

PREDICTION AND MEASUREMENT OF HEAT TRANSFER IN AIR-COOLED DISC-TYPE ELECTRICAL MACHINES

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Index Terms—Axial flux permanent magnet machine, disc type machine, stator heat transfer, thermal analysis, CFD

Abstract—Accurate thermal analysis of axial flux permanent magnet (AFPM) machines is crucial in predicting maximum power output. Stator convective heat transfer is one of the most important and least investigated heat transfer mechanisms and is the focus of this paper. Experimental measurements were undertaken using a thin-film electrical heating method, providing radially resolved steady state heat transfer data from an experimental rotor-stator system designed as a geometric mockup of a through-flow ventilated AFPM machine. The measurements are compared with computational fluid dynamics (CFD) simulations using both 2D axisymmetric and 3D models. These were found to give a conservative estimate of heat transfer, with inaccuracies near the edge and in the transitional flow regime. Predicted stator heat transfer was found to be relatively insensitive to the choice of turbulence model and the SST model was used for most of the simulations.

I. INTRODUCTION

Disc type electrical machines such as the axial flux permanent magnet (AFPM) machine offer high torque density and high efficiency in a compact package and are of great interest for applications such as electric and hybrid vehicles and portable generator sets [1]. Their electromagnetic design is well understood, but far less research has been undertaken on thermal aspects. This is despite the fact that power density is usually limited by maximum temperature. Since the stator temperature that is achieved depends on the amount of specific internal heat generation and the convection heat transfer from the stator surface, an understanding of the latter is essential in machine design.

Significant research, e.g. [2] has been undertaken on the thermal modelling of radial flux (cylindrical) electrical machines. However, the air flow within a radial flux geometry is completely different to the air flow within a disc type machine, and the convective heat transfer results which are relevant to the former case cannot be applied to the latter. Recent work on the thermal aspects of disc type machines [3]–[5] has tended to focus on one-dimensional lumped parameter (LP) networks to represent both the solid domain and the fluid domain. These are fast to solve and widely used in electrical machine design. However, the LP method is only as good as the input correlations used for thermal contact resistances, convective

heat transfer coefficients and air flow modelling and therefore CFD and finite element analysis (FEA) are being increasingly applied to investigate the thermal design of electrical machines in more detail, combined with experimental validation.

AFPM machines are inherently more three dimensional in nature than radial flux machines and the air flow in the rotor-stator gap sees an expanding cross sectional area from inside to outside, unlike in a radial flux geometry. All authors comment that suitable stator convective heat transfer coefficients could not be found in the literature, and therefore they either apply rotor correlations (which are likely to lead to an over-optimistic assessment of heat transfer), or use CFD simulations to predict heat transfer coefficients. Few studies have applied CFD to AFPM machines. Airoidi et al. [6], [7] undertook CFD modelling of the fluid flow in an AFPM machine, finding good correspondence between CFD and experiment for mass flow rates and temperatures, although a limited experimental data set is presented.

We have recently reported direct experimental measurements of stator heat transfer in a rotor-stator system using a flat rotor and flat stator as an idealized geometry [8], [9]. Heat transfer in the form of non-dimensional Nusselt number was measured at stator radii $0.6 < r/R < 1$, at three gap ratios relevant to disc type electrical machines. Transition from laminar to turbulent flow was observed to begin at rotational Reynolds numbers $Re_\theta > 3e5$. It was also shown that the ingress of ambient air at the periphery of the stator has a significant effect on stator heat transfer.

