

Fermion mass prediction from Infra-red fixed points

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Abstract

We argue that in a wide class of theories the fermion and soft supersymmetry breaking mass structure is largely determined by the infra-red fixed point structure of the theory lying beyond the Standard Model. We show how knowledge of the symmetries and multiplet content of this theory is sufficient to determine the infra-red structure, illustrating the idea for the case of a simple abelian family symmetry. The resulting structure determines the fermion masses and mixing angles in terms of a restricted number of parameters.

The origin of the pattern of fermion masses and mixing angles has long fascinated physicists yet it remains one of the outstanding questions raised by the structure of the Standard Model. Although there are many ideas based on broken symmetries capable of generating structures with the general hierarchical structures observed, detailed predictions still remain elusive with a few gallant exceptions. The basic problem is that in order to calculate masses and mixing angles the Yukawa couplings of the Standard Model must be determined. If this is to be done via symmetries then there must be introduced a family symmetry group and this has proved elusive and beset by problems associated with flavour changing processes. Another possibility is that the Yukawa couplings are determined by the underlying (super)string theory. Again this has proved difficult to realise particularly because in string theories the couplings are determined by the vacuum expectation values (vevs) of moduli fields and to know these the vacuum structure of the full theory including supersymmetry breaking effects must be calculated, a task currently beyond our capability although general features in a specific models have been investigated.

In this paper we explore a third possibility, namely that the Yukawa couplings are determined by the infra red stable fixed point (IRSFP) structure of the theory lying beyond the Standard Model. There are several attractive features of such a proposal. In the first place the predictions rely only on a knowledge of the dynamics of the theory, which follow from the symmetries, and do not require knowledge of the initial conditions.

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As a result the uncertainties associated with the final “Theory of Everything”¹ are avoided or reduced. Secondly the flavour changing problems associated with a family symmetry are under control in a class of theories with a family independent fixed point structure[1]. Indeed we may take this as a strong hint that the IRSFP structure plays an important role for it seems likely that there *must* be new family dependent interactions if the pattern of fermion masses is to be understood and then it is essential to explain why flavour changing processes are small. Finally the known structure of the quark and lepton masses and mixing angles may be used to infer the symmetry structure needed for the theory beyond the Standard Model. Given the symmetries the dynamics of the theory and thus the renormalisation group (RG) equations are determined and so one may try to develop a systematic and *quantitative* investigation of the idea that IRSFP structure determines the masses and mixing angles.

Before turning to this we first comment on why we think that IRSFP are likely to play a role in the theory beyond the Standard Model. We suppose there is some extension of the Standard Model at a scale M . It is plausible (but not essential - see later) to identify this with the scale, M_X , at which the gauge couplings unify. However as there is only a small gap between the gauge unification scale $O(10^{16}GeV)$ and the compactification or Planck scale there seems little room for logarithmic corrections to drive couplings to their fixed points. Now we can envisage two general possibilities for generating the hierarchy of fermion masses. The first possibility is that the Yukawa interactions themselves have an hierarchical structure. This is certainly possible in string theories and there have been attempts [2] to develop a theory of fermion masses this way although to date these have not been very successful. The second possibility is that the small Yukawa interactions arise due to small mixing between states, the quarks and leptons and/or the Higgs². In this case there must be *many* more states than are currently observed, the mixing being between these (heavy) states and those observed at low energies. However the addition of such heavy states typically has the effect of causing the effective couplings to run quickly at scales above their mass. This has been discussed in [5] where we showed that in many theories beyond the Standard Model the rate of approach to the fixed point is very fast. Thus we consider it quite plausible that in such models of fermion masses the IRSFP structure plays an important role in determining the couplings.

We turn now to the construction of a theory beyond the Standard Model which will realise the ideas discussed above. The starting point is to identify a symmetry capable of giving the observed pattern of fermion masses and mixing angles. In [6] we analysed the simplest possible gauge extension of the Standard Model in which an abelian family symmetry generates an acceptable pattern of fermion masses and mixings. For the case of symmetric mass matrices we considered the most general possibilities for the charge assignments under this group. The structure turns out to be determined by a combination, a , of these charges and we found the remarkable result that for any value $a > 0$ the fermion mass matrix has two texture zeros leading to excellent predictions for two combinations of the CKM matrix elements. Further we found a particular choice for a generates the observed hierarchical pattern of the non-zero matrix elements in the quark sector. The extension to the lepton sector is straightforward and depends on another parameter, b , determined in terms of the charges assigned to the leptons. Again we found that there were choices for b capable of explaining the hierarchy of lepton masses and even the relation of charged lepton to down quark masses. The extension of these ideas to neutrinos is also straightforward[7]. One important conclusion of this study was that the abelian symmetry could only be made anomaly free through the Green Schwarz mechanism[8]. This led to a prediction for the value of $\sin\theta_W$ at the unification scale because the Green Schwarz

¹For example the uncertainties associated with the moduli vevs in string theories.

²This opens the attractive possibility that the mixings are ordered by a (spontaneously) broken family symmetry[3] giving textures[4] as discussed below.

term relates the ratio of gauge couplings to the ratio of anomalies associated with the new abelian family symmetry. Thus the value of $\sin\theta_W$ is related to the pattern of fermion masses and we showed that this constrained the value of $\sin\theta_W$ to be $\frac{3}{8}$! Further we found that the resulting pattern of mass matrices were consistent with a larger symmetry, extending the $SU(2)_L \otimes U(1)$ of the Standard Model to $SU(2)_L \otimes SU(2)_R$.

