# Modelling Alternative Energy in the Electricity Market




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To my parents and girlfriend, for their infinite support.

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# Abstract

In this thesis, we try to analyse the consequences to the electricity market when we try to introduce wind farms. We start by following [1], considering a large number of fuel-fired power plants bidding according to a Ornstein-Uhlenbeck process and building a bid-stack curve from these. To introduce wind power generation, we model the wind speed at each location by a similar process and extract the total power generated by wind farms from it. We then consider the implications this power has on the bid-stack curve, and we try to anticipate how the market might evolve and adapt.

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# Chapter 1

# Introduction

In 2010, the European Union set the goal of achieving "a 20% share of energy from renewable sources by 2020" [2]. In order to achieve such a goal, certainly a big part of this share will have to come from electricity consumption. In that year, $19,9\%$ of the electricity consumed in the whole EU was already produced from renewable energies. However, different regions were performing very differently, with countries such as Portugal with at least 50% of electricity consumed having its origins in a renewable source, while the UK was still below 10% [3].

Most of this effort in electricity production are expected to come from the installation of wind farms, since in the previous decade "the quantity of electricity generated from wind turbines increased almost seven-fold", while "the relative shares of wind turbines in the total quantity of electricity generated from renewable energy sources rose to 21.8%" [3].

In this thesis we try to understand the impact of introducing wind power generation into the open market, and we make some guesses as to what direction we think the market will take. Chapter 2 is where all the theoretical work is done, while in Chapter 3 we bring some plots to try to visually verify what we claimed early on.

In Section 2.1 we follow the work done in [1] to construct a bid-stack curve from a large number of fuel-fired power plants that bid following a SDE. After this the electricity price is calculated by matching the curve to a demand process. In Section 2.2 we introduce a large number of wind farms in the market, and after figuring out how they transform wind speed into electricity, we compute the total amount of power produced this way and feed it into the early model. We discuss the possible existence of wind derivatives in Section 2.3, specifically the benefits that exchange traded derivatives could bring into the market. Finally, in Section 2.4 we briefly highlight possible improvements.

As for Chapter 3, in Section 3.1 we plot possible sample paths in order to visualize what is actually happening, while in Section 3.2 we value a wind farm according to our model and try to understand what happens when we change some parameters.

In the Appendix we have some computations that are thought to be better placed outside of the main text as well as the MatLab code used to generate all the plots present through this thesis.

# Chapter 2

# Wind Power Generation

## 2.1 Fuel Only Model

In this section we follow the work done in [1]. We want to create a bid-stack curve from the various bids made by fuel-fired power plants.

Suppose there are $N$ producers in the market, and each producer $i$ places a bid offering to produce $a_i$ of electric power for a unit price of $p_i$. This price depends on the evolution of the underlying fuel price (according to [4] this is typically gas, as usually gas-fired Power Plants are the ones that sets the price), but also on the specific conditions of the producer. Hence it is assumed $p_i$ follows the following SDE:

$$dp_i(t) = \kappa(\theta - p_i(t))\, dt + \sigma(\rho\, dW_t^f + \sqrt{1-\rho^2}\, dW_t^i)$$

where $W^f$ is a Brownian Motion driving the fuel price, $W^i$ is a Brownian Motion describing the individual characteristics of producer $i$, independent of $W^f$ and $W^j$ for $j \neq i$, $\mu \in \mathbb{R}$, $\kappa > 0$ and $\sigma > 0$ are constants representing respectively the mean, mean-reversion rate and volatility of the bids and $\rho \in (0, 1)$ is a constant representing the correlation between the bids and the fuel price.

One of the features of this model is that, no matter how big $\mu$ and $\kappa$ are, there is always a non-zero probability of placing a negative bet. One possible explanation for this is that either old gas-fired or nuclear power plants may incur in significant costs to shut down, and so they are probably willing to pay in order to avoid those costs.

We now build an empirical measure with all the bids and try to understand what happens as our homogeneous market gets larger in number of producers. Let $A = \sum_{i=1}^{N} a_i$ be the total capacity of all producers, $\bar{a} = \frac{A}{N}$ the average production capacity and denote $\tilde{a}_i = \frac{a_i}{\bar{a}}$. Now define:

$$\nu_{N,t} = \frac{1}{A}\sum_{i=1}^{N} a_i\ \delta_{p_i(t)} = \frac{1}{N}\sum_{i=1}^{N} \tilde{a}_i\ \delta_{p_i(t)}\ .$$

Our goal is to keep $A$ fixed while increasing the number of producers up to infinity, treating each bid as an infinitesimal bid. Hence we want to study the limit measure $\nu_t$, defined by:

$$\nu_t = \lim_{N\to+\infty} \nu_{N,t}\ .$$

To do this, we are going to be using test function $\phi \in C_\infty$, the infinitely differentiable functions with compact support on $\mathbb{R}$. Define:

$$< \phi, \nu_{N,t} > = \frac{1}{N}\ \sum_{i=1}^{N} \tilde{a}_i\ \phi(p_i(t)) \tag{2.1}$$

and

$$< \phi, \nu_t > = \int_{-\infty}^{+\infty} \phi(x)\ \nu_t(dx)\ . \tag{2.2}$$

We will now proceed the study of (2.1) and then, by taking limits, extract information about (2.2). So apply Itô Lemma to (2.1):

$$\begin{aligned}
d < \phi, \nu_{N,t} > =& \frac{1}{N}\ \sum_{i=1}^{N} \tilde{a}_i(\phi'(p_i)\ dp_i + \frac{1}{2}\ \phi''(p_i)\ d < p_i >) \\
=& \frac{1}{N}\ \sum_{i=1}^{N} \tilde{a}_i(\kappa(\theta - p_i)\ \phi'(p_i) + \frac{1}{2}\ \sigma^2\ \phi''(p_i))\ dt \\
&+ \frac{1}{N}\ \sum_{i=1}^{N} \tilde{a}_i\ \sigma\ \rho\ \phi'(p_i)\ dW_t^f + \frac{1}{N}\ \sum_{i=1}^{N} \tilde{a}_i\ \sigma\sqrt{1-\rho^2}\ \phi'(p_i)\ dW_t^i \\
=& < \kappa(\theta - \bullet)\ \phi' + \frac{1}{2}\ \sigma^2\ \phi'', \nu_{N,t} > dt + < \sigma\ \rho\ \phi', \nu_{N,t} > dW_t^f + R_N(t)
\end{aligned} \tag{2.3}$$

where:

$$R_N(t) = \frac{1}{N}\ \sum_{i=1}^{N} \tilde{a}_i\ \sigma\sqrt{1-\rho^2}\ \phi'(p_i)\ dW_t^i\ .$$

$R_N$ is a martingale since $\phi \in C_\infty$, and its quadratic variation is:

$$d < R_N >_t = \frac{1}{N^2} \sum_{i=1}^{N} \tilde{a}_i^2 \, \sigma^2 \, (1-\rho^2) \, \phi'(p_i)^2 \, dt \, .$$

Now if we assume the market is homogeneous, in opposition to a monopoly or an oligopoly, then $\tilde{a}_i$, which represents how much bigger than average producer $i$ is, should be bounded. If that is the case, then since $\phi \in C_\infty$, $\phi'$ is also in $C_\infty$, hence it is bounded, and so:

$$< R_N >_t \leq \frac{C}{N} \; \to \; 0, \qquad \text{as} \quad N \to +\infty$$

which implies:

$$R_N(t) \xrightarrow{\text{a.s.}} 0, \qquad \text{as} \quad N \to +\infty \, .$$

Therefore, when we take the limit of (2.3) as $N \to +\infty$, we have the following:

$$d < \phi, \nu_t > = < \kappa(\theta - \bullet)\phi' + \frac{1}{2}\sigma^2\phi'', \nu_t > \; dt + < \sigma \, \rho \, \phi', \nu_t > \; dW_t^f$$

writing in integral form:

$$< \phi, \nu_t > = < \phi, \nu_0 > + \int_0^t < \kappa(\theta - \bullet)\phi' + \frac{1}{2}\sigma^2\phi'', \nu_s > \; ds + \int_0^t < \sigma \, \rho \, \phi', \nu_s > \; dW_s^f \, . \tag{2.4}$$

If we make the further assumption that $\nu_t$ is an absolutely continuous measure, i.e., there exists $V_t$ such that $\nu_t(dx) = V_t(x) \, dx$, then (2.4) translates into:

$$\begin{aligned} \int_{-\infty}^{+\infty} \phi(x) V_t(x) dx = & \int_{-\infty}^{+\infty} \phi(x) V_0(x) dx + \int_0^t \int_{-\infty}^{+\infty} \kappa(\theta - x)\phi'(x) V_s(x) dx \, ds \\ & + \int_0^t \int_{-\infty}^{+\infty} \frac{1}{2}\sigma^2 \phi''(x) V_s(x) dx \, ds + \int_0^t \int_{-\infty}^{+\infty} \sigma \, \rho \, \phi'(x) \, V_s(x) dx \, dW_s^f \, . \end{aligned}$$

Using Fubini and integration by parts (keeping in mind that $\phi$, $\phi'$ and $\phi''$ have a compact support), we have:

$$\begin{aligned} \int_{-\infty}^{+\infty} \phi(x) V_t(x) dx = & \int_{-\infty}^{+\infty} \phi(x) V_0(x) dx - \int_{-\infty}^{+\infty} \int_0^t \phi(x) \frac{\partial}{\partial x}(\kappa(\theta - x) V_s(x)) ds \, dx \\ & + \int_{-\infty}^{+\infty} \int_0^t \phi(x) \frac{1}{2}\sigma^2 \frac{\partial^2 V_s(x)}{\partial x^2} ds \, dx - \int_{-\infty}^{+\infty} \int_0^t \phi(x) \sigma\rho \frac{\partial V_s(x)}{\partial x} dW_s^f \, dx \, . \end{aligned}$$

Since this has to be true for all $\phi \in C_\infty$, we can conclude:

$$V_t(x) = V_0(x) + \int_0^t -\frac{\partial}{\partial x}(\kappa(\theta - x)V_s(x)) + \frac{1}{2}\sigma^2\frac{\partial^2 V_s(x)}{\partial x^2}ds - \int_0^t \sigma\rho\frac{\partial V_s(x)}{\partial x}dW_s^f$$

or, in differential form,

$$dV_t(x) = (-\frac{\partial}{\partial x}(\kappa(\theta - x)V_t(x)) + \frac{1}{2}\sigma^2\frac{\partial^2 V_t(x)}{\partial x^2})dt - \sigma\rho\frac{\partial V_t(x)}{\partial x}dW_t^f \ . \tag{2.5}$$