In this paper, measurements are compared with CFD simulations using both 2D axisymmetric and 3D models. Results have been non-dimensionalised using various dimensionless groups: rotational Reynolds number Re_θ , Nusselt number Nu , gap ratio G and non-dimensional air mass flow rate C_w . These are defined as follows:

$$Re_\theta = \frac{\omega R^2}{\nu} \quad (1)$$

$$Nu = \frac{hR}{k}, \quad \bar{Nu} = \frac{\bar{h}R}{k} \quad (2)$$

$$G = \frac{g}{R} \quad (3)$$

$$C_w = \frac{\dot{m}}{\mu R} \quad (4)$$

where R is the rotor outside radius, ω is the rotor speed, ν is the fluid kinematic viscosity at ambient temperature, h is the convective heat transfer coefficient, k is the fluid conductivity at ambient temperature, g is the gap size between rotor and stator, \dot{m} is the air mass flow rate and μ is the dynamic viscosity. Also, radial position is non-dimensionalised to r/R .

The flow between a rotating and a stationary disk can be divided into: Batchelor flow, where there is a rotating core of fluid between separate laminar or turbulent boundary layers; and Stewartson flow, where the tangential velocity drops away from the rotor surface to nearly zero at the stator surface, with no core rotation. The condition at the periphery determines the flow type. Batchelor flow is observed in enclosed systems where a recirculation of fluid from rotor to stator occurs at the periphery. In systems which are open at the periphery and have radial through-flow, Stewartson flow is seen. There is a smooth transition from one flow type to the other, as discussed by Poncet et al. [10], depending on the amount of radial through-flow. This may vary from one type of AFPM machine to another.

Stator heat transfer has been investigated experimentally by Owen et al. [11], Bunker et al. [12] and Yuan et al. [13], as well as in CFD by Iacovides and Chew [14]. However to date only a very limited range of parameters have been investigated. Generalised correlations that might be applied in the design of AFPM machines are not available in the literature.

II. EXPERIMENTAL RIG

Given the wide variation in available geometries of AFPM machines, it was decided in the present work to focus mainly on a simplified through-flow ventilated rotor-stator system with a flat rotor and a flat stator. Real electrical machines are more complex than this, but studying this simpler system allows comparison with historical work on rotor-stator systems and provides a basic understanding of stator convective heat transfer, as well as tending to give a conservative estimate of stator heat transfer. A brief exploration of the effect of rotor protrusions (such as permanent magnets) on the air flow has also been undertaken.

An experimental rig was constructed to measure stator heat transfer, consisting of a 471 mm diameter rotor driven by a servo motor, and a heated stator surface allowing direct measurement of stator heat transfer, with 14 channels of spatial resolution and a measurement accuracy of about $\pm 8\%$. The measurement technique used a thin-film electrical heating method and is fully described in [8], [9]. Two rotor configurations were tested: a flat rotor (R1), and a rotor with 16 protrusions designed to mimic the magnets on an AFPM machine (R2), shown in figure 1. The protrusions act like crude fan blades. The gap ratio G was adjustable from 0.01 to 0.09 by insertion of accurately machined spacers. Air was pumped by the movement of the rotor from an inlet at the stator centre

to four exits at the edge of the rotor-stator system. The air mass flow rate was measured using a calibrated bellmouth entry.



Fig. 1. Photograph of R2, aluminium rotor with protrusions

III. CFD MODEL

Computational fluid mechanics is concerned with the numerical solution of the equations governing fluid flow. The commercial software Ansys CFX was used in the present study. Rotor-stator flows exhibit a number of phenomena which are challenging for CFD, such as rotation, confinement, heat transfer, separation, transition and turbulence.

It has been found [15] that 2D axisymmetric CFD simulations are able to capture the important characteristics of rotor-stator flow. Since 2D models are substantially faster to solve, but give useful results, their use in the present work allowed a number of flow speeds and gap sizes to be investigated. Subsequent 3D modelling was then informed by the 2D results but only one gap size was investigated. In order to predict the mass flow rate correctly, modelling of the air inlet geometry could not be neglected, and this was only possible using the 2D model. Because the inlet geometry was not modelled in 3D, the inlet total pressure boundary condition for the 3D model was derived from the 2D models.

Table I gives a brief summary of the various CFD simulations that were undertaken.