While this family symmetry is clearly not the only possibility it does represent the simplest extension of the Standard Model. As such it is a good starting point for our programme seeking to explore the possibility that the IRSFP structure of the theory determines fermion masses and that the Standard Model is entirely given by an effective low-energy theory insensitive to the physics of the ultimate ‘‘Theory of Everything’’. However at first sight it seems unacceptable just to consider the simplest gauge group extension $SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)_X$ because we know that in the MSSM the gauge unification scale, M_X , is only $3.10^{16} GeV$, roughly an order of magnitude below the string unification scale, M_S . It has been suggested that (string) threshold corrections[9] will increase M_X but in the present case this seems unlikely because the evolution of couplings above the scale M of new physics is very fast and would require very large threshold corrections. To stop the relative evolution of the gauge couplings above M it appears necessary to consider extensions of the Standard Model which do this by embedding the gauge group in a larger structure with a single gauge coupling (in such models we should identify M with M_X). While it is possible to use IRSFP to determine the couplings in such unified models here we will consider a simpler possibility that does *not* extend the gauge group beyond the abelian flavour group. This model evades the problems just discussed by adding massive states in vectorlike representations which form complete $SU(5)$ representations (even though the gauge group is not $SU(5)$). As a result at one loop order above the scale M (*not* now to be identified with M_X) the *relative* evolution of the $SU(3) \otimes SU(2) \otimes U(1)$ couplings is unchanged. At two loop order the effects are to increase M_X close to the string unification scale so in this case it is quite reasonable to suppose the remaining string threshold corrections will bring the string and gauge unification scale into agreement. The reason we choose to analyse this model is that it provides the simplest example of a theory in which the ISFP structure determines aspects of the fermion mass and soft supersymmetry breaking mass spectrum. While we think it is interesting in its own right it also demonstrates the structure that may result in (more complicated) schemes in which the infra-red structure of the theory below the compactification scale also determines the effective low energy theory. It also illustrates how flavour changing effects need not be large even in a theory involving a family dependent gauge interaction.

Fermion masses from an Abelian family symmetry

To discuss this question we first turn to a review of the construction of the model[6] of quark and charged lepton masses. The structure of the mass matrices is determined by a family symmetry, $U(1)_{FD}$, with charge assignment of the Standard Model states given in Table 1. The need to preserve $SU(2)_L$ invariance requires (left-handed) up and down quarks (leptons) to have the same charge. This plus the requirement of symmetric matrices then requires that all quarks (leptons) of the same i -th generation transform with the same charge $\alpha_i(a_i)$. Allowing for a family independent component, $U(1)_{FI}$ considered below, allows us to make $U(1)_{FD}$ traceless without any loss of generality, the full family symmetry being $U(1)_X = U(1)_{FI} + U(1)_{FD}$. Thus $\alpha_3 = -(\alpha_1 + \alpha_2)$ and $a_3 = -(a_1 + a_2)$.

The $U(1)_{FD}$ charge of the quark-antiquark pair has the form

$$\begin{pmatrix} -2(\alpha_1 + \alpha_2) & -\alpha_1 & -\alpha_2 \\ -\alpha_1 & 2\alpha_2 & \alpha_1 + \alpha_2 \\ -\alpha_2 & \alpha_1 + \alpha_2 & 2\alpha_1 \end{pmatrix} \quad (1)$$

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_2	H_1
$U(1)_{FD}$	α_i	α_i	α_i	a_i	a_i	a_i	$-2\alpha_1$	$-2\alpha_1$

Table 1: $U(1)_{FD}$ symmetries.

This matrix neatly summarises the allowed Yukawa couplings for a Higgs boson coupling in a definite position. They should have charge minus that shown for the relevant position.

For the leptons we have a similar structure of lepton-antilepton charges

$$\begin{pmatrix} -2(a_1 + a_2) & -a_1 & -a_2 \\ -a_1 & 2a_2 & a_1 + a_2 \\ -a_2 & a_1 + a_2 & 2a_1 \end{pmatrix} \quad (2)$$

If the light Higgs, H_2 , H_1 , responsible for the up and down quark masses respectively have $U(1)$ charge so that only the (3,3) renormalisable Yukawa coupling to H_2 , H_1 is allowed, only the (3,3) element of the associated mass matrix will be non-zero as desired. The remaining entries are generated when the $U(1)$ symmetry is broken. We assume this breaking is spontaneous via Standard Model singlet fields, θ , $\bar{\theta}$, with $U(1)_{FD}$ charge -1, +1 respectively, which acquire approximately equal vacuum expectation values (vevs) along a “D-flat” direction³. After this breaking all entries in the mass matrix become non-zero. For example, the (3,2) entry in the up quark mass matrix appears at $O(\epsilon^{|\alpha_2 - \alpha_1|})$ because $U(1)$ charge conservation allows only a coupling $c^c t H_2 (\theta/M_2)^{\alpha_2 - \alpha_1}$, $\alpha_2 > \alpha_1$ or $c^c t H_2 (\bar{\theta}/M_2)^{\alpha_1 - \alpha_2}$, $\alpha_1 > \alpha_2$ and we have defined $\epsilon = (\langle \theta \rangle / M_2)$ where M_2 is the unification mass scale which governs the higher dimension operators. As discussed in reference[6] one may expect a different scale, M_1 , for the down quark mass matrices (it corresponds to mixing in the H_2 , H_1 sector with M_2 , M_1 the masses of heavy H_2 , H_1 fields). Thus we arrive at mass matrices of the form

