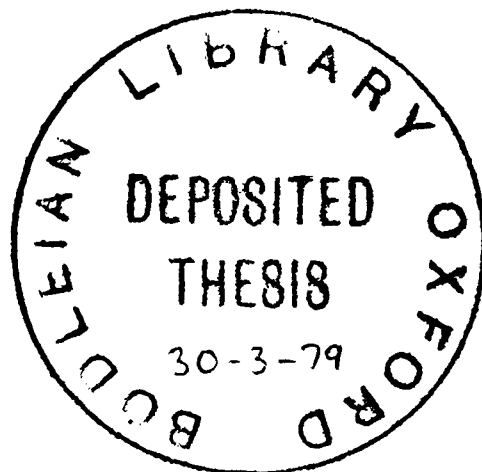




STABILITY OF SPHERICAL SHELLS

by

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ABSTRACT

Recent application of using spheres as carriers of LNG on ships has highlighted a lack of guidance given to the inclusion of imperfections and to the operating conditions imposed as a result of the motion of the ship.

Spheres with a radius to thickness ratio of between 450 and 1700 were made by electrodeposition of copper on a wax former. Imperfections of three types were introduced:

- a) A local flat.
- b) A zone of reduced curvature.
- c) A bulge.

These shells were tested under partial vacuum and the deformation at selected points monitored by proximity gauges. Buckling pressures were found to range between $1/5$ and $1/15$ of the theoretical value for a perfect shell. The experimental results were analysed on the basis of Koiter's imperfection theory and by a numerical computer solution. The former showed good agreement with types a) and b) and no agreement with type c); the latter showed little agreement with the experimental results. The ASME design code was found to be adequate for all imperfection sizes considered, BS 5500 only partly.

Cast epoxy models with a radius to thickness ratio of 900, supported on an equatorial ring and partially filled with water, were subjected to vertical accelerations both normal and inclined to their equatorial rings. Buckling was detected by proximity gauges, and occurred as a result of tensile meridional and compressive circumferential

stresses. The sphere was found to be imperfection insensitive and to be more susceptible to buckling at approximately $1/3$ and $2/3$ full for respectively the normal and inclined loading cases. Both a numerical computer solution and an analytical analysis were found to be in good agreement with the experimental results. Both the ASME and BS5500 design codes were however, found for this mode of failure to be too conservative for design purposes.

P R E F A C E

This dissertation is an account of the research carried out by the author in the Department of Engineering Science in the University of Oxford. The work was carried out under the supervision of Dr. C. Ruiz. No part of the dissertation has been submitted for a degree at any other University. The research described here is original, though the work of others has been drawn upon freely, with due acknowledgement in the text. A complete list of references appears at the end of the thesis.

The author would like to thank Dr. C. Ruiz for his guidance and advice throughout the work. He would also like to thank the many other people who have given him assistance, but especially the following:

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CHAPTER I

STABILITY OF THIN-WALLED SPHERICAL TANKS: INTRODUCTION

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CHAPTER 1

STABILITY OF THIN-WALLED SPHERICAL TANKS : INTRODUCTION

1.1 INTRODUCTION

The aim of this investigation is to study the elastic stability of thin-walled spheres of the type used for the transportation of liquefied natural gas (LNG).

The most economical method found of storing the gas has been to operate the spheres at a pressure slightly above atmospheric pressure and at a temperature of -162°C , the resulting liquefied state being at approximately 1/600th of its volume as a gas. Their radius to thickness ratios are high in order to make the most economical use of the expensive material that must be specified at these low temperatures. Furthermore, the use of thin walls allows fast welding rates thereby shortening the construction time.

It will subsequently be shown, however, that using partially filled spheres on ships may give rise to stability problems and therefore the possibility of catastrophic failure. This occurrence would not only endanger operating personnel but would also produce economic consequences as to the viability of using thin-walled spheres for this particular application.

1.2 DESIGN CONDITIONS

The loading conditions encountered are the following:

- a) Internal pressure due to a combination of hydrostatic loading and a small vapour pressure of 40 mbar.

b) Thermal strains due to operation of the spheres at -162°C and at room temperature during emptying.

c) External pressure due to;

1) emptying of the container which may introduce a vacuum,

2) sinking of the ship,

3) design conditions which may be different from the operating conditions, i.e. design codes have stipulated that the sphere must be able to withstand an external pressure of 2 KN/m^2 .

d) External loading which is experienced due to the interaction between the hull and sphere and to an acceleration when the tank is in the vertical position and when it is tilted as a result of the motion of the ship, as shown in Figs. 1.1a and b. In the latter loading case, the design stipulation is a maximum expected vertical acceleration of 1.43 g .

1.3 DESIGN OF THE SPHERICAL LNG TANK

It was originally proposed that the independent spherical tank design be used since it was assumed it could be classed as a pressure vessel, which implied that an accurate stress analysis could be performed. It was therefore thought that the tank would not suffer catastrophic failure, and so no emergency containment system was envisaged. However, insurance companies have since concluded that a secondary safety barrier should be provided between the sphere and the hull structure, although it is stipulated that it need only be designed for small leakage and therefore its size is somewhat reduced, which still

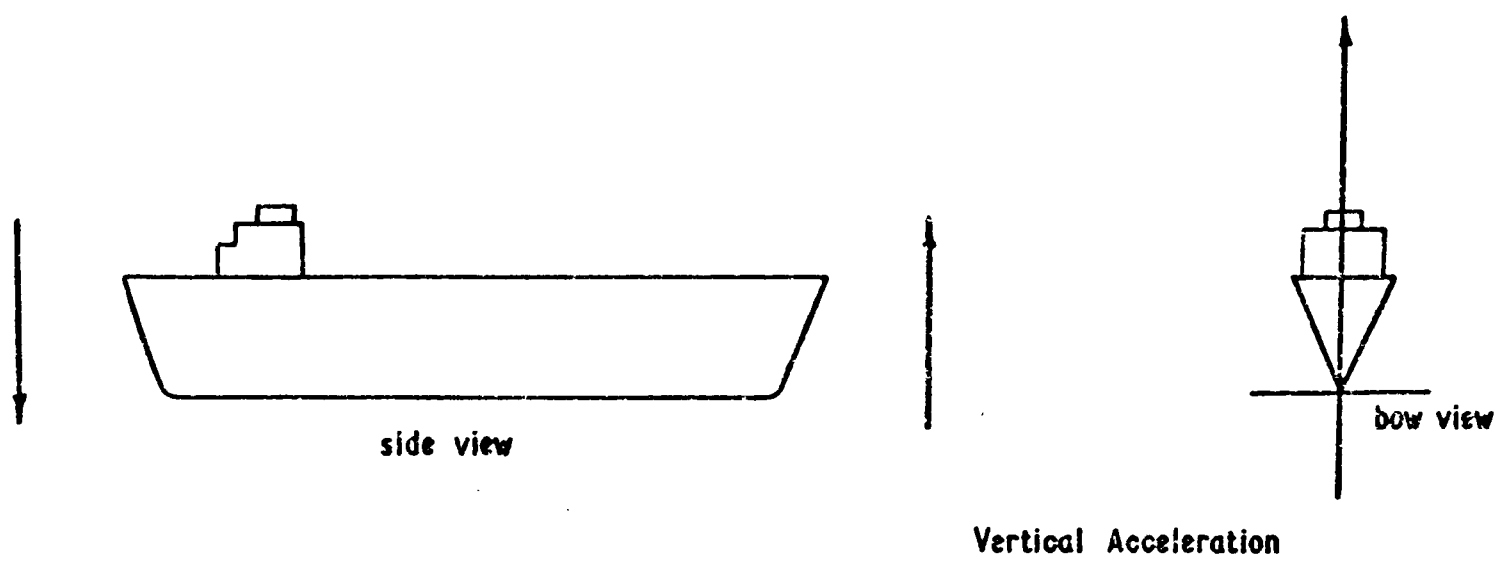


Fig.1.1a. Pitching motion of ship

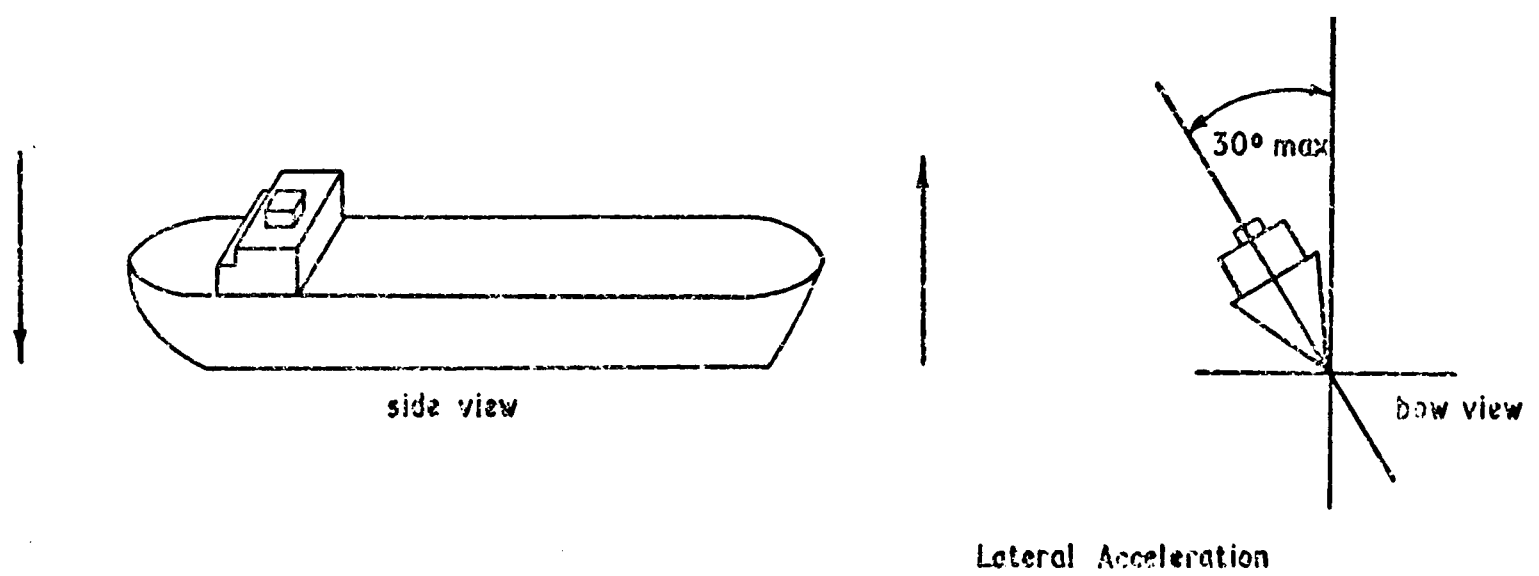


Fig.1.1b. Pitching and tilting motion of ship

enables savings in weight and cost to be made. This barrier is designed to be external to the sphere, consisting of a drip tray catchment underneath, in addition to a system of splash guards at the sides, as shown in Fig. 1.2, which is a typical section of a spherical tank design.

The thin-walled spheres used have diameters of between 30 and 35 m with plate thicknesses varying in each sphere from approximately 25 to 45 mm. At present, ships have been designed to contain up to five of these tanks, each of which holds 20000 m³ of gas. This enables ships of typically 250 m in length, with a breadth of 40 m and a draught of 10 m to transport 100000 m³ of LNG. It is thought, however, in the growing field of LNG transportation, yet larger carriers will be needed and capacities of 200000 m³ and beyond are envisaged. Shown in Fig. 1.3 is a typical profile of a free-standing spherical tank cargo system.

Insulation for the spheres is provided either by filling the space between the sphere and the ship's structure with perlite or by installing either PVC, polyurethane foam, or mineral wool, their use being to limit heat loss across the walls of the tank. The extensive use of this insulation is shown in Fig. 1.2.

Where deviations between particular designs of spherical tanks do occur, is in the method of support. A number of support designs are given in Fig. 1.4, each design's object being to provide stiffness at the junction between the cylindrical skirt and the sphere while minimising stress concentration effects which may give rise to fracture.

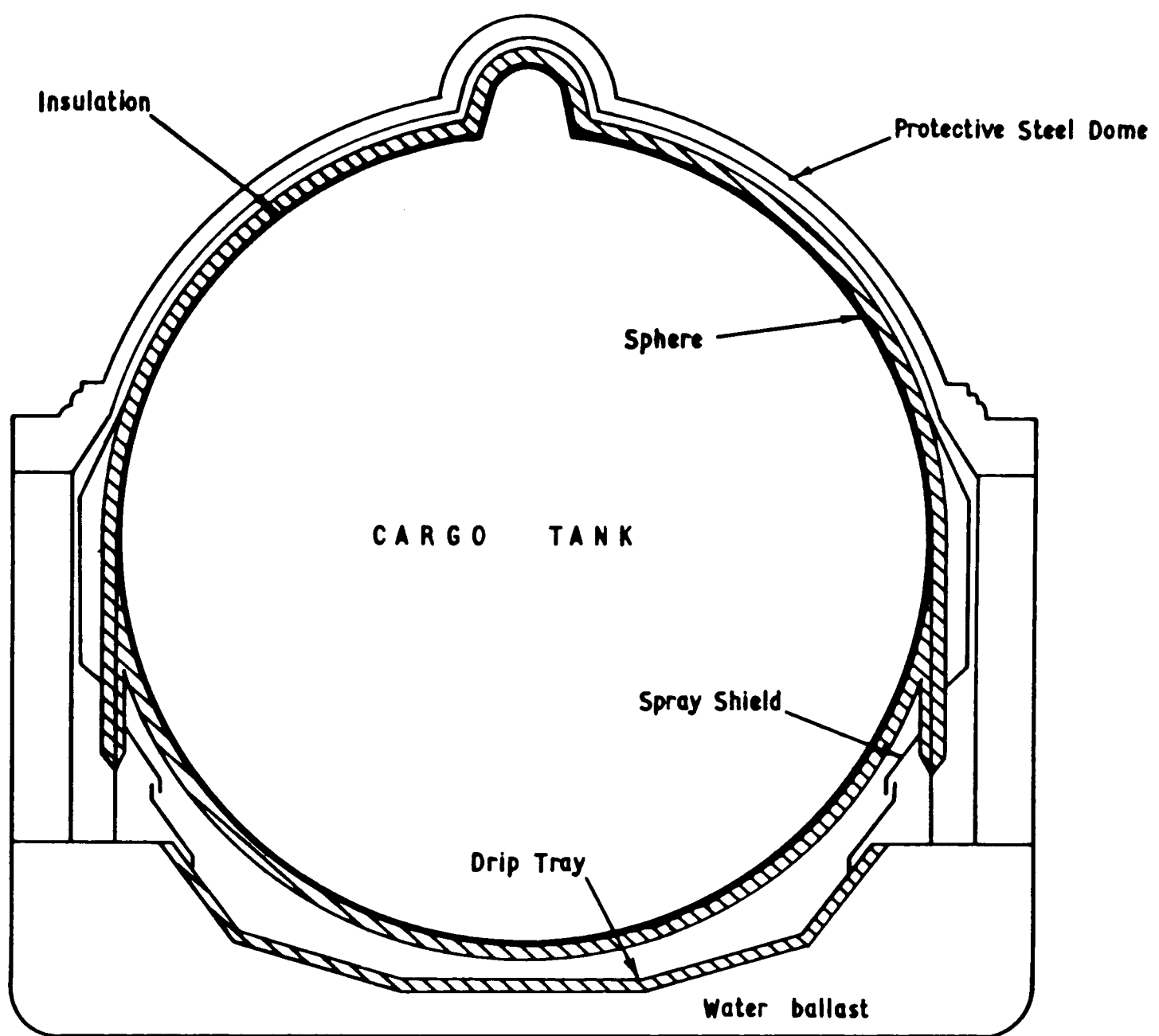


Fig.1.2. Section of a spherical tank design (Ref. 79.)

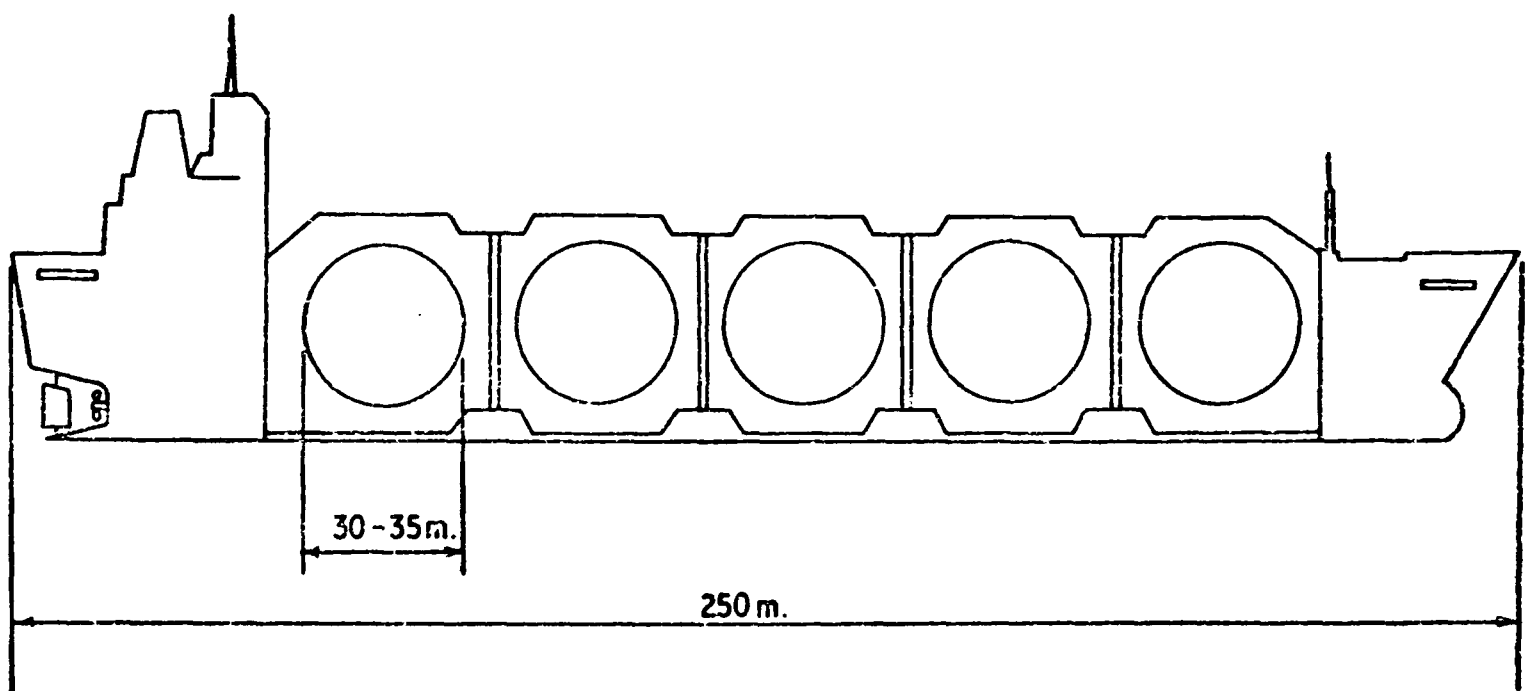


Fig. 1.3. Typical profile of a free standing spherical tank cargo system.

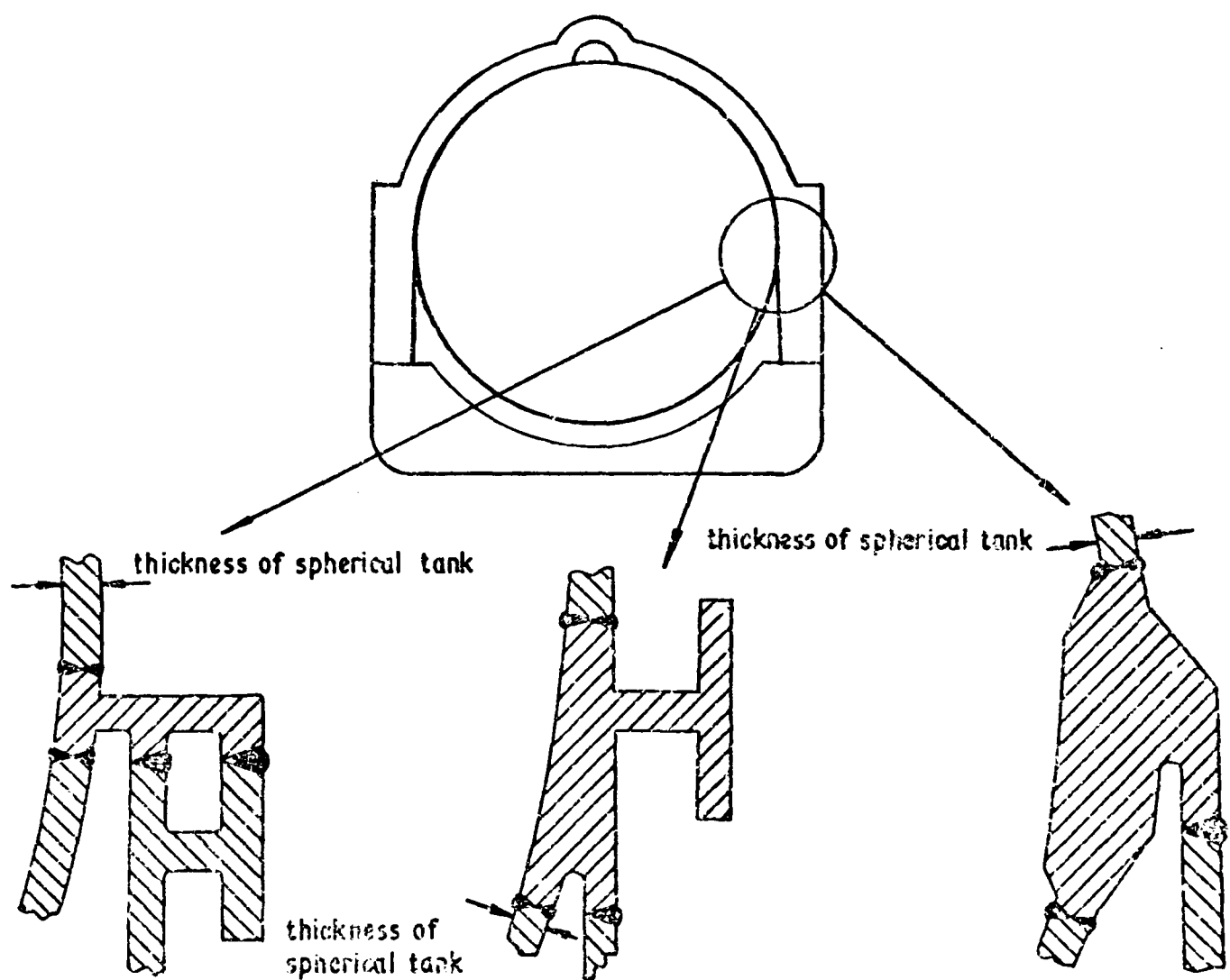


Fig. 1.4. Three different types of support designs for the spherical tank.

1.4 MATERIAL

The materials considered at these low temperatures are 9% Nickel steel, Aluminium alloy 5083 (Al-4.5% Mg-0.7% Mn), austenitic stainless steel and Invar (Fe-36% Ni), their chemical composition and mechanical properties being supplied to design codes ASTM A353-74,^[5] ASTM 553-74,^[6] ASTM B209-77,^[7] and BS 1501-509,^[14].

Although all these materials offer satisfactory resistance to brittle fracture and high yield and tensile strength, it is 9% Ni steel that predominates in the construction of thin-walled spheres used as containers of LNG. The principal reasons for its preferred use are that both austenitic stainless steel and Invar are prohibitively expensive and that Al alloy has a lower yield stress than 9% Ni steel which results in the need to increase the wall thickness. On a typical 30 m diameter sphere this necessitates using a radius-to-thickness ratio of 350 for Al alloy as compared to 800 for spheres made of 9% Ni steel. Furthermore, thermal contraction of the Al alloy at -162°C is approximately twice that of 9% Ni steel, which results in a circumferential contraction of respectively 0.3 m and 0.15 m for the same 30 m diameter sphere.

For these reasons given, between 60 and 70% of LNG containers either operational or under construction are of 9% Ni steel.

However, the use of 9% Ni steel also presents user difficulties. These are its non-linear expansion property which causes distortion difficulties during welding and that the weld material, usually high Ni alloy (50 to 70% Ni), has tended to have lower mechanical properties thus

preventing the ultimate strength of the steel from being used.

1.5 CONSTRUCTION

The recommended rules for the design and construction of thin-walled spheres are given in design codes, API 620 [3], Det. Norske Veritas^[28], Nippon Kaiji Kyokai^[69], BS 5500^[13], and ASME^[8]. The former three codes cover the specific use of spheres as containers of LNG whereas the latter two are concerned with the more general application of spheres as pressure vessels.

It will subsequently be shown, that the spherical shape is an extremely imperfection sensitive structure under uniform external pressure. The method of construction governs the extent and magnitude of these imperfections and so the correct erection procedure is of paramount importance if their inclusion is to be minimised.

The most satisfactory method found, is to fabricate the middle, upper and lower sections as cylindrical sub-assemblies consisting of welded doubly curved swaged plates, usually 5 m x 3 m for 9% Ni steel and 10 m x 3 m for Al alloy. The middle section is then temporarily supported on columns while the upper and lower sub-assemblies are welded into place. Spherical caps are then welded at both apexes, the upper one allowing the necessary piping to be attached and so enable the tank to be filled and emptied. A diagram showing the positioning of the plates relative to each other and the temporary supports used is given in Fig. 1.5a.

Imperfections can be introduced into the finished tank by numerous means, e.g. uneven heating of the welded joint;

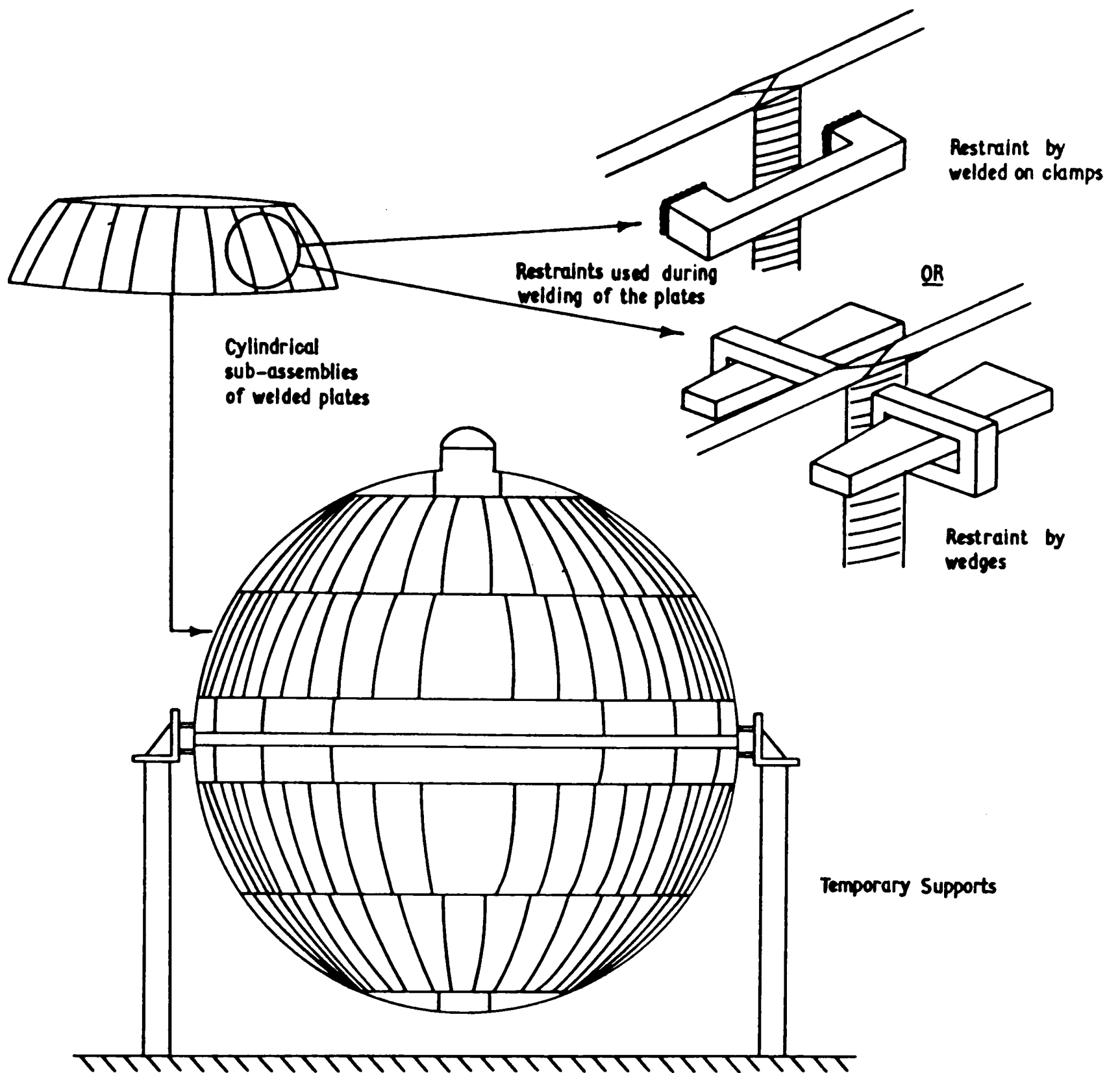


Fig.1.5a. Fabrication of the spheres from welded double curvature plates

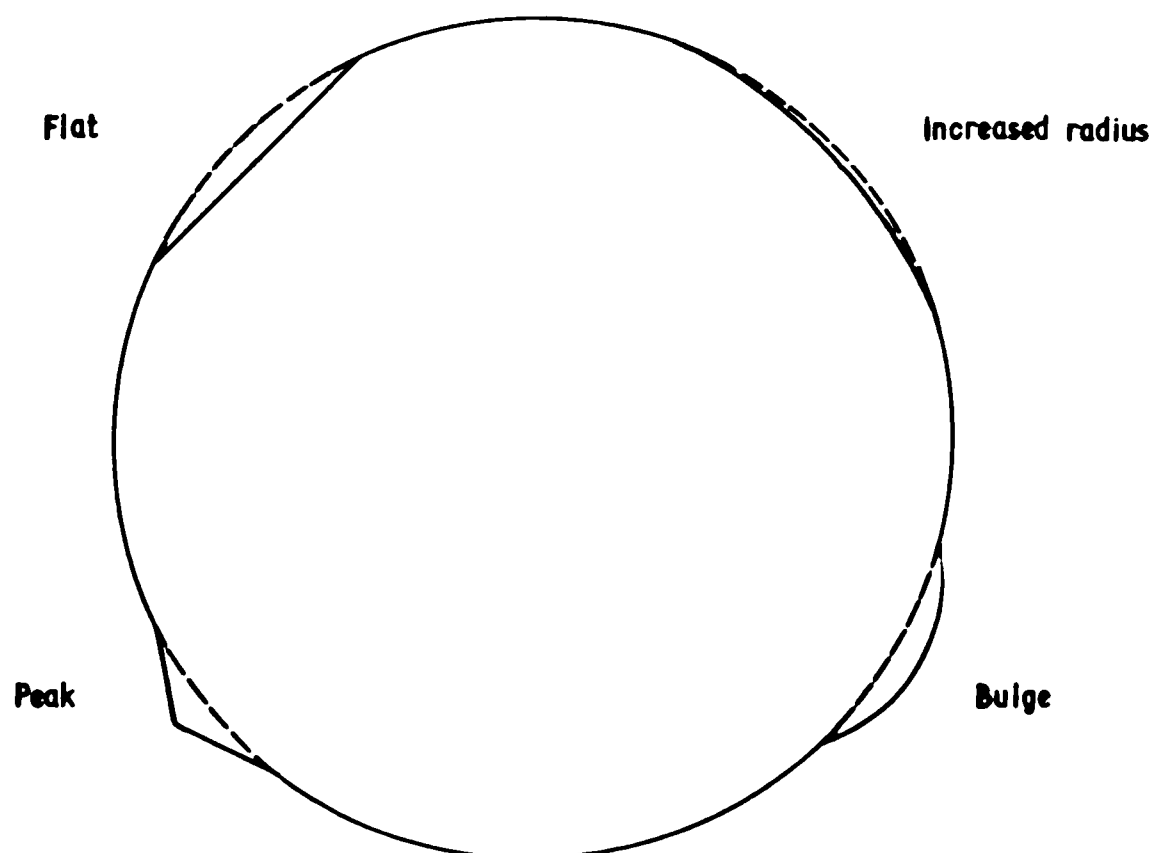


Fig.1.5b. Common geometrical imperfections introduced during fabrication

the action of restraints during welding; thermal expansion properties of the weld and plate metal; dimensional accuracy of the doubly curved plates. These potential areas for imperfections will now be discussed.

Due to uneven heating being produced during the welding of the plates, the weld and plate metal will on cooling contract unevenly and so introduce stresses, which relieve themselves by distorting the plates, and hence a tank of non-spherical shape is produced. This distortion can be reduced, however, by adopting a welding sequence such as shown in Fig. 1.6, whereby the difference in heat input is balanced on either side of the V butt joint such that contraction and expansion of the metal effectively nullify each other. Furthermore, if an automatic welding method is chosen in preference to a manual one, for the usual processes of submerged arc or metal inert gas (M.I.G.), then distortion can be further reduced. This is due to the former's ability of depositing a larger volume of the weld metal in one run and therefore the number of runs required for one joint is reduced. In addition, it avoids the inclusion of random stresses that occur in manual welding due to the shrinkage of the cooling weld and plates during the interval between the deposition of each electrode.

Distortion of the plates can still be further reduced by imposing a restraint on their movement during the welding sequence. Designers are, however, reluctant to provide too rigid a restraint, even though it allows very little distortion of the plates, because of the introduction of high stresses on cooling of the weld and plate metal, which may give rise to yielding. This could be in addition to

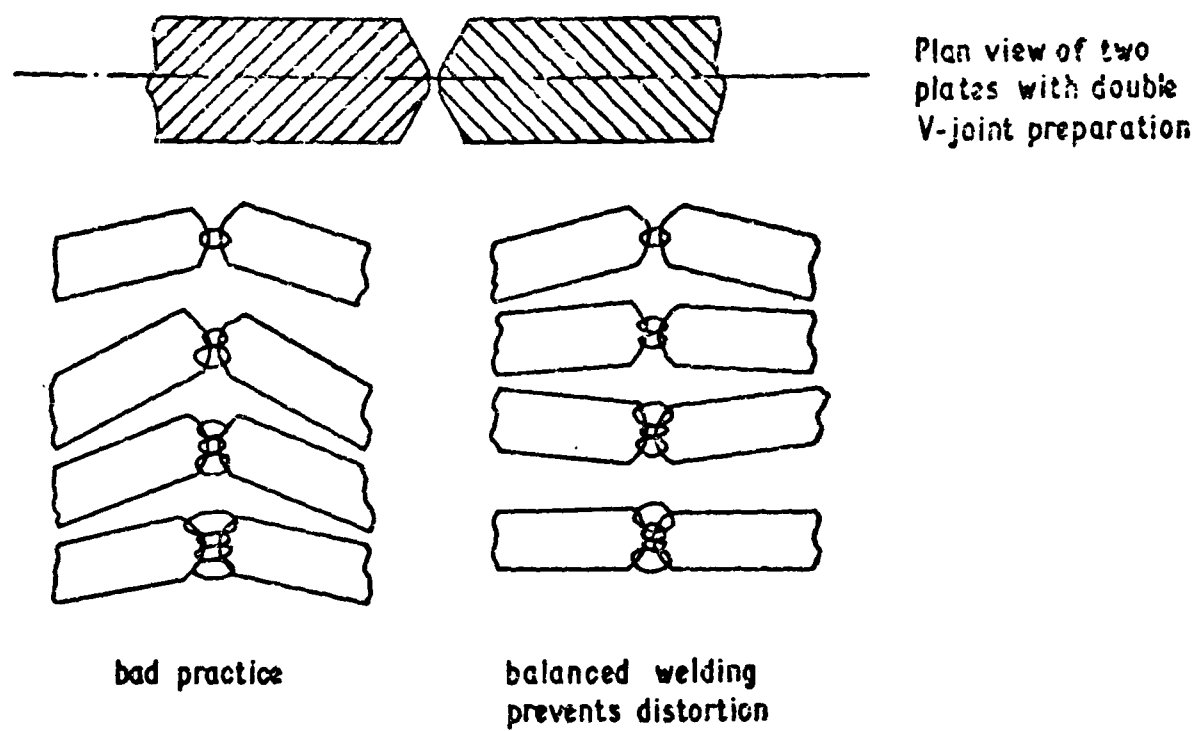


Fig.1.6. Effect of welding procedure (Ref.47).

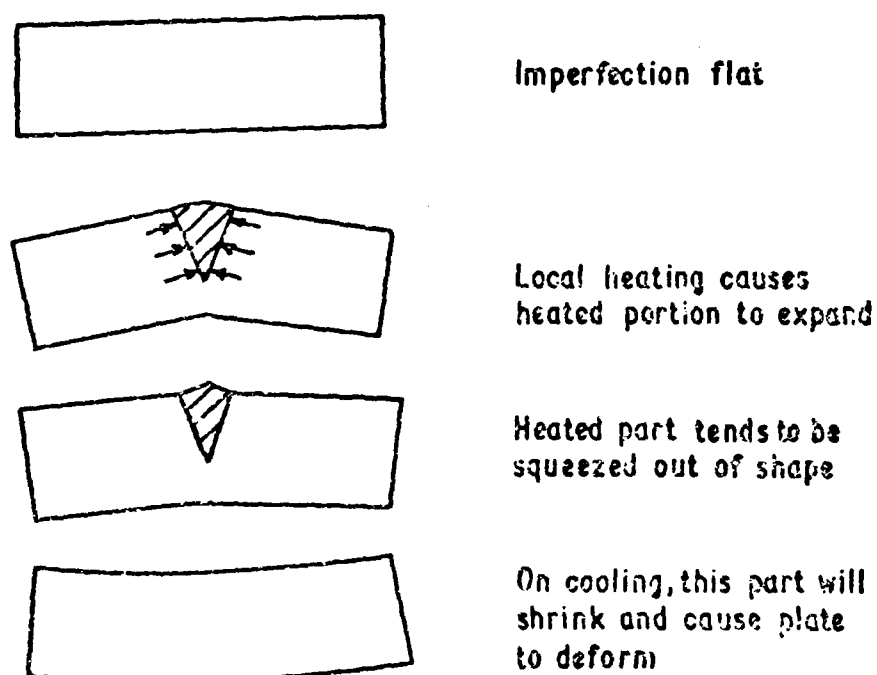


Fig.1.7. Effect of local heating in order to remove imperfections (Ref. 47).

the inherent stresses in the plates prior to welding or conversely these same stresses could have the opposite effect, and thus tend to nullify the stresses introduced by the restraints. The two most popular methods of restraint are by means of welded-on clamps or by the use of wedges as shown in Fig. 1.5a. Welding is then initially made either side of these restraints, the final weld being made after their removal.

Thermal expansion of the plate metal affects the extent to which it is influenced by the input of heat during welding. Al alloy as stated earlier, has a high coefficient of expansion, which causes it to distort much more than 9% Ni steel, although the latter metal has the disadvantage of a non-linear thermal expansion property which makes it difficult to accurately assess the extent of deformation.

The cutting and weld joint preparation are usually performed by an oxy-acetylene torch. If the plates are the incorrect size or shape, then this would cause an error in their location to each other. If it is found that there is too large a gap at the butt joint, then more weld material is required which therefore increases the heat input to the plates and so causes the further possibility of distortion. Conversely, if the gap is too small, then there may be a lack of penetration by the weld and a weak joint may be produced.

Although every effort is made to minimise distortion of the plates caused by the foregoing manufacturing problems, geometric imperfections do occur, the most common being peaking, flats, increased radius and bulges, as shown in Fig. 1.5b.

Attempts made to rectify these imperfections range from the simple expedient use of a hammer to deform the affected area into correct shape, to the more satisfactory method of applying concentrated heat, usually by propane burner and so cause the deformed part to expand. The surrounding colder metal opposes this expansion and tends to deform the heated part such that on cooling, it contracts, and produces a force sufficient to deform the plate into the correct shape, as shown in Fig. 1.7. It is possible, however, to further exaggerate the distorted area, by heating too large an area, or heating to too high a temperature and so extreme caution is necessary. Although this local heating method is usually successful in reducing the peaking effect, only costly substitution of the imperfect plates provides an effective solution; a task which the construction companies are obviously reluctant to undertake.

1.6 PRESENT STUDY

The foregoing presentation has outlined the particular application of thin-walled spheres considered in this thesis.

In order to ascertain what previous analytical and experimental work has been performed, a bibliographical survey was conducted, the findings of which are given in Chapter II. From this survey it was possible to identify the problem areas still remaining in the design of these shells. These were found to be the following:

a) The effect of imperfection shapes and size typically introduced in the construction of a thin-walled sphere, on its ability to avoid collapse under uniform

external pressure.

b) The effect on the stability of a thin-walled sphere of introducing a vertical acceleration normal and inclined to its equatorial ring when it is full/partially full.

In Chapter III, the guidance given by the design codes for both these problems outlined is determined, and in addition an approximate analytical analysis using membrane shell equations is given, from which it was possible to define the experimental programme, which was divided into two parts.

The first part concerned the modelling of thin-walled copper spheres of radius to thickness ratio in the range 450 to 1700, each of which contained one of three imperfection shapes. The effect on the buckling pressure of varying the size and shape of imperfection was examined. A photograph of a model, under uniform external pressure, is given in Fig. 1.8. A detailed description of the manufacture of these spheres is given in Chapter IV and in addition an explanation of the experimental apparatus and testing procedure. The geometry of the 25 models and their buckling pressures are presented in Chapter V. Chapter VI contains a discussion of these results, the object of which was to present the designer with specific guidelines on acceptable imperfection sizes and shape.

The second part of the investigation concerned the manufacture of large diameter nominally perfect, thin-walled plastic spheres of radius to thickness ratio of 900, a description of which is given in Chapter VII. The experimental procedure was to fill the models to various levels of water

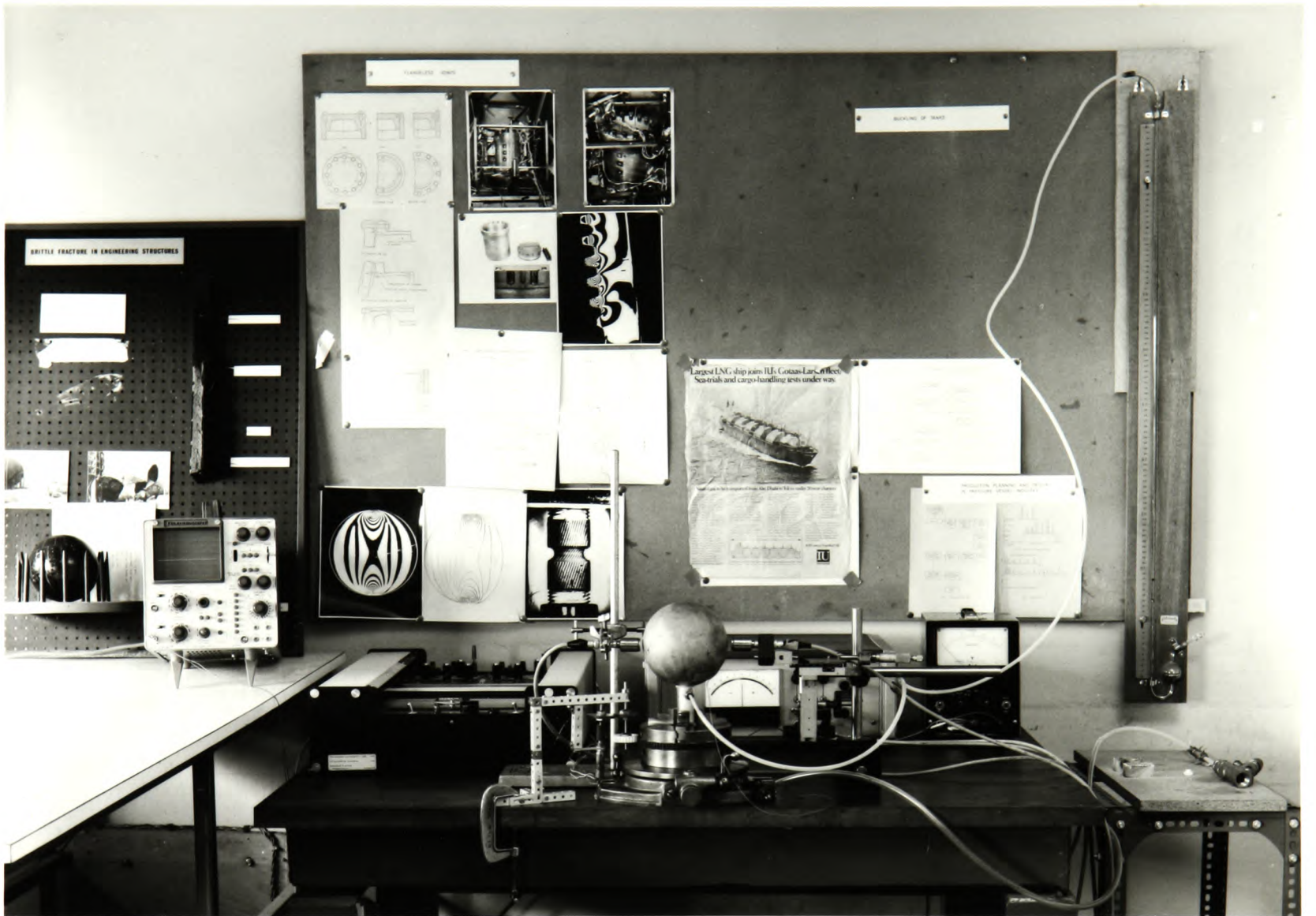


Fig. 1.8 Photograph of an imperfect copper sphere under uniform external pressure.

and to accelerate vertically, both normally to their equatorial ring supports and at the envisaged worst tilting condition of 30 degrees, in order to introduce buckling. A photograph of a model in the inclined loading condition is shown in Fig. 1.9. A description of the method of manufacture, the experimental apparatus and testing technique is given in Chapter VII. Chapter VIII contains a description of the geometry of the six spheres used, together with their buckling results. A discussion of these results is given in Chapter IX and as in the first part of the investigation, guidelines were presented to the designer to ensure that the optimum design, as regards expense and stability of the structure, is used in the construction of thin-walled spheres as carriers of LNG.

Finally in Chapter X, the overall conclusions from the two investigations are presented, and in addition, suggestions for further work proposed.

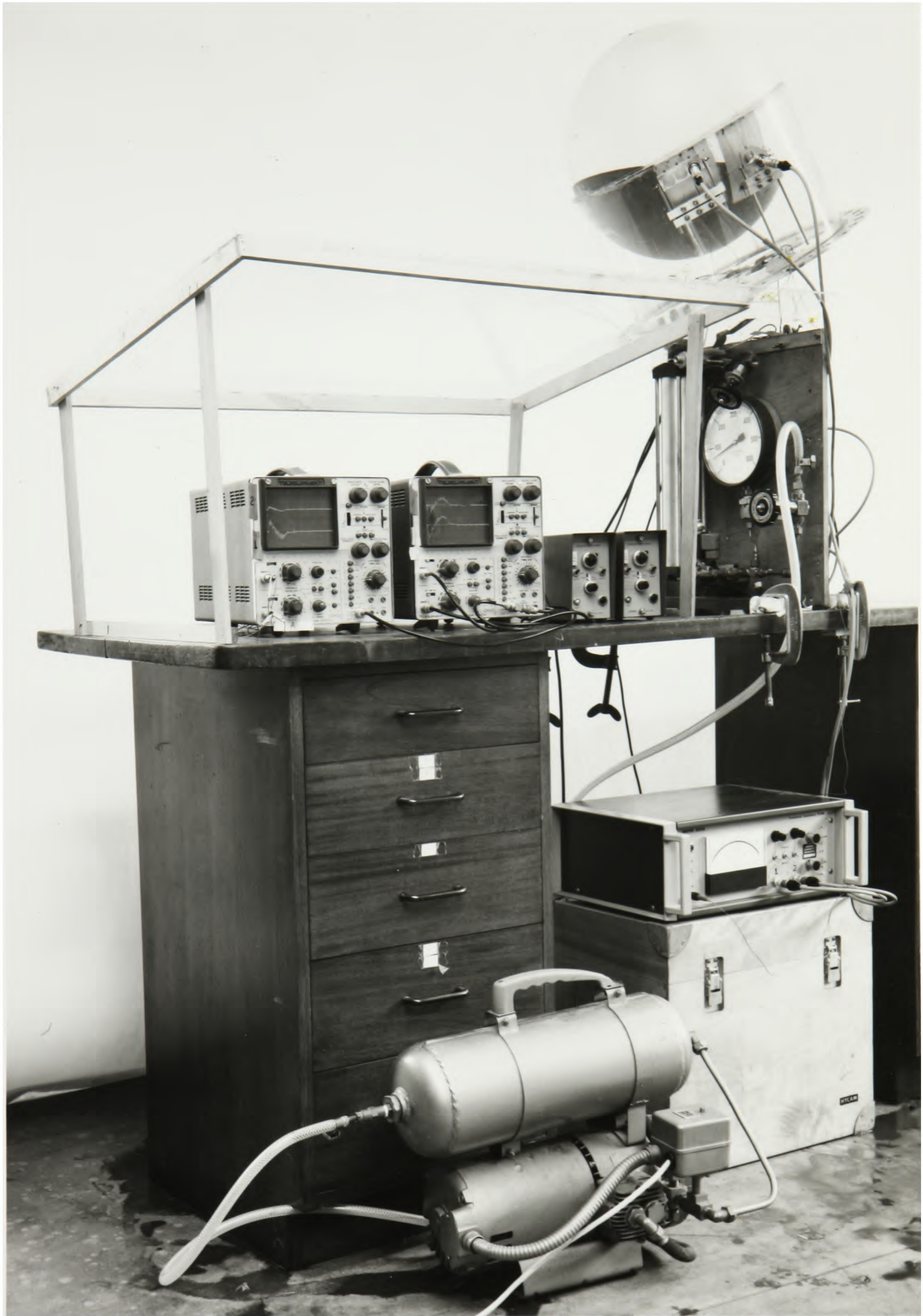


Fig. 1.9 Photograph of a partially filled epoxy sphere at rest after being subjected to a vertical acceleration at 30 degrees to its equatorial ring.

CHAPTER II

BIBLIOGRAPHICAL SURVEY

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CHAPTER II

BIBLIOGRAPHICAL SURVEY

2.1 INTRODUCTION

From the turn of this century, many investigations both theoretical and experimental have been made to further the understanding of the behaviour of thin* shells.

Much of this work has been directed towards their buckling behaviour. Whilst some shell structures, such as a very shallow shell, have shown good agreement between experimentally observed buckling pressures and theoretically predicted ones, results for structures such as circular cylinders under compressive loadings and spherical shells under uniform external pressure have shown large differences between experiment and theory and indeed within both experimental and theoretical investigations.

Although various reasons were given for these discrepancies, and these will be discussed for the spherical shell subsequently, it has only recently been known that for some structures, the inclusion of imperfections can produce deleterious effects on the buckling pressure. The thin-walled sphere has been shown to belong to this category.

* A thin shell is usually defined as a shell whose radius to thickness ratio is greater than 20.

2.2 THEORETICAL WORK

2.2.1 Early work

The first theoretical solution to the problem of buckling of thin-walled spheres under uniform external pressure, was that obtained by Zoelly (1915).

This classical solution was based on linear shell theory, i.e. a change in displacement will cause a corresponding change in the dependent variables (force, stress, strain), the solution of which may be obtained by superposition. It was confined to buckling of spheres experiencing small elastic deformations.

He predicted that buckling would give rise to a series of waves over the whole surface and his buckling criterion was defined as follows: "at a pressure slightly less than the critical pressure the shell on being given a small displacement will move away to a new equilibrium position; hence at the critical pressure, the shell must remain displaced and hence there must be an adjacent equilibrium position under the same pressure".

The classical buckling pressure was found to be given by the following expression,

$$P_C = \frac{2E}{(3(1-\nu^2))^{\frac{1}{2}}} \left(\frac{t}{R}\right)^2 \quad (2.1)$$

where E and ν are Young's modulus of elasticity and Poisson's ratio respectively; t and R are the shell thickness and radius respectively.

The detailed derivation of equ. 2.1 need not be included here, since it is given in many reference textbooks, e.g. Flugge (1973).

It was found, however, that large discrepancies existed between the P_C values, and experimentally observed ones, e.g. Sechler and Bollay (1939) reported that a copper hemisphere with a radius to thickness ratio of 900 had buckled at $\frac{1}{4} P_C$. Furthermore, collapse was observed to initiate with a rotationally symmetric inward dimple which quickly increased in size and was therefore in contradiction with Zoelly's (1915) predicted overall waveform pattern.

2.2.2 Post-buckling behaviour

This apparent discrepancy between theory and experiment initiated an investigation into the post-buckling behaviour of the sphere.

The change in equilibrium configuration in the collapse of a sphere entailed a large increase in the deflection of the shell. The linear theory was therefore thought to be no longer applicable for these large deflections, and so a non-linear theory was proposed. This theory took into account the new equilibrium equations that had to be formulated for the deformed shell and so retained the non-linear terms in the expression for strains as functions of the displacements (see Appendix D).

2.2.3 Lower buckling load

Using this new non-linear theory, von Kármán and Tsien (1939) considered a shallow region of a sphere, deforming axisymmetrically and obtained a lower buckling load P_L which they found had the following expression,

$$P_L = 0.365 E (t/R)^2 \approx 0.3 P_C \quad (2.2)$$

after making the following simplifying assumptions to the complicated non-linear equations:

a) The solid angle of the segment is small, which implies that the deflection of any element of the shell is parallel to the axis of rotational symmetry.

b) The deflection is rotationally symmetric, which thus reduces the problem to two dimensions.

c) The effect of lateral contraction is neglected, i.e. Poisson's ratio is assumed to be zero.

An energy method was chosen to solve the resulting simplified equations. The energy associated with the deformation is:

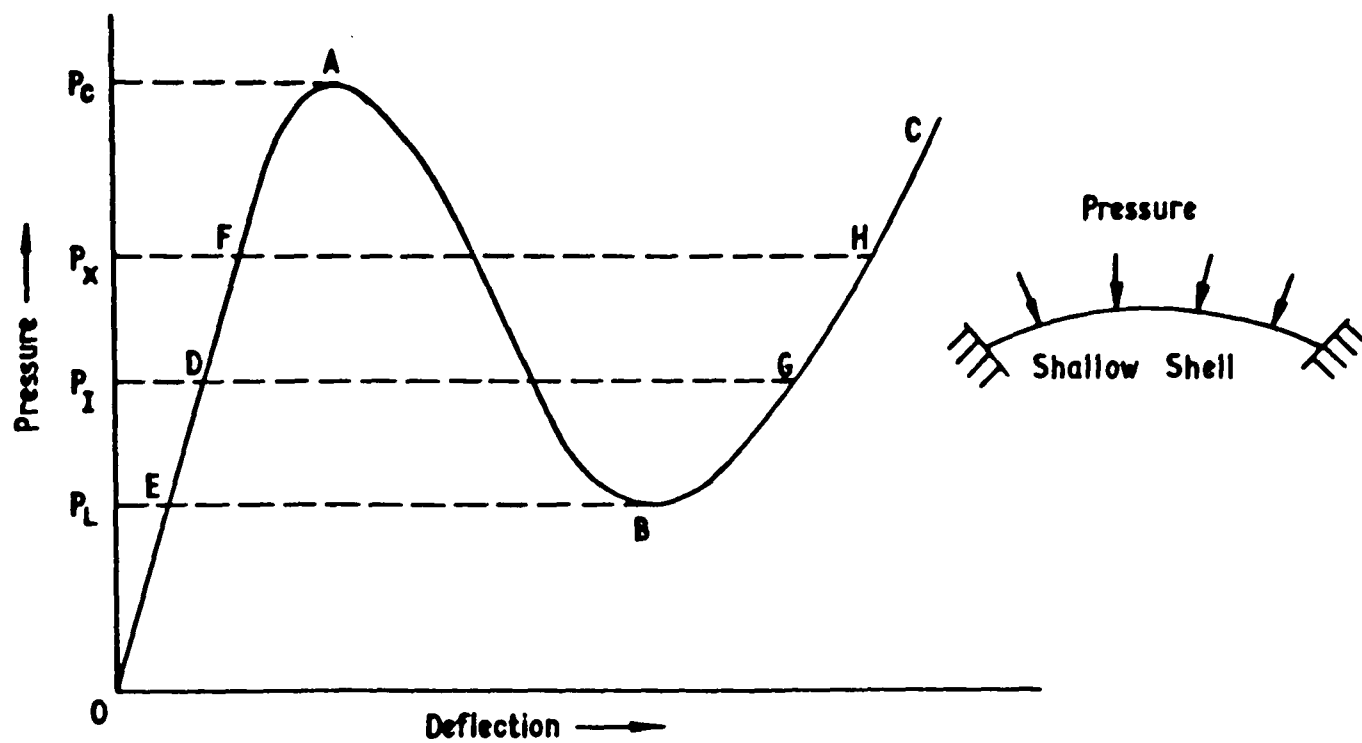
a) The incremental increase in strain energy, ΔU_s

b) The incremental decrease in potential energy of the external forces, ΔU_p

The total potential energy, ΔU , gained or lost during the exchange of the equilibrium forces is constant, and from the equilibrium condition, $\Delta U = \Delta U_s + \Delta U_p$, the stability criterion was expressed as,

$$\partial^2 U = 0 \quad (2.3)$$

P_L as shown in Fig. 2.1, was defined as the minimum load necessary to keep the shell in a buckled state with finite deformations. It was suggested that initial imperfections or disturbances may cause the shell to buckle at this lower load. It was also suggested in order that the designer may account for imperfections introduced during fabrication of thin-walled spheres and spherical shells,



OA = Stable unbuckled state

BC = Stable buckled state

AB = Unstable buckled state

Fig.2.1. Symmetric equilibrium path of shallow shell.

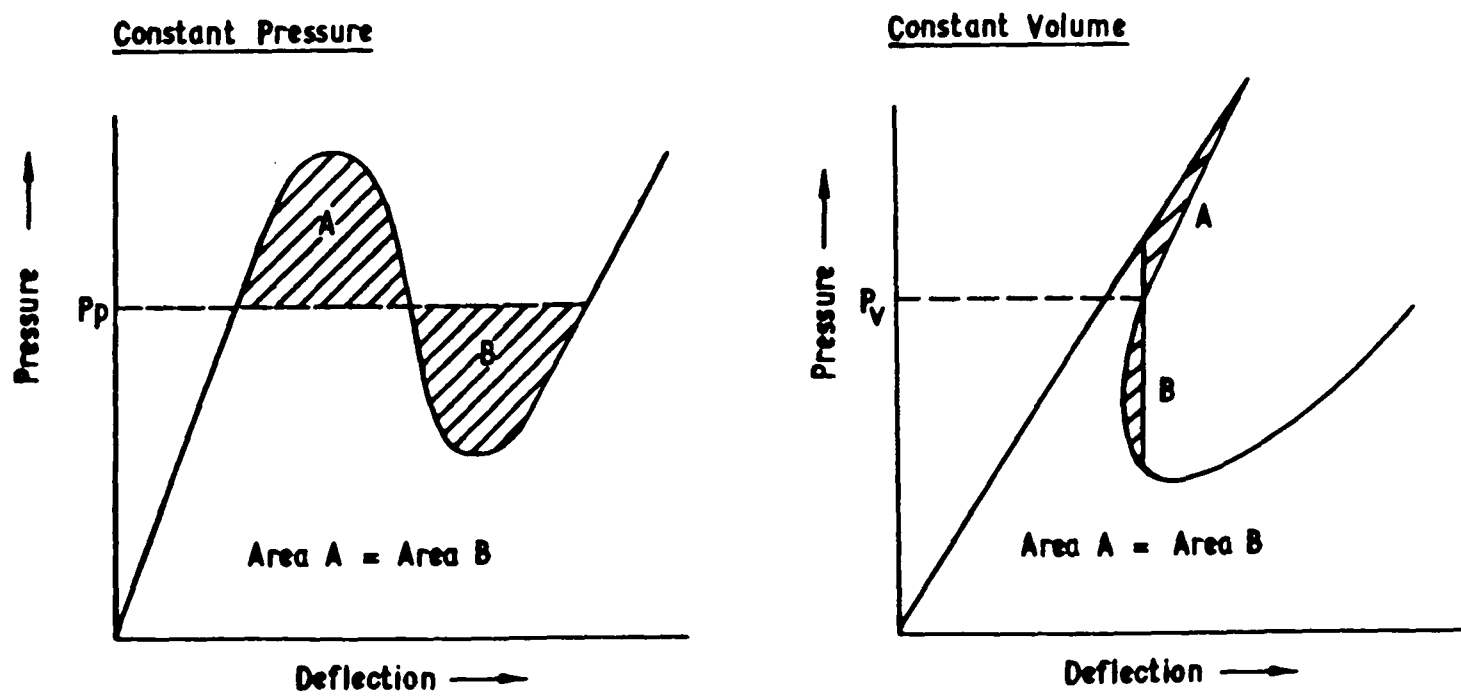


Fig.2.2. The two intermediate buckling loads defined by Tsien (1942)

that P_L may be used as a safe recommended buckling load.

2.2.4 Energy criterion

Friedrich (1941) argued that the buckled state of the shell shown as position B in Fig. 2.1, corresponding to P_L , involves a higher potential energy level than the unbuckled state, shown as position E at the same pressure. Consideration of this fact thus prompted him to define an intermediate collapse load P_I , $P_L < P_I < P_C$ as: "the lower limit of all values of pressure for which a buckled state of equilibrium exists with an energy level below that of the unbuckled state".

In Fig. 2.1, this corresponds to the total potential energy of state B being greater than that of state E; the total potential energy of state D being equal to that of state G; for pressures greater than P_I , P_X say, the total potential energy of state F being greater than that of state H.

Tsien (1942) next considered the effect of the rigidity of the testing medium. By assuming a deflected form of a rotationally symmetric dimple, he calculated, on the basis of equal potential energy in the buckled and unbuckled states, two intermediate buckling loads. These were P_p , which was defined as the collapse pressure obtained at constant pressure and P_v which was defined as the collapse pressure obtained at constant volume. These two pressures are shown in Fig. 2.2.