TABLE I
CFD SCENARIOS

	2D simulations	3D simulations
Gap ratio	$G = 0.0106, 0.0212$	$G = 0.0106$
Outer edge	open	open, partially blocked
Turbulence	laminar, turbulent	laminar, transitional, turbulent
Rotor	R1 only	R1 and R2

The Shear Stress Transport (SST) turbulence model was chosen for modelling turbulent flow. This is an improved two-equation model available in CFX, developed by Menter [16]. It is a combination of the $k-\omega$ and $k-\epsilon$ models. Near a wall, two-equation turbulence models need to be modified to account for molecular viscosity μ . This is achieved either by using a wall function, which does not require a fine mesh by the

wall, or by using a fine mesh and adding additional terms in the near wall region and integrating directly through the viscous sub-layer. The CFX software includes algorithms that automatically select the latter formulation near the wall if the grid is fine enough. This requires $y^+ \approx 1$ (y^+ is the non-dimensional distance from the wall to the first grid point, it depends on the flow solution). In the present work, modelling was undertaken wherever possible with $y^+ < 1$ to avoid the use of wall functions, although in the case of rotor R2 simulations, this was not possible.

Whether the flow is laminar or turbulent has a large influence on the wall heat transfer and therefore assumptions must be made about transition. In the case of the free rotor, transition is known to begin at $1.8e5 < Re_\theta < 2.1e5$ and completely turbulent flow is found for $Re_\theta > 3e5$ [17]. It was decided initially to assume the flow was completely laminar for simulations below 600 rpm and fully turbulent for speeds of 1800 rpm and above. Between these speeds, both fully turbulent SST modelling and transition modelling were attempted using the Langtry and Menter γ - θ transition model [18]. Transition modelling for the 3D model of R1 was attempted using the γ - θ model with default settings.

In all simulations it was assumed that air was an ideal gas with constant viscosity, thermal conductivity and specific heat capacity. At the temperatures and speeds of interest these properties do not vary considerably and do not have a large affect on the parameters of interest.

Figure 2 shows the simulated 2D fluid domain, superimposed on a drawing of the experimental rig. The air inlet geometry was modelled excluding the bellmouth itself (which is expected to have low losses).

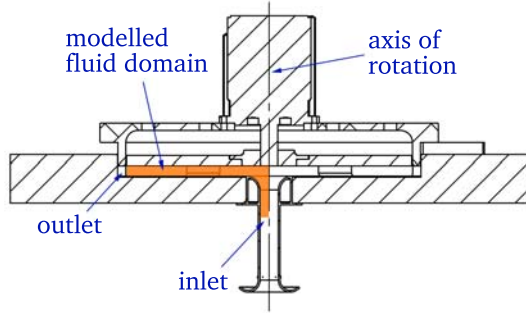


Fig. 2. Fluid domain used for 2D CFD simulation (shaded), relative to rig

One quarter of the entire rotor-stator domain was modelled in 3D for rotor R1, since there are four air exits in the experimental rig. The inlet geometry was not modelled, instead the domain begins at a radius of 38.1 mm; this radius was chosen since it is the point where the curved inlet from the bellmouth and pipe comes to an end. Figure 3 shows the 3D modelled fluid domain in the context of the experimental rig.

In addition to the simplified geometry with a flat rotor (R1), a brief CFD study was also carried out on a rotor with

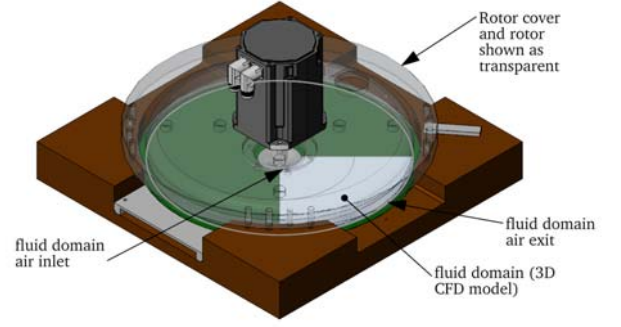


Fig. 3. 3D Fluid domain geometry in context

protrusions (R2), mimicking the permanent magnets of an AFPM machine with gap ratio 0.0106.

