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**DEPLETION AND DEVELOPMENT: NATURAL
RESOURCE SUPPLY WITH ENDOGENOUS FIELD
OPENING**

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Depletion and development: natural resource supply with endogenous field opening

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Abstract

This paper develops a model in which supply of a non-renewable resource can adjust through two margins: the rate of depletion and the rate of field opening. Faster depletion of existing fields means that less of the resource can ultimately be extracted, and optimal depletion of open fields follows a (modified) Hotelling rule. Opening a new field involves sinking a capital cost, and the timing of field opening is chosen to maximize the present value of the field. Output dynamics depend on both depletion and field opening, and supply responses to price changes are studied. In contrast to Hotelling, the long run equilibrium rate of growth of prices is independent of the rate of interest, depending instead on characteristics of demand and geologically determined supply.

Keywords: natural resource, depletion, Hotelling, fossil fuel, carbon tax.

JEL classification: Q3, Q5

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1. Introduction:

The supply of a non-renewable resource can adjust through intensive and extensive margins; the rate of depletion of existing fields (or mines) can change, and new fields can be opened. The objective of this paper is to provide a tractable model that captures these two supply margins and permits investigation of the interaction between them. The work-horse model of resource supply remains that of Hotelling (1931) which looks exclusively at the intensive margin; how do the insights of the Hotelling model hold up when resource owners are making decisions on both depletion and development?

The approach taken in this paper has several key features. First, the extensive margin is captured by the development of new fields, rather than (as is common in the literature) incremental expansion of the existing (single) field. The key feature of opening a new field is that it requires sunk capital, and this is central in our approach. We assume that fields differ in capital cost per unit reserve, and it is this that produces the ordering of fields according to date of opening. Capital has to be sunk before a new field is opened, an approach which we think accords with reality, and is a quantitatively important feature of major mining developments and oil investments in offshore and very deep fields. This modelling approach is in contrast with much of the literature, where additions to stock are typically modelled as the outcome of a continuous variable (exploration) that adds to the capacity and reduces extraction costs of the existing field (as in Pindyck 1978, Dasgupta and Heal 1979).¹

A second feature concerns the modelling of extraction costs on existing fields, the intensive margin. Extraction costs are typically modelled as a function of extraction and the stock of resource remaining.² We assume that they depend on the ratio of these variables, i.e. on the *rate* of extraction, and that a faster rate of extraction (depletion) raises costs. Furthermore, these costs are ‘iceberg’, using up the resource itself. Both these assumptions seem to be supported in the technical literature on oil extraction (discussed in section 2.2) which suggests that a faster rate of depletion means that less of the resource is ultimately recoverable. They are also convenient modelling simplifications which make for a tractable characterisation of the intensive margin and, by allowing aggregation over fields, facilitate analysis of field opening, the extensive margin, and aggregate resource supply.

¹ See Krautkraemer (1998) for a survey.

² For example, Pindyck (1978) assumes that they are proportional to extraction and increasing in remaining stock.

This combination of features enables characterisation of the resource supply through time, showing how supply depends on both the level and rate of change of prices, as well as on the geology of supply. We then endogenise the price path, showing how it depends on parameters of the model and responds to shocks such as resource or carbon taxation. This goes beyond previous work that models field set-up costs such as Hartwick et al (1986), who assume zero extraction costs, in which case only one field is operated at any time. Holland (2003) looks at cases where marginal extraction costs are constant or infinite. Livernois and Uhler (1987) look at the rate of discovery of new fields with field-specific extraction costs, characterising first order conditions for the problem but doing little subsequent analysis of the equilibrium.

The next section of the paper sets out decision making for a single field; the owner has to choose when to open and how fast to deplete. In discussion of these decisions we link our approach to existing literature from the specialist fields of resource and energy economics. We then (section 3) place this in the context of a continuum of fields and derive aggregate supply. Supply depends on both the rate of change of price (relative to the interest rate), as in the Hotelling model, and on the level of price, operating through the extensive margin and the timing of field openings. Thus, a permanent proportional price reduction postpones field opening, reducing the quantity produced in the short run, raising it in the long run, and reducing the cumulative quantity produced at all future dates. A permanent reduction in the rate of growth of price increases production in the short run (bringing forward depletion of existing fields and, temporarily, field opening), but has a long run negative effect on cumulative quantity supplied.

Section 4 turns from the path of supply, given the path of price, to the full market equilibrium with price endogenous. The long run rate of change of price is determined by the rate of growth of demand, the price elasticity of demand, and a parameter summarising the geology of supply. In contrast to the Hotelling model, it is completely independent of the rate of interest. Reductions in the level or the rate of growth of demand both have the effect of reducing the cumulative quantity supplied, even though they may increase the rate of extraction of existing fields. This has important implications for our understanding of climate change policy. Anticipated slower demand growth brings forward production on existing fields (as pointed out by Sinn 2008), but this is offset by the lower level of prices postponing field opening. Carbon taxes do not have a ‘paradoxical’ effect.

2. Field depletion and development:

There is a continuum of fields all of which are known at date 0, and are owned by price-taking profit maximizing agents. Each field contains one unit of the resource, but cannot produce until a field specific fixed cost $Ke^{-\theta T}$, $\theta \geq 0$, has been paid, where $e^{-\theta T}$ captures technical progress in field development that has taken place by date T , when the cost is paid. K varies across fields, and we will use K as the index of field types, with K running to plus infinity. The number (measure) of fields of type K is $S(K)$.³

2.1 Depletion and development

Focusing on a particular field (i.e. taking a particular value of K), output at date t is $xq(z)$, where x is the stock remaining and z is the *rate* of depletion, defined as the proportionate rate of decline of remaining stock, so $\dot{x} = -xz$. While z is the rate of depletion of the field and xz is the reduction in the recoverable stock, $xq(z)$ is the useable output of the resource. The function $q(z)$ is increasing, has $q(z) \leq z$, is concave in z and, if strictly concave, increases in the rate of depletion yield less than proportionate increases in output, perhaps as too rapid pumping from an oil-field reduces the capacity of the field. The implied extraction cost, expressed as a proportion of the stock in the field, is therefore $z - q(z) \geq 0$. We give some examples and further discussion of this relationship in section 2.2.

Profit maximization in a field with fixed cost K requires that the opening date, T , and subsequent time paths of z and x are chosen to maximize the present value of profits (evaluated at date $t = 0$ with interest rate r),

$$PV \equiv e^{-rT} \int_0^\infty p(T + \tau)x(\tau)q(z(\tau))e^{-r\tau}d\tau - Ke^{-(\theta+r)T} \quad (1)$$

subject to

$$\dot{x}/x = -z, \quad \text{and } x(0) = 1, \quad x \geq 0. \quad (2)$$

The integral in (1) runs over dates τ measured from when the field is opened, so $t = T + \tau$, and $p(t)$ is the (exogenous) price at date t . We assume that, as $t \rightarrow \infty$, $p(t)$ converges to

³ Assuming each field contains one unit of resource is without loss of generality as K can be interpreted as capital cost per unit capacity. The total stock of resource in fields with capital cost K is $S(K)$.

constant exponential growth at rate less than or equal to r , as is necessary for the objective to be bounded. We denote this limiting rate of change of price \hat{p}_∞ .

The profit maximizing depletion path in an open field is given by the Euler equation

$$\dot{z} = \left[r - \frac{\dot{p}}{p} + z - \frac{q(z)}{q'(z)} \right] \frac{q'(z)}{q''(z)}. \quad (3)$$

This depends on the difference between the rate of interest and rate of price increase, and also on the curvature of $q(z)$, indicating the cost penalty from increasing the rate of depletion.

Notice that z can jump, while the stock variable x cannot. The stock remaining in a field that has been open for τ periods is

$$x(\tau) = \exp \left[- \int_0^\tau z(\chi) d\chi \right]. \quad (4)$$

Concavity of $q(z)$ ensures that differential equation (3) is locally stable (since $z - q(z)/q'(z)$ is decreasing in z), so z converges to stationary value z^* implicitly defined by

$$\hat{p}_\infty - r = z^* - q(z^*)/q'(z^*), \quad \text{or} \quad z^* = \varsigma(r - \hat{p}_\infty), \quad \varsigma' > 0, \quad (5)$$

where the function $\varsigma(r - \hat{p}_\infty)$ summarizes the long-run relationship. We discuss this further in section 2.2.