Fortunately, this SDE can be solved explicitly, as shown in the next Lemma:

**Lemma 2.1** *The solution to the SDE* (2.5) *is:*

$$V_t(x) = u(t, x - \sigma\ \rho\ M_t) \tag{2.6}$$

*where*

$$dM_t = -\kappa M_t dt + dW_t^f, \qquad M_0 = 0$$

*and* $u$ *is the solution to the following PDE:*

$$\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}(\kappa(\theta - x)u) + \frac{1}{2}\sigma^2(1 - \rho^2)\frac{\partial^2 u}{\partial x^2}, \qquad u(0, x) = V_0(x) \ . \tag{2.7}$$

*Proof:* Applying Itô formula to (2.6) results into:

$$\begin{aligned} dV_t(x) =& \frac{\partial u}{\partial t}dt - \sigma\rho\frac{\partial u}{\partial x}dM_t + \frac{1}{2}\sigma^2\rho^2\frac{\partial^2 u}{\partial x^2}d < M >_t \\ =& (-\frac{\partial}{\partial x}(\kappa(\theta - x)u) + \frac{1}{2}\sigma^2(1 - \rho^2)\frac{\partial^2 u}{\partial x^2})dt \\ & - \sigma\rho\frac{\partial u}{\partial x}(-\kappa M_t dt + dW_t^f) + \frac{1}{2}\sigma^2\rho^2\frac{\partial^2 u}{\partial x^2}dt \\ =& (-\frac{\partial}{\partial x}(\kappa(\theta - x)u) + \kappa\frac{\partial u}{\partial x}\sigma\rho M_t + \frac{1}{2}\sigma^2\frac{\partial^2 u}{\partial x^2})dt - \sigma\rho\frac{\partial u}{\partial x}dW_t^f \ . \end{aligned} \tag{2.8}$$

At this point we need to be careful. In order to prevent overload of notation, the evaluation point of each function ($u$ and its partial derivatives) was omitted, but they are to be evaluated at $(t, x - \sigma\ \rho\ M_t)$. Formally we have that:

$$\frac{\partial}{\partial x}(\kappa(\theta - x)u) = -\kappa\ u + \kappa(\theta - x)\frac{\partial u}{\partial x}$$

but since this has to be evaluated at $(t, x - \sigma\ \rho\ M_t)$, we actually get:

$$\frac{\partial}{\partial x}(\kappa(\theta - x)u)(t, x - \sigma\ \rho\ M_t) = -\kappa\ u + \kappa(\theta - x + \sigma\ \rho\ M_t)\frac{\partial u}{\partial x}$$

we can now make use of the following equalities:

$$\frac{\partial u(t, x - \sigma\ \rho\ M_t)}{\partial x} = \frac{\partial V_t(x)}{\partial x}$$
$$\frac{\partial^2 u(t, x - \sigma\ \rho\ M_t)}{\partial x^2} = \frac{\partial^2 V_t(x)}{\partial x^2}$$

and substituting everything into (2.8) yields:

$$\begin{aligned}
dV_t(x) =&(\kappa\ V_t(x) - \kappa(\theta - x + \sigma\rho M_t)\frac{\partial V_t(x)}{\partial x} + \kappa\frac{\partial V_t(x)}{\partial x}\sigma\rho M_t)dt \\
&+ \frac{1}{2}\sigma^2\frac{\partial^2 V_t(x)}{\partial x^2}dt - \sigma\rho\frac{\partial V_t(x)}{\partial x}dW_t^f \\
=&(\kappa\ V_t(x) - \kappa(\theta - x)\frac{\partial V_t(x)}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2 V_t(x)}{\partial x^2})dt - \sigma\rho\frac{\partial V_t(x)}{\partial x}dW_t^f \\
=&(-\frac{\partial}{\partial x}(\kappa(\theta - x)V_t(x)) + \frac{1}{2}\sigma^2\frac{\partial^2 V_t(x)}{\partial x^2})dt - \sigma\rho\frac{\partial V_t(x)}{\partial x}dW_t^f
\end{aligned}$$

which is, indeed, the SDE in (2.5). ■

Furthermore, according to [5], (2.4) is a Zakai Equation with correlation noise, where $W^f$ is the observable noise and $p_i$ the signal. Since both drift and volatility of $p_i$ satisfy the linear growth condition, (2.4) has a unique solution, (2.6).

Our progress is now dependent on us finding the solution to (2.7). Luckily this PDE is the forward Kolmogorov equation for the Ornstein-Uhlenbeck (OU) process, and so its solution is well known. The solution is given by:

$$u(t, x) = \int_{-\infty}^{+\infty} V_0(y)k(t, y, x)dy \tag{2.9}$$

where

$$k(t, y, x) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2(1 - e^{-2\kappa t})}}\exp(-\frac{(x - \theta - (y - \theta)e^{-\kappa t})^2}{2\hat{\sigma}^2(1 - e^{-2\kappa t})})$$

where $\hat{\sigma}^2 = \frac{\sigma^2(1-\rho^2)}{2\kappa}$. We know that the OU process has a stationary distribution, which is $N(\theta, \hat{\sigma}^2)$. If we assume the bids are stationary at time 0, i.e. if we assume that $V_0$ is the density of $N(\theta, \hat{\sigma}^2)$:

$$V_0(y) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp(-\frac{(y-\theta)^2}{2\hat{\sigma}^2})$$

then, substituting this into (2.9):

$$\begin{aligned} u(t,x) &= \int_{-\infty}^{+\infty} V_0(y)k(t,y,x)dy \\ &= \int_{-\infty}^{+\infty} \frac{1}{2\pi\hat{\sigma}^2\sqrt{1-e^{-2\kappa t}}} \exp(-\frac{(y-\theta)^2}{2\hat{\sigma}^2} - \frac{(x-\theta-(y-\theta)e^{-\kappa t})^2}{2\hat{\sigma}^2(1-e^{-2\kappa t})})dy \\ &= \int_{-\infty}^{+\infty} \frac{1}{2\pi\hat{\sigma}^2\sqrt{1-e^{-2\kappa t}}} \exp(-\frac{(y-\theta)^2+(x-\theta)^2-2(x-\theta)(y-\theta)e^{-\kappa t}}{2\hat{\sigma}^2(1-e^{-2\kappa t})})dy \ . \end{aligned}$$

We can now notice that inside the integral we have the density of a bivariate normal distribution, with mean $\theta$ and variance $\hat{\sigma}^2$ in both variables, with correlation equal to $e^{-\kappa t}$. Hence when we integrate with respect to $y$ we get the marginal distribution of $x$, i.e., $u$ is the density of a $N(\theta, \hat{\sigma}^2)$:

$$u(t,x) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp(-\frac{(x-\theta)^2}{2\hat{\sigma}^2}) \ .$$

This is in fact a very natural answer, as we get the stationary distribution when we start from the stationary distribution. Getting back to Lemma 2.1, we finally have an explicit expression for $V_t$:

$$V_t(x) = u(t, x-\sigma\rho M_t) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp(-\frac{(x-\sigma\rho M_t-\theta)^2}{2\hat{\sigma}^2})$$

which is the density of a $N(\theta+\sigma\rho M_t, \hat{\sigma}^2)$.

Now coming back to $\nu_{N,t}$, if we have $N$ producers bidding into the bid-stack then the electricity we can get for a unit price of $x$ is:

$$\sum_{i=1}^{N} a_i \, \mathbb{1}_{p_i(t)\leq x} = A < \mathbb{1}_{\bullet\leq x}, \nu_{N,t} > \ .$$

Hence, when we extend the number of producers to infinity (while keeping the total capacity fixed) we get that for a unit price of $x$ we can produce:

$$A < \mathbb{1}_{\bullet\leq x}, \nu_t > = A \int_{-\infty}^{x} V_t(y)dy = A \ \Phi(\frac{x-\theta-\sigma\rho M_t}{\hat{\sigma}}) \ . \tag{2.10}$$

The electricity price is the price that will produce the amount of electricity we

need, i.e., the demand $\tilde{D}$, that also needs to be modelled. To avoid dealing with shortages we assume the demand can never exceed the total capacity $A$, and so for our convenience we model the proportion of total power used $D = \frac{\tilde{D}}{A}$. Our choice is to consider $D$ is ruled by the following SDE:

$$dD_t = \tau(\omega - D_t)dt + \xi\sqrt{D(1-D)}dW_t^D$$

where $W^D$ is a Brownian Motion that drives the demand independent of $W^f$ and $\omega \in (0,1)$, $\tau > 0$ and $\xi > 0$ are constants representing respectively the mean, mean-reversion rate and volatility of the demand. This is a reasonable choice because it forces $D$ to stay in $(0,1)$, as expected.

Now that we have the bid-stack and the demand process, we have all the ingredients necessary to get the electricity price. Hence we have the following Lemma:

**Lemma 2.2** *The electricity price $\pi_t$ satisfies:*

$$\pi_t = \theta + \sigma\rho M_t + \hat{\sigma}\ \Phi^{-1}(D_t)\ .$$

*Proof:* The electricity price comes from matching the bid-stack with the demand process. From (2.10) we know how much we can produce for a certain price, therefore the electricity price comes from solving:

$$\tilde{D}_t = A\ \Phi(\frac{\pi_t - \theta - \sigma\rho M_t}{\hat{\sigma}}) \quad \Leftrightarrow \quad D_t = \Phi(\frac{\pi_t - \theta - \sigma\rho M_t}{\hat{\sigma}})\ .$$

Therefore Lemma 2.2 comes simply from the inversion of the normal CDF. ■

## 2.2 Introducing Wind Power Generation

We now want to expand the work from last section by trying to understand what happens when we introduce wind farms in the market.

Firstly, we need a model for wind speed. According to [6], in [7] "ARMA(5,4) is employed to calculate the 1-month average wind speed and the 1 h average wind speed". One would think using a continuous analogous of ARMA(5,4) would be a good choice for our model, but that is unfortunately not true. If we try to get a continuous version of ARMA(5,4) (following [8] for example), we realize a nice, simple SDE is not easy to find. The problem may be that ARMA(n,m) models with $n > 1$ are not Markovian (of course maybe this can be overcome, since when we let the time step go to 0 all the previous points collapse on the current point). We will then stick to the Ornstein-Uhlenbeck process, which is the continuous analogue of AR(1). Let the wind speed $v$ be ruled by the following SDE:

$$dv_t = \eta(\mu - v_t)dt + \zeta dW_t$$

where $W$ is a Brownian Motion and $\mu \in \mathbb{R}$, $\eta > 0$ and $\zeta > 0$ are constants representing respectively the mean, mean-reversion rate and volatility of the wind speed.

