One for you, Two for me: Quantitative Sharing by Young Children

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Thesis Submitted for the Degree of DPhil Education

Michaelmas Term, 2014

Lincoln College

Acknowledgments

I would first like to thank all of the principals, teachers, parents and students from Bristol, Eynsham and Toronto as well as the Toronto District School Board for making this research possible and for participating with such enthusiasm. It was a pleasure working with all of you.

I must also thank my supervisor, Professor Terezinha Nunes, for her support and guidance over the last four years. Your help and insightful input on my work has been invaluable and I will begin my career a better researcher and critical thinker because of your training. I would also like to thank all of those people who have contributed to my thinking and learning over the last four years including Laura and my research group, my peers in the DPhil group, and other teachers and professors in the Education Department in Oxford, particularly Professor Peter Bryant.

I would not have made it through the sometimes stressful process of completing a DPhil without the love, laughter and encouragement of my friends and my three sisters both here in the UK and at home in Toronto. Thank you for reminding me that there is life outside of this work and for pushing me to carry on when times were tough. I am very lucky to have such a wonderful support system.
My parents also deserve a huge thank you for providing me with the opportunity to come to Oxford to pursue this research and for providing never-ending love and emotional support.

To my husband Mike, thank you for taking this journey with me over the last four years. Your love, statistics skills and unwavering support kept me going the whole way through. Thank you for always being there when I needed you.

Abstract

The current research aimed to examine children's understanding of cardinality by looking at their ability to use several quantitative concepts that underpin this understanding: correspondence, counting and equivalence in the context of sharing. Understanding cardinality requires children to develop knowledge about the relations between these quantitative concepts which is important for the development of mathematical reasoning.

The first study aimed to investigate how flexibly children can use correspondence to build equivalent sets in different types of sharing scenarios: equal sharing, reciprocity and equity. In some situations two characters each received one object at a time, and in others one character received double units while the other character received single units. After children shared blocks between the two characters, they were asked to make a number inference about the cardinal of one set after counting a second, equivalent set. Children had more difficulty sharing in the reciprocity and equity conditions than the equal sharing condition. The majority of children were able to make number inferences in the equal sharing and reciprocity conditions where both characters received equivalent shares in the end.

A second study with new groups of four and five- year-olds investigated whether children were using visual cues about the relation between double and single blocks to help build equivalent sets and make number inferences. It was predicted that the use of coins would be difficult and would increase the difference between the equal sharing and reciprocity conditions. In half of the trials children shared Canadian $2 and $1 coins and in half they shared blocks. There are no visual cues about the relation between $2 and $1 coins because they are the same size. Children were allowed to use counting or correspondence to build equivalent sets to compare their use of both strategies. Contrary to the first study, the reciprocity and equal sharing conditions were not significantly different. This may be due to the appearance of a new sharing strategy in the reciprocity condition termed "equalizing" where children first counted each set, dealt singles to make the two sets equal and then shared blocks or coins on a one-to-one basis. There was also no significant difference between the trials using coins and trials using blocks. The majority of children were able to answer the number inference questions correctly, however 25% of children made the number inference after sharing all singles but not after sharing doubles and singles, suggesting that using different units did impact their understanding of the equivalence of the two sets.

A third study aimed to investigate children's ability to coordinate cardinal and ordinal information to determine the cardinal of a single set, and their ability to coordinate counting principles with knowledge of equivalence to determine the cardinal of an equivalent set. Children in this study were asked to make a numerical inference about a set of blocks after watching a puppet correctly or incorrectly count an equivalent set of blocks. Many children were able to identify that the puppet did not count correctly, but struggled to correct the mistake. This indicates a gap in their knowledge about ordinality and cardinality in the context of a single set. The miscount also impacted their ability to make a correct number inference. Children performed significantly better on trials where the puppet counted correctly than trials where he made a counting error. This suggests that while children have good knowledge of counting principles in isolation, they are still developing an understanding of how to coordinate these principles with ordinal information and knowledge of equivalence to establish the cardinal of one set and to infer the cardinal of an equivalent set.

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1 Introduction

The present research aims to explore the development of young children's early quantitative concepts in the context of sharing. This research examines how quantitative knowledge develops within a social context. Before children are taught any formal mathematics in school, they begin to think about numbers and quantities and begin to build mathematical models of the world around them (Saxe, Gearhart and Guberman, 1984; Saxe, 1988). This informal mathematical knowledge is a critical part of their understanding of mathematical concepts and their ability to solve mathematical tasks. The specific context of sharing was chosen because it is an activity all young children engage in and understand. Many researchers have suggested it is an important social context where children are able to learn and practice mathematical concepts for building equivalence, such as using counting or correspondence to divide objects between people, and it also provides them with the opportunity to develop knowledge of quantitative concepts for examining the relations between sets, such as equivalence and cardinality (Squire and Bryant, 2002). The main focus of this thesis is on young children's developing understanding of cardinality: the different methods four and five-year-olds use for constructing equivalent sets and their understanding of the equivalent relation between sets of the same quantity. Cardinality is a relation between quantities that are equivalent and therefore have the same number. The understanding of relations between quantities is an essential development in mathematical understanding. A child's main aim when sharing is most often to establish equivalent sets between people, which makes it an ideal context for studying children's understanding of the concepts of equivalence and cardinality as well as the strategies they use to construct equivalent sets, in particular counting and correspondence. These concepts are a fundamental part of mathematical reasoning, and lay the foundation for solving more complex mathematical problems as children progress through school and into adult life.

This research examines the quantitative knowledge of four and five-year-old children attending kindergarten in the U.K. and in Canada. The age group of four and five-year-olds was selected to participate in this research because a great deal of social and cognitive development takes place during these years, including significant developments in the use and understanding of quantitative skills and concepts. It was necessary to select children who have a good knowledge of counting and the ability to establish and use cardinal numbers, which is too advanced for most three-year-olds. It has also been suggested in previous studies that three-year-olds' mathematical knowledge has not developed enough for them to fully appreciate the quantitative link between sharing and cardinality (Muldoon, Lewis and Towse, 2005; Davis and Pitkethly, 1990).

This chapter will first outline the rationale for the present research including a brief discussion of the importance of sharing as a social and quantitative context, a description of the social concepts related to sharing that are developing in this age group and how these connect to the quantitative knowledge children exhibit in sharing scenarios. The next section will discuss the general development of mathematical understanding at four and five years of age. This will be followed by a section defining the specific mathematical concepts explored throughout this thesis: correspondence, equivalence, cardinality and numerical inferences. The final section presents an overview of the research strategy, as well as an outline of the other five chapters included in this thesis.

1.1 Rationale

The current thesis looks at the progression of children's quantitative understanding from a developmental perspective. It takes as its basis the idea that in order to think mathematically, children must develop a combination of logical-mathematical reasoning and acquire knowledge of the cultural signs and symbols used to represent mathematical concepts. Here it is important to make explicit what is meant by the term concept. Carey (2004) defines a concept as a "unit of thought […] individuated on the basis of two kinds of considerations: [its] reference to different entities in the world and [its] role in distinct mental systems of inferential relations" (p. 60). Children acquire new concepts through their experiences and interactions with their environment and through making connections between pre-existing and new knowledge. In the context of mathematical thinking, children must learn how to use mathematical signs and symbols to represent these thought processes as well as apply logical reasoning to determine how and when different concepts should be applied. For example, in the case of cardinality, once a child learns to count, they could use counting as a tool to establish that two sets have the same number of items and are equivalent. This knowledge begins to develop before children are taught formal mathematics in school, through a child's everyday interactions with the people and objects in their environment. Social interactions are therefore a crucial part of children's early mathematics learning, and it is important to examine what quantitative knowledge children are developing and how they are learning in these informal contexts.

In particular the focus of this thesis is on the different ways that young children are able to establish equivalence, and whether they understand cardinality-the relation between equivalent sets. Knowledge of the relation between equivalent quantities is a very important step for the development of children's logical reasoning and one that several researchers have discovered is quite difficult for many four and five-year-olds to make (Frydman and Bryant, 1988; Sophian, 1988). As was stated earlier, children first develop quantitative knowledge in the context of their everyday actions, including in interactions such as sharing. Sharing is an important social context for the development of ideas about fairness and equality, but these ideas are also linked to quantitative actions. When a child decides how to share fairly, they have to determine the best way to divide objects between people, and can also compare the sets they have created to judge whether fairness or equality have been accomplished. Thus sharing allows children to practice working with numbers and quantities, to create equivalent sets in different ways and to compare the amount in each set to ensure fair sharing was achieved. These are all important mathematical concepts that build the foundation for solving more complex problems. It is for this reason that this thesis chose to look at counting, correspondence, equivalence, and cardinality through children's sharing actions.

1.2 Social Concepts in Sharing

Socially, young children are most familiar with and commonly use the strategy of sharing objects on a one-for-you, one-for-me basis to achieve equal amounts, particularly because equality in sharing is often emphasized as the fairest way to share in school (Miller, 1984). However, sharing is not a singular action and there are many sharing norms that could be considered fair depending on the context: allocating based on need, proportional to amount of work, based on how another person has previously shared with you, etc. and children must learn when and how to apply these different concepts. This research focuses on establishing and comparing equivalent sets, which is usually the primary aim of a child sharing between friends. For this reason the scenarios used in the design of the sharing paradigm that is the basis for the three studies in this thesis have been adopted from literature on sharing as a social context. A brief overview of how young children understand different sharing norms has been included in order to show the connection between the social implications of the tasks and the quantitative skills children exhibit when sharing. The following paragraphs will briefly outline how four and five-year-old children understand and use some of the social concepts in sharing and will also suggest how these different norms of fair sharing might influence children's sharing actions and therefore the quantitative concepts and skills they would use.

There have been many studies examining young children's ideas about sharing and fairness. Much of this literature reports stage-like developments in children's sharing allocations and reasoning, drawing parallels between sharing behaviour and Piaget's theory of cognitive development or Kohlberg's stages of moral development (Piaget, 1932; Damon, 1975; Leventhal and Anderson, 1970; Enright, Franklin and Manheim, 1980). It has been reported that four and five-year-olds most often allocate resources self-interestedly or equally among recipients (Damon, 1980). The consideration of other factors such as need, and the ability to consider multiple or competing claims for resources becomes progressively more evident as children get older (Zinser, Starnes and Wild, 1991; Hook and Cook, 1979). Many studies have found that young children have a strong preference to share resources equally and many do not consider sharing fair if each person does not receive the same amount. Hook (1978), for example, found that five, nine and thirteen-year-olds' answers to allocation questions were related to their answers to proportional reasoning questions, and that these could be organized into age-related stages progressing from the use of equality in the five-year-olds, to the use of ordinal equity in the nine-year-olds and finally to the use of proportional equity in the thirteen-year-olds. Sigelman and Waitzman (1991) asked five, nine and thirteen-year-olds to distribute resources between 3 campers in different scenarios designed to elicit sharing based on a need, equity or equality norm. They also found that five-year-olds preferred to allocate resources equally regardless of the sharing scenario, whereas the nine and thirteen-year-olds modified their sharing based on the context. However, their tasks relied heavily on verbal skills, which are not always well developed in young children, and this may have impacted the five-year-olds' ability to understand the tasks. Their results still clearly demonstrate that even young children are very familiar with the concept of equal sharing and will use this norm readily to divide resources. Equal sharing allows children to deal out objects to each person on a one-to-one basis to create equal sets, usually using one-to-one correspondence between people and objects. Many young children may use a counting strategy as well to establish the amount in each set or after the sets are created to check whether they are equal. Thus, correspondence, counting and equivalence are important mathematical concepts for children to understand in the context of sharing using an equality norm.

Reciprocity and equity are more difficult concepts than equality for children to learn and use, because instead of only considering what resources are available, people's inputs or contributions must also be taken into account (Sin and Singh, 2005). While there are studies examining young children's thoughts about moral reciprocity in terms of reciprocating behaviours like aggression, stealing or lying, which show that most young children think it is fair to act towards another child as they have been treated (Piaget, 1932; Durkin, 1960), there has been less research into how young children use a reciprocity norm in the context of sharing. Findings seem to suggest that the design of the study influences whether children exhibit reciprocal sharing or not. For example, whether the sharing situation is hypothetical or the child is physically asked to share and whether they are sharing with a friend, acquaintance or the experimenter all seem to influence the child's sharing behaviour (Staub and Sherk, 1970; Birch and Billman, 1986). Staub and Sherk (1970) observed pairs of children in two scenarios: first, one of the two children was given candies while the two listened to a story, and later the child who did not receive candies in the initial interaction was given a crayon when the two children were asked to draw a picture of something that happened in the story. They recorded whether and how often the first child shared candies with the second, and whether and how often the second child shared the crayon with the first. They found that children were more likely to share the crayon if they had received candies earlier, but also that how often the second child shared was related to how generous the first child had been with the candies. There are problems with the method of this study, for instance the researcher told both children to draw a picture in the crayon condition, which may have influenced the child's decision to share the crayon with their peer, whereas no instructions from the experimenter were given regarding the candies. A significant relationship between candy sharing and crayon sharing was still uncovered, which suggests that children are able to use a reciprocity norm to share, however the children in this study were in fourth grade so it is unclear whether younger children would have used an evaluation of how fair their peer was to them when deciding whether to share the crayon. Olson and Spelke (2008) wanted to examine children's use of reciprocity when the child was not personally involved as a recipient to remove any self-interest that might bias their sharing. They asked three-and-a-half-year-olds to help a protagonist doll share resources with other dolls that had either previously shared with the protagonist or previously shared with a different doll. The children shared more with the dolls that had previously shared with the protagonist than the dolls who had previously shared with another doll. They argue that this shows that even very young children can take reciprocity into account when sharing. As was stated earlier, reciprocity is a more difficult norm for young children to use because they need to remember information over time that they then need to factor into their sharing decision. In order to create equivalent shares, a child would therefore need to remember how much the first person gave, and then reverse the sharing procedure to give the second person the same amount. This is considerably more social and quantitative information to take into account than sharing using an equality norm where objects are allocated to people from an undistributed pile of resources.

Equity is the most difficult norm to study amongst four and five-year-olds, as it is the most advanced in terms of reasoning skills. Equity is traditionally discussed in two ways: *proportional equity*, where equity is only achieved if the ratios of individual's outcomes to inputs are equivalent (Adams, 1965), or *ordinal equity*, where equity allocations are based on rank, with high performers receiving more resources than low performers (Elliot and Meeker, 1984). Proportional equity requires greater precision in the distribution of resources, and is therefore thought to develop at a later age than ordinal equity (Sin and Singh, 2005). Most of the research into the use of equity in young children has used a reward allocation design, where the child and a partner or two characters perform a task. The child is then told that one person or character has done more, less, or the same amount of work as the other and is asked to distribute rewards accordingly. Lane and Coon (1972), for example, had a child paste stickers to a worksheet as fast as they could during a certain time, and told the child that another researcher was testing their partner (a child of the same age and sex) in a separate room. When the child finished, the researcher showed the child their partner's worksheet which had considerably more, considerably less or the same amount of stickers on it. They found that children did not use equity to share the rewards. Most five-year-olds distributed the rewards equally between themselves and their partner, and most four-year-olds acted self-interestedly, regardless of whether their worksheet had more, less or the same amount of stickers as their partner's. In contrast, using the same paradigm as Lane and Coon (1972), Nelson and Dweck (1977) found that four-year-olds were able to use equity to divide resources if the researcher's instructions cued them to do so by saying 'share these candies so that you get the right amount for doing this much work, and they get the right amount for doing this much work' rather than 'give out the candies however you want to'. They also found that most children used equity to share resources when they were asked to divide the reward for two other children who had completed worksheets. Anderson and Butzin (1978) found that children as young as four could divide rewards using equity based on a story of how much two children helped their respective mothers wash dishes. However, they had a very small sample size so their results should be taken with caution. Finally, using a work paradigm similar to Lane and Coon (1972), Leventhal, Popp and Sawyer (1973) found that four and five-year-olds could use an equity norm to distribute rewards, however most children in their study gave only marginally more reward (about 60%) to the person who had performed more work, deviating only slightly from an equal distribution, so their results cannot completely confirm that children as young as four and five truly understand the concept of equity. But, children did demonstrate at least some ability to take into account factors like the amount of work each person had done when making their allocation decisions. Thus, there is some evidence that young children may be able to use equity to share fairly, but it is far from conclusive and further research is necessary. Equity again requires children to take into account more social and quantitative information to make a decision about how to distribute resources. They must remember how much each person has done and relate that to a quantitative amount to decide what is fair in terms of allocating portions. Equity has the additional difficulty of resulting in unequal but fair shares for each person. This norm, therefore, requires children to build proportional rather than equivalent sets and was included in this research to see how far four and five-year-old children could stretch their mathematical reasoning skills- how do they apply correspondence or counting strategies when the aim is to create unequal rather than equivalent sets and how to they evaluate the relation between sets?

It is clear from the literature that there is at least some evidence to suggest that four and five-year-old children can engage in sharing using equality, reciprocity and equity norms. These social norms differently impact not only the social demands of a task, but also the quantitative information children need to take into account and the quantitative strategies they might use to distribute resources. Sharing is an activity where children often have a clear goal of establishing equivalent sets between people, and is therefore an early social interaction that allows them to practice and learn many quantitative skills and concepts, particularly cardinality and number. If a child builds two sets of six objects and truly understands equivalence and cardinality, they should know that if they count one set of six they automatically know without counting that the second set also has six, regardless of any perceptual differences, such as the sets being built from different types of objects or the objects being more spread out. It is this relational knowledge and the coordination of different quantitative concepts that is extremely important for the growth of children's logical-mathematical reasoning skills. The next section will discuss a theory of the development of mathematical reasoning in more detail.

1.3 The Development of Mathematical Reasoning

A discussion of the development of mathematical reasoning must first begin with a brief examination of the difference between numbers, quantities, relations and operations. These are the basic units that children need to learn how to use in order to think about problems mathematically (Nunes and Bryant, 1996). An important distinction is the difference between numbers and quantities. Numbers can be used to represent quantities (for example Hannah has 4 apples) and relations between quantities (for example, Hannah has 4 more apples than Tom), but they also have an intrinsic value defined by their place in the conventional number system, for example 4 is one more than 3 and means 3+1, 2+2 and 1+3. While the words used to represent numbers and the systems for counting vary by culture, the logic behind counting systems, that the next number counted is one larger than the number before and that each number name is said one time only in the counting string, is consistent (Nunes and Bryant, 1996). Quantities, on the other hand, can be represented numerically but do not have to be, and one can reason about quantities or the relations between quantities without measuring them numerically. For example, Julie is taller than Alice and Alice is taller than Harriet. Without knowing the actual height of the three girls we know that Julie must be the tallest, then Alice and then Harriet. Quantities are transformed by operations (i.e. adding or taking things away will change the quantity) (Nunes and Bryant, 1996). Understanding the relations between these operations is also a crucial development for logical-mathematical reasoning (Piaget, 1952). In order to share fairly, children need to coordinate numbers and counting with knowledge of the relations between items. For example, children can learn how to use non-counting strategies such as correspondence to create sets. To use one-to-one correspondence properly, they must understand the relations between two or more items to decide how to construct equivalent shares, which is different knowledge than knowing number names for counting. They can also use counting and numbers to establish equivalence by counting out the same number of objects for each person. Sharing therefore requires keeping track of both the relations between the items in each set and quantifying the sets in order to know whether equivalence has been established (Frydman and Bryant, 1988). Connecting this knowledge about correspondence and counting is important for understanding equivalence and to developing an understanding of cardinality. Children must develop an understanding of both quantities and quantitative relations as well as learn how to use numbers in order to model mathematical problems (Nunes et al., 2007), and sharing is a good example of an early context where this learning happens.

Beyond developing an understanding of the basic units of mathematics, three ideas are also important for children's development of mathematical reasoning. First, children need to learn the conventional system of signs used to represent mathematical concepts. Once they are familiar with these culturally developed signs and symbols, they can use them to solve mathematical problems. However, they must also use logic to reason about numbers, quantities and the relations between them. Learning certain routines, for example how to count, is not sufficient. Children must learn to make inferences using information they know to solve problems they do not yet know the answer to. Related to this, the third important aspect of mathematical reasoning is learning when and how to coordinate different mathematical principles in appropriate and meaningful ways based on the problem to be solved (Nunes and Bryant, 1996; Klein and Starkey, 1988). The following paragraphs will discuss these three aspects of mathematical reasoning in greater detail.

The first thing important for mathematical reasoning is developing knowledge of the conventional systems used to represent mathematical concepts, for example learning the system for representing numbers. Children can then use these conventional tools for furthering their understanding of mathematical principles (Nunes and Bryant, 1996). While there are signs and symbols used to represent many mathematical concepts, learning how to count and use numbers is a very important part of this development for young children, particularly in the context of sharing where they can learn to use counting as a strategy for building equivalent sets. Gelman and Gallistel (1978) established several core counting principles that children must understand how to use in order to count correctly: the one-to-one correspondence principle (count each object only once), the order irrelevance principle (the order you count the objects in the set does not matter, it will not change the number in the set), the fixed order principle (number words must be produced in a specific order each time one counts), the abstraction principle (sets do not need to be made up of uniform objects), and the cardinal principle (the last number word counted tells you the total number of objects in a set). Learning individual skills is also not enough to be able to build mathematical models, and a second important component is learning to co-ordinate different skills to solve a problem. In the case of counting, while each of these principles must be respected in order to count correctly, it is the co-ordination of these principles that is essential for children to start understanding the meaning of number. The context of sharing provides children with the opportunity to learn how to use counting in different ways. At four and five children are quite good at using counting as a strategy to create equal sets, however, they do not often think to use counting in a situation where they are asked to compare sets (Cowan and Daniels, 1989; Cowan, Foster and Al-Zubaidi, 1993). To share fairly children need to understand the information that counting sets provides them (for instance that if one set has five, each equivalent set must also have five objects) in order to be able to use it as a tool not only for building sets, but also for comparing them. If they cannot first use counting to establish equivalent sets, they will not be able to further this knowledge to understand both that if two sets are equivalent they must have the same number of objects, and conversely that if they know that two sets have the same number of objects they must be equivalent. Counting knowledge is therefore an important part of sharing interactions and is an example of a conventional tool that children can learn to employ in the context of understanding more complex concepts like equivalence and cardinality.

Logic is also an important part of learning mathematics. The idea that logic is necessary in addition to knowledge about counting and using numbers was first championed by Piaget (1952), and has been built upon by more recent psychologists, for example Nunes and Bryant (1996). Working memory and intelligence play a role in mathematical development, but longitudinal studies have shown that logical-mathematical reasoning predicts children's mathematical achievement after controlling for working memory, arithmetic skills and general intelligence (Nunes et al., 2009; Nunes et al., 2012), and that children trained in logical reasoning outperformed those who were not trained when solving mathematical problems (Nunes et al., 2007). Piaget demonstrated that many children know how to count, but do not necessarily understand the quantitative information that counting provides them- that a set with a certain number has only that number of objects in it, and that this also means that any other equivalent set also has the same number of items. For example, Piaget (1952) performed an experiment with children in which he traded them pennies for sweets in one-to-one correspondence. Once several pennies and sweets had been exchanged, Piaget asked the child how many pennies they had (a set which they could see and count) and then how many sweets he had (which required them to understand that if there was one penny for every sweet then he must have had the same number). Some children found this inference quite difficult to make despite having no trouble stating how many pennies they had. Thus, just because a child is able to demonstrate strong counting abilities, this is not sufficient proof that they truly have a mathematical understanding of number. The use of logical reasoning about quantities and the relations between them is a critical component for learning mathematics, and is extremely important in the context of sharing. A child must learn that when they divide resources between people if they have given each person the same amount this means that they all have the same number of objects. Sharing allows children not only to create equivalence through counting and correspondence, but gives them the opportunity to work with quantities in equivalent sets which is helpful for building an understanding of what information counting provides with respect to equivalence and cardinality.

A third essential component is the ability to understand what mathematical tools or strategies are appropriate for solving a specific mathematical question. Sophian (1997) suggests that a child's conceptual knowledge consists of more than just the competency to complete a task. It also involves the ability to select which process is relevant for the specific problem. She also hypothesises a dynamic interaction between a child's cognitive competencies and performance outcomes such that both influence each other to provide new or more advanced knowledge for the child. These kinds of interactions develop over time as children continue to discover new concepts or ways of using concepts from their social interactions. Vergnaud (1977) expressed similar views, theorizing that children progressively learn about the meaning of concepts and ways of using them through their actions. They may not at first be conscious of all of the ways a concept can be used or of all of the contexts in which it is appropriate to use a concept- this develops over time. A concept a child knows how to use in one circumstance may not seem relevant in another circumstance where they could use it. Vergnaud (1999) argues that children develop their own representations of concepts related to how they view and interact with the world around them. In other words children develop different "action schemas": a sequence of organized behaviour specific to a problem (or in Vergnaud's terms a situation). These are not only related to mathematics learning, children can and do develop schemas in many aspects of their life, but the term captures an important aspect of the dynamic nature of children's learning of mathematics. "Action schemas" incorporate not just mathematical signs and symbols, but also gestures, actions and words which may be unique to the individual child (Vergnaud, 1999). The procedures for sharing involve different gestures, for example repeated cycles of giving out one item per person until resources are all distributed, or giving back to a person the same amount that they have previously given to you. A child does not need to know how to count or even to necessarily explicitly think in terms of quantitative strategies to be able to share; they learn the gestures, actions and words associated with sharing through social interaction. However, the words related to sharing- fair, equal, same, etc. have quantitative implications, as do the gestures. For example, when you express a desire to share equally you are expressing a desire to give each person the same amount. While the concepts of fair and equal may at first mean the same thing to a child, over time they come to understand that fair sharing and equal sharing are not always identical (Hook and Cook, 1979), and there may be other ways of sharing that can be considered fair (for example building shares proportionally). The action schema of sharing clearly provides children with many contextual factors that can challenge their thinking and enhance their quantitative development. It is the iterative process of learning and practicing concepts in contexts like sharing, encountering new problems to solve that require developing further skills or learning to use a previously acquired concept in a different way that allows children's mathematical reasoning to progress as they get older, even before they are given any formal instruction in mathematics.

It is clear from this discussion of mathematical reasoning that learning systems of conventional mathematical signs and symbols, applying logic and learning when to apply specific concepts or strategies to different problems are all important developments that begin at an early age, specifically in social contexts like sharing. This thesis narrows in on some specific quantitative skills developing at four and five years of age: counting, correspondence, equivalence, cardinality and number inferences. These were chosen as a focus because they are all mathematical concepts that children begin to develop through social interactions in their environment, and these concepts are at the core of a child's understanding of number and quantitative relations. Sharing is an ideal social context in which to study this quantitative development. Due to the importance of these concepts as a foundation for later mathematical understanding, an examination of how children develop and practice them in different contexts is essential for understanding how their development can be supported.

1.4 Definition of Key Terms

There are several concepts that are at the core of this research. Four terms from literature on the development of children's quantitative concepts are particularly important: correspondence, equivalence, cardinality and numerical inference. Defining quantitative concepts in the context of developmental psychology can be difficult because all children do not share a single understanding of mathematical concepts and a child's definition of a mathematical concept will change over time as their knowledge increases (Cooper, Meyenn, and Nason, 1982). Although not all researchers may define these terms in the same way, the definitions employed in this research have been derived from previous studies of children's quantitative development and are appropriate definitions of mathematical concepts in the context of developmental psychology. These definitions aim to reflect the fact that a child's understanding of mathematical concepts is not static, and they may not learn all of the aspects of a concept at one time. As they continue to get older and their thinking is challenged and extended they will develop more sophisticated understandings and learn new ways of using their quantitative understanding (Vergnaud, 1977).

Correspondence refers to the principle that each number name can only be applied to one item to be counted in a set (Gelman and Gallistel, 1978), and to the idea that when sets are in correspondence, for every item in one set there must be an equivalent item in the other set (Bryant, 1995). This can be extended to apply to multiple sets in one-to-one correspondence as well. Equivalence is a term referring to a specific relation between two or more things. This can be in terms of using fingers or other manipulatives to represent the amount of objects in one set, or in terms of the relations between two or more sets of objects. Items or sets can be considered equivalent if for each object in one set there is a corresponding object in the other set (Sarnecka and Gelman, 2004). The term numerical inference refers to the ability to infer that if two sets are equivalent and one knows the number of objects in one set, the other set will have the same number of objects and both sets therefore share the same cardinal. Conversely, if one knows that two sets share the same cardinal, they must be equivalent (Frydman and Bryant, 1988). Finally, cardinality is a term describing a relation between equivalent sets that share the same number. The cardinal number is the last number counted in a set, and represents the number of items in the whole set (Muldoon, Lewis and Francis, 2007) and cardinals are a tool for the comparison of multiple sets because "any set of a given number of objects will have the same quantity of objects in it as any other set with the same number" (Bryant, 1995, p. 6). This also means that any set with a different quantity cannot have the same number.

1.5 The Current Research

The current research aims to examine the development of children's understanding of quantitative concepts, specifically equivalence, counting, correspondence, and cardinality as well as how these concepts are related. To address this aim three studies were conducted using the context of sharing in order to present children with scenarios they are familiar with from their everyday lives where they use quantitative reasoning to solve mathematical problems. The first study investigates the impact of varying social and quantitative demands in a sharing task on children's ability to create equivalent sets and make number inferences about two equivalent sets. The second study investigates how removing visual cues affects children's sharing strategies and their ability to build equivalent sets and to make a number inference about one set after counting an equivalent set. The final study further examines children's understanding of cardinality by investigating their ability to coordinate ordinality and cardinality to identify and correct a miscount when the number of objects in one set is counted and also examines their ability to then make a numerical inference about a second equivalent set. Taken together these three studies provide a more detailed picture of important quantitative concepts that four and five-year-old children are developing through their everyday interactions. These studies will show that while many four and five-year-olds are quite good at using counting or correspondence strategies to construct equivalent sets using all single blocks, these same children often find it difficult to use unequal units (such as double and single blocks) to establish equivalent sets and have difficulty using counting or correspondence strategies to make a number inference about one set based only on knowing the amount of a second, equivalent set. These studies will also show that children can flexibly apply their quantitative knowledge depending on the context in which they are asked to share. Five-year-olds and at least some four-year-olds recognize that sharing fairly is not tied to a single procedure. The different sharing tasks given to children in these studies have varied social and quantitative demands. As a result, children will not necessarily use the same concepts to solve each problem and they may apply the same quantitative concept in different ways.

1.6 Thesis Outline

This thesis is divided into six chapters, including the present one. The next chapter will provide a review of the literature related to the quantitative development of four and five-year-olds. Following that are three chapters outlining the hypotheses, designs, procedures, and results of three studies examining different ways young children are able to establish equivalent sets and the extent of their knowledge about equivalence and cardinality based on their ability to make a number inference about two equivalent sets. The final chapter includes a general discussion of the findings of these three studies, including the empirical, theoretical and methodological contributions of this research, the strengths and limitations of this work and suggestions for future research.

Chapter 2- Literature Review:

This chapter will review literature on young children's mathematical abilities at age four and five. What has become clear from examining young children's quantitative knowledge is that well before they are taught mathematics in school, children possess early skills and conceptual knowledge, which develop into more formal mathematical concepts over time. This knowledge develops from their interactions with and exploration of the world around them. This chapter will provide an overview of early quantitative skills and concepts concentrating on studies that have specifically examined children's developing understanding of cardinality, correspondence, equivalence and counting. These concepts were chosen because they are frequently used in the context of sharing and are crucial building blocks for the development of more complex mathematical reasoning.

Chapter 3- Establishing Equivalence and Comparing Equivalent Sets Across Sharing Contexts

This chapter will outline the first of three sharing studies presented in this thesis. This study aims to investigate how children create equivalent sets and make numerical inferences when the social and quantitative demands of a sharing task are varied. No previous studies have examined the same quantitative concepts across different sharing contexts to examine whether children are able to flexibly apply their knowledge of correspondence, equivalence and cardinality. Children were asked to share either all single blocks or doubles and singles between two puppets without using counting in an equal sharing, reciprocity and equity condition. The use of double and single blocks allowed for a comparison of children's ability to use equal and unequal units to build equivalent sets. This study also aims to investigate children's understanding of equivalence and cardinality. Children were asked to make a number inference about one set after counting a second, equivalent set. Finally this study aims to investigate any developmental differences in four and five-year-olds performance on the sharing conditions.

Chapter 4- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting

This chapter will outline the second sharing study which is designed to examine how children share to create equivalent sets and make number inferences about equivalent sets when there is no longer a visual indication of the relation between double and single units. This study will use the same sharing paradigm as the first study (without the equity condition) and introduce Canadian $2 and $1 coins, which have the same relation as double and single blocks (a $2 coin is twice the value of a $1) but are identical in size and shape. The second aim of this study is to replicate the trends from the previous study's findings in a new sample of Canadian four and five-year-olds. The third aim of this study is to examine whether introducing coins instead of blocks increases the cognitive demands of sharing in the equal sharing and reciprocity conditions. The fourth aim of this study is to extend the findings of the previous study to compare children's ability to make a number inference after sharing all single blocks versus double and single blocks. This study will allow children to use counting and the final aim of this study is to examine how children use a counting strategy to build equivalent sets and make a numerical inference about one set after counting an equivalent set.

Chapter 5- The Relations Between Counting, Equivalence and Cardinality

This chapter will outline the final sharing study presented in this thesis. This study aims to examine children's understanding of the relations between counting principles, equivalence, and cardinality. This study aims to examine four and five-year-olds' ability to coordinate ordinality and cardinality to correct a counting error and establish the cardinal of a single set, as well as their ability to then make a numerical inference about a second, equivalent set. In the previous two studies, the majority of children answered the number inference questions correctly when two sets were equivalent. However, it is possible that they were able to do so simply by repeating the total of the set that was stated by the puppet. Trials where the puppet does not count correctly force children to reason about more than one number to determine the total of the first set and then infer the correct cardinal of the second set. Only equal sharing trials will be used in this study. Children will share all single, double and single, or triple and single blocks between two puppets. After they share, on half of the trials one puppet will count his set incorrectly by skipping one block and in half of the trials he will count his set correctly. Children will then be asked how many blocks the puppet thinks he has, how many blocks are in the set and to make a number inference about a second, equivalent set. The miscount trials allow for a more stringent test of children's understanding of counting principles as well as the concepts of equivalence and cardinality, as they cannot make a correct number inference simply by repeating the number that the puppet gives as the total for the set.

Chapter 6- General Discussion and Conclusions

In the final chapter the results of these three studies will be discussed, including a summary of the findings in relation to the main hypotheses of this research. These studies will show how the understanding of cardinality and equivalence, as well as the ability to flexibly use correspondence and counting to establish equivalent sets, are developing in four and five-year-olds. As well, the chapter will highlight the idea that sharing is a useful and meaningful context both for practicing quantitative concepts like counting and equivalence and for furthering quantitative knowledge by pushing children to think about relations between quantities rather than only dealing with discrete units. The theoretical contributions of this research will be outlined in relation to the theory of mathematical development put forward by researchers such as Vergnaud (1997) and Nunes and Bryant (1996). The methodological contribution of this research will be discussed in terms of the design and use of the three different sharing conditions used in this research to examine children's conceptual knowledge and quantitative reasoning skills. The strengths and limitations of the present research will also be discussed and suggestions for future research will be made. This chapter will show how the three studies included in this thesis provide an in-depth picture of several important mathematical concepts that young children learn and practice in sharing interactions long before they are taught any formal mathematics.

2 Literature Review

The main focus of this research is the development of the concept of cardinality. Cardinality means different things for mathematicians and developmental psychologists. In mathematics, quantitative concepts have a singular, formal definition such that those who understand some parts but not all parts of the definition would not be considered to have acquired that concept. However, a definition of cardinality in developmental psychology must capture the evolving nature of how children learn concepts: they will not understand all aspects of a concept at once and will learn over time how and when it is appropriate to use it (Vergnaud, 1999). Therefore an appropriate definition of cardinality for developmental psychology must reflect both the consistencies of the concept across time and the differences over time as children learn new things. Within developmental psychology different criteria have been used to define cardinality. Rochel Gelman and her colleagues speak of the cardinal principle, which states that a child understands cardinality if they know that the last number name counted represents the total number of items in that set. However, this is a weak criterion for understanding cardinality, as it deals solely with ordinal information about cardinals and does not capture the unique property of cardinal numbers: they can be added together (Vergnaud, 2009). A definition of cardinality must also include the idea that cardinal numbers represent a relation between quantities. Other researchers, such as Piaget (1952), have therefore argued that the development of relational knowledge about how to compare based on quantity is critical to a true understanding of cardinality (Fuson, 1988; Nunes and Bryant, 2012). A child may know that if the last word they say is six it means that set has six objects, but they may not be able to connect this knowledge to how it applies to multiple sets in correspondence, and therefore a child can only be said to fully understand cardinality if they understand that sets in correspondence also share the same number.

More recent studies have suggested that each of these definitions of cardinality taps different developmental steps and both are important stages in a child's understanding of number (Izard, Pica, Spelke and Dehaene, 2008; Sarnecka and Wright, 2013). Muldoon, Lewis and Freeman (2009) suggest that children need to learn to use both item-to-word and item-to-item correspondences. These terms are behavioural descriptions of what children need to learn conceptually: that when only one set is involved children are dealing with a quantity, whereas when two or more sets are involved they must establish a relation between quantities. Children must learn to monitor item-to-word correspondence in order to understand how to establish the quantity in one set correctly and to monitor item-to-item correspondence between sets to understand that equivalent sets must have the same cardinal and unequal sets must have different cardinals.

The difference between quantities and relations is crucial to a complete understanding of cardinality. Children need both a good understanding of how to build equivalence using counting and correspondence, and to extend this knowledge to develop an understanding of the relation between cardinals in multiple sets to develop a theory of number. This knowledge of the concepts of equivalence and cardinality is critically important for later learning in many aspects of mathematics including algebra and geometry, and the current chapter will look at what evidence exists of four and five-year-olds knowledge of these concepts and the related concepts of counting and correspondence.

The previous chapter discussed how four and five-year-old children develop mathematical reasoning concepts prior to being taught mathematics in school. It highlighted how children must develop knowledge of many quantitative concepts and symbols as well as develop their logical reasoning skills to determine how and when specific tools are appropriate for solving a mathematical problem. Young children gain a wealth of knowledge about quantities and the relations between quantities through the exploration of the world around them and as their cognitive skills develop. For example, if you tell a four or five-year-old child that Sally has 4 sweets then her mother gives her 3 more, and then ask how many sweets Sally has now, they can solve this problem without having received any formal mathematics training by counting on their fingers or using objects to signify the sweets (Nunes and Bryant, 1996). Solving this problem requires a child to understand that each set of sweets is one specific number, that only one number name can be attached to each finger or object, that each finger or object is equivalent to one pretend sweet and that the last number name counted represents the total of that set. This knowledge is not trivial, and it takes time for these concepts to become explicit to the child so that they can manipulate and apply them to fit different mathematical problems (Vergnaud, 2009).

The current thesis focuses on how children share discrete rather than continuous quantities. There is a large body of literature, beyond the scope of this thesis, which has developed around how young children learn to divide and share continuous quantities in the context of developing early fraction and division knowledge (for example Nunes, 2008, Kornilaki and Nunes, 2005, Miller, 1984, or Hunting and Sharpley, 1988). These studies show that young children find creating equivalent quantities from continuous quantities particularly difficult. Due to the young age of children in this study and the difficulty of establishing equivalence with continuous quantities, the current research chose to focus on discrete quantities because the ability to examine how children make number inferences about equivalent sets after sharing was of primary interest to the researcher.

The literature reviewed in this chapter is organized according to the research areas that guided the development of the current studies. The studies discussed in this literature review will outline how children learn to share or deal out discrete objects in one-to-one correspondence or using larger correspondences like two-to-one, how they use counting to build sets, how they learn to compare sets using correspondence or counting strategies and how they make number inferences about two equivalent sets. These studies will highlight developments in children's understanding of equivalence and cardinality between the ages of four and five years, and the methods previous studies have used to measure these concepts. Some previous research into young children's mathematical reasoning has focused on the limitations of their abilities, comparing young children's skills or understanding to that of older children (Briars and Siegler, 1984), but this literature review aims to focus on what evidence there is of children's understanding of number. Given that children will not know every aspect of a concept at one time and that their knowledge of a concept develops as they get older, this research will look at what aspects of cardinality four and five-year-olds do know as well as where there are gaps in their understanding. The first section of this chapter will review studies that have examined how children build sets using one-to-one correspondence, particularly in the context of sharing. It will also examine their ability to make number inferences about equivalent sets that have already been built or to make number inferences after sharing to build equivalent sets. The second section will highlight research examining how children build sets when asked to use other forms of correspondence (2-to-1, 3-to-1, 2-to-3, etc.) rather than one-to-one and how this affects their ability to make number inferences between sets. The third section will discuss literature related to the role of counting in establishing equivalence, how counting and correspondence strategies interact in sharing and how counting relates to children's understanding of number and making number inferences. This section will also discuss differences in the understanding of cardinality in the context of a single set and between equivalent sets. The final section will provide a summary of the research reviewed and will discuss these findings in relation to the aims and design of the current research.

2.1 Sharing using One-to-One Correspondence, Understanding Equivalence and Making Number Inferences

This first section will examine children's thinking about equivalence and fairness through their sharing actions. It will look at how four and five-year-olds use correspondence to build sets, and whether there is a difference between knowing how to establish equivalent sets and knowing how to compare equivalent sets. Finally, it will examine whether knowledge of a process for establishing equivalent sets in turn enables children to understand that sets that are equivalent must share the same cardinal, whereas sets that are not equivalent must have different cardinals by looking at children's ability to make number inferences.

2.1.1 Sharing Using One-to-one Correspondence

From a young age children begin to share and divide objects between people. Developmental research has shown that by 2 years of age children can share out objects equally on a one-to-one basis (Cooper, 1980), referred to in the literature as one-to-one correspondence, dealing or distributive counting, as long as the sets are small numbers (Miller, 1984; Davis and Hunting, 1990). This early use of correspondence to perceptually pair items helps children learn how to establish equivalent sets, and lays a basic foundation for learning the more complex schemas of action needed to solve multiplication and division problems (Nunes, Bryant, Evans and Bell, 2009).

Cowan and Biddle (1989) and Desforges and Desforges (1980) examined how flexibly children could use one-to-one correspondence in the context of sharing. Desforges and Desforges (1980) gave three to six-year-old children mints to share between 2, 3 or 5 dolls. There were eight different set sizes: 5, 6, 9, 10, 11, 15, 20 or 30, and all children were given all combinations of set sizes and divisors. Many children in each of the age groups used a one-to-one correspondence strategy to share out the mints. They found that the children who were five and a half or older had fewer problems with set size when sharing using one-to-one correspondence such that they made significantly less errors than younger children when dividing up to 30 objects between 2, 3 and 5 dolls. They also found that the three and a half to four and a half year-olds found sharing when there were remainders considerably more difficult than did older children. Desforges and Desforges (1980) identified two other types of strategies which were also used to make equivalent sets but were less common: dividing the mints into equal portions and then giving one portion to each doll, or sharing 2 or 3 mints at a time to each doll. Desforges and Desforges (1980) also found age trends such that younger children made more procedural errors when there were more than 3 recipients, when set sizes were larger and when there were remainders. Cowan and Biddle (1989) addressed the issue of procedural errors by having the researcher share on behalf of the child and used fewer trials to eliminate the possibility of fatigue impacting children's responses. Like Desforges and Desforges (1980), they also examined the effect of set size on children's ability to say that sets were equal, but in addition they included the effects of set visibility and homogenous versus heterogeneous objects. They found that most children were able to state correctly whether sets were equal or unequal, however, they performed better when sets were small and visible and when heterogeneous objects were used. The findings of both Desforges and Desforges (1980) and Cowan and Biddle (1989) provide clear evidence that even very young children can understand and frequently use one-to-one correspondence to achieve equal sharing, but their skills are limited by set size, object visibility and remainders.

Several other researchers have also investigated children's understanding of correspondence and equivalence through examining the strategies they use to share fairly. Davis and Pepper (1992), for example, classified four and five-year-old children into three groups: poor, developing or good counters based on their answers to questions that required them to count in different ways to establish the total number of items in sets of animals on a farm. Children were then given the task of dividing 12 crackers between two dolls. Once they completed the first distribution, a third doll arrived who also wanted a fair share. Children had to re-distribute the crackers amongst the three dolls. They found that 61% of children (including all of the good counters) were able to solve the problem, most of them using a dealing or one-to-one correspondence strategy. Davis and Pepper (1992) suggest these results indicate that while counting ability appeared helpful to children insofar as the only group where all of the children solved the problem was the good counters, a large number of poor or developing counters were also efficient at re-distributing the crackers. Some children used counting to check the portions they allocated, but very few children used counting as an initial strategy to allocate the crackers. This suggests that even before children develop strong counting skills they are able to use one-to-one correspondence to build equivalent sets and see correspondence as an effective strategy for establishing equivalence. Frydman and Bryant (1988) also found that the majority of four and five-year-olds in their sample were able to distribute 12 or 24 items between 2, 3 or 4 dolls using one-to-one correspondence. These studies show that young children are quite capable of using one-to-one correspondence to build equivalent sets (through the action of dealing objects out one per person) and are generally very accurate in their allocations before their counting skills are strongly developed and even when they are asked to redistribute resources to an additional recipient (Davis, 1990; Davis and Pitkethly, 1990, Pitkethly and Hunting, 1996). This suggests that a grasp of correspondence as an appropriate strategy for establishing equivalent quantities begins to develop quite early on.

2.1.2 Comparing Equivalent Sets in One-to-one Correspondence

While by the age of four children have developed knowledge of how to use correspondence to build equivalent sets, they must also coordinate this knowledge of equivalence with number knowledge to develop an understanding of cardinality. To grasp the concept of cardinality as it has been defined and used in this research, children need both an understanding of how to create equivalent sets and an understanding of the relation between two sets of the same number.

A key concept for understanding cardinality is equivalence. Several different types of tasks have been designed to study children's understanding of equivalence by asking children to make a comparison based on a number of items in two rows. These tasks include number conservation, relative number judgments and static row tasks. The current research examines children's understanding of cardinality through the use of number inferences, however number conservation and static row comparisons reveal important findings about four and five-year-olds understanding of equivalence, the types of information that children take into account and the types of strategies they use when comparing sets. Therefore a brief overview of these findings will be included before a discussion of number inference tasks.

2.1.3 Number Conservation and Static Row Tasks

Piaget (1952) suggested that young children do not fully understand cardinality: that if two sets are equal they will always share the same number name regardless of whether they are perceptually identical. He based this idea on findings from his conservation of number tasks. Piaget (1952) presented a child with two perceptually identical sets, then spread out or condensed the objects in one set without altering the number of objects and asked children whether the two sets were still the same. Most young children judged the longer row as being a larger number than the smaller row, despite agreeing that both were equal when they were perceptually identical. This suggests that children did not base their equivalence judgments on number and were heavily influenced by perceptual differences between sets. Gelman (1982) conducted a training study to see whether children's poor performance on conservation tasks could be improved. She first gave three and four-year-old children practice trials with small sets of 3 and 4 blocks. Children had to count equal and unequal sets before and after one set was transformed in order to explicitly show children that sets in one-to-one correspondence have the same cardinal and sets that are unequal have different cardinals. On the experimental conservation tasks with larger set sizes the majority of three and four-year-olds were able to both give a correct judgment and explain their reasoning for the judgment on at least one trial with small sets and one trial with large sets. Gelman (1982) suggested that this shows that children do have some knowledge of correspondence but may have difficulty accessing this knowledge when asked to compare sets. If children were forced to focus on number to make the set comparison they could do so, but they did not choose to do so spontaneously.

Bryant (1972) theorized that young children have some understanding of equivalence, because in situations where small numbers of objects are placed in one-to-one correspondence even very young children recognize that they are equal. However, young children also tend to base judgments of equivalence on other cues that are not always correct indicators of number, such as length. He suggests that children have not yet developed a system for determining which cues are relevant to deciding whether two sets are equivalent. To test whether children were able to learn to use knowledge of equivalence in situations where they previously made mistakes, Bryant (1972) designed three different types of displays with unequal counters. Display A showed sets in one-to-one correspondence with a gap where the more numerous set did not have a matching counter in the smaller set. Display B showed sets where the less numerous set was spread out to be longer than the more numerous set. Display C showed sets where some of the counters in each set were spaced close together and some were spread further apart. At baseline children performed well above chance with A, well below chance with B and at chance level with C. Experimental trials showed children display A and then transformed the display into either display B or C. Children performed above chance on display C after the experimental trials, which Bryant (1972) concluded shows they were able to transfer their understanding of equivalent sets from display A and use this knowledge when comparing sets in display C. Their answers were relatively unchanged with display B. They still relied heavily on a length cue to decide the numerosity of the two sets. However, Bryant's (1972) method has been criticized by some researchers who suggest that because the dots are not arranged in a way that allows children to see the one-to-one correspondence between pairs, children may make judgments about which set is larger in the displays based on relative density rather than correspondence (Klahr and Wallace, 1973; Cowan, 1984).

Brainerd (1973) developed a similar task for examining children's knowledge of correspondence and equivalence, asking 5 and 6-year-old children to compare two rows of dots (without counting) that were printed on cards. One row was red and one blue, and the rows were sometimes the same length but different numbers, same number but different lengths or same length and same number. The child had to determine whether both rows had the same number of dots or one row had more. He found that many of the five-year-olds answered based on the length of the lines rather than the number of dots, whereas many of the six-year-olds were able to answer based on correspondence.

The consistent finding that children rely heavily on perceptual cues when making judgments of relative number led several researchers to examine whether children would make more accurate judgments if guidelines were drawn to highlight the correspondence between objects in each set. Avesar and Dickerson (1987) conducted a series of studies to try and separate children's judgments of equivalence based on spatial cues and judgments based on one-to-one correspondence. They discussed children's ability to compare sets in the context of whether or not they knew a "plan" for using one-to-one correspondence, in other words whether they held in mind a procedure or series of steps for making the comparison with correspondence. They suggested that children's errors on one-to-one comparison tasks could be the result of three different things: they do not yet have a plan for using one-to-one correspondence for comparisons, they have a plan but do not retrieve it, or they have a plan and retrieve it but cannot properly execute it. In their first study, children were asked to compare one set of squares and one set of circles arranged in six different ways: equal length and density, unequal length, unequal density, conflict between length and density (one row is longer and has more objects but the other row is more dense, or one row is denser and has more but the other row is longer), or conflict equal (one row is longer, one is denser but both are equal). Four and five-year-old children were tested with and without perceptual support (a grid denoting pairs of items) in order to see the effect this had on whether they selected a correspondence strategy or not. Consistent with many other studies, they found that the majority of children made judgments about the sets using a "length-density" plan regardless of age or problem type. In a second study, children were asked to create pairs of objects on a grid to see whether all of the objects could be paired or if some were left over in order to determine whether children did not have one-to-one correspondence plans or were simply not retrieving them in the previous study. Most children answered consistently using one-to-one correspondence, even if they previously used length-density or other methods of comparison in the first study. In a final study, children were given the same comparison problems as study one but with guidelines drawn between pairs rather than the grid used for perceptual support in study one to see whether this kind of perceptual support would trigger the use of a one-to-one correspondence plan over other strategies. In this study children again responded consistently with a length-density plan when no guidelines were drawn, but 92% of children used a one-to-one correspondence plan when guidelines were provided. They suggest this shows that four and five-year-olds do know how to use one-to-one correspondence for comparing equivalence in sets, but fail to see it as an appropriate strategy unless salient perceptual cues highlight the pairs of items in the set. While there are studies where guidelines have not improved children's performance on number conservation or static row tasks (Miller and West, 1976), the majority of research in this area suggests that guidelines cue children to the correspondence between pairs of items in two sets and help them see how to use one-to-one correspondence to determine equivalence (Whiteman and Peisach, 1970; Cowan, 1984). The usefulness of guidelines may increase with age, however, as Cowan (1987) found that five-year-old children continued to rely heavily on a length cue, even after drawing lines between objects in one-to-one correspondence between two sets, but seven-year-old children performed much better, making far fewer errors when comparing the size of the two sets with the help of the guidelines.