2.2.5 Summary of early investigations

Von Kármán and Tsien's (1939) evaluation of P_L , has shown a thin-walled spherical shell to be highly unstable under uniform external pressure. The use of this buckling load as the minimum load a sphere can sustain before collapse is however incorrect, since their result was based on static equilibrium equations and therefore preclude the dynamic considerations which may give rise to a totally different buckled configuration.

The use of loads, P_I , P_p , and P_v , which are based on the energy criterion, in order to explain the low observed buckling values is also incorrect since these loads all assume the spherical shell to physically jump from the pre-buckled to the post-buckled state. In order for this to occur there must be another equilibrium path connecting the pre-buckled and post-buckled states in addition to line OABC shown in Fig. 2.1. Since the only other equilibrium path to cross the stable pre-buckled path is that when bifurcation occurs (this term will be explained subsequently), then it can be concluded that the energy criterion cannot be used to represent the phenomenon of buckling. This conclusion is further supported by experimental investigations conducted by Kaplan and Fung (1954), Almroth et al. (1964), Thurston and Penning (1966), Carlson et al. (1967) and Berke (1967), where no differences were found in the buckling pressure of a spherical shell between a constant pressure type loading medium, say air, and a constant volume type loading medium, say water.

2.2.6 Further work

Further theoretical work was then conducted in order to obtain solutions to the non-linear equations for symmetric and asymmetric deformation.

Von Kármán and Kerr (1965) have commented that: "the principal difficulty encountered in this type of problem is that for simultaneous non-linear differential equations of this type, no ways are known to obtain exact general solutions. This is mainly the reason why, until recently, a large number of papers written on this subject restricted their investigations to axially symmetrical deformations, shallow shells, and the solutions were only approximations obtained by means of a variational method by Rayleigh - Ritz or Galerkin. The general procedure is to choose a deflection expression W and substitute it into the compatibility equation. The resulting differential equation in F is then solved. The unknown parameters are then determined by applying the Galerkin method to the equilibrium equation or using the Ritz method".

The equilibrium and compatibility equations referred to are respectively found to take (Hutchinson (1967)) for the shallow spherical shell the following form;

$$D\nabla^4 W + \frac{1}{R} \nabla^2 F - F_{,xx} W_{,yy} - F_{,yy} W_{,xx} + 2F_{,xy} W_{,xy} = -P \quad (2.4)$$

$$+ \frac{1}{Et} \nabla^4 F - \frac{1}{R} \nabla^2 W + W_{,xx} W_{,yy} - W_{,xy}^2 = 0 \quad (2.5)$$

where F and W are respectively the stress function and displacement expression; x and y are cartesian co-ordinates chosen in the base plane of the spherical shell; P is the uniform external pressure; $D = \frac{Et^3}{12(1-\nu^2)}$; ∇^4 and ∇^2 are

the two-dimensional biharmonic and Laplacian operators; and where a comma followed by subscripts represents partial differentiation in turn with respect to each subscript variable.

It was found, [15,85,98], using these non-linear equations and assuming a symmetric buckling shape, that the theoretical predictions were in reasonable agreement with experimental tests performed on spherical caps. However, considerable difference still existed between theory and experiment for the deeper shells.

It was thought the reason for this lack of agreement was due to the asymmetric or bifurcation buckling behaviour which occurs at some load, called the bifurcation load, when the symmetric and asymmetric load equilibrium paths intersect. This is the point Z shown in Fig. 2.3.

Theoretical results for asymmetric collapse [42,71], suggested that while shallow shells will deform symmetrically for the geometric parameter (λ) (defined in Table 2.1) < 5.5 , asymmetric collapse will occur for $(\lambda) > 5.5$, i.e. for deeper shells.

However, although the bifurcation load was approximately $0.8 P_c$, it was not sufficient in itself an explanation for the discrepancies between the experimental results on presumed identical specimens or between these experimental values and bifurcation theory. Attention was then turned to Koiter's (1945) imperfection theory.

2.2.7 Effect of imperfections

Koiter (1945) had shown that the initial post-buckling analysis for spheres yields an asymptotically exact relation

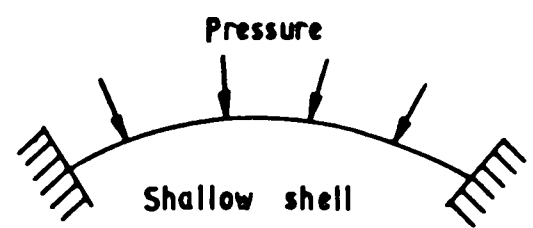
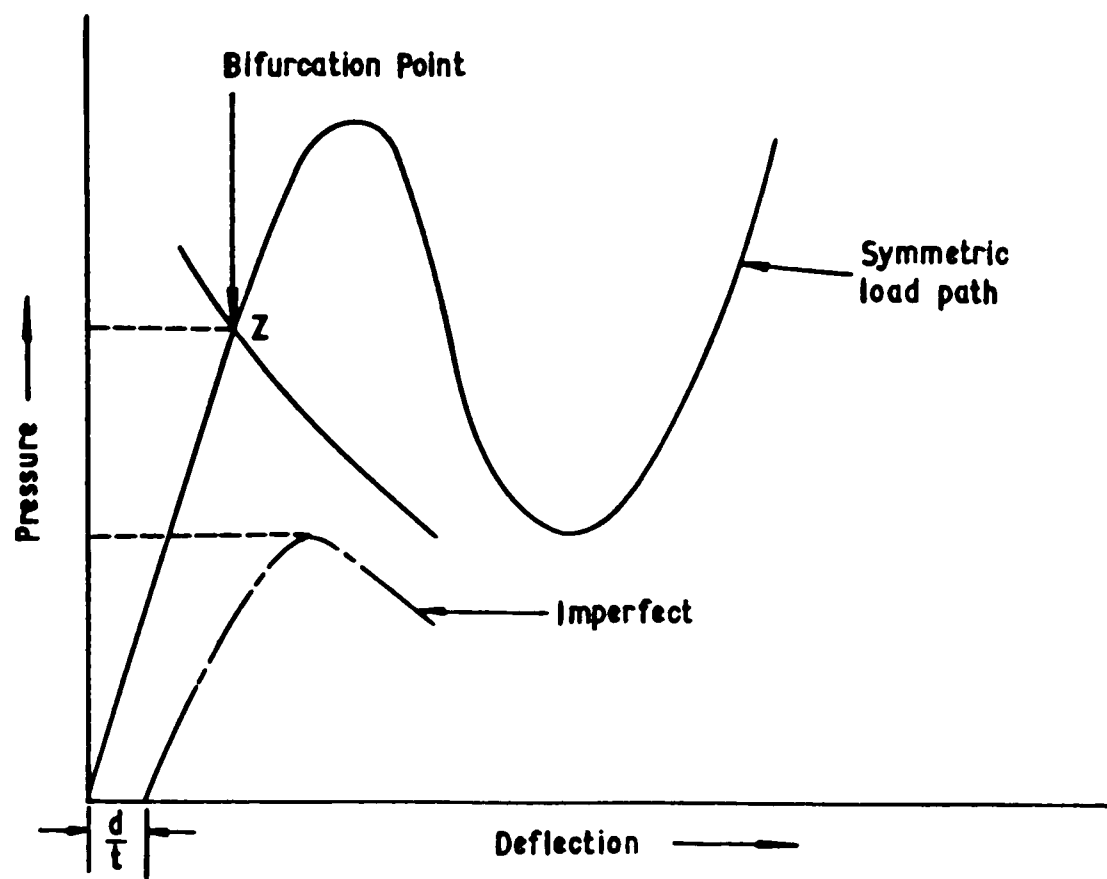


Fig.2.3. Intersection of symmetric load path with an asymmetric load path

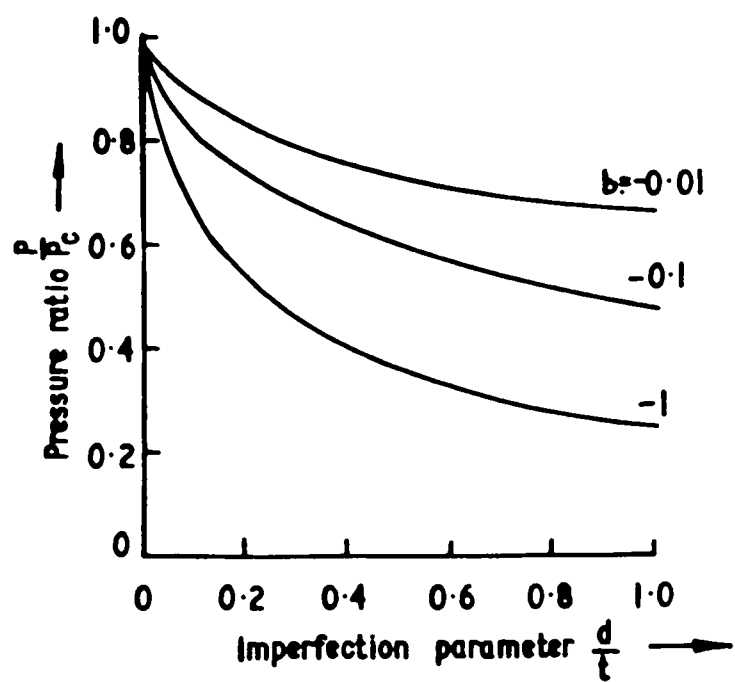


Fig.2.4. Effect of increasing the imperfection parameter b in Koiter's (1945) equation.

between the load parameter P and an imperfection of amplitude $(\bar{d})$ normalized with respect to thickness (t) and in the shape of the buckled mode, valid in the neighbourhood of the bifurcation point of the shell.

The following expression was obtained,

$$P = P_c (1 + A(\bar{d}/t) + b(\bar{d}/t)^2 + \dots) \quad (2.6)$$

where the coefficients A and b determine the initial post-buckling behaviour.

For spherical shells, the linear term was shown to be zero and the post-buckling path was determined by the value of the second order coefficient (b) . For spherical shells under uniform external pressure (b) was found to be negative which indicated that the shell had an unstable post-buckling behaviour.

Koiter (1945) showed that the relationship P/P_c was governed by the following equation,

$$(1 - P/P_c)^{3/2} = \frac{3\sqrt{3}}{2} \left| \frac{\bar{d}}{t} \right| (-b)^{1/2} (P/P_c) \quad (2.7)$$

The dotted line in Fig. 2.3 refers to an imperfect spherical shell. The degree of sensitivity of the shell to imperfections is dictated by the magnitude of (b) as illustrated in Fig. 2.4.

From these observations, Koiter (1945) concluded that the buckling pressure of spherical shells was greatly affected by the inclusion of imperfections which were thus imputed to be the cause of the wide scattering of experimental results and the poor correlation between theory and experiment for the deeper spherical shells. Based on Koiter's (1945) theory, both Hutchinson (1967) and

Kalnins and Biricikoglu (1970) have proposed varying degrees of imperfection sensitivity. In some instances, the effect of specific imperfection shapes have been analysed. Dimple type imperfections have been theoretically treated by Thompson (1962), Sabir (1964), Thurston and Penning (1966) and Koga and Hoff (1969). Imperfections in the form of an increased radius have been treated by Wedellsborg (1962), Bushnell (1967) and Koga and Hoff (1969).

Kaplan (1974) has shown graphically the imperfection sensitivities of a number of these theories, and his results are shown in Fig. 2.5.

2.2.8 Computer programmes

The advent of the computer had an appreciable effect on the investigations into non-linear shell theory. Approximate numerical techniques such as numerical integration, finite difference and finite element type solutions were now made available.

The most popular present-day programmes are K-SHELL [48], STARS [79], which employ numerical integration, BOSOR 4 [20] and STAGS [2] which use finite difference, and PAFEC [72] which uses finite element techniques to solve the governing shell equations. The former four programmes have similar capabilities in that they all perform stress, stability and free vibration analyses of thin-walled structures, although STAGS [79] has the added facility of being able to treat asymmetric problems, whereas PAFEC [72] is limited to the determination of solely the stiffness and deformation values.

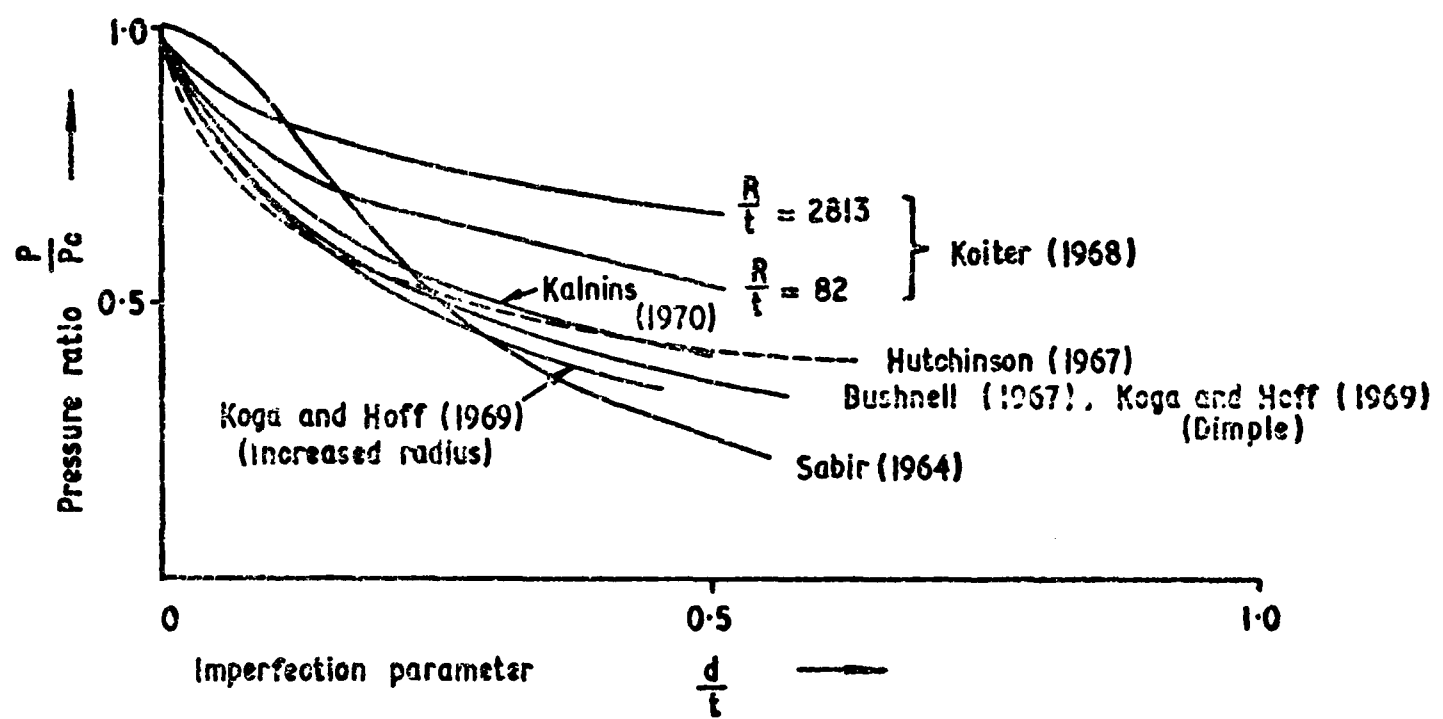


Fig.2.5. Analytical predictions for the imperfection sensitivity of thin-walled spheres (Kaplan [1974]).

2.3 EXPERIMENTAL WORK ON SPHERICAL SHELLS UNDER UNIFORM EXTERNAL PRESSURE

A brief description of the experimental programmes performed on spherical caps, deep shells including hemispheres and complete spheres, are given in Table 2.1. The majority of the experimental results from these programmes are given in Fig. 2.6, where their buckling pressures relative to P_c are plotted against the commonly used geometric parameter (λ).

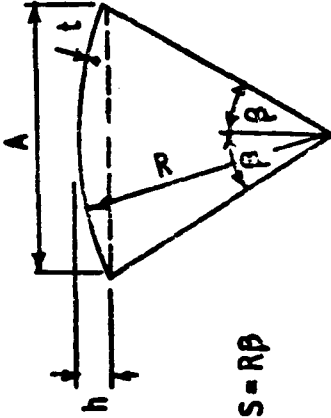
Considering the wide scattering of results from Fig. 2.6, particularly so for the early experimental tests and for the deeper shells, it is not surprising that doubt existed as to the validity of using P_c for real structures. The later experimental results for shallow shells are, however, in more satisfactory agreement with the predicted symmetric buckling values, although the bifurcation load for a perfect spherical shell still vastly over-estimated the stability of deep spherical shells and complete spheres with typical buckling pressures $\frac{1}{2} P_c$ being common.

This scattering of experimental results for presumed identical models has since been attributed to the manufacturing techniques which have resulted in the inclusion of imperfections and to the inadequate boundary conditions used for the clamped spherical shells.

If one excludes Thompson's (1962) work on two spheres of radius to thickness ratio of 20 on the grounds that their behaviour may not be representative of the truly elastic behaviour of thin-walled spheres, then it would appear that Carlson, Sendlebeck and Hoff (1967) are the

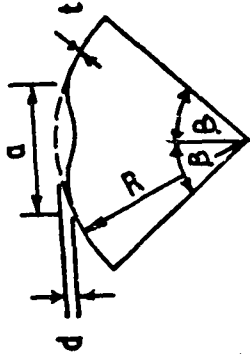
TABLE 2.1

List of experimental investigations conducted on spherical shells under uniform external pressure

a) Shallow and deep spherical shells		Geometric parameter, $\lambda = 2^4 \sqrt{3(1-\nu^2)} \sqrt{(h/t)}$ for shallow shells - $\frac{2h}{A} < \frac{1}{8}$ $= \sqrt[5]{12(1-\nu^2)} \sqrt[5]{Rt}$ for deep shells	<u>Description of Specimens</u>
von Karman and Tsien (1939)			Copper hemisphere, pressurized in a bath of mercury, A = 36 in., R/t = 900. Performed by Sechler and Bollay (1939)
Tsien (1942)			Clamped dome, constant volume loading, $\beta = 18^\circ$.
von Klöppel and Jungbleth (1953)			Deep steel shells, A = 16 in., $\beta = 23^\circ, 53^\circ$. Shells experienced asymmetric behaviour.
Kaplan and Fung (1954)			Hot spun magnesium, edges clamped, A = 8 in., $\beta = 8^\circ, 11^\circ$.
Goerner (1956)			Spun aluminium alloy shells, representative of ballistic missile bulkheads, R = 27 in. and 35 in. Elastically supported edges.
Homewood, Brine, Johnson (1961)			Spun steel and aluminium shells, edges clamped between rings. A = 34 in., $\beta = 13^\circ, 25^\circ$ R/t = 160 to 1250.
Bellinfante (1962)			Hydroformed aluminium, edges clamped between rings. A = 10 in., $\beta = 13^\circ, 39^\circ$.

<u>Reference</u>	<u>Description of Specimens</u>
Evan-Iwanowski and Loo (1962)	Hot die-pressed PVC, edges clamped between rings, variable β .
Krenzke (1962)	Aluminium alloy machined hemispheres connected to cylinders. $R/t = 11$ to 107 .
Krenzke (1963)	Aluminium alloy machined hemispheres. $R/t = 165$.
Parmeter (1963)	Electroplated copper, edges bonded between clamping rings, $A = 8$ in., $\beta = 60^\circ, 110^\circ$.
Little (1964)	Hot vacuum formed PVC bonded to clamping ring, $A = 18$ in., 36 in., $\beta = 30^\circ$.
Sabir (1964)	Copper and aluminium alloy hemispheres. $R/t = 150$ to 200 . Small circular region etched to $R/t = 600$. Also tested the shell with a dimple imperfection introduced by point load.
Adam and King (1964)	Hot vacuum formed PVC with machined internal surface, bonded to clamping ring, $A = 16$ in., $\beta = 230^\circ, 330^\circ, 53^\circ, 90^\circ$.
Wang, Rodrigues-Agrait, Little (1966)	Hot vacuum formed PVC, simply supported and bonded edges, $A = 12, 18, 25.75$ in., $\beta = 190^\circ, 300^\circ, 46^\circ$.
Tillman (1970)	Hot vacuum formed PVC, $A = 4$ in., $\beta = 5^\circ, 9.5^\circ$. Constant volume loading.
Kruger and Glockner (1971)	Hot vacuum formed PVC with machined internal surface, simply supported and clamped edges. $A = 15$ in., $\beta = 20^\circ, 280^\circ, 440^\circ$.

b) Shallow and deep spherical shells with a dimple imperfection



Reference

Thurston and Penning (1966)

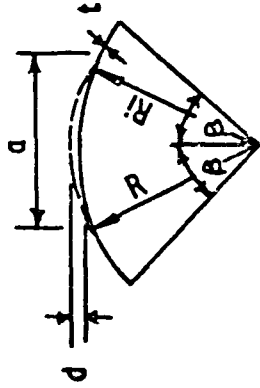
Description of specimens

Explosively formed aluminium shells, $R = 20, 30 \text{ in.}$, $\beta = 11.50$ and 80° . Chemically etched to t between 0.005 and 0.05 in. Axisymmetric dimple formed by inserting 0.001 in. aluminium foil at the apex of the forming die. Buckled between $0.1 P_C$ for $R/t = 2000$ to $0.65 P_C$ for $R/t = 400$.

Sabir (1964)

See section a)

c) Shallow and deep spherical shells with an increased radius imperfection



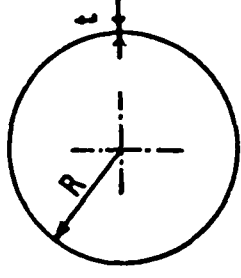
Reference

Krenzke and Kiernan (1965)

Description of specimens

Machined aluminium hemispheres connected to a circular cylinder. $R = 0.81 \text{ in.}$, $\beta = 50^\circ, 100^\circ, 150^\circ$. $R_i = 1.15 R$ and $1.4 R$. R/t varied from 10 to 200 .

d) Complete Spheres



Reference

Thompson (1962)

Description of Specimens

Two PVC spheres $R = 2.1$ in., $R/t = 21$. Buckled at approx. 0.5 P_c . Also introduced a dimple imperfection by hand pressure.

Carlson, Sandelbeck, Hoff (1967)

Electroplated nickel spheres, with wax sphere as the former, $R = 4.25$ in., $R/t = 1570$ to 2120.

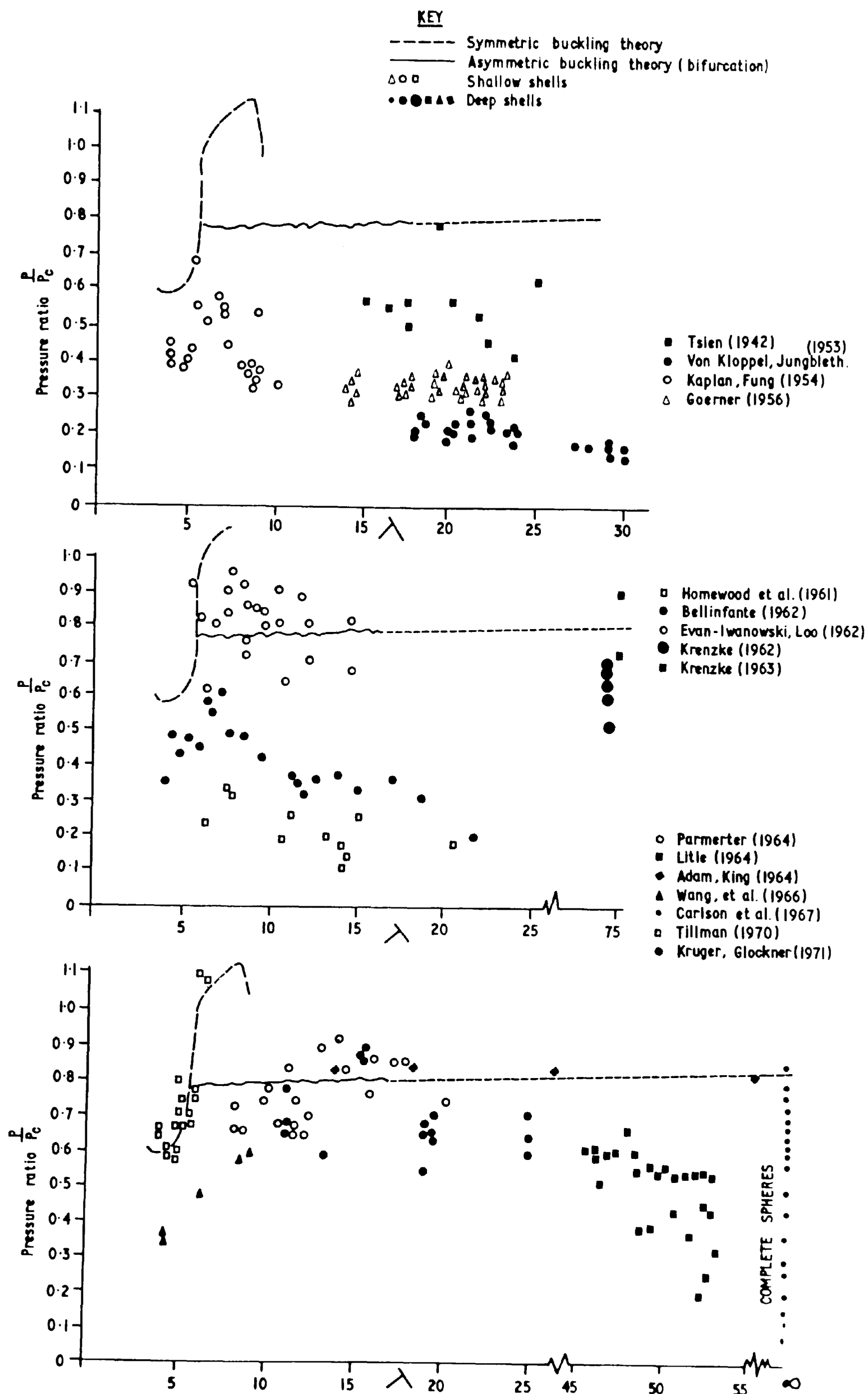


Fig. 2.6. Summary of experimentally obtained buckling pressures for shallow and deep shells compared to analytical predictions.

only ones to have conducted experiments on thin-walled spheres. (Berke (1967) used spheres manufactured by these authors). They attempted to produce the perfect thin-walled sphere using an electroplating method, a process which had earlier been used by Thompson (1960). However, a wide scattering of buckling pressures, as shown in Fig. 2.6, was obtained with buckling values ranging from 0.05 to $0.86 P_c$ even though these P_c values were based on the thinnest section in spheres which had thickness variations from 20 to 30% higher than this value.

Few experimental programmes have been conducted on spherical shells containing a measured imperfection. Of these, Sabir (1964), in addition to performing experimental tests on nominally perfect hemispheres, tested various sizes of dimple type imperfections, introduced by impressing the shells with varying inward point loads. The results obtained are shown in Fig. 2.7. They show that the inclusion of an imperfection size of amplitude(d) equal to only half the wall thickness(t) can reduce a shell's buckling pressure to as little as $\frac{1}{5}P_c$.

Included in Fig. 2.7, is a result obtained by Thompson (1962), who also formed the dimple-type imperfection, this time by hand pressure in a region of the shell of near perfect sphericity. The result obtained agrees well with an extrapolation of Sabir's (1964) results. This implies that for a dimple-type imperfection, once the minimum buckling pressure has been reached, which was found to be approximately $0.12 P_c$ for $(d/t) = 1$, then larger imperfections no longer produce any further deleterious effect.

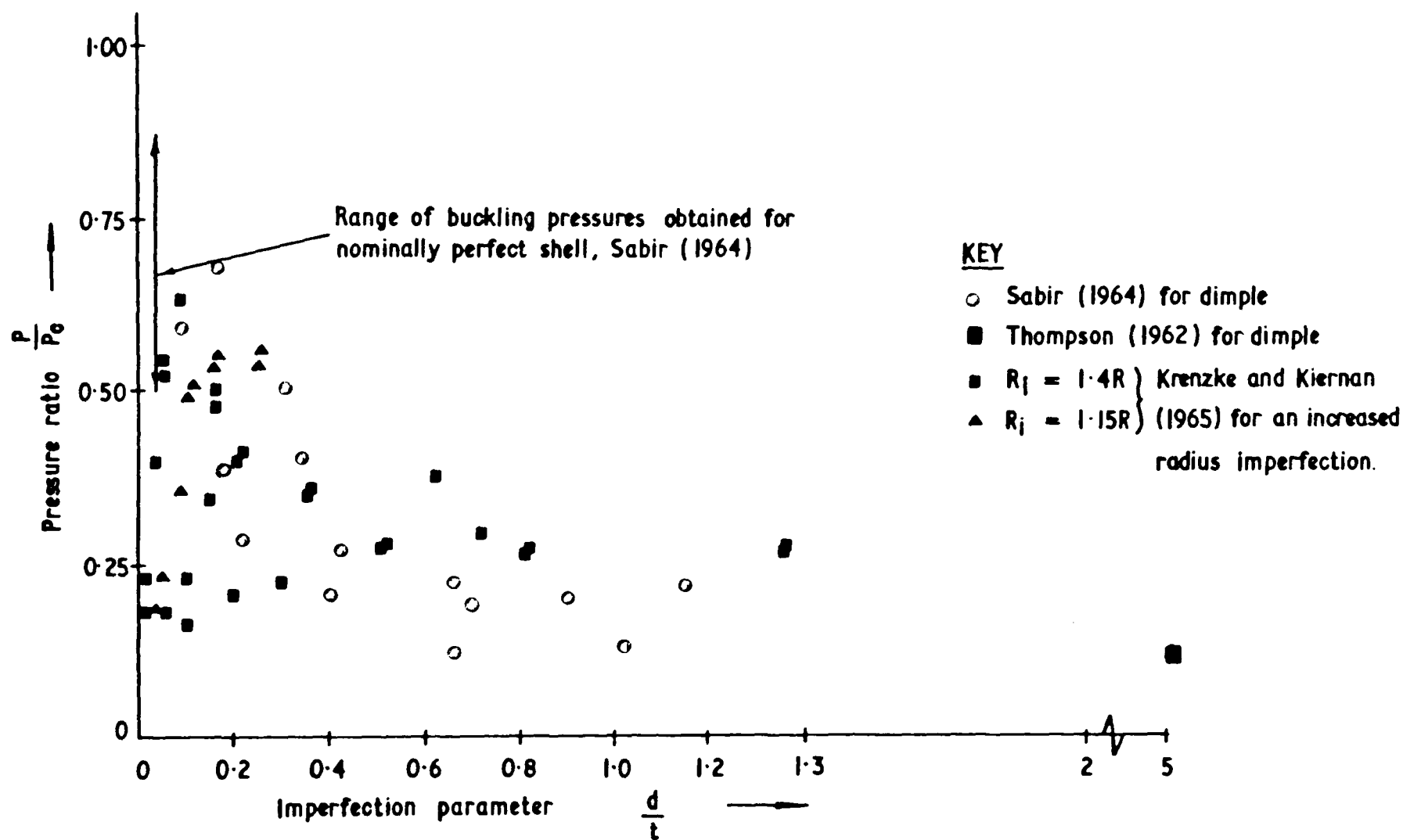


Fig. 2.7. Experimental investigations into the imperfection sensitivity of a spherical shell containing a dimple type imperfection.

Fig. 2.7 also shows the results obtained by Krenzke and Kiernan (1965) for shallow spherical shells which contained an increased radius imperfection. They are in approximate agreement with the general trend shown by the other experiments, although a great deal of scatter in the results for the very small imperfection amplitudes suggests an inadequacy in the experimental models. It is, however, necessary to take them into account given the paucity of well documented experimental values obtained for spherical shells containing a controlled imperfection.

2.4 APPLICATION OF SPHERES ON SHIPS

So far the review has contained only information concerning spheres under uniform external pressure. Since the commercial application of spherical LNG containers on ships is relatively new (1971), it is not surprising that there is a paucity of information. Only one publication, that by Pedersen and Jensen (1976), was found. These authors analysed the axisymmetric (i.e. equatorial ring normal to accelerating force) loading case using the quasi-shallow shell approximation to find a value of the vertical acceleration required to produce buckling for various fluid levels, concluding that the worst condition occurred when the sphere was about third full. In that case, the stress system below the equatorial ring support consisted of tension in the meridional direction and compression in the hoop direction. The membrane shell equations were used to define the pre-buckling situation. A similar analysis was also presented for the 30 degrees inclined loading case and, in both, Koiter's (1945) imperfection theory was

applied to establish the imperfection sensitivity of the shell. In practice, it was assumed that this was the same as for a shallow spherical shell under point load. No experimental work to validate the analysis was presented, the only experiments being on hemispheres under concentrated loading^[74].

2.5 SUMMARY

The theoretical classical analysis for the buckling of thin-walled spheres under uniform external pressure has been found to be in disagreement with experimental results. This disagreement has been explained by introducing in the theoretical analysis, the effect of imperfections.

However, when applying these ideas to practical problems, the following conclusions can be made.

a) The imperfection size exceeds the amplitude size studied analytically.

b) Real structures contain imperfection shapes of several forms, not necessarily in the shape of the buckled mode stipulated by Koiter (1945).

c) It is inevitable in any experimental work that random imperfections will be introduced during the manufacture of models, i.e. departures from sphericity, variations in thickness, residual stresses, and for spherical shells used to depict the behaviour of complete spheres, the introduction of boundary problems.

A review of the information concerning the operation of spheres on ships has highlighted:

a) Paucity of information.

b) No experimental validation that a sphere would

be more susceptible to buckle for certain filling ratios when under the imposition of an acceleration both normal and inclined to its equatorial ring support.

c) No information, either analytical or experimental concerning the imperfection sensitive/insensitive behaviour of spheres under these aforementioned loading conditions.

CHAPTER III

APPLICATION TO DESIGN

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CHAPTER III

APPLICATION TO DESIGN

3.1 INTRODUCTION

Chapter I has reviewed present day LNG tank design and their operating conditions on ships; Chapter II, the theoretical and experimental investigations conducted. This chapter considers the application of both to the safe design of these tanks, the purpose being to highlight those areas where further experiments are desirable and to help in the design of these experiments.

3.2 DESIGN AGAINST EXTERNAL PRESSURE

The ASME^[8] code is based on von Kármán and Tsien's (1939) formula, equ. 2.2,

$$P_L = 0.365 E \left(\frac{t}{R}\right)^2$$

or

$$\sigma_L = \frac{P_L R}{2t} = 0.1825 E \left(\frac{t}{R}\right); \text{ see equ. 3.3c.} \quad (3.1)$$

The ASME^[8] code expresses the maximum allowable compressive stress as,

$$\sigma_{ASME} \approx \frac{E}{32} \frac{t}{R} \approx \frac{\sigma_L}{6}$$

i.e. a safety factor of approximately 18 is assumed compared to P_c , equ. 2.1. This is a somewhat approximate value since the operation temperature and hence the change in the modulus of elasticity will have a bearing on

the design strength of the material. Thus for a typical 35 m diameter, 25 mm thick, 9% Ni steel sphere, the ASME^[8] code predicts the buckling pressure to be, by interpolation, 22.5 KN/m², or approximately 0.045 P_c . BS 5500^[13] is much less conservative in its safety factor and gives for this example a value of approximately 0.125 P_c , i.e. a safety factor of 8.

These predicted buckling pressures are however independent of the imperfections in the sphere. The manufacturing tolerances specified for spherical shells by the design codes are in the form of allowable departures from a circular template. Since the length of the template is limited for practical reasons, the comparison can only be made over an arc. The tolerances specified by the ASME^[8] code are in fact, the same as those developed for cylindrical shells and represent widely accepted standards of manufacture rather than an attempt to quantify their likely effect on the buckling strength. The BS 5500^[13] code implies that the design method is only acceptable for those shells which are "spherical" within 1% on radius and in which the radius of curvature, based on an arc length of $2.4 \sqrt{Rt}$ does not exceed the nominal value by more than 30%.

The code does not specify a limit to the area of the imperfection, nor does it allow for the effect of a bulge since it only sets an upper limit to the radius of curvature. Likewise other codes,^[3,28,69] contain little that provides any specific guidance on the "fitness-for-purpose" of imperfect shells.

3.3 DESIGN AGAINST EXTERNAL ACCELERATION

3.3.1 Tank full

If the tank is completely filled, then the pressure normal to the shell can be given by the expression,

$$P = k\rho R(1-\cos\phi) \quad (3.2)$$

where k is the vertical acceleration in m/s^2 ; ρ is the density of the LNG, and ϕ is as defined in Fig. 3.1.

The following membrane equations, derived by Flugge (1973) are used;

$$\frac{\partial N_{\phi}}{\partial \phi} \sin\phi + N_{\phi} \cos\phi + \frac{\partial N_{\phi\theta}}{\partial \theta} - N_{\theta} \cos\phi = 0 \quad (3.3a)$$

$$\frac{\partial N_{\phi\theta}}{\partial \phi} \sin\phi + 2N_{\phi\theta} \cos\phi + \frac{\partial N_{\theta}}{\partial \theta} = 0 \quad (3.3b)$$

$$N_{\phi} + N_{\theta} = PR \quad (3.3c)$$

the nomenclature of which is explained in Fig. 3.2.

They yield, for this symmetrical loading problem,

$$N_{\phi} = \frac{R}{\sin^2\phi} (\int P \cos\phi \sin\phi d\phi + c) \quad (3.4)$$

where the constant(c) to be evaluated, depends on the boundary conditions.

Substitution of equ. 3.2, into equ. 3.4, and integration yields,

$$N_{\phi} = \frac{k\rho R^2}{6\sin^2\phi} [(2\cos\phi-3)\cos^2\phi+6c] \quad (3.5)$$

At the vertex, $\phi = 0$, the denominator vanishes, which implies that $c = \frac{1}{6}$, giving,

$$N_{\phi} = \frac{k\rho R^2 (1-\cos\phi) (1+2\cos\phi)}{6 (1+\cos\phi)}, \quad (3.6a)$$

and from equ. 3.3c,

$$N_{\theta} = \frac{k\rho R^2 (1-\cos\phi) (5+4\cos\phi)}{6 (1+\cos\phi)} \quad (3.6b)$$

These formulae are valid above the equatorial support. Another constant(c) is applied which makes N_{ϕ} finite at $\phi = \pi$ for the lower part of the shell. It is $c = \frac{5}{6}$, giving,

$$N_{\phi} = \frac{k\rho R^2 (5-5\cos\phi+2\cos^2\phi)}{6 (1-\cos\phi)}, \quad (3.7a)$$

and from equ. 3.3c,

$$N_{\theta} = \frac{k\rho R^2 (1-7\cos\phi+4\cos^2\phi)}{6 (1-\cos\phi)} \quad (3.7b)$$

Two curves are given in Fig. 3.3 for σ_{ϕ} and for σ_{θ} , one for the shell below the support, the other for the section above the support. When the support is at the equator, $\phi = 90^{\circ}$, all stresses are tensile. When the support is below the 120° parallel, compressive meridional stresses appear, while when it is above the 80° parallel, the hoop stress becomes compressive. Ideally the support should be between these limits, if compressive stresses are to be avoided. As an example, suppose the support is at the 100° parallel. The σ_{ϕ} distribution would proceed from A to B, at which point, the curve becomes discontinuous, and change to path CO. Likewise the σ_{θ} distribution would proceed from A to D, become discontinuous and change to path EO.

The usual practice is to design to $\frac{2}{3} \sigma_{\text{yield}}$. Therefore maximum acceptable acceleration is given by the expression,

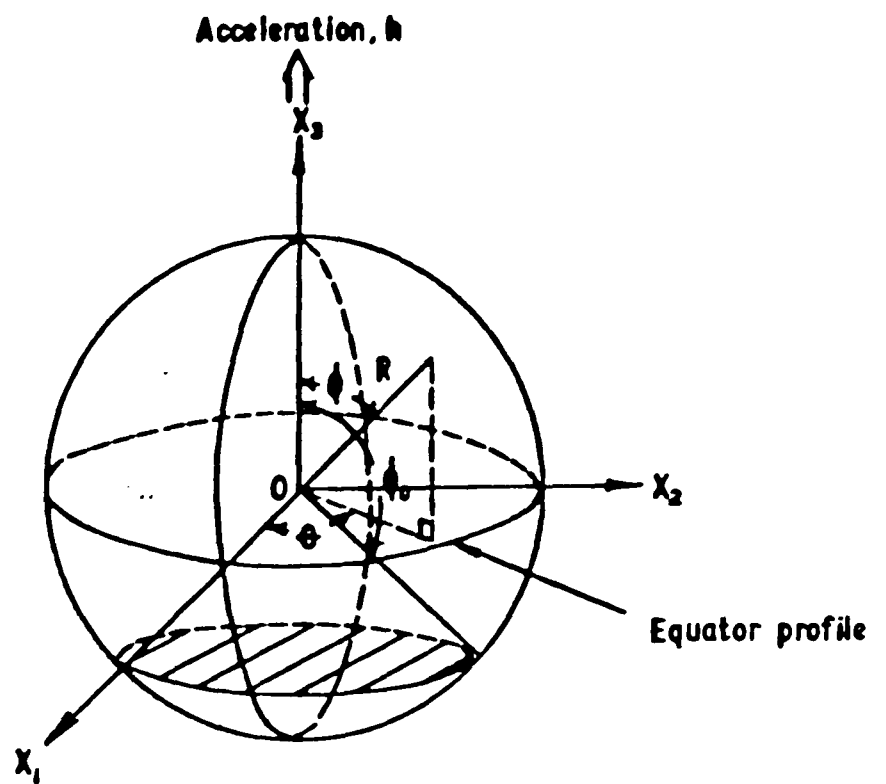
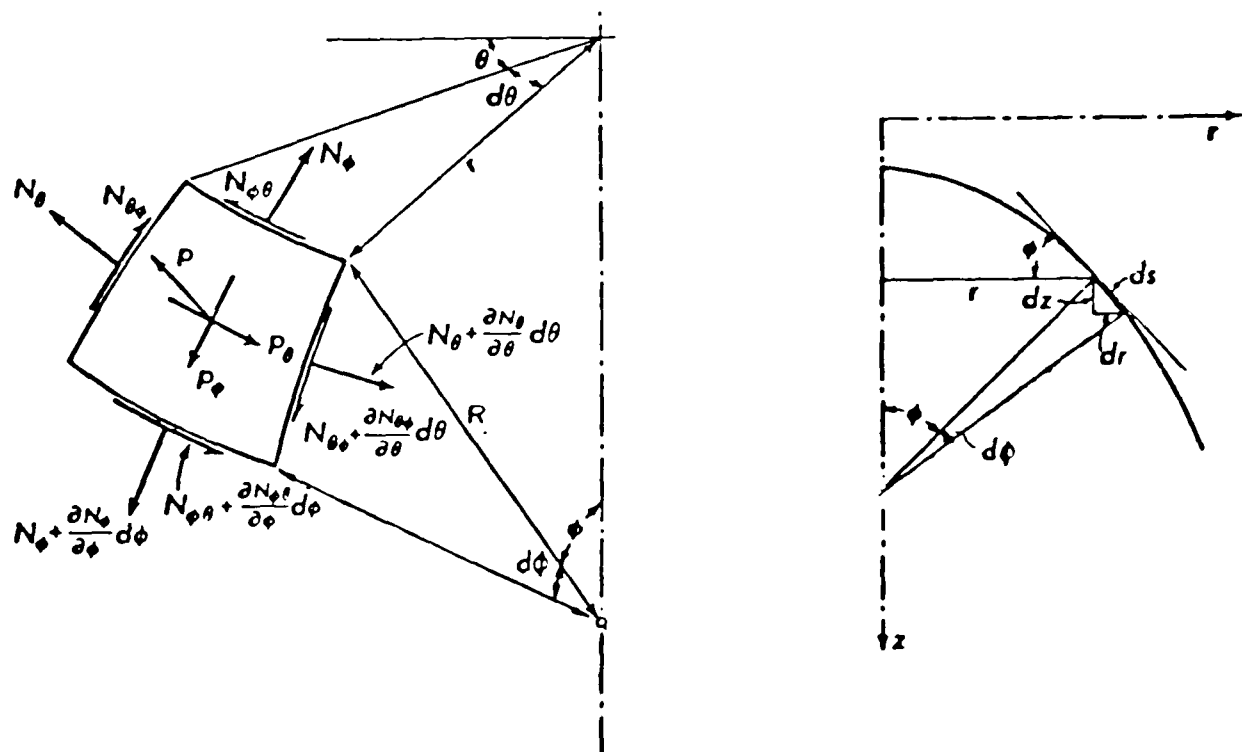


Fig. 3.1. Spherical tank



$$\begin{aligned}
 N_{\theta} &= \text{stress resultant in circumferential direction} = \frac{\sigma_{\theta}}{t} \\
 N_{\phi} &= \text{stress resultant in meridional direction} = \frac{\sigma_{\phi}}{t} \\
 N_{\theta\phi} &= \text{shear stress resultant} = \frac{\sigma_{\theta\phi}}{t} \\
 N_{\phi\theta} &= \text{shear stress resultant} = \frac{\sigma_{\phi\theta}}{t}
 \end{aligned}$$

Fig. 3.2 Shell element and meridian of shell of revolution

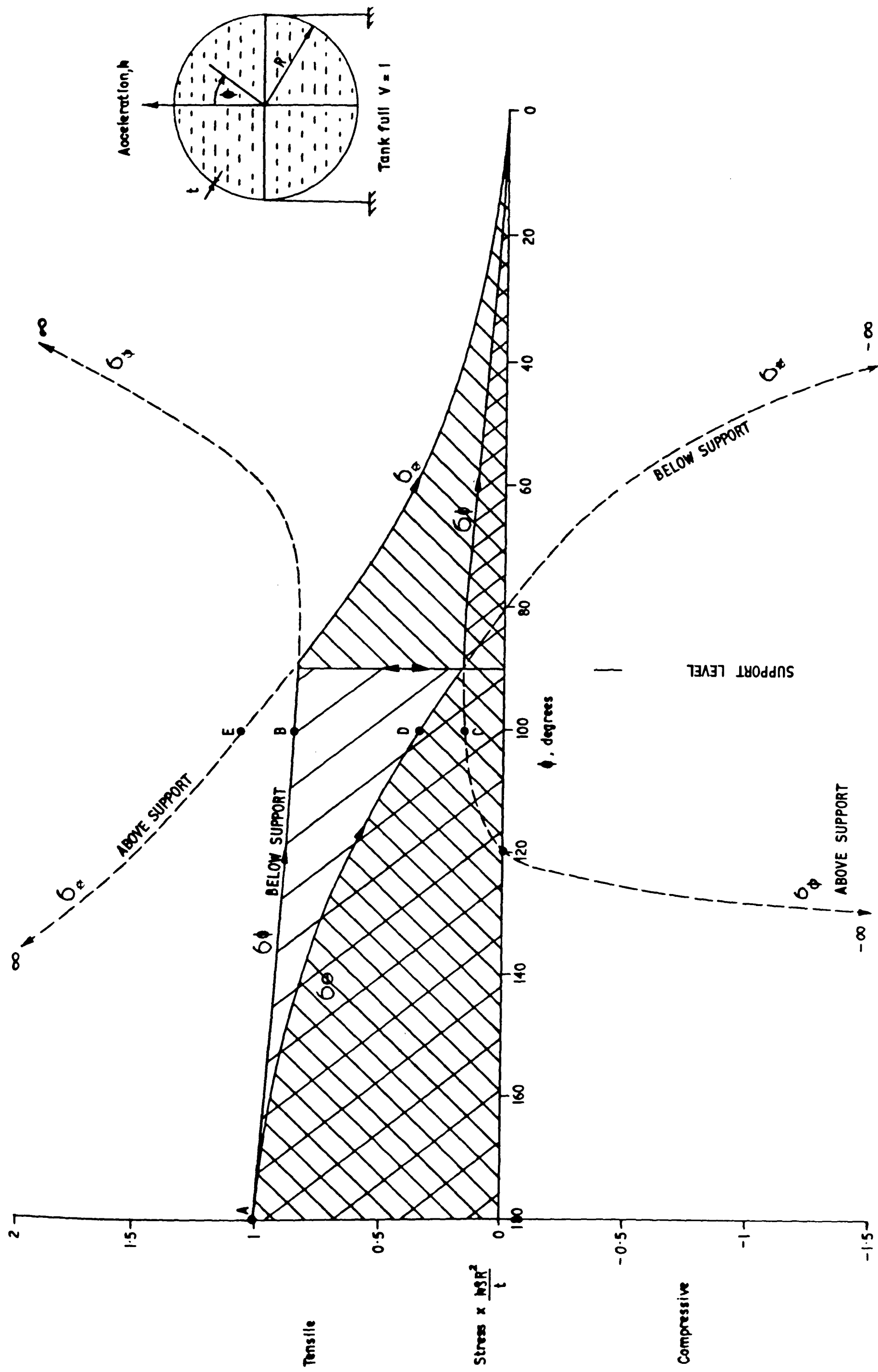


Fig. 3.3. Stress distribution of full spherical tank; support at $\phi = 90^\circ$

$$k < \frac{\frac{2}{3} \sigma_{\text{yield}}}{\frac{\rho R^2}{t}}$$

If the same tank dimensions as given in section 3.2 are assumed, ρ of methane = $0.72 \times 10^3 \text{ kg/m}^3$, and σ_{yield} of 9% Ni steel = $4.5 \times 10^8 \text{ N/m}^2$, then maximum acceptable k ,
 $= 3.5 \text{ g}$

i.e. a safety factor of ≈ 2.5 when compared to the envisaged maximum acceleration stipulated of 1.43 g , (see section 1.2).

3.3.2 Tank partly empty and its equatorial ring horizontal

From Fig. 3.1,

$$\begin{aligned} P &= k\rho R [(1-\cos\phi) - (1-\cos\phi_0)] \\ &= k\rho R (\delta - \cos\phi), \end{aligned} \tag{3.8}$$

where $\delta = \cos\phi_0$, determines the liquid filling ratio V , the relationship of which can quite easily be shown to be,

$$V = 1 - (1-\delta)^2 (2+\delta) / 4 \tag{3.9}$$

Substitution of equ. 3.8 into equ. 3.4 then yields,

$$N_\phi = \frac{k\rho R^2}{\sin^2\phi} \left[\frac{\cos^3\phi}{3} - \frac{\delta}{2} (\cos^2\phi - \frac{1}{2}) + c \right] \tag{3.10}$$

For the bottom hemisphere, $\phi > 90^\circ$ and for points in the shell wall submerged in the liquid, $\phi_0 < \phi < 90^\circ$, a finite solution of N_ϕ is possible at $\phi = \pi$, if

$$c = \frac{1}{3} + \frac{\delta}{4}$$

Substitution of (c) into equ. 3.10, yields,

$$(N_{\phi})_{\phi_0 < \phi < 90^\circ} = k\rho R^2 \left(\frac{\delta}{2} + \frac{1+\cos^3\phi}{3\sin^2\phi} \right) \quad (3.11a)$$

and from equ. 3.3c,

$$(N_{\theta})_{\phi_0 < \phi < 90^\circ} = k\rho R^2 \left(\frac{\delta}{2} - \cos\phi - \frac{1+\cos^3\phi}{3\sin^2\phi} \right) \quad (3.11b)$$

For the top hemisphere, $\phi < 90^\circ$ and for points in the shell wall submerged in the liquid, $\phi_0 < \phi < 90^\circ$, the constant(c) is obtained from equilibrium considerations at the free surface.

i.e. at $\phi = \phi_0$, $N_{\phi} = N_{\theta} = 0$, giving,

$$c = \frac{\delta^3}{6} - \frac{\delta}{4} \quad ; \text{ from equ. 3.10,}$$

$$(N_{\phi})_{\phi_0 < \phi < 90^\circ} = \frac{k\rho R^2}{\sin^2\phi} \left[\frac{\delta}{2} \left(\frac{\delta^2}{3} - \cos^2\phi \right) + \frac{\cos^3\phi}{3} \right] \quad (3.12a)$$

and from equ. 3.3c,

$$(N_{\theta})_{\phi_0 < \phi < 90^\circ} = \frac{k\rho R^2}{\sin^2\phi} \left(\delta - \frac{\delta}{2}\cos^2\phi - \cos\phi + \frac{2}{3}\cos^3\phi - \frac{\delta^3}{6} \right) \quad (3.12b)$$

For the bottom hemisphere, $\phi > 90^\circ$ and for points in the shell wall above the liquid level, $\phi_0 > \phi > 90^\circ$, N_{ϕ} may be found from equilibrium considerations.

$$\text{i.e.} \quad N_{\phi} \sin\phi R \sin\phi \, d\phi = \frac{4}{3} \pi R^3 \rho V k$$

Re-arranging and using the expression for V given in equ. 3.9 yields,

$$(N_{\phi})_{\phi_0 > \phi > 90^\circ} = \frac{k\rho R^2}{6\sin^2\phi} (1+\delta)^2 (2-\delta) \quad (3.13a)$$

and since the fluid pressure is zero above the liquid level,

it follows from equ. 3.3c, that,

$$(N_{\theta})_{\phi_0 > \phi > 90^\circ} = \frac{-k_p R^2}{6 \sin^2 \phi} (1+\delta)^2 (2-\delta) \quad (3.13b)$$

Finally, for points on the shell wall in the top hemisphere, $\phi < 90^\circ$ and above the liquid level, $\phi_0 > \phi < 90^\circ$,

$$(N_{\phi})_{\phi_0 > \phi < 90^\circ} = 0 \quad (3.14a)$$

$$(N_{\theta})_{\phi_0 > \phi < 90^\circ} = 0 \quad (3.14b)$$

Using equs. 3.11,12,13, and 14, it is possible to determine the values of σ_{ϕ} and σ_{θ} at any point in the shell for a specified value of V.

An example of the variation in stress levels is shown in Fig. 3.4 for the sphere nearly half full. It shows σ_{θ} to become compressive and reach a maximum value at the liquid level, whereas σ_{ϕ} remains always tensile. This graph is typical for $V < \frac{1}{2}$. For $V > \frac{1}{2}$, both σ_{θ} and σ_{ϕ} are tensile at the liquid level although a somewhat smaller compressive stress can occur at and below the equator, as shown in Fig. 3.5.

The maximum compressive values of σ_{θ} from these graphs have been plotted in Fig. 3.6 against V in order to obtain an approximation to the most likely position at which buckling would occur.

The maximum compressive stress is seen to occur when the tank is half full.

$$\begin{aligned} \sigma_{\theta} = -\sigma_{\phi} &= \frac{-k_p 2\pi R^3}{3 \times 2\pi R t} \\ &= \frac{-k_p R^2}{3t} \end{aligned} \quad (3.15)$$

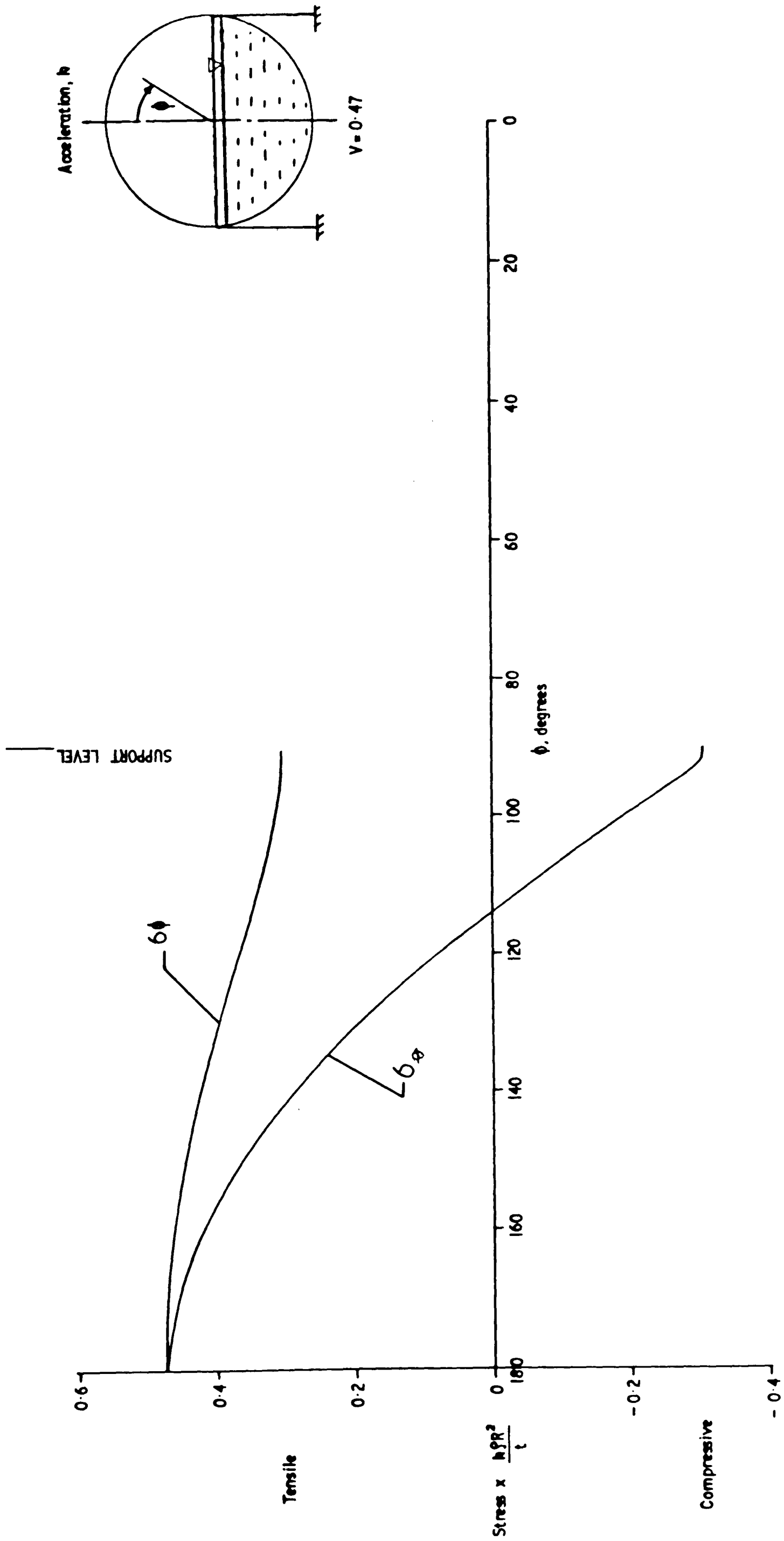


Fig. 3.4. Stress distribution of sphere nearly a half full.

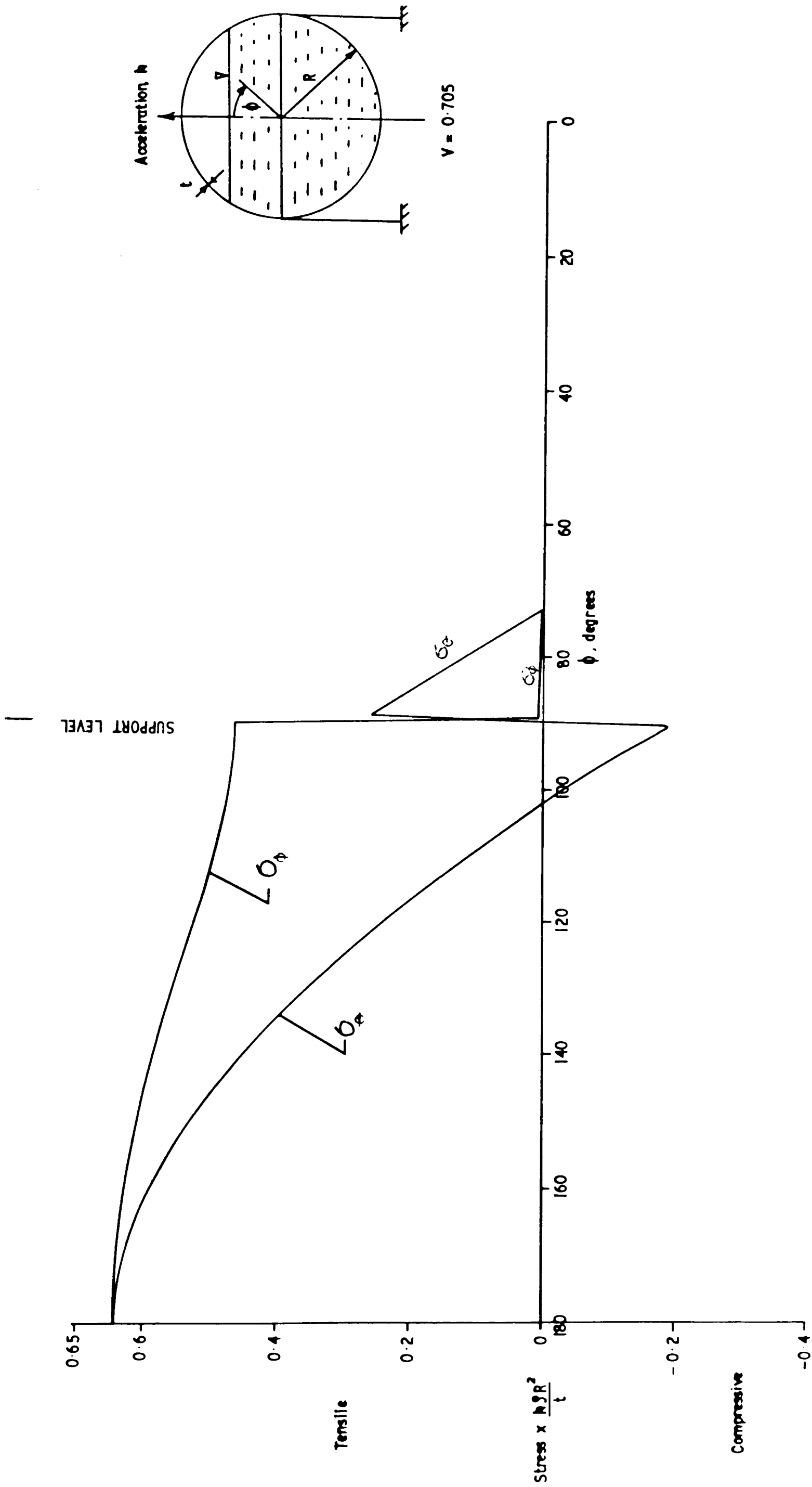


Fig 3.5. Stress distribution of a sphere approximately three quarters full.