Secondly we need to roughly understand how wind farms transform wind speed into electricity. According to [9], there is a minimum threshold of wind speed needed in order to produce electricity. After that lower boundary has been reached, a fairly good approximation is that the electricity produced grows with $v^3$ until it reaches an upper boundary, producing a constant amount from there on (there is also a third threshold, that if hit shuts down the wind farm for security reasons, which we will ignore in order to simplify our framework). We set 0 as our lower boundary and 1 as our upper boundary for our convenience (shifting and rescaling the wind speed variable as necessary). Although the OU process is not a perfect fit for the wind speed (OU can be arbitrarily low and wind speed cannot be negative), values below a certain threshold can be thought as "no wind" and giving a indication of how likely it is to get windy again.

Following this reasoning, the electricity $X_t$ generated by a wind farm at time $t$ is:

$$X_t = v_t^3 \, \mathbb{1}_{0 \leq v_t \leq 1} + \mathbb{1}_{v_t > 1} \, .$$

We want to follow the same reasoning as in last section, and instead of looking at a single wind farm, we suppose there are plenty of independently owned wind farms,

and see what effects this has on the market.

If we make the rather raw assumption that the wind speed in different locations is affected by a universal process (global noise), representing the big movements in the atmosphere, and an individual process (local noise) representing the local irregularities in each location, then we can model the wind speed $v_i$ at location $i$ by:

$$dv_i(t) = \eta(\mu - v_i(t))dt + \zeta(\varrho\, dW_t^{\text{wind}} + \sqrt{1-\varrho^2}\, dW_t^i)$$

where $W^{\text{wind}}$ is a Brownian Motion driving the big movements in the athmosphere, $W^i$ is a Brownian Motion describing the unique characteristics of region $i$, independent from $W^{\text{wind}}$ and $W^j$ for $j \neq i$ and $\varrho \in (0,1)$ is a constant representing the correlation between the various locations.

Now suppose we have $N$ wind farms, and that each wind farm can produce (at full speed) a certain amount of power, say $b_i$ for wind farm $i$. Then our total power output from wind farm generation would be:

$$\sum_{i=1}^{N} b_i(v_i^3(t)\ \mathbb{1}_{0\leq v_i(t)\leq 1} + \mathbb{1}_{v_i(t)>1})\ .$$

Following what was done in the last section, let $B = \sum_{i=i}^{N} b_i$ be the maximum amount of power that can be produced by wind generation, $\bar{b} = \frac{B}{N}$ be the average maximum capacity and denote $\tilde{b}_i = \frac{b_i}{\bar{b}}$. We can then define the following measure:

$$\nu_{N,t} = \frac{1}{N}\sum_{i=1}^{N} \tilde{b}_i\ \delta_{v_i(t)}\ .$$

We proceed this way because we can express the proportion of total capacity $B$ we are currently producing $X$ as:

$$X_{N,t} = \int_{-\infty}^{+\infty} (x^3\ \mathbb{1}_{0\leq x\leq 1} + \mathbb{1}_{x>1})\ \nu_{N,t}(dx)\ . \tag{2.11}$$

Hence to understand what happens as $N \to +\infty$ we need to understand $\nu_t = \lim_{N\to+\infty} \nu_{N,t}$. If $\phi \in C_\infty$, then using the definition (2.1) (substituting $p$ for $v$ and $a$ by $b$) and (2.2) from last section. By applying Itô Lemma to the first:

$$
\begin{aligned}
d<\phi,\nu_{N,t}> =& \frac{1}{N}\sum_{i=1}^{N}\tilde{b}_i(\phi'(v_i)\,dv_i+\frac{1}{2}\,\phi''(v_i)\,d<v_i>)\\
=& <\eta(\mu-\bullet)\,\phi'+\frac{1}{2}\,\zeta^2\,\phi'',\nu_{N,t}>dt+<\zeta\,\varrho\,\phi',\nu_{N,t}>dW_t^f+R_N(t)
\end{aligned}
\tag{2.12}
$$

where:

$$
R_N(t)=\frac{1}{N}\sum_{i=1}^{N}\tilde{b}_i\,\zeta\sqrt{1-\varrho^2}\,\phi'(v_i)\,dW_t^i\,.
$$

If we now assume $\tilde{b}_i$'s are bounded (in order to prevent any singular wind farm from having a decisive influence in production), we have:

$$
\begin{aligned}
d<R_N(t)>_t \;&=\frac{1}{N^2}\sum_{i=1}^{N}\tilde{b}_i^2\,\zeta^2\,(1-\varrho^2)\,\phi'(v_i)^2\,dt\\
&\leq\frac{C}{N}\,dt\;\to\;0,\qquad\text{as}\quad N\to+\infty\,.
\end{aligned}
$$

Since $R_N$ is a martingale, this implies:

$$
R_N(t)\xrightarrow{\text{a.s.}}0,\qquad\text{as}\quad N\to+\infty\,.
$$

So by taking (2.12) to infinity we get:

$$
d<\phi,\nu_t>=<\eta(\mu-\bullet)\,\phi'+\frac{1}{2}\,\zeta^2\,\phi'',\nu_t>dt+<\zeta\,\varrho\,\phi',\nu_t>dW_t^f\,.
$$

If we assume $\nu_t$ is absolutely continuous, i.e., there exists a function $V_t$ such that $\nu_t(dx)=V_t(x)dx$, then, after integrating by parts and using Fubini like we did in the last section, we get:

$$
V_t(x)=V_0(x)+\int_0^t-\frac{\partial}{\partial x}(\eta(\mu-x)V_s(x))+\frac{1}{2}\zeta^2\frac{\partial^2V_s(x)}{\partial x^2}ds-\int_0^t\zeta\varrho\frac{\partial V_s(x)}{\partial x}dW_s^f\,. \tag{2.13}
$$

This is exactly the same SDE in integral form (only with different constants) as last time so we get a similar Lemma:

**Lemma 2.3** *The solution to the equation* (2.13) *is:*

$$
V_t(x)=w(t,x-\zeta\,\varrho\,N_t)
$$

*where*

$$dN_t = -\eta N_t dt + dW_t^f, \qquad N_0 = 0$$

*and $w$ is the solution to the following PDE:*

$$\frac{\partial w}{\partial t} = -\frac{\partial}{\partial x}(\eta(\mu - x)w) + \frac{1}{2}\zeta^2(1-\varrho^2)\frac{\partial^2 w}{\partial x^2}, \qquad w(0,x) = V_0(x)\ . \tag{2.14}$$

*Proof:* Same as Lemma 2.1. ■

Moreover, we also know that the solution to (2.14) is the forward Kolmogorov Equation of the Ornstein-Uhlenbeck process. If we assume the initial distribution of wind speed is also stationary, i.e., $V_0$ is the density of a $N(\mu, \hat{\zeta}^2)$, where $\hat{\zeta}^2 = \frac{\zeta^2(1-\varrho^2)}{2\eta}$, then $w(t,x) = V_0(x)$ (as shown in the last section) and:

$$V_t(x) = \frac{1}{\sqrt{2\pi\hat{\zeta}^2}} \exp(-\frac{(x - \zeta\varrho N_t - \mu)^2}{2\hat{\zeta}^2})\ .$$

$V_t$ is then the density of a random variable with distribution $N(\mu + \zeta\varrho N_t, \hat{\zeta}^2)$. We are now in position to see what happens to the total amount of energy generated by wind farms as $N \to \infty$. Define $X_t = \lim_{N\to+\infty} X_{N,t}$. Then, from taking the limit of (2.11) we have the following:

$$X_t = \int_{-\infty}^{+\infty} (x^3\ \mathbb{1}_{0\le x\le 1} + \mathbb{1}_{x>1})\ V_t(x)dx = \mathbb{E}[Z_t^3\ \mathbb{1}_{0\le Z_t\le 1}|N_t] + P(Z_t > 1|N_t)$$

where $Z_t|N_t \sim N(\mu + \zeta\varrho N_t, \hat{\zeta}^2)$. Then there exists a closed form expression for $X_t$ in terms of $N_t$, and this can be found in Lemma A.1 in the Appendix.

How does this influence the electricity price? There are at least two ways of looking into this: we can assume wind farms emerge to supress increasing demand, and so high levels of demand can only be fulfilled if the wind speed is favourable; or we can assume wind farms emerge due to political and climatic reasons, with high levels of demand still able to be covered by all the fuel-fired power plants. In this case wind farms are used to reduce the usage of fuel-fired power plants, preventing the usage of the end of the bid-stack, hence providing more stability to prices.

The first option has one problem: since demand is independent of wind speed, high demand during a calm day could lead to severe shortage, as there are not enough fuel-fired power plants to cover it. This is not a desirable market, and one I do not believe we are evolving to. Hence we are going to stick with the second option.

Another question is what the bid price of these wind farms should be. My impression is that nowadays wind farms are subsidized, and so all the power they produce is automatically introduced in the grid. Even in the future, I think it is a good guess to assume wind farm owners will just bid 0, since there is no cost in producing electricity when wind is available. If they bided a positive value, no matter how small, there is always the chance that at some point they could be selling free power for half their bid but are not. On the other hand, it should be fairly easy to stop producing if needed be, so there is no incentive to bid negative values.

We are now going to analyse what happens to the electricity prices. Let $B = \alpha A$, i.e., write the total capacity of wind generation power as a fraction $\alpha > 0$ of the total capacity of gas-fired power plants. Hence the demand-supply equation turns into:

$$D_t = \Phi(\frac{\pi_t - \theta - \sigma\rho M_t}{\hat{\sigma}}) + \alpha X_t \mathbb{1}_{\pi_t \geq 0} \, . \tag{2.15}$$

So we can conclude that when we add wind farms into the system the price is:

**Lemma 2.4** *If the electricity price $\pi_t$ is bigger than 0, then:*

$$\pi_t = \theta + \sigma\rho M_t + \hat{\sigma} \, \Phi^{-1}(D_t - \alpha X_t) \, .$$

*Proof:* Just invert (2.15). ■

Note that in this framework demand has to be extraordinary small in order to achieve negative prices, making the above Lemma useful most of the times.

## 2.3 Derivatives on Wind Speed

With the expansion of wind farms, and their integration into the liberalized market, it is natural to think derivatives on wind speed will start emerging, just like temperature derivatives did.

