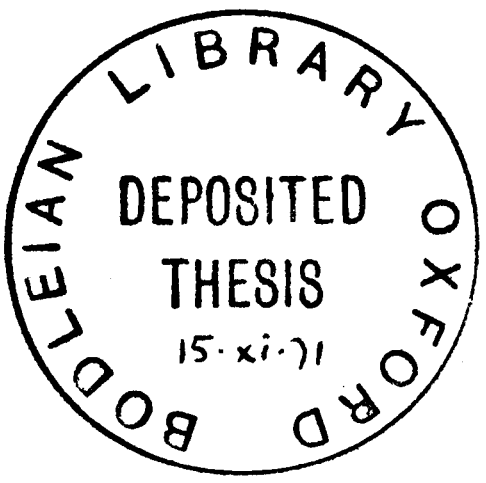
ON THE STRUCTURE OF

STOCHASTIC CONTROL LAWS

by

John W. Patchell



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Abstract

Stochastic control theory is introduced and its importance relative to control science in general is discussed. It is noted that although most practical control problems are stochastic in some sense, stochastic control theory has found few successful applications.

One of the major reasons for this dearth of practical applications is considered to be the difficulty of finding optimal solutions for most stochastic control problems. Consequently, suboptimal solutions are usually necessary and it is shown that the determination of useful suboptimal solutions depends largely on an understanding of the structure of stochastic control laws.

It is observed that the basic result about the structure of stochastic control laws is that stochastic controllers perform two functions, estimation and control implementation. The mathematical form of the first function is updating equations while the mathematical form of the second function is the control policy. It is observed that it is the possible interaction between the two functions of estimation and control implementation which causes stochastic control policies to differ from deterministic control policies. It is shown that this interaction can occur in two ways:

1) the accuracy of the estimation can affect the control policy;

2) the control policy can affect the rate at which uncertainty is reduced.

The effects of the possible interaction on stochastic control policies is investigated by studying three classes of stochastic control problems, those that are certainty equivalent, neutral and separable. A consideration of the relationship between these three properties leads to the conclusion that stochastic control policies have three components:

1) the certainty equivalent policy

2) caution

3) probing

and that the functions of these components are

1) to produce the required plant behaviour,

2) to take into account the fact that the accuracy of the estimation can affect the control policy,

3) to take advantage of the fact that the control policy can affect the rate at which uncertainty is reduced.

Two of the components of stochastic control policies, caution and probing, describe the effect of the possible interaction between estimation and control implementation. The properties of these components are determined through several simple examples. It is noted that the caution component is more easily determined exactly than is the probing component; however, the exact form of the probing component is possibly not very important.

Two simulations of simple stochastic control problems are reported. The first of these concerns the 'known gain problem' and clearly demonstrates that the advantages given by the application of stochastic control theory increase rapidly as the length of the process controlled increases. The second simulation is of the more complex 'unknown constant gain problem' and demonstrates the importance of caution. Caution is necessary if large and very costly errors are to be avoided while probing is necessary particularly for longer processes to reduce uncertainty.

It is shown that a control policy called the neutral control policy which includes the optimal amount of caution but which has no probing component is more easily derived than is the optimal control policy. It is seen from the second simulation that the neutral control policy with a very simple additive probing term is a good suboptimal control policy. It is

conjectured that neutral-plus-probing control policies may prove very useful as suboptimal policies for a large number of stochastic control problems.

Symbols

The page numbers refer to where symbols are first introduced.

Symbols found exclusively in Appendix D are at the end of the list.

Symbol	Description	Page
a	Cost parameter, intentional error	96, 124
$\underline{a}$	Vector of unknown plant constants	29
$\underline{a}'$	Deterministic equivalent of $\underline{a}$	40
b	Gain in plant equation	11
b(i)	Stochastic gain in plant equation	104
$\underline{b}$	Vector of unknown measurement constants	30
c, c', d	Variables	83, 93, 94
$E\{\cdot\}$	Expected value of $\cdot$	16
$f_N[\ ]$	Minimum expected cost function	15
$f_N^{nt}[\ ]$	Minimum expected cost function for related neutral problem	49
$f_N'[\ ]$	Minimum expected cost function for symmetrical problem	125
$\underline{G}_1$	Plant dynamics	29
$\underline{G}_2$	Measurement process	38
$g^*(\ )$	Functional form of optimal control policy	39
$g_{c.e.}(\ )$	Functional form of certainty equivalent control policy	40
$g_{caut}(\ )$	Functional form of caution component of optimal control policy	52
$g_{nt}(\ )$	Functional form of neutral control policy	49
$g_{prob}(\ )$	Functional form of probing component of optimal control policy	52

Symbol	Description	Page
$g_s(\ \ )$	Functional form of separable control policy	43
$g_N^{c.e.}$	Certainty equivalent control parameter for N stages to go	116
g_N^{nt}	Neutral control parameter for N stages to go	116
$H(\ \ )$	Cost at a particular time	31
h_N^{nt}	Neutral cost parameter for N stages to go	116
I_N	Cost of a process N stages long	12
i	Discrete time integer	11
j	Alternative discrete time integer	74
M	Number of trials	21
m_x	Mean value of the state	12
m_b	Mean value of the gain	61
m_s	Mean value of symmetrical state	75
N	Number of stages to process	12
$N(m_x, s_x^2)$	Normal probability density for mean (m_x) and variance (s_x^2)	12
P_1, P_2, P_3, P_4	Probability density parameters	119
$p(\cdot)$	Probability density of $\cdot$	12
$p(\cdot \mid *)$	Probability density of $\cdot$ conditional on $*$	13
q	Probability density parameter	92
r_i	ith value of noise generator output	74
$s[p(\cdot)]$	Information (negative entropy) associated with probability density $p(\cdot)$	41
s_x	Standard deviation of state	12
s_b	Standard deviation of gain	61
s_z	Standard deviation of stochastic variable	12
$S.D.\{\cdot\}$	Standard deviation of $\cdot$	20
u(i)	Control variable at time i	2

Symbol	Description	Page
u^*	Optimal control policy	15
$u^{c.e.}$	Certainty equivalent policy	52
$u_{min.inf.gain}$	Control policy which gives minimum information gain	64
$u^{no\ meas.}$	No-measurement control policy	19
$u^{no\ mem.}$	No-memory control policy	20
u^{nt}	Neutral control policy	49
u^{ntP}	Neutral-plus-probing control policy	74
$u_N^*[\cdot]$	Optimal control policy when there are N stages left to go and the state is $\cdot$	15
$u_N^{*'}[\cdot]$	Optimal control policy of symmetrical problem when there are N stages left to go and the state is $\cdot$	125
$u_N^{nt}[\cdot]$	Neutral control policy when there are N stages left to go and the state is $\cdot$	117
$u_N^*(OP)$	Realization of optimal control policy for program OPCON	74
$u_2^*(OP)$	Realization of optimal control policy for two stages to go by program OPCON numerically derived	74
$u_N^*(Theory)$	Optimal control policy for unknown constant gain problem	76
$Var\{\cdot\}$	Variance of $\cdot$	21
$\underline{x}(i), x(i)$	State at time i	29,11
$\underline{y}(i), y(i)$	Measurement at time i	30,11
$\underline{y}^i, y^i$	Set of all past and present measurements and all past control inputs	32,13
$\underline{z}(i), z(i)$	Stochastic variable at time i	29,11
$\underline{z}'(i)$	Deterministic equivalent of $\underline{z}(i)$	40

Greek Symbols

Symbol	Description	
β	Nondimensional gain	126
δ	Dirac delta function	92
$\mathcal{J}$	Nondimensional measurement noise	126
η	Nondimensional measurement	126
μ_x	Nondimensional mean of state	126
μ_s	Nondimensional symmetrical mean of state	72
μ_b	Nondimensional mean of gain	72
ν	Nondimensional control value	128
$\nu_N^*[\cdot]$	Nondimensional optimal control policy when there are N stages left to go and the state is	129
$\nu_N^{*'}[\cdot]$	Nondimensional optimal control policy for symmetrical problem when there are N stages left to go and the state is ·	128
ξ	Nondimensional state	128
ρ	Nondimensional correlation coefficient	61
σ_x	Nondimensional standard deviation of state	72
$\phi_N[\cdot]$	Nondimensional minimum expected cost when there are N stages to go and the state is ·	129
$\phi_N'[\cdot]$	Nondimensional minimum expected cost for nonsymmetrical problem when there are N stages to go and the state is ·	129
Ψ_N	Nondimensional cost for a process N stages long	129

Symbols Used Exclusively in Appendix D

Symbol	Description	
$A_1(\quad) - A_9(\quad)$	Coefficients for approximations of ϕ_N'	13
a_i	Gauss-Hermite weighting coefficient	134
a, b	Intervals in Simpson's rule	134
F	Partial sum in Gauss-Hermite integration	134a
f(x)	Unspecified part of integrand in Gauss-Hermite formula	134
f(x)	Integrand in Simpson's formula	134
$f^{2n}(\xi)$	The 2^{n+1} derivative of f evaluated at ξ	134
i	Integer series	134
n	Order of Gauss-Hermite integration	134
w(i)	Weighting coefficients in Gauss-Hermite formula	134
Δ	Increment in minimizing procedure	135a
μ_{s_i}	Nondimensional mean of state grid points	138
μ_{b_i}	Nondimensional mean of gain grid points	138

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B.2	Caution in Example A.1	10[illegible]a
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Chapter 1

Introduction

1.1 Introduction

Stochastic control theory is a very important branch of control science which during its existence has not received as much attention as it deserves. A certain absence of enthusiasm has resulted from the fact that great difficulties have been encountered whenever practical applications have been attempted. This thesis demonstrates that these difficulties are in large part a result of a lack of understanding of the underlying structure of stochastic control laws, and it contributes towards removing that misunderstanding. Studies such as the one reported in this thesis should give a much broader practical value to stochastic control theory in the future.

Control science has from its very beginnings been concerned with reducing the effects of gaps in our knowledge about how the world we live in operates. Unpredictable loads meant that James Watt had to use governors on his steam engines instead of a device which opened the throttle at certain times and closed it at others in anticipation of changes in the load. By measuring the speed at which the engine was running and using this to regulate the throttle (the principle of feedback), it was found that the sensitivity of the engine speed to load variations was greatly reduced. Classical control theory has been concerned almost entirely with applying the principle of feedback to reasonably simple systems. The fact that most of the controllers designed were based on this principle was a tacit acknowledgement that almost all systems controlled were of a stochastic or partially unpredictable nature. However, this stochastic fact was never formally acknowledged by classical control theory. Also, design procedures used were always of an indirect nature and one of the main efforts of control engineers was to determine under

what conditions feedback systems work at all.

In modern control theory a much more direct approach has been used. Instead of merely ensuring that the governor worked in some acceptable fashion, it was decided to try to 'optimize' its performance. The speed of the engine was to be kept as close as possible to the desired level and this requirement was wrapped up in a performance criterion which was a direct measure of how well this was being achieved.

At about the same time as modern control theory was coming into existence, digital computers were becoming available for control applications. The combination of modern control theory and the digital control computer should have produced outstanding results. Systems much more complicated than had yet been tackled could in theory be handled by this combination.

However, the perversity of nature in the form of uncertainty ensured that such systems did not operate nearly as well in practice as in theory. Modern control theory which took no account at all of uncertainty made no use of the principle of feedback. Thus this important method of overcoming the effects of uncertainty was ignored and modern control theory was found to be almost useless for any systems for which an exact description of the system and of the disturbances acting on it were not available. Stochastic control theory was introduced to take these uncertainties into account.

Stochastic control theory is a combination of modern control theory and the classical theory of probabilities. Early attempts to apply it almost invariably met with failure because to be of value, stochastic control theory must be applied either 1) to systems that cannot be solved by any other means, 2) to systems in which a large improvement is both feasible and desirable or 3) to systems in which small improvements are very important. Using stochastic control theory for problems which do not satisfy one of these requirements can result in very expensive control systems that have little advantage over the

conventional controllers they replace. Stochastic controllers are expensive because their implementation almost always requires some form of on-line digital computer so that using a stochastic controller for a problem such as the steam engine is not necessarily an economic proposition. Also, the improvement in performance given by the stochastic controller would be very marginal as the governor designed on classical principles already does an entirely adequate job.

Stochastic control theory has also failed to find many applications because of its complexity. Two stages in the application of stochastic control theory can be distinguished: the first is the determination of the stochastic control law to be used, and the second is using it to control the process. Both stages present considerable difficulties. It is rarely possible to solve stochastic control problems completely* even with the help of the largest computers available today and consequently optimal solutions are seldom obtained. Also, the implementation of any solution that is found is generally restricted by the speed and size of the computer available. The complexity of the calculations that the computer has to perform while it is actually controlling the process must be limited so that the computer can keep pace. Obtaining solutions which can be handled quickly by the small control computers and which can be obtained with a reasonable amount of effort has necessitated the use of suboptimal controllers which are approximations of the optimal controller. Unfortunately most of these suboptimal controllers have ignored the very properties of the system which they are supposedly designed to take into account - uncertainty. These controllers have been designed on the principle of replacing any unknowns by their estimated values

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* The exception is a very special group of problems called LQP problems which are linear, have quadratic performance criteria and normal random variables.

and deriving a control that is optimal for these estimated values.* Consequently the way in which uncertainties should influence controllers has been eliminated and the whole reason for using stochastic control theory in the first place is negated. Considering this, it is not surprising that conventional controllers which at least take uncertainties into account in a roundabout way can often outperform their more complicated rivals.

This misapplication of stochastic control theory has resulted from a lack of understanding of the structure of stochastic control laws. It is the failure to understand the fundamental differences between stochastic controllers and conventional controllers which has brought stochastic control theory into disrepute. In this thesis these fundamental differences are defined and discussed and their practical importance is considered. If stochastic control theory is applied to the right sort of problems and if the principles outlined in this thesis are used in the design of stochastic controllers then there should be a very useful future for stochastic control theory.

The rest of this chapter is devoted to stating more precisely what is meant by a stochastic control problem, to giving a brief outline of known results about stochastic control theory, to introducing the basic mathematical tools used in this thesis and to explaining the organization, content and exact contributions of this thesis.

1.2 Stochastic Control Problems

Since the admission of uncertainty is the feature which distinguishes stochastic control theory from the modern and classical theories, the formulation and characteristics of uncertainty are of considerable importance.

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* One reason for the popularity of this method of designing controllers was that it gave optimal solutions for the limited group of LQP problems. The method does not however give optimal solutions to non-LQP problems.

Uncertainties fall into two general but well-defined categories:[1]

1) Uncertainties that cannot be reduced.

An example of this type of uncertainty is the unpredictable error of any measurement of the speed of the steam engine. Uncertainties that cannot be reduced can be modelled as random processes.

2) Uncertainties that can be reduced.

Examples of uncertainties of this type which occur in the steam engine problem are uncertainties about the speed of the engine and the relationship between the throttle setting and the torque exerted by the engine. Uncertainty about the speed of the engine exists because of the inaccuracies inherent in tachometers. These uncertainties can be reduced by averaging a series of tachometer readings. However, if the speed changes, the value of these readings will be partially or completely nullified and thus the uncertainty about the speed may actually increase. The other example of uncertainty, the relationship between the throttle setting and torque produced, can be reduced by the direct experiment of measuring the torque for a series of throttle settings; it can also be reduced through a combination of measurements of the speed, knowledge of the load and knowledge of the throttle setting. Once the relationship between throttle setting and torque has been established it is known for all future times.

The uncertainty about the speed is an example of uncertainty about a state variable. This is modelled by a probability distribution which is updated as the uncertainty about the speed either increases or decreases. The uncertainty about the throttle setting vs. torque characteristic, because the characteristic does not change, is an example of uncertainty about an unknown constant or constants. This is also modelled by a probability distribution which is updated as more is learnt about the unknown characteristic.

With this understanding of uncertainty it is possible to define the term 'stochastic control problem'. For the purposes of this thesis any control problem in which there is uncertainty which can be described in terms of the classical theory of probabilities and for which a direct performance criterion can be found is called a stochastic control problem.

It was due to the efforts of Kolmogorov[2] and Wiener[3] in the 1940's that stochastic processes were introduced to control. Little advance occurred after this until Laning and Battin's works[4] on r.m.s. filtering and prediction theory appeared in 1956. The following year Newton, Gould and Kaiser[5] considerably extended the work of Wiener to the control science area in a book which considered the use of mean-squared error criteria for linear stochastic control problems. The beginning of the 1960's marked an increased interest in stochastic control theory. Both Fel'dbaum[6] who introduced the concept of dual control and Bellman[7] who discussed stochastic control problems using dynamic programming considered nonlinear as well as linear stochastic problems. At the same time Kalman[8,9] described a general method of sequential estimation for multivariable linear problems based on the Wiener-Kolmogorov theory. The Kalman filter as it is called was of particular significance because it described how parameter estimates could be updated as data was obtained. Lee[10] discussed the use of the Kalman filter in his work on estimation and control. Recent important works on the general theory of stochastic control are by Fel'dbaum[11], Aoki[12] and Sage[13]. This thesis is based in particular on the works of Fel'dbaum and of Aoki.

On a more specific level a considerable amount of work has been centered around the certainty equivalence principle and the separation theorem[14-30]. Also many particular cases of stochastic control problems of interest because of some deviation from the LQP problem have been discussed[31-88]. However, only in a very few cases have optimal solutions been obtained and then either

the examples are nearly trivial[53,60,83] or the solution has been obtained by extensive numerical work[51,56]. Only a very few practical applications of stochastic control theory have been published[89-92].

1.3 Tools Available for the Study of Stochastic Control Theory

Two important mathematical tools are used in this thesis in the solution of stochastic control problems. The first, from the classical theory of probabilities, is Bayes' rule which describes how the knowledge contained in probability distributions for the unknowns is combined with measurements of process variables in order to reduce uncertainty about the unknowns. The second tool, dynamic programming[93], is a technique for finding the optimal solutions of stochastic control problems. The use and importance of these tools is discussed by means of the stochastic control problem considered in Chapter 2.

1.4 Organization and Scope of this Thesis

The purpose of this thesis is to summarize existing results about stochastic control theory and to present new material which is the specific contribution of this thesis to the science of control engineering. In order to clarify the presentation of this work, much of the detailed mathematics and numerical work is presented in appendices and only material of general interest has been retained in the body of the thesis.

In Chapter 2 the methods used in the solution of stochastic control problems are introduced by means of a simple example. A numerical simulation of this example illustrates under what conditions optimal stochastic controllers are likely to give superior results to conventional controllers.

In Chapter 3 a general formulation of stochastic control problems is introduced and using this formulation the significance of presently available results about the structure of stochastic control laws is discussed.

In Chapter 4, by using the general formulation presented in Chapter 3 existing confusion about three properties - certainty equivalence, separability and neutrality - is clarified and three suboptimal control policies of general importance are derived from these properties.

In Chapter 5 it is shown how the study of certainty equivalence, separability and neutrality lead to the discovery of two important concepts called caution and probing. The significance of these concepts is discussed with the help of ~~two~~ six simple examples and it is shown that caution and probing are the key to an understanding of the structure of stochastic control laws.

In Chapters 6 and 7 a slightly more complex example is discussed and a simulation of this example is reported. Chapter 6 deals with the theoretical aspects of this problem, it is shown that the optimal solution cannot be obtained analytically and a numerical calculation of the optimal control law is described in Appendix D. These results are of special interest since optimal policies are known for very few problems of this general level of complexity, and knowledge of an optimal policy is a very useful tool for studying the structure of stochastic control laws.

The simulation of the example introduced in Chapter 6 is described in Chapter 7. The importance of understanding the structure of stochastic control laws is clearly illustrated by this simulation and the theory presented in Chapter 5 is used to derive a very useful suboptimal policy for this problem. The procedure used to derive this suboptimal policy is not specific to the problem studied and is shown to have a broad general application.

1.5 Contributions of this Thesis

This thesis represents a significant contribution to the science of control engineering in that it adds to the existing knowledge about the theory of stochastic control problems and demonstrates how this theory can be applied

to practical systems. Knowledge of stochastic control theory is augmented by the clarification of the meaning of the three properties, certainty equivalence, separability and neutrality, and by the introduction of the concepts of caution and probing which describe the structure of stochastic control laws. Knowledge of how to apply stochastic control theory to practical problems is advanced by the observations made in Chapter 2 and by the results about how to apply the new ideas about structure that are given in Chapter 7.

Chapter 2

The Known Gain Problem

2.1 Introduction

In this chapter a simple example of a stochastic control problem is used to introduce the mathematical techniques of stochastic control theory. The importance of Bayes' rule and the value of the dynamic programming approach are discussed. The problem is an LQP one and some of the properties that make the solution of LQP problems so much simpler than that of other stochastic control problems are discussed.

A simulation of the problem is reported which investigates the appropriateness of optimal stochastic control for various cases. It is shown that because of the stochastic nature of the problem, even the optimal control policy cannot produce consistent performances. Sometimes the performance may be very good while sometimes it may be very bad and it is only the average performance that is optimized. Because of this, if the problem is only to be controlled for a short period, much simpler controllers may be very nearly as good as the optimal stochastic controller.

Derivations of the optimal policy for this example have appeared before[12,16] but the detailed discussion and the simulation are original.

2.2 The Problem and its Optimal Solution

In this section the problem is defined, Bayes' rule is applied and the optimal stochastic control law is derived.

i) Statement of the problem

Four individual elements must be specified to give a complete statement of a stochastic control problem. The first element that must be described in mathematical form is the object that is to be controlled. For the steam engine, this is the relationship that exists between the throttle setting (the

variable that can be manipulated or the control input) and the speed at which the engine runs. The change in speed from one time instant to another is proportional to the torque applied which in turn is directly proportional to the throttle setting. A very simplified version of the steam engine might have this relationship expressed by the plant or state equation

$$x(i+1) = x(i) + bu(i) \qquad 2.1$$

where i is an integer representing time, x(i) is the speed or state at time i, u(i) is the control input or throttle setting at time i and b is the constant of proportionality between throttle setting and the torque applied. Equation 2.1 states that the speed at time i + 1 is equal to what the speed was at time i plus the effect of the effort used to either increase (u(i) positive) or decrease (u(i) negative) the speed.*

The second element of the problem which must be described is how the state (in the case of the steam engine, the speed) is measured. For this problem, this description is given by the measurement equation

$$y(i) = x(i) + z(i) \qquad 2.2$$

Here y(i) is the measurable output and z(i) is the measurement error. For the steam engine, y(i) could be regarded as the tachometer reading at time i and z(i) could be regarded as the error in that reading or the difference between that reading and the true speed.

The third element that must be given is a mathematical description of the uncertainties in the problem. For this problem the uncertainties are two in number. The first, the value of the measurement error, is an example of uncertainty that cannot be reduced. The measurement error at any parti-

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* This is not intended to be a realistic description of a steam engine; it is used instead only as an analogy to give an intuitive grasp of the problem considered.

cular time is independent of the measurement error at any other time and can therefore be modelled as a stochastic process. In this problem, this process is described by a normal probability density

$$p(z(i)) = N(0, s_z^2)$$

where s_z^2 is the variance of the measurement error. Because of the measurement error the state is also uncertain and this uncertainty can be reduced by a series of measurements. The initial state (the speed at time i = 0) is not known exactly and is represented by a normal probability density

$$p(x(0)) = N(m_x(0), s_x^2(0))$$

where $m_x(0)$ is the expected value of the state at time zero and $s_x^2(0)$ is the variance of the state at time zero.

The fourth element of a stochastic problem that must be specified is the performance criterion. If it is desired to keep the state (the speed) at some desired constant level, this requirement can be expressed as the desire to minimize a cost which is proportional to the square of the difference between the state and the desired level. In this problem the required state is taken as zero* and thus the performance criterion is expressed as a cost function

$$I_N = \sum_{i=0}^{N} x^2(i) \qquad 2.3$$

where the control extends over the period from time zero to time i = N. The object of the control is to minimize this cost function. However, because of the stochastic nature of the problem in that the state is not exactly known,

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* The desired speed could equally well have been chosen to be other than zero and the state x(i) would then be regarded as the deviation from the required speed rather than the absolute speed.

only the average or expected cost can be minimized.

ii) Bayes' rule

As the control process progresses, the uncertainty about the state can be reduced by combining the series of measurements y(i) with the knowledge about the state and the knowledge of past control inputs. The set of measurements and control inputs is denoted by

$$y^i = \{y(1), \ldots, y(i), u(0), \ldots, u(i-1)\}$$

The knowledge about the state at time i which is based on the set y^i can be expressed in a conditional probability density

$$p(x(i)|y^i)$$

This probability density is updated from time i to i + 1 by using Bayes' rule to combine the knowledge about the state given by $p(x(i)|y^i)$ with the new measurement y(i + 1) and control input u(i). For this problem Bayes' rule is[12]

$$p(x(i+1)|y^{i+1}) = \frac{p(y(i+1)|x(i+1))p(x(i+1)|y^1, u(i))}{p(y(i+1)|y^i, u(i))} \qquad 2.4$$

Because the state and measurement equations are linear and because the initial information and measurement errors are described by normal probability densities, repeated application of Bayes' rule gives that the probability density of the state at time i is normal:

$$p(x(i)|y^i) = N(m_x(i), s_x^2(i))$$

This allows the knowledge about the state at any time i to be described completely by the mean and variance of the state.* They are the sufficient

---

* The fact that a similar convenient summary of the knowlege about the process is available for any system with linear equations and normal random variables is one of the prime reasons that this type of system is so frequently substituted for more exact descriptions of reality.

statistics (see Chapter 3) for this particular problem.

Detailed calculation of Bayes' rule for this problem gives updating equations for the mean and variance of the state:

$$m_x(i+1) = \frac{s_x^2 y(i+1) + s_z^2 (m_x(i) + bu(i))}{s_x^2(i) + s_z^2} \qquad 2.5a$$

$$s_x^2(i+1) = \frac{s_x^2(i) s_z^2}{s_x^2(i) + s_z^2} \qquad 2.5b$$

Because the mean and variance of the state describe completely the knowledge about the system, the equations 2.5 describe the manner in which learning occurs in this problem. They are the Kalman filter equations for this problem and could have been obtained by direct substitution.[22] However, they were derived using the Bayes' rule formula because it is a fundamental result from which most other estimation procedures can be derived. This point is considered further in Chapter 3.

The significance of the equations 2.5 for control theory in general is that they are in a form which is particularly suitable for use in an on-line digital computer. The entire sequence of past measurements and control inputs does not have to be stored by the computer so that new estimates of the state can be computed at each stage. Instead, the knowledge about the state can be summarized by two variables, the sufficient statistics, which moreover can be conveniently updated by Equations 2.5 as each new piece of knowledge becomes available. Unfortunately, similar equations are not always easily found for problems more general than the one considered here.*

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* See Chapter 3 for a further discussion of this point.

iii) Dynamic programming - determination of the optimal control policy

The optimal control policy is the series of control variables $u^*(0), \ldots, u^*(N-1)$ which produces the minimum expected value of the cost function. The control decision at any particular time (u(i)) can only be based on the knowledge about the state available at that particular time and not on any future knowledge of the state. To emphasize this dependence of the optimal policy on the available knowledge, the notation

$$u_N^*\left[m_x(0),\ s_x^2(0)\right]$$

is used. This denotes the optimal control policy at the beginning of a process N stages in length when the knowledge about the initial state is summarized by the sufficient statistics $m_x(0)$ and $s_x^2(0)$.

Dynamic programming is important to the determination of the optimal control policy because it allows the formulation of the problem in terms of a recurrence equation which is suitable for digital computers.* Only the aspects of dynamic programming which are of particular relevance to stochastic control problems are discussed here.

The first step in the application of dynamic programming is to define the minimum expected cost function:

$f_N\left[m_x(0),\ s_x^2(0)\right]$ $\triangleq$ expected cost using the optimal control u^* over N stages. The expectation is taken at stage i = 0 when the knowledge y^0 is summarized by the sufficient statistics $m_x(0)$ and $s_x^2(0)$.

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* A useful text which introduces dynamic programming in a similar notation to that used here and in which the advantages of the dynamic programming approach over classical methods of optimization are discussed is by Jacobs.[94]

The expected cost is considered in two sections given by

$$E\{I_N\} = E\{x^2(1)\} + E\left\{\sum_{i=2}^{N} x^2(i)\right\}$$

The expected cost of the first stage of a process is

$$E\{x^2(1)\} = E\{(x(0) + bu(0))^2\}$$

$$= (m_x(0) + bu(0))^2 + s_x^2(0) \qquad 2.6$$

and if the optimal control policy is used, the cost to be expected from the remaining stages is

$$E\left\{\sum_{i=2}^{N} x^2(i)\right\} = \int_{-\infty}^{\infty} f_{N-1}[m_x(1), s_x^2(1)]\, p(y(1)|y^o, u(0))\, dy(1) \qquad 2.7$$

where $m_x(1)$ and $s_x^2(1)$ are determined according to Equations 2.5. The probability density for $y(1)$ is found from considering that

$$y(1) = x(0) + bu(0) + z(1)$$

and hence

$$p(y(1)|y^o, u(0)) = N(m_x(0) + bu(0),\ s_x^2(0) + s_z^2)$$

Since the expected cost of the stages from $i = 2$ to N (Equation 2.7) has been minimized with respect to the control $u(i)$ at each time i except for $i = 0$, it and the expected cost of the first stage (Equation 2.6) depend only on the first control ($u(0)$). Therefore the two expected costs can be added to give the total expected cost and this can be minimized with respect to the first control ($u(0)$) to give the minimum expected cost. In this way the functional recurrence equation is derived

$$f_N[m_x(0), s_x^2(0)] = \min_{u(0)} \left\{(m_x(0) + bu(0))^2 + s_x^2(0) + \int_{-\infty}^{\infty} f_{N-1}[m_x(1), s_x^2(1)]\, p(y(1)|y^o, u(0))\, dy(1)\right\} \qquad 2.8$$

The great advantage of functional recurrence equations is that they

can be solved recursively. The first step is to consider the equation for the case in which there is only one stage left to go and for this problem this gives

$$f_1\left[m_x(0), s_x^2(0)\right] = \min_{u(0)}\left\{(m_x(0) + bu(0))^2 + s_x^2(0)\right\} \qquad 2.9$$

Using the standard calculus technique of minimization gives

$$f_1\left[m_x(0), s_x^2(0)\right] = s_x^2(0) \qquad 2.10$$

with the optimal control policy

$$u_1^*\left[m_x(0), s_x^2(0)\right] = - m_x(0)/b \qquad 2.11$$

The next step is to consider the case in which there are two stages to go which gives

$$f_2\left[m_x(0), s_x^2(0)\right] = \min_{u(0)}\left\{(m_x(0) + bu(0))^2 + s_x^2(0) + \int_{-\infty}^{\infty} f_1\left[m_x(1), s_x^2(1)\right] p(y(1)\,|\,y^o, u(0))\,dy(1)\right\}$$

and substitution from 2.10 gives

$$f_2\left[m_x(0), s_x^2(0)\right] = \min_{u(0)}\left\{(m_x(0) + bu(0))^2 + s_x^2(0) + s_x^2(1)\right\} \qquad 2.12$$

Since $s_x^2(1)$ is independent of the control $u(0)$, minimization in Equation 2.12 gives

$$f_2\left[m_x(0), s_x^2(0)\right] = s_x^2(0) + s_x^2(1) \qquad 2.13$$

with the optimal control policy

$$u_2^*\left[m_x(0), s_x^2(0)\right] = - m_x(0)/b \qquad 2.14$$

In this problem the optimal control policy and minimum expected cost for the case in which there are N stages to go can be found by using mathematical induction.* If it is assumed that

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* Most LQP problems can be completely solved in this way. However the method works very rarely for other stochastic control problems.

$$f_{N-1}\left[m_x(1), s_x^2(1)\right] = \sum_{i=1}^{N-1} s_x^2(i) \qquad 2.15$$

then substitution in Equation 2.8 gives

$$f_N\left[m_x(0), s_x^2(0)\right] = \min_{u(0)}\left\{(m_x(0) + bu(0))^2 + s_x^2(0) + \sum_{i=1}^{N-1} s_x^2(i)\right\} \qquad 2.16$$

where the expectation integral has been dropped because the sum of the variances is independent of the measurement y(1). Since the sum of the variances is also independent of the control u(0), minimization in Equation 2.16 gives

$$f_N\left[m_x(0), s_x^2(0)\right] = s_x^2(0) + \sum_{i=1}^{N-1} s_x^2(i)$$

$$= \sum_{i=0}^{N-1} s_x^2(i) \qquad 2.17$$

with the optimal policy

$$u_N^*\left[m_x(0), s_x^2(0)\right] = - m_x(0)/b \qquad 2.18$$

Equation 2.15 can be derived from Equation 2.17 and since 2.17 is known to be true for N = 2 (Equation 2.13), the proof by induction is complete. Equations 2.17 and 2.18 give the minimum expected cost and the optimal control for the case in which there are N stages to go.

2.3 Realization of the Optimal Control Law

Once the optimal control law has been derived, the second step in applying stochastic control theory is the realization of the control law. The main restriction in the realization is that the calculations that must be done between one stage of a process and the next must be able to be done by an on-line computer. If the computer cannot keep pace with the process controlled, either a faster computer must be used or the calculations must be simplified.

The realization of the optimal control law for the known gain problem

is illustrated by a flow diagram, Figure 2.1. From this flow diagram it can be seen that two calculations must be completed at each stage: 1) the measurement must be combined with the previous estimate of the state to produce the next estimate and 2) the optimal control must be determined on the basis of this estimate. For this problem, since the optimal control policy has such a simple form, the most complex and therefore the most time-consuming calculations are for the Bayes' rule updating equation. Therefore the minimum possible time between stages is determined primarily by how quickly this calculation can be done.

2.4 Empirical Control Policies

Either because of the expense of realizing the optimal stochastic control law which requires a digital computer or because of the time factor discussed in the previous section, it may be desirable to use a simplified control law. In this section two such control laws are derived.

For the first of these control policies the measurements are not used at all and the control policy is based entirely on the prior knowledge of the state. Because the measurements are not used, the Bayes' rule calculation is not necessary, the realization of this policy is considerably simpler than for the optimal policy, and a digital computer is not required at all.

This no-measurement control policy is defined as

$$u^{\text{no meas.}}(i) = -m_x(0)/b \quad \text{for } i = 0$$
$$= 0 \quad \text{for } i > 0 \qquad 2.19$$

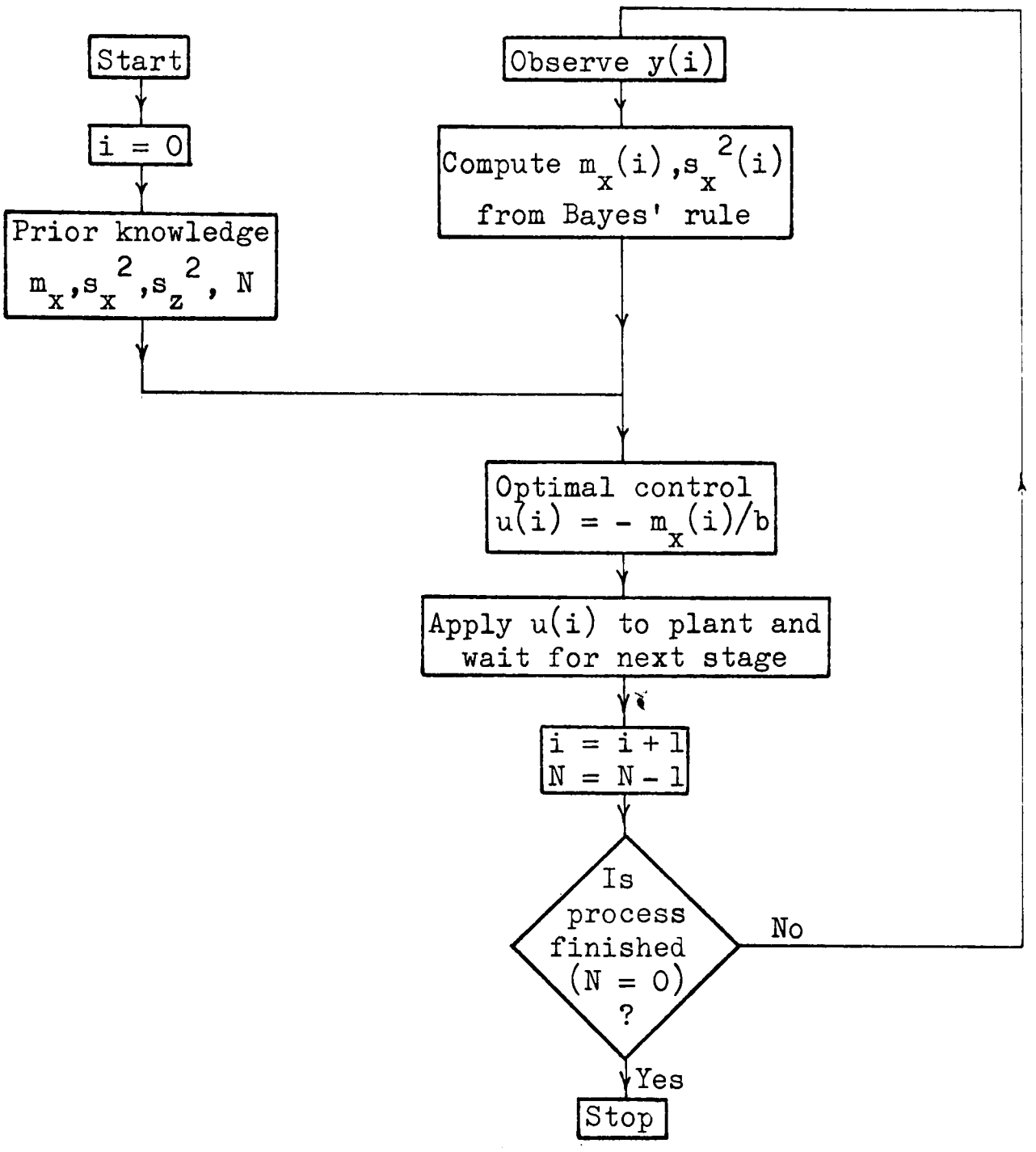
The initial knowledge is used by the first control and from that time on the control is zero. Because of this the state is constant,

$$x(i) = x(1) = x(0) - m_x(0)$$

the expected value of the state is zero, and the variance of the state remains constant at its initial value. Therefore the probability density

Figure 2.1

Implementation of Optimal Control for the Known Gain Problem



of the cost (I_N) is similar to the chi-squared density:[95]

$$p(I_N) = \frac{1}{\sqrt{2\pi s_x^2(0) N I_N}} \exp(-I_N/(2 N s_x^2(0)))$$

From this the expected cost and the standard deviation of the cost can be derived:

$$E\{I_N\} = N s_x^2(0) \qquad 2.20$$

$$S.D.\{I_N\} = \sqrt{2} N s_x^2(0) \qquad 2.21$$

The significance of the cost probability density and the standard deviation of the cost are discussed later.

The second of the empirical control laws operates without the use of memory and the control policy is based only on the present measurement. Because memory is not used, the Bayes' rule calculation is not required to combine previous knowledge with new measurements. As for the first empirical controller, a digital computer is not required to realize the no-memory controller.

The no-memory control policy is defined as

$$u^{\text{no mem.}}(i) = -m_x(0)/b \quad \text{for } i = 0$$

$$= -y(i)/b \quad \text{for } i > 0 \qquad 2.25$$

The initial knowledge is used by the first control and from that time on only the presently available measurement is used. Because of this the state becomes a stochastic variable:

$$x(i) = x(i-1) - y(i-1)$$

$$= z(i-1) \quad \text{for } i > 1$$

The expected value of the state is always zero and the variance is a constant equal to the variance of the measurement error. Uncertainty about the state is not reduced.

Because the state is a stochastic variable it is not easy to find the probability density of the cost when the no-measurement policy is used. However, the expected cost is

$$E\{I_N\} = E\left\{\sum_{i=1}^{N} x^2(i)\right\}$$

$$= s_x^2(0) + (N-1)s_z^2 \qquad 2.26$$

and the variance of the cost is

$$\mathrm{Var}\{I_N\} = E\left\{\left(\sum_{i=1}^{N} x^2(i)\right)^2\right\} - \left(E\left\{\sum_{i=1}^{N} x^2(i)\right\}\right)^2$$

$$= 2s_x^4 + 2(N-1)s_z^4 \qquad 2.27$$

from which the standard deviation of the cost is

$$\text{S.D.}\{I_N\} = \sqrt{2(s_x^4 + (N-1)s_z^4)}$$

The performance given by the two controllers described in this section was compared with that given by the optimal control law by means of the simulation outlined in the next section.

2.5 Simulation of the Known Gain Problem

The object of the simulation was to compare the performances of the three control laws described above. The expected cost given by each of the control laws had been determined theoretically but, as the simulation results showed, a comparison could not be made on that basis alone.

The flow diagram of the simulation which was done on an IBM 1130 digital computer appears in Figure 2.2. The process was repeated M times (each time is called a trial) for the same initial values of m_x and s_x^2 but for a different initial state. The initial state for each trial was a random variable with probability density

Figure 2.2 Flow Diagram for Known Gain Problem Simulation

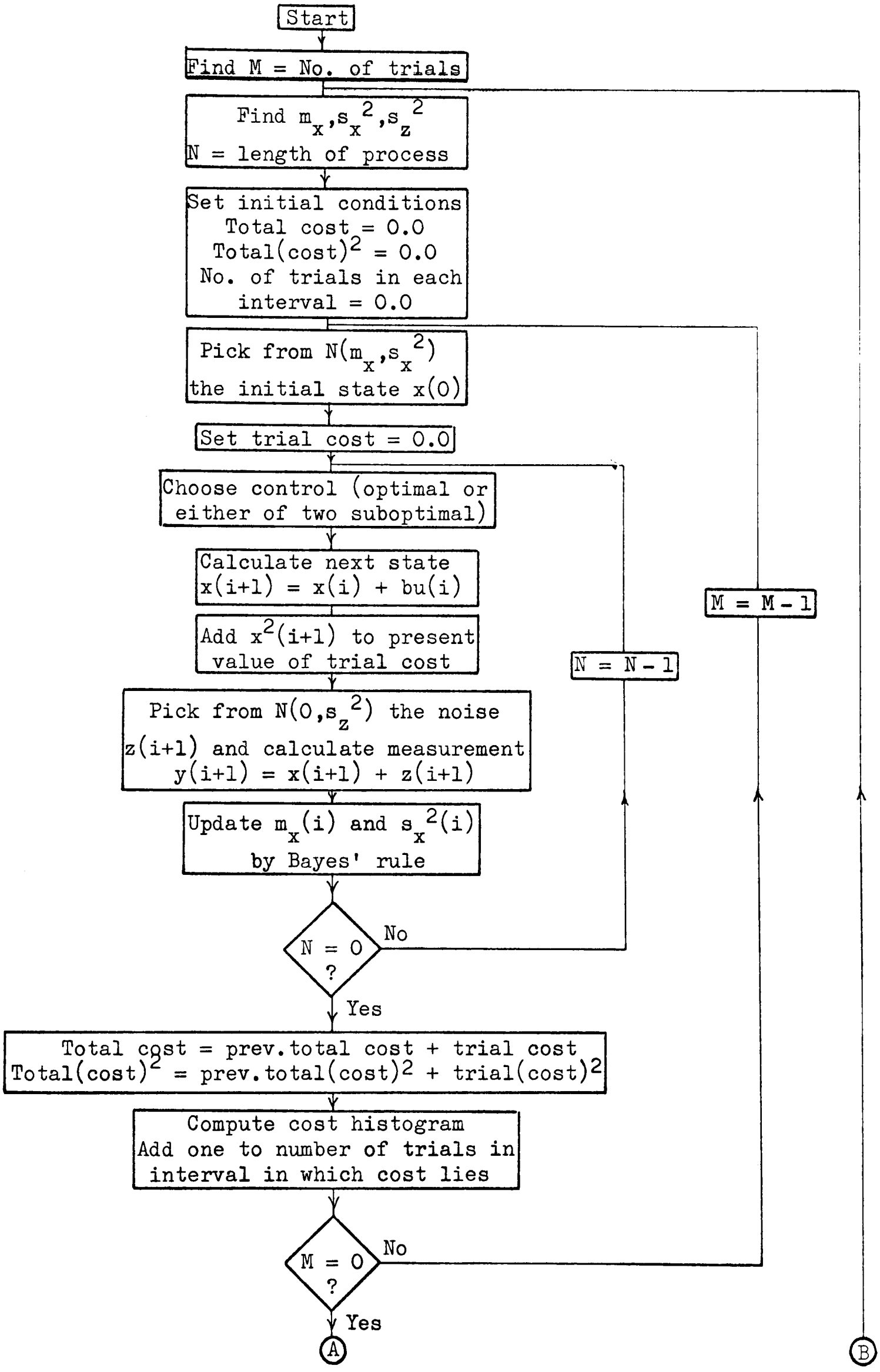
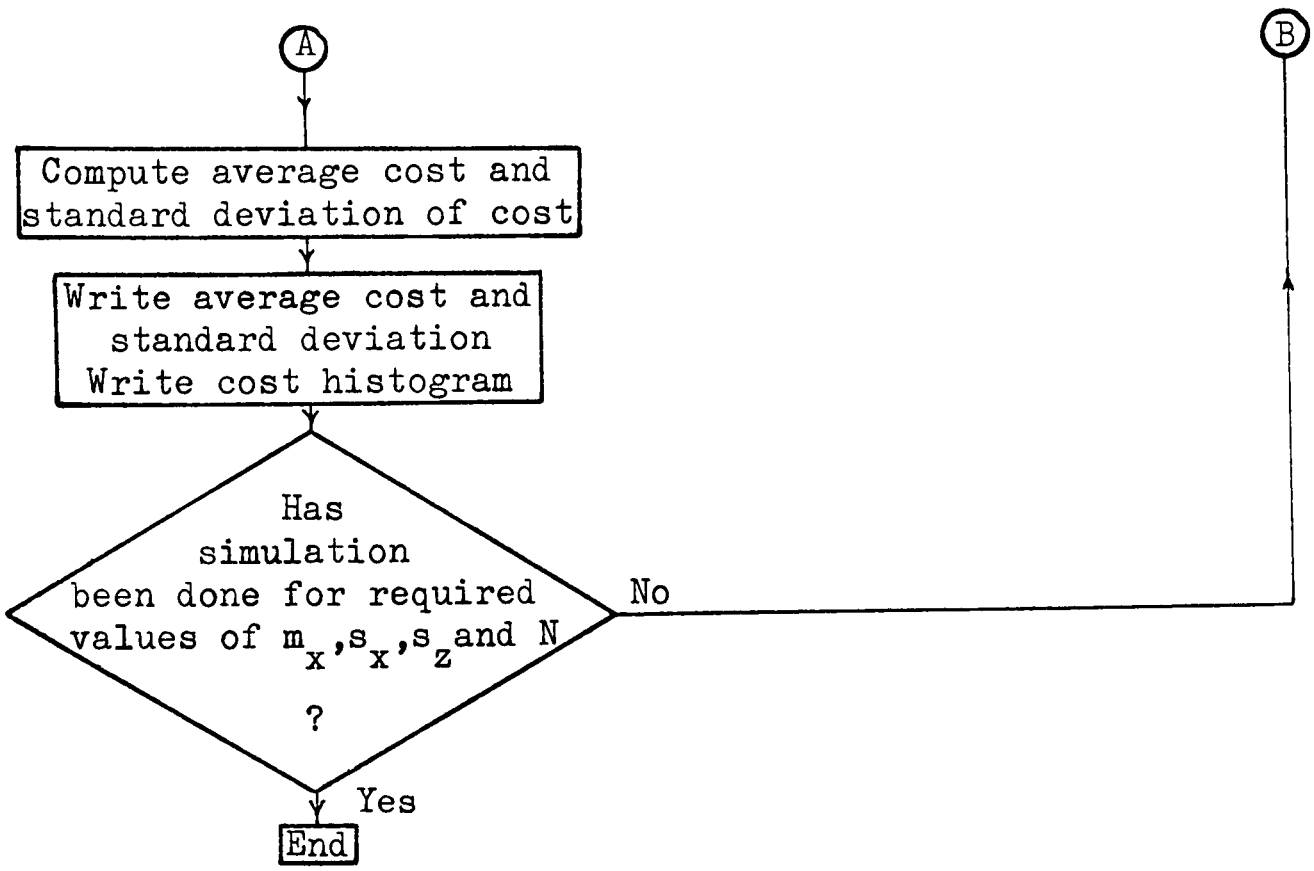


Figure 2.2 (cont.)



$$p(x(0)) = N(m_x(0), s_x^2(0))$$

and was independent of the initial state of other trials. Thus the trials were independent of each other and averages were unbiased. The results of the M trials were averaged to give the expected cost and the standard deviation of this cost. A histogram of the cost was produced by counting the number of trials in which the cost was within a particular interval for each of a specified number of intervals. Normal random numbers for the initial state and the measurement errors were generated within the computer by the Fortran-supplied subprogram GAUSS.

In every case the number of trials used was one thousand for two reasons: the first is that the accuracy with which the expected costs are determined is proportional to the square root of the number of trials[96] and hence a large number of trials is desirable. The second reason is that GAUSS produces a sequence of numbers which repeats itself periodically. It was determined experimentally that for N = 10 (the case in which the largest number of random numbers is used), this produced no adverse effects for one thousand trials but for three thousand trials the simulation began to repeat. Since there is little difference in accuracy between using one thousand and three thousand trials ($\sqrt{3000/1000} = 1.73$), the lower number was used to avoid the possibility of the simulation repeating.

The results of the simulation in terms of the expected costs and the standard deviations of the costs are given in Table 2.1. In this table the initial value of the mean of the state is always zero because this value does not affect the cost in any way. The value of the gain is not specified since this does not affect the cost either. Also the variance of the noise is always one because variation with respect to this parameter can be considered by an appropriate change of scale. No theoretical results for the

Table 2.1 Results of Known Gain Problem Simulation

$s_x^2 = 10.0$, $s_z^2 = 1.0$

Control	N	Theoretical expected cost	Simulation average cost	Theoretical standard dev.	Simulation standard dev.
u^*	5	11.95	12.1 ± 0.4		13.9
	10	12.69	12.9 ± 0.5		14.2
$u^{\text{no meas.}}$	5	50.0		70.5	
	10	100.0		141.4	
$u^{\text{no mem.}}$	5	14.0	14.1 ± 0.4	14.1	14.0
	10	19.0	19.2 ± 0.4	14.2	14.1

$s_x^2 = 1.0$, $s_z^2 = 1.0$

u^*	5	2.28	2.30 ± 0.08		2.36
	10	2.92	2.94 ± 0.09		2.70
$u^{\text{no meas.}}$	5	5.0		7.05	
	10	10.0		14.1	
$u^{\text{no mem.}}$	5	5.0	5.04 ± 0.09	3.16	2.92
	10	10.0	10.11 ± 0.13	4.95	4.15

$s_x^2 = 0.1$, $s_z^2 = 1.0$

u^*	5	0.422	0.42 ± 0.02		0.54
	10	0.718	0.72 ± 0.03		0.86
$u^{\text{no meas.}}$	5	0.50		0.71	
	10	1.00		1.41	
$u^{\text{no mem.}}$	5	4.10	4.16 ± 0.08	2.83	2.55
	10	9.10	9.22 ± 0.12	4.25	3.90

standard deviation of the cost are given for the optimal policy because this was not determined theoretically. (To do so would be very complex, if not impossible). No experimental results are given for the no-measurement policy because simulations using this policy were not done. This was because the expected cost, the standard deviation of the cost and the cost probability density had all been determined theoretically for this control policy.

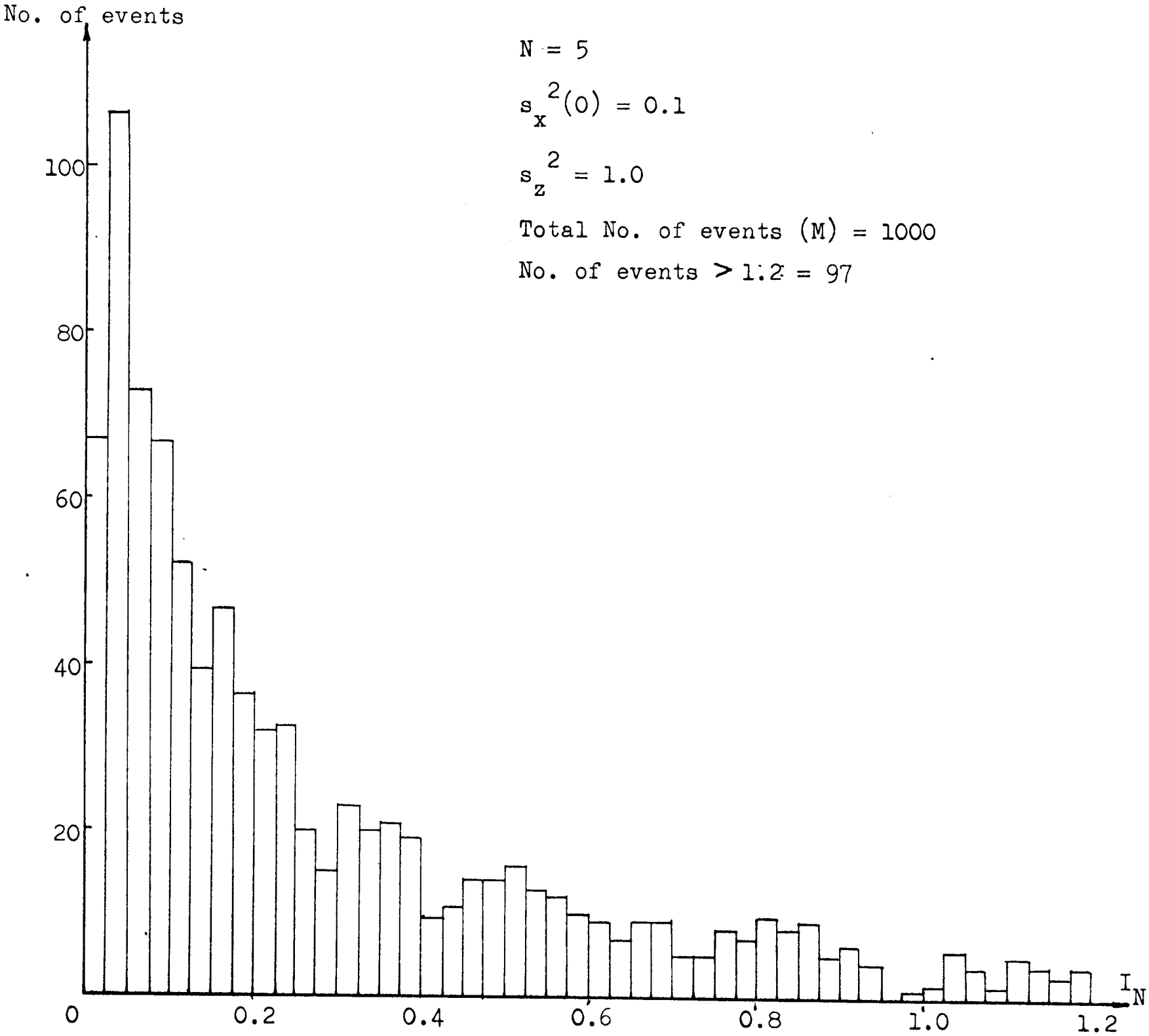
Further simulation results are given in Figures 2.3 to 2.13. Four sample histograms are produced in Figures 2.3 to 2.6. Figure 2.7 is the theoretically determined cost probability density when the no-measurement control is used. Figures 2.8 to 2.13 are comparisons of this cost probability density and the cost probability densities derived from histograms such as Figures 2.3 to 2.6.

2.6 Comparison of the Three Control Laws

In Figures 2.14 to 2.16 the expected costs given by the three control laws are compared. It can be seen from these figures that the relative improvement in expected cost given by the optimal control over the other two controllers increases rapidly as the length of the process increases. For a one-stage process all three control policies give identical performances but for a ten-stage process there is a considerable advantage to be gained in terms of the expected cost by using the optimal control policy.

From Table 2.1 it can be seen that for all three control policies the standard deviation of the cost is larger than the expected cost. This large variation of the cost from its expected value is also confirmed by the cost probability densities, Figures 2.8 to 2.13. Because of the skewness of the probability densities there is a large probability that the cost on any particular trial will be smaller than the average while there is a small probability that it will be much larger than the average. The standard

Figure 2.3 Histogram of Cost Using Optimal Policy



$N = 5$

$s_x^2(0) = 0.1$

$s_z^2 = 1.0$

Total No. of events $(M) = 1000$

No. of events $> 1.2 = 97$

Figure 2.4 Histogram of Cost Using No-memory Policy

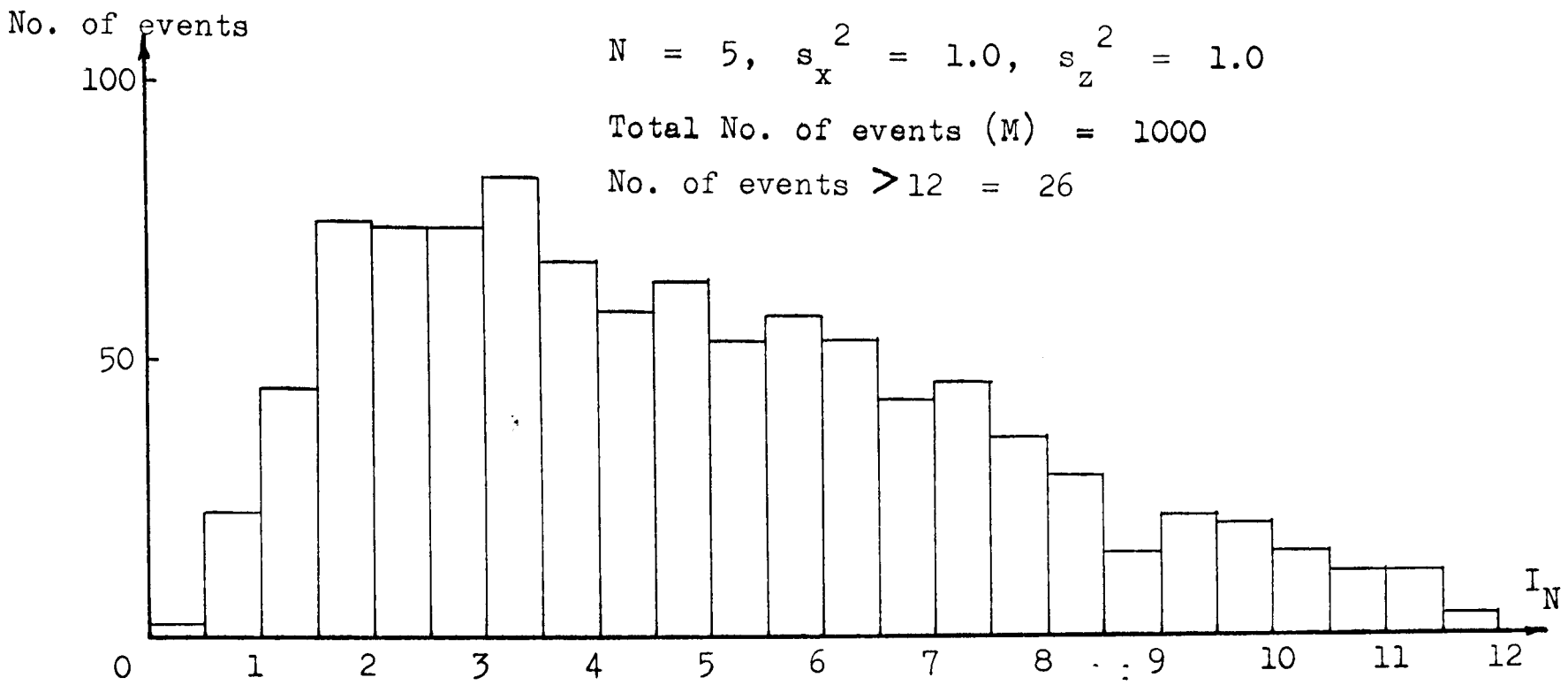


Figure 2.5 Histogram of Cost Using Optimal Policy

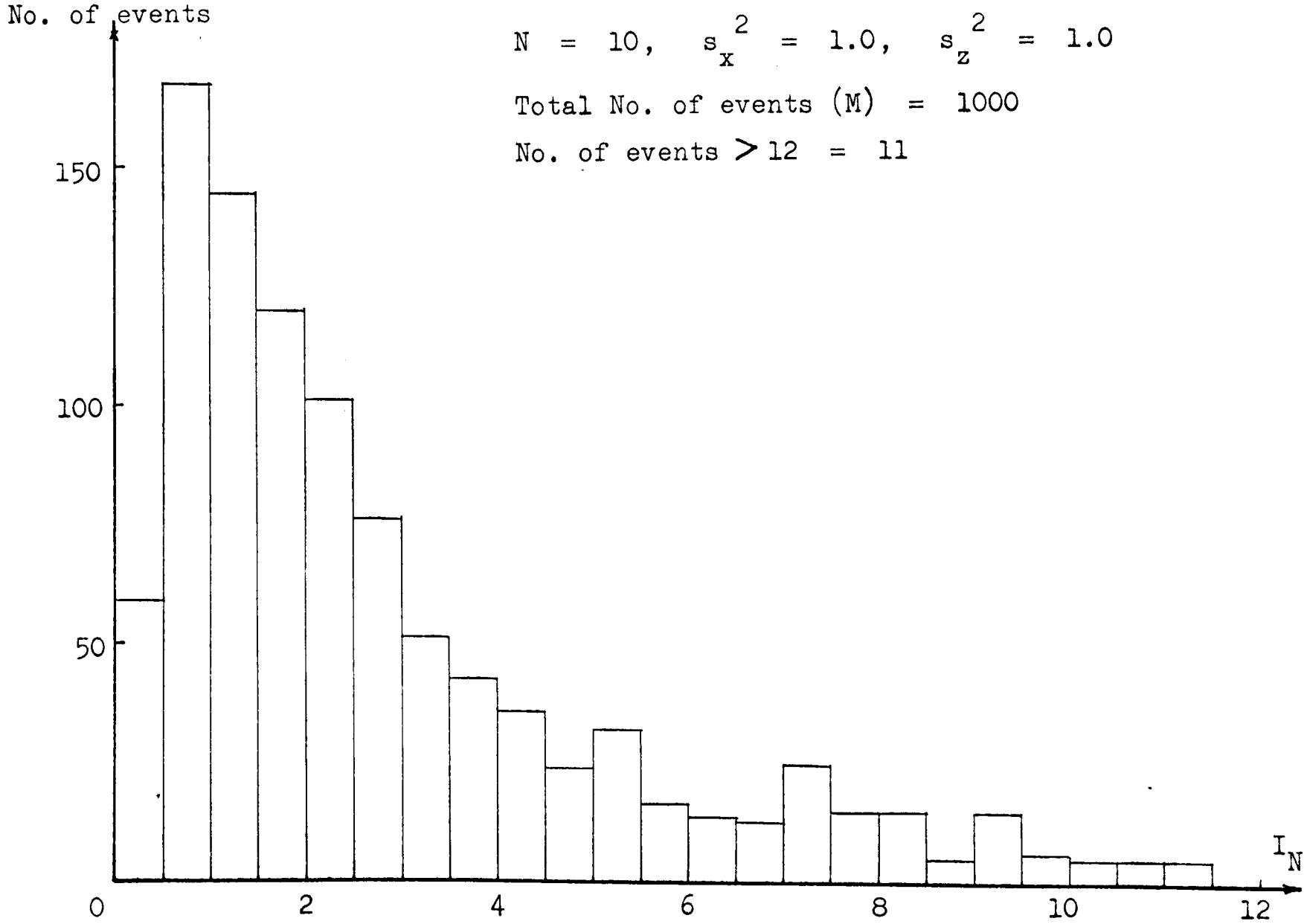
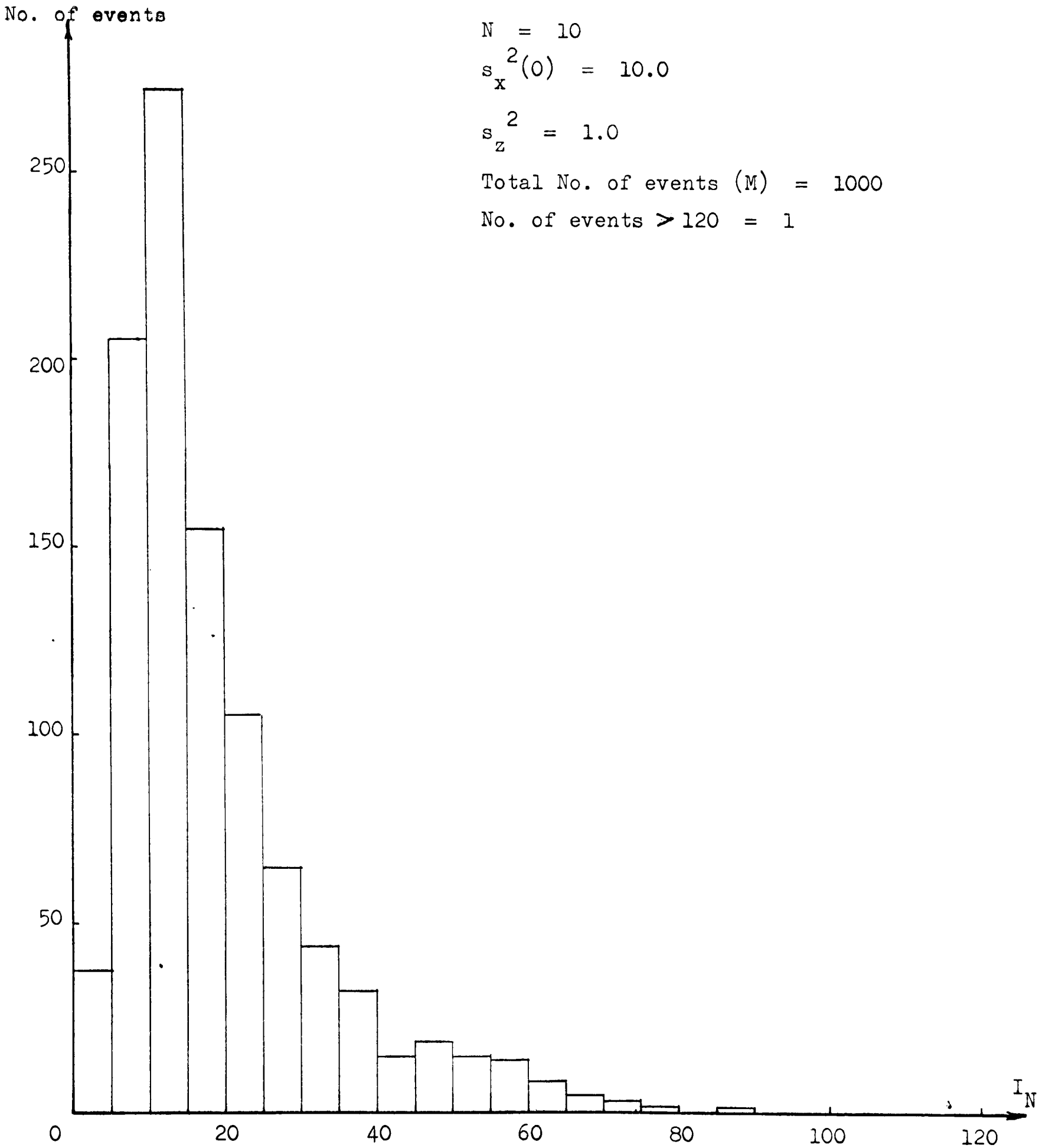


Figure 2.6 Histogram of Cost Using No-memory Policy



$N = 10$

$s_x^2(0) = 10.0$

$s_z^2 = 1.0$

Total No. of events (M) = 1000

No. of events > 120 = 1

Figure 2.7 Probability Density of Cost

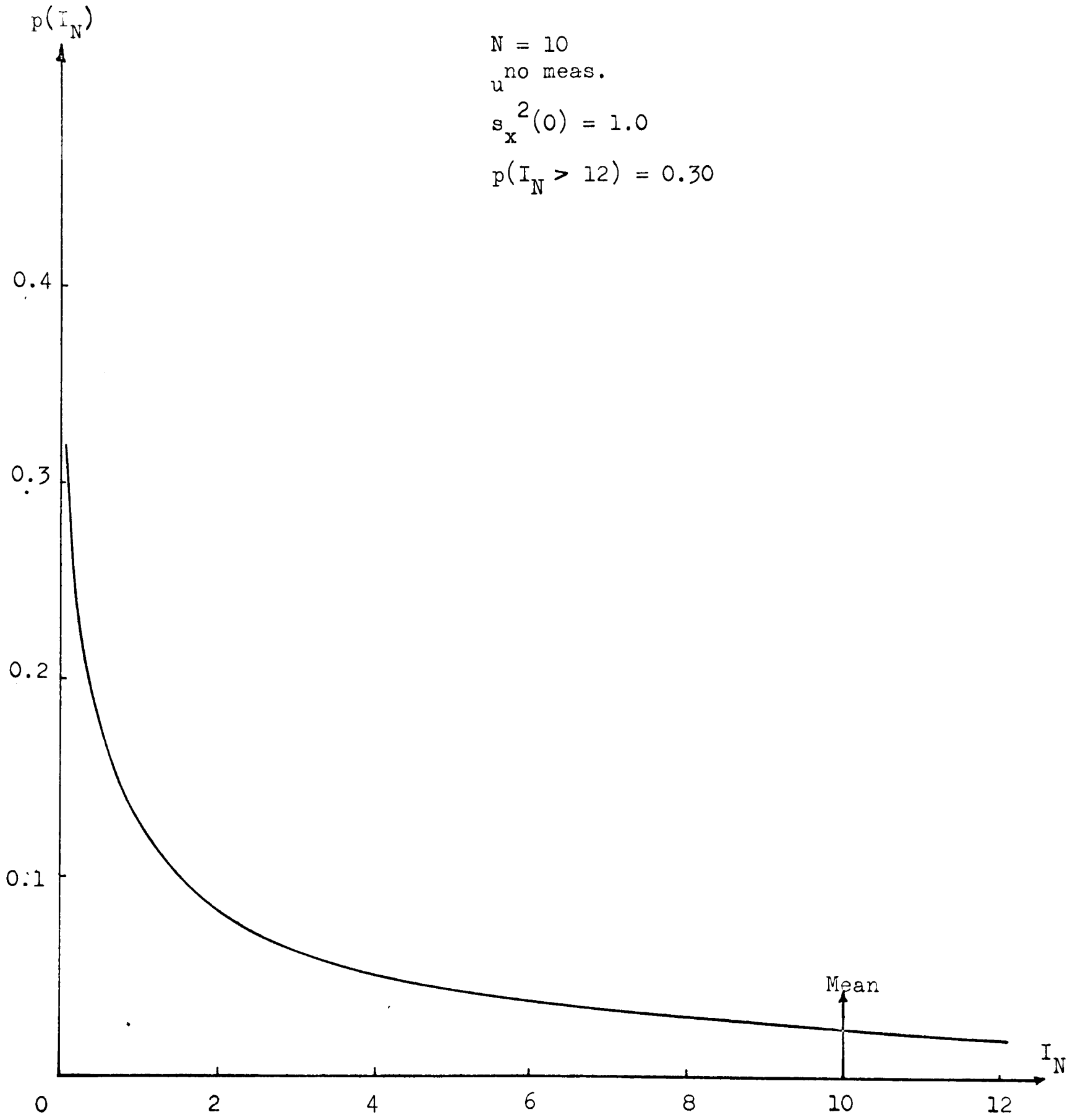
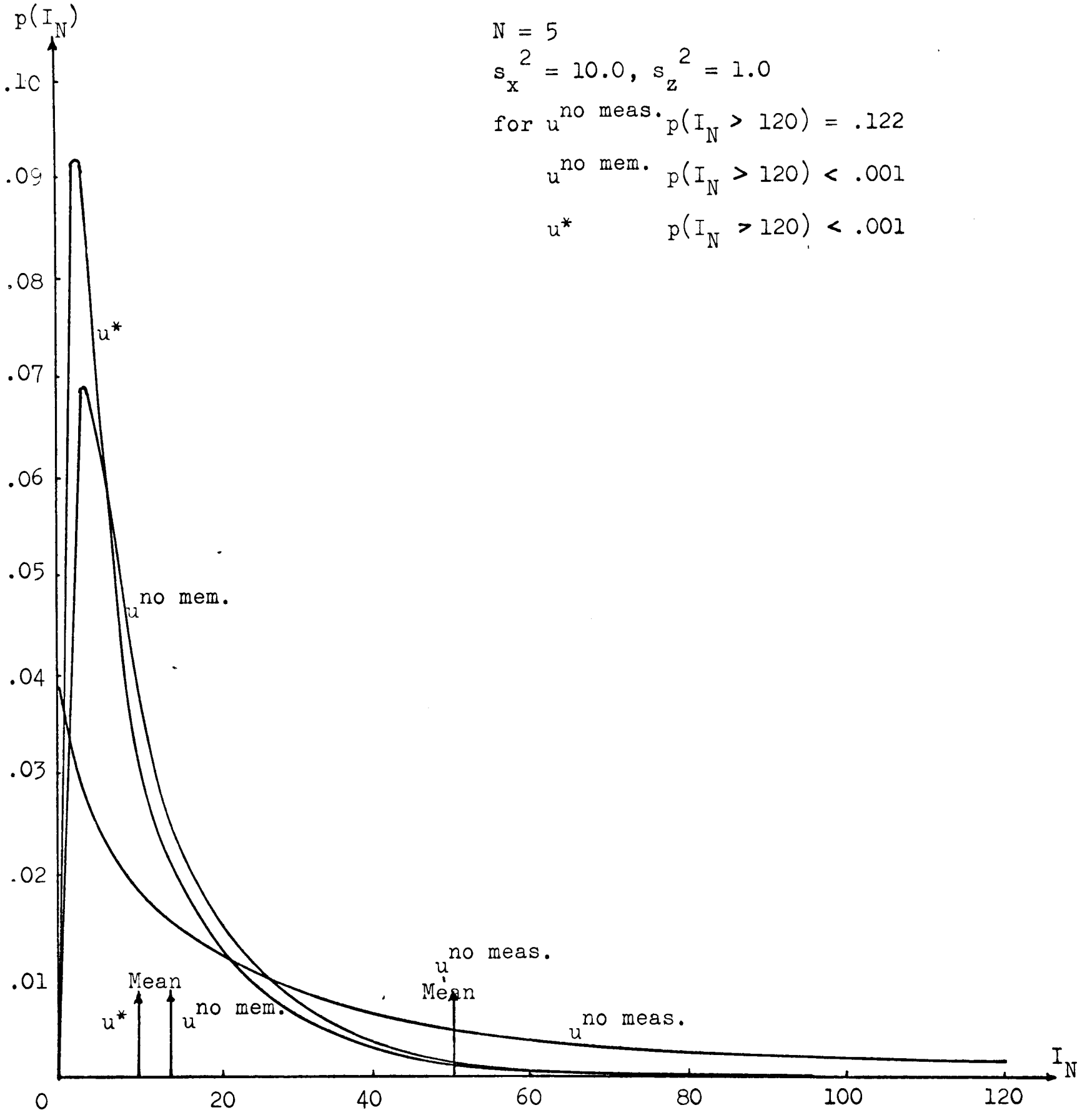