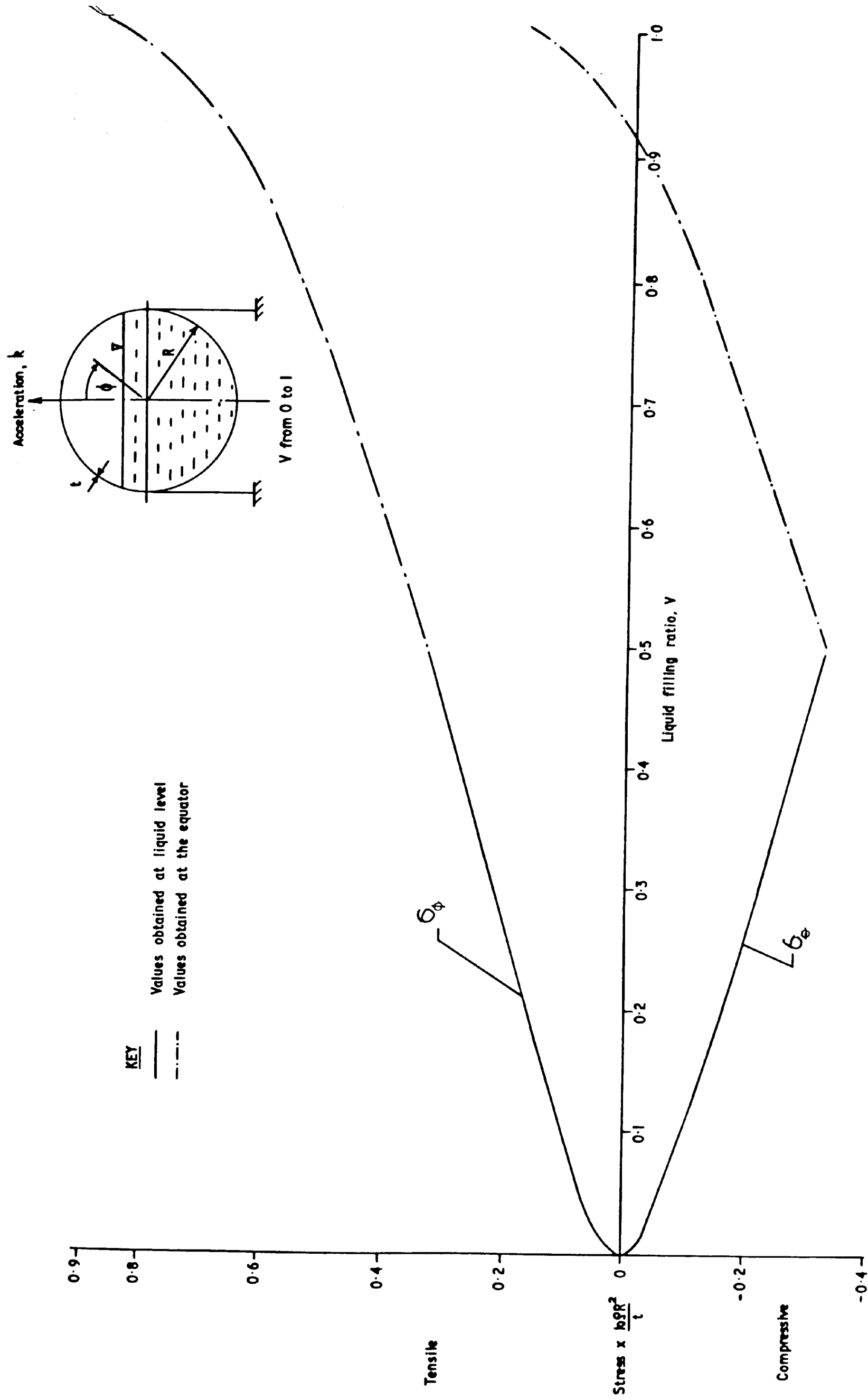
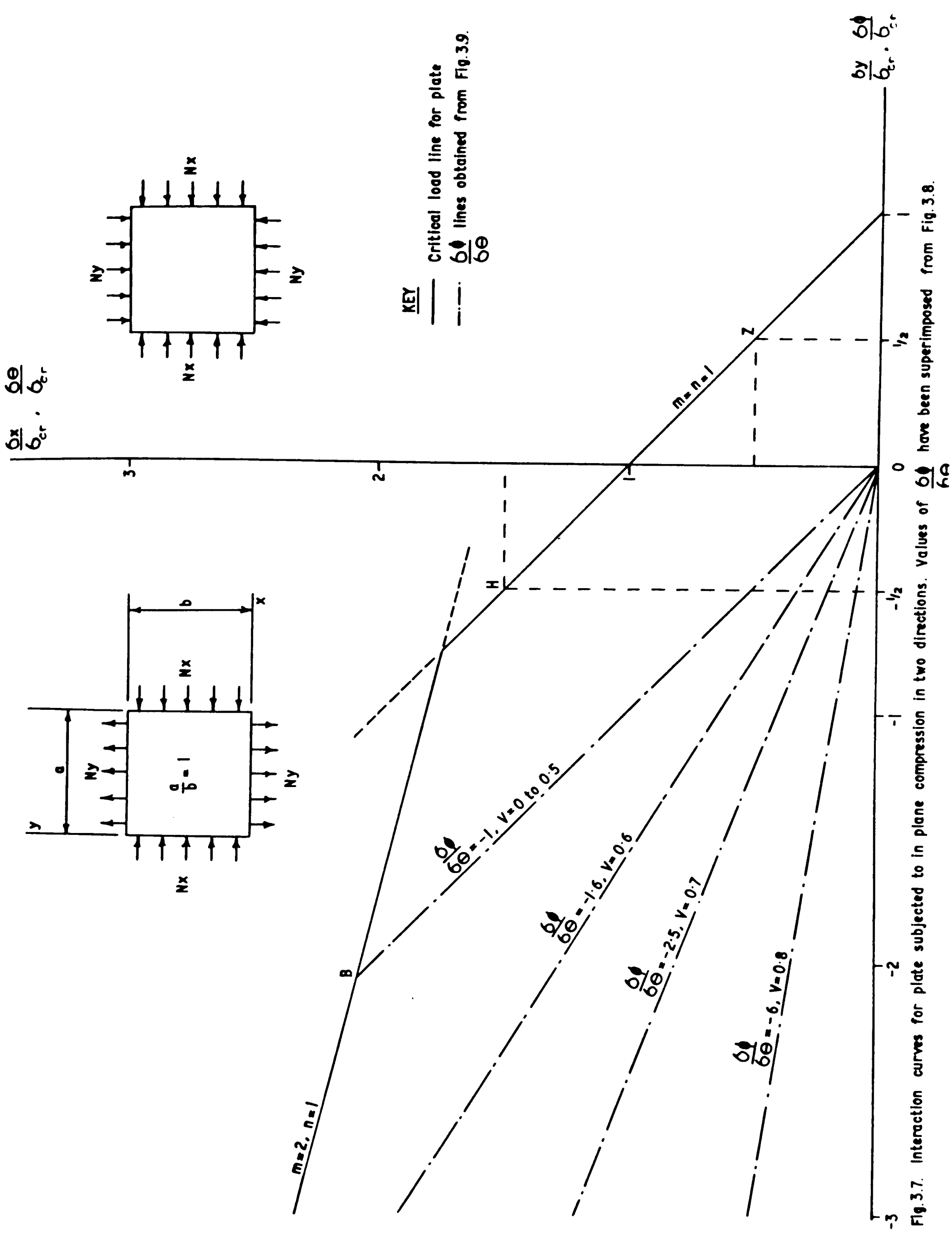


Fig. 3.6. Variation of circumferential, σ_ϕ and meridional, σ_θ stresses with liquid filling ratio, V; acceleration normal to equatorial ring support.

However, since this condition corresponds to the position of the equator, this stress would be transferred to the shell's support, and buckling would not occur. Intuitively buckling would be most likely to occur at positions of high compressive stress and some distance from the equatorial region. Although in Fig. 3.6, the variation in σ_θ is approximately symmetrical with respect to V , the variation in σ_ϕ is not. For $V > \frac{1}{2}$, it may be assumed that the large tensile σ_ϕ would have the effect of inhibiting buckling and it can also be assumed that if buckling is to occur, it will do so at a position in the lower hemisphere for $V < \frac{1}{2}$, i.e. at a position of high compressive σ_θ and comparatively low tensile σ_ϕ .

The most direct approach to the establishment of an acceptable level for the compressive component σ_θ is to ignore the tensile component σ_ϕ entirely, limiting σ_θ to the value derived from the classical buckling equ. 2.1, or equ. 3.1, with some safety factor. In this case, the value of the safety factor may be less than the one specified by both the ASME^[8] and BS 5500^[13] since the state of stress is not isotropic but one of tension combined with compression.

The interaction between compressive and tensile components may be considered by comparing the spherical shell to an assemblage of square plates. For these, interaction curves have been presented by a number of authors, e.g. Brush and Almroth (1975), Timoshenko and Gere (1972). In a square plate with simply supported edges under loads per unit length N_x , N_y , the interaction curves shown in Fig. 3.7 are obtained.



The co-ordinates of the graph are stresses in the x and y directions normalized with respect to the critical stress at buckling, σ_{cr} . The curve represents a plot of the critical loads for half wavelength parameters m and n in both x and y directions respectively. Negative values of the parameter σ_y/σ_{cr} , represent tensile loading in the y direction, i.e. similar to the meridional loading condition shown in Fig. 3.6, for a partially filled sphere.

The graph in Fig. 3.7 can be interpreted as follows. Suppose the plate experiences an initial compressive stress of $\sigma_y/\sigma_{cr} = \frac{1}{2}$. If the plate is then loaded compressively in the x direction, it will not buckle until the critical load line is crossed (Z). This position will correspond to a compressive σ_x one-half that if σ_x was acting alone. If, however, σ_y/σ_{cr} is tensile and equal to $-\frac{1}{2}$ say, a compressive value of σ_x greater than that if σ_x was acting alone would be needed to produce buckling (H), i.e. the plate will sustain far higher loads in the x direction if there is a tensile force in the y direction.

The applicability of this graph to the partially filled sphere is seen from the $\sigma_\phi/\sigma_\theta$ versus V graph given in Fig. 3.8. For $V < \frac{1}{2}$, $\sigma_\phi/\sigma_\theta = -1$; for $V > \frac{1}{2}$, the line becomes progressively steeper until σ_θ changes sign and becomes tensile near the full position. These $\sigma_\phi/\sigma_\theta$ ratios are given in Fig. 3.7.

Considering the case $V = \frac{1}{2}$, $\sigma_\theta = -\sigma_\phi$ and, for the equivalent plate, the critical condition corresponds to point B in Fig. 3.7. On the other hand, when the sphere is loaded under uniform external pressure, $\sigma_\theta = \sigma_\phi$ and the corresponding point in Fig. 3.7, is Z, it follows that the

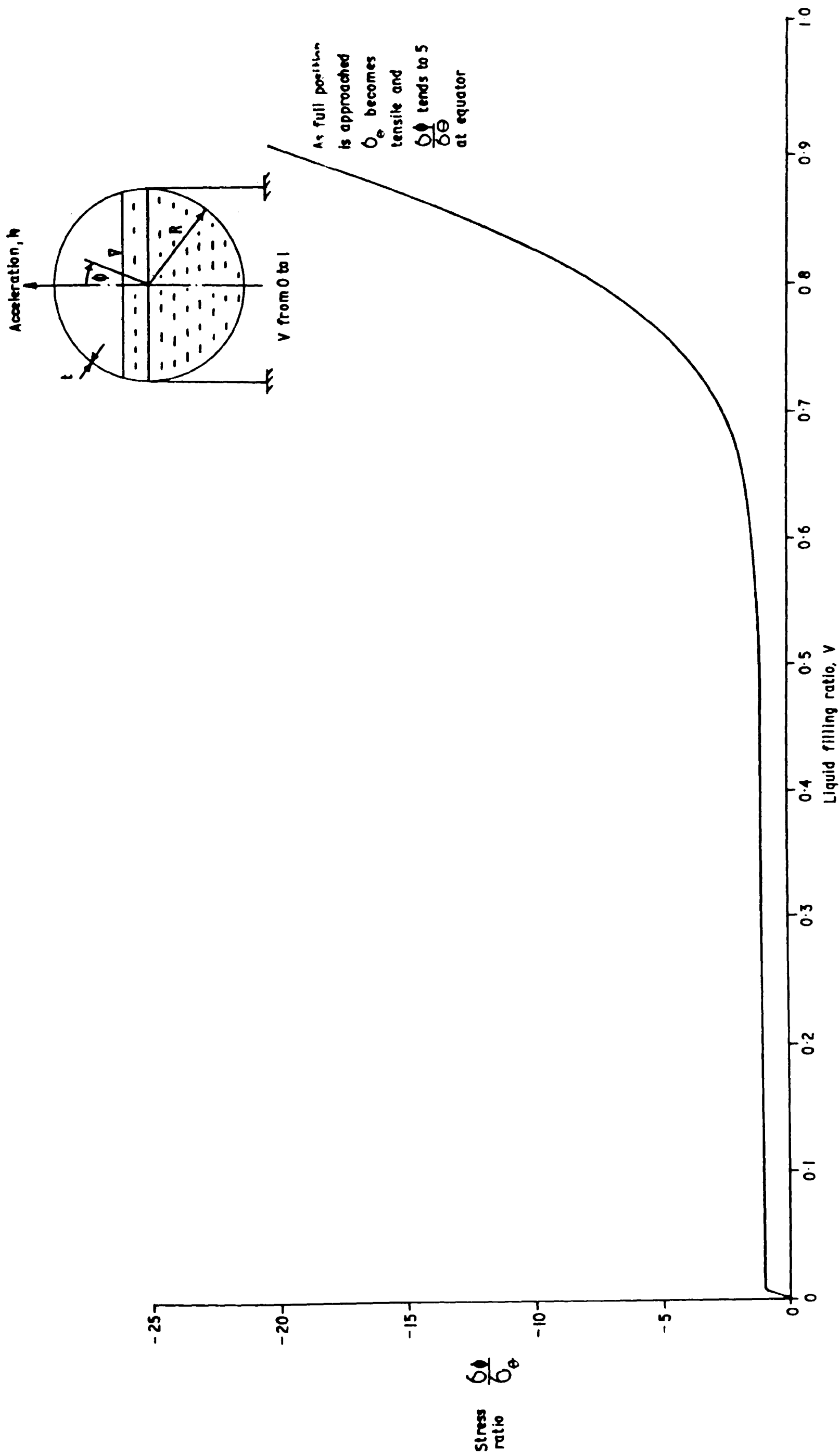


Fig. 3.8. Graph of ratio $\frac{\sigma_p}{\sigma_e}$ versus V . Values of σ_p and σ_e are taken from Fig.3.6.

maximum compressive stress may be allowed to reach the classical value given by equ. 2.1, with a safety factor of approximately 4 still in hand.

$$P_c = \frac{2E}{(3(1-\nu^2))^{\frac{1}{2}}} \left(\frac{t}{R}\right)^2 \approx 1.21E\left(\frac{t}{R}\right)^2 \text{ for } \nu = 0.3$$

$$\sigma_c = \frac{P_c R}{2t} = 0.605E\left(\frac{t}{R}\right) \quad (3.16)$$

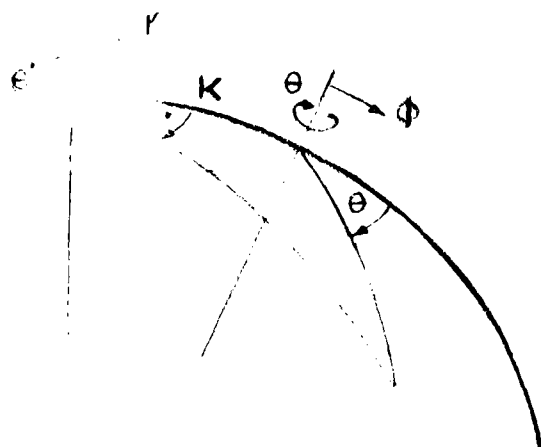
and from Fig. 3.6, for $\nu = \frac{1}{2}$,

$$\sigma_\theta = \frac{k\rho R^2}{3t} < 0.605E\left(\frac{t}{R}\right)$$

or $k < 1.815 \frac{E}{\rho} \frac{t^2}{R^3} \quad (3.17)$

The acceleration thus obtained may be taken as a safe design value. Failure would occur at an acceleration roughly equal to four times the calculated value, which for the same tank dimensions as considered in section 3.2, equals 6.3 g. This is some 4.4 times that of the envisaged maximum vertical acceleration of 1.43 g. If the safety factor of 4 is included, then this results in an overall safety factor of 17.6.

3.4.3 TANK PARTLY FULL AND ITS EQUATORIAL RING 30 DEGREES INCLINED



From the diagram above,

$$\cos\Psi = \cos K \cos\phi - \sin K \sin\phi \cos\theta \quad (3.18)$$

For the bottom hemisphere, liquid level above the equator:

$$\Psi < \Psi_0, \quad P = 0$$

$$\Psi > \Psi_0, \quad P = k\rho R(\cos\Psi_0 - \cos\Psi) \quad (3.19)$$

Substitution of equ. 3.18 into equ. 3.19 and noting

$$\delta = \cos\Psi_0,$$

$$P = k\rho R(\delta - \cos K \cos\phi + \sin K \sin\phi \cos\theta) \quad (3.20)$$

Equ. 3.21 contains those terms which are symmetrical and asymmetrical;

$$P^S = k\rho R(\delta - \cos K \cos\phi) \quad (3.21a)$$

$$P^a = k\rho R(\sin K \sin\phi) \cos\theta \quad (3.21b)$$

where superscripts (s) and (a) refer to the symmetrical and asymmetrical terms respectively.

Solution of equ. 3.21a is as shown in section 3.3.2, with the inclusion of the $\cos K$ term, hence,

$$N_\phi^S = k\rho R^2 \left[\frac{\delta}{2} + \frac{\cos K (1 + \cos^3 \phi)}{3 \sin^2 \phi} \right] \quad (3.22a)$$

$$N_\theta^S = k\rho R^2 \left[\frac{\delta}{2} - \cos\phi - \frac{\cos K (1 + \cos^3 \phi)}{3 \sin^2 \phi} \right] \quad (3.22b)$$

Solution of equ. 3.21b is given by Flugge (1973) as,

$$N_\phi^a = \frac{k\rho R^2 \sin K}{3} \frac{(2 - \cos\phi)(1 + \cos\phi)\cos\phi}{(1 - \cos\phi)\sin\phi} \cos\theta \quad (3.23a)$$

$$N_{\theta}^a = \frac{-k_{\rho} R^2 \sin K}{3} \frac{(3-4\cos\phi+2\cos^2\phi)(1+\cos\phi)\cos\theta}{(1-\cos\phi)\sin\phi} \quad (3.23b)$$

Superposition of equs. 3.22 and 3.23, then yields the following solution for the lower hemisphere;

$$(N_{\phi})_{\phi_0 < \phi < 90^\circ} = k_{\rho} R^2 \left[\frac{\delta}{2} + \frac{\cos K (1+\cos^3\phi)}{3\sin^2\phi} + \frac{\sin K (2-\cos\phi)\cos\phi \cos\theta}{3(1-\cos\phi)\tan\frac{\phi}{2}} \right] \quad (3.23a)$$

$$(N_{\theta})_{\phi_0 < \phi < 90^\circ} = k_{\rho} R^2 \left[\frac{\delta}{2} - \cos\phi - \frac{\cos K (1+\cos^3\phi)}{3\sin^2\phi} - \frac{\sin K (3-4\cos\phi+2\cos^2\phi)\cos\theta}{3(1-\cos\phi)\tan\frac{\phi}{2}} \right] \quad (3.24b)$$

A similar solution may be obtained for the bottom hemisphere when the liquid level is below the equator. Here, however it is not within the range of buckling interest since a liquid level at the equator corresponds to a filling ratio of just 0.16 and so the solution has not been included.

Neither has the solution been given for the top hemisphere when the liquid level is above the equator since it is apparent from section 3.3.2 that buckling will not occur here, due to the presence of both tensile meridional and circumferential stresses. The validity of this assumption will be determined in the experimental investigation.

Finally, for positions where the liquid level crosses the equator, the setting up of the equations would represent a major undertaking since boundary conditions at the equator would have to be satisfied. Since significant

bending stresses would be introduced at these boundaries it is doubtful whether a membrane analysis would be adequate. Consequently the membrane solution for this case is of questionable value.

A typical graph obtained showing the variation in stress in the lower hemisphere and in both ϕ and θ directions for V above the equator is given in Fig. 3.9.

The compressive hoop stress σ_{θ} is seen to reach a maximum at $\phi = 90^{\circ}$, $\theta = 0^{\circ}$, whereas σ_{ϕ} remains always tensile. These maximum compressive values of σ_{θ} have been plotted in Fig. 3.10 against V , together with the corresponding σ_{ϕ} values. Although it is not possible from this graph to determine the absolute maximum compressive value of σ_{θ} or the corresponding critical V , it can be used to give approximations to these values, i.e. extrapolating from the curve gives a maximum compressive value of σ_{θ} of approximately $0.6 \frac{k\rho R^2}{t}$

Following the same argument as in section 3.3.2,

$$k < \frac{E}{\rho} \frac{t^2}{R^3} \quad (3.25)$$

which results in an acceleration of 3.5 g being needed to introduce buckling of the sphere, i.e. a safety factor of 2.5 based on the maximum envisaged vertical acceleration of 1.43g. Including the safety factor of 4 found in section 3.3.2, results in an overall safety factor of approximately 10 being obtained. A value just over one-half of that for the case of accelerating normal to the sphere's equatorial ring support.

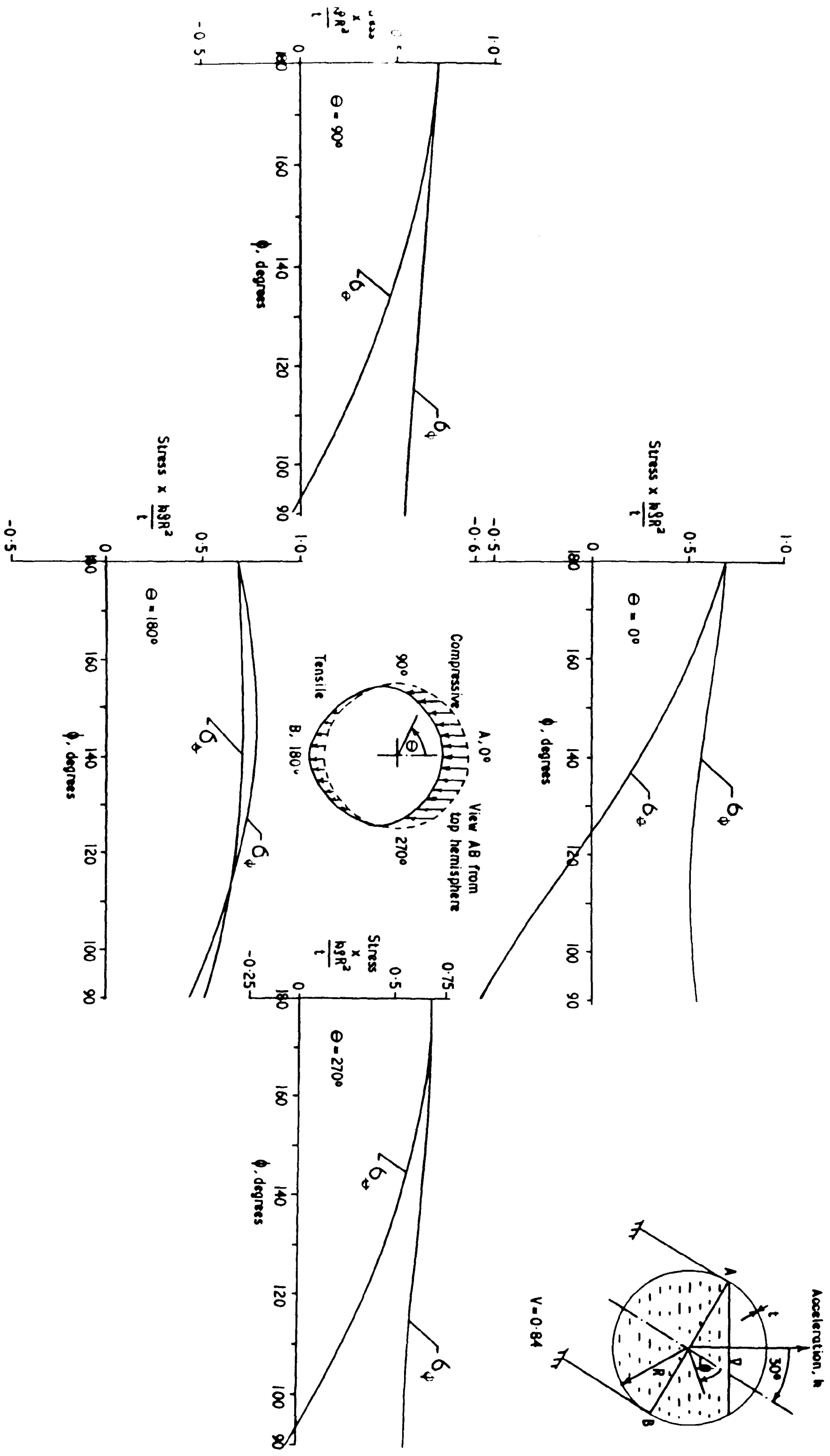


Fig 3.9. Stress distribution of the bottom hemisphere, for the spheres' equatorial ring inclined at 30°.

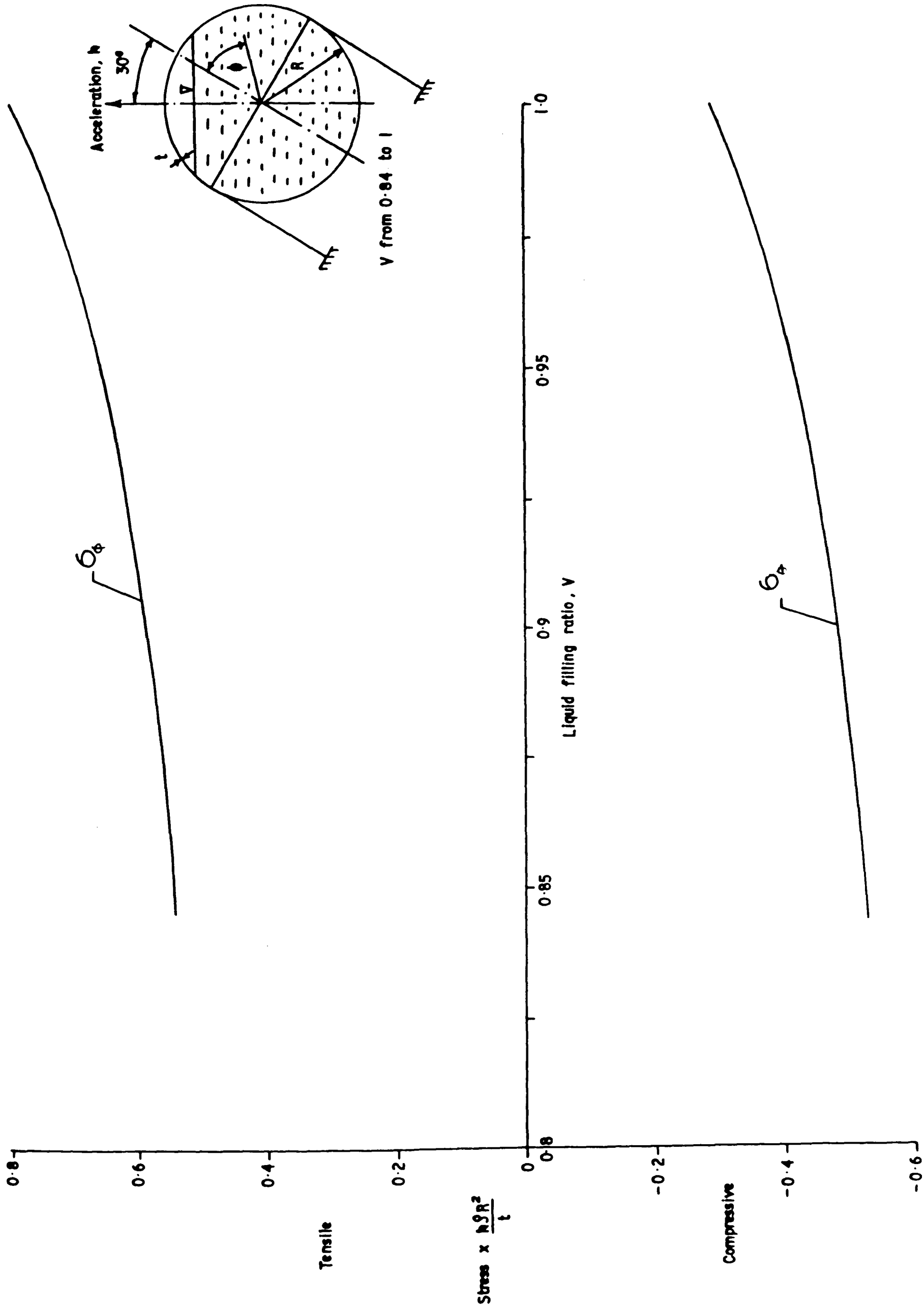


Fig 3.10. Variation of circumferential, σ_ϕ and meridional, σ_α stresses with liquid filling ratio, V ; acceleration 30° to equatorial ring support.

3.4 SUMMARY

3.4.1 Design against external pressure

Imperfections of shapes different from those already tested and analysed, of uncertain magnitude, but likely to be several times the shell thickness and thus larger than the limits normally accepted by the theory are possible in large spherical storage tanks. Their influence is likely to become increasingly important as the use of high strength materials enables an increase in design stresses and thus in flexibility or with the use of low Young's modulus aluminium alloys for the cryogenic conditions imposed by using LNG.

3.4.2 Design against external acceleration

From the preceding membrane analysis safety factors of 17.5 and 10 have been found for the cases of accelerating a partially filled sphere respectively normal and 30° inclined to its equatorial ring support. Whether this approximate analysis is strictly valid considering the approximations made from the interaction curves for a square plate and the omission of bending effects, or whether the sphere will suffer catastrophic collapse at buckling, remain to be answered.

CHAPTER IV

STABILITY OF IMPERFECT THIN-WALLED SPHERES:

MODELS AND EXPERIMENTAL METHOD

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CHAPTER IV

STABILITY OF IMPERFECT THIN-WALLED SPHERES:

MODELS AND EXPERIMENTAL METHOD

4.1 INTRODUCTION

The first part of the investigation was concerned with the effect on stability, under uniform external air pressure, of the inclusion of one of three imperfection shapes in the manufacture of thin-walled spheres; these shapes were the flat, increased radius and bulge.

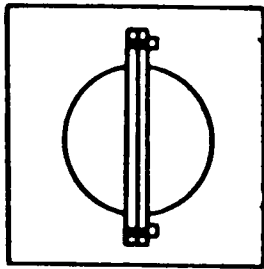
The stages necessary to make, measure and test these shells are shown in Fig. 4.1 and a description of each stage will now be given.

4.2 PROCEDURE

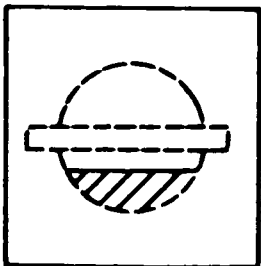
4.2.1 Spherical mould

The spherical mould in conjunction with the rotating mechanism was used to cast wax into a spherical shape of uniform thickness. This method was chosen in preference to using a solid spherical former since a hollow one would facilitate its subsequent removal from the electroplated spheres. Fig. 4.2 shows both halves of the mould, a description of its design being given in Appendix A.1.

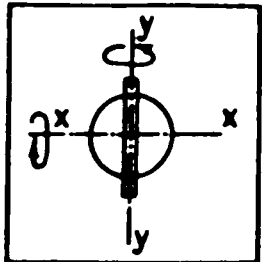
Both hemispheres were treated with two release agents. Initially a grease-like release agent was applied, the moulds then placed in an oven for 12 hours at 100° C in order to allow it to soak into the metal's surface. A thin liquid release agent was next used, and this was liberally



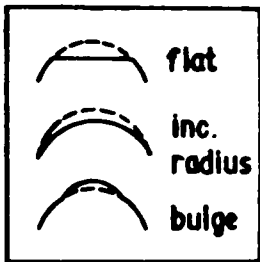
spherical mould



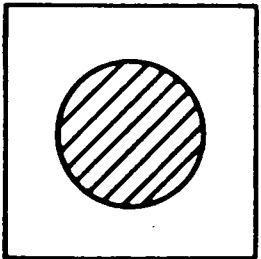
insertion of wax
into mould



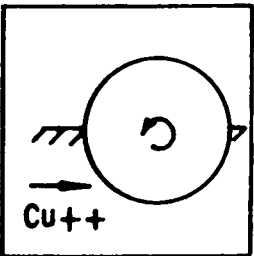
rotating mechanism
used



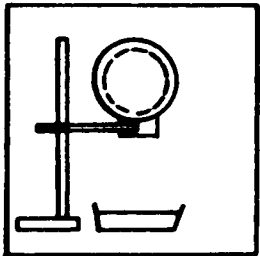
imperfection introduced



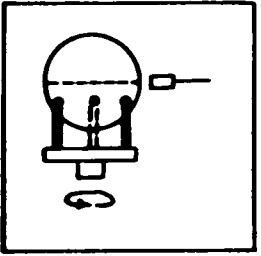
conducting paint
applied



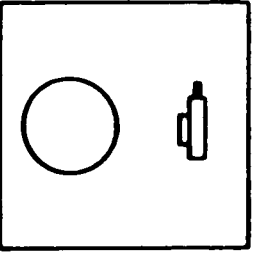
electroplating of
spheres



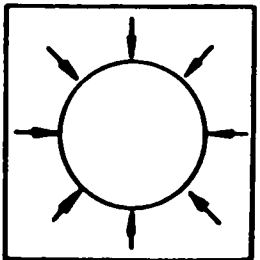
removal of wax



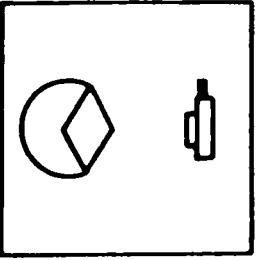
sphericity determined



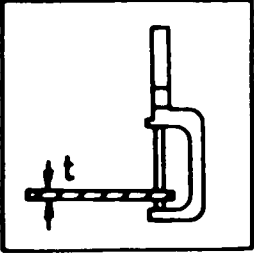
photographs.



pressurize



photographs



thickness, Young's
modulus,
microstructure
determined

Fig. 4.1. Steps made in the manufacture, testing and measuring of the electroplated spheres



Fig. 4.2 Photograph of the two halves of the spherical mould used to cast wax into a spherical shape.

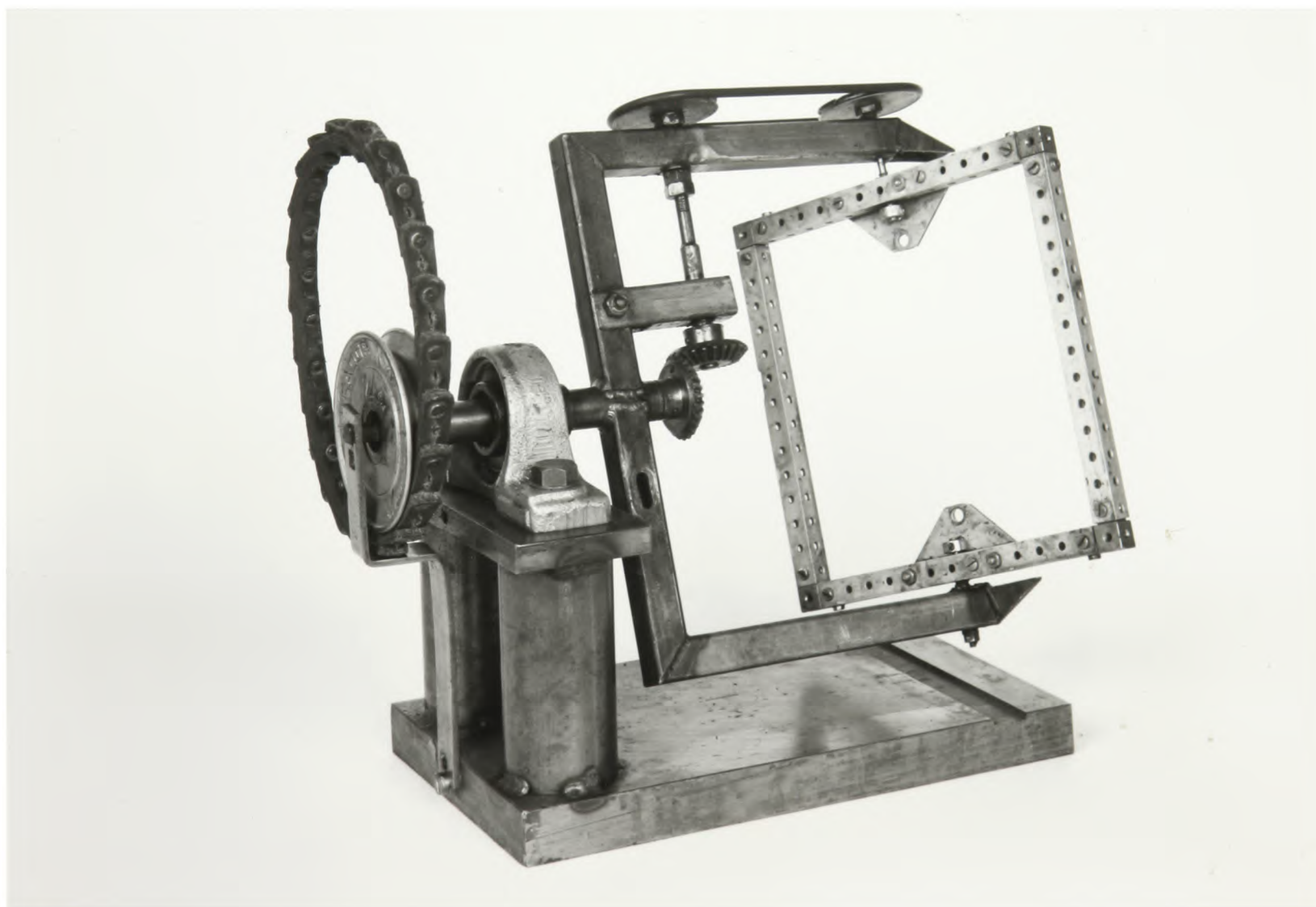


Fig. 4.3 Photograph of the rotating mechanism which provided rotation of the spherical mould about two mutually orthogonal axes.

applied before each casting.

4.2.2 Insertion of wax into mould

Wax was chosen as the casting material since it could be removed from the electroplated spheres without disturbing their spherical shape.

Two thermosetting type waxes were used, and both possessed the following properties:

- a) They had excellent surface smoothness and definition.
- b) They were sufficiently resilient in retaining spherical shape during the electroplating process.
- c) They could be melted at a temperature much less than the melting temperature of copper.

The reason for the change in wax type arose, due to the poor conducting performance of the thin layer of conducting paint applied to the spheres before electroplating. This problem is discussed in section 4.2.5.

Method

- a) The wax was broken into small pieces and 230 g weighed. This gave the cast sphere a thickness of approximately 5 mm.
- b) Wax inserted into mould.
- c) Mould attached to rotating mechanism.

4.2.3 Rotating mechanism

The rotating mechanism's function was to rotate the mould about two mutually orthogonal axes and so enable the liquefied wax, on cooling, to solidify into a spherical shape. Fig. 4.3 shows the apparatus used, a description of

its design being given in Appendix A.2.

Method

- a) Rotating mechanism placed in oven.
- b) Oven temperature of 90°C selected (used for both wax types).
- c) After approximately 2 hours, rotation of mould commenced. Approximate speed: 60 rev/min.
- d) After spinning for $1\frac{1}{2}$ hours, the mould was removed from the rotating mechanism.
- e) Removal of cast wax sphere.

4.2.4 Imperfection

Three types of imperfection of various sizes were introduced on the surface of the wax spheres.

1) Flat

This imperfection was formed by holding the sphere firmly and drawing a millenicut file horizontally a number of times over the wax surface. The surface was then lightly rubbed with fine sandpaper and the resulting surface finish was found to be nearly as smooth as the rest of the cast sphere.

The overall diameter of the flat was not too critical, since the copper thickness could be adjusted in the electroplating bath to give the required imperfection size to thickness ratio. However, variations in diameter from the overall diameter and flatness of the imperfection were important. Imperfection diameters ranging from 12 mm to 47 mm were produced on the 126 mm diameter wax spheres. Using a pair of dividers, diameter readings at 8 positions on the imperfection were taken. If a variation from the

mean diameter of more than 2.5% was found, then the imperfection was not used. The flatness was determined by an engineer's square.

If the imperfection was found to be outside the specified tolerances, then it was increased in size and another attempt made to conform. If this could not be achieved, then the sphere was discarded.

2) Increased radius (spherical region whose radius of curvature is larger than that of the sphere).

The manufacture of this imperfection necessitated the use of a 100 mm radius dural spherical mould, which was used to cast plano-convex wax segments and the procedure adopted is as follows:

- a) Application of release agent to the mould.
- b) Mould heated in an oven to 100° C.
- c) Small quantities of wax gradually added. The wax was allowed to melt and the diameter of the imperfection was measured. If the diameter was found to be too small, then more wax was added and the procedure repeated until the required diameter was achieved. (This method was found to be preferable to that of weighing the wax since the average sized imperfection was only of the order of 1 g).
- d) The mould was left to cool for approximately 1 hour, until the wax had been seen to solidify.
- e) The removal of the wax from its mould was facilitated by placing the mould in a refrigerator for approximately 5 minutes, and so allow the wax to contract a little relative to the mould.

In this way, plano-convex segments with diameters ranging from 18.5mm to 40 mm were produced.

A flat of the same diameter was next made on the wax sphere, and the segment then tack glued into position. At this stage, the transition at the junction between the perfect sphere and the segment was not smooth. This difficulty was overcome by heating a small quantity of wax in a container, and distributing a very small amount via a small bore tube around the periphery of the segment. By lightly rubbing the area with fine sandpaper, a smooth and accurate curvature transition was obtained.

3) Bulge (spherical region whose radius of curvature is smaller than that of the sphere).

The same procedure as was adopted in the making of the increased radius imperfection was used.

For the bulge, a 48 mm radius spherical mould was used to cast plano-convex segments of diameters ranging from 12 mm to 47 mm. The task of obtaining a smooth transition between the bulge segment and the sphere was however more difficult than with the increased radius imperfection, due to the more pronounced change in curvature at the junction.

4.2.5 Conducting paint

Before the wax sphere could be electroplated, it had to be coated with a thin layer of conducting paint. A commercially available high conductivity silver conducting paint was used.

The first method tried was to spray the paint, which, necessitated the addition of a thinner. Although some success was achieved continual blocking of the spraying nozzle occurred and in addition, since the silver particles tended

to settle at the bottom of the solution, the paint applied was in fact mostly thinner and so was no longer conducting. Therefore these problems, together with the toxic conditions for the operator, tended to make this an unsuitable method.

The second method tried, and the one used for all the spheres tested, was to brush paint the sphere. Since the paint dried almost immediately on application it was important to initially fully charge the paintbrush and then to use one stroke only in a north-south manner. Although the surface finish was not as smooth as was found by spraying, with practice it was soon possible to deposit an even coating with no prominent brush marks.

In Section 4.2.2, it was stated that the change in wax occurred because of the poor performance of the conducting paint. Initially a number of reasons were assumed, such as contamination by grease, inadequate mixing of the paint and the possibility of bad electrical contact in the electroplating bath.

Precautions were therefore taken in order to eliminate these potential trouble areas. This amounted to always wearing plastic gloves, mixing the paint vigorously for approximately two minutes before application and to increasing the spring pressure on the brass electrical contact used during electroplating.

It was found however that these precautions made a marginal difference, although it was always possible to obtain a uniform coating of copper by intermittently stopping the electroplating process and repainting the non-conducting parts on the sphere. This occurred in the very

early stages of electroplating and so did not produce an uneven thickness coating.

After considerable delay it was concluded that a chemical reaction had occurred between the wax surface and the conducting paint, which had caused the conducting property of the paint to be reduced. As an interim stage before another wax could be found, a very thin diluted water based paint layer was used as an interface between the wax and the paint. The method proved a success and initial overall electroplating was achieved. However, a paraffin based wax was later obtained which was found not to react with the conducting properties of the paint.

4.2.6 ELECTROPLATING

Shown in Fig. 4.4, is a plan and side elevation of the apparatus used, a description of its design being given in Appendix A.3.

The rectangular glass tank was first cleaned and thoroughly washed with hot water. The electrolytic solution was then prepared, the chemicals used and their relative proportions being shown below.

Crystalline CuSO_4	200 g
Concentrated H_2SO_4	31 cc
De-ionised H_2O	1000 cc
$\text{Al.K}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$	12.5 g

(this was added in order to give a fine grain structure to the electro-deposited copper).

Approximately 15 litres were needed in order to immerse half the wax sphere in the electrolytic solution. The electrical contact for the conducting paint was a 0.8 mm

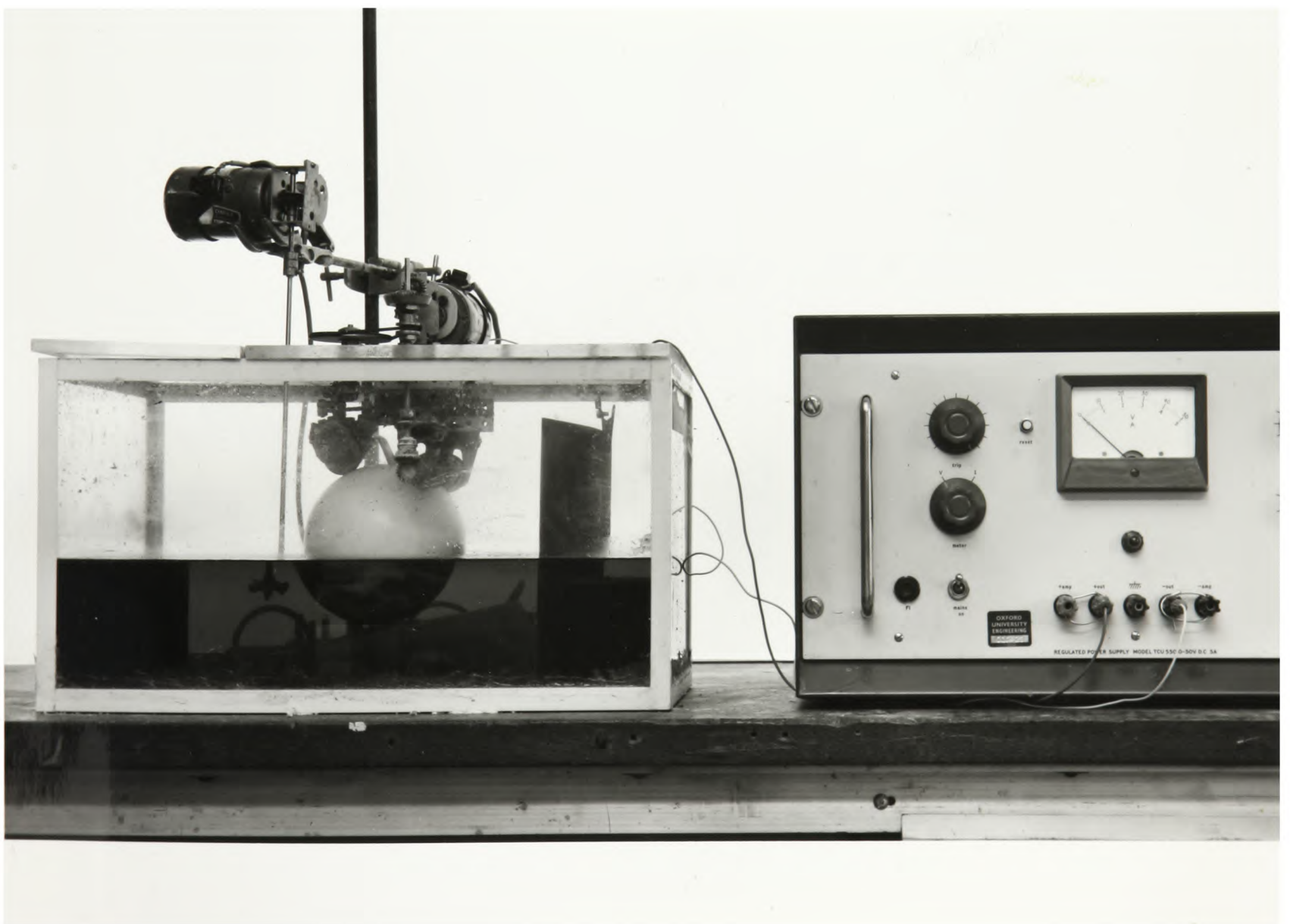
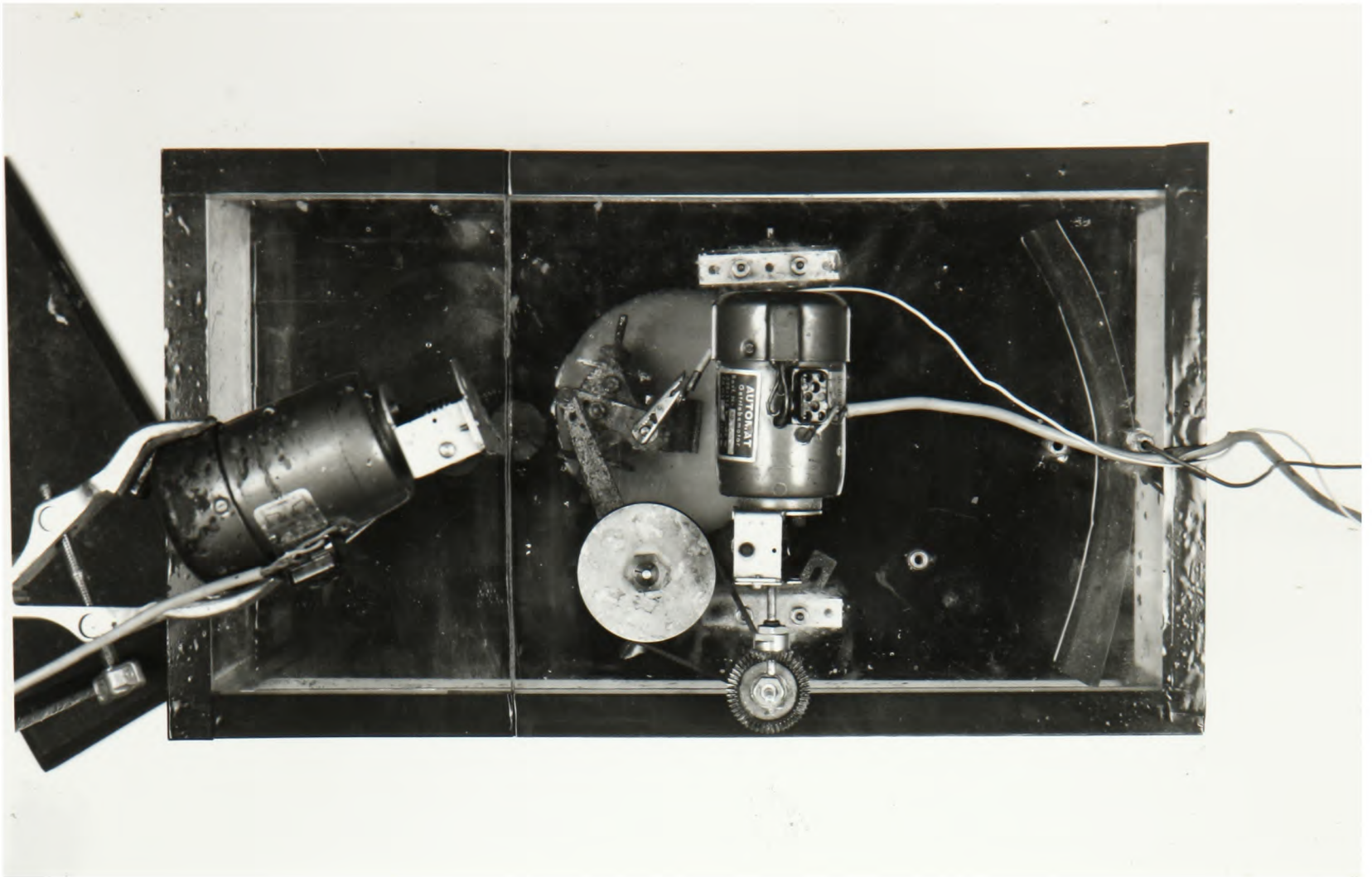


Fig. 4.4 Photograph of the electroplating apparatus. (Plan view shown above; side view shown below).

thick, 50 mm x 25 mm brass shim. This was found to be satisfactory although scraping of the rather soft conducting paint occurred if dirt was entrapped between the contact and the paint. This problem was overcome by:

a) Starting the electroplating process while the sphere was stationary and then after approximately 30 seconds inverting the sphere such that the newly deposited, very thin, hard copper layer was in contact with the brass shim. The half resting in solution was then electroplated for approximately 30 seconds. This ensured that the sphere had an overall covering of a very thin, hard layer of copper which successfully deterred the electrical contact from removing the conducting paint.

b) Ensuring the bath was kept clean. This involved filtering the solution and removing the collected dirt at the bottom of the tank. Additional electrolytic solution was added in order to compensate for solution lost.

c) Stirring the electrolytic solution vigorously near the liquid surface and close to the sphere so as to thoroughly clean its surface before it passed underneath the electrical contact.

Faraday's law was used to estimate the thickness of copper deposited. This law states that the mass M , deposited, is given the expression,

$$M = Z I T \quad (4.1)$$

where Z is the electro-chemical equivalent of the deposit and $I T$ are the ampere-hours used during the deposition. The constant Z was obtained by first determining experimentally the thickness deposited on a test sphere after 20 ampere-hours, and then using the equation,

$$(R_C^3 - R_W^3) \frac{4}{3} \pi \rho = 20Z \quad (4.2)$$

where R_W is the radius of the wax sphere and R_C is the external radius of the copper. Knowing both R_W and R_C and assuming a density, ρ , of $9 \times 10^3 \text{ kg/m}^3$, Z was calculated to be 1.685 g/ampere-hours.

However this formula was used only as a guide in determining the actual amount deposited, since the current level was gradually built up to a maximum value of 4 ampere over a period of 15 minutes, in order to ensure that the conducting paint and the wax were not melted, through the use of too high a current. Furthermore this was a mean value since fluctuations of approximately ± 0.2 ampere occurred in practice. Therefore the method was improved. This amounted to first weighing the wax sphere before it was placed in the electroplating bath. The thickness of the deposit was then calculated using equ. 4.1, and when this had been determined to be approximately 0.005 mm less than the required thickness, the sphere was taken out of solution and weighed. By subtracting the original weight of the sphere it was then possible to determine the actual thickness deposited. If it was found that the copper coating was too thick, then the current polarity was reversed, i.e. the cathode was then the anode and the anode was then the cathode.

4.2.7 Removal of wax

After removing the sphere from the electroplating bath it was first washed with cold water.

The sphere with the wax former inside was quite robust and so it was possible to use an electric drill to make the opening ($\frac{1}{8}$ in.) for the removal of the wax. The only requirement on its location was that it should be approximately perpendicular to the imperfection, in order to facilitate the measuring of the deflection at the imperfection when the sphere was being pressurized.

The sphere was then rested on a cup-shaped attachment and placed in an oven and left at a temperature of 100° C for approximately 2 hours until all the wax had been seen to drain away into the collecting tray.

As a precaution, checks were made at this stage to determine if the imperfection size had altered. However this was not found to be the case.

4.2.8 Sphericity

The accuracy of the sphere's sphericity was determined by measuring the deviation from circularity of a number of circularity plots. The apparatus used, together with a schematic diagram is shown in Fig. 4.5.

The sphere was mounted on a three point base which was in turn attached to a dividing head. A rubber toothed belt was glued to this dividing head and then by a series of gears connected to a potentiometer. The potentiometer recorded the angular rotation of the dividing head and the signal thus generated was fed into the X channel of an XY plotter. The Y channel input was connected to a proximity gauge located horizontal and normal to the surface of the sphere.

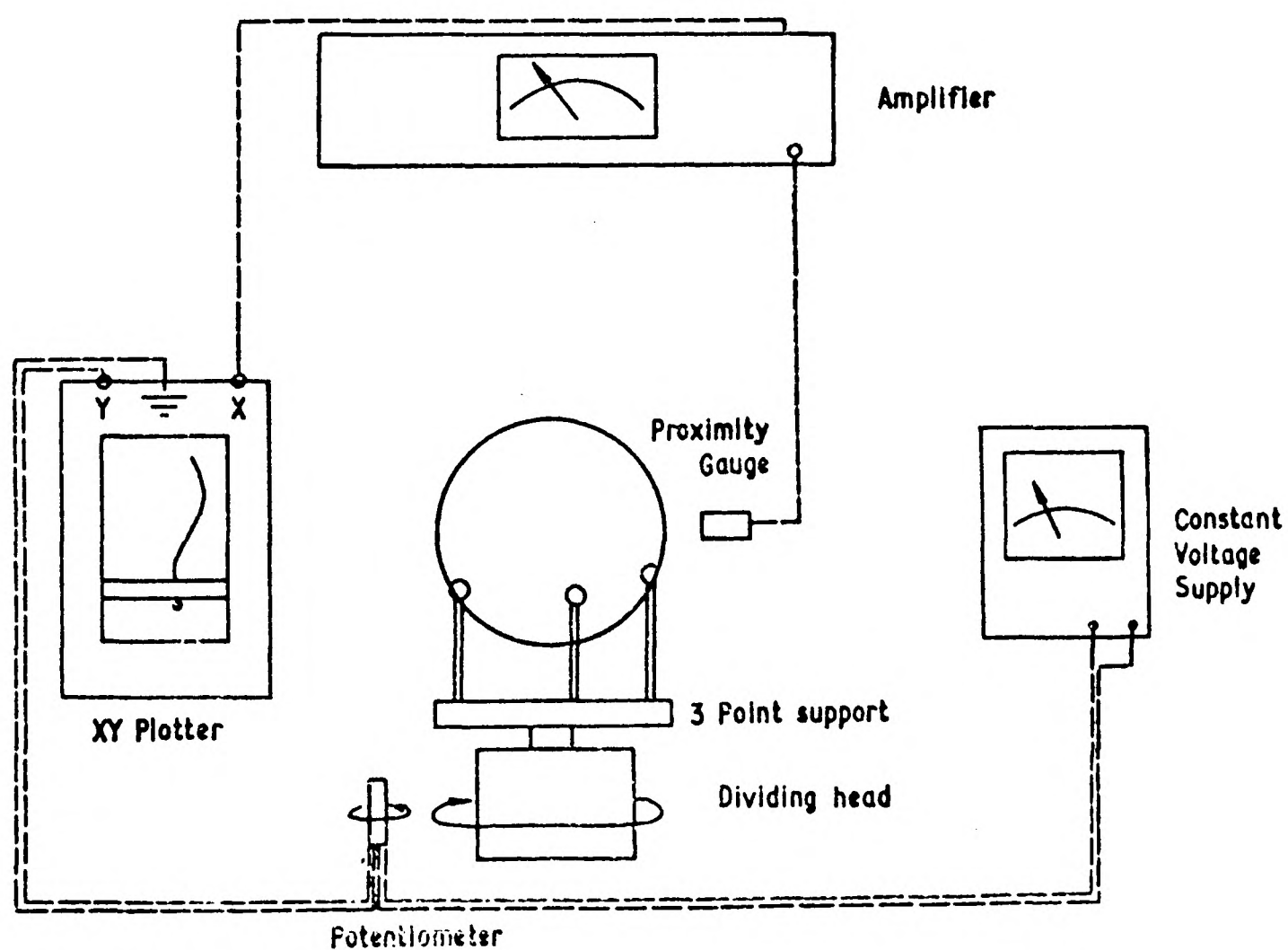
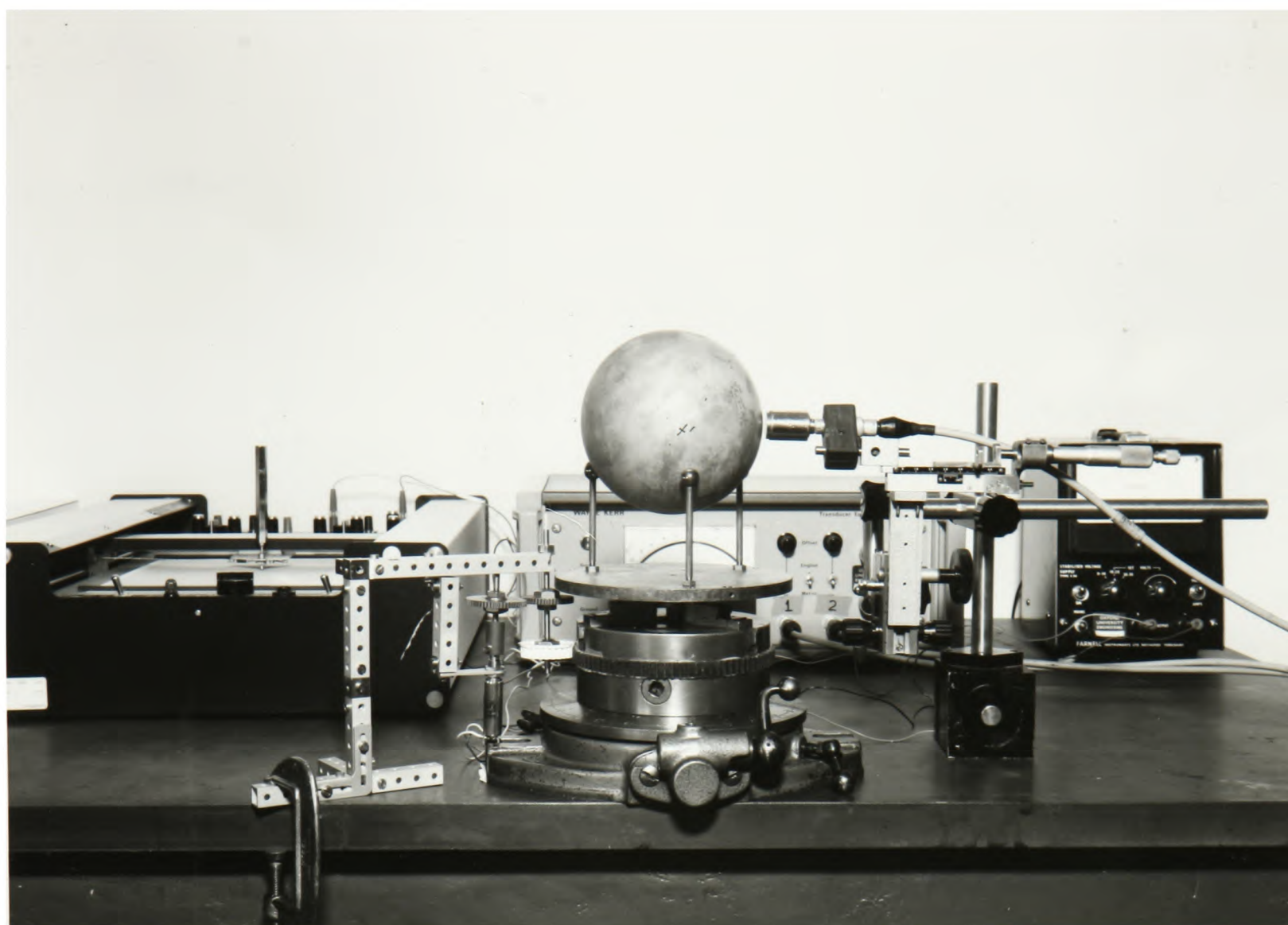


Fig.4.5. Photograph and schematic diagram of apparatus used to judge each spheres sphericity. This was obtained by rotating the sphere relative to a fixed proximity gauge.