$$\frac{M_u}{m_t} \approx \begin{pmatrix} h_{11}\rho_{11}\epsilon^{|2+6a|} & h_{12}\rho_{12}\epsilon^{|3a|} & h_{13}\rho_{13}\epsilon^{|1+3a|} \\ h_{21}\rho_{21}\epsilon^{|3a|} & h_{22}\rho_{22}\epsilon^2 & h_{23}\rho_{23}\epsilon^1 \\ h_{31}\rho_{31}\epsilon^{|1+3a|} & h_{32}\rho_{32}\epsilon^1 & h_{33} \end{pmatrix} \quad (3)$$

$$\frac{M_d}{m_b} \approx \begin{pmatrix} k_{11}\sigma_{11}\bar{\epsilon}^{|2+6a|} & k_{12}\sigma_{12}\bar{\epsilon}^{|3a|} & k_{13}\sigma_{13}\bar{\epsilon}^{|1+3a|} \\ k_{21}\sigma_{21}\bar{\epsilon}^{|3a|} & k_{22}\sigma_{22}\bar{\epsilon}^2 & k_{23}\sigma_{23}\bar{\epsilon}^1 \\ k_{31}\sigma_{31}\bar{\epsilon}^{|1+3a|} & k_{32}\sigma_{32}\bar{\epsilon}^1 & k_{33} \end{pmatrix} \quad (4)$$

where $\bar{\epsilon} = (\frac{\langle \bar{\theta} \rangle}{M_1})^{|\alpha_2 - \alpha_1|}$, $\epsilon = (\frac{\langle \theta \rangle}{M_2})^{|\alpha_2 - \alpha_1|}$ and $a = \alpha_1 / (\alpha_2 - \alpha_1)$. The light Higgs states are given by $H_{33}^2 + \sum \rho_{ij} H_{ij}^2 \epsilon^{n_{ij}}$ and $H_{33}^1 + \sum \sigma_{ij} H_{ij}^1 \bar{\epsilon}^{n_{ij}}$ where the powers n_{ij} are those appearing in eq(4) and ρ , σ are related to Yukawa couplings in the Higgs sector in a manner discussed below. These and the Yukawa couplings h_{ij} , k_{ij} are all assumed to be of $O(1)$. As discussed above, for $a > 0$, there are two approximate texture zeros in the (1,1) and (1,3), (3,1) positions. These give rise to excellent predictions for two combinations of the CKM matrix. Choosing $a = 1$ the remaining non-zero entries have

³The spontaneous breaking of non-anomalous gauge symmetries at high scales in supersymmetric theories must proceed along such flat directions to avoid large vacuum energy contributions from D-terms, giving $\langle \theta \rangle = \langle \bar{\theta} \rangle$. In the case there is an anomalous current the Fayet Iliopoulos mechanism can generate a range of vevs for θ and $\bar{\theta}$. In the limit the θ vev driven by the Fayet Iliopoulos term alone is much larger than the vevs that would have been induced by radiative breaking without the Fayet Iliopoulos term we will have $\langle \theta \rangle \gg \langle \bar{\theta} \rangle$; in the other limit we will have $\langle \theta \rangle \approx \langle \bar{\theta} \rangle$. In what follows we assume the second possibility but this is not necessary to achieve a viable mass matrix as will be shown elsewhere.

magnitude in excellent agreement with the measured values. To a good approximation we have the relation [6]

$$\epsilon = \bar{\epsilon}^2 \quad (5)$$

which also implies that $M_2 > M_1$.

The charged lepton mass matrix may similarly be determined. In [6] we restricted the lepton charges by requiring the good relation $m_b = m_\tau$ at unification scale $\alpha_1 = a_1$ giving

$$\frac{M_l}{m_\tau} \approx \begin{pmatrix} l_{11}\sigma_{11}\bar{\epsilon}^{|2+6a+2b|} & l_{12}\sigma_{12}\bar{\epsilon}^{|3a|} & l_{13}\sigma_{13}\bar{\epsilon}^{|1+3a+b|} \\ l_{21}\sigma_{21}\bar{\epsilon}^{|3a|} & l_{22}\sigma_{22}\bar{\epsilon}^{|2(1+b)|} & l_{23}\sigma_{23}\bar{\epsilon}^{|1+b|} \\ l_{31}\sigma_{31}\bar{\epsilon}^{|1+3a+b|} & l_{32}\sigma_{32}\bar{\epsilon}^{|1+b|} & l_{33} \end{pmatrix} \quad (6)$$

where $b = (a_2 - \alpha_2)/(\alpha_2 - \alpha_1)$ and again the Yukawa couplings, l_{ij} , are assumed of $O(1)$.

At this stage b is not determined but for any value of $b > 0$ there are texture zeros in the (1,1) and (1,3) positions giving rise to the phenomenologically successful prediction $Det(M_l) \approx Det(M_d)$. To proceed further a value of b must be chosen and two viable choices were explored. For $b = 0$ the lepton charges are the same as the down quark sector, and so the structure of the down quark and lepton mass matrices are identical. In order to explain the detailed difference between down quark and lepton masses it is necessary in this case to assume that the constants of proportionality determined by Yukawa couplings which we have so far taken to be equal (and of $O(1)$) differ slightly for the lepton case. A factor 3 in the (2,2) entry is sufficient to give excellent charged lepton masses. An alternative which does not rely on different Yukawa couplings is to choose b half integral. For $a = 1$, $b = 1/2$ we found excellent agreement for the charged lepton masses (in this case there is a Z_2 symmetry forcing the (1,3), (3,1), (2,3), (3,2) matrix elements to vanish).