CFX-mesh was used for the meshing of the 2D geometries. There were approximately 40 elements across the gap in the z -direction and approximately 100,000 elements in total. Meshing for the 3D geometry with rotor R1 was accomplished using Ansys ICEM to produce a fan-shaped regular hexahedral mesh with inflated layers on the rotor and stator with approximately 500,000 elements. Meshing for R2 was accomplished using Ansys v12 meshing software to produce an irregular mesh with inflated boundary layers with approximately 425,000 elements. In all R1 simulations, $y^+ < 1$ was achieved. In R2 simulations, the largest value of y^+ was approximately 14. Further mesh refinement for R2 in an attempt to achieve $y^+ \approx 1$ led to failure due to computer memory limitations.

The boundary conditions for rotor R1 (flat) are given in table II. In the case of the 3D simulations, the inlet total pressure was decreased to a negative value to reflect losses in the inlet. In the simulations using rotor R2, the inlet mass flow rate was specified directly from experimental results, instead of specifying inlet total pressure. Additionally, R2 simulations used a rotating reference frame for the fluid domain, with boundary conditions specified in relation to this.

TABLE II
BOUNDARY CONDITIONS USED FOR R1 SIMULATIONS

Boundary	Condition
Inlet	$p_{total} = 0 \text{ Pa}$, $T = 293 \text{ K}$
Outlet	$p_{static} = 0 \text{ Pa}$
Rotor	No-slip rotating wall
Stator	No-slip stationary adiabatic wall
Heater	No-slip stationary isothermal wall (333 K)

Grid independence tests were undertaken at $Re_\theta = 9.6e5$ to investigate the effect of grid size on $Nu(r)$, and grid independence at both gap sizes was achieved.

Solver controls were specified as (1) a convergence criterion and (2) a domain balance criterion. The convergence criterion was that the maximum residual should be less than $1e-4$. A domain imbalance (of momentum and enthalpy) of less than 1% was also specified as a requirement. In general, 2D and 3D

simulations converged well to these conditions in both laminar and turbulent cases, with the maximum residuals usually less than $1e-5$ and sometimes approaching $1e-6$.

IV. EXPERIMENTAL RESULTS

Figure 4 shows measured average stator heat transfer versus rotational Reynolds number for various gap sizes using R1 (flat rotor). Higher speed tests were conducted at two gap sizes $G = 0.0106, 0.0212$ as shown.

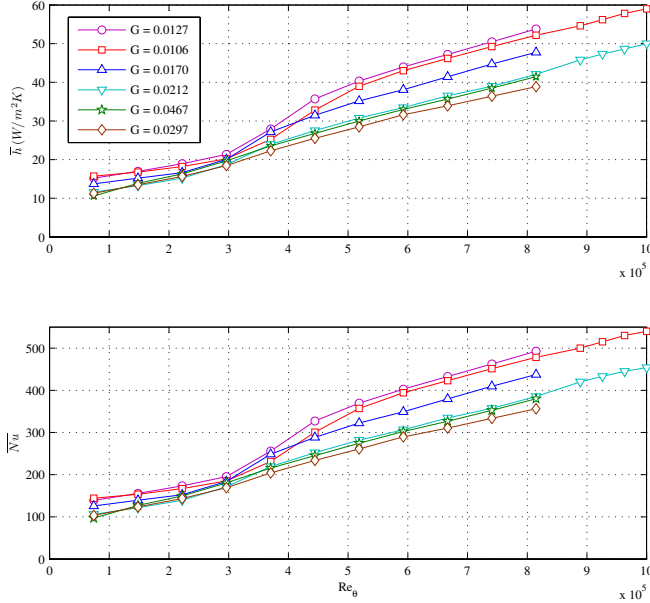


Fig. 4. Average heat transfer coefficients and Nusselt numbers, rotor R1

The average heat transfer results can be correlated in the turbulent regime according to:

$$\overline{Nu}_{turb} = A Re_{\theta}^B \quad (5)$$

The constants A and B are given in table III; these are valid in the range $Re_{\theta} \geq 5.19e5$ for the data shown. All values of B are similar, $B = 0.673 \pm 0.028$. Values of A differ according to gap size G .