The profit maximizing date, T , at which to spend $Ke^{-\theta T}$ and open the field is given by first order condition

$$\frac{\partial PV}{\partial T} = e^{-rT} \left[-r \int_0^\infty pxq(z)e^{-r\tau} d\tau + (\theta + r)Ke^{-\theta T} + \int_0^\infty \dot{p}xq(z)e^{-r\tau} d\tau \right] = 0. \quad (6)$$

The intuition is that if the profile of production and costs is shifted back by dT , then the first term is the cost of pushing revenues further away, the second the benefit of moving costs, and the final term is the change in revenue from the fact that output $xq(z)$ is now valued at prices dT later. Rearranging, T is given by first order condition

$$\int_0^\infty (\dot{p} - rp)xq(z)e^{-r\tau} d\tau + (\theta + r)Ke^{-\theta T} = 0. \quad (7)$$

To see the implications of this it is easiest to look at (1) and (7) with the assumptions that price is growing at constant rate \hat{p} (taking value p_0 at $t = 0$) and z is at its stationary value z^* . The integral in (1) can then be evaluated as

$$PV = p_0 e^{(\hat{p}-r)T} q(z^*) \int_0^\infty e^{(\hat{p}-r-z^*)\tau} d\tau - Ke^{-(\theta+r)T} = e^{-(\theta+r)T} \left[\frac{p_0 e^{(\theta+\hat{p})T} q(z^*)}{z^* + r - \hat{p}} - K \right]. \quad (8)$$

The first and second order conditions for choice of T are

$$\frac{\partial PV}{\partial T} = (\hat{p} - r)PV + (\theta + \hat{p})Ke^{-(\theta+r)T} = 0, \quad \frac{\partial^2 PV}{\partial T^2} = -(\theta + \hat{p})(\theta + r)Ke^{-(\theta+r)T} < 0. \quad (9)$$

If $\theta + r > 0$, the second order condition requires that $\hat{p} + \theta > 0$, and we assume this to be satisfied. From the first order condition, an interior solution requires $r > \hat{p}$, as already assumed; if not it would pay to postpone entry indefinitely getting the dual benefit of later capital cost and higher present value of revenue flow.⁴ These conditions imply that the

higher is K the later is the field opened, since $\frac{dT}{dK} = -\frac{\partial^2 PV / \partial T \partial K}{\partial^2 PV / \partial T^2}$ and

$$\frac{\partial^2 PV}{\partial T \partial K} = e^{-(\theta+r)T} (\theta + \hat{p}) > 0.$$

2.2 The rate of depletion: discussion

We draw the modeling of extraction costs from the technical literature on resource depletion, particularly in the oil sector. In this literature the benchmark assumption is that output from a field follows an exponential rate of decline (Adelman 1990, 1993); in our framework, z is

⁴ And, with prices endogenous, competitive equilibrium would not exist, see Holland (2003).

constant.⁵ Varying the rate of depletion has a cost primarily by its impact on total recoverable reserves. This variation is typically achieved by altering the rate of water or gas injection which pressurizes the well, and its effects are geology dependent; Nystad (1985, 1987) categorises fields as ‘Hotelling’, ‘intermediate’, and ‘geosensitive’, in increasing order according to loss of recoverable reserves from faster depletion. We capture this in relationships $q(z)$ and $\dot{x} = -zx$. Concavity means that an increase in output, q , involves a greater than proportionate decrease in remaining (recoverable) reserves, x .

The intuition behind Euler equation (3) for optimal depletion is as follows. Suppose that the price is growing at constant rate \hat{p} , so $z = z^*$, and consider a perturbation at some date (say date 0) which is an instantaneous increase in extraction δ , offset by a reduction in the next instant which puts the resource stock back on its previous path. If δ is small, the value of the perturbation is

$$p_0 x_0 [q(z^* + \delta) - q(z^*)] + \frac{P_0}{1 + r - \hat{p}} [(1 - z^* - \delta)x_0 q(z^* - \delta) - (1 - z^*)x_0 q(z^*)]$$

The first term is the value of increasing extraction by δ . Stock carried through into the next instant changes from $(1 - z^*)x_0$ to $(1 - z^* - \delta)x_0$ and its value is discounted by the interest rate minus the rate of price growth. To undo the perturbation, the rate of extraction must fall to $z^* - \delta$.⁶ Differentiating with respect to δ and evaluating at $\delta = 0$, this expression is

$$p_0 x_0 \delta \left[q' - \frac{q + (1 - z^*)q'}{1 + r - \hat{p}} \right], \text{ so the perturbation has zero value if the term in square brackets is}$$

zero, this being equation (5).

Understanding these relationships is facilitated by working with a particular functional form that will be used in simulations later in the paper. We suppose $q(z)$ is iso-elastic, taking the form

$$q(z) = a(z - b\lambda)^{1-\lambda}, \text{ with parameters } a > 0, b \geq 0, \text{ and } \lambda \leq 1. \quad (10)$$

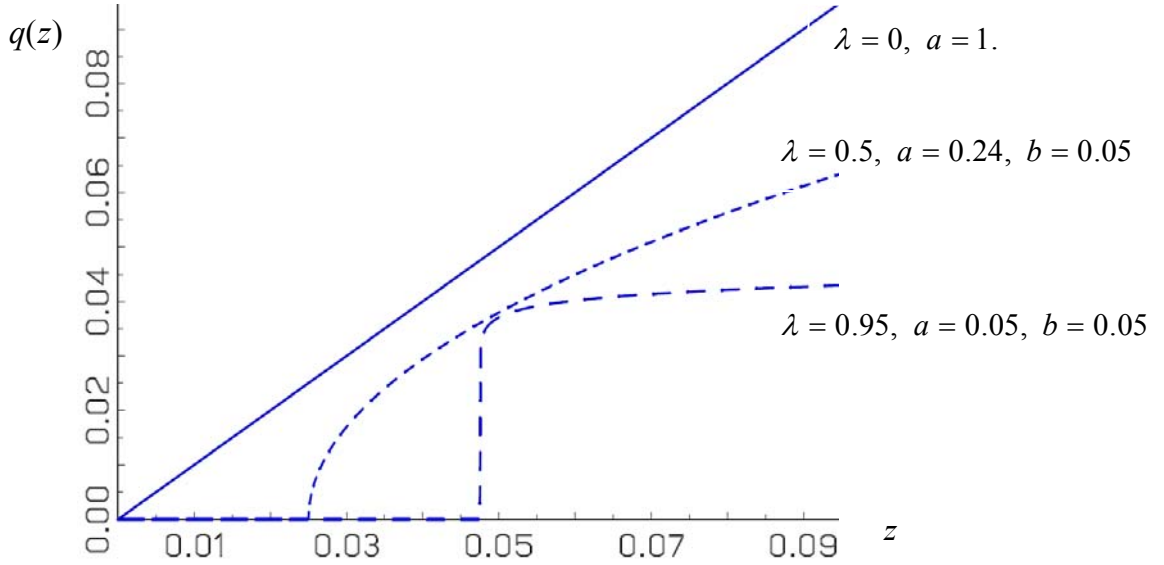
With this specification the Euler condition and long-run value of the rate of depletion are,

⁵ A constant rate of depletion means exponential decline in remaining stock x , and hence in output $q(z)x$.

⁶ Accurate, if δ is small enough for δ^2 to be ignored

$$\dot{z} = \left(\frac{z - b\lambda}{\lambda} \right) \left[\frac{\dot{p}}{p} - r + \frac{\lambda(z - b)}{1 - \lambda} \right], \quad z^* = b + \frac{(1 - \lambda)(r - \hat{p}_\infty)}{\lambda}. \quad (11)$$

Figure 1: Examples of extraction costs, $q(z)$



Examples are given in figure 1. Parameter b gives a minimal rate of extraction below which marketable output is zero. The key parameter is λ , which captures the extent to which faster depletion leads to loss of reserves, and hence also the extent to which optimal depletion is sensitive to price. The pure Hotelling case is $\lambda = 0$, (solid line in figure 1) in which the rate of depletion is infinitely sensitive to the gap between the rate of price increase and the rate of interest, and continuing extraction over an interval of time is possible only if these are equal. At the other extreme, as $\lambda \rightarrow 1$ with $b > 0$, the optimal rate of depletion is equal to b , and completely independent of the rate of price increase or rate of interest (the long-dashed line has $\lambda = 0.95$). This is consistent with the work of Adelman (1990), who argues that the rate of depletion from a particular reservoir is quite insensitive to price, and well approximated by a constant exponential rate of decline (at rate b in this specification). For cases with intermediate degrees of ‘geosensitivity’, any value of $r - \hat{p}_\infty$ will induce an extraction path which is more tilted towards the present the larger is the rate of interest relative to the rate of price increase.