To complete the discussion we turn to a consideration of the anomaly structure of the model. Although the choice of charges for the quarks and leptons given above has no $SU(3)^2U(1)_{FD}$, $SU(3)^2U(1)_{FD}$ or $U(1)^2U(1)_{FD}$ anomalies it is clear that the Higgs charges are not anomaly free. To construct an anomaly free theory it is necessary to modify the additional abelian gauge factor as follows

$$U(1)_X = U(1)_{FD} + U(1)_Z \quad (7)$$

where $U(1)_{FD}$ is the original family dependent symmetry discussed above, and $U(1)_Z$ is a family independent symmetry. As we have seen if the fermion mass matrix is to be symmetric $U(1)_{FD}$ must act the same way on left- and right handed components but $U(1)_Z$ is not so constrained.

As discussed in [6] the general structure of $U(1)_Z$ is given by

$$U(1)_Z = z U(1)_H + x U(1)_X + y U(1)_{XX} . \quad (8)$$

where the various component factors are defined in Table 2 giving the charges of Table 3 for $U(1)'$. This choice is anomalous but the anomaly can be cancelled through the GS mechanism by an appropriate shift of the axion present in the dilaton multiplet of four-dimensional strings. This happens because such an axion has a direct coupling to $F\tilde{F}$. For the GS mechanism to be possible, the coefficients A_i , $i = 3, 2, 1$ of the mixed anomalies of the $U(1)$ with $SU(3)$, $SU(2)$ and $U(1)_Y$ have to be in the ratio $A_3 : A_2 : A_1 = k_3 : k_2 : k_1$. Here k_i are the Kac-Moody levels of the corresponding gauge factors and they determine the boundary condition of the gauge couplings at the string scale by the well-known equation $g_3^2 k_3 = g_2^2 k_2 = g_1^2 k_1^2$. For the general choice of eq(8) the mixed anomalies of the $U(1)$ with the SM gauge factors are in the ratio $A_3 : A_2 : A_1 = 1 : 1 : 5/3$ [10] and hence one recovers the usual (GUT) canonical values for these normalization factors (corresponding to the successful result $\sin^2(\theta_W) = 3/8$) $k_3 : k_2 : k_1 = 1 : 1 : 5/3$.

	Q	u	d	L	e	H_2	H_1
$U(1)_H$	0	0	0	0	0	1	-1
$U(1)_{XX}$	0	0	1	1	0	0	0
$U(1)_X$	1	1	0	0	1	0	0

Table 2: Anomaly-free $U(1)_{FI}$ symmetries.

$U(1)$	$\alpha_i + x$	$\alpha_i + x$	$\alpha_i + y$	$a_i + y$	$a_i + x$	$z - 2\alpha_1$	$-z + w\alpha_1$
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Table 3: Anomaly-free $U(1)'$ symmetries.

One can easily check that for $z = -2x$ one gets the results of 3 for the u-quark mass matrix. If one further has $3x + y = -4\alpha_1$ (and $w = -2$) one gets the results of eq.4 for the d-quark masses.

The $SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)_X$ model

Let us now construct the model that realizes this (a=1,b=0) solution for the fermion mass structure. This will illustrate how a knowledge of the symmetries of the theory largely determine the infra red structure of the theory. In order to implement the structure of eq(3) it is necessary to introduce at least 5 additional Higgs doublets H_i^2 with the appropriate $U(1)_X$ charges, i, to allow them to couple to the quarks in the (2,3), (2,2), (1,3), (1,2) and (1,1) directions respectively. These Higgs doublets will belong to massive supermultiplets pairing up with their partners \bar{H}_i^2 in the conjugate representation through a term in the superpotential $r_i^2 \Phi^2 H_i^2 \bar{H}_i^2$ where Φ^2 is a singlet chiral superfield whose scalar component acquires a vacuum expectation value, V_2 , giving the Higgs mass $r_i^2 V_2$. We also need Standard Model singlet fields Θ^2 , $\bar{\Theta}^2$ as discussed above to break $U(1)_X$. When $\bar{\Theta}^2$ acquires a vev it causes mixing between the massive Higgs fields and the original $H_{2,0}$ field coupling in the (3,3) position. For example the $U(1)_X$ symmetry allows the term $s_0^2 H_0^2 \bar{H}_{-1}^2 \bar{\Theta}^2$ giving the massive state proportional to $H_{-1}^2 + s_0^2 < \bar{\Theta}^2 > / (r_1^2 V_2) H_2^2$ leaving the massless state $H_0^2 - < \bar{\Theta}^2 > s_0^2 / (r_1^2 V_2) H_{-1}^2$. This generates $\rho_{23} = s_0^2 / r_1^2$ in eq(3) and $\epsilon = < \Theta^2 > / V_2$. While this multiplet structure may appear complicated it is of the type that appears in compactified string theories where in addition to the three generations of states left massless at the compactification scale there are typically a large number of additional states in vectorlike representations. In what follows we will assume that the five copies of H_i^2 plus \bar{H}_i^2 appear together with five copies of D_i^{2c} plus \bar{D}_i^{2c} where D_i are additional chiral superfields in a down quark representation of the Standard Model. As discussed above this will ensure the successful gauge unification of the MSSM persists in this model at one loop order. Given that the families fill out complete representations of $SU(5)$ it is perhaps not so surprising that the vectorlike matter should do so too and presumably just reflects the underlying GUT structure of the string even though the low energy theory has a reduced (non-GUT) gauge group.