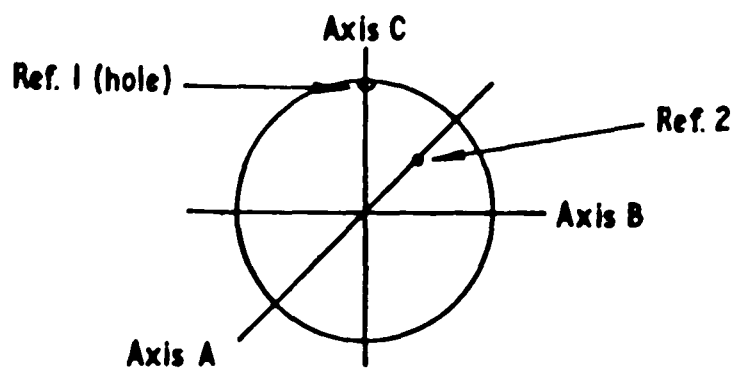
The hole in the sphere was chosen as a reference point. The sphere was positioned such that a plumb line remained central in the hole during rotation of the dividing head. The position of the gauge relative to the sphere was then marked at 90° intervals, in order to define the 3 axes of the sphere. In order that each sphere had the same circularity plots made and that each plot was from a different position, the sequence shown in Fig. 4.6 was adhered to. This prevented random circularity plots from being taken, plots that could conceivably be for only one particular area of the sphere. The accuracy in measuring the angles was not too critical since interest was directed to an area, not a specific point, and circular plots at 43 degrees or 47 degrees would be just as informative as one at 45 degrees. As the dividing head was rotated, a sinusoidal trace due to the eccentricity of rotation and to inaccurate positioning of the three point supports, was obtained from the XY plotter. A trace for a perfect sphere is shown in Fig. 4.7. Any deviations from this trace give an indication of the non-circularity of one circular plot.

4.2.9 Photographs

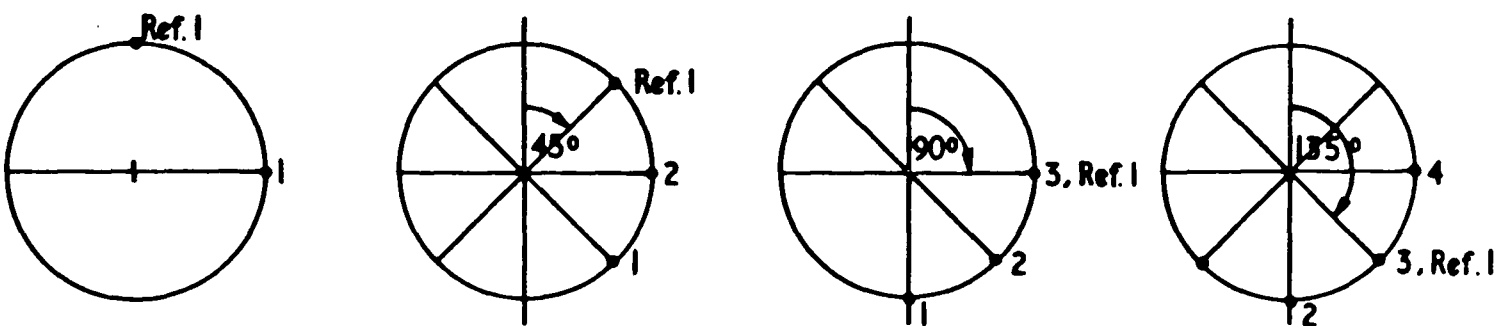
Each sphere was photographed before testing to collapse. Six photographs of each sphere were taken in order to document all of the sphere's surface.

4.2.10 Pressurizing of sphere

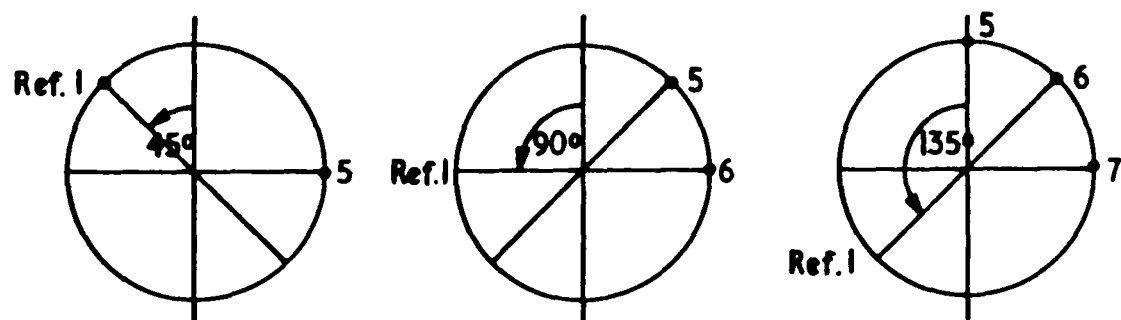
Using a vacuum pump it was possible to evacuate air from the sphere and hence apply uniform external pressure. Deflections at the imperfection were recorded for each



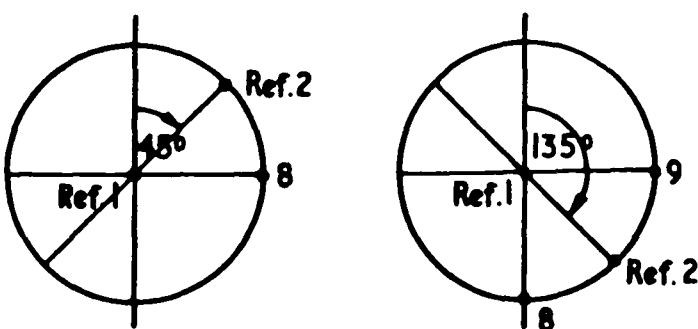
Rotation about axis A, (drawn into the paper).



Rotation about axis B, (drawn into the paper).



Rotation about axis C, (drawn into the paper).



Total 9 Circularity plots

Fig.4.6. Sequence adopted in the determination of 9 circularity plots for each sphere.

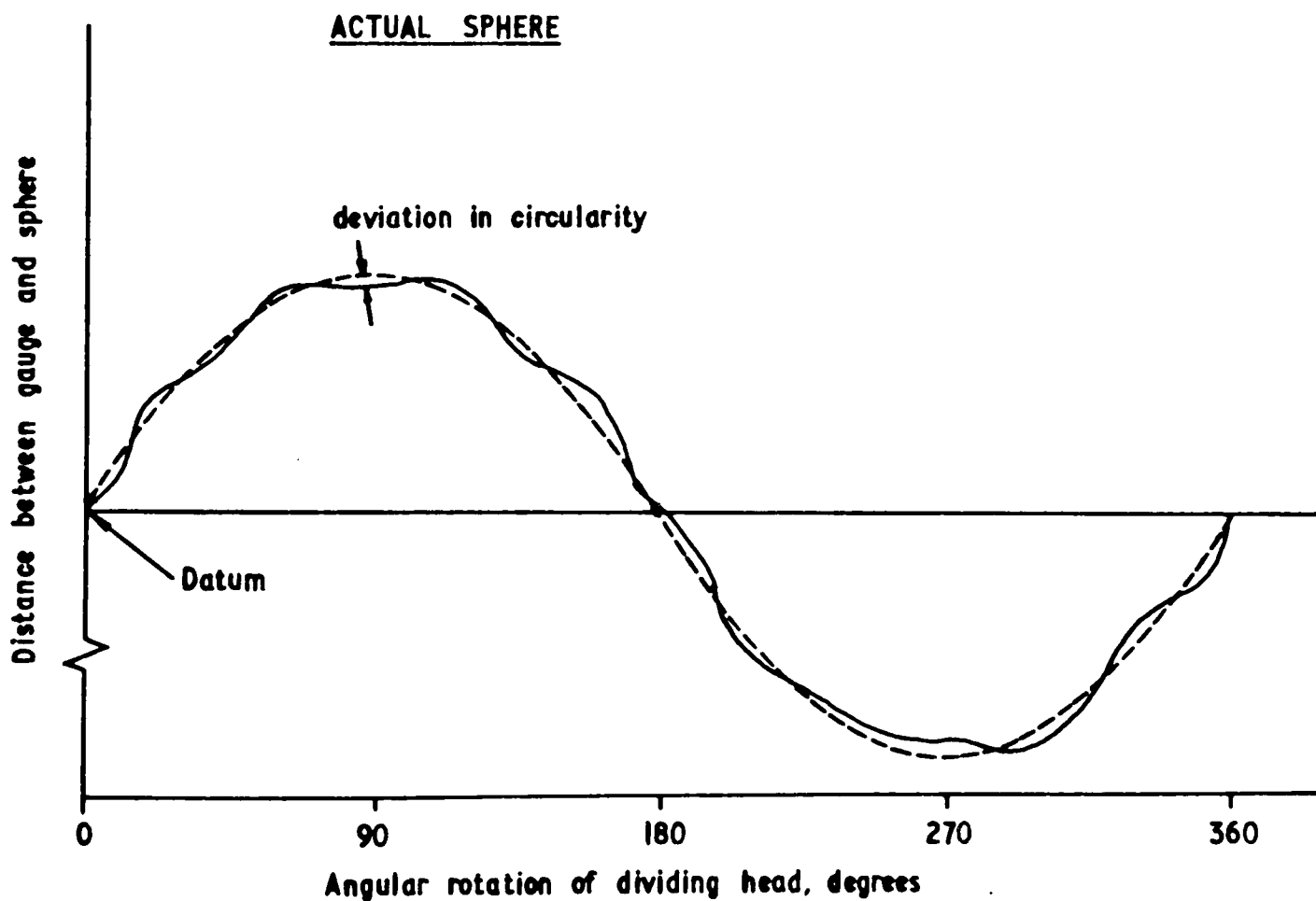
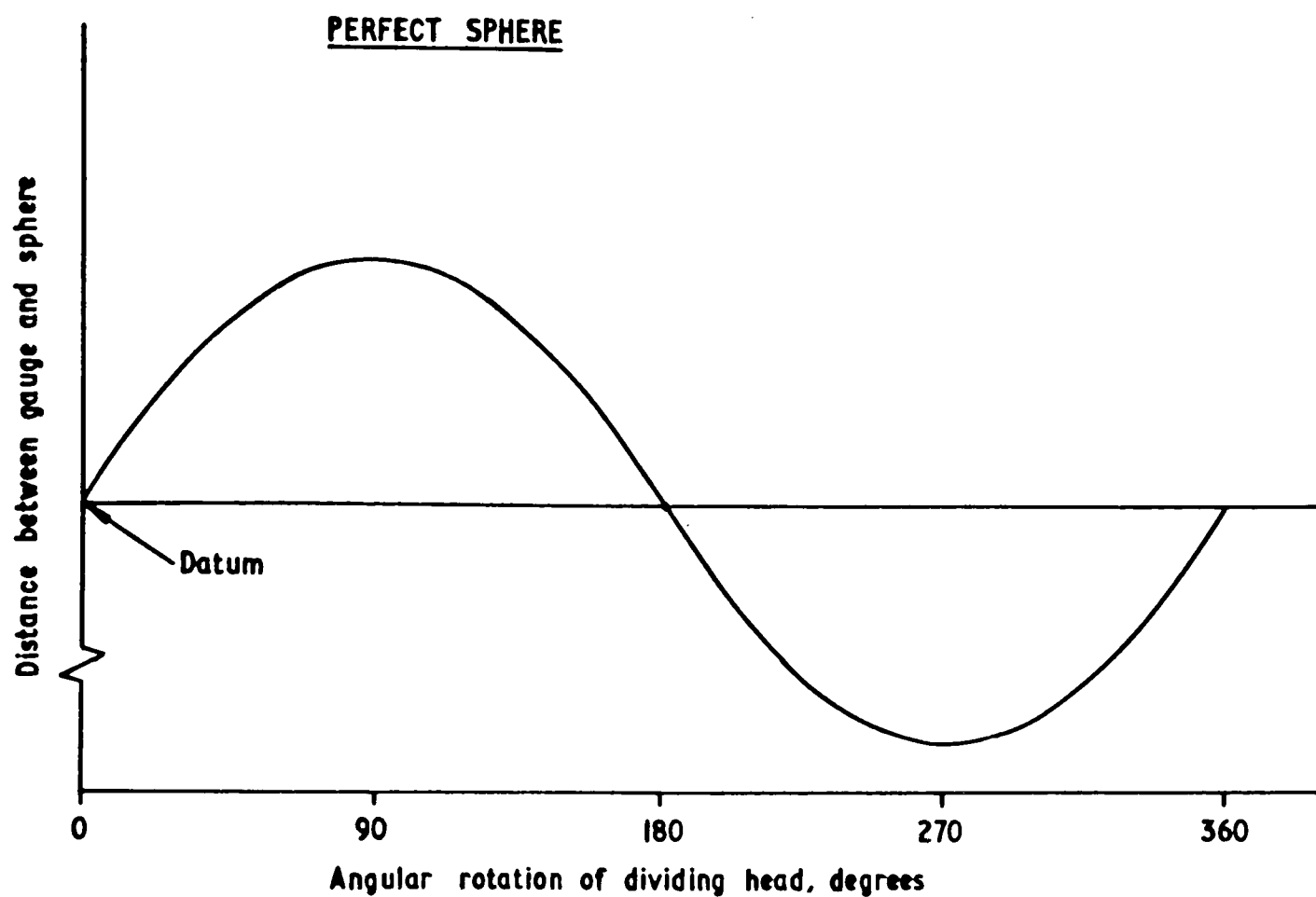


Fig.4.7. Determination of sphericity by comparing a perfect sinusoidal trace with the actual trace from an imperfect sphere. Traces obtained from XY plotter.

increase in pressure up to the buckling pressure. The apparatus used together with a schematic diagram is shown in Fig. 4.8.

The sphere was glued to a perspex holder, which in turn was located vertically in the dividing head. The proximity gauge (A) was used to measure deflections at the imperfection and another (B) was positioned 180 degrees to (A) which enabled any movement of the sphere towards (A) to be detected and so confirm whether gauge (A) was measuring solely the deflection at the imperfection. This gauge was positioned centrally and normal to the imperfection and was connected to the Y channel of an XY plotter. Gauge (B) was positioned normal to the sphere and was connected to an oscilloscope. The procedure for determining the buckling pressure and the documentation taken at each pressure interval is as follows.

The mercury manometer was adjusted to read zero on the 100 cm (mm divisions) rule. Datum points were recorded for both proximity gauges. The vacuum pump was then started and its pressure adjusted in order to apply a given uniform external air pressure to the sphere. The Y channel of the XY plotter automatically recorded deflection at the imperfection. At the same time a reading from the oscilloscope was taken. The X controller on the XY plotter was then moved manually, ready for the next increase in pressure. This procedure was repeated until collapse of the sphere occurred.

4.2.11 Photographs of collapse mode

A photographic record of each sphere's buckled form

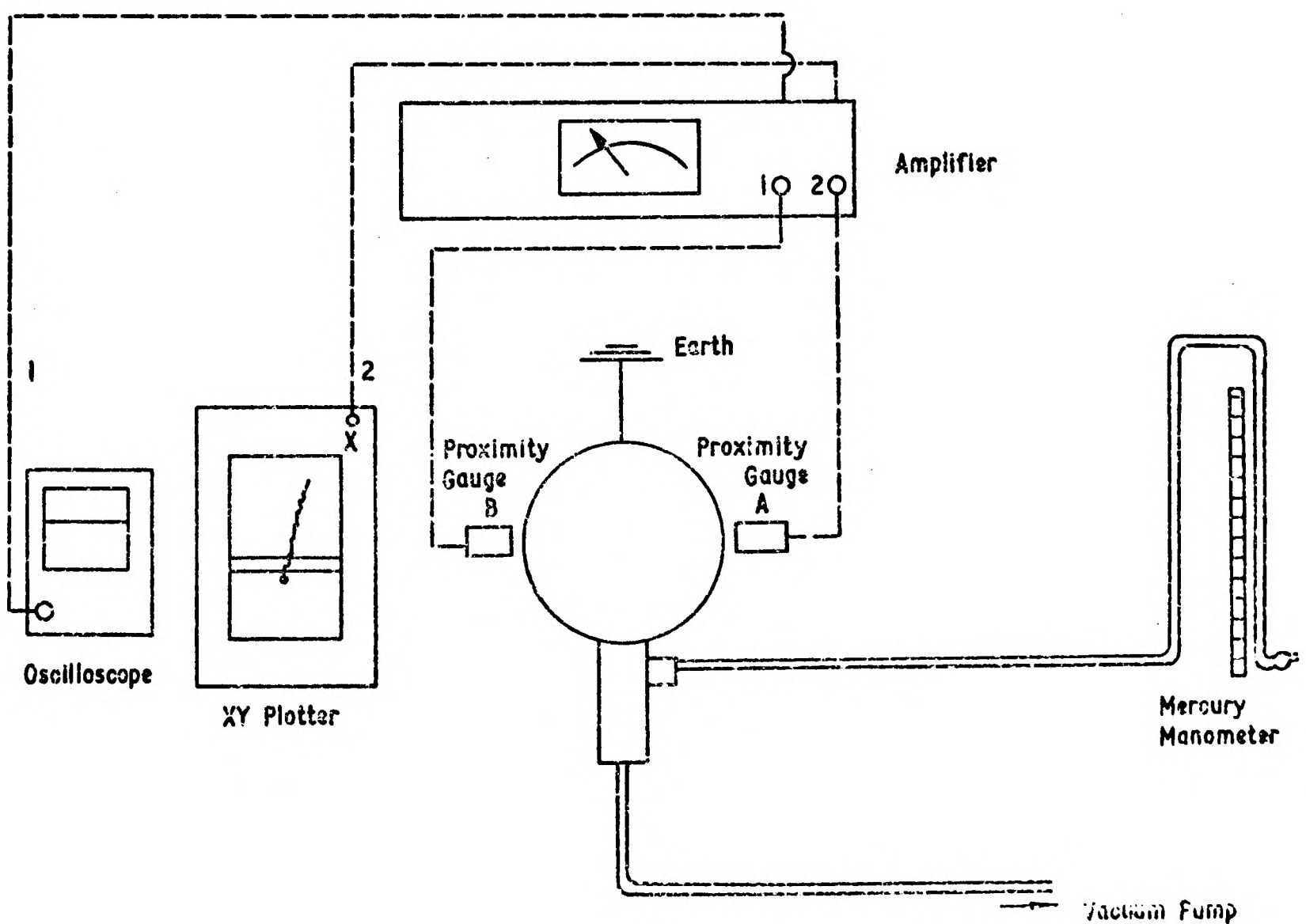
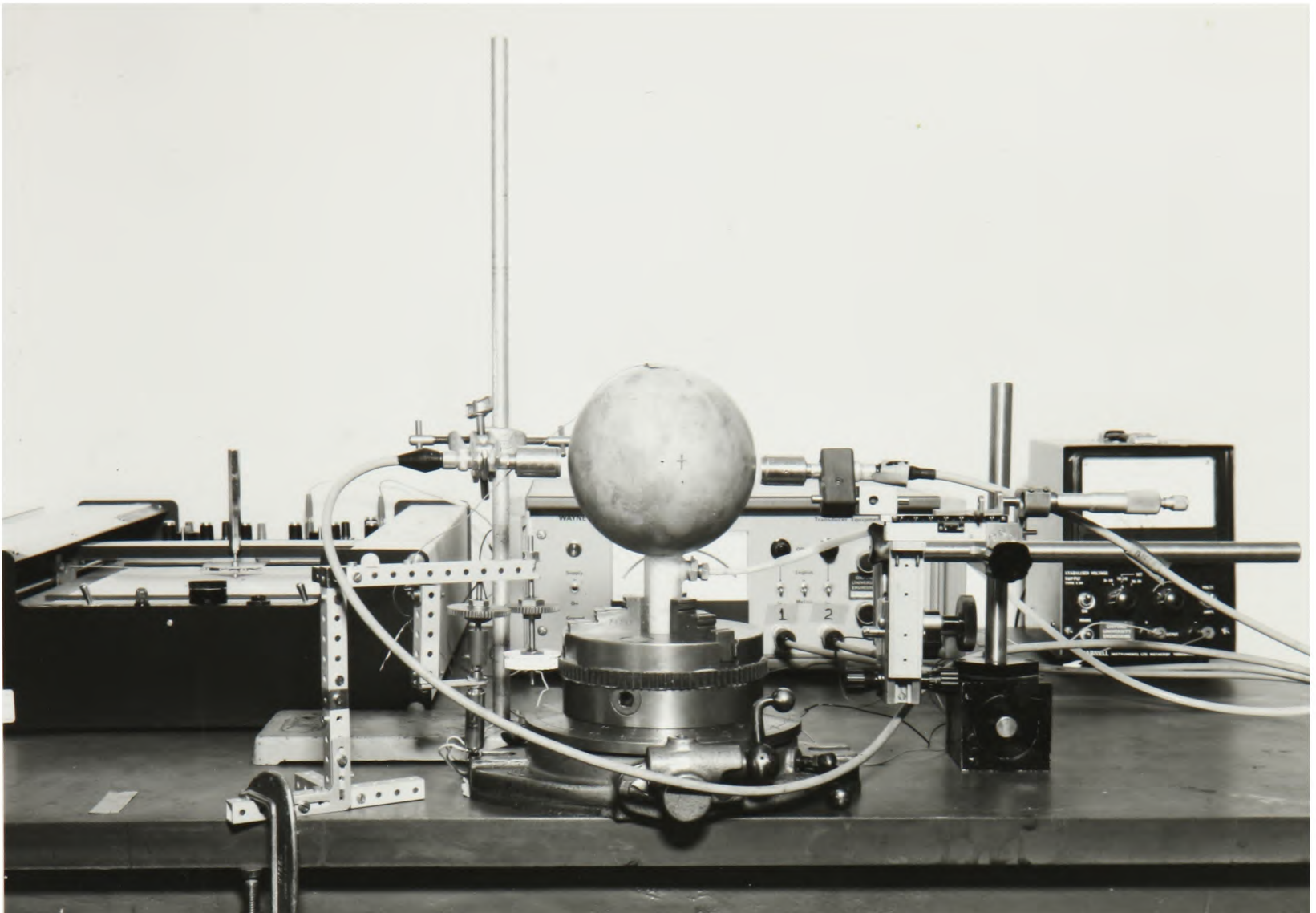


Fig. 48. Photograph and schematic diagram of the pressurizing of an imperfect copper sphere. Proximity gauge A recorded deflection at imperfection. Uniform external pressure applied by vacuum pump.

was made.

4.2.12 Thickness, Young's modulus of elasticity, microstructure

The experimental buckling pressures were to be compared with P_c , equ. 2.1, for a perfect sphere. The calculation of this equation required the evaluation of the shell thickness, Young's modulus of elasticity and Poisson's ratio.

Since any error in the shell thickness would make an appreciable difference to the value of P_c , it was important to accurately measure with a micrometer the buckled sphere's thickness extensively over the whole of its surface and record what deviations were found from the mean value.

The evaluation of Young's modulus of elasticity was determined from tensile tests on strips cut from spherical shells in areas away from the buckled region and so the inclusion of kinks was avoided. A strip was cut from two spheres which represented the early and later models electroplated. A small width of 7 mm was chosen in order to reduce the curvature across the strip's width and a length of 60 mm was used. Two strain gauges were attached on either side of each strip. Several initial loadings were then made before any results were taken in order to compensate for the straightening effect of the strip. The results obtained represented the variation in Young's modulus of elasticity within each strip and variation between the two strips.

Since the magnitude of Poisson's ratio is not too critical in equ. 2.1, it was assumed that a value of 0.3

for copper was applicable.

A microstructure examination was conducted in order to determine whether the copper shells were typical of electroplated copper or whether the metal surface was porous and contained voids. A microstructure examination was made for three of the copper spheres, these shells representing the early, middle and later stages of manufacture. The etching agent used^[64] was:-

25 ml NH_4OH + 50 ml H_2O_2 + 25 ml H_2O .

CHAPTER V

STABILITY OF IMPERFECT THIN-WALLED SPHERES:

EXPERIMENTAL RESULTS

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CHAPTER V

STABILITY OF IMPERFECT THIN-WALLED SPHERES:

EXPERIMENTAL RESULTS

5.1 INTRODUCTION

The electroplating method allowed good control of thickness variation and radius in the production of 25 spheres of radius 63 mm with radius-to-thickness ratios ranging from 463 to 1703.

A presentation of the results will now be given.

5.2 PROPERTIES OF THE MODELS

5.2.1 Geometry

The two imperfection parameters used to define the size of imperfection were the diameter (a) and the distance at mid-point (d) between the imperfection and the sphere. The former was convenient to use during the manufacture of the various sizes of imperfection, although it is the amplitude (d) which is normally used in theoretical imperfection studies, (see section 2.2.1).

The imperfection geometrical data for the 25 spheres tested is given in Table 5.1.

5.2.2 Thickness variations

Although the sphere's rotating axis during the electroplating process was continually changed relative to a stationary anode, some sections of the sphere were, on average, a closer distance to this anode and hence received

Table 5.1 Geometrical data for the 25 spheres tested

Shell No.	a	d	t	d/t	R/t	R	R _{IR}	R _B
Units	mm	mm	mm			mm	mm	mm
						63	100	48
FLAT	1F	44.5	4.0600	0.087	46.7	724		
	2F	41	3.4290	0.063	54.4	1000		
	3F	35	2.4793	0.065	38.1	969		
	4F	31	1.9365	0.038	51.0	1658		
	5F	28.5	1.6300	0.075	21.8	840		
	6F	20	1.7987	0.068	11.7	926		
	7F	19	0.7204	0.088	8.2	716		
INCREASED RADIUS	1IR	40	1.2385	0.071	17.4	887		
	2IR	39	1.1741	0.099	11.9	636		
	3IR	38.5	1.1427	0.092	12.4	685		
	4IR	36	0.9928	0.056	17.7	1125		
	5IR	35.5	0.9643	0.077	12.5	818		
	6IR	33	0.8284	0.037	22.4	1703		
	7IR	26	0.5073	0.076	6.7	829		
	8IR	18.5	0.2540	0.066	3.8	955		
BULGE	1B	47	1.6000	0.088	18.2	716		
	2B	40	1.1060	0.086	12.9	733		
	3B	40	1.1060	0.053	20.9	1119		
	4B	39	1.0460	0.081	12.9	778		
	5B	28	0.5118	0.077	6.7	818		
	6B	26	0.4381	0.083	5.3	759		
	7B	22.5	0.3244	0.079	4.1	797		
	8B	18	0.2051	0.086	2.4	733		
	9B	13	0.1059	0.105	1.0	600		
	10B	12	0.0901	0.136	0.66	463		

Flat imperfection

Increased radius imperfection

bulge imperfection

a higher deposition of copper.

The maximum variation in thickness, Δt , from the mean thickness (t) for each sphere is shown in Table 5.2. A maximum $(\Delta t/t)$ value of 6% was found. This value compares satisfactorily with the rather lower value of 2.5% obtained by Thompson (1960) for his electroplated copper spheres and very favourably when compared to Calson et al's (1967) maximum value of 14% for their electroplated nickel spheres. (This latter value, however, was further increased to 30% by chemically polishing an area of the sphere in order to contain buckling to a narrow region of the shell, this being necessary since the method of suspension of holding the sphere by two diametrically opposite electrical contacts, created holes which necessitated reinforcement and hence introduced imperfection areas, as witnessed by Fig. 2.6).

5.2.3 Sphericity

A typical circularity plot for one sphere is shown in Figs. 5.1a, b and c.

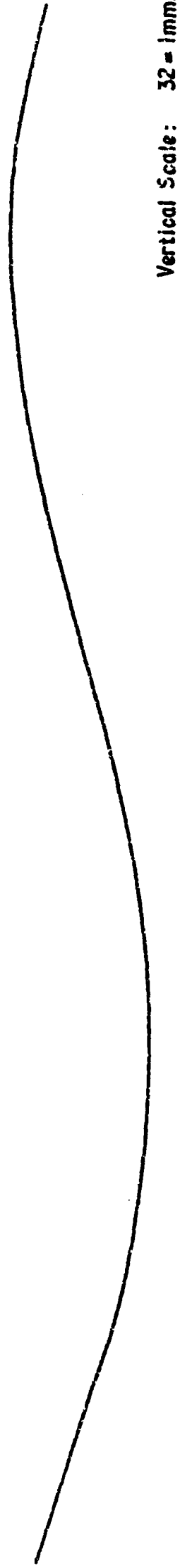
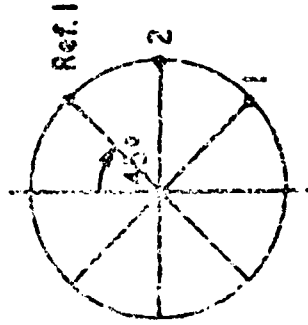
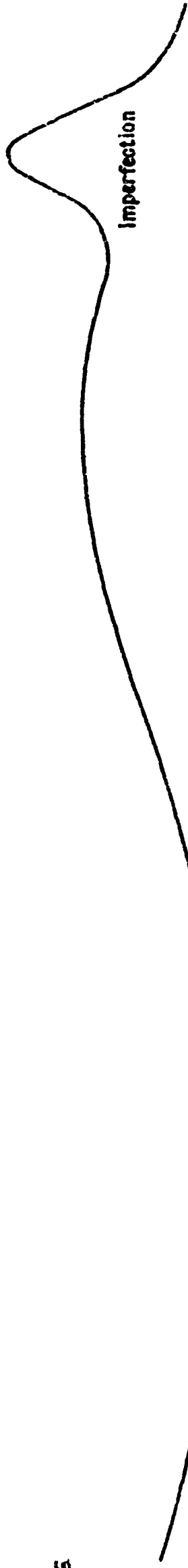
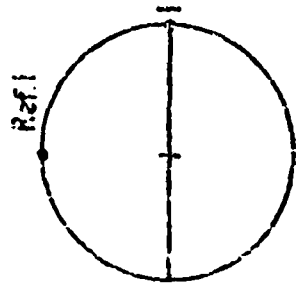
Deviations from the sinusoidal curves represented local variations in radius. The method used to measure the variations in circularity, was to compare the experimental curves with sinusoidal curves of equal wavelength and various amplitudes.

The maximum deviation (e) from the radius for each sphere was then measured from the sinusoidal traces and the results for the 25 spheres tested are given in Table 5.2. The maximum variation in radius for all the spheres tested was 0.2%.

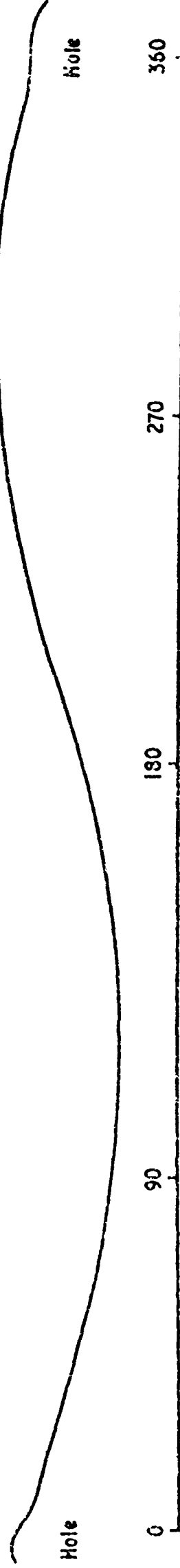
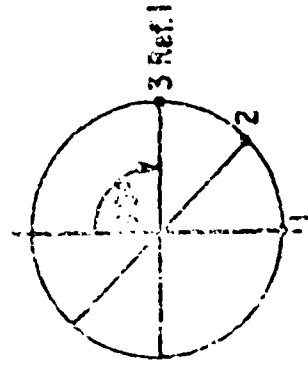
Table 5.2 Maximum variation in thickness, Δt , from the mean thickness, t , and maximum deviation e , obtained from sphericity measurements as a fraction of the radius, R , and mean thickness, t .

	Shell No.	$\Delta t/t$	e/R	e/t
Units		%	%	
FLAT	1F	5	0.15	1.08
	2F	4	0.15	1.50
	3F	4	0.15	1.44
	4F	5	0.15	2.47
	5F	3	0.17	1.46
	6F	4	0.13	1.19
	7F	5	0.15	1.07
INCREASED RADIUS	1IR	4	0.15	1.32
	2IR	5	0.18	1.15
	3IR	6	0.10	0.71
	4IR	4	0.15	1.67
	5IR	3	0.18	1.48
	6IR	6	0.13	2.20
	7IR	3	0.15	1.23
	8IR	4	0.15	1.42
BULGE	1B	3	0.13	0.92
	2B	3	0.18	1.32
	3B	6	0.13	1.53
	4B	5	0.15	1.16
	5B	4	0.13	1.06
	6B	3	0.15	1.13
	7B	3	0.20	1.58
	8B	4	0.13	0.94
	9B	3	0.13	0.77
	10B	4	0.13	0.60

Position as given in fig. 4.6



Vertical Scale: 32 = 1mm.



Rotation of dividing head, degrees

Fig. 5.1a. Circularity plots (shell 31R).

Position as given in fig. 4.6

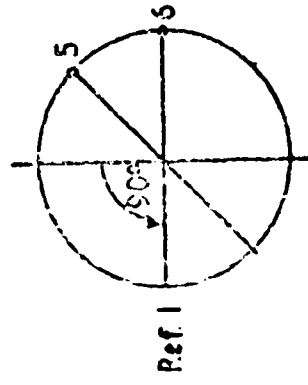
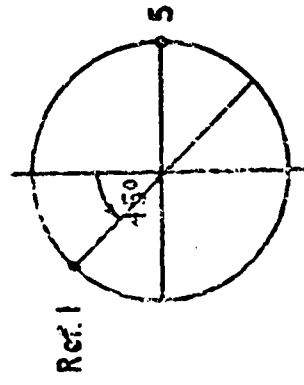
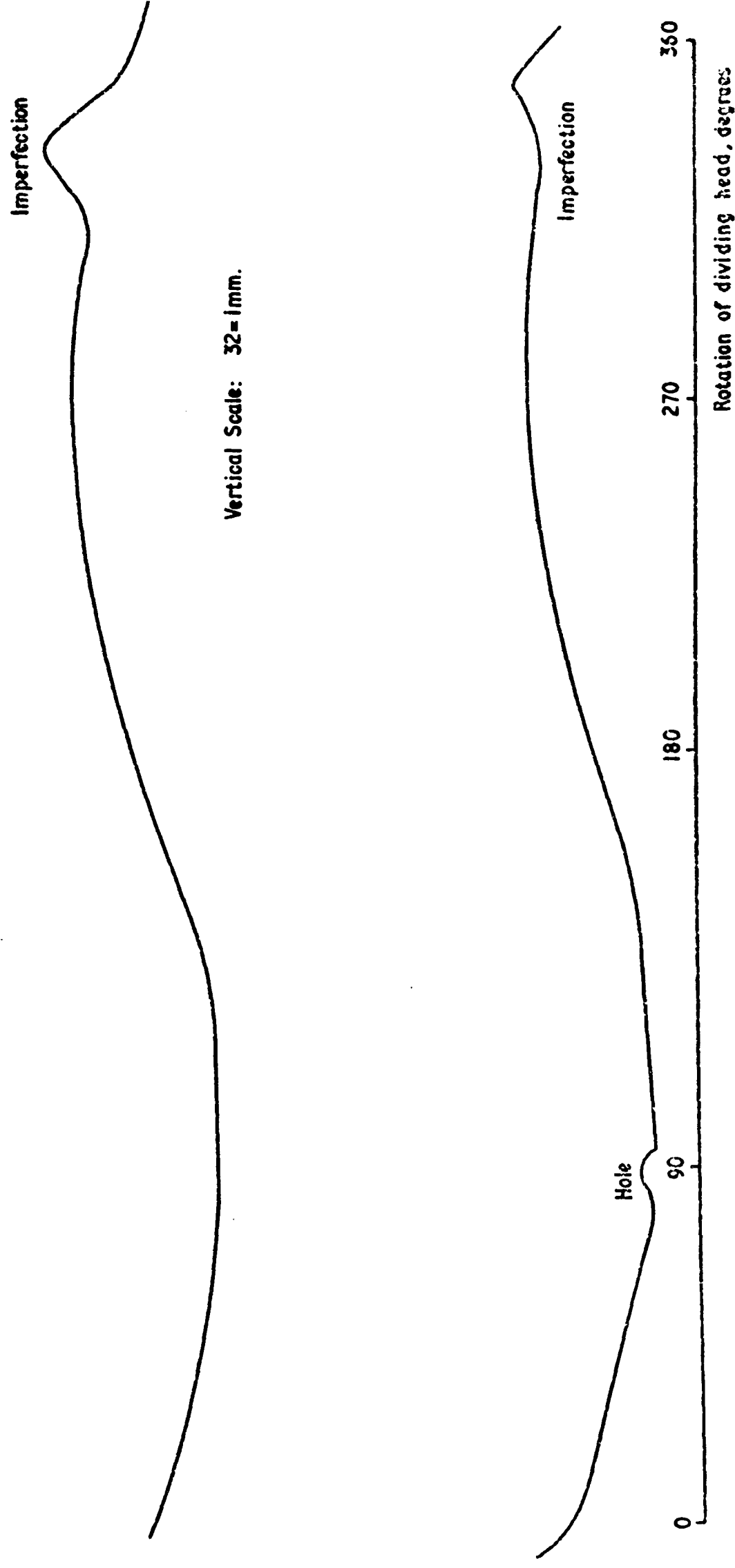
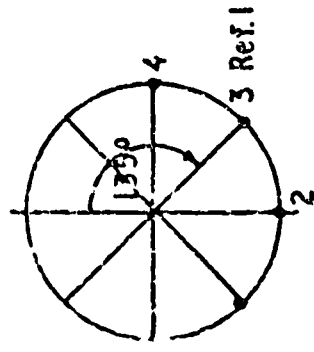
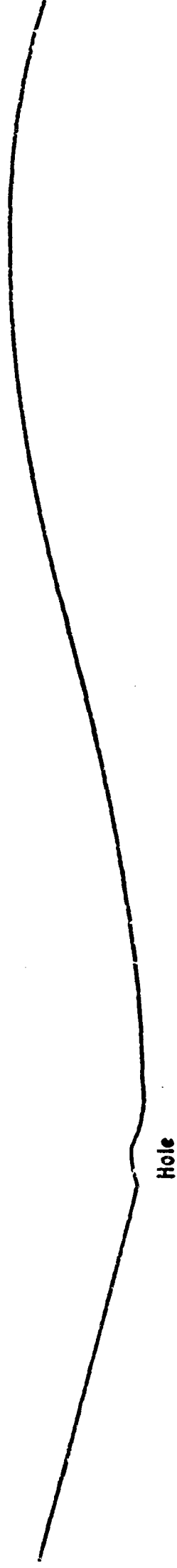
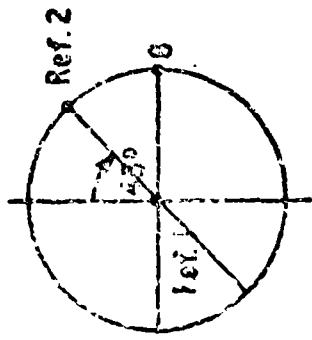
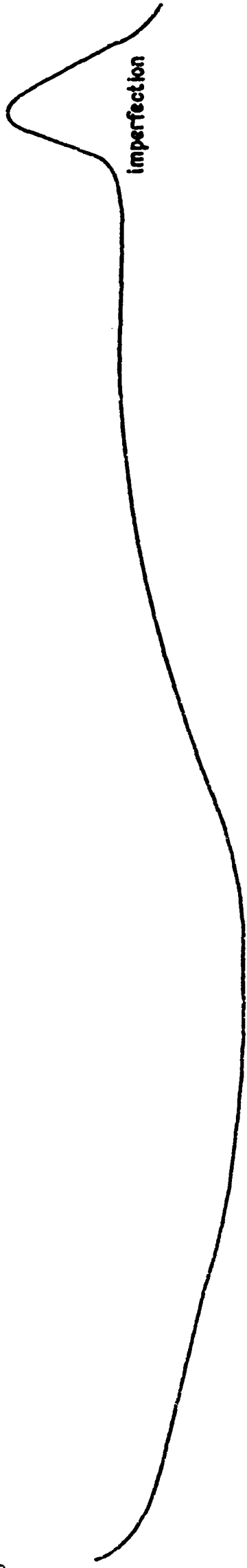
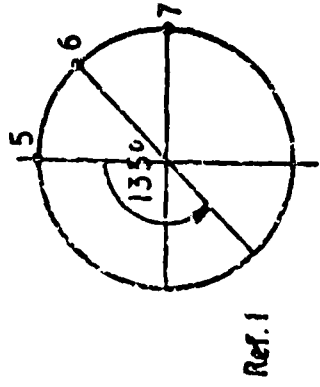


Fig. 5.1b. Circularity plots (shell 31R).

Position as given in fig. 4.6



Vertical Scale: 32=1mm

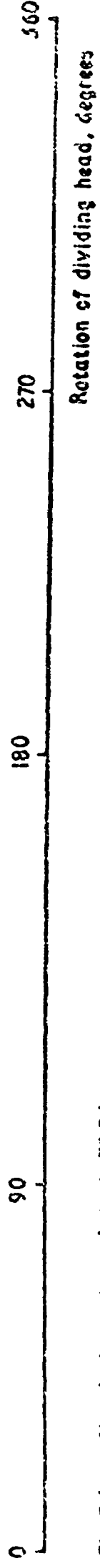
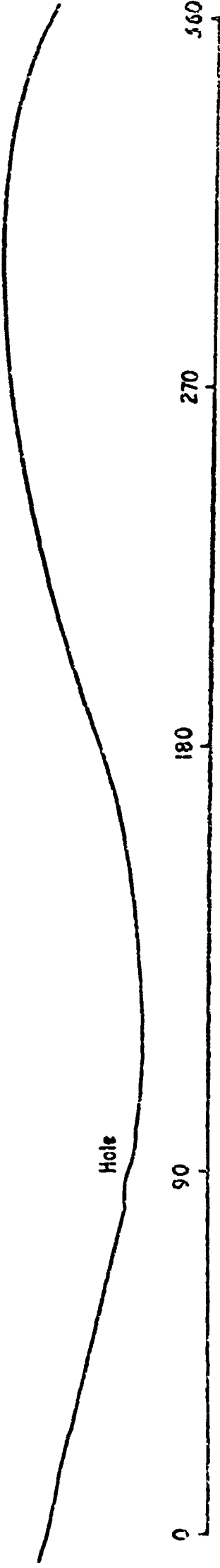
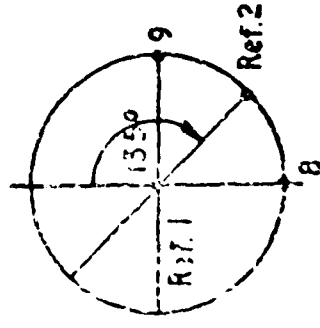


Fig. 5.1c. Circularity plots (shell 31 R).

In order to determine if geometrical imperfections were introduced by the removal of the wax former, circularity plots were taken for a sphere before and after its removal. No differences were noticed in their sinusoidal curves, and so it can be concluded that the stresses in the copper caused by electroplating onto a wax former were too low to introduce geometrical imperfections when the stresses were allowed to relieve themselves on removal of the wax.

5.2.4 Young's modulus of elasticity

The variation in Young's modulus of elasticity was determined to be $108 \text{ KN/mm}^2 \pm 2.7 \text{ KN/mm}^2$ i.e. a variation of 2.5% was found.

The tensile stress versus strain curves for the two electroplated copper specimens are shown in Fig. 5.2.

The maximum strain imposed on the shells can be derived as follows.

$$\sigma_c \text{ from equ. 3.16} = 0.605\left(\frac{t}{R}\right)E$$

From Hooke's law,

$$\epsilon_\theta = \frac{1}{E} (\sigma_\theta - \nu\sigma_\phi)$$

$$\text{Since } \sigma_\phi = \sigma_\theta, \epsilon_\theta = \epsilon_\phi = 0.605(1 - \nu) \left(\frac{t}{R}\right)$$

$$\text{Minimum } \frac{R}{t} \text{ from table 5.1} = 463; \nu = 0.3$$

$$\text{Maximum } \epsilon_\theta = \epsilon_\phi = \frac{0.605 \times 0.7}{463} = 0.09\%$$

i.e. a strain within the linear part of the stress-strain curve shown in Fig. 5.2.

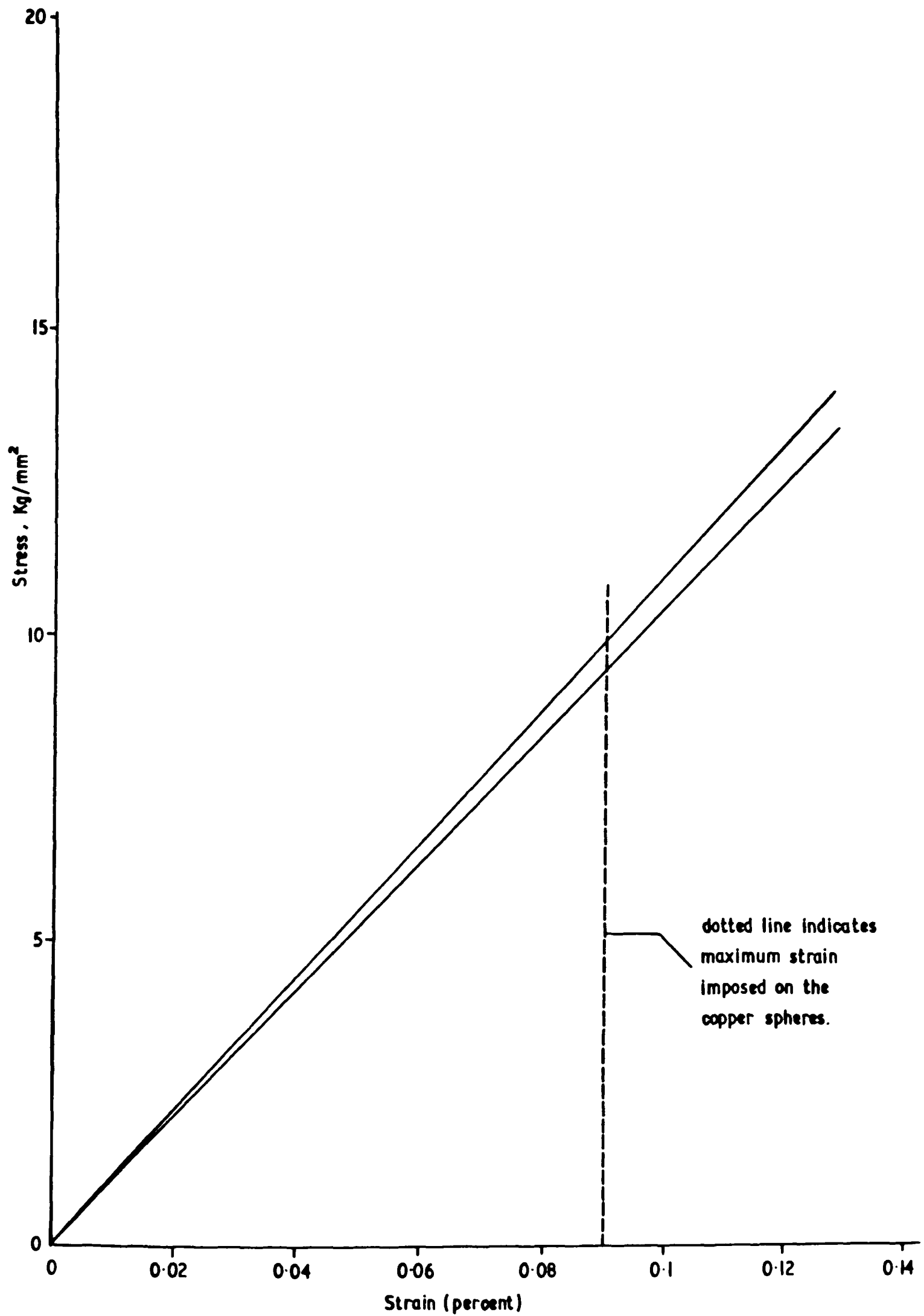


Fig.5 2. Tensile stress versus strain curves for the electrodeposited copper. The two curves shown represented the early and later stages of manufacture.

5.2.5 Microstructure

The results of the microstructure examination showed the shells to be free of voids and to have the same typical acicular structure as that for electroplated copper. [64]

5.3 PRE-BUCKLING DEFLECTIONS AT IMPERFECTION

Step-type graphs of the type shown in Fig. 5.3 were obtained from the XY plotter, the more familiar pressure versus deflection curves being obtained from those graphs, of which an example is given in Fig. 5.4. This is a typical graph for the flat and increased radius imperfection shapes. Linear pre-buckling behaviour occurs until near the onset of buckling where the curve becomes increasingly non-linear. In contrast, zero deflection was recorded for the bulge imperfections which implies that the deflections were of the order of the pre-buckling uniform contraction, i.e. the gauge opposite the imperfection was found to be insensitive to these very small deflections.

The deflection versus pressure results for each imperfection shape are summarised in Fig. 5.5. It is clearly seen that the pre-buckling deflections are dependent on the type of imperfection. Relatively large deflections occur for the flat imperfection in comparison to the two other imperfection shapes. The expected result of the gradient becoming steeper as the imperfection parameter, (d/t) , is decreased (i.e. approaching the perfect sphere), is clearly seen for the flat imperfection. Although this was also the case for the upper and lower (d/t) values of the increased radius imperfection, the deflections from the step type graphs recorded from the XY plotter were too

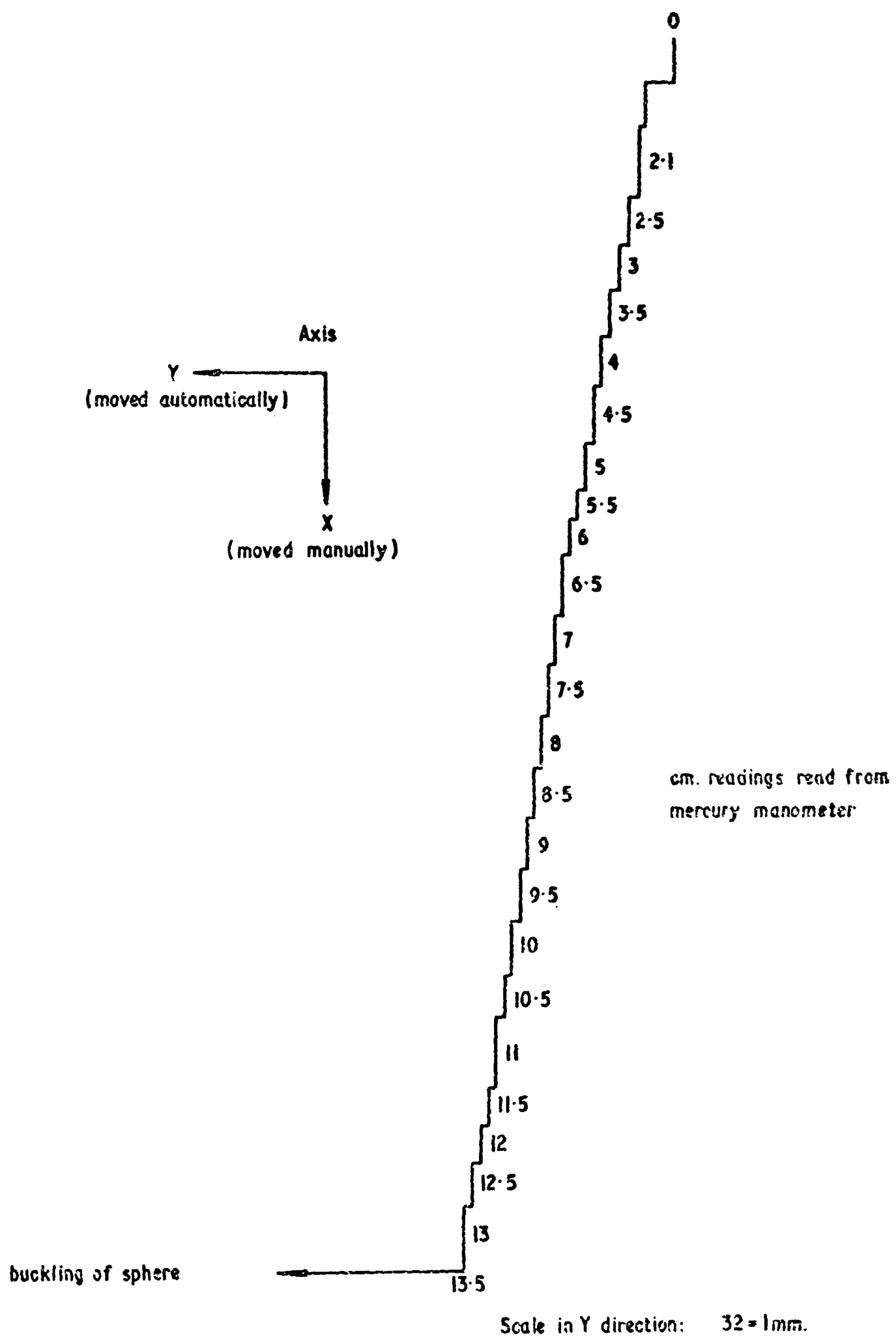


Fig.5.3. Step type graph of deflection at imperfection during pressurizing of a sphere (shell 2F) obtained from an XY plotter.

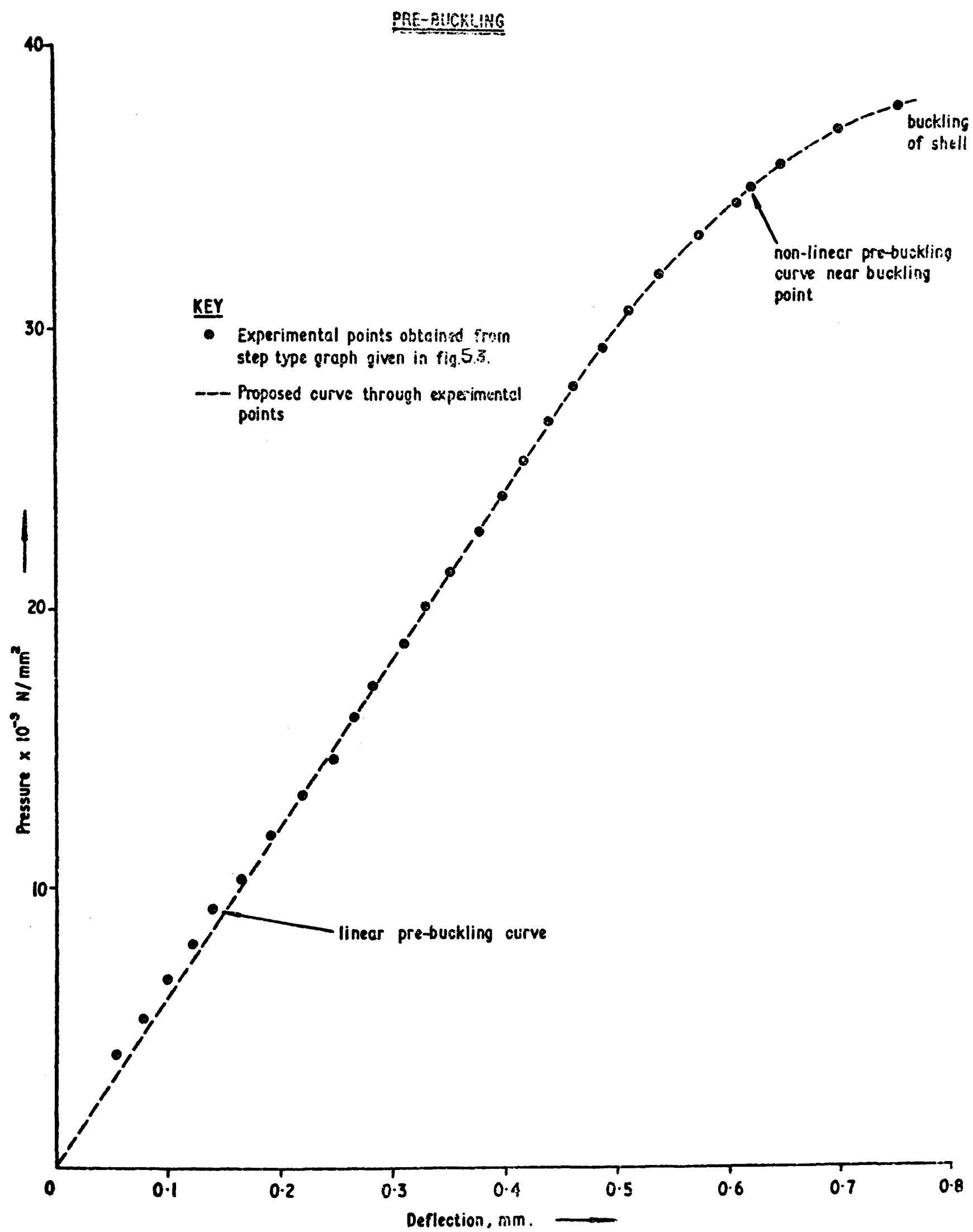


Fig.5.4. Pressure versus deflection graph (shell 7F) obtained from the step type graphs of the type shown in fig.5.3.

PRE - BUCKLING

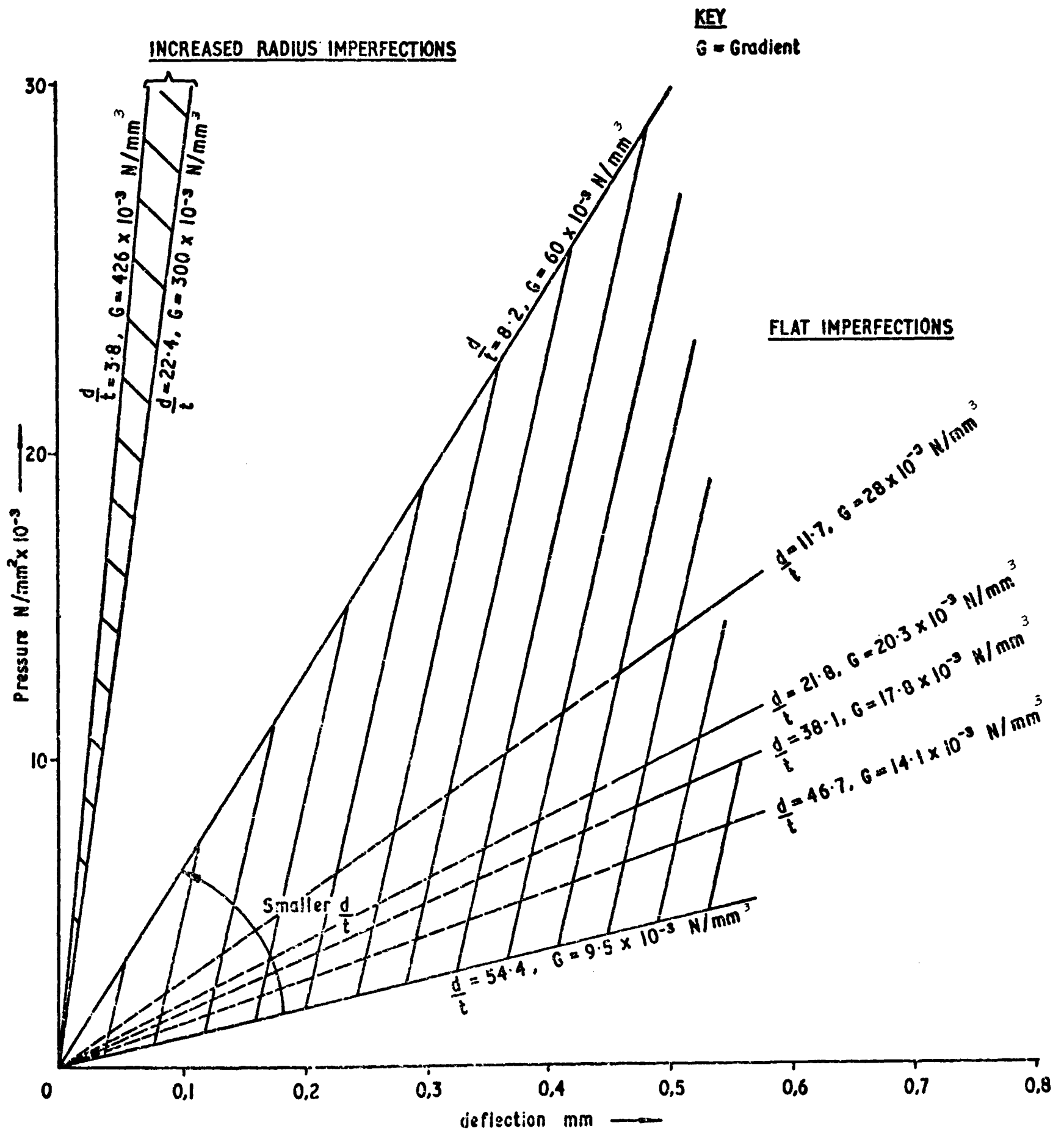


Fig.5.5. Summary of pre-buckling deflections taken at the centre of the increased radius and flat imperfections during pressurizing of the spheres. Zero deflection was recorded for the bulge imperfection indicating a deflection of the order of the uniform contraction of the sphere.

small to accurately confirm that the gradient became steeper as (d/t) was decreased.