Figure 2.8 Comparison of Cost Probability Densities



$N = 5$

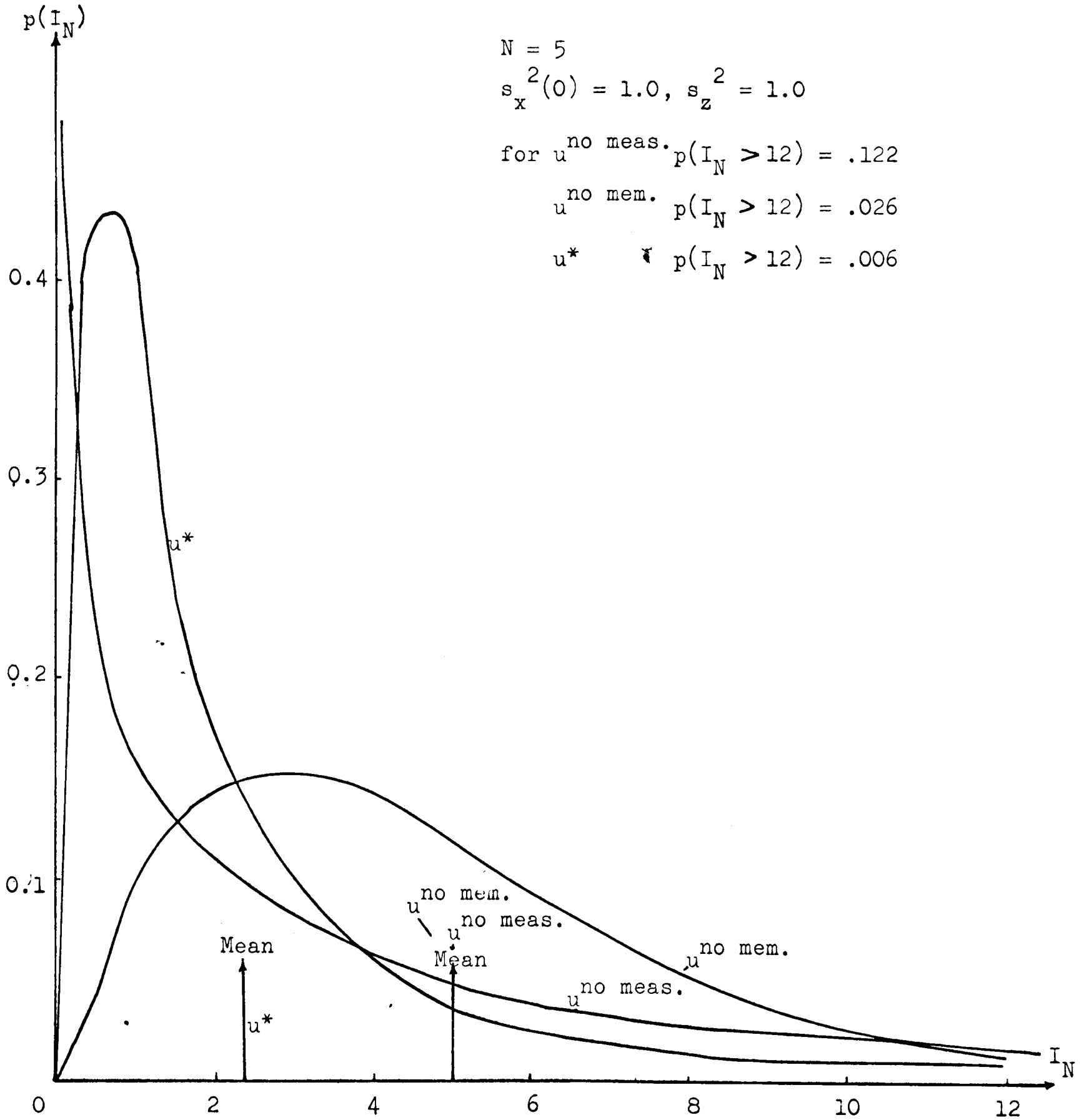
$s_x^2 = 10.0$, $s_z^2 = 1.0$

for $u^{\text{no meas.}}$ $p(I_N > 120) = .122$

$u^{\text{no mem.}}$ $p(I_N > 120) < .001$

u^* $p(I_N > 120) < .001$

Figure 2.9 Comparison of Cost Probability Densities



$N = 5$

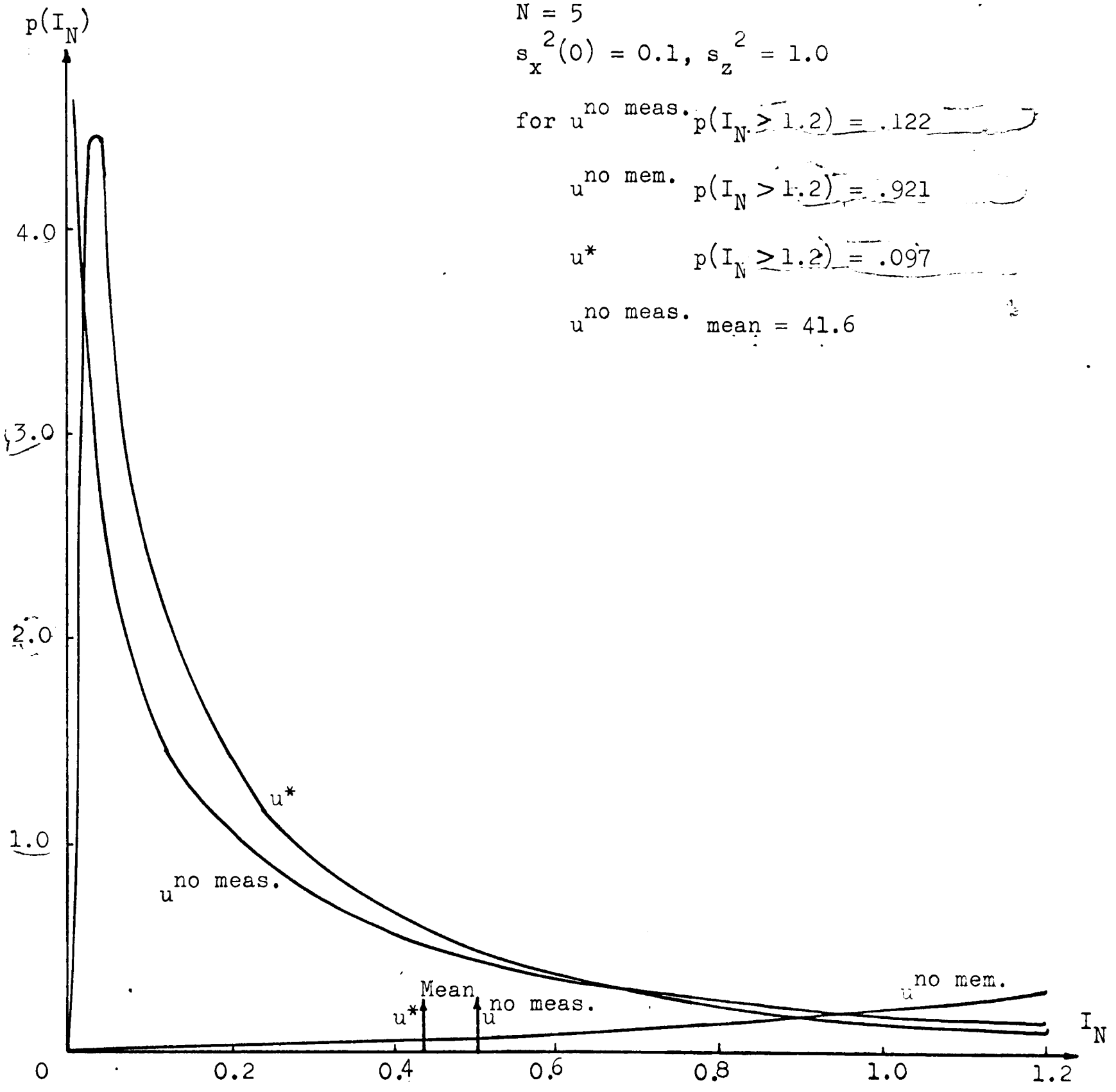
$s_x^2(0) = 1.0,\ s_z^2 = 1.0$

for $u^{no\ meas.}$ $p(I_N > 12) = .122$

$u^{no\ mem.}$ $p(I_N > 12) = .026$

u^* $p(I_N > 12) = .006$

Figure 2.10 Comparison of Cost Probability Densities



$N = 5$

$s_x^2(0) = 0.1,\ s_z^2 = 1.0$

for $u^{no\ meas.}$ $p(I_N \geq 1.2) = .122$

$u^{no\ mem.}$ $p(I_N > 1.2) = .921$

u^* $p(I_N \geq 1.2) = .097$

$u^{no\ meas.}$ mean = 41.6

Figure 2.11 Comparison of Cost Probability Densities

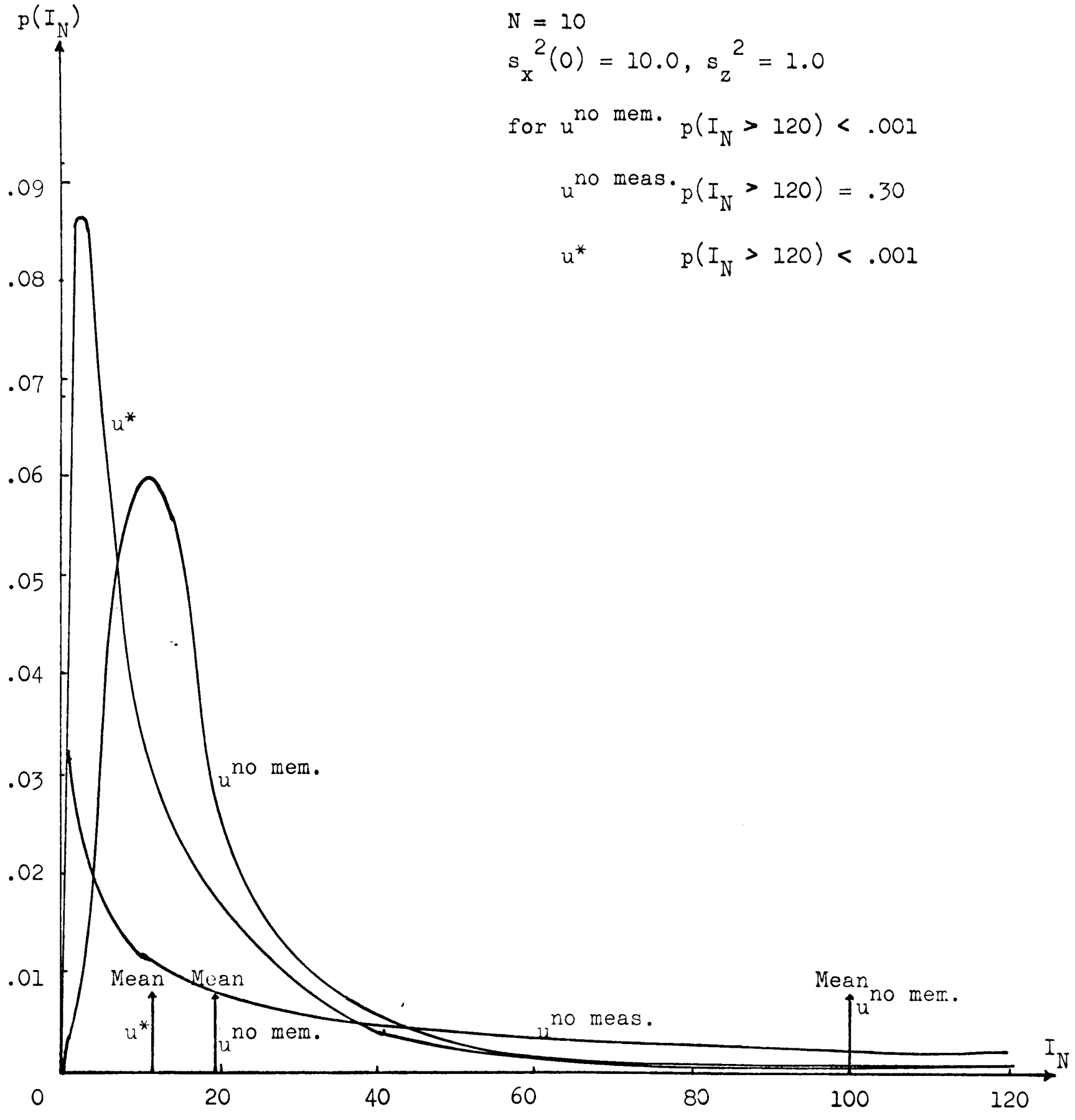


Figure 2.12 Comparison of Cost Probability Densities

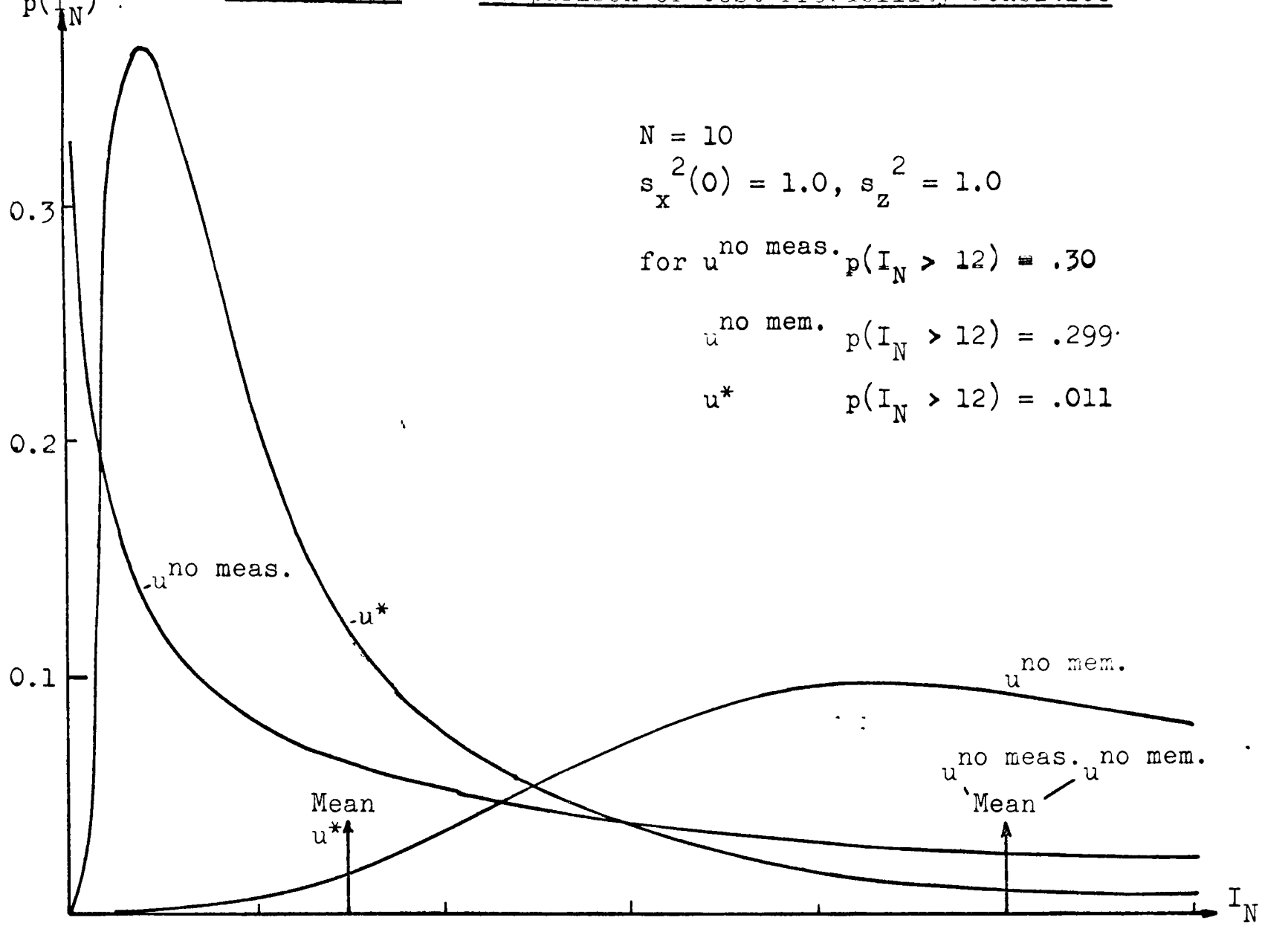


Figure 2.13 Comparison of Cost Probability Densities

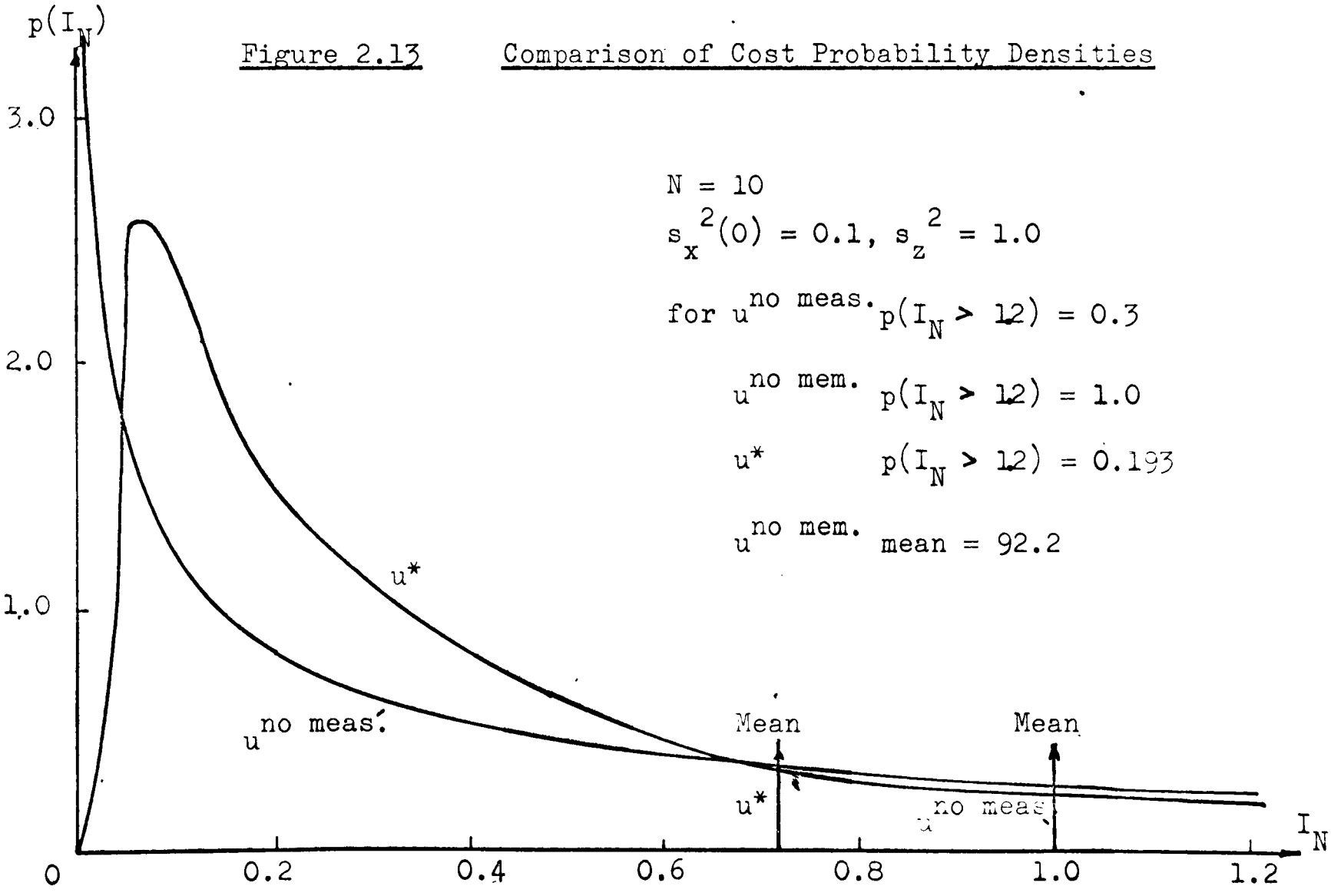


Figure 2.14 Expected Cost vs. Number of Stages To Go

$s_x^2 = 10.0,\ s_z^2 = 1.0$

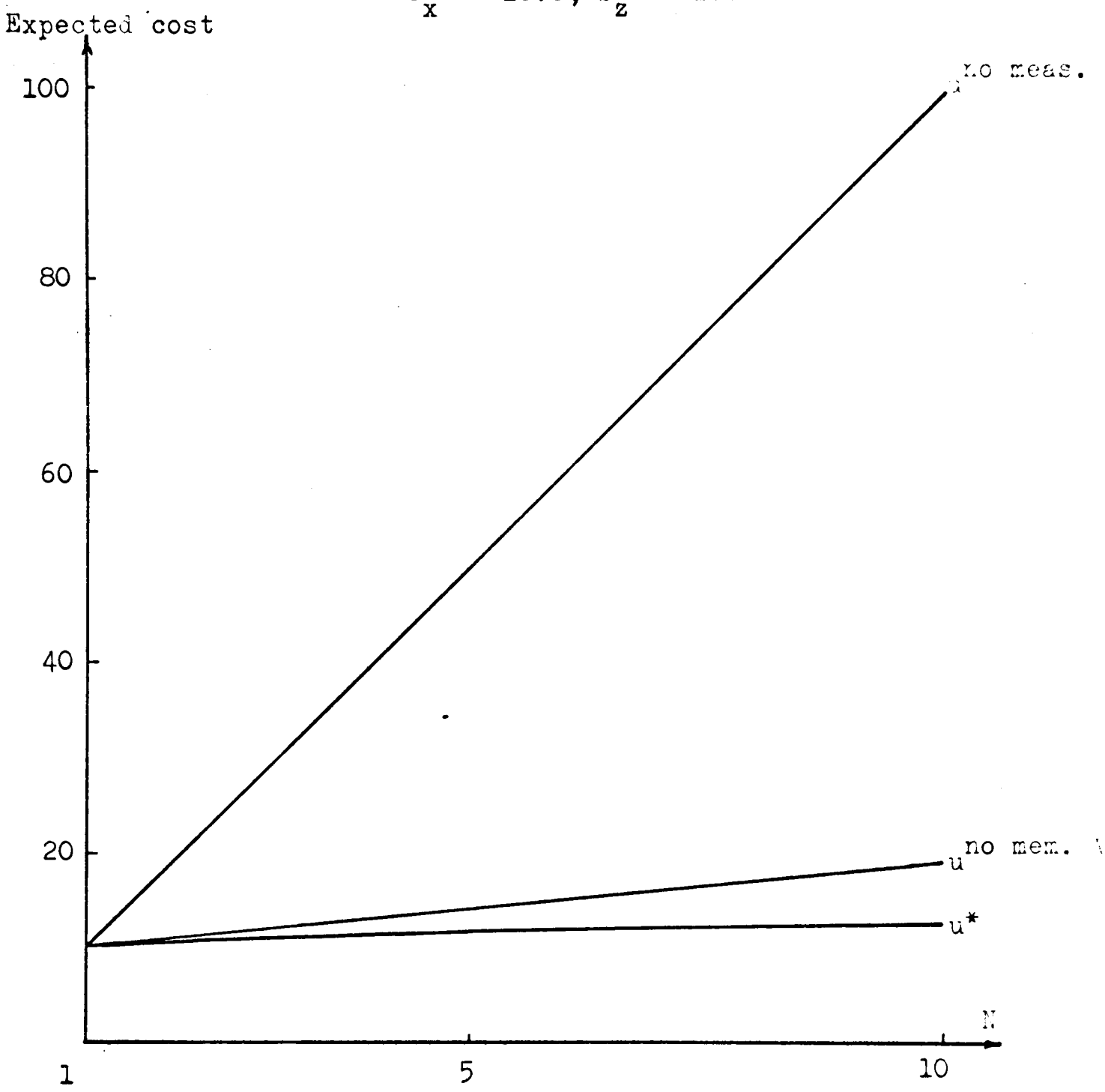


Figure 2.15 Expected Cost vs. Number of Stages To Go

$$s_x^2 = 1.0,\ s_z^2 = 1.0$$

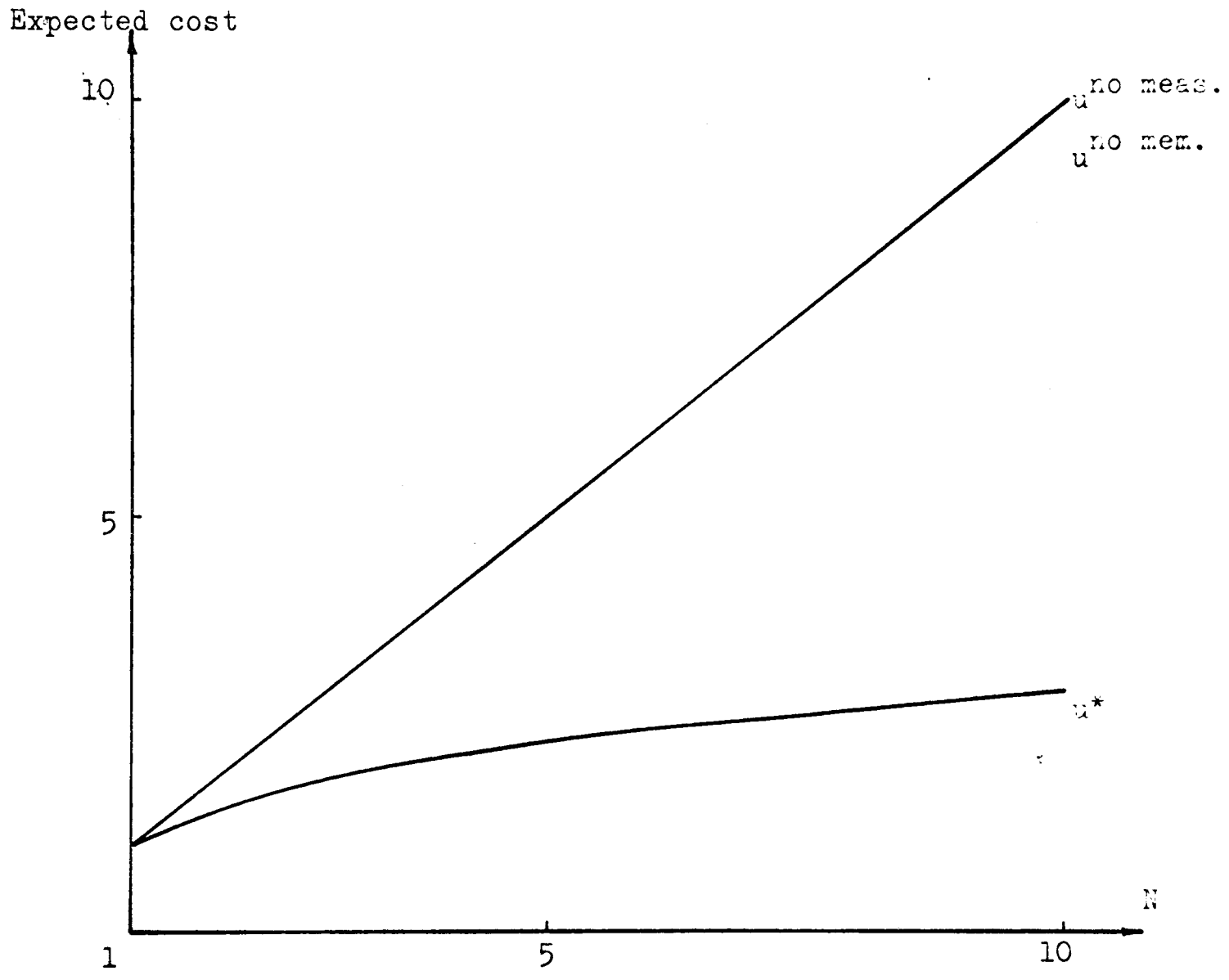
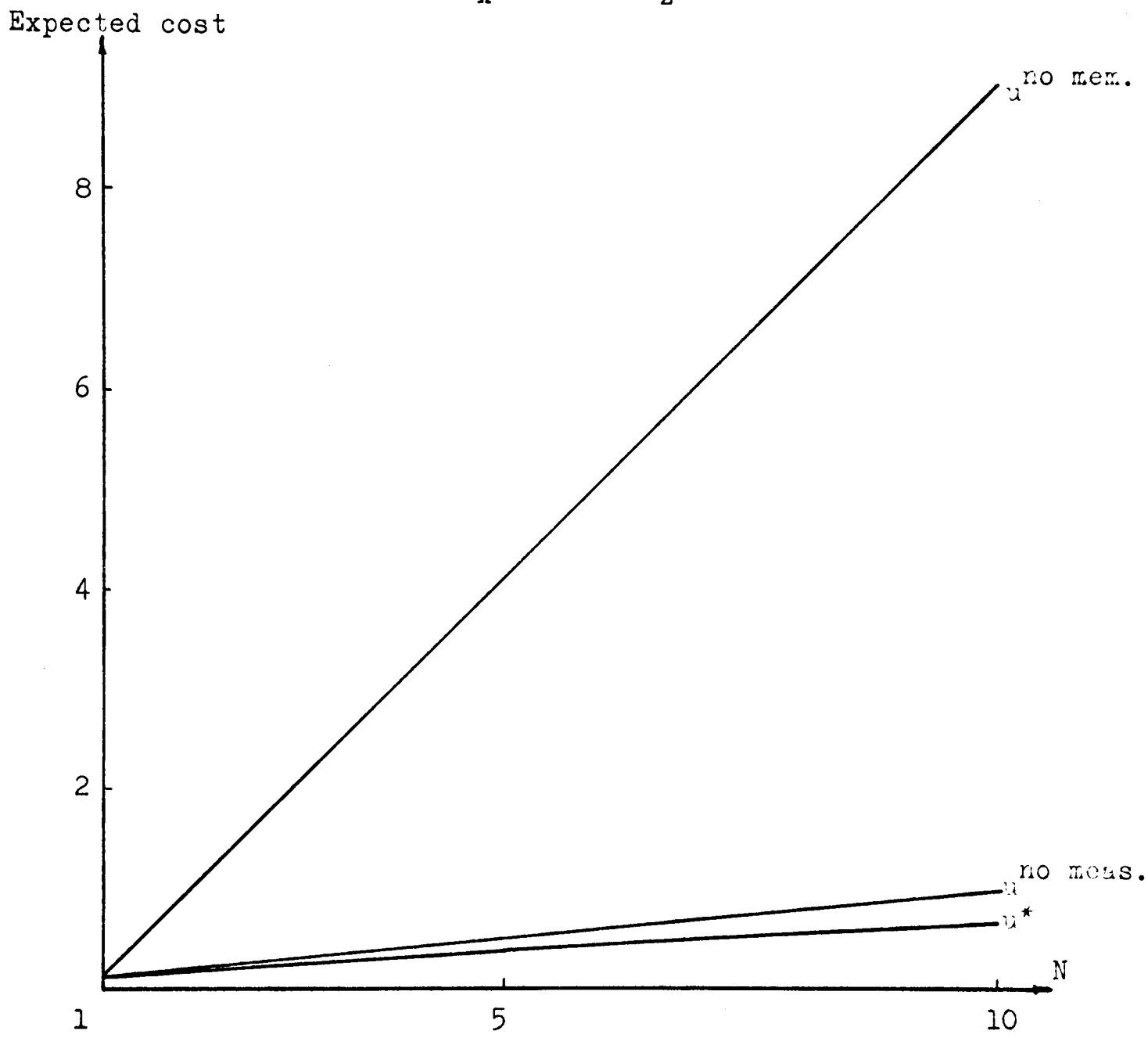


Figure 2.16 Expected Cost vs. Number of Stages To Go

$s_x^2 = 0.1,\ s_z^2 = 1.0$



deviation does vary with the control policy used and of the policies tried the optimal one gives the lowest standard deviation. This does not necessarily mean that there does not exist a control policy which would give an even smaller standard deviation for the cost, although it seems unlikely that the standard deviation could be reduced by very much. Thus this relative unpredictability of individual results is a feature of the problem.

It can also be seen from Table 2.1 that for the optimal control policy and the no-memory control policy the ratio of the standard deviation of the cost to the expected cost decreases as the length of the process (N) increases.* This can also be confirmed for the no-memory control policy by a comparison of Equations 2.26 and 2.27. Therefore the relative unpredictability of the results decreases as the length of the process increases.

Three points of relevance to the particular problem considered in this chapter have been made above:

1) The advantage in terms of expected cost given by the optimal control over the other two control policies increases rapidly as the length of the process increases.

2) This advantage may be obscured by the large standard deviation of the cost.

3) This obscuring effect is less for long processes than for short processes.

The relevance of these points to stochastic control problems in general is now considered. For the particular problem studied in this chapter, uncertainties account for the entire cost incurred. If there were no uncertainties, perfect control would be possible and the cost would be zero. This is not true,

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* The ratio is independent of the length of the process for the no-measurement control policy because this is a special case in which the state is a constant.

however, for stochastic problems in general, as dynamic effects (the inability of the state to change instantaneously) or time lags could also contribute to the cost. For this reason the improvements in expected cost gained by applying stochastic control theory are not necessarily as large in all stochastic control problems as they are in this one. Also the magnitude of the standard deviation of the cost relative to the expected cost is not necessarily as large as for this problem. However, for any stochastic control problem in which uncertainties do account for a large part of the cost, the three points made above will be of some importance when the decision is being made as to whether or not to apply stochastic control theory. If the process to be controlled is relatively short, then very probably the advantage gained by using stochastic control theory will be small and moreover this small advantage will be masked by the large variations in the results obtained. However for a longer process the advantages are greater and the relative effect of variations in the results are smaller and therefore stochastic control will be more likely to produce significant gains.

One further point of general interest can be made from the cost probability densities, Figures 2.8 to 2.13. If instead of minimizing the expected cost it was desired to maximize the probability of the cost being less than a certain value, the problem would be considerably altered. For example, if in Figure 2.12 it was desired to maximize the probability of the cost being less than 0.1, then conversely to what might be expected, the no-measurement policy would give a much better result than would the so-called optimal policy. This illustrates that when stochastic control theory is being applied it is very important to ensure that the performance criterion used accurately describes the desired result. Costly mistakes can result from paying too little attention to this point.

2.7 Conclusion

From the study of the simple stochastic control problem reported in this chapter the following observations on stochastic control problems in general can be made:

1) There are four requirements for the complete formulation of a stochastic control problem. They are a mathematical description of i) the process controlled, ii) the measurement process, iii) the uncertainties involved and iv) the performance criterion.

2) There are two stages in the determination of an optimal stochastic control law. The first is to solve the learning problem of combining previous knowledge of the problem with knowledge which becomes available during the course of the process; the second is the formulation and solution of the problem of minimizing the cost function.

3) Solution of both of these problems is greatly simplified if a convenient summary of the knowledge about the system is available.

4) Realization of an optimal stochastic control law may well require a digital computer which must be large enough and fast enough to update the summary of the knowledge and to compute the optimal control between one stage and the next.

5) Empirical control laws may be preferable to the optimal control law, one reason being that they are often more easily realized than the optimal control law.

6) Optimal stochastic control laws show more advantage over empirical control laws for long processes than for short. This is because the relative difference in expected costs is greater for long processes and this cost improvement is not masked as much by the high standard deviation of the cost.

7) It is important to formulate the performance criterion carefully so

that it accurately describes the desired performance.

In the light of the introduction to stochastic control theory presented in this chapter, a general formulation of stochastic control problems is given and discussed in Chapter 3.

Chapter 3

Stochastic Control Laws

3.1 Introduction

In theory, most stochastic control problems once formulated can be handled by techniques similar to those used in Chapter 2. However, because for most problems complete solutions in the form of updating equations and optimal control policies are not available, it is necessary to study stochastic control problems in general so that techniques for deriving approximate solutions can be developed.

A general stochastic control problem is formulated in this chapter and the solution of this problem is discussed in two parts, estimation and control implementation. It is shown that neither of these parts of the general problem can be handled by presently available mathematical techniques and that for most stochastic control problems some approximations must be made to obtain a solution. The possible interaction between estimation and control implementation is discussed and it is shown that an understanding of this interaction is essential to the derivation of suboptimal control laws.

3.2 Formulation

In this section a general formulation of stochastic control problems is presented. The formulation described is for discrete-time systems as opposed to continuous time systems for three reasons:

1) Digital computers operate in discrete time rather than in continuous time and thus a discrete time formulation is a more realistic representation of a system which includes a digital computer.

2) Working in discrete time makes the mathematics of the problems much

simpler and leads to a better intuitive understanding of the concepts involved.

3) The concepts can then be applied to continuous time problems for which the formulation is more complex.

As was seen from the statement of the known gain problem in Chapter 2, four elements must be expressed in mathematical form to completely specify a stochastic control problem. These four elements, the plant, the measurement process, the uncertainties and the performance criterion are considered in turn.

The first element which must be described in mathematical terms is the system or plant that is to be controlled. This is done by vector difference equations which define the way in which the state of the plant changes from one time to the next. The general form of these state equations is

$$\underline{x}(i+1) = \underline{G}_1(\underline{x}(i), \underline{u}(i), \underline{a}, \underline{z}_1(i), i) \qquad 3.1$$

where $\underline{x}(i)$ is a vector describing the state of the plant at time i. The state at the next time ($\underline{x}(i+1)$) in general depends on

$\underline{G}_1$ - a nonlinear vector-valued function describing the known dynamics of the plant

$\underline{x}(i)$ - the state at the previous time

$\underline{u}(i)$ - the control inputs used

$\underline{a}$ - a vector of unknown constants which describe any uncertainty about the plant dynamics that can be reduced.

$\underline{z}_1(i)$ - a random vector which describes any uncertainties about the plant dynamics that cannot be reduced

i - the discrete time integer.

The state of the plant cannot generally be measured exactly and therefore the second element of a stochastic control problem that must be described is the measurement process. In this thesis this is described by the vector equations

$$\underline{y}(i) = \underline{G}_2(\underline{x}(i), \underline{b}, \underline{z}_2(i), i) \qquad 3.2$$

Here $\underline{y}(i)$ is the measurable output of the plant and it depends on

$\underline{G}_2$ - a vector-valued nonlinear function representing the known form of the measurement process

$\underline{b}$ - a vector of unknown constants which describes any uncertainty about the measurement process that can be reduced

$\underline{z}_2(i)$ - a random vector which describes any uncertainty about the measurement process that cannot be reduced

i - the discrete time integer.

The third element which must be described in mathematical terms is the uncertainties that exist about the problem. In the formulation used here these are

i) uncertainty about random vectors $\underline{z}_1$ and $\underline{z}_2$

ii) uncertainty about unknown constant vectors $\underline{a}$ and $\underline{b}$

iii) uncertainty about the initial value of the state $\underline{x}(0)$.

These uncertainties are described by probability densities for the random vectors

$$p(\underline{z}_1(i)), \; p(\underline{z}_2(i))$$

and by a joint probability density for the state and unknown constants

$$p(\underline{x}(0), \underline{a}, \underline{b})$$

The fourth element that must be expressed in mathematical form is the performance criterion or the object of the control. In this thesis

performance criteria are restricted to being in the form of a cost function

$$I = \sum_{i=1}^{N} H(\underline{x}(i), \underline{u}(i-1), i) \qquad 3.3$$

where

H is a positive scalar function representing the cost incurred at a particular time

I represents the total cost incurred by a process lasting from time $i = 1$ to time $i = N$.

The object of the control is to minimize the total cost.

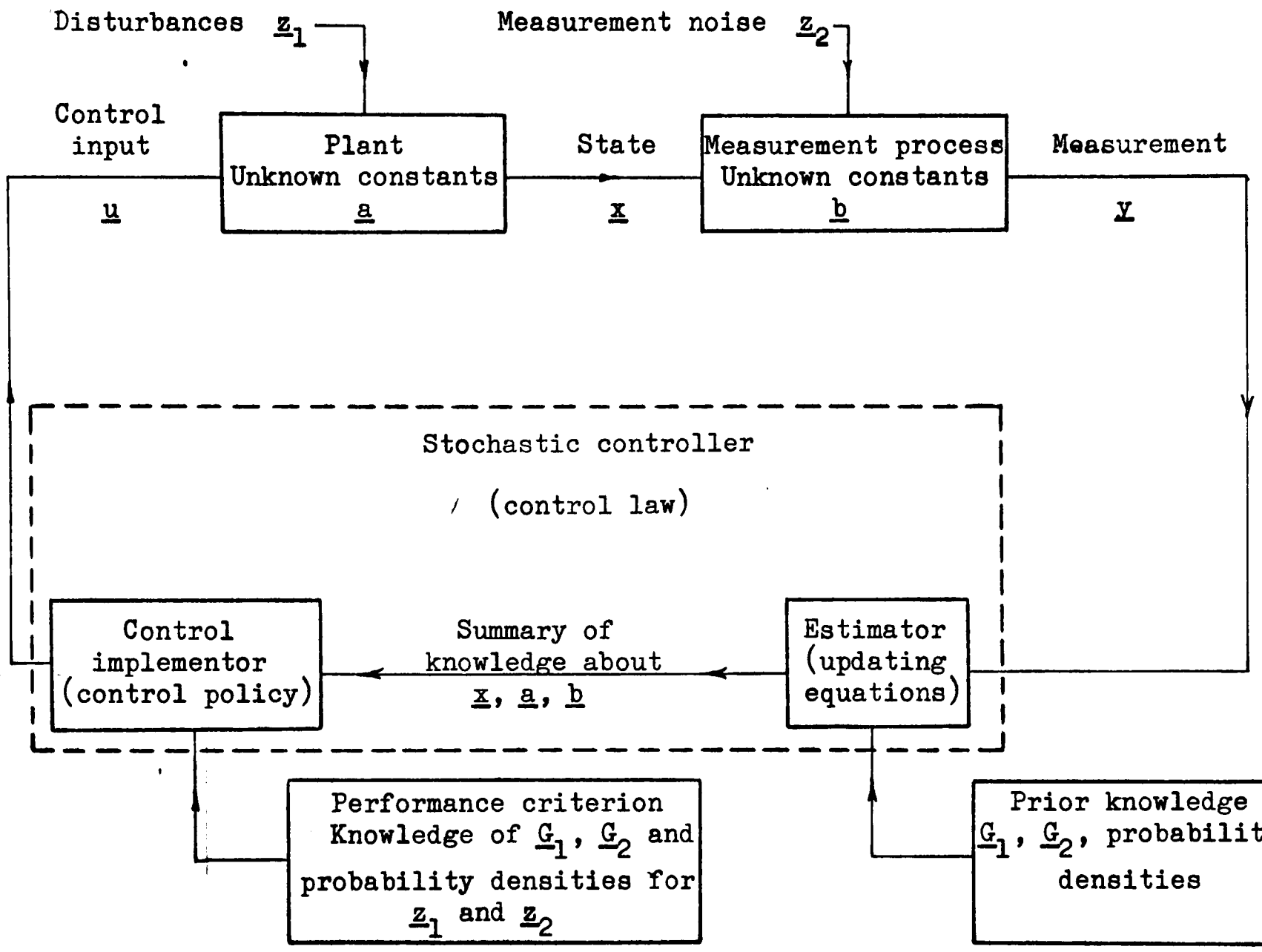
The stochastic control problem formulated in this section is illustrated in Figure 3.1. The purpose of this thesis is to study the structure of the control law denoted by the dashed box marked 'control law'.* The fundamental observation about the structure of stochastic control laws is that a stochastic controller performs two functions: it processes the facts given to it (i.e. the measurements and the prior knowledge of the system) in order to reduce them to a usable form, and it determines the control output on the basis of these processed facts. In this thesis the first function is called estimation and is accomplished by an estimator while the second function is called the control implementation and is accomplished by a control implementor. The first function takes the mathematical form of updating equations while the mathematical form of the second function is the control policy. The two taken together constitute a stochastic control law and the realization of the control law is a stochastic controller.

The remainder of this chapter is devoted to considering estimation and control implementation and the interaction between these two functions.

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* Other problems such as the modelling of the plant and the determination of suitable performance criteria are beyond the scope of this thesis.

Figure 3.1 Block Diagram of a General Stochastic Control Problem



3.3 Estimation

The object of the estimator is to provide the control implementor with a summary of the presently available knowledge about the state and unknown constants. The knowledge available at time i consists of

the prior knowledge given by $p(\underline{x}(0), \underline{a}, \underline{b})$

the past and present measurements $\underline{y}(1), \ldots, \underline{y}(i)$

the past control values $\underline{u}(0), \ldots, \underline{u}(i-1)$

and is symbolized by $\underline{y}^i$. This knowledge about the state and unknown constants can always be summarized by a conditional probability density for these quantities:

$$p(\underline{x}(i), \underline{a}, \underline{b} | \underline{y}^i)$$

Furthermore, this summary can in theory be updated by Bayes' rule:

$$p(\underline{x}(i+1), \underline{a}, \underline{b} | \underline{y}^{i+1}) = \text{const.} p(\underline{y}(i+1) | \underline{x}(i+1), \underline{a}, \underline{b}) p(\underline{x}(i+1), \underline{a}, \underline{b} | \underline{y}^i, \underline{u}(i))$$

Bayes' rule shows how to combine the previous knowledge (contained in $p(\underline{x}(i+1), \underline{a}, \underline{b} | \underline{y}^i, \underline{u}(i))$) with knowledge given by the latest measurement (contained in $p(\underline{y}(i+1) | \underline{x}(i+1), \underline{a}, \underline{b})$) to give an updated summary of the knowledge (contained in $p(\underline{x}(i+1), \underline{a}, \underline{b} | \underline{y}^{i+1})$). The important feature of Bayes' rule is its sequential nature in that only the latest measurement and control value and the previous summary are needed to produce the updated summary.

Unfortunately, the direct application of Bayes' rule to a stochastic control problem is seldom possible because in general the probability densities involved do not have analytic forms. As a result the continuous density functions must be stored as sets of points and the operation of Bayes' rule has to be done by a digital computer, usually a lengthy process since at least one numerical integration is involved. Some work is in progress on numerical techniques for dealing with Bayes' rule[97] but it will

be some time yet before general problems can be handled in this way.

For a small class of stochastic control problems the estimation problem can be solved analytically. These are the problems in which sufficient statistics[29] exist. Sufficient statistics are a finite set of variables which at any time summarize the joint conditional probability density for the state and unknown constants. As was done for the problem discussed in Chapter 2, updating equations for the sufficient statistics can usually be found by a straightforward application of Bayes' rule. The most important class of problems for which sufficient statistics exist are the LQP problems (linear equations, quadratic performance criteria, and normal probability densities).* Application of Bayes' rule to these problems gives the Kalman filter equations which are updating equations for the sufficient statistics of a general LQP problem. Nonlinear filtering techniques related to the Kalman filter equations have been applied to some non-LQP problems although they usually provide only approximate solutions in these cases.[98]

The problem of estimation is not discussed further in this thesis but it must be pointed out that a solution to this problem whether it be by Bayes' rule approximations or by non-linear filtering techniques is one of the essential elements in the solution of stochastic control problems.

3.4 Control Implementation

The objective of the control implementor is to use the processed facts about the state and unknown constants provided by the estimator together with prior knowledge about the plant and measurement process to

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* Examples of non-LQP problems for which sufficient statistics exist are found in Appendices A and B of this thesis.

minimize the total cost. However, because of the stochastic nature of the problem, the cost is a stochastic variable and only the average or expected cost can be minimized. The optimal control policy is defined as the series of control values which minimizes the expected total cost.

The basic tool for finding the optimal control policy is dynamic programming and, as was shown in Chapter 2, dynamic programming leads to a functional recurrence equation which must then be solved to find the optimal control policy. A functional recurrence equation can easily be derived for any stochastic control problem which comes within the general formulation given in Section 3.2 but the solution of these equations is another matter. For most problems sufficient statistics do not exist and in these cases the minimum expected cost cannot be expressed as a function of a finite number of variables as it was in Chapter 2, but must be expressed as a <u>functional</u> of the joint conditional probability density for the state and unknown constants. Mathematical techniques for dealing with functionals are virtually nonexistent and consequently any attempt to determine the optimal control policy for a stochastic problem which does not have sufficient statistics fails at this point. The only possible recourse is to use a set of approximate sufficient statistics and a numerical approach but because of the immense complexity of such an approach this has never been attempted.

For problems in which sufficient statistics do exist the picture is somewhat brighter, although even for these problems functional recurrence equations have been solved only in a few simple cases. Complete solutions can be obtained for the class of LQP problems mentioned previously (the known gain problem in Chapter 2 is an example of this) but for other problems numerical procedures are almost invariably necessary. For these problems the difficulty of finding the optimal control policy increases in

proportion to the number of sufficient statistics involved, so that only relatively simple cases have been tackled. The unknown constant gain problem (Chapter 6) is possibly the most complex stochastic control problem that has been approached in this way and the difficulties encountered are discussed in that chapter and its associated appendices.

Because of the difficulty in determining optimal control policies, suboptimal policies which are approximations of the optimal policy may in many cases be the best answer. Some general methods for finding suboptimal policies and their advantages and disadvantages are considered in Chapter 7. Some of the difficulties of realizing stochastic control laws and in particular of realizing the control policy part of the control law are also considered in Chapter 7.

3.5 Interaction between Estimation and Control Implementation

The fundamental interaction between estimation and control implementation is that the control policy is based on the estimates of the unknown constants and of the state produced by the estimator. There are two further ways in which the estimator and control implementor may interact:

1) The accuracy of the estimated values of the unknown constants and the state may affect the control policy.

2) Because it acts on the plant and hence on the measurement, the control policy may affect the rate at which uncertainty about the unknown constants and the state is reduced.

Both of these effects can result in modifications in the control policy, and in this thesis the modification due to the first effect is called caution while that due to the second is called probing.

In the next chapter three groups of stochastic control problems in which either there is no interaction (certainty equivalent problems)

or the interaction is limited in some way (neutral and separable problems) are defined and discussed. The work presented in that chapter allows the precise definition of caution and probing given in Chapter 5 to be made.

Caution and probing are discussed through examples in Chapters 5, 6 and 7 and it is shown that the lack of understanding of these concepts is in large part responsible for the present dearth of successful practical applications of stochastic control theory. It is also shown that when sub-optimal control policies are being derived, caution and probing should be used to take into account the possible interaction between the estimator and the control implementor.

3.6 Summary

Stochastic control laws perform two functions which in this thesis are called estimation and control implementation. The ability to produce mathematical representations of both functions, in the form of updating equations for estimation and a control policy for the control implementor, depends largely on the existence of sufficient statistics. For problems in which sufficient statistics do exist, the estimation problem can be completely solved by a straightforward application of Bayes' rule but the control implementation problem is not usually so easily dealt with and often must be solved by numerical means. For problems in which sufficient statistics do not exist the estimation problem can occasionally be solved approximately, but solution of the control implementation problem is rarely, if ever, available. Thus approximate solutions must be used for almost all stochastic control problems.

Knowledge of the possible interaction between estimator and control implementor is essential to the derivation of approximate control policies and to a lesser extent to the derivation of approximate estimation procedures.

Lack of awareness of this interaction has been one of the main reasons that the application of stochastic control theory has not been more successful.

Chapter 4

Certainty Equivalence, Neutrality and Separability

4.1 Introduction

In the previous chapter it was noted that few stochastic control problems can be solved completely and that for most problems suboptimal control laws must be used. It will also be shown in Chapter 7 that the design of suboptimal control laws must be based on a firm understanding of the structure of stochastic control laws and as is noted in Chapter 3, a study of the structure of stochastic control laws is a study of the possible interaction between the functions of estimation and control implementation.

The groundwork for the study of this interaction is given in this chapter by considering three classes of stochastic control problems in which the interaction between estimation and control implementation is restricted in some way. By studying problems in these classes, the effects of the interaction on control policies can be observed and the precise definition of the interaction given in Chapter 5 can be made.

In this chapter certainty equivalence, neutrality and separability are defined and discussed and some existing confusion about the difference between them is clarified. The relationships between the three properties are investigated and the relevance of these relationships to the interaction between estimation and control implementation is discussed. Three control policies derived from the three properties are defined. These are used in Chapter 5 to define caution and probing and in Chapter 7 as a basis for deriving suboptimal control policies.

4.2 Definition of Certainty Equivalence, Separability and Neutrality

In this section the three properties - certainty equivalence, neutrality and separability - are introduced, defined and discussed. For these definitions the notation

$$\underline{u}^*(i) = \underline{g}^*(p(\underline{x},\underline{a},\underline{b}\,|\underline{y}^i),p(\underline{z}_1(i)),p(\underline{z}_2(i)),\ldots p(\underline{z}_1(N)),p(\underline{z}_2(N))) \qquad 4.1$$

is used to denote the optimal control policy for the general problem formulated in Section 3.2.* This notation describes the dependence of the optimal policy on

1) the existing knowledge of the state and unknown constants

2) the known probability densities for the random inputs from time i to time N.

4.2.1 Certainty equivalence

Certainty equivalence was the first of the three properties to be described in the literature and has been the most widely discussed. The certainty equivalence principle was first demonstrated by Simon[25] in connection with a specific problem taken from economics. This principle has since been more exactly defined and proved to hold for certain general classes of stochastic control problems.[12,16,17,19,24,26] The definition of certainty equivalence given here is the one described in Patchell and Jacobs.[23]

A stochastic control problem is defined as being certainty equivalent if the functional form of its optimal control policy is the same as that of the optimal control policy for an equivalent deterministic problem. The deterministic equivalent to the problem of Equations 3.1,

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* In precise mathematical terms $\underline{g}^*$ is a functional as opposed to a function.

3.2 and 3.3 has dynamics described by

$$\underline{x}(i+1) = \underline{G}_1(\underline{x}(i), \underline{u}(i), \underline{a}', \underline{z}'(i), i) \qquad 4.2$$

where $\underline{G}_1$ is the same as in Equation 3.1 but $\underline{a}'$ and $\underline{z}'(i)$ are now deterministic variables replacing the random variables $\underline{a}$ and $\underline{z}_1(i)$.

The optimal deterministic control policy to minimize the scalar function I of Equation 3.3 can be written

$$\underline{u}^*(i) = \underline{g}_{c.e.}(\underline{x}(i), \underline{a}', \underline{z}'(i), \ldots, \underline{z}'(N), i) \qquad 4.3$$

The stochastic problem of Equations 3.1, 3.2 and 3.3 is then certainty equivalent if and only if the optimal control law of Equation 4.1 can be written

$$\underline{u}^*(i) = \underline{g}_{c.e.}(E\{\underline{x}(i)|\underline{y}^i\}, E\{\underline{a}|\underline{y}^i\}, E\{\underline{z}_1(i)\}, \ldots, E\{\underline{z}_1(N)\}, i) \qquad 4.4$$

where $\underline{g}_{c.e.}$ in Equation 4.4 is the same as $\underline{g}_{c.e.}$ in Equation 4.3.

This definition means that if a stochastic control problem is certainty equivalent, the optimal control policy can be derived as if the problem were completely deterministic. This design procedure can be used because in a certainty equivalent problem the degree of uncertainty about the state and the unknowns does not affect the control policy. Also any affect that the control policy may have on the uncertainty about the state and unknowns is not relevant to the optimal control of the problem. In other words there is no interaction between estimation and control implementation in a certainty equivalent problem.

Because they are the most easily solved, LQP stochastic control problems were quite naturally the first problems to be considered when stochastic control theory was in its infancy. From the sufficient conditions for certainty equivalence* it is known that all LQP problems are

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* These, as well as sufficient conditions for neutrality and separability, are summarized in Patchell and Jacobs[23], a copy of which is bound into the

certainty equivalent. Because there is therefore no interaction between estimation and control implementation the determination of stochastic control laws for LQP problems consists of two completely independent steps: the determination of the estimator and the determination of the control policy. However, the assumption that this design procedure applies to non-certainty equivalent problems as well could result in a stochastic controller which worked very poorly, if at all. In most stochastic control problems the functions of estimation and control implementation are intimately coupled and this fact must be allowed for when stochastic controllers are being designed.

4.2.2 Neutrality

The property of neutrality was introduced by Fel'dbaum.[11] He noted that control implementors have a 'dual' character. The rate at which uncertainty is reduced may depend on the control policy and consequently the control policy may have a probing function as well as its primary function of producing the required plant behaviour. Depending on the particular problem considered, probing may or may not be needed and Fel'dbaum called problems in which probing was not needed, 'neutral'. Surprisingly, this idea of neutrality has not been devloped subsequent to Fel'dbaum's work.

Before defining neutrality precisely it is first necessary to define a measure of uncertainty and for this purpose the well-known information measure, negative entropy, is used. The information associated with the stochastic control problem of Section 3.2 at time i is defined as

$$s\left[p(\underline{x}(i),\underline{a},\underline{b}\,|\underline{y}^i)\right] = -\int_{(\underline{x}(i),\underline{a},\underline{b})} p(\underline{x}(i),\underline{a},\underline{b}\,|\underline{y}^i)\,\log\left(p(\underline{x}(i),\underline{a},\underline{b}\,|\underline{y}^i)\,d(\underline{x}(i),\underline{a},\underline{b}\right) \quad 4.5$$

---

back of this thesis for easy reference.

The stochastic control problem of Section 3.2 is then defined as being neutral if and only if the information associated with the problem at time i is independent of all past control inputs $\{\underline{u}(0), \ldots, \underline{u}(i-1)\}$.

In a neutral problem the rate at which uncertainty is reduced is independent of the control policy used and therefore in neutral problems the estimator and the control implementor interact in only one way: the accuracy of the estimation may influence the control policy. The reason that neutral problems are of interest is because one of the possible ways in which estimation and control implementation can interact can be examined on its own.

4.2.3 Separability

The idea that the two functions of stochastic controllers, estimation and control implementation, could in some way be separated[17,19,28,29] was, like certainty equivalence, introduced in the infancy of stochastic control theory. Since certainty equivalence describes a form of separation (complete independence) of the two functions, it is not surprising that a considerable amount of confusion has occurred about the meaning of the two terms. Although many definitions of separability are possible, that given here is perhaps the most popular one and is based mainly on the work of Striebel[29] and Wonham.[30]

A stochastic control problem is separable when the optimal control policy is a function only of the expected values of the state and of the unknown constants and not of any other parameters which cannot be considered as prior ~~information~~ knowledge.* The problem of Equations 3.1, 3.2 and 3.3 is defined

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* For example the variance of the state ($s_x^2(i)$) in the known gain problem could be considered as prior knowledge since from Equation 2.5b it could be computed for all future times.

as being separable if and only if the optimal control policy can be written

$$\underline{u}^*(i) = g_s(E\{\underline{x}(i)|\underline{y}^i\}, E\{\underline{a}|\underline{y}^i\}, E\{\underline{b}|\underline{y}^i\}, p(\underline{z}_1(i), p(\underline{z}_2(i)), \ldots, p(\underline{z}_1(N)), p(\underline{z}_2(N)), i) \quad 4.6$$

The difference between separability and certainty equivalence can be seen from a comparison of Equations 4.4 and 4.6. In Equation 4.4 the form of $g_{c.e.}$ is specified while in Equation 4.6 the form of g_s is not specified. Also in a separable problem the control implementor is allowed prior knowledge (such as the probability densities of the z's) which is unavailable to the control implementor of a certainty equivalent problem. Since the function $g_{c.e.}$ in Equation 4.4 is of the form required for g_s in Equation 4.6 it is true that all certainty equivalent problems are separable (which is only logical since in certainty equivalent problems the functions of estimation and control implementation are completely independent or separable). However separable problems are not necessarily certainty equivalent. This last point is considered further in the next section.

The way in which the interaction between estimation and control implementation is limited in separable problems is not quite as clear as for the other two properties. The restrictions are that

1) the effect of the control policy on the estimator is not restricted in any way

2) the effect of the accuracy of the estimation on the control policy is restricted in that no knowledge other than expected values and prior knowledge is available to the control implementor.

4.3 Relationships between Certainty Equivalence, Neutrality and Separability

Two questions are considered in this section:

1) What is the relationship between certainty equivalence and neutrality,

i.e. are there any examples of problems which are neutral but not certainty equivalent or certainty equivalent but not neutral?

2) What is the relationship between certainty equivalence and separability, i.e. are there any examples of problems that are separable but not certainty equivalent? (It is known that all certainty equivalent problems are separable).

A third less important question is about the relationship between separability and neutrality. Answers to the question follow from the answers to the two previous questions.

These questions are important because if any one of the properties is equivalent to another then one of the two need not be considered at all. Also, if one is a sufficient condition for another then a problem that is known to have the first property need not be tested for the second.

Two examples in Appendix A (Examples A.1 and A.2) illustrate that neutrality is not a sufficient condition for certainty equivalence. Since in both certainty equivalent and neutral control problems the control policy cannot affect the rate at which uncertainty is reduced, it must be the possible effect of the accuracy of the estimation on the control policy which causes some neutral problems to be non-certainty equivalent.

As is noted above (Section 4.2.2), Fel'dbaum[11] postulated that optimal control policies perform two functions: probing to increase the rate at which uncertainty is reduced and producing the required plant behaviour. In problems which are neutral but not certainty equivalent the optimal control policy uses no probing yet it does not concentrate entirely on producing the required plant behaviour (as it would if the problem were certainty equivalent). Therefore the existence of a third function of the optimal control policy is indicated. This function of the control policy

is to take into account the uncertainties about the plant controlled. It is called caution and is precisely defined in Chapter 5.

It might be assumed that all certainty equivalent problems are neutral since the presence of no interaction (certainty equivalence) ought to be a sufficient condition for a restricted interaction (neutrality). However, this is not necessarily true because in certain very special problems, such as Example A.3 in Appendix A, the three functions of the control implementor are all optimized by the same control policy. As is noted in Appendix A, such cases are very rare and for the most part can safely be ignored. Therefore it is usually true that certainty equivalent problems are also neutral.

The answer to the second question is important because of presently existing confusion about the differences and similarities between separability and certainty equivalence. Examples A.2 and A.4 in Appendix A and B.1 in Appendix B demonstrate that separability and certainty equivalence are not synonymous and that problems which are separable but not certainty equivalent are relatively common. It is therefore important when writing about either of these properties to distinguish which definitions are being used for which terms. This distinction is not always made (see, for example, Reference 14).

The answer to the third question about the relationship between neutrality and separability is, from Example A.1, that neutral problems are not necessarily separable and, from Example A.4, that separable problems are not necessarily neutral.

These results are summarized in Table 4.1.

Table 4.1

Relationships between Certainty Equivalence, Separability and Neutrality

Relationship	Comments
Neutrality not sufficient for certainty equivalence	Examples A.1 and A.2. This shows the existence of caution.
Certainty equivalence not sufficient for neutrality	Example A.3. This is an unusual case and most certainty equivalent problems are neutral.
Certainty equivalence is sufficient for separability	Shown to hold in Section 4.2.3.
Separability is not sufficient for certainty equivalence	Examples A.2, A.4 and B.1. Separability and certainty equivalence are not synonymous.
Neutrality not sufficient for separability	Example A.1.
Separability not sufficient for neutrality	Example A.4.

4.4 Certainty Equivalent, Neutral and Separable Control Policies

It is useful to define control policies which for a given problem are derived on the assumption that the problem is certainty equivalent, neutral or separable (it may or may not have any of these properties). Two of these policies, the certainty equivalent and neutral policies are used to define caution and probing in Chapter 5 and all three are tried as suboptimal control policies in Chapter 7.

4.4.1 The certainty equivalent control policy

The certainty equivalent control policy for a given problem is a control policy derived on the assumption that the problem is certainty equivalent. For the problem of Equations 3.1, 3.2 and 3.3 the certainty equivalent control policy is defined by Equation 4.4. The certainty equivalent control policy is a suboptimal policy for a problem that is not certainty equivalent and is the optimal policy if the problem is certainty equivalent. For any problem whether it be certainty equivalent or not there is only one certainty equivalent control policy.

4.4.2 The neutral control policy

A neutral control policy differs from the optimal control policy in that it uses no probing to reduce uncertainty. Although the concept of a neutral control policy is simple, because of the nature of the property of neutrality the definition is rather complex. For the understanding of the remainder of this thesis it is sufficient to know that the neutral control policy of a particular problem is derived on the assumption that that problem is neutral. The detailed definition which follows here, although unnecessary to the understanding of this thesis, is included because it describes a method for determining the neutral control policy for any given stochastic control problem.*

Information gained on the first stage of a process is of no use in controlling the first stage of the process because of the feedback (as opposed to feedforward) nature of the problem. Since information gained on the first stage of a process is only of use in controlling the remaining stages, neutrality is a property of these remaining stages. The sense in which neutrality is a property of all but the first stages of the process is that depending on whether or not the problem is neutral, the control will affect the information gained on the first stage which in

*The definition which follows is strictly true only for non-neutral problems; however, it is thought to apply to neutral

turn will be reflected only in the cost of the remaining stages. Thus Equation 3.3 is considered in two parts:

$$E\{I\} = E\{H(\underline{x}(1), \underline{u}(0), 1)\} + E\{\sum_{i=2}^{N} H(\underline{x}(i), \underline{u}(i-1), i)\} \qquad 4.7$$

Neutrality is a property of the second term on the right hand side of Equation 4.7. Even if the problem is neutral, this term depends on the initial control decision because the state at time $i = 2$ depends on $\underline{u}(0)$. If the problem is not neutral then the second term will depend on $\underline{u}(0)$ for the same reason and also because $\underline{u}(0)$ affects the amount of information gained at time $i = 1$. It is not possible to separate these two effects of $\underline{u}(0)$ on the second term for the general case and thus a related neutral problem is considered from which a neutral second term is derived.

The neutral problem related to the problem of Equations 3.1, 3.2 and 3.3 has dynamics described by

$$\underline{x}(i+1) = \underline{G}_1(\underline{x}(i), \underline{u}(i), \underline{z}_3(i), \underline{z}_1(i), i) \qquad 4.8a$$

Here $\underline{z}_3(i)$ is an independent random variable with a given probability density $p(\underline{z}_3(i))$ and replaces the unknown constant $\underline{a}$. $\underline{G}_1$ and $\underline{z}_1(i)$ are the same as in Equation 3.1. The state $\underline{x}$ is measured exactly:

$$\underline{y}(i) = \underline{x}(i) \qquad 4.8b$$

The cost function is given by Equation 3.3.

Since the unknown constant has been replaced by an unpredictable random variable and since the state is measured exactly at every stage, the problem of Equations 4.8 and 3.3 contains no uncertainty which can be reduced. Thus it is a neutral problem.

The minimum expected cost for the neutral problem is defined as

$f_N^{nt}[p(\underline{x}(0),\underline{z}_3(0))]$ $\triangleq$ the minimum expected value of the cost for an N stage process. The initial information about the state and random parameter $\underline{z}_3$ is described by the joint probability density $p(\underline{x}(0),\underline{z}_3(0))$.

Note that this particular functional form of the minimum expected cost function is used even though $\underline{x}(0)$ is known exactly and $\underline{z}_3$ is an unpredictable random variable because this form will be needed in Equation 4.9.

The neutral control policy for the problem of Equations 3.1, 3.2 and 3.3 is derived by replacing the second (non-neutral) term of Equation 4.7 with an expected cost term from the related neutral problem. This replacement term retains the dependence on $\underline{u}(0)$ caused by the dependence of the state at time $i=2$ on $\underline{u}(0)$ but because it is derived from a neutral problem it does not retain the dependence on $\underline{u}(0)$ due to information gained at time $i=1$. The neutral control for the problem of Equations 3.1, 3.2 and 3.3, when the problem is not neutral and there are N stages left to go and the initial condition is given by the joint probability density for the state and unknown constants, is then defined as the value of $\underline{u}(0)$ which minimizes the expression

$$E\{H(\underline{x}(1), \underline{u}(0), 1)\} + f_{N-1}^{nt}[p(\underline{x}(1), \underline{a}|\underline{y}^o, \underline{u}(0)] \qquad 4.9$$

Note that no expectation is needed for the second term since $p(\underline{x}(1),\underline{a}|\underline{y}^o,\underline{u}(0))$ is known at time $i=0$.* If the problem under consideration is neutral the neutral control policy is defined as the optimal control policy. The neutral policy is written

$$\underline{u}^{nt}(i) = \underline{g}_{nt}p(\underline{x}(i), \underline{a}|\underline{y}^i), p(\underline{z}_1(i)), \ldots, p(\underline{z}_1(N)), i) \qquad 4.10$$

As with the certainty equivalent policy there is only one neutral control policy for any given problem and for a neutral problem the neutral control

* The procedure outlined above defines the neutral control policy for non-neutral problems only, although this procedure works in many cases for neutral problems as well.

policy is (by definition) the optimal control policy.

4.4.3 The separable control policy

A separable control policy is a function only of the expected values of the state and unknown constants. When a separable policy is used, there is thus no need to know the probability densities or even the variances of the state and unknown constants. A separable control policy for the problem of Equations 3.1, 3.2 and 3.3 is defined by Equation 4.6. In contrast to the certainty equivalent and neutral control policies there is no unique separable policy. This is because the function g_s is not specified. For a separable problem any particular separable policy may or may not be the optimal policy.

4.5 Summary

Certainty equivalence, neutrality and separability are three possible properties of stochastic control problems. They are of interest in this thesis because they describe classes of control problems in which the interaction between estimation and control implementation is limited in some specified way. These are:

1) In certainty equivalent problems there is no interaction at all.

2) In neutral problems the control policy does not affect the estimation.

3) In separable problems the effect of the accuracy of the estimation on the control is limited.

Definition of these three properties and their related control policies permits the definition of caution and probing given in Chapter 5.

A study of the relationships between certainty equivalence, neutrality and separability gave two important results:

1) Stochastic control implementors generally have three functions: producing the required plant behaviour, probing and caution.

2) Separability and certainty equivalence are not synonymous.

Chapter 5

Caution and Probing

5.1 Introduction

At the end of the previous chapter it was noted that stochastic control implementors generally have three functions:

1) producing the required plant behaviour

2) probing to speed the rate at which uncertainty is reduced

3) being cautious in order to take into account the fact that the plant is not precisely known.

Three components of stochastic control policies can be associated with these three functions and in this chapter these components are defined and their significance is discussed.

These definitions are important because one of the main obstacles to the successful application of stochastic control theory, the difficulty of determining suitable control policies, is overcome by considering the control policies as being the sum of the three components discussed here. As is shown in Chapter 7, the knowledge of the existence of these components makes the determination of suboptimal control policies much less of a hit-and-miss affair than it has been in the past.

Probing is a term which has been used before in the control literature[11,51] but it has never been precisely defined. The concept of caution as presented here has not been previously considered.

5.2 Definition of Caution and Probing

As was noted in Section 4.4.2, the neutral control policy for a given problem performs all the functions of the optimal control policy except that it does not use probing in order to increase the rate at which uncertainty is reduced. Therefore probing is defined as the difference

between the optimal control policy and the neutral control policy:

$$\text{Probing} = \underline{u}^{*}(i) - \underline{u}^{nt}(i) \qquad 5.1$$

For the problem of Equations 3.1, 3.2 and 3.3

$$\text{Probing} = \underline{g}_{prob}(p(\underline{x}(i),\underline{a},\underline{b}|\underline{Y}^{i}),\ p(\underline{z}_1(i)),\ p(\underline{z}_2(i)),$$
$$\ldots,\ p(\underline{z}_1(N)),\ p(\underline{z}_2(N)),\ i)$$

i.e. probing depends on the same quantities as the optimal control policy does. For a neutral problem the probing is zero since in this case the neutral control policy is identical to the optimal policy, i.e. probing occurs only in non-neutral stochastic control problems.

Caution on the other hand may occur in any stochastic control problem whether it is neutral or not. For a stochastic problem the neutral control policy and the certainty equivalent control policy are generally different. Although the neutral policy uses no probing it can take into account the degree of uncertainty about the state and plant parameters and as a result is cautious. Since the neutral control policy is cautious and the certainty equivalent policy is not (it does not make allowances for uncertainty), caution is defined as the difference between the neutral policy and the certainty equivalent policy:

$$\text{Caution} = \underline{u}^{nt}(i) - \underline{u}^{c.e.}(i) \qquad 5.2$$

For the problem of Equations 3.1, 3.2 and 3.3

$$\text{Caution} = \underline{g}_{caut}(p(\underline{x}(i),\underline{a}|\underline{Y}^{i}),\ p(\underline{z}_1(i)),\ \ldots,\ p(\underline{z}_1(N)),\ i)$$

i.e. caution depends on the same quantities as the neutral policy does. For a problem which is certainty equivalent both caution and probing are zero.

Using these definitions of caution and probing the three components of stochastic control policies can be distinguished. Combining Equations

5.1 and 5.2 gives

$$\underline{u}^{*}(i) = \underline{u}^{c.e.}(i) + \text{caution} + \text{probing} \qquad 5.3$$

The first component is the certainty equivalent control policy and its function is to produce the required plant behaviour. The second component is caution and its function is to take into account the fact that the estimates of the state and unknowns are not exact. Caution is necessary because control actions based on inaccurate knowledge could produce costly errors. The third component is probing and its function is to probe the process in order to increase the rate at which uncertainty about the state and unknown parameters is reduced. The caution and probing components of the optimal control policy describe the two ways in which the functions of estimation and control implementation can interact. Caution is the effect of the accuracy of the estimation on the control implementor while probing is the effect the control implementor has on the estimation.

For most stochastic control problems all three components are essential if stochastic controllers are to operate properly.* A suboptimal control policy using two of the components, the certainty equivalent policy and probing, has been successfully applied to problems in which caution was not necessary (see Appendix B, Example B.5), but the three components together have never before been used to derive suboptimal control policies.

5.3 Examples Illustrating Caution and Probing

Six examples that illustrate caution and probing are discussed in Appendix B. The most important results demonstrated by these examples are considered in this section.

The most obvious statement that can be made about the examples in Appendix B is that none of the non-neutral problems could be solved analy-

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* This is demonstrated in Chapter 7.

tically. The estimation part of the problem was solved in each case but attempts to find the optimal control policy succeeded only in finding the optimal control policy for the case in which there is one stage left to go in the process. Determination of the optimal control policy for a larger number of stages to go was not possible because the necessary expectation integral could not be evaluated in analytic form. It cannot be categorically stated that optimal control policies cannot be determined analytically for any non-neutral control problem but this is true in most cases.

