premises but not as conclusions of arguments. Axioms are thought to be necessary truths or logical truths and are arguably self-evident. See logical truth, necessary, tautology, theorem.

axiomatic development of a calculus. The generation of proofs by axioms rather than by rules of inference.

'because'. ' q because p ' is true if and only if $p \rightarrow q$. ' q because p ' means ' p therefore q '.

belief. (1) Psychological attitude entailing a disposition to allocate the truth-value 'true' towards a truth-valued content. The believing of what is believed. (2) The truth-valued content of a propositional attitude. That which is believed. Doxastic logic is the logic of belief. See proposition, propositional attitude.

both. Both p and q are true if and only if $p \& q$. ' $\text{Both } p \text{ and } q$ ' is true if and only if p is true and q is true, so false if p is false or (vel) q is false. Conjunction defined by the truth table defining '&':

<u>$p \& q$</u>			
q		q	
		T	F
p	T	T	F
p	F	F	F

$p \& q$: TFFF

biconditional. The logical operator ' $\leftrightarrow$ '. ' $p \leftrightarrow q$ ' is true if and only if if p is true then q is true and if p is false then q is false, so p and q have the same truth-value. See logical equivalence.

Def. ' $\leftrightarrow$ ': $p \leftrightarrow q = (p \rightarrow q) \& (q \rightarrow p)$

<u>$p \leftrightarrow q$</u>			
		q	q
		T	F
p	T	T	F
p	F	F	T

$p \leftrightarrow q$: TFFT

box. The name of the necessary operator: ' $\Box$ '. ' $\Box p$ ' is read 'necessarily p ' or 'box p '.

box arrow. The name of the necessary implication operator: ' $\Box \rightarrow$ '.

' $p \Box \rightarrow q$ ' is read ' p necessarily implies q ' or ' p box arrow q '.

bivalence. The property of being either true or false.

brackets. Arc-shaped signs deployed in pairs to show the scope of logical connectives: ' $(\ ' , \ ')$ '. Curly brackets, ' $\{ \ ' , \ ' \}$ ', show set membership. See scope.

C

calculus. Formal language, containing constants and variables, for the determination of the validity of patterns of arguments.

'can'. Modal term expressing possibility. p can be true if and only if: $\Diamond p$. See possibility.

canonical. Accepted. Established.

category. Class. Set.

character. The properties of an object, especially as sufficient for the object's belonging to a type.

characterising. Of properties: pertaining to an object, especially as sufficient for the object's belonging to a type.

circular argument. Argument in which the conclusion features as a premise.

circular definition. Definition in which the defined concept features as a defining concept.

class. Plurality defined by a common property.

class-variable. class-variable. Letters used in the theory of classes to refer to classes: $\checkmark$, $\checkmark$, $\blacksquare$

coherence. p and q are coherent if and only if the truth of p does not preclude the truth of q and the truth of q does not preclude the truth of p .

Consistency. See truth, consistency.

coherence theory of truth. The thesis that the truth of a proposition (belief, theory) is its consistency with endorsed propositions (beliefs, theories). The theory cannot be an analysis of truth because consistency with endorsed propositions is neither necessary nor sufficient for truth. As a definition of truth it is circular because the concept of truth is presupposed by consistency. Nevertheless, the theory points towards a pragmatic test for the endorsement or repudiation of beliefs.

comparative. Adjective used to express a relational property: 'greater than', 'less than'.

coimplicant. p and q are coimplicant if and only if p implies q and q implies p . See biconditional, logical equivalence.

common noun. Noun admitting of a plural. Noun that is not abstract. See abstract noun.

commutative laws. Logical truths concerning '&' and 'v': (1) $p \& q \leftrightarrow q \& p$. (2) $p \vee q \leftrightarrow q \vee p$.

complement. The class of objects that are not members of a given class. For example, the complement of set $\checkmark$ is all the things that are not members of $\checkmark$. The sign ' $a \mid$ ' means 'the complement of a ': 'complement' def : $a \mid = \{x: \neg(x \in a)\}$. The complement of a set is confined to a relevant universe of discourse. See universe of discourse.

completeness of propositional calculus. The property the propositional calculus possesses if and only if any tautologous sequent is provable by its rules.

completeness of predicate calculus. The property the predicate calculus possesses if and only if any logically true sequent is provable by its rules.

complex conjunction. Conjunction with conjunctions other than itself as parts.

complex disjunction. Disjunction with disjunctions other than itself as parts.

conclusion. (1) Proposition proven by an argument's premises. (2) Proposition putatively proven by an argument's premises. Any proposition in an argument is either a conclusion or a premise or both.

conditional. Complex proposition in which the antecedent implies the consequent. Proposition of the form 'if p then q ': $p \rightarrow q$. It is controversial whether conditionals have truth conditions. See antecedent, consequent, implication.

Conditional Proof (CP). Rule legitimising inferences with a conditional conclusion. For example: $p \rightarrow q \vdash \neg q \rightarrow \neg p$. The antecedent of the conclusion to be proven is assumed and its consequent is derived as a conclusion. That antecedent and its consequent jointly comprise the proven conclusion. In $p \rightarrow q \vdash \neg q \rightarrow \neg p$, assuming $\neg q$, we may derive $\neg p$ from $\neg q$ and $p \rightarrow q$ by *modus tollens*. $\neg q$ is the antecedent and $\neg p$ the consequent of $\neg q \rightarrow \neg p$.

concept. (1) Discriminatory ability: A has the concept of x if and only if A can discriminate x 's from non x 's. By this criterion, a robot that, say, sorts boxes has the concept of a box. (2) Mental representation.

conclusion. Proposition putatively proven by an argument's premises. Any proposition in an argument is either a conclusion or a premise or both.

condition, necessary. See necessary condition.

condition, sufficient. See sufficient condition.

condition, truth. See truth condition.

conjunct. Proposition related to another proposition by '&'. In ' $p \& q$ ', p is a conjunct and q is a conjunct.

conjunction. (1) Word used to relate two propositions which are jointly asserted: 'and', 'both', 'but'. (2) The proposition that both p and q are true: $p \& q$. '&' is defined by this truth-table:

$p \ \& \ q$

		q	q
		T	F
p	T	T	F
p	F	F	F

' $p \ \& \ q$ ' is true when p is true and q is true but false if either conjunct is false or if both conjuncts are false: $p \ \& \ q$: TFFF

connective. Proposition forming operator on propositions. In propositional calculus: ' $\neg$ ', ' $\&$ ', ' $\vee$ ', ' $\rightarrow$ '. ' $\leftrightarrow$ '. See negation, conjunction, disjunction, implication, biconditional.

consequent. The implied proposition in a conditional proposition. In ' $p \rightarrow q$ ', q is the consequent. See antecedent.

consistency of predicate calculus. The property the predicate calculus possesses if and only if any sequent provable by its rules is a logical truth. See completeness.

consistency of propositional calculus. The property the propositional calculus possesses if and only if any sequent provable by its rules is tautologous. See completeness.

consistent. [If the truth of p does not preclude the truth of q and if the truth of q does not preclude the truth of p then p and q are consistent.] p and q are consistent if and only if the truth of p does not preclude the truth of q and the truth of q does not preclude the truth of p : $\neg(p \rightarrow \neg q) \ \& \ \neg(q \rightarrow \neg p)$. See contrary.

consistent *wff*. A *wff* is consistent if and only if it is tautologous or contingent. That is, either: in every allocation of truth values to its component propositions it takes the truth value 'true' or: for at least one assignment it takes the truth value 'true' and for at least one assignment it takes the truth value 'false'. See inconsistent *wff*.

constituent. Logical or semantic part.

contingency. The property of being contingent. See contingent.

contingent. If p is contingent then if p is true then p could have been false and if p is false then p could have been true. If p is contingent then p is not necessary. Contingent propositions do not have their truth values essentially. See necessary.

contingent *wff*. *wff* that is neither a tautology nor a contradiction as determined by the truth tables of the propositional calculus.

contradiction. The conjunction of a proposition and its negation. Proposition of the form $(p \ \& \ \neg p)$ ('both p and not p '). See contradiction, law of.

contradiction, law of (or law of non-contradiction). The rule that the conjunction of a proposition and its negation is false. The law that any proposition of the form $(p \ \& \ \neg p)$ ('both p and not p ') is false.

contradictory. p and q are contradictory if and only if: if p is true then q is false and if p is false then q is true and if q is true then p is

false and if q is false then p is true. If p and q are contradictory they differ in truth value.

'contradictory' (def.): p and q are contradictory iff $\neg(p \leftrightarrow q)$ is a tautology.

contrary. p and q are contrary if and only if: if p is true then q is false and if p is false then q is true. If p and q are contraries they are not both true but may both be false. 'contrary' (def.): p and q are contrary iff $\neg(p \& q)$ is a tautology. See inconsistent, subcontrary.

conventionalism. The doctrine that necessary truths are true by linguistic convention. The thesis that the meaning of a necessary truth is sufficient for its truth, or all necessary truths are analytic.

converse. The proposition that results from exchanging the propositions on the left hand and right hand side of a (main) connective. The converse of $p \rightarrow q$ is $q \rightarrow p$.

correspondence. The relation between a proposition and a state of affairs such that if the state of affairs obtains, the proposition is true and if the state of affairs does not obtain, the proposition is false.

correspondence theory of truth. The doctrine that p is true if and only if p . The thesis that the obtaining or non-obtaining of states of affairs makes propositions true or false. See correspondence.

counter example. Particular truth falsifying a generalisation. True proposition of the form $(\exists x) \neg(Fx)$ so inconsistent with $(\forall x) (Fx)$, or of the form $(\exists x) (Fx)$ so inconsistent with $(\forall x) \neg(Fx)$.

counterfactual. *adj.* Contrary to fact. Only hypothetical. *n.* *abbrev.* counterfactual conditional.

counterfactual conditional. Conditional with hypothetical antecedent. For example: If kangaroos did not have tails they would fall over.

counterpart theory.

D

decidable. Demonstrable with regard to truth value. Provable or refutable. See proof, refutation.

decision procedure. Method of proof or refutation. Method for demonstrating truth value.

declarative. Apt for making an assertion. Truth-valued.

deduce. Infer. Logically derive. Prove. See deduction.

deduction. Kind of argument in which, putatively, the conclusion logically follows from the premises. Argument type in which, putatively, it is impossible for the premises to be true but the conclusion false. Kind of argument in which, putatively, the conjunction of the premises with the negation of the conclusion entails a contradiction. Logical inference. Logical derivation. Proof. No deduction is induction. See induction.

deductive. See deduction.

defeasible. Vulnerable to refutation. Open to counter example.

definite article. 'The'. Word used to refer to anything countable, specifying it as that particular thing but without thereby mentioning what it is. See indefinite article.

definite description. Expression with sense used to denote exactly one individual: 'the tallest man in the room'. Definite descriptions are typically introduced by the definite article 'the'. Definite descriptions are distinguished from proper names. See proper name.

definite reference. Reference to a specific thing. Reference to one and only one individual.

degree. (1) Of application of a predicate: Fully applying or not fully applying but not not applying. For example if a is F to some degree, a is F to some degree more than 0 but less than 1, for example : a is F to degree 0.7 but not F to degree 0.3. (2) Of truth of a proposition: Fully true or not fully true but not not true. If p is to some degree true then p true to some degree between 0 and 1, for example, true to degree 0.6 but false to degree 0.4. Fuzzy logic is the logic of degrees of truth. See fuzzy logic.

degrees of truth. See fuzzy logic.

demonstration. Valid argument. Sound argument. If p is demonstrated, then p is true. If p is not true, then p is not demonstrated.

demonstrative. Referring pronoun: 'that', 'this', 'I', 'now'.

denial. The assertion that a proposition is false. The denial that p is the assertion that $\neg p$.

deny. To assert of a proposition that it is false. To deny that p is to assert that $\neg p$.

denying the antecedent. The fallacy of taking as valid the invalid inference: $(p \rightarrow q) \ \& \ \neg p \rightarrow \neg q$.

deontic. Pertaining to obligation.

deontic logic. The logic of obligation.

designator. Referring expression. See rigid designator.

discourse. Language. Portion of language characterised by its subject matter.

disjunct. Proposition expressing one alternative in a disjunction. Proposition related to another proposition by ' $\vee$ '. In $p \vee q$, p is a disjunct and q is a disjunct. See complex disjunction, disjunction.

disjunction. The proposition that either p is true or q is true or both: $p \vee q$. Logical relation defined by the truth table for ' $\vee$ ':

<u>$p \vee q$</u>			
		q	q
		T	F
p	T	T	T
p	F	T	F

' $p \vee q$ ' is true if p is true, p if q is true, q is both p and q are true but false if both p and q are false: $p \vee q$: TTTF

v-introduction (v-I). Rule of the propositional calculus legitimating the inference: $p \vdash p \vee q$. Given p as premise, $p \vee q$ follows as conclusion.

v-elimination (v-E). Rule of the propositional calculus legitimating the inference: $p \vee q, p \rightarrow r, q \rightarrow r, \vdash r$. Given $p \vee q$ as assumption, and given a proof of r from p as assumption and a proof of r from q as assumption, r follows as conclusion.

distinct. a and b are distinct if and only if a is not b :

$\neg(a = b)$. See identity.

distinction. The fact of two things not being the same thing. What their being distinct consists in.

domain. Set of objects referred to. The domain of a relation, R , is the set of objects that stand in relation R to something. See range, universe.

double negation. The negation of the negation of a proposition. $\neg\neg p$ is the double negation of p . The double negation of a proposition is logically equivalent to that proposition.

double negation, rule of (DN). Rule of the propositional calculus legitimating inferences of the form: $p \vdash \neg\neg p$ and $\neg\neg p \vdash p$.

dyadic. Of relations: two term. 'is next two' is a two term relation.

'E Shriek'. ' $\exists!$ '. Quantifier with the meaning: 'There exists one and only one...', 'There exists exactly one...', or 'There is a unique...'.

'each'. See quantifier.

'either...or...'. See disjunction.

elementary. Without parts.

elementary conjunction. Conjunction which has no conjunction except itself as a part.

elementary disjunction. Disjunction which has no disjunction except itself as a part.

elimination, rules of. See $\&$ -elimination, $\vee$ -elimination, $\forall$ -elimination, $\exists$ -elimination, identity-elimination.

empirical. Knowable through experience.

empty class. Class with no members. The null set.

empty field. The field of a relation with no relata. A relation that holds between nothing and nothing has an empty field. See field.

empty universe. Universe containing no objects.

entailment. (1) Implication. The relation between p and q expressed by ' $\rightarrow$ '. p entails q if and only if: $p \rightarrow q$. ' p entails q ' is true unless p is true and q is false. (2) Strict implication. The relation between p and q expressed by ' $\Box \rightarrow$ '. p entails q if and only if: $p \Box \rightarrow q$. ' p entails q ' if and only if p necessarily implies q . This

relationship is sometimes called 'strict implication. See material implication.

entails. See entailment.

equivalence. The relation between p and q if and only if: if p is true then q is true and if q is true then p is true, that is, if they have the same truth value. The relation between p and q if and only if their biconditional is true: $p \leftrightarrow q$. Sometimes equivalence is called 'logical equivalence'. See biconditional.

equivalence relation. A relation, ' R ', is an equivalence relation if and only if R is transitive, symmetric and reflexive. See reflexive, relation, symmetric, transitive.

'equivalence relation'. A relation, ' R ', is an equivalence relation if and only if: R is transitive, symmetric and reflexive. 'equivalence relation' (def.): R is an equivalence relation iff $(\forall x) (\forall y) (\forall z) (Rxy \ \& \ Ryz \rightarrow Rxz) \ \& \ (Rxy \rightarrow Ryx) \ \& \ (\exists x) Rxx$

equivalent. p and q are logically equivalent if and only if: if p is true q is true and if q is true p is true and if p is false q is false and if q is false p is false that is, if they have the same truth value. Two propositions, p , q , are logically equivalent if and only if their biconditional is true: $p \leftrightarrow q$. The meaning of ' $\leftrightarrow$ ' is given by this truth table:

$p \leftrightarrow q$

$q \quad q$

		T	F
p	T	T	F
p	F	F	T

' $p \leftrightarrow q$ ' is true if p is true and q is true and if p is false and q is false. ' $p \leftrightarrow q$ ' is false if p is true and q is false and if q is true and p is false:

$p \leftrightarrow q$: TFFT

erotetic logic.

ersatz world.

ersatzism.

essential property. A property something could not lack and either: (1) remain the thing it is, or, (2) remain the sort of thing it is, or, (3) exist. See property, inessential property.

'every'. See quantifier.

'everything'. See quantifier.

evidence. p is evidence that q (or for q) if and only if p 's truth would make the truth of q more likely than p 's not being true.

'exactly one'. No more than one and no fewer than one. See numerically definite quantifier.

'except if'. And not if. ' p is true except if q is true' means ' $q \rightarrow \neg p$ ' which is logically equivalent to ' $p \rightarrow \neg q$ '.

exception. See counter example.

excluded middle, law of. Rule that any proposition of the form ' p or not p ' is a necessary truth. The claim that the disjunction of a proposition and its negation, $p \vee \neg p$, is a tautology. The law of excluded middle is not admitted by intuitionism. Fuzzy logic admits degrees of truth and falsity. Free logics admit truth values other than 'true' and 'false'. See free logics, fuzzy logics, intuitionism, proposition.

exclusive. (1) Of a distinction: admitting of no other term or kind. (2) A sense of 'or'. ' p or q ' is true in the exclusive sense of 'or' if and only if at least one and at most one of p and q is true. The meaning of 'or' in the exclusive sense is defined by this truth table:

p or q (exclusive)

		q	q
		T	F
p	T	F	T
	F	T	F

' p or q ' is true if p is true but q is false and if p is false but q is true but false if p and q are both false and false if p and q are both true: p or q : FTTF (exclusive). The exclusive sense of 'or' is not the one depicted by ' $\vee$ ' in ordinary propositional calculus. See disjunction, 'or'.

existence. (1) What there is. (2) The being of what there is.

existential. Pertaining to existence.

existential assumption. Proposition governed by the existential quantifier, featuring in an argument typically as a premise but never as a conclusion.

existential generalisation. Proposition expressing the claim that all members of a class have a certain property. Proposition of the form: $\forall x (Fx)$.

existential 'is', the. See 'is'.

existential proposition. Proposition apt for asserting existence. Proposition governed by the existential quantifier.

existential quantifier. The symbol ' $\exists$ ' for 'there is a' or 'there exists at least one'. A sentence governed by the existential quantifier is logically equivalent to a complex disjunction. For example, in the universe of discourse m, n, o , $(\exists x Fx)$ means $(Fm \vee Fn \vee Fo)$.

Existential Quantifier Introduction ($\exists I$). Rule of inference of the predicate calculus legitimating inferences of the form: $Fm \vdash (\exists x) (Fx)$. Intuitively, if a particular object is F then there exists something or other that is F .

Existential Quantifier Elimination ($\exists E$). Rule of inference of the predicate calculus legitimating inferences of the form: $(\exists x) (Fx), Fa \rightarrow p, \vdash p$. Intuitively, If something specific is F then p , and something is F , then p .

extension. The set of objects to which an expression applies. The reference of a proposition. The set of objects falling under a concept. See set theoretic extension.

F

false. p is false if and only if not p . (no italics) p is false if and only if the truth-conditions sufficient for p 's falsity obtain. If a proposition is not true, and not neither true nor false, it is false.

falsity. The property of being false. The truth of $\neg p$ is necessary and sufficient for the falsity of p .

fallacies. Invalid patterns of inference commonly taken to be valid.

field. The field of a relation R is the set of objects either in the domain of R or in the converse domain of R . See converse domain, domain, empty field, relation.

finite. A set is finite if and only if for some natural number, 0, 1, 2..., the number of its members is that number. Equivalently, a set is finite if and only if a one-one mapping obtains between its members and a subset of the natural numbers. A set is finite if and only if it is not infinite. See infinite.

finite axiomatization.

fixed. Unchanging, in particular with regard to referent.

fixed universe of discourse. Set of objects, or type of objects, referred to that does not gain or lose members.

follows from. q follows from p if and only if p entails q . See entailment.

'for any x '. See Universal Quantifier.

form of argument. If one argument is a substitution instance of another they have the same logical form.

formal implication. Conditional generalisation. Universal quantification over material implication. Proposition of the form: $(\forall x) (Fx \rightarrow Gx)$.

formal language. Symbolic system comprising rules, operators and variables, for the development of proofs. Language with its content subtracted. Abstract system in which meaning, truth and validity depend upon logical form. See logical form.

formation rules. The rules governing what counts as a *wff*. in a formal language.

formula. A sentence or putative sentence of a formal language. Formulae are not well-formed (that is, ill-formed) or well-formed. '*wff*' abbreviates 'well formed formula'. See *wff*.

formulated. Translated into symbolic notation. See formula, *wff*.

foundation. (1) Necessary condition. (2) Ultimate necessary condition, that is: necessary condition with no necessary condition. See necessary condition.

free logic. Logic in which truth values other than 'true' and 'false' are admitted. For example:

'neither true nor false', 'both true and false',
'undecidable'.

free variable. Variable not bound by a quantifier.

function. See propositional function, truth
function.

fuzzy logic. logic in which degrees of truth and
falsity are admitted. For example, if p is true to
degree 0.6 then p is false to degree 0.4. Classical
logic is the limiting case in which p is true to
degree 1 or true to degree 0, and so false.

G

gap, truth value. See truth value gap.

given. (1) Unargued. For example, assumptions are
given because they are justified by no stretch of
argument. (2) Uninterpreted.

grammar. The set of implicit or explicit rules
governing what may be expressed in a language.

H

I

idempotence, laws of. $p \ \& \ p \leftrightarrow p$, $p \ \vee \ p \leftrightarrow p$.

identity. (1) The relationship between a and b
which holds if and only if a is b : $a = b$. (2) The
relationship between a and b which holds if and
only if a is like b : $Fa \leftrightarrow Fb$. See identical.

identity and class. Two classes are identical if and only if they have the same members. Any two empty classes are identical.

identity elimination (=E). Rule legitimating inferences of the form:

$a = b, Fa \vdash Fb$.

identity introduction (=I). Rule legitimating the introduction of a proposition the form $x = x$ at any stage of a proof. Because $x = x$ is a logical truth, it rests on no assumptions.

identity statements. See 'is'.

identity, law of. See law of identity.

identical. The same. 'Same' is ambiguous between 'same one' and 'same sort'. a is the same one as b if and only if a is numerically identical to b . a is of the same sort as b if and only if a is qualitatively similar to b . a is of only the same sort as b if and only if a is qualitatively identical to b . (1) a and b are numerically identical if and only if a is b : $a = b$. (2) a and b are qualitatively identical if and only if a is (exactly) like b : $Fa \leftrightarrow Fb$.

a is numerically identical to b if and only if a is b . a is numerically identical to b if and only if a is the same one as b . a is qualitatively similar to b if and only if a is like b in at least one respect. a is qualitatively identical to b if and only if a is like b in every respect. a is qualitatively similar to b if and only if a and b are of the same sort (kind, type).

In predicate calculus, 'a is numerically identical to b' is symbolised ' $a = b$ '. Numerical identity is a transitive relation: $(\forall x) (\forall y) (\forall z)$

$(x = y \ \& \ y = z \rightarrow x = z)$ If a is numerically identical to b and b is numerically identical to c , then a is numerically identical to c .

Numerical identity is a symmetrical relation:
 $(\forall x) (\forall y) (x = y) \leftrightarrow (y = x)$ If a is numerically identical to b then b is numerically identical to a .

Numerical identity is a reflexive relation:
 $(\forall x) (x = x)$ a is numerically identical to a .

Numerical identity is an equivalence relation because it is transitive, symmetrical and reflexive : $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z)$ & $(\forall x) (\forall y) (x = y) \leftrightarrow (y = x)$ & $(\forall x) (x = x)$.

'if'. ' q if p ' means ' p is a sufficient condition for q ': $p \rightarrow q$. See implication, necessary condition, sufficient condition.

'if and only if'. See biconditional.

iff. abbrev. 'if and only if'. See biconditional.

imperative. Sentence appropriate for expressing a command or suggestion. Imperatives are not capable of being true or false, only obeyed, disobeyed, ignored.

imperative logic. The logic of imperatives. Even though imperatives are not truth-valued, logical relations may obtain between them, For example: 'All A 's do x ', 'This is an A ', ' A , do x '.

implication. The relation between two propositions expressed by ' $\rightarrow$ '. ' $p \rightarrow q$ ' is true unless p is true and q false.

implication, formal. See formal implication.

implication, material. See material implication.

implicature, conversational. Unstated or non literal meaning assumed in ordinary conversation. For example: 'Oh, you are awful'.

implies. p implies q if and only if $p \rightarrow q$. $p \rightarrow q$ is true unless p is false and q is true. See material implication.

included in. Set $\mathcal{V}$ is included in set $\mathcal{V}'$ if and only if every member of $\mathcal{V}$ is a member of $\mathcal{V}'$, that is, if and only if $\mathcal{V} \subseteq \mathcal{V}'$ is true. The inclusion sign, ' $\subseteq$ ', is defined as follows: ' $\subseteq$ ' def: $\mathcal{V} \subseteq \mathcal{V}' \leftrightarrow (\forall x) (x \in \mathcal{V} \rightarrow x \in \mathcal{V}')$

(Inclusion is not membership. It is a category mistake to say set $\mathcal{V}$ is a member of set $\mathcal{V}'$ if $\mathcal{V}$ includes $\mathcal{V}'$, except if $\mathcal{V}$ is a set of sets. Even then, the inclusion relation is not the membership relation.)

inclusion. The relation which holds between two sets $\mathcal{V}$ and $\mathcal{V}'$ if and only if ' $\mathcal{V} \subseteq \mathcal{V}'$ ' is true. See included in.

inconsistency. The relation which holds between p and q if and only if $(p \rightarrow \neg q) \ \& \ (q \rightarrow \neg p)$. See inconsistent.

inconsistent. If the truth of p precludes the truth of q or if the truth of q precludes the truth of p , or both, then p and q are inconsistent.

p and q are inconsistent if and only if: if p then not q and if q then not p , that is: $(p \rightarrow \neg q) \ \& \ (q \rightarrow \neg p)$.

inconsistent *wff*. *wff* that for all possible allocations of truth values to its component propositions takes the truth value 'false'. *wff* that is neither tautologous nor contingent. See consistent *wff*, truth table.

indefinite article. 'a', 'an'. Word used to refer to anything countable without specifying it as that particular thing. See definite article, indefinite reference.

indefinite reference. Reference to anything countable without specifying what it is. Reference using, or equivalent to using, the indefinite article: 'a', 'an'. See indefinite article.

independent. See logically independent.

indeterminacy of translation. The thesis that any knowledge of behaviour is insufficient for the correct interpretation of utterances.

indexical. Expression requiring knowledge of the context of utterance for the determination of its reference. For example, demonstrative and personal pronouns: 'I', 'here', 'this', 'that', 'now', 'yesterday', 'today', 'tomorrow', and all tensed verbs. To understand who 'I' refers to' it is necessary to know who is saying 'I'. To know which time 'now' refers to, it is necessary to know when 'now' is uttered. To know where 'here' refers to, it is necessary to know where 'here' is uttered. Indexicals are sometimes called 'token reflexives'.

indexicality. The property of being indexical. See **indexical**.

indicative. Capable of being true or false.

indirect proof. See *reductio ad absurdum*.

individual. Particular thing, as opposed to type, sort or universal.

individual variable. Symbolic place holder for an individual. In predicate calculus: ' x ' stands for any individual, ' y ' stands for any individual distinct from x and so on. See **individual**, **predicate calculus**, **variable**.

individual variable substitution. The replacement of all the occurrences of one variable in a *wff*. by another variable.

induction. The putative derivation of a conclusion from premises by probabilistic rather than deductive means. ' The drawing of conclusions of the form ' $(\forall x) (Fx)$ ' from premises of the form ' $Fa, Fb...$ ' All inductive arguments are formally invalid.

inductive. Of an inference: probabilistic rather than deductive. See **induction**.

intransitive. A relation, ' R ', is intransitive if and only if: if a is related by R to b , and if b is related by R to c , then a is not related by R to c . '**intransitive**' (def.): R is intransitive iff $(\forall x) (\forall y) (\forall z) (Rxy \ \& \ Ryz \rightarrow \neg Rxz)$

inessential property. A property something could lack and nevertheless, either: (1) remain the thing

it is, or, (2) remain the sort of thing it is, or, (3) exist.

inference. (1) The relation between premises and conclusion in a valid argument. (2) The relation between premises and conclusion in a putatively valid argument.

infinite. A set is infinite if and only if there is a one-one mapping between its members and a proper subset of that set. Intuitively: a set is infinite if and only if, for any natural number $0, 1, 2, \dots$, the number of its members is greater than that number. A set is infinite if and only if it is not finite. See finite, one-one mapping.

infinite conjunction. Conjunction with infinitely many conjunctions as parts: $p \ \& \ q \ \& \ r \dots n$.

infinite disjunction. Disjunction with infinitely many disjunctions as parts: $p \vee q \vee r \dots n$.

infinity. See infinite.

informative. (1) synthetic. (2) non-tautological.

integer. Number.

integer, non-negative. Number in the series $0, 1, 2, 3 \dots n$.

integer, positive. Number in the series $1, 2, 3 \dots n$.

intension. (1) Sense, especially the sense of a sentence or proposition. (2) Property in virtue of which the members of a class belong to that class.

interderivability. If q is provable from p and p is provable from q then p and q are interderivable. If $q \vdash p$ and $p \vdash q$ then $p \dashv\vdash q$. See proof, assertion sign.

internal structure. The parts of a proposition related in ways that affect meaning, truth and validity. In propositional calculus, the parts of propositions are only propositions. For example, ' p ' and ' q ' are parts of ' $p \& q$ '. An atomic proposition, p , has no parts (except itself). In predicate calculus the parts of propositions are names, predicates and quantifiers. For example, the parts of ' $\exists a (Fa)$ ' are ' $\exists a$ ', ' F ' and ' a '. What could only be treated as an atomic proposition in propositional calculus may be 'opened up' in predicate calculus and inferences that depend upon relations between parts of the proposition depicted.

interpretation. The assignment of truth values to the component propositions of a proposition in propositional calculus. The assignment of objects, properties and relations to propositions of the predicate calculus.

intersection. The set of objects common to two sets. The intersection of $\checkmark$ and $\checkmark$ is the set of objects that are members of both $\checkmark$ and $\checkmark$. The sign for intersection, ' $\cap$ ', is defined in this way: ' $\cap$ '
def. $\checkmark \cap \checkmark = \{x: x \in \checkmark \& x \in \checkmark\}$.

introduction rules. See $\&$ -introduction ($\&I$), $\vee$ -introduction ($\vee I$), $\exists$ -introduction ($\exists I$), $\forall$ -introduction ($\forall I$), $=$ introduction ($=I$).

intransitive. The kind of relation ' R ' such that if x is related by R to y and y is related by R to z

then x is not related by R to z : $(\forall x) (\forall y) (\forall z)$
 $(xRy \ \& \ yRz) \rightarrow \neg(xRz)$.

intuitionism. (1) Logic not admitting the law of excluded middle, the law of double negation and *reductio ad absurdum*. (2) The anti-realist thesis that truth entails provability. (3) The meta-mathematical thesis that proofs are not discovered but invented or 'constructed'. See anti-realism, double negation, excluded middle, *reductio ad absurdum*.

invalid. Of a deductive argument: the truth of the conclusion is consistent with the falsity of the premises. If an argument is invalid, then even if the premises are true, the conclusion could still be false. If an argument is invalid it cannot be sound. If an argument is not invalid then it is valid.

invalidity. The property a deductive argument possesses if and only if the truth of the conclusion is consistent with the falsity of the premises. Invalidity is inconsistent with soundness.

irreflexive. A relation is irreflexive if it does not hold between anything and itself. ' R ', is irreflexive if and only if, a is not related by R to a . 'irreflexive' (def.): R is irreflexive iff $(\forall x) \neg Rxx$

'is'. (1) 'is the same thing as'. 'is identical to' The 'is' of identity: ' a is b ' is true if and only if ' $a = b$ ' is true. (2) 'is' in ' $\text{is } F$ ', ' $\text{is } G$ ' (where ' F ' and ' G ' denote properties). The 'is of predication': ' a is F ' is true if and only if ' Fa '

is true. (3) 'exists'. The existential 'is': ' x exists' is true if and only if ' $\exists x Fx$ ' is true.