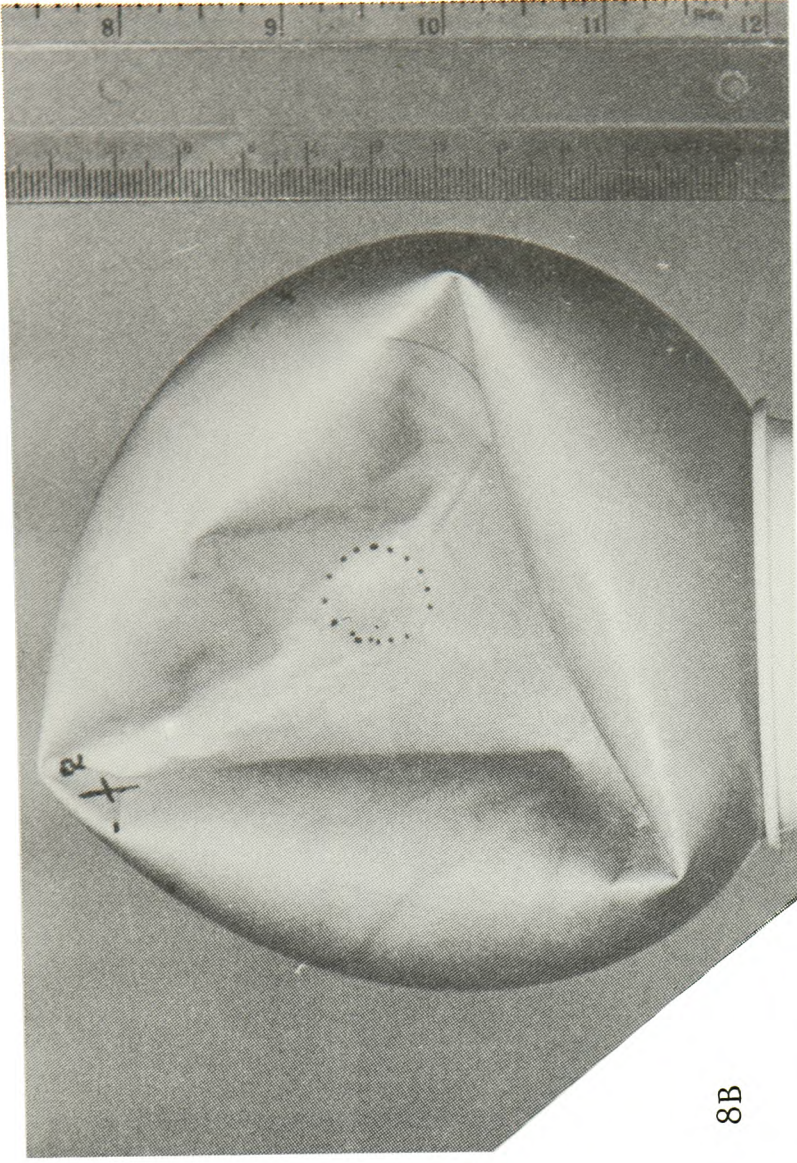
5.4 BUCKLING PRESSURES AND BUCKLED MODE SHAPES

The buckling pressures were obtained by increasing the pressure slowly, in order to not only take deflection readings at the imperfection, but also to avoid any surge of pressure from the vacuum pump, up to the point at which the spheres buckled.

The use of a constant pressure type loading system resulted in rapid inward displacement in the vicinity of the imperfection and the resulting crushed spheres adopted one of 4 mode shapes. These were either 2-lobed, 3-lobed, 4-lobed, or 5-lobed buckled configurations, of which an example of each is shown in Fig. 5.6.

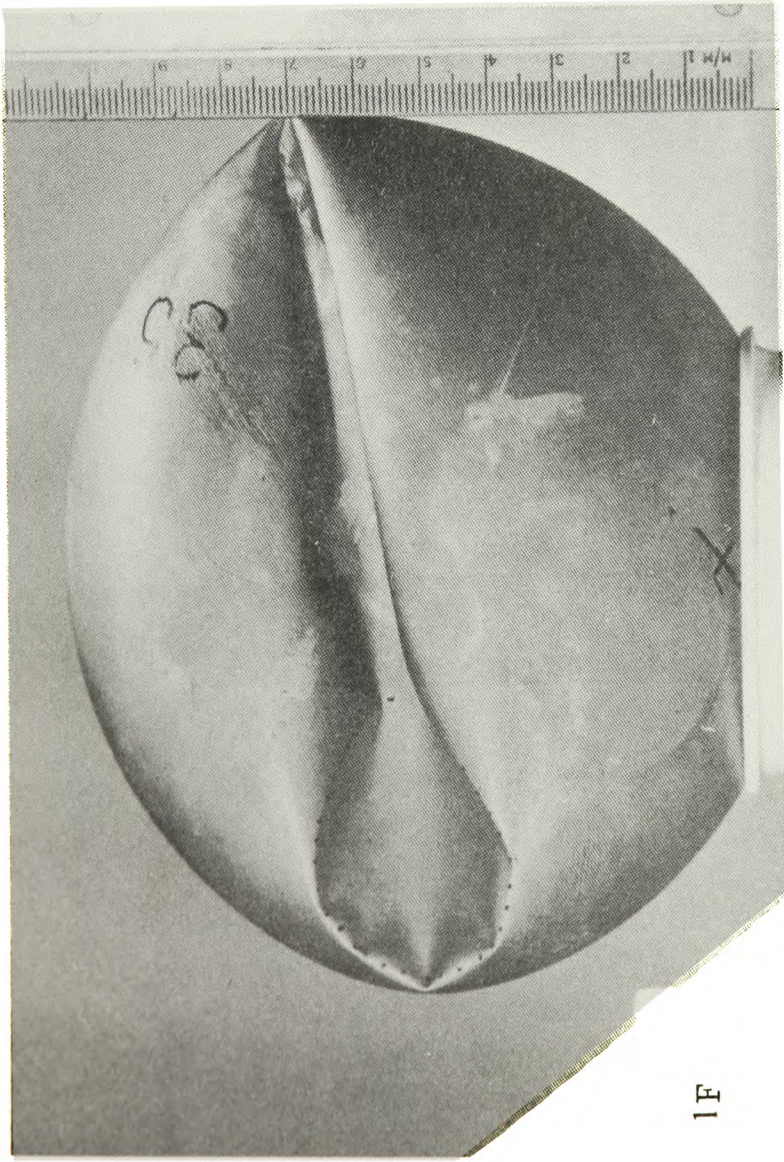
It is of interest to compare these buckled mode shapes with experiments performed by Berke (1967). He was able to control the deflection of the sphere using a constant volume loading system and obtain the post-buckled states shown in Fig. 5.7, which show the sphere to be weakly stable between the post-buckled 2-lobed, 3-lobed, 4-lobed and 5-lobed mode states. Similar results have been obtained for cylindrical shells^[60]. There does not seem to be any correlation between the type of imperfection and the number of lobes as seen from Table 5.3.

The experimental buckling pressure was clearly defined by a sudden increase in deformation. It is tabulated in Table 5.3 together with the classical value of equ. 2.1, obtained using values of the mean thickness given in Table 5.1.



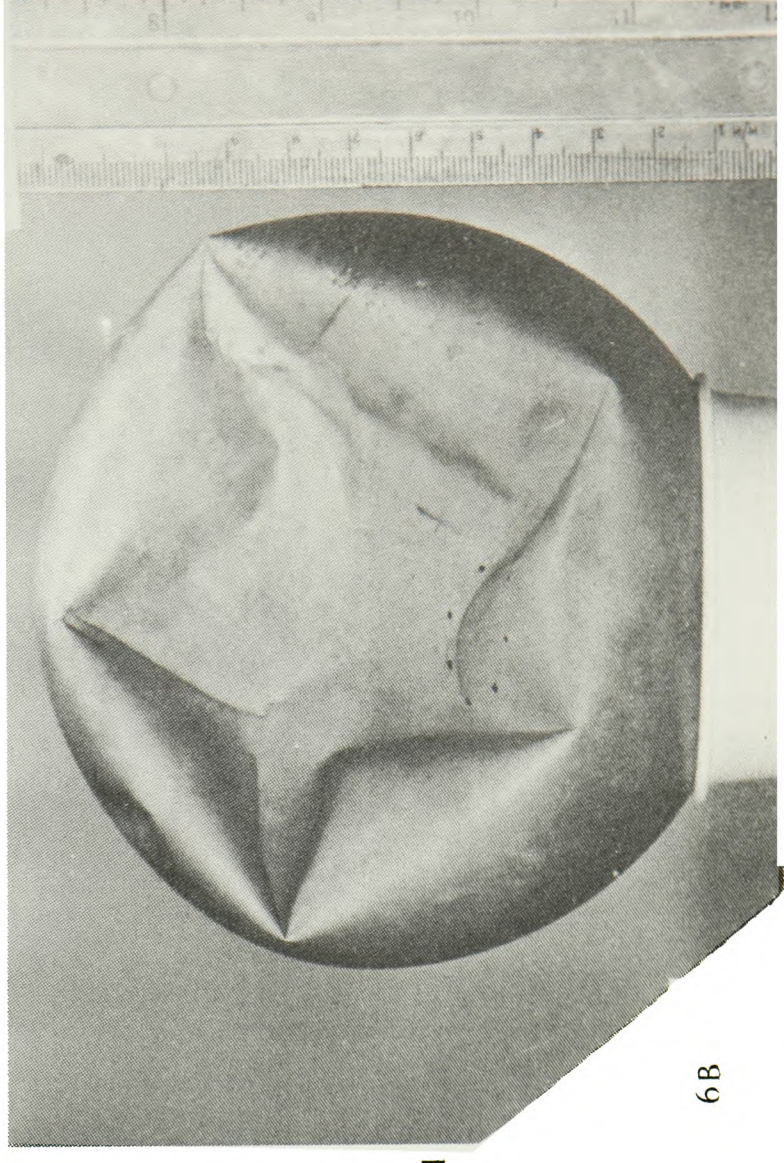
3 - lobed

8B



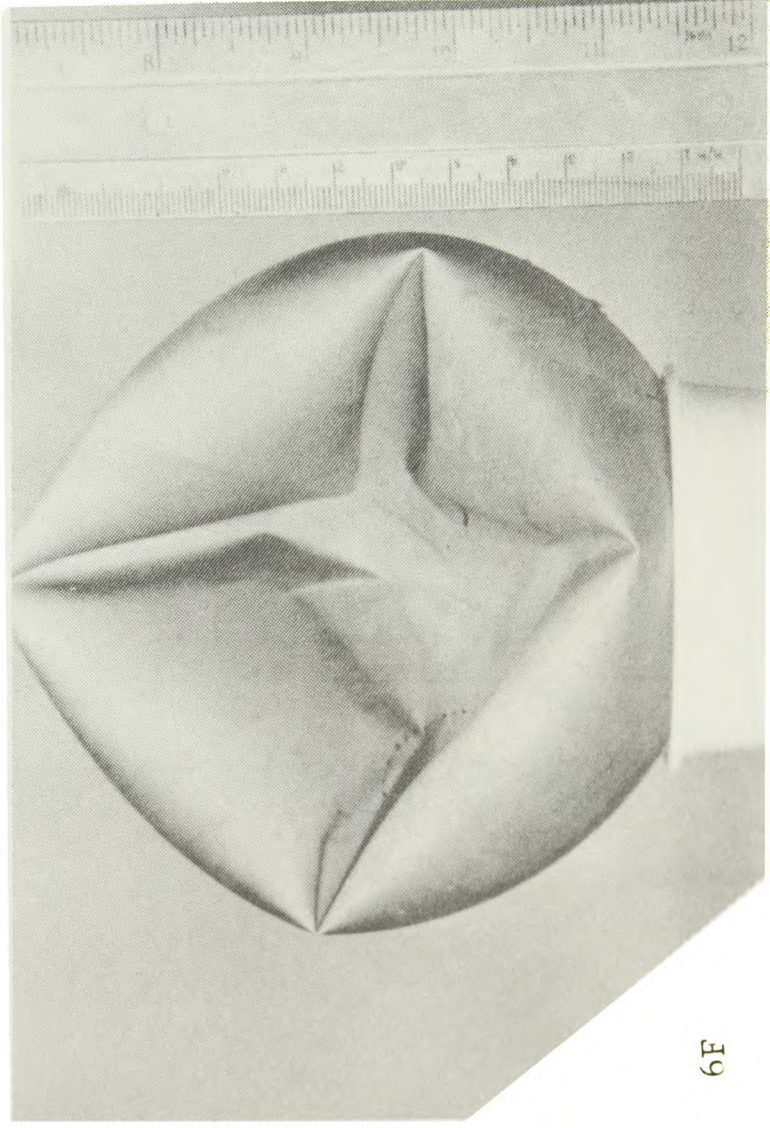
2 - lobed

1F



5 - lobed

6B



4 - lobed

6F

Fig. 5.3. Post-buckled mode states

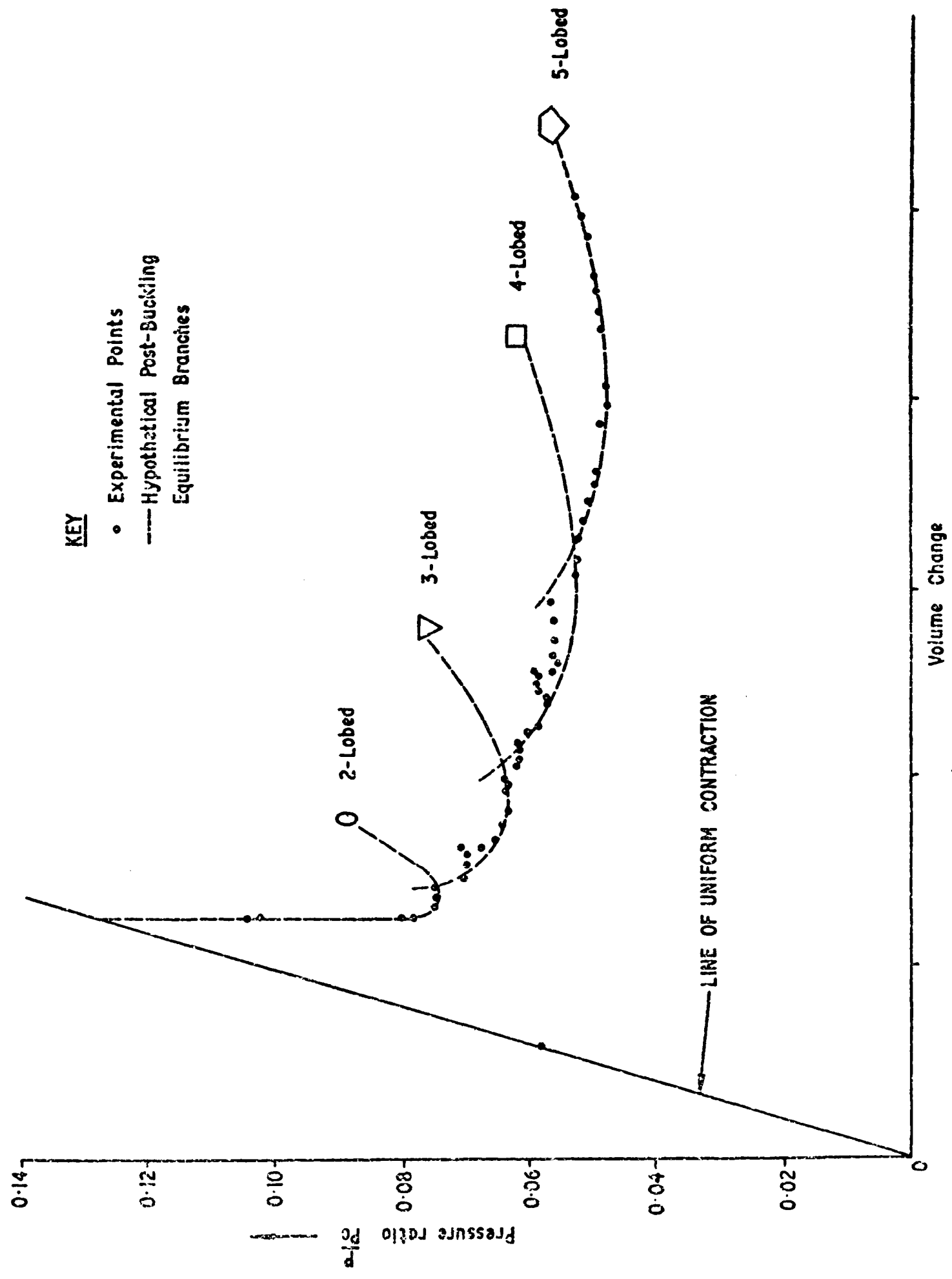


Fig. 5.7. Pressure-Volume curve taken during unloading (Berke [1967])

Table 5.3 Table shows number of lobes for the buckled
shell configurations, m, and a comparison
between the experimental buckling results, P_e ,
and the classical values for a perfect shell P_c .

	Shell No.	m	P_e	P_c	P_e/P_c
Units			N/mm ²	N/mm ²	
FLAT	1F	2	0.0188	0.2493	0.075
	2F	3	0.0096	0.1307	0.073
	3F	2	0.0115	0.1392	0.083
	4F	3	0.0036	0.0480	0.074
	5F	3	0.0190	0.1853	0.103
	6F	4	0.0171	0.1523	0.112
	7F	3	0.0395	0.2550	0.155
INCREASED RADIUS	1IR	3	0.0174	0.1660	0.105
	2IR	3	0.0493	0.3228	0.153
	3IR	5	0.0374	0.2788	0.134
	4IR	4	0.0101	0.1033	0.098
	5IR	4	0.0267	0.1953	0.137
	6IR	4	0.0037	0.0451	0.082
	7IR	4	0.0260	0.1902	0.137
	8IR	4	0.0275	0.1435	0.192
BULGE	1B	4	0.0426	0.2551	0.167
	2B	3	0.0382	0.2436	0.157
	3B	4	0.0142	0.0925	0.153
	4B	3	0.0296	0.2161	0.137
	5B	4	0.0263	0.1953	0.135
	6B	5	0.0295	0.2269	0.130
	7B	4	0.0267	0.2056	0.130
	8B	3	0.0307	0.2436	0.126
	9B	4	0.0420	0.3631	0.116
	10B	4	0.0771	0.6092	0.127

CHAPTER VI

STABILITY OF IMPERFECT THIN-WALLED SPHERES:

INTERPRETATION OF RESULTS

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CHAPTER VI

STABILITY OF IMPERFECT THIN-WALLED SPHERES:

INTERPRETATION OF RESULTS

6.1 INTRODUCTION

The objective of this chapter is to compare both existing theory and a numerical method with the imperfection sensitivity of the imperfect models and hence determine the adequacy of the analysis.

6.2 ANALYSIS OF RESULTS USING IMPERFECTION SENSITIVITY THEORY

The experimental results have been plotted in Figs. 6.1a, b and c. The magnitude of the defect is characterized by means of a single parameter, (d/t) i.e. the ratio between defect amplitude or depth and the shell thickness.

Koiter (1945) has given in equ. 2.7 an expression for the stability of elastic systems, although its limitations should be borne in mind, (see also Appendix C):

a) It applies in general only if the imperfections are of small amplitude and only in the immediate neighbourhood of the bifurcation point on the load-deflection curve.

b) The assumption is made that the growth of the post-buckling displacement distribution is proportional to a buckling mode of the perfect shell.

Koiter (1968) found that the imperfection sensitivity parameter (b) of equ. 2.7 was equal to -0.05 for a spherical shell under uniform external pressure, with $(R/t) = 2813$,

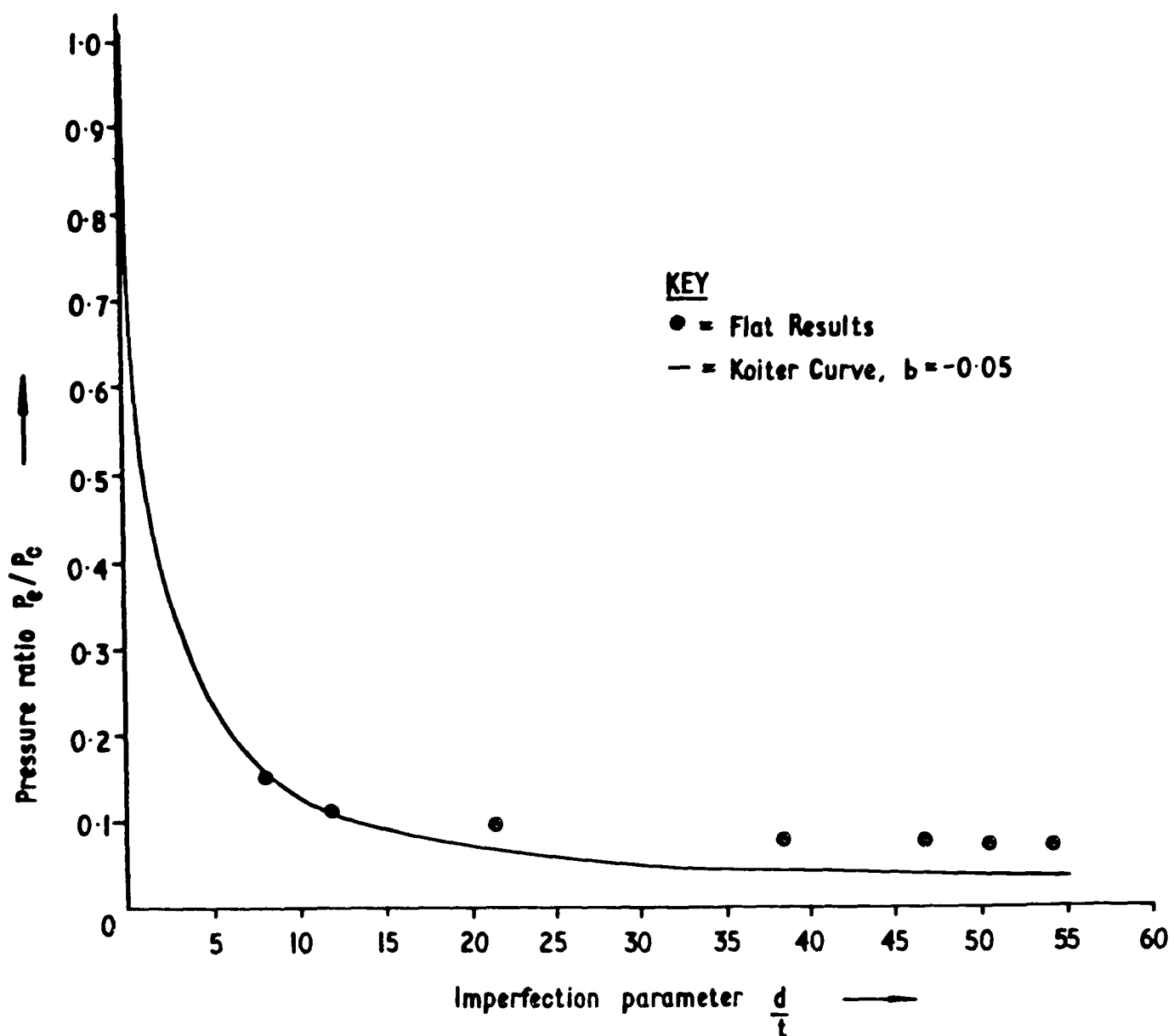


Fig. 6.1a. Experimental buckling pressures for the FLAT imperfection normalized with respect to the classical value versus imperfection amplitude normalized with respect to shell thickness. Comparison made with Koiter's (1945) equation.

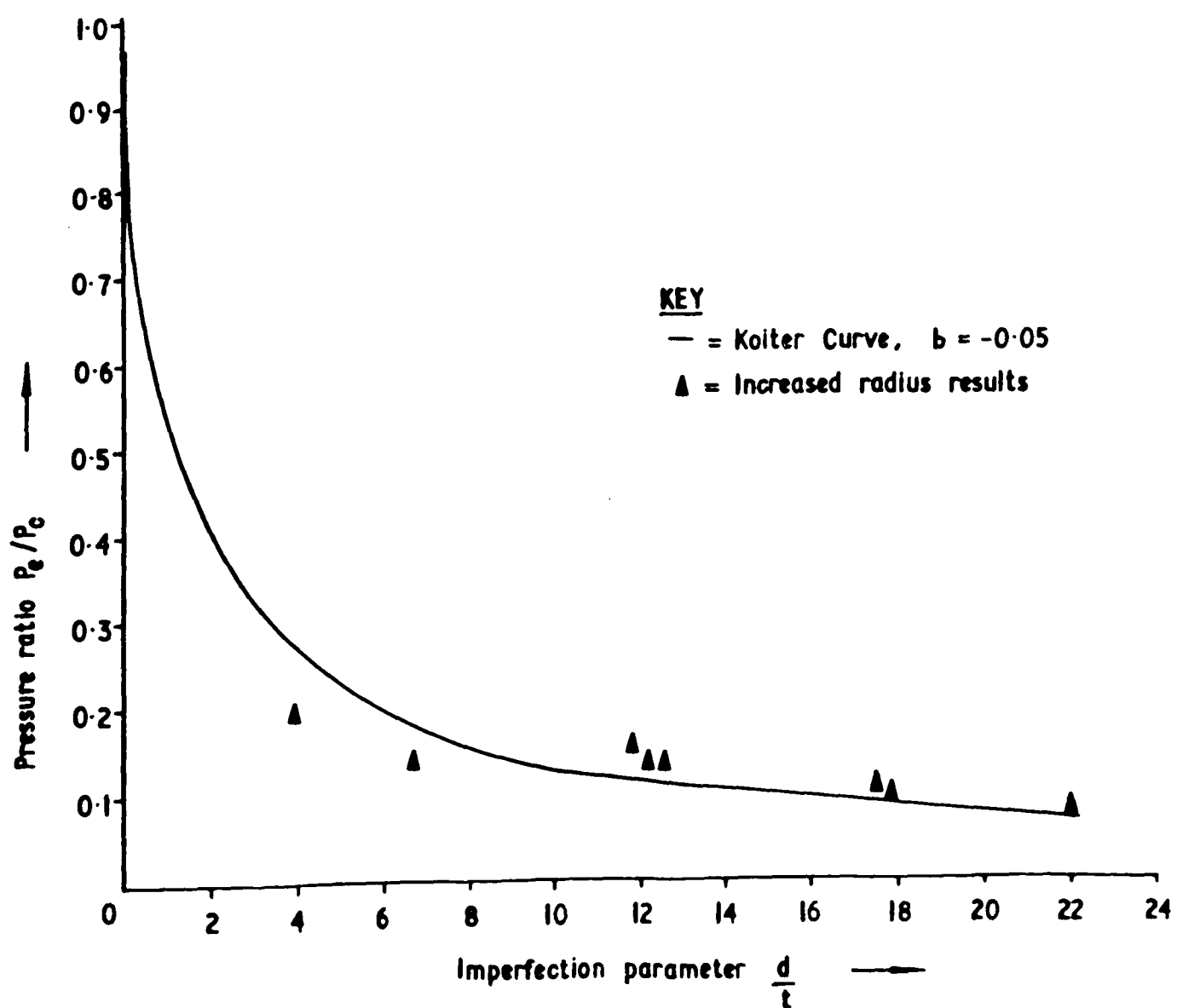


Fig. 6.1b. Experimental buckling pressures for the INCREASED RADIUS imperfection normalized with respect to the classical value versus imperfection amplitude normalized with respect to the shell thickness. Comparison made with Koiter's (1945) equation.

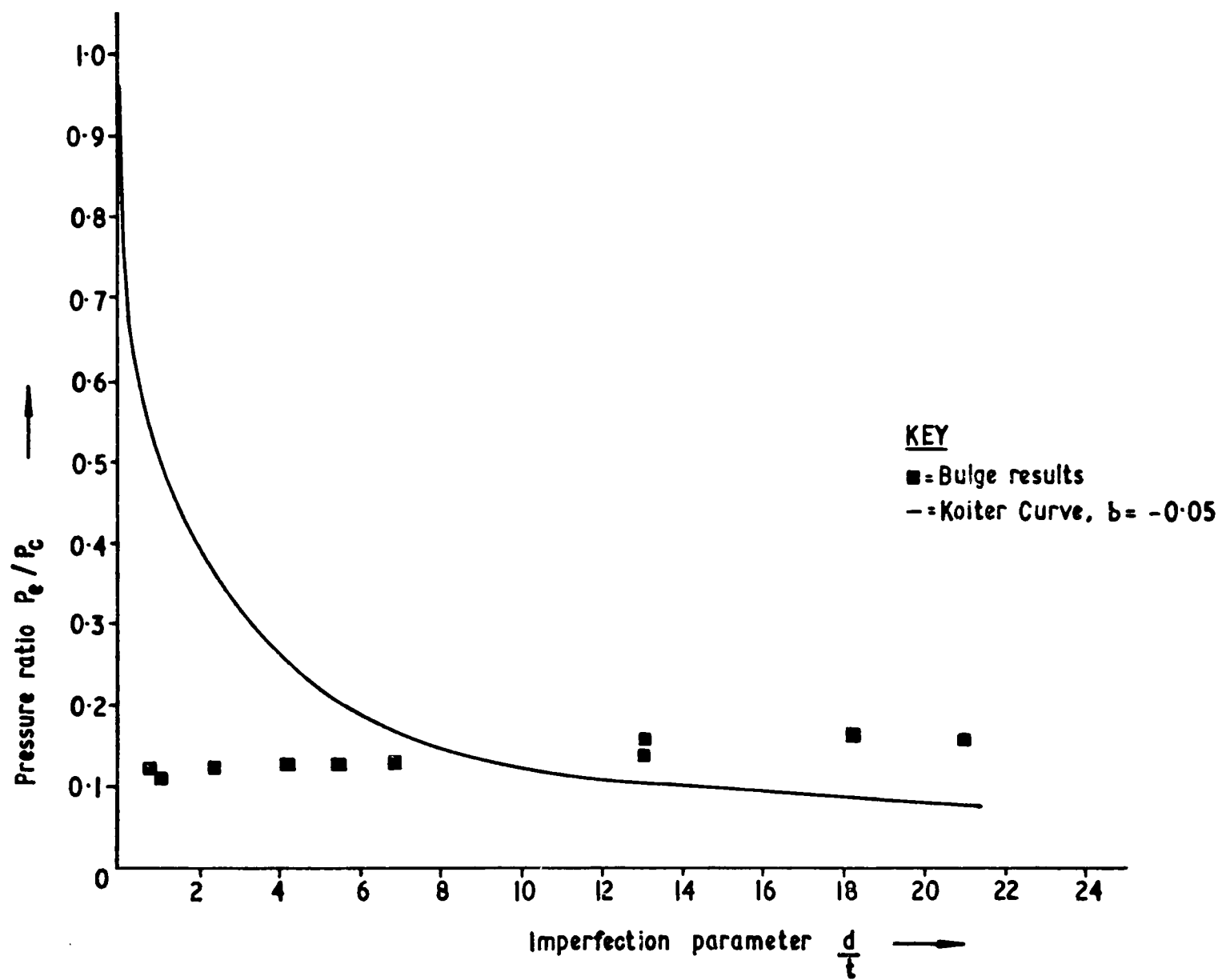


Fig. 6.1c. Experimental buckling pressures for the BULGE imperfection normalized with respect to the classical value versus imperfection amplitude normalised with respect to the shell thickness. Comparison made with Koiter's (1945) equation.

rather thinner than the ones reported here: see Fig. 2.5. To look into the possibility of applying equ. 2.7 to explain the present results, the values predicted with $(b) = -0.05$ have also been plotted in Figs. 6.1a, b and c. The following conclusions can be drawn:

(a) The results for the flat imperfection and for the increased radius imperfection, up to $(d/t) = 25$ are in excellent agreement with the theoretical curve.

(b) For larger values of (d/t) , the only experimental results, for flat imperfections, lie above the curve but there is still fair agreement, particularly considering the very low values of the pressure compared to the theoretical buckling pressure.

(c) There is no agreement at all between the theoretical curve and the results for imperfections in the form of a bulge; for these, the experimental pressure is always between 0.116 and 0.167 of the theoretical value. Models with small imperfections, 5B to 10B, failed at values slightly below those reached by the more imperfect models 1B to 4B.

Failure of all models was clearly localised at the imperfection, whose magnitude overshadowed any other unwanted errors in the model. However, in models 5B to 10B and in models 7IR and 8IR, the combination of general and localised imperfection may have resulted in an increased reduction of the buckling load since both imperfections are of the same order. In Table 5.2, the general imperfection has been expressed in terms of the radius, (e/R) . Expressed in terms of the thickness, (e/t) , it varies between 0.6 and 2.47. The difference in shape between both imperfections

makes a direct comparison impossible but it may be surmised that there will be some interaction when (e/t) and (d/t) are of the same order of magnitude. Adding (e/t) to (d/t) to the points corresponding to models 5B to 10B, 7IR and 8IR would merely displace them to the right in Fig. 6.1b and c, without changing the previous conclusions. That the effect of general imperfections is in fact an important cause for the reduction of the buckling pressure of nominally perfect spheres with a small controlled imperfection follows from Sabir's (1964) experimental results with dimple-type imperfections; see Fig. 2.7. Nominally perfect spheres failed at between 0.5 and 0.85 of P_c . Those with dimples of magnitude (d/t) more than 1.2 failed at pressures as low as 0.12 P_c , comparable to the pressures here obtained for bulges with (d/t) of 0.66 or more.

6.3 ANALYSIS OF RESULTS BY MEANS OF A NUMERICAL METHOD

Having established the inadequacy of the existing imperfection theory to explain the results of the spheres with bulges, it was decided to analyse all models with a numerical method. Amongst the existing finite element and numerical integration computer programmes, BOSOR 4^[20] (see Appendix D), was selected since it has been extensively used and validated by experiments^[37,40,60,61,65]. The procedure consisted of dividing the shell into segments, subdivided into nodes for the numerical integration of the shell equations, expressed in finite difference form. One form that suggested itself was to take one segment for the imperfection and one for the rest of the shell, as shown in Fig. 6.2a. The use of 75 nodes for

Fig.6.2a. First computer model of imperfect sphere, 2 segments.

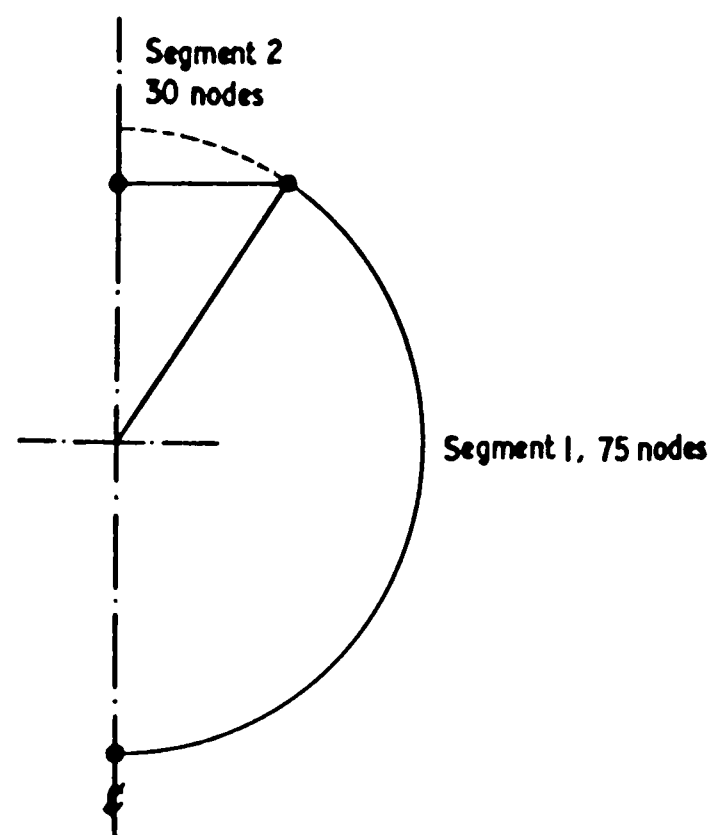


Fig.6.2b. Second computer model of imperfect sphere, 3 segments.

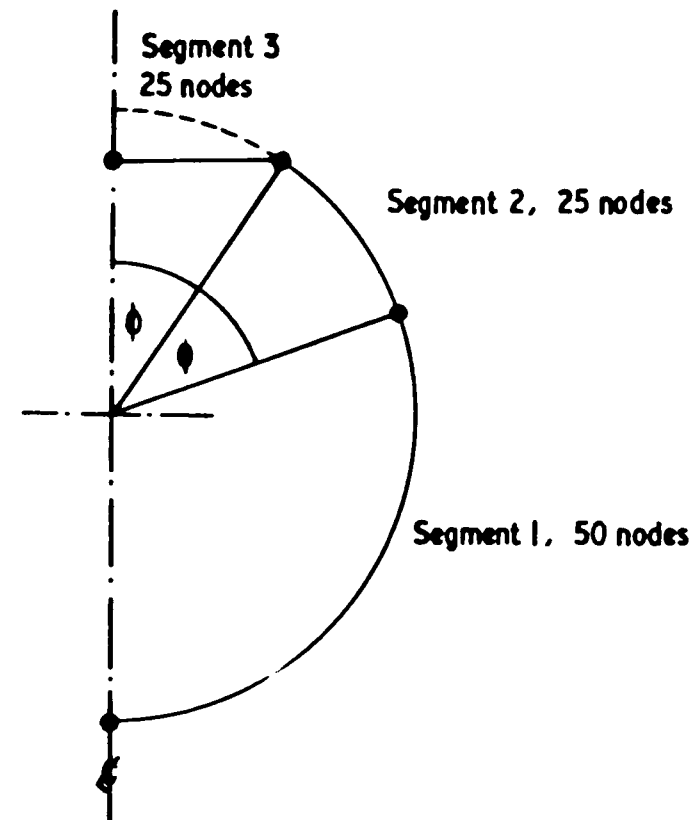
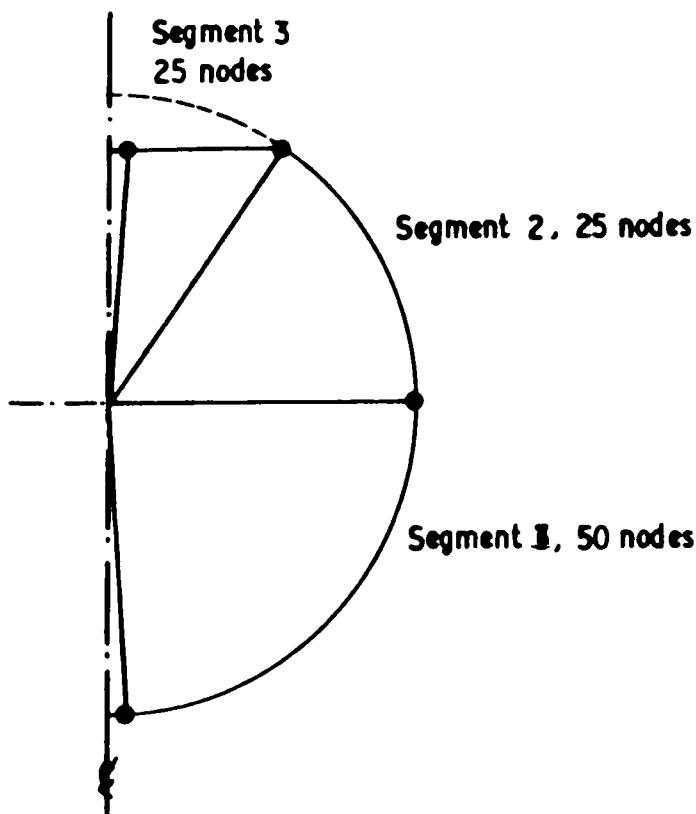


Fig.6.2c. Third computer model of imperfect sphere, 3 segments.
Inclusion of small diameter holes (approx. 3 degrees) at apexes.



segment 1 and 30 nodes for segment 2 is arbitrary, since previous experience (see the second part of the investigation on the stability of partially filled spheres), had shown a negligible difference in the computed results when changing the number of nodes between 25 and 200. It was however found that, in this case, the programme was not only grid-sensitive but also sensitive to the value of the scaling factor which operates on the initial value of the load, taken as a reference for the calculation. At the very low values of the final buckling pressure, rounding-off errors become important.

After considerable trial-and-error, the segment positions and nodes of Fig. 6.2b were found to be successful in eliminating the sensitivity of the programme and resulted in computed buckling pressures that were roughly in agreement with the experimental results for the increased radius and flat imperfections while still failing to agree with the bulge-type imperfections (Fig. 6.3). A disturbing factor to emerge was the initial imperfection insensitivity of the programme for small imperfections in comparison with Koiter's (1945) theory and with all known experimental results on quasi-perfect shells. Another observation that tended to cast some doubt on the value of the computed results was made when plotting the circumferential and meridional stress resultants given by the numerical analysis at each node. As shown, in Fig. 6.4a for the circumferential component, physically meaningless spikes appear at the junction between segments and at the top and bottom of the shell. In order to determine the cause of these high peaks, the position of the segments was changed. The results

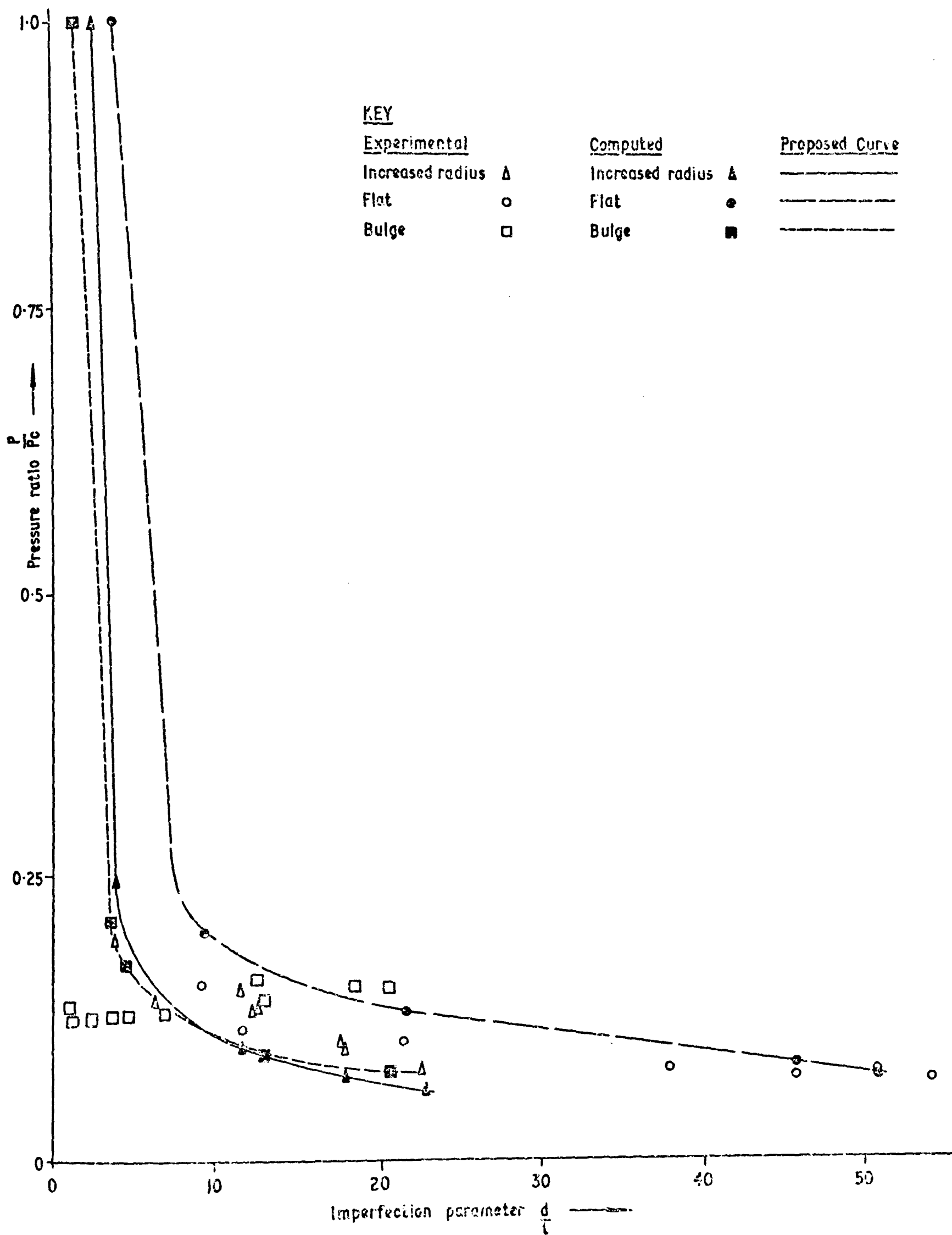


Fig. 6.3. Computed results for the imperfect spheres using segment positioning and number of nodes as shown in fig. 6.2b.

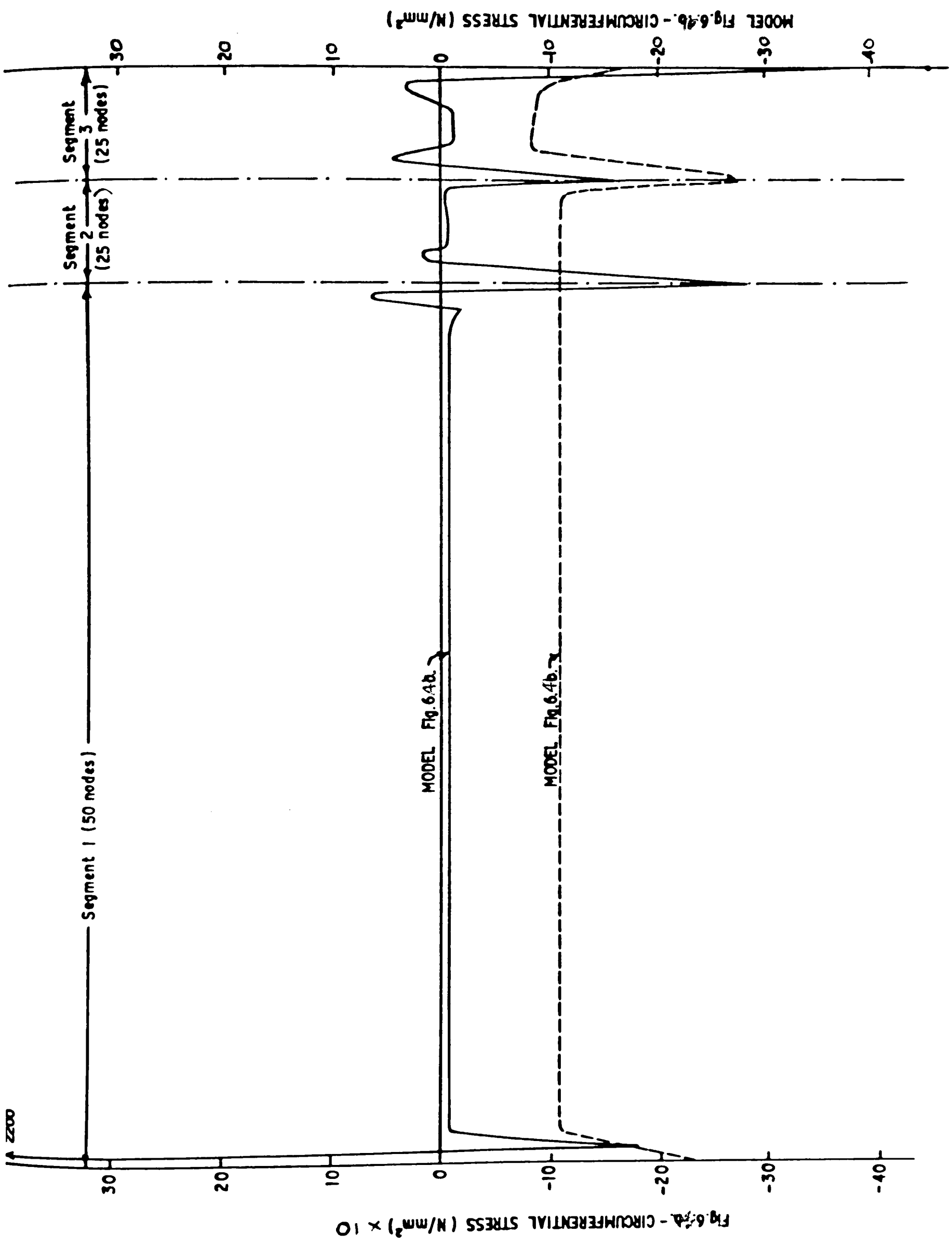


Fig. 6.4. Circumferential stress curve for positioning of nodes and segments as shown in Fig. 6.2b. and c.

showed the high stresses to be indeed fictitious, although the buckling pressures depended very much on the positioning of the segments. Attention was then focussed on the very high stresses at the apex since it was thought that the apex at the imperfection could be the cause of the variation in the calculated buckling pressure.

In order to eliminate the high stresses, stress relieving holes were introduced at the top and bottom, as shown in Fig. 6.2c. This is normal practice for a part where the meridian crosses the axis of symmetry and is termed a singular point, i.e. the shell surface becomes an infinitely small area at the pole positions thus causing σ_θ , σ_ϕ which will be indistinguishable from each other, to become infinite; a point discussed by Kalnins (1969) in the KSHELL^[48] computer programme - "in order to analyse a part which is actually closed on the axis, a small hole must be inserted about the axis of symmetry. The opening of the hole should be about three thicknesses of the part, or approximately 2 degrees"; an action which results in $\sigma_\phi = 0$ and from equ. 3.3c,

$$\sigma_\theta = \frac{PR}{t}$$

This comparatively high circumferential stress will have an effect over a distance in the meridional direction of approximately $\sqrt{Rt}$ from the hole. The stress curve now showed the more reasonable shape of Fig. 6.4b. The meridional component varied smoothly, down to zero at the free edges created by the holes. There was no stress change across the junction between segments 1 and 2 in the spherical part and, from this encouraging result it was

decided to use model 6.2c, to analyse all three shapes. It was found that the new results were in total disagreement with the experimental values and with the previous computation, as evidenced by the curves of Fig. 6.5. The programme failed to predict any sensitivity to the flat imperfection, at variance with all observations. Thus, a paradoxical situation has been reached wherein computed results that agree with the experimental buckling pressure give a meaningless stress distribution while those that agree with the static stress distribution are in total disagreement with the experimental results.

The BOSOR 4^[20] programme has been shown to predict accurately the buckling pressure of cylindrical shells under external pressure^[60,61,65] and as will be shown subsequently in the second part of the investigation, spherical shells supported on the equator and partly filled with liquid under vertical acceleration. It has also been reported to agree with experiments on torispherical heads and on transitions between spheres and cylinders^[37,40]. It would be expedient but, as has been shown, not always justified, to apply it to any other situation without first checking that the segments into which the shell is divided, the number of nodes within each segment, the initial value of the reference load and the conditions at the terminal nodes in closed shells do not influence the result. This requires "testing" a number of numerical models until the one giving a minimum value of the buckling load is found. Violation of the conditions of compatibility at the junction between segments need not invalidate the final result obtained for that load.

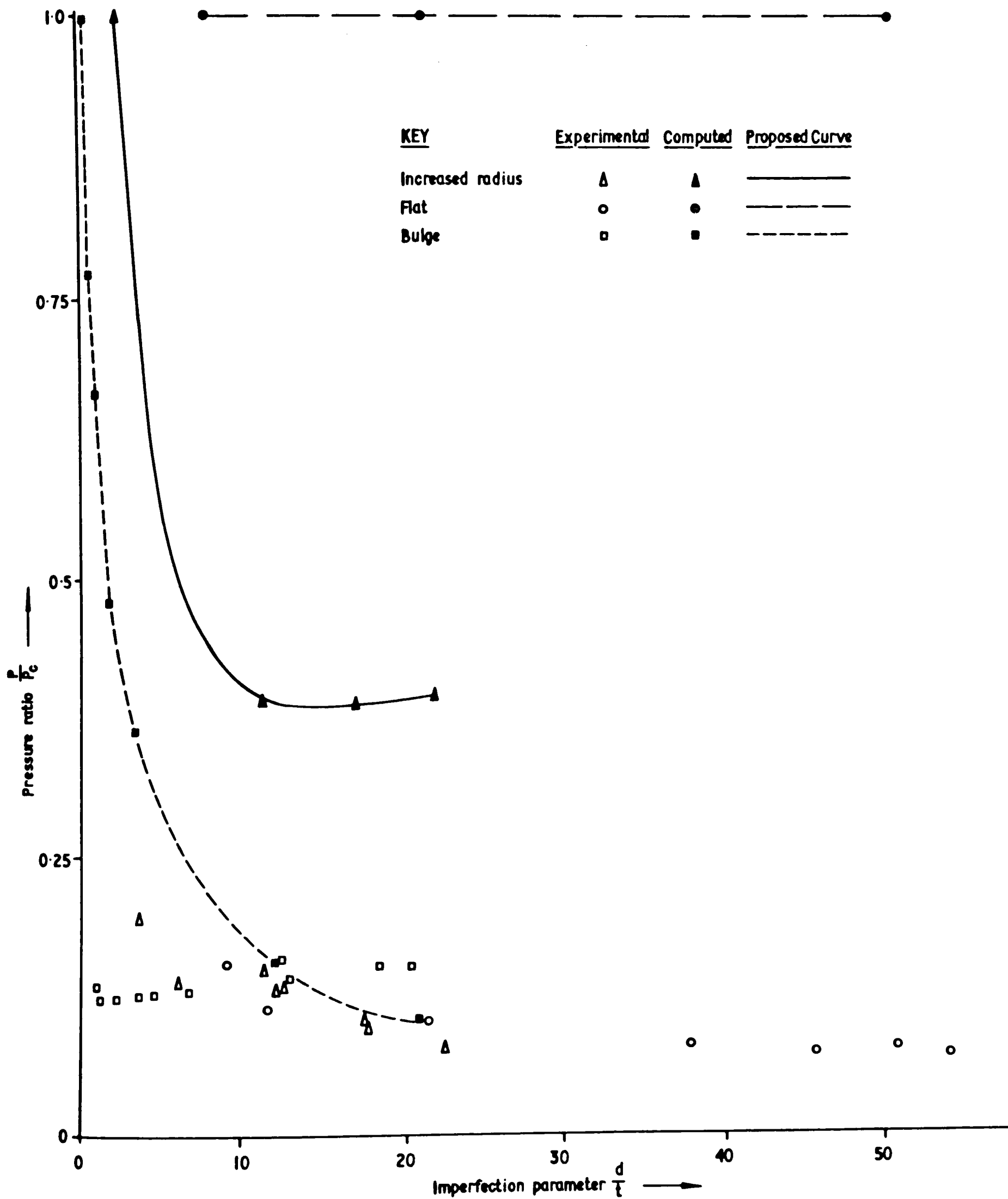


Fig.6.5. Computed results for the imperfect spheres using segment positioning and number of nodes as shown in fig.6.2c.

6.4 CONCLUSIONS

The imperfection sensitivity treatment of Koiter (1945) has been shown to predict the buckling pressure of large imperfections, well outside Koiter's (1945) original specified limitation where the amplitude of the imperfection was of the order of the thickness, for increased radius and local flat types in spherical shells under uniform external loading. The value of the imperfection sensitivity parameter (b) in equ. 2.7 has been found to be -0.05. The strength of quasi-perfect shells, as tested by other investigators appears to be less than predicted. This may be due to difficulties in producing such shells, with a sufficiently accurate control of the imperfection.

In contrast, the experimental results obtained for the bulge imperfection cannot be explained by Koiter's (1945) treatment. The same remark applies to the results quoted in Fig. 2.7 for spheres with small dimples^[76]. Comparing the behaviour of the three types of imperfection in Figs. 6.6a, b and c, both the flat and increased radius imperfections deform by bending about a circular band defined by the junction between the imperfection and the rest of the shell. Large departures from the initial linear behaviour of Fig. 5.4 occur without any change in the wavelength, which is virtually fixed by the diameter of the imperfection (a). The increase in amplitude, detected by the proximity gauge, continued until the shell "snapped-through" into a dimple shape which would eventually lead to the formation of yield hinges and the final plastic collapse of Fig. 5.6. No deformation was detected, on the other hand, for the bulge-type imperfection until it suddenly collapsed into the

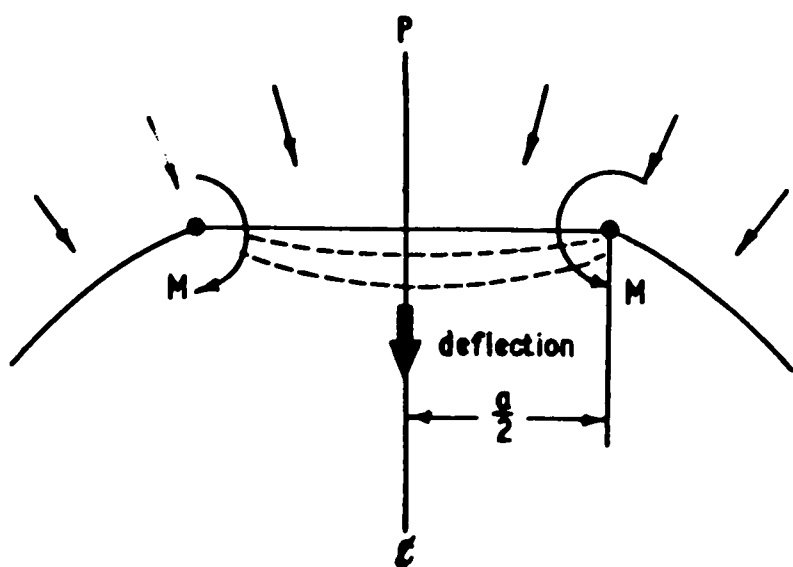


Fig.6.6a. Suggested deformation of flat imperfection.

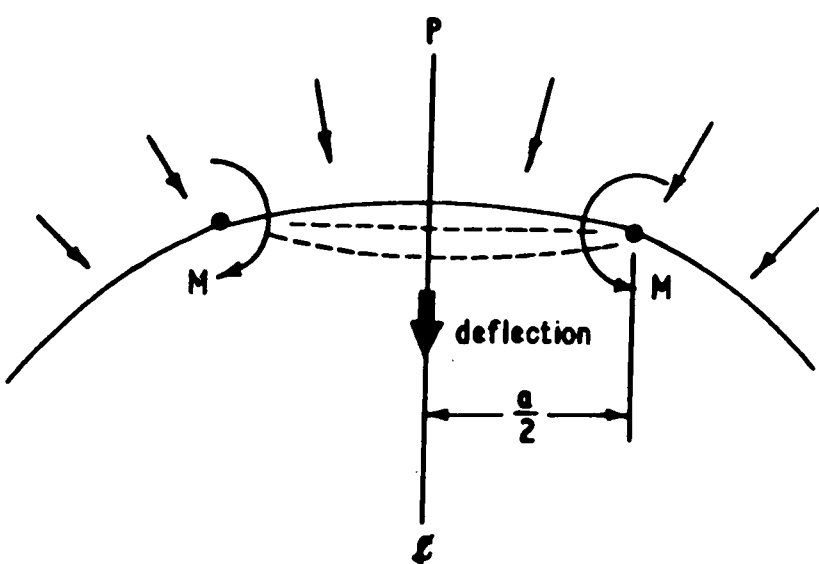


Fig.6.6b. Suggested deformation of increased radius imperfection.

KEY

- P. External pressure
- M. Bending moment
- , Hinge site

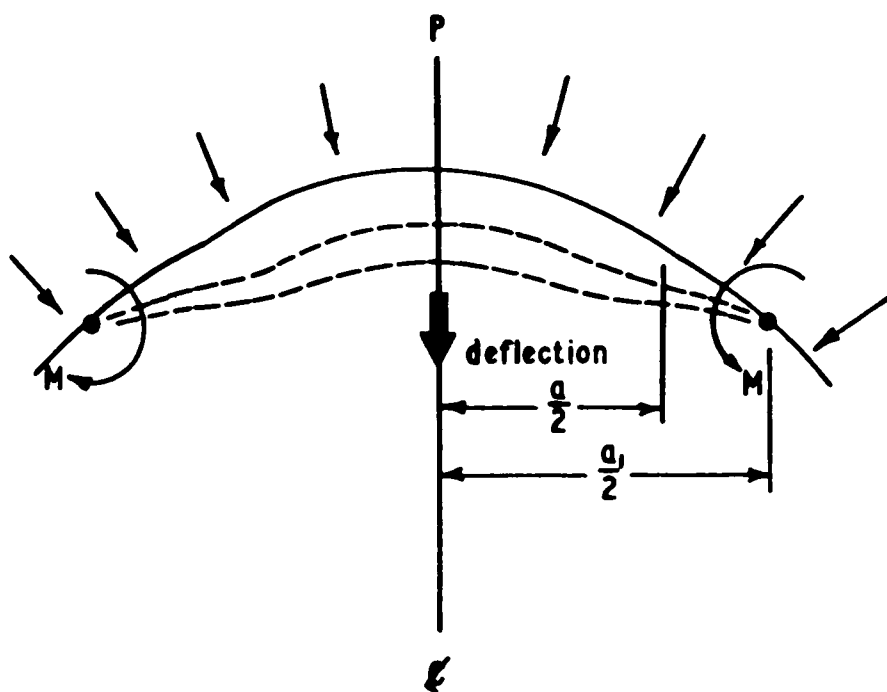


Fig.6.6c. Suggested deformation of bulge imperfection.