The remainder of the spectrum is chosen to complete the structure needed to generate the mass matrices of eq(4) and (6) in the manner just discussed. Thus we add five further representations $H_i^1 + \bar{H}_i^1 + D_i^{1c} + \bar{D}_i^{1c}$. We also introduce further Standard Model singlet fields Φ^1 , Θ^1 , $\bar{\Theta}^1$ which acquire vevs generating masses and mixing for these vectorlike representations in the manner just discussed. In particular we have $\sigma_{23} = s_0^1 / r_1^1$ in eq(3) and $\bar{\epsilon} = < \Theta^1 > / V_1$ etc.

This completes the multiplet structure of the model. A summary of the multiplet content and their $U(1)_X$ charges is given in Table 4. To complete the model the couplings

	$(Q, u^c, d^c, L, e^c)_i$		$(H^{1,2}, D^{1,2c})_j$	$(\bar{H}^{1,2}, \bar{D}^{1,2c})_j$	$H_0^{1,2}$	$\Phi^{1,2}$	$\Theta^{1,2}$	$\bar{\Theta}^{1,2}$
i=1	0	j=-2	-2	2	0	0	1	-1
i=2	1	j=-1	-1	1				
i=3	-4	j=3	3	-3				
		j=4	4	-4				
		j=8	8	-8				

Table 4: The $U(1)_X$ charges of the chiral supermultiplets.

must be specified. We will allow all trilinear couplings consistent with the symmetries of the theory. Apart from the gauge symmetries as usual there must be discrete symmetries to inhibit nucleon decay. We choose the simplest possibility, matter parity, under which the three generations of quarks and lepton supermultiplets are odd and the Higgs and the additional D quarks are even. With this symmetry the allowed couplings are just the normal Yukawa couplings plus the singlet couplings discussed above together with further singlet couplings of the singlets to D^c and \bar{D}^c which will give all these states mass at the scale $V_{1,2}$. Note that the normal quarks cannot mix with the D^c quarks because of the R symmetry. In addition we assume there is a symmetry preventing the coupling of the H^1 to the H^2 fields in order to solve the μ problem i.e. such a coupling occurs only after supersymmetry breaking[11]. Again a simple Z_2 symmetry suffices. With this the allowed couplings are given in eq(9).

$$\begin{aligned}
L_{Yuk} = & h_{ijk} Q_i u_j^c H_k^2 + k_{ijk} Q_i d_j^c H_k^1 + l_{ijk} L_i l_j^c H_k^2 \\
& + \sum_{l,j} (r_l^j \Phi^j H_l^j \bar{H}_l^j + s_l^j \Theta H_l^j \bar{H}_{l+1}^j + \bar{s}_l^j \bar{\Theta} H_l^j \bar{H}_{l-1}^j) \\
& + \sum_{l,j} (t_l^j \Phi^j D_l^{jc} \bar{D}_l^{jc} + u_l^j \Theta D_l^{jc} \bar{D}_{l+1}^{jc} + \bar{u}_l^j \bar{\Theta} D_l^{jc} \bar{D}_{l-1}^{jc})
\end{aligned} \tag{9}$$

Renormalisation Group equations

Having specified the couplings the renormalisation group equations are now specified. If they turn out to determine the Yukawa couplings and soft masses of the theory we will have a self contained low-energy theory (low relative to the compactification scale!) in which all the parameters are determined independent of the underlying string theory. To investigate this possibility we turn to a discussion of the renormalisation group equations for the gauge and Yukawa couplings.

The renormalisation group equations for the gauge couplings $\tilde{\alpha}_i \equiv g_i^2/(4\pi)^2$ are

$$\frac{d\tilde{\alpha}_i}{dt} = -b_i \tilde{\alpha}_i^2 \tag{10}$$

where $b_i = -3 + n_V, 1+n_V, 11+n_V$, and 1197 for the group factors of $SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)_X$ respectively. Here n_V is the number of vectorlike representations making up complete 5 dimensional representations of $SU(5)$; for the model just discussed $n_V = 10$. The reason the $U(1)_X$ factor is so large is mainly due to the fact that the colour and $SU(2)$ multiplicity factors are large. Note that none of the gauge couplings are asymptotically free due to the profusion of matter fields. We may see that the evolution of all the gauge couplings is fast, which is why the fixed point structure dominates the infra-red behaviour. Moreover, assuming rough equality at the compactification scale the relative size of $b_{U(1)^\nu}$ ensures the Abelian coupling will be quite negligible at the gauge unification scale. This means that family violating effects coming from the family gauge interaction become negligible, an important feature for a viable model where small variations in the

scalar mass spectrum can lead to large flavour changing neutral currents at low energies [1]. Given this in what follows we will neglect the tiny effects of the Abelian family gauge interactions. These renormalisation group equations apply in the region between the gauge or Yukawa unification scale (which to avoid the need for a further GUT threshold we assume are the same and correctly given by the string scale, M_S , plus (small) threshold corrections) and the scale, $V_{1,2}$, at which the additional vectorlike multiplets acquire their mass. This scale is determined by radiative breaking and due to the large number of multiplets can happen quite close to the compactification scale. However as it is sensitive to the initial values of the soft masses we will treat it as a parameter together with the vevs for the θ and $\bar{\theta}$ fields which arise by a combination of radiative breaking and the Fayet Iliopoulos term.

Below the scale $V_{1,2}$ the renormalisation group equations revert to the MSSM form. The value of the gauge unification scale is sensitive at two loop order to the vectorlike representations. We find it is increased by a factor of 4 bringing it closer to the string unification scale without string threshold contributions. In addition there will be threshold corrections associated with the vectorlike states if the doublet and triplet components are split. However the fixed point structure of the Yukawa couplings does not entirely determine these couplings, there being some residual dependence on the initial values. Thus we cannot determine these threshold effects at this stage although for some values of the initial parameters these can also drive the unification scale closer to the string unification scale.