'is a member of'. Some object x is is a member of some set $\forall$ if and only if ' $x \in \forall$ ' is true.

is implied by. q is implied by p if and only if $p \rightarrow q$. See implication, material implication.

'is the same object as'. See identical.

'is' of identity, the. See identical, 'is'.

'is' of predication, the. See 'is'.

J

jointly exhaustive. Two propositions are jointly exhaustive if and only if: if p is not true then q is true and if q is not true then p is true: $(\neg p \rightarrow q) \ \& \ (\neg q \rightarrow p)$.

K

L

law. See associative, commutative, conversion, de Morgan's, distributive, double negation, excluded middle, idempotence, obversion.

law of excluded middle. Rule that any proposition of the form p or not p is a necessary truth. Rule that at least and at most one of the disjunction of a proposition and its negation is true. Claim that $p \vee \neg p$ is a tautology. The law of excluded middle is not admitted by intuitionism.

law of identity. Rule that if a proposition is true then it is true. The entailment of a proposition by that proposition. The claim that $p \rightarrow p$ is a tautology.

law of non-contradiction. The rule that the conjunction of a proposition and its negation is necessarily false. The claim that $(p \ \&\neg \ p)$ is a contradiction.

least. The smallest. If n is the least number in some domain then there is no number smaller than n in that domain.

lemma. Auxiliary result used in a proof.

logic. The distinguishing of valid from invalid patterns of argument.

logic, many valued. See free logic.

logical connective. Proposition forming operator on propositions that logically relates propositions by its application. For example, ' $\vee$ ' but not ' $\neg$ '.

logical constant. Logical operator that is either a connective or not a connective. For example ' $\neg$ ' is a logical constant but not a connective, ' $\&$ ' both a logical constant and a logical connective. 'Constant' contrasts with 'variable'. See logical connective, variable.

logical equivalence. The relationship that holds between p and q if and only if p and q have the same truth value.

$p \leftrightarrow q$

		q	q
		T	F
p	T	T	F
p	F	F	T

$p \leftrightarrow q$: TFFT

logical form. What remains of a proposition if its content is subtracted. The properties of propositions bestowed only by relations between variables and connectives, in virtue of which meaning, truth and validity obtain. If one argument is a substitution instance of another they have the same logical form.

logical independence. p and q are logically independent if and only if: p and q are not contrary, not subcontrary, p does not imply q , q is not implied by p , p and q are not equivalent and p and q are not contradictory. See contradictory, contrary, subcontrary.

logically follows from. See implication, proof.

logically true. See logical truth.

logical truth. Proposition true only for logical reasons. Proposition for whose truth the principles of logic are sufficient. Truth made true by logic. No logical truth is synthetic or contingent.

M

main column. In a truth table for a *wff*, the list of possible truth values of the whole *wff* which shows whether the *wff* is a tautology, a

contradiction or contingent. See connective, main connective, truth table, *wff*.

main connective. That connective in a *wff* to which all others are subordinate in scope. The connective in a *wff* with the widest scope. In any *wff* there is one and only one main connective. In a truth table for a *wff*, the possible truth values of the *wff* as a whole appear in the column under the main connective. See connective, main column, truth table, *wff*.

many-valued logics. See free logic.

material implication. The relation between p and q that holds if and only if $p \rightarrow q$ is true. The conditional $p \rightarrow q$ is true unless p is true and q is false. Material implication falls short of entailment, or strict implication, because if p entails q then necessarily $p \rightarrow q$, ie: $(p \Box \rightarrow q)$. The material implication of q by p differs from a formally valid inference from p to q because that requires the impossibility of p 's falsity given p . If q is validly derivable from p then ' p but not q ' entails a contradiction. If p materially implies q ' p but not q ' does not entail a contradiction even though $p \rightarrow q$ is in fact true. The meaning of the logical connective ' $\rightarrow$ ' is given by this truth table:

<u>$p \rightarrow q$</u>		
	q	q
	T	F
p T	T F	
p F	T T	

So $p \rightarrow q$ is true under all allocations of truth values except if the antecedent, p , is true and the consequent, q , is false: $p \rightarrow q$: TFFT.

material implication, paradoxes of. If p is false then $p \rightarrow q$, so a false proposition implies any proposition, which is counter-intuitive. Paradoxically, if p implies any proposition, p implies q and p implies $\neg q$. If q is true then $p \rightarrow q$, so a true proposition is implied by any proposition, which is counter-intuitive. Paradoxically, if q is true, q is implied by p and q is implied by $\neg p$. See material implication.

materially implies. p materially implies q if and only if $p \rightarrow q$. See material implication.

mathematics. The study of reality through the medium of number.

matrix. Truth table for the definition of connectives or determination of sequents as tautologous, contradictory or contingent:

$\neg p$

		$\neg p$
p	T	F
p	F	T

$\neg p$: FT

$p \rightarrow q$

		q	q
		T	F
p	T	T	F
p	F	T	T

$p \rightarrow q$: TFFT

$p \ \& \ q$

		q	q
		T	F
p	T	T	F
p	F	F	F

$p \ \& \ q$: TFFF

$p \vee q$

		q	q
		T	F
p	T	T	T
p	F	T	F

$p \vee q$: TTTF

$p \leftrightarrow q$

		q	q
		T	F
p	T	T	F
p	F	F	T

$p \leftrightarrow q$: TFFT

meaning. (1) Sense. An expression has a sense if and only if it has a verbal definition. (2) Reference. An expression has reference if and only if it has a referent.

mechanical proof check. See: algorithm.

mechanical proof-discovery. Proof discovery that could be conducted by a machine.

mechanical test. Test that could be conducted by a machine. For example: the truth table test for the logical status of *wffs* of the propositional calculus.

member. Item possessing a property necessary and sufficient for set membership.

membership-relation. Property possession sufficient for set membership. See *member*.

metatheorem. Theorem about a theorem. See *theorem*.

metalogical variable. Variable used to describe variables.

modal. Pertaining to necessity, possibility or probability.

modal idioms. Informal terms expressing necessity, possibility or probability: 'it is necessary that', 'it cannot be false that', 'probably'.

modal logic. The logic of necessity, possibility and probability.
Modal logic includes propositional and predicate calculus as parts.

modal realism. The view that all possible worlds exist. The actual world does not differ from other worlds by existing rather than not existing but by our being in it. Modal realism entails the existence of an infinite number of possible worlds. See *actual*, *ersatz world* *ersatzism*, *possible world*,.

modus ponens (MP). Rule of inference of the propositional calculus legitimating inferences of the form: $p \rightarrow q, p \vdash q$.

modus tollens (MT). Rule of inference of the propositional calculus legitimating inferences of the form: $p \rightarrow q, \neg q, \vdash \neg p$.

monotonic. Property a valid deductive argument possesses just in case it remains valid no matter what is added to it.

'most'. More than half.

mutually exclusive. Two propositions are mutually exclusive if and only if: if one is true then the other is false: $(p \rightarrow \neg q) \ \& \ (q \rightarrow \neg p)$. See jointly exhaustive.

N

naïve. Pre theoretical.

name. Term used to refer to an individual.
Substantive. Noun. See arbitrary name, proper name.

name-letter. Letter that replaces a proper name:
'm', 'n'... .

name substitution. The replacement of all the occurrences of one name in a *wff*. by another name.

natural deduction. The discovery of proofs using rules of inference, rather than truth tables.

natural kind. Type of thing that exists, and is the way it is, independently of any classificatory system.

natural number. Member of the set of numbers: $\{0, 1, 2, 3 \dots n\}$.

Non-negative integer.

natural structure. That which is indicated by the relations of scope and subordination in a *wff*. See rank, scope, subordination.

necessary. If p is necessary, then if p is true p could not have been false and if p is false then p could not have been true. If p is necessary then p has its truth value essentially, or could not have had a different truth value. See analytic, logical truth, theorem, truth value.

necessary condition. If the truth of p is a necessary condition for the truth of q then if not p then not q (ie if p is not true then q is not true). If p is a necessary condition for q then q is a sufficient condition for p . If p is a sufficient condition for q then q is a necessary condition for p . If the truth of p is both necessary and sufficient for the truth of q then q if and only if p . Everything entails its own prerequisites. See sufficient condition.

necessary and sufficient condition[s].

[If the truth of p is a necessary condition for the truth of q then if not p then not q (ie if p is not true then q is not true). If the truth of p is a sufficient condition for the truth of q then if p then q (ie if p is true then q is true).] If the truth of p is both necessary and sufficient for the truth of q then q if and only if p . If p is a

necessary condition for q then q is a sufficient condition for p . If p is a sufficient condition for q then q is a necessary condition for p .

necessary falsehood. False proposition that could not have been true. Proposition that entails any proposition.

necessary truth. True proposition that could not have been false.
Proposition that follows from any proposition.

negation. The result of prefacing a proposition by the negation sign ' $\neg$ '. Contradictory of a proposition. Proposition, including a negation sign, used to assert that a proposition is false. ' $\text{not } p$ ' is true if and only if p is false. The meaning of the negation sign ' $\neg$ ' is given by the following truth table:

$\neg p$

	$\neg p$
p T	F
p F	T

' $\neg p$ ' is true if p is false, false if p is true:
 $\neg p$: FT

Negation, Double, See: Double Negation (DN).

negative sentence. Sentence including a negative expression.

negative proposition. Proposition including a negative expression.

'neither...nor...'. ' $\text{Neither } p \text{ nor } q$ ' is true if and only if $\neg p \ \& \ \neg q$.

'no'. Not any. See quantifier.

non-contradiction, law of. The rule that the conjunction of a proposition and its negation is false: $\neg (p \ \&\ \neg p)$.

non-denumerable.

non-empty term. A term ' F ' is empty if and only if there is nothing that has the property of being F : $\neg(\exists x) (Fx)$.

non-empty universe of discourse. A universe which can be referred to, in which at least one thing exists.

non-negative integer. Member of the set: $\{0, 1, 2, 3...n\}$. Natural number.

non-particular. General or abstract.

non-symmetric. 'non symmetric' (def.): R is non symmetric iff $(\exists x)(\exists y) (Rxy \ \&\ \neg Ryx)$

non-tautologous. (1) In propositional calculus: contingent or contradictory. (2) Not analytic. A non-tautological proposition's meaning is not sufficient for its truth.

non-transitive. A relation, R , is non-transitive if and only if R is not transitive and R is not intransitive. See intransitive, relation, transitive.

'none'. Not any. See quantifier.

'not'. See negation.

'not any'. None. See quantifier.

'not both...and...'. 'Not both p and q ' is true if and only if $\neg(p \ \& \ q)$ is true, that is, if either p is false or q is false.

notation.

'nothing'. Not anything. See quantifier.

noun. Word used to refer to an object rather than ascribe a property. Substantive. See name, proper name.

null class. The class with no members. There is only one null class because all classes with no members are identical.

number. Abstract object that has mathematical relations essentially. (Unity or) plurality. Set. See natural number, rational number, real number.

numeral. Symbol used to refer to a number. For example, '1' refers to the number one.

numerically definite quantifier. Quantifier used to assert the existence of some exact number of objects. For example ' $(\exists x^3) (Fx)$ ' is true if and only if there are exactly three things that have the property of being F .

numerical identity. The relationship between a and b if and only if a is b : $a = b$. Numerical identity is a transitive relation: $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z)$. If a is numerically identical to b and b is numerically identical to c , then a is numerically identical to c . Numerical identity is a symmetrical relation: $(\forall x) (\forall y) (x = y) \leftrightarrow (y = x)$

If a is numerically identical to b then b is numerically identical to a . Numerical identity is a reflexive relation: $(\forall x) (x = x)$ a is numerically identical to a . Numerical identity is an equivalence relation: The relation 'is numerically identical to' is transitive, symmetrical and reflexive. Therefore it is an equivalence relation.
 $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z) \ \& \ (\forall x) (\forall y) (x = y) \leftrightarrow (y = x) \ \& \ (\forall x) (x = x)$. See identity, identical, qualitative identity.

0

0-form proposition. Particular negative proposition, of the form:

$(\exists x) (Fx \ \& \ \neg Gx)$.

one-one mapping. Relation holding between two sets, $\checkmark$ and $\checkmark$  if and only if there is a relation, R , such that each member of $\checkmark$ bears R to exactly one member of $\checkmark$ and each member of $\checkmark$ bears R to exactly one member of $\checkmark$.

one-many mapping. Relation, ' R ', between two sets, $\checkmark$ and $\checkmark$  such that each member of $\checkmark$ bears R to at least one member of $\checkmark$ and each member of $\checkmark$ bears R to at least one member of $\checkmark$.

'only'. 'Only if p then q ' means ' p is necessary for q '. If p is necessary for q then q is sufficient for p , so 'Only if p then q ' is true if and only if 'if q then p ' is true. 'Only if p then q ' therefore means: $q \rightarrow p$.

'Only things that are F are G ' means being F is necessary for being G . If being F is necessary for being G then being G is sufficient for being F , so 'Only things that are F are G ' is true if and

only if 'Anything that is G is F ' is true. 'Only things that are F are G ' therefore means: $(\forall x) (Gx \rightarrow Fx)$

'only, the'. 'The only things that are F are G ' means 'Anything that is F is G ': $(\forall x) (Fx \rightarrow Gx)$

'only if...then...'. 'Only if p then q ' means ' p is necessary for q '. If p is necessary for q then q is sufficient for p , so 'Only if p then q ' means if q then p : $q \rightarrow p$.

ontological. Pertaining to existence.

ontological commitment. The existence entailed by the truth of an existential proposition. The existence entailed by then use of the existential quantifier.

opaque. A context in a sentence is referentially opaque if and only if the truth value of that sentence is not necessarily retained through substitution of propositions in that context. For example, 'believes that' is referentially opaque because in 'A believes that p ', q cannot be substituted for p without the possibility of changing truth value.

operator. Logical constant. Logical connective. See Logical constant. Logical connective.

'or'. See disjunction.

order of proper names. Changing the order of proper names in a sentence changes its meaning and might change its truth value. For example: Rmn (m bears relation R to n) might be true but Rnm (n bears R to m) might be false.

order of quantifiers. Changing the order of quantifiers in a sentence containing both existential and universal quantifiers might change its truth-value. For example: $(\forall y) (\exists x) (Rxy)$ (For any y , there exists an x such that x bears relation R to y) might be true but $(\exists y) (\forall x) (Rxy)$ (There exists a y such that for all x , x bears relation R to y) might be false. Changing the order of quantifiers in a sentence containing only existential or only universal quantifiers does not change its truth values. See quantifier shift, quantifier shift principles.

ordered n -tuple. Sequence essentially ordered such that its members are the same members as another sequence. For example, $\langle 1, 2, 3, 4 \rangle$ is an ordered sequence in relation to $\langle 1, 2, 3, 4 \rangle$ but not in relation to $\langle 1, 4, 3, 2 \rangle$. See ordered pair.

ordered pair. A pair essentially ordered such that its members are the same members as another pair. For example, $\langle 1, 2 \rangle$ is an ordered pair in relation to $\langle 1, 2 \rangle$ but not in relation to $\langle 2, 1 \rangle$. See ordered sequence.

ostensive definition. Defining an expression by pointing to an object it refers to and naming it using that expression.

P

pair class. The class whose members are either a or b : $\{a, b\}$. If x is a member of the pair class $\{a, b\}$ then either x is a or x is b .

paradigm. Example.

paradigm case. Typical example.

paradox. Apparent contradiction or inconsistency generated by otherwise uncontroversial logical means.

paradoxes of formal implication. Given universal quantification over material implication: $(\forall x) (Fx \rightarrow Gx)$, it follows from $(\forall x) (Fx)$ that $(\forall x) (Fx \rightarrow Gx)$ and from $(\forall x) \neg(Fx)$ that $(\forall x) (Fx \rightarrow Gx)$. But if $(\forall x) (Fx) \vdash (\forall x) (Fx \rightarrow Gx)$ then if anything has a certain property it has a certain distinct property, which is counter intuitive and if $(\forall x) \neg(Fx) \vdash (\forall x) (Fx \rightarrow Gx)$ then if anything lacks a certain property it follows that it has a certain property, which is counter intuitive. If anything either possesses or lacks a certain property then it has a certain distinct property, which is paradoxical.

paradoxes of material implication. If p is false then $p \rightarrow q$, so a false proposition implies any proposition, which is counter-intuitive. Paradoxically, if p implies any proposition, p implies q and p implies $\neg q$. If q is true then $p \rightarrow q$, so a true proposition is implied by any proposition, which is counter-intuitive. Paradoxically, if q is true, q is implied by p and q is implied by $\neg p$. See material implication.

particle. Grammatical connective, typically a preposition: 'if', 'on', 'in', or a conjunction: 'and', 'but'.

particular. (1) *n.* Individual as opposed to universal. Token rather than type. (2) *adj.* Referring to one and only one thing. Appropriate

for referring to only one thing. See individual, universal.

particular affirmative. Proposition of the form:
 $(\exists x) (Fx \ \& \ Gx)$.

particular negative. Proposition of the form: $(\exists x)$
 $(Fx \ \& \ \neg Gx)$.

positive term. Term definable without the use of negative terms.

positive integer. Member of the set of numbers 1, 2, 3... n .

possessive. Expressive of ownership or belonging. Expressible by the genitive case.

pattern of argument. See form of argument, logical form.

polyadic. Many term. A polyadic relation is a relation that holds between more than two relata. See dyadic, relation.

possibility. That which is not contradictory to describe. The modality expressed by ' $\Diamond$ '.

possible. Not contradictory to describe. In modal logic possibility and necessity are interdefinable: possibly p if and only if not necessarily not p : $\Diamond p \leftrightarrow \neg \Box \neg p$ and necessarily p if and only if not possibly not p : $\Box p \leftrightarrow \neg \Diamond \neg p$.

predicate. The result of subtracting the proper name from an indicative sentence. A word or string of words used to ascribe a property. The sense of a

word, or senses of a string of words, used to ascribe a property.

predicate calculus. Formal language consisting in individual variables ($x, y, \dots$), predicate letters ($F, G, \dots$) individual constants ($m, n, \dots$), propositional connectives ($\&, \leftrightarrow, \rightarrow, \neg, \vee$) and either the universal quantifier ($\forall$) or the existential quantifier ($\exists$) or both and governed by both propositional calculus rules and quantifier rules.

predicate calculus with identity. See identical.

predicate calculus, intuitionist. Predicate calculus in which the law of excluded middle is not admitted.

predicate calculus and propositional calculus. The predicate calculus includes the propositional calculus as a part.

predicate, grammatical. That which remains of an indicative sentence if a proper name is deleted. A word or set of words used to ascribe a property. Adjectival expression. See proper name, subject.

predicate letter. In the predicate calculus, an upper case letter: ' F ', ' G '..., which replaces and stands for a predicate.

predicate-letter substitution. The replacement of all the occurrences of one predicate letter in a *wff*. by another predicate letter.

predicate logic. Predicate calculus. See predicate calculus.

premise. Proposition that is putative evidence for the conclusion of an argument. In a valid argument it is not possible for the premises to be true but the conclusion false because the conjunction of the premises and the conclusion entails a contradiction. Any proposition in an argument is either a premise or a conclusion or both. See argument, conclusion, sound, valid.

presupposition. Necessary condition. If q is a presupposition of p then if not q then not p , so p entails q . If p presupposes q then: $(\neg q \rightarrow \neg p)$ but $(\neg q \rightarrow \neg p) \leftrightarrow (p \rightarrow q)$ so if p presupposes q then: $(p \rightarrow q)$. Everything entails its own presuppositions. See necessary condition, sufficient condition.

primitive rule. Rule of inference that is not derived. Axiomatic rule. See derived rules.

principle of identity. If anything is the very same thing as anything then any property of one is a property of the other and if any property of one thing is a property of the other then they are the very same thing: $(\forall x) (\forall y) (x = y) \leftrightarrow (Fx \leftrightarrow Fy)$.

principle of quantifier shift. See quantifier-shift principles.

progressive. The application of a proven result in further proofs.

pronoun. Word that may replace a noun in a sentence without changing its truth value.

proof. Demonstration of truth. Valid argument. Sound argument. If p is proven, then p is true. If p is not true, then p is not proven. See

completeness proof, conditional proof, consistency proof, indirect proof.

proof by induction. From Fx it follows that $(\forall x)(Fx)$ on the assumption that any successor of x is F . For example, if it can be shown that 0 is F and that the successor to any number has F , it may be validly concluded that all numbers are (or any number is) F .

proof, mechanical check of. See mechanical check of proof.

proof procedure. Any method of exhibiting the validity of valid arguments by deploying rules of inference. See argument, rule of inference, valid.

proof, stage of. Any of: the making of assumptions; the deploying of rules of inference; the drawing of a conclusion; the use of the conclusion of one application of a rule of inference as a premise for the next.

proposition.

(1) What is expressed by an indicative (declarative) sentence. (2) The meaning of an indicative (declarative) sentence. (3) What is asserted when something is asserted. (4) What is expressed by an indicative sentence.

A proposition is capable of being true or false. The letter ' p ' stands for any proposition. ' q ' stands for any proposition distinct from p ' r ' for any proposition distinct from both p and q , etc. If p is true then p is not false. If p is false then p is not true. It makes no sense to talk of propositions as valid or invalid, sound or unsound but true or false.

provability sign. The assertion sign: ' $\vdash$ ' meaning 'given...it is provable that...'. See assertion sign.

propositional variable. Lower case letter, ' p ', ' q ', ' r '... used to replace and stand for a proposition. ' p ' stands for a proposition. ' q ' stands for a proposition distinct from ' p ' etc.

proper name. Referring expression that is a name. Name used to refer to just one thing or kind of thing: 'London', 'New York', 'Jesus', 'oxygen'. In predicate calculus, the lower-case letters ' m ', ' n '..., replace and symbolise proper names.

property. Characteristic. Anything true of anything. (There are arguable exceptions: It is true of any object that exists that it exists, but existing might not be a property.) Property possession determines set membership.

property-expression. Predicate. Adjectival term. See predicate, property.

proposition. (1) What is expressed by an indicative (declarative) sentence. (2) The meaning of an indicative (declarative) sentence. (3) What is asserted when something is asserted. (4) What is expressed by an indicative sentence. A proposition is capable of being true or false. The letter ' p ' stands for any proposition. ' q ' stands for any proposition distinct from p ' r ' for any proposition distinct from both p and q , etc. If p is true then p is not false. If p is false then p is not true. It makes no sense to talk of propositions as valid or invalid, sound or unsound but true or false. See affirmative, conditional, existential proposition, internal structure of proposition, negative,

particular, sentence, statement, universal proposition.

propositional attitude. Psychological relation a subject bears to some propositional content. For example, as reported in: 'A hopes that p ', 'A doubts that p ', 'A wonders whether p ', 'A desires that p ', 'A believes that p '. In propositional attitude sentences it is possible to vary the subject, the attitude or the proposition. For example: 'A desires that p ', 'B hopes that q ', 'C doubts that r '.

propositional calculus. Formal language for the proof of sequents whose validity depends on relations between propositions rather than on the internal structure of propositions. See completeness of the propositional calculus, consistency of the propositional calculus, intuitionism, intuitionist propositional calculus propositional calculus connectives, propositional calculus rules, propositional calculus variables.

propositional calculus rules. See Rule of Assumptions (A), *modus ponens* (MP), *modus tollens* (MT), Double Negation (DN), Conditional Proof (CP), &-Introduction (&-I), &-Elimination (&-E), v-Introduction (v-I), v-Elimination (v-E), *reductio ad absurdum* (RAA) and biconditional.

propositional calculus connectives. The proposition-forming operators on propositions: ' $\neg$ ', ' $\vee$ ', '&', ' $\rightarrow$ ', ' $\leftrightarrow$ '.

propositional calculus variables. Individual letters used to stand for propositions: ' p ', ' q ', ' r '... n . p is any proposition. q is any proposition distinct from p . r is any proposition distinct from

p and q . The number of variables available in the propositional calculus is assumed to be infinite.

propositional function. Formula such as: ' Fx ', or ' $Fx \rightarrow Gx$ ', which falls short of being well-formed, and so does not express a proposition, either because it contains no names or its variables are not bound by quantifiers. A propositional function may be turned into a *wff*. by the addition of names or quantifiers. For example: Fx might become Fm or $(\exists x) (Fx)$, or $Fx \rightarrow Gx$ might become $Fm \rightarrow Gn$ or $(\forall x) (Fx \rightarrow Gx)$. See function, well-formed formula, *wff*.

propositional logic. Propositional calculus. See propositional calculus.

proved. (1) Demonstrated to validly follow from premises. (2) Demonstrated to soundly follow from premises and so true.

proven. (1) Demonstrated to validly follow from premises. (2) Demonstrated to soundly follow from premises and so true.

proven sequent. Sequent either: (1) Demonstrated to validly follow from premises, or: (2) Demonstrated to soundly follow from premises and so true. See derivable sequent, sequent, valid sequent.

qualitative identity. a is qualitatively identical to b if and only if a is like b in every respect: $(Fa \leftrightarrow Fb)$. Qualitative identity logically follows from numerical identity: $(\forall x) (\forall y) (x = y) \rightarrow (Fx \leftrightarrow Fy)$ and (more controversially) numerical

identity follows from qualitative identity: $(\forall x) (\forall y) (Fx \leftrightarrow Fy) \rightarrow (x = y)$. See identical.

quality. Property. That which is ascribed by the use of a true sentence containing a predicate.

quantification theory. See predicate calculus.

quantification. The application of quantifiers (for example: 'all', 'some', 'a few' 'more than half'). In predicate logic, the prefacing of a sentence by either the universal quantifier $(\forall x)$, 'For any x ', or the existential quantifier $(\exists x)$, 'There exists at least one x such that'. Quantification turns a sentence with free variables into a sentence with bound variables, for example: Fx , 'Something is F ', into $(\exists x) Fx$, 'There exists at least one x that is F '. The propriety of the universal and existential quantifiers is not beyond philosophical question. The use of the universal quantifier assimilates 'all', 'any', and 'every' which are arguably distinct concepts. The use of the existential quantifier enshrines formally the doctrine that 'exists' is not a first-order predicate, but 'exists' might be some kind of first-order predicate. Quantification theory is predicate logic. See predicate calculus.

quantifier. Term used to express a quantity, such as: 'all', 'each', 'every', 'any', 'some', 'most', 'many', 'fewer than half', 'none', 'at least one', 'nothing'. See existential quantifier, universal quantifier.

quantifier, existential. See existential quantifier.

quantifier, numerically definite. See numerically definite quantifier.

quantifier phrase. Expression employing quantifiers short of a sentence: 'most of them', 'not all', 'many'. See existential quantifier, quantifier, universal quantifier.

quantifier rules. See $\forall$ -introduction, $\forall$ -elimination, $\exists$ -introduction, $\exists$ -elimination.

quantifier, scope of. See scope.

quantifier, universal. See universal quantifier.

quantifier, vacuous. See vacuous quantifier.

quantifier-shift. The changing of the order of quantifiers in a sequent. For example: changing; ' $(\forall x) (\forall y) (Fxy)$ ' to ' $(\forall y) (\forall x) (Fxy)$ ' or changing; ' $(\forall x) (\exists y) (Fxy)$ ' to ' $(\exists y) (\forall x) (Fxy)$ '. See order of quantifiers, quantifier shift principles.

quantifier-shift principles. The rules that the order of only universal quantifiers in a sequent and the order of only existential quantifiers in a sequent do not affect meaning, validity and truth. The rules depend upon two interderivability results: $(\forall x) (\forall y) (Fxy) \vdash (\forall y) (\forall x) (Fxy)$ and $(\exists x) (\exists y) (Fxy) \vdash (\exists y) (\exists x) (Fxy)$. $(\forall x) (\forall y) (Fxy)$ and $(\forall y) (\forall x) (Fxy)$ mean the same, are true or false under the same conditions and are mutually entailing. $(\exists x) (\exists y) (Fxy)$ and $(\exists y) (\exists x) (Fxy)$ mean the same, are true or false under the same circumstances and are mutually entailing.