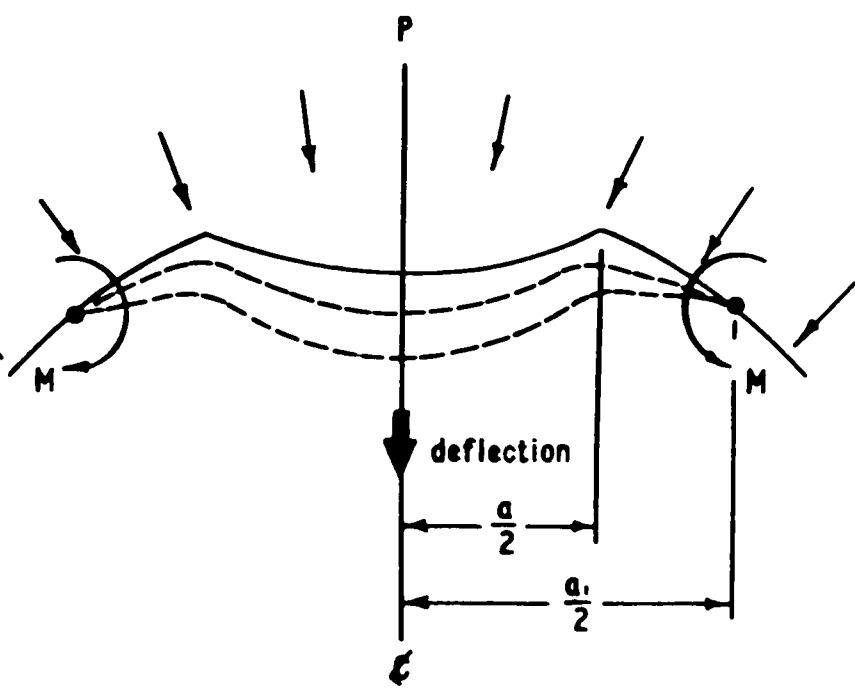


Fig.6.6d. Suggested deformation of dimple imperfection.

shell. The bulge, being of a smaller radius of curvature than the shell, was more rigid. Not only did it not deform during the elastic loading period but it retained its shape and size after the complete collapse of the model. This suggests that collapse initiated at some distance from the junction between bulge and sphere, as shown in Fig. 6.6c. It could then be argued that the severity of the bulge depends not only on the parameter (d/t) but also on the undetermined value of the radius $(a_{1/2})$ which in turn will vary with the general shape of the shell and the presence of other flaws or defects. A similar argument could be used to explain the behaviour of dimples, Fig. 6.6d, since here uniform external pressure of the shell would cause the dimple to be in tension and would shift the position of the final plastic hinge away from the junction between dimple and sphere. In either case, the virtually flat response of the experimental results for bulges in Fig. 6.1c and the very rapid drop of the results for dimples quoted from Sabir (1964) in Fig. 2.7, down to the level of Fig. 6.1c, would indicate that (d/t) is not the critical parameter that governs failure in these cases. To calculate the buckling pressure on the radius of the bulge rather than on that of the shell would bring the 4 experimental points above the theoretical curve of Fig. 6.1a just level with the curve but it would also bring down the remaining 6 points. The only conclusion that can be drawn is that the severity of a bulge or dimple cannot be characterised by a single parameter.

The results obtained from the computer programme BOSOR 4^[20] do not suggest that it can be used by a designer

to assess the effect of geometrical imperfections in spheres under uniform external pressure since they depend on the mesh configuration selected.

The safety factor of 3, introduced in von Kármán and Tsien's (1939) formula, equ.2.2, was not sufficient to guard against buckling in the models tested. An additional factor of 2 would still leave most experimental points below the load considered to be safe while unduly penalizing the quasi-perfect shell. The safety factor of approximately 8 given by the BS 5500^[13] code is also seen to be optimistic for $(d/t) > 12$ for the flat and increased radius imperfections and for nearly all sizes of the bulge. However the ASME^[8] code's safety factor of about 18 is seen to be adequate even for (d/t) values as large as 55. Designers are however advised to estimate the buckling pressure from equ. 2.7 for all imperfections of increased radius of curvature. Bulges and dimples should be avoided, since their presence causes the buckling pressure to drop by about 85%, regardless of their size.

CHAPTER VII

STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES:

MODELS AND EXPERIMENTAL METHOD

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7.2.3 Casting	69
7.2.4 Separation of moulds	70
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CHAPTER VII

STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES:

MODELS AND EXPERIMENTAL METHOD

7.1 INTRODUCTION

This part of the investigation concerned the manufacture of plastic spheres of the required modulus of elasticity and radius to thickness ratio. They were to be filled with water and accelerated vertically, normal and inclined to their equatorial rings in order to introduce buckling at different filling ratios.

The stages necessary to make, test and measure the models are shown in Fig. 7.1, and a description of each stage will now be given.

7.2 PROCEDURE

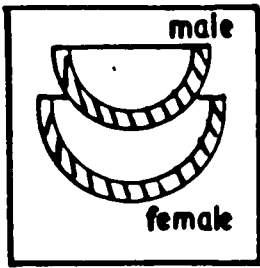
7.2.1. Male and female hemispherical moulds

Photographs of the stages necessary in the making of the aluminium moulds used in the casting of the plastic hemispheres, are shown in Figs. 7.2, 3, 4 and 5, a description of which can be found in Appendix B.1.

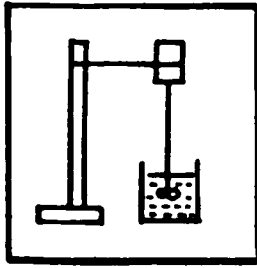
7.2.2 Preparation of epoxy

An epoxy of the required modulus of elasticity, was prepared by the addition of a liquid resin and liquid hardener.

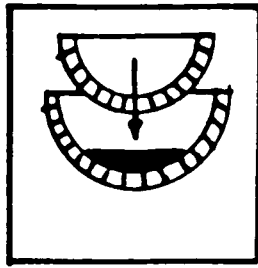
For the first test samples, the resin and hardener were at room temperature and the mechanical mixing of the two



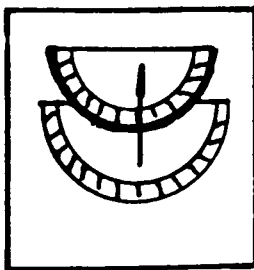
male and female hemispherical moulds



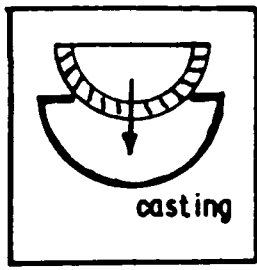
preparation of epoxy



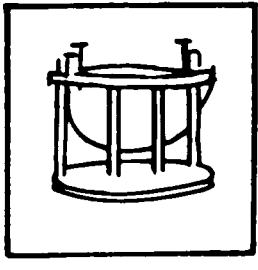
casting



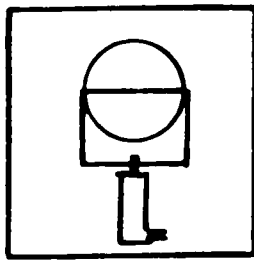
separation of moulds



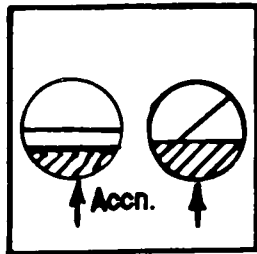
release of epoxy from mould



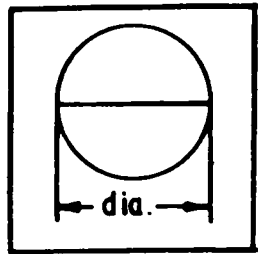
removal of excess epoxy



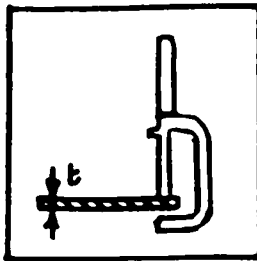
preparation of castings



buckling of spheres



sphericity



thickness

Fig.7.1. Steps made in the manufacturing, testing and measuring of the cast epoxy spheres.



Fig. 7.2 Photographs of male (above) and female (below) sacrificial polystyrene moulds made from cutting 1 inch thick polystyrene rings and gluing them together.



Fig. 7.3 Photographs of the cast aluminium male (above) with dural location plate attached, and both halves of the female (below) mould.

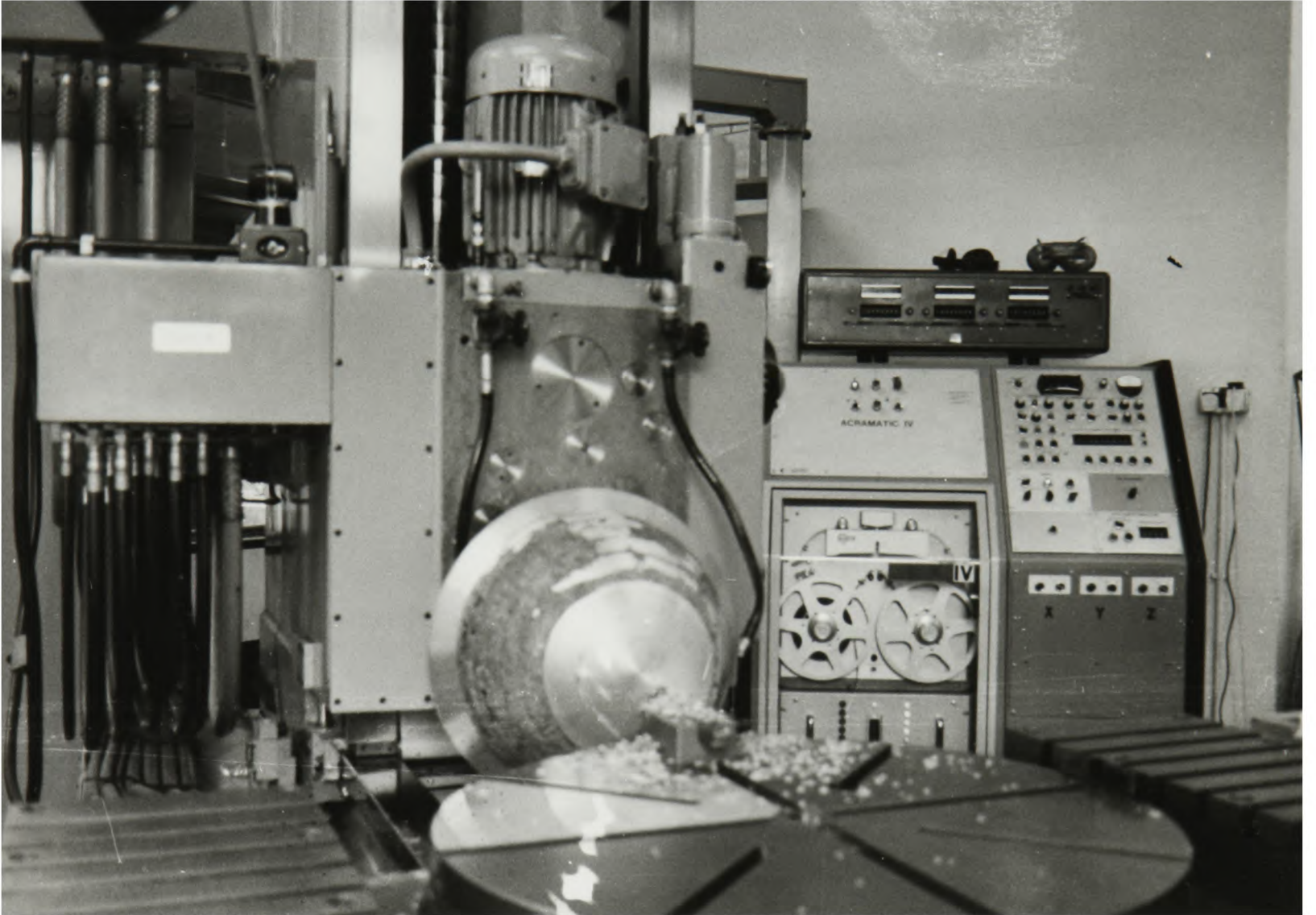


Fig. 7.4 Machining of the aluminium moulds to dimension was performed on a numerical controlled machine. This photograph was taken at the early stage in the machining of the male mould.

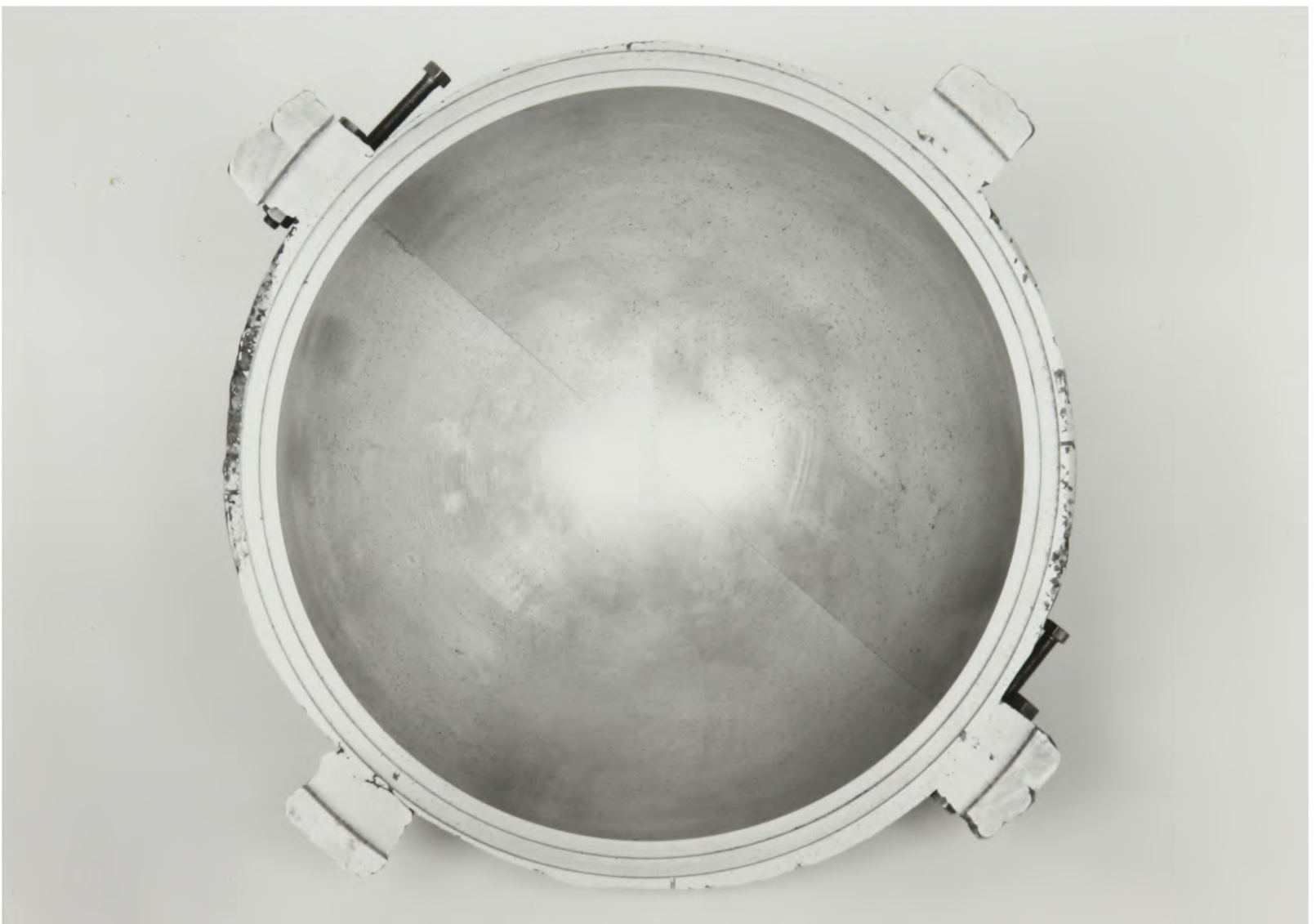


Fig. 7.5 Photographs of the machined male (above) and female (below) aluminium moulds used to cast epoxy into a hemispherical shape.

resulted in the formation of air bubbles. Therefore in order to reduce the viscosity of the solution the resin was heated before mixing, and whilst this action was successful in removing the air bubbles, it did reduce the time taken to begin curing to approximately 20 minutes.

The following procedure was adopted in the preparation of a quantity of epoxy for casting into a hemisphere:

- a) 130 g of resin weighed, (Araldite MY 753).*
- b) Resin placed in a preheated oven (38° C) for 20 minutes.
- c) 15 g of hardener weighed, (Araldite HY 951).*
- d) Resin added to hardener.
- e) Mechanically mixed for 7 minutes. Care was taken to avoid lifting the stirrer out of solution in order to minimize the formation of air bubbles.
- f) Mixture left to stand for 2 minutes, and so allow the air bubbles to reach the surface and escape.

7.2.3 Casting

The epoxy's behaviour as it cured was the same as a glue in so much as it would bond to a surface if insufficient, or inferior, release agent was applied. The application of both the grease-like and liquid release agent was as given in section 4.2.1.

The moulds were then ready for casting the epoxy solution made in section 7.2.2. Both the moulds were placed under a press in order to ensure that they were fully seated. This small pressure was held for approximately

* Obtained from manufacturer's Ciba-Geigy

12 hours, by which time the epoxy had nearly finished curing. The curing process was then accelerated by placing the moulds in a preheated oven (60°C) for approximately one hour. After allowing to stand and thereby cool for approximately two hours, the moulds were ready to be parted.

7.2.4 Separation of moulds

The two moulds were inverted, such that the legs of the female pointed upwards. It was found that it was not possible to separate the moulds by hand pressure alone, this probably being due to the high dimensional tolerancing between the moulds, and also to the clinging action of the casting. To this end, the device shown in Fig. 7.6 was used. This was simply a rectangular bar with a threaded hole at one end through which a threaded bar was screwed. A separating force was then applied by gradually tightening this bar, which resulted in the male mould moving relative to the female. By tapping lightly with a soft faced hammer on the opposite side to this force it was then possible to separate the moulds.

7.2.5 Release of epoxy from mould

The release agent that had previously been applied to the moulds, effectively prevented the casting from bonding to either of them. However, it was not possible to simply remove the cast hemisphere from the mould, whether male or female, because of the vacuum interface between the two.

The preference was for the casting to remain on the male mould; of the 18 hemispheres manufactured, 12 were found to remain on the male. If this was so, the following

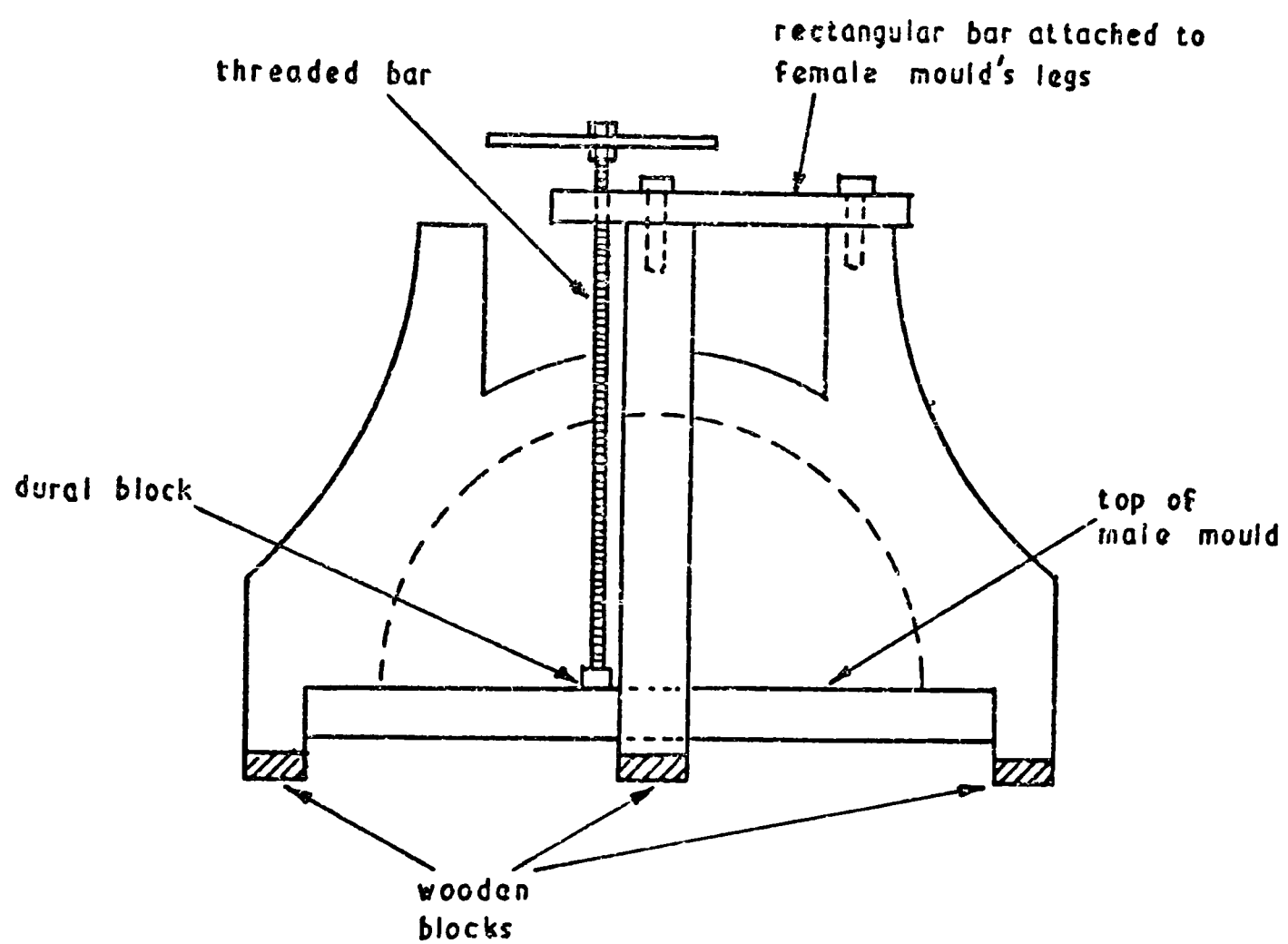


Fig. 7.6 Schematic diagram showing way in which moulds were separated.

removal procedure was adopted.

The dural top of the male was first removed and a record of the previous locations of the retaining bolts was made in addition to marking the relative positioning of the top plate in the mould. Next, with the male apex upwards, one inch wide brass shim was carefully inserted between the epoxy and the mould. Great care was needed, since past experience had shown the thin epoxy tended to crack if roughly handled. Pushing the shim upwards towards the apex then allowed air to enter. The shim was then removed and inserted at another position on the hemisphere's periphery. It was difficult to push the shim up as far as the apex at this stage and so hand pressure was used to encourage the newly formed air gap to flow upwards. After obtaining an air gap, over all of the hemisphere's surface, it was then possible to remove the cast hemisphere.

To summarise, removing the casting from the male mould was not an easy task. However, after learning to handle the epoxy very carefully, this method chosen proved to be satisfactory.

On the occasions when the casting was attached to the female mould, then the removal procedure proved to be less exacting in the skill required. The two halves of the female mould were parted by first removing the four clamping bolts on the female's legs, drifting out the two dowel pins and lastly tightening the two separating bolts. The casting was usually found to be attached to one of the mould's halves. By applying hand pressure, an air film was encouraged to flow between the casting and the mould, facilitating its removal.

7.2.6 Removal of excess epoxy

It was necessary to remove the overflow from the cast hemispheres before they could be joined together to form a sphere. The stand shown in Fig. 7.7 was used for this purpose. The sphere was clamped to the stand's steel ring, and a small bandsaw was used to cut away the majority of the excess epoxy. This cut section was firstly rough filed and secondly smooth filed in order to carefully bring the two surfaces A and B flush: see Fig. 7.7.

7.2.7 Preparation of castings

This stage was necessary in order to prepare the spherical models for their subsequent attachment to the acceleration rig.

The first requirement was to determine which models had minor damage. The term minor is used for models which contained very small holes and/or had not been too severely cracked during the removal process. Models which were found to have more severe damage were discarded. An epoxy glue was used to fill the holes. For the cracks, a $\frac{1}{32}$ in. hole was drilled at both ends which effectively prevented further growth, the hemisphere then being placed over the male mould, in order to ensure spherical accuracy and glue sparing applied over the length of the crack.

Since proximity gauges were used to measure the shell deflection, this necessitated painting the area of the shell opposite the gauge positions with conducting paint. (The use of a high speed cine camera in the preliminary testing stage, necessitated painting the shells with both translucent matt paint and a grid pattern. This method

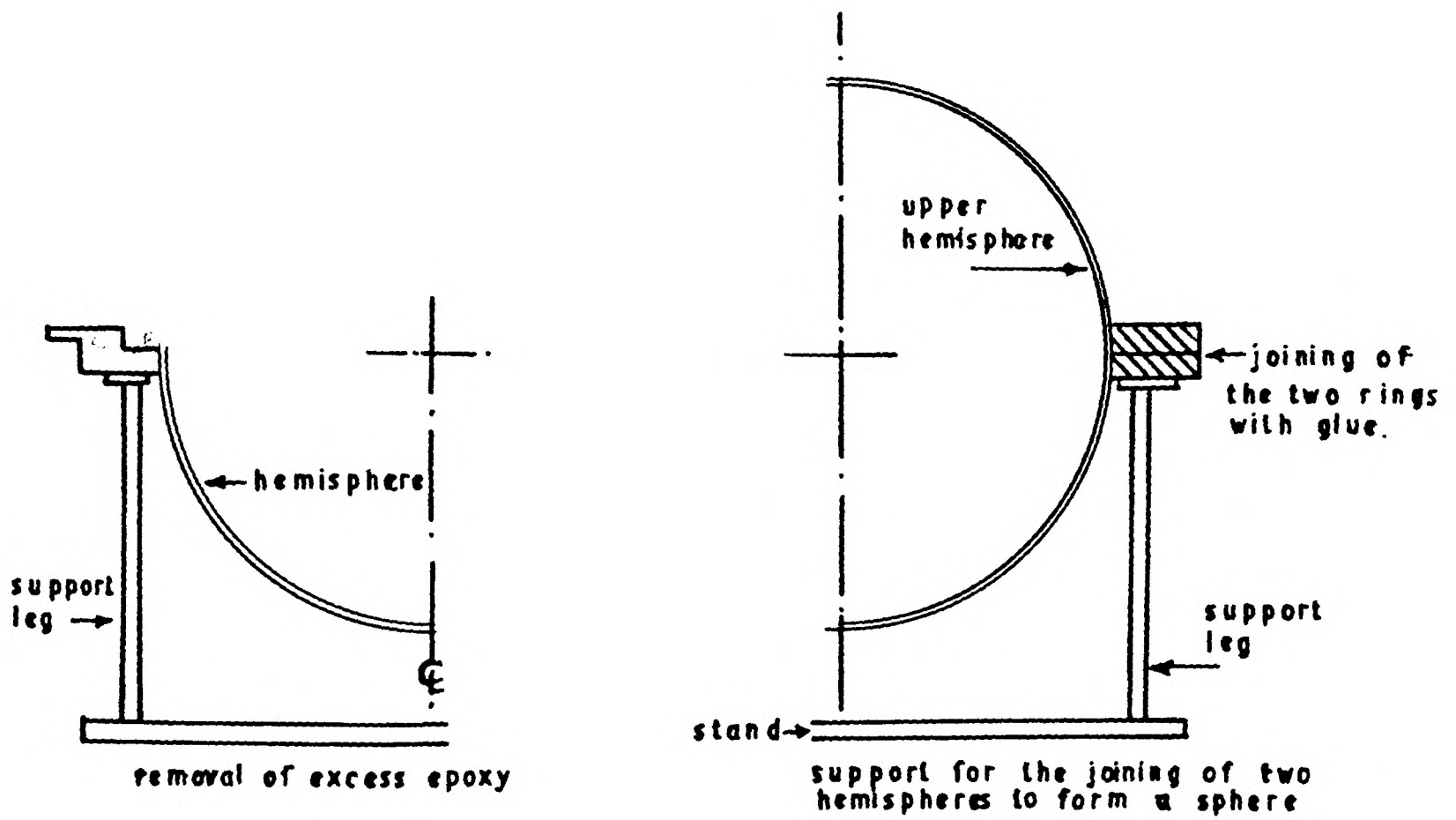


Fig.7.7. Photograph and schematic diagrams of the stand used in sections 7.2.6. and 7.2.7.

was later discontinued however, due to the difficulty experienced in determining the onset of buckling).

As a precautionary measure to prevent the possibility of small leaks when the sphere was being accelerated, the grease-like release agent was smeared over the inside of the hemispheres. It is difficult to estimate what value this had in preventing leaks occurring, but every sphere tested proved to be leakproof. It was also easier to introduce a precaution at this stage, since the hemispheres were to be glued together to form spheres and if any leaking was detected during testing it would necessitate removal of the sphere from the acceleration rig and the separation of the moulds, and so risk the possibility of damage to the models.

The stand shown in Fig. 7.7 was used in the joining of the hemispheres to form spheres. To produce a rigid joining an epoxy glue was chosen, the accuracy of the joining being judged by holding an engineer's square against the outside of the two rings.

The opening necessary for water to be added was made by melting the plastic at the apex of the top hemisphere with a soldering iron.

The final stage of glueing (epoxy) the sphere to the cylindrical support was then made.

7.2.8 Buckling of spheres

This cylinder, of 3 mm thick transparent plastic was used to support the sphere at its equator: see Fig. 7.8. The use of proximity gauges, required slots to be cut, their locations being limited to positions which allowed

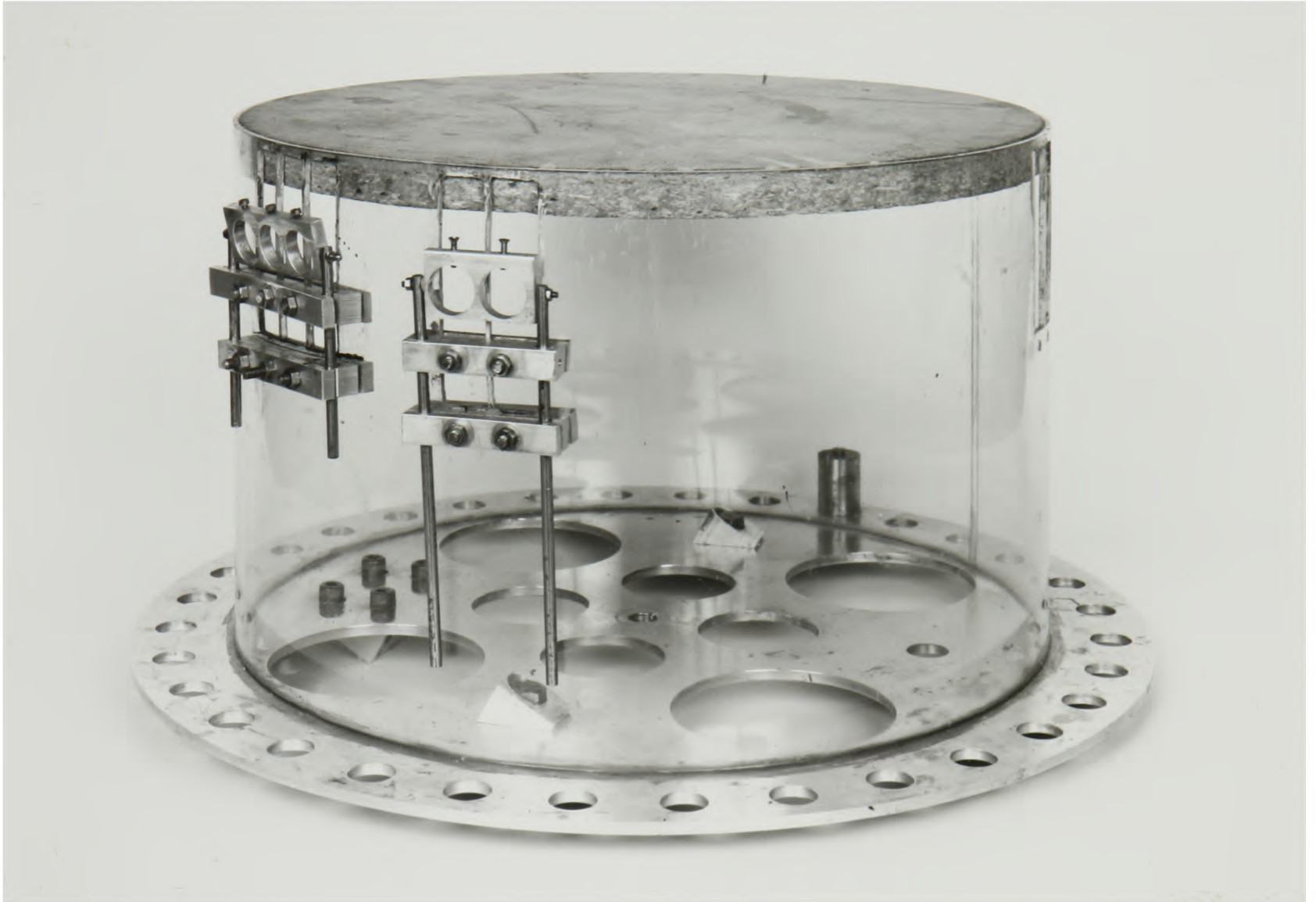


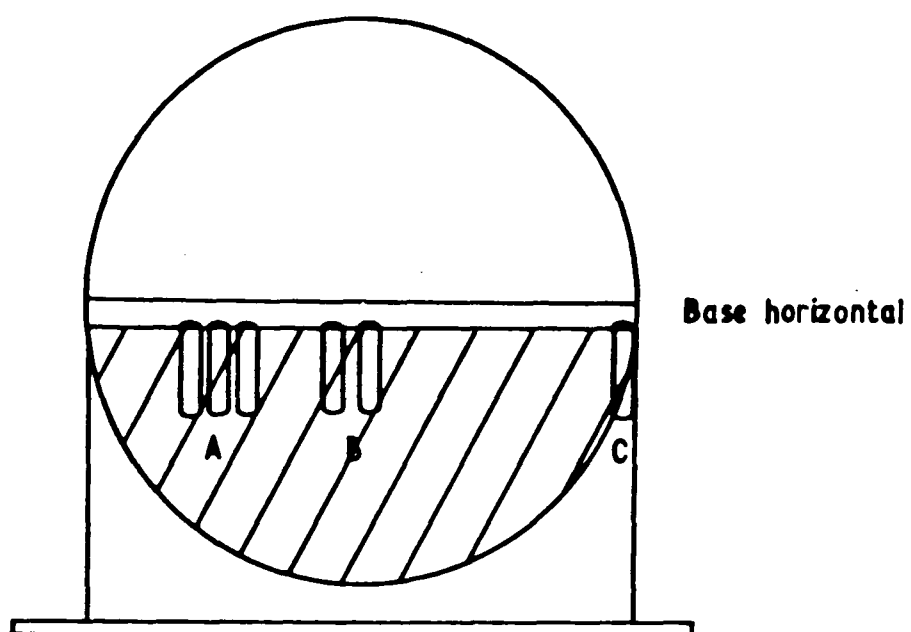
Fig. 7.8 Photograph showing the cylindrical support for the sphere, dural holders for the proximity gauges, a counterbalanced weight positioned on the opposite side to these holders, a lightened dural base plate and the three 30 degrees wedges that were used when an acceleration inclined to the spheres' equatorial ring was imposed.

the gauges to extend to the required distance and be normal to the sphere's surface. This was a requirement with the base plate horizontal and inclined at 30 degrees. The positions used allowed deflections to be measured at several places on the sphere's surface, and so it was possible to determine if buckling was localized due to the possibility of geometrical inaccuracies in the spherical models, for filling ratios in the range 0.23 to 0.8, as shown in Fig. 7.9. Note the positioning of the gauges was an important factor in the detection of buckling, since it could be envisaged that they might be facing a peak, a trough or a node of the waveform in which case their reading would fail to indicate any abnormally large deflection. It is also possible for the wave to shift from one loading to the next. Some scattering of results was therefore expected.

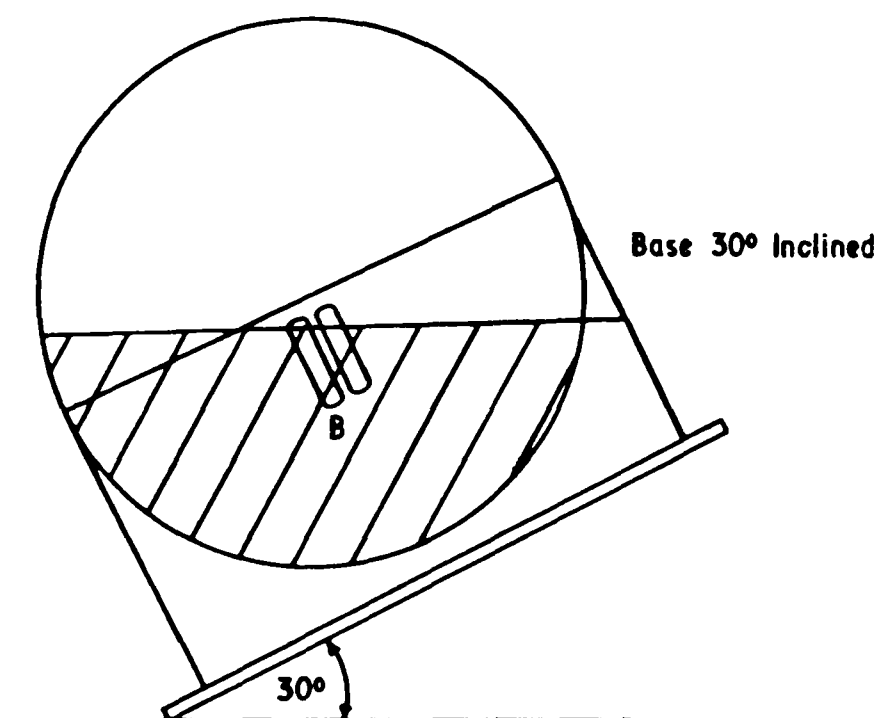
The dural holders used to provide rigid location for the gauges were designed to tilt and so enable the gauges to be always normal to the sphere at the waterline, where the buckling pattern was usually most pronounced. Their leads were always securely taped to the base plate in order to avoid exerting a tilting force during the accelerating period. The combined weight of the holders and gauges was counterbalanced by a weight added to the opposite side of the base plate. This base was a 5 mm thick, 500 mm diameter dural plate, suitably lightened by drilling a number of holes.

The three 30 degree wedges bolted to the base plate were used for the inclined loading condition. The larger of the three was positioned such that when attached to the

A,B,C positions used
for liquid filling ratios
from 0.23 to 0.42



B position used for
liquid filling ratios
from 0.23 to 0.48



C position used for
liquid filling ratios
from 0.61 to 0.8

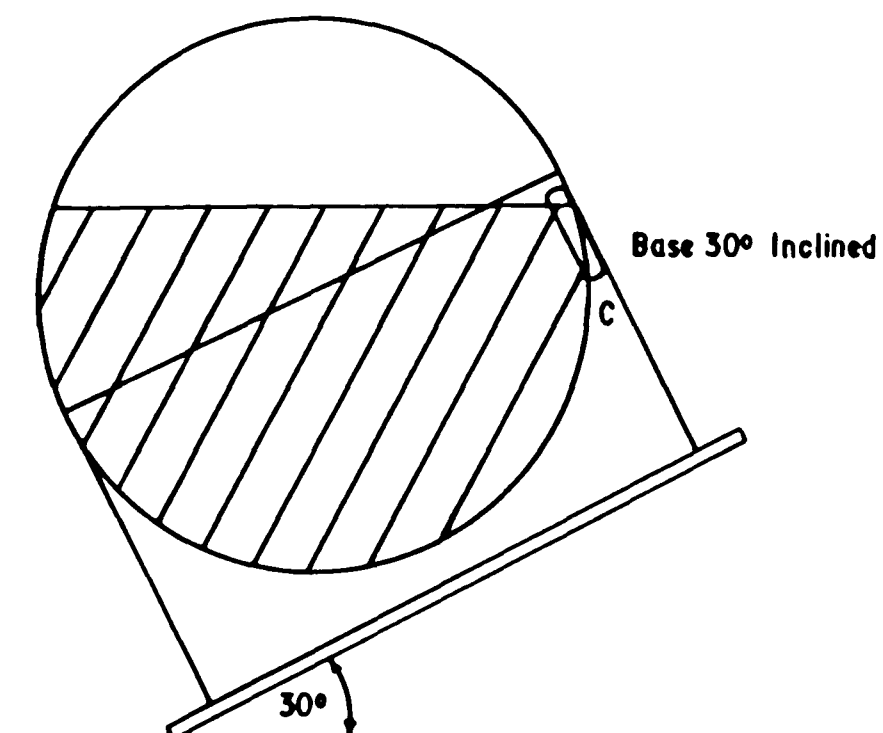


Fig.7.9. Schematic diagram of the partially filled sphere showing the positions of the proximity gauges used for the range of liquid filling ratios considered.

pneumatic piston, shown in Fig. 7.10, the liquid mass remained in line with the accelerating force. The two smaller wedges were mounted in order that the accelerometers attached, remained vertical. Using two accelerometers mounted diametrically opposite on the base plate, enabled a check to be made that the sphere received a uniform acceleration loading. This piston was used to accelerate the partially filled spheres up to an acceleration of approximately 5 g at a maximum air pressure of 120 psi, the required pressure being supplied by a compressor. By momentarily opening the solenoid operated spool valve, the piston accelerated with a high initial acceleration, and then slowly retarded until it came to rest near the top of its stroke. The one-way valve stopped the piston returning to its initial position and so prevented a sudden acceleration from being imposed. In practice, the on/off valve was opened slightly, which allowed the piston to automatically return slowly to its initial position after each acceleration run.

The signals from both the gauges and accelerometers were fed into amplifiers and thence to two oscilloscopes, as shown in Figs. 7.11 and 12. The oscilloscopes therefore recorded the acceleration of the sphere, calibrated by charge amplifiers in N/m^2 , and the corresponding deflection of the spherical shell. A triggering mechanism was used to simultaneously actuate the spool valve and the oscilloscopes.

The experimental procedure was as follows. Measured one litre amounts of water were added in order to give the required filling ratio. Vertical accelerations were then

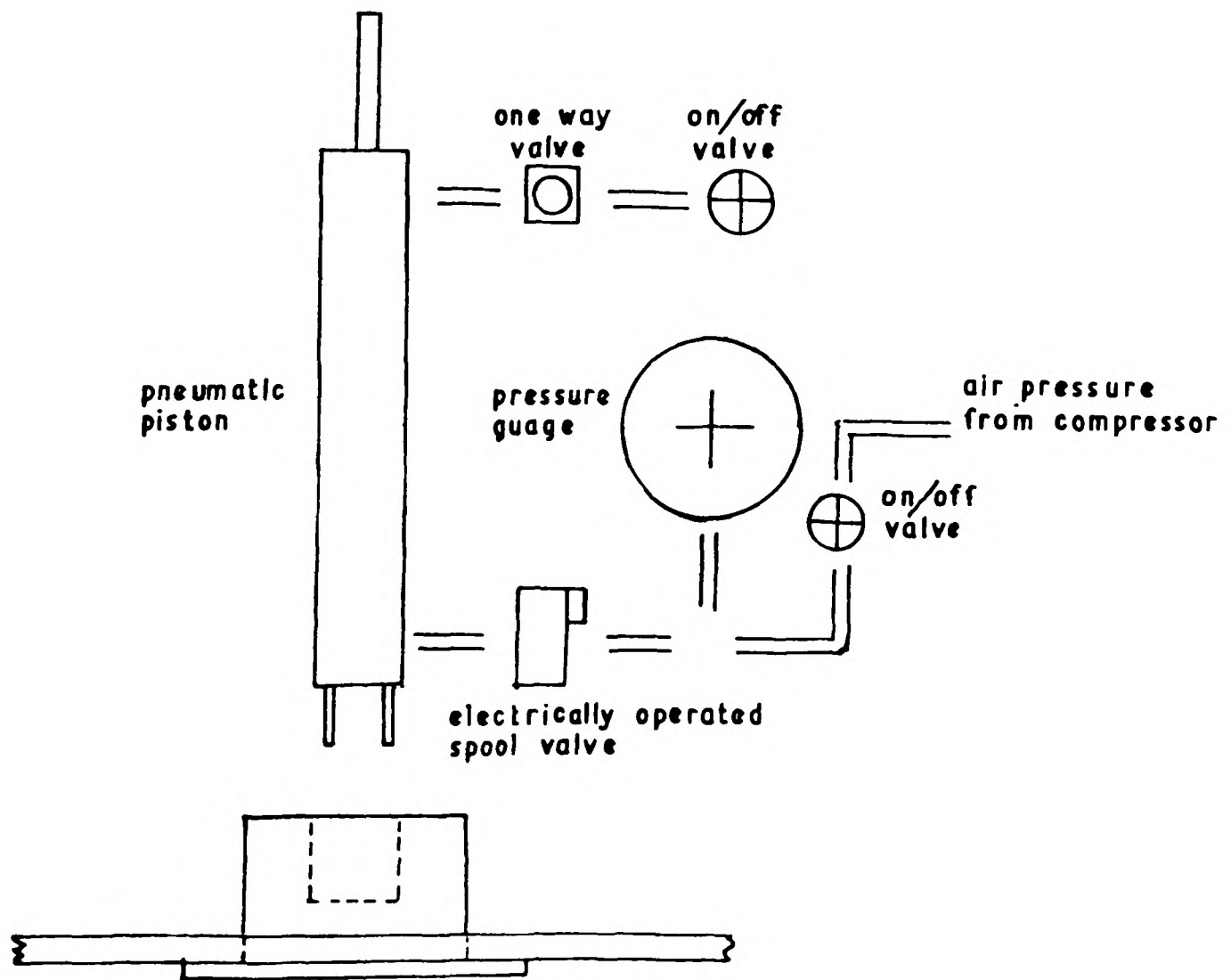
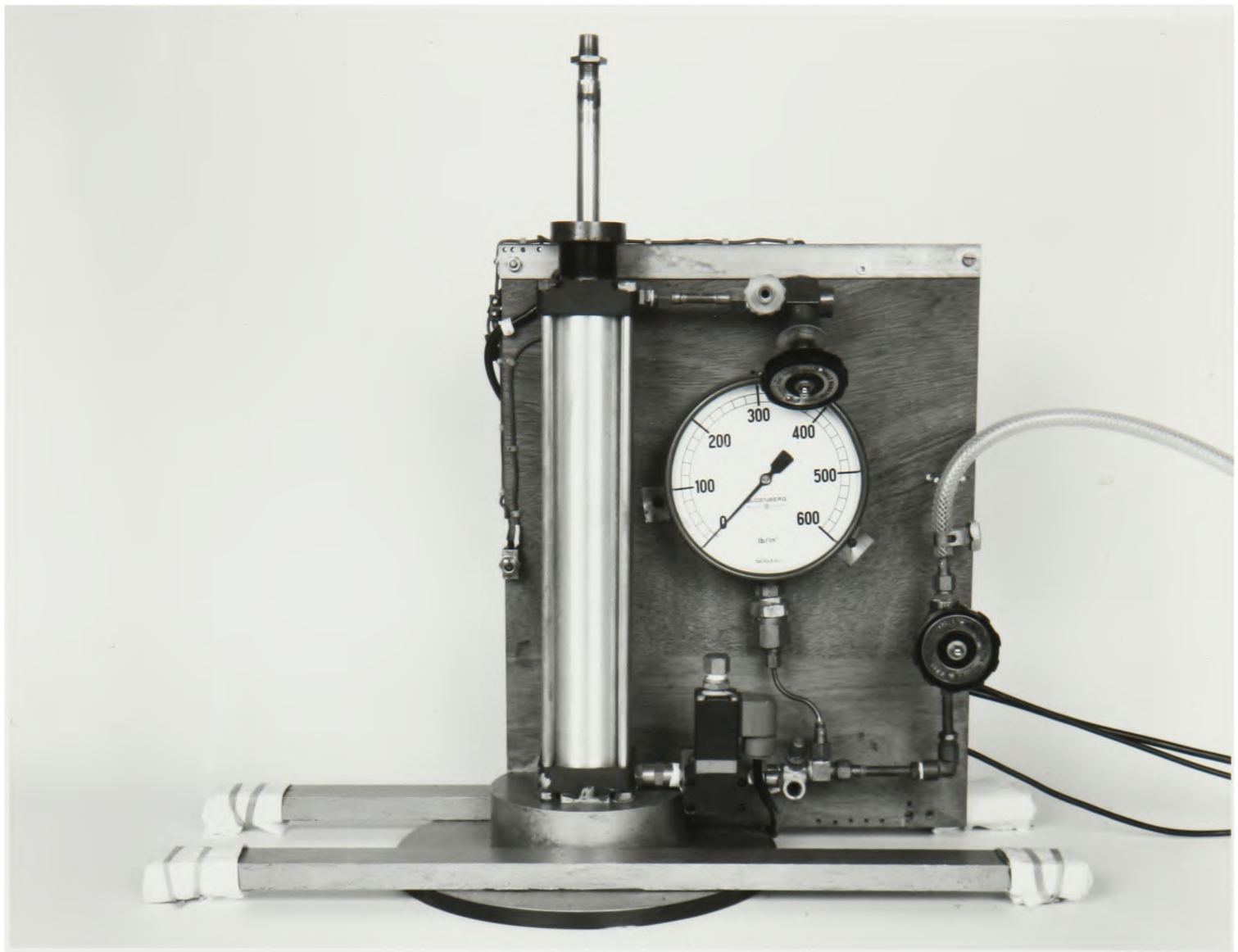


Fig.7.10. Photograph and schematic diagram of the pneumatic piston and ancillaries.



Fig. 7.11 Photograph taken of a partially filled sphere after undergoing a vertical acceleration normal to its equatorial ring. The two oscilloscopes were used to record the deflection of the shell (left) and the acceleration imposed (right).

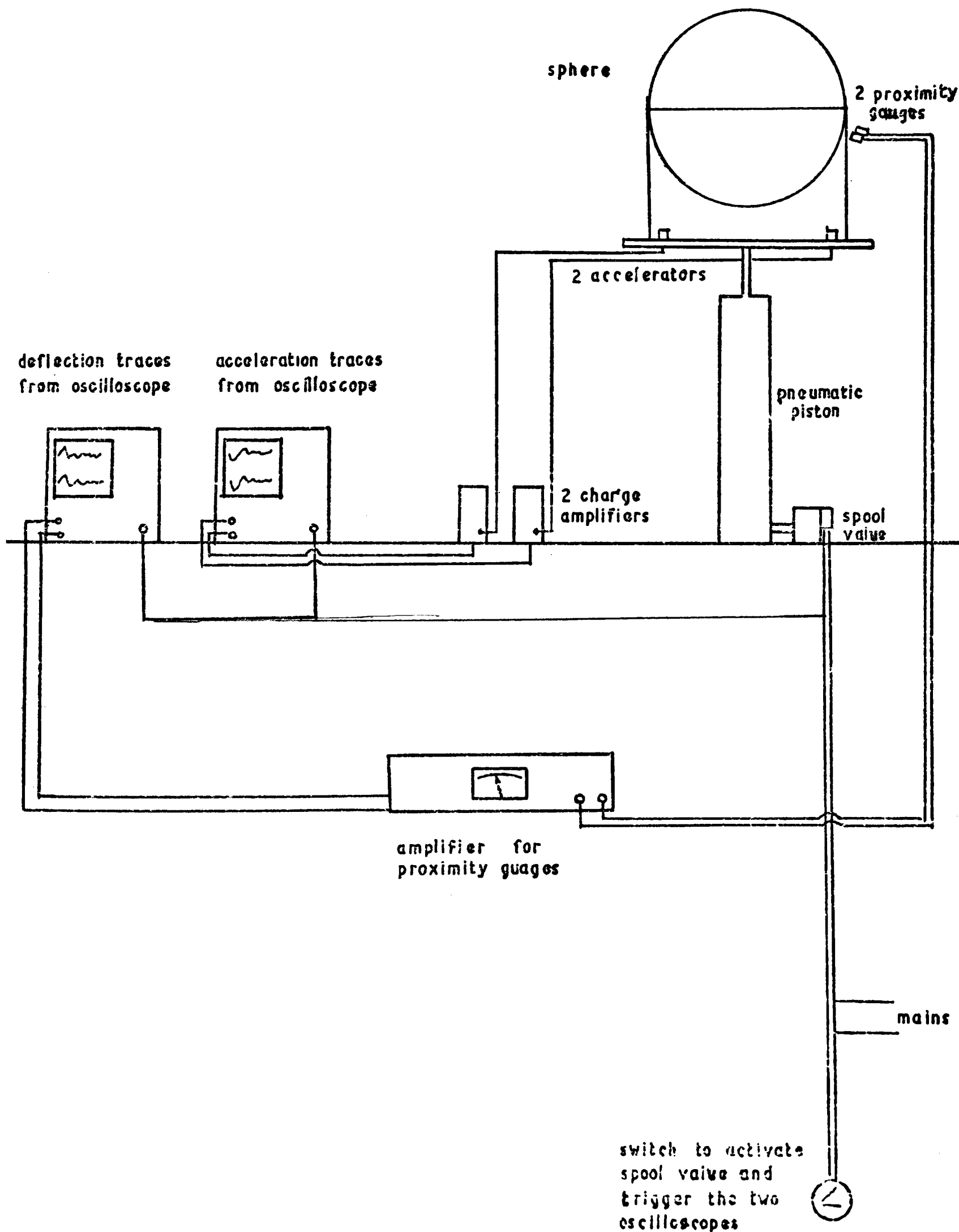


Fig.7.12 Schematic diagram of photograph shown in fig 7.11.

imposed, the magnitude of which was controlled by the air pressure. As the acceleration was increased, so did the maximum detected radial shell deformation, until a sudden change occurred with considerably larger displacements. This point was defined as the buckling acceleration.

7.2.9 Sphericity

Since the same aluminium moulds were used to cast each hemisphere, it was not envisaged that there would be differences in sphericity between the 6 models used. Therefore only one sphere's sphericity was determined, using the same apparatus as was used for the electroplated spheres: see section 4.2.8.

The method had to be amended however, due to the inclusion of the equatorial ring. This necessitated only taking circularity plots either side of this horizontally positioned ring. It was found necessary to glue the sphere to the three point support, since rotation of the dividing head caused it to move relative to its support.

In order to further assess the spherical accuracy of the model, diameter measurements were taken by mounting the sphere on a milling table and traversing the sphere such that it moved relative to a fixed datum position.

7.2.10 Thickness

Four strips approximately 50 mm wide and equidistantly spaced around each hemisphere were cut, starting from the hemisphere's equatorial ring and finishing at its apex. After cleaning each strip of grease, a micrometer was used to measure the thickness along the length of each strip.

In addition to obtaining the mean thickness of the shells, it was also possible to detect variations in both meridional and circumferential directions.

CHAPTER VIII

STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES:

EXPERIMENTAL RESULTS

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CHAPTER VIII

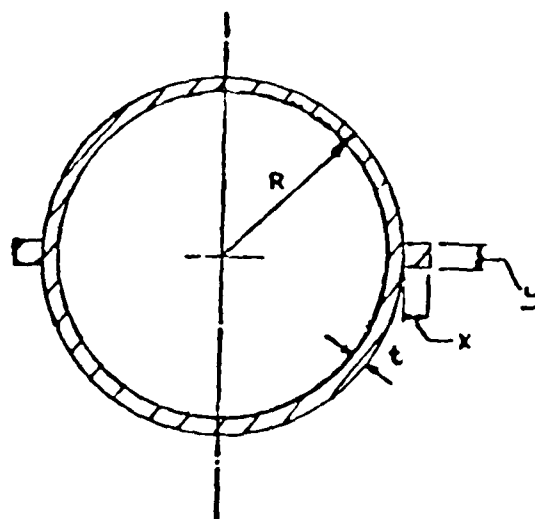
STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES: EXPERIMENTAL RESULTS

8.1 INTRODUCTION

The results of the investigation for the 6 spheres manufactured to radius to thickness ratios representative of current LNG carriers, will now be discussed.

8.2 PROPERTIES OF THE MODELS

8.1 Geometry



R , radius of sphere	= 195 mm
t , mean thickness of sphere	= 0.22 mm
x , width of ring	= 8.5 mm
y , depth of ring	= 6 mm
E , Young's Modulus of Elasticity	= 2×10^9 N/m ² *
ν , Poisson's ratio	= 0.39 *
ρ , density of the epoxy plastic	= 1200 kg/m ³ *

* Obtained from manufacturer's Ciba-Geigy

8.2.2 Thickness variations

The thickness variations found for each model are shown in Table 8.1. There is an approximate variation in thickness in the meridional direction of 0.24 mm at the apex to 0.20 mm just below the equator. A closer variation in thickness was found in the circumferential direction where only random variations were responsible for a deviation of 0.01 mm from the mean.

The thickness of the 6 cast spherical models was determined to be within 0.02 mm of the mean thickness of 0.22 mm, i.e. $0.22\text{mm} \pm 9\%$.

8.2.3 Sphericity

Four circularity plots were obtained from the XY plotter, of the same type as shown in Figs. 5.1a, b and c, for the electroplated copper spheres.

The maximum deviation (e) from the radius, was determined from these sinusoidal traces to be 0.05%. This value corresponds to a (d/t) ratio of 4.4%.

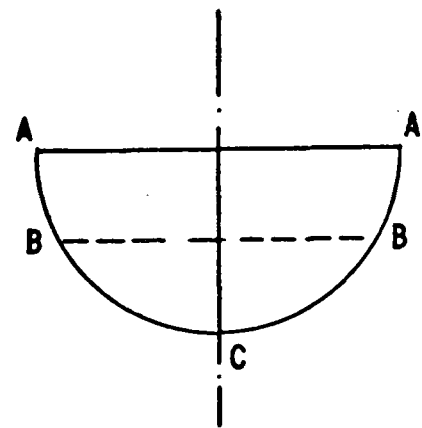
From the four diameter readings taken at random positions on the sphere's periphery, a maximum variation of 0.1% was found.

8.3 ACCELERATION AND DEFLECTION TRACES

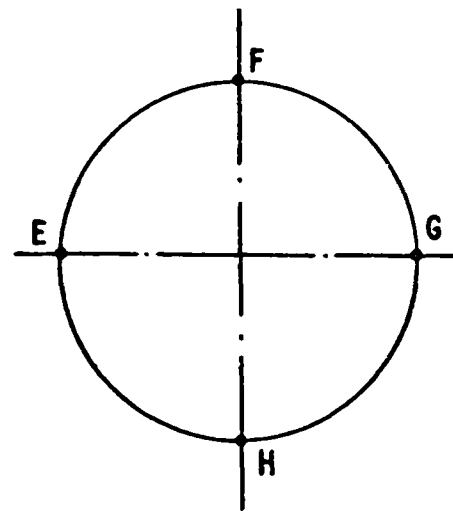
An example of a set of acceleration and deflection traces from the two oscilloscopes is given in Fig. 8.1. (Note the acceleration traces were not always smooth curves due to spurious signals from the accelerometer cables and to a certain amount of vibration of the base plate).

Table 8.1 Thickness variations in the lower hemisphere of each epoxy sphere tested

Sphere number	Meridional Mean Value			Circumferential				Average thickness of shell
	AA	BB	C	E	F	G	H	
1	0.195			0.19	0.20	0.20	0.19	0.213
		0.215		0.21	0.22	0.22	0.20	
			0.23					
2	0.215			0.21	0.21	0.22	0.22	0.234
		0.238		0.23	0.24	0.23	0.25	
			0.25					
3	0.19			0.20	0.19	0.18	0.19	0.206
		0.208		0.22	0.20	0.20	0.21	
			0.22					
4	0.203			0.21	0.18	0.21	0.21	0.224
		0.23		0.23	0.23	0.23	0.23	
			0.24					
5	0.198			0.20	0.20	0.19	0.20	0.210
		0.203		0.20	0.21	0.20	0.20	
			0.23					
6	0.193			0.20	0.18	0.20	0.19	0.215
		0.213		0.22	0.20	0.21	0.22	
			0.24					
Av.value	0.199	0.218	0.235					0.217



side view



plan view

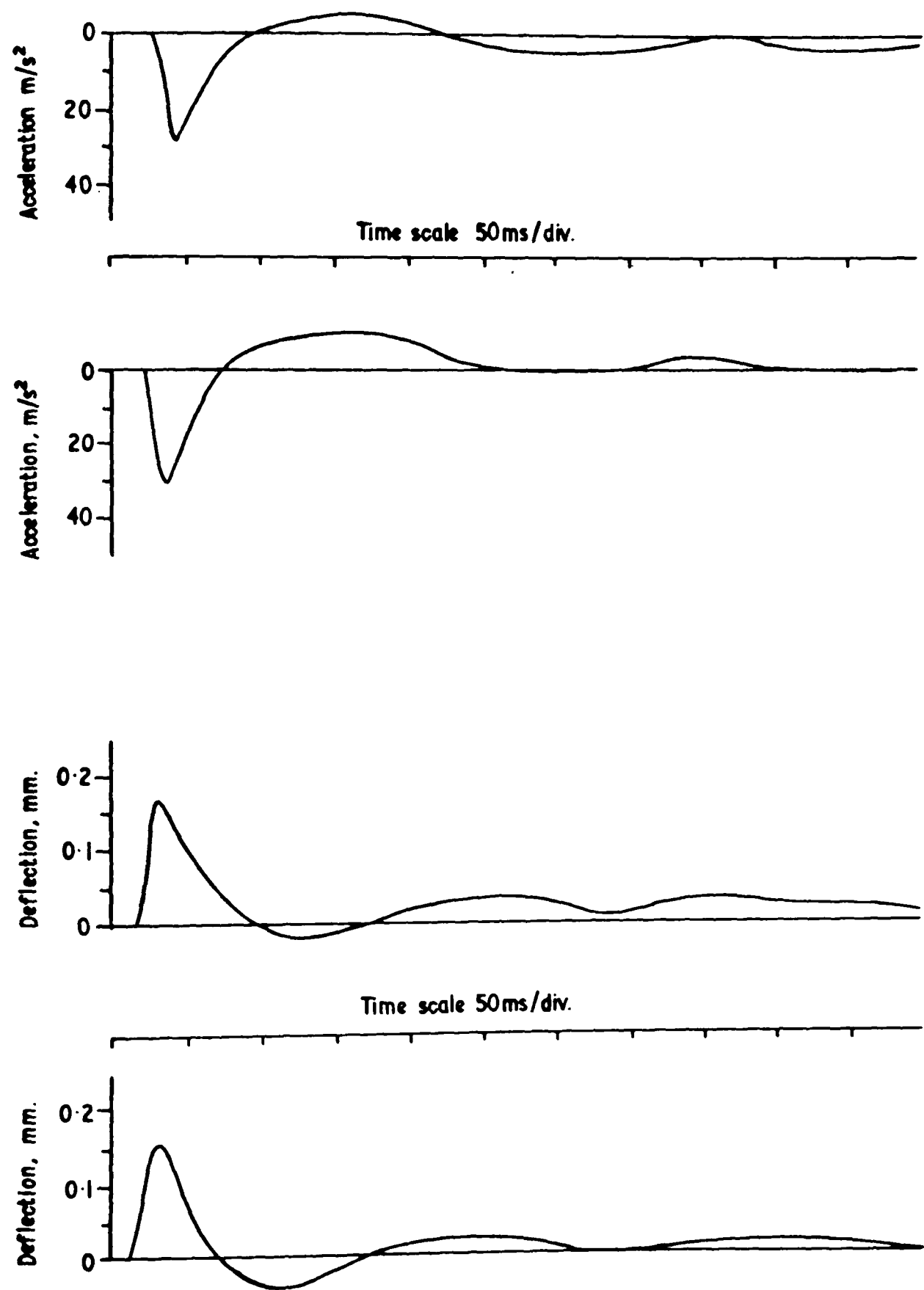


Fig.8.1. Acceleration and deflection traces recorded by the two oscilloscopes

The maximum deflection recorded was found to co-incide with the maximum acceleration imposed, an expected result from a material displaying the linear relationship of an elastic system and not of one under shock loading where a time lag would be noticed. Although the time taken to build up to these maximum values is only of the order of 50 ms, it is in fact very large indeed when compared to the time taken for longitudinal wave propagation through the plastic.