In a first moment they will probably be over-the-counter (OTC), and hence specific to each location $i$. Since the market is incomplete (wind speed is not traded) the price of the derivative will depend on the market price of risk. The most natural derivative to be considered is a call option on the wind speed. If the market price of risk $\lambda$ is constant, then under the risk neutral measure $\mathbb{Q}$:

$$dv_i(t) = \eta(\mu - \frac{\zeta\,\lambda}{\eta} - v_i(t))dt + \zeta dW_t^{\mathbb{Q}}\,.$$

The OU process has a closed form solution, so we can express $v$ as:

$$v_i(t) = (\mu - \frac{\zeta\,\lambda}{\eta}) + e^{-\eta t}(v_i(0) - \mu + \frac{\zeta\,\lambda}{\eta}) + \zeta\int_0^t e^{-\eta(t-s)}\,dW_s^{\mathbb{Q}}\,.$$

We can now price a call option with strike $K$ and maturity $T$ at time 0 by computing $e^{-rT}\,\mathbb{E}^{\mathbb{Q}}[(v_i(T) - K)^+]$, where $r$ is the constant risk-free rate. Since under $\mathbb{Q}$ $v_i(T) - K \sim N((\mu - \frac{\zeta\,\lambda}{\eta}) + e^{-\eta T}(v_i(0) - \mu + \frac{\zeta\,\lambda}{\eta}) - K, \frac{\zeta^2}{2\eta}(e^{-2\eta T} - 1))$, this is a direct application of Lemma A.2 (that also shows it is possible to retrieve the market price of risk from the price of the option).

As the market expands, it is natural to think these options will get more and more traded, to the point where they will start getting traded in an exchange rather than OTC. At this point it is likely they are not tailored for each location anymore, but more instead are written on some sort of index of various locations' wind speed. From our point of view, we can interpret this index as a generic wind speed only affected by $W^{\text{wind}}$, ruled by the following SDE:

$$dv_t = \eta(\mu - v_t)dt + \zeta dW_t^{\text{wind}}\,.$$

With call options on this index being traded on an exchange for a wide range of maturities and strikes, we can not only better calibrate our model's volatility structure, but we can also extract the market price of risk associated with $W^{\text{wind}}$, effectively making the market of wind speed derivatives complete, since we now can eliminate the risk in $W^{\text{wind}}$ by trading in these call options. This allows us to consider

interesting derivatives that can be fully hedged. Consider the following, that we call a *wind proxy*:

*Given our index of wind speed through various locations $v$, a* wind proxy *with maturity $T$ pays, at time $T$:*

$$\int_0^T e^{r(T-t)} \left(v_t^3 \, \mathbb{1}_{0 \leq v_t \leq 1} + \mathbb{1}_{v_t > 1}\right) \pi_t \, dt \, .$$

A *wind proxy* is mimicking a generic wind farm, with the slight difference we only get paid at maturity (for simplicity). In real life this could be seen as leasing a wind farm for a fixed period of time, much in the same way a tolling agreement represents leasing a power plant, as specified in [4]. Of course this is also a great way for purely financial investors to have access to the market.

Apart from the call options on wind, which we assume are liquidly traded, there are other instruments currently traded we can use to complete our market. $\pi_t$ is influenced by $W^{\text{wind}}$, $W^f$ and $W^D$. We can hedge the first one by trading in wind speed call options, the second by trading in fuel options (or the fuel itself), while the last one can be hedged by trading in electricity derivatives, like forwards. Hence we can hedge all sources of risk and our market is completed, making the price of a *wind proxy* uniquely determined by hedging arguments.

## 2.4 Possible Extensions

### 2.4.1 Seasonality

We have been working within a simple framework, but we can do some adjustments to introduce more complexity. One of them is introducing seasonality. While we have been dealing with constant parameters all along, some of these are expected to change through time. Seasonality usually manifests itself through the means, so we should analyse what happens when we make these parameters time dependent:

- $\omega$, the mean of the demand process $D$, is clearly one of the most influenced by seasonality, since the demand behaves differently at night/day, weekend/weekday, summer/winter .... In our previous work, we can change $\omega$ to $\omega(t)$ without any further consequence. In fact, since $D$ is only used to match supply-demand and there is no actual computation involving its SDE, all parameters could be time-dependent/stochastic, and every result would still be valid.

- $\mu$, the wind speed mean, could also be affected by seasonality, as it is my perception that usually there is more wind during the night than during the day and more wind during the winter than during the summer. If we changed it from $\mu$ to $\mu(t)$, Lemma 2.3 would still be valid, but (2.14) would now be the forward Kolmogorov equation of a generalized OU process (with time dependent mean), and so the solution would be a different $w$, leading to a different $V$.

- $\theta$, the mean of the bids made by gas-fired power plants, is probably not affected by seasonality, but likely it has a non-zero trend. Much like what we said about $\mu$, we can change $\theta$ to $\theta(t)$, and everything up to Lemma 2.1 would still be correct, but another solution to (2.7) is needed.

From my experience, usual choices for seasonality functions are linear combinations of sines and cosines, while trends are frequently represented by linear functions. Of course we may want to characterize these or other phenomena with a different set of functions we feel are more appropriate to the occasion.

Another parameter that could be made time dependent is $\alpha$, the proportion of wind power in the market. It is expected that this will increase in time, so we can change $\alpha$ to $\alpha(t)$. All the results are still valid.

There are of course other ways to introduce more complexity into the models. Other parameters, like mean-reversion rate, volatility and correlation, could be made

time dependent or even stochastic. A careful analysis has to be made, case to case, to see which results hold and which need to be reformulated.

### 2.4.2 Different Models

When we were modelling the bids from fuel-fired power plants, we discussed there was always a slight chance of getting a negative bid. If we want to completely avoid this, one way to do it is to consider $p_i$ not as the bid itself, but as the logarithm of the bid, as suggested in [1]. This would mean that in (2.10) we would not get $\mathbb{1}_{\bullet \leq x}$ but $\mathbb{1}_{\bullet \leq log(x)}$ instead, and so Lemma 2.2 would be:

$$\pi_t = \exp(\theta + \sigma\rho M_t + \hat{\sigma}\ \Phi^{-1}(D_t))$$

thus preventing negative prices.

As for the wind speed model, we chose a OU process to model the wind speed due to its connections to ARMA models, but we could have picked other models. Here we look at two alternatives:

***Model 1:***

$$dv_i = \eta(\mu - v_i)dt + \zeta\ \sqrt{v_i(1-v_i)}\ (\varrho\ dW_t^{\text{wind}} + \sqrt{1-\varrho^2}\ dW_t^i)\ . \tag{2.16}$$

Why use this model? Mainly because it provides a massive simplification when we analyse the power generated by a wind farm, as this process is always in $(0, 1)$. It might also be useful for locations/countries where wind is quite uniform - it is windy enough to always produce power, but never strong enough to reach high capacity.

Now what happens when we try to repeat the previous analysis with this model? Everything works smoothly with the necessary changes, until we get to (2.13), were we get:

$$\begin{aligned} dV_t(x) =& (-\frac{\partial}{\partial x}(\eta\ (\mu - x)\ V_t(x)) + \frac{\partial^2}{\partial x^2}(\frac{1}{2}\ \zeta^2\ x\ (1-x)\ V_t(x))\ )dt \\ & - \frac{\partial}{\partial x}(\zeta\ \varrho\ \sqrt{x\ (1-x)}\ V_t(x))\ dW_t^{\text{wind}}\ . \end{aligned} \tag{2.17}$$

Ideally, this would have a nice explicit solution like Lemma 2.3 provides. However, if we are unable to find it, the next best thing is to prove a solution actually exists. According to [5], the above equation is a **Zakai Equation** in the correlated noise framework, where $W^{\text{wind}}$ is the noise in the observations and $v_i$ is the signal. Since our

observation process is only the noise $W^{\text{wind}}$, the existence of a solution is guaranteed. Moreover, since both the drift and volatility of $v_i$ are functions of $v_i$ only (in opposition to function of $v_i$ and $t$) and satisfy the linear growth condition ($f(x) \leq K(1+|x|)$ for some $K \in \mathbb{R}$), the solution exists and is unique.

***Model 2:***

$$dv_i = -\theta\, v_i(v_i - \mu)(v_i - 1)\, dt + \sigma(\rho\, dW_t^{\text{wind}} + \sqrt{1-\rho^2}\, dW_t^i)\ .$$

Why use this model? It gives us a different perspective of how the wind speed behaves in terms of what was offered in the last model. While in Model 1 the wind speed moves around the mean, varying only slightly and never getting too high or too low, this model offers us a more abrupt approach, with the wind speed varying between periods of extreme intensity, where our wind farms achieve full capacity or very close to that, and periods of calmness, where the wind speed leads to very small or even inexistent power generation.

Following Section 2.2 like we did in the previous model, the SDE we encounter in (2.13) is:

$$dV_t(x) = (\theta\, \frac{\partial}{\partial x}(x(x-\mu)(x-1)\, V_t(x)) + \frac{1}{2}\, \sigma^2\, \frac{\partial^2 V_t(x)}{\partial x^2})dt - \sigma\, \rho\, \frac{\partial V_t(x)}{\partial x}\, dW_t^{\text{wind}}\ .$$

Again, this is a Zakai Equation and the existence of a solution is once again assured by the overlapping of the noise and observation process. However, the uniqueness of solution is not clear, as the drift of $v_i$ does not satisfy the linear growth condition. It is my opinion however that a close analysis to [10] with this specific case in mind would probably prove uniqueness of solution to this equation, as the high order drift is not causing the SDE to explode, but to return to the mean faster.

### 2.4.3 Multiple Sources

As suggested in [1], a more realistic way to look into the electricity market is to consider a vast number of players using a wide range of sources. We can split the various bids in terms of what fuel they use (or what renewable source), in a similar way to [11]. For example consider bids made from gas-fired, coal-fired and oil-fired power plants, as well as from nuclear plants. On the other hand we also have renewable producers of power, like wind farms, hydroelectric dams and photovoltaic power stations.

Suppose coal and oil bids are modelled in the same way gas bids were, only with different parameters. For the hydro and solar energy, if we wanted to follow the same approach as Section 2.2, we would have to model the river speed/solar intensity and then understand how hydroelectric dams and photovoltaic power stations transform these quantities into power. If we got this, then hopefully we can follow the rest of the section and by the end of it get the proportion of total capacity that is currently generating from each source, $X^h$ and $X^s$. Like in the wind power case, I think there are no costs to produce electricity, so we can assume all these bids are made at 0. I believe nuclear power plants can produce power at a very low price, but have to deal with a significant amount of costs if they have to shut down. Hence, for lack of better understanding of the topic, we make the assumption that nuclear energy is always producing at constant rate, bidding at 0.