The examples in Appendix B confirm the belief expressed elsewhere in this thesis that most stochastic control problems are not certainty equivalent, or even neutral. Any non-linearity* or other departure from the LQP condition almost inevitably means the problem is not neutral and since almost all stochastic control problems contain some such nonlinearity very few are neutral. Since non-neutral problems can almost never be solved analytically, the point made in Chapter 1 that approximate controllers must be accepted for almost all stochastic control problems is confirmed (for reasons discussed in Chapter 6 and 7 a numerically determined solution is considered to be an approximate solution).

From the examples in Appendix B it is seen that both caution and probing do occur. Caution is a moderation of the control effort to offset the effects of uncertainties about the plant. In Examples B.1, B.3 and B.4, uncertainty about the gain through which the control operates results in a reduction of control effort from that which would be used if the gain

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* In this thesis non-linearities are taken to include such things as the random gains in Examples B.1 and B.3 and the unknown gain in B.4 as well as more obvious nonlinearities such as the one in Example B.5.

were precisely known; the control implementor is cautious. Probing is used in each of the non-neutral problems and is manifested by a modification in the control effort, usually small, which tends to increase the rate at which information about the problem is gained.

The possible occurrences of caution and probing are summarized in Table 5.1. From this table it can be seen that since most stochastic control problems are not neutral, most stochastic control policies will use probing or both probing and caution.

Table 5.1 Occurrence of Caution and Probing

	Certainty Equivalent	Neutral but not certainty equivalent	Not neutral or certainty equivalent
No caution or probing	Known gain problem Chapter 2		
Caution only		Example B.1	
Probing only			Example B.5
Caution and probing			Example B.3, B.4 and unknown constant gain problem Chapter 6

From Section B.2 it is observed that caution may exist for rather obscure reasons. Non-normal random variables (Example A.1) and non-quadratic performance criteria (Example A.2) can produce caution in otherwise linear problems and the need for this caution is not at all obvious from an examination of the problem. It is never safe to assume for any non-LQP problem that caution does not form an important component of the optimal control policy.

Some idea of the existence and form of the probing term is more

easily obtained than for the caution term. One reason for this is that probing almost certainly exists in any non-neutral problem. Also the information gain equation (see Examples B.3, B.4 and B.5) is very useful in determining the form of the probing term (e.g. whether it increases or decreases the control effort, etc.).

Therefore on an intuitive or approximate basis, probing is more easily determined than is caution. However, on a theoretical or exact basis, exactly the opposite is true. For all of the examples considered in Appendix B the neutral control policy and hence the caution component of the optimal policy has been determined in analytical form with relative ease. On the other hand the optimal control policy (and hence the probing component of the optimal policy) could not be determined for any of the non-neutral problems. For all stochastic control problems it is generally true that exact determination of the caution component (dependent on determination of the neutral policy) is considerably easier than is the exact determination of the probing component (dependent on determination of the optimal policy). This is because determination of the neutral control policy depends largely on the solution of the related neutral problem defined in Section 4.4.2. Because uncertainties that can be reduced have been eliminated, this related neutral problem is much simpler to solve than the original problem. For one thing, an estimator is not necessary and this is a great help particularly in cases in which sufficient statistics do not exist. Also in these cases there is no need to express the minimum expected cost function of the neutral problem as a functional as there is for the original problem.

No simulation of problems in which both caution and probing occur have been reported in the literature but a simulation of a problem with

probing only (Example B.5) has been done. This simulation demonstrates two important points:

1) Probing may be an essential component of a control policy not only to improve the performance but also to ensure that the controller works at all.

2) The exact form of the probing term for this problem at least, is not very important. An empirical probing term gives a performance indistinguishable from that of the optimal probing term.

Whether or not these factors apply to all stochastic control problems is not known, although there is a good chance that they apply to a large proportion of problems, especially in light of the simulation of the unknown constant gain problem reported in Chapter 7.

For problems in which the above two points do hold the following design procedure for suboptimal control policies should prove useful. The steps in the design procedure are:

1) Determine the neutral control policy. As is noted above, this is much more easily achieved than is the determination of the optimal policy.

2) Estimate the form of probing required using the information gain equation or ordinary intuition.

3) Simulate the problem to find the best magnitude for the probing term.

This procedure produced excellent results in Example B.5 and consequently it was tested for another example (the unknown constant gain problem) with good results in Chapter 7. At present no reason is known why it should not be equally successful for other stochastic control problems.

5.4 Summary

Optimal control policies have been shown to have three components:-

1) the certainty equivalent control policy which is used to produce the desired plant behaviour

2) caution which is used to allow for the fact that the plant is imprecisely known

3) probing which is used to increase the rate at which uncertainty is reduced.

The second and third of these components have been defined in this chapter.

Caution and probing describe the two ways in which the functions of estimation and control implementation can interact. Caution is the effect of the accuracy of the estimation on the control implementor while probing is the effect the control implementor has on the estimation.

The examples in Appendix B demonstrate several points:

1) Most stochastic control problems are not neutral and cannot be solved analytically.

2) Caution and probing exist in stochastic problems.

3) Most stochastic controllers use probing or both caution and probing.

4) Caution is difficult to determine empirically but relatively easy to determine analytically.

5) Probing is relatively easy to determine empirically but difficult to determine analytically. Moreover the exact form of the probing component of the stochastic control policy may not be important.

On the basis of these points a general procedure for designing sub-optimal control policies is proposed. One of the purposes of the next two chapters is to investigate this proposal.

Chapter 6

The Unknown Constant Gain Problem

6.1 Introduction

The purpose of this chapter and the next is to investigate the importance of the caution and probing components of stochastic control policies in an application of stochastic control theory. Because no solutions of general non-neutral stochastic control problems are available, a particular example, the unknown constant gain problem, is used for the investigation.

There are two important requirements which the example must fulfil. The first is that both caution and probing must be non-zero components of the optimal control policy and the second is that the certainty equivalent, neutral and optimal control policies must be known. These requirements ensure that it will be possible to compare the performance given by 1) the certainty equivalent policy (no probing or caution), 2) the neutral policy (caution only) and 3) the optimal policy (both caution and probing) in order to discover the relative importance of caution and probing.

Unfortunately no examples of non-neutral stochastic control problems which fulfil these two requirements were known to exist. The only two examples known which come close are described in Appendix B but both are unsatisfactory because for Example B.4 the optimal control policy is not accurately known and for Example B.5 the optimal policy does not have a caution component; consequently it was necessary to start with a fresh example. The unknown constant gain problem was chosen for four reasons:

1) Sufficient statistics exist and therefore there is no difficulty with the estimation part of the problem.

2) Both certainty equivalent and neutral policies are easily derived and have very simple forms.

3) The optimal control policy can be derived by numerical methods.

4) Uncertainty that can be reduced exists about two variables instead of one as in other possible examples. Because of this the effects of caution and probing on the knowledge about two unknowns and how they interact can be observed.

In this chapter the unknown constant gain problem is introduced and the equations defining the optimal control are formulated. In Appendix C the development necessary for the numerical calculation of the optimal policy is given,,the actual numerical work is described in Appendix D and the complete numerical results are given in Appendix E. These results are discussed in Section 6.3.

Generalized versions of the problem studied in this chapter have appeared in the literature[38,52,67] but only suboptimal control policies have been described. A slightly simpler version (gain restricted to two values) is considered by Sworder[79] who points out that the control is not available in analytic form and also describes suboptimal policies.

6.2 The Problem

In this section the problem is defined, Bayes' rule is applied, the information gain equation is derived, the certainty equivalent and neutral control policies are derived and the dynamic programming equations defining the optimal control policy are introduced. The development follows that given for the known gain problem in Chapter 2.

i) Statement of the problem

The plant equation is the same as for the known gain problem

$$x(i+1) = x(i) + bu(i) \qquad 6.1a$$

except that here the gain (b) is considered to be an unknown constant. If the steam engine analogy used in Chapter 2 is considered, this would

be equivalent to saying that the relationship between the throttle setting and the torque applied is initially uncertain. It might not even be known which way to move the throttle to produce a torque increase but this uncertainty could gradually be reduced by a trial and error method. This is exactly what a stochastic controller for this problem must do - try various control inputs (throttle settings), observe the effect on the state (the speed of the engine) and from this deduce the unknown b (the constant of proportionality between throttle setting and torque applied).

As in Chapter 2, measurements of the state are not exact but are corrupted by measurement noise:

$$y(i) = x(i) + z(i) \qquad 6.1b$$

Consequently uncertainty about the state exists in addition to the uncertainty about the unknown gain.

Uncertainties are described by probability densities and for this problem uncertainty about the measurement (the measurement noise) is described by a normal probability density

$$p(z(i)) = N(0, s_z^2)$$

The uncertainty about the state and the uncertainty about the gain are generally coupled in some way and hence the initial uncertainty about them is described by a joint normal probability density

$$p(x(0), b) = N(m_x, m_b, s_x^2, s_b^2, \rho)$$

Here m_x is the expected value of the state, m_b is the expected value of the unknown constant gain, s_x^2 is the variance of the state, s_b^2 is the variance of the gain and ρ is the correlation coefficient which describes the degree of coupling between the knowledge about the state and the gain.

As in Chapter 2, the performance criterion is in the form of a

cost function

$$I = \sum_{i=1}^{N} x^2(i) \qquad 6.2$$

where the control extends over the period from time $i = 0$ to $i = N$. The object of the controller is to minimize this cost function.

ii) Bayes' rule

The application of Bayes' rule follows the same pattern as in Chapter 2. For this problem Bayes' rule is

$$p(x(i+1),b|y^{i+1}) = \text{const.}\, p(y(i+1)|x(i+1),b)\; p(x(i+1),b|y^{i},u(i))$$

where y^i is the set of all past control inputs and past and present measurements:

$$y^i = \{y(1), \ldots, y(i), u(0), \ldots, u(i-1)\}$$

Because the state and measurement equations are effectively linear as far as the estimation problem is concerned, and because the initial knowledge of the state and unknown gain and the measurement errors are described by normal probability densities, repeated applications of Bayes' rule gives that the conditional probability density of the state and gain at time i is normal, i.e.

$$p(x(i),b|y^i) = N(m_x, m_b, s_x^2, s_b^2, \rho)$$

Since the variables $(m_x, m_b, s_x^2, s_b^2, \rho)$ summarize the joint conditional probability density for the state and unknown gain, they are the sufficient statistics for this problem (c.f. Section 3.3). Detailed calculation of Bayes' rule gives updating equations for the sufficient statistics:

$$m_x(i+1) = \frac{y(i+1)(s_x^2(1-\rho^2) + (us_b + \rho s_x s_b)^2) + (m_x + m_b u)s_z^2}{s_x^2(1-\rho^2) + (us_b + \rho s_x s_b)^2 + s_z^2} \qquad 6.3a$$

$$m_b(i+1) = m_b + (y(i+1) - m_x - m_b u)\frac{(\rho s_x s_b + us_b^2)}{s_x^2(1-\rho^2) + (us_b + \rho s_x)^2 + s_z^2} \qquad 6.3b$$

$$s_x^2(i+1) = \frac{s_z^2(s_x^2(1-\rho^2) + (us_b + \rho s_x)^2)}{s_z^2 + s_x^2(1-\rho^2) + (us_b + \rho s_x)^2} \qquad 6.3c$$

$$s_b^2(i+1) = \frac{s_z^2 + s_x^2(1-\rho^2)}{s_z^2 + s_x^2(1-\rho^2) + (us_b + \rho s_x)^2} \qquad 6.3d$$

$$\rho(i+1) = \left(\frac{s_z^2}{s_z^2 + (1-\rho^2)s_x^2}\right)^{\frac{1}{2}} \cdot \frac{s_x + us_b}{(s_x^2(1-\rho^2) + (us_b + \rho s_x)^2)^{\frac{1}{2}}} \qquad 6.3e$$

Although these equations are algebraically much more complex than the corresponding equations in Chapter 2 (Equations 2.5), they have the same essential feature in that they allow the estimation to be done sequentially.

iii) Information

In order to determine whether or not the problem is neutral it is necessary to consider the information associated with this problem. From Equation 4.5 this is

$$s\left[p(x(i),b|y^i)\right] = -\tfrac{1}{2}\log\left[s_b^2 s_x^2(1-\rho^2)\right] + \text{const.} \qquad 6.4$$

Substitution from Equations 6.3 shows that the information at time i does depend on the past control inputs and thus the problem is shown to be non-neutral.

Since the problem is not neutral the next step is to consider the information change from one time to the next. From Equations 6.3 and 6.4

this can be shown to be

$$s\left[p(x(i+1),b|y^{i+1})\right] - s\left[p(x(i),b|y^{i})\right] = \frac{1}{2}\log\frac{s_z^2 + s_x^2(1-\rho^2) + (s_x\rho + us_b)^2}{s_z^2} \qquad 6.5$$

From Equation 6.5 it can be seen that the information change is always positive (information gain) and that the information gain is largest when

$$u_{max.inf.gain} = \pm\infty$$

and is smallest when*

$$u_{min.inf.gain} = -\rho s_x/s_b$$

From this knowledge of the control policies which give maximum and minimum information gains it is possible to get some idea of what form of probing is appropriate for this problem. The larger the difference between the control policy and the minimum information gain policy, the greater will be the information gain and thus probing has the effect of increasing the absolute value of

$$(u - u_{min.inf.gain})$$

This result is used to derive an empirical probing term in Chapter 7.

There is no reason why this method of determining the form of the probing term should not be applicable to many stochastic control problems. In many cases (Example B.5) it would be necessary to consider expected information gain. Also in some cases in which the relationship between information gain and control policy is more complex it might be necessary

---

* There is no special significance attached to the fact that the control for minimum information gain has this particular value rather than zero. It is a result of the correlation between the state and the gain.

to consider the complete relationship rather than just the control policies which give maximum and minimum information gains. In problems without sufficient statistics the situation would be more complex, but some form of information measure could probably be applied.

iv) Certainty equivalent and neutral control policies

The certainty equivalent control policy for this problem is derived by considering the related deterministic problem (see Section 4.2.1) and is

$$u^{c.e.}(i) = - m_x/m_b \qquad 6.6$$

The neutral control policy for this problem is derived by considering the related neutral problem* (see Section 4.4.2). The neutral policy is

$$u^{nt}(i) = - (m_x m_b + \rho s_x s_b)/(m_b^2 + s_b^2) \qquad 6.7$$

Thus two of the components of the optimal control policy are readily available in analytic form. They are the certainty equivalent policy which has the function of producing a desired performance and the caution component (the difference between the certainty equivalent and neutral policies) which has the function of allowing for plant uncertainties. These components and their functions are discussed further in Chapter 7.

v) The optimal control policy

It is necessary to determine the optimal control policy in order to find the third component, probing. As in Chapter 2, this is achieved by the application of dynamic programming.

First the minimum expected cost is defined:

---

* The related neutral problem for the unknown constant gain problem is the example considered in Section B.1.

$f_N[m_x, m_b, s_x^2, s_b^2, \rho] \triangleq$ the expected cost using the optimal control u^* over N stages when the initial condition is specified by m_x, m_b, s_x^2, s_b^2, ρ.

Then using the same procedure as in Chapter 2 the functional recurrence equation is derived:

$$f_N[m_x, m_b, s_x^2, s_b^2, \rho] = \min_u \{E\{x^2(1)\} + E\{f_{N-1}[m_x(1), m_b(1), s_x^2(1), s_b^2(1), \rho(1)]\}\} \quad 6.8$$

Considering the case in which there is one step to go (N = 1) gives

$$f_1[m_x, m_b, s_x^2, s_b^2, \rho] = \min_u \{m_x^2 + s_x^2 + u^2(m_b^2 + s_b^2) + 2u(m_x m_b + \rho s_x s_b)\}$$

Using the standard calculus technique of minimization gives

$$f_1[m_x, m_b, s_x^2, s_b^2, \rho] = \frac{s_b^2(m_x^2 + s_x^2(1-\rho^2)) + m_b^2 s_x^2 - 2m_x m_b s_x s_b \rho}{m_b^2 + s_b^2} \quad 6.9$$

$$u_1^*[m_x, m_b, s_x^2, s_b^2, \rho] = -\frac{m_x m_b + \rho s_x s_b}{m_b^2 + s_b^2} \quad 6.10$$

This is as far as the analytic determination of the optimal control policy can progress. Determination of the optimal control policy for a larger number of steps to go is prevented by the fact that the expectation integral needed in the second term on the right hand side of Equation 6.8 cannot be evaluated in terms of analytic functions. This necessitates a numerical solution.

This is precisely the point at which the determination of the optimal control policy is brought to a standstill in most non-neutral stochastic control problems. In this problem there is a possibility of a numerical solution but in most cases the functional recurrence equation

is so complex that this is impossible. One of the main difficulties is dimensionality. In this problem there are five sufficient statistics plus the noise variance which the control policy depends on but in more complex problems the number is much larger. As is shown in Appendix D, the difficulty of determining the optimal policy increases more or less exponentially with the number of arguments of the control policy. Therefore it is important to reduce the number of significant variables as much as possible. Two methods of doing this, symmetry conditions and dimensional analysis (cf. Reference 99) are described in Appendix C.

6.3 Discussion of Numerical Results

The results of the numerical work reported in Appendix D are listed in Appendix E and an example of how the results about the optimal control policy can be applied is considered in Chapter 7. The accuracy of the results is confirmed by the simulation reported in Chapter 7.

From Appendix E it can be seen that a large amount of complex numerical work was necessary to produce the optimal control policy and optimal expected cost functions. Even then they were only calculated for processes of six stages or less in length. Because the accuracy of the results decreases as the number of steps to go increases, computing was terminated at this point. Fortunately the results produced are sufficient to have a meaningful simulation based on them.

The difficulty of determining the optimal policy for this simple problem is conclusive evidence that with presently available computers this approach cannot be applied to most stochastic control problems. Consequently some form of approximate control is necessary for most stochastic control problems.

Figures 6.1 to 6.16 are graphical representations of some of the

Figure 6.1 Minimum Expected Cost vs. Mean of State

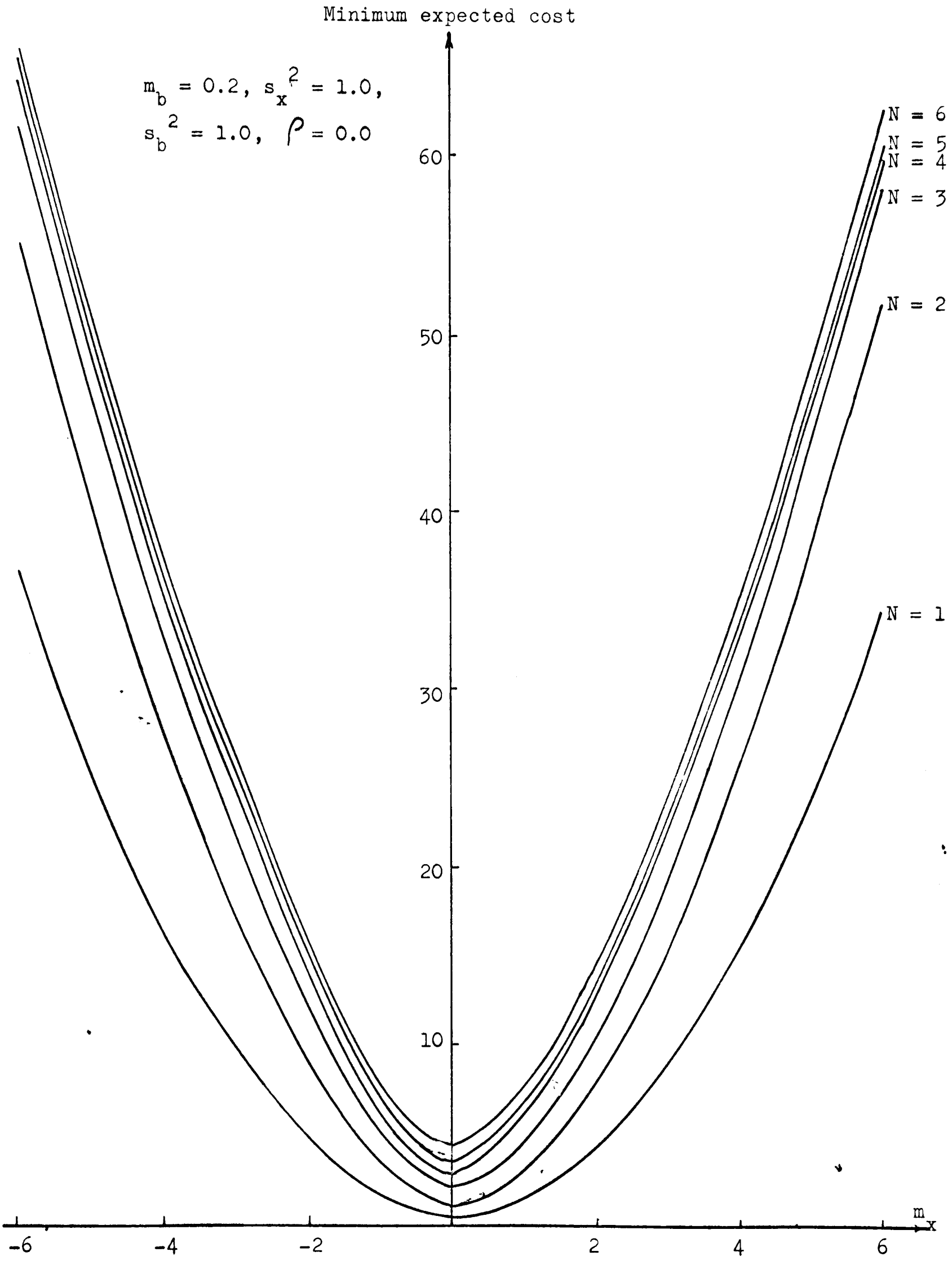


Figure 6.2 Minimum Expected Cost vs. Mean of State

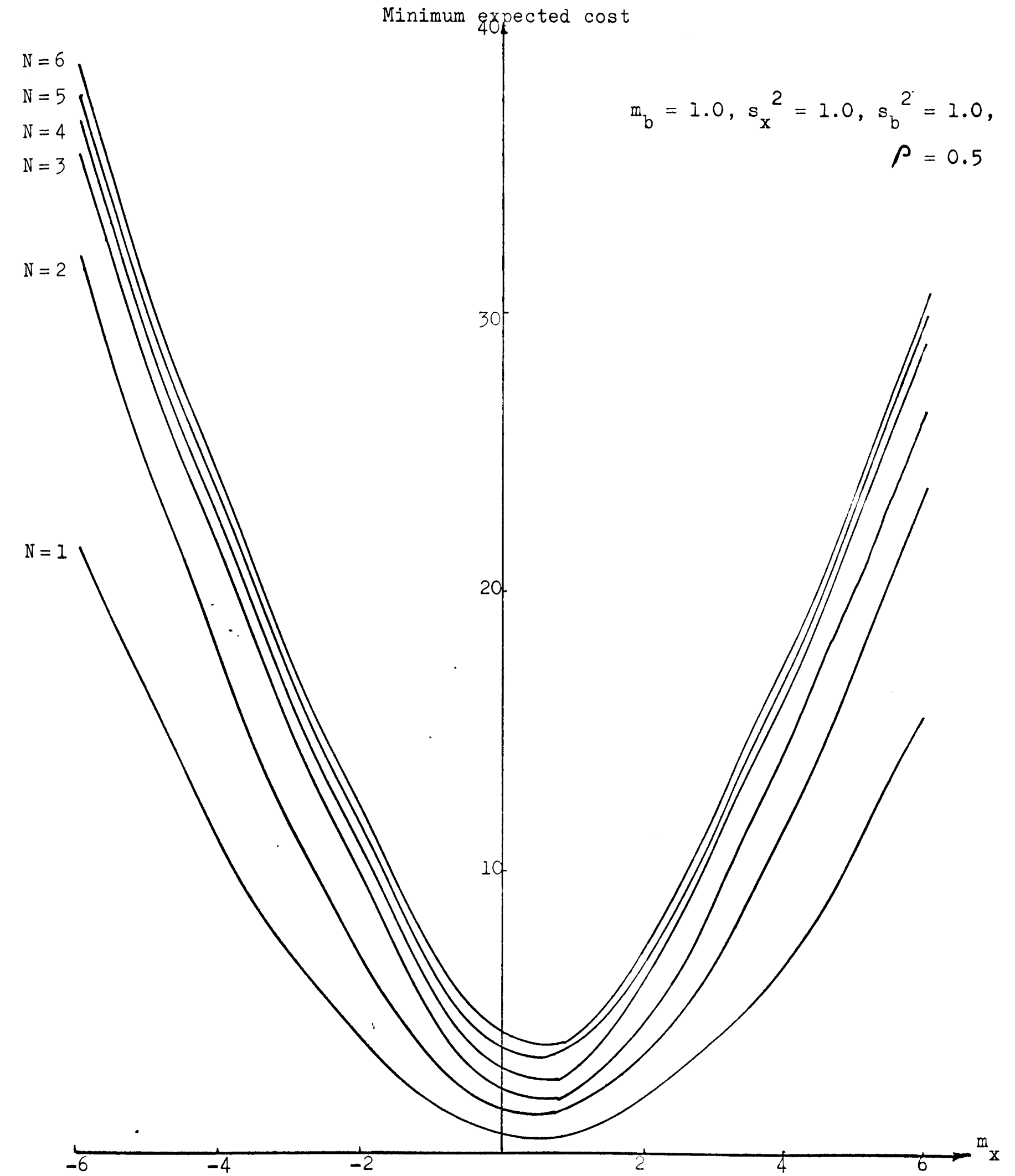


Figure 6.3 Minimum Expected Cost vs. Mean of State

$$m_b = 5.0,\ s_x^2 = 1.0,\ s_b^2 = 1.0,\ \rho = 0.0$$

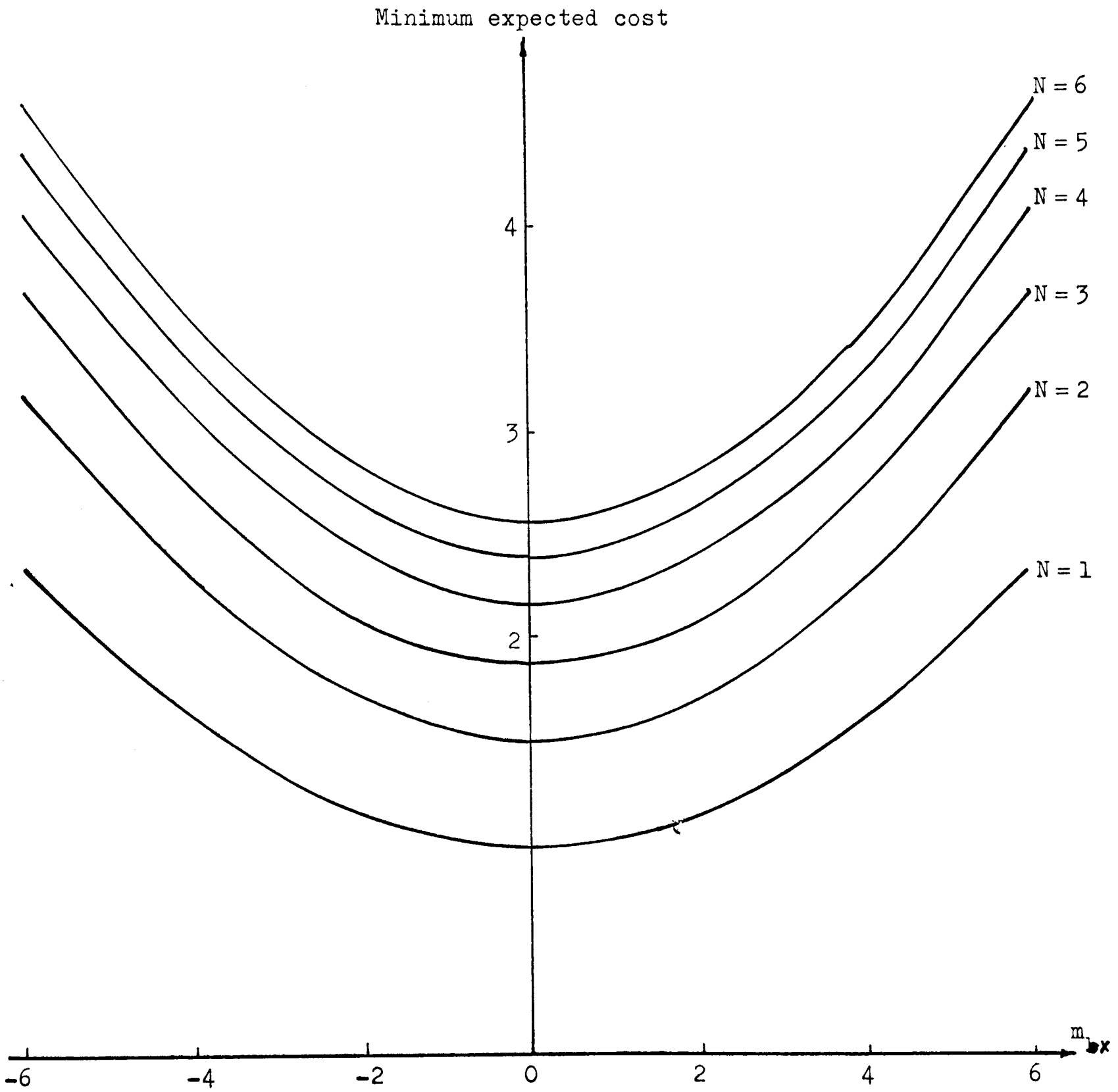


Figure 6.4 Control vs. Mean of State

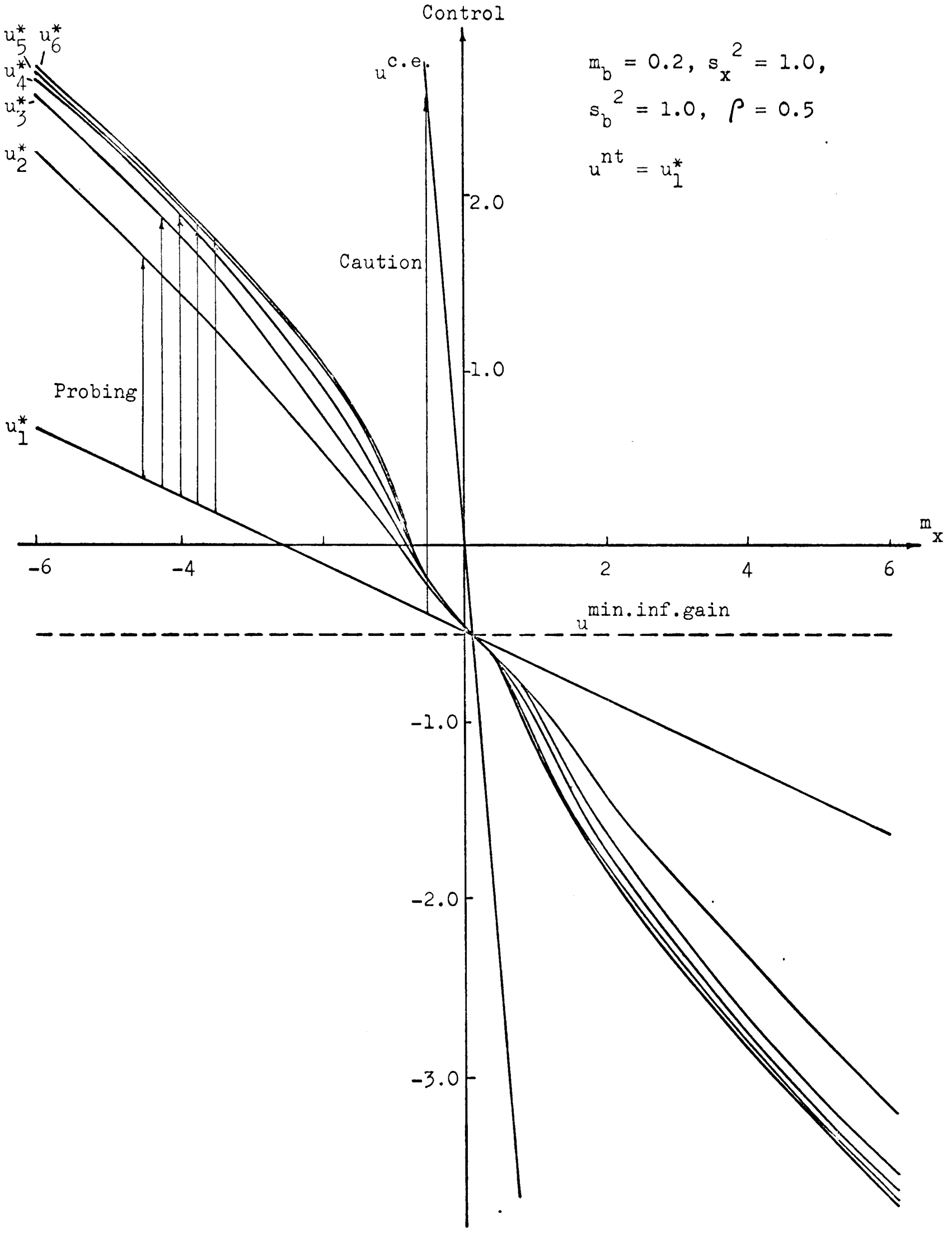


Figure 6.5 Control vs. Mean of State

$m_b = 1.0,\ s_x^2 = 1.0,\ s_b^2 = 1.0,\ \rho = 0.5$

$u^{nt} = u_1^*$

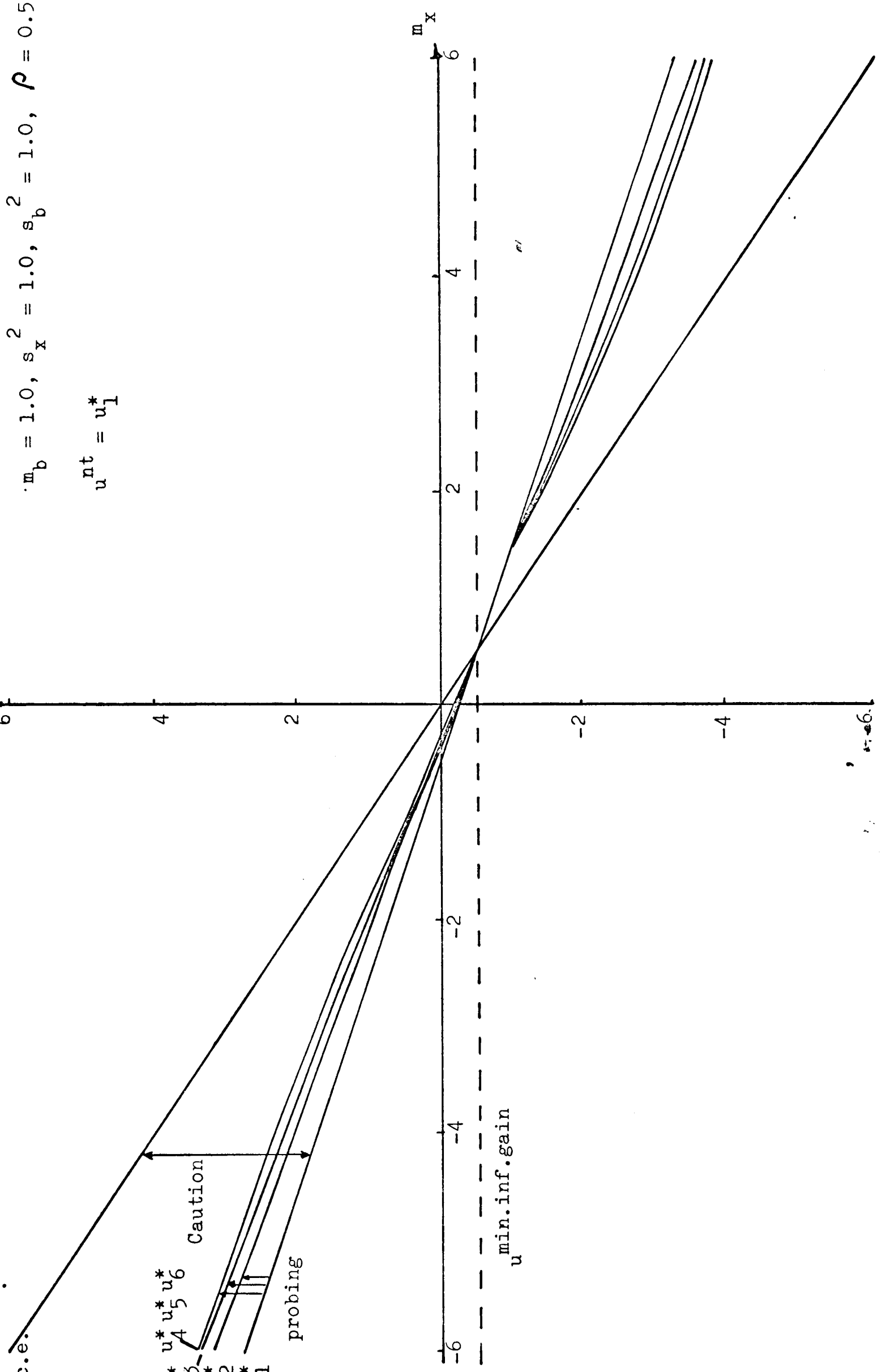


Figure 6.6 Control vs. Mean of State

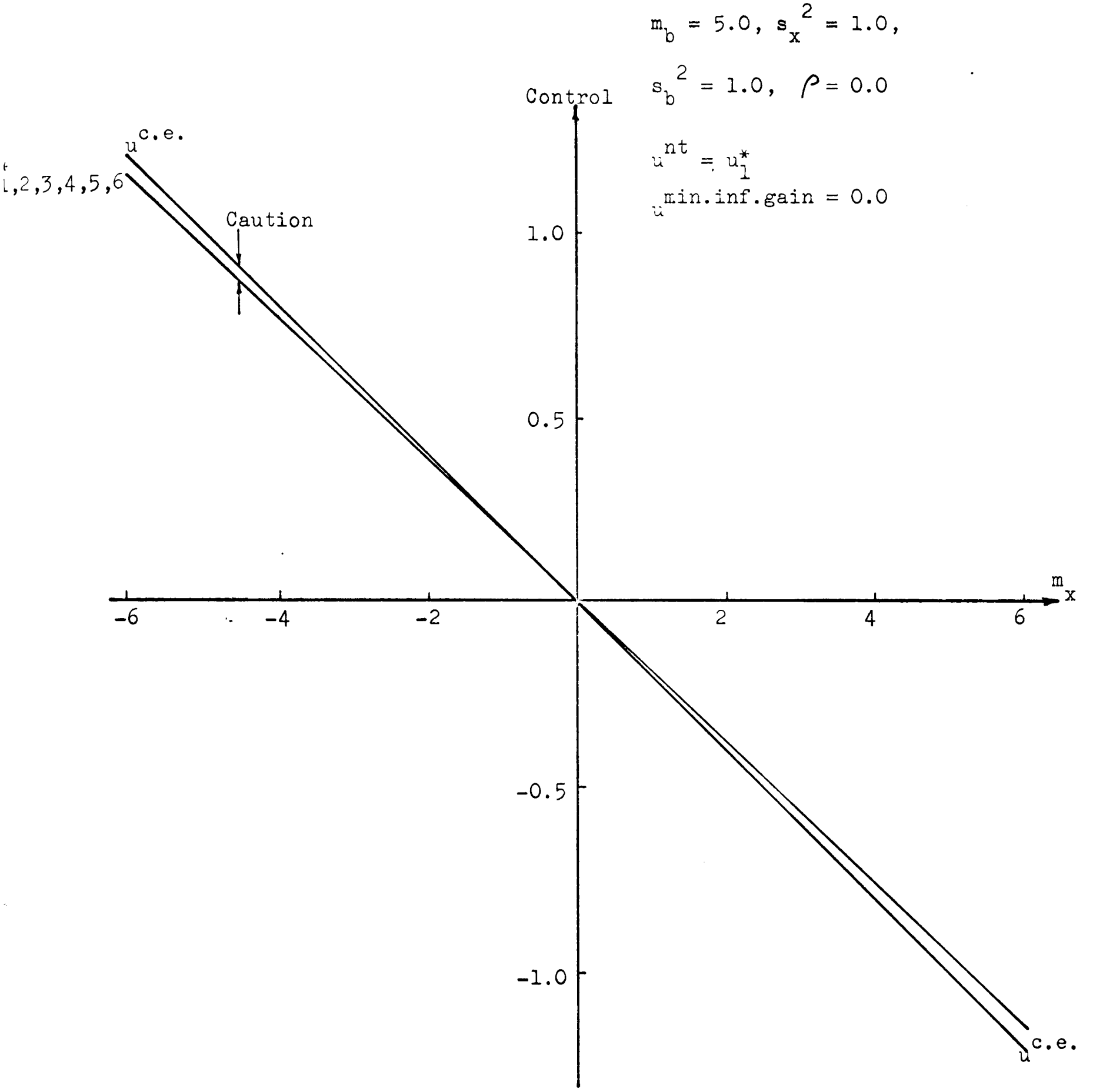


Figure 6.7 Minimum Expected Cost vs. Mean of Gain

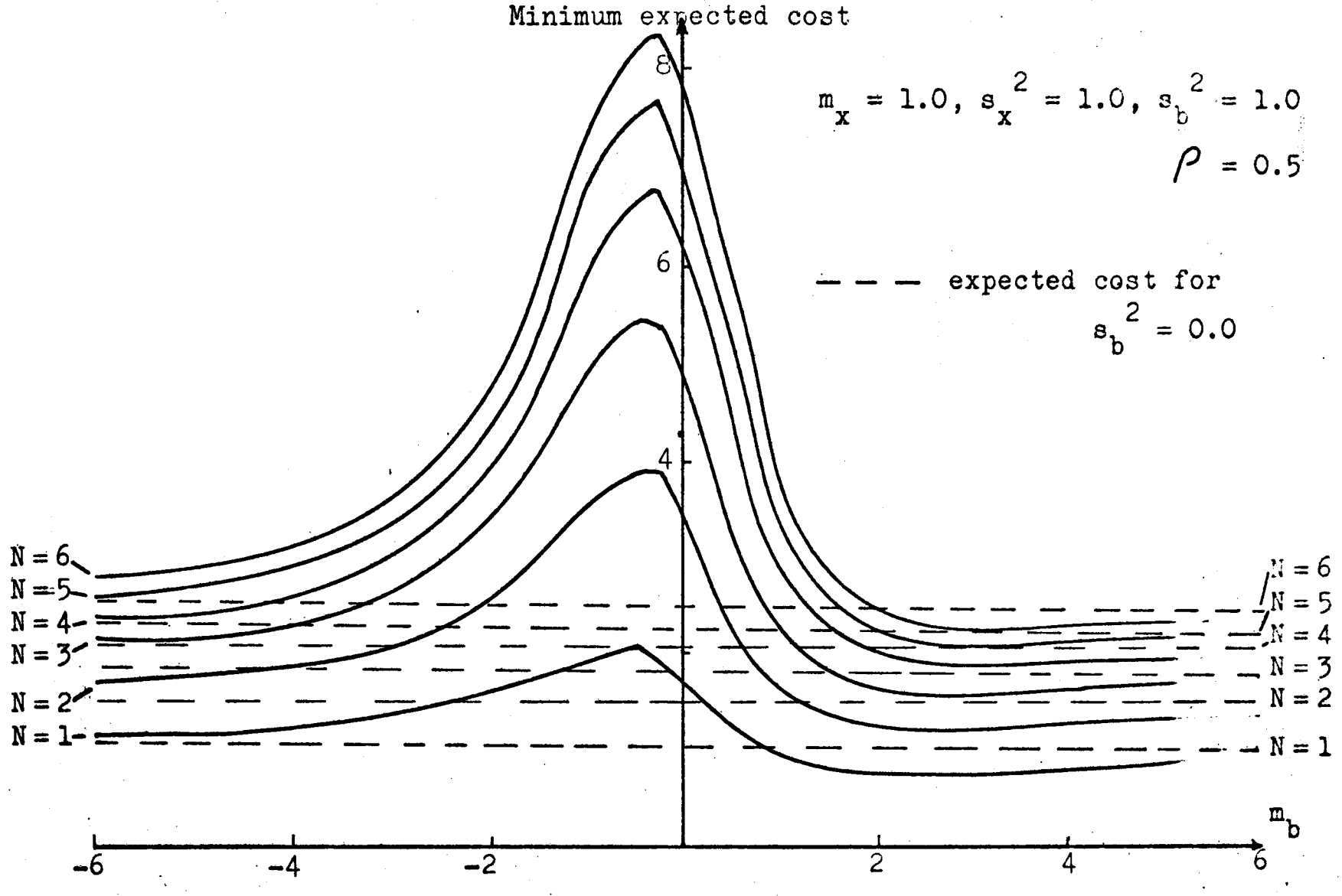


Figure 6.8 Minimum Expected Cost vs. Mean of Gain

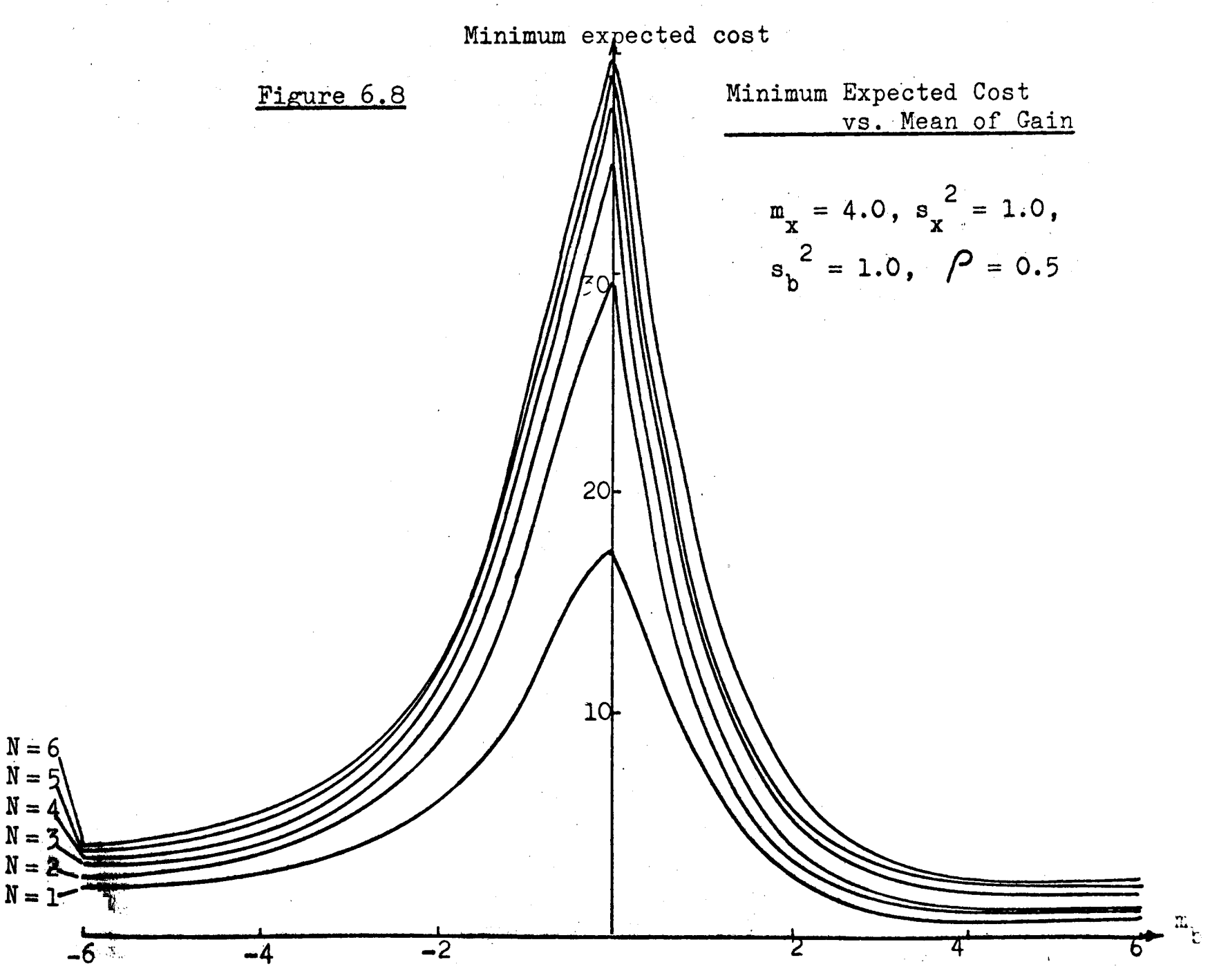


Figure 6.9 Minimum Expected Cost vs. Mean of Gain

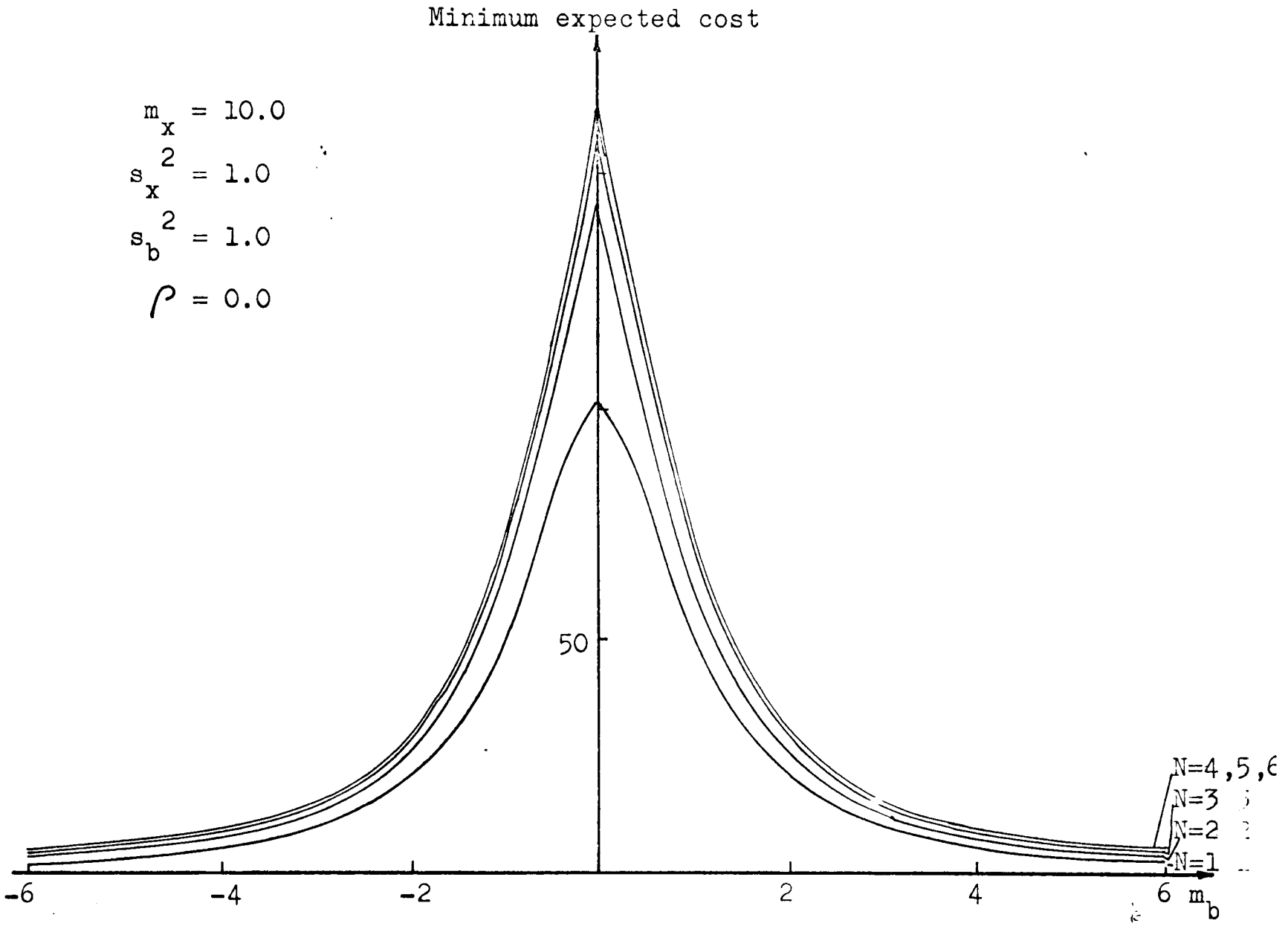


Figure 6.10

Control vs. Mean of Gain

$m_x = 1.0$, $s_x^2 = 1.0$, $s_b^2 = 1.0$, $\rho = 0.5$

$u^{nt} = u_1^*$

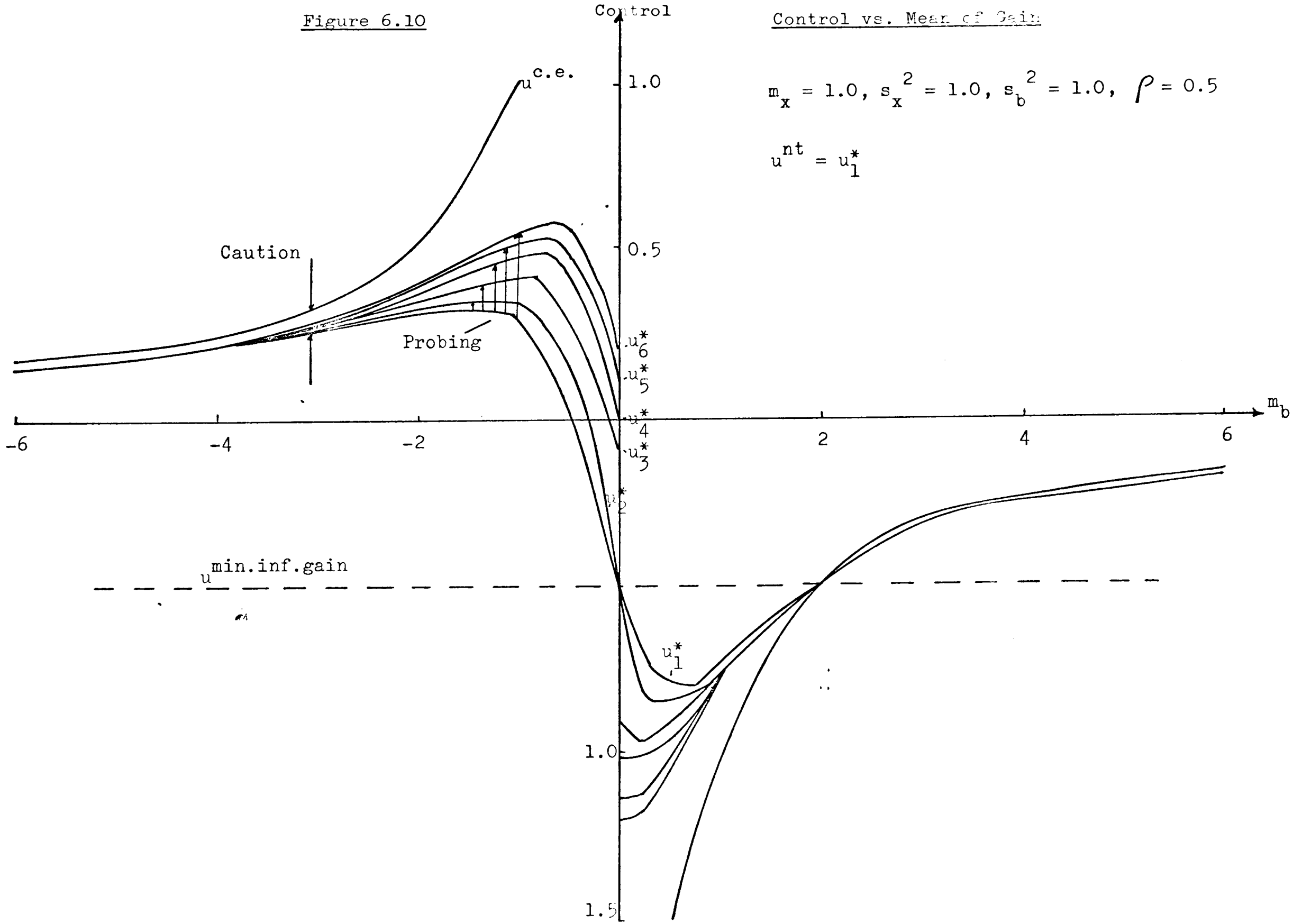


Figure 6.11

Control vs. Mean of Gain

$m_x = 4.0,\ s_x^2 = 1.0,\ s_b^2 = 1.0,\ \rho = 0.5$

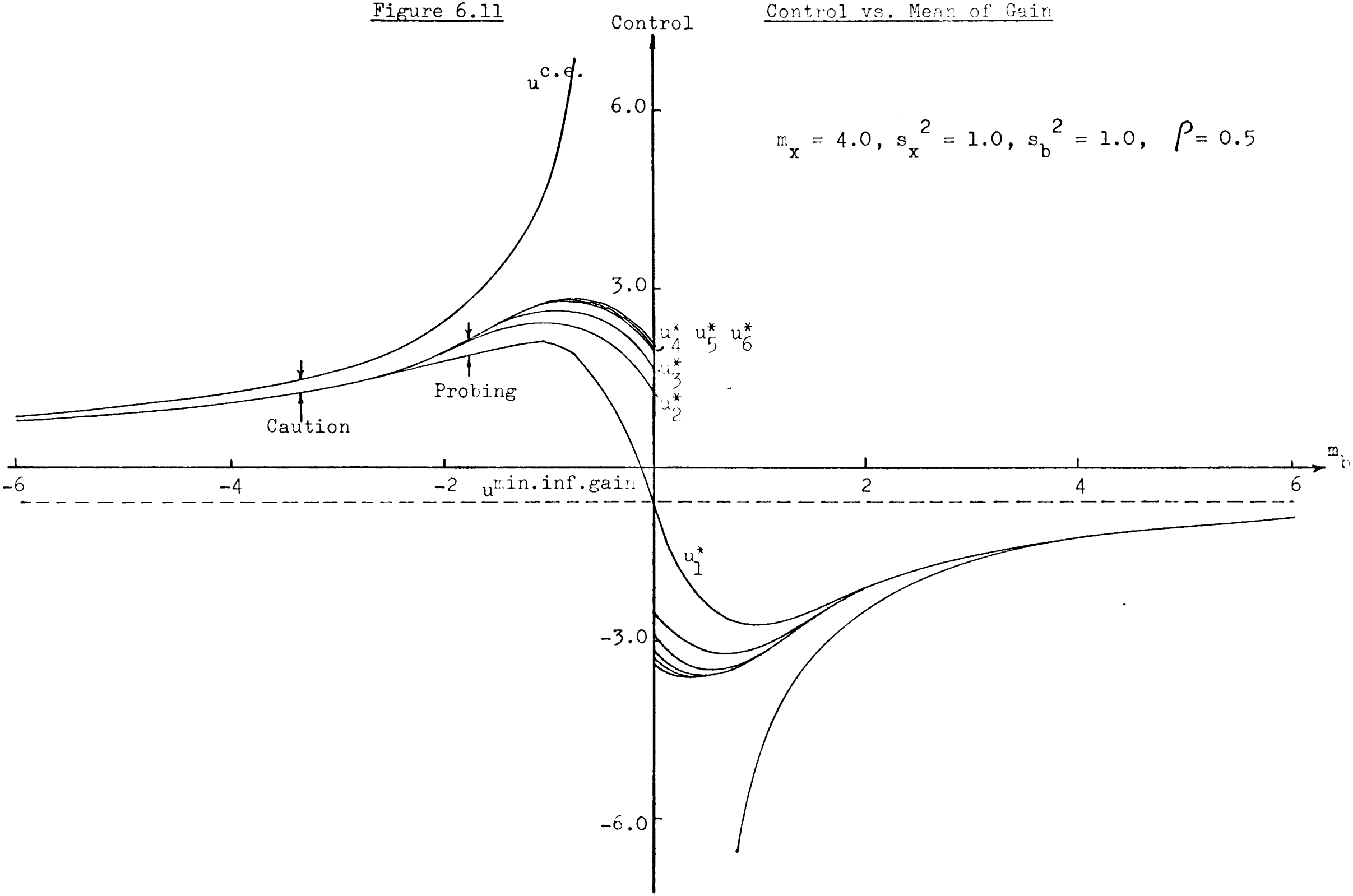
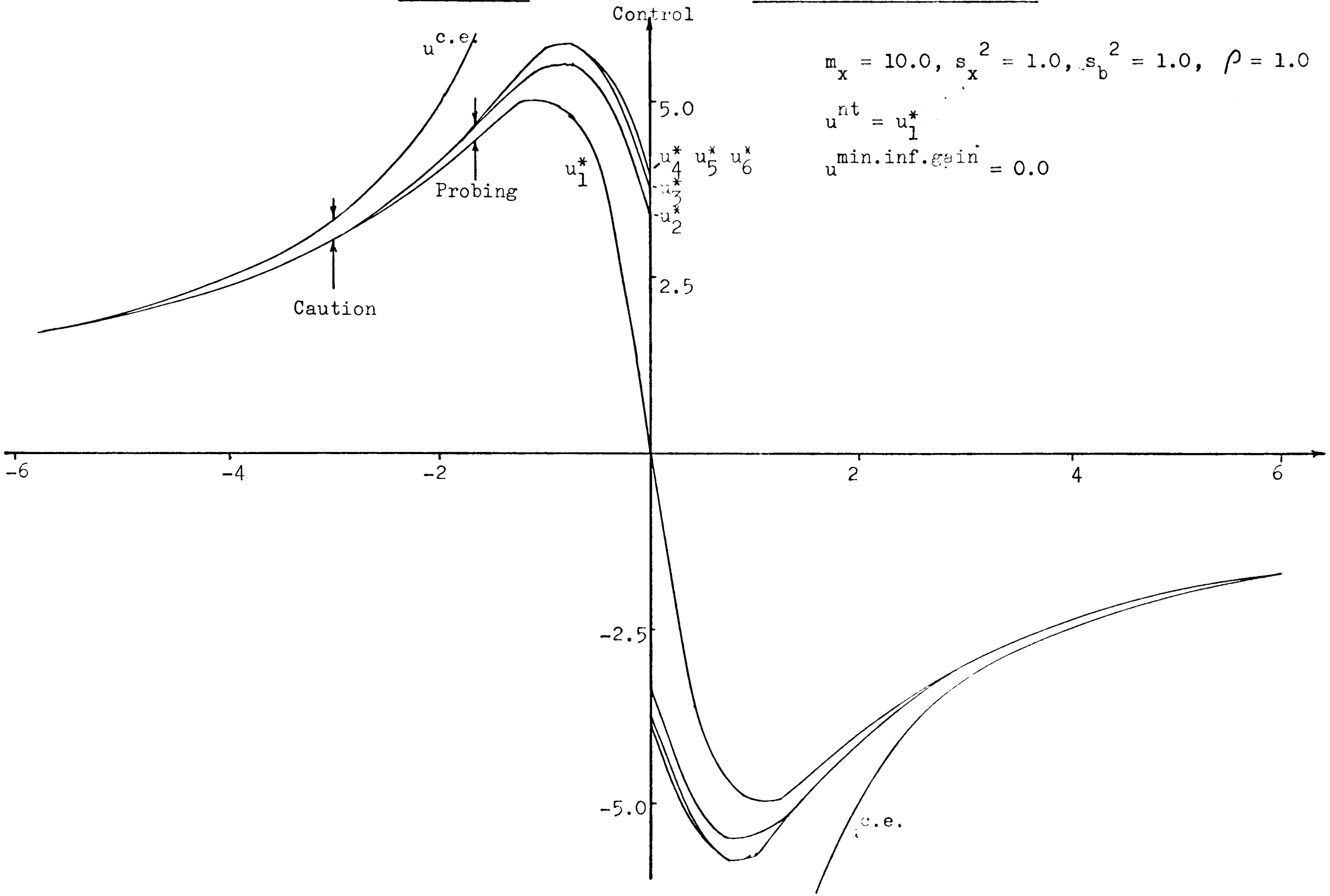


Figure 6.12

Control vs. Mean of Gain



$m_x = 10.0$, $s_x^2 = 1.0$, $s_b^2 = 1.0$, $\rho = 1.0$

$u^{nt} = u_1^*$

$u^{\text{min.inf.gain}} = 0.0$

Figure 6.14 Control vs. Variance of State

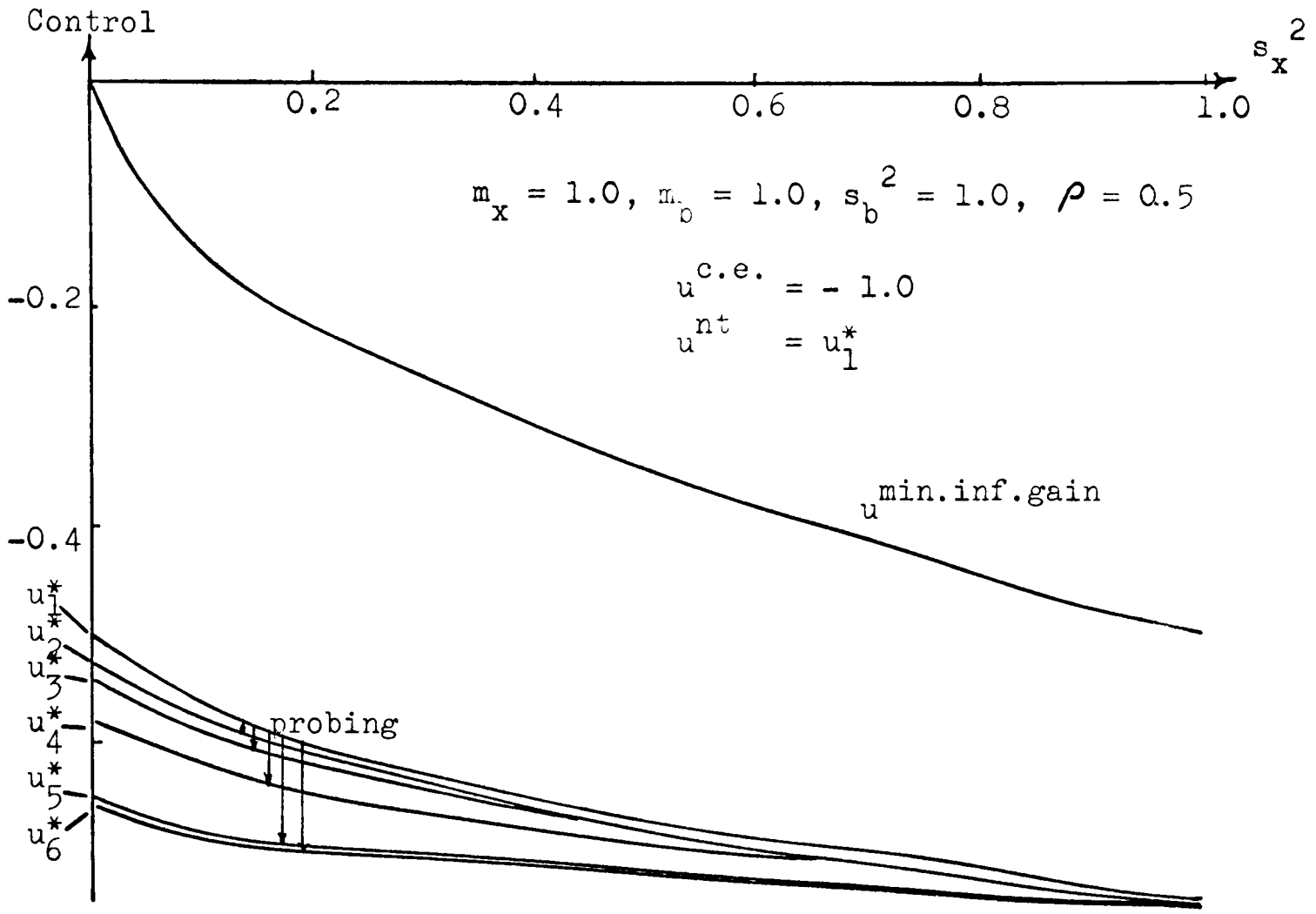


Figure 6.13 Minimum Expected Cost vs. Variance of State

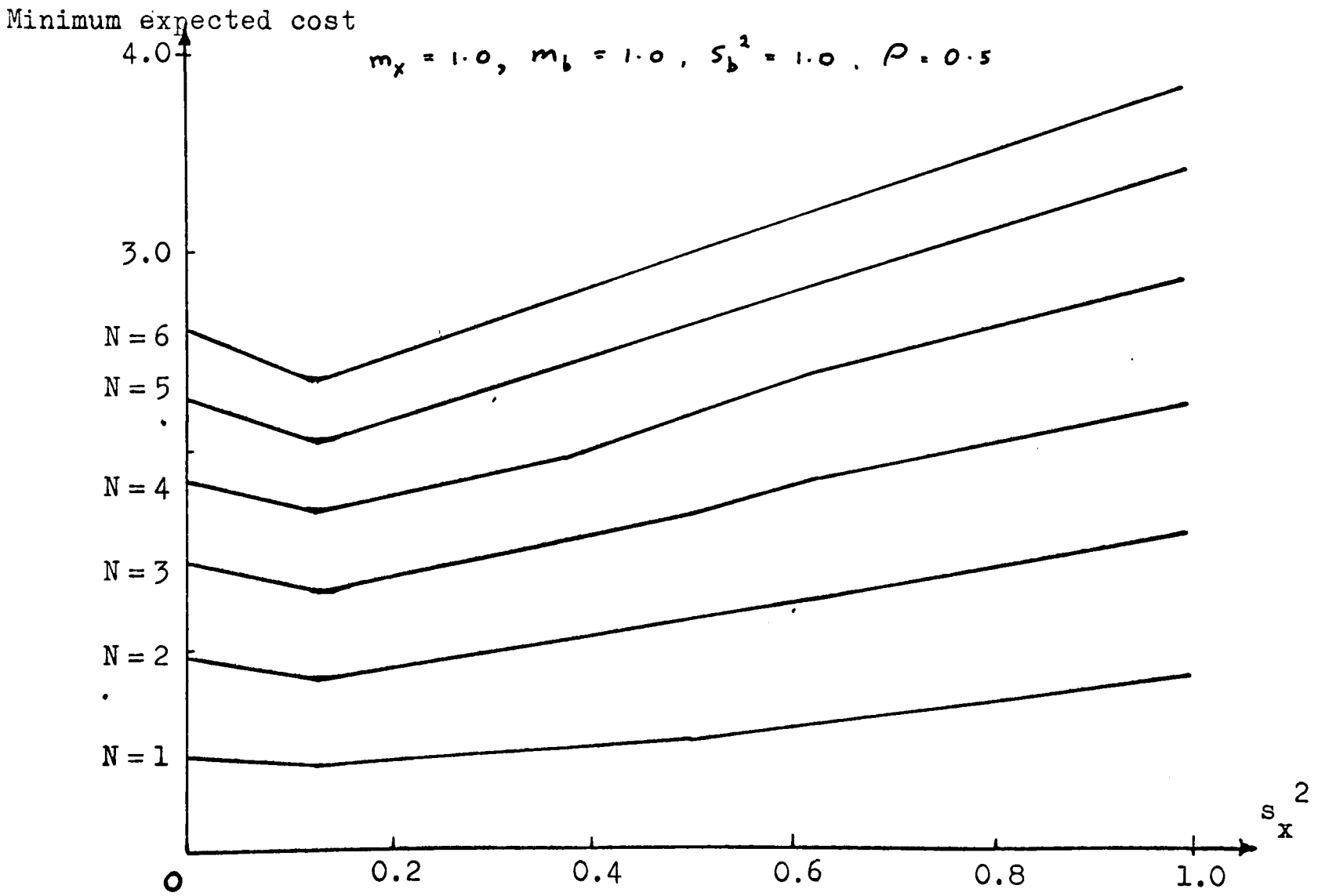
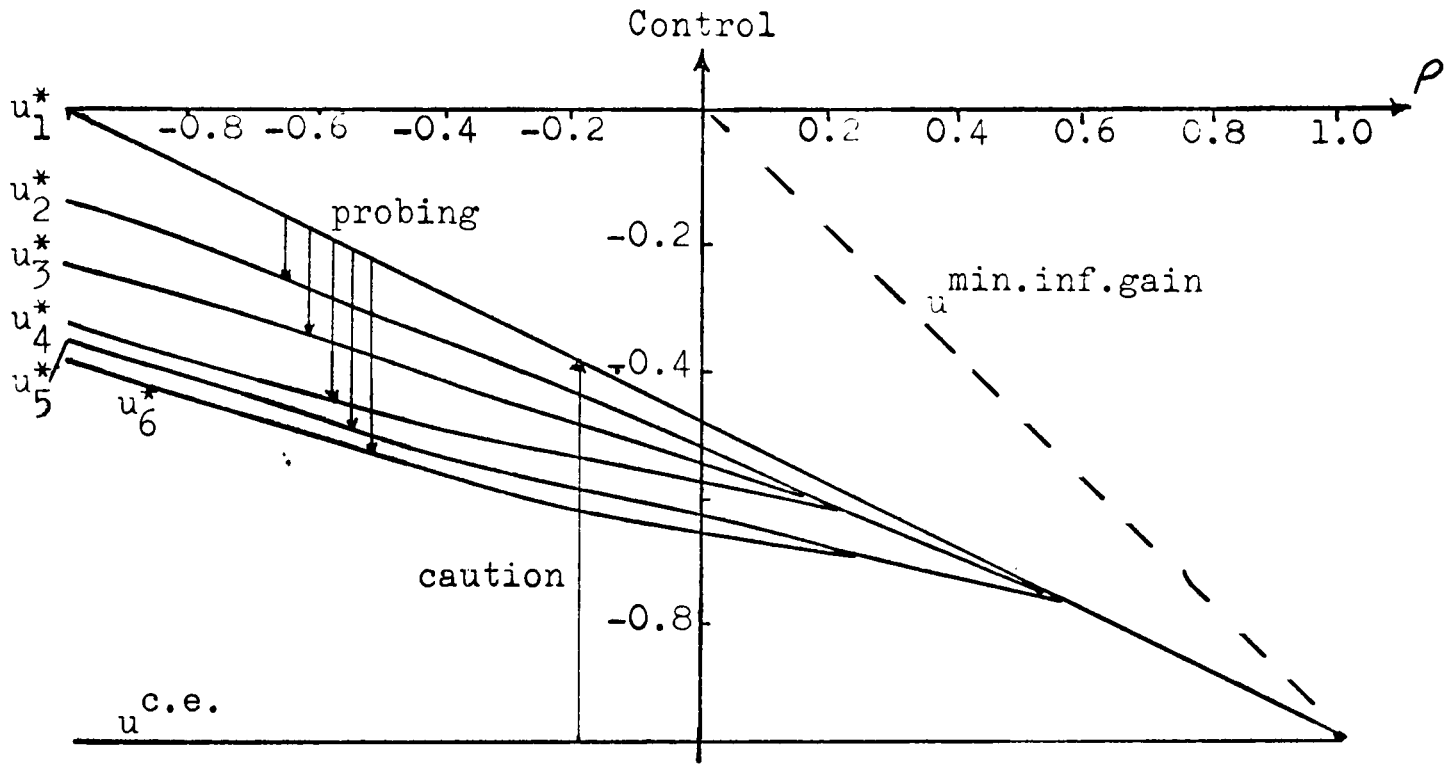
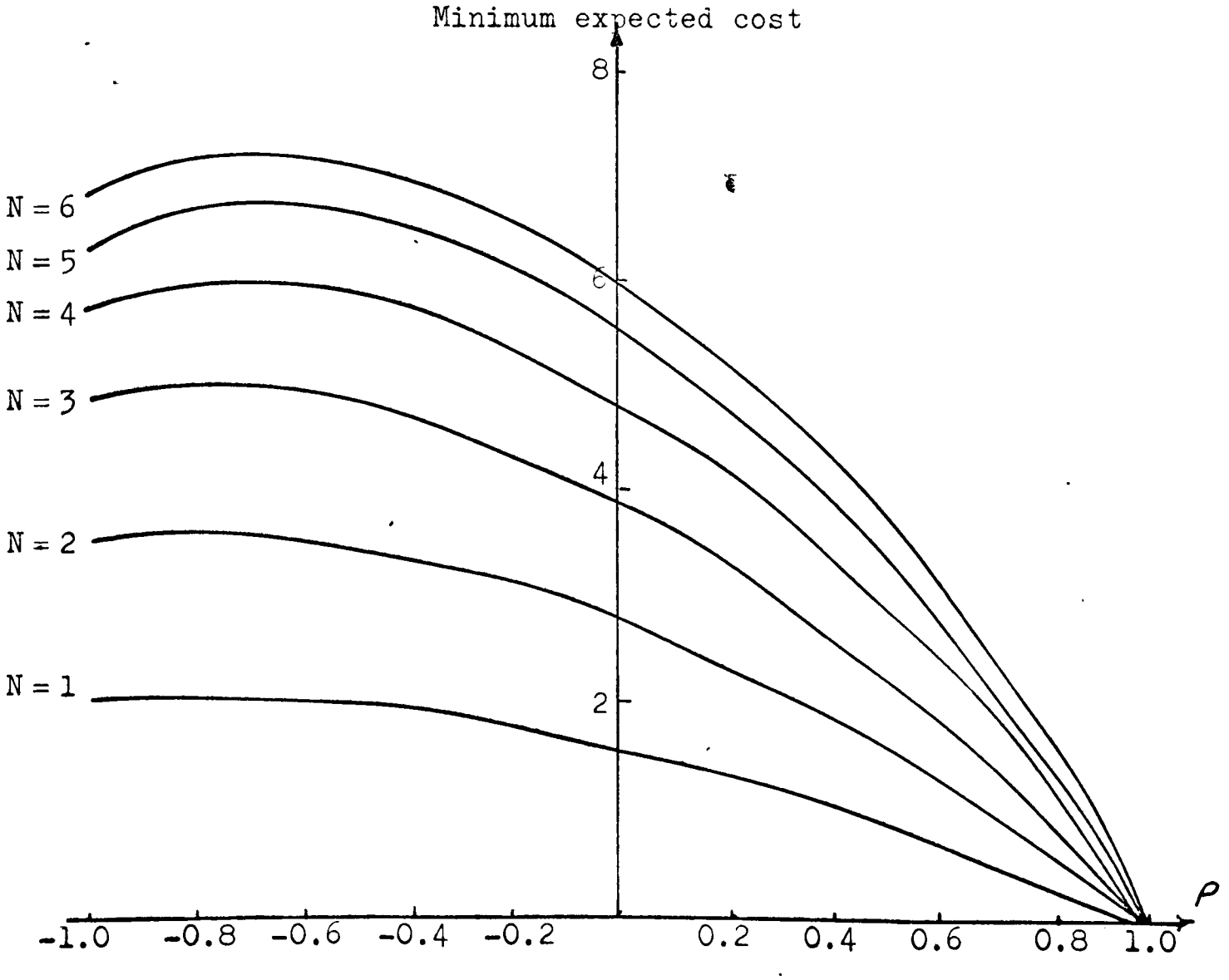


Figure 6.16 Control vs. Correlation Coefficient



$m_x = 1.0$, $m_b = 1.0$, $s_x^2 = 1.0$, $s_b^2 = 1.0$

Figure 6.15 Minimum Expected Cost vs. Correlation Coefficient



results contained in Appendix E. They show how the certainty equivalent, neutral* and optimal control policies as well as the optimal expected cost depend on the statistics m_x, m_b, s_x^2 and ρ.** Some facts of general importance can be deduced from these figures:

1) The sign of the probing component of the optimal control policy was accurately predicted from the information gain equation 6.5. Probing is positive when the control policy is positive with respect to the control for minimum information gain and is negative when the control policy is negative with respect to the control for minimum information gain. This gives some confidence that a similar method of determining the sign of the probing component will be effective for other stochastic control problems.