Thus from the rigid equatorial support the wave would travel a maximum meridional distance of about $0.14 \pi R$ to the water level where maximum deflection occurs for the smallest filling ratio of $\frac{1}{5}$ used.

∴ time taken to travel this distance,

$$= \frac{0.14 \pi \times 0.195}{\sqrt{(E/\rho)}} = 0.07 \text{ ms}$$

If the maximum values of acceleration and deflection are approximated from Fig. 8.1 to be acting for 5 ms, then this time would be some 70 times that of the wave propagation time. For larger filling ratios, the difference between these two times would be substantially larger.

The justifications in choosing to record the maximum values and not the lower ones owing to the fact that the maximum did not occur in an infinite time but over a period of approximately 5 ms are that:

a) The maximum acceleration occurred at the same time as the maximum deflection and so was the cause of this displacement.

b) If a 5 ms period is assumed at the maximum values, then from the approximately triangular waveform of base

time 50 ms, shown in Fig. 8.1, the maximum values would be reduced by only $\frac{1}{30}$, or 3.33%.

8.4 PRE-BUCKLING AND POST-BUCKLING CURVES

These maximum values obtained from similar traces to the ones shown in Fig. 8.1, have been plotted to produce acceleration versus deflection graphs as shown for example in Fig. 8.2. The pre-buckling curves were, as expected, linear, since the strain imposed was well within the linear region of the stress-strain curve for epoxy.

Thus from Fig. 3.10, for the 30° inclined loading condition,

$$\sigma_\theta \approx -0.6 \frac{k\rho R^2}{t} \quad \sigma_\phi \approx 0.5 \frac{k\rho R^2}{t}$$

$$\therefore \epsilon_\theta = \frac{k\rho R^2}{tE} (-0.6 - 0.5\nu) = 0.14\%$$

for the maximum acceleration of about 2g imposed in the region of the critical filling ratio.

From Fig. 3.6 for loading normal to the sphere's equatorial ring,

$$\sigma_\theta = -\frac{k\rho R^2}{3t} \quad \sigma_\phi = \frac{k\rho R^2}{3t}$$

$$\therefore \epsilon_\theta = \frac{k\rho R^2}{tE} (-0.33 - 0.33\nu) = 0.12\%$$

for the maximum acceleration of about 3 g imposed in the region of the critical filling ratio.

For the epoxy used, the yield strain is approximately 2.5%, a value which is ≈ 20 times that experienced by the models.

From Fig. 8.2, the initial slope, for the pre-buckling situation indicates a strain of,

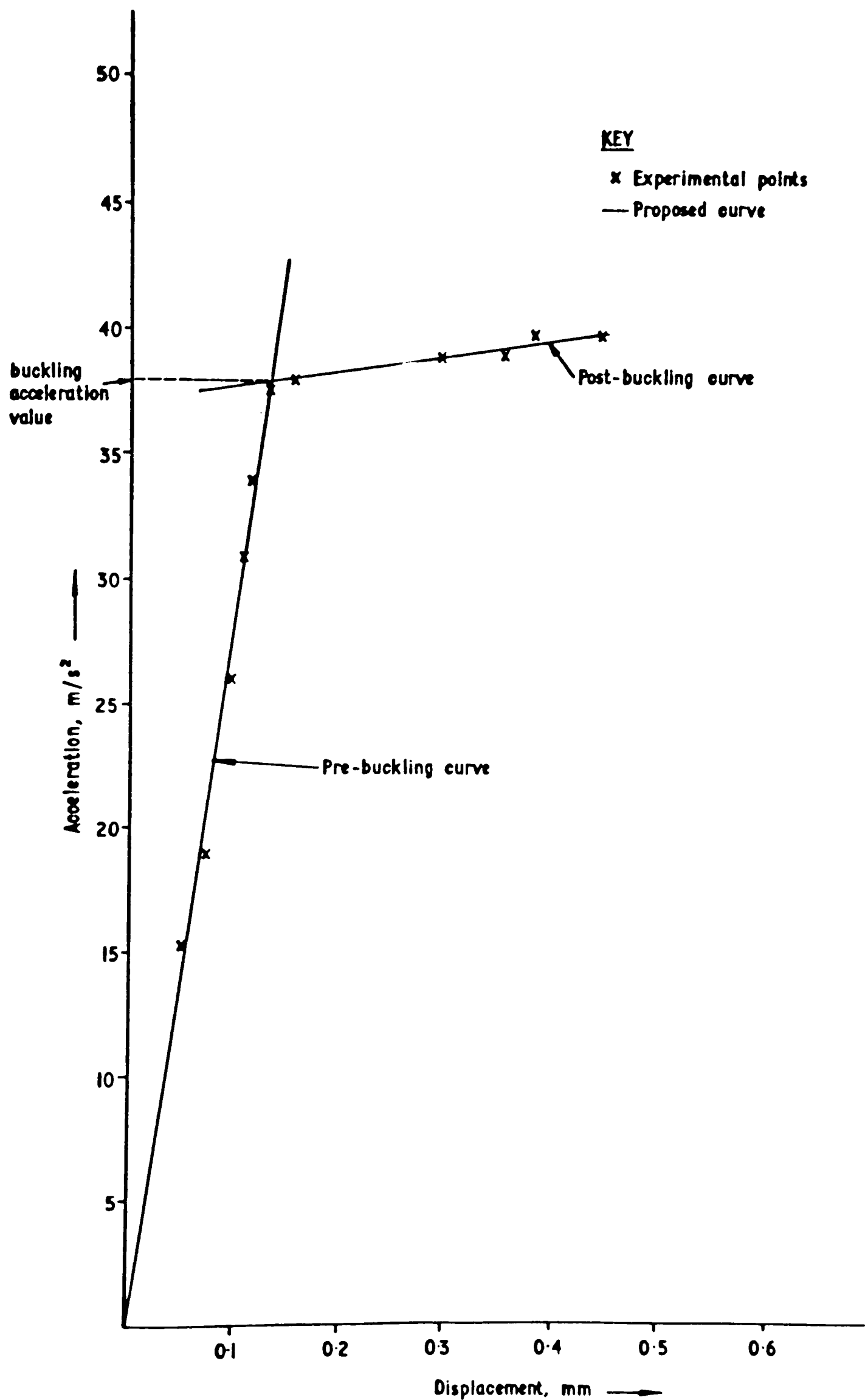


Fig. 8.2. Acceleration versus displacement graph of partially filled sphere no.3., $V = 0.225$, with its equatorial ring horizontal to the accelerating force. The buckling acceleration is taken to be given by the intersection between the tangent lines to the pre- and post-buckling regimes.

$$\epsilon_{\theta} = \frac{u}{R}$$

assuming that the radial displacement (u) is perfectly uniform and no waves are formed.

The filling ratio of 0.225 in Fig. 8.2 corresponds, from Fig. 3.6, to,

$$\sigma_{\theta} = -\sigma_{\phi} = -0.18$$

∴ strain imposed ϵ_{θ} ,

$$= \frac{k\rho R^2}{tE} (-0.18 - 0.18\nu) = 2.16 \times 10^{-5}k$$

where k is in m/s^2

Thus for $k = 30 m/s^2$ the predicted radial displacement is,

$$u = R\epsilon_{\theta} = 0.126 \text{ mm}$$

A value which is in agreement with the "as measured" one from Fig. 8.2 of 0.1 mm, although as expected, the experimental value is a little lower due to the effect of the restraint at the equatorial support.

The behaviour of the acceleration/displacement curve, in Fig. 8.2, above about $38 m/s^2$, suddenly changes. Whilst previously the slope corresponded to uniformly distributed radial deflection with strain well below yielding, now the slope is such that yield stress would be exceeded if the same uniform pattern of deformation was still assumed. For the material used, once the yield stress is reached, fracture immediately follows. Epoxy, at room temperature behaves in a brittle manner, introducing a permanent deformation. Moreover, upon unloading the structure

retained its original shape, thus confirming that no plastic stress had been introduced. It follows that the only way in which a large radial displacement can occur is when this is in the form of inextensional deformation, i.e. forming a series of waves. Furthermore, since this buckling pattern disappeared upon unloading, this proves that the process is not one of true instability and, as an important corollary, that the shell is not imperfection sensitive since post-buckling deformation only proceeds under increasing loads, (see section 2.2.7).

8.5 BUCKLING ACCELERATIONS

The buckling accelerations, taken to be given by the intersection between the tangent lines to the pre- and post-buckling regimes, have been used to obtain buckling acceleration versus filling ratio graphs for the six spheres tested. These graphs are shown in Figs. 8.3a,b,c, d,e and f.

All results consistently point out that a filling ratio of 0.34 to 0.38 and 0.55 to 0.7 constitutes the most severe operational condition under vertical acceleration, respectively normal and 30 degrees inclined to the sphere's equatorial ring. In the former case, the acceleration required to cause buckling varies between 26.25 and 31 m/s², in the latter, between 15.5 and 18.75 m/s². The experimental values listed in Table 8.2 are all consistent. Except two - numbers 2 and 3 - all models survived the test. Model 2 failed due to inadvertent overloading resulting in fracture of the shell when its acceleration versus filling ratio curve was about to level off, as shown

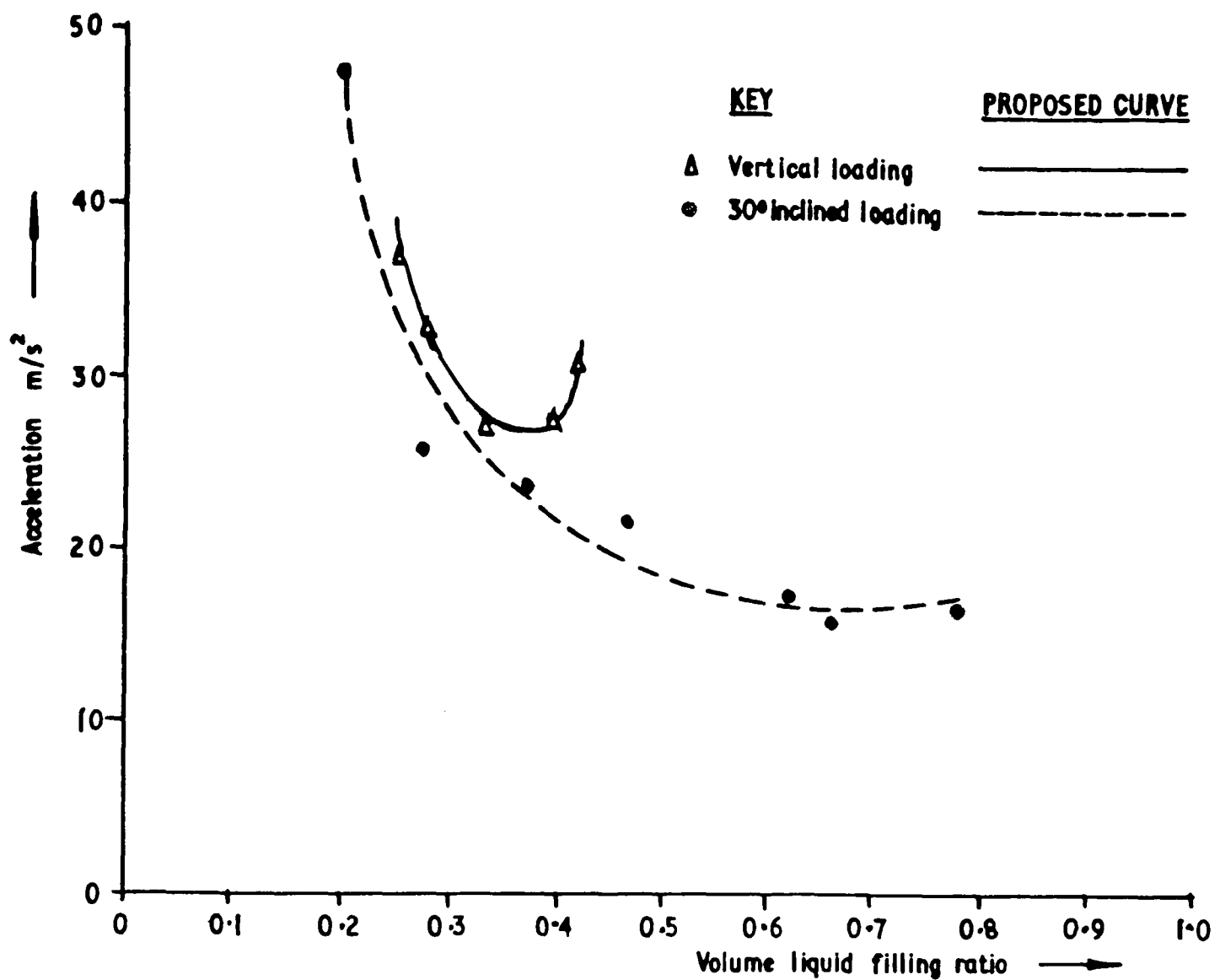


Fig. 8.3a. Buckling results for sphere no.1

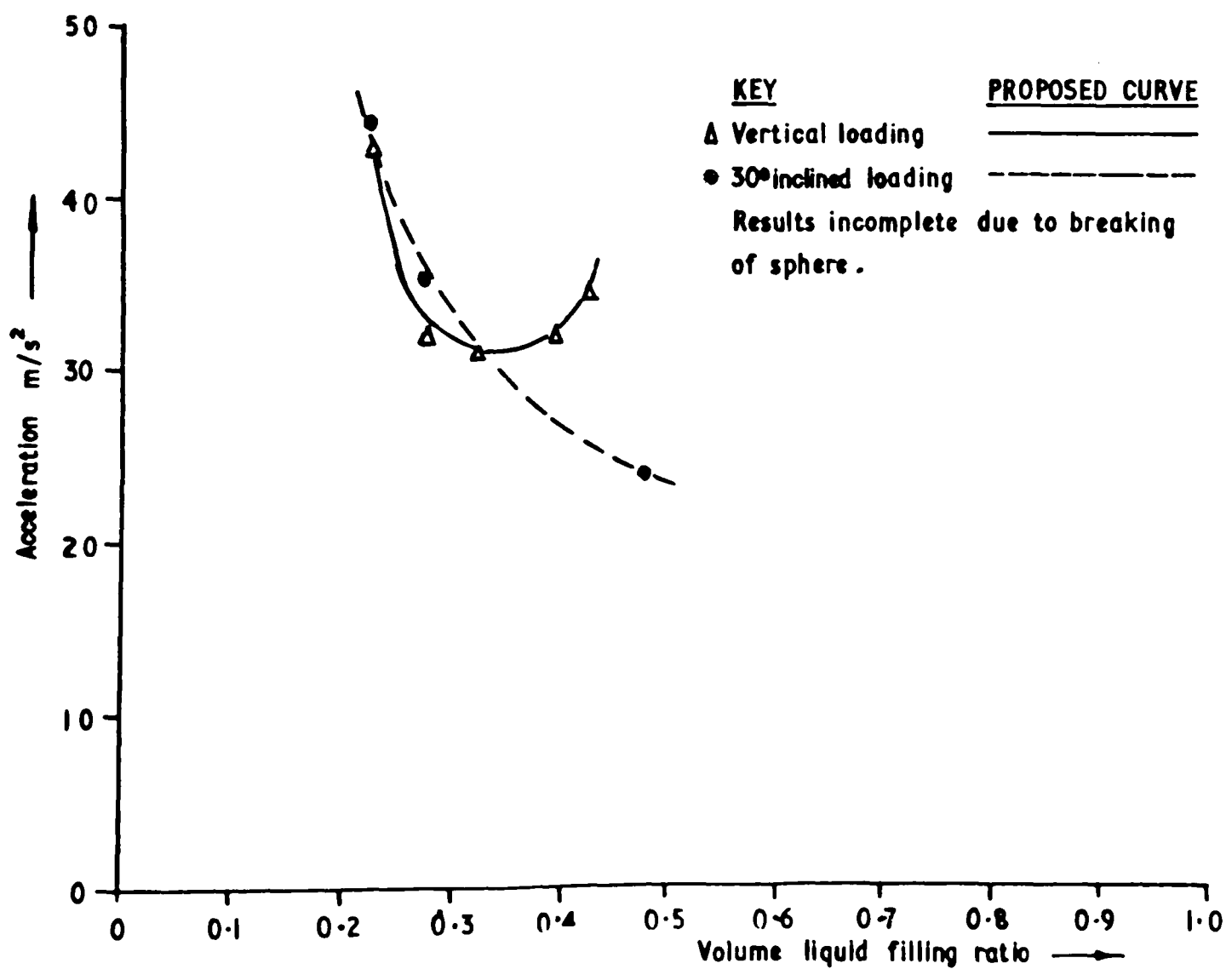


Fig. 8.3b. Buckling results for sphere no.2

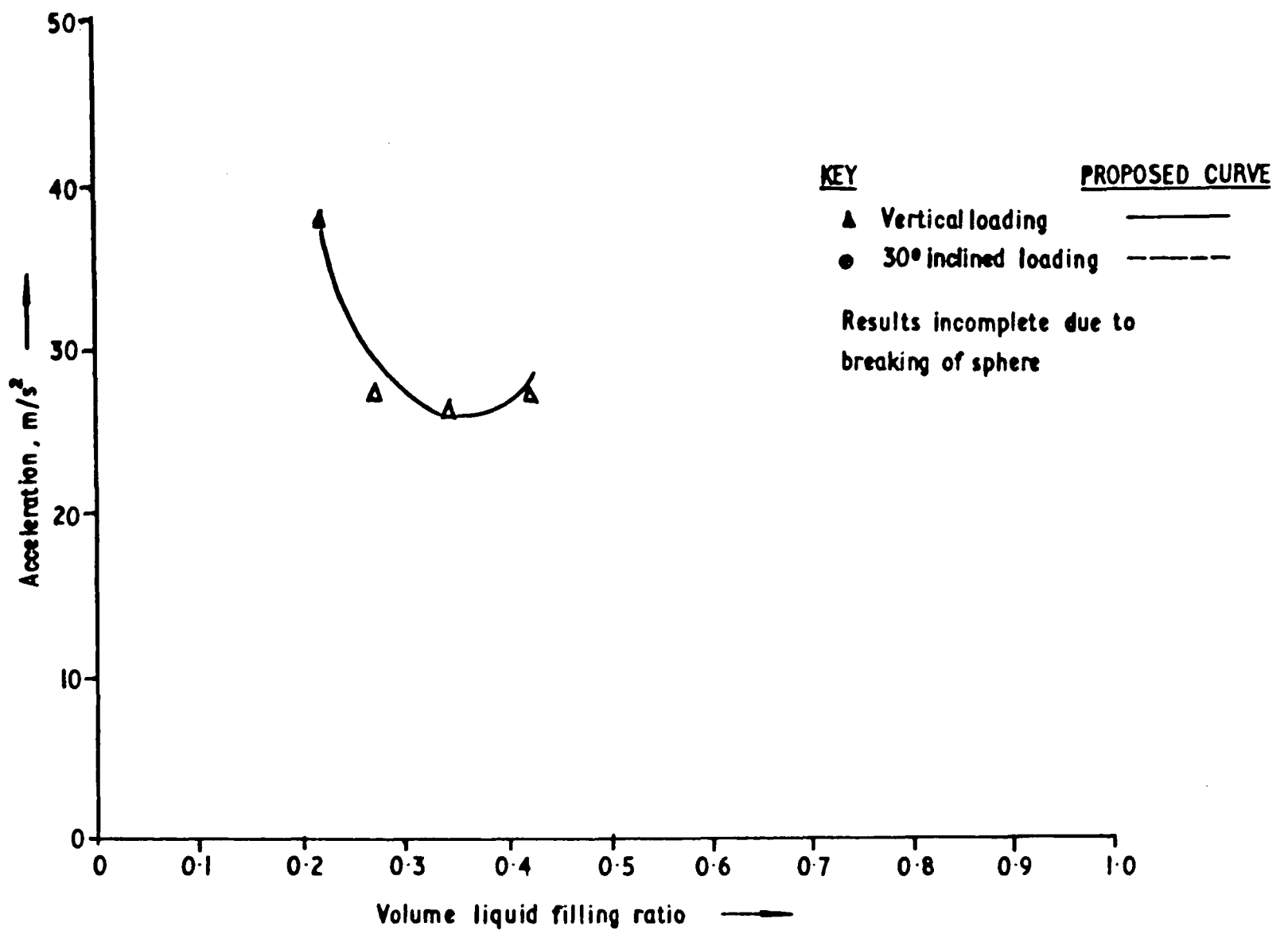


Fig.8.3c. Buckling results for sphere No.3

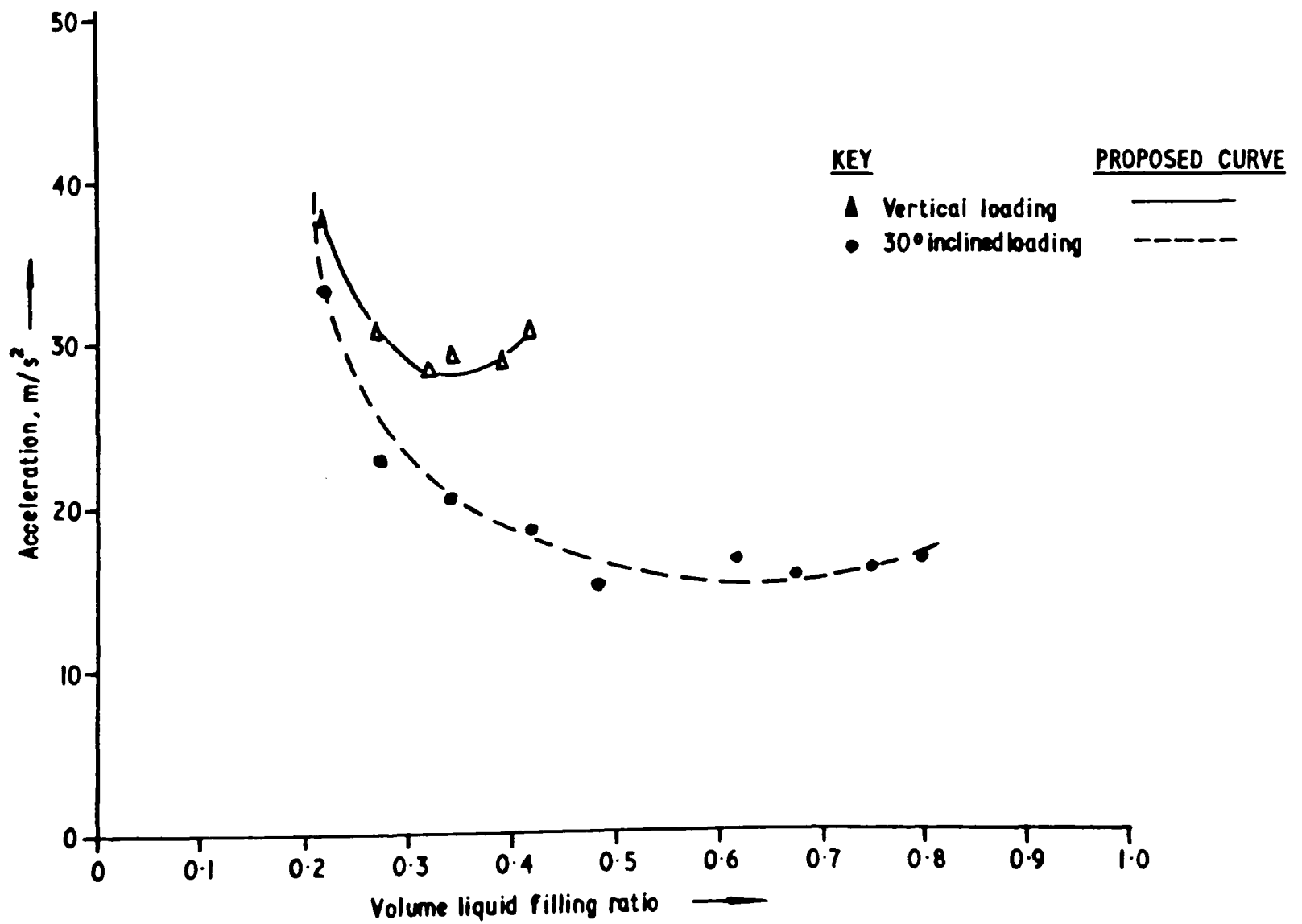


Fig.8.3d. Buckling results for sphere No.4

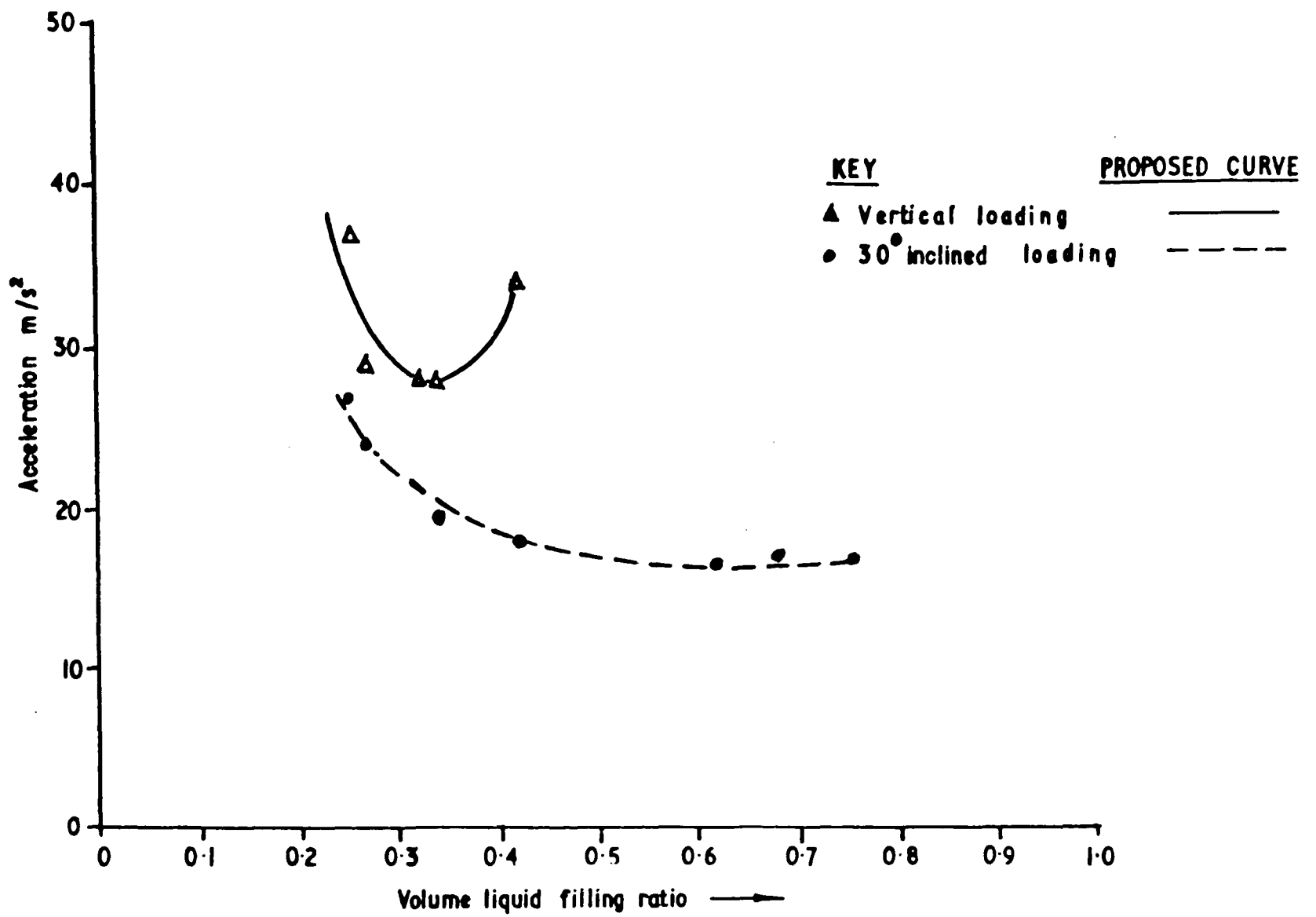


Fig. 8.3e. Buckling results for sphere No. 5.

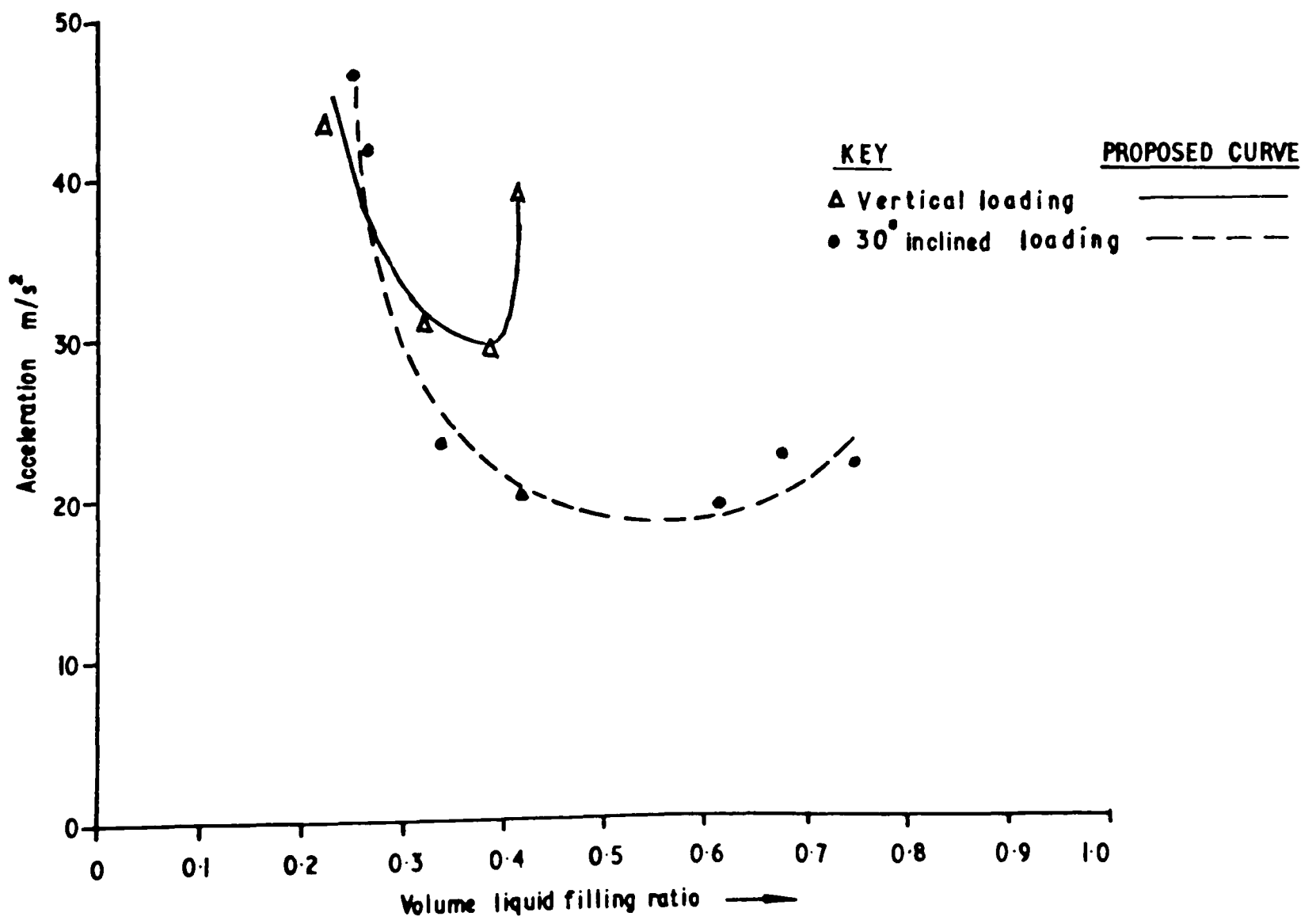


Fig. 8.3f. Buckling results for sphere No. 6.

Table 8.2 Summary of critical liquid filling ratio and critical buckling acceleration values obtained for the six spheres tested. Note no results available for sphere nos. 2 and 3 for the 30° loading case due to fracture of the shells before the experiment could be completed

a) Vertical loading normal to equatorial ring

Sphere No.	V _c	k _c ,m/s ²
1, Fig. 8.3a	0.37	27
2, Fig. 8.3b	0.335	31
3, Fig. 8.3c	0.35	26.25
4, Fig. 8.3d	0.34	28.5
5, Fig. 8.3e	0.345	28
6, Fig. 8.3f	0.38	29.25
Mean value	0.35	28.3
Maximum variation from mean	8.6%	9.5%

b) Vertical loading at 30 degrees to equatorial ring

Sphere No.	V _c	k _c ,m/s ²
1, Fig. 8.3a	0.7	16
2, Fig. 8.3b	-	-
3, Fig. 8.3c	-	-
4, Fig. 8.3d	0.6	15.5
5, Fig. 8.3e	0.675	16.5
6, Fig. 8.3f	0.55	18.75
Mean value	0.63	16.7
Maximum variation from mean	12.7%	12.4%

Ratio of the mean critical accelerations for the loading cases inclined and normal to the sphere's equatorial ring,

= $\frac{16.7}{28.3}$ = 59%

in Fig. 8.3b. Although no results can be quoted for this model under 30 degrees loading in Table 8.2, it may be assumed that it does not fall out of line with the rest. Model 3 failed also as a result of overloading, before it could be tested under the 30 degrees loading condition.

CHAPTER IX

STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES:

INTERPRETATION OF RESULTS

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CHAPTER IX

STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES:

INTERPRETATION OF RESULTS

9.1 INTRODUCTION

From the general standpoint of the design against failure, several modes of failure may be identified for these spherical vessels. These are: buckling under external pressure; excessive stresses which would lead to yielding or rupture; vibration (outside scope of present study); and the present investigation which considers local buckling in a partially filled tank. Following an experimental programme based on model tanks, results will now be reviewed in the light of their possible application to design.

9.2 THEORETICAL ANALYSIS

One approximate form of analysis was presented in Chapter III, where the peak compressive stress under uniform external pressure was compared to the maximum compressive membrane stress component in the partially filled sphere. In addition, the effect of the interaction between the compressive and tensile stress components was also considered. The adequacy of this analysis will now be discussed.

The scaling factor between the model and prototype sphere will first be determined. Based on the peak compressive stress under uniform external pressure, the calculated buckling accelerations obtained from equ. 3.17 correspond to 1.95 g and 6.3 g for the model and prototype respectively. The scaling factor between the two can thus be expressed as,

$$\frac{6.3g}{1.95g} = 3.2 \quad (9.1)$$

The average buckling acceleration of 2.9 g (28.3 m/s²) obtained from the experimental investigation (see Fig. 9.1), for loading normal to the spheres equatorial ring, therefore corresponds to an acceptable acceleration for the prototype sphere of,

$$2.9 \text{ g} \times 3.2 = 9.4 \text{ g}$$

This buckling acceleration, as expected lies between the two limits of 6.3 g and 25.2 g (i.e. based on the suggested effect of the interaction curves discussed in section 3.3.2). The buckling acceleration of 9.4 g thus corresponds to a safety factor of

$$\frac{9.4\text{g}}{1.43\text{g}} = 6.6$$

of the design maximum acceleration of 1.43 g (see section 1.2).

For the 30 degrees inclined loading condition, an average buckling acceleration of 1.7 g (16.7 m.s⁻²) was found for the model (see Fig. 9.1), which from equ. 3.1, corresponds to an acceleration for the prototype sphere of,

$$1.7 \text{ g} \times 3.2 = 5.4 \text{ g}$$

As expected this buckling acceleration lies between the value 3.5 g, which was based on the peak compressive stress under uniform external pressure (see equ. 3.25) and the value 14 g which includes the suggested effect of the interaction curves. The buckling acceleration of 5.4 g thus corresponds to a safety factor of $\frac{5.4\text{g}}{1.43\text{g}} = 3.8$ of the design maximum acceleration of 1.43 g.

It can therefore be concluded from the above that even for the worst envisaged loading condition of 30 degrees inclined, an adequate safety factor is present over the design maximum acceleration of 1.43 g. However, the important point to be noted is that even if incipient buckling were to occur, say due to freak loading conditions,

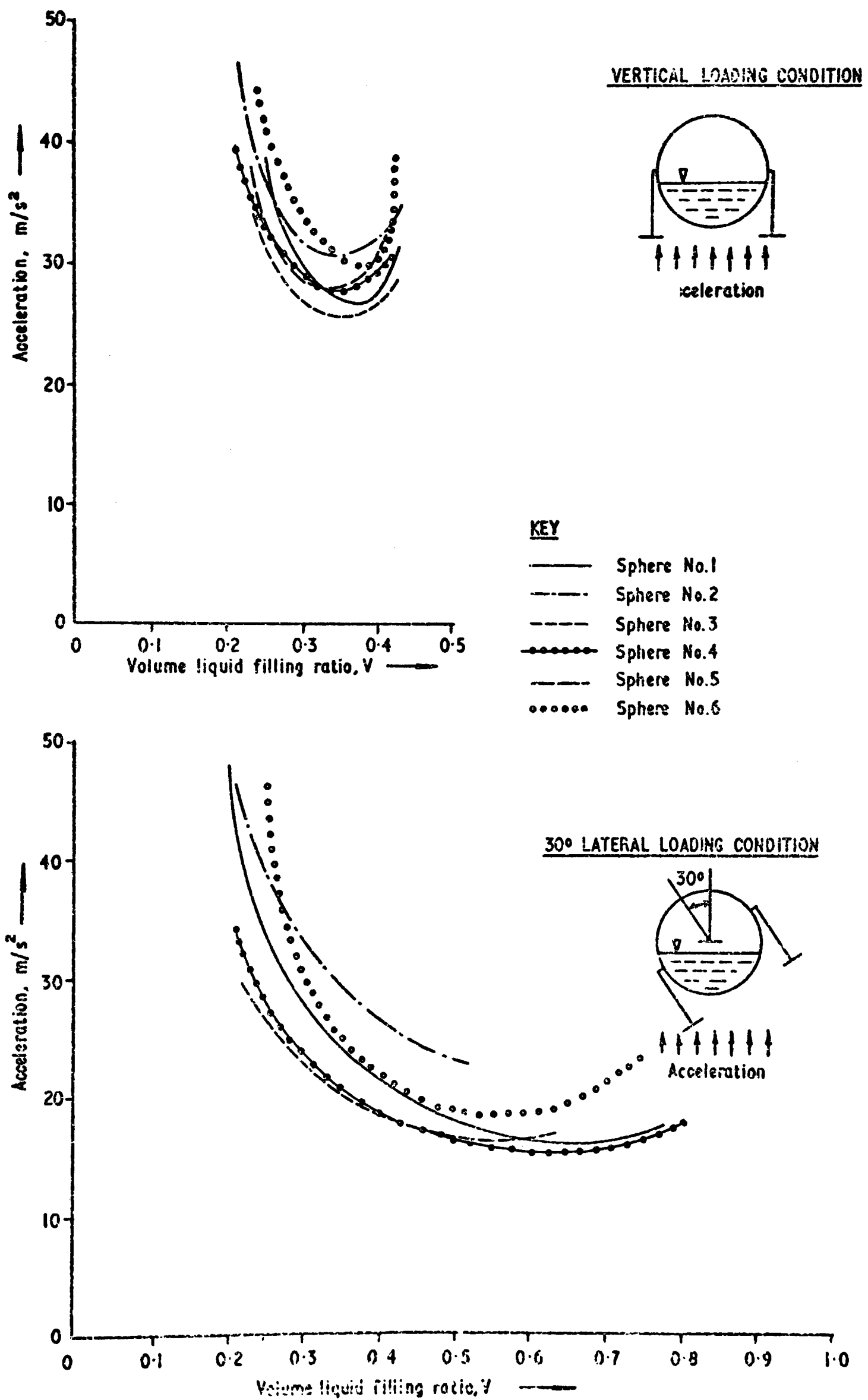


Fig. 9.1. Summary of experimental results for the six spheres tested under vertical and lateral loading conditions.

or to the use of small ships which would be more inclined to suffer high acceleration loadings, collapse of the shells would not occur.

The adequacy of the design codes will now be discussed. It was shown in section 3.2 that the ASME^[8] and BS 5500^[13] design codes introduce safety factors of respectively 18 and 8 of the classical value determined by Zoelly (1915) for the loading case of uniform external pressure. (It should be emphasised that these safety factors have been interpreted from the graphical information supplied by the codes). For this type of loading, imperfections of shape were seen from the first part of the investigation to produce markedly deleterious effects on the shells' stability to avoid buckling and the high safety factors introduced by the design codes were seen to be justified. For the acceleration loading case however, if these safety factors are applied it is seen that the allowable acceleration will be limited to very much below the design condition of 1.43 g. However it was seen in Chapter V that typical imperfection sizes reduced the buckling pressure of the shells to the level of and even below that allowed by the BS 5500^[13] code, equivalent to an actual safety factor of about 2 for the ASME^[8] code. Given these revised safety factors therefore, for the pitching and tossing motion experienced on a ship, this limits the allowable acceleration for the prototype sphere for the most severe loading case of 30 degrees inclined, to 1.75 g (i.e. $\frac{3.5g}{2}$) for the ASME^[8] code and 3.5 g (i.e. $\frac{3.5g}{1}$) for the BS 5500^[13] code, and the design condition of 1.43 g is seen in both cases to be satisfied.

In conclusion therefore, the design codes may be used for this imperfection insensitive acceleration loading condition, only if the safety factors interpreted from the codes are amended to take account of the reduction in buckling pressure caused by imperfections and are replaced by factors of say 2 and 1 for respectively the ASME^[8] and BS 5500^[13] codes.

The BOSOR 4^[20] computer programme was used to analyse the axisymmetric loading case. To this end the shell was modelled as the bottom hemisphere, clamped at the supporting ring round the equator and consisting of two segments, one above the water line, the other below. Since the mean critical filling ratio was in the vicinity of the thinner section of the shell just under the equator, it was thought expedient to use a thickness of 0.2 mm in the programme and not the mean thickness found, of 0.22 mm. Each segment was subdivided into 40 nodes although to check that the programme was not mesh sensitive, test cases were run with up to 200 segments in all. It was indeed found that the number of segments did not affect the results. The theoretical curve obtained is plotted in Fig. 9.2., and compares well with the experimental results in Fig. 9.1. - critical acceleration of 28.8 m/s^2 , 2% higher, at $V = 0.41$, 14% higher - as do the circumferential and meridional stress curves with those given in Fig. 3.4, obtained from

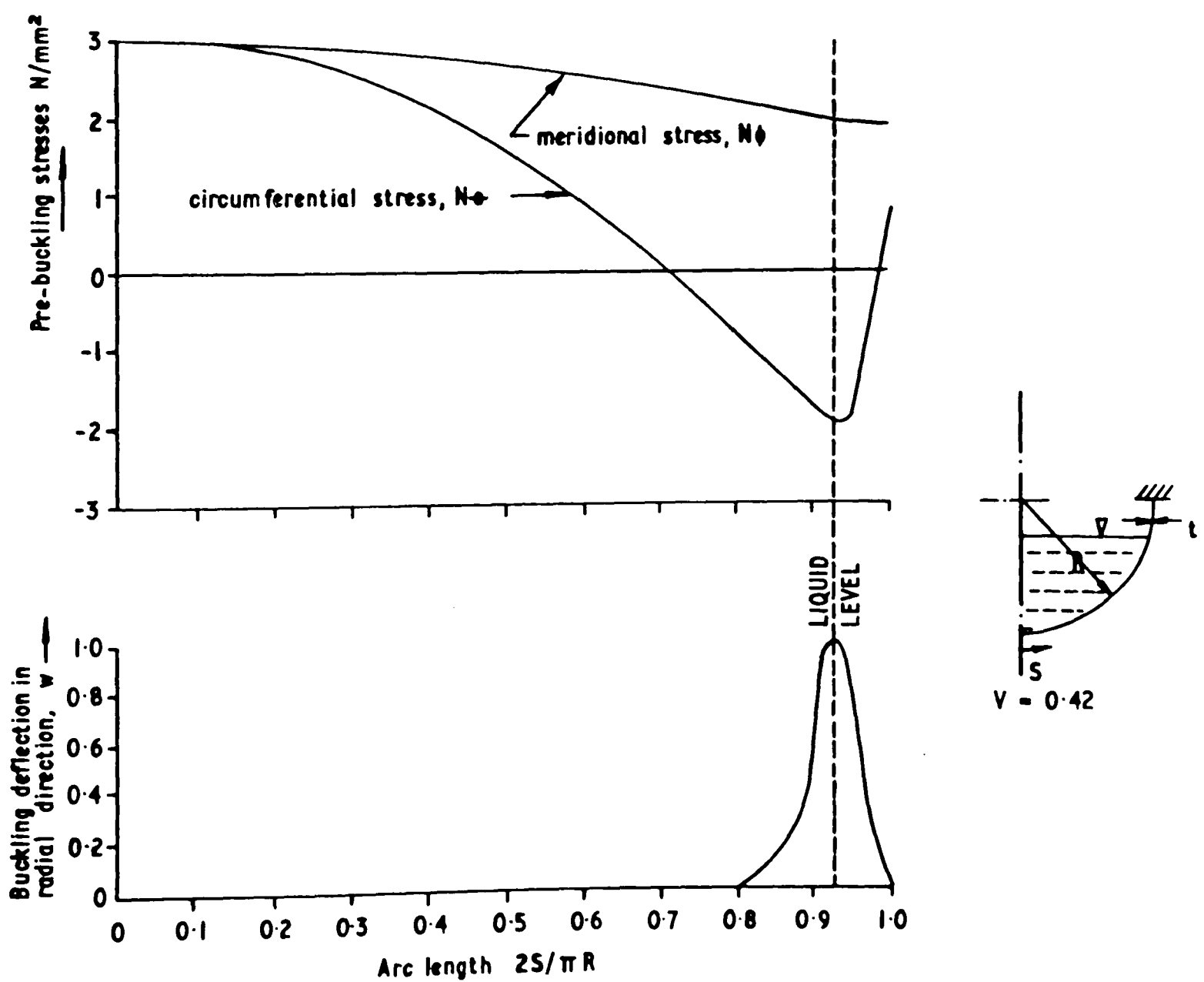
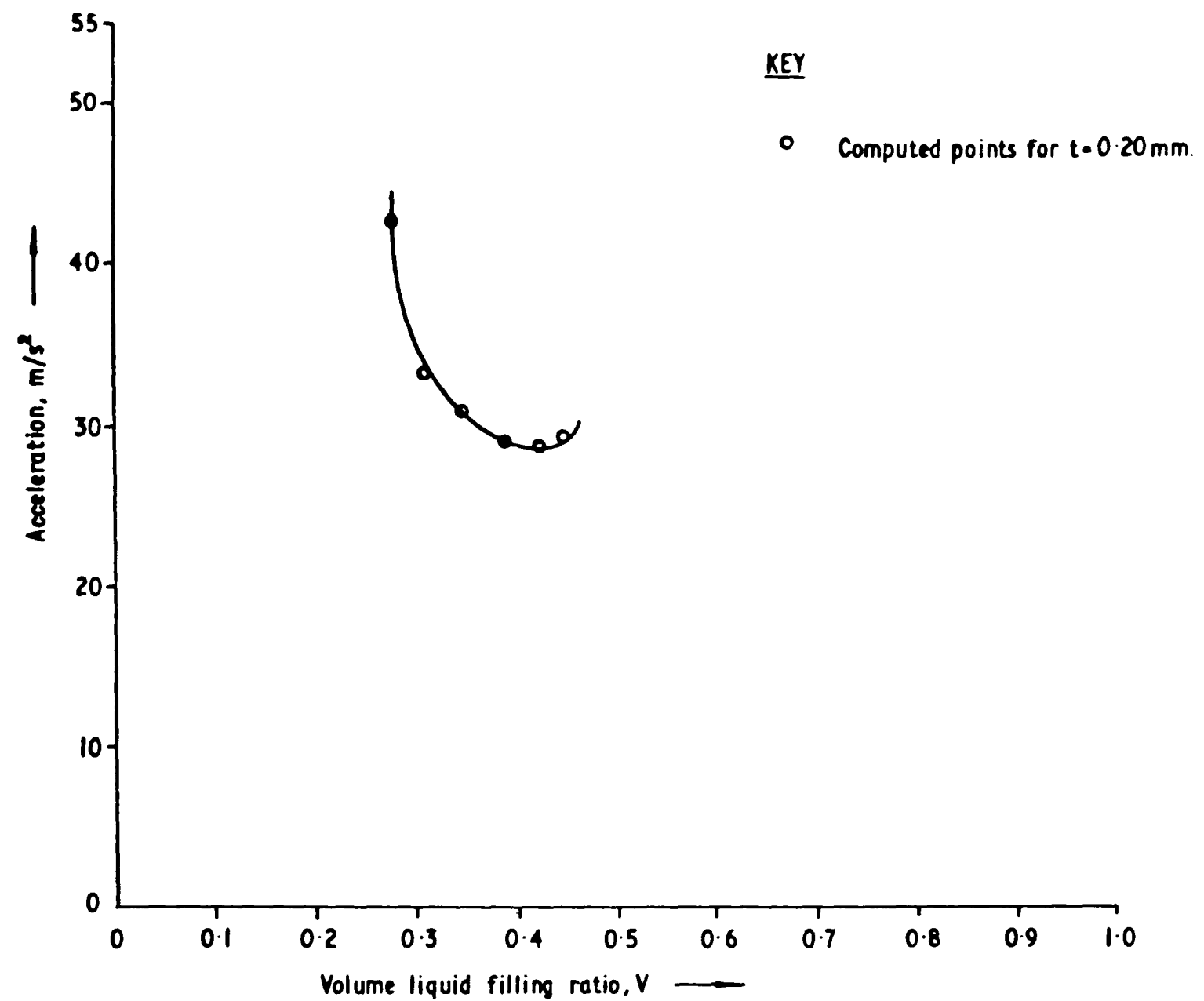


Fig.9.2. Computed buckling results given by BOSOR 4 for the axisymmetric vertical loading case on the same epoxy spheres as were used in the experimental programme.

the membrane analysis.

The numerical analysis could not be applied to problems without rotational symmetry. For the 30 degrees loading case, the method chosen was the one followed by Pedersen and Jensen (1976), based on the solution of the shallow shell equations, with the membrane solution as the reference static equilibrium condition. The displacements are chosen in the usual double trigonometric form, and an undefined wavelength in the circumferential direction which is modified until a minimum is found for the associated buckling pressure. The critical filling ratios were found to be 0.36 for the vertical loading case and 0.71 for the 30 degrees case. These compare well with the values given in Table 8.2 of 0.35 and 0.63 respectively. The ratio between the critical accelerations was 0.55 as against 0.59 for the experimental results.

9.3 IMPERFECTION SENSITIVITY

An expression for the imperfection sensitivity parameter (b) given in Koiter's (1945) equ. 2.7, has been found by Fitch (1968) for shallow spherical caps under point loading. This imperfection sensitive expression was the one adopted by Pedersen and Jensen (1976) even though Bushnell (1974) has shown the hemisphere/sphere to be imperfection insensitive to this loading condition. They attempted to justify their decision by conducting a number of experiments using vacuum formed plastic shells under a combination of external pressure and ring loading; see Fig. 9.3 . In their experiments, the variation in shell thickness was as much as $\pm 50\%$, with a corresponding

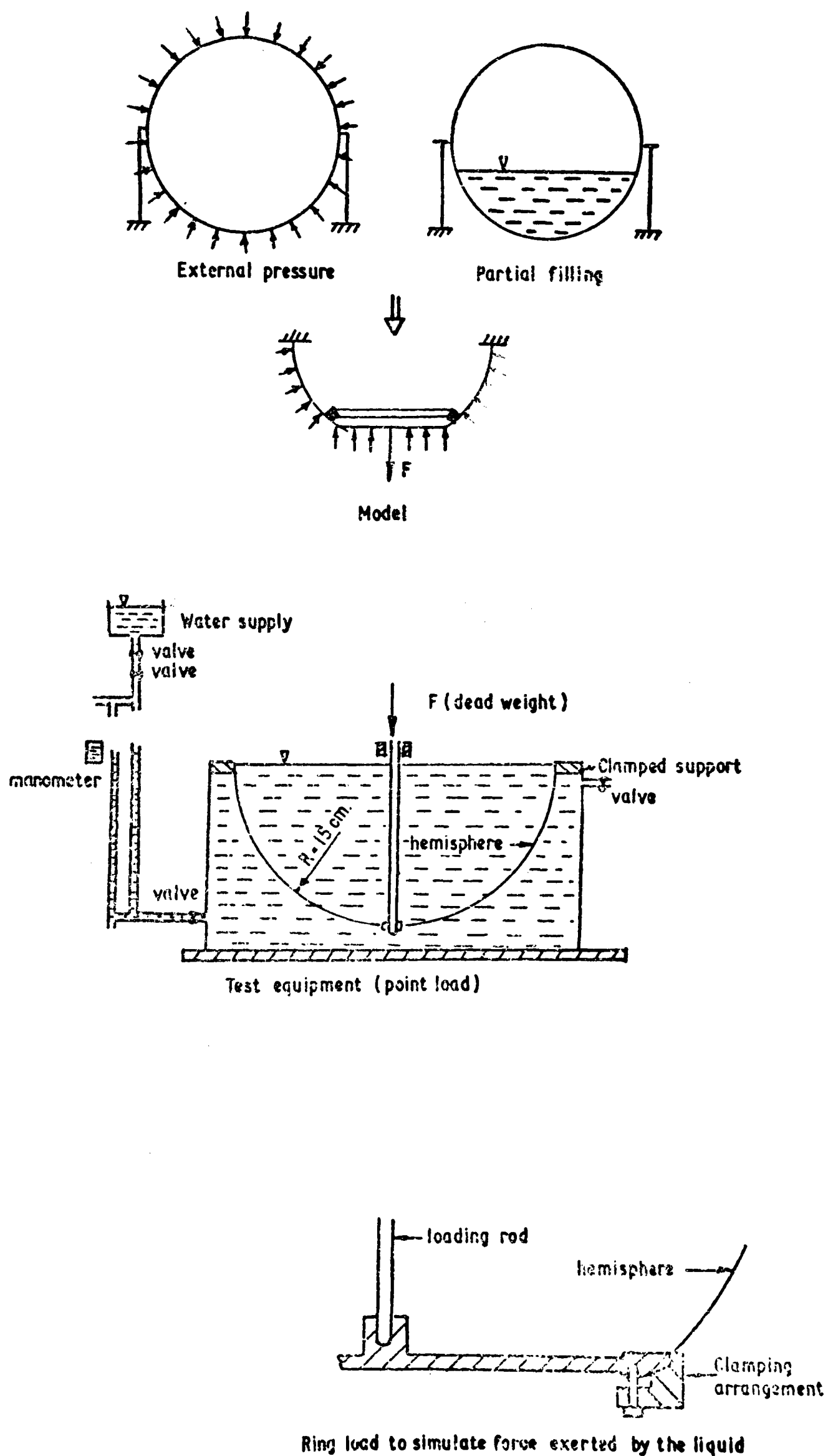


Fig. 9.3 Pedersen and Jensen's (1976) experimental programme conducted to simulate the two loading conditions of partial filling and external pressure.

anisotropy introduced by the forming process. At the same time, a ring loading was introduced by clamping the shell to a rigid plate.

It is known that the end conditions in cylindrical shells are an important factor in deciding the buckling load [67] and it may be surmised that the same will hold in the case of spheres. Neither do these tests simulate the operating conditions, for two important reasons:

- a) An artificial constraint of no movement at the junction between the ring and the hemisphere is introduced.
- b) An artificial bending moment is introduced at this same junction, a condition which is extremely unfavourable for thin-walled structures.

For these reasons, it is not considered that the evidence presented by Pedersen and Jensen (1976b) is conclusive. In the experiments reported here, the stress system was such that stresses in the meridional direction were tensile while those in the hoop direction were compressive, a situation that may be compared to that arising in the toroidal transition between a cylindrical shell and a spherical cap, in which buckling can develop under internal pressure. In that case, as in the one described here, it has been shown [37] that the agreement between the BOSOR 4^[20] predictions and the experimental results is good, thus confirming that imperfections in the model do not affect the buckling load. When testing plastic models, all deformations remain elastic, up to the point of failure. Models made of a more ductile material behave differently, in that large deformations lead to plasticity and thus remain in the model after unloading.

9.4 CONCLUSIONS

The present investigation has shown the possibility of making accurate models of thin-walled shells and has provided results that have validated a theoretical analysis, justifying the simplifying assumptions necessary for the 30 degrees inclined loading case.

It is also concluded that the loading considered does not lead to a reduction of buckling load through imperfection sensitivity. Imperfections of shape would not cause an instability. The distinction between an unstable situation and a stable bifurcation is particularly relevant in this case.

Both the ASME^[8] and BS5500^[13] design codes were found, for this mode of failure, to be too conservative for design purposes.

CHAPTER X

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

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CHAPTER X

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

10.1 INTRODUCTION

The research in this thesis was concerned with the use of spheres as carriers of liquefied natural gas (LNG) on ships. The operating conditions are such that loads may be introduced in these spherical tanks due to uniform external pressure and to an acceleration both when the tank is in the vertical position and when it is tilted as a result of the motion of the ship.

Manufacture, by welding doubly curved plates, either of 9% Nickel steel or Aluminium alloy, does however result in the inclusion of geometrical imperfections. In order to ascertain their effect on the stability of a sphere and also to determine the adequacy of guidance given in the use of spheres on ships, a review was initiated.

This review showed that while there had been much experimental activity on shallow spherical shells and to a lesser extent on deep ones for the loading condition of uniform external pressure, little agreement was found between observed and theoretical buckling pressures, with observed values often $\frac{1}{4}$ of those predicted being common. The little experimental work that had been done to determine the effect of specific imperfection shapes and sizes was limited to imperfection amplitudes of not much more than the shell thickness, much less than the as fabricated ones. In addition, all previous experimental

work apart from one, had been performed on spherical shells and not complete spheres, thus resulting in further imperfections being introduced due to the restraints imposed at the boundaries. The only analytical expression offered to depict the imperfection sensitive behaviour of spherical shells, was that given by Koiter (1945), although its adequacy here, considering its limitations in the size and shape of imperfection, was not known. The paucity of information available to the designer concerning the stability of partially filled spheres under the operating conditions experienced on a ship also became apparent.

Little guidance was found to be given by the design codes, with safety factors as diverse as 18 (ASME^[8]) and 8 (BS 5500^[13]) being applied to the theoretical predicted values for perfect spheres in order to account for included imperfections, and they were also found to contain little that provided specific guidance on the "fitness-for-purpose" of imperfect shells. Furthermore, the possibility of buckling due to the motion of the ship was not discussed, only the maximum envisaged vertical acceleration being specified.

This lack of knowledge prompted an approximate analysis based on membrane shell theory in order to determine if buckling was possible at this maximum envisaged acceleration. It showed, for the tank full, supported at its equator, both the meridional and circumferential stresses to be tensile, failure then only occurring as a result of yielding of the material. For partially full tanks however, based on the compressive stress at buckling under uniform external pressure and on assumptions

regarding the interaction of tensile meridional and compressive circumferential stresses, it was found buckling was possible for both the normal and inclined loading conditions. In addition, this analysis helped in defining more fully the investigation to be conducted, which was divided into two parts:

a) The effect of specific imperfection shapes and size typically found in the construction of a thin-walled sphere, on its stability under uniform external pressure.

b) The effect on the stability of a thin-walled sphere of imposing vertical accelerations both normal and at the worst envisaged tilted condition of 30 degrees to its equatorial ring support.

10.2 STABILITY OF IMPERFECT THIN-WALLED SPHERES UNDER UNIFORM EXTERNAL PRESSURE

The first conclusion to be made was that it was possible to manufacture seamless thin-walled spheres with radius to thickness ratios between 450 to 1700 by an electroplating method such that the intended imperfection was the sole contributor to the lowering of the buckling pressure and not random imperfections that had previously been the cause of the wide scattering of buckling results in many previous experimental investigations. Thickness and sphericity (with respect to radius) variations were kept to within respectively 6 and 0.2% for the 25 copper spheres used.

Buckling appeared in 2, 3, 4 and 5 lobed form, although their buckled shape did not appear to be influenced by either the type of imperfection or the

buckling pressure, values of which varied between $\frac{1}{5}$ and $\frac{1}{15}$ of the theoretical buckling values for a perfect sphere.

It was clear that the theoretical expression for a perfect shell was not applicable and neither was the BS 5500^[13] code for some imperfection shapes and sizes. The ASME^[8] code however, was found to be adequate for all imperfection shapes and sizes considered, although unduly conservative for amplitudes < 15 times the shell thickness. The ability of the spheres to remain stable under uniform external pressure was found to be influenced by both the shape and size of imperfection. For the flat and increased radius shapes, the relationship between buckling pressure and imperfection size was found to be in good agreement with Koiter's (1945) imperfection sensitivity expression, in contrast to no agreement found between this equation and the buckling results for the bulge imperfection, or indeed with reported work on the dimple shape.