As we did previously, we assume nuclear, coal, oil and gas joint capacity are enough to cover demand, as to avoid shortage due to inadequate climatic conditions. Hence we denote $\alpha_n$, $\alpha_c$, $\alpha_o$ and $\alpha_g$ their proportions in the market such that $\alpha_n + \alpha_c + \alpha_o + \alpha_g = 1$. Therefore we view the renewable sources as extras, and denote their extra proportion as $\alpha_h$, $\alpha_s$ and $\alpha_w$. Hence the supply-demand equation that sets electricity price $\pi$ becomes:

$$D_t = \alpha_c \, \Phi\Big(\frac{\pi_t - \theta_c - \sigma_c \rho_c M_t}{\hat{\sigma_c}}\Big) + \alpha_o \, \Phi\Big(\frac{\pi_t - \theta_o - \sigma_o \rho_o M_t}{\hat{\sigma_o}}\Big) + \alpha_g \, \Phi\Big(\frac{\pi_t - \theta_g - \sigma_g \rho_g M_t}{\hat{\sigma_g}}\Big) + \big(\alpha_h \, X_t^h + \alpha_s \, X_t^s + \alpha_w \, X_t^w + \alpha_n\big) \, \mathbb{1}_{\pi_t \geq 0} \, .$$

### 2.4.4 Improving Demand

In the same way we have been analysing electricity production by modelling each individual power plant/wind farm, we can also do the same with demand. Each individual can be modelled by itself, having a unique noise but also featuring a common one. On one hand, if it is raining outside, everyone will probably need to use the drying machine instead of drying cloths in the sun. If on the other hand it is sunny, it is likely that everyone stays less at home, lowering the energy consumption.

From my perspective a good choice of a model is (2.16), obviously with different parameters. This is because usually each consumer has a limit to what he can consume, making a process that always stays in $(0, 1)$ a natural choice. We would then have to make the same analysis we have been doing, eventually reaching an equation similar to (2.17). From what we said previously, we know this equation has a unique

solution $V$, so if we could not find it we could at least try to simulate it. After all this, we define our demand process $D$ as:

$$D_t = \int_0^1 x\, V_t(x)\, dx$$

and then use it in the same way we have been using demand.

Another possible extension is to split the demand into various profiles (similar to what we said previously about splitting power generation into various sources). There are many profiles of energy consumption, some of which are exemplified in [11]. Industrial demand has a very different profile than Residential demand, and so it is a good idea to model, or at least do parameter estimation, separately. Another significant profile is the Service sector, which also consumes a huge amount of electricity. According to [12], this 3 profiles where responsible for almost 92% of the world power consumption.

Once we have the demand process for each profile we just need to average them (weighting each according to their significance in the market) before inputting it to our supply-demand curve.

# Chapter 3

# Numerical Implementation

## 3.1 Model Simulation

We are now going to simulate the electricity price with and without wind power generation, and we will try to understand what is happening. We will use artificial parameters, but these were picked when trying to have a realistic output, particularly in comparison with Nord Pool's hourly prices per MW for Oslo, found in [13].

First thing we have to do is simulate the various stochastic processes, in particular $M$, $N$ and $D$. For $M$ and $N$ we use a simple Euler-Maruyama time step. For $D$, however, we need a little modification: the process is supposed to stay in $(0, 1)$, and this is not guaranteed by the numeric discretization. The way we solve this is by reflecting on the boundaries if it happens to fall off. We used years as our timescale, and the set of parameters used were:

| | | | | | |
|---|---|---|---|---|---|
| $\kappa$ | 0.5 | $\tau$ | 10 | $\eta$ | 2 |
| $\theta$ | 35 | $\omega(t)$ | $0.8 + 0.1sin(2\pi t)$ | $\mu$ | 0.8 |
| $\sigma$ | 10 | $\xi$ | 1 | $\zeta$ | 4 |
| $\rho$ | 0.7 | $\alpha$ | 0.3 | $\varrho$ | 0.5 |

After computing the electricity prices, we can plot them. There are a few phenomenon we are hoping to see: when we incorporate wind power, the electricity price should be lower and more stable; spikes, especially without wind power or when wind production is low, should be justified by the demand approaching the total capacity; and the more the wind blows the lower the prices should be.

One possible sample plot is the following:

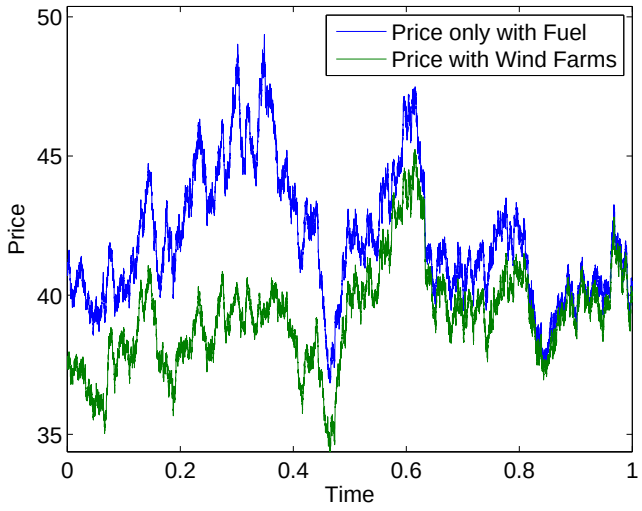



Figure 3.1: Electricity Prices with and without Wind Power.

When wind power is incorporated, prices seem indeed to be lower and spikes seem to be smaller, as expected. To get a better understanding, the following plot shows the bid-stack curve at a given moment:

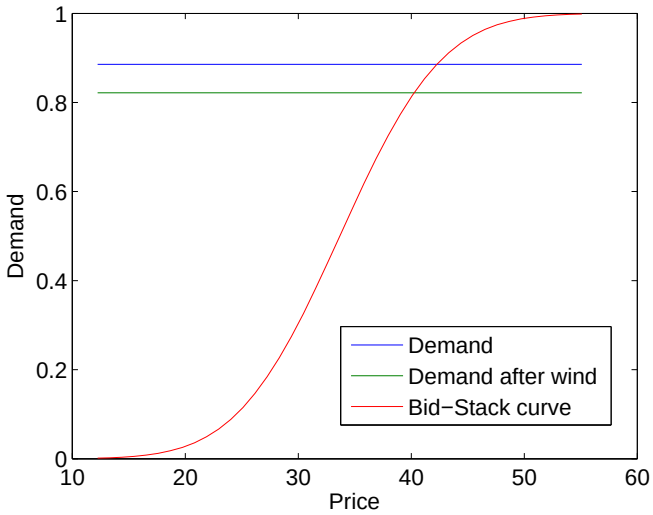



Figure 3.2: Bid-Stack curve at $t = 0.5$.

The reason the prices are lower is obviously because we are retrieving them from lower points of the bid-stack curve. But the reason we have less spikes is exactly the same: since we are in a lower and steadier region of the bid-stack, the same changes in demand lead to smaller changes in prices.

Now we can consider the demand process, to see if we can observe high volatility in prices when demand approaches the total capacity:

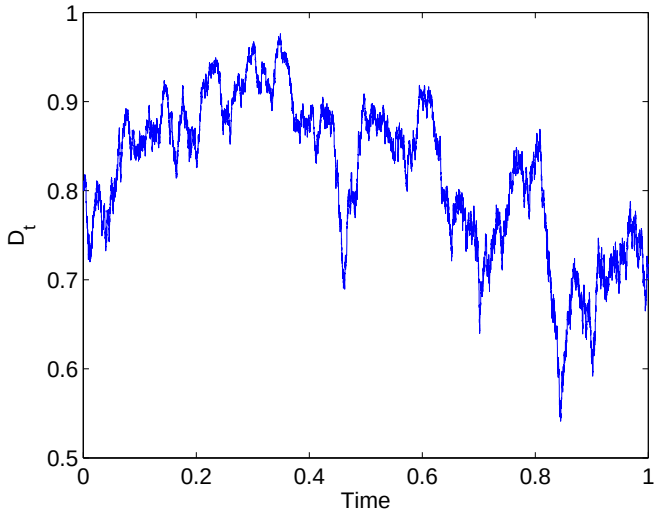



Figure 3.3: Demand Process.

We can in fact see that when $D$ gets closer to 1, small changes in demand cause bigger changes in price than when levels of demand are lower. The influence of demand in prices is also observable when the former changes abruptly, causing significant changes in the latter.

The last phenomenon we hope to understand is the evolution of the spread in prices. We do this by looking into the amount of wind power generated:

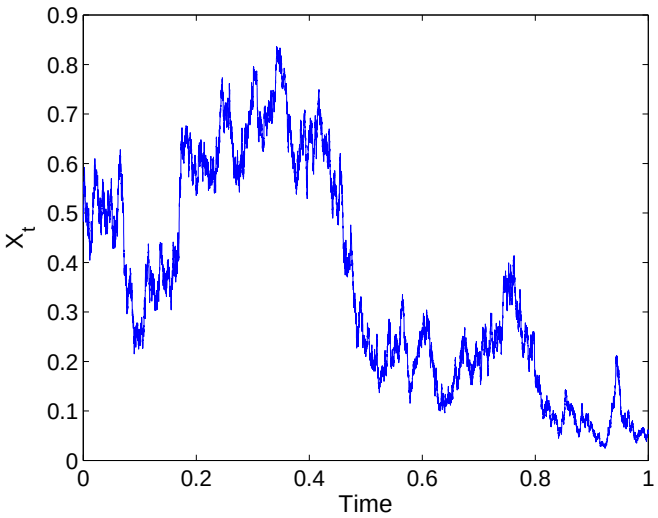



Figure 3.4: Wind Power Generated.

Here we can clearly see that periods of greater wind power generation coincide with periods where the price reduction is greater. On one hand, the reason is that more demand is absorbed, leaving less to be inputted into the bid-stack curve. On the other hand, in periods of weak wind, the prices are approximately the same, as almost all demand continues to be fed into the curve.

Finally, one could notice we are not observing spikes as large as can be witnessed in real life. If we look at the bid-stack curve, we can see how hard it is to have huge variations in price, due to the shape of the normal distribution cumulative density function. This seems to hint that in practice bid-stacks have a fatter tail to the right, which would mean the bids are not normally distributed.