2) In this problem caution is independent of the number of stages to go. This is probably true for most stochastic control problems in which the cost function incurred at each stage (the function H in Equation 3.3) is independent of the time i and of the control input. The reason for this is that caution is a result of the controller's attempt to avoid a large cost on the following stage as a result of the existing uncertainty.

3) Probing on the other hand is used to reduce the effects of uncertainty on the cost incurred on all future stages except the next one and is consequently dependent on the number of stages left to go in the process.

---

* From comparing Equations 6.7 and 6.10 it can be seen that for this problem the neutral control policy is identical to the optimal control policy for one step to go. See Appendix B, Example B.3 for a discussion of this point.

** The dependence on s_b^2 and s_z^2 is not shown because it is known from the dimensional analysis (Appendix C) that this would add no new knowledge to that given by all the figures presented.

For this problem it is seen from the figures that the probing component of the optimal control policy increases with the number of stages to go. This occurs because there is a cost associated with the use of probing. Any deviation from the neutral policy results in a greater expected cost on the following stage than is necessary, but to offset this the increased knowledge gained by using probing allows more accurate control of the later stages. The controller must balance these two effects to determine the best amount of probing to use and consequently more probing is used when there is a large number of stages to go because the extra knowledge gained will be of benefit over a longer time. Since this effect almost certainly occurs in all non-neutral control problems its importance is considered further in Chapter 7.

One other point of general interest can be noted from Figures 6.1 to 6.16 and this is the frequent asymmetry of the results. These asymmetries occur whenever the correlation coefficient (ρ) is not zero and are in fact solely a result of the correlation between the state and the gain. An interesting effect in Figure 6.7 is that in some cases the expected cost for the case in which the gain is exactly known may be <u>greater</u> than the expected cost when there is some uncertainty about the gain. Correlation effects such as these occur in any stochastic control problem in which there is uncertainty that can be reduced about more than one unknown. Ignoring such possible effects could be as important as ignoring the fact that uncertainty existed about one of the unknowns; consequently the temptation to treat unknown parameters as being uncorrelated is a dangerous one.

6.4 Summary

In this chapter the unknown constant gain problem has been introduced and the derivation of the optimal stochastic control law has been

described. The estimation part of the problem was solved analytically but numerical methods were necessary to find the optimal control policy. It was noted that the required numerical calculations were complex and did not give a complete result. Consequently this type of numerical approach is not recommended as a practical method of determining optimal policies. On the other hand, both the certainty equivalent and neutral control policies were very easily determined and had quite simple analytic forms.

The numerical results show that the caution component of the optimal control policy is independent of the number of stages to go but that the probing component increases in magnitude as the number of stages to go increases. This is because caution is used to reduce the effects of uncertainty on the cost incurred at the next stage while probing is used to reduce the effects of uncertainty on the cost incurred on all future stages except the next. Thus the relative importance and hence the magnitude of probing increase as the length of the process increases. These two properties of caution and probing probably hold for ~~all~~ many stochastic control problems.

Chapter 7

Simulation of the Unknown Constant Gain Problem

7.1 Introduction

This chapter describes a simulation of the unknown constant gain problem discussed in Chapter 6. The simulation had three objectives: to investigate the importance of caution and probing to the performance of stochastic control policies, to check the accuracy of the numerical determination of the optimal control policy described in Appendix D, and to evaluate some methods of deriving suboptimal control policies. The first objective was achieved by simulating the problem using the certainty equivalent, neutral and optimal control policies in order to investigate the effect of caution (the difference between the certainty equivalent and neutral policies) and probing (the difference between the neutral and optimal policies) respectively. The second objective was achieved by comparing the theoretical minimum expected cost from Appendix E with that given when the problem was simulated using the optimal control policy. The third objective was achieved by comparing the performance given by

1) a suboptimal policy derived from the optimal policy,

2) a suboptimal policy derived using the procedure outlined at the end of Chapter 5,

3) a separable suboptimal policy.

Details of the simulation are reported in Section 7.2, and the results are discussed in Sections 7.3 and 7.4. The simulation was done on the Oxford University Systems Engineering Group IBM 1130 digital computer.

7.2 Procedure

The simulation procedure used is similar to that reported in Chapter 2. In this case the average cost, the initial information content and the average final information content were produced for the various control policies noted above. To do this the computer program had to simulate a particular case (a case being a set of specific values for N, the number of stages to go, and m_x, m_b, s_x^2, s_b^2, ρ , the sufficient statistics) M times (one simulation is called a trial so that there are M trials) and average the results of the trials.

The main program is presented in the flow diagram (Figure 7.1). For each trial the initial value of the state x(0) and the value of b are chosen from a normal joint probability density

$$p(x(0), b) = N(m_x(0), m_b(0), s_x^2, s_b^2, \rho)$$

Thus although the sufficient statistics are the same for each trial, the actual values of x(0) and b are statistically independent so that each trial is statistically independent of the rest and the averages are therefore unbiased.

When the optimal control policy is to be used it must be obtained by interpolating the table of control values (Appendix E). This was accomplished by the program described by the flow diagram of Figure 7.2. Because of limited computer storage the entire range of values of control inputs in Appendix E was not used but was restricted to*

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* Here μ_s, μ_b, and σ_x^2 are non-dimensional parameters corresponding approximately to the mean of the state, the mean of the gain and the variance of the state respectively (see Appendix C).

Figure 7.1 Unknown Constant Gain Problem Simulation

Flow Diagrám for Main Program

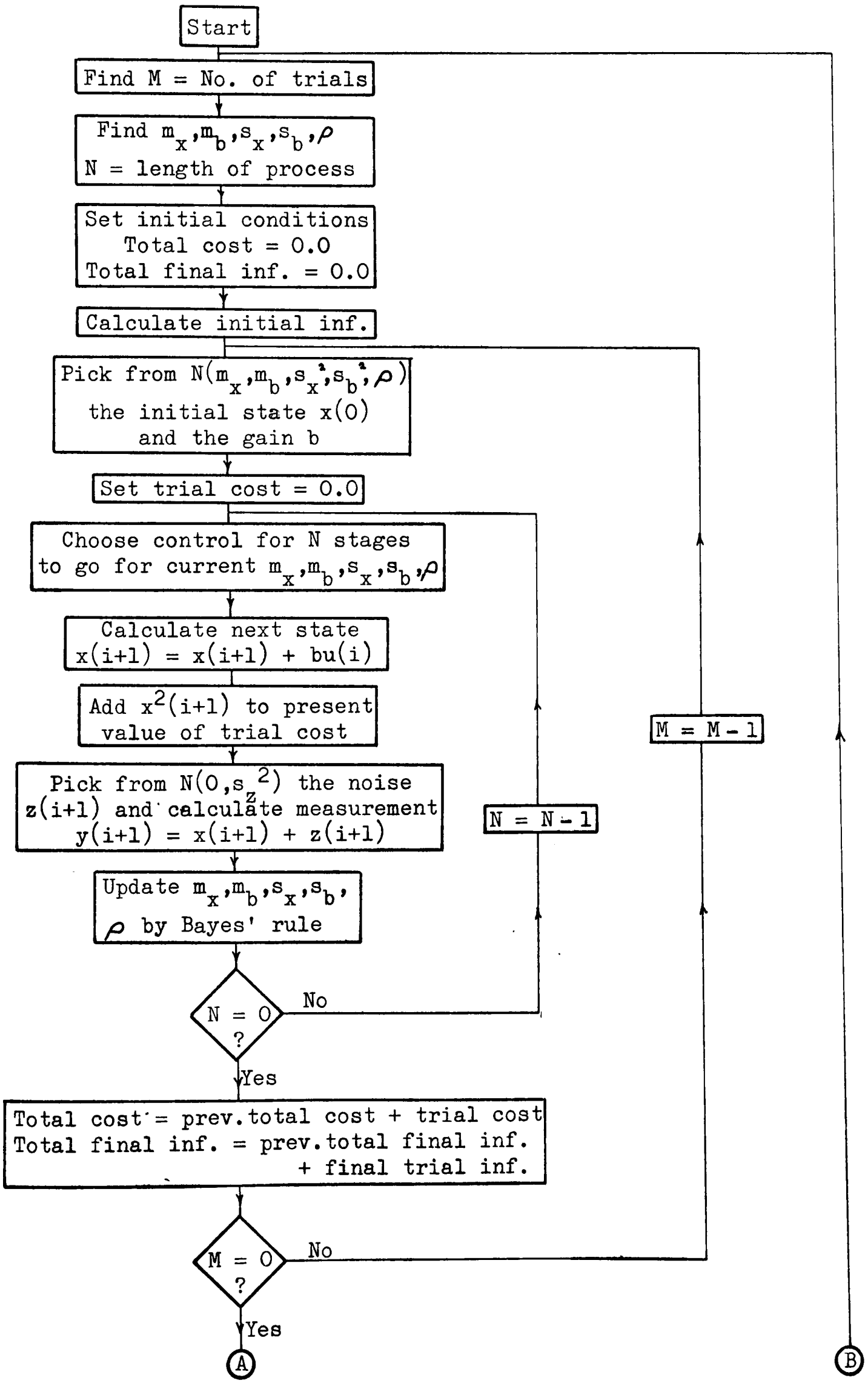


Figure 7.1 (cont.)

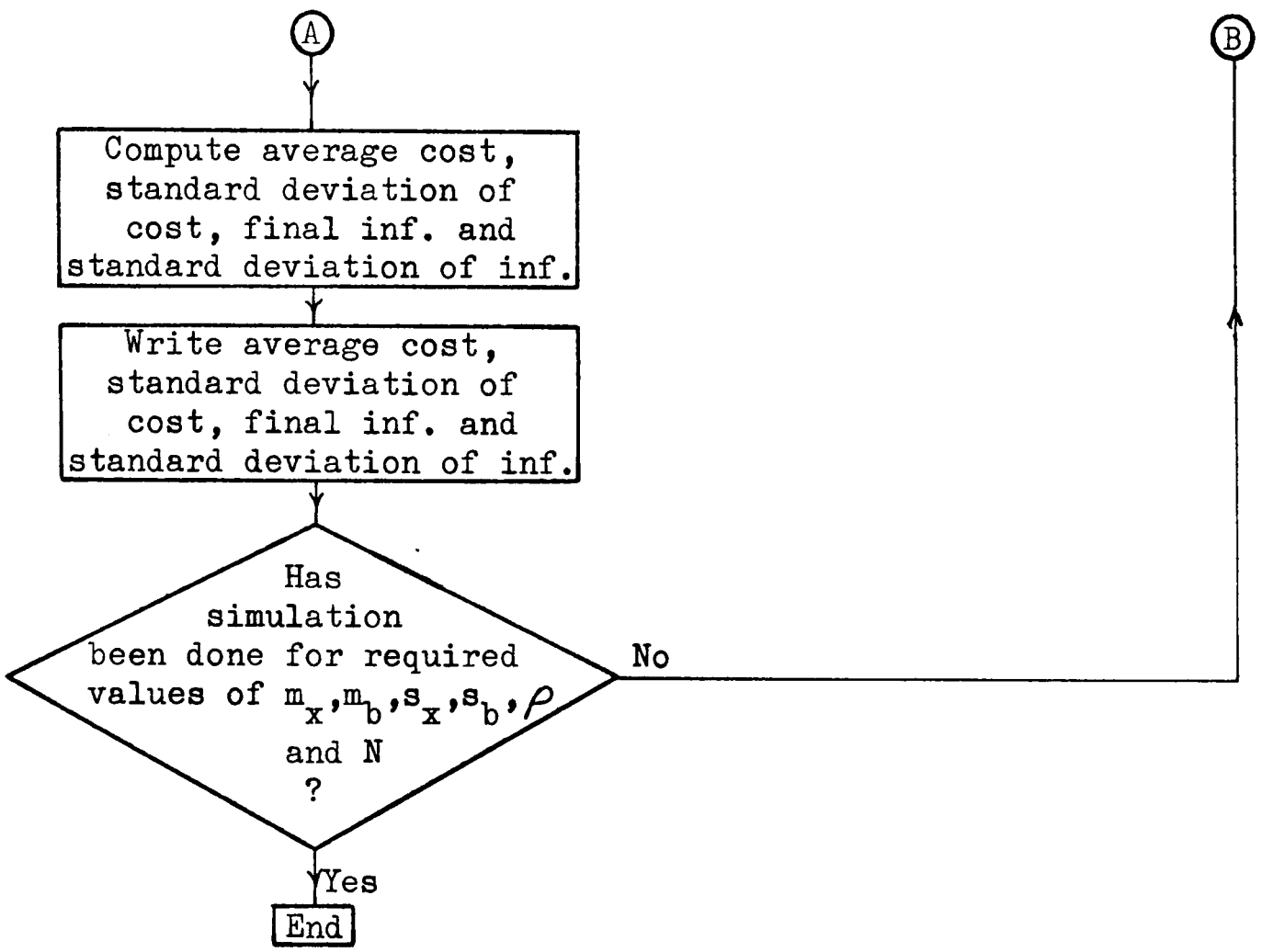
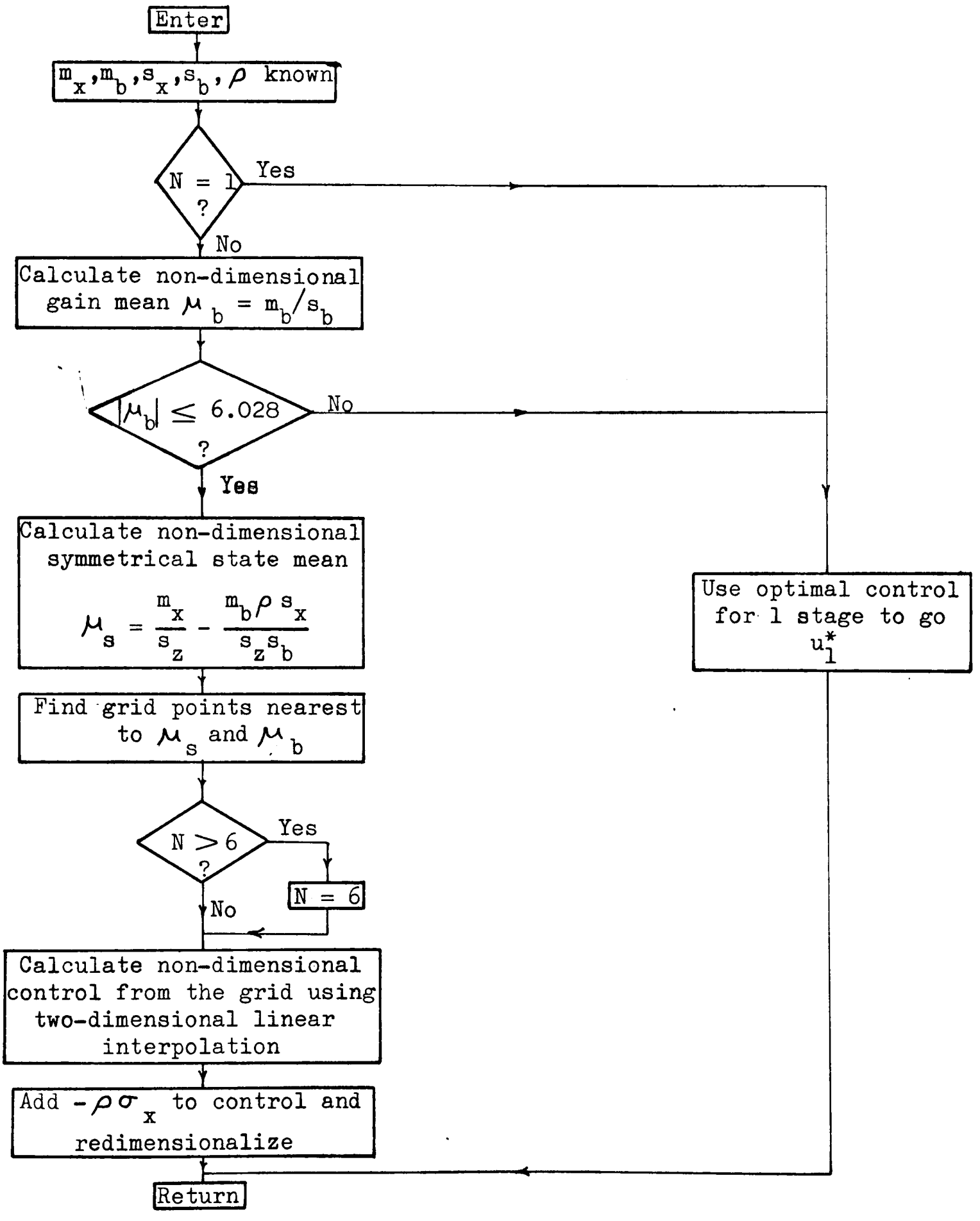


Figure 7.2 Unknown Constant Gain Problem Simulation

Optimal Control Subprogram (OPCON)



μ_s = 0 to 16.0 (8 values)

μ_b = 0 to 6.028 (14 values)

N = 2 to 6

In Appendix F an approximation which describes the variation of the optimal control policy with respect to σ_x and ρ is derived. The use of this approximation means that only the variation of the optimal control policy with respect to μ_s and μ_b needs to be tabulated. Thus the table of control values contains (8 x 14 x 5 =) 560 entries. Since μ_s tends to decrease during a process (as the state approaches zero), it can be arranged by choosing small enough initial conditions that the optimal control is never required for $\mu_s \geq 16.0$. However μ_b increases during a process (as the variance of the gain decreases) and consequently values of μ_b greater than 6.028 are sometimes required, particularly in longer processes. From Figures 6.10 to 6.12 it can be seen that the optimal policy for one stage to go (the neutral policy) is a good approximation of the optimal policy whenever $\mu_b \geq 6.028$ and therefore this approximation was used. The optimal control policy has only been determined for cases in which there are no more than six stages to go and in this simulation it is approximated by u_6^* for the cases in which there are more than six stages to go. If either μ_s, μ_b or ρ is negative the symmetry properties (Appendix C) allow the use of the table for positive μ_s and μ_b.

The main program needed a source of normally distributed random numbers for choosing x(0) and b for each trial and for choosing the value of the measurement noise z(i) at each stage. To avoid the difficulty of repeating random numbers mentioned in Section 2.5, a noise generator was used. A program was designed to check its characteristics by measuring the

mean, the standard deviation and the autocorrelation function, and by producing a histogram of the noise. A typical result is given in Figure 7.3 where the histogram is compared with an equivalent normal distribution. The noise-check program sampled at the rate of once every 15 msec. whereas the main simulation program used random numbers at a rate of one every 50 msec. so that since the noise program showed the noise to be uncorrelated, it also appeared to be uncorrelated to the simulation program. It was arranged that the mean of the noise generator output was zero and the standard deviation one, so that x(0), b and z(i) could be calculated from the externally generated random numbers r_i according to

$$x(0) = r_i s_x(0) + m_x(0) \qquad 7.2a$$

$$b = r_{i+1}\sqrt{s_b^2(0)(1-\rho^2(0))} + m_b(0) + \rho(0)s_b(0)(x(0) - m_x(0))/s_x(0) \qquad 7.2b$$

$$z(i) = r_j s_z \qquad 7.2c$$

Here Equation 7.2b takes into account the correlation between x(0) and b.

Six different control policies were used in the simulation:

Control policy	Notation	Equation
1) The certainty equivalent policy	$u^{c.e.}$	$= -m_x/m_b$
2) The neutral policy	u^{nt}	$= -(m_x m_b + \rho s_x s_b)/(m_b^2 + s_b^2)$
3) The optimal policy	$u_N^*(Op)$	Produced by OPCON
4) The optimal policy for N = 2	$u_2^*(Op)$	Produced by OPCON
5) A neutral-plus-probing policy	u^{ntP}	$= u^{nt}$ + probing
6) A separable policy	u^s	$= -m_x(m_b + 0.1)/(m_b^2 + 1.0)$

Figure 7.3 Unknown Constant Gain Problem Noise Histogram

Measured Statistics

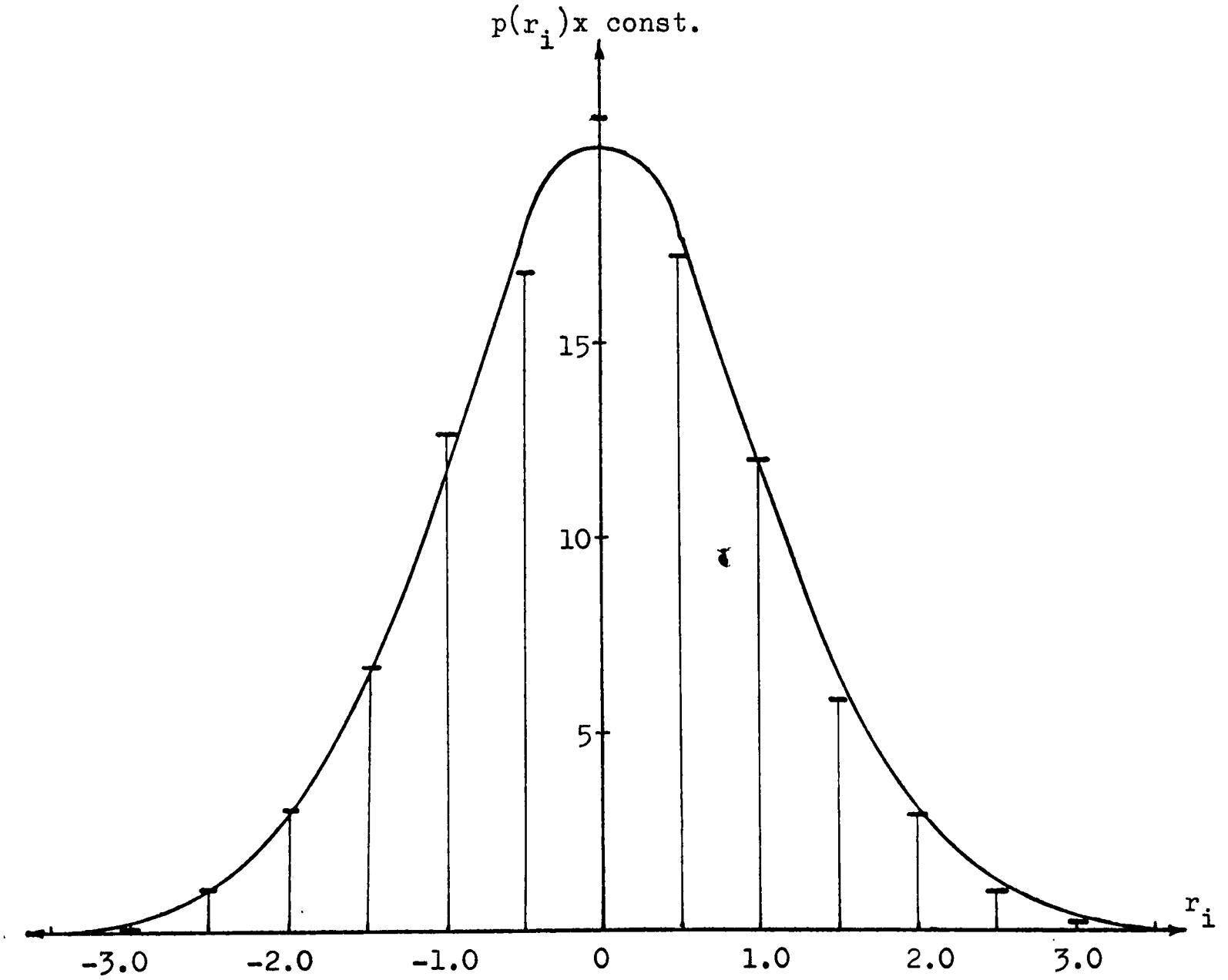
Average = -0.028

Standard deviation = 0.994

Number of samples = 10,000

Autocorrelation

j	$\frac{1}{N-j}\sum_{i=1}^{N} r_i r_{i+j}$
0	0.944
1	-0.0209
2	0.0078
3	-0.0127
4	-0.0056
5	-0.0069



Average costs and information gains were computed for a representative set of cases. Two values of N were chosen (6 and 20), the first because the optimal control had only been computed for up to six stages to go and the second because it showed up the properties of the controllers for longer length processes without requiring too lengthy computer runs. The values of the initial state chosen were designed to show how each controller coped with a fairly wide variety of initial states. Thus computations were done for a small, medium and large value of both m_s and m_b.* These were chosen also to coincide with grid points used in the optimal control policy derivation (Appendix D) so that a direct comparison could be made with the theoretical results of Appendix E. For N = 6 the averages were also calculated at each value of m_s and m_b for three different sets of values of s_x and ρ. This was not done for N = 20 because it was seen from the N = 6 results that with $s_x = 1.0$ and $\rho = 0.0$ a representative sample was obtained. Thus averages were computed for the following cases:

N = 6 (27 cases)	N = 20 (9 cases)
$m_s = 0.5, 2.0, 9.0$	$m_s = 0.5, 2.0, 9.0$
$m_b = 0.0, 1.116, 6.028$	$m_b = 0.0, 1.116, 6.028$
$(s_x, \rho) = (0.0,0.0), (1.0,0.0), (1.0,0.5)$	$(s_x, \rho) = (1.0,0.0)$.

These results appear in Tables 7.1 and 7.2. Also in Table 7.1 the numerically determined optimal expected cost from Appendix E is listed opposite

---

* The symmetric mean m_s (see Appendix C, Equation C.1) is used here rather than m_x so that direct comparisons can be made with the numerical results in Appendix E. There is little significant difference between the two and none at all for $\rho = 0.0$.

Table 7.1 The Unknown Constant Gain Problem

Average Costs and Final Information for Six Stages To Go

$m_s = 0.5$, $m_b = 0.0$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	1.46.		
u_N^*(Op)	1.45 ± 0.02		
$u^{c.e.}$			
u^{s}	1.65 ± 0.03		
u^{nt}	1.50 ± 0.00		
u_2^*(Op)	1.50 ± 0.00		
u^{ntP}	1.54 ± 0.02		

$m_s = 0.5$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	6.26		
u_N^*(Op)	6.28 ± 0.25	0.0	1.32 ± 0.01
$u^{c.e.}$	1,025 ± 396	0.0	2.83 ± 0.04
u^{s}	7.24 ± 0.30	0.0	1.03 ± 0.00
u^{nt}	7.44 ± 0.30	0.0	0.97 ± 0.00
u_2^*(Op)	7.31 ± 0.30	0.0	1.16 ± 0.01
u^{ntP}	5.66 ± 0.23	0.0	1.36 ± 0.01

$m_s = 0.5$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	5.26		
u_N^*(Op)	5.22 ± 0.22	0.1438	1.27 ± 0.01
$u^{c.e.}$	33,205 ± 26,797	0.1438	2.74 ± 0.04
u^{s}	5.84 ± 0.24	0.1438	1.22 ± 0.01
u^{nt}	5.87 ± 0.26	0.1438	1.00 ± 0.00
u_2^*(Op)	5.92 ± 0.23	0.1438	1.13 ± 0.00
u^{ntP}	5.57 ± 0.21	0.1438	1.36 ± 0.01

Table 7.1 (cont.)

$m_s = 0.5$, $m_b = 1.116$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	0.66		
u_N^*(Op)	0.68 ± 0.03		
$u^{c.e.}$	695 ± 553		
u^{s}	0.91 ± 0.04		
u^{nt}	0.66 ± 0.03		
u_2^*(Op)	0.70 ± 0.03		
u^{ntP}	0.74 ± 0.03		

$m_s = 0.5$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	4.31		
u_N^*(Op)	4.16 ± 0.20	0.0	1.26 ± 0.01
$u^{c.e.}$	2,879 ± 2,007	0.0	1.63 ± 0.03
u^{s}	4.44 ± 0.22	0.0	1.26 ± 0.01
u^{nt}	4.32 ± 0.23	0.0	1.17 ± 0.01
u_2^*(Op)	4.50 ± 0.21	0.0	1.23 ± 0.01
u^{ntP}	4.30 ± 0.18	0.0	1.28 ± 0.01

$m_s = 0.5$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	3.59		
u_N^*(Op)	3.61 ± 0.18	0.1438	1.22 ± 0.01
$u^{c.e.}$	167 ± 99	0.1438	1.61 ± 0.03
u^{s}	3.75 ± 0.17	0.1438	1.25 ± 0.01
u^{nt}	3.58 ± 0.17	0.1438	1.16 ± 0.00
u_2^*(Op)	1.71 ± 0.17	0.1438	1.21 ± 0.01
u^{ntP}	3.91 ± 0.17	0.1438	1.29 ± 0.01

Table 7.1 (cont.)

$m_s = 0.5$, $m_b = 6.028$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	0.0397		
u_N^*(Op)	0.0414 ± 0.0017		
$u^{c.e.}$	0.0382 ± 0.0016		
u^{s}	0.0357 ± 0.0014		
u^{nt}	0.0413 ± 0.0017		
u_2^*(Op)	0.0416 ± 0.0017		
u^{ntP}	0.0424 ± 0.0017		

$m_s = 0.5$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	2.59		
u_N^*(Op)	2.56 ± 0.09	0.0	0.992 ± 0.001
$u^{c.e.}$	2.48 ± 0.09	0.0	0.993 ± 0.001
u^{s}	2.45 ± 0.08	0.0	0.992 ± 0.001
u^{nt}	2.47 ± 0.08	0.0	0.992 ± 0.001
u_2^*(Op)	2.49 ± 0.08	0.0	0.992 ± 0.001
u^{ntP}	2.56 ± 0.08	0.0	0.993 ± 0.001

$m_s = 0.5$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	2.16		
u_N^*(Op)	2.11 ± 0.07	0.1438	1.01 ± 0.00
$u^{c.e.}$	1.95 ± 0.06	0.1438	1.01 ± 0.00
u^{s}	2.13 ± 0.07	0.1438	1.01 ± 0.00
u^{nt}	2.15 ± 0.08	0.1438	1.01 ± 0.00
u_2^*(Op)	2.29 ± 0.08	0.1438	1.01 ± 0.00
u^{ntP}	2.12 ± 0.07	0.1438	1.01 ± 0.00

Table 7.1 (cont.)

$m_s = 2.0$, $m_b = 0.0$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	12.56		
u_N^*(Op)	12.71 ± 0.31		
$u^{c.e.}$	6,700 ± 4,744		
u^s	20.27 ± 0.30		
u^{nt}	24.00 ± 0.00		
u_2^*(Op)	13.55 ± 0.28		
u^{ntP}	14.00 ± 0.29		

$m_s = 2.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	17.03		
u_N^*(Op)	17.04 ± 0.53	0.0	2.34 ± 0.02
$u^{c.e.}$	5,772 ± 5,337	0.0	3.16 ± 0.05
u^s	26.12 ± 0.74	0.0	1.39 ± 0.01
u^{nt}	30.44 ± 0.82	0.0	0.97 ± 0.00
u_2^*(Op)	18.67 ± 0.56	0.0	2.14 ± 0.02
u^{ntP}	17.02 ± 0.50	0.0	2.20 ± 0.02

$m_s = 2.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	16.13		
u_N^*(Op)	15.84 ± 0.49	0.1438	2.40 ± 0.02
$u^{c.e.}$	3,412 ± 2,204	0.1438	3.19 ± 0.04
u^s	25.31 ± 0.67	0.1438	1.52 ± 0.01
u^{nt}	26.90 ± 0.66	0.1438	1.00 ± 0.00
u_2^*(Op)	17.97 ± 0.51	0.1438	2.20 ± 0.02
u^{ntP}	17.05 ± 0.49	0.1438	2.28 ± 0.02

Table 7.1 (cont.)

$m_s = 2.0$, $m_b = 1.116$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	6.25		
u_N^*(Op)	6.59 ± 0.27		
$u^{c.e.}$	318.12 ± 170.71		
u^s	8.46 ± 0.37		
u^{nt}	7.64 ± 0.30		
u_2^*(Op)	6.26 ± 0.25		
u^{ntP}	7.02 ± 0.27		

$m_s = 2.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	9.22		
u_N^*(Op)	9.71 ± 0.45	0.0	1.91 ± 0.02
$u^{c.e.}$	2,431 ± 2,139	0.0	2.23 ± 0.04
u^s	10.89 ± 0.59	0.0	1.63 ± 0.01
u^{nt}	10.90 ± 0.53	0.0	1.63 ± 0.01
u_2^*(Op)	10.02 ± 0.47	0.0	1.85 ± 0.02
u^{ntP}	10.13 ± 0.48	0.0	1.89 ± 0.02

$m_s = 2.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	8.58		
u_N^*(Op)	9.27 ± 0.42	0.1438	2.02 ± 0.02
$u^{c.e.}$	9,488 ± 6,650	0.1438	2.32 ± 0.04
u^s	11.52 ± 0.59	0.1438	1.70 ± 0.01
u^{nt}	11.40 ± 0.55	0.1438	1.70 ± 0.01
u_2^*(Op)	9.62 ± 0.47	0.1438	1.90 ± 0.02
u^{ntP}	9.12 ± 0.41	0.1438	1.97 ± 0.02

Table 7.1 (cont.)

$m_s = 2.0$, $m_b = 6.028$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	0.53		
u_N^*(Op)	0.51 ± 0.02		
$u^{c.e.}$	0.56 ± 0.02		
u^{s}	0.53 ± 0.02		
u^{nt}	0.52 ± 0.02		
u_2^*(Op)	0.57 ± 0.02		
u^{ntP}	0.55 ± 0.02		

$m_s = 2.0$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	2.75		
u_N^*(Op)	2.70 ± 0.09	0.0	1.036 ± 0.001
$u^{c.e.}$	2.76 ± 0.10	0.0	1.035 ± 0.001
u^{s}	2.61 ± 0.09	0.0	1.034 ± 0.001
u^{nt}	2.57 ± 0.08	0.0	1.034 ± 0.001
u_2^*(Op)	2.69 ± 0.09	0.0	1.034 ± 0.001
u^{ntP}	2.64 ± 0.09	0.0	1.035 ± 0.001

$m_s = 2.0$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	2.34		
u_N^*(Op)	2.26 ± 0.08	0.1438	1.064 ± 0.001
$u^{c.e.}$	2.30 ± 0.08	0.1438	1.069 ± 0.001
u^{s}	2.34 ± 0.08	0.1438	1.067 ± 0.001
u^{nt}	2.24 ± 0.08	0.1438	1.065 ± 0.001
u_2^*(Op)	2.26 ± 0.08	0.1438	1.066 ± 0.001
u^{ntP}	2.43 ± 0.08	0.1438	1.072 ± 0.001

Table 7.1 (cont.)

$m_s = 9.0$, $m_b = 0.0$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	126.64		
u_N^*(Op)	129.87 ± 2.92		
$u^{c.e.}$			
u^{s}	269.86 ± 5.05		
u^{nt}	486.00 ± 0.06		
u_2^*(Op)	128.74 ± 2.80		
u^{ntP}	133.2 ± 3.4		

$m_s = 9.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	140.27		
u_N^*(Op)	145.48 ± 3.38	0.0	4.03 ± 0.04
$u^{c.e.}$	1,000 ± 253	0.0	4.28 ± 0.05
u^{s}	237.74 ± 5.38	0.0	3.00 ± 0.01
u^{nt}	490.05 ± 3.38	0.0	0.97 ± 0.00
u_2^*(Op)	149.12 ± 3.39	0.0	4.04 ± 0.04
u^{ntP}	146.2 ± 3.7	0.0	4.08 ± 0.03

$m_s = 9.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	137.12		
u_N^*(Op)	136.88 ± 3.07	0.1438	4.06 ± 0.03
$u^{c.e.}$	45,352 ± 44,394	0.1438	4.28 ± 0.04
u^{s}	333.99 ± 5.12	0.1438	2.98 ± 0.01
u^{nt}	490.25 ± 2.92	0.1438	1.00 ± 0.00
u_2^*(Op)	135.52 ± 2.90	0.1438	4.06 ± 0.03
u^{ntP}	142.5 ± 3.6	0.1438	4.12 ± 0.03

Table 7.1 (cont.)

$m_s = 9.0$, $m_b = 1.116$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	51.52		
u_N^*(Op)	57.90 ± 2.47		
$u^{c.e.}$	311.46 ± 100.39		
u^s	115.37 ± 5.52		
u^{nt}	56.61 ± 2.57		
u_2^*(Op)	53.33 ± 2.37		
u^{ntP}	59.7 ± 2.5		

$m_s = 9.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	57.63		
u_N^*(Op)	56.41 ± 2.52	0.0	3.35 ± 0.03
$u^{c.e.}$	502.90 ± 281.37	0.0	3.52 ± 0.04
u^s	114.82 ± 5.49	0.0	2.89 ± 0.01
u^{nt}	62.11 ± 2.88	0.0	3.30 ± 0.03
u_2^*(Op)	61.03 ± 2.68	0.0	3.39 ± 0.04
u^{ntP}	67.2 ± 2.9	0.0	3.43 ± 0.03

$m_s = 9.0$, $m_b - 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	56.35		
u_N^*(Op)	56.03 ± 2.4?	0.1438	3.48 ± 0.03
$u^{c.e.}$	620.56 ± 278.06	0.1438	3.65 ± 0.04
u^s	107.59 ± 5.43	0.1438	3.00 ± 0.01
u^{nt}	64.04 ± 2.93	0.1438	3.45 ± 0.03
u_2^*(Op)	57.25 ± 2.58	0.1438	3.47 ± 0.03
u^{ntP}	68.5 ± 3.09	0.1438	3.56 ± 0.03

Table 7.1 (cont.)

$m_s = 9.0$, $m_b = 6.028$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
u_N^*(Theory)	4.08		
u_N^*(Op)	4.22 ± 0.15		
$u^{c.e.}$	4.11 ± 0.13		
u^{s}	4.12 ± 0.13		
u^{nt}	4.08 ± 0.14		
u_2^*(Op)	4.33 ± 0.13		
u^{ntP}	4.33 ± 0.12		

$m_s = 9.0$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

u_N^*(Theory)	5.29		
u_N^*(Op)	5.09 ± 0.16	0.0	1.525 ± 0.004
$u^{c.e.}$	5.37 ± 0.18	0.0	1.533 ± 0.004
u^{s}	5.12 ± 0.17	0.0	1.530 ± 0.003
u^{nt}	5.21 ± 0.18	0.0	1.524 ± 0.003
u_2^*(Op)	5.28 ± 0.17	0.0	1.526 ± 0.004
u^{ntP}	5.35 ± 0.16	0.0	1.534 ± 0.003

$m_s = 9.0$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

u_N^*(Theory)	5.00		
u_N^*(Op)	4.65 ± 0.15	0.1438	1.626 ± 0.004
$u^{c.e.}$	4.99 ± 0.16	0.1438	1.635 ± 0.004
u^{s}	5.20 ± 0.17	0.1438	1.635 ± 0.004
u^{nt}	4.86 ± 0.16	0.1438	1.620 ± 0.004
u_2^*(Op)	4.84 ± 0.16	0.1438	1.631 ± 0.004
u^{ntP}	5.98 ± 0.19	0.1438	1.642 ± 0.003

Table 7.2 The Unknown Constant Gain Problem

Average Costs and Final Information Contents for 20 Stages To Go

$m_s = 0.5$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
$u_N^*(0p)$	9.65 ± 0.33	0.0	2.50 ± 0.03
u^s	19.39 ± 0.78	0.0	2.15 ± 0.02
u^{nt}	24.10 ± 1.02	0.0	1.52 ± 0.00
$u_2^*(0p)$	13.38 ± 0.45	0.0	2.14 ± 0.03
u^{ntP}	11.27 ± 0.45	0.0	2.39 ± 0.02

$m_s = 0.5$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$	6.69 ± 0.29	0.0	2.16 ± 0.03
u^s	9.38 ± 0.61	0.0	2.10 ± 0.02
u^{nt}	9.28 ± 0.51	0.0	1.96 ± 0.02
$u_2^*(0p)$	7.69 ± 0.33	0.0	2.10 ± 0.02
u^{ntP}	7.77 ± 0.35	0.0	2.16 ± 0.02

$m_s = 0.5$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$	3.58 ± 0.10	0.0	1.557 ± 0.001
u^s	3.62 ± 0.10	0.0	1.559 ± 0.001
u^{nt}	3.64 ± 0.10	0.0	1.559 ± 0.001
$u_2^*(0p)$	3.65 ± 0.11	0.0	1.558 ± 0.001
u^{ntP}	3.51 ± 0.10	0.0	1.558 ± 0.001

Table 7.2 (cont.)

$m_s = 2.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
$u_N^*(0p)$	19.56 ± 0.62	0.0	3.38 ± 0.04
u^s	56.10 ± 1.87	0.0	2.91 ± 0.02
u^{nt}	97.15 ± 2.62	0.0	1.52 ± 0.00
$u_2^*(0p)$	22.26 ± 0.78	0.0	3.30 ± 0.04
u^{ntP}	23.26 ± 0.80	0.0	3.28 ± 0.03

$m_s = 2.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$	10.96 ± 0.45	0.0	2.74 ± 0.03
u^s	23.89 ± 1.53	0.0	2.52 ± 0.02
u^{nt}	18.80 ± 1.06	0.0	2.58 ± 0.03
$u_2^*(0p)$	12.12 ± 0.54	0.0	2.73 ± 0.03
u^{ntP}	13.59 ± 0.64	0.0	2.78 ± 0.03

$m_s = 2.0$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$	3.90 ± 0.11	0.0	1.606 ± 0.002
u^s	3.76 ± 0.11	0.0	1.604 ± 0.002
u^{nt}	3.92 ± 0.12	0.0	1.604 ± 0.002
$u_2^*(0p)$	3.89 ± 0.12	0.0	1.606 ± 0.002
u^{ntP}	3.80 ± 0.11	0.0	1.606 ± 0.002

Table 7.2 (cont.)

$m_s = 9.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

	Average cost	Initial inf.	Av.final inf.
$u_N^*(0p)$	150.76 ± 4.50	0.0	4.83 ± 0.06
u^s	545.47 ± 17.72	0.0	4.36 ± 0.02
u^{nt}	1,650.10 ± 10.93	0.0	1.52 ± 0.00
$u_2^*(0p)$	151.95 ± 5.25	0.0	4.87 ± 0.08
u^{ntP}	162.37 ± 6.18	0.0	4.89 ± 0.07

$m_s = 9.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$	64.00 ± 2.90	0.0	4.05 ± 0.04
u^s	212.30 ± 14.44	0.0	3.76 ± 0.02
u^{nt}	65.52 ± 3.08	0.0	4.03 ± 0.04
$u_2^*(0p)$	59.84 ± 2.59	0.0	4.04 ± 0.04
u^{ntP}	67.09 ± 2.82	0.0	4.04 ± 0.03

$m_s = 9.0$, $m_b = 6.028$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$	6.33 ± 0.19	0.0	2.123 ± 0.004
u^s	6.33 ± 0.19	0.0	2.123 ± 0.004
u^{nt}	6.29 ± 0.18	0.0	2.121 ± 0.004
$u_2^*(0p)$	6.68 ± 0.19	0.0	2.125 ± 0.005
u^{ntP}	6.63 ± 0.19	0.0	2.128 ± 0.004

u_N^*(Theory) so that it may be compared with the simulation results.

For six sets of initial sufficient statistics average cost and information trajectories were computed. These consisted of computing the average cost and final information contents for N = 2, 4, 6, 10 and 20 for the following initial conditions:

$m_s(0)$	$m_b(0)$	$s_x^2(0)$	$s_b^2(0)$	$\rho(0)$
0.5	0.0	1.0	1.0	0.0
0.5	1.116	1.0	1.0	0.0
0.5	1.116	0.0	1.0	0.0
0.5	1.116	1.0	1.0	0.5
2.0	0.0	1.0	1.0	0.0
2.0	1.116	1.0	1.0	0.0

The choice of initial conditions was designed to represent small and medium values of m_s and m_b and to show variations caused by different values of (s_x^2, ρ) for one value of (m_s, m_b). These results appear in Table 7.3 and Figures 7.4 to 7.9. The values of s_z and $s_b(0)$ were always the same $(s_z = 1.0$ and $s_b(0) = 1.0)$ for all the tables and figures because the effect of a change in s_z or $s_b(0)$ could be seen by considering proportional changes in the other parameters (see Appendix C, Dimensional Analysis).

Some of the entries in Tables 7.1, 7.2 and 7.3 are missing because in some cases both the initial and final information contents are infinite. This occurs whenever $s_x^2(0) = 0.0$ and in these cases the entries under initial and final information contents are omitted. Also entries for the average cost given by the certainty equivalent control policy are omitted occasionally in Table 7.1 because in these cases the average cost was

Table 7.3 The Unknown Constant Gain Problem

Average Cost and Final Information Trajectories

$m_s = 0.5$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

Stages To Go	$u_N^*(0p)$		u^s	
	Average cost	Av.final inf.	Average cost	Av.final inf.
2	2.53 ± 0.10	0.55 ± 0.00	2.50 ± 0.10	0.55 ± 0.00
4	4.38 ± 0.17	0.91 ± 0.01	4.86 ± 0.20	0.82 ± 0.00
6	6.28 ± 0.24	1.32 ± 0.01	7.23 ± 0.30	1.03 ± 0.00
10	8.00 ± 0.30	1.84 ± 0.02	11.76 ± 0.46	1.40 ± 0.01
20	9.65 ± 0.33	2.51 ± 0.03	19.38 ± 0.77	2.14 ± 0.02

	u^{nt}		$u_2^*(0p)$	
2	2.53 ± 0.11	0.54 ± 0.00	2.36 ± 0.09	0.55 ± 0.00
4	4.79 ± 0.19	0.80 ± 0.00	4.37 ± 0.19	0.86 ± 0.00
6	7.44 ± 0.30	0.97 ± 0.00	7.30 ± 0.30	1.15 ± 0.01
10	12.57 ± 0.53	1.20 ± 0.00	9.72 ± 0.34	1.54 ± 0.02
20	24.10 ± 1.02	1.52 ± 0.00	13.38 ± 0.44	2.14 ± 0.03

	u^{ntP}	
2	2.82 ± 0.13	0.65 ± 0.00
4	4.30 ± 0.20	1.08 ± 0.01
6	5.66 ± 0.23	1.36 ± 0.01
10	7.69 ± 0.29	1.76 ± 0.02
20	11.27 ± 0.45	2.39 ± 0.02

Table 7.3 (cont.)

$m_s = 0.5$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

	$u_N^*(0p)$		u^s	
Stages To Go	Average cost	Av.final inf.	Average cost	Av.final inf.
2	1.82 ± 0.08	0.61 ± 0.00	2.05 ± 0.08	0.63 ± 0.00
4	3.03 ± 0.13	0.96 ± 0.01	3.29 ± 0.16	0.99 ± 0.01
6	4.16 ± 0.20	1.26 ± 0.01	4.44 ± 0.22	1.26 ± 0.01
10	5.56 ± 0.24	1.65 ± 0.02	5.45 ± 0.29	1.59 ± 0.01
20	6.69 ± 0.29	2.16 ± 0.03	9.38 ± 0.61	2.10 ± 0.02

	u^{nt}		$u_2^*(0p)$	
2	1.82 ± 0.07	0.61 ± 0.01	1.85 ± 0.08	0.61 ± 0.00
4	3.34 ± 0.14	0.94 ± 0.00	3.38 ± 0.16	0.96 ± 0.01
6	4.31 ± 0.23	1.17 ± 0.01	4.51 ± 0.21	1.23 ± 0.01
10	6.22 ± 0.32	1.49 ± 0.01	5.81 ± 0.25	1.60 ± 0.02
20	9.28 ± 0.51	1.96 ± 0.02	7.69 ± 0.33	2.13 ± 0.02

	u^{ntP}	
2	2.06 ± 0.08	0.67 ± 0.00
4	3.24 ± 0.15	1.03 ± 0.01
6	4.30 ± 0.18	1.28 ± 0.01
10	5.43 ± 0.23	1.64 ± 0.01
20	7.77 ± 0.35	2.16 ± 0.02

Table 7.3 (cont.)

$m_s = 0.5$, $m_b = 1.116$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

	$u_N^*(0p)$		u^s	
Stages To Go	Average cost	Av.final inf.	Average cost	Av.final inf.
2	0.22 ± 0.01		0.24 ± 0.01	
4	0.47 ± 0.02		0.62 ± 0.03	
6	0.67 ± 0.03		0.96 ± 0.04	
10	1.11 ± 0.04		1.29 ± 0.06	
20	2.14 ± 0.08		2.38 ± 0.10	

	u^{nt}		$u_2^*(0p)$	
2	0.24 ± 0.01		0.23 ± 0.01	
4	0.47 ± 0.02		0.48 ± 0.02	
6	0.66 ± 0.03		0.70 ± 0.03	
10	1.06 ± 0.04		1.16 ± 0.04	
20	2.02 ± 0.08		2.32 ± 0.08	

u^{ntP}

2	0.22 ± 0.01	
4	0.47 ± 0.02	
6	0.74 ± 0.03	
10	1.13 ± 0.04	
20	2.17 ± 0.08	

Table 7.3 (cont.)

$m_s = 0.5$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

	$u_N^*(0p)$		u^s	
Stages To Go	Average cost	Av.final inf.	Average cost	Av.final inf.
2	1.57 ± 0.07	0.66 ± 0.00	1.53 ± 0.06	0.65 ± 0.00
4	2.56 ± 0.11	0.97 ± 0.00	2.61 ± 0.13	1.01 ± 0.01
6	3.60 ± 0.18	1.22 ± 0.01	3.75 ± 0.17	1.25 ± 0.01
10	4.58 ± 0.20	1.60 ± 0.02	5.61 ± 0.29	1.59 ± 0.01
20	5.49 ± 0.23	2.11 ± 0.02	8.73 ± 0.52	2.10 ± 0.01

	u^{nt}		$u_2^*(0p)$	
2	1.49 ± 0.06	0.65 ± 0.00	1.48 ± 0.06	0.66 ± 0.00
4	2.81 ± 0.12	0.95 ± 0.00	2.73 ± 0.12	0.98 ± 0.00
6	3.57 ± 0.17	1.15 ± 0.00	3.71 ± 0.17	1.21 ± 0.01
10	5.16 ± 0.29	1.45 ± 0.01	4.71 ± 0.23	1.55 ± 0.01
20	9.22 ± 0.55	1.91 ± 0.01	6.67 ± 0.32	2.06 ± 0.02

	u^{ntP}	
2	1.63 ± 0.07	0.72 ± 0.00
4	2.85 ± 0.13	1.06 ± 0.01
6	3.91 ± 0.17	1.29 ± 0.01
10	5.11 ± 0.26	1.62 ± 0.01
20	6.53 ± 0.29	2.08 ± 0.02

Table 7.3 (cont.)

$m_s = 2.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

$u_N^*(0p)$ u^s

Stages To Go	Average cost	Av.final inf.	Average cost	Av.final inf.
2	9.7 ± 0.3	0.80 ± 0.01	10.3 ± 0.3	0.59 ± 0.01
4	14.8 ± 0.4	1.78 ± 0.01	19.0 ± 0.5	1.00 ± 0.01
6	17.0 ± 0.5	2.34 ± 0.02	26.1 ± 0.7	1.39 ± 0.01
10	18.6 ± 0.6	2.88 ± 0.03	40.6 ± 1.2	2.05 ± 0.01
20	19.6 ± 0.6	3.37 ± 0.04	56.1 ± 1.9	2.90 ± 0.02

u^{nt} $u_2^*(0p)$

2	9.5 ± 0.2	0.55 ± 0.00	9.9 ± 0.3	0.81 ± 0.01
4	19.8 ± 0.6	0.80 ± 0.00	15.5 ± 0.5	1.61 ± 0.01
6	30.4 ± 0.8	0.97 ± 0.00	18.6 ± 0.6	2.13 ± 0.02
10	47.5 ± 1.2	1.99 ± 0.00	20.5 ± 0.6	2.69 ± 0.03
20	97.1 ± 2.6	1.52 ± 0.00	22.2 ± 0.8	3.29 ± 0.04

u^{ntP}

2	10.1 ± 0.3	1.11 ± 0.01
4	15.1 ± 0.5	1.80 ± 0.02
6	17.0 ± 0.5	2.20 ± 0.02
10	20.7 ± 0.7	2.69 ± 0.03
20	23.3 ± 0.8	3.28 ± 0.03

Table 7.3 (cont.)

$m_s = 2.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$

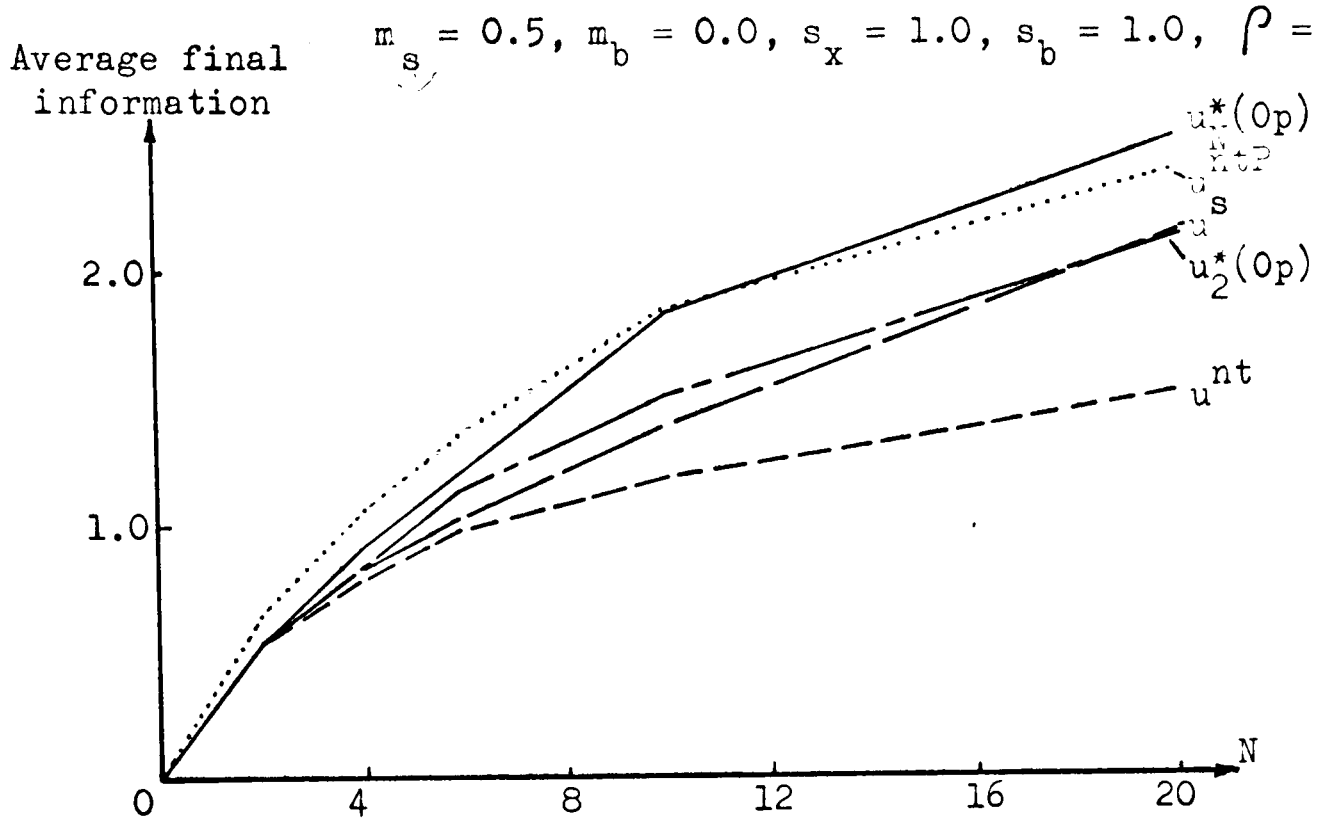
	$u_N^*(0p)$		u^s	
Stages To Go	Average cost	Av.final inf.	Average cost	Av.final inf.
2	4.78 ± 0.21	0.91 ± 0.00	5.02 ± 0.23	0.93 ± 0.00
4	7.71 ± 0.35	1.52 ± 0.01	9.35 ± 0.46	1.37 ± 0.01
6	9.72 ± 0.43	1.91 ± 0.02	10.94 ± 0.63	1.63 ± 0.01
10	9.94 ± 0.49	2.26 ± 0.02	15.19 ± 0.88	2.02 ± 0.02
20	10.96 ± 0.45	2.74 ± 0.03	24.00 ± 1.55	2.52 ± 0.02

	u^{nt}		$u_2^*(0p)$	
2	5.20 ± 0.23	0.88 ± 0.00	4.89 ± 0.22	0.92 ± 0.00
4	8.39 ± 0.39	1.32 ± 0.01	8.05 ± 0.36	1.45 ± 0.01
6	10.90 ± 0.53	1.61 ± 0.01	10.01 ± 0.47	1.84 ± 0.02
10	14.22 ± 0.82	2.05 ± 0.02	10.96 ± 0.50	2.22 ± 0.03
20	18.84 ± 1.00	2.53 ± 0.03	12.10 ± 0.53	2.72 ± 0.03

	u^{ntP}	
2	4.89 ± 0.22	1.08 ± 0.01
4	8.11 ± 0.36	1.58 ± 0.01
6	10.13 ± 0.48	1.89 ± 0.02
10	11.77 ± 0.52	2.27 ± 0.03
20	13.59 ± 0.64	2.78 ± 0.03

Figure 7.4 Average Cost and Information vs. No. of Stages

$m_s = 0.5$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$



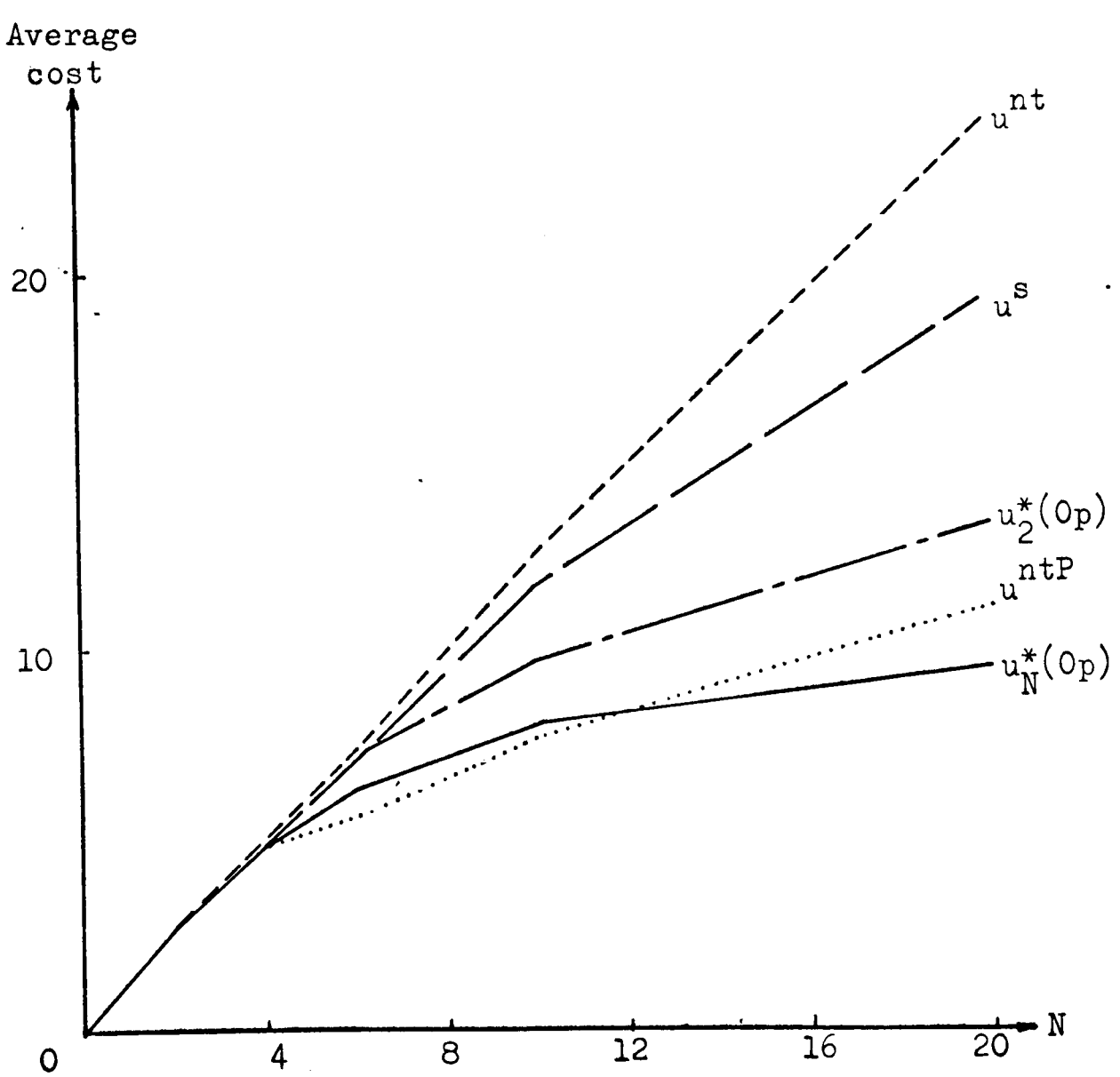


Figure 7.5 Average Cost and Information vs. No. of Stages To Go

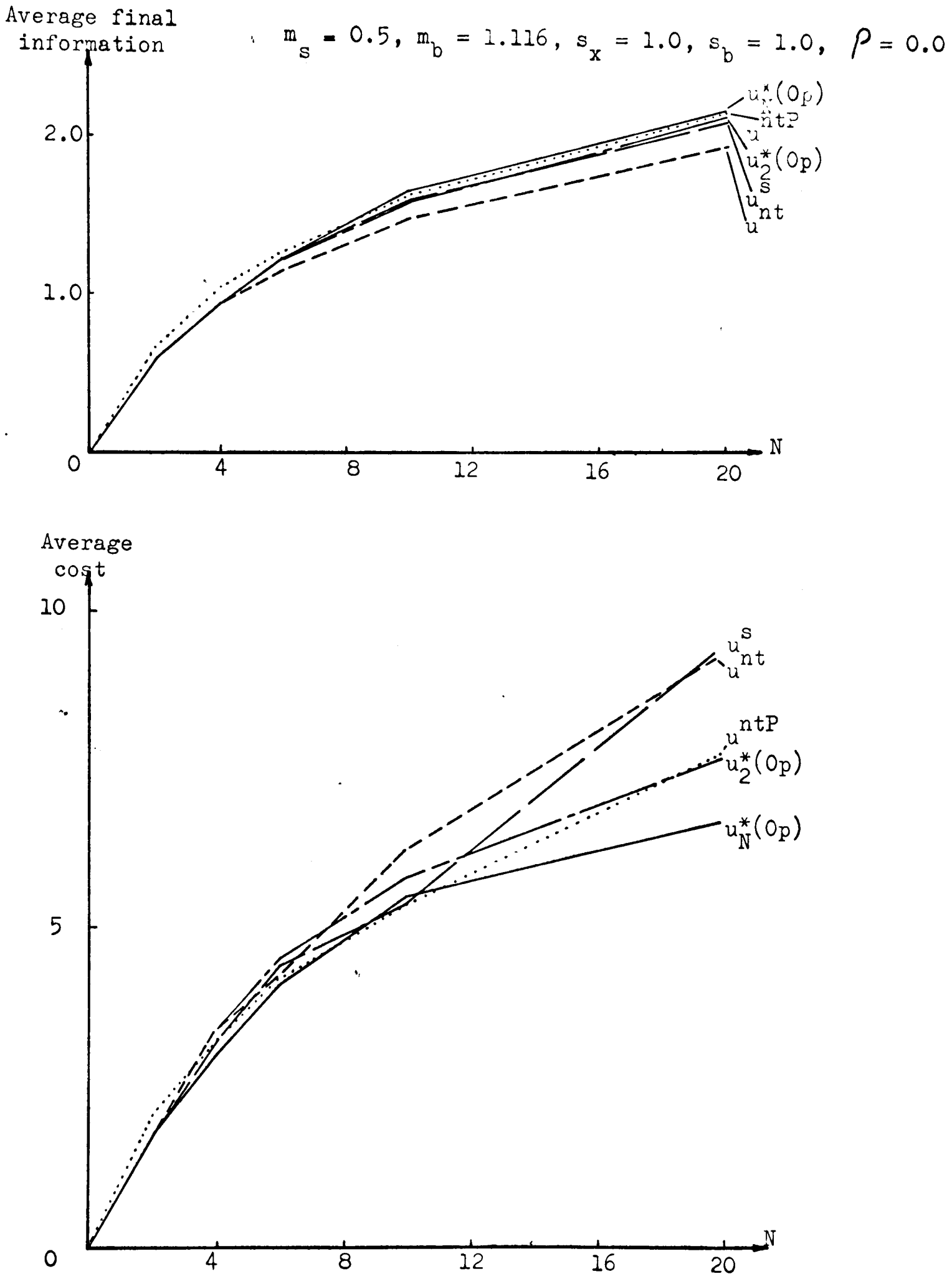


Figure 7.6 Average Cost vs. Number of Stages To Go

$m_s = 0.5$, $m_b = 1.116$, $s_x = 0.0$, $s_b = 1.0$, $\rho = 0.0$

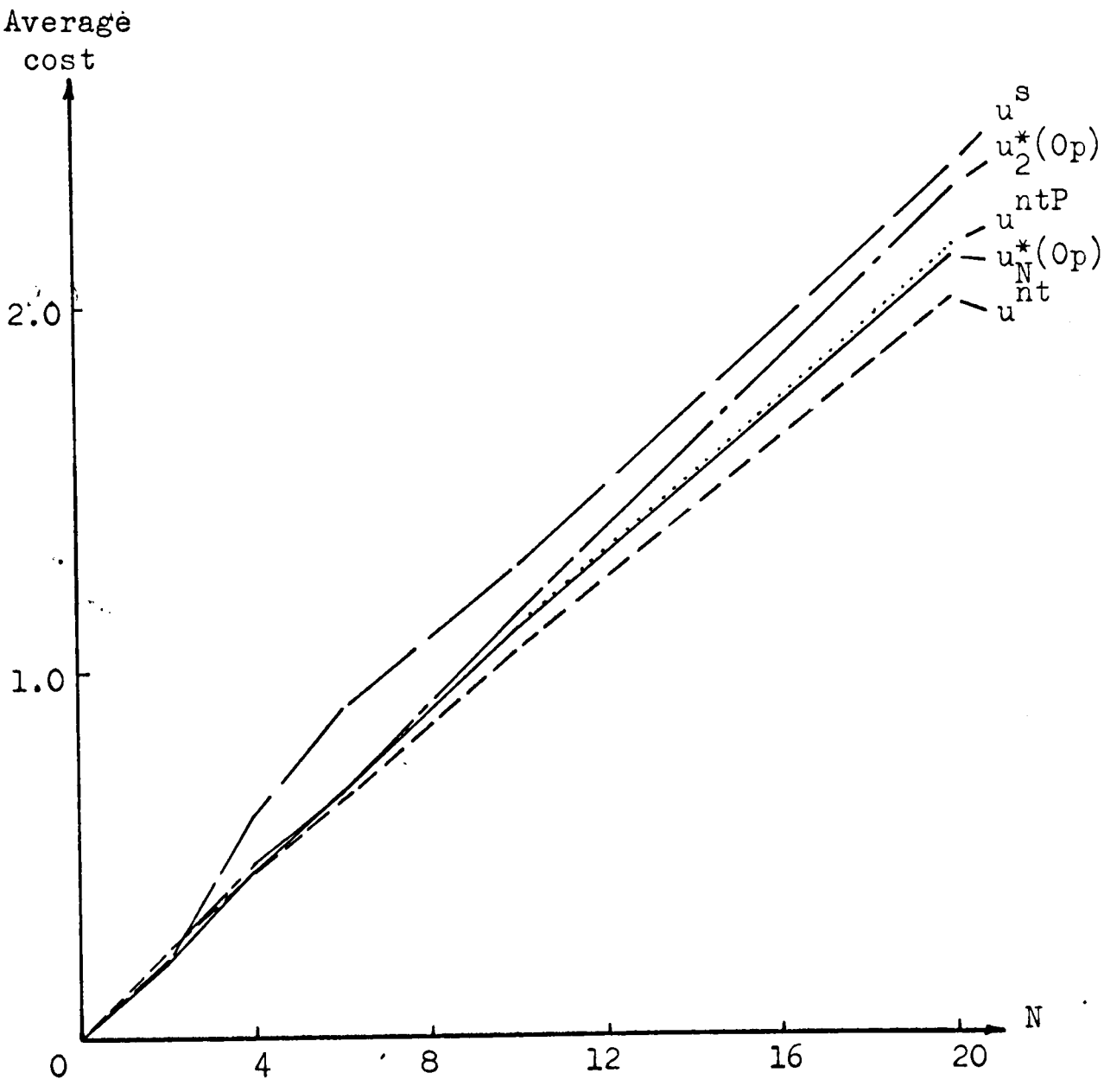


Figure 7.7 Average Cost and Information vs. No. of Stages To Go

$m_s = 0.5$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.5$

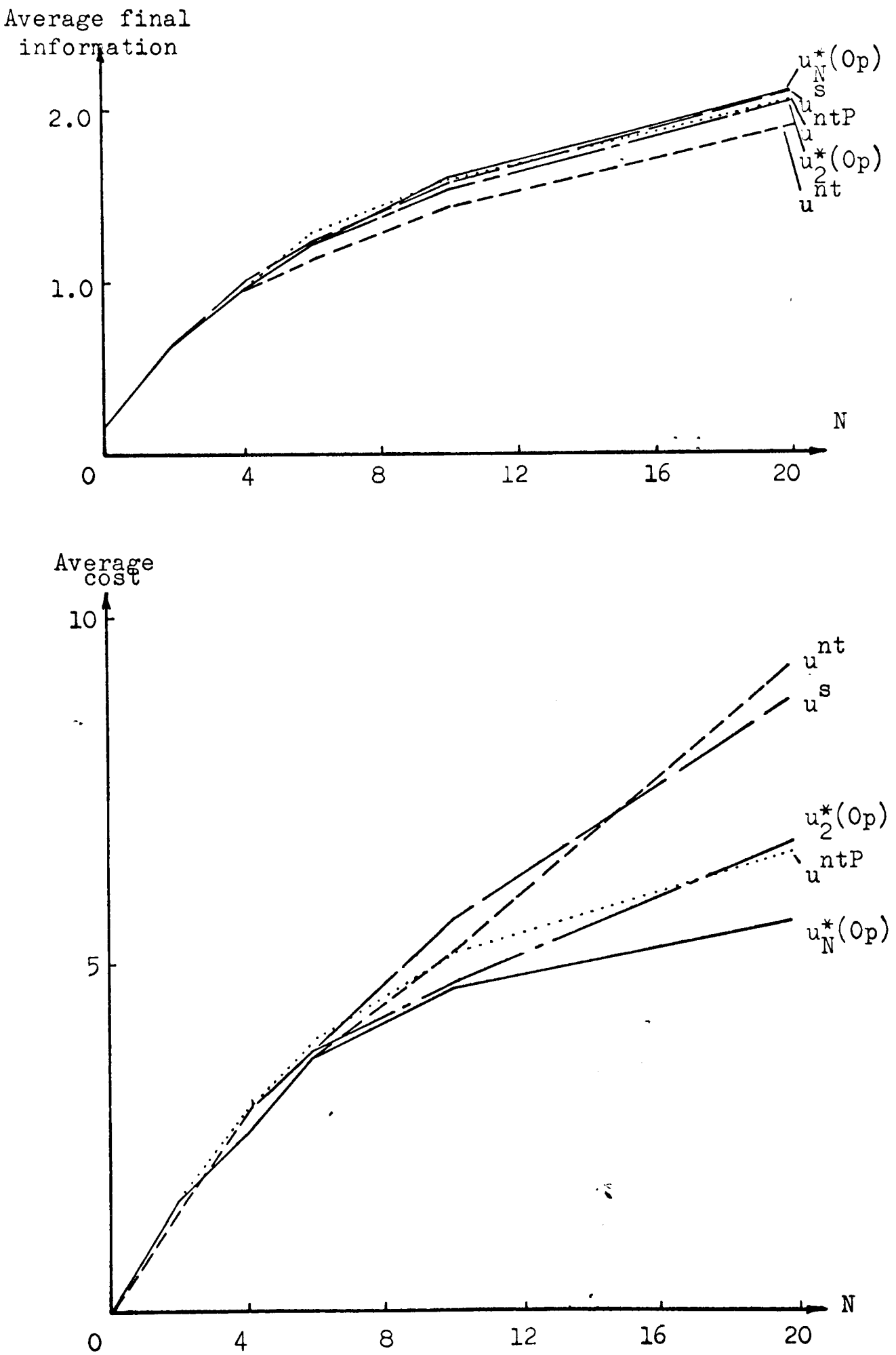
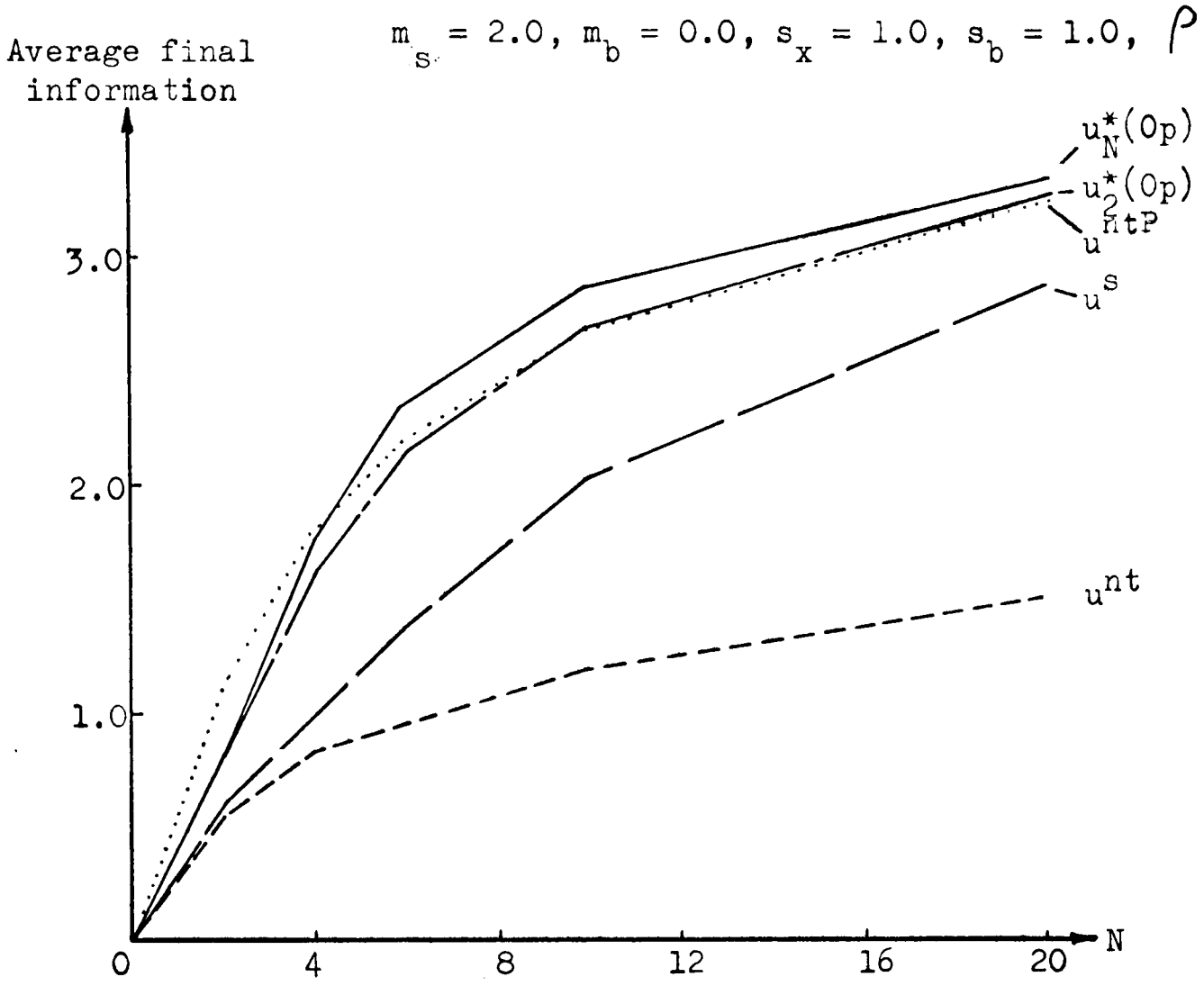