Other studies have also found that young children are generally unwilling to infer the cardinal of a set based on number name, and will deny that sets are equivalent if they are not perceptually identical, even when they have a strong grasp of counting and if they have previously stated that sets have the same number (Cowan and Daniels, 1989; Piaget, 1965). These findings suggest that children do not yet consistently rely on correspondence as a better strategy for judging whether sets are equivalent than perceptual indicators such as length, despite frequently using correspondence to build equivalent sets. Young children are capable of using one-to-one correspondence for comparing the relative number of two sets, but tend not to use it as a comparison strategy if other methods of comparison, such as length or density, are more salient.

When perceptual cues indicate the relation between pairs of items in the sets, they are considerably more likely to use a one-to-one strategy for determining equivalence.

Some researchers have suggested that children may perform poorly on static row correspondence tasks like Bryant's (1972) and Brainerd's (1973) because when rows are shown in a static presentation where they are already different lengths or different amounts, children do not have the opportunity to see the sets in correspondence before the manipulation as they do in a conservation task. This may make it more difficult for children to mentally or spatially pair items together to establish the cardinal of each set (Russac, 1978).

Mix (1999) examined children's ability to make equivalence judgments based on static rows or sequentially presented sets. In each trial three and four-year-old children were presented with a target set and were then asked to choose an equivalent set from a selection of two other sets. Sets ranged from 1-5 dots and the set that was not equivalent always varied from the target set by 1 dot (either 1 more or 1 less). In the static row condition, children were presented with the target array and the two choices for equivalent set all at once and had to point to which set they thought was the same amount as the target. In the sequential set condition, the target set was built one item at a time by sliding each item under a cover, and then children were presented with the two other sets to decide which was equivalent to the target. Mix (1999) found that older children performed better than younger children, but in general children had significantly better scores on the static row condition than the sequential row condition. This suggests that the way equivalent sets are produced or presented does influence children's ability to compare them. Mix (1999) also conducted a second experiment where children were asked to produce an equivalent set to the target rather than pick one that matched from pre-built sets. In this study children either watched the experimenter move a puppet a certain number of times (for example, jump up and down twice) and had to emulate the movements with their own puppet, or watched the experimenter place a certain number of objects under a cover and then had to put the same number of objects on a mat in front of them. In this way because the target set was not continuously visible, children had to match the experimenter only on number and not any other feature of a set (length, density, etc.). This study found that in general children's performance on both tasks improved with age, and they performed better when they were asked to match objects than when they were asked to match the puppet's movements. However, it may be that the youngest children in her sample had difficulty being able to retain information about the target set in the sequential condition when they had no visual reminders about set size, which would have caused poor performance, but not because they did not know how to solve the task. This is supported by her finding that the errors children made in both types tasks were random rather than close approximations of the target number. It seems likely that if children remembered the target number and were simply unable to count the right number for the target set, their answers would cluster close to the target number, whereas a finding of random responses suggests there may have been problems understanding the task or remembering target numbers. Despite this potential methodological issue, both of Mix's (1999) studies raise the point that children may think about equivalent sets differently depending on the way they are presented.

These studies clearly demonstrate that while most four and five-year-old children are able to create equivalent sets through the use of one-to-one correspondence or counting, this is not sufficient to demonstrate an understanding of what equivalence means. Many four and five-year-olds are distracted by perceptual dissimilarities in sets and make mistakes on set comparisons when length or spatial features are misleading. They have not yet learned that sets can be equivalent and not perceptually identical or to rely on correspondence or counting as tools for comparing sets. Children learn to use counting and correspondence as strategies to build equivalent sets before they learn to use the same strategies reliably for comparing sets (Nunes and Bryant, 1996; Sophian 1988[B]). This suggests that an understanding of the relation between two or more equivalent sets based on number is not as well developed in most four and five-year-olds, hence the large variation in understanding and performance on the set comparison tasks.

It is possible that when children are given the chance to construct equivalent sets themselves they can better see how to apply knowledge of strategies for building equivalent sets for comparison purposes. Building sets themselves mimics the social contexts from their lives where they learn about and use correspondence, for example in sharing scenarios and may also help limit potential distractions from misleading length or spatial cues that may cause a child to give a wrong answer because building the sets themselves forces them to focus on achieving matched pairs of items.

2.1.4 Inferring Cardinals- Number Inference Tasks

The studies outline above examined children's understanding of equivalence or nonequivalence through the use of rows presented side-by-side but manipulated in some way so children could base a decision on number or another salient cue such as length. These studies suggest that children rely heavily on perceptual information to make decisions about equivalence, despite knowing how to count or use correspondence strategies. Another group of studies has looked at children's understanding of number and cardinality by asking them to make an inference about one set based on receiving information about the numerosity of a second, equivalent set (number inference tasks). These studies examine whether children are able to state the number of items in a set that they cannot see or count when they have been given information about the number and equivalence of another visible set.

Working with conceptually paired items is a helpful and important way for young children to begin to understand numerical equivalence (Muldoon, Lewis and Freeman, 2009) because it makes it easier to see the correspondence between items in each set. However, some of the studies that have looked at children's ability to make numerical inferences have used very small set sizes (3-5 items) and have kept sets visible (Sophian, Wood and Vong, 1995; Gelman, 1972; Muldoon, Lewis and Towse, 2005; Mix, Huttenlocher and Levine, 1996). While it is possible to prevent children from counting out loud, they may still be able to use perceptual strategies to assist their comparisons. This makes it difficult to support the claim that children are truly making number inferences based on conceptual knowledge of the relation between equivalent sets in these studies. A great deal of research shows how well three and four-year-olds are able to use subitizing to make comparisons with small numbers (less than 5) (Klein and Starkey, 1988; Starkey, 1992), therefore it is possible that children were able to see that the two sets were equivalent just by looking at them without actually understanding why they were equal or how to employ appropriate comparison strategies.

Studies that have used larger set sizes provide evidence that five-year-olds and some four-year-olds can make numerical inferences. Sophian (1988[A]) examined three and four-year-olds' knowledge of one-to-one correspondence and cardinality. Children were given two different tasks using sets of conceptually related items (for example balloons and clowns). The first task gave children information about the number of items in each set and asked them about the correspondence between items, and the second task asked children to make a number inference about one set after they were told the number of the first set and that it was equivalent to the second set. The two sets in these tasks were never visible at the same time removing the possibility of children making judgments based on length or perceptual cues. There were four trials with equal sets and four trials with unequal sets, using set sizes 3, 4, 6 and 7. Similar to other studies, Sophian (1988[A]) found that children's performance improved with age and was more accurate with smaller than larger set sizes. She also found that children performed better with equal rather than unequal sets. A minority of three-year-olds and majority of four-year-olds showed reliably correct response patterns for both types of task. While this study demonstrates that even very young children have some knowledge of one-to-one correspondence which they can use to compare sets, there are problems with the method used in this study that Sophian (1988[A]) herself raises. The largest difficulty is that in order to avoid the possibility of children making comparisons based on length or perceptual matching, one set was piled together in a cup while the other was presented in a row. Children therefore could not see or count how many items were in the set in the cup and had to take the amount the researcher told them was in the cup and her description of the relationship between the sets (either each object does have a corresponding one or one object does not have a corresponding one) as true. This allowed for two possibilities: some children answered incorrectly because they could not retain or did not understand all of the verbal information given, alternatively some may have been able to answer questions about the equality and numerosity of the sets correctly based on verbal cues from the researcher rather than an implicit understanding of correspondence.

A second study was conducted to address this issue. Here the researcher used positive and negative terms to describe both equal and unequal sets so that there was no longer a straightforward relationship between description terms and type of set. Sets were colour coded to see whether children would base their judgments about the sets more on colour or number information. Finally, sets were presented in pairs while they were being described to minimize the amount of verbal information the child had to understand and retain. Not all of the elements of the sets remained visible once children were asked to make a decision about equality to make sure they were not basing their decision on direct perceptual information. Children were again presented with sets of conceptually paired items and asked questions about the correspondence between the sets or to make a number inference. Sophian (1988[A]) found that as in the first study many three and four-year-olds performed well on both types of tasks. There were no age differences in this study, suggesting that the performance differences between the three and four-year-olds in the first study were due to difficulty with the heavy verbal information about the sets by the youngest children which was eliminated by the use of supporting visual information in this study. As in the first study children also performed better on tasks where the sets were equal rather than unequal. This may mean young children understand one-to-one correspondence when sets are equal, but are still learning what it means for sets to be unequal. Sophian (1988[A]) suggests this makes sense because there are many different ways for sets to be unequal and the specific type of inequality has implications for what information can be obtained about the relation between sets.

While Sophian's (1988[A]) studies demonstrate that three and four-year-olds understand how to compare sets in one-to-one correspondence, at least in the case of conceptually paired items and smaller set sizes, these studies still constructed sets for the child. Presenting the sets together in the second study also raises the possibility that children performed better on the equal sets simply because they knew from a brief visual comparison that they looked the same whereas the unequal sets looked different.

The question of whether children are able to make number inferences about equivalent sets is more clearly addressed by other research that has asked children to create two sets themselves using one-to-one correspondence through sharing on a one-for-A, one-for-B basis. These studies have generally found that by five-years-old children are sensitive to the relation between sets in correspondence and perform quite well on number inference tasks. For example, Frydman and Bryant (1988) asked four and five-year-olds to share different numbers of single blocks evenly between 2, 3 or 5 dolls. They found that virtually all of the children could establish equivalent sets between the dolls. Children then completed two further sharing trials using only 2 dolls. Once they finished sharing, they were asked to count how many blocks one doll had and then to tell the researcher without counting how many blocks the second doll had. They found that almost all of the children could share the blocks equally, and that the majority of their five-year-olds could make the numerical inference. A large portion (42%) of the four-year-olds were also able to make the inference that the two sets shared the same number.

Squire and Bryant (2002) used a procedure similar to that of Frydman and Bryant (1988) for the screening measure of their study. Five to eight-year-old children were asked to divide 8 chocolates evenly between 4 dolls. After the children finished sharing, the sets for the second, third and fourth dolls were removed and the child was asked to count how many chocolates the first doll had. The child was then asked to infer how many chocolates the other three dolls had received based on counting the first doll's set. Of the 92 five to eight-year-olds recruited for the study, 87 passed this screening trial. Although the other dolls' sets were removed from the child's view before they were asked to infer the number each doll received, the set size was very small (2 chocolates per doll), which means the possibility that at least some children used subitizing to help provide the answer cannot be eliminated. However, this performance is still impressive as the majority of children were able to infer the set of not only a second doll, but a third and fourth as well. There is also evidence that in the absence of conflicting perceptual cues, by five-years-old children will make a numerical inference based on equivalent sets (Muldoon, Lewis and Berridge, 2007).

These studies suggest that the connection between knowledge about correspondence, equivalence and cardinality is still developing in three to five-year-olds, but that even young children are capable of demonstrating the knowledge that two equivalent sets share the same cardinal in some situations. However, they have to learn to effectively use a reliable strategy for comparing sets, namely counting or correspondence, and overcome what may be more salient information such as length or density. How sets are presented and whether or not the child participates in creating sets are both important factors for how they will think about comparing equivalent sets.

2.1.5 Summary

The studies above have examined four and five-year-old children's developing understanding of number through their use of the concept of correspondence to build and compare equivalent sets. Studies which have looked at the use of one-to-one correspondence to build sets show that even from a very young age children can use one-to-one correspondence to distribute resources equally between recipients, however factors such as set size and object visibility affect their accuracy. The ability to build equivalent sets using this strategy is robust despite variations in the strength of their counting abilities and number of participants. While their understanding of how to use correspondence to build sets is quite well developed, the use of correspondence as a strategy for comparing sets is significantly less consistent. Studies that have used conservation of number or static row comparison tasks show that four and five-year-olds rely heavily on perceptual indicators of equivalence such as length or density, and rarely spontaneously use other strategies for comparing sets. However, studies that have asked children to make a number inference about equivalent sets, asking them to count one set and then infer the cardinal of a second, unseen set, have found relatively positive results. Many four and five-year-olds were able to make inferences after building equivalent sets. This suggests that when children focus their attention on pairing items in the sets instead of perceptual features of the sets, they are able to use appropriate strategies for determining the equivalence of the sets, knowledge that is crucial for understanding relations between quantities and for understanding the concept of cardinality.

2.2 Sharing Unequal Portions and Making Number Inferences

The previous section outlined children's ability to build and compare sets using one-to-one correspondence. This research showed that by four and five children are very familiar with the use of a one-for-you, one-for-me strategy in sharing and will often attempt to use this strategy to create equivalent sets. Research has also shown that while comparing sets based on one-to-one correspondence is more difficult than simply learning the procedure for building equivalent sets, many five-year-olds and some four-year-olds are able to do so. While this knowledge is a critical first step, children must also learn how to extend their knowledge of correspondence and equivalence to build and compare sets that are not constructed from equal amounts but aim to result in equal portions. This section will review research into four and five-year-olds' understanding of correspondences larger than one-to-one.

2.2.1 Constructing Equivalent Sets using One-for-N Correspondences

While most four and five-year-old children can use one-to-one correspondence quite well, using correspondences between one and a larger number (2-to-1 or 3-to-1) is more difficult. An early study by Piaget (1952) demonstrates the increase in quantitative understanding that children need to be able to use one-to-many correspondences correctly. He presented children with a row of vases and asked them to put one blue flower in each vase. He then took out the blue flowers and asked them to put one pink flower in each vase. The pink flowers were then removed so that only the original vases remained on the table. Children were then shown a box of plastic tubes to put on the stems of each flower. The child's task was to place the correct number of plastic tubes in front of the vases so that there would be one for each flower (i.e. twice as many tubes as vases since there were 2 flowers in each vase). While statistical results were not reported, Piaget (1952) classified children's answers into three stages: those who made a one-to-one correspondence between tubes and vases and could not understand how to fix the mistake, those who made a one-to-one correspondence between tubes and vases but realized the problem and corrected their mistake, and those who understood the two-to-one correspondence and were able to generalize this knowledge to other relationships, i.e. three-to-one. There were clear developments in children's reasoning and mathematical understanding across the three stages.

Frydman and Bryant (1988) also examined the use of two-to-one correspondence and three-to-one correspondences in the context of sharing. Four and five-year-old children were asked to share blocks evenly between two dolls, only some blocks were stuck together as doubles (or triples) and some were left as singles (creating a two-to-one (or three-to-one) correspondence). The five-year-olds were able to adjust their sharing procedure to deal out the doubles and singles equally, but the four-year-olds continued to share using one-to-one correspondence, giving one double and one single to each doll without adjusting for the fact that one double is equivalent to two singles. This finding led to a further study exploring whether training would help the four-year-olds understand how to solve the problem. This study repeated the procedure from the previous study, and then split children into a control and experimental group. In the experimental group, the double and single blocks were coloured so that one double was made up of one blue and one yellow block, while some of the singles were blue and others were yellow. In this way the children had a visual colour cue that one double was the same as two singles. The children in the control group did not improve, but the children in the experimental group were able to solve the sharing questions when the blocks had colours and at post-test when the colour cues were removed. These results support the conclusion that certainly five-year-olds can solve tasks that go beyond using one-to-one correspondence, as long as the ratios are small and the larger number is a multiple of the smaller number (i.e. 1:2 or 1:3). There is also evidence that four-year-olds can demonstrate the ability to use larger correspondences to share, at least after they have had practice with the concept and if they are given training about the relationship between the larger and smaller numbers.

2.2.2 Constructing Equivalent Sets using M-for-N Correspondences

Frydman and Bryant (1994) also extended the findings of their 1988 study to examine how well four and five-year-olds are able to share using correspondences between numbers that are not multiples of each other (for example 2-to-3). In this instance the child would have to use a common multiple to create equivalent sets, which is even more difficult than the "one-for-n" correspondence tasks discussed above where the child can match the smaller units to the larger unit. In an earlier study by Piaget, Kaufmann and Bourquin (1978) children were asked to build two equal sets of blue and yellow counters by giving 2 blue and 3 yellow counters each turn. In this study children performed very poorly until the age of seven or eight, sharing using one-to-one correspondence rather than a common multiple. However, Frydman and Bryant (1994) point out that children had to remember the number of each type of counter to give each turn as well as keep track of the overall number of counters given, which may have simply been too much information for younger children to pay attention to, rather than a reflection of their actual ability to share using correspondence between numbers that are not multiples of each other. In their first study, Frydman and Bryant (1994) gave four to six-year-old children two different types of correspondence tasks: "one-for-n" tasks similar to their 1988 study except using numbers larger than one (using 2s and 4s or 2s and 6s), and what they termed "m-for-n" tasks where the smaller number was not a multiple of the larger number (using 2s and 3s and 3s and 4s). Blocks were stuck together into different sized units and the two unit types in each trial were different colours. The experimenter asked the child to count how many blocks were in each size and then children were asked to share the different units between two stuffed animals so that they both ended up with the same number. They found that most children performed very well on the “one-for-n” tasks, suggesting that by five years-of-age most children are capable of adjusting the one-to-one correspondence rule to share unequal quantities if one number is a multiple of the other and they can match the smaller unit to the larger unit. Most children who gave a correct response to these tasks were also able to answer a question about how many times they had to take the smaller units to build the equivalent sets. Conversely, the majority of children did not perform well on the “m-for-n” tasks primarily because children could not match the smaller units to the larger units. Children who were successful on this task either used a correspondence strategy, putting large and small units together until they matched, or counted the blocks and matched the number. These children were able to use counting or correspondence as strategies for constructing equivalent sets. There was an increase in the use of a counting strategy to solve the problem from five to six-year-olds and for the most difficult task (3s and 4s). This suggests that children learn increasingly well how to use counting to build equivalent sets as they get older, and may see it as a more appropriate strategy than correspondence when faced with a difficult problem. The poor performance of children on the “m-for-n” tasks led Frydman and Bryant (1994) to wonder whether children would perform better if they were also asked to share a third set of blocks to a third person that consisted of units of the common multiple between the two smaller units (for example the 2s and 3s task would also include a set of 6s). What they found was that children performed well when the common multiplier was readily available to them, but only if they were allowed to put the smaller units together. Their results suggest that four to six-year-old children have a good working knowledge of how to share unequal portions to build equal sets, but find it much more difficult when the portions to be shared are not multiples of each other. Counting strategies and a reliance on correspondence to match the smaller units to the common multiple allowed children in this study to have success on the “m-for-n” tasks.

2.2.3 Comparing Equivalent Sets built from Unequal Portions

While all of these studies asked children to build sets using correspondences larger than one-to-one, none of them went further to examine children’s ability to make a number inference or compare sets after they were built. The same logical rules for making comparisons apply to these sets as to sets in one-to-one correspondence, so in theory at least some children should be able to make the inferences if they are able to construct the equivalent sets. If a child has learned to understand that sets in one-to-one correspondence must share the same cardinal and therefore if you know the number in one set, you know the number in the second set, they may be able to extend this knowledge to apply it to two equivalent sets constructed from different sized units, as long as they have a reliable strategy for determining the cardinal of each set. This issue therefore needs further investigation and the current research aims to address this gap in the literature.

2.2.4 Summary

The studies reviewed above show that four and five-year-old children are capable of flexibly applying their knowledge of correspondence to accommodate sets made up of unequal units. Frydman and Bryant (1988) showed that five-year-olds can easily adjust a one-to-one correspondence routine to share double or triples and singles into equal sets, and four-year-olds can also do so once their attention has been drawn to the relation between double and single units. Frydman and Bryant (1994) also showed that in general children do not perform as well when they are not able to use a matching strategy to make the smaller units the same size as the larger units, however some children were capable of using counting or correspondence strategies to build equal sets when the common multiple of the two numbers was available to them. This shows a great deal of flexibility in their quantitative reasoning- they can see how one sharing strategy (one-to-one correspondence) can be altered and used in different ways when the size of individual units in each set changes. Based on these findings it is clear that children are capable of understanding and using more complex forms of correspondence. Combined with the results from research into making number inferences based on sets in one-to-one correspondence from the previous section, which suggests that many five-year-old children can accurately infer that two equivalent sets share the same cardinal, it seems possible that children would also be able to make number inferences based on sets with units larger than one as long as children are capable of remembering the relation between the larger and smaller units and can use counting or correspondence strategies effectively. However, because sets using correspondences larger than one-to-one are built from units that are not perceptually identical, it is also possible that children's tendency to base equivalence judgments on visual cues would hinder their ability to make a number inference. One of the aims of the current research will be to examine four and five-year-olds' ability to make number inferences based on sets in two-to-one correspondence to extend the findings on young children's number inferences and set comparisons.

2.3 The Role of Counting in Sharing and Understanding Equivalence

The previous two sections have focused on four and five-year-old children's developing understanding of how to construct and compare equivalent sets using correspondence. While knowledge of how to pair or match items is important for understanding how to establish equivalent quantities and for understanding the relation between equivalent quantities, another crucial piece of quantitative knowledge that children must develop to understand cardinality is counting. This section will examine how children use counting as a strategy for building equivalent sets and for comparing equivalent sets.

2.3.1 Counting to Construct a Set of a Given Number

Counting and learning to use numbers is crucial to young children's mathematical reasoning (Frydman, 1995). Children must learn how to use the signs and symbols that represent numbers in order to solve mathematical tasks (Nunes and Bryant, 1996). By the time four and five-year-old children enter preschool they are able to produce a counting sequence up to at least twelve or so without making a mistake (Resnick, 1984; Gelman and Gallistel, 1978). They also have a fairly good grasp of the rules for counting correctly including that each number name can be said only once in a counting string and that each number name must correspond to one and only one object being counted (Rodriguez et al., 2012). It is clear that young children grasp how to count. However, children must also learn how to relate this procedure to establishing and comparing quantities.

By the age of four most children are exhibiting success on the Give-N task. This task asks children to give a specified number of items to a doll (for example: give three sweets). Even though children younger than four may be able to recite number names to ten or higher, they may not fully grasp what these number names mean or understand the information that counting provides them. Research suggests that children first go through the process of attributing meaning to specific number names, linking them to mathematical concepts like cardinality before developing an overarching theory about numbers generally (Carey, 2009). Young children learn slowly that you should give only one object when asked to give one, two objects for two and three objects for three. Once they reach an understanding of number three, their performance advances rapidly so that when they reach the stage where they can correctly give five when asked for five objects they are able to give the right amount not just for five but also for larger numbers (Wynn, 1992; Sarnecka and Carey, 2008). Sarnecka and Carey (2008) examined 73 two to four-year-old children's counting knowledge using a Give-N task. Children were asked to give a dinosaur a specific number of objects starting with one and moving up the counting sequence by one if the child was able to give the correct number of objects. They classified 49% of their sample as cardinal principle knowers, meaning children were able to give the right amount for all numbers asked (one-six). Cardinal principle knowers were also significantly more likely to use counting correctly to build a set than other children. This result is evidence of an increased understanding of number and how to use counting. Once children are able to perform well on numbers up to five or six on the Give-N task they are also able to understand that adding an item to a set means the set is now the next number name in the sequence (Sarnecka and Carey, 2008) and more generally that if you add objects to a set or take objects away, that set no longer has the same number (Sophian, 1987; Stojanov, 2011). Similarly, children that have reached this level understand that if two sets are equivalent and you add items or take items away from one set, those sets are no longer the same (Bryant, Christie and Rendu, 1999). These studies show an increase in the understanding of counting and cardinal numbers in the context of a single set over four and five years-of-age, and this knowledge is a critical development for understanding cardinality, as it represents a shift in understanding the information that counting provides. However, when children are not explicitly instructed to use counting, there is less clear evidence that they understand when the use of counting is appropriate. Schaeffer, Eggleston and Scott (1974), for example, gave children a Give-N task in which they asked two to five-year-old children to put between one and seven candies into a cup. They found that 33 of 65 children (51%) either did not use counting at all or used counting incorrectly, even though they demonstrated the ability to count when they were directed to do so on other tasks. Sophian (1987) gave three and four-year-olds two different tasks where they had to build sets of a specific number. The first task asked children to place a certain number of items onto a tray and the second asked them to stamp a certain number of pictures onto a paper. While virtually all children counted on tasks where they were asked to quantify a set, 38% of children did not count at all or only counted on one trial when they were asked to generate a set of objects and 50% did not count or only counted on one trial when they were asked to generate a number of pictures. These results suggest that although children of this age can and will use counting when they are directed to, they may not spontaneously do so. However, in both studies the appropriate use of counting for Give-N tasks increased with age suggesting that children learn over time how and when to count to build sets.

2.3.2 Counting to Build Equivalent Sets

While the studies above examined how children use counting to build one set, several researchers have also looked at how children use counting when building sets in one-to-one correspondence. These studies examine whether children can extend knowledge of how to build a set of a single quantity to building a relation between two quantities with two sets in correspondence. Russac (1978) compared kindergarten, first grade and second grade children's ability to establish equivalence using counting versus correspondence. He presented children with cards with seven, eight, nine or ten dots drawn onto them in random order. Children were then asked to establish equivalent sets in one of two ways: counting or correspondence. In the counting condition he asked children to count the number of dots on the card and then put the same number of chips as dots into a bucket in front of them. In the correspondence condition he asked children if they could put the same amount of chips as dots onto the card without using counting. He found that in general the counting task was easier for children than the correspondence task, and that the kindergarten age children performed significantly worse on the correspondence task than the first or second graders with an increase in accuracy related to age. However, it is possible that differences in the researcher's instructions exacerbated the difference in performance between the two types of tasks.

Children were explicitly told what they had to do to achieve a correct response in the counting condition because the experimenter told them to count the number of dots, but in the instructions for the correspondence condition children were not given the same kind of guidance. The tasks children were asked to complete in order to create equivalent sets in each condition were also different- putting chips in a bucket versus placing them on the card with the first set. A more direct comparison of strategies could have been achieved if children were asked to build the sets the same way in both tasks. This study shows that young children have the ability to use both correspondence and counting as strategies for creating equivalent sets, however they may be more likely to use one concept over the other depending on the context.

Fuson (1988) examined children's ability to build a set when no comparison set was given, to construct an equivalent set based on an existing set and the strategies children used to do so. She found that in three to six-year-olds, most children four and older were able to give the correct number of tokens to make a set of a specific number and virtually all of the children who were successful in this task used counting to do so. However, when the same children were asked to build an equivalent set many, even some of the older children, had difficulty. Very few children consistently or reliably used a counting strategy to build the second set (25%) and most attempted to match the tokens to the first set. This study highlights the difference between establishing a quantity and establishing a relation between quantities. Children in this study easily used counting to establish a set of a given quantity; however, it was much more difficult for the same children to see how to use counting when they were asked to build a second set based on an existing set. This second scenario is a relation between quantities, which requires children to think about equivalence between items in a set. This requires greater quantitative knowledge than establishing the quantity of one set, and highlights the extension of quantitative concepts that children must achieve in order to understand cardinality. They have to develop relational knowledge to learn that counting and correspondence are strategies not only for building single quantities, but for comparing quantities across sets as well.

Saxe et al. (1987) examined whether groups of two and four-year-old children were able to build an equivalent set based on one set that had already been constructed. They were presented with two puppets side-by-side and the first puppet had a square of cardboard with either three or nine pennies on it. The second puppet had a blank piece of cardboard and the child was given a pile of pennies and asked to give puppet two the same number of pennies as puppet one. The set sizes used were deliberately within the counting range of the children so that failure on the task would not be a result of using numbers that were too large. They found that in general all children performed better on the set of three than nine, with four-year-olds outperforming two-year-olds. When the strategies used to build the set of nine pennies were examined, they found that only 10% of children used counting and only those children who used a counting strategy obtained a correct response on the task. They also found that as children got older they were more likely to use counting both to establish the number in the first set and to check or modify the second set they built. This study shows that children's ability to apply counting appropriately to different types of quantitative problems increases with age. Many of the four-year-olds could see that counting was a relevant strategy for establishing an equivalent set, whereas the younger children did not consistently do so.

In a later study, Saxe et al. (1987) also asked mothers to show their children two different boards, one with three cookie monster pictures on it and one with nine cookie monster pictures on it and then told the mothers to instruct their child to pick up the same number of pennies as pictures on the board and place them in a cup. They measured children's accuracy as well as their use of counting or correspondence to check that the number of pennies they placed in the cup was correct. Children's answers were divided into four categories: children who did not succeed on either set size and used no visible counting strategies, children who succeeded on set size three but not nine and used no visible counting strategies, children who succeeded on set size three but not nine and did show some evidence of using counting and children who succeeded on both set sizes and used counting. The use of counting and rate of success again increased with age. They also found that most children would use at least some counting when completing the task with their mother's assistance even if they did not use counting when completing the task alone, but that four-year-olds were significantly more likely to do so than two-year-olds. Thus, children knew how to count but did not think to use it as a strategy for building equivalent sets of pennies and pictures until their mothers assisted them. These studies suggest that although children are proficient counters by the age of four, as with correspondence they do not always readily turn to counting as an effective strategy for building or comparing sets. Children increasingly learn with age to connect their procedural knowledge of counting with an understanding of the information that counting produces and this increase in knowledge allows them to learn to use counting as a strategy for establishing whether sets are equivalent and whether they share the same cardinal.

2.3.3 Counting to Compare Sets

Counting behaviour may be learned by rote without reflecting true understanding that numbers represent quantities that can be related to other quantities or manipulated to solve mathematical tasks (Bryant, 1995). Children must learn the quantitative significance of counting in order to truly be able to use it for mathematical modeling (Muldoon, Lewis and Freeman, 2003), and several studies have looked at children's use of counting to compare sets. Sophian (1987) gave three and four-year-old children tasks where they were asked to build sets of a specific number and to compare sets to examine their use of counting. The results of the tasks used in this study to build a set of a specific number were reviewed above (see section 2.3.1), while children's use of counting in the compare set tasks will be outlined here. Children were given two types of conditions. In the paired set condition, one set was presented in a circular formation and the corresponding objects from the second set were immediately to the left of each object in the first set. In the separate set condition, the first set was still presented in a circular formation, but the corresponding objects were presented in a straight line to the right of the circle. She found that children performed better on the paired set than separate set task, with smaller set sizes, and in general performance improved with age. She also found that despite many children performing well on counting tasks, children rarely used counting in either of the comparison tasks. When they did use counting it tended to be on the smaller rather than larger set sizes and often they would only count one set rather than both. Sophian (1987) suggests the processing demands of keeping track of two counts when comparing sets compared to one count for establishing the number in one set may account for children's poor counting performance, but that does not explain why they refrained from using counting at all in most of the set comparison tasks. Sophian (1987) modified the tasks in a second study so that the child was asked to build an equivalent set to the first set, rather than compare two pre-built sets. This modification was made to see whether children would be more likely to use counting to counter the possibility that in the first study children did not count simply because they did not think they were supposed to. Other than the change in building sets rather than comparing, the tasks remained the same. In the paired task children were asked to pair a second group of objects with the first set of objects already arranged in a circle, and in the separate set task children were asked to put the same number of objects on one tray as they saw in a set already built on a separate tray. The trays were different shapes to avoid children being able to simply duplicate or match the shape of the first set. Sophian (1987) found that as in the first study, children were unlikely to use counting in the paired set task and did not often use counting in the separate set task either. As before, when they did use counting they tended to count one set not both sets. These results suggest that while young children have good knowledge of how to count, they do not often use it when they are asked to compare sets, even when it would be the most accurate way to establish equivalence. Children must learn to extend their conceptual knowledge of counting to not only establish quantities but also compare them. This extension of logic is important for a complete understanding of number and cardinality.

Fuson, Secada and Hall (1983) compared children's use of a matching or counting strategy on a conservation of number task. Four and five-year-old children were divided into three groups: a counting group, a matching group and a control group given no additional instructions for the task. In the control group children were shown a line of seven toy lions with a line of seven peanuts in front of them. After judging the two rows as equal, the lions were then spread out and the child was asked whether there were now more, less or the same amount of lions as peanuts. In the counting condition children were told to count the number of lions and peanuts after the lions had been spread out. In the match condition the child placed a string between each lion and peanut so that one-to-one correspondence was visually maintained before and after the lions were spread out. The majority of children responded correctly in the count and matching conditions, compared with only 14% in the control condition. Virtually all of the children in the counting condition were able to give a correct explanation for their equivalence judgment, whereas only half of the children who answered the matching condition correctly were able to provide a sensible explanation for their judgment. Children in the experimental conditions were also given a traditional conservation task with no additional instructions either before or after completing the counting or matching condition to see whether these successful strategies would be employed again without prompting from the researcher. A significantly higher proportion of children performed well on the conservation task without additional instructions if it came after the counting condition than if it came before. The difference in correct responses was not significant for the order of trials in the matching condition.

In a later study, Fuson (1988) also examined how well four and five-year-old children were able to compare the amount of objects in two sets. Children were presented with three different tasks and were asked whether one row was larger than the other or if both had the same number. The number of objects in both sets was always the same and there was a colour correspondence between objects in the two sets, however the length of the sets differed. The first task allowed children to compare static sets of objects that could not be moved or touched, the second task involved sets of toys that were movable and touchable and the third task mirrored the first task except that the child was also explicitly told they could be tricked into thinking one set had more than the other when they actually had the same amount. The results of Fuson's (1988) first task support those of earlier studies: the majority of children used the length cue to decide whether the sets were equal. However, in the second task when children were prompted to show how they came up with their judgment of equivalence, the vast majority of children (70% of four-year-olds and 80% of five-year-olds) used counting to compare sets. Similarly, when told that they might be misled into thinking the sets were different when they were not many children also used counting (63% of four-year-olds and 60% of five-year-olds). However, Fuson (1988) kept the amount of objects in each set consistent across trials and also did not vary the order of her tasks. This leads to the possibility that children could perform better on the latter two tasks simply because they learned that the two sets would always be the same. Nunes and Bryant (1996) also point out that children were only asked to give one judgment in the first task, whereas in the second two tasks they were given the opportunity to prove or revise their judgments after prompting from the researcher, which may have also led to better performance.

Despite the flaws in her method, Fuson's (1988) results show that children can be prompted to use counting even though they often will not do so spontaneously.

Cowan (1987) examined several possible reasons for why children fail to use counting to compare sets. Three to five-year-old children who were able to count well were given a set comparison task with two rows of dots (yellow and blue) that were either the same amount or one row had one more dot than the other (sets of 3:3 or 3:4 and sets of 8:8 or 8:9). Children were divided into a self-count group where they were asked to count each row of dots and say how many there were in each, or an experimenter-count group where the experimenter counted each set and said how many there were in each. After both counting conditions children were asked whether both sets had the same number of dots. Cowan (1987) found that despite children using very accurate counting to establish the number in each set, their judgments of the equivalence of the sets remained based on length. Whether the child or the experimenter did the counting had no effect on the accuracy of their judgments. The results of this study were also consistent with a further study conducted with five-year-olds using larger numbers within their counting range. However, when the study was repeated once more using six-year-olds, children consistently and accurately based their comparison judgments on the results of their counting. This suggests that the tendency to rely on length or perceptual cues despite having other more accurate information for set comparisons is very persistent in young children. Even children with strong counting skills may not choose to use counting as a basis for determining equivalence. These studies again show the persistence of four and five-year-old children's reliance on inaccurate strategies for establishing and comparing equivalence in sets. They must learn that counting or correspondence are reliable indicators of quantity and can also be used to examine the relations between quantities. This conceptual knowledge develops with age and as children have more and more opportunities to use their quantitative knowledge in different ways.

Cowan, Foster and Al-Zubaidi (1993) examined whether children with strong counting ability but who failed to spontaneously use counting to compare sets could be trained to see the value of using a counting strategy. They randomly assigned five to seven-year-old children into four groups: an “agreement between strategies” group where counting and matching strategies were both used on the same sets to show that both yielded correct responses, an “agreement between strategies” control group where either counting or matching strategies were used but never both on the same set so they could not compare the usefulness of counting to any other strategy, a feedback group where children were encouraged to count to compare the two sets and were given positive feedback after counting the sets to indicate whether the rows were equal or unequal, and a feedback control group where children were still encouraged to count but were not given explicit feedback about whether sets were equal or unequal. Children were then given a post-test after training in their respective groups where they received no feedback or instructions about comparing the sets. They found that children in both experimental conditions counted significantly more often on post-test trials than children in either control group (57% compared to 32%). The difference between the two types of experimental groups was not significant. Thus, a large number of children who did not think to use counting before training relied on counting in every post-test trial to make comparisons. Between children who counted to solve the tasks, there was a significant difference in the accuracy of children’s answers by age with older children performing better than younger children. Children in the experimental groups also significantly improved the accuracy of their judgments relative to children in the control groups. This suggests that children can be taught to see the relevance of counting as a strategy for set comparison, and are able to use it accurately. Clearly by five-years-old, children do not lack the skills for using counting to compare sets. Rather, they do not seem to have extended their thinking about counting to see it as an appropriate strategy for comparing equivalent sets without prompting.

Based on their results with older children, Cowan, Foster and Al-Zubaidi (1993) went on to examine an even younger group of children (three to four-year-olds) to see whether they too could be taught to use a counting strategy. The conditions of this experiment were the same as the first: children who could count but did not spontaneously use it for set comparisons were randomly assigned to the two experimental or two control groups and were given training trials followed by post-test trials. They found that as with their older children training did succeed in getting children to use counting to compare sets (approximately 42% of children in the experimental groups counted on every post-test trial compared to 12.5% of children in the control groups). However, even children who used counting were not able to consistently judge the numerosity of two sets correctly. Children who used counting performed better than non-counters, but 46% of children who used counting also did not make a single correct judgment on the post-test trials. It is unclear why children still made errors in the post-test trials, however this may reflect the fact that despite counting both sets they did not actually use this information as the basis for their judgments about equivalence. These studies suggest that even younger children have the capacity to use counting as a strategy for set comparison, but struggle to see its usefulness for this purpose rather than simply for determining an overall amount.

Zhou (2002) conducted two studies in China to examine several hypotheses for why young children are so hesitant to use counting to compare sets: children's counting skills (in particular their understanding of cardinal numbers) may not be strong enough to support the use of counting for comparing sets, they may not yet have the cognitive capacity to process counting two separate sets, they may not trust the accuracy of their own counting, they may be able to use the strategy but fail to perceive it as a useful one in the context of comparing sets or they may just lack experience using counting for comparisons rather than to find out how many items are in one set. In her first study, three and four-year-old children were divided into three groups: a group that was consistently reminded that they could use counting on the tasks, a group that was given a brief training period using counting to compare sets, and a control group that was given no instructions with the tasks. Zhou (2002) found no difference between the groups amongst three-year-olds and suggests that their understanding of cardinal concepts (as measured by a Give-N task) may have been too weak to benefit from the counting intervention. Using the same conditions with a slightly older sample, Zhou (2002) found that there was no significant difference between the two types of intervention group, but children in the counting reminder and training groups were significantly more likely to use counting to compare sets than children in the control group. Zhou (2002) also found that despite efforts to reduce reliance on visual cues, for example by putting objects into piles rather than lines, many children in all three groups persisted in using visual comparisons. Children's performance on the Give-N task was positively and significantly correlated with their use of counting on the comparison task, whereas their performance on a how many task was not. This suggests that a strong understanding of cardinal number was related to the ability to use counting to compare sets. In a second study, Zhou (2002) compared children's performance on the tasks from the first study with their performance on a static row test using rows of dots printed on cards to see whether experimental conditions impacted children's use of counting. She found that children who were asked to compare sets that had been constructed with real objects were significantly more likely to use counting and to do so correctly than children who were given the static row test. Even so, only 26% of children used counting in the real object condition with the majority still relying on visual cues to make comparisons. These two studies reveal three important things: first, children's counting abilities in the context of one set are closely related to whether they will use counting as a comparison strategy, second, children can be taught to use counting for comparing sets, and third, the design of a comparison task does appear to have an impact on the type of strategy children will choose for comparing sets.

In another training study in the UK, Muldoon, Lewis and Francis (2007) worked with four to six-year-old children who were strong counters, but consistently made set comparisons based on length cues to see whether children could be trained to use counting to compare rather than length. They presented children with static rows of blocks (in one set the two rows were unequal in length but equal in number, and in the other set the rows were equal in length but unequal in number). Children were asked to compare the sets after they counted one and a puppet counted the other. In half of the trials the puppet counted his row correctly, and in half he miscounted. This allowed for an examination both of their ability to detect counting accuracy and ability to use cardinals rather than length cues to compare sets. Children took part in 5 training sessions and a post-test session and were divided into five groups to examine whether counting sets to be compared, receiving feedback about their judgments or being asked to explain their reasoning impacted performance on the post-test. They found that children did learn over the training phase that relying on a length cue when comparing sets produced unreliable results compared to relying on cardinals and many children began comparing using cardinals during training. However, on the post-test 67% of children failed to use any counting when comparing the sets which suggests they did not in fact fully grasp the conceptual shift they were being taught. Another factor related to whether they performed well on the post-test was sensitivity to the puppet's miscounts. Some children did begin to use counting strategies to compare sets with cardinals, but were not able to correct the puppet's miscounts. These children were more likely to revert to using length cues on the post-test than children who were able to correct the puppet's mistake.

All of these studies indicate that even though young children know how to count and are capable of using it compare sets (Becker, 1989), they are not likely to use counting spontaneously. Even with training to use counting for set comparisons it is difficult for many children to move away from using perceptual cues such as length or density to make judgments about equivalence. Young children seem to continue to rely on inaccurate methods for comparison such as length or density even when provided with other strategies or information, and need to learn that correspondence or counting strategies are more reliable ways of determining the quantity of a set or for comparing equivalent sets, if they are applied correctly. This leads one to ask what knowledge could help children achieve this extension of quantitative reasoning. Several researchers have suggested that children's counting abilities and their understanding of the principles that underlie counting correctly play a large role in whether they will use counting effectively as a comparison strategy, which will be explored next.

2.3.4 Sensitivity to Counting Errors

In examining why some children use counting and others do not, some researchers have suggested that children's sensitivity not only to their own but also to other's counting actions and mistakes is important for furthering their understanding of what information counting provides them- the cardinal number of sets (Freeman, Antonucci and Lewis, 2000; Muldoon, Lewis and Freeman, 2009). As stated earlier, children need to connect the knowledge that word-to-item correspondence must be maintained when counting one set, and that item-to-item correspondence must be maintained when counting to compare sets. Some researchers have suggested that how sensitive children are to errors made during another person's counting is related to how well children understand cardinality. Seeing counting errors and being able to correct them is important because it indicates that a child understands what counting principles are crucial for counting correctly and how these principles interact. For example, to count correctly children must remember that you can only give one number name to each object in the set and that each object in the set can only be counted once.

Before examining studies that have looked at children's sensitivity to other's counting errors, there is evidence that children become more attuned to their own counting errors as they get older. Saxe and Sicilian (1981) looked at children's actual counting performance compared with children's judgments of their counting accuracy in five, seven and nine-year-old children. Children were presented with three types of tasks involving 28-30 objects per task. The first was a task where several objects were placed inside a beaker and the child had to count them without being able to touch or move them, the second was a task where several objects were glued to a board so the child could touch but not move them, and the third was a task where the child was presented with a pile of loose objects that they could touch and move. In each task the child was asked to count how many objects there were and then the experimenter questioned whether they were sure they were right or if they thought they might be wrong. They found that five-year-olds showed no difference in their judgments of how accurate their counting was across the three types of trials. They most often thought their counting was correct regardless of their actual performance, whereas seven and nine-year-olds were more often uncertain about their accuracy in the tasks where they could not touch or move the objects. However, this study had a small sample size (15 children per age group) and used relatively large set sizes where younger children were more likely to make counting errors. Still, the results suggest that children's awareness of the accuracy of their counting is an important step in their ability to use counting effectively as a tool for comparing or building sets.

Most studies of children's sensitivity to other's counting errors have employed an error detection task where a child is asked to evaluate the accuracy of a puppet's counting. In some trials the puppet counts correctly and in other trials they miscount in different ways (for example skipping or double counting one object). Gelman and Meck (1983) found that using this type of design three and four-year-old children correctly identified when an object was skipped or double counted in a set on 75% of trials and using a similar design Briars and Siegler (1984) found their sample detected around 72% of errors. Rodriguez et al. (2012) found that children's error detection skills increased with age, but the majority of children from kindergarten to grade two were able to detect when violations of the logical rules of counting had occurred.

The above studies suggest that young children know the procedure for counting correctly quite well and are good at identifying when mistakes have been made in the context of establishing the amount in one set. However, counting to obtain a total and counting for comparing sets are two different tasks. Sophian (1988[B]) asked 20 three-year-olds and 20 four-year-olds to judge a puppet's counting accuracy when he either counted the total amount for two sets or used counting to compare the amounts in each set. Children were shown conceptually paired sets: six toy cows and six calves, or six horses and six colts. There were sixteen different tasks involving all the combinations of conditions. Trials were mixed between whether the puppet was asked to find the total amount or to compare the amount in each set, type of counting, whether the puppet counted all the objects or each set individually, whether the set size was 5 or 6 vs. 11 or 12, and whether the sets were equal or unequal. Importantly, Sophian (1988[B]) found that three and four-year-olds were able to discriminate between appropriate counting strategies for each type of task. They deemed counting all of the objects appropriate in the context of finding out the total amount and counting each set appropriate for comparing the amount in each set suggesting some understanding that the way one counts differs depending on the purpose of counting. However, while most children were able to assess correctly the accuracy of counting the total number of objects, they were at chance level for assessing the accuracy of counting to compare the sets. Sophian (1988[B]) suggests that this result parallels the fact that children frequently use counting as a method for obtaining a total but do not often use it as a comparison tool. When children's response patterns were analysed she found that many children relied mainly on incorrect strategies for determining counting accuracy, and that even when occasional correct judgments were made these strategies were not yet strong enough to completely overcome the use of incorrect ones.

Freeman, Antonucci and Lewis (2000) examined 22 four-year-old children's understanding of miscounts and incorrect cardinals. Children were presented with a situation where a doll (Molly) counted items that had been placed in a row in front of her. Molly sometimes counted the row correctly and sometimes skipped one item or double counted one item to produce an incorrect count. Children were asked to say whether Molly counted correctly or incorrectly, how many items Molly thought there were and how many items there really were in the set. Seven of the 22 children (32%) consistently identified the miscount and were able to say what the false and true cardinals of the sets were across both set sizes (3 or 5 items). They also found that miscount identification increased with age. In a second study with three to five-year-olds using set sizes 3-6, there was a sharp increase in performance on the miscount identification task between four and five years-of-age. Most of the five-year-olds used counting and 23 of 28 passed every trial. While the five-year-olds were at ceiling levels, half of the four-year-olds passed the miscount detection trials but a quarter failed entirely. It is clear from these two studies that four-year-olds are still learning how to identify miscounts and what miscounts mean for the total of a set, whereas five-year-olds have a much more developed understanding of the cardinal principle.

Muldoon, Lewis and Freeman (2003) furthered the results of Freeman, Antonucci and Lewis (2000) by looking at the relation between children's sensitivity to miscounts and their understanding that counting can be used to compare sets. Three and four-year-olds were given an extensive battery of tests measuring their counting skills, ability to detect miscounts and to state that the miscount produced an incorrect cardinal. They also received two different tests aimed at examining how they use counting in the context of building an equivalent set and comparing equivalent sets. In the set creation test children were told that frogs (set sizes 3 to 6) wanted to sail down a river in boats (one frog per boat) to attend a party. Once the frogs were deposited at the party and arranged in random order, the boats were sailed away and docked with other boats up river. The child was then told that the party was finishing and was asked to count the frogs and then get the correct number of boats ready so that the frogs could sail home. In the compare a set test children watched as a puppet got the boats ready for the frogs at the end of the party. She either took one too many or one too few boats for the number of frogs. Muldoon, Lewis and Freeman (2003) found that 15 of 23 children (65%) used counting to build an equivalent set of boats for the frogs at the party. Several children who succeeded in using counting to build a single set did not perform well on the set creation task, showing that children need to know more than just how to count to build equivalent sets. All of the children who were able to identify miscounts in a puppet's counting and say how this affected the cardinal of a set passed the set creation test, and this factor was highly significant in predicting success on the set creation test. They also found that children who were able to use counting to construct equivalent sets were also likely to know how to use counting to compare equivalent sets, in other words set creation success predicted set comparison success. A second study was also run where half of the children were prompted and half were not prompted to count the frogs to see whether children would spontaneously use counting. There was not a significant difference between children who were prompted to count and those who were not. This is interesting because many other studies have shown that children do not often spontaneously use counting on their own without suggestion from the researcher. 43% of children gave correct responses on the set creation task. Again the largest predictor of children's success on the set creation test was the ability to detect violations to the cardinality principle. Muldoon, Lewis and Freeman (2003) suggest this highlights a link between knowing the consequences of counting incorrectly ("conceptualisation of error") and being able to understand the link between cardinals of multiple sets in correspondence. Spotting counting errors and knowing what they mean appears to be an important step for learning about cardinals of equivalent or non-equivalent sets, possibly because it draws children's attention to the importance of cardinal numbers for set comparison.

Muldoon, Lewis and Berridge (2007) investigated the relation between knowledge of procedures for counting and sharing and their relation to mathematical understanding. Five-year-old children who had just started school were given a series of tasks designed to test their counting and sharing proficiency in three sessions over a six-month period. They were asked to compare two sets of 10 or 20 items either when the experimenter counted both for them or when neither were counted, and they were asked to infer the number in one set when the other was counted. They were also asked to share 10 or 14 items between two puppets. An error detection task was also used to test children's conceptual knowledge of counting and sharing where a child was asked to examine a puppet's counting and sharing behaviours for any errors. Muldoon, Lewis and Berridge (2007) found that in their first session over half of their sample had strong counting and sharing skills with small sets and over half were also sensitive to a puppet's miscounts, however very few expressed knowledge of the consequences of miscounting. By the third session approximately 50% could say that miscounting meant the cardinal number was not the same as the last number counted. They also found that those children who were sensitive to another's counting and sharing errors were more likely to understand the importance of cardinal numbers for comparing sets and to pass the number inference task by the third session. They suggest that there is a difference between learning to monitor word-to-item correspondence in the context of one set necessary for counting and monitoring item-to-item correspondence in the context of two sets necessary for sharing. The former knowledge is important for predicting whether a child will rely on cardinals instead of perceptual cues when comparing sets, because if a child knows that a miscount produces an incorrect cardinal they should then know that the puppet will think there is a different amount in the set than the true amount. The latter knowledge seems to be more important for learning how to extend cardinal knowledge to multiple sets in correspondence. Their study also confirms that children can master counting and sharing procedures before truly understanding the mathematical implications of their actions. This learning is vitally important for their development of mathematical concepts like equivalence, correspondence and cardinality.

2.3.5 Summary

The studies outlined above show that both correspondence and counting are important strategies for determining how many objects to give to each person to establish equivalent sets, and for comparing equivalent sets. While children learn early on the procedures for counting correctly and the order of number names, it takes time for them to join this knowledge with deeper mathematical concepts like succession and cardinality and to become more competent at using counting to establish the amount in a set. Once they have a more developed understanding of counting in the context of one set, they must also extend their knowledge to understand how counting can be used to compare sets in correspondence. Several studies have shown that although many young children have a strong grasp of counting and will readily employ it as a strategy to build or check sets, they do not as often seem to rely on counting for comparing sets. They must learn the relevance of counting for comparing equivalent sets and this is where social interactions like sharing become important because they provide children with the opportunity not only to build, check and compare sets in correspondence, but also to watch how others do so and how others employ counting strategies.

2.4 Summary of Research Findings

This chapter reviewed research into the development of counting, correspondence, equivalence, cardinality and number inferences in children four and five years-of-age. These are important quantitative concepts in children's mathematical development and are ones they begin to develop before receiving formal mathematical training, through social interactions.