With this multiplet content all couplings evolve so fast that if there is an IRSFP it will determine the low energy structure even though the vectorlike multiplets have mass very close to the gauge unification scale (in practice they need differ by less than two orders of magnitude). However there is not an IRSFP determining the gauge couplings which are rapidly driven to small values. Thus in this picture the expectation is that the gauge unification occurs at a *large* gauge coupling, a value which is easier to obtain from string theories since it is close to the duality invariant point. At one loop order the numerical analysis of gauge unification remains the same as for the MSSM as the differences between the gauge group factor beta functions is unchanged. Thus to this order the gauge unification scale remains at $2.10^{16} GeV$ and the prediction for $\sin^2(\theta)$ at low energies is unchanged. However as just noted the magnitude of the gauge coupling at the unification scale is increased by an amount dependent on the scale $V_{1,2}$. At two loop order there is an effect due to the increase in the beta functions. We find that these effects decrease $\sin^2(\theta)$ by 0.0002 and marginally increase M_X [16].

We turn now to the main point of the paper namely the study of the renormalisation group equations for the Yukawa couplings. These are given by the general form

$$\frac{dY_{ijk}^a}{dt} = Y_{ijk}^a(N_i + N_j + N_k) \quad (11)$$

where $Y_{ijk}^a = \frac{|f_{ijk}^a|^2}{4\pi^2}$ and f_{ijk}^a refers to one of the couplings in eq(9), $f^a = h, k, l, r, s, \bar{s}, t, \bar{t}$ labeled by $a = h, k, l, r, s, \bar{s}, t, u, \bar{u}$ respectively. The quantities N_i are wave function normalisation coefficients given by

$$\begin{aligned} N_i^Q &= \frac{8}{3}\tilde{\alpha}_3 + \frac{3}{2}\tilde{\alpha}_2 + \frac{1}{18}\tilde{\alpha}_1 - \sum_{jk} (Y_{ijk}^h + Y_{ijk}^k) \\ N_i^{U^c} &= \frac{8}{3}\tilde{\alpha}_3 + \frac{8}{9}\tilde{\alpha}_1 - 3 \sum_{jk} Y_{ijk}^h \\ N_i^{D^c} &= \frac{8}{3}\tilde{\alpha}_3 + \frac{2}{9}\tilde{\alpha}_1 - 3 \sum_{jk} Y_{ijk}^k \end{aligned}$$

$$\begin{aligned}
N_i^L &= \frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1 - \sum_{jk} Y_{ijk}^l \\
N_i^{E^c} &= 2\alpha_1 - 2 \sum_{jk} Y_{ijk}^l \\
N^{\Theta^j} &= -2 \sum_l (Y_l^{s,j} + Y_l^{u,j}) \\
N^{\bar{\Theta}^j} &= -2 \sum_l (Y_l^{\bar{s},j} + Y_l^{\bar{u},j}) \\
N^{\Phi^j} &= -2 \sum_l (Y_l^{r,j} + Y_l^{t,j}) \\
N_l^{H^1} &= \frac{3}{2}\tilde{\alpha}_2 + \frac{1}{2}\tilde{\alpha}_1 - \sum_{jk} Y_{ijk}^l - 3 \sum_{jk} Y_{jkl}^k - \sum_{m=r, s, \bar{s}, t, u, \bar{u}} Y_l^{m,1} \\
N_l^{H^2} &= \frac{3}{2}\tilde{\alpha}_2 + \frac{1}{2}\tilde{\alpha}_1 - \sum_{jk} Y_{ijk}^l - 3 \sum_{jk} Y_{jkl}^k - \sum_{m=r, s, \bar{s}, t, u, \bar{u}} Y_l^{m,2} \\
N_l^{\bar{H}^j} &= \frac{3}{2}\tilde{\alpha}_2 + \frac{1}{2}\tilde{\alpha}_1 - Y_l^{r,j} - Y_{l-1}^{s,j} - Y_{l+1}^{\bar{s},j}
\end{aligned} \tag{12}$$

In the approximation of ignoring the small differences between the gauge couplings, the renormalisation group equations have infra-red-stable fixed points (IRSFP) determining the Yukawa couplings in terms of the gauge coupling, g.. For the couplings of the quarks and leptons to the Higgs we find

$$\begin{aligned}
h_2 &= 1.79 \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \\
k_2 &= \begin{pmatrix} 3.06 - x - y & y & x \\ y & 3.06 - y - z & z \\ x & z & 3.06 - z - x \end{pmatrix} \\
l_2 &= \begin{pmatrix} -3.12 + 3x + 3y & 3.02 - 3y & 3.02 - 3x \\ 3.02 - 3y & -3.12 + 3y + 3z & 3.02 - 3z \\ 3.02 - 3x & 3.02 - 3z & -3.12 + 3x + 3z \end{pmatrix}
\end{aligned} \tag{13}$$