Figure 7.8 Average Cost and Information vs. No. of Stages To Go

$m_s = 2.0$, $m_b = 0.0$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$



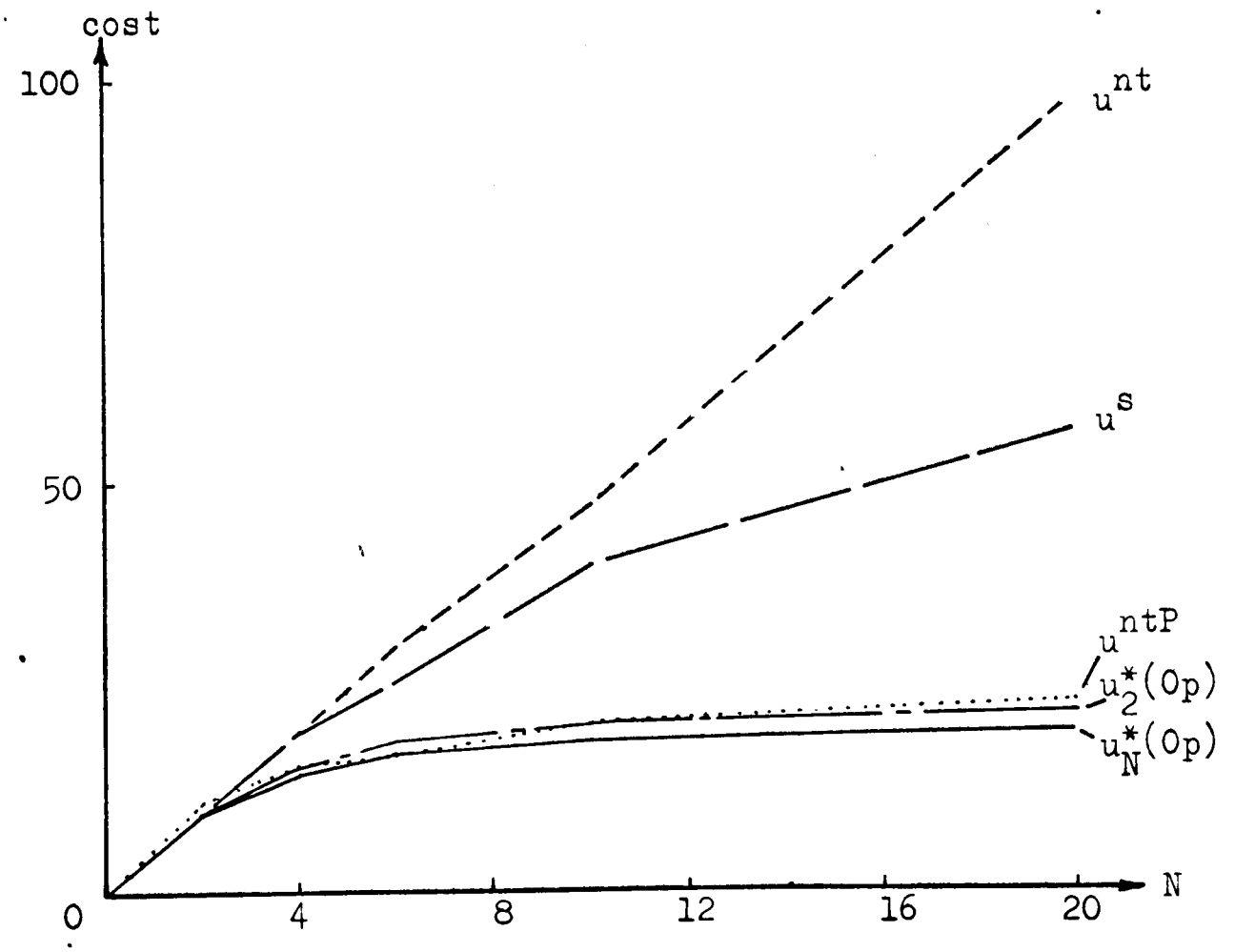
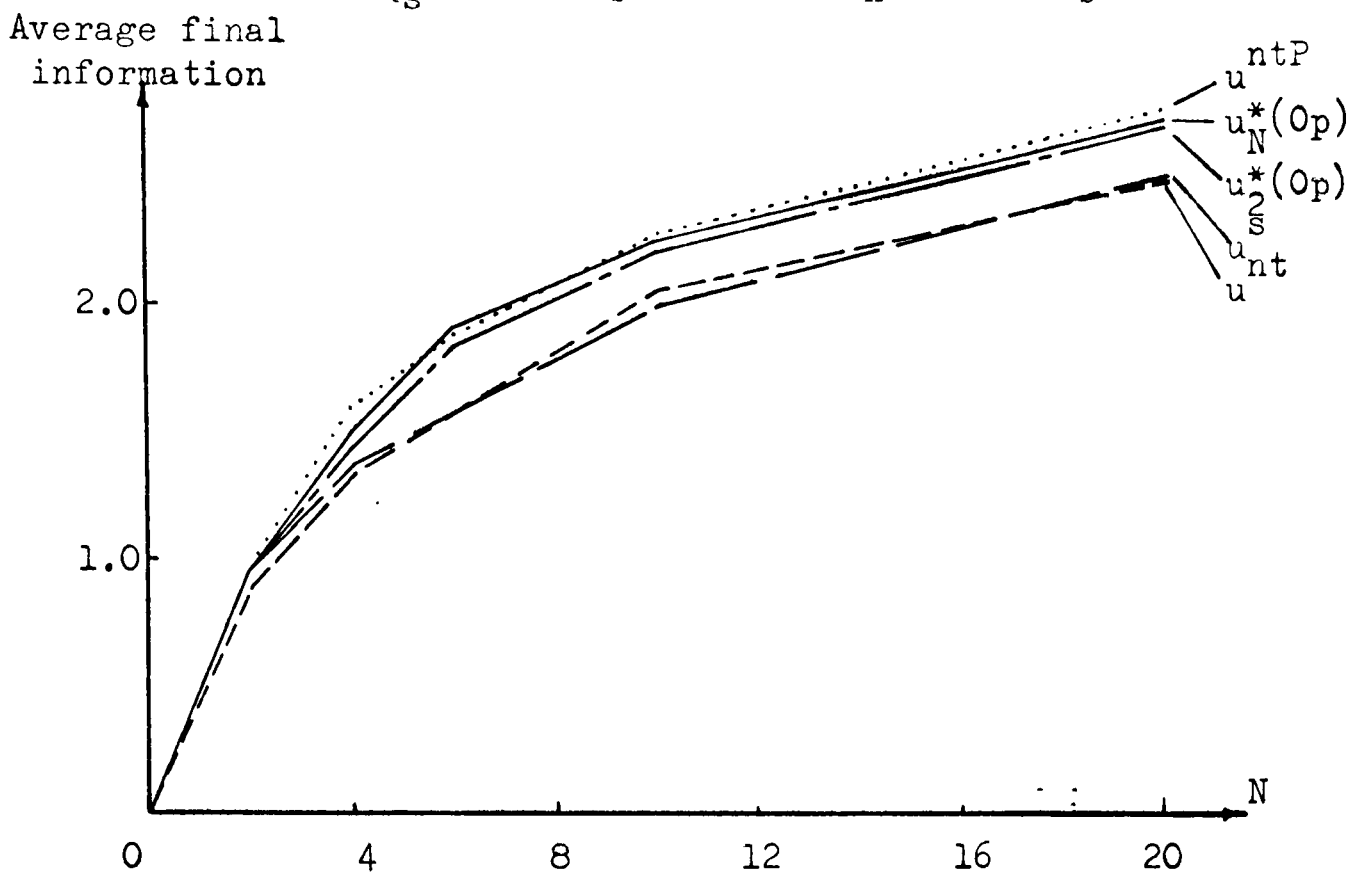
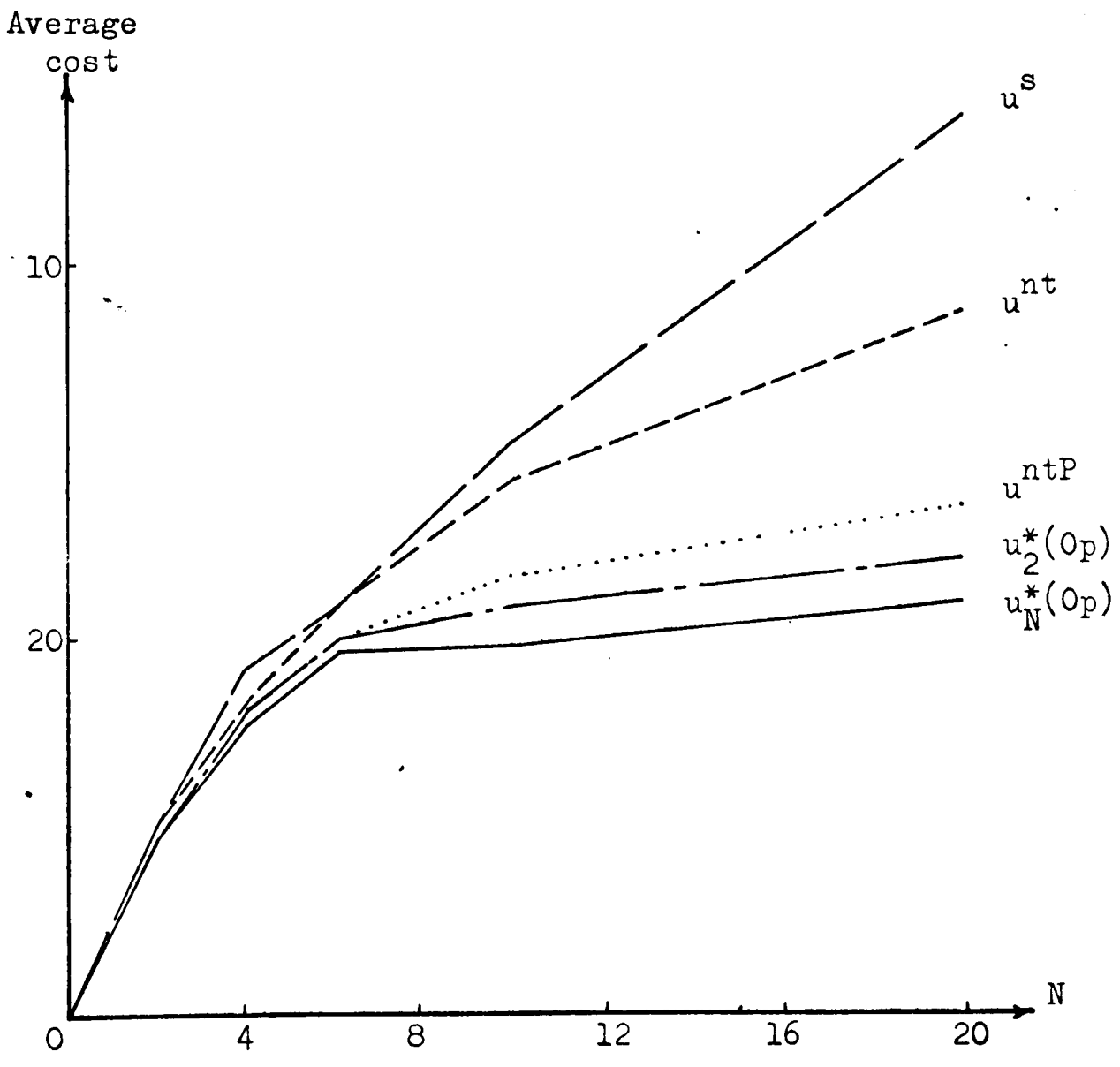


Figure 7.9 Average Cost and Information vs. No. of Stages To Go

$m_{-s} = 2.0$, $m_b = 1.116$, $s_x = 1.0$, $s_b = 1.0$, $\rho = 0.0$





greater than 100,000 and therefore exceeded the computer program output format. Little significance other than that they are large is attached to these results and their omission is not considered to be important.

7.3 Accuracy of the Numerically Determined Optimal Control Policy and Minimum Expected Cost

From Table 7.1 it can be seen that there is agreement between the average cost given by the simulation when the optimal control policy (OPCON) is used and the numerically determined minimum expected cost from Appendix E. There is also agreement for the few cases listed in Table 7.3 in which simulations were done for N = 2 and N = 4.* The agreement between the theoretical and simulation results is summarized in Tables 7.4a and 7.4b. From these tables it can be seen that the results show little bias (Table 7.4a) and that the distribution is close to what was expected. The approximations made in OPCON (Section 7.2) are also justified by this agreement.

7.4 Discussion

In this section the performance given by each of the control policies is discussed in turn. Some general observations on the relative importance of the caution and probing components of stochastic control policies are made and the value of the various control policies tried as general suboptimal policies is discussed.

7.4.1 The certainty equivalent control policy

The certainty equivalent control policy is the component of the optimal policy whose function is to produce the desired plant performance (see Chapter 5). It can be seen from Table 7.1 that this component on its

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* The numerical results are not listed in Table 7.3 but comparison can be made directly with Appendix E.

Table 7.4 The Unknown Constant Gain Problem

Agreement between Theoretical and Experimental Optimal Costs

Table 7.4a

Number of cases in which

	the theoretical cost is greater than the average cost	the theoretical cost is less than the average cost	Total
N = 2	3	3	6
N = 4	5	1	6
N = 6	14	13	27
Total	22	17	39

Table 7.4b

Number of cases in which the expected cost agrees

	within 1 standard deviation	between 1 and 2 standard deviations	between 2 and 3 standard deviations	Total
N = 2	4	2	0	6
N = 4	2	3	1	6
N = 6	17	8	2	27
Total	23	13	3	39
Actual % agreement	59.0%	33.3%	7.7%	
Theoretical % agreement	68.3%	27.2%	4.28%	

own gives very poor performance and the costs incurred are many times those given by other policies.*

The reason that the certainty equivalent policy gives poor performance is that it does not allow for the fact that the estimate of the gain it uses may be relatively inaccurate. For example, if $s_b^2 = 1.0$ then it is perfectly feasible that while the gain is one ($b = 1.0$) the estimate of the gain may be much smaller, e.g. $m_b = 0.001$. Because the certainty equivalent policy makes no allowance for this inaccuracy, it uses here a control effort which is one thousand times greater than that which is required. Consequently a very large cost is incurred on the next stage.

When the certainty equivalent policy makes a large error like this it also produces a large information gain since, as was observed in Section 6.2, part iii), information gain increases as the magnitude of the control input increases. Thus as is seen from Table 7.1 the certainty equivalent policy gives final information contents which are large compared with those given by the other control policies. This demonstrates that in this problem the poor performance given by the certainty equivalent policy would not be improved by the addition of a probing component designed to increase information gain.

The certainty equivalent control policy does not perform as badly for all non-neutral control problems as it does for this one but neither does it always perform as well. In the problem discussed in Section B.5, for example, the certainty equivalent policy is effectively equivalent to no control at all. However, although no general conclusion can be made

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* Except for cases in which the mean of the gain is large and there is therefore little relative uncertainty about the gain.

about the performance of certainty equivalent control policies, this example and the one discussed in Section B.5 demonstrate that in some cases at least, because it lacks caution or probing or both, the certainty equivalent policy gives very poor performance.

The certainty equivalent policy is not in general a reliable sub-optimal policy.

7.4.2 The neutral control policy

The difference between the neutral policy and the certainty equivalent policy is caution. From Table 7.1 it can be seen that the neutral control policy produced a much better performance than did the certainty equivalent policy and in fact gave expected costs which are in most cases at least of the same order as those given by the optimal policy.

It is important for this problem that the caution component of the control policy be exact since it must cancel as nearly as possible the large errors made by the certainty equivalent policy. Subtracting one large number from another to produce a result which is several orders of magnitude smaller is an inherently inaccurate operation. Using the neutral control policy overcomes this problem since it incorporates both the certainty equivalent control component and the caution component and thus eliminates this subtraction difficulty. No reason can be seen why this should not be true for many stochastic control problems and it is therefore recommended that the caution component be derived exactly by determining the neutral control policy whenever this is possible.

Two faults can be observed in the performance of the neutral control policy for this problem. From Figures 7.4 to 7.9 it can be seen that the neutral policy produces low expected costs for short processes but for long processes it produces expected costs considerably larger than those given by

the optimal policy. Also, whenever the mean of the gain and the variance of the state are both zero, the neutral control 'sticks' or it ceases to operate. The reason this occurs in this problem can be seen from Equations 6.7 and 6.3. Whenever $m_b = 0$ and $s_x^2 = 0$ then the neutral control is zero. This results in $m_b(i+1)$ and $s_x^2(i+1)$ also being zero and thus the control is always zero from that time onwards.

These two faults, the poor performance for long processes and the sticking in some cases, are a result of the absence of a probing term. Not enough information is gained by the neutral policy (compare information trajectories in Figures 7.4 to 7.9) and hence it is not able to control the later stages of a process as well as some of the other policies. A probing term would also remove any possibility of sticking.

It is almost certainly true that for most non-neutral stochastic control problems the neutral control policy produces better performance for short processes than for long ones as is the case here. It is not recommended therefore that neutral control policies be used as suboptimal control policies without some form of probing term added.

7.4.3 The optimal control policy

The optimal control policy combines all three components of stochastic control laws in their 'optimal' form. As was expected, this control policy gave the best performance of any of the control policies tried. This is particularly true for the longer processes even though the policy used was not strictly optimal for these cases (see Section 7.2). It is not possible to predict how much improvement would be obtained by using the true optimal policy for these cases ($N > 6$) but from Figures 7.4 to 7.9 it can be seen that in many cases it would not be large.

Using the optimal control policy for this problem and in fact for

any non-neutral stochastic control problem has two important drawbacks. First of all the optimal control policy must be determined and this is seldom possible or else is very difficult as is discussed in Chapters 5 and 6. Secondly the optimal control policy must be realized by an optimal control implementor. For this problem this is represented by the computer program OPCON. As is the case for OPCON, the optimal control policy for most stochastic control problems would need to be stored as a set of points and in most cases so many points would be required that a fairly large digital computer would be necessary. Also the calculations necessary to interpolate from the table of points are usually complex and therefore involve a significant amount of computer time. Since the rate at which the controller can operate is dependent on how quickly the computer operates (see Chapter 2), this can be a very important factor.

For stochastic control problems in which the optimal policy cannot be used for the above reasons, one of the two following design procedures (Sections 7.4.4 and 7.4.5) is recommended as a possible alternative.

7.4.4 The optimal policy for N = 2

A possible simplification of the optimal policy is to use the optimal control policy for the case in which there are two stages to go. At every stage of the process the control policy applied is the optimal policy that would be applied if there were only two steps left to go. This policy uses the first two components of the optimal control policy - the certainty equivalent component and the caution component - with the optimal probing component for the case in which there are two stages to go. It is a simplification of the optimal policy for two reasons:

1) It is easier to calculate the optimal control policy for two steps to go than for any larger number of steps to go (see Appendix D).

2) The optimal policy for N = 2 requires less computer storage than does the complete optimal policy for any number of steps to go. However, this policy still retains one of the disadvantages of the optimal policy in that an interpolating program such as OPCON is required.

The optimal control policy for two stages to go (u_2^*) gives very good performance in general and in many cases it is indistinguishable from that given by the optimal policy. This would seem to indicate, for this problem at least, that the variation in the magnitude of the probing term with the number of stages to go observed in the optimal control policy (see Section 6.3) is not very important. This might not be true however for processes longer than those considered here, or for other stochastic control problems.

7.4.5 The neutral-plus-probing policy

This is the control policy that is based on the design procedure outlined at the end of Chapter 5. It consists of the first two components of the optimal policy as combined in the neutral policy together with an empirical probing term:

$$u^{ntP}(i) = u^{nt} + \text{probing}$$

where the value of the probing term is determined experimentally. From the discussion in Section 6.2, part iii) it can be seen that the sign of the probing term should be such that it increases the absolute value of the control effort*. Therefore a probing term of the form

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* This is not strictly true since it is the absolute value of the difference of the control policy from the control policy for minimum information gain that should be increased. However the discrepancy is very small and the form of the probing term is considerably simplified.

$$\text{Probing} = c\, u^{nt}/|u^{nt}|$$

was tried. The best value of c was determined for three cases with the following results:

m_x	m_b	s_x^2	s_b^2	ρ	Best value of c
0.5	0.0	1.0	1.0	0.0	0.2
2.0	1.116	1.0	1.0	0.0	0.1
9.0	0.0	1.0	1.0	0.0	2.0

From these results it was seen that some variation of c with respect to m_x and m_b was necessary and the following was tried:

$$c = c' \, m_x \, /(m_b^2 + 1.0)$$

It was determined that for most cases a value of $c' = 0.5$ gave very good results. Consequently the exact form of the neutral-plus-probing control policy used was

$$u^{ntP}(i) = u^{nt} + \frac{0.5 \; m_x \; u^{nt}}{(m_b^2 + 1.0)\; |u^{nt}|}$$

The neutral-plus-probing control policy gave a very good performance for this problem, as good as that given by u_2^* and very close to that given by the optimal policy. More effort spent on determining the best value of c could no doubt have produced an even better performance. Moreover, because of its relatively simple functional form the neutral-plus-probing policy is much easier to realize than either the optimal control policy or u_2^* which, as is noted above, must be interpolated from tables.

Because of the good performance given by the neutral-plus-probing control policy, it is known that for this problem at least the exact form

of the probing component of the control policy used is not important. Only the approximate magnitude of the probing term seems to be of much significance. This same fact was observed in connection with Example B.5 and therefore it is thought possible that this may be true for a large number of stochastic control problems. If so, the procedure used here could be a very useful method of deriving suboptimal control policies.

7.4.6 A separable control policy*

For some stochastic control problems there might be some advantage in using a separable control policy since in this case the estimator would only be required to produce estimations of the unknowns and nothing about the accuracy of these estimates. A separable policy was tried for this problem in order to establish whether or not separable policies might be useful suboptimal control policies.

In Section 4.4 it is shown that the separable policy is not unique, the only restriction being the dependence on expected values alone. In this problem a separable policy may depend only on the mean values of the state and the gain and the noise variance (which is constant and therefore satisfies the condition of being known in advance). But since for the purpose of this simulation the variance of the noise is always one, this dependence was not considered.

The separable policy used was

$$u^s(i) = - \frac{m_x(i)(m_b(i) + 0.1)}{m_b^2(i) + 1.0}$$

The 1.0 in the denominator was included to avoid the large errors of the

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* The definition of separable used here is the one given in Chapter 4 and not any of the other possible definitions.

certainty equivalent policy while the 0.1 in the numerator was included to avoid the possibility of the controller sticking as the neutral policy does. Different values of these numbers might produce better controllers but these parameters could only be optimized for one value of the initial variance of the gain. Other initial gain variances would produce different optimal parameters and thus there would be little point in doing this.

The main disadvantage of this control is that it does not allow for possible dependence on the variance of the gain and as a result the amount of caution (determined by the value of the constant that is 1.0 here) cannot vary with the uncertainty about the gain. Consequently even though it gives much better performance than the certainty equivalent policy, the separable policy cannot be considered as successful in this problem. This is especially true since its one advantage is eliminated by the fact that the updating equations 6.3 do give the accuracy of the estimation as well as the estimates.

It is doubtful that separable policies would be very useful for many stochastic control problems. As is noted above, obtaining the correct amount of caution is fairly important and this cannot be achieved without some knowledge of the accuracy of the estimation of the unknowns. Moreover, it seems reasonable to assume that most types of estimators need some knowledge of the accuracy of the estimation they produce in order to modify their own performance.

7.5 Conclusions

From the results of the simulation reported in this chapter the following general conclusions can be made:

1) Certainty equivalent control policies are not likely to be good suboptimal control policies for non-certainty equivalent control problems.

2) Neutral control policies are not likely to be good suboptimal control policies for non-neutral control problems. Apparently all three components

of control policies, the certainty equivalent policy, caution and probing are required for most stochastic control problems.

3) Optimal control policies as well as being difficult to derive are also comparatively difficult to realize.

4) Variations of the optimal control policy (specifically the optimal control policy for two steps to go) may be easier to derive and realize than the optimal policy and may in some cases give nearly as good a performance.

5) The design procedure outlined at the end of Chapter 5 was successful for this problem and ought to be successful for many other stochastic control problems.

6) The exact form of the caution component of a control policy appears to be fairly important but the exact form of the probing component appears to be relatively unimportant.

7) Separable control policies do not appear to be very useful as sub-optimal control policies.

Chapter 8

Conclusions

Stochastic control problems are control problems in which the admission of the possibility of uncertainty is the essential feature of their formulation that distinguishes them from deterministic control problems. Although most practical ~~practical~~ problems are stochastic in some way or another, stochastic control theory has found successful applications only infrequently.

One of the reasons for this is that stochastic control theory is expensive to use in practice. Often on-line digital computers are required and hence it is very important to establish in advance whether or not the improvements achieved will justify the expenditure involved. One general rule is that the improvement given by using stochastic control theory increases rapidly as the length of time over which the process is controlled increases.

A second reason that stochastic control theory has found few applications is that optimal stochastic control laws can be easily derived only for a very limited class of control problems. Generally it is necessary to use a suboptimal or approximate control law and successful derivation of such suboptimal control laws depends largely on an understanding of the structure of stochastic control laws. In the past this understanding has been incomplete.

The basic result about the structure of stochastic control laws is that stochastic controllers perform two functions which in this thesis are called estimation and control implementation. The mathematical form of the first function is updating equations for the estimation of unknown parameters and the mathematical form of the second is the control policy.

Stochastic control policies differ from deterministic control policies because of the possible interaction between the functions of estimation and control implementation. This interaction can occur in two ways:

1) The accuracy of the estimation can affect the control policy used.

2) The control policy can affect the rate at which uncertainty is reduced.

Some idea of how this possible interaction affects stochastic control policies can be obtained by studying three classes of stochastic control problems in which the interaction is restricted. In certainty equivalent problems there is no interaction, in neutral problems the interaction is restricted to the effect the accuracy of the estimation can have on the control policy while in separable problems the interaction is restricted in that the effect the accuracy of the estimation has on the control policy is limited. Knowledge of the relationship between these three properties leads to the conclusion that stochastic control policies have three components whose functions are

1) to produce the required plant behaviour

2) to take into account the fact that the accuracy of the estimation may affect the control policy

3) to take advantage of the fact that the control policy can affect the rate at which uncertainty is reduced.

The three components of stochastic control policies are

1) the certainty equivalent control policy

2) caution

3) probing

The caution and probing components describe the two ways in which the functions of estimation and control implementation can interact. Caution

is the effect of the accuracy of the estimation on the control implementation while probing is the effect the control implementation has on estimation.

For most stochastic control problems caution and probing are important components of the control policy. Without caution, performance can be very erratic while without probing, performance over a long period of time can be poor and in some cases control action can cease altogether. Therefore when designing stochastic controllers it is important to consider the use of both of these components.

The caution component is more easily derived on theoretical grounds than is the probing component. This is because caution depends on the neutral control policy which is more easily derived than is the optimal control policy which is needed to determine the probing component of the optimal control policy. Moreover it seems to be more important for some stochastic control problems to use the correct amount of caution than it is to use the correct amount of probing.

It is sensible therefore to design a suboptimal control policy that uses the neutral control, which contains the correct amount of caution and which has an empirical probing term added on. Such a suboptimal control policy is known to give very good results in the two cases in which it has been tried. It has two advantages over the optimal policy:

1) It is easier to derive than is the optimal policy.

2) It is easier to realize than is the optimal policy. Optimal policies often must be stored as a table of values while neutral control policies can in many cases be stored in functional form and this can represent a considerable saving in terms of computer speed and storage requirements.

It is not possible to state at the present time how long it will be before successful applications of stochastic control theory are common. One of the main problems which remains to be solved is the estimation problem

for cases in which sufficient statistics do not exist. Given a suitable estimator it ought to be possible to design useful suboptimal controllers similar to the neutral-plus-probing control policy described in this thesis. Some difficulties that may arise are:

1) It may be difficult in some cases to derive neutral control policies (even though they are more easily derived than optimal control policies). It would be useful to determine under what circumstances neutral control policies can be found and in what cases numerical or approximate methods are necessary.

2) It may be difficult to determine what form of probing is required. If an information gain equation is available, determination of a probing term appears to be fairly simple but the existence of information gain equations depends at the present time on the existence of sufficient statistics.

Despite these remaining difficulties, it is believed that this thesis represents a significant advance both in terms of stochastic control theory and of its application. Contributions to the advance of stochastic control theory are

1) the clarification of existing results about the two functions of stochastic control laws, estimation and control implementation,

2) the observation that it is the possible interaction between estimation and control implementation which distinguishes stochastic from deterministic control laws,

3) precise definitions of certainty equivalence, neutrality and separability and an understanding of the relationships between them,

4) the observation that stochastic control policies have three components and the precise definition of these components,

5) the observation that two of these components, caution and probing, describe the possible interaction between estimation and control implementation,

6) the calculation of the optimal control policy and expected cost function for the unknown constant gain problem.

Contributions to the possible application of stochastic control theory are

1) the observation that stochastic controllers are more useful for long processes than for short processes,

2) the observations about how to derive suboptimal control policies based on the three components of stochastic control policies.

Appendix A

Counterexamples Demonstrating the Relationships Between Certainty Equivalence, Separability and Neutrality

In this appendix the four counterexamples listed in Table 4.1 are described. They are:

A.1 Neutral but not separable or certainty equivalent

A.2 Neutral and separable but not certainty equivalent

A.3 Separable and certainty equivalent but not neutral

A.4 Separable but not certainty equivalent or neutral

None of these counterexamples is intended to be a description of a realistic control problem and in particular Counterexamples A.3 and A.4 are very carefully contrived.

A.1 Counterexample: Neutral but not Certainty Equivalent or Separable

This example demonstrates that neutral problems are not necessarily certainty equivalent or separable i.e. neutrality is not a sufficient condition for certainty equivalence or for separability.

i) Statement

The state equation is

$$x(i+1) = x(i) + u(i) \qquad \text{A.1}$$

and observations are made according to

$$y(i) = x(i) + z(i) \qquad \text{A.2}$$

where $z_1(i)$ is an independent random variable with

$$p(z_1(i)) = N(0, s_z^2)$$

The probability density of the state is given initially as

$$p(x(0)) = q(0)\,\delta(x - x_1(0)) + (1 - q(0))\,\delta(x - x_2(0))$$

where δ is the Dirac delta function i.e. the state is known to have initially one of two values $x_1(0)$ or $x_2(0)$. Measurements are made during the course of

the process to determine more accurately which is the actual value of the state.

The cost function is

$$I = \sum_{i=1}^{N} x^4(i) \qquad A.3$$

ii) Bayes' rule is

$$p(x(i+1)|y^{i+1}) = \text{const } p(x(i+1)|y^i,u(i))p(y(i+1)|x(i+1))$$

Detailed calculation of Bayes' rule gives

$$p(x(i)|y^i) = q(i)\,\delta(x(i) - x_1(i)) + (1 - q(i))\,\delta(x(i) - x_2(i)) \qquad A.4$$

with updating equations for $q(i)$, $x_1(i)$ and $x_2(i)$:

$$x_1(i+1) = x_1(i) + u(i)$$

$$x_2(i+1) = x_2(i) + u(i)$$

$$q(i+1) = \frac{q(i)}{q(i) + (1 - q(i))\exp(-(x_2^2(i) - x_1^2(i) + 2(x_1(i) - x_2(i))(y(i+1) - u(i)))/2s_z^2)} \qquad A.5$$

Because there is no input disturbance it is known at each time i that the state has one of the two values $x_1(i)$ or $x_2(i)$ and thus the probability density for the state retains the same discrete form throughout the process. As a result, sufficient statistics exist.

iii) Information associated with the problem at time i is given by

$$s[p(x(i)|y^i)] = -\int p(x(i)|y^i)\log(p(x(i)|y^i))dx(i)$$

$$= q(i)\log q(i) + (1 - q(i))\log(1 - q(i)) \qquad A.6$$

The information at time i is thus dependent only on $q(i)$. Consider the term

$$c = (x_2^2(i) - x_1^2(i) + 2(x_1(i) - x_2(i))(y(i+1) - u(i))$$

from the denominator of Equation A.5 and let

$$d = x_1(i) - x_2(i)$$

Then

$$c = -d^2 + 2d(y(i+1) - x_2(i) - u(i))$$

Since

$$y(i+1) = x(0) + \sum_{j=0}^{i} u(j) + z(i+1)$$

$$x_2(i) + u(i) = x_2(0) + \sum_{j=0}^{i} u(j)$$

gives

$$y(i+1) - (x_2(i) + u(i)) = x(0) - x_2(0) + z(i+1)$$

Then

$$c = -d^2 + 2d(x(0) - x_2(0) + z(i+1))$$

is independent of all past control decisions and thus $q(i+1)$ is independent of all past control decisions and the problem is neutral. There is no way in which the control policy can affect the rate at which $q(i)$ tends to either one or zero (corresponding to knowing that the state is actually $x_2(i)$ or $x_1(i)$ respectively.)

iv) Optimal control policy

The optimal expected cost is defined

$f_N[x_1(0), x_2(0), q(0)] \triangleq$ the expected cost of the process when the optimal control u^* is used over N stages and the initial set of sufficient statistics is $x_1(0)$, $x_2(0)$ and $q(0)$.

Then a functional recurrence equation is derived:

$$f_N[x_1(0), x_2(0), q(0)] = \min_{u(0)} \{E\{x^4(1)\} + E\{f_{N-1}[x_1(1), x_2(1), q(1)]\}\} \quad A.7$$

From A.7 when there is only one step to go

$$f_1[x_1(0),\ x_2(0),\ q(0)] = \min_{u(0)}\left\{q(0)(x_1(0)+u(0))^4 + (1-q(0))(x_2(0)+u(0))^4\right\}$$

which gives

$$u_1^*[x_1(0),\ x_2(0),\ q(0)] = -\frac{x_2(0)\sqrt[3]{1-q(0)} + x_1(0)\sqrt[3]{q(0)}}{\sqrt[3]{q(0)} + \sqrt[3]{1-q(0)}}$$

$$f_1[x_1(0),\ x_2(0),\ q(0)] = \left(q(0)\left(\frac{\sqrt[3]{1-q(0)}}{\sqrt[3]{1-q(0)} + \sqrt[3]{q(0)}}\right)^4\right.$$

$$\left. + (1-q(0))\left(\frac{\sqrt[3]{q(0)}}{\sqrt[3]{1-q(0)} + \sqrt[3]{q(0)}}\right)^4\right)(x_1(0) - x_2(0))^4$$

When there are two stages to go

$$f_2[x_1(0),\ x_2(0),\ q(0)] = \min_{u(0)}\left\{E\left\{x^4(1)\right\} + E\left\{f_1[x_1(1),\ x_2(1),\ q(1)]\right\}\right\}$$

The second expectation in this equation does not result in an analytic function but since $f_1[x_1(1),\ x_2(1),\ q(1)]$ contains $x_1(1)$ and $x_2(1)$ in the form $x_2(1) - x_1(1) = d$ and since $q(1)$ is independent of $u(0)$, the expected cost of all but the first stage $E\left\{f_1[x_1(1),\ x_2(1),\ q(1)]\right\}$ is independent of $u(0)$ and thus

$$u_2^*[x_1(0),\ x_2(0),\ q(0)] = u_1^*[x_1(0),\ x_2(0),\ q(0)]$$

It is not possible to derive an analytic expression for $f_2[x_1(0),\ x_2(0),\ q(0)]$ but this will have the form $f(q(0))(x_1(0) - x_2(0))^4$ and using an inductive argument it can be seen that the optimal control is independent of N.

$$u^*(i) = -\frac{x_2(i)\sqrt[3]{1-q(i)} + x_1(i)\sqrt[3]{q(i)}}{\sqrt[3]{1-q(i)} + \sqrt[3]{q(i)}} \qquad \text{A.8}$$

v) Discussion

It is easy to show that the certainty equivalent control for this example is

$$u^{c.e.}(i) = -E\{x(i)\} = -(q(i)x_1(i) + (1-q(i))x_2(i)) \qquad A.9$$

and it can be seen from comparing Equations A.8 and A.9 that the problem is not certainty equivalent. The example is also not separable since $u^*(i)$ cannot be expressed as a function of $E\{x(i)\}$ and predetermined quantities (the sufficient statistic $q(i)$ cannot be predetermined since, as is shown in iii) above, it depends on the random sequence $z(i)$). This example is not certainty equivalent or separable because the probability density for the state is not symmetrical about the mean and the cost function is of high enough order ($x^4(i)$) that this asymmetry is taken into account.

A.2 Counterexample: Neutral and Separable but not Certainty Equivalent

This example demonstrates that a separable problem is not necessarily certainty equivalent. It also shows that the two properties neutrality and separability taken together are not sufficient to guarantee certainty equivalence.

i) Statement

The state equation is

$$x(i+1) = x(i) + u(i) \qquad A.10a$$

and observations are made according to

$$y(i) = x(i) + z(i) \qquad A.10b$$

where $z(i)$ is an independent random variable with

$$p(z(i)) = N(0, s_z^2)$$

The probability density of the state given initially is normal:

$$p(x(0)) = N(m_x(0), s_x^2(0))$$

The cost function is

$$I = \sum_{i=1}^{N} ax^3(i) + x^4(i) \qquad A.11$$

This example is identical to the known gain problem (Chapter 2) except that here the gain is equal to one (not significant) and a different cost function is used.

ii) Bayes' rule

This gives the same updating equations as in Chapter 2:

$$m_x(i+1) = \frac{s_z^2(m_x(i)+u(i)) + s_x^2(i)y(i+1)}{s_z^2 + s_x^2(i)} \qquad A.12$$

$$s_x^2(i+1) = \frac{s_x^2(i)s_z^2}{s_x^2(i) + s_z^2}$$

As in the known gain problem (Chapter 2), the variance of the state can be predicted for all times before the process starts.

iii) Information associated with the problem at time i is given by

$$s[p(x(i)|y^i)] = -\int p(x(i)|y^i)\log(p(x(i)|y^i))dx(i)$$

$$= \tfrac{1}{2}\log s_x^2(i) + \text{constant} \qquad A.13$$

Since s_x^2 is independent of all past control decisions the example is neutral.

iv) Optimal control policy

The optimal expected cost is defined:

$f_N[m_x(0),s_x^2(0)] \triangleq$ the expected cost of the process when the optimal control $u^*(i)$ is used over N stages and the initial sufficient statistics are $m_x(0)$ and $s_x^2(0)$.

The functional recurrence equation is derived as in Equation 2.2:

$$f_N[m_x(0),s_x^2(0)] = \min_{u(0)}\{E\{ax^3(1)+x^4(1)\} + E\{f_{N-1}[m_x(1),s_x^2(1)]\}\} \qquad A.14$$

From A.12 when there is one stage to go

$$f_1[m_x(0),s_x^2(0)] = \min_{u(0)}\{a(m_x(0)+u(0))^3 + 3(m_x(0)+u(0))s_x^2(0))$$

$$+ (m_x(0)+u(0))^4 + 3s_x^4(0) + 6(m_x(0)+u(0))^2 s_x^2(0)\}$$

from which

$$u_1^*[m_x(0),s_x^2(0)] = -m_x(0) - \frac{a}{4} - \tfrac{1}{3}((\tfrac{3}{4}a)^3$$

$$+ (s_x^2(0)(729s_x^4(0) - 243s_x^2(0)(\tfrac{3}{4}a)^2 + 27(\tfrac{3}{4}a)^4)^{\frac{1}{2}})^{\frac{1}{3}}$$

$$- \tfrac{1}{3}((\tfrac{3}{4}a)^3 - (s_x^2(0)(729s_x^4(0) - 243s_x^2(0)(\tfrac{3}{4}a)^2 + 27(\tfrac{3}{4}a)^4)^{\frac{1}{2}})^{\frac{1}{3}}$$

A.15a

Reasoning similar to that used in Section A.1 shows that the optimal policy is independent of the number of stages to go and thus

$$u_N^*[m_x(0),s_x^2(0)] = u_1^*[m_x(0),s_x^2(0)] \qquad \text{A.15b}$$

v) Discussion

Deriving the certainty equivalent control policy for this problem gives

$$u^{c.e.}(i) = -m_x(i) - \tfrac{3}{4}a \qquad \text{A.16}$$

From comparing A.16 with A.15 it can be seen that this example is not certainty equivalent. However, since the variance of the state is a predictable parameter, the only unpredictable parameter on which the control depends is the expected value of the state and thus this example is separable. Even though it depends on the variance of the state which is one of the sufficient statistics, the optimal control policy is a separable policy because that variance can be determined in advance.

A.3 Counterexample: Certainty Equivalent and Separable but not Neutral

The examples of this section and the next have been very carefully contrived. It seems possible that in some cases the control policy which maximizes information gain may be the same as the certainty equivalent policy. In these cases there would be no conflict between the demands of reducing

uncertainty and realizing desired performance and the problem would be certainty equivalent even though it was not neutral. However, most examples which might have this property do not because of certain 'second order' effects. This example and the next (A.4) are arranged so that these effects are eliminated.

This example shows that neither certainty equivalence nor separability nor the two taken together are sufficient to guarantee that a problem is neutral. It is however a very special case and most problems which are certainty equivalent are also neutral.

i) Statement

$$x_1(i+1) = x_1(i) + u_1(i) + z_1(i)$$

$$x_2(i+1) = z_2(i)x_1(i) + x_2(i) + u_2(i) \qquad \text{A.17a}$$

$$x_3(i+1) = (s_{z_2}^2 x_1^2(i) + x_3(i))s_{z_3}^2/(1+s_{z_3}^2)$$

where $z_1(i)$ and $z_2(i)$ are independent random variables with

$$p(z_1(i)) = N(0, s_{z_1}^2)$$

$$p(z_2(i)) = N(0, s_{z_2}^2)$$

and the measurements are made according to

$$y_1(i) = x_1(i)$$

$$y_2(i) = x_2(i) + z_3(i)(x_3(i)(1+s_{z_3}^2)/s_{z_3}^2)^{\frac{1}{2}} \qquad \text{A.17b}$$

$$y_3(i) = x_3(i)$$

where $z_3(i)$ is an independent random variable with

$$p(z_3(i)) = N(0, s_{z_3}^2)$$

The initial condition is given by

$$p(x_2(0)) = N(m_x(0), s_x^2(0))$$

$$x_1(0) = x_1(0)$$

$$x_3(0) = s_x^2(0)$$

The cost function is

$$I = \sum_{i=1}^{N} x_1^2(i) + x_2^2(i) \qquad \text{A.18}$$

ii) Bayes' rule gives for $x_2(i)$ that

$$p(x_2(i)|y^i) = N(m_x(i), s_x^2(i))$$

with updating equations for the statistics

$$m_x(i+1) = (y_2(i+1) + s_{z_3}^2(m_x(i) + u_2(i)))/(1 + s_{z_3}^2) \qquad \text{A.19a}$$

$$s_x^2(i+1) = s_{z_3}^2(s_x^2(i) + s_{z_2}^2 x_1^2(i))/(1 + s_{z_3}^2) \qquad \text{A.19b}$$

By comparing A.19b with the third part of A.17a it can be seen that it has been arranged that

$$s_x^2(i) = x_3'(i) \text{ for all } i$$

This is one sense in which this example is a special case.

iii) Information is given by

$$s\left[p(x_2(i)|y^i)\right] = \tfrac{1}{2} \log\left[s_x^2(i)\right] + \text{constant}$$

Since $s_x^2(i)$ depends on $x_1^2(i-1)$ which in turn depends on $u_1(i-2)$, the example is not neutral.

iv) Optimal control policy

The minimum expected cost is defined

$f_N[x_1(0), m_x(0), s_x^2(0)]$ $\triangleq$ the expected cost when the optimal control $u^*(i)$ is used over N stages and the initial condition is specified by $x_1(0)$, $m_x(0)$ and $s_x^2(0)$.

The functional recurrence equation is

$$f_N[x_1(0),m_x(0),s_x^2(0)] = \min_{u(0)}\left\{E\left\{x_1^2(1)+x_2^2(1)\right\} + E\left\{f_{N-1}[x_1(1),m_x(1),s_x^2(1)]\right\}\right\} \qquad \text{A.20}$$

When there is one stage left to go

$$f_1[x_1(0),m_x(0),s_x^2(0)] = \min_{u(0)}\left\{(m_x(0)+u_2(0))^2 + s_{z_2}^2\, x_1^2(0) + s_x^2(0) + (x_1(0)+u_1(0))^2\right\}$$

giving

$$f_1[x_1(0),m_x(0),s_x^2(0)] = s_{z_2}^2\, x_1^2(0) + s_x^2(0)$$

and

$$u_1^*[x_1(0),m_x(0),s_x^2(0)] = \begin{bmatrix} -x_1(0) \\ -m_x(0) \end{bmatrix}$$

When there are two stages to go

$$f_2[x_1(0),m_x(0),s_x^2(0)] = \min_{u(0)}\left\{(m_x(0)+u_2(0))^2 + s_{z_2}^2\, x_1^2(0) + s_x^2(0) + (x_1(0)+u_1(0))^2 + E\left\{s_{z_2}^2\, x_1^2(1) + s_x^2(1)\right\}\right\}$$

giving

$$f_2[x_1(0),m_x(0),s_x^2(0)] = (1+s_{z_3}^2/(1+s_{z_3}^2))(s_x^2(0)+s_{z_2}^2\, x_1^2(0)) + s_{z_2}^2\, s_{z_1}^2$$

$$u_2^*[x_1(0),m_x(0),s_x^2(0)] = \begin{bmatrix} -x_1(0) \\ -m_x(0) \end{bmatrix}$$

Inductive reasoning gives for the case when there are N stages left to go that

$$f_N[x_1(0),m_x(0),s_x^2(0)] = (s_{z_2}^2 x_1^2(0) + s_x^2(0))\left(\sum_{i=0}^{N-1}(s_{z_3}^2/(1+s_{z_3}^2))^i\right) + s_{z_2}^2\, s_{z_1}^2\left(\sum_{i=0}^{N-2}(s_{z_3}^2/(1+s_{z_3}^2))^i\right) \qquad \text{A.21}$$

$$u_N^*[x_1(0),m_x(0),s_x^2(0)] = \begin{bmatrix} -x_1(i) \\ -m_x(i) \end{bmatrix} \qquad \text{A.22}$$

A second sense in which this example is a very special case is that it has been arranged that $f_{N-1}[x_1(1), m_x(1), s_x^2(1)]$ is quadratic in $x_1(1)$ (this can be seen from Equations A.21 and A.19b). This is the cause of the example being certainty equivalent.

v) Discussion

The certainty equivalent control policy for this example is derived as

$$u^{c.e.}(i) = \begin{bmatrix} - x_1(i) \\ - m_x(i) \end{bmatrix} \qquad A.23$$

and thus the example is certainty equivalent. It follows that it is also separable.

A.4 Counterexample: Separable but not Neutral or Certainty Equivalent

This example shows that the fact that a problem is separable is not sufficient to guarantee that the problem will be either certainty equivalent or neutral. This example is identical to the previous one except that a non-linearity has been introduced into the first state equation to make the problem not certainty equivalent.

i) Statement

The state equations are

$$\left.\begin{aligned} x_1(i+1) &= x_1(i) + b(i)u_1(i) \\ x_2(i+1) &= z_2(i)x_1(i) + x_2(i) + u_2(i) \\ x_3(i+1) &= (s_{z_2}^2 x_1^2(i) + x_3(i))s_{z_3}^2/(1+s_{z_3}^2) \end{aligned}\right\} \qquad A.24a$$

where $z_3(i)$ is as in Section A.3 and $b(i)$ is an independent random variable with $p(b(i)) = N(m_b, s_b^2)$. The measurements are made according to

$$\left.\begin{aligned} y_1(i) &= x_1(i) \\ y_2(i) &= x_2(i) + z_3(i)(x_3(i)(1+s_{z_2}^2/s_{z_3}^2))^{\frac{1}{2}} \\ y_3(i) &= x_3(i) \end{aligned}\right\} \qquad A.24b$$

The noise $z_3(i)$, the initial condition and the cost function are as in Example A.3.

ii) Bayes' rule gives the same updating equation for $m_x(0)$ and $s_x^2(0)$ as Example A.3.

iii) Information

The information equation is the same as in Example A.3 and thus this example also is not neutral.

iv) Optimal control policy

Derivation of the optimal control follows the same pattern as for Example A.3. The optimal control is

$$u_N^*[x_1(0), m_x(0), s_x^2(0)] = \begin{bmatrix} -x_1(0)m_b/(m_b^2 + s_b^2) \\ -m_x(0) \end{bmatrix} \qquad \text{A.25}$$

v) Discussion

The certainty equivalent control policy for this example is

$$u^{c.e.}(i) = \begin{bmatrix} -x_1(i)/m_b \\ -m_x(i) \end{bmatrix} \qquad \text{A.26}$$

and since this is not the optimal policy this example is not certainty equivalent. It is however separable since the optimal policy depends only on expected values and other predictable variables.

Although this example and the previous one are very special cases, they do serve the purpose for which they are intended. They show that not all certainty equivalent or separable problems are necessarily neutral (even though this seems to be true in the majority of cases).

Appendix B

Simple Examples Illustrating Caution and Probing

In this appendix four more examples as well as two examples from Appendix A are discussed in terms of caution and probing. Three of the six examples are neutral and thus display caution only, so that the various causes of caution can be demonstrated. The other three are not neutral and thus display probing as well as caution.

B.1 A Neutral Problem

In this simple neutral example caution which is caused by a gain with a random component is an easily observed phenomenon.

i) Statement

The plant equation is

$$x(i+1) = x(i) + b(i)u(i) \qquad \text{B.1}$$

where $b(i)$ is a stochastic variable with

$$p(b(i)) = N(m_b, s_b^2)$$

Measurements of the state are exact:

$$y(i) = x(i) \qquad \text{B.2}$$

The initial value of the state is given as $x(0)$ and the cost function is

$$I = \sum_{i=1}^{N} x^2(i) \qquad \text{B.3}$$

ii) Bayes' rule and information

In this problem there is no possibility of any reduction of uncertainty because the state is measured exactly at each time and the gain is a stochastic variable. Thus the problem is neutral as there is no information gain at all.

iii) Optimal control policy

The optimal expected cost is defined

$f_N[x(0)]$ $\triangleq$ the expected cost when the optimal control is used over N stages and the initial state is x(0).

The functional recurrence equation is

$$f_N[x(0)] = \min_{u(0)} \left\{ E\left\{x^2(1)\right\} + E\left\{f_{N-1}[x(1)]\right\} \right\} \qquad \text{B.4}$$

from which the optimal control policy and minimum expected cost are derived:

$$u_N^*[x(0)] = - \frac{x(0) m_b}{\bar{m}_b^{\,2} + s_b^{\,2}} \qquad \text{B.5}$$

$$f_N[x(0)] = x^2(0) \sum_{i=1}^{N} \left(\frac{s_b^{\,2}}{m_b^{\,2} + s_b^{\,2}} \right)^i \qquad \text{B.6}$$

iv) Neutral and certainty equivalent policies

Since the example is neutral the neutral control policy is also the optimal control policy, i.e.

$$u^{nt}(i) = u^*(i)$$

The certainty equivalent policy can be derived as

$$u^{c.e.}(i) = - x(i)/m_b$$

and since it is different from the optimal policy this problem is not certainty equivalent.

v) Discussion

This is another example (cf. Section A.2) of a problem that is separable but not certainty equivalent. The problem is separable despite the dependence of the optimal control on the variance of the gain because this variance is a constant which is part of the prior information given to the controller, i.e. it is known in advance.

The caution in this problem is given by

$$\text{Caution} = -\frac{x(i)m_b}{m_b^2 + s_b^2} + \frac{x(i)}{m_b}$$

$$= \frac{s_b^2 x(i)}{m_b(m_b^2 + s_b^2)} \qquad \text{B.7}$$

and is illustrated in Figure B.1. The optimal control is cautious because the gain b(i) is not exactly known and thus there is uncertainty about the result of the control decisions. If the gain is known exactly ($s_b^2 = 0$), then caution is zero and the certainty equivalent policy is used, but if on the other hand nothing is known about the gain ($s_b^2 = \infty$), the caution exactly cancels out the certainty equivalent action and no control at all is used. The effect of caution in this example is to reduce the absolute value of the control action because as the absolute value decreases, the uncertainty introduced decreases.

B.2 Caution in Examples A.1 and A.2

Neither of these problems is certainty equivalent and thus both display caution. Both are neutral and thus probing occurs in neither.

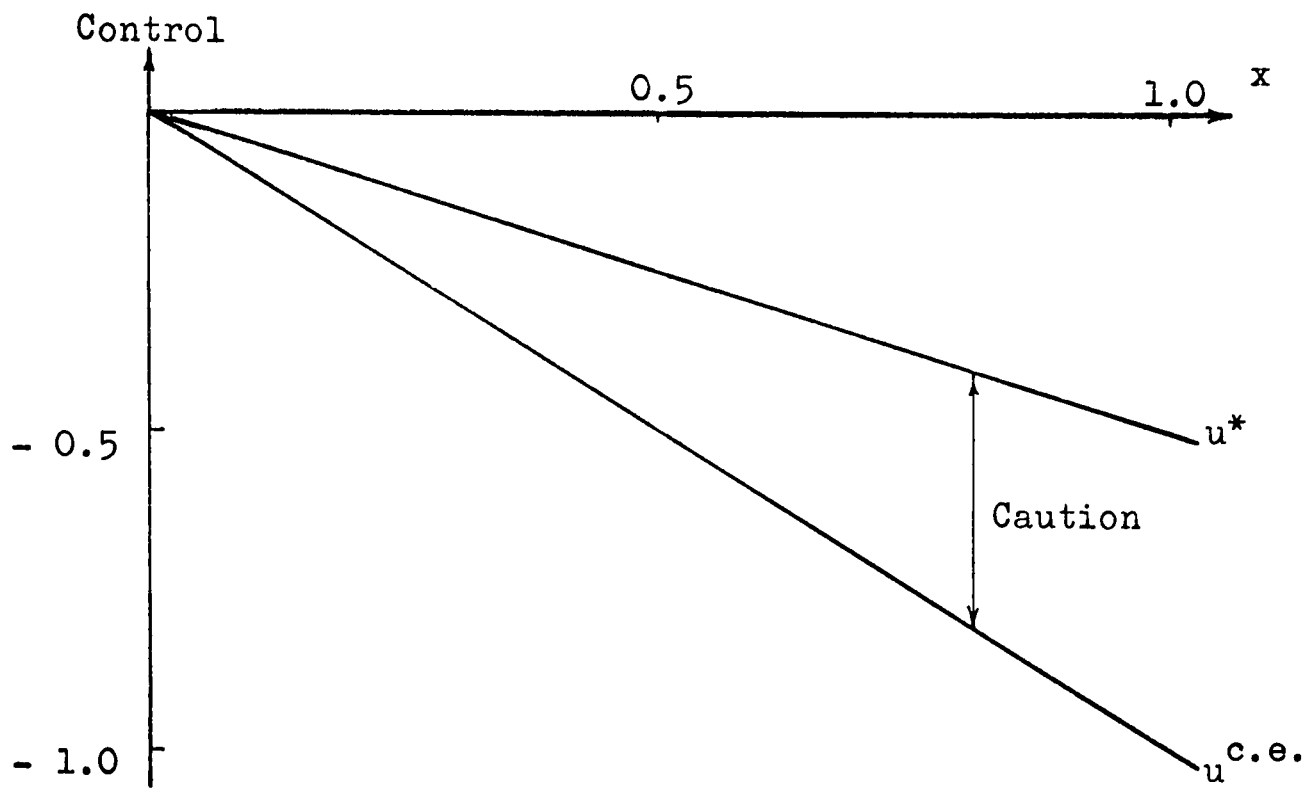
In Example A.2, caution is given by

$$\text{Caution} = u^*(i) - u^{c.e.}(i)$$

$$= (x_1(i) - x_2(i)) \frac{q(i)\sqrt[3]{1-q(i)} - \sqrt[3]{q(i)}(1-q(i))}{\sqrt[3]{1-q(i)} + \sqrt[3]{q(i)}}$$

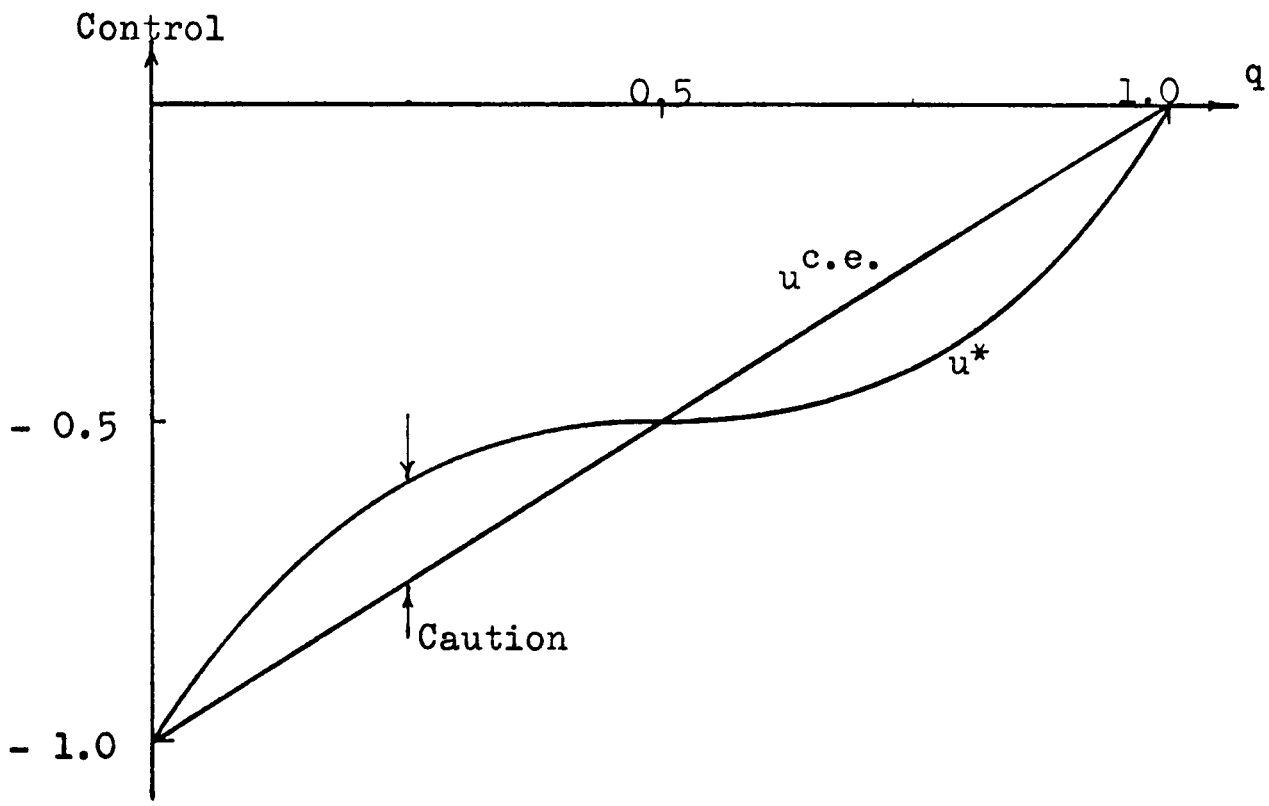
In this example, caution is dependent only on $d = x_1(i) - x_2(i)$, a constant, and on q(i). It is zero when q(i) = 0, 0.5, 1.0 and is of a different sign on either side of q(i) = 0.5 (see Figure B.2). Caution is used because at any time i the state at time i+1 is not known exactly and because the probability density for this unknown state is not symmetrical (except for q = 0.5 when caution is zero). Since the cost function is fourth order $x^4(i)$, this non-symmetrical property has an effect on the expected cost which the optimal

Figure B.1 Caution in Example B.1



$m_b = 1.0,\ s_b^2 = 1.0$

Figure B.2 Caution in Example A.1



$x_1 = 0.0,\ x_2 = 1.0$

control allows for as caution. Thus two factors cause the control to be cautious: the non-symmetrical probability density for the state and the high order of the cost function.

In Example A.2, caution is given by

$$\text{Caution} = u^*(i) - u^{c.e.}(i)$$

$$= \tfrac{1}{2}a - \tfrac{1}{3}\left(\left(\tfrac{3}{4}a\right)^3 + \left(s_x^2(i)\left(729 s_x^4(i) - 243 s_x^2(i)\left(\tfrac{3}{4}a\right)^2 + 27\left(\tfrac{3}{4}a\right)^4\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right.$$

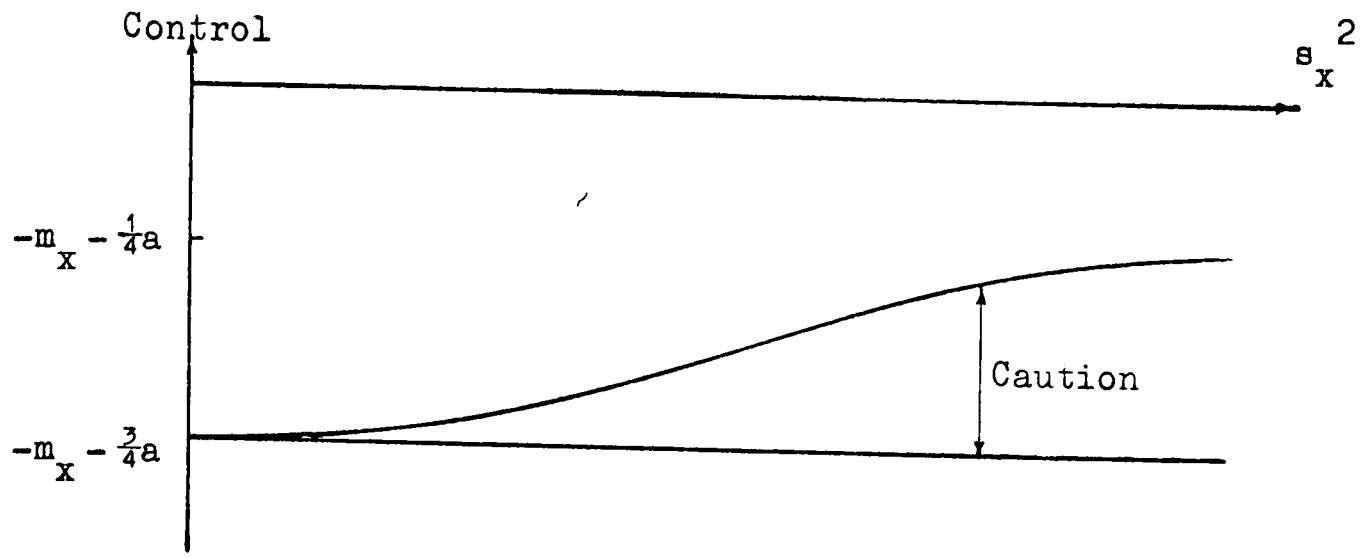
$$\left. - \tfrac{1}{3}\left(\left(\tfrac{3}{4}a\right)^3 - \left(s_x^2(i)\left(729 s_x^4(i) - 243 s_x^2(i)\left(\tfrac{3}{4}a\right)^2 + 27\left(\tfrac{3}{4}a\right)^4\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)$$

In this example, caution is dependent only on the variance of the state ($s_x^2(i)$) and on the constant a. It is zero when $s_x^2(i) = 0$ (i.e. when the state is exactly known) and it approaches the value $\frac{1}{2}a$ as $s_x^2(i)$ approaches infinity (i.e. when nothing is known about the state)(see Figure B.3). The caution is caused by the fact that the cost function is not symmetrical (Figure B.4). The cost rises more steeply for values of the state to the left of the minimum than it does for values to the right of the minimum and thus the optimal control tends to keep the state, because it is not known exactly, slightly to the right rather than exactly at the minimum as the certainty equivalent policy does. The degree to which it is kept to the right increases as uncertainty about the state increases. Thus it is seen that caution is caused by the fact that the state is not known exactly and the cost function is not symmetrical.

B.3 A Non-neutral Problem

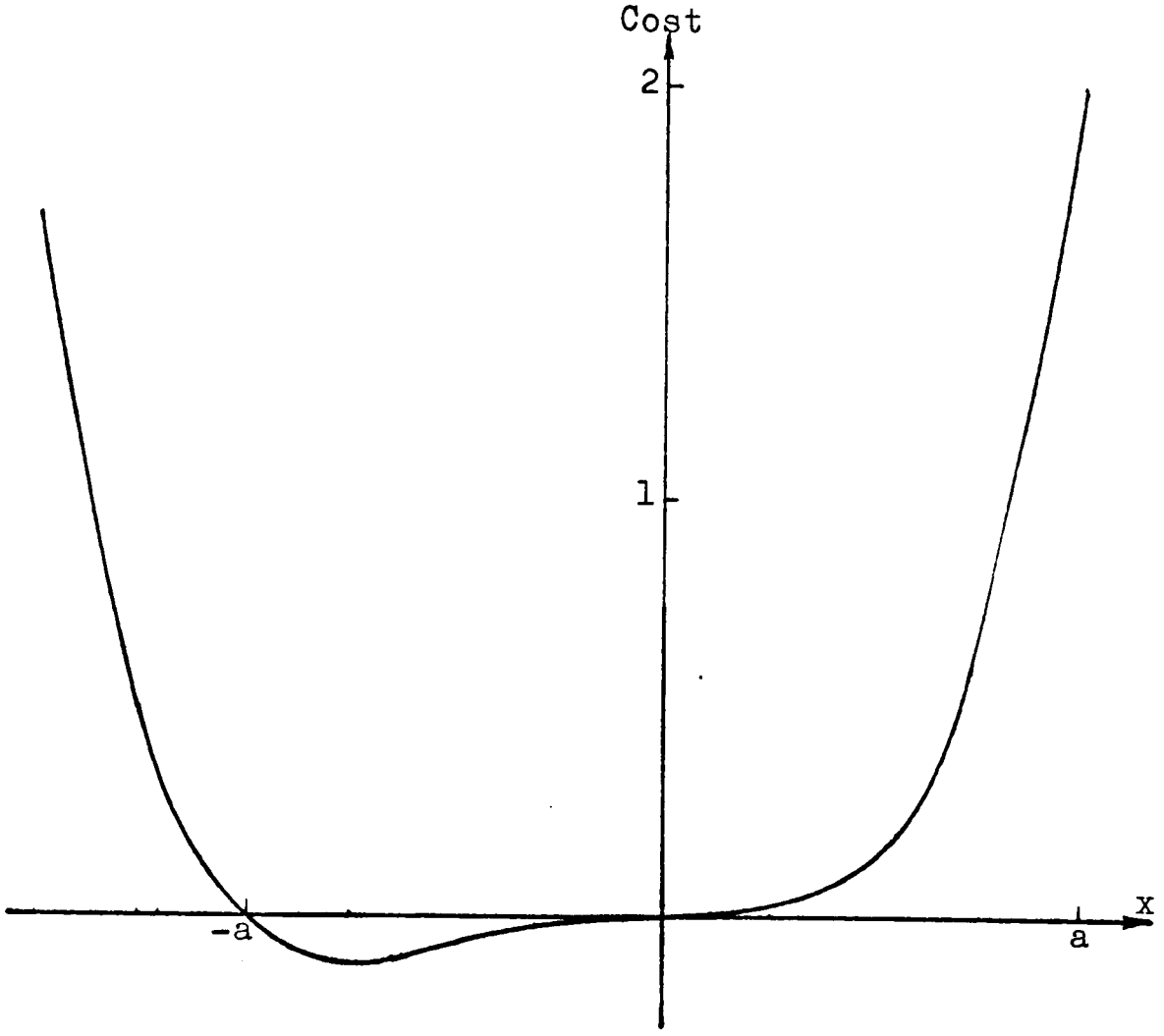
This example is identical to Example B.1 except that the state is not measured precisely. As a result, there is uncertainty about the state which can be reduced during the course of the process. Because of the random gain, the control policy used affects the rate at which uncertainty is reduced and the problem is not neutral.

