Noise induced suppression of spiral waves in a hybrid FitzHugh–Nagumo neuron with discontinuous resetting

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Abstract

A modified FitzHugh-Nagumo neuron model with sigmoid function-based recovery variable is considered with electromagnetic flux coupling. The dynamical properties of the proposed neuron model are investigated and as the excitation current becomes larger, the number of fixed points decreases to one. The bifurcation plots are investigated to show the chaotic and periodic regimes for various values of excitation current and parameters. A $N \times N$ network of the neuron model is constructed to study the wave propagation and wave re-entry phenomenon. Investigations are conducted to show that for larger flux coupling values, the spiral waves are suppressed but for such values of the flux coupling the individual nodes are driven into periodic regimes. By introducing Gaussian noise as an additional current term we showed that when noise is introduced for entire simulation time, the dynamics of the nodes are largely altered while the noise exposure for 200-time units will not alter the dynamics of the nodes completely.

Keywords: FitzHugh-Nagumo neuron; hybrid neuron; resetting; spiral waves; noise effects

FitzHugh-Nagumo neuron models are simplest yet one of the effective mathematical models used to define the bursting and spiking phenomenon of neurons. We modified the model by including sigmoid based recovery variable and flux coupling. Dynamical behaviours and properties of the modified model is investigated. A network model of the neuron model is constructed, and wave propagation phenomenon is studied. We could effectively show the emergence of spiral waves in the network and by including the noise effects we could show that introducing noise can suppress spiral waves.

1. Introduction

The behaviours of dynamical systems were considered rather smooth and continuous. On contrary impact, switching, friction and sliding prologue intricate response, abrupt changes and unpredictable regions. These discontinuities and instabilities inaugurate fertile field of research for the scientific community and the yields categorized into two promising branches namely “Catastrophe theory”[1]and “Chaos theory” [2]. The Catastrophe theory describes the sudden or abrupt changes for the parameter variation by considering not only first derivative of the potential function rather higher derivatives also zero[3]. Chaos theory elucidate the absence of order in a system and provides a graphical representation called strange attractors[4, 5]. Chaos can be predictable only locally but for long term it is unpredictable. Horde of literature discussed the coexistence of discrete and continuous dynamics of the nonlinear systems [6-9] and penned a technical term “Hybrid systems”[10-12]. The challenge persists to investigate nonlinear dynamical behaviours of the hybrid systems since it consists different bifurcation phenomena compared with smooth, continuous systems. Because of the rich dynamical nature most of the real time systems are considered as hybrid and throng of research articles generated[11-15].

The switching property in Biochemical systems and gene regulatory networks are well-handled using hybrid modelling [16-18].Advanced computational techniques are demanded for investigating state changes and to capture chaotic behaviours[11, 19-21]. The corresponding network properties also subjected to redefine based on the discrete scenario[10, 11, 22-24]. Simultaneously the non-homogeneity of the cell and hormonal stimuli are investigated with hybrid models[16, 25]. Particularly the cancer cell division, spread and proliferation along with tumour growth. Peculiar bifurcation structures were identified and this study give a new raise to intermittent hormone therapy[25]. Firing phenomena of neuron modelled as a continuous system while it reaches a threshold potential and reset the states which results with spiking[11, 12, 14, 15, 17]. In 2010,Izhikevich formulated a hybrid model which expose rich dynamic behaviours similar to real neurons[26]. Basically, neurons are connected through electrical and chemical synapse; the spike generation mechanism drives the synapse with discontinuity as switching function. A simplified hybrid model was configured to investigate spiking and bursting mechanism[27] during impulsive perturbation[21, 26, 28]. Chaotic spiking and bursting were identified and significant abrupt changes simulated. Chaotic maps are used to describe various network phenomena of neurons articulated with hybrid models[11, 12, 14, 17, 29]. Coupling strength, external excitation and topology plays vital role to describe dynamics of network[30-32].