The first section of this chapter looked at children's use of one-to-one correspondence both to build sets and to compare sets. Studies showed that from a very young age children become able to deal out resources on a one-to-one basis to create equivalent shares. While set size, object visibility and other factors will influence the accuracy of their sharing, in general children's ability to use one-to-one correspondence is very robust. Other studies that have examined children's ability to compare equivalent sets have found that children are very susceptible to spatial or perceptual differences in sets and will make inaccurate judgments of equivalence based on those features rather than using counting or correspondence to compare sets. A large array of studies using conservation of number and static row comparison tasks have repeatedly found that children fail to use counting or correspondence to compare sets despite having the capacity to do so. Some studies using number inference tasks have found more positive results: that four and five-year-olds are able to make an inference about one set based on counting the number of items in an equivalent set. This suggests that children in this age range do have knowledge of cardinality and equivalence, but that they need to build connections between these concepts and counting and correspondence in order to have accurate methods for creating equivalence and comparing equivalent sets.

The second section reviewed studies that asked children to use more difficult correspondence strategies with numbers larger than one-to-one. This research shows that children are quite flexible in their ability to apply correspondence strategies and are able to adjust their sharing to accommodate unequal units when the goal is to build equivalent sets, as long as the numbers are multiples of each other and they can see how to match the smaller units to the larger ones. These findings suggest that children are able to see how to use more than one kind of strategy to achieve the goal of creating equivalent sets. However, current research has not taken this one step further to see whether asking children to build sets using different correspondence strategies in turn affects their ability to compare the equivalent sets based on number. The findings from the static row and number conservation studies suggest that children tend to base judgments about equivalence on perceptual information. If equivalent sets are built using unequal units (for example twos and ones), they may not appear perceptually identical which may lead the child to believe they do not share the same number. Conversely, based on the findings of studies that have asked children to make number inferences after building sets in one-to-one correspondence, it seems possible that if a child knows how to construct equivalent sets they could also make the inference that they share the same number.

The third section of this chapter examined children's use of counting as a strategy to build and compare equivalent sets. Much research has investigated how children learn to count and shows that by four and five children generally have good working knowledge of the procedural rules of counting and produce counting words up to 12 or so without errors. Children also begin to show success on the Give-N task and can use counting fairly well to build a set of a specified number when prompted to do so. However, they do not often spontaneously use counting to build sets. The results of studies that have examined how children use counting to compare sets also show that while they are capable of using counting to make judgments about equivalence, they rarely do so without explicitly being asked to. Even children with strong counting abilities are more likely to use spatial or perceptual cues like length or density to compare sets. These studies suggest that children with a strong grasp of cardinal numbers (as measured by counting tasks like the Give-N test) and children who are able to identify other's miscounts and state how they affect the cardinal of a single set are more likely to use a counting strategy to compare sets. Thus, children need to extend their knowledge of counting to see not only how it can be used to establish quantity, but also how it can be used to compare quantities in correspondence. This relational knowledge is crucial for a true understanding of cardinality.

In general, research into the development of quantitative concepts in four and five-year-olds shows that many children know how to use a concept, such as correspondence or counting in one context, but may not use this same concept in another context where it would also be useful. For example, many children did not use correspondence or counting strategies for comparing sets despite demonstrating the ability to use these concepts to build equivalent sets. Children need strong knowledge of counting and correspondence and the ability to flexibly apply these strategies to determine the amount in one set and to make an inference about one set based on the number of units in an equivalent set. With a wealth of knowledge about correspondence and counting available to them, children must learn to select these strategies as reliable indicators of quantity and use them appropriately to compare quantities in order to successfully develop their understanding of cardinality.

2.5 The Current Research

Based on the literature reviewed above, the current research aims to examine children's understanding of cardinality and equivalence, as well as how and when four and five-year-olds will use correspondence or counting in the context of sharing to create equivalent sets and make a number inference about one set based on counting a second, equivalent set. The studies presented in this thesis are based on three main hypotheses derived from the literature about the development of young children's quantitative reasoning. The main hypotheses of this research are as follows:

1) Four and five-year-olds' knowledge of counting and correspondence is not tied to one single procedure for sharing, they can flexibly use counting or correspondence strategies in different contexts to build equivalent sets
2) Knowledge of how to construct equivalent sets is not sufficient to demonstrate a full understanding of cardinality and children must also be able to infer the number of one set based on counting a second, equivalent set which requires knowledge about the relation between the cardinals of two or more sets in correspondence
3) There is a progression between four and five-year-olds in their understanding of cardinality

The previous chapter argued that sharing provides an excellent way to study the development of this early quantitative knowledge, as it provides a context for the child to learn from their own and others' use of mathematical concepts. This chapter documented the connections and sometimes lack of articulation between children's understanding of correspondence, equivalence, numerical representations for sets and counting principles. This research examines children's quantitative knowledge in the context of sharing because it is familiar to children from everyday life. This familiarity and the social information included in the tasks may help support their understanding of how to use quantitative concepts to solve the problems.

Using the context of sharing for all experimental tasks, the three studies presented in the subsequent chapters investigate the following questions which have been raised in this review of the literature:

1. Can four and five-year-olds employ different sharing procedures to construct equivalent shares in contexts with varied social and cognitive demands?
2. Can four and five-year-olds coordinate knowledge of counting and knowledge of the equivalent relation between sets in correspondence to make a numerical inference about one set after first counting an equivalent set?
3. Does using units of different sizes (for example single blocks and double blocks) affect children's ability to establish equivalent quantities?
4. Does using units of different sizes affect children's ability to make numerical inferences when quantities are equivalent?
5. Do four and five-year-olds assume that fair sharing must always result in equal shares?
6. Do four and five-year-olds rely on visual cues between units to establish equivalent sets and to make numerical inferences about equivalent sets?
7. Do four and five-year-olds know how to coordinate their knowledge of counting principles, specifically knowledge of ordinality and cardinality to identify and correct a counting error to establish the total of a single set?
8. Do four and five-year-olds understand how to coordinate knowledge of counting principles and knowledge of the equivalent relation between sets in correspondence to make an appropriate number inference about one set if an equivalent set is not counted correctly?

This research will ask children to use correspondence or counting to construct equivalent sets in different types of sharing scenarios, allowing for an examination of how flexibly they are able to apply these concepts across varying contexts. It also aims to assess the depth of their understanding of cardinality and equivalence by examining whether four and five-year-olds can make a numerical inference about one set after counting an equivalent set under different circumstances, for example when the quantities in two sets are equivalent but each set is built from units of different sizes.

This research will examine whether four and five-year-olds are able to identify and correct a miscount to determine the cardinal of a single set and whether this impacts their ability to make a number inference about a second, equivalent set. Combined the three studies included in this research aim to fill some of the gaps in the literature about the development of children's mathematical reasoning. Each of these studies targets children's understanding of individual quantitative concepts, but also their ability to understand the relations between these different quantitative concepts to solve mathematical tasks.

3 Establishing Equivalence and Comparing Equivalent Sets Across Sharing Contexts

3.1 Introduction and Aims

The previous chapter reviewed literature which examined four and five-year-old children's understanding of how counting and correspondence can be used to establish or compare equivalent sets, and their understanding of the concepts of equivalence and cardinality. Those studies indicated that although most four and five-year-olds are able to use counting or correspondence to build equivalent sets, knowledge of equivalence and cardinality as well as how counting and correspondence could be used to compare sets is not as developed. The studies also showed that many children of this age are very good at counting and use it when sharing as a way of achieving equal resources or making things fair. Other studies showed that they can use correspondences in order to build equivalent shares. However, they do not seem to put these two pieces of knowledge about the number in a set and about correspondence together in a single schema. Two studies were designed to investigate the development of this coordination. In the first, the children were discouraged from counting in order to see how they used correspondences to share the resources. In the second, the children were allowed to count.

This chapter reports on the first study, in which the children's counting efforts were inhibited. The study builds upon the previous work that has examined children's understanding of counting, correspondence and number inferences in several ways. First, this study aims to extend the literature about children's use of one-to-one correspondence to compare how children share equal units (single blocks) between two characters in three sharing conditions: equal sharing, reciprocity and equity. Second, it aims to examine how flexibly children are able to use knowledge of correspondence to establish equivalence in the equal sharing, reciprocity and equity conditions by asking them to use double and single units to build equivalent sets in addition to trials with only single blocks. Third, it aims to examine whether the use of correspondences larger than one-to-one affects children's understanding of equivalence and cardinality by asking them to make number inferences after sharing using double and single blocks. Finally, this study aims to investigate any developmental differences in four and five-year-olds' understanding of correspondence, equivalence and cardinality.

In order to address the first and second aims of this study, children will be asked to share all single or double and single blocks between two puppets in an equal sharing, reciprocity and equity condition. Previous research indicates that four and five-year-olds are very good at sharing single blocks fairly between two or more recipients (Desforges and Desforges, 1980; Cowan and Biddle, 1989). It is therefore expected that the children in this study will find this type of sharing easy. This study aims to examine whether four and five-year-olds can push their quantitative skills further by including trials in which children will be asked to share using double and single blocks. Previous research has shown that four and five-year-olds can share doubles and singles (Frydman and Bryant, 1988), but it is expected that these trials will be much harder because children have to take into account that one double block is equal to two single blocks each time they share in order to construct equivalent sets. The amount of difficulty they have sharing doubles and singles can therefore be compared to their performance using all singles.

In order to share fairly in the equal sharing condition with double and single blocks, children need to give one double to one character and two singles to the other each time they share blocks in order to attain equivalent sets because the individual quantities being shared are not equivalent. In the equal sharing condition, children can use correspondence to construct two equivalent sets at the same time. In the reciprocity condition, before children are asked to share blocks, they first observe one character share doubles to his friend and keep singles for himself, resulting in the friend receiving twice as many blocks. In order to establish equivalence in this condition, children have to invert the sharing procedure of the first character by giving him double blocks and giving single blocks to the friend. In this condition they have to construct equivalent sets over time taking into account the blocks that have already been distributed. The equity condition examines four and five-year-olds' quantitative knowledge in the context of sharing in a different way than the previous two conditions. In the equity condition, children are told that one character works twice as hard or wins twice as many times as the other and are then given blocks to divide fairly between the two. This condition examines whether four and five-year-old children can differentiate between the concepts of fairness and equivalence because this task asks them to construct unequal but fair sets. Equity is an example of a context in which fair sharing does not mean establishing equivalent sets. In order to share fairly, the children must focus on the amount of work each character completes and construct sets based on that information. If they use a correspondence of one double for the character that did twice as much work and one single for the other character, they will produce sets that match the amount of work or number of times each character wins: unequal but fair shares. Asking children to use both one-to-one and two-to-one correspondences in these three sharing conditions examines whether children can share flexibly and understand correspondence as a broader concept than just tied to the actions of sharing on a one-for-you, one-for-me basis. Each of these three sharing conditions asks children to use knowledge of the relation between doubles and singles to construct sets appropriate to the sharing context.

In order to address the third aim of this study, children will be asked to make a number inference after sharing double and single blocks. Previous research into children's ability to make number inferences indicates that by age five many children are able to say the amount in one set after counting the objects in a second, equivalent set (Sophian, 1988[A], Frydman and Bryant, 1988). These studies have all used sets of conceptually paired items or single blocks. None of these studies has pushed children's knowledge even further and asked them to compare equivalent sets that were built from unequal units (such as double and single blocks). In each of the three sharing conditions in this study, after children have dealt out double and single blocks to establish two sets, they will be asked to say how many blocks are in one set after they have counted the blocks in the other set. The target set will be covered so children cannot see or count the number of blocks, and thus this is an inference task. If four and five-year-olds have a strong understanding of equivalence and cardinality, they should understand that even though the individual blocks that make up each set are perceptually different, if the total number of blocks in each set is the same then both sets share the same number. The number inference question examines whether if they agree that both sets are equivalent, they understand that this means that both sets share the same cardinal number. In the equal sharing and reciprocity tasks, if one character has six blocks, the second character also has six blocks. The development of this understanding is crucial to later quantitative development with more complex concepts. In the equity condition, sharing results in one set having twice as many blocks as the other, in which case children cannot make the inference that both sets are equivalent.

Finally, knowledge of the concepts of equivalence, correspondence and cardinality are still developing in this age group and the last aim of this study is to shed light on whether there are any developmental differences between four and five-year-olds in their understanding of correspondence as a way to build and compare sets as well as their understanding of equivalence and cardinality. Five-year-olds have had more practice sharing and have more developed cognitive skills, and performance differences were found between four and five-year-olds in previous studies where children were asked to make number inferences about two equivalent sets (Sophian, 1988[A]), thus it seems plausible that there would be differences in performance between the four and five-year-olds in this study.

3.2 Hypotheses and Predictions

It is hypothesised that young children are sensitive to both the social and quantitative information within a sharing context. Varying the amount of social and quantitative information changes the cognitive demands of a sharing task. It is predicted that four and five-year-olds' ability to use appropriate sharing strategies to build equivalent sets and to compare the equivalence of two sets will be affected by manipulations to the demands of the task that change the amount of social and quantitative information that must be considered in order to decide how to share fairly.

It is predicted that four and five-year-olds will find the trials where they are asked to share all single blocks easier than the trials where they are asked to share double and single blocks. It is also predicted that children will have more difficulty sharing in the reciprocity condition than the equal sharing condition. In the reciprocity condition, children need to take into account one character's previous sharing actions and compensate for those actions in order to establish two equivalent sets, whereas in the equal sharing condition children share to both recipients at the same time and only need to consider the total amount of resources available. It is predicted that children will have the most difficulty with the equity condition because they have to establish unequal but fair sets by taking into account even more information: the amount of work each recipient completes or the number of times each character wins a game. It is therefore predicted that children will find it easier to use double and single blocks to build and compare equivalent portions for each character (in the equal sharing and reciprocity conditions) than in the equity condition where sharing does not result in equivalent sets.

It is hypothesised that four and five-year-olds' ability to compare equivalent sets is related to their ability to build equivalent sets. It is predicted that children's ability to compare equivalent sets will be impacted by the type of sharing condition in which they are asked to construct the equivalent sets. In the equity condition children cannot make the inference that both sets share the same number and therefore this condition is predicted to be the most difficult. In the reciprocity and equal sharing conditions children can make the inference that both sets share the same number, but the demands of the reciprocity condition in terms of constructing equivalent sets are more difficult than the equal sharing condition. It is therefore predicted that children will have less difficulty making the number inference in the equal sharing than the reciprocity condition. If a child is unable to use an appropriate strategy to build two equivalent sets, it is unlikely that they will then have the knowledge to infer the cardinal of an equivalent sets.

Finally, it is hypothesised that children learn over time how to adapt and apply their knowledge of quantitative concepts like correspondence and equivalence to share fairly across different contexts. It is predicted that five-year-olds will perform better than four-year-olds in terms of being able to share double and single blocks fairly to build equivalent sets. It is also predicted that five-year-olds will be better than four-year-olds at comparing equivalent sets because they have more developed cognitive and quantitative skills. Five-year-olds may also perform better than four-year-olds on the equity condition, as their more developed cognitive skills may allow them to understand and accept that in some cases sharing fairly does not mean sharing equally. In order to examine this, four and five-year-olds will be asked to share in three experimental conditions in which they are asked to build sets out of double and single blocks in different ways.

3.3 Methods

3.3.1 Participants

Participants were 57 four and five-year-old children from reception classes at two schools in Bristol, UK (25 male, M= 61.4 months, SD= 4.04 months). In every school, all children from each reception classroom were invited to participate unless they had a documented learning disability, behavioural disorder or mental disability. Because the schools were reluctant to release this type of sensitive information about their students, the class teacher applied the exclusion criteria on behalf of the researcher. Of a possible 125 students, 68 children did not participate either because the teacher determined that they did not meet the above criteria or because their parents did not return the consent forms.

3.3.2 Ethics

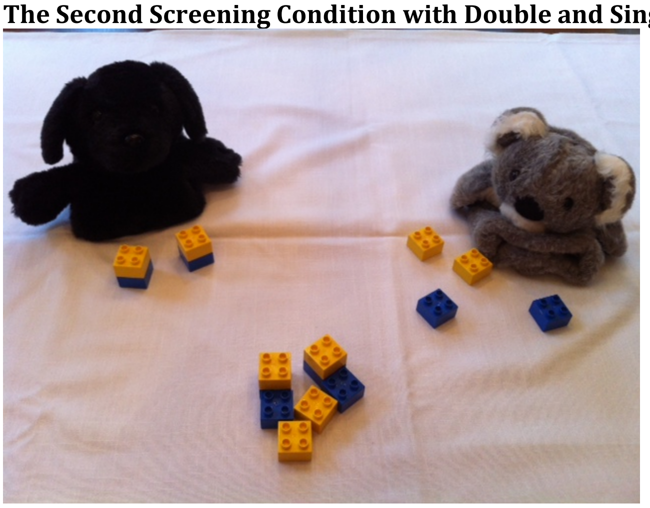
Parents and head teachers were given detailed information about the study before they gave consent to allow children to participate, and were given a chance to contact the researcher for further information or with questions or concerns. The researcher obtained ethics clearance from the Central University Research Ethics Committee (CUREC) at the University of Oxford and a Criminal Records Bureau (CRB) clearance in the UK, and has complied with CUREC guideline 2.1.2, British Educational Research Association (BERA) and British Psychological Society (BPS) ethical guidelines. Only children whose parents returned consent forms were allowed to participate. The largest concern from parents was about who would have access to their child’s data and how it would be used. Parents were reassured that all data would be safely stored in a secure location that only the researcher has access to and only the researcher would view individual children’s data. They were also told that no individual data would be included in this doctoral thesis or any writing or presentations related to this research to protect their child's identity. The researcher was concerned that while parents might agree to allow their child to participate in the research, the child themselves may not want to take part. In order to ensure that only children who wished to take part were included, all children were asked if they would like to participate prior to starting any tasks in order to obtain verbal assent from them.

3.3.3 Screening Conditions

Before receiving any experimental trials, all children were given two screening conditions to ensure they were able to understand how to share all singles and singles and doubles in a very basic way. In the first screening condition only single blocks were used. The researcher introduced two animal puppets by saying "Mr. Giraffe and Mr. Rabbit are very hungry after a long day at school and want a snack. I have some [12 pieces] biscuits here for them to eat. Can you help me share the biscuits between them so it's fair? " Children would then have to share out 12 orange Lego blocks between the puppets. In the second task a 3rd puppet was introduced who also wanted biscuits, so children would have to gather the blocks and share again between the three puppets. Children who did not pass at least 1 of the 2 trials in this condition did not move on to the second screening condition.

The second screening condition used a task from the Frydman and Bryant (1988) study. Children were asked to share out Legos where some of the blocks were stuck together as double units (one yellow and one blue) while others were left as single units (blues and yellows) (see figure 3.1). This trial was given because the researcher wanted to ensure that children who received the experimental conditions had previously demonstrated some ability to share doubles and singles with colour cues to reduce the likelihood that any mistakes made while distributing double and single blocks between characters were due to not understanding the relation between doubles and singles. Prior to starting these trials, children were shown that one double block (blue and yellow) was the same amount as one single blue and one single yellow and were also asked to count one double and two singles to show that they were the same number of blocks. The double and single blocks were shared between 2 animal puppets different from those used in the first screening condition. The researcher introduced the new puppets saying, "These two friends are having a picnic and need help dividing up their treats. They have some sweets to share. Mr. Dog only likes to get the big sweets each time while Mr. Koala only likes to have the little ones, but they both want to have the same amount. Can you help them share their sweets so it's fair? Keep sharing until I tell you to stop." Children were stopped as soon as they had shared 6 single blocks to Mr. Koala who only liked singles. As in condition 1, if children did not correctly complete at least 1 of the 2 trials, they did not complete the 3 experimental conditions because they had not shown the basic understanding necessary to move on to more difficult sharing distributions. All children who passed the screening conditions (at least 2 out of 4 trials) received all 3 experimental conditions regardless of their performance on each one.

Figure 3-1 The Second Screening Condition with Double and Single Blocks



3.4 Design of the Sharing Conditions

The sharing paradigm used in this study was based upon methods used in previous sharing studies with young children, primarily Frydman and Bryant (1988). Children were asked to share blocks between two characters based on different sharing scenarios. Once the blocks were shared, they were asked to make an inference about the number of blocks in one set based on counting the objects in a second, equivalent set.

This study consisted of three experimental conditions: equal sharing, reciprocity and equity. These were compared conceptually in the introduction; the procedure that defined the differences between the tasks operationally is presented in the next section. Each of the three experimental conditions contained 6 questions: 2 questions with all single blocks and 4 with single and double blocks (see table 3.1 for an overview of the design of the sharing conditions). All singles and single and double questions were structured in exactly the same way, except that children were asked to share out either all single blocks or double and single blocks between the two characters.

After half of the double and singles questions, children were asked to make a number inference. After children shared out the blocks, they were asked whether they thought each friend had a fair share as a way of gauging whether they thought they had shared correctly and to ensure that incorrect answers to the number inference question were not simply due to thinking each character had a different amount. If children shared correctly and thought both friends had a fair share, they were given the number inference question. If children made a mistake in sharing and did not think each friend had a fair share, the researcher said "I think it is fair if we do this" and added blocks to make both sets equivalent. If children made a mistake but stated that both friends did have a fair share, they were asked the number inference question. In this instance the sets would not be equivalent, so they could not infer the number of blocks in the covered set from counting the blocks in the first set, but the researcher wanted to see whether children would make this realization. After the first number inference, in order to provide all of the children with the opportunity to make a correct number inference, the researcher corrected the sharing distribution in the same way as the others by saying “I think it is fair if we do this” and adding blocks to make the sets equivalent. For the number inference question, children were asked to count how many blocks the character that received singles had in order to avoid procedural counting errors. Which character received singles was alternated in each trial. The researcher then covered the second pile and asked them to say without counting how many blocks were in the covered set.

The questions within each of the experimental conditions were always presented in the same order:

1. Share all single blocks
2. Share double and single blocks
3. Share double and single blocks then answer a number inference question
4. Share all single blocks
5. Share double and single blocks
6. Share double and single blocks then answer a number inference question

The overall structure of the questions in each condition was the same, and the order in which children received the three conditions (equal sharing, reciprocity or equity) was varied using a Latin square. Completing the screening and the experimental tasks took approximately 30 minutes per child.

The sharing conditions were piloted by the researcher on 19 four and five-year-old children at a school in Oxfordshire prior to their use in this study to make sure young children understood the language used and what they were being asked to do in each condition.

Table 3-1 Design of the Three Sharing Condition

Condition:	Total Number of Trials:	Number of trials with only single blocks:	Number of trials with double and single blocks:	Number of trials with a number inference:
Equal Sharing	6	2	4	2
Reciprocity	6	2	4	2
Equity	6	2	4	2

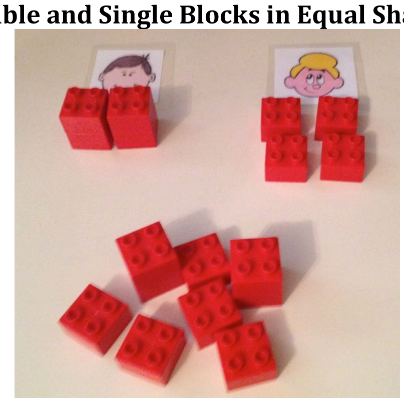
3.5 Procedure

Students were called out of class and seen individually by the researcher in a quiet area designated by the school. All of the materials for the study were brought into the schools and set up by the researcher on a table. Children sat beside the researcher at the table to complete all sharing trials. The researcher followed a script so that each trial was delivered in exactly the same way to each child. For each trial, the researcher recorded whether or not children were able to establish equivalent sets of blocks (a score of 1 or 0), the method of sharing used, whether children thought the distribution was fair, and whether the number inference question was answered correctly (a score of 1 or 0).

The experiment conditions used laminated cards with cartoon children's faces on them to represent the characters in each sharing story. For each sharing task, two of these cards were randomly drawn out of a white envelope by the researcher to represent the two characters in each story. The characters in each task were same-gender pairs of friends and the names were changed to match the gender of each child to avoid any potential issues of preferential sharing due to the gender of the recipient (Berndt, 1981; Leman et al., 2009).

The equal sharing condition was very similar to the second screening condition, except that the Lego blocks were all the same colour so there was no longer a colour cue about how doubles related to singles. Singles-only questions in the equal sharing condition asked children to share out single Lego blocks equally between two characters (similar to the first screening condition). Double and single blocks questions involved sharing double and single blocks equally between two characters; however, children were told that one character only wanted to receive double units, the other character only wanted to receive single units, but both wanted the same amount in the end (see figure 3.2). These questions presented children with scenarios where a one-for you, one for me correspondence strategy would not result in equivalent sets, because the character getting double blocks would receive twice as many as the character receiving singles. Therefore, in this condition children had to share two singles for each double to achieve equal amounts for each character.

Figure 3-2 Double and Single Blocks in Equal Sharing Condition



In the reciprocity condition, children were shown with orange Lego blocks by the researcher how one character shared his or her blocks with a second character. Singles-only trials showed the first character sharing by giving singles to their friend and keeping singles for themselves, resulting in equal amounts. On these trials children were then asked to share the remaining single blocks fairly between the two friends. Doubles and singles trials showed the first character giving doubles to their friend and keeping singles for themselves, thus giving their friend twice as many as they kept for themselves. For these trials, children were asked to share the remaining double and single blocks fairly between the two friends (see figure 3.3). On trials with double and single blocks, children could achieve equivalent shares by inverting the researcher's sharing procedure. In this condition, children had to take into account how the first character shared with the second in order to distribute the remaining blocks correctly.

Figure 3-3 Double and Single Blocks in Reciprocity Condition

The equity condition asked children to base their sharing on two kinds of information: how much work each character had done or how many times each character won a game. In order to remind children of the details of what each character had completed, laminated cards related to the objects in each sharing story were placed beside each character. On trials that asked children to share using only single blocks, they were told that both characters worked or won the same amount, and the same number of laminated cards were laid beside each character. Children were then given all single blocks to share fairly between the two characters. Trials involving double and single blocks told children that one character worked twice as much or won a game twice as many times as the other character. On these trials the researcher placed twice as many cards beside one character as the other. Children were then given double and single blocks and were asked to decide how the blocks could be shared fairly between the two characters (see figure 3.4). For these trials, in order to share fairly, children had to take into account that one character did twice as much as the other and could achieve fair shares by giving double the resources to the character that had worked or won twice as much as the other, for example by giving one double to the character who did more and one single to the character who did less each time.

Figure 3-4 Double and Single Blocks in Equity Condition

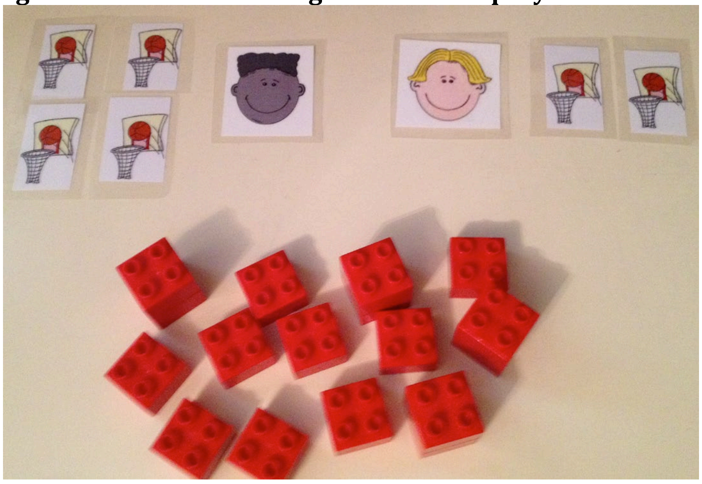


Table 3.2 contains examples of doubles and singles questions and number inference questions from each of the three conditions.

Table 3-2 Examples of Sharing Questions Using Double and Single Blocks

Condition:	Example:
Equal Sharing	Stuart and David want to share crisps. Stuart likes to get large crisps each time while David likes to have little ones each time, but they both want to have the same amount. Can you help them share these crisps so it's fair? (children have a pile of double and single red Legos to share; children must share 2 singles to David and 1 double to Stuart each time). Keep sharing until I tell you to stop. Once children have shared….Do you think each friend has a fair share? <u>Number inference:</u> Can you count how many crisps David has? Can you tell me without counting how many crisps Stuart has?
Reciprocity	Ralph and Tom have some coins to share. Ralph shares his coins with Tom like this: (researcher puts a double orange block by Tom's picture and a single orange block by Ralph's picture, another double by Tom's picture and another single by Ralph's (this is done 3 times)). Now it's Tom's turn to share his coins. Can you help him share with Ralph so it's fair? (children have a pile of double and single red Legos to share and must invert the researcher's sharing procedure to give one double to Ralph and one single to Tom each time). Keep sharing until I tell you to stop. Once children have shared…Do you think each friend has a fair share? <u>Number Inference:</u> Can you count how many coins Ralph has? Can you tell me without counting how many Tom has?
Equity	Kelly and Amy are playing a card game where if you draw a card with a gold star you win a lollipop. Kelly picked 2 gold stars each time Amy picked 1 (researcher places 2 gold stars by Kelly's face and 1 by Amy's (3x)). Now they want to share the lollipops they won. Can you help them share these lollipops so it's fair? (children have a pile of double and single red Legos to share and must give a double to Kelly each time Amy gets a single to give Kelly double the amount) Keep sharing until I tell you to stop. Once children have shared… Do you think each friend has a fair share? <u>Number Inference:</u> Can you count how many lollipops Amy has? Now can you tell me without counting how many Kelly has?

3.6 Results

First the results of the screening conditions will be presented, followed a preliminary analysis of children's sharing scores by gender and by school attended to see whether these factors were related to children's performance on the sharing conditions. This will be followed by an analysis of children's understanding of fairness and equivalence as separate concepts. The predictions are then tested by an examination of sharing scores and the strategies children used to share double and single blocks in each condition. Finally, an analysis of sharing scores compared between four and five-year-olds will be presented, as well as an analysis of children's answers to the number inference questions by condition and by age.

3.6.1 Screening Conditions

All of the children answered both questions in the first screening condition (sharing all single blocks) correctly. On the second screening condition, sharing double and single blocks with colour cues, 10 children answered the first question incorrectly but answered the second question correctly, which suggests that children responded to the feedback given to them after attempting the first question and were able to then implement an appropriate sharing strategy on the following doubles and singles sharing question. 1 child answered the first question correctly but the second question incorrectly. The majority (64%) of mistakes were due to sharing on a one-to-one basis and not accounting for the double block. Two four-year-olds and one five-year-old did not pass the screening condition and did not move on to the experimental conditions. They are therefore excluded from all further analyses.

3.6.2 Analysis of Sharing Scores by Gender and School

It was not anticipated that gender would be related to whether children successfully employed correspondence strategies to share in the equal sharing, reciprocity and equity conditions, but Mann-Whitney tests were conducted to examine this. As expected, the Mann-Whitney tests were not significant $U= 360$, $z= -0.13$, n.s.

for equal sharing, *U*= 319, z=-0.78, n.s. for reciprocity, and *U*= 346, z= -0.31, n.s. for equity. These results support the conclusion that gender is not significantly related to sharing score and therefore further analyses collapsed the data across gender.

It was also not expected that which school children attended would be related to their sharing scores, but Kruskal-Wallis tests were conducted to examine this. School was not significantly related to scores on any of the sharing conditions: *H*(1)= 0.13, n.s. for equal sharing, *H*(1)= 0.34, n.s. for reciprocity, and *H*(1)= 0.22, n.s. for equity. Therefore further analyses collapsed data across schools.

3.6.3 Differentiating Between Fairness and Equivalence

The equity condition presented children with a scenario where fair sharing did not mean creating equivalent sets. This condition examined whether four and five-year-olds understood the difference between the concepts of fairness and equivalence, which can be seen from an analysis of their answers to the number inference question. The percentages of children shown in table 3.3 suggest that the vast majority of children did not accept that it was fair if shares were unequal; however, there were some children who both agreed that the sets were fair after sharing unequally, and gave different numbers for each set on the number inference question. This appears to show a developmental shift in understanding that sharing two unequal sets can still be fair in some situations. Approximately 41% of the five-year-olds thought it was fair that the sets were unequal, whereas the majority of four-year-olds (81%) did not accept that the shares were fair if they were not equal. For most of the children, particularly the four-year-olds in this study, fairness and equivalence were not differentiated. In the older group of children, many more could accept that shares could be unequal and still be fair. This shift in understanding is important for the concept of equivalence and for the understanding of cardinality. This finding is in agreement with previous sharing literature which has found that young children do not tend to use an equity norm to share even when it would be most appropriate to do so, but increasingly see equity as a fair way to share as they get older (Hook and Cook, 1979).

Table 3-3 Percentage of 4 and 5-year-olds who thought Shares were Unequal and Fair or Unequal and Not Fair

	% Fair and unequal	% **Not** fair if unequal
4-year-olds	11	81
5-year-olds	41	52

3.6.4 Performance on the Doubles and Singles Questions by Condition

It was hypothesised that children's ability to share fairly would be affected by the amount of social and quantitative information that had to be considered. It was predicted that children would find sharing all single blocks easy and sharing double and single blocks more difficult because they had to consider the relation between a double and two single blocks each time. Every child except 1 (who answered one question incorrectly) answered all of the questions involving only single blocks correctly in each condition. As expected from their performance on the screening condition, children were at ceiling on these trials. All of them were able to share single blocks fairly regardless of sharing condition. This suggests that increasing the cognitive demands of the task by asking children to share doubles and singles instead of all singles did impact their performance and made it more difficult for children to share fairly in the three sharing conditions. Due to the ceiling effect on the all singles trials the analyses below examine children's sharing using doubles and singles and exclude the singles-only trials.

Tables 3.4, 3.5 and 3.6 show the percentages of children who correctly shared double and single blocks in each condition. It is clear from tables 3.4 and 3.5 that children were at ceiling in the equal sharing condition. Only one four-year-old (who answered only 1 question correctly) and one five-year-old (who answered 3 questions correctly) were unable to answer all four double and singles distribution questions correctly for the equal sharing condition.

The percentages of children sharing correctly on the reciprocity and equity conditions reveal bi-modal distributions. In these conditions the sample is split between children who were unable to share doubles and singles correctly on any questions and children who performed well on all of them.

Table 3-4 Percentage of Children who Shared Doubles and Singles Correctly on 0,1,2,3 or 4 Questions in Equal Sharing vs. Reciprocity Conditions

	Reciprocity					
Equal sharing	**Number correct**	**0**	**1**	**2**	**3**	**4**
	1	1 (2%)	0	0	0	0
	3	0	0	0	1 (2%)	0
	4	15 (28%)	3 (6%)	5 (9%)	13 (24%)	16 (30%)

Table 3-5 Percentage of Children who Shared Doubles and Singles Correctly on 0,1,2,3 or 4 Questions in Equal Sharing vs. Equity Conditions

	Equity					
Equal sharing	**Number correct**	**0**	**1**	**2**	**3**	**4**
	1	0	1 (2%)	0	0	0
	3	0	0	0	1 (2%)	0
	4	28 (52%)	0	2 (4%)	6 (11%)	16 (30%)

Table 3-6 Percentage of Children who Shared Doubles and Singles Correctly on 0,1,2,3 or 4 Questions in Reciprocity vs. Equity Conditions

	Equity					
Reciprocity	**Number correct**	**0**	**1**	**2**	**3**	**4**
	0	10 (19%)	0	0	2 (4%)	3 (6%)
	1	2(4%)	1 (2%)	0	1 (2%)	0
	2	3 (6%)	0	0	0	2 (4%)
	3	3 (6%)	0	1 (2%)	3 (6%)	5 (9%)
	4	9 (17%)	0	1 (2%)	1 (2%)	5 (9%)

Figures 3.5, 3.6 and 3.7 show the distributions of children's sharing scores on the equal sharing, reciprocity and equity conditions. All of the distributions are non-normal. Komolgorov-Smirnov tests confirmed that all distributions were non-normal

D(54)= .532, p<.001 for equal sharing, *D*(54)= .242, p<.001 for reciprocity and *D*(54)= .337, p<.001 for equity. Therefore non-parametric measures were used to examine children's sharing scores.

Figure 3-5 Distribution of Sharing Scores for Equal Sharing Condition

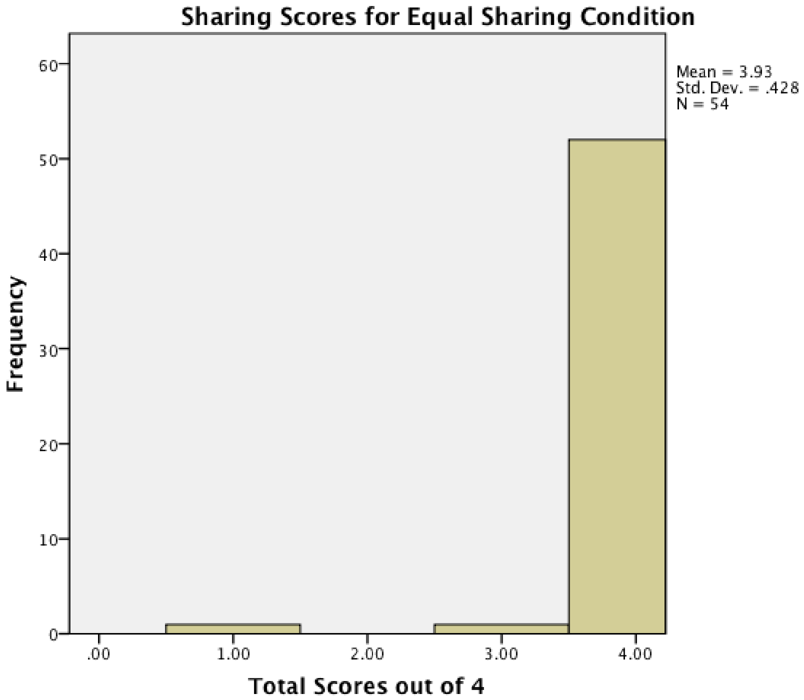


Figure 3-6 Distribution of Sharing Scores for Reciprocity Condition

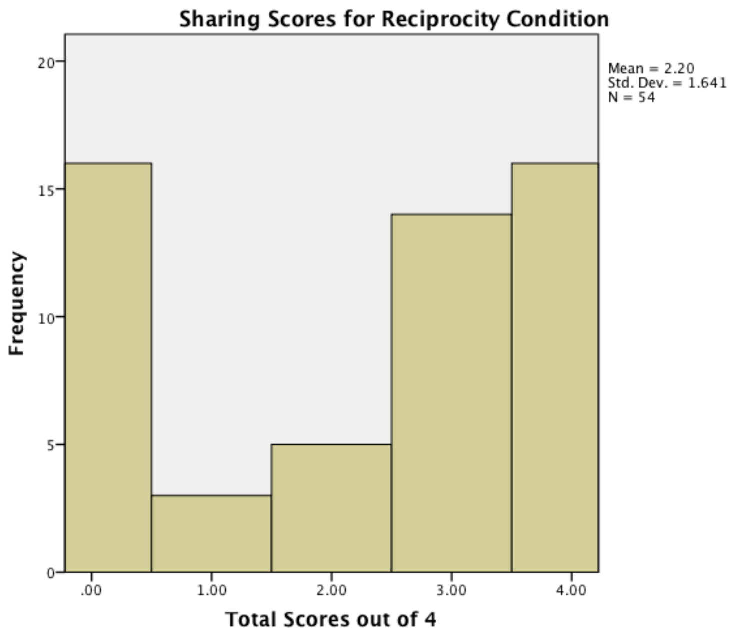
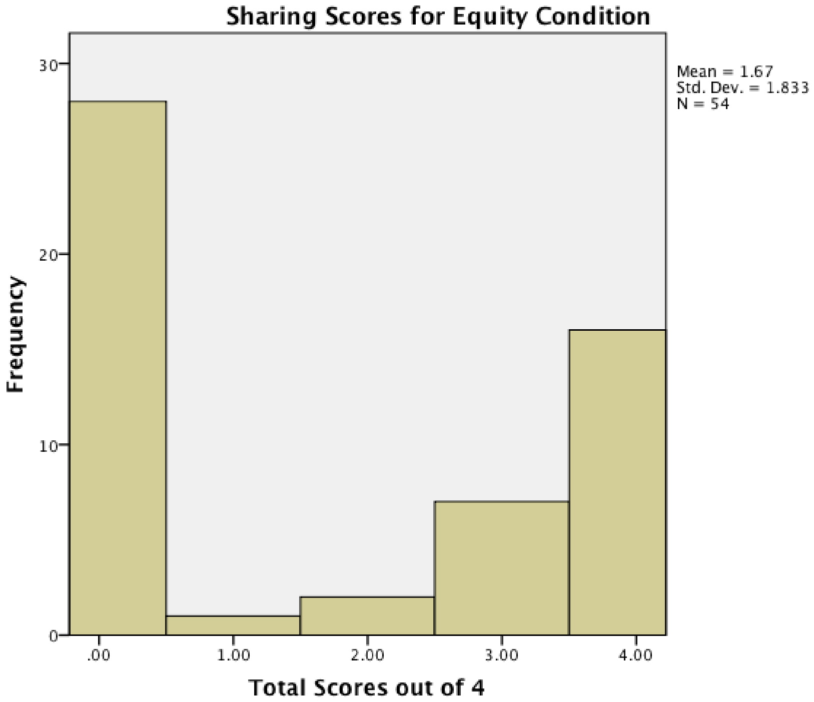


Figure 3-7 Distribution of Sharing Scores for Equity Condition



It was predicted that children would have more difficulty with the reciprocity and equity sharing conditions where they had to consider both the amount of resources to share and the previous actions of each recipient to decide how to share fairly, as opposed to the equal sharing condition where only the amount of resources had to be considered. It was also predicted that the equity condition would be the most difficult of all three conditions because it did not result in equivalent shares at the end.

A Friedman's ANOVA was conducted to see whether the differences in children's sharing of doubles and singles for each condition were significant. The Friedman's ANOVA was significant $\chi^2(2)= 49.53$, $p<.01$. Wilcoxon signed-rank tests were used to follow up this finding. Children performed significantly better in the equal sharing than the reciprocity condition $T= 0$, $p<.01$ and in the equal sharing than the equity condition $T= 1$, $p<.01$. They did not perform significantly differently between the reciprocity and equity conditions $T= 15$, n.s. Thus, overall children shared double and single blocks correctly significantly more in the equal sharing than in the reciprocity or equity conditions, suggesting that as was predicted they did have more difficulty sharing in the latter two conditions.

3.6.5 Sharing Scores on the Doubles and Singles Questions by Age

It was predicted that the five-year-olds would perform better on the sharing conditions than the four-year-olds, as they have had more experience with sharing from having been in school longer and would have more developed cognitive skills. Kruksal-Wallis tests were conducted to examine whether five-year-olds were better at sharing using double and single blocks than four-year-olds in each of the three conditions. No significant differences were detected between four and five-year-olds on any of the three conditions: $H(1)= .001$, n.s. for equal sharing, $H(1)= 2.38$, n.s. for reciprocity, and $H(1)= 3.65$, n.s. for equity (see table 3.7). This suggests that contrary to what was predicted, both four and five-year-olds performed similarly on the double and singles questions in each condition.

Table 3-7 Percentage of Children who Shared Doubles and Singles Correctly on 0, 1-2 or 3-4 Questions in each Condition by Age

Condition:		0 Questions	1-2 Questions	3-4 Questions
Equal Sharing	4 year-olds	0	2	48
	5 year-olds	0	0	50
Reciprocity	4 year-olds	20	7	24
	5 year-olds	9	7	33
Equity	4-year-olds	32	4	15
	5-year-olds	20	2	28

3.6.6 Number Inference Scores

The analyses in this section aimed to examine whether children's ability to answer the number inference question differed according to condition, between age groups and whether the researcher had to correct any mistake in sharing to establish the two equivalent sets.

It was predicted that children's ability to answer the number inference question would reflect the same pattern of results as their ability to share double and single blocks: they would be most successful on the equal sharing condition, then the reciprocity condition and would not be able to make the number inference on the equity condition where sharing did not result in equivalent sets. A Wilcoxon signed rank test compared children's answers to the number inference questions on the equal sharing condition and reciprocity conditions where sharing resulted in equivalent sets. More children were able to make number inferences in the equal sharing condition than the reciprocity condition (see table 3.8), however this difference was not significant T=53, n.s., d=0.16.

Table 3-8 Percentage of Children who Answered 0, 1 or 2 Number Inference Questions Correctly

Condition:	0 correct	1 correct	Both correct
Equal Sharing	30	7	63
Reciprocity	32	19	50

It was predicted that five-year-olds would be better than four-year-olds at answering the number inference questions. More five-year-olds than four-year-olds answered the number inference questions correctly in the equal sharing and reciprocity conditions (see table 3.9). Mann-Whitney tests were run to see whether there was a significant difference in four and five-year-old children's performances on the number inference questions. The Mann-Whitney test for the equal sharing condition was not significant: U= 287.5, z= -1.57, n.s. The Mann-Whitney test for the reciprocity condition was also not significant, but approached significance at the .05 level: U= 279.5, z= -1.61, p=.06. Unexpectedly four and five-year-olds did not perform significantly differently on the number inference questions. However, the trend observed in this case suggests that older children were performing better than younger children on the number inference questions in the more difficult reciprocity condition.

Table 3-9 Percentage of Children who Answered the Number Inference Questions Correctly in each Condition by Age

	Equal Sharing	Reciprocity
4-year-olds	52	37
5-year-olds	74	63

It was also predicted that children's ability to share doubles and singles fairly to construct equivalent sets would be related to their ability to compare equivalent sets. Children whose distributions were first corrected by the researcher would not perform as well on the number inference questions as children who were able to build equivalent sets themselves. This prediction was tested by chi square analyses. In the equal sharing condition it was not possible to draw any general conclusions about whether or not sharing correctly was significantly related to children's scores on the number inference questions because there were so few sharing mistakes made (only 3 children shared incorrectly). Table 3.10 gives the percentage of children who answered the number inference correctly or incorrectly based on whether they shared correctly themselves or if their sharing was corrected by the researcher. This was done to see whether children's sharing ability was associated with their success on the number inference questions in each condition. Similar amounts of children answered the equal sharing and reciprocity number inference questions correctly, however, considerably more children did so after they had been corrected by the researcher in the reciprocity condition than in the equal sharing condition. Nevertheless, the largest percentage of children both shared correctly and answered the number inference question correctly in both conditions. For each of the reciprocity questions there was a significant association between whether or not children shared correctly and whether or not they answered the number inference questions correctly: $\chi^2\,(1)= 12.35$, $p<.001$ for the first question and $\chi^2\,(1)= 13.67$, $p<.001$ for the second question. The odds ratios for the first and second reciprocity number inference questions were 8.57 and 10.34 respectively. Based on these odds ratios, the odds of a child answering the first number inference correctly were 8.57 times higher if they shared correctly than if they shared incorrectly and 10.34 times higher for the second number inference question. It is evident that in both conditions, if children shared correctly, they were more likely to give a correct answer to the number inference question than to give an incorrect answer. The opposite appears to be true for children who shared incorrectly and had the mistake corrected by the researcher before making the number inference, they were more likely to give an incorrect answer. It seems likely that the ability to share correctly was related to the ability to answer the number inferences correctly. There is a logical connection between these two types of knowledge: the ability to understand how to create equivalent sets is related to the ability to understand that two equal sets share the same cardinal.

Table 3-10 Percentage of Children who Answered the Number Inference Questions Correctly Based on Whether they Shared Doubles and Singles Correctly or Incorrectly

Number inference question:	Was a mistake in sharing made?	Were the number inferences answered correctly?	
		Correct	Incorrect
Reciprocity question 1	No	44	9
	Yes	17	30
Reciprocity question 2	No	50	17
	Yes	7	26

3.7 Conclusions and Discussion

This study aimed to examine the quantitative knowledge that children exhibit while sharing in three different conditions: equal sharing, reciprocity and equity. Previous research indicated that four and five-year-olds were able to use counting and correspondence to build equivalent sets, but did not always apply these strategies when asked to compare equivalent sets. Children in this study were prevented from counting in order to focus on their use of correspondences to build sets and infer the cardinal of an equivalent set.

It was hypothesised that children were sensitive to both the social and quantitative demands of sharing. The amount of social and quantitative information was manipulated in each sharing condition to increase the cognitive demands of the sharing tasks. It was predicted that four and five-year-olds' ability to use appropriate sharing strategies would be affected by the amount of social and quantitative information they had to consider in order to share fairly. All of the children were able to share all single blocks regardless of which condition they were in, which suggests that sharing on a one-to-one basis is a familiar and easy way of sharing resources. Both four and five-year-olds appear to have a strong grasp of how to establish equivalence using one-to-one correspondence and can apply this strategy in different sharing contexts. Children's performance sharing using double and single blocks was considerably more varied than with all singles which supports the prediction that they were affected by the increased amount of quantitative information they had to take into account when sharing.

It was predicted that children's ability to share fairly would be affected by the type of sharing condition: the reciprocity and equity conditions required children to consider more social and quantitative information than the equal sharing condition. As was predicted, children's ability to build equivalent amounts using doubles and singles varied by condition. In the equal sharing condition children were allowed to build two equivalent sets at the same time. In contrast, the reciprocity condition presented children with a situation in which they had to build equivalent sets over time factoring in the blocks that had already been shared by the first character to determine how to help the second character share fairly. The equity condition also presented children with a scenario where they had to take additional information into account to decide how to share: how many times each character had won or how much work each had completed. The demands of these latter two conditions were higher than those of the equal sharing condition. Children's ability to share blocks correctly was in line with the increasing difficulty of these conditions: they had significantly higher scores on the equal sharing condition than on the reciprocity or equity conditions. Children were nearly at ceiling in the equal sharing condition. Over half of the four and five-year-olds were able to carry out three or four of the distributions with double and single blocks correctly in the reciprocity condition where the outcome was equal amounts for each character.

It was predicted that the equity condition would be the most difficult for all children because this condition asked them to share doubles and singles into unequal but fair sets, requiring them to think about the relation between doubles and singles in a different way than the reciprocity or equal sharing conditions. However, even in the equity condition 43% of children were able to share doubles and singles correctly on 3 or 4 trials. It was not surprising that children's lowest scores were in the equity condition, because it required children to give characters unequal sets in the end. It was predicted that five-year-olds would perform better than four-year-olds on the equity condition due to their more developed cognitive skills and there is evidence that the older children were beginning to understand equivalence and fairness as separate concepts: 41% of five-year-olds thought that shares could be fair even if they were not equal compared with only 11% of four-year-olds.

The findings from this study suggest that when the amount of social and quantitative information needed to determine how to share fairly in each question was increased, children found it more difficult to share correctly. However, many four and five-year-olds had developed a sufficient understanding of how to share using correspondences between double and single units to answer the majority of questions correctly in each condition. This is in line with the findings of Frydman and Bryant's 1988 study, where the majority of five-year-olds were able to share double and single blocks in a task similar to the equal sharing condition of this study. Many of their four-year-olds were also able to share doubles and singles once they had practice sharing blocks with colours that cued them to the relation between doubles and singles.

The fact that there were many children who were able to share doubles and singles correctly, at least in both the equal sharing and reciprocity conditions, shows that even as young as four and five-years, children have begun to realize that you can use different sharing actions to achieve the same outcome- in this case two equivalent sets. However, preventing children from using counting in order to focus on their knowledge of correspondence excluded a sharing strategy that is very commonly employed by four and five-year-olds when they are asked to share things fairly. Including the use of counting with these sharing conditions in further research would be useful to see how many children use the strategy and to see whether some children rely on counting instead of or in addition to correspondence when faced with the more difficult reciprocity or equity sharing conditions instead of using correspondences, as has been suggested in some previous research (Russac, 1978).