Figure B.3 Caution in Example A.2



$m_x = 0.25$, $a = 1.0$

Figure B.4 Cost vs. State in Example A.2



i) Statement

The plant equation is as in B.1

$$x(i+1) = x(i) + b(i)u(i) \qquad \text{B.8}$$

where $b(i)$ is as in B.1 but the measurements are corrupted by noise:

$$y(i) = x(i) + z(i) \qquad \text{B.9}$$

where $z(i)$ is an independent random variable with

$$p(z(i)) = N(0, s_z^2)$$

The initial value of the state is given by a probability density

$$p(x(0)) = N(m_x(0), s_x^2(0))$$

and the cost function is quadratic:

$$I = \sum_{i=1}^{N} x^2(i) \qquad \text{B.10}$$

ii) Bayes' rule gives that the probability density for the state is always normal:

$$p(x(i)|y^i) = N(m_x(i), s_x^2(i))$$

and the sufficient statistics are updated by the equations

$$m_x(i+1) = \frac{y(i+1)(s_x^2(i) + u^2(i)s_b^2) + (m_x(i) + m_b u(i))s_z^2}{s_x^2(i) + u^2(i)s_b^2 + s_z^2} \qquad \text{B.11a}$$

$$s_x^2(i+1) = \frac{s_z^2(s_x^2(i) + u^2(i)s_b^2)}{s_x^2(i) + u^2(i)s_b^2 + s_z^2} \qquad \text{B.11b}$$

The equations B.11 are similar to the updating equations 2.5 for the known gain problem, the only difference being the change of $s_x^2(i)$ to $s_x^2(i) + u^2(i)s_b^2$. The term $u^2(i)s_b^2$ is added because with the uncertainty about the gain, any control action increases the uncertainty about the state by this amount. Because the series of control actions $u(i)$ are unpredictable, the variance of the state cannot in this problem be determined for all times before the process

starts. This is a fundamental difference between this example and the known gain problem.

iii) Information associated with the problem at time i is given by

$$s\left[p(x(i)|y^i)\right] = -\int p(x(i)|y^i)\,\log(p(x(i)|y^i))\,dx(i)$$

$$= \tfrac{1}{2}\log(s_x^2(i)) + \text{const.} \qquad \text{B.12}$$

Since $s_x^2(i)$ is dependent upon $u(i-1)$, the problem is not neutral. It is also useful to consider the information change from one stage to the next:

$$s\left[p(x(i+1)|y^{i+1})\right] - s\left[p(x(i)|y^i)\right] = \tfrac{1}{2}\log\frac{s_x^2(i)}{s_x^2(i+1)}$$

$$= \tfrac{1}{2}\log\frac{s_x^2(i)(s_z^2 + s_x^2(i) + u^2(i)s_b^2)}{s_z^2(s_x^2(i) + u^2(i)s_b^2)} \qquad \text{B.13}$$

Since in this example $s_x^2(i+1)$ is independent of $y(i+1)$ (Equation B.11b), the information change is not a stochastic quantity. In general, however, the expected value of the information change would have to be used (see Example B.5).

In this example, the information change may be positive or negative, i.e. there may be information gain or information loss. There is information loss (i.e. uncertainty about the state increases) if $s_x^2(i) < s_z^2$ and $u^2(i) > s_x^4(i)/s_b^2(s_z^2 + s_x^2(i))$. This means that if $s_x^2(i)$ is small ($x(i)$ is known fairly accurately) and if a large enough control is used through the random gain, then uncertainty about the state will increase. From Equation B.13, information loss is greatest when $u(i) = \pm\infty$ while information gain (positive information change) is greatest when $u(i) = 0$. A zero control means the random gain has no effect and no information is destroyed.

iv) Optimal control policy

The optimal expected cost is defined

$f_N[m_x(0), s_x^2(0)] \triangleq$ the expected cost when the optimal control $u^*(i)$ is used over N stages and the probability density for the initial state is $p(x(0)) = N(m_x(0), s_x^2(0))$.

The functional recurrence equation is derived as

$$f_N[m_x(0), s_x^2(0)] = \min_{u(0)} \{E\{x^2(1)\} + E\{f_{N-1}[m_x(1), s_x^2(1)]\}\} \quad \text{B.14}$$

When there is one stage left to go

$$f_1[m_x(0), s_x^2(0)] = \min_{u(0)} \{m_x^2(0) + s_x^2(0) + u^2(0)(m_b^2 + s_b^2) + 2u(0)m_x(0)m_b\} \quad \text{B.15}$$

giving

$$u_1^*[m_x(0), s_x^2(0)] = -\frac{m_x(0)m_b}{m_b^2 + s_b^2} \quad \text{B.16}$$

$$f_1[m_x(0), s_x^2(0)] = \frac{m_x(0)s_b^2}{m_b^2 + s_b^2} + s_x^2(0) \quad \text{B.17}$$

When there are two stages to go, minimizing the expected cost leads to a 5th order polynomial equation for $u_2[m_x(0), s_x^2(0)]$. Although this equation has not been solved, it is known that $f_2[m_x(0), s_x^2(0)]$ is a very complicated function and that because of this, u_N^* and f_N^* are not available as analytic functions for $N > 2$. Thus numerical methods similar to those discussed in Chapter 6 would be necessary to find the optimal control for the general case in which there are N stages to go.

v) Certainty equivalent and neutral control policies

The certainty equivalent control policy can be derived as

$$u^{c.e.}(i) = -m_x(i)/m_b \quad \text{B.18}$$

The neutral policy is found by considering the related neutral problem (see Section 4.4.2). For this problem, the related neutral problem is Example B.1. From Example B.1 and Equation B.15, the neutral policy is the value of u(0) which minimizes

$$m_x^2(0) + s_x^2(0) + u^2(0)(m_b^2 + s_b^2) + 2u(0)m_x(0)m_b$$

$$+ \int_{-\infty}^{\infty} f_{N-1}^{nt}\left[x(1), m_b, s_b^2\right] p(x(1)|\, y^o, u(0))dx(1)$$

$$= m_x^2(0) + s_x^2(0) + u^2(0)(m_b^2 + s_b^2) + 2u(0)m_x(0)m_b$$

$$+ (m_x^2(0) + s_x^2(0) + u^2(0)(m_b^2 + s_b^2) + 2u(0)m_x(0)m_b)\sum_{i=1}^{N-1}(s_b^2/(m_b^2 + s_b^2))^i \qquad \text{B.19}$$

The cost function $f_{N-1}^{nt}\left[x(1), m_b, s_b^2\right]$ used here is that found as the minimum expected cost for the related neutral problem (Example B.1). From Equation B.19

$$u^{nt}(i) = -\frac{m_x(i)m_b}{m_b^2 + s_b^2} \qquad \text{B.20}$$

Although the optimal control policy is not available in analytic form, both the certainty equivalent and neutral policies have very simple forms. This happens in many problems and is one reason that the certainty equivalent and neutral policies are often considered as suboptimal policies (see Chapter 7). The neutral policy for this problem is the same as the optimal control when there is only one step to go. This is because when there is only one step to go, information gained on the next step is of no value and thus when there is one step to go the problem is effectively neutral. Since the neutral policy is independent of N, the one-step-to-go controller ($u_1^*\left[m_x(0), s_x^2(0)\right]$) and $u^{nt}(i)$ are identical. This occurs for any problem in which the neutral policy is not explicitly dependent upon the number of steps to go and can be a simpler method of finding the neutral policy than that described in Section 4.4.2.

vi) Discussion

This problem is not neutral and thus there is probing. Since the certainty equivalent and neutral policies are different there is also caution, which is the same as in Example B.1 and exists for the same reason. There is no analytic expression for the probing since the optimal policy is not available. However, it can be shown from the fifth order polynomial equation for u_2^* that

$$\left|u_2^*\left[m_x(0),\ s_x^{\,2}(0)\right]\right| \leq \left|u_1^*\left[m_x(0),\ s_x^{\,2}(0)\right]\right|$$

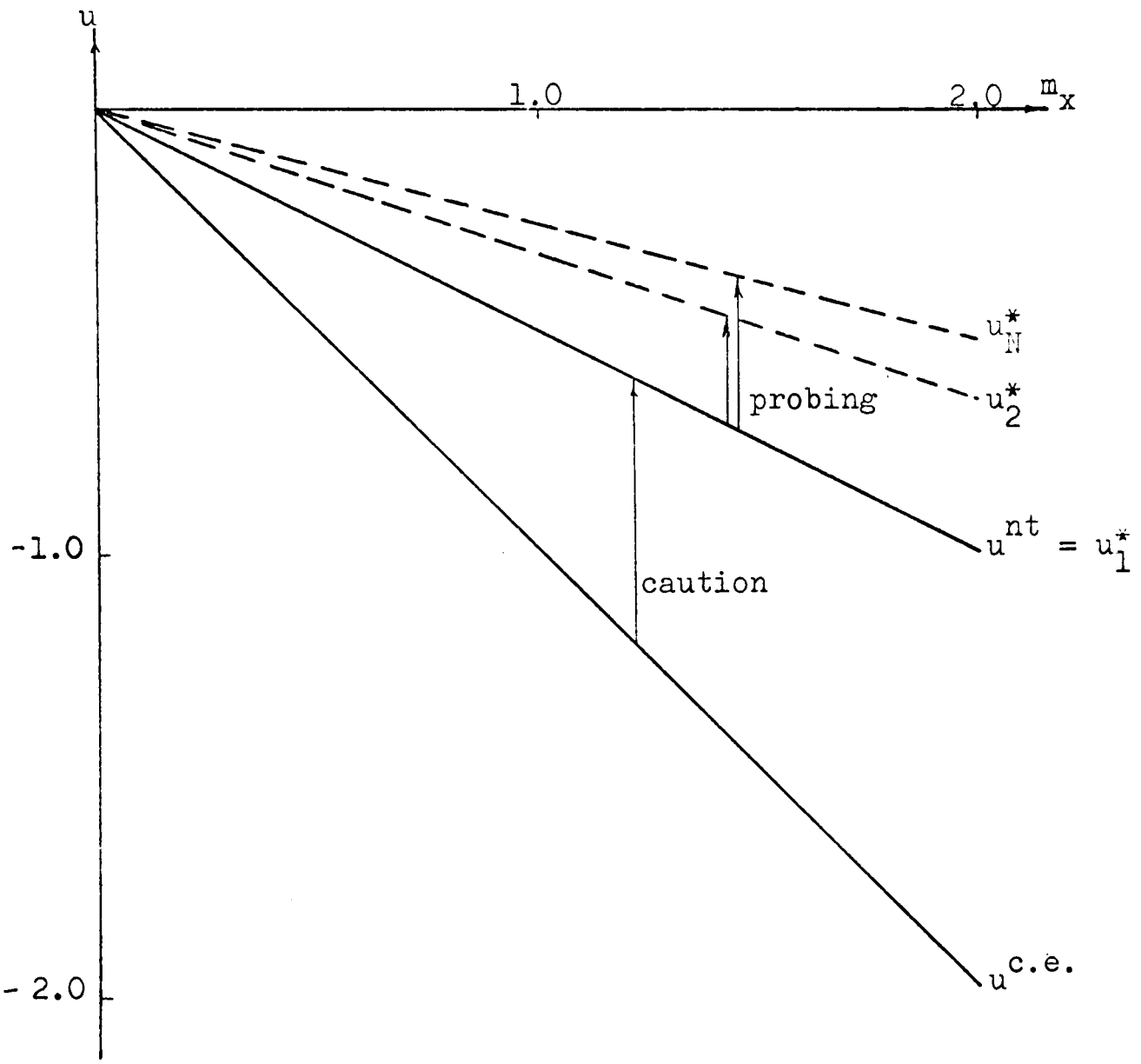
which enables Figure B.5 to be drawn.

Because information gained at the beginning of a long process is more useful than information gained at the beginning of a short process, it is expected that when the number of stages to go is greater than two, the optimal control policy will use more probing than when there are only two stages to go. Thus the optimal control will try to obtain more information at the beginning of the process at the expense of incurring a slightly higher cost over the first few stages. This has not been confirmed for this problem by numerically calculating the optimal control because a similar effect is shown to exist in the problem discussed in Chapters 6 and 7. The expected type of optimal policy is illustrated in Figure B.5.

From Figure B.5 it can be seen that both caution and probing tend to reduce the magnitude of the control effort. The reason caution reduces the control effort is explained above and the reason probing reduces the control effort is that decreasing the control effort decreases the amount of information destroyed due to the random gain.

The introduction of a random gain has made the solution of this problem much more complex than that of the known gain problem (Chapter 2). The variance of the state is not a predictable sequence and as a result the

Figure B.5 Caution and Probing in Example B.3



problem is not neutral. Because of the nonlinear nature of the state equation (see Chapter 1) and the fact that the problem is not neutral, the functional recurrence equation (B.14) cannot be solved by analytic means and the optimal control is not available in analytic form. It is for problems such as this where nonlinearities mean that the optimal control policy cannot be easily found that the value of the concepts such as caution and probing become apparent. This is shown in Example B.5 and in the problem discussed in Chapters 6 and 7.

B.4 A Second Non-neutral Problem

Florentin[51] discusses a non-neutral problem and describes a numerical calculation of the optimal control policy. This problem is considered here in terms of caution and probing.

i) Statement

The state equation is

$$x(i+1) = x(i) + bu(i) + z(i) \qquad \text{B.21a}$$

Here b is an unknown constant rather than a stochastic variable as in Section B.3 and thus uncertainty about b can be reduced during the course of the process. The input disturbance $z(i)$ is an independent random variable with probability density

$$p(z(i)) = N(0, s_z^2)$$

The measurements are considered to be exact

$$y(i) = x(i) \qquad \text{B.21b}$$

and thus the only uncertainty that can be reduced is uncertainty about the gain. The initial probability density for the gain is

$$p(b) = N(m_b(0), s_b^2(0))$$

and the cost function is taken as

$$I = \sum_{i=1}^{N} (x^2(i) + u^2(i-1)) \qquad \text{B.22}$$

ii) Bayes' rule gives that

$$p(b|y^i) = N(m_b(i), s_b^2(i))$$

with updating equations for the mean and variance of the gain:

$$m_b(i+1) = \frac{m_b(i)s_z^2 + (x(i+1) - x(i))u(i)s_b^2(i)}{s_z^2 + s_b^2(i)u^2(i)} \qquad \text{B.23a}$$

$$s_b^2(i+1) = \frac{s_b^2(i)s_z^2}{s_z^2 + s_b^2(i)u^2(i)} \qquad \text{B.23b}$$

In this problem as in Example B.3 the variance of the unknown parameter cannot be predicted in advance. Because of the dependence on $u^2(i)$, it is a random sequence.

iii) Information associated with the problem at time i is from Equation 4.5

$$s[p(b|y^i)] = -\tfrac{1}{2} \log s_b^2(i) + \text{const.} \qquad \text{B.24}$$

As in Section B.3 it is useful to calculate the information change from time i to time i + 1:

$$s[p(b|y^{i+1})] - s[p(b|y^i)] = \tfrac{1}{2} \log (s_b^2(i)/s_b^2(i+1))$$

$$= \tfrac{1}{2} \log((s_z^2 + s_b^2(i)u^2(i))/s_z^2) \qquad \text{B.25}$$

In this example the information change is always positive or zero, i.e. only information gain is possible. Information gain is maximum for $u = \pm \infty$ and is minimum (zero) for $u = 0$. In other words, if no control is used, nothing is learnt about the gain, whereas if an infinite control is used, uncertainty due to the input disturbance becomes negligible and the gain is determined exactly. Because it is the gain that is learnt about and not the state, information gain is increased by a larger control effort rather than by a smaller one as in Section B.3.

iv) Optimal control policy

The optimal expected cost is defined

$$f_N[x(0),m_b(0),s_b^2(0)] \triangleq \text{the expected cost when the optimal control } u^*(i) \text{ is used over N stages and the initial condition is specified by } x(0),\ m_x(0) \text{ and } s_x^2(0).$$

The functional recurrence equation is

$$f_N[x(0),m_b(0),s_b^2(0)] = \min_{u(0)}\left\{E\left\{x^2(1)\right\} + u^2(0) + E\left\{f_{N-1}[x(1),m_x(1),s_x^2(1)]\right\}\right\} \qquad \text{B.26}$$

When there is one stage to go

$$f_1[x(0),m_b(0),s_b^2(0)] = \min_{u(0)}\left\{x^2(0) + (m_b^2(0) + s_b^2(0))u^2(0) + s_z^2 + 2xm_b(0)u(0) + u^2(0)\right\}$$

which gives

$$f_1[x(0),m_b(0),s_b^2(0)] = \frac{x^2(0)(s_b^2(0) + 1.0)}{m_b^2(0) + s_b^2(0) + 1.0} + s_z^2$$

$$v_1^*[x(0),m_b(0),s_b^2(0)] = -\frac{x(0)m_b(0)}{m_b^2(0) + s_b^2(0) + 1.0} \qquad \text{B.27}$$

Because the expectation integral

$$\int_{-\infty}^{\infty} f_1[x(1),m_b(1),s_b^2(1)]p(x(1)|y^o,u(0))dx(1)$$

cannot be evaluated analytically, the optimal control and cost functions are not available in analytic form for $N>1$. Florentin[51] gives some numerical results which approximate the control policy when there are N stages to go.

v) Certainty equivalent and neutral control policies

The certainty equivalent control policy, when there are N stages to go is given by

$$u_N^{c.e.}[x(0),m_b(0),s_b^2(0)] = -\frac{x(0)m_b(0)(1+g_{N-1}^{c.e.})}{m_b^2(0)(1+g_{N-1}^{c.e.})+1} \qquad \text{B.28a}$$

where $g_N^{c.e.}$ is calculated recursively according to

$$g_N^{c.e.} = \frac{1+g_{N-1}^{c.e.}}{(m_b^2(0)(1+g_{N-1}^{c.e.})+1)^2} \qquad \text{B.28b}$$

with

$$g_0^{c.e.} = 0$$

The neutral control policy is found by considering the related neutral problem (see Section 4.4.2) which has the plant equation

$$x(i+1) = x(i) + b(i)u(i) + z(i)$$

where b(i) is an independent random variable with

$$p(b(i)) = N(m_b, s_b^2)$$

The measurements are perfect

$$y(i) = x(i)$$

and the cost function is given by Equation B.22. The optimal expected return for this problem can be shown to be

$$f_N^{nt}[x(0), p(b)] = g_N^{nt}x^2(0) + h_N^{nt}$$

where

$$g_N^{nt} = (1+g_{N-1}^{nt})\frac{(s_b^2m_b^2(1+g_{N-1}^{nt})^2 + (s_b^2(1+g_{N-1}^{nt})+1.0)^2 + m_b^2(1+g_{N-1}^{nt})+1.0)}{((1+g_{N-1}^{nt})(m_b^2+s_b^2)+1.0)^2}$$

$$h_N^{nt} = (1+g_{N-1}^{nt})s_z^2 + h_{N-1}^{nt}$$

The quantities g_N^{nt} and h_N^{nt} are calculated recursively and for N = 0:

$$g_0^{nt} = 0$$

$$h_0^{nt} = 0$$

Then from Equation 4.9 for the case when there are N stages left to go, the neutral control policy for the problem of this section is the value of u(0) which minimizes the expression

$$x^2(0) + (m_b^2(0) + s_b^2(0))u^2(0) + s_z^2 + 2x(0)m_b(0)u(0) + u^2(0)$$
$$+ f_{N-1}^{nt}[x(1),\ p(b|y^o,\ u(0))]$$

which gives the neutral control when there are N stages to go as

$$u_N^{nt}[x(0), m_x(0), s_x^2(0)] = \frac{-x(0)m_b(0)(1+g_{N-1}^{nt})}{(m_b^2(0)+s_b^2(0))(1+g_{N-1}^{nt})+1.0} \qquad \text{B.29a}$$

where g_N^{nt} is calculated recursively from

$$g_N^{nt} = (1+g_{N-1}^{nt})[s_b^2(0)m_b^2(0)(1+g_{N-1}^{nt})^2 + (s_b^2(0)(1+g_{N-1}^{nt})+1.0)^2$$
$$+ m_b^2(0)(1+g_{N-1}^{nt})+1.0]/((1+g_{N-1}^{nt})(m_b^2(0)+s_b^2(0))+1.0)^2 \qquad \text{B.29b}$$

$$g_0^{nt} = 0$$

Because the cost function is explicitly dependent upon $u^2(i)$, the certainty equivalent and neutral policies are explicitly dependent on the number of stages to go (N). The appearance of $u^2(i)$ in the cost function is responsible for the 1.0 in the denominators of the right hand sides of Equations B.28a and B.29b. If it is removed, the certainty equivalent and neutral control policies revert to forms similar to those of the previous problem. Because the neutral policy depends on the number of stages to go, it is not the same as the optimal policy for one step to go as was the case in Example B.4.

vi) Discussion

It is not simple to write down general expressions for caution and probing in this problem because of the complicated form of the neutral and certainty equivalent policies and because the optimal policy is not in analytic

form. For this reason, the neutral and certainty equivalent policies have been determined for a particular case and are compared with the optimal policy for this case as determined by Florentin[51](Figure B.6). The optimal policy is taken from Figure 3 of Florentin. Florentin states: "After about three stages the control became a stationary function of state, computing was terminated at the fourth stage"[51] and thus the neutral and certainty equivalent policies are also computed for N = 4 in Figure B.6.

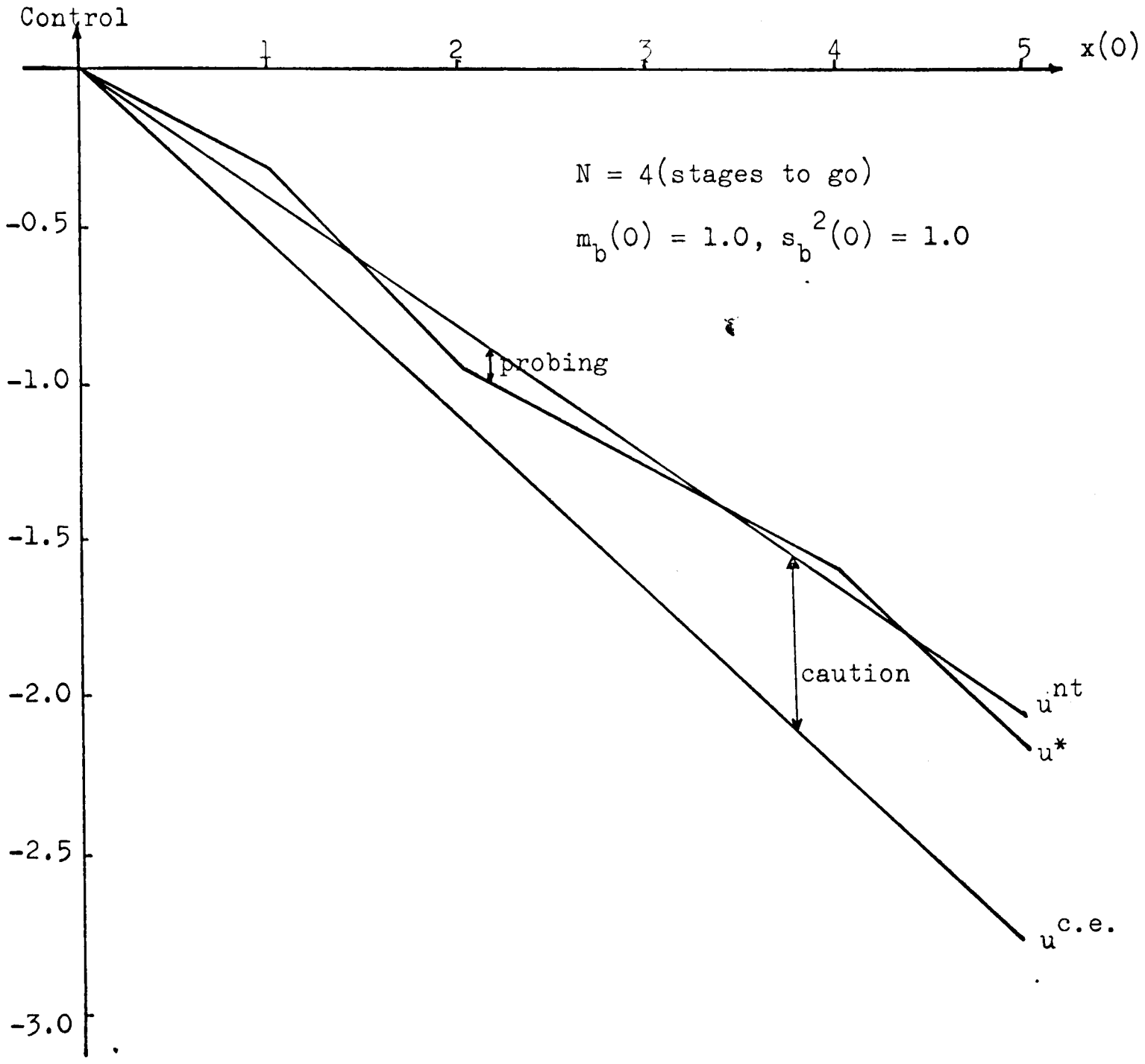
From Figure B.6 the caution is seen as a decrease in absolute value of the control effort. This occurs for the same reason (unknown gain) as in Sections B.1 and B.3. Probing, however, is not as easily observed because of the inaccuracies in the knowledge of the optimal policy.. Because information gain increases with increasing magnitude of control effort, it is expected that the optimal policy would be larger in absolute value than the neutral policy, but the effect is small enough that it is masked by the error in the optimal policy. Note that what Florentin has termed probing is in fact what is called caution here.

From Equations B.28a and B.29a it can be seen that the effect of including some control cost (the $u^2(i)$ term) in the cost function has been to decrease the magnitude of the certainty equivalent and neutral policies. Thus in this problem the cost-of-control term has produced a cautious type of effect. However, in other problems (e.g. Examples A.1 and A.2), a cost-of-control term would not have a similar effect to caution because in these examples, caution can either increase or decrease the magnitude of the control while the cost-of-control term always decreases the magnitude of the control.

B.5 A Third Non-Neutral Problem

The example considered in this section is discussed by Jacobs[55] who describes the derivation of the functional recurrence equation as well as a

Figure B.6 Caution and Probing in Example B.4



simulation using an 'intentional error' suboptimal policy. The calculation of the optimal policy and further simulations are reported in Jacobs and Langdon.[56] This example differs from the previous examples considered in this chapter in that the optimal policy is precisely known and some simulation results are available.

i) Statement

The plant equation is linear

$$x(i+1) = x(i) + u(i) + z(i) \qquad \text{B.30a}$$

with a normally distributed independent random variable input disturbance

$$p(z(i)) = N(0, s_z^2)$$

The output measurement is nonlinear.

$$y(i) = |x(i)| \qquad \text{B.30b}$$

and thus only the sign of the state is not determined by the measurement. The initial probability density of the state is

$$p(x(0)) = q(0)\,\delta(x(0) - y(0)) + (1 - q(0))\,\delta(x(0) + y(0))$$

where $q(0)$ and $y(0)$ are given quantities and δ is the Dirac delta function.

The cost function is quadratic

$$I = \sum_{i=1}^{N} x^2(i) \qquad \text{B.31}$$

ii) Bayes' rule gives

$$p(x(i)|y^i) = q(i)\,\delta(x(i) - y(i)) + (1 - q(i))\,\delta(x(i) + y(i)) \qquad \text{B.32}$$

where the updating equation for the sufficient statistics $y(i)$ and $q(i)$ are

$$y(i+1) = y(i+1) \qquad \text{B.33a}$$

$$q(i+1) = \frac{P_1 q(i) + P_2(1 - q(i))}{(P_1 + P_3)q(i) + (P_2 + P_4)(1 - q(i))} \qquad \text{B.33b}$$

with

$$P_1 = \exp\left\{-\left(y(i+1) - y(i) - u(i)\right)^2/2s_z^2\right\}$$

$$P_2 = \exp\left\{-\left(y(i+1) + y(i) - u(i)\right)^2/2s_z^2\right\}$$

$$P_3 = \exp\left\{-\left(y(i+1) + y(i) + u(i)\right)^2/2s_z^2\right\} \qquad \text{B.33c}$$

$$P_4 = \exp\left\{-\left(y(i+1) - y(i) + u(i)\right)^2/2s_z^2\right\}$$

Because of the discrete nature of the probability density for the state (the state is either plus or minus the measurement), the sufficient statistics exist. Both sufficient statistics (q(i) and y(i)) depend on the measurement and on the control and thus neither can be predicted in advance even if the control policy is known.

iii) Information

The information associated with the problem is at time i

$$s\left[p(x(i)|y^i)\right] = q(i)\log q(i) + (1-q(i))\log(1-q(i)) + \text{const.} \qquad \text{B.34}$$

Thus information is infinite (the state is known exactly) when q(i) is zero or one. Least is known when q(i) = 0.5.

The information change between time i and i+1 is

$$s\left[p(x(i+1)|y^{i+1})\right] - s\left[p(x(i)|y^i)\right] =$$

$$q(i+1)\log q(i+1) + (1-q(i+1))\log(1-q(i+1) - q\log q$$

$$-(1-q)\log(1-q) \qquad \text{B.35}$$

From Equation B.35 there are three possibilities:

1) Information change may be positive (information gain) and this occurs when

$$|q(i+1) - 0.5| > |q(i) - 0.5| \qquad \text{B.36a}$$

2) There may be no information change which occurs when

$$|q(i+1) - 0.5| = |q(i) - 0.5| \qquad \text{B.36b}$$

3) There may be information loss which occurs when

$$|q(i+1) - 0.5| < |q(i) - 0.5| \qquad \text{B.36c}$$

Because information change from time i to i + 1 is dependent upon the measurement at time i + 1, the information change is a random variable. The expected information change could be considered but this is not available in analytic form. However, it is possible to deduce some general properties of the information process by examining Equations B.35 and B.33:

1) Information at time i + 1 is infinite ($q(i+1) = 0$ or 1) if $u(i) = \pm\infty$. This is shown by the fact that for $u(i) = \pm\infty$, $P_1 = P_2$ and $P_3 = P_4 = 0$. The uncertainty due to the input disturbance $z(i)$ and the unknown state is negligible compared to the infinite control policy and the state is determined exactly.

2) For $q(i) = 0$, 0.5 or 1.0, the certainty equivalent control policy (from Part v of this section)

$$u^{c.e.}[y(i), q(i)] = -(2q(i) - 1)y(i) \qquad B.37$$

produces the minimum possible information change (i.e. $q(i+1) = 0.5$) which is in fact information loss for $q(i) = 0$ or 1.0. This is verified by substitution in Equations B.3.

3) If $y(i+1) = 0$ then independently of what $u(i)$ is, $q(i+1) = 0.5$ and information change is minimum. Also, if the certainty equivalent policy is used, then from Equation B.37

$$u^{c.e.}(i+1) = 0$$

and

$$q(i+2) = 0.5$$

and thus the controller effectively ceases to operate. The uncertainty about the state is never reduced. This failure of the certainty equivalent control was first noted by Jacobs.[55]

iv) Optimal control policy

The optimal expected cost is defined:

$f_N[y(0),q(0)]$ $\triangleq$ the expected cost when the optimal control $u^*(i)$ is used over N stages and the initial condition is specified by $y(0)$ and $q(0)$.

The functional recurrence equation is

$$f_N[y(0),q(0)] = \min_u \{E\{x^2(1)\} + \int_{-\infty}^{\infty} f_{N-1}[y(1),q(1)]p(y(1)|y^o,u(0))dy(1)\} \quad \text{B.38}$$

When there is only one step to go

$$f_1[y(0),q(0)] = \min_u \{(y(0)(2q(0)-1) + u(0))^2 + y^2(0)(2q(0)-1) + s_z^2\}$$

from which

$$u_1^*[y(0),q(0)] = -y(0)(2q(0)-1) \quad \text{B.39}$$

and

$$f_1[y(0),q(0)] = y^2(0)(2q(0)-1) + s_z^2 \quad \text{B.40}$$

Equation B.38 has been solved numerically[100] and Langdon has shown that for N = 7 the cost per stage and optimal control policy become independent of N. The optimal policy is shown in Figure B.7. The optimal expected cost per stage for the case $y(0) = 0.0$, $q(0) = 0.0$ is given as

$$f_N[y(0),q(0)]/N = 2.2\, s_z^2 \quad \text{B.41}$$

v) <u>Certainty equivalent and neutral control policies</u>

The certainty equivalent policy is from Jacobs[55]

$$u^{c.e.}(i) = (1 - 2q(i))y(i) \quad \text{B.42}$$

The neutral policy is found by considering the related neutral problem. The plant equation is

$$x(i+1) = x(i) + u(i) + z(i) \quad \text{B.43a}$$

where $z(i)$ is an independent random variable with

$$p(x(i)) = N(0, s_z^2)$$

Figure B.7 Optimal Control Law in Example B.5

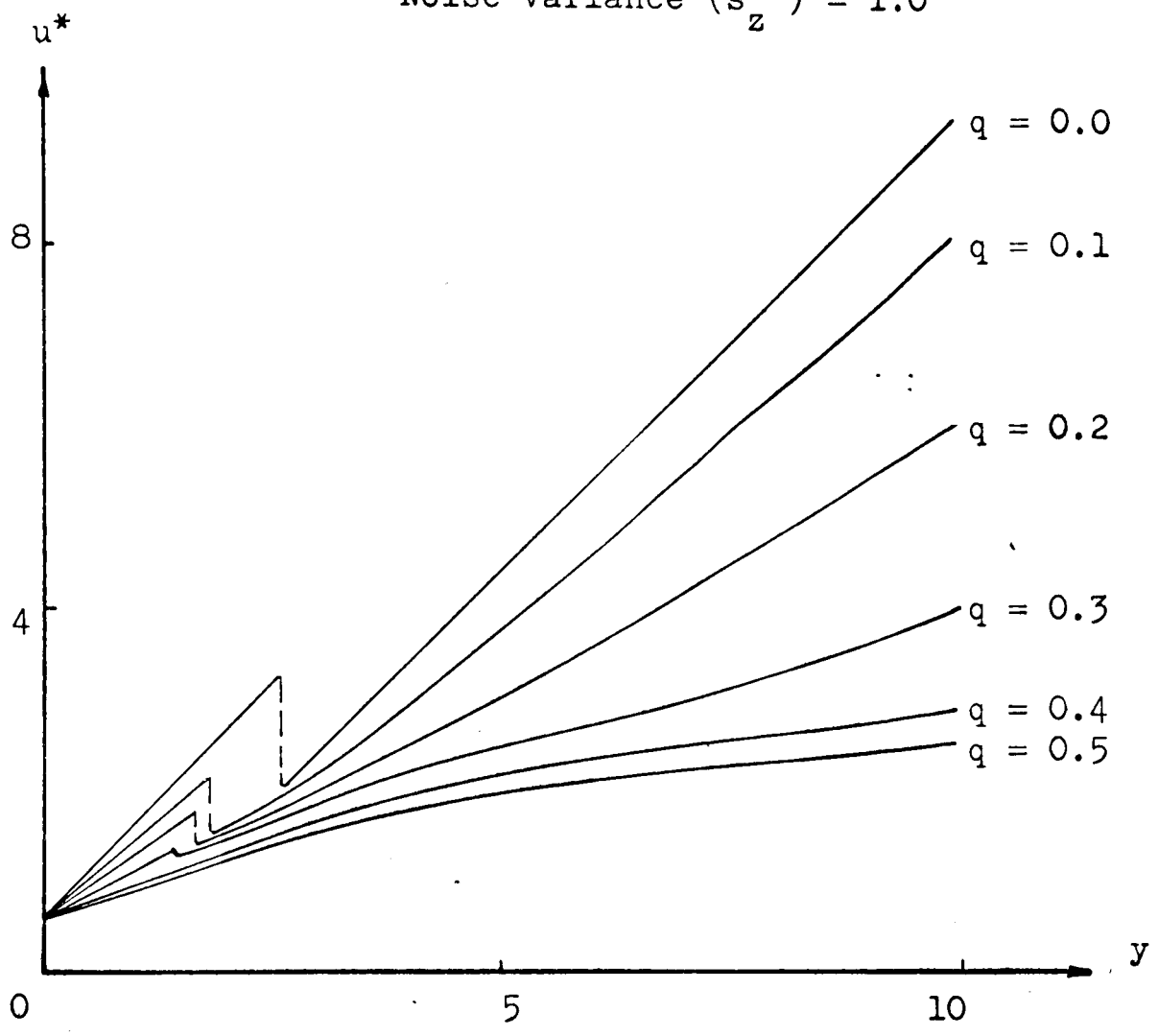
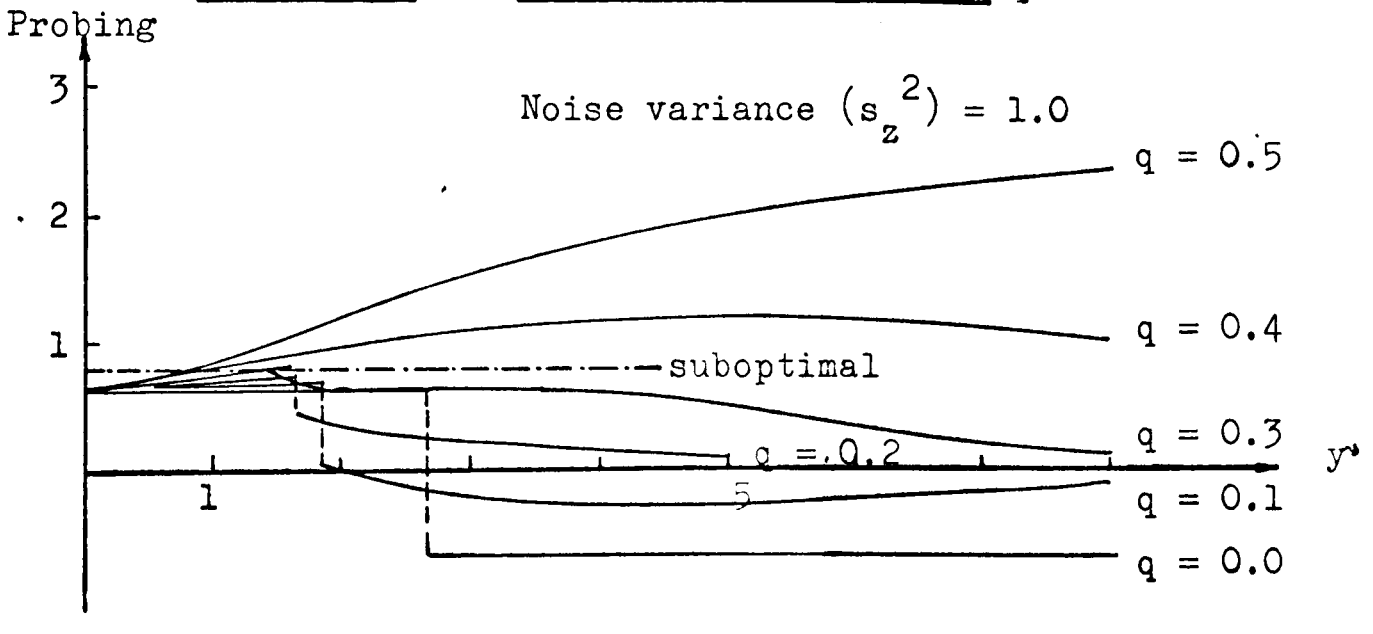


Figure B.8 Probing in Example B.5



Measurements are exact

$$y(i) = x(i) \qquad \text{B.43b}$$

and the cost function is given by Equation B.31.

Without resorting to dynamic programming it is easily seen that the optimal cost of the related neutral problem is

$$f_N^{nt}[x(0)] = Ns_z^2$$

Thus the optimal neutral control policy for the problem of Equations B.30 and B.31 is the value of u which minimizes the expression

$$E\{x^2(1)\} + (N-1)s_x^2$$

which gives

$$u_N^{nt}[y(0), q(0)] = -(2q(0)-1)y(0) \qquad \text{B.44}$$

The neutral policy is independent of the number of stages to go and as in Example B.3 it is identical to the optimal policy when there is only one step to go. It is also identical to the certainty equivalent policy.

vi) Discussion

The neutral and certainty equivalent policies are identical and consequently caution is zero. The only reason that the optimal control policy is different from the certainty equivalent policy is the need to reduce uncertainty by probing. This probing is given by

$$\text{Probing} = u^*(i) - u_N^{nt}(i)$$

$$= u^*(i) - u^{c.e.}(i) \quad \text{(for this problem only)}$$

and is shown in Figure B.8.[56] Probing in this problem is more important than in the unknown gain problem because here, as is pointed out above, without probing the control ceases to operate whenever the output becomes zero. As the certainty equivalent policy has the sole object of making the output zero,

this case might be expected to occur fairly often and in fact simulation results[55] confirm this. Jacobs[55] states that when the problem was simulated using the certainty equivalent policy, the process soon degenerated to the case where the control was zero and q(i) was equal to one-half. It was shown above that for the cases q = 0.0, 1.0 or 0.5, the certainty equivalent policy produces maximum information loss (zero for q = 0.5). These simulation results show that for any q (except q = 0.5) and for any y, the certainty equivalent policy on average produces information loss and thus for this problem the certainty equivalent (neutral) policy is a very bad control policy if not the worst possible control policy.

From Figure B.8 it can be seen that for small values of the output, probing increases as the uncertainty increases. The effect is to place the expected value of the next state slightly beyond zero so that there will be some prior knowledge as to whether or not the next state is positive or negative. For some values of q the probing shows a discontinuity and decreases as y(i) increases because control becomes more important than information gain if y(i) is large. For values of q close to one-half, however, probing continues to be the dominating factor for large y(i).

Simulations are reported[56,100] using the optimal control policy and these confirm the average expected cost as that given in Equation B.41. A suboptimal policy was also tried[55] which used the certainty equivalent (or neutral) policy with an additive 'intentional error' of the form

$$(-1)^i a$$

It was found that for a = 0.8 the suboptimal policy gave the same average expected cost as the optimal policy. It appears that the exact form of the probing is not important but only the existence of probing and its approximate magnitude are important. This effect is also observed in Chapter 7 in connection with the unknown gain problem.

Appendix C

Simplification of the Unknown Constant Gain Problem for Numerical Solution

Two basic methods of simplifying problems for numerical solution are the determination of some symmetry relations and the application of dimensional analysis. Both methods are considered in this appendix.

C.1 Symmetry Conditions

One of the most useful methods of reducing the difficulty of determining optimal control policies numerically is to establish some symmetry conditions. In this way the range of values of the parameters over which the control policy must be determined can be greatly reduced.

To establish symmetry conditions in this problem it is first necessary to define a 'symmetric mean' by

$$m_s \triangleq m_x - m_b \rho s_x / s_b \qquad \text{C.1}$$

Then with

$$f_N'[m_s, m_b, s_x^2, s_b^2, \rho] = f_N[m_x, m_b, s_x^2, s_b^2, \rho] \qquad \text{C.2a}$$

$$u_N^{*\prime}[m_s, m_b, s_x^2, s_b^2, \rho] = u_N^*[m_x, m_b, s_x^2, s_b^2, \rho] \qquad \text{C.2b}$$

the following symmetry conditions can be shown to hold:

1) $$f_N'[m_s, m_b, s_x^2, s_b^2, \rho] = f_N'[-m_s, m_b, s_x^2, s_b^2, \rho]$$

$$u_N^{*\prime}[m_s, m_b, s_x^2, s_b^2, \rho] = -u_N^{*\prime}[-m_s, m_b, s_x^2, s_b^2, \rho] - 2\rho s_x/s_b \qquad \text{C.3a}$$

2) $$f_N'[m_s, m_b, s_x^2, s_b^2, \rho] = f_N'[-m_s, -m_b, s_x^2, s_b^2, \rho]$$

$$u_N^{*\prime}[m_s, m_b, s_x^2, s_b^2, \rho] = u_N^{*\prime}[-m_s, -m_b, s_x^2, s_b^2, \rho] \qquad \text{C.3b}$$

3) $$f_N'[m_s, m_b, s_x^2, s_b^2, \rho] = f_N'[-m_s, m_b, s_x^2, s_b^2, -\rho]$$

$$u_N^{*\prime}[m_s, m_b, s_x^2, s_b^2, \rho] = -u_N^{*\prime}[-m_s, m_b, s_x^2, s_b^2, -\rho] \qquad \text{C.3c}$$

These symmetry conditions mean that if f_N' and $u_N^{*\prime}$ are determined for positive m_s, m_b and ρ they are also known for any negative values of these parameters. The variances s_x^2, s_b^2 and s_z^2 are always positive.

An updating equation for m_s and a new updating equation for m_b in terms of m_s can be derived from Equations C.1 and 6.3. They are

$$m_s(i+1) = \frac{y(i+1)s_x^2(1-\rho^2) + s_z^2 m_s}{s_z^2 + s_x^2(1-\rho^2)} \qquad \text{C.4a}$$

$$m_b(i+1) = \frac{m_b(s_z^2 + s_x^2(1-\rho^2)) + (y(i+1) - m_s)(\rho s_x s_b + u s_b^2)}{s_z^2 + s_x^2(1-\rho^2) + (us_b + \rho s_x)^2} \qquad \text{C.4b}$$

A functional recurrence equation for f_N' is derived from Equation 6.6:

$$f_N'[m_s, m_b, s_x^2, s_b^2, \rho] = \min_u \left\{(m_s + m_b(\rho s_x/s_b + u))^2 + (\rho s_x + s_b u)^2 + s_x^2(1-\rho^2) \right.$$
$$\left. + \int_{-\infty}^{\infty} f_{N-1}[m_s(1), m_b(1), s_x^2(1), s_b^2(1), \rho(1)]\, p(y(1)|y^o, u)\, dy(1)\right\} \qquad \text{C.5}$$

where

$$p(y(1)|y^o, u) = N(m_s + m_b(\rho s_x/s_b + u), s_x^2(1-\rho^2) + (us_b + \rho s_x)^2 + s_z^2)$$

The symmetry relations (Equations C.3) are first shown to hold for $N = 2$. From Equation C.5, f_2' can be written

$$f_2'[m_s, m_b, s_x^2, s_b^2, \rho] = \min_{u(0)} \left\{(m_s + m_b(\rho s_x/s_b + u))^2 + (\rho s_x + s_b u)^2 + s_x^2(1-\rho^2) \right.$$
$$\left. + \int_{-\infty}^{\infty} f_1[m_s(1), m_b(1), s_x^2(1), s_b^2(1), \rho(1)]\, p(y(1)|y^o, u)\, dy(1)\right\} \qquad \text{C.6}$$

where

$$p(y(1)|y^o, u) = N(m_x + m_b(\rho s_x/s_b + u), s_x^2(1-\rho^2) + (us_b + \rho s_x)^2 + s_z^2) \qquad \text{C.7}$$

Consider in Equation C.6 the part within the brackets before the integral

$$(m_s + m_b(\rho s_x/s_b + u))^2 + (\rho s_x + s_b)^2 + s_x^2(1-\rho^2) \qquad \text{C.8}$$

For the 1st symmetry condition (Equation C.3a), m_s is replaced by $-m_s$ and u by $-u - 2\rho s_x/s_b$ and when these changes are made, Expression C.8 remains unchanged. This is also true for the 2nd and 3rd symmetry conditions (Equations C.3b and C.3c respectively).

Consider the integral term in Equation C.6. This depends on the probability density of Equation C.7. When the changes of the 1st symmetry condition take place, this density changes in that the sign of y(1) is changed. Now consider the effect of the symmetry changes and the change of sign of y(1) in the updating equations C.4, 6.3c, 6.3d and 6.3e. From Equations C.4, $m_s(1)$ changes sign while $m_b(1)$ does not. From Equation 6.3, $s_x^2(1)$ and $s_b^2(1)$ are unaffected but $\rho(1)$ changes sign. From Equation C.5 it can be shown that

$$f_1'[m_s(1), m_b(1), s_x^2(1), s_b^2(1), \rho(1)] = \frac{s_b^2(1) m_s^2(1)}{s_b^2(1) + m_b^2(1)} + s_x^2(1)(1 - \rho^2(1))$$

and it is seen that the changes in $m_s(1)$, $m_b(1)$, $s_x^2(1)$, $s_b^2(1)$ and $\rho(1)$ make no change in f_1'. Thus the integral term remains unchanged under the 1st symmetry condition. Similarly it also remains unchanged under the 2nd and 3rd symmetry conditions.

Since both parts of Equation C.6 remain unchanged under each of the symmetry conditions, $f_2'[m_s, m_b, s_x^2, s_b^2, \rho]$ remains unchanged and thus all the symmetry conditions hold for N = 2. It can then be shown by induction that the symmetry conditions also hold for any value of N.

The range of variables for which it is necessary to determine the optimal control has been reduced by a factor of 8 (a factor of 2 for each symmetry condition). This represents a very considerable saving in terms of computer time necessary to determine the optimal control policy.

C.2 Dimensional Analysis

Dimensional analysis[99,101] is a very useful tool because it generally allows the number of significant variables to be reduced. In this problem, since the dimensions of the problem are two in number (state and gain), the number of significant parameters can be reduced by two. First the non-dimensional parameters are defined, then a non-dimensional version of the problem is introduced, Bayes' rule is applied and a functional recurrence equation is derived.

i) Non-dimensional parameters

The parameters that are chosen to be eliminated are the noise variance s_z^2 and the variance of the gain $s_b^2(i)$. In the non-dimensional problem both of them become effectively one. The noise variance was chosen because it is constant during the course of a process. However, it was necessary to eliminate either the mean or variance of the gain (the only parameters with the dimension of gain) and thus the variance of the gain which does vary during the course of a process was eliminated.

The non-dimensional parameters are defined as

State	ξ	$= x/s_z$
Gain	β	$= b/s_b$
Measurement	η	$= y/s_z$
Noise	ζ	$= z/s_z$
Control	ν	$= us_b/s_z$
State mean	μ_x	$= m_x/s_z$
State variance	σ_x^2	$= s_x^2/s_z^2$
Gain mean	μ_b	$= m_b/s_b$
Correlation	ρ	$= \rho$

Symmetric mean $$\mu_s = m_s/s_z$$

Cost $$\Psi_N = I_N/s_z^2$$

Optimal control $$\nu^*[\mu_x, \mu_b, \sigma_x^2, \rho] = u^*[m_x, m_b, s_x^2, s_b^2, \rho]\, s_b/s_z \qquad \text{C.9}$$

Optimal cost $$\phi_N[\mu_x, \mu_b, \sigma_x^2, \rho] = f_N[m_x, m_b, s_x^2, s_b^2, \rho]/s_z^2$$

Optimal symmetric control

$$\nu^{*'}[\mu_s, \mu_b, \sigma_x^2, \rho] = u^{*'}[m_s, m_b, s_x^2, s_b^2, \rho]\, s_b/s_z$$

Optimal symmetric cost

$$\phi_N'[\mu_s, \mu_b, \sigma_x^2, \rho] = f_N'[m_s, m_b, s_x^2, s_b^2, \rho]/s_z^2$$

ii) Statement

The plant equation becomes

$$\xi(i+1) = \xi(i) + \beta(i)\,\nu(i) \qquad \text{C.10a}$$

Here the gain changes with time (instead of being an unknown constant) because of the way it is defined. This is one of the consequences of eliminating a variable that is time-dependent.

The measurement equation is

$$\eta(i) = \xi(i) + \zeta(i) \qquad \text{C.10b}$$

where $\zeta(i)$ is an independent random variable with

$$p(\zeta(i)) = N(0, 1)$$

Because there are two unknowns, the gain and the state, the initial condition is a joint probability density

$$p(\xi(0), \beta(0)) = N(\mu_x, \mu_b, \sigma_x^2, 1, \rho)$$

The cost function is

$$\Psi = \sum_{i=1}^{N} \xi^2(i) \qquad \text{C.11}$$

iii) Bayes' rule gives that the joint conditional probability density for the state and gain at time i is normal

$$p(\xi(i),\beta(i)|\eta^i) = N(\mu_x,\mu_b,\sigma_x^2,1,\rho)$$

Here the variance of the gain is always equal to one.

From Equations 6.3 the following updating equations for the statistics can be derived:

$$\mu_x(i+1) = \frac{\eta(i+1)(\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2)+(\mu_x+\mu_b\nu)}{\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2+1.0} \qquad C.12a$$

$$\mu_b(i+1) = \frac{\mu_b(1.0+\sigma_x^2+\nu\rho\sigma_x)+(\eta(i+1)-\mu_x)(\rho\sigma_x+\nu)}{((1.0+\sigma_x^2(1-\rho^2))(1.0+\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2))^{\frac{1}{2}}} \qquad C.12b$$

$$\sigma_x^2(i+1) = \frac{\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2}{\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2+1.0} \qquad C.12c$$

$$\rho(i+1) = \frac{\nu+\rho\sigma_x}{((1.0+(1-\rho^2)\sigma_x^2)(\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2))^{\frac{1}{2}}} \qquad C.12d$$

iv) Symmetry conditions

The symmetry conditions also apply to the non-dimensional problem.

From Equations C.1 and C.9

$$\mu_s = \mu_x - \mu_b\rho\sigma_x \qquad C.13$$

and

$$\phi_N'[\mu_s,\mu_b,\sigma_x^2,\rho] = \phi_N[\mu_x,\mu_b,\sigma_x^2,\rho] \qquad C.14a$$

$$\nu_N^{*'}[\mu_s,\mu_b,\sigma_x^2,\rho] = \nu_N^*[\mu_x,\mu_b,\sigma_x^2,\rho] \qquad C.14b$$

and the following hold:

$$1)\quad \phi_N'[\mu_s,\mu_b,\sigma_x^2,\rho] = \phi_N'[-\mu_s,\mu_b,\sigma_x^2,\rho]$$

$$\nu_N^{*\prime}[\mu_s,\mu_b,\sigma_x^2,\rho] = -\nu_N^{*\prime}[-\mu_s,\mu_b,\sigma_x^2,\rho] - 2\rho\sigma_x \qquad \text{C.15a}$$

$$2)\quad \phi_N'[\mu_s,\mu_b,\sigma_x^2,\rho] = \phi_N'[-\mu_s,-\mu_b,\sigma_x^2,\rho]$$

$$\nu_N^{*\prime}[\mu_s,\mu_b,\sigma_x^2,\rho] = \nu_N^{*\prime}[-\mu_s,-\mu_b,\sigma_x^2,\rho] \qquad \text{C.15b}$$

$$3)\quad \phi_N'[\mu_s,\mu_b,\sigma_x^2,\rho] = \phi_N'[-\mu_s,\mu_b,\sigma_x^2,-\rho]$$

$$\nu_N^{*\prime}[\mu_s,\mu_b,\sigma_x^2,\rho] = -\nu_N^{*\prime}[-\mu_s,\mu_b,\sigma_x^2,-\rho] \qquad \text{C.15c}$$

An updating equation for μ_s and a new updating equation for μ_b in terms of μ_s can be derived from Equations C.12a, C.12b and C.15:

$$\mu_s(i+1) = \frac{\eta(i+1)\sigma_x^2(1-\rho^2)+\mu_s}{1.0+\sigma_x^2(1-\rho^2)} \qquad \text{C.16a}$$

$$\mu_b(i+1) = \frac{\mu_b(1+\sigma_x^2(1-\rho^2))+(\eta(i+1)-\mu_x)(\rho\sigma_x+\nu)}{((1.0+(1-\rho^2)\sigma_x^2)(1.0+\sigma_x^2(1-\rho^2)+(\nu+\rho\sigma_x)^2))^{\frac{1}{2}}} \qquad \text{C.16b}$$

v) Functional recurrence equation

A functional recurrence equation for ϕ_N' can be derived from Equation C.5:

$$\phi_N'[\mu_x,\mu_b,\sigma_x^2,\rho] = \min_{\nu(0)}\Big\{(\mu_x+\mu_b(\nu+\rho\sigma_x))^2+(\nu+\rho\sigma_x)^2+\sigma_x^2(1-\rho^2) + \int_{-\infty}^{\infty}\phi_N'[\mu_x(1),\mu_b(1),\sigma_x^2(1),\rho(1)]p(\eta(1)|\eta^\circ,\nu(0))d\eta(1)\Big\} \qquad \text{C.17}$$

where

$$p(\eta(1)|\eta^\circ,\nu(0)) = N(\mu_x+\mu_b(\rho\sigma_x+\nu),\sigma_x^2(1-\rho^2)+(\rho\sigma_x+\nu)^2+1.0)$$

This is the equation that is solved numerically in Appendix D.

When N = 1 the solution is analytic

$$\phi_1'[\mu_s, \mu_b, \sigma_x^2, \rho] = \mu_s^2/(\mu_b^2 + 1.0) + \sigma_x^2(1-\rho^2) \qquad \text{C.18a}$$

$$\nu_1^{*\prime}[\mu_s, \mu_b, \sigma_x^2, \rho] = -\mu_s \mu_b/(\mu_b^2 + 1.0) - \rho\,\sigma_x \qquad \text{C.18b}$$

but for $N \geq 2$ the expectation integral of Equation C.17 cannot be evaluated in terms of known functions and thus Equation C.17 must be solved numerically for $N \geq 2$.

vi) Comments

Since the number of significant variables has been reduced from six to four and since the range over which the equation must be solved has been reduced by a factor of eight due to the symmetry conditions, Equation C.17 is considerably easier to solve than was the original functional recurrence equation (Equation 6.8).

Appendix D

Numerical Calculation of the Optimal Control Policy for the Unknown Constant Gain Problem

The object of this numerical work is to obtain ϕ'_N and ν'_N of Equation C.17 for as wide a range of μ_s, μ_b, σ_x and ρ, and as many values of N as is practical. The general approach to the problem is illustrated by the flow diagram Figure D.1.

D.1 Numerical Procedures

The various operations indicated in the flow diagram Figure D.1 are considered in turn.

D.1.1 Integration

It is required to perform the integration

$$\int \phi'_{N-1}\left[\mu_s(1), \mu_b(1), \sigma_x(1), \rho(1)\right] p(\eta(1) | \eta^o, \nu) d\eta(1) \qquad \text{D.1}$$

numerically for each value of ν chosen by the minimization procedure for the complete grid of values of $\mu_s(0), \mu_b(0), \sigma_x(0), \rho(0)$ for each value of N. Thus the integration must be performed

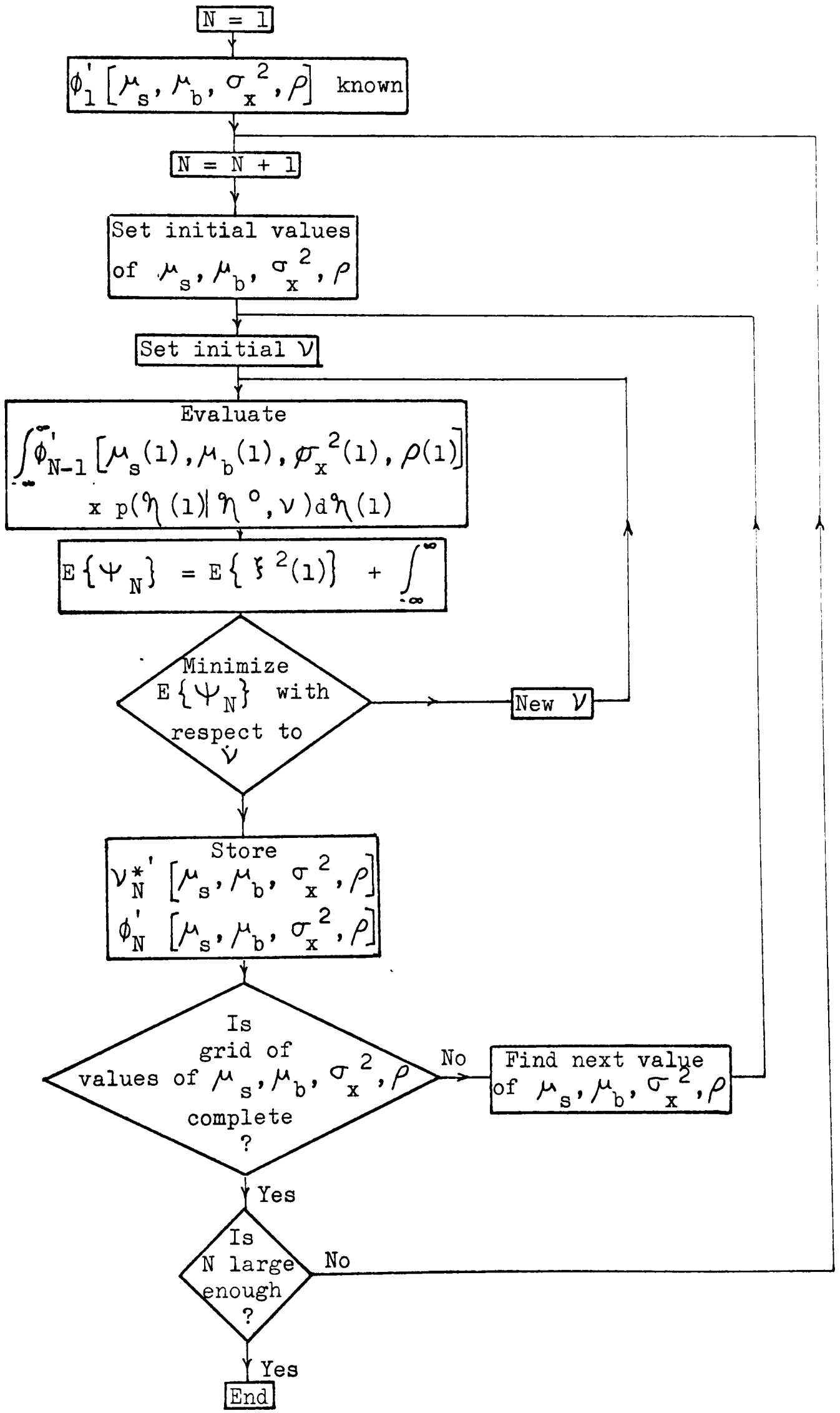
(number of values of ν used) x (grid size) x (number of values of N used)

times. This is a large number and thus the time required for the integration is important if computing time is to be reasonable. For N = 2 the argument of the integration is known as an analytic function but for larger values of N the argument must be interpolated from a table of values.

A relatively fast type of numerical integration which is suited to this problem because of the infinite range of the integral and the normal probability density is known as the Gauss-Hermite quadrature method. The

Figure D.1

Flow Diagram of Numerical Solution of the Unknown Constant Gain Problem



Gauss-Hermite formula is[102]

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx = \sum_{i=1}^{N} w(i)f(a_i) + \frac{n!\sqrt{\pi}}{2^n(2n)!} f^{(2n)}(\zeta) \qquad D.2$$

where the w(i) are weighting coefficients and the a_i are abscissae which are available for n up to twenty in table form.[103] The term ζ in the error term is restricted to the integration interval $(-\infty, \infty)$ and thus for large 2n the higher order derivatives of f(x) are to be avoided. The implementation of the Gauss-Hermite method is illustrated by a flow diagram Figure D.2.

Accuracy is checked by increasing the value of n and checking for convergence. Usually the accuracy of this method is very good but for some cases of large μ_x and small μ_b it is poor. The reason for this is that the denominator of ϕ'_{N-1} contains terms in powers of $\eta(1)$ and for some values of μ_b the 2nth derivation can be very large and this integration method fails.

Therefore a second method of integration was tried which is known as Simpson's rule integration.[102] Because the interval size can be varied at will it is more suited to handling large peaks than is the Gauss-Hermite method. Simpson's rule as used here is

$$\int_{x(0)}^{x(0)+a+b} f(x)dx \simeq ((a+b)/6)(f(x(0)) + 4f(x(0)+a) + f(x(0)+a+b)) + ((b-a)/3)(f(x(0)+a+b) - f(x(0))) \qquad D.3$$

and its implementation is illustrated by the flow diagram in Figure D.3.

The interval (i.e. a or b) is adjusted before each new calculation of the integrand so that it is inversely proportional to the last value of the integrand. Thus the interval becomes smaller for large peaks. It is calculated according to

$$\text{Interval} = \text{const} \times \frac{\text{estimated value of the integral}}{\text{last calculated value of the integrand}}$$

The estimated value of the integral is produced by the Gauss-Hermite method

Figure D.2 Flow Diagram for Gauss-Hermite Integration

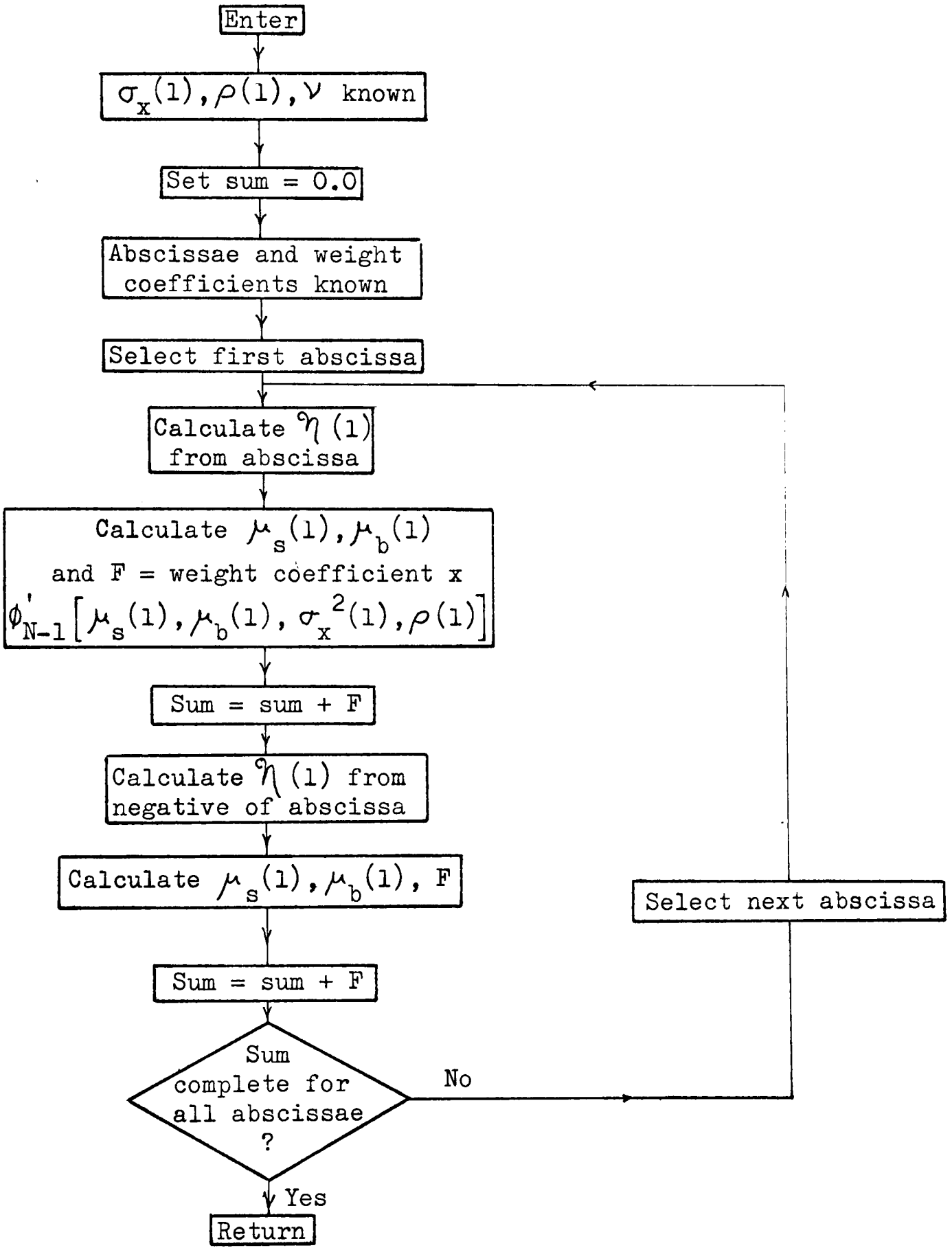


Figure D.3 Simpson's Rule Integration Flow Diagram

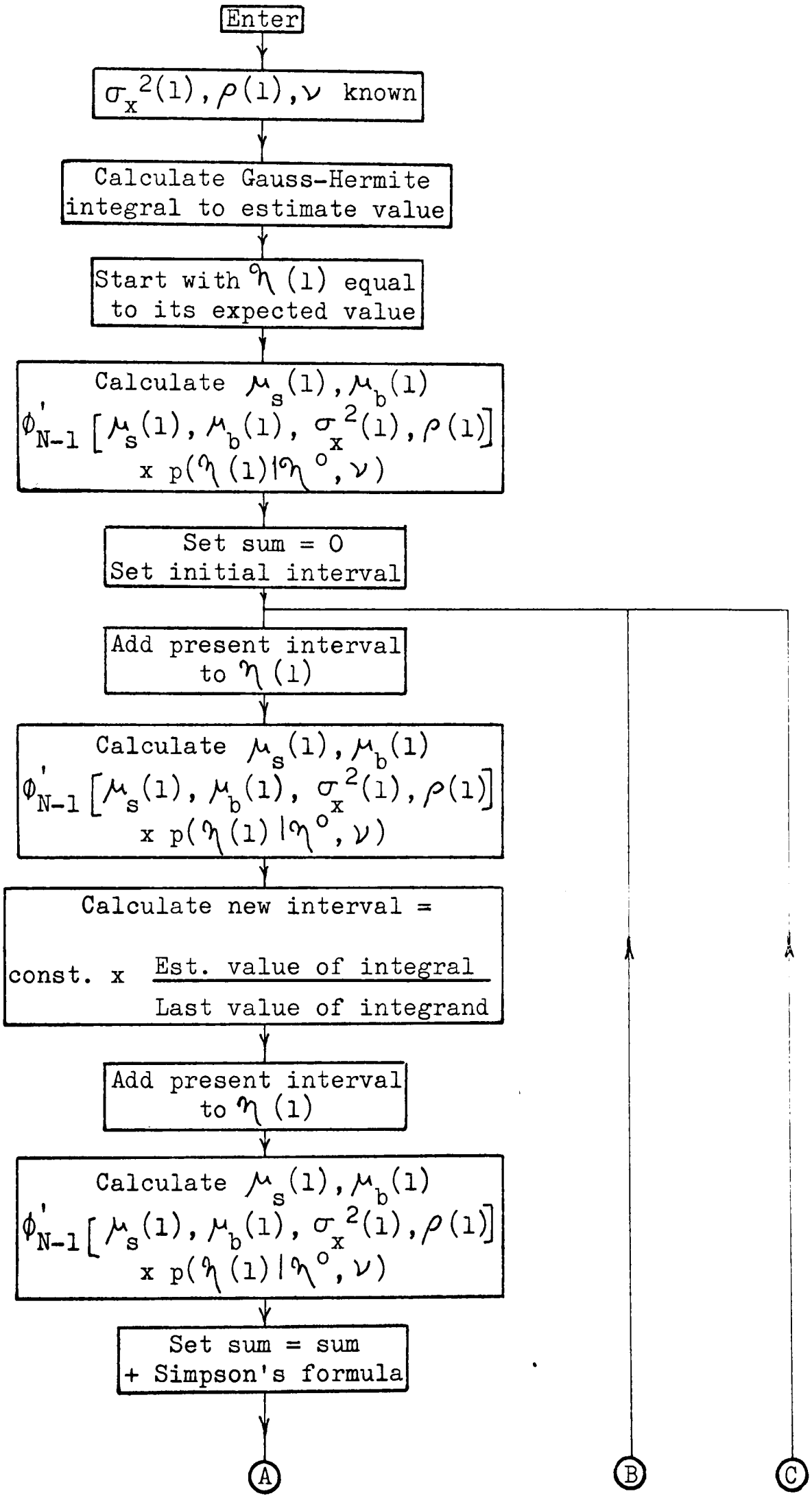
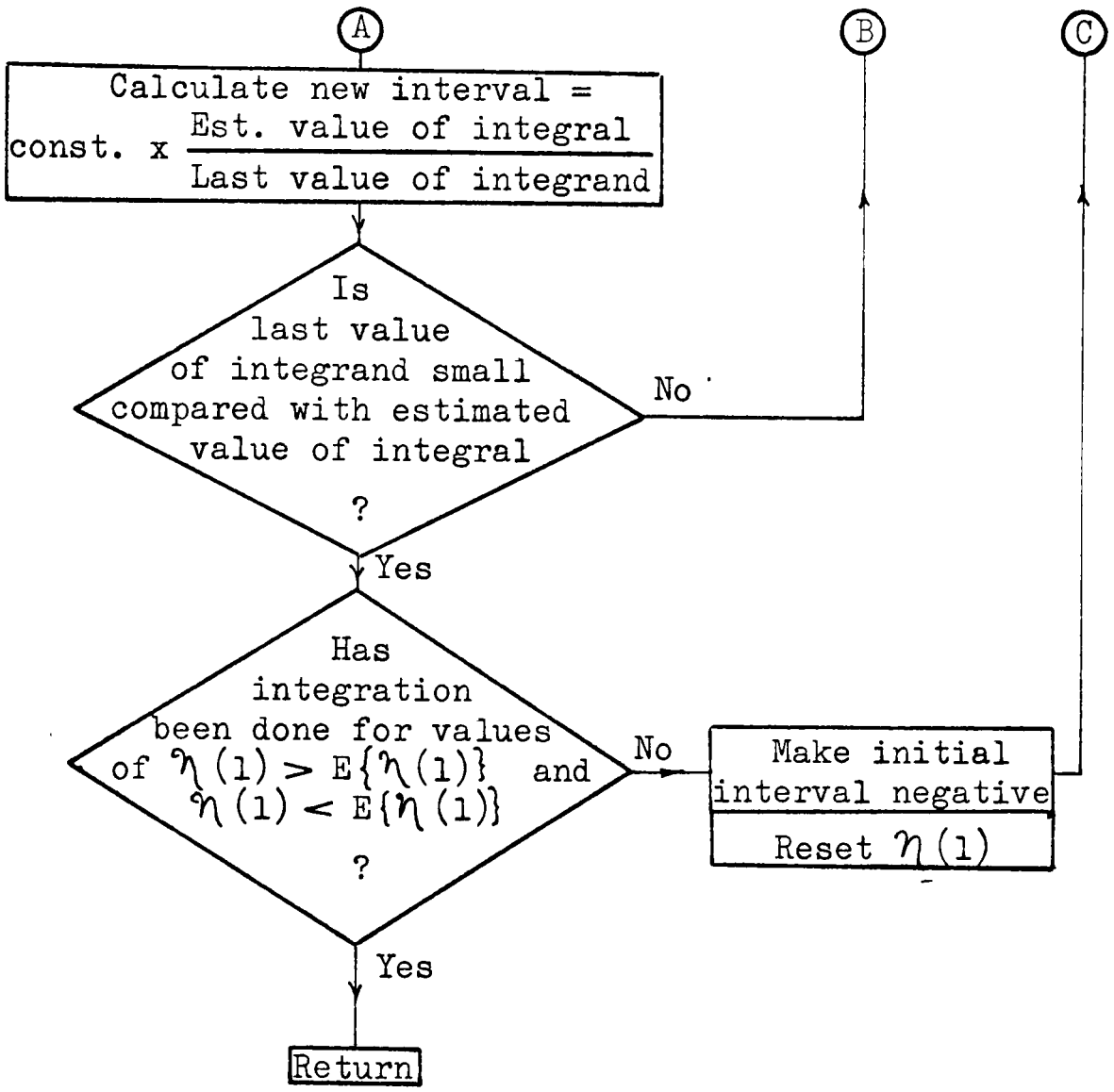


Figure D.3 (cont.)



and is used as a normalisation factor. The constant is varied to give different step sizes.