Order does matter to validity and truth in sequents mixing types of quantifier. For example, ' $(\forall x) (\exists y) (Fxy)$ ' and ' $(\exists y) (\forall x) (Fxy)$ ' have different meanings, are true or false under different conditions and are not mutually entailing. See order of quantifiers, quantifier-shift.

R

radical reference failure. Putative reference in which nothing is referred to.

range. The converse domain of a relation. The set of entities such that something stands to them in the relation R.

rank of connectives. The order of logical connectives according to their scope: ' $\neg$ ' has a narrower scope than ' $\&$ ' and ' $\vee$ '. ' $\&$ ' and ' $\vee$ ' have a narrower scope than ' $\rightarrow$ ', and ' $\rightarrow$ ' has a narrower scope than ' $\leftrightarrow$ '. The order ' $\neg$ ', ' $\&$ ', ' $\vee$ ', ' $\rightarrow$ ', ' $\leftrightarrow$ ' makes brackets redundant from, for example: $\neg p \vee q, p \rightarrow p \& q$ or $p \& q \rightarrow p$.

rational number.

real number.

realism. The thesis that something exists whether or not it is: (1) believed to exist, (2) perceived to exist, (3) proven to exist, or (4) could be proven to exist, or any combination of these. Realism about truth is this thesis that any proposition is true or false, that is, that the principle of excluded middle holds. See anti-realism, excluded middle. See also modal realism.

recursive. Pertaining to the first and subsequent members of a series.

recursive definition. Application of a property to the first term of a series and the inference that any successor in the series has that property.

reductio ad absurdum, rule of. Rule legitimising inferences of the form

$p \rightarrow q, p \rightarrow \neg q \vdash \neg p$. If a contradiction may be derived from p then it may be concluded that not p . If q follows from p and if $\neg q$ follows from p then q & $\neg q$ follows from p . q & $\neg q$ is a contradiction, so p is false.

redundancy theory of truth. The thesis that ' p is true' means nothing more or less than ' p '.

reference. Semantically picking out in the world. Relation between a referring expression and its referent.

reference failure. (1) Reference, but not by the use of a term to refer to what it conventionally refers to. For example: 'That man drinking black currant juice' might be used successfully to refer to that man, even though he is drinking red wine and 'man drinking blackcurrant juice' is not conventionally used to refer to a man drinking red wine. (2) Failure to refer to anything. Radical reference failure. See radical reference failure.

referent. What is referred to when a referring expression is used correctly or when a sentence including a referring expression is true.

referential. Pertaining to reference.

referentially opaque. A context in a sentence is referentially opaque if and only if substitution of one referring expression by another might change the sentence's truth value.

referring. Semantically picking out.

referring expression. (1) Part of language correctly used to pick something out. (2) Term whose sense is insufficient for its reference.

reflexive. A relation, ' R ', is reflexive if and only if, a is related by R to a : 'reflexive'
(def.): R is reflexive iff $(\exists x) Rxx$. See
irreflexive, relation,

reflexivity. The property of being reflexive. See
irreflexive, reflexive relation, relation.

result. Conclusion validly derived.

relation. That kind of property holding between two or more objects or an object and itself.

relation of identity. a is numerically identical to b if and only if a is b . a is numerically identical to b if and only if a is the same one as b . a is qualitatively similar to b if and only if a is like b in at least one respect. a is qualitatively identical to b if and only if a is like b in every respect. a is qualitatively similar to b if and only if a and b are of the same sort (kind, type).

relational expression. Expression correctly used to refer to a relation: 'next to', 'larger than', 'on top of', 'is a brother of'.

relational sentence. (1) Sentence used to report or describe a relation. Sentence containing at least one relational predicate.

relationship, logical. Two or more propositions being logically dependent or logically independent. See logically dependent, logically independent.

'rests on'. A conclusion rests on some assumption used as a premise if and only if the truth of that assumption is necessary for the truth of that conclusion. If q rests on p then p is necessary for q . In a valid argument, the premises are singularly necessary but jointly sufficient for the conclusion so the conclusion is necessary for the set of its premises. 'rests on' means 'depends on' or 'based on'. See argument, conclusion, entailment, premise, presupposition.

'reverse E'. A name for ' $\exists$ ', the existential quantifier. See existential quantifier.

rigid designator. Expression which picks out the same object in all possible worlds. Expression which changes its meaning if it changes its referent.

robust. Difficult to refute.

rule recorded in proof. The citation on the right hand side of a proven line of argument of the rule of inference used to prove it, usually in abbreviated form.

rule, derived. A rule that is not primitive but logically derived from a primitive rule. See primitive.

rule, primitive. See primitive rule.

rule, quantifier. See quantifier rule.

rule, safe. Rule of inference that ensures validity, that is, from true premises yields only true conclusions.

rule of assumptions (A). Rule of inference legitimising the introduction of any proposition at any stage of an argument as an assumption. Such an assumption typically features as a premise. The conclusion of the argument rests on any such assumptions. See argument, assumption, premise, 'rests on', conclusion, valid.

rule of conditional proof (CP). See conditional proof.

rule of inference. Rule legitimising the valid drawing of a conclusion from premises. See double negation (DN), existential quantifier elimination ($\exists$ E), existential quantifier introduction ($\exists$ I), identity elimination ($=$ E), identity introduction ($=$ I), *modus ponens* (MP), *modus tollens* (MT), *reductio ad absurdum* (RAA), sequent introduction (SI), theorem introduction, universal quantifier elimination ($\forall$ E), rule of universal quantifier introduction ($\forall$ I), $\&$ -elimination ($\&$ -E), $\&$ -introduction ($\&$ -I), v-elimination (v-E), v-introduction (v-I).

S

safe rule. Rule of inference that never yields a false conclusion from true premises. Rule that legitimises only valid inferences.

salva veritate. Latin for: saving (the) truth, truth preserving. See *substitution salva veritate*.

same. (1) a is the same as b if and only if a is b : $a = b$. (2) a is the same as b if and only if a is like b : $Fa \leftrightarrow Fb$. See identity.

schematic letter. Letter replacing and standing for an indicative sentence ' p ', ' q '..., name, ' m ', ' n '... or property ' F ', ' G '... . Variable. See variable.

scope. The scope of a logical connective is the *wff*. it operates upon. This is always the shortest *wff*. in which it features. For example, in ' $(p \vee q) \rightarrow r$ ', the scope of ' $\vee$ ' is ' $p \vee q$ '. In ' $p \leftrightarrow \neg p$ ' the scope of ' $\leftrightarrow$ ' is that entire *wff*. The scope of one connective may be contained within the scope of another. For example in ' $(p \ \& \ q) \vee r$ ' the scope of ' $\&$ ' is included in the scope of ' $\vee$ '. The connective with the contained scope is said to be 'subordinate' to the connective with the containing scope. The scope of a quantifier is always the shortest function in which it occurs. For example, in ' $(\forall x) (Fx) \rightarrow (\exists y) (Gy)$ ' the scope of ' $(\exists y)$ ' is ' $(\exists y) (Gy)$ '. In ' $(\forall x) (Fx)$ ' the scope of ' $(\forall x)$ ' is the entire function ' $(\forall x) (Fx)$ '. See rank.

second order. Applying to a language that applies to the world. If a language applies to the world it is first order.

self-contradictory. Contradictory. Entailing a contradiction. See contradiction.

self-evident. p is self evident if and only if perceiving the truth of p is a necessary condition for understanding p .

self-membered class. A class that is a member of itself. The class of classes is a member of itself because it is a class. The existence of a class of classes that are not members of themselves gives rise to a paradox: That class is and is not a member of itself, which is contradictory. It is nevertheless intuitively plausible that there is a class of classes that are not members of themselves because the most classes are not members of themselves. See paradox.

semantic. Pertaining to meaning.

semantic equivalence. Synonymy. See synonymous.

semantic theory of truth. Thesis that truth is the truth of true sentences.

sentence. A grammatically well-formed portion of language appropriate for making assertions, asking questions or giving orders, so indicative, interrogative or imperative in mood. See atomic sentence, sentence connective. binary, unary, declarative sentence, indicative sentence, proposition, relational sentence, subject-predicate sentence, subjunctive.

sentence, analysis of. (1) Grammatically, the exhibition of the parts of speech in a sentence: nouns, pronouns, adjectives, verbs, adverbs, prepositions, articles and the distinguishing of subject from predicate. (2) Logically, the translation of a sentence into the language of propositional or predicate calculus, exhibiting its logical form in order to determine its logical status or exhibit its role in argument.

sentence-forming operator. See logical connective.

sentential calculus. See propositional calculus.

sequence. Set of things ordered by some principle.

sequent. An argument form consisting in the provability sign ' $\vdash$ ' flanked on the left hand side by assumptions and on the right hand side by a conclusion which putatively follows from them.

sequent, derivable. See derivable.

sequent expression. Any series of *wffs*. that separated by an occurrence of the assertion sign. See assertion sign, proof.

sequent-expression substitution. The replacement of one sequent expression by another with the same logical form. Any substitution instance of a proven sequent can be proven.

sequent introduction (SI). Derived rule permitting the introduction of a proven sequent, or substitution instance of a proven sequent, at any stage of a proof.

set. Plurality of objects with a common property. Class.

set theoretic extension. The members of a set.

singular. Concerning one and only one item. Not plural.

singular term. Expression correctly used to refer to one and only one thing. Proper name. Definite description. See definite description, proper name.

'so'. Conclusion marker in ordinary English.
Therefore. 'p so q' means 'q because p'.

'some'. (1) In logic: at least one. If 'one a is F' is true then 'some a's are F' is true. If 'more than one a is F' is true then 'some a's are F' is true. If 'all a's are F' is true then 'some a's are F' is true. If 'no a's are F' is true, then 'some a's are F' is false. (2) In ordinary English: at least two but not all. See quantifier.

sorites ($? \parallel X \bullet \blacktriangle \sqcap ?$). Greek for 'heap'.

sorites paradox. See vague.

sortal. Term correctly used to pick out things that may be counted: 'book', 'house', 'molecule' but not 'butter', 'wine', 'oxygen'. See universal.

sound. Of an argument: valid with true premises and so with a true conclusion. If an argument is sound then it is valid but a valid argument is not sound if any of its premises is false. All the propositions in a sound argument are true. If an argument is not sound it is unsound. (It is a category mistake to speak of sentences or propositions as sound.) See unsound.

soundness. See sound, unsound.

statement. (1) Sentence expressing a proposition. (2) Token assertion of a sentence expressing a proposition. See proposition.

stipulative definition. Verbal explanation of meaning that is invented or decided rather than discovered.

strategy. Technique for proof discovery.

subalternate. Subimplicant. See subimplicant.

subcontrary. p and q are subcontrary if and only if: if p is false then q is true and if q is false then p is true. If p and q are subcontraries: (1) p and q are not both false (2) p and q may both be true and (3) at least one of p and q is true.
'subcontrary' (def.): p and q are subcontrary iff $p \vee q$ is a tautology.

subimplicant. If q is implied by p then q is subimplicant to p . See implication.

subject. Grammatical subject. Expression within a sentence denoting an object to which a property is ascribed by a predicate. What remains of an indicative sentence when its predicate is deleted. A subject is typically a noun or pronoun or nominal or pronominal phrase. (Confusingly, the term 'subject' is also used to refer to the object the grammatical subject denotes. The grammatical subject is a word or string of words. The subject in this second sense is (typically) a non-linguistic item. No parallel confusion exists between 'predicate' and 'property'.) See predicate.

subject-predicate sentence. Sentence in which a property is ascribed to an object. See predicate, subject.

subordinate. See scope.

substitution. The replacement of one expression by another of the same logical form, especially to provide an assumption for an argument.

substitution-instance. One sequent is a substitution-instance of another if they have the same logical form. Any substitution-instance of a tautology is a tautology. Any substitution instance of an inconsistent proposition is an inconsistent proposition. However, a substitution instance of a contingent proposition might be a tautologous, contingent or inconsistent proposition. Any substitution instance of a proven theorem can be proven.

substitutivity *salve veritate*. The retention of truth value despite substitution. See *salva veritate*, substitution.

sufficient. If p is sufficient for q then if p then q : $p \rightarrow q$. If p is sufficient for q then q is necessary for p : $(\neg q \rightarrow \neg p)$, so $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$. See sufficient condition.

sufficient condition. If the truth of p is a sufficient condition for the truth of q then if p then q (ie if p is true then q is true). If the truth of p is both necessary and sufficient for the truth of q then q if and only if p . If p is a necessary condition for q then q is a sufficient condition for p . If p is a sufficient condition for q then q is a necessary condition for p . See necessary condition

superalternate. Superimplicant. See superimplicant.

superimplicant. If $p \rightarrow q$ is tautologous then p is superimplicant to q .

syllogism. Argument comprising two premises and one conclusion usually relying on quantifiers.

symmetric. A relation, ' R ', is symmetric if and only if: if a is related by R to b , then b is related by R to a . ' symmetric ' (def.): R is symmetric iff $(\forall x) (\forall y) (Rxy \rightarrow Ryx)$. See not symmetric, asymmetric.

synonymous. With the same meaning.

syntax. Grammar.

synthetic. (1) The meaning of a synthetic proposition is insufficient for its truth. (2) The truth of a synthetic proposition is not necessary for its meaning. (3) A synthetic proposition is true or false but not by definition. (4) A synthetic proposition is true or false but not in virtue of the meaning of its constituent terms. (5) The negation of a synthetic proposition is internally consistent. (6) In a synthetic proposition the predicate is not ' contained ' in the subject. (7) Synthetic propositions might provide information about non-linguistic reality. It is a mistake to define ' synthetic ' as ' contingent ' even if controversial thesis that all and only synthetic propositions are contingent is true. See analytic.

T

tautologous sequent. A sequent is tautologous if and only if, for every allocation of truth values to its component propositions, it takes the truth value ' true '.

tautologous *wff*. *wff* that for every possible allocation of truth values to its component propositions, takes the truth value 'true'. Every theorem of the propositional calculus is a tautology. See consistent, contingent, truth table, *wff*.

tautology. (1) Proposition taking the truth value 'true' under all possible allocations of truth values to its component propositions. (2) Logical truth. (3) Substitution instance of a logical law. See consistent, inconsistent, logical law, necessary.

tense. A grammatical form of a verb, determining whether it refers to past, present or future. Not all natural languages include tensed verbs.

tense logic. The study of the validity and invalidity of arguments depending on the tenses of verbs in their component propositions.

'the'. The definite article in English. Word used to refer to some specific thing without thereby specifying what it is.

theorem. Logical truth. The conclusion of a provable sequent in which the number of assumptions is zero. Substitution instance of a logical law. For example: $p \rightarrow p$, $\neg(p \ \& \ \neg p)$.

theorem introduction (TI.) Rule legitimising the introduction of a proven theorem, or substitution instance of a proven theorem, as an assumption at any stage of a proof.

theory of (definite) descriptions. The depiction of the logical form of sentences such as 'The king of

France is bald' as $(\exists x) (\text{bald } x)$ to do justice to the fact that there are two ways in which 'The king of France is bald' can be false: either because there is no king of France or because there is a king of France but he is not bald. ' $(\exists x) (\text{bald } x)$ ' is false if $\neg(\exists x) (\text{bald } x)$ or if $(\exists x) \neg(\text{bald } x)$.

theory of presupposition. Thesis that a subject-predicate sentence does not express a proposition unless the subject refers. ie: A subject-predicate sentence is not true or false unless what the subject refers to exists.

theory of types. Putative solution to Russell's paradox which prohibits claims that a set is a member of itself as ill-formed.

'there is an x such that...'. See existential quantifier.

'therefore'. ' p therefore q ' means ' q is provable from p ': $p \vdash q$.
See assertion sign, proof.

tilde. A name of the negation sign: ' $\sim$ ', pronounced 'not'.

token. Particular. Individual. See type.

token-reflexive. Expression overtly or covertly referring to the sentence in which it occurs. For example: 'Now' refers to when the sentence it occurs in is uttered. 'Here' refers to where the sentence it occurs in is uttered. Indexical. See indexical.

transitive. A relation, ' R ', is transitive if and only if: if a is related by R to b , and if b is

related by R to c , then a is related by R to c .

'transitive' (def.): R is transitive iff $(\forall x) (\forall y) (\forall z) (Rxy \ \& \ Ryz \rightarrow Rxz)$

true. p is true if and only if p . (no italics) p is true if and only if the truth conditions sufficient for p 's truth obtain. If a proposition is not false, and not neither true nor false, nor both true and false, it is true.

true, logically. p is logically true if and only if p is: (1) a tautology (2) a theorem (3) a logical law or a substitution instance of a logical law, or if (4) the meaning of the logical constants in p are sufficient for p 's truth.

truth conditions. The conditions under which a proposition is true or false.

truth-function. A proposition whose truth value is wholly determined by the truth value of its constituent propositions. For example, if p is true and if q is true, then $p \ \& \ q$ is true. If either or both of p and q is false then $p \ \& \ q$ is false. The truth value of a truth function's constituent propositions is necessary and sufficient for its truth value.

truth table. Matrix for the definition of connectives or proof of sequents.

$\neg p$

$\neg p$

p T	F
p F	T

$\neg p$: FT

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$p \rightarrow q$

		q	q
		T	F
p	T	T	F
p	F	T	T

$p \rightarrow q$: TFFT

$p \& q$

		q	q
		T	F
p	T	T	F
p	F	F	F

$p \& q$: TFFF

$p \vee q$

		q	q
		T	F
p	T	T	T
p	F	T	F

$p \vee q$: TTTF

$p \leftrightarrow q$

		q	q
		T	F
p	T	T	F
p	F	F	T

$p \leftrightarrow q$: TFFT

truth-value. The property of being true or false.
 If p is true then p has the truth value 'true'. If
 p is false then p has the truth value 'false'.
 Intuitionism admits truth values other than 'true'
 and 'false', for example: 'neither true nor false',

'indeterminate'. Fuzzy logic admits 'to some degree true and to some degree false'. See fuzzy logic, intuitionism.

turnstile. A name of the provability symbol or assertion sign: $\vdash$. See assertion sign.

type. Some objects are of the same type if and only if they share a property. Category. Sort. Set. Class.

U

union. The union of two sets $\mathcal{V}$ and $\mathcal{V}$ is set of objects which are either members of $\mathcal{V}$ or members of $\mathcal{V}$.

unique. If x is unique one and only one x exists. Nothing like x exists except x .

unit class. Class with only one member.

universal. n . Type. Sort. Kind. *adj.* Without exception.

universal affirmative. Proposition of the form:
 $(\forall x) (Fx \ \& \ Gx)$.

universal negative. Proposition of the form: $(\forall x)$
 $(Fx \ \& \ \neg Gx)$.

universal proposition. Proposition either of the form $(\forall x) (Fx \ \& \ Gx)$ or of the form $(\forall x) (Fx \ \& \ \neg Gx)$.

universal quantifier. The sign: $(\forall x)$ which is read 'for all x ' or 'for every x ', or 'everything' or 'anything'. See predicate calculus.

Universal Quantifier Elimination ($\forall E$). Rule legitimising the inference: $(\forall x) (Fx) \vdash Fm$. If everything is F , then a particular thing is F . If each object in a universe of discourse has a property, then a specific object in that universe has that property.

Universal Quantifier Introduction ($\forall I$). Rule legitimising the inference: $Fa \vdash (\forall x) (Fx)$. If an arbitrarily selected object, a , is F then everything is F . If any arbitrarily selected object in a domain is F then any object in that domain is F .

universals. Types. Sorts. Kinds.

universe. Set of objects.

universe class. The class of which everything is a member.

universe of discourse. Set of objects referred to.

universe, empty. Set not containing any objects.

universe, non-empty. Set containing objects.

'unless'. ' q unless p ' means 'if not p then q ': $\neg p \rightarrow q$. See necessary condition, only, unless...then...
.

'unless...then...'. ' q unless p then q ' means 'if not p then q ': $\neg p \rightarrow q$.

See necessary condition, only, unless.

unsound. Argument that is either invalid or possesses at least one false premise or both. If an argument is valid then it is either sound or unsound. If an argument is invalid then it is unsound. See sound, valid, invalid.

vacuous. Without referent.

vacuous quantifier. Quantifier governing no variable.

vague. Indefinite, especially in reference. For example, it might be thought that there is no exact number of items enough to comprise a heap, so 'heap' is vague in its application. It is not clear under exactly which conditions it should be applied and under which conditions withheld. See sorites.

valid. If an argument is valid, then the truth of the premises is inconsistent with the falsity of the conclusion. If an argument is valid then, if the premises are true, then the conclusion is true. If an argument is not valid it is invalid.

vel. Latin for 'or' in the inclusive sense: either p or q or both: $p \vee q$.
(See aut.)

variable. Symbol used to replace an expression. Propositional variables $p, q, r...$, replace propositions. In predicate calculus, $x, y...$, replace names.

vel. Latin for 'or' in the inclusive sense of: 'either...or...or both'.

wff. abbrev. well-formed formula.

verb. Word used to denote an action or event.

verb, intransitive. Verb not taking a direct object. See verb.

verb, transitive. Verb taking a direct object. See verb.

W

well-formed formula (*wff*). Member of the class of formulae obeying the syntactic formation rules of a calculus.

wff. abbrev. well-formed formula.

'when'. If.

'when and only when'. If and only if.

word. The smallest unit of meaning in a natural language.

LOGICAL ATOMISM

Logical atomism is a theory of meaning designed to solve philosophical problems. Russell's version of the theory is strongly influenced by that of his pupil and colleague, Wittgenstein, and is presented in the eight lectures *The Philosophy of Logical Atomism*, and the short essay entitled 'Logical Atomism'.

Russell tells us that his work in the philosophy of mathematics first suggested to him the correctness of logical atomism. It is not too misleading to say that in the philosophy of mathematics Russell attempts to reduce mathematics to logic, while in

the philosophy of language he attempts to reduce language to logic: that is, to reduce ordinary language to a logical language. His ambition is to produce a logically perfect language which will not generate philosophical problems, the thought being that the imprecise and misleading nature of natural language leads us into those problems.

The title 'logical atomism' is intended to capture two aspects of the doctrine it names. Atomism is the thesis that the universe (and in logical atomism the language which names it) is intrinsically a plurality. There really are discrete elements constituting what exists, even if the set of such elements is infinite, and even if the elements are thoroughly interrelated. Atomism is thus opposed to the monism of, say, the neo-Hegelians, who think that the pluralistic nature of what exists is an illusion, or mere appearance.¹⁰ Russell calls the theory 'logical atomism' because the atomic constituents of what exists are logical rather than physical, or, for that matter, mental. A physical atom could in principle be divided, but a logical atom could not even in principle be analysed into constituents.

According to Russell, 'the world contains facts, which are what they are whatever we may choose to think about them' (LA 35). Logical atomism thus entails a kind of realism. Something either is or is not the case, regardless of whether we believe it to be the case. Propositions are either true or false, irrespective of whether they are believed to be true or false. A fact is whatever makes a proposition true or false, and a fact is a state of affairs that obtains quite independently of thought and language. No fact is any particular thing or person: 'a fact is the sort of thing that is expressed by a whole sentence, not by a single name

like "Socrates" ' (LA 36), so that A has property F, or that A bears relation R to B, are facts. It does not make sense to speak of true or false facts: 'A fact cannot be either true or false' (LA 38). The bearers of truth values are propositions, and we may talk of beliefs, for example, as true or false because the propositions we believe are true or false. A proposition is treated by Russell as an indicative sentence - that is, a sentence used to assert what is or is not the case. Clearly, not all sentences are propositions because not all sentences are indicative. Some are imperative, interrogative, or exclamatory. Another way of explaining the concept of a proposition is to say that it is a sentence which is always capable of being prefaced by the propositional 'that', so ' "that Socrates is alive" and "that two and two are four" ' (LA 39) are propositions in Russell's terms.

It follows that no facts are propositions and every proposition is an indicative sentence. Every proposition is a complex symbol, a symbol that has symbols for its parts. For example, a sentence containing more than one word is a complex symbol and its constituent words are the simple symbols which are its parts. It is essential to clarity, and to the avoidance of error, not to mistake the properties of words and sentences for properties of objects and states of affairs. Meaning is a relationship between language and non-linguistic reality: nouns mean objects, adjectival expressions name qualities, and whole indicative sentences mean facts. Although names name objects, propositions assert or deny facts and do so truly or falsely, in virtue of those facts.

Facts are the objects of analysis in logical atomism. Complex facts are analysed into atomic facts, but the only way of doing this is via the

analysis of propositions: the analysis of molecular (or complex) propositions into atomic (or simple) propositions, and the analysis of atomic propositions into simple symbols.

Simple symbols are the ultimate constituents of propositions, and they denote the simple components of the facts which make the propositions true or false. 'Red', for example, is a simple symbol because it cannot be analysed into constituent symbols, yet it may be a constituent of a symbol. Although 'red' can in one sense be defined (a description of what 'red' is can be given in terms of physics), it cannot be defined in the sense of 'analysed'. No verbal definition of 'red' could make a person unacquainted with the colour red know what red is. In this sense of 'red', ' "red" could not be understood except by seeing red things' (LA 49) .

Atomic propositions are analysed into proper names and predicates. Proper names name particulars. A particular is a component of a fact which exists logically independent of other components and yet bears some relation to them. A predicate designates a quality, so 'red', for instance, is a predicate, but 'John' is a proper name. Russell says that 'to understand a name you must be acquainted with the particular of which it is the name' (LA 60) .

Molecular propositions are truth functional. This means their truth or falsity depends upon the truth or falsity of their constituent propositions.

Propositional functions are sentences with at least one uninterpreted constituent, and they are expressed in a symbolism. 'P', 'Q', 'R', etc. are propositional variables, and '-', '·', 'v', '→', '≡' are logical constants. The logical constants may be defined by truth tables as follows:

P - P

T F

Stephen Priest *Remarks on Logic* (2012 Draft)

F T

' $\neg P$ ' (read 'not P') is true if and only if P is false.

P . Q

T T T

T F F

F F T

F F F

'P . Q' (read 'P and Q') is true if and only if both P and Q are true.

P $\vee$ Q

T T T

T T F

F T T

F F F

'P $\vee$ Q' (read 'P or Q') is true if and only if either both P and Q are true or if only one of P and Q is true.

P $\rightarrow$ Q

T T T

T F F

F T T

F T F

'P $\rightarrow$ Q' (read 'if P then Q') is true if and only if either not P is true or Q is true or both.

P = Q

T T T

T F F

F F T

F T F

'P = Q' (read 'P if and only if Q') is true if and only if either both P and Q are true or both P and Q are false. The analysis of a propositional function may proceed in a purely logical, indeed algorithmic, manner. A complex propositional function is a tautology, a contradiction, or contingent, depending on the truth values of its constituent atomic propositional variables. A

proposition is atomic if, and only if, it has no constituent propositions.

The truth-functional analysis of propositions leaves entirely on one side questions about the semantic constituents of propositions. Indeed, propositional functions are undetermined in the sense that P , Q , etc. may stand for any (mutually distinct) propositions whatsoever. Russell says that 'One may call a propositional function necessary, when it is always true possible, when it is sometimes true impossible, when it is never true' (LA 87) but it does not make sense to ascribe these properties to propositions: 'Propositions are only true or false' (LA 88).