Since Koiter's (1945) equation was used well outside its range of validity, it was decided to determine the shells imperfection sensitivity using one of the commercially available computer programmes, most of which perform stress, vibration and buckling analyses of thin-walled structures. BOSOR 4^[20] was chosen since it was widely used and representative of the programmes currently available. It became apparent, however, after considerable trial and error, that this programme was not only grid sensitive but also sensitive to the value chosen for the scaling factor which operates on the reference load. Meaningless spikes appeared at the junctions between the

segments on the meridional and circumferential stress curves and particularly so at the apexes. Even after the normal practice of introducing small diameter stress relieving holes at these apexes, the predicted results were found to be in total disagreement with not only the experimental results presented here, but also with all known experimental results and analytical imperfection sensitivity theories. It is thus concluded that BOSOR 4 [20] is of little value in the assessment of the effect of geometrical imperfections on spheres under uniform external pressure.

10.3 STABILITY OF PARTIALLY FILLED THIN-WALLED SPHERES UNDER VERTICAL ACCELERATIONS NORMAL AND INCLINED TO THEIR EQUATORIAL RINGS

The second part of the investigation showed the casting method employed, which made use of aluminium moulds machined to high tolerances on a numerical controlled machine, resulted in the manufacture of accurate large diameter plastic hemispheres, of radius to thickness ratio of 900 which were glued together to form spheres. Thickness and sphericity (with respect to radius) variations were kept to within respectively 9 and 0.05% for the 6 spheres used.

The introduction of vertical accelerations to the partially filled spheres by means of a pneumatic piston, both normal and at 30 degrees inclined to their equatorial rings, resulted in deflections being introduced, the maximum of which invariably occurred at the water line. By measuring these deflections for each imposition of acceleration, it was possible to detect the position at

which a sudden increase in the rate of deformation occurred. Since the strain imposed was well within the elastic region of the epoxy and since no permanent deformation remained upon unloading, this was defined as the point at which buckling had occurred. All the experimental results consistently showed a partially filled sphere to be more susceptible to buckling at particular filling ratios, this occurring below the equatorial ring support at approximately $\frac{1}{3}$ full for loading normal to the equator and at $\frac{2}{3}$ full for the 30 degrees inclined loading case. As a consequence it can be concluded, that increasing the shell thickness in those areas which correspond to the critical filling ratios and thinning in those areas away from these regions would certainly produce valuable savings in the expensive material which needs to be specified in their construction.

Both the ASME and BS5500 design codes were found for this mode of failure to be too conservative for design purposes. A reduction in the safety factor was proposed. Comparison of the experimental results with the only available published data for this problem, that given by Pedersen and Jensen (1976), whose analysis was based on shallow shell equations, showed there to be good agreement in the critical buckling acceleration values for the normal and inclined loading cases.

However the predicted result, given by these authors, that a partially filled sphere would be imperfection sensitive under these loading conditions was found to be

incorrect. That the shells were in fact found experimentally to be imperfection insensitive and furthermore that good agreement was found between the computer programme BOSOR 4^[20] and the experimental results confirms that imperfections in spheres under these loading conditions do not affect the buckling load and that the shell does not experience catastrophic failure at buckling. Hence it can be concluded that if unnecessarily high safety factors are to be avoided, it is necessary to not only specify the shell geometry but also the type of loading. In the case considered here, a combination of tensile meridional and compressive circumferential stresses has been found to result in a sphere behaving imperfection insensitive, in contrast to the imperfection sensitive behaviour of a sphere under uniform external pressure, where a combination of both compressive meridional and circumferential stresses occur.

10.4 SUGGESTIONS FOR FURTHER WORK

Whilst the foregoing has made an important contribution to the design of spherical LNG carriers, some problems still remain, including those which have appeared as a result of this research.

Of these, the most disturbing one, particularly so since the current trend is to present the designer with computer packages often reputed to deal with all envisaged shell problems, is the inadequacy of the programme BOSOR 4^[20] in coping with the inclusion of imperfections. It would appear that it is most unusual to obtain disagreement since it has been reported to be valid for many other shell

problems, including the one presented here for partially filled spheres. It is therefore recommended that the programme at present, only be used in conjunction with a series of pilot tests in order to validate it for families of problems. These tests could form the basis of further research. Its inability to deal with asymmetric problems could also be tackled, however, it is suggested that the designer may use the STAGS^[2] finite difference programme for these problems although he should be prepared to accept the increased effort needed in data preparation and in running time.

The research conducted has also highlighted the inadequacy of characterization of an imperfection shape with a single parameter, i.e. amplitude of imperfection. Expressing the imperfection shape in a Fourier series could be the answer, although choosing the harmonic term of the series which corresponds to that of buckling of a perfect shell (Koiter (1945)) is clearly inadequate since it is quite possible for an imperfection shape to be such that the amplitude of this harmonic term is zero.

Certainly progress on the theoretical side would be preferable at this stage to continuation of more experimental studies which can be both expensive and time consuming. The determination of the lower bound of the buckling pressure for different radii of the bulge and dimple imperfections could then be included as could a study of other typical imperfection shapes not treated here such as peaks and lack of plate alignment. That of residual stresses would lend itself more to a computer analysis since the imperfection could be formulated in

terms of energy. Here is an example where an experimental analysis is impossible.

The problems remaining in the particular operation of spheres on ships can be confined to the effect of interaction from the hull structure via the cylindrical support to the sphere's equatorial ring and to an analytical confirmation of the imperfection insensitive behaviour of a partially filled sphere under the aforementioned loading conditions. For the former case the analysis should not only include axial deformation but also ovalisation of the top of the cylindrical support.

APPENDIX A

ELECTROPLATED COPPER SPHERES

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APPENDIX A

ELECTROPLATED COPPER SPHERES

A.1. SPHERICAL MOULD

The object was to produce two dimensionally accurate hemispheres such that they could be used to cast wax into a spherical shape.

Two important requirements had to be satisfied by the design. These were that the hemispheres should have high dimensional accuracy and be located properly to each other when seated together. The former requirement was achieved by using a numerical controlled machine. Location of the two moulds was obtained by using one mould as the male and the other as the female. The design of the two moulds is shown in Fig. A.1.

A.2 ROTATING MECHANISM

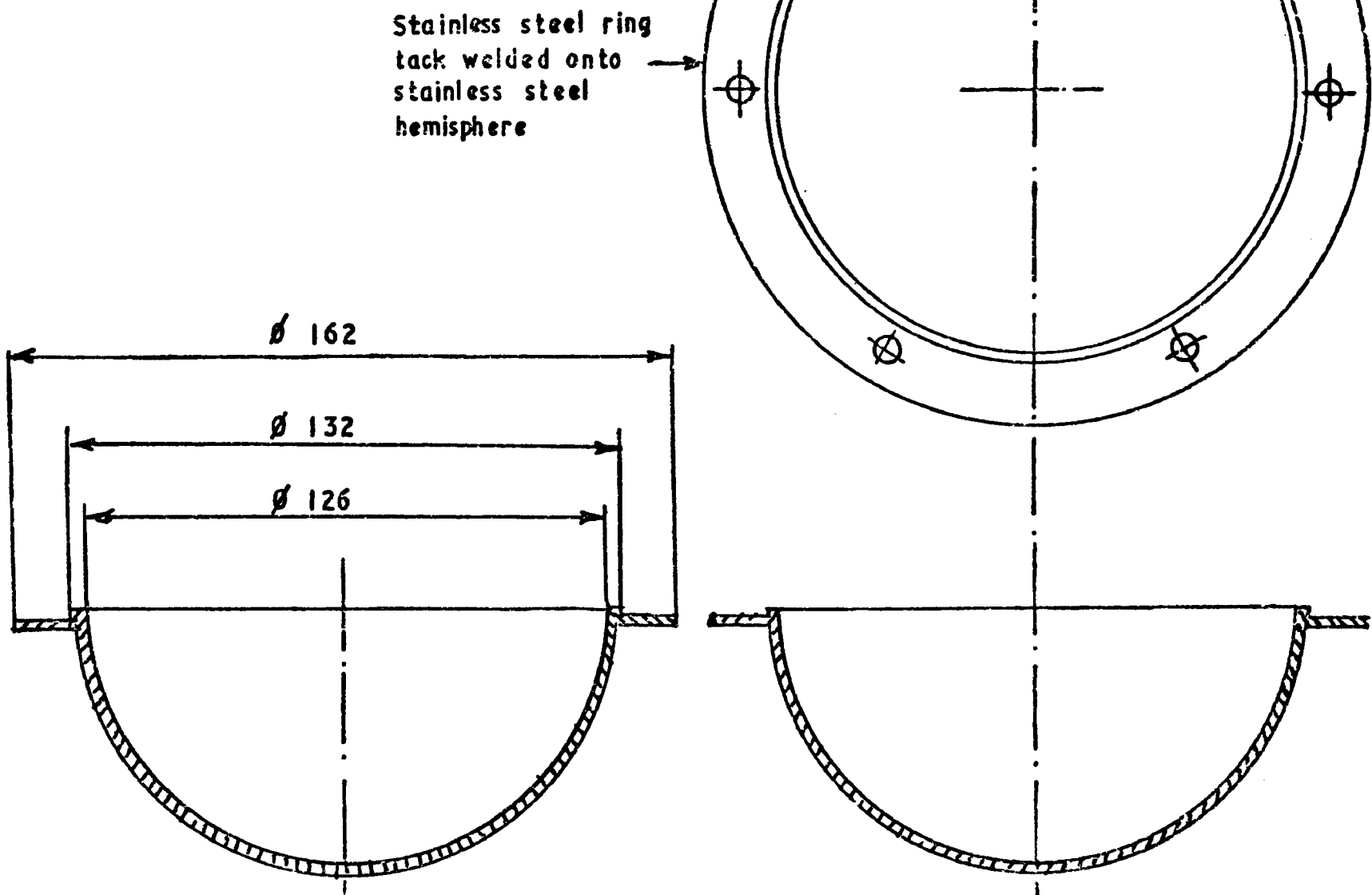
A.2.1 Introduction

The design had to satisfy the following requirements:

- a) To be able to rotate the spherical mould about two mutually orthogonal axes.
- b) To be used inside an oven up to temperatures of 100°C .
- c) To be driven from inside the oven.
- d) To be heavy, in order that it would remain stable during rotation.

A schematic diagram of the apparatus used is shown in Fig. A.2; a photograph is given in Fig. 4.3.

MALE MOULD



FEMALE MOULD

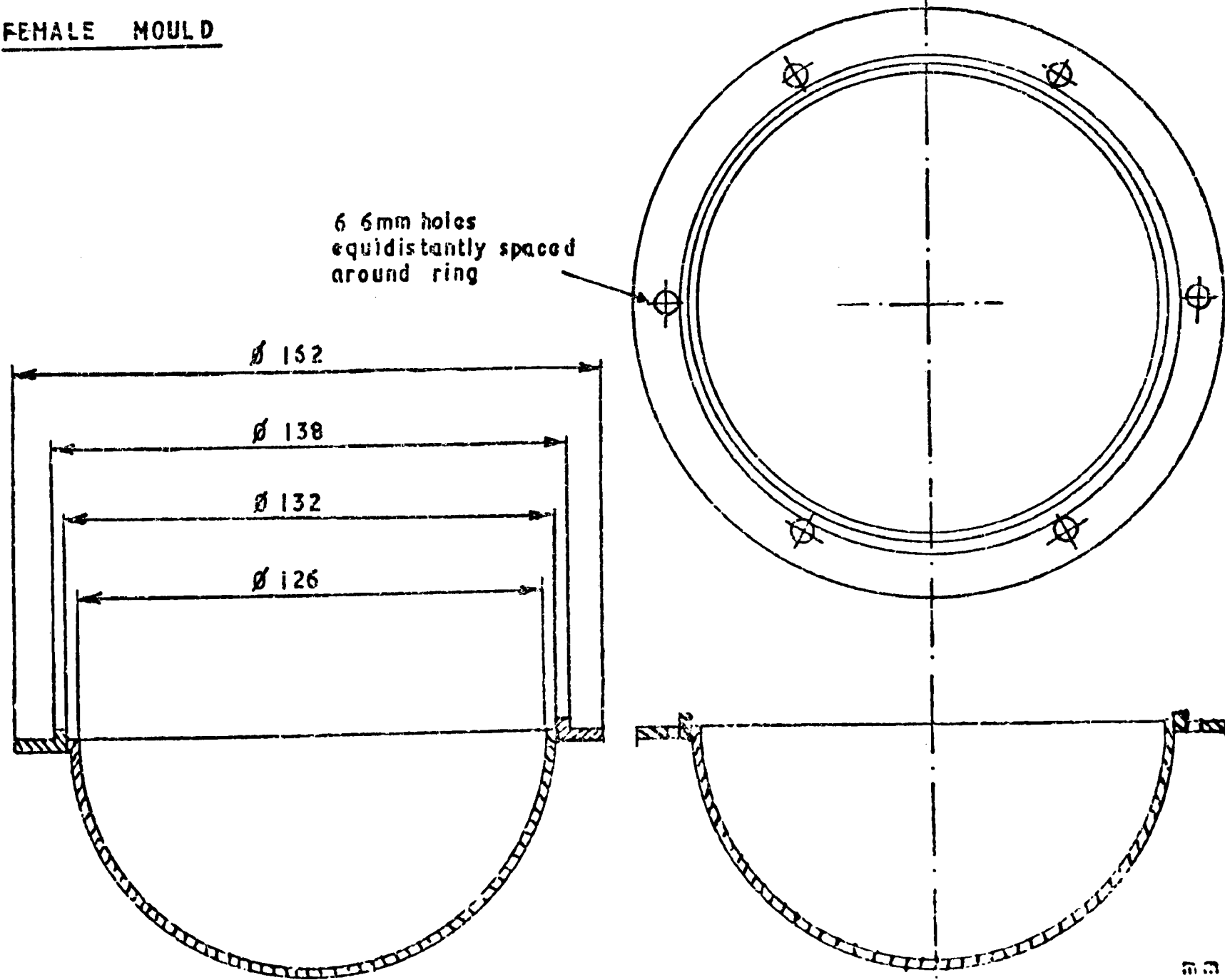


Fig.A.1. Male and Female hemispherical moulds.

mm.
Scale 1:2

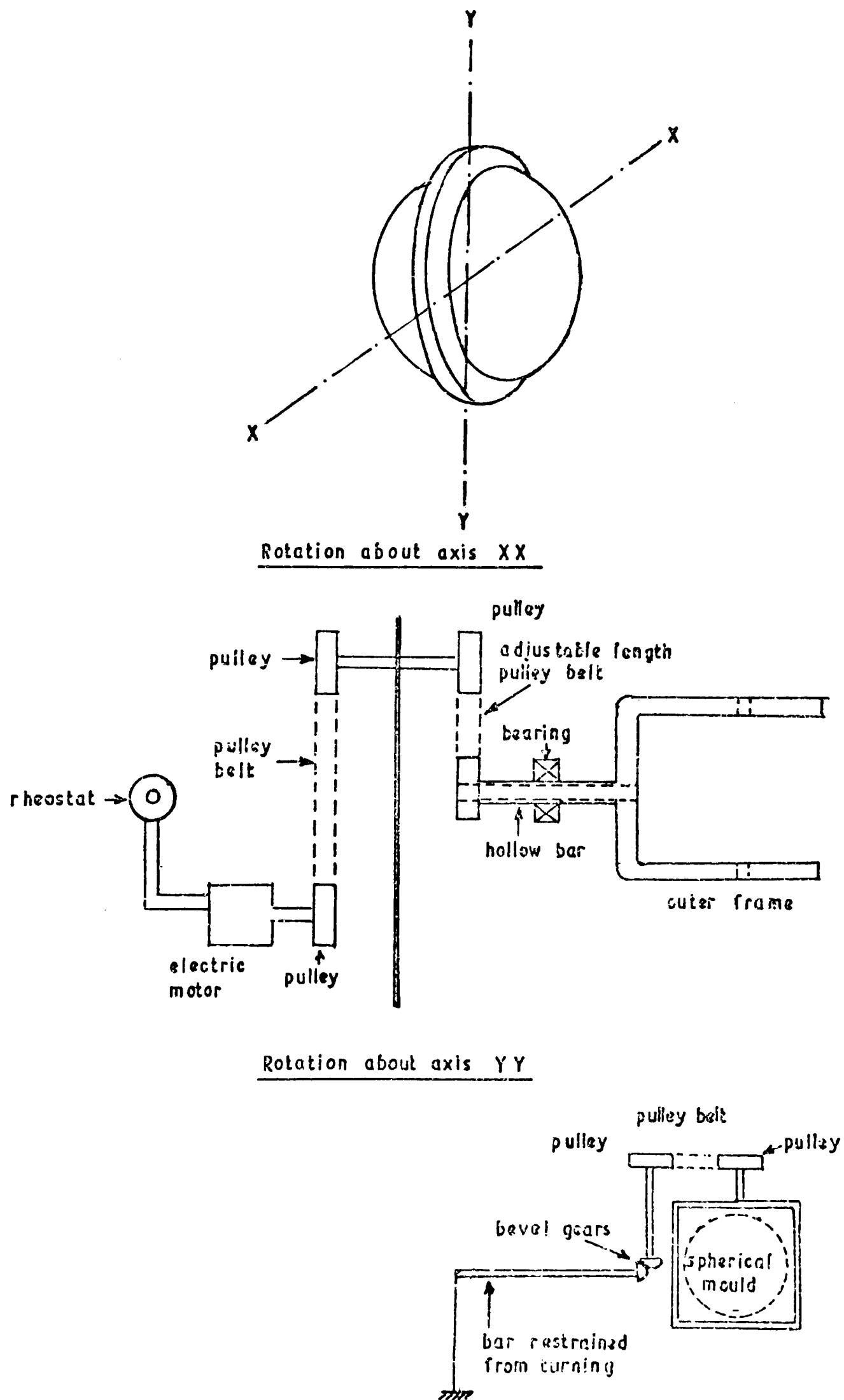


Fig. A.2. Schematic diagram of rotating mechanism (photograph of apparatus given in fig.4.3.).

A.2.2 Description

Rotation about axis XX was obtained by rotating the outer frame; this in turn was attached to a hollow bar running in a bearing. A pulley wheel was secured to the end of this bar and this was driven via an adjustable length belt by another pulley wheel situated inside the oven. This wheel was driven by an electric motor, whose speed was regulated by a rheostat.

Rotation about axis YY was obtained by rotating a square cage, to which was bolted the spherical mould. Two bevel gears were used to rotate this cage. One was held stationary while the other was allowed to freely rotate around it. This rotation was then transmitted via pulley wheels to the cage.

A.3 ELECTROPLATING APPARATUS

A.3.1 Introduction

The problem of electroplating spheres has received attention from Carlson et al. (1967) and Thompson (1960).

The former investigators rotated their spheres about a fixed axis relative to a nickel anode. A lot of difficulties were reported and even after modifications to the design, thin-walled spheres with thickness variations of up to 30% were produced.

The earlier design presented by Thompson (1960) used the principle that a wax sphere being less dense than the electrolytic solution, can be rotated relative to a fixed anode without the need to damage the surface of the wax, as was necessary in the former case. This elegant method was chosen as the basis for the electroplating method used

in this thesis, although important modifications were included, in order to refine the basic design.

A schematic diagram of the apparatus used is shown in Fig. A.3; photographs of the plan and side view are given in Fig. 4.4.

A.3.2. Description

The sphere was located by 3 rubber tyred wheels, A, B, C. The rear two, B and C, were used to provide rotation of the sphere at approximately 10 rev/min; the free wheeling front one, A, was used to steer the sphere. This steering was appropriately geared to turn the sphere slowly such that the rotational axis of the sphere was being continually changed. This action ensured that during the 4 hours average plating time, each part of the sphere passed approximately an equal number of times through point D.

The negative electrical contact for the conducting paint which covered the sphere was a 0.08 mm thick brass shim. This provided a light spring pressure, and yet did not scrape away the rather soft conducting paint. The anode was a 22 guage copper sheet curved such that its surface was approximately equidistant from the sphere, which hence ensured that no area of the sphere would receive a thicker plating of copper.

In order to ensure no dirt contaminated the electrolytic solution, the rotating mechanism was attached to the perspex top of the rectangular glass tank.

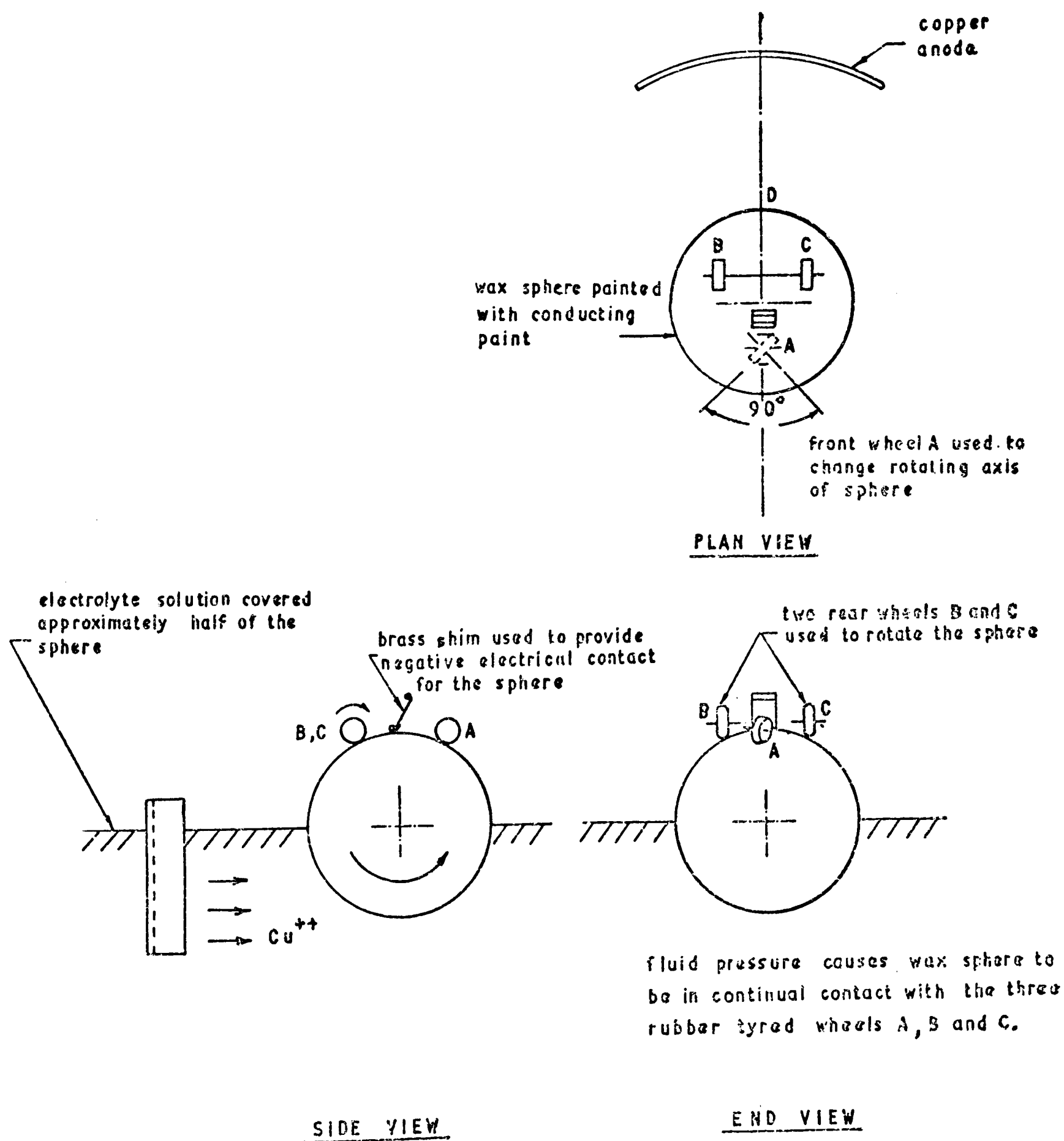


Fig. A.3. Schematic diagram of electroplating apparatus
(photographs of apparatus given in Fig 4.4.).

APPENDIX B

CAST EPOXY SPHERES

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APPENDIX B

CAST EPOXY SPHERES

B.1 MOULDS

B.1.1 Introduction

Three stages were necessary in the manufacture of the accurately dimensioned hemispherical male and female moulds:

- a) The preparation of polystyrene male and female moulds.
- b) The aluminium castings from these moulds.
- c) The accurate machining of these castings.

Photographs of the sequence are shown in Chapter VII, Figs. 7.2, 3, 4 and 5.

B.1.2 Polystyrene moulds

Various diameter polystyrene rings were marked out on a one inch thick polystyrene sheet. These rings were then cut out and tack glued to each other to form the rough shape of the male and female moulds. The female mould was made in two parts since it was envisaged that this mould would need to be parted in order to facilitate removal of the cast epoxy hemispheres.

In order to minimise machining time and also to reduce the cost of the aluminium castings, the polystyrene moulds were rubbed down both inside and outside with coarse sandpaper.

The shape of the four legs for the female mould were

marked out and cut from the polystyrene sheet. These legs were later tack glued onto the female mould.

B.1.3 Aluminium castings

The polystyrene models were used as sacrificial moulds for the aluminium castings. The small residue of burnt polystyrene was skimmed off the molten aluminium at the end of the casting procedure.

The aluminium castings had a rough texture since these castings adopted the same appearance as the polystyrene moulds.

B.1.4 Machining of castings

The design of the male and female moulds were to satisfy the following requirements:

- a) To be accurately machined such that a constant thickness hemispherical epoxy could be cast.
- b) To incorporate space for an overflow, and therefore inhibit any excess epoxy from interfering with the seating of the two moulds.
- c) To have accurate location of the two moulds with each other.
- d) To incorporate into the design, an equatorial ring.
- e) To make provision for parts of the moulds to be separated and so allow removal of the casting but still retain high dimensional accuracy when the moulds were re-assembled.

The design of the male and female moulds is shown in Fig. B.1.

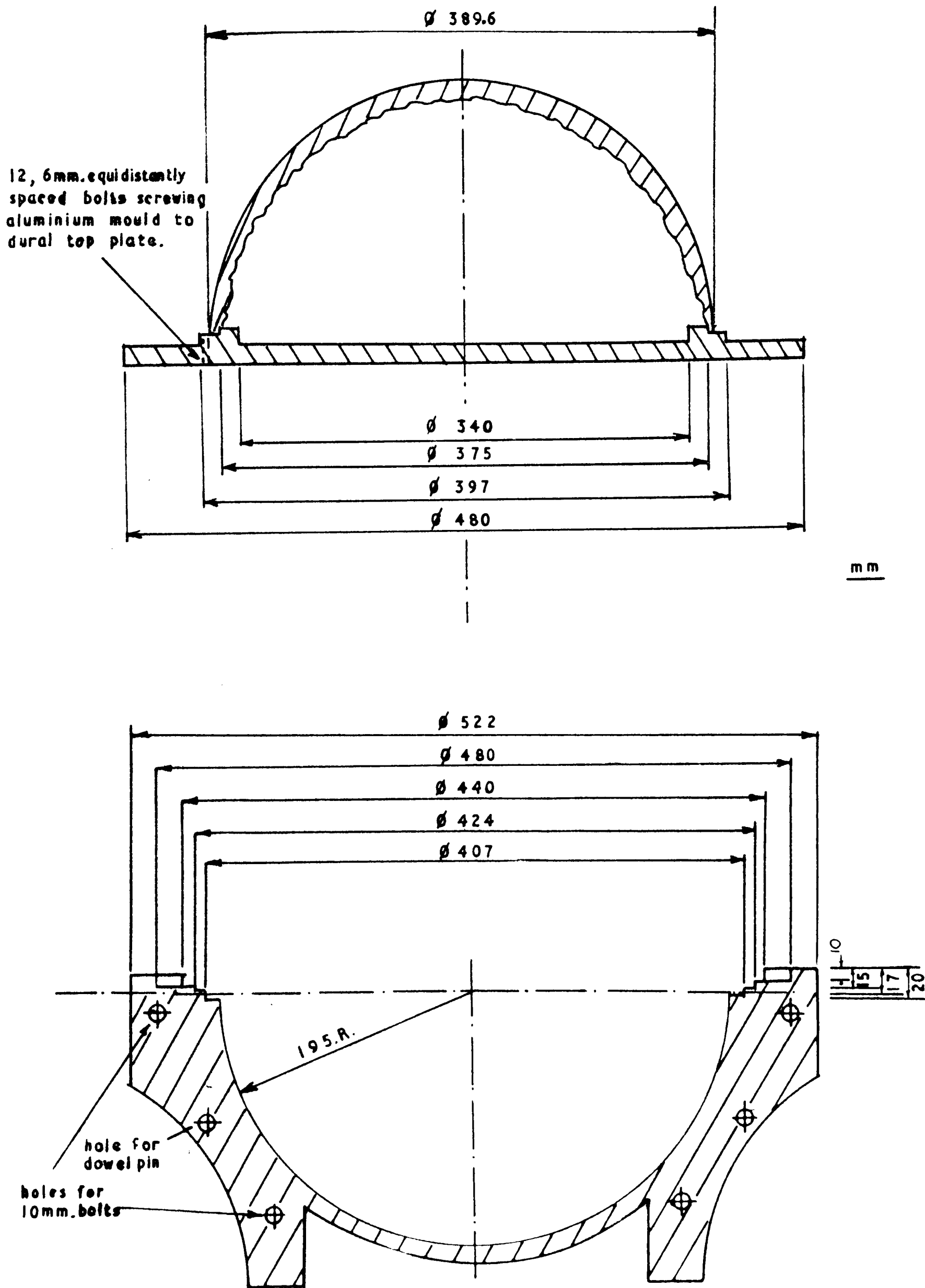


Fig.B.1. Machining of aluminium moulds to dimensions

All machining was performed on a numerical controlled machine. Both halves of the female mould were located by dowel pins and held together by 4 bolts located in two of its four legs. The top dural plate for the male mould was located on the inside of the mould and was secured by 12, 6 mm diameter bolts. Provision was also made in the design for 6, 8 mm diameter bolts to be used to separate the top plate from the male mould.

After machining of the moulds to design dimensions, it was noticed that the aluminium had a certain porosity. Since these small holes would have caused thickness variations in the cast epoxy hemispheres as well as hindering removal of the epoxy from the moulds, it was essential that these holes were filled. This requirement was met by covering the moulds with filler paste and then carefully using fine wet and dry paper (600 grade) in order to remove the excess.

APPENDIX C

KOITER'S IMPERFECTION THEORY

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APPENDIX C

KOITER'S IMPERFECTION THEORY

C.1 PERFECT STRUCTURE: PRE-BUCKLING, BUCKLING AND POST-BUCKLING BEHAVIOUR

The summarised version given by Budiansky (1965) is given.

Suppose that under a loading q the elastic structure acquires displacements u , strains ϵ , and stresses σ . These variables will be required to satisfy the strain-displacement relation,

$$\epsilon = L_1(u) + \frac{1}{2} L_2(u) \quad (C.1)$$

where L_1 and L_2 are linear and quadratic functionals respectively; the stress-strain relation,

$$\sigma = H(\epsilon) \quad (C.2)$$

where H is a linear functional; and the variational equation of equilibrium,

$$\sigma \cdot \delta \epsilon = q \cdot \delta u \quad (C.3)$$

In equ. C.3 the 'dot' operation is a shorthand notation that in a term $a \cdot b$, means the virtual work or stresses (or loads) a , acting through strains (or displacements) b , integrated over the whole surface.

$\sigma \cdot \delta \epsilon$ denotes strain energy and $q \cdot \delta u$ denotes the work done.

If the bilinear functional, L_{11} , is defined by the relation,

$$L_2(u+v) = L_2(u) + 2L_{11}(u,v) + L_2(v)$$

then the strain variation compatible with δu may be written (from equ. C.1) as,

$$\delta \epsilon = L_1(\delta u) + L_{11}(u, \delta u) \quad (C.4)$$

It is assumed that $L_{11}(u,v) = L_{11}(v,u)$ and that,

$$L_{11}(u,u) = L_2(u). \quad (C.5)$$

The additional notation of $e = L_1(u)$ is used for convenience giving,

$$\delta \epsilon = \delta e + L_{11}(u, \delta u) \quad (C.6)$$

Finally the reciprocal relation

$$H(\epsilon_1) \cdot \epsilon_2 = H(\epsilon_2) \cdot \epsilon_1 \quad (C.7)$$

will be assumed valid for all ϵ_1 and ϵ_2 . Apply the external loads $q = Pq_0$ to the perfect structure; it will be supposed that before buckling occurs the response of the structure is simply

$$\begin{aligned} u &= Pu_0 \\ \epsilon &= P\epsilon_0 \\ \sigma &= P\sigma_0 \end{aligned} \quad (C.8)$$

where the 'trivial' displacement satisfied the conditions

$$L_{11}(u_0, v) = 0 \quad (C.9)$$

for all v . Thus a linear theory is assumed valid before buckling with $\epsilon_0 = e_0 = L_1(u_0)$, $\sigma_0 = H(e_0)$, and

$$\sigma_0 \delta e = q_0 \delta u \quad (C.10)$$

The occurrence of buckling can be detected by substituting

$$\begin{aligned} u &= P_c u_o + \xi u_1, \\ \epsilon &= P_c e_o + \xi \epsilon_1, \\ \sigma &= P_c \sigma_o + \xi \sigma_1, \end{aligned} \tag{C.11}$$

into equs. C.1, 2 and 3, linearising with respect to the scalar parameter ξ , and simplifying the results using equs. C.9 and 10. Then the eigenvalue problem for the classical buckling load P_c and the buckling mode $u_1, \epsilon_1, \sigma_1$ is found to be governed by the variational equilibrium equation,

$$P_c \sigma_o \cdot L_{11} (u_1, \delta u) + \sigma_1 \cdot \delta e = 0 \tag{C.12}$$

the strain-displacement relation $\epsilon_1 = e_1 = L_1 (u)$ and the stress-strain relation $\sigma_1 = H(e_1)$.

It will be assumed now that only one buckling mode is associated with the lowest eigenvalue P_c .

Next in order to discover how the structure behaves after buckling as P deviates from P_c , the magnitude of the eigenfunction u_1 is assumed to be normalised in some convenient fashion,

$$\begin{aligned} u &= P u_o + \xi u_1 + \xi^2 u_2 + \xi^3 u_3 + \dots \\ \epsilon &= P \epsilon_o + \xi \epsilon_1 + \xi^2 \epsilon_2 + \xi^3 \epsilon_3 + \dots \\ \sigma &= P \sigma_o + \xi \sigma_1 + \xi^2 \sigma_2 + \xi^3 \sigma_3 + \dots \end{aligned} \tag{C.13}$$

where $u_2, u_3 \dots$ are all orthogonalised to u_1 in the sense

$$\sigma_o \cdot L_{11} (u_1, u_n) = 0 \quad (n = 2, 3, \dots) \tag{C.14}$$

As a consequence of equ. C.12 this orthogonality condition implies that,

$$\sigma_1 \cdot e_n = 0 \quad (n \neq 1) \quad (C.15)$$

which by equ. C.7 implies that,

$$H(e_n) \cdot e_1 = 0 \quad (n \neq 1) \quad (C.16)$$

Also, from equ. C.12 with $\delta u = u_1$, $\delta e = e_1$,

$$\sigma_o \cdot L_2(u_1) = \frac{1}{P_c} \sigma_1 \cdot e_1 \quad (C.17)$$

Substituting equ. C.13 into equ. C.3 noting that,

$$\delta \epsilon = \delta e + \xi L_{11}(u_1, \delta u) + \xi^2 L_{11}(u_2, \delta u) + \dots$$

and using equs. C.10 and C.12 to simplify the result gives,

$$\begin{aligned} & \xi(1-P/P_c) \sigma_1 \cdot \delta e + \xi^2 [P \sigma_o \cdot L_{11}(u_2, \delta u) + \\ & + \sigma_1 \cdot L_{11}(u_1, \delta u) + \sigma_2 \cdot \delta e] + \xi^3 [P \sigma_o \cdot L_{11}(u_3, \delta u) + \\ & + \sigma_1 \cdot L_{11}(u_2, \delta u) + \sigma_2 \cdot L_{11}(u_1, \delta u) + \sigma_3 \cdot \delta e] + \\ & + \dots = 0 \end{aligned} \quad (C.18)$$

The choice $\delta u = u_1$, $\delta e = e_1$, $\delta \epsilon = \epsilon_1$ in equ. C.18 together with the use of the orthogonality relations equs. C.14 and 15 gives ,

$$\begin{aligned} & \xi(1-P/P_c) \sigma_1 \cdot \epsilon_1 + \xi^2 \left[\frac{3}{2} \sigma_1 \cdot L_2(u_1) \right] + \xi^3 [2 \sigma_1 \cdot L_{11} \\ & (u_1, u_2) + \sigma_2 \cdot L_2(u_1)] + \dots = 0 \end{aligned} \quad (C.19)$$

This result provides the desired information concerning the post-buckling variation of ξ with P ; thus after

buckling,

$$P/P_c = 1 + A\xi + b\xi^2 + \dots \quad (C.20)$$

where $A =$

$$\frac{\frac{3}{2}\sigma_1 \cdot L_2(u_1)}{\sigma_1 \cdot \epsilon_1} \quad (C.21)$$

$$\text{and } b = [2\sigma_1 \cdot L_{11}(u_1, u_2) + \sigma_2 \cdot L_2(u_1)] / \sigma_1 \cdot \epsilon_1 \quad (C.22)$$

The functions u_2 and σ_2 needed in the evaluation of b , together with the strain function ϵ_2 satisfy the strain-displacement and stress-strain relations,

$$\epsilon_2 = L_1(u_2) + \frac{1}{2} L_2(u_2)$$

$$\sigma_2 = H(\epsilon_2)$$

and from equ. C.18 with δu chosen orthogonal to u_1 in the sense of eqs. C.14 and 15, the variational equations of equilibrium,

$$P\sigma_0 \cdot L_{11}(u_2, \delta u) + \sigma_2 \cdot \delta e + \sigma_1 \cdot L_{11}(u_1, \delta u) = 0$$

The variation of P/P_c with ξ immediately after buckling is shown by the solid curves in Figs. C.1a, b and c for respectively the three cases

$A \neq 0$; $A = 0, b > 0$; and $A = 0, b < 0$.

The imperfection sensitivity of these curves will now be discussed.

C.2 IMPERFECT STRUCTURES AND IMPERFECTION SENSITIVITY

In order to study the influence of imperfections, an initial displacement $\bar{u}$ is assumed to exist in the unloaded,

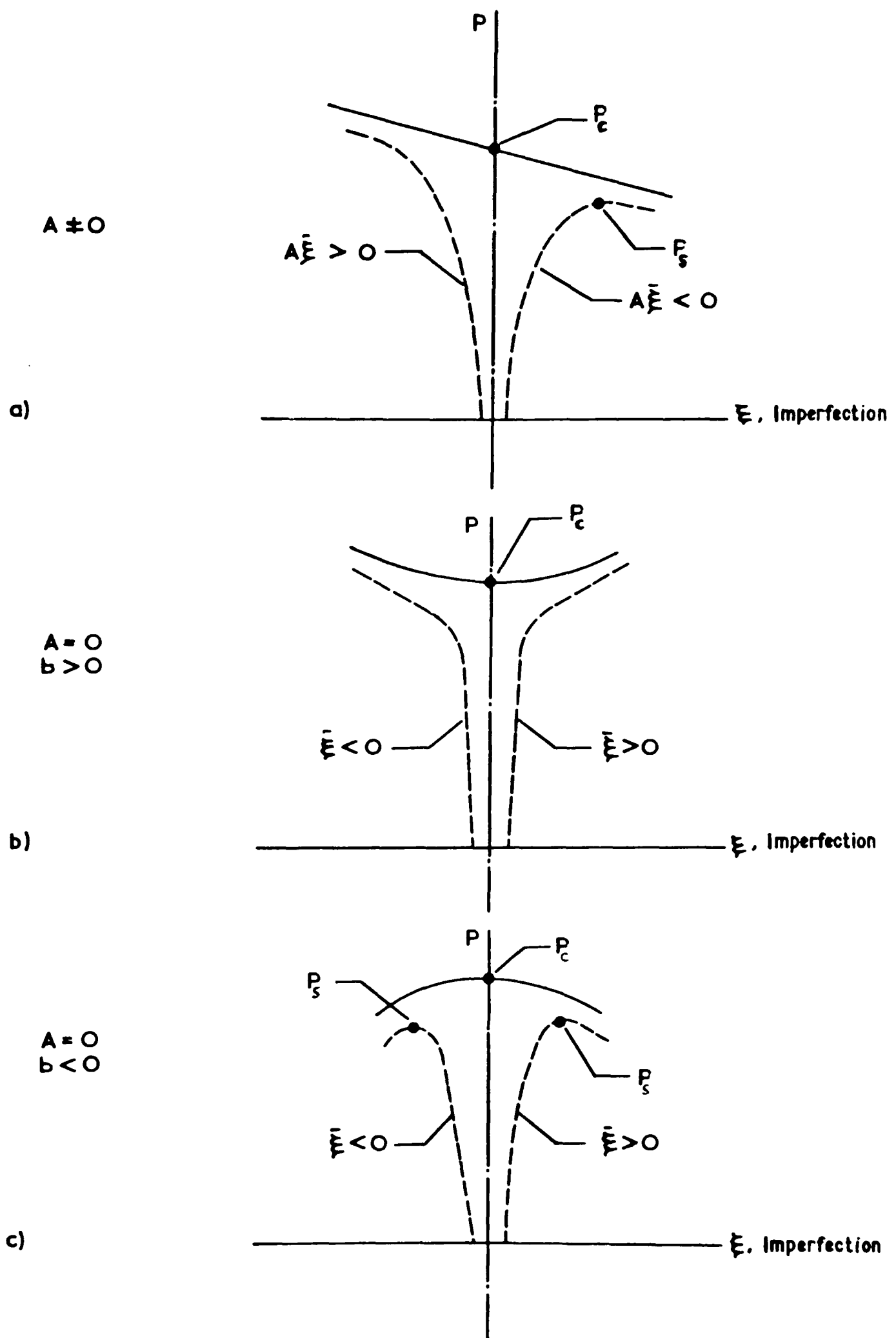


Fig.C.1. Load deflection curves obtained from Kolter's (1945) equation, [Budiansky (1965)].

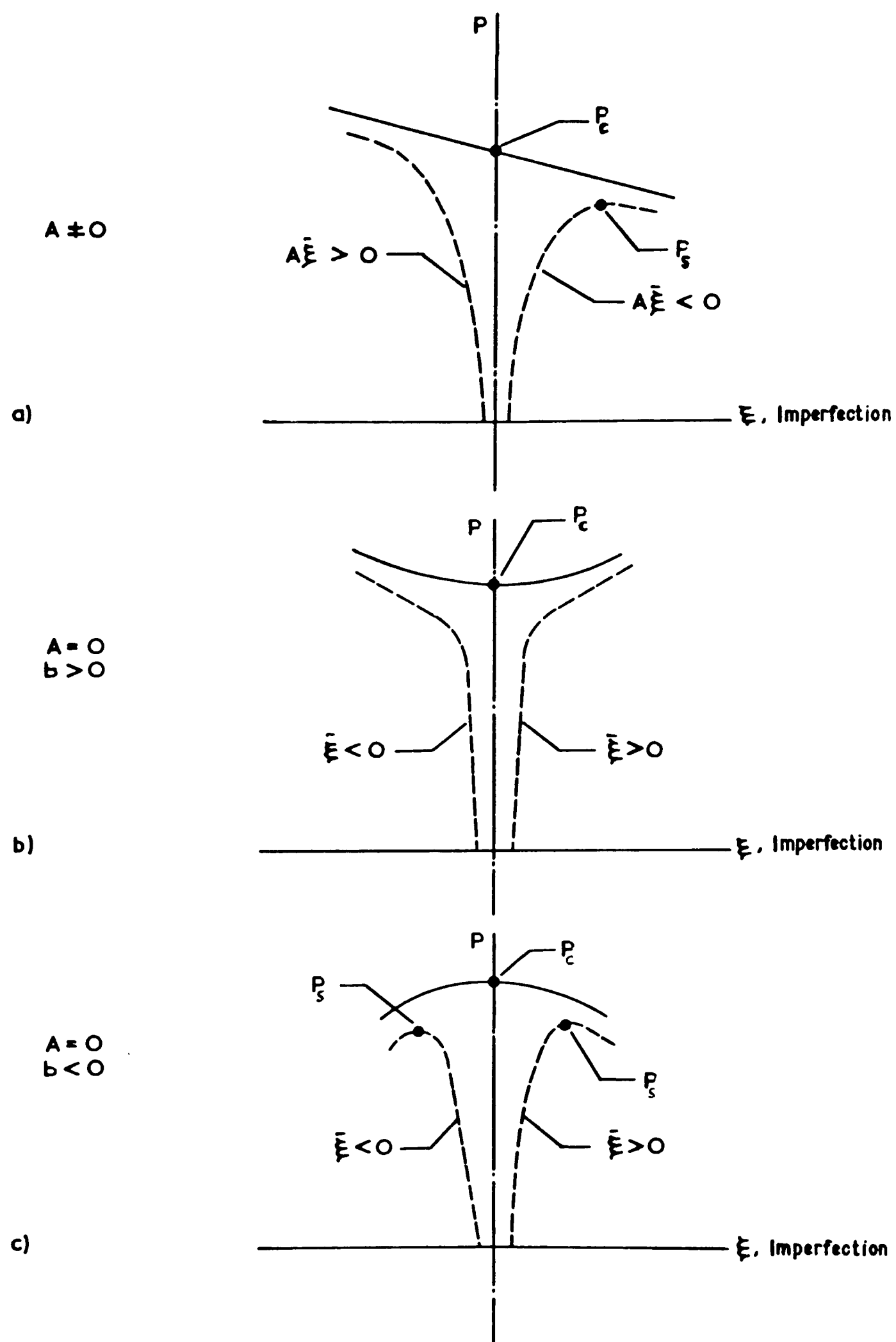


Fig.C.1. Load deflection curves obtained from Koiter's (1945) equation, [Budiansky (1965)].

stress-free structure; the strain ϵ in terms of the additional displacement $\bar{u}$ is now,

$$\begin{aligned}\epsilon &= L_1(u + \bar{u}) + \frac{1}{2} L_2(u + \bar{u}) - L_1(\bar{u}) + \frac{1}{2} L_2(\bar{u}) \\ &= L_1(u) + \frac{1}{2} L_2(u) + L_{11}(u, \bar{u})\end{aligned}$$

The stress-strain relation equ. C.2 and the strain energy-work done relationship, equ. C.3, are as before.

Equ. C.13 is then used for the perfect case with $\delta u = u_1 \delta \xi_1$ and $\delta \epsilon = [\epsilon_1 + L_{11}(u, u_1) + L_{11}(\bar{u}, u_1)] \delta \xi$; this gives,

$$(1 - P/P_C) \xi + A \xi^2 + b \xi^3 + \dots = \frac{-\sigma \cdot L_{11}(\bar{u}, u_1)}{\sigma_1 \cdot \epsilon_1}$$

Let $\bar{u} = \xi u_1$, and keep only the lowest order term in ξ to obtain,

$$(1 - P/P_C) \xi + A \xi^2 + b \xi^3 + \dots = (P/P_C) \xi \quad (C.23)$$

as a modification of equ. C.20 that accounts for an initial imperfection in the shape of the buckled mode. The dotted curves in Fig. C.1 are based on equ. C.23 in the case of Figs. C.1a and c, the structure is considered to be imperfection sensitive and buckling will occur at the lower pressure, P_s . This latter figure has been shown to depict the behaviour of a sphere under uniform external pressure. Maximising P and keeping terms of the third degree in equ. C.23 leads to

$$(1 - P_s/P_C)^{3/2} = \frac{3\sqrt{3}}{2} (-b)^{1/2} |\xi| (P_s/P_C) \quad (C.24)$$

APPENDIX D

DESCRIPTION OF COMPUTER PROGRAMME BOSOR 4

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APPENDIX D

DESCRIPTION OF COMPUTER PROGRAMME BOSOR 4

D.1 PRE-BUCKLING

Bushnell's (1972) computer programme BOSOR 4^[20] analyses thin shell structures by the use of the energy method. This method is based on the principle of minimum potential energy.

The total potential energy of a shell can be expressed as the summation of:

a) Strain energy of the shell, U_s , given by the integro-differential expression (for simple shell structures of one geometric segment),

$$U_s = \frac{1}{2} \int_s \int_\theta \{\epsilon\}^T [C] \{\epsilon\} r d\theta ds \quad (D.1)$$

in which $\{\epsilon\}$ is the strain vector, $\{\epsilon\}^T$ is the transpose of $\{\epsilon\}$, $[C]$ is the constitutive matrix which relates the stress resultants to strains and r , θ and s are radial, circumferential and meridional co-ordinates respectively. The non-linear kinematic relationship derived by Novozhilov (1959) is used,

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \\ K_1 \\ K_2 \\ 2K_{12} \end{Bmatrix} = \begin{Bmatrix} u' + w/R_1 + 1/2 (\psi^2 + \gamma^2) \\ v'/r + ur'/r + w/R_2 + 1/2 (\psi^2 + \gamma^2) \\ u'/r + r(v'/r)' + \underline{\psi\psi} \\ \psi' \\ \psi'/r + r'\psi/r \\ 2(-\psi/r + r'\psi/r + v'/R_2) \end{Bmatrix} \quad (D.2)$$

where $\psi = w' - u/R_1$

$$\psi = \dot{w}/r - v/R_2$$

$$\gamma = \frac{1}{2} (\dot{u}/r - v' - r'v/r)$$

and $()' = \partial / \partial s$, $()^\cdot = \partial / \partial \theta$

R_1 and R_2 are the radii of curvature and r is the parallel circle radius. The non-linear terms arise from the products of the rotation terms underlined.

b) Potential energy of the applied load, U_p , given by the expression,

$$U_p = - \int_s \int_\theta [pw - \frac{1}{2} p(\frac{1}{R_1} + \frac{1}{R_2})w^2 + \frac{1}{2} p(\frac{u^2}{R_1} + \frac{v^2}{R_2}) + \text{(D.3)} \\ + uwp'] r d\theta ds$$

c) The energy associated with constraint conditions imposed at boundaries and segment junctions, U_c , which ensures continuity of u , w and meridional slope ψ .

The total potential energy of the shell can then be expressed as,

$$U = U_s + U_p + U_c \quad \text{(D.4)}$$

The equilibrium condition is then obtained by setting

$$\delta U = 0 \quad \text{(D.5)}$$

If the analysis is non-linear, then this equation is solved iteratively (Newton-Raphson method). From this equilibrium condition, the equilibrium equation is derived,

$$[K] \underline{\delta} = \underline{F} \quad \text{(D.6)}$$

where $\underline{F}$ is a vector of applied loads and $\underline{\delta}$ represents the

displacements which result. The stiffness matrix $[K]$ is a function of geometry and physical properties only if the analysis is linear.

The BOSOR 4 code develops finite difference approximations for the differential co-efficients which appear in the energy formulation. Thus segments of the body are divided into a number of mesh points n ($5 < n < 98$). The displacements at each of the mesh points are then the unknowns in equ. D.6. Hence the larger the number of mesh points, the larger $[K]$ and the greater the computing time required, and more so if the analysis is of the non-linear type. However the restriction of axisymmetric structures tends to keep $[K]$ to a manageable size since the finite difference mesh is one-dimensional.

D.2 STABILITY ANALYSIS

The buckling analysis is also based on the energy method. The total potential energy is expanded as,

$$U = U_0 + \delta U + \frac{1}{2} \delta U^2 + \dots \quad (D.7)$$

by putting displacements $u = u_0 + \delta u$, $v = v_0 + \delta v$, $w = w_0 + \delta w$, into the energy expressions. $()_0$ represents the equilibrium or pre-stress displacement and $\delta()$ a small perturbation which varies circumferentially as $\cos m\theta$ where m is the circumferential wave mode. The condition $\delta U = 0$ indicates equilibrium while critical equilibrium occurs when,

$$\delta^2 U = 0 \quad (D.8)$$

as shown in Fig. D.1. The expansions for u , v , and w are

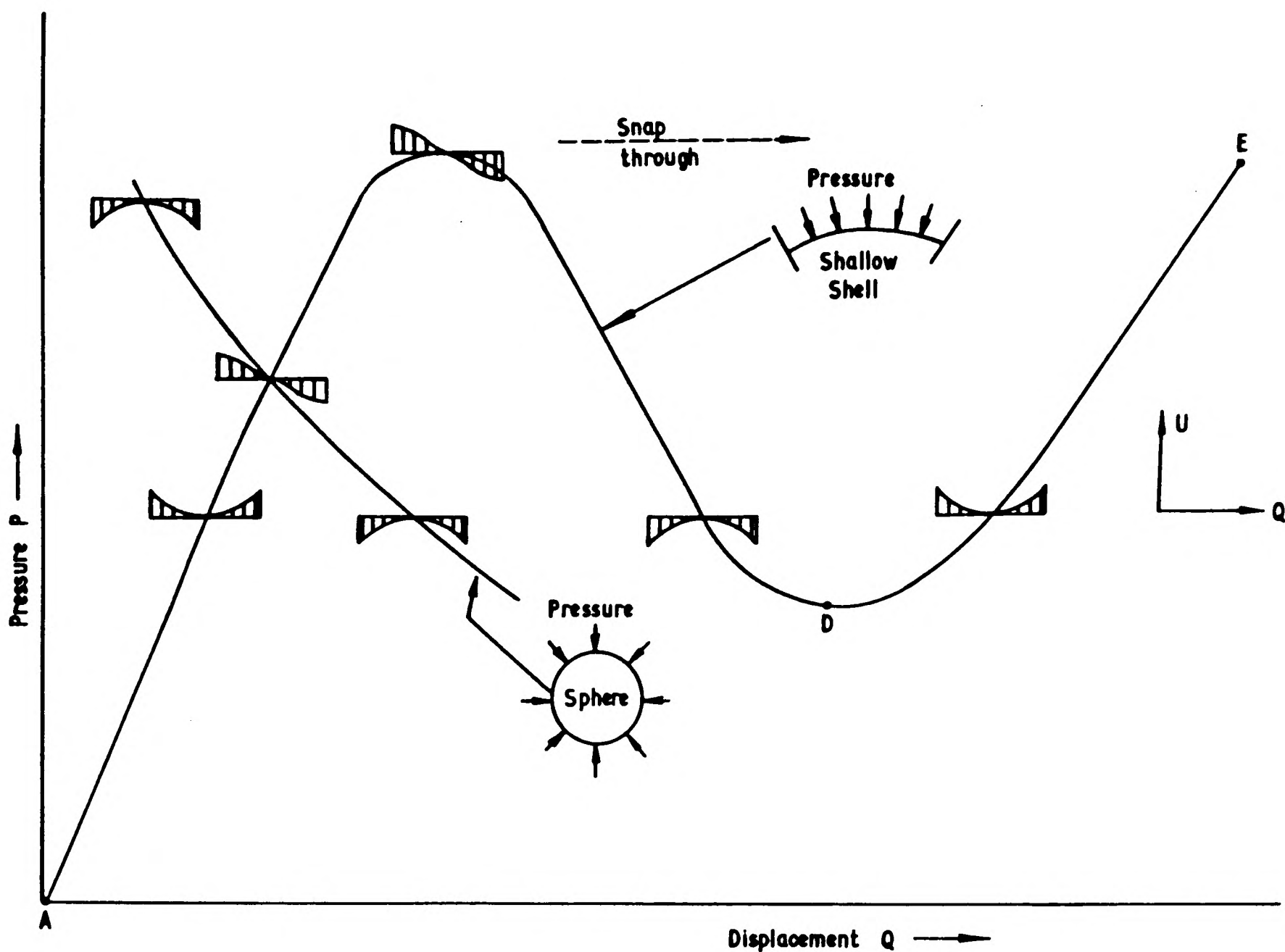


Fig.D.1.. Equilibrium paths of the shallow shell (symmetric) and of the deep shells and complete spheres (asymmetric). Buckling occurs when the equilibrium path changes from being a plot of minimum values of potential energy U , to a plot of the maximum values.

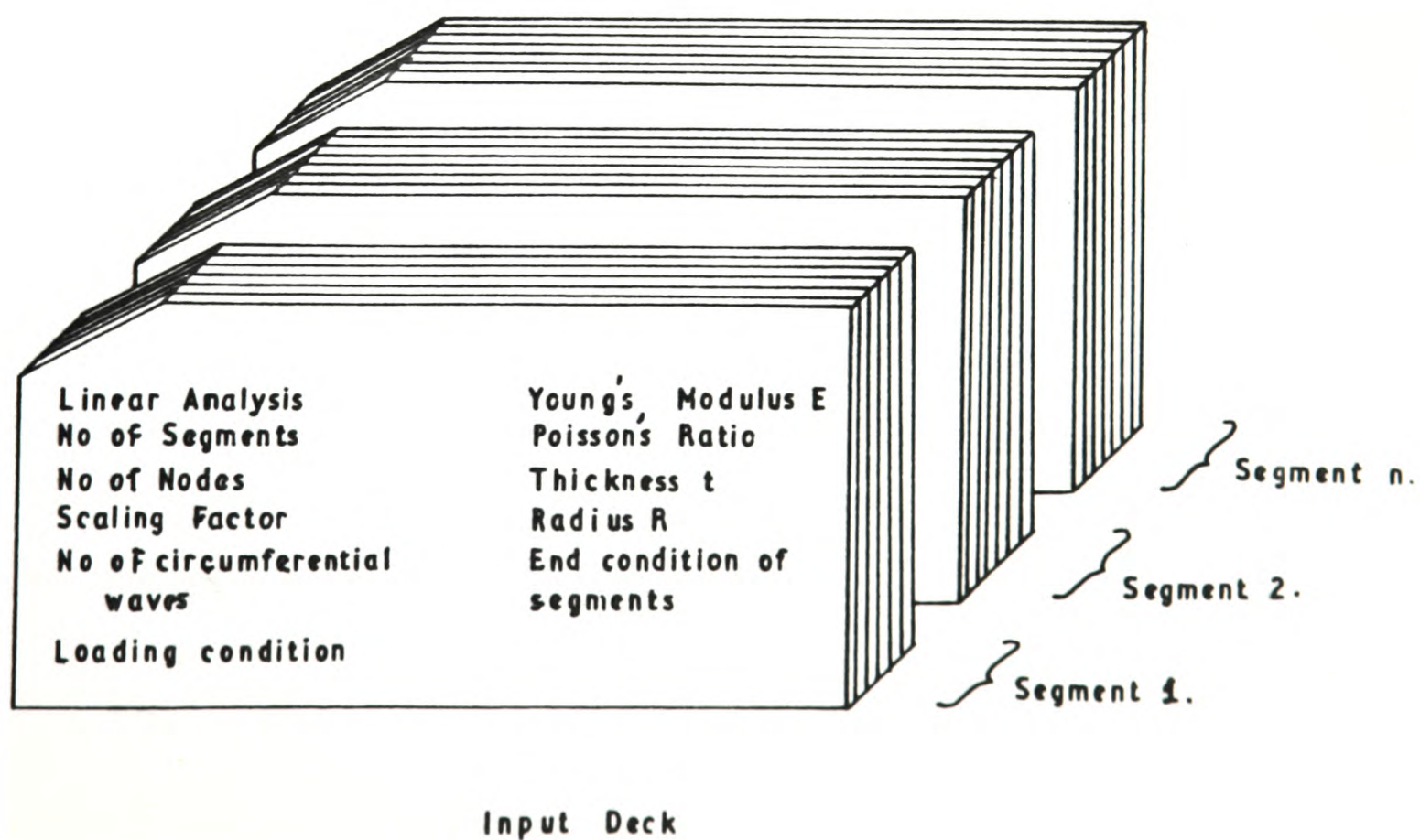


Fig D.2. Data required for a buckling analysis of a shell.

then substituted into the kinematic relationships equ. D.2, and then into the energy expressions (equs. D.1 and 3). Terms in $(\delta u)^2$, $(\delta v)^2$, $(\delta w)^2$ are then collected to determine $\delta^2 U$. For the shell of one geometric parameter, this becomes,

$$\delta^2 U = \int_S \int_\theta [\{\epsilon^1\}^T [c] \{\epsilon^1\} + 2\{\epsilon^2\}^T [c] \{\epsilon^0\} + \{\delta u, \delta v, \delta w\} [P] \begin{Bmatrix} \delta u \\ \delta v \\ \delta w \end{Bmatrix}] r d\theta ds \quad (D.9)$$

$$\text{where } [P] = \begin{bmatrix} -p/R_1 & 0 & -p' \\ 0 & -p/R_1 & 0 \\ -p' & 0 & p(1/R_1 + 1/R_2) \end{bmatrix}$$

where ϵ^0 , ϵ^1 , ϵ^2 represent, respectively, the zeroth, first and second order terms in variations $\delta u, \delta v, \delta w$.

In general there will only be a solution of equ. D.8 for each value of m . As before, the buckling condition is written in terms of the stiffness matrices,

$$[K + \lambda K_1] \underline{\delta}_1 = 0 \quad (D.10)$$

where $[K_1]$ is a matrix which depends upon m as well as the geometry and physical properties, $\underline{\delta}_1$ represents the buckling deflection (measured from the equilibrium state), and λ is a load scaling factor. Non-trivial values of the buckling displacement are obtained when $[K + \lambda K_1] = 0$. This is an eigenvalue problem. The buckling loads are determined from λ . The applied load p is defined as,

$$p = p_0 + \lambda \Delta p_0 \quad (D.11)$$

D.3 PRACTICAL CONSIDERATIONS

a) The structure must be divided into segments of shells of revolution which are geometrically different or are loaded in a different way from other segments. The maximum number of segments is 25. The number of mesh points for each segment must then be chosen.

b) The user must decide on whether a non-linear analysis is necessary.

c) The user must choose the range of m to be considered in the buckling option.

d) The scaling factor λp_0 in equ. D.11 has to be correctly chosen.

e) Geometrical data for each segment needs to be supplied. These foregoing requirements are summarised in Fig. D.2.

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