TABLE III
AVERAGE HEAT TRANSFER TURBULENT CORRELATIONS

G	A	B
0.0106	0.0790	0.640
0.0127	0.0888	0.633
0.0170	0.0406	0.682
0.0212	0.0315	0.691
0.0297	0.0347	0.679
0.0467	0.0234	0.712

In the laminar regime ($Re_{\theta} \leq 3e5$), the correlation is expected to have the same functional form i.e. $\overline{Nu}_{lam} = A Re_{\theta}^B$. However, there were a limited number of measured data points in this regime (only four per gap ratio) and additionally the

situation is complicated by the non-adiabatic rotor condition (discussed below). Nonetheless the results are approximately correlated according to table IV.

TABLE IV
AVERAGE HEAT TRANSFER LAMINAR CORRELATIONS

G	A	B
0.0106–0.0127	14.02	0.204
0.0170	7.249	0.252
0.0212–0.0297	2.359	0.336
0.0467	0.739	0.434

V. COMPARISON AND DISCUSSION

A. Flat rotor (R1)

CFD simulations using the R1 geometry showed a Stewartson flow profile in the laminar regime, but a Batchelor profile with core rotation in the fully turbulent regime. At low speeds an inbound flow recirculation at the stator was observed in all simulated configurations, but at higher speeds this is only prevalent at $G > 0.0212$. The simulated temperature profiles in general show the fluid temperature increasing with radius as the fluid is pumped across the heated region. The turbulent thermal boundary layers are much thinner than the laminar layers. The influence of ambient temperature air at the periphery can be seen at both gap ratios, particularly near the edge where there is a drop in fluid temperature near the stator. This is much more prevalent at the larger gap ratio, in both laminar and turbulent results. Some features of the velocity profiles are illustrated in figure 5. In general, there is a distinct difference between the laminar and turbulent regimes, but the non-dimensionalised turbulent profiles are all quite similar across different speeds.

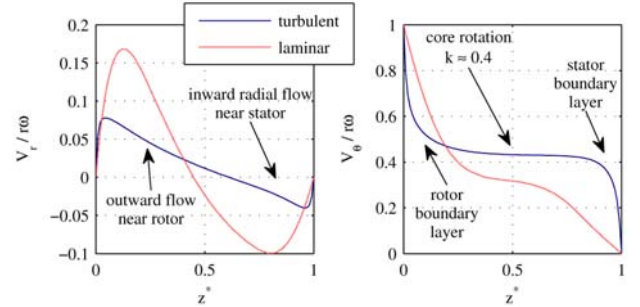


Fig. 5. Velocity profiles, $r/R = 0.99$, $G = 0.0212$

Radially resolved heat transfer simulations and measurements for R1 exhibited certain common features for both gap sizes in the 2D and 3D cases. Firstly, Nusselt number increases with increasing Re_{θ} . At the outer radii the Nusselt number increases due to ingress of ambient temperature air. Average measured heat transfer decreases with increased gap size, although there is a small increase from $G = 0.0106$ to $G = 0.0127$ due to the slightly greater ingress of ambient temperature air at the periphery. Transition from laminar to turbulent flow is

observed at $Re_\theta > 3e5$, but this is only pronounced at gap ratios $G < 0.02$ as shown in figure 4.