While this paper deals with supply coming from many fields, it is worth briefly connecting with the standard model of market equilibrium with a single field. If demand for

the resource is iso-elastic, $Q_D = Dp^{-\eta}e^{gt}$ where D is a constant, η is the price elasticity of demand, and g the exogenous rate of growth of demand, then along the equilibrium path output, $q(z)x$, must grow at rate $g - \eta\hat{p}$. The rate of growth of output is simply $\dot{x}/x = -z^*$, so, using (11), the equilibrium rate of growth of price is

$$\hat{p} = \frac{\lambda(b + g) + (1 - \lambda)r}{\lambda\eta + 1 - \lambda} . \quad (12)$$

This is a simple generalization of the Hotelling model, in which the role of the interest rate depends on parameter λ . $\lambda = 0$ this gives the pure Hotelling case, and when $\lambda > 0$ the rate of price increase is greater the faster the growth of demand, g , the smaller the price elasticity, η , and the larger the base rate of depletion, b .

2.3 Field development: discussion

The firm's objective, equation (1), was written in terms of a field of size one ($x(0) = 1$) developed at cost K . Setting the size of each field at unity is a normalization, and the key measure is size per unit capital cost. Uncertainty about recoverable reserves in a new field can be incorporated, providing firms are risk neutral and there are a large number of fields of each type, simply by letting expected field size be unity. K , the capital cost of a unit of reserve, has empirical counterpart in the oil sector of 'finding and development' (F&D) costs per barrel, and data indicates that these are now the largest part of the sector's costs. F&D costs have risen sharply in recent years, with global average of \$21 per barrel over the period 2006-09 (EIA 2011); they are of course field specific and in some cases go much higher (e.g. US F&D costs on offshore projects were \$64 per barrel in 2006-08). These costs are several times greater than other production costs ('lifting' costs), running at global average of \$11 per barrel (EIA 2011).⁷ Furthermore, from an economic standpoint some elements of lifting cost should probably be classified as F&D; e.g. some capital equipment may be highly specific to a field but is rented by the firm and counted as 'lifting' not F&D costs.

Central to the model is the idea that there are multiple fields, and we capture this by indexing fields by their capital cost, K , and letting the measure of fields of type K be $S(K)$.

⁷ Reported lifting and F&D costs both include some tax element, EIA 2011.

The date of opening a field of type K is as given in equations (7) – (9), this giving relationship $K(T)$. Hence, in any instant of time, the total stock of capacity that is ‘opened’ is $\dot{K}S(K)$ and total costs incurred are $\dot{K}S(K)K$.

In order to model the evolution of total supply the relationship $S(K)$ needs to be specified. In some of what follows we assume that it is iso-elastic, with $S(K) = K^{\sigma-1}$. Parameter σ may be positive or negative, but we shall generally interpret results taking $\sigma < 0$, meaning that the remaining resource stock is finite, while $\sigma > 0$ means it is infinite.⁸ This relationship can easily be given a micro-foundation. The size distribution of oil fields is well approximated by a power law (see the discussion in Laherrere 2000). If the elasticity of capital costs with respect to field size is less than unity and greater than the absolute value of the exponent in the power law, then the relationship $S(K) = K^{\sigma-1}$ with $\sigma < 0$ follows (see appendix for derivation).

3. Resource supply

We now move to analysis of total supply from all open fields. This is simplified by the fact that, once open, all fields are identical apart from the scale factor x giving the remaining stock. The rate of depletion, z , (equation 3) is the same for all fields, not depending on x or any other field specific characteristics. We will henceforth denote the stock remaining at date t in a field opened at date T as $x(t, T)$, while the rate of depletion (not depending on T) will be written simply $z(t)$, with

$$x(t, T) = \exp\left[-\int_T^t z(\chi) d\chi\right], \quad x(t, t) = 1. \quad (13)$$

The date of opening depends on K , as we saw in discussion of equations (7) – (9). Using equation (7) we can express $K(T)$, the type of field that opens at date T , as⁹

⁸ K runs to plus infinity; the stock remaining is finite iff $\sigma < 0$, since

$$\int_{\bar{K}}^{\infty} S(K) dK = \int_{\bar{K}}^{\infty} K^{\sigma-1} dK = \left[K^{\sigma} / \sigma \right]_{\bar{K}}^{\infty}$$

⁹ Notice that the variable of integration is changed from τ to $t = T + \tau$; this changes the lower limit of integration

$$\begin{aligned}
K(T) &= \frac{e^{\theta T}}{(r + \theta)} \int_T^\infty (rp - \dot{p})q(z)x(t, T)e^{-r(t-T)} dt \\
&= \frac{e^{\theta T}}{(r + \theta)} \int_T^\infty \left\{ (rp - \dot{p})q(z) \exp\left[-\int_T^t z(\chi)d\chi\right] e^{-r(t-T)} \right\} dt.
\end{aligned} \tag{14}$$

We define open reserves at date t , $R(t)$, as the stock remaining in fields that have been opened by that date, i.e.

$$R(t) = \int_{-\infty}^t \dot{K}(T)S(K(T))x(t, T)dT. \tag{15}$$

This is the integral over all previous dates of the set of field types that opened at each date, $\dot{K}(T)$, times the number of fields of type K , $S(K(T))$, times stock remaining, $x(t, T)$. R moves according to differential equation

$$\dot{R} = \dot{K}S(K) - zR, \tag{16}$$

derived by differentiating (15) with respect to t and using $x(t, t) = 1$ and $\dot{x} = -zx$. The interpretation is straightforward: fields are opened at rate $\dot{K}S(K)$ and depleted at rate z .

Total output at each date is the sum of current extraction from all open fields. The fact that all open fields are identical, except for the scalar difference in the size of stock remaining, makes this aggregation over open fields straightforward. Total supply, Q_s , is simply depletion from open fields,

$$Q_s = q(z)R, \tag{17}$$

and its rate of growth is

$$\hat{Q}_s = \frac{q'(z)\dot{z}}{q(z)} + \frac{\dot{K}S(K)}{R} - z = \frac{\dot{K}S(K)}{R} + \frac{q'(z)q'(z)}{q(z)q''(z)} \left[r - \hat{p} + z - \frac{q(z)}{q'(z)} \right] - z, \tag{18}$$

derived by differentiating (17) and using (16) and (3).

With these ingredients in place, we now investigate the path of total supply associated with an exogenous path of the resource price. Given such a price path, $p(t)$, the supply side is characterized by three variables. The first is z , the rate of depletion, this inducing values of $x(t, T)$ in each field. The second is $K(T)$, the time path of field openings, and the third is $R(t)$, the stock left in open fields this, together with the rate of depletion, determining supply. z and K are forward looking decision variables that can jump in response to a shock, although K can only jump upwards (capital costs in field openings are sunk). R is a state variable, depending on both new field openings and past history.

3.1 The long run path

An analytical characterization of the path of supply can be found if price grows at a constant exponential rate \hat{p} for all future dates. The rate of depletion is then constant with value $z^* = \varsigma(r - \hat{p})$, and stocks decline exponentially, $x(t, T) = e^{-z^*(t-T)}$ (from (5) and (12)). The path of field openings through time is given by equation (14) and, with a constant future rate of growth of price and constant depletion rate, the integral can be evaluated as,

$$K(T) = \frac{p_0 e^{(\theta + \hat{p})T} q(z^*)(r - \hat{p})}{(r + \theta)} \int_T^\infty e^{(\hat{p} - z^* - r)(t-T)} dt = \frac{p_0 e^{(\theta + \hat{p})T} q(z^*)(r - \hat{p})}{(r + \theta)(z^* + r - \hat{p})} \quad (19)$$

This equation gives $K(0)$ proportional to initial price p_0 , and the rate of growth of K equal to the constant, $\hat{K} = \hat{p} + \theta$.

While constant future growth of prices implies that $z = z^*$ is constant and K grows at a constant rate, the behaviour of R depends on the history of past prices and field opening, as given by equation (16). Even if price and field openings grow at a constant rate at all future dates, R need not. To proceed, we have to specify the distribution of the number of fields associated with each level of capital costs, $S(K)$ and we will make this iso-elastic, with $S(K) = K^{\sigma-1}$, as discussed above. With this specification, the differential equation for open reserves becomes

$$\dot{R} = \dot{K} K^{\sigma-1} - z^* R. \quad (20)$$

With a constant z^* and \hat{K} , this differential equation has explicit solution,

$$R = \frac{\hat{K}K^\sigma}{z^* + \sigma\hat{K}} + e^{-z^*t} \left[R_0 - \frac{\hat{K}K_0^\sigma}{z^* + \sigma\hat{K}} \right] \quad (21)$$

where K_0 and R_0 are the values of K and R at date zero. The effect of these initial values goes to zero with e^{-z^*t} , so R converges asymptotically to value given by $R/K^\sigma = \hat{K}/(z^* + \sigma\hat{K})$.