Spatiotemporal patterns are substantial dynamical characteristics of complex network systems with excitable media such as neuronal systems. Investigating network dynamics is one of the fruitful research topics in the last few decades. Spiral waves are typical spatiotemporal pattern found harmful and undesirable in most of the network systems[33-38]. In 1997, Spiral waves are captured experimentally in neuron system while investigating turtle visual cortex[39]. Multitude of theoretical models were developed to study the spiral wave formation[35, 40-44] and its dynamics[45-50], considerable literatures found in control scheme[48, 51-56].Periodic excitation of local neuron could be a potential method but recent literatures conceived few more methods to control or suppress spiral waves[48, 51, 53-55, 57-59]. Introduction of channel noise and additive noise found promising method to disintegrate the spiral waves smoothly[38, 60-66].

The FitzHugh-Nagumo model (FHN) is a simplified model to replicate substantial dynamical properties of neurons under external excitation[67]. On the other hand, Morris-Lecar (ML) model exhibit fold/Hopf bursting bifurcations with two different classes of neuronal excitability[24, 68-71]. Based on the discussion we arrive at a direction to investigate the dynamical response of a spiking neuron model with discontinuous function and including electromagnetic flux coupling[21, 26, 28]. The paper is organised as follows (i) Formulation of the model and definition about sigmoid function and magnetic flux coupling. (ii) Dynamic analysis and numerical simulations (iii) Network formulation, dynamics of spiral waves and its propagation based on coupling properties. (iv) Spiral wave suppression by introducing Box-Mueller type white noise[72]. Finally, a discussion on results obtained and concluding remarks are presented.

2. The model

The FitzHugh–Nagumo neuron model [67, 73] is a 2D system of differential equations defined by

$\begin{aligned}\dot{v} &= v(v-1)(a-v) - u + I_{ext} \\ \dot{u} &= bv - cu\end{aligned}$	(1)
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where v represents the membrane potential of a neuron and u represents the membrane recovery variable. The parameter a determines the shape of a $\dot{v}=0$ nullclines, while b and c represents the time-constant and sensitivity of the recovery variable. In physical neurons, the state variables transits from resting to spiking through both Hopf and saddle-node bifurcations[74], whereas the neuron model in (1) transits through only Hopf bifurcation. There are many famous spiking neuron models like the Morris–Lecar neuron model[75] which exhibits transition through both bifurcations and this model uses a sigmoidal function for v as the state equation for the recovery variable given by

$\dot{u} = \mu((1 + \exp(-(v-b)/\epsilon))^{-1} - u)$	(2)
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Using (2) in (1) the modified form of (1) called the FitzHugh–Nagumo-Morris-Lecar (FNML) neuron model defined in[21],

$\begin{aligned}\dot{v} &= v(v-1)(a-v) - u + I_{ext} \\ \dot{u} &= \mu((1 + \exp(-(v-b)/\epsilon))^{-1} - u)\end{aligned}$	(3)
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To derive the continuous spiking neuron model a resetting method is defined, using[75], as:

$\text{if } v \geq 0.4 \Rightarrow \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$	(4)
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The dynamical behaviour of the FNML neuron was discussed in detail in (6) however many literatures have shown that by including the effects of electromagnetic flux coupling we could explore a complete dynamical behaviour of a physical neuron. Hence, we modify (4) by including another flux variable as:

$\begin{aligned}\dot{v} &= v(v-1)(a-v) - u + I_{ext} - k_0 M(\phi)v \\ \dot{u} &= \mu((1 + \exp(-(v-b)/\epsilon))^{-1} - u) \\ \dot{\phi} &= k_1 v - k_2 \phi\end{aligned}$	(5)
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where ϕ denotes the magnetic flux across the membrane and $M(\phi) = \alpha + 3\beta\phi^2$ defines the memductance. The significance of the memductance function and its properties such as V-I curves are considered as in [76, 77].