It is a source of philosophical error to confuse the properties of propositions with those of propositional functions, as it is to confuse properties of both of those with properties of facts. Being clear about grammar makes progress possible in philosophy. Indeed, 'practically all traditional metaphysics is filled with mistakes due to bad grammar' (LA 128-9).¹¹

8 MEANING AND TRUTH

Meaning

Russell subscribes to a referential theory of meaning; that is, he thinks the meaning of a word is the thing the word refers to: 'Words all have meaning, in the simple sense that they are symbols that stand for something other than themselves' (PM 47). Russell's referential theory is, however, more sophisticated than most and contains attempts to forestall philosophical objections.¹²

If the meaning of a word is held to be the object the word refers to, then certain difficulties arise. An object may be referred to by a number of non-synonymous expressions: for example, 'the

person with open eyes' and 'the person understanding this sentence' both refer to you but their meaning is not the same. Conversely, one description, say 'the head of department', may be used to refer to more than one person: a university professor at Cambridge and one at Edinburgh, for instance. If meaning were only reference, both these facts would be impossible. Again, if the meaning of 'The person writing this sentence is me' were me, then if I ceased to exist the meaning of that sentence would cease to exist. Clearly, however, the meaningfulness of that sentence does not depend so closely on my existence.

If you assert that 'Scott is the author of *Waverley*' (LA 105), what you assert is true but not tautologically true, as it would be if meaning were only reference. It is contingently true because a definite human being, Scott, wrote a specific book, *Waverley*. It is not true by definition but in virtue of a certain fact. 'Scott' is a name and 'the author of *Waverley*' is a definite description. The definite description is true of the person named by the proper name, and that is what makes the whole proposition true. If one were to substitute a proper name for the definite description in this example the result would be a different proposition, and if that proper name referred to Scott the new proposition would be a tautology.

The distinction between proper names and definite descriptions explains how identity statements may be informative. Although 'George IV wished to know if Scott was the author of *Waverley*', it is not true that George IV wished to know if Scott was Scott' (LA 105).

In *The Philosophy of Logical Atomism* Russell argues that 'the notion of meaning is always more or less psychological' (LA 40). Nevertheless, as we have

seen, names name particulars, predicates denote qualities or relations, and indicative sentences express facts; thus there are three different kinds of meaning. Giving the meaning of an expression involves analysing that expression, and ideally the meanings of a logically perfect language would be clear. Ordinary language is riddled with ambiguities but this is not necessarily a disadvantage for non-philosophical work. Ambiguity gives ordinary language a certain richness and allows for originality in expression.

Russell's most influential contribution to the theory of meaning is his 'theory of descriptions'.¹³ A singular definite description, a sentence of the form 'the so-and-so', may be true or false, but if it is false because what it putatively describes does not exist, then two philosophical problems arise. If we say, 'the present king of France is bald' is false, we seem to be covertly asserting the existence of what does not exist, and that is manifestly false. This is a problem concerning existence. If we claim that names name objects and propositions name facts, then objects and facts are the meanings of names and propositions. However, it then follows that if the king of France does not exist, the meaning of 'the present king of France' does not exist and the expression is meaningless. Similarly, if the king of France is not bald, then the fact that the king of France is not bald does not exist, so the meaning of 'the present king of France is bald' does not exist and that proposition is meaningless. This is a problem concerning meaning. The point of the theory of descriptions is to show in what way false existential claims are meaningful. Russell's solution is to claim that 'a name has got to name something or it is not a name' (LA 100). If a putative name names nothing, it is a truncated

description of the logical form 'x has such and such properties' (LA 100). The proposition 'the present king of France is bald' is in fact two propositions:

(1) 'There is a c such that c is now king of France'

(2) 'c is bald'.

Analysing the proposition this way avoids construing its denial, 'The present king of France is not bald', as entailing 'There is such a person as the king of France and that person is not bald' (LA 109), which is false. There are in fact two ways of denying the proposition: by denying the king of France exists and by denying the king of France is bald. So the denial of the proposition is not

(3) 'There is a c such that c is now king of France and c is not bald'

but

(4) 'Either there is not a c such that c is now king of France, or, if there is such a c, then c is not bald'. (LA 109)

If we wish to deny the proposition we have a choice of asserting either of the disjuncts of (4). In these ways we will not have said something meaningless and not have covertly asserted the existence of what does not exist.

Truth

Truth and falsity are properties of beliefs; but if I believe something, what I believe is a sentence; thus truth and falsity are derivatively properties of sentences. However, the truth or falsity of a sentence is logically independent of its being believed. Our beliefs may be true or false, and what is the case and what is not the case in no way depends on our beliefs.

A belief is true or false in virtue of the occurrence or non-occurrence of extra-linguistic

states of affairs - 'facts' as Russell calls them in *The Philosophy of Logical Atomism*, or 'verifiers' as he calls them in *An Inquiry into Meaning and Truth*. For example, the truth of 'Caesar was assassinated' (IMT 215) consists in the occurrence of a particular event - the assassination of Caesar. This event is a fact which verifies the sentence.

Russell rejects the view that truth can be explained in terms of verifiability, perhaps for the very good reason that to 'verify' p means to demonstrate the truth of p , so any such explanation would be circular. Later, Russell subscribes to a correspondence theory of truth: p is true if, and only if, the fact expressed by p exists: 'The "verifier" is defined as that occurrence in virtue of which my assertion is true (or false)' (IMT 219).

A series of philosophical problems exists which concern what the truth of propositions about unobserved facts consists in: propositions about other minds, historical events, theoretical particles, for example. Russell, correctly I think, wishes to maintain that the truth of such propositions consists in the existence of these facts: 'on what may be called the realist view of truth, there are "facts", and there are sentences related to these facts in ways which make the sentences true or false, quite independently of deciding the alternative' (IMT 232), but 'the difficulty is to define the relation which constitutes truth if this view is adopted' (IMT 234). I should say the truth of the proposition lies simply in the existence of the fact and the proposition, and there is no residual relation and so no residual problem.¹⁴

Russell argues in the massive and influential three-volume work *Principia Mathematica* that mathematics is reducible to logic. Indeed he maintains that essentially there is no difference between mathematics and mathematical logic because any putative distinction between the two is arbitrary. As he puts it in *The Principles of Mathematics*, 'All mathematics is symbolic logic ... all mathematics is deduction by logical principles from mathematical principles' (PM 5), and in the *Introduction to Mathematical Philosophy*: Logic has become more mathematical and mathematics has become more logical. The consequence is that it has now become wholly impossible to draw a line between the two; in fact the two are one. They differ as boy and man: logic is the youth of mathematics and mathematics is the manhood of logic. (IMP 194)

Russell wrote an enormous amount of outstanding quality on the philosophy of mathematics, but here I shall confine attention to one fundamental question: 'What is a number?' (IMP 11).

A number is not identical with the number of things there are that number of. For example, the number two is not identical with any two objects, the number three is not identical with any three objects, and so on. Rather, the number two is what is common to all pairs of objects, the number three is what is common to all trios, and so forth. All pairs of objects form a class of objects, the class of pairs; all trios form a class, the class of trios. Russell maintains that a number is a class of such classes. The number two, for example, is the class of pairs, the number three the class of trios, etc., but further argument is needed to reach this conclusion on pain of using 'number' in a question-begging way.

To analyse the concept of number Russell uses the notions 'class', 'member' and 'similar'. Classes may be defined extensionally or intensionally. To define a class extensionally is to enumerate the members of that class (a, b, c, ... n). To define a class intensionally is to mention a defining property of the members of the class, a property such that if an individual possesses that property then it logically follows that it is a member of that class, and such that if it lacks that property it logically follows that it is not a member of that class. Intension is logically prior to extension for Russell because extension can always be reduced to intension but intension cannot always be reduced to extension. For example, we could not in principle enumerate all the members of an infinite set, but we could state the defining characteristic of all members. The series of natural numbers (0, 1, 2, 3 ... n) is infinite, so any definition of 'natural number' in terms of classes must be by intension and not by extension. To define 'number' in terms of classes, it is necessary to know when two classes have the same number of members and so belong to the same class of classes. This is possible without knowing how many members those two classes contain. For example, if there were only monogamy the number of wives would be the number of husbands. Clearly, this is knowable a priori because the relation is 'one-one' (IMP 15). Russell distinguishes relations that are 'one-one', 'one-many', and 'many-one'. A relation is one-one if, and only if, 'if x has the relation in question to y, no other term x' has the same relation to y, and x does not have the same relation to any term y' other than y' (IMP 15). Clearly, a relation is one-many if, and only if, x has a relation to y and to y' but no term x' distinct from x has that relation to either y or y

'; and a relation is many-one if, and only if, both y and some term distinct from y , y' , has a relation to x but does not have that relation to x' , some term distinct from x . Notice the concept of number is nowhere employed in these definitions.

Russell uses the idea of a one-one relation to provide a criterion for the similarity of classes: 'two classes are said to be "similar" when there is a one-one relation which correlates the terms of the one class each with one term of the other class' (IMP 15-16). If a is related to b by relation R , then a is the 'domain' of R , and b is the 'converse domain' of R . The relation 'similar' between classes has three logical properties:

- (1) Reflexivity: 'Every class is similar to itself' (IMP 16).
- (2) Symmetry: 'If a class a is similar to a class b , then b is similar to a ' (IMP 16).
- (3) Transitivity: 'If a is similar to b , and b to c , then a is similar to c ' (IMP 16).

Russell defines 'number' in this way: 'A number is anything which is the number of a class' (IMP 19). There is no circularity here because he uses 'class' and 'similar' to define 'the number of a class' without using 'number': 'The number of a class is the class of all those classes that are similar to it' (IMP 18). Thus the number two comes out as the class of all couples, the number three is the class of all classes of three objects, and so on. Notice 'the class of all couples will be the number 2' (IMP 18). Numbers, then, are not Platonic, metaphysical or other-worldly, but classes of classes.¹⁵

Russell and Whitehead's attempt in *Principia Mathematica* to derive the order of mathematics from certain logical axioms was not entirely successful, as Gödel demonstrated.¹⁶ However, it is in large

part because of their efforts that mathematical logic is the thriving subject it is today.¹⁷

Philosophy

Lent Week 9, Friday

Mr.

Zeno

Zeno

Zeno was a follower of Parmenides. Parmenides denied the existence of nothingness or non-being, thought that being is one, and that it is changeless. We can understand Zeno as defending the last two of these claims, and extending Parmenides' view of being to objects and events. Zeno regards objects and events as paradoxical and contradictory, and thus they cannot exist. He felt that many of our commonsensical views of the world are thoroughly mistaken.

Zeno seems to assume that if motion is an illusion, he can show that change is an illusion. In order to establish the fact that all change is motion, we would have to show that everything is physical (materialism). However, this assumption may be fallacious.

The Paradox of the Arrow (Barnes p. 155) says (1) what is travelling is always in a place at a time and (2) anything in a place is at rest (in a space that is no smaller or larger than the physical dimensions of the thing. From this Zeno concludes that there is no motion.

Given a billion years of science, what we perceive as motion might turn out to be something else. After all, people used to see the world was flat, and were convinced that there were witches. Even

something as central to our worldview as motion could be called into question in time. Zeno is saying that something moving from a place (P1) goes through numerous places (P2) to a destination (P3). Objects moving through these places are either in a particular place at a certain time (in which case it is either at rest, or partly outside that place) or not in that place at that time.

In order to show Zeno is wrong, we would have to argue that time is not composed of time instants (which have no duration). What it means to say that x has moved is to suggest that it was in P1 at t_1 and P2 at t_2 . Zeno is right that our notions of motion depend on assumptions about time and places. He points out that there might not be time or places.

The Paradox of Achilles goes as follows. Achilles runs from a place (P) to a destination (D). A necessary prerequisite for getting to D is getting to the halfway point between P and D (M). But a prerequisite for getting to M is getting to the halfway point between P and M. Thus travel over finite distances is impossible because one must travel through an infinite number of points. Zeno himself does not discuss time travel but this example raises interesting questions. If time travel into the past were possible, one could go back and eliminate the causes of one's existence (killing one's parents). If we go back to a destination in past time, we seem to have arrived at this destination, but we have not truly arrived at the same destination because we are present. Objections to time travel are based on paradoxes, and Zeno's attack on motion works in a similar way. We could say that it is empirically obvious that things move, and that Zeno is wrong. But time travel seems paradoxical, contradictory and

impossible, until someone does it. In this sort of situation, we would find time travel remarkable but would accept it. The fact that we have found contradiction in theory does not mean the theory is impossible.

On p. 157, Zeno says that what is moving is neither in the place in which it is, nor in the place in which it is not. These two possibilities are jointly exhaustive and mutually exclusive according to Zeno. Therefore, motion, he concludes, is an illusion. We might suggest the following example. Supposing there is a fly on a table, and it flies off the table. It is clear what it is to say that the fly is on the table, and it is clear what it is to say that the fly has left the table. But what is it to say that fly is leaving the table?

If we say that if one atom of one leg of the fly is in contact with the table, then it is on the table, and if it some distance off the table it has left the table. There must be some event where the fly leaves the table. You can image an object being in some minute contact with the ground, or you can imagine a gap or a distance between the surface and the object. But there is a difficulty about what it is to change from one state to another - this applies to any change. Our rather static view of time and motion may be mistaken.

Zeno thinks there are no places, places being regions or areas of space. Zeno argues that any region of space is located in a region of space that is greater than it. Paradoxically, this process can continue *ad infinitum* (p. 158), therefore no place can exist. In arguing against the existence of places, Zeno is arguing that there are no such things as places, and also denying

plurality, affirming the one and denying the many (like Parmenides).

On the same page there may be a early version of the Paradox of the Heap, which is designed to show that many of our concepts are vague. Consider a heap of sand, and remove one grain. The heap of sand is still a heap of sand. However, if we continue this process we will reach a point where the heap ceases to be a heap. Is there a particular point where this change takes place?

ZENO'S PARADOXES OF MOTION

The Paradox of the Arrow

- (a) Anything is at a place at a time.
- (b) If something is in a place at a time it is not in motion.

Conclusion: Nothing is in motion.

- (1) 'Everything is always at rest when it is in a space equal to itself'
- (2) 'What is travelling is always in such a space at any instant'
- (3) 'The travelling arrow is motionless' (155)
- (4) 'Zeno does away with motion by saying:

"What is moving is moving neither in the place in which it is nor in the place in which it is not"
Diogenes Laertius *Lives of the Philosophers* IX 72 (157)

Reply to the Paradox of the Arrow

- (1) 'the travelling arrow stands still' (156)
- (2) 'It depends on the assumption that time is composed of instants'

(3) 'That is false; for time is not composed of indivisible instants'

Achilles and the Tortoise

Achilles gives the tortoise a head start (in distance but not time) in a race. Of course Achilles can move much faster than the tortoise.

- (a) T moves from place P1 to place P2.
 - (b) A move from P to P1.
 - (c) T moves from P2 to P3
 - (d) A moves from P1 to P2 and so on *ad infinitum*.
- Conclusion: Achilles never catches up with the tortoise.

'The pursuer must first reach the point from which the pursued set out, so that the slower must always be ahead of it' (155)

Reply to Achilles and the Tortoise

- (1) '[...] it is false to claim that the one ahead is not caught'
- (2) 'it is not caught while it is ahead' (156)
- (3) 'nonetheless it is caught (provided you grant that they can cover a finite distance)'

The Paradox of the Stadium (or Race-course)

- (a) x has to traverse a finite distance, ' d '.
 - (b) To traverse distance d , x has to traverse half distance d , d^*
 - (c) To traverse distance d^* , x has to traverse half distance d^* , distance d^{**} and so on *ad infinitum*.
- Conclusion: x cannot traverse distance d .

'Nothing moves because what is travelling must first reach the half-way point before it reaches the end' (155)

What is motion?

x moves if and only if:

- (1) *x* is at some place P1 at some time t1.
- (2) *x* is at some numerically distinct place P2, at some later time t2.
- (3) *x* exists at some juxtaposed series of places between P1 and P2 at all times between t1 and t2.

Motion is a kind of change.

What is change?

x changes if and only if *x* is F at t1 and *x* is not F at t2 or *x* is not F at t1 and *x* is F at t2.

Questions about motion:

- (1) Can there be change without motion?
- (2) Can there be motion without change?
- (3) What is it to begin to move? (How long does beginning to move last?)

NB: It seems to be a materialist thesis that if there is change there is motion.

It seems to be a dualist or idealist thesis that there can be change without motion.

References to: Jonathan Barnes (ed.) *Early Greek Philosophy*

(Penguin)

conceivability. 160 Admissibility by the mind. Thinkability. If imagination entails the generation of mental imagery, then everything imaginable is conceivable but not everything conceivable is imaginable. Something is conceivable if and only if it is possible to form a concept of it, but a concept need not involve an image.

Logical possibility is necessary for conceivability, so logical impossibilities are inconceivable, but conceivability is not what logical possibility consists in. A putative state of affairs is logically impossible if and only if its description entails a contradiction, logically possible if it has at least one consistent description. A putative state of affairs is conceivable if and only if it can be thought. Nevertheless, conceivability without contradiction is important to the epistemology of logical possibility.

On some empiricist views, if I have experience of *a* and experience of *b* then I can conceive of some new item, *c*, composed of *a* and *b*, but not of any item I have neither experienced nor experienced the constituents of.

On some materialist or anti-metaphysical views, if I think I am thinking of something non-physical, say spiritual or abstract, I have only in fact only succeeded in thinking of something physical. On this view, to conceive of something is to think of it as possessing at least some primary qualities.

Stephen Priest *Remarks on Logic* (2012 Draft)

Hume *An Enquiry Concerning Human Understanding*
(Oxford, 1955)

Hobbes *Leviathan* (Harmondsworth, 1985)

The British Empiricists (Penguin, 1990)

Anaximander and the Infinite

Lecture by

1. There are a number of ways in which the infinite enters into philosophy. It seems to be a presupposition of mathematics that there are infinite numbers. It seems to be a presupposition of theology that there is an infinite God. It seems

to be a presupposition of certain systems of metaphysics that some things are infinite. If there was no first event, perhaps the universe is infinite in backward time. If there will be no last event, the universe is infinite in forward time.

2. Does infinity exist? Do mathematics, theology and some kinds of metaphysics in fact rest upon an illusion? Is the appearance of infinity an illusion? Anaximander believes infinity does exist. Possibly he is the earliest Western thinker to suggest this. Why does Anaximander believe the infinite exists? He believes the finite presupposes the infinite. He thinks there are infinite processes or infinite number which, in a logical sense, presuppose infinity.

3. Anaximander thinks of infinity as a presupposition. There has to be infinity for there to be the world as it exists. It is a mistake to think of Anaximander as postulating infinity as an extra abstract reality. Infinity is not metaphysically grounded.

4. All of Anaximander's arguments for infinity are given in Aristotle's *Physics*. Anaximander's claims.
i) Infinite nature is the first principle of the things that exist. "First principle" is the translation of the Greek word *arche*, which can also mean "first cause" or "ultimate necessary condition." A necessary condition is a prerequisite for something that has to be the case in order for something else to be the case. There must be infinity in order for there to be anything. An ultimate necessary condition (a first principle) is a necessary condition that has no necessary condition. Nothing is necessary for infinity.

5. "Things that exist" could mean empirical objects, or Anaximander might be saying that infinity is necessary for Being. In this case, Being would be infinite. This is not the thought that

there is an infinite number of empirical objects, but that the Being of the things that are is infinite, much in the way that someone might think that space or Nothingness are infinite. It makes sense to think of a physical object as finite, but not to think in this way of Being. Being is not bounded.

6. ii) Infinity is eternal. Infinity did not begin and it will not end. There are sequences, i.e. the positive number sequence beginning from zero, which could be regarded as infinite, while still having an origin, i.e. zero. This is not the kind of infinity Anaximander means. He means infinity that did not begin and will not end.

7. iii) Infinity is ageless. If x has three properties -it exists, it did not begin and it will not end-, we can raise some philosophical questions about this. Does it last? Arguably it does. If x exists, then it lasts, because existing takes time. Normally, it would seem that if x exists, then it lasts. x at a time t_2 would seem to be older than at a time t_1 . Anaximander seems to be denying this in the case of infinity.

8. There are possible grounds for endorsing Anaximander's view. One could take the view that it is always now. There are plausible metaphysical reasons for thinking this. The past did exist, but it doesn't exist, because it is over. The future will exist, but it doesn't exist, because it hasn't begun. Therefore only the present exists. If someone asked what the time was, it would be philosophically correct to reply that it is now. In a sense it is always now despite the fact that clock time changes and different things happen. There is something motionless or non-temporal in the midst of time, and that is now. Nobody knows what now is. It is an unsolved philosophical problem.

9. iv) Infinity contains all the worlds. It is tempting to read Anaximander as saying something similarly to the Princeton philosopher David Lewis, who thinks there are possible world. This world is not the only world. A possible world is any world that has a consistent description, one that is coherent or does not entail a contradiction. Lewis invokes this notion in order to explain the difference between contingency and necessity. To say that p is necessary is to say that p is true in all necessary world, and to say that p is contingent is to say that p is true in some but not all possible worlds. It seems to some people that these possible worlds do not exist; they are only logical postulates. Lewis says they all exist. In fact there is an infinity of possible worlds. What is the difference between this world and all the other possible worlds? Lewis thinks this world, the one in which we exist, is the actual world? What does it mean to say that this is the actual world? It means we are in it, we occupy it. Lewis thinks we must not confuse the following two points: i) This is the world we occupy. ii) Only this world exists. There is nothing to prevent one from reading Anaximander as referring to possible worlds.

10. v) There is eternal motion. Distinguish change and motion. Cf. Heraclitus. There is a profound understanding of motion which is more fundamental than Aristotle's understanding of motion. According to Aristotle, if x has property f at t_1 , and does not have property f at t_2 , or does not have property f at t_1 , but has property f at t_2 , this is sufficient for change. Change is the gaining or losing of properties. On this view, x could be anything. If anything changes, it either gains or loses a property. Something is true of it at a later time that was false of it an earlier time, or

vice versa. Heraclitus can be interpreted in a strong or a weak way, as saying either that there is only change, or that everything changes. Anaximander seems to hold a view rather like that attributed to Heraclitus on the weak interpretation: everything changes. Suppose that x is at a place p_1 at time t_1 , and at the same place p_1 at time t_2 , x has moved.

11. On the Aristotelian view, motion is a kind of change. If there is motion, then there is change. Is the converse true? If there is change, then is there motion? The view that if there is change then there is motion is a materialist view. If there is change, then there is a change of spatio-temporal position, i.e. physical change. It is an unsolved philosophical problem whether all change is motion. It is not clearly that psychological changes involve motion. Perhaps atoms in the brain are in motion. According to Anaximander, if there is eternal motion, then at any time there is change of spatio-temporal position. Choose any time you like, there is change of spatio-temporal position.

12. There are two kinds of pre-Socratic philosophers: philosophers of change or process philosophers (Anaximander, Heraclitus) and philosophers of the changeless or static philosophers (Parmenides, Zeno).

13. One could construct an argument that everything is in motion. i) Everything is changing (an empirical premise). ii) All change is motion (a materialist premise).

14. vi) Worlds are infinite in number. This corresponds to what Lewis says.

15. vii) Infinity is one. It is clear that Anaximander thinks there is a plurality of finite things. Things that can be considered as grouped into sets with a finite number of members. It seems that if we ask how many things exist, the answer is

that the number of things that exist is infinite. In what sense can it be that infinity is one? Distinguish between infinity and an infinite number of things. A difference between the things which are infinite in number and their infinity. The property of being infinite is what is one, or a unity.

16. Another way of understanding vii). There are different infinite phenomena, eg. mathematical phenomena, theological phenomena etc. If this is true, Anaximander can be read as saying that it is the same infinity that is found in mathematics and theology. Interestingly, since the Renaissance, a split emerges between religion and science, in particular between mathematics and theology. We have inherited this very strong division. The explanations provided by religion and science and by science are different, and seem to make each other redundant. One explanation makes the other superfluous. This distinction is not to be found in the time of Anaximander. The pre-Socratics are attempting to understand everything, without limitations. Since the Renaissance, mathematics has been thought of as scientific. The scholastics thought of as mathematics as divine. It contains perfect truths, therefore it appears to reflect the workmanship of God. Above the portal of his Academy, Plato set the words: "Let no one enter here who has not studied geometry." We are living at a moment in history when there a sharp distinctions between academic disciplines, whereas the pre-Socratics were inventing ways of thinking for explaining the universe. Understanding this should make it less odd that Anaximander thinks there is only one kind of infinity.

17. An argument for infinity. i) If something is finite, then something is infinite. ii) Something is finite. iii) Something is infinite. There are

two ways of critising such an argument: to say that either the first or the second premise is false, or to accept the premises and try to show that the conclusion does not follow. Premises are called premises because they are reasons for believing the conclusion.

18. Distinguish between the validity and the soundness of an argument. To say that an argument is valid is to say hypothetically: "If these premises are true, then the conclusion is true." If an argument is sound, it has two features: i) it is valid, and ii) the premises are true. Clearly if an argument is sound, the conclusion is true.

Lecture delivered in the University of Edinburgh, David Hume Tower, Lecture Theatre A, Monday 8th February 1999 10.00-11.00 am.

ANAXIMANDER

Infinity

- (1) 'infinite nature is [the] first principle of the things that exist' (71)
- (2) 'It is eternal'
- (3) 'ageless'
- (4) 'it contains all the worlds' (7) 'one' (74)
- (5) 'there is an eternal motion' (8) 'moving'
- (6) 'worlds, infinite in number' (9) 'infinite'

Arguments For Infinity

NB: All these are derived from Aristotle's *Physics* (203b6-11). It is historically uncertain whether Anaximander subscribed to them but Anaximander is mentioned in the same passage.

- (1) 'it is ungenerated and indestructible' (75)

Stephen Priest *Remarks on Logic* (2012 Draft)

(a) What is did not begin.

(b) What is cannot, so will not, end.

Conclusion: What is is infinitely extended
backwards and forwards
in time.

(2) 'it is [...] the divine' (75)

(a) If what ultimately is is divine then what
ultimately is
is infinite.

(b) What ultimately is is divine.

Conclusion: What ultimately is is infinite.

(3) 'time [...] this is infinite'

(a) Time exists.

(b) Time is infinite

Conclusion: infinity exists.

(4) 'division of magnitudes'

(a) If x is infinitely divisible, something is
infinite.

(b) x is infinitely divisible.

Conclusion: Something is infinite.

(5) 'mathematicians actually use the infinite'

(a) Mathematics presupposes the infinite.

(b) Mathematics is true/works.

Conclusion: the infinite exists.

(6) 'generation and destruction will give out
unless there is

something infinite'

(a) Beginning and ending would cease if nothing were infinite.

(b) Beginning and ending do not (ever) cease.

Conclusion: Something is infinite.

(7) 'what is finite is always limited by something'
(76)

(a) If something is finite, something is infinite.

(b) Something is finite.

Conclusion: Something is infinite.

Questions about Anaximander's arguments:

- (1) Did what is begin?
- (2) Is what is ultimately divine?
- (3) Could time end?
- (4) Is anything infinitely divisible?
- (5) Does mathematics presuppose the existence of the infinite?
- (6) Could beginning and ending end?
- (7) Is the limited necessarily limited by something?

Aristotle appears to ascribe these views to Anaximander:

'Because they do not give out in thought, numbers seem to be infinite [...] so do mathematical magnitudes and the region outside the heavens. If void and space are infinite, body too must be infinite'

Physics 203b

Aristotle himself holds that no infinity is *actual*. Any infinity is only *potential*. What might this mean?

coherence. p and q are coherent if and only if the possible *truth of p does not preclude the possible truth of q and the possible truth of q does not preclude the possible truth of p . It follows that the concept of coherence presupposes the concept of truth, so truth cannot be explained in terms of coherence without circularity. Nevertheless, coherence is necessary for truth because if $\{p, q\}$ form an incoherent set then at least one of p and q is false. Coherence also provides a test for truth because if it can be shown that $\{p, q\}$ form an incoherent set, and it is known that one of p, q is true, then it follows that the other is false, and if it can be known which is true, then it can be known which is false. However, coherence is not sufficient for truth because the coherence of $\{p, q\}$ is consistent with the falsity of both p and q .

**coherence theory of truth*

SP

Sybil Wolfram *Philosophical Logic* (London, 1989)

conceptual analysis. The attempt to solve philosophical problems, or exhibit them as illusory, by defining words or being clear about how concepts are used. In practice, conceptual analysis involves logical deduction because it requires showing the entailments of definitions. The logical status of true sentences of philosophy

is then 'analytic'. Philosophy is a second order subject in so far as conceptual analysis is its method because it is about language not the world or what language is about. However, the philosophical results of solving problems of the form 'What is the definition of "x"'? are not independent of those obtained from answering 'What is x?' or 'What is the essence of x?'.

Some concepts seem recalcitrant to analysis or, to put it another way, some words seem resistant to verbal definition. 'Red' is hard to define if construed phenomenologically even though readily definable in terms of physics. 'Exists' resists definition, at least once we realise that being *F* is not sufficient for being.

Conceptual analysis, if not under that name, has been intermittently practised in philosophy at least since Plato. For example it is recognisable in the work of Aquinas, Hobbes, Leibniz, and Kant. In the last century it was the philosophical method advocated by the Logical Positivists.