The long run rate of change open reserves is therefore $\hat{R} = \sigma\hat{K} = \sigma(\hat{p} + \theta)$ so, with $\sigma < 0$, open reserves decline exponentially. Furthermore, since $Q_s = q(z^*)R$, output is declining at the same rate. We summarize these properties as follows.

Proposition 1:

If price is growing at constant rate \hat{p} at all future dates and $\hat{p} + \theta > 0, r > \hat{p}$, then:

- i) z , the rate of depletion of each field is constant, and is faster the larger is $r - \hat{p}$.
- ii) K , the sunk cost per unit reserve incurred on fields opened at each date, is increasing at rate $\hat{K} = \hat{p} + \theta$.
- iii) Values of z^* and K are given by

$$z^* = \varsigma(r - \hat{p}), \quad K = \frac{p_0 e^{(\theta + \hat{p})T} q(z^*)(r - \hat{p})}{(r + \theta)(z^* + r - \hat{p})}. \quad (5), (19)$$

If, additionally, the number of fields of type K is $S(K) = K^\sigma$, $\sigma < 0$ corresponding to a finite stock of the resource, then:

- iv) The rate of growth of open reserves and of supply converge asymptotically to $\hat{Q}_s = \hat{R} = \sigma(\hat{p} + \theta) < 0$.
- v) On the long run (asymptotic) growth path values of R and Q are given by

$$R = \frac{K^\sigma(\hat{p} + \theta)}{z^* + \sigma(\hat{p} + \theta)}, \quad Q_s = q(z^*)R. \quad (22)$$

Proposition 1 makes an important point about the supply side of the model. The intensive margin (the rate of depletion) depends on the rate of change of the price, not the price level. The extensive margin, the date at which new fields are opened, depends on both the level of the price and its rate of change. So too therefore do open reserves and the level of output at

each date. Comparative dynamics across asymptotic growth paths indicates that a higher level of initial prices, p_0 , is associated with more fields having been opened (higher K , equation 19) and, if $\sigma < 0$, lower open reserves and supply of output at each date (equation 22). The intuition is that a higher level of prices means that more fields have been opened; the large number of low K fields have been open a long time, and are largely depleted.

More interesting – and more insightful – than the asymptotic behaviour of supply, is the response of supply to unanticipated permanent changes in p_0 and in \hat{p} to which we now turn. To investigate this we suppose that the economy is initially on the long run path described above, this determining values of z , K , R and Q_S as given in proposition 1. How does supply respond to unanticipated change in p_0 and in \hat{p} occurring at date $t = 0$? z and K are choice variables which can jump (the latter, upwards only). The motion of R , the stock of open reserves, is given by (20); it cannot jump independently, although a jump in K at date zero will cause a discrete change in the stock of open reserves.

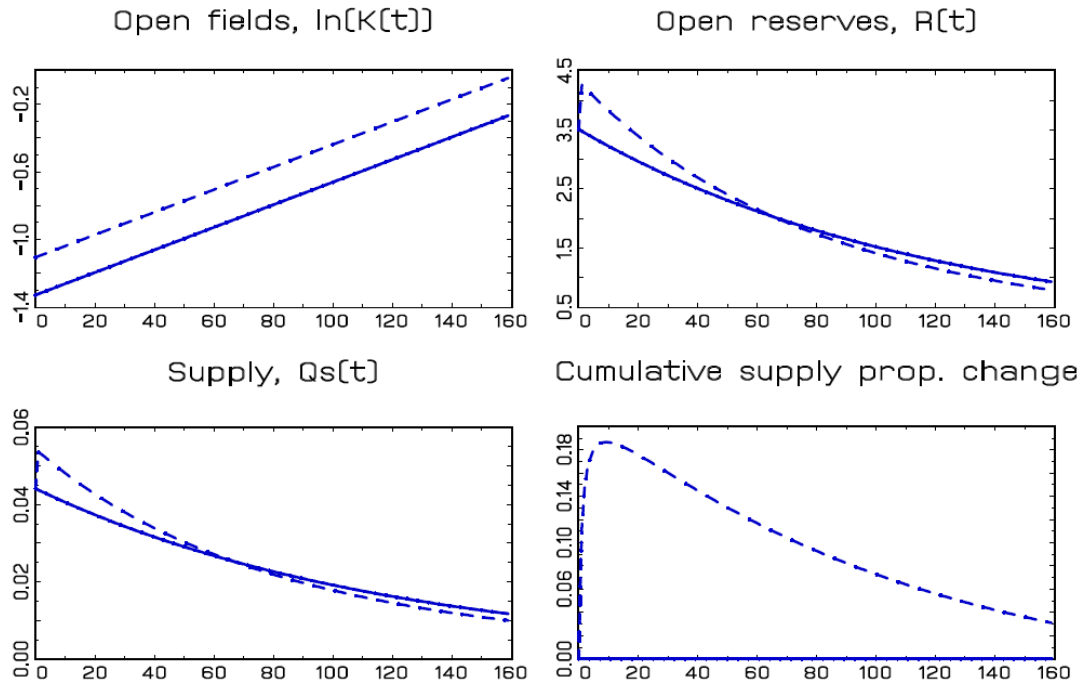
3.2 Price level changes.

We look first at the effect of an unanticipated upwards jump in p occurring at date 0 and lifting the price path by the same proportion at all future dates. Since this is a price level (not growth) effect it has no effect on the rate of depletion (intensive margin, equations (3), (5)), in which price enters only in the form of expected future price growth, in the usual Hotelling manner. However, an increase in p_0 affects the extensive margin through the timing of field openings, causing an equi-proportionate increase in K as given by equation (19). This is illustrated in the top left panel of figure 2a below, for a jump of 20% in the price level (parameter values are in the appendix). The horizontal axis is time and the vertical is $\ln(K)$. The solid line is the path without the price change, and the dashed is with the change. There is an upwards shift but no change in the subsequent rate of growth of K .

An upwards jump in K means that a discrete number of new fields are opened as the shock occurs and this causes an upwards jump in R through equation (20), and sets new values K_0 , R_0 . Thereafter, the evolution of R is given by equation (21), and it converges asymptotically to $R / K^\sigma = \hat{K} / (z * + \sigma \hat{K})$. The right hand side of this expression is unchanged but K is larger at each date and hence (with $\sigma < 0$) the asymptotic value of R is smaller than it otherwise would have been. After its initial jump, R therefore falls to below its previous level

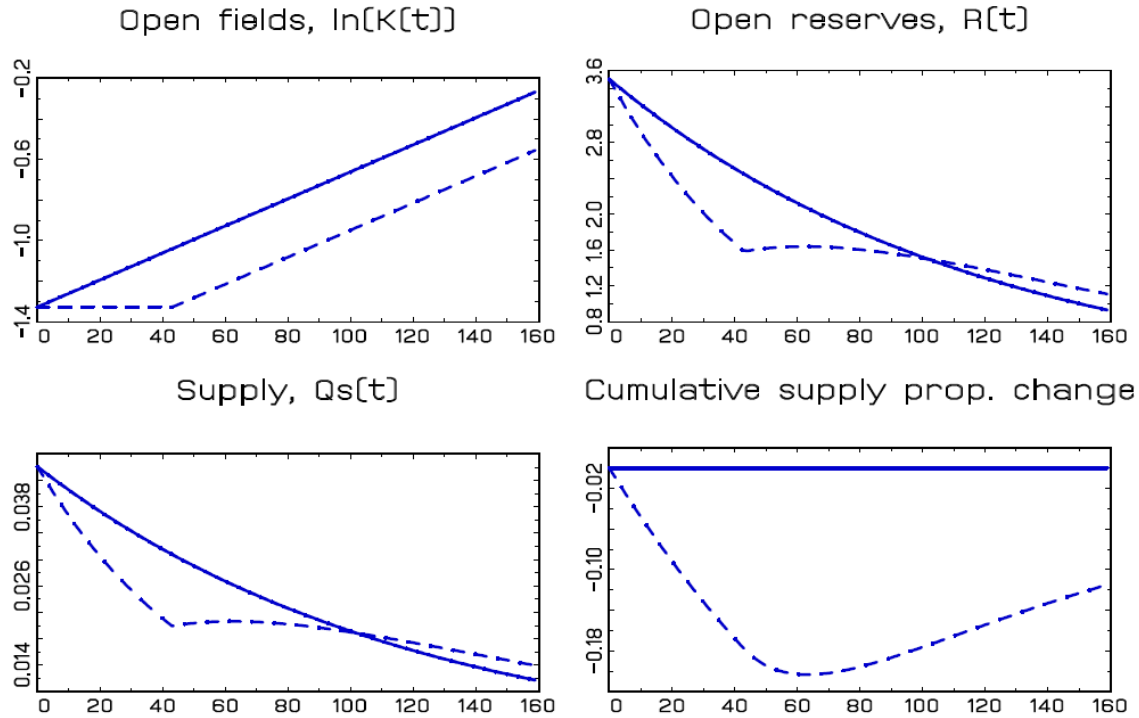
as it converges to its previous rate of growth. This is illustrated in the top right hand panel of figure 2a. The intuition is that, following the discrete jump in K and in open reserves, the higher value of K means that fewer new fields are opened at each date, since $S(K)$ is a decreasing function of K .