The estimated (Gauss-Hermite integral) value of the integral is also used in determining the range of integration. Integration starts at the expected value of $\eta(1)$ and runs first for values of $\eta(1)$ greater than the expected value until a point is reached where the integrand is a specified (small) fraction of the estimated integral. Then a similar procedure is used for values of $\eta(1)$ less than the expected value.

The accuracy of Simpson's rule integration increases by a factor of 64 when the step size is doubled and this produces a very useful estimate of accuracy that is not available when the Gauss-Hermite method is used.

In practice it was found that a typical Gauss-Hermite integration required about 100 to 200 msec while a Simpson's rule integration required from one to six seconds. Therefore it was desirable to use the Gauss-Hermite method as often as possible. The accuracy of the Gauss-Hermite method decreases as μ_s increases and μ_b decreases. The point at which it was necessary to change over to Simpson's rule was determined by checking the consistency of the Gauss-Hermite method between high and low orders.

D.1.2 Minimization procedure

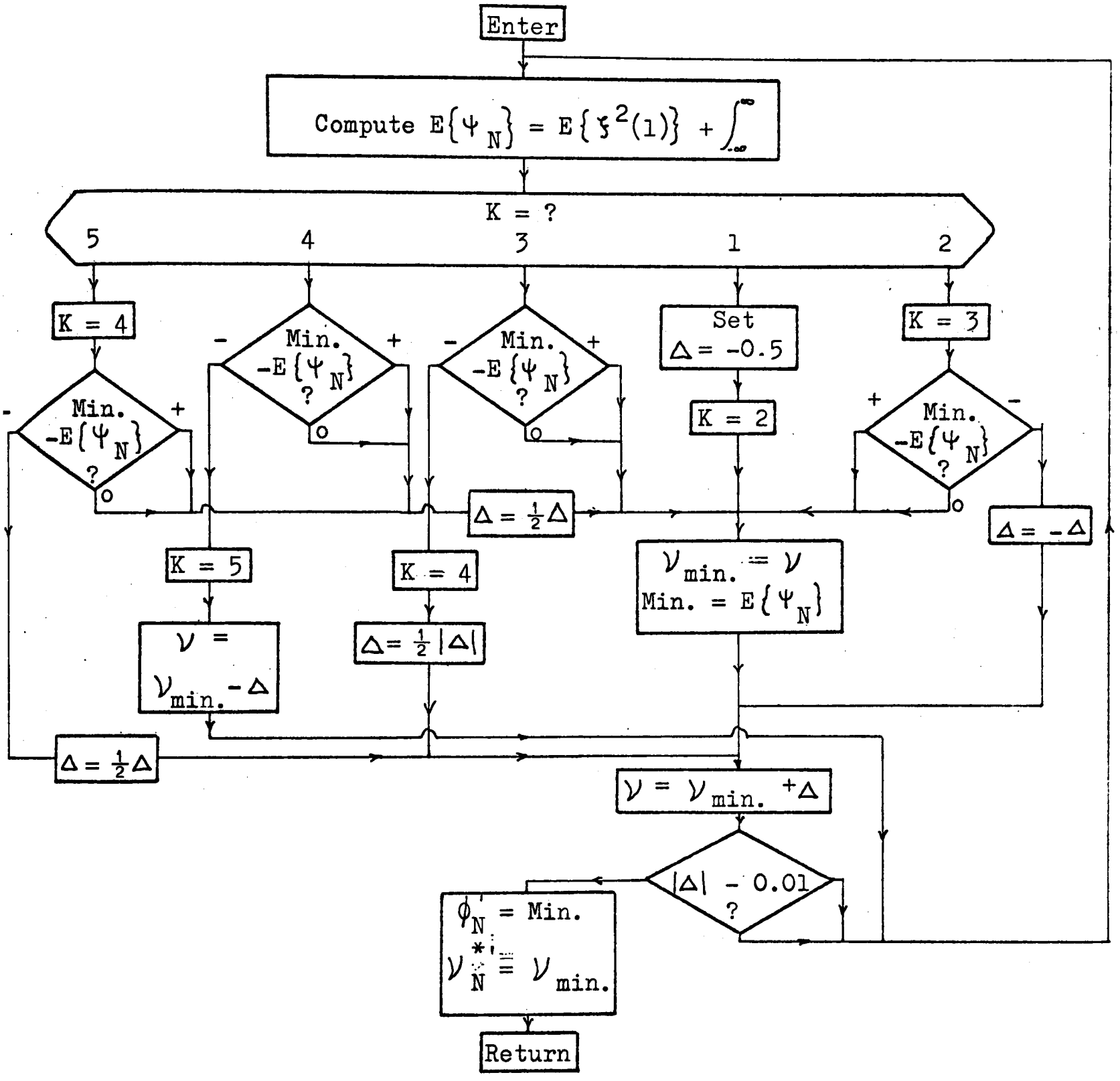
The minimization procedure used is a stepping method. The step size is halved and the sign is changed whenever the mimimum is passed. The procedure is illustrated by a flow diagram Figure D.4.

The minimizing value of ν is determined within ± 0.01 if the integrations are perfectly accurate. On average twelve steps are required to find the minimum.

D.1.3 Storage and retrieval of ϕ'_{N-1}

The minimum expected cost function ϕ'_{N-1} (for $N > 2$) must be stored as it is calculated in some convenient form so that it may be used in determining

Figure D.4 Minimization Procedure Flow Diagram



ϕ'_N. It may be stored either in functional form or as a set (grid) of points. If it is stored as a set of points an interpolating procedure is necessary but on the other hand functional approximations are difficult to determine.

In either case the prime consideration is the accuracy of the representation of ϕ'_{N-1} and the range of values of μ_s, μ_b, σ_x and ρ for which it is represented.

Given a set range of values accuracy is basically determined by the number of points within that range for which ϕ'_{N-1} is calculated and thus is largely dependent on the time available to compute values of ϕ'_{N-1}. Ultimately the number of values of ϕ'_{N-1} used is restricted by storage requirements since it is generally convenient to store ϕ'_{N-1} for its entire range of values while determining ϕ'_N. This was the limiting factor in this calculation.

Consider now the range of values of μ_s, μ_b, σ_x and ρ. It is known from the symmetry relations (Equation C.15) that only positive values of these need be considered. Also it is known that $|\rho| \leq 1$ and that after the first step in any process $\sigma_x^2(i) \leq 1$. (Thus only values of $\sigma_x^2 \leq 1$ are considered since tables of values for larger σ_x^2 would not need to be calculated recursively.) However, the other two parameters μ_s and μ_b have an infinite range which must in practice be curtailed. The range of these two requires some consideration because of the 'contracting grid problem' i.e. given a range of values for which ϕ'_{N-1} is known, the range of values for which ϕ'_N can be accurately determined will in general be smaller than the original. Thus it is necessary to start with the largest range possible.

Two properties about the variation of ϕ'_N with respect to σ_x^2 and ρ are known:

1) If $\sigma_x^2 = 0$ then $\phi'_N[\mu_s, \mu_b, 0, \rho]$ is independent of ρ.

2) If $\rho = 1$ then $\phi'_N[\mu_s, \mu_b, \sigma_x^2, 1]$ is independent of σ_x^2.

These two facts can be verified by substitution in Equations C.16 and C.17.

In the view of these two facts it was decided to use a polynomial approximation for the variation of ϕ_N' with respect to σ_x^2 and ρ. Since ϕ_N' is an even function of ρ only even order approximations are needed. Assume

$$\begin{aligned}\phi_N'[\mu_s, \mu_b, \sigma_x^2, \rho] = {} & A_1(\mu_s, \mu_b)\sigma_x^4\rho^4 + A_2(\mu_s, \mu_b)\sigma_x^2\rho^4 \\ & + A_3(\mu_s, \mu_b)\rho^4 + A_4(\mu_s, \mu_b)\sigma_x^4\rho^2 \\ & + A_5(\mu_s, \mu_b)\sigma_x^2\rho^2 + A_6(\mu_s, \mu_b)\rho^2 \\ & + A_7(\mu_s, \mu_b)\sigma_x^4 + A_8(\mu_s, \mu_b)\sigma_x^2 + A_9(\mu_s, \mu_b)\end{aligned}$$

Using the facts 1) and 2) above, this can be reduced to

$$\begin{aligned}\phi_N'[\mu_s, \mu_b, \sigma_x^2, \rho] = {} & \sigma_x^2(\rho^2-1)\{(A_5(\mu_s, \mu_b)\sigma_x^2 + A_1(\mu_s, \mu_b)(\rho^2+1) \\ & + A_2(\mu_s, \mu_b)\sigma_x^2 + A_3(\mu_s, \mu_b)) + A_4(\mu_s, \mu_b)\end{aligned}$$

where the A's have been renumbered. However, when the A's are calculated, A_5 is always insignificant and therefore the following approximation is used:

$$\begin{aligned}\phi_N'[\mu_s, \mu_b, \sigma_x^2, \rho] = {} & \sigma_x^2(\rho^2-1)(A_1(\mu_s, \mu_b)(\rho^2+1) \\ & + A_2(\mu_s, \mu_b)\sigma_x^2 + A_3(\mu_s, \mu_b)) + A_4(\mu_s, \mu_b) \qquad \text{D.4}\end{aligned}$$

This approximation is carefully checked (see Section D.3) for each value of N.

To determine the A's for any μ_s, μ_b and N the positive values of $\phi_N'[\mu_s,0,\mu_b,0]$, $\phi_N'[\mu_s,0.5,\mu_b,0]$, $\phi_N'[\mu_s,1.0,\mu_b,0]$ and $\phi_N'[\mu_s,1.0,\mu_b,0.5]$ are calculated. Four equations then give the values of the A's. The choice of the points is arbitrary (except for the first which gives A_4 directly) and the justification for their use is that they produce good approximations.

The A's are calculated for a set of values of μ_s and μ_b for each value of N. These points (the grid) are chosen so that interpolation errors are uniform over the entire range of values. Thus the points become farther apart as ϕ_N' becomes easier to approximate. The grid points are determined by

$$u_{s_i} = \tfrac{1}{2}(i - 1) \text{ for } i = 1, \ldots,5$$

$$= (i - 4)^2 \text{ for } i = 6, \ldots,11$$

$$u_{b_i} = \exp((i - 1)0.15) - 1.0 \text{ for } i = 1, \ldots,26$$

resulting in (11 x 26 =) 286 values of each A for each N. These particular formulae are used because they give something close to the desired distribution of grid points and because the inverse operations (necessary for interpolation) are well defined.

As N increases the grid size gradually decreases (because of the contracting grid problem) as is shown in Table D.1. The size reaches a steady state after N = 4. This is because some extrapolation over the limits of the range of μ_s and μ_b is allowed and the accuracy of this extrapolation is sufficient to maintain the grid size.

The interpolation procedure used to produce values of the A's between grid points is illustrated by the flow diagram Figure D.5. $A_4(\mu_s, \mu_b)$ is interpolated by a two-dimensional (μ_s x μ_b) quadratic Lagrange formula while $A_3(\mu_s, \mu_b)$, $A_2(\mu_s, \mu_b)$ and $A_1(\mu_s, \mu_b)$ are interpolated by two-dimensional linear Lagrange formulae. The accuracy of this interpolation is checked for each N.

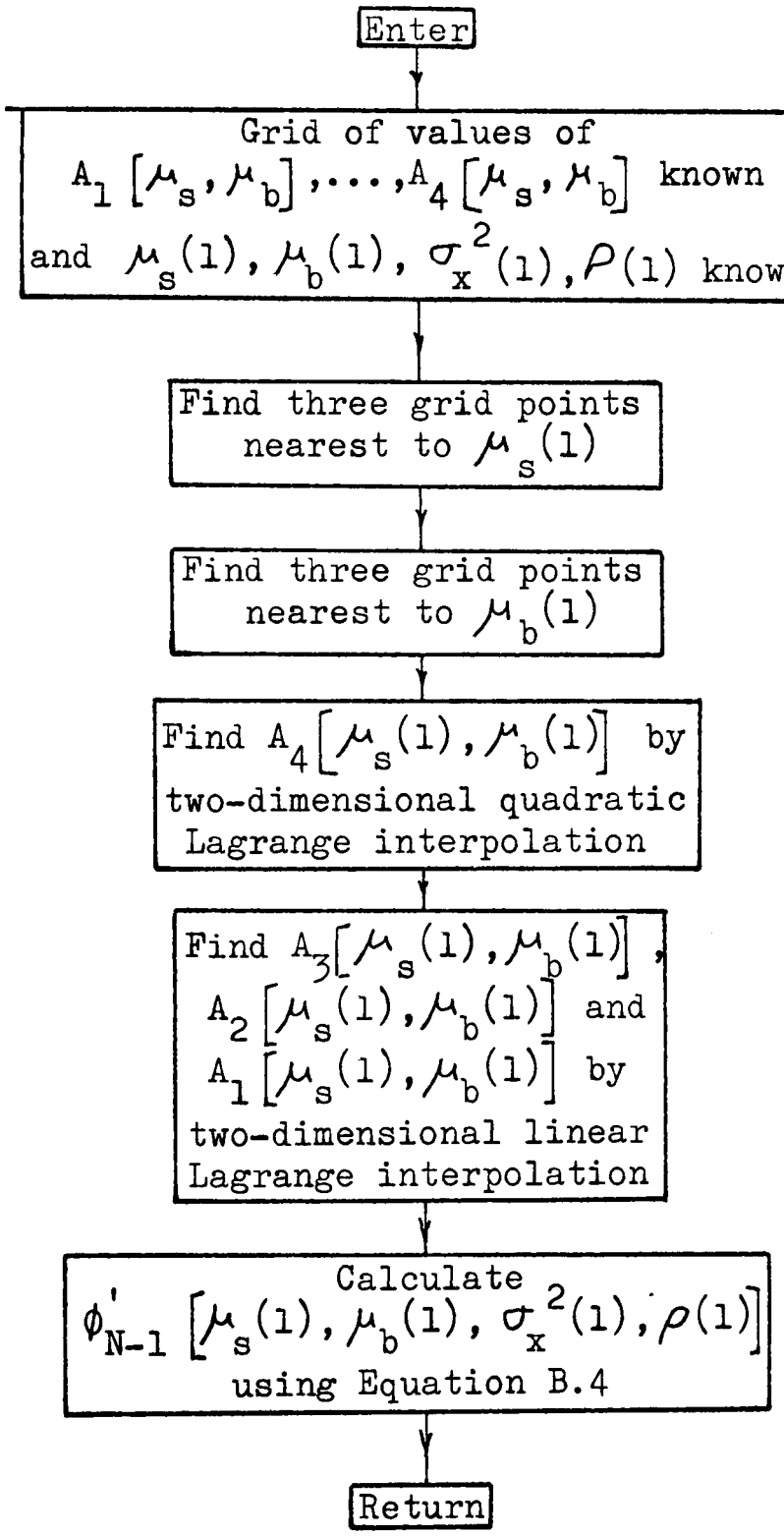
D.2 Error Checking Procedures

For each value of N several checks on the numerical procedure were made to ensure the best possible accuracy for ν_N' and ϕ_N'. These checks are

Table D.1 Unknown Constant Gain Problem Grid Size

N	Number of values of μ_s	Maximum value of μ_s	Number of values of μ_b	Maximum value of μ_b	Grid Size
2	11	49.0	26	41.5	286
3	9	25.0	25	35.6	225
4	8	16.0	25	35.6	200
5	8	16.0	25	35.6	200
6	8	16.0	25	35.6	200

Figure D.5 Interpolation Procedure Flow Diagram



described here.

D.2.1 Integration checking

A typical table of results used for checking the integration method is given in Table D.2. Figure D.6 is derived from this table and shows for which values of μ_s, μ_b Simpson's rule was used and for which values the Gauss-Hermite method was used. This procedure was used for each value of N.

D.2.2 Minimization procedure checking

For each value of N the minimization procedure was checked by computing the expected cost ($E\{\Psi_N\}$) as a function of the control ν' for several values of μ_s, μ_b, σ_x and ρ and observing whether the computed minimum was in fact the actual minimum. Some of these results are shown in Figures D.7 and D.8. No false minima were observed except for the case when $\mu_b = 0.0$ when there are two identical minima, one for positive and one for negative ν'. The minima become shallower as N increases which makes it more difficult to determine the minimizing ν' accurately (because of other possible integration and interpolating errors) but accuracy is of course less important for shallow minima.

D.2.3 Interpolation and extrapolation

Checks on the interpolation procedure and on the effects of extrapolation were made for all values of N but to clarify the discussion the procedures used after the N = 2 grid had been computed and before the N = 3 grid was computed are described.

Three checks are necessary. It must be established that the interpolation procedure gives an accurate representation of ϕ_2' for values of μ_s, μ_b, σ_x and ρ that are within the grid range but are not grid points. Next the extent to which extrapolation beyond the grid range of μ_s and μ_b, is accurate must be established and thirdly the amount of extrapolation beyond the ϕ_2' grid limits that is used in determining the ϕ_3' grid for a given ϕ_3'

Table D.2 Numerical Integration Checking

ϕ_2' is determined using

1) the Gauss-Hermite method for n = 20 — G-H_{20}

2) the Gauss-Hermite method for n = 6 — G-H_6

3) Simpson's rule: — Simpson 0.1, 1.0, 0.001

step-size constant = 0.1

maximum step size = 1.0

minimum integrand cutoff point = 0.001

4) Simpson's rule: — Simpson 0.2, 2.0, 0.002

step-size constant = 0.2

maximum step size = 1.0

minimum integrand cutoff point = 0.001

In theory, 3) is at least 64 times as accurate as 4) (see Section B.1.1).

μ_s	μ_b	σ_x	ρ	G-H_{20}	G-H_6	Simpson 0.1, 1.0, 0.001	Simpson 0.2, 2.0, 0.002
0.5	0.8	0.0	0.0	0.3039	0.3039		
0.5	0.8	1.0	0.5	1.6924	1.6924		
0.5	39.2	0.0	0.0	0.0003	0.0003		
0.5	39.2	1.0	0.5	1.1789	1.1789		
1.5	0.8	0.0	0.0	2.5627	2.5586	2.5633	2.5234
1.5	0.8	1.0	0.5	4.0229	4.0085	4.0232	4.0064
1.5	28.8	0.0	0.0	0.0054	0.0054		
1.5	28.8	1.0	0.5	1.1825	1.1825		
9.0	0.2	0.0	0.0	104.65	95.25	103.84	103.69
9.0	0.2	1.0	0.5	111.43	103.48	110.17	110.15

Table D.2 (cont.)

μ_s	μ_b	σ_x	ρ	G-H_{20}	G-H_6	Simpson 0.1, 1.0, 0.001	Simpson 0.2, 2.0, 0.002
9.0	33.8	0.0	0.0	0.1370	0.1370		
9.0	33.8	1.0	0.5	1.2720	1.2720		
16.0	0.8	0.0	0.0	170.30	186.37	182.48	182.36
16.0	0.8	1.0	0.5	178.93	196.05	190.06	189.91
25.0	0.0	0.0	0.0	725.43	696.59	773.88	773.61
25.0	0.0	1.0	0.5	756.11	720.09	803.14	802.77
36.0	3.2	0.0	0.0	119.37	118.52	117.98	117.93
36.0	3.2	1.0	0.5	120.38	120.20	119.21	119.18
36.0	28.8	0.0	0.0	2.1723	2.1723		
36.0	28.8	1.0	0.5	3.0111	3.0111		
49.0	0.2	0.0	0.0	2461.1	2363.6	2527.2	2528.4
49.0	0.2	1.0	0.5	2530.0	2396.5	2582.7	2582.4

Figure D.6 Choice of Integration Method

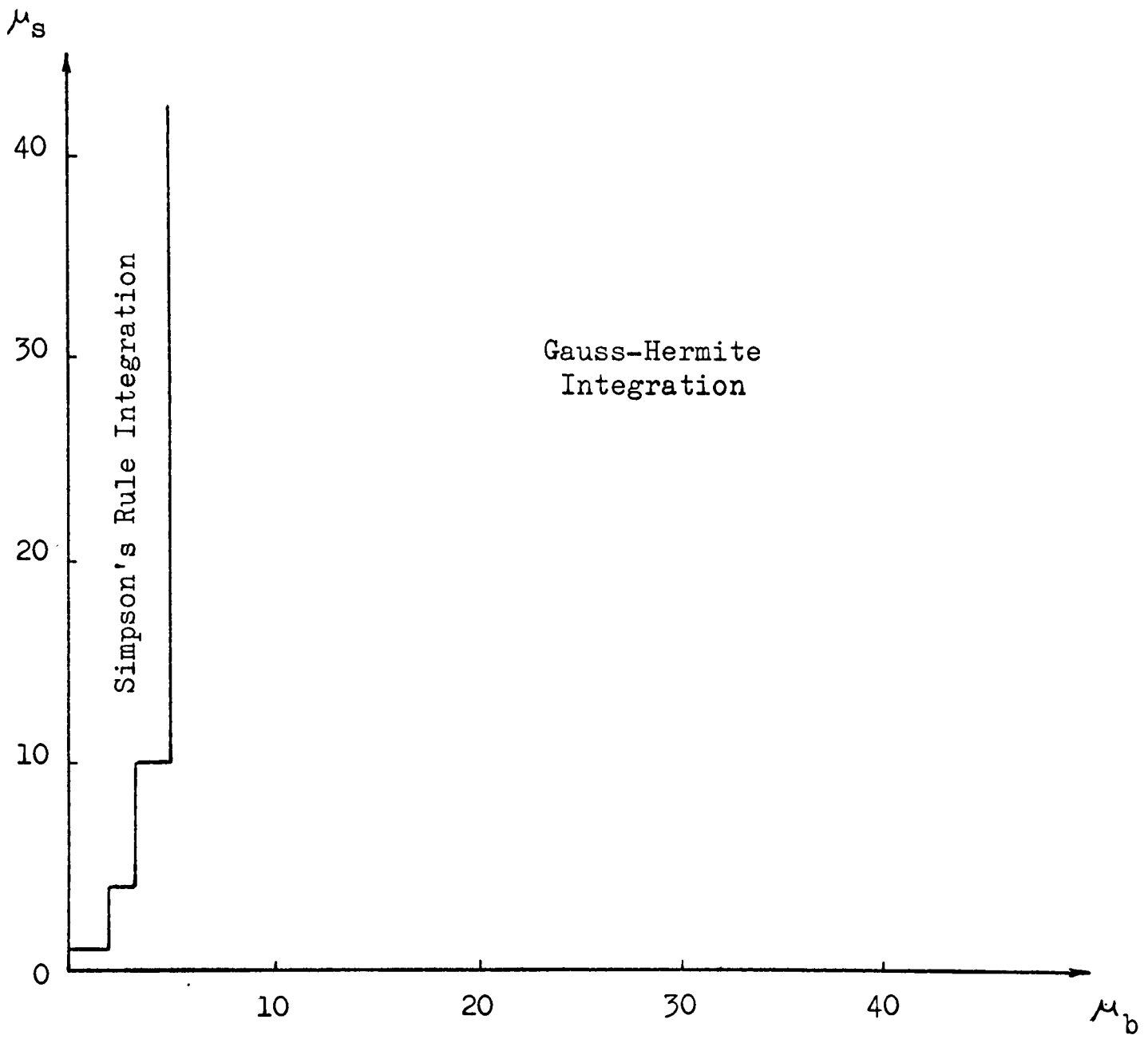


Figure D.7 Minimizing Procedure Check

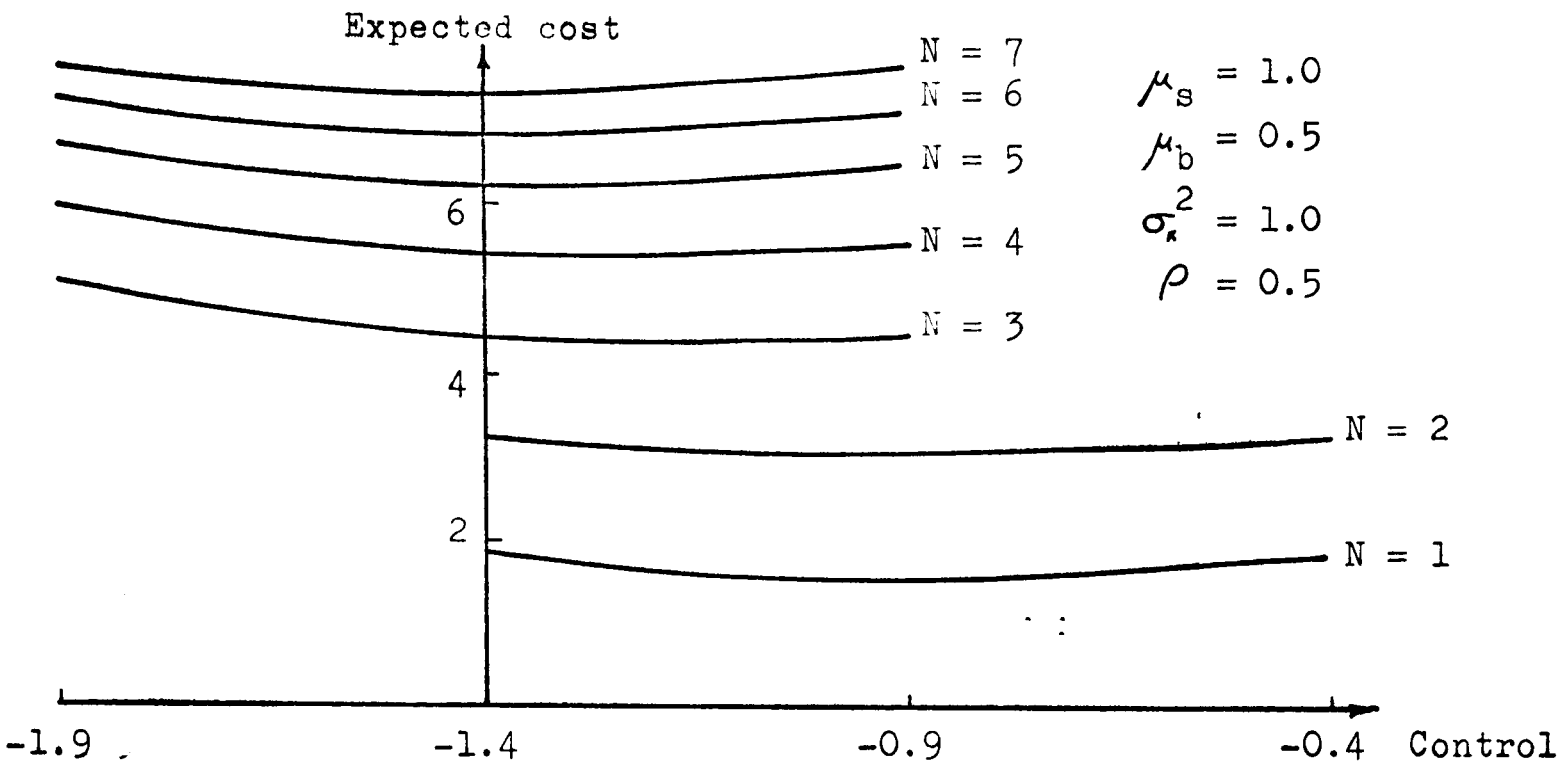
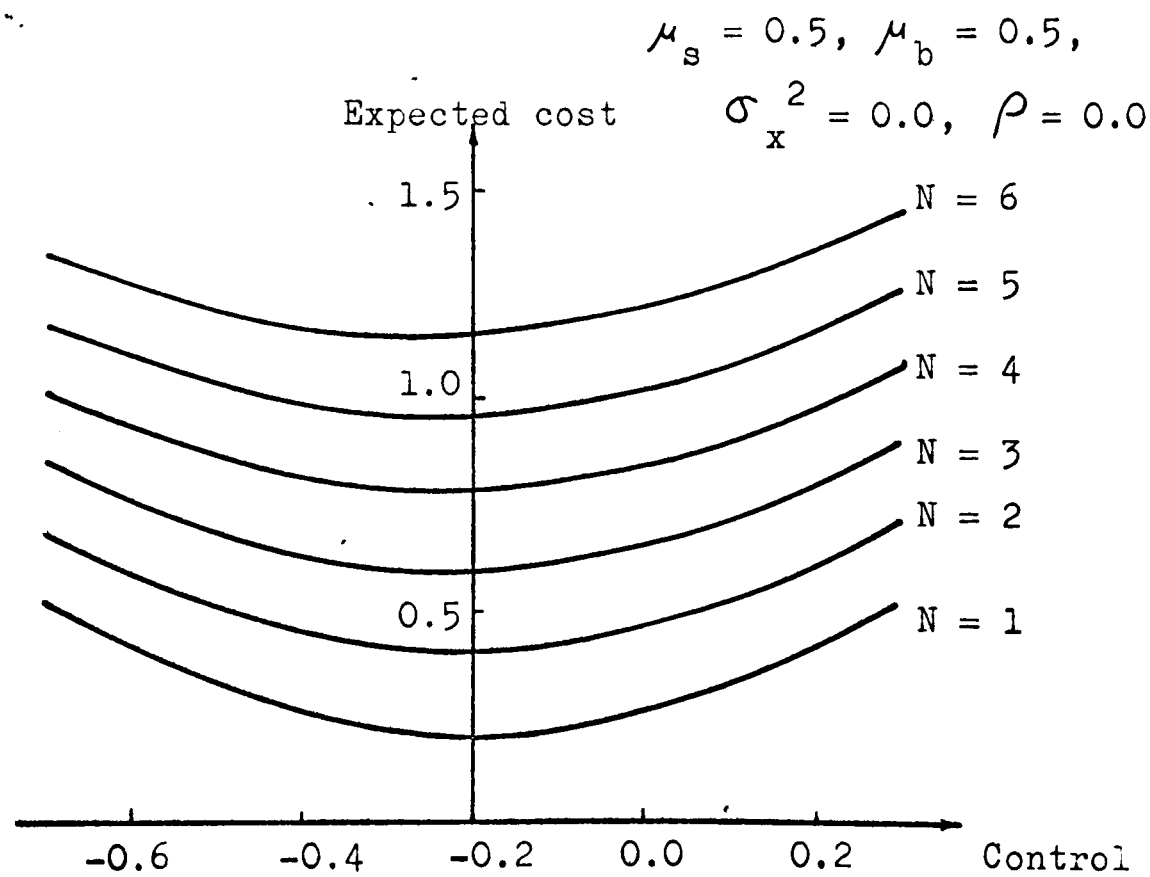


Figure D.8 Minimizing Procedure Check



grid size must be determined.

Results of the interpolation and extrapolation tests for the ϕ_2' grid are given in Table D.3. These results were obtained by computing ϕ_2' at the points shown using both the original integration-minimization procedure and the ϕ_2' grid interpolation procedure and comparing the results. From this table it was determined that extrapolation was sufficiently accurate if $\mu_s \leq 55.0$ and $\mu_b \leq 45.0$ and then ϕ_3' was calculated for several values of μ_s, μ_b, σ_x and ρ with a modified program which showed whenever $\mu_s \leq 55.0$ or $\mu_b \leq 45.0$ was required from the interpolating program. These results are shown in Figure D.9. The ϕ_3' grid size was determined on the basis of Figure D.9.

D.2.4 Overall accuracy

Overall accuracy of the grids of values of ϕ_N' and $\nu^{*'}_N$ have not been considered in detail from a numerical analysis point of view because the results given in Chapter 7 give sufficient confirmation of accuracy on their own. The simulation reported in Chapter 7 is in fact a Monte Carlo integration of the overall cost for a given policy and thus is a check on both integration and interpolation errors. The minimization errors are small (less than 1%) and the optimal policy as simulated in Chapter 7 was in fact the best policy tried. This confirms the accuracy of the minimization procedure.

Table D.3 Interpolation Procedure Checking

ϕ_2' is calculated using both the original program and the interpolation program for σ_x = 0.25, 0.75 and 1.0 for each of ρ = 0.0, 0.25, 0.5 and 0.75 (i.e. 3 x 4 = 12 sets of values of σ_x and ρ) for each combination of μ_s and μ_b listed below. The maximum percentage error (the percentage difference between the two values of ϕ_2') for any value of σ_x and ρ is quoted opposite the appropriate value of μ_s and μ_b.

μ_s	μ_b	Max.error	μ_s	μ_b	Max.error
0.7	0.45	1.08%	7.75	45.0	3.8%
0.7	9.2	3.03%	36.0	7.75	0.54%
0.7	45.0	3.37%	36.0	35.598	0.15%
0.7	50.0	3.33%	40.0	0.45	0.16%
1.0	8.487	2.65%	40.0	50.0	16.6%
1.0	41.521	3.26%	55.0	0.45	0.18%
2.0	41.521	3.26%	55.0	9.2	0.5%
3.0	3.7	0.63%	55.0	37.0	0.57%
3.0	15.0	1.99%	55.0	45.0	3.62%
7.75	0.45	0.26%	55.0	50.0	18.7%
7.75	9.2	0.79%	60.0	50.0	19.5%
7.75	37.0	2.2%			

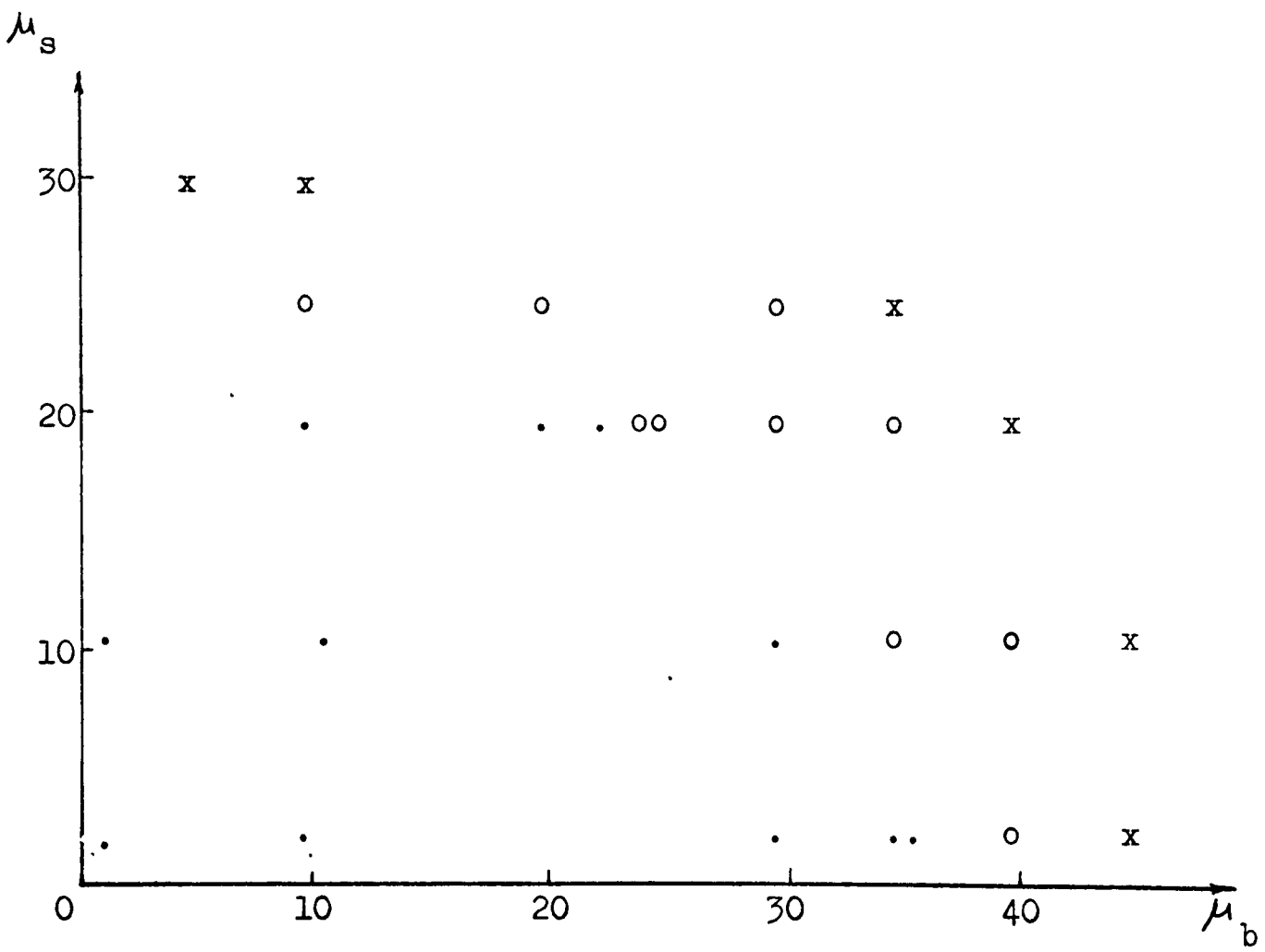
If 4% errors are accepted, it can be seen from this table that extrapolation up to μ_s = 55.0 and μ_b = 45.0 can be allowed.

Figure D.9 Determination of Grid Size for N = 3

Key

- . ϕ_2' grid limits not exceeded
- o ϕ_2' grid limits seldom exceeded and not when $\nu = \nu_3^{*\prime}$
- x ϕ_2' grid limits often exceeded

ϕ_2' grid limits are from Table D.3, $\mu_s = 55.0$, $\mu_b = 45.0$



On the basis of this figure the ϕ_3' grid size was chosen to be $\mu_s \leq 25.0$ and $\mu_b \leq 35.598$

Appendix E

Table of the Optimal Control Policy and Cost Function for the Unknown Constant Gain Problem

This appendix contains the results of the numerical calculation of the optimal control policy and minimum expected cost function. The results are for the symmetrical non-dimensional problem (see Appendix C) and cover the range listed in Table D.1. The symbols used represent:

Symbol	Quantity
MUS	μ_s
MUB	μ_b
SIGX	σ_x
RO	ρ
V2*',...,V6*'	$\nu_2^{*'},\ldots,\nu_6^{*'}$
PH2*',...,PH6*'	$\phi_2',\ldots,\phi_6'$

Computer programs for interpolating and redimensionalizing the results to give the optimal control policy and minimum expected cost are described in Section 7.2 and Section D.1.3.

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.499		1.999		1.499

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.498		1.987		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.494		1.945		1.464

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.569	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.487		1.877		1.421

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.922	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.479		1.798		1.370

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.472		1.722		1.321

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.465		1.659		1.281

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.461		1.612		1.250

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.457		1.578		1.228

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.455		1.554		1.213

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.453		1.538		1.203

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.452		1.526		1.195

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.451		1.518		1.190

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.451		1.513		1.187

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.509		1.184

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.506		1.182

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.504		1.181

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.503		1.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.502		1.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.501		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.501		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.500		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.500		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.500		1.178

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.500		1.178

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.000		0.450		1.500		1.178

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		0.499		0.999		2.499		1.999

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.094		-0.078		-0.094		-0.578
	PH2*'		0.485		0.986		2.476		1.981

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.171		-0.155		-0.155		-0.655
	PH2*'		0.441		0.943		2.401		1.919

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.230		-0.214		-0.183		-0.699
	PH2*'		0.375		0.871		2.274		1.815

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.245		-0.245		-0.229		-0.729
	PH2*'		0.297		0.782		2.111		1.680

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.248		-0.248		-0.248		-0.748
	PH2*'		0.222		0.693		1.945		1.543

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.233		-0.233		-0.233		-0.733
	PH2*'		0.159		0.618		1.805		1.428

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.208		-0.208		-0.208		-0.708
	PH2*'		0.111		0.564		1.705		1.346

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.181		-0.181		-0.181		-0.681
	PH2*'		0.077		0.527		1.638		1.290

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.155		-0.155		-0.155		-0.655
	PH2*'		0.054		0.502		1.593		1.254

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.132		-0.132		-0.132		-0.632
	PH2*'		0.037		0.486		1.564		1.230

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.112		-0.112		-0.112		-0.612
	PH2*'		0.026		0.475		1.544		1.214

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.095		-0.095		-0.095		-0.595
	PH2*'		0.018		0.467		1.531		1.203

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.080		-0.080		-0.080		-0.580
	PH2*'		0.013		0.462		1.522		1.196

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.068		-0.068		-0.068		-0.568
	PH2*'		0.009		0.458		1.515		1.191

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.058		-0.058		-0.058		-0.558
	PH2*'		0.006		0.456		1.511		1.187

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.049		-0.049		-0.049		-0.549
	PH2*'		0.004		0.454		1.508		1.185

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.042		-0.042		-0.042		-0.542
	PH2*'		0.003		0.453		1.505		1.183

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.035		-0.035		-0.035		-0.535
	PH2*'		0.002		0.452		1.504		1.181

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.030		-0.030		-0.030		-0.530
	PH2*'		0.001		0.451		1.503		1.181

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.026		-0.026		-0.026		-0.526
	PH2*'		0.001		0.451		1.502		1.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.022		-0.022		-0.022		-0.522
	PH2*'		0.000		0.450		1.501		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.019		-0.019		-0.019		-0.519
	PH2*'		0.000		0.450		1.501		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.016		-0.016		-0.016		-0.516
	PH2*'		0.000		0.450		1.500		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	35.599	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.014		-0.014		-0.014		-0.514
	PH2*'		0.000		0.450		1.500		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.012		-0.012		-0.012		-0.512
	PH2*'		0.000		0.450		1.500		1.178

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		0.000		0.000		0.000		-0.500
	PH2*'		1.994		2.499		4.001		3.499

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.345		-0.329		-0.360		-0.845
	PH2*'		1.900		2.418		3.921		3.423

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.467		-0.467		-0.483		-0.967
	PH2*'		1.712		2.230		3.717		3.231

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.523		-0.523		-0.523		-1.023
	PH2*'		1.451		1.965		3.409		2.940

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.537		-0.537		-0.521		-1.021
	PH2*'		1.166		1.661		3.022		2.580

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.512		-0.512		-0.496		-1.012
	PH2*'		0.879		1.346		2.606		2.201

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.466		-0.481		-0.481		-0.981
	PH2*'		0.629		1.073		2.244		1.869

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.417		-0.432		-0.432		-0.932
	PH2*'		0.438		0.869		1.983		1.629

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.363		-0.379		-0.379		-0.879
	PH2*'		0.304		0.732		1.815		1.474

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.311		-0.311		-0.327		-0.811
	PH2*'		0.212		0.642		1.710		1.376

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.265		-0.265		-0.265		-0.765
	PH2*'		0.148		0.582		1.642		1.312

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.224		-0.224		-0.224		-0.724
	PH2*'		0.104		0.541		1.597		1.270

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.190		-0.190		-0.190		-0.690
	PH2*'		0.074		0.514		1.568		1.242

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.161		-0.161		-0.161		-0.661
	PH2*'		0.052		0.495		1.547		1.223

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.136		-0.136		-0.136		-0.636
	PH2*'		0.037		0.482		1.533		1.210

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	8.497	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.116		-0.116		-0.116		-0.616
	PH2*'		0.027		0.473		1.524		1.201

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.098		-0.098		-0.098		-0.598
	PH2*'		0.019		0.466		1.517		1.194

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.084		-0.084		-0.084		-0.584
	PH2*'		0.014		0.462		1.512		1.190

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.071		-0.071		-0.071		-0.571
	PH2*'		0.010		0.458		1.509		1.187

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.061		-0.061		-0.061		-0.561
	PH2*'		0.007		0.456		1.506		1.184

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.052		-0.052		-0.052		-0.552
	PH2*'		0.005		0.454		1.504		1.183

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.044		-0.044		-0.044		-0.544
	PH2*'		0.003		0.453		1.503		1.181

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.038		-0.038		-0.038		-0.538
	PH2*'		0.002		0.452		1.502		1.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.032		-0.032		-0.032		-0.532
	PH2*'		0.002		0.451		1.501		1.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.028		-0.028		-0.028		-0.528
	PH2*'		0.001		0.451		1.501		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.024		-0.024		-0.024		-0.524
	PH2*'		0.001		0.450		1.501		1.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.468		-0.437		-0.421		-0.921
	PH2*'		4.389		4.932		6.452		5.952

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.658		-0.658		-0.658		-1.142
	PH2*'		4.098		4.654		6.210		5.697

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.795		-0.795		-0.826		-1.311
	PH2*'		3.650		4.205		5.751		5.252

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.863		-0.878		-0.894		-1.394
	PH2*'		3.110		3.647		5.138		4.648

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.845		-0.876		-0.876		-1.376
	PH2*'		2.512		3.013		4.422		3.961

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.807		-0.807		-0.792		-1.307
	PH2*'		1.921		2.382		3.661		3.248

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.730		-0.730		-0.715		-1.230
	PH2*'		1.385		1.812		2.967		2.595

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.641		-0.641		-0.641		-1.141
	PH2*'		0.961		1.365		2.444		2.098

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.560		-0.560		-0.560		-1.060
	PH2*'		0.663		1.062		2.107		1.774

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.467		-0.483		-0.483		-0.983
	PH2*'		0.462		0.869		1.900		1.573

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.397		-0.397		-0.397		-0.897
	PH2*'		0.325		0.737		1.771		1.446

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.337		-0.337		-0.337		-0.837
	PH2*'		0.230		0.650		1.685		1.363

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.285		-0.285		-0.285		-0.785
	PH2*'		0.164		0.591		1.629		1.307

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	6.029	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.242		-0.242		-0.242		-0.742
	PH2*'		0.117		0.550		1.590		1.268

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.205		-0.205		-0.205		-0.705
	PH2*'		0.084		0.521		1.564		1.242

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.174		-0.174		-0.174		-0.674
	PH2*'		0.060		0.501		1.545		1.224

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.148		-0.148		-0.148		-0.648
	PH2*'		0.043		0.486		1.532		1.211

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.126		-0.126		-0.126		-0.626
	PH2*'		0.031		0.476		1.523		1.202

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.107		-0.107		-0.107		-0.607
	PH2*'		0.023		0.469		1.517		1.195

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	16.297	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.091		-0.091		-0.091		-0.591
	PH2*'		0.016		0.464		1.512		1.191

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.078		-0.078		-0.078		-0.578
	PH2*'		0.012		0.460		1.509		1.187

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.067		-0.067		-0.067		-0.567
	PH2*'		0.008		0.457		1.506		1.185

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.057		-0.057		-0.057		-0.557
	PH2*'		0.006		0.455		1.504		1.183

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.049		-0.049		-0.049		-0.549
	PH2*'		0.004		0.454		1.503		1.182

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.042		-0.042		-0.042		-0.542
	PH2*'		0.003		0.452		1.502		1.181

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.036		-0.036		-0.036		-0.536
	PH2*'		0.002		0.452		1.501		1.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.750		-0.718		-0.671		-1.187
	PH2*'		7.518		8.139		9.755		9.236

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.940		-0.940		-0.940		-1.424
	PH2*'		6.966		7.579		9.239		8.700

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.092		-1.092		-1.123		-1.623
	PH2*'		6.180		6.780		8.420		7.894

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.171		-1.202		-1.218		-1.718
	PH2*'		5.247		5.810		7.377		6.872

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.199		-1.215		-1.215		-1.715
	PH2*'		4.241		4.767		6.224		5.749

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.103		-1.118		-1.118		-1.618
	PH2*'		3.266		3.734		5.044		4.621

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.995		-0.995		-0.979		-1.495
	PH2*'		2.376		2.796		3.947		3.574

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.865		-0.865		-0.865		-1.365
	PH2*'		1.655		2.035		3.081		2.742

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.742		-0.742		-0.758		-1.258
	PH2*'		1.136		1.506		2.511		2.187

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.639		-0.639		-0.639		-1.139
	PH2*'		0.790		1.166		2.163		1.844

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.530		-0.546		-0.546		-1.046
	PH2*'		0.558		0.946		1.947		1.630

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.449		-0.449		-0.449		-0.949
	PH2*'		0.398		0.797		1.807		1.490

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.381		-0.381		-0.381		-0.881
	PH2*'		0.285		0.695		1.713		1.395

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.322		-0.322		-0.322		-0.822
	PH2*'		0.205		0.624		1.650		1.331

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.273		-0.273		-0.273		-0.773
	PH2*'		0.147		0.575		1.606		1.287

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.232		-0.232		-0.232		-0.732
	PH2*'		0.106		0.539		1.576		1.256

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.197		-0.197		-0.197		-0.697
	PH2*'		0.077		0.514		1.554		1.234

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.168		-0.168		-0.168		-0.668
	PH2*'		0.056		0.496		1.539		1.218

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.143		-0.143		-0.143		-0.643
	PH2*'		0.040		0.484		1.528		1.207

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.122		-0.122		-0.122		-0.622
	PH2*'		0.029		0.474		1.520		1.199

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	19.095	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.104		-0.104		-0.104		-0.604
	PH2*'		0.021		0.468		1.515		1.194

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.089		-0.089		-0.089		-0.589
	PH2*'		0.015		0.463		1.511		1.189

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.076		-0.076		-0.076		-0.576
	PH2*'		0.011		0.459		1.508		1.186

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.065		-0.065		-0.065		-0.565
	PH2*'		0.008		0.457		1.505		1.184

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.056		-0.056		-0.056		-0.556
	PH2*'		0.006		0.455		1.504		1.183

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.048		-0.048		-0.048		-0.548
	PH2*'		0.004		0.453		1.503		1.181

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.578		-1.562		-1.593		-2.093
	PH2*'		26.656		27.699		30.259		29.472

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.880		-1.896		-1.943		-2.427
	PH2*'		24.419		25.406		27.884		27.118

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.163		-2.184		-2.231		-2.715	
	PH2*'	21.498		22.407		24.717		23.996	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.358		-2.374		-2.421		-2.890	
	PH2*'	18.076		18.884		20.983		20.314	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.399		-2.399		-2.430		-2.930	
	PH2*'	14.516		15.208		17.051		16.465	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.269		-2.269		-2.284		-2.784	
	PH2*'	11.134		11.720		13.292		12.784	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.036		-2.036		-2.036		-2.536	
	PH2*'	8.166		8.638		9.925		9.506	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.763		-1.763		-1.763		-2.263	
	PH2*'	5.733		6.099		7.138		6.796	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.516		-1.500		-1.500		-2.000	
	PH2*'	3.904		4.217		5.126		4.826	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.278		-1.278		-1.278		-1.778	
	PH2*'	2.685		2.985		3.860		3.571	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.076		-1.076		-1.076		-1.576	
	PH2*'	1.901		2.209		3.092		2.802	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-0.915		-0.915		-0.915		-1.415	
	PH2*'	1.376		1.699		2.601		2.308	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-0.762		-0.762		-0.762		-1.262	
	PH2*'	1.008		1.348		2.273		1.976	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.645		-0.645		-0.645		-1.145
	PH2*'		0.740		1.099		2.047		1.745

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.547		-0.547		-0.547		-1.047
	PH2*'		0.544		0.921		1.890		1.584

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.464		-0.464		-0.464		-0.964
	PH2*'		0.400		0.792		1.779		1.470

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.395		-0.395		-0.395		-0.895
	PH2*'		0.294		0.699		1.701		1.388

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	11.907	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.336		-0.336		-0.336		-0.836
	PH2*'		0.216		0.632		1.645		1.330

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.286		-0.286		-0.286		-0.786
	PH2*'		0.159		0.582		1.605		1.289

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	16.297	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.244		-0.244		-0.244		-0.744
	PH2*'		0.116		0.547		1.576		1.259

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.209		-0.209		-0.209		-0.709
	PH2*'		0.085		0.521		1.556		1.237

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.178		-0.178		-0.178		-0.678
	PH2*'		0.063		0.502		1.540		1.221

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.152		-0.152		-0.152		-0.652
	PH2*'		0.046		0.488		1.530		1.210

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.131		-0.131		-0.131		-0.631
	PH2*'		0.034		0.478		1.522		1.201

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-0.112		-0.112		-0.112		-0.612	
	PH2*'	0.025		0.470		1.516		1.195	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-0.096		-0.096		-0.096		-0.596	
	PH2*'	0.018		0.465		1.511		1.191	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.156		-3.218		-3.328		-3.828	
	PH2*'	116.346		119.105		125.584		123.646	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.716		-3.809		-3.981		-4.419	
	PH2*'	106.391		108.827		114.659		112.904	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.383		-4.461		-4.602		-5.055	
	PH2*'	93.400		95.503		100.585		99.039	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.897		-4.959		-5.069		-5.538	
	PH2*'	77.972		79.735		84.083		82.754	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-5.086		-5.118		-5.196		-5.680	
	PH2*'	61.807		63.261		66.869		65.757	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.894		-4.925		-4.972		-5.456	
	PH2*'	46.716		47.862		50.764		49.861	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.461		-4.477		-4.493		-4.993	
	PH2*'	33.830		34.683		36.877		36.188	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.912		-3.912		-3.928		-4.428	
	PH2*'	23.540		24.141		25.714		25.212	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.349		-3.365		-3.365		-3.865	
	PH2*'	15.927		16.332		17.461		17.101	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.852		-2.852		-2.852		-3.352
	PH2*'		10.766		11.082		11.982		11.687

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.419		-2.419		-2.419		-2.919
	PH2*'		7.497		7.770		8.586		8.315

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.040		-2.040		-2.040		-2.540
	PH2*'		5.368		5.636		6.436		6.169

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.730		-1.730		-1.730		-2.230
	PH2*'		3.930		4.199		5.000		4.734

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.468		-1.468		-1.468		-1.968
	PH2*'		2.920		3.196		4.010		3.741

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.231		-1.231		-1.231		-1.731
	PH2*'		2.191		2.478		3.311		3.037

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.045		-1.045		-1.045		-1.545
	PH2*'		1.654		1.956		2.814		2.534

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.889		-0.889		-0.889		-1.389
	PH2*'		1.252		1.571		2.456		2.170

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	11.907	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.756		-0.756		-0.756		-1.256
	PH2*'		0.947		1.285		2.198		1.905

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.645		-0.645		-0.645		-1.145
	PH2*'		0.715		1.071		2.010		1.711

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.550		-0.550		-0.550		-1.050
	PH2*'		0.538		0.912		1.873		1.570

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.470		-0.470		-0.470		-0.970
	PH2*'		0.403		0.793		1.773		1.466

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.402		-0.402		-0.402		-0.902
	PH2*'		0.301		0.704		1.700		1.390

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.344		-0.344		-0.344		-0.844
	PH2*'		0.224		0.638		1.647		1.334

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.294		-0.294		-0.294		-0.794
	PH2*'		0.167		0.589		1.608		1.293

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.252		-0.252		-0.252		-0.752
	PH2*'		0.123		0.552		1.579		1.262

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.216		-0.216		-0.216		-0.716
	PH2*'		0.091		0.525		1.558		1.240

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-4.890		-5.031		-5.328		-5.750
	PH2*'		335.966		341.973		355.968		351.787

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-5.913		-6.070		-6.351		-6.773
	PH2*'		308.300		313.394		325.501		321.886

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-7.143		-7.252		-7.518		-7.909
	PH2*'		271.338		275.547		285.705		282.625

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-8.232		-8.310		-8.482		-8.919
	PH2*'		226.073		229.619		238.142		235.553

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-8.677		-8.739		-8.864		-9.286
	PH2*'		178.413		181.208		188.048		185.956

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-8.451		-8.482		-8.576		-9.060	
	PH2*'	133.895		136.040		141.348		139.720	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-7.772		-7.803		-7.850		-8.335	
	PH2*'	96.195		97.736		101.638		100.437	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-6.834		-6.849		-6.881		-7.365	
	PH2*'	66.914		67.936		70.546		69.731	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-5.893		-5.909		-5.909		-6.409	
	PH2*'	45.345		45.970		47.617		47.090	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-5.035		-5.035		-5.035		-5.535	
	PH2*'	30.683		31.069		32.165		31.808	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.276		-4.276		-4.276		-4.776	
	PH2*'	21.158		21.459		22.343		22.052	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.615		-3.615		-3.615		-4.115	
	PH2*'	14.953		15.224		16.033		15.764	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.064		-3.064		-3.064		-3.564	
	PH2*'	10.755		11.017		11.800		11.539	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.598		-2.598		-2.598		-3.098	
	PH2*'	7.843		8.101		8.877		8.618	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.190		-2.190		-2.190		-2.690	
	PH2*'	5.701		6.051		6.826		6.568	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.859		-1.859		-1.859		-2.359	
	PH2*'	4.326		4.589		5.373		5.112	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.580		-1.580		-1.580		-2.080
	PH2*'		3.265		3.535		4.332		4.068

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	11.907	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.345		-1.345		-1.345		-1.845
	PH2*'		2.484		2.763		3.579		3.310

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.146		-1.146		-1.146		-1.646
	PH2*'		1.899		2.192		3.031		2.756

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.978		-0.978		-0.978		-1.478
	PH2*'		1.456		1.764		2.630		2.348

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.836		-0.836		-0.836		-1.336
	PH2*'		1.115		1.441		2.334		2.046

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.714		-0.714		-0.714		-1.214
	PH2*'		0.851		1.196		2.116		1.822

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.611		-0.611		-0.611		-1.111
	PH2*'		0.648		1.010		1.955		1.655

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.524		-0.524		-0.524		-1.024
	PH2*'		0.490		0.869		1.836		1.531

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.449		-0.449		-0.449		-0.949
	PH2*'		0.369		0.763		1.748		1.440

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.385		-0.385		-0.385		-0.885
	PH2*'		0.277		0.683		1.683		1.371

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-6.796		-6.984		-7.453		-7.828
	PH2*'		773.874		784.930		810.880		803.138

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-8.411		-8.598		-9.051		-9.426	
	PH2*'	713.233		722.486		744.212		737.670	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-10.464		-10.542		-10.964		-11.355	
	PH2*'	630.087		637.478		655.190		649.852	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-16.848		-12.754		-12.739		-13.192	
	PH2*'	496.561		529.824		545.885		541.282	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-17.123		-13.826		-13.560		-14.076	
	PH2*'	304.810		416.506		429.716		425.940	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-13.361		-13.345		-13.314		-13.830	
	PH2*'	308.002		311.835		321.207		318.375	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-13.734		-13.765		-13.875		-14.344	
	PH2*'	48.338		74.737		123.504		110.447	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-10.590		-10.606		-10.637		-11.137	
	PH2*'	154.748		156.337		160.350		159.110	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-9.165		-9.165		-9.180		-9.680	
	PH2*'	105.429		106.322		108.651		107.909	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-7.825		-7.825		-7.841		-8.341	
	PH2*'	71.672		72.172		73.537		73.097	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-6.648		-6.648		-6.648		-7.148	
	PH2*'	49.475		49.803		50.761		50.446	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-5.640		-5.640		-5.640		-6.140	
	PH2*'	34.774		35.053		35.883		35.607	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.779		-4.779		-4.779		-5.279	
	PH2*'	24.764		25.028		25.818		25.555	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.035		-4.035		-4.035		-4.535	
	PH2*'	17.818		18.076		18.849		18.591	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.421		-3.421		-3.421		-3.921	
	PH2*'	12.950		13.206		13.972		13.716	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.905		-2.905		-2.905		-3.405	
	PH2*'	9.510		9.765		10.530		10.275	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.469		-2.469		-2.469		-2.969	
	PH2*'	7.058		7.314		8.080		7.825	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.102		-2.102		-2.102		-2.602	
	PH2*'	5.292		5.551		6.324		6.066	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.791		-1.791		-1.791		-2.291	
	PH2*'	4.006		4.269		5.053		4.793	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.529		-1.529		-1.529		-2.029	
	PH2*'	3.057		3.328		4.127		3.863	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.306		-1.306		-1.306		-1.806	
	PH2*'	2.347		2.629		3.448		3.178	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-1.117		-1.117		-1.117		-1.617	
	PH2*'	1.808		2.103		2.946		2.670	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	26.110	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-0.955		-0.955		-0.955		-1.455	
	PH2*'	1.394		1.705		2.574		2.292	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.818		-0.818		-0.818		-1.318
	PH2*'		1.073		1.401		2.298		2.009

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.701		-0.701		-0.701		-1.201
	PH2*'		0.823		1.169		2.092		1.797

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.601		-0.601		-0.601		-1.101
	PH2*'		0.628		0.992		1.939		1.639

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-8.796		-9.109		-9.718		-10.046
	PH2*'		1542.161		1560.340		1603.335		1590.450

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-11.161		-11.427		-12.052		-12.349
	PH2*'		1427.458		1442.286		1477.402		1466.856

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-14.393		-14.549		-14.955		-15.315
	PH2*'		1264.901		1277.093		1305.189		1296.764

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-18.823		-20.542		-18.527		-23.464
	PH2*'		906.701		931.690		1085.017		987.462

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-19.050		-20.175		-22.347		-22.253
	PH2*'		497.992		530.182		614.975		539.099

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-19.093		-19.609		-20.656		-20.859
	PH2*'		108.586		166.358		282.408		250.318

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-18.426		-18.722		-19.379		-19.691
	PH2*'		433.320		438.374		451.411		447.385

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-15.212		-15.228		-15.290		-15.759
	PH2*'		310.996		313.223		318.858		317.114

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-13.163		-13.163		-13.179		-13.679	
	PH2*'	213.170		214.395		217.522		216.534	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-11.255		-11.255		-11.270		-11.755	
	PH2*'	145.689		146.300		147.939		147.411	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	3.491	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-9.567		-9.567		-9.567		-10.067	
	PH2*'	100.765		101.115		102.141		101.798	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-8.115		-8.115		-8.115		-8.615	
	PH2*'	70.695		70.980		71.824		71.543	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-6.860		-6.860		-6.860		-7.360	
	PH2*'	50.120		50.385		51.182		50.916	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-5.811		-5.811		-5.811		-6.311	
	PH2*'	35.823		36.082		36.858		36.599	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.927		-4.927		-4.927		-5.427	
	PH2*'	25.813		26.069		26.835		26.580	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-4.183		-4.183		-4.183		-4.683	
	PH2*'	18.755		19.009		19.770		19.517	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.556		-3.556		-3.556		-4.056	
	PH2*'	13.744		13.998		14.757		14.504	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-3.027		-3.027		-3.027		-3.527	
	PH2*'	10.162		10.415		11.175		10.922	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-2.580		-2.580		-2.580		-3.080	
	PH2*'	7.582		7.837		8.600		8.346	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.201		-2.201		-2.201		-2.701
	PH2*'		5.709		5.966		6.736		6.490

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.881		-1.881		-1.881		-2.381
	PH2*'		4.336		4.598		5.377		5.118

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.608		-1.608		-1.608		-2.108
	PH2*'		3.319		3.587		4.380		4.118

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.376		-1.376		-1.376		-1.876
	PH2*'		2.555		2.833		3.645		3.377

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.179		-1.179		-1.179		-1.679
	PH2*'		1.975		2.265		3.099		2.825

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.010		-1.010		-1.010		-1.510
	PH2*'		1.529		1.832		2.692		2.412

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
36.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-0.866		-0.866		-0.866		-1.366
	PH2*'		1.180		1.502		2.388		2.102

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-10.953		-11.197		-12.140		-12.375
	PH2*'		2776.206		2804.132		2869.829		2850.229

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-14.196		-14.539		-15.305		-15.571
	PH2*'		2580.050		2602.117		2654.724		2638.870

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-20.101		-19.273		-19.523		-19.867
	PH2*'		2286.790		2309.823		2352.984		2340.388

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-19.199		-22.048		-27.580		-26.580
	PH2*'		1646.770		1621.939		1677.048		1649.108

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-26.771		-28.099		-25.349		-24.678	
	PH2*'	1467.199		1488.292		962.174		943.513	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-25.944		-26.663		-28.319		-28.319	
	PH2*'	1088.436		1098.998		1131.512		1120.878	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-22.956		-23.049		-23.471		-23.784	
	PH2*'	769.646		770.863		776.343		774.225	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-22.513		-22.544		-22.607		-23.091	
	PH2*'	-585.632		-443.200		-187.475		-255.234	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-17.888		-17.888		-17.920		-18.404	
	PH2*'	389.184		390.679		394.617		393.401	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-15.308		-15.308		-15.308		-15.808	
	PH2*'	267.160		267.879		269.809		269.199	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	3.491	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-13.016		-13.016		-13.016		-13.516	
	PH2*'	185.124		185.503		186.583		186.228	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-11.024		-11.024		-11.040		-11.524	
	PH2*'	129.742		130.030		130.890		130.600	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-9.337		-9.337		-9.337		-9.837	
	PH2*'	91.835		92.101		92.901		92.634	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	6.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-7.910		-7.910		-7.910		-8.410	
	PH2*'	65.429		65.689		66.466		66.207	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'	-6.707		-6.707		-6.707		-7.207	
	PH2*'	46.941		47.197		47.964		47.708	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-5.694		-5.694		-5.694		-6.194
	PH2*'		33.911		34.165		34.926		34.672

.MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-4.840		-4.840		-4.840		-5.340
	PH2*'		24.670		24.923		25.681		25.428

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-4.120		-4.120		-4.120		-4.620
	PH2*'		18.077		18.330		19.086		18.834

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-3.512		-3.512		-3.512		-4.012
	PH2*'		13.346		13.599		14.355		14.103

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.997		-2.997		-2.997		-3.497
	PH2*'		9.932		10.185		10.943		10.690

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.560		-2.560		-2.560		-3.060
	PH2*'		7.451		7.706		8.468		8.214

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-2.189		-2.189		-2.189		-2.689
	PH2*'		5.637		5.894		6.663		6.407

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.873		-1.873		-1.873		-2.373
	PH2*'		4.299		4.560		5.339		5.081

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.604		-1.604		-1.604		-2.104
	PH2*'		3.301		3.569		4.362		4.100

MUS	MUP	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.375		-1.375		-1.375		-1.875
	PH2*'		2.549		2.826		3.638		3.370

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
49.000	41.521	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V2*'		-1.179		-1.179		-1.179		-1.679
	PH2*'		1.974		2.264		3.098		2.824

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.203		-0.406
	PH3*'		0.000		0.750		2.973		2.248

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.031		-0.468
	PH3*'		0.000		0.750		2.953		2.231

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.015		-0.484
	PH3*'		0.000		0.741		2.853		2.167

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.015		-0.484
	PH3*'		0.000		0.723		2.707		2.069

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.698		2.530		1.948

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.674		2.349		1.824

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.655		2.191		1.718

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.642		2.075		1.641

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.633		1.998		1.590

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.627		1.949		1.557

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.622		1.917		1.536

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.620		1.896		1.521

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.618		1.882		1.512

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.616		1.872		1.505

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.616		1.865		1.501

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.615		1.861		1.498

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.615		1.858		1.496

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.614		1.855		1.494

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.614		1.854		1.493

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.614		1.853		1.492

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.614		1.852		1.492

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'	0.000		0.000		0.000		-0.500
		PH3*'	0.000		0.614		1.851		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.614		1.851		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.614		1.850		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.000		0.000		0.000		-0.500
	PH3*'		0.000		0.614		1.850		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		0.015		0.031		0.265		-0.296
	PH3*'		0.749		1.504		3.682		2.974

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.141		-0.094		0.139		-0.406
	PH3*'		0.723		1.485		3.648		2.943

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.202		-0.187		-0.046		-0.562
	PH3*'		0.656		1.415		3.510		2.832

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.230		-0.214		-0.152		-0.667
	PH3*'		0.558		1.300		3.274		2.641

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.245		-0.245		-0.214		-0.714
	PH3*'		0.444		1.156		2.976		2.399

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.248		-0.248		-0.248		-0.748
	PH3*'		0.332		1.008		2.669		2.149

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.233		-0.233		-0.233		-0.733
	PH3*'		0.238		0.883		2.401		1.932

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.208		-0.208		-0.224		-0.724
	PH3*'		0.166		0.791		2.203		1.773

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.181		-0.181		-0.197		-0.681
	PH3*'		0.116		0.731		2.074		1.671

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.155		-0.155		-0.155		-0.655
	PH3*'		0.080		0.692		1.996		1.609

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.132		-0.132		-0.132		-0.632
	PH3*'		0.056		0.667		1.947		1.570

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.112		-0.112		-0.112		-0.612
	PH3*'		0.039		0.650		1.916		1.544

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.095		-0.095		-0.095		-0.595
	PH3*'		0.028		0.639		1.896		1.528

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.080		-0.080		-0.080		-0.580
	PH3*'		0.019		0.632		1.882		1.517

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.068		-0.068		-0.068		-0.568
	PH3*'		0.014		0.626		1.872		1.509

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.058		-0.058		-0.058		-0.558
	PH3*'		0.010		0.623		1.866		1.504

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.049		-0.049		-0.049		-0.549
	PH3*'		0.007		0.620		1.861		1.500

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.042		-0.042		-0.042		-0.542
	PH3*'		0.005		0.618		1.858		1.497

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.035		-0.035		-0.035		-0.535
	PH3*'		0.003		0.617		1.856		1.495

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.030		-0.030		-0.030		-0.530
	PH3*'		0.002		0.616		1.854		1.494

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.026		-0.026		-0.026		-0.526
	PH3*'		0.002		0.615		1.853		1.493

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.022		-0.022		-0.022		-0.522
	PH3*'		0.001		0.615		1.852		1.492

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.019		-0.019		-0.019		-0.519
	PH3*'		0.001		0.614		1.851		1.492

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.016		-0.016		-0.016		-0.516
	PH3*'		0.000		0.614		1.851		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.014		-0.014		-0.014		-0.514
	PH3*'		0.000		0.614		1.850		1.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.406		-0.390		-0.468		-0.937
	PH3*'		2.872		3.652		5.720		5.051

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.548		-0.548		-0.626		-1.095
	PH3*'		2.705		3.487		5.578		4.904

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.608		-0.639		-0.717		-1.186
	PH3*'		2.428		3.224		5.295		4.628

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.632		-0.632		-0.695		-1.179
	PH3*'		2.089		2.857		4.864		4.216

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.568		-0.584		-0.521		-1.053
	PH3*'		1.696		2.422		4.261		3.682

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.528		-0.528		-0.496		-1.028
	PH3*'		1.293		1.954		3.589		2.531

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.481		-0.481		-0.481		-0.981
	PH3*'		0.927		1.536		3.000		2.543

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.432		-0.432		-0.432		-0.948
	PH3*'		0.644		1.219		2.569		2.151

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.363		-0.379		-0.379		-0.879
	PH3*'		0.445		1.010		2.294		1.901

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.311		-0.327		-0.327		-0.827
	PH3*'		0.310		0.879		2.132		1.753

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.265		-0.265		-0.265		-0.765
	PH3*'		0.218		0.795		2.036		1.665

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.224		-0.224		-0.224		-0.724
	PH3*'		0.154		0.740		1.976		1.609

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.190		-0.190		-0.190		-0.690
	PH3*'		0.110		0.702		1.937		1.572

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.161		-0.161		-0.161		-0.661
	PH3*'		0.078		0.676		1.910		1.548

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.136		-0.136		-0.136		-0.636
	PH3*'		0.056		0.658		1.892		1.531

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.116		-0.116		-0.116		-0.616
	PH3*'		0.040		0.645		1.880		1.519

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.098		-0.098		-0.098		-0.598
	PH3*'		0.029		0.636		1.871		1.511

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.084		-0.084		-0.084		-0.584
	PH3*'		0.021		0.630		1.865		1.505

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.071		-0.071		-0.071		-0.571
	PH3*'		0.015		0.626		1.861		1.501

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.061		-0.061		-0.061		-0.561
	PH3*'		0.011		0.622		1.858		1.498

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.052		-0.052		-0.052		-0.552
	PH3*'		0.008		0.620		1.856		1.496

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.044		-0.044		-0.044		-0.544
	PH3*'		0.005		0.618		1.854		1.494

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.038		-0.038		-0.038		-0.538
	PH3*'		0.004		0.617		1.853		1.493

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.032		-0.032		-0.032		-0.532
	PH3*'		0.003		0.616		1.852		1.493

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.028		-0.028		-0.028		-0.528
	PH3*'		0.002		0.615		1.851		1.492

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.765		-0.765		-0.750		-1.250
	PH3*'		5.903		6.777		8.886		8.217

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.908		-0.908		-0.955		-1.439
	PH3*'		5.491		6.339		8.495		7.815

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.998		-1.014		-1.076		-1.545
	PH3*'		4.919		5.746		7.866		7.187

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.034		-1.066		-1.097		-1.581
	PH3*'		4.238		5.017		7.056		6.404

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.970		-0.985		-1.001		-1.517
	PH3*'		3.492		4.210		6.092		5.490

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.870		-0.870		-0.854		-1.339
	PH3*'		2.716		3.356		5.007		4.485

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.746		-0.761		-0.746		-1.246
	PH3*'		1.975		2.539		3.957		3.509

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.657		-0.657		-0.657		-1.157
	PH3*'		1.372		1.884		3.158		2.757

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.560		-0.576		-0.576		-1.076
	PH3*'		0.941		1.441		2.649		2.273

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.483		-0.483		-0.483		-0.983
	PH3*'		0.656		1.167		2.353		1.989

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.397		-0.413		-0.413		-0.913
	PH3*'		0.466		0.994		2.181		1.821

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.337		-0.337		-0.337		-0.837
	PH3*'		0.333		0.880		2.074		1.716

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.285		-0.285		-0.285		-0.785
	PH3*'		0.239		0.802		2.004		1.646

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	6.028		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.242		-0.242		-0.242		-0.742
		PH3*'		0.172		0.748		1.958		1.599

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	7.166		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.205		-0.205		-0.205		-0.705
		PH3*'		0.124		0.710		1.926		1.567

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	8.487		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.174		-0.174		-0.174		-0.674
		PH3*'		0.090		0.683		1.904		1.545

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	10.023		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.148		-0.148		-0.148		-0.648
		PH3*'		0.065		0.663		1.888		1.530

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	11.807		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.126		-0.126		-0.126		-0.626
		PH3*'		0.047		0.650		1.878		1.519

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	13.879		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.107		-0.107		-0.107		-0.607
		PH3*'		0.034		0.640		1.870		1.511

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	16.287		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.091		-0.091		-0.091		-0.591
		PH3*'		0.025		0.633		1.864		1.505

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	19.085		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.078		-0.078		-0.078		-0.578
		PH3*'		0.018		0.627		1.860		1.501

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	22.336		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.067		-0.067		-0.067		-0.567
		PH3*'		0.013		0.624		1.857		1.498

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	26.112		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.057		-0.057		-0.057		-0.557
		PH3*'		0.009		0.621		1.855		1.496

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	30.500		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.049		-0.049		-0.049		-0.549
		PH3*'		0.007		0.619		1.854		1.495

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.042		-0.042		-0.042		-0.542
	PH3*'		0.005		0.618		1.853		1.493

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.031		-1.062		-1.046		-1.546
	PH3*'		9.654		10.629		12.949		12.235

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.190		-1.237		-1.252		-1.752
	PH3*'		8.920		9.866		12.179		11.453

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.326		-1.357		-1.404		-1.873
	PH3*'		7.949		8.848		11.086		10.381

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.390		-1.406		-1.452		-1.937
	PH3*'		6.831		7.647		9.774		9.103

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.340		-1.356		-1.387		-1.871
	PH3*'		5.623		6.363		8.306		7.683