## 3.2 Wind Farm Value

Although they can produce electricity at zero cost, wind farms require a big initial investment and also maintenance costs. So before an investor decides on the construction of a wind farm, he has to make sure that investment is likely to pay out. The typical lifespan of wind farm is 20 years, so in order for it to be appealing the expected income over a 20 years period has to be bigger than the costs.

The income for a wind farm $i$ over a twenty year period is, at terminal time $T = 20$ (assuming a constant interest rate $r = 0$ for simplicity):

$$\int_0^T (v_i(t)^3 \, \mathbb{1}_{0 \leq v_i(t) \leq 1} + \mathbb{1}_{v_i(t) > 1}) \, (24 \times 365 \times \pi_t) \, dt \, . \tag{3.1}$$

Since our time-scale is years and $\pi$ is priced in hours. According to [14], in 2012 a wind turbine cost between 1 and 1.7 million euros per MW, while our simulation points to an expected return of 3.3 million euros. Of course we are using artificial data and so we can take no conclusions, but nevertheless we can take a look and see what happens to the value of a wind farm as we vary some parameters.

The first two we want to take a look at are the correlation $\varrho$ and the proportion of the market $\alpha$. The first because it is not a very natural quantity, especially since we assumed it was constant for all wind farms and the second because we have the expectation that will actually increase over time. The following plots aren't perfect, since calculating (3.1) is computationally expensive.

We will start by looking into the effects of changes in $\varrho$:

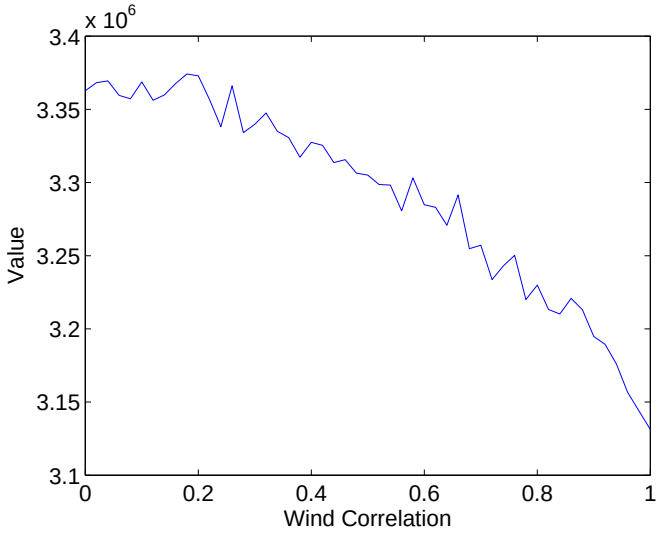



Figure 3.5: Value of a wind farm as a function of $\varrho$.

As we can see, the value of the wind farm lowers as the correlation increases. If we look at the two extremes, we can see some reason for this. On one hand, if the correlation is 0, then our wind speed doesn't affect the electricity price at all. On the other hand, if the correlation is 1, then our wind speed is in fact the index wind speed that directly affects electricity prices, and so as the wind speed goes up, we get our prices from lower points in the bid-stack curve, and so we get lower prices and lower income.

Notice that unlike the *wind proxy* from Section 2.3, an investor can't hope to fully hedge his risk even when there exists a liquid exchange in derivatives on the wind speed index, because of the risk of his location $W^i$. However, if $\varrho = 1$, then building a wind farm is essentially the same as buying a *wind proxy*, and so its value can be determined by hedging arguments.

Next we will analyse the changes in $\alpha$:

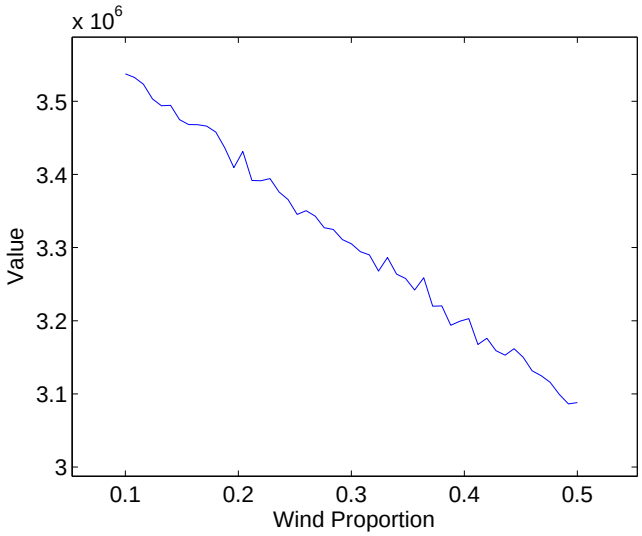



Figure 3.6: Value of a wind farm as a function of $\alpha$.

It is clear that the value of a wind farm decreases as a function of $\alpha$. The justification is simple: the more wind farms there is in the market, the bigger levels of demand they'll absorb, leading to less demand in the bid-stack curve, and hence lower prices, that imply lower income.

Finally we can try to figure out what happens as $\mu$ and $\zeta$ change. While at first glance these parameters shouldn't be subject to a long term change, in recent years many studies showed evidence supporting the claim that climate is changing, which could directly affect wind speed, and hence these parameters.

We can start by $\mu$:

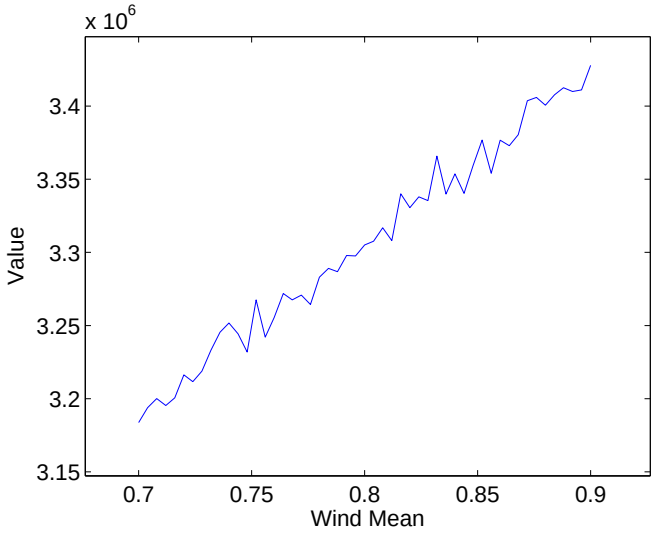



Figure 3.7: Value of a wind farm as a function of $\mu$.

It is again clear that the value of a wind farm increases as $\mu$ increases. This is not surprising at all, if $\mu$ increases we will be producing more electricity, hence selling more, hence increasing our income.

Finally we look at $\sigma$:

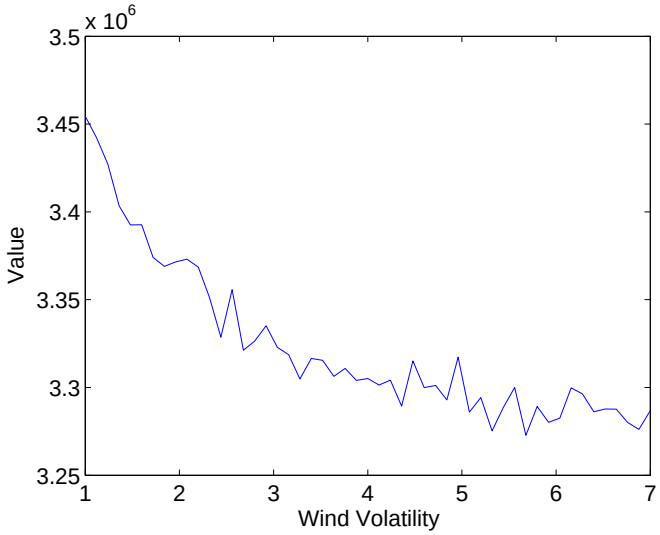



Figure 3.8: Value of a wind farm as a function of $\sigma$.

We notice the value of a wind farm decreases as $\sigma$ gets larger. Since $\sigma$ affects both our wind speed and the index wind speed in the same way, the explanation has to be that increasing volatility decreases electricity production. Since we are dealing with $\mu = 0.8$, this does indeed make sense. By increasing the volatility, we make the wind speed more likely to reach higher values, but also more likely to reach lower

values. Since $\mu$ is so much closer to 1 than to 0, the increase in volatility will make it more likely to get lower or almost no production, but it won't increase as much the probability of having a full production, as this is achieved at 1. Hence the value of a wind farm decreases, as it starts to have periods without sufficient wind that are not compensated by an increase in periods with full power.

# Appendix A

# Computations

Let $X \sim N(\mu, \sigma^2)$. Define:

$$\alpha_k = \mathbb{E}[X^k \, \mathbb{1}_{0 \le X \le 1}] = \int_0^1 \frac{x^k}{\sqrt{2\,\pi\,\sigma^2}} \, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx \, .$$

**Lemma A.1** *With $\alpha$ defined as above, we have the following relations:*

$$\alpha_0 = \Phi(\frac{1-\mu}{\sigma}) - \Phi(-\frac{\mu}{\sigma}) \, ,$$

$$\alpha_1 = \sqrt{\frac{\sigma^2}{2\pi}}(e^{-\frac{\mu^2}{2\sigma^2}} - e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + \mu \, \alpha_0 \, ,$$

$$\begin{aligned} \alpha_2 &= -\sqrt{\frac{\sigma^2}{2\pi}}((1-\mu) \, e^{-\frac{(1-\mu)^2}{2\sigma^2}} + \mu \, e^{-\frac{\mu^2}{2\sigma^2}}) + 2 \, \mu \, \alpha_1 + (\sigma^2 - \mu^2) \, \alpha_0 \\ &= \sqrt{\frac{\sigma^2}{2\pi}}(\mu \, e^{-\frac{\mu^2}{2\sigma^2}} - (1+\mu) \, e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + (\sigma^2 + \mu^2) \, \alpha_0 \quad \textit{and} \end{aligned}$$

$$\begin{aligned} \alpha_3 &= \sqrt{\frac{\sigma^2}{2\pi}}(\mu^2 \, e^{-\frac{\mu^2}{2\sigma^2}} - (1-\mu)^2 \, e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + 3 \, \mu \, \alpha_2 + (2\sigma^2 - 3\mu^2)\alpha_1 + (\mu^3 - 2 \, \mu \, \sigma^2)\alpha_0 \\ &= \sqrt{\frac{\sigma^2}{2\pi}}((\mu^2 + 2\sigma^2) \, e^{-\frac{\mu^2}{2\sigma^2}} - (1 + \mu + \mu^2 + 2\sigma^2) \, e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + (\mu^3 + 3 \, \mu \, \sigma^2)\alpha_0 \, . \end{aligned}$$