Figure 2a: Price increase



The corresponding path of output is in the lower left panel, proportional to the path of open reserves since the rate of depletion is constant. A permanent proportional price increase therefore elicits a positive short to medium run supply response which turns negative because, at each date t , fewer new fields are being opened. The elasticity of asymptotic supply with respect to the price level is σ , as can be seen by noting that K is proportional to the price level (equation (19)), while asymptotic R and Q_S are proportional to K^σ (equation (22)). While the short run price elasticity of supply is positive, the long run supply elasticity is therefore negative (if $\sigma < 0$). The short and long run supply responses can be combined by looking at the cumulative change in output, final panel of figure 2a, expressed as a proportion of cumulative output on the initial path. An increase in price causes a permanent increase in cumulative output, although the proportionate increase goes asymptotically to zero.

Figure 2b: Price decrease



A downwards price jump (-20% all dates) is illustrated on figure 2b, and is not completely symmetric to a price increase because there is no possibility of field closure. The shift in K (top left panel) is therefore a horizontal shift, and there is a period in which no new fields are opened. During this period open reserves fall, as does output. Once field openings resume output and open reserves recover, coming to lie above what they otherwise would have been. This captures the fact that remaining fields are larger, so mirrors the long run effects of a price increase. The price decrease reduces cumulative output at all dates. We summarize these effects in proposition 2.

Proposition 2:

A permanent proportionate change in the price (leaving the rate of growth of prices thereafter constant and unchanged) has no effect on the rate of depletion or the long run rate of growth of output. A price increase brings forward the opening of fields. Supply increases before eventually falling below what it otherwise would have been (with long run price elasticity of supply of σ). Cumulative supply is increased at all dates. A price decrease has reverse effects, leading to a permanent reduction in cumulative supply.

3.3 Price growth:

We now turn from a change in the level of price, to a change in its rate of growth. At the intensive margin, a permanent increase in price growth causes an immediate and permanent fall in the rate of depletion, z (equation (5)). The corresponding intensive margin effects are illustrated by the short-dashes in figure 3a, constructed with the decrease in z but holding the time path of K unchanged; solid lines are the original paths. Slower depletion means less supply from a given quantity of open reserves but more open reserves at all future dates, so a short run reduction in supply is followed by higher supply in future, the Hotelling-like response that would be expected. Cumulative output is reduced for a period but then becomes larger than it otherwise would have been; (this long run effect is entirely due to the fact that slower depletion uses up less of the resource in extraction costs).

At the extensive margin – letting K change – both the price growth effect and the price level effect operate. Faster price growth means faster growth of K (since $\hat{K} = \hat{p} + \theta$) but, on impact, makes it profitable to postpone field openings. The tension between these forces can be seen by using equation (5), $z^* + r - \hat{p} = q(z^*)/q'(z^*)$, in equation (19) to give

$$K(T) = \frac{p_0 e^{(\theta + \hat{p})T} q(z^*)(r - \hat{p})}{(r + \theta)(z^* + r - \hat{p})} = \frac{p_0 e^{(\theta + \hat{p})T} [q(z^*) - z^* q'(z^*)]}{(r + \theta)} \quad (23)$$

and differentiating with respect to T giving

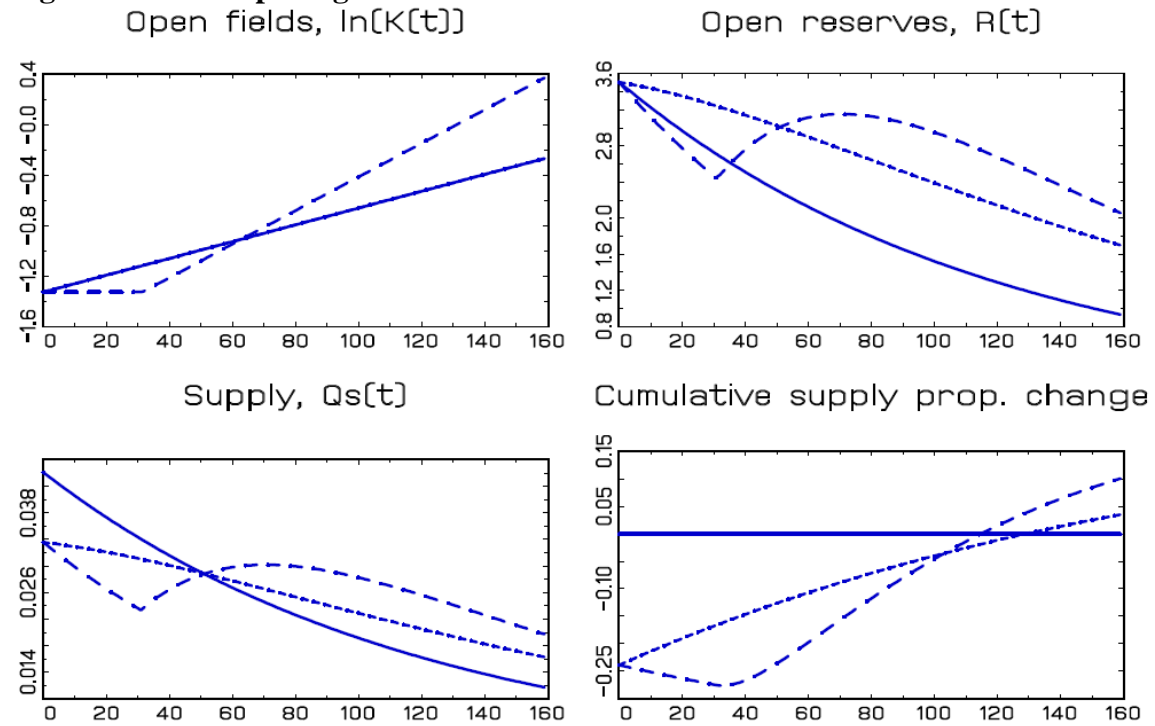
$$\frac{dK(T)}{d\hat{p}} = \frac{p_0 e^{(\theta + \hat{p})T}}{(r + \theta)} \left[\{q(z^*) - z^* q'(z^*)\}T - z^* q''(z^*) \cdot \frac{dz^*}{d\hat{p}} \right]. \quad (24)$$

This expression is negative for small T (since $q'' < 0$ and $dz^*/d\hat{p} < 0$) and positive for large T , when the first term in the square brackets comes to dominate. There is therefore a period in which field openings are reduced (or cease altogether), following which more fields are opened at each date and the new path overtakes the old.

This is illustrated in the top left panel of figure 3a. Faster price growth increases the value of opening fields in the future, causing opening to pause for a period but then to

continue more rapidly giving the crossing identified in equation (24). The long-dashes in the top right and bottom left hand panels give paths of R and Q_s that are followed with both intensive and extensive margin effects operating. The pause in field opening causes a decline in open reserves and larger initial fall in output. But following the pause, faster field opening eventually leads to higher open reserves, higher output, and a positive effect on cumulative supply. The effect of the extensive margin change is therefore to amplify intensive margin changes, as seen most clearly for the bottom right hand panel, giving the cumulative supply response.