where $(h_2)_{ij} = |h_{ij}/g|^2$ etc. Note that there are three undetermined parameters due to a degeneracy in the coupled differential equations. The fact that not all the Yukawa couplings are determined by the fixed point structure means that some information remains of the underlying theory at the compactification scale; i.e. not every piece of the fermion mass structure is determined by the long distance physics after compactification. In fact we have already limited the number of parameters by demanding a symmetric form for the couplings. Since the renormalisation group equations respect this symmetry this form will result if the initial values of the couplings are symmetric but there are also non symmetric solutions possible. At this stage we could proceed to use these parameters to determine the lepton masses and follow the implications for the quark masses. This would amount to the statement that the underlying string theory determines the lepton masses and the IRSFP only determine aspects of the quark mass matrix. However, following our inclination to look for the most symmetric solution consistent with the observed fermion mass spectrum, here we will follow the more ambitious approach of limiting the Yukawa structure through further symmetries. In particular we consider two possibilities. The first follows the route discussed in the second example given above and assumes the difference between leptons and quarks is due to a Z_2 symmetry under which the third generation of leptons is odd while all other states are even. As a result the lepton mass matrix has zeros in the (3,1), (1,3), (3,2), (2,3) positions. The resulting Yukawa couplings have

the form

$$\begin{aligned}
h_2 &= 1.79 \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \\
k &= \begin{pmatrix} 2.05 - x & x & 1.01 \\ x & 2.05 - x & 1.01 \\ 1.01 & 1.01 & 1.04 \end{pmatrix} \\
l &= \begin{pmatrix} -0.10 + 3x & 3.02 - 3x & 0 \\ 3.02 - 3x & -0.10 + 3x & 0 \\ 0 & 0 & 2.92 \end{pmatrix}
\end{aligned} \tag{14}$$

We see there remains one undetermined parameter.

The second case we consider is that the boundary conditions have a larger symmetry leading to a more symmetric solution at the fixed point. To investigate this we need to look for enhanced symmetries commuting with the RG equations. This is most readily done by relating ratios of couplings which have the same wave function renormalisation. The mildest extension of the symmetry we can envisage is that there is a relation between the up and down quark Yukawas perhaps coming from an underlying $SU(2)_R$ symmetry broken at the compactification scale. The RG equations do not change the equality of *ratios* of up quark couplings to the equivalent down quark couplings so we can determine the effects of this symmetry by requiring in eq(13) the equality of these ratios (the renormalisation group equations respect this symmetry up to the terms involving the $U(1)_X$ gauge interactions and the latter are expected to be small because the coupling is driven to be negligibly small close to the compactification scale). In this case we get

$$(h_2, k_2, l_2) = (1.79, 1.53, 1.46) \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \tag{15}$$

Note that the fixed points are relating lepton and quark couplings even though there is no underlying symmetry; “Grand Unification” without “Grand Unification”! It may appear that there are no free parameters left undetermined but this is not the case for what is determined is the modulus squared of the Yukawa couplings, their phase is not determined and will depend on the initial values i.e. they depend on the underlying (string) theory.

Before we can confront these forms with experiment we must determine the Yukawa couplings associated with the $SU(3) \otimes SU(2) \otimes U(1)$ singlet couplings for they in turn determine the composition of the light Higgs which, as discussed above, generates the quark and lepton masses. In fact the alert reader will have questioned why it was not necessary to include these when determining the IRSFP values for the Yukawa couplings just discussed. The answer to this is that the solution to the IRSFP for the quark and lepton Yukawa couplings requires the contribution to $N_I^{H_{1,2}}$ from these couplings to be independent of i . Thus the RG equations for the IRSFP factor into two parts, one determining the quark and lepton Yukawa couplings and one determining the Yukawa couplings involving the singlet fields. There are two IRSFP structures depending on the initial relative values of the Yukawa couplings relative to the gauge couplings. For large initial values the fixed point structure gives

$$\begin{aligned}
(h_8, h_{-2}, h_{-1}) &= \frac{1}{22}(4, 2, 1) \\
(s_3, s_{-2}) &= \frac{1}{44}(9, 4) \\
(\bar{s}_4, \bar{s}_0, \bar{s}_{-1}) &= \frac{1}{44}(7, 4, 4)
\end{aligned} \tag{16}$$

where, as above, we have given the square of the couplings. The couplings h_4 and h_3 are driven smaller than these at the fixed point. Using these couplings we may determine the light Higgs eigenstates to be

$$\begin{aligned} H_{light}^1 &= H_0^1 + H_{-1}^1 \epsilon + \frac{1}{\sqrt{2}} H_{-2}^1 \epsilon^2 + \sigma_{22} H_{1,2}^1 \epsilon^3 \\ H_{light}^2 &= H_0^2 + H_{-1}^2 \bar{\epsilon} + \frac{1}{\sqrt{2}} H_{-2}^2 \bar{\epsilon}^2 + \rho_{22} H_{1,2}^2 \epsilon^3 \end{aligned} \quad (17)$$

where we have absorbed a factor of $\sqrt{2}$ in the original definition of ϵ , $\bar{\epsilon}$. Note that the IRSFP structure does not determine σ_{22} or ρ_{22} because we have not included light vectorlike states coupling $H_0^{1,2}$ to $H_3^{1,2}$. These terms must be driven by higher dimension operators or we must modify the Standard Model singlet Higgs sector to include some such coupling. While we have constructed schemes to implement this we will not pursue them here but concentrate on a discussion of the phenomenological implications of eq(17) as it stands. Using it in eqs(3), (4) and (6) determines the quark and lepton masses and mixing angles. Consider first the implications for quark and lepton masses. We find for the third generation at the scale M

$$\begin{aligned} \frac{m_b}{m_\tau} &= 1.02 \\ \frac{h_t}{h_b} &= 1.08 \\ h_t &= 1.34 g \end{aligned} \quad (18)$$

After including radiative corrections the $b-\tau$ ratio is in good agreement with experiment. The initial value of the top Yukawa coupling implies the top mass at low energies will be close to the quasi fixed point $m_t \approx 195 \text{ GeV}$ where the number follows since from eq(18) the bottom Yukawa coupling is large and $\tan\beta \approx 50$.