It was hypothesised that there would be developmental differences between four and five-year-olds in their understanding of correspondence and equivalence. It was predicted that age would be a factor that would impact children's sharing scores and ability to infer the cardinal of an equivalent set because five-year-olds have had more practice sharing in school and have more developed cognitive skills. Unexpectedly, there was not a significant main effect of age on sharing scores or on number inference scores. It is possible that the practice children received on the screening condition helped the younger children achieve a similar performance to that of the older children, eliminating any large differences in scores. This idea is supported by findings from the Frydman and Bryant (1988) study where their five-year-olds switched from sharing all singles to sharing double and single blocks easily, but their four-year-olds initially had a great deal of difficulty sharing double and single blocks. After the four-year-olds were given practice with coloured blocks that reminded them of the relation between doubles and singles the four-year-olds were able to share as well as the five-year-olds. It could be that the four-year-olds learned from the screening session using coloured double and single blocks, which then eliminated any differences in performance between the two age groups on the experimental conditions.

It was hypothesised that children's ability to build equivalent sets would be related to their ability to compare equivalent sets. If a child understands how to construct equivalence using counting or correspondence strategies, they have the potential to use these same strategies to compare equivalents sets. If a child lacks the ability to build equivalent sets, it is unlikely that they would then have the knowledge to compare equivalent sets. The idea that knowledge of how to create equivalence is tied to knowledge of understanding the relation between equivalent sets is supported by findings from a study by Muldoon, Lewis and Freeman (2003) who found that children's performance on a set creation task where they were asked to use counting to build equivalent sets predicted their success on a set comparison task where they were asked to compare two equivalent sets. In this study, regardless of age, if children shared correctly, they were more likely to answer the number inference question correctly than to give an incorrect answer. If children shared incorrectly and were corrected by the researcher, they were more likely to give an incorrect answer to the number inference question. While there were too few sharing mistakes made in the equal sharing condition to draw general conclusions about the association between sharing correctly and answering number inferences, chi squares for the two reciprocity questions support the idea that how well children were able to share was related to their ability to answer the number inference questions. This suggests that as was hypothesised, children's ability to use correspondence to create two equivalent sets is related to their understanding of the relation between the two sets, which was reflected in their answers to the number inference questions.

It was predicted that the type of sharing condition would impact children's ability to make an inference about the two equivalent sets based on number. Only children's scores on the equal sharing and reciprocity conditions were considered, because the equity condition did not allow children to make the inference that both sets shared the same cardinal since it resulted in unequal but fair sets in the end. Children's scores on the number inference questions were not significantly different between the equal sharing and reciprocity conditions. The percentages of children able to answer the number inference questions in the equal sharing and reciprocity conditions were similar to the percentage of children who were able to answer the number inference questions in the Frydman and Bryant (1988) study (42%). Frydman and Bryant (1988) asked children in their sample to make an inference after sharing all single blocks, and this study asked children to make an inference after establishing equivalence using double and single blocks. This suggests that contrary to what was predicted, the use of a more difficult correspondence procedure did not significantly impact children's ability to detect the equivalent relationship between the two sets.

The findings from children's sharing on the three different conditions (equal sharing, reciprocity and equity) as well as their answers to the number inference questions raise further questions about children's understanding of correspondence, equivalence, counting and cardinality. First, children's strong performance sharing doubles and singles on the equal sharing and reciprocity conditions leads to a question about how children were creating equivalence: were children truly using only knowledge of correspondence to build equivalent sets or were they using cues from the perceptual difference between doubles and singles to help construct sets? A great deal of previous research suggests that young children have difficulty using correspondence or counting strategies when perceptual cues are very salient (Piaget, 1952; Gelman, 1982; Bryant, 1972). It is possible that at least some children achieved correct sharing scores by using visual cues to match blocks rather than conceptual knowledge of correspondence. Further research removing perceptual cues between doubles and singles should be done to determine whether children's performance on the sharing conditions is impacted by the type of object they are asked to share. Second, as was stated above, children in this study were prohibited from counting in order to solely examine their ability to use correspondences to build and compare equivalent sets; however, this excluded a very common strategy that children employ when asked to share fairly. Including counting as a possible strategy would allow for a comparison of children's performance based on the type of sharing strategy they employed and to see whether allowing the use of counting has any impact on children's ability to share fairly in the different conditions. There is some research which suggests that children will preferentially select a counting or correspondence strategy based on the situation in which they are asked to share (Russac, 1978). The variation between the three sharing conditions provides an ideal context to examine whether children would opt for different strategies to achieve equivalent sets depending on the situation. Third, while many children from both age groups performed well on the sharing conditions and were able to both share doubles and singles correctly and answer the number inference questions, there was a group of children in each condition who could share fairly but who were not able to answer the number inference questions (between 9 and 17%). Frydman and Bryant (1988) found a similar result with some of the four and five-year-olds in their study. They posited that the children in their study who could use one-to-one correspondence to build equivalent sets but could not state the number of objects in one set after counting how many were in the other set showed a difficulty with using number names to describe the relation between equivalent sets. They suggest that while these children clearly knew how to build equivalent sets, they did not know how to express an understanding of equivalence using the terms of cardinal numbers. Much previous research supports the idea that children first base comparisons of sets on perceptual cues before basing their judgments on number which is a more reliable strategy (Bryant, 1972; Brainerd, 1973). The small group of children in this study may have understood how to establish equivalence, but clearly have not connected this knowledge to an understanding of how to express the equivalent relation between the sets using number names. It would be worthwhile to compare children's answers to number inference questions after sharing double and singles and number inference questions after sharing all singles to see whether there are any differences in their ability to compare the equivalent sets. A second study using the same sharing conditions as the current study was conducted to address each of these questions.

4 Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting

4.1 Introduction and Aims

The previous sharing study examined children's ability to use correspondence strategies to build two equivalent sets and their ability to make numerical inferences about the two sets. It was found that most four and five-year-olds were quite good at using double and single blocks to build equivalent shares, and those who were able to build equivalent sets were also more likely to give a correct response to the number inference questions than those who made the inference after the researcher corrected their distributions. However, there was a minority of children in each condition who could share fairly but who could not answer the number inference questions, a result that shows that understanding equivalence may be necessary but not sufficient for understanding that the cardinal for two equivalent sets is the same. This study aims to address several questions raised by the results of the first study.

The first aim of the current study is to address the question of whether some children in the previous study were building equivalent sets using visual cues about quantity (a double is twice the size of a single), rather than using knowledge of correspondence. It is possible that the visual difference between blocks helped children remember how to distribute doubles and singles correctly, as opposed to knowledge of two-to-one correspondence. Thus, their understanding of correspondence could have been over-estimated in the first study. Previous research has shown that young children often use perceptual cues, such as the lengths of two lines or the heights of two piles of objects, to help make judgments about equality and often do so instead of using more reliable strategies like counting or correspondence (Steffe, Cobb, and von Glasersfeld, 1988; Hunting and Davis, 1991). When visual cues coincide with the correct correspondence, children might give the right answer for the wrong reason. In Frydman and Bryant's (1988) study, four-year-olds only performed well on two-to-one correspondence tasks when the double and single blocks were different colours and therefore were perceptually distinct. The study reported in this chapter examines whether removing all visual indications of quantity will make the task of building equivalent sets using double and single units too difficult even for the five-year-olds. In this study the same equal sharing and reciprocity sharing trials with blocks as the previous study will be used, but other sharing trials with Canadian $2 and $1 coins will be included. Canadian $2 and $1 coins are the same size and therefore do not provide a visual indication of the value of each coin. Using coins instead of Lego blocks eliminates the possibility of visually comparing double and single units, which changes the cognitive demands of the tasks. Thus, children's sharing and number inference performance when visual cues are available will be compared with trials where there are no visual cues.

The second aim of the current study is to replicate the trends from the findings of the previous study using a new sample of four and five-year-olds in Kindergarten classrooms from Toronto, Canada. The ability to replicate the results of the previous study with a new sample from a different country is important for examining the generalisability of these findings for four and five-year-olds. A sample from another Western culture was selected because there is some research which suggests that children from Eastern and Western cultures share differently due to differences in the importance of group versus individual needs (Berman, Murphy-Berman and Singh, 1985; Leung and Iwawaki, 1981; Rao and Stewart, 1999). It was expected that the trends observed in the first study would be replicated in the trials using block in this study- children would perform better on the equal sharing condition than the reciprocity condition. Significant differences in the trends were not expected between the UK and

Canadian samples, as the two samples speak the same language, were both drawn from state supported schools, and represent a mixture of social backgrounds. While Canada is a bilingual country, the province of Ontario is not on the whole a bilingual environment and children did not speak French at home. In both the UK and Ontario English was the dominant language in children's lives. While the trends of each sample will be examined, a direct comparison of scores between the samples would be inappropriate because the children went through different screening conditions and thus this will not be made.

The third aim of this study is to examine whether the increased cognitive demands of sharing coins instead of blocks will increase the difference in children's sharing scores on the equal sharing and reciprocity conditions. In the equal sharing task, the equivalent sets are built in a step-by-step manner as children share doubles and singles at the same time to each character. In the reciprocity task, children must invert the sharing procedure that the first character uses to build the equivalent sets. When blocks are used children could use visual comparison to keep track of their sharing, but with coins they need to think about the equivalence between two $1 coins and one $2 coin rather than look at the number of units in front of them. It was felt that coins would be appropriate to use in this context because money has social significance to children: they are exposed to people using coins and discussing the value of different coins in their everyday lives. Some previous studies have also used coins or money in sharing tasks with four and five-year-olds and the children in these studies were able to understand how to use money to solve the tasks (McCrink, Bloom and Santos, 2010; Handlon and Gross, 1958; Pepper and Hunting, 1998; Hook, 1978). Nunes et al. (2007) successfully used a similar sharing procedure to the equal sharing condition of this study, asking children to share UK 2p and 1p coins between two dolls so that they could buy the same amount of sweets. Based on this research it seemed likely that the four and five-year-olds in this study would be able to understand how to use money in this paradigm. However, to maximize the likelihood that children would understand how to use coins of different values in this context, they were given practice trials prior to starting any experimental trials to familiarize them with the relation between $2 and $1 coins.

The fourth aim of this study is to compare children's ability to make the number inference question when the sharing involves only single units versus when it involves singles and doubles. In the previous study children were only asked to make a numerical inference after sharing using doubles and singles. There was a group of children who were able to share double and single blocks correctly but were unable to make the number inference in each condition. These children knew how to build equivalent sets but were not able to express an understanding of the equivalence between sets using number names. They may have had an understanding of equivalent relations between the sets, but the physical dissimilarity of the sets built using doubles and singles caused them difficulty in the number inference task. The question is whether the same children would have succeeded in the number inference task if all the blocks were singles. Virtually all of the children in the previous study found it easy to share all single blocks and the resulting sets are perceptually identical. This group of children, who were able to share doubles and singles correctly but not make the number inference, might be able to make a number inference when the two sets were built using all singles. In this study therefore the children were asked to make number inferences after sharing only singles and after sharing doubles and singles to compare any differences in performance.

The fifth aim of this study is to examine whether the exclusion of counting as a possible sharing strategy had an impact on children's performance in the previous study. Muldoon, Lewis and Berridge (2007) suggest that using counting while sharing could "act as a conceptual bridge between the cardinality of a single set and the way cardinal numbers define the numerical relationship between two sets" (p. 555). Not allowing children to count in the previous study may have caused some children to have difficulty sharing and may have been a factor in why some children were unable to make the number inferences. Children in this sample will therefore be allowed to use counting as a possible method of sharing. This study will examine how children use counting as a sharing strategy compared with correspondence to build equivalent sets in the equal sharing and reciprocity conditions.

Finally, this study aims to explore other variables that may have an impact on children's sharing abilities as well. To investigate this possibility, whether or not a child has siblings will be recorded along with what school they attend and their gender. These variables will be controlled for in the subsequent analyses if a significant relation with sharing performance is found. One study by Hunting and Davis (1991) did find a significant relationship between having siblings and sharing behaviour. In a sample of 21 children, Hunting and Davis (1991) found that having siblings was one of several home and social factors significantly related to how successfully children shared. However, other previous research that has investigated the impact of having siblings on sharing behaviour suggests that, at least for children of school age, having or not having siblings has no significant impact (Dreman and Greenbaum, 1973; Berndt and Bulleit, 1985; Dunn and Munn, 1986). These studies all used observations of children's sharing behaviour at home and at school and focused on the social development of sharing, which is very different from looking at the use of quantitative concepts in sharing in this study and therefore the researcher will obtain further information about whether having siblings is related to children's performance on these sharing conditions.

4.2 Hypotheses and Predictions

It is hypothesised that many four and five-year-old children use visual cues rather than solely counting or correspondence strategies to facilitate sharing fairly. This may have been the case with some children's solutions to the sharing tasks in the previous study. This hypothesis will be tested by increasing the cognitive demands of the sharing tasks through the introduction of Canadian $1 and $2 coins. There is no direct relation between the size of the coins and their value, therefore children will have to base their decision about how to share purely on number. It is predicted that children's sharing scores on the trials using blocks will be higher than scores on trials using coins. It is also predicted that the introduction of the coin trials will increase the difference in sharing scores between the equal sharing and reciprocity conditions on the singles and doubles trials, because children cannot rely on any visual comparisons to create equivalent sets.

It is hypothesised that children's ability to create equivalent sets is related to their ability to compare equivalent sets. In the previous study children were able to share all single blocks easily in all three conditions, whereas their ability to share double and single blocks varied in each condition. The previous study only asked children to make a number inference after sharing double and single blocks. This study predicts that children will find it easier to make a number inference after they share all singles to create equivalent sets compared to making a number inference after sharing using doubles and singles.

Although the findings of the previous study did not support the prediction that five-year-olds would perform better than four-year-olds in terms of building and comparing equivalent sets using blocks, this study does predict developmental differences in children's performance at least on the coin trials. The use of coins instead of blocks increases the cognitive demands of the sharing conditions, and it is predicted that five-year-olds will have stronger knowledge of counting, correspondence and equivalence which they can employ to build and compare sets on the coin tasks.

4.3 Methods

In order to investigate how children perform on trials when the relation between doubles and singles is visible compared to when it is not visible, children were asked to share Canadian $2 and $1 coins (called toonies and loonies, see figure 4.1) as well as double and single Lego blocks in an equal sharing and a reciprocity condition. Using block trials in the equal sharing and reciprocity conditions allowed for an attempt to replicate the trends of the previous study with a new sample.

Figure 4-1 Photograph of a Loonie ($1) and Toonie ($2)

4.3.1 Participants

There are currently over 91 schools in the Toronto District School Board (TDSB) with all-day Kindergarten programs for four and five-year-olds. Participants were recruited only from these schools so that the sample was comparable to the four-year-old children from the UK sample. 40 schools were approached and 5 agreed to participate in the study, giving a total sample of 96 children (42 four-year-olds and 54 five-year-olds, 61 male, M= 63.8 months, SD= 7.29 months) from 10 different classrooms. All children from each Kindergarten classroom were invited to participate unless they had a documented learning disability, behavioural disorder or mental disability. Preliminary discussion about the study with principals and teachers suggested that schools would be reluctant to release sensitive information about their students to the researcher, and so in order to ensure schools felt comfortable participating in the research, the decision to exclude children based on these criteria was left to the class teachers' discretion.

4.3.2 Ethics

As in the previous study, parents and head teachers were given detailed information about the study and a chance to contact the researcher for further information before giving consent to allow children to participate. The researcher obtained ethics clearance from CUREC and a CRB clearance in the UK, as well as ethics clearance from the External Research Review Committee (ERRC) of the Toronto District School Board (TDSB) and clearance from the Toronto Police Service to work with children in schools. As requested by the ERRC of the TDSB, classroom teachers returned a consent form allowing their students to be excused from lessons to participate in this research prior to the start of the study. Similar to the previous study, parents' main concern when considering whether their child should participate in the research was what would happen to their child's data. Parents were assured that individual children's data would only be viewed by the researcher and would be stored in a secure location. Neither classroom nor head teachers were given access to individual children's data, only group summaries of all children who took part in the research. Only children whose parents returned consent forms were allowed to take part in the study. Again, to ensure that only children who truly wanted to take part in the research were included, children were asked if they would like to participate prior to starting any tasks in order to obtain verbal assent from them. This study also complied with CUREC guideline 2.1.2, BERA and BPS ethical guidelines.

4.3.3 Screening Session Design

Due to the large number of questions when both block and coin trials were included, children were seen in two sessions on two separate days. The first day was a screening session including practice trials using the $2 and $1 coins, followed on the second day by a session of the experimental conditions. As the two sessions involved different procedures and types of sharing trials, they are explained in separate sections.

Prior to receiving any experimental conditions, children were given a 30-minute screening session. The aims of this session were to implement the screening trials as in the previous study as well as familiarize the children with the idea that one $2 coin is worth two $1 coins. Two types of practice trials were used:

1. Practice trials using only double or only single blocks and only toonies or only loonies gave children an opportunity to become familiar with all of the sharing objects before the concept of using doubles and singles to build equivalence was introduced
2. Trials in which the children were asked to match a target amount of toonies (double blocks) with loonies (single blocks) (or vice versa) were designed to test whether they understood the relation between $2 and $1 coins and double and single blocks, as this understanding was the key to sharing fairly in the equal sharing and reciprocity conditions in the experimental session.

First, children completed sharing trials using only one type of object: all single blocks, all double blocks, all loonies or all toonies for a total of 4 trials (2 with coins, 2 with blocks). Children were given a pile of blocks or coins (all identical) and were asked to share the blocks (coins) fairly between two puppets. Once the sharing was complete a third puppet joined who also wanted blocks (coins) and the child was asked to redistribute the blocks (coins) fairly between all three puppets. The order in which children received the trials was counterbalanced so that half of the children received coin trials then block trials and half received block trials and then coins. Children first completed trials using all single blocks (or loonies), then all double blocks (or toonies).

Any children who did not pass at least half of these trials sharing all singles did not go further with the screening and were sent back to class, because if they could not establish equivalent sets using one-to-one correspondence between objects, building equivalence with doubles and singles would be too difficult.

The second screening task in this study used exchange trials rather than sharing trials, because there are no visual cues about the relation between doubles and singles when $2 and $1 coins are used. The exchange trials were put in place to test whether children understood that in the questions in which they had to build sets with toonies and loonies they were being asked to make sets based on the value of the $1 and $2 coins rather than the number of coins.

Children were given two sets of exchange trials: one with double and single blocks and one with $2 and $1 coins. For both types of exchange trials (coins and blocks) there was a pair of puppets, one with $2 coins (or double blocks) and one with $1 coins (or single blocks). In the coin trials children were told that the puppet with $1 coins needed $2 coins to buy different things (or vice versa) so she wanted to trade her coins with her friend's. The target number of $2 coins was placed in front of the puppet that owned $2 coins and the child was asked to move over the right number of $1 coins from a pile in front of the other puppet for the exchange to be fair. A similar procedure was carried out when the exchange occurred in the opposite direction: the target number of $1 coins was placed on the table and the child was asked to collect the right number of $2 coins from the pile for the exchange to be fair. The blocks trials were identical to the coin trials except that children were told that the characters wanted to trade double and single blocks. The story about what the pairs of friends wanted to buy changed in each trial to maintain the child's interest in the task.

Before the first block trial with doubles and singles, the relation between one double and two singles was explained. The researcher lay one double on its side (one blue and one yellow block stuck together) and placed 2 singles (one blue, one yellow) beside the double to show the child how two singles were equivalent to one double. The researcher then helped the child to point to and count the amount in one double and two singles saying, “the double is 1, 2 blocks and we have 1, 2 single blocks, so they are the same.” The researcher then handed the child one double block and asked them to show her how many singles were needed to be the same amount as the double. If the child was unsure how many singles were needed, or was incorrect, the child was given the correct answer by the researcher and the explanation of two singles to one double was repeated one more time. An identical procedure was used before the first trial using toonie and loonie coins as well. Children were told that a $2 coin is worth two dollars by the researcher placing 2 fingers on top of the coin while counting 1, 2, and a $1 coin is worth 1 dollar (the researcher placed 1 finger on the coin while saying 1). The researcher then took one $2 coin (with 2 fingers on it) and moved two $1 coins beside the toonie (placing 1 finger on each coin side-by-side) to explain that two loonies are equivalent to one toonie. Children were then given a toonie and asked to show the researcher how many loonies they needed to have the same amount of money. Again if the child was unsure or incorrect they were given the correct answer by the researcher and the procedure was repeated one more time.

There were a total of 20 exchange trials (10 coin and 10 block). A cut-off point of 8 out of 10 trials correct was used for the screening because it is significantly different from chance at the .05 level. If a child was able to answer at least 8 trials in a row correctly it was extremely unlikely that they could have done so by chance. This allowed the researcher to discontinue screening if a child passed the first 8 trials in a row or when it became clear that a child was not going to pass so that they did not become frustrated having to go through all 20 trials. See table 4.1 for an overview of the design of the screening session.

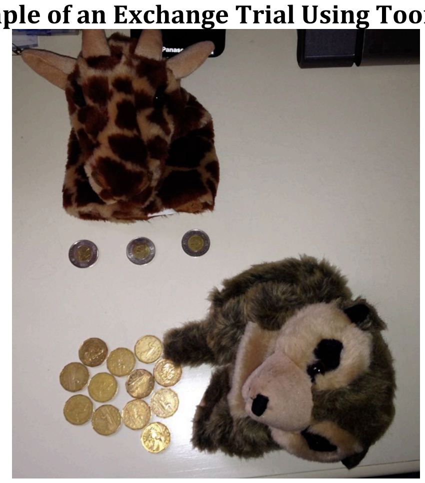
Table 4-1 Design of Screening Session

Condition:	Total Number of Trials:	Number of trials with coins:	Number of trials with blocks:
One-to-one sharing	4	2	2
2-to-1 Exchange	20	10	10

4.3.4 Screening Session Procedure

Students were called out of class and seen individually by the researcher in a quiet area designated by the school (most often a corner of the Kindergarten classroom). All of the materials for the study were brought into the schools and set up by the researcher on a table. The child sat beside the researcher at the table to complete all sharing trials. Children first completed trials sharing all single blocks, all double blocks, all loonies and all toonies between two puppets and then completed the exchange trials. Figure 4.2 is an example of an exchange trial with toonies and loonies. In this trial the giraffe puppet has 3 toonies and the child would have to help the meerkat puppet select the right number of loonies from the pile to be the same amount of money as the toonies that the giraffe puppet has.

Figure 4-2 Example of an Exchange Trial Using Toonies and Loonies



The researcher followed a script to make sure all trials were delivered to each child in the same way. For each child the researcher recorded the child's birth date, gender, whether or not a child had siblings and if they did have siblings, number of siblings. Whether the child correctly shared the blocks or coins (a score of 1 or 0), the method of sharing the child used, and whether they thought each character had a fair share was also recorded. Once all trials were completed (or the child failed to meet the screening criteria by passing less than half of trials sharing all singles or less than 8 out of 10 exchange trials) they were given a sticker and sent back to class.

4.3.5 Experiment Session Design

Children who passed the screening conditions were given a second session lasting approximately 30-minutes, during which they completed all of the questions from the equal sharing and reciprocity conditions using blocks and coins. The total number of trials was 24 (12 with coins, 12 with blocks). Every effort was made to have the second session as soon after the screening session as possible (normally the next day of class unless the child was sick or unable to see the researcher for another reason). This maximized the likelihood that children would remember the information about the relation between doubles and singles that was practiced in the screening session. Regardless of when the second session fell, the researcher always went through the same procedure as before the exchange trials: explaining how many single blocks were the same as a double block and how many $1 coins were the same as a $2 coin. To refresh their memory from the screening session, the child was also asked to show how many singles or $1s were needed to be the same amount as a double block or $2 coin, before starting any experimental trials.

The order in which children received the experimental trials was counterbalanced: half of the children received coin trials then block trials and half received block trials and then coins. Whether they received the equal sharing or reciprocity condition first was also counterbalanced. For each question children were told to keep sharing until the experimenter asked them to stop.

The order of the trials within each condition was always the same:

1. Share all single blocks (one-to-one correspondence)
2. Share double and single blocks (two-to-one correspondence)
3. Share double and single blocks then answer a number inference question
4. Share all single blocks (one-to-one correspondence)
5. Share double and single blocks (two-to-one correspondence)
6. Share double and single blocks then answer a number inference question

There were 6 number inference question per condition (see Table 4.2) to allow for a comparison between children's ability to make the inference that both sets shared the same cardinal after sharing in the singles only and in the singles and doubles conditions.

Table 4-2 Design of Experimental Session

Condition:	Total Number of Trials:	Number of singles-only trials:	Number inferences after singles-only:	Number of doubles and singles trials:	Number inferences after doubles and singles:
Equal Sharing	6	2	1	4	2
Reciprocity	6	2	1	4	2

4.3.6 Experiment Session Procedure

Students were called out of class and seen individually by the researcher. The child sat beside the researcher at the table to complete all sharing trials. The experimental trials used laminated cards with cartoon children's faces (either pairs of boys or girls depending on the gender of the child).

In the equal sharing condition, children were asked to share all single or double and single red Lego blocks as well as all loonies or loonies and toonies fairly between two puppets. Singles-only questions in the equal sharing condition asked children to share out single Lego blocks or loonies equally between two characters. Doubles and singles questions involved sharing double and single blocks or toonies and loonies equally between two characters; however, the child was told that one character only wanted to receive double blocks or toonies, while the other character only wanted to receive single blocks or loonies but both wanted the same amount. In this condition children had to share two singles or loonies for each double or toonie to build two equivalent sets. Figure 4.3 shows an example of the start of a child's correct sharing procedure on the equal sharing condition with toonies and loonies- they have given one toonie to one character and two loonies to the other character resulting in equal shares.

Figure 4-3 Example of the Equal Sharing Condition with Toonies and Loonies

In the <u>reciprocity condition,</u> on the singles-only trials the first character shared by giving a single block or loonie to their friend and keeping a single block or loonie for themselves each time, resulting in equal amounts. On these trials children were then asked to share the remaining single blocks or loonies fairly between the two characters. On doubles and singles trials, children were shown that the first character gave a double block or toonie to their friend and kept a single block or loonie for themselves each time, thus giving their friend twice as much as they kept for themselves. For these trials, the child was asked to share the remaining double and single blocks or loonies and toonies fairly between the two friends. In this condition, children had to take into account how the first character shared with the second in order to distribute the remaining blocks correctly. Figure 4.4 is an example of what a reciprocity with coins trial looked like after the first character finished sharing their toonies and loonies, before the child began to share the rest of the coins. The first character gave a toonie to her friend and kept a loonie for herself each time she shared resulting in her friend having twice as much money. The child now has to determine how to share the remaining toonies and loonies fairly between the two characters so they both end up with the same amount.

Figure 4-4 Example of the Reciprocity Condition with Toonies and Loonies

The researcher continued to follow a script to ensure every trial was identically delivered. Table 4.3 contains examples of equal sharing and reciprocity questions involving double and single blocks or toonie and loonie coins.

Table 4-3 Examples of Doubles and Singles Sharing Questions with Coins and Blocks

Condition:	Example:
Equal Sharing with Coins	Distribution: Frank and Luke want to buy some toy cars. Frank and Luke want to have the same, but Frank needs toonies for his cars and Luke needs loonies for his cars. Can you help them share these coins so it's fair? (child has a pile of loonies and toonies to share.) Keep sharing until I tell you to stop…Do you think each friend has a fair share? Do you think both friends have the same amount of money to buy toy cars? Number Inference: Can you count how much money Luke has? Now can you tell me how much money Frank has?
Equal Sharing with Blocks	Distribution: Francine and Lisa want to share marbles. Francine and Lisa want to have the same, but Francine likes to get 2 marbles, while Lisa likes to get one at a time. Can you help them share these marbles so it's fair? (child has a pile of double and single blocks to share.) Keep sharing until I tell you to stop…Do you think each friend has a fair share? Number Inference: Can you count how many marbles Francine has? Now can you tell me how many marbles Lisa has?
Reciprocity with coins	Distribution: Hannah and Michelle have some coins that they want to use to buy chocolates. Hannah and Michelle want to have the same. Hannah shares her coins with Michelle like this: (researcher gives Michelle a toonie while Hannah gets a loonie (3x)). Now it's Michelle's turn to share her coins. Can you help her share with Hannah so it's fair? (child has a pile of toonies and loonies to share.) Keep sharing until I tell you to stop. Once the child has shared…Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy chocolates? Number Inference: Can you count how much money Michelle has? Now can you tell me how much money Hannah has?
Reciprocity with blocks	Distribution: Ralph and Tom have some sweets to share. Ralph and Tom want to have the same. Ralph shares his sweets with Tom like this: (researcher gives Tom a big sweet (double block) each time Ralph gets a small sweet (single block) (3x)). Now it's Tom's turn to share his sweets. Can you help him share with Ralph so it's fair? (child has a pile of double and single blocks to share.) Keep sharing until I tell you to stop… Do you think each friend has a fair share? Number Inference: Can you count how many sweets Ralph has? Now can you tell me how many Tom has?

After sharing, children were asked two questions: whether they thought each character had the same amount and whether they thought each character received a fair share. These questions were designed to make sure that if children made a mistake on the number inference question it was not simply because they did not think that the two sets were equivalent. Children were asked to make a number inference once after sharing using all single blocks (or coins) in each condition as well as twice after sharing using double and single blocks (or coins) in each condition for a total of three number inferences per condition. If a child made a mistake while sharing, the experimenter said "actually I think they have the same if we do this" and added blocks (or coins) to correct any mistakes and make the distribution even before asking them the number inference. This ensured that all children were asked to make number inferences based on equivalent sets of blocks (or coins). The number inference questions alternated between covering the blocks (or coins) of the character that received doubles and the character that received singles so that children were asked to count a set made up of either singles or doubles. In the block condition children could point to each block contained in a double to establish the amount in the set. On the coin condition, when the child counted doubles they were asked to tap twice on each toonie to establish the amount of money the character had in their set. The researcher corrected any procedural counting errors and guided the child through the counting if it was necessary before asking the child to make the number inference about the second set.

For each trial, the researcher recorded whether or not the child correctly distributed the blocks or coins (a score of 1 or 0) and whether they answered the number inference questions correctly (a score of 1 or 0). The experimenter also made a note of the method of sharing the child used, whether the child thought each character had the same amount, and whether the child thought both characters had a fair share, but these were not used as part of a scoring procedure. In this study children were not prohibited from using counting to build equivalent sets in order to see whether the addition of counting as a sharing strategy impacted the results of children's sharing in the two conditions.

Children were awarded a point for answering the number inference questions correctly if they were able to state the correct number of objects that the second character had without counting the second set. Stating that the second character had more, less or the same amount as the first was not enough for a correct score.

Each child was given a total score for sharing (a score for equal sharing with blocks, equal sharing with coins, reciprocity with blocks and reciprocity with coins) as well as a total score for answers to the number inference questions in each of the experimental tasks. Once the children completed all tasks, they were given a sticker, thanked for their time and sent back to class.

4.4 Results

First the results of the screening conditions will be presented, followed by a summary of preliminary analyses by gender, school and whether or not a child had siblings. Second, an examination of the scores by condition (equal sharing versus reciprocity) will be presented, followed by a comparison of sharing scores in each condition by age group. Third, an examination of whether allowing the use of counting impacted children's ability to share fairly will be conducted. Fourth, a comparison of the trends in the Canadian and UK samples' sharing scores will be presented. Next, an analysis of sharing scores by the type of sharing object (block versus coin) will be presented, follow by a comparison of scores using each type of sharing object by age. Finally, an analysis of the answers to the number inference questions by condition and a comparison of trends in the Canadian and UK samples' number inference scores will be presented.

4.4.1 Screening Conditions

The majority of four and five-year-olds passed all four screening trials using all single blocks or coins. Only four children made mistakes sharing on these trials. Two children made mistakes on one of the two trials sharing coins but passed the other three trials, one child did not pass any trials, and one child passed one block trial but did not pass the other three trials. The two children who did not pass at least half of the all singles sharing trials did not move on to the exchange trials.

On the exchange trials, there was initial concern that the criteria of passing 8 out of 10 trials would be too difficult and too many children would be lost. However, 71 out of 94 children passed 8 out of 10 exchange trials. This suggests that 8 out of 10 trials was not too difficult for many of the five-year-olds and some of the four-year-olds. Many children learned quickly after the first two practice trials and figured out successful strategies to solve the exchange tasks. If one looks at the 23 children who did not pass the exchange trials, 13 did not do well on either type of exchange trial, but the other 10 failed to answer any questions on the exchange trials with coins despite passing 8 out of 10 trials on the exchange trials with blocks. There were no children who passed the exchange trials with coins but did not pass the exchange trials with blocks. Almost half of the four and five-year-olds that failed to pass the exchange trials therefore found the coin trials more difficult than the block trials. The children who passed the exchange trials were able to employ successful sharing strategies in both types of trials, regardless of whether they had to base their exchanges on the value of the coins or the amount of blocks. This suggests that children who were successful on the screening trials were using an understanding of counting or correspondence to answer the questions rather than visual comparisons. Most children who did not pass the screening trials treated both types of coins as the same and exchanged them on a one-to-one basis not taking into account that the value of the $2 coin is twice that of the $1 coin. While the children who passed the screening showed the ability to exchange blocks or coins without the assistance of visual comparisons between doubles and singles, this study wanted to see whether there would be any differences between block and coin trials with the introduction of the more difficult equal sharing and reciprocity conditions where children had to build equivalent sets instead of just exchange pairs of doubles and singles.

Older children were more successful on the screening measure: 45% of four-year-olds and 11% of five-year-olds did not pass the screening and were therefore not given the experimental conditions. They are not included in any further analyses. It is clear that the exchange trials were significantly harder for the four-year-olds than the five-year-olds. The lower pass rate of the four-year-olds could have an effect on the comparison of children's performance between the age groups because the sample of four-year-olds who moved on to the experimental conditions were the most advanced, however it is not anticipated that this difference in passing the screening trials will affect the trends in other comparisons.

4.4.2 Preliminary Analyses

Several variables were analysed to see whether they were related to children's ability to share doubles and singles. It was not anticipated that gender or which school a child attended would influence sharing scores. No prediction was made regarding the impact of having siblings on performance on the sharing conditions.

Mann-Whitney tests were used to examine the effects of gender. As was predicted, gender was not significantly related to sharing scores: U= 507.5, z= -0.56, n.s. for equal sharing and U= 512, z= -0.50, n.s. for reciprocity.

Kruskal-Wallis tests were used to examine the variables of which school the child attended and whether or not a child had siblings. The results for having versus not having siblings were $H(4)$= 4.64, n.s. and $H(4)$= 2.33, n.s. for equal sharing and reciprocity. The results for which school the child attended were $H(4)= 8.81$, n.s. for equal sharing and $H(4)= 5.54$, n.s. for reciprocity.

None of these variables were significantly related to sharing scores, therefore data was collapsed across these variables in further analyses.

4.4.3 Comparison of Sharing Scores by Condition (Equal Sharing vs. Reciprocity)

Whether children were able to attain fair shares in each sharing condition was examined as well as the strategies children used to share blocks or coins. As expected children were at ceiling on the singles-only trials in each condition, regardless of whether coins or blocks were used. All 71 children who passed the screening trials shared correctly on all 8 singles-only trials. Therefore the analyses in all of the sections below consider only children's performance on trials using double and single blocks, except for the number inference section which compares their performance on the number inference question after sharing only singles versus sharing doubles and singles.

The distributions of children's sharing scores were non-normal (see figures 4.5, 4.6, 4.7 and 4.8). If the distribution of scores had been normal, parametric analyses could have been used to analyse differences as well as interactions, such as the interaction between age and type of sharing condition, however the non-normal distributions meant that these analyses had to be run separately. Komolgorov- Smirnov tests for the equal sharing with blocks and coins and reciprocity with blocks and coins distributions were all significantly non-normal ($D(71)= 0.287$, $p<.001$; $D(71)= 0.280$, $p<.001$; $D(71)= 0.266$, $p<.001$; and $D(71)= 0.251$, $p<.001$, respectively.)

Figure 4-5 Distribution of Sharing Scores for Equal Sharing with Blocks

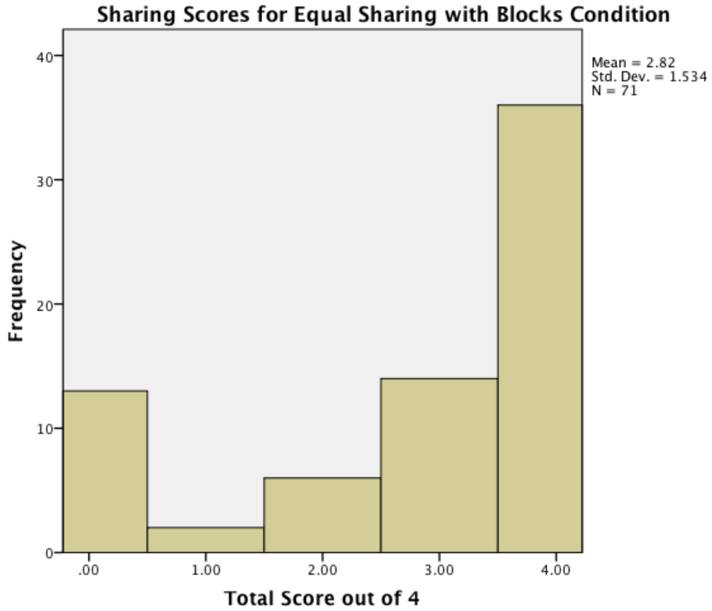


Figure 4-6 Distribution of Sharing Scores for Equal Sharing with Coins

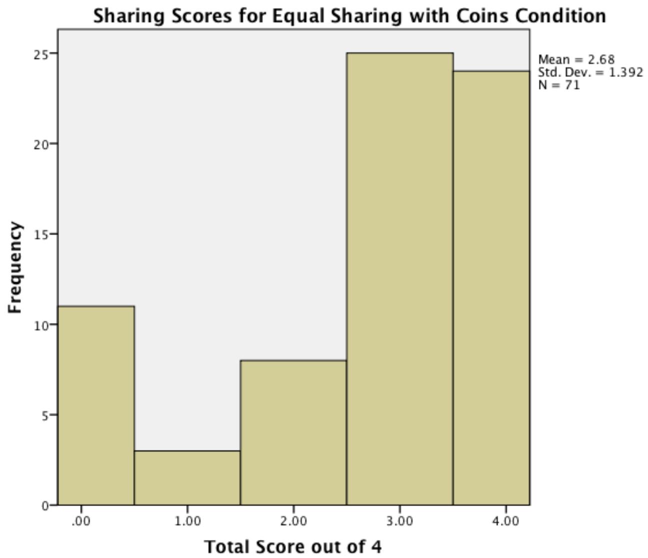


Figure 4-7 Distribution of Sharing Scores for Reciprocity with Blocks

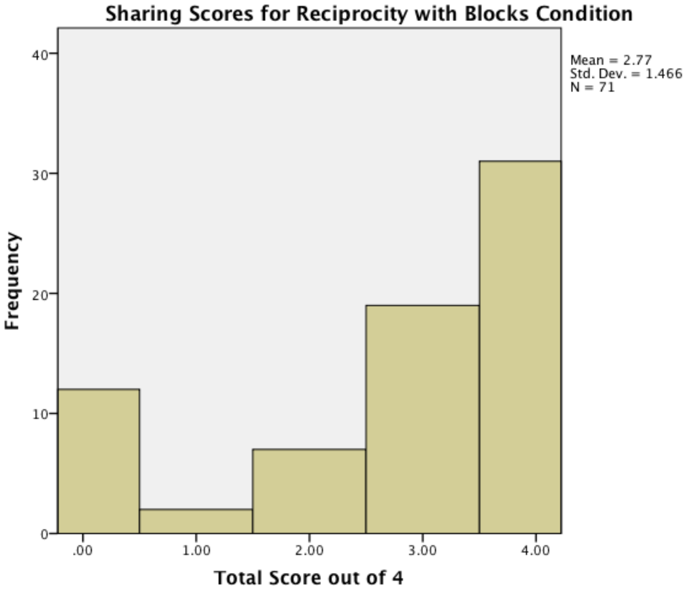
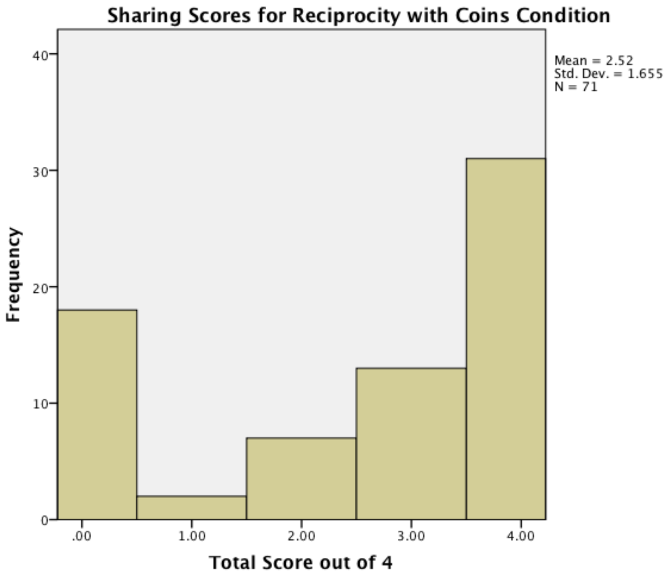


Figure 4-8 Distribution of Sharing Scores for Reciprocity with Coins



The reciprocity condition required children to establish equivalence over time, because they had to factor in the blocks or coins that were shared by the first character when deciding how to share for the second character. In contrast, the equal sharing condition allowed them to establish equivalent sets for both characters at the same time. Thus the demands of the task were higher in the reciprocity condition. It was predicted that the reciprocity condition would be more difficult for children than the equal sharing condition. Table 4.4 shows the percentage of correct sharing scores in the equal sharing and reciprocity conditions. A Friedman's ANOVA was conducted to examine scores by condition. Contrary to what was predicted, sharing scores in the equal sharing and reciprocity conditions were not significantly different $\chi^2(1)= 0.31$, n.s.

Table 4-4 Percentage of Children who Shared Doubles and Singles Correctly on 0-1, 2-4, 5-6 or 7-8 Questions in Each Condition

Condition:	0-1 Questions	2-4 Questions	5-6 Questions	7-8 Questions
Equal Sharing	9.9	16.9	30.9	42.2
Reciprocity	15.5	15.5	22.5	46.4

4.4.4 Comparison of Sharing Scores by Age

It was also predicted that there would be a difference between four and five-year-olds for scores on the two conditions which involved sharing coins because of the increased cognitive demands of building sets based on the value of $2 and $1 coins rather than the number of blocks in doubles and singles. While five-year-olds did not perform significantly better than four-year-olds on the block conditions in the previous study, a comparison of four and five-year-olds scores on these conditions was run to examine whether any differences in performance appeared in this study. Table 4.5 shows the percentage of correct sharing scores in each condition by age group. Kruskal-Wallis tests revealed that contrary to what was predicted, sharing scores for the equal sharing and reciprocity conditions were not significantly different by age: $H(1)= 0.52$, n.s. for equal sharing and $H(1)= 0.24$, n.s. for reciprocity. It is possible that there was no significant difference between conditions by age because the screening conditions eliminated any children who were unable to demonstrate the ability to share all single blocks or coins and make exchanges between double and single blocks and coins. A much larger percentage of four-year-olds than five-year-olds were lost at the screening stage, thereby excluding all but the top performers in the younger age group. It is possible that if the four-year-olds who did not pass the screening had been included, a significant difference would have been found.

Table 4-5 Percentage of Children who Shared Doubles and Singles Correctly on 0-1, 2-4, 5-6 or 7-8 Questions in Each Condition by Age Group

Condition:		0-1 Questions	2-4 Questions	5-6 Questions	7-8 Questions
Equal Sharing	4-year-olds	13.2	21.7	26.1	39.0
	5-year-olds	8.3	14.6	33.3	43.8
Reciprocity	4-year-olds	26.1	17.4	8.7	47.8
	5-year-olds	10.4	14.6	29.2	45.8

4.4.5 The Effect of Counting on Children's Ability to Share

Children in this study were not prohibited from counting, and the researcher found that many children employed counting in the reciprocity conditions. After the first character shared the blocks (or coins) with the second character by giving two to the friend and keeping one for themselves, some children would first count how much the second character had and then give the first character enough single blocks (or $1 coins) to make the shares even. If pressed to keep sharing they would then distribute doubles or singles ($2 or $1 coins) on a one-to-one basis to both characters. The researcher termed this strategy "equalizing". 28 children (39%) used this strategy on one or more trials in the reciprocity with blocks condition and 23 children (32%) on one or more trials in the reciprocity with coins condition. It is noteworthy that many children relied on a counting strategy in the reciprocity conditions, but used a correspondence strategy in the equal sharing conditions. This suggests that children may be able to use either counting or correspondence as strategies for building equivalent sets in the equal sharing condition, but may find correspondence more difficult to apply in the reciprocity condition and thus choose to use counting. A comparison with the previous study was not possible. Children in the UK study were not able to use this "equalizing" strategy because they were not allowed to count.

For this study, it appears that the equalizing strategy helped children share successfully in the reciprocity condition, eliminating any significant differences in performance between reciprocity and equal sharing. This possibility was investigated by

Mann-Whitney tests comparing children's scores on the reciprocity conditions based on whether they used a correspondence or equalizing strategy. Children who equalized in the reciprocity with blocks condition had significantly higher scores than children who did not (Mean= 5.75, Median= 6.00 for equalizers and Mean= 4.14, Median= 5.00 for children who did not equalize), U= 217, z= -4.79, p<.01. Children who equalized in the reciprocity with coins condition also had significantly higher scores than children who did not (Mean= 5.39, Median= 6.00 for equalizers and Mean= 4.10, Median= 5.00 for children who did not equalize), U=334.5, z= -2.83, p<.01. A Wilcoxon signed rank test excluding the scores from children who used the equalizing strategy was run comparing scores on the equal sharing and reciprocity conditions. It approached significance at the .05 level (*T*= 173, p<.09, d=-0.24), which supports the theory that the ability to use counting through the equalizing strategy helped children perform well on the reciprocity conditions.

4.4.6 Comparison of Trends in the Canadian and UK Samples

One of the aims of this study was to see whether the Canadian sample replicated the trends from the data of the UK children. Children in the UK sample performed significantly better on the equal sharing condition than the reciprocity condition. An examination of children's scores on the equal sharing with blocks and reciprocity with blocks conditions of this study (excluding children who used an equalizing strategy) showed that unlike the UK sample, they did not perform significantly differently on the two conditions, although the analysis did approach significance at the .05 level: *T*= 29.50, p<.08. The Canadian sample did not replicate the performance of the UK sample on the equal sharing and reciprocity conditions with blocks, but one can say that there is a trend in that direction.

4.4.7 Sharing Scores Based on the Type of Object Shared (coins vs. blocks)

As was stated above, all of the children performed at ceiling on the singles-only trials and on these trials all of the objects were the same size. Singles-only trials are therefore not included in these analyses. It was thought that some children might have been performing well on the doubles and singles trials because they were using perceptual cues from the size difference of doubles and singles to help create and compare equivalent sets. The trials sharing toonies and loonies were predicted to be more difficult because the difference between the quantities represented by the coins was represented symbolically and could not be verified perceptually. Table 4.6 shows the percentage of correct sharing scores in each condition by type of sharing object (blocks vs. coins). A Wilcoxon signed rank test was conducted to examine this hypothesis. Contrary to predictions, there was no significant difference between sharing scores when children shared using blocks or using coins in each condition (T= 289, n.s. for equal sharing and T= 218, n.s. for reciprocity). This means that children shared equally well whether they could make visual comparisons between the objects or not. However, the screening session eliminated any children who were unable to understand the relation between \$2 and \$1 coins and make exchanges with both blocks and coins. As well, practice working with the coins during the screening trials may have helped some children better understand the relation between doubles and singles in both blocks and coins prior to completing experimental trials.

Table 4-6 Percentage of Children who Shared Double and Single Blocks or $2 and $1 Coins Correctly on 0-1, 2, 3 or 4 Questions

Condition:	0-1 Questions	2 Questions	3 Questions	4 Questions
Equal Sharing with Blocks	21.1	8.5	19.7	50.7
Reciprocity with Blocks	19.6	9.9	26.8	43.7
Equal Sharing with Coins	19.7	11.3	35.2	33.8
Reciprocity with Coins	28.2	9.9	19.6	42.3

4.4.8 Comparison of Sharing Scores Using Blocks or Coins by Age Group

It was predicted that five-year-olds would perform better trials than the four-year-olds, at least on the trials using coins, because they have had more practice sharing in school, and have more developed cognitive skills. Table 4.7 shows the percentage of correct sharing scores in each condition by type of sharing object (blocks vs. coins) and age group. Kruskal-Wallis tests were used to examine this prediction. They revealed no significant differences between age groups $H(1)= 1.09$, n.s. for sharing scores using blocks or for sharing using coins $H(1)= .01$, n.s. Again it is possible that if the children, particularly four-year-olds, with the weakest correspondence and equivalence skills who were screened out prior to the experimental conditions had been included, an age difference with type of sharing object may have been seen as many of these children had more difficulty with the trials using coins than using blocks.

Table 4-7 Percentage of Children who Shared Double and Single Blocks or $2 and $1 Coins Correctly on 0-1, 2, 3 or 4 Questions by Age Group

Condition:		0-1 Questions	2 Questions	3 Questions	4 Questions
Equal Sharing with Blocks	4-year-olds	30.4	13.1	13.1	43.4
	5-year-olds	16.7	6.3	22.9	54.2
Reciprocity with Blocks	4-year-olds	30.4	8.7	13.1	47.8
	5-year-olds	14.6	10.4	33.3	41.7
Equal Sharing with Coins	4-year-olds	21.7	8.7	34.8	34.8
	5-year-olds	18.8	12.5	35.4	33.3
Reciprocity with Coins	4-year-olds	39.1	4.3	13.1	43.5
	5-year-olds	22.9	12.5	20.8	43.8

4.4.9 Number Inference Scores

The number of correct answers each child gave to the number inference questions after sharing using singles only versus the number inference questions after sharing using doubles and singles was compared (see tables 4.8, 4.9, 4.10 and 4.11). It was hypothesised that children would have more difficulty responding after sharing using doubles and singles compared with all singles as the sharing procedure is more demanding: the child has to remember that the objects one character is receiving are worth twice as much as the objects the other character is receiving, thereby using unequal units to construct equivalent sets.

Table 4-8 Percentage of Children with Correct Answers after Sharing Singles-Only vs. Doubles and Singles (Equal Sharing with Blocks)

	Doubles and Singles:		
Singles-Only:	0 correct	1 correct	2 correct
0 correct	7	0	0
1 correct	6	7	80

Table 4-9 Percentage of Children with Correct Answers after Sharing Singles-Only vs. Doubles and Singles (Equal Sharing with Coins)

	Doubles and Singles:		
Singles-Only:	0 correct	1 correct	2 correct
0 correct	7	1	0
1 correct	13	11	68

Table 4-10 Percentage of Children with Correct Answers after Sharing Singles-Only vs. Doubles and Singles (Reciprocity with Blocks)

	Doubles and Singles:		
Singles-Only:	0 correct	1 correct	2 correct
0 correct	7	1	0
1 correct	10	10	72

Table 4-11 Percentage of Children with Correct Answers after Sharing Singles-Only vs. Doubles and Singles (Reciprocity with Coins)

	Doubles and Singles:		
Singles-Only:	0 correct	1 correct	2 correct
0 correct	8	1	0
1 correct	17	4	69

There were several children (between 6-17%) in each condition who were able to correctly answer the number inference after sharing all singles but not after sharing doubles and singles. In each condition, number inference questions were asked after two different trials where children shared doubles and singles and once on a trial after children shared all singles. Using only the first number inference question asked after they shared doubles and singles allowed the children the same opportunity to achieve a correct response after sharing all singles or singles and doubles. Wilcoxon signed ranks tests compared children's answers to the number inference question after they shared all singles and their answers to the first number inference question after they shared doubles and singles to see whether there was any significant difference. The Wilcoxon signed ranks tests were significant $T = 0$, $p<.01$ for equal sharing and for the reciprocity $T = 0$, $p<.01$. Children answered more number inference questions correctly after sharing all singles than after sharing doubles and singles. This suggests that for at least some children, they understood the equivalent relation between the two sets when they shared on a one-to-one basis, but even when they shared doubles and singles correctly they did not seem to recognize the same relation.