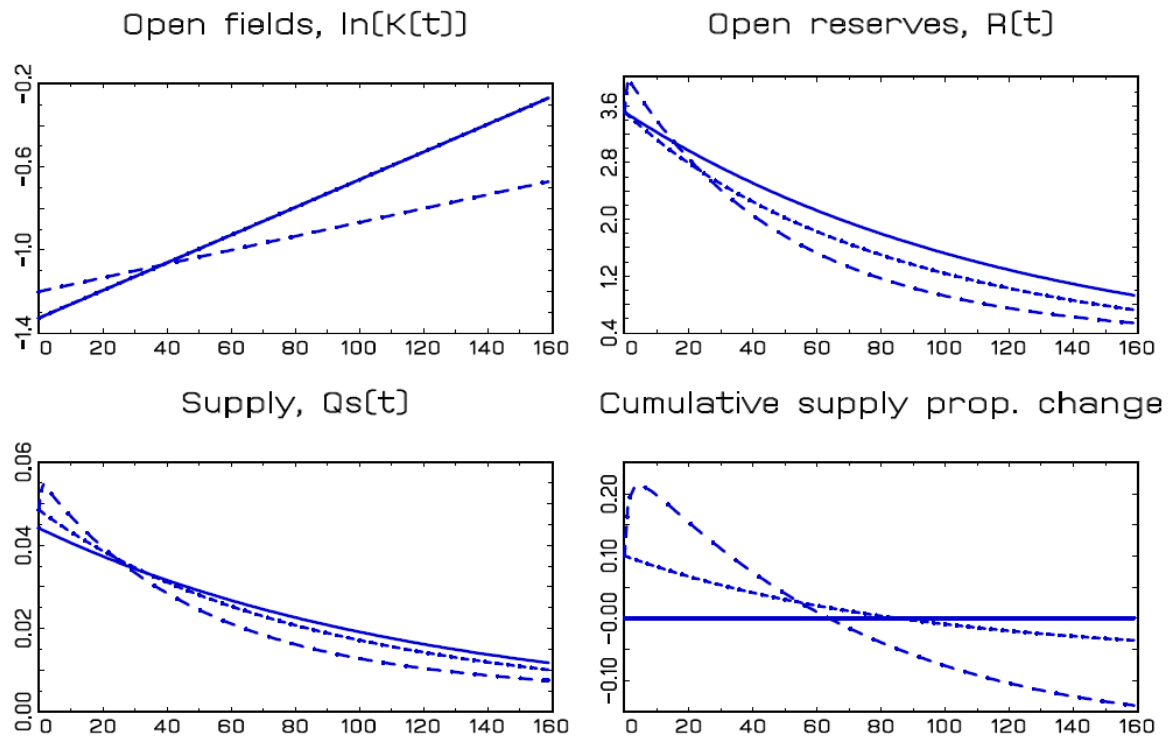
Figure 3a: Faster price growth



Short dash: Intensive margin, K constant, z adjusts:

Long dash: Intensive and extensive margin, K and z adjust

Figure 3b. Slower price growth



Short dash: Intensive margin, K constant, z adjusts:
Long dash: Intensive and extensive margin, K and z adjust.

Figure 3b gives the effects of a permanent reduction in the rate of growth of price. This increases the rate of depletion and brings forward field opening, giving the K crossing that we noted above (top left hand panel). The top right and bottom left panels give the paths of R and Q_s , once again giving initial path (solid), intensive margin only (K constant, short dash) and full adjustment (long dash). Faster depletion alone (short dash) gives a fall in open reserves at all dates, associated with higher output in the short run and lower output in the long run. Combining this with the change in field openings (long dash), the effect is magnified with a larger output increase in the short run, but a sharper fall in the long run. Cumulative output is raised for a short period, but then permanently reduced as lower prices have a major impact in reducing field openings (bottom left panel). We summarize results in proposition 3:

Proposition 3:

A permanent increase in the rate of growth of price tilts production to the future.
Depletion of existing fields is slowed down, and opening of new fields postponed. Supply

is reduced for a period, after which it overtakes its previous level. The converse holds for a permanent decrease in the rate of price growth.

4. Market equilibrium:

We now go from looking just at the response of supply to price, to the full market equilibrium with price endogenous. The demand curve is assumed to have constant price elasticity $\eta \geq 0$, exogenous rate of growth g , and level parameter D ,

$$Q_D = Dp^{-\eta} e^{gt}, \text{ so } \hat{Q}_D = g - \eta\hat{p}. \quad (25)$$

The equilibrium price path comes from equating Q_D to Q_S .

4.1 Constant growth.

We saw in section 3 that if price is growing at a constant rate the long run rate of growth of supply is constant at $\hat{Q}_S = \sigma(\hat{p} + \theta)$ (proposition 1). Equating this with the rate of growth of demand, the equilibrium rate of growth of price is

$$\hat{p} = \frac{g - \sigma\theta}{\eta + \sigma}. \quad (26)$$

Recalling that σ is the (asymptotic) price elasticity of supply, this expression links a demand shift (demand growth g) to price change via elasticities of supply and demand in the usual way. In the present context, a number of points are noteworthy.

First, in contrast to the standard Hotelling approach, the equilibrium rate of price increase is completely independent of the rate of interest. Our model gives a Hotelling-like result (equation (12)) if the extensive margin is completely fixed (no new fields open, and supply response comes only from altering depletion of existing fields). However, once the extensive margin is included in the supply response the long run rate of growth of price depends on demand and supply elasticities in a familiar way, and not at all on the interest rate.

Second, the necessary condition for our characterization of the date of field opening to be a profit maximum is that $\hat{p} + \theta > 0$ (section 2.1). With \hat{p} given endogenously by (26), this condition could fail for two distinct reasons. One is that g is substantially negative (with denominator of (26) positive) in which case demand is falling too fast to support the positive price growth necessary to induce delay in field opening.¹⁰ The other is that $\eta + \sigma < 0$ (with numerator of (26) positive). This could arise if $\sigma < 0$ in which case, as already noted, the *long run* price elasticity of supply is negative. We impose the condition that $\eta + \sigma > 0$, failing which the second order condition for field opening is not satisfied.

Equilibrium values of other variables in the system follow directly from the price growth given by (26) together with proposition 1. The long run rates of growth of open fields ($\hat{K} = \hat{p} + \theta$) open reserves, and output are

$$\hat{K} = \frac{g + \eta\theta}{\eta + \sigma}, \quad \hat{Q} = \hat{R} = \frac{\sigma(g + \eta\theta)}{\eta + \sigma}. \quad (27)$$

The initial price equates supply and demand so, using (19) and (22) in (25), satisfies

$$D = p_0^{\eta+\sigma} \frac{(\hat{p} + \theta)q(z^*)}{z^* + \sigma(\hat{p} + \theta)} \left[\frac{q(z^*)(r - \hat{p})}{(r + \theta)(z^* + r - \hat{p})} \right]^\sigma. \quad (28)$$

The following proposition summarises these properties of the long run equilibrium.

Proposition 4:

On the long run (asymptotic) path the rate of growth of price is independent of the rate of interest, and given by $\hat{p} = (g - \sigma\theta)/(\eta + \sigma)$. The elasticity of the level of the price with respect to the level of demand is $1/(\eta + \sigma)$. On this path the rate of depletion is constant, and output is declining at rate $\sigma(g + \eta\theta)/(\eta + \sigma)$.

¹⁰ A high value of θ , the rate of technical change on K , supports postponement of field opening.

This describes the long run equilibrium path but, as before, it is more interesting to investigate responses to exogenous changes. We look first at shocks to the level of demand, and then to its rate of growth.