The parameter values of system (5) are

$I_{ext} = 0.008, a = 0.1, b = 0.5, c = 0.33, d = 0.01, \mu = 0.1, \varepsilon = 0.05$ $\alpha = 0.1, \beta = 0.001, k_0 = 0.1, k_1 = 0.1, k_2 = 0.01$	(6)
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3. Numerical analysis

Because of the exponential term in the equation for $u(t)$, it is not possible to determine exact expressions for the equilibrium states. Instead we follow Sou et al (2018)[21] and plot the v

and u nullclines, after substituting for the equilibrium value of $\phi = \frac{k_1 v}{k_2}$ into evolution equation for $v(t)$. Since the objective of this study is to ascertain the effects of magnetic induction on the FNML neuron, we also include the nullclines for various values of I_{ext} . The v -nullcline and u -nullcline are given in terms of u by

$u_1 = v(v-1)(a-v) + I_{ext} - k_0 M(\phi)v$ $u_2 = \mu \left(\frac{1}{1 + e^{\left(-\frac{(v-b)}{\varepsilon} \right)}} \right)$	(7)
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The parameter choices are taken to be

$a = 0.1; b = 0.5; c = 0.33; d = 0.01; \mu = 0.1; \varepsilon = 0.05;$ $\alpha = 0.1; \beta = 0.001, k_0 = 0.1; k_1 = 0.1; k_2 = 0.01$	(8)
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The equilibrium state of equation (5) is obtained by plotting the nullclines and noting their points of intersection. It is of interest to see how varying I_{ext} affects these fixed points. Fig.1 shows the nullclines for the Sou et al (2018) case, where ϕ is absent. When $I_{ext} = 0$ Sou et al (2018)[21] note that the locations of the fixed points are at $(v, u) \approx (0, 0), (0.1, 0), (0.35, 0.06)$, and $(v, u) = (0, 0)$ is stable. When I_{ext} increases the intersections of the two nullclines change. For $I_{ext} \approx 0.002376$, the v -nullcline is tangential to the u -nullcline at $(v, u) = (0.049, 0.00012)$. (This is found by finding the solutions to $\frac{du_1}{dv} = 0$). The remaining fixed point is at $(v, u) \approx (0.366, 0.0641)$. For values of $I_{ext} > 0.002376$, there is only one fixed point: the unstable one, bounded away from $(0, 0)$.

When ϕ is included, we obtain Figs.2 and 3. For $I_{ext} = 0$, the three fixed points are located at $(v, u) \approx (0, 0)$, $(v, u) \approx (0.112, 0.000426)$, $(v, u) \approx (0.377, 0.0541)$. From Fig.3 for $I_{ext} = 0.004$,

the only fixed point is at $(v, u) \approx (0.362, 0.595)$, while for $I_{ext} = 0.008$ it is at $(v, u) \approx (0.366, 0.0645)$. From Figure 2, we see that the

Fig.1: The nullclines for the neuron equations when ϕ is absent as I_{ext} varies. The red line corresponds to the u-nullcline, which is independent of I_{ext} , while the v -nullcline varies in a series of tight curves. The curves for $I_{ext} = 0.004$ and $I_{ext} = 0.008$ are in distinguishable

Minimum of the v -nullcline occurs for $I_{ext} \approx 0.00289$. The other fixed point is at $(v, u) \approx (0.360, 0.0577)$. This means that for larger values of I_{ext} , there is only one fixed point, shown in Figure3. Moreover, such fixed points are unstable, as can be verified by computing the characteristic equation from the Jacobian matrix:

$J = \begin{pmatrix} fac & -1 & -k_0 v M_\phi \\ \mu g' & -\mu & 0 \\ k_1 & 0 & -k_2 \end{pmatrix}$	(9)
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Fig.2: The nullclines for the neuron equations when φ is included as I_{ext} varies. The red line corresponds to the u-nullcline, which is independent of I_{ext} . The curves for $I_{ext}=0.00289$, $I_{ext}=0.004$ and $I_{ext}=0.008$ are also included.

where

$$fac = -3v^2 + 2(a+1)v - a, \quad g' = \frac{f}{\varepsilon(\det(J)^2 \lambda_1)} \quad (10)$$

The cubic characteristic equation, $\varepsilon(\det(J)^2 \lambda_1)$ is therefore

$$\lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 = 0 \quad (11)$$

Fig.3: Behaviour away from the minima of the nullclines as I_{ext} varies.