It was predicted that age would be a factor in whether children were able to answer the number inference questions correctly. The majority of children answered all three number inferences correctly, but a Kruskal-Wallis test was run to check whether age had an impact on children's scores on the number inference questions. It was significant $H(1)= 5.75$, $p<.05$ and Mann-Whitney tests revealed that five-year-olds performed better than four-year-olds in the reciprocity with coins condition: $U= 341$, $z= -3.18$, $p<.01$ (see table 4.12). There were no differences by age in the ability to share blocks or coins in each condition, but more older children were able to say that the two sets shared the same number in the most difficult condition where they had to both remember how the first character shared to decide how the second character should share and then build the equivalent sets out of unequal units with no visual cues to remind them of the relation between $2 and $1 coins.

Table 4-12 Percentage of Children who Answered Neither or Both Number Inference Questions Correctly after Sharing Singles-Only or Doubles and Singles in Each Condition by Age Group

Condition:		0 Questions Correct (Singles-Only)	2 Questions Correct (Singles-Only)	0 Questions Correct (Doubles and Singles)	2 Questions Correct (Doubles and Singles)
Equal Sharing	4-year-olds	4.3	78.3	17.4	52.2
	5-year-olds	4.2	93.8	8.3	70.8
Reciprocity	4-year-olds	8.7	73.9	26.1	39.1
	5-year-olds	4.2	93.8	8.3	68.8

4.4.10 Trends for Number Inference Scores in Canadian and UK Samples

Children's scores on the number inference questions asked after sharing double and single blocks were examined in each sample to see whether children in the

Canadian sample performed similarly to the UK sample. The majority of children in the UK sample were able to answer a number inference question after sharing double and single blocks in both the equal sharing and reciprocity conditions (63% answered both questions correctly in the equal sharing condition and 50% answered both questions correctly in the reciprocity condition). In this study, after sharing double and single blocks, 80.3% of children answered both questions correctly in the equal sharing condition and 71.8% of children answered both questions correctly in the reciprocity condition. Although trends were not replicated with sharing scores in terms of performing significantly differently on the equal sharing and reciprocity conditions, in the case of number inferences the Canadian sample did perform similarly to the UK sample. In both cases the majority of children were able to make the number inference regardless of condition. In the UK sample the difference in performance on the number inference questions between the equal sharing and reciprocity conditions was not significant. A Wilcoxon signed rank test comparing number inference scores on the equal sharing and reciprocity conditions revealed that the difference in performance was also not significant for the Canadian sample $T = 54$, n.s.

4.5 Conclusions and Discussion

This study aimed to examine children's ability to use counting or correspondence strategies to build equivalent sets when they could not rely on visual cues about the relation between doubles and singles. It also aimed to replicate the trends of the block trials from previous study in a new group of four and five-year-olds. Trials using coins were compared with children's performance on trials using blocks to see whether increasing the cognitive demands of the sharing tasks by asking them to build sets with toonies and loonies would make establishing equivalent sets more difficult. Children's performance was also compared between two different sharing conditions: equal sharing and reciprocity. In the equal sharing condition a pile of blocks or coins was to be divided evenly between two characters. In the reciprocity condition, one character shared with another character by giving their friend doubles and keeping singles for themselves, and then the child was asked to help the second character share back to the first so that both ended up with the same amount.

It was hypothesised that some children in the previous study may have used visual cues based on the size difference of double and single blocks to facilitate sharing fairly. The introduction of the coin trials in this study eliminated the ability to make visual comparisons between the objects being shared because toonies and loonies are not different in size, only in value. The introduction of coin trials increased the cognitive demands of the sharing tasks. It was predicted that the coin trials would be harder for children than the block trials because children would not be able to use a visual cue from the size difference of a double and single block to remind them of the relation between doubles and singles. Unexpectedly, there was no significant difference between children's total scores for the coin trials and block trials within the equal sharing and reciprocity conditions. This means that despite having to remember that one double is the same as two singles without a visual cue, children in both age groups were able to share equally well with coins and with blocks. It was also predicted that the introduction of the coin trials in this study would result in an even larger significant difference in performance between the equal sharing and reciprocity conditions than was seen in the previous study. Contrary to what was predicted, there was no significant difference between children's sharing scores in each condition and both age groups did not perform significantly differently.

The screening session in this study was more demanding than the previous study and excluded any children who could not pass 8 out of 10 exchange trials using coins and using blocks, a statistically rigorous inclusion criterion. This meant that any children who struggled to grasp the relation between double and single blocks and toonies and loonies in the exchange trials did not continue on to the experimental trials, and indeed many of the younger children at the lower end of the performance spectrum did not pass the screening. The idea that the coin trials were in fact more difficult than the block trials for at least these children is supported by the fact that nearly half of the children (43.5%) who did not pass the screening trials were able to pass the 8 out of 10 trials with blocks but could not complete the exchange trials with coins. The 8 out of 10 trial screening criterion was used because the addition of the coin trials increased the cognitive demands of the experimental conditions, and this was an attempt to ensure that the children who made it to the experimental session had enough knowledge of correspondence and the relation between doubles and singles to be able to answer at least some of the experiment questions correctly. However, it also made it impossible to achieve a broader sample of children at all different levels of knowledge. It would be interesting in the future to see how a more randomly selected sample of children would perform on the sharing tasks.

Age differences were predicted because it was thought that the four-year-olds would have more difficulty than the five-year-olds at least with the coin tasks where no perceptual cues could help with building and comparing the two sets. The screening trials proved significantly more difficult for four-year-olds than five-year-olds and as a result 45% of four-year-olds did not continue on to the experimental conditions compared with 11% of five-year-olds. It was predicted that four-year-olds might have more difficulty with the coin trials than the five-year-olds due to the increased cognitive demands of the tasks. Contrary to what was predicted, there were no significant differences between age groups for their ability to share blocks or coins in the equal sharing or reciprocity conditions. Developmental differences in children's ability to answer the number inference questions were also examined. While no significant differences in performance were found between the sharing scores of each age group on the equal sharing and reciprocity conditions, there was a significant difference between four and five-year-olds number inference scores in the reciprocity with coins condition. This was the most difficult condition for children to make a number inference: they had to remember how the first character shared in order to decide how to share fairly and to construct the two equivalent sets out of unequal units. In this condition the older five-year-olds performed better than the four-year-olds on the number inference questions.

This study also aimed to examine whether allowing children to use counting as a possible sharing strategy would have an impact on their ability to share doubles and singles. The sharing strategies that children employed were examined and it was found that 39% used counting to share on at least one trial in the reciprocity with blocks condition and 32% used counting on at least one trial in the reciprocity with coins condition. These children would use counting to establish how many blocks or coins each character had, would distribute single blocks or coins to each character to make the two sets equal and would then continue sharing on a one-to-one basis. The researcher termed this strategy "equalizing". This strategy is based on counting rather than sharing, and children achieved a correct response by evening out one character's set instead of creating a two-to-one correspondence through reversing the sharing actions of the first character. This strategy was an unexpected finding of this study, but nonetheless was an intuitive way of solving the reciprocity problem using counting. It shows that children can solve mathematical tasks using sharing/correspondence based strategies or counting strategies, which suggests great flexibility in their ability to model simple mathematical problems. It would be interesting to further explore children's use of this equalizing strategy in future research. This strategy was not observed in the first sharing study because children were prohibited from counting. Children who equalized had significantly higher scores in the reciprocity condition than children who did not, and an analysis of the equal sharing and reciprocity conditions excluding children who equalized approached significance at the .05 level. This suggests that reciprocity was indeed more difficult for children whose answers were based on sharing using correspondence, but the children who used counting performed very well on the reciprocity condition which eliminated any significant differences between the reciprocity and equal sharing conditions. The appearance of this counting strategy shows that many children chose to use two-to-one and one-to-one correspondences to share in the equal sharing condition, but then chose to answer the more difficult reciprocity questions using counting to establish equivalence rather than correspondence. These children turned the reciprocity sharing task into a version of a give-N task, counting the target number from the larger set and then giving the first character more blocks until they reached the same target number. It appears that, for these children at least, the sharing situation impacts the type of sharing strategy that is most apparent to them. On the equal sharing tasks where they were asked to build two equivalent sets simultaneously correspondence seemed most appropriate, and on the reciprocity tasks where they had to factor in blocks or coins that had already been shared counting seemed more appropriate. This finding is in agreement with the findings of Frydman and Bryant (1988) who found that approximately 20% of their sample opted to use a counting strategy rather than a correspondence strategy when asked to share using doubles and singles or triples and singles to make equivalent sets. These children would allocate a portion of blocks to one character, count that portion, and then allocate the same amount of blocks to a second character. It is clear from this study and that of Frydman and Bryant (1988) that four and five-year-old children are able to use both counting and correspondence strategies to establish equivalent sets, but some may rely on one type of strategy over the other depending on context. When a portion of blocks or coins had already been distributed in the reciprocity conditions, some children found counting a more reliable strategy to build equivalence than correspondence. The equalizing strategy used by some children in this study is not as difficult as implementing a correspondence procedure in the reciprocity condition where the first character's sharing actions would need to be inverted to create equivalent sets, but it represents another way of using number to build equivalence.

This study aimed to replicate the trends in children's sharing scores on the block trials in the equal sharing and reciprocity conditions from the previous study. Children's performance on the block trials in both conditions was compared. The previous study found a significant difference between children's sharing scores on the equal sharing and reciprocity conditions. In this study, children did not perform significantly differently between the two conditions. This sample therefore did not replicate the trends of the previous study. There was no significant difference in the ages of children between the two samples. It is possible that the more rigorous screening condition given to the Canadian sample excluded more children at the lower end of the performance spectrum than did the screening condition given to the UK sample, particularly for the younger age group. The exchange trials between double and single blocks that the Canadian children received were also a different way of familiarizing children with the materials of the study than the two-to-one correspondence trials using colour cues that were given to the UK sample. It is possible that the differences in the screening procedure impacted children's understanding of the relation between double and single blocks resulting in a significant difference in the first sample and none in this sample.

Children's number inference scores after sharing doubles and singles were also compared between the Canadian and UK samples to see whether both groups performed similarly. In the case of the UK sample the majority of children were able to answer the number inference questions after sharing doubles and singles in both the equal sharing and reciprocity conditions. The Canadian children in this sample performed similarly to the previous sample with the majority of children also able to answer the number inference questions after sharing doubles and singles. There was no significant difference between number inference scores on the equal sharing and reciprocity conditions in the UK sample, and this was also true for the Canadian sample. While the Canadian sample did not replicate the trends of the UK sample's sharing scores, both groups did perform similarly on the number inference questions.

Finally, in this study children were asked to make a number inference after sharing all singles and after sharing doubles and singles. It was predicted that children would find making a number inference after sharing all singles easier than making an inference after sharing doubles and singles. There was a minority (between 6-17% in each condition) of children who were able to answer the number inference question after sharing all singles but not after sharing doubles and singles. These percentages are very similar to the percentage of children who were unable to answer the number inferences in the first study (between 9 and 17% in each condition). This suggests that for these children, even though they were able to share all single blocks or coins and double and single blocks or coins correctly, the units they were sharing affected their ability to view both sets as equivalent. They could not flexibly apply their knowledge of equivalence across contexts, and could only see the equivalent relation between the two sets in the singles-only conditions. Once unequal units were used to build the equivalent sets, they no longer thought the sets shared the same number even after agreeing that both sets were fair and equal. This finding is supported by other studies which suggest that children first begin to create equivalent sets using one-to-one correspondence and first learn to understand cardinal numbers in the context of two sets in one-to-one correspondence before being able to extend this knowledge to more difficult tasks like comparing sets that have been built from double and single units (Muldoon, Lewis and Francis, 2007; Bryant, 1996). This finding is also in line with Vergnaud's (1999) idea of a spectrum of ability when it comes to understanding and applying different concepts. Some of the four and five-year-olds in this study had not mastered all aspects of the concepts of cardinality and equivalence. In line with Vergnaud's (1999) theory, it appears children learn and develop different aspects of a concept over time and gradually understand how a concept can be applied in different contexts. These children mastered how to build and compare equivalent sets when all single blocks were used, but could not yet see how the same concept could be applied when double and single units were used. They were not yet able to completely coordinate their knowledge of equivalence, number and cardinality in different contexts.

The majority of children in this study were able to answer number inference questions after sharing all singles or doubles and singles. This is significant, as it means that most children were able to flexibly use different sharing strategies to create equivalent sets, and despite using different means to achieve equivalence, they also demonstrated the knowledge that equivalent sets share the same number name. These children were able to see that the relation between equivalent sets is consistent regardless of the method used to build the equivalent sets. These results clearly show how four and five-year-olds knowledge of equivalence and cardinality progressively increases as they develop.

The current study addressed the possibility that visual cues about the relation between doubles and singles had an impact on children's ability to build equivalent sets and make number inferences. While it was predicted that children would have more difficulty sharing using coins because they had no visual cue about the relation between doubles and singles, children did not perform significantly differently between the equal sharing and reciprocity conditions or when using blocks versus using coins to share. It appears that children were able to construct equivalent sets using correspondence or counting strategies regardless of condition or the type of object they shared. In this study, children who used counting in an equalizing strategy in the reciprocity condition performed better than children who did not use counting. These children raise questions about the role that counting plays in children's understanding of equivalence and cardinality. Previous research suggests that a strong understanding of how to count, while not the sole factor that contributes to a child's understanding of equivalence and cardinality, does play a vital role (Gelman and Meck, 1983; Rodriguez et al., 2012; Freeman, Antonucci and Lewis, 2000). Further examination of how children use counting to build and compare equivalent sets in these sharing conditions is important for understanding the development of their understanding of equivalence, cardinality and correspondence. This study and the previous one have both indicated that if a child was able to build the equivalent sets, most were also able to make a number inference. For the number inference questions, one character's blocks or coins were covered while the other character counted the number of blocks or amount of coins he had. The child then had to state how many blocks or coins the character whose set was covered had. The puppet always counted his set correctly and stated the correct number of blocks or coins that they had. A child could simply repeat this number and achieve a correct response to the number inference question, rather than truly understanding that the two sets were equivalent. The majority of children in this study and the previous study were able to answer the number inference questions regardless of whether they shared all single blocks or double and single blocks and in both the equal sharing and reciprocity conditions. The next study will examine children's understanding of cardinality in two ways. First, it will examine children's ability to coordinate their knowledge of counting principles, particularly the cardinal principle (the last number word represents the total of a set), with ordinality to determine the cardinal of a single set that is counted correctly or miscounted by skipping one block. Second, children will be asked to make a number inference about one set after first watching a puppet count an equivalent set correctly or incorrectly. The miscount trials force children to reason about multiple cardinals in the context of a single set and equivalent sets. Unlike the number inference procedure used in this study and the previous study, where children could obtain a correct response by repeating the last number word the puppet stated, children must use knowledge of counting and equivalence to correct the miscount and obtain the true cardinal of the set in order to make a correct numerical inference.

5 The Relations Between Counting, Equivalence and Cardinality

5.1 Introduction and Aims

This thesis has argued that an understanding of cardinality is critical to the development of young children's mathematical reasoning. This research has defined cardinality as a concept describing the equivalent relation between sets that share the same number. To understand cardinality, children need both knowledge of the equivalent relation between sets in one-to-one correspondence as well as an understanding of how to coordinate counting principles to determine the cardinal of a set (Piaget, 1952; Nunes, Bryant, Evans and Barros, under review). This study will examine both of these aspects of understanding cardinality: children's knowledge of equivalence and their knowledge of how to coordinate counting principles with ordinal information about numbers.

Gelman and Gallistel (1978) proposed five counting principles that guide children's understanding of how to count: only one counting word can be assigned to an item to be counted, count words must be used in a repeatable order, the last number word counted represents the total number of objects in the set, any collection of objects can be counted and the order in which objects are counted does not matter. These five principles teach children how to determine the number of items in one set. However, there is another aspect of learning about counting and number words not addressed by Gelman and Gallistel's (1978) principles that is also very important: ordinality. Children must learn that each number word in the counting sequence represents the cardinal related to the previous number word in the sequence plus one. Knowing that count words must be used in a repeatable order is not the same as understanding that the next number in the counting string is the label of the previous number plus one (for example that 4 is 3+1). Children must develop two different kinds of knowledge about counting: knowledge of the procedures for counting correctly and knowledge of how to use counting to represent things numerically (Nunes, Bryant, Evans and Barros, under review). While many children can demonstrate good procedural knowledge of how to count correctly (Gelman and Gallistel, 1978; Gelman and Meck, 1986; Rodriguez et al., 2012), it is the latter knowledge of how to represent things numerically that is critical for understanding cardinality (Nunes and Bryant, 2015). In order to use counting effectively, children must learn to coordinate their knowledge about ordinality and cardinality to determine the cardinal of a set.

Susan Carey (2004) puts forward a "bootstrapping hypothesis" for how children develop this knowledge. She suggests that children first learn to recite the verbal count words but initially this knowledge is numerically meaningless. Children then begin to create connections between the count word and representations of that number which in turn supports the development of the counting principles (Le Corre and Carey, 2007). This theory depends on the idea that children first learn to connect count words to small sets that they can subitize (one, two, three and then four) and then learn to generalize this rule about counting and apply it to larger set sizes. According to Carey's theory, children eventually realize how to combine two pieces of information they have developed for ordering numbers: that number words are recited in an ordered list and that the models they have developed for cardinal numbers up to number four are related by adding one to the previous set. They are then able to apply this rule (if x has a cardinal of n then the next number in the counting string must be n+1) to other numerals. When children learn to apply this rule they are considered to be "cardinal principle knowers" who know that "a numeral's cardinal meaning is determined by its ordinal position in the list" (Sarnecka and Carey, 2008, p. 665). This idea is controversial. Rips, Asmuth and Bloomfield (2006) and (2008), for instance, argue that the development of such a rule does not intrinsically teach children about natural numbers and children would not know how to apply the rule to count to larger numbers if they did not inherently have knowledge of a counting sequence that takes them beyond the smaller numbers they have already learned. In this way the bootstrap hypothesis presupposes the development of the advanced counting knowledge that it is trying to explain. It is clear from this controversy that there is still much we do not understand about how young children develop an understanding of counting and numbers.

Several studies of cardinality have examined four and five-year-olds' ability to establish the quantity in a single set in relation to their counting abilities (Freeman, Antonucci and Lewis, 2000; Bermejo, Morales and deOsuna, 2004; Fuson, 1988). Freeman, Antonucci and Lewis (2000) conducted a study where a single set of blocks was counted by a puppet who sometimes counted correctly and sometimes miscounted by skipping one block or counting one block twice. They examined whether children understood that a single set has only one true cardinal and if the set is not counted correctly this leads to an incorrect cardinal. They found that children older than four were generally good at identifying when a miscount occurred and many recounted the set to correct the miscount and give the true total of the set. Conversely, children four years old or younger did not perform well on their counterfactual cardinal test, with only about 50% of younger children able to identify and correct the miscount. Bermejo, Morales and deOsuna (2004) also found that when a puppet began counting a set from the number two instead of one, most four to six-year-olds accepted the cardinal of the set produced by the incorrect counting instead of adjusting for the counting error. This research suggests that while children may understand the individual rules for counting, including the cardinal principle (the last number word counted represents the total of the set), if they do not understand the need for all counting principles to be used together and have an understanding of ordinality they do not truly understand the meaning of counting. This research tells us much about children's ability to determine the amount of objects in a set, and trials similar to those used by Freeman, Antonucci and Lewis (2000) will be used in the current study to assess children's knowledge of cardinal numbers in the context of one set. The first aim of the current study is therefore to examine the development of the coordination of cardinal and ordinal information in four and five-year-olds' by investigating their ability to establish the quantity of a single set (of various set sizes from 3-12) when a puppet counts correctly or when the puppet miscounts by skipping one block. When a miscount occurs children will be asked to identify, explain and correct the counting error without recounting the set. Correcting a miscount without counting the set indicates an understanding of the significance of the order of numbers (ordinality) for cardinality.

The above research tells us about children's knowledge of cardinals in the context of one set. However, an examination of cardinality must also include the importance of the property of equivalence. The current study will therefore also look at children's understanding of counting and cardinality in the context of two equivalent sets. Knowing how to count only gives children knowledge about how many items are in one set, whereas establishing equivalent sets and making numerical inferences about the cardinal numbers of equivalent sets requires children to coordinate knowledge of several quantitative concepts. In the current study children will be asked to demonstrate their knowledge of the equivalent relation between quantities by making a numerical inference about one set after first watching a puppet count an equivalent set.

In order to examine whether four and five-year-olds truly can reason about the relation between two equivalent sets, a procedure must be implemented where more than one number is introduced when establishing the total of each set. In the first study in this thesis this was the case in the equity condition where the two sets were constructed based on the amount of work each puppet completed and were not equivalent in the end. In this instance children could not say that the number in both sets was the same. In the other two sharing conditions where sets were equivalent in the end it was possible for children to achieve a correct response on the number inferences simply by repeating the last number word stated by the puppet when he counted his set. This raises the question of whether the previous number inference tasks were truly tapping children's understanding of equivalence. While the equity condition proved too difficult for four and five-year-olds, other research into children's understanding of the relations between quantities has successfully used a miscount procedure to include both correct counting trials and trials in which the first set is counted incorrectly to produce an improper total for the set before asking children to make a number inference about a second set. The miscount trials force children to reason about two different cardinals for the first set: the one produced by the puppet's miscount and the true cardinal of the set before making the numerical inference. Muldoon, Lewis and Freeman (2003) examined whether children's ability to identify and correct a miscount was related to their ability to create and compare equivalent sets. They found that children who could identify miscounts and understood what miscounts meant for the total of a single set performed significantly better on a set creation task. These same children in turn performed better on a set comparison task than children who were unable to identify and correct a miscount. This study will use a task similar to their miscount task. After asking children to construct two equivalent sets, a puppet will count one set either correctly or make a mistake by skipping one block before the child is then asked to determine the amount of blocks in the second, equivalent set. The use of both a single row task and equivalent set tasks addresses the second aim of this study, which is to investigate whether children find it easier to coordinate counting principles to determine the cardinal of a single set or to determine the cardinal of an equivalent set. If four and five-year-olds show that their knowledge of cardinality is restricted to obtaining the cardinal of a single set, this means that they have not yet achieved the understanding that sets in one-to-one correspondence are equivalent and share the same cardinal.

Knowing the procedure for counting and understanding the information that counting gives you with respect to cardinality are two different but related matters. This study aims to examine both knowledge of how to establish the quantity in a set using counting as well as knowledge of the relation between quantities in the same children. The current study will examine four and five-year-olds' ability to identify, explain and correct a miscount when a puppet skips one block while counting single sets of different sizes to see whether they understand how to coordinate counting principles and ordinal information to determine the cardinal of a single set. The current study will also investigate four and five-year-olds' ability to coordinate counting principles and knowledge of equivalence to infer the number of one set after first watching a puppet count an equivalent set either correctly or incorrectly. Children's ability to detect, explain and correct a miscount to obtain the total of single set and infer the cardinal of an equivalent set will provide a clear indication of their ability to coordinate knowledge about counting principles and ordinality with their knowledge about equivalence, all of which contributes to an understanding of cardinality.

5.2 Hypotheses and Predictions

It is hypothesised that making an inference about the cardinal of a single set requires coordinating knowledge of counting principles and ordinality. It is further predicted that a counting error will make it more difficult for children to coordinate this knowledge to establish the total of a single set than if the set is counted correctly. It is also predicted that the miscount trials will be more difficult for four and five-year-olds than the correct count trials. It is also predicted that children who can identify that a counting error has occurred and explain why the counting was incorrect will be more likely to give the correct cardinal for a single set than children who cannot identify and explain the mistake.

It is hypothesised that coordinating knowledge of equivalence and counting principles to infer the cardinal of an equivalent set is a greater cognitive demand than coordinating knowledge of counting principles and ordinality to determine the cardinal of a single set. It is predicted that children will perform better when they are asked to determine the cardinal of a single set than when they are asked to infer the cardinal of an equivalent set.

In three conditions in this study, children will be asked to share different sized units to create two equivalent sets. In order to share fairly in these conditions, children have to remember the relation between the large and small units (for example double blocks and single blocks) as well as monitor the amount in each set. Differences in children's ability to share fairly are predicted based on the type of units they are asked to share. It is predicted that the triples and singles condition will be significantly more difficult than the doubles and singles or all singles conditions because they have to remember the relation between a triple unit and three single units. No significant difference is predicted between the all singles and doubles and singles conditions based on the performance of children in the previous two studies.

It is hypothesised that children's ability to coordinate knowledge of counting principles and ordinality, as well as other quantitative concepts like equivalence and cardinality develops over time. Significant differences are predicted between five-year-olds' and four-year-olds' ability to both establish the cardinal of a single set and to infer the cardinal of an equivalent set.

5.3 Method

5.3.1 Participants

The five schools that participated in the previous study were approached about conducting a follow-up study. Three of the five schools agreed to take part in this study, and a fourth school was recruited through contacting the principals of several other schools within the TDSB with an all-day Kindergarten program for four and five-year-olds. This resulted in a sample of 87 children (38 four-year-olds and 49 five-year-olds, 44 male, M= 61.0 months, SD= 6.52 months) from 9 different classrooms. All children from each Kindergarten classroom were invited to participate unless they had a documented learning disability, behavioural disorder or mental disability. As in the previous study, the decision to exclude children based on these criteria was left to the class teacher's discretion.

5.3.2 Ethics

As in the previous study, both parents and head teachers were given detailed information about the study and a chance to contact the researcher for further information or with questions or concerns before giving consent for children to participate. The researcher obtained ethics clearance from CUREC and a CRB clearance in the UK, as well as ethics clearance from the External Research Review Committee (ERRC) of the Toronto District School Board (TDSB) and clearance from the Toronto Police Service to work with children in schools. Again, as requested by the ERRC of the TDSB, classroom teachers returned a consent form allowing their students to be excused from lessons to participate in this research prior to the start of the study. This study had fewer parent concerns about the safety of their child's data, likely because three of the four schools took part in the previous study. Classroom and head teachers were able to discuss the research in greater detail with any new parents who had not been asked to give consent for their child to take part in the previous study. Only children whose parents returned consent forms were allowed to take part in the study.

To satisfy the researcher's concern that only children who wished to be a part of the research participated, children were asked if they would like to participate prior to starting the study in order to obtain verbal assent from them. This study also complied with CUREC guideline 2.1.2, BERA and BPS ethical guidelines.

5.3.3 Design

This study had four different conditions. The first condition used identical trials to those in the Freeman, Antonucci and Lewis (2000) study. In these trials a single row of blocks was constructed for children by the researcher and then a puppet (Mr. Dog) counted the row of blocks. The other three conditions asked children to share all single, double and single or triple and single blocks between two characters (Mr. Dog and Mr. Koala) to create two equivalent sets. The triples and singles condition was added because the majority of children were able to create equivalent sets in the singles-only and doubles and singles conditions in the previous studies, so this study aimed to see whether four and five-year-olds could extend their knowledge of creating equivalence to even larger numbers.

There were 8 trials in each of the four conditions: 4 when Mr. Dog miscounted and 4 when he counted correctly and four different set sizes per condition (4, 6, 8 and 10 for the single row, all singles and doubles and singles conditions and 3, 6, 9 and 12 for the triples and singles condition). This makes a total of 32 trials (see table 5.1). The order that children received the single row and sharing conditions was randomized. The order of correct count and miscount trials within each condition was also randomized.

Table 5-1 General Design

	Number of trials	
Condition:	Miscount	Correct Count
Single row	4	4
Share all singles	4	4
Share doubles and singles	4	4
Share triples and singles	4	4

In the three conditions where children shared blocks to create two equivalent sets, they were asked to make a number inference about one set after the puppet first counted an equivalent set. The current study will modify the number inference task from the one used in the previous two studies to include trials where the puppet makes a mistake when counting his set. In order to make the numerical inference in the previous studies one set was counted while the second set was covered so it could not be seen or counted. It is possible that instead of demonstrating an understanding of the equivalence of the two sets, children's strong performance on the number inference questions in the previous two studies was simply due to them repeating the only number that was mentioned as the total for a set. The miscount trials offer a way to separate children who understand the equivalent relation between the two sets and children who are basing their answers on the last number word stated.

Prior to starting any trials, children were asked to count on a counting line consisting of two rows of 10 smiley faces on a page. They were asked to count to as high a number as they could reach up to 20. This was done in order to be certain that the numbers used in the study were within the counting range of these four and five-year-olds. It was not anticipated that many children would have difficulty counting, both because counting is practiced frequently in Kindergarten classrooms and also because previous research by Miller and Stigler (1987) indicated that 80% of their sample of four-six year-olds could count to 20. If children were able to count to at least 12 (the highest number used in the study) with no problems, they went on to the experimental trials.

5.3.4 Procedure

In the single row condition, the procedure was virtually identical to that of Freeman, Antonucci and Lewis (2000). The researcher told children that her friend Mr. Dog wanted some sweets (or other object, varied by trial). She then laid out 4, 6, 8 or 10 identical blocks in front of the puppet. The researcher made sure the blocks were evenly spaced so children could easily see if the puppet made a counting mistake by skipping one block. There was one correct counting and one miscount trial for each set size (4, 6, 8 and 10).

For the all singles, doubles and singles, and triples and singles conditions children were asked to share blocks between two characters and were then asked to make a number inference about the second set after first watching Mr. Dog count his set. The sharing procedure for these conditions was the same as in the equal sharing with blocks condition from the other two studies. Children were asked to share different numbers of blocks equally between Mr. Dog and his friend Mr. Koala. In the all singles and doubles and singles conditions children were asked to create sets of 4, 6, 8, or 10. In the triples and singles condition children were asked to create sets of 3, 6, 9 or 12. As in the previous two studies, the researcher pretended the blocks were sweets or other objects that Mr. Dog and Mr. Koala wanted to share fairly. Children were stopped after they shared the correct number to each character. If children made a mistake in sharing, the researcher said “I think it’s fair if we do this”, and added or took away blocks as necessary. Identical to the single row condition, the all singles, doubles and singles and triples and singles conditions also had both one correct counting and one miscount trial for each set size.

In the three sharing conditions, after children finished sharing blocks between Mr. Dog and Mr. Koala, Mr. Koala's blocks were covered by a large paper so children could not see or count them. The researcher also made sure the blocks ended up evenly spaced and that children agreed that both characters had a fair share and the same amount before continuing to the number inference question. This made sure that no incorrect answers to the number inference questions were given simply because they did not think both characters had the same amount.

In the three sharing conditions, Mr. Dog counted his set after Mr. Koala's blocks were covered. In the single row condition Mr. Dog counted his set after the researcher set out the target number of blocks. For all four conditions, the procedure for correct counting or miscounting was kept as close to the original script from Freeman, Antonucci and Lewis (2000) as possible. The researcher said, "Mr. Dog wants to know how many blocks he has, so he is going to count his blocks now. Sometimes he does a good job and counts right but sometimes he makes mistakes and counts wrong, so watch carefully when he is counting. When he is finished I want you to tell me if his counting was ok or not ok." Mr. Dog then counted his blocks at 1 item per second. If the trial was a miscount trial, one block was skipped during the counting. The block that was skipped was either the first or last block in the set, and was alternated in each trial. The first or last block was chosen to make it more likely that children saw the counting mistake being made. If the trial was a correct counting trial, Mr. Dog simply counted all the blocks correctly. In the doubles and singles and triples and singles conditions, whether Mr. Dog received double, triple or single blocks was alternated so that on half of the trials he counted the double (or triple) blocks and half of the trials he counted the single blocks. In order to keep the counting procedure consistent across all trials and to make sure that obtaining the correct number of blocks for the set was not too difficult for four and five-year-olds, on the trials in which Mr. Dog received double or triple blocks the procedure for miscounting remained the same as with single blocks. Mr. Dog only skipped one of the blocks that made up the double or triple unit, rather than skipping two or three blocks at a time (again either the first block or the last block of the set), so children would have to say the next number in the counting string to obtain the correct number of blocks in the set.

In all four conditions, after Mr. Dog finished counting his set, children were asked a series of three questions. They were always asked first, did Mr. Dog count ok? Then children were asked how many blocks Mr. Dog thought he had, and how many blocks there were in the set. Freeman, Antonucci and Lewis (2000) were worried about mentalistic burdening by asking the child to think about the puppet's state of mind with the question "how many do you think he has?". However, they found no association between a theory of mind task and children's performance on this task, so empirically it seemed acceptable to use the question in this research. They suggested that children may actually interpret the question as "how many did the puppet get it to come out as" rather than actually considering the puppet's state of mind. Still, future research should examine alternative ways to ask children about the counterfactual cardinal.

When children were asked about the true and counterfactual cardinals of the set, on correct count trials they should give the same number for both questions and on miscount trials Mr. Dog should have one less block than the true cardinal of the set because he skipped one. The order of the latter two questions was alternated so on half of the trials children were first asked how many blocks there were in the set and then how many blocks Mr. Dog thought he had and on the other half the order of the two questions was reversed. Children were prohibited from recounting Mr. Dog's set to establish how many blocks there were because the researcher wanted them to correct the counting error without counting. On the miscount trials, if children answered correctly that Mr. Dog did not count ok, they were asked an additional question: what did Mr. Dog do wrong, to see whether they could explain his counting mistake before being asked how many blocks were in the set and how many blocks Mr. Dog thought he had. If children said they could not remember how many blocks Mr. Dog counted on the miscount trials because of the intermediary question- what did he do wrong- the researcher asked Mr. Dog to recount the blocks in exactly the same way as the first time before asking children again how many blocks there were and how many Mr. Dog thought he had.

In the three sharing conditions (all singles, doubles and singles and triples and singles), after children answered the questions about Mr. Dog's counting and how many blocks were in the set, children were asked a number inference question. This followed the same procedure as in the first two studies: the researcher first double-checked that children agreed that Mr. Dog and Mr. Koala had the same amount in order to make sure that no incorrect answers were given due to forgetting how many blocks Mr. Dog had or thinking the characters had different amounts. The researcher then pointed to the paper covering the second character's blocks and asked children how many blocks Mr. Koala had. On both the miscount and correct count trials, a correct number inference score was only achieved if children were able to state the correct number of blocks in Mr. Dog's set and inferred the correct number of blocks in the second character's set as well. A number inference was not marked as correct in a miscount trial if children accepted the cardinal that Mr. Dog produced through skipping a block as the cardinal for his set and inferred that the second set had that number as well. Children were asked to make a number inference on a miscount trial and a correct count trial for each set size, which makes a total of 8 number inferences in each condition.

5.4 Results

First a brief overview of children's performance on counting items in a line will be presented, followed by a summary of preliminary analyses by gender and school. The remaining analyses address the main hypotheses of this study. Children in this study were asked to make two different kinds of inferences: one about the number of a single set when counting was either done correctly or a mistake was made, and the second about the cardinal of one set based on knowing the cardinal of an equivalent set that was either counted correctly or incorrectly. The analyses in the first section investigate whether a miscount made it more difficult for children to determine the cardinal of a set, whether being able to explain why someone's counting actions were incorrect impacted their ability to state the correct cardinal of a set and whether set size had any impact on children's ability to determine the cardinal of a single set. The analyses in the next section examine whether making an inference about two equivalent sets involves a greater cognitive demand than the coordination of counting principles and ordinal information to establish the total of a single set. The final sections contain further exploration of the data examining whether children's ability to share fairly was impacted by the whether they were asked to share single units or units of different sizes, as well as whether performance on the single set inference and equivalence inference tasks differed between age groups.

5.4.1 Counting Items in a Line

12 four-year-olds and 7 five-year-olds had problems counting after they reached 15, either confusing or omitting number names, but no children had to be excluded from the study because they could not count to 12 without making a mistake. Therefore the researcher could be confident that all of the set sizes used in this study were within the counting range of all the four and five-year-olds who participated.

5.4.2 Preliminary Analyses

It was not anticipated that gender would have an effect on children's scores, but Mann-Whitney tests were conducted to investigate this. As predicted, gender was not significantly related to total scores: $U= 911$, $z=-0.32$, n.s. for the single row trials, $U= 832.5$, $z= -0.98$, n.s. for the all single sharing trials, $U= 859$, $z= -0.75$, n.s. for the doubles and singles sharing trials, and $U= 900$, $z= -0.39$, n.s. for the triples and singles sharing trials. Therefore data is collapsed across this variable in further analyses.

It was also not anticipated that the school children attended would be related to their scores in each condition, however it was important to examine this variable. While the majority of children in this sample were different children from those in the previous study, there were three schools that participated in both the previous study and this study. Some of the children who were four-year-olds in the previous study were now five-year-olds in the same classrooms and took part in this study. It was therefore important to examine whether there were any differences in performance between schools that had taken part in the previous sharing study and schools that had not. Kruskal-Wallis tests were conducted. As predicted school was not significantly related to total scores: $H(3)= 4.06$, n.s. for the single row trials, $H(3)= 2.68$, n.s. for the all singles sharing trials, $H(3)= 3.78$, n.s. for the doubles and singles sharing trials, and $H(3)= 5.92$, n.s. for the triples and singles sharing trials. Based on these results, one can be confident that there were no significant differences in performance between children from the three schools who previously participated in the sharing study and children with no prior experience with this sharing paradigm. Data is collapsed across this variable in subsequent analyses.

The subsequent analyses examine the hypotheses in this study. The children were asked a set of questions that must be analysed separately in order for the research questions to be examined. In the first section, the analyses aim to establish whether children found it easier to state the cardinal of a set counted by a puppet than state the cardinal for an equivalent set. A comparison will be made between children's answers on the single row task and children's answers when they were asked to build and compare two equivalent sets. In both tasks children's answers were based only on single units. Sometimes the puppet counted the set correctly and sometimes he skipped one block. In the second section, the analysis examines whether counting single, double or triple units affected children's ability to state the correct cardinal for the set either when the puppet counted correctly or made a mistake. This analysis considers children's responses to the question "how many blocks are there?" after children shared all single, double and single or triple and single blocks. This analysis does not include the single row trials where no sharing took place.

The distribution of children's scores on each of the conditions: single row and the three sharing conditions: all singles, doubles and singles and triples and singles were non-normal (see figures 5.1, 5.2, 5.3 and 5.4). Komolgorov-Smirnoff tests for each of the four conditions were also significantly non-normal: $D(87)= .29$, $p<.001$ for the single row trials, $D(87)= .16$, $p<.001$ for the all singles sharing trials, $D(87)= .15$, $p<.001$ for the doubles and singles sharing trials, $D(87)= .11$, $p<.05$ for the triples and singles sharing trials. Therefore non-parametric analyses were used.

Figure 5-1 Distribution of Scores for Single Row Trials

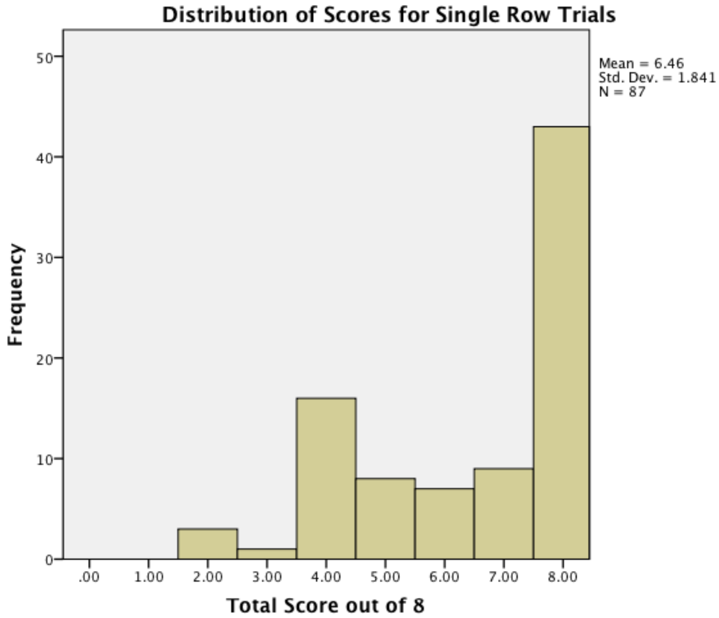


Figure 5-2 Distribution of Scores for All Singles Sharing Trials

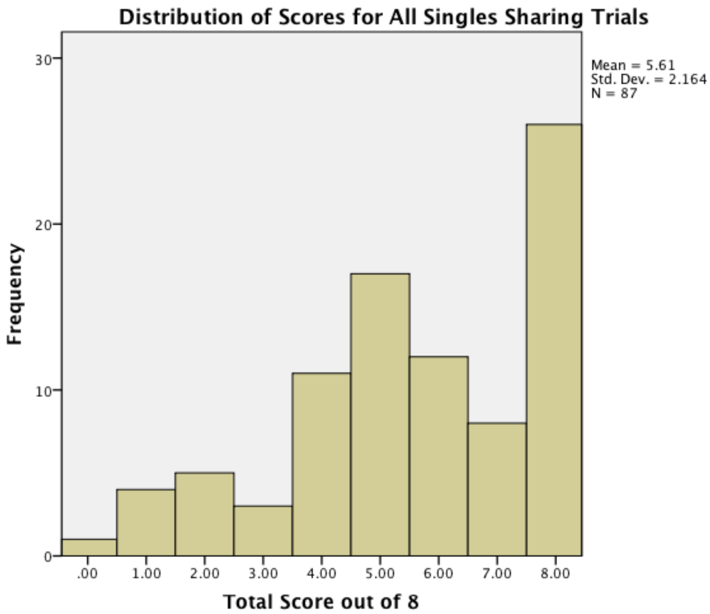


Figure 5-3 Distribution of Scores for Doubles and Singles Sharing Trials

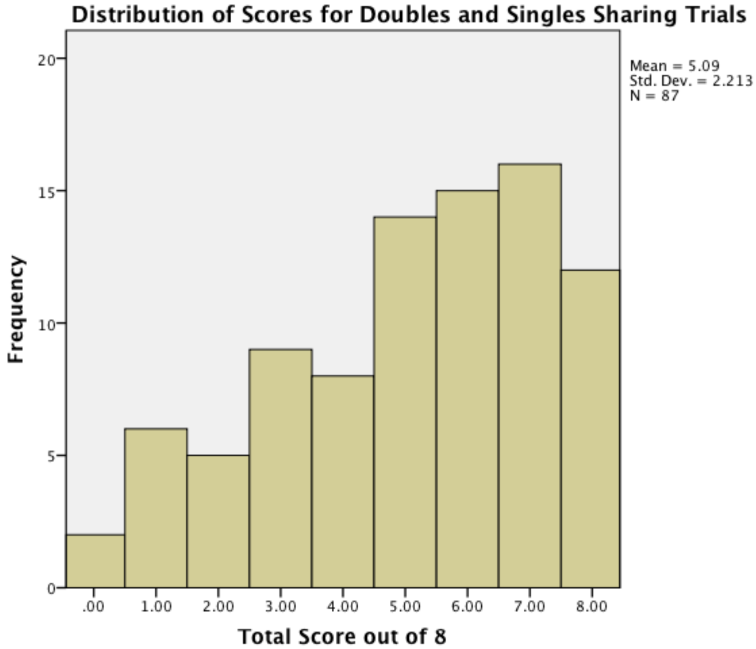
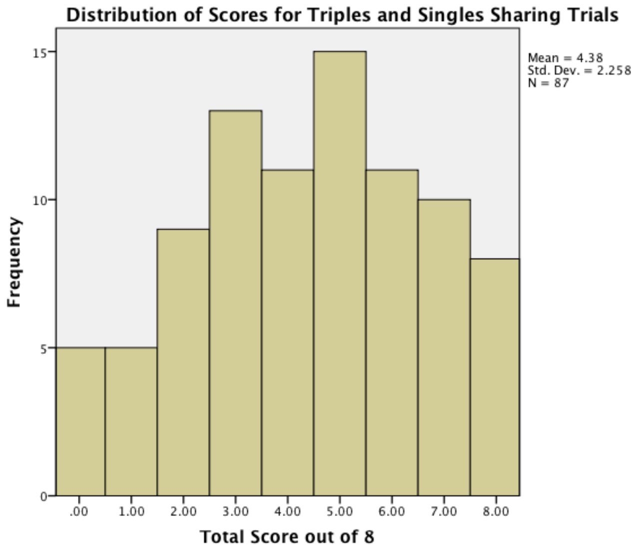


Figure 5-4 Distribution of Scores for Triples and Singles Sharing Trials



5.4.3 Coordinating Ordinality and Cardinality in order to Establish the Cardinal of a Single Set: Does a Miscount make it Harder to Determine the Cardinal of a Set?

It was hypothesised that inferring the cardinal of a single set requires knowledge of cardinal and ordinal information. Based on this hypothesis, it was predicted that trials where the puppet miscounted would be significantly more difficult than trials where the puppet counted correctly. On the correct counting trials, in which no mistakes were made, the child should give the same cardinal as the total for the set and as the total for how many blocks Mr. Dog thinks are in the set. On the miscount trials in which Mr.

Dog skips one block, the child should give a cardinal that is one larger than the number Mr. Dog states as the total for the set. This requires children to know that by skipping one item Mr. Dog states a number that is smaller by one than the correct number and that the next number in the counting sequence is one more than the number that Mr. Dog attributed to the set. In other words, they must know ordinal information about the number sequence. Children must coordinate this knowledge with knowledge that the last number word stated represents the total for a set (cardinal information), but if counting is not done correctly this number word will not be correct.

Table 5-2 Percentages of Children who Gave the Correct Cardinal of a Single Set on 0,1, 2, 3, or 4 Questions on the Correct Counting or Miscount Trials

	Correct Counting Trials				
	Correct on 0 Trials	Correct on 1 Trial	Correct on 2 Trials	Correct on 3 Trials	Correct on 4 Trials
All Singles	1.1	9.2	8.0	17.2	64.4
Doubles and Singles	3.4	9.2	16.1	19.5	51.7
Triples and Singles	5.7	9.2	17.2	21.8	46.0
	Miscount Trials				
	Correct on 0 Trials	Correct on 1 Trial	Correct on 2 Trials	Correct on 3 Trials	Correct on 4 Trials
All Singles	13.8	21.8	19.5	13.8	31.0
Doubles and Singles	14.9	23.0	25.3	18.4	18.4
Triples and Singles	24.1	37.9	17.2	10.3	10.3

Table 5.2 shows the percentages of children who stated the correct cardinal of a single set on 0, 1, 2, 3, or 4 trials when Mr. Dog counted correctly or miscounted. The majority of children gave the correct answer on the correct count trials. Children's performance on the miscount trials shows that they had significantly more difficulty giving the correct cardinal for the set in all conditions compared to the correct count trials. Children who were above chance level were able to give the correct cardinal on 3 or 4 out of 4 for each type of trial (correct count or miscount) in each condition. A large

portion of the sample was below chance level for stating the correct cardinal in the miscount trials, answering one or none of the questions correctly when Mr. Dog made a counting error. This suggests that although the majority of children performed well on the correct count trials, many were not reliable when asked to correct the counting error and guessed their answers to the questions. A Wilcoxon signed-rank test was used to compare children's performance on the correct count and miscount trials. As was predicted, children performed significantly better on the correct count trials than the miscount trials $T= 4.50$, $p<.001$. 68-82% of the sample was able to give the correct cardinal for the set on the correct counting trials on 3 or 4 trials in all three conditions, whereas only 21-45% of children were able to do so when a miscount occurred. This suggests that as was predicted many four and five-year-olds had difficulty coordinating cardinal and ordinal information to correct the counting mistake.

5.4.4 Coordinating Ordinality and Cardinality in order to Establish the Cardinal of a Single Set: Does Being Able to Explain the Miscount Make a Difference?

It was predicted that children who could explain Mr. Dog's counting error would be more likely to give the correct cardinal for the set after Mr. Dog miscounted because they had a stronger understanding of how to coordinate counting principles to count correctly, which could also mean a stronger understanding of ordinality as well. Table 5.3 shows the percentages of children who gave a correct or incorrect cardinal for a single set on the miscount trials based on whether they were able to explain why Mr. Dog's counting was wrong when he made a counting error. Wilcoxon signed-rank tests were run for each condition to examine whether children who explained Mr. Dog's miscount performed better than children who could not. The results were as follows: $T= 30$, $p<.001$ for the single row condition, $T= 133$, $p<.001$ for the all singles condition, $T= 17.5$, $p<.001$ for the doubles and singles condition and $T= 93.5$, $p<.001$ for the triples and singles condition. As was predicted, children who could explain Mr. Dog's counting mistake more often gave a correct cardinal than children who could not explain what Mr. Dog did wrong. These children demonstrated a better understanding of how to coordinate cardinal and ordinal information about numbers to determine the cardinal of the set. These children demonstrated knowledge that the correct cardinal of the set was Mr. Dog's cardinal plus one when he miscounted. However, table 5.3 again shows that there was a large portion of the sample who were unable to explain Mr. Dog's counting error and who struggled to correct the error.

Table 5-3 Percentages of Children who Gave a Correct or Incorrect Cardinal on the Miscount Trials Based on whether they could Explain why Mr. Dog's Counting was Incorrect

	% 0-1 Correct	% 2 Correct	% 3-4 Correct
Did Not Explain Counting Error			
All Singles	21	7	6
Doubles and Singles	23	6	6
Triples and Singles	34	3	4
Did Explain Counting Error			
All Singles	15	12	39
Doubles and Singles	15	20	30
Triples and Singles	28	14	17

5.4.5 Children's Answers for the Cardinal of a Single Set by Set Size

It was predicted that children who could identify that Mr. Dog made a mistake and explain what Mr. Dog did wrong would be able to correct Mr. Dog's counting error by adding one to his total. Less than half of the sample was able to give the correct cardinal for the set on the miscount trials in each condition. Based on previous research, it was thought that set size might have played a role in children's performance. Children's answers when asked to give the correct total of Mr. Dog's set were therefore examined by set size to see whether there were any differences between when he counted smaller or larger numbers of blocks. When one looks at the number that children gave as the cardinal of the set by set size, their answers are only consistently correct in the smallest set sizes (3 and 4) (see table 5.4). In all of the larger set sizes (6, 8, 9, 10 and 12) many children were not able to give the correct total. These children demonstrated the ability to count as high as at least 12 on the counting line task, identified that a counting error occurred and many even explained why Mr. Dog's counting actions were incorrect and but could not give the correct cardinal for the set. When children made a mistake giving the cardinal of the set, it was most often stating the same cardinal that Mr. Dog gave as the total after he skipped one block. It appears that despite identifying that Mr. Dog made an error in counting, many children had trouble correcting the mistake and proceeded to accept Mr. Dog's number as the cardinal for the set. Four and five-year-olds' ability to coordinate their knowledge of ordinality and cardinality does not appear to be very strong when the correct procedure for counting is not followed. Children's success on the smaller set sizes could also be a result of subitizing rather than using knowledge of ordinality and counting principles. Overall four and five-year-olds' performance, particularly on the larger set sizes, suggests that while many appear to have good knowledge of counting principles in that they were able to identify and explain why Mr. Dog's counting actions were not correct when he skipped one block, their ability to coordinate this knowledge with ordinal information about numbers is not fully developed and they struggled to correct the mistake to determine the true cardinal of the set.