*Proof:* $\alpha_0$ is immediate.

$$
\begin{aligned}
\alpha_1 &= \int_0^1 \frac{x}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx \\
&= \int_0^1 \frac{x-\mu}{\sqrt{2\,\pi\,\sigma^2}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}\, dx + \mu\,\alpha_0 \\
&= [-\sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}]_0^1 + \mu\,\alpha_0 \\
&= \sqrt{\frac{\sigma^2}{2\pi}}(e^{-\frac{\mu^2}{2\sigma^2}} - e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + \mu\,\alpha_0\ ,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2 &= \int_0^1 \frac{x^2}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx \\
&= \int_0^1 \frac{(x-\mu)^2}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx + 2\,\mu\,\alpha_1 - \mu^2\,\alpha_0 \\
&= [-(x-\mu)\,\sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}]_0^1 + \int_0^1 \sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx + 2\,\mu\,\alpha_1 - \mu^2\,\alpha_0 \\
&= -\sqrt{\frac{\sigma^2}{2\pi}}((1-\mu)\, e^{-\frac{(1-\mu)^2}{2\sigma^2}} + \mu\, e^{-\frac{\mu^2}{2\sigma^2}}) + 2\,\mu\,\alpha_1 + (\sigma^2-\mu^2)\,\alpha_0 \\
&= \sqrt{\frac{\sigma^2}{2\pi}}(\mu\, e^{-\frac{\mu^2}{2\sigma^2}} - (1+\mu)\, e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + (\sigma^2+\mu^2)\,\alpha_0 \quad \text{and}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_3 &= \int_0^1 \frac{x^3}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx \\
&= \int_0^1 \frac{(x-\mu)^3}{\sqrt{2\,\pi\,\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx + 3\,\mu\,\alpha_2 - 3\,\mu^2\,\alpha_1 + \mu^3\,\alpha_0 \\
&= [-(x-\mu)^2\sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}]_0^1 + \sqrt{\frac{2\sigma^2}{\pi}}\int_0^1 (x-\mu)e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx + 3\mu\alpha_2 - 3\mu^2\alpha_1 + \mu^3\alpha_0 \\
&= \sqrt{\frac{\sigma^2}{2\pi}}(\mu^2\, e^{-\frac{\mu^2}{2\sigma^2}} - (1-\mu)^2\, e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + 3\,\mu\,\alpha_2 + (2\sigma^2 - 3\mu^2)\alpha_1 + (\mu^3 - 2\,\mu\,\sigma^2)\alpha_0 \\
&= \sqrt{\frac{\sigma^2}{2\pi}}((\mu^2+2\sigma^2)\, e^{-\frac{\mu^2}{2\sigma^2}} - (1+\mu+\mu^2+2\sigma^2)\, e^{-\frac{(1-\mu)^2}{2\sigma^2}}) + (\mu^3 + 3\,\mu\,\sigma^2)\alpha_0\ .\ \blacksquare
\end{aligned}
$$

**Lemma A.2** *When pricing call options on normally distributed random variables, the following is useful:*

$$
\mathbb{E}[X^+] = \sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{\mu^2}{2\sigma^2}} + \mu\,\Phi(\frac{\mu}{\sigma})\ .
$$

*Moreover there is a one-to-one correspondence between values of $\mu$ and values of $\mathbb{E}[X^+]$ (in $(0,+\infty)$).*

*Proof:*

$$\begin{aligned}
\mathbb{E}[X^+] &= \int_0^{+\infty} \frac{x}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx \\
&= \int_0^{+\infty} \frac{x-\mu}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx + \mu\, P(X>0) \\
&= [-\sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}]_0^{+\infty} + \mu(1-\Phi(-\frac{\mu}{\sigma})) \\
&= \sqrt{\frac{\sigma^2}{2\pi}}\, e^{-\frac{\mu^2}{2\sigma^2}} + \mu\, \Phi(\frac{\mu}{\sigma})\,.
\end{aligned}$$

If we now differentiate this with respect to $\mu$ we get:

$$-\frac{\mu}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{\mu^2}{2\sigma^2}} + \frac{\mu}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{\mu^2}{2\sigma^2}} + \Phi(\frac{\mu}{\sigma}) = \Phi(\frac{\mu}{\sigma}) > 0\,.$$

So as a function of $\mu$ it is an increasing, hence injective function. ■

# Appendix B

# Code

## B.1 Simulation

```
% Created from code found in in the course page for
% Numerical Methods 1: Finite Difference Methods (lectured by Christoph Reisinger) and
% Numerical Methods 2: Monte Carlo Methods (lectured by Mike Giles and Jeff Dewynne


function simulation

close all
clear all
clc

T     = 1;
N    = 24*6*100*T;


h = T/N;
hh = sqrt(h);
samplen = 1;

thetafuel = 35;
corrfuel = 0.7;
kfuel = 0.5;
sigmafuel = 10;


windprop = 0.3;
windrate = 2;
windmean = 0.8;
windvol = 4;
windcor = 0.5;


demandrate = 10;
demandvol = 1;

function u = season(x)

u = 0.8 + 0.1*sin(2*pi*x);

end

Mfuel = zeros(N+1,samplen);
Nwind = zeros(N+1,samplen);
Demand = ones(N+1,samplen)*season(0);
```

```
Electricitypricefuel = ones(N,samplen);
Electricitypricewind = ones(N,samplen);

mu = zeros(1,samplen);
A1 = zeros(1,samplen);
A0 = zeros(1,samplen);
volume = zeros(N,samplen);

windvolhat = windvol*sqrt(1-windcor*windcor)/(2*windrate);

for t = 1:N

    Mfuel(t+1,:) = Mfuel(t,:).*(1-kfuel*h)+randn(1,samplen).*hh;

    Nwind(t+1,:) = Nwind(t,:).*(1-windrate*h)+randn(1,samplen).*hh;

    Demand(t+1,:) = Demand(t,:) + demandrate.*(season(t/N)*ones(1,samplen)-Demand(t
        ,:))*h + demandvol*sqrt(Demand(t,:).*(1-Demand(t,:))).*randn(1,samplen).*hh;

    mu = windmean+windvol*windcor*Nwind(t+1,:);
    A0 = sqrt(windvolhat^2/(2*pi)).*exp(-(mu.^2)./(2*windvolhat^2));
    A1 = sqrt(windvolhat^2/(2*pi)).*exp(-((1-mu).^2)./(2*windvolhat^2));

    volume(t,:) = (mu.^2+2*windvolhat^2).*A0-(1+mu+mu.^2+2*windvolhat^2).*A1+(mu
        .^3+3*windvolhat^2.*mu-1).*normcdf((1-mu)./windvolhat)-(mu.^3+3*windvolhat
        ^2.*mu).*normcdf(-mu./windvolhat)+1;

    for k = 1:samplen

        if(Demand(t+1,k)>1)
            Demand(t+1,k) = 2 - Demand(t+1,k);
        end

        if(Demand(t+1,k)<0)
            Demand(t+1,k) = - Demand(t+1,k);
        end

        if(Demand(t+1,k) > windprop*volume(t,k))

            Electricitypricewind(t,k) = thetafuel+sigmafuel*corrfuel.*Mfuel(t+1,k)+
                sigmafuel*sqrt(1-corrfuel*corrfuel).*norminv(Demand(t+1,k)-windprop.*
                volume(t,k))/sqrt(2*kfuel);

        else Electricitypricewind(t,k) = Electricitypricewind(t,k)*0.8;

        end

    end

    Electricitypricefuel(t,:) = thetafuel+sigmafuel*corrfuel.*Mfuel(t+1,:)+sigmafuel*
        sqrt(1-corrfuel*corrfuel).*norminv(Demand(t+1,:))/sqrt(2*kfuel);

end

 figure(1)

 plot((1:N)/N,Electricitypricefuel,(1:N)/N,Electricitypricewind)
 xlabel('Time','fontsize',13)
 ylabel('Price','fontsize',13)
 legenda1=legend('Price only with Fuel','Price with Wind Farms');
 set(legenda1,'FontSize',13)
 set(gca,'fontsize',13)


 x = thetafuel + sigmafuel*corrfuel*Mfuel(floor(N/2),1) + (-1:0.05:1)*3*sigmafuel*
     sqrt(1-corrfuel*corrfuel)/sqrt(2*kfuel);

 figure(2)
```

```
106
107 plot(x,Demand(floor(N/2),1)*ones(41,1),x,Demand(floor(N/2),1)*ones(41,1)-windprop*
        volume(floor(N/2),1),x,normcdf((-1:0.05:1)*3));
108 xlabel('Price','fontsize',13)
109 ylabel('Demand','fontsize',13)
110 legenda2=legend('Demand','Demand after wind','Bid-Stack curve');
111 set(legenda2,'FontSize',13)
112 set(gca,'fontsize',13)
113
114
115
116 figure(3)
117
118 plot((0:N)/N,Demand)
119 xlabel('Time','fontsize',13)
120 ylabel('D_t','fontsize',13)
121 set(gca,'fontsize',13)
122
123
124
125 figure(4)
126
127 plot((1:N)/N,volume)
128 xlabel('Time','fontsize',14)
129 ylabel('X_t','fontsize',14)
130 set(gca,'fontsize',13)
131
132 end
```