where,

$$\begin{aligned} A_2 &= \mu + k_2 - fac \\ A_1 &= k_0 k_1 M_\varphi v + \mu(g' + k_2) - fac(\mu + k_2) \\ A_0 &= k_0 k_1 \mu M_\varphi v + \mu k_2 (g' - fac) \end{aligned} \quad (12)$$

The eigen values are then determined by substitution of the appropriate equilibrium

state for v . When $I_{ext} = 0$, the eigen values for the stable trivial state $(v, u, \varphi) = (0, 0, 0)$ are $(-0.0100, -0.1 \pm 0.00953i)$, while for $(v, u, \varphi) = (0.112, 0.00042, 1.12)$, we have $(0.1, -0.0958, -0.0993)$ space (a saddle) and for $(v, u, \varphi) = (0.357, 0.054, 3.57)$, we have $(-0.01, 0.1015 \pm 0.2488i)$ (a saddle focus).

The corresponding characteristic equation for the $I_{ext} = 0$ limit of the 2DSouet al (2018) model gives Eigen values of $(-0.1 \pm 0.00953i)$ for the stable state $(v, u) = (0, 0)$.

Fig.4 Time series of (a) $v(t)$ and (b) $u(t)$ for the 2D Sou et al system for $I_{ext}=0.004$. Also shown are time series of (c) $v(t)$ and (d) $u(t)$ for the 3D system (1).

For $I_{ext}=0.008$, the unstable fixed point at $(v, u, \phi) = (0.367, 0.0648, 3.67)$ has Eigenvalues $(-0.01088, 0.1017 \pm 0.2857i)$ and for $I_{ext}=0.004$, the fixed point at $(v, u, \phi) = (0.362, 0.0597)$ has Eigenvalues $(-0.100, 0.10167 \pm 0.267i)$.

We find it of interest to investigate the effects of the magnetic induction ϕ on the nonlinear behaviour of system (7) in comparison with the Sou et al model (2018) when resetting is imposed. Throughout, we use $\beta=0.5$. Fig.4 shows time series of $v(t)$ and $u(t)$ for $I_{ext}=0.004$ (the choice of Sou et al (2018)[21]). In the absence of magnetic induction, we observe regular periodic spiking behaviour for the 2D system, whereas we have irregular bursts and spiking in the 3D model.

Fig.5. (a) $u(t)$ time series (b) First return map of successive maxima of $u(t)$ for the 3D system

We can plot successive maxima of $u(t)$ in a return map. For the Sou et al system we obtain a periodic map with just 1 point. Fig.5 shows the corresponding first return map for the 3D system over a longer time scale, while Fig.6 shows the corresponding first return map when $I_{ext} = 0.008$.

Fig.6. (a) $u(t)$ time series (b) First return map of successive maxima of $u(t)$ for the 3D system

Following Sou et al (2018)[21], we can produce the bifurcation transition plot of the maxima of $u(t)$ over each cycle, as the resetting parameter c varies. This is shown in Fig.7, and we could note the system taking period doubling route to chaos. For a brief range of c

between [3.25,3.75] the chaotic regime of the FNML system is seen in Fig.7 with multiple cascaded perodic-chaotic regions.

Fig.7. Bifurcation transition plot of successive maxima of $u(t)$ for the 3D system as c increases.

To demonstrate the effect of the external current I_{ext} , we plotted the bifurcation diagram as shown in Fig.8. The range of I_{ext} is taken between [4e-3,12e-3] and the local maxima of $u(t)$ is plotted. We could note two distinct chaotic regions and period halving exit from chaos.

Fig.8. Bifurcation transition plot of successive maxima of $u(t)$ for the 3D system as I_{ext} increases.

4. Network behaviour of the FNML model

To investigate the network performance of the FNML model, we construct a lattice array of the media whose local nodes are the FNML neurons. The mathematical model of the lattice array can be defined as

$\begin{aligned}\dot{v}_{ij} &= v_{ij}(v_{ij} - 1)(a - v_{ij}) - u_{ij} + I_{ext} - k_0 M(\phi_{ij})v_{ij} + D(v_{i+1,j} + v_{i-1,j} + v_{i,j+1} + v_{i,j-1} - 4v_{ij}) \\ \dot{u}_{ij} &= \mu((1 + \exp(-(v_{ij} - b)/\varepsilon))^{-1} - u_{ij}) \\ \dot{\phi}_{ij} &= k_1 v_{ij} - k_2 \phi_{ij}\end{aligned}$	(13)
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The stimuli function is a periodic sinusoid defined by $\varphi(t) = A \cos(\omega t)$ which is applied to the network when $i=m$ and $j=n$. The size of the network is 110×110 and thus $i \in [1, 110]$ and $j \in [1, 110]$. The stimuli is applied from the left centre of the network by setting $i=0$ and $j=55$. In our discussion we consider the coupling strength D , stimuli frequency ω and the flux coupling coefficient k_0 as the control variables and discuss the spiral waves and wave propagation in the network. We have used a periodic boundary conditions with random initial condition settings for the nodes.

4.1 Spiral waves in the network:

Firstly, we consider $\omega=0.0001$ and $k_0=0.01$ with coupling strength D as the control parameter and capture the snapshot of the lattice network for the applied stimuli as in Fig.9. For the coupling strength $D < 0.5$, the network is of lowly excited nodes with no wave propagation seen. Increasing the coupling strength over $D=0.5$, the nodes are of different excitability with the media showing wave reentry and spiral waves. The change in the node excitability near the right boundary for $D=0.8$ results in a simple travelling periodic wave seen near the right boundary.

Fig.9. The snapshots captured at 5000-time units of the network (13) for s applied stimuli of amplitude $A=0.01$ and $\omega=0.0001$. The flux coupling coefficient $k_0=0.01$ and considering coupling strength as the control parameter.

Now we consider the applied stimuli frequency as the control parameter with $k_0=0.01$ and $D=1$. We have tried from very low frequency of $\omega=0.0001$ to as high as $\omega=1 \text{ KHz}$. Irrespective of the frequency of the stimuli, the snapshot confirms the emergence of spiral waves in the network and doesn't seem to be affected by the stimuli. The captured snapshots are shown in Fig.10 and most of the literatures have shown that the stimuli frequency either drives the network unstable or makes the media highly excitable thus eliminating spiral waves. But the FNML network differs as the spiral waves initiated in the network are not affected by the stimuli frequency.

Fig.10. The snapshots captured at 5000-time units of the network (13) for applied stimuli of amplitude $A=0.01$ considering the flux coupling coefficient $k_0=0.01$. The stimuli frequency is considered as the control parameter.

To show the effects of flux coupling on the wave propagation in the network, we considered k_0 as the control parameter. When there is no flux coupling, the network is idle with no excitability and thus no wave propagation is supported as in Fig.11. When flux coupling is applied by fixing $k_0=0.01$, spiral waves are formed, and these spiral waves are seen for the range $0.01 \leq k_0 \leq 0.3$. By increasing $k_0=0.5$ the spiral waves are eliminated in the network because of the network excitability is minimum with the nodes in an idle state.

Fig.11. The snapshots captured at 5000-time units of the network (13) for s applied stimuli of amplitude $A=0.01$ and $\omega=0.0001$. The flux coupling coefficient is considered as the control parameter.

4.2 Suppressing spiral waves using Box-Mueller noise:

To study the effects of noise on the excitability and spiral waves in the network and for this we have considered the Box-Mueller type white [78, 79] defined by

$N(t) = \left(\frac{-4\delta}{\Delta t} \ln(a) \right)^{0.5} \cos(2\pi b)$	(14)
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where $[a, b]$ are random chosen in the range $[0, 1]$, δ is the variance of the noise and Δt is the step size of the solver. This noise term is considered as an additional current and added to the state equation (v) and the network model with noise is defined as

$\begin{aligned} \dot{v}_{ij} &= v_{ij}(v_{ij} - 1)(a - v_{ij}) - u_{ij} + I_{ext} - k_0 M(\phi_{ij}) v_{ij} \\ &\quad + D(v_{i+1,j} + v_{i-1,j} + v_{i,j+1} + v_{i,j-1} - 4v_{ij}) + N(t) \\ \dot{u}_{ij} &= \mu((1 + \exp(-(v_{ij} - b)/\epsilon))^{-1} - u_{ij}) \\ \dot{\phi}_{ij} &= k_1 v_{ij} - k_2 \phi_{ij} \end{aligned}$	(15)
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Depending on the time duration of the inclusion of noise to the network, we divide the discussion in to two sections. In the first discussion we consider that the noise is applied for the entire simulation time of 5000-time units and captured the snapshot as in Fig.12. For the simulation we considered the stimuli parameters as $A=0.01$ and $\omega=0.0001$ with the coupling strength $D=1$ and flux coupling constant $k_0=0.01$. We started by applying a noise variance of $\delta=0.0001$ and kept increasing the variance as shown in Fig.12. When the variance is increased to $\delta=0.002$ the spiral waves are compressed in the network and we could see travelling waves originating from the centre of the network. It should be noted that the stimuli are applied to the centre of the network for this simulation by fixing $\alpha=55$ and $\beta=55$. Thus, we could confirm that by applying a noise with variance $\delta=0.002$, the spiral waves are

suppressed in the network and the network shows target waves for the selected noise variance. This is because of the loss of heterogeneity and the nodes becoming highly excitable.

Fig.12: The snapshots of the network with noise considering the noise variance as the control parameter while the stimuli settings are $A=0.01$ and $\omega=0.0001$ with network coupling strength at $D=1$ and flux coupling at $k_0=0.01$. The noise is applied to the network for the entire simulation time.

In Fig.12 we have shown the snapshots considering the noise being applied for the entire simulation time. In Fig.13, we considered noise applied for 200-time units between 2500- and 2700-time units. In this case we need a higher noise variance of $\delta=0.005$ to make the media highly excitable and thus suppressing the spiral waves in the network. This is again because of the loss of heterogeneity in the network making the nodes highly excitable. It should be noted that the noise is applied only for a shorter duration which means that the dynamical behaviour of the nodes is not much affected by the insertion of noise which was not the case when noise is applied for the whole simulation time wherein the dynamics of the neurons are completely affected by noise.

Fig.13: The snapshots of the network with noise considering the noise variance as the control parameter while stimuli settings are $A=0.01$ and $\omega=0.0001$ with network coupling strength at $D=1$ and flux coupling at $k_0=0.01$. The application of noise to the network for the time 2500-time units to 2700-time units.

5. Conclusion

In this paper we have considered the modified Fitzhugh – Nagumo neuron model with discontinuous resetting and considering the recovery variable as the sigmoid function of voltage. This new modified model is now extended by considering electromagnetic flux coupling. In the numeric analysis of the FNML nodes we could show that the model has three fixed points for $I_{ext}=0$ and one fixed point for $I_{ext}=0.008$. By using the nullclines we have shown that for a larger value of I_{ext} there is only one fixed point for the FNML model. When the flux coupling is not considered and keeping $I_{ext}=0.004$, the FNML model shows periodic spiking but when flux coupling is introduced, we could note irregular bursts and spiking in the FNML model.

To understand the network behaviour and wave re-entry a $N \times N$ lattice array of FNML neurons are constructed and a stimuli is applied from the left boundary. Interestingly the stimuli frequency has no effects except the initiation of wave re-entry and spiral waves and even for larger values of the stimuli the network shows different excitability. Hence as many literatures showed, we couldn't suppress the spiral turbulence with periodic stimuli. But when introducing the flux coupling, we can control the excitability of the nodes in the network and thus we could suppress spiral waves in the network. But it should be noted that for the values of flux coefficient which suppresses spiral waves in the network, the nodes are in periodic state. But our interest is suppressing the spiral waves when these nodes are in different excitability (irregular bursting).

Finally, we introduced Gaussian white noise in the network. During application of noise for the entire simulation time, we could show that the noise variance needed to support spiral waves is $\delta = 0.002$. But if it restricted to only 200-time units the variance needed is $\delta = 0.005$. In this case as the application of noise for only a shorter period, the effect of noise on individual nodes is much less compared to the effect of noise when applied for entire simulation time.

Conflict of Interest: The authors declare that they have no conflict of interest.

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