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.197		-1.212		-1.212		-1.712
	PH3*'		4.417		5.053		6.742		6.206

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.010		-1.057		-1.026		-1.526
	PH3*'		3.254		3.795		5.205		4.755

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.881		-0.897		-0.881		-1.381
	PH3*'		2.278		2.738		3.949		3.563

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.758		-0.758		-0.758		-1.258
	PH3*'		1.558		1.998		3.128		2.770

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.957	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.639		-0.639		-0.654		-1.154
	PH3*'		1.086		1.537		2.654		2.303

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.546		-0.546		-0.546		-1.046
	PH3*'		0.774		1.250		2.377		2.029

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.449		-0.449		-0.465		-0.965
	PH3*'		0.560		1.063		2.209		1.859

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.381		-0.381		-0.381		-0.881
	PH3*'		0.406		0.934		2.097		1.747

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.322		-0.322		-0.322		-0.822
	PH3*'		0.295		0.843		2.023		1.670

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.273		-0.273		-0.273		-0.773
	PH3*'		0.215		0.779		1.972		1.618

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.232		-0.232		-0.232		-0.732
	PH3*'		0.156		0.733		1.937		1.581

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.197		-0.197		-0.197		-0.697
	PH3*'		0.114		0.700		1.912		1.556

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.168		-0.168		-0.168		-0.668
	PH3*'		0.083		0.677		1.895		1.538

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.143		-0.143		-0.143		-0.643
	PH3*'		0.060		0.659		1.882		1.525

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.122		-0.122		-0.122		-0.622
	PH3*'		0.044		0.647		1.873		1.515

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.104		-0.104		-0.104		-0.604
	PH3*'		0.032		0.638		1.867		1.508

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.089		-0.089		-0.089		-0.589
	PH3*'		0.023		0.631		1.862		1.504

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.076		-0.076		-0.076		-0.576
	PH3*'		0.017		0.627		1.859		1.500

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.065		-0.065		-0.065		-0.565
	PH3*'		0.012		0.623		1.856		1.497

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.056		-0.056		-0.056		-0.556
	PH3*'		0.009		0.621		1.854		1.496

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.906		-1.968		-2.031		-2.515
	PH3*'		30.830		32.437		36.117		35.014

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.208		-2.255		-2.333		-2.818
	PH3*'		28.174		29.659		33.145		32.082

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.434		-2.496		-2.574		-3.043
	PH3*'		24.822		26.153		29.366		28.390

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.562		-2.624		-2.687		-3.187
	PH3*'		20.999		22.173		25.053		24.168

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.540		-2.571		-2.634		-3.134
	PH3*'		17.040		18.033		20.556		19.775

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.378		-2.409		-2.425		-2.925
	PH3*'		13.254		14.060		16.168		15.507

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.115		-2.115		-2.115		-2.615
	PH3*'		9.826		10.449		12.094		11.576

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.810		-1.810		-1.794		-2.310
	PH3*'		6.911		7.368		8.614		8.216

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.532		-1.532		-1.532		-2.032
	PH3*'		4.708		5.057		6.067		5.743

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.293		-1.293		-1.293		-1.793
	PH3*'		3.251		3.568		4.504		4.197

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.092		-1.092		-1.092		-1.592
	PH3*'		2.338		2.663		3.605		3.299

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.915		-0.915		-0.915		-1.415
	PH3*'		1.733		2.084		3.055		2.744

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.762		-0.762		-0.762		-1.262
	PH3*'		1.305		1.691		2.697		2.379

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.645		-0.645		-0.645		-1.145
	PH3*'		0.984		1.404		2.449		2.124

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.547		-0.547		-0.547		-1.047
	PH3*'		0.741		1.196		2.277		1.945

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.464		-0.464		-0.464		-0.964
	PH3*'		0.557		1.044		2.156		1.818

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.395		-0.395		-0.395		-0.895
	PH3*'		0.416		0.931		2.071		1.728

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.336		-0.336		-0.336		-0.836
	PH3*'		0.310		0.848		2.009		1.663

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.286		-0.286		-0.286		-0.786
	PH3*'		0.230		0.786		1.966		1.616

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.244		-0.244		-0.244		-0.744
	PH3*'		0.170		0.741		1.934		1.582

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.209		-0.209		-0.209		-0.709
	PH3*'		0.126		0.707		1.911		1.557

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.178		-0.178		-0.178		-0.678
	PH3*'		0.093		0.683		1.895		1.539

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.152		-0.152		-0.152		-0.652
	PH3*'		0.068		0.664		1.882		1.526

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.131		-0.131		-0.131		-0.631
	PH3*'		0.050		0.651		1.874		1.517

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.112		-0.112		-0.112		-0.612
	PH3*'		0.037		0.641		1.867		1.510

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-3.453		-3.609		-3.828		-4.265
	PH3*'		123.666		127.420		135.943		133.422

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-4.075		-4.200		-4.403		-4.825
	PH3*'		112.737		116.060		123.753		121.449

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-4.664		-4.758		-4.961		-5.383
	PH3*'		98.778		101.640		108.367		106.365

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-5.100		-5.178		-5.350		-5.788
	PH3*'		82.523		84.944		90.686		88.944

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-5.227		-5.274		-5.399		-5.863
	PH3*'		65.636		67.619		72.433		70.966

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-5.003		-5.035		-5.081		-5.597
	PH3*'		49.839		51.410		55.271		54.088

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-4.524		-4.540		-4.586		-5.071
	PH3*'		36.212		37.370		40.259		39.376

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-3.943		-3.959		-3.975		-4.475
	PH3*'		25.199		25.965		27.970		27.346

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-3.380		-3.380		-3.380		-3.880
	PH3*'		16.999		17.473		18.809		18.392

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.868		-2.868		-2.868		-3.368
	PH3*'		11.505		11.818		12.795		12.479

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.419		-2.419		-2.419		-2.919
	PH3*'		8.062		8.330		9.192		8.909

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.056		-2.056		-2.056		-2.556
	PH3*'		5.872		6.132		6.967		6.690

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.715		-1.715		-1.715		-2.215
	PH3*'		4.406		4.665		5.498		5.224

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.452		-1.452		-1.452		-1.952
	PH3*'		3.354		3.629		4.476		4.197

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.231		-1.231		-1.231		-1.731
	PH3*'		2.588		2.879		3.755		3.469

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.045		-1.045		-1.045		-1.545
	PH3*'		2.010		2.327		3.239		2.944

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.889		-0.889		-0.889		-1.389
	PH3*'		1.567		1.914		2.866		2.562

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.756		-0.756		-0.756		-1.256
	PH3*'		1.220		1.600		2.594		2.281

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.645		-0.645		-0.645		-1.145
	PH3*'		0.946		1.362		2.396		2.074

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.550		-0.550		-0.550		-1.050
	PH3*'		0.729		1.180		2.251		1.921

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.470		-0.470		-0.470		-0.970
	PH3*'		0.558		1.040		2.144		1.808

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.402		-0.402		-0.402		-0.902
	PH3*'		0.425		0.934		2.066		1.725

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.344		-0.344		-0.344		-0.844
	PH3*'		0.321		0.854		2.009		1.663

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.294		-0.294		-0.294		-0.794
	PH3*'		0.241		0.793		1.967		1.618

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-0.252		-0.252		-0.252		-0.752
	PH3*'		0.180		0.747		1.936		1.584

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-5.203		-5.390		-5.750		-6.156
	PH3*'		345.811		353.275		370.278		365.165

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-6.226		-6.398		-6.757		-7.163	
	PH3*'	316.571		322.940		337.668		333.292	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-7.393		-7.518		-7.846		-8.252	
	PH3*'	278.116		283.389		295.811		292.133	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-8.419		-8.435		-8.716		-9.138	
	PH3*'	231.628		236.035		246.403		243.314	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-8.770		-8.864		-9.005		-9.473	
	PH3*'	183.065		186.552		194.905		192.387	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-8.513		-8.576		-8.685		-9.154	
	PH3*'	137.611		140.316		146.884		144.875	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-7.803		-7.835		-7.897		-8.382	
	PH3*'	99.058		100.991		105.790		104.327	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-6.865		-6.881		-6.912		-7.396	
	PH3*'	68.753		70.009		73.178		72.209	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-5.909		-5.925		-5.940		-6.425	
	PH3*'	46.494		47.220		49.117		48.521	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-5.035		-5.035		-5.050		-5.550	
	PH3*'	31.407		31.836		33.044		32.668	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-4.276		-4.276		-4.276		-4.776	
	PH3*'	21.741		22.045		22.958		22.659	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-3.615		-3.615		-3.615		-4.115	
	PH3*'	15.486		15.755		16.581		16.309	

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	5.049		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-3.048		-3.048		-3.048		-3.548
		PH3*'		11.271		11.527		12.324		12.061

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	6.028		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-2.582		-2.582		-2.582		-3.082
		PH3*'		8.338		8.588		9.376		9.117

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	7.166		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-2.190		-2.190		-2.190		-2.690
		PH3*'		6.263		6.520		7.308		7.049

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	8.487		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-1.859		-1.859		-1.859		-2.359
		PH3*'		4.779		5.038		5.838		5.576

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	10.023		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-1.580		-1.580		-1.580		-2.080
		PH3*'		3.693		3.963		4.783		4.515

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	11.807		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-1.345		-1.345		-1.345		-1.845
		PH3*'		2.884		3.169		4.016		3.741

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	13.879		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-1.146		-1.146		-1.146		-1.646
		PH3*'		2.268		2.574		3.455		3.171

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	16.287		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.978		-0.978		-0.978		-1.478
		PH3*'		1.789		2.120		3.041		2.748

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	19.085		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.836		-0.836		-0.836		-1.336
		PH3*'		1.410		1.772		2.734		2.431

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	22.336		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.714		-0.714		-0.714		-1.214
		PH3*'		1.107		1.502		2.506		2.194

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	26.112		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.611		-0.611		-0.611		-1.111
		PH3*'		0.863		1.293		2.337		2.015

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-0.524		-0.524		-0.524		-1.024	
	PH3*'	0.669		1.131		2.211		1.882	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-0.449		-0.449		-0.449		-0.949	
	PH3*'	0.512		1.004		2.116		1.781	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-7.078		-7.328		-7.843		-8.187	
	PH3*'	785.942		798.896		828.586		819.808	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-8.661		-8.864		-9.380		-9.770	
	PH3*'	723.171		733.894		758.922		751.446	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-10.495		-10.823		-11.230		-11.636	
	PH3*'	637.881		646.611		667.060		660.946	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-12.723		-12.786		-12.911		-13.379	
	PH3*'	528.991		537.385		555.306		550.049	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-13.342		-13.435		-13.592		-14.045	
	PH3*'	418.836		424.483		437.922		433.823	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-13.017		-13.080		-13.220		-13.673	
	PH3*'	313.926		318.089		328.261		325.160	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-12.047		-12.109		-12.187		-12.672	
	PH3*'	224.677		227.570		234.854		232.560	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-10.606		-10.621		-10.668		-11.152	
	PH3*'	156.742		158.599		163.254		161.812	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'	-9.165		-9.165		-9.196		-9.696	
	PH3*'	106.616		107.636		110.249		109.419	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-7.825		-7.841		-7.841		-8.341
	PH3*'		72.438		72.968		74.413		73.952

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-6.648		-6.648		-6.664		-7.164
	PH3*'		50.064		50.397		51.379		51.054

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-5.640		-5.640		-5.640		-6.140
	PH3*'		35.314		35.595		36.433		36.154

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-4.764		-4.764		-4.764		-5.264
	PH3*'		25.288		25.552		26.347		26.081

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-4.035		-4.035		-4.035		-4.535
	PH3*'		18.328		18.585		19.361		19.102

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-3.421		-3.421		-3.421		-3.921
	PH3*'		13.450		13.702		14.472		14.213

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.905		-2.905		-2.905		-3.405
	PH3*'		9.997		10.249		11.017		10.761

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.469		-2.469		-2.469		-2.969
	PH3*'		7.531		7.786		8.557		8.300

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-2.102		-2.102		-2.102		-2.602
	PH3*'		5.750		6.008		6.789		6.529

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.791		-1.791		-1.791		-2.291
	PH3*'		4.446		4.711		5.507		5.243

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V3*'		-1.529		-1.529		-1.529		-2.029
	PH3*'		3.474		3.751		4.570		4.300

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	19.085		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-1.306		-1.306		-1.306		-1.806
		PH3*'		2.738		3.030		3.878		3.601

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	22.336		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-1.117		-1.117		-1.117		-1.617
		PH3*'		2.169		2.482		3.365		3.079

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	26.112		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.955		-0.955		-0.955		-1.455
		PH3*'		1.718		2.059		2.982		2.686

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	30.500		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.818		-0.818		-0.818		-1.318
		PH3*'		1.357		1.727		2.695		2.389

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
25.000	35.598		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V3*'		-0.701		-0.701		-0.701		-1.201
		PH3*'		1.080		1.475		2.479		2.165

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		-0.250		-0.328
	PH4*'		0.000		1.014		3.851		2.960

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.015		0.265		-0.484
	PH4*'		0.000		1.016		3.833		2.944

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.015		0.031		-0.468
	PH4*'		0.000		0.999		3.697		2.846

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.015		-0.484
	PH4*'		0.000		0.961		3.475		2.689

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.913		3.204		2.494

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.865		2.913		2.286

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.827		2.656		2.105

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.800		2.462		1.972

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.783		2.337		1.887

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.771		2.263		1.836

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.764		2.217		1.804

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.759		2.188		1.783

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.756		2.169		1.770

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.754		2.156		1.761

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.752		2.147		1.755

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.751		2.141		1.751

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.750		2.137		1.748

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.750		2.134		1.746

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.750		2.132		1.744

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.749		2.131		1.743

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.749		2.129		1.742

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.749		2.129		1.742

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.749		2.128		1.741

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.749		2.128		1.741

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.000		0.000		0.000		-0.500
	PH4*'		0.000		0.749		2.127		1.741

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		0.078		0.093		-0.375		-0.781
	PH4*'		0.997		2.009		4.697		3.846

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.172		0.030		-0.422		-0.344
	PH4*'		0.956		1.981		4.657		3.806

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.218		-0.124		-0.030		-0.562
	PH4*'		0.866		1.883		4.486		3.663

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.246		-0.214		-0.136		-0.652
	PH4*'		0.738		1.720		4.158		3.394

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.260		-0.245		-0.229		-0.729
	PH4*'		0.590		1.519		3.742		3.052

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.248		-0.248		-0.248		-0.748
	PH4*'		0.443		1.310		3.304		2.693

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.233		-0.233		-0.248		-0.748
	PH4*'		0.317		1.128		2.914		2.376

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.208		-0.208		-0.224		-0.724
	PH4*'		0.221		0.994		2.618		2.137

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.320		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.181		-0.181		-0.197		-0.697
		PH4*'		0.153		0.908		2.428		1.985

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.857		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.155		-0.155		-0.155		-0.655
		PH4*'		0.107		0.854		2.316		1.896

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	3.481		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.132		-0.132		-0.132		-0.632
		PH4*'		0.075		0.820		2.251		1.843

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	4.206		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.112		-0.112		-0.112		-0.612
		PH4*'		0.052		0.797		2.210		1.810

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	5.049		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.095		-0.095		-0.095		-0.595
		PH4*'		0.037		0.783		2.184		1.788

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	6.028		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.080		-0.080		-0.080		-0.580
		PH4*'		0.026		0.773		2.167		1.774

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	7.166		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.068		-0.068		-0.068		-0.568
		PH4*'		0.019		0.766		2.155		1.764

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	8.487		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.058		-0.058		-0.058		-0.558
		PH4*'		0.013		0.761		2.147		1.757

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	10.023		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.049		-0.049		-0.049		-0.549
		PH4*'		0.009		0.757		2.141		1.752

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	11.807		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.042		-0.042		-0.042		-0.542
		PH4*'		0.007		0.755		2.137		1.749

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	13.879		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.035		-0.035		-0.035		-0.535
		PH4*'		0.005		0.753		2.134		1.746

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.030		-0.030		-0.030		-0.530
	PH4*'		0.003		0.752		2.132		1.745

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.026		-0.026		-0.026		-0.526
	PH4*'		0.002		0.751		2.130		1.744

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.022		-0.022		-0.022		-0.522
	PH4*'		0.001		0.751		2.129		1.743

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.019		-0.019		-0.019		-0.519
	PH4*'		0.001		0.750		2.129		1.742

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.016		-0.016		-0.016		-0.516
	PH4*'		0.001		0.750		2.128		1.742

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.014		-0.014		-0.014		-0.514
	PH4*'		0.000		0.749		2.128		1.741

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.562		-0.546		-0.593		-1.046
	PH4*'		3.590		4.605		7.051		6.274

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.657		-0.688		-0.751		-1.220
	PH4*'		3.355		4.391		6.861		6.077

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.717		-0.764		-0.842		-1.311
	PH4*'		3.045		4.056		6.501		5.733

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.726		-0.742		-0.820		-1.288
	PH4*'		2.649		3.621		5.984		5.235

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.631		-0.646		-0.568		-1.146
	PH4*'		2.176		3.094		5.257		4.585

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.559		-0.559		-0.512		-1.028
	PH4*'		1.675		2.499		4.397		3.814

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.497		-0.497		-0.497		-0.997
	PH4*'		1.210		1.953		3.627		3.114

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.432		-0.448		-0.448		-0.948
	PH4*'		0.839		1.531		3.048		2.587

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.379		-0.379		-0.379		-0.879
	PH4*'		0.578		1.251		2.677		2.250

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.311		-0.327		-0.327		-0.827
	PH4*'		0.403		1.081		2.465		2.057

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.265		-0.265		-0.280		-0.780
	PH4*'		0.285		0.976		2.346		1.947

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.224		-0.224		-0.224		-0.724
	PH4*'		0.202		0.906		2.274		1.880

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.190		-0.190		-0.190		-0.690
	PH4*'		0.144		0.859		2.228		1.836

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.161		-0.161		-0.161		-0.661
	PH4*'		0.103		0.827		2.197		1.807

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.136		-0.136		-0.136		-0.636
	PH4*'		0.074		0.805		2.176		1.787

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.116		-0.116		-0.116		-0.616
	PH4*'		0.053		0.789		2.162		1.774

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.098		-0.098		-0.098		-0.598
	PH4*'		0.038		0.778		2.152		1.764

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.084		-0.084		-0.084		-0.584
	PH4*'		0.028		0.770		2.145		1.758

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.071		-0.071		-0.071		-0.571
	PH4*'		0.020		0.764		2.140		1.753

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.061		-0.061		-0.061		-0.561
	PH4*'		0.014		0.760		2.136		1.749

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.052		-0.052		-0.052		-0.552
	PH4*'		0.010		0.757		2.133		1.747

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.044		-0.044		-0.044		-0.544
	PH4*'		0.007		0.755		2.132		1.745

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.038		-0.038		-0.038		-0.538
	PH4*'		0.005		0.753		2.130		1.744

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.032		-0.032		-0.032		-0.532
	PH4*'		0.004		0.752		2.129		1.743

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.028		-0.028		-0.028		-0.528
	PH4*'		0.003		0.751		2.128		1.742

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.921		-0.921		-0.906		-1.421
	PH4*'		7.004		8.089		10.586		9.823

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.049		-1.049		-1.095		-1.580
	PH4*'		6.504		7.575		10.077		9.303

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.349		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.139		-1.155		-1.217		-1.701
		PH4*'		5.866		6.876		9.338		8.575

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.568		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.128		-1.159		-1.206		-1.706
		PH4*'		5.091		6.059		8.398		7.663

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.822		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.079		-1.095		-1.110		-1.595
		PH4*'		4.259		5.127		7.292		6.609

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.116		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.917		-0.932		-0.901		-1.417
		PH4*'		3.368		4.132		6.004		5.427

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.459		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.777		-0.808		-0.761		-1.277
		PH4*'		2.477		3.142		4.723		4.234

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.857		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.672		-0.672		-0.672		-1.172
		PH4*'		1.732		2.320		3.721		3.287

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.320		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.576		-0.576		-0.576		-1.076
		PH4*'		1.188		1.758		3.073		2.671

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.857		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.483		-0.483		-0.483		-0.983
		PH4*'		0.830		1.417		2.706		2.318

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	3.481		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.397		-0.413		-0.413		-0.913
		PH4*'		0.593		1.208		2.500		2.117

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	4.206		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.337		-0.337		-0.337		-0.837
		PH4*'		0.428		1.072		2.379		1.996

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	5.049		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.285		-0.285		-0.285		-0.785
		PH4*'		0.309		0.979		2.300		1.917

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.242		-0.242		-0.242		-0.742
	PH4*'		0.224		0.913		2.247		1.864

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.205		-0.205		-0.205		-0.705
	PH4*'		0.163		0.867		2.212		1.828

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.174		-0.174		-0.174		-0.674
	PH4*'		0.118		0.834		2.187		1.802

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.148		-0.148		-0.148		-0.648
	PH4*'		0.086		0.811		2.170		1.785

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.126		-0.126		-0.126		-0.626
	PH4*'		0.062		0.794		2.158		1.772

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.107		-0.107		-0.107		-0.607
	PH4*'		0.045		0.781		2.149		1.764

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.091		-0.091		-0.091		-0.591
	PH4*'		0.033		0.773		2.143		1.757

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.078		-0.078		-0.078		-0.578
	PH4*'		0.024		0.766		2.138		1.753

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.067		-0.067		-0.067		-0.567
	PH4*'		0.017		0.761		2.135		1.749

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.057		-0.057		-0.057		-0.557
	PH4*'		0.013		0.758		2.133		1.747

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.049		-0.049		-0.049		-0.549
	PH4*'		0.009		0.756		2.131		1.745

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	35.598		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.042		-0.042		-0.042		-0.542
		PH4*'		0.007		0.754		2.130		1.744

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.000		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.187		-1.218		-1.218		-1.734
		PH4*'		11.019		12.241		14.940		14.127

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.161		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.331		-1.377		-1.424		-1.909
		PH4*'		10.186		11.350		14.027		13.220

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.349		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.435		-1.482		-1.545		-2.045
		PH4*'		9.127		10.212		12.797		12.006

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.568		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.484		-1.515		-1.577		-2.046
		PH4*'		7.895		8.887		11.319		10.560

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.822		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.434		-1.449		-1.481		-1.965
		PH4*'		6.578		7.461		9.673		8.986

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.116		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.290		-1.275		-1.275		-1.775
		PH4*'		5.237		5.993		7.896		7.304

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.459		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-1.073		-1.088		-1.073		-1.557
		PH4*'		3.919		4.535		6.102		5.612

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.857		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.912		-0.912		-0.912		-1.412
		PH4*'		2.770		3.284		4.595		4.182

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.320		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.773		-0.773		-0.773		-1.273
		PH4*'		1.904		2.387		3.597		3.218

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.857		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.654		-0.654		-0.654		-1.154
		PH4*'		1.333		1.832		3.026		2.657

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.546		-0.546		-0.546		-1.046
	PH4*'		0.960		1.496		2.707		2.339

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.449		-0.465		-0.465		-0.965
	PH4*'		0.703		1.280		2.518		2.148

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.381		-0.381		-0.381		-0.881
	PH4*'		0.517		1.131		2.397		2.025

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.322		-0.322		-0.322		-0.822
	PH4*'		0.380		1.026		2.316		1.940

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.273		-0.273		-0.273		-0.773
	PH4*'		0.279		0.950		2.260		1.882

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.232		-0.232		-0.232		-0.732
	PH4*'		0.204		0.895		2.222		1.842

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.197		-0.197		-0.197		-0.697
	PH4*'		0.149		0.855		2.195		1.813

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.168		-0.168		-0.168		-0.668
	PH4*'		0.109		0.826		2.176		1.793

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.143		-0.143		-0.143		-0.643
	PH4*'		0.080		0.805		2.162		1.778

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.122		-0.122		-0.122		-0.622
	PH4*'		0.058		0.790		2.152		1.768

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.104		-0.104		-0.104		-0.604
	PH4*'		0.043		0.779		2.145		1.760

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.089		-0.089		-0.089		-0.589
	PH4*'		0.031		0.771		2.140		1.755

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.076		-0.076		-0.076		-0.576
	PH4*'		0.023		0.765		2.137		1.751

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.065		-0.065		-0.065		-0.565
	PH4*'		0.017		0.761		2.134		1.748

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.056		-0.056		-0.056		0.000
	PH4*'		0.012		0.758		2.132		1.746

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.046		-2.109		-2.200		0.000
	PH4*'		32.738		34.622		38.804		37.565

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.318		-2.380		-2.458		-2.943
	PH4*'		29.904		31.642		35.593		34.402

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.559		-2.606		-2.715		-3.199
	PH4*'		26.385		27.955		31.598		30.503

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.671		-2.718		-2.812		-3.296
	PH4*'		22.403		23.776		27.055		26.059

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.634		-2.680		-2.727		-3.212
	PH4*'		18.286		19.457		22.302		21.432

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.456		-2.456		-2.503		-2.987
	PH4*'		14.322		15.275		17.648		16.920

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.146		-2.161		-2.161		-2.661
	PH4*'		10.694		11.421		13.266		12.695

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.825		-1.825		-1.825		-2.341
	PH4*'		7.576		8.096		9.463		9.035

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.547		-1.547		-1.532		-2.047
	PH4*'		5.202		5.576		6.660		6.319

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.309		-1.309		-1.293		-1.793
	PH4*'		3.624		3.956		4.939		4.621

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.092		-1.092		-1.092		-1.592
	PH4*'		2.646		2.985		3.970		3.653

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.915		-0.915		-0.915		-1.415
	PH4*'		2.000		2.370		3.390		3.067

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.762		-0.762		-0.762		-1.262
	PH4*'		1.538		1.952		3.016		2.684

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.645		-0.645		-0.645		-1.145
	PH4*'		1.183		1.643		2.757		2.416

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.547		-0.547		-0.547		-1.047
	PH4*'		0.907		1.416		2.577		2.227

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.464		-0.464		-0.464		-0.964
	PH4*'		0.693		1.247		2.450		2.092

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.395		-0.395		-0.395		-0.895
	PH4*'		0.526		1.121		2.360		1.996

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.336		-0.336		-0.336		-0.836
	PH4*'		0.397		1.026		2.295		1.926

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.286		-0.286		-0.286		-0.786
	PH4*'		0.297		0.955		2.249		1.876

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.244		-0.244		-0.244		-0.744
	PH4*'		0.222		0.902		2.216		1.839

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.209		-0.209		-0.209		-0.709
	PH4*'		0.165		0.862		2.192		1.813

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.178		-0.178		-0.178		-0.678
	PH4*'		0.122		0.833		2.174		1.793

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.152		-0.152		-0.152		-0.652
	PH4*'		0.090		0.811		2.161		1.779

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.131		-0.131		-0.131		-0.631
	PH4*'		0.067		0.795		2.152		1.769

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.112		-0.112		-0.112		-0.612
	PH4*'		0.049		0.783		2.145		1.761

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-3.578		-3.703		-3.937		-4.359
	PH4*'		125.616		129.688		138.858		136.173

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-4.122		-4.263		-4.497		-4.934
	PH4*'		114.488		118.102		126.330		123.912

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-4.727		-4.789		-5.055		-5.445
	PH4*'		100.330		103.440		110.657		108.491

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-5.163		-5.241		-5.428		-5.850
	PH4*'		83.876		86.506		92.692		90.833

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-5.274		-5.321		-5.446		-5.915
	PH4*'		66.805		68.970		74.152		72.582

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-5.019		-5.066		-5.144		-5.613
	PH4*'		50.831		52.544		56.718		55.451

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-4.540		-4.555		-4.602		-5.102
	PH4*'		37.005		38.286		41.415		40.447

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-3.959		-3.975		-3.990		-4.475
	PH4*'		25.818		26.659		28.813		28.148

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-3.380		-3.380		-3.396		-3.896
	PH4*'		17.464		17.996		19.417		18.962

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.868		-2.884		-2.868		-3.368
	PH4*'		11.869		12.222		13.251		12.924

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.434		-2.434		-2.434		-2.934
	PH4*'		8.413		8.685		9.577		9.288

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-2.056		-2.056		-2.056		-2.556
	PH4*'		6.203		6.465		7.326		7.043

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.715		-1.715		-1.715		-2.215
	PH4*'		4.724		4.984		5.844		5.565

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.452		-1.452		-1.452		-1.952
	PH4*'		3.656		3.933		4.810		4.524

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.231		-1.231		-1.231		-1.731
	PH4*'		2.872		3.167		4.079		3.785

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-1.045		-1.045		-1.045		-1.545
	PH4*'		2.274		2.599		3.554		3.249

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.889		-0.889		-0.889		-1.389
	PH4*'		1.807		2.170		3.173		2.856

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.756		-0.756		-0.756		-1.256
	PH4*'		1.435		1.841		2.895		2.567

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.645		-0.645		-0.645		-1.145
	PH4*'		1.134		1.587		2.692		2.353

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.550		-0.550		-0.550		-1.050
	PH4*'		0.890		1.391		2.542		2.194

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.470		-0.470		-0.470		-0.970
	PH4*'		0.693		1.239		2.432		2.076

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.402		-0.402		-0.402		-0.902
	PH4*'		0.535		1.121		2.352		1.988

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.344		-0.344		-0.344		-0.844
	PH4*'		0.410		1.031		2.292		1.924

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.294		-0.294		-0.294		-0.794
	PH4*'		0.309		0.961		2.248		1.875

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-0.252		-0.252		-0.252		-0.752
	PH4*'		0.235		0.909		2.217		1.841

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'		-5.250		-5.437		-5.828		-6.171
	PH4*'		347.558		355.301		372.828		367.643

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-6.257		-6.429		-6.835		-7.210	
	PH4*'	318.125		324.745		339.963		335.451	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-7.440		-7.565		-7.877		-8.299	
	PH4*'	279.463		284.935		297.786		293.941	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-8.419		-8.529		-8.748		-9.185	
	PH4*'	232.773		237.374		248.095		244.896	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-8.786		-8.864		-9.036		-9.489	
	PH4*'	184.080		187.703		196.365		193.733	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-8.529		-8.591		-8.638		-9.154	
	PH4*'	138.451		141.271		148.072		146.014	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-7.803		-7.835		-7.897		-8.382	
	PH4*'	99.751		101.809		106.778		105.262	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-6.865		-6.881		-6.912		-7.412	
	PH4*'	69.318		70.635		73.894		72.884	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-5.909		-5.925		-5.940		-6.425	
	PH4*'	46.936		47.693		49.653		49.040	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-5.035		-5.035		-5.050		-5.550	
	PH4*'	31.798		32.250		33.446		33.058	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-4.276		-4.276		-4.276		-4.776	
	PH4*'	22.092		22.403		23.329		23.029	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V4*'	-3.615		-3.615		-3.615		-4.115	
	PH4*'	15.829		16.103		16.934		16.661	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-3.048		-3.048		-3.048		-3.548
		PH4*'	11.607		11.868		12.669		12.405

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-2.582		-2.582		-2.582		-3.082
		PH4*'	8.666		8.920		9.713		9.454

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-2.190		-2.190		-2.190		-2.690
		PH4*'	6.584		6.844		7.639		7.379

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-1.859		-1.859		-1.859		-2.359
		PH4*'	5.090		5.353		6.163		5.899

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-1.580		-1.580		-1.580		-2.080
		PH4*'	3.993		4.268		5.101		4.831

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-1.345		-1.345		-1.345		-1.845
		PH4*'	3.170		3.463		4.329		4.050

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-1.146		-1.146		-1.146		-1.646
		PH4*'	2.538		2.855		3.763		3.474

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-0.978		-0.978		-0.978		-1.478
		PH4*'	2.039		2.387		3.344		3.044

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-0.836		-0.836		-0.836		-1.336
		PH4*'	1.638		2.024		3.033		2.721

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-0.714		-0.714		-0.714		-1.214
		PH4*'	1.311		1.740		2.802		2.477

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'	-0.611		-0.611		-0.611		-1.111
		PH4*'	1.040		1.516		2.629		2.293

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	30.500		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.524		-0.524		-0.524		-1.024
		PH4*'		0.816		1.337		2.499		2.153

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	35.598		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V4*'		-0.449		-0.449		-0.449		-0.949
		PH4*'		0.650		1.211		2.408		2.055

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		-0.296		-0.281
	PH5*'		0.000		1.299		4.631		3.624

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.031		0.296		-0.265
	PH5*'		0.000		1.302		4.607		3.606

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.015		0.015		-0.484
	PH5*'		0.000		1.269		4.443		3.477

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.015		-0.484
	PH5*'		0.000		1.206		4.157		3.265

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.015		-0.500
	PH5*'		0.000		1.126		3.804		2.999

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		1.047		3.416		2.709

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.984		3.063		2.452

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.940		2.795		2.261

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.912		2.623		2.139

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.894		2.523		2.068

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.882		2.464		2.026

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.875		2.428		1.999

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.871		2.404		1.982

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.867		2.389		1.971

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.865		2.378		1.963

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.864		2.371		1.958

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.863		2.366		1.954

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.862		2.362		1.952

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.862		2.360		1.950

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.861		2.358		1.949

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.861		2.357		1.948

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.861		2.356		1.947

MUS	MUR	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.861		2.355		1.947

MUS	MUR	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.861		2.354		1.946

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.000		0.000		0.000		-0.500
	PH5*'		0.000		0.861		2.354		1.946

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		0.171		0.171		-0.406		-0.859
	PH5*'		1.236		2.491		5.551		4.606

MUS	MUR	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.235		-0.235		-0.547		-0.922
	PH5*'		1.182		2.453		5.498		4.558

MUS	MUR	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.265		-0.124		-0.562		-0.546
	PH5*'		1.072		2.329		5.303		4.391

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.261		-0.214		-0.152		-0.667
	PH5*'		0.916		2.122		4.907		4.054

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.260		-0.245		-0.229		-0.745
	PH5*'		0.734		1.864		4.397		3.627

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.248		-0.248		-0.264		-0.764
	PH5*'		0.552		1.595		3.849		3.173

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.233		-0.233		-0.248		-0.748
	PH5*'		0.395		1.355		3.355		2.764

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.208		-0.208		-0.224		-0.724
	PH5*'		0.275		1.177		2.972		2.452

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.181		-0.181		-0.197		-0.697
	PH5*'		0.191		1.062		2.723		2.251

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.155		-0.155		-0.155		-0.655
	PH5*'		0.133		0.992		2.581		2.135

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.132		-0.132		-0.132		-0.632
	PH5*'		0.093		0.949		2.500		2.069

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.112		-0.112		-0.112		-0.612
	PH5*'		0.065		0.921		2.451		2.028

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.095		-0.095		-0.095		-0.595
	PH5*'		0.046		0.902		2.420		2.002

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.080		-0.080		-0.080		-0.580
	PH5*'		0.033		0.890		2.400		1.985

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.068		-0.068		-0.068		-0.568
	PH5*'		0.023		0.881		2.386		1.973

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.058		-0.058		-0.058		-0.558
	PH5*'		0.017		0.875		2.376		1.965

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.049		-0.049		-0.049		-0.549
	PH5*'		0.012		0.871		2.370		1.959

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.042		-0.042		-0.042		-0.542
	PH5*'		0.008		0.868		2.365		1.955

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.035		-0.035		-0.035		-0.535
	PH5*'		0.006		0.866		2.362		1.953

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.030		-0.030		-0.030		-0.530
	PH5*'		0.004		0.865		2.359		1.951

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.026		-0.026		-0.026		-0.526
	PH5*'		0.003		0.863		2.358		1.949

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.022		-0.022		-0.022		-0.522
	PH5*'		0.002		0.863		2.356		1.948

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.019		-0.019		-0.019		-0.519
	PH5*'		0.001		0.862		2.355		1.947

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.016		-0.016		-0.016		-0.516
	PH5*'		0.001		0.862		2.355		1.947

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.014		-0.014		-0.014		-0.514
	PH5*'		0.000		0.861		2.354		1.946

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.640		-0.671		-0.687		-1.171
	PH5*'		4.150		5.375		8.065		7.227

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.751		-0.782		-0.845		-1.329
	PH5*'		3.914		5.118		7.823		6.983

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.780		-0.842		-0.921		-1.389
	PH5*'		3.559		4.737		7.409		6.578

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.757		-0.835		-0.882		-1.382
	PH5*'		3.125		4.242		6.828		6.026

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.724		-0.724		-0.662		-1.209
	PH5*'		2.606		3.649		6.027		5.301

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.621		-0.621		-0.543		-1.059
	PH5*'		2.031		2.974		5.038		4.414

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.497		-0.513		-0.513		-1.013
	PH5*'		1.474		2.318		4.136		3.589

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.432		-0.448		-0.448		-0.948
	PH5*'		1.024		1.806		3.443		2.952

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.379		-0.379		-0.394		-0.894
	PH5*'		0.704		1.462		2.994		2.542

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.311		-0.327		-0.327		-0.827
	PH5*'		0.492		1.255		2.738		2.308

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.265		-0.265		-0.280		-0.780
	PH5*'		0.349		1.129		2.599		2.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.224		-0.224		-0.224		-0.724
	PH5*'		0.249		1.047		2.518		2.102

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.190		-0.190		-0.190		-0.690
	PH5*'		0.178		0.992		2.466		2.053

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.161		-0.161		-0.161		-0.661
	PH5*'		0.128		0.953		2.431		2.020

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.136		-0.136		-0.136		-0.636
	PH5*'		0.092		0.927		2.408		1.998

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.116		-0.116		-0.116		-0.616
	PH5*'		0.066		0.908		2.392		1.983

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	10.023		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.098		-0.098		-0.098		-0.598
		PH5*'		0.048		0.895		2.381		1.972

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	11.807		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.084		-0.084		-0.084		-0.584
		PH5*'		0.035		0.885		2.373		1.965

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	13.879		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.071		-0.071		-0.071		-0.571
		PH5*'		0.025		0.879		2.368		1.959

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	16.287		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.061		-0.061		-0.061		-0.561
		PH5*'		0.018		0.874		2.364		1.955

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	19.085		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.052		-0.052		-0.052		-0.552
		PH5*'		0.013		0.870		2.361		1.953

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	22.336		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.044		-0.044		-0.044		-0.544
		PH5*'		0.009		0.867		2.359		1.951

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	26.112		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.038		-0.038		-0.038		-0.538
		PH5*'		0.007		0.866		2.357		1.949

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	30.500		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.032		-0.032		-0.032		-0.532
		PH5*'		0.005		0.864		2.356		1.948

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	35.598		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.028		-0.028		-0.028		-0.528
		PH5*'		0.003		0.863		2.355		1.947

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.000		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.984		-1.031		-1.015		-1.531
		PH5*'		7.769		9.044		11.742		10.927

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.161		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-1.111		-1.174		-1.205		-1.689
		PH5*'		7.252		8.460		11.174		10.353

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.155		-1.264		-1.295		-1.780
	PH5*'		6.556		7.714		10.359		9.547

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.206		-1.238		-1.284		-1.784
	PH5*'		5.752		6.811		9.337		8.556

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.095		-1.142		-1.157		-1.673
	PH5*'		4.846		5.821		8.134		7.414

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.964		-0.979		-0.948		-1.448
	PH5*'		3.869		4.720		6.740		6.113

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.840		-0.793		-0.777		-1.293
	PH5*'		2.897		3.620		5.304		4.785

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.688		-0.688		-0.688		-1.188
	PH5*'		2.042		2.681		4.162		3.708

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.576		-0.576		-0.592		-1.076
	PH5*'		1.407		2.025		3.415		2.995

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.483		-0.498		-0.498		-0.983
	PH5*'		0.987		1.629		2.991		2.587

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.413		-0.413		-0.413		-0.913
	PH5*'		0.710		1.388		2.759		2.359

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.337		-0.337		-0.353		-0.853
	PH5*'		0.516		1.234		2.626		2.226

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.285		-0.285		-0.285		-0.785
	PH5*'		0.376		1.127		2.541		2.139

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.242		-0.242		-0.242		-0.742
	PH5*'		0.275		1.052		2.484		2.081

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.205		-0.205		-0.205		-0.705
	PH5*'		0.200		0.999		2.445		2.041

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.174		-0.174		-0.174		-0.674
	PH5*'		0.146		0.961		2.418		2.014

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.148		-0.148		-0.148		-0.648
	PH5*'		0.106		0.933		2.400		1.994

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.126		-0.126		-0.126		-0.626
	PH5*'		0.077		0.913		2.387		1.981

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.107		-0.107		-0.107		-0.607
	PH5*'		0.056		0.899		2.377		1.971

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.091		-0.091		-0.091		-0.591
	PH5*'		0.041		0.889		2.371		1.964

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.078		-0.078		-0.078		-0.578
	PH5*'		0.030		0.881		2.366		1.959

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.067		-0.067		-0.067		-0.567
	PH5*'		0.022		0.876		2.362		1.955

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.057		-0.057		-0.057		-0.557
	PH5*'		0.016		0.872		2.360		1.952

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.049		-0.049		-0.049		-0.549
	PH5*'		0.012		0.869		2.358		1.950

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.042		-0.042		-0.042		-0.542
	PH5*'		0.008		0.866		2.357		1.949

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.265		-1.328		-1.328		-1.828
	PH5*'		11.927		13.285		16.215		15.351

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.440		-1.471		-1.518		-2.002
	PH5*'		11.045		12.331		15.212		14.341

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.529		-1.560		-1.639		-2.107
	PH5*'		9.913		11.125		13.886		13.040

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.546		-1.593		-1.640		-2.140
	PH5*'		8.631		9.728		12.317		11.525

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.481		-1.512		-1.543		-2.012
	PH5*'		7.250		8.222		10.575		9.846

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.290		-1.322		-1.322		-1.822
	PH5*'		5.827		6.646		8.682		8.051

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.120		-1.104		-1.073		-1.588
	PH5*'		4.404		5.084		6.731		6.216

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.928		-0.928		-0.912		-1.428
	PH5*'		3.149		3.701		5.076		4.647

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.789		-0.789		-0.773		-1.273
	PH5*'		2.184		2.698		3.962		3.569

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.654		-0.654		-0.654		-1.154
	PH5*'		1.542		2.074		3.325		2.940

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.546		-0.546		-0.546		-1.046
	PH5*'		1.121		1.701		2.973		2.590

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.465		-0.465		-0.465		-0.965
	PH5*'		0.830		1.462		2.770		2.384

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.381		-0.381		-0.381		-0.881
	PH5*'		0.617		1.297		2.642		2.252

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.322		-0.322		-0.322		-0.822
	PH5*'		0.458		1.179		2.555		2.161

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.273		-0.273		-0.273		-0.773
	PH5*'		0.339		1.093		2.496		2.099

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.232		-0.232		-0.232		-0.732
	PH5*'		0.250		1.030		2.455		2.055

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.197		-0.197		-0.197		-0.697
	PH5*'		0.184		0.985		2.426		2.024

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.168		-0.168		-0.168		-0.668
	PH5*'		0.135		0.951		2.405		2.002

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.143		-0.143		-0.143		-0.643
	PH5*'		0.099		0.927		2.391		1.987

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.122		-0.122		-0.122		-0.622
	PH5*'		0.073		0.909		2.381		1.975

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.104		-0.104		-0.104		-0.604
	PH5*'		0.053		0.896		2.373		1.967

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.089		-0.089		-0.089		-0.589
	PH5*'		0.039		0.887		2.368		1.961

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.076		-0.076		-0.076		-0.576
	PH5*'		0.028		0.880		2.364		1.957

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.065		-0.065		-0.065		-0.565
	PH5*'		0.021		0.875		2.361		1.954

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.056		-0.056		-0.056		-0.556
	PH5*'		0.015		0.871		2.359		1.952

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.109		-2.171		-2.281		-2.750
	PH5*'		33.670		35.703		40.128		38.838

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.380		-2.443		-2.552		-3.037
	PH5*'		30.770		32.654		36.836		35.600

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.559		-2.653		-2.778		-3.231
	PH5*'		27.192		28.890		32.735		31.590

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.702		-2.749		-2.812		-3.327
	PH5*'		23.129		24.628		28.079		27.044

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.665		-2.696		-2.774		-3.259
	PH5*'		18.933		20.211		23.234		22.316

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.456		-2.487		-2.519		-3.019
	PH5*'		14.901		15.933		18.456		17.687

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.161		-2.177		-2.177		-2.693
	PH5*'		11.186		11.971		13.938		13.338

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.841		-1.856		-1.841		-2.341
	PH5*'		7.985		8.544		9.993		9.541

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.563		-1.563		-1.547		-2.047
	PH5*'		5.524		5.941		7.057		6.705

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.309		-1.309		-1.309		-1.809
	PH5*'		3.893		4.244		5.264		4.937

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.108		-1.108		-1.092		-1.592
	PH5*'		2.882		3.236		4.256		3.929

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.931		-0.931		-0.915		-1.415
	PH5*'		2.212		2.599		3.657		3.325

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.762		-0.762		-0.762		-1.262
	PH5*'		1.729		2.164		3.273		2.931

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.645		-0.645		-0.645		-1.145
	PH5*'		1.349		1.840		3.005		2.652

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.547		-0.547		-0.547		-1.047
	PH5*'		1.050		1.598		2.819		2.456

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.464		-0.464		-0.464		-0.964
	PH5*'		0.813		1.416		2.688		2.315

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.395		-0.395		-0.395		-0.895
	PH5*'		0.625		1.279		2.595		2.214

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.336		-0.336		-0.336		-0.836
	PH5*'		0.476		1.174		2.528		2.142

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.286		-0.286		-0.286		-0.786
	PH5*'		0.360		1.096		2.480		2.089

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.244		-0.244		-0.244		-0.744
	PH5*'		0.271		1.036		2.446		2.050

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.209		-0.209		-0.209		-0.709
	PH5*'		0.202		0.991		2.421		2.022

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.178		-0.178		-0.178		-0.678
	PH5*'		0.151		0.958		2.403		2.002

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.152		-0.152		-0.152		-0.652
	PH5*'		0.112		0.932		2.389		1.987

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.131		-0.131		-0.131		-0.631
	PH5*'		0.083		0.914		2.380		1.976

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.112		-0.112		-0.112		-0.612
	PH5*'		0.061		0.900		2.373		1.968

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-3.593		-3.718		-3.953		-4.406
	PH5*'		126.292		130.489		139.819		137.100

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-4.138		-4.247		-4.497		-4.934
	PH5*'		115.128		118.829		127.250		124.752

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-4.742		-4.852		-5.086		-5.492
	PH5*'		100.904		104.101		111.482		109.283

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-5.178		-5.256		-5.444		-5.834
	PH5*'		84.392		87.093		93.433		91.537

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-5.290		-5.336		-5.477		-5.946
	PH5*'		67.268		69.517		74.824		73.215

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-5.035		-5.066		-5.160		-5.628
	PH5*'		51.256		53.032		57.304		56.019

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-4.540		-4.571		-4.618		-5.102
	PH5*'		37.398		38.709		41.913		40.939

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-3.959		-3.975		-3.990		-4.490
	PH5*'		26.162		27.028		29.242		28.566

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-3.300		-3.396		-3.396		-3.896
	PH5*'		17.773		18.305		19.765		19.320

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.868		-2.868		-2.884		-3.368
	PH5*'		12.170		12.508		13.559		13.223

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.434		-2.434		-2.434		-2.934
	PH5*'		8.668		8.952		9.866		9.573

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.056		-2.056		-2.056		-2.556
	PH5*'		6.451		6.721		7.604		7.319

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.715		-1.715		-1.715		-2.215
	PH5*'		4.965		5.234		6.116		5.833

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.452		-1.452		-1.452		-1.952
	PH5*'		3.888		4.173		5.074		4.784

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.231		-1.231		-1.231		-1.731
	PH5*'		3.094		3.397		4.336		4.036

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.045		-1.045		-1.045		-1.545
	PH5*'		2.483		2.818		3.805		3.493

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.889		-0.889		-0.889		-1.389
	PH5*'		2.002		2.377		3.420		3.094

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.756		-0.756		-0.756		-1.256
	PH5*'		1.613		2.036		3.137		2.798

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.645		-0.645		-0.645		-1.145
	PH5*'		1.293		1.771		2.930		2.578

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.550		-0.550		-0.550		-1.050
	PH5*'		1.029		1.564		2.778		2.414

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.470		-0.470		-0.470		-0.970
	PH5*'		0.813		1.402		2.666		2.293

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.402		-0.402		-0.402		-0.902
	PH5*'		0.635		1.276		2.583		2.202

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.344		-0.344		-0.344		-0.844
	PH5*'		0.490		1.177		2.523		2.135

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.294		-0.294		-0.294		-0.794
	PH5*'		0.373		1.100		2.478		2.086

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.252		-0.252		-0.252		-0.752
	PH5*'		0.288		1.045		2.446		2.050

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-5.265		-5.453		-5.843		-6.125
	PH5*'		348.152		355.869		373.410		368.270

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-6.288		-6.429		-6.882		-7.226	
	PH5*'	318.619		325.261		340.504		336.012	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-7.440		-7.565		-7.893		-8.315	
	PH5*'	279.906		285.406		298.286		294.454	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-8.435		-8.529		-8.748		-9.185	
	PH5*'	233.236		237.840		248.586		245.381	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-8.802		-8.817		-9.052		-9.489	
	PH5*'	184.468		188.126		196.818		194.203	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-8.529		-8.591		-8.701		-9.154	
	PH5*'	138.813		141.651		148.501		146.437	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-7.803		-7.835		-7.913		-8.350	
	PH5*'	100.114		102.170		107.166		105.632	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-6.865		-6.896		-6.912		-7.412	
	PH5*'	69.619		70.945		74.227		73.235	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-5.925		-5.925		-5.940		-6.440	
	PH5*'	47.209		47.988		49.966		49.358	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-5.035		-5.050		-5.050		-5.550	
	PH5*'	32.076		32.525		33.726		33.336	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-4.276		-4.276		-4.276		-4.776	
	PH5*'	22.351		22.674		23.605		23.301	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'	-3.615		-3.615		-3.631		-4.131	
	PH5*'	16.084		16.366		17.203		16.928	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-3.048		-3.048		-3.048		-3.548
	PH5*'		11.858		12.128		12.934		12.669

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.582		-2.582		-2.582		-3.082
	PH5*'		8.914		9.176		9.974		9.713

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-2.190		-2.190		-2.190		-2.690
	PH5*'		6.827		7.095		7.896		7.633

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.859		-1.859		-1.859		-2.359
	PH5*'		5.327		5.599		6.416		6.149

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.580		-1.580		-1.580		-2.080
	PH5*'		4.224		4.507		5.351		5.077

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.345		-1.345		-1.345		-1.845
	PH5*'		3.393		3.695		4.575		4.293

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-1.146		-1.146		-1.146		-1.646
	PH5*'		2.751		3.078		4.006		3.712

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.978		-0.978		-0.978		-1.478
	PH5*'		2.239		2.602		3.584		3.277

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.836		-0.836		-0.836		-1.336
	PH5*'		1.823		2.229		3.271		2.951

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.714		-0.714		-0.714		-1.214
	PH5*'		1.477		1.933		3.038		2.704

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V5*'		-0.611		-0.611		-0.611		-1.111
	PH5*'		1.187		1.697		2.863		2.516

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	30.500		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.524		-0.524		-0.524		-1.024
		PH5*'		0.955		1.511		2.731		2.372

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	35.598		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V5*'		-0.449		-0.449		-0.449		-0.949
		PH5*'		0.797		1.389		2.644		2.278

MUS	MUB	SIGX	RO,	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		-0.359		-0.750
	PH6*'		0.000		1.602		5.309		4.228

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.031		-0.359		-0.234
	PH6*'		0.000		1.598		5.277		4.204

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.015		0.031		-0.468
	PH6*'		0.000		1.549		5.097		4.053

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.015		-0.484
	PH6*'		0.000		1.455		4.750		3.786

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.015		-0.500
	PH6*'		0.000		1.338		4.325		3.454

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		1.224		3.856		3.093

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		1.130		3.419		2.762

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		1.065		3.083		2.514

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		1.024		2.865		2.356

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.998		2.740		2.264

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.982		2.670		2.211

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.972		2.627		2.179

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.966		2.599		2.158

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.962		2.581		2.145

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.959		2.569		2.135

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.957		2.560		2.129

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.956		2.554		2.125

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.955		2.550		2.122

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.954		2.547		2.120

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.954		2.545		2.118

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.954		2.544		2.117

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.953		2.543		2.116

MUS	MUP	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.953		2.542		2.116

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.953		2.541		2.115

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.000		0.000		0.000		-0.500
	PH6*'		0.000		0.953		2.541		2.115

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		0.218		0.187		-0.500		-0.906
	PH6*'		1.461		2.939		6.262		5.260

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.281		-0.281		-0.610		-1.047
	PH6*'		1.397		2.891		6.193		5.197

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.312		-0.358		-0.608		-1.046
	PH6*'		1.270		2.745		5.959		4.988

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.277		-0.214		-0.152		-0.667
	PH6*'		1.089		2.495		5.536		4.624

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.260		-0.245		-0.245		-0.745
	PH6*'		0.875		2.185		4.945		4.121

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.248		-0.248		-0.264		-0.764
	PH6*'		0.660		1.860		4.314		3.591

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.233		-0.233		-0.248		-0.748
	PH6*'		0.472		1.566		3.732		3.103

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.208		-0.208		-0.224		-0.724
	PH6*'		0.329		1.342		3.275		2.725

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.181		-0.181		-0.197		-0.697
	PH6*'		0.228		1.198		2.975		2.479

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.155		-0.155		-0.171		-0.655
	PH6*'		0.159		1.111		2.803		2.338

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.132		-0.132		-0.132		-0.632
	PH6*'		0.111		1.058		2.708		2.258

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.112		-0.112		-0.112		-0.612
	PH6*'		0.078		1.024		2.651		2.210

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.095		-0.095		-0.095		-0.595
	PH6*'		0.055		1.002		2.616		2.180

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.080		-0.080		-0.080		-0.580
	PH6*'		0.039		0.987		2.592		2.160

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.068		-0.068		-0.068		-0.568
	PH6*'		0.028		0.977		2.577		2.146

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.058		-0.058		-0.058		-0.558
	PH6*'		0.020		0.970		2.566		2.137

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.049		-0.049		-0.049		-0.549
	PH6*'		0.014		0.965		2.558		2.131

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.042		-0.042		-0.042		-0.542
	PH6*'		0.010		0.962		2.553		2.126

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.035		-0.035		-0.035		-0.535
	PH6*'		0.007		0.959		2.549		2.123

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.030		-0.030		-0.030		-0.530
	PH6*'		0.005		0.958		2.547		2.120

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.026		-0.026		-0.026		-0.526
	PH6*'		0.004		0.956		2.545		2.119

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.022		-0.022		-0.022		-0.522
	PH6*'		0.002		0.955		2.543		2.117

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.019		-0.019		-0.019		-0.519
	PH6*'		0.002		0.955		2.542		2.117

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.016		-0.016		-0.016		-0.516
	PH6*'		0.001		0.954		2.542		2.116

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
0.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.014		-0.014		-0.014		-0.514
	PH6*'		0.001		0.954		2.541		2.116

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.703		-0.734		-0.765		-1.234
	PH6*'		4.619		5.985		8.835		7.951

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.798		-0.860		-0.907		-1.392
	PH6*'		4.359		5.687		8.557		7.676

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.858		-0.921		-0.983		-1.452
	PH6*'		3.983		5.285		8.100		7.233

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.835		-0.882		-0.960		-1.429
	PH6*'		3.523		4.754		7.475		6.630

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.740		-0.740		-0.709		-1.271
	PH6*'		2.966		4.114		6.617		5.856

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.621		-0.621		-0.543		-1.075
	PH6*'		2.338		3.363		5.547		4.890

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.513		-0.528		-0.513		-1.028
	PH6*'		1.720		2.636		4.546		3.974

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.448		-0.464		-0.464		-0.964
	PH6*'		1.197		2.046		3.769		3.257

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.379		-0.394		-0.394		-0.894
	PH6*'		0.824		1.646		3.257		2.786

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.327		-0.327		-0.327		-0.827
	PH6*'		0.577		1.404		2.966		2.518

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.265		-0.265		-0.280		-0.780
	PH6*'		0.410		1.260		2.810		2.372

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.224		-0.224		-0.224		-0.724
	PH6*'		0.294		1.166		2.720		2.287

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.190		-0.190		-0.190		-0.690
	PH6*'		0.211		1.103		2.662		2.233

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.161		-0.161		-0.161		-0.661
	PH6*'		0.152		1.059		2.625		2.197

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.136		-0.136		-0.136		-0.636
	PH6*'		0.110		1.029		2.600		2.172

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.116		-0.116		-0.116		-0.616
	PH6*'		0.079		1.007		2.582		2.155

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.098		-0.098		-0.098		-0.598
	PH6*'		0.057		0.992		2.570		2.144

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.084		-0.084		-0.084		-0.584
	PH6*'		0.042		0.981		2.561		2.135

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.071		-0.071		-0.071		-0.571
	PH6*'		0.030		0.974		2.555		2.130

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.061		-0.061		-0.061		-0.561
	PH6*'		0.022		0.968		2.551		2.125

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.052		-0.052		-0.052		-0.552
	PH6*'		0.016		0.964		2.548		2.122

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.044		-0.044		-0.044		-0.544
	PH6*'		0.011		0.961		2.546		2.120

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.038		-0.038		-0.038		-0.538
	PH6*'		0.008		0.959		2.544		2.119

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.032		-0.032		-0.032		-0.532
	PH6*'		0.006		0.957		2.543		2.117

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.028		-0.028		-0.028		-0.528
	PH6*'		0.004		0.956		2.542		2.117

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.046		-1.093		-1.093		-1.593
	PH6*'		8.371		9.708		12.552		11.711

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.174		-1.220		-1.252		-1.736
	PH6*'		7.810		9.106		11.936		11.078

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.264		-1.295		-1.342		-1.826
	PH6*'		7.085		8.313		11.068		10.226

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.253		-1.284		-1.331		-1.816
	PH6*'		6.236		7.380		9.999		9.197

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.173		-1.188		-1.204		-1.688
	PH6*'		5.294		6.327		8.741		7.998

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.979		-1.011		-0.979		-1.479
	PH6*'		4.282		5.181		7.268		6.634

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.840		-0.840		-0.793		-1.308
	PH6*'		3.225		3.999		5.747		5.215

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.704		-0.704		-0.704		-1.188
	PH6*'		2.303		2.975		4.512		4.045

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.576		-0.592		-0.592		-1.092
	PH6*'		1.600		2.250		3.693		3.260

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.483		-0.498		-0.498		-0.998
	PH6*'		1.128		1.808		3.227		2.810

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.413		-0.413		-0.413		-0.913
	PH6*'		0.816		1.541		2.974		2.560

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.337		-0.353		-0.353		-0.853
	PH6*'		0.599		1.371		2.830		2.416

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.285		-0.285		-0.285		-0.785
	PH6*'		0.440		1.253		2.740		2.323

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.242		-0.242		-0.242		-0.742
	PH6*'		0.323		1.170		2.679		2.260

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.205		-0.205		-0.205		-0.705
	PH6*'		0.236		1.110		2.638		2.218

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.174		-0.174		-0.174		-0.674
	PH6*'		0.173		1.067		2.609		2.188

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.148		-0.148		-0.148		-0.648
	PH6*'		0.126		1.036		2.590		2.167

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.126		-0.126		-0.126		-0.626
	PH6*'		0.092		1.013		2.576		2.152

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.107		-0.107		-0.107		-0.607
	PH6*'		0.067		0.997		2.566		2.142

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.091		-0.091		-0.091		-0.591
	PH6*'		0.049		0.985		2.559		2.134

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.078		-0.078		-0.078		-0.578
	PH6*'		0.036		0.976		2.553		2.129

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.067		-0.067		-0.067		-0.567
	PH6*'		0.026		0.970		2.550		2.125

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.057		-0.057		-0.057		-0.557
	PH6*'		0.019		0.965		2.547		2.122

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.049		-0.049		-0.049		-0.549
	PH6*'		0.014		0.962		2.545		2.120

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
1.500	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.042		-0.042		-0.042		-0.542
	PH6*'		0.010		0.960		2.544		2.118

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.328		-1.375		-1.390		-1.890
	PH6*'		12.555		13.990		17.030		16.130

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.471		-1.518		-1.565		-2.049
	PH6*'		11.614		13.002		15.972		15.086

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.529		-1.639		-1.685		-2.170
	PH6*'		10.467		11.752		14.601		13.743

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.593		-1.624		-1.702		-2.187
	PH6*'		9.141		10.304		12.990		12.168

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.481		-1.527		-1.559		-2.074
	PH6*'		7.720		8.750		11.183		10.440

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.353		-1.353		-1.353		-1.853
	PH6*'		6.249		7.123		9.220		8.578

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.104		-1.151		-1.120		-1.604
	PH6*'		4.768		5.477		7.191		6.665

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.944		-0.944		-0.928		-1.428
	PH6*'		3.445		4.020		5.441		4.999

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.789		-0.789		-0.773		-1.289
	PH6*'		2.412		2.948		4.252		3.847

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.654		-0.654		-0.654		-1.154
	PH6*'		1.719		2.275		3.568		3.172

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.546		-0.546		-0.546		-1.046
	PH6*'		1.262		1.872		3.193		2.798

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.465		-0.465		-0.465		-0.965
	PH6*'		0.945		1.615		2.978		2.579

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.381		-0.381		-0.381		-0.881
	PH6*'		0.710		1.438		2.843		2.440

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.322		-0.322		-0.322		-0.822
	PH6*'		0.531		1.309		2.752		2.344

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.273		-0.273		-0.273		-0.773
	PH6*'		0.396		1.214		2.690		2.278

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.232		-0.232		-0.232		-0.732
	PH6*'		0.294		1.145		2.646		2.231

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.197		-0.197		-0.197		-0.697
	PH6*'		0.217		1.093		2.616		2.199

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.168		-0.168		-0.168		-0.668
	PH6*'		0.160		1.056		2.595		2.175

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.143		-0.143		-0.143		-0.643
	PH6*'		0.118		1.028		2.580		2.158

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.122		-0.122		-0.122		-0.622
	PH6*'		0.087		1.008		2.569		2.146

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.104		-0.104		-0.104		-0.604
	PH6*'		0.064		0.994		2.561		2.138

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.089		-0.089		-0.089		-0.589
	PH6*'		0.047		0.983		2.555		2.131

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.076		-0.076		-0.076		-0.576
	PH6*'		0.034		0.975		2.551		2.127

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.065		-0.065		-0.065		-0.565
	PH6*'		0.025		0.969		2.548		2.123

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
2.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.056		-0.056		-0.056		-0.556
	PH6*'		0.018		0.965		2.546		2.121

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.125		-2.203		-2.296		-2.781
	PH6*'		34.177		36.297		40.846		39.533

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.380		-2.443		-2.583		-3.052
	PH6*'		31.250		33.219		37.523		36.256

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.621		-2.668		-2.762		-3.278
	PH6*'		27.641		29.414		33.360		32.202

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.733		-2.780		-2.890		-3.358
	PH6*'		23.553		25.115		28.663		27.607

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.665		-2.727		-2.790		-3.274
	PH6*'		19.325		20.665		23.768		22.833

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.472		-2.503		-2.534		-3.034
	PH6*'		15.255		16.349		18.939		18.153

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.177		-2.177		-2.193		-2.693
	PH6*'		11.506		12.337		14.355		13.749

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.856		-1.856		-1.856		-2.356
	PH6*'		8.263		8.845		10.344		9.888

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.563		-1.563		-1.563		-2.063
	PH6*'		5.772		6.194		7.355		6.985

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.309		-1.309		-1.309		-1.809
	PH6*'		4.102		4.471		5.517		5.184

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.108		-1.108		-1.092		-1.592
	PH6*'		3.071		3.441		4.487		4.154

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.931		-0.931		-0.915		-1.415
	PH6*'		2.386		2.789		3.876		3.537

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.762		-0.762		-0.762		-1.262
	PH6*'		1.889		2.343		3.484		3.134

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.645		-0.645		-0.645		-1.145
	PH6*'		1.493		2.005		3.210		2.848

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.547		-0.547		-0.547		-1.047
	PH6*'		1.176		1.752		3.019		2.645

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.464		-0.464		-0.464		-0.964
	PH6*'		0.921		1.559		2.884		2.500

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.395		-0.395		-0.395		-0.895
	PH6*'		0.716		1.412		2.788		2.395

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.336		-0.336		-0.336		-0.836
	PH6*'		0.551		1.299		2.720		2.319

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.286		-0.286		-0.286		-0.786
	PH6*'		0.420		1.214		2.671		2.264

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.244		-0.244		-0.244		-0.744
	PH6*'		0.318		1.148		2.635		2.224

MUS	MUP	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.209		-0.209		-0.209		-0.709
	PH6*'		0.239		1.099		2.609		2.195

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.178		-0.178		-0.178		-0.678
	PH6*'		0.179		1.062		2.591		2.173

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.152		-0.152		-0.152		-0.652
	PH6*'		0.133		1.034		2.577		2.158

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.131		-0.131		-0.131		-0.631
	PH6*'		0.098		1.013		2.567		2.146

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
4.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.112		-0.112		-0.112		-0.612
	PH6*'		0.073		0.997		2.560		2.138

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-3.593		-3.718		-3.968		-4.406
	PH6*'		126.639		130.882		140.273		137.531

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-4.138		-4.278		-4.513		-4.950
	PH6*'		115.463		119.215		127.687		125.170

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-4.742		-4.852		-5.055		-5.524
	PH6*'		101.221		104.457		111.884		109.675

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-5.147		-5.272		-5.459		-5.881
	PH6*'		84.684		87.431		93.806		91.908

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-5.290		-5.352		-5.477		-5.946	
	PH6*'	67.541		69.831		75.173		73.556	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-5.035		-5.066		-5.144		-5.628	
	PH6*'	51.516		53.327		57.631		56.345	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-4.540		-4.571		-4.618		-5.102	
	PH6*'	37.649		38.978		42.222		41.246	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-3.959		-3.975		-3.990		-4.490	
	PH6*'	26.390		27.283		29.518		28.836	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-3.380		-3.396		-3.396		-3.896	
	PH6*'	17.991		18.535		20.021		19.567	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-2.868		-2.868		-2.868		-3.384	
	PH6*'	12.359		12.731		13.816		13.472	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-2.434		-2.434		-2.434		-2.934	
	PH6*'	8.871		9.170		10.099		9.805	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-2.056		-2.056		-2.056		-2.556	
	PH6*'	6.649		6.933		7.830		7.544	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-1.715		-1.715		-1.715		-2.215	
	PH6*'	5.160		5.440		6.338		6.053	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-1.452		-1.452		-1.452		-1.952	
	PH6*'	4.077		4.372		5.291		4.998	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-1.231		-1.231		-1.231		-1.731	
	PH6*'	3.275		3.588		4.548		4.244	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.045		-1.045		-1.045		-1.545
	PH6*'		2.656		3.001		4.012		3.695

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.889		-0.889		-0.889		-1.389
	PH6*'		2.165		2.551		3.622		3.290

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.756		-0.756		-0.756		-1.256
	PH6*'		1.764		2.200		3.336		2.989

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.645		-0.645		-0.645		-1.145
	PH6*'		1.431		1.926		3.126		2.764

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.550		-0.550		-0.550		-1.050
	PH6*'		1.152		1.710		2.971		2.596

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.470		-0.470		-0.470		-0.970
	PH6*'		0.920		1.539		2.857		2.471

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.402		-0.402		-0.402		-0.902
	PH6*'		0.726		1.405		2.773		2.378

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.344		-0.344		-0.344		-0.844
	PH6*'		0.564		1.299		2.711		2.309

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	30.500	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.294		-0.294		-0.294		-0.794
	PH6*'		0.433		1.216		2.666		2.258

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
9.000	35.598	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.252		-0.252		-0.252		-0.752
	PH6*'		0.341		1.158		2.634		2.222

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.000	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-5.265		-5.453		-5.843		-6.234
	PH6*'		340.446		356.175		373.722		368.592

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.161	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-6.210		-6.413		-6.851		-7.226	
	PH6*'	318.910		325.555		340.808		336.320	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.349	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-7.440		-7.581		-7.909		-8.315	
	PH6*'	280.170		285.706		298.576		294.748	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.568	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-8.419		-8.513		-8.748		-9.185	
	PH6*'	233.534		238.126		248.882		245.672	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	0.822	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-8.786		-8.880		-9.052		-9.505	
	PH6*'	184.730		188.392		197.102		194.475	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.116	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-8.545		-8.576		-8.701		-9.170	
	PH6*'	139.067		141.915		148.772		146.702	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.459	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-7.819		-7.850		-7.897		-8.382	
	PH6*'	100.354		102.414		107.445		105.913	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	1.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-6.881		-6.881		-6.912		-7.412	
	PH6*'	69.844		71.178		74.480		73.471	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.320	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-5.909		-5.925		-5.940		-6.440	
	PH6*'	47.423		48.215		50.197		49.578	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	2.857	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-5.019		-5.050		-5.050		-5.550	
	PH6*'	32.295		32.737		33.956		33.560	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	3.481	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-4.276		-4.276		-4.276		-4.776	
	PH6*'	22.554		22.896		23.832		23.521	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	4.206	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'	-3.615		-3.631		-3.631		-4.131	
	PH6*'	16.285		16.578		17.423		17.145	

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	5.049	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-3.048		-3.048		-3.048		-3.548
	PH6*'		12.058		12.338		13.150		12.882

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	6.028	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.582		-2.582		-2.582		-3.082
	PH6*'		9.111		9.383		10.186		9.924

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	7.166	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-2.190		-2.190		-2.190		-2.690
	PH6*'		7.022		7.299		8.106		7.841

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	8.487	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.859		-1.859		-1.859		-2.359
	PH6*'		5.519		5.800		6.623		6.353

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	10.023	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.580		-1.580		-1.580		-2.080
	PH6*'		4.411		4.704		5.554		5.278

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	11.807	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.345		-1.345		-1.345		-1.845
	PH6*'		3.574		3.886		4.775		4.489

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	13.879	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-1.146		-1.146		-1.146		-1.646
	PH6*'		2.924		3.264		4.203		3.905

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	16.287	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.978		-0.978		-0.978		-1.478
	PH6*'		2.403		2.780		3.779		3.468

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	19.085	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.836		-0.836		-0.836		-1.336
	PH6*'		1.976		2.399		3.464		3.138

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	22.336	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.714		-0.714		-0.714		-1.214
	PH6*'		1.620		2.094		3.229		2.888

MUS	MUB	SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	26.112	0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
	V6*'		-0.611		-0.611		-0.611		-1.111
	PH6*'		1.326		1.849		3.053		2.697

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	30.500		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V6*'		-0.524		-0.524		-0.524		-1.024
		PH6*'		1.098		1.658		2.921		2.551

MUS	MUB		SIGX	RO	SIGX	RO	SIGX	RO	SIGX	RO
16.000	35.598		0.000	0.000	0.500	0.000	1.000	0.000	1.000	0.500
		V6*'		-0.449		-0.449		-0.449		-0.949
		PH6*'		0.961		1.541		2.835		2.459

Appendix F

An Approximation for the Optimal Control Policy of the Unknown Constant Gain Problem

From Appendix E it can be seen that in many cases (generally whenever $\mu_b \geq 1.0$) the control policies for $\sigma_x = 0.0$, 0.5 and 1.0 with $\rho = 0.0$ are identical and that for $\sigma_x = 1.0$ and $\rho = 0.5$ the control policy is exactly 0.5 less than the other three. This led to the investigation of the variation of the control policy with respect to σ_x and ρ (see Table F.1). From Table F.1 it can be seen that for large μ_b

$$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho] \doteq \nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\,\sigma_x \qquad \text{F.1}$$

is a very accurate approximation while for small μ_b this approximation gives a value of $\nu^{*'}$ that is within 7% of the true value of $\nu_2^{*'}$. From Figures D.1 and D.2 it can be seen that because of the shallow minima when μ_b is small, a 7% error in the minimizing value of the control gives at most a 2% error in the expected cost and this error diminishes as μ_s and N increase. The approximation of Equation F.1 was checked for other values of N with similar results to those given in Table F.1. Therefore the approximation

$$\nu_N^{*'}[\mu_s, \mu_b, \sigma_x, \rho] \doteq \nu_N^{*'}[\mu_s, \mu_b, 0, 0] - \rho\,\sigma_x \qquad \text{F.2}$$

used in Chapter 7 is justified since the accuracy claimed there for the expected costs is not greater than 2%. From Equation C.18b it can be seen that for N = 1 Equation F.2 is exact.

Table F.1 Comparison of the Optimal Policy with an Approximation

μ_s, μ_b	Control (N = 2)	σ_x, ρ 0.25, 0.0	σ_x, ρ 0.75, 0.0	σ_x, ρ 1.0, 0.0	σ_x, ρ 0.25, 0.25	σ_x, ρ 0.75, 0.25	σ_x, ρ 1.0, 0.25
0.7, 0.45	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 0.309	- 0.293	- 0.277	- 0.371	- 0.480	- 0.528
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 0.301	- 0.301	- 0.301	- 0.361	- 0.489	- 0.551
7.75, 0.45	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 4.10	- 4.19	- 4.26	- 4.16	- 4.36	- 4.49
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 4.01	- 4.01	- 4.01	- 4.07	- 4.51	- 4.55
0.7, 9.2	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 0.075	- 0.075	- 0.075	- 0.138	- 0.263	- 0.325
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 0.075	- 0.075	- 0.075	- 0.138	- 0.263	- 0.325
7.75, 9.2	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 0.833	- 0.833	- 0.833	- 0.895	- 1.02	- 1.08
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 0.833	- 0.833	- 0.833	- 0.895	- 1.02	- 1.08

		σ_x, ρ 0.25, 0.5	σ_x, ρ 0.75, 0.5	σ_x, ρ 1.0, 0.5	σ_x, ρ 0.25, 0.75	σ_x, ρ 0.75, 0.75	σ_x, ρ 1.0, 0.75
0.7, 0.45	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 0.434	- 0.668	- 0.793	- 0.496	- 0.856	- 1.04
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 0.426	- 0.676	- 0.801	- 0.489	- 0.864	- 0.97
7.75, 0.45	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 4.23	- 4.54	- 4.71	- 4.28	- 4.70	- 4.92
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 4.44	- 4.70	- 4.82	- 4.59	- 4.97	- 5.16
0.7, 9.2	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 0.200	- 0.450	- 0.575	- 0.263	- 0.638	- 0.825
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 0.200	- 0.450	- 0.575	- 0.263	- 0.638	- 0.825
7.75, 9.2	$\nu_2^{*'}[\mu_s, \mu_b, \sigma_x, \rho]$	- 0.958	- 1.21	- 1.33	- 1.02	- 1.40	- 1.58
	$\nu_2^{*'}[\mu_s, \mu_b, 0, 0] - \rho\sigma_x$	- 0.958	- 1.21	- 1.33	- 1.02	- 1.40	- 1.58

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