simulation.m

## B.2 Wind Farm

```
1 % Created from code found in in the course page for
2 % Numerical Methods 1: Finite Difference Methods (lectured by Christoph Reisinger)
      and
3 % Numerical Methods 2: Monte Carlo Methods (lectured by Mike Giles and Jeff Dewynne
4
5
6 function windfarm
7
8 close all
9 clear all
10 clc
11
12 T     = 20;
13 N    = 52*T;
14
15
16 h = T/N;
17 hh = sqrt(h);
18 samplen = 51;
19 M = 500;
20
21 x = ((-(samplen-1)/2):((samplen-1)/2))/((samplen-1)/2);
22
23 thetafuel = 35;
24 corrfuel = 0.7;
25 kfuel = 0.5;
26 sigmafuel = 10;
27
28
29 windprop = 0.3;
30 varwindprop = windprop + 0.2.*x;
```

```
windrate = 2;
windmean = 0.8;
varwindmean = windmean + 0.1.*x;
windvol = 4;
varwindvol = windvol + 3.*x;
windcor = 0.5;
varwindcor = 0.5 + 0.5.*x;


demandrate = 10;
demandvol = 1;

function u = season(x)

u = 0.8 + 0.1*sin(2*pi*x);

end

windvolhat = windvol*sqrt(1-windcor*windcor)/(2*windrate);
varvolwindvolhat = varwindvol.*sqrt(1-windcor.*windcor)/(2.*windrate);
varcorwindvolhat = windvol.*sqrt(1-varwindcor.*varwindcor)/(2.*windrate);

valueprop = zeros(1,samplen);
valuemean = zeros(1,samplen);
valuevol = zeros(1,samplen);
valuecor = zeros(1,samplen);

Electricitypriceprop = zeros(1,samplen);
Electricitypricemean = zeros(1,samplen);
Electricitypricevol = zeros(1,samplen);
Electricitypricecor = zeros(1,samplen);

for n = 1:M

farmprop = zeros(1,samplen);
farmmean = zeros(1,samplen);
farmvol = zeros(1,samplen);
farmcor = zeros(1,samplen);
Mfuel = zeros(N+1,samplen);
Nwind = zeros(N+1,samplen);
Demand = ones(N+1,samplen)*season(0);
Windspeedprop = ones(N+1,samplen)*windmean;
Windspeedmean = repmat(varwindmean,N+1,1);
Windspeedvol = ones(N+1,samplen)*windmean;
Windspeedcor = ones(N+1,samplen)*windmean;

for t = 1:N

    Mfuel(t+1,:) = Mfuel(t,:).*(1-kfuel*h)+randn(1,samplen).*hh;

    windnoise = randn(1)*hh;
    locationnoise = randn(1)*hh;

    Nwind(t+1,:) = Nwind(t,:).*(1-windrate*h)+windnoise;



    Windspeedprop(t+1,:) = Windspeedprop(t,:) + windrate.*(windmean*ones(1,samplen)-
        Windspeedprop(t,:))*h + windvol.*(windcor.*windnoise+sqrt(1-windcor*windcor)
        .*locationnoise);

    Windcaughtprop = max(Windspeedprop(t+1,:),0);
    Windcaughtprop = min(Windcaughtprop,1);
    Windcaughtprop = Windcaughtprop.^3;

    muprop = windmean+windvol*windcor*Nwind(t+1,:);
    A0prop = sqrt(windvolhat^2/(2*pi)).*exp(-(muprop.^2)./(2*windvolhat^2));
    A1prop = sqrt(windvolhat^2/(2*pi)).*exp(-((1-muprop).^2)./(2*windvolhat^2));
```

```
97
98      volumeprop = (muprop.^2+2*windvolhat^2).*A0prop-(1+muprop+muprop.^2+2*windvolhat
            ^2).*A1prop+(muprop.^3+3*windvolhat^2.*muprop-1).*normcdf((1-muprop)./
            windvolhat)-(muprop.^3+3*windvolhat^2.*muprop).*normcdf(-muprop./windvolhat)
            +1;
99
100
101
102     Windspeedmean(t+1,:) = Windspeedmean(t,:) + windrate.*(varwindmean-Windspeedmean(
            t,:))*h + windvol.*(windcor.*windnoise+sqrt(1-windcor*windcor).*locationnoise
            );
103
104     Windcaughtmean = max(Windspeedmean(t+1,:),0);
105     Windcaughtmean = min(Windcaughtmean,1);
106     Windcaughtmean = Windcaughtmean.^3;
107
108     mumean = varwindmean+windvol*windcor*Nwind(t+1,:);
109     A0mean = sqrt(windvolhat^2/(2*pi)).*exp(-(mumean.^2)./(2*windvolhat^2));
110     A1mean = sqrt(windvolhat^2/(2*pi)).*exp(-((1-mumean).^2)./(2*windvolhat^2));
111
112     volumemean = (mumean.^2+2*windvolhat^2).*A0mean-(1+mumean+mumean.^2+2*windvolhat
            ^2).*A1mean+(mumean.^3+3*windvolhat^2.*mumean-1).*normcdf((1-mumean)./
            windvolhat)-(mumean.^3+3*windvolhat^2.*mumean).*normcdf(-mumean./windvolhat)
            +1;
113
114
115
116     Windspeedvol(t+1,:) = Windspeedvol(t,:) + windrate.*(windmean*ones(1,samplen)-
            Windspeedvol(t,:))*h + varwindvol.*(windcor.*windnoise+sqrt(1-windcor*windcor
            ).*locationnoise);
117
118     Windcaughtvol = max(Windspeedvol(t+1,:),0);
119     Windcaughtvol = min(Windcaughtvol,1);
120     Windcaughtvol = Windcaughtvol.^3;
121
122     muvol = windmean+varwindvol.*windcor.*Nwind(t+1,:);
123     A0vol = sqrt(varvolwindvolhat.^2/(2*pi)).*exp(-(muvol.^2)./(2*varvolwindvolhat
            .^2));
124     A1vol = sqrt(varvolwindvolhat.^2/(2*pi)).*exp(-((1-muvol).^2)./(2*
            varvolwindvolhat.^2));
125
126     volumevol = (muvol.^2+2.*varvolwindvolhat.^2).*A0vol-(1+muvol+muvol.^2+2.*
            varvolwindvolhat.^2).*A1vol+(muvol.^3+3.*varvolwindvolhat.^2.*muvol-1).*
            normcdf((1-muvol)./varvolwindvolhat)-(muvol.^3+3.*varvolwindvolhat.^2.*muvol)
            .*normcdf(-muvol./varvolwindvolhat)+1;
127
128
129
130     Windspeedcor(t+1,:) = Windspeedcor(t,:) + windrate.*(windmean*ones(1,samplen)-
            Windspeedcor(t,:))*h + windvol.*(varwindcor.*windnoise+sqrt(1-varwindcor.*
            varwindcor).*locationnoise);
131
132     Windcaughtcor = max(Windspeedcor(t+1,:),0);
133     Windcaughtcor = min(Windcaughtcor,1);
134     Windcaughtcor = Windcaughtcor.^3;
135
136     mucor = windmean+windvol.*varwindcor.*Nwind(t+1,:);
137     A0cor = sqrt(varcorwindvolhat.^2/(2*pi)).*exp(-(mucor.^2)./(2*varcorwindvolhat
            .^2));
138     A1cor = sqrt(varcorwindvolhat.^2/(2*pi)).*exp(-((1-mucor).^2)./(2*
            varcorwindvolhat.^2));
139
140     volumecor = (mucor.^2+2.*varcorwindvolhat.^2).*A0cor-(1+mucor+mucor.^2+2.*
            varcorwindvolhat.^2).*A1cor+(mucor.^3+3.*varcorwindvolhat.^2.*mucor-1).*
            normcdf((1-mucor)./varcorwindvolhat)-(mucor.^3+3.*varcorwindvolhat.^2.*mucor)
            .*normcdf(-mucor./varcorwindvolhat)+1;
141
142
```

```
143
144     Demand(t+1,:) = Demand(t,:) + demandrate.*(season(t/N)*ones(1,samplen)-Demand(t
            ,:))*h + demandvol*sqrt(Demand(t,:).*(1-Demand(t,:))).*randn(1,samplen).*hh;
145
146     for k = 1:samplen
147
148         if(Demand(t+1,k)>1)
149             Demand(t+1,k) = 2 - Demand(t+1,k);
150         end
151
152         if(Demand(t+1,k)<0)
153             Demand(t+1,k) = - Demand(t+1,k);
154         end
155
156         if(Demand(t+1,k) > varwindprop(k)*volumeprop(k))
157             Electricitypriceprop(k) = thetafuel+sigmafuel*corrfuel.*Mfuel(t+1,k)+
                    sigmafuel*sqrt(1-corrfuel*corrfuel).*norminv(Demand(t+1,k)-
                    varwindprop(k).*volumeprop(k))/sqrt(2*kfuel);
158
159             else Electricitypriceprop(k) = 0;
160
161         end
162
163         if(Demand(t+1,k) > windprop*volumemean(k))
164             Electricitypricemean(k) = thetafuel+sigmafuel*corrfuel.*Mfuel(t+1,k)+
                    sigmafuel*sqrt(1-corrfuel*corrfuel).*norminv(Demand(t+1,k)-windprop.*
                    volumemean(k))/sqrt(2*kfuel);
165
166         else Electricitypricemean(k) = 0;
167
168         end
169
170         if(Demand(t+1,k) > windprop*volumevol(k))
171             Electricitypricevol(k) = thetafuel+sigmafuel*corrfuel.*Mfuel(t+1,k)+
                    sigmafuel*sqrt(1-corrfuel*corrfuel).*norminv(Demand(t+1,k)-windprop.*
                    volumevol(k))/sqrt(2*kfuel);
172
173         else Electricitypricevol(k) = 0;
174
175         end
176
177         if(Demand(t+1,k) > windprop*volumecor(k))
178             Electricitypricecor(k) = thetafuel+sigmafuel*corrfuel.*Mfuel(t+1,k)+
                    sigmafuel*sqrt(1-corrfuel*corrfuel).*norminv(Demand(t+1,k)-windprop.*
                    volumecor(k))/sqrt(2*kfuel);
179
180         else Electricitypricecor(k) = 0;
181
182         end
183
184     end
185
186     farmprop = farmprop + Windcaughtprop.*Electricitypriceprop*h*365*24;
187     farmmean = farmmean + Windcaughtmean.*Electricitypricemean*h*365*24;
188     farmvol = farmvol + Windcaughtvol.*Electricitypricevol*h*365*24;
189     farmcor = farmcor + Windcaughtcor.*Electricitypricecor*h*365*24;
190
191 end
192
193 valueprop = valueprop + farmprop/M;
194 valuemean = valuemean + farmmean/M;
195 valuevol = valuevol + farmvol/M;
196 valuecor = valuecor + farmcor/M;
197
198 end
199
200  figure(1)
201
```

```
 plot(varwindprop,valueprop)
 xlabel('Wind Proportion','fontsize',13)
 ylabel('Value','fontsize',13)
 set(gca,'fontsize',13)


 figure(2)

 plot(varwindmean,valuemean)
 xlabel('Wind Mean','fontsize',13)
 ylabel('Value','fontsize',13)
 set(gca,'fontsize',13)



 figure(3)

 plot(varwindvol,valuevol)
 xlabel('Wind Volatility','fontsize',13)
 ylabel('Value','fontsize',13)
 set(gca,'fontsize',13)



 figure(4)

 plot(varwindcor,valuecor)
 xlabel('Wind Correlation','fontsize',13)
 ylabel('Value','fontsize',13)
 set(gca,'fontsize',13)

end
```

windfarm.m

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