For the lighter quarks and leptons we have the results

$$\begin{aligned} m_d m_s &= 1.04 m_\mu m_e \\ \frac{m_c}{m_t} &= \frac{1}{2} \sqrt{3 - 2\sqrt{2} \cos\theta_u} \epsilon^2 \\ \frac{m_s}{m_b} &= \frac{1}{2} \sqrt{3 - 2\sqrt{2} \cos\theta_d} \bar{\epsilon}^2 \\ \frac{m_\mu}{m_\tau} &= \frac{1}{2} \sqrt{3 - 2\sqrt{2} \cos\theta_e} \bar{\epsilon}^2 \end{aligned} \quad (19)$$

where the angles $\theta_{u,d,e}$ [12] come from the undetermined phases of the Yukawa couplings. After including the radiative corrections which increase the quark masses at low energies relative to the lepton masses by a factor approximately 3 the first relation is in excellent agreement with the experimental measurement. For arbitrary angles the last two relations imply $0.18 < m_\mu/m_\tau < 5.4$. Finally the CKM matrix elements are given by

$$\begin{aligned} |V_{us}| &= |V_{su}| = \left(\frac{m_d}{m_s} + \frac{m_u}{m_c} + 2\sqrt{\frac{m_d m_u}{m_s m_c}} \cos\phi \right)^{1/2} \\ \left| \frac{V_{ub}}{V_{cb}} \right| &= \sqrt{\frac{m_u}{m_c}} \\ \left| \frac{V_{bu}}{V_{ub}} \right| &= \sqrt{\frac{m_u m_s}{m_c m_d}} \\ |V_{cb}| &= |V_{bc}| = \left(\frac{m_s}{m'_b} + \frac{m_c}{m'_t} + 2\sqrt{\frac{m_s m_c}{m'_b m'_t}} \cos\phi' \right)^{1/2} \end{aligned} \quad (20)$$

$\frac{m_d}{m_s}$	$\frac{m_\mu}{m_e}$	$\frac{m_s}{m_b}$	V_{cb}	$\frac{V_{ub}}{V_{cb}}$
0.04-0.067	0.003-0.005	0.03-0.07	0.038 ± 0.003	0.08 ± 0.02

Table 5: Experimental limits on quark masses and mixing angles

where

$$\begin{aligned}
m'_b &= m_b \sqrt{3 - 2\sqrt{2}\cos\theta_d} \\
m'_\tau &= m_\tau \sqrt{3 - 2\sqrt{2}\cos\theta_e} \\
m'_t &= m_t \sqrt{3 - 2\sqrt{2}\cos\theta_u}
\end{aligned} \tag{21}$$

and the angles ϕ, ϕ' come from the undetermined phases. All but the last result of eq(20) comes entirely from the texture zero structure and are in excellent agreement with experiment[13] (cf Table 5). The last result is in agreement with the experimental measurement only if there is a strong cancellation between the terms. Using $m_\mu/m'_\tau = m_s/m'_b$, together with the requirement that m_s/m_μ should be small, requires extremum values for the top and lepton phases, $\phi' \approx \pi, \theta_u \approx 0, \theta_e \approx \pi$, giving $|V_{cb}| \approx 0.045$. Because we are in the large $\tan\beta$ domain this value is subject to significant finite supersymmetric threshold corrections estimated in [14] to be of $O(10\%)$. The value of m_s is determined by θ_d . Allowing for radiative corrections this gives a range for the strange quark mass approximately $(140 - 300)MeV$.

We have not space to discuss the second solution of eq(14) fully but want to stress that the structure of the fermion masses is sensitive to the underlying symmetry. In this example the leptons differ from quarks because they have a Z_2 symmetry. As a result the fixed point structure also changes. For example the ratio $m_\mu/m_d = \sqrt{3}$ and the ratio $m_t/m_b = 1.3$. What the fixed point structure does is to translate a symmetry structure into a quantitative prediction for the parameters of the theory.

In summary we have examined the possibility that the Yukawa couplings determining the quark and lepton masses and mixing angles are given by the IRSFP structure of the theory beyond the Standard Model. The idea was illustrated by a very simple extension of the Standard Model which had a single abelian family symmetry plus additional states in vector representations of the gauge group which acquire their mass beneath the unification scale. We found that many of the couplings were indeed fixed by the IRSFP structure. Indeed allowing for a simple extension of the symmetry at the compactification scale we found cases in which the magnitude of all the couplings were fixed. The resultant structure for the masses and mixing angles is entirely consistent with measurement in terms of the remaining parameters. It is notable that apart from the phase θ_d the undetermined phases must take extremal values for an acceptable phenomenology. As string theories will identify these phases with the phases of moduli this suggests they may be driven to their extremum values through minimisation of the effective potential below the scale M when the phases first play a role[15]. We will discuss this possibility elsewhere but it suggests that at least some of the remaining parameters of the model may be determined by *low* energy physics too. While we consider the model interesting in its own right and worth developing further we would like to stress the generality of the approach; in many extensions of the Standard Model the multiplet structure grows dramatically making the IRSFP structure phenomenologically important. A particularly promising aspect of the fixed point structure is that symmetries are enhanced at low energies offering an explanation for the “family problem” because it is easy to arrange for the enhanced symmetry to include a family symmetry that protects the low energy theory from large flavour changing neutral processes. It remains to be seen whether the IRSFP structure is responsible for *all* the quark and lepton masses and mixing angles. If so the “physics” of

the string may be largely hidden from we observers of the low-energy world and only the multiplet structure and symmetries will be directly given by the string.

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