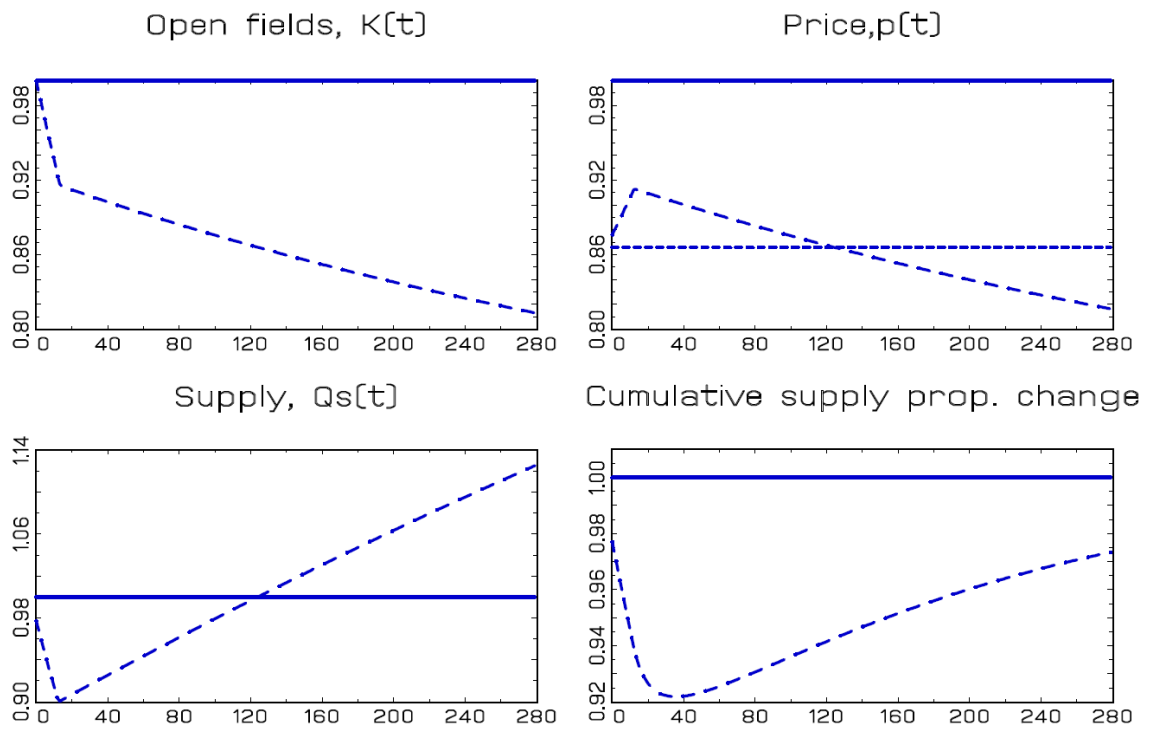
4.2 Proportional change in demand:

We look first at a permanent proportional shift in the level of demand at all dates, i.e. a change in D . We know from the results of the preceding sub-section that there is no effect on long rate rates of growth of p , Q_s , R , or on z , although there is a change in the price level. If there were no extensive margin effects (the path of K held constant) then there would be no short-run effects either; all quantities would be unaffected and the demand change would be shifted wholly to the price level. However, the extensive margin is sensitive to the level of prices, as well as their rate of change; a change in the price level changes the timing of field opening, this changing supply and inducing a transitional dynamic response.

Figure 4 illustrates the effect of a permanent decrease in demand (D falling to 75% of its previous value), with all variables expressed proportional to the initial constant growth path. The top right hand panel gives the price path. The short dashed line gives the price path in the absence of extensive margin effects: a one-off drop to $0.866 = 0.75^{1/\eta}$ of its previous value. Including extensive margin effects, the long dashed line indicates a larger ultimate price drop (going asymptotically to $0.68 = 0.75^{1/(\eta+\sigma)}$ of its previous value. There is a temporary cessation of field opening (top left), followed by a slower rate of opening thereafter (equation 19). Supply falls, both because of the change in open reserves and a change in the rate of extraction in response to the new price path. As field opening is pushed into the future supply eventually comes to exceed its previous level, although cumulative supply is lower at all dates since, at all dates, fewer fields have been opened.

The main message concerns the equilibrium path of supply, particularly cumulative supply. Without the extensive margin, a demand change would have no effect whatsoever on output. With the extensive margin operating, a reduction in demand cuts supply in the short run, raises it in the long run, and has a negative impact on cumulative quantity supplied at all dates.

Figure 4: Decrease in demand: relative to constant growth path



Short dash: Intensive margin, K constant, z adjusts:
Long dash: Intensive and extensive margin, K and z adjust.

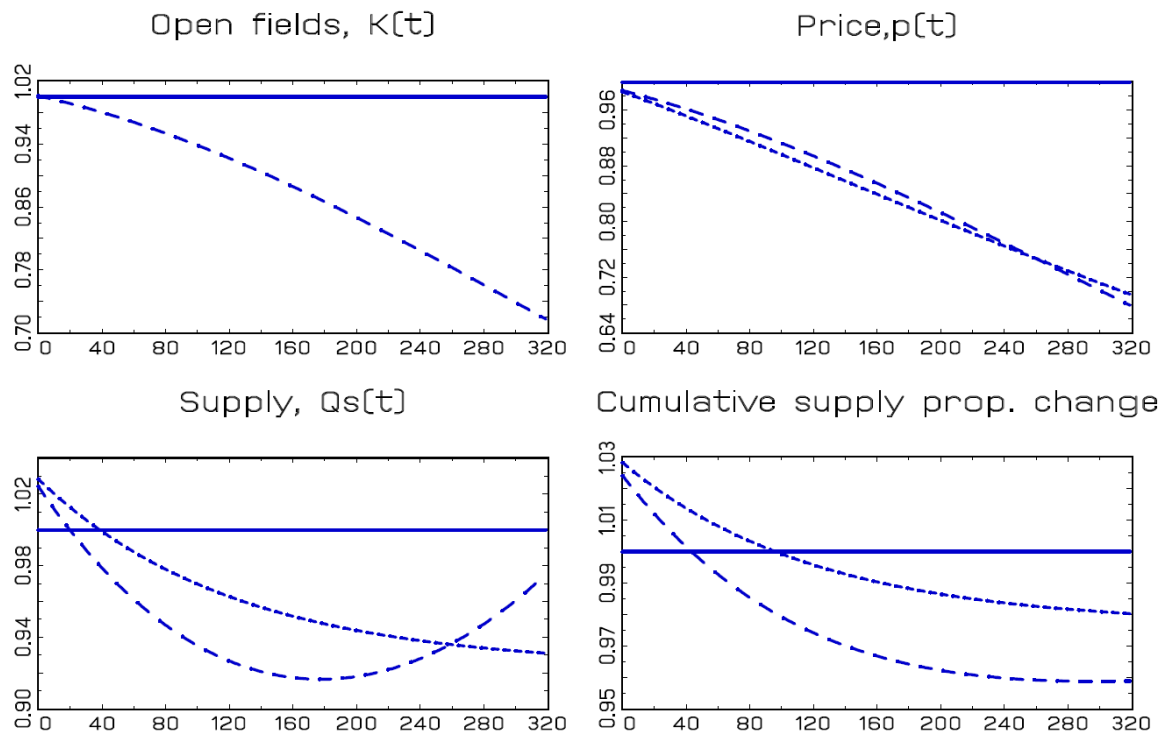
4.3 Change in rate of growth of demand:

A permanent change in the rate of growth affects the long run growth of variables, as well as transitional dynamics. Long run growth rates can be found explicitly, and are given in table 1 of the appendix. The full dynamic story is given in figure 5. The short dashes give adjustment at the intensive margin only. Slower growth of demand reduces the rate of price increase (equation 12), this increasing the rate of depletion and shifting the supply towards the present, lower left panel, as would be expected from Hotelling.

Adding the extensive margin response (endogenous K , long dashes) means that the price level matters, as well as its rate of growth. The lower price means that fewer fields are open at each date, as illustrated in the top left panel. This pushes supply into the future. The U shaped response of supply (bottom left) is interpreted as follows. In the short run, the Hotelling effect of faster extraction dominates, this giving the supply increase. In the medium run supply is lower because open fields have been depleted faster and because fewer fields have been opened. In the long run supply turns up, because the high $S(K)$ field types,

opening of which was postponed, are coming on stream. Looking at cumulative supply, we see that adding the extensive margin effect mitigates the Hotelling effect; slower demand growth increases cumulative supply for a shorter period, beyond which it is associated with larger reductions in the cumulative stock of resource extracted and output produced.

Figure 5: Slower growth of demand: relative to constant growth path



Short dash: Intensive margin, K constant, z adjusts:
Long dash: Intensive and extensive margin, K and z adjust.