The correspondence between radially resolved heat transfer measurements vs. CFD is good at low speeds (laminar) and $G = 0.0212$. At gap ratio $G = 0.0106$ low speed correspondence is less good and most of the experimental heat transfer results are higher than the CFD predictions by approximately 50%. This is probably because the CFD boundary condition assumption of an adiabatic rotor becomes invalid at low rotational speeds and small gap sizes, where in experiments there is significant heat transfer into the aluminium rotor resulting in higher measured heat transfer.

It was found that the fully turbulent regime CFD simulations tended to always over-predict the air mass flow rate through the system compared with experimental measurements. This is probably because the total pressure loss from ambient conditions through the bellmouth and entry into the rotor-stator gap in the experimental rig is higher than predicted using CFD. As a result of this, a number of fully turbulent CFD simulations were re-run using the experimentally measured mass flow rates as inlet boundary conditions so that heat transfer could be compared in a like-for-like manner. Figure 6 shows the resulting comparison. As can be seen, the simulated results are now further from the measurements. The reason for the discrepancy is due to edge effects as will now be discussed.

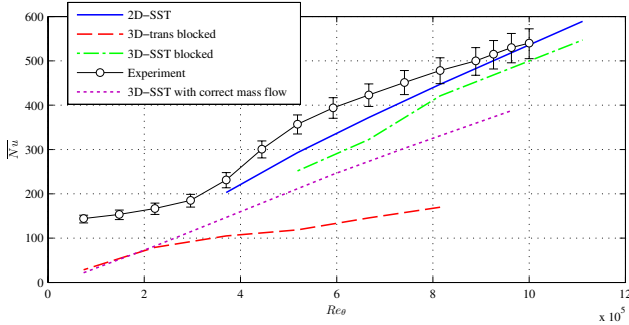


Fig. 6. $G = 0.0106$ comparison CFD and experiment

Figure 7 shows a comparison of the radially resolved predictions at $G = 0.0106$ and $Re_\theta = 9.6e5$ with the experimentally determined inlet mass flow, using the baseline Reynolds stress model (BSLRSM), SST model with and without curvature correction (SST curv. corr.) and $k-\epsilon$ model, in both 2D and 3D (blocked periphery) cases. As can be seen, choice of turbulence model does not have a significant influence on heat transfer predictions. In the region $0.6 < r/R < 0.85$ the match between CFD and experiment is quite close. In the region $0.85 < r/R < 1$ all results deviate substantially from the experimental measurements and this is the cause of the deviation seen in the average results. It is generally accepted that modelling of edge effects is difficult; this deviation is probably caused by the differences in geometry at the boundary in the CFD model compared with the experimental rig.

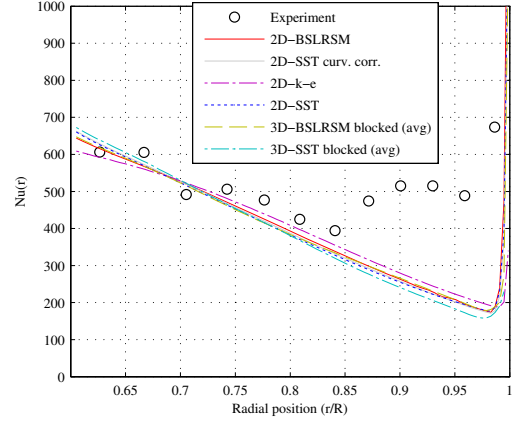


Fig. 7. $G = 0.0106$ comparison CFD & experiment at $Re_\theta = 9.6e5$

The radially resolved comparison results for $G = 0.0212$ at this speed showed a closer correspondence to experiment than the smaller gap ratio particularly in the region $0.85 < r/R < 1$. In the transitional regime, the correspondence between CFD and experiment is not as good as in the fully turbulent regime. Transition modelling does not conclusively provide a better fit.