The later Wittgenstein argues that, for at least some concepts, there are no necessary and sufficient conditions for being *F*, because there is nothing that all and only those items truly called 'F' have in common. Rather than engage in definition, then, philosophers should attend to the vast diversity of linguistic use.

The idea that non-linguistic reality is too changing, too phenomenological or too paradoxical to be realistically depicted by terms admitting of precise definition is as old as Heraclitus but also salient in the thought of Hegel and Nietzsche. In philosophy, though, we should be as clear as we can for as long as we can if we are to begin to understand its problems. If we define our terms as clearly as we are able, we can avoid beginning by arguing past one another. A philosopher should be

concerned about definition in the way that a historian is concerned about putting dates in the right order. *Ayer, *family resemblance, *implicature *Wittgenstein.

SP

A. J. Ayer *Language, Truth and Logic* 2nd ed.
(Harmondsworth, 1976)

John Hospers *An Introduction to Philosophical Analysis* (London and New York, 1997)

Ludwig Wittgenstein *Philosophical Investigations*
(Oxford, 1952)

conceivability. 160 Admissibility by the mind. Thinkability. If imagination entails the generation of mental imagery, then everything imaginable is conceivable but not everything conceivable is imaginable. Something is conceivable if and only if it is possible to form a concept of it, but a concept need not involve an image.

Logical possibility is necessary for conceivability, so logical impossibilities are inconceivable, but conceivability is not what logical possibility consists in. A putative state of affairs is logically impossible if and only if its description entails a contradiction, logically possible if it has at least one consistent description. A putative state of affairs is conceivable if and only if it can be thought. Nevertheless, conceivability without contradiction is important to the epistemology of logical possibility.

On some empiricist views, if I have experience of *a* and experience of *b* then I can conceive of some new item, *c*, composed of *a* and *b*, but not of

any item I have neither experienced nor experienced the constituents of.

On some materialist or anti-metaphysical views, if I think I am thinking of something non-physical, say spiritual or abstract, I have only in fact only succeeded in thinking of something physical. On this view, to conceive of something is to think of it as possessing at least some primary qualities.

SP

Hume *An Enquiry Concerning Human Understanding* (Oxford, 1955)

Hobbes *Leviathan* (Harmondsworth, 1985)

The British Empiricists (Penguin, 1990)

future contingents. Either: predictions neither true nor false at the time they are made, or, future states of affairs that might or might not obtain. On the first definition, a future contingent is a claim about the future, or is the content of a future tense indicative sentence. On the second, it is the possible truth condition for such a claim. If there are future contingents, in either sense, then predictions are neither true nor false until the states of affairs they are used to predict obtain or fail to obtain. *Sea-battle argument.

SP

Aristotle's Categories and De Interpretatione ed. J. L. Ackrill (Oxford, 1963): *de Interpretatione* Book IX

*Guillelmi de Ockham opera philosophica et
theological ad fidem codicum manuscriptorum edita*
ed. S. Brown et. al. (New York, 1967 ff.)

fuzzy logic. Logical system which allows degrees of truth. For example, where '1' denotes truth and '0' falsity, p might be true to degree 0.7 and so false to degree 0.3. In general, if p is true to degree n then p is false to degree $1 - n$. Fuzzy logic is a departure from classical logic because a proposition may be to some extent both true and false, and a proposition and its negation to some extent both true. Classical logic, in a sense, is a version of fuzzy logic, that version in which the only admissible truth values in the range from 0 to 1 are 0 and 1.

Arguably, fuzzy logic does justice to the intuitive idea that some indicative sentences are not wholly true and not wholly false. For example, the claim 'He is in the room' seems not wholly true and not wholly false but partly true and partly false if he is leaving the room at the time of utterance.

SP

A. Kandel *Fuzzy Mathematical Techniques with Applications* (Addison Wesley, 1986)

L. A. Zadeh 'Fuzzy Sets' in *Information and Control* (1965)

formal and material mode. A distinction between two ways of using language. In the formal mode, the expressions of a language refer only to linguistic items: concepts and the logical relations between

them. In the material mode the expressions of a language refer to non-linguistic items: objects, events and the relations between them. The question 'What is x ?' is in the material mode but the question 'What is the meaning of " x "?' is in the formal mode.

The distinction is analogous to the
*use/mention distinction and due to *Carnap.
*conceptual analysis, *Logical Positivism,
*semantic ascent.

SP

Rudolf Carnap *The Logical Syntax of Language* (1934)

free logics. Logical systems in which existential commitment is not assumed. Logic in which names and predicates need not refer to anything. Logic whose sentences are not made true or false by what does or does not exist. So, in free predicate logic, $(\exists x) (Fx)$, 'There exists at least one x that is F ', does not have to follow from Fa , ' a is F '. Free logic is called 'free logic' because, in contrast with standard predicate calculus, the use of names is free from commitment to the existence of a referent.

Arguably, free logics do justice to the intuitive view that from the fact that the concept of something exists it does not follow that that thing exists: From 'A unicorn has a single horn' it does not follow that there are any unicorns. The intuitive view is not beyond question. Arguably, unicorns do exist but are fictional objects or exist in story books or people's imaginations. 'Exists' does not mean the same as 'is a four dimensional publicly observable object'. Arguably, in a world in which unicorns did not exist story

books and people's imaginations would differ from the actual world.

There exists both modal and nonmodal free logic. In free modal logic, the use of a name implies commitment to existence in at least one possible world, but this is commitment only to the possible existence of a referent not to the existence of a referent.

SP

Ermanno Bencivenga 'Free Logics' in Dov A. Gabbay and Franz Guenther (eds.) *Handbook of Philosophical Logic* vol. 3 (Dordrecht, 1986) pp. 373-426

Graeme Forbes *Modern Logic* (Oxford, 1994) pp. 322-5

Mark Sainsbury *Logical Forms* (Oxford, 1991) pp. 204-10

logical laws. Propositions true on logical grounds alone. Logical truths. For example, the laws of non-contradiction, identity, excluded middle, and double negation. In propositional calculus the law of non-contradiction is: $\neg(p \ \& \ \neg p)$, 'It is not the case that both p and not p ', in predicate calculus: $(\forall x) \neg(Fx \ \& \ \neg Fx)$ 'For any x , it is not the case that x is F and x is not F '. In propositional calculus the law of identity is: $(p \rightarrow p)$, 'If p then p ', in predicate calculus: $(\forall x) (Fx \rightarrow Fx)$, 'For any x , if x is F then x is F ', in predicate calculus with identity: $(\forall x) (x = x)$, 'For any x , x is x ', in modal predicate calculus with identity: $\Box(\forall x) (x = x)$, 'Necessarily, for any x , x is x '. In propositional calculus the law of excluded middle is: $p \vee \neg p$, 'Either p or not p ', in predicate calculus: $(\forall x) (Fx \vee \neg Fx)$, 'For any x , either x is F or x is not F '. In propositional

calculus the laws of double negation are: $--p \rightarrow p$, 'If not not p then p ', and $p \rightarrow --p$, 'If p then not not p ', and in predicate calculus: $(\forall x) (--Fx \rightarrow Fx)$ 'For any x , if x is not not F then x is F ' and $(\forall x) (Fx \rightarrow --Fx)$, 'For any x , if x is F then x is not not F '.

Aristotle does not distinguish sharply between logical laws, laws of thought and laws of being, so the consistent, the *conceivable and what could exist coincide, and the inconsistent, the inconceivable and what could not exist coincide. Aristotle's informal statements of the law of non-contradiction include: 'For the same thing to hold good and not to hold good simultaneously of the same thing and in the same respect is impossible' (*Metaphysics* ¹ 1005b): $(\forall x) -(Fx \cdot -Fx)$ or arguably: $(\forall x) -\Diamond(Fx \cdot -Fx)$, and 'Nor [...] is it possible that there should be anything in the middle of a contradiction' (1011b): $-\Diamond(p \cdot -p)$. His statement of the law of excluded middle is 'but it is necessary either to assert or deny any one thing of one thing' (1011b), $(\forall x) (Fx \vee -Fx)$ or arguably; $\cdot(\forall x) (Fx \vee -Fx)$. Aristotle says it shows a lack of education to demand a proof of logical laws. He does however bring a self-refutation argument against their putative denial by his pre-Socratic predecessors: Protagoras, who thinks that every claim is true but there is no truth over and above belief by or appearance to persons, and Heraclitus, who thinks that everything is changing in every respect so there is no truth. Aristotle points out that saying anything meaningful or true, for example making Protagorean or Heraclitian claims, presupposes logical laws.

*Mill maintains that logical laws are not a *priori* or necessary, but empirical generalisations

confirmed by all experience but, so far, refuted by none. He thinks all deduction is really induction.

*Quine has suggested revision of the law of excluded middle to simplify quantum mechanics.
*Plantinga has commented that this is rather like revising a law of arithmetic to simplify the doctrine of the Holy Trinity.

It is widely taken as axiomatic that if the description of a putative phenomenon entails a violation of a logical law then that phenomenon cannot exist. However, if we are persuaded, for example, that Zeno has found contradictions in the concept of motion (for example: If x moves then x is at a place at a time and x is not at that place at that time) we do not thereby conclude that nothing moves; "Foolish, foolish us! We thought things *moved*. But no. That philosopher Zeno has shown that the concept of motion entails a contradiction. Clearly we should give up this widespread, perceptually compelling but incoherent assumption! Motion is *logically impossible*." Rather, we retain the view that things move and look for a consistent theory of motion. The implications for philosophy, science and theology are wide. Perhaps time travel is not logically impossible, it is just that we so far lack a consistent theory of it. Arguably, something is possible if and only if there is at least one consistent description of it.

Perhaps nothing is logically impossible, because contradictions do not pick out any putative states of affairs. If not, they do not pick out any impossible putative states of affairs. "Ah yes, 'Both $(p \text{ . } - p)$ ', it is the putative state of affairs picked out by *that* sentence that could not come about!" But what state of affairs could not come about?

E. J. Lemmon Beginning Logic (London, 1967)

Aristotle's Metaphysics Books 📖 📖 📖
translated with notes by
Christopher Kirwan (Oxford, 1971) Book Gamma.

John Stuart Mill *A System of Logic* 2 vols. (London, 1879)

Alvin Plantinga *The Nature of Necessity* (Oxford, 1974)

W. V. O. Quine 'Two Dogmas of Empiricism' in his
From a Logical Point of View (Harvard, 1953) pp.
20-46

logic of discovery. *Deduction in the testing of scientific theories. For example, the exhibiting of logical relations between the sentences of a theory (such as equivalence, derivability, consistency, inconsistency) or between a theory and established theories; the logical inferring of predictions from a theory. *Popper argues against the view that scientific theories are conclusively inductively verifiable but argues for their deductive and empirical falsifiability. A claim of the form $(\forall a)(Fa)$, 'Every a is F ', cannot be confirmed by any finite number of observations of a 's that are F , because there could always in principle exist an undiscovered a that is not F , but $(\forall a)(Fa)$ can be refuted by the discovery of just one a that is not F .

Popper has an evolutionary epistemology of scientific discovery. The formulation of theories is analogous to genetic mutation in evolutionary

theory. Theories and mutations arise randomly as putative solutions to environmental problems and only those conducive to the survival of the species in that environment themselves survive through trial and error. Popper adopts a Platonist view of logic, on the grounds that proofs are (sometimes surprising) discoveries not unsurprising inventions. *Popper

SP

Karl R. Popper *The Logic of Scientific Discovery*
(London, 1980)
Ch. 1 Sect. 3 pp. 32-4.

paralogisms. In Kant's critical philosophy, a fallacious argument. In the *Critique of Pure Reason* Kant identifies three paralogisms in the first edition and a fourth in the second edition. They are invalid inferences to conclusions expressing: the simplicity of the soul, the personality of the soul, the immortality of the soul and the existence of the external world. In each case, the mistake is to try to derive a tenet of transcendental realism from premises expressing only transcendental idealism, that is, conclusions about a putative non-spatio-temporal reality from premises only about possible appearances.

SP

Immanuel Kant's Critique of Pure Reason trans.
Normal Kemp Smith (London, 1978)

quantification. The application of quantifiers (for example: 'all', 'some', 'a few' 'more than half'). In predicate logic, the prefacing of a sentence by

either the universal quantifier ($\forall x$), 'For any x ', or the existential quantifier ($\exists x$), 'There exists at least one x such that'. Quantification turns a sentence with free variables into a sentence with bound variables, for example: Fx , 'Something is F ', into $(\exists x) Fx$, 'There exists at least one x that is F '. The propriety of the universal and existential quantifiers is not beyond philosophical question. The use of the universal quantifier assimilates 'all', 'any', and 'every' which are arguably distinct concepts. The use of the existential quantifier enshrines formally the doctrine that 'exists' is not a first-order predicate, but 'exists' might be some kind of first-order predicate. Quantification theory is predicate logic.

SP

Martin Davies *Meaning, Quantification, Necessity: Themes in Philosophical Logic* (London, 1981)

Mark Sainsbury *Logical Forms* (Oxford, 1991) Ch. 4

sea-battle argument. Argument criticised by *Aristotle in *de Interpretatione* Book IX. Either there will be a sea-battle tomorrow or there will not. These possibilities are mutually exclusive and jointly exhaustive. It seems, therefore, to be a necessary truth that there will be a sea-battle tomorrow or there will be no sea-battle. The future is inevitable now, it is just that we are presently ignorant of which future events will happen. The argument putatively establishes the fatalistic conclusions that everything that happens happens necessarily, and it was always true that the those events that do happen would happen.

In criticism, Aristotle argues that from the necessity of the disjunction $(p \vee \neg p)$ it does not

follow that the disjunct p is necessary nor that the disjunct $\neg p$ is necessary. Although it is necessary that a sea-fight take place tomorrow or not, it is not necessary that it should take place tomorrow, and is it not necessary that it should not take place tomorrow. Therefore the argument does not validly establish that future events are inevitable. This is consistent with its being a necessary truth that either there will or will not be a sea-battle tomorrow. *future contingents

Aristotle's Categories and De Interpretatione
trans. J. L. Ackrill (Oxford, 1963)

semantics, possible worlds. The meanings of *modal logic. Possible worlds are the referents of the sentences of modal logic. For example, $[]p$ is true if and only if p is true in all possible worlds, $\langle \rangle p$ is true if and only if p is true in some possible world, $\neg \langle \rangle p$ is true if and only if p is true in no possible world. A possible world is a way things might be. One possible world is the way things are. According to the modal realism advanced by David Lewis, all possible worlds exist. No possible world is spatio-temporally related to any other possible world. The difference between the present world and other possible worlds is not that this one exists and the others do not, but that it is actual. Actuality is indexed to a world because, for example, this world being actual consists in our occupying it. It is debatable whether there is a possible world in which nothing exists. It seems possible that there should have been nothing, yet existence would seem necessary to anything's being a world.

SP

Saul Kripke 'Semantical Considerations on Modal Logic' *Acta Philosophica Fennica* 16 (1963) 83-94
reprinted in Leonard Linsky (ed.) *Reference and Modality* (Oxford, 1971) pp. 63-87)

David Lewis On the Plurality of Worlds (Oxford, 1986)

NECESSITY

The fundamental philosophical problem about modality [remains] unsolved: saying exactly what new information is added to 'If x is red then x is extended' by prefacing the whole sentence with the necessity operator. Saying that this would add that if x is red then x cannot not be extended just shifts the problem of the unanalysed modal notion onto 'can'.

Royal Institute of Philosophy Supplement 44, 1999
p. 211

It is hard to see how modality can be analysed in non-modal terms.

Royal Institute of Philosophy Supplement 44, 1999
p. 211

Suppose someone wished to maintain that the world is all that is the case. It is hard to see what extra information is provided about a portion of the world when it is claimed that it is not only the case but could not not be the case or although the case might not have been the case.

Stephen Priest *Remarks on Logic* (2012 Draft)

Royal Institute of Philosophy Supplement 44, 1999
p. 211-12

MATHEMATICS AND THE MIND OF GOD

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1. Necessity
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Use the quotations from *The God of Mathematics* but arranged thematically not by mathematician.

Pythagoras, Descartes, Newton, Leibniz,

THE VIENNA CIRCLE

Stephen Priest *Remarks on Logic* (2012 Draft)

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Notes

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I

WHAT WAS THE VIENNA CIRCLE?

The philosophy of the Vienna Circle is essentially Kantian. If the Kantian components of Logical Positivism were subtracted from it, the residue could not be correctly called 'Logical Positivism'. By 'Kantian' I mean: semantically or logically equivalent to concepts or claims deployed by Kant in his philosophical writings. Most of the anticipations are in the *Critique of Pure Reason*.

I

Synthetic *A Priori* Propositions

The feature which the Logical Positivists think most sharply distinguishes their own philosophy from Kant's is their refusal to allow that there are any meaningful synthetic *a priori* propositions, because any putative claim that is neither analytic and *a priori* nor synthetic and *a posteriori* is nonsensical. As Arthur Pap puts it: 'A metaphysical proposition might be defined as a synthetic *a priori* proposition'. (Pap: 19)

Kant agrees that doing metaphysics entails the attempt to discover synthetic *a priori* truths but thinks the attempt fails. Still, Kant insists there are many meaningful, true and informative

non-metaphysical synthetic *a priori* propositions in, for example: theoretical physics, geometry, arithmetic, and his own philosophy; statements which do not simply explicate definitions.

Kant and the Vienna Circle share this concept of the synthetic *a priori*: *p* is synthetic *a priori* if and only if *p* is knowable, observation is not necessary for the knowledge that *p*, and *p* is nonetheless informative. If *p* is synthetic *a priori* then *p* is a subject-predicate proposition, and 'informative' may be unpacked as either: '*p* is informative if and only if, the concept of *p*'s predicate is not already contained in *p*'s subject' or, less metaphorically, as '*p* is informative if and only if in defining the subject of *p*, recourse does not have to be made to the (concept expressed by) the predicate in *p*'.

Because the conjunction of Kant's claim there are synthetic *a priori* propositions and the Logical Positivist claim that there are no synthetic *a priori* propositions entails a contradiction, at most and at least one of Kant and the Vienna Circle can be right.

Suppose it is true that there is a class of statements which are genuinely informative or synthetic, and suppose there is another class that is knowable to be true or false without observations. Then there exists a class of *a priori* statements and there exists a class of synthetic statements. If we look through the writings of the Logical Positivists it is noticeable that many of the statements there do, in fact, fall into both categories. In other words they are both synthetic and *a priori*. For example, the following tenets of Logical Positivism are both synthetic and *a priori*:

'there is nothing which could not become an object of science'

(Carnap, 1967: 293)

'[...] every statement has a meaning only insofar as it can be verified'

(Schlick, 1979 II: 264)

These claims are neither true by definition nor open to empirical confirmation so they are neither analytic nor *a posteriori*. On the other hand, their content is supposed to come as news to the reader so they are putatively informative or synthetic. If true but not *a posteriori* they are knowable independently of experience so *a priori*. It follows that they are synthetic *a priori*.

Many substantial philosophical claims are synthetic *a priori*, for example, Carnap's neo-Humean doctrine of the self in the *Aufbau*:

'The self is the class (not the collection) of the experiences (or autopsychological states).'

(Carnap, 1967: 299)

This cannot be decided by experience, nor is it a tautology. If it can be known it is knowable purely intellectually and is informative, so synthetic *a priori*.

The logical positivists show no overt awareness that the status of many of their claims is synthetic *a priori* and that their philosophy is thereby self-refuting. Waismann does see a threat in construing sentences criterial of Logical Positivism as truth valued, or expressing propositions, perhaps because they can not be plausibly construed as either both analytic and *a priori* or both synthetic and *a posteriori*. He says

'the meaning of a statement is its method of verification'. To say this is not to hold a theory

which can be true or false – but it is rather a recommendation as to the way in which we should deal with certain sentences when they have no conventional meaning' (Waismann 1965:)

However, this will not do. The logical form of 'the meaning of a statement is its method of verification' is not imperative but indicative because it is an identity statement and all identity statements are indicative. Having indicative entailments is not sufficient for not being an imperative, but although 'the meaning of a statement is its method of verification' entails 'S is meaningful if and only if S is verifiable' it does not clearly entail anything imperative.

Waismann thinks it is covertly imperative but this is an assimilation of an imperative to the putative truth that would make it worth obeying. Only someone who holds, or ought to hold, a belief ought to obey an imperative to act on that belief, but the contents of beliefs are indicative.

The logical status of 'the meaning of a statement is its method of verification' is synthetic *a priori*. It is knowable without empirical observation and it is informative so the Vienna Circle is therefore overtly committed to 'there are no synthetic *a priori* propositions' but logically committed to at least one synthetic *a priori* proposition. By 'overtly committed' I mean 'professes'. By 'logically committed to' I mean, 'The conjunction of the sentences of Logical Positivism entails...'. So, catastrophically, Logical Positivism entails that there are and are not synthetic *a priori* propositions, which is a contradiction. The philosophy of the Vienna Circle is therefore either meaningless or false.

The only viable remedy is to give up the second conjunct and endorse the first conjunct of

'there are and there are not synthetic *a priori* propositions'. In this sense the Logical Positivists have not recognised how thoroughly Kantian they are. Like Kant's, the sentences essential to their own theory are synthetic *a priori*. Kant recognised this and admitted a meaningful non-metaphysical middle class of synthetic *a priori* statements. The Logical Positivists should do the same and recognise their thorough Kantianism on pain of self-refutation. (5) 'There are no synthetic *a priori* propositions' is synthetic *a priori*.

Metaphysics, Science and Common Sense

The conclusion of *The Critique of Pure Reason* presents the central doctrine of Logical Positivism. Metaphysical knowledge is impossible. Both Kant and the Logical Positivists define 'metaphysics' as the putative study of reality either as it is independently of our experience of it, or as a whole, or both.

For example, on the first page of the Preface to the first edition of *The Critique of Pure Reason*, Kant says the principles of metaphysics are not subject to any empirical test (CPR 7 A viii), and it employs 'principles which overstep all possible empirical employment', and these 'transcend the limits of experience'. On the first page of Chapter One of *Language Truth and Logic* Ayer says his criticism of metaphysics will take as its starting point 'those who believe that it is possible to have knowledge of a transcendent reality' (LTL 33). The metaphysician, as the object of Ayer's attack, 'claimed to have knowledge of a reality which transcended the phenomenal world.' (LTL 33) So Kant and the Vienna Circle hold that

Kxp → - (p is metaphysical)

'Transcendent' and 'phenomenal' are Kantian terms, and Ayer uses both of them in a very Kantian way. Kant explicitly defines the 'transcendental employment' of our understanding as 'employment extending beyond the limits of experience (*über die Erfahrungsgrenze*)' (CPR 299 A 296/B 353). Ayer says:

'from empirical premisses nothing whatsoever concerning the properties, or even the existence of anything super-empirical can legitimately be inferred.' (LTL 33)

'Super empirical' means for Ayer, as it would for Kant, 'transcendent'. Ayer has just asked rhetorically in the same passage,

'Must he [the metaphysician] not begin as other men do with the evidence of his senses? And, if so, what valid process of reasoning can possibly lead him to the conception of a transcendent reality?' (LTL 33)

For both Ayer and Kant 'all our knowledge begins with experience' (CPR 41). (1) By 'the phenomenal world' both Kant and Ayer mean the empirical world; the world as it appears to us through the five senses. Ayer, for example, contrasts the 'phenomenal world' with 'anything super-empirical' (LTL 33) and frequently uses 'phenomenal' and 'empirical' synonymously. Kant contrasts 'phenomenal' with 'noumenal', which means 'super-empirical' at least in part, and Kant too often uses 'empirical' and 'phenomenal' interchangeably. So both Kant and Ayer endorse this empiricist tenet:

$\forall x \forall p (Kxp \rightarrow x \text{ has experience})$

If anybody knows anything then they have experience.

Both Ayer and Kant not only contrast metaphysics with common sense but with science. In the resulting definitions, metaphysics is found disasterously wanting and for both thinkers science is the paradigm of knowledge-yielding inquiry. Ayer says his first aim in his book is to

'criti[cise] the metaphysical thesis that philosophy affords us knowledge of a reality transcending the world of science and common sense' (LTL 33)

In the Preface to the second edition, Kant says metaphysics 'has not yet had the good fortune to enter upon the secure path of a science [...] *den sichern Gang einer Wissenschaft* [...]' (CPR 21 B xiv) and this for precisely the same reason as Ayer: 'Metaphysics is a completely speculative science of reason, which soars far above the teachings of experience *einer ganz isolierten spekulativen Vernunftkenntnis, die sich gänzlich über Erfahrungsbelehrung erhebt* [...]' (CPR 21 B xiv)

According to Kant, the metaphysician purportedly employs a special non-empirical faculty called 'pure reason'. According to Ayer,

'the metaphysician [...] would say that he was endowed with a faculty of intellectual intuition which enabled him to know facts that could not be known through sense experience.' (LTL 33-4)

The term 'intellectual intuition' (*intellektuelle Anschauung*) is Kant's. (CPR 35 Bx1 fn.) He explicitly says human beings do not possess it. If a being has intellectual intuition then if they believe something then it is true:

$$Bxp \rightarrow p$$

And because $Kxp \rightarrow Bxp$ the empiricist principle

$$\forall x \forall p (Kxp \rightarrow x \text{ has experience})$$

is violated by:

$$\exists x \ \& \ \exists p \ Kxp \ \& \ -(x \text{ has experience})$$

which is logically equivalent to

$$\neg \forall x \forall p (Kxp \rightarrow x \text{ has experience})$$

Both Kant and Ayer repudiate the view that philosophy should study the 'totality' or reality as a whole. For Ayer the metaphysician is someone who seeks 'first principles' but

'the belief that it is the business of the philosopher to search for first principles is bound up with the familiar conception of philosophy as the study of reality as a whole' (LTL 47)

A first principle, p^* , is a truth that is necessary for any truth

$$(\forall p) (Ep^*) \neg p^* \rightarrow \neg p$$

so any truth is sufficient for a first principle:

$$(\forall p) p \rightarrow p^*$$

Kant also thinks it a mistake to think that philosophy can study 'the whole', but this is how the metaphysician conceives metaphysics. Indeed both Kant and Ayer attach very little sense to the expression 'the world as a whole'. For example, Kant says

'The world does not exist in itself, independently of the regressive series of my (re)presentations, it exists *in itself* neither as an *infinite* whole nor as a *finite* whole (*weder als ein an sich unendliches, noch als ein an sich endliches Ganzes*)' (CPR 448 A 505/B 533)

and claims, 'the world is not an unconditioned whole (*kein unbedingtes Ganzes*), and does not exist as such [as a whole]' (CPR 448 A 505/B 533). Ayer says of 'reality as a whole': 'This conception is one which it is difficult to criticise, because it is so vague' (LTL 47). For both Kant and Ayer, the attempt to study the world as a whole, in any putative metaphysical inquiry, results in nonsense. For Ayer, the metaphysician produces sentences which are devoid of meaning by his verificationist criteria, for Kant the metaphysician produces the conjunction of statements and their contradictories: and for Kant to contradict oneself is to produce nonsense.

II

The World as a Whole

If Kant and Ayer do attach any significance to the expressions 'reality as a whole' or 'the world as a

whole', then it is to denote the world as studied by science. Ayer:

'The study of reality as a whole [...] if it is taken to imply, as it sometimes is, that the philosopher somehow projects himself outside the world, and takes a bird's eye view of it, then it is plainly a metaphysical conception. And it is also metaphysical to assert, as some do, that "reality as a whole" is somehow generically different from the reality which is investigated by the special sciences.' (LTL 47)

This image of the metaphysician as a bird flying is in Kant. For example, at, Kant speaks of metaphysics as speculation 'which soars above the teachings of experience (*die sich gänzlich über Ehrfarungsbelehrung erhebt*)' (CPR 21), and says,

'Plato left the world of the senses, as setting too narrow limits to the understanding, and ventured out beyond it on the wings of the Ideas, in the empty space of the pure understanding (*in den leeren Raum des reinen Verstandes*).' (CPR 47 A 5/B 9)

Kant and Ayer share the same metaphysical targets and the same anti-metaphysical metaphors.

For Kant, as we have seen above, the world 'as a whole' can only mean something if it denotes the 'series of my (re)presentations (*Vorstellungen*)' (CPR 448). This is just the world of nature, the empirical world with which each of us is directly acquainted through sense perception. Ayer's word is not 'presentations' but 'sense contents', but the conjunction of the two theses is analytic. (5)

Kant thinks science is a genuine means of inquiry into this one empirical world, and distinguishes it from 'random groping', which is the category into which metaphysics falls. He says:

'[...] the study of nature has entered on the secure path of a science after having for so many centuries been nothing but a process of merely random groping (*bloßes Herumtappen*)'.
(CPR 20-21)

Both Kant and Ayer agree that the natural science of their day is the most reliable source of knowledge, and if, for either, there is a rival for this place, it is common sense. They reject metaphysics partly because it is not commonsensical. Kant, for example, claims that metaphysics (comparing her to a queen) 'tried to trace her lineage to vulgar origins in common experience' but 'this genealogy has as a matter of fact been fictitiously invented' (CPR 8). Ayer says that we should not despise common sense and it is an important part of both the Kantian and the positivist attacks on metaphysics that the metaphysician uses concepts in abstraction from any commonsensical empirical application.

It is controversial whether the *Critique of Pure Reason* entails a version of phenomenalism but there is at least considerable consistency between the positivist and the Kantian conceptions of the empirical world. Both agree it is the one world of our experience that science investigates. For Kant it is a 'series of presentations (*Vorstellungen*)', for Ayer a series of 'sense contents'. For Kant and Ayer, respectively, 'presentations' and 'sense contents' are not clearly or straightforwardly either mental or physical. As Ayer puts it: 'The

distinction between what is mental and what is physical does not apply to sense contents' (LTL 123). He says 'they are neither' (LTL 123). Kant sometimes talks about 'presentations' in a direct realist way; as though we were directly acquainted with aspects of physical objects in perception. At other times he talks as though they were sense contents. The most coherent way to read Kant is to allow that presentations are 'neutral' between physical and mental descriptions of them.

Kant and Ayer, then, share broadly similar conceptions of metaphysics, its relation to the empirical world, and to science.

III

Sense and Sense

There is an anti-metaphysical argument which is essential to both the Kantian and the Logical Positivist critiques of metaphysics. I shall call this the 'sense and sense' argument, it's conclusion is that concepts only have meaning if, in the last resort, they or the sub-concepts they 'contain' have an empirical use.

Kant and the Logical Positivists share the view that the concepts we use to make our experience intelligible have their only legitimate application in the spatio-temporal world we inhabit empirically. Hence it is quite illegitimate to use those concepts to make claims about what supposedly lies in principle outside our experience. This position is articulated most clearly by Moritz Schlick in his paper 'Meaning and Verification' (6). Schlick distinguishes two sorts of definition: verbal and ostensive. To define a word verbally is

to explain its meaning in other words. To define a word ostensively is to point to the object it refers to and pronounce the word, to name the object for the benefit of the listener or learner. The 'sense and sense' argument appears in its positivist form in this passage:

'It is clear that [...] to understand a verbal definition we must know the signification of the explaining words beforehand, and that the only explanation which can work without any previous knowledge is the ostensive definition. We conclude that there is no way of understanding any meaning without ultimate reference to ostensive definitions and this means, in an obvious sense, to 'experience' or 'possibility of verification'.

If sound, this argument works with devastating force against metaphysical language. Metaphysical terms can be defined verbally but not ostensively. This would not prove fatal if those words used to define them verbally could in turn be defined ostensively. But this, it seems, is not the case. Take the word 'monad', for example. (8) Monads according to Leibniz are the non-physical ultimate constituents of reality. Each is a non-spatial and non-temporal atomic substance that 'mirrors' or 'reflects' all the others. It is clear that none of the terms of this definition can be defined ostensively. We cannot point to entities outside space and time. Similarly the word 'God' admits of verbal definition - 'omniscient, omnipotent, transcendent, spiritual being', perhaps. But none of these terms can be defined ostensively, or so it seems. Similar remarks could be made about Plato's 'Ideas', Hegel's 'Geist', or the concept of an immaterial soul, as used by Plato, Augustine or Descartes. (~)

Schlick's potentially powerful argument was used by Kant. Indeed, as for the positivists, it is essential to his theory of meaning. Compare this passage with Schlick's:

'There is something strange (*Befremdliches*) and even absurd (*Widersinniges*) in [the assertion] that there should be a concept which possesses a meaning (*Bedeutung*) and yet is not capable of any explanation (*der aber keiner Erklärung fähig wäre*).' (CPR 263 A 244 fn.)

Kant uses 'explanation' here to mean 'definition' ('that is, of *defining it*' *d.i ihn zu definieren* [...] 'they cannot themselves be defined' *sie also nicht definiert werden* CPR 263 A 246) so his view is logically equivalent to Schlick's claim about 'definition'. Although Kant distinguishes categories from empirical concepts, neither has any non-empirical use. Even though categories are psychologically imposed on experience and empirical concepts are abstracted from it, the function of both is non-metaphysical. He says about the categories:

'The categories have this peculiar feature, that only in virtue of the general conditions of sensibility (*der allgemeinen sinnlichen Bedingung*) can they possess a determinate meaning (*eine bestimmte Bedeutung*) and relation to any object (*Gegenstand*).' (CPR 263)

As we have seen the 'conditions of sensibility' are space and time. So any possible object that can give a category meaning has to be a spatio-temporal one. This is exactly Schlick's premiss about ostensive definition.

There are important differences between Kant and the positivists on concepts. For example, Ayer and Schlick would either deny that any is a *priori* or (aut) regard 'a *priori* concept' as a senseless concatenation. Yet Kant's categories are a *priori*.

But their anti-metaphysical accounts nevertheless have it in common that no concept has any transcendent meaning or reference. Schlick would have been in complete agreement with Kant's insistence that:

'[...] the understanding can never transcend those limits of sensibility within which alone objects can be given to us.' (CPR 264)

and Kant, like the Logical Positivists, thinks that the empirical objects we perceive, in the last resort, enable all our concepts to be meaningful.

By 'intuition' (*Anschauung*) Kant means something very close to the positivist's 'sense experience' or even 'sense content'. Each of space and time is a pure intuition (*eine reine Anschauung* CPR 69 A 24/B 39) and Kant says 'the only intuition possible to us is sensible ([...] *ist alle uns mögliche Anschauung sinnlich*)' (CPR 162 B 146) but he talks about 'a *priori* intuitions (*Anschauungen a priori*)' entailed by the sort of thinking used for example in geometry or mental arithmetic. Kant's view is that even the most abstract thinking would not be possible unless its concepts could be applied to the empirical world. For example he says

'[...] the pure concepts of understanding, even when they are applied to a *priori* intuitions (*Anschauungen a priori*), as in mathematics, yield knowledge (*Erkenntnis*) only in so far as these intuitions [...] can be applied to empirical

intuitions (*auf empirische Anschauungen angewandt werden können*) (CPR 162 B147)

Schlick and Kant argue that a putatively non empirical concept only has a use if it can have an empirical use. Metaphysical concepts, although putatively non-empirical, can have no empirical use so they have no use. Kant deploys this very positivist argument:

'Even [...] with the aid of [pure] intuition the categories do not afford us any knowledge of things (*keine Erkenntnis von Dingen*). In other words they serve only for the possibility of empirical knowledge (*zur Möglichkeit empirischer Erkenntnis*), and such knowledge is what we entitle experience (*Erfahrung*). (CPR 162 B 147)

For Schlick too, as we have just seen, the explanation of meanings requires reference 'in an obvious sense to experience'.

The sense-sense argument is a general positivist tenet. It is not confined to Schlick. For example, Ayer says: 'No statement is literally meaningful unless it describes what could be experienced'. Although he thinks the argument needs revising so as not to eliminate the sentences of scientific theory, he says metaphysical statements are still excluded by it. Ayer calls it an old empiricist principle and this is right. The positivists hold a critical empiricism in common with Kant. Although Kant and the positivists think thinking is only possible because there is sense-experience, neither is thereby committed to the view that metaphysics is impossible as a psychological activity: just that it is nonsensical and so yields no knowledge. Both agree there is metaphysical thinking, and this is the common object of their attack. Kant:

'If no intuition could be given corresponding to the concept, the concept would still indeed be a thought, so far as its form is concerned, but would be without any object, and no knowledge of anything would be possible by means of it.' (CPR 162)

It is the positivists' view that the concepts 'God', 'angel', 'Monad', although evidently possible as ideas, do not refer to anything.

Because the logical positivists reject the theory of innate ideas, they need a special account of how it is possible for us to possess mathematical concepts which *prima facie* are not empirical. On this difficulty, Ayer says:

'It is necessary to make a distinction which is perhaps already enshrined in Kant's famous dictum that although there can be no doubt that all our knowledge begins with experience, it does not follow that it all arises out of experience. (LTL 74)

Ayer says of mathematical statements that although in the first instance 'we may come to discover them through an inductive process', it is still true that they are *a priori* because they are 'independent of experience in the sense that they do not owe their validity to empirical verification.' (LTL 75)

Despite the fact that Kant and Ayer differ over the logical status of mathematical statements, it remains common to both that sense experience is initially necessary for the use of mathematical concepts. As we have seen, there can be no mathematical thinking, according to Kant, without the forms of 'pure intuition', space and time.

Ayer and the Logical Positivists' concern to show that

'[...] it cannot be significantly asserted that there is a non-empirical world of values, or that men have immortal souls or that there is a transcendent God' (LTL 31).

entails the conclusion of Kant's attacks on God and the soul. Kant spends much of the *Critique of Pure Reason*, but especially the 'Paralogisms' chapter, condemning metaphysics as the attempt to apply concepts outside possible experience: essentially the positivist line. Kant claims that the category of substance has been used illegitimately by his rationalist predecessors. 'Substance' is legitimate and indeed indispensable in so far as it helps transform our passively received sensory input into the world of enduring spatio-temporal objects, but the rationalists' mistake is to apply the category of substance to the self: to assume that because the self is not a spatial object it must be a non-spatial object. They allegedly failed to realise that the concatenation of predicates 'non-spatial substance' is incoherent.

IV

The Verification Principle

The Logical Positivists' test of whether a statement is meaningful depends on Kant's analytic/synthetic distinction. Kant and the Logical Positivists agree that every statement is either analytic or synthetic, and that no statement is both analytic and synthetic. Their explanations of

these terms are similar even though they sometimes disagree about which statements are analytic and which synthetic.

v

Analytic Statements

Ayer says a statement is 'analytic when its validity depends solely on the definitions of the symbols it contains' (LTL 78). He says Kant's account of analyticity is not 'clear', and contains the mistaken assumption that every statement has the subject-predicate grammatical form. However, Kant too possesses a linguistic criterion for analyticity. He says the predicate is 'contained' in the subject in an analytic statement. Although this is a metaphorical way of putting it, it is clear his concept of analyticity is in fact very close to Ayer's. Kant means by 'contained' here: If one were to define the word or concept which is the grammatical subject of the sentence, then one would have to use that word or concept which is the grammatical predicate of the sentence. His example 'All bodies are extended' (*alle Körper sind ausgedehnt*) (or, as we should say, 'All physical objects have size') (CPR 48 A 7/B 11) makes this clear because, if analytic, is true by definition. The idea of 'size' is already contained in the idea of a physical object, so the statement provides no new information to someone who knows the definition of 'physical object'. Ayer's examples are, if analytic, are also true by definition and give no new information in the predicate not already contained in the subject. He says it is analytic that: 'Nothing can be coloured in different ways at

the same time with respect to the same part of itself.' (LTL 79)

Ayer and Kant share the view that denials, or negations, of analytic truths are self-contradictory so, if someone were to say, 'This is red but not coloured' they would have contradicted themselves, just because 'If this is red then it is coloured' is analytic. According to Kant and Ayer, the mistake is in logic and in semantics, and is not a mistake about empirical colours. They also think that all analytic truths are necessary truths and that none is contingent.

VI

Synthetic Statements

Synthetic statements are informative; for Kant in the sense that they provide in the predicate information not already contained in the subject. Or, as Ayer would put it, no inspection of the meaning of the statement will decide its truth or falsity. He says the proposition

'"There are ants which have established a system of slavery" is a synthetic proposition. For we cannot tell whether it is true or false merely by considering the definitions of the symbols which constitute it.' (LTL 78-9)

Ayer and Kant agree that the denials or negations of synthetic statements do not entail contradictions. For example, 'That object is red', said when that object is not red, does not entail a contradiction but expresses an empirical mistake about the colour of the object. For Ayer as well as Kant, 'analytic and synthetic' is incoherent because it is contradictory to both affirm and deny

that some subject contains some predicate so
'analytic' and 'synthetic' are opposites.

VII

A Priori Statements

For Kant and the Logical Positivists '*a priori*' means 'knowable to be true or false independently of sense experience'. For example, if it is true that 'there are three heavy chairs in that room' and I am told this, then I can know *a priori* that 'there are at least three physical objects in that room' is true. This is *a priori* because I don't have to go and carry out any observations to decide it is true.

Kant and Ayer have very similar definitions of '*a priori*'. Ayer says *a priori* statements 'cannot be confuted by experience' (LTL 31). Kant says

'we shall understand by *a priori* knowledge, not knowledge independent of this or that experience but absolutely independent of all experience (*sondern schlechterdings von aller Erfahrung unabhängig stattfinden*)' (CPR 43 A 2/B 2).

Thus Ayer holds the Kantian views that the statements of arithmetic are *a priori*, and that all analytic statements are *a priori*.

VIII

A Posteriori Statements

All and only a *posteriori* statements are empirical because *p* is a *posteriori* if and only if *p* can be known to be true or false only through sense experience. Kant puts it this way:

'Opposed to [a *priori*] is empirical knowledge, which is knowledge possible a *posteriori*, that is, through experience' (CPR 43).

This Kantian epistemological category is essential to the formulation of Logical Positivism.

The Logical Positivists believe there are two and only two classes of meaningful statement. On the one hand are those which are analytic and a *priori*, on the other hand are those which are synthetic and a *posteriori*. Into the first category, according to them, fall all truths of mathematics and logic, all definitions and in general any statement true in virtue of the definitions of its terms. Into the second category fall all empirical claims, which are those shown to be true or false by observation. These two groups are mutually exclusive and jointly exhaustive of meaningful statements. If there is any sentence which appears to be a statement but which on inspection does not fall into either category, then it is really a piece of nonsense.

Metaphysical language is full of would-be statements which in fact fall into neither category. Claims that the empirical world is an illusion, that there exists a transcendent God, that heaven is populated with immaterial souls or resurrected persons, that really the world is composed of monads, that physical objects are mere reflections of Platonic Ideas, all these and many more besides are dismissed as utterly nonsensical through the deployment of the verification principle. On the one hand, these metaphysical

sentences are not analytic and *a priori*; on the other hand, they are not synthetic and *a posteriori*. As these are the only two classes of meaningful statement, it follows that the metaphysical sentences do not really mean any thing at all. The Positivists are not saying they are false, but that they are literally nonsensical and so are not even capable of being true or false.

IX

Meaning and Truth

We should consider how close the verification principle is to certain ideas of Kant, but putative divergences between Kant and the Logical Positivists also need to be considered.

Even though Ayer is acutely aware of many of the similarities between his own philosophy and Kant's, there are nevertheless several points at which he points out what he takes to be important dissimilarities. One such concerns the verification principle. Kant and the Logical Positivists are closer even than Ayer allows.

Ayer distinguishes his position from Kant's by claiming that Kant has employed a psychological, and so contingent, criterion for the rejection of metaphysics while he, Ayer, employs a logical criterion. Ayer is worth quoting at length:

'It may be suggested that this (that metaphysics is nonsense) is a proposition which has already been proved by Kant. But although Kant also condemned transcendent metaphysics, he did so on different grounds. For he said that the human understanding was so constituted that it lost itself in contradictions when it ventured out beyond the

limits of possible experience and attempted to deal with things in themselves. And thus he made the impossibility of a transcendent metaphysic not, as we do, a matter of logic, but a matter of fact. (LTL 34)

I should say this distinction is overdrawn and that Kant, just as much as the Logical Positivists, makes the impossibility of metaphysics a matter of logic. As Ayer recognises, metaphysics for Kant generates contradictions. Contradictions are precisely illogical sentences. They entail the conjunction of a proposition and its negation and that is a sufficient condition for their being meaningless. A putative state of affairs which is self-contradictory to describe is logically impossible, whatever the contingencies of our psychology or our conceptual scheme. So, in that sense, the impossibility of metaphysics is a matter of logic for Kant, not psychology.

It is wrong to see Kant's attack as resting on purely psychological grounds, and wrong to see the positivist attack as resting on purely logical grounds. Central to both attacks on metaphysics is the idea of truth. And crucially for both, the possibility of meaning is tied logically to the possibility of truth or falsity.

To verify a statement is to demonstrate its truth. To falsify a statement is to demonstrate its falsity. Here Moritz Schlick establishes the positivist connection between meaning and truth:

'Stating the meaning of a sentence amounts to stating the rules according to which the sentence is to be used, and this is the same as stating the way in which it can be verified (or falsified). The meaning of a statement is its method of verification.' (1)

For example, empirical statements can be verified by observation, analytic statements can be verified by the inspection of meanings; so those two groups are meaningful. There is no such thing as the 'method of verification' of a metaphysical claim, so no such thing as its meaning either.

Ayer formulates the verification principle like this:

'A sentence has literal meaning if and only if the proposition it expresses is either analytic or empirically verifiable. (LTL 5)

Now, it is an essential part of Kant's account of meaning that in order for something to count as a genuine statement it must not be contradictory. Kant's judgements have psychological as well as syntactic and semantic properties, but there is sufficient overlap with the positivists' 'proposition' to warrant our counting judgements as statements or expressing propositions. Kant spells out this necessary condition for meaningfulness:

'The universal, though merely negative condition of our judgements in general, whatever be the content of our knowledge, and however it may relate to the object (*Objekt*), is that they be not self-contradictory (*nicht selbst widersprechen*); for if self-contradictory, these judgements are in themselves even without reference to the object, null and void ((*auch ohne Rücksicht aufs Objekt*) *nichts sind*).' (CPR 189 A 150/B 189)

If a sentence is self-contradictory that is sufficient for its not expressing a genuine statement. Contradictions are senseless. It is not just that they lack referents, though that is also

true, they are not 'judgements' because they do not assert or deny anything.

It is Kant's view that if something can be used to genuinely assert or deny something, then that is a sufficient condition of its being a judgement, or at least of its being identical with that non-psychological component of a judgement called a 'statement' or 'proposition'. As he says: 'Truth and error therefore [...] are only to be found in the judgement' (CPR 297). So, if something is genuinely capable of truth or falsity it is a judgement in Kant's view unless it is contradictory and so logically false. If something is not capable of truth or falsity it cannot count as a judgement. Verifiability is the demonstrability of truth, and falsifiability the demonstrability of falsehood. I conclude then that the following positivist theory of meaning (here again stated by Schlick) is Kant's: 'Verifiability is the sufficient and necessary condition of meaning.' () The conjunction of the Kantian and positivist theories of meaning is analytic.

It is Kant's view that metaphysics leads to Antinomies: Two perfectly valid arguments can be produced, yielding a purportedly metaphysical statement, and its contradictory, as their conclusions. We can list premisses implying that the world did have a beginning, and that it did not, that it is finite, and that it is infinite, that God exists and that he does not. Now, straight away it logically follows from Kant's theory, as from the positivists', that there is no way of deciding whether any of these putative statements is true. In other words, they are not verifiable or falsifiable.

The fact that these sentences feature as conclusions of valid arguments is no proof whatsoever of their truth. The conclusion of a

valid argument is true only if the premisses it follows from are also true. The putative statements and their logical negations, which are the conclusions of the antinomies, seem equally plausible, but of course they cannot both be true; so, in this way, metaphysical thinking leads to contradictions, and as we have just seen, not being self-contradictory is a necessary condition for being meaningful for Kant. It follows that metaphysical statements are not meaningful for him, or as Kant would put it, they are not judgements.

I conclude then that Kant, like the positivists, finds metaphysics meaningless. It is not Kant's view, as it is not the Positivists' view, that metaphysics contains sentences that are meaningful but false. This too, is for very similar reasons. They are meaningless because there is no way of showing them to be genuine assertions or denials; no way of verifying them or falsifying them. The conjunction of this tenet of Logical Positivism with the lesson of the antinomies is analytic. (4) Both Kantianism and Logical Positivism entail *metaphysical anti-realism*. (Indeed, 'idealism' is Kant's 'transcendental idealism' is best understood as 'anti-realism'.) Kant thinks there are no metaphysical propositions. There only seem to be.

Kant thinks coherence is a necessary condition not only for meaning but for truth, so if a statement is self-contradictory, it cannot be true. That a statement 'be in complete accordance with logical demands, that is, may not contradict itself' Kant calls 'the negative condition of all truth (*die negative Bedingung aller Wahrheit*)' (CPR 98 A 59-60/B 84). Ayer of course agrees with this but Kant and the Positivists have further in common that logic is entirely *a priori*: it gives in itself no information whatsoever about the empirical

world. Referring to the exhibition of the internal coherence or incoherence of our knowledge Kant says: 'But further than this logic cannot go (*weiter aber kann die Logik nicht gehen*). It has no touchstone (*Probierstein*) for the discovery of such error as concerns not the form but the content' (CPR 98 A 60/B 84). These two passages show the similarity. Kant:

'Although our knowledge [...] may not contradict itself, it is still possible that it may be in contradiction with its object.' (CPR 98 A 59/B 84)

Ayer:

'An empirical proposition, or a system of empirical propositions may be free from contradiction and still be false. It is said to be false not because it is formally defective, but because it fails to satisfy some material criterion.
(LTL 90)

So Kant and the Logical Positivists both accept:

$$\forall p (p \text{ is empirical}) \rightarrow \Diamond \neg [p \rightarrow (p \ \& \ \neg p)] \ \& \ \Diamond \neg p$$

But they both accept that in the case of purely logical propositions, if they are not contradictory then they are true:

$$\forall p (p \text{ is purely logical}) \rightarrow \neg [p \rightarrow (p \ \& \ \neg p)] \rightarrow p$$

Kant and the Logical Positivists draw a sharp distinction between form and content. Both agree that logic is the study of the form but not the content of putative knowledge. Logic, Kant says, concerns the 'mere form *bloßen Form*' of knowledge 'leaving aside all content *mit Beiseitesetzung*

alles Inhalts' (CPR 98 A 59/B 83). 'It is characteristic of empirical propositions', Ayer says, 'that their validity is not purely formal'. (LTL 90) Both philosophers agree that coherence is not sufficient for what they both call material truth. As Kant puts it:

'Agreement with logical laws is far from being sufficient to determine the material (objective) truth of knowledge (*materielle (objective) Wahrheit dem Erkenntnisse*). ' (CPR 98 A 60/B 85)

The Logical Positivists believe tautologies (or analytic statements or logical truths) give no empirical information whatsoever so logic only makes explicit what in a sense we know already. Kant's view also is:

'no one can venture with the help of logic alone to judge regarding objects, or to make any assertion (*etwas zu behaupten*). ' (CPR 98 A 60/B 85)

What then is 'material' truth for Kant and Ayer? Logic operates only *a priori* on statements but we need to know now what makes statements true or false in virtue of content not form.

Kant and the Logical Positivists have a correspondence theory of empirical truth. They hold that logical coherence is a necessary condition for truth, but coherence conjoined with correspondence with the empirical world is sufficient for empirical truth. Ayer says:

'Empirical propositions are one and all hypotheses, which may be confirmed or discredited in actual sense experience.' (LTL 93-4)

States of affairs which are in principle observable make *a posteriori* statements true or false. Kant puts it this way:

'Truth consists in agreement of knowledge with its object [...] knowledge is false if it does not agree with the object to which it is related (*mit dem Gegenstande, worauf sie bezogen wird, nicht ubereinstimmt*) ' (CPR 97 A 58/B 83).

The 'agreement' is correspondence. Kant thinks our concepts and categories have genuinely only an empirical use so 'object' here is 'empirical object' or even 'physical object'. Kant and the Logical Positivists both hold:

$\forall p (p \text{ is empirical}) \rightarrow [p \leftrightarrow \exists x (x \text{ is an empirical state of affairs with which } p \text{ 'agrees'})]$

I conclude that the positivist theory of meaning and truth is essentially Kantian. (1)

X

Existence

Kant's theory of existence is adopted wholesale by the Logical Positivists and employed, as it was by Kant in the *Critique of Pure Reason*, in attacking arguments for the existence of God. Indeed, the positivists have only one reservation about Kant's attack on theism; they think he should have pushed his attack still further. Arthur Pap puts it this way:

'Kant, though he demolished the traditional proofs of the existence of God and the immortality of the soul, gave a curious ethical proof of these dogmas of Christian theology. (Pap 1949: 18)

Kant, on one construal, criticises the classical proofs of the existence of God in order to 'make room for faith', not to open the way to atheism. Kant's view in the *Critique of Practical Reason* is that people have a notion of their own self-perfection, and as this cannot be realised on earth, there must exist an afterlife to be enjoyed by an immortal soul which, there, does indeed reach perfection. () Such claims cannot make any sense within the positivist programme. Pap says 'Kant's ethical proof cannot be taken seriously'. To find the parallels between Kant and the Positivists on existence and on God we need to confine the search to the *Critique of Pure Reason*.

Kant presents his theory of existence in the context of his criticism of the Ontological Argument for the existence of God. The Ontological Argument, used in various versions by Anselm, Descartes and Leibniz, is construed by Kant and the Logical Positivists as having two premisses

1. God is, by definition, that than which nothing greater can be (conceived).

2. Existence is a perfection.

which purportedly entail the conclusion:

3. God exists. ()

Kant's attacks the second premiss. A perfection (*perfectio*) is a property which a perfect being must possess. But Kant says it is a drastic mistake

to think that existence is any kind of property, or that 'exists' a predicate. Saying something exists is not saying it has a special characteristic but saying it is instantiated, that there is one. If Kant is right, then the conclusion that God exists begs the question.

The Logical Positivists agree Kant's move is effective against the Ontological Argument and make further use of it themselves. Here for example Rudolf Carnap acknowledges Kant's theory as correct:

'It has been known for a long time that existence [...] is not a property (cf Kant's refutation of the ontological proof of the existence of God)' (Ayer 1959: 74)

and then proceeds to discuss its wider applications:

'Most metaphysicians since antiquity have allowed themselves to be seduced into pseudo-statements by the verbal and therewith predicative form of the verb 'to be', e.g., 'I am', 'God is'. (Ayer 1959: 74)

Ayer attacks 'the postulation of real non-existent entities' for example by 'Heidegger who bases his metaphysics on the assumption that "Nothing" is a name which is used to denote something peculiarly mysterious.' (LTL 43-4)

Like Carnap, Ayer accepts that this anti-metaphysical view of existence is derived from Kant:

'As Kant pointed out [...] existence is not an attribute [...] so those who raise questions about Being which are based on the assumption that

existence is an attribute are guilty of following grammar beyond the boundaries of sense.' (LTL 43)

There are indications that the positivists think Kant has only successfully said what 'existence' does not mean. The question still remains: what does 'exists' mean if it does not denote a property?

In fact Kant does have a positive account of existence and, like the Positivists' account, it is empiricist. Both Kant and the Logical Positivists, in the last resort, take the question 'What does exists mean?' to require just the same sort of answer as 'What exists?' or 'What is real?' Kant and the positivists telescope the questions together. For example Schlick says of the statement 'It is real':

'This means that there exists a very definite connection between perceptions or other experiences, that under certain conditions data appear.' (Ayer 1959: 97)

Schlick seems to be echoing Kant's remark that when we

'ascribe existence to an object' [...] in the case of objects of the senses this takes place through their connection with some one of our perceptions, in accordance with empirical laws. (CPR 506)

Paradigmatically at least, for both philosophers, existence is empirical existence.

The term 'exists' has only an empirical use and no metaphysical function. Schlick speaks for the Logical Positivists in general when he says:

'For us (as for Kant; and the same applies to every philosopher who knows his business) it is simply a matter of saying what it means in everyday life or in science to ascribe real existence to a thing.'
(Ayer 1959: 97)

Existence and Non-Existence feature in Kant's list of categories (under 'Modality') and so, in common with all the other categories, have no non-empirical application. So the Logical Positivists and Kant, at his most consistent moments, share:

$$\forall x (Fx) \rightarrow (x \text{ is empirical})$$

which implies that if something is not empirical it does not exist. However, neither Kant nor the Logical Positivists can attach any sense to the denial that there are non-empirical objects, Kant because a category is misused beyond the bounds of its application, the Logical Positivists because such a putative claim is not in principle verifiable. It follows that although ' $\exists x (Fx) \rightarrow (x \text{ is empirical})$ ' is an entailment common to Kantianism and Logical Positivism, by their criteria for sense entailment of it is nonsensical:

$$- (x \text{ is empirical}) \rightarrow - \exists x (Fx)$$

If a proposition entails a senseless proposition that casts doubt on the sense of that proposition.

It follows that the positivist and Kantian views on existence are fundamentally identical. They share essentially the same merits and limitations and stand or fall together.

Science

Kant and the Logical Positivists draw a sharp distinction between science and metaphysics. Both wish philosophy to share some of the sophistication that characterises scientific method. Ayer says about his own movement:

'The Vienna Circle [...] thought that they had succeeded where Kant failed, in finding a way to set philosophy upon the sure path of a science. (Ayer 1959: 9)

It is not clear that the Logical Positivists have been successful. What they have done is to continue the critical switch away from philosophy as an *a priori* means of inquiry into ultimate reality and towards philosophy as a conceptual inquiry into scientific and other forms of thinking. This was the effect of the critique of pure reason; to abolish rationalist metaphysics. As Ayer puts it:

One of his (Kant's) most important contentions was that the rationalists had been entirely mistaken in supposing that they could discover the nature of things merely by the exercise of reason. ()

The Logical Positivists agree with Kant that philosophy should not compete with science and agree over what scientific method is and should be. Of course science has changed in all sorts of ways since Kant's time. Kant thought Newtonian mechanics was the whole of physics. We know now that Kant was wrong about that. Kant also thought Euclid's the only possible geometry. We now know there are geometries other than Euclid's. These are massive

and mistaken assumptions that Kant subscribed to and which, with the benefit of witnessing these new developments, the Logical Positivists did not share.

Despite Kant's understandable ignorance about the future of science, he agrees with the Logical Positivists that it should be mathematically rigorous, clear in its definitions and make essential use of empirical experimental methods. These, for Kant and the positivists, are amongst the hallmarks of scientific method. They both think that for a statement to be scientific it must be verifiable and that science is distinguished by consensus and progress. The Vienna-Circle absorbed the implications of the Copernican Revolution for what scientific method must be. Ayer says:

'It is true no doubt that the procedure of science is rather, as Kant said, to ask nature questions than to make random collections of empirical facts.' (*I)

It has been judged impossible since Kant to take seriously the idea that the scientific investigator just passively receives information about the empirical world realistically; just as it really is. The scientist is a questioner with an order of priorities which he brings to bear on his chosen subject matter. He makes a selection from the welter of information available to him and interprets it according to agreed, scientific, criteria. The Logical Positivists recognise the Kantian point that the preconceptions of the subject or observer makes a difference to what he observes.

Two questions concerning the status of science are crucially important for the success of the project of Logical Positivism. The first is: Is it

not true that science contains a host of claims about the world as it really is independently of our experience of it? and was this not one of the definitions of 'metaphysical'? The evident danger here is that it might be as nonsensical to talk about atoms and molecules as about God or the monads. In other words, what stops scientific theory being metaphysics? The second question is: Does not science rest on non-scientific, indeed, metaphysical assumptions? In other words, might not unscientific and perhaps ultimately incoherent presuppositions have to be made in order to do science?

The Logical Positivists are quite aware of these two problems. To try to resolve the first one Schlick draws a very Kantian distinction between:

'simple nature which surrounds us in everyday life' (Ayer 1959: 101)

and

'the transcendent world of metaphysics.' (Ayer 1959: 101)

Schlick argues that the relationship between the world described by the theoretical physicist and the world we perceive is identity. There are not two worlds but two vocabularies for describing one and the same world: one practical, one scientific. The putative world of the metaphysician is transcendent; that is, it is not part of the world at all. As Schlick readily acknowledges, this is exactly Kant's view. Kant thinks the single world ordered by the categories is perceived and discussed commonsensically and investigated by the natural sciences.

This doctrine of the unity of nature must not be confused with any remarks Kant makes about things in themselves. Schlick recognises this too:

'Atoms in Kant's system have no transcendent reality, they are not things in themselves.' (Ayer 1959: 101)

In Kant's view it is just the putative claims of the metaphysician which seem to create another reality. That the unity of nature obtains is one of the conclusions of the Analogies chapter of the *Critique of Pure Reason*. He says: 'By nature, in the empirical sense, we understand the connection of appearances' (CPR 237 A 216/B 263), so nature which is governed by 'empirical laws' (CPR 237 A 216/B 263) is not anything different from the everyday world we perceive. As he puts it: 'All appearances lie and must lie in one nature' and 'Our analogies [...] really portray the unity of nature'. (CPR 237 A 216/B 263)

Clearly the Logical Positivists cannot accept that the law abidingness of the empirical world is the result of the *a priori* conceptual framework of perceiving subjects, any more than that space and time are 'ideal'. Still, bearing in mind Kant's remarks above we can appreciate that Schlick is quite correct when he says:

'The distinction between the two [worlds] is [...] especially clear in Kant's philosophy. Nature and everything of which the physicist can and must speak, belongs according to Kant to empirical reality, and what that means is (as we have already said) explained by him in just the way it must be by us.'
(Ayer 1959: 101)

The Logical Positivists answer the first objection by adopting the Kantian argument that there is only one world, no other world of metaphysics, and so no danger of science being metaphysical.

The threat that science might rest on metaphysical assumptions is answered by recourse to a Kantian argument. Arthur Pap agrees that science rests on presuppositions but empirical not metaphysical ones. He gives the example: 'There are physical constants' which he says is 'a proposition a follower of Kant may wish to call a presupposition of empirical science'. (Pap 1949: 23) But as a presupposition it is to be justified by the fact that the regularities reported in, say, physics, are exemplified in our everyday experience. Of course Kant, construed as an idealist, would give a reason for agreeing with this which Pap would reject: Those regularities are in the last resort due to the subjective ordering activities of the categories. Despite this, both Kant and the Logical Positivists see the justification of science in the order and regularity of experience. It is not any metaphysical truth that vindicates science or makes it possible but the sense experiences of the human subject. Science has no metaphysical entailments:

$$\forall p (p \text{ is scientific}) \rightarrow \neg [\exists q (q \text{ is metaphysical}) \& (p \rightarrow q)]$$

where the reason why the conjunction that is the consequent is false is that its first conjunct is false. But science does have empirical entailments:

$$\forall p (p \text{ is scientific}) \rightarrow [\exists q (q \text{ is empirical}) \& (p \rightarrow q)]$$

For these purposes I assume that if q is a presupposition of p then p entails q . It follows that the positivists' conception of philosophy and scientific method is essentially Kantian, despite their rejection of transcendental idealism, and despite the multiplicity of developments in science which Kant could not foresee.

XII

Ethics

The Logical Positivists allow other sorts of meaning than 'literal' or 'factual' meaning. Ethical sentences might provoke or express emotions. What such sentences cannot do is provide any information other than signal the affective state of the utterer. They have no factual moral content. Although this emotivist theory of ethics fundamentally exhausts the positivists' position on the subject, it has rarely been recognised that, for Schlick at least, emotivism is an extension of Kant's account of ethics.

Schlick says Kant discovered the 'formal characteristic' of ethics. That is, Kant correctly interpreted ethical sentences as imperatives, and not as statements. Schlick credits Kant with noticing that:

'the good always appears as something that is demanded or commanded; the evil as something forbidden' (Ayer 1959: 252).

According to Schlick, one of the worst mistakes of modern ethics is the assumption that Kant's theory is complete. The categorical imperative provides only a formal criterion for morality, so Kant's

ethics lack content. Schlick says Kant 'took only a preliminary step in the characterisation of the content of the good'. (Ayer 1959: 253) The task of the Logical Positivists is to provide Kant's ethics with content.

The proposed content could not possibly be factual, because ethical sentences are not statements. The positivist solution is to propose that the content is emotional. Thus Ayer says:

The presence of an ethical symbol in a proposition adds nothing to its factual content [...] it merely serves to show that the expression of it is attended by certain feelings in the speaker. (LTL 17)

So, ethical language reveals the speaker's emotional attitudes. It also provokes them, in the speaker or others: 'Ethical statements are expressions and excitants of feeling'. (LTL 110) (17)

I think this is a development of his ethics which would have appalled Kant. He, after all, positively insists that the categorical imperative is purely formal and in the *Groundwork of the Metaphysic of Morals*, for example, is at pains to deny that a will can be purely good if determined by 'subjective conditions' and 'impulsions' (ML 77) .

We could try to soften the Positivists' blow to Kant by saying they are just explaining what meaning ethical statements in fact have; not saying what it consists in to be good. This will hardly do though. Although, *prima facie*, emotivism is descriptive and Kant's ethics prescriptive, Kant too is trying to provide an account of what ethics is: in terms of the conditions for a truly good will (ie, intention). This is only possible,

according to Kant, in complete abstraction from the emotional states of the agent. The positivist view, however, is that ethical sentences are not just causes but consequences of emotional states. Kant could not possibly accept such sentences as 'ethical' at all.

It seems then that although Schlick thinks of positivist ethics as providing a content for Kant's, this is an extension of his theory he would have found unacceptable, despite Ayer's use of the Kantian term 'critique' for his treatment of ethics. (LTL 1~ 2) Nevertheless, Kant and Ayer share the view that that moral sentences do not express propositions.

XIII

I conclude that the philosophy of the Vienna Circle is essentially a version of Kant's critical philosophy. Tenets essential to Logical Positivism about meaning, truth, existence, science and ethics either entail or are logically equivalent to Kantian claims. Because some entailments essential to Logical Positivism are synthetic *a priori* the movement is more thoroughly Kantian than it acknowledges.

Notes

(1) Kemp Smith's 'empirical test' is Kant's *Probierstein der Erfahrung* (A viii), and 'transcend the limits of experience' is *über die Grenze aller Erfahrung hinaus gehen*. So the conjunction of Kant's claim that metaphysics putatively 'transcend(s) the limits of experience' (CPR 7) and Ayer's claim that the metaphysician claims 'to know facts that could not be known through sense experience' (LTL 34) is analytic. Some identity of

semantic content obtains between Kant's *Erfahrung* and Ayer's 'experience' because neither philosopher subscribes to the view that 'experience' just means 'sense experience' but both hold the empiricist theory that if there is non sensory experience then that is only possible on condition sense experience is possible. Kant, for example, has it as the conclusion of the Refutation of Idealism that inner sense is only possible on condition that outer sense is possible. Ayer's view, articulated in Ayer (1973) is that 'what happens when one is self-conscious is that one claims some present or past experience as one's own, where its being one's own is a matter of its being related to other experiences and to one's body'. (LTL 120) Kant and the Vienna Circle admit a passively received or 'given' content of experience, Intuitions *Anschauungen* and sense data respectively.

It follows that when Kant asserts 'Das alle unsere Erkenntnis mit der Erfahrung anfangt ist gar kein Zweifel' (Ai / Bi), even though sense experience is only part of the extension of 'experience', he means that knowledge is only possible on condition there is sense experience. This is exactly the view of the Logical Positivists as can be shown by the fact that the denial of only one of the conjuncts of the conjunction formed from the first sentence of Kant's introduction and the assertion formed from answering 'yes' to Ayer's rhetorical question yields a contradiction.

2 For Ayer's use of 'empirical' and 'phenomenal' see LTL passim but especially chapters 1 and 2. For Kant on phenomena and noumena see CPR especially pp. 265, 266 ff., 291 ff., 382, 469, and 471. Kemp Smith's 'phenomena' is just Kant's

Phaenomena (see for example B294/A235 ff).

Kant shares Ayer's conception of metaphysics as a spurious search for 'first principles'. He calls them *Grundsätzen* (Aviii, CPR 7), where *Grundsatz* is the ordinary German for 'principle' or 'axiom'. I take it *p* is just such a putative 'first principle' on Ayer's account, or a 'Grundsatz' on Kant's if and only if *p* cannot be derived even in principle from any empirical proposition but *p* is allegedly informative about a purported transcendent reality. Kant and the Logical Positivists hold the view that no proposition falls under that description (even though *Grundsatz* also has legitimate, non-metaphysical uses according to Kant).

4 On Kant's theory of meaning a contradiction does not express a genuine proposition. This is the force of:

Von welchem Inhalt auch unsere Erkenntnis sei, und wie sie sich auf das Objekt beziehen mag, so ist doch die allgemeine, obzwar nur negative Bedingung aller unserer Urteile überhaupt, das sie sich nicht selbst widersprechen; widrigenfalls diese Urteile an sich selbst (auch ohne Rücksicht aufs Objekt) nichts sind. (B 189/A 150).

If 'propositions' is read for 'Urteile', it follows that a contradiction cannot feature as part of a genuine assertion or a genuine denial, so that if (as Kant believes) the putative propositions of metaphysics entail contradictions, then metaphysics cannot be genuinely informative.

5 Just as long as the 'two worlds' ontology is not foisted on Kant then the world as it is in itself is not numerically distinct from the world of appearances, which is itself identical with that

studied by the natural sciences. Similarly the Logical Positivists reject any suggestion that the world studied by the sciences is numerically distinct from or in any way duplicates ontologically the world as it appears to human perceivers.

6 Moritz Schlick 'Meaning and Verification' in Feigl and Sellars 1949.

7 Schlick in Feigl and Sellars 1949:148.

8 See Leibniz 'Monadology' in G. W. F. Leibniz *Selected Philosophical Writings* ed. Parkinson (Everyman, London, 1976).

9 For the Ideas see Plato, *Republic*, translated by Desmond Lee (Penguin, Harmondsworth, 197). For Hegel's conception of *Geist*, see G.W.F. Hegel, *Lectures on the Philosophy of World History: Introduction* translated by H. B. Nisbet, with an introduction by Duncan Forbes (Cambridge University Press, Cambridge, 1975), especially pp. 30, 52 ff., 63, 76, 82, 92, 124, 313 ff. For Descartes on the soul see *Descartes Meditations* and *Descartes Philosophical Letters* ed. Anthony Kenny (Blackwell, Oxford, 1970).

10 The 'sense and sense' argument common to Kant and Schlick may be formulated thus: For any genuine but not straight-forwardly empirical concept, there must exist, at least in principle, some empirical definition into sub- concepts at least one of which admits of ostensive definition. In this formulation a concept is empirical if and only if its only use is empirical, so Kant's categories are empirical in this sense even though (according to Kant) they are also *a priori*.

11 Ironically Ayer deploys a version of the 'sense and sense' argument against what he takes to be a reification of the self by Kant. He says that

Kant postulate(s) a self which is always a subject and never an object of consciousness. It is to this "transcendental" self, as Kant called it, that the task is assigned of processing the raw material of experience, so that a world emerges in which one's everyday self or selves can find a place. In Kant's system, the transcendental ego stands outside this world. Its labours make our experiences possible, but because it is not a possible object of experience, it is not itself in space or time. (Ayer, 1976, P-118)

Admittedly, many passages in all three Critiques can be read to substantiate this view, but it is at variance with Kant's warnings in the Paralogisms chapter (CPR) that it is a mistake to think of the self as a substance. Indeed, part of the anti-metaphysical message of the first *Critique* is that the transcendental ego should not be hypostatized metaphysically. Philosophers from Hegel to Husserl have attributed a substantial subject to Kant's theory, and this is a part of the 'two worlds ontology' attributed to Kant. In fact it is possible to understand myself as I am in myself as not numerically distinct from myself as I appear to myself, and this is really Kant's considered view. Ayer's verdict on the reified transcendental subject is: 'Clearly there can be no empirical evidence for the existence of any such entity'. (Ayer, 1976: 119) Kant would have wholeheartedly agreed.

12 Schlick in Feigl and Sellars 1949: 148

13 Reading 'proposition' for 'judgement', 'a sentence expresses a genuine proposition if and only if it possesses truth conditions' is a formulation of the necessary and sufficient conditions for factual meaning which feature in Kant's concept of a judgement, and the positivists concept of a genuine proposition.

14 The view of Kant and the positivists of any putative metaphysical statement is captured by: If p cannot in principle be verified or falsified, then p is not genuinely informative. Both hold this principle consistently with the theory that analytic statements and contradictions (which are necessary truths and necessary falsehoods respectively) are conclusively verifiable and falsifiable, but are still not thereby informative because they are not empirical (nor synthetic *a priori* either, in Kant's view).

15 In this passage, Kant's word for 'to judge' is *urteilen* and Kemp Smith's 'to make any assertion' is a translation of *irgend etwas zu behaupten* (A 60/B 84), so clearly Kant is denying that logic can be genuinely informative: exactly the positivist view.

16 So the Kantian and positivist theory of truth for some empirical proposition may be formulated as follows: ' p is true if and only if p ', where this sentence is given a realist rather than a redundancy reading. The coherence condition, which is necessary for truth for Kant and the positivists, is catered for because that p not be a contradiction is a necessary condition for p 's being an empirical proposition.

17 Ayer, like Kant, repudiates a Utilitarian analysis of ethical judgements. The reason he gives is: 'It is not self contradictory to say that it is sometimes wrong to perform the action which would actually or probably cause the greatest happiness'. (LTL 105) Although Kant would have found Ayer's emotivism unacceptable, and even though Ayer devotes most of the first half of Chapter VI of LTL to showing that ethieal sentenees 'are simply expressions of emotion which ean be neither true nor false', he allows that exhortations to moral virtue are not propositions at all but ejaculations or commands which are designed to provoke the reader to aetion of a eertain sort' (LTL 103), whieh is at least consistent with Kant's view that moral principles are best understood as action guiding imperatives.

18 Pap 1949: 54.

19 For the original and full statement of the argument see the extraet from St. Anselm's *Proslogion* in John Hiek (ed.) *The Existence of God* (Macmillan, London, 1964). For rigorous diseussion of the argument see Jonathan Barnes *The Ontologieal Argument* (Macmillan, London, 1972).

20 Ayer 1976: 9

21 Ayer 1940: 206

22 So, on the Kantian and positivist theory of scientifie ontology any true sentence or set of sentences of the natural sciences eontains terms which denote objeets, or properties, or constituents of objects, which are numerically identical with items which could in principle be observed.

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AYER

1. AYER IN HISTORY

Ayer is the author of what may be regarded as the manifesto of logical positivism, perhaps the most radically anti-metaphysical movement Western philosophy has ever seen. For this reason, Ayer's place in the history of philosophy is assured. That manifesto is called *Language, Truth and Logic*, first published in 1936 but most effective in its impact in the immediate aftermath of the Second World War.¹

What is logical positivism? The logical positivists were a group of philosophers and scientists who met in Vienna in the late 1920s and early 1930s to form what became known as the Vienna Circle.

'Positivism' is the name of the thesis that every problem may in principle be solved using the methods of the natural sciences. Logic is the exact study of inference. The logical positivists were impressed by the explanatory power of science and the rigour and precision of mathematics and logic. They hoped to make philosophy at least as rigorous as mathematics and put philosophy at the service of science. Their conception of philosophy was the Lockean one of clearing intellectual obstacles from the path of scientific progress.²

Traditionally, philosophy had been thought of as metaphysical, or at least as the study of metaphysical problems. By 'metaphysics' I mean 'the study of reality as a whole' - perhaps in its essential properties. Metaphysical philosophers often think it is the role of philosophy to tell us what the world is really like, with its intrinsic nature undistorted by the contingencies of our possible perceptions of it. Hence Plato's thesis that there exists a non-spatio-temporal, non-mental world of perfect universals called 'forms' is a metaphysical idea. So also is Hegel's claim that reality is ultimately spiritual and that the paradoxical nature of the world can be 'overcome' in Spirit's 'absolute knowing'. Similarly, the claims that what is is fundamentally material, mental, or both are metaphysical, as is Leibniz's postulation of non-spatio-temporal constituents of reality called 'monads'. Western philosophy contains innumerable metaphysical projects.³

The radical case advanced by the logical positivists is that the whole of metaphysics is meaningless: literally nonsensical. They do not think it false that forms exist, or that God exists, or that monads exist. Rather these putative claims were devoid of any literal meaning whatsoever. They did not even have the merit of being false. Metaphysics was nonsense.

In order to demonstrate the impossibility of metaphysics, the logical positivists advocated a criterion to distinguish meaningful from meaningless statements (or, to be accurate, putative statements which are in fact meaningless). This criterion became known as the 'verification principle'. It asserted that the meaning of a statement is the method of verifying it (or falsifying it) and there are two, and only two, methods of verification: empirical observation or

logical analysis. By this criterion the logical positivists claimed that the various special sciences emerged as meaningful - genuinely capable of truth or falsity, but the whole of metaphysics and of theology were classified as nonsensical. For the logical positivists, many of the traditional questions of philosophy, such as: Does God exist? What are minds? What are numbers? Was there a first event? turned out to be meaningless questions to ask. At least, if there were meaningful questions thrown up by philosophy these were either in the last resort empirical, and so scientific, or else logical and so to be solved by producing definitions. Indeed, a major task of philosophy was to define the meanings of words precisely. 'What is x?' became 'What does x mean?' It is clear, then, that if logical positivism is a viable position the consequences for much else in philosophy are disastrous. The whole metaphysical tradition from Plato to Heidegger is called into question.⁴ Admittedly, the logical positivists thought that many past philosophers were engaged in genuine logical analysis, but that many too were engaged in meaningless metaphysics.⁵ Logical positivism is a movement of potentially enormous eliminatory power, so I shall say a little now about its most important exponent.

Alfred Jules Ayer was born in 1910. He was a King's scholar at Eton, and a classical scholar at Christ Church, Oxford - Locke's old college. At Oxford he studied under Gilbert Ryle, who was later to publish the brilliant critique of Cartesian mind-body dualism, *The Concept of Mind* (1949). After graduating, Ayer considered going to Cambridge to study under the influential Austrian philosopher, Ludwig Wittgenstein.⁶ However, Ryle had recognized Ayer's ability and suggested that instead he should go to Vienna to report back on the activities of

the Vienna Circle, whose anti-metaphysical philosophy was then little known in England. This Ayer decided to do.

Ayer arrived in Vienna in 1932, and quickly learnt enough German to follow the debates of the Vienna Circle. The chairman of the Vienna Circle was Moritz Schlick, then Professor of the Philosophy of Science at the University of Vienna. Other prominent members of the Vienna Circle included Rudolf Carnap (1891–1970), whose work *Der Logische Aufbau der Welt* had been published in 1928; the Marxist philosopher and sociologist Otto Neurath (1882–1945); and Friedrich Waismann (1896–1959), whose specialities were the philosophies of mathematics and of language. The circle was also attended by Willard O. Van Quine, the American philosopher and logician. Quine and Ayer were in Vienna at the same time.

It is sometimes maintained that logical positivism is a conservative movement politically and philosophically, in the history of ideas. This view is wholly mistaken. In the 1940s and 1950s logical positivism was a radical movement self-consciously iconoclastic in its devastation of received philosophical orthodoxy. Central among its targets were the concepts of God and the soul. Also, as noted, Neurath was a socialist in his political views and Ayer himself always remained a liberal humanist, implacably opposed, for example, to any erosion of freedom of information by the British state. Finally, the Nazis mercilessly hunted down the logical positivists, and the Vienna Circle had to be abandoned in 1938 in the face of Hitler's Anschluss. It is to be noticed that the phenomenologist and existentialist Martin Heidegger was happy to co-operate with the Nazis in their dark closure of the freedom of thought. When Schlick was shot to death by one of his students in

1936, the right-wing press in Austria thought this something of a joke – that it was, after all, what one might expect if one taught students logical positivism. The student may not have been a Fascist, but the apologists for totalitarian dictatorship fully appreciated that the logical positivists' claim that knowledge was tentative and provisional was incompatible with their own nationalistic Fascist mysticism.

Ayer took up a lecturing post at Christ Church in 1933, and became a research student in the same college in 1935. He served as an officer in the Welsh Guards at the beginning of the Second World War and was subsequently employed in military intelligence. In 1945 he became Fellow and Dean of Wadham College, Oxford. He was Grote Professor of the Philosophy of Mind and Logic in the University of London from 1946 to 1959, and Wykeham Professor of Logic in the University of Oxford from 1959 to 1978. He was knighted in 1970. Sir Alfred died in 1989, while this chapter was being written.⁷

2 MEANING AND METAPHYSICS

'Metaphysics' may be defined in two ways. It is sometimes thought of as the study of reality as a whole, where it usually means the study of what exists in its essential properties; and it is sometimes thought of as the study of what is as it really is in itself, independently of how it appears to us in experience. Ayer's logical positivism is a repudiation of metaphysics in both senses.

Ayer does not argue that metaphysical sentences are false; rather he seeks to establish that they do not even have the merit of being false. They are meaningless, or devoid of literal significance. Therefore, in Ayer's view, if someone claims that God exists; that there was a first event; that the empirical world is unreal; that what exists is

ultimately mental; or if they make any similar metaphysical claim, then they do not make a false assertion. They produce sentences which are utterly nonsensical, notwithstanding their precise grammatical construction.

If Ayer's attack on metaphysics is sound, many of the celebrated philosophical systems of the past are discredited. Plato's theory of forms, Spinoza's view that there is only one substance, Leibniz's view that there are many substances, Hegel's concept of Geist, Heidegger's Question of Being and many others must turn out to be so much nonsense.⁸ Logical positivism is thus a crucially important movement in modern philosophy, because its potential eliminatory power is so great. We should examine next Ayer's grounds for maintaining that metaphysics is meaningless.

Ayer argues that there are two, and only two, classes of meaningful statement. On the one hand there are sentences which may be confirmed or refuted by observation; on the other hand there are sentences whose truth or falsity may be decided by purely intellectual procedures. Into the first category fall the claims of science and the vast multiplicity of commonsensical claims we make in the course of our everyday lives. Into the second category fall the sentences of mathematics and logic, all definitions, and any sentence that is of a formal or self-proving nature. There are thus two, and only two, broad sorts of procedure for confirming and refuting statements: empirical observation and the inspection of meanings. If a sentence's truth or falsity is decidable in one of these two ways, then it is meaningful. If it is not decidable in one of these two ways it is not meaningful. Hence 'Water boils at 100°C' is the sort of claim that counts as meaningful on this criterion because it may be confirmed or refuted by

observation. ' $2 + 2 = 4$ ' and 'A square has four sides' also count as meaningful because they may be decided purely intellectually, by inspecting the meanings of their constituent terms. 'God exists' or 'The Absolute exists' do not count as meaningful because they are not decidable by observation, nor are they decidable by any formal system or analysis of terms.

Ayer expresses this position in a traditional philosophical terminology which I shall now explain. The indicative sentences of science and commonsense express propositions - that is, they are genuinely capable of truth or falsity. They are a posteriori or empirical sentences, meaning that the propositions they express may be confirmed or refuted by observation. They are also contingent, meaning that if they are true they could in principle have been false and if they are false they could in principle have been true. If something is contingently the case, then it is the case but it could possibly have not been the case. Finally, the sentences of science and commonsense are synthetic, because if true they are not true by definition but are empirically informative.

The sentences of logic and mathematics also express propositions, because they too are genuinely capable of truth or falsity. They are a priori sentences, meaning that the propositions they express may be confirmed or refuted independently of observation. They are necessary rather than contingent, meaning that if true they are not only true but could not have been false, and if false they are not only false but could not have been true. If something is necessarily the case, then it is not only the case but could not possibly have not been the case. Finally, the sentences of logic and mathematics are analytic or tautologous, because if true they are true by definition or in

virtue of the meanings of their constituent terms and so are not empirically informative.

The two classes of meaningful statement are mutually exclusive and collectively exhaustive. Every meaningful statement is either a posteriori, contingent and synthetic, or a priori, necessary and analytic, but not both. Either some observation is relevant to deciding the statement's truth or falsity, or some inspection of meanings is relevant, but not both.

If a sentence falls into one of these two categories then it logically follows that that sentence is literally meaningful. If a sentence does not fall into one of those two categories, it logically follows that that sentence is literally meaningless. Thus verifiability by one or other of these two procedures is both logically necessary and sufficient for meaningfulness.

Devastatingly, metaphysical sentences fall into neither category. Claims about a putative reality transcending our experience, or reality as a whole, cannot be confirmed or refuted by observation. Nor, on the other hand, are they definitions or logical truths. It follows that they are literally nonsensical. As Ayer puts it, 'As tautologies and empirical hypotheses form the entire class of significant propositions, we are justified in concluding that all metaphysical assertions are nonsensical' (LTL 56). Notice that once again Ayer is not claiming that they are false but that they are meaningless.⁹

Why does Ayer speak of empirical hypotheses? He thinks that no empirical claim may be absolutely certain because no sentence may be conclusively confirmed or refuted by observation. Although for practical purposes we count many empirical claims as certain, it is logically possible that any empirical claim we hold to be true may be false,

and any we hold to be false may be true. Only analytic propositions, or tautologies, are conclusively verifiable because they are true by definition. An empirical claim has only a probability (less than one) of being true, even if this probability is very high. Ayer makes a distinction between strong and weak verifiability to accommodate this fact. If a proposition is weakly verifiable, observation makes its truth probable. If a proposition is strongly verifiable, analysis makes its truth certain. Either weak or strong verifiability is sufficient for meaningfulness.

Ayer also makes a distinction between verifiability in principle and in practice. Clearly, if someone says there are mountains on the far side of a distant planet, we may lack the practical means to verify or falsify that proposition. The proposition is still meaningful, however, because it is verifiable in principle. We may imagine without contradiction empirical procedures which would confirm or refute it. 'Verifiable' need not mean 'verifiable by us now', so propositions about remote regions of time and space are meaningful because some observer could in principle confirm or refute them. In this way Ayer protects the claims of historians about the past, or the wide empirical generalizations of the natural scientists, from the eliminatory power of his criterion.

Ayer calls his criterion for meaningfulness 'the verification principle' because it is the thesis that 'a sentence [has] literal meaning if and only if the proposition it expresses [is] either analytic or empirically verifiable' (LTL 7).¹⁰ Claims about God, about Being, about the Absolute and so on have no literal significance. They are at best of some emotional comfort to those who utter

them, but they cannot be used to assert or deny anything of what is. Metaphysics is meaningless.¹¹

3 PHILOSOPHICAL ANALYSIS

If philosophy is not metaphysics, what is it?

Philosophy still has a critical role in demonstrating the meaninglessness of metaphysical sentences and in rewriting philosophical problems so that they may be solved scientifically. If a problem is not amenable to this treatment, and cannot be resolved by analysis either, it is not a genuine problem at all but a pseudo-problem; a piece of nonsense.

It is no part of philosophy's role to compete with the natural sciences. It is science's function to tell us what the world is like and philosophy's to clear conceptual obstacles from the path of science's progress. Ayer has enormous confidence in the explanatory power of the natural sciences:

There is no field of experience which cannot, in principle, be brought under some form of scientific law, and no type of speculative knowledge about the world which it is, in principle, beyond the power of science to give. (LTL 64)

It is no part of philosophy's function to discover facts about the universe. Philosophy's function is critical for Ayer, and it is concerned not with the world but with the language we use to describe the world. Thus philosophy differs radically from any of the special sciences. It deals with the language we use to make the world intelligible, not with the world itself. This point is sometimes made by saying that philosophy is a second order subject, not a first order subject. For Ayer, philosophy is, in fact, a part of logic:

The propositions of philosophy are not factual, but linguistic in character – that is, they do not describe the behaviour of physical, or even mental, objects; they express definitions, or the formal

consequences of definitions. Accordingly we may say that philosophy is a department of logic. (LTL 76) Thus the role of philosophy is not to generate empirical truths about, say, physical objects, but to define 'physical object'; or, to put it another way, to analyse the concept of a physical object. Similarly, philosophy does not discover causes and effects but states the logically necessary and sufficient conditions for the truth of sentences of the form 'A causes B'. In other words, it analyses the concept of causation. Philosophy analyses the concepts of science and commonsense, so that our thinking about the world gains precision and is shorn of metaphysical illusions. Instead of asking 'What is x?' or 'What is the nature of x?', the philosopher should ask 'What is the definition of x?' or 'What is the analysis of x?' Instead of asking 'What are numbers?', the philosopher asks for an analysis of the concept of number, perhaps in terms of sets.

As Ayer says, it follows from this conception of philosophy that there can be no contradiction between the claims of philosophy and those of science, and 'It follows that philosophy does not in any way compete with science' (LTL 76).

Ayer maintains that many major figures in the history of philosophy anticipated the practice of conceptual analysis in their work. Even though Plato was guilty of generating much metaphysical nonsense he also produced valuable linguistic analyses. Aristotle and Kant were conceptual analysts, but Ayer reserves his greatest approbation for the British empiricists Locke, Berkeley and Hume. Indeed, he quotes with approval Locke's conception of philosophy as the under-labourer for science, clearing obstacles from the path of knowledge.¹²

A central example of linguistic analysis in action is provided by Ayer's solution to the problem of perception: the question of what the relation is between our sense experiences and the 'external world' of physical objects. Ayer's account also provides an analysis of the concept of a physical object.

Ayer's theory of perception is phenomenalism, the thesis that any sentence or set of sentences about physical objects may be translated without loss of meaning into a sentence or set of sentences about actual or hypothetical sense contents. As Ayer intends it, this is not an answer to the questions 'What is perception?' and 'What is a physical object?', but an analysis of the concepts 'perception' and 'physical object'.

To make sense of phenomenalism we have to understand the idea of a logical construction. A is a logical construction out of Bs if, and only if, A is nothing over and above Bs, or if all meaningful talk about A may be expressed in meaningful talk about Bs: 'The English state, for example, is a logical construction out of individual people' (LTL 85) in the sense that anything said about the English state could in principle be expressed by a set of (long and complex) sentences about a set of people. To say that the state is a logical construction is not to say that it does not exist, or is a fiction, but to say that 'state' is a shorthand term for a complex of individuals. In a parallel way, 'physical object' is a shorthand term for a complex of sense contents. This is not a factual, empirical, claim but 'a linguistic assertion to the effect that the symbol "table" is definable in terms of certain symbols which stand for sense contents' (LTL 85). To say, for example, that a table exists is to say that certain sense contents of hardness, brownness and squareness are

either actual or possible. Those two ways of speaking are logically equivalent.

Another way of expressing the same claim is to say that physical objects may be reduced to sense contents. If A is a logical construction out of Bs, then A may be reduced to Bs. As Ayer puts it: ' "I am now sitting in front of a table" can, in principle, be translated into a sentence which does not mention tables, but only sense contents' (LTL 86).

A physical object does not have sense contents as its parts. What the parts of a physical object are is not a philosophical but an empirical or scientific matter. Rather, the mentioning of sense contents is necessarily required in saying what it is for something to be a physical object. It is in this definitional sense that we must understand Ayer's thesis that 'a material thing is constituted by both actual and possible sense contents' (LTL 88).

The phenomenalist language has to contain both categorial and hypothetical statements because it does not follow from any single set of categorial statements about sense contents that a physical object exists. However, to say that certain sense contents are possible, even though not actual, allows translation into the sense, datum language of sentences about absent physical objects and the unperceived properties of present physical objects. To say that a physical object exists but is not present to perception, or to say that a present perception is of a physical object, is to say that certain sense contents are possible.¹³

5 CAUSATION

Typically, Ayer's approach to the problem of causation is not to ask 'What sorts of things are causes?' but 'What is it that we are asserting when we assert that one event is causally connected with

another?' (LTL 73). His answer falls centrally within the empiricist tradition.

Ayer accepts much of Hume's account of causation.¹⁴ For example, Ayer thinks Hume was correct to assert that no causal connections are necessary because the sentences reporting them are not necessary truths, and that in turn is because no negation of any causal statement is or entails a contradiction, even if it is false. He also agrees with Hume that knowledge of causal relations is a posteriori, not a priori, and that sentences expressing natural laws are not analytic but synthetic. It follows that Ayer's account of causation is essentially Humean, and we should think of Ayer as developing Hume's theory.

Ayer improves on Hume in two respects. Ayer's account of causation is less psychological than Hume's, so that causal relations may be safely asserted to hold between events which never have been, and may never be, observed. Though perhaps not impossible for Hume to maintain, this position would have been difficult because for him the idea of causal necessity is acquired from the impression of expectation habitually developed from the observation of constantly conjoined events. For Ayer, even though the observed constant conjunction of two events is excellent evidence for a causal relation between them, 'there is no self-contradiction involved in asserting the proposition "C is the cause of E" and at the same time denying that any events like C or E ever have been observed' (LTL 74).

The other improvement on Hume is the thesis that the making of a specific causal claim 'C causes E' is the invocation of a natural law that 'Cs cause Es', so that the sentences reporting particular causal connections are logically derivable from sentences expressing natural laws.

6 INDUCTION

The problem of induction is a pseudo-problem for Ayer because there is no possible method of solving it. No amount of empirical observation or inspection of meanings will resolve the putative sceptical issue, so 'it is a fictitious problem, since all genuine problems are at least theoretically capable of being solved' (LTL 67). It follows that the problem of induction is not a problem for the logic of the natural sciences because induction is not a problem at all.

Ayer describes the apparent problem of induction as that of proving that empirical generalizations about past experience will hold for the future.

This is in fact a limited statement of the issue, because the problem of induction also includes the question of the rationality of non-deductive inferences from present to past and from some to many at least. If we assume the problem is genuine, according to Ayer, there are two, and only two, ways of trying to solve it. A claim about the future may putatively be derived from some formal, a priori claim or from some a posteriori, empirical claim, but neither of these is satisfactory. Ayer's view is that all true a priori propositions are tautologies, but no empirical claim may be derived from a tautology, therefore no claim about the future may be derived from a tautology, so no claim about the future may be derived from any a priori proposition. Nor, on the other hand, may a claim about the future be derived from empirical propositions about the past, on pain of circularity, because then 'one simply assumes what one is setting out to prove' (LTL 66). Ayer means that induction is at work in any such derivation but the justification of inductive inference is precisely what is at issue. If we assume, for example, the uniformity of nature, that begs the

question because it is itself an inductive principle.

It follows from Ayer's premises that 'there is no possible way of solving the problem of induction' (LTL 67). Both the a priori and the a posteriori attempts at solution have failed, but as any meaningful statement is either a priori or a posteriori there remains no meaningful solution to the problem.

Ayer believes we should have confidence in induction as a practical principle and that this is all that is necessary for the conduct of science. It does not deductively follow from the past explanatory success of science that science will continue to be successful in the future, but it is wrong to hold this as an objection to scientific method because 'it is a mistake to demand a guarantee where it is logically impossible to obtain one' (LTL 67).

Nor does it follow from the fact that the problem of induction cannot be solved that it is irrational to suppose that the future will resemble the past. It follows, rather, that the problem of induction is a pseudo-problem. It is rational to believe scientific predictions because part of what it is to be rational is to be guided by the past: 'being rational entails being guided in a particular fashion by past experience' (LTL 67).

Scientific method is justified in practice and by experience, not by philosophy. Scientific prediction, for example, is justified in so far as scientific predictions turn out to be true, and no more than this. It is not the role of philosophy to construct a priori justifications of scientific method but to remove from its path pseudo-problems of which the problem of induction is one.¹⁵

Ayer rejects the traditional positions in the philosophy of mind – dualism, idealism and materialism – as solutions to a pseudo-problem. His own thesis is that the distinction between mental and physical is a distinction between two kinds of logical construction out of sense contents; a view which approximates in its essentials to Russell's 'neutral monism' (see pp. 211–13 above).

As we have seen, for Ayer 'the existence of a material thing is defined in terms of the actual and possible occurrence of sense contents which constitute it as a logical construction' (LTL 162). The existence of mental items is equally to be analysed in terms of sense contents, such that something is mental if constructed out of introspective sense contents rather than out of sense contents which are parts of experience through the five senses. It follows that Ayer makes an epistemological distinction between mental and physical. We know about mental items through introspection, but we know about physical objects through sense perception. But these two modes of access are constitutive of the distinction, not just ways in which we become acquainted with items already distinguished. Ayer also says that mental items are constructed out of sense contents 'which are elements of one's own body' (LTL 163), but this cannot be criterial of 'mental' because one's body may have both mental and physical characteristics. If we ask whether sense contents are mental or physical, this question makes no sense for Ayer: 'The answer to the question whether sense contents are mental or physical is that they are neither; or rather, that the distinction between what is mental and what is physical does not apply to sense contents' (LTL 162). This is because the mental-physical distinction only applies to logical constructions out of sense contents, not to the

contents themselves, much as what is true of 'the British state' may not be true of each British citizen taken singly, or what is true of 'the average man' may not be true of each man considered individually.

Thus Ayer follows Russell in maintaining that sense contents are intrinsically neither mental nor physical. Depending on the epistemological relation a perceiver stands in with regard to them, and on the relation they have to one another, the constructions out of them are either mental or physical.

There is a complication in this account. A sense content is not an experience for Ayer, yet any sense content is part of some experience. It would seem to follow that sense contents are mental if experiences are mental, but that is not a conclusion Ayer wishes to draw. He does not in fact offer a solution to the problem, but one solution would seem to be that experiences are not intrinsically mental or physical but count as either mental or physical depending on whether their sense-content parts are accessible introspectively or through sense perception.

From Ayer's account it follows that the universe is intrinsically neither mental nor physical. Indeed, the putative claim that it is either is meaningless because it is neither analytic and a priori nor synthetic and a posteriori. Ayer's empiricism entails an explanation of 'mental' and 'physical' in terms of sense contents which fall under neither designation, so I should say that the most accurate characterization of Ayer's philosophy of mind is 'neutral monism'.

The so-called 'mind-body problem' is the problem of whether, and if so in what sense, mind and matter exist, and of the relation between them. This problem is clearly meaningless by Ayer's criteria.

It may, however, be reformulated as a meaningful linguistic question which may be resolved through conceptual analysis: 'there is no philosophical problem concerning the relationship of mind and matter, other than the linguistic problems of defining certain symbols which denote logical constructions in terms of symbols which denote sense contents' (LTL 164).

It follows that there is no meaning in the claim that there are mental or physical substances. That claim is metaphysical and therefore a pseudo-claim, because it is not decidable a priori by analysis, nor a posteriori, empirically. Our commonsensical talk about mental and physical is empirical and depends upon our acquaintance with sense contents, so our ordinary language mental-physical distinction is meaningful.¹⁶

8 PERSONAL IDENTITY

If we ask the question 'What is the self?', Ayer's answer is that the self, like mental states and physical objects, is a logical construction out of sense contents. The claim that a self exists is logically equivalent to the claim that a set of relations obtains between certain sense contents. It follows that Ayer has a phenomenalist theory of the self which is, in its essentials, the same as Hume's. Like Hume, Ayer rejects the Cartesian notion of the self as a mental substance.¹⁷ Self-consciousness reveals no such substantial self, but as self-consciousness is the only plausible experiential route to such a putative self the existence of the self cannot be known empirically. There is no empirical knowledge of the Cartesian self or soul, and it is not true by definition or true a priori that such a self exists. It follows that, if every meaningful claim is either a priori or empirical, the claim that Cartesian selves exist

is devoid of literal significance. The claim is meaningless.

Any meaningful notion of the self must be grounded in experience and Ayer's phenomenalist account is designed to do just that.¹⁸ The question 'What is the self?' is the same for Ayer as the question 'What are the conditions under which some experiences belong to one and the same self?' He says:

To answer this question is that for any two sense-experiences to belong to the sense-history of the same self it is necessary and sufficient that they should contain organic sense contents which are elements of the same body. (LTL 165)

A body, like any physical object, is a logical construction out of sense contents and the criteria for the identity of a body are the same as the criteria for the identification of any physical object.¹⁹

9 RELIGION

The claim that God exists is not false but meaningless within the conceptual framework of Ayer's verificationism. Ayer's position therefore needs to be distinguished from traditional atheism. Atheism is the view that it is false that God exists; that, however, presupposes it is meaningful to claim that God exists and it is precisely this presupposition that Ayer denies.

The claim that God exists is not analytic and a priori, nor is it synthetic and a posteriori, but those two classes are mutually exclusive and collectively exhaustive of meaningful statements; thus the claim that God exists is meaningless: 'to say that "God exists" is to make a metaphysical utterance which cannot be either true or false' (LTL 152). It follows from this that there cannot possibly be any religious knowledge because knowledge is of what is the case - of truths - and

religious claims are not truths because they are not true or false. They do not express propositions, so they cannot be known. Therefore there is no religious knowledge. It follows that religious talk is not meaningful talk, however emotionally significant it may be to the speaker. Atheism and agnosticism are as meaningless as theism because they depend upon religious language and that language is meaningless. If it is true that 'God exists' is meaningless, then it is true that 'God does not exist' and 'It is possible that God exists' are also meaningless.

If per impossible the concept of God were significant then it would still be the case, according to Ayer, that no proof of God's existence is possible. This is because the claim that God exists would have to be derived either from a priori analytic premises or from a posteriori synthetic premises, these being the only two meaningful sorts of premise. However, the claim that God exists does not follow from any analytic truth, because only tautologies logically follow from tautologies and the claim that God exists is not a tautology. Nor does the claim that God exists follow from any empirical premise because no transcendent (and so non-empirical) claim follows from any empirical premises, and, in any case, all non-empirical claims that are not tautologies are nonsensical.

If someone were to claim that the truth of a particular set of empirical claims is both necessary and sufficient for the existence of God - for example, some set of claims about causal regularities in the universe - then it would follow that 'God' is being used in an unconventional sense. 'God' would then be a shorthand term for certain regularities in nature. This, Ayer maintains, would not prove God's existence as

traditionally conceived: as the transcendent cause of the universe. The traditional concept of God is in any case incoherent: 'the notion of a person whose essential attributes are non-empirical is not an intelligible notion at all' (LTL 154).

It follows that all the claims of theology are senseless. Theology is either the study of God, or the study of human relations with God. On either account, the concept of God is incoherent and putative claims about God's existence or non-existence are literally nonsensical. So, as with the metaphysician, the theist 'says nothing at all about the world', so 'he cannot justly be accused of saying anything false or anything for which he has insufficient grounds' (LTL 153). Theology, like metaphysics, does not even have the merit of being false. It is meaningless.²⁰

10 ETHICS

Value judgements may be neither true nor false for Ayer, so they, like metaphysical sentences, are literally senseless. To condemn someone or something morally is not to say anything informative but merely to express an emotion, and perhaps to cause another to have an emotion. For example, 'Stealing money is wrong' is 'a sentence which has no factual meaning - that is, [it] expresses no proposition which can be either true or false' (LTL 142). Ayer is not claiming that stealing is right or wrong - it is not the job of the philosopher to engage in value judgements; he is providing an analysis of what it is to judge something right or wrong. To say something is right is to express a feeling of approval, and to say something is wrong is to express a feeling of disapproval. It follows that if someone thinks something is wrong, say abortion or capital punishment, and someone else thinks that thing is right, then there is no contradiction between those

two persons' ethical positions. Contradictions may only obtain between propositions - items capable of truth or falsity - but value judgements are not and do not express propositions. It follows that there can be no contradictions between value judgements. All that follows from moral disagreements between persons is that those persons have different emotional responses to some phenomenon:

For in saying that a certain type of action is right or wrong, I am not making any factual statement, not even a statement about my own state of mind. I am merely expressing certain moral sentiments. (LTL 142)

Notice that ethical judgements are not to be construed as covertly autobiographical. It is not Ayer's position that in evaluating something ethically one is really making a report on one's emotions. Rather, one is expressing one's emotions, and causing emotions in others. Hence he claims that 'ethical terms do not serve only to express feeling. They are calculated also to arouse feeling, and so to stimulate action' (LTL 143).

Ayer's emotivist theory of ethics has two important consequences. Firstly, value judgements are in a sense relative; they are not truth-valued, so there is no rational way of arbitrating in ethical disputes. If two people disagree ethically there is no fact of the matter about who is right. Secondly, Ayer has the outline of a theory of how ethics function in social contexts that partially anticipates the theory of ethics called 'prescriptivism'. Prescriptivism is the view that value judgements have some of the logical features of commands and, indeed, that they logically entail imperatives. Ayer says that some value judgements have the effect of commands. This is clearly a claim about the causal efficacy of value judgements and not a claim about their logical status.

Nevertheless, Ayer has it in common with prescriptivists like Richard Hare to maintain that value judgements are like imperatives. For example: 'The sentence "You ought to tell the truth" ... involves the command "Tell the truth" ' (LTL 143). Clearly, the imperative-like status of some value judgements is consistent with a theory of action which regards such value judgements as action-guiding and as action-causing. It follows that Ayer's theory of ethics is emotivist but contains prescriptivist elements. Ayer makes a fourfold distinction between the kinds of claim to be found in philosophical writings about ethics. There are definitions of ethical words; descriptions and causal explanations of moral experience; prescriptions, and value judgements. Ayer says that only claims of the first sort count as genuinely philosophical. It is the business of the moral philosopher to analyse concepts like 'good', 'wrong', 'moral' and so on, and to justify these definitions. It is also the role of the philosopher to separate value judgements from factual statements. It is no part of the philosopher's business to pronounce on ethical issues, to exhort people to virtue, or to produce causal explanations of moral experience. This latter task is an empirical one which belongs to the sciences, perhaps to psychology and sociology. Philosophy is a second-order inquiry into the logic of ethical concepts, not a first-order inquiry into what is right or wrong: A strictly philosophical treatise on ethics should ... make no ethical pronouncements. But it should, by giving an analysis of ethical terms, show what is the category to which all such pronouncements belong. (LTL 137)

Ayer

1. See A. J. Ayer, *Language, Truth and Logic* (2nd ed., Harmondsworth, 1976). Ayer added an Introduction to the second edition, which appeared ten years after the first edition, in 1946. I strongly advise the reader to read this introduction only after reading the main body of the text, because 'introduction' is something of a misnomer here. It is Ayer's defence against criticism and is largely unintelligible without prior acquaintance with the book.
2. The most useful survey of the movement is Leszek Kolakowski's *Logical Positivism* (Harmondsworth, 1972). For a forceful summary of its central tenets, see Herbert Feigl, 'Logical Empiricism', in D. D. Runes (ed.), *Twentieth-Century Philosophy* (Philosophical Library, New York, 1943), reprinted in H. Feigl and W. Sellars (eds.), *Readings in Philosophical Analysis* (New York, 1949), pp. 3-26.
3. See Plato, *The Republic*, translated by Desmond Lee (Harmondsworth, 1977), Books VI-VII, especially pp. 299-325; Hegel's *Phenomenology of Spirit*, translated by A. V. Miller (Oxford, 1979), Chapter 8, 'Absolute Knowing', pp. 479-93; G. H. R. Parkinson (ed.), *Leibniz: Philosophical Writings*, translated by Mary Morris and G. H. R. Parkinson (London, 1984), 'Monadology', pp. 179-94. Plato, Leibniz and Hegel are ably discussed by Father Frederick Copleston in his *A History of Western Philosophy* (New York, 1963), in Vols. 1, 4 and 7 respectively. On materialist and idealist metaphysics, see Stephen Priest, *Theories of the Mind* (Penguin Books, forthcoming).
4. According to Ayer, 'Heidegger ... bases his metaphysics on the assumption that "Nothing" is a name which is used to denote something peculiarly mysterious' (*Language, Truth and Logic*, p. 59). This assumption involves two kinds of mistake: the

error of assuming that all nouns denote entities or 'things', and the error of using a vocabulary in a fashion which precludes its translation into a set of primitive empirical concepts. The claim that nouns, or pronouns, may mislead the thinker into the postulation of illusory metaphysical objects is a leitmotif of empiricist and critical philosophy. It is to be discerned, for example, in Berkeley's critique of Locke's doctrine of abstract ideas, Kant's attack on the Cartesian theory of the soul, Wittgenstein's Private Language Argument, and Ryle's notion of a category mistake. See Berkeley, *The Principles of Human Knowledge* (London, 1977), pp. 57-9, and pp. 116-19 of the present work. See also Kant's *Critique of Pure Reason*, op. cit., 'Transcendental Dialectic', Chapter 1, 'paralogisms', pp. 328-83; Wittgenstein, *Philosophical Investigations*, op. cit., paras. 243ff.; Gilbert Ryle, *The Concept of Mind*, op. cit., Chapter 1, 'Descartes' Myth', section 3, 'The Absurdity of the Official Doctrine', pp. 15-18, and 'The Origin of the Category Mistake', pp. 18-23. The idea that any meaningful term must ultimately and in principle be translatable into some empirical term or terms is discussed on pp. 30-31 of the present work. It is anticipated by Hobbes. In his criticism of Heidegger, Ayer is influenced by Rudolf Carnap in his 'Überwindung der Metaphysik durch logische Analyse der Sprache', *Erkenntnis*, II (1932). It seems to me that Heidegger is at pains not to reify 'nothingness', at least in *Being and Time*, and Heidegger was of course steeped in pre-Socratic writings and would have been fully apprised of the Parmenidean view that nothingness does not exist. I have in mind Heidegger's claim that nothing ('Nichts') is the object of anxiety, but that 'That in the face of which one has anxiety is not an entity within-the-world' (Martin

Heidegger, *Being and Time*, translated by J. Macquarrie and E. Robinson (Oxford, 1973), p. 231). Ayer and Carnap, however, have in mind the use of 'nothing' in Heidegger's essay 'What is Metaphysics?'. (See D. F. Krell (ed.), *Martin Heidegger: Basic Writings* (London, 1978), pp. 91-112.) For Heidegger's use of 'nothing', see *Being and Time*, op. cit., pp. 43, 128, 186-9, 266, 273, 276f., 279, 308, 341, 343, 352; for 'not-Being' ('Nichtsein'), see pp. 170, 243, 431, 434; for 'notness' ('Nichtheit'), see pp. 285ff.; and for 'the "not" ' ('das Nicht'), see pp. 29, 283-6. References are to Macquarrie and Robinson's marginal numbers taken from the later German editions of *Sein und Zeit*.

5. Philosophers who, according to Ayer, were largely engaged in philosophical analysis rather than metaphysics include Plato, Aristotle, Hobbes, Locke, Berkeley, Hume, Kant, Mill and Bentham (*Language, Truth and Logic*, op. cit., pp. 70-75). A clear-cut case of a philosopher whose utterances are neither empirically verifiable nor analytic is the British neo-Hegelian idealist philosopher, F. H. Bradley. Quoting 'at random' from Bradley's *Appearance and Reality*, Ayer says: 'Such a metaphysical pseudo-proposition as "the Absolute enters into, but is itself incapable of, evolution and progress" is not even in principle verifiable' (ibid., p. 49). For Bradley's use of the term 'Absolute', see his *Appearance and Reality: A Metaphysical Essay* (London, 1902), pp. 144, 159, 172, 182, 195, 204, 411-12, 487-9, 519ff., 536ff., 556.

6. The best introduction to Wittgenstein's philosophy is Anthony Kenny's *Wittgenstein* (Harmondsworth, 1973). For Ryle, see note 4, above.

7. My biographical remarks on Ayer are based on the two volumes of his autobiography, *Part of My*

Life (Oxford, 1977) and *More of My Life* (Oxford, 1985).

8. See notes 3 and 4 above. Spinoza's thesis that only one substance exists, and that it has two attributes, thought and extension, is argued for in his *Ethics*, translated by A. Boyle (London, 1948), especially pp. 38-40.

9. Clearly, then, logical positivism is a kind of radical empiricism. Ayer's claim that a sentence is meaningful only if the proposition it expresses is either analytic and a priori or synthetic and a posteriori is semantically similar to Hume's distinction between 'relations of ideas' and 'matters of fact' (Hume's *Enquiries*, ed. L. A. Selby-Bigge and P. H. Nidditch (Oxford, 1975), IV, 1, p. 25). Notice too that relations of ideas for Hume and a priori truths for Ayer are necessary, and that matters of fact for Hume and a posteriori claims for Ayer are contingent. For Hume as well as Ayer, the consequences for some putative claim which falls into neither logical category are devastating. Hume invites us to inspect the sentences of any book of 'divinity' or 'metaphysics' and decide whether it contains 'any abstract reasoning concerning quantity or number' or 'any experimental reasoning concerning matter of fact and existence'. If the answer is 'no', then Hume's advice is 'Commit it to the flames: for it can contain nothing but sophistry and illusion' (E XII, III, 165). Notice that Hume is so confident of his empiricism that he recommends book-burning - something characteristic of the associates of Heidegger rather than Mill and Ayer. Hume's intolerance is misplaced on at least three counts: he might be wrong and the views he burns might be true. If he is right and the views he burns are false or meaningless, it is not clearly morally right to deny people the right to hold views which

are mistaken or meaningless. Finally, it is educative to understand empathetically views which are not one's own, irrespective of their truth value or meaningfulness. At worst, one's own truths are tested against falsity and nonsense. Notice that Ayer cannot consistently share Hume's practical intolerance. This is not just because of Ayer's liberal politics but also because Ayer thinks it is always in principle possible that we could be mistaken (except, maybe, about logical truths).

10. Moritz Schlick says: 'Stating the meaning of a sentence amounts to stating the rules according to which the sentence is to be used, and this is the same as stating the way in which it can be verified (or falsified). The meaning of a proposition is the method of its verification' ('Meaning and Verification', in H. Feigl and W. Sellars (eds.), *Readings in Philosophical Analysis* (New York, 1949), p. 148).

11. The merits of the verification principle are debated critically by Sir Isaiah Berlin in 'Verification', and the verification principle is defended by Friedrich Waismann in 'Verifiability', both in G. H. R. Parkinson (ed.), *The Theory of Meaning* (Oxford, 1976), pp. 15-34 and 35-60 respectively. Berlin brings three broad objections to verificationism. He suggests that it must be possible to understand a sentence in order to decide whether it is the sort of sentence that could be verified or falsified. Hence, meaning is prior to, and so not the same as, possibility of verification. Secondly, he doubts that sentences about the past or future may be wholly translated into sentences about the present (in particular because this generates two senses of 'present'). Thirdly, he claims there are meaningful sentences which are not verifiable even in principle:

universal generalizations, unfulfilled singular hypothetical statements, and statements about physical objects and other minds. Ayer defends verificationism against its critics in the Introduction to the second edition of *Language, Truth and Logic*, op. cit., pp. 7–35.

12. *Language, Truth and Logic*, op. cit., p. 70.

13. Ayer's phenomenalism is in the tradition of that of Mill and Russell. See the chapters 'Mill' and 'Russell' in the present work, pp. 189 and 211–13 respectively. Ayer devotes Chapter 3 of *The Problem of Knowledge* (Harmondsworth, 1956) to the philosophy of perception (pp. 84–133). This chapter is an improvement on the more rudimentary epistemological discussion in his *The Foundations of Empirical Knowledge* (London, 1940).

Induction is essential to our commonsensical and scientific conceptual schemes.

The commonsensical distinction between mental and physical is meaningful, even if, a metaphysical distinction between mental and physical substance is meaningless. It is not quite right then to say that the science-metaphysics distinction is identical with the meaningful-meaningless distinction. This misleadingly conceals the meaningfulness of commonsense, even though the world of commonsense is not numerically distinct from the objects of science for the logical positivists. It is clearly right to imply that the sentences of science are meaningful and the sentences of metaphysics meaningless on the logical positivists' account.

Materialism about personal identity entails that B at t_2 is numerically identical with A at t_1 if, and only if, B is a body which is spatio-temporally

continuous with a body A at t_1 . But if physical objects are logical constructions out of sense contents there is a *prima facie* problem of circularity here. To establish what the identity of the person over time consists in, we need good criteria for the identity of physical objects. But to obtain criteria for the identity of physical objects we need good criteria for a set of sense contents being logical constituents of one and the same physical object, and that in turn may require good criteria for distinguishing the sense contents of more than one subject. If that is right, then criteria for personal identity are needed to establish the identity of physical objects, and criteria for the identity of physical objects are needed to establish personal identity. We need to escape from this difficulty. I am not saying one could not be devised.

Logic defends theism against the charge that religious language is nonsensical.

What is reference?

Despite the immense literature on reference we have next to no idea what the relation is between some sounds or marks and some object if they are used to refer to that object. There does not seem to be room in the psycho-physical universe for a referring relation. No shadowy piece of string connects a sound or a mark to its referent. Whatever these properties are, they seem unlikely to make the world necessarily the way it is or possibly otherwise. Speaking, writing or even thinking does not have that power.