Table 5-4 Percentage of Children on the Miscount Trials who stated the Same Cardinal as Mr. Dog, the Correct Cardinal or a Different Number than the True Cardinal of the Set by Set Size and Condition

	Set Size:											
Condition	% Same Cardinal Set size 4	% Correct Set size 4	% Other Number Set size 4	% Same Cardinal Set size 6	% Correct Set size 6	% Other Number Set size 6	% Same Cardinal Set size 8	% Correct Set size 8	% Other Number Set size 8	% Same Cardinal Set size 10	% Correct Set size 10	%Other Number Set size 10
Single Row	5.7	94.3	0	82.8	16.1	1.1	18.4	77.0	4.6	82.6	14.9	2.5
All Singles	4.6	94.3	1.1	74.7	23.0	2.3	23.0	73.6	3.4	28.7	67.8	3.4
Doubles and Singles	2.2	96.6	1.1	74.7	25.3	0	72.4	26.5	1.1	40.2	55.2	4.6
	Set Size:											
	% Same Cardinal Set size 3	% Correct Set size 3	% Other Number Set size 3	% Same Cardinal Set size 6	% Correct Set size 6	% Other Number Set size 6	% Same Cardinal Set size 9	% Correct Set size 9	% Other Number Set size 9	% Same Cardinal Set size 12	% Correct Set size 12	% Other Number Set size 12
Triples and Singles	1.1	98.9	0	75.9	19.5	4.6	72.4	25.3	2.3	51.8	43.7	4.5

5.4.6 Comparing the Single Row Inference Task and the Number Inference Task

It was hypothesised that coordinating knowledge of equivalence and counting principles to infer the cardinal of an equivalent set was a greater cognitive demand than coordinating knowledge of counting principles and ordinality to determine the cardinal of a single set. It was predicted that children would perform better on the single set inference task than on the number inference task. Only trials involving single blocks are used in this analysis.

Tables 5.5 and 5.6 show the percentages of children who gave correct cardinals on 0, 1, 2, 3 or 4 trials on the single row task and on the all singles sharing task in the correct count trials and miscount trials. When one looks at children's answers, it is clear that they gave the correct cardinal for the set significantly more often in the single row condition than the all singles sharing (equivalence) condition.

Table 5-5 Percentage of Children who Gave the Correct Cardinals for the Single Set and Equivalent Set on 0, 1, 2, 3 or 4 Correct Count Trials in the Single Row and the All Singles Sharing Conditions

	% Correct on 0 Trials	% Correct on 1 Trial	% Correct on 2 Trials	% Correct on 3 Trials	% Correct on 4 Trials
Single Set Inference	0	0	5.7	16.1	78.2
Equivalent Set Inference	2.3	8.0	4.6	16.1	69.0

Table 5-6 Percentage of Children who Gave the Correct Cardinals for the Single Set and Equivalent Set on 0, 1, 2, 3 or 4 Miscount Trials in the Single Row and the All Singles Sharing Condition

	% Correct on 0 Trials	% Correct on 1 Trial	% Correct on 2 Trials	% Correct on 3 Trials	% Correct on 4 Trials
Single Set Inference	9.2	19.5	12.6	16.1	42.5
Equivalent Set Inference	2.3	12.6	24.1	19.5	41.4

To test the prediction that performance would be lower on the equivalence inference than on the single row trials, Wilcoxon signed-rank tests were run to compare children's scores when they were asked to give the cardinal of a single set and their scores when they were asked to give the cardinal of an equivalent set on trials where Mr. Dog counted his set correctly and trials where Mr. Dog made a mistake counting his set. Analyses revealed that on correct count trials, as was predicted, children's scores were significantly higher on the single row trials than the number inference trials for equivalent sets $T= 285$, $p<.05$. Children's performance was not significantly different between the single row and equivalent set inferences on miscount trials $T= 725$, n.s. In both conditions, children had more difficulty determining the amount in Mr. Dog's set when a miscount occurred which is reflected in lower scores both on the single row trials and equivalence trials compared to correct count trials. The results on correct counting trials suggest that as was predicted making a number inference about the cardinal of an equivalent set is more difficult than determining the cardinal of a single set. However, children's performance on the miscount trials suggests that coordinating counting principles and ordinal information about numbers is also difficult for four and five-year-olds when the procedure for counting correctly is not followed. Their inability to correct the miscount on the first set impacted their ability to give the correct cardinal of an equivalent set resulting in lower scores on both types of cardinal inference. This may suggest that the correct count trials overestimated children's knowledge of cardinality. If their ability to coordinate counting principles, ordinality and equivalence was strong, children should have been able to determine the cardinal of the set regardless of the counting error and then infer the correct cardinal of the second, equivalent set.

Children's poor performance giving the correct cardinal of a single set or an equivalent set when Mr. Dog miscounted prompted an examination of the types of errors they made. If children did not give the correct cardinal, two other types of answers occurred: the child either gave the same cardinal for the set as the number Mr. Dog stated or the child gave a smaller cardinal for the set than the number Mr. Dog stated. Table 5.7 shows the percentages of children who made each type of error or gave the correct cardinal of the set when asked about a single set compared with an equivalent set. Virtually identical percentages of the sample gave the correct cardinal for the single set and the equivalent set. More children stated the same number that Mr. Dog gave as the total when they were asked to make an inference about a single set compared to an equivalent set. Fewer children made the error of stating a smaller cardinal than the number Mr. Dog gave as the total for the set in the single set condition compared to the equivalent set condition. The virtually identical percentages of children with correct responses suggest that many children who could identify and correct Mr. Dog's counting error to obtain the total of a single set also understood that the total of Mr. Dog's set was the same cardinal as the second set resulting in a correct equivalence inference. When children were asked to make a number inference about two equivalent sets, Mr. Dog counted his set and the second set of blocks was covered so children could not see or count them. No children opted to recount Mr. Dog's set in order to help them make the numerical inference about the second set. It is possible that because the single row and equivalence trials were mixed together and children were prohibited from counting on the single row trials they assumed they could never use recounting as a strategy. More children stated a smaller number than the number Mr. Dog gave as the total for the set on the number inference tasks. It is possible that if children had recounted Mr. Dog's set they would have been able to obtain the correct total for the set and would then have been able to make an appropriate number inference.

Table 5-7 Percentage of Children who Gave a Correct or Incorrect Cardinal for the Single Set and Equivalent Set on Miscount Trials

	Condition:	All Singles
% correct cardinal as Mr. Dog	Single set	43
	Equivalent sets	41
% same cardinal for the set	Single set	37
	Equivalent sets	25
% smaller cardinal than Mr. Dog's	Single set	20
	Equivalent sets	34

5.4.7 Analysis of Sharing Strategies- Do Children Depend on Their Sharing Schema to Establish Equivalent Sets?

In three conditions in this study children were asked to share blocks to create equivalent sets. It was predicted that children's ability to share fairly would be impacted by the units they were asked to share: all singles, doubles and singles or triples and singles. The analyses below examine how well children were able to share to create two equivalent sets in each condition.

Based on previous findings from Frydman and Bryant (1988), it was predicted that children would find it more difficult to share in the triples and singles condition than in the doubles and singles and all singles conditions because they would have to remember to give three singles for one triple to build equivalent sets. It was therefore expected that the triples and singles condition would be the most difficult for children in both age groups. No significant difference was predicted between the all singles and doubles and singles conditions based on the performance of children in the previous two studies.

Table 5.8 shows the percentages of children who used a successful sharing strategy in each condition. The majority of children shared quite well on the all singles trials with 90% able to create two equivalent sets in 7 or 8 trials. This was chosen as a cut-off point because it was above chance. Children did much worse in the doubles and singles condition, with only 46% able to share correctly. In the triples and singles condition only a quarter of the sample (25%) shared correctly on 7 or 8 trials.

Table 5-8 Percentage of Children who Used a Successful Sharing Strategy in 7 out of 8 Trials by Condition

	Correct Sharing	Incorrect Sharing
All Singles	90	10
Doubles and Singles	46	54
Triples and Singles	25	75

These large differences in performance are visible within each age group as well. Approximately 18% of four-year-olds and 31% of five-year-olds were correct in 7 or 8 of the trials in the triples and singles condition and 37% of four-year-olds and 55% of five-year-olds were correct in the doubles and singles condition. In contrast, 81.5% of four-year-olds and 95.9% of five-year-olds were correct on 7 or 8 trials in the all singles condition. Frydman and Bryant (1988) had similar percentages of four-year-olds sharing correctly in their doubles and singles and triples and singles conditions (25% and 20% respectively), but considerably higher percentages of five-year-olds sharing correctly (75% on doubles and singles and 70% on triples and singles). In general, children in the previous two studies also performed better than the children in this study, and the five-year-olds in particular performed worse than those from the previous two studies. While the lower performance could be for many reasons, one strong possibility may be the fact that children in this study received no screening trials other than the counting line. Children in the previous two studies received more extensive screening which asked them to share doubles and singles and work with the concepts of one-to-one and two-to-one correspondence prior to completing any further trials. It is therefore possible that children benefited from the practice that the screening sessions afforded them in the other two studies in terms of learning the relation between a double block and two single blocks and how to share doubles and singles to create equivalent sets, and they were then able to transfer this knowledge to the experiment conditions.

A Friedman's ANOVA was run to compare sharing performance between the all singles, doubles and singles and triples and singles conditions. This was significant $\chi^2(2)= 92.81$, $p<.001$. Wilcoxon signed-rank tests were used to follow up this finding. A Bonferroni correction was applied so all effects are reported at a .0167 level of significance. These revealed that scores were significantly different between the all singles and doubles and singles conditions $T= 103.5$, $p<.001$, between the all singles and triples and singles conditions $T= 48.5$, $p<.001$ and between the doubles and singles and triples and singles conditions $T= 119.5$, $p<.001$. As was predicted children had the most difficulty sharing in the triples and singles condition.

The significant difference between the doubles and singles and all singles conditions was not predicted based on the findings of the previous two studies. However, in general children did not perform as well in the doubles and singles condition in this study compared with the previous two studies. It is possible that the practice children received during the screening sessions in the previous studies contributed to this marked difference in performance. The fact that even without any additional practice children shared quite well in the all singles condition is in line with much previous research which shows that this is the first type of sharing young children learn and they are quite good at dividing up objects in this way (Frydman and Bryant, 1988; Cowan and Biddle, 1989; Desforges and Desforges, 1980).

The poor performance of both four and five-year-olds in terms of being able to share fairly in the doubles and singles and triples and singles conditions was examined further. It was thought that if children in the previous study had learned to share doubles and singles from practicing in the screening sessions, some children in this study may also have learned over the course of the 8 trials to use a correct sharing strategy because they would gain a better understanding of the relation between doubles and singles or triples and singles from repeated attempts at sharing and from watching Mr. Dog count his set of blocks. On the first doubles and singles sharing trial, 25 children (29% of the sample) used a correct correspondence strategy- giving two singles for each double block. On the final doubles and singles sharing trial, 42 children (48% of the sample) used a correspondence strategy. On the first triples and singles sharing trial, 8 children (9% of the sample) used a correct correspondence strategy- giving three singles for each triple. On the final triples and singles sharing trial, 25 children (29% of the sample) used a correspondence strategy. Wilcoxon signed-rank tests revealed a significant difference between children's scores on the first four trials and the last four trials of the doubles and singles condition $T= 18.5$, $p<.001$, and between their scores on the first four and last four trials of the triples and singles condition $T= 0$, $p<.001$. This suggests that children did in fact learn over time to use an appropriate strategy for creating equivalent sets in both the doubles and singles and triples and singles condition. This could be for two reasons: because watching Mr. Dog count the doubles or triples made the relation between the doubles and singles or triples and singles more salient for these children and because they learned from their own previous unsuccessful attempts to share the blocks fairly.

5.4.8 Age and Ability to Determine the Cardinal of a Single Set

Children's understanding of counting principles and their ability to coordinate different principles with ordinal information to count correctly increases with age. It was therefore predicted that age would be related to children's ability to determine the cardinal of a single set in this study, with the five-year-olds performing better than the four-year-olds.

Table 5.9 shows the percentages of children who gave the correct cardinal for a single set on 3 or 4 correct count or miscount trials in each condition by age group. Mann-Whitney tests were used to compare four and five-year-olds performance when asked to give the cardinal of a single set in each condition. All four Mann-Whitney tests in the miscount conditions were significant: U= 628, z= -2.78, $p<.01$ for the single row condition, U= 447, z= -4.23, $p<.001$ for the all singles condition, U= 683, z= -2.14, $p<.05$ for the doubles and singles condition, and U= 643.5, z= -2.48, $p<.05$ for the triples and singles condition. Significantly more five-year-olds than four-year-olds were able to state the correct cardinal for the set in both the correct count and miscount trials. For the miscount trials, this was particularly true on the more difficult doubles and singles and triples and singles conditions. Freeman, Antonucci and Lewis (2000) found that their five-year-olds were near ceiling on their miscount discrimination task whereas many of their four-year-olds showed only partial success on the task. The results in the correct count condition also support the findings of Frydman and Bryant (1988). Children found it much more difficult to establish equivalence and then identify the cardinal of the set when units of different sizes were used to construct the sets (in the doubles and singles and triples and singles conditions) than they did when only single units were used. It is clear from the results of this study that an age difference in the overall ability to identify, explain and correct a miscount in the context of a single set is consistent across conditions.

Table 5-9 Percentages of 4 and 5-year-olds who Gave the Correct Cardinal for a Single Set on 3 or 4 Correct Count or Miscount Trials in Each Condition

	4-year-olds		5-year-olds	
	Correct Count	Miscount	Correct Count	Miscount
Single Row	92	50	96	73
All Singles	68	21	92	63
Doubles and Singles	63	29	76	43
Triples and Singles	50	13	82	27

5.4.9 Age and Ability to Make a Number Inferences

It was predicted that five-year-olds would perform better than four-year-olds on the number inference questions because they have more developed knowledge of how to count correctly, and should have a greater understanding of how counting relates to the total of a set. Five-year-olds should have more developed knowledge of equivalence and a greater ability to coordinate their knowledge of counting, equivalence and cardinality to answer the number inference questions. Table 5.10 shows the percentages of children who gave the correct cardinal for the number inference question on 3 or 4 correct count or miscount trials in each condition by age group. Mann-Whitney tests were run to examine any differences in performance on the number inference questions by age. Unexpectedly none of the Mann-Whitney tests were significant (U= 857, z= -0.66, n.s. for the all singles condition, U= 828, z= -0.89, n.s for the doubles and singles condition, and U= 926.5, z= -0.04, n.s for the triples and singles condition). Contrary to what was predicted, four and five-year-olds did not perform significantly differently on the number inference questions. It seems that although there was an age difference in children's ability to give the cardinal of a single set, for an equivalent set the biggest factor for whether children in both age groups stated the correct cardinal was whether Mr. Dog counted correctly or miscounted.

Table 5-10 Percentages of 4 and 5-year-olds who Gave the Correct Cardinal for an Equivalent Set on 3 or 4 Correct Count or Miscount Trials in Each Condition

	4-year-olds		5-year-olds	
	Correct Count	Miscount	Correct Count	Miscount
All Singles	80	55	92	68
Doubles and Singles	65	39	79	66
Triples and Singles	67	43	71	32

5.5 Conclusions and Discussion

The current study examined four and five-year-olds' understanding of counting and equivalence, two concepts that are important for understanding cardinality. This study had two main aims: to assess four and five-year-olds ability to coordinate knowledge of counting principles and ordinality to infer the cardinal of a single set when it was counted correctly or a counting error was made, and to assess their knowledge of the equivalent relation between quantities by asking them to make a numerical inference about one set after first watching a puppet count an equivalent set correctly or incorrectly. Trials in which the puppet miscounted allowed for an examination of whether four and five-year-olds could identify, explain and correct a counting error in the context of counting a single set. Children were asked to correct the counting error without recounting the set, which meant children had to use their knowledge of ordinal and cardinal information about numbers to determine the cardinal of the set. Asking children to make a numerical inference about one set after watching a puppet either correctly or incorrectly count an equivalent set provided a more stringent test of children's understanding of equivalence than was employed in the previous two studies. In the previous studies children could obtain a correct response on the number inference questions simply by repeating the last number word spoken by the puppet. On the miscount trials in this study, when the puppet skipped one block while counting, the number that he stated as the cardinal for the set was incorrect. If children accepted this cardinal for the first set they would in turn make an incorrect inference about the number of blocks in the second set. In order to make a correct inference under these conditions, children need a strong understanding of counting principles and the relation between cardinals of equivalent sets, as well as an understanding of ordinality- that the true cardinal of the set is the cardinal created by Mr. Dog’s miscount plus one. Coordinating this knowledge of counting and numbers is critical to understanding cardinality.

In relation to the first aim of this study, it was hypothesised that inferring the cardinal of a single set required an understanding of both cardinal and ordinal information about numbers. To determine the cardinal of a set correctly, children had to understand that the next number in the number sequence was the previous number plus one and that the last number word counted in a sequence represented the total of the set. It was predicted that identifying and correcting a counting error would be an additional cognitive demand that would make it more difficult for children to establish the total of a single set. It was predicted that children would find the miscount trials significantly more difficult than correct count trials. The majority of children gave the correct cardinal of the set on correct count trials. As was predicted, children had significantly more difficulty obtaining the correct cardinal of the set on miscount trials in all three conditions. A large portion of the sample was at or below chance level for giving the correct cardinal of the set on the miscount trials, answering 1 or 0 questions correctly. Only 21-45% of the sample was able to give the correct cardinal of Mr. Dog's set when he miscounted his blocks compared to 68-82% when he counted correctly. The results of the single set task lend support to the findings of Freeman, Antonucci and Lewis (2000) who also found that many of the younger children in their study were inconsistent in their ability to identify and correct a counting error when a single row of blocks was counted. The five-year-olds in this study did not perform as well as those in the Freeman, Antonucci and Lewis (2000) study, but this may be due to the use of larger set sizes (3-12 items instead of 3-6 items). It is clear from the findings of the current study that children found it significantly more difficult to establish the cardinal of Mr. Dog's set when he did not count his blocks accurately and many children lacked an understanding that Mr. Dog's miscount represented one less than the true cardinal of the set.

It was predicted that children who could identify and explain Mr. Dog's miscount would be more likely to give the correct cardinal for the set compared to children who could not. These children would be demonstrating stronger knowledge of the counting principles that dictate how to count correctly, and this knowledge also supports an understanding of ordinal information about the counting sequence. As was predicted, children who were able to explain why Mr. Dog's counting actions were not correct when he skipped one block were more likely to give the correct cardinal of a single set. Still, children who identified, explained and corrected the counting error on the miscount trials represented less than half of the sample. This again suggests that many children had difficulty coordinating knowledge of cardinality and ordinality to determine the cardinal of a single set when Mr. Dog miscounted.

Children's answers for the cardinal of a single set were examined by set size to see whether this had any effect. Their answers were only consistently correct on the smallest set sizes (3 and 4), and many children made mistakes on all of the larger set sizes (6-12). Children's strong performance on the smallest set sizes may have been the result of subitizing rather than actual knowledge of counting principles. While there is debate in the literature about the exact nature of subitizing, it is generally agreed that by age four children can use this perceptual process to quickly determine the amount of objects in a set of four objects or less (Starkey and Cooper, 1980; Klein and Starkey, 1988). Similar to the findings of this study, Frye et al. (1989) found that children were consistently better at judging whether a set was counted correctly and whether the cardinal of a set was accurate on smaller compared to larger set sizes (5 vs. 14). Therefore it seems possible that children's accuracy on the smallest set sizes was not necessarily due to knowledge of ordinality or counting principles. In this study, the most frequent mistake on larger set sizes was stating the same cardinal as Mr. Dog. This shows a clear lack of understanding the impact that Mr. Dog's miscount had on the cardinal of the set. It would appear that many children were not able to use ordinal information about the counting sequence to give a cardinal that was one larger than the cardinal given by Mr. Dog. This finding has potential implications for the bootstrapping hypothesis put forth by Carey (2004). One would not expect children's performance on the larger set sizes to be so different from the smaller set sizes if children in this study had developed and were applying the rule that a number's cardinal meaning is related to it's ordinal position in the counting sequence. It would appear that if such a rule is formed, it is not completely in place by four or five-year-of-age.

The findings of this study have so far demonstrated that four and five-year-olds do have knowledge of counting principles, at least in isolation, and know the procedure for counting correctly, but many do not yet understand how counting relates to the cardinal of a set. The juxtaposition in this study between children's demonstrated strong counting skills and their difficulty correcting a counting error is in line with other research which indicates that children can learn the routine of how count as a rote procedure without understanding the purpose of counting and its relation to more complex concepts such as equivalence and cardinality (Muldoon, Lewis and Freeman, 2003; Fuson, 1988; Nunes and Bryant, 2015). Fuson et al. (1985), for example, found that three-to-five-year-olds could count sets ranging in size from 2-19 accurately, but very few demonstrated an understanding that obtaining the cardinal of a set relies on accurate counting. This research and the findings of this study suggest that there is a gap in children's understanding of counting and ordinality.

The second aim of this study was to examine children's ability to make a number inference about one set based on knowing the amount in an equivalent set that was counted correctly or incorrectly. It was hypothesised that making a number inference about the cardinal of an equivalent set requires knowledge of the equivalent relation between sets above and beyond the knowledge of how to coordinate counting principles and ordinality necessary to establish the cardinal of a single set. It was predicted that children's performance on the equivalence inference questions would be lower than their performance on the single set inference questions. As was predicted, children gave the correct cardinal for the set significantly more often in the single set task than the equivalence task when the puppet counted his set correctly.

When the puppet made a counting error, children's scores on the single set and equivalence tasks were not significantly different. The puppet's miscount made both types of inference more difficult for children resulting in lower scores on both the single set and equivalence inference tasks. Children's performance on the correct counting trials suggest that the equivalence inference was a more difficult task than the single set inference due to needing knowledge of equivalence in addition to knowledge of how to coordinate counting principles and ordinality. For many of the four and five-year-olds in this study, however, their ability to coordinate counting principles and use ordinal information about the number sequence to determine the cardinal of a single set was not yet strong enough to correct a counting error, which impacted their ability to make a number inference about an equivalent set as well. These findings suggest that coordinating counting principles and ordinality to give the correct cardinal of a single set is just as difficult as coordinating counting knowledge and knowledge of equivalence to give the cardinal of an equivalent set when a set is not counted correctly. Nunes and Bryant (2015) suggest that children require an explicit understanding of the relation between numbers and quantities to be able to compare equivalent sets solely using counting or correspondence. From the results of the number inference tasks, it seems that this level of understanding was not fully developed in most children in this study. It is also clear from the results on the single set tasks that knowing a procedure for how to count is not in itself sufficient knowledge for understanding the information that counting gives you in terms of the total of a set. Most of the four and five-year-olds in this study required stronger knowledge of how to coordinate counting principles and ordinality, as well as how to coordinate this knowledge with knowledge of equivalence in order to establish the cardinal of a single set and to make a correct number inference about an equivalent set when the first set was not counted correctly.

Children's poor performance on both the single row and equivalence tasks in the miscount condition led to an examination of the types of errors they made. More children made mistakes than gave the correct cardinal on both types of trials (57% for the single set task and 59% on the equivalence task). In both cases, less than half of the sample gave a correct response. The almost identical percentages of children who were able to do so in both conditions suggest that there was a group of children in the sample who did understand how to coordinate knowledge of counting principles, ordinality and equivalence to give the cardinal of a single set and infer the cardinal of an equivalent set even when counting was not done correctly. This subset of children demonstrated an integration of their knowledge of how to coordinate counting principles and ordinality as well as knowledge of the equivalent relation between sets in correspondence. For the majority of children who did make errors in the miscount condition, more stated the same cardinal as Mr. Dog for the total of the set on the single set task, whereas more children stated a smaller cardinal than Mr. Dog's on the equivalence task. To determine the cardinal of the single set, children had to realize that Mr. Dog's counting error produced a cardinal that was smaller by one than the true cardinal of the set. It would appear from this pattern of errors that many did not recognize the impact his counting error had on the total of the set. To determine the cardinal of the equivalent set, children had to establish the cardinal of the first set and then recognize that both sets shared the same number name. Their errors suggest that their understanding of the equivalent relation between sets in correspondence is not yet strong enough. These results are in line with previous research which has suggested that four and five-year-olds' knowledge of equivalence is not fully developed and they are reluctant to make an inference about the amount in a set solely based on number (Frydman and Bryant, 1988). Another factor which may have impacted children's performance on the equivalence tasks is that no children opted to recount the set before making the numerical inference. Children were asked to give the cardinal of the single set without recounting, and it is possible that because the single row and equivalence trials were mixed together in the study they assumed they were never allowed to count. It seems likely that children's number inference responses would have been considerably more accurate if they had used this strategy. However, the lack of recounting in this study is also consistent with previous research (Cowan and Daniels, 1989; Michie, 1984; Schaeffer, Eggleston and Scott, 1974) which has shown that despite being proficient counters, four and five-year-olds often do not spontaneously use counting to establish the number of items in two equivalent sets.

It was predicted that children's ability to build equivalent sets would be affected by the type of units they were asked to share- all singles, doubles and singles or triples and singles. Based on the previous two studies no significant difference was predicted between the all singles and doubles and singles conditions because the majority of children were able to share fairly in both conditions. However, it was predicted that children would find sharing blocks in the triples and singles condition more difficult than sharing doubles and singles or all single blocks. There were large differences in children's ability to share fairly in each condition: 90% shared correctly in the all singles condition whereas only 46% shared correctly on the doubles and singles condition and only 25% on the triples and singles condition. As was predicted the triples and singles condition was the most difficult for children. This supports Frydman and Bryant's (1988) finding that building equivalent sets using triples and singles was more difficult than building sets using doubles and singles or all singles for four and five-year-olds in their study because children have to remember the relation between one triple and three singles. Unexpectedly there was a significant difference between the all singles and the doubles and singles conditions. All children, but particularly the 5-year-olds, performed significantly worse on the doubles and singles condition in this study than in the previous two studies. Children's poor performance in this study was possibly due to the lack of a screening session prior to starting the experimental tasks, so they did not have a chance to practice with double and single units and work on the concept of two-to-one correspondence beforehand. Children in the previous two studies were able to practice on the screening conditions which may have helped them better understand the relation between doubles and singles They could have transferred the knowledge that they gained from the screening session to the experimental conditions boosting their performance sharing doubles and singles on the experimental conditions. This idea led the researcher to examine whether it was possible that children learned from repeated attempts to share fairly in the doubles and singles and triples and singles conditions. Children's sharing strategies were compared between their performance on the first four trials and the last four trials in both conditions. There is evidence that children learned over the course of 8 trials to use more effective sharing strategies in the doubles and singles and triples and singles conditions: significantly more children used a correct sharing strategy on the last four trials in the doubles and singles and triples and singles conditions than the first four trials. 29% compared with 48% did so in the doubles and singles condition and 9% compared to 29% did so in the triples and singles condition representing roughly a 20% increase in the use of correct sharing strategies in both conditions. Children may have learned from watching Mr. Dog repeatedly count his blocks which reinforced the relation between double or triple and single blocks, and they may also have learned from their own previously unsuccessful attempts to share fairly on earlier trials.

The final hypothesis of this study was that children learn over time to coordinate their knowledge of counting principles and ordinality as well as their knowledge of other quantitative concepts like equivalence and cardinality. Significant differences in the ability to determine the correct cardinal for a single set and to determine the correct cardinal of an equivalent set were predicted between four and five-year-olds. As was predicted, significantly more five-year-olds than four-year-olds stated the correct cardinal for the single set in all four conditions (single row, sharing all singles, sharing doubles and singles and sharing triples and singles) on both correct count and miscount trials, particularly on the more difficult doubles and singles and triples and singles conditions. There were developmental differences in children's ability to coordinate counting principles and ordinal information to obtain the total of a single set. Freeman, Antonucci and Lewis (2000) also found age differences on their miscount discrimination task with their five-year-olds performing significantly better than their four-year-olds. In their study and in this study, five-year-olds appear to have a more developed understanding of how to count, of the ordinal position of numbers in the number sequence and how this relates to the cardinal of a set. It is not surprising that children would develop this knowledge over time with more practice counting and developing more connections between quantitative concepts.

Age differences in performance on the number inference questions were also predicted. Unexpectedly, four and five-year-olds did not perform significantly differently on the number inference questions in any of the conditions (all singles, doubles and singles or triple and singles). It appears that although five-year-olds did perform significantly better than four-year-olds when asked to give the cardinal of a single set, suggesting a better understanding of how to coordinate ordinal and cardinal information about numbers, their knowledge of equivalence was not significantly greater than that of the four-year-olds in this sample. The failure of many children in this study to make a correct numerical inference also suggests that the previous two studies may have overestimated children's level of understanding the equivalent relation between sets in correspondence.

For both age groups, being able to identify and correct Mr. Dog's miscount was the key to performing well on the single set and on the number inference questions. Many children in this study found it difficult to state the correct amount of blocks in the set after Mr. Dog's miscount. Most four and five-year-olds clearly did not know that the cardinal of the set should be Mr. Dog's cardinal plus one, demonstrating a disconnect between their knowledge of counting principles and ordinality. Many children also found it difficult to make a number inference about a second equivalent set if Mr. Dog had not counted his blocks correctly. In the previous two studies children could achieve a correct response on the number inference questions simply by stating the only number mentioned as a cardinal. In the miscount trials in this study, children could not simply repeat one number or they would not know the correct number in the first set and then could not infer the correct cardinal of the second set. The poor performance on the miscount trials corroborates the findings of other studies that have asked children in this age group to infer the number in one set based solely on knowing the number in an equivalent set (Frydman and Bryant, 1988; Cowan and Daniels, 1989; Saxe, Guberman and Gearhart, 1987). These studies found that many four and five-year-olds are unable or unwilling to make the inference, despite strong knowledge of counting or correspondence procedures for establishing equivalent sets. The ability to correct a counting error has been shown to be significant for comparing equivalent sets as well. Muldoon, Lewis and Francis (2007) found that children in their study were less likely to correctly use counting to compare two sets if they were unable to correct a puppet's counting mistake. Muldoon, Lewis and Freeman (2003) found that children's ability to identify a miscount and know that this produced an incorrect cardinal for the set predicted their ability to build and compare sets in correspondence. Finally, the findings of this research support those of Muldoon, Lewis, and Berridge (2007) who found that error-detection tasks were important for predicting whether children could use cardinals to compare sets.

It is clear from the findings of the miscount trials in this study that there is a gap in four and five-year-old children's understanding of the information that counting gives you with respect to the total of a set and in their understanding of how to coordinate counting principles, ordinality and equivalence. Many children were able to share blocks to create equivalent sets, although sharing units of different sizes did impact their performance, and these same children showed a strong understanding of how to count correctly when they were asked to identify whether Mr. Dog made a mistake while counting his set. However, when Mr. Dog made a counting mistake, these same children found it difficult to correct the error by adding one to Mr. Dog's cardinal to determine the true cardinal of a single set and in turn make an inference about the cardinal of an equivalent set. Less than half of the sample was able to correct Mr. Dog's counting error. In contrast to their poor performance on the miscount trials, very few children made errors identifying the cardinal of a single set or an equivalent set when Mr. Dog counted correctly. This suggests that correct counting trials may overestimate the degree of children's understanding of counting, equivalence and cardinal numbers. Age was an important factor in children's performance, particularly on the single set task, with five-year-olds generally performing better than four-year-olds in terms of identifying, explaining and correcting Mr. Dog's counting error. This suggests that, as would be expected, older children demonstrated greater understanding of how to coordinate ordinal and cardinal information to determine the cardinal of a set. Knowledge of the relation between equivalent sets appears to be difficult for children in both age groups, as both groups did not perform well on the number inference questions if Mr. Dog did not count his blocks correctly.

While most four and five-year-olds have good knowledge of the procedure for counting correctly (in other words the principles for how to count correctly), it is clear from this study that many did not yet fully understand how to coordinate these principles with ordinal information about numbers to establish the total of a single set, or how to coordinate this knowledge with knowledge of equivalence in order to infer the cardinal of an equivalent set when the correct procedure for counting was not followed. On the miscount questions, children had to reason about the cardinal produced by the incorrect count and the true cardinal of the set, whereas in correct count trials they could achieve correct responses simply by repeating the last count word stated. Relational knowledge of how quantitative concepts work together is critical for understanding how to solve more difficult mathematical problems, and it is clear from this study that four and five-year-olds are still developing this understanding. In the context of a single set, four-year-olds in particular found coordinating knowledge of ordinality and cardinality difficult when they were asked to correct a counting mistake to determine the cardinal of the set. Ordinality and cardinality are related concepts and the children in this study were still developing an understanding of how to coordinate them. Similarly, both four and five-year-olds in this study struggled with making a numerical inference when a counting error was made suggesting that their understanding of how to coordinate knowledge about counting with the equivalent relation between sets is also still developing. Many children had a strong grasp of certain quantitative concepts in isolation but failed to coordinate this knowledge.

All three studies in this thesis have shown that children learn how to use counting and correspondence strategies in different ways to construct equivalence, but they must also develop an understanding of what equivalence means in terms of cardinal numbers in order to have an understanding of cardinality. Providing an environment for children to practice identifying and correcting counting errors and asking them to build equivalent sets and then infer the cardinal of one set based on knowing the cardinal of an equivalent set encourages them to expand their thinking about how to use individual quantitative concepts such as counting, correspondence, equivalence, and ordinality to see how these concepts can be used together to solve mathematical tasks. The next chapter will reflect on the findings of all three studies presented in this thesis, and will discuss the practical implications of this research as well as ideas for future research into young children's quantitative understanding.

6 General Discussion and Conclusions

The aim of this chapter is to discuss and interpret the findings of the previous chapters with respect to the main hypotheses outlined in this thesis. This chapter will begin with a summary of the findings of this research and how they support the main hypotheses of this work. This chapter will also reflect on the theoretical and methodological contributions of these findings and on potential practical applications for parents and teachers of four and five-year-olds. It will then go on to discuss the strengths and limitations of the current research and finally, provide suggestions for future research.

6.1 Summary of Main Findings

The focus of this thesis has been to examine four and five-year-old children's understanding of cardinality as well as the concepts of equivalence, correspondence, and counting and the relations between these concepts. These terms were discussed theoretically in the introduction to this thesis, and the definitions of these concepts will be briefly revisited here in relation to the findings of this research. This research focused primarily on children's developing understanding of cardinality. Sets can be described as equivalent if for each object in one set there is a corresponding object in the second set, and cardinality is a concept that describes the equivalent relation between sets that share the same number. Some researchers discuss the cardinal principle, which states that the last number word counted represents the total of a set, however it is the understanding of the relation between equivalent quantities that is critical to the development of knowledge of cardinality. In order to tap this relational knowledge, this research asked children to make numerical inferences about equivalent sets in different contexts, for example when the two sets were equivalent but constructed from units of different sizes or when one set was counted incorrectly. The concepts of correspondence and counting are related to cardinality in that they can be used as tools for establishing equivalence or determining whether sets are equivalent. It was anticipated that there would be evidence of children's developing understanding of the concepts of counting and correspondence. For the concept of correspondence, this would be reflected in a growing ability to use units of different sizes to create equivalent sets. In counting, this would be reflected in the ability to coordinate counting principles and ordinality to establish the cardinals of equivalent sets. Evidence for the development of strong knowledge of equivalence and cardinality would be reflected in their ability to infer the number of objects in one set based solely on knowing the quantity of a second, equivalent set.

The introduction to this research outlined three main overall hypotheses regarding four and five-year-old children's quantitative knowledge. The findings of this research in relation to these hypotheses will be reviewed here. The three hypotheses are as follows:

1. Four and five-year-olds' understanding of the concepts of counting and correspondence is no longer tied to only one single procedure for creating equivalent sets, they can apply their knowledge of counting and correspondence flexibly to suit the demands of different contexts.

2. Knowledge of how to build equivalent sets is not sufficient to demonstrate a full understanding of cardinality. Children must understand how to infer the number in one set based on knowing the number in an equivalent set, which is a more difficult cognitive demand because it requires knowledge about the relation between the cardinal numbers of equivalent sets.

3. There is a progression in children's understanding of cardinality between four and five-year-olds.

In all three studies, several four and five-year-olds demonstrated an understanding of how to use quantitative concepts like correspondence and counting in different ways, but also revealed a lack of understanding the relations between these concepts. The literature suggests that knowing how to build equivalence is an important step, but children also need to extend this knowledge to explicitly understand the connection between counting or correspondence and equivalence to truly understand cardinality (Nunes and Bryant, 2012). The following paragraphs will untangle the findings of this research regarding the development of children's understanding of the concepts of correspondence, counting, equivalence and cardinality.

6.1.1 Four and Five-year-olds can Flexibly Apply their Knowledge of Correspondence and Counting

All three studies provide some support for the hypothesis that four and five-year-old children are able to think about counting and correspondence in different ways and that they can apply this knowledge to suit different contexts. Counting and correspondence are different methods that children can use to create equivalent sets. Due to the varied social and cognitive demands of the sharing tasks, applying only one kind of sharing strategy would not have enabled children to establish equivalent sets in every condition. Children in the first study were prohibited from counting, but many showed good knowledge of how to use correspondence to construct equivalent sets in the equal sharing and reciprocity conditions. Children in the second study were allowed to count, and many opted to use a counting strategy to create equivalent sets in the reciprocity condition. In order to share successfully in either condition, children had to consider the relation between the individual units being used to build the sets, as well as the relation between the two sets as a whole. The use of a counting strategy in the reciprocity condition demonstrates that many children were able to succeed on a Give-N task when some of the blocks had already been shared. These children had to count on from the number that the first character shared in order to construct equivalent sets, which is a sign that their understanding of counting was progressing. The fact that when they were allowed to, children used both counting and correspondence strategies in the different sharing conditions suggests that they adjusted their sharing strategy to meet the demands of the task and thus to some extent showed an understanding of the connection between correspondence and counting procedures.

While many children in the first two studies were able to share single blocks as well as double and single blocks between two puppets to create equivalent sets, children in the third study had considerably more difficulty sharing using doubles and singles. This may suggest that the screening sessions completed before experimental trials in the first two studies helped children understand the relation between doubles and singles. The practice they received working with units of different sizes may have helped them see how to use counting or correspondence to construct equivalent sets during the experimental trials. It is therefore possible that the first two studies overestimated children's ability to use counting and correspondence strategies by screening out children who were not able to demonstrate a basic understanding of these concepts.

Overall the findings of all three studies suggest that many four and five-year-olds did not think of sharing as tied to a single procedure for building equivalent sets. When they were permitted to do so, many children used a mix of both counting or correspondence strategies depending on the context in which they were asked to share. These children were able to flexibly apply their knowledge of both quantitative concepts and used different strategies to suit the sharing scenario. Their ability to use both concepts to create equivalence also demonstrates some understanding of the connection between these two concepts. These are significant steps in the development of children's logical-mathematical reasoning.

6.1.2 Knowledge of how to Build Equivalence is not Sufficient to Demonstrate Knowledge of Cardinality

The hypothesis that knowledge of how to build equivalence is not sufficient to demonstrate an understanding cardinality was also supported. Previous literature suggests that, while four and five-year-olds often do well when asked to construct equivalent sets, they fail to use appropriate strategies to compare equivalent sets

(Bryant, 1972; Gelman, 1982; Fuson, 1988). When children create equivalent sets using correspondence, they do not necessarily know how to make a number inference about one set based on knowing the number in an equivalent set. Children's understanding of the connection between counting and correspondence is developing at the same time as their understanding of how to use the two concepts. Their ability to create equivalent sets therefore does not inherently show an understanding of what equivalence really means. The first two studies found that children who were able to build equivalent sets using correspondence or counting performed better on the number inference questions than children who could not construct equivalent sets. However, this association cannot be interpreted as causal because the direction of the association could go either way, or it could indicate that children are getting better at coordinating their ability to create equivalent sets using counting and correspondence. As well, the number inference task in those studies may have overestimated their abilities by allowing children to achieve a correct response simply by repeating the last number word stated by the puppet rather than using knowledge of equivalence. These two studies also contained a large minority of children who were able to share fairly using doubles and singles but who did not go on to answer any of the number inference questions correctly. These children demonstrated knowledge of how to create equivalence but not a deeper understanding of the relation between equivalent quantities when they were asked to make a numerical inference. Children in the third study provide further support for the hypothesis that knowledge of how to create equivalence is not sufficient for understanding the relation between equivalent sets. Most children in the third study did not perform well on the number inference questions when the first set was miscounted. Many children identified that a counting error occurred but were unable to correct the mistake to obtain the true cardinal of the set. These children had difficulty coordinating their knowledge of ordinality and the cardinal principle. In this study when children were dealing with single blocks only, there was also a clear difference between their performance on the single row task where they had to determine the quantity of a single set and their performance when they were asked to determine the quantity of an equivalent set. This difference disappeared when the task became more difficult and units of different sizes were involved. This suggests an interaction between these two aspects of task difficulty.

It is clear from these results that knowing how to establish equivalence is not on its own evidence of understanding what equivalence means. While many four and five-year-olds were able to construct equivalent sets using different sharing strategies, making a numerical inference about the amount in one set based on knowing the amount in another, equivalent set was much more difficult. Children who were able to establish equivalent sets using correspondence or counting performed better on the number inference questions than children who could not. Knowing how to create equivalence thus reflects at least some understanding of what equivalence means, however, there were also groups of children in each study who could construct equivalent sets but who did not perform well on the number inference questions. The third study in this thesis also found that children were better able to establish the cardinal of a single set than they were at making an inference about the cardinal of a second, equivalent set. These findings suggest that four and five-year-olds have some knowledge of equivalence, but they need to extend this knowledge to understand the equivalent relation between sets. This knowledge develops as their understanding of how to use the concepts of correspondence and counting develops.

6.1.3 There is a Progression in Children’s Understanding of Cardinality between Four and Five years-of-age

Finally, there is evidence from the three studies to support the hypothesis that children’s understanding of cardinality progresses between four and five-year-olds. Many children were able to build equivalent sets using correspondence and were also able to make numerical inferences in the first two studies. As a result, there were few significant differences in the performance of four and five-year-olds. However, as was previously discussed, children at the lowest end of the performance spectrum were lost at the screening stage which may have biased the sample towards children who were strong performers. This is possibly true for the four-year-olds as more four-year-olds than five-year-olds did not pass the screening trials in both of the first two studies. Despite this, five-year-olds did perform better than four-year-olds on the conditions with the greatest cognitive demands, such as making number inferences in the reciprocity with coins condition in the second study. There was also a clear developmental shift in understanding equivalence and fairness as separate concepts in the equity condition in the first study. For the equity condition in the first study, the vast majority of four-year-olds did not think that sharing was fair if it did not result in equal sets, whereas 41% of five-year-olds stated that sharing could be fair even if the sets were unequal in the end. This suggests that at least in some circumstances older children in the sample were demonstrating greater understanding of how to use counting or correspondence to construct equivalent sets and of the equivalent relation between sets.

The third study provides clearer evidence of the development of understanding cardinality with age. Children's understanding of counting develops as they become better able to coordinate different counting principles, which is a key criterion for understanding cardinality. Children in this study who were better able to coordinate counting principles were more likely to make a correct inference about the number in a single set and the number in an equivalent set. Only children who were able to coordinate ordinality and cardinality were able to make the inference in the equivalent set task because only these children established the number in the first set correctly, and in the third study, more five-year-olds than four-year-olds were able to explain and correct a counting error to determine the cardinal of a single set.

Overall the findings of these studies suggest that a great deal of quantitative knowledge related to the understanding of cardinality does develop between the age of four and five. Exposure to procedural counting errors, such as when the puppet skipped one block when counting his set in the third study, as well as practice using counting and correspondence to create equivalence can increase children's awareness of the quantitative concepts underlying their actions. This increased awareness of the mathematical significance of their actions can then lead to greater understanding of different ways to use quantitative concepts and the relations between these concepts. This learning is important for the development of children's logical reasoning, which is a critical component for later mathematical success (Nunes and Bryant, 1996).

6.2 Theoretical Contributions

The findings of the current research support the theory of how children learn mathematics that has been developed by Vergnaud (1997) and Nunes and Bryant (1996). In the introduction it was proposed that in order to be able to use mathematical models, children must develop logical-mathematical reasoning skills, must learn to understand and use the culturally developed signs and symbols that represent mathematical quantities and relations, and must learn to apply different combinations of quantitative skills and concepts to suit different mathematical problems. A key component of this learning is developing knowledge of the relations between different quantitative concepts. The studies conducted in this thesis required children to consider not only how to use certain quantitative concepts, such as correspondence or counting, in isolation, but also investigated their ability to coordinate their knowledge of different concepts to solve mathematical tasks. Establishing equivalent sets required the coordination of different quantitative concepts to decide how to share fairly, particularly in the reciprocity and equity conditions. Many children in these studies were able to build equivalent sets using all single units, double and single units, and even triple and single units. These four and five-year-olds were able to use the concepts of correspondence and counting flexibly in order to use different strategies to establish equivalence. For instance, some children used correspondence strategies when they were asked to share blocks equally between the two characters, but found counting strategies more effective in the reciprocity condition where some blocks had already been distributed. This suggests that even at four and five children are learning that quantitative concepts are not defined in a singular way; they can be used and applied in different ways depending on the context. While children's knowledge of how to establish equivalence was quite strong, their understanding of how to compare equivalent quantities was weaker. In order to answer the numerical inference questions children had to have knowledge of counting as well as an understanding of equivalence in order to be able to infer the amount in one set after an equivalent set was counted. Children who were successful on this task were able to coordinate their understanding of counting correctly with an understanding of the equivalent relation between sets of the same quantity. The third study in this thesis demonstrated that many four and five-year-olds do not yet fully understand how counting impacts the cardinal of a single set or the relation between cardinals of sets of the same quantity. These four and five-year-olds demonstrated some knowledge of correspondence and counting through their ability to create equivalent sets and also agreed that both sets had the same amount, but they failed to connect the number in each set to the relation between the two equivalent quantities and therefore could not infer the amount in the second set. This step of comprehending the connections between how numbers and quantities are related is critical to the understanding of cardinality.

This research also supports Vergnaud's (1999) assertion that children do not learn every aspect of a concept or skill at once. They learn how to use things in one context and may not see how that same concept could be used in a different context. This learning takes place over time and is built upon their interactions with the world around them. In this research some children were able to see the equivalent relationship between the sets when they had shared using one-to-one correspondence, but were unable to make the number inference after sharing using two-to-one correspondence. These children are not yet able to extend their knowledge of correspondence and equivalence to understand that sets built from unequal units can be made equal and that this means both sets would share the same cardinal number. They are lacking greater knowledge of the relation between equivalent sets. There were also children who were able to use one-to-one correspondence quite well but had difficulty using two-to-one or three-to-one correspondence to build equivalence. These children again showed some knowledge of the concept of correspondence but had not yet mastered how it applied to sets built from unequal units. Some children who initially were incorrect began to use appropriate sharing strategies on the two-to-one and three-to-one correspondence questions by the end of the trials. These children also lend support to the idea that new quantitative knowledge develops over time as a result of practice and learning from experience.

6.3 Methodological Contributions

In the literature review chapter, previous research indicated that four and five-year-olds are able to use counting and correspondence to build equivalent sets, at least when only single units are involved. However, they are not very good at comparing equivalent quantities and do not often spontaneously use counting or correspondence strategies to do so (Fuson, 1988; Cowan, Foster and Al-Zubaidi, 1993; Muldoon, Lewis and Francis, 2007). The current research aimed to see whether four and five-year-olds understood how to build equivalent sets and make an inference about one set after counting an equivalent set in the context of sharing. The tasks had to be both developmentally appropriate and sensitive to the fact that children would be at different stages of understanding the quantitative concepts being measured. Different sharing contexts were designed in order to see how flexibly they could apply their quantitative knowledge across conditions with varying social and quantitative demands. A major part of carrying out this research was therefore designing the sharing conditions that were used to examine four and five-year-olds' quantitative knowledge.

The current sharing conditions were carefully designed based on methods used in previous studies of children's quantitative knowledge (Frydman and Bryant, 1988). Frydman and Bryant (1988) varied children's sharing actions by asking them to share single units, double and single units or triple and single units between different numbers of recipients. This study incorporated the use of sharing units of different sizes between two recipients, but the different sharing conditions in this research also offer another option for varying children's sharing actions to construct equivalent sets. The equal sharing condition was very similar to the one used by Frydman and Bryant (1988)- children were given a pile of blocks and were asked to share the blocks fairly between two recipients. In this condition, children constructed both sets of blocks simultaneously. The reciprocity condition asked children to use different actions to share, but the goal of sharing was still the same outcome: equivalent sets. In the reciprocity condition, one character gave his friend twice as many blocks as he kept for himself. The child was then given a pile of blocks and was asked to help the second character share with the first so that both ended up with the same amount. Children therefore had to account for the blocks that had already been shared when determining how to create equivalent sets for both recipients.

The current research offers a way of examining children's understanding of quantitative concepts that places children's quantitative actions within the framework of a social interaction- in the case of these studies, sharing. Sharing is an interaction that four and five-year-old children frequently engage in and is a context where the goal of their actions is usually equivalence. Several previous studies of quantitative understanding simply asked children to build two rows or presented them with sets that were already created by the researcher. The studies in this research aimed to provide children with a context for their actions by asking them to help the puppets share blocks or coins. It was thought that building a context around their actions might help support the extent of the knowledge they would exhibit, because the tasks attempted to mimic the real life scenarios where they are beginning to learn and use quantitative concepts such as counting, correspondence and equivalence. It was also hoped that removing the child as a recipient of the sharing would eliminate any attempts to share self-interestedly rather than in a way that was appropriate to the demands of the sharing task.

As well, the ability to vary different quantitative and social demands in each condition allowed for an examination of children's understanding of correspondence, counting, equivalence and cardinality across several different contexts. The flexibility of these conditions for measuring different types of quantitative knowledge is extremely helpful. It allows one to examine several different quantitative skills and concepts within the same child to see how well they are able to adapt and extend their knowledge to suit changing contexts. This provides a good developmental picture of a child's quantitative knowledge.

6.4 Practical Implications

This research clearly indicated that knowing how to use quantitative concepts in isolation, while it is a good starting point, is not sufficient evidence that a child has fully developed their mathematical reasoning skills. All three studies showed that learning the relations between quantitative concepts and learning how to coordinate the use of these concepts in different ways were essential steps for developing an understanding of cardinality. In particular these studies demonstrated that knowing how to count was not sufficient knowledge for understanding how numbers relate to quantities and the relations between quantities. Children who were strong counters did not necessarily perform well on the miscount identification tasks and were not always able to understand the equivalent relation between the two sets in order to answer the number inferences. Children who demonstrated good knowledge of how to count also did not always use this strategy to help them construct equivalent sets. Teachers and parents need to focus on more than just teaching children how to count. They must also focus on helping children develop an understanding of what counting means and the relation between different numbers. This research adds to previous literature which suggests that as well as learning the procedure for counting correctly, understanding miscounts is an important part of learning about coordinating counting principles and understanding cardinality, which in turn is critical knowledge for later mathematics learning. Providing children with opportunities to learn from their own or another person's counting errors can help foster this knowledge.

Another finding with practical implications for parents and teachers is that children who received screening sessions in the first and second studies performed much better when they were asked to use more difficult forms of correspondence to build and compare equivalent sets than children in the third study who did not receive a screening session. Most children in the first and second studies were able to share well using one-to-one or two-to-one correspondence, whereas many children in the third study had difficulty using two-to-one correspondence despite being able to use one-to-one correspondence well. It seems possible that children in the first two studies learned about the relation between doubles and singles during the screening session and benefited from the practice trials they were given before the experiment session. The screening sessions were only half an hour long and in the case of the second study were even on a different day from the experiment sessions. This suggests that in a relatively short period of time children were able to learn and retain information about how to use correspondence in different ways. It therefore seems possible that parents or teachers could work with children even just for a half-hour period of time using the same materials from this research which many families and schools already have available: puppets and blocks or coins and can help foster four and five-year-olds' quantitative knowledge.

6.5 Strengths and Limitations of Current Research

One of the strengths of this research is that all of the studies conducted in this thesis used large sample sizes in an attempt to ensure a wide array of children were included across a variety of different classrooms. A limitation of the research in terms of sampling is that in some studies, particularly the second study in this thesis, children at the lower end of the developmental range (mostly four-year-olds) did not pass the screening stage to move on to the experimental tasks. The screening session was designed to ensure that children who did not exhibit at least a preliminary understanding of the concepts being studied would not move on to tasks that were beyond their understanding. However, excluding children with less understanding also biased the sample towards children with a good grasp of the concepts being examined. As a result the studies did not have a randomly selected group of children, which means the results do not necessarily reflect the wider population of four and five-year-olds.