B. Rotor with protrusions (R2)

The CFD simulated non-dimensional velocity and temperature profiles in the gap for rotor R2 were found to be similar in some respects to the profiles obtained for R1. For example, the turbulent tangential velocity profiles exhibited core rotation in the gap, but the laminar profiles showed no core rotation. However, the radial velocity profiles showed a very different flow pattern to R1. It was found that the main route for radial outflow of fluid was in the slot between protrusions and only a small amount of fluid flows out near the rotor underneath the protrusions; this agrees well with the simulation results of Airoldi et al. [7]. The streamlines, figure 8, show that fluid is entrained into the channel between magnets/ protrusions and flung outwards, like a crude radial fan.

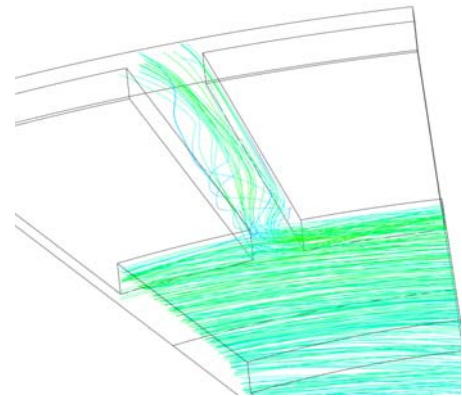


Fig. 8. Streamlines of rotor R2 in the rotating reference frame

The experimentally measured results for R2 show that the gap

ratio seems to have much less effect on the heat transfer than for R1. Additionally, the clear regime change from laminar to turbulent flow at $Re_\theta > 3e5$ that occurs with R1 cannot be seen for R2; transition probably occurs at a much lower rotational speed.

A comparison between measured results for R1 and R2 shows that at a comparable speed, average heat transfer with R2 is about 20–30% higher than R1. This result is similar to the CFD comparison but slightly less marked, figure 9.

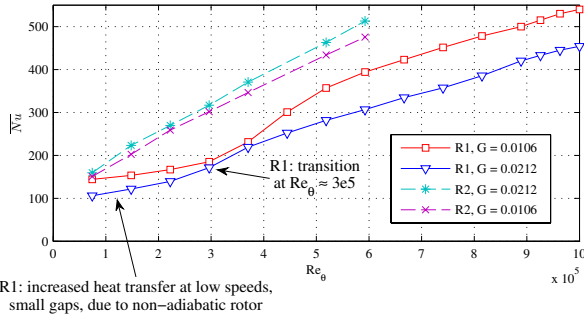


Fig. 9. Comparison of measured average heat transfer R1 vs. R2

Comparison of radially resolved simulated and measured heat transfer for rotor R2 show significant differences at all radial locations. The most likely cause is that the assumption of an open periphery in the case of R2 is completely invalid; in the experimental rig the fluid in fact pulses through the radial slots in R2 as the rotor moves past consecutive open and closed sections of the periphery. This could be heard during testing as an audible hum at sixteen times the rotor frequency. In order to resolve this more accurately in CFD, a transient simulation with a moving mesh would be required, since there is interference between the slots in R2 and the open and blocked areas of the periphery in much the same way as the nozzle guide vanes and rotor blades interact in a turbine. This was outside the scope of the present work but is a worthwhile area of future work.

VI. CONCLUSIONS

This work has compared CFD simulations with experimental measurements of stator heat transfer in a rotor-stator disc system, with two rotor types. It was found in general that stator heat transfer is always lower than rotor heat transfer. It was also found that CFD modelling is relatively insensitive to the choice of turbulence model for this problem, but accuracy in modelling boundary conditions is very important. There is some comfort to be gained in the result that the CFD is generally conservative. Investigation of the effects of outlet geometry and magnet geometries in the form of rotor protrusions is an area where further research is needed. Correlations have been given for average stator heat transfer in a range of conditions; these should be useful to AFPM

machine designers in providing conservative estimates of stator heat transfer.

VII. ACKNOWLEDGEMENTS

The authors would like to thank Mr Jim Ford and Mr Gary Austin at City University for their assistance with construction of the experimental rig.

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