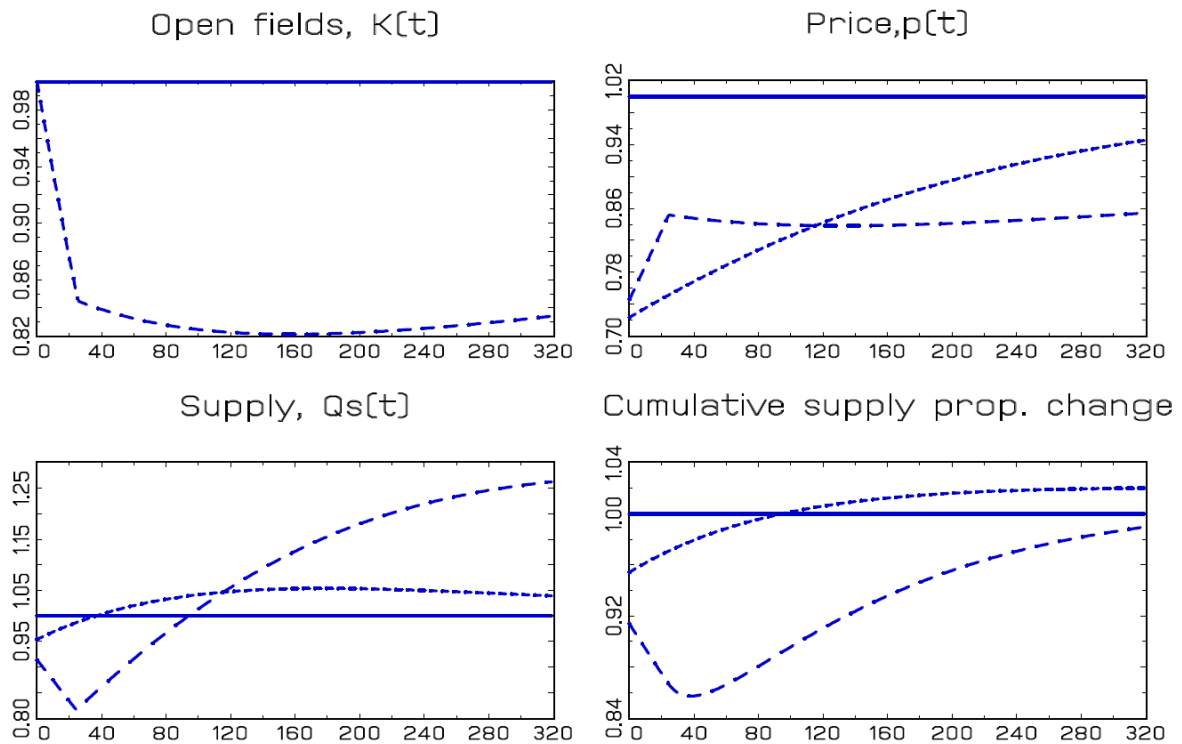
5. Carbon taxation with endogenous field opening:

The equilibrium impact of climate change measures such as a carbon tax depend on both the demand and supply responses of fossil fuel. Much of the climate change literature has concentrated on demand reduction, while Sinn (2008) has used a simple model of resource depletion to argue that there may be a ‘green paradox’; carbon taxes or other measures to reduce demand might be ineffective or even, if they are expected to become more severe in future, have perverse effects, bringing forwards extraction from the far future to the nearer future. How does this work when both extensive and intensive margin effects are present?

Policy measures that lead to permanent proportionate demand reduction cause an immediate and continuing reduction in the cumulative quantity of the resource supplied (section 4.2, figure 4), as the lower price delays field opening and postpones production. This is in contrast to the case when the extensive margin effect is absent in which policy has no effect on quantities produced.

Policy measures that reduce the rate of growth of demand (section 4.3, figure 5) bring forward extraction from existing fields, this raising current output. This is offset by the price level effect which postpones field opening. Output therefore falls faster, and the cumulative output increase is smaller, and positive for a shorter period of time, when the extensive margin effect is present.

Figure 6: Constant specific resource tax: relative to constant growth path



Short dash: Intensive margin, K constant, z adjusts:
Long dash: Intensive and extensive margin, K and z adjust.

Finally, it is worth relating the policy experiments to a CO_2 tax. A proportionate demand reduction could be achieved by a constant ad valorem tax, i.e. a tax per unit CO_2 increasing at the same rate as the resource price. A policy that achieved slower demand

growth would be equivalent to an exponentially increasing ad valorem tax rate. By way of contrast, figure 6 looks at a CO₂ tax imposed at date 0 and then constant in perpetuity, (i.e. declining relative to the resource price). As before, short-dashed lines give the effect when only the intensive margin operates. The producer price falls on impact, but then converges back to its previous level (as the relative value of the tax diminishes). This reduces the rate of extraction, giving the short run fall in supply followed by long run increase. However, when the extensive margin operates (long dashes) the producer price fall leads to a period in which no new fields are opened, and hence a much larger fall in supply. As usual, this is a postponement of field opening, so supply rises in future. Once again, the key point is that the price level effect postpones field openings and therefore leads to lower cumulative supply at all future dates.

6. Concluding comments

The paper has developed a model of supply of a non-renewable resource in which the empirically compelling fact that large sunk costs are associated with the development of new mines or fields is put centre stage. Supply responses then depend on both the rate of change of prices and the level of prices, this eliminating some ‘paradoxes’ that have gained recent attention. For example, a permanent proportionate demand reduction pushes production into the future, so unambiguously reduces cumulative output and any associated stock of emissions. The Hotelling relationship between the rate of price increase and the rate of interest ceases to hold in the long run as underlying supply considerations (the geology of available fields) become decisive.

Much remains to be done developing and applying the line of research proposed in this paper. We have assumed throughout that future price paths are known with certainty and that owners of fields will postpone opening until the date at which the present value of the field is maximized. Allowing price uncertainty and placing the field opening decision in a stochastic context is clearly important. On the applied side, the model provides a framework for thinking about taxation of resources, both resource use (as with carbon taxes) and a resource rents, as with royalties, production sharing agreements, and corporate income taxes.

Appendix:

In the text field size is normalized at unity, fields vary in capital cost K , with the number of fields of type K denoted $S(K)$. This can be derived from the following alternative set up. Suppose that fields are ordered by size, s , with $m(s)$ fields of size s , $m' < 0$. $m(s)$ follows a power law, so $m(s) = s^\alpha$, $\alpha < 0$. The total capacity of fields of size s is $sm(s) = s^{1+\alpha}$. The capital cost of a field of size s is $k(s)$, and we suppose $k(s) = s^\kappa$, $0 < \kappa < 1$, so costs are increasing and strictly concave in field size; the capital cost of one unit of capacity on a field of size s is $s^{\kappa-1}$, i.e. $K = s^{\kappa-1}$. Since the capacity associated with fields of size s is $S = s^{1+\alpha}$, we have, eliminating s , $S(K) = K^{(1+\alpha)/(\kappa-1)}$. Thus, $\sigma - 1 = (1+\alpha)/(\kappa-1)$ and hence $\sigma = (\kappa + \alpha)/(\kappa-1)$, which is negative if $\kappa < 1$ and $\kappa + \alpha > 0$.

Appendix:

Parameter values, figures 2, 3, and 4:

$r = 0.02$; $g = 0.005$; $\eta = 2$; $\sigma = -1.25$; $a = 0.1$; $b = 0.005$; $\lambda = 0.5$.

Long run equilibrium $\hat{p} = 0.067$ (exogenous in figures 2 and 3).

Figure 2: initial price p_0 raised by 20%, reduced by 20%.

Figure 3: \hat{p} doubled to 0.01, halved to 0.0025

Figure 4: demand, D , cut by 25%

Figure 5: growth rate g halved to 0.0025

Figure 6: Constant specific tax at 30% of initial price (eg carbon price \$50, oil price \$70, 0.43 tonnes of CO₂ per barrel of oil).

Table 1: Asymptotic growth rates for a reduction in the rate of growth of demand $g_N < g_I$

	Initial, g_I		New, g_N Intensive margin only		New, g_N Intensive & extensive margin	
\hat{K}	$\frac{g_I + \eta\theta}{(\eta + \sigma)}$	=	$\frac{g_I + \eta\theta}{(\eta + \sigma)}$	>	$\frac{g_N + \eta\theta}{(\eta + \sigma)}$	> 0
\hat{Q}_S	$\frac{\sigma(g_I + \eta\theta)}{(\eta + \sigma)}$	=	$\frac{\sigma(g_I + \eta\theta)}{(\eta + \sigma)}$	<	$\frac{\sigma(g_N + \eta\theta)}{(\eta + \sigma)}$	< 0
\hat{Q}_D	$g_I - \eta\hat{p}$	=	$g_N - \eta\hat{p}$	<	$g_N - \eta\hat{p}$	< 0
\hat{p}	$\frac{g_I - \sigma\theta}{\eta + \sigma}$	>	$\frac{g_N - \sigma\theta + \sigma(g_N - g_I)/\eta}{\eta + \sigma}$	>	$\frac{g_N - \sigma\theta}{\eta + \sigma}$	> 0

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