A strength of this research is that children's use of quantitative concepts was examined across different contexts because the social and quantitative demands of each sharing task were varied within and across the three studies. Previous research has not examined children's quantitative knowledge in a variety of contexts within the same study. This approach allowed the researcher to see whether children's knowledge of correspondence, counting, equivalence and cardinality was flexible or tied to a single understanding of how to apply these concepts.

A limitation of the current research is that although the sharing conditions were created to mimic a real-life sharing scenario as much as possible, in order to ensure consistency across participants, it was still a controlled experimental task. This may have influenced how children responded to the tasks to a certain extent. It would be interesting to collect observational data from school or home interactions where sharing takes place in order to compare the quantitative knowledge children exhibit in this setting with their performance on the sharing conditions.

The results from the third sharing study included in this thesis revealed another limitation of this research. The first two studies in this thesis attempted to assess four and five-year-olds' understanding of the relation between knowledge of counting or correspondence and equivalence by asking them to make a number inference about two equivalent sets. However, in these studies children could have achieved a correct response simply by repeating the only number stated as the total for the first set. The third sharing study revealed that many four and five-year-olds had difficulty when faced with correcting a counting error to obtain the correct total of one set and in turn were unable to make a correct number inference about a second, equivalent set. This would suggest that the strong performance of four and five-year-olds on the number inference task used in the first two studies did not accurately reflect their understanding that all counting principles must be coordinated with knowledge of equivalence in order to correctly infer the cardinal of one set based on knowing the amount in an equivalent set.

6.6 Suggestions for Future Research

Future research should examine children's use of quantitative concepts both through observational data collection at school or home and on structured conditions like the sharing paradigm employed in this research in order to achieve an even more complete picture of the types of quantitative skills and concepts that are developing from everyday interactions with their environment. This type of research would be able to examine the role that individual differences in sharing play in four and five-year-olds ability to make number inferences in order to build on the results from this thesis. If sharing is indeed an important part of children's developing quantitative knowledge, observing differences in the types of quantitative strategies they use to share in real life should predict their understanding of cardinality and performance on a sharing measure such as the one used in this research.

The children in the first two studies who received a practice session performed better sharing singles and doubles than children in the third study who did not receive this practice. Further research should examine whether explaining the relations between units of different sizes (for example doubles and singles) and giving children practice working with the materials, even for as little as half an hour, does in fact significantly improve children's understanding of the concepts of equivalence, correspondence and counting as measured by their performance on the different sharing conditions. If it is found that there is an improvement, research should examine whether this gain in understanding is long term using a delayed post-test. It would also be worthwhile to examine whether the children who did not pass the screening in the first two studies could have reached a point where they demonstrated the necessary quantitative knowledge to move on to the experiment tasks if they received more practice. Each of these studies should be an intervention with a control group in order to test whether practicing creating equivalence and practicing how to count units of different sizes both lead to a better understanding of equivalence and how to make number inferences. Bryant and Alegria (1989) outline the strengths and weaknesses of a longitudinal-correlational versus a training study approach to studying the causal mechanisms of developmental change in children. They suggest that both types of evidence are necessary to support a causal explanation. Future research examining developments in children's understanding of cardinality and other quantitative skills should follow the methods outlined by Bryant and Alegria (1989).

The third study in this thesis found that understanding how to identify and correct a counting error when one block in a set was skipped significantly impacted children's ability to create and compare equivalent sets. Children struggled to coordinate their knowledge of different counting principles (specifically cardinality) and ordinality in order to correct the mistake and then make an inference about the amount in a second, equivalent set. Future research could expand on this to examine how other types of counting errors impact children's ability to establish the total of a single set and then make an inference about an equivalent set.

Finally, replicating the second and third studies from this thesis with samples of four and five-year-olds from different places would also provide a useful comparison of different groups to see whether there is consistency in four and five-year-olds' quantitative development. This thesis was unable to run direct comparisons between the Canadian and UK samples because both groups received different screening sessions prior to the experimental conditions.

6.7 Concluding Remarks

The three studies that make up this thesis focused on four and five-year-old's understanding of counting, correspondence, equivalence and cardinality in the context of sharing. Children's main aim when sharing is typically to establish equivalent sets between people, which made it an ideal context to examine their quantitative knowledge using a familiar context from everyday life.

All three studies revealed the depth of quantitative knowledge that four and five-year-olds have developed prior to receiving any formal mathematics training. Children were able to use both counting and correspondence strategies to construct equivalent sets when the demands of the sharing tasks were varied. Children who were able to construct equivalent sets were also better at inferring the cardinal of one set based on knowing the amount in an equivalent set. All four and five-year-olds had more difficulty making a numerical inference about equivalent sets than establishing equivalent sets, particularly when one set was not counted correctly. These studies forced children to think about how different quantitative concepts such as counting, correspondence or equivalence must be coordinated in order to fully understand cardinality. The development of an understanding of the relations between different concepts is crucial for further mathematical development, and many of the four and five-year-olds in these studies, while they showed good knowledge of some concepts in isolation, were not yet able to coordinate their knowledge of different quantitative concepts.

Understanding numbers, quantities and quantitative relations is critical to being able to solve complex mathematical tasks. The sharing conditions used in this study are a good example of a way to examine the extent of a child's mathematical understanding. The different sharing conditions also have the potential to be useful tools for teachers and parents in terms of helping four and five-year-olds develop their understanding of individual quantitative concepts such as counting, correspondence, equivalence and cardinality as well as extending their knowledge to encompass the different relations between these concepts.

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Appendices

A) TDSB Ethics Application

Toronto District School Board

APPLICATION TO CONDUCT A RESEARCH STUDY

Please refer to the TDSB External Research Review Guidelines when completing this application.
Adherence to the guidelines and criteria will facilitate the review process by minimizing the frequency of conditional approvals and requests for modification.

IDENTIFYING INFORMATION Date: March 22, 2011
Name of Investigator(s): Sarah Walter
Institution/Agency: University of Oxford, Department of Education, 15 Norham Gardens, Oxford, UK, OX2 6PY, tel: +44 01865 274018

Mailing Address: 9 Ridgefield Rd., Toronto, ON, M4N 3H7

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Check all that apply.

☐ Doctoral Thesis ☐ Masters Thesis ☐ Other (please describe):
☐ Institutionally funded project ☐ Contractual project
☐ TDSB staff ☐ Externally funded project

Give details, if necessary

TITLE OF STUDY: The Relationship Between Young Children's Sharing Behaviours and Early Mathematical Concepts

Preferred start date: October 2011

This is to certify that this research proposal has been vetted for its academic soundness. We have also given consideration to ethical, legal and moral questions arising from the proposal.

Sponsoring Professor (Type name and sign)
Professor Terezinha Nunes (academic supervisor)

Chairperson of Department (Type name and sign)
For Toronto District School Board staff (if appropriate)
Dr. Lars-Erik Malmberg (head of the Department of Education Research Ethics Committee)

School Principal (Type name and sign)

1. OVERVIEW OF THE STUDY

The purpose of this doctoral project is to examine young children's cognitive abilities in the realm of sharing and early mathematics. Both sharing and early mathematical procedures (for example counting or correspondence) use similar cognitive skills, but it remains unclear exactly what the relationship between these two areas is (for example, how do these skills interact and influence each other). While some previous research has addressed this area, there is still much to be learned and this project hopes to provide a clearer developmental picture of the skills that children do or do not have at certain ages and the ways they use these skills to answer different questions (social vs. purely mathematical). This is the first study to use a measure of mathematical and social sharing where both tasks are structured in identical ways to allow for comparison between the two areas. The project will involve 4 small-scale studies with groups of 30 4-year olds and 30 5-year olds recruited from several different schools participating in each. The four studies will all use the same measure of sharing as a base, but the questions posed in each study will be slightly different, and will derive from the results of the previous study or studies. It is thought that differences will be found in how 4 and 5-year olds perform on the mathematical and sharing tasks. The researcher has obtained ethics approval from the Central University Research Ethics Committee at the University of Oxford, has a basic police background check completed (see appendix p. 19 and 20) and is in the process of completing the vulnerable sector check as well. Certificates of ethics approval and police background clearance will be provided to schools prior to starting the study.

2. RELEVANT LITERATURE

There is a great deal of literature that has explored young children's understanding of sharing and number concepts from Piaget (1932, 1941) onwards. There is a plethora of studies examining children's responses to moral reasoning or distributive justice tasks (Sigelman and Waitzman, 1991; Vikan, 1986; Damon, 1975; Cook and Hegtvedt, 1983; Thomson, 2005; Huntsman, 1984), as well as studies of children's actual sharing behaviours in different contexts (Benenson et al., 2003; Hay, 1979). Other sharing research has looked at various contextual and cross-cultural differences in how, when and why children share (Enright et al., 1984; Enright, Lapsley and Enright, 1981; McGillicuddy-De Lisi et al., 1994; Wong and Nunes, 2003; Rao and Stewart, 1999; Leung and Iwawaki, 1988; Berman, Murphy-Berman and Singh, 1985; Singh et al., 2002). Still another body of literature has looked at children's understanding and use of equity and/or reciprocity norms (McCrink, Bloom and Santos, 2010; Gummerum et al., 2010; Hook and Cook, 1979; Lucas, Wagner and Chow, 2008; Keil, 1986; Dreman and Greenbaum, 1973; Harris, 1970). This research generally suggests that while young children (around ages 3, 4 or 5 years) seem to understand the concepts of reciprocity and equity in the context of social sharing, they are more likely to employ an equality-based sharing strategy. Whether this is due to difficulties with the cognitive demands of using a non-equality based sharing strategy or simply because they automatically employ a learned routine that sharing fairly means sharing equally is unclear.

In addition to the literature on sharing, there are also a large number of studies that have examined children's early maths knowledge in areas such as counting (Sophian, 1987; Sophian, 1995; Muldoon, Lewis and Towse, 2005), proportion (Sophian and Wood, 1997), understanding number (Gelman and Gallistel, 1978), problem solving (Davis and Pepper, 1992), multiplication and division tasks (Mulligan, 1992) and early knowledge of fractions (Pitkethley and Hunting, 1996). This literature has established that well before children begin to learn maths in school, many have developed a wealth of skills that can be conceived of as precursors to the more abstract formulas and concepts taught in formal mathematics.

While the research into sharing and maths both look at similar cognitive skills in children, current theories of the development of an understanding of number (for example Gelman and Gallistel, 1978; Dowker, 2008) do not tend to consider the role that social experiences like sharing play in shaping a child's learning about mathematical principles (for example cardinality) and social sharing literature does not tend to examine the mathematical strategies that children often employ in decisions about when things have been shared fairly. As well, the way in which studies are designed and analysed in these two bodies of literature have traditionally been quite different. Many of the social sharing studies have used qualitative approaches, such as observation or analyzing children's verbal responses to moral/reward scenarios, whereas maths studies have generally used more quantitative approaches, comparing scores for children's solutions to maths reasoning or number problems. However, several researchers have identified a crossover between these two research areas, and there is some literature that has explored the overlap between sharing and maths. This literature posits that sharing may be an early schema for division and other maths reasoning skills, as well as a way that children gain real world experience of using maths concepts, such as distribution and counting (Muldoon, Lewis and Francis, 2007; Piaget and Szeminska, 1941; Dickson et al., 1984; Hook, 1978; Squire and Bryant, 2002; Correa, Nunes and Bryant, 1998; Kouba, 1989). These studies have examined several aspects of children's early quantitative knowledge using sharing tasks, such as counting, cardinality, number inferences, and one-to-one correspondence (Pepper and Hunting, 1998; Davis and Pitkethly, 1990; Frydman and Bryant, 1988; Muldoon et al., 2007; Desforges and Desforges, 1980; Cowan and Biddle, 1989; Norris, 1993).

In order to extend previous findings, the current study has developed a measure of social and mathematical sharing skills. These tasks are structured identically, the only difference being that one task requires purely mathematical reasoning, whereas the social sharing tasks require the children to make a social judgment about how to share resources (using the concepts of reciprocity and equity). These tasks will be used with a group of 4 and 5-year old children in schools the Oxfordshire, UK area before they are given in Toronto. Children's testing sessions are still underway for the UK study, but it is not anticipated that any changes will be made to the sharing measure.

3. PROBLEM TO BE INVESTIGATED

This project intends to investigate the cognitive skills that young children use in relation to sharing and math, in particular, how children respond to situations where sharing fairly involves the distribution of unequal resources. The relationship between children's math reasoning skills and social reasoning skills is of particular interest.

Research questions:

1. How do 4 and 5 year old children respond to sharing tasks involving unequal distribution of resources? Are there differences in how they respond?
2. Can 4 and 5-year old children learn to use and apply two-to-one correspondence to solve mathematical tasks? Are there differences in their ability to use a two-to-one sharing strategy in a mathematical context?
3. Are there differences between 4 and 5-year old children's abilities to take reciprocity and equity into account when sharing?
4. Are there differences between 4 and 5-year old children's abilities to use quantitative knowledge (apply a two-to-one correspondence strategy) to a social context?

Hypothesis: It is hypothesized, based on previous research, that both 4 and 5 year old children will be able to learn and apply a two-to-one sharing strategy to the mathematical tasks, but that differences will be seen in 4 and 5 year-olds abilities to apply this strategy to social tasks due to the extra cognitive demands these tasks place on their reasoning skills. It is hypothesized that 5 year-olds will perform better than 4 year-olds on the social sharing tasks.

4. IMPORTANCE OF THE STUDY

It is hoped that this study will provide a clearer developmental picture of the cognitive skills that 4 and 5 year-old children have and how they use these in the context of sharing and mathematics tasks. This information could help to clarify the relationship between math reasoning and social reasoning: how these skills work together and influence each other. Further research into how sharing and mathematical skills are related could provide important information for parents and educators about how to foster informal quantitative skills in a social context that are beneficial to understanding more abstract mathematical concepts later in their education. As well, the tasks in the sharing measure are designed to be fun and interactive for the child, using brightly coloured Legos, animal puppets and different characters in tasks they are familiar with (i.e. sharing an after-school snack), thus it is anticipated that they will enjoy participating in the study and that few, if any risks are involved for either the child or school.

5. RESEARCH DESIGN

a) Describe the general research design and methods of the study.

Children will be seen individually by the researcher in an area designated by the school (either within the child's classroom or in another space designated by the school). Children will be asked if they would like to help the researcher share some objects with friends, and if they say yes, will asked to complete five sharing tasks. All five tasks involve the child distributing Lego Duplo blocks between puppets or characters in response to short scenarios that are set up by the researcher. The tasks should take 35-40 minutes per child and each child only needs to participate in one session. Sessions will be kept as short and minimally disruptive as possible. The questions in the task ask the child to share items in a one-to-one manner or in a two-to-one manner, and some two-to-one questions ask the child to count how many objects one character has and then infer how many another has without counting to see whether they can understand set equivalence. The gender of the characters will be changed to match the gender of each child participating and each child will be given a sticker as a reward for their efforts at the end of the session. The order of experimental tasks will be varied using a Latin square.

b) Describe the key concepts being measured and the specific data collection tools to be used. **Include final copies of all tests1 , questionnaires, interviews or measures.**

The five sharing tasks are designed to measure children's reasoning while sharing in a purely mathematical context versus a social context. Other than asking for the child's date of birth from the school, no other information will be gathered about individual children. ID numbers will be assigned to each child to keep all data anonymous. The measure consists of two warm up tasks designed to see what basic sharing skills the child has, and three experimental tasks that are structured in exactly the same way to allow for comparisons across the tasks. The three experimental tasks are one mathematical and two social tasks modeled from questions used in previous sharing literature.

c) Indicate the date(s) for data collection.

April 2011- June 2012 (this will encompass 4 small-scale studies conducted at different schools with different children participating in each one. The same sharing measure will be used in all the studies, however the research questions posed in each one will be slightly different, and will be derived from the results of the previous study or studies.) For example, in further studies the researcher may make the child a recipient in the sharing stories, rather than the allocator of resources, or may give the tasks to groups of 2 or 3 children in the classroom instead of seeing children individually, but the measure itself will not differ.

d) Describe where the study will be conducted and what facilities will be required.

The study will be conducted within each school that agrees to participate in whatever space they would like the researcher to work (i.e. within the Kindergarten classroom or in another room.) The only thing needed from each school to set up the study would be a desk and two chairs. All other materials will be brought in by the researcher (i.e. puppets, Lego and laminated pictures to accompany the sharing stories).

e) Provide a copy of an Information & Invitation Letter to School Principals that outlines the purpose and scope of the study, and includes the commitments required of all potential participants.

** Attached in the appendix.**

6. DATA ANALYSIS

Total scores, task scores, method of sharing and answers to the number inference questions will be examined by age group to see whether there were differences in performance in the four and five year old groups. Although it is not expected that there will be any effects, results will be analyzed by gender as well. It is likely that a mixed ANOVA will be the main statistical test used.

7. PARTICIPANT INFORMATION

a) Specify the number of elementary, middle and/or secondary schools requested:
2 or 3 schools per study (although this depends on Kindergarten class size)

b) Names of preferred schools (if known): Unknown.

c) Number of students (Include grade levels, number of classes etc.):
30 4-year olds and 30 5-year olds in Kindergarten (per study)

d) Number of teachers (Include grade levels, etc.): None.

e) Number of school administrators: None.

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1 Please note that under the Education Act (Reg.298) where it is proposed to administer a test of intelligence or personality to a student, it is the responsibility of the Principal to inform and obtain written permission from the students and/or parents when the pupil is a minor (under 18 years of age).

f) Number of other school support staff (e.g. for administrative tasks etc): None.

g) Number of parents, if applicable: None.

h) Describe how participants will be selected (schools, students, staff etc.):
Schools in the GTA that offer the full day Kindergarten program will be approached about participating first so that all the children will be receiving the same curriculum, although other schools may also be approached if the desired sample size of 30 4 year olds and 30 5 year olds per study cannot be obtained. All 4 and 5 year old children at each participating school will be invited to participate in the study, unless they have a documented learning or mental disability, a behavioural disorder, or the child's teacher feels they would not handle participation in the study well for any reason (i.e. extreme anxiety). The researcher will attempt to recruit roughly equal numbers of 4 and 5 year olds from each school.

i) Indicate how much time will be required of participants:
Each child will need to spend approximately one or two 30-minutes sessions with the researcher, however if the child requests a break between some of the tasks, this can be facilitated.

8. CONFIDENTIALITY AND CONSENT
<u>NOTE</u>: Since the inception of Freedom of Information legislation, it is not possible to isolate individuals or groups and provide names to the

researcher. Information that is collected as a routine part of school records is not obtained with the expectation of disclosure to independent researchers.

a) Describe how participants will be prepared prior to the study and debriefed after their involvement (e.g. including provisions for follow-up support where applicable).

School principals and parents will provide consent for the child to participate in the study prior to any sessions being scheduled. When each session will take place will be arranged with the child's teacher, and at the time of their session, the child will be asked at the start whether or not they would like to participate. After each session they will be thanked for their time and given a sticker for their hard work. If they have any questions relating to the tasks, the researcher will answer them at that time. Similarly, if parents or school staff have questions about the research in general either before or after children begin to participate, the researcher will provide contact information and be available to answer these as well. The researcher will not answer any questions regarding a particular child's performance on the tasks.

b) Describe the method to be used to obtain informed participant consent (refer to the ERRC Guidelines). Translations may be necessary in some cases. **Copies of all consent letters must be included.**

School principals will first be provided with an information letter about the study, and asked whether they would like their school to participate. If the school agrees, through verbal (over the phone) or written (via e-mail) consent from the principal, information letters for parents will then be provided for class teachers to send home which will have a consent form attached. Only children whose parents send back completed consent forms will be allowed to participate in the study. Once children have signed consent from their parents, they will also be asked to give verbal assent to participate at the start of the testing session. If all of these criteria are satisfied, the child will complete the study. Copies of the principal information letter and parent information letter are attached in the appendix.

c) Describe the provisions and safeguards that will be taken to ensure security and confidentiality.

All children will be assigned ID numbers so that all data will be anonymous and non-identifiable. As well, all scores from the sharing measure will be kept confidential and stored in a locked storage cupboard only accessible by the researcher. No information about individual children will be released to parents, schools or any other party and all information gathered in the study will only be used for the purposes of this research project. Finally, all data reported in the final thesis will be non-identifiable. Data will be stored in accordance to the UK Data Protection Act and according to the guidelines of the ERRC of the TDSB.

d) Describe procedures and timeframes for the use, retention, disclosure and disposal of data.

Data collection will be ongoing for approximately 1 full school year from Oct 2011-June 2012. Completion of the entire project, including analyses and write up is anticipated for approximately May 2013. All data will be securely stored during this time and will be used only for the purposes of this research project.

Only non-identifiable data will be reported in the final thesis. A report of the results will also be provided to interested school and the TDSB, again with only non-identifiable data. In accordance with the UK Data Protection Act, data (using ID numbers, not children's names) will be securely stored for 10 years following completion of the research project at which time paper data will be shredded and any information stored on the computer will be erased.

9. PROVIDING FEEDBACK

a) Describe the procedures for providing feedback to participating schools. Schools will be informed that if they are interested, they will be able to obtain a brief write up of the results of the individual study that involved their school (since each small study will involve different groups of children and different schools). This would be available one or two months after the completion of the study. As well, if they are interested, they would be provided with a copy of the final results of the entire thesis, which would be available in approximately May or June 2013 after the final write up is complete. All write ups would contain only non-identifiable data.

b) What are your intended plans for the future use and/or publication of results?
At this point, the researcher only intends to use the results for the purposes of a doctoral research project. If, at a later date, it seemed possible that a research paper would be published using the data, only non-identifiable data would be published, and the TDSB as well as any individual schools involved would be informed of this prior to the publication of the paper.

c) It is required that an electronic copy and a hard copy of the completed report be submitted to the Toronto District School Board. (Please note that this report may be circulated within the board to interested staff and/or posted on our internal ERRC webpage.)

Indicate an expected date for submission of completed report May 2013

I agree that:

- **Information collected as part of this study will not be used for any purpose other than that described in the application without written authorization from the Toronto District School Board.**
- **All individual identifiers will be destroyed after completion of the data analysis.**
- **No individual to whom personal information relates will be contacted directly or indirectly after completion of the research described in the application.**
- **No Toronto District School Board schools, teachers or students or parents will be identified in any report emanating from this research.**

March 22, 2011

Signature of principal investigator Date of submission:

Please address any questions to: Chair, Sally Erling
External Research Review Committee
E-mail: ERRC@tdsb.on.ca

Revised: February 2011

Research Application Checklist

I have checked to see that my application complies with the following requirements:

1. The application must have the prior approval of the agency or institution with whom the researcher is affiliated. If you are associated with a university or hospital, it is required that your proposal successfully completes the ethical review process in your organization *before* submission to TDSB. A copy of the ethics approval must be submitted with this application.	X
2. The TDSB research application form must be completed; it is not acceptable to fill out various sections of the application by referring to other documents or materials submitted. The application is submitted in a typed form (handwritten applications will not be accepted).	X
3. FINAL copies of <u>all</u> data collection instruments are included.	X
4. A copy of an Information & Invitation Letter to School Principals that outlines the purpose and scope of the study, and includes the commitments required of all potential participants is included.	X
5. Parental permission is required for any research involving students under 18 years of age. Copies of information letters and consent forms for staff, students or parents must be included with the application materials. In some cases, the Research Review Committee may request the translation of these materials.	X
6. The demands on staff and students have been considered. The number of schools and participants and the amount of time requested have been minimized. (Large-scale studies or studies that are intrusive will not be supported.)	X
7. The names of preferred schools have been provided. (This is not required but may streamline the application process.)	X
8. Certification of a Criminal Records Background Check will be submitted for any person(s) who will have direct involvement with students.	X
9. Nine (9) copies of the research application form and support materials have been included.	

Send your submission to:
Sally Erling,
Chair External Research Review Committee
Toronto District School Board
1 Civic Centre Court LL, Etobicoke, ON, M9C 2B3

B) <u>TDSB Approval Letter:</u>

May 4, 2011

Dear Sarah Walter,

Re: The Relationship Between Young Children's Sharing Behaviours and Early Mathematical Concepts

On behalf of the External Research Review Committee (ERRC), final approval for the above- mentioned research study is granted, given that the following issues have now been addressed:

· The addition of a Teacher Information and Consent Form

· Vulnerable sector screening in process through the TDSB Police Check office (please forward a copy to me once that is in place)

· Confirmation by David Griffiths of your University of Oxford Research Ethics approval

· A reduction in the number of small-scale studies from 4 to 3, if recruiting sufficient schools becomes problematic

You should be aware that ERRC approval does not obligate schools to participate and individual principals and teachers make the final decision about their own involvement. For our record- keeping, we would appreciate receiving the names of the schools agreeing to participate in your research once that has been confirmed.

As a condition of this approval we will also look forward to receiving an electronic and hard copy of your findings upon completion, which you estimated would be available by May 2013.

Sincerely,

Sally Erling, Chair
External Research Review Committee, TDSB E-mail: ERRC@tdsb.on.ca

2010-2011-70

C) CUREC 1A Application:

University of Oxford
CENTRAL UNIVERSITY RESEARCH ETHICS COMMITTEE (CUREC)
CUREC/1A Checklist for the Social Sciences and Humanities

The University of Oxford places a high value on the knowledge, expertise, and integrity of its members and their ability to conduct research to high standards of scholarship and ethics. The research ethics clearance procedures have been established to ensure that the University is meeting its obligations as a responsible institution. They start from the presumption that all members of the University will take their responsibilities and obligations seriously and will ensure that their research on human subjects is conducted according to the established principles and good practice in their fields and in accordance, where appropriate, with legal requirements. Since the requirements of research ethics review will vary from field to field and from project to project, the University accepts that different guidelines and procedures will be appropriate. Please check the CUREC website to ensure that you have the correct form for your project.

This form does not cover research governance, satisfactory methodology, or the health and safety of employees and students. As principal investigator, it is your responsibility to ensure that requirements in these areas are met. Please carry out a risk assessment of the project, in consultation with all researchers involved, using the checklist and CUREC's other documentation.

The use of an asterisk in this form indicates a phrase defined in the glossary. The glossary and further information on the University's research ethics procedures are available from the CUREC website:

www.admin.ox.ac.uk/curec

This form is designed largely for research that falls within the Divisions of Social Sciences and Humanities and which does not involve a high-level of risk to the subjects. Elite interviews, field work and oral history are included in the CUREC process. Please take a moment to read through it and if you have any questions or doubts as to whether it is the appropriate form, please review Section A or consult the CUREC website.

Note on anonymised data: If you are using previously collected anonymised data about people which neither you nor anyone else involved in your study can trace back to the individuals who provided them (e.g. census data, administrative data, secondary analysis), you do not need to obtain ethical approval for your study. Please refer to the definition of *personal data in the glossary and FAQ no. 6 for further guidance.

Note on audit: If you are conducting research on behalf of or at the request of a service provider that matches the definition of *audit in the glossary, you do not need to obtain ethical approval for your study.

SECTION A

	Yes	No
1) Are you using research methodologies commonly used in biomedical or behavioural laboratory sciences?		X
2) Is there a significant risk that the research will induce anxiety, stress or other harmful psychological states in participants that might persist beyond the duration of any test or interview in which they are participating?		X
3) Will the research involve human participants recruited by means of their status as present or past NHS patients or their relatives or carers or present or past NHS staff?		X
4) Does the research involve *human participants aged 16 and over who do not have *capacity to consent for themselves? See the Mental Capacity Act 2005		X
5) Is the study to be funded by the US National Institutes of Health or another US federal funding agency?		X

Office use only: IDREC Ref. No. _______________
Date of confirmation that checklist accepted on behalf of IDREC: // //

If you answered 'yes', please stop work on this checklist and
for questions 1 and 2, complete CUREC/1 instead (available from www.admin.ox.ac.uk/curec);
for questions 3 and 4, submit your proposal to the appropriate NHS ethics committee (see www.nres.npsa.nhs.uk and www.admin.ox.ac.uk/rso/clinical for further information);
for question 5, or **if you answered 'yes' to questions 1, 2 or 4 and your research will take place outside the EU**, submit your proposal to OXTREC, which uses separate documentation (www.tropicalmedicine.ox.ac.uk/oxtrec).
If you have answered 'no' to all questions in Section A, please complete Sections B-E. This form and any supporting materials should be typewritten.

SECTION B

****Principal investigator/ supervisor/student researcher (title and name):***	Sarah Walter (doctoral student)
Name of supervisor (STUDENT RESEARCH PROJECTS ONLY):	Terezinha Nunes
Department or institute:	Education 15 Norham Gardens, Oxford, OX2 6PY
Address for correspondence (if different):	Lincoln College, Turl Street, Oxford, OX1 3DR
Email and phone contact:	sarah.walter@lincoln.ox.ac.uk; 07504590658
Title of research project:	The Relationship Between Young Children's Sharing Behaviours and Early Mathematical Concepts
Brief description of research methods and goals plus description of the nature of participants (including the criteria for inclusion/exclusion, method of recruitment), explanation of how professional guidelines and/or CUREC protocol(s) will be applied (if relevant) and expected use to which the results/data will be put. Please describe how you will obtain informed consent. Approx 400 words.	

Sharing has been studied as a social behaviour for a long period of time, and a large body of literature exists on topics such as distributive justice and fairness (Piaget, 1932; Damon, 1975; Enright et al., 1984; Benenson et al., 2003; Guroglu et al., 2009; etc.) There is also a large body of literature examining children's use and understanding of early mathematical concepts (Piaget and Szeminska, 1941; Dickson et al., 1984; Frydman and Bryant, 1988; Correa et al., 1998; etc.). More recent research is beginning to address the conceptual crossover between these areas. Sharing provides different contexts for young children to use mathematical strategies, such as counting and correspondence, to distribute resources and decide when things are fairly divided. While there are some studies that have examined young children's quantitative knowledge in the context of sharing, for example Desforges and Desforges, 1980; Hook, 1978; Klein and Langer, 1987; Frydman and Bryant, 1987; Frydman, 1990; Cowan and Biddle, 1989, there are still many things uncover in this area. The current study aims to further clarify the relationship between early mathematical and sharing concepts by examining whether young children can learn and use strategies beyond one-to-one correspondence to share resources in situations where giving equal shares is not appropriate or will not result in fair sharing (for example, if one person has done more work than another). Participants will be 4 and 5 year old children selected from classrooms in schools in the Oxford area and schools in the Toronto, Ontario, Canada area. Head teachers will be approached and asked if they would be willing to allow the researcher to conduct a study. Once schools agree to partake in the study, the researcher will contact the classroom teacher to set up a schedule and to provide material about the study for the children to take home. All efforts will be made to minimize any disruption to the classrooms and schools. An opt-in procedure will be used so that only those children whose parents return written consent to their child's form teacher will be able to participate. Parents will have the opportunity to contact the researcher with questions or concerns at any point before, during or after the study. Information about participating children's dates of birth will be collected from the school once consent has been received. The study will take place at the child's school either in their classroom or in another quiet space designated by the child's teacher. Children will be seen individually by the researcher and will be asked whether or not they want to participate before the researcher begins any tasks. There will be four sharing tasks during one session, which should last no longer than 30 minutes. All four tasks will ask the child to share objects fairly between two puppets. Stories and pictures will be used to set the context of the sharing behaviour. Each child will only need to participate in one session. All data collected will be anonymised, confidential, and will be securely stored. The results of the study will only be used for the purposes of a doctoral thesis. The researcher has obtained all requirements for working with children in schools in the UK and Ontario, and will comply with any other legal requirements, as well as the BERA and BPS ethical guidelines and CUREC protocol 2.1.2.	
List actual or probable location(s) where project will be conducted, if known:	Schools in the Oxfordshire area and schools in the Toronto, Ontario, Canada area (particular schools are unknown at this time).
Anticipated duration of project:	__35__ months
Anticipated start date:	01 / February / 2011
Anticipated end date:	31 / Dec / 2013
Name and status (e.g. 3rd year undergraduate; post-doctoral research assistant) of others taking part in the project:	N/A

Please indicate what training on research ethics the researchers involved with this study have received, e.g. the title of the online or in-person course, and date completed (online training available at www.admin.ox.ac.uk/rso/integrity/#Training):	The researcher received ethics training during her masters (MSc Child Development and Education) from the education department at the University of Oxford, both within a weekly course entitled the Foundations of Educational Research, and when filling out CUREC and other ethics forms for her masters research. As well, she attended a class on ethics as part of the Strategies for Educational Research class on November 3rd, 2010.

SECTION C

Methods to be used in the study (**tick** as many as apply: this information will help the committee understand the nature of your research and may be used for audit).

	Please
Interview	
Questionnaire	
Analysis of existing records	
Participant performs verbal/paper and pencil/computer based task	X
Measurement/recording of motor behaviour	
Audio recording of participant	
Video recording or phtotography of participant	
Physiological recording from participant	
Participant observation	
Covert observation	
Systematic observation	
Observation of specific organisational practices	
Other (please specify)	

SECTION D

Have you read one or more of the following professional guidelines and do you undertake to use the principles listed there as a guide for your own work? Please note that this is not intended to be an exhaustive list. Links to the guidelines listed below are included on the CUREC website.

	Please tick
British Society of Criminology: Code of Ethics for Researchers in the Field of Criminology [britsoccrim.org/ethical.htm]	
British Educational Research Association Ethical Guidelines for Educational Research [www.bera.ac.uk/publications/guidelines/]	X
Academy of Management's Professional Code of Ethics [www.aomonline.org]	

Association of American Geographers Statement on Professional Ethics [www.aag.org/Info/ethics.htm]	
Oral History Society of the UK Ethical Guidelines [www.oralhistory.org.uk/ethics/index.php]	
American Political Science Association (APSA) Guide to Professional Ethics in Political Science (Section H) [www.apsanet.org/content_9350.cfm]	
British Psychological Society Code of Ethics and Conduct [www.bps.org.uk/the-society/code-of-conduct/code-of-conduct_home.cfm]	X
Ethics Guidelines of the Association of Social Anthropologists of the UK and Commonwealth [www.theasa.org/ethics/guidelines.htm]	
Social Research Association: Ethical Guidelines [www.the-sra.org.uk/ethical.htm]	
Statement of Principles of Ethical Research Practice from the Socio-Legal Studies Association [www.slsa.ac.uk]	
Statement of Ethical Practice for the British Sociological Association [www.britsoc.co.uk/equality/Statement+Ethical+Practice.htm]	
Other professional guidelines (please specify):	

SECTION E

Please put a tick in the yes/no column as appropriate to indicate your response.

1) Will you obtain informed consent according to good practice in your discipline before participation?	**Yes**	**No**
	X	
2) Will you ensure that *personal data collected directly from participants or via a *third party is held and processed in accordance with the provisions of the Data Protection Act?	**Yes**	**No**
	X	
3) Does the research involve as participants *people whose ability to give free and informed consent is in question? (This includes those under 18 and vulnerable adults.)	**Yes**	**No**
	X	
4) As a consequence of taking part in the research, will participants be at serious risk of rendering themselves liable to criminal prosecution (e.g. by providing information on drug abuse or child abuse)?	**Yes**	**No**
		X
5) Does the research involve the *deception of participants, as part of the investigation/experiment?	**Yes**	**No**
		X
If any of your answers above are in a shaded box, please indicate whether those aspects of your project are fully covered by the following.		
6) Research protocol(s) which has/ve received IDREC/CUREC approval? ***If yes, please give protocol number(s): 2.1.2***	**Yes**	**No**
	X	
7) Professional guidelines that you will be following, as noted under Section D?	**Yes**	**No**
	X	

If any of your answers in Section E are in a shaded box and are not covered by a protocol or by professional guidelines, please complete CUREC/2, available to download from the CUREC website. Then submit both this form (you need not complete section F) and the CUREC/2 to the Social Sciences and Humanities IDREC.
If all your answers in Section E are in the unshaded boxes or your answers in shaded boxes are covered by a protocol or professional guidelines, complete Section F and submit this form and any accompanying documents to the Social Sciences and Humanities IDREC (see notes and address below).

SECTION F
Complete this section only if you do not need to submit form CUREC/2.
I understand my responsibilities as principal researcher/supervisor/student researcher as outlined in the CUREC glossary and guidance on the CUREC website. I declare that the answers above accurately describe my research as presently designed and that I will submit a new checklist should the design of my research change in a way which would alter any of the above responses so as to require completion of CUREC/2 (involving full scrutiny by an IDREC). I will inform the relevant IDREC if I cease to be the principal researcher on this project and supply the name and contact details of my successor if appropriate.
Signed by principal researcher/supervisor/student researcher:……………………………………
Date:…………………
Print name (block capitals)………Sarah Walter……………………………
Signed by supervisor:……………………………………………………………(for student projects)
Date:…………………
Print name (block capitals)……………Terezinha Nunes……………………………………

I understand the questions and answers that have been entered above describing the research, and I will ensure that my practice in this research complies with these answers, subject to any modifications made by the principal researcher properly authorised by the CUREC system.
Signed by associate/other researcher:
…………………………………………………………….
Print name (block capitals)…………………………………………………………………………
Date ………………
I have read the research project application named above. On the basis of the information available to me, I:
(i) consider the principal researcher/supervisor/student researcher to be aware of her/his ethical responsibilities in regard to this research;
(ii) consider that any ethical issues raised have been satisfactorily resolved or are covered by relevant professional guidelines and/or CUREC approved protocols, and that it is appropriate for the research to proceed without further formal ethical scrutiny at this stage (noting the principal researcher's obligation to report should the design of the research change in a way which would alter any of the above responses so as to require completion of a CUREC/2 full application);
(iii) am satisfied that the proposed project has been/will be subject to appropriate *peer review and is likely to contribute something useful to existing knowledge and/or to the education and training of the researcher(s) and that it is in the *public interest.

(iv) [FOR DEPARTMENTS/FACULTIES WITH A DEPARTMENTAL RESEARCH ETHICS COMMITTEE (DREC) OR EQUIVALENT BODY - PLEASE DELETE IF NOT APPLICABLE] confirm that this checklist (and associated research outline) has been reviewed by the Department's Research Ethics Committee (DREC)/equivalent body, and attach the associated report from that body.

Signed:……………………………………..
(Head of department or nominee e.g Chair of DREC, Director of Graduate Studies for postgraduate student projects)
Print name (block capitals)…………………………………………………………………………
Date:……………………
Please send an electronic copy and a signed paper copy of this completed checklist, together with any supporting documentation, to the following addresses, keeping a copy for yourself:
Secretary of the Social Sciences and Humanities IDREC Email: ethics@socsci.ox.ac.uk
University of Oxford Social Sciences Division
Hayes House, George Street
Oxford, OX1 2BQ
Forms may be sent by email (without signature), where both the note of submission from the researcher and the note of endorsement from the supervisor/Head of Department are sent from a University of Oxford email address.
IDRECs and/or CUREC will review a sample of completed checklists and may ask for further details of any project.
FINAL CHECK
To prevent delay please check each of the following before submitting the application.

Have you completed Sections A-E? ☒
Have you defined all technical terms and abbreviations used? ☒
Have you included any supporting documentation, including – as appropriate – questionnaires and participant information, consent forms/form or note of procedure for recording oral consent, advertisements and surveys to be used? ☒
Are all pages (including appendices and attachments) numbered? ☒
Are all relevant declarations in Section F complete and any necessary authorisations
obtained (by email or by signing the form)? ☒

D) CUREC Approval Letter:

15/02/2011

Dear Sarah Walter,

Application Approval

Title: The Relationship Between Young Children's Sharing Behaviours and Early Mathematical Concepts

The above application has been considered on behalf of the Departmental Research Ethics Committee (DREC) in accordance with the procedures laid down by the University for ethical approval of all research involving human participants.

I am pleased to inform you that, on the basis of the information provided to DREC, the proposed research has been judged as meeting appropriate ethical standards, and accordingly approval has been granted.

Should there be any subsequent changes to the project, which raise ethical issues not covered in the original application, you should submit details to DREC for consideration.

Yours sincerely,

Justina Kurkova

Research Office Assistant

E) Toronto Police Clearance Letter:

Toronto Police Service

40 College Street, Toronto, Ontario, Canada. M5G 2J3
TEL 416-808-2222 FAX 416-808-8202
Website: www.TorontoPolice.on.ca

TORONTO

William Blair
Chief of Police

File Number 11-557

January 13, 2011

Ms. Sarah Walter

To Whom It May Concern:

RE:WALTER, Sarah (dob:1987.02.11)

Please be advised that a search of the National Criminal Records repository maintained by the RCMP did not identify any records for a person with the name(s) and date of birth as indicated.

Positive identification that a criminal record may or may not exist at the National Criminal Records repository can only be confirmed by fingerprint comparison.

Issuing Clerk: Gilon El-Walsh (82013) 2011-01-13
Signature, Badge No. & Date Issued

NOTE: This screening is not intended for individuals being employed and/or volunteering with vulnerable* person(s). A more comprehensive process is available through the Police Reference Check Program.

For further information please contact Cathy Blair, Co-ordinator, Records Management Service at 416 808 - 8239

*Vulnerable person means a person who, because of their age, a disability, or other circumstances, whether temporary or permanent, are (a) in a position of dependence on others or (b) are otherwise at a greater risk than the general population of being harmed by a person in a position of authority or trust relative to them.

To Serve and Protect - Working with the Community

THIS IS AN ORIGINAL DOCUMENT AND CONTAINS SECURITY FEATURES - SEE REVERSE FOR DETAILS

F) CRB Clearance Letter:

Enhanced Disclosure

Page 1 of 2

disclosure

Disclosure Number 001311729924

Date of Issue: 09 FEBRUARY 2011

Applicant Personal Details

Surname: WALTER

Forename(s): SARAH ELIZABETH

Other Names: NONE DECLARED

Date of Birth: 11 FEBRUARY 1987

Place of Birth: NORTH YORK CANADA

Gender: FEMALE

Employment Details

Position applied for:
RESEARCHER WITH CHILDREN

Name of Employer:
UNIVERSITY OF OXFORD

Countersignatory Details

Registered Person/Body:
UNIVERSITY OF OXFORD DEPARTMENT OF EDUCATION

Countersignatory:
ERICA OAKES

Police Records of Convictions, Cautions, Reprimands and Warnings

NONE RECORDED

Information from the list held under Section 142 of the Education Act 2002

NONE RECORDED

ISA Children's Barred List information

NONE RECORDED

ISA Vulnerable Adults' Barred List information

NOT REQUESTED

Other relevant information disclosed at the Chief Police Officer(s) discretion

NONE RECORDED

Enhanced Disclosure

This document is an Enhanced Criminal Record Certificate within the meaning of sections 113B and 116 of the Police Act 1997.

THIS DISCLOSURE IS NOT EVIDENCE OF IDENTITY

CRB PO Box 165, Liverpool, L69 3JD Helpline: 0870 90 90 844

Continued on page 2

G) Study 1: Head Teacher Information Letter- Establishing Equivalence and Comparing Equivalent Sets Across Sharing Contexts:

Department of Education
15 Norham Gardens,
Oxford, OX2 6PY
Tel: +44(0)1865 27402

X Primary School
Address
Address

Dear Mr/Mrs. Xyz,

The Relationship Between Young Children's Sharing Behaviors and Early Mathematical Concepts
Sarah Walter
DPhil in Education
University of Oxford

I am writing to ask if you would be interested in your school being involved in a doctoral thesis project looking at the relationship between sharing and early mathematics. The project is supervised by Professor Terezinha Nunes and has received ethics clearance from the University of Oxford Central University Research Ethics Committee.

The purpose of this study is to gain important information about the ways that 4 and 5-year-old children share. Sharing gives young children the opportunity to use mathematical concepts in real world situations, which could help them develop their knowledge of more abstract mathematical operations when they are taught later on. Sharing fairly can be complicated, particularly in situations where, for example, one person has done more work than another and therefore deserves a larger share. This study aims to investigate the strategies that young children use in these types of situations where sharing fairly does not mean giving equal shares.

Children will be asked if they would like to participate, and will be free to withdraw at any time they wish without explanation. The study would involve the researcher working with 4 and 5 year old students from the reception year for approximately 30 minutes per child. Each child would be asked to complete four sharing trials with the researcher, for example sharing sweets between two puppets, and will only need to participate in one session. The researcher would aim to visit at times convenient for the class teacher, and to work outside the classroom if possible so as to cause minimal distraction. I would be happy to produce a short report of the findings for staff or parents who may be interested in the research.

All data will be securely stored, and kept confidential and anonymous in accordance with UK data protection requirements. No information about individual children will be made available to anyone outside of the study. The researcher has also received the required clearance to work with children in schools.

I hope to start work in March, as soon as possible and convenient after half term. If you are interested in the project, I can be contacted at: 07504590658 or at sarah.walter@lincoln.ox.ac.uk. As well, I will telephone one week after half term to see if any teachers may wish to be involved, and to provide more information as required. I hope I will have the opportunity of working with your school. Thank you very much for your time.

Yours Sincerely,
Sarah Walter
Doctoral Student, Department of Education, University of Oxford

H) Study 1: Parent Information Sheet/Consent Form- Establishing Equivalence and Comparing Equivalent Sets Across Sharing Contexts:

X Primary School
Address
Address

Date:

UNIVERSITY OF
OXFORD

Headteacher: Mr/Mrs. Xyz

Dear Parents/Carers:

Your child has been invited to participate in a research study.

The purpose of this study is to gain important information about how 4- and 5-year-old children solve sharing tasks to help us understand better the relationship between early maths and how they work with others.

If you agree to allow your child to take part, a time will be arranged with the teacher and the researcher will work with your child in a quiet area at the school.

The tasks will involve your child sharing objects between two or more puppets with the help of pictures and stories. Your child will only need to participate in one session lasting no more than 40 minutes. It is your decision whether or not your child takes part in this study.

All information collected will be kept strictly confidential in accordance with the UK data protection requirements and will be used only for the purposes of this study. No information about individual children will be made available to anyone outside of this study.

If you wish to allow your child to participate in the project, please complete and return the attached sheet to your child's form teacher.

If you have any questions, please contact: Sarah Walter (research student) at sarah.walter@lincoln.ox.ac.uk or at 07504590658 or Dr. Terezinha Nunes (academic supervisor) at terezinha.nunes@education.ox.ac.uk.

Thank you for your help!

By signing below, I give permission for my child to participate in this study.

Child’s School:____

Child’s Teacher:_____________________________________

Child’s Name (Printed): ________________________________

Parent/Guardian Name (Printed):___________________________________

Signature: ____________________________________

Date:_____________

I) Study 1: Researcher's Script- Establishing Equivalence and Comparing Equivalent Sets Across Sharing Contexts:

Screening Conditions:

- Use orange legos.

1. These two friends are very hungry after a long day at school and want a snack. I have some biscuits here for them to eat. Can you help me share the biscuits between them so they each have the same? Give one to this puppet and then one to this puppet, like this….(Share one to one)….Now can you do the rest?

2. Oh look, another one of their friends has come to join them! They want him to have some biscuits as well. Let's gather up the biscuits so we can hand them out again. Can you share the biscuits between all three friends this time?

STOP. Thank child if they did not get 1 of 2 trials right.

- Now we're going to help some other friends. Some yellow and blue blocks are stuck together like this, and some yellows and blues are on their own. There is one yellow and one blue in this unit, which is the same as having one yellow and one blue on their own. Can you tell me how many pieces are in this (double) unit?

1. I have two friends who are having a picnic and need help sharing out the treats they have brought to eat. Let's see if you can help them. The first thing they have to eat is sweets. They both want to have the same, but Mr. Koala likes to get 2 sweets at a time, while Mr. Dog likes to get one at a time. So, only give Mr. Koala these blocks and only give Mr. Dog these blocks. Keep sharing with them until I tell you stop. Ok? (Stop child and remind them to give the same number of portions to each puppet if they have not given any shares to one). After Mr. Dog has 6 sweets…Do you think each friend has a fair share?

2. Now Mr. Koala and Mr. Dog want to share pieces of chocolate. Mr. Koala likes to get 2 pieces at a time and Mr. Dog only likes to get 1 at a time, so only give Mr. Koala these blocks and only give Mr. Dog these blocks. Keep sharing with them until I tell you to stop. Ok? After Mr. Dog has 6 pieces of chocolate….Do you think each friend has a fair share?

STOP. Thank child if they did not get 1 of 2 trials right.

Equal Sharing:

- Now we're going to play with these blocks. See how some reds are stuck together and some reds are on their own? It's the same as the blue and yellow ones before, the two reds stuck together are the same as having two reds on their own. Can you tell me how many pieces are in this (double) unit?

- Thanks for helping Mr. Koala and Mr. Dog! I have some more friends here who need help sharing!

1. Jeff/Jessica and Alex want to share some blueberries. Jeff/Jessica likes to get 1 berry at a time and Alex like to get 1 berry at a time too, so give one block at a time to Jeff/Jessica and one block to Alex. Can you help them share these blueberries? Keep sharing with them until I tell you to stop. Ok? After Alex receives 6 berries….Do you think each friend has a fair share?

2. Now Bob/Briana and Joe/Joanna want to share some biscuits. Bob/Briana likes to get 2 biscuits while Joe/Joanna like to have one at a time, so only give doubles to Bob/Briana and singles to Joe/Joanna. Can you help them share these biscuits? Keep sharing until I tell you to stop. Ok? After Joe/Joanna has 6 biscuits….Do you think each friend has a fair share?

3. Sally/Stuart and Danielle/David want to share crisps this time. Sally/Stuart likes to get 2 crisps while Danielle/David likes to have one at a time, so only give doubles to Sally/Stuart and singles to Danielle/David. Can you help them share these crisps? Keep sharing until I tell you to stop. Ok? After Danielle/David has 6 crisps…Do you think each friend has a fair share? Can you count how many crisps Sally/Stuart has? Now can you tell me how many crisps Danielle has? (cover the pile if they try to count). Can you tell me without counting?
4. Alice/Albert and Ruth/Roger have some balloons to share. Alice/Albert likes to get 1 balloon at a time and Ruth/Roger likes to get 1 balloon at a time as well, so give one block at a time to Alice/Albert and one block to Ruth/Roger. Can you help them share these balloons? Keep sharing until I tell you to stop. Ok? After Ruth/Roger has received 6 balloons…Do you think each friend has a fair share?

5. Now Jack/Jenna andTheo/Tracy want to share stickers. Jack/Jenna likes to get 2 stickers at a time, while Theo/Tracy likes to get one at a time, so only give doubles to Jack/Jenna and singles to Theo/Tracy. Can you help them share these stickers? Keep sharing until I tell you to stop. Ok? After Theo/Tracy has received 6 stickers…Do you think each friend has a fair share?

6. This set of friends has some marbles they want to share. Frank/Francine likes to get 2 marbles, while Luke/Lisa likes to get one at a time, so only give doubles to Frank/Francine and singles to Luke/Lisa. Can you help them share these marbles? Keep sharing until I tell you to stop. Ok? After Luke/Lisa has 6 marbles…Do you think each friend has a fair share? Can you count how many marbles Frank/Francine has? Now can you tell me how many marbles Luke/Lisa has? (cover pile if necessary) Can you tell me without counting?

Reciprocity:

- Now let’s see what some of my other friends are up to!

1. Emma/Eric and Lily/Larry have toy ponies to share. Emma/Eric shares her ponies with Lily like this: Lily/Larry gets one pony and Emma/Eric gets one pony. Now it’s Lily’s turn to share her ponies. Can you help her

share with Emma? Keep sharing until I tell you to stop. Ok? After Lily/Larry has 6 ponies…Do you think each friend has a fair share?

2. Now Rosie/Ron and Olivia/Oliver would like to share some bubble gum. Rosie shares her bubble gum with Olivia like this: Olivia/Oliver gets 2 pieces at a time while Rosie/Ron gets 1 piece at a time. Now it's Olivia/Oliver's turn to share her bubble gum. Can you help her share with Rosie/Ron? Keep sharing until I tell you to stop. Ok? After Olivia/Oliver has 6 pieces….Do you think each friend has a fair share?

3. Hannah/Harry and Michelle/Michael have chocolates to share. Hannah/Harry shares her chocolates with Michelle/Michael like this: Michelle/Michael gets 2 pieces at a time while Hannah/Harry gets 1 piece at a time. Now it's Michelle/Michael's turn to share her chocolates. Can you help her share with Hannah/Harry? Keep sharing until I tell you to stop. Ok? After Michelle/Michael has 6 pieces…Do you think each friend has a fair share? Can you count how many chocolates Hannah/Harry has? Now can you tell me how many Michelle/Michael has? (cover pile if necessary) Can you tell me without counting?

4. Now Mark/Mandy and Jim/Jessica have lollipops to share. Mark/Mandy shares his lollipops with Jim/Jessica like this: Jim/Jessica gets 1 at a time while Mark/Mandy gets 1 at a time. Now it's Jim/Jessica's turn to share his lollipops. Can you help him share with Mark/Mandy? Keep sharing until I tell you to stop. Ok? After Jim/Jessica has 6 pieces….Do you think each friend has a fair share?

5. Melissa/Matthew and Nicole/Nick are sharing a snack together after school. Melissa/Matthew shares her crackers with Nicole/Nick like this: Nicole/Nick gets 2 crackers each time while Melissa/Matthew gets 1 piece at a time. Now it's Nicole/Nick's turn to share her crackers. Can you help her share with Melissa/Matthew? Keep sharing until I tell you to stop. Ok? After Nicole/Nick has 6 crackers…Do you think each friend has a fair share?

6. Ralph/Ruby and Tom/Tina have some coins to buy sweets. Ralph/Ruby shares his coins with Tom/Tina like this: Tom/Tina gets 2 coins at a time while Ralph/Ruby gets 1 coin at a time. Now it's Tom/Tina's turn to share his coins. Can you help him share with Ralph/Ruby? Keep sharing until I tell you to stop. Ok? After Tom/Tina has 6 pieces…Do you think each friend has a fair share? Can you count how many coins Ralph/Ruby has? Now can you tell me how many Tom/Tina has? (cover pile if necessary) Can you tell me without counting?

Equity:

1. Sophia/Steve and Alanna/Allen's neighbour says he will give them coins if they plant flowers in his garden. Sophia/Steve plant and Alanna/Allen plant the same number of flowers (place 6 by each) and got these coins. Can you help them share these coins? Keep sharing until I tell you to stop. Ok? After Sophia/Steve has 6 coins…Do you think each friend has a fair share? (If they made a mistake) Did Alanna/Allen and Sophia/Steve plant the same number of flowers? (remind them they did if they can't remember).

2. John/Jane and Ezra/Elizabeth drew picture to sell at a school art show. John/Jane made 2 pictures each time Ezra/Elizabeth made 1 picture. They want to share the coins they got from the art show. Can you help them share these coins? Keep sharing until I tell you to stop. Ok? After Ezra/Elizabeth has 6 coins…Do you think each friend has a fair share? (If they made a mistake) Can you tell me who drew more pictures? (remind them John/Jane did) Is it fair that John/Jane and Ezra/Elizabeth get the same amount when John/Jane made more pictures? Can you try sharing the coins again so they each get a fair share?

3. Samantha/Sam and Anne/Andrew's mother says she will give them some sweets if they clean up their toys first. Samantha/Sam cleans puts 2 toys in the basket each time Anne/Andrew puts in 1. They want to share the sweets they got. Can you help them share these sweets? Keep sharing until I tell you to stop. Ok? After Anne/Andrew has 6 sweets….Do you think each friend has a fair share? Can you count how many sweets Anne/Andrew has? Now can you tell me how many Samantha/Sam has? (cover pile if necessary) Can you tell me without counting? (If they made a mistake) Can you show me who put away more toys? (remind them Samantha/Sam did) Is it fair that Samantha/Sam and Anne/Andrew get the same amount when Samantha/Sam put away more toys? Can you try sharing the sweets again so they each get a fair share?

4. Lauren/Luke and Victoria/Vince are playing a game where you win a coin if you draw a red card out of the pile. Lauren/Luke and Victoria/Vince get the same number of red cards and got these coins. Can you help them share these coins? Keep sharing until I tell you to stop. Ok? After Lauren/Luke has 6 coins…Do you think each friend has a fair share? (If they made a mistake) Did Lauren/Luke and Victoria/Vince get the same number of red cards? (remind them they did if they can't remember).

5. Felicity/Fred and Polly/Paul are playing a basketball game where they win a sticker each time they get the ball in the hoop. Felicity/Fred gets the ball in 2 times each time Polly/Paul gets the ball in 1 time. Now they want to share the stickers from the game. Can you help them share these stickers? Keep sharing until I tell you to stop. Ok? After Polly/Paul has 6 stickers…Do you think each friend has a fair share? (If they made a mistake) Can you tell me who got the ball in the hoop more times? (remind them Felicity/Fred) Is it fair that Felicity/Fred and Polly/Paul get the same amount when Felicity/Fred got the ball in more times? Can you try sharing the stickers again so they each get a fair share?

6. Kyle/Kelly and Adam/Amy are playing a game where picking a gold star means they win a lollipop. Kyle/Kelly picks 2 gold stars each time Adam/Amy picks 1. They want to share the lollipops they won. Can you help them share these lollipops? Keep sharing until I tell you to stop. Ok? After Adam/Amy has 6 sweets….Do you think each friend has a fair share? Can you count how many lollipops Adam/Amy has? Now can you tell me how many Kyle/Kelly has? (cover pile if necessary) Can you tell me without counting? (If they made a mistake) Can you show me who picked more gold stars? (remind them Kyle/Kelly did) Is it fair that Kyle/Kelly and Adam/Amy get the same amount when Kyle/Kelly picked more gold stars? Can you try sharing the lollipops again so they each get a fair share?

J) **Study 1: Answer Sheet- Establishing Equivalence and Comparing Equivalent Sets Across Sharing Contexts:**

Answer Sheet (version **A**)

School: Gender:

Child ID: DOB:

Age:

Screening Task 1 (1-to-1):

a) ______________; method of sharing:

Notes:

b) ______________; method of sharing:

Notes:

Screening Task 2 (2-to-1 w/ colour cues):

a) _________________; method of sharing:

Notes:

b) _________________; method of sharing:

Notes:

Task 3 (2-to-1 no colour cues):

a) (1-to-1) ________________; method of sharing:

Notes:

b) (2-to-1) ________________; method of sharing:

Notes:

c) (2-to-1, inference) ____________; method of sharing:

Inference 1:

Inference 2:

Notes:

d) (1-to-1) _______________; method of sharing:

Notes:

e) (2-to-1) ________________; method of sharing:

Notes:

f) (2-to-1, inference) ____________; method of sharing:

Inference 1:

Inference 2:

Notes:

Task 4 (2-to-1 reciprocity):

a) (1-to-1) _________________; method of sharing:

Notes:

b) (2-to-1) _________________; method of sharing:

Notes:

c) (2-to-1, inference) _____________; method of sharing:

Inference 1:

Inference 2:

Notes:

d) (1-to-1) ________________; method of sharing:

Notes:

e) (2-to-1) ________________; method of sharing:

Notes:

f) (2-to-1, inference) ____________; method of sharing:

Inference 1:

Inference 2:

Notes:

Task 5 (2-to-1 equity):

a) (1-to-1) _________________; method of sharing:

Notes:

b) (2-to-1) ________________; method of sharing:

Notes:

c) (2-to-1, inference) ____________; method of sharing:

Inference 1:

Inference 2:

Notes:

d) (1-to-1) _______________; method of sharing:

Notes:

e) (2-to-1) _______________; method of sharing:

Notes:

f) (2-to-1, inference) ___________; method of sharing:

Inference 1:

Inference 2:

Notes:

K) **Study 2: Head Teacher Information Letter- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting:**

UNIVERSITY OF OXFORD

Department of Education
15 Norham Gardens,
Oxford,
OX2 6PY

X Elementary School
Address
Address
A1B 2C3

Dear Mr/Mrs. Xyz,

One for you, two for me: Quantitative Sharing by Young Children
Sarah Walter
DPhil in Education
University of Oxford

I am a doctoral candidate at the University of Oxford, and am writing to ask if your school would be interested in participating in a research project looking at how young children share. The project is supervised by Professor Terezinha Nunes (Department of Education, University of Oxford). It has received ethics clearance from the University of Oxford Central University Research Ethics Committee and the External Research Review Committee of the TDSB.

Sharing gives young children the opportunity to use mathematical concepts in real world situations, and is a chance to develop their quantitative reasoning skills. Sharing fairly can be complicated, particularly in situations where, for example, one person has done more work than another and therefore deserves a larger share. This study aims to investigate the strategies that young children use in these types of situations where sharing fairly does not mean giving equal shares; knowledge which could help them understand more abstract mathematical operations when they are taught later on.

Should you agree to participate, class teachers will be given consent forms to send home for parents to sign. Children whose parents return the form will then be able to participate. Each child will be free to withdraw from the study at any time they wish without explanation. The researcher would work individually with 4 and 5-year-old students from Kindergarten approximately 30-minutes per child. Each child would be seen for two 30-minute sessions across two days (i.e. 1 session per day). The first is a training session to get them used to using the coins in the study and in the second session they will be asked to complete three sharing trails with the researcher, for example sharing coins between two puppets. The researcher would arrange to visit at times convenient for the class teacher, and will aim to cause minimal distraction. I would be happy to produce a short report of the findings for staff or parents who may be interested in the research.

All data will be securely stored, and kept confidential and anonymous in accordance with the TDSB and UK data protection requirements. No information about individual children will be made available to anyone outside of the study. The researcher has also received the required clearance to work with children in schools.

I hope to start work in January 2012. I will get in touch with your school in a couple of weeks to see whether you are able to participate. In the meantime, please let me know if you are interested in participating or would like more information. I can be contacted at: (416) 565-9166 or at sarah.walter@lincoln.ox.ac.uk. I hope I will have the opportunity of working with your school. Thank you very much for your time.

Yours Sincerely,

Sarah Walter
Doctoral Student
Department of Education
University of Oxford

L) Study 2: Class Teacher Consent Form- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting:

Department of Education
15 Norham Gardens,
Oxford,
OX2 6PY
Tel: +44(0)1865 27402

X Elementary School
Address
Address
A1B 2C3

Dear Mr/Mrs. Xyz,

One for you, two for me: Quantitative Sharing by Young Children
Sarah Walter
DPhil in Education
University of Oxford

I am a doctoral candidate at the University of Oxford working on a research project looking at the relationship between sharing and early mathematics. The project is supervised by Professor Terezinha Nunes (Department of Education, University of Oxford) and has received ethics clearance from the University of Oxford Central University Research Ethics Committee and the External Research Review Committee of the TDSB.

The purpose of this study is to gain important information about the ways that 4 and 5-year-old children share. Sharing gives young children the opportunity to use mathematical concepts in real world situations, and this study aims to investigate the mathematical strategies that young children use in situations where sharing fairly does not mean giving equal shares.

The study involves the researcher working individually with students from your classroom for 2 sessions of approximately 30 minutes each. The sessions will take place on 2 different days. The study will take place during school hours and within the school, so children will need to be briefly excused from regular activities to participate. The researcher will aim to visit at times convenient for you and your class, and to work outside the classroom if possible so as to cause minimal distraction.

All data will be securely stored, and kept confidential and anonymous in accordance with the TDSB and UK data protection requirements. No information about individual children will be made available to anyone outside of the study.

At the bottom of this sheet you will find a consent form asking for your permission to allow students to leave class to participate in the study. Please fill this in and return it to me (Sarah Walter). Your help and cooperation is greatly appreciated.

If you would like more information, I can be contacted at: (416) 565-9166 or at sarah.walter@lincoln.ox.ac.uk. Thank you very much for your time.

Yours Sincerely,

Sarah Walter
Doctoral Student
Department of Education
University of Oxford

UNIVERSITY OF OXFORD

One for you, two for me: Quantitative Sharing by Young Children

I agree to permit those students whose parents have returned consent forms to leave regular class activities when they are called by the researcher to participate in order to complete the study tasks.

By signing below, I give participating students permission to leave class and take part in this study:

Teacher Name:____

Teacher School:____

Teacher Signature:____Date:__________________________

M) Study 2: Parent Information Sheet- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting:

Department of Education
15 Norham Gardens, Oxford, OX2 6PY
Tel: +44(0)1865 274024
Fax: +44(0)1865 274027

One for you, two for me: Quantitative Sharing by Young Children
Sarah Walter
DPhil in Education
University of Oxford

Information for Parents:
Your child has been invited to participate in a research study. Before you decide whether or not you would like them to participate, it is important to understand why the study is being conducted and what their participation entails. Please read the following information carefully. If you have any questions or would like more information please ask.

This study has received ethics clearance from the University of Oxford Central University Research Ethics Committee and the External Research Review Committee of the TDSB. The researcher also has the required clearance to work with children in schools.

Purpose:
I am a researcher from the University of Oxford interested in investigating the relationship between early mathematical and social concepts by looking at how 4 and 5 year old children share. Children often use math reasoning to decide how to share resources fairly between people. But fair sharing may be complicated sometimes, for example when one person does more work than another and might deserve a larger share. Sharing gives young children the opportunity to use mathematical concepts in real world situations and in different ways. This could help them develop their knowledge of more abstract mathematical concepts that they are taught later on. The purpose of this study is to gain important information about 4- and 5-year-old children's math reasoning and ability to manipulate numbers through solving sharing tasks.

Participation:
Your child's school has kindly agreed to participate in this study. All parents with children in Kindergarten are being asked if they would give permission for their child to participate in a research project. If you agree to allow your child to take part, a time will be arranged with the teacher and the researcher will work with your child in a quiet area at the school. The children will be asked if they would like to help the researcher share out some objects. The tasks will involve your child sharing objects (coins) between two or more puppets with the help of pictures and stories. Your child will only need to participate in two sessions each lasting approximately 30 minutes. It is your decision whether or not your child takes part

in this study. Their participation is entirely voluntary, and they may stop participating in the study at any time without having to explain why.

Confidentiality:
The results of this study will be reported in an Oxford doctoral thesis. All information collected will be kept strictly confidential in accordance with the TDSB and UK data protection requirements and will be used only for the purposes of this study. No information about individual children will be made available to anyone outside of this study. A report of the results of the study will be made available to the TDSB and your child's school.

Contacts for Questions or Concerns:
If you have any further questions or concerns about this study, please don't hesitate to contact:

Sarah Walter, doctoral student, by e-mail: sarah.walter@lincoln.ox.ac.uk;
by phone: (416) 565-9166
OR
Dr. Terezinha Nunes, academic supervisor, by e-mail:
terezinha.nunes@education.ox.ac.uk

If you wish to allow your child to participate in the project, please complete and return the attached sheet to your child's teacher.

If you do not want your child to participate then you do not need to return anything.

Thank you for your help!

N) Study 2: Parent Consent Form- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting:

Department of Education
15 Norham Gardens, Oxford, OX2 6PY
Tel: +44 (0)1865 274024
Fax: +44(0)1865 274027

One for you, two for me: Quantitative Sharing by Young Children
Sarah Walter, Doctoral Student, University of Oxford

Please read the following and return this sheet to your child's teacher.

1. I have read the information related to this study, have had the opportunity to ask questions about the research and have considered any risks.

2. I understand that I can withdraw from the study at any time without penalty simply by informing the researcher of my decision.

3. I understand who will have access to the data collected for the study and what will happen to the data at the end of the study.

4. I understand whom to contact should I have questions after participating in the study.

5. I understand that this project has received ethics clearance from the University of Oxford Central University Research Ethics Committee and External Research Review Committee of the TDSB.

By signing below, I give permission for my child to participate in this study.

Child's School:____

Child's Teacher:______________________________________

Child's Name (Printed): __________________________________

Parent/Guardian Name (Printed):_____________________________________

Signature: _______________________________________Date:__________

O) Study 2: Researcher's Script- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting:

Screening Conditions (Session 1)

1-to-1 blocks:

3. These two friends are very hungry after a long day at school and want a snack. I have some biscuits here for them to eat. Can you help me share the biscuits between them so they each have the same? Give one to this puppet and then one to this puppet, like this….(Share one to one)….Now can you do the rest? (get child to share singles)

4. Oh look, another one of their friends has come to join them! They want him to have some biscuits as well. Let's gather up the biscuits so we can hand them out again. Can you share the biscuits between all three friends this time? (get child to share doubles)

1-to-1 coins:

5. These two friends want to buy a snack at the store. I have some loonies here for them to use. Can you help me share the loonies between them so they each have the same? Give one to this puppet and then one to this puppet, like this….(Share one to one)….Now can you do the rest? When the child finishes ask: Do you think each friend has a fair share? Do you think each friend has the same amount? (get child to share loonies)

6. Another one of their friends has come to join and they want him to have some loonies for a snack as well. Let's gather up the loonies so we can hand them out again. Can you share the loonies between all three friends this time? When the child finishes: Do you think each friend has a fair share? Do you think each friend has the same amount? (get child to share toonies)

Stop child if they haven't been able to answer at least one of the coin and one of the block trials.

Block Familiarization Phase:

This is a double block made up of one blue blocks and one yellow block. It is the same as one single yellow and one single blue. Can you count how many one double is? Now can you count how many two singles are? So they are the same aren't they? So, if I ate a double block I would have to eat ….pause to see if child will answer….two single blocks to have the same.

Block Exchange Trials:

1. Haley/Harris and Jane/Jonathan want to share these blocks. Hayley would like to have 3 doubles, but Haley only has singles. Her friend Jane has doubles so Haley wants to trade some blocks with her. Can you help Hayley give Jane the right amount of singles for 3 doubles?

2. Sophia/Steve and Alanna/Allen are building towers with blocks. Sophia would like to have 4 singles for her tower, but Sophia only has doubles. Her friend Alanna has singles so Sophia wants to trade some blocks with her. Can you help Sophia give Alanna the right amount of doubles for 4 singles?

3. Samantha/Sam and Anne/Andrew are using blocks to make castles. Samantha would like to have 4 doubles for her castle, but Samantha only has singles. Her friend Anne has doubles so Samantha wants to trade some blocks with her. Can you help Samantha give Anne the right amount of singles for 4 doubles?

4. Lauren/Luke and Victoria/Vince want to build some fences out of blocks. Lauren would like to have 10 singles to finish her fence, but Lauren only has doubles. Her friend Victoria has singles so Lauren wants to trade some blocks with her. Can you help Lauren give Victoria the right amount of doubles for 10 singles?

5. Sally/Stuart and Danielle/David want to share these blocks. Sally would like to have 2 doubles, but Sally only has singles. Her friend Danielle has doubles so Sally wants to trade some blocks with her. Can you help Sally give Danielle the right amount of singles for 2 doubles?

6. Bob/Briana and Joe/Joanna are building towers with blocks. Bob would like to have 12 singles for his tower, but Bob only has doubles. His friend Joe has singles so Bob wants to trade some blocks with him. Can you help Bob give Joe the right amount of doubles for 12 singles?

7. Jeff/Jessica and Alex are using blocks to make houses. Jessica would like to have 5 doubles for her house, but Jessica only has singles. Her friend Alex has doubles so Jessica wants to trade some blocks with her. Can you help Jessica give Alex the right amount of singles for 5 doubles?

8. Alice/Albert and Ruth/Roger want to build some castles out of blocks. Alice would like to have 6 singles to finish her castle, but Alice only has doubles. Her friend Ruth has singles so Alice wants to trade some blocks with her. Can you help Alice give Ruth the right amount of doubles for 6 singles?

9. Jack/Jenna and Theo/Tracy are using blocks to make forts. Jenna would like to have 6 doubles for her fort, but Jenna only has singles. Her friend Tracy has doubles so Jenna wants to trade some blocks with her. Can you help Jenna give Tracy the right amount of singles for 6 doubles?

10. Frank/Francine and Luke/Lisa want to build some palaces out of blocks. Francine would like to have 8 singles to finish her palace, but Francine only has doubles. Her friend Lisa has singles so Francine wants to trade some blocks with her. Can you help Francine give Lisa the right amount of doubles for 8 singles?

Stop child if it is no longer possible for them to answer 8 trials in a row correctly.

Coin Familiarization Phase:
This coin is a toonie and is worth 2 and this coin is a loonie and is worth 1. So, one toonie (put 2 fingers on the coin) is the same as two loonies (put 1 finger on each loonie). Can you show me with your fingers how much a toonie is worth? And how much two loonies are worth? So if I wanted to buy this really nice big sticker that costs a toonie, I would need to give….pause to see if child answers….two loonies.

Coin Exchange Trials:

11. John/Janet and Ezra/Elizabeth are at a fair. John would like to buy some cotton candy. Cotton candy costs 3 toonies, but John only has loonies. His friend Ezra has toonies so John wants to trade some coins with him. Can you help John give Ezra the right amount of loonies for 3 toonies?

12. Felicity/Fred and Polly/Paul are at the store. Felicity wants to buy a popsicle. A popsicle costs 4 loonies, but Felicity only has toonies. Her friend Polly has loonies so Felicity wants to trade some coins with her. Can you help Felicity give Polly the right amount of toonies for 4 loonies?

13. Kyle/Kelly and Adam/Amy are at the movies. Kyle wants to buy a bag of popcorn. Popcorn costs 4 toonies, but Kyle only has loonies. His friend Adam has toonies so Kyle wants to trade some coins with him. Can you help Kyle give Adam the right amount of loonies for 4 toonies?

14. Charlie/Caitlin and Tim/Theresa want to draw pictures. Charlie wants to buy a box of crayons. A box of crayons costs 10 loonies, but Charlie only has toonies. His friend Tim has loonies so Charlie wants to trade some coins with him. Can you help Charlie give Tim the right amount of toonies for 10 loonies?

15. Emma/Eric and Lily/Larry are at a store. Eric would like to buy a teddy bear. The bear costs 2 toonies, but Eric only has loonies. His friend Larry has toonies so Eric wants to trade some coins with him. Can you help Eric give Larry the right amount of loonies for 2 toonies?

16. Rosie/Ron and Olivia/Oliver are at a toy store. Rosie wants to buy a video game. The game costs 12 loonies, but Rosie only has toonies. Her friend Olivia has loonies so Rosie wants to trade some coins with her. Can you help Rosie give Olivia the right amount of toonies for 12 loonies?

17. Harry/Hannah and Michael/Michelle are in a store. Harry want to buy a sticker book. The book costs 5 toonies, but Harry only has loonies. His friend Michael has toonies so Harry wants to trade some coins with him. Can you help Harry give Michael the right amount of loonies for 5 toonies?

18. Patrick/Phoebe and Travis/Tanya are at the toy store. Patrick wants to buy some bubbles. Bubbles costs 6 loonies, but Patrick only has toonies. His friend Travis has loonies so Patrick wants to trade some coins with him. Can you help Patrick give Travis the right amount of toonies for 6 loonies?

19. Mason/Melissa and Noah/Noelle are at a book store. Mason would like to buy a comic book. The book costs 6 toonies, but Mason only has loonies. His friend Noah has toonies so Mason wants to trade some coins with him. Can you help Mason give Noah the right amount of loonies for 6 toonies?

20. Vanessa/Victor and Isabelle/Isaac are at an ice cream truck. Vanessa wants to buy an ice cream sandwich. The ice cream sandwich costs 8 loonies, but Vanessa only has toonies. Her friend Isabelle has loonies so Vanessa wants to trade some coins with her. Can you help Vanessa give Isabelle the right amount of toonies for 8 loonies?

Stop child if it is no longer possible for them to answer 8 trials in a row correctly.

Experimental Conditions (Session 2)

Coin Familiarization Phase:
Remember the toonies and loonies from last time? Can you show me with your fingers how much a toonie is worth? Can you show me with your fingers how much a loonie is worth? So, one toonie (put 2 fingers on the coin) is the same as how many loonies? (Confirm they are the same by getting the child to put their fingers on the coins if they give the correct answer) If child doesn't give correct answer say I think it's two loonies. Can you show me with your fingers how much a toonie is again? Now can you show me how much two loonies are? They are the same aren't they? So if I wanted to buy this really nice big sticker that costs a toonie, I would need to give….pause to see if child answers….two loonies.

Children will be told to stop sharing when they have completed 6 correct turns in a row. If a child shares only to one character, they will be stopped after distributing 3 blocks (or coins) and asked to also share with the other character in order to give them the possibility of evening out the shares without it becoming too difficult.

Equal Sharing Condition (Coins):

Can you show me again with your fingers how much a toonie is? Can you show me how much a loonie is? (Remind child how much each is if they cannot remember).

7. Jeff/Jessica and Alex want to buy some blueberries at the grocery store. Jeff/Jessica and Alex both want to have the same. Jeff/Jessica needs loonies to buy blueberries and Alex needs loonies to buy blueberries too, so give loonies to both Jeff/Jessica and to Alex. Can you help them share these coins so it's far? Keep sharing with them until I tell you to stop. Ok?….Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy blueberries?

8. Now Bob/Briana and Joe/Joanna want to buy some ice cream cones. Bob/Briana and Joe/Joanna want to have the same, but Bob/Briana needs toonies for their ice cream while Joe/Joanna needs loonies for their ice cream, so only give toonies to Bob/Briana and loonies to Joe/Joanna. Can you help them share these coins so it's fair? Keep sharing until I tell you to stop. Ok?….Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy ice cream cones?

9. Sally/Stuart and Danielle/David want to buy some chips to eat. Sally/Stuart and Danielle/David want to have the same, but Sally/Stuart needs toonies for their chips while Danielle/David needs loonies for their chips, so only give toonies to Sally/Stuart and loonies to Danielle/David. Can you help them share these coins so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy chips? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add coins to make the distribution even. Now they have the same. Can you count how much money

Sally/Stuart has? Now can you tell me how much money Danielle/David has? (cover pile)

10. Alice/Albert and Ruth/Roger are buying some balloons for their birthday party. Alice/Albert and Ruth/Roger want to have the same. Alice/Albert needs loonies for their balloons and Ruth/Roger needs loonies for their balloons as well, so give loonies to both Alice/Albert and Ruth/Roger. Can you help them share these coins so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy balloons? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add coins to make the distribution even. Now they have the same. Can you count how much money Alice/Albert has? Now can you tell me how much money Ruth/Roger has? (cover pile)

11. Now Jack/Jenna and Theo/Tracy want to buy some stickers. Jack/Jenna and Theo/Tracy want to have the same, but Jack/Jenna needs toonies for their stickers and Theo/Tracy needs loonies for their stickers, so only give toonies to Jack/Jenna and loonies to Theo/Tracy. Can you help them share these coins so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy stickers?

12. Frank/Francine and Luke/Lisa have some toy cars (or dolls) they want to buy at the toy store. Frank/Francine and Luke/Lisa want to have the same, but Frank/Francine need toonies for their cars/dolls while Luke/Lisa need loonies for their cars/dolls, so only give toonies to Frank/Francine and loonies to Luke/Lisa. Can you help them share these coins so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy toy cars? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add coins to make the distribution even. Now they have the same. Can you count how many much money Luke/Lisa has? Now can you tell me how much money Frank/Francine has? (cover pile)

Block Familiarization Phase:

Remember the blue and yellow double and single blocks from last time? These are the same except they are all red. Can you count how many one double is? Now can you tell me how many singles are the same as one double? (Confirm they are the same by getting child to count if they give the correct answer) If child doesn't give correct answer say I think it's two singles. Let's count how many are in the double again and how many two singles are. They are the same aren't they? So, if I ate a double block I would have to eat ….pause to see if child will answer….two single blocks to have the same.

Equal Sharing Condition (Blocks):

13. Derek/Daisy and Anthony/Alyssa want to share some candies. Derek and Anthony both want to have the same. Derek likes to get 1 candy at a time and Anthony likes to get 1 candy at a time too, so give singles to Derek and

singles to Anthony. Can you help them share these candies so it's fair? Keep sharing until I tell you to stop. Ok?….Do you think each friend has a fair share? Do you think each friend has the same amount of candies?

14. Now Bill/Beverly and Jamie/Janine want to share some cookies. Bill and Jamie want to have the same, but Bill likes to get 2 cookies while Jamie likes to have one at a time, so only give doubles to Bill and singles to Jamie. Can you help them share these cookies so it's fair? Keep sharing until I tell you to stop. Ok?….Do you think each friend has a fair share? Do you think each friend has the same amount of cookies?

15. Sarah/Stan and Daphne/Dale want to share French fries this time. Sarah and Daphne want to have the same, but Sarah likes to get 2 fries while Daphne likes to have one at a time, so only give doubles to Sarah and singles to Daphne. Can you help them share these fries so it's fair? Keep sharing until I tell you to stop. Ok?…Do you think each friend has a fair share? Do you think each friend has the same amount of French fries? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add blocks to make the distribution even. Now they have the same. Can you count how many fries Sarah has? Now can you tell me how many fries Daphne has? (cover pile)

16. Alicia/Anton and Rayna/Raoul have some bouncy balls to share. Alicia and Rayna want to have the same. Alicia likes to get 1 bouncy ball at a time and Rayna likes to get 1 bouncy ball at a time, so give singles to Alicia and singles to Rayna. Can you help them share these bouncy balls so it's fair? Keep sharing until I tell you to stop. Ok?…Do you think each friend has a fair share? Do you think each friend has the same amount of bouncy balls? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add blocks to make the distribution even. Now they have the same. Can you count how many balls Alicia has? Now can you tell me how many balls Rayna has? (cover pile)

17. Now Nadia/Nathan and Tessa/Troy want to share stamps. Nadia and Tessa both want to have the same, but Nadia likes to get 2 stamps at a time, while Tessa likes to get one at a time, so only give doubles to Nadia and singles to Tessa. Can you help them share these stamps so it's fair? Keep sharing until I tell you to stop. Ok?…Do you think each friend has a fair share? Do you think each friend has the same amount of stamps?

18. Fred/Faye and Leonard/Liza have some marbles to share. Fred and Leonard want to have the same, but Fred likes to get 2 marbles at a time, while Leonard likes to get one at a time, so only give doubles to Fred and singles to Leonard. Can you help them share these marbles so it's fair? Keep sharing until I tell you to stop. Ok?…Do you think each friend has a fair share? Do you think each friend has the same amount of marbles? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add blocks to make the distribution even. Now they have the same. Can you count how

many marbles Leonard has? Now can you tell me how many marbles Fred has?

Reciprocity Condition (Coins):

7. Emma/Eric and Lily/Larry each have some coins to buy movies from the store. Emma/Eric and Lily/Larry want to have the same. Emma/Eric shares her coins with Lily like this: Lily/Larry gets one loonie and Emma/Eric gets one loonie. Now it's Lily's turn to share her coins. Can you help her share with Emma/Eric so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy movies?

8. Now Rosie/Ron and Olivia/Oliver would like to buy some bubble gum from the candy store. Rosie/Ron and Olivia/Oliver want to have the same. Rosie/Ron shares her coins with Olivia/Oliver like this: Olivia/Oliver gets a toonie while Rosie/Ron gets a loonie. (place 1 toonie next to Olivia and 1 loonie next to Rosie (5x)). Now it's Olivia/Oliver's turn to share her coins. Can you help her share with Rosie/Ron so it's fair? Keep sharing until I tell you to stop. Ok? ….Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy bubble gum?

9. Hannah/Harry and Michelle/Michael want to buy some chocolates. Hannah/Harry and Michelle/Michael want to have the same. Hannah/Harry shares her coins with Michelle/Michael like this: Michelle/Michael gets a toonie while Hannah/Harry gets a loonie (3x) Now it's Michelle/Michael's turn to share her coins. Can you help her share with Hannah/Harry so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy chocolates? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add coins to make the distribution even. Now they have the same. Can you count how much money Hannah has? Now can you tell me how much money Michelle has? (cover pile)

10. Mark/Mandy and Jim/Jessica are at the candy store and want to buy lollipops. Mark/Mandy and Jim/Jessica want to have the same. Mark/Mandy shares his coins with Jim/Jessica like this: Jim/Jessica gets a loonie while Mark/Mandy gets a loonie. Now it's Jim/Jessica's turn to share his coins. Can you help him share with Mark/Mandy so it's fair? Keep sharing until I tell you to stop. Ok? ….Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy lollipops? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add coins to make the distribution even. Now they have the same. Can you count how much money Mark has? Now can you tell me how much money Jim has? (cover pile)

11. Melissa/Matthew and Nicole/Nick have some money to buy popcorn at the movies. Melissa/Matthew and Nicole/Nick want to have the same. Melissa/Matthew shares her coins with Nicole/Nick like this:

Nicole/Nick gets a toonie while Melissa/Matthew gets a loonie (6x). Now it's Nicole/Nick's turn to share her coins. Can you help her share with Melissa/Matthew so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy popcorn?

12. Ralph/Ruby and Tom/Tina have some coins to buy crayons so they can draw pictures. Ralph/Ruby and Tom/Tina want to have the same. Ralph/Ruby shares his coins with Tom/Tina like this: Tom/Tina gets a toonie while Ralph/Ruby gets a loonie (4x). Now it's Tom/Tina's turn to share his coins. Can you help him share with Ralph/Ruby so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of money to buy crayons? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add coins to make the distribution even. Now they have the same. Can you count how much money Tom has? Now can you tell me how much money Ralph has? (cover pile)

Reciprocity Condition (Blocks):

13. Eleanor/Earl and Lola/Lance have some stuffed animals to share. Eleanor and Lola want to have the same. Eleanor shares her animals with Lola like this: Lola gets one animal and Eleanor gets one animal. Now it's Lola's turn to share her stuffed animals. Can you help her share with Eleanor so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of stuffed animals?

14. Now Rachel/Robert and Karen/Kevin would like to share some smarties. Rachel and Karen want to have the same. Rachel shares her smarties with Karen like this: Karen gets 2 at a time while Rachel keeps 1 at a time (5x). Now it's Karen's turn to share her smarties. Can you help her share with Rachel so it's fair? Keep sharing until I tell you to stop. Ok?….Do you think each friend has a fair share? Do you think each friend has the same amount of smarties?

15. Harriet/Howard and Monica/Marco have chocolate cookies to share. Harriet and Monica want to have the same. Harriet shares her chocolate cookies with Monica like this: Monica gets 2 cookies at a time while Harriet gets 1 cookie at a time (3x). Now it's Monica's turn to share her chocolate cookies. Can you help her share with Harriet so it's fair? Keep sharing until I tell you to stop. Ok? …Do you think each friend has a fair share? Do you think each friend has the same amount of cookies? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add blocks to make the distribution even. Now they have the same. Can you count how many cookies Harriet has? Now can you tell me how many Monica has? (cover pile)

16. Now Patrick/Phoebe and Travis/Tanya have candy canes to share. Patrick and Travis want to have the same. Patrick shares his candy canes with Travis like this: Patrick gets 1 at a time while Travis gets 1 at

a time. Now it's Travis' turn to share his candy canes. Can you help him share with Patrick so it's fair? Keep sharing until I tell you to stop. Ok? ....Do you think each friend has a fair share? Do you think each friend has the same amount of candy canes? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add blocks to make the distribution even. Now they have the same. Can you count how many candy canes Patrick has? Now can you tell me how many Travis has? (cover pile)

17. Melanie/Mason and Noelle/Noah are sharing some crackers. Melanie and Noelle want to have the same. Melanie shares her crackers with Noelle like this: Noelle gets 2 crackers each time while Melanie gets 1 piece at a time (6x). Now it's Noelle's turn to share her crackers. Can you help her share with Melanie so it's fair? Keep sharing until I tell you to stop. Ok? ...Do you think each friend has a fair share? Do you think each friend has the same amount of crackers?

18. Vanessa/Victor and Isabelle/Isaac have some mints to share. Vanessa and Isabelle want to have the same. Vanessa shares her mints with Isabelle like this: Isabelle gets 2 mints at a time while Vanessa keeps 1 each time (4x). Now it's Isabelle's turn to share her mints. Can you help her share with Vanessa so it's fair? Keep sharing until I tell you to stop. Ok?...Do you think each friend has a fair share? Do you think each friend has the same amount of mints? If child says no but has shared correctly just say yes, they do. If child has shared incorrectly say: I think they have the same if we do this and add blocks to make the distribution even. Now they have the same. Can you count how many mints Isabelle has? Now can you tell me how many Vanessa has? (cover pile)

P) <u>Study 2: Answer Sheet- Establishing Equivalence and Comparing Equivalent Sets Using Correspondence and Counting:</u>

Experimental Answer Sheet (version **A**)

Child ID:

Task 1 (equal sharing coins):

a) (1-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

b) (2-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

c) (2-to-1, inference) ____________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

d) (1-to-1, inference) _______________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

e) (2-to-1) _______________; method of sharing:

Fair share?:
Same amount?:

f) (2-to-1, inference) ___________; method of sharing:

Fair share?:
Same amount?:

Number inference:

Task 2 (equal sharing blocks):

a) (1-to-1) ________________; method of sharing:

Fair share?:
Same amount?

b) (2-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

c) (2-to-1, inference) ____________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

d) (1-to-1, inference) _______________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

e) (2-to-1) _______________; method of sharing:

Fair share?:
Same amount?:

f) (2-to-1, inference) ___________; method of sharing:

Fair share?:
Same amount?:

Number inference:

Task 3 (reciprocity coins):

a) (1-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

b) (2-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

c) (2-to-1, inference) ____________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

d) (1-to-1, inference) _______________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

e) (2-to-1) _______________; method of sharing:

Fair share?:
Same amount?:

f) (2-to-1, inference) ___________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

Task 4 (reciprocity blocks):

a) (1-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

b) (2-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

c) (2-to-1, inference) ____________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

d) (1-to-1, inference) _______________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

e) (2-to-1) ________________; method of sharing:

Fair share?:
Same amount?:

f) (2-to-1, inference) ____________; method of sharing:

Fair share?:
Same amount?:

Number Inference:

Q) Study 3: Head Teacher Information Letter- Relations Between Counting, Equivalence and Cardinality:

Department of Education
15 Norham Gardens,
Oxford, OX2 6PY
Tel: +44 1865 27402

One for You, Two for me: Quantitative Sharing by Young Children

Sarah Walter
PhD Programme Education
University of Oxford

I am a doctoral candidate at the University of Oxford, and am writing to ask if your school would be interested in participating in a research project looking at how young children share. The project is supervised by Professor Terezinha Nunes (Department of Education, University of Oxford). It has received ethics clearance from the University of Oxford Central University Research Ethics Committee and the External Research Review Committee of the TDSB.

Sharing gives young children the opportunity to use mathematical concepts in real world situations and a chance to develop their quantitative reasoning skills. This study aims to investigate the math concepts that children practice in sharing situations and how this can help them understand more abstract mathematical concepts like cardinality.

Should you agree to participate, class teachers will be given consent forms to send home for parents to sign. Children whose parents return the form will then be able to participate. Each child will be free to withdraw from the study at any time they wish without explanation. The researcher would work individually with 4 and 5-year-old students from Kindergarten. Each child would be seen for one 30-minute session. In this session they will be asked to complete three sharing trials with the researcher, sharing blocks between two puppets. They will also be asked to help one of the puppets count their blocks. The researcher would arrange to visit at times convenient for the class teacher, and will aim to cause minimal distraction. A short report of the findings will be produced for staff or parents who may be interested in the research.

All data will be securely stored, and kept confidential and anonymous in accordance with the TDSB and UK data protection requirements. No information about individual children will be made available to anyone outside of the study. The researcher has also received the required police clearance to work with children in schools.

I hope to start work in January 2013. I will get in touch with your school in a couple of weeks to see whether you are able to participate. In the meantime, please let me know if you are interested in participating or would like more information. I can be contacted by e-mail at sarah.walter@lincoln.ox.ac.uk. I hope I will have the opportunity of working with your school. Thank you very much for your time.

Yours Sincerely,

Sarah Walter
Doctoral Student
Department of Education
University of Oxford

R) <u>Study 3: Parent Information Sheet- Relations Between Counting, Equivalence and Cardinality:</u>

Department of Education
15 Norham Gardens, Oxford, OX2 6PY
Tel: +44(0)1865 274024
Fax: +44(0)1865 274027

One for you, two for me: Quantitative Sharing by Young Children

Sarah Walter
DPhil in Education
University of Oxford

Information for Parents:
Your child has been invited to participate in a research study. Before you decide whether or not you would like them to participate, it is important to understand why the study is being conducted and what their participation entails. Please read the following information carefully. If you have any questions or would like more information, please ask: Sarah Walter, doctoral student, by e-mail: sarah.walter@lincoln.ox.ac.uk or by phone: (416) 565-9166.

This study has received ethics clearance from the University of Oxford Central University Research Ethics Committee and the External Research Review Committee of the TDSB. The researcher also has the required clearance to work with children in schools.

Purpose:
I am a doctoral student from the University of Oxford interested in investigating the relationship between early mathematical concepts by looking at how 4 and 5-year-old children use sharing and counting in play. Sharing gives young children the opportunity to use mathematical concepts in real world situations and in different ways. This could help them develop their knowledge of more abstract mathematical concepts that they are taught later on. In this study, the children will play games in which they will share different objects between puppets and reason about number in these games.

Participation:
Your child's school has kindly agreed to participate in this study. All parents with children in Kindergarten are being asked for permission for their child to participate in a research project. If you agree to allow your child to take part, a time will be arranged with the teacher and the researcher will work with your child in a quiet area at the school. The games will involve your child sharing objects between two puppets with the help of stories. The games take only one session of approximately 30 minutes. It is your decision whether or not your child takes part in this study. Their participation is entirely voluntary, and they may stop participating in the study at any time without having to explain why.

Confidentiality:
The results of this study will be reported in an Oxford doctoral thesis. All information collected will be kept strictly confidential in accordance with the TDSB and UK data protection requirements and will be used only for the purposes of this study. Only collective results will be reported and no information about individual children will be made available to anyone besides the researcher, including your child's teacher or any other staff at the school. A report of the results of the study will be made available to the TDSB and your child's school.

Contacts for Questions or Concerns:
If you have any further questions or concerns about this study, please don't hesitate to contact:

Sarah Walter, doctoral student, by e-mail: sarah.walter@lincoln.ox.ac.uk;
by phone: (416) 565-9166
OR
Dr. Terezinha Nunes, academic supervisor, by e-mail:
terezinha.nunes@education.ox.ac.uk

If you wish to allow your child to participate in the project, please complete and return the attached sheet to your child's teacher.

If you do not want your child to participate then you do not need to return anything.

Thank you for your help!

S) Study 3: Parent Consent Form- Relations Between Counting, Equivalence and Cardinality:

Department of Education
15 Norham Gardens, Oxford, OX2 6PY
Tel: +44 (0)1865 274024
Fax: +44(0)1865 274027

One for you, two for me: Quantitative Sharing by Young Children
Sarah Walter, Doctoral Student, University of Oxford

Please read the following and return this sheet to your child's teacher.

1. I have read the information related to this study, have had the opportunity to ask questions about the research and have considered any risks.

2. I understand that I can withdraw from the study at any time without penalty simply by informing the researcher of my decision.

3. I understand who will have access to the data collected for the study and what will happen to the data at the end of the study.

4. I understand whom to contact should I have questions after participating in the study.

5. I understand that this project has received ethics clearance from the University of Oxford Central University Research Ethics Committee and External Research Review Committee of the TDSB.

By signing below, I give permission for my child to participate in this study.

Child's School:____
Child's Teacher:____
Child's Name (Printed): __________________________________
Parent/Guardian Name (Printed):_____________________________________
Signature: _______________________________________Date:__________

T) **Study 3: Researcher's Script- Relations Between Counting, Equivalence and Cardinality:**

Single Row Trials:

Researcher says "I am going to give my friend Mr. Dog some sweets" (or cookies, or other type of food; object will be different in each trial).

One row of 4, 6, 8 or 10 blocks is placed in front of the puppet, evenly spaced and in a straight line.

Researcher says "Now Mr. Dog wants to know how many sweets (or other food item) he has, so he is going to count his sweets now. Sometimes he does a good job and counts right, but sometimes he makes mistakes and counts wrong, so watch carefully while he is counting. When he is finished I want you to tell me if his counting was ok or not ok."

Mr. Dog counts his blocks at 1 item per second. On miscount trials he skips one block (this is randomly varied in each trial). The researcher covers the blocks when Mr. Dog is finished.

Researcher says "Did Mr. Dog count ok?" (on miscount trials, if the child answers no, move to the next question; if the child answers yes, she says "I'm not sure. Watch carefully while Mr. Dog counts again" and will give the child a second chance to say no)(on correct counting trials, the reverse happens).

On miscount trials she asks, "What did Mr. Dog do wrong?"

On trial 1 and 3 she will first ask "How many blocks does Mr. Dog think he has?" then "How many blocks are there" and on 2 and 4 the order of the questions will be reversed.

In the miscount trials, if the child cannot remember how many blocks Mr. Dog thinks he has because of the extra question asked, the researcher will ask Mr. Dog to count one more time for the child and will then re-ask the last two questions.

One-to-one correspondence trials:

Researcher says, "Mr. Dog and his friend Mr. Rabbit want to share some (crisps, chocolates, stickers or coins depending on the trial). I have some (crisps) here. Can you help them share so they both get the same? Keep sharing until I tell you to stop."

Child will be stopped after they have shared 4, 6, 8 or 10 blocks to each character. If the child has made a mistake, the researcher will say, "I think it's fair if we do this" and will move blocks to make both sets equal. The researcher will also ask the child, "does each friend have a fair share" (researcher will agree with child's answer if they say yes, or will say I think they do if the child says no) and will then ask "do they have the same amount….(to eat, play with, etc. depending on the object)" to confirm that they think both puppets have equal shares. Mr. Rabbit will then put his blocks in a bag so the child can no longer see them.

Next, the researcher says "Now Mr. Dog wants to know how many crisps (or other item) he has, so he is going to count his crisps now. Sometimes he does a good job and counts right, but sometimes he makes mistakes and counts wrong, so watch carefully while he is counting. When he is finished I want you to tell me if his counting was ok or not ok."

Mr. Dog counts his blocks at 1 item per second, skipping one block (either the first or last block, alternated in each trial). The researcher then covers Mr. Dog's blocks when he is finished counting.

Researcher says "Did Mr. Dog count ok?" (if the child answers no, move to the next question; if the child answers yes, she will say "I'm not sure. Watch carefully while Mr. Dog counts again" and will give the child a second chance to say no) (on correct counting trials the reverse happens).

On miscount trials she will then ask, "What did Mr. Dog do wrong?"

On trial 1 and 3 she will first ask "How many blocks does Mr. Dog think he has?" then "How many blocks are there" and on 2 and 4 the order of the questions will be reversed.

In the miscount trials, if the child cannot remember how many blocks Mr. Dog thinks he has because of the extra question asked, the researcher will ask Mr. Dog to count one more time for the child and will then re-ask the last two questions.

After the last question above, the researcher says, "Now, Mr. Rabbit has the same amount of crisps as Mr. Dog because they shared fairly didn't they? (gesture to the bag with Mr. Rabbit;s blocks inside), How many crisps (or other item) does Mr. Rabbit have?"

If the child answers correctly move to the next trial. If the child says I don't know or is incorrect, the researcher will say, "I've forgotten, can you remind me how many does Mr. Dog have, do you remember?" Then try asking the child about Mr. Rabbit again.

<u>Two-to-one correspondence trials:</u>

The researcher will hold up one double block made of a single blue and single yellow and will show the child that it is the same amount as a single yellow and single blue not pushed together. The child will then be asked, "so if I give you this (a double block), what should I get (pointing to single blues and yellows) to be the same?" If the child says they don't know or gives an incorrect answer, the researcher will lay a double block on it's side and say there's a yellow and blue here, so I need a yellow and blue to be the same. She will then repeat the question about how many singles would be the same as one double block. Then the experimental trials will begin.

Researcher says, "Mr. Dog and his friend Mr. Rabbit want to share some (stickers, crackers, sweets or biscuits depending on the trial). Mr. Dog and Mr. Rabbit want to have the same, but Mr. Dog likes to get these ones (doubles) each time, while Mr. Rabbit likes to get these ones (singles) each time, so only give these (doubles) to Mr. Dog and these (singles) to Mr. Rabbit (Mr. Dog will get doubles for the first 2 trials and then singles for the second 2 trials to vary whether he counts single or double blocks). I have some (stickers) here. Can you help them share so they both get the same? Keep sharing until I tell you to stop."

Child will be stopped after they have shared 4, 6, 8 or 10 single blocks to one character and 2, 3, 4 or 5 doubles to the other character. If the child makes a mistake, the researcher will say, "I think it's fair if we do this" and will move blocks to make both sets equal. After any adjustments are made, the researcher will also ask the child, "does each friend have a fair share" and "do they have the same amount" to confirm that they think both puppets have equal shares. The blocks of the second puppet (Mr. Rabbit) are then covered.

Next, the researcher says "Now Mr. Dog wants to know how many stickers (or other item) he has, so he is going to count his stickers now. Sometimes he does a good job and counts right, but sometimes he makes mistakes and counts wrong, so watch carefully while he is counting. When he is finished I want you to tell me if his counting was ok or not ok."

Mr. Dog counts his blocks at 1 item per second, skipping one block (either the first or last depending on the trial) (in the trials where Mr. Dog counts doubles skipping one block refers to skipping one block in the double block to remain consistent with the other 2 conditions where the cardinal and Mr. Dog's count are only different by 1 number, the researcher will gesture very carefully to make sure it is clear that Mr. Dog is counting each block in the double blocks (i.e. pointing twice at each block)). The researcher then covers Mr. Dog's blocks when he is finished counting.

Researcher says "Did Mr. Dog count ok?" (if the child answers no, move to the next question; if the child answers yes, she will say "I'm not sure. Watch carefully while Mr. Dog counts again" and will give the child a second chance to say no)(on correct counting trials the reverse happens).

On miscount trials she will then ask, "What did Mr. Dog do wrong?"

On trial 1 and 3 she will first ask "How many blocks does Mr. Dog think he has?" then "How many blocks are there" and on 2 and 4 the order of the questions will be reversed.

In the miscount trials, if the child cannot remember how many blocks Mr. Dog thinks he has because of the extra question asked, the researcher will ask Mr. Dog to count one more time for the child and will then re-ask the last two questions.

After the last question above, the researcher says, "Now, Mr. Rabbit has the same amount of stickers as Mr. Dog because they shared fairly, didn't they? (gesture to the bag of blocks in front of Mr. Rabbit). How many stickers (or other item) does Mr. Rabbit have?"

If the child answers correctly move to the next trial. If the child says I don't know or is incorrect, the researcher will say, "I've forgotten, can you remind me how many does Mr. Dog have, do you remember?" Then try asking the child about Mr. Rabbit again.

U) Study 3: Answer Sheet- Relations Between Counting, Equivalence and Cardinality:

Answer Sheet Version: (single row, 1-to-1, 2-to-1, 3-to-1)

Child ID:

School:

Age:

Gender:

DOB:

Task 1 (single row):

a) (correct- 10 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

b) (correct- 6 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

c) (miscount- 8 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

d) (miscount- 6 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

e) (correct- 4 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

f) (miscount- 10 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

g) (correct- 8 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

h) (miscount- 4 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

Task 2 (1-to-1 correspondence):

a) (miscount- 10 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

b) (correct- 8 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

c) (miscount- 4 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

d) (correct- 4 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

e) (miscount- 8 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

f) (correct- 6 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

g) (correct- 10 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

h) (miscount- 6 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

Task 3 (2-to-1 correspondence):

a) (miscount- 4 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

b) (correct- 6 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

c) (correct- 4 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

d) (miscount- 8 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

e) (correct- 10 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

f) (correct- 8 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

g) (miscount- 10 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

h) (miscount- 6 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Task 4 (3-to-1 correspondence):

a) (miscount- 3 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

b) (correct- 6 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

c) (miscount- 6 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

d) (miscount- 9 blocks)

Did Mr. Dog count ok?

What did Mr. Dog do wrong?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

e) (correct- 12 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

f) (correct- 9 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes:

g) (miscount- 12 blocks)

Did Mr. Dog count ok?

How many blocks does Mr. Dog think he has?

How many blocks are there?

Notes:

h) (correct- 3 blocks)

Did Mr. Dog count ok?

How many blocks are there?

How many blocks does Mr. Dog think he has?

Notes: