

PHYSICAL AND NUMERICAL MODELLING OF OFFSHORE FOUNDATIONS UNDER COMBINED LOADS

by

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ABSTRACT

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In addition to vertical loads, the foundations of offshore structures are subjected to horizontal loads and overturning moments as a result of environmental (wind and wave) loading. The behaviour of circular footings on cohesive soil under conditions of combined vertical, horizontal and moment (V, H, M) loading is the primary concern of this thesis. A programme of physical model tests, involving combined loading of circular footings on reconstituted Speswhite kaolin, is reported. The shape of footing used is typical of the "spudcan" foundations of independent leg jack-up drilling platforms. Previous experience with combined loading of footings on sand has revealed that the observed load:displacement behaviour is best understood, and theoretically modelled, in terms of work hardening plasticity theory. The present tests on clay confirm this, and the results are interpreted to give empirical expressions for (i) the combined load yield surface in $V:H:M$ space, and (ii) a suitable flow rule to allow prediction of the corresponding footing displacements (z, h, θ) during yielding. Extension to a complete plasticity model is achieved using theoretical stiffness factors to define elastic behaviour, and theoretical lower bound bearing capacity factors (derived specifically for this work) to define the size of the yield surface as a function of vertical penetration. The predictive capabilities of the numerical model are evaluated by retrospective simulation of various footing tests. Finally some plane frame structural analyses of a representative jack-up unit are described; some of these analyses incorporate the plasticity-based numerical model of spudcan footing behaviour under combined loads.

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Mr Bob Earl machined all non-proprietary components of the model testing apparatus, and made numerous welcome suggestions in the course of its design.

NOTATION

SYMBOLS USED IN THESIS (excluding those which only appear in one location)

a_u	undrained footing/soil adhesion
A	plan area of footing
A'	effective plan area of footing under eccentric load
A	member cross-sectional area
A_s	member cross-sectional area effective in resisting shear
A, B	least squares fitting parameters (undrained strength profile)
A, B, C	least squares fitting parameters (rotated ellipse)
b_1, b_2	elastic bowing functions
b'_1, b'_2	derivatives of b_1 and b_2 with respect to q
$\bar{b}_1, \bar{b}_2$	modified b_1, b_2 including shear deformation effects
b_{ij}	entries in apparatus flexibility matrix
B	($= R\sqrt{\pi}$) side length of square footing with same area as circular footing
B	width of strip footing
B'	width of equivalent rectangular footing
$[B]$	7×7 matrix for solution of elastic or elastoplastic increment
B_{ij}	entries in matrix $[B]$
c_1, c_2	elastic stability functions
c'_1, c'_2	derivatives of c_1 and c_2 with respect to q
$\bar{c}_1, \bar{c}_2$	modified c_1, c_2 including shear deformation effects
c_b	length correction factor accounting for bowing action
$\bar{c}_b$	modified c_b including shear deformation effects
$C_1 - C_4$	dimensionless elastic flexibility factors
D	depth below soil surface; embedment of footing reference point below soil surface
e	($= M/V$) eccentricity of footing load
e	eccentricity parameter of least squares ellipse
e_1, e_2	coefficients in parabolic fit to variation of eccentricity parameter e with V/V_0
E	Young's modulus
f	yield function
g	plastic potential function
G, G'	elastic shear modulus (for soil, in terms of effective stress)
G/s_u	rigidity index
G_s	specific gravity of solids
h	horizontal displacement
$\dot{h}$	horizontal displacement rate

Notation

H	horizontal load
H_0	maximum horizontal load capacity when $M = 0$
H_1, H_2	horizontal footing loads in orthogonal directions
H'	"horizontal" load measured by inclined load cell
H_{env}	environmental load (per leg) applied to jack-up unit
H_{int}	H intercept of least squares ellipse
I	member second moment of area
k_{ULRL}	vertical unload-reload stiffness (chord of unload-reload loop)
k_{VIR}	vertical virgin penetration stiffness
K_{onc}	coefficient of lateral earth pressure at rest for normally consolidated soil
$K_1 - K_4$	dimensionless elastic stiffness factors
L	undeformed member length
$L(1 + \Delta)$	deformed member length
L'	length of equivalent rectangular footing
M	moment load
M_0	maximum moment capacity when $H = 0$
M_1, M_2	footing moment loads about orthogonal axes
M_1, M_2	member end moments
M'	moment load measured by inclined load cell
M_{int}/R	M/R intercept of least squares ellipse
n_p	($= p'_0/p'_i$) overconsolidation ratio in terms of p'
N_c	($= Q/As_u$) vertical bearing capacity factor
N_{c0}	($= Q/As_{u0}$) vertical bearing capacity factor with respect to s_{u0}
OCR	($= \sigma'_{v0}/\sigma'_{vi}$) overconsolidation ratio in terms of σ'_v
p'	mean normal effective stress
p'_0	mean normal effective stress during isotropic normal consolidation; reference size of $p':q$ yield locus
P	point load on structure
P_{cr}	buckling load of encastré strut
$\{P\}$	vector of external applied loads
$\{P_{\text{int}}\}$	vector of internal loads
q	deviator stress
q	($= Q/Q_{\text{Euler}}$) dimensionless member axial force
Q	vertical bearing capacity
Q	member axial force
Q_{Euler}	($= \pi^2 EI/L^2$) critical axial force for pin-ended column
$\{\delta Q\}$	vector of out-of-balance loads
r^2	linear regression correlation coefficient
R	footing radius
R_{equiv}	radius of equivalent conical footing
$R_a - R_c$	load cell circuit outputs

Notation

$s_{::}$	undrained shear strength
s_{um}	s_u at mudline level
s_{u0}	s_u at footing reference point level
$[S]$	3×3 footing tangent stiffness matrix
SE	standard error of individual regression parameter
SE_{fit}	standard error of overall regression
tol	yield function convergence tolerance
$[t]$	3×3 member tangent stiffness matrix in local coordinates
$[T]$	6×6 member tangent stiffness matrix in global coordinates
T	torque load
T_0	maximum torque capacity when $H_1 = M_1 = H_2 = M_2 = 0$
u	member axial deflection
v	specific volume
V	vertical load
V_0	maximum vertical load capacity when $H = M = 0$
ΔV	load amplitude of vertical unload-reload loop
V'	"vertical" load measured by inclined load cell
w	water content
w_i	weight assigned to i th regression point
$\{x\}$	vector of joint displacements
z	vertical displacement
Δz	displacement amplitude of vertical unload-reload loop
$\dot{z}$	vertical displacement rate
α	($= a_u/s_u$) footing/soil roughness factor
α	inclination (from vertical) of footing load
α_s	inclination (from vertical) of footing load at transition to sliding failure
β	conical footing tip angle
β_{equiv}	tip angle of equivalent conical footing
β	exponent in equation for modified parabola
β_1, β_2	exponents in equation for doubly modified parabola; exponents in equation for yield surface
β_s	($= 12EI/AGL^2$) member shear area factor
γ'	submerged unit weight
δ	(as prefix) incremental value
Δ	(as prefix) change from initial or reference value
ϵ_i	regression error (<i>fitted</i> – <i>actual</i>) associated with i th point
ϵ_p	volumetric strain
ϵ_q	triaxial shear strain
ζ_a	association parameter for determining vertical plastic displacement
η	($= ML/2\pi EI$) dimensionless cantilever end moment
θ	rotational displacement

Notation

$\dot{\theta}$	rotational displacement rate
θ_1, θ_2	member end rotations
$\bar{\theta}_1, \bar{\theta}_2$	member end rotations including shear deformations
θ_{add}	additional member end rotation due to shear deformations
θ_T	torsional displacement
κ	slope of swelling lines in $v:\ln p'$ space
λ	slope of normal consolidation line in $v:\ln p'$ space
Λ	plastic volumetric strain ratio, $(\lambda - \kappa)/\lambda$
μ	coefficient in equation for modified parabola
M	slope of critical state line in $p':q$ space (triaxial compression)
ν	Poisson's ratio
N	specific volume v at $p' = 1$ kPa (isotropic consolidation)
N_{1D}	specific volume v at $p' = 1$ kPa (one dimensional consolidation)
ξ	scalar determining size of plastic displacement increment
ρ	rate of undrained strength increase with depth
σ'_v	vertical effective stress
σ'_{v0}	vertical effective stress during normal consolidation
$\sigma_{v\text{max}}$	maximum vertical consolidation pressure
$[\tau]$	global tangent stiffness matrix assembled for whole structure
χ^2	least squares merit function
ω	direction of displacement increment in $h:R\theta$ space (Fig. 4.13)
Ω	direction of load increment in $H:M/R$ space (Fig. 4.13)

SUBSCRIPTS

i. init	initial
nc	normally consolidated
OB	out-of-balance
RP	reference point
sec	secant
tang	tangent
ULRL	unload-reload

SUPERSCRIPTS

e	elastic component
p	plastic component
ep	combined elastic and plastic components
(i)	initial

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ABSTRACT

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INTRODUCTION

1.1 Jack-Up Units

1.1.1 General

A large proportion of the world's offshore oil and gas exploration in water depths of 100 m or less is carried out by a fleet of several hundred self-elevating ("jack-up") drilling units. These are mobile structures consisting of a floatable hull, and legs which may be moved up and down by machinery on the platform. Jack-up units may be classified into two categories according to their foundation type: those with individual pad footings at the end of each leg, and those with all legs supported by a large mat structure. These latter units are designed for use on very soft clay or silt deposits which have a fairly level surface. The vast majority of present day jack-ups are, however, of the independent leg type; Fig. 1.1 shows a typical example. Although early independent leg jack-ups were supported by as many as twelve footings, the three legged configuration illustrated in Fig. 1.1 has become the current industry standard. Fig. 1.2[a] shows the manner in which a jack-up is towed to a drilling site with its legs elevated. As the legs are moved down, the footings make contact with the seabed and continue penetrating until there is adequate bearing capacity for the hull to climb out of the water. Fig. 1.2[b] shows the process of preloading, which is a means of proof testing the foundations. Water is pumped into ballast tanks in the hull, forcing the footings deeper into the seabed until the bearing capacity of each footing safely exceeds the maximum vertical reaction expected during the design storm. Once the ballast tanks have been emptied, the hull is jacked well clear of the maximum calm water level to ensure that the largest anticipated storm waves do not interfere with operation of the unit (Fig. 1.2[c]).

The foundations of independent leg jack-ups are commonly known as "spudcan" footings, and some typical examples are shown in Fig. 1.3. Spudcan diameters in excess of 20 m have become quite common in post-1980 designs due to the increasing trend of using jack-ups for year-round drilling in harsh, deep water environments (particularly in the North Sea). Most spudcans are approximately circular in plan, typically with a shallow conical underside profile and a sharp protruding spigot to facilitate initial location and to provide extra horizontal stability. During installation, and while the unit is operating in perfectly calm conditions, the footings of a jack-up are essentially subjected to purely vertical loading. In general, however, the footings must also resist horizontal loads, overturning moments and changes in vertical load caused by environmental loading of the unit (*i.e.* wave and current action on the legs, and wind loads acting on the non-submerged parts of the structure). Foundation performance under these conditions is critical to the overall stability of the unit, yet the behaviour of footings under combined vertical, horizontal and moment loading is still not understood in sufficient detail.

1.1.2 Modelling of Foundation Behaviour

In assessing the suitability of a jack-up for operation at a particular site, common practice is to perform a structural analysis (usually for the 50 year design storm) in which the foundations are modelled as pin joints (Reardon, 1986). The maximum vertical footing reaction predicted by this analysis, suitably factored for safety, is used to determine the amount of preload required during installation of the unit. More advanced calculations of the necessary preload may account for the detrimental effect of concurrent horizontal load on vertical bearing capacity, typically using the semi-empirical methods of Brinch Hansen (1970) or Vesic (1975). The assumption that spudcan footings behave as pin joints, providing no rotational fixity to the structure, is evidently quite unrealistic. Since the mid 1980s there has been considerable interest in the moment:rotation behaviour of spudcan foundations, largely because:

- an accurate estimate of rotational stiffness is required for assessing the natural period of a jack-up, and for determining structural stress ranges in fatigue design;
- the ultimate capacity of a jack-up unit under extreme environmental loading is strongly dependent on the ultimate moment resistance of its footings.

These issues motivated a major industry-funded study of spudcan footing behaviour (Noble Denton and Associates, 1987) which attempted to quantify moment resistance as a function of rotation, using both scale model experiments and finite element analysis. The results of this study have allowed more rational structural analyses which take account of spudcan moment resistance and its highly nonlinear dependence on rotation.

Even with the important effect of rotational spudcan fixity included, present methods of structural analysis remain unsatisfactory because of their assumptions about vertical and horizontal footing displacements. In general these movements are either constrained to be zero (as in the pinned footing model) or taken to be linearly related to the corresponding spudcan reactions. In reality both vertical and horizontal load:displacement relationships are highly nonlinear and, as with the moment:rotation response, there are dramatic reductions in vertical and horizontal stiffnesses as the spudcan approaches failure. Furthermore, there is a strong coupling between the vertical, horizontal and rotational degrees of freedom at loads which approach the combined load bearing capacity of a footing. The complex nature of this behaviour has undoubtedly been something of a deterrent to geotechnical engineering researchers. Although there has been significant progress in developing a theoretical framework for modelling the important features of spudcan footing behaviour under combined loads, very few experimental investigations of the problem (particularly for clay soils) have been undertaken. Practical jack-up structural analyses, which are now highly sophisticated in most other respects, continue to adopt grossly idealised models of foundation behaviour.

1.1.3 The Need for More Research

The widespread and successful use of jack-up drilling platforms in the North Sea has prompted several operators to consider launching a new generation of "harsh environment"

units able to operate for extended periods in water depths of approximately 120 m (Hambly & Nicholson, 1991). Both the initial design and subsequent site-specific assessments of these units would clearly require a high level of confidence in the results of structural analyses. It is generally agreed that this confidence would be greatly enhanced if analyses were to incorporate a realistic numerical model of nonlinear spudcan behaviour under combined vertical, horizontal and moment loads. Fig. 1.4 shows the three key jack-up failure modes identified by Leijten & Efthymiou (1989). Foundation performance obviously plays a critical role in Figs 1.4[a] and [b], while in Fig. 1.4[c] the loads developed at the leg/hull interface are directly related to the spudcan displacements and reaction forces. Leijten & Efthymiou also note that, compared with fixed production platforms, jack-up units have historically suffered (and continue to suffer) a much higher number of operational accidents which can be directly attributed to foundation inadequacy. Several factors are responsible for this situation, including the very limited site investigation which is undertaken before installation of a jack-up, and the relatively serious economic implications of major damage to a fixed platform (which is thought by some to have led to overdesign of these structures). It is certain, however, that the number of jack-up accidents and failures could be reduced by an improved understanding of spudcan footing behaviour under combined loading conditions.

1.2 Aims of Present Work

In order to define the load:displacement response of a spudcan for use in three dimensional structural analysis, a numerical model with six degrees of freedom would be required (Fig. 1.5[a]). The present research is, however, confined to an investigation of loading applied within a single vertical plane, reducing the problem to one of three loads (V, H, M) and corresponding displacements (z, h, θ) as shown in Fig. 1.5[b]. At the outset of this project, the principal objective was to develop a complete numerical model capable of predicting the incremental displacements $(\delta z, \delta h, \delta \theta)$ arising from a general combined load increment $(\delta V, \delta H, \delta M)$ applied to a spudcan footing on clay soil.

Recent research into the behaviour of footings on sand (reviewed in Chapter 2) has shown that classical work hardening plasticity theory provides a very useful framework for the prediction of footing displacements associated with an increment of combined loading, and *vice versa*. In a typical plasticity-type model, linear elastic footing behaviour is assumed when the (V, H, M) force point lies inside a three dimensional yield surface, or bearing capacity envelope. Elastoplastic deformation takes place when the force point lies on the yield surface, with plastic footing displacements determined by a flow rule. The $V:H:M$ yield surface expands with vertical footing penetration (work hardening) and contracts with vertical footing heave (work softening). Within the last few years, several experimental studies involving combined loading of footings on sand have been used to calibrate plasticity-based numerical models which are capable of describing the observed behaviour reasonably well. Prior to the commencement of this project, however, no such work had been conducted with footings on cohesive soils.

The ultimate objective of developing (and using) a realistic numerical model for spudcan footing behaviour on clay was approached in stages:

1. Theoretical and experimental investigations of the manner in which vertical bearing capacity varies with spudcan penetration; comparison with currently used methods.
2. A thorough experimental study of the shape of the combined load yield surface in $V:H:M$ space; comparison with available semi-empirical theories.
3. Empirical determination of a suitable flow rule.
4. Mathematical formulation and computer implementation of the plasticity model.
5. Incorporation of the complete footing model in structural analysis of a representative jack-up platform.

Given the availability of theoretical and numerical elastic solutions, it was decided that a detailed experimental investigation of spudcan behaviour within the $V:H:M$ yield surface would not be attempted as part of this project. The consideration of cyclic foundation loading is also beyond the scope of the present research. Realistic numerical modelling of cyclic behaviour under combined loads is extremely complex, and even the relatively advanced work on sands has not yet addressed this issue in any detail. This thesis is concerned with the development of a plasticity-based numerical model which can accurately describe the *monotonic* load:displacement response of spudcan footings on clay under combined loading.

1.3 Outline of Thesis

Chapter 2 includes a review of literature relevant to the behaviour of jack-up footings on cohesive soil, for both purely vertical and combined loading. A new set of vertical bearing capacity factors, produced specifically for use in jack-up foundation design, is presented. On the basis of previously published work a preliminary work hardening plasticity model for spudcan footings on clay is postulated, and various experiments suitable for refining the features of this model are devised.

Chapter 3 describes the design and operation of a novel experimental apparatus capable of performing displacement controlled tests on model spudcan footings. The testing rig allows precise and fully independent control over vertical, horizontal and rotational footing displacements, and is equipped with a special load cell to measure the resulting vertical, horizontal and moment loads. It is also possible to conduct tests in which horizontal and/or rotational displacements are applied at constant vertical load. The method used to prepare clay specimens is described, and various measured soil properties are presented.

Chapter 4 contains a description of the experimental testing programme, which consisted of model footing tests on 33 clay samples. Full procedural details are given, along with a preliminary discussion of the results obtained. **Chapter 5** is devoted to an in-depth analysis of the experimental results. A reasonably simple mathematical expression is found to provide an excellent fit to the shape of the $V:H:M$ yield surface suggested by the test data. This experimentally determined yield surface shape is compared with the predictions of Brinch

Hansen's (1970) widely used semi-empirical bearing capacity formula. An appropriate flow rule for the incremental plasticity model is derived from the relevant experimental results.

Chapter 6 summarises the component features of the footing plasticity model, and gives a full derivation of the incremental load:displacement formulation. Implementation of the model in the FORTRAN program *OXSPUD* is described in some detail, and numerical simulations of several footing tests are compared with the actual experimental results.

Chapter 7 describes an advanced plane frame structural analysis technique suitable for analysing a two dimensional idealisation of a jack-up platform. The effects of both large displacements and shear deformations are included in the analysis method, which is implemented in the FORTRAN program *2DJUAN*. The important feature of this program is that it allows foundation behaviour to be specified using the realistic plasticity-based numerical model developed in Chapter 6. By way of comparison, analyses of a typical large jack-up unit are also performed using pinned footings and encastré (fully fixed) footings.

Chapter 8 summarises the main findings of the thesis and suggests some areas requiring further work.

BACKGROUND

2.1 Introduction

This chapter incorporates a survey of literature relevant to the prediction of jack-up unit footing performance, with particular reference to clay soils. Both elastic behaviour and ultimate bearing capacity are considered, for the cases of purely vertical loading and combined vertical, horizontal and moment loading. A new set of lower bound vertical bearing capacity factors for conical footings, obtained using the Method of Characteristics, is presented. Recent attempts to describe foundation behaviour under combined loads using nonlinear elastic and work hardening plasticity models are discussed, and a simple plasticity-based model for spudcan footings on clay is proposed. Experimental techniques for investigating the overall suitability, and specific shortcomings, of this tentative model are discussed.

2.2 Elastic Behaviour of Spudcan Footings

The installation of a jack-up unit on clay causes extensive plastic deformation and remoulding of the soil surrounding the footings, and additional plastic deformation occurs if the combined load bearing capacity is approached during operation of the unit. Most of the time, however, the loads applied to a spudcan footing are remote from those required for foundation failure, and the load:displacement response is governed by essentially elastic soil behaviour (Mommaas & Dedden, 1989). Structural analyses of jack-up units under non-extreme loads (*e.g.* fatigue studies or investigations of natural frequency) invariably represent foundation stiffness with equivalent translational and rotational elastic springs. While the vertical and horizontal elastic stiffnesses are frequently assumed to be infinite in such analyses, a good quantitative estimate of rotational elastic stiffness is essential (Carlsen *et al.*, 1986).

2.2.1 Available Elastic Solutions

Although spudcan footings are polygonal in plan (see Fig. 1.3) they are usually assumed to be circular for the purposes of analysis, with an appropriate equivalent diameter. Solutions for the deflection of a rigid circular footing on the surface of a homogeneous elastic half space are summarised by Poulos & Davis (1974):

$$V = \left[\frac{4GR}{1-\nu} \right] z; \quad H = \left[\frac{32GR(1-\nu)}{7-8\nu} \right] h; \quad M = \left[\frac{8GR^3}{3(1-\nu)} \right] \theta \quad (2.1; 2.2; 2.3)$$

where the loads and displacements are as defined in Fig. 1.5[b], G and ν are the elastic shear modulus and Poisson's ratio of the soil, and R is the footing radius. In the American

Petroleum Institute's widely used offshore platform design guidelines (API, 1993), these equations are recommended for estimating short term footing movements due to static loading. A thorough investigation by Bell (1991) revealed that, for the special case of a rough footing on incompressible soil ($\nu = 0.5$), Eqns 2.1-2.3 are exact solutions. According to Poulos (1988), spudcans on clay seabeds should be treated as having full footing/soil roughness. For rough footings on soil with $\nu < 0.5$ Bell (1991) showed that none of Eqns 2.1-2.3 is exact, though the details will not be discussed here. The present work is limited to the undrained loading of clays ($\nu = 0.5$), and Eqns 2.1-2.3 are correct under these conditions.

Although the Poulos & Davis (1974) solutions are only applicable to a surface footing, they are frequently used in practice for spudcan footings which undergo considerable penetration below the mudline during preloading (Fig. 1.2[b]). Endley *et al.* (1981) state that in practice the ratio of spudcan penetration to spudcan diameter rarely exceeds 2.5, even on very soft clays. On this basis Bell (1991), using three dimensional finite element analysis, studied the elastic response of rough circular footings embedded at depths of 0.25, 0.5, 1.0 and 2.0 diameters. Significant increases in vertical, horizontal and rotational stiffnesses (compared with the values for a surface footing) were observed, confirming the previous findings of Butterfield & Banerjee (1971) and Gazetas *et al.* (1985). The numerical investigations by Bell (1991) also highlighted the existence of elastic cross-coupling between the horizontal and rotational degrees of freedom. In general, a horizontal load applied at the footing level causes not only horizontal displacement, but rotation as well. Similarly a pure applied moment results in some horizontal displacement, not just rotation. When $\nu = 0.5$ the cross-coupling effect is absent for a flat footing at the clay surface, but it becomes increasingly important at deeper embedments. Bell's (1991) results may be conveniently expressed in matrix form:

$$\begin{Bmatrix} V \\ H \\ M/R \end{Bmatrix} = GR \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & K_4 \\ 0 & K_4 & K_3 \end{bmatrix} \begin{Bmatrix} z \\ h \\ R\theta \end{Bmatrix} \quad (2.4)$$

Values of K_i when $\nu = 0.5$ are listed in Table 2.1, which clearly shows the stiffer elastic behaviour and the greater significance of the cross-coupling term K_4 at deep embedments. The small negative K_4 values indicate that a positive horizontal load applied at the level of the reference point RP causes a *positive* (clockwise) elastic rotation of the footing. This is not immediately obvious from Eqn 2.4, but becomes more apparent when the equation is inverted.

Eqns 2.1-2.3 refer to a flat circular footing, so the conventional treatment of spudcan elasticity ignores the shallow conical footing profile shown in Fig. 1.3. In an extension to his original finite element work, Bell (personal communication, 1991) determined K_i values for rigid conical footings with apex angles 150° , 120° and 90° , although the effect of embedment was not considered in these later analyses (see Table 2.2). As would be expected, vertical elastic stiffness is not particularly sensitive to the cone angle β , but sharper

cones have considerably higher K_2 and K_3 values than flat footings. Surface conical footings also have non-zero K_4 factors, with the elastic cross-coupling effect becoming quite strong for sharper cones. For the purposes of analysis, a spudcan-shaped footing may be treated as an equivalent cone having a radius R_{equiv} equal to the maximum radius of the spudcan in contact with the soil, and a tip angle β_{equiv} such that the volume of the equivalent cone is equal to the embedded volume of the spudcan underside (see Fig. 2.1). Finite element analyses reported by Noble Denton & Associates (1987), using a typical spudcan shape, confirmed that the rotational stiffness K_3 of a fully embedded perfectly rough spudcan is very close to K_3 of the rough equivalent cone.

2.2.2 Elastic Shear Modulus G

The greatest source of uncertainty in applying Eqns 2.1-2.4 lies in the choice of a suitable elastic shear modulus G . In practice this value is usually estimated from the undrained shear strength s_u , via an assumed rigidity index G/s_u . The recent "Recommended Practice for Site-Specific Assessment of Mobile Jack-Up Units" (SNAME, 1993) references an earlier Joint Industry Study (Noble Denton & Associates, 1987) in which a value $G/s_u = 39$ is suggested for use with spudcan footings on clay. In SNAME (1993) this is acknowledged to be a lower bound value representative of large strains in the clay beneath the spudcan, and $G/s_u = 39$ is only recommended for the analysis of extreme loading events. For small strain applications such as fatigue analyses, a higher figure of $G/s_u = 150$ to 200 is suggested in SNAME (1993). At very small strains G/s_u may vary from 200 up to 800, depending on overconsolidation ratio and cyclic shear strain level (SNAME, 1993; Ladd, 1964; Wroth *et al.*, 1984). It is clear that any value of G used for predicting the elastic behaviour of spudcan footings can only be approximate, and the situation is not helped by the difficulty of obtaining reliable s_u measurements in offshore locations. Poulos (1988) points out that an appropriate elastic shear modulus is best determined by "laboratory tests in which the field stress state and stress path (including cyclic loading) are reproduced as closely as possible".

2.3 Bearing Capacity of Spudcan Footings

In determining whether a jack-up unit may be safely used at a particular site, two separate analyses of foundation bearing capacity must generally be undertaken:

- prediction of footing penetration during preloading (vertical load only);
- assessment of footing stability under design storm conditions (combined vertical, horizontal and moment loading).

These two aspects of bearing capacity analysis will be considered in some detail, with particular emphasis on the undrained loading of clay soils.

2.3.1 Central Vertical Loading: Existing Theories

A reasonably accurate prediction of footing penetration under preload is essential before any jack-up installation because the combined air gap, water depth and estimated footing

penetration must not exceed the available leg length (see Fig. 1.2[c]). Although there is a vast quantity of published research on vertical bearing capacity (both theoretical and experimental), a full review would not be appropriate here. In the jack-up industry only a few methods are used to estimate spudcan penetration into cohesive soils, notably the empirical bearing capacity equation of Skempton (1951) and the semi-empirical formulae of Brinch Hansen (1970) and Vesic (1975). In the application of all these techniques the spudcan is tacitly modelled as a flat circular footing.

The empirical equation proposed by Skempton (1951) has been widely used to predict jack-up footing penetrations into clay ever since Gemenhardt & Focht (1970) first proposed its use in this context. Skempton's equation is

$$Q/A = 6s_u(1 + 0.2D/2R) + \gamma'D \leq 9s_u + \gamma'D \quad (2.5)$$

where the various quantities are as defined in Fig. 2.2. Note that for a partially penetrated spudcan $R = R_{\text{equiv}}$, A is the plan area of the footing in contact with the soil (πR_{equiv}^2), and $D = 0$. As pointed out by Gemenhardt & Focht (1970), clay seabeds commonly exhibit a profile of increasing undrained strength with depth. In these circumstances the value of s_u to be used in Eqn 2.5 (which was derived from laboratory tests on homogeneous clay) is not obvious. Gemenhardt & Focht suggested that a representative s_u be taken as the average strength over one diameter below the widest embedded cross-section of the footing (Fig. 2.2); this was found to give the best correlation of Eqn 2.5 with numerous field observations. Current practice is, however, to take an s_u averaged over a depth of one *radius* below the widest embedded cross-section (see Fig. 2.2). This change was proposed by Young *et al.* (1984), on the basis of many additional comparisons between observed and predicted footing penetrations in clay. Adoption of a "critical strength zone" one radius deep is also supported by the analytical computations of slip-line fields presented below. The surcharge component of bearing capacity $\gamma'D$ in Eqn 2.5 is only applicable to cases where the hole above the footing remains open, but in practice (particularly on soft clays) there is usually at least partial backfilling during installation of a jack-up. Endley *et al.* (1981) state that Eqn 2.5 gives better predictions of penetrations observed in the field if complete backfilling is assumed and the $\gamma'D$ term is omitted.

The semi-empirical bearing capacity formulae of Brinch Hansen (1970) and Vesic (1975) are well known. For footings on clay, both methods are based on the universally accepted bearing capacity for a strip foundation on the surface of a weightless Tresca material:

$$Q/A = (\pi + 2)s_u \quad (2.6)$$

In the Brinch Hansen and Vesic methods this exact solution is modified with empirical shape factors and depth factors to account for different footing geometries and the effect of embedment. Circular foundations are treated as square footings of equal area, with side $B = R\sqrt{\pi}$. For the case of a square footing under purely vertical loading on undrained clay, the Brinch Hansen (1970) method gives

$$Q/A = (\pi + 2)s_u(1.2 + 0.4D/B) + \gamma'D, \quad D/B \leq 1 \quad (2.7a)$$

$$Q/A = (\pi + 2)s_u(1.2 + 0.4 \tan^{-1}(D/B)) + \gamma'D, \quad D/B > 1 \quad (2.7b)$$

although Eqn 2.7b was only proposed tentatively. In fact there is not a smooth transition between the two formulae at $D/B=1$ because $\tan^{-1}(1) \neq 1$; this may be corrected by replacing $\tan^{-1}(D/B)$ in Eqn 2.7b with $\tan^{-1}[\tan(1)D/B]$. For square footings on clay Vesic's (1975) method gives

$$Q/A = (\pi + 2)s_u \left(1 + \frac{1}{\pi + 2}\right) (1 + 0.4D/B) + \gamma'D, \quad D/B \leq 1 \quad (2.8a)$$

$$Q/A = (\pi + 2)s_u \left(1 + \frac{1}{\pi + 2}\right) (1 + 0.4 \tan^{-1}(D/B)) + \gamma'D, \quad D/B > 1 \quad (2.8b)$$

Here the shape factor and depth factor are introduced as multiplicative rather than additive factors, though it is clear that Brinch Hansen's oversight regarding the transition at $D/B=1$ has been reproduced.

Fig. 2.3[a] shows vertical bearing capacity plotted against footing penetration for the various methods described so far. Complete backfilling (so that Q/As_u is simply the familiar bearing capacity factor N_c) and a uniform profile of s_u with depth have been assumed. The correction mentioned above has also been incorporated in the Brinch Hansen and Vesic formulae to ensure continuity at $D/B=1$ (corresponding to $D/2R=0.886$). For completeness Fig. 2.3[a] also includes the solution obtained by Meyerhof (1951) using approximate slip-line fields. The figure shows that, up to embedments of about two diameters, Skempton's empirical formula is significantly conservative with respect to the other three methods (*i.e.* for a given vertical preload, the Skempton formula predicts a larger spudcan penetration). The curve obtained from Vesic's (1975) method closely resembles the Brinch Hansen (1970) prediction, but is slightly less conservative over the full range of $D/2R$. All methods predict similar limiting bearing capacities at very deep penetration, as indicated by the table in Fig. 2.3[a]. The API (1993) guidelines recommend Vesic's formula for predicting spudcan penetration under preload, though in Europe the Brinch Hansen method seems to be more popular (Reardon, 1986). It is surprising that no detailed study comparing field observations with the predictions of Eqns 2.7 and 2.8 has been undertaken. Young *et al.* (1984) mention that although they prefer Skempton's (1951) equation to the methods of Meyerhof (1951) or Hansen (1970), accurate predictions of footing penetration are more dependent on the selection of an undrained shear strength value than on the chosen analytical technique. This comment is likely to be even more applicable to sites where s_u increases with depth.

2.3.2 Central Vertical Loading: Other Relevant Research

At locations where there is a significant increase of s_u with depth, the approximate "critical strength zone" method illustrated in Fig. 2.2 allows any of the above bearing capacity methods to be used for predicting spudcan penetration. A number of researchers have,

however, undertaken rigorous investigations of bearing capacity on soils having a linearly increasing profile of undrained strength. Davis & Booker (1973) obtained solutions for an infinitely long strip footing (rough or smooth) on the surface of such a soil, while Salençon & Matar (1983) solved the corresponding problem for a rough circular footing. Houlsby & Wroth (1983) used a computer implementation of the Method of Characteristics to obtain lower bound bearing capacity factors for both circular and strip footings (rough or smooth) resting on clays having increasing strength with depth.

All of the vertical bearing capacity theories discussed in Section 2.3.1 assume a flat footing base, and the validity of their predictions for conical or spudcan-shaped footings has not been extensively tested. Meyerhof (1961) conducted vertical loading tests on rough cones in uniform clay. He found that all footings with cone angles $90^\circ \leq \beta \leq 180^\circ$ had very similar bearing capacities, whereas sharper cones showed a rapid increase in Q/As_u with decreasing β . These results agreed well with an approximate theory developed in the same paper. Houlsby & Wroth (1983) made a rigorous lower bound analysis of conical footing bearing capacity in a cohesive soil, and their results for rough cones were in close agreement with Meyerhof's observations. Recent experimental work by Santa Maria (1988) also confirmed the Meyerhof (1961) finding that, provided $90^\circ \leq \beta \leq 180^\circ$, the vertical bearing capacity of a surface conical footing on clay is not significantly different from that of a flat-based footing having the same radius.

2.3.3 Central Vertical Loading: Calculations with Program *FIELDS*

Although the references cited in Section 2.3.2 address two important aspects of vertical bearing capacity relevant to spudcan foundations (increasing strength with depth and conical shape), all of these investigations were confined to the case of a surface footing. As has already been mentioned, spudcan penetration under preload may well exceed two footing diameters; furthermore, accurate pre-installation estimates of these large penetrations are required. In order to provide a comprehensive set of bearing capacity factors for predicting spudcan penetrations on clay, the numerical work of Houlsby & Wroth (1983) has been extended to examine the effect of footing embedment in combination with those of cone angle, roughness and increasing strength with depth. The calculations were performed with Prof. G. T. Houlsby's existing FORTRAN program *FIELDS*, which uses the Method of Characteristics to compute lower bound (conservative) collapse loads for either axisymmetric or plane strain foundations. For undrained clay the Tresca yield criterion ($\sigma_{\text{maj}} - \sigma_{\text{min}} = 2s_u$) is assumed, and a linear variation of undrained strength with depth may be specified. Further details of the theoretical method are not appropriate here but may be found in Houlsby (1982) and Houlsby & Wroth (1983).

Lower bound bearing capacity factors were computed for all 1296 combinations of the following dimensionless parameters:

- cone apex angle: $\beta = 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ, 180^\circ$
- footing embedment depth: $D/2R = 0.0, 0.1, 0.25, 0.5, 1.0, 2.5$
- roughness factor: $\alpha = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$
- undrained strength increase factor: $\rho 2R/s_{um} = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0$.

These parameters are defined in Fig. 2.4[a]. A roughness factor $\alpha = 0$ corresponds to a smooth footing, while $\alpha = 1$ corresponds to full footing/soil roughness (the usual assumption for spudcans on clay, though an intermediate roughness may be appropriate for very stiff clays). The six values of the increasing strength parameter $\rho 2R/s_{um}$ give adequate coverage of the range likely to be encountered in practice (Gemenhardt & Focht, 1970), even for a very large ($R = 10$ m) spudcan. The results of the numerical parametric study, in terms of dimensionless bearing capacity factors $N_{c0} = Q/As_{u0}$, appear in Tables 2.3-2.8 (one table for each cone angle β). The full set of tables also appears in SNAME (1993). Note that the average collapse pressure Q/A is normalised by the undrained strength at the footing level, s_{u0} , not by the mudline strength s_{um} . The dimensionless strength increase factor is, however, defined in the usual manner of Davis & Booker (1973) as $\rho 2R/s_{um}$. In Table 2.8 ($\beta = 180^\circ$) the bearing capacity factors for smooth and rough surface footings ($D/2R = 0$) on uniform clay ($\rho 2R/s_{um} = 0$) are 5.69 and 6.05 respectively, in agreement with the published lower bound solutions of Shield (1955) and Eason & Shield (1960). The dependence of N_{c0} on the various parameters β , $D/2R$, α and $\rho 2R/s_{um}$ is considered in some detail by Martin (1991) and will not be repeated here.

All of the tabulated factors are derived for a general shear failure mechanism of the type shown in Fig. 2.4[b], in which a field of slip lines forms between the underside of the footing and the horizontal free soil surface (the field shown corresponds to $\beta = 150^\circ$, $D/2R = 2.5$, $\alpha = 1.0$ and $\rho 2R/s_{um} = 0$; N_{c0} is 8.99). This type of mechanism is appropriate for the relatively shallow embedments $D/2R \leq 2.5$ considered in the parametric study. If further calculations are made for deeper embedments, the bearing capacity continues to increase but it becomes less likely that the "general shear" mode represents a realistic failure mechanism. With *FIELDS* it is also possible to compute bearing capacities associated with "local shear" failure mechanisms of the type shown in Fig. 2.4[c]. For the case of $\beta = 150^\circ$, $\alpha = 1.0$ and $\rho = 0$, *FIELDS* predicts a transition from general shear failure to local shear failure at a penetration of 3.62 diameters, with a cutoff (upper limit) bearing capacity of $N_{c0} = 9.60$ imposed by the local shear mechanism for all deeper penetrations.

Fig. 2.3[b] illustrates some theoretical predictions, obtained from program *FIELDS*, of circular footing vertical bearing capacity as a function of embedment. To allow comparison with the semi-empirical predictions of Fig. 2.3[a], the two cases considered are $\alpha = 1.0$ (fully rough) and $\alpha = 0.0$ (smooth), with $\beta = 180^\circ$ and $\rho 2R/s_{um} = 0$. Skempton's (1951) equation is, in general, slightly conservative with respect to the *FIELDS* calculations. Over the range of spudcan penetrations most likely to be encountered in practice, $0.5 \leq D/2R \leq 1.5$, the other three semi-empirical methods (Brinch Hansen, 1970; Vesic, 1975; Meyerhof, 1951) all predict bearing capacities which are significantly unconservative with respect to the *FIELDS* predictions. This is, however, consistent with the widespread

preference for using Skempton's method to predict jack-up penetrations during preload (Young *et al.*, 1984; Poulos, 1988), and it must also be remembered that the *FIELDS* curves are lower bound (rather than exact) solutions. At very deep penetrations the approximate methods all approach limiting bearing capacities close to the *FIELDS* cutoff values for local shear failure. The theoretical cutoff value for a rough footing ($N_{c0} = 9.60$) is best matched by the Brinch Hansen prediction of 9.40 (-2.1%). The fact that local shear failure becomes critical at embedments of between 3 and 4 diameters means that, as suggested in SNAME (1993), the general shear bearing capacity factors of Tables 2.3 and 2.8 should not be extrapolated to penetrations $D/2R > 2.5$.

2.3.4 Combined Loading: Vertical Load and Moment ($H = 0$)

Bearing capacity under combined loads V and M has traditionally been studied as the statically equivalent problem of an eccentric vertical load (Fig. 2.5[a]). This interpretation led Meyerhof (1953) to the concept of effective area, in which the eccentric vertical load is assumed to act centrally on a foundation of reduced area. For a strip footing of width B the effective width is $B' = B - 2e$ as shown in Fig. 2.5[b], and on clay soil the $V:M$ bearing capacity interaction diagram is evidently a simple parabola:

$$\frac{M}{M_0} = 4 \cdot \frac{V}{V_0} \left(1 - \frac{V}{V_0} \right) \quad (2.9)$$

where V_0 is the bearing capacity under central vertical loading and $M_0 = 0.25BV_0$ is the maximum moment capacity. Brinch Hansen (1970) was the first to propose that eccentrically loaded circular footings be treated as shown in Fig. 2.5[c], though he did not specify how the length to width ratio L'/B' of the equivalent rectangle should be determined. The usual procedure is that suggested in API (1993), where taking L'/B' as the ratio of the line lengths l to b (see Fig. 2.5[c]) gives

$$A' = B'L' = \pi R^2 - 2e\sqrt{R^2 - e^2} - 2R^2 \sin^{-1}(e/R); \quad (2.10a)$$

$$L'/B' = \sqrt{\frac{R+e}{R-e}} \quad (2.10b)$$

Brinch Hansen's (1970) method then gives the following prediction of $V:M$ bearing capacity interaction for a circular footing on clay:

$$\frac{V}{V_0} = \left(\frac{1 + 0.2(B'/L') + 0.4D/B'}{1.2 + 0.4D/B} \right) \cdot \frac{A'}{A} \quad (2.11)$$

where $B = R\sqrt{\pi}$ and A is the gross footing area πR^2 . This formula applies for $D/B \leq 1$ and is modified in the manner of Eqns 2.7 for greater embedments. The equation obtained from Vesic's (1975) method is independent of embedment:

$$\frac{V}{V_0} = \left(\frac{\pi + 2 + (B'/L')}{\pi + 3} \right) \cdot \frac{A'}{A} \quad (2.12)$$

Fig. 2.5[d] shows, in dimensionless $V/V_0:M/RV_0$ form, the Vesic $V:M$ interaction and the corresponding Brinch Hansen predictions for a surface circular footing and an infinitely deep circular footing. All three curves are very similar and approximately parabolic, with the peak normalised moment around $M_0/RV_0 = 0.19$ in every case.

Meyerhof (1953) reported the results of eccentric vertical loading tests with circular footings on clay. He observed behaviour very similar to that of strip footings on clay, namely that "the average bearing capacity decreases linearly, with increase in eccentricity, to zero for $[e/R=1]$ ". This implies a simple parabolic variation analogous to Eqn 2.9, with a peak normalised moment of $M_0/RV_0 = 0.25$. As shown in Fig. 2.5[d], this is significantly higher than the peak moments predicted by the semi-empirical bearing capacity formulae. Vesic (1975) does state, however, that a theoretical prediction of $V:M$ interaction using the effective area method always gives conservative results. Brinch Hansen (1970) mentions that he has confirmed the validity of the effective area approach, but no specific tests are reported. Meyerhof (1963) states that, for strip footings, "detailed model tests have shown that [the effective area] procedure is on the safe side", but since Meyerhof's (1953) paper there appears to have been no published experimental research dealing with eccentrically loaded foundations on clay. By contrast there have been numerous investigations of eccentrically loaded strip and square footings on sand, and a comprehensive review appears in Ingra & Baecher (1983). More recent experimental research by Georgiadis & Butterfield (1988), Nova & Montrasio (1991) and Gottardi & Butterfield (1993) has revealed that the $V:M$ interaction locus for strip footings on sand is well fitted by a parabolic expression having the same form as Eqn 2.9.

2.3.5 Combined Loading: Vertical Load and Horizontal Load ($M = 0$)

This loading combination is statically equivalent to a single load acting at the footing centre line, inclined at an angle α to the vertical (Fig. 2.6[a]). An exact bearing capacity theory exists for the case of a rough strip footing on homogeneous clay (Bolton, 1979). The maximum horizontal load is

$$H_0 = A s_u = \left(\frac{1}{\pi + 2} \right) \cdot V_0 \quad (2.13a)$$

and sliding failure at this load occurs if $V/V_0 \leq 0.5$. For larger vertical loads the $V:H$ failure envelope is

$$\frac{V}{V_0} = \frac{\pi + 1 + \sqrt{1 - (H/H_0)^2} - \sin^{-1}(H/H_0)}{\pi + 2} \quad (2.13b)$$

which defines an approximately parabolic variation (see Fig. 2.6[b]). In terms of the angle of inclination, a conventional bearing capacity failure occurs for $\alpha < \alpha_s = 21.26^\circ$, while loads inclined at greater α result in failure by sliding.

Meyerhof (1953) carried out inclined loading tests on both strip and square footings, finding that vertical bearing capacity was significantly reduced as the angle of inclination α

increased. The original results were only presented graphically, but Meyerhof (1956) subsequently introduced algebraic expressions which he termed "inclination factors". For a footing on clay the predicted $V:H$ failure envelope is

$$\frac{V}{V_0} = \left(1 - \frac{\alpha^\circ}{90^\circ}\right)^2 \quad (2.14)$$

Although it is not stated explicitly by Meyerhof in either paper, he does imply that for rough footings on clay Eqn 2.14 only holds for angles of load inclination

$$\alpha \leq \alpha_s = \tan^{-1}(H_0/V) \quad (2.15)$$

For greater inclinations it is assumed that the footing fails by sliding at the maximum possible horizontal load $H_0 = As_u$ (cf. Fig. 2.6[b]). The Meyerhof (1953) concept of an empirical inclination factor was retained by both Brinch Hansen (1970) and Vesic (1975). For a circular footing on undrained clay, Brinch Hansen's formula for a central inclined load reduces to

$$\frac{V}{V_0} = \frac{1.2 - 0.7(1 - \sqrt{1 - H/H_0}) + 0.4 D/B}{1.2 + 0.4 D/B}, \quad H \leq H_0 = As_u \quad (2.16)$$

where $B = R\sqrt{\pi}$ and A is the gross footing area πR^2 . For $D/B > 1$, Eqn 2.16 is modified in the usual way (see Eqns 2.7). The corresponding Vesic (1975) expression for inclined loading is simpler, being independent of penetration:

$$\frac{V}{V_0} = 1 - \frac{1.5H}{(\pi + 2)H_0}, \quad H \leq H_0 = As_u \quad (2.17)$$

Fig. 2.6[c] compares the normalised $V/V_0:H/V_0$ failure envelopes obtained from the Meyerhof, Brinch Hansen and Vesic methods for a surface circular footing on uniform clay. Because they all make very similar predictions of V_0 ($=Q$ in Fig. 2.3[a]) for a surface footing, the three methods agree closely in their values for the peak normalised horizontal load $H_0/V_0 = As_u/V_0$. They differ considerably, however, in their predictions of the vertical load level at which the transition from bearing failure to sliding failure occurs. Brinch Hansen's (1970) method gives the most conservative envelope; it also predicts a changeover to sliding at a load inclination of $\alpha_s = 21.26^\circ$, the same as the theoretically correct value for a strip footing. The Vesic and Meyerhof curves agree quite closely, but both predict sudden transitions to sliding at relatively high vertical loads. At very deep penetrations in uniform clay all three methods assume that V_0 increases to a limiting value (see Fig. 2.3[a]), but that H_0 remains unchanged at As_u . The result is a considerable drop in the normalised sliding load H_0/V_0 , as shown in Fig. 2.6[d]. In this plot Vesic's (1975) interaction diagram is slightly more conservative than the Brinch Hansen (1970) prediction, but again Meyerhof's inclination factor (Eqn 2.14) gives the least conservative result at high vertical loads.

The above semi-empirical methods for predicting bearing capacity under inclined loading are widely used and generally thought to be conservative. There are, however, very few published experimental results relating to inclined loading of foundations on clay; the

studies which have been undertaken (Meyerhof, 1953; Hanna & Meyerhof, 1981; Meyerhof & Koumoto, 1987) have largely concentrated on strip footings located at the soil surface. On the other hand, many investigators (reviewed by Ingra & Baecher, 1983) have performed inclined load bearing capacity tests on sand using a variety of footing shapes. Experimental data obtained by Georgiadis & Butterfield (1988), Nova & Montrasio (1991) and Gottardi & Butterfield (1993) suggest that the $V/V_0:H/V_0$ interaction diagram for strip footings on sand is approximately parabolic (*cf.* Eqn 2.9). Note that when $V = 0$, the horizontal load capacity of a footing on cohesionless soil is of course zero, in contrast to the theoretical undrained sliding capacity $H_0 = As_u$ of a footing on clay.

2.3.6 Combined Loading: Vertical, Horizontal and Moment Loads

This case has usually been approached in the manner first suggested by Meyerhof (1953): an inclined load of magnitude $\sqrt{V^2 + H^2}$ is assumed to act centrally on a reduced foundation area determined by the eccentricity $e = M/V$. For a surface circular footing on clay, Brinch Hansen's (1970) method may be used to define the following $V:H:M$ failure envelope:

$$\frac{V}{V_0} = \frac{1 + 0.2 B'/L' - 0.5(1 - \sqrt{1 - H/A's_u})(1 + 0.4 B'/L')}{1.2} \cdot \frac{A'}{A}, \quad H \leq A's_u \quad (2.18)$$

where $V_0 = 1.2(\pi + 2)As_u$ (see Eqn 2.7a), $A = \pi R^2$ and A' , B' and L' are functions of eccentricity as defined in Eqns 2.10. The Vesic (1975) method gives

$$\frac{V}{V_0} = \left(1 - \frac{\left(\frac{2+B'/L'}{1+B'/L'} \right) H}{(\pi + 2)A's_u} \right) \left(\frac{\pi + 2 + B'/L'}{\pi + 3} \right) \cdot \frac{A'}{A}, \quad H \leq A's_u \quad (2.19)$$

where, from Eqn 2.7b, $V_0 = (1 + \frac{1}{\pi+2})(\pi + 2)As_u$. By nominating V/V_0 values, Eqns 2.18 and 2.19 may be used to draw failure envelope contours in $H/V_0:M/RV_0$ space (see Figs 2.7[a] and 2.7[b] respectively). At high vertical loads both methods predict an almost linear $H/V_0:M/RV_0$ interaction, but sliding failure begins to complicate matters for $V/V_0 \leq 0.417$ (Brinch Hansen) and $V/V_0 \leq 0.708$ (Vesic). With decreasing V/V_0 the locus of bearing/sliding transition points follows a spiral-like track across the interaction surface, terminating at the origin $(V, H, M) = (0, 0, 0)$. The Brinch Hansen method gives smooth transitions between bearing and sliding, whereas the contours of Vesic's failure envelope are quite pointed at the transition points. Analogous surfaces for infinitely deep circular footings may also be drawn; the equations will not be given here but the relevant Brinch Hansen (1970) and Vesic (1975) failure envelopes have been plotted in Figs 2.8[a] and 2.8[b]. Both interaction surfaces reflect the earlier observations that footing embedment does not significantly affect the predicted peak moment M_0/RV_0 (see Fig. 2.5[d]), but the maximum normalised horizontal load H_0/V_0 is significantly reduced (Figs 2.6[c], [d]).

The complex nature of Eqns 2.18 and 2.19 arises principally from the superposition of shape factors and inclination factors which both depend on the eccentricity of the applied load. For a circular footing, the area A' and aspect ratio L'/B' of the equivalent rectangle are also rather elaborate functions of eccentricity (Eqns 2.10). Because of their heavy

reliance on empiricism and multiplicative factors, the interaction surfaces depicted in Fig. 2.7 should obviously be used with extreme caution in the design of real circular foundations on clay. Predictions of bearing capacity for embedded footings require the introduction of yet another empirical factor; further uncertainty is entailed in using equations for flat footings to design conical or spudcan foundations (particularly with respect to the horizontal load capacity). In the absence of other available techniques, however, the Brinch Hansen (1970) and Vesic (1975) methods have become the standard means of checking spudcan footings for combined load bearing capacity (Poulos, 1988; Reardon, 1986).

It is a matter of some concern that the various semi-empirical predictions of circular footing bearing capacity under combined $V:H:M$ loading have not been verified experimentally. It should also be recalled that the general bearing capacity formulae of Brinch Hansen and Vesic are primarily intended for use in conventional (onshore, non-seismic) shallow foundation design. Here horizontal loads and moments are usually small in comparison with the vertical load; it would be most unusual to find a design in which sliding was the critical failure mode. By contrast, the spudcan foundations of jack-up units are subjected to relatively large horizontal loads and moments, and sliding failure is frequently critical for windward foundations where the vertical footing reaction is reduced by environmental loading (Poulos, 1988; Hambly & Nicholson, 1991; see also Fig. 1.4[a]). Remarkably, however, there has been no direct experimental investigation of the actual shape of the $V:H:M$ failure surface for any type of foundation on clay (let alone the specialised spudcan shapes used on jack-up units).

2.3.7 Alternative $V:H:M$ Failure Envelopes

For strip footings on sand, the shape of the $V:H:M$ failure envelope *has* been thoroughly investigated over the full range of V/V_0 . Butterfield & Ticof (1979), on the basis of several hundred load controlled tests performed by various researchers at Southampton University, suggested the use of an elliptical $H:M$ locus at constant V . Furthermore, by combining this shape with parabolic $V:M$ and $V:H$ loci (also supported by numerous tests), they proposed the simple three dimensional failure surface shown in Fig. 2.9[a]. For a surface footing, Butterfield & Ticof (1979) recommended that the size of their $V:H:M$ failure surface should be fixed using dimensionless peak loads $M_0/BV_0 \approx 0.10$ and $H_0/V_0 \approx 0.12$ (occurring at $V/V_0 = 0.5$). Somewhat larger peak loads were suggested for embedded footings.

Georgiadis & Butterfield (1988) and Nova & Montrasio (1991) have verified that a cigar-shaped failure surface, as depicted in Fig. 2.9[a], does indeed provide a good description of combined load bearing capacity for surface strip footings on sand. The peak loads M_0/BV_0 and H_0/V_0 observed in their studies were broadly similar to those suggested by Butterfield & Ticof (1979). Work at Cambridge University (Noble Denton & Associates, 1987; Shi, 1988; summarised in Dean *et al.*, 1992) has shown that a similarly shaped envelope is also very suitable for modelling the combined load bearing capacity of conical and spudcan footings on sand. Dean *et al.* (1992) proposed a cigar-shaped failure surface for

spudcans on sand, with empirically determined peak loads of $M_0/RV_0 = 0.175$ and $H_0/V_0 = 0.14$.

For spudcan footings on clay, suitable parameter values for a cigar-shaped $V:H:M$ interaction diagram were proposed by Houlsby & Martin (1992). Based on theoretical expectations and a limited number of model footings tests, they suggested peak horizontal and moment loads (occurring at $V/V_0 = 0.5$) of $M_0/RV_0 = 0.2$ and $H_0 = As_u$. The equation of their envelope was thus

$$\left(\frac{H}{As_u}\right)^2 + \left(\frac{M}{0.2RV_0}\right)^2 = 16\left(\frac{V}{V_0}\right)^2\left(1 - \frac{V}{V_0}\right)^2 \quad (2.20)$$

Houlsby & Martin (1992) show that, despite its relative simplicity, this expression gives a better overall description of the combined load bearing capacity of a spudcan than either Brinch Hansen's method (Eqn 2.18) or Vesic's method (Eqn 2.19).

2.3.8 Combined Loading: Sense of Eccentricity

None of the failure surfaces discussed so far has distinguished between the two possibilities of positive and negative eccentricity under combined $V:H:M$ loading (see Figs 2.9[b] and [c]). It is, admittedly, difficult to envisage how a real structure (particularly an offshore structure) could exert a load combination with negative eccentricity on any of its foundations. The problem is nevertheless one of considerable theoretical interest, since there is no physical reason why the cross-sectional interaction locus at constant V should be symmetrical with respect to the H and M coordinate axes; it is also desirable that modelling of footing behaviour for all possible $V:H:M$ load paths be achieved.

Meyerhof (1953) drew attention to the fundamental difference between the two cases illustrated in Figs 2.9[b] and [c], but gave no clear directions as to how backward (negative) eccentricity should be treated. The first detailed investigation of the sense of eccentricity was undertaken by Zaharescu (1961) using flat strip footings on sand. He found that with H and V fixed, a significantly larger moment could be supported in the negative eccentricity sense (Fig. 2.9[c]). Despite this finding, neither Brinch Hansen (1970) nor Vesic (1975) makes any distinction between positive and negative eccentricity. A recent study by Gottardi & Butterfield (1993) has confirmed Zaharescu's (1961) observation and quantified it in terms of an $H/V_0:M/BV_0$ failure locus. Gottardi & Butterfield's tests involved combined loading of strip footings on very dense sand, using a variety of load paths to establish points on the normalised $V/V_0:H/V_0:M/BV_0$ failure surface at a vertical load level $V/V_0 = 0.5$. Fig. 2.9[d] shows the failure points obtained, with half of the points being deduced from symmetry. The $H/V_0:M/BV_0$ failure locus is clearly not symmetrical with respect to the coordinate axes, but a *rotated* elliptical shape provides a very good description of the data. The clockwise rotation implies, in accordance with further observations made by Zaharescu (1961), that the maximum normalised horizontal and moment load capacities are located in the negative eccentricity quadrants (Nos. 2 and 4) of $H/V_0:M/BV_0$ space. Note that the $V:H:M$ sign

convention shown in Fig. 2.9[b] (which is, incidentally, used throughout this thesis) differs from that adopted in the paper of Gottardi & Butterfield (1993).

2.3.9 Combined Loading: Finite Element Analyses

From the preceding sections it is clear that, in terms of experimental studies, the problem of combined $V:H:M$ bearing capacity has been investigated much more thoroughly for footings on sand than for footings on clay. On the other hand, numerical studies using finite element analysis appear to be more advanced for the clay case. A series of runs undertaken by Fugro Ltd (Noble Denton & Associates, 1987) for rough cones on clay gave the normalised $H:M$ interaction loci shown in Fig. 2.10[a]. For sharper cones there is a significant increase in the maximum horizontal load capacity $H_{\max}$, but only a small increase in the maximum moment capacity. At the vertical load level used ($V/V_0 \approx 0.6$), Brinch Hansen's (1970) prediction of the $H:M$ locus is shown to be conservative, but its shape is in poor agreement with the numerical results. The finite element runs suggest instead that horizontal loads remote from $H_{\max}$ have very little effect on moment capacity, and *vice versa*. There is some doubt about the quality of these numerical results with respect to the adequacy of mesh refinement around the footing, and the fact that no attempt was made to model the partial loss of footing/soil contact which occurs under large moment loads (Fig. 2.5[c]).

Bell (1991) performed a more thorough finite element analysis of a rough, flat circular footing on clay subjected to combined $V:H:M$ loading. Bell's investigations were truly three dimensional (the Fugro work used the method of harmonic analysis), and he also allowed for the loss of footing/soil contact under tension. One further feature was his consideration of both positive and negative eccentricity (Figs 2.9[b] and [c]), but unfortunately only vertical load levels $V/V_0 > 0.5$ were investigated. Contours of the normalised $V:H:M$ failure surface obtained by Bell (1991) are shown in Fig. 2.10[b]. At $V/V_0 \approx 0.7$ (the highlighted contour) Bell's $H:M$ failure locus is less conservative than the corresponding Brinch Hansen (1970) prediction. There is, however, fair agreement with the Brinch Hansen shape, particularly in the positive eccentricity quadrants (Nos. 1 and 3). It is interesting that Bell's contours for high V/V_0 closely resemble the shape obtained by Gottardi & Butterfield for a strip footing on sand (Fig. 2.9[d]), namely an ellipse rotated such that the peak horizontal and moment loads are located in the negative eccentricity quadrants. In Fig. 2.10[b] there is clearly a change of shape as V/V_0 decreases, however, with the $H:M$ interaction locus taking on a more circular appearance. By contrast Gottardi & Butterfield (1993) observed similarly shaped $H:M$ loci over a range of V/V_0 values, though only a limited number of failure points were obtained at vertical load levels $V/V_0 \neq 0.5$.

2.4 Numerical Models of Foundation Behaviour

In three dimensional structural analysis of a jack-up unit, each footing has six degrees of freedom (Fig. 1.5[a]) for which stiffnesses must be specified. In practice, however, an equivalent two dimensional (plane frame) structural analysis is frequently used to consider loading along the axes of symmetry of a jack-up, as shown in Fig. 2.11[a]. The problem of defining spudcan load:displacement behaviour is thus reduced to three degrees of freedom (Fig. 1.5[b]). There has been considerable progress in the numerical modelling of spudcan response under $V:H:M$ loading, and this section contains a brief review.

2.4.1 Simple Models; Foundation Fixity

Reardon (1986) gave a description of typical jack-up design and analysis methods at that time. In structural analyses spudcan footings were modelled as pin joints, *i.e.* they were assumed to have infinite vertical and horizontal stiffnesses, but zero rotational stiffness (Fig. 2.11[b]). Although the pinned foundation is clearly a simplistic model, it is still useful in structural analyses which focus on other aspects of jack-up response (Kjeøy *et al.*, 1989, for example, investigate the effect of using different wave and current models). As shown in Fig. 2.11[c], employing a footing model with no rotational fixity is evidently conservative as far as stresses in the vicinity of the hull are concerned. An obvious refinement of the pinned footing is one with finite rotational stiffness, since an analysis in which moments are allowed to develop at the foundations gives a reduction in bending moments at the leg/hull interface (Fig. 2.11[c]). The economic implications of this are considerable, because a given jack-up unit is able to operate in deeper water (or harsher environmental conditions) if rotational spudcan fixity can be confidently accounted for (Santa Maria, 1988; Chiba *et al.*, 1986).

The assumption of a linear elastic moment:rotation response (Fig. 2.11[d]) is appropriate when small footing displacements are expected. Various authors have assumed linear elastic $M:\theta$ behaviour for cyclic fatigue studies (Hambly & Nicholson, 1989), investigations of non-extreme dynamic behaviour (Carlsen *et al.*, 1986), and predictions of jack-up response to moderate impact loading (Ellinas *et al.*, 1989). The choice of a suitable rotational spring stiffness has already been addressed in Section 2.2. Field measurements of small displacement rotational stiffness (deduced from measurements of natural period) have been undertaken by various researchers for jack-ups on both clay (Brekke *et al.*, 1989) and sand (Brekke *et al.*, 1990; Chiba *et al.*, 1986). The measured rotational stiffnesses have generally been of the same order of magnitude as those expected from theory (Eqn 2.3).

2.4.2 Nonlinear Elastic Models

Under extreme storm conditions, spudcan behaviour can no longer be realistically treated as linear elastic (Arnesen *et al.*, 1988). Footing reactions approach the combined load failure envelope and the $M:\theta$ response, in particular, becomes highly nonlinear. If horizontal and vertical displacements are still assumed to be negligible, an improved model of spudcan behaviour is provided by a nonlinear rotational spring of the type shown in Fig. 2.12[a].

Several experimental studies have investigated the nonlinear moment:rotation response of circular footings. Hattori *et al.* (1986) tested flat circular foundations on sand, both as individual footings and when attached to a model four-legged jack-up. The Joint Industry Study entitled "Foundation Fixity of Jack-up Units", coordinated by Noble Denton & Associates (1987), included experimental investigations of the $M:\theta$ response for conical and spudcan footings on both clay (Santa Maria, 1988) and sand (laboratory and centrifuge tests performed by various workers at Cambridge University). For spudcans on clay, the main outcome of the Joint Industry Study was an empirical, non-dimensional expression for the moment:rotation response. A two parameter hyperbola was used to model the transition from initial elastic behaviour (Eqn 2.3) at small rotations to fully plastic (*i.e.* constant M) behaviour at very large rotations. Empirical factors were given to adjust the reference $M:\theta$ curve for the effects of (i) cone angle, (ii) vertical load level V/V_0 , (iii) horizontal load to moment ratio, and (iv) embedment depth. An iterative procedure is needed to incorporate this predicted nonlinear moment:rotation response into the structural analysis of a jack-up under monotonic storm loading. For each footing an initial guess of the likely spudcan rotation is made, and an appropriate *secant* rotational stiffness (see Fig. 2.12[a]) is obtained from the empirical $M:\theta$ curve. Structural analysis with this spring stiffness yields a new set of spudcan rotations, and an improved estimate of secant stiffness may then be obtained for each footing. Iteration continues until satisfactory convergence is achieved.

Arnesen *et al.* (1988) present a spudcan model featuring independent nonlinear springs for all three degrees of freedom (Fig. 2.12[b]). Vertical, horizontal and rotational load:displacement responses are modelled, as above, using hyperbolic curves with initial elastic stiffnesses determined from Eqns 2.1-2.3. Structural analysis using the Arnesen *et al.* (1988) footing model again requires iteration to obtain convergence of the secant stiffnesses. Hambly *et al.* (1990) propose a model in which the horizontal spudcan stiffness is assumed to be infinite, but vertical and rotational stiffnesses (K_V and K_M) depend on the position of the (V, M) force point within an idealised failure envelope (see Fig. 2.12[c]). K_V and K_M are equal to their theoretical elastic values when the force point is at the centre of the failure envelope ($V = 0.5V_0$, $M = 0$), and transition functions are used to reduce both stiffnesses to zero as the force point approaches the failure envelope. This is an example of a coupled nonlinear model since the moment load affects the vertical stiffness and *vice versa*. Note also that the Hambly *et al.* (1990) model does not give secant stiffness values but *tangent* stiffnesses suitable for a structural analysis where loading is applied incrementally. Hambly *et al.* (1990) accommodate an outward-moving (V, M) force point by allowing the failure envelope of Fig. 2.12[c] to expand. This expansion is associated with an increase in V_0 and additional spudcan penetration (determined by a so-called vertical plastic stiffness).

One final nonlinear elastic model worth mentioning is that described in the SNAME (1993) recommended practice document. For spudcans on clay the cigar-shaped $V:H:M$ failure envelope of Eqn 2.20 is assumed, and rotational stiffness K_M is taken as the full elastic value (*cf.* Eqn 2.3) when the force point lies on the V axis ($H = M = 0$). For other locations of the force point, a reduction multiplier

$$f_r = 1 - \frac{\left[\left(\frac{H}{As_u} \right)^2 + \left(\frac{M}{0.2RV_0} \right)^2 \right]^{1/2}}{4 \left(\frac{V}{V_0} \right) \left(1 - \frac{V}{V_0} \right)} \quad (2.21)$$

is applied to the initial elastic K_M , giving a secant rotational stiffness $f_r K_M$ which is deliberately intended to be a conservative estimate. Note that $f_r K_M \rightarrow 0$ as the (V, H, M) force point approaches the failure envelope. For spudcans on clay, SNAME (1993) states that the vertical and horizontal load:displacement behaviour may be modelled as linear elastic, with stiffness values corresponding to Eqns 2.1 and 2.2. Structural analysis must be undertaken iteratively until the reduction factor f_r converges for each spudcan. SNAME (1993) gives a very similar nonlinear elastic procedure for estimating spudcan rotational fixity on sand.

2.4.3 Work Hardening Plasticity Models

Certain aspects of the nonlinear models described in the previous section, particularly those of Hambly *et al.* (1990) and SNAME (1993), are strongly reminiscent of classical work hardening plasticity theory. The failure envelope in load space corresponds to a yield locus defining permissible stress states. A high rotational stiffness under small moments is progressively reduced as the force point approaches the failure envelope; this corresponds to the (admittedly more sudden) transition from elastic to elastoplastic stress:strain behaviour which characterises yielding in classical plasticity. The size of the $V:H:M$ failure envelope is determined by V_0 , the pure vertical load capacity, which is a function of spudcan penetration; herein lies an analogous hardening law. Although implemented rather crudely, the vertical plastic stiffness of Hambly *et al.* (1990) is evidently motivated by the desire to incorporate some form of "work hardening" into their model. There is, however, no feature of any of the nonlinear elastic models which could be seen as corresponding to a flow rule (or, equally, to a plastic potential function). This final necessary ingredient of a work hardening plasticity model for spudcan behaviour would define the relative magnitudes of vertical, horizontal and rotational plastic displacements arising from any (V, H, M) load combination causing yield.

Butterfield (1981) appears to be the first author to consider applying the ideas of classical plasticity to the prediction of irreversible footing displacements. It will be recalled that Butterfield & Ticoft (1979) established the validity of a simple cigar-shaped $V:H:M/B$ interaction diagram for strip footings on sand, based on numerous laboratory tests. Butterfield (1981) analysed the displacements observed in some of the $V:H$ (inclined load) tests by plotting the work-conjugate displacement increments δz and δh on the corresponding load axes. In $V:H (\delta z:\delta h)$ space, he observed that the plastic potential did not coincide with the simple parabolic failure envelope (*i.e.* an associated flow rule was not appropriate). In fact an elliptical $V:H$ construction was found to provide a very satisfactory plastic potential, but further details will not be given here.

As discussed above, Noble Denton & Associates (1987) proposed that spudcan fixity on clay be estimated using an empirical $M:\theta$ response defined by a two parameter hyperbola. The same Joint Industry Study contains a more advanced calculation method for spudcans on sand which, although it relies heavily on experimental results, incorporates a number of plasticity-based features. First of all there is a semi-empirical definition of the virgin load:penetration curve for the spudcan, so that the vertical bearing capacity V_0 and the vertical plastic stiffness may be evaluated at any embedment. The combined load yield surface is taken to be cigar-shaped, with peak horizontal and moment loads of $H_0/V_0 = 0.125$ and $M_0/RV_0 = 0.25$ (see Fig. 2.13[a]). At a given vertical load level V/V_0 , it is then straightforward to compute the yield moment M_y for any radial load path in a direction $H/(M/R) = \text{constant}$. Combining M_y with the theoretical elastic stiffness of Eqn 2.3, the first portion of the moment:rotation curve shown in Fig. 2.13[b] may be obtained. The post-yield $M:\theta$ response is derived by assuming that the incremental plastic displacement vectors $\langle \delta z, \delta h, R\delta\theta \rangle$ are normal to the $V:H:M/R$ yield surface of Fig. 2.13[a]. Noble Denton & Associates (1987) does not contain a full mathematical formulation of the elastoplastic response, but instead gives a series of design curves which are used to estimate the plastic rotational stiffness K_{θ_p} in Fig. 2.13[b]. Note that for $V/V_0 < 0.5$ a negative slope K_{θ_p} is predicted (shown dashed) since yielding at low vertical loads causes vertical plastic heave of the spudcan, as opposed to vertical plastic settlement for $V/V_0 > 0.5$. At low vertical loads, yielding results in a reduction of V_0 and therefore contraction of the $V:H:M/R$ yield surface; at high vertical loads yielding results in an increase in V_0 and yield surface expansion. The full predicted $M:\theta$ curve in Fig. 2.13[b] is bilinear, and may be incorporated into a structural analysis using secant stiffnesses and iteration in the manner outlined above.

The work described in the preceding paragraph, due to R.G. James of Cambridge University, was the first practical application of plasticity concepts to the modelling of jack-up foundation behaviour. Using a similar load:displacement analogue of stress:strain plasticity theory, Schotman (1989) develops a three degree of freedom ($V:H:M, z:h:\theta$) numerical model of spudcan behaviour. Furthermore, he incorporates this model into the incremental structural analysis of a representative plane frame jack-up unit. A yield surface $f(V, H, M, z^p) = 0$ and plastic potential surface $g(V, H, M, z^p) = 0$ are first defined for the spudcan/soil system. The vertical plastic penetration z^p determines V_0 and hence controls the size of the $V:H:M$ yield surface. For elastic behaviour (*i.e.* (V, H, M) force point inside the yield surface, $f < 0$) a diagonal elastic stiffness matrix corresponding to Eqns 2.1-2.3 is used:

$$\begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix}^e = GR \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^e \quad (2.22)$$

For elastoplastic behaviour (force point on the yield surface, $f = 0$) the incremental stiffness matrix is, in general, fully populated:

$$\begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix}^{\text{cp}} = \begin{bmatrix} GR \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^{\text{cp}} \quad (2.23)$$

The terms k_{ij} are derived (using standard plasticity theory) from the plastic potential function g , the yield function f , and the condition that $\delta f = 0$ during an increment of plastic yielding. Unfortunately Schotman (1989) only gives sketchy details of the actual f and g adopted in his analyses. For spudcans on clay he appears to have used Brinch Hansen's (1970) method to determine the $V:H:M$ yield surface and the $V_0:z^p$ relationship, combining them with a non-associated flow rule based on very limited finite element analyses of a plane strain "spudcan" under inclined loading (Fig. 2.14[a]). No finite element calibration was performed for moment loads. Schotman does emphasise, however, that the yield surface and plastic potential used in his numerical model are only preliminary, and that a more realistic analysis would require physical model footing tests to determine f and g .

Even though the details of his spudcan model were very simplistic, Schotman's (1989) structural analysis results were the first obtained using genuine incremental plasticity to describe footing behaviour (*cf.* the "pseudo-plasticity" of the Hambly *et al.* (1990) nonlinear elastic model). As discussed in Section 2.4.1, a conventional jack-up structural analysis is performed with pinned footings and the resulting (V, H) spudcan reactions are compared with, for example, the Brinch Hansen (1970) failure envelope for inclined loads (Figs 2.6[c], [d]). Jack-up load carrying capacity is thus dictated by the first set of spudcan reactions to reach the failure envelope, suitably factored for design. The possibility that subsequent footing displacements may lead to advantageous load redistribution, or stable spudcan yielding with further penetration, is not considered. With a plasticity-type footing model, however, loading of a jack-up can continue beyond spudcan yield. The structural analyses of Schotman (1989) confirmed that plasticity theory is well suited to modelling the complex load:displacement response of a yielding spudcan attached to a jack-up structure, particularly with respect to coupling between the different degrees of freedom. One example of the interesting numerical results obtained is shown in Fig. 2.14[b], for a plane frame jack-up on sand with its single leg to windward. After vertical preloading and unloading to $V = 0.5V_0$, horizontal load is incrementally applied to the upper part of the structure. All three footings yield at about the same applied load. The two leeward footings undergo further penetration as they yield, so their horizontal load capacity increases in a stable manner. Conversely the windward footing suffers upward plastic vertical displacement and its yield surface contracts; horizontal load capacity begins to fall and large horizontal sliding displacements occur. It is important to note that, because of load redistribution to the leeward spudcans, the load carrying capacity of the jack-up structure as a whole continues to increase. The ultimate capacity of the unit would probably be governed by structural failure at the leg/hull interface.

Nova & Montrasio (1991) presented a three degree of freedom numerical model for strip footings on sand. Although not rigorously formulated as a full plasticity model, the Nova & Montrasio method has features which make it broadly analogous to a rigid / work

hardening plastic constitutive law. A series of laboratory tests, designed to allow full calibration of the numerical model, was reported in the same paper. The observed virgin load:penetration curve was fitted with an empirical relationship, thus defining the hardening law. Load controlled bearing capacity tests were used to define a three dimensional yield surface in $V/V_0:H/V_0:M/BV_0$ space, though most of the data refer to the $M = 0$ plane. A cigar-shaped surface with peak loads $H_0/V_0 \approx 0.125$ and $M/BV_0 \approx 0.075$ was found to provide a good fit to the observed yield points. Displacements during yield indicated, however, that an associated flow rule would result in overpredictions of vertical settlement (or, at low V/V_0 , overpredictions of heave). Two additional model parameters were therefore introduced to adjust the normality rule predictions of vertical displacement: one for loading in the $H = 0$ plane, one for the $M = 0$ plane. To test the capabilities of their model, Nova & Montrasio (1991) conducted constant vertical load tests in which H and M were increased (from zero) at a constant ratio. Fig. 2.15 shows experimental results and numerical simulations for two such tests (one at $M/BH = 1$, one at $M/BH = 0.3$), and the comparison is clearly very good. Note that even though the vertical load remains constant throughout each test, the footing does undergo significant vertical settlement. This response is clearly not compatible with elastic theory, but (as shown in Fig. 2.15[d]) the numerical model is well suited to the prediction of coupled plastic load:displacement behaviour. Further tests by Nova & Montrasio (1991) confirmed that their model gave good predictions of footing displacements under horizontal and moment load reversals. They point out that "displacement and rotation consequent to virgin loading are essentially irreversible, and if the foundation is unloaded the permanent displacement is almost as large as the maximum observed". Again this implies that, for an advanced model of footing behaviour, the theoretical framework of plasticity is much more suitable than that of elasticity.

Tan (1990) made a detailed investigation of the $V:H$ yield loci for various conical and spudcan footings on sand. His work combined both physical model testing in a geotechnical centrifuge and a theoretical (Method of Characteristics) study of plane strain wedge-shaped footings subjected to inclined loading. Tan found that, for spudcans, the $V:H$ yield locus was not symmetrical about $V/V_0 = 0.5$; the peak horizontal load was in fact located at $V/V_0 \approx 0.4$. The $V:H$ yield locus could, however, be well fitted using a shape derived from the Original Cam-Clay constitutive model (Roscoe & Schofield, 1963; Schofield & Wroth, 1968), as shown in Fig. 2.16. Experimental work revealed that a non-associated flow rule was required, and the chosen family of plastic potentials is also shown in Fig. 2.16. Note that at $V/V_0 = 0.135$, foundation yielding results in purely horizontal plastic displacements. Tan (1990) compared this with the critical state eventually attained by a Cam-Clay soil during triaxial testing, in which unlimited plastic shear strains develop without any plastic volumetric strain. Experiments proved that an analogous "critical state" does indeed exist for footings on sand subjected to $V:H$ (*i.e.* inclined) loading. Tan (1990) combined the yield locus and plastic potential of Fig. 2.16 with theoretical elastic behaviour (defined by Eqns 2.1-2.2) and a semi-empirical definition of vertical bearing capacity as a function of plastic vertical

penetration. The resulting work hardening plasticity model was used to obtain very good retrospective simulations of both load and displacement controlled footing tests.

2.5 Preliminary Numerical Model for Spudcans on Clay

It is evident from the previous section that plasticity-based numerical modelling of load:displacement behaviour is reasonably well advanced for foundations on sand. There is, however, no work hardening plasticity model for spudcan footings on clay which has been experimentally calibrated. The major aim of this thesis is, of course, to provide such a numerical model. To assist with identifying the type of physical calibration tests which should be carried out, it is useful if a preliminary numerical model is described to provide a framework for discussion. The features of this hypothetical model, referred to hereafter as Model A, are described in this section.

2.5.1 Model A: Elastic Response

Elastic cross-coupling between the horizontal and rotational degrees of freedom will not be considered in the preliminary model. Elastic behaviour within the yield surface is therefore given by Eqns 2.1-2.3, and may be expressed as a diagonal stiffness matrix:

$$\begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix}^e = GR \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^e \quad (2.22 \text{ bis})$$

2.5.2 Model A: Yield Surface

Given its success in modelling footing behaviour on sand, a cigar-shaped $V/V_0:H/V_0:M/RV_0$ yield surface will be adopted for Model A (see Fig. 2.17[a]):

$$\left(\frac{H}{H_0}\right)^2 + \left(\frac{M}{M_0}\right)^2 = 16 \left(\frac{V}{V_0}\right)^2 \left(1 - \frac{V}{V_0}\right)^2 \quad (2.24)$$

The semi-empirical methods of Brinch Hansen (1970) and Vesic (1975) may be used to establish peak horizontal and moment loads for the preliminary model. From Fig. 2.5[d] it seems that $M_0/RV_0 = 0.19$ is a suitable choice for the maximum normalised moment. The semi-empirical methods all predict a maximum normalised horizontal load H_0/V_0 of about 0.16 for a surface footing (Fig. 2.6[c]) and 0.10 for a very deep footing (Fig. 2.6[d]). In Model A it is assumed, for simplicity, that the maximum horizontal load capacity remains proportional to the pure vertical load capacity at all embedments: $H_0/V_0 = 0.13$.

The most obvious difference between the cigar-shaped yield surface of Fig. 2.17[a] and the Brinch Hansen/Vesic interaction surfaces (Figs 2.7, 2.8) concerns the horizontal load capacity at low V/V_0 . According to the parabolic $V:H$ variation of the cigar-shaped surface, horizontal load capacity approaches zero as the normalised vertical load approaches zero. This is clearly realistic for footings on sand, but its application to footings on clay has no

theoretical basis. In fact the semi-empirical methods are strictly correct in predicting that, provided there is no moment, the full theoretical sliding capacity $H_0 = As_u$ is available right down to $V/V_0 = 0$. Intuitively, however, this seems rather optimistic. At small vertical loads there will be a strong tendency for the footing to break contact with the clay before full horizontal load capacity is mobilised, particularly on stiff clays. One of the main objectives of the experimental programme was to check whether a cigar-shaped surface can provide accurate modelling of $V:H$ interaction at low V/V_0 .

2.5.3 Model A: Flow Rule

Despite other researchers' experience with footings on sand, a straightforward associated flow rule will be assumed for the preliminary Model A (see Fig. 2.17[b]). This may well prove to be a realistic assumption at high vertical loads, but at low V/V_0 an overprediction of plastic footing heave seems likely (as observed on sand by both Nova & Montrasio, 1991 and Tan, 1990). From symmetry the incremental plastic displacement vectors at $V/V_0 = 1$ and $V/V_0 = 0$ must clearly be directed along the δz^p axis, as shown in Fig. 2.17[b]. At each of these two points on the yield surface, however, there is no uniquely defined normal vector (the surface is pointed). A method of rounding off the yield surface at its extremities is discussed in Chapter 5.

2.5.4 Model A: Hardening Law

For simplicity, Skempton's (1951) empirical formula for the variation of vertical bearing capacity with embedment (Eqn 2.5; Fig. 2.3[a]) will be used to define the virgin load:penetration curve in Model A. Fig. 2.17[c] is a schematic illustration of the dependence of yield surface size on the vertical plastic penetration z^p . The (non-normalised) yield surface expands with plastic settlement, and contracts with plastic heave. With the yield surface and associated flow rule assumed above, yielding at vertical loads $V/V_0 > 0.5$ results in work hardening, while yielding at $V/V_0 < 0.5$ causes work softening.

2.6 Experimental Strategy

The goal of the laboratory testing programme was to check the whether the assumed features of Model A were satisfactory, or whether refinements were necessary. It was decided that experimental investigations would focus on determining (for monotonic loading):

- the actual shape of the $V/V_0:H/V_0:M/RV_0$ yield surface for spudcan footings on clay;
- whether this shape varies with depth of embedment, up to $D/2R \approx 2$;
- if an associated flow rule is applicable;
- the validity of the assumed $V_0:z^p$ hardening law.

As discussed in Chapter 1, a detailed examination of elastic behaviour within the yield surface was seen as somewhat less important. Noble Denton & Associates (1987) states that, for extreme loading events, "the precise form and size of the yield surface in relation to the application of further loads completely dominates the footing's behaviour".

2.6.1 Probing Tests

Previous research into the combined load bearing capacity of foundations has made extensive use of load controlled probing tests to define a yield surface. Typically a footing is loaded vertically until it yields (*i.e.* $V = V_0$), and unloaded to a given V/V_0 level. Horizontal and moment loads (in a given proportion) are then applied, usually at constant V , and examination of the $H:h$ and $M:\theta$ relationships allows a single yield point on the combined $V:H:M$ interaction surface to be determined (see Fig. 2.18[a]). A series of such tests conducted at various V/V_0 values, and over a range of footing penetrations, could be used to build up the complete three dimensional yield surface and trace its evolution with embedment. By considering the incremental displacement vectors $\langle \delta z, \delta h, R\delta\theta \rangle$ associated with foundation yielding in various probing tests (Fig. 2.18[b]), an appropriate flow rule could also be determined. An exhaustive programme of probing tests giving full coverage of the yield surface would clearly be extremely time consuming, and furthermore a great many clay samples would be required.

It is worth considering a close analogy which exists between type of footing test just described and the critical state soil mechanics interpretation of drained triaxial testing. Vertical loading of a footing to yield and subsequent unloading corresponds to isotropic normal consolidation of a clay sample (to $p' = p'_0$) and unloading to a specified overconsolidation ratio $n_p = p'_0/p'_i$. Application of horizontal and/or moment load at constant V is analogous to triaxial shearing of the clay sample at constant mean stress $p' = p'_i$. The incremental plastic displacement vector observed at yield in a footing test corresponds to the incremental plastic strain vector $\langle \delta\epsilon_p^p, \delta\epsilon_q^p \rangle$ at yield in a triaxial test. Figs 2.19[a] and 2.19[b] illustrate the two related forms of yield surface probing. The upper figure shows three constant p' tests performed on an overconsolidated Modified Cam-Clay soil (Roscoe & Burland, 1968) with $\lambda = 0.25$, $\kappa = 0.05$, $M = 0.9$, $N = 3.50$ and $G' = 1000$ kPa. After isotropic normal consolidation to $p'_0 = 100$ kPa (Point A), three possible drained test paths are illustrated at $p' = 75$ kPa (B→C→D), $p' = 50$ kPa (P→Q→R) and $p' = 25$ kPa (X→Y→Z). Note that at $p' = 75$ kPa there is positive plastic volumetric strain, with expansion of the initial yield surface to $p'_0 = 150$ kPa at critical state. Conversely, Path X→Y→Z is associated with plastic volumetric expansion of the sample, and the initial yield surface eventually shrinks to $p'_0 = 50$ kPa. The $q:\epsilon_q$ plot clearly shows that in all three tests the initial behaviour is elastic, but upon yielding there are three different responses: work hardening (C→D), work softening (Y→Z) and perfect plasticity (Q→R).

Fig. 2.19[b] shows a schematic representation of the corresponding probing-type tests on a Model A footing at $V/V_0 = 0.75$ (B'→C'→D'), $V/V_0 = 0.5$ (P'→Q'→R') and $V/V_0 = 0.25$ (X'→Y'→Z'). Note that a strict analogy with the soil in Fig. 2.19[a] has been maintained, in that the ratio of vertical unload-reload stiffness to virgin penetration stiffness is $k_{ULRL}/k_{VIR} = 5$. Obviously for a real footing/soil system the corresponding ratio would be much higher, perhaps around 100 or 200, so the amount of work hardening and work softening shown in Fig. 2.19[b] is greatly exaggerated. Nevertheless, Fig. 2.19[b] gives a good illustration of Tan's (1990) concept of a "critical state" (and indeed a "critical state

line") for footings yielding under combined loads. Tan (1990) also raises an important point with regard to experimental probing tests at low V/V_0 . Because of the expected work softening response after yield, load controlled application of H or M would be susceptible to sudden failure of the footing (making measurement of the incremental displacements very difficult). Tan therefore advises that, if possible, probing tests at constant V should be carried out using horizontal or rotational *displacement* control.

2.6.2 Tracking Tests

Having explored the similarity between constant p' triaxial testing and constant V footing tests, a logical question is "what type of footing test is analogous to an *undrained* triaxial test?". Because the specific volume v corresponds to the vertical penetration z , the required footing experiment is one similar to a probing test but performed at constant vertical *penetration*, not at constant vertical load. This analogy was first presented by Tan (1990), who used the term "sideswipe test" to describe the application of horizontal displacement to a footing while its vertical penetration is held constant. Tan exploited the analogy with undrained triaxial testing very successfully, deducing that the load paths followed in just two sideswipe tests (one commencing from $V/V_0 \approx 1$, the other from $V/V_0 \approx 0$) could be combined to define the entire $V/V_0:H/V_0$ yield locus of a spudcan footing on sand (for a given embedment). The reasoning behind this important discovery is discussed in the following paragraphs.

Taking the same Modified Cam-Clay soil as the one used in Fig. 2.19[a] ($\lambda = 0.25$, $\kappa = 0.05$, $M = 0.9$, $N = 3.50$ and $G' = 1000$ kPa), it is a fairly straightforward task to compute the $p':q$ stress path and $q:\epsilon_q$ response in any undrained triaxial test. Two particular undrained tests are plotted in Fig. 2.20[a]. The first involves isotropic normal consolidation to $p'_0 = 100$ kPa, followed by immediate shearing at constant volume (Path A→B); the other involves normal consolidation to $p'_0 = 100$ kPa (Point A) and unloading to $p' = 1$ kPa (Point X) before undrained shearing is commenced (X→Y→Z). Path A→B in $p':q$ space represents a transition between the initial ($p'_0 = 100$ kPa) yield locus and the expanded yield locus ($p'_0 = 115.1$ kPa) prevailing at critical state. Between Points X and Y the response is elastic, with increasing shear stress but no change in mean stress. The initial ($p'_0 = 100$ kPa) yield locus is intersected at Point Y; along Path Y→Z continuous yielding and very significant work softening take place, with critical state eventually attained on a yield locus with $p'_0 = 46.0$ kPa. In Fig. 2.20[b] exactly the same pair of undrained tests is performed, with just one change to the soil properties: κ is reduced (by a factor of 10) to 0.005. Now $\lambda/\kappa = 50$ is not a realistic ratio for a clay, but it is clear from Fig. 2.20[b] that if such a soil did exist then the paths A'→B' and X'→Y'→Z' in $p':q$ space would show a much smaller deviation from the initial ($p'_0 = 100$ kPa) yield locus than Paths A→B and X→Y→Z in Fig. 2.20[a]. This is because, during undrained yielding, the elastic and plastic volumetric strains must be equal and opposite ($\delta\epsilon_p^p + \delta\epsilon_p^e = 0$). A greater unload-reload stiffness $1/\kappa$ implies a smaller elastic volumetric strain for a given $\delta p'$, which in turn implies a smaller plastic volumetric strain; if the virgin compression stiffness $1/\lambda$ remains unchanged, a smaller

amount of yield locus hardening or softening will occur during undrained yielding. Experimentally, only two undrained triaxial tests would be needed to define the complete $p':q$ yield locus for a particular clay, *provided the ratio λ/κ was known to be very high* for the clay concerned. Fig. 2.20[a] indicates that, unfortunately, this strategy would not be very successful for a real clay having $\lambda/\kappa \approx 5$.

Considering now the analogy with a sideswipe footing test, the Cam-Clay ratio λ/κ corresponds to the vertical stiffness ratio k_{ULRL}/k_{VIR} (note that λ and κ are both flexibility, rather than stiffness, parameters). For a typical footing penetrating into clay, it was suggested above that the ratio k_{ULRL}/k_{VIR} might be of the order of 100. By analogy with Fig. 2.20[b], a sideswipe (h at constant z) test on such a footing would be expected to give a $V:H$ load path closely resembling a portion of the initial $V:H$ yield locus. (Note that during a sideswipe test, elastic and plastic vertical displacements must be equal and opposite, $\delta z^p + \delta z^e = 0$, just as $\delta \epsilon_p^p + \delta \epsilon_p^e = 0$ in an undrained triaxial test). A sideswipe footing test commencing from $V/V_0 = 1$ ("normally consolidated") should give a good indication of the $V:H$ yield locus shape for high normalised vertical loads (the "wet side of critical"), while a sideswipe started after unloading to very small V/V_0 ("heavily overconsolidated") should produce a load path with increasing V and H which closely follows the yield locus on the "dry side of critical".

Some of Tan's (1990) experimental results for a flat circular footing on sand are shown in Fig. 2.21. Fig. 2.21[a] shows the procedure followed in a "normally consolidated" sideswipe test (*i.e.* one commencing from high V/V_0). In vertical load:vertical displacement space Path A→B shows the initial vertical penetration of the footing, while Path B→C→D is a vertical unload-reload loop (note the very high ratio k_{ULRL}/k_{VIR}). Upon stopping the footing penetration at Point D, the vertical load immediately drops by some 15% to Point E due to creep relaxation of the sand. The sideswipe test itself is then commenced: horizontal displacement is applied with the vertical penetration remaining locked (Path E→F). In vertical load:horizontal load space the sideswipe load path closely resembles a suitably scaled version of Tan's proposed yield locus (Fig. 2.16). The (V, H) load state remains virtually unchanged at large horizontal displacements ($h > 2$ mm), providing confirmation of the theoretically expected "critical state" for footings subjected to combined loading. After unloading the footing to $H = 0$ and $V = 0.015V_0$, another sideswipe test is performed (Path G→H, Fig. 2.21[b]). The resulting path in vertical load:horizontal load space again reaches a "critical" (V, H) load state, although this end point does not lie particularly close to that attained in the previous sideswipe (Point F). Tan (1990) confirmed that the yield locus suggested by the sideswipe test load paths in $V:H$ space did indeed give good predictions of the (V, H) yield points observed in a limited number of conventional (constant V/V_0) probing tests on flat circular footings. He then proceeded to use the sideswipe method for investigating the $V:H$ yield loci of several conical footings on sand.

Sideswipe tests on the hypothetical Model A footing ($k_{ULRL}/k_{VIR} = 50$) would be expected to give load paths of the type shown in Fig. 2.22. The experiments shown involve interrupting vertical penetration of the footing for several sideswipe tests, allowing the

evolution of the (non-normalised) $V:H$ yield locus with depth to be investigated. For a real footing the yield locus shape may not remain constant with depth, so there would be interest in comparing the normalised $V/V_0:H/V_0$ load paths obtained at several embedments. Normalisation by V_0 would also allow the comparison of sideswipes commencing from $V/V_0 = 1$ (Fig. 2.22[a]) and from low V/V_0 (Fig. 2.22[b]) even if they were conducted on separate clay samples. This would eliminate the considerable soil disturbance associated with Tan's (1990) method of performing a sideswipe from high V/V_0 , unloading horizontally and vertically, then conducting a sideswipe from low V/V_0 at the same location in the soil bed.

The centrifuge tests by Tan (1990) were concerned only with inclined (*i.e.* $V:H$) loading, but the term "moment sideswipe test" has since been introduced by other researchers at Cambridge (*e.g.* Dean *et al.*, 1992) to refer to the application of footing rotation θ at constant penetration z . There is also the further possibility of applying combined displacements h and θ simultaneously at constant z . In this latter type of generalised sideswipe test, the load path in three dimensional $V:H:M/R$ space would be expected to follow a track lying very close to the initial yield surface (by extension from the two dimensional $V:H$ case discussed above). For this reason, any footing test conducted at constant z (whether it involves horizontal displacement, rotation or a combination of the two) will be referred to in this thesis as a *tracking test*. A series of tracking tests, conducted with various $h:R\theta$ ratios, should allow the complete yield surface to be defined in the manner shown in Fig. 2.23. Tests from both high V/V_0 and low V/V_0 are shown; note also that, from symmetry, only half of the $H:M/R$ plane needs to be covered. Comparing Fig. 2.23 with Fig. 2.18[a], it is clear that a single tracking test provides a great deal more information about the yield surface shape than a single probing test. The former gives a path across the yield surface (spanning a range of V/V_0 values), while the latter gives only a single $(V, H, M/R)$ point. This makes the tracking-type test particularly attractive for experimental work, where a long time must invariably be spent on the careful preparation of soil specimens (especially clays). It is interesting that the similarly time consuming nature of three dimensional finite element runs led Bell (1991) to use the tracking test philosophy in his numerical study of a surface circular footing on clay subjected to combined loading. The yield surface contours of Fig. 2.10 were deduced from several runs which involved vertically loading the footing to failure ($V/V_0 = 1$), then imposing $h:R\theta$ displacements in various ratios with the vertical footing penetration z held constant.

2.6.3 Loop Tests

Another type of footing test may be envisaged, producing a $V:H:M/R$ load path which tracks directly around the cross-sectional yield surface shape at constant V . This type of test might involve vertical penetration and unloading to a certain V/V_0 value, followed by the application of a circular loop in $h:R\theta$ displacement space (Fig. 2.24, inset). For the hypothetical Model A footing, Fig. 2.24 shows the load path that would be expected from this type of "loop test". Initially the response is elastic ($A \rightarrow B$), but from Point B the load path begins to follow an elliptical track around the yield surface cross-section ($B \rightarrow C \rightarrow D \rightarrow$

E). If an experimental loop test were to be conducted on a Model A footing, the vertical load level of $V/V_0 = 0.5$ (as shown in Fig. 2.24) would be preferred because:

- the cross-sectional $H/V_0 : M/RV_0$ yield locus is at its largest;
- neither hardening nor softening of the initial yield surface would be expected during the continuous yielding on Path $B \rightarrow C \rightarrow D \rightarrow E$ (since $\delta z^p = 0$ at $V/V_0 = 0.5$).

As with the tracking tests described above, vertical penetration of the footing could be interrupted for a series of loop tests, thus investigating any change of the cross-sectional yield surface shape with embedment. Once again a single loop test should provide considerably more information about the yield surface shape than the single $(V, H, M/R)$ yield point derived from a conventional probing test at constant V .

CLAY SPECIMENS & APPARATUS

3.1 Introduction

The first part of this chapter is concerned with the soil used for the physical modelling programme. Clay sample preparation is described, and the results of shear strength and water content measurements are presented. The observed variation of undrained shear strength with depth is in good agreement with theoretical expectations. The second part of this chapter outlines the design, construction and operation of a testing apparatus allowing independent control of the vertical, horizontal and rotational displacements of a model footing. Modifications to permit testing at constant vertical load are also described. Finally the arrangements for load and displacement instrumentation, data acquisition and computerised control of the apparatus are summarised.

3.2 Clay Specimens

3.2.1 Speswhite Kaolin

This clay has been used in numerous laboratory studies involving small-scale physical model tests, including work on:

- piles (Randolph *et al.*, 1979; Steenfelt *et al.*, 1981; Martins, 1983; Gue, 1984);
- tunnels (Mair, 1979; Kim, 1994);
- shallow foundations (Santa Maria, 1988; Craig and Chua, 1990);
- reinforced soil (Love, 1984; Fannin, 1986).

At Oxford, Speswhite kaolin has also been used for calibration chamber studies of the piezocone (May, 1987; Nyirenda, 1989) and Marchetti dilatometer (Smith, 1993).

The principal reason for the popularity of Speswhite kaolin is that, for a clay, it has a relatively high coefficient of permeability (of the order of 10^{-9} m/s according to Al-Tabbaa, 1987 and Smith, 1993), allowing rapid consolidation of large clay samples from a reconstituted slurry. Because of its widespread use in research, several key properties of Speswhite kaolin are well established: plastic limit $w_p \approx 34\%$, liquid limit $w_L \approx 65\%$, specific gravity of solids $G_s \approx 2.61$. Although the framework of critical state soil mechanics has provided a useful means of describing the behaviour of Speswhite kaolin, strength and consolidation properties reported in the literature show considerable variation (see Table 3.1). In particular there are some significant differences in the measured values of λ , and κ and N_{ID} , mainly because (for a real soil) the normal consolidation and swelling lines are not perfectly straight in $v:\ln p'$ space. As noted by Butterfield (1979), Martins (1983), Airey (1984) and many others, the "instantaneous" value of λ reduces steadily with increasing mean stress p' . Similarly during swelling, the local value of κ is initially very small but increases with further overconsolidation (Al-Tabbaa, 1987). For a given set of experimental

observations, the values of λ , κ and N_{1D} which provide the best Cam-Clay fit will clearly be strongly dependent on the mean stress range and the degree of overconsolidation used for the measurements. Smith (1993) points out that different batches of Speswhite kaolin may well have slightly different properties; this is another likely source of the variations within each row of Table 3.1.

3.2.2 Consolidation Apparatus

The soil mechanics laboratory at Oxford is equipped with a clay consolidation system designed and built by Gue (1984). Santa Maria (1988) used the same equipment to produce soil samples for his model footing tests. Both workers obtained good quality homogeneous clay specimens with repeatable profiles of undrained shear strength and water content with depth. In the interests of both economy and continuity with previous work at Oxford, it was decided that Gue's consolidation apparatus would be used for this research. The design and operation of the equipment is described and illustrated by Gue (1984) and again by Santa Maria (1988), so only some essential details are appropriate here.

There are three identical consolidation frames and slurry tanks (Fig. 3.1 shows a single setup). A double action hydraulic ram is mounted on top of each reaction frame, with compressive load applied to the clay *via* a rigid loading platen. All three hydraulic rams are connected to a single pressurisation system. Each slurry tank consists of a top cylinder, a bottom cylinder and a base plate, all bolted together and sealed with silicone sealant. This sealant was found to be more reliable than the greased rubber gaskets employed by previous users of the apparatus. The top and bottom cylinders are identical: 450 mm high, 450 mm inside diameter, 6 mm wall thickness. Like the loading platen, they are made of the corrosion-resistant Cu/Al alloy HE15 ("dural"); the piston is made of stainless steel. Holes in the loading platen and base plate provide the slurry with two-way drainage, and two "Vyon" porous plastic discs (2.5 mm thick) are used as filters. The inside surface of each tank was lightly coated with Shell "Barbatia" grease before being filled with slurry, helping to create a low-friction, watertight seal between the loading platen and the tank wall (see detail in Fig. 3.1).

3.2.3 Slurry Preparation and Consolidation

The Speswhite kaolin was obtained from the English China Clay Company in powdered form, with an initial water content w of approximately 1%. It was mixed for two hours with an accurately measured quantity of water to form a homogeneous kaolin slurry ($w = 120 \pm 1\%$). Gue (1984) gives full details of the 110 litre ribbon blade mixer. A vacuum of approximately 0.8 bar was applied throughout the mixing period, ensuring the removal of most air bubbles. At the completion of a mix the slurry was pumped out of the mixer and deposited in the fully assembled consolidation tanks under 100 mm of water. This prevented desiccation and air entrainment, and also gave an accurate indication of the volume of slurry in each tank. Four mixes were needed to fill a batch of three consolidation tanks to the required initial slurry height. Fig. 3.2 shows the progress of one dimensional consolidation

and unloading for a typical batch of three samples. The sequence of vertical pressures (25, 50, 100, 200, 100, 50, 0 kPa) was the same as that employed by Gue (1984) and Santa Maria (1988), with only the final loading stage of 200 kPa being allowed to reach the end of primary consolidation. Pore pressure measurements by Gue (1984) indicated that a final swelling period of 36 hours at 0 kPa provided ample time for the dissipation of pore water suctions throughout the sample.

A fully unloaded sample, ready for testing, is shown in Fig. 3.3. In order to leave room for a 15 mm layer of water above the clay surface, the final sample height had to be about 410 mm. For the first batch an initial slurry height was computed using the λ , κ , N_{ID} and K_{onc} values of Martins (1983) from Table 3.1. This batch gave final sample heights of around 405 mm ($\approx 1\%$ too low), so the initial height of slurry was increased accordingly (to 735 mm) for subsequent batches. Very good repeatability of the final sample height, 410 ± 2 mm, was observed throughout the remainder of the testing programme.

3.2.4 Measured Sample Properties

At the conclusion of each footing test, the tank assembly was dismantled and a miniature site investigation was carried out. Fig. 3.4[a] shows the plan layout of shear vane tests and water content sampling performed at each of four depths (60, 140, 220, 300 mm). Since the footing tests only involved a maximum penetration of 200 mm, soil properties in the bottom 100 mm of the sample were not of particular interest. By conducting all of the soil tests 70 mm in from the circumference of the kaolin sample (Fig. 3.4[a]) it was thought that the results would not be unduly influenced by the proximity of either the sample boundary or the central zone of cracked and heavily disturbed soil created by a footing test.

A standard "Pilcon" laboratory shear vane was used, fitted with the larger of two the two vanes supplied (50 mm high, 33 mm diameter). The handle was rotated at 1 r.p.m. in all tests, as recommended by the manufacturer. Fig. 3.4[b] illustrates how a consistent amount of shaft embedment was maintained in vane tests at the various levels. A soil auger was used to drill the 80 mm deep boreholes, and the clay sample was cut with thin wire for the tests at depths of 220 and 300 mm. Careful calibration of the shear vane (Fig. 3.5) was performed using existing apparatus in the laboratory. The undrained shear strength corresponding to a given applied torque was computed from the usual BS 1377 (1975) formula

$$\text{torque} = s_u \cdot \frac{\pi(D_{\text{vane}})^2}{2} \cdot H_{\text{vane}} \left(1 + \frac{D_{\text{vane}}}{3H_{\text{vane}}} \right) \quad (3.1)$$

with dimensions $D_{\text{vane}} = 33$ mm and $H_{\text{vane}} = 50$ mm as mentioned above. Water content specimens (typically of about 80 grams) were taken and weighed to the nearest 0.01 gram as quickly as possible after a fresh vertical clay surface had been exposed by cutting the sample. Each specimen was then oven dried and re-weighed in accordance with BS 1377 to determine its water content. The miniature site investigation commenced immediately after a footing test, and took around two to three hours to complete.

Eleven batches of three samples were prepared in the course of the experimental programme, so there are 132 shear vane results and 66 water content measurements for each of the four testing depths. Fig. 3.6 presents this information in histogram form, along with the mean and standard deviation for each set of values. As would be expected for such a heavily overconsolidated clay sample, there is a very marked increase of undrained strength with depth (and a corresponding decrease in water content with depth). In general there are few outlying values in either the strength or water content plots, indicating that good repeatability was achieved in preparing the kaolin specimens. A plot of water content (linear scale) against undrained strength (log scale) appears in Fig. 3.7[a]. Despite considerable scatter, there appears to be general verification of the expected straight line relationship (Muir Wood, 1990). In Fig. 3.7[b] the points from Fig. 3.7[a] have been plotted together with data presented by Smith (1993) for much stronger samples of Speswhite kaolin ($s_u = 20$ to 140 kPa). The results obtained in the present study seem to be quite consistent with Smith's corresponding (*i.e.* "vane - unstressed") data. Smith (1993) also performed vane tests on clay subjected to vertical and radial stresses in a 1 m diameter calibration chamber, and for a given water content these values of s_u are slightly higher than those obtained from the unstressed clay. Undrained strengths determined from Smith's (1993) triaxial tests are (for a given w) significantly greater than the shear vane measurements, confirming the experience of Love (1984) and Santa Maria (1988) with soft Speswhite kaolin (vane $s_u = 6$ to 15 kPa).

3.2.5 Theoretical Variation of Undrained Strength with Depth

Wroth (1984) derives the following relationship between undrained shear strength and overconsolidation ratio for a Modified Cam-Clay (Roscoe & Burland, 1968) soil:

$$\frac{s_u}{p'_i} = \frac{M}{2} \left(\frac{n_p}{r} \right)^\Lambda \quad (3.2)$$

where

$$\Lambda = \frac{\lambda - \kappa}{\lambda}$$

is the plastic volumetric strain ratio, n_p is the isotropic overconsolidation ratio (defined as $n_p = p'_0/p'_i$) and r is the Cam-Clay spacing ratio (defined in Wroth, 1984). Eqn 3.2 may be rewritten in the more convenient form

$$\frac{s_u}{p'_i} = \left(\frac{s_u}{p'_i} \right)_{nc} \cdot n_p^\Lambda \quad (3.3)$$

where $(s_u/p'_i)_{nc}$ is the undrained strength ratio for a normally consolidated sample (*i.e.* $n_p = 1$). For a soil history of one dimensional consolidation and swelling, the analogous expression in terms of vertical effective stresses is

$$\frac{s_u}{\sigma'_{vi}} = \left(\frac{s_u}{\sigma'_{vi}} \right)_{nc} \cdot OCR^\Lambda \quad (3.4)$$

where $OCR = \sigma'_{v0}/\sigma'_{vi}$. Muir Wood (1990) discusses the theoretical justification for Eqn 3.4 at some length. Although this expression cannot be rigorously derived from Eqn 3.3 because the ratio n_p/OCR is not a constant (even for a particular soil), Eqn 3.4 could still be said to have some theoretical basis. In any case, there is a wealth of experimental evidence (*e.g.* Ladd *et al.*, 1977; Andresen *et al.*, 1979) in support of Eqn 3.4 for clays of widely varying plasticity. According to Houlsby (1993), typical values of the undrained strength ratio $(s_u/\sigma'_{vi})_{nc}$ and the exponent Λ are 0.25 and 0.8 respectively.

It would be reasonable to expect an expression of the form of Eqn 3.4 to provide a good fit to the observed variation of undrained strength with depth (Fig. 3.6[a]). Consider a soil element at a depth D below the surface of a typical clay sample with submerged unit weight γ' . During the consolidation procedure described above, this element is subjected to a maximum vertical effective stress of

$$\sigma'_{v0} = 200 \text{ kPa} + \gamma'D \quad (3.5)$$

Upon total removal of the compressive load, and after pore suctions have dissipated, the vertical effective stress is simply $\sigma'_{vi} = \gamma'D$. The OCR at depth D is therefore

$$OCR = \frac{200 \text{ kPa} + \gamma'D}{\gamma'D} \quad (3.6)$$

This function is plotted in Fig. 3.8[a], from which it is clear that even the bottom of the sample is very heavily overconsolidated. The submerged unit weight of the unloaded clay sample may be estimated from

$$\gamma' = \left(\frac{G_s - 1}{wG_s + 1} \right) \gamma_w \quad (3.7)$$

where γ_w is the unit weight of water. Taking a representative value for w of 50% (see Fig. 3.6[b]), Eqn 3.7 yields $\gamma' = 6.85 \text{ kN/m}^3$. Fig. 3.8[b] shows the observed variation of s_u with depth, together with the results of a nonlinear fit

$$s_u = A \cdot \sigma'_{vi} \cdot OCR^B \quad (3.8)$$

with $\sigma'_{vi} = \gamma'D$ and OCR as defined by Eqn 3.6. An expression of this form clearly provides an excellent fit to the measured strengths over the depth range considered; furthermore, the values of the regression coefficients A and B (see Fig. 3.8[b]) imply

$$\left(\frac{s_u}{\sigma'_{vi}} \right)_{nc} = 0.253 \quad \text{and} \quad \Lambda = 0.754$$

which are both close to the typical values (Houlsby, 1993) quoted above. It is interesting that Ladd *et al.* (1977) obtained the best fit to their data by varying Λ from 0.85 to 0.75 with increasing overconsolidation, which is consistent with the fitted value of 0.754 obtained here for heavily overconsolidated clay. A simple linear regression provides a reasonable

approximation to the observed strength profile (see Fig. 3.8[b]), but the standard error of regression SE_{fit} is more than three times greater than that for nonlinear regression using Eqn 3.8.

3.3 Apparatus for Model Footing Tests

3.3.1 General Design Considerations

Section 2.6.2 introduced the concept of the yield surface tracking test, in which a model foundation is subjected to either horizontal displacement h , rotation θ , or a combination of the two, at constant vertical penetration z . For a particular model footing, experimental exploration of the complete $V:H:M$ yield surface using tracking tests (Fig. 2.23) would require a testing apparatus capable of subjecting the footing to combined h and θ - in various ratios - with the vertical displacement z locked. Furthermore, some means of simultaneously measuring all three load components V , H and M would be required.

A concept design drawing of the system developed for this research is shown in Fig. 3.9. Vertical, horizontal and rotational displacements of the footing are guided by separate bearing arrangements (two linear and one rotary), allowing each degree of freedom to be controlled independently of the others. The entire rotary system is mounted on a plate which moves horizontally with respect to a second, larger plate; this second plate moves vertically with respect to a rigid frame attached to the cylinder containing the soil sample. In this way any desired footing displacement path in $z:h:\theta$ space may be achieved by superimposing the relevant components of movement. The shaft connecting the footing to the motion system incorporates a load cell which can measure the vertical, horizontal and moment loads acting on the footing. Additional instrumentation (not shown in Fig. 3.9) measures the three components of displacement.

3.3.2 Preliminary Design Development

Since spudcan penetrations of two diameters are rarely exceeded in jack-up operations (Endley *et al.*, 1981), this value was thought to be suitable maximum for investigating the evolution of the $V:H:M$ yield surface with footing embedment. The overall dimensions of the clay samples (Fig. 3.3) clearly placed a limit on the size of footing which could be driven to a penetration of two diameters without excessive interference from the tank boundaries. Program *FIELDS* (Section 2.3.3) was used to compute theoretical slip line fields for a rough conical footing ($\beta = 150^\circ$) embedded in uniform clay. From graphical output resembling Fig. 2.4[b], it was determined that footings of diameter 100 mm or less could be driven to a depth of two footing diameters in a 450 mm diameter tank without the possibility of interference from the boundaries. A model footing diameter of 100 mm was therefore settled upon for the preliminary design. Having established the footing diameter and maximum penetration, the largest expected values of the loads V , H and M were estimated from Brinch Hansen's (1970) semi-empirical bearing capacity method and from previous experimental observations (Meyerhof, 1953; Santa Maria, 1988). For these calculations a design profile of undrained

shear strength with depth was derived from the vane test results of Gue (1984), the original user of the consolidation apparatus. The maximum expected *working* loads for the 100 mm diameter footing were obtained as follows: $V = 1000 \text{ N}$, $H = 300 \text{ N}$, $M = 12.5 \text{ Nm}$. A safety factor of 1.5 was adopted for the design of the motion system and the load cell.

The required vertical stroke of the apparatus was determined by the need for a 200 mm footing embedment. After allowing for an extra 30 mm of penetration and the need to retract the footing tip well clear of the clay surface before and after a test, the design vertical travel was fixed at 300 mm. The maximum required horizontal displacement was more difficult to estimate, but Tan (1990) found that only small movements (approximately 0.1 footing diameters) were needed to trace out the $V:H$ yield locus in a horizontal displacement tracking test. Because of the uncertainty involved, and the likelihood that the apparatus would be used for completely different tests in the future, a very conservative design horizontal travel of $\pm 25 \text{ mm}$ was adopted. Similarly for rotational displacements, the testing rig was designed to provide footing tilts of $\pm 15^\circ$ even though Santa Maria (1988) found that, in general, no more than 6° was needed to reach an ultimate moment. Table 3.2[a] summarises the design travel information for the three axes.

Design rates for applying the three displacements also had to be established. In attempting to model undrained soil behaviour it is important that any loading increment is conducted reasonably quickly; this is especially true when working with a very permeable clay like Speswhite kaolin. On the other hand, excessively fast displacement rates may introduce unwanted complications such as viscous effects. The scanning capabilities of the datalogging system (discussed below) also had to be borne in mind at the design stage to ensure that adequate sampling resolution of rapidly changing loads and displacements could be achieved. A useful guide in determining design rates of displacement was the previous experience of Santa Maria (1988), who used a vertical penetration rate of 0.33 mm/s and a rotation rate of 0.067 deg/s for his combined load testing of 100 mm diameter footings (also on Speswhite kaolin). The displacement rates adopted for design of the apparatus are listed in Table 3.2[b].

One very important design criterion was that the vertical footing penetration be easily "lockable" to cater for tracking tests at constant z . A highly rigid drive and bearing mechanism, free from backlash, was seen as essential for each of the three displacement axes. The apparatus had to be constructed within a very limited budget (£7000, excluding technician time, in 1991) since external funding of the project was not forthcoming. This tight cost control was, however, a useful incentive for keeping the equipment both simple and easy to maintain. Figs 3.10[a] and 3.10[b] show the completed testing rig in operation. The following sections are devoted to a detailed look at the various components of the system, namely the:

- reaction frame and main back plate;
- vertical, horizontal and rotational motion systems;
- model spudcan footing and connecting shaft;
- data acquisition unit;

- load and displacement instrumentation;
- control equipment.

3.3.3 Reaction Frame and Back Plate

As shown in Fig. 3.10, the main back plate of the apparatus (labelled PB) is made of dural, $360 \times 675 \times 25$ mm thick. The lower end of this plate is bolted to a 25 mm thick base plate B1. The reaction frame RF, constructed from 100×50 mm dural box sections mounted to a second 25 mm thick base plate B2, provides support for the top of the main back plate PB. Box sections have the advantage of being very light, yet they still provide excellent resistance to both torsion and bending. In order to make handling of the rig as simple as possible, the main back plate, reaction frame and base plates were designed to remain permanently connected together. The whole testing rig is fixed to the top flange of a clay tank with four M12 bolts, two through each of the base plates B1 and B2.

3.3.4 Vertical Displacement Control System

The main back plate PB provides support for the vertical motion system. This consists (Figs 3.10, 3.11[a]) of a second plate PV ($365 \times 480 \times 19$ mm thick) which moves up and down on a linear guide GV, powered by the stepper motor MV and ballscrew SV.

The linear guide system, manufactured by Hepco Slide Systems Ltd of London, is attractively simple and substantially cheaper than linear bearings which rely on a recirculating ball system. The Hepco system consists of a track (or "slideway") and four wheels (known as "journals") which are mounted on a carriage plate. The slideway has two precision ground male vee-shaped edges, and the journals have a corresponding female profile. An extremely rigid bearing system is possible because the journals are supplied in pairs, eccentric and concentric. As shown in Fig. 3.11[b], each eccentric journal is positioned opposite a concentric one and tightened up against the slideway to produce a very light preload on the bearings, just enough to eliminate play. This tightening process, requiring one wrench to hold the eccentric journal in the correct position and another to lock up the fixing nut, must be performed very carefully (too much preload results in excessive friction and reduced bearing life). The journal bearings are grease packed for life and do not require any maintenance, nor is lubrication of the slideway needed for smooth operation of the system. An occasional spray with WD40 was, however, found to be useful in preventing excessive corrosion.

The steel used for the slideway plate is a carbon chromium alloy type with a nominal tensile strength of 800 MPa. The centre of the slideway is left untreated (for ease of machining holes, *etc.*) but the vee tracks are induction hardened. Opposing faces of the vee tracks are guaranteed parallel to within 0.013 mm per metre of travel. The main body of each journal is made of mild steel, with standard chromium steel ball bearings inside. The necessary journal and slideway sizes were determined from the manufacturer's charts, using the design working loads determined in Section 3.3.2 with a safety factor of 1.5. It was realised that all three components of load would never be at their design maxima

simultaneously, so a number of probable worst case combinations were investigated. The length of the vertical slideway (600 mm) was determined by the need to accommodate a 300 mm stroke (Table 3.2[a]), plus the 300 mm vertical spacing of the journal pairs. This latter figure was determined from Hepco's design charts as the minimum journal spacing needed to resist the worst expected combination of footing loads V , H and M .

Linear vertical motion is generated by a ballscrew and nut arrangement. Manufacturer's design charts, incorporating checks on buckling, ball bearing deformation and fatigue life, were used to select the screw diameter of 16 mm. The smallest available lead (2 mm) was chosen since this maximises the mechanical advantage of the screw and also optimises the resolution (linear travel per step) of the stepper motor. Identical bearing blocks BB (Fig. 3.11[a]), incorporating both radial and thrust ball bearings, receive the specially machined ends of the ballscrew. To eliminate play, the lower bearing block is tightened hard up against the ballscrew using slotted fixing holes in the main back plate PB. An M25 thread on the ballnut housing allows it to be screwed into a small dural block, which is in turn securely bolted to the vertical sliding plate. The ballnut has a guaranteed axial play of no more than 0.03 mm.

Referring to Fig. 3.11[a], torque is applied to the ballscrew SV by the small stepper motor MV, *via* a 50:1 reduction gearbox RV and a "Panamech" flexible coupling CV to cope with any slight offset between the axes of the gearbox output shaft and the ballscrew. The choice of a stepper motor (rather than a DC servo motor) was determined by the need to maintain constant vertical displacement z when performing a tracking test. An expensive closed loop feedback arrangement would be needed to achieve this with a DC servo motor, and the supposedly fixed vertical displacement would actually be fluctuating about a target value. On the other hand an energised stepper motor automatically stays locked in position if it is not being driven, making it a natural choice for this application. While stepper motors themselves are very cheap, the necessary control hardware is costly (particularly the modern intelligent controllers which must be linked to a host computer - see Section 3.3.12 below). The stepper motor solution was, nevertheless, substantially cheaper than the DC servo motor option.

Stepper motors running at low speed frequently suffer from a drop in power output caused by resonance; they usually operate most satisfactorily at 400-1000 r.p.m.. Clearly this sort of speed is not suitable for direct application to the chosen ballscrew as it would produce a vertical penetration rate of around 20 mm/s (*cf.* the design velocity of 0.3 mm/s, Table 3.2[b]). The 50:1 gearbox reduction allows the motor to run within its optimum speed range and yet still produce the desired range of linear displacement rates. Another advantage of choosing the smallest available lead is now apparent, since this minimises the required gearbox reduction ratio. With knowledge of the maximum vertical load, ballscrew lead and gearbox ratio, the necessary torque output could be determined and a suitable stepper motor selected. The small lead and high gearbox reduction ratio meant that a size 23 motor could provide more than enough power. The 23HS-108, manufactured by McLennan Servo Supplies Ltd, is a standard 4 phase stepper motor (maximum 3.9 A/phase) providing 400

steps of 0.9° per revolution. The phases are wired in parallel rather than series (to give optimum torque output at high speed) and driven by a McLennan PM164C bipolar translator. To minimise electromagnetic interference with the instrumentation, power supply wiring to the stepper motor uses shielded cable and the motor housing is earthed. An intelligent stepper motor controller (McLennan PM301) accepts position, velocity and acceleration commands from the host IBM compatible PC *via* a 3 wire RS 232 interface. These control commands are discussed in more detail in Section 3.3.12. Two limit switches, one of which appears (labelled LV) in Fig. 3.11[a], are wired directly to the stepper controller and cause immediate motor stoppage if tripped. Although only an emergency measure, this is a certain means of ensuring that the vertical sliding plate cannot run off its track.

3.3.5 Vertical Load Control System

About halfway through the experimental programme the apparatus was modified to allow yield surface probing tests at constant vertical load V (Section 2.6.1) in addition to tracking tests at constant z . The improved rig permits quick and simple disconnection of the vertical sliding plate from its ballscrew drive, leaving the plate free to move up and down on its slideway. The footing load V is then just the self-weight of the vertical sliding plate (and the other mechanisms it transports), minus the friction of the Hepco linear guide system. If a larger V is desired, extra weight is added using a simple hanger. Vertical loads less than the self-weight value are achieved by applying an upwards force to the vertical sliding plate *via* a counterweight system.

Fig. 3.12[a] shows a general view of the apparatus set up for a vertical load control test. To accommodate the hanger HA and its weights WW, the clay tank is placed on a 1 m high stand ST made of mild steel angle sections. The hanger, also made of mild steel, is designed to support up to 100 kg. For convenience its knife edge rests in a groove on top of the horizontal sliding plate (rather than the vertical sliding plate), but of course this does not affect the vertical load V which is applied to the footing. The plastic water tank WT provides a convenient means of making fine adjustments to the vertical hanger load. It has a supply hose (13 mm i.d.) connected to mains pressure, and a dump hose (30 mm i.d.) which siphons into a laboratory drain. Valves in both hoses are located adjacent to the host computer, so the amount of water in the tank may be increased or decreased by the operator as he watches the vertical footing load V displayed on the screen in real time. Because the vertical slideway edges are not perfectly parallel (and because the journals are not perfectly round), the upward vertical load due to bearing friction changes appreciably as the footing moves up or down during a load controlled test. Adjustments to the level in the water tank can compensate very effectively for these variations and ensure that the V applied to the footing remains very close to its target value. The quantitative performance of this load control method is discussed in Sections 4.5 and 4.6 with the relevant test results.

A rear view of the apparatus set up for a test under vertical load control appears in Fig. 3.12[b]. The counterweight pulleys CP (made of brass) are supported by two steel angle sections bolted to the reaction frame of the rig, and a steel cable connects the weights

CW to the vertical sliding plate. In the original (vertical displacement control) design described above, the ballnut was fixed to a small dural block which was permanently fastened to the vertical sliding plate using M6 capscrews. With the original apparatus fully assembled, these screws were obscured by the horizontal sliding plate and were not accessible during a footing test. In the modified apparatus the ballscrew nut block is fixed from the rear, as shown in Fig. 3.12[b], with access to the four M6 capscrews CS through two 15 mm wide slots in the main back plate PB. By removing these capscrews and sliding out the spacing plate SP, the apparatus may be easily converted to vertical load control mode. Returning to the original ballscrew-driven, displacement controlled operation of the vertical motion system is equally straightforward.

3.3.6 Horizontal Motion System

As shown in Fig. 3.13[a], the entire horizontal motion system is mounted on the vertical sliding plate PV. The operating principle is identical to that of the vertical displacement control system described in Section 3.3.4. The horizontal sliding plate PH (395 × 270 × 19 mm thick) is driven back and forth along its linear guide GH by the stepper motor MH and ballscrew SH. For this axis a smaller ballscrew (10 mm diameter × 2 mm lead) is adequate for the design horizontal load, but a bigger reduction ratio for the gearbox RH (250:1) is needed to allow optimum motor operation at the design horizontal displacement rate of 0.05 mm/s (Table 3.2[b]). The horizontal stepper motor is of the same type as the vertical one (23HS-108); all wiring and control hardware details are also identical to those of the vertical stepper motor.

3.3.7 Rotational Motion System

A slightly different approach is needed to generate tilting of the footing. As shown in Fig. 3.13[b], rotation with respect to the horizontal sliding plate PH is generated with a 60° segment of Hepco slide ring PR (30 mm thick), a variation on the linear slideways described above. This slide ring arrangement is ideal for the testing apparatus because it allows rotary motion without the need for a fixed pivot at the centre of rotation. Note that here the journals remain fixed to the horizontal sliding plate, and the slide ring segment is the moving part.

MR in Fig. 3.13[b] is another 23HS-108 stepper motor (with reduction gearbox RR), and rotational movement of the slide ring segment is generated with a rack and pinion system. The blue rack FR is a tough injection moulded plastic product called "Flexi-Rack", supplied by the Japanese company Nirei. Bending a metal rack around the relatively tight curve would almost certainly have resulted in localised yielding, but the plastic Flexi-Rack is specifically designed for such treatment. The 1.0 Module rack size has ample capacity for resisting the maximum expected shear force at the rack/pinion interface. The outside face of the dural mounting block MB was accurately cut to the required 425 mm radius by an NC machine in the main Departmental workshops. The Flexi-Rack is held in position on this auxiliary block by 22 L-shaped steel clips, and the block itself is rigidly bolted to the slide

ring segment. A specially machined 10-tooth pinion gear (not visible in Fig. 3.13[b]) is attached to the gearbox output shaft using a standard metric keyway. This is the smallest available 1.0 Module spur gear, giving the greatest possible reduction effect on the footing rotation rate. The design footing rotation rate of 0.05 deg/s (Table 3.2[b]) is equivalent to less than 0.01 r.p.m.. Although a reduction of about 80:1 is achieved with the rack and pinion system alone, a further 500:1 reduction with the gearbox RR is needed to allow the stepper motor to run within its optimum speed range (*i.e.* at several hundred r.p.m.). An alternative scheme using a worm and wheel was investigated, as this would have offered much greater direct reduction and eliminated the need for a gearbox. Unfortunately standard wormwheels are only available up to a radius of around 100 mm; a wormwheel segment with the very large radius required here could have been custom made, but the prices quoted by various manufacturers were prohibitive.

3.3.8 Model Spudcan Footing and Connecting Shaft

Figs 3.14[a] and [b] show the two model spudcan footings used in the experimental research. The original spudcan design (100 mm diameter) was used only for the first batch of tests; the remaining footing tests (Batches 2 to 11) were conducted on the slightly larger 125 mm diameter spudcan. Reasons for this change of footing size are discussed in Section 4.4.2. Both spudcans have the shape adopted as "typical" for the Joint Industry Study of Jack-up Foundation Fixity (Noble Denton and Associates, 1987). Santa Maria's (1988) model spudcan footing resembled the one drawn in Fig. 3.14[a] except that it had a vertical upstand of 10 mm and a flat upper surface. The footings shown in Fig. 3.14 have a smaller 2.5 mm vertical upstand and an inverted conical upper surface, making them more representative of recent jack-up footing shapes. As illustrated in Fig. 1.3, real spudcan footings are actually polygonal in plan, sometimes with diameters in excess of 20 m. The use of circular model footings in this research is thus a convenient idealisation, and the linear scaling factor is of the order of 80 to 180. It should also be noted that, compared with the model footing used in the testing, the conical tip on many modern spudcans is smaller in proportion to the overall footing diameter.

The shaft which connects the model footing to the motion system is made of mild steel and is bolted to the Hepco slide ring segment (Fig. 3.13[b]) through slightly oversized clearance holes. This allows fine adjustment of the footing position such that the load and displacement reference point of the footing (marked RP in Fig. 3.14) coincides precisely with the centre of rotation of the slide ring. Adjustment is performed by repositioning the shaft until point RP does not move perceptibly (with respect to an adjacent fixed spike) when the slide ring is rotated from one side to another. In the interests of maximum horizontal and rotational stiffness, the connecting shaft is braced by two small struts (see Fig. 3.13[b]) which reduce its unsupported length to just over 275 mm.

Fig. 3.14[c] shows the attachment of the model spudcan to the connecting shaft. The waterproofing shield, also visible in Figs 3.10 and 3.13[b], is made of welded 13 gauge mild steel plate (2.34 mm thick). Its purpose is to protect the strain gauged load cell (Section

3.3.10) from contact with moisture, recalling that the footing tests are conducted with a layer of water covering the clay sample. The waterproofing shield has a 6 mm wide flange welded around its base; this flange slots into the top of the model footing and is secured by 16 M3 screws. To ensure a watertight joint, the exterior metal surfaces were thoroughly cleaned with 1-1-1 trichloroethane and a bead of silicone sealant was applied over the top of the M3 screws. Fig. 3.15 illustrates the various steps involved in fitting the waterproofing shield to the model spudcan. Painting of the exterior of the waterproofing shield not only prevented corrosion, but was an effective means of sealing pinhole leaks in the welded joints. The plan dimensions of the waterproofing shield clearly had to be kept as small as possible in order to maximise the amount of uninhibited footing rotation and sliding which could occur at deep embedments (Fig. 3.10[b]). On the other hand the interior dimensions of the shield had to provide adequate clearance around the load cell (36×36 mm in plan); as shown in Fig. 3.14[c], the interior shield size of 39×39 mm gives 1.5 mm clearance at each edge. The connecting shaft is as wide as possible while still maintaining a reasonable 2.0 mm clearance from the waterproofing shield on either side. Note that there is no contact between the waterproofing shield and the connecting shaft at any point.

3.3.9 Data Acquisition Unit

The laboratory was equipped with an existing Mowlem Microsystems "Autonomous Datalogging Unit". The ADU's analogue input module accepts up to eight DC signals and also provides a precision DC transducer supply of $10 \text{ V} \pm 10 \text{ mV}$. Analogue to digital conversion is performed to 12 bit accuracy (1 in 4096), with a full scale input range of $\pm 10 \text{ V}$. The A/D converter module also contains a preamplifier with eleven software selectable gain ranges ($\times 1$, $\times 2$, ... , $\times 1024$), allowing even a $\pm 10 \text{ mV}$ signal to be fed directly to the ADU. Once digitised, channel readings may (if desired) be stored in the ADU's battery-backed internal memory, expanded from 16 kByte to 256 kByte specifically for this research. A built in microcomputer supervises all aspects of the ADU's operation and handles RS 232 communications with the host PC. Before a test the host sends a series of simple setup commands to the ADU (see Section 3.3.12). Thereafter data acquisition becomes the responsibility of the ADU, although the host computer may request a reading of any channel at any time. At the completion of a test, the contents of the ADU's memory are downloaded to the host PC for permanent storage on floppy disk.

3.3.10 Load Cell

A special "Cambridge" type load cell measures the vertical, horizontal and moment loads acting on the model footing. The evolution of this load cell design from the original Stroud (1971) cell is described by Bransby (1973), who also gives a detailed explanation of how these devices operate. Fig. 3.16 shows the cell designed for the present research. Loads applied to the lower, or active, face of the cell are transmitted to the main upper block through thin strain gauged webs. A compressive vertical load causes compression in both sets of vertical webs (marked "a" and "b"), while a pure moment induces compression in one

set of vertical webs and tension in the other set. Horizontal load applied to the active face is mainly transmitted to the upper block through the four horizontal webs (marked "c"). Every web is strain gauged on both sides, with each pair of gauges connected in series to maximise sensitivity to axial web deflections. These strain gauge pairs are connected to form a full Wheatstone bridge circuit for each set of webs ("a", "b" and "c"). Each vertical circuit utilises four dummy gauges (mounted on the main upper block of the cell) to complete a full bridge, while all eight gauges in the horizontal load circuit are active. Sensitivity is further enhanced by the use of $350\ \Omega$ strain gauges in preference to the more common $120\ \Omega$ type, and by the choice of strain gauges which cover as much of their respective web areas as possible. Excitation for all three strain gauge circuits is provided by the precision 10 V DC supply incorporated in the ADU. Sensitivity at full design working load is 0.75 mV/V for each vertical load circuit, and 1.5 mV/V for the horizontal load circuit. These very small output signals require considerable preamplification by the ADU ($\times 256$) before A/D conversion. The load cell is made of dural ($E = 70000\ \text{MPa}$, 0.1% proof stress = 220 MPa) and the webs are sized to give deflections of about 1000 microstrain under the worst anticipated combination of working loads, in accordance with Bransby (1973). Permanent strain of 0.1% in the webs would be expected at an overload factor of about 3.0, while the safety margin on web buckling is much greater. The cell was supplied (complete with strain gauges and wiring) by Cambridge Insitu Ltd.

Calibration of the load cell was performed twice, once (on the brand new cell) before assembly of the rig, and again after the apparatus was dismantled at the end of testing. Despite intensive use of the cell over a period of 14 months, the two sets of calibration constants agreed to within 1%. Since the second calibration was performed somewhat more carefully, and because most experiments were conducted in the latter half of the testing period, experimental $V:H:M$ load data reported in the following chapters are based on the more recent calibration. Both Stroud (1971) and Bransby (1973) give full accounts of the calibration procedure and the precautions which must be taken, but in essence the technique involves holding two of the applied loads V , H and M constant while varying the other. Fig. 3.17 includes a picture of the load cell calibration apparatus. Vertical load V is varied by adding weights to the hanger HH when it is positioned centrally on the calibration block CB. Moment M is changed by moving the hanger (with a constant V) to a series of different eccentricities on the calibration block. Horizontal load H is applied by hanging weights on the end of the steel cable SC which runs over a pulley (not shown). Separate calibrations for positive H and negative H must be performed, although the same calibration constants would be expected. The height at which the horizontal load acts is clearly of considerable importance. As shown in Fig. 3.14[c], the footing/load cell interface is located 7.5 mm above the load and displacement reference point level, so during calibration the line of action of the horizontal force is 7.5 mm above the active face of the cell (see Fig. 3.17). Note that during calibration the cell is upside down, with its active face uppermost.

Results of the final load cell calibration are plotted in Fig. 3.17, and it is clear that all three strain gauge circuits give highly linear responses over the range of loads applied. The gradients of the nine regression lines become the matrix entries in

$$\begin{Bmatrix} R_a \\ R_b \\ R_c \end{Bmatrix} = \begin{bmatrix} 3.821 \times 10^{-1} & 2.642 \times 10^{-1} & -2.352 \times 10^1 \\ 3.815 \times 10^{-1} & -2.606 \times 10^{-1} & 2.404 \times 10^1 \\ 6.890 \times 10^{-4} & 2.682 & -2.408 \end{bmatrix} \begin{Bmatrix} V \\ H \\ M \end{Bmatrix} \quad (3.9)$$

where R_a , R_b and R_c are the strain gauge bridge outputs (in bits, after amplification and A/D conversion) and V , H and M are the applied loads (in N and Nm). Inversion of this equation allows V , H and M to be deduced from the strain gauge bridge outputs (again using units of bits, N and Nm):

$$\begin{Bmatrix} V \\ H \\ M \end{Bmatrix} = \begin{bmatrix} 1.324 & 1.295 & -4.605 \times 10^{-3} \\ -1.940 \times 10^{-2} & 1.875 \times 10^{-2} & 3.766 \times 10^{-1} \\ -2.122 \times 10^{-2} & 2.125 \times 10^{-2} & 4.155 \times 10^{-3} \end{bmatrix} \begin{Bmatrix} R_a \\ R_b \\ R_c \end{Bmatrix} \quad (3.10)$$

Although measurement errors due to nonlinearity and hysteresis may be estimated from the calibration results, they are only really applicable to the particular loadings used during calibration (*i.e.* pure V , pure H , and M at constant V). A better idea of the overall load cell accuracy may be gained by applying various pre-determined combinations of V , H and M , then comparing the known loads with those calculated from Eqn 3.10. Ten such accuracy tests were performed, using the calibration apparatus to apply typical (V, H, M) load combinations which had been encountered during the experimental programme. As shown in Table 3.3, most of the measured loads were accurate to within ± 1.5 N, ± 0.5 N and ± 0.05 Nm for V , H and M respectively. In terms of precision the load cell also performed well, particularly on vertical load measurement ($\pm 0.2\%$ or better in all cases). Relative errors in measuring H and M did not exceed 1% in any of the checks.

Load measurement errors due to drift and random A/D conversion noise may be estimated from Fig. 3.18. The load cell circuit outputs are very stable, and over a period of one hour there is no discernible drift in any of the three measured loads V , H or M . Most footing tests actually lasted well under one hour. When configuring input channels on the ADU (with commands from the host computer) there is an option for using or bypassing a 50 Hz filter designed to reduce interference from the mains power supply. The right hand column of plots in Fig. 3.18 shows that, *without* the 50 Hz filter, random fluctuations in the measured loads V , H and M are approximately ± 5 N, ± 0.5 N and ± 0.1 Nm respectively. With the 50 Hz filter option activated, random noise is reduced by a factor of approximately 10 for each load reading. The penalty for this vastly improved performance is that the A/D conversion process for a single channel takes 20 milliseconds, because the "filtered" reading is in fact just the average of many A/D conversions performed over one 50 Hz cycle. With the filter option switched on for all channels, however, it was still possible to perform a complete scan of the load and displacement instrumentation in just under 0.25 seconds. A

logging frequency of 4 Hz was therefore adopted as the standard data acquisition rate for the experimental programme.

3.3.11 Displacement Measurement: LVDTs

The four LVDTs (specifications in Table 3.4) are of the AC type, but they do not have any self-contained electronics. A separate signal conditioning and power supply unit converts the output of each transducer to DC and also supplies the AC excitation voltage (5 V r.m.s. @ 5 kHz). LVDT Nos. 2, 3 and 4 were calibrated against a micrometer, LVDT No. 1 against a steel ruler. The calibration factors were checked four times during the research, with observed fluctuations in the range $\pm 0.5\%$.

As shown in Fig. 3.11[a], both of the vertical LVDTs (labelled L1 and L2) are attached to the main back plate PB with dural brackets. The long stroke LVDT No. 1 remains fixed in position throughout a test and provides a coarse measure of vertical displacement over the full travel distance. More accurate measurements during significant parts of a test, such as vertical unload-reload loops, are obtained by fixing the bracket for LVDT No. 2 at the appropriate height (note in Fig. 3.11[a] that there are several pairs of fixing holes FH at vertical spacings of 50 mm). Both displacement transducers measure the deflection of the top of the vertical sliding plate PV. This is clearly not ideal since it is the movement of the model footing with respect to the clay sample that is of interest. Unfortunately the need to accommodate large horizontal and rotational footing movements meant that a more direct measurement of the vertical footing displacement was not practicable.

Horizontal displacement and rotation of the footing also had to be measured somewhat remotely. In Fig. 3.13[a], LVDT No. 3 (labelled L3) measures the movement of the horizontal sliding plate PH relative to the vertical sliding plate PV. Referring to Fig. 3.13[b], footing rotation is deduced from the displacement of a pin PN mounted to the connecting shaft, as measured by the LVDT L4. The other end of LVDT No. 4 (not shown in Fig. 3.13[b]) is attached to a second pin which is fixed with respect to the horizontal sliding plate, and since the footing reference point RP coincides with the centre of rotation, it too is fixed relative to the horizontal sliding plate. The cosine rule may therefore be applied to the triangle formed by RP and the two ends of LVDT No. 4 to calculate the footing rotation θ .

Displacement measurement errors due to drift and noise may be estimated from Fig. 3.18. Random fluctuations for the four displacements are very small with the 50 Hz filter activated, well inside the design targets of ± 0.01 mm, ± 0.005 mm and $\pm 0.005^\circ$ for fine vertical, horizontal and rotational displacement measurements. All of the LVDTs show excellent stability over a one hour period. It must be remembered that Fig. 3.18 does not give any indication of errors arising from LVDT nonlinearity (see Table 3.4), but this is not likely to be a serious source of uncertainty when the measured displacements only involve a small fraction of the total LVDT stroke (as they do for LVDT Nos. 2, 3 and 4).

In order to deduce footing displacements from the LVDT readings, a correction must be made for the flexibility of the apparatus. This is achieved most easily with an equation of the form

$$\begin{Bmatrix} z \\ h \\ \theta \end{Bmatrix}_{\text{footing}} = \begin{Bmatrix} z \\ h \\ \theta \end{Bmatrix}_{\text{LVDT}} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{Bmatrix} V \\ H \\ M \end{Bmatrix} \quad (3.11a)$$

where the b_{ij} are experimentally determined apparatus flexibility coefficients. Just as with the load cell calibration, the matrix entries were determined by holding two of the applied loads constant and varying the third. Fig. 3.19[a] shows the miniature hydraulic jack HJ used to apply vertical load (at various eccentricities) to a special footing attachment FA. The jack was placed on a pile of dural plates DP inside an empty clay tank; the testing rig was firmly bolted to the tank with all three stepper motors energised but stationary. Horizontal loads were applied to the footing attachment using weights and a pulley, as shown in Fig. 3.19[b]. For each combination of load, three sensitive dial gauges (± 0.001 mm) were used to measure the actual footing displacements z , h and θ with respect to the clay tank; any small changes in the LVDT readings were also noted. For units of mm, degrees, N and Nm, the apparatus flexibility matrix in Eqn 3.11a was found to be

$$\begin{bmatrix} -2.75 \times 10^{-4} & -2.69 \times 10^{-5} & 7.67 \times 10^{-5} \\ -1.22 \times 10^{-4} & -2.14 \times 10^{-3} & 5.39 \times 10^{-3} \\ 6.54 \times 10^{-6} & 3.49 \times 10^{-4} & -4.63 \times 10^{-3} \end{bmatrix} \quad \text{for } H > 0 \quad (3.11b)$$

and

$$\begin{bmatrix} -2.75 \times 10^{-4} & 1.31 \times 10^{-4} & 7.67 \times 10^{-5} \\ -1.22 \times 10^{-4} & -2.25 \times 10^{-3} & 5.39 \times 10^{-3} \\ 6.54 \times 10^{-6} & 3.61 \times 10^{-4} & -4.63 \times 10^{-3} \end{bmatrix} \quad \text{for } H < 0 \quad (3.11c)$$

Note that both positive and negative horizontal loads result in an upward (*i.e.* negative z) flexibility-related movement of the footing. There is a considerable difference in magnitude between the two b_{12} coefficients, however, possibly because of the eccentric location of the vertical ballscrew drive with respect to the vertical slideway (see Fig. 3.11[a]).

3.3.12 Control and Monitoring of Tests

Fig. 3.20[a] shows the electronic equipment set up. The ADU and the three axis stepper motor control unit MC are linked to the same host computer, an IBM compatible 80286 PC. Also shown in Fig. 3.20[a] is the signal conditioning and power supply unit for the LVDTs, labelled SC. During a footing test the host computer executes program *CONTROL.BAS*, written in compiled QuickBASIC. This program oversees the running of a test and handles all communication over the two separate RS 232 serial interfaces (running at 9600 baud). Both the ADU and stepper drive unit are controlled and queried with simple, meaningful ASCII character strings. Note that since each stepper motor has its own intelligent

controller, truly independent control of the three axes is possible. For example, commands may be issued to the horizontal motor controller while the vertical motor is running. The generation of simultaneous horizontal and rotational footing displacements (an essential capability of the apparatus) is also straightforward. The stepper motor drive unit allows full digital specification of displacement, velocity and acceleration details for each axis.

Motor step counts are related to their respective footing displacements by the conversion factors listed in Table 3.5. These numbers were determined by moving each sliding plate a precisely known distance (monitored with a recently calibrated LVDT) and noting the number of motor steps required. Accuracy of the factors in Table 3.5 is not of crucial importance since the LVDTs described in Section 3.3.11 are the primary means of measuring footing displacements. The most important use of the conversion factors is in the QuickBASIC control program, where footing velocities specified in mm/s (or deg/s) need to be expressed in step/s when issuing instructions to the motor control unit.

Program *CONTROL.BAS* performs a standard initialisation routine for all footing tests, but the sequence of displacement- or load-controlled events to be executed thereafter must be specified in a separate test control file. The syntax used in these test control files is very simple, and several examples are given in the following chapter. During a test, *CONTROL.BAS* updates load and displacement information on the host computer screen four times a second (see Fig. 3.20[b]). Every time the screen is updated, all load and displacement information is written to a hard disk file. Displacements logged to this file are derived from the motor step counts. When the contents of the ADU's memory are retrieved by the host computer at the end of a test, this information is written to a second disk file. Displacement information in this second file is, of course, provided by the LVDT readings. The file derived from step counts provides a supplementary test record and is useful as a check on the displacements measured by the LVDTs. Given the frequency of datalogging, both output files are inevitably rather large: approximately 1 MByte each for a typical 40 minute test.

EXPERIMENTAL PROCEDURES & RESULTS

4.1 Introduction

This chapter describes the programme of model footing tests performed using the apparatus described in Chapter 3. Each clay sample was used to conduct a number of separate displacement- or load-controlled footing experiments at different levels of embedment. In this chapter, the word "test" refers to the group of individual footing experiments ("events") carried out with a single clay specimen. Eleven batches of three kaolin samples were prepared; each test is identified by the letter "T" followed by two digits indicating the batch number (01 to 11), with a third digit to specify the sample number (1 to 3) within that batch. Thus T072 refers to the second clay sample of the seventh batch.

The three types of footing experiment described in Section 2.6, namely tracking, probing and loop events, were allocated to the different clay batches as follows:

- **Batch 1:** trial tracking events;
- **Batches 2 - 6:** main series of tracking events;
- **Batch 7:** loop events;
- **Batches 8 - 10:** probing events;
- **Batch 11:** further tracking events, investigating displacement rate effects.

The following pages contain a detailed description of the footing experiments carried out on each clay sample, along with a preliminary discussion of the results obtained. Chapter 5 will be devoted to further interpretation and analysis of the data.

4.2 Procedures Common to All Tests

Every test began with a standard equipment check and initialisation routine. Similarly at the completion of any test, the same method was used to process the results and plot them in a standardised format. These pre- and post-test operations are described in this section.

4.2.1 Test Initialisation

When a kaolin sample was ready for use, the testing rig was bolted onto the top flange of the clay tank. At this stage the footing was in its fully retracted position, such that the tip of the spudcan was some 35 mm clear of the clay surface. As mentioned in Chapter 3, the running of each footing test was computer controlled. The QuickBASIC program *CONTROL.BAS* began by establishing communications with the stepper motor controller and the ADU datalogger. The initial footing position was then adjusted (if necessary) by manual control of the relevant stepper motors. First the footing was levelled with respect to the clay surface,

then positioned horizontally until it was directly over the centre of the clay tank. The stepper motor control unit was then instructed to define these initial horizontal and rotational motor positions as datums (*i.e.* step count = 0). Using a steel ruler, the vertical distance from the footing reference point level (Fig. 3.14) to the clay surface was carefully measured. This distance, typically around 60 mm, was entered into the computer, thus establishing the program's frame of reference for vertical footing penetration.

After resetting the datalogger, all seven channels (three load cell circuits and the four LVDTs) were initialised with appropriate gain settings and checked for correct operation. The datalogger was then instructed to start sampling and storing channel readings in its memory at suitable intervals (determined by the memory capacity of the ADU). For the majority of tests, which lasted between 30 and 60 minutes, data was logged at 4 Hz (*i.e.* a complete scan of all channels every 0.25 seconds). This logging frequency was adjusted accordingly for the tests in Batch 11 which used non-standard displacement rates.

4.2.2 Data Processing

After each test, a raw data file of load and displacement information was retrieved from the datalogger. FORTRAN programs were written to process the load and displacement data and perform various corrections. The first steps in processing a row of raw data were to compute the footing loads V' , H' and M' (using Eqn 3.10) and the displacements z_{RP} , h and θ (using the appropriate LVDT calibration coefficients). These loads and displacements are defined as shown in Figs 4.1[a] and 4.2. Note that loads are calculated at the reference point RP, and that displacements also refer to the movement of this point. The choice of reference point is of course somewhat arbitrary, and other sensible locations might be the footing centroid or the spudcan tip. Transforming loads and displacements from one reference point to another is quite straightforward; an example is given by Dean *et al.* (1992). Vertical displacement of the footing is written as z_{RP} to emphasise that it refers to the penetration of the reference point, not of the tip.

The most important correction to the raw data concerns the transformation of V' , H' and M' to equivalent loads in the $V:H:M$ system. The "vertical" and "horizontal" loads V' and H' registered by the load cell may in fact be inclined by up to 10° when the footing is being rotated (Fig. 4.1[a]). On the other hand the directions of V and H are defined with respect to the undisturbed soil surface (Fig. 4.1[b]), providing a consistent frame of reference which is independent of footing rotation. The necessary transformations are simply

$$\begin{aligned} V &= V' \cos \theta + H' \sin \theta \\ H &= H' \cos \theta - V' \sin \theta \end{aligned} \quad (4.1a;b)$$

The moment M is not affected by spudcan rotation and does not require any tilt correction. With the loads and rotations encountered in this project, V was invariably dominated by its $V' \cos \theta$ term. This was in contrast to the value of H , where footing rotations of just a few degrees were enough to make the contribution of $-V' \sin \theta$ very significant.

Raw displacements measured by the LVDTs had to be corrected for apparatus flexibility, as discussed in the previous chapter (see Eqn 3.11). Corrections to the vertical and horizontal displacements caused no concern, but the adjustment to θ meant that it was slightly different from the θ used in Eqn 4.1 to obtain V and H from V' and H' . This problem was overcome by re-evaluating Eqn 4.1 with the updated θ , obtaining new loads V and H , and repeating the apparatus flexibility correction to the original raw displacements measured by the LVDTs (Eqn 3.11). It was found that, in most cases, only two iterations were needed to achieve 6 digit precision on all loads and displacements.

In the processed and corrected load and displacement data file for each test, moment load is divided by the footing radius R to provide dimensional consistency with V and H . Similarly the rotation θ is multiplied by R to give a displacement with dimensions of length. It is worth noting that since the processed loads and displacements are all referred to the point RP on the spudcan and have consistent, fixed directions, they form conjugate pairs in the incremental work equation

$$\delta W = V\delta z + H\delta h + M\delta\theta \quad (4.2a)$$

or

$$\delta W = V\delta z + H\delta h + (M/R)\delta(R\theta) \quad (4.2b)$$

4.2.3 Standardised Plotting of Test Data in Martin & Houlsby (1994)

The fully corrected results of all 33 footing tests appear, in graphical form, in Martin & Houlsby (1994). Each test record is laid out on two facing pages, and the space required did not allow these figures to be included in the present volume. The first page shows the variation of the three loads and three displacements with time; the second page contains 13 of the 15 possible load:load and load:displacement plots (only $H:z_{RP}$ and $M/R:z_{RP}$ are excluded). An additional graph in each test record shows the measured undrained strength and water content values plotted against depth. Finally there is a text block listing the various events conducted during the test, and stating the displacement rates used. Martin & Houlsby (1994) is intended to act as a comprehensive reference for the entire experimental programme, providing information about the sequence of events in each footing test and a broad overview of the results obtained. The remainder of this chapter will provide a more detailed explanation of the tests performed, with analysis of the significant events in each one.

4.3 Tests T011-T013: Trial Tests

The first three clay samples were devoted to a series of trial tests involving tracking events. These tests were intended to check the performance of the apparatus in applying horizontal footing displacements h or rotations θ , at constant embedment z . As discussed in Section 2.6.2, there are sound theoretical reasons for expecting this type of event to produce a load

path which tracks closely across the $V:H:M/R$ yield surface established by footing penetration to the level z .

4.3.1 Test T011: Procedure and General Comments

The initial stages of this test will be described in some detail to illustrate the procedures involved and the results obtained; the control aspects of subsequent tests have much in common with this first trial. As mentioned in Section 3.12, the sequence of events to be carried out in any particular test was specified in a test control file read as input by the main QuickBASIC control program. Table 4.1 shows an extract from the T011 test control file, illustrating the type of syntax used to generate various events.

The complete test record for T011 may be found on pages 1 and 2 of Martin & Houlsby (1994), but for explanatory purposes the first 19 minutes have been replotted (on facing pages) in Figs 4.3 and 4.4. The first instruction in the test control file (Table 4.1, Line 1) sets the spudcan descending vertically at a speed of 20 mm/min (0.33 mm/s). Note that the 100 mm diameter spudcan, sketched in Fig. 3.14[a], is used in this test. The origin for time is taken at the moment when $z_{RP} = -25$ mm, *i.e.* when the footing reference point RP is 25 mm above the surface of the clay. At this point the tip of the spudcan is still 1.7 mm above the soil surface, but at $t \approx 5$ s footing penetration begins (Point A in Figs 4.3, 4.4). Because of the small contact area, very little vertical load is developed during embedment of the sharp 76° conical spigot. When the main shallow conical part of the footing starts penetrating at Point B ($z_{RP} \approx -9$ mm, $t \approx 50$ s), the footing/soil contact area begins to increase rapidly with further embedment. This accounts for the characteristic kink in the vertical load-penetration curve (Point B, Fig. 4.4[f]) and the ensuing steep rise in bearing capacity. Just when the footing attains full embedment ($z_{RP} = 0$ mm) its penetration is automatically interrupted (Line 2, Table 4.1). From a peak vertical load of 328.1 N (Point C), an immediate 14% drop to $V = 282.6$ N at Point D is observed when the motor stops. This relaxation of load is thought to be caused by:

- rapid dissipation of transient excess pore pressures in the soil just below the footing;
- creep behaviour of the soil skeleton.

It will be recalled from Section 2.6.2 that Tan (1990) observed corresponding load drops of between 10 and 25% prior to his sideswipe tests on flat circular footings. He attributed the loss of V to sand creep, but made no attempt to restore the vertical load to its previous peak level. His sideswipe test load paths therefore commenced from values of V/V_0 somewhat less than unity (see Fig. 2.21[a]), meaning that no direct evidence about the shape of the yield surface tip was obtained.

For the present tests on clay, however, the QuickBASIC control program was equipped with a simple feedback control routine to provide vertical load holding before significant events during a test. The command HOLD (Table 4.1, Line 3) brings this feedback routine into operation, and its effectiveness is shown in Fig. 4.3[a]. Initially a penetration rate of about 0.05 mm/s is needed to regain and hold the vertical load at its target level of 328.1 N. Over two minutes of holding, however, the required rate of vertical

displacement settles to a steady value of around 0.01 mm/s. At this stage the <F10> key on the computer keyboard is pressed, signalling that the test should now proceed. The vertical stepper motor is stopped, and Line 4 of the control file (Table 4.1) commences horizontal translation of the footing at a rate of 5 mm/s (Points E→F in Figs 4.3 and 4.4). Figs 4.4[c] and [e] show that H and M/R both increase rapidly over the first millimetre of horizontal movement, but for $h \geq 3$ mm they maintain more or less constant values ($H \approx 50$ N, $M/R \approx 37.5$ N). Meanwhile Path E→F in Fig. 4.4[a] shows a rapid decrease in vertical load, followed by a more gradual tendency towards $V \approx 100$ N at large h . The three-dimensional $V:H:M/R$ load path for the tracking event may be visualised from its projections in $H:V$ space and $M/R:V$ space (E→F in Figs 4.4[b] and [d]). Note that in both of these plots the force point moves most of the way from Point E to Point F during the first 2 mm of horizontal movement. For $h \geq 5$ mm the $(V, H, M/R)$ force point remains almost stationary, despite continued horizontal displacement.

Once the steady load state at Point F is well established, pressing the <F10> key signals that the horizontal stepper motor should be stopped and the reversed (Table 4.1, Lines 5 and 6). As shown in Fig. 4.3[c] (Points F→G), the vertical displacement z remains fixed while the footing is moved back to its central position. Both the $H:h$ and $M/R:h$ plots (Figs 4.4[c] and [e]) reveal a pronounced asymmetry of yielding (analogous to the Bauschinger effect) during this phase. Initially there is a stiff unloading response, followed by reverse yielding when $|H|$ and $|M/R|$ are both considerably smaller than the corresponding steady state values attained at Point F. The vertical load (Fig. 4.4[a]) shows an initial decrease when the horizontal displacement is reversed, but then increases gradually as the footing approaches its original central position ($h = 0$).

On the completion of the horizontal displacement cycle E→F→G, vertical driving of the footing is resumed at the previous rate of 0.33 mm/s (Line 7, Table 4.1). The vertical load quickly regains its maximum past value, then exhibits a gradual re-yielding with further penetration (Path G→H, Fig. 4.4[f]). Although there is no further increase in footing/soil contact area (since $z_{RP} > 0$), vertical bearing capacity continues to rise quite steeply as a result of embedment below the soil surface, and because of increasing clay strength with depth. The residual horizontal and moment loads from the h cycle are quickly erased by the time z_{RP} reaches +5 mm (G→H in Figs 4.3[c], [d]). In accordance with Lines 8 and 9 of the test control file, vertical penetration continues until $z_{RP} = 25$ mm; the vertical load at this point becomes the target value for a second load holding phase. It is clear from Fig. 4.3[b] that this feedback stage (H→I→J) is very similar to the previous one (C→D→E), although the higher vertical load means that soil creep effects are now more pronounced. The vertical displacement rate takes slightly longer to reach a steady value, and the total creep settlement during the holding phase is greater (Fig. 4.3[e]).

The <F10> key is pressed to end the load holding routine, and Lines 10-12 of Table 4.1 then generate the vertical unload-reload loop shown (enlarged) in Fig. 4.4[f]. As soon as penetration is stopped (Point J), the footing is immediately moved upwards at a rate of 10 mm/min (0.17 mm/s) until the vertical load reaches zero (Point K). Retraction is then

stopped and downward vertical movement is resumed at the usual rate of 20 mm/min (0.33 mm/s). The slower rate of vertical retraction allows the datalogger to obtain better definition of the very stiff unloading response just after Point J. The reloading path intersects the unloading path at about $V/V_0 = 0.87$, forming a hysteretic loop. There is a reasonably well defined yield point (Point L) as the maximum past vertical load is approached and the virgin load:penetration curve is rejoined.

Lines 13 and 14 of Table 4.1 instruct the apparatus to enter another load holding stage as soon as z_{RP} reaches 50 mm. This holding phase (M→N→O in Figs 4.3 and 4.4) lasts for 3½ minutes and is terminated manually with the <F10> key. Lines 15-17 of the control file then generate a cycle of horizontal displacement at constant z , just like the earlier cycle specified by Lines 4-6. The only difference is that this time the footing is moved to the left, *i.e.* in the negative h direction (Path O→P in Fig. 4.3[f]). This change of direction between successive h cycles is to avoid excessive soil disturbance on one side of the kaolin sample as a test progresses. The curved load paths O→P in $H:V$ space and $M/R:V$ space resemble scaled up versions of those obtained in the tracking event at $z_{RP} \approx 0$ mm, bearing in mind the change of horizontal displacement direction and the much higher value of V_0 . The plots of V , H and M/R against h reveal load:displacement responses which are similar to those observed earlier. V and H do not, however, approach constant values at large h in the second tracking event; this is discussed further in Section 4.3.2. At Point P the horizontal displacement is halted, as usual, by the operator. For clarity, only the first few millimetres of the return to $h = 0$ are shown in Figs 4.3 and 4.4. As would be expected, the load response to this reversal of horizontal displacement is very similar to that observed on Path F→G.

The subsequent progress of T011 may be followed on pages 1 and 2 of Martin & Houlsby (1994), which contain the complete test record. After the h cycle at $z_{RP} \approx 50$ mm, vertical penetration of the footing was interrupted another six times. Three further vertical unload-reload loops were carried out at embedments of $z_{RP} \approx 75$, 125 and 175 mm. Three further horizontal displacement cycles (all with the vertical displacement z locked as before) were performed at $z_{RP} \approx 100$, 150 and 200 mm. Each of these events was preceded by a vertical load holding phase, and each successive V_0 value was higher than the previous one.

After the last h cycle the footing was extracted from the soil at a speed of 20 mm/s, and large negative values of V were registered during this phase. This pullout resistance was partly a result of spudcan/clay adhesion on the footing underside, and partly because of drag on the top side of the footing as it moved back up through clay which had partially backfilled the hole created by virgin penetration. At the completion of the pullout process, the footing was thoroughly cleaned of accumulated clay and a check was taken to confirm that the measured loads V , H and M all returned to zero; a similar check was made on the measured displacements h and θ .

4.3.2 Test T011: Tracking Event Results

In Figs 4.3 and 4.4, the Paths E→F and O→P are printed in bold since they represent the most significant events, namely the "outward" horizontal displacements applied at fixed

penetration z . The "return" horizontal displacement paths $F \rightarrow G$ and $P \rightarrow$ fall into the realm of stress reversal and cyclic loading, and a detailed consideration of this behaviour is beyond the scope of the present work. Fig. 4.5 shows the results of all five outward h events conducted in T011. As mentioned earlier, the initial direction of horizontal displacement alternates between successive events. To facilitate comparison of all the tracking events, the results for embedments of 50 mm and 150 mm have been plotted with the signs of H , h and M/R reversed. Figs 4.5[b] and [d] clearly illustrate the way in which combined load bearing capacity increases with footing penetration. Not only does each successive load path begin from a higher vertical load V_0 , but higher values of H and M/R are also attained. Assuming (for the time being) that these load paths do indeed track closely across their respective yield surfaces, there is encouraging evidence that the *shape* of the yield surface remains reasonably constant as it expands with deeper footing penetration.

In the horizontal displacement event at $z_{RP} = 0$ mm, all three loads maintain fairly constant values at large h (as discussed above). It is clear from Fig. 4.5 that this constant load state is not achieved in any of the subsequent tracking events; in fact the $H:h$ and $M/R:h$ results at deeper embedments are characterised by the onset of a distinct *upward* curvature (Points X, Y and Z in Figs 4.5[b]-[e]). The explanation for this unexpected behaviour is illustrated in Fig. 4.6, for an embedment of $z_{RP} = 50$ mm. Only a certain amount of uninhibited horizontal footing displacement (Fig. 4.6[b]) can take place before the silicone sealing bead, and eventually the metal waterproofing shield, start becoming embedded in the clay sidewall (Fig. 4.6[c]). Because the waterproofing shield is rigidly connected to the model spudcan, any passive pressure applied to the shield and its silicone sealant are registered by the load cell (in addition to genuine footing loads). The plots of Fig. 4.5 indicate that, unfortunately, this shield embedment problem becomes much more pronounced at greater depths of embedment as clay starts to flow into the hole above the penetrating footing (see Figs 4.7[a] and [b]). In T011, the horizontal displacement event at $z_{RP} = 200$ mm was terminated at $h = 3.4$ mm when a distinct upward curvature became apparent in the $H:h$ and $H:V$ plots (Point Z in Fig. 4.5). By this stage it was clear that a large proportion of the measured horizontal and moment loads were not being applied directly to the spudcan, but were reaching the footing *via* the waterproofing shield.

Another matter of interest in Fig. 4.5 concerns the concave-up shape of the $H:V$ and $M/R:V$ load paths at Points A, B and C. At each of these points there is a sudden drop in vertical load just as the horizontal and moment loads begin to increase, but no such phenomenon is apparent in the two earlier tracking events at embedments of 0 and 50 mm. Although each of the horizontal displacement events is preceded by a vertical load holding routine, problems from transient pore pressure dissipation and soil creep become significantly more pronounced under higher vertical loads. For example, a vertical displacement rate of 0.12 mm/s was required to maintain $V = 993.6$ N prior to the h event at 150 mm embedment, compared with the 0.01 mm/s needed to maintain $V = 328.1$ N for the first h event at the soil surface. Unsurprisingly, stoppage of the vertical stepper motor was accompanied by a much larger drop in V in the former case. The load path shapes at A, B and C in Fig. 4.5 are thus

associated with the abrupt halting of vertical displacement just before each tracking event. A method of avoiding such sudden drops in V was implemented in the next test, and is discussed below.

Note that although the vertical stepper motor remains locked in position throughout each tracking event, some very small downward vertical movements of the footing still occur (see Path E→F in Figs 4.3[a] and [e]). These displacements are a result of apparatus flexibility. Since the coefficient b_{11} in Eqn 3.11 is equal to -2.75×10^{-4} mm/N, every 100 N drop in vertical load results in a downward footing movement of 0.0275 mm (in addition to any flexibility-related movement registered directly by the vertical LVDTs). The outward stage of each h cycle is accompanied by a significant fall in vertical load, so these are not strictly constant z events; for a typical ΔV of -500 N, however, the deviation from the constant z ideal is only $\Delta z = +0.14$ mm. Apparatus flexibility also means that each horizontal displacement event does not begin precisely at $h = 0$ (see Figs 4.5[a], [c] and [e]), although again the deviations are very minor.

4.3.3 Test T012

This second trial test was very similar to the first except that footing penetration was interrupted for cycles of rotation, θ , rather than horizontal displacement, h . A number of procedural changes were also implemented in the light of results obtained in T011. Figs 4.5[b] and [d] indicate that there is a very large increase in combined load bearing capacity between $z_{RP} = 0$ and $z_{RP} = 50$ mm. It was therefore decided that, at $z_{RP} = 25$ mm, a rotation cycle would be of more interest than a vertical unload-reload loop. Secondly, a rotational displacement rate of $R\dot{\theta} = 10$ mm/min ($\dot{\theta} = 11.46^\circ/\text{min}$) was adopted, compared with $\dot{h} = 5$ mm/min in T011. A higher displacement rate ensures that the ideal of undrained behaviour is more closely modelled, although if datalogging continues at the same rate there is inevitably a loss of definition in the measured load and displacement data. There appeared to be adequate resolution in the T011 data, however, so a doubling of future h and θ displacement rates was not expected to cause problems in this area.

The final change for T012 involved a small modification of the QuickBASIC control program, designed to give a smoother transition from vertical load holding phases into subsequent h or θ tracking events at constant z . As discussed above, each load holding routine in T011 was terminated with a sudden "locking up" of the vertical stepper motor. At high vertical loads, this caused a sharp creep-related drop in V which distorted the ensuing $H:V$ and $M/R:V$ load paths (Fig. 4.5, Points A, B and C). For T012 it was decided that, at the end of each load holding phase, the prevailing rate of vertical creep penetration would be linearly reduced to zero over the first $2\frac{1}{2}$ seconds of footing rotation (see Fig. 4.8). This ramping down of the vertical displacement rate is obviously associated with a small additional footing penetration (in what is supposed to be an event at constant z), but even from a fairly high creep velocity of 0.1 mm/s the additional penetration is only $\Delta z = +0.125$ mm. This is similar to the flexibility-related settlement during a typical tracking event (as discussed

above), and any influence on the observed $V:H:M/R$ load path was again expected to be negligible.

A full test record for T012 appears in Martin & Houlsby (1994), pages 3 and 4, showing the six rotational displacement events and three vertical unload-reload loops performed in the course of spudcan penetration. The results of the "outward" phases of the rotation events appear in Fig. 4.9, and a strong similarity with Fig. 4.5 is immediately apparent in the $H:V$ and $M/R:V$ load spaces. It is not surprising that each θ event of T012 produces larger moments and smaller horizontal loads than the h event of T011 conducted at the same embedment. Just as in Fig. 4.5, each successive load path in Fig. 4.9 not only commences from a larger V_0 , but exhibits higher H and M/R capacities as well (although the probe at $z_{RP} = 100$ mm inexplicably develops a rather low horizontal load). Figs 4.9[a], [c] and [e] also resemble the corresponding plots from T011. In the rotation events at shallow embedments ($z_{RP} = 0, 25$ and 50 mm), a virtually constant $(V, H, M/R)$ load state is reached and maintained despite continued application of θ . The footing rotations at greater embedments begin to show misleading upturns in horizontal load and moment capacity; as before, this is because the waterproofing shield becomes embedded in clay above the footing level (see Fig. 4.7[c]). The load paths in the vicinity of A' , B' and C' in Fig. 4.9 demonstrate that a 2.5 second ramp-down of the vertical creep displacement rate successfully avoids the sudden drop in V associated with an abrupt motor stoppage after vertical load holding (cf. A, B and C in Fig. 4.5).

4.3.4 Test T013

T013 was, in many respects, very similar to T011. Both tests involved horizontal displacement events at nominally constant z ; they differed only in the level of vertical load at which the h cycles were commenced. In T013, each h event began after vertical unloading and subsequent load holding at $V \approx 10$ N. Fig. 4.10 shows the first 300 seconds of T013, with all loads and displacements plotted against time. From Point A, spudcan penetration begins as usual at $\dot{z} = 20$ mm/min. As soon as the footing reaches $z_{RP} = 0$ mm (Point B), it is immediately stopped and retracted at 10 mm/min. Unloading takes place until $V = 10$ N (Path B→C). When the footing is stopped at Point C, V jumps back up to 24.0 N (Point D); this mirrors the drop in V which was observed between Points C and D in Fig. 4.3. To regain the target load of $V = 10$ N and maintain this, the load holding routine must generate additional upward displacements. Just as with load holding in the previous tests, however, the required displacement rate quickly decays from its initial value and settles to a steady creep velocity. Hitting the <F10> key on the control computer signals that the test should proceed, and horizontal displacement is commenced at $\dot{h} = 10$ mm/min (Path E→F). As in T012, the vertical creep velocity is linearly reduced to zero over the first 2.5 seconds of horizontal displacement. The horizontal and moment loads (Figs 4.10[c], [d]) respond much as they did in T011, with an initial rapid increase followed by a steady state phase in which H and M/R both remain virtually constant despite continued horizontal displacement. The vertical load also responds with an initial increase and approaches a reasonably constant

value as the h event progresses (with z fixed). At Point F in Fig. 4.10 the horizontal stepper motor is stopped and reversed, in response to manual intervention. The V , H and M/R load responses between Points F and G are all very similar to those observed during the corresponding reversal in T011 (Path F→G, Fig. 4.3). Downward driving of the footing is resumed at Point G to allow further tracking events (and vertical unload-reload loops) at greater embedments.

The results of the six outward h events have been extracted and plotted in Fig. 4.11 (the complete test record for T103 appears in Martin & Houlsby, 1994). Figs 4.11[c] and [e] are very similar to their $H:h$ and $M/R:h$ counterparts in Fig. 4.5 (although there was no h probe at $z_{RP} = 25$ mm in T011). Horizontal and moment load capacities both increase with embedment, and, as before, the effects of contact between the waterproofing shield and the clay sidewall (Fig. 4.7[b]) become progressively more conspicuous at $z_{RP} = 100, 150$ and 200 mm. The major difference between T013 and T011 is best highlighted by comparing their $V:h$ plots, Figs 4.11[a] and 4.5[a]. In T011, each horizontal displacement event commences from $V = V_0$ and causes a reduction in V . Fig. 4.11[a] shows that, in T013, every h event commences from $V \approx 10$ N and brings an *increase* in V . Both plots do, however, have an important common feature: the largest changes in V occur in the first few moments of horizontal displacement, and thereafter the vertical load tends to approach a constant value as h is increased.

4.3.5 Comments on Trial Batch Results

The three trial tests were considered successful because they confirmed many aspects of behaviour predicted by the qualitative Model A hypothesis (Section 2.5). Horizontal or rotational displacement events at fixed embedment z did indeed support the notion of an analogous "critical state", characterised by a virtually stationary $(V, H, M/R)$ force point at large h or θ . The tracking events in T011, commencing from $V = V_0$, gave a series of approximately parabolic $H:V$ load paths (Fig. 4.5[b]) with an encouraging resemblance to the qualitative predictions of Fig. 2.22[a]. In T013 the horizontal displacements were applied after unloading to a very small V , and again the $H:V$ load paths (Fig. 4.11[b]) resembled those expected from the qualitative model (Fig. 2.22[b]). The most surprising aspect of Tests T011 and T013 was the highly significant magnitude of the footing moments developed during horizontal displacement (see Figs 4.5[d], 4.11[d]). Model A would, of course, predict $M/R = 0$ in the absence of any footing rotation.

Turning to Test T012, Fig. 4.9[d] shows that footing rotation events (at constant z) produce more or less parabolic $M/R:V$ load paths, as would be expected from Model A. The hypothetical numerical model would not, however, be capable of predicting the small but significant horizontal loads (Fig. 4.9[b]) arising from pure rotation of the footing.

One of the positive findings to emerge from the testing of Batch 1 was the very stiff response observed during vertical unloading and reloading. As discussed in Section 2.6.2, a high ratio of unload-reload stiffness to virgin penetration stiffness (k_{ULRL}/k_{VIR}) is desirable for effective yield surface tracking tests. The higher the value of k_{ULRL}/k_{VIR} , the greater the

confidence that the yield surface prevailing at the start of a tracking event undergoes only negligible expansion (or contraction) during the horizontal or rotational displacement at constant z . Under these conditions the observed $V:H:M/R$ load path may be considered, for all intents and purposes, to represent a track across the initial yield surface.

Fig. 4.12 shows the vertical unload-reload stiffnesses observed in T011, together with an approximate indication of the virgin penetration stiffnesses at corresponding depths. k_{ULRL} appears to increase fairly steadily with footing penetration; k_{VIR} , on the other hand, peaks just before the spudcan makes full contact with the soil (at $z_{RP} = 0$) and decreases with depth thereafter. Table 4.2 lists the load and displacement amplitudes of the four unload-reload loops, and also shows how the ratio k_{ULRL}/k_{VIR} evolves with depth. The implication from Table 4.2 is clearly that tracking events undertaken at greater depths of embedment are to be preferred for their higher k_{ULRL}/k_{VIR} ratios. From extrapolation, the vertical unload-reload stiffness for a footing at the clay surface ($z_{RP} = 0$) might be estimated at about 300 N/mm. Since the virgin penetration stiffness at this level is relatively high (≈ 19.2 N/mm, see Fig. 4.12), the ratio k_{ULRL}/k_{VIR} is only of the order of 15. Any h or θ tracking events conducted at $z_{RP} = 0$ would therefore be expected to give load paths reflecting a substantial deviation from the initial yield surface; the use of these load paths to define the $V:H:M/R$ yield surface for $z_{RP} = 0$ would not be advisable. For tracking events at deeper penetrations, however, the very high k_{ULRL}/k_{VIR} ratios in Table 4.2 suggest that the resulting load paths should suffer only minimal influence from yield surface expansion or contraction.

On the other hand, there is no doubt that in Tests T011-T013 load paths at deeper embedments were seriously influenced by the waterproofing shield embedment effect discussed above and illustrated in Figs 4.6 and 4.7. As the shield started becoming embedded in the clay sidewall, the rapid increase in H and M/R caused a sudden upturn in the $H:V$ and $M/R:V$ load paths for $z_{RP} \geq 100$ mm in all three trial tests (see Figs 4.5, 4.9 and 4.11). An analogous type of leg embedment probably occurs quite frequently with real jack-ups on soft clay, and is of course beneficial to the overall foundation capacity (although local stresses in the leg members may be increased). In this research, however, any embedment of the waterproofing shield was undesirable since it interfered with observation of the true spudcan/soil interaction. In an attempt to minimise the effects of shield embedment, it was decided that a larger model footing would be used for subsequent tests.

4.3.6 Definition of Angles ω and Ω

Tests T011 to T013 gave a reasonable indication that, as assumed in Model A, the combined load yield surface intersects a plane of constant $H/(M/R)$ with an approximately parabolic locus. The load path results also suggested that the $V:H:M/R$ yield surface maintains reasonably constant proportions between the three load axes as it expands with footing penetration. The Batch 1 test results did show, however, that a simple elliptical shape for the yield surface cross-section at constant V (Figs 2.9[a], 2.17[b]) would be inadequate for a realistic numerical model.

Before continuing with discussions of the cross-sectional yield surface shape, and related aspects of footing behaviour, two useful angles will be defined as shown in Fig. 4.13: Ω (in the $H:M/R$ plane), and ω (in the $h:R\theta$ plane). Model A uses an elliptical yield surface cross-section, together with an associated flow rule (Fig. 2.17[b]) and a diagonal elastic stiffness matrix. For any footing test in which no rotation is applied ($\omega = 0^\circ$), Model A predicts that there cannot be any moment (*i.e.* $\Omega = 0^\circ$). Similarly, if horizontal displacement is inhibited ($\omega = 90^\circ$), the load path predicted by Model A lies in the $M/R:V$ plane. Fig. 4.13 shows $H:M/R$ projections of the load paths obtained in [a] T011 (pure horizontal displacement, $\omega = 0^\circ$) and [b] T012 (pure footing rotation, $\omega = 90^\circ$). It is apparent that applied displacement paths with $\omega = 0^\circ$ consistently result in load paths in a direction $\Omega \approx 30^\circ$, while displacements applied at $\omega = 90^\circ$ give load paths directed at about $\Omega = 70^\circ$. These significant departures from the predictions of Model A indicated the need for a much more detailed exploration of the yield surface shape, particularly the cross-sectional shape at constant V . Further footing tests described in the next section feature tracking (constant z) events with horizontal and rotational displacements combined to give different values of ω .

4.4 Tests T021-T063: Tracking Events

The object of this main series of tracking events was a thorough definition of the three-dimensional $V:H:M/R$ yield surface shape at various footing penetrations. Details of the tests carried out, and an initial discussion of the results obtained, appear in this section.

4.4.1 Choice of ω Values for T021-T063

Under the (yet unproven) assumption that an associated flow rule applies in the $H:M/R$ plane, the trial batch of tests suggested that a constant V cross-section of the yield surface might be as shown in Fig. 4.14. The displacement increment vectors shown in this plot represent *total* displacements, but the elastic components of δh and $\delta(R\theta)$ are likely to be negligible compared with the plastic contributions during a tracking event (in which continuous yielding takes place). This assertion is discussed further in Section 5.6.3.

Fig. 4.14 shows that the trial tests only gave information about the two quadrants in which H and M/R have the same sign (Nos. 1 and 3). With these results it was only possible to estimate the likely yield surface shape in quadrants 2 and 4, but some speculation was necessary for determining which displacement directions (ω) should be adopted for obtaining full yield surface coverage in the main series of tracking tests. As shown in Fig. 4.15, four additional directions were chosen to complement the pure h and pure θ displacement modes already used in Batch 1. It was hoped that the two directions $\omega = 112.5^\circ$ and $\omega = 157.5^\circ$ would produce load paths in $V:H:M/R$ space lying very close to the $M/R:V$ and $H:V$ planes respectively. With $\omega = 135^\circ$ the target was an excursion into the negative eccentricity load quadrant No. 2. The other $h:R\theta$ displacement direction, $\omega = 45^\circ$, was designed to explore the shape of the yield surface in quadrant No. 1 of the $H:M/R$ plane.

So that the yield surface shape could be explored at both high and low levels of vertical load, tracking events commencing from both $V/V_0 = 1$ and very small V/V_0 were planned for each direction ω (*cf.* Fig. 2.23). Table 4.3 shows how the different test types were allocated in Batches 2 to 6. Three tests were performed twice to provide a check on repeatability.

4.4.2 Changes to Apparatus and Procedure after Trial Tests

The most important change for T021 and subsequent tests was the change from a 100 mm diameter spudcan (Fig. 3.14[a]) to a 125 mm version (Fig. 3.14[b]). As discussed above, it was hoped that the larger hole created during footing penetration would alleviate the waterproofing shield embedment problems encountered in the trial tests. The amount by which the footing size could be increased was, however, limited by the design capacity of the load cell and by the 450 mm diameter of the clay tanks. A 25% increase in diameter was chosen as a suitable compromise figure, giving a 56% increase in plan footing area.

In T012, all of the horizontal displacement events commenced from a small vertical load of approximately 10 N. In T021, and in Batches 5 and 6, the displacement events were commenced from a vertical load of $V = 0.01V_0$ rather than an absolute, pre-determined value of V . This was seen as more rational, making each tracking event analogous to an undrained triaxial test on a clay sample isotropically overconsolidated to $n_p = 100$.

After considering the trial batch results, it was decided that vertical penetration of the footing would be interrupted as follows:

- for h , θ or combined $h:\theta$ displacement events at $z_{RP} = 0, 20, 50, 100$ and 200 mm;
- for vertical unload-reload loops at $z_{RP} = 35, 75$ and 150 mm.

This schedule took account of the very rapid increase in bearing capacity (and hence the rapid expansion of the whole $V:H:M/R$ yield surface) which was observed during the early stages of spudcan penetration. One consequence of the change to a larger footing was that a penetration of $z_{RP} = 200$ mm was only equivalent to a 1.6 diameter embedment (compared with 2.0 diameters for the original footing). There was, however, only a small increase in bearing capacity between 1.6 and 2.0 diameters in the trial tests (see Fig. 4.12), so the loss of relative embedment was not considered to be a major drawback.

The vertical displacement rates $\dot{z}$ used in the trial tests were kept unchanged for Tests T021-T063. For pure horizontal and pure rotational displacements, rates of 10 mm/min (0.167 mm/s) were also retained. For a combined $h:\theta$ displacement event in the direction ω , the individual stepper motor speeds were set to give

$$\dot{h} = 10 \cos \omega \text{ mm/min} \quad \text{and} \quad R\dot{\theta} = 10 \sin \omega \text{ mm/min} \quad (4.3a;b)$$

such that

$$\sqrt{(\dot{h})^2 + (R\dot{\theta})^2} = 10 \text{ mm/min} \quad (4.4)$$

This ensured that, regardless of the direction of a combined displacement event, $h:R\theta$ space would still be traversed at the same speed.

4.4.3 Tests T021-T063: Load Path Results

For each of these tests, a full record of load and displacement data, and clay sample properties, appears in Martin & Houlsby (1994). As with the trial batch, however, the essential results from Tests T021-T063 are the $V:H:M/R$ load paths arising from the tracking events at fixed vertical penetration. The relevant data have been extracted from each test record and plotted in Figs 4.16 to 4.23.

Fig. 4.16 shows the load paths followed during the horizontal displacement events (at constant z) in Tests T023 and T052. The $H:V$ and $M/R:V$ load path projections closely resemble those obtained earlier with the 100 mm diameter footing (see Figs 4.5[b],[d]), though all three load components in Fig. 4.16 reflect the 56% increase in footing area. A gratifying feature of Fig. 4.16 is that, with the larger footing, waterproofing shield embedment has a relatively minor influence on the results. Unlike the heavily distorted load paths of Figs 4.5 and 4.11, in Fig. 4.16 only the h event at $z_{RP} = 200$ mm betrays an obvious onset of shield embedment. Events at shallower depths all tend to approach a steady load state as horizontal displacement progresses. The results of Test T041, plotted in Fig. 4.17, show a fair repeatability compared with the T023 load paths of Fig. 4.16. In T041 there was only a small increase in bearing capacity between $z_{RP} = 20$ mm and $z_{RP} = 50$ mm, and consequently the $H:V$ and $M/R:V$ load path projections for these two events lie very close together. The H and M/R values from which each tracking event commences also vary considerably between T023 and T041. Such differences are only to be expected from variations in undrained strength and water content between kaolin specimens, and from localised non-uniformities within a given sample.

The results of the four tests involving displacements directed at $\omega = 90^\circ$ (*i.e.* pure rotation) are summarised in Figs 4.18 and 4.19, and excellent repeatability is apparent. Only the footing rotations at $z_{RP} = 200$ mm show evidence of waterproofing shield embedment, giving a characteristic upturn to their $H:V$ and $M/R:V$ load paths. In all of the tracking events at shallower depths, the $(V, H, M/R)$ force point becomes almost stationary after a few degrees of rotation.

Test T031 was the first involving a simultaneous application of rotational and horizontal displacements at constant z . The $h:t$ and $R\theta:t$ plots on page 13 of Martin & Houlsby (1994) clearly show the way in which the two stepper motors were started, stopped and reversed in unison to produce displacement paths with $h = R\theta$ (*i.e.* $\omega = 45^\circ$). As with all of the tracking events, vertical displacement z remained fixed for the duration of the combined $h:\theta$ displacement cycle. The load path results for T031, and its corresponding low V/V_0 test, T053, are shown in Fig. 4.20. As would be expected, values of H and M/R developed with $\omega = 45^\circ$ lie between those obtained from the tracking events using pure horizontal displacement and pure rotation. In T053, the load paths for $z_{RP} = 50$ mm and $z_{RP} = 100$ mm both display peculiar sharp peaks at Points A and B in Fig. 4.20. These peaks are characterised by a sudden drop in both horizontal load and moment, accompanied by a sudden increase in vertical load. This is perhaps more easily recognisable in the load:time plots of the full test record (page 29 of Martin & Houlsby, 1994). The load discontinuities

are probably caused by a sudden loss of adhesion between the rising side of the spudcan and the clay during footing rotation (see Fig. 4.7[c]). Such a loss of tensile force on one side of the footing would explain both the drop in moment and the increase in V . Intuitively, the combination of rotation and horizontal displacement at $\omega = 45^\circ$ would be expected to increase the tendency for contact-breaking, although there are signs of a similar occurrence in the pure rotation event at $z_{RP} = 20$ mm in T021 (see Fig. 4.18). All of these incidents are associated with events commencing from low vertical load; no such contact-breaking problems seem to be evident in the tracking events commencing from high V/V_0 .

Tests T032 and T061 involved combined $h:\theta$ events at $\omega = 135^\circ$ (*i.e.* $h/R\theta = -1$). The footing was simultaneously tilted clockwise and moved to the *left* at constant vertical penetration, giving rise to positive moments combined with negative horizontal loads as shown in Fig. 4.21. The load path projections obtained from $h:\theta$ tracking events at $\omega = 112.5^\circ$ appear in Fig. 4.22 (T042, from high V/V_0 ; T062, from low V/V_0). As mentioned above, the goal with these tests was to define the shape of the yield surface in, or very close to, the $M/R:V$ plane ($\Omega = 90^\circ$). The small horizontal loads shown in Fig. 4.22[a] verify that this was achieved. The $M/R:V$ plot (Fig. 4.22[b]) shows that the yield surface shape in this plane is approximately parabolic, although the peak moments seem to occur at around $V/V_0 = 0.4$. Fig. 4.22[b] gives a good illustration of the very significant yield surface expansion which occurs with footing penetration.

Data from the two tests involving $h:\theta$ events at $\omega = 157.5^\circ$ appear in Fig. 4.23. The small moments developed indicate that, as intended, the load paths in $V:H:M/R$ space lie very close to the $V:H$ plane. Furthermore, yield surfaces for the various different footing embedments appear to intersect this plane with approximately parabolic loci (as assumed for the preliminary Model A in Section 2.5).

The shape of the three-dimensional $V:H:M/R$ yield surface will not be considered in any more detail at this stage. Section 5.2 is devoted to appropriate normalisation of the load paths obtained in Tests T021-T063, and Section 5.3 explains the curve fitting strategy used to define a single normalised yield surface.

4.5 Tests T071-T073: Loop Events

Batch 7 was devoted to the loop type events described in Section 2.6.3, the object being a direct definition of the cross-sectional ($H:M/R$) yield surface shape by applying a circular $h:R\theta$ displacement path to the footing at constant vertical load V .

4.5.1 Generation of Closed Loops in $h:R\theta$ Displacement Space

The loop displacement path suggested in Fig. 2.24 is circular in $h:R\theta$ space, implying a sinusoidal variation of both displacements with time (and, of course, a 90° phase difference between the $h:t$ and $R\theta:t$ traces). Generating such a loop would therefore require sinusoidal variations of velocity with time, but the stepper motor control hardware is only capable of generating constant motor accelerations (*i.e.* linear variations of velocity with time).

Although a sinusoidal velocity variation could have been simulated as accurately as desired with a piecewise linear approximation, it was found that the simple method illustrated in Fig. 4.24 gave acceptable results. Figs 4.24[a] and [b] show the variation of the two displacement rates $\dot{h}$ and $R\dot{\theta}$ with time. At the start of the loop (Point A) the horizontal motor is accelerated sharply, at 2.0 mm/s/s, to its peak velocity of 0.167 mm/s (Point B). As soon as this velocity is reached the motor is decelerated at 0.0056 mm/s/s, such that it takes exactly 30 seconds to reach zero velocity (Point C). Over the same 30 seconds, the rotational motor is started at Point B and accelerated at 0.0056 mm/s/s. It thus reaches its peak velocity (also 0.167 mm/s) at Point C, just as the horizontal motor stops. Between Points C and D the rotational motor is decelerated to zero velocity, while the horizontal motor is restarted with constant acceleration in the negative direction. A similar process of linear velocity variations continues until the completion of the cycle at Point F, when the horizontal motor is brought to a stop at 2.0 mm/s/s (F→G). The initial acceleration (A→B) and final stoppage (F→G) of the this motor result in a small residual horizontal displacement; the negative h "correction" G→H→I returns the horizontal motor to its datum position ($h = 0$).

The piecewise linear variations of velocity with time result in piecewise parabolic displacement:time plots (Figs 4.24[c] and [d]). These traces provide a reasonable approximation to the sinusoidal variations which would produce a circle in $h:R\theta$ space. In fact the resulting loop (Fig. 4.24[e]) is given by

$$h^2 = \pm R\theta(\sqrt{10} - \sqrt{R\theta})^2 \quad (4.7a)$$

for the lower half ($0 \leq R\theta \leq 2.5$ mm), and

$$h^2 = \pm(5 - R\theta)(\sqrt{10} - \sqrt{5 - R\theta})^2 \quad (4.7b)$$

for the upper half ($2.5 \leq R\theta \leq 5$ mm). The loop is also offset by a small amount (0.035 mm) to the right of the $h = 0$ axis because of the displacement involved in initial acceleration of the horizontal motor to its peak velocity (A→B).

The Type 1 loop illustrated in Fig. 4.24 and in Fig. 4.25[a] begins with pure horizontal displacement; Fig. 4.25[b] shows a corresponding Type 2 loop in which rotation is now the leading displacement. In fact, as shown in Fig. 4.25, there are eight possible loop types. Types 3 and 4 are similar to the Type 1 and Type 2 loops already detailed, except that the directions of traversal are reversed. Types 5 to 8 are simply reflected versions of the first four. In the test control file for a loop event, the two peak velocities $\dot{h}$ and $R\dot{\theta}$ (in mm/s), the loop period (in seconds) and the loop type (1-8) must be specified. The velocity and period data automatically determine the dimensions of the loop in $h:R\theta$ space. For T071-T073 peak velocities of 10 mm/min were used. The chosen two minute period allowed a loop of reasonably large "diameter" (5 mm) to be traversed in a fairly short time, bearing in mind the need to minimise drainage in the soil around the footing. A very small loop of, say, 1 mm diameter would probably not have allowed full horizontal or moment loads to be

mobilised, so the resulting load path in $H:M/R$ space would not have represented a true path around the cross-sectional yield locus.

4.5.2 Vertical Load Control

The important difference between Tests T071-T073 and previous ones was that the $h:R\theta$ loops had to be generated under vertical load control (constant V) rather than vertical displacement control (constant z). The vertical load holding routine used in all of the tests described so far was software-driven. A simple feedback algorithm in the program *CONTROL.BAS* checked the vertical load against a target value and generated appropriate stepper motor displacements to correct any discrepancy. As illustrated in Figs 4.3 and 4.10, the load holding algorithm performed very well whether it was maintaining a peak vertical load, or holding a very small value of V after vertical unloading. With these good results in the absence of horizontal or rotational displacements, it was initially thought that the same software routine might be able to provide satisfactory vertical load control during an $h:R\theta$ loop. To check whether this was the case, several trial loop events were performed using pieces of kaolin discarded from earlier tests. Unfortunately it turned out that the 4 Hz rate of data scanning was not sufficient for the algorithm to "react" to the sudden change of V caused by the initiation of horizontal or rotational displacement. In early trials the vertical load proceeded to oscillate wildly about the target V as the $h:R\theta$ loop was traversed, although modifications to the QuickBASIC program did allow a stable recovery of the target vertical load in some of the later trials. A fair degree of control ($V = \text{target } V \pm 10\%$) could then be maintained throughout the loop.

In light of these results it was decided that an alternative, and rather more direct, system of vertical load control would be developed. The final version of this system, in which the vertical sliding plate is disconnected from its stepper motor drive, has already been described in detail in Section 3.3.5. Trial tests with the modified apparatus indicated that an excellent degree of vertical load control (generally target $V \pm 1\%$) could be achieved during execution of an $h:R\theta$ loop. Furthermore, there were no difficulties with maintaining constant vertical load during the first few seconds of a loop event, whereas the software-based load control method remained susceptible to instability in this critical period. The direct load control system was not without its drawbacks, particularly in terms of the extra apparatus required and the additional time needed for disconnecting and re-attaching the vertical sliding plate several times during a test. Its superior reliability and load holding performance meant, however, that the direct method was used for vertical load control in the loop events of T071-T073.

4.5.3 Typical Loop Event Procedure: Test T071

Fig. 4.26 shows plots of all three loads and displacements against time during the early stages of Test T071 (Martin & Houlsby, 1994, contains a full record). Straightforward vertical driving of the footing occurs until full penetration is reached at $z_{RP} = 0$ mm (Point B). The control program notes the maximum vertical load attained, in this case $V_0 = 541.5$ N.

Unloading then takes place as the footing is retracted at 10 mm/min, stopping when a vertical load of $V = 0.4V_0$ is reached (Point C). The usual software-driven load holding routine maintains V at this target level (C→D→E). The choice of the vertical load level V/V_0 was made after studying Figs 4.16-4.23. These plots suggested that a loop event at $V = 0.4V_0$ would be expected to trace out the largest possible $H:M/R$ cross-section of the yield surface for a particular footing penetration. In the analogy with triaxial testing described in Section 2.6, the ratio $V/V_0 = 0.4$ corresponds to a moderate overconsolidation ratio of 2.5.

The method of disconnecting the vertical sliding plate from its ballscrew drive is discussed and illustrated in Section 3.3.5. In order to ensure a smooth transition between displacement controlled and load controlled modes, the correct vertical dead weight had to be prepared and applied to the vertical sliding plate before disconnection took place. As shown in Fig. 4.26, the required dead load for the first loop event was $0.4V_0 = 216.6$ N. The footing load V resulting from self-weight of the vertically released rig would have been 610 N, so a load of 393.4 N (40.1 kg) was therefore applied to the counterweight hanger (see Fig. 3.12[b]). During testing this required dead weight was computed and displayed by the control program to save time and eliminate the risk of error. The disturbance to the vertical load trace at Point D in Fig. 4.26[c] reflects the placement of weights on the counterweight hanger, although the rig is still in displacement controlled mode. At Point E the vertical sliding plate is finally disconnected from its stepper motor drive mechanism, and (as desired) there is only a very slight change in V as the rig is converted into load controlled mode. There is, however, a sudden jump in H at the instant of disconnection, almost certainly indicating a small horizontal displacement of the footing as the vertical sliding plate is released. The moment load (Fig. 4.26[e]) also shows a small increase at Point E. These changes to H and M/R , while unexpected, are of such a small magnitude that the $(V, H, M/R)$ force point remains well within the yield surface for the start of the loop event itself.

After a period of direct load control between Points E and F, a Type 1 $h:R\theta$ loop is executed between Points F and G (see the $h:t$ and $R\theta:t$ traces, Figs 4.26[g] and [h]). The load response is best viewed in cross-sectional $H:M/R$ load space, and such results will be presented below. The $H:t$ and $M/R:t$ traces do, however, reveal a very stiff initial response to the loop event beginning at Point F. Fig. 4.26[a] shows, in detail, the variations of vertical load and vertical displacement during the first $h:R\theta$ event. Vertical load control is very good for the majority of the loop, although it does drift significantly from the target V of 216.6 N to a maximum of 225.7 N and a minimum of 205.7 N. As described in Section 3.3.5, the level of water in the ballast tank was manually adjusted throughout the loop event to compensate for friction variations in the vertical linear guide system. In early constant V tests the author's control of the inflow and dump valves was not particularly good, and the +4/-5% performance on vertical load holding in Fig. 4.26[a] is not typical of later achievements with the same apparatus. Vertical displacement z remains roughly constant during the first minute of the loop, but significant footing settlement occurs during the latter half. At the completion of the loop event, residual horizontal displacement is eliminated

(Points G→H) and the vertical sliding plate is re-connected to its ballscrew drive mechanism (Point I). Note that this re-connection is accompanied by drops in H and M/R which mirror the increases observed at Point E.

At Point J in Fig. 4.26, vertical driving of the footing is resumed under displacement control. Horizontal and moment loads remaining from the loop event are quickly "erased", as shown in Figs 4.26[d] and [e]. When the vertical penetration z_{RP} reaches 20 mm (Point K), the footing is again retracted until $V = 0.4V_0 = 316.7$ N (Point L). Compared with the first loop, the higher target V means that a smaller counterweight is required for the switch into load controlled mode at Point M. Here sudden jumps in H and M/R are observed, just as at Point E prior to the previous loop. Path N→O→P shows a Type 5 $h:R\theta$ loop (a reflection of the Type 1 loop executed at $z_{RP} = 0$ mm). As with Tests T021-T063, alternating the initial directions of $h:R\theta$ loop events at successive embedments avoids asymmetrical disturbance of the clay sample. In T071 (and in all of the ensuing tests with events performed under vertical load control), this alternating strategy is also advantageous since the small jumps in H and M/R which accompany every switch into load controlled mode are consistently positive in sign (see Figs 4.26[d] and [e]). Any systematic influence on the subsequent $H:M/R$ load path results should therefore be avoided.

Fig. 4.26[b] shows that an improved vertical load holding performance is achieved during the second loop event of T071. The maximum deviations from the target V are +4.7 and -5.8 N (+1.5/-1.8%). Vertical displacement over the course of the loop follows a very similar pattern to that observed in the previous event: very slight heave in the first half of the loop followed by more substantial settlement in the second. Path O→P in Fig. 4.26 shows the return of the horizontal stepper motor to its central datum, with the slight changes in H and M/R which accompany this adjustment. The remainder of T071 followed a similar pattern to that of earlier tests. Further Type 1 $h:R\theta$ loops were performed at embedments of 50 and 200 mm, with a Type 5 loop at $z_{RP} = 100$ mm. Vertical unload-reload events were carried out at the usual depths of 35, 75 and 150 mm, but in Tests T071-T073 unloading was not performed all the way to $V = 0$ (as it was in T021-T063). Instead loops of a smaller amplitude were used, with unloading to $V/V_0 = 0.5$ followed by reloading. Vertical unload-reload loops from all tests are discussed in Section 4.8 below.

4.5.4 Summary of Results: T071-T073

Details of the tests performed in Batch 7 are shown in Table 4.4. The first two samples were devoted to h -leading loop events, traversed in both directions. Five θ -leading loops were performed on the third sample, with anticlockwise and clockwise directions of traversal at successive embedments.

The parts of Fig. 4.26 printed in bold are clearly the most interesting, since they show load and displacement data from the actual $h:R\theta$ loop events. Fig. 4.27 presents this essential information for all five loops performed in T071. The four plots used are:

- vertical load against time (a check on the load holding performance at $V = 0.4V_0$);
- change in vertical displacement with time;

- $h:R\theta$ (loop displacement paths);
- $H:M/R$ ("cross-sectional" load paths).

Vertical load holding and heave/settlement have already been considered in detail for the first two loop events (at $z_{RP} = 0$ mm and 20 mm). Fig. 4.27[a] confirms that very good control of vertical load was achieved at the subsequent embedments of 50, 100 and 200 mm. Fig. 4.27[b] shows that, regardless of embedment, the footing undergoes 0.1 to 0.2 mm of heave during the first half of a loop. In the second 60 s, when downward displacement occurs, there is a clear trend of progressively less settlement during loop events at greater embedments. Fig. 4.27[c] shows that apparatus flexibility (Eqn 3.11) has quite a significant effect on the displacements actually applied to the footing, considering that the $h:R\theta$ displacement path generated by the stepper motors is identical for each loop.

In $H:M/R$ load space (Fig. 4.27[d]), the results are broadly consistent with the Model A load path prediction of Fig. 2.24. Although the experimental load paths do follow roughly elliptical paths, the axes of these ellipses are rotated by some 30 to 35° (clockwise) with respect to the H and M/R axes. This is in agreement with the indication of cross-sectional yield surface shape obtained from the early tracking events (see Fig. 4.14). As would be expected, in Fig. 4.27[d] there is a well defined pattern of increasing horizontal load and moment capacity as the yield surface expands (and V_0 increases) with further penetration. The $H:M/R$ load paths obtained at different embedments seem to maintain a reasonably constant shape with depth, as observed in the $H:V$ and $M/R:V$ planes in Tests T021-T063. In the latter stages of each loop event, particularly in the last 30 seconds, there is a marked softening of the load path as both horizontal and moment loads drop towards small values. It should be borne in mind that by this stage of an $h:R\theta$ loop the soil around the footing has suffered a large amount of disturbance. Indeed, only the first quarter of a loop can really be classed as a monotonic displacement event; from this point on there are displacement reversals. A larger diameter of $h:R\theta$ loop would probably have decreased the load softening tendency by keeping the force point "pushed out" against the yield surface for longer. As mentioned above, however, a larger loop would have taken longer to execute and confidence in the undrained nature of the loading would have been diminished.

Considering now the initial stages of each loop event, Model A predicts a stiff, elastic load:displacement response until the yield surface is intersected. As mentioned above, the measured $H:h$ and $M/R:h$ responses are indeed very stiff just after a loop is commenced. But even though the initial displacements of each $h:R\theta$ loop are directed at an angle $\omega = 0^\circ$ (pure horizontal displacement), all five load paths in Fig. 4.27[d] head off at an angle of $\Omega \approx 30^\circ$ before intersecting, and tracking around, the yield surface. By contrast, the diagonal elastic stiffness matrix of Model A predicts an initial load path direction of $\Omega = 0^\circ$. The effect of conducting Type 5 loops at depths of 20 and 100 mm may also be studied in Fig. 4.27[d]. Because their actual horizontal and moment loads have been reversed for the purposes of plotting, these two loops commence from significant negative horizontal loads in Fig. 4.27[d] (unlike the three Type 1 loops which start with $H > 0$). The overall results suggest, however, that the initial position of the force point inside the $H:M/R$ yield locus

does not have any influence on the subsequent load path which traces out this locus at constant V .

Five clockwise $h:R\theta$ loops (Types 3 and 7) were performed in T072, and the results appear in Fig. 4.28. Fig. 4.28[c] shows that the symmetrical displacement path generated by the horizontal and rotational stepper motors is considerably distorted as a result of apparatus flexibility, as in T071. Vertical load holding performance (Fig. 4.28[a]) is considerably better than that in the previous test, with the vast majority of holding time spent within $\pm 1\%$ of the target V . The heave/settlement plot (Fig. 4.28[b]) reveals very similar behaviour at all embedments. Each loop initially results in a small amount of heave, but footing settlements occur over the final 90 seconds. Settlements in T072 are a little greater than those observed in the anticlockwise loop events of T071. Fig. 4.28[d] shows that the initial $H:M/R$ load path response is very similar to that observed in T071. At the start of each loop, the load paths consistently head in a direction $\Omega \approx 30^\circ$, just as in Fig. 4.27[d]. Upon reaching the yield surface, however, there is a much sharper change of direction as the load paths begin to track around in a clockwise direction. Although the moments developed in T071 and T072 are roughly comparable, the horizontal loads in T072 are noticeably smaller, giving rise to a much thinner looking series of ellipse-like load paths. The amount of ellipse rotation with respect to the H and M/R axes also appears to be slightly smaller (about 25°). The use of a wider $h:R\theta$ loop (*i.e.* one with a greater maximum h) may well have mobilised a greater proportion of the horizontal load capacity observed in T071. Another interesting discrepancy concerns the load softening which occurred towards the end of each loop in Fig. 4.27[d]; this effect is not nearly as pronounced for the T072 load paths in Fig. 4.28[d]. It is not easy to explain the significant differences between these two $H:M/R$ plots, differences which arise only from changing the direction of loop traversal. The very existence of such differences does, however, provide further evidence that the cross-sectional yield surface shape cannot be symmetrical with respect to the H and M/R axes (as assumed in Model A).

The results of T073a appear in Fig. 4.29. Fig. 4.29[c] shows the anticlockwise, θ -leading $h:R\theta$ loops executed at embedments of 0, 50 and 200 mm. The vertical footing displacements during these loops (Fig. 4.29[b]) follow a very similar pattern to those of T071: initial heave of the footing is followed by settlement in the second half of each loop, with a greater settlement observed for the loop at $z_{RP} = 0$ mm. There are also some clear similarities between the two sets of results in $H:M/R$ load space (compare Figs 4.27[d], 4.29[d]). Once again the T073a load paths are strongly suggestive of a cross-sectional yield surface shape that is elliptical, but not symmetrical with respect to the coordinate axes. As before, loop events at increasing embedments trace out progressively larger $H:M/R$ loci (indicating hardening of the entire $V:H:M/R$ yield surface with increasing penetration). The cross-sectional shape of the yield surface appears to remain reasonably constant during this expansion. The initial load path direction is about $\Omega = 80^\circ$, with very well defined corners in the load paths as they approach (and begin tracking around) their respective $V:H:M/R$ yield surfaces at constant vertical load.

Fig. 4.30 presents the data from two clockwise, θ -leading loop events performed in T073b. The vertical displacements occurring during these loops (Fig. 4.30[b]) follow a similar pattern to those of T072. As was found with the two sets of h -leading loops, the $H:M/R$ load paths in Fig. 4.30[d] are significantly thinner and more pointed than their anticlockwise counterparts in Fig. 4.29[d]. Once again, this appears to be the result of an under-development of horizontal load during the clockwise $h:R\theta$ loop events. Despite this difference, the general pattern of the load paths from both T073a and T073b is fairly consistent with the emerging picture of a "slice" through the yield surface at constant V - an elliptical cross-section with an M/R to H aspect ratio in the region of 2:1, but with the ellipse rotated approximately 30° clockwise with respect to the H and M/R coordinate axes.

4.6 Tests T081-T103: Probing Events

The probing event tests conducted on Batches 8, 9 and 10 had much in common with the loop event tests (T071-T073) just described. Tests T081-T103 all involved unloading from a peak vertical load V_0 followed by the application of horizontal, rotational or combined $h:R\theta$ footing displacements at constant V . But whereas the loop events involved a complex $h:R\theta$ displacement path, Tests T081-T103 saw a return to the straightforward $\omega = \text{constant}$ displacement paths used in Tests T021-T063. Table 4.5 summarises the testing programme for Batches 8, 9 and 10.

The loop tests were performed at $V/V_0 = 0.4$, so it was not thought necessary to conduct any further tests at this vertical load level. For Batches 8-10, V/V_0 values of 0.75 and 0.2 were chosen to allow probing of the $V:H:M/R$ yield surface at one moderately high and one moderately low level of vertical load. As discussed in Section 2.6.1, these constant V probing events are closely analogous to drained triaxial tests conducted at constant mean stress p' on clay specimens isotropically overconsolidated to $n_p = 1.33$ and $n_p = 5$ respectively.

4.6.1 Typical Probing Test Procedure: T081

Fig. 4.31 shows an excerpt from the full test record for T081 which appears in Martin & Houlsby (1994). Vertical driving of the footing (Path A→B) is automatically interrupted at $z_{RP} = 0$ mm, and unloading is (again automatically) performed to the required vertical load level of $V = 0.75V_0 = 330.1$ N (B→C). The QuickBASIC control program holds V close to this value while appropriate weights are added to the counterweight hanger, ready for the conversion of the rig into load controlled mode at Point D. At Point E the horizontal stepper motor is started at the usual displacement rate of 10 mm/s. The horizontal load H initially increases rapidly (E→F) but then the rate of increase slows down (F→G). The moment load M/R shows a rapid increase from E to F, followed by a slight drop between F and G. After 7.5 mm of horizontal displacement the footing is stopped and returned to its central ($h = 0$) position. Both H and M/R exhibit a stiff unloading response and subsequent reverse yielding along Path G→H. The vertical displacement occurring during the h cycle is best observed on

the first detail plot (Fig. 4.31[a]). This shows that the footing settles at a remarkably constant rate during both the outward and inward phases of the horizontal displacement event. The total increase in z is a little over 3 mm. At Point I the vertical sliding plate is reconnected to its ballscrew mechanism; penetration is resumed under vertical displacement control (Point J) until the next testing level is reached at $z_{RP} = 20$ mm (Point K). Then follows the usual sequence of displacement controlled unloading to $V = 0.75V_0$ (Point L) and software-driven load holding in preparation for the switch to direct vertical load control at Point M. The second horizontal displacement cycle ($N \rightarrow O \rightarrow P \rightarrow Q$) is performed at constant V , with the loads H and M/R again showing well defined yielding (at Point O) after an initially stiff response. Footing settlement during the complete h cycle is, once more, a little over 3 mm (see Fig. 4.31[b]). After reconnection of the vertical sliding plate (Point R), the test proceeds with further probing events at embedments of 50, 100 and 200 mm, and vertical unload-reload loops at $z_{RP} = 35, 75$ and 150 mm.

4.6.2 Summary of Results: Probing Events at $V/V_0 = 0.75$

A standard plotting format has been devised for showing the essential data from each of these tests. For purposes of clarity, only the results from the outward portion of each probing event are plotted, as with the earlier tests involving tracking events (T021-T063). Fig. 4.32 shows this data, for T081, on five plots:

- $V:H$ and $V:M/R$ (showing that the events occurred at virtually constant V);
- $H:h$ and $M/R:h$ (load:displacement curves for observing yield points);
- $\Delta z:h$ (for monitoring the change of vertical displacement with horizontal displacement).

The two load:load plots in Fig. 4.32 confirm that excellent control of the vertical load was achieved throughout each horizontal displacement probing event. The load:displacement plots show, for each event, extremely stiff behaviour at small h leading quickly to a well defined yield point; after yield the horizontal and moment loads remain virtually constant as h is increased. At deeper footing penetrations the values of H and M/R at yield are greater, as would be expected. Finally, the $\Delta z:h$ plot shows that footing settlements of about 1.5 mm occur during each 7.5 mm application of horizontal displacement at constant V . There appears to be some evidence of a systematic reduction in settlement with increasing footing embedment, but the trend is by no means conclusive.

Fig. 4.33 shows the essential results of T082 (pure footing rotations at $V = 0.75V_0$). There is good vertical load control during each of the rotation events, but the load:displacement yield points are not as well defined as those of T081. At every embedment the moment continues to increase steadily after yield, while the horizontal load drops steadily. In the $M/R:R\theta$ plot (Fig. 4.33[d]) there is a clear pattern of increasing moment capacity with greater footing embedment. The upturn at Point A during the rotation event at $z_{RP} = 200$ mm is caused by embedment of the waterproofing shield in the clay sidewall, an effect already familiar from Tests T021-T063. The $\Delta z:R\theta$ plot in Fig. 4.33 indicates that a fairly consistent footing settlement occurs during each rotation, regardless of the depth of embedment. Except at $z_{RP} = 0$ mm, the rate of settlement appears to increase slightly as each

rotation event proceeds; a corresponding effect can also be detected in the $\Delta z:h$ plot for T081 (Fig. 4.32[e]).

In Figs 4.34 and 4.35 the main results of T101 and T102 are summarised. As shown in Table 4.5, these two tests were intended as a repeatability check on T081 and T082. A minor difference with T101 and T102 was that the displacements h and $R\theta$ were taken a little further, to 10 mm instead of 7.5 mm. Comparing Figs 4.32 and 4.34, the general trend of results is very similar. There is especially good repeatability with the settlement plots. Figs 4.33 and 4.35 do not compare quite so well; in particular the moments developed in T102 are considerably smaller than those of T082. This can be explained by the fact that the clay sample for T082 was somewhat stronger than that used for T102, see Martin & Houlsby (1994). This difference is also reflected by the vertical load levels, $V = 0.75V_0$, at which rotation events in the two tests were carried out.

Tests T083 and T103 involved combined horizontal and rotational displacement events in directions $\omega = 45^\circ$ and $\omega = 135^\circ$ respectively. The pertinent results of the two tests appear in Figs 4.36 and 4.37. The load:displacement plots feature fairly well defined yield points after a very stiff response at small displacements, and confirm a definite increase in combined load bearing capacity with footing penetration. Figs 4.36[e] and 4.37[e] indicate that vertical settlement of the footing does not depend on embedment, but that it does depend strongly on the displacement direction ω . At the same "radial" displacement $\sqrt{\Delta h^2 + (R\Delta\theta)^2}$, settlements with $\omega = 45^\circ$ are about three times greater than those for $\omega = 135^\circ$.

4.6.3 Summary of Results: Probing Events at $V/V_0 = 0.2$

Fig. 4.38 shows the five horizontal displacement events from T091 plotted in the usual format. The vertical loads (held constant at $V = 0.2V_0$) are obviously much smaller than those used in Batches 8 and 10, but the horizontal and moment loads developed are of comparable magnitude. The $H:h$ and $M/R:h$ plots show initial stiff behaviour followed by yielding, and as horizontal displacement continues both loads remain fairly constant. The most important feature of the behaviour observed at low vertical load is shown by the $\Delta z:h$ plot, Fig. 4.38[e]. In T091 the footing undergoes upward vertical movement (*i.e.* heave) during horizontal displacement at constant V , as opposed to the settlement observed in Tests T081 and T101. This behaviour is also predicted by the simple yield surface and associated flow rule of Model A. Note, however, that the amount of footing heave in T091 (0.5-0.8 mm) is substantially less than the 1.2-1.8 mm settlement observed in T081 for the same horizontal displacement of 7.5 mm. This suggests that the assumption of associated flow at all levels of V/V_0 may well require revision in the refinement of Model A.

Fig. 4.39 shows the essential data from T092, involving footing rotations at $V/V_0 = 0.2$. Moments and horizontal loads are of similar magnitude to those developed in T082 and T102; the absence of well defined yield points is also reminiscent of the corresponding rotation events at $V/V_0 = 0.75$. Fig. 4.39[e] shows a pattern of footing heave

similar to that observed in T091, with the exception of a very large upward movement during the rotation event at the soil surface ($z_{RP} = 0$ mm).

The final probing events at $V/V_0 = 0.2$ were performed in Test T093. As shown in Figs 4.40 and 4.41, combined $h:R\theta$ displacements were applied in directions of both $\omega = 45^\circ$ (three embedments) and $\omega = 157.5^\circ$ (two embedments). Yield points for the probes at $\omega = 45^\circ$ are less well defined than those for $\omega = 157.5^\circ$, but for both directions there is a clear increase in combined load bearing capacity with footing penetration. Figs 4.40[e] and 4.41[e] confirm the dependence of footing heave on ω , noted previously with the settlements depicted in Figs 4.36 and 4.37. At a given vertical load level V/V_0 , probes into the negative eccentricity quadrants of the $H:M/R$ plane seem to be associated with much less vertical displacement than probes in which H and M/R have the same sign.

4.7 Displacement Rate Effects: Tests T111-T113

Footing tests on Batches 2 to 9 were conducted with a set of standard displacement rates established after the trial tests (see Section 4.4.2). The eleventh and final batch was used to repeat some earlier tests (T022 and T023), but varying the displacement rates as shown in Table 4.6. Tests T022 and T023 both involved tracking (constant z) events commencing from $V/V_0 = 1$ (rotational displacements were applied in T022, horizontal displacements in T023). In Tests T111-T113, rotational displacement tracking events were executed at three embedments ($z_{RP} = 0, 50, 200$ mm) and horizontal displacement events at the other two ($z_{RP} = 20, 100$ mm).

The effect of the rate variations on vertical load:displacement behaviour is shown in Fig. 4.42. The upper graph shows V plotted against z_{RP} for all three tests in Batch 11. For comparison, appropriate data points have been taken from the 27 standard rate $V:z_{RP}$ curves obtained from tests on Batches 2-10 using $\dot{z} = 20$ mm/min. A decrease in vertical displacement rate clearly leads to a *larger* vertical load at a given depth. Test T112, 16 times slower than standard, gave the largest V recorded in the whole experimental programme (1890 N) while T111, 4 times slower than standard, gave vertical loads well above the standard rate average. The test conducted at twice the normal speed, however, gave a vertical load:penetration curve fairly typical of those obtained in earlier tests, peaking around 1500 N.

Any variations in undrained shear strength between clay samples would be expected to have a direct effect on vertical bearing capacity in Fig. 4.42[a]. The vertical loads of this plot have therefore been normalised with respect to a local value of s_u , as defined in Fig. 4.42[b]. The normalised results indicate that there is a highly significant increase in bearing capacity associated with the 16-fold decrease in displacement rate. The 4-fold decrease in rate gives a slightly higher than average normalised load at embedments greater than 100 mm. The 2-fold rate increase in T113 continues this trend, giving lower than average normalised vertical loads beyond 100 mm penetration. These displacement rate effects may be explained by the occurrence of drainage in the clay around the penetrating footing. At

very slow displacement rates there is ample time for drainage to occur, so the highly stressed kaolin underneath the footing undergoes significant consolidation. In the process its water content is reduced and its shear strength increases. At higher displacement rates there is less time for consolidation to occur as the footing penetrates; in the limiting case of truly undrained behaviour, there would be no increase in vertical load capacity as a result of clay consolidation. At very high penetration rates, however, viscous effects might be expected to reverse the above trend and start causing an increase in vertical bearing capacity with increasing displacement rate.

In summary, then, Fig. 4.42[b] suggests that the vertical loads observed in Tests T021-T103 may be very slightly influenced by drainage, and thus a little higher than those which would be observed under truly undrained conditions. The effects of varying $\dot{h}$ and $R\dot{\theta}$ are best discussed in terms of the normalised horizontal and moment loads to be introduced in the next chapter (see Section 5.4).

4.8 Vertical Unload-Reload Loops

Apart from the trial tests (T011-T013), the footing tests on every clay sample incorporated vertical unload-reload events at the standardised embedments of $z_{RP} = 35, 75$ and 150 mm. Unloading in Tests T021-T063 was performed all the way to zero vertical load. In Batch 7, however, a smaller load amplitude was used: unloading to $V/V_0 = 0.5$, followed by reloading. Further variations were adopted for Batches 8 and 9, as shown in Table 4.7, with the object of studying how the chord stiffness of an unload-reload loop varies with the amplitude of the loop.

Fig. 4.43 shows some typical unload-reload events of different amplitudes. All of the loops were executed at $z_{RP} \approx 75$ mm, commencing from vertical loads of approximately 1100 N. The highly nonlinear unloading and reloading paths result in hysteretic loops whose chord slopes k_{ULRL} reveal a strong dependence of unload-reload stiffness on loop amplitude. The loop with unloading to $V/V_0 = 0.75$ has a chord stiffness nearly 10 times as great as the one with unloading to $V = 0$; any meaningful record of observed vertical unload-reload stiffnesses must clearly include some reference to loop amplitude.

A FORTRAN program was used to analyse all of the vertical unload-reload loops detailed in Table 4.7. The results are presented in Fig. 4.44, as plots of chord slope against displacement amplitude. Fig. 4.44[a] confirms the trend highlighted in Fig. 4.43, namely a very marked increase in chord stiffness with decreasing loop amplitude. For a given loop amplitude Δz , the chord stiffness appears to increase with footing embedment; this would, however, be expected for a profile of increasing undrained strength with depth. Comparing Figs 4.44[a] and [b], the limited data suggest that vertical unload-reload stiffness (for a given amplitude Δz) is independent of the displacement rate used to generate the unload-reload loop.

ANALYSIS OF EXPERIMENTAL RESULTS

5.1 Introduction

This chapter presents a unified analysis of the experimental results reported in Chapter 4. The tracking event load paths are used to define a normalised three-dimensional yield surface in $V/V_0:H/V_0:M/RV_0$ space. Normalised yield points observed in the probing events are checked for proximity to this surface, and displacements at yield are interpreted with a view to the calibration of a complete incremental plasticity model (Chapter 6). Appropriately normalised results from the Batch 7 loop events, and from the vertical unload-reload loops, are presented. Finally the observed form of the $V/V_0:H/V_0:M/RV_0$ yield surface is compared with some of the interaction surfaces discussed in Chapter 2.

5.2 Tracking Events: Normalised Load Paths

The tracking event load paths of Tests T021-T063 (Figs 4.16-4.23) indicate very clearly that the $V:H:M/R$ yield surface expands with increasing footing embedment. These plots also suggest that the yield surface maintains approximately the same proportions between the three load axes as it grows. A more stringent test of this constant shape hypothesis is a comparison of the *normalised* load paths obtained from events at different embedments in a particular test. Normalisation is achieved by dividing all three footing loads by the pure vertical load capacity, V_0 , appropriate to each embedment. For events commencing from high vertical load, V_0 is defined as shown in Fig. 4.3[a]. Note that it is taken as the vertical load at the start of the tracking event proper, and not as V_{peak} (which is the target load for the preceding holding stage). This definition means that each event commences from a normalised vertical load of $V/V_0 = 1$ precisely. With tracking events commencing from low vertical load the situation is clearer; the only logical definition of V_0 is that shown in Fig. 4.10[a].

Figs 5.1-5.8 contain normalised versions of the load path plots appearing in Figs 4.16-4.23. Fig. 5.1 shows that the five normalised load paths obtained in T052 (pure horizontal displacement from low V/V_0) are virtually identical. Dashed lines in the event at $z_{RP} = 200$ mm indicate that passive soil pressures on the waterproofing shield are judged to be interfering with this part of the load path (see Section 4.3.2). The horizontal displacement events commencing from high vertical load (T023) all follow very consistent normalised load paths, again with the exception of the event at $z_{RP} = 200$ mm. In this case, negative starting values of H/V_0 and M/RV_0 appear to have affected the subsequent load path, which lies noticeably below the others (until the onset of shield embedment).

One very pronounced feature of Fig. 5.1 is that the tracking events commencing from high V produce a substantial drop in normalised vertical load, reaching steady load states around $V/V_0 = 0.35$, while the events starting from low vertical load only show a small increase (to around $V/V_0 = 0.25$). It will be recalled from Section 2.6.2 that Tan (1990) observed very similar behaviour in his sideswipe tests (see Fig. 2.21[b]). From an overall consideration of Fig. 5.1 it could reasonably be inferred that the two sets of normalised load paths track across the yield surface and meet up at a point in the vicinity of $(V/V_0, H/V_0, M/RV_0) = (0.3, 0.13, 0.11)$. Both the $H/V_0:V/V_0$ and $M/RV_0:V/V_0$ projections of the load paths are approximately parabolic.

Fig. 5.2 shows normalised load path data from T041 (the repeat of T023). There is good agreement with the results of the original test, though with slightly more scatter. The tracking event at $z_{RP} = 0$ mm results in larger normalised horizontal and moment loads than those observed at deeper penetrations, but apart from this there is no evidence that depth of embedment has a systematic effect on the normalised load path in $V/V_0:H/V_0:M/RV_0$ space.

Normalised load paths from the tracking tests involving pure footing rotation appear in Fig. 5.3 (original tests) and Fig. 5.4 (repeated tests). As would be expected, normalised moment loads are noticeably greater than those in Figs 5.1 and 5.2, while the normalised horizontal loads are smaller. Once again, however, load paths originating from high and low vertical loads seem to meet up at about $V/V_0 = 0.3$ to define a roughly parabolic intersection between the $V/V_0:H/V_0:M/RV_0$ yield surface and a plane of constant $H/(M/R)$. The normalised load paths obtained at different embedments are remarkably similar, with the obvious exception of the events at $z_{RP} = 0$ mm. These rotation events at the clay surface give noticeably higher normalised moments in tests commencing from $V/V_0 = 1$, but give smaller moments in the tests originating from low V . This anomalous behaviour is by no means unexpected, because at $z_{RP} = 0$ mm the ratio of vertical elastic stiffness to virgin penetration stiffness is relatively low (see Section 4.3.5 and Fig. 4.12). Considering first the tracking events starting from $V/V_0 = 1$ in Figs 5.3 and 5.4, it is likely that the irregular load paths for $z_{RP} = 0$ arise from significant expansion of the initial yield surface during the rotation events themselves. In these circumstances, normalisation of the entire load path with a single (initial) value of V_0 is not particularly meaningful. The normalised $V/V_0:H/V_0:M/RV_0$ yield surface at $z_{RP} = 0$ may indeed be slightly larger than that at depth (and possibly differently shaped), but in any case a tracking test load path cannot be relied upon to follow the initial yield surface closely when the stiffness ratio $k_{UL,RL}/k_{VIR}$ is only of the order of 10 (see Section 2.6.2). The rotation events commencing from low vertical load seem to confirm the theoretical expectation that, when $z_{RP} = 0$ mm, the initial yield surface undergoes significant contraction in the course of a footing rotation event at constant vertical displacement z . Tracking events at all of the deeper embedments, when $k_{UL,RL}/k_{VIR}$ is much higher, follow consistent normalised load paths. Returning briefly to Fig. 5.1, it is perhaps slightly puzzling that both of the horizontal displacement tracking events at $z_{RP} = 0$ mm give normalised load paths entirely consistent with those observed at deeper penetrations. In Fig. 5.2, however,

there is a slight suggestion that the $z_{RP} = 0$ event gives a normalised load path which deviates from those obtained at subsequent embedments.

Fig. 5.5 presents normalised load path results from Tests T031 and T053, involving tracking events with $\omega = 45^\circ$ in $h:R\theta$ space. The observed pattern of behaviour resembles that of Figs 5.1-5.4. Load paths commencing from $V/V_0 = 1$ all follow very similar paths across the normalised $V/V_0:H/V_0:M/RV_0$ yield surface, reaching end points around $V/V_0 = 0.4$. Events starting from $V/V_0 = 0.01$ also follow fairly consistent normalised load paths, approaching $V/V_0 \approx 0.4$ from below (provided the effects of waterproofing shield embedment do not interfere, as at $z_{RP} = 200$ mm in T053). Normalised horizontal and moment loads at the common end point reflect the combination of horizontal displacement and rotation (*cf.* the preceding plots for pure h and pure θ events). In Fig. 5.5[a] it can be seen that the load path for the first event in T031 ($z_{RP} = 0$ mm) cuts across the other load paths as V/V_0 reduces; the same effect is apparent in Fig. 5.5[b], though to a lesser extent. As discussed above, anomalous behaviour of this type is expected when a yield surface tracking event is attempted with an insufficiently large vertical stiffness ratio k_{ULRL}/k_{VIR} (as is the case at $z_{RP} = 0$). The sharp discontinuities at Points A and B in the T053 load paths, and the similar peak at Point A in Fig. 5.3, have already been discussed in Section 4.4.3.

Fig. 5.6 reveals that the two sets of normalised load paths for $\omega = 135^\circ$ do not meet at a common value of V/V_0 . The vertical loads in T061 reach $V/V_0 \approx 0.3$ (as in earlier tests from low V), but the load paths for T032 only drop to around $V/V_0 = 0.5$ before maintaining a reasonably steady load state. The reasons for this are not clear, though some differences in footing behaviour might be expected in the negative eccentricity quadrants of $H/V_0:M/RV_0$ space. Fig. 5.6 shows that consistent normalised load paths are followed in both tests, with the exception of the two events at $z_{RP} = 0$ mm. The load path projections in both the $H/V_0:V/V_0$ and $M/RV_0:V/V_0$ planes appear to be roughly parabolic, despite the gap in V/V_0 between the two sets of normalised results.

The final two sets of normalised tracking test results, for combined $h:R\theta$ displacements at $\omega = 112.5^\circ$ and $\omega = 157.5^\circ$, are presented in Figs 5.7 and 5.8. All of the load paths in Fig. 5.7[a] are characterised by very small H/V_0 values, so Fig. 5.7[b] gives a good indication of the locus with which the normalised yield surface intersects the $M/RV_0:V/V_0$ plane. The simple parabolic $M/RV_0:V/V_0$ variation assumed for Model A appears to be well matched by the tracking event results, although in Fig. 5.7[b] the peak moment is located closer to $V/V_0 = 0.4$ than to $V/V_0 = 0.5$. The maximum normalised moment M_0/RV_0 also appears to be a little lower than the value of 0.19 adopted in Model A. In both T042 and T062, as in several previous tests, the load paths for the events at $z_{RP} = 0$ deviate very noticeably from the consistent results obtained at the four deeper embedments. The deviations are once more in accordance with theoretical expectations: In Fig. 5.7[b] the $z_{RP} = 0$ event commencing from $V/V_0 = 1$ develops a larger-than-average moment, while among the events beginning from low V/V_0 the load path for $z_{RP} = 0$ lies well below the others.

Fig. 5.8[b] shows that normalised moments M/RV_0 are generally close to zero in the tracking events of T063 and T033. In Fig. 5.8[a] the two sets of load paths thus confirm that the normalised yield surface intersects the $H/V_0:V/V_0$ plane in a parabolic locus very similar to that assumed for Model A. The peak value of $H_0/V_0 = 0.13$ assumed in the preliminary numerical model is supported by the experimental results, although the tests once more suggest that this peak occurs at a vertical load level V/V_0 slightly lower than 0.5. In T033 the event at $z_{RP} = 0$ mm develops larger normalised moments than those obtained at deeper penetrations, in keeping with the established pattern of behaviour discussed above.

5.3 Tracking Event Results: Surface Fitting Strategy

5.3.1 The $H/V_0:M/RV_0$ Plane

From Figs 5.1-5.8 it is clear that the $V/V_0:H/V_0:M/RV_0$ yield surface intersects the planes $H/V_0 = 0$ and $M/RV_0 = 0$, and indeed any plane of constant $H/(M/R)$, with approximately parabolic loci. The cross-sectional shape with which the yield surface intersects a plane of constant V/V_0 is rather more difficult to visualise from Figs 5.1-5.8. By plotting the same normalised load path results in $H/V_0:M/RV_0$ space, however, a very clear picture emerges. Fig. 5.9[a] presents the data from all 40 tracking events commencing from $V/V_0 = 1$. For completeness the reflected versions of each event (deduced from symmetry) have also been plotted, giving a total of 80 load paths. It is immediately clear that the values of ω chosen for the experimental programme (see Fig. 4.15) have given very good coverage of the yield surface in the $H/V_0:M/RV_0$ plane. The expected load path directions of Fig. 4.15, characterised by the angle Ω , are generally confirmed by Fig. 5.9[a].

Two data markers representing "spot heights" have been added to each curve in Fig. 5.9[a]: one at the start of the load path ($V/V_0 = 1$, $H/V_0 \approx 0$, $M/RV_0 \approx 0$) and one where the load path intersects the plane $V/V_0 = 0.7$. If all of the latter data points are extracted and re-plotted (Fig. 5.9[b]) it may be seen that the cross-sectional shape of the yield surface at $V/V_0 = 0.7$ is that of a rotated ellipse ($M/R:H$ aspect ratio about 1.5:1, clockwise rotation approximately 25°). This shape is in accordance with the load paths observed in the loop events at $V/V_0 = 0.4$ (Figs 4.27[d]-4.30[d]) and it also confirms the qualitative shape postulated after the trial footing tests T011-T013 (see Fig. 4.14).

In Fig. 5.9[b] the data points obtained from tracking events at $z_{RP} = 0$ mm have been excluded from the curve fitting procedures described below. The motivation for this exclusion has already been discussed in detail in the previous section: there is insufficient confidence that any of these $z_{RP} = 0$ points lies on the same yield surface which existed at the start of the tracking event (*i.e.* the yield surface to which the normalising load V_0 refers). In Fig. 5.9[b] the data for $z_{RP} = 0$ fall outside the general trend of the other results, but with the $z_{RP} = 0$ points removed there is a well defined shape suitable for regression analysis. The dashed ellipse in Fig. 5.9[b] is a least squares fit of the form

$$A(H/V_0)^2 + B(M/RV_0)^2 + C(H/V_0)(M/RV_0) = 1 \quad (5.1)$$

with $A = 103.5$, $B = 61.64$ and $C = -51.84$. A more useful interpretation of these parameters is possible if Eqn 5.1 is written in a more standard form:

$$\left(\frac{H/V_0}{H_{\text{int}}/V_0}\right)^2 + \left(\frac{M/RV_0}{M_{\text{int}}/RV_0}\right)^2 - 2e\left(\frac{H/V_0}{H_{\text{int}}/V_0}\right)\left(\frac{M/RV_0}{M_{\text{int}}/RV_0}\right) = 1 \quad (5.2)$$

H_{int}/V_0 and M_{int}/RV_0 are the points at which the ellipse intersects the coordinate axes, while e is an eccentricity parameter. If $e = 0$ the principal axes of the ellipse coincide with the coordinate axes, and at the extremes of $e = \pm 1$ the ellipse degenerates into two parallel lines. The negative sign in Eqn 5.2 simply means that an ellipse rotated clockwise with respect to the coordinate axes has a positive value of e . The two sets of regression parameters are related by

$$H_{\text{int}}/V_0 = \frac{1}{\sqrt{A}}; \quad M_{\text{int}}/RV_0 = \frac{1}{\sqrt{B}}; \quad e = \frac{-C}{2\sqrt{A}\sqrt{B}} \quad (5.3a;b;c)$$

so the rotated ellipse in Fig. 5.9[b] has intercepts of $H_{\text{int}}/V_0 = 0.098$ and $M_{\text{int}}/RV_0 = 0.127$, with an eccentricity $e = 0.325$ (clockwise rotation = 25.5°). The dash-dot curve in Fig. 5.9[b] is the best fitting non-rotated ellipse, with $H_{\text{int}}/V_0 = 0.109$ and $M_{\text{int}}/RV_0 = 0.142$. The standard error of this regression is 52% greater than that of the rotated ellipse fit, confirming the visual impression that a non-rotated ellipse cannot represent the data very satisfactorily.

Fig. 5.10[a] corresponds closely to Fig. 5.9[a], except that it shows normalised data from the 35 tracking events commencing from low vertical load. Once more it is apparent that good radial coverage of the $H/V_0:M/RV_0$ plane has been achieved in these footing experiments. Each normalised load path begins close to the origin of $H/V_0:M/RV_0$ space, with $V/V_0 \approx 0.01$; the outer data markers indicate points at which the plane $V/V_0 = 0.2$ is intersected. Note, however, that because of waterproofing shield embedment there are three normalised load paths on which the vertical load does not increase enough for this intersection to occur (events at $z_{\text{RP}} = 200$ mm in T021, T051 and T053). Upon extracting and re-plotting the data markers as before (Fig. 5.10[b]), the points obtained from events at the clay surface ($z_{\text{RP}} = 0$ mm) lie inside the general trend of the other results, as expected from Figs 5.1-5.8. Their exclusion from the curve fitting to follow is once more justified by the theoretical expectation that, for tracking events commencing from low V/V_0 , a load path obtained at $z_{\text{RP}} = 0$ probably undergoes substantial inward deviation from the initial yield surface it is meant to track. The bracketed data points in Fig. 5.10[b] are also excluded because they are derived from a portion of load path deemed to be unacceptably distorted by embedment of the waterproofing shield (dashed lines in Fig. 5.10[a]).

Regression analysis on the remaining points in Fig. 5.10[b] gives a rotated ellipse with $H_{\text{int}}/V_0 = 0.093$, $M_{\text{int}}/RV_0 = 0.139$ and $e = 0.378$ (20.9° rotation). The best fitting non-rotated ellipse ($H_{\text{int}}/V_0 = 0.107$ and $M_{\text{int}}/RV_0 = 0.159$) has a standard error of regression nearly twice that of the rotated ellipse fit. No doubt the addition of terms in $(H/V_0)^4$, $(M/RV_0)^4$ and $(H/V_0)^2(M/RV_0)^2$ to Eqn 5.1 would allow a marginally better fit to the points in Figs 5.9[b] and 5.10[b], but the added complexity of the equation would not be justified.

The rotated ellipse defined by three coefficients appears to describe the data quite adequately, while maintaining an attractive simplicity.

The regression procedure described above for $V/V_0 = 0.7$ and $V/V_0 = 0.2$ has been repeated at 17 other levels of vertical load to obtain best fit rotated ellipses for $V/V_0 = 0.05, 0.1, \dots, 0.95$. Nine of these regressions (for $V/V_0 = 0.1, 0.2, \dots, 0.9$) are illustrated in Fig. 5.11, showing the data points used and the least squares ellipses obtained. Adjacent to each plot are numerical details of the fit: the three parameters H_{int}/V_0 , M_{int}/RV_0 and e , the standard error of regression SE_{fit} , and the number of (independent) data points N . One noticeable feature of Fig. 5.11 is the variation of N with V/V_0 . Because there are tracking events commencing from both $V/V_0 = 1$ and $V/V_0 = 0.01$, data points are plentiful at both high and low normalised vertical loads. On the other hand, as shown in Figs 5.1-5.8, many tracking events attain a steady load state before the intermediate vertical load levels of $V/V_0 = 0.3$ or $V/V_0 = 0.4$ are reached. This explains why there are relatively few data points available for ellipse fitting in Figs 5.11[f] and [g].

Fig. 5.12 shows the variation of N , SE_{fit} and the three regression parameters H_{int}/V_0 , M_{int}/RV_0 and e with vertical load level. Fig. 5.12[a] confirms that the least squares ellipses for $V/V_0 = 0.3, 0.35$ and 0.4 are based on an undesirably small number of data points, although the standard error of regression is greatest for ellipses fitted at high V/V_0 . This latter observation may be explained by the fact that tracking events commencing from $V/V_0 = 1$ have initial locations in $H/V_0 : M/RV_0$ space which are relatively scattered about the origin (compare Figs 5.9[a] and 5.10[a]). Figs 5.12[b] and [c] confirm that H_{int}/V_0 and M_{int}/RV_0 both vary approximately parabolically with V/V_0 , as expected from the earlier discussions of Figs 5.1-5.8. The variation of the eccentricity parameter e with V/V_0 (Fig. 5.12[d]) is not particularly well defined, although ellipses fitted at intermediate vertical load levels ($V/V_0 \approx 0.4$) clearly have the smallest eccentricities.

The error bars appearing in Figs 5.12[b], [c] and [d] show the uncertainty associated with each of the regression parameters at different vertical load levels. Unfortunately the least squares fit of each ellipse does not yield these standard errors directly. Eqn 5.2 is not linear in the parameters H_{int}/V_0 , M_{int}/RV_0 and e , so it cannot be used for multiple linear regression. Instead Eqn 5.1 (which is linear in A , B and C) must be used, returning parameters and standard errors $A \pm SE_A$, $B \pm SE_B$ and $C \pm SE_C$. Eqns 5.3 show how H_{int}/V_0 , M_{int}/RV_0 and e are obtained from A , B and C . From Eqns 5.3 and the usual law of propagation of errors - see, for example, Rektorys (1969) - the standard errors of H_{int}/V_0 , M_{int}/RV_0 and e may be derived as:

$$SE_{H_{\text{int}}/V_0} = \frac{A^{-3/2} SE_A}{2}; \quad SE_{M_{\text{int}}/RV_0} = \frac{B^{-3/2} SE_B}{2};$$

$$SE_e = \frac{1}{4\sqrt{AB}} \left[C^2 \left(\frac{SE_A^2}{A^2} + \frac{SE_B^2}{B^2} \right) + 4SE_C^2 \right]^{1/2} \quad (5.4a;b;c)$$

Returning to Figs 5.12[b] and [c], the error bars verify the intuitive expectation of much greater uncertainty in H_{int}/V_0 and M_{int}/RV_0 when the number of data points N is low. Fig. 5.12[d] shows that confidence in the eccentricity parameter is highest for low V/V_0 , where there is little scatter in the data. The small number of data points around $V/V_0 = 0.4$ means that virtually no significance can be attached to the best fit e values obtained in this region. At high values of V/V_0 , the scattered nature of the data results in another area of considerable uncertainty with regard to the fitted eccentricity parameter e .

5.3.2 The $V/V_0:H/V_0$ and $V/V_0:M/RV_0$ Planes

The final stage of the yield surface fitting procedure involves a second round of regressions, using the data in Fig. 5.12 to define the variations of H_{int}/V_0 , M_{int}/RV_0 and e with V/V_0 . Recent work on determining the $V/V_0:H/V_0$ interaction diagram for strip footings on sand (notably that of Nova & Montrasio, 1991) has revealed that a modified parabola of the form

$$\frac{H_{\text{int}}}{V_0} = \mu \frac{V}{V_0} \left(1 - \frac{V}{V_0}\right)^\beta \quad (5.5)$$

offers certain advantages over the simple ($\beta=1$) parabola. The exponent β controls the vertical load level at which the peak horizontal load occurs, and Nova & Montrasio report an improved fit to experimental failure points with $\beta=0.95$. Not only does this choice of β shift the peak to $V/V_0 = 0.513$, but it has the theoretically desirable effect (see Section 6.2.3) of rounding off the tip of the parabola, *i.e.* making the tangent at $V/V_0 = 1$ vertical. Both Nova & Montrasio (1991) and Gottardi & Butterfield (1993) state that an expression analogous to Eqn 5.5, say

$$\frac{M_{\text{int}}}{BV_0} = \mu' \frac{V}{V_0} \left(1 - \frac{V}{V_0}\right)^\beta \quad (5.6)$$

provides a good fit to experimental strip footing data in the $V/V_0:M/BV_0$ plane (B is the width of the strip footing).

Eqns 5.5 and 5.6 are limited in that for the rounding off effect to occur at $V/V_0 = 1$, β must be less than unity. This in turn means that the location of the peak horizontal (or moment) load can only be shifted from $V/V_0 = 0.5$ towards $V/V_0 = 1$. The present experimental data for spudcan footings on clay suggest that the peak values of both H_{int}/V_0 and M_{int}/RV_0 occur when $V/V_0 < 0.5$ (see Figs 5.12[b] and [c]), so it is unlikely that Eqns 5.5 and 5.6 could model these results satisfactorily. A completely general "parabola" may, however, be obtained by introducing a second variable exponent. The following modified form of Eqn 5.5 is proposed:

$$\frac{H_{\text{int}}}{V_0} = \mu \left(\frac{V}{V_0}\right)^{\beta_1} \left(1 - \frac{V}{V_0}\right)^{\beta_2} \quad (5.7)$$

As long as $\beta_1 < 1$ and $\beta_2 < 1$ the modified parabola is rounded off at both $V/V_0 = 0$ and $V/V_0 = 1$. Furthermore, with judicious variation of the two exponents the peak of the

modified parabola may be shifted to any desired vertical load level $0 < V/V_0 < 1$. Unfortunately, however, each change to β_1 or β_2 affects the magnitude as well as the location of the peak, meaning that the parameter μ must also be adjusted to preserve the desired peak horizontal load. Substituting an appropriate function for μ into Eqn 5.7 gives

$$\frac{H_{\text{int}}}{V_0} = \frac{H_0}{V_0} \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1} (\beta_2)^{\beta_2}} \right] \left(\frac{V}{V_0} \right)^{\beta_1} \left(1 - \frac{V}{V_0} \right)^{\beta_2} \quad (5.8)$$

With this expression, β_1 and β_2 may both be varied to shift the peak, while leaving the maximum normalised horizontal load in the $V/V_0: H/V_0$ plane unchanged at H_0/V_0 . An exactly analogous expression may be formed for the $V/V_0: M/RV_0$ interaction locus:

$$\frac{M_{\text{int}}}{RV_0} = \frac{M_0}{RV_0} \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1} (\beta_2)^{\beta_2}} \right] \left(\frac{V}{V_0} \right)^{\beta_1} \left(1 - \frac{V}{V_0} \right)^{\beta_2} \quad (5.9)$$

Eqns 5.8 and 5.9 provide suitable regression models for the data in Figs 5.12[b] and [c] respectively. It is clear from these plots, however, that some form of weighted regression is needed to account for the considerable fluctuations in error bar magnitude with V/V_0 . The topic of weighted least squares is covered in most advanced texts on statistics, for example Hines & Montgomery (1980). In terms of the present notation, the method involves assigning the i th point of the regression data set a weight

$$w_i = \frac{1}{SE_i^2} \quad (5.10)$$

and minimising the merit function

$$\chi^2 = \sum_i w_i \epsilon_i^2 \quad (5.11)$$

rather than the usual (equally weighted) least squares function

$$\chi^2 = \sum_i \epsilon_i^2 \quad (5.12)$$

Fig. 5.13[a] shows the results of weighted least squares fits to the H_{int}/V_0 and M_{int}/RV_0 data, with both β_1 and β_2 forced to take values of unity. This reduces Eqns 5.8 and 5.9 to simple second-order parabolas, and the best fitting curves of this type can be seen to provide a reasonably good interpretation of the data. If the exponents β_1 and β_2 are both forced to take values of 0.5, Eqns 5.8 and 5.9 are reduced to semi-ellipses. Weighted regressions on the H_{int}/V_0 and M_{int}/RV_0 data produce the curves shown in Fig. 5.13[b]. The elliptical curves seem to provide a good fit to the data in the vicinity of $V/V_0 = 0$ and $V/V_0 = 1$, but they fail to capture the peaks in both horizontal load and moment data at intermediate values of V/V_0 .

With β_1 and β_2 specified *a priori*, curve fitting using Eqn 5.8 (or 5.9) is a simple matter of linear regression to find the single parameter H_0/V_0 (or M_0/RV_0). If β_1 and β_2 are allowed to vary as additional fitting parameters, nonlinear regression must be used. A problem arises here because independent nonlinear regressions with Eqn 5.8 (to the H_{int}/V_0

data) and Eqn 5.9 (to the M_{int}/RV_0 data) would produce different values of β_1 and β_2 . Accommodation of all four β values in a single expression for the $V/V_0:H/V_0:M/RV_0$ yield surface would, however, require an equation of unwarranted complexity. On the other hand Eqns 5.8 and 5.9 may be very easily incorporated into a three-dimensional yield surface expression provided the *same pair* of exponents β_1 and β_2 is used in each equation. A rational curve fitting criterion under this constraint is minimisation of the combined merit function, *i.e.* finding H_0/V_0 , M_0/RV_0 , β_1 and β_2 such that

$$\chi^2 = \left[\sum_i w_i \varepsilon_i^2 \right]_{\text{Eqn 5.8}} + \left[\sum_i w_i \varepsilon_i^2 \right]_{\text{Eqn 5.9}} \quad (5.13)$$

is a minimum. This type of simultaneous regression on separate data sets is not a standard feature of statistics software packages, but it was reasonably simple to program such a method in *Mathematica*. The result of the four parameter nonlinear regression is shown in Fig. 5.13[c]. Very good modelling of both the horizontal load and moment data is achieved, although both curves look to have slightly low peak values compared with the experimental points. It should be recalled, however, that with weighted least squares the data points around $V/V_0 = 0.4$ to 0.5 exert much less influence on the fitted curves because of their larger error bars. With the best fit parameters $\beta_1 = 0.764$ and $\beta_2 = 0.882$, the peak values of H_{int}/V_0 and M_{int}/RV_0 occur when $V/V_0 = 0.464$. One final point worth noting in Fig. 5.13 is the substantial reduction in the overall regression error SE_{fit} obtained by introducing β_1 and β_2 as fitting parameters.

Fig. 5.14 illustrates various attempts at modelling the variation of the ellipse eccentricity parameter e with normalised vertical load. Once again the standard errors (derived from the Eqn 5.4c) show considerable variation with V/V_0 , making the weighted least squares method essential. The simplest possible regression, namely $e = \text{constant}$, is shown in Fig. 5.14[a]. Provided the same (β_1, β_2) pair is used in Eqns 5.8 and 5.9, a constant eccentricity parameter implies that the elliptical $H/V_0:M/RV_0$ yield surface cross-section is rotated through the same angle at all vertical load levels. Fig. 5.11 confirms that this is approximately the case, though there is evidence that ellipse rotation is greater at the extremes of normalised vertical load than at intermediate values ($V/V_0 = 0.3$ to 0.6). Such a variation of eccentricity with V/V_0 can be described quite well by a simple two parameter parabola, symmetrical about $V/V_0 = 0.5$:

$$e = e_1 + e_2 \left(\frac{V}{V_0} \right) \left(\frac{V}{V_0} - 1 \right) \quad (5.14)$$

The weighted least squares fit using this regression model is shown in Fig. 5.14[c]. The addition of the parabolic term allows a much more convincing fit (compared with that in Fig. 5.14[a]) and gives a worthwhile reduction in the standard error of regression. On the other hand, a fitted straight line model (Fig. 5.14[b]) offers no real improvement over the $e = \text{constant}$ regression. The extra linear term does not reduce χ^2 (Eqn 5.11) enough to offset the loss of one degree of freedom, so SE_{fit} actually increases slightly between Figs

5.14[a] and [b]. A two parameter equation for describing the $e:V/V_0$ variation should clearly be of parabolic rather than straight line form, though a fair representation of the data is possible simply with $e = 0.316$.

5.3.3 Three Dimensional Yield Surface Equation

Upon substituting Eqns 5.8, 5.9 and 5.14 into Eqn 5.2, an expression for the full normalised $V/V_0:H/V_0:M/RV_0$ yield surface is obtained:

$$f(V/V_0, H/V_0, M/RV_0) = \left(\frac{H}{H_0}\right)^2 + \left(\frac{M/R}{M_0/R}\right)^2 - 2\left[e_1 + e_2\left(\frac{V}{V_0}\right)\left(\frac{V}{V_0} - 1\right)\right]\left(\frac{H}{H_0}\right)\left(\frac{M/R}{M_0/R}\right) - \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1}(\beta_2)^{\beta_2}}\right]^2 \left(\frac{V}{V_0}\right)^{2\beta_1} \left(1 - \frac{V}{V_0}\right)^{2\beta_2} = 0 \quad (5.15a)$$

The best fit parameter values, from Figs 5.13[c] and 5.14[c], are:

$$\begin{aligned} H_0/V_0 &= 0.127; & M_0/RV_0 &= 0.166; & \beta_1 &= 0.764; & \beta_2 &= 0.882; \\ e_1 &= 0.518; & e_2 &= 1.18 \end{aligned} \quad (5.15b)$$

It is by no means imperative that the full six parameter model be used in all situations. Fig. 5.14[a] suggests, for example, that taking $e_1 = 0.316$ and $e_2 = 0$ would still allow a good representation of the experimental results (with only five parameters). Eqn 2.24 is a simplified form of Eqn 5.15a, with $\beta_1 = \beta_2 = 1$ and $e_1 = e_2 = 0$.

Fig. 5.15[a] shows a perspective view of the normalised yield surface described by Eqns 5.15. Comparing this with the simple cigar-shaped surface adopted for Model A (see Fig. 2.17[a]), the more rounded tips of the modified surface can be readily discerned, particularly at low V/V_0 . The more revealing plot is, however, the view down the V/V_0 axis shown in Fig. 5.15[b]. Comparison with Fig. 2.17[b] reveals that the cross-sectional dimensions chosen for the Model A yield surface (*i.e.* H_0/V_0 and M_0/RV_0) were approximately correct. On the other hand, the non-rotated constant V ellipses of Model A clearly provide a rather poor description of the cross-sectional yield surface shape deduced from Tests T021-T063. In Figs 5.15[b] and 2.17[b] this shortcoming is most apparent at high V/V_0 , where the experimentally determined ellipse rotation is greatest.

5.4 Tests T111-T113: Normalised Load Paths

It will be recalled from Section 4.7 that the final batch of kaolin samples was devoted to an investigation of displacement rate effects. Details of the tracking events performed in T111-T113, some involving horizontal displacement and others rotation, are given in Section 4.7, and the displacement rates used in each test are summarised in Table 4.6. The effects of displacement rate on vertical load:penetration behaviour have already been investigated (Fig. 4.42); this section deals with the tracking event load paths observed in Batch 11.

The procedure for analysing one of these load paths is identical to that used with the standard rate events commencing from $V/V_0 = 1$. All three footing loads are divided by V_0 , the initial vertical load, and the points in $H/V_0:M/RV_0$ space at which the load path intersects the planes $V/V_0 = 0.95, 0.9, \text{etc.}$ are determined. For brevity, however, only two typical sets of results (for $V/V_0 = 0.8$ and $V/V_0 = 0.6$) will be considered. Fig. 5.16[a] shows data points from the normalised load paths of T112, obtained with displacement rates $\dot{h}$ and $\dot{\theta}$ 16 times slower than the standard (T021-T063) rates. At each vertical load level the data markers generally lie a little outside the Eqn 5.15 ellipse (which is representative of standard rate behaviour), indicating that very slow rates of horizontal and rotational displacement result in a slight increase of the corresponding normalised load capacities. The likely explanation is localised consolidation of the highly stressed clay adjacent to the footing during an h or θ event (*cf.* the discussion of $V:z$ behaviour in Section 4.7). Fig. 5.16[b] shows that displacement rates 4 times slower than the standard have no discernible effect on the normalised loads H/V_0 and M/RV_0 developed during tracking events from high V/V_0 . Similarly with displacement rates $\dot{h}$ and $\dot{\theta}$ at twice their standard values, no effect on normalised horizontal or moment load capacity is apparent (Fig. 5.16[c]), though there is an increase in scatter. Fig. 4.42 shows that very slow vertical displacement rates $\dot{z}$ lead to higher vertical loads V . Fig. 5.16 indicates, however, that the normalised $V/V_0:H/V_0:M/RV_0$ load paths followed in the tracking events of T111-T113 closely resemble those observed at standard displacement rates.

5.5 Loop Events: Normalised Load Paths

The various $h:R\theta$ loop events of Tests T071-T073, discussed in Section 4.5.4, provide convincing evidence that the $H:M/R$ cross-section of the yield surface undergoes significant expansion with footing penetration (Figs 4.27-4.30). It is of interest to consider the same $H:M/R$ load path results after normalisation by V_0 , to see if the $H/V_0:M/RV_0$ cross-section suggested by these events is consistent with that obtained from the normalised tracking test results.

Fig. 5.17[a] shows normalised load path data from the loop events of T071 (*cf.* Fig. 4.27). The results obtained at $z_{RP} = 20, 50, 100$ and 200 mm show remarkably good agreement, but the event at the clay surface ($z_{RP} = 0$ mm) suggests a somewhat larger $H/V_0:M/RV_0$ cross-section. Like the anomalous behaviour observed in tracking events at $z_{RP} = 0$ mm, this effect probably arises because the virgin load:penetration curve is relatively steep at $z_{RP} = 0$. Even the small amount of settlement during the T071 loop event at this embedment (see Fig. 4.27[b]) would be expected to cause significant yield surface expansion, making normalisation of the entire load path by the initial V_0 value inappropriate. Fig. 5.17[b] shows the $H/V_0:M/RV_0$ load paths observed in the clockwise, h -leading loop events of T072 (*cf.* Fig. 4.28). There is once again a very consistent response at all embedments except for $z_{RP} = 0$ mm, where larger normalised horizontal and moment loads are attained in the early part of the loop. Figs 5.17[c] and [d], showing the $H/V_0:M/RV_0$ load paths traced

out in T073, are normalised versions of Figs 4.29 and 4.30. The three anticlockwise loops in Fig. 5.17[c] conform with the pattern of Figs 5.17[a] and [b]: the load path for $z_{RP} = 0$ mm lies outside the ones obtained at $z_{RP} = 50$ mm and 200 mm (which agree well). The two clockwise θ -leading loops of T073b (Fig. 5.17[d]) give almost identical load paths in the normalised $H/V_0:M/RV_0$ load space.

The load paths from Figs 5.17[a] and [b] have been superimposed in Fig. 5.18[a], though for clarity only the first half (*i.e.* the first 60 seconds) of each loop event is shown. Also plotted is the cross-sectional yield surface shape of Eqn 5.15, for the appropriate vertical load level of $V/V_0 = 0.4$. Fig. 5.18[a] confirms that the normalised loop event load paths are reasonably consistent with the constant V yield locus derived from the tracking events of T021-T063. It is noticeable, however, that most of the load paths do not mobilise all of the available horizontal load capacity indicated by the dashed ellipse. As mentioned in Section 4.5.4, the use of a wider displacement controlled loop in $h:R\theta$ space (*i.e.* one with a greater h dimension) would probably have resulted in larger horizontal loads, and consequently a more faithful tracing out of the true cross-sectional yield surface shape. In Fig. 5.18[b] the loop event load paths of T073 (first halves only) have been superimposed, together with the elliptical locus of Eqn 5.15 for $V/V_0 = 0.4$. Agreement between the loop load paths and Eqn 5.15 is not as good as that in Fig. 5.18[a]; the main source of disparity is, once again, an apparent under-development of normalised horizontal load during the loop events. Nevertheless, Fig. 5.18 gives the overall impression that both the size and shape of the yield surface cross-section at $V/V_0 = 0.4$, as deduced from Tests T021-T063, are satisfactorily verified by the normalised loop event load paths obtained in Tests T071-T073.

5.6 Probing Events: Normalised Results

It remains to be seen whether the yield points observed in the probing events of T081-T103 give further confirmation of Eqn 5.15, the surface fitted to the tracking event data. To this end, a normalised yield point in $H/V_0:M/RV_0$ space must be determined for each of the probing events at both $V/V_0 = 0.75$ and $V/V_0 = 0.2$.

5.6.1 Normalised Yield Points: $V/V_0 = 0.75$

Test T081 involved horizontal displacement probing events at a vertical load level of $V/V_0 = 0.75$. The results have already been discussed in some detail in Section 4.6.2, and the relevant load:load and load:displacement curves (non-normalised) appear in Fig. 4.32. For determining normalised yield points in T081, the relevant plots are $H/V_0:h/R$ and $M/RV_0:h/R$ (as shown in Figs 5.19[a] and [b]). Inspection of these load:displacement curves allows an $(H/V_0, M/RV_0)$ yield point to be selected for each of the five probing events, though the choice is obviously rather subjective. Yield can usually be recognised as the point of greatest curvature in a load:displacement plot, but problems arise in Fig. 5.19 if this maximum curvature occurs at noticeably different horizontal displacements in the $H/V_0:h/R$ and $M/RV_0:h/R$ diagrams. In these circumstances some compromise is required, because

only one point on the three dimensional $V/V_0:H/V_0:M/RV_0$ load path may be selected as the representative yield point for a particular event.

With the exception of the event at $z_{RP} = 0$ mm (which shows a significant post-yield increase in horizontal load capacity), the normalised load paths for T081 all agree closely. The repeated Test T101 exhibits a very similar set of normalised load:displacement responses for the various footing embedments (Figs 5.19[d] and [e]). The development of larger horizontal load H/V_0 in the probing events at $z_{RP} = 0$ mm is reminiscent of the anomalous behaviour observed at $z_{RP} = 0$ in Tests T071-T073 (Fig. 5.17). The effect may be explained along similar lines: as a result of footing settlement (see Figs 5.19[c], [f]) the initial value of V_0 increases significantly over the course of a probing event at $z_{RP} = 0$. At deeper penetrations, although the settlements observed are much the same, there is only a negligible increase in V_0 during an event because the virgin load:penetration stiffness is much lower. Normalisation of the whole load path by the initial V_0 value is thus somewhat misleading for probing events at $z_{RP} = 0$, giving the impression of greater combined load capacity at the *initial* footing embedment. Dimensionless plots of vertical displacement against horizontal displacement ($\Delta z/R:h/R$) are included in Fig. 5.19 for completeness; detailed comments have already been made about the corresponding $\Delta z:h$ plots in Section 4.6.2.

Fig. 5.20 shows normalised data from the pure rotation ($\omega = 90^\circ$) events conducted at $V/V_0 = 0.75$. As noted in Section 4.6.2, the onset of yield in a footing rotation event is not particularly well defined, so the visually determined yield points in Fig. 5.20 are heavily dependent on subjective judgement. The original (non-normalised) plots of the two tests - Figs 4.33 and 4.35 - did not show particularly good repeatability because the kaolin sample used for T082 was markedly stronger than that used for T102. It can be seen from Fig. 5.20, however, that after normalisation of M/R by V_0 there is good agreement between the two sets of load:displacement data. The irregular $M/RV_0:\theta$ behaviour in the events at $z_{RP} = 0$ mm is more pronounced than the corresponding deviations of Fig. 5.19, though the same hardening effect is responsible.

Normalised data from Tests T083 ($\omega = 45^\circ$) and T103 ($\omega = 135^\circ$) appear in Fig. 5.21 (cf. Figs 4.36 and 4.37). In T083 the combined $h:R\theta$ events all produce a consistent $H/V_0:M/RV_0$ load response, with the familiar exception of the event at $z_{RP} = 0$ mm. Yield points in T083 are reasonably well defined. Fig. 5.21[e] shows that in T103 the normalised moment loads do not agree particularly well at large displacements; in fact there are signs of a systematic dependence on embedment as the events at both $z_{RP} = 0$ mm and 20 mm exhibit significant post-yield hardening. The normalised horizontal loads obtained in T103 are, however, consistent at all five depths (see Fig. 5.21[d]).

5.6.2 Normalised Yield Points: $V/V_0 = 0.2$

The results of Tests T091-T093 have been discussed in Section 4.6.3 and plotted (without normalisation) in Figs 4.38-4.41. Normalised load and displacement data from T091 ($\omega = 0^\circ$) appear in Fig. 5.22, showing very consistent H/V_0 and M/RV_0 load responses at all embedments. The yield points are not very well defined, but under footing rotation in T092

(Figs 5.22[d], [e]) yielding is even more gradual: the choice of yield points is clearly very subjective. Load:rotation response in T092 is similar at all depths apart from $z_{RP} = 0$ mm, where a clear case of post-yield softening (*i.e.* a reduction in H/V_0 and M/RV_0) occurs. This behaviour is closely related to the large post-yield increase in M/RV_0 exhibited by the $z_{RP} = 0$ events of Fig. 5.20. The occurrence of heave during probing events at low vertical load (Figs 5.22[c], [f]) causes contraction of the non-normalised $V:H:M/R$ yield surface, and the influence of this is most noticeable at $z_{RP} = 0$ where even a small amount of heave during an event causes a substantial reduction of the initial V_0 .

In T093a ($\omega = 45^\circ$ at $V/V_0 = 0.2$), the normalised load:displacement response is virtually identical at depths of 50 and 200 mm (see Fig. 5.23[a] and [b]). The event at $z_{RP} = 0$ mm, however, shows post-yield softening very similar to that observed at the same embedment in Fig. 5.22 during pure rotation at $V/V_0 = 0.2$. As shown in Figs 5.23[d] and [e], the two probing events at $\omega = 157.5^\circ$ produce consistent normalised load:displacement curves.

5.6.3 Incremental Plastic Displacements at Yield

Each probing event produces a single $(V/V_0, H/V_0, M/RV_0)$ yield point, as depicted in Figs 5.19-5.23. Also of interest are the relative magnitudes of vertical, horizontal and rotational displacement occurring at each yield point. In particular, incremental *plastic* displacements at yield are needed to define a suitable flow rule for use in conjunction with the experimentally determined yield surface.

The total displacement vector at yield in a probing event is determined from changes in the three measured displacements over a small interval centred on the chosen yield point. This interval corresponds to about 0.25 mm of displacement in the $h:R\theta$ plane. Plastic displacements are determined by deducting the estimated elastic components from the measured total displacements, using Eqn 2.4 to obtain the approximate elastic displacements:

$$\begin{Bmatrix} \delta z^p \\ \delta h^p \\ R\delta\theta^p \end{Bmatrix} = \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix} - \frac{1}{GR} \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & K_4 \\ 0 & K_4 & K_3 \end{bmatrix}^{-1} \begin{Bmatrix} \delta V \\ \delta H \\ \delta(M/R) \end{Bmatrix} \quad (5.16)$$

in which the measured load increment vector refers to the same interval as the displacements. For each depth of model spudcan embedment ($z_{RP} = 0$ mm, 20 mm, *etc.*) appropriate values of the elastic stiffness factors K_i may be obtained from the tables of Bell (1991), in the manner described in Section 6.2.1. The major source of uncertainty in Eqn 5.16 is the value chosen for the shear modulus G . If an expression similar to Eqn 3.8 is fitted to the shear vane strength data from a particular kaolin sample, s_u at the various depths of embedment may be estimated with reasonable confidence. An indicative value of G may then be computed using a rigidity index $G/s_u = 100$. This latter value is obviously approximate, but it is of the correct order of magnitude for Speswhite kaolin at moderate strain levels (Davidson, 1980; Fannin, 1986; Santa Maria, 1988; Smith, 1993). With $G/s_u = 100$, elastic displacements are

found to constitute about 3-4% of the total displacement increments δz , δh and $R\delta\theta$ in Eqn 5.16.

5.6.4 Comparison with Eqn 5.15: Yield Points

Fig. 5.24[a] summarises the results of all the probing events conducted at $V/V_0 = 0.75$. It is clear that the majority of the observed yield points lie some way inside the dashed ellipse of Eqn 5.15. At first sight, then, the surface fitted to the tracking event data appears to overestimate the combined load bearing capacity of the model footing at $V/V_0 = 0.75$. It is worth bearing in mind, however, that with increasing $h:R\theta$ displacements nearly all of the probing events go on to attain horizontal and moment loads significantly greater than those at their subjectively assigned yield points (see Figs 5.19-5.21). In Fig. 5.24[b] most yield points from the probing events of Batch 9 ($V/V_0 = 0.2$) are also located slightly inside the relevant locus of Eqn 5.15. Once again, however, the load:displacement plots of Figs 5.22 and 5.23 show that, in the majority of cases, post-yield increases in horizontal load and moment cause the normalised load point to move outwards in $H/V_0:M/RV_0$ space (and thus into better agreement with the Eqn 5.15 ellipse). The probing event yield points of Figs 5.24[a] and 5.24[b] do provide general confirmation that the normalised yield surface intersects a plane of constant V/V_0 with a rotated elliptical shape, as deduced from the tracking events of Tests T021-T063.

5.6.5 Comparison with Eqn 5.15: Displacements at Yield

The incremental plastic displacement vectors in Figs 5.24[a] and [b] give a convincing demonstration that the normality rule is indeed applicable in the $H/V_0:M/RV_0$ plane. In an attempt to quantify this general impression, the direction ω^p of plastic displacement observed at each yield point may be compared with a "theoretical" direction defined by normality at the closest point on the yield surface of Eqn 5.15. This process is illustrated schematically in Fig. 5.25[a]. Note that, for simplicity, the closest point is determined within the relevant plane $V/V_0 = 0.2$ or $V/V_0 = 0.75$, and not by a strict minimum perpendicular distance between the $(V/V_0, H/V_0, M/RV_0)$ yield point and the three-dimensional surface. The direction of the normal at any point $(V/V_0, H/V_0, M/RV_0)$ on the yield surface of Eqn 5.15a may be found from the gradient vector

$$\nabla f(V/V_0, H/V_0, M/RV_0) = \left\langle \frac{\partial f}{\partial (V/V_0)}, \frac{\partial f}{\partial (H/V_0)}, \frac{\partial f}{\partial (M/RV_0)} \right\rangle \quad (5.17)$$

The "theoretical" work-conjugate plastic displacement vector (Fig. 5.25[a]) is parallel to this gradient vector:

$$\langle \delta z^p, \delta h^p, R\delta\theta^p \rangle = \xi \left\langle \frac{\partial f}{\partial (V/V_0)}, \frac{\partial f}{\partial (H/V_0)}, \frac{\partial f}{\partial (M/RV_0)} \right\rangle \quad (5.18)$$

where ξ is a scalar. Computation of the partial derivatives from Eqn 5.15[a] is straightforward, but the expressions will not be stated explicitly here. The procedure shown

in Fig. 5.25[a] was implemented in a short FORTRAN routine and applied to all of the observed probing event yield points; the results appear in Fig. 5.25[b]. A similar number of data points appears above the solid line as below, though the scatter is not random: the sense of the deviation in direction is mildly dependent on ω^p . Results for the two vertical load levels, however, show no discrepancy. Overall it is clear from Fig. 5.25[b] that experimental data confirm the applicability of an associated flow rule in $H/V_0:M/RV_0$ load space.

In Figs 5.24 and 5.25[a], vertical plastic displacements do not appear because they are perpendicular to the page. In Fig. 5.26, however, it is possible to compare the vertical components of the observed and "theoretical" plastic displacement vectors at yield. The quantity plotted on each axis is the vertical direction cosine of the incremental displacement vector:

$$\frac{\delta z^p}{\sqrt{(\delta z^p)^2 + (\delta h^p)^2 + (R\delta\theta^p)^2}} \quad (5.19)$$

It is immediately clear from Fig. 5.26 that the assumption of normality to the Eqn 5.15 yield surface leads, in general, to an *overprediction* of the observed vertical plastic displacements at yield. The notable exceptions are the two groups of probing events conducted with $\omega = 45^\circ$: there is very good agreement between "theoretical" and observed settlements at $V/V_0 = 0.75$, and only a slight overprediction of the observed heaves at $V/V_0 = 0.2$. Conversely the negative eccentricity probes ($\omega = 135^\circ$, $\omega = 157.5^\circ$) give respective settlements and heaves at yield which are significantly less than those predicted by the normality rule. In probing tests using pure horizontal displacement ($\omega = 0^\circ$) and pure rotation ($\omega = 90^\circ$) the observed vertical displacements at yield are generally about 60% of those expected from normality to the Eqn 5.15 yield surface. For $V/V_0 = 0.75$ the probing events conducted at $z_{RP} = 0$ mm and 20 mm result in settlements at yield which are larger than those observed at subsequent depths, particularly when $\omega = 0^\circ$. The hatched area adjacent to the origin indicates that initial yielding in a loop event is associated with very slight footing heave (see Figs 4.27-4.30), giving qualitative agreement with the associated flow prediction for $V/V_0 = 0.4$. Unfortunately the nature of the loop events is such that well defined yielding is very hard to identify, and a quantitative analysis has not been undertaken. The dashed and dash-dot lines in Fig. 5.26 are separate best fit lines (forced to pass through the origin) for the heave and settlement data points. These regression lines are only intended to give a very general idea of the way in which the normality rule, applied to Eqn 5.15, overpredicts the vertical plastic displacements occurring at yield in the experimental probing events. A more refined modelling procedure would need to account for the pronounced dependence of spudcan heave and settlement on the displacement direction ω .

5.7 Vertical Unload-Reload Loops: Equivalent Rigidity Index

A preliminary analysis of all the vertical unload-reload loops has already been undertaken in Chapter 4 (Section 4.8; Table 4.7; Figs 4.43, 4.44). Figs 4.44[a] and [b] relate the chord

unload-reload stiffness, in N/mm, to the vertical displacement amplitude of the loop Δz (in mm). A useful dimensionless form of these two plots involves computation of the equivalent rigidity index G/s_u implied by each unload-reload loop:

$$(G/s_u)_{ULRL} = \frac{\Delta V}{K_1 \Delta z R s_u} = \frac{k_{ULRL}}{K_1 R s_u} \quad (5.20)$$

For each test, ΔV and Δz are obtained directly from the unload-reload loops performed at embedments z_{RP} of 35, 75 and 150 mm. The required values of s_u are determined from a least squares fit to the measured sample vane strengths, as described in Section 5.6.3 above. K_1 , the elastic stiffness factor, takes values of 8.66, 9.36 and 10.22 at the three embedments (interpolated from Tables 2.1 and 2.2 as described in Section 6.2.1, using the equivalent cone angle for the model spudcan of 152.7°). The $(G/s_u)_{ULRL}$ values obtained from Eqn 5.20 are too high, however, because the footing experiments were performed in a relatively small tank. Bell (1991) experimented with different mesh sizes in his finite element work and found that, for a flat circular footing, a soil depth of z_{max} gave dimensionless factors K_1 that were overstiff by a factor of about $(1 + R/z_{max})$ relative to the exact infinite half space solution. For the present analysis, therefore, $(G/s_u)_{ULRL}$ values from Eqn 5.20 have been divided by the correction factor $(1 + 62.5/400) \approx 1.16$.

Corrected $(G/s_u)_{ULRL}$ values are plotted against the dimensionless loop amplitude $\Delta z/R$ (on a log scale) in Fig. 5.27. It is clear that equivalent rigidity indices from unload-reload events in all of the standard rate tests (Fig. 5.27[a]) fall close to a single curve. The systematic dependence of k_{ULRL} on embedment depth (see Fig. 4.44[a]) appears to be greatly reduced in the dimensionless version: for a given loop amplitude $\Delta z/R$, the equivalent G/s_u is not markedly dependent on z_{RP} . The dashed curve in Fig. 5.27[a] is the result of a nonlinear fit to all the data points, and it describes the overall variation quite well. Very small amplitude loops were used in Batch 9 (unloading only to $V/V_0 = 0.75$ before reloading), but the high stiffnesses obtained should be treated with caution since each loop contains just a few data points (see Fig. 4.43[d]). The dash-dot curve in Fig. 5.27[a] comes from a nonlinear regression with the data from Tests T091-T093 excluded. Fig. 5.27[b] suggests that, for the range of amplitudes covered, the displacement rate used in a vertical unload-reload loop does not affect the equivalent rigidity index $(G/s_u)_{ULRL}$.

5.8 Comparisons with Other Yield Surfaces

Normalised results from the experimental loop events and probing events provide satisfactory confirmation of both the size and shape of the $V/V_0:H/V_0:M/RV_0$ yield surface fitted to the tracking event data (Eqn 5.15). It is of interest to compare various planar sections through the surface of Eqn 5.15 with the predictions of some other $V/V_0:H/V_0:M/RV_0$ interaction surfaces described in Chapter 2. Note that the Brinch Hansen (1970) and Vesic (1975) formulae assume a flat footing and a uniform undrained strength profile, whereas the present experiments were performed with spudcan-shaped footings on clay samples having a significant increase in s_u with depth. The widespread use of general bearing capacity

formulae in the design and assessment of real jack-up foundations is, however, subject to exactly the same inconsistencies, making the comparisons which follow particularly relevant.

5.8.1 The $V/V_0 : H/V_0$ Plane

Fig. 5.28[a] shows various predictions of normalised vertical load:horizontal load interaction for circular footings on clay. Two extremes of the Brinch Hansen solution (surface footing and deeply embedded footing) are plotted. It is clear that, contrary to widely held opinion, the Brinch Hansen method significantly overpredicts the horizontal load capacity of a surface spudcan footing. For vertical load levels $0.5 < V/V_0 < 1$ the disparity is not serious, but for $V/V_0 < 0.5$ the Brinch Hansen assumption of full sliding capacity ($H = As_u$) is too optimistic. Brinch Hansen's deep footing solution shows better agreement with Eqn 5.15. For $0.7 < V/V_0 < 1$ the two curves are almost coincident, and for $0.25 < V/V_0 < 0.7$ the Brinch Hansen prediction is conservative. When $V/V_0 < 0.25$, however, the theoretical sliding load (again $H = As_u$) assumed by Brinch Hansen gives an unsafe estimate of the actual horizontal load capacity. The symmetrical parabola of Model A (peak $H/V_0 = 0.13$) shows very good overall agreement with Eqn 5.15 (peak $H/V_0 = 0.127$ at $V/V_0 = 0.464$). The largest discrepancy between these two curves occurs at about $V/V_0 = 0.15$, where Model A underpredicts the measured horizontal load capacity by some 25%.

5.8.2 The $V/V_0 : M/RV_0$ Plane

As shown in Fig. 5.28[b], the two Brinch Hansen (1970) predictions of moment load:vertical load interaction (for surface and deep circular footings) agree closely with the symmetrical parabola of Model A. In the region of $V/V_0 = 0.5$, however, all three curves are unconservative by about 15% with respect to Eqn 5.15. It should be remembered that normalised moments M_{int}/RV_0 up to 0.185 were obtained in the rotated ellipse fitting process (see Fig. 5.13[c]) but the subsequent weighted least squares fit gave a lower peak moment ($M/RV_0 = 0.166$ at $V/V_0 = 0.464$). Brinch Hansen's formula and Model A are thus in somewhat better agreement with the experimental data than Fig. 5.28[b] suggests. In any case, it is clear from Fig. 5.28 that the Brinch Hansen (1970) method provides much more realistic modelling of $V/V_0 : M/RV_0$ interaction than of $V/V_0 : H/V_0$ behaviour for spudcan footings on clay.

5.8.3 The $H/V_0 : M/RV_0$ Plane

Predictions of the yield surface cross-sectional shape are considered at three representative vertical load levels: $V/V_0 = 0.8, 0.5$ and 0.2 . Fig. 5.29[a] shows that the non-rotated ellipses of Model A agree quite well with the realistic $H/V_0 : M/RV_0$ interaction loci of Eqn 5.15. The non-rotated ellipses, however, fail to model the important experimental finding that maximum horizontal load capacity is actually obtained in conjunction with a small moment, and not when $M = 0$. Similarly the moment capacity when $H = 0$ is not the *maximum* moment capacity of the footing. Fig. 5.29[b] compares the Brinch Hansen solution for a surface footing with Eqn 5.15. At $V/V_0 = 0.8$ there is excellent agreement in the negative

eccentricity quadrants of $H/V_0 : M/RV_0$ space, but in the other two quadrants Brinch Hansen's method is conservative. This is a little surprising because, as mentioned in Section 2.3.8, both the Brinch Hansen and Vesic methods appear to be primarily intended for use with positive eccentricities when a horizontal loading component is present. In Fig. 5.29[b] the overall agreement between the Brinch Hansen (surface footing) prediction and Eqn 5.15 is best when $V/V_0 = 0.5$. At $V/V_0 = 0.2$ Brinch Hansen's presumption of full sliding capacity at low vertical loads (Fig. 5.28[a]) results in a poor modelling of the true $H/V_0 : M/RV_0$ interaction. In Fig. 5.29[c], showing Brinch Hansen's prediction of combined load bearing capacity for a deep footing, there is again better agreement with Eqn 5.15 in the two negative eccentricity quadrants (particularly at $V/V_0 = 0.8$ and $V/V_0 = 0.2$). As in Fig. 5.29[b], the Brinch Hansen formula is generally conservative in the other two quadrants but its shape is in poor agreement with the experimentally determined interaction loci.

5.8.4 Sense of Eccentricity in the $H/V_0 : M/RV_0$ Plane

As discussed in Section 2.3.8, experiments with flat strip footings on sand (Zaharescu, 1961; Gottardi & Butterfield, 1993) have indicated that the bearing capacity interaction diagram in $H/V_0 : M/BV_0$ space is not symmetrical with respect to the coordinate axes. For a given horizontal load, moment capacity is greater if the sense of eccentricity is negative (see Fig. 2.9[d]). Numerical studies by Bell (1991) confirmed this observation for a rough flat circular footing on clay (Fig. 2.10[b]). Clearly the present experimental results for spudcan footings on clay are in direct conflict with these previous findings, since - for a given H/V_0 - moment capacity is greater if the sense of eccentricity is *positive* (see, for example, Fig. 5.11).

This contrasting behaviour is almost certainly attributable to the use of a spudcan-shaped footing in the present tests (all three references quoted above deal with footings having *flat* undersides). Some simple experiments using a flat circular footing and a 150° conical footing have confirmed that a relatively minor change in underside profile should indeed have a profound influence on the shape of the $H/V_0 : M/RV_0$ yield surface cross-section. These experiments, carried out on loose Leighton Buzzard sand with 100 mm diameter model footings, are briefly described below.

When a flat circular footing on sand is subjected to a combination of central vertical load and horizontal load acting to the right, it fails (Fig. 5.30[a]) by settling vertically, sliding horizontally to the right and rotating clockwise about the centre of its base. This failure mode may be represented in Fig. 5.30[c] by a yield point on the H/V_0 axis (Point A) with an incremental plastic displacement vector directed, as shown, with positive δh^p and positive $\delta \theta^p$. If in a separate test the flat circular footing is subjected to an eccentric vertical load, it fails in the manner of Fig. 5.30[b]: vertical settlement, clockwise rotation and positive horizontal displacement. The corresponding yield point and plastic displacement vector are shown at Point B in Fig. 5.30[c]. After deducing points A' and B' and their displacement vectors from symmetry, an assumed $H/V_0 : M/RV_0$ normality rule (*cf.* Fig. 5.24) suggests the dashed yield surface shown. This shape is consistent with the findings of Zaharescu (1961), Gottardi & Butterfield (1993) and Bell (1991) for flat-based footings, and interestingly the

shape in the positive eccentricity quadrants also shows good qualitative agreement with Brinch Hansen's (1970) method (see Figs 5.29[b] and [c]).

If the same two loading tests are performed on a 150° conical footing, the displacements at failure are rather different. Under a central inclined load (Fig. 5.30[d]) the footing slides to the right as before, but this time rotates anticlockwise. Under eccentric vertical loading the conical footing rotates clockwise but slides horizontally to the left (Fig. 5.30[e]). These two failure modes may be plotted as yield points and plastic displacement vectors in Fig. 5.30[f], Points D and E. If the normality rule is again assumed to be correct, the yield surface shape for a 150° conical footing on sand may be sketched in as shown. This qualitative $H/V_0 : M/RV_0$ interaction locus agrees with the present experimental observations for spudcan footings on clay. Plasticity theory with an associated flow rule thus implies that flat footings and shallow conical footings would indeed be expected to have constant V/V_0 yield surfaces rotated into different quadrants of $H/V_0 : M/RV_0$ load space. Furthermore, there should be some cone angle $150^\circ < \beta < 180^\circ$ for which the cross-sectional yield surface is symmetrical with respect to the coordinate axes.

In closing it should be mentioned that the choice of reference point for footing loads and displacements is extremely important when determining the cross-sectional yield surface shape. Dean *et al.* (1992) give a lucid explanation of how, by changing the height of the reference point on the footing centreline, a rotated elliptical $H:M/R$ yield locus may be transformed into a simple non-rotated ellipse (see Fig. 5.31). Note that the normality rule, if applicable for one reference point, is unaffected by such a transformation. In Eqn 5.15 the ellipse eccentricity is dependent on V/V_0 , so any attempt to simplify Eqn 5.15 with a change of reference point would not be worthwhile. If, however, the constant eccentricity value $e = 0.316$ is chosen as a best fit (see Fig. 5.14[a]), then an upwards movement of the reference point by a distance $0.413R$ reduces the yield surface cross-section to non-rotated elliptical form, with parameters $H_0/V_0 = 0.134$ and $M_0/RV_0 = 0.166$.

NUMERICAL MODELLING

6.1 Introduction

This chapter describes the formulation of an incremental work hardening plasticity model (Model B) for predicting the load:displacement response of conical or spudcan footings on clay. The model caters for combined loading with three degrees of freedom ($V:H:M$, $z:h:\theta$), and is essentially a refined version of the hypothetical Model A presented in Chapter 2. Some features of Model B are based on analytical solutions, while others are determined empirically from the physical footing test results. The theory and procedures used in *OXSPUD*, a computer implementation of the numerical model, are described in some detail. Program *OXSPUD* is used to obtain retrospective predictions of several footing tests, allowing a thorough evaluation of the capabilities of the numerical model.

6.2 Features of Model B

Model B is intended for use with spudcan or conical footings on clay under conditions of undrained loading ($\nu = 0.5$). Positive loads and displacements are as defined Fig. 1.5[b]. The method of treating a spudcan as an equivalent conical footing is retained from Fig. 2.1; note also that in the following sections, R and β always refer to the current contact radius R_{equiv} and equivalent cone angle β_{equiv} .

6.2.1 Model B: Elastic Response

The elastic stiffness factors obtained by Bell (1991) are used to determine load:displacement behaviour inside the yield surface:

$$\begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix}^e = GR \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & K_4 \\ 0 & K_4 & K_3 \end{bmatrix} \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^e \quad (6.1) \quad (2.4 \text{ bis})$$

Note that the cross-coupling coefficient K_4 , omitted from Model A, is now included. Values of K_i for a flat circular footing ($\beta = 180^\circ$) at embedments $0 \leq D/2R \leq 2$ appear in Table 2.1. Here the depth of embedment D refers to the *plastic* vertical footing penetration z^p . The elastic vertical displacement z^e is added to z^p to obtain the total vertical displacement, but it is most important to realise that the elastic stiffness factors K_i are a function of z^p , not of $(z^p + z^e)$. The values of K_i for conical footings ($\beta = 150^\circ, 120^\circ, 90^\circ$) located at the soil surface ($D = 0$) are given in Table 2.2. Stiffness factors for embedded cones were not obtained directly by Bell (1991), but may be estimated by assuming that conical footings are subject to the same changes $\Delta K_i = K_i(\text{embedded}) - K_i(\text{surface})$ as the flat circular footing of Table 2.1.

6.2.2 Model B: Yield Surface

In Chapter 5 it was found that experimental results could be represented very satisfactorily by a yield surface of the form

$$f\left(\frac{V}{V_0}, \frac{H}{H_0}, \frac{M/R}{M_0/R}\right) = \left(\frac{H}{H_0}\right)^2 + \left(\frac{M/R}{M_0/R}\right)^2 - 2\left[e_1 + e_2\left(\frac{V}{V_0}\right)\left(\frac{V}{V_0} - 1\right)\right]\left(\frac{H}{H_0}\right)\left(\frac{M/R}{M_0/R}\right) - \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1}(\beta_2)^{\beta_2}}\right]^2 \left(\frac{V}{V_0}\right)^{2\beta_1} \left(1 - \frac{V}{V_0}\right)^{2\beta_2} = 0 \quad (6.2a) \text{ (5.15a bis)}$$

where V_0 is the bearing capacity under purely vertical loading, and H_0 and M_0/R are proportional to V_0 . From the yield surface tracking tests, the following best fit parameter values were derived for spudcan footings on clay:

$$H_0/V_0 = 0.127; \quad M_0/RV_0 = 0.166; \quad \beta_1 = 0.764; \quad \beta_2 = 0.882; \\ e_1 = 0.518; \quad e_2 = 1.18 \quad (6.2b) \text{ (5.15b bis)}$$

Inside the yield surface $f < 0$ and behaviour is elastic, governed by Eqn 6.1. For $(V, H, M/R)$ force points lying on the yield surface, $f = 0$ and footing behaviour is elastoplastic. Because β_1 and β_2 are not integers, there is a minor difficulty in that f is undefined when $V/V_0 < 0$ or $V/V_0 > 1$. Physically, of course, V can never exceed V_0 , but the numerical solution method (Section 6.4) frequently requires evaluation of f when $V/V_0 > 1$ (e.g. when checking if an increment is elastic or elastoplastic). It is therefore desirable to have an equivalent expression for the yield function f which is defined for all $V/V_0 \geq 0$. This may be achieved by rewriting Eqn 6.2a as

$$\left(\frac{H}{H_0}\right)^2 + \left(\frac{M/R}{M_0/R}\right)^2 - 2\left[e_1 + e_2\left(\frac{V}{V_0}\right)\left(\frac{V}{V_0} - 1\right)\right]\left(\frac{H}{H_0}\right)\left(\frac{M/R}{M_0/R}\right) = \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1}(\beta_2)^{\beta_2}}\right]^2 \left(\frac{V}{V_0}\right)^{2\beta_1} \left(1 - \frac{V}{V_0}\right)^{2\beta_2} \quad (6.3)$$

and raising both sides to the power of $1/2\beta_2$:

$$\left[\left(\frac{H}{H_0}\right)^2 + \left(\frac{M/R}{M_0/R}\right)^2 - 2\left[e_1 + e_2\left(\frac{V}{V_0}\right)\left(\frac{V}{V_0} - 1\right)\right]\left(\frac{H}{H_0}\right)\left(\frac{M/R}{M_0/R}\right)\right]^{\frac{1}{2\beta_2}} = \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1}(\beta_2)^{\beta_2}}\right]^{\frac{1}{\beta_2}} \left(\frac{V}{V_0}\right)^{\frac{\beta_1}{\beta_2}} \left(1 - \frac{V}{V_0}\right) \quad (6.4)$$

The revised version of the yield function is therefore

$$f\left(\frac{V}{V_0}, \frac{H}{H_0}, \frac{M/R}{M_0/R}\right) = \left[\left(\frac{H}{H_0}\right)^2 + \left(\frac{M/R}{M_0/R}\right)^2 - 2 \left[e_1 + e_2 \left(\frac{V}{V_0}\right) \left(\frac{V}{V_0} - 1\right) \right] \left(\frac{H}{H_0}\right) \left(\frac{M/R}{M_0/R}\right) \right]^{\frac{1}{2\beta_2}} - \left[\frac{(\beta_1 + \beta_2)^{\beta_1 + \beta_2}}{(\beta_1)^{\beta_1} (\beta_2)^{\beta_2}} \right]^{\frac{1}{\beta_2}} \left(\frac{V}{V_0}\right)^{\frac{\beta_1}{\beta_2}} \left(1 - \frac{V}{V_0}\right) = 0 \quad (6.5)$$

Note that f remains undefined for $V/V_0 < 0$ unless β_1/β_2 happens to be an integer value.

6.2.3 Model B: Flow Rule

As discussed in Section 5.6.5, a straightforward assumption of associated flow

$$\langle \delta z^p, \delta h^p, R \delta \theta^p \rangle = \xi \left\langle \frac{\partial f}{\partial V}, \frac{\partial f}{\partial H}, \frac{\partial f}{\partial (M/R)} \right\rangle \quad (6.6) \text{ (cf. 5.18)}$$

leads to an overprediction of the vertical footing displacements (whether settlement or heave) occurring during yield. On the other hand, experimental evidence does support the applicability of associated flow in the $H: M/R$ plane (Figs 5.24, 5.25). A rigorous plasticity model would require the definition of a plastic potential function g , but for the purposes of Model B a convenient modification of Eqn 6.6 will suffice for the flow rule:

$$\langle \delta z^p, \delta h^p, R \delta \theta^p \rangle = \xi \left\langle \zeta_a \cdot \frac{\partial f}{\partial V}, \frac{\partial f}{\partial H}, \frac{\partial f}{\partial (M/R)} \right\rangle \quad (6.7a)$$

ζ_a is a scalar multiplier which allows empirical adjustment of the vertical plastic displacements predicted from normality. This adjustment of the vertical displacement component is closely analogous to Burd's (1986) adjustment of plastic volumetric strains in the formulation of a constitutive law for sand. In Eqn 6.7a the horizontal and rotational displacements are not altered from their associated flow values, so a view of Model B down the $V/V_0, \delta z^p$ axis still gives the impression of normality to the yield surface (see Fig. 6.1). The dashed and dash-dot regression lines in Fig. 5.26 imply that, for spudcan footings on clay, appropriate values of the "association parameter" ζ_a are

$$\begin{aligned} \zeta_a &= 0.580 \quad \text{when} \quad \frac{\partial f}{\partial V} < 0 \quad (\text{i.e. when } V/V_0 < 0.464); \\ \zeta_a &= 0.639 \quad \text{when} \quad \frac{\partial f}{\partial V} \geq 0 \quad (\text{i.e. when } V/V_0 \geq 0.464) \end{aligned} \quad (6.7b)$$

Figs 6.2[a] and [b] show the yield surface of Model B projected onto the $V/V_0: M/RV_0$ plane. In Fig. 6.2[a] the incremental plastic displacement vectors are normal to the yield surface (*i.e.* $\zeta_a = 1$ for all V/V_0). Fig. 6.2[b] shows the actual Model B displacement vectors, with the associated flow predictions of δz^p reduced in accordance with Eqns 6.7. Note that at $V/V_0 = 0.464$ the displacement vectors are perpendicular to the δz^p axis; the actual value of ζ_a is not relevant at this vertical load level since $\partial f / \partial V = 0$. Similarly at $V/V_0 = 0$ and $V/V_0 = 1$ the incremental plastic displacement vectors are directed along the δz^p axis

($\delta h^p = R\delta\theta^p = 0$) so the particular value of ζ_a does not have any significance at these two points either. An important feature of Model B is that, because $\beta_1 < 1$ and $\beta_2 < 1$, all three load derivatives of f exist at both $V/V_0 = 0$ and $V/V_0 = 1$. It will be recalled that the gradient vector $\langle \partial f / \partial V, \partial f / \partial H, \partial f / \partial (M/R) \rangle$ was undefined at the pointed ends of the Model A yield surface (which had $\beta_1 = \beta_2 = 1$); in Model B, however, the ends of the yield surface are rounded off and Eqn 6.7a may be used for all $0 \leq V/V_0 \leq 1$.

6.2.4 Model B: Hardening Law

The size of the non-normalised $V:H:M/R$ yield surface is controlled by the maximum vertical load capacity

$$V_0 = N_{c0} \cdot \pi R^2 \cdot s_{u0} \quad (6.8)$$

In Model B, N_{c0} is determined from the set of theoretical bearing capacity factors presented in Table 2.3 (see also Section 2.3.3). The undrained strength s_{u0} is as defined in Fig. 2.4[a]. It is important to note that, as with the elastic stiffness factors K_1 to K_4 , the embedment D in Tables 2.3-2.8 and Fig. 2.4[a] refers to the *plastic* footing penetration z^p and not to the total vertical displacement ($z^p + z^e$). This ensures that purely elastic load increments do not result in expansion or contraction of the $V:H:M/R$ yield surface.

6.3 Mathematical Formulation of Model B

It is easier to approach development of the model in terms of flexibility (rather than stiffness). Considering first the case of a purely elastic loading increment inside the yield surface, we have from Eqn 6.1

$$\begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^e = \frac{1}{GR} \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & K_3 \\ 0 & K_4 & K_3 \end{bmatrix}^{-1} \begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & C_4 \\ 0 & C_4 & C_3 \end{bmatrix} \begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix} \quad (6.9)$$

where the C_i are flexibility factors given by

$$\begin{aligned} C_1 &= \frac{1}{GR} \cdot \frac{1}{K_1}; & C_2 &= \frac{1}{GR} \cdot \frac{K_3}{K_2 K_3 - K_4^2}; \\ C_3 &= \frac{1}{GR} \cdot \frac{K_2}{K_2 K_3 - K_4^2}; & C_4 &= \frac{1}{GR} \cdot \frac{K_4}{K_4^2 - K_2 K_3} \end{aligned} \quad (6.10a-d)$$

When the footing is yielding ($f = 0$, force point on the yield surface), there are additional plastic displacements given by

$$\begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^p = \xi \begin{Bmatrix} \zeta_a \cdot \partial f / \partial V \\ \partial f / \partial H \\ \partial f / \partial (M/R) \end{Bmatrix} \quad (6.11) \quad (6.7a \text{ bis})$$

with the non-negative scalar ξ determining the magnitude of the plastic displacement increment. Finally there are "coupled" displacements which arise during yielding because:

- the dimensionless elastic stiffness factors K_i depend on the plastic vertical penetration z^p ;
- the shear modulus G will, in general, also vary with footing embedment;
- for a partially penetrated spudcan, the contact radius R is a function of z^p .

From these three points and Eqns 6.10 it is clear that the flexibility factors C_i are functions of z^p , giving a third incremental displacement component which must be included when the footing is yielding:

$$\begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^{\text{coup}} = \delta z^p \begin{bmatrix} dC_1/dz^p & 0 & 0 \\ 0 & dC_2/dz^p & dC_4/dz^p \\ 0 & dC_4/dz^p & dC_3/dz^p \end{bmatrix} \begin{Bmatrix} V + \delta V \\ H + \delta H \\ M/R + \delta M/R \end{Bmatrix} \quad (6.12)$$

where $V + \delta V$, $H + \delta H$ and $M/R + \delta M/R$ are the loads at the end of the elastoplastic increment. Eqn 6.12 is based on a corresponding expression derived by Maier & Hueckel (1977) for a material with elastic moduli dependent on plastic strains.

The incremental displacement vector during yielding is determined from the sum of its elastic, plastic and coupled components (Eqns 6.9, 6.11 and 6.12):

$$\begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}^{\text{ep}} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & C_4 \\ 0 & C_4 & C_3 \end{bmatrix} \begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix} + \xi \begin{Bmatrix} \zeta_a \cdot \partial f / \partial V \\ \partial f / \partial H \\ \partial f / \partial (M/R) \end{Bmatrix} + \xi \cdot \zeta_a \cdot \frac{\partial f}{\partial V} \begin{bmatrix} dC_1/dz^p & 0 & 0 \\ 0 & dC_2/dz^p & dC_4/dz^p \\ 0 & dC_4/dz^p & dC_3/dz^p \end{bmatrix} \begin{Bmatrix} V + \delta V \\ H + \delta H \\ M/R + \delta M/R \end{Bmatrix} \quad (6.13)$$

noting that, from Eqn 6.11,

$$\delta z^p = \xi \cdot \zeta_a \cdot \partial f / \partial V \quad (6.14)$$

The presence of the unknown scalar ξ in Eqn 6.13 means that one further equation is required for the solution of an elastoplastic increment. This equation is derived from the consistency condition

$$\delta f = 0 \quad (6.15)$$

which ensures that the $(V, H, M/R)$ force point remains on the yield surface during plastic yielding. Now $\delta f = 0$ requires that

$$\begin{aligned} \frac{\partial f}{\partial V} \delta V + \frac{\partial f}{\partial H} \delta H + \frac{\partial f}{\partial (M/R)} \delta (M/R) + \\ \frac{\partial f}{\partial V_0} \delta V_0 + \frac{\partial f}{\partial H_0} \delta H_0 + \frac{\partial f}{\partial (M_0/R)} \delta (M_0/R) = 0 \end{aligned} \quad (6.16)$$

But V_0 , H_0 and M_0/R all depend on the plastic vertical penetration z^p , so from the chain rule

$$\frac{\partial f}{\partial V} \delta V + \frac{\partial f}{\partial H} \delta H + \frac{\partial f}{\partial (M/R)} \delta (M/R) + \xi \cdot \zeta_a \cdot \frac{\partial f}{\partial V} \left[\frac{\partial f}{\partial V_0} \cdot \frac{dV_0}{dz^p} + \frac{\partial f}{\partial H_0} \cdot \frac{dH_0}{dz^p} + \frac{\partial f}{\partial (M_0/R)} \cdot \frac{d(M_0/R)}{dz^p} \right] = 0 \quad (6.17)$$

again noting that $\delta z^p = \xi \cdot \zeta_a \cdot \partial f / \partial V$ (Eqn 6.14). The overall incremental form of Model B may be expressed in 7×7 matrix form as

$$\begin{bmatrix} B_{11} & B_{12} & \dots & & & & B_{17} \\ B_{21} & \ddots & & & & & \\ \vdots & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ B_{71} & & & & & & B_{77} \end{bmatrix} \begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \\ \delta z \\ \delta h \\ R\delta\theta \\ \xi \end{Bmatrix} = \begin{Bmatrix} inc_1 \\ inc_2 \\ inc_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (6.18)$$

where the matrix $[B]$ is given by:

$$[B] = \begin{bmatrix} 1 \text{ or } 0 & 0 & 0 & 0 \text{ or } 1 & 0 & 0 & 0 \\ 0 & 1 \text{ or } 0 & 0 & 0 & 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 & 1 \text{ or } 0 & 0 & 0 & 0 \text{ or } 1 & 0 \\ C_1 & 0 & 0 & -1 & 0 & 0 & \zeta_a \frac{\partial f}{\partial V} \left(1 + [V + \delta V] \frac{dC_1}{dz^p} \right) \\ 0 & C_2 & C_4 & 0 & -1 & 0 & \frac{\partial f}{\partial H} + \zeta_a \frac{\partial f}{\partial V} \left([H + \delta H] \frac{dC_2}{dz^p} + [M/R + \delta M/R] \frac{dC_4}{dz^p} \right) \\ 0 & C_4 & C_3 & 0 & 0 & -1 & \frac{\partial f}{\partial (M/R)} + \zeta_a \frac{\partial f}{\partial V} \left([H + \delta H] \frac{dC_4}{dz^p} + [M/R + \delta M/R] \frac{dC_3}{dz^p} \right) \\ B_{71} & B_{72} & B_{73} & B_{74} & B_{75} & B_{76} & B_{77} \end{bmatrix}$$

The seventh row of $[B]$ is either $\{0 \mid 0 \mid 0 \mid 0 \mid 0 \mid 0 \mid 1\}$ for an elastic increment, or, from Eqn 6.17,

$$\left\{ \frac{\partial f}{\partial V} \mid \frac{\partial f}{\partial H} \mid \frac{\partial f}{\partial (M/R)} \mid 0 \mid 0 \mid 0 \mid \zeta_a \frac{\partial f}{\partial V} \left(\frac{\partial f}{\partial V_0} \cdot \frac{dV_0}{dz^p} + \frac{\partial f}{\partial H_0} \cdot \frac{dH_0}{dz^p} + \frac{\partial f}{\partial (M_0/R)} \cdot \frac{d(M_0/R)}{dz^p} \right) \right\}$$

for an elastoplastic increment. An increment may be performed under full displacement control, full load control or mixed control (*e.g.* a vertical load increment with horizontal and rotational displacement increments). The three values

$$\begin{aligned} inc_1 &= \delta V \text{ or } \delta z \\ inc_2 &= \delta H \text{ or } \delta h \\ inc_3 &= \delta M/R \text{ or } R\delta\theta \end{aligned} \quad (6.19)$$

are entered in the right hand side vector of Eqn 6.18, and the first three rows of matrix $[B]$ are adjusted accordingly.

By differentiation of Eqn 6.5, expressions for $\partial f/\partial V$, $\partial f/\partial H$ and $\partial f/\partial(M/R)$ may be evaluated in closed form. This process is straightforward but the rather lengthy results will not be given here. The other required partial derivatives of f follow simply as

$$\frac{\partial f}{\partial V_0} = -\frac{V}{V_0} \cdot \frac{\partial f}{\partial V}; \quad \frac{\partial f}{\partial H_0} = -\frac{H}{H_0} \cdot \frac{\partial f}{\partial H}; \quad \frac{\partial f}{\partial(M_0/R)} = -\frac{M/R}{M_0/R} \cdot \frac{\partial f}{\partial(M/R)} \quad (6.20a;b;c)$$

In Model B the elastic stiffness factors K_i and the bearing capacity factors N_{c0} are obtained by interpolation from tables, so the derivatives

$$\frac{dC_1}{dz^p}, \frac{dC_2}{dz^p}, \frac{dC_3}{dz^p}, \frac{dC_4}{dz^p} \text{ and } \frac{dV_0}{dz^p}$$

must be evaluated numerically. Since H_0 and M_0/R are assumed proportional to V_0 (Eqn 6.2b), the derivatives dH_0/dz^p and $d(M_0/R)/dz^p$ are simple multiples of dV_0/dz^p .

The inclusion of coupled displacements (Eqn 6.12) means that the incremental loads δV , δH and $\delta M/R$ appear in the seventh column of matrix $[B]$ in Eqn 6.18. If an elastoplastic increment is completely load controlled then no problems arise because δV , δH and $\delta M/R$ are specified; all of the terms in $[B]$ may be evaluated and Eqn 6.18 solved directly for δz , δh and $R\delta\theta$. If, however, displacement control is used for one or more degrees of freedom, the corresponding incremental loads are not known prior to the increment. An iterative solution of Eqn 6.18 is evidently required in these circumstances, and this is discussed in more detail below.

The incremental plasticity solution method which has been outlined in this section is (with the exception of the coupled displacement aspects) due to Prof. G. T. Houlsby. In particular the formulation of Eqn 6.18 is considerably more versatile than conventional numerical approaches.

6.4 Program OXSPUD

Model B has been implemented in a FORTRAN program entitled *OXSPUD*. This section describes the essential features of the program and demonstrates its performance in a simple test problem.

6.4.1 Input and Output Data Files

Each run of *OXSPUD* is driven by an input data file containing information about footing geometry, soil properties and the sequence of load or displacement increments to be executed. Footings may be either conical or spudcan-shaped, and the dimensions required by *OXSPUD* are shown in Figs 6.3[a] and [b]. The footing/soil roughness factor α must also be specified as one of the footing properties. In *OXSPUD* the variation of undrained strength with depth may be specified by either a linear function

$$s_{u0} = s_{um} + \rho D \quad (6.21) \text{ (see Fig. 2.4[a])}$$

or a power law expression of the form

$$s_{u0} = A \cdot \gamma' D \cdot OCR^B, \quad OCR = \left(\frac{\sigma_{vmax} + \gamma' D}{\gamma' D} \right) \quad (6.22) \text{ (3.8, 3.6 bis)}$$

An approximately linear increase of s_u with depth is very common in offshore clay deposits (Poulos, 1988) and one would expect to use Eqn 6.21 in most design situations. Eqn 6.22 is only included in program *OXSPUD* to facilitate the retrospective simulation of experimental results reported in Chapter 4 (it will be recalled from Fig. 3.8[b] that - for $\gamma' = 6.85 \text{ kN/m}^3$ and $\sigma_{vmax} = 200 \text{ kPa}$ - the average undrained strength profile was modelled very well by Eqn 6.22 with $A = 0.253$ and $B = 0.754$; by contrast, a linear regression using Eqn 6.21 did not provide a satisfactory fit to the observed sample strengths). At a given footing embedment, *OXSPUD* assumes that the elastic shear modulus G is proportional to the undrained strength s_{u0} . The rigidity index G/s_u is therefore required by the program.

Instructions concerning load and/or displacement increments complete the input file. These instructions are quite similar to those used for controlling the experimental apparatus during a physical footing test (see Section 4.3.1 and Table 4.1). In *OXSPUD* the initial footing position is assumed to be as shown in Fig. 6.3[c] (cone) or Fig. 6.3[d] (spudcan), with tip penetration just about to commence. The input data file lists a sequence of load or displacement controlled stages which are executed one after the other, just as in the physical model tests. For each degree of freedom (vertical, horizontal and rotational), the required action over the course of a stage is specified with either an increment of load or an increment of displacement.

Default output from *OXSPUD* is simply a data file listing all of the calculated loads and displacements ($V, H, M/R, z, h, R\theta$) at the end of each stage. The user may, however, request that each stage be subdivided into a number of equal steps for computation, with additional output generated after each step.

6.4.2 Key *OXSPUD* Subroutines

SUBROUTINE VOCAL

The main purpose of this routine is the calculation of V_0 , the bearing capacity under pure vertical load, at any plastic footing penetration z^p . As with the experimental results, vertical footing penetrations are referred to the level of the reference point RP (Fig. 4.2) rather than to the tip level. Fig. 6.4 illustrates the method used in VOCAL to compute V_0 for a spudcan at different stages of embedment. The undrained strength profile is the same as that in Fig. 3.8[b], and the footing dimensions are those of the 125 mm spudcan used in the experimental programme (Fig. 3.14[b]). Considering the partially penetrated spudcan in Fig. 6.4 ($z_{RP}^p = -7.5 \text{ mm}$), there are a number of steps involved in evaluating V_0 . First of all VOCAL computes the maximum contact radius ($R_{equiv} = 30.0 \text{ mm}$, in this case) and equivalent cone angle ($\beta_{equiv} = 142.6^\circ$), thus simplifying the problem to one of finding V_0 for a surface conical footing. Although the variation of undrained strength with depth is nonlinear, the tabulated bearing capacity factors only deal with linear rates of increase (Eqn 6.21). VOCAL therefore fits a least squares straight line to the strength profile over a depth of one radius (*i.e.*

$R_{\text{equiv}} = 30.0$ mm) below the equivalent conical footing. As shown in Fig. 6.4, this regression gives an intercept of $s_{\text{um}} = s_{\text{u0}} = 4.77$ kPa and a slope $\rho = 0.177$ kPa/mm. The dimensionless strength increase parameter is therefore $\rho 2R/s_{\text{um}} = 2.23$, so together with $D = 0$, $\beta = 142.6^\circ$ and a roughness factor $\alpha = 1.0$, VOCAL performs a 4-way linear interpolation on the full set of bearing capacity factors (stored in a $6 \times 6 \times 6 \times 6$ array) to find $N_{c0} = 7.98$. A vertical load capacity of $V_0 = 107.7$ N for the partially embedded spudcan is then obtained from Eqn 6.8, noting that $R = R_{\text{equiv}} = 30.0$ mm.

For the fully embedded spudcan of Fig. 6.4 ($z_{\text{RP}}^p = 100$ mm), the procedure for finding V_0 is very similar. The equivalent conical footing has $R_{\text{equiv}} = 62.5$ mm and $\beta_{\text{equiv}} = 152.7^\circ$, and the dimensionless embedment is $D/2R = 100/125 = 0.8$. A representative undrained strength and rate of increase ρ are again obtained from a straight line fit to the nonlinear profile, this time over a depth of $R_{\text{equiv}} = 62.5$ mm below the footing reference point (see Fig. 6.4). Note that the dimensionless rate of strength increase is determined using the projected strength at the mudline, $s_{\text{um}} = 10.1$ kPa (so $\rho 2R/s_{\text{um}} = 0.32$). With $\alpha = 1$ as before, VOCAL interpolates a bearing capacity factor $N_{c0} = 7.57$, and with $s_{\text{u0}} = 12.7$ kPa a vertical bearing capacity $V_0 = 1174.9$ N is obtained from Eqn 6.8. Because of adhesion (skin friction) on the 2.5 mm vertical upstand of the spudcan, a small extra load must be added to the basic bearing capacity:

$$V_{\text{upst}} = 2\pi R \cdot H_{\text{upst}} \cdot \bar{s}_u \cdot \alpha \quad (6.23)$$

where H_{upst} is the total height of the upstand, $\bar{s}_u$ is the undrained strength at the mid-height of the upstand and α is the footing/soil roughness factor. For the embedded spudcan of Fig. 6.4, $V_{\text{upst}} = 12.3$ N; at just over 1% of the basic V_0 this quantity is almost negligible. For footings with larger vertical upstands, however, the effects of skin friction are potentially significant.

With the equivalent cone angle β_{equiv} and the dimensionless embedment $D/2R$ ($= z_{\text{RP}}^p/2R$), subroutine VOCAL performs linear interpolation on the tabulated values of the elastic stiffness factors K_1 to K_4 . Using the equivalent radius R_{equiv} and an elastic shear modulus $G = (G/s_u) \cdot s_{\text{u0}}$, VOCAL is able to return the four flexibility factors C_1 to C_4 (Eqns 6.10) required by the incremental plasticity solution. Note that Bell (1991) derived his K_i values for a soil with uniform shear modulus G , so at present OXSPUD does not account for the effects of increasing stiffness with depth on the elastic stiffness factors K_1 to K_4 .

SUBROUTINES YIELD and YIELD2

These routines take input consisting of a $(V, H, M/R)$ force point and a value for V_0 . Using the yield surface defined by Eqn 6.5 and the Model B parameters of Eqn 6.2b, YIELD simply returns the value of the yield function f . YIELD2 returns values for the partial derivatives of f which can be evaluated in closed form, namely $\partial f/\partial V$, $\partial f/\partial H$, $\partial f/\partial(M/R)$, $\partial f/\partial V_0$, $\partial f/\partial H_0$ and $\partial f/\partial(M_0/R)$.

SUBROUTINE NUMBER

As mentioned above, the other derivatives required for solution of an elastoplastic increment (dC_1/dz^p to dC_4/dz^p and dV_0/dz^p) must be found numerically. Prior to solving Eqn 6.18 the incremental plastic penetration δz^p is, of course, not known. In this situation NUMBER uses a finite difference formula to evaluate tangent derivatives at the initial plastic penetration z^p . For example in computing dV_0/dz^p , VOCAL is called to find V_0 at $z^p - 2\varepsilon$, $z^p - \varepsilon$, $z^p + \varepsilon$ and $z^p + 2\varepsilon$. With the standard 5-point finite difference formula,

$$\left(\frac{dV_0}{dz^p} \right)_{\text{tang}} \approx \frac{V_0|_{z^p-2\varepsilon} - 8 \cdot V_0|_{z^p-\varepsilon} + 8 \cdot V_0|_{z^p+\varepsilon} - V_0|_{z^p+2\varepsilon}}{12\varepsilon} \quad (6.24)$$

In NUMBER ε is initially set to $0.001 \times 2R$, then reduced by factors of 10 until $(dV_0/dz^p)_{\text{tang}}$ converges. (It was found that, although they are simpler, the usual three point and difference/quotient formulae suffered from serious round-off errors and generally did not converge as ε was reduced). Eqn 6.18 may therefore be solved, for a new increment, using tangent numerical derivatives evaluated for the initial footing state. This first solution yields an approximate value of δz^p (from Eqn 6.14) which is refined by iteration (see Section 6.4.3 below). During the iteration process, NUMBER is called upon to calculate secant numerical derivatives relevant to the known, converging value of δz^p for the increment. The difference/quotient formula is of course the appropriate one here:

$$\left(\frac{dV_0}{dz^p} \right)_{\text{sec}} = \frac{V_0|_{z^p+\delta z^p} - V_0|_{z^p}}{\delta z^p} \quad (6.25)$$

Numerical derivative expressions analogous to Eqns 6.24 and 6.25 are used to calculate tangent or secant values for dC_1/dz^p to dC_4/dz^p .

6.4.3 Solution Control

When considering a new load or displacement increment, *OXSPUD* always assumes that the increment is elastic and makes a trial solution of Eqn 6.18 on this basis. There are, however, several possible cases of footing behaviour which may arise (see Fig. 6.5[a]):

CASE 1: $f_{\text{init}} < 0$, $f_{\text{trial}} < 0$

Both the initial force point and the trial force point are located inside the yield surface. The whole increment is elastic, and the trial state of the system is accepted as the final state.

CASE 2: $f_{\text{init}} < 0$, $f_{\text{trial}} = 0$

The trial force point is located on the yield surface, but again the whole increment is elastic. The trial state of the system is accepted as the final state.

CASE 3: $f_{\text{init}} < 0$, $f_{\text{trial}} > 0$

The trial force point is outside the yield surface, so the increment must be subdivided into elastic and elastoplastic parts. A straightforward bisection algorithm is used to determine the

proportion $0 < \psi < 1$ which is elastic (*i.e.* a CASE 2 increment); the remaining fraction $1 - \psi$ is treated as an elastoplastic increment (CASE 6), with the force point finishing on the yield surface ($f_{\text{final}} = 0$).

CASE 4: $f_{\text{init}} = 0$, $f_{\text{trial}} < 0$

Unloading from the yield surface takes place. The whole increment is elastic, and the trial state is accepted as the final state.

CASE 5: $f_{\text{init}} = 0$, $f_{\text{trial}} = 0$

Both initial and trial force points lie on the yield surface, but the increment is in fact elastic. The trial state corresponds to the final state.

CASE 6: $f_{\text{init}} = 0$, $f_{\text{trial}} > 0$

The entire increment is elastoplastic; computation is straightforward.

In Section 6.3 above, and in the discussion of subroutine NUMDER, the need for an iterative solution to Eqn 6.18 (in the case of an elastoplastic increment) was mentioned. δV , δH and $\delta M/R$ all appear in the matrix $[B]$, and these load changes are not known before an increment if displacement control is used. Similarly the appropriate value of δz^p for evaluating the secant numerical derivatives (*e.g.* Eqn 6.25) is initially unknown. Both problems are overcome by the use of iteration. Once it has been determined that an elastoplastic increment is required, matrix $[B]$ in Eqn 6.18 is formulated for the initial state of the system ($V, H, M/R, z^p$). In evaluating the seventh column of $[B]$, tangent numerical derivatives with respect to z^p are determined using NUMDER and if any of δV , δH and $\delta M/R$ have not been specified as control increments they are set to zero. After the first solution of Eqn 6.18, which in *OXSPUD* is performed by Gauss-Jordan elimination, matrix $[B]$ is reformulated using:

- load increments δV , δH and $\delta M/R$ obtained from the first solution;
- secant numerical derivatives obtained (from NUMDER) using the first estimate of δz^p .

Iterative solution of Eqn 6.18 continues with progressively more accurate estimates of δV , δH , $\delta M/R$ and δz^p . Note, however, that the closed form derivatives $\partial f / \partial V$, $\partial f / \partial H$, $\partial f / \partial (M/R)$, $\partial f / \partial V_0$, $\partial f / \partial H_0$ and $\partial f / \partial (M_0/R)$ are evaluated for the initial state of the system (*i.e.* at the start of the increment) and they remain unchanged throughout the iteration process. Fig. 6.5[b] shows a schematic representation of the iteration process, for the simple case of an applied load increment δV . As shown in this figure, iterating δz^p allows the solution method to cope with sudden slope changes in the exact $V_0:z^p$ relationship. Pronounced slope discontinuities arise at various points during the initial penetration of a spudcan footing, as will be seen in Section 6.4.4. There are also very slight kinks in the theoretical $V_0:z^p$ curve at embedments of $D/2R = 0.1, 0.25, 0.5$ and 1.0 because linear interpolation is used on the tabulated bearing capacity factors. During an elastoplastic increment, the convergence criterion for the iterative solution of Eqn 6.18 is simply

$$|f| \leq tol \quad (6.26)$$

where tol is a user-defined tolerance set in the input data file (typically $tol = 10^{-6}$ for the sample *OXSPUD* runs presented below). If Eqn 6.26 is satisfied, the force point is deemed to lie on the yield surface and the solution proceeds to the next increment.

For one reason or another, the iteration process occasionally fails to converge to a sufficiently small value of f . In this situation *OXSPUD* attempts to reapply the increment in smaller substeps, using an automatic step size division algorithm. The strategy used is shown schematically in Fig. 6.5[c]. $A \rightarrow B$, the default step, gives a value of f lying outside the acceptable range. The first of two equal substeps is successful ($A \rightarrow C$), but the second one ($C \rightarrow D$) is not. *OXSPUD* therefore begins subdivision on $C \rightarrow D$, and two subsubsteps $C \rightarrow E \rightarrow F$ are sufficient to satisfy Eqn 6.26 in this case.

Despite the iterative solution method and the facility for automatic step size division, initial runs of *OXSPUD* showed that the absolute value of f tended to increase very slightly with every elastoplastic step (despite the consistency condition $\delta f = 0$). In a very long sequence of elastoplastic increments, $|f|$ eventually reached a level where even very fine step size division could no longer satisfy the convergence criterion of Eqn 6.26. This situation was considerably improved by modifying the consistency condition from

$$\delta f = 0 \quad (6.15 \text{ bis})$$

to

$$\delta f = -f_{OB} \quad (6.27)$$

where f_{OB} is the initial ("out-of-balance") value of f at the start of an elastoplastic increment. Solution of the elastoplastic version of Eqn 6.18 is thus performed with $-f_{OB}$ in the seventh row of the right hand side vector instead of zero. This has the effect of correcting the force point back towards the yield surface with every elastoplastic step, rather than letting it drift away gradually with $\delta f = 0$.

6.4.4 Test Problem

For the case of purely vertical loading it is possible to perform some simple verifications of the *OXSPUD* incremental plasticity formulation. To this end a second FORTRAN program, *VZPLOT*, was written around the subroutine V0CAL discussed in Section 6.4.2 above. *VZPLOT* takes the same footing geometry and undrained strength input data as *OXSPUD*, but its output is simply a series of points on the $V_0:z_{RP}$ load:penetration curve. Each point is generated by submitting a nominated plastic penetration z_{RP}^p to subroutine V0CAL, which returns the corresponding vertical bearing capacity V_0 and elastic flexibility factors C_1 to C_4 . *VZPLOT* then computes the total vertical penetration due to V_0 as

$$z_{RP}^{ep} = z_{RP}^p + z_{RP}^e = z_{RP}^p + C_1 V_0 \quad (6.28)$$

Referring to Fig. 6.6, the crosses are points produced by *VZPLOT* for the undrained strength profile of Fig. 3.8[b] and the fully rough ($\alpha = 1.0$) spudcan footing of Fig. 3.14[b]. The solid

bold line is the incremental $V_0:z_{RP}$ prediction obtained from program *OXSPUD*, commencing from the start of tip penetration ($z_{RP} = -29.14$ mm) and using displacement controlled steps of 0.25 mm. Fig. 6.6 shows that the incremental *OXSPUD* curve passes straight through each of the points obtained directly from *VZPLOT*. In fact the agreement can be made as close as desired simply by reducing *tol* in Eqn 6.26. This confirms, at least for the vertical degree of freedom, that the coupled incremental displacements of Eqn 6.12 are correctly implemented in program *OXSPUD*. The curve shown was produced for $tol = 10^{-6}$, and the computation took around 80 seconds on a 486DX2/33 MHz PC (*i.e.* slightly under 0.09 seconds per 0.25 mm step). Another check of the *OXSPUD* plasticity implementation was performed by using incremental load control (steps of δV) to generate the $V_0:z_{RP}$ relationship. By specifying sufficiently small values of *tol*, the load controlled incremental plasticity results could again be made to agree with *VZPLOT* to any desired accuracy. The dashed line in Fig. 6.6 represents the plastic component of z_{RP} , and it can be seen that for $G/s_u = 100$ the elastic vertical displacement z_{RP}^e remains reasonably constant (around 1.5 to 1.8 mm) for most of the spudcan penetration.

6.5 Retrospective Prediction of Selected Tests

In order to investigate the predictive capabilities of program *OXSPUD*, five representative tests from the experimental programme were chosen for numerical simulation:

T022: tracking events commencing from $V/V_0 = 1$, $\omega = 90^\circ$ (pure rotation)

T052: tracking events commencing from $V/V_0 = 0.01$, $\omega = 0^\circ$ (pure horizontal displacement)

T082: probing events at $V/V_0 = 0.75$, $\omega = 90^\circ$ (pure rotation)

T091: probing events at $V/V_0 = 0.2$, $\omega = 0^\circ$ (pure horizontal displacement)

T071: anticlockwise loop events at $V/V_0 = 0.4$, horizontal displacement leading

All of these tests used the 125 mm diameter spudcan of Fig. 3.14[b], so obviously the same dimensions were used in the *OXSPUD* predictions. Fig. 6.7 shows the five $s_u:D$ profiles supplied as input for the various numerical test simulations; these curves were obtained by fitting an expression of the form of Eqn 6.22 (with $\gamma' = 6.85$ kN/m³ and $\sigma_{vmax} = 200$ kPa) to the vane strength data for each particular clay sample. The individual shear vane test results may be found in Martin & Houlsby (1994). For all of the *OXSPUD* test predictions full footing/soil roughness ($\alpha = 1.0$) and a rigidity index $G/s_u = 100$ were assumed.

6.5.1 T022: *OXSPUD* Simulation

Fig. 6.8 shows the experimental results obtained in T022. The full test appears in the $V:z_{RP}$ plot (Fig. 6.8[f]), but for clarity only the results of the tracking events are shown in the other plots (in the $V:z_{RP}$ plot these events are shown in bold). In all seven plots, data marker spacing corresponds to 2 mm intervals of rotational displacement $R\theta$. An identical plotting system is used on the facing page (Fig. 6.9) to present the *OXSPUD* simulation of T022. Again the full test prediction appears in $V:z_{RP}$ space, but only the rotation events are shown in the remaining plots.

Like the experimental sequence of moves in T022, the *OXSPUD* test prediction in Fig. 6.9 is basically displacement controlled. The depths at which tracking events take place, and the amplitude of rotation used in each one, are determined directly from the experimental results. Similarly the numerically predicted vertical unload-reload loops are made to begin from the same depths as those in the actual experiment. Table 6.1 summarises the load and displacement control information used to obtain Fig. 6.9. From an initial spudcan position as shown in Fig. 6.3[d], Stage 1 (Table 6.1) penetrates the footing to the requisite embedment for the first rotation event. Stage 2 generates the appropriate amount of rotation (at constant z) and Stage 3 returns the footing to $\theta = 0$. Stage 4 resumes pure vertical penetration to the level of the next rotation event; Stages 5 and 6 produce an $R\theta$ cycle of the required amplitude. After some more vertical penetration (Stage 7), the first half of a vertical unload-reload loop is generated by Stage 8. Note that the unloading to $V = 0$ is performed under load control. Stage 9 resumes displacement controlled footing penetration, and the *OXSPUD* run continues with further rotation cycles and vertical unload-reload loops at greater embedments. After the last rotation cycle, Stage 22 retracts the numerical "footing" to $V = 0$ under load control.

Table 6.1 shows that there is no attempt to reproduce, for example, the small flexibility-related changes in vertical penetration which occur during tracking events at nominally constant z (see the discussion in Section 4.3.2). Similarly, horizontal displacement is forced to be zero throughout the numerical prediction, even though small values of h were registered during the experimental T022 (again as a result of apparatus flexibility). The *OXSPUD* simulation depicted in Fig. 6.9 is therefore slightly idealised, but it is still instructive to make comparisons with Fig. 6.8. The two $R\theta:z_{RP}$ plots (Figs 6.8[g], 6.9[g]) are indistinguishable because the numerical simulation is displacement controlled. Comparing the two $V:z_{RP}$ plots (Figs 6.8[f], 6.9[f]), *OXSPUD* clearly provides an excellent overall prediction of the observed increase in vertical bearing capacity with depth. The most serious difference between the two plots concerns the manner in which the virgin load:penetration curve is rejoined after a rotation event or vertical unload-reload loop. *OXSPUD* predicts a very sudden transition from elastic to elastoplastic behaviour each time the force point arrives on the yield surface, whereas the experimental observations show a much more gradual re-yielding when vertical penetration is resumed (particularly after the rotation cycles). There is no facility in *OXSPUD* for modelling time-dependent behaviour, so the vertical load holding stages of T022 are completely ignored in the numerical simulation. Another shortcoming of the *OXSPUD* model is the restriction to vertical loads $V \geq 0$; in fact very significant tensile loads were developed during the final footing retraction.

Turning to the load:load plots in Fig. 6.9, *OXSPUD* predictions of both the $V:H$ and $V:M/R$ load paths are in very good agreement with the experimental results. The predicted moments are of about the right magnitude, but the predicted horizontal loads are slightly low. *OXSPUD* is of course a model for footing/soil interaction, and cannot be expected to predict the effect of waterproofing shield embedment on the experimental results at $z_{RP} \approx 200$ mm (dashed lines in Fig. 6.8). Similar comments apply to the predicted $H:R\theta$ and $M/R:R\theta$

curves (Figs 6.9[c], [e]), though in general *OXSPUD* captures the character of the observed responses very satisfactorily. The numerically predicted $V:R\theta$ curves (Fig. 6.9[a]) all approach a well defined constant vertical load at large rotations, but in Fig. 6.8[a] only the event at $z_{RP} \approx 0$ mm exhibits this behaviour. In the subsequent experimental tracking events V continues to drop as rotation is increased, even though virtually constant values of H and M/R have been attained. There is, however, good overall agreement between the two $V:R\theta$ plots, and in general it is clear that *OXSPUD* performs fairly well in its simulation of T022.

6.5.2 T052: *OXSPUD* Simulation

Experimental results and corresponding numerical predictions for T052 appear on facing pages in Figs 6.10 and 6.11. The *OXSPUD* predictions of Fig. 6.11 were obtained using control instructions of the type shown in Table 6.2. Just as in the experimental T052, displacement controlled vertical penetration (Stage 1) is interrupted for load controlled unloading to $V = 0.01V_0$ (Stage 2) followed by a displacement controlled h cycle at constant z (Stages 3 and 4). Because vertical embedments and horizontal displacement amplitudes for the numerical simulation were determined directly from the experimental results, Figs 6.10[g] and 6.11[g] are virtually identical. There is good general agreement between the two $V:z_{RP}$ plots (Figs 6.10[f] and 6.11[f]), but as in T022 the idealised re-yielding behaviour obtained from *OXSPUD* is not particularly realistic. The numerically predicted $H:h$ and $M/R:h$ curves agree reasonably well with those actually obtained in T052, though the predicted moment loads (Fig. 6.11[e]) are somewhat lower than the observed ones in Fig. 6.10[e]. In the $V:H$, $V:M/R$ and $V:h$ plots, there are quite serious discrepancies between Fig. 6.10 and Fig. 6.11 because (except at $z_{RP} = 0$ mm) *OXSPUD* substantially overpredicts the increase in V caused by the horizontal displacement events. Generally speaking the *OXSPUD* modelling of T052 is satisfactory, but not as good as the simulation of T022.

6.5.3 T082: *OXSPUD* Simulation

As shown in Table 6.3, this *OXSPUD* simulation makes use of mixed control for the rotation events at constant vertical load. In Stages 3 and 4, V is load controlled but the other two degrees of freedom are displacement controlled; the same approach was, of course, used to control the experimental test. Figs 6.12 and 6.13 show the experimental results and the numerical prediction of T082. In $V:z_{RP}$ space the observed vertical bearing capacity generally exceeds the *OXSPUD* prediction by about 12.5%, for the full range of penetration. The kaolin sample for T082 was not unusually strong (see Fig. 6.7), so there is no apparent explanation for the larger-than-average vertical loads developed in this test. The difference between the observed and predicted vertical bearing capacity curves is reflected in the $V:H$, $V:M/R$ and $V:R\theta$ plots of Fig. 6.12 and Fig. 6.13, all of which show the loads $V = 0.75V_0$ used during the rotation events.

A comparison of Figs 6.12[e] and 6.13[e] shows that *OXSPUD* gives a reasonable overall prediction of the observed moment:rotation behaviour. A rigidity index of

$G/s_u = 100$ is, however, clearly too low for modelling the initial rotational stiffness. A better prediction of this aspect of the footing test is obtained with $G/s_u \approx 400$, but the quality of the experimental data did not warrant a detailed investigation of stiffness at very small rotations (where apparatus flexibility is a source of considerable uncertainty). At large rotations ($R\theta > 5$ mm) the predicted moments are lower than the observed ones, but this is only to be expected given the differences between predicted and observed $V:z_{RP}$ curves. For the rotation event at $z_{RP} \approx 0$, *OXSPUD* predicts a significant post-yield increase in moment capacity. This effect is much less noticeable in subsequent predicted rotations because the virgin load:penetration stiffness k_{VIR} is not as great (*cf.* Fig. 4.12). In the experimental results of Fig. 6.12[e] it is difficult to separate post-yield hardening from the gradual process of yielding itself. It does seem, however, that rotations at large embedments do exhibit a significant rotational plastic stiffness which is underpredicted by *OXSPUD*.

The initial elastic portions of the predicted $H:R\theta$ curves (Fig. 6.13[c]) are characterised by negative horizontal loads. This is because the cross-coupling elastic stiffness factor K_4 (Eqn 6.9) is negative for loads and displacements evaluated at the footing reference point RP (see Tables 2.1, 2.2). *OXSPUD* thus predicts that, for elastic behaviour inside the yield surface, a positive rotation $R\theta$ results in both a positive moment and a negative horizontal load. Once yielding occurs, however, elastoplastic rotation causes the horizontal load to increase again; this gives rise to the slope discontinuities associated with yielding in Fig. 6.13[c]. At small rotations $R\theta < 0.5$ mm, the experimental results of Fig. 6.12[c] are obviously in poor agreement with the numerically predicted curves. There is no experimental evidence that a positive footing rotation (even a very small one) leads to a decrease in horizontal load, though it does appear that the initial $H:R\theta$ stiffnesses in Fig. 6.12[c] are considerably smaller than the initial moment:rotation stiffnesses of Fig. 6.12[e]. At rotations $R\theta > 0.5$ mm, agreement between the observed and calculated $H:R\theta$ responses is reasonably good. While *OXSPUD* predicts that continued rotation leads to an almost constant horizontal load, the experimental results show a tendency for H to decrease gradually at large $R\theta$.

OXSPUD correctly predicts that each of the rotation events at constant V causes footing settlement (Fig. 6.13[g]). Note, however, that in each event the predicted settlement is zero until the yield surface is intersected and elastoplastic deformation begins. The calculated settlements agree quite well with the observed ones (Fig. 6.12[g]) until $R\theta \approx 1.5$ mm, but at larger rotations the *OXSPUD* predictions of Δz are too small. This is because the flow rule for Model B (Eqns 6.7a, 6.7b) was determined empirically on the basis of measured incremental plastic displacements *at first yield* (see Figs 5.19-5.23 and Fig. 5.26). Fig. 6.12[g] shows that the experimental $\Delta z:R\theta$ curves are far from linear, and accurate modelling into the large rotation range would therefore require an association parameter ζ_a (Eqn 6.7a) which was a function of rotation. *OXSPUD* predicts that the $R\theta$ event at $z_{RP} \approx 0$ mm results in rather less settlement than the subsequent rotations at deeper embedments (Fig. 6.12[g]), the difference becoming more pronounced with increasing $R\theta$.

In Fig. 6.12[g] it can be seen that the experimental $\Delta z:R\theta$ curve for $z_{RP} \approx 0$ mm does exhibit similar qualitative behaviour.

6.5.4 T091: OXSPUD Simulation

Figs 6.14 and 6.15 present the experimental results of T091 (h events at $V/V_0 = 0.2$) and the *OXSPUD* prediction of the same test. The calculated $V:z_{RP}$ plot (Fig. 6.15[f]) shows good quantitative agreement with the experimental curve, so the predicted and observed $V:H$, $V:M/R$ and $V:h$ plots also agree well. In Figs 6.15[c] and 6.15[e] there is post-yield softening (*i.e.* a reduction in H and M/R) at large h in all five simulated events, though this is only discernible for the $z_{RP} \approx 0$ curves. The predicted initial elastic $H:h$ stiffnesses are considerably lower than those obtained experimentally (Fig. 6.14[c]), implying that a larger value of G/s_u would be needed for accurate *OXSPUD* modelling of small displacement behaviour. Fig. 6.15[e] resembles Fig. 6.13[c], with the negative K_4 in Eqn 6.9 predicting that positive horizontal displacement should result in a negative footing moment prior to yield. This behaviour is certainly not manifested in the experimental curves of Fig. 6.14[e], which suggest that a positive elastic cross-coupling coefficient K_4 would be more appropriate than the theoretical negative value. At larger (post-yield) horizontal displacements the *OXSPUD* predictions of $H:h$ and $M/R:h$ behaviour agree well with the experimental results. The ultimate horizontal loads in Fig. 6.15[c] are generally quite close to the observed ones, while the predicted ultimate moments of Fig. 6.15[e] are evidently a little low. In $\Delta z:h$ space (Fig. 6.15[g]) *OXSPUD* correctly predicts that yielding at low vertical load levels is associated with upward vertical displacement. The calculated values of heave agree well with the T091 results (Fig. 6.14[g]) up to $h \approx 2$ mm, but thereafter too much heave is predicted. As discussed in Section 6.5.3, this is because Model B was calibrated using measured incremental plastic displacements occurring at first yield. *OXSPUD* predictions of Δz at large h are thus effectively based on extrapolation of the behaviour at small horizontal displacements, and Fig. 6.14[g] indicates that this will indeed lead to an overprediction of heave at large h .

6.5.5 T071: OXSPUD Simulation

Experimental data from T071 and the corresponding numerical predictions appear in Figs 6.16 and 6.17 respectively. The two $V:z_{RP}$ plots agree very well, and both show that the loop events were conducted at a vertical load level of $V/V_0 = 0.4$. In *OXSPUD* each loop event was generated by subdividing the nominal displacement path of Fig. 4.24[e] into 400 displacement controlled $h:R\theta$ increments, applied at constant V . As would be expected, *OXSPUD* produces a series of rotated elliptical $H:M/R$ load paths (Fig. 6.17[b]) which reflect the expansion of the non-normalised $V:H:M/R$ yield surface with embedment. The initial elastic portion of each predicted load path lies (at least in part) in quadrant No. 4 of $H:M/R$ space. This is because, as discussed above, the initial displacement direction ($\omega = 0^\circ$, *i.e.* pure h) results in negative footing moments. It has already been noted in Section 4.5.4 that the experimental results of Fig. 6.16[b] consistently follow an initial

direction $\Omega \approx 30^\circ$ in $H:M/R$ space. A superficial improvement to the predictions of program *OXSPUD* could obviously be achieved by using an empirically determined elastic cross-coupling factor $K_4 > 0$, rather than Bell's (1991) negative theoretical value. There would, however, be little justification for this course of action. The discrepancy in initial load path directions most probably arises because, in Fig. 6.16[b], significant plastic deformations occur from the very start of each loop event. Model B, with its simple elastic / plastic formulation, is not capable of modelling this gradual onset of full plasticity. Improvements to this aspect of the numerical model would certainly allow much more realistic predictions of loop event yielding than the sharp load path corners of Fig. 6.17[b]. After the initial elastic phase of each $H:M/R$ load path, there is good qualitative (and reasonable quantitative) agreement between Figs 6.16[b] and 6.17[b]. The agreement does, however, deteriorate once more in the latter stages of each loop event; the softening behaviour exhibited by the experimental load paths has already been discussed in Section 4.5.4.

6.5.6 Specific Shortcomings of Model B

From the preceding sections it is clear that Model B, as implemented in program *OXSPUD*, generally performs well in retrospective simulations of various footing tests. The comparisons between measured and predicted response have, however, concentrated on primary (monotonic) loading; no detailed comments have been made about *OXSPUD*'s handling of vertical unload-reload loops or reversals of h or $R\theta$. Fig. 6.18 contains an extract from the T082 footing test (rotation events at $V/V_0 = 0.75$), showing one vertical unload-reload loop (at $z_{RP} \approx 35$ mm) and one complete rotation cycle (at $z_{RP} \approx 50$ mm). Corresponding predictions obtained using program *OXSPUD* appear in Figs 6.19 ($G/s_u = 100$) and 6.20 ($G/s_u = 500$).

Fig. 6.18[c] shows that neither the vertical unloading response nor the reloading response is linear, with the result that a hysteretic $V:z$ loop is formed. *OXSPUD* models behaviour inside the yield surface as linear elastic, so the numerically predicted vertical unloading and reloading paths are identical (see Figs 6.19[c], 6.20[c]). By varying G/s_u the slope of the predicted unload-reload line may be varied to give the best overall approximation to the observed hysteretic loop, and Fig. 5.27[a] indicates that $G/s_u \approx 200$ would be appropriate for simulating the loop shown in Fig. 6.18[c]. Model B is, however, not capable of predicting the observed nonlinear behaviour which results in a hysteretic loop. Analogous comments may be made about modelling of the primary $M/R:R\theta$ response (Fig. 6.18[b], shown in bold). Of the two *OXSPUD* predictions, Fig. 6.19[b] clearly underpredicts the observed initial stiffness with $G/s_u = 100$, while Fig. 6.20[b] ($G/s_u = 500$) gives a much better simulation. Although there is scope for adjusting the initial linear elastic stiffness in *OXSPUD*, there is no way of avoiding the sudden transition from elastic to elastoplastic behaviour which is evident in both Figs 6.19[b] and 6.20[b]. In reality the process of yielding is rather more gradual, and Fig. 6.18[b] shows that this gradual yielding effect is even more apparent when the direction of rotation is reversed. Although *OXSPUD* makes quite a good prediction of the ultimate moment reached during the return to $R\theta = 0$ (i.e. $M/R \approx -125$ N),

the linear elastic unloading and sudden reverse yielding of Figs 6.19[b] and 6.20[b] compare poorly with the shape of the observed unloading curve. If further large amplitude cycles of rotation were conducted, similar discrepancies between experimental and calculated results would be expected to continue. On the other hand *OXSPUD* would probably give a satisfactory prediction of footing behaviour under *small amplitude* cycles of rotation (in which the effects of reverse plasticity would not be so dominant). In $z_{RP}:R\theta$ space, the chosen value of G/s_u does not visibly affect the vertical displacements predicted during the rotation cycle (Figs 6.19[d] and 6.20[d]). Qualitatively, the agreement between calculated and measured settlement response is very good; the reason for *OXSPUD*'s significant underprediction of the observed settlements (Fig. 6.18[d]) has already been discussed in Section 6.5.3.

Model B and program *OXSPUD* ignore the effects of time-dependent behaviour such as creep and consolidation. Fig. 6.18[c] shows the vertical load holding stage executed prior to the unload-reload loop, with vertical penetration occurring at constant V as a result of soil creep. For purely vertical loading, Model B predicts that any change in vertical displacement must be accompanied by a change in vertical load, as shown in Figs 6.19[c] and 6.20[c]. (Note, however, that during the rotation cycle Model B *does* predict vertical settlement at constant V). In Fig. 6.18[a] soil creep is again responsible for the relaxation of moment load which occurs at the end of the rotation cycle. M/R drops from -125 N to about -60 N during the $2\frac{1}{2}$ minutes taken to prepare the apparatus for a resumption of displacement controlled vertical penetration. Once vertical penetration is commenced and the virgin load:penetration curve is rejoined, the magnitude of the moment load in Fig. 6.18[a] immediately reduces towards zero. *OXSPUD* cannot model the creep-induced relaxation of moment which follows the return to $R\theta = 0$, but as shown in Figs 6.19[a] and 6.20[a] the numerical model does predict that M/R quickly returns to zero upon the resumption of vertical penetration.

SOIL/STRUCTURE INTERACTION

7.1 Introduction

This chapter describes the theory and operation of program *2DJUAN* (Two Dimensional Jack-Up ANalysis), which performs structural analysis of jack-up units on clay. The program includes the effects of both shear deformations and geometric $P \cdot \Delta$ nonlinearities, which are essential for realistic modelling of jack-up behaviour (SNAME, 1993). Dynamic effects are also important, particularly in cyclic analyses, but the present version of *2DJUAN* is restricted to static loading. A quasi-static analysis does, however, provide an acceptable means of modelling jack-up response to a single extreme loading event, and the results of a number of *2DJUAN* runs (using simplified representations of environmental loading and rig self-weight) are reported. Analyses are performed using different numerical models of spudcan behaviour, including the conventional assumption of pinned footings and the work hardening plasticity formulation (Model B) described in Chapter 6.

7.2 Structural Analysis Method

7.2.1 Introduction

Prior to about 1970, the principal analytical methods available to structural designers were (Harrison, 1973):

- linear elastic analysis;
- elastic stability (buckling) analysis;
- rigid / plastic analysis.

With the development of digital computers during the 1960s it became possible to analyse complex structures using these techniques, but the results remained highly idealised representations of the actual nonlinear structural response (see Fig. 7.1). Because it provides an acceptable model of structural behaviour for small loads and displacements, linear elastic analysis is still widely used today. Classical elastic stability analysis based on the linearised eigenvalue approach is also an important analysis tool, but the actual buckling load of a structure may be significantly lower because of initial imperfections and geometric nonlinearities. Furthermore, an elastic stability analysis does not give any information about the nature of the post-buckling response, which is not necessarily catastrophic and may be stable as the structure deforms further. Within the last two decades there has been an increasing amount of research into methods of structural analysis which can make realistic predictions of nonlinear load:displacement response up to (and including) the point of failure. Some methods even allow analysis to continue beyond the point of ultimate load capacity (*i.e.* to trace the falling portion of the nonlinear curve in Fig. 7.1).

Reviews of research into nonlinear structural analysis may be found in reports by Meek & Tan (1983) and Chan & Kitipornchai (1987); a detailed literature survey will not be undertaken here. Most available techniques are formulated for incremental loading with some form of equilibrium iteration at each load step. There is considerable variation in the number of nonlinear effects taken into account, although the two most important ones are included in all methods:

- finite joint deflections and rotations;
- the influence of member axial forces on flexural stiffness.

By contrast, in conventional linear elastic analysis equilibrium and compatibility requirements are formulated for the initial (undeformed) configuration of the structure, and flexural and axial stiffnesses are assumed to be independent. Many nonlinear structural analysis techniques have ignored the effect of flexure on member axial strains (*i.e.* bowing action), but some recent methods have taken this into account. Member shear deformations introduce further complexity and have generally been neglected in nonlinear analysis methods, although Chugh (1977) and Bhatt (1986) have both developed beam-column elements which include the effect of shear deformations on flexural stiffness. Material nonlinearity, accounted for by plastic hinge formation, has been incorporated in methods of large deformation structural analysis described by Kassimali (1983) and Al-Bermani & Kitipornchai (1990a).

7.2.2 Details of Beam-Column Element Formulation

The elastic beam-column theory outlined by Kassimali (1983) - with some minor enhancements - is used for the jack-up structural analyses presented below. In earlier work (Oran, 1973; Oran & Kassimali, 1976) the method is shown to be highly accurate, even in the presence of substantial deflections. Kassimali's (1983) method caters for two dimensional frames composed of prismatic members, with loads applied at the joints. Loads are assumed to move with their respective joints as the structure deforms. An Eulerian formulation allows accurate treatment of arbitrarily large joint translations (and hence arbitrarily large rigid body rotations of the members). On the other hand, conventional beam-column theory is used to define the member stiffness relationships in local coordinates, so the *relative* rotations of the ends of a member with respect to its chord are assumed to be small. The influence of axial force on flexural stiffness and the effect of flexural bowing on member chord length are both included. A summary of the Kassimali (1983) method appears below, and an extension to include the effect of shear deformations is presented in Section 7.2.3.

Fig. 7.2[a] shows a beam-column element with relative member deformations θ_1 , θ_2 and u and corresponding end forces M_1 , M_2 and Q referred to a local coordinate system. According to conventional beam-column theory (Timoshenko and Gere, 1961; Saafan, 1963) these forces and deformations are related by

$$M_1 = \frac{EI}{L}(c_1\theta_1 + c_2\theta_2); \quad M_2 = \frac{EI}{L}(c_2\theta_1 + c_1\theta_2); \quad Q = EA\left(\frac{u}{L} - c_b\right) \quad (7.1a;b;c)$$

where L is the undeformed member length, c_1 and c_2 are elastic stability functions and c_b is a dimensionless correction factor for bowing action. c_b is given by

$$c_b = b_1(\theta_1 + \theta_2)^2 + b_2(\theta_1 - \theta_2)^2 \quad (7.2)$$

where b_1 and b_2 are elastic bowing functions. Now c_1 , c_2 , b_1 and b_2 are all functions of the axial force Q , and it is convenient to define a dimensionless axial force parameter

$$q = \frac{Q}{Q_{\text{Euler}}} = \frac{QL^2}{\pi^2 EI} \quad (7.3)$$

The expressions for the stability functions c_1 and c_2 are well known. For compression members ($q > 0$, $\phi^2 = \pi^2 q$):

$$c_1 = \frac{\phi \sin \phi - \phi^2 \cos \phi}{2 - 2 \cos \phi - \phi \sin \phi}; \quad c_2 = \frac{\phi^2 - \phi \sin \phi}{2 - 2 \cos \phi - \phi \sin \phi} \quad (7.4a;b)$$

while for tension members ($q < 0$, $\psi^2 = -\pi^2 q$):

$$c_1 = \frac{\psi^2 \cosh \psi - \psi \sinh \psi}{2 - 2 \cosh \psi + \psi \sinh \psi}; \quad c_2 = \frac{\psi \sinh \psi - \psi^2}{2 - 2 \cosh \psi + \psi \sinh \psi} \quad (7.5a;b)$$

and for no axial force ($q = 0$)

$$c_1 = 4; \quad c_2 = 2 \quad (7.6a;b)$$

Note that Kassimali's (1983) version of Eqns 7.5 is incorrect (he has $-\psi \sinh \psi$ in the denominator for both c_1 and c_2). The bowing functions b_1 and b_2 are given by

$$b_1 = \frac{(c_1 + c_2)(c_2 - 2)}{8\pi^2 q}; \quad b_2 = \frac{c_2}{8(c_1 + c_2)} \quad (7.7a;b)$$

The incremental form of Eqns 7.1 (derived in full by Oran, 1973) is

$$\begin{Bmatrix} \delta M_1 \\ \delta M_2 \\ L \delta Q \end{Bmatrix} = [t] \begin{Bmatrix} \delta \theta_1 \\ \delta \theta_2 \\ \delta u/L \end{Bmatrix}, \quad [t] = \frac{EI}{L} \begin{bmatrix} c_1 + \frac{G_1^2}{\pi^2 H} & c_2 + \frac{G_1 G_2}{\pi^2 H} & \frac{G_1}{H} \\ c_2 + \frac{G_1 G_2}{\pi^2 H} & c_1 + \frac{G_2^2}{\pi^2 H} & \frac{G_2}{H} \\ \frac{G_1}{H} & \frac{G_2}{H} & \frac{\pi^2}{H} \end{bmatrix} \quad (7.8)$$

where

$$G_1 = c_1' \theta_1 + c_2' \theta_2, \quad G_2 = c_2' \theta_1 + c_1' \theta_2 \quad \text{and} \quad H = \frac{\pi^2 I}{AL^2} + b_1'(\theta_1 + \theta_2)^2 + b_2'(\theta_1 - \theta_2)^2$$

Primes denote differentiation with respect to the axial force parameter q (Eqn 7.3). The four required derivatives c_1' , c_2' , b_1' and b_2' may be conveniently expressed as functions of c_1 , c_2 , b_1 and b_2 , eliminating the need for direct differentiation of Eqns 7.4, 7.5 or 7.7:

$$c_1' = -2\pi^2(b_1 + b_2); \quad c_2' = -2\pi^2(b_1 - b_2) \quad (7.9a;b)$$

$$b'_1 = -\frac{(b_1 - b_2)(c_1 + c_2) + 2c_2b_1}{4q}; \quad b'_2 = \frac{\pi^2(16b_1b_2 - b_1 + b_2)}{4(c_1 + c_2)} \quad (7.10a;b)$$

In Kassimali (1983) there is a misprint in the expression for b'_1 (he has $-2c_2b_1$ in the numerator).

Eqn 7.8 defines the member tangent stiffness relationship in local coordinates. For use in a general plane frame analysis program, the incremental relationship between member end forces and end displacements must be re-expressed in terms of a global coordinate system:

$$\{\delta F\} = [T]\{\delta v\} \quad (7.11)$$

where $[T]$ is the 6×6 tangent stiffness matrix in global coordinates and $\{\delta F\}$ and $\{\delta v\}$ are 6×1 vectors of the incremental forces and displacements defined in Fig. 7.2[b]. $[T]$ is given by

$$[T] = [A][t][A]^T + M_1[g^{(1)}] + M_2[g^{(2)}] + QL[g^{(3)}] \quad (7.12)$$

in which $[t]$ is the 3×3 tangent stiffness matrix in local coordinates (Eqn 7.8), $[A]$ is the 6×3 statics matrix satisfying

$$\begin{Bmatrix} F_1 \\ \vdots \\ F_6 \end{Bmatrix} = [A] \begin{Bmatrix} M_1 \\ M_2 \\ QL \end{Bmatrix} \quad (7.13)$$

for the deformed member configuration at the start of a load increment, and the $[g^{(i)}]$ are second order geometric matrices which account for changes to the elements of matrix $[A]$ during an increment. Derivation of the matrices $[A]$ and $[g^{(i)}]$ is straightforward. Full details are given by both Oran (1973) and Kassimali (1983), but the results will not be quoted here. $[A]$ is a function of the angle α (Fig. 7.2[b]) which refers to the deformed member configuration; the matrices $[g^{(i)}]$ are functions of α and of the deformed chord length parameter $(1 + \Delta)$, again defined in Fig. 7.2[b].

7.2.3 Extension to Include Shear Deformations

The usual Bernoulli theory of bending assumes that plane sections remain plane, but this assumption is only valid in regions of constant bending moment (where there is no transverse shear force). It is possible, however, to account for shear deformations within the framework of simple bending theory. As shown in Fig. 7.3 (after Harrison, 1980), the effect of member shear deformation is to augment both of the anticlockwise end rotations (in local coordinates) by an angle

$$\theta_{\text{add}} = \gamma = \frac{V_{12}}{A_s G} = \frac{M_1 + M_2}{A_s GL} \quad (7.14)$$

where G is the member shear modulus, γ is the shear strain, V_{12} is the transverse shear force and A_s is the cross-sectional member area effective in resisting shear. It is realised that some

approximation is involved in making the convenient assumption $V_{12} = (M_1 + M_2)/L$, rather than $V_{12} = (M_1 + M_2)/L(1 + \Delta)$, but the errors involved are likely to be negligible. The derivations which follow could be performed using the more exact expression if desired.

Let overbars denote the inclusion of shear deformations in the beam-column element formulation, such that

$$\bar{\theta}_1 = \theta_1 + \theta_{\text{add}}; \quad \bar{\theta}_2 = \theta_2 + \theta_{\text{add}} \quad (7.15\text{a;b})$$

Because of the changes to θ_1 and θ_2 , new stability functions and bowing functions $\bar{c}_1$, $\bar{c}_2$, $\bar{b}_1$ and $\bar{b}_2$ are required for modified versions of Eqns 7.1 and 7.2:

$$M_1 = \frac{EI}{L}(\bar{c}_1\bar{\theta}_1 + \bar{c}_2\bar{\theta}_2); \quad M_2 = \frac{EI}{L}(\bar{c}_2\bar{\theta}_1 + \bar{c}_1\bar{\theta}_2); \quad Q = EA\left(\frac{u}{L} - \bar{c}_b\right) \quad (7.16\text{a;b;c})$$

$$\bar{c}_b = \bar{b}_1(\bar{\theta}_1 + \bar{\theta}_2)^2 + \bar{b}_2(\bar{\theta}_1 - \bar{\theta}_2)^2 \quad (7.17)$$

Introducing the parameter

$$\beta_s = \frac{12EI}{A_s GL^2}, \quad (7.18)$$

θ_{add} may be expressed as a function of θ_1 and θ_2 by substituting Eqns 7.1a and 7.1b into Eqn 7.14:

$$\theta_{\text{add}} = \frac{EI[(c_1 + c_2)\theta_1 + (c_1 + c_2)\theta_2]}{A_s GL^2} = \frac{\beta_s}{12}(c_1 + c_2)(\theta_1 + \theta_2) \quad (7.19)$$

Now from Eqns 7.15, 7.16a and 7.19:

$$\begin{aligned} M_1 &= \frac{EI}{L} \left[\bar{c}_1 \left(\theta_1 + \frac{\beta_s}{12}(c_1 + c_2)(\theta_1 + \theta_2) \right) + \bar{c}_2 \left(\theta_2 + \frac{\beta_s}{12}(c_1 + c_2)(\theta_1 + \theta_2) \right) \right] \\ &= \frac{EI}{L} \left[\theta_1 \left(\bar{c}_1 \left\{ 1 + \frac{\beta_s}{12}(c_1 + c_2) \right\} + \bar{c}_2 \left\{ \frac{\beta_s}{12}(c_1 + c_2) \right\} \right) + \right. \\ &\quad \left. \theta_2 \left(\bar{c}_1 \left\{ \frac{\beta_s}{12}(c_1 + c_2) \right\} + \bar{c}_2 \left\{ 1 + \frac{\beta_s}{12}(c_1 + c_2) \right\} \right) \right] \end{aligned} \quad (7.20)$$

so comparing Eqns 7.20 and 7.1a we have

$$c_1 = \bar{c}_1 \left\{ 1 + \frac{\beta_s}{12}(c_1 + c_2) \right\} + \bar{c}_2 \left\{ \frac{\beta_s}{12}(c_1 + c_2) \right\} \quad (7.21)$$

and

$$c_2 = \bar{c}_1 \left\{ \frac{\beta_s}{12}(c_1 + c_2) \right\} + \bar{c}_2 \left\{ 1 + \frac{\beta_s}{12}(c_1 + c_2) \right\} \quad (7.22)$$

Solving Eqns 7.21 and 7.22 for $\bar{c}_1$ and $\bar{c}_2$ yields

$$\bar{c}_1 = \frac{12c_1 + \beta_s(c_1^2 - c_2^2)}{12 + 2\beta_s(c_1 + c_2)}; \quad \bar{c}_2 = \frac{12c_2 + \beta_s(c_2^2 - c_1^2)}{12 + 2\beta_s(c_1 + c_2)} \quad (7.23\text{a;b})$$

The modified elastic stability functions in Eqns 7.16a and 7.16b may thus be evaluated very easily using the unbarred c_1 and c_2 values (from Eqns 7.4 to 7.6) and the parameter β_s (Eqn 7.18, constant for a given member). It is a straightforward matter to show that the stability functions $\bar{c}_1$ and $\bar{c}_2$ (Eqns 7.23) are in exact agreement with the expressions for beam-column flexural stiffness, including shear deformations, obtained by Chugh (1977) and Bhatt (1986). Both of these authors use different methods, but their derivations are longer and more complicated than the one presented above. Note that the assumption of ignoring shear deformation effects corresponds to taking $A_s = \infty$ (*i.e.* $\beta_s = 0$), in which case Eqns 7.23 reduce to $\bar{c}_1 = c_1$ and $\bar{c}_2 = c_2$, as they should.

Although the beam-column stiffness matrices of Chugh (1977) and Bhatt (1986) include the effects of shear deformations, both authors ignore the influence of flexural bowing on the member axial strains. This is equivalent to assuming $\bar{c}_b = 0$ in Eqn 7.16c, or $\bar{h}_1 = \bar{h}_2 = 0$ in Eqn 7.17. The following derivation of bowing functions $\bar{h}_1$ and $\bar{h}_2$ (which account for shear deformations) is thought to be a new contribution to beam-column theory. From Eqns 7.15,

$$\bar{\theta}_1 + \bar{\theta}_2 = \theta_1 + \theta_2 + 2\theta_{\text{add}} \quad (7.24)$$

and

$$\bar{\theta}_1 - \bar{\theta}_2 = \theta_1 - \theta_2 \quad (7.25)$$

Eqn 7.17 therefore becomes

$$\begin{aligned} \bar{c}_b &= \bar{h}_1(\theta_1 + \theta_2 + 2\theta_{\text{add}})^2 + \bar{h}_2(\theta_1 - \theta_2)^2 \\ &= \bar{h}_1(\theta_1 + \theta_2)^2 \left[1 + \frac{4\theta_{\text{add}}(\theta_1 + \theta_2) + 4\theta_{\text{add}}^2}{(\theta_1 + \theta_2)^2} \right] + \bar{h}_2(\theta_1 - \theta_2)^2 \end{aligned} \quad (7.26)$$

Using Eqn 7.19 to substitute for θ_{add} ,

$$\begin{aligned} \bar{c}_b &= \frac{\bar{h}_1(\theta_1 + \theta_2)^2}{36} \left[36 + \frac{12\beta_s(c_1 + c_2)(\theta_1 + \theta_2)^2 + \{\beta_s(c_1 + c_2)(\theta_1 + \theta_2)\}^2}{(\theta_1 + \theta_2)^2} \right] + \bar{h}_2(\theta_1 - \theta_2)^2 \\ &= \frac{\bar{h}_1(\theta_1 + \theta_2)^2}{36} [6 + \beta_s(c_1 + c_2)]^2 + \bar{h}_2(\theta_1 - \theta_2)^2 \end{aligned} \quad (7.27)$$

Now shear deformations affect only the member end rotations, not the overall length (see Fig. 7.3), so $\bar{c}_b$ (Eqn 7.27) must be equal to c_b (Eqn 7.2). Comparing the coefficients of $(\theta_1 + \theta_2)^2$ and $(\theta_1 - \theta_2)^2$ in these two expressions, it must be the case that

$$b_1 = \frac{\bar{h}_1}{36} [6 + \beta_s(c_1 + c_2)]^2; \quad b_2 = \bar{h}_2 \quad (7.28a;b)$$

i.e. that

$$\bar{h}_1 = \frac{36b_1}{[6 + \beta_s(c_1 + c_2)]^2}; \quad \bar{h}_2 = b_2 \quad (7.29a;b)$$

The bowing functions $\bar{h}_1$ and $\bar{h}_2$ (including shear deformation effects) are thus simple expressions involving the unbarred bowing functions b_1 and b_2 (Eqns 7.7), the unbarred stability functions c_1 and c_2 (Eqns 7.4 to 7.6) and the parameter β_s (Eqn 7.18). Note that Eqn 7.29a correctly reduces to $\bar{h}_1 = h_1$ when shear deformations are neglected ($\beta_s = 0$).

In addition to the modified stability and bowing functions $\bar{c}_1$, $\bar{c}_2$, $\bar{h}_1$ and $\bar{h}_2$, derivatives with respect to the axial force parameter q (Eqn 7.3) are also required for the incremental structural analysis method described in Section 7.2.2. Letting a prime denote differentiation with respect to q as before, from Eqns 7.23 and 7.29 we obtain (after simplification):

$$\bar{c}'_1 = \frac{c'_1 - c'_2}{2} + \frac{18(c'_1 + c'_2)}{[6 + \beta_s(c_1 + c_2)]^2}; \quad \bar{c}'_2 = \frac{c'_2 - c'_1}{2} + \frac{18(c'_1 + c'_2)}{[6 + \beta_s(c_1 + c_2)]^2} \quad (7.30a;b)$$

$$\bar{h}'_1 = \frac{36}{[6 + \beta_s(c_1 + c_2)]^2} \cdot \left[b'_1 - \frac{2b_1\beta_s(c'_1 + c'_2)}{[6 + \beta_s(c_1 + c_2)]} \right]; \quad \bar{h}'_2 = b'_2 \quad (7.31a;b)$$

where the unbarred derivatives c'_1 , c'_2 , b'_1 and b'_2 are as defined in Eqns 7.9 and 7.10. This completes the extension of Kassimali's (1983) beam-column element formulation to include shear deformations.

7.3 Program 2DJUAN

The static plane frame analysis program *2DJUAN* is described in this section, with particular emphasis on the incorporation of Model B footings (Chapter 6) into the structural analysis. Several test problems involving large deflections and rotations are used to validate the implementation of Kassimali's (1983) beam-column theory.

7.3.1 Input and Output Data Files

Most of the input for *2DJUAN* is quite standard for a plane frame analysis program. First of all the input data file must specify the initial location of each joint in $x:y$ space, and whether displacement is constrained or unconstrained with respect to the x , y and rotational degrees of freedom. Frame connectivity data follows, along with the necessary structural properties of each member (I , A , E , A_s and G). Each member is assumed to be rigidly connected to joints at either end; there is no allowance for internal hinges or rotational springs in the present version of *2DJUAN*. Incremental loading of the structure takes place in stages, and each stage may be divided into steps if desired. Loads may be either point forces or concentrated moments, but must be applied at the joints. If any Model B foundations are to be attached to the frame, all three degrees of freedom have to be specified as unconstrained at the relevant joint(s). For each Model B footing the input data file must contain information about clay properties and footing geometry, using the same format as the program *OXSPUD* input file (see Section 6.4.1). Finally the *2DJUAN* input file must specify

whether large displacement or small displacement (linear elastic) analysis is required, and whether shear deformations are to be included in the analysis or not.

By default *2DJUAN* generates a block of output data at the end of each loading stage, although the user may request additional output at the end of each step. An output block consists of a list of joint translations and rotations, together with the stress resultants (end moments and axial force) for each member. Finally the net external forces acting on each joint are listed, allowing the reactions at constrained degrees of freedom to be obtained.

7.3.2 Key *2DJUAN* Subroutines

SUBROUTINE BOWFAC

For a given dimensionless axial force q (Eqn 7.3) this subroutine computes the elastic stability functions c_1 and c_2 , the bowing functions b_1 and b_2 and their derivatives with respect to q . If shear deformations are to be included, the corresponding barred values are returned. There is a limit on the compressive axial force which an elastic member can sustain. The theoretical buckling load for a column with both ends completely restrained against rotation is

$$Q = \frac{4\pi^2 EI}{L^2} = 4Q_{\text{Euler}} \quad (7.32)$$

Although it possible to evaluate most of the stability functions and bowing functions for $q \geq 4$, the results are meaningless. A call to BOWFAC with $q \geq 4$ therefore results in an error warning, but this situation only arises when catastrophic buckling failure of the structure is imminent. Kassimali (1983) points out that for very small absolute values of axial force, say $|q| < 0.01$, there are potential numerical problems in evaluating the closed form expressions for the stability and bowing functions and their derivatives (Eqns 7.4, 7.5, 7.7, 7.9 and 7.10, all of which become singular at $q = 0$). When $|q| < 0.01$ program *2DJUAN* bypasses the closed form expressions and instead uses the polynomial series expansions (valid for small q) given by Kassimali (1983).

SUBROUTINE SPUD

This routine is a slightly modified version of program *OXSPUD* (Section 6.4). It performs two functions which allow a Model B footing to be incorporated into the structural analysis. The first is computation of the 3×3 tangent stiffness matrix which defines the incremental response of the footing in its current $(V, H, M/R)$ load state. The second is computation of the load changes $(\delta V, \delta H, \delta M/R)$ resulting from a specified displacement increment $(\delta z, \delta h, R\delta\theta)$. Calls to SPUD contain a flag which indicates which of the two modes should be executed.

Depending on the location of the $(V, H, M/R)$ force point for a particular footing (inside or on the yield surface), the first mode of subroutine SPUD returns either an elastic or an elastoplastic incremental stiffness matrix. In program *OXSPUD* the incremental

load:displacement response (whether elastic or elastoplastic) was formulated as a 7×7 matrix $[B]$ (cf. Eqn 6.18):

$$\begin{bmatrix} 1 \text{ or } 0 & 0 & 0 & 0 \text{ or } 1 & 0 & 0 & 0 \\ 0 & 1 \text{ or } 0 & 0 & 0 & 0 \text{ or } 1 & 0 & 0 \\ 0 & 0 & 1 \text{ or } 0 & 0 & 0 & 0 \text{ or } 1 & 0 \\ B_{41} & B_{42} & B_{43} & -1 & 0 & 0 & B_{47} \\ B_{51} & B_{52} & B_{53} & 0 & -1 & 0 & B_{57} \\ B_{61} & B_{62} & B_{63} & 0 & 0 & -1 & B_{67} \\ B_{71} & B_{72} & B_{73} & 0 & 0 & 0 & B_{77} \end{bmatrix} \begin{Bmatrix} \delta V \\ \delta H \\ \delta(M/R) \\ \delta z \\ \delta h \\ R\delta\theta \\ \xi \end{Bmatrix} = \begin{Bmatrix} inc_1 \\ inc_2 \\ inc_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (7.33)$$

Since program *2DJUAN* uses the conventional displacement (*i.e.* stiffness) method of structural analysis, the incremental load:displacement relationship defined by Eqn 7.33 must be expressed in the form of a 3×3 footing tangent stiffness matrix, $[s]$, defined by:

$$\begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix}; \quad \begin{Bmatrix} \delta z \\ \delta h \\ R\delta\theta \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}^{-1} \begin{Bmatrix} \delta V \\ \delta H \\ \delta M/R \end{Bmatrix} \quad (7.34a;b)$$

The entries in $[s]^{-1}$ are easily obtained from Eqn 7.33 by eliminating the seventh row of matrix $[B]$:

$$[s]^{-1} = \frac{1}{B_{77}} \begin{bmatrix} B_{41} - B_{71} \cdot B_{47} & B_{42} - B_{72} \cdot B_{47} & B_{43} - B_{73} \cdot B_{47} \\ B_{51} - B_{71} \cdot B_{57} & B_{52} - B_{72} \cdot B_{57} & B_{53} - B_{73} \cdot B_{57} \\ B_{61} - B_{71} \cdot B_{67} & B_{62} - B_{72} \cdot B_{67} & B_{63} - B_{73} \cdot B_{67} \end{bmatrix} \quad (7.35)$$

Inversion of Eqn 7.35 then gives the required 3×3 incremental stiffness matrix $[s]$, though Row 3 and Column 3 of $[s]$ must be adjusted because the main *2DJUAN* structural analysis program uses M and θ rather than M/R and $R\theta$. The resulting foundation stiffness matrix $[S]$ returned by subroutine SPUD satisfies

$$\begin{Bmatrix} \delta V \\ \delta H \\ \delta M \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{Bmatrix} \delta z \\ \delta h \\ \delta\theta \end{Bmatrix} \quad (7.36)$$

which is directly compatible with the overall stiffness matrix of the structure, as discussed in Section 7.3.3 below.

In its second mode of operation, subroutine SPUD receives a displacement increment $(\delta z, \delta h, \delta\theta)$ as input and returns a calculated load increment $(\delta V, \delta H, \delta M)$. The change in vertical plastic displacement δz^p is also returned, though this is of course zero if the increment happens to be entirely elastic. The FORTRAN code and solution strategy used for this computing $(\delta V, \delta H, \delta M)$ from $(\delta z, \delta h, \delta\theta)$ are virtually identical to those of program *OXSPUD*. As discussed in Section 6.4.3, an increment may be wholly elastic, wholly elastoplastic, or yielding may occur at some point during the increment. If any elastoplastic deformation is involved, automatic iteration and (if required) step size division algorithms are

applied until the user-specified yield function tolerance is satisfied, just as in program *OXSPUD*.

7.3.3 Solution Control in Program *2DJUAN*

Large displacement analysis of a structure without any Model B footings is performed using the standard Newton-Raphson nonlinear solution strategy described by Kassimali (1983). Fig. 7.4 shows a qualitative representation of this technique for a single degree of freedom system. In the initial state (i) the applied loads $\{P\}^{(i)}$ are in equilibrium with the so-called internal loads $\{P_{\text{int}}(\{x\}^{(i)})\}$ corresponding to the deformed configuration of the structure, $\{x\}^{(i)}$. Upon application of some external load increment $\{\delta P\}$, the task is to find the deformed configuration $\{x\}^{(i+1)}$ which gives equilibrium with the new external loads $\{P\}^{(i+1)} = \{P\}^{(i)} + \{\delta P\}$.

The first step is to compute, for each member, the tangent stiffness matrix $[T]$ in global coordinates (Eqn 7.12). These calculations are based on the deformed configuration of the structure and the member stress resultants in the initial state (i). The overall tangent stiffness matrix for the structure $[\tau]$ is then assembled in the usual way, and by solving

$$[\tau]\{\delta x\} = \{\delta P\} \quad (7.37)$$

an approximate deformed configuration may be obtained from

$$\{x\}_1 = \{x\}^{(i)} + \{\delta x\} \quad (7.38)$$

In general this linearised incremental solution is unsatisfactory because the corresponding internal loads $\{P_{\text{int}}(\{x\}_1)\}$ are not in equilibrium with the new external loads $\{P\}^{(i+1)}$. For each member, the relative deformations θ_1 , θ_2 and u in the configuration $\{x\}_1$ follow directly from the updated frame geometry. It is then straightforward to compute M_1 , M_2 and Q from Eqns 7.1, and the end forces in global coordinates follow from Eqn 7.13. After assembling the internal load vector $\{P_{\text{int}}(\{x\}_1)\}$ the out-of-balance loads may be obtained from (see Fig. 7.4)

$$\{\delta Q\}_1 = \{P\}^{(i+1)} - \{P_{\text{int}}(\{x\}_1)\} \quad (7.39)$$

The overall tangent stiffness matrix $[\tau]_1$ is then re-assembled, based on the approximate deformed configuration $\{x\}_1$ and the corresponding member stress resultants. The out-of-balance loads are applied as a new load increment:

$$[\tau]_1\{\delta x\}_1 = \{\delta Q\}_1 \quad (7.40)$$

which is solved to give the first correction displacement vector $\{\delta x\}_1$. If this correction vector is not sufficiently small a new approximate configuration $\{x\}_2 = \{x\}_1 + \{\delta x\}_1$ is determined, along with a new out-of-balance force vector $\{\delta Q\}_2 = \{P\}^{(i+1)} - \{P_{\text{int}}(\{x\}_2)\}$ and structure tangent stiffness matrix $[\tau]_2$. Iteration continues until the elements of the correction displacement vector $\{\delta x\}_j$ become sufficiently small compared with their cumulative values for the inequality

$$\sqrt{\frac{\sum (\delta x)^2}{\sum (x)^2}} \leq tol_2 \quad (7.41)$$

to be satisfied. In accordance with the recommendations of Kassimali (1983), joint translations and joint rotations are grouped separately, and Eqn 7.41 must be satisfied simultaneously and independently by each group before the structure is deemed to be in equilibrium. Most of the *2DJUAN* analyses discussed below were performed with $tol_2 = 10^{-9}$. A convergence criterion involving the out-of-balance forces $\{\delta Q\}$ could of course have been used instead of Eqn 7.41.

7.3.4 Solution Control with Model B Footings Present

It is straightforward to incorporate one or more Model B type footings into the procedure just described. Fig. 7.5 shows a three member frame ABCD with two Model B foundations represented by coupled nonlinear springs. When a new load increment is to be applied, subroutine SPUD is called to obtain the 3×3 incremental footing stiffness matrices $[S]^{(A)}$ and $[S]^{(D)}$ (see Eqn 7.36). If a footing is in an elastic state at the start of the increment ($f < 0$, force point inside the yield surface) then SPUD simply returns an elastic stiffness matrix $[S]$. If a footing begins the increment with $f = 0$ (force point on the yield surface), SPUD returns an elastoplastic incremental stiffness matrix $[S]$ based on the initial footing reactions $V^{(i)}$, $H^{(i)}$, $M^{(i)}$ and the initial plastic vertical penetration $z^{p(i)}$ (which is used to determine $V_0^{(i)}$). In assembling the structure tangent stiffness matrix $[\tau]$, the 6×6 contributions $[T]$ from each member are combined with the 3×3 footing tangent stiffness matrices $[S]^{(A)}$ and $[S]^{(D)}$ in the usual additive manner, as shown in Fig. 7.5. The linearised incremental displacements $\{\delta x\}$ and the approximate deformed configuration of the structure $\{x\}_1$ are then determined from Eqns 7.37 and 7.38.

In order to determine the out-of-balance loads $\{\delta Q\}_1$ from Eqn 7.39, the contribution of the Model B footings to the internal load vector $\{P_{int}(\{x\}_1)\}$ must obviously be taken into account. This is where the second mode of subroutine SPUD is needed, to compute the changes to the initial footing reactions caused by the linearised incremental displacements $\{\delta x\}$ determined from Eqn 7.37. For each Model B footing, the displacements $(\delta z, \delta h, \delta \theta)$ of its associated joint are passed to SPUD, which returns the corresponding footing reaction changes $(\delta V, \delta H, \delta M)$ and the change in vertical plastic penetration δz^p (if any). The so-called internal loads contributed by each footing to its associated joint are:

$$(V^{(i)} + \delta V, \quad H^{(i)} + \delta H, \quad M^{(i)} + \delta M) \quad (7.42)$$

and these loads are used to assemble the internal load vector $\{P_{int}(\{x\}_1)\}$ corresponding to the deformed configuration of the structure $\{x\}_1$. Eqn 7.39 may then be used to find the out-of-balance loads $\{\delta Q\}_1$, and the next task is re-assembly of the structure tangent stiffness matrix $[\tau]_1$ to allow the first equilibrium iteration (Eqn 7.40). For the structural members new matrices $[T]$ are formed on the basis of the approximate $\{x\}_1$ configuration and the associated internal stress resultants, as outlined in Section 7.3.3. For the Model B footings

new incremental stiffness matrices are obtained from subroutine **SPUD**, based on the approximate reactions of Eqn 7.42 and the vertical plastic penetration $z^{p(i)} + \delta z^p$. After assembly of $[\tau]_i$ Eqn 7.40 is solved for $\{\delta x\}_i$, and if these correction displacements are not sufficiently small then iteration continues in the manner described above. Note that, because of the path-dependent nature of the footing load:displacement response, the iteration procedure must be conducted very carefully. For a Model B footing which is elastic ($f < 0$) at the start of a load increment, yielding may be detected in one of the iteration cycles when **SPUD** is called (in its second mode) to find the reaction changes $(\delta V, \delta H, \delta M)$ resulting from displacements $(\delta z, \delta h, \delta \theta)$. If this happens then the footing is forced to remain in an elastoplastic state ($f = 0$) for all subsequent equilibrium iterations of that particular load increment, even if negative values of ξ arise in the solution of Eqn 6.18 (a negative ξ usually indicates attempted elastic unloading from the yield surface). Similarly if a Model B footing is elastoplastic at the start of a load increment and a non-negative ξ is obtained in the first equilibrium iteration, that footing is forced to remain elastoplastic for the remainder of the iteration process. Only if a negative value of ξ is obtained in the *first* iteration cycle of a load increment does program **2DJUAN** allow the yielded footing to become elastic again. In this way the initial yielding, elastic unloading and subsequent re-yielding of Model B foundations may be accommodated within the incremental/iterative structural analysis procedure, if required.

7.3.5 Test Problems

The problem of a cantilever subjected to an end moment (Fig. 7.6[a]) is a popular means of validating new methods of large displacement elastic structural analysis. Closed form expressions for the tip deflections are simple because curvature is constant along the entire length of the cantilever (*i.e.* the deflected shape is a circular arc). These expressions, valid for all $\eta \neq 0$, are

$$\frac{u}{L} = 1 - \frac{1}{2\pi\eta} \sin(2\pi\eta); \quad \frac{v}{L} = \frac{1}{2\pi\eta} [1 - \cos(2\pi\eta)]; \quad \frac{\phi}{2\pi} = \eta \quad (7.43a;b;c)$$

where

$$\eta = \frac{ML}{2\pi EI} \quad (7.44)$$

is the dimensionless applied moment. Note that when $\eta = 1$ the deflected shape is a full circle; thereafter the cantilever begins to coil up on itself. Fig. 7.6[b] shows the performance of Kassimali's (1983) large displacement beam-column method (as implemented in program **2DJUAN**) in analysing a cantilever having $L = 100$ in, $I = 0.01042$ in⁴, $A = 0.5$ in² and $E = 1.2 \times 10^4$ psi, the properties used by several previous researchers. Using just one element to model the cantilever, **2DJUAN** gives very good agreement with theory up to $\eta = 0.3$ but the numerical solution breaks down just after $\eta = 0.75$. If the cantilever is modelled with two beam-column elements there are no numerical problems for the range of loading shown, and agreement with the analytical results is excellent for $0 \leq \eta \leq 1$. With four beam-column

elements the calculated tip displacements show almost exact agreement with Eqns 7.43 up to $\eta = 1.5$. For comparison, Fig. 7.6[c] shows some previously published numerical analyses of the same problem. Ramm's (1977) analysis with five shell (curve) elements shows reasonable agreement with the exact solution up to $\eta = 1$, while the results of Al-Bermani & Kitipornchai (1990b), using two elements to model the cantilever, are somewhat inferior. It is clear that program *2DJUAN*, using just two Kassimali (1983) beam-column elements, performs better than either of the earlier analyses.

Fig. 7.7 shows the performance of *2DJUAN* in predicting the buckling behaviour of an encastré strut. For an ideal (perfectly straight) strut the critical load is

$$P_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{P_{Euler}}{4} \quad (7.45)$$

An exact solution for the post-buckling response ($P > P_{cr}$) has been obtained by Frisch-Fay (1962) using elliptic integrals. The tip deflections u and v , as defined in Fig. 7.7, are given by

$$\frac{u}{L} = \frac{2p}{K(p^2)}; \quad \frac{v}{L} = 2 - \frac{2E(p^2)}{K(p^2)} \quad (7.46a;b)$$

where p is a dimensionless parameter satisfying

$$\frac{P}{P_{cr}} = \frac{4[K(p^2)]^2}{\pi^2} \quad (7.47)$$

and $K(p^2)$ and $E(p^2)$ are, respectively, complete elliptic integrals of the first and second kind:

$$K(p^2) = \int_0^{\pi/2} (1 - p^2 \sin^2 \theta)^{-1/2} d\theta; \quad E(p^2) = \int_0^{\pi/2} (1 - p^2 \sin^2 \theta)^{1/2} d\theta \quad (7.48)$$

The analytical curves shown in Fig. 7.7 were obtained using *Mathematica*. For nominated values of P/P_{cr} , Eqn 7.47 was solved for p and the tip deflections evaluated from Eqns 7.46. In the numerical analyses with program *2DJUAN*, the compressive load was applied with a small eccentricity ($e = 0.001L$ in Fig. 7.7) to initiate instability of the strut. As shown in Fig. 7.7, the buckling load P_{cr} is correctly predicted with either one or two beam-column elements used to model the strut. Agreement with the theoretical post-buckling deflections is, however, much better with two elements. To improve the accuracy of the numerical predictions at very large displacements, further subdivision of the strut is required. The sharpness of the curvature in the numerical *2DJUAN* curves near $P/P_{cr} = 1$ may be improved by reducing the eccentricity e .

One final test problem involving very large displacements is the four member square frame subjected to equal and opposite compressive loads as shown in Fig. 7.8[a] (note that, from symmetry, only one quarter of the frame needs to be considered). This structure has been analysed by Mattiasson (1981) using elliptic integrals, and the data markers in Fig. 7.8[b] represent his exact results. Using only two beam-column elements per member (*i.e.* eight for the whole frame), program *2DJUAN* gives more accurate results than the four

element per member analysis of Al-Bermani & Kitipornchai (1990b). With four elements per member, *2DJUAN* gives results which are indistinguishable from the exact solution. Fig. 7.8[c] illustrates the deflected shape of the frame at $PL^2/EI = 2$ and $PL^2/EI = 4$; it is clear that the elastic beam-column theory of Kassimali (1983) is indeed highly accurate in the presence of large joint deflections and large rigid body element rotations.

Program *2DJUAN* can be used for standard small displacement structural analysis, with or without shear deformation effects. Programming of the small displacement analysis was based on the matrix method outlined by Harrison (1980), and example problems from this reference (some including shear deformations) were used to verify the implementation. In order to check the correct operation of subroutine *SPUD*, several *2DJUAN* analyses were performed on a trivial structure consisting of a single joint with a Model B footing attached. One such analysis involved the successful reproduction of the vertical elastoplastic load:penetration curve shown in Fig. 6.6. Other *2DJUAN* analyses of the trivial structure involved the application of horizontal load and moment at constant vertical load; as would be expected, the footing displacements were in exact agreement with those predicted by program *OXSPUD* for the same sequence of load controlled stages.

7.4 Structural Analysis of a Typical Jack-Up Unit

7.4.1 Representative Jack-Up Rig and Soil Conditions

Fig. 7.9 shows a plane frame idealisation of a three-legged jack-up unit, with representative dimensions, structural properties and preloading details appropriate for a large present day rig. These values were chosen after considering the properties of "typical" jack-up units analysed by Schotman (1989), Hambly & Nicholson (1991) and SNAME (1993). The leg spacing of 51.96 m indicates that, in plan, the three legs of the rig form an equilateral triangle of side 60 m. A more realistic structural model than that shown in Fig. 7.9 would obviously need to account for such things as the finite stiffness of the leg/hull interface (assumed rigid here) and leg self-weight. For reasons of computational economy, the modelling of latticework legs as equivalent beams is standard practice in the structural analysis of jack-up units. J. J. Osborne of Noble Denton and Associates (personal communication, 1993) confirmed that the values proposed in Fig. 7.9 are reasonable for a large North Sea jack-up, although a detailed check was not performed. The undrained strength profile chosen for the clay seabed ($s_{um} = 10$ kPa, $\rho = 2$ kPa/m) is typical for a soft marine clay deposit (Gemenhardt and Focht, 1970); a nominal rigidity index of $G/s_u = 100$ was adopted for the analyses involving Model B footings. Spudcan diameters of 20 m are expected to become typical for the next generation of harsh environment jack-up units, and several existing rigs have footings of this size (Fig. 1.3). Although not shown in Fig. 7.9, the 20 m diameter spudcans have the same underside shape as the footings used in the experimental programme (see Fig. 3.14).

Figs 7.9[a] and [b] show the two possible plane frame reductions of a three legged jack-up unit for environmental (wind and wave) loading directed along an axis of symmetry

(see also Fig. 2.11[a]). In reality the wind load acts mainly on the hull, and wave loading on the legs is of course distributed over a considerable depth; there may also be current loading on the legs to consider. The modelling of environmental loading with two point loads as shown in Fig. 7.9 is evidently a gross simplification, and the possibility of wave phase effects (an uneven distribution of load between windward and leeward legs) is completely ignored. Nevertheless, the idealised two dimensional structures of Fig. 7.9 are adequate for a preliminary investigation of the effects of using different numerical models of foundation behaviour.

7.4.2 Prediction of Initial Penetration

As shown in Fig. 7.9, it is assumed that vertical loads due to hull weight and preload are evenly distributed between the three legs, and that these loads act directly above each leg. At maximum preload a pure vertical load of 100 MN is therefore applied to each spudcan, with a subsequent reduction to 50 MN per spudcan in the initial operating condition. The resulting footing penetrations into the seabed may be predicted using either program *OXSPUD* (Section 6.4) or the vertical load:penetration program *VZPLOT* (Section 6.4.4). Alternatively the vertical preloading and unloading stages may be incorporated directly into the *2DJUAN* structural analysis, provided Model B footings are used to define the foundation behaviour. These three methods all give the same results, which are shown in Fig. 7.10. Under full preload the predicted penetration is 14.945 m (approximately 0.75 diameters), and during the elastic unloading to $V = V_0/2$ there is a small amount (0.128 m) of upward footing movement. The analyses which follow assume that monotonic, quasi-static environmental loading commences with the rig in this initial operating condition.

7.4.3 Analyses with Pinned and Encastré Footings

As discussed in Section 2.4, structural analyses of jack-up units frequently ignore spudcan rotational fixity and model the footings as pin joints. Under small to moderate environmental loads this assumption is certainly conservative (as far as stresses in the upper part of the structure are concerned), but under extreme storm conditions the pin joint assumption is thought to be quite realistic (Arnesen *et al.*, 1988). For small loads an equally simple assumption of infinite moment:rotation stiffness (encastré footings) is likely to be more appropriate. This section discusses the analyses of the representative jack-up rig performed using these two extremes of foundation behaviour.

All of the analyses presented below made full use of the capabilities of program *2DJUAN* (*i.e.* large displacement and shear deformation effects were included). For the case of pinned footings, however, the consequences of using small displacement analysis and of ignoring shear deformations were examined. Fig. 7.11 shows how two key quantities - hull sway and the bending moment at the leeward lower guide (leg/hull connection) - are influenced by the type of structural analysis adopted. Taking the linear elastic small displacement analysis as the reference case, shear deformations alone result in a significant 12% increase in hull sway, but a negligible 0.2% increase in leeward lower guide moment

(percentage differences taken at $H_{env} = 10$ MN). Large displacement analysis (ignoring shear deformations) gives a hull sway 16.5% greater than the reference prediction and a bending moment 15% greater, emphasising the importance of including $P \cdot \Delta$ effects in jack-up analyses. The full analysis including both large displacements and shear deformations gives 34% and 18% increases in hull sway and leeward lower guide moment respectively, compared with the reference linear elastic analysis. There seems little doubt that a conventional small displacement analysis, ignoring shear deformations, is quite inadequate for predicting the behaviour of jack-up units subjected to extreme environmental loads.

Fig. 7.12[a] shows the spudcan reactions predicted by *2DJUAN* under the assumptions of pinned footing behaviour and a single leg to windward. All footings start with $V = 50$ MN and $H = 0$ MN. Upon the application of environmental load (H_{env} values, in MN, are shown in brackets) the vertical load on the two leeward footings increases, and the vertical reaction on the windward footing decreases. Reardon (1986) states that the load paths obtained from a pinned jack-up analysis are frequently compared with the $V:H$ interaction locus of Brinch Hansen (1970) to estimate the point at which bearing capacity failure or spudcan sliding might be expected. The dashed line in Fig. 7.12[a] is the Brinch Hansen $V:H$ failure envelope (Eqn 2.16) established by the spudcan penetration under full preload. Based on load path intersections with this locus the windward footing is critical, with sliding failure predicted at $H_{env} = 8.50$ MN. Note that the windward footing is very close to uplift (*i.e.* $V = 0$) at this point. In practice it is usually accepted that the development of full sliding capacity under small vertical loads is overly optimistic, and a linear cutoff of the type shown in Fig. 7.12[a] is often imposed. The dash-dot line is drawn for $H/V = 0.45$, and if H/V ratios are not allowed to exceed this quantity then sliding failure of the single windward spudcan is predicted for $H_{env} = 6.19$ MN. The choice of $H/V = 0.45$ as the limiting load ratio is related to consideration of a drained sliding failure, which according to API (1993) occurs when

$$H = V \tan \phi' \quad (7.49)$$

for a soil with zero drained cohesion (by analogy with the Mohr-Coulomb failure criterion). A typical effective friction angle for a clay ($\phi' = 24^\circ$) gives $H/V = 0.45$. In jack-up operation a single extreme wave loading event would not be expected to last more than a few seconds, allowing little time for drainage to occur. The drained H/V cutoff nevertheless provides a useful lower bound for design against sliding behaviour at low vertical loads. By contrast the Brinch Hansen prediction of horizontal load capacity at low V/V_0 is probably quite optimistic, so in Fig. 7.12[a] the windward footing would be expected to fail at some load $6.19 < H_{env} < 8.50$ MN.

For the case of a pinned jack-up loaded with its single leg to leeward, a similar analysis of the $V:H$ load paths (Fig. 7.12[b]) reveals that the single spudcan is again critical. A bearing capacity failure is predicted for $H_{env} = 6.18$ MN, well before the windward footings would be expected to undergo sliding (at $H_{env} = 9.53$ MN). In actual jack-up design the nominal Brinch Hansen $V:H$ failure envelopes of Fig. 7.12 would of course be reduced in

size by appropriate safety factors. The API (1993) design guidelines suggest that respective reduction multipliers of 0.80 and 0.67 be applied to the calculated horizontal load and vertical load capacities.

If the representative jack-up unit of Fig. 7.9 is analysed with encastré footings, the infinite rotational fixity means that very large moment reactions develop at relatively small values of H_{env} . For the single leg to windward case, Fig. 7.13[a] shows the windward and leeward spudcan load paths projected into the $V:M/R$ plane. Comparing this plot with Fig. 7.12[a], there are clearly much smaller changes in the vertical footing reactions for a given environmental load H_{env} (small displacement theory gives $\Delta V_{enc.} = 0.5\Delta V_{pinned}$, for the case of an infinitely stiff hull). Because only small horizontal reaction forces are developed in the encastré analysis (compared with those in Fig. 7.12[a]) there is not a serious error in assuming that spudcan capacity can be estimated with the Brinch Hansen failure envelope for $H = 0$ (Eqn 2.11). Under this assumption the single windward footing is marginally critical, yielding at $H_{env} = 3.67$ MN. Fig. 7.13[b] shows that in the encastré analysis with a single leg to leeward, the first yielding spudcan (at $H_{env} = 3.57$ MN) is again the one attached to the single leg. The use of the word "yield" (rather than "failure") in the previous two sentences is deliberate. As has already been stated, the assumption of encastré footings is likely to be a fairly realistic means of estimating footing reactions for small environmental loads, but for extreme events the loss of rotational spudcan moment fixity means that pinned footings provide a more realistic model. Taking the single leg to leeward case as an example, a very stiff structural response would be expected up to an applied load of $H_{env} = 3.57$ MN (Fig. 7.13[b]), followed by a progressive decrease in stiffness up to an ultimate load of $H_{env} = 6.18$ MN (Fig. 7.12[b]) with the gradual loss of rotational foundation fixity. Although it is obviously rather crude, this use of Brinch Hansen's bearing capacity theory with "interpolation" between two extremes of footing behaviour does allow a very approximate prediction of jack-up response to an extreme environmental loading event. Section 7.4.6 compares some results inferred from this approximate method with the results of realistic structural analyses performed using Model B type footings.

7.4.4 Analyses with Model B Footings: Reactions

As for the pinned and encastré cases, the representative jack-up unit was analysed with its single leg to windward and to leeward using Model B (Chapter 6) to describe the load:displacement response of each spudcan. Fig. 7.14[a] shows the $V:H:M$ footing reactions obtained from the single leg to leeward analysis. Environmental load H_{env} is plotted along the horizontal axis. By coincidence, both sets of footings (windward and leeward) yield at the same applied load, $H_{env} = 4.32$ MN; prior to this the six reactions vary almost linearly with H_{env} . While the footings are elastic the single windward spudcan takes slightly more horizontal load than each of the leeward spudcans, but the overturning moments are nearly equal at all three footings. Immediately after yield the moment reactions start to drop, *i.e.* rotational fixity begins to reduce. The loss of moment resistance accelerates with further environmental loading, particularly for the single windward footing.

Yielding causes a noticeable increase in the rate of change of both vertical load reactions with H_{env} ; there is also a tendency for the windward footing to take a slightly greater proportion of the applied horizontal load H_{env} . The windward spudcan eventually suffers a catastrophic sliding failure at $H_{env} = 7.74$ MN (footing displacements are discussed in the next section) and the incremental structural analysis breaks down. Just prior to failure the windward horizontal reaction undergoes a small but sudden drop, and horizontal load redistribution to the leeward footings (which are yielding in a stable manner) takes place. It is also interesting to note that, at failure, the two leeward spudcans are still providing a significant moment fixity of 123.4 MNm each; this is equal to 19.9% of the net overturning moment per leg (*i.e.* $7.74 \text{ MN} \times 80 \text{ m}$). By contrast, the 39.2 MNm of moment fixity provided by the windward footing when $H_{env} = 7.74$ MN corresponds to only 6.3% of the net overturning moment per leg.

Fig. 7.14[b] is a similar plot of the reactions obtained from the single leg to leeward analysis using Model B footings. The leeward spudcan yields at $H_{env} = 4.09$ MN, slightly before the two windward footings at $H_{env} = 4.37$ MN. During the period of elastic footing behaviour the horizontal and moment reactions are quite similar for all three footings, just as in Fig. 7.14[a]. After yield there is a progressive loss of moment fixity, and once again it is the single footing which displays the greater drop in M . As the applied load approaches $H_{env} = 8$ MN there is a dramatic drop in the horizontal load capacity of the leeward footing, and redistribution of H to the windward spudcans is accelerated. Failure occurs at $H_{env} = 8.62$ MN, with the vertical load on the single leeward footing closely approaching the preload value of 100 MN. At failure the net overturning moment per leg is $8.62 \text{ MN} \times 80 \text{ m} = 689.4 \text{ MNm}$; the moment fixity of 77.2 MNm provided by each of the windward footings at failure is 11.2% of this value. The single leeward spudcan, with a moment reaction of 24.9 MNm, resists only 3.6% of the net overturning moment on its leg at failure. Comparing Figs 7.14[a] and [b], program *2DJUAN* predicts that (in terms of ultimate capacity) the representative jack-up unit is rather more vulnerable to environmental loading with a single leg to windward.

As was noted while analysing the experimental results in Chapter 5, a useful way of visualising three dimensional load paths in $V:H:M/R$ space is an $H:M/R$ projection with "spot height" values of V . Figs 7.15 [a] and [b] show plots of this type for windward and leeward footings respectively, from the single leg to windward analysis. For comparison the load paths obtained with pinned and encastré footings are included as well. In Fig. 7.15[a] it is clear that the initial $H:M/R$ load path direction Ω of the windward Model B footing is very similar to that obtained in the encastré analysis, although the Model B analysis predicts a more rapid fall in V with increasing H . Yield of the Model B footing occurs at $V = 37.5$ MN (Point Y); the $(V, H, M/R)$ force point then remains on the yield surface until failure of the footing at Point F ($V = 11.2$ MN). The two dash-dot curves represent the constant V cross-sections of the $V:H:M/R$ yield surface which pass through points Y and F. Although there is a small amount of softening because of vertical plastic heave on Path Y→F (V_0 drops from its preload value of 100 MN to 99.2 MN), Fig. 7.15[a] shows that the post-yield loss of

moment fixity is basically a consequence of the reduction in vertical load V . Note that at failure, the windward footing is providing the maximum possible horizontal reaction force for its vertical load of 11.2 MN. At Point F there is also a little remaining moment fixity, as discussed above; the pinned footing ($M = 0$) assumption is an unduly pessimistic model of footing behaviour, even at ultimate load.

Fig. 7.15[b], showing the leeward footing reactions associated with Fig. 7.15[a], indicates that the initial elastic response of each leeward Model B spudcan is again quite close to the encasté prediction. After yield the elastoplastic path $Y' \rightarrow F'$ shows only a slight loss of moment fixity because the vertical load only increases from 56.2 to 69.4 MN; the reduction in size of the "available" $H:M/R$ yield locus is therefore not nearly as pronounced as that observed for the single windward footing in Fig. 7.15[a]. Note that Point F' merely shows the leeward reactions at the ultimate applied load $H_{env} = 7.74$ MN, and is not intended to imply failure of the leeward footings. As shown in Fig. 7.14[a], sliding of the windward footing is responsible for failure of the structure at $H_{env} = 7.74$ MN.

Fig. 7.16 shows the footing load paths obtained in the various single leg to leeward analyses. After yielding of the leeward Model B footing (Point Y, Fig. 7.16[a]) there is an almost total loss of moment fixity as the vertical load increases to 96.7 MN at failure (Point F). Vertical plastic settlement on Path $Y \rightarrow F$ causes V_0 to increase from the preload value of 100 MN to 103.0 MN, but this yield surface expansion is not enough to offset the loss of horizontal load and moment capacity caused by the increasing vertical reaction V . Up to $V = 85$ MN the leeward horizontal reaction increases, but in order to accommodate further vertical load a reduction in H becomes necessary (this is particularly noticeable for $V > 90$ MN). Fig. 7.16[b] shows the considerable post-yield drop in the moment fixity provided by the two windward footings in the Model B analysis. Despite a significant decrease in vertical load from 43.6 to 26.7 MN, each windward footing is still able to accommodate an increasing horizontal reaction force along the whole of Path $Y' \rightarrow F'$. Once more Point F' does not indicate failure of the pair of footings; it simply shows the windward spudcan reactions prevailing when the leeward footing fails at $H_{env} = 8.62$ MN (see Fig. 7.14[b]).

7.4.5 Analyses with Model B Footings: Displacements

One of the major advantages of using the Model B type footings in structural analysis of a jack-up unit is that realistic predictions of all spudcan displacements (Δz , h and θ) are obtained. With pinned footings the vertical and horizontal displacements are constrained to be zero, and of course with encasté footings no displacements are allowed at all. Fig. 7.17[a] shows the Model B footing displacements calculated in the single leg to windward analysis. Considering first the vertical displacements Δz , heave of the single windward footing (dashed) shows a noticeable acceleration after yield and a very rapid increase just prior to failure (at which point $\Delta z = -0.26$ m). Settlement of the leeward footings (solid Δz line) proceeds in a more stable manner throughout the analysis, reaching only 0.09 m at the ultimate applied load H_{env} . Horizontal displacements h of the windward and leeward

spudcans are almost identical prior to yield, but by failure the single windward spudcan has undergone 0.56 m more sliding than the leeward footings ($h = 0.75$ m as opposed to 0.19 m). The result is a significant 1.1% reduction of the original 51.96 m spacing between the windward and leeward footings. Windward and leeward spudcan rotations remain very similar throughout the analysis. At yield both rotations are about 0.33° , and both show a marked acceleration with the post-yield reduction in moment fixity. At failure the greater horizontal displacement of the windward spudcan means that its rotation is slightly smaller than that of the leeward footings ($\theta = 1.71^\circ$ versus $\theta = 2.00^\circ$). It is of course assumed that the jack-up structure remains elastic throughout these analyses; it is doubtful whether a real rig could tolerate such large footing rotations.

Fig. 7.17[b] presents the calculated Model B spudcan displacements from the single leg to leeward analysis. As failure is approached and the vertical load on the single leeward footing increases more rapidly, the rate of vertical penetration increases as well. Leeward spudcan settlement reaches a substantial 0.70 m at failure, while the heave predicted for the pair of windward spudcans is only 0.13 m at this point. Horizontal footing movements remain reasonably small up to about $H_{env} = 8$ MN, but large sliding displacements then take place. The single leeward footing suffers a very large 2.16 m horizontal displacement (more than 0.1 diameters) before it fails, while the windward footings both move 0.71 m. This causes the windward/leeward footing spacing to increase from 51.96 m to 53.41 m at failure (+2.8%), in contrast to the 1.1% spacing reduction which occurred with a single leg to windward. It is interesting to note that, for $5 < H_{env} < 8$ MN, rotation of the leeward spudcan significantly exceeds that of the windward footings, even though their horizontal displacements are almost the same. The difference in rotations arises because the large compressive axial force in the leeward leg reduces its flexural stiffness. Once the applied load exceeds $H_{env} = 8$ MN, however, the dramatic sliding displacement of the leeward footing means that its rotation at failure (2.10°) is somewhat smaller than the ultimate windward rotation of 3.06° . Again these rotations are unrealistic for a real jack-up unit, and in practice the possibilities of local buckling and/or plastic yielding within the structure at a lower applied load H_{env} would need to be considered, particularly at the leg/hull interfaces (where critical bending stresses would be expected)

The deflected shape of the rig at failure is drawn in Fig. 7.18 for the single leg to windward case. All deflections of the structure and of the Model B footings are drawn to scale, though for simplicity the spudcans are shown as 150° cones. Note the clearly visible reduction in the windward/leeward footing spacing and the large sway displacement of the hull. The 0.50° hull rotation is mainly a result of the differential vertical footing displacements. Fig. 7.19 shows the deflected shape at failure in the single leg to leeward analysis with Model B footings. In this case the hull sway at ultimate load is 5.26 m and the hull rotation is 0.97° . These very large values are, once again, based on the unrealistic assumption that the rig structure remains elastic. The large sliding displacement of the single leeward footing leads to a very conspicuous widening of the gap between windward and leeward spudcans. Comparing Figs 7.18 and 7.19, it is clear that although the ultimate load

capacity of the jack-up is slightly higher with a single leg to leeward, development of this capacity entails much larger displacements of both the jack-up structure and its foundations.

7.4.6 Comparisons of Model B Results with Approximate Methods

Perhaps the most important values obtained from quasi-static structural analysis of a jack-up unit are the hull sway and the maximum bending moments in each leg. Fig. 7.20[a] shows, for the jack-up of Fig. 7.9 with a single leg to windward, predictions of hull sway obtained from program *2DJUAN* using encastré footings, pinned footings and Model B footings. With encastré footings the approximate method shown in Fig. 7.13[a] predicts first yield of the windward footing at an applied load of $H_{env} = 3.67$ MN, at which point the hull sway is 0.31 m (Fig. 7.20[a]). Using Model B footings, simultaneous yield of all spudcans is predicted at $H_{env} = 4.32$ MN with a corresponding hull sway of 0.74 m. A simple structural analysis with encastré footings therefore seems to be a reasonable means of estimating the applied load which causes spudcan yielding, but the assumption of infinite rotational fixity gives unconservative predictions of hull sway. Fig. 7.20[a] shows that, by contrast, the pinned footing assumption of zero rotational fixity significantly overpredicts the hull sway for low values of H_{env} . The ultimate load obtained with Model B footings ($H_{env} = 7.74$ MN) does, however, fall in the range $6.19 < H_{env} < 8.50$ MN inferred from the pinned footing analysis and Brinch Hansen's (1970) $V:H$ failure envelope (see Fig. 7.12[a]). In practice the pessimistic value of $H_{env} = 6.19$ MN would probably be used for design, so the 1.90 m hull sway at failure predicted by the pinned footing analysis at this H_{env} would be conservative with respect to the more realistic sway estimate of 1.57 m obtained from the Model B analysis (for the same applied load). The available load capacity with Model B footings ($H_{env} = 7.74$ MN) exceeds the pessimistic pinned footing prediction by 25%, but the 3.20 m hull sway at this ultimate load is rather more than a pinned jack-up analysis would suggest.

In Fig. 7.20[b] hull sway predictions from the various single leg to leeward analyses are plotted. While the Model B footings remain elastic at low H_{env} , the hull sway is about twice that obtained from the encastré analysis. As shown in Fig. 7.13[b], the use of Brinch Hansen's $V:M/R$ failure locus in conjunction with the encastré footing load paths correctly predicts that the single leeward footing is the first to yield. The corresponding applied load of $H_{env} = 3.57$ MN agrees reasonably well with the Model B analysis, in which the leeward and windward footings yield at loads of $H_{env} = 4.09$ and 4.37 MN respectively. In Fig. 7.12[b] bearing capacity failure of the leeward footing is predicted at $H_{env} = 6.18$ MN from the pinned footing $V:H$ load paths and Brinch Hansen's $V:H$ failure envelope. If this were adopted as the ultimate design capacity then the hull sway from the pinned analysis (1.90 m) would be greater than the realistic Model B prediction of a 1.57 m sway at that load. The results of the Model B analysis indicate, however, that there is a considerable reserve of capacity beyond $H_{env} = 6.18$ MN. In fact the structure, assuming it remains elastic, can resist an extra 39% of environmental loading before leeward footing failure is predicted at $H_{env} = 8.62$ MN. As this ultimate load is approached, it is interesting that the hull sway greatly exceeds that obtained from the pinned footing analysis (at a comparable H_{env}). This

difference arises from the horizontal and vertical spudcan displacements which are accounted for in the Model B analysis but constrained to be zero in the pinned footing analysis.

Fig. 7.21 illustrates the different predictions of lower guide (leg/hull interface) bending moments obtained from the various structural analyses. For the single leg to windward case (Fig. 7.21[a]), the pinned footing assumption of zero moment fixity clearly gives conservative results compared with the Model B analysis. The difference is most marked while the Model B footings are elastic ($H_{env} < 4.32$ MN), during which time the lower guide moments predicted with encastré footings provide a better (but unconservative) approximation. After yield the moment fixity provided by the Model B footings reduces (see Fig. 7.14[a]) so agreement with the pinned analysis gradually improves. As noted above, an approximate design based on the pinned footing load paths of Fig. 7.12[a] would give a conservative ultimate load of $H_{env} = 6.19$ MN. Fig. 7.21[a] shows that the estimated lower guide moments at failure would also be significantly conservative, overpredicting the Model B moments for the same load level by 41% (windward lower guide) and 37% (leeward lower guides). The most interesting feature of the Model B analysis in Fig. 7.21[a] is the redistribution of bending moment from the single windward leg to the two leeward legs which occurs just prior to failure. This is associated with the large sliding displacement of the windward spudcan which causes the footings to move closer together, as shown in Fig. 7.18. The effect of this change in geometry is to augment the bending moment at both leeward lower guides, while relieving bending stresses at the windward lower guide.

Fig. 7.21[b], illustrating the single leg to leeward results, is broadly similar to Fig. 7.21[a]. The approximate analysis using pinned footings and Brinch Hansen's (1970) bearing capacity method would predict jack-up failure at $H_{env} = 6.18$ MN, with lower guide bending moments of 603.3 MNm (leeward leg) and 582.0 MNm (windward legs). At the same applied load level in the Model B analysis, the corresponding lower guide moments are much lower (426.2 and 418.8 MNm) because of spudcan moment fixity. It will be recalled from Fig. 7.17[b] that loading beyond $H_{env} = 8$ MN in the Model B analysis caused a very large sliding displacement and a large vertical penetration of the single leeward footing. Fig. 7.21[b] shows that the combined effect of these displacements is a dramatic reduction in bending moment at the leeward lower guide, with redistribution to the windward legs. In fact when the leeward spudcan fails at $H_{env} = 8.62$ MN, the bending moment at the leeward lower guide (472.9 MNm) is less than half that at the top of each windward leg (977.5 MNm). This serious departure from the pinned footing prediction emphasises once more that a realistic numerical model of spudcan behaviour allows greatly enhanced analysis of a plane frame jack-up structure on clay.

CONCLUDING REMARKS

This thesis has been concerned with the development of a numerical model able to describe the load:displacement behaviour of jack-up unit "spudcan" footings subjected to combined vertical, horizontal and moment loading on cohesive soil. Experimental and theoretical studies have indicated that classical work hardening plasticity theory provides a highly suitable framework for such numerical modelling. In this final chapter the principal conclusions of the work are summarised, and some possible directions for future research are suggested.

8.1 Conclusions

8.1.1 Main Findings

- A theoretical study of the vertical bearing capacity of conical footings on clay was conducted, using the Method of Characteristics to obtain lower bound solutions. Computations were performed for all combinations of a range of cone angles, footing roughness factors, depths of embedment and linear rates of undrained strength increase with depth.
- As part of the experimental programme, scale model spudcan footings were driven into soft clay samples having a pronounced increase of undrained strength with depth. The observed variation of vertical bearing capacity with embedment was well matched by theoretical predictions.
- A purpose built testing apparatus was used to apply horizontal and rotational displacements (both individually and in combination) to a model spudcan while the vertical penetration was held constant at a series of embedments in soft clay. The resulting load paths were used to define the shape of the combined load yield surface in $V:H:M/R$ space and investigate its evolution with vertical footing penetration.
- It was found that while the three dimensional $V:H:M/R$ yield surface underwent significant expansion with spudcan penetration, its overall shape remained very nearly constant. Definition of a single normalised $V/V_0:H/V_0:M/RV_0$ yield surface was therefore possible, with V_0 being the pure vertical load capacity at any depth of embedment.
- The shape of the empirically determined yield surface is essentially parabolic in the $H/V_0 = 0$ and $M/RV_0 = 0$ planes, and elliptical in the cross-sectional planes of constant V/V_0 . A particularly interesting feature of these cross-sectional ellipses is that their major and minor axes do not coincide with the M/RV_0 and H/V_0 coordinate axes.
- A further series of physical model tests involved the application of horizontal, rotational and combined $h:\theta$ displacements at various spudcan embedments, but with the vertical

load (rather than the vertical penetration) held constant. Tests were performed using a number of different vertical load levels V/V_0 , and the results confirmed the yield surface shape deduced from the wholly displacement controlled tests.

- The tests at constant vertical load V provided additional information in the form of incremental displacement vectors associated with spudcan yielding. Horizontal and rotational plastic displacements at yield were found to obey the normality law. On the other hand an associated flow rule predicted excessive amounts of footing heave and settlement at yield, necessitating an empirical adjustment to the vertical plastic displacements obtained from an assumption of normality to the $V:H:M/R$ yield surface.
- Ancillary footing tests performed using a variety of displacement rates confirmed that the standard rates adopted for the main experimental programme were sufficiently high to allow realistic modelling of undrained behaviour.
- A complete incremental plasticity model for describing spudcan behaviour under combined loading (Model B) has been implemented in the FORTRAN program *OXSPUD*. Model B uses the experimentally determined yield surface and flow rule, together with the theoretical lower bound bearing capacity factors to define V_0 as a function of embedment. Elastic behaviour inside the yield surface is modelled with a set of stiffness factors for conical footings derived from three dimensional finite element analyses (Bell, 1991).
- Program *OXSPUD* was used to make retrospective simulations of several footing experiments involving both load- and displacement- controlled stages. The primary (monotonic) phases of horizontal displacement and rotation events were modelled convincingly, as was the variation of vertical bearing capacity with embedment. By contrast, only the initial stages of cyclic horizontal displacement or rotation reversals were realistically predicted by *OXSPUD*.
- The specially written plane frame structural analysis program *2DJUAN* is particularly suitable for jack-up unit analysis because it accounts for both large displacement and shear deformation effects (although quasi-static behaviour is assumed). Most importantly, the program allows spudcan load:displacement response to be specified with the incremental plasticity formulation of Model B.
- A two dimensional idealisation of a typical jack-up unit was analysed with program *2DJUAN*, using realistic dimensions and structural properties but somewhat simplified representations of environmental loading and self-weight. Analyses performed with pinned footings and encastré footings were shown to be less informative than those employing the plasticity-based model of foundation behaviour.
- In the structural analyses with Model B footings, large horizontal and vertical spudcan displacements were predicted as the ultimate load capacity of the jack-up unit was approached. These displacements (which have in the past been constrained by modelling spudcan footings as pin joints) were shown to have a significant influence on both predicted hull displacements and critical bending moments at the leg/hull connections.

8.1.2 Additional Comments

The use of work hardening plasticity theory for the numerical modelling of foundation behaviour under combined loads is not in itself an original concept. Model B is, however, the first such method to have been calibrated by (and validated against) an extensive programme of physical model tests involving footings on clay. Previous experimental work of this nature has been confined to footings on sand, while the numerical model for spudcans on clay proposed by Schotman (1989) was not derived from, or compared with, any experimental results. The substantial body of experimental data presented in this thesis and in Martin & Houlsby (1994) is relevant to spudcan-shaped footings, but it is likely that corresponding tests with conical, flat circular, square or strip footings on clay would give broadly comparable results. The overall plasticity-based concepts of Model B could certainly be used to develop analogous numerical models for other footing types subjected to combined vertical, horizontal and moment loading on cohesive soil.

Previous empirically calibrated plasticity models of footing behaviour have not only been confined to sands, but their use has invariably been restricted to the retrospective simulation of tests on a single scale model footing. Program *2DJUAN* represents a significant advance in that it demonstrates the feasibility of incorporating a rational, experimentally verified plasticity model of foundation load:displacement response into a sophisticated method of plane frame structural analysis. It should be mentioned that Schotman (1989) reported the results of one plane frame jack-up analysis which used a three degree of freedom plasticity model of spudcan response, but the case described was for a unit on sand. Furthermore a small displacement structural analysis method (which also neglected shear deformations) was used, and only the case of a single leg to windward was considered. Although Schotman's (1989) paper must be acknowledged as pioneering the use of a plasticity-type footing model in jack-up unit structural analysis, the results reported in Chapter 7 of this thesis were obtained using a much more sophisticated structural analysis method and, more importantly, using an empirically calibrated plasticity model of spudcan load:displacement response.

The extension of Kassimali's (1983) elastic beam-column theory to include the effects of finite shear deformations (Section 7.2.3) represents a useful contribution to the area of large displacement plane frame structural analysis. Similarly the set of theoretical lower bound bearing capacity factors for conical footings on clay (Tables 2.3-2.8) is likely to prove quite useful in practice. For a jack-up on clay these factors are ideally suited to the prediction of spudcan penetration caused by preloading, and Tables 2.3-2.8 appear in the recommended practice document recently published by SNAME (1993). Finally, the capabilities of the experimental testing apparatus are presently being used to advantage in a new research project investigating the behaviour of circular skirted plate foundations subjected to combined vertical, horizontal and moment loading on sand. The requirements of this project have included the need for tensile vertical loads, higher magnitudes of load, much finer resolution of measured displacements, faster displacement rates and faster data logging rates. These have all been achieved through minor modifications to the apparatus described

in Chapter 3, principally a new load cell (of similar design), a new displacement measurement system and new control software.

8.2 Areas for Future Research

8.2.1 Numerical Model with Kinematic Hardening

Load:displacement results obtained in several footing experiments were compared with the numerical predictions of Model B in Section 6.5. In general it was found that the numerical model performed well, reproducing the important features of the observed behaviour in a satisfactory manner (both qualitatively and quantitatively). A number of shortcomings of the numerical predictions were identified in Section 6.5.6, the most serious of which were:

- in reality the process of spudcan yielding entails a gradual reduction in stiffness, not a sudden one as predicted by Model B;
- the observed load:displacement response upon unloading and reloading is hysteretic, whereas Model B predicts recoverable elastic behaviour.

A consequence of the second point is that the chord stiffness of an experimental unload-reload loop is strongly dependent on the amplitude of the loop; with its assumption of linear elasticity inside the yield surface, Model B is incapable of simulating this behaviour.

Very similar problems arise in the formulation of constitutive laws for describing the stress:strain behaviour of soils. To date, the most successful extensions of linear elastic / work hardening plastic models have been those based on the "multiple yield surface" concept. In its simplest form, this method uses a small inner yield surface (bounding a region of truly elastic behaviour) which is carried around by the current stress state inside an outer limiting yield surface. Translation of the inner yield surface is associated with some small irrecoverable strains together with kinematic hardening (or softening) of both inner and outer yield surfaces, while full elastoplastic yielding occurs when the stress point reaches the outer yield locus. Al-Tabbaa (1987) formulated a two surface extension of the Modified Cam Clay constitutive model and presented some encouraging retrospective simulations of gradual yielding and hysteretic unload-reload behaviour in both oedometer and triaxial tests on Speswhite kaolin. Tan (1990) successfully adapted Al-Tabbaa's model to obtain credible qualitative predictions of spudcan load:displacement response under cycles of combined vertical and horizontal loading, but no attempt was made to calibrate the two surface numerical model with data from physical footing tests.

For spudcan footings on clay, the extension of Model B to a two surface, kinematic hardening model is suggested as one area of research requiring immediate attention. In its present form, Model B is incapable of adequately modelling a single stress reversal, let alone an extended series of combined loading cycles. Realistic structural analyses involving the cyclic environmental loading of jack-up units on clay cannot be undertaken with confidence until appropriate improvements have been made to the existing numerical model of spudcan behaviour. Such improvements would, of course, require a more complex mathematical formulation, and an extensive series of experimental footing tests would be needed to

determine the additional model parameters. In the long term, a highly refined model of spudcan footing behaviour under combined loading might make use of several nested yield surfaces (Muir Wood, 1990, discusses such an extension of the Modified Cam Clay constitutive model).

8.2.2 Extension to Six Degrees of Freedom

As noted in Chapter 1, the problem of combined vertical, horizontal and moment loads addressed in this thesis is in fact a special case of the six load system depicted in Fig. 1.5[a]. It is therefore of interest to consider how Model B might be extended to allow for horizontal and moment loads in two orthogonal directions, and to include the torsional degree of freedom. A realistic plasticity model of footing behaviour could then be incorporated into a full three dimensional structural analysis of a jack-up unit, not just an idealised plane frame analysis of the type presented in Chapter 7.

Defining elastic behaviour within the six dimensional $V:H_1:M_1:H_2:M_2:T$ yield surface presents no problem; according to Poulos and Davis (1974) the torsional displacement of a rough circular footing on an elastic half space is given by

$$\theta_r = \frac{3}{16GR^3} \cdot T \quad (8.1)$$

Determining the likely form of the yield surface itself is a little more difficult. The three dimensional yield surface equation of Model B takes the form

$$\left(\frac{H}{H_0}\right)^2 + \left(\frac{M/R}{M_0/R}\right)^2 - a\left(\frac{H}{H_0}\right)\left(\frac{M/R}{M_0/R}\right) - b\left(\frac{V}{V_0}\right)^{2\beta_1} \left(1 - \frac{V}{V_0}\right)^{2\beta_2} = 0 \quad (8.2) \text{ (cf. 6.2a)}$$

The following expression for a six dimensional yield surface with loads V , H_1 , H_2 , M_1 , M_2 and T is suggested:

$$\begin{aligned} &\left(\frac{H_1}{H_0}\right)^2 + \left(\frac{M_1/R}{M_0/R}\right)^2 - a\left(\frac{H_1}{H_0}\right)\left(\frac{M_1/R}{M_0/R}\right) + \\ &\left(\frac{H_2}{H_0}\right)^2 + \left(\frac{M_2/R}{M_0/R}\right)^2 - a\left(\frac{H_2}{H_0}\right)\left(\frac{M_2/R}{M_0/R}\right) + \left(\frac{T/R}{T_0/R}\right)^2 - b\left(\frac{V}{V_0}\right)^{2\beta_1} \left(1 - \frac{V}{V_0}\right)^{2\beta_2} = 0 \end{aligned} \quad (8.3)$$

This relatively simple extension of Eqn 8.2 is possible because, from considerations of symmetry, there can be no terms in H_1H_2 , M_1M_2 , H_1M_2 , M_1H_2 , nor any cross-products involving the torque T . The only model parameter which cannot be deduced directly from the Model B parameters obtained already is T_0 , and since in theory

$$T_0 = \frac{2}{3} \pi R^3 s_u \quad (8.4)$$

it seems probable that

$$T_0/R = \frac{2}{3} \pi R^2 s_u \approx 0.1V_0 \quad (8.5)$$

Experiments would be needed to check whether the shape of the $Q:V$ interaction diagram was compatible with the same values of β_1 and β_2 which define the approximately parabolic $H:V$ and $M/R:V$ interaction loci. Further experiments involving combined vertical, torsional, horizontal and moment loads would then be required to ascertain whether an extended six dimensional flow rule

$$\langle \delta z^p, \delta h_1^p, R\delta\theta_1^p, \delta h_2^p, R\delta\theta_2^p, R\delta\theta_T^p \rangle = \xi \left\langle \zeta_a \cdot \frac{\partial f}{\partial V}, \frac{\partial f}{\partial H_1}, \frac{\partial f}{\partial (M_1/R)}, \frac{\partial f}{\partial H_2}, \frac{\partial f}{\partial (M_2/R)}, \frac{\partial f}{\partial (T/R)} \right\rangle \quad (8.6) \text{ (cf. 6.7a)}$$

could be used with reasonable accuracy.

8.2.3 Physical Modelling of Complete Jack-up Units

In Chapter 7 a plane frame idealisation of a typical jack-up unit was analysed using various numerical models of foundation behaviour. The predictions obtained with Model B footings are thought to be quite realistic, but if such analyses are to be used for real jack-up assessments then some physical tests on a complete three-legged model platform would be highly desirable. In particular the numerically predicted spudcan $V:H:M$ load paths and $z:h:\theta$ displacement paths would need to be checked against experimental observations; the agreement between calculated and measured bending moments at various locations in the structure would also be of great interest. At Cambridge University some of these "whole rig" physical tests have recently been performed on sand in a geotechnical centrifuge (Murff *et al.*, 1991).

8.2.4 Dynamic Structural Analysis

The most serious deficiency of the structural analyses carried out with program *2DJUAN* was the assumption of quasi-static behaviour. In reality the highly flexible nature of a jack-up unit means that significant dynamic amplification of stresses and deflections can be expected under environmental loading. For the future generation of deep water jack-ups, accurate modelling of dynamic effects will be even more important because the natural period of such platforms will begin to exceed 5 or 6 seconds. Wave energy starts to become quite significant at these periods (L  seth *et al.*, 1990), and resonance effects would be expected to have a considerable influence on the structural response. Computer-based research currently in progress at Oxford (Thompson, 1993) is investigating the dynamic response of a plane frame jack-up structure to simulated wave loading. It is intended that these dynamic analyses will eventually incorporate the Model B description of spudcan behaviour under combined vertical, horizontal and moment loads.

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TABLE 2.1 Elastic stiffness factors for a rough flat circular footing embedded in incompressible soil, obtained by Bell (1991) using 3D finite element analysis

β	$D/2R$	K_1	K_2	K_3	K_4
180°	0.0	8.00	5.33	5.33	0.00
180°	0.25	8.58	7.00	6.30	-0.32
180°	0.5	9.21	7.63	7.03	-0.52
180°	1.0	9.98	8.03	7.50	-0.69
180°	2.0	11.18	8.46	8.14	-0.93

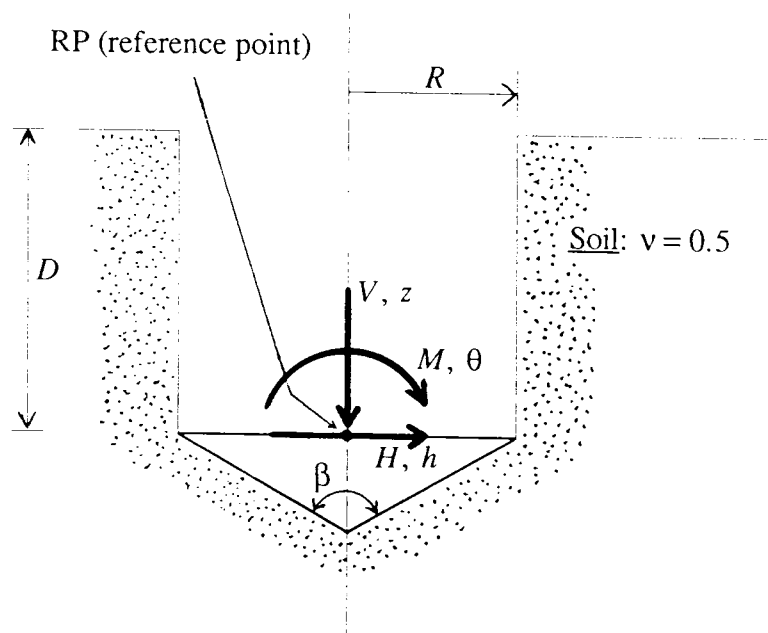


TABLE 2.2 Elastic stiffness factors for rough conical footings on incompressible soil, obtained by Bell (personal communication, 1991) using 3D finite element analysis

β	$D/2R$	K_1	K_2	K_3	K_4
180°	0.0	8.00	5.33	5.33	0.00
150°	0.0	8.00	5.74	5.35	-0.06
120°	0.0	8.05	6.45	5.71	-0.52
90°	0.0	8.22	7.68	7.47	-1.84

$\frac{\rho 2R}{s_{um}}$	$\frac{D}{2R}$	$\frac{s_{u0}}{s_{um}}$	Roughness factor α						
			0.0	0.2	0.4	0.6	0.8	1.0	
0.0	0.0	1.00	4.61	5.51	6.38	7.22	8.03	8.78	
0.0	0.1	1.00	4.80	5.70	6.56	7.40	8.20	8.95	
0.0	0.25	1.00	5.05	5.94	6.80	7.63	8.43	9.18	
0.0	0.5	1.00	5.41	6.29	7.14	7.97	8.76	9.50	
0.0	1.0	1.00	5.98	6.85	7.70	8.51	9.29	10.03	
0.0	2.5	1.00	7.12	7.98	8.81	9.61	10.38	11.10	
1.0	0.0	1.00	7.53	9.02	10.46	11.84	13.19	14.46	
1.0	0.1	1.10	7.45	8.89	10.27	11.61	12.90	14.13	
1.0	0.25	1.25	7.38	8.73	10.05	11.32	12.55	13.72	
1.0	0.5	1.50	7.28	8.55	9.78	10.98	12.14	13.24	
1.0	1.0	2.00	7.20	8.38	9.51	10.61	11.68	12.68	
1.0	2.5	3.50	7.34	8.39	9.39	10.37	11.30	12.19	
2.0	0.0	1.00	10.45	12.51	14.51	16.44	18.31	20.10	
2.0	0.1	1.20	9.65	11.53	13.33	15.08	16.79	18.40	
2.0	0.25	1.50	8.89	10.58	12.19	13.76	15.27	16.72	
2.0	0.5	2.00	8.20	9.67	11.09	12.47	13.80	15.08	
2.0	1.0	3.00	7.60	8.87	10.10	11.30	12.45	13.54	
2.0	2.5	6.00	7.37	8.44	9.48	10.48	11.44	12.35	
3.0	0.0	1.00	13.36	15.98	18.56	21.03	23.42	25.71	
3.0	0.1	1.30	11.51	13.76	15.92	18.02	20.05	22.00	
3.0	0.25	1.75	9.98	11.89	13.72	15.49	17.21	18.85	
3.0	0.5	2.50	8.74	10.33	11.87	13.36	14.80	16.18	
3.0	1.0	4.00	7.79	9.11	10.39	11.64	12.83	13.98	
3.0	2.5	8.50	7.40	8.46	9.51	10.52	11.49	12.42	
4.0	0.0	1.00	16.27	19.46	22.57	25.62	28.52	31.32	
4.0	0.1	1.40	13.10	15.68	18.14	20.54	22.86	25.08	
4.0	0.25	2.00	10.83	12.87	14.86	16.79	18.66	20.44	
4.0	0.5	3.00	9.11	10.77	12.38	13.96	15.47	16.91	
4.0	1.0	5.00	7.91	9.26	10.57	11.84	13.06	14.23	
4.0	2.5	11.00	7.40	8.47	9.52	10.54	11.52	12.46	
5.0	0.0	1.00	19.18	22.94	26.61	30.20	33.63	36.92	
5.0	0.1	1.50	14.48	17.33	20.06	22.72	25.29	27.75	
5.0	0.25	2.25	11.46	13.64	15.75	17.80	19.78	21.68	
5.0	0.5	3.50	9.37	11.08	12.77	14.38	15.94	17.43	
5.0	1.0	6.00	7.98	9.35	10.68	11.97	13.21	14.40	
5.0	2.5	13.50	7.40	8.47	9.53	10.56	11.55	12.48	

TABLE 2.3 Bearing capacity factors $N_{c0} = \frac{Q}{A s_{u0}}$ for cone angle $\beta = 30^\circ$

$\frac{\rho 2R}{s_{um}}$	$\frac{D}{2R}$	$\frac{s_{u0}}{s_{um}}$	Roughness factor α						
			0.0	0.2	0.4	0.6	0.8	1.0	
0.0	0.0	1.00	4.45	4.96	5.45	5.90	6.32	6.69	
0.0	0.1	1.00	4.68	5.19	5.67	6.12	6.53	6.90	
0.0	0.25	1.00	4.98	5.50	5.96	6.40	6.81	7.18	
0.0	0.5	1.00	5.41	5.90	6.37	6.81	7.21	7.57	
0.0	1.0	1.00	6.07	6.55	7.01	7.43	7.84	8.18	
0.0	2.5	1.00	7.33	7.81	8.25	8.66	9.05	9.39	
1.0	0.0	1.00	5.81	6.51	7.15	7.77	8.34	8.87	
1.0	0.1	1.10	5.92	6.59	7.23	7.83	8.38	8.89	
1.0	0.25	1.25	6.04	6.70	7.30	7.88	8.42	8.91	
1.0	0.5	1.50	6.20	6.84	7.41	7.96	8.47	8.94	
1.0	1.0	2.00	6.43	7.05	7.58	8.12	8.59	9.03	
1.0	2.5	3.50	6.97	7.55	8.08	8.54	8.98	9.39	
2.0	0.0	1.00	7.14	8.02	8.84	9.60	10.32	10.99	
2.0	0.1	1.20	6.92	7.73	8.49	9.21	9.88	10.50	
2.0	0.25	1.50	6.74	7.50	8.18	8.84	9.46	10.03	
2.0	0.5	2.00	6.59	7.29	7.91	8.53	9.09	9.61	
2.0	1.0	3.00	6.55	7.20	7.76	8.33	8.83	9.30	
2.0	2.5	6.00	6.99	7.49	8.03	8.50	8.95	9.37	
3.0	0.0	1.00	8.49	9.54	10.50	11.42	12.29	13.10	
3.0	0.1	1.30	7.77	8.70	9.56	10.38	11.14	11.85	
3.0	0.25	1.75	7.24	8.03	8.80	9.53	10.20	10.82	
3.0	0.5	2.50	6.82	7.56	8.21	8.86	9.45	10.00	
3.0	1.0	4.00	6.60	7.27	7.85	8.44	8.94	9.43	
3.0	2.5	8.50	6.99	7.47	8.01	8.49	8.94	9.36	
4.0	0.0	1.00	9.83	11.02	12.16	13.24	14.26	15.18	
4.0	0.1	1.40	8.51	9.52	10.48	11.38	12.22	13.00	
4.0	0.25	2.00	7.61	8.44	9.26	10.04	10.75	11.41	
4.0	0.5	3.00	6.97	7.74	8.41	9.08	9.69	10.26	
4.0	1.0	5.00	6.64	7.31	7.90	8.49	9.01	9.51	
4.0	2.5	11.00	6.86	7.45	8.00	8.48	8.94	9.35	
5.0	0.0	1.00	11.17	12.52	13.83	15.06	16.20	17.26	
5.0	0.1	1.50	9.14	10.23	11.26	12.25	13.15	13.99	
5.0	0.25	2.25	7.90	8.78	9.63	10.43	11.17	11.87	
5.0	0.5	3.50	7.08	7.84	8.55	9.24	9.86	10.45	
5.0	1.0	6.00	6.66	7.32	7.94	8.53	9.06	9.56	
5.0	2.5	13.50	6.85	7.44	7.99	8.47	8.93	9.35	

TABLE 2.4 Bearing capacity factors $N_{c0} = \frac{Q}{A s_{u0}}$ for cone angle $\beta = 60^\circ$

$\frac{p2R}{s_{um}}$	$\frac{D}{2R}$	$\frac{s_{u0}}{s_{um}}$	Roughness factor α						
			0.0	0.2	0.4	0.6	0.8	1.0	
0.0	0.0	1.00	4.64	5.02	5.36	5.67	5.95	6.17	
0.0	0.1	1.00	4.90	5.28	5.61	5.91	6.18	6.41	
0.0	0.25	1.00	5.22	5.59	5.93	6.23	6.49	6.71	
0.0	0.5	1.00	5.68	6.03	6.36	6.66	6.92	7.14	
0.0	1.0	1.00	6.37	6.71	7.05	7.32	7.58	7.79	
0.0	2.5	1.00	7.65	8.03	8.32	8.60	8.86	9.05	
1.0	0.0	1.00	5.57	6.05	6.47	6.87	7.22	7.53	
1.0	0.1	1.10	5.74	6.21	6.62	7.00	7.36	7.65	
1.0	0.25	1.25	5.94	6.38	6.79	7.16	7.50	7.79	
1.0	0.5	1.50	6.16	6.61	6.99	7.36	7.68	7.97	
1.0	1.0	2.00	6.50	6.93	7.30	7.64	7.95	8.21	
1.0	2.5	3.50	7.25	7.57	7.94	8.25	8.53	8.78	
2.0	0.0	1.00	6.46	7.03	7.54	8.01	8.45	8.82	
2.0	0.1	1.20	6.41	6.94	7.43	7.88	8.28	8.65	
2.0	0.25	1.50	6.41	6.88	7.35	7.76	8.14	8.46	
2.0	0.5	2.00	6.40	6.88	7.29	7.69	8.03	8.35	
2.0	1.0	3.00	6.54	6.99	7.37	7.73	8.06	8.33	
2.0	2.5	6.00	7.16	7.49	7.86	8.18	8.47	8.72	
3.0	0.0	1.00	7.36	8.00	8.59	9.14	9.65	10.08	
3.0	0.1	1.30	6.99	7.57	8.10	8.60	9.05	9.45	
3.0	0.25	1.75	6.70	7.24	7.73	8.17	8.59	8.94	
3.0	0.5	2.50	6.54	7.04	7.47	7.88	8.24	8.57	
3.0	1.0	4.00	6.56	7.02	7.41	7.78	8.11	8.39	
3.0	2.5	8.50	7.12	7.46	7.83	8.15	8.44	8.69	
4.0	0.0	1.00	8.22	8.96	9.64	10.25	10.82	11.33	
4.0	0.1	1.40	7.49	8.11	8.68	9.22	9.70	10.14	
4.0	0.25	2.00	6.94	7.50	8.01	8.48	8.92	9.29	
4.0	0.5	3.00	6.63	7.15	7.58	8.01	8.38	8.72	
4.0	1.0	5.00	6.57	7.03	7.43	7.80	8.14	8.42	
4.0	2.5	11.00	7.05	7.44	7.81	8.13	8.42	8.67	
5.0	0.0	1.00	9.11	9.93	10.66	11.35	12.00	12.56	
5.0	0.1	1.50	7.87	8.55	9.17	9.74	10.26	10.75	
5.0	0.25	2.25	7.12	7.71	8.24	8.72	9.17	9.57	
5.0	0.5	3.50	6.70	7.22	7.67	8.09	8.47	8.82	
5.0	1.0	6.00	6.57	7.04	7.44	7.82	8.16	8.44	
5.0	2.5	13.50	7.03	7.42	7.80	8.12	8.41	8.66	

TABLE 2.5 Bearing capacity factors $N_{c0} = \frac{Q}{As_{u0}}$ for cone angle $\beta = 90^\circ$

$\frac{p2R}{s_{um}}$	$\frac{D}{2R}$	$\frac{s_{u0}}{s_{um}}$	Roughness factor α						
			0.0	0.2	0.4	0.6	0.8	1.0	
0.0	0.0	1.00	4.96	5.25	5.51	5.73	5.92	6.05	
0.0	0.1	1.00	5.23	5.52	5.77	5.99	6.17	6.30	
0.0	0.25	1.00	5.57	5.85	6.10	6.31	6.49	6.62	
0.0	0.5	1.00	6.04	6.31	6.55	6.76	6.93	7.05	
0.0	1.0	1.00	6.74	7.01	7.24	7.44	7.61	7.72	
0.0	2.5	1.00	8.07	8.32	8.55	8.75	8.90	8.99	
1.0	0.0	1.00	5.69	6.04	6.36	6.65	6.89	7.09	
1.0	0.1	1.10	5.89	6.24	6.55	6.82	7.07	7.26	
1.0	0.25	1.25	6.12	6.45	6.76	7.02	7.26	7.45	
1.0	0.5	1.50	6.39	6.72	7.01	7.27	7.48	7.66	
1.0	1.0	2.00	6.80	7.10	7.37	7.61	7.82	7.97	
1.0	2.5	3.50	7.52	7.82	8.08	8.29	8.49	8.61	
2.0	0.0	1.00	6.38	6.79	7.16	7.50	7.80	8.04	
2.0	0.1	1.20	6.41	6.80	7.16	7.47	7.75	7.97	
2.0	0.25	1.50	6.46	6.83	7.17	7.46	7.72	7.94	
2.0	0.5	2.00	6.56	6.91	7.22	7.49	7.74	7.92	
2.0	1.0	3.00	6.80	7.12	7.40	7.65	7.87	8.03	
2.0	2.5	6.00	7.43	7.72	7.99	8.21	8.41	8.53	
3.0	0.0	1.00	7.04	7.51	7.93	8.31	8.66	8.93	
3.0	0.1	1.30	6.84	7.27	7.65	8.00	8.31	8.57	
3.0	0.25	1.75	6.71	7.09	7.45	7.76	8.05	8.27	
3.0	0.5	2.50	6.66	7.02	7.34	7.63	7.88	8.08	
3.0	1.0	4.00	6.81	7.11	7.41	7.67	7.89	8.06	
3.0	2.5	8.50	7.38	7.68	7.95	8.17	8.38	8.51	
4.0	0.0	1.00	7.70	8.22	8.69	9.11	9.49	9.81	
4.0	0.1	1.40	7.20	7.66	8.07	8.44	8.77	9.03	
4.0	0.25	2.00	6.88	7.28	7.65	7.98	8.27	8.53	
4.0	0.5	3.00	6.72	7.08	7.42	7.71	7.97	8.18	
4.0	1.0	5.00	6.80	7.12	7.41	7.68	7.90	8.08	
4.0	2.5	11.00	7.39	7.66	7.93	8.15	8.36	8.49	
5.0	0.0	1.00	8.35	8.91	9.43	9.89	10.31	10.67	
5.0	0.1	1.50	7.52	7.99	8.43	8.82	9.18	9.45	
5.0	0.25	2.25	7.01	7.43	7.81	8.15	8.45	8.72	
5.0	0.5	3.50	6.77	7.13	7.47	7.77	8.03	8.25	
5.0	1.0	6.00	6.80	7.12	7.42	7.69	7.91	8.09	
5.0	2.5	13.50	7.34	7.64	7.91	8.14	8.34	8.48	

TABLE 2.6 Bearing capacity factors $N_{c0} = \frac{Q}{As_{u0}}$ for cone angle $\beta = 120^\circ$

$\frac{\rho 2R}{s_{um}}$	$\frac{D}{2R}$	$\frac{s_{u0}}{s_{um}}$	Roughness factor α						
			0.0	0.2	0.4	0.6	0.8	1.0	
0.0	0.0	1.00	5.32	5.55	5.74	5.89	6.01	6.05	
0.0	0.1	1.00	5.60	5.82	6.00	6.16	6.26	6.30	
0.0	0.25	1.00	5.94	6.16	6.34	6.49	6.59	6.61	
0.0	0.5	1.00	6.41	6.62	6.80	6.94	7.03	7.05	
0.0	1.0	1.00	7.13	7.32	7.49	7.62	7.71	7.72	
0.0	2.5	1.00	8.46	8.65	8.81	8.93	8.99	8.99	
1.0	0.0	1.00	5.94	6.22	6.46	6.67	6.84	6.97	
1.0	0.1	1.10	6.16	6.43	6.67	6.87	7.04	7.15	
1.0	0.25	1.25	6.41	6.67	6.90	7.09	7.25	7.36	
1.0	0.5	1.50	6.71	6.96	7.18	7.36	7.51	7.60	
1.0	1.0	2.00	7.13	7.36	7.57	7.73	7.86	7.95	
1.0	2.5	3.50	7.91	8.12	8.31	8.44	8.56	8.61	
2.0	0.0	1.00	6.50	6.82	7.11	7.35	7.57	7.73	
2.0	0.1	1.20	6.59	6.90	7.16	7.40	7.59	7.74	
2.0	0.25	1.50	6.69	6.98	7.23	7.45	7.63	7.76	
2.0	0.5	2.00	6.84	7.10	7.34	7.54	7.70	7.82	
2.0	1.0	3.00	7.11	7.35	7.57	7.74	7.89	7.99	
2.0	2.5	6.00	7.81	8.01	8.21	8.35	8.47	8.53	
3.0	0.0	1.00	7.03	7.40	7.72	7.98	8.24	8.43	
3.0	0.1	1.30	6.94	7.27	7.56	7.81	8.03	8.21	
3.0	0.25	1.75	6.88	7.18	7.45	7.68	7.88	8.03	
3.0	0.5	2.50	6.91	7.18	7.43	7.63	7.81	7.94	
3.0	1.0	4.00	7.10	7.35	7.57	7.75	7.90	8.00	
3.0	2.5	8.50	7.76	7.97	8.16	8.31	8.43	8.49	
4.0	0.0	1.00	7.55	7.94	8.30	8.58	8.88	9.10	
4.0	0.1	1.40	7.23	7.58	7.89	8.16	8.40	8.59	
4.0	0.25	2.00	7.02	7.34	7.62	7.86	8.07	8.23	
4.0	0.5	3.00	6.95	7.23	7.49	7.70	7.88	8.01	
4.0	1.0	5.00	7.09	7.34	7.56	7.75	7.90	8.00	
4.0	2.5	11.00	7.72	7.94	8.13	8.29	8.41	8.47	
5.0	0.0	1.00	8.05	8.48	8.86	9.19	9.48	9.74	
5.0	0.1	1.50	7.46	7.83	8.16	8.44	8.69	8.90	
5.0	0.25	2.25	7.13	7.45	7.74	7.99	8.20	8.37	
5.0	0.5	3.50	6.99	7.27	7.53	7.74	7.93	8.07	
5.0	1.0	6.00	7.09	7.34	7.56	7.75	7.91	8.01	
5.0	2.5	13.50	7.70	7.93	8.12	8.27	8.40	8.46	

TABLE 2.7 Bearing capacity factors $N_{c0} = \frac{Q}{A_{s_{u0}}}$ for cone angle $\beta = 150^\circ$

$\frac{\rho 2R}{s_{um}}$	$\frac{D}{2R}$	$\frac{s_{u0}}{s_{um}}$	Roughness factor α						
			0.0	0.2	0.4	0.6	0.8	1.0	
0.0	0.0	1.00	5.69	5.86	5.97	6.03	6.05	6.05	
0.0	0.1	1.00	5.97	6.13	6.24	6.29	6.30	6.30	
0.0	0.25	1.00	6.31	6.47	6.57	6.61	6.61	6.61	
0.0	0.5	1.00	6.79	6.93	7.02	7.05	7.05	7.05	
0.0	1.0	1.00	7.49	7.63	7.70	7.71	7.71	7.71	
0.0	2.5	1.00	8.82	8.94	8.99	8.99	8.99	8.99	
1.0	0.0	1.00	6.25	6.47	6.65	6.79	6.90	6.95	
1.0	0.1	1.10	6.48	6.69	6.87	7.00	7.10	7.14	
1.0	0.25	1.25	6.74	6.94	7.11	7.23	7.32	7.35	
1.0	0.5	1.50	7.05	7.24	7.39	7.51	7.58	7.60	
1.0	1.0	2.00	7.47	7.64	7.79	7.88	7.93	7.94	
1.0	2.5	3.50	8.26	8.32	8.52	8.60	8.61	8.61	
2.0	0.0	1.00	6.73	6.98	7.20	7.39	7.53	7.63	
2.0	0.1	1.20	6.85	7.08	7.30	7.46	7.59	7.68	
2.0	0.25	1.50	6.98	7.20	7.39	7.55	7.66	7.72	
2.0	0.5	2.00	7.15	7.36	7.53	7.67	7.76	7.80	
2.0	1.0	3.00	7.45	7.63	7.78	7.90	7.96	7.98	
2.0	2.5	6.00	8.16	8.27	8.43	8.50	8.53	8.53	
3.0	0.0	1.00	7.16	7.45	7.69	7.91	8.08	8.21	
3.0	0.1	1.30	7.13	7.40	7.62	7.81	7.96	8.07	
3.0	0.25	1.75	7.15	7.37	7.58	7.75	7.88	7.96	
3.0	0.5	2.50	7.21	7.42	7.61	7.75	7.86	7.91	
3.0	1.0	4.00	7.43	7.62	7.78	7.90	7.97	7.99	
3.0	2.5	8.50	8.13	8.23	8.38	8.46	8.49	8.49	
4.0	0.0	1.00	7.56	7.87	8.15	8.38	8.58	8.73	
4.0	0.1	1.40	7.38	7.64	7.89	8.09	8.26	8.39	
4.0	0.25	2.00	7.26	7.50	7.71	7.89	8.03	8.13	
4.0	0.5	3.00	7.25	7.46	7.65	7.80	7.92	7.98	
4.0	1.0	5.00	7.44	7.61	7.77	7.89	7.97	8.00	
4.0	2.5	11.00	8.09	8.19	8.36	8.44	8.47	8.47	
5.0	0.0	1.00	7.94	8.27	8.57	8.83	9.05	9.23	
5.0	0.1	1.50	7.56	7.85	8.10	8.32	8.50	8.64	
5.0	0.25	2.25	7.34	7.59	7.81	8.00	8.15	8.25	
5.0	0.5	3.50	7.27	7.49	7.68	7.84	7.96	8.02	
5.0	1.0	6.00	7.43	7.60	7.77	7.89	7.97	8.00	
5.0	2.5	13.50	8.07	8.18	8.35	8.43	8.46	8.46	

TABLE 2.8 Bearing capacity factors $N_{c0} = \frac{Q}{A_{s_{u0}}}$ for cone angle $\beta = 180^\circ$

TABLE 3.1 Properties of Speswhite kaolin measured by various researchers

	Steenfelt <i>et al.</i> (1981)	Martins (1983)	Fannin (1986)	Al-Tabbaa (1987)	Santa Maria (1988)	Smith (1993)
λ	0.25	0.25-0.18*	0.25	0.187	0.22	0.174
κ	0.05	0.04	0.04	0.028	0.03	0.05
N_{ID}	3.58	3.52	3.51	3.13	3.34	3.10
M	0.9	0.9	0.88	0.9	-	0.8
K_{0nc}	-	0.65	0.64	0.69	-	-

* $\lambda = 0.25$ at $p' = 5$ kPa, decreasing to $\lambda = 0.18$ at $p' = 350$ kPa

TABLE 3.2 Design displacement ranges and displacement rates for experimental apparatus

[a]	Maximum Travel	Probable Required Travel
z	300 mm	250 mm
h	± 25 mm	± 10 mm
θ	$\pm 15^\circ$	$\pm 6^\circ$

[b]	Design Range	Typical
$\dot{z}$	0 - 1 mm/s	0.3 mm/s
$\dot{h}$	0 - 0.25 mm/s	0.05 mm/s
$\dot{\theta}$	0 - 0.25 deg/s	0.05 deg/s

TABLE 3.3 Checking of load cell accuracy and precision

Test	Applied Loads			Measured Loads			Absolute Error			Relative Error (%)		
	<i>V</i> (N)	<i>H</i> (N)	<i>M</i> (Nm)	<i>V</i> (N)	<i>H</i> (N)	<i>M</i> (Nm)	<i>V</i> (N)	<i>H</i> (N)	<i>M</i> (Nm)	<i>V</i>	<i>H</i>	<i>M</i>
1	507.4	0.0	0.00	508.2	0.0	-0.01	+0.8	0.0	-0.01	+0.15	-	-
2	1108.2	0.0	0.00	1109.0	0.1	0.04	+0.8	+0.1	+0.04	+0.07	-	-
3	795.5	0.0	-11.93	794.9	0.3	-11.87	-0.6	+0.3	+0.06	-0.07	-	-0.52
4	646.0	98.1	0.00	646.9	98.4	-0.03	+0.9	+0.3	-0.03	+0.14	+0.31	-
5	1242.0	49.1	-6.21	1241.3	49.3	-6.15	-0.7	+0.2	+0.06	-0.06	+0.41	-0.94
6	937.9	147.2	9.38	936.7	147.7	9.42	-1.2	+0.5	+0.04	-0.13	+0.32	+0.46
7	795.5	0.0	11.93	794.1	-0.1	11.94	-1.4	-0.1	+0.01	-0.17	-	+0.08
8	646.0	-98.1	0.00	646.5	-98.2	0.03	+0.5	-0.1	+0.03	+0.08	+0.07	-
9	1242.0	-49.1	-6.21	1242.1	-48.7	-6.22	+0.1	+0.4	-0.01	+0.01	-0.73	+0.15
10	937.9	-147.2	9.38	936.0	-146.9	9.39	-1.9	+0.3	+0.01	-0.20	-0.19	+0.09

TABLE 3.4 LVDT specifications (all transducers supplied by RDP Electronics)

LVDT No.	Displacement	LVDT Type	Linear Range (mm)	Rated Nonlinearity
1	Vertical (coarse)	ACT6000C	± 150	0.27% of L.R.
2	Vertical (fine)	D5/1000A	± 25	0.25%
3	Horizontal	D5/1000A	± 25	0.25%
4	Rotational	ACT2000C	± 50	0.10%

TABLE 3.5 Approximate motor step counts associated with unit footing displacements

Motor No.	Displacement	Step Count Conversion
1	Vertical, <i>z</i>	1 mm $\approx$ 10000 steps
2	Horizontal, <i>h</i>	1 mm $\approx$ 50340 steps
3	Rotational, θ	1 degree $\approx$ 47240 steps

TABLE 4.1 Excerpt from test control file for T011

Line	Command	Action
1	*V, CV, 20	commence moving vertically downwards at a constant velocity of 20 mm/minute
2	SCAN, DV, 0	keep checking the vertical LVDT reading, and stop when $z_{RP} = 0$ mm
3	HOLD, F10	use a feedback routine to maintain the vertical load just attained, until the <F10> key is pressed
4	*H, CV, 5	lock the vertical displacement and start moving horizontally (to the right) at 5 mm/min
5	SCAN, F10	check keyboard and stop when <F10> is pressed
6	*H, ABS, 5, 0	return to an absolute stepper motor position of $h = 0$ mm (the central datum) at 5 mm/min
7	*V, CV, 20	resume vertical penetration at 20 mm/minute
8	SCAN, DV, 25	stop when $z_{RP} = 25$ mm
9	HOLD, F10	maintain the vertical load just reached, waiting for the <F10> key
10	*V, CV, -10	move back up (<i>i.e.</i> unload vertically) at 10 mm/min
11	SCAN, V, 0	unload to a load of $V = 0$
12	*V, CV, 20	immediately resume vertical penetration as before...
13	SCAN, DV, 50	...until $z_{RP} = 50$ mm
14	HOLD, F10	hold vertical load again, waiting for the <F10> key
15	*H, CV, -5	this time move to the <i>left</i> at 5 mm/min (with the vertical displacement locked)
16	SCAN, F10	stop when <F10> is pressed
17	*H, ABS, 5, 0	move back to the central ($h = 0$ mm) position at 5mm/min
18	*V, CV, 20	resume vertical penetration at 20 mm/min
19...	↓	perform more horizontal displacement events and vertical unload-reload loops at deeper penetrations

TABLE 4.2 Vertical unload-reload loop summary: Test T011

Nom. z_{RP} (mm)	ΔV (N)	Δz (mm)	$k_{UL,RL}$ (N/mm)	k_{VIR} (N/mm)	$k_{UL,RL}/k_{VIR}$
25	469.7	1.20	390.8	≈ 5.8	≈ 67
75	706.7	1.54	460.2	≈ 4.5	≈ 102
125	785.8	1.64	476.3	≈ 2.8	≈ 170
175	895.8	1.67	535.8	≈ 2.5	≈ 214

TABLE 4.3 Test allocation: T021-T063

	h , θ and combined $h:\theta$ displacement events at constant z			
	Main Tests		Repeat Tests	
ω (deg)	from high V/V_0	from low V/V_0	from high V/V_0	from low V/V_0
0	T023	T052	T041	-
90	T022	T021	T043	T051
45	T031	T053	-	-
135	T032	T061	-	-
112.5	T042	T062	-	-
157.5	T033	T063	-	-

TABLE 4.4 Test allocation: T071-T073

Note: vertical load level for all loop events is $V/V_0 = 0.4$

	$h:R\theta$ Loop Event Types	Leading Displacement	Direction of Loop Traversal
T071	Types 1, 5	h	anticlockwise
T072	Types 3, 7	h	clockwise
T073a ($z_{RP} = 0, 50, 200$ mm)	Types 4, 8	θ	anticlockwise
T073b ($z_{RP} = 20, 100$ mm)	Types 2, 6	θ	clockwise

TABLE 4.5 Test allocation: T081-T103 (cf. Table 4.3)

	<i>h</i> , θ and combined <i>h</i> : θ displacement events at constant <i>V</i>			
	Main Tests		Repeat Tests	
ω (deg)	$V/V_0 = 0.75$	$V/V_0 = 0.2$	$V/V_0 = 0.75$	$V/V_0 = 0.2$
0	T081	T091	T101	-
90	T082	T092	T102	-
45	T083	T093a	-	-
135	T103	-	-	-
112.5	-	-	-	-
157.5	-	T093b	-	-

TABLE 4.6 Displacement rates: T111-T113

Note: units are mm/minute throughout

	T022 (Std Rates)	T023 (Std Rates)	T111 (4 times slower)	T112 (16 times slower)	T113 (2 times faster)
$\dot{z}$ (penetration)	20	20	5	1.25	40
$\dot{z}$ (retraction)	10	10	2.5	0.625	20
$\dot{h}$	-	10	2.5	0.625	20
$R\dot{\theta}$	10	-	2.5	0.625	20

TABLE 4.7 Vertical unload-reload loop amplitudes: T021-T113

Unloading to:	Test Nos	Total Loops
$V/V_0 = 0$	T021-T063 (standard displacement rates); T111-T113 (modified displacement rates)	45 9
$V/V_0 = 0.25$	T081-T083; T101-T103	18
$V/V_0 = 0.5$	T071-T073	9
$V/V_0 = 0.75$	T091-T093	9
		90

TABLE 6.1 OXSPUD simulation of T022: control instructions

Stage Number	Degree of Freedom		
	Vertical	Horizontal	Rotational
1	target $z_{RP} = 0.04$ mm	$\delta h = 0$	$R\delta\theta = 0$
2	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = 9.84$ mm
3	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = -9.84$ mm
4	target $z_{RP} = 21.55$ mm	$\delta h = 0$	$R\delta\theta = 0$
5	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = 10.93$ mm
6	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = -10.93$ mm
7	target $z_{RP} = 35.21$ mm	$\delta h = 0$	$R\delta\theta = 0$
8	target $V = 0$ N	$\delta h = 0$	$R\delta\theta = 0$
9	target $z_{RP} = 51.17$ mm	$\delta h = 0$	$R\delta\theta = 0$
10	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = 16.50$ mm
11	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = -16.50$ mm
12	target $z_{RP} = 75.77$ mm	$\delta h = 0$	$R\delta\theta = 0$
13	target $V = 0$ N	$\delta h = 0$	$R\delta\theta = 0$
14	target $z_{RP} = 99.51$ mm	$\delta h = 0$	$R\delta\theta = 0$
15	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = 15.07$ mm
16	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = -15.07$ mm
17	target $z_{RP} = 158.77$ mm	$\delta h = 0$	$R\delta\theta = 0$
18	target $V = 0$ N	$\delta h = 0$	$R\delta\theta = 0$
19	target $z_{RP} = 203.19$ mm	$\delta h = 0$	$R\delta\theta = 0$
20	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = 9.03$ mm
21	$\delta z = 0$	$\delta h = 0$	$R\delta\theta = -9.03$ mm
22	target $V = 0$ N	$\delta h = 0$	$R\delta\theta = 0$

TABLE 6.2 OXSPUD simulation of T052: control instructions

Stage Number	Degree of Freedom		
	Vertical	Horizontal	Rotational
1	target $z_{RP} = 1.65$ mm	$\delta h = 0$	$R\delta\theta = 0$
2	target $V = 0.01V_0$	$\delta h = 0$	$R\delta\theta = 0$
3	$\delta z = 0$	$\delta h = 7.48$ mm	$R\delta\theta = 0$
4	$\delta z = 0$	$\delta h = -7.48$ mm	$R\delta\theta = 0$
5	target $z_{RP} = 22.23$ mm	$\delta h = 0$	$R\delta\theta = 0$
<i>etc.</i>			

TABLE 6.3 OXSPUD simulation of T082: control instructions

Stage Number	Degree of Freedom		
	Vertical	Horizontal	Rotational
1	target $z_{RP} = 0.05$ mm	$\delta h = 0$	$R\delta\theta = 0$
2	target $V = 0.75V_0$	$\delta h = 0$	$R\delta\theta = 0$
3	$\delta V = 0$	$\delta h = 0$	$R\delta\theta = 7.59$ mm
4	$\delta V = 0$	$\delta h = 0$	$R\delta\theta = -7.59$ mm
5	target $z_{RP} = 20.01$ mm	$\delta h = 0$	$R\delta\theta = 0$
<i>etc.</i>			

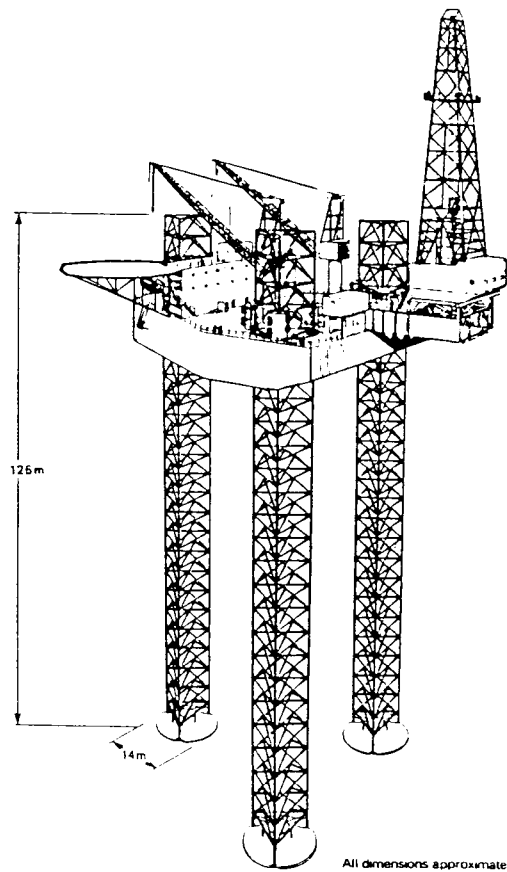


FIGURE 1.1

Typical three legged jack-up platform (after Reardon, 1986)

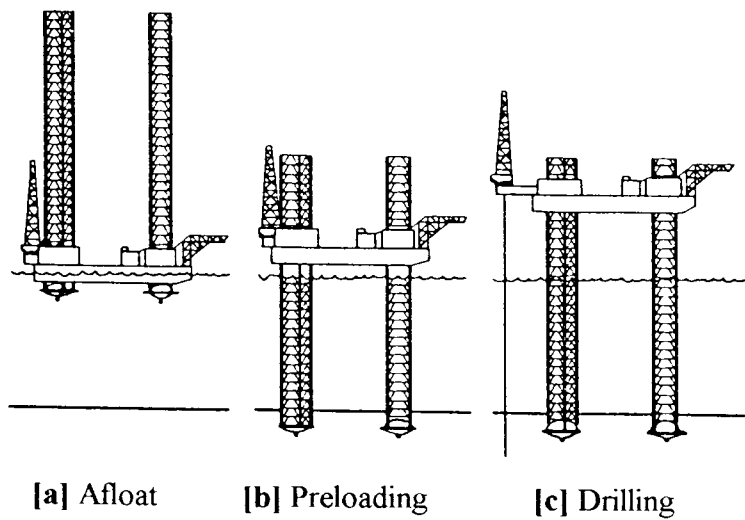


FIGURE 1.2

Jack-up installation procedure (after Young *et al.*, 1984)

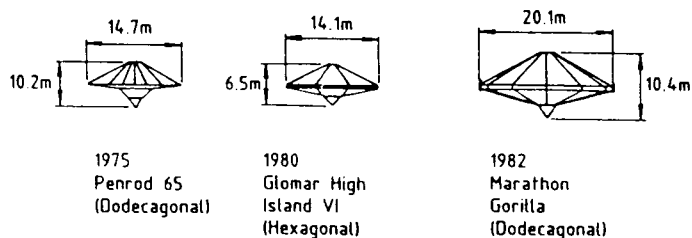


FIGURE 1.3

Typical "spudcan" footings (after Young *et al.*, 1984)

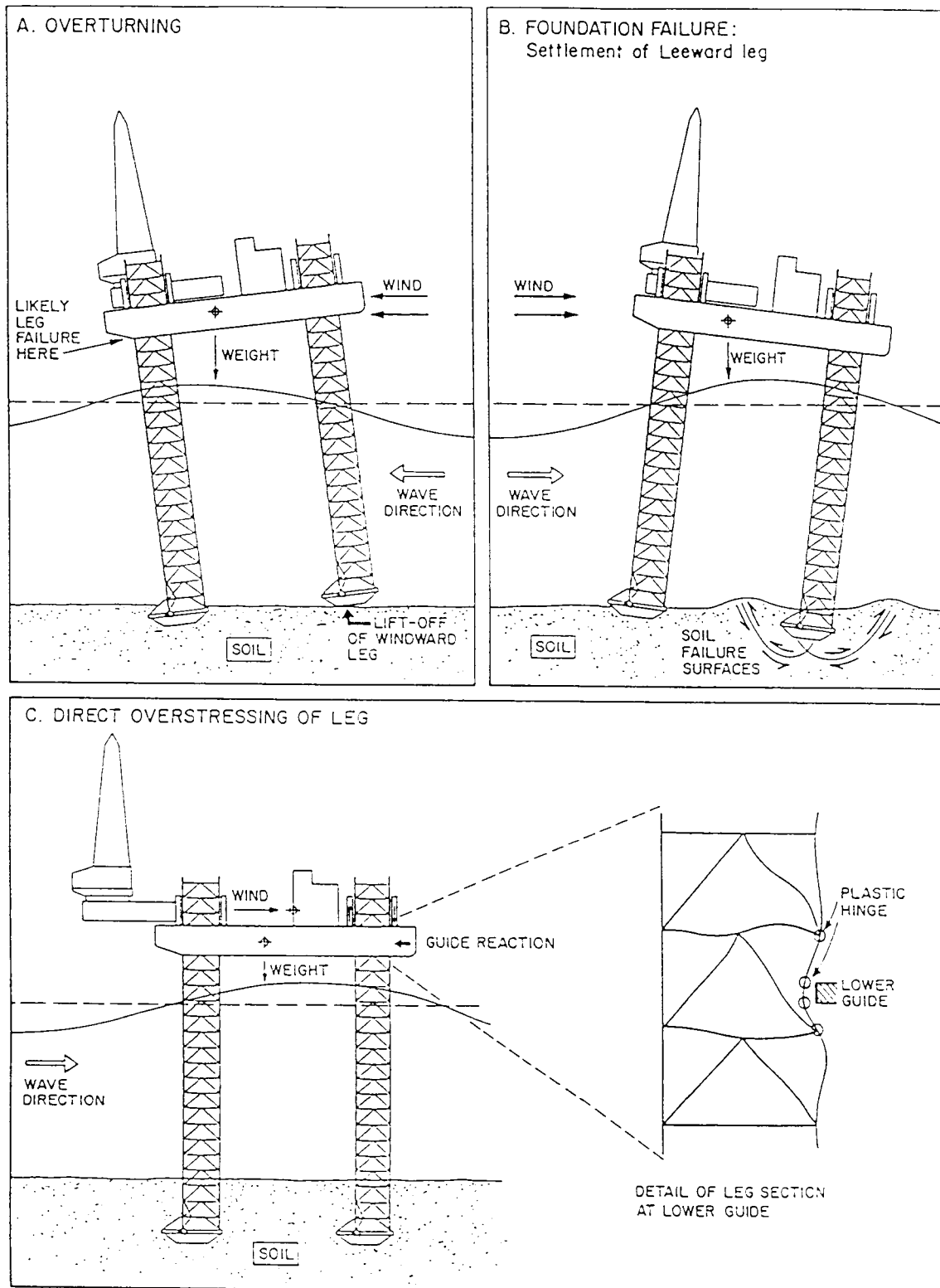
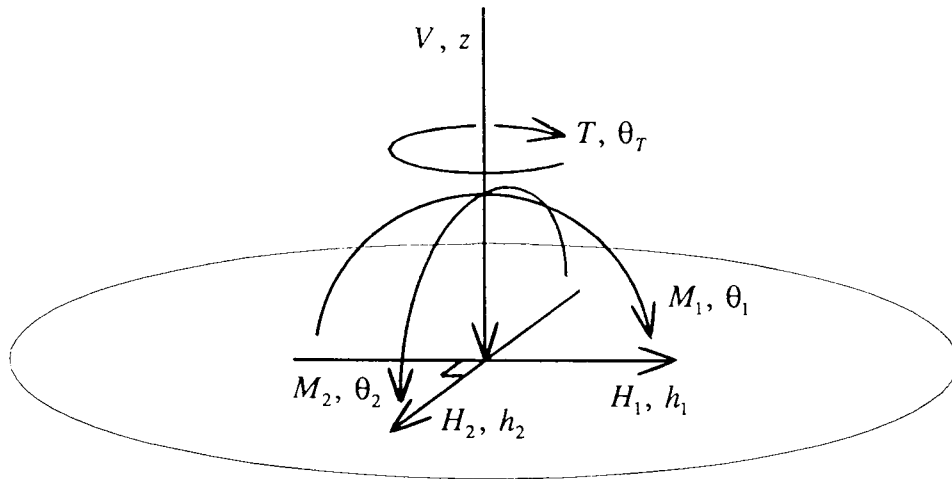
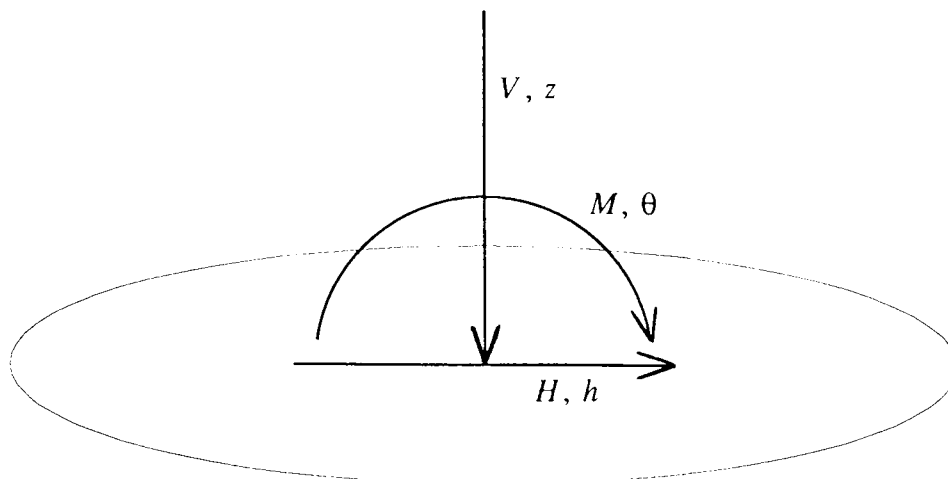


FIGURE 1.4 Key jack-up failure modes (after Leijten & Efthymiou, 1989)



[a] Six degrees of freedom



[b] Three degrees of freedom (planar loading)

FIGURE 1.5 Combined loading of a circular footing, with corresponding displacements

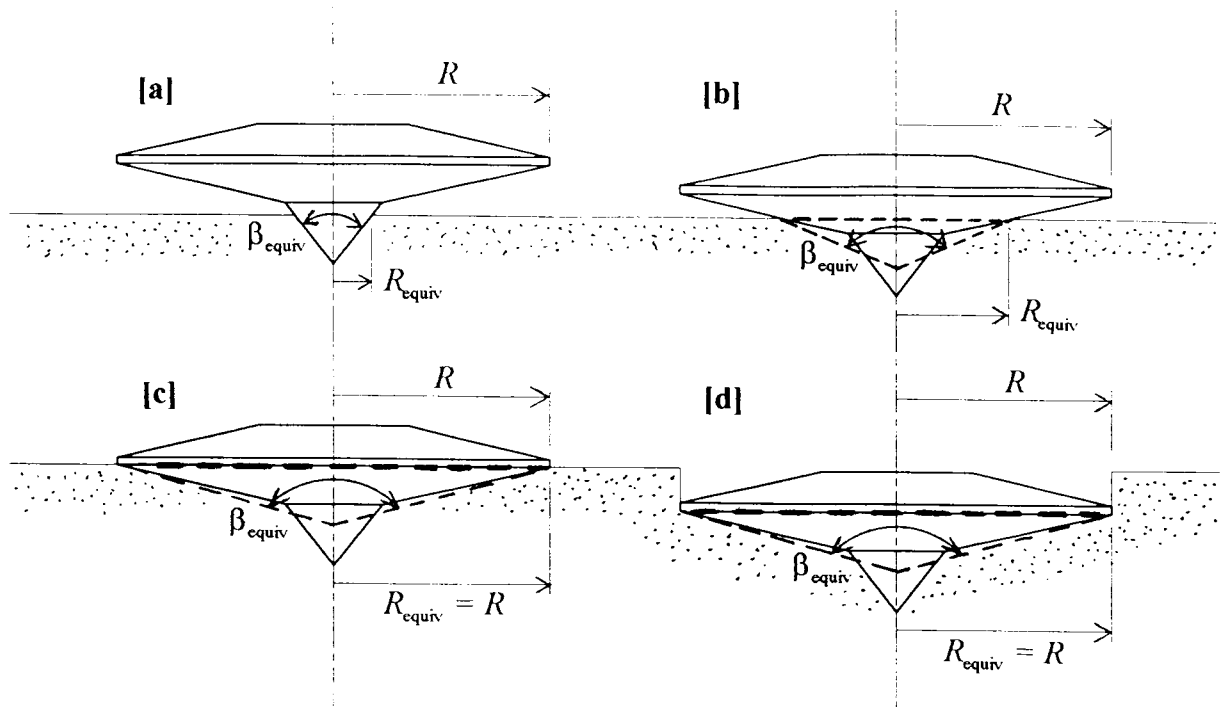


FIGURE 2.1 Definition of equivalent cone for penetrating spudcan
(volume of equivalent cone = volume of penetrated portion of spudcan)

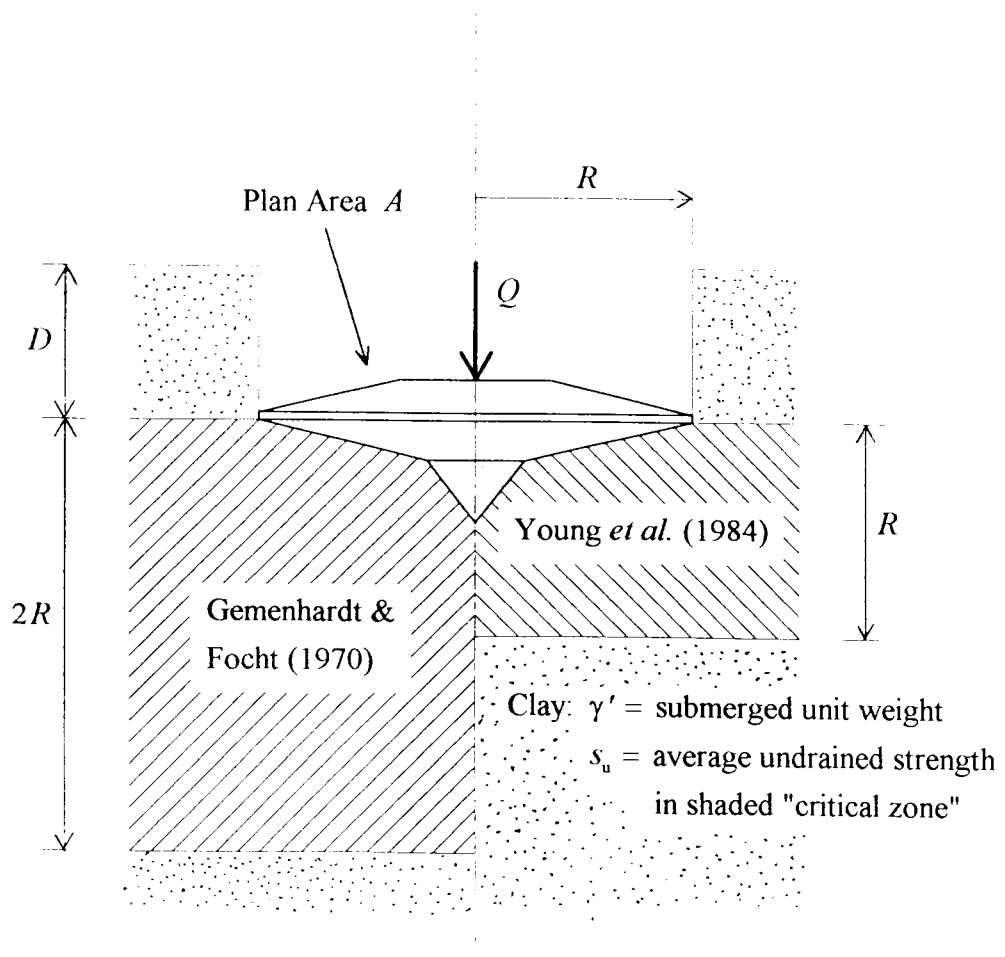
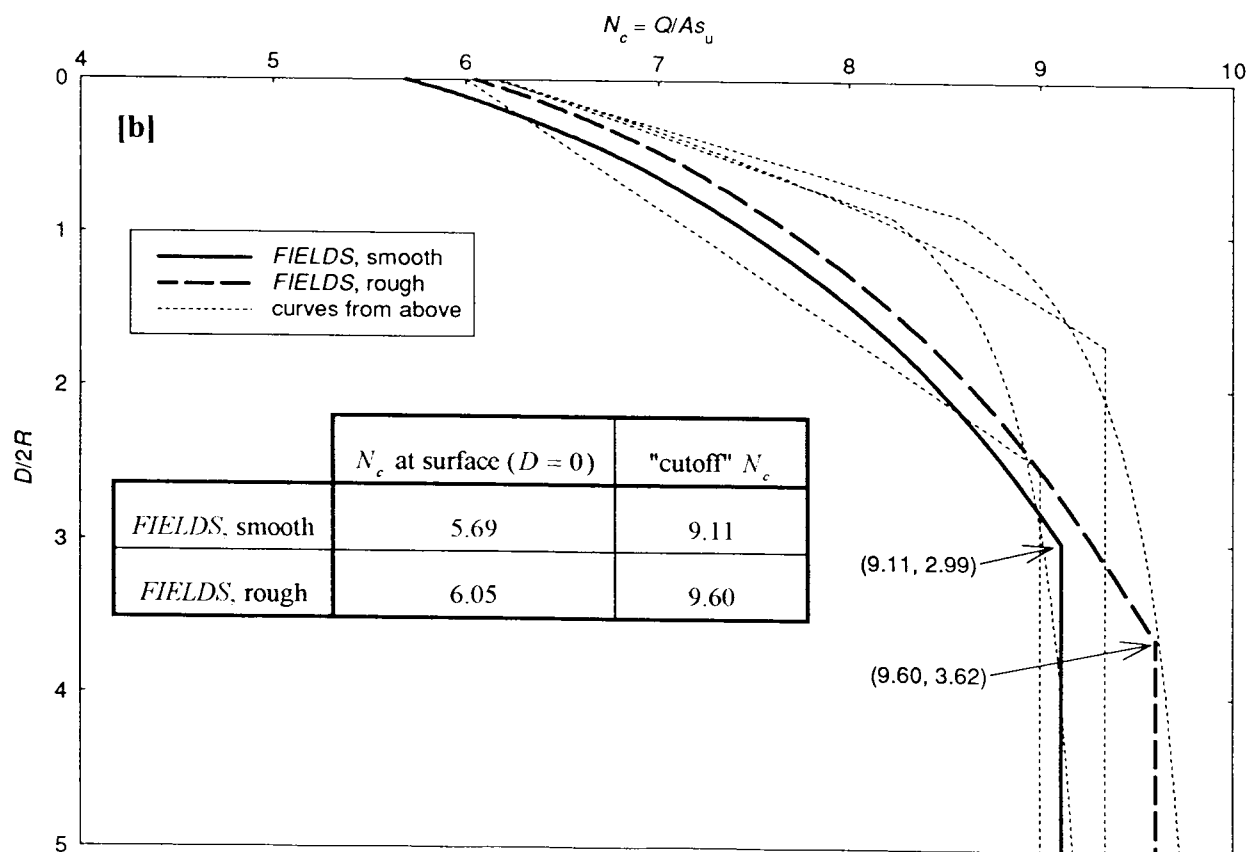
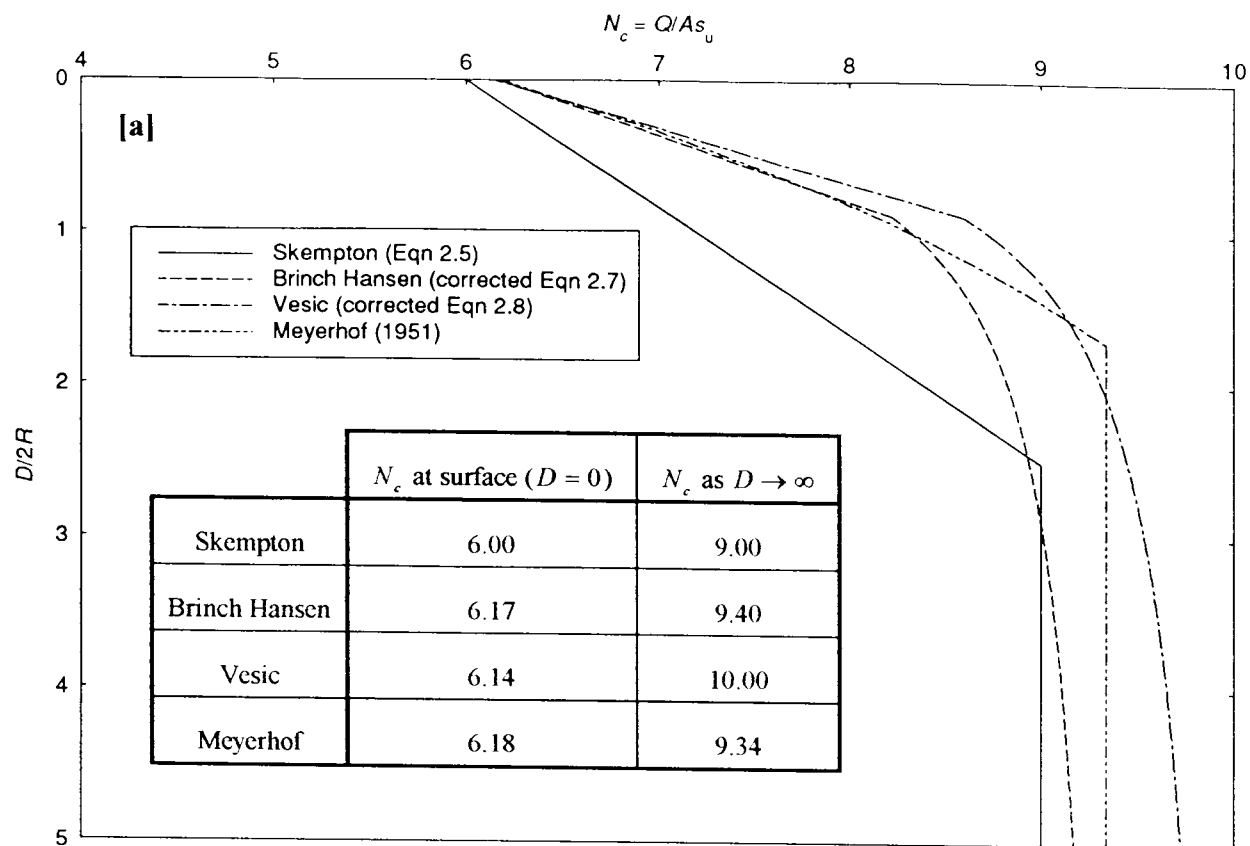
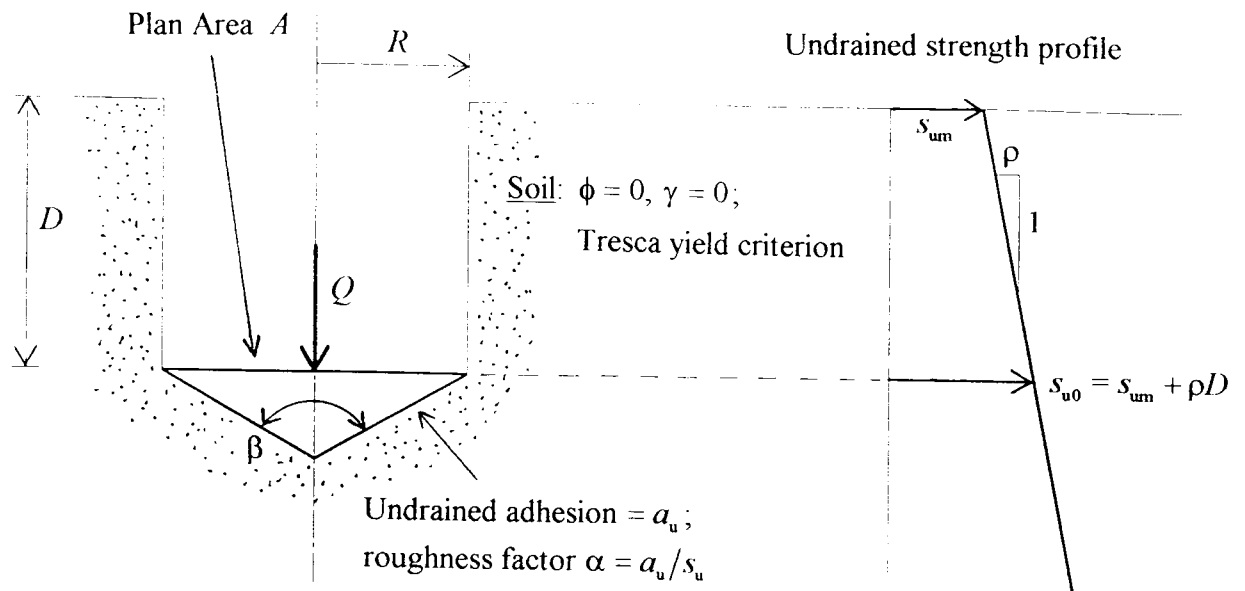


FIGURE 2.2 Vertical bearing capacity: terminology



N.B. relevant penetration range for jack-ups is approx. $D/2R \leq 2.5$

FIGURE 2.3 Variation of vertical bearing capacity factor with embedment (flat circular footing)



[a] Problem definition and notation

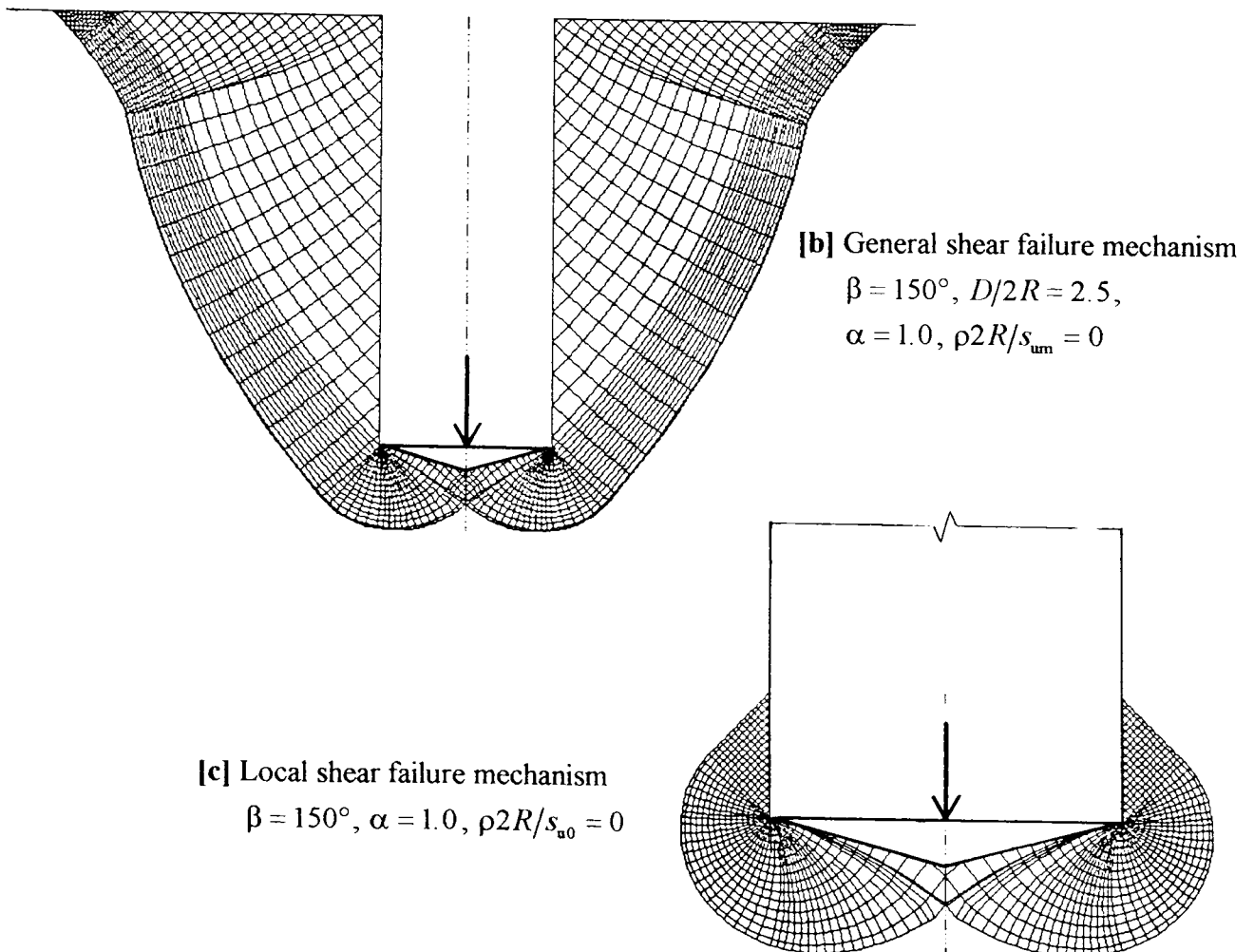
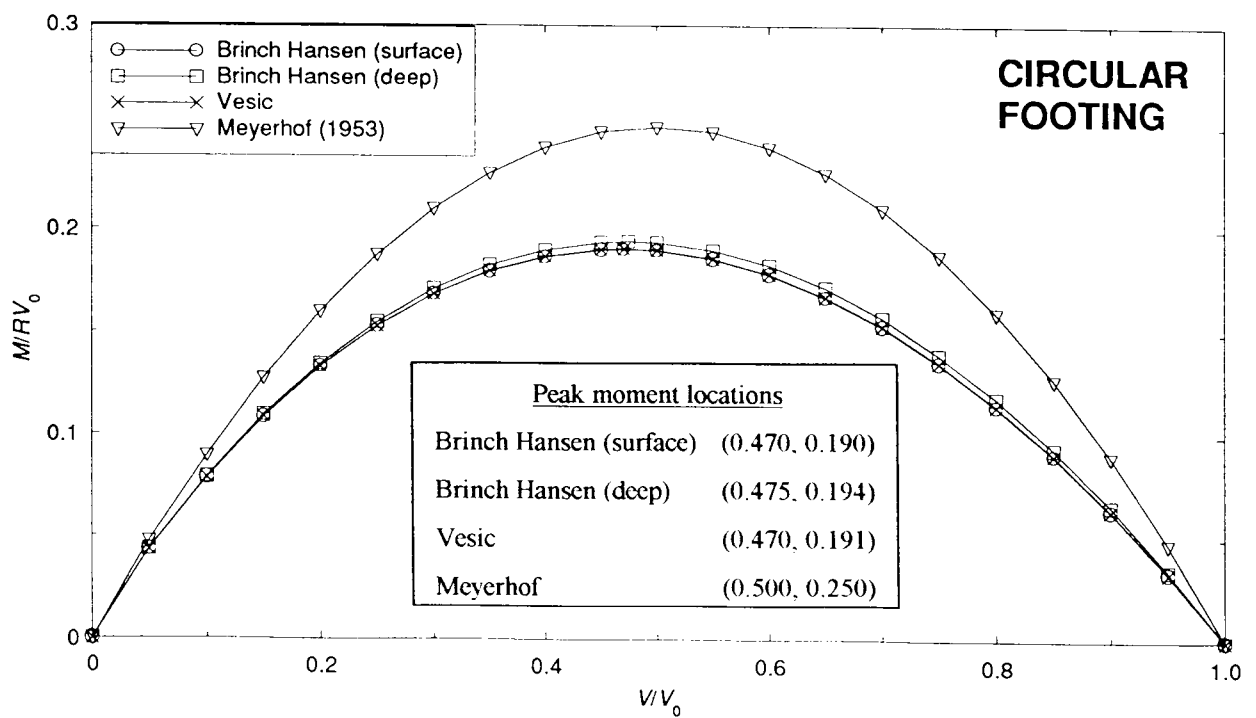
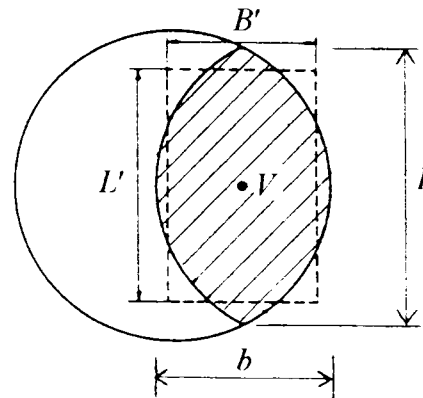
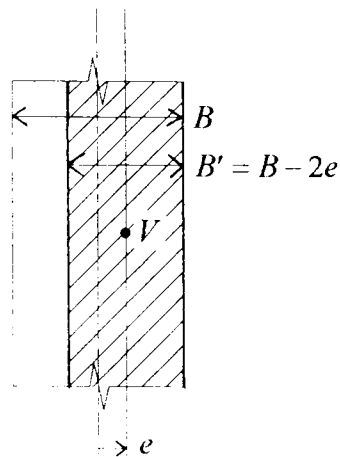
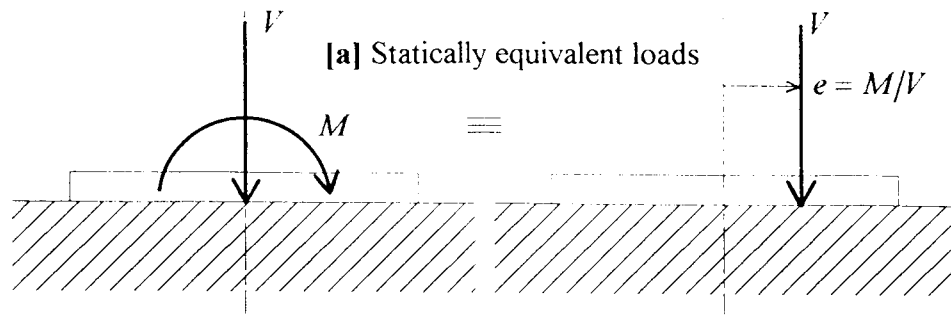
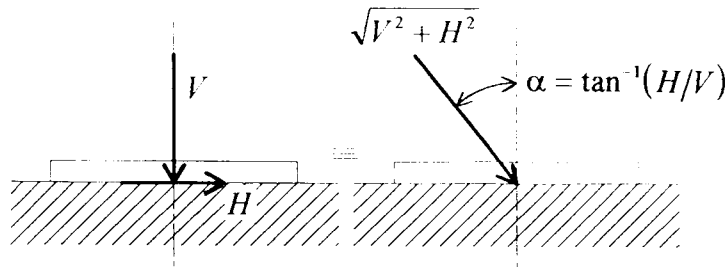


FIGURE 2.4 Vertical bearing capacity calculations with program *FIELDS*



[d] V - M bearing capacity interaction

FIGURE 2.5 Combined vertical load and moment (eccentric loading)



[a] Statically equivalent loads

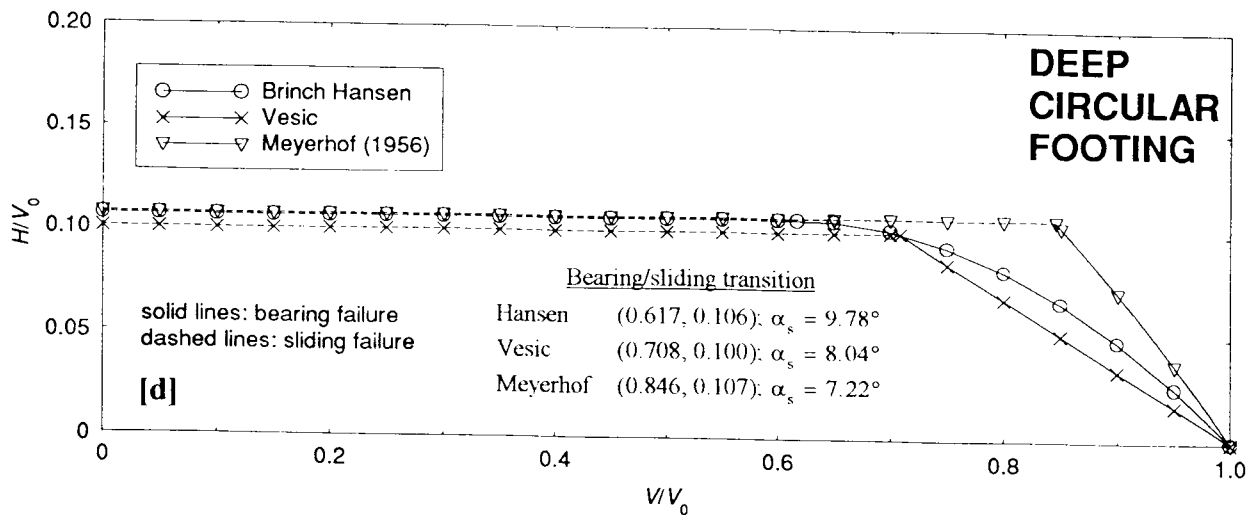
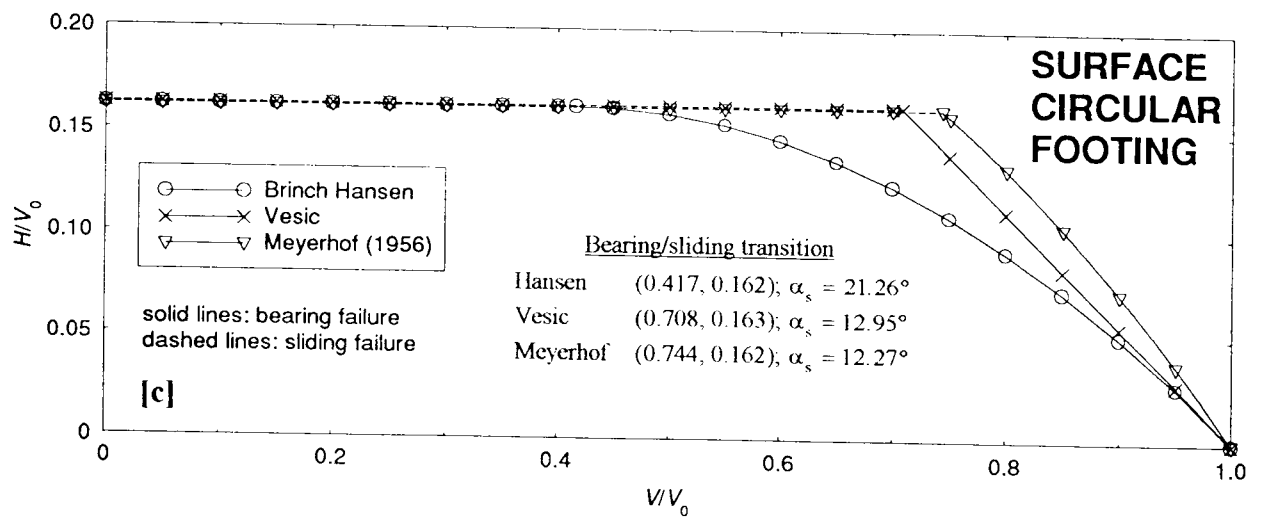
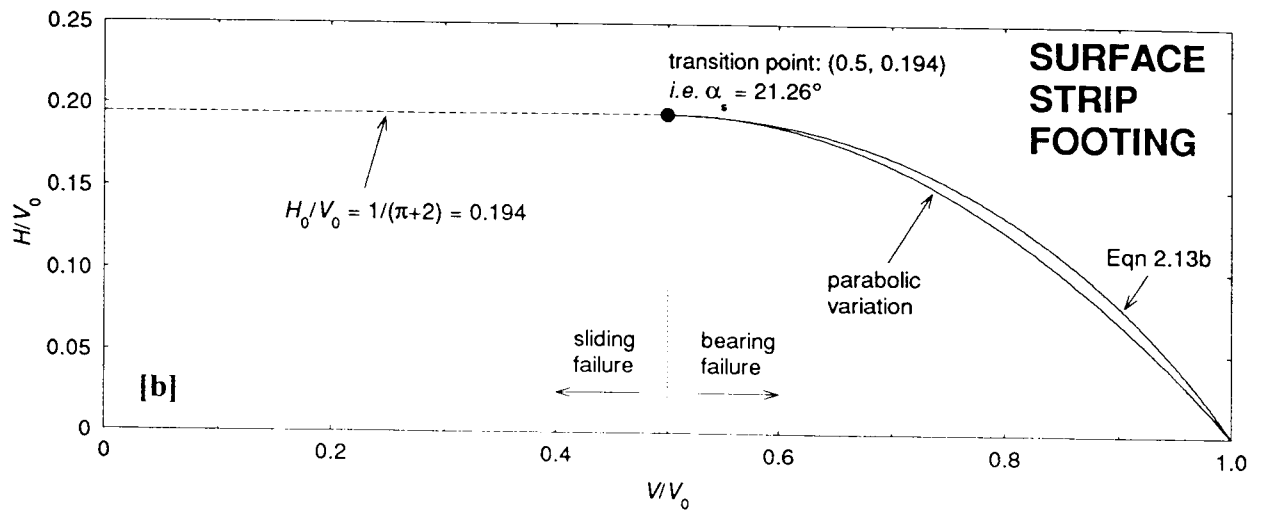


FIGURE 2.6 Combined vertical load and horizontal load (inclined loading)

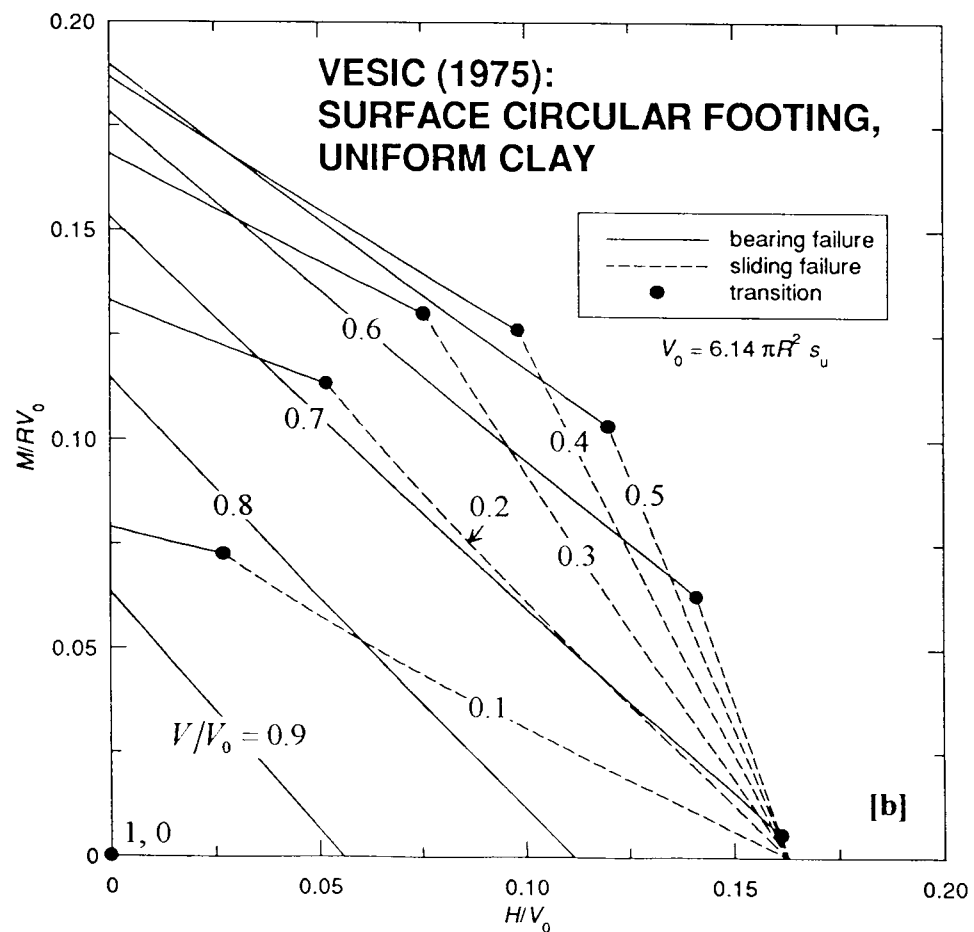
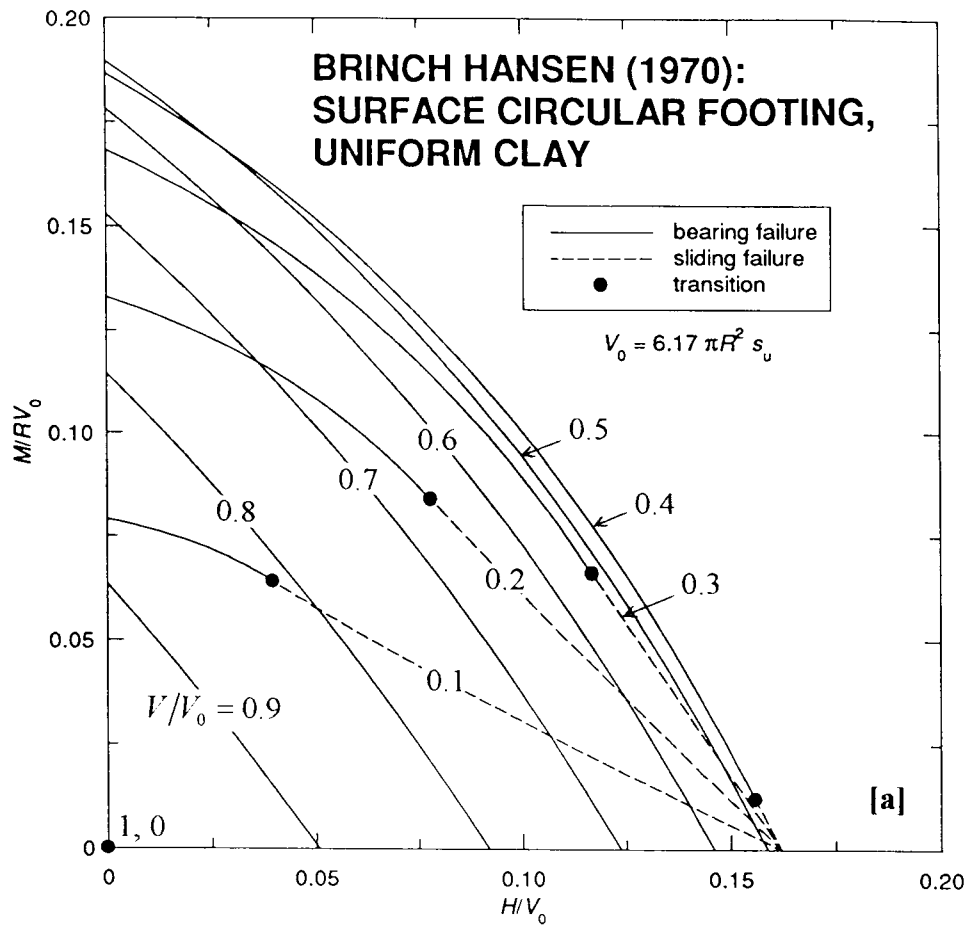


FIGURE 2.7 $V:H:M$ failure envelope contours: surface circular footing ($D = 0$)

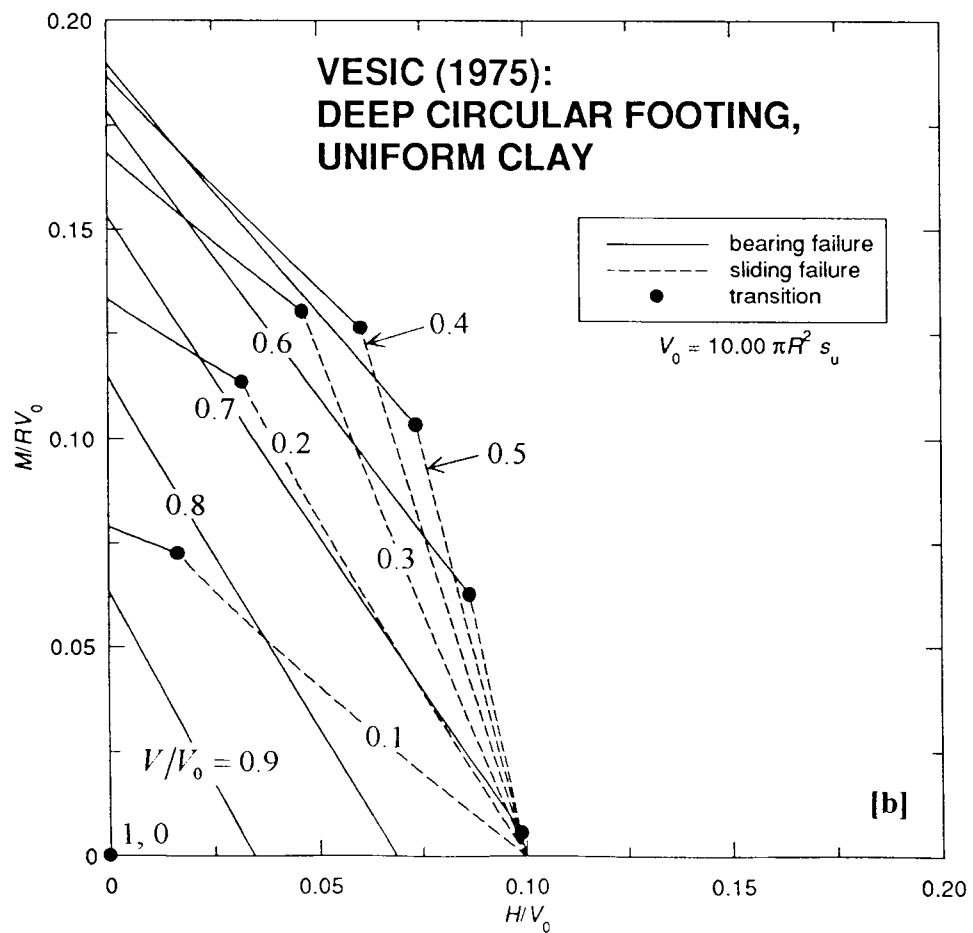
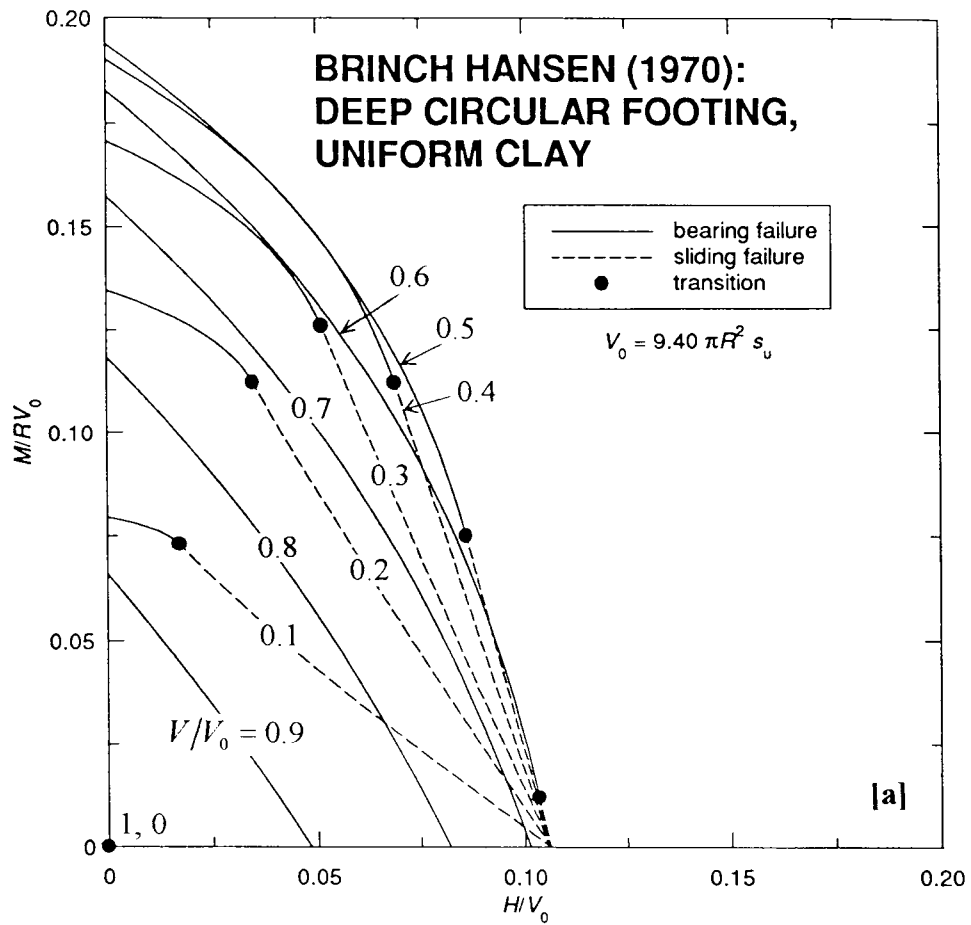


FIGURE 2.8 $V:HM$ failure envelope contours: deep circular footing ($D \rightarrow \infty$)

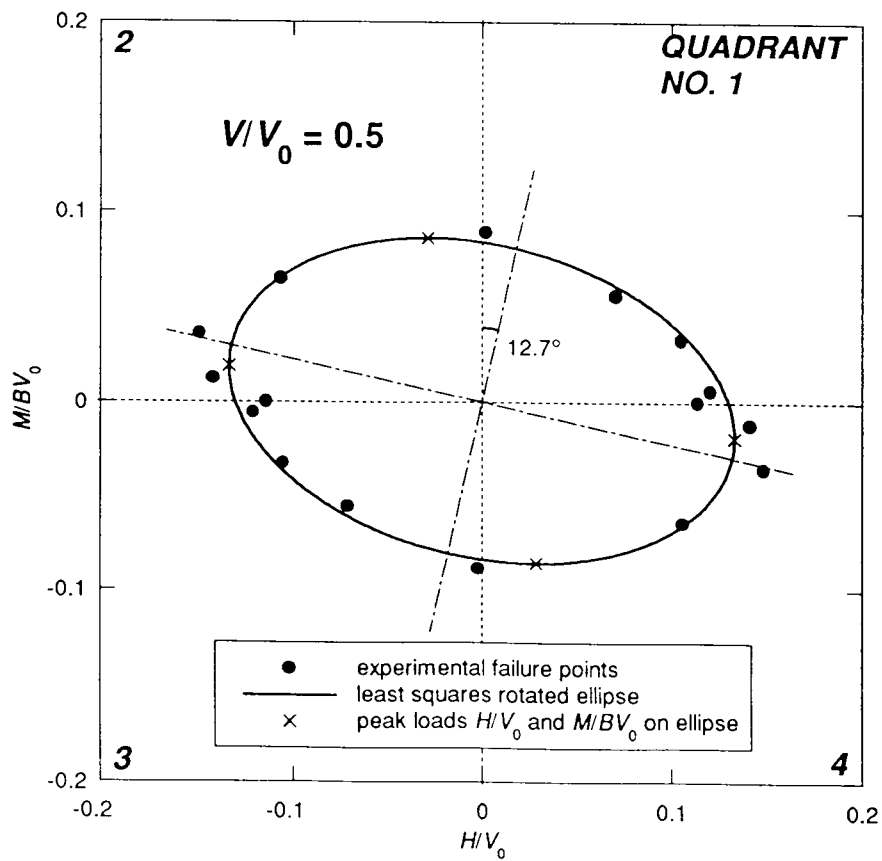
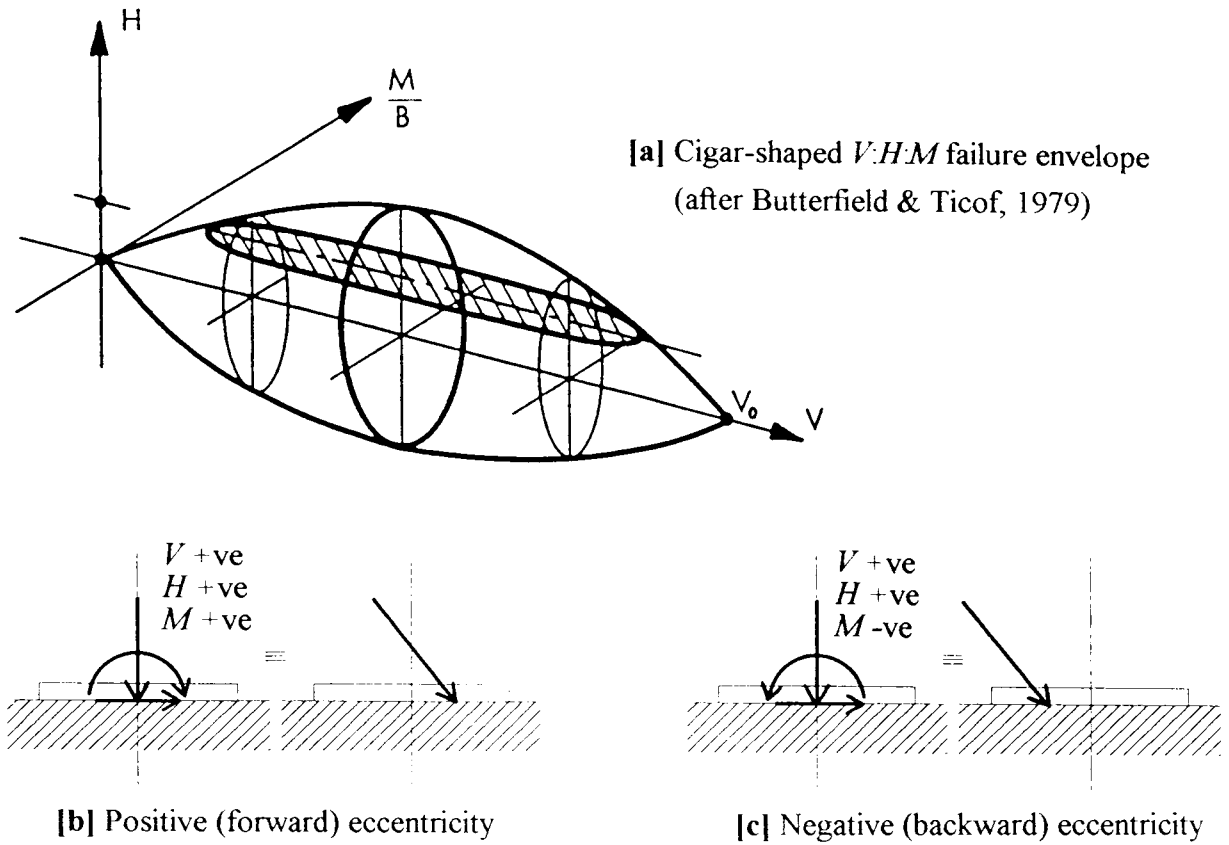
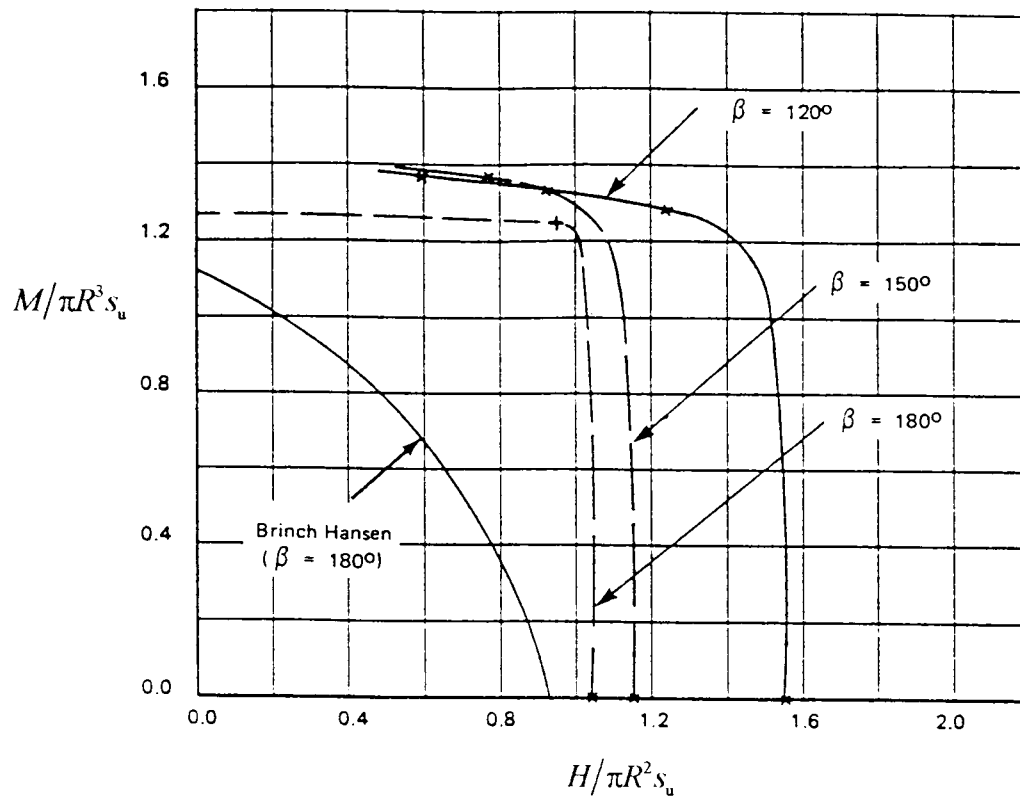
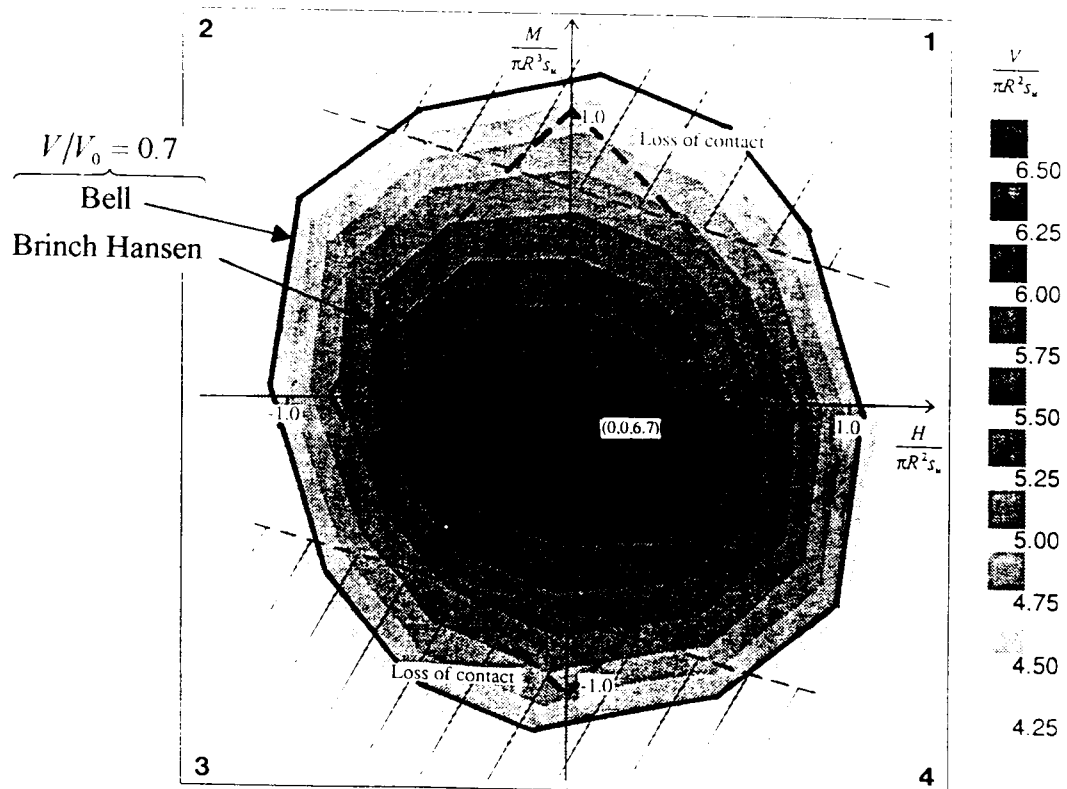


FIGURE 2.9

[d] $H/V_0:M/BV_0$ failure locus at $V/V_0 = 0.5$ for surface strip footing on dense sand (data from Gottardi & Butterfield, 1993)

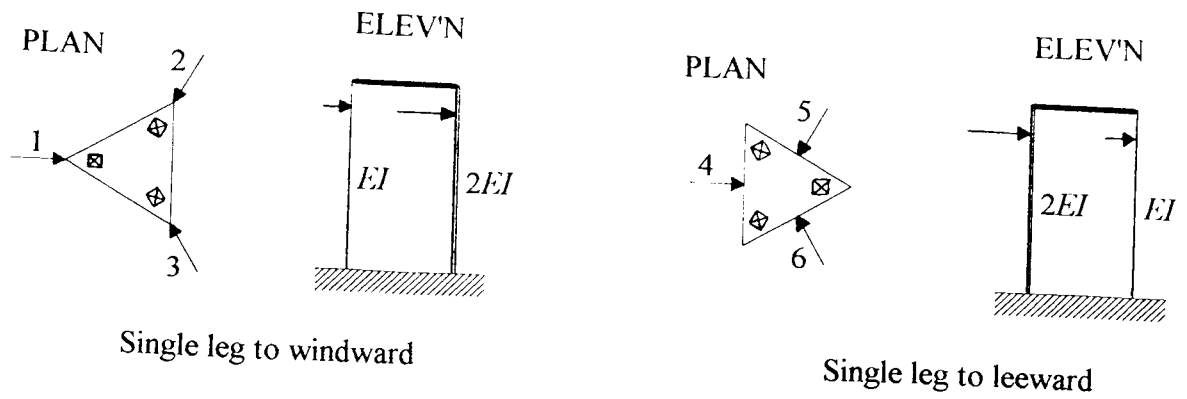


[a] H - M failure loci at $V/\pi R^2 s_u = 3.6$ ($V/V_0 \approx 0.6$) for conical footings (after Noble Denton and Associates, 1987)

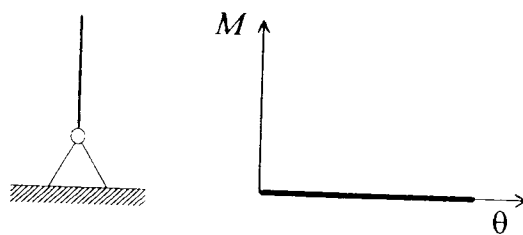


[b] V - H - M failure envelope contours for a flat circular footing (after Bell, 1991)

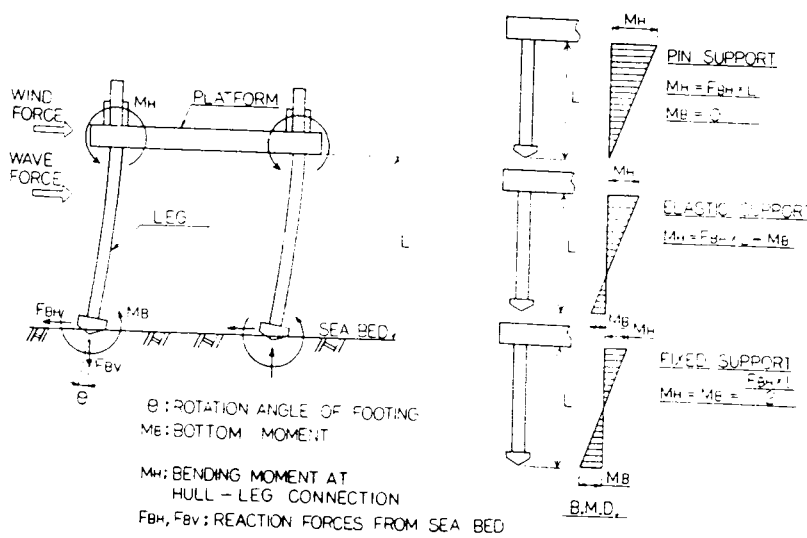
FIGURE 2.10 Investigations of combined load bearing capacity using finite element analysis



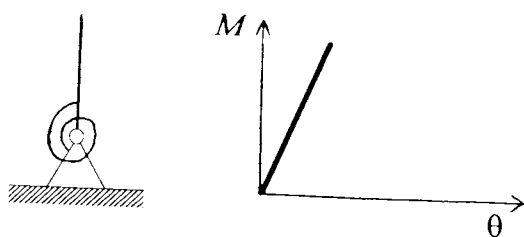
[a] Plane frame reductions for symmetrical loading directions



[b] Simple pin joint

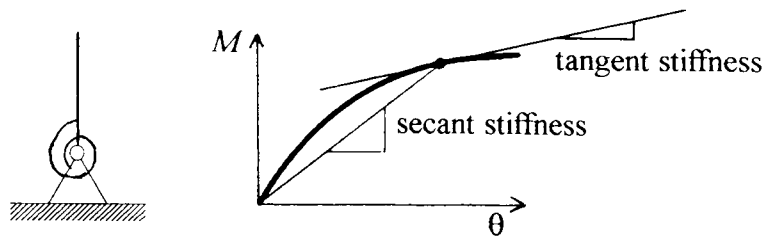


[c] Influence of rotational fixity on leg bending moments (after Chiba *et al.*, 1986)

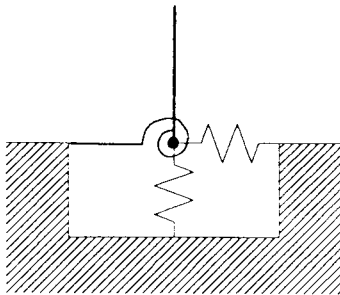


[d] Joint with linear elastic rotational spring

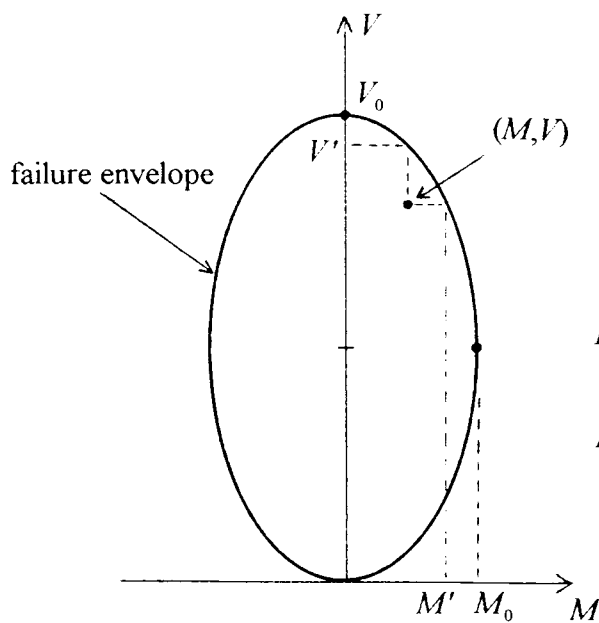
FIGURE 2.11 Two dimensional jack-up rig reductions; rotational foundation fixity



[a] Joint with nonlinear elastic rotational spring



[b] Joint with three nonlinear elastic springs



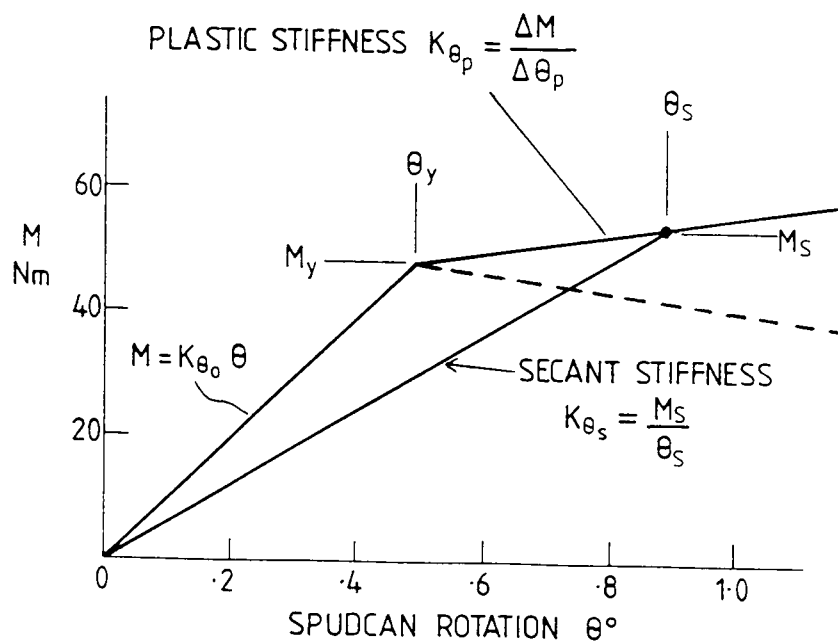
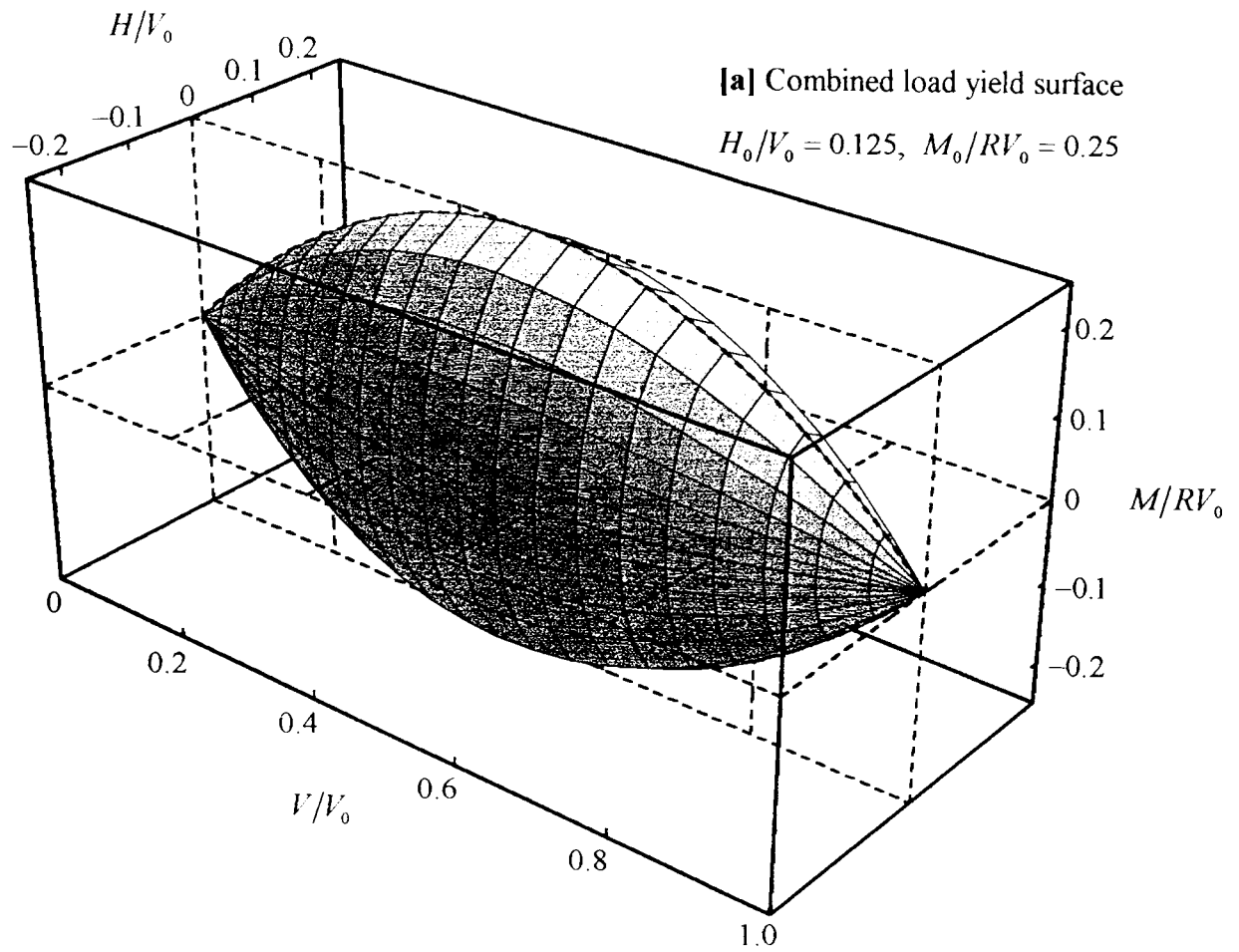
Nonlinear elastic stiffnesses

$$K_{V'} = K_{V'}^e \left(\frac{V' - V_0/2}{V_0/2} \right) \left(1 - \frac{V - V_0/2}{V' - V_0/2} \right)$$

$$K_M = K_M^e \left(\frac{M'}{M_0} \right) \left(1 - \frac{M}{M'} \right)$$

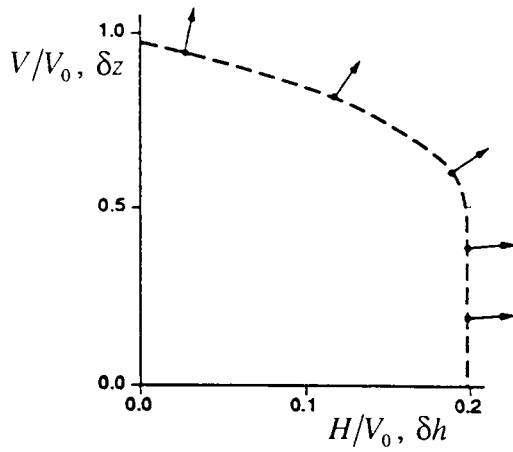
[c] Simple VM failure envelope and stiffness transition equations
(after Hambly *et al.*, 1990)

FIGURE 2.12 Nonlinear elastic footing models

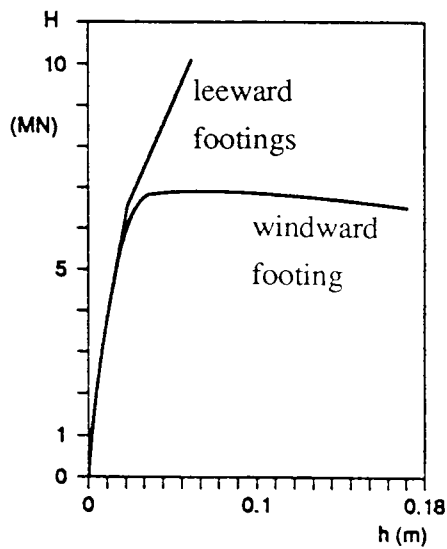


[b] Bilinear moment:rotation curve (after Noble Denton and Associates, 1987).

FIGURE 2.13 Calculation of design moment:rotation response for spudcan footings on sand (Noble Denton and Associates, 1987).



[a] incremental plastic displacement vectors at yield, from finite element analysis of a plane strain "spudcan" on clay



[b] Horizontal load:displacement responses obtained from structural analysis of a typical jack-up

FIGURE 2.14 Two plots from Schotman (1989)

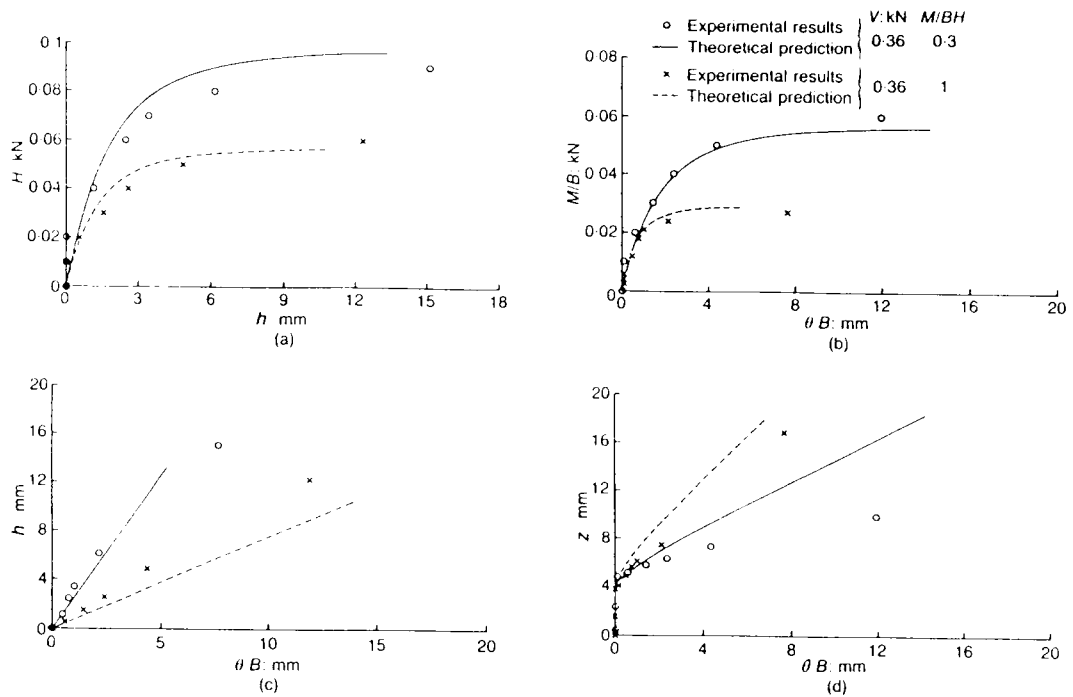


FIGURE 2.15 Experimental results and numerical predictions for load controlled tests of a strip footing on sand (after Nova & Montrasio, 1991)

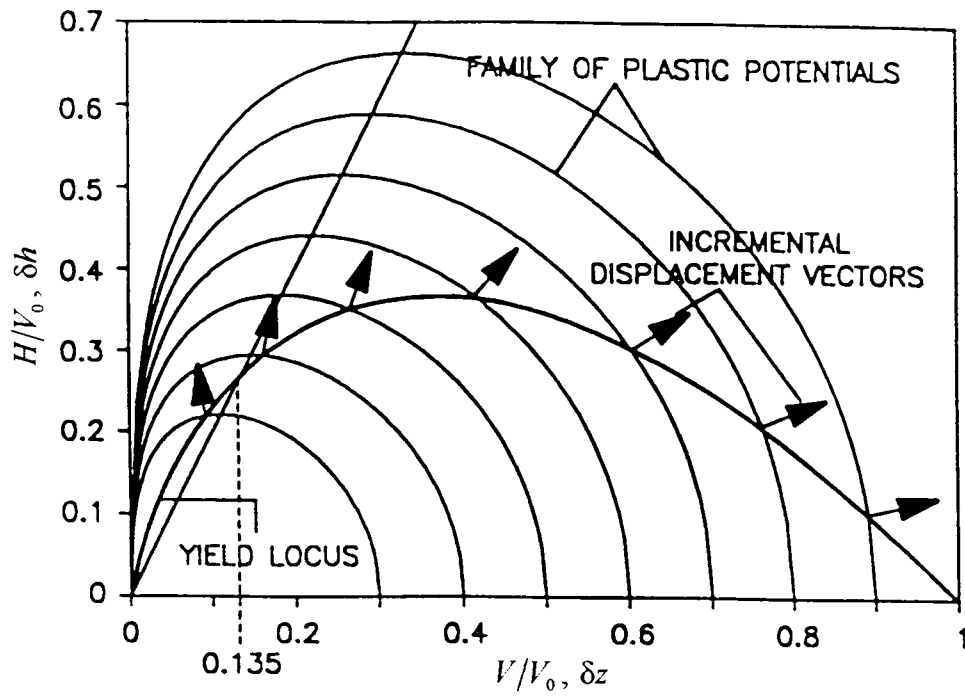


FIGURE 2.16 Proposed form of $V:H$ yield locus and plastic potential for conical or spudcan footings on sand (after Tan, 1990)

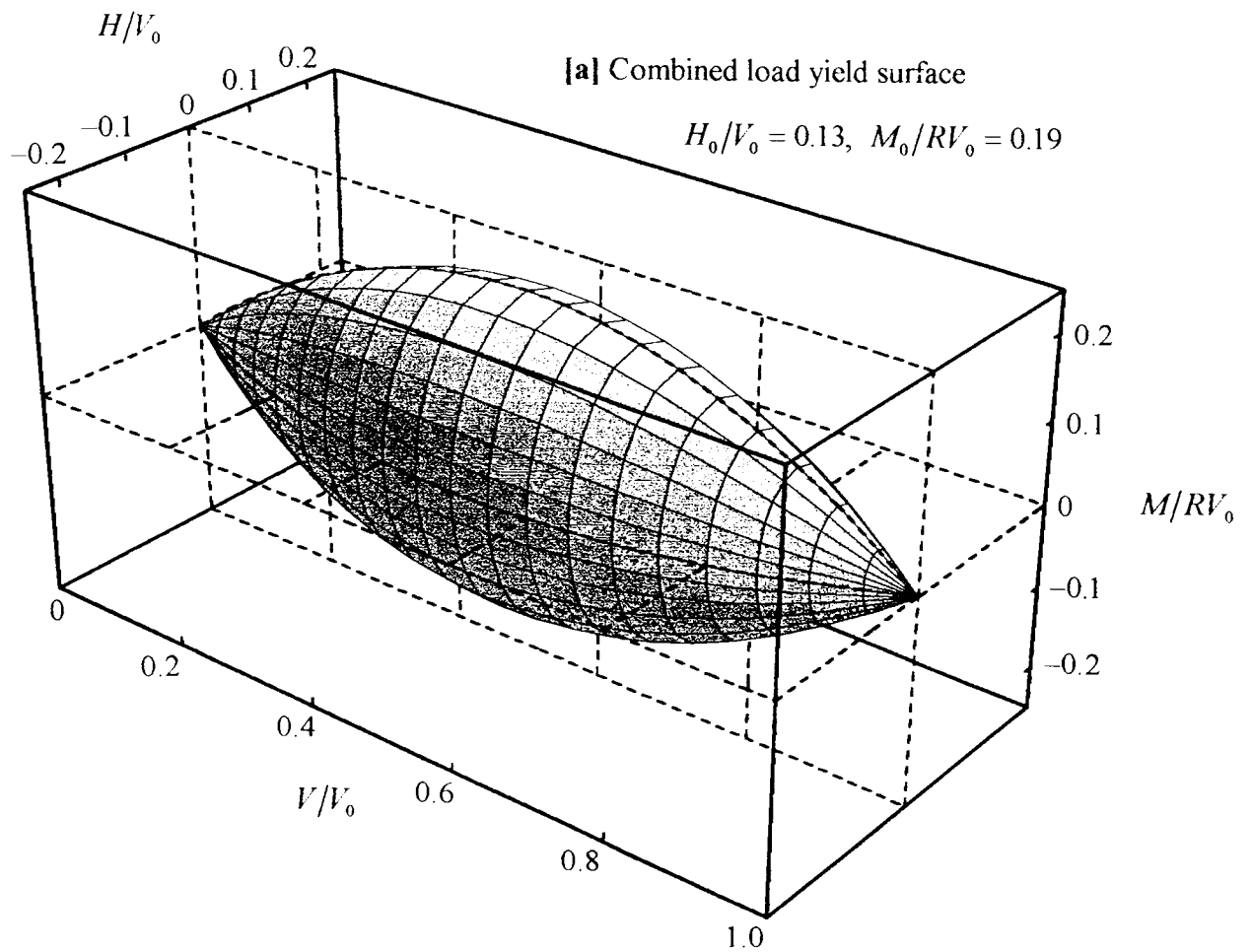


FIGURE 2.17 Features of Model A

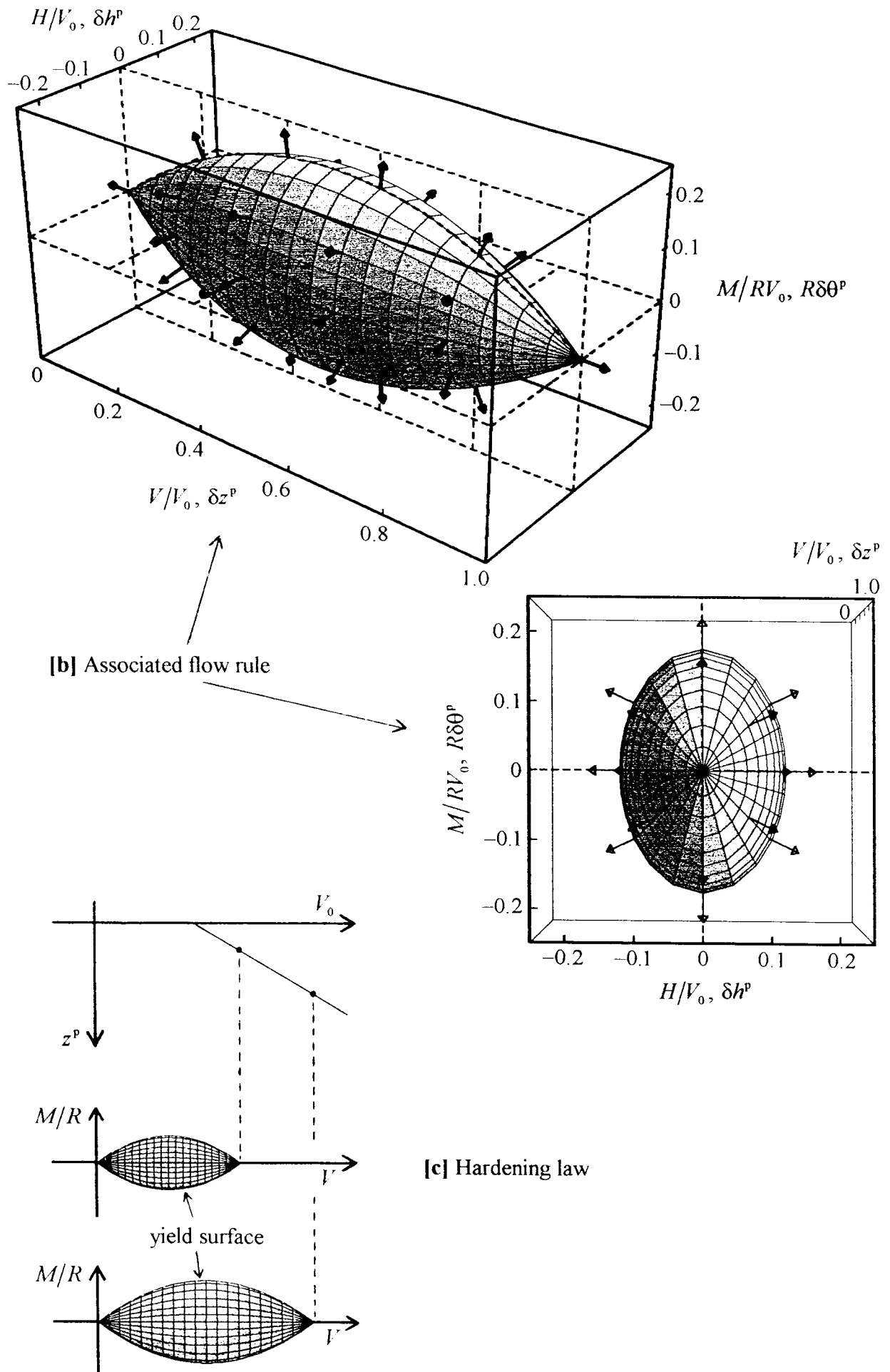
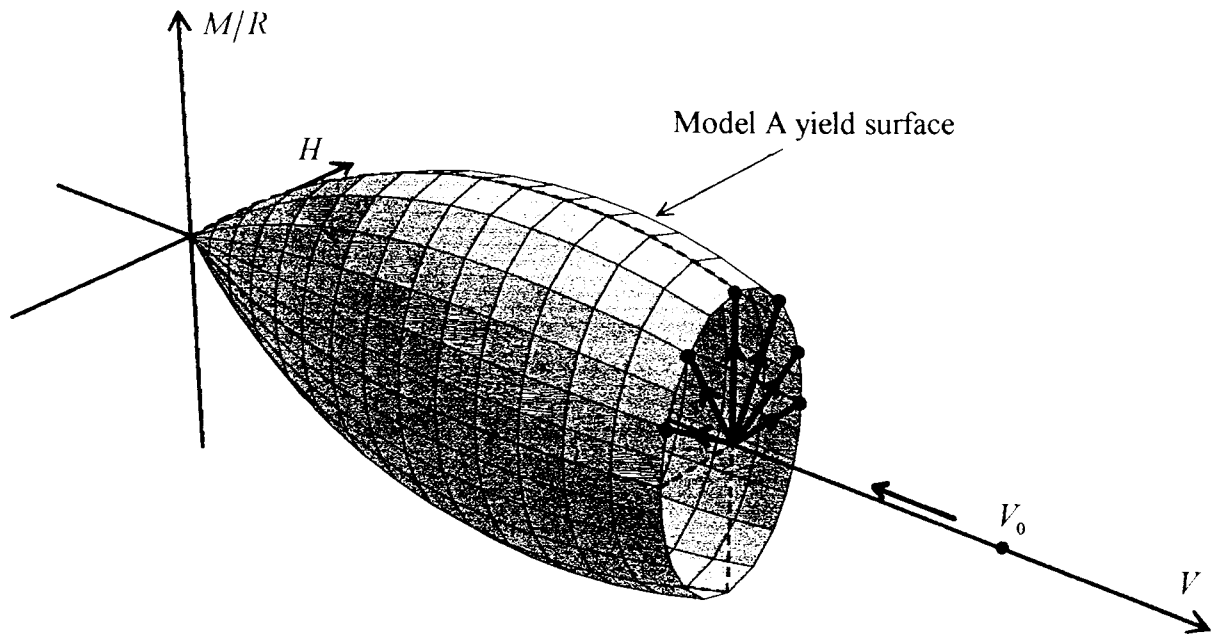
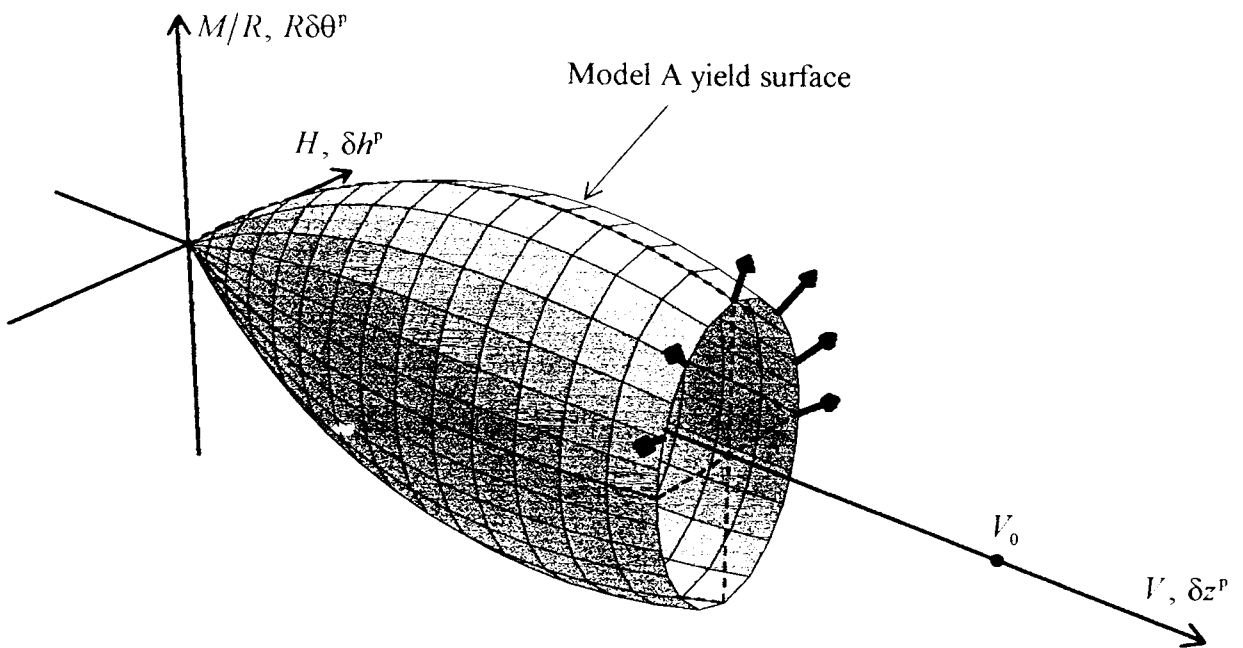


FIGURE 2.17 Features of Model A (continued)



[a] Determination of points on yield surface (one point per probing test)



[b] Incremental plastic displacement vectors at yield

FIGURE 2.18 Schematic representation of probing tests at $V/V_0 = 0.7$

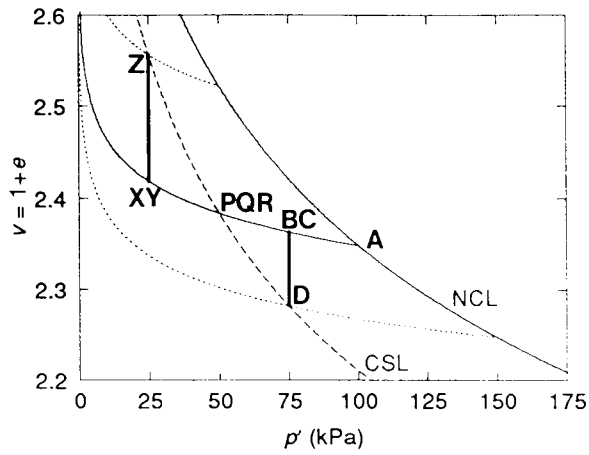
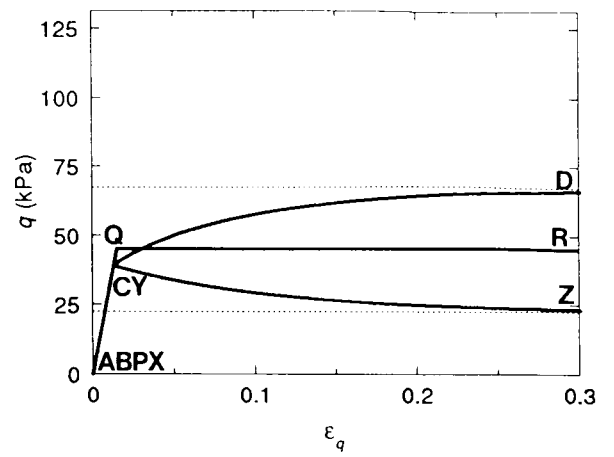
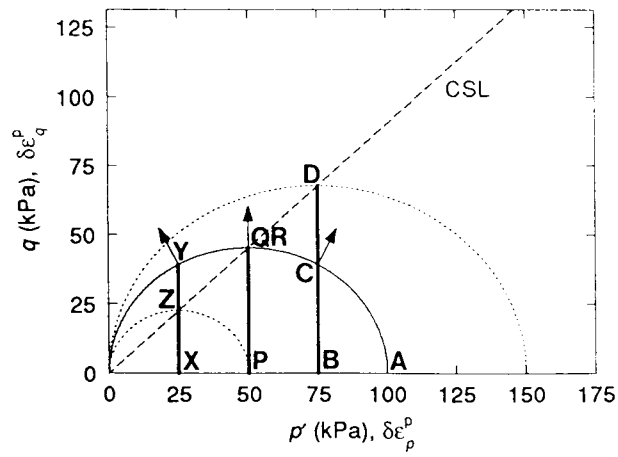


FIGURE 2.19 [a]

Drained constant p' triaxial tests
on a Modified Cam-Clay soil
($\lambda = 0.25$, $\kappa = 0.05$, $M = 0.9$,
 $N = 3.50$, $G' = 1000$ kPa)

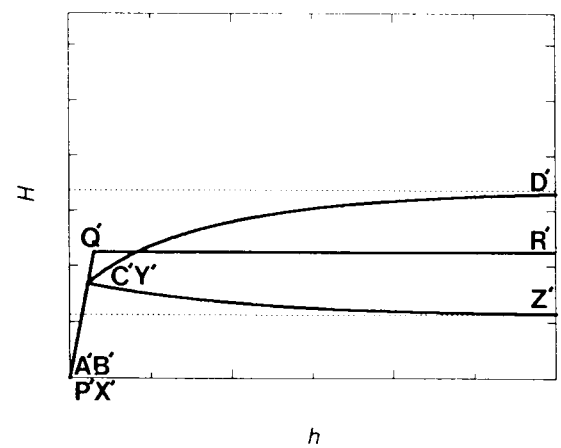
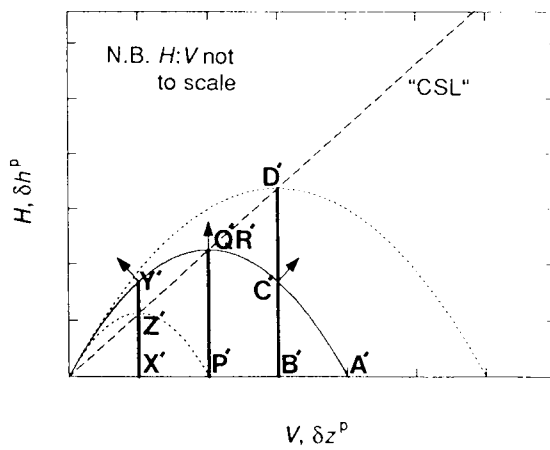
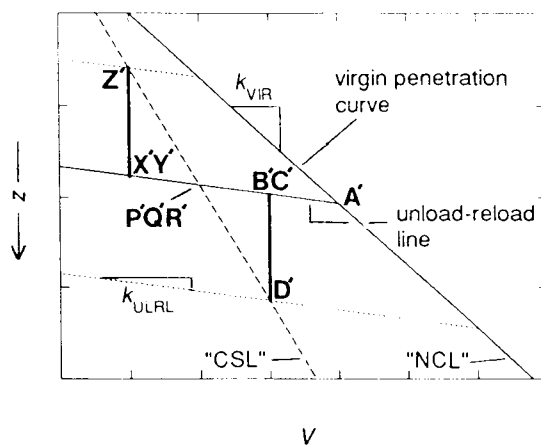


FIGURE 2.19 [b]

Schematic representation of probing
tests on a Model A footing
(h at constant V , $k_{ULRL}/k_{VIR} = 5$)



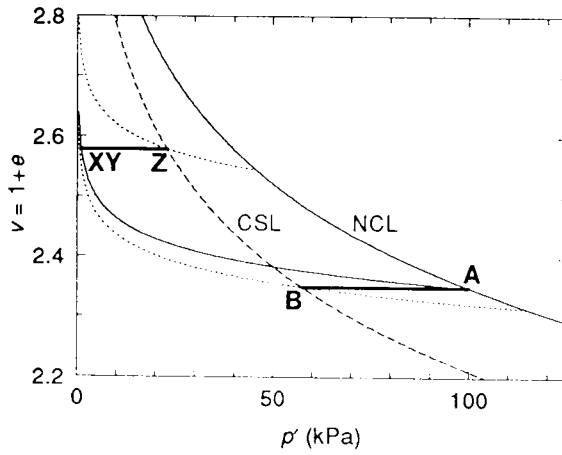
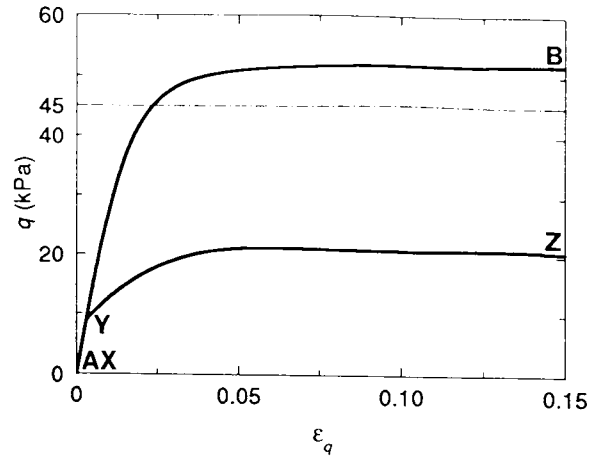
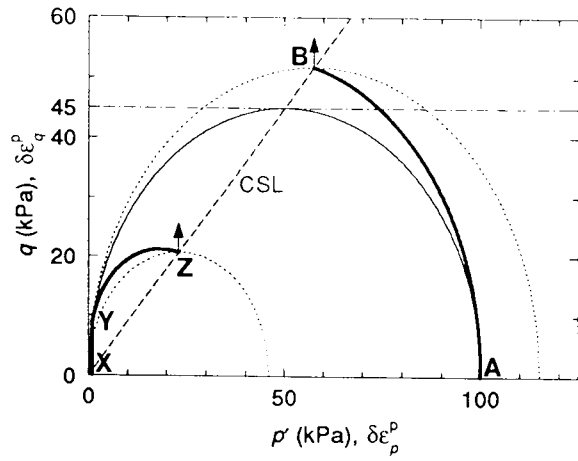


FIGURE 2.20 [a]

Undrained triaxial tests on a Modified Cam-Clay soil with $\lambda/\kappa = 5$
 $(\lambda = 0.25, \kappa = 0.05, M = 0.9,$
 $N = 3.50, G' = 1000 \text{ kPa})$

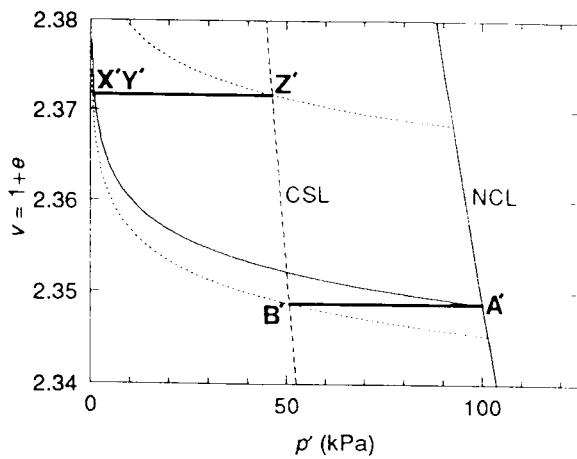
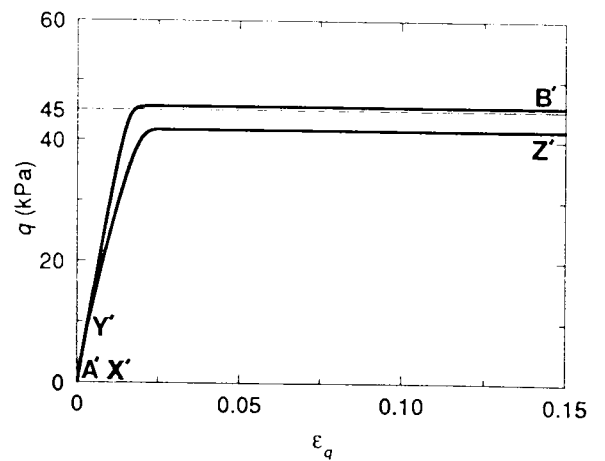
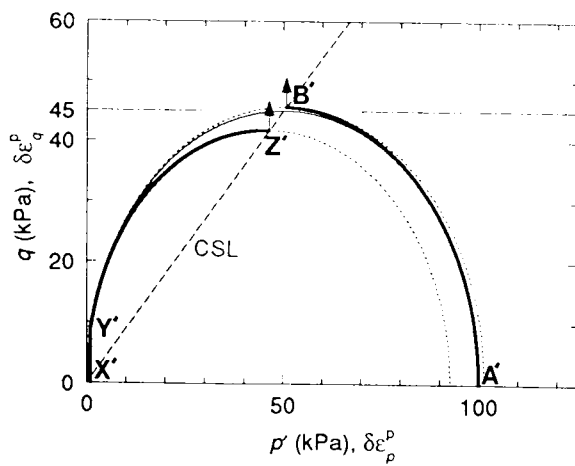
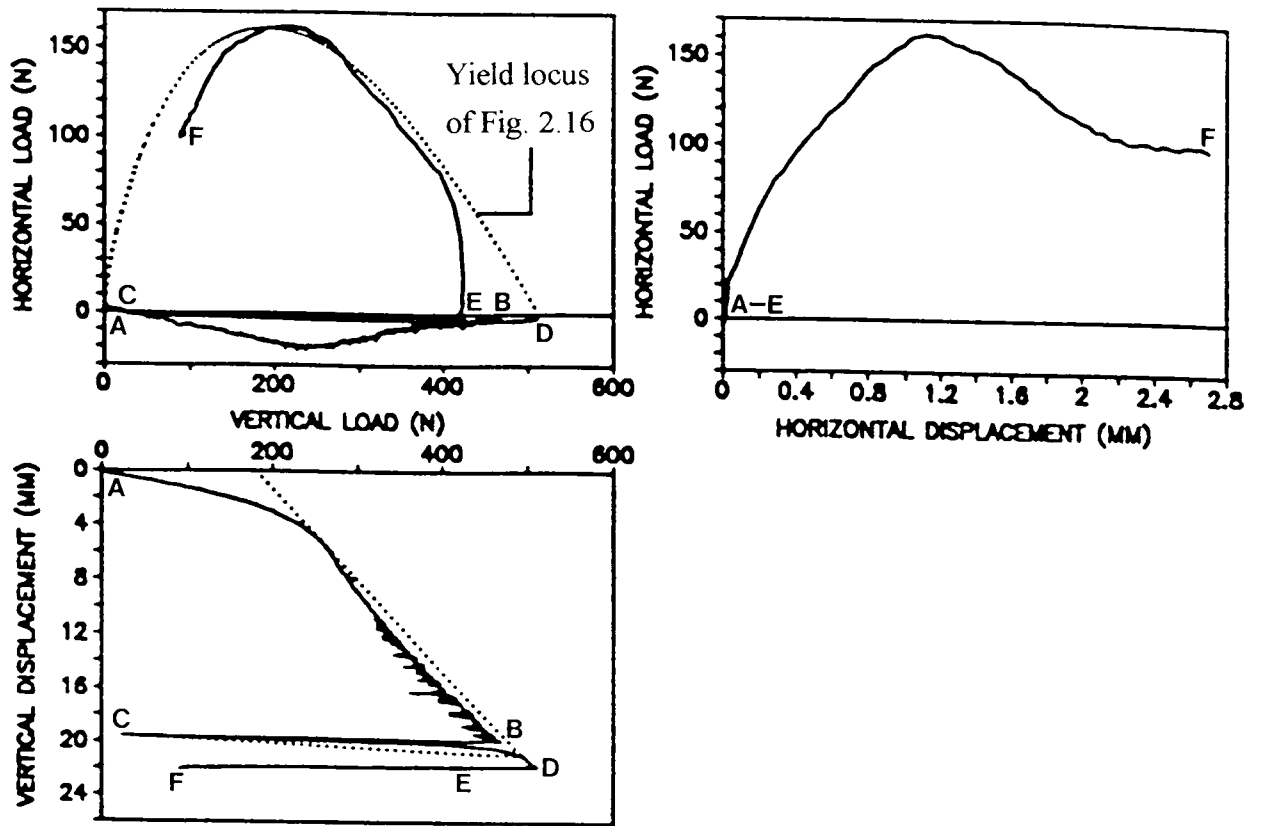
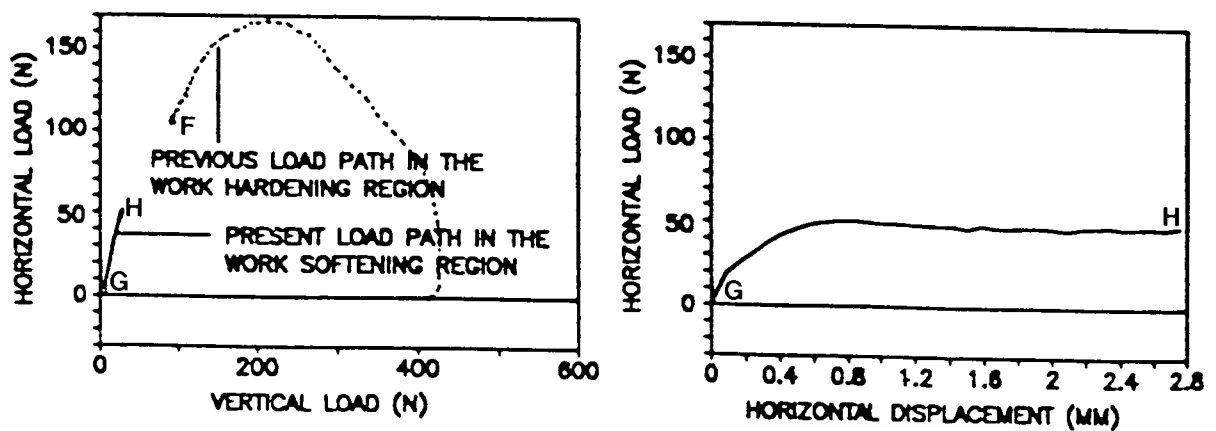


FIGURE 2.20 [b]

Undrained triaxial tests on a Modified Cam-Clay soil with $\lambda/\kappa = 50$
 $(\lambda = 0.25, \kappa = 0.005, M = 0.9,$
 $N = 3.50, G' = 1000 \text{ kPa})$



[a] Initial penetration of footing, followed by sideswipe test from high V/V_0



[b] Sideswipe test from low V/V_0

FIGURE 2.21 Sideswipe (h at constant z) test results for a flat circular footing on sand (after Tan, 1990)

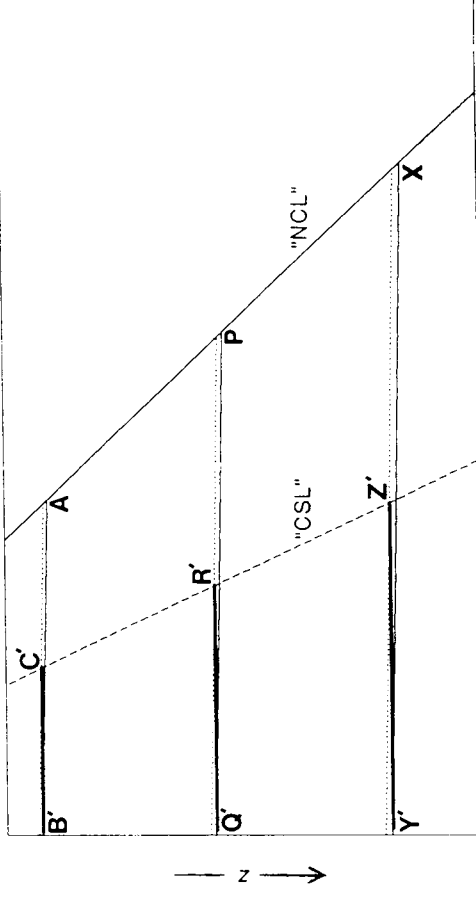
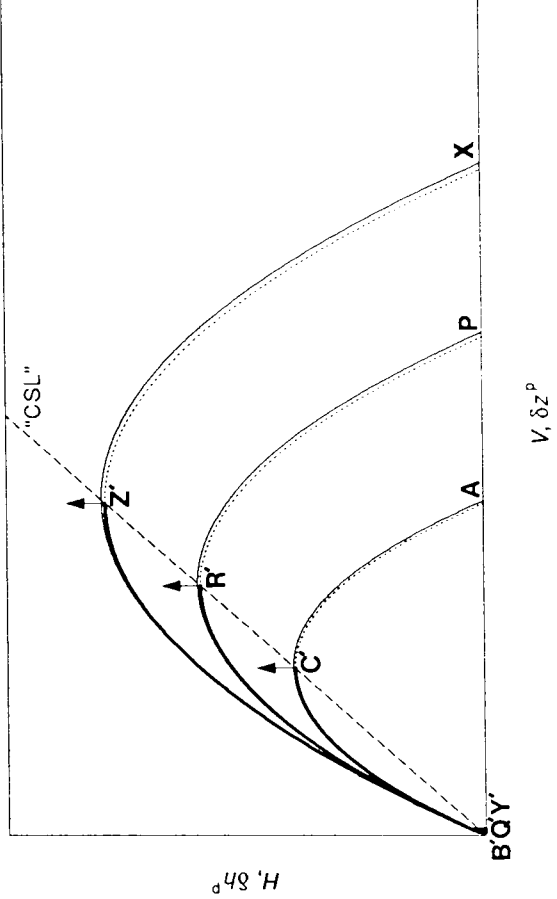


FIGURE 2.22 [a]

Schematic representation of sideswipe tests on a Model A footing (h at constant z , commencing from $V/V_0 = 1.0$; $k_{ULRL}/k_{VIR} = 50$)

FIGURE 2.22 [b]

Schematic representation of sideswipe tests on a Model A footing (h at constant z , commencing from $V/V_0 = 0.01$; $k_{ULRL}/k_{VIR} = 50$)

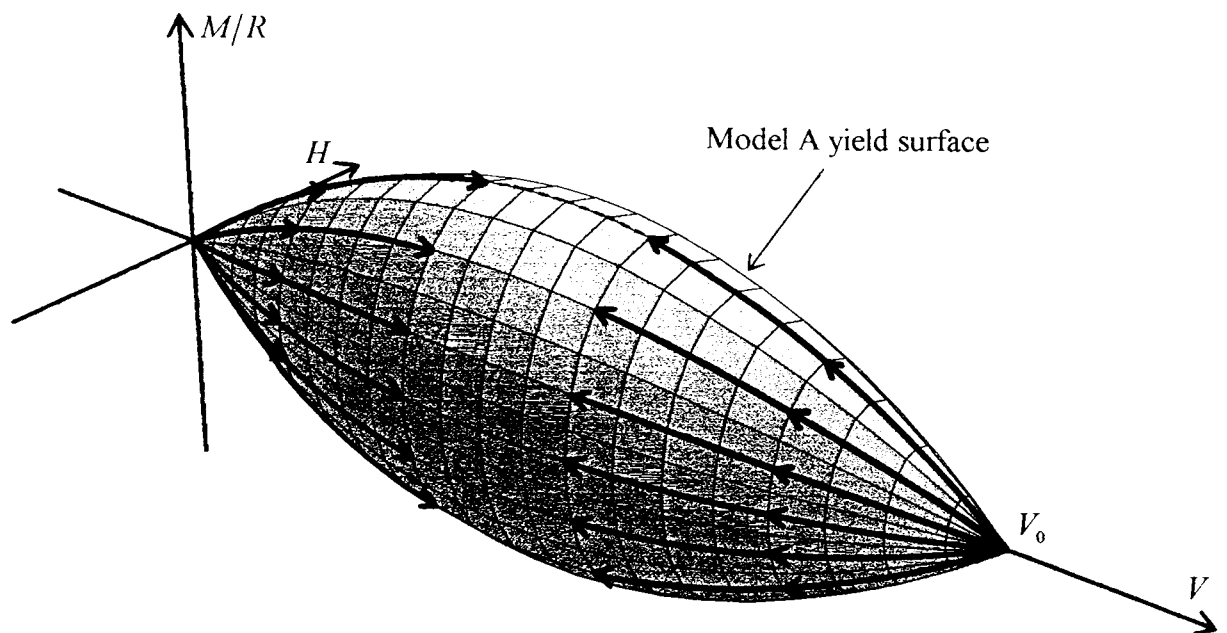


FIGURE 2.23 Schematic representation of tracking (constant z) tests commencing from $V/V_0 = 1.0$ and from very small V/V_0

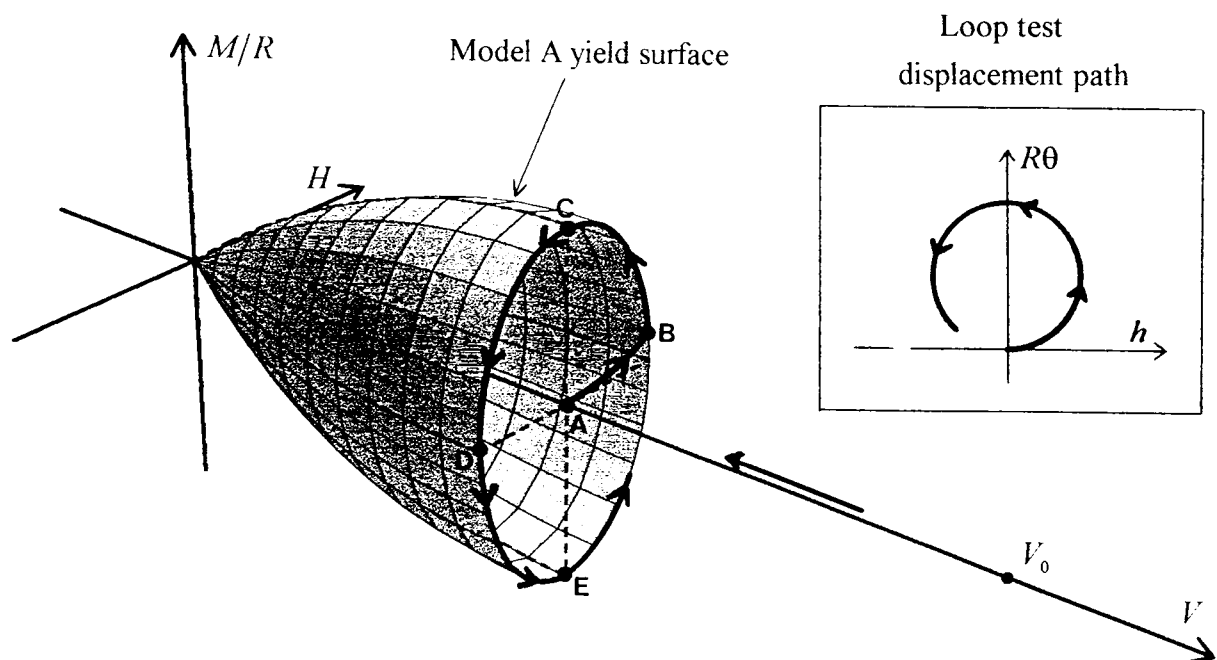


FIGURE 2.24 Schematic representation of loop test at $V/V_0 = 0.5$

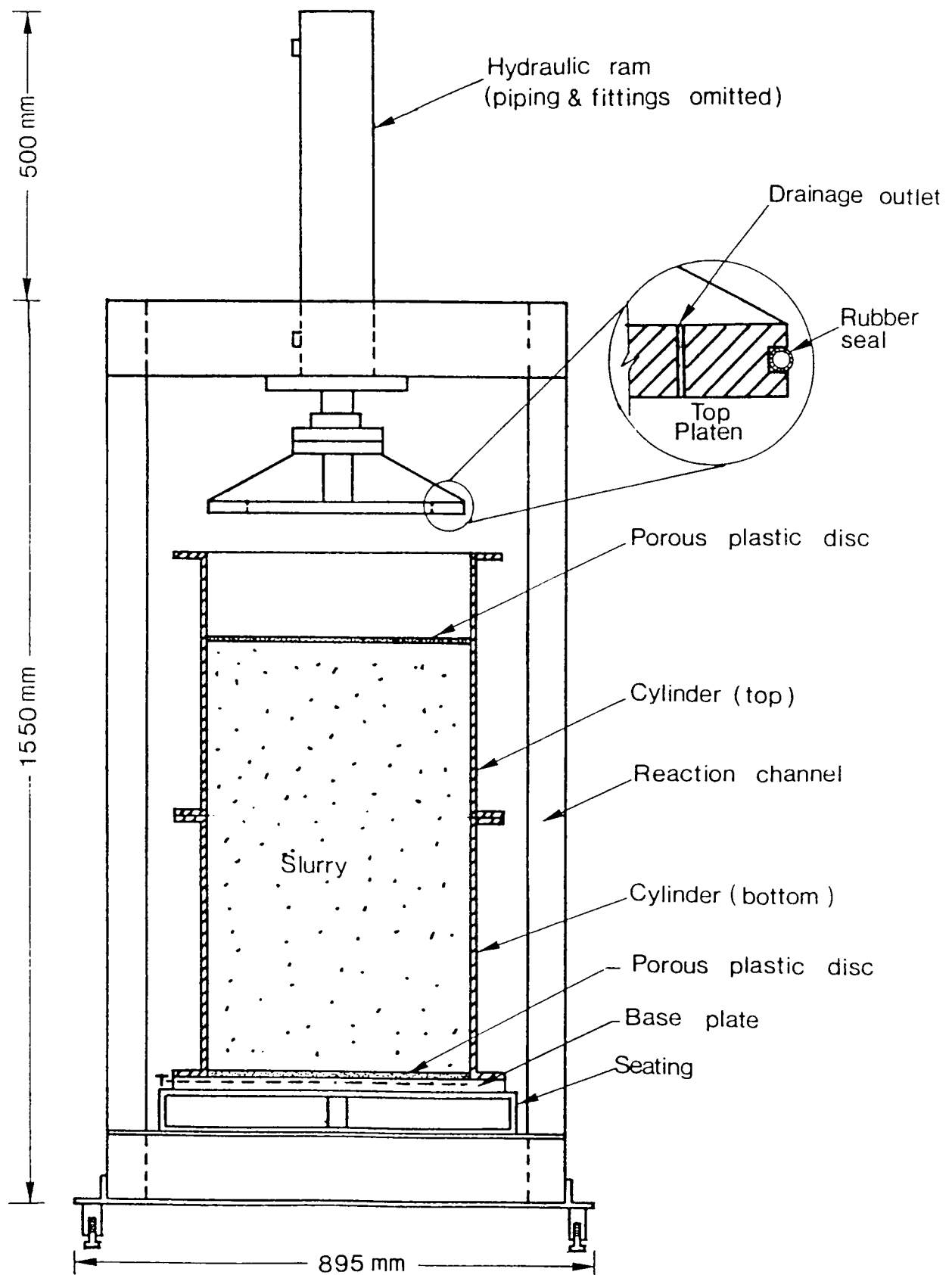


FIGURE 3.1 Clay consolidation apparatus: one of the three identical units
(after Gue, 1984)

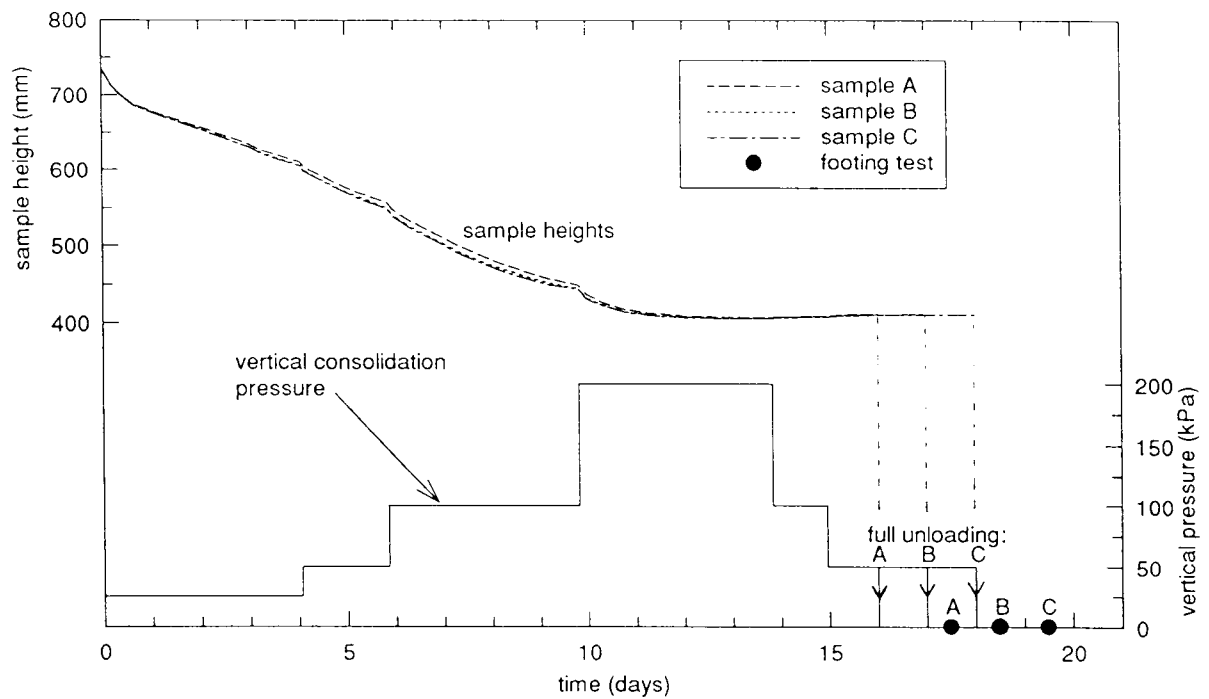


FIGURE 3.2 Consolidation and unloading schedule for a typical batch

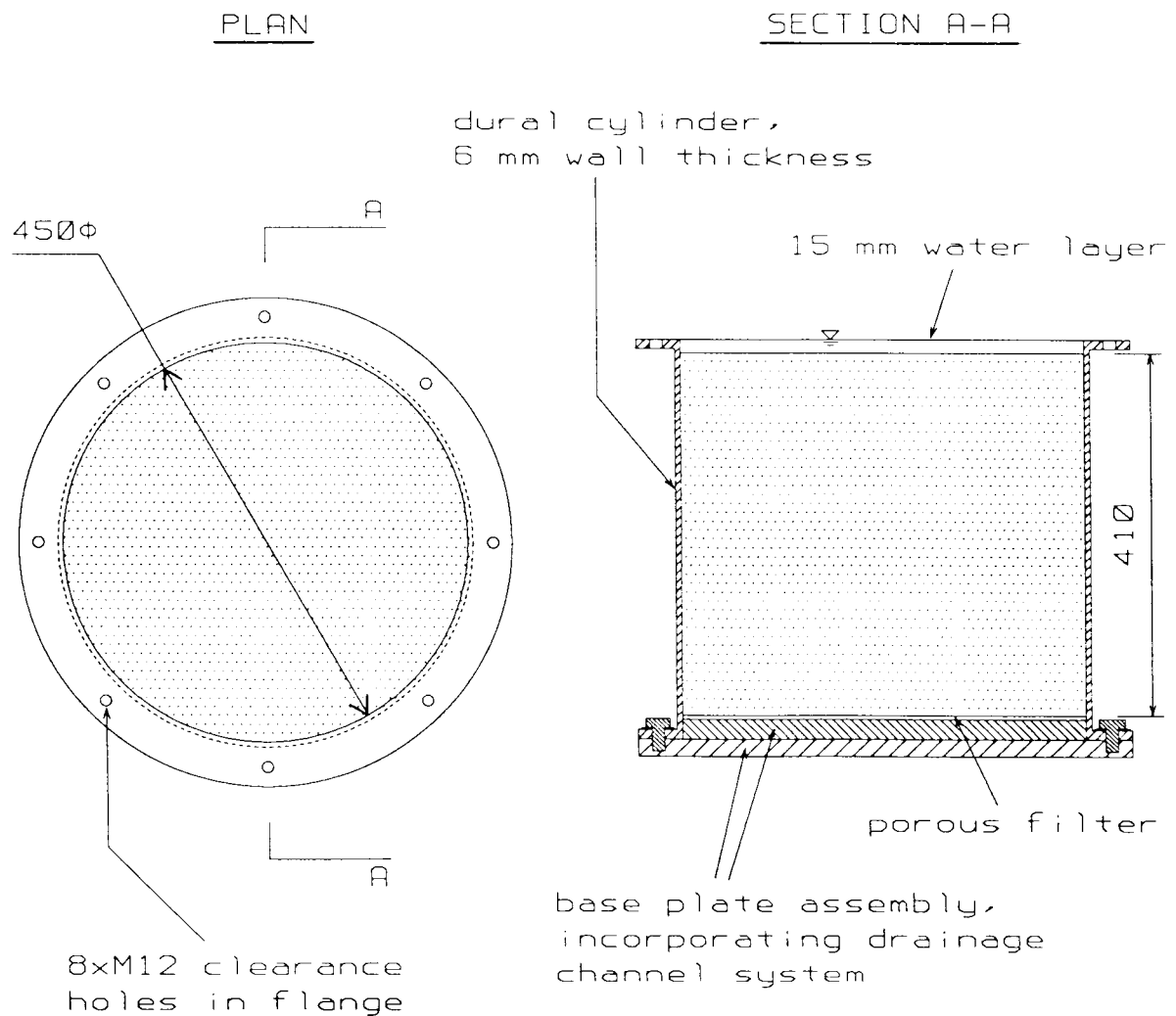


FIGURE 3.3 Kaolin sample ready for a model footing test

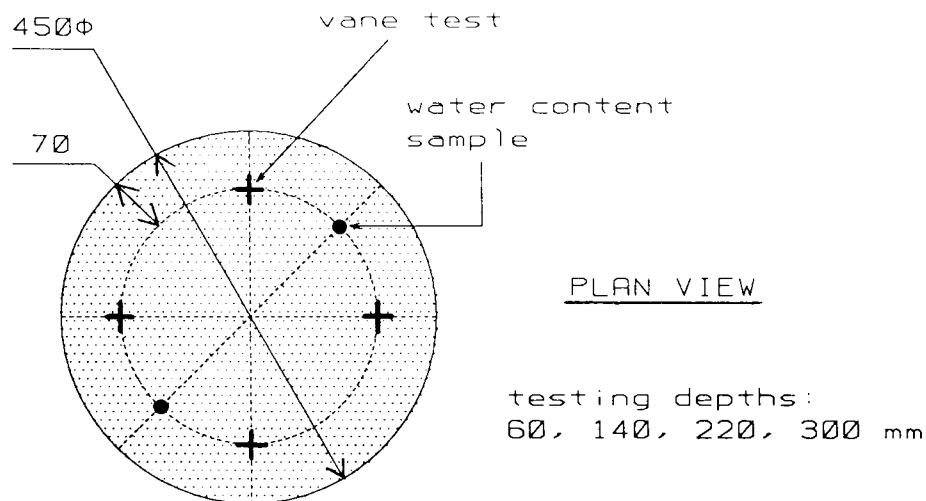


FIGURE 3.4 [a] Miniature site investigation layout

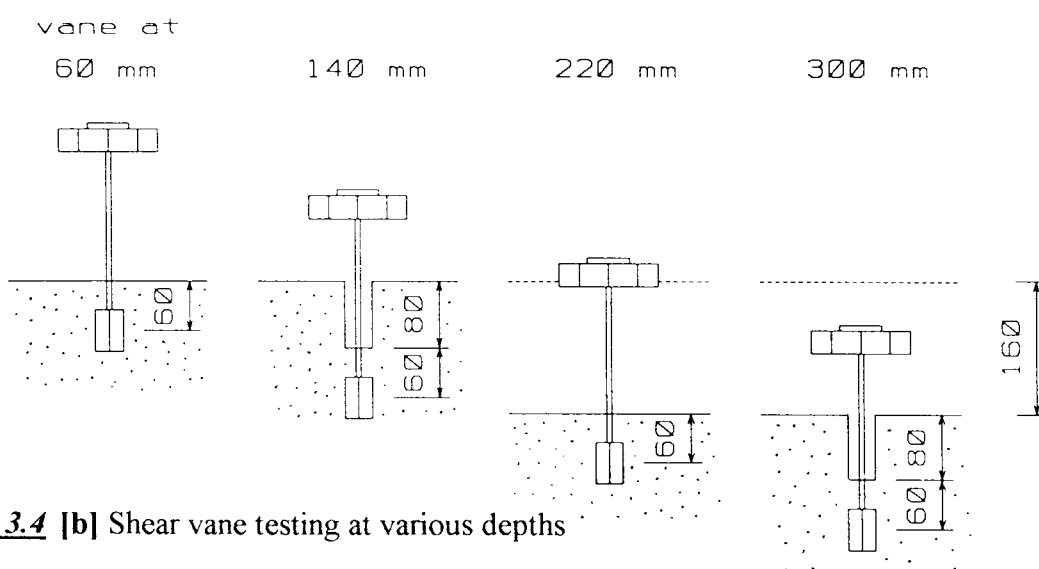


FIGURE 3.4 [b] Shear vane testing at various depths

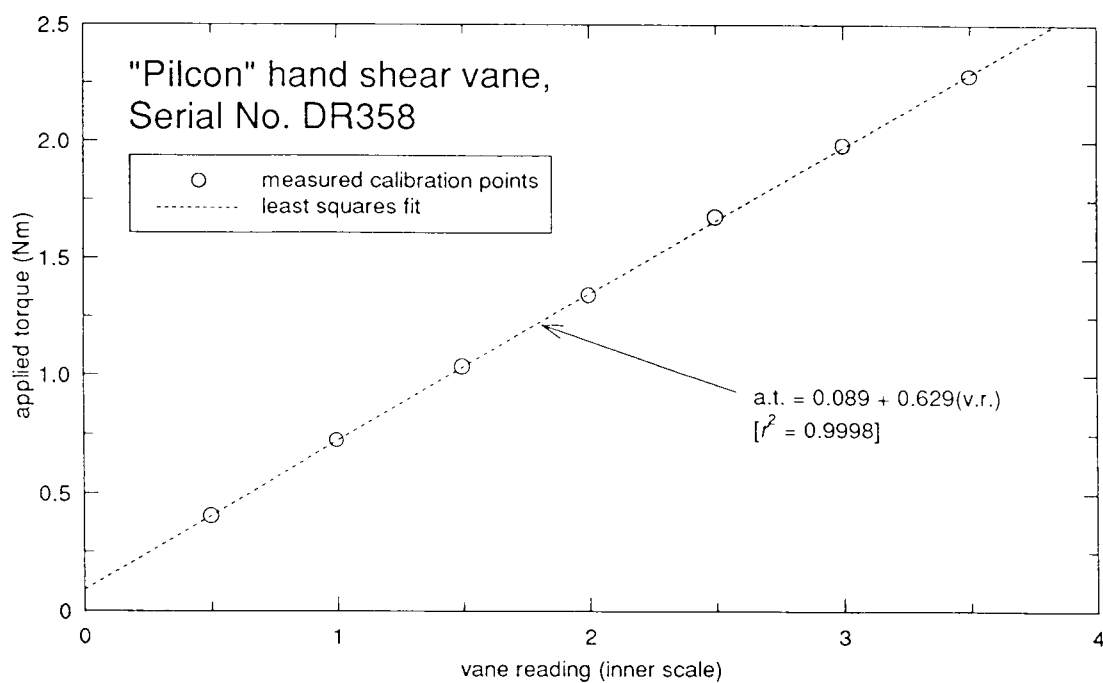


FIGURE 3.5 Shear vane calibration

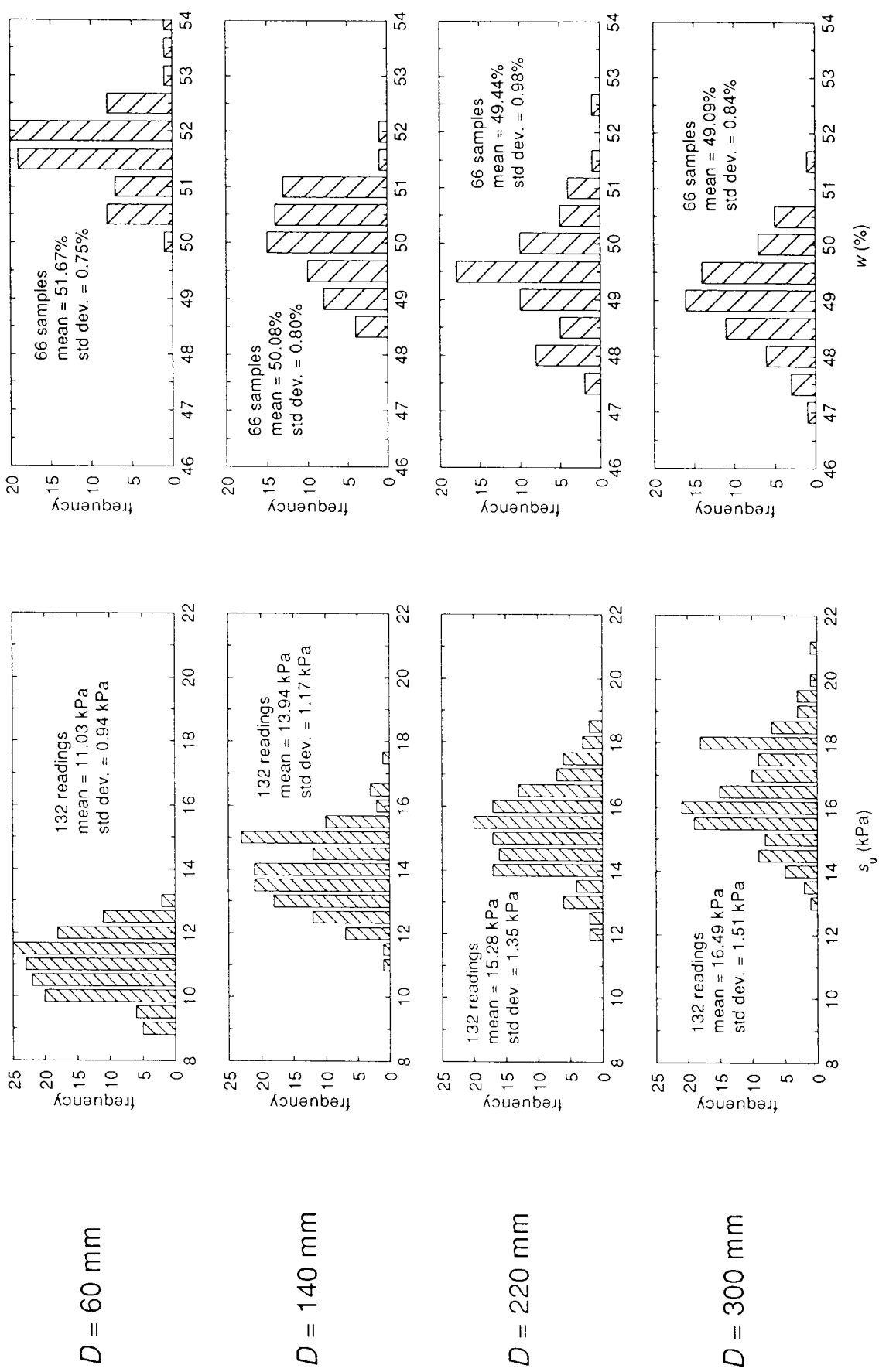


FIGURE 3.6 [a] Variation of undrained strength with depth [b] Variation of water content with depth

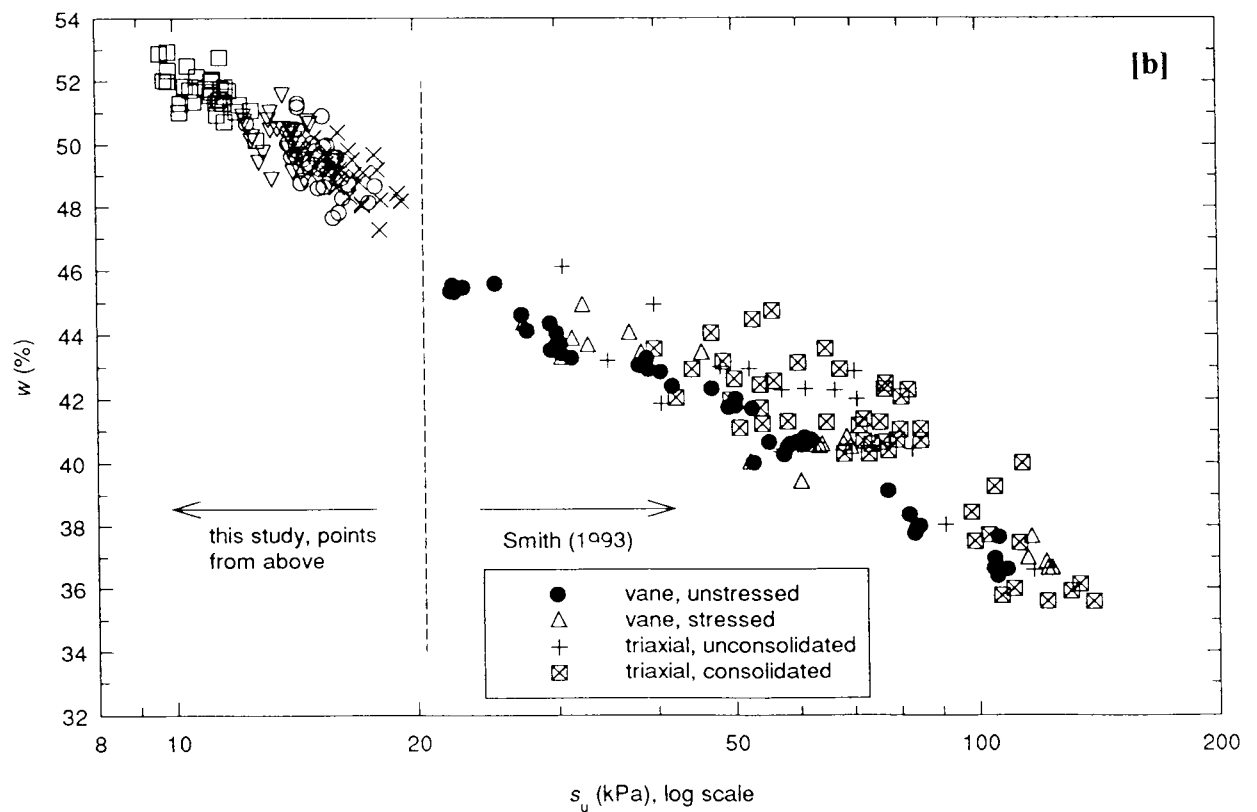
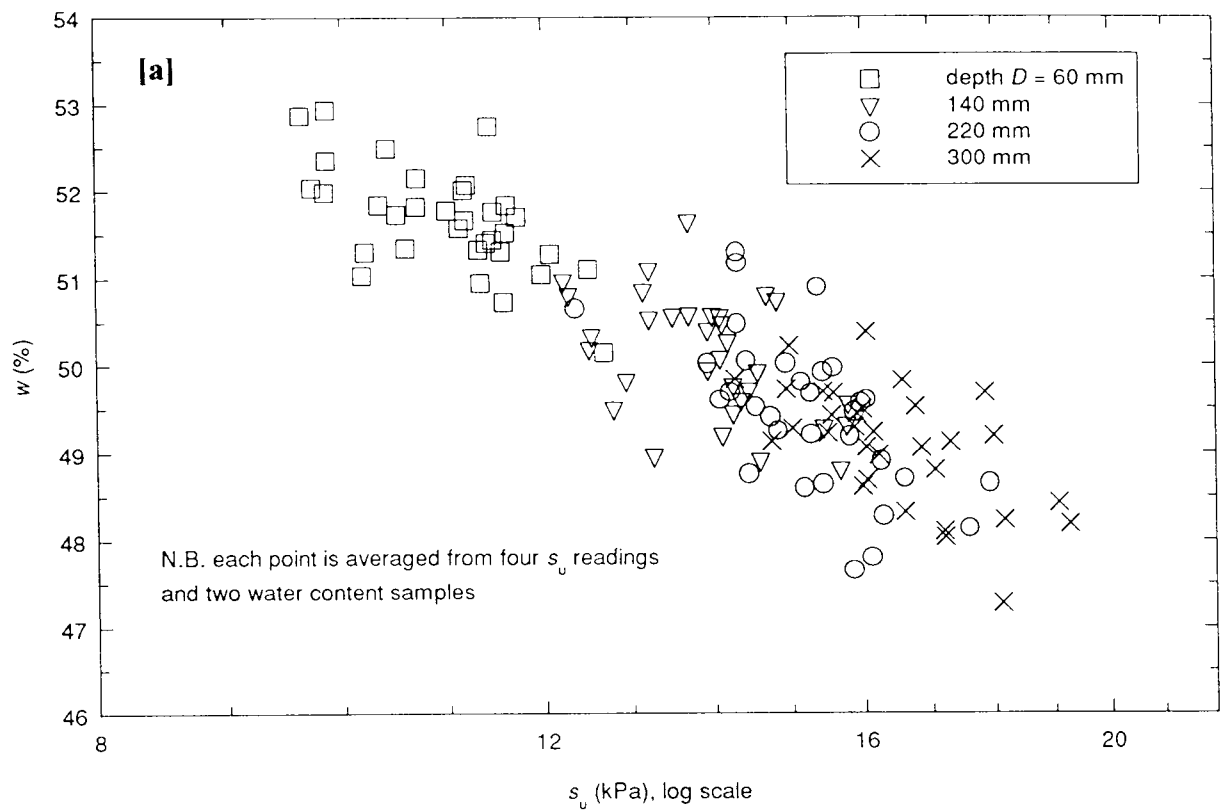


FIGURE 3.7 [a] Undrained strengths and corresponding water contents
[b] Comparison with w : s_u data from Smith (1993)

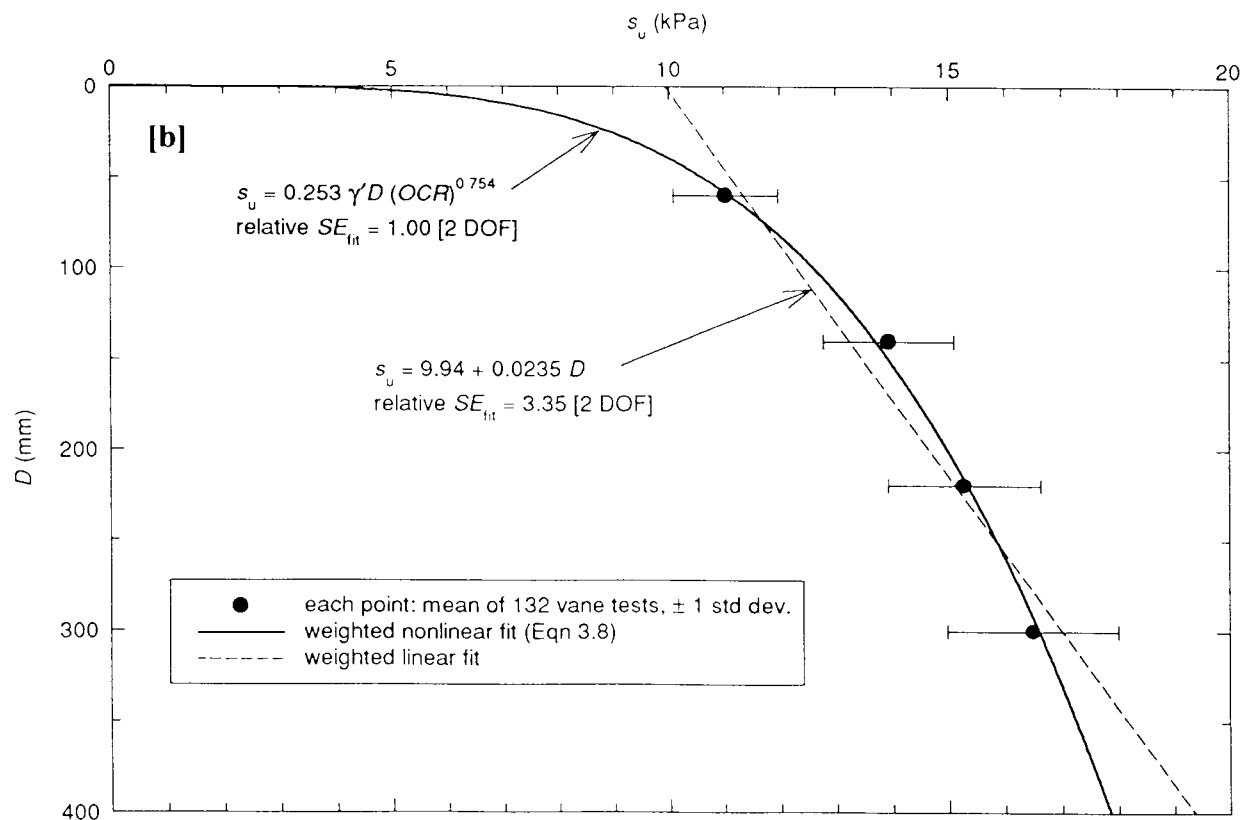
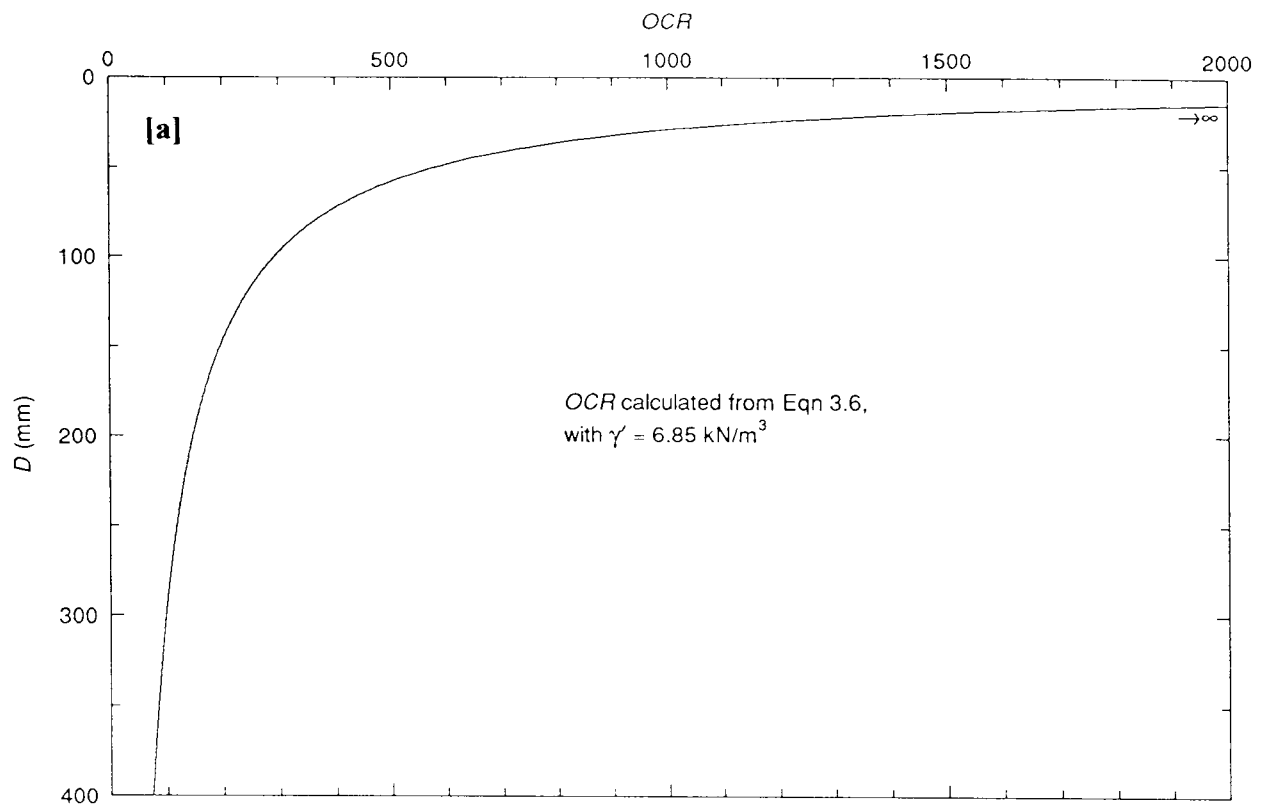
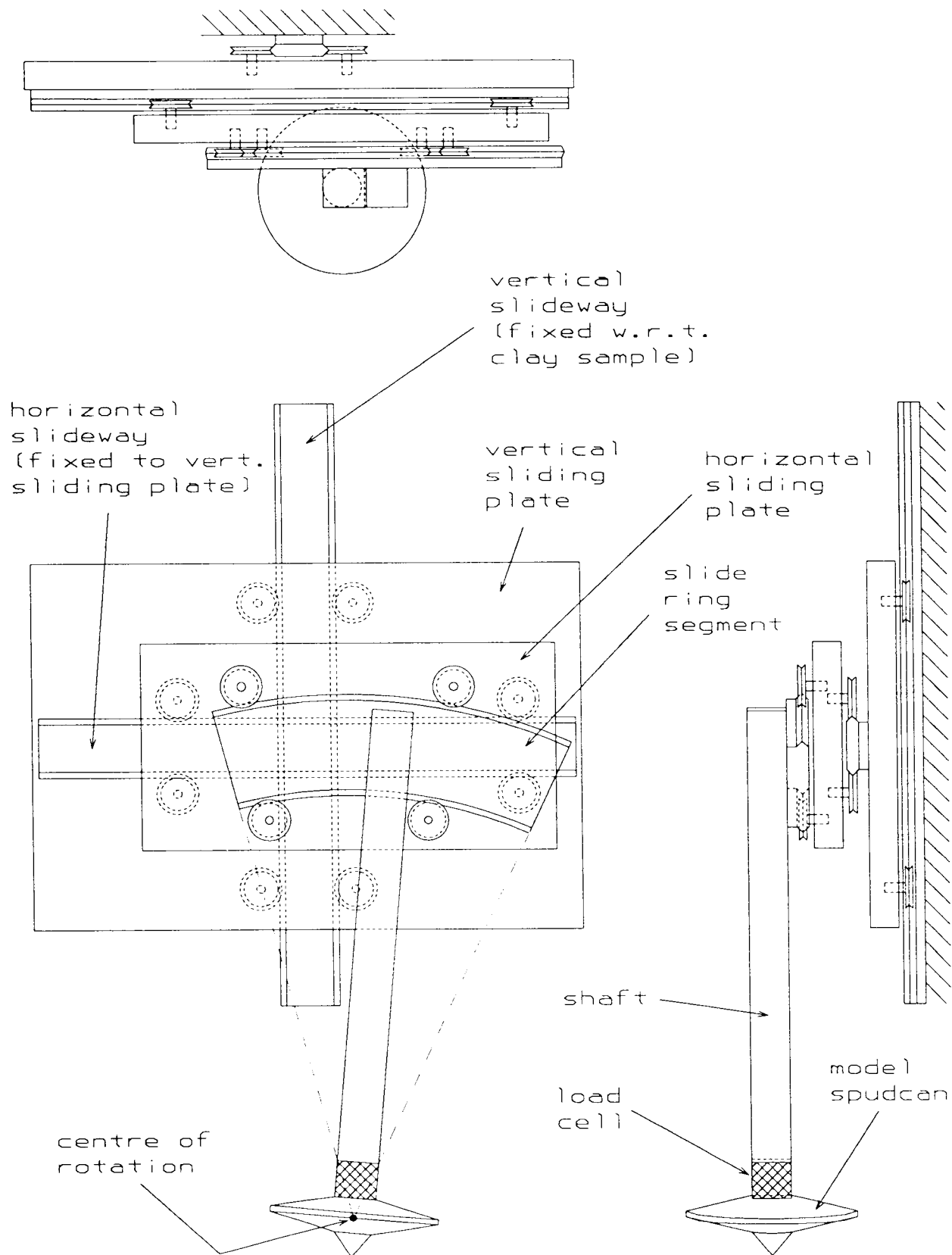


FIGURE 3.8 [a] Variation of OCR with depth

[b] Curve fitting of measured undrained strength profile



N.B. some hidden lines omitted for clarity

FIGURE 3.9 Concept design of experimental apparatus: model footing with independently controlled vertical, horizontal and rotational displacements.

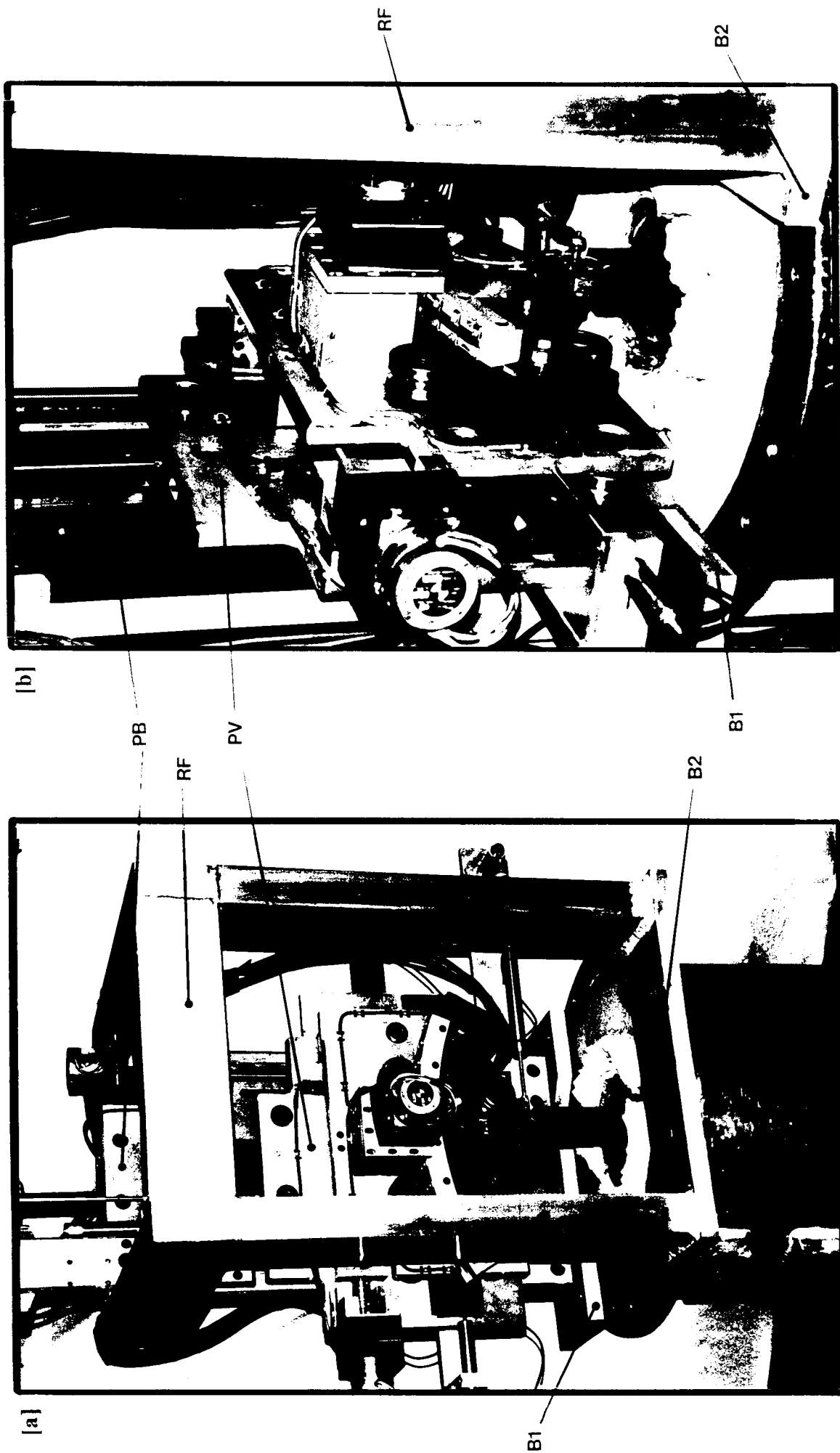


FIGURE 3.10 Overall views of experimental apparatus during a footing test

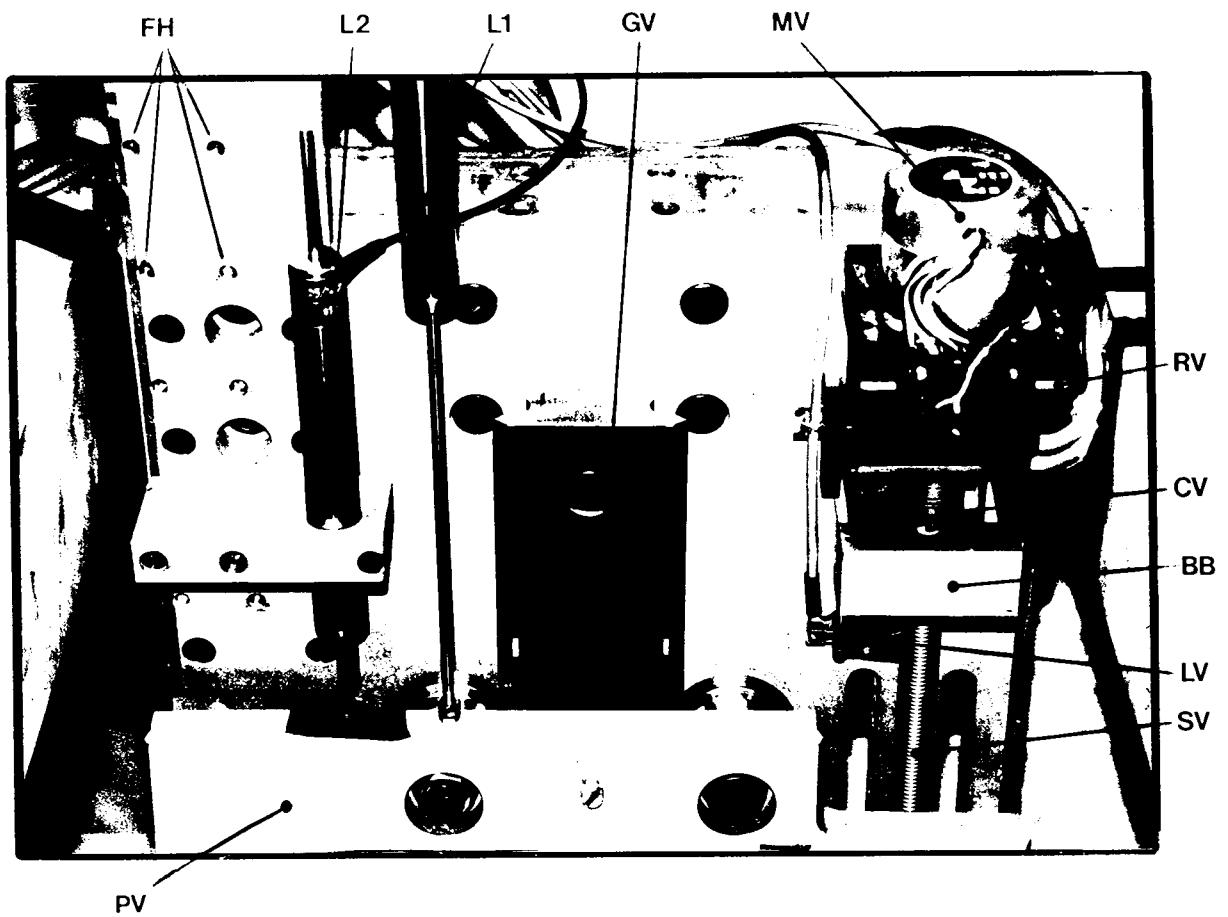


FIGURE 3.11 [a] Vertical displacement control system

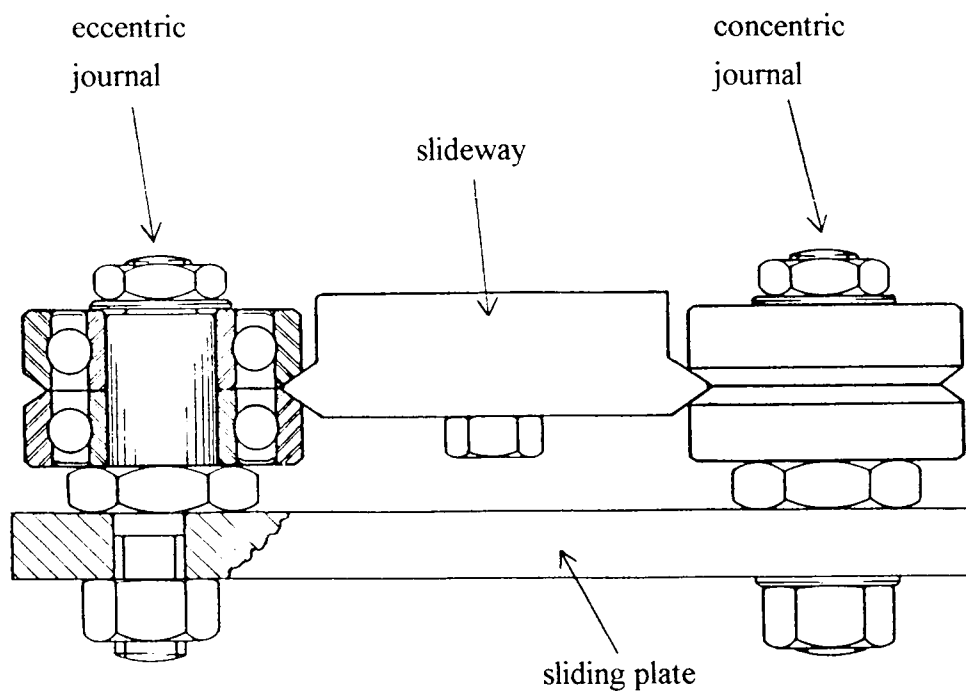


FIGURE 3.11 [b] "Hepco" linear bearing system (drawing from manufacturer's literature)

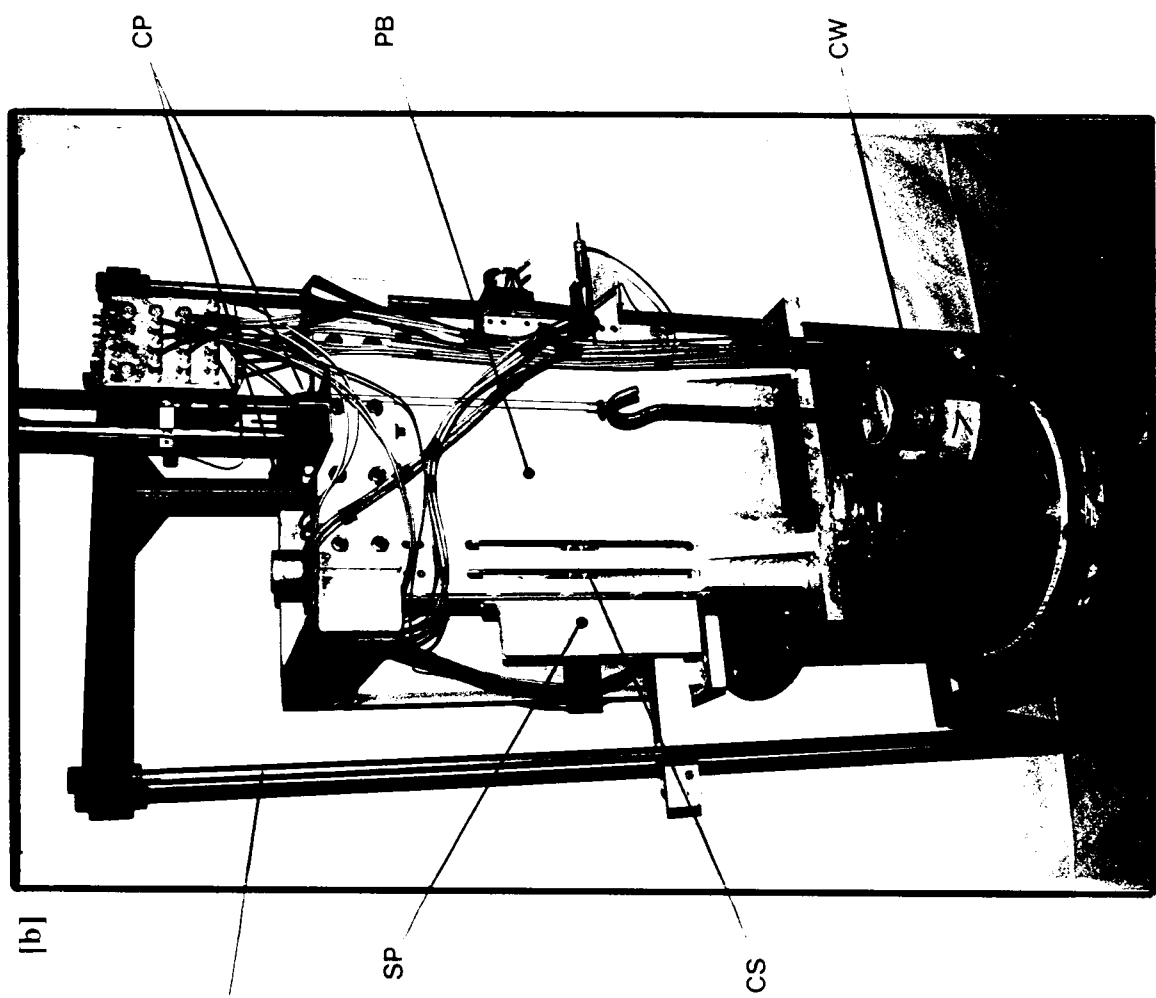
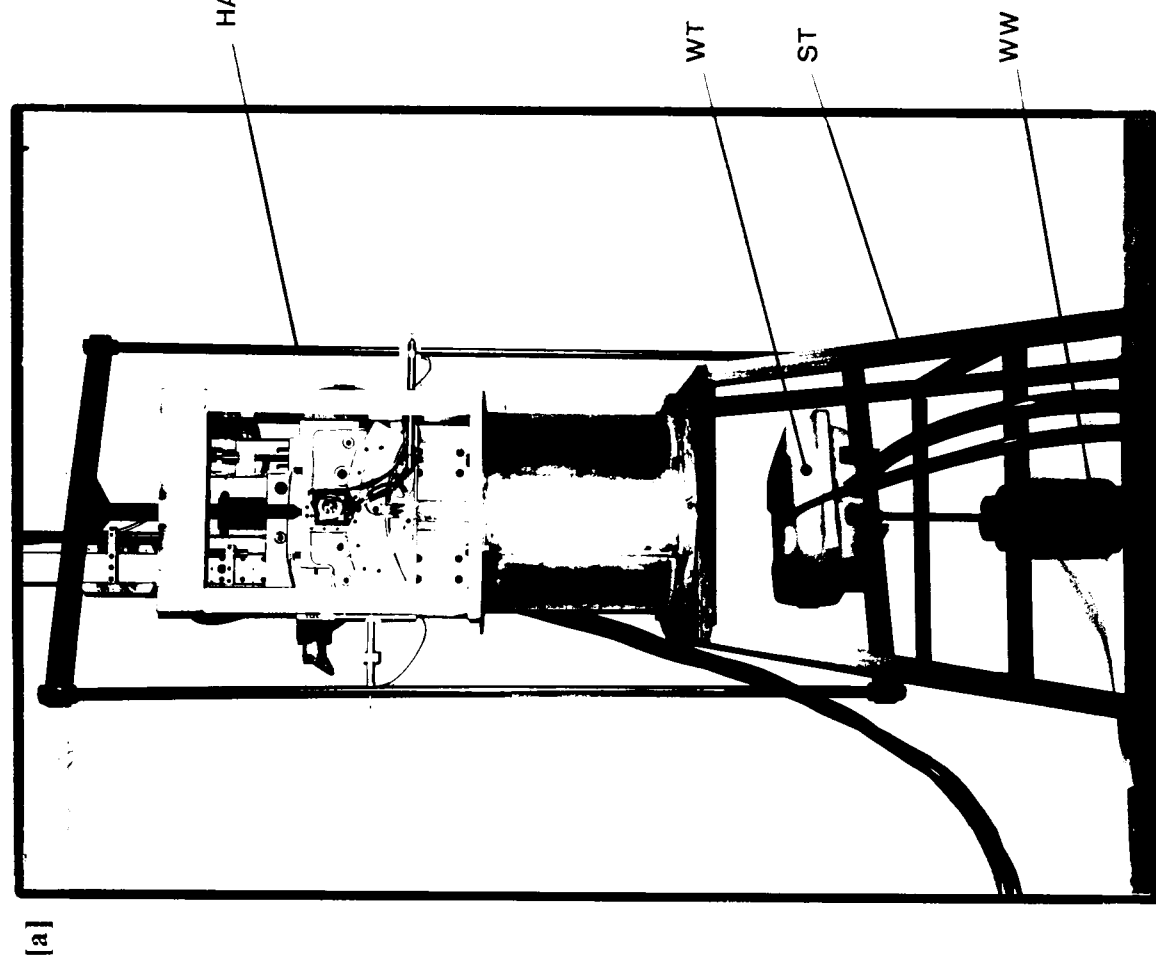


FIGURE 3.12 Modified apparatus for footing tests with vertical load control

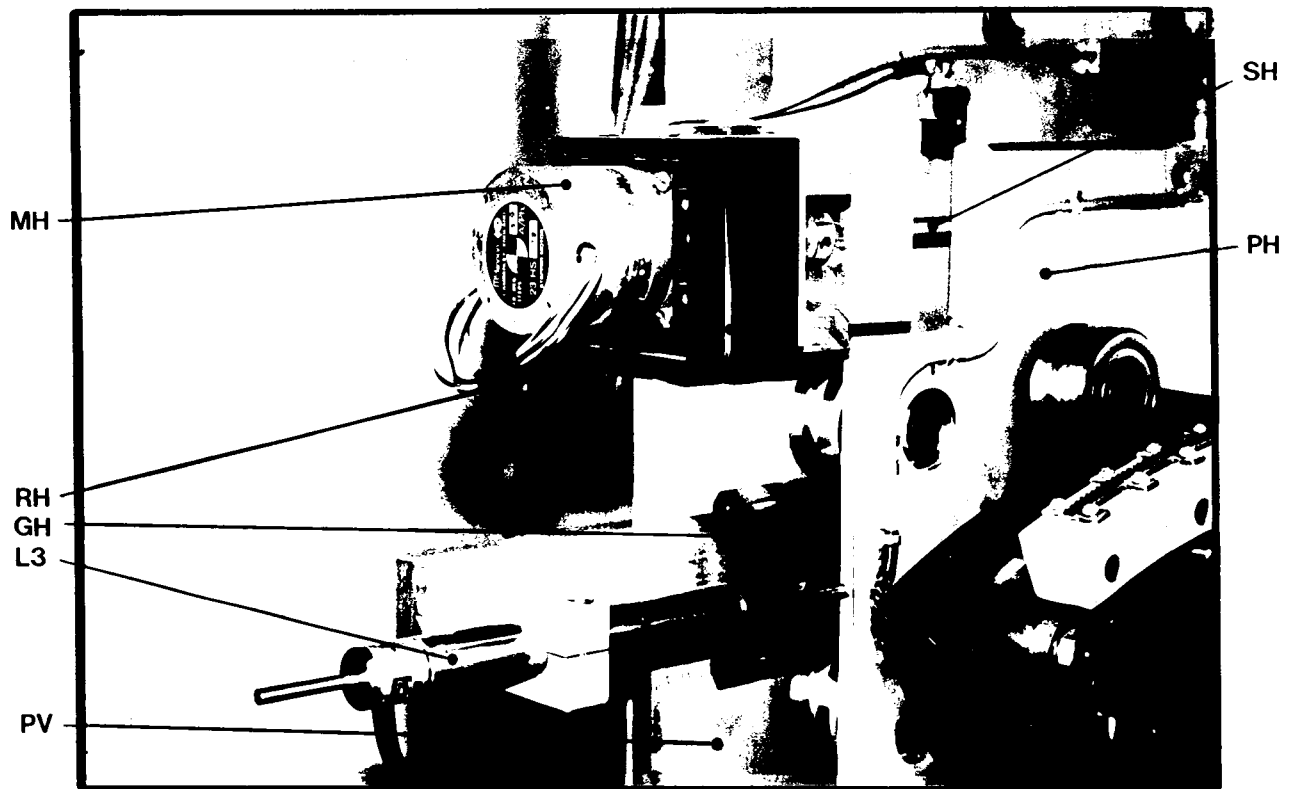


FIGURE 3.13 [a] Horizontal displacement control system

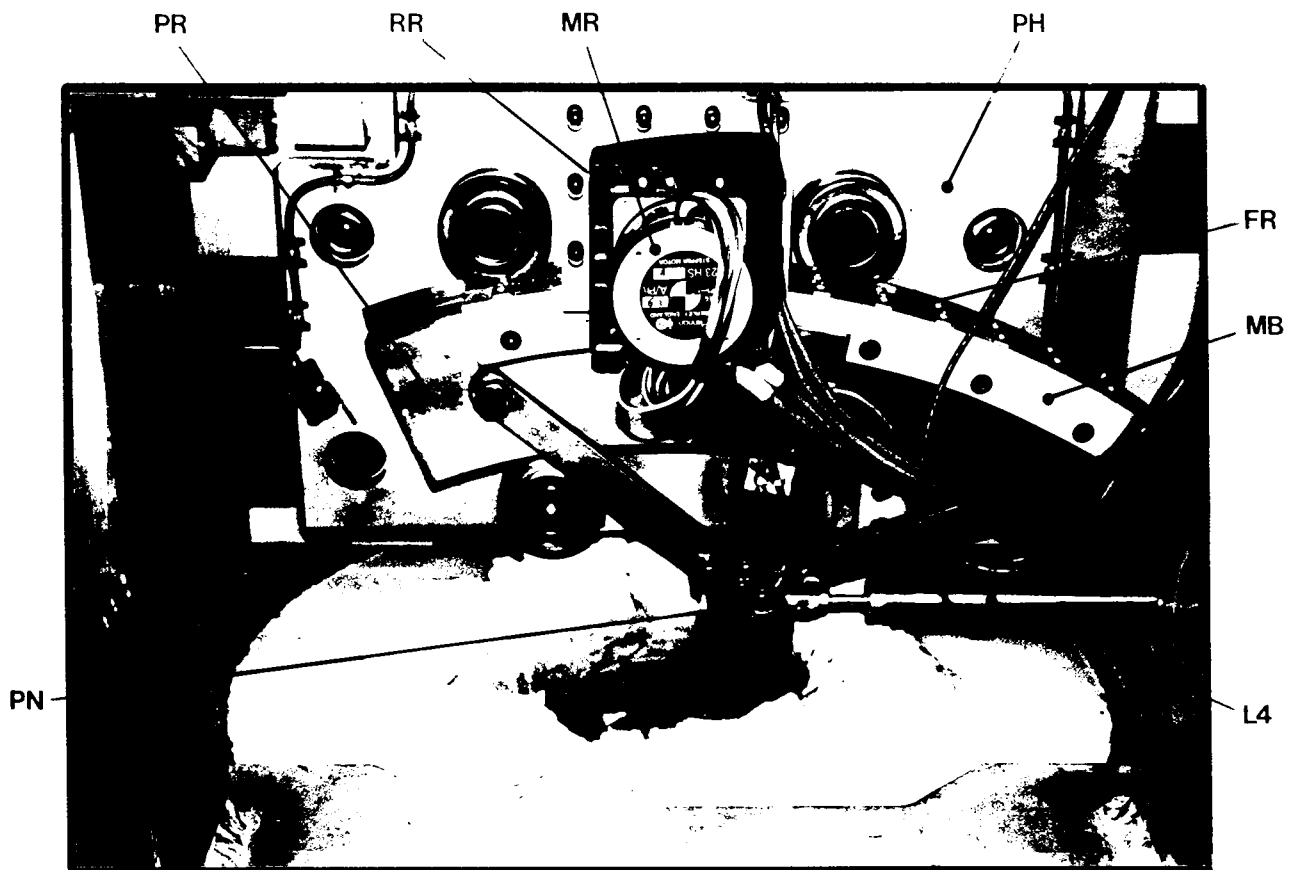


FIGURE 3.13 [b] Rotational displacement control system

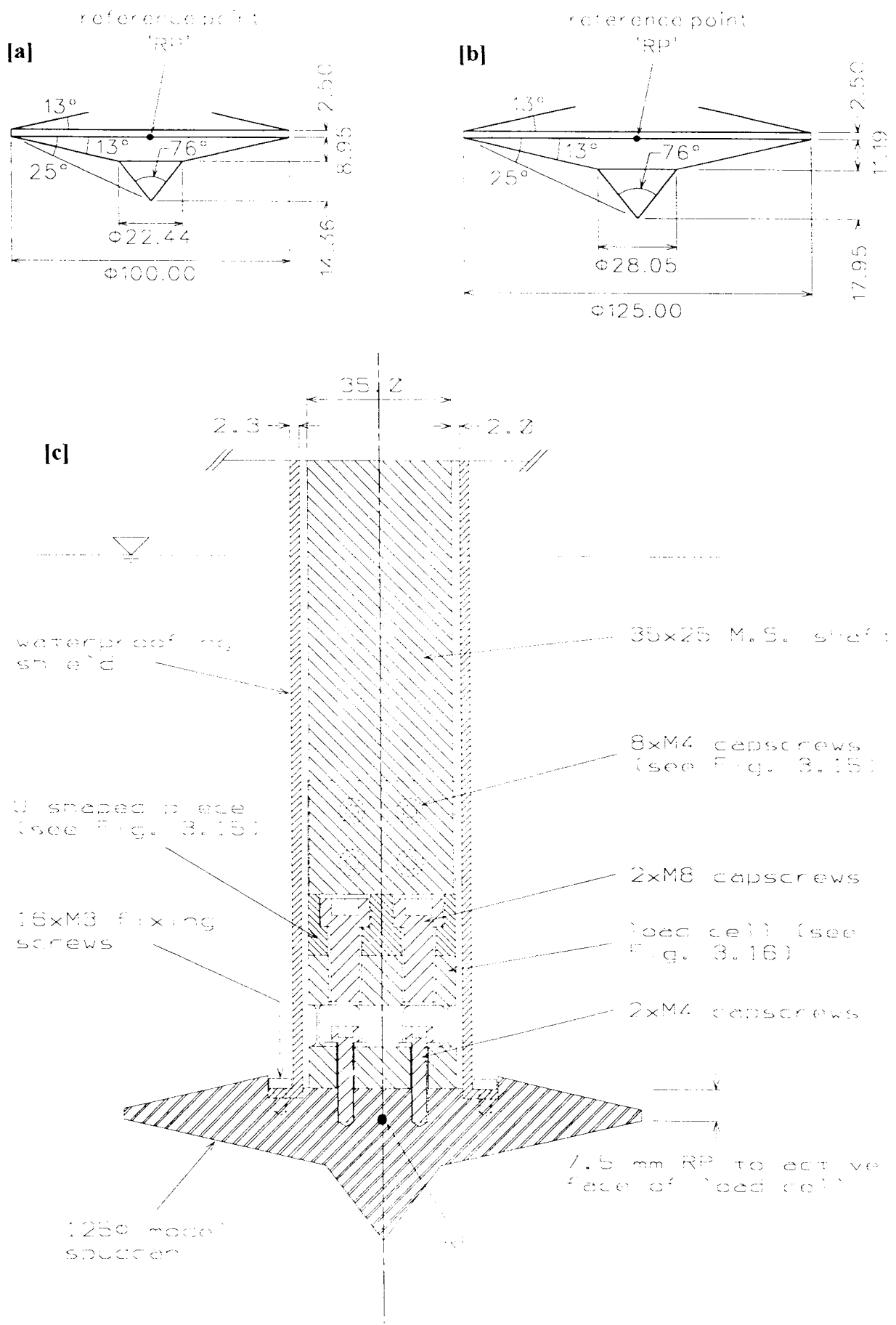
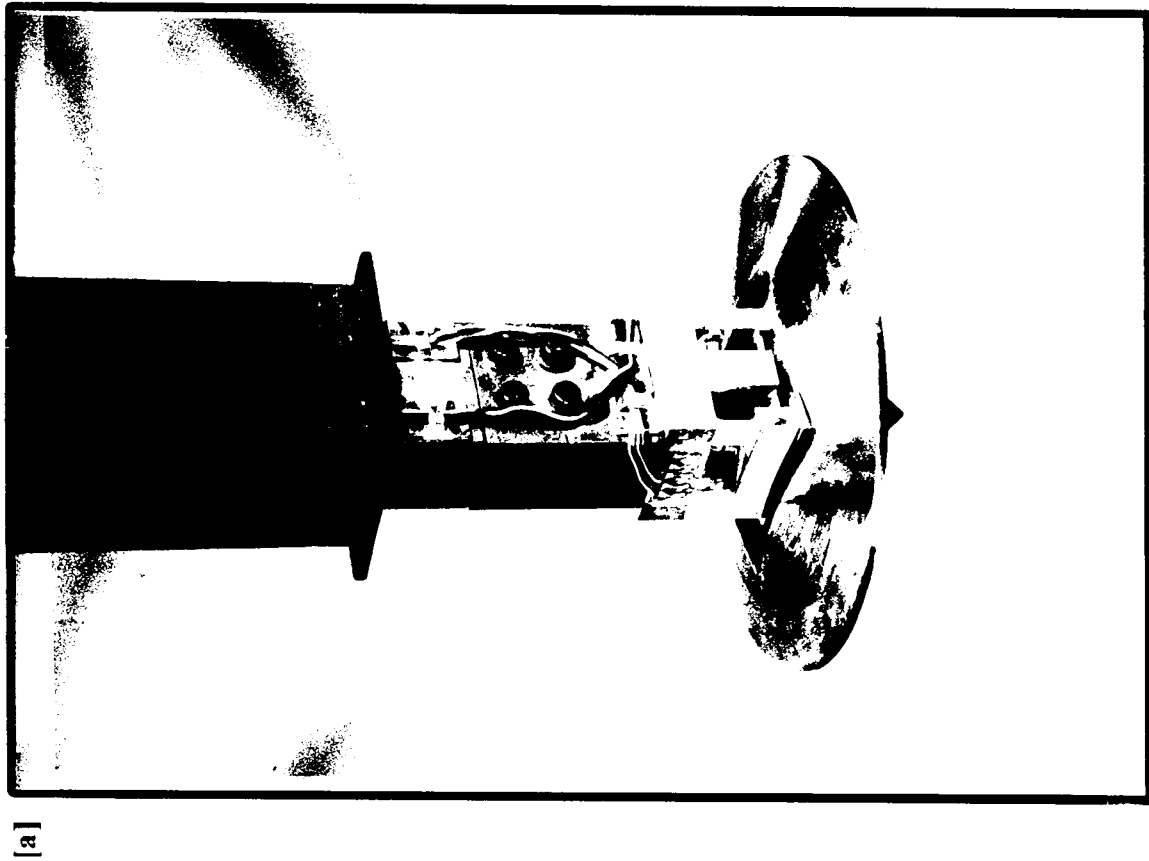
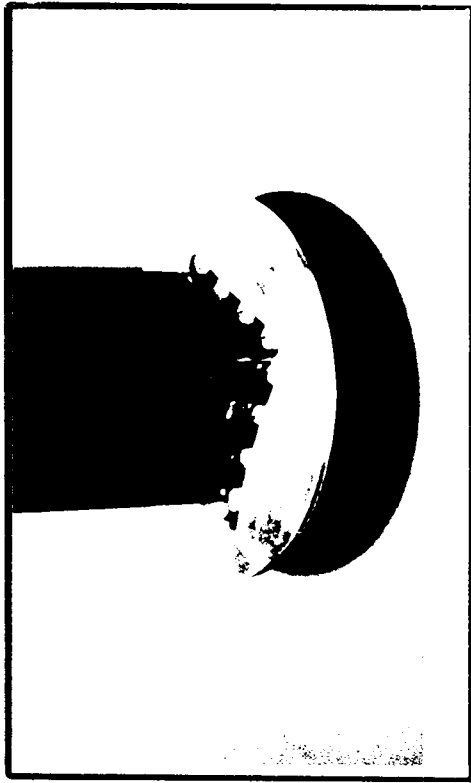


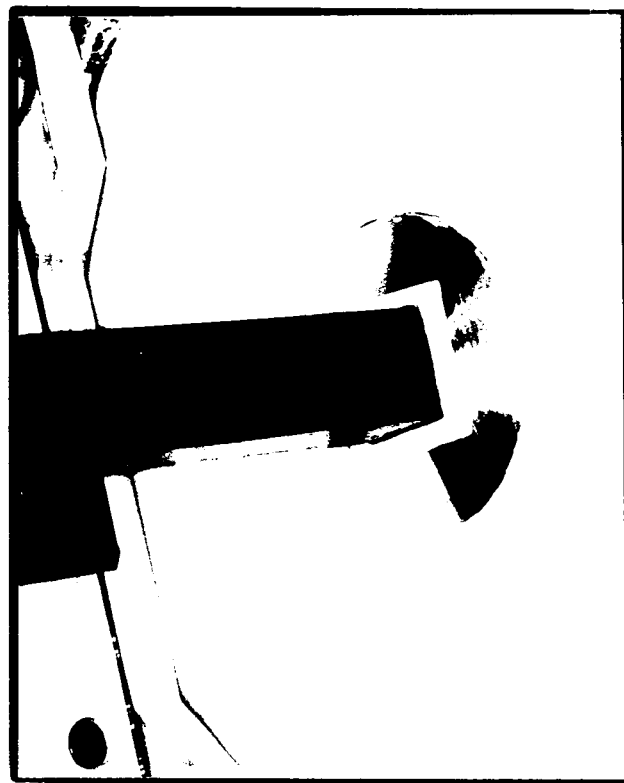
FIGURE 3.14 [a] & [b] Model spudcan footings used in experimental programme
[c] Cross-sectional detail of spudcan to shaft connection



[a]



[b]



[c]

FIGURE 3.15 Attachment of waterproofing shield to model spudcan

NOTES

1. material: dural (HE15)
2. 1 mm radii at all web ends
3. all dims ± 0.1 mm U.N.O.
4. strain gauges not shown

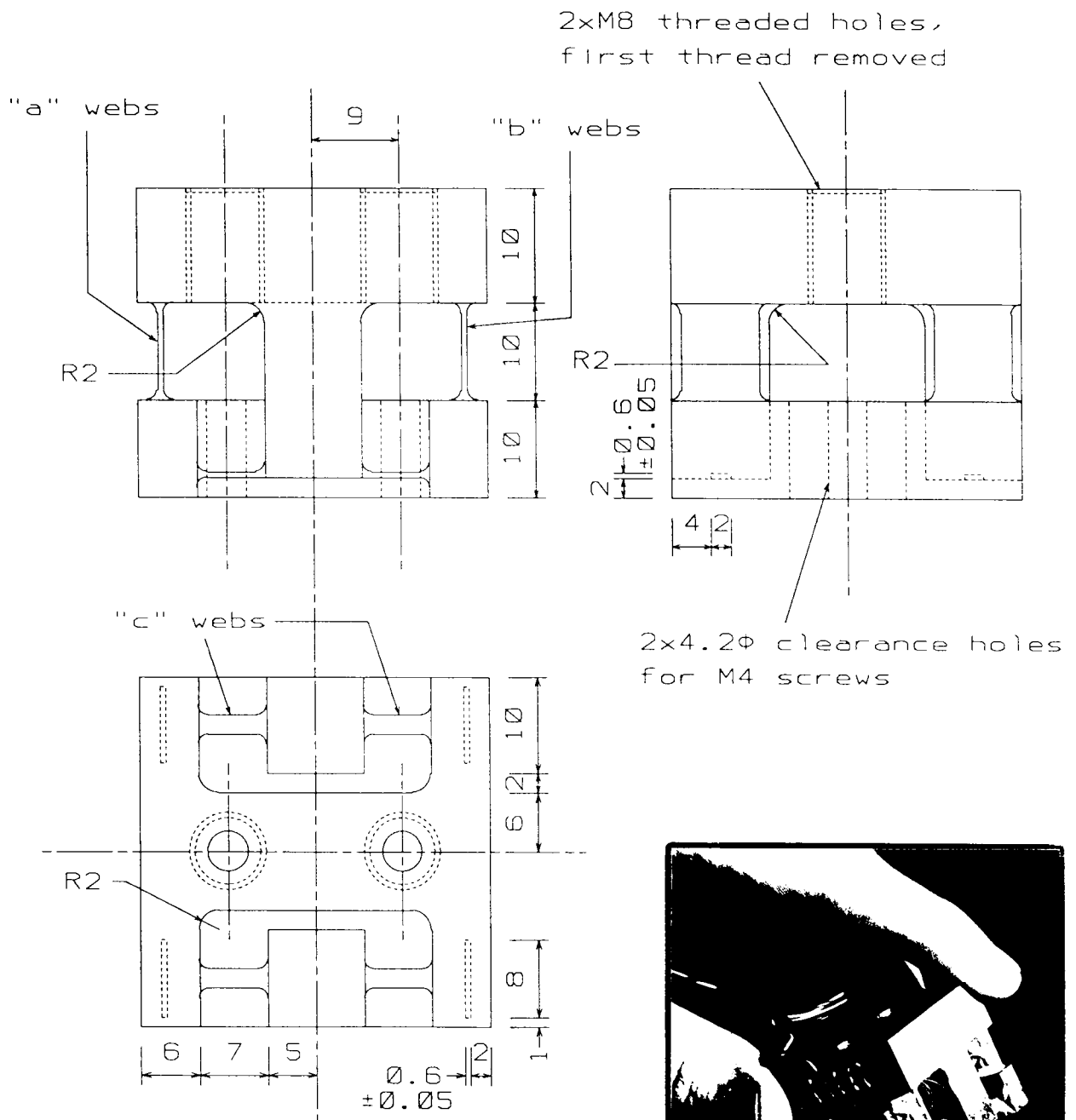
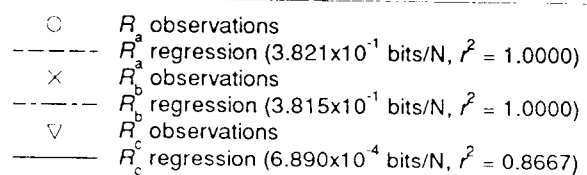
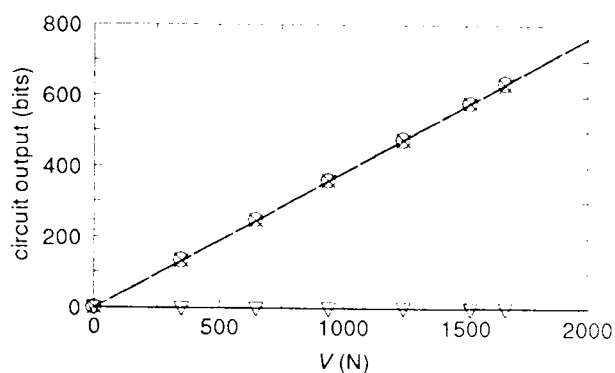
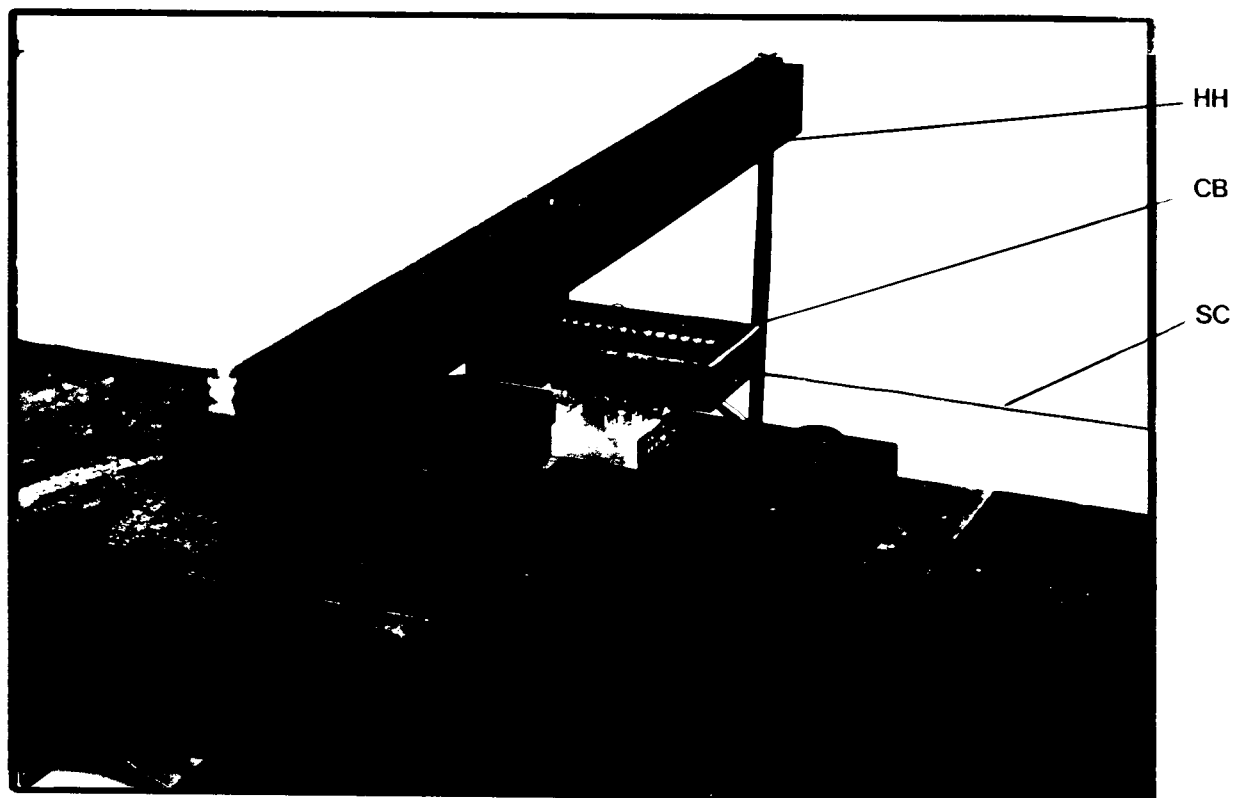


FIGURE 3.16 "Cambridge" type load cell for measuring combined vertical, horizontal and moment loads



N.B. each observation is the average of two readings

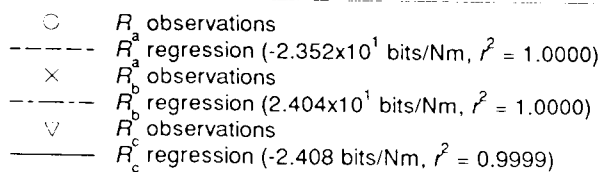
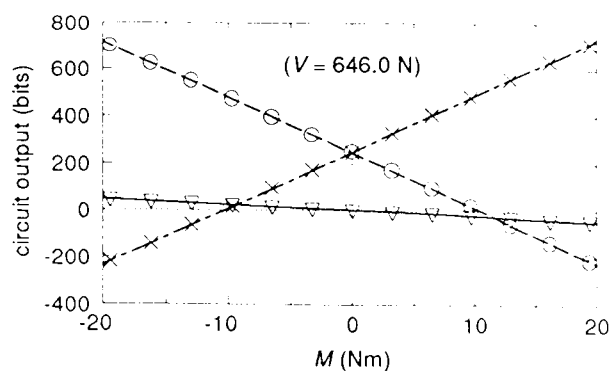
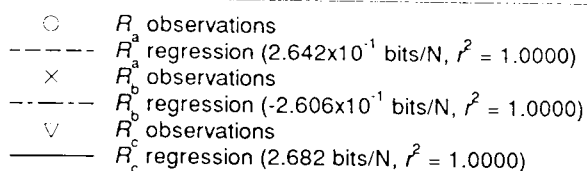
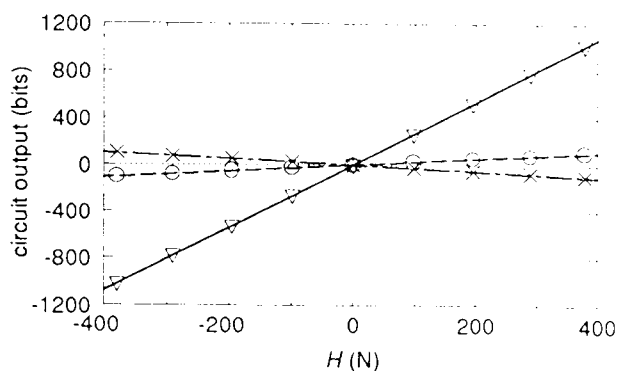


FIGURE 3.17 Calibration of the load cell

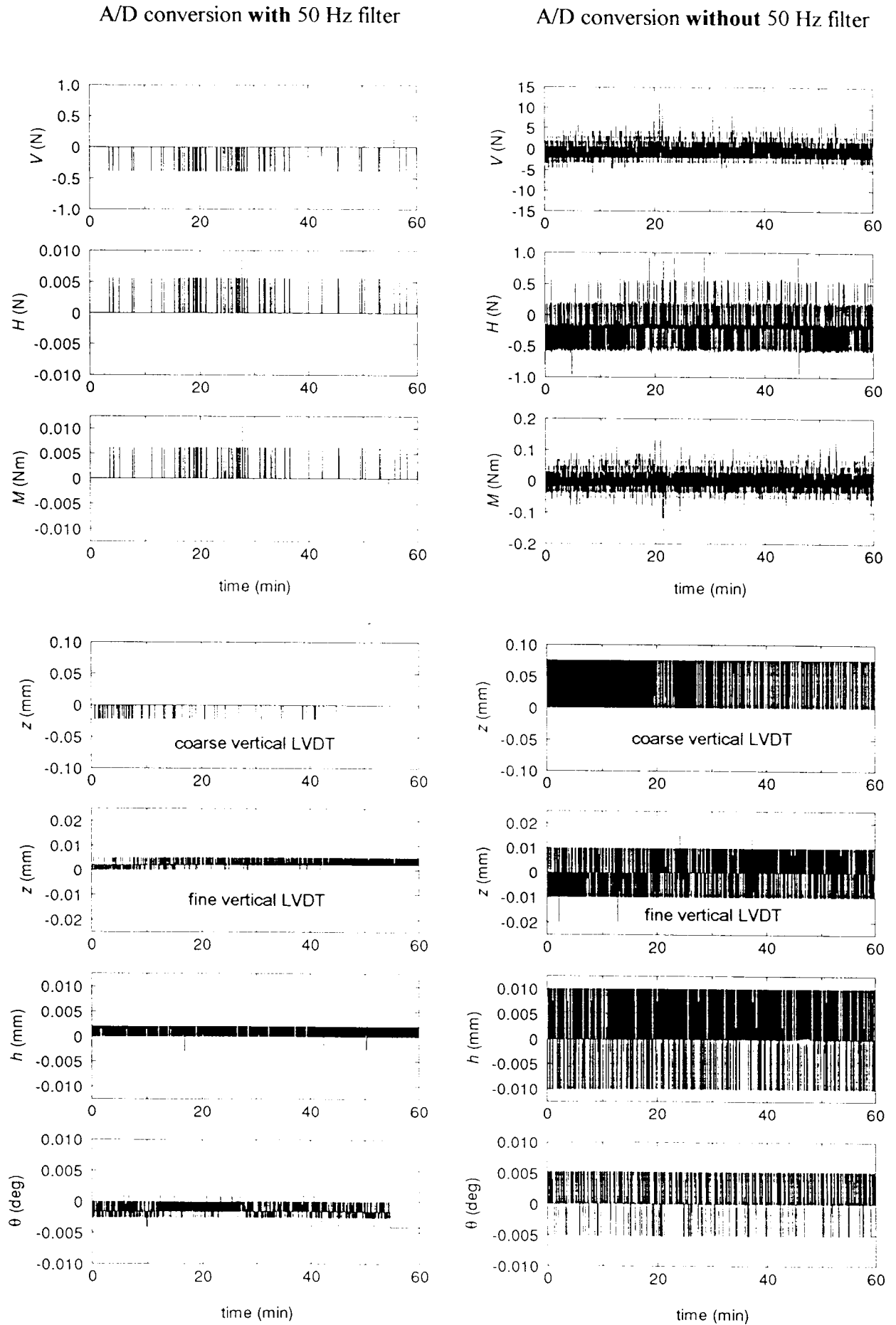
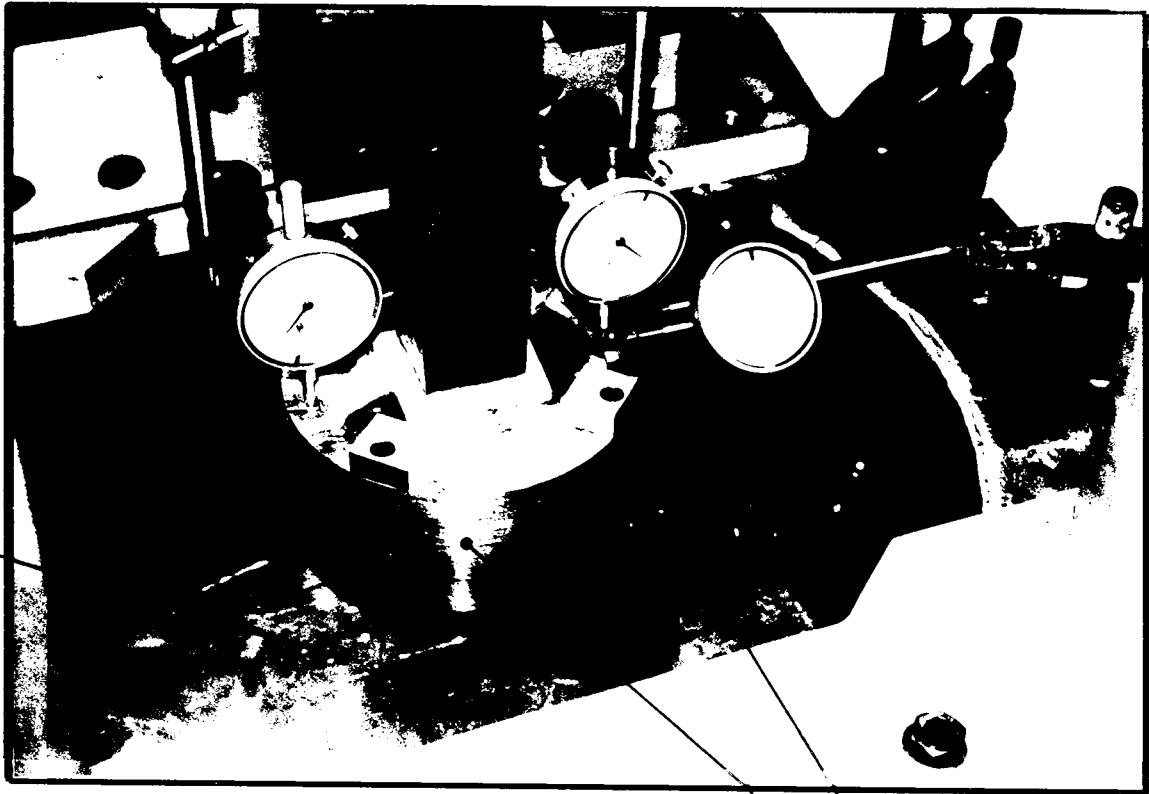


FIGURE 3.18 One hour assessment of errors due to random noise and drift
(logging frequency: 2 Hz).

[a]

DP



FA HJ

[b]

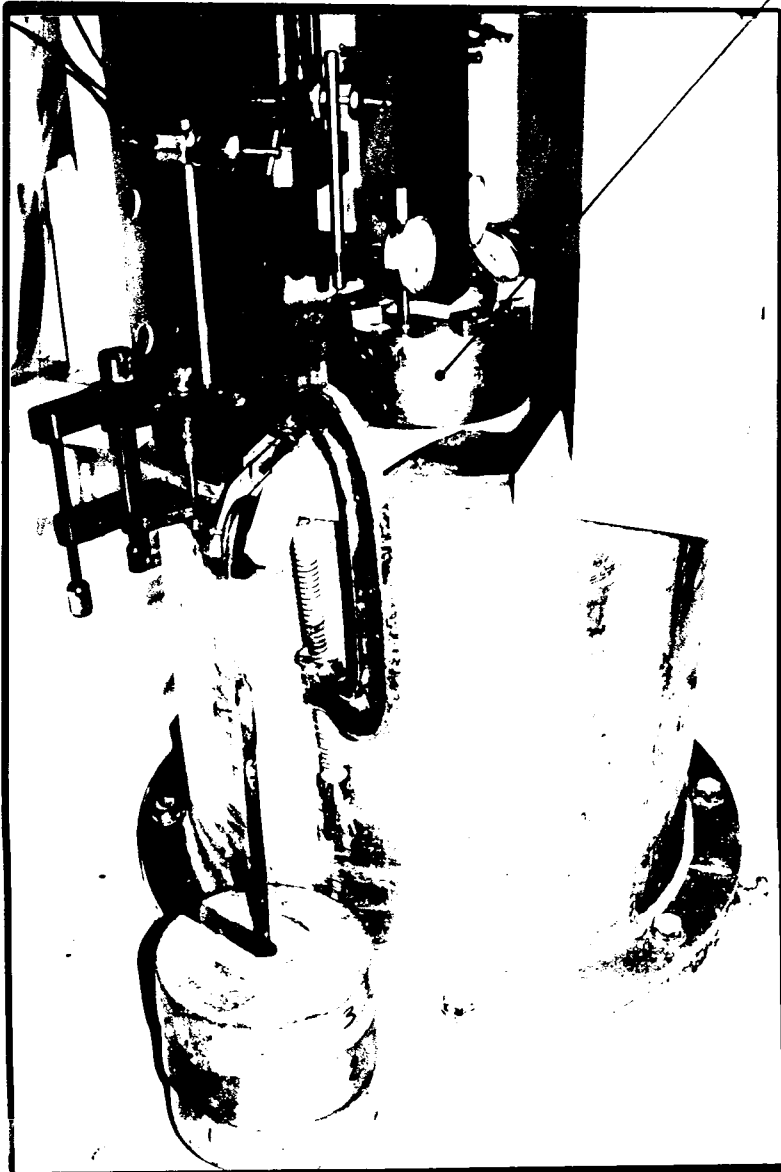


FIGURE 3.19

Direct measurement of
apparatus flexibility
under [a] vertical and
moment loads;
[b] horizontal load

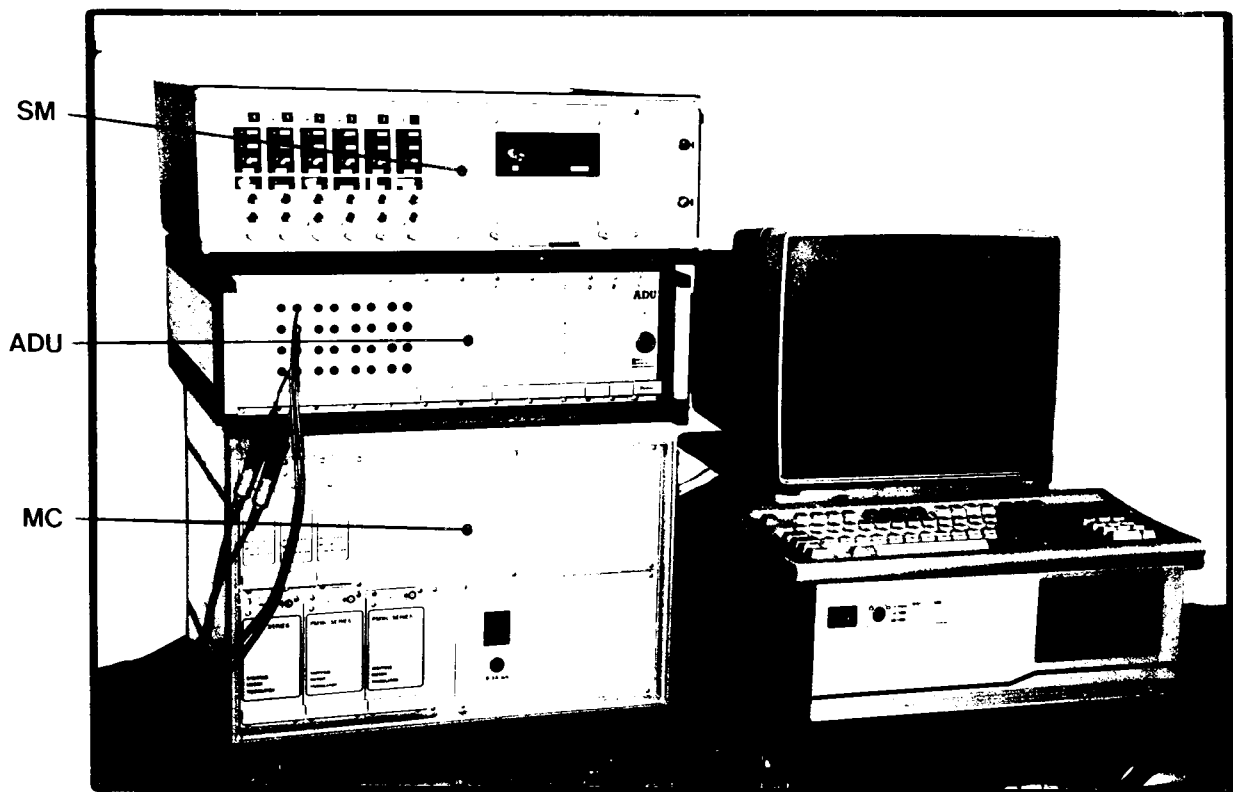


FIGURE 3.20 [a] Control and instrumentation equipment

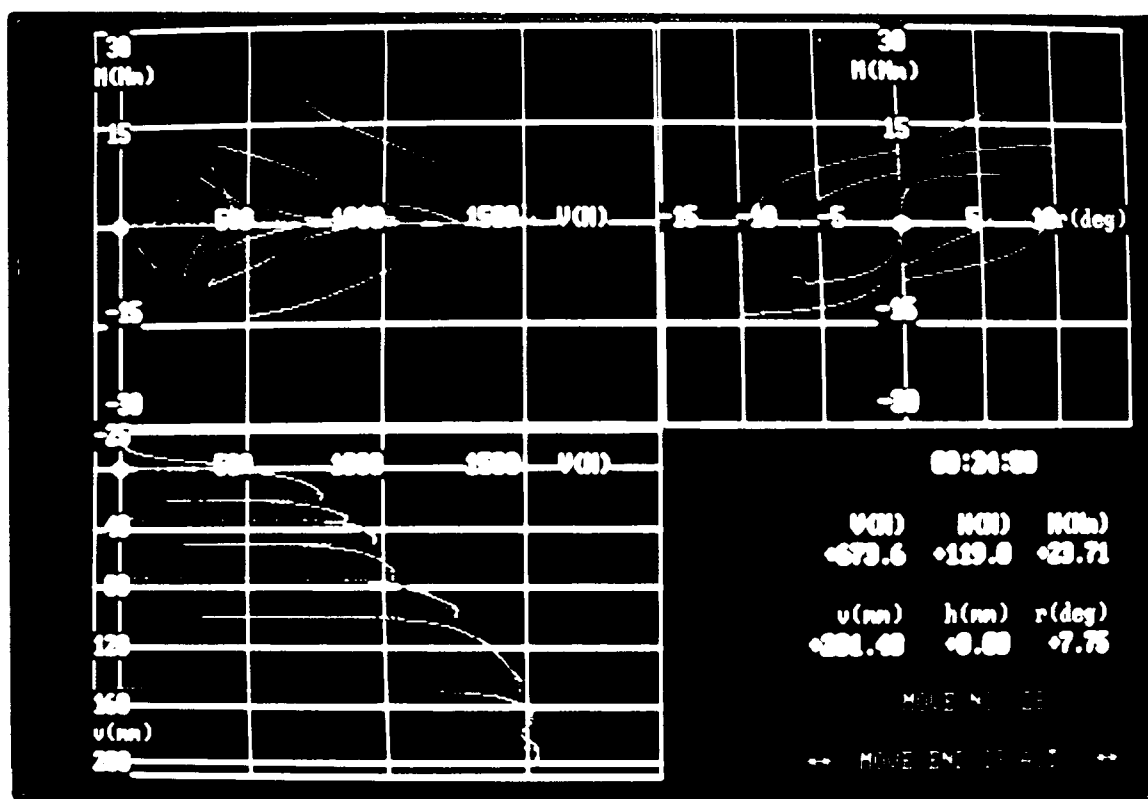


FIGURE 3.20 [b] Computer screen display during a footing test

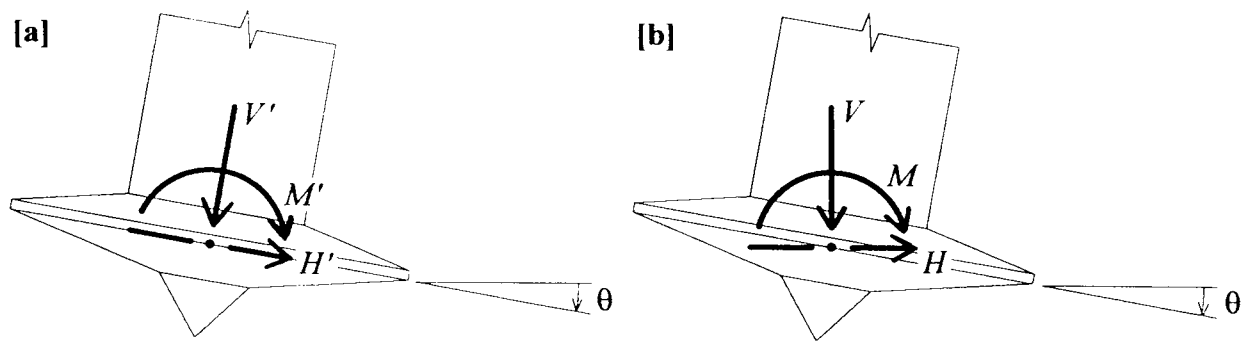


FIGURE 4.1 Correction of loads registered by load cell

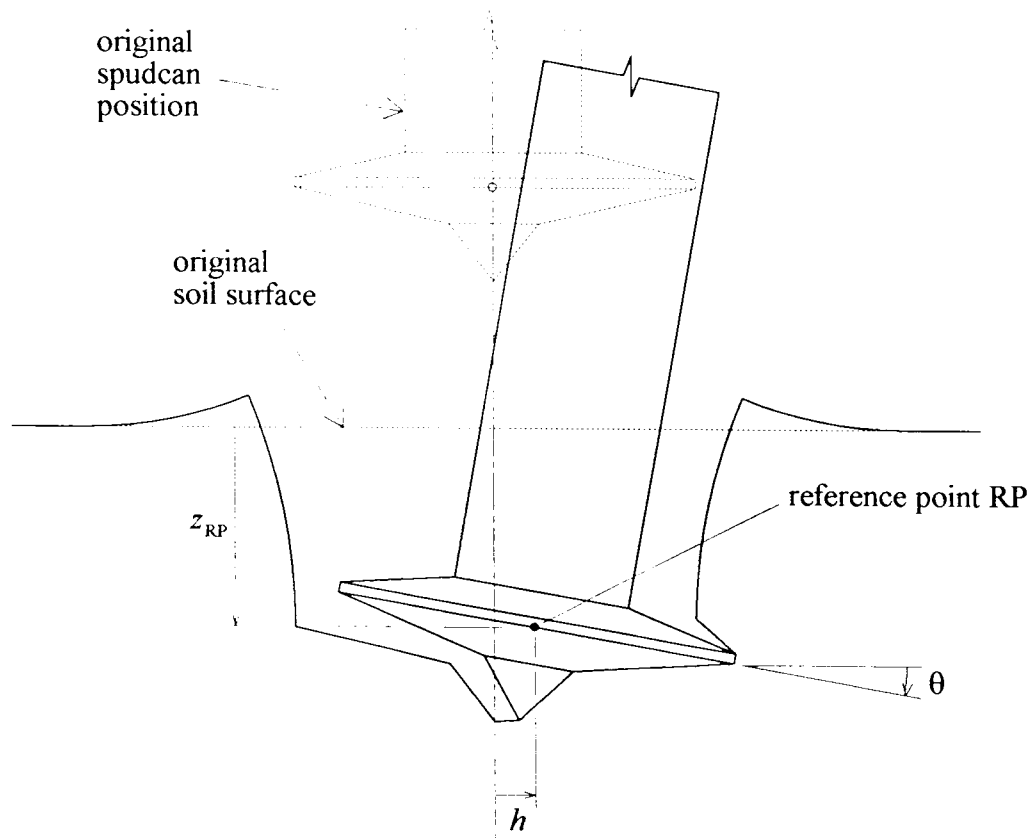


FIGURE 4.2 Definition of spudcan displacements

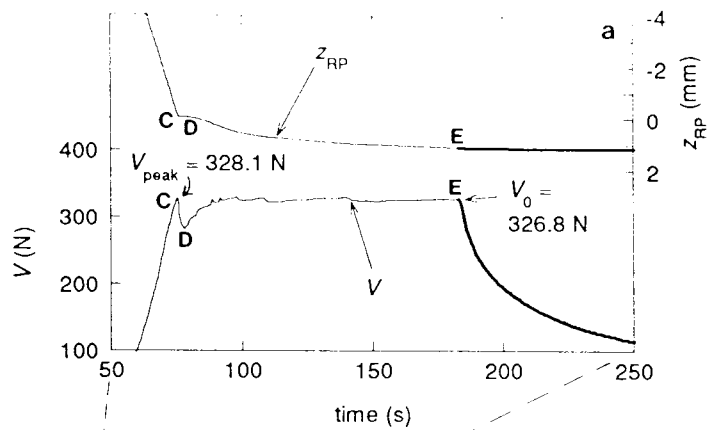


FIGURE 4.3

Extract from test record for **T011**
(variation of loads and
displacements with time)

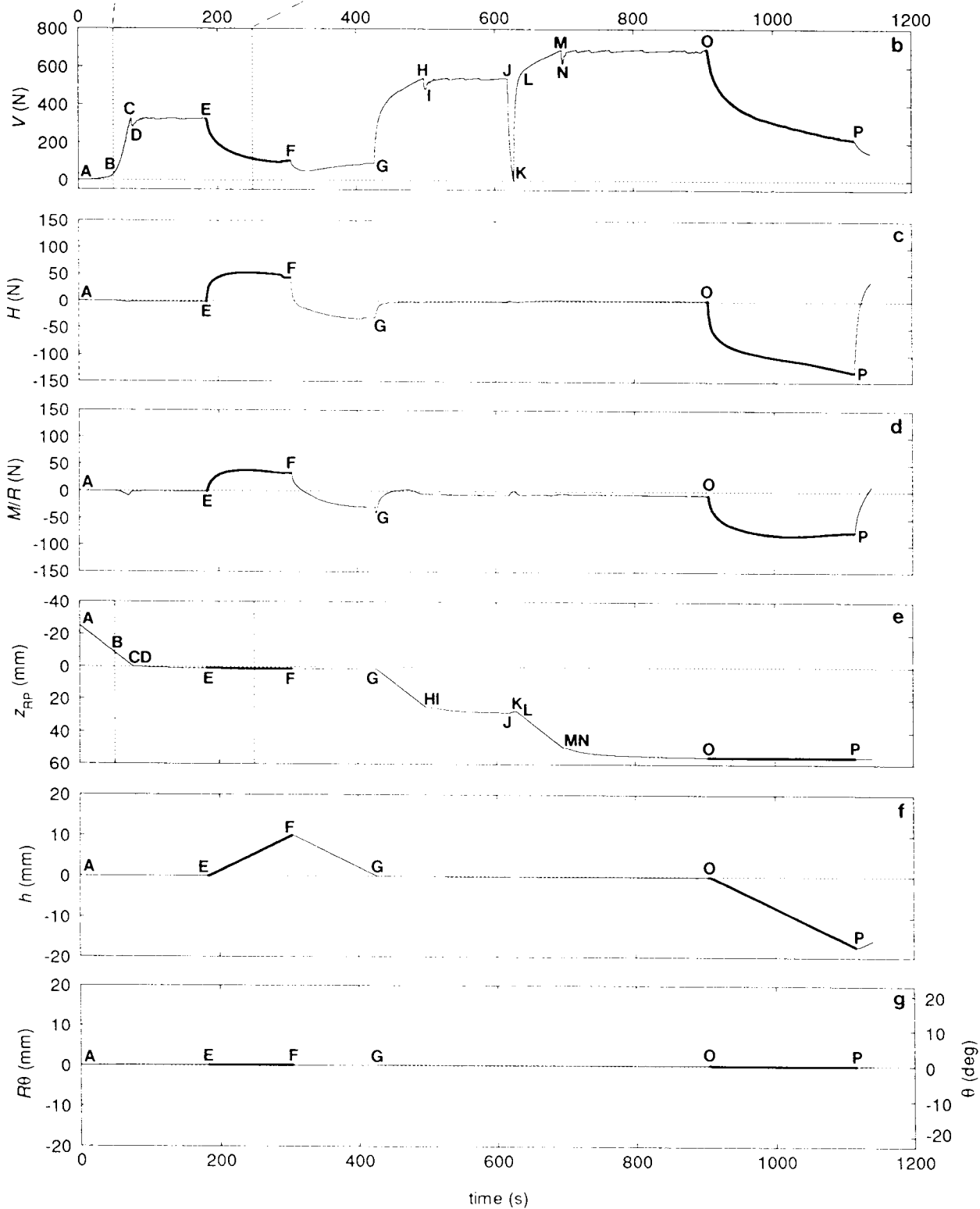
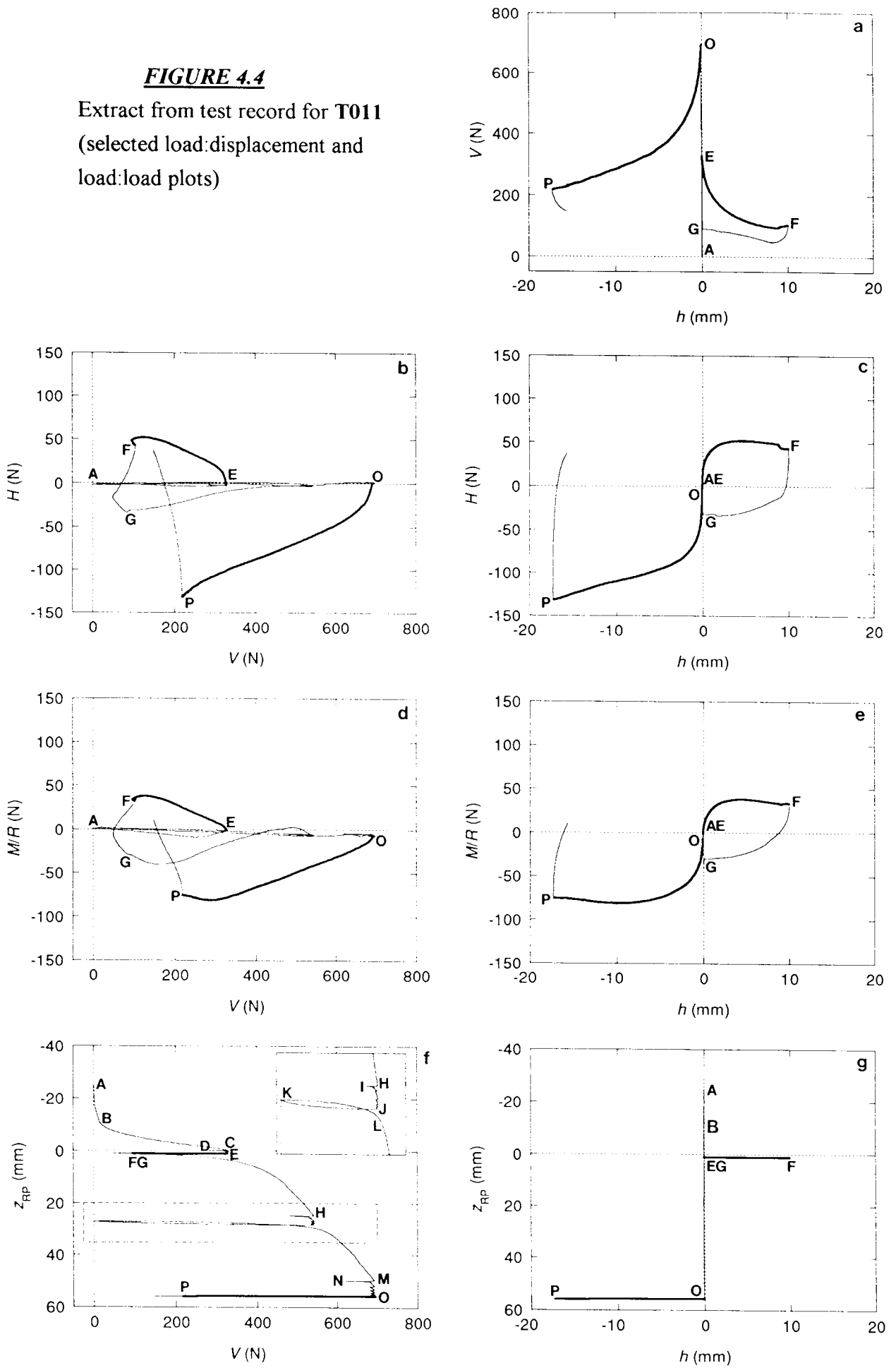


FIGURE 4.4

Extract from test record for **T011**
(selected load:displacement and
load:load plots)



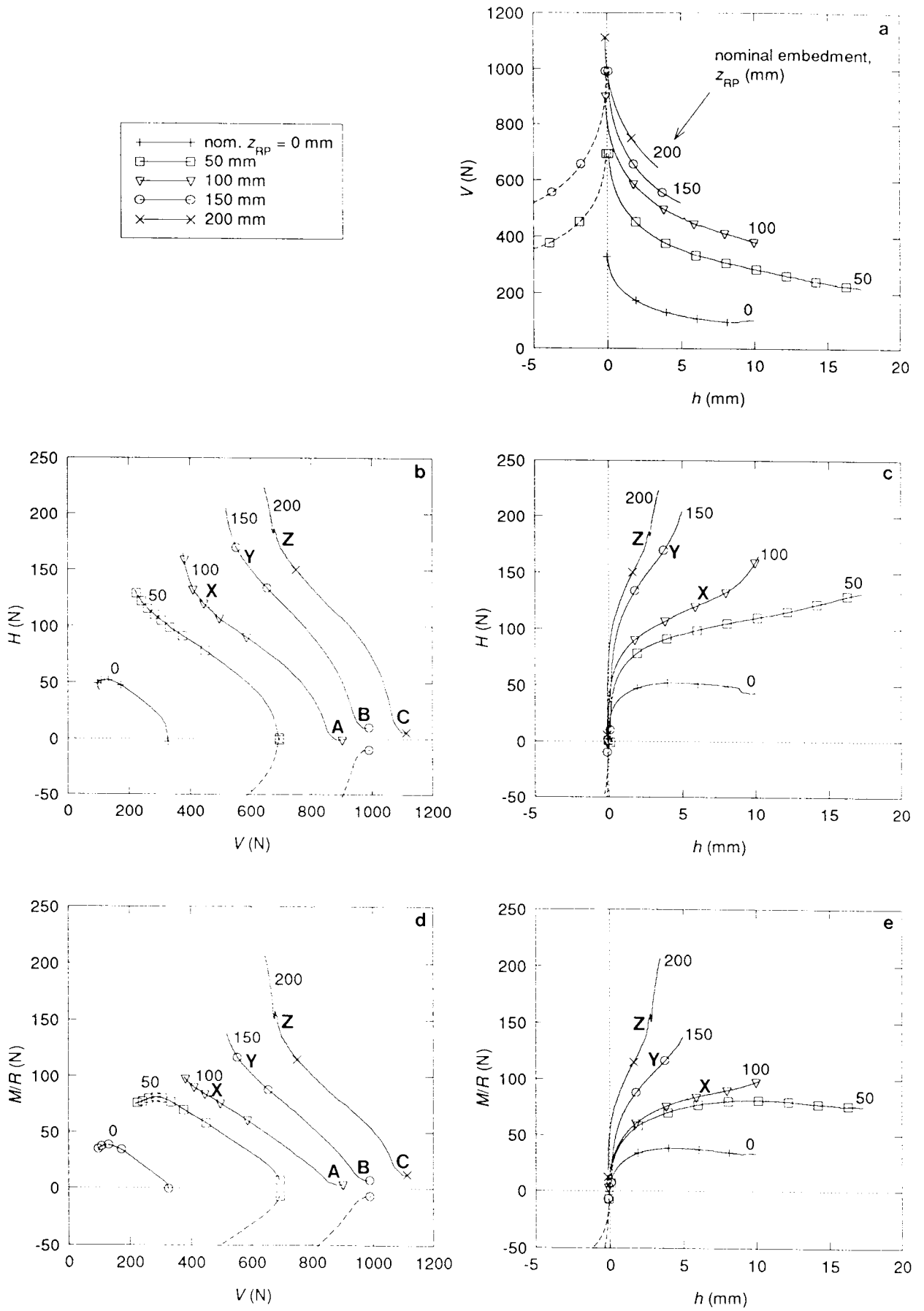


FIGURE 4.5 Results of tracking events in T011 (h from high V/V_0)

Note: dashed lines for events at $z_{RP} = 50$ mm and $z_{RP} = 150$ mm indicate actual loads and displacements obtained in T011 (*i.e.* before sign reversal for plotting)

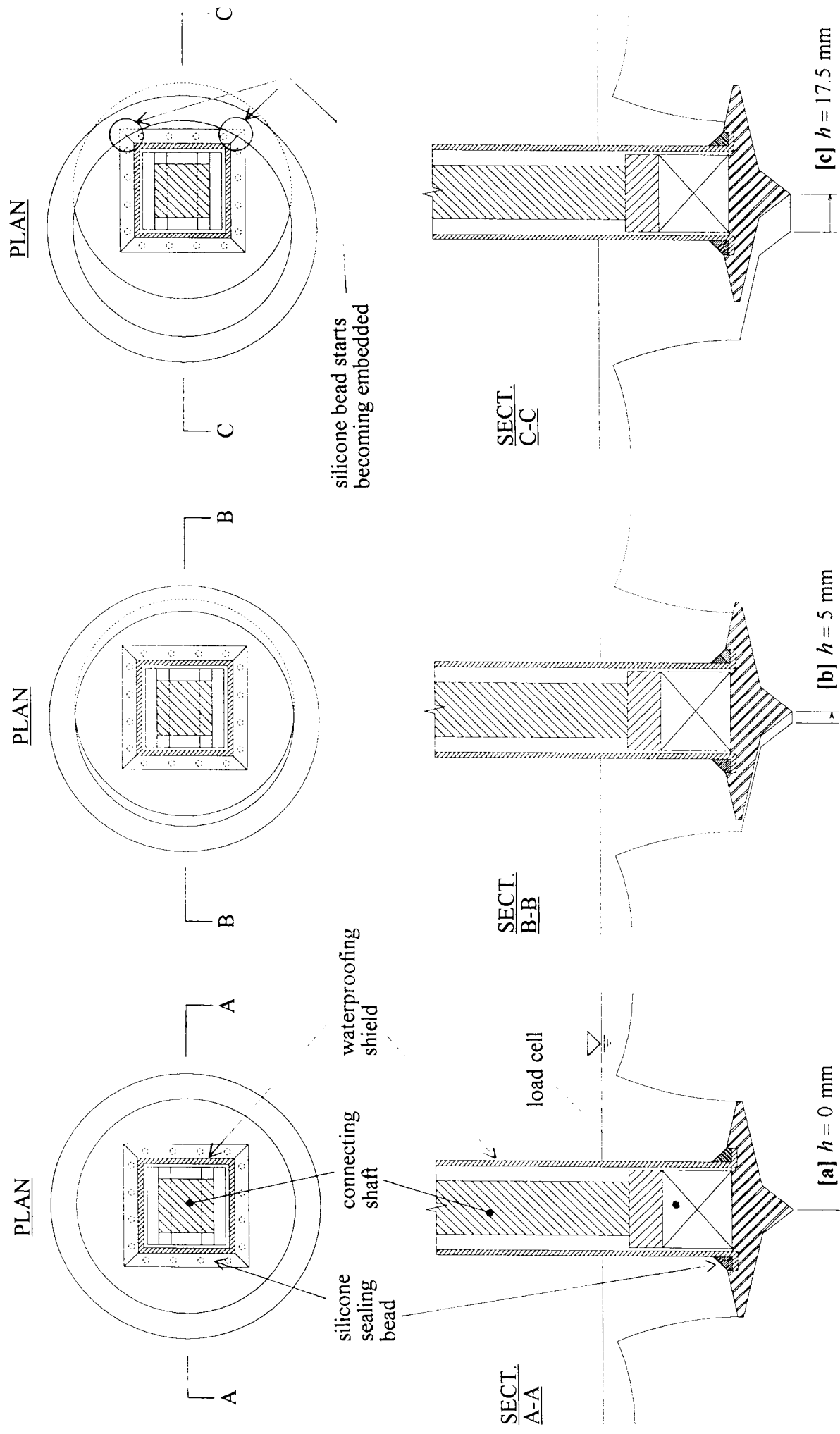


FIGURE 4.6 Waterproofing shield embedment during horizontal displacement (drawn for 100 mm diameter footing)

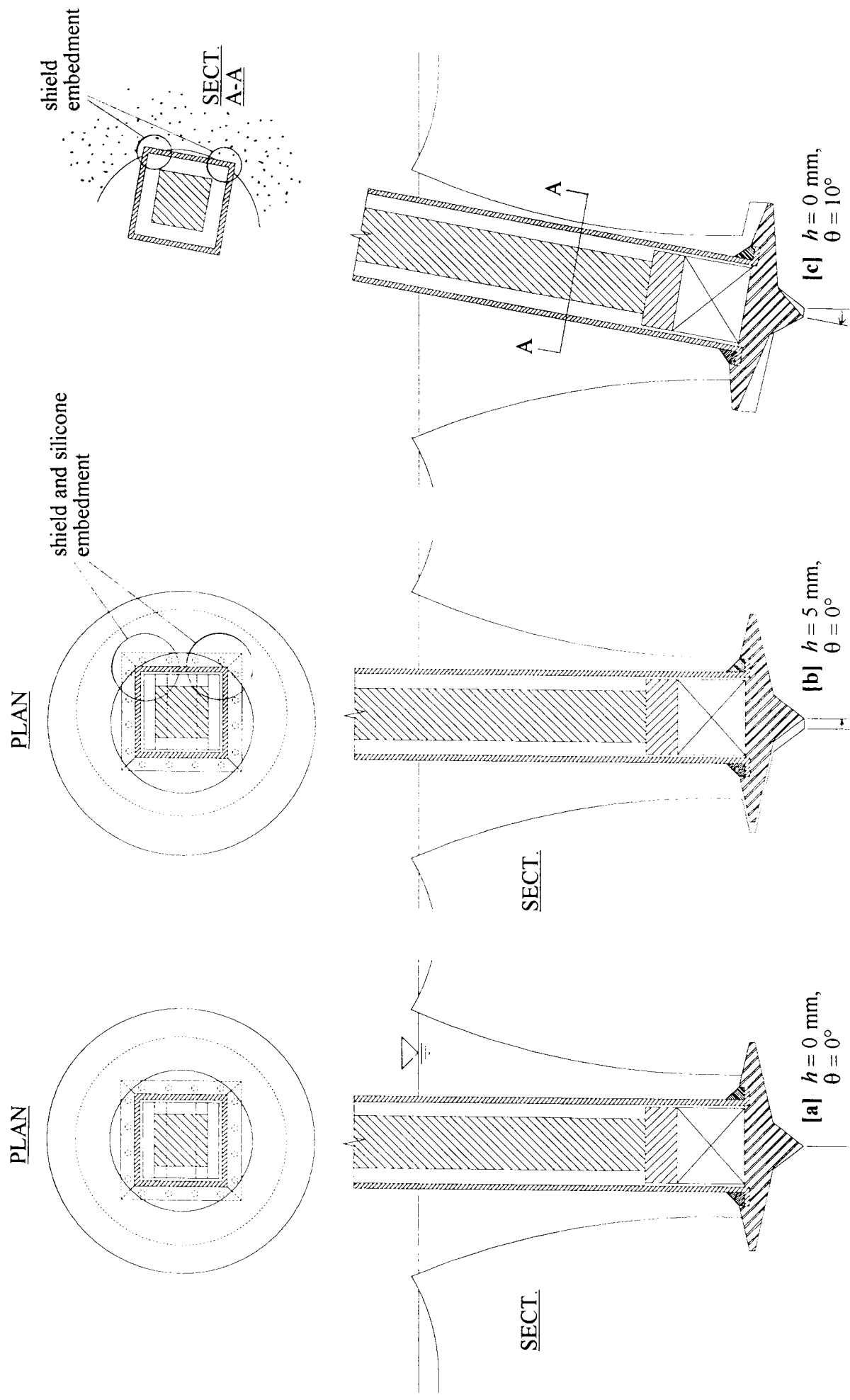


FIGURE 4.7 Waterproofing shield embedment with deeply embedded 100 mm diameter footing

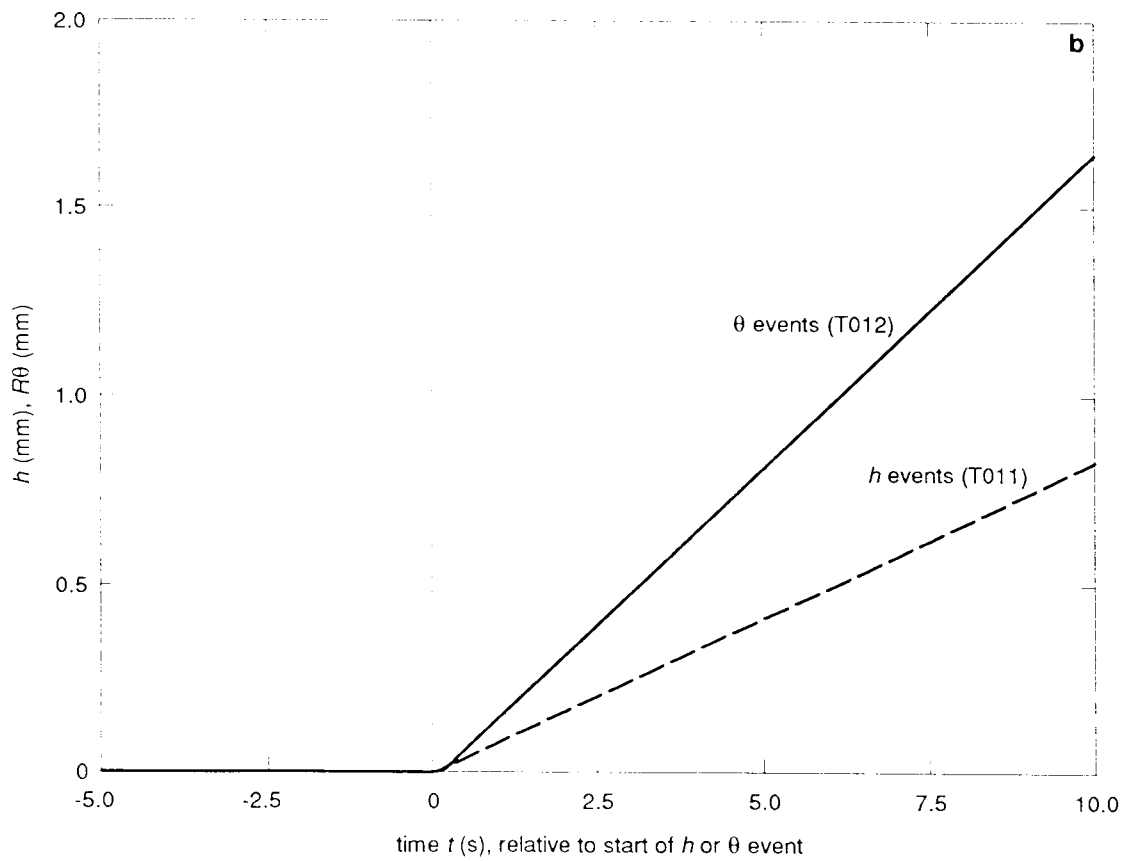
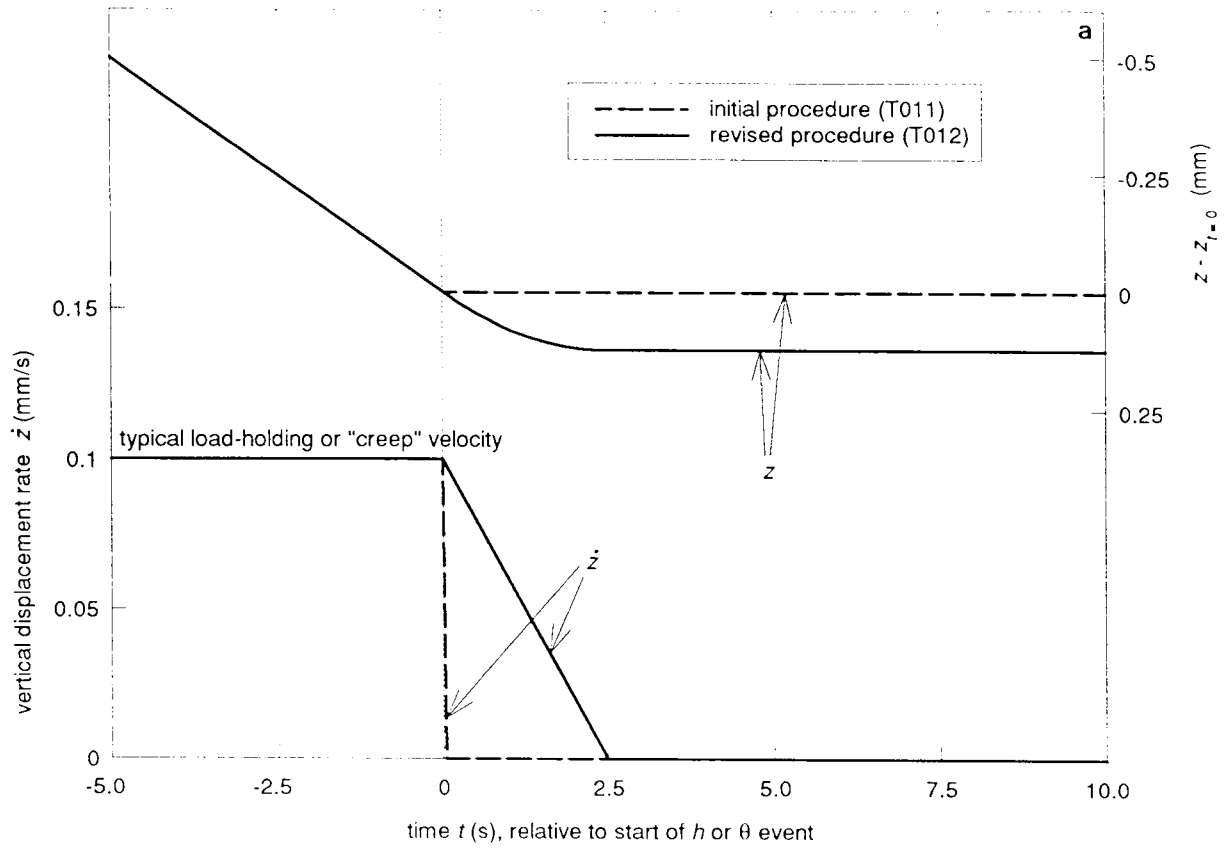


FIGURE 4.8 Ramping down of vertical displacement rate at the end of a load holding stage

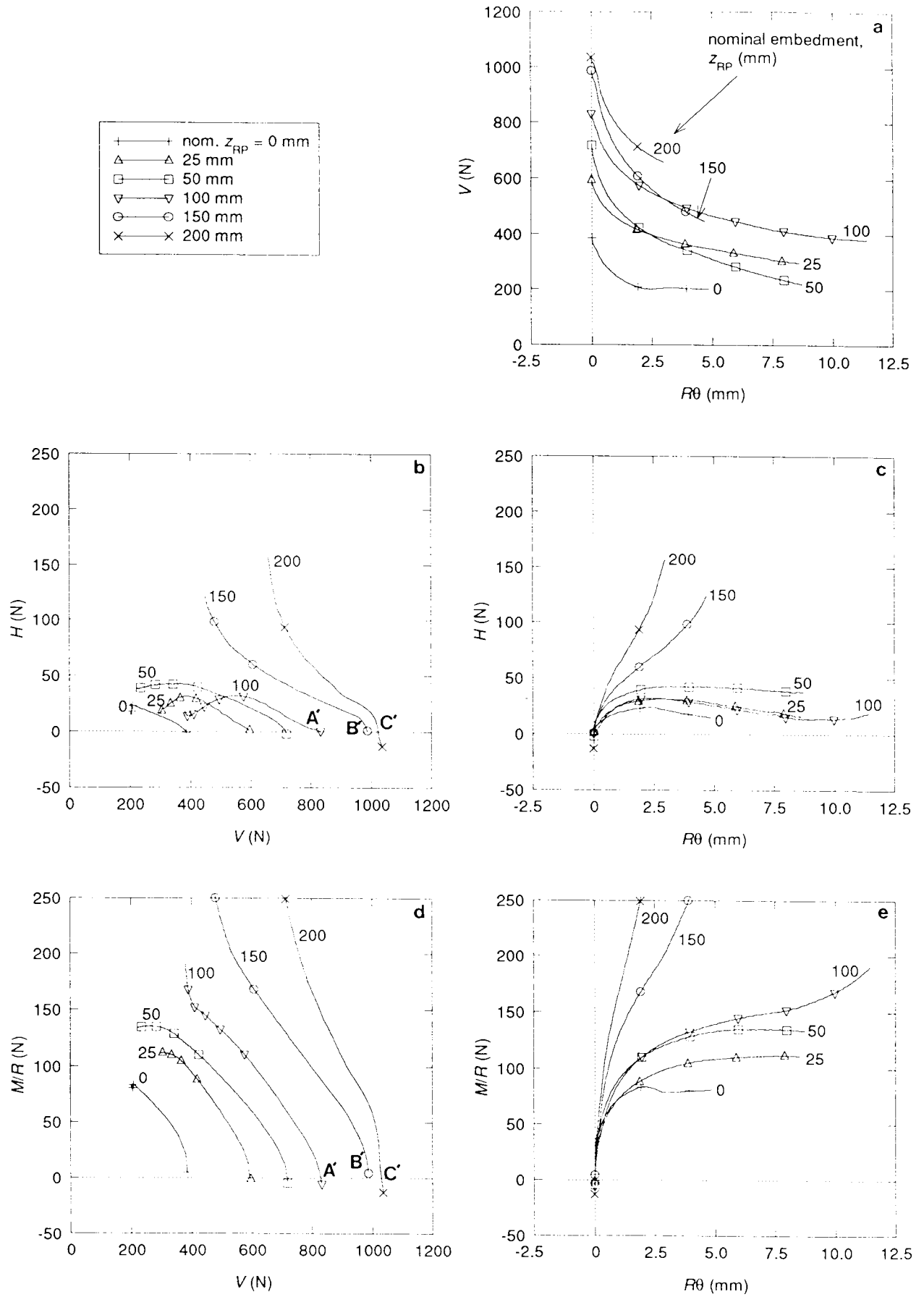
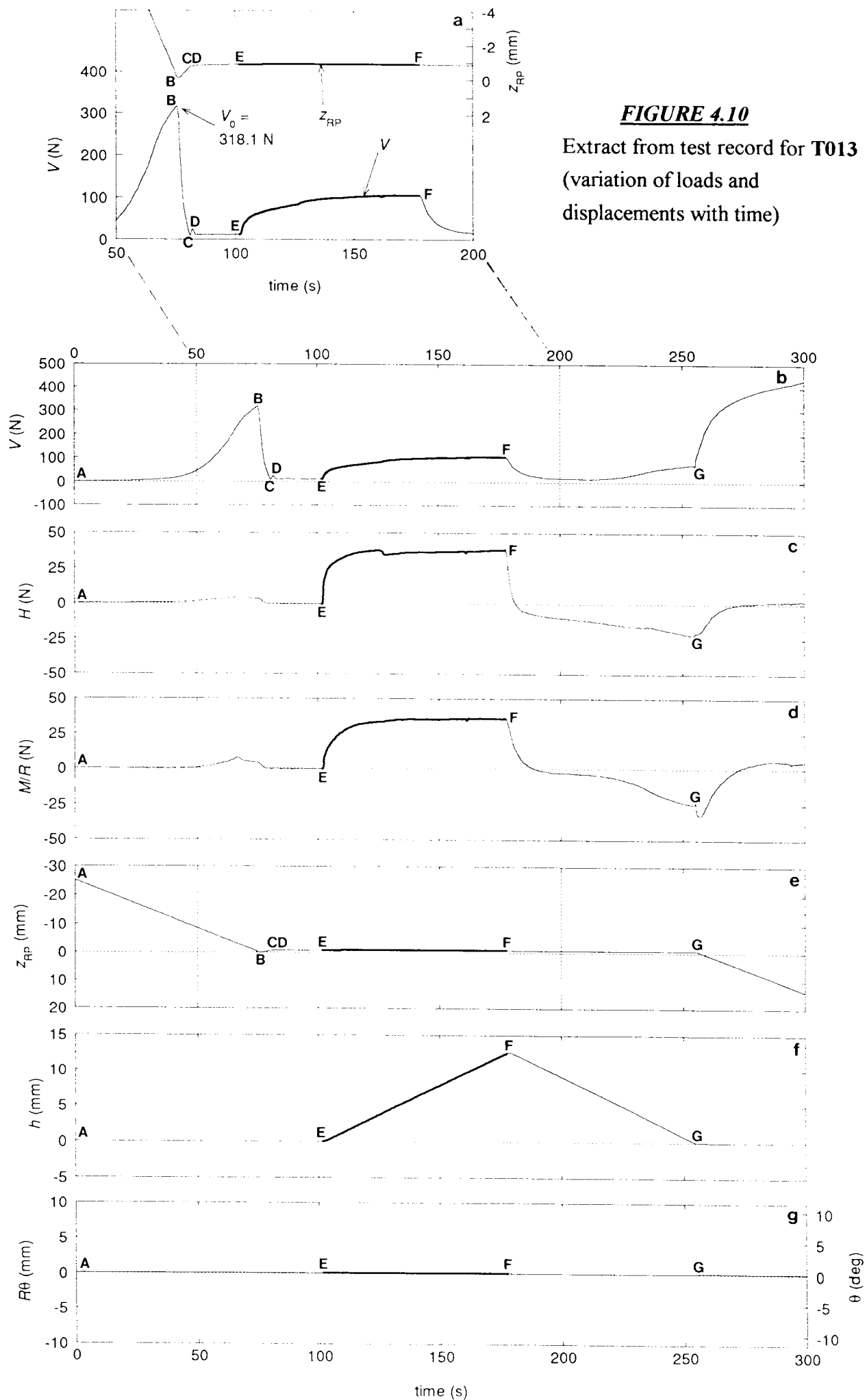


FIGURE 4.9 Results of tracking events in T012 (θ from high V/V_0)

Note: since $R = 50$ mm, 1 mm of $R\theta$ displacement corresponds to a rotation of 1.15°



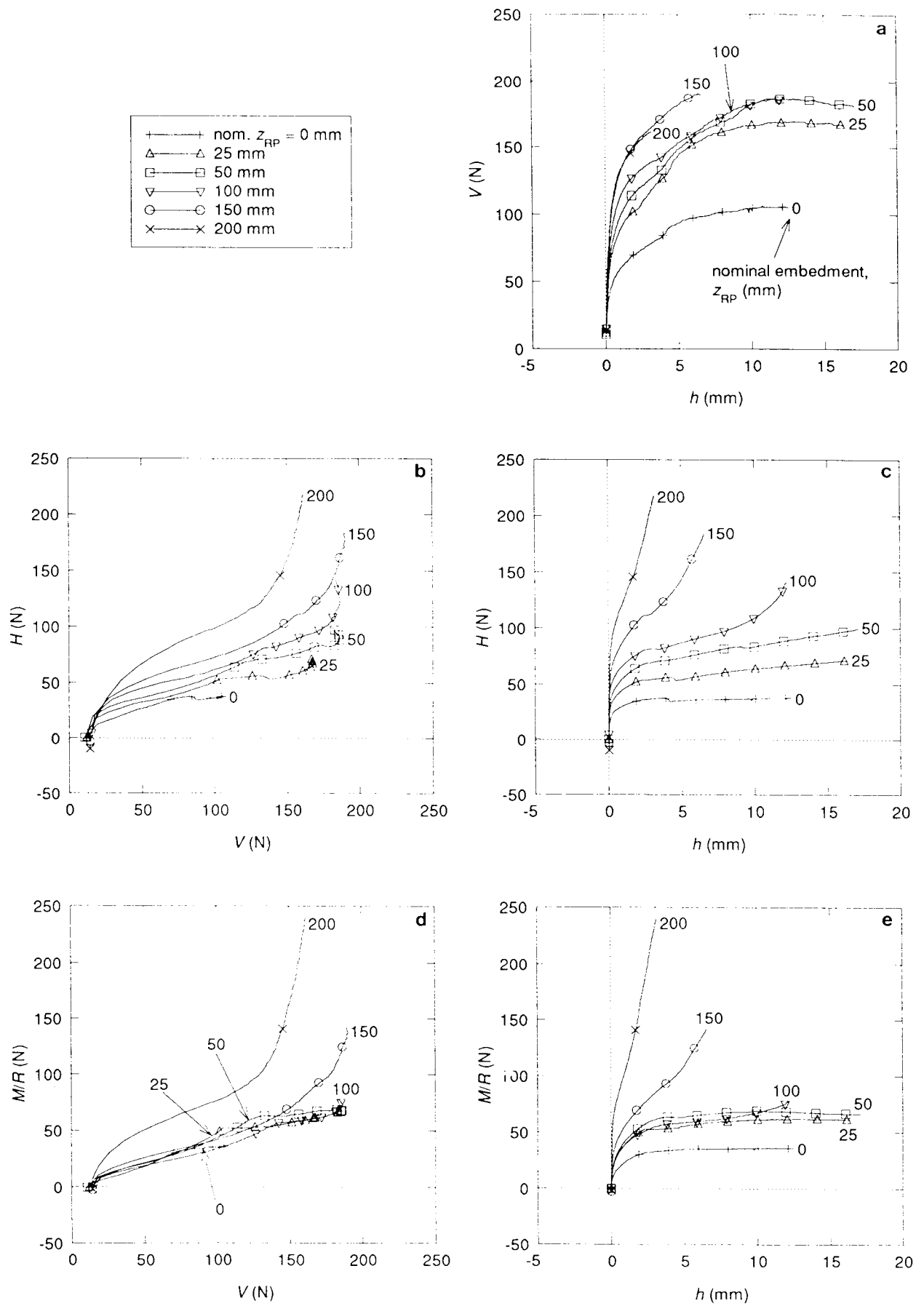


FIGURE 4.11 Results of tracking events in T013 (h from low V/V_0)

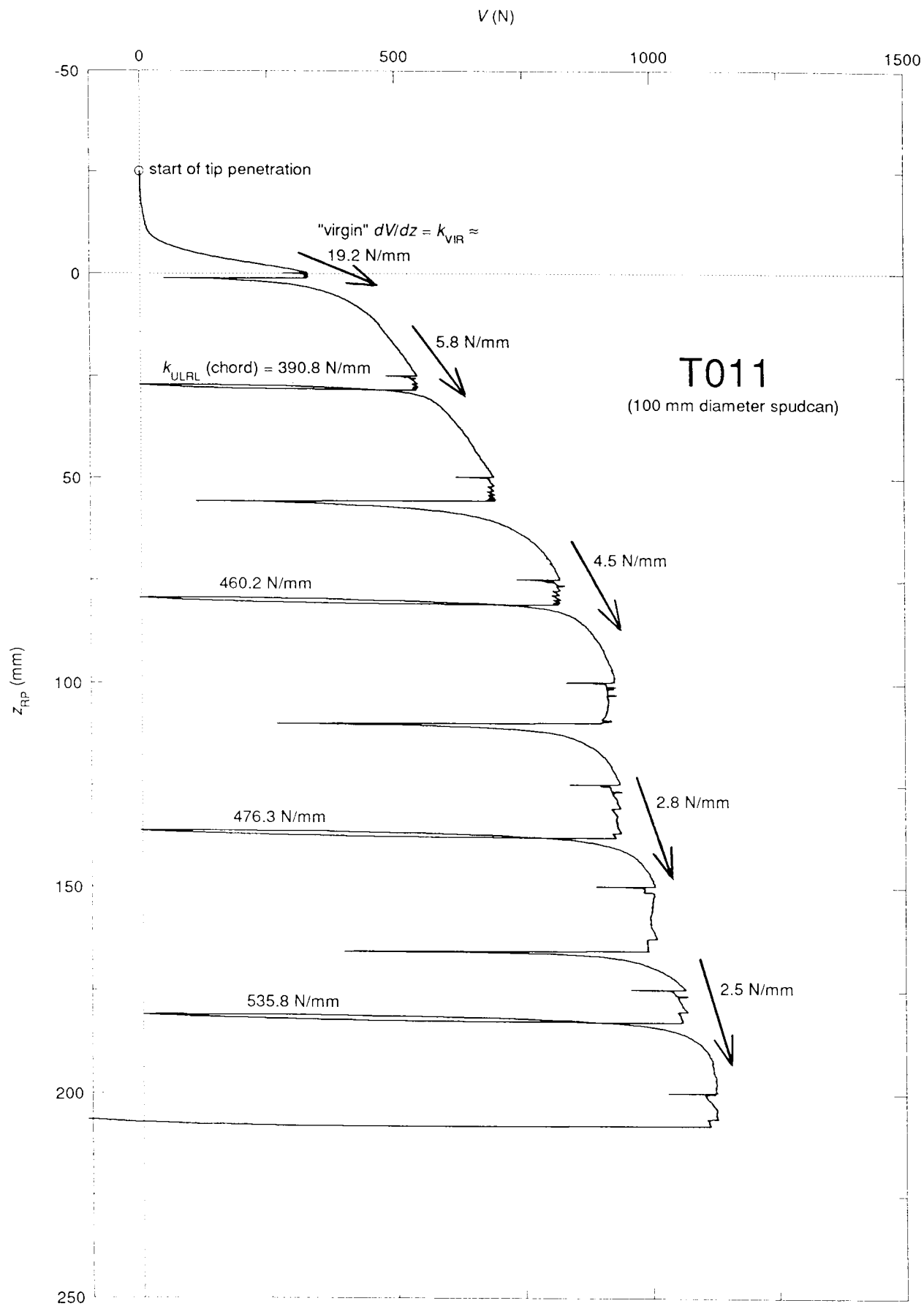


FIGURE 4.12 Unload-reload and (approximate) virgin penetration stiffnesses in T011

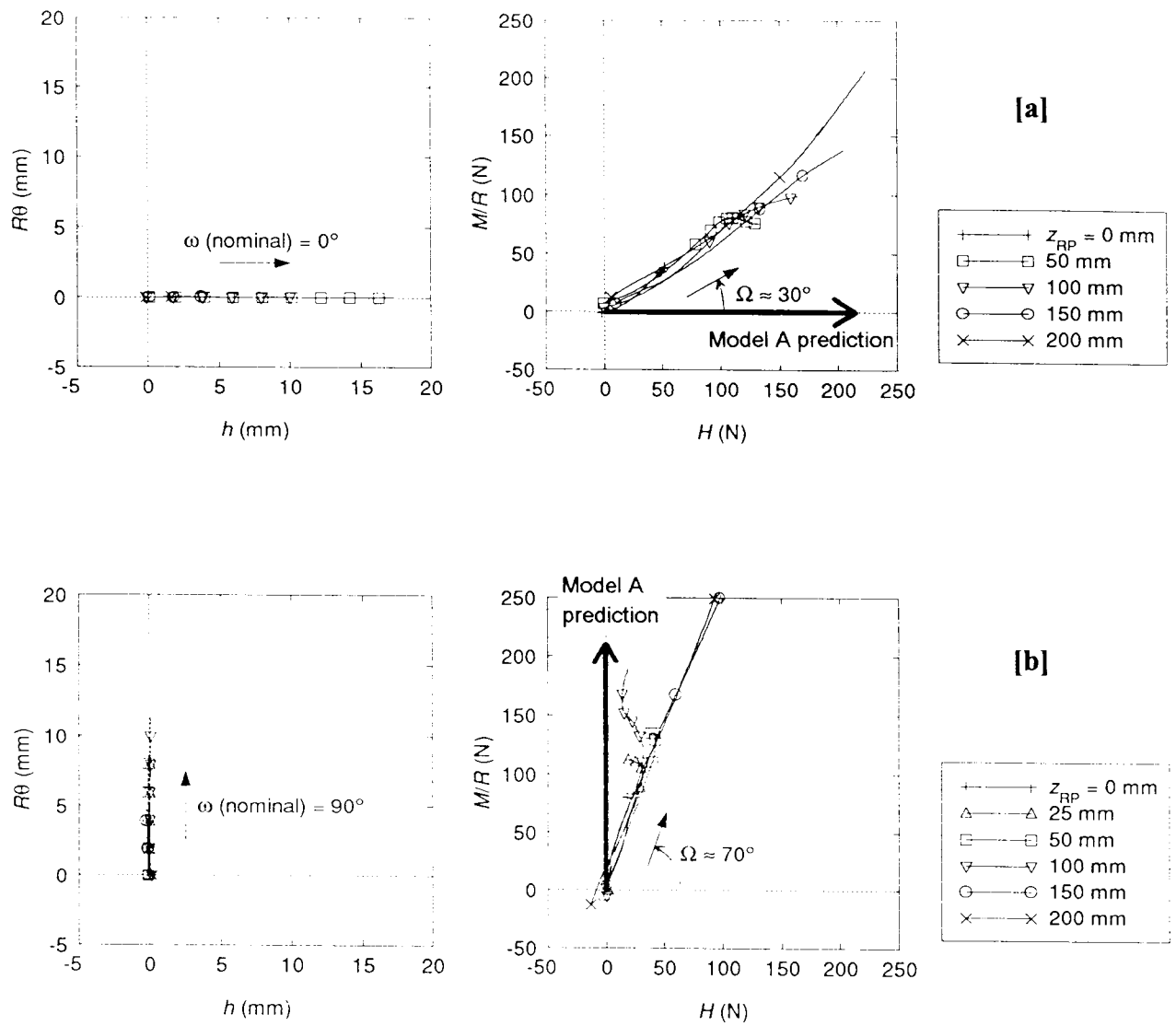


FIGURE 4.13 Imposed $h: R\theta$ displacement paths and observed $H: M/R$ load paths in [a] T011; [b] T012

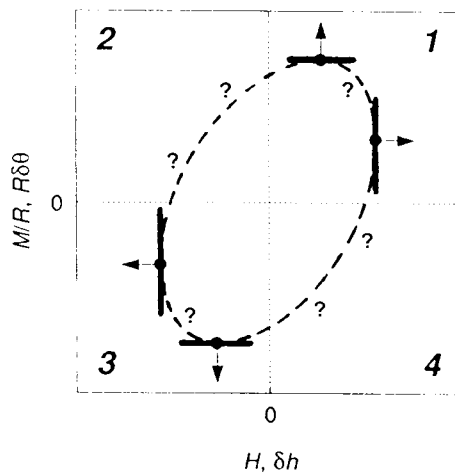


FIGURE 4.14 Cross-sectional yield surface shape suggested by Tests T011-T013, assuming normality in the $H: M/R$ plane

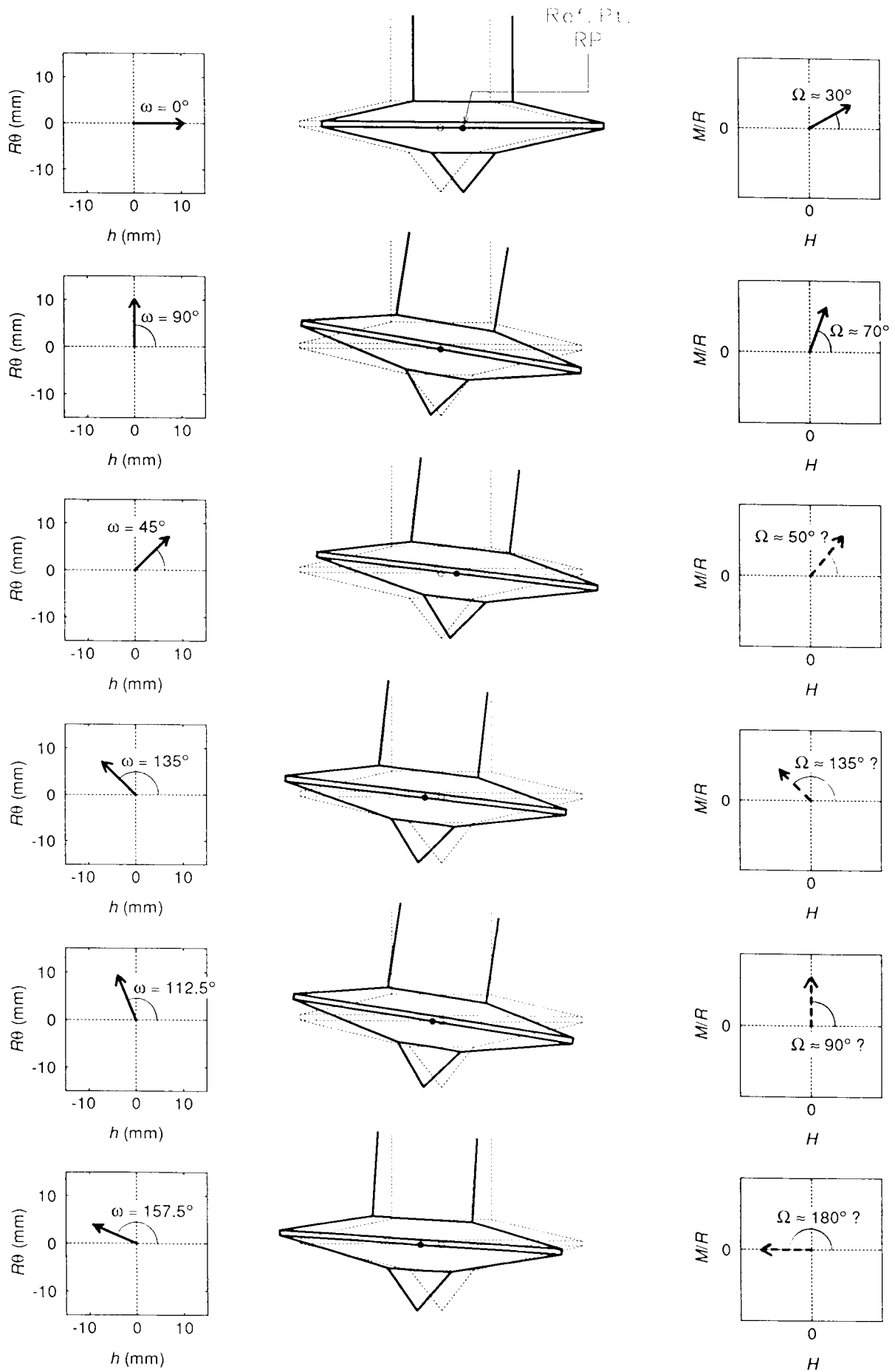


FIGURE 4.15 Combinations of horizontal and rotational displacement used in the experimental programme, together with anticipated load path directions Ω (each sketch shows, for a 125 mm diameter spudcan, 10 mm of displacement in h : $R\theta$ space at constant z)

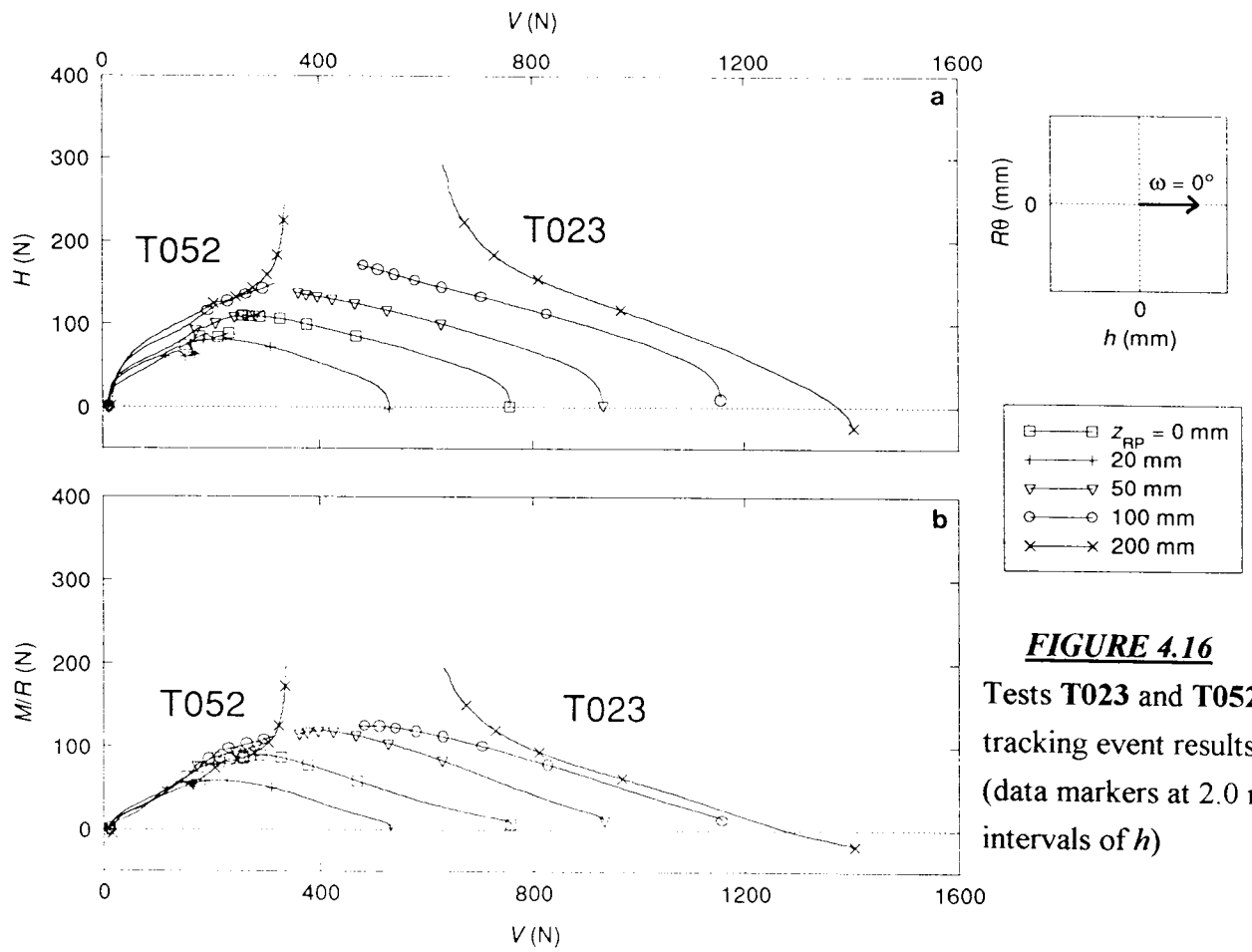


FIGURE 4.16

Tests **T023** and **T052**:
tracking event results
(data markers at 2.0 mm
intervals of h)

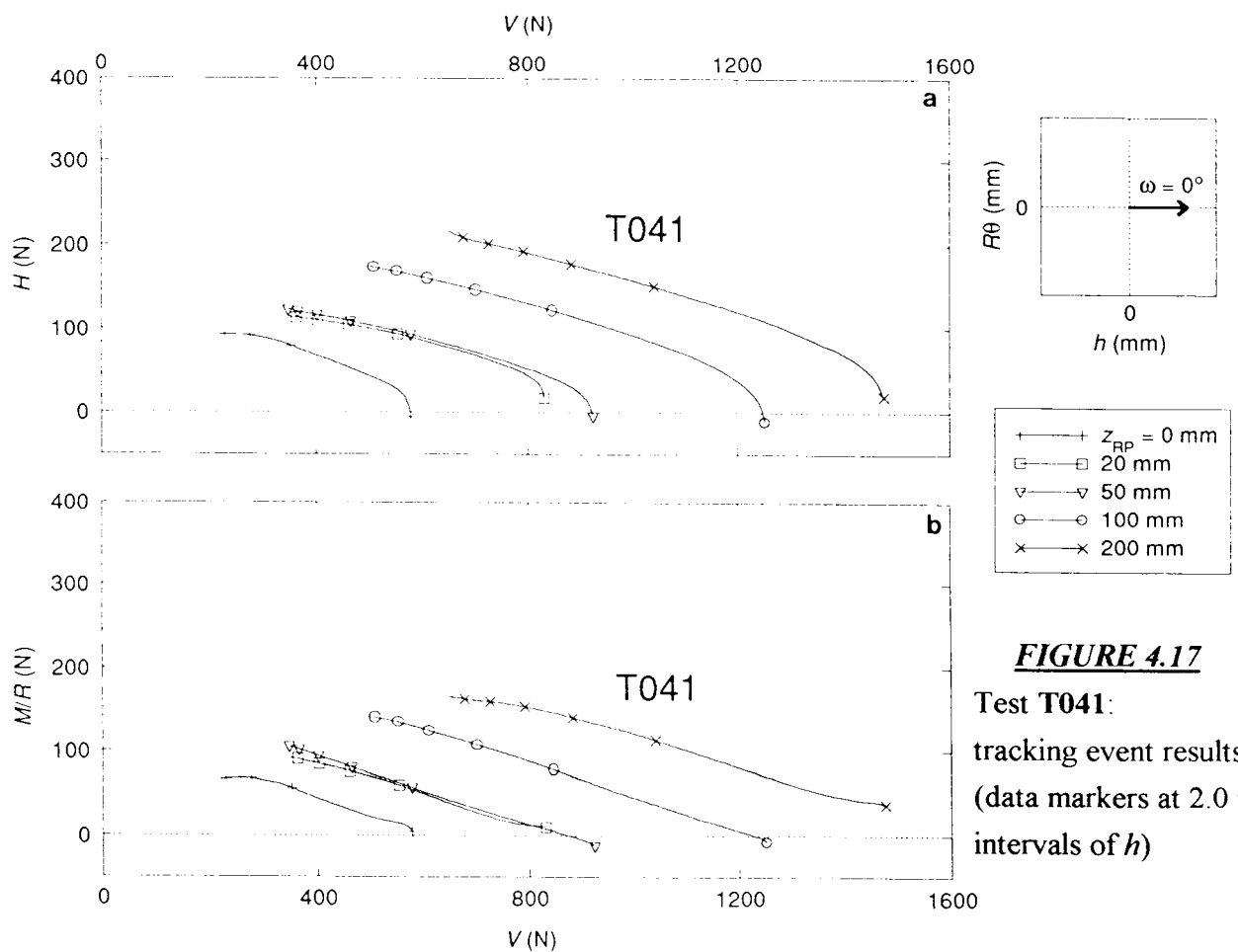


FIGURE 4.17

Test **T041**:
tracking event results
(data markers at 2.0 mm
intervals of h)

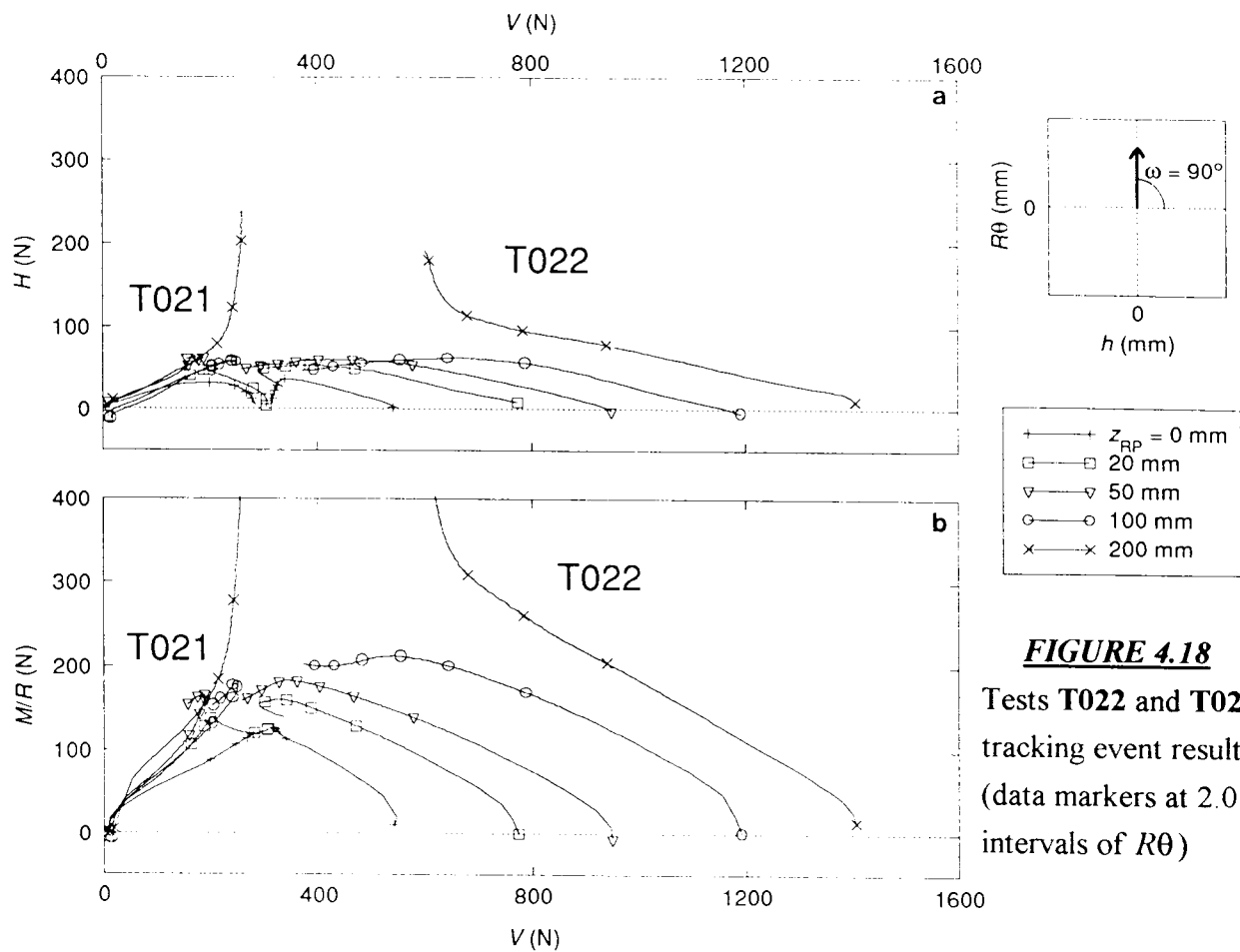


FIGURE 4.18
Tests T022 and T021:
tracking event results
(data markers at 2.0 mm
intervals of $R\theta$)

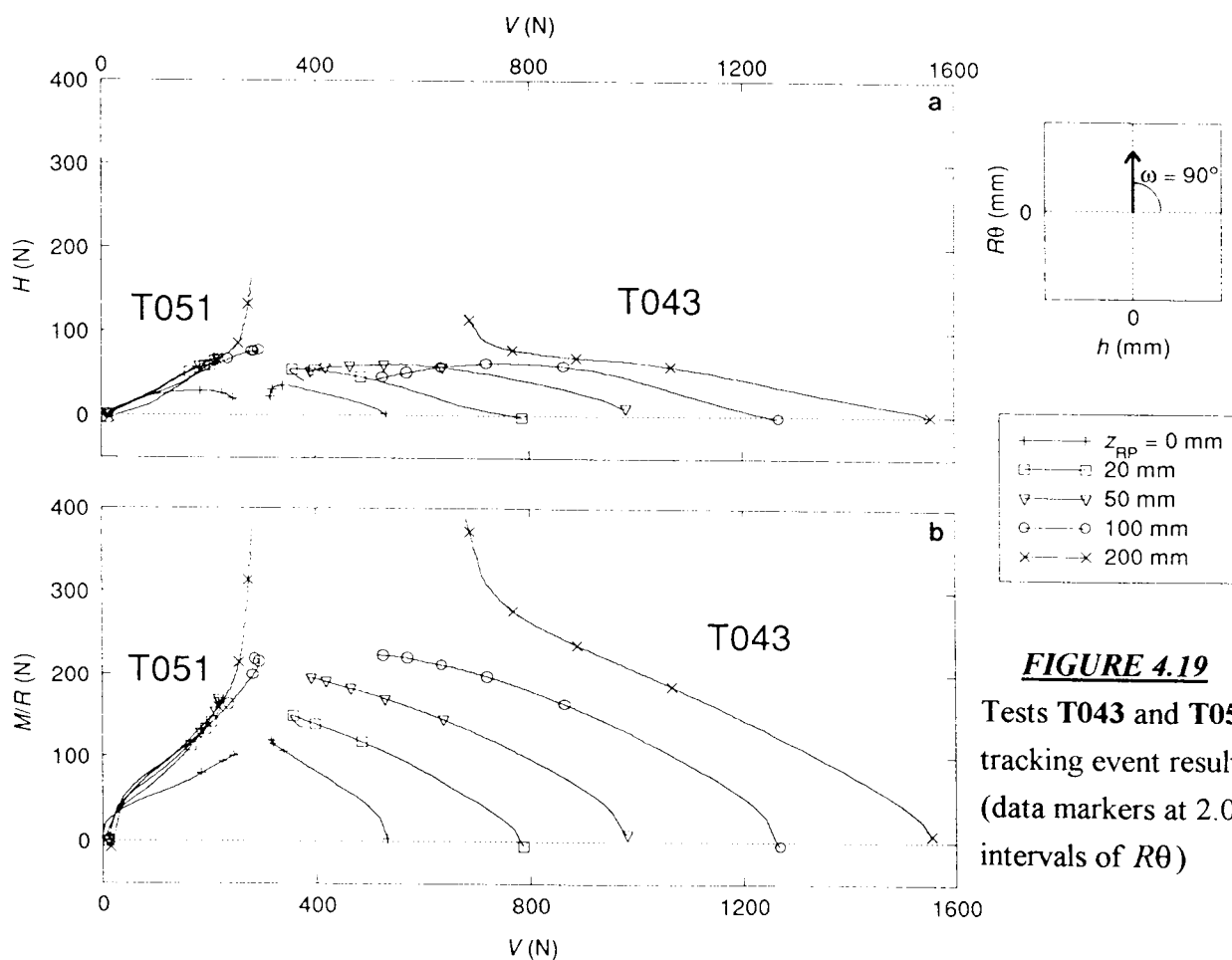


FIGURE 4.19
Tests T043 and T051:
tracking event results
(data markers at 2.0 mm
intervals of $R\theta$)

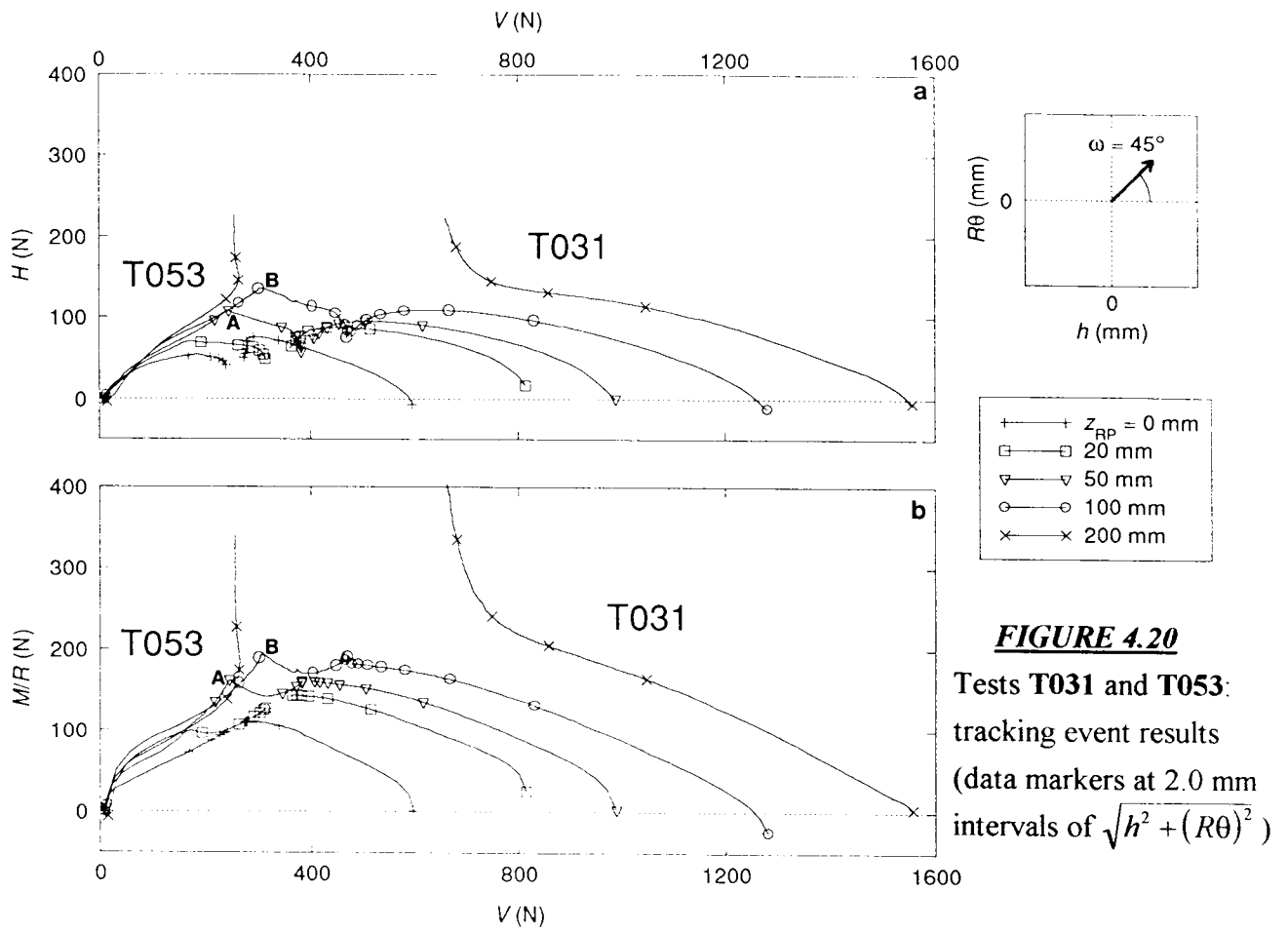


FIGURE 4.20

Tests T031 and T053:
tracking event results
(data markers at 2.0 mm
intervals of $\sqrt{h^2 + (R\theta)^2}$)

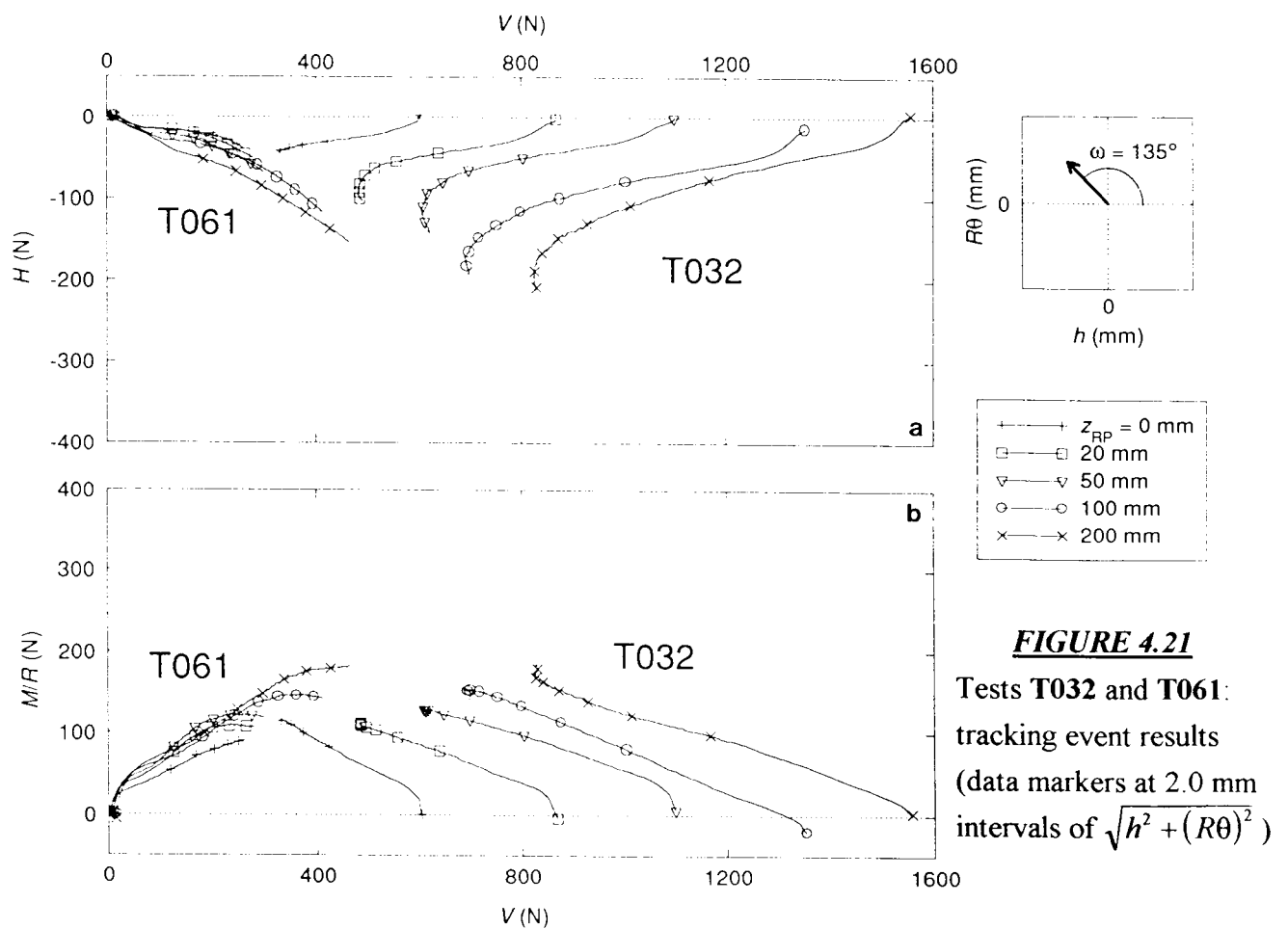


FIGURE 4.21

Tests T032 and T061:
tracking event results
(data markers at 2.0 mm
intervals of $\sqrt{h^2 + (R\theta)^2}$)

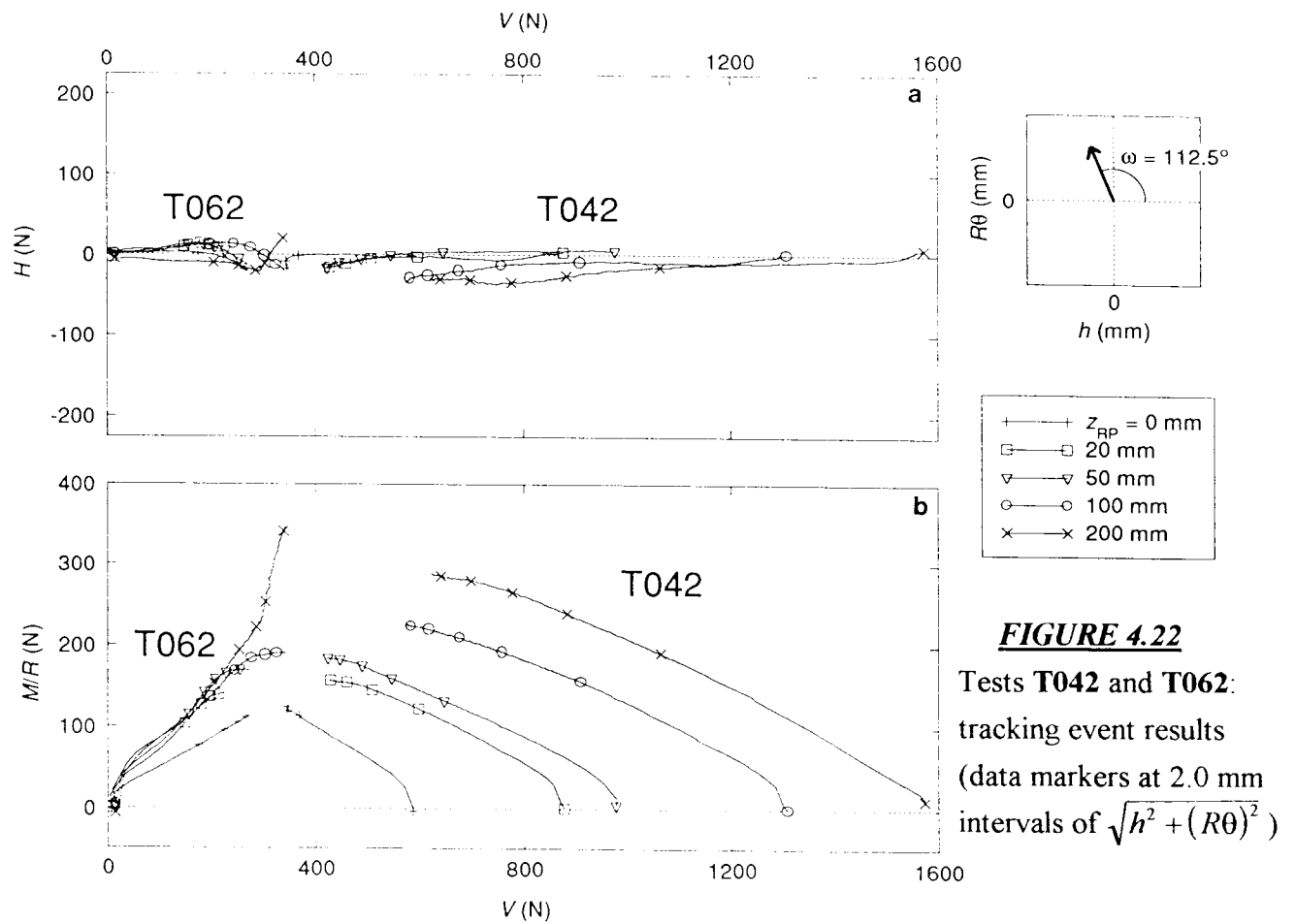


FIGURE 4.22

Tests **T042** and **T062**:
tracking event results
(data markers at 2.0 mm
intervals of $\sqrt{h^2 + (R\theta)^2}$)

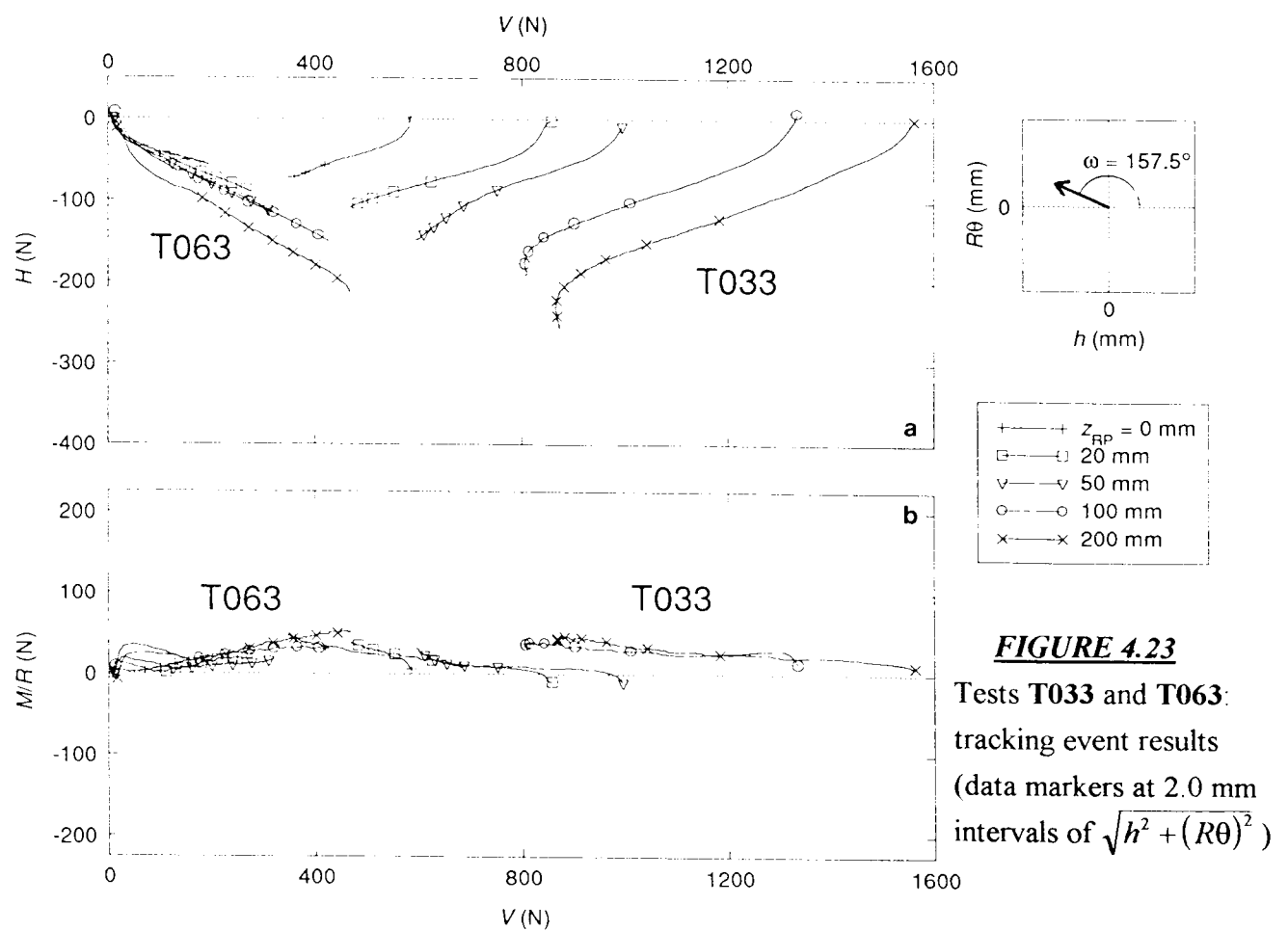


FIGURE 4.23

Tests **T033** and **T063**:
tracking event results
(data markers at 2.0 mm
intervals of $\sqrt{h^2 + (R\theta)^2}$)

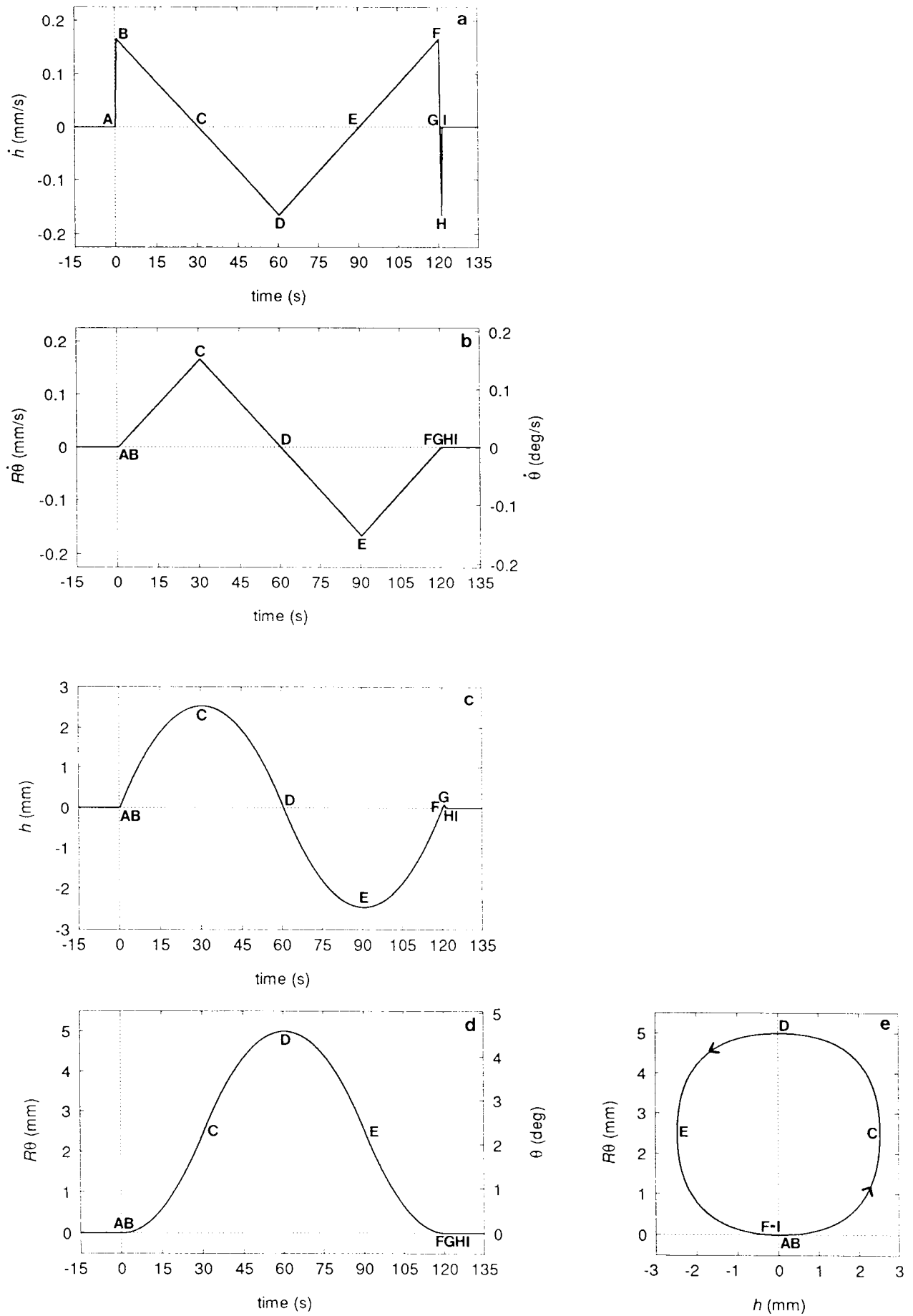


FIGURE 4.24 Generation of a closed loop in $h:R\theta$ displacement space

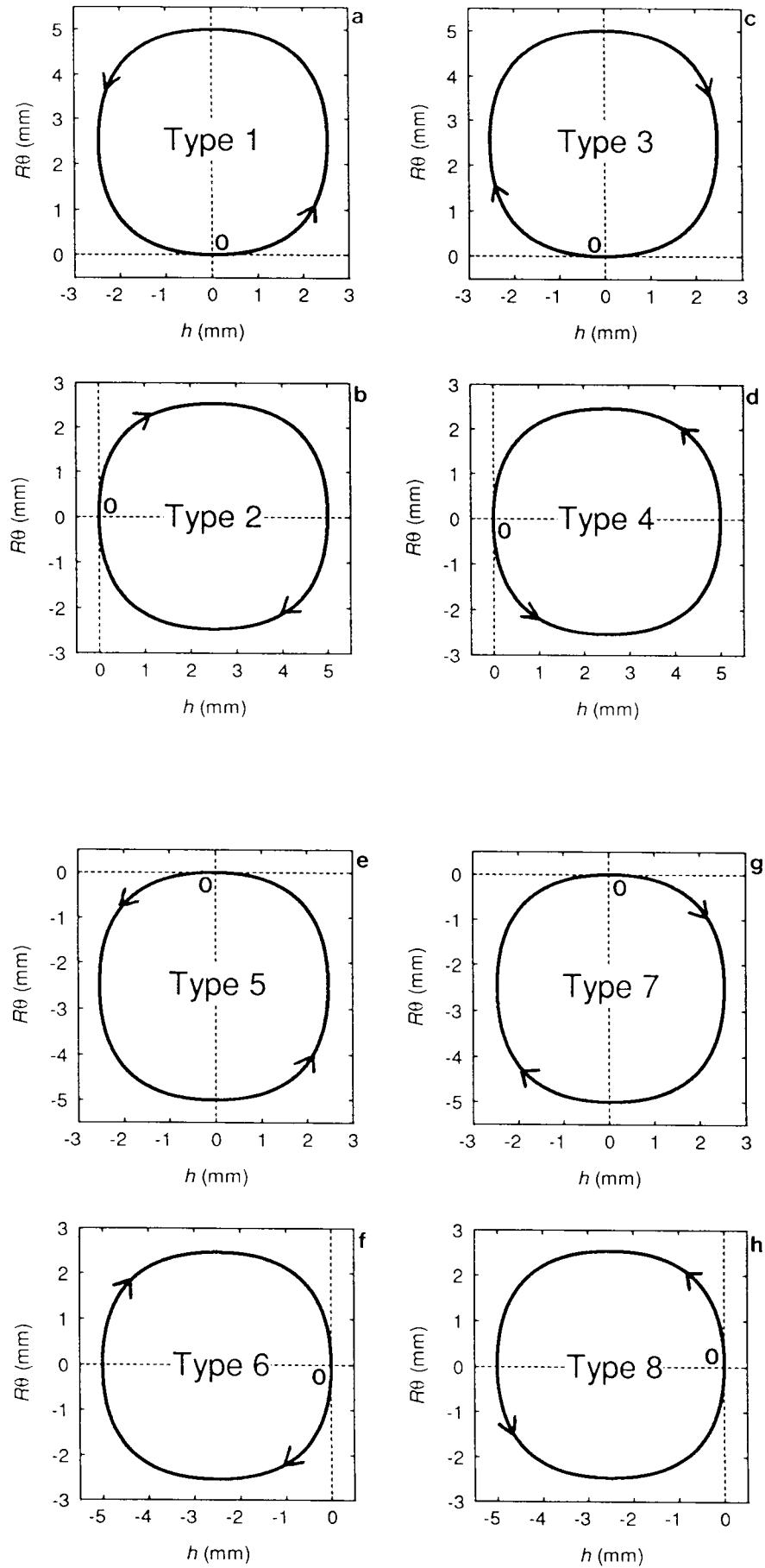
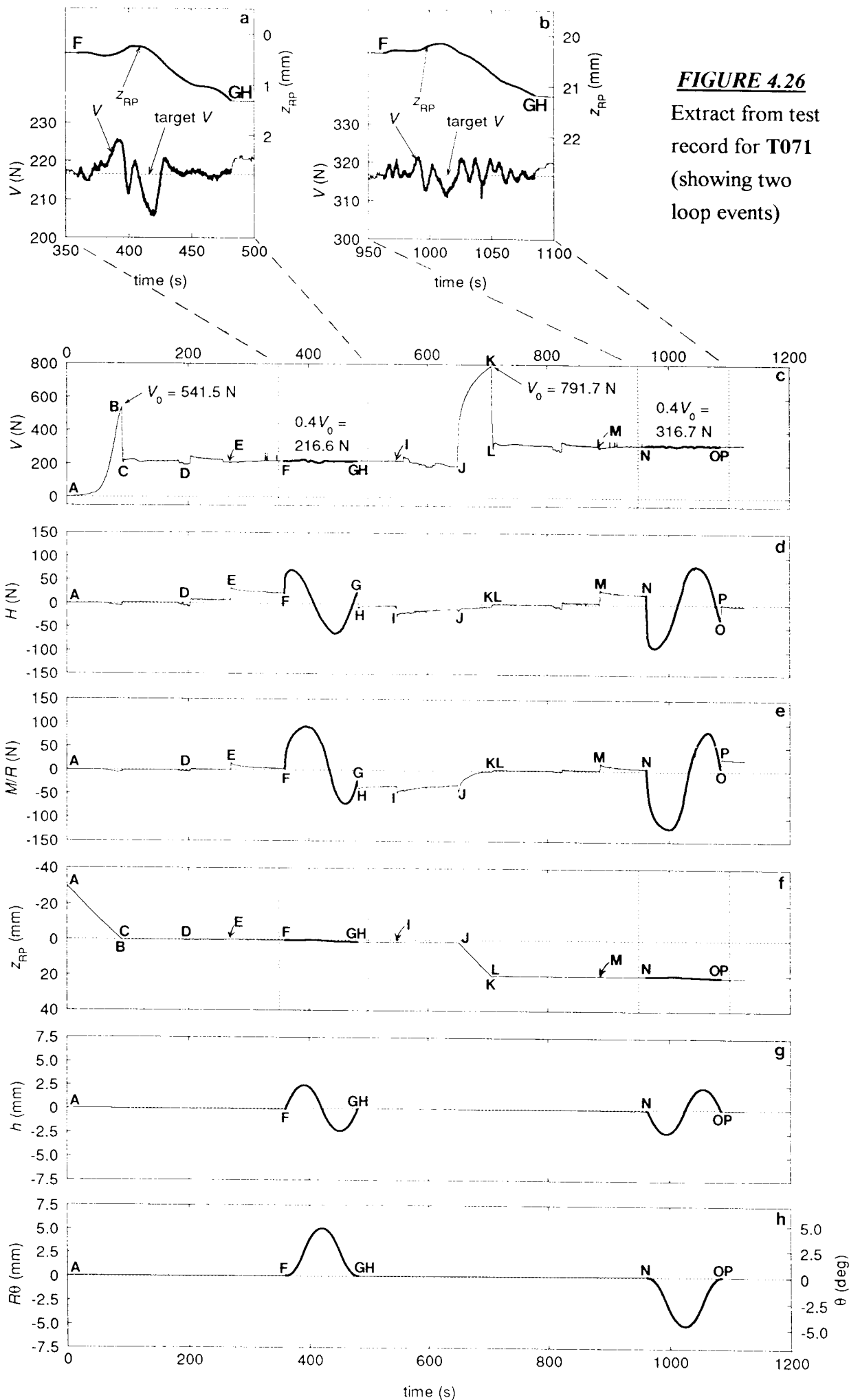


FIGURE 4.25 Definition of $h:R\theta$ loop types



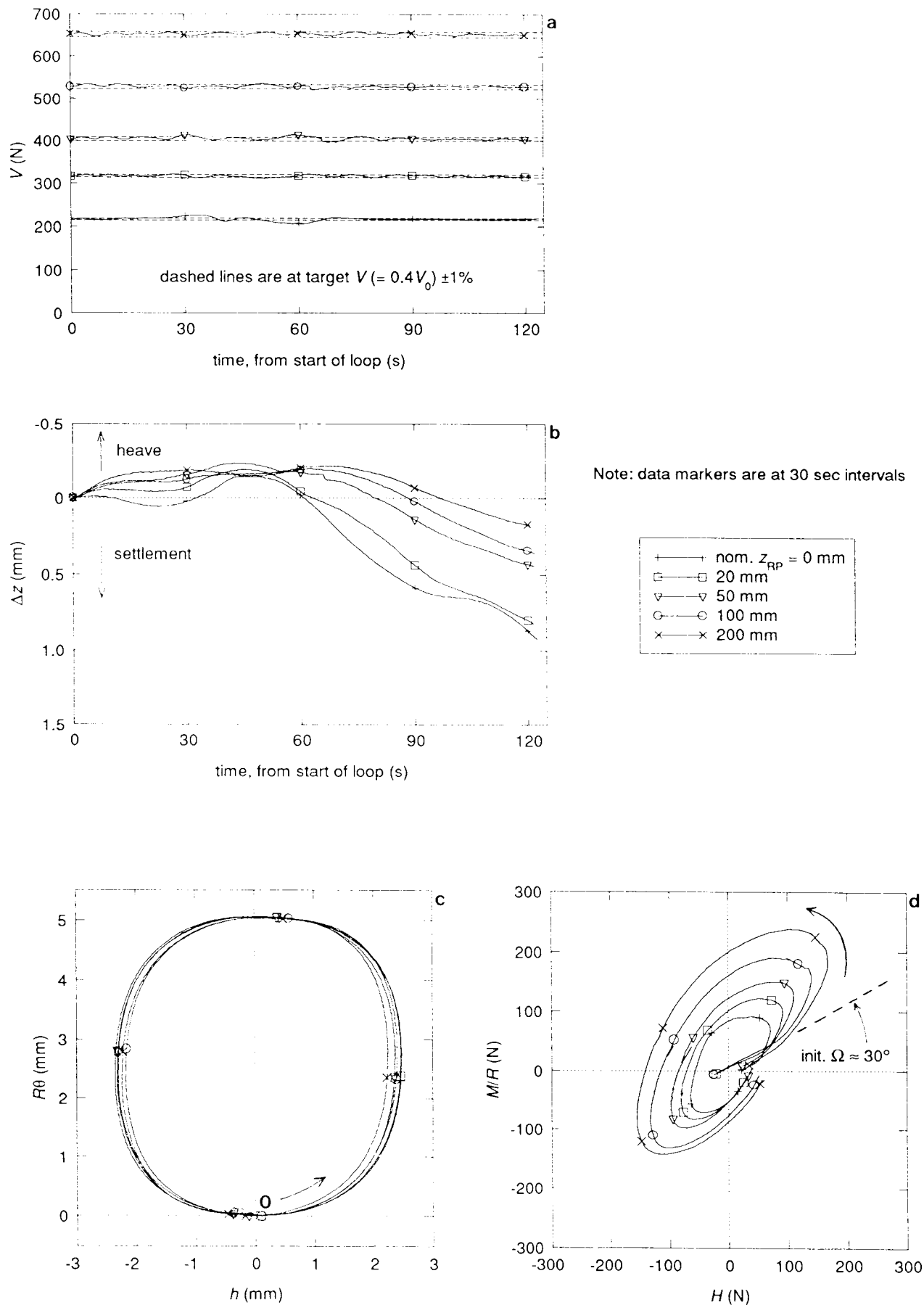


FIGURE 4.27 Results of loop events in T071 (anticlockwise, h leading)

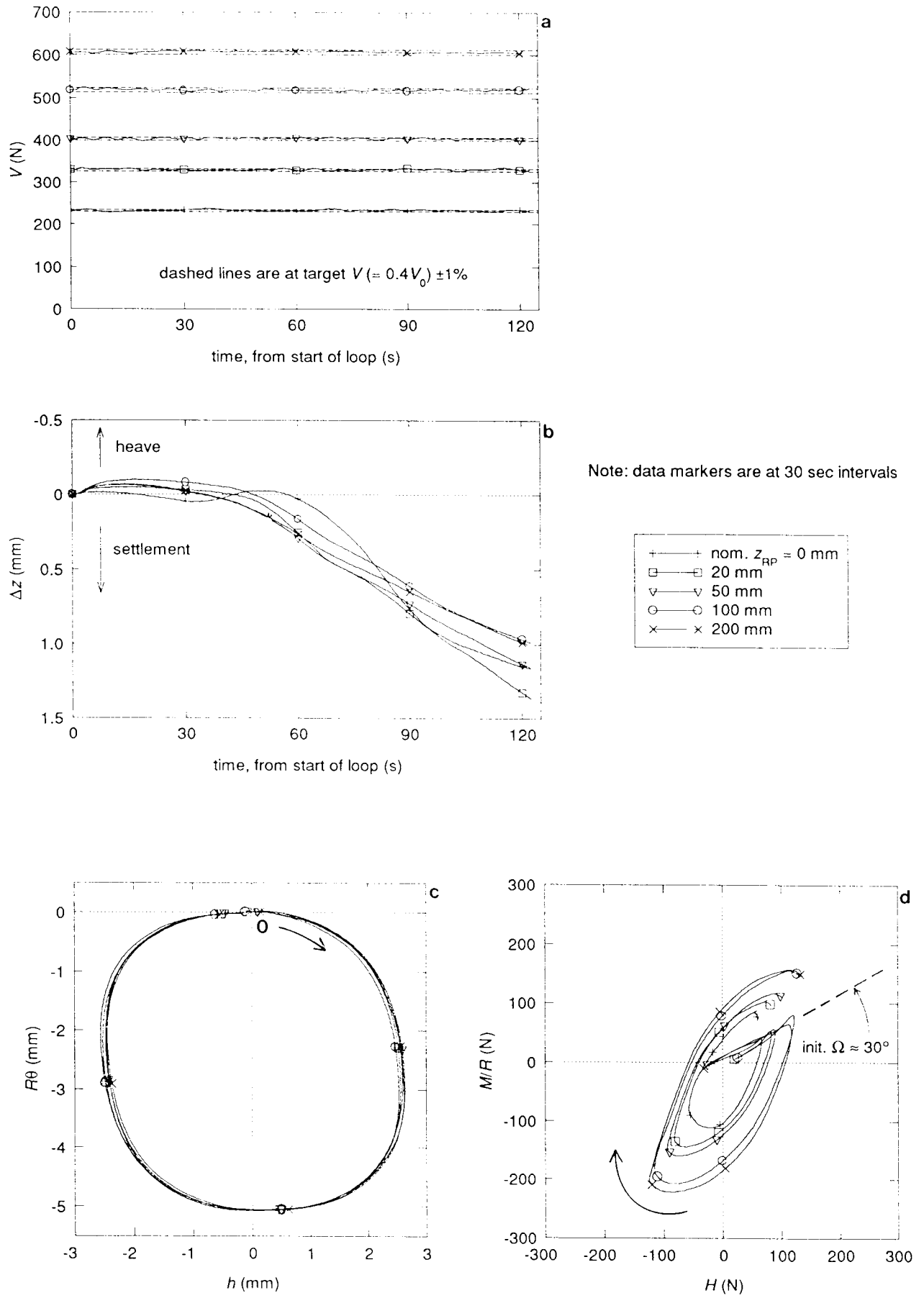


FIGURE 4.28 Results of loop events in T072 (clockwise, h leading)

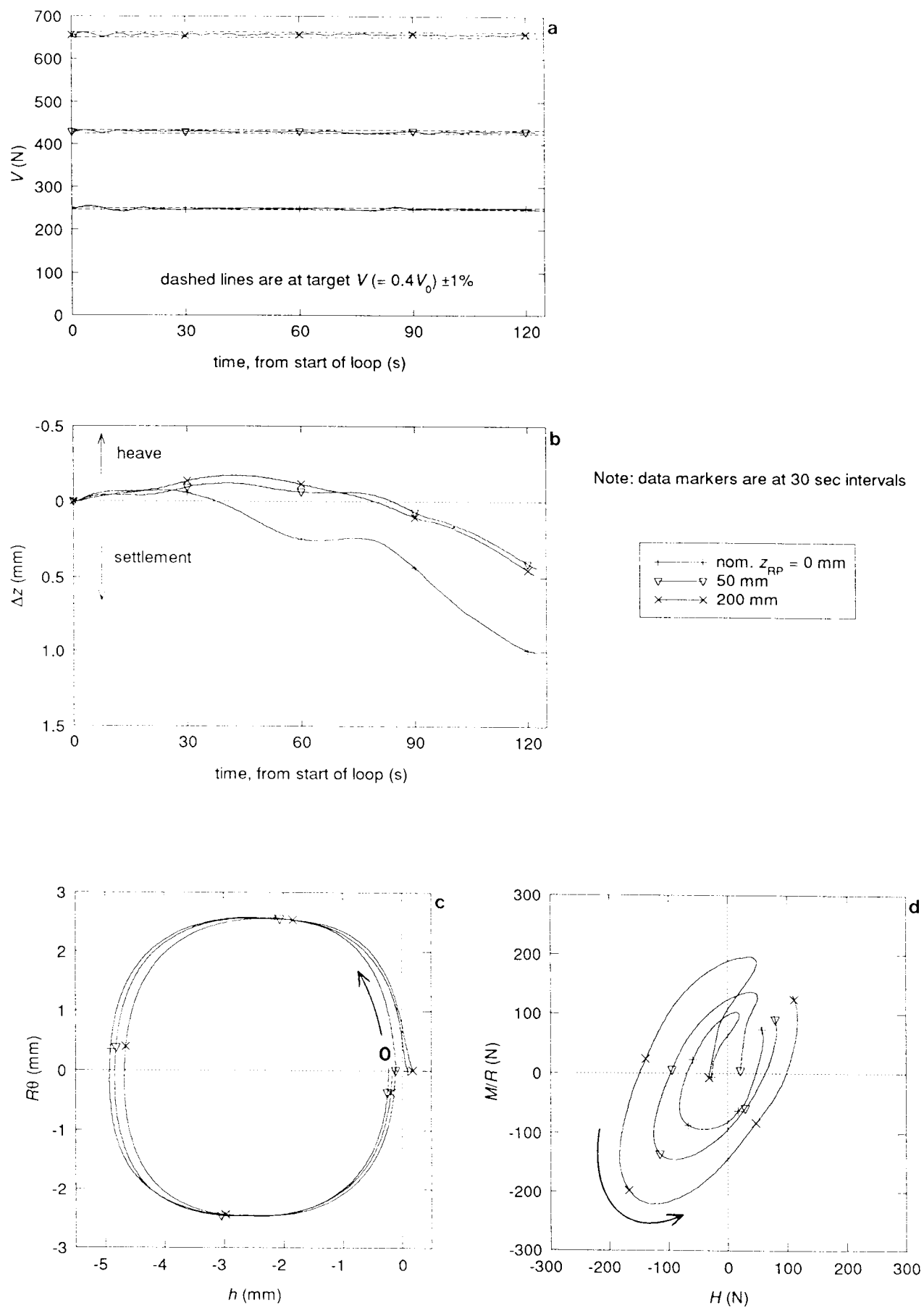


FIGURE 4.29 Results of loop events in T073a (anticlockwise, θ leading)

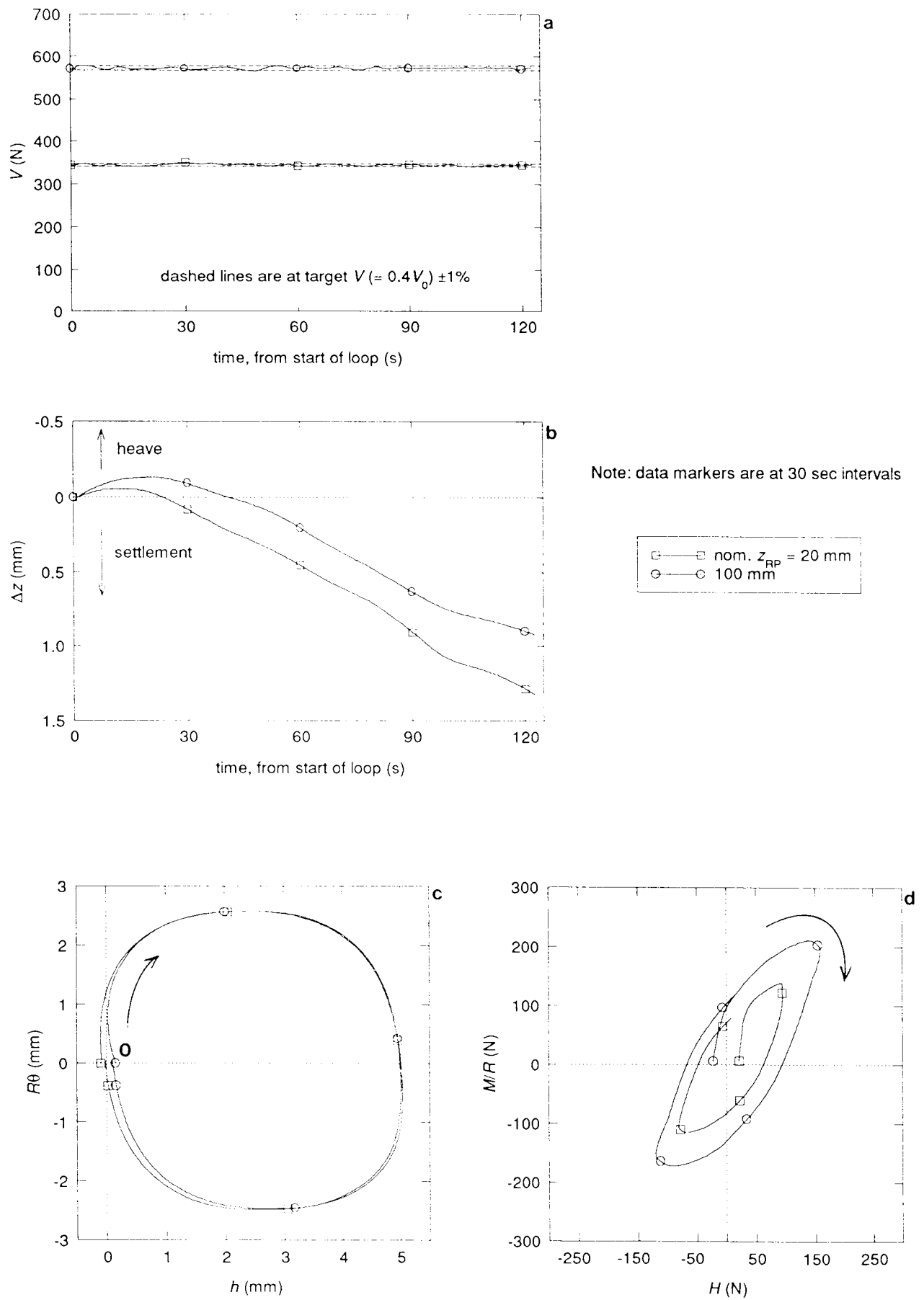


FIGURE 4.30 Results of loop events in **T073b** (clockwise, θ leading)

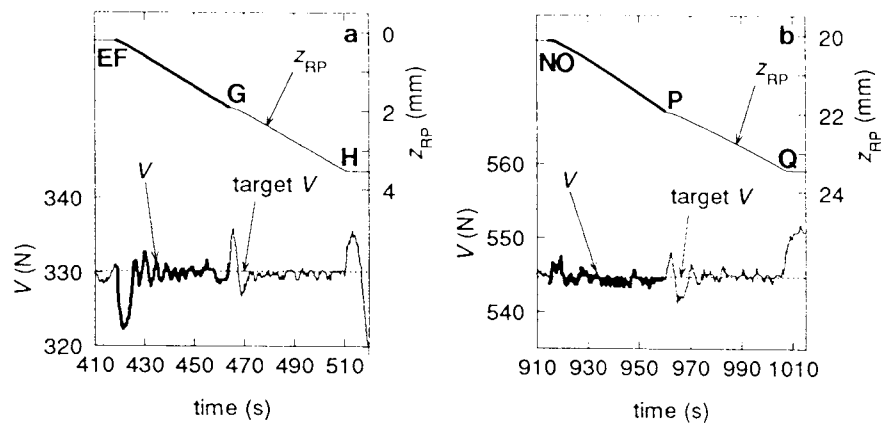
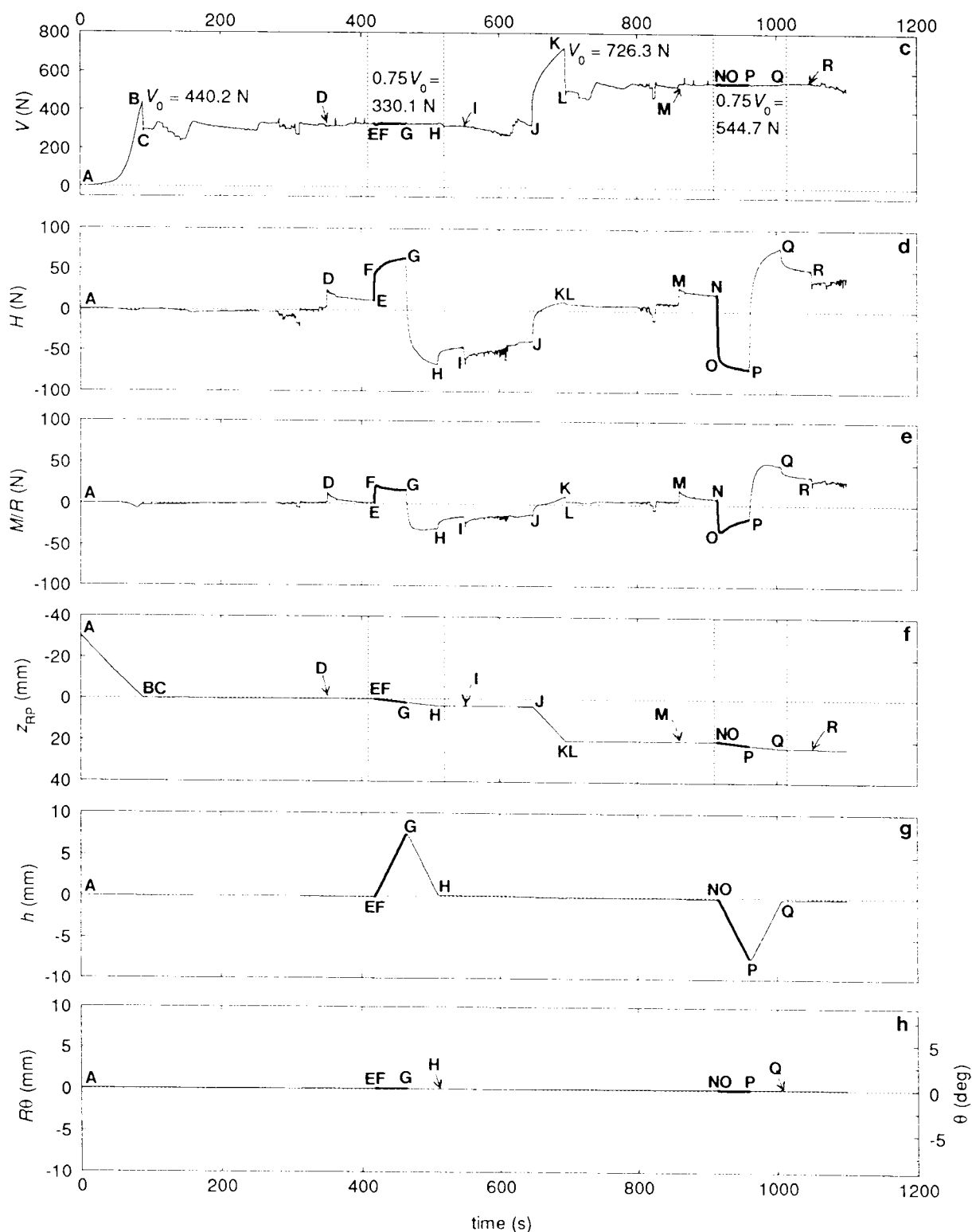
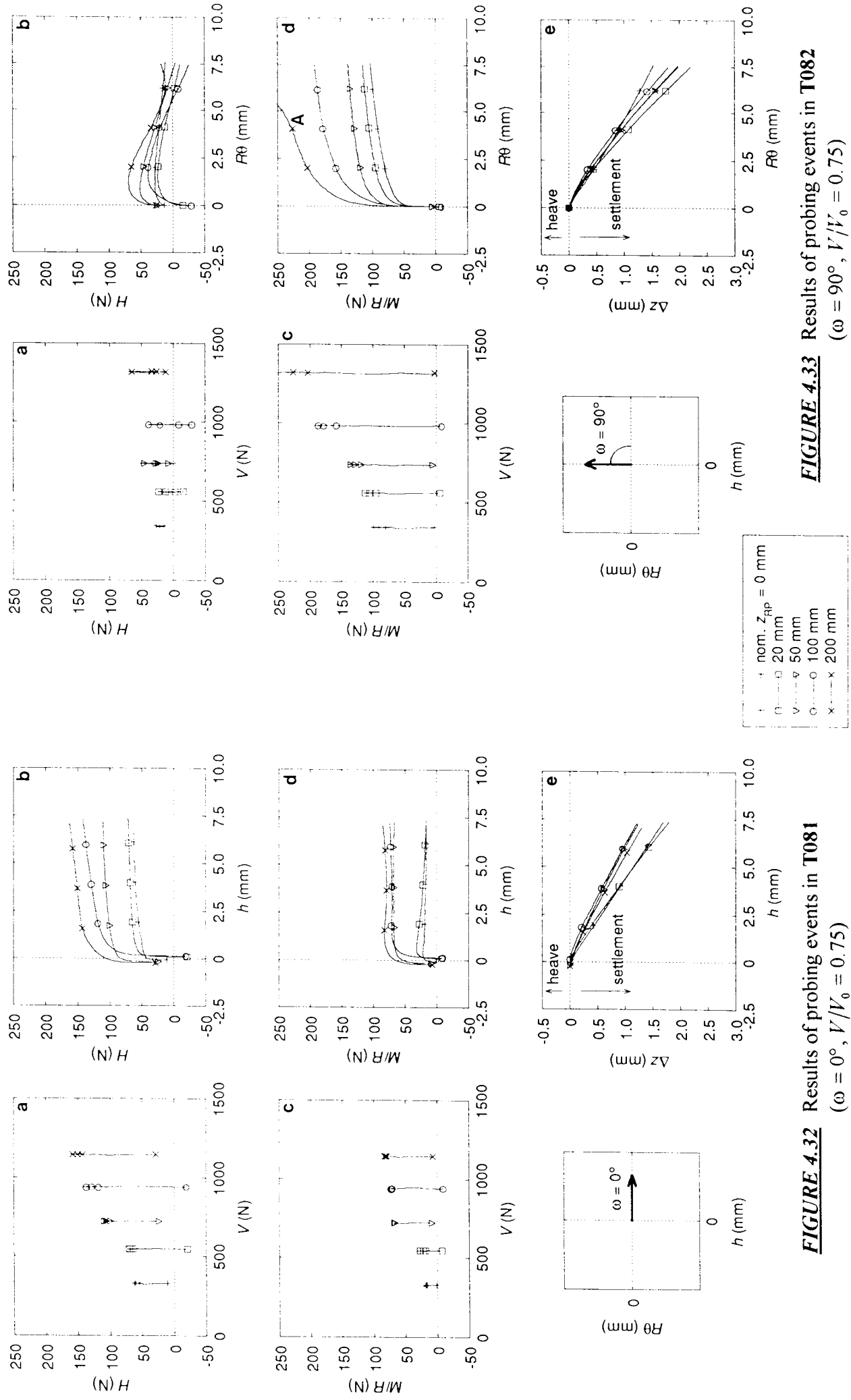


FIGURE 4.31

Extract from test record for T081 (showing two probing events)





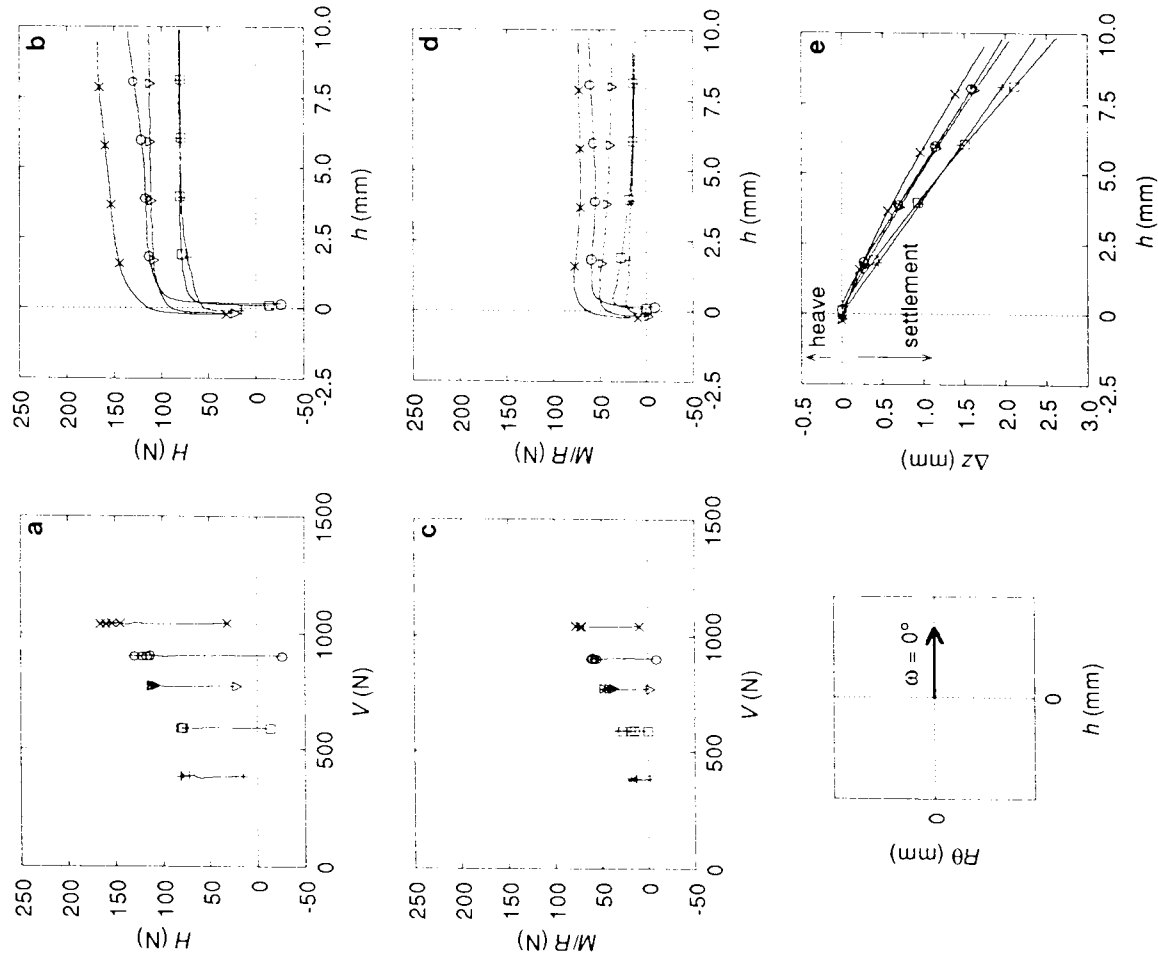


FIGURE 4.34 Results of probing events in T101
 $(\omega = 0^\circ, V/V_0 = 0.75)$

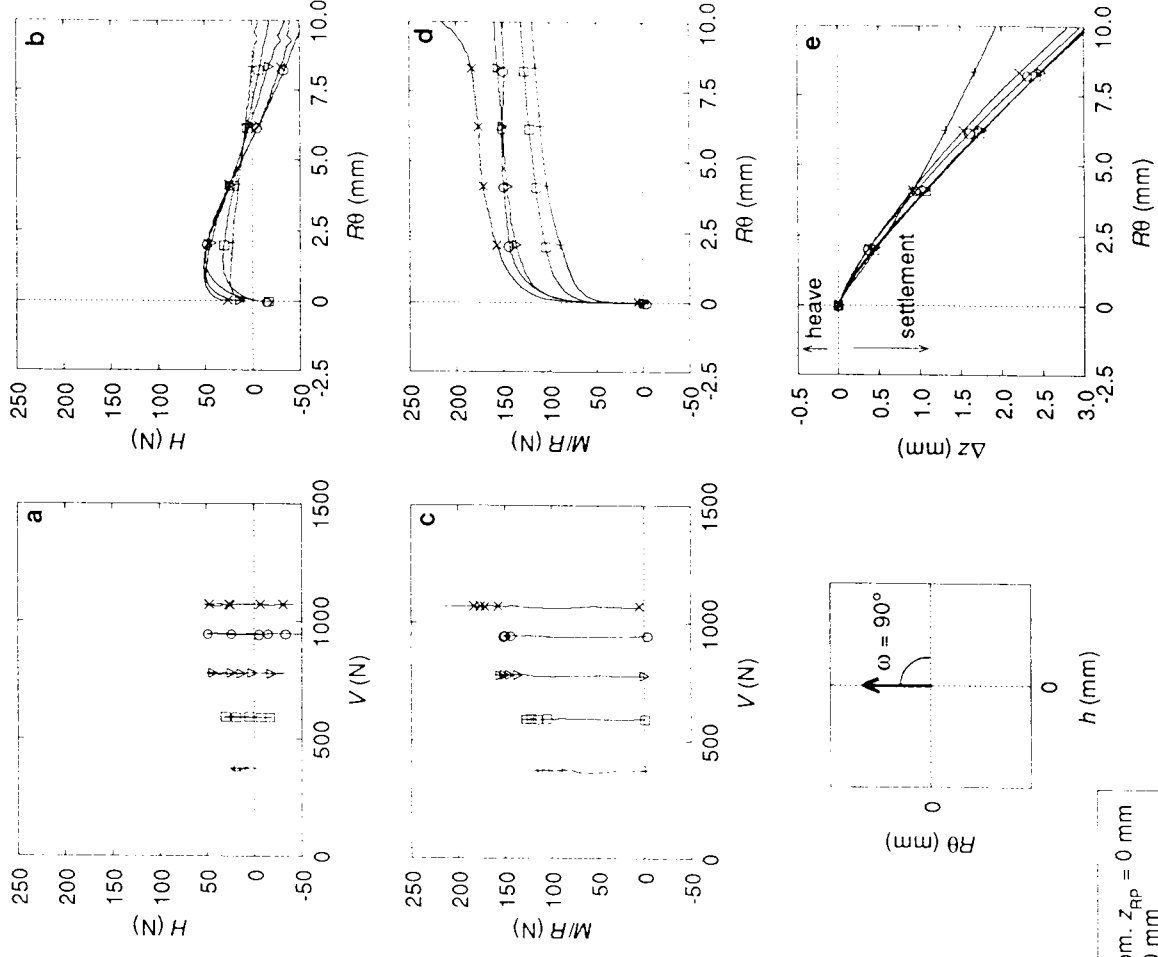


FIGURE 4.35 Results of probing events in T102
 $(\omega = 90^\circ, V/V_0 = 0.75)$

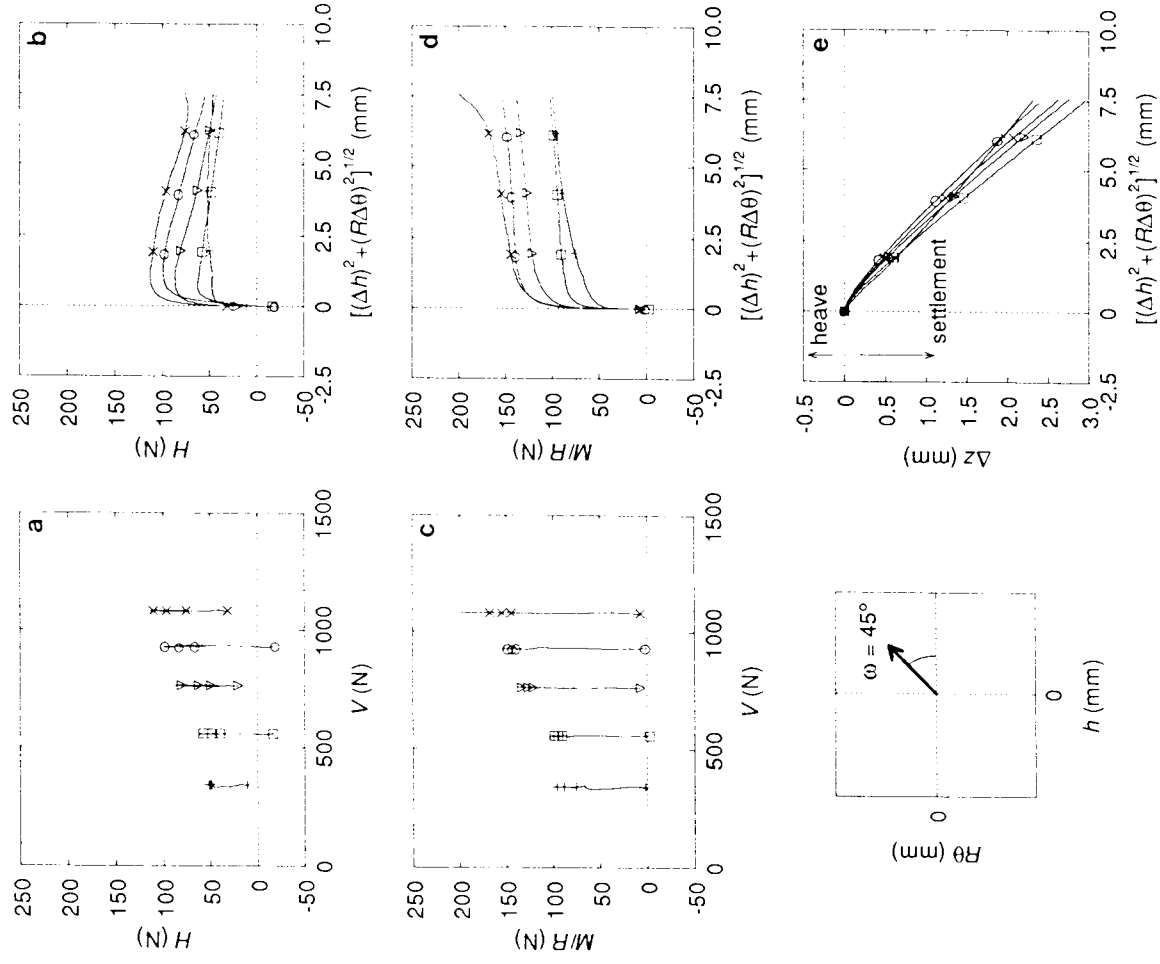


FIGURE 4.36 Results of probing events in T083
($\omega = 45^\circ$, $V/V_0 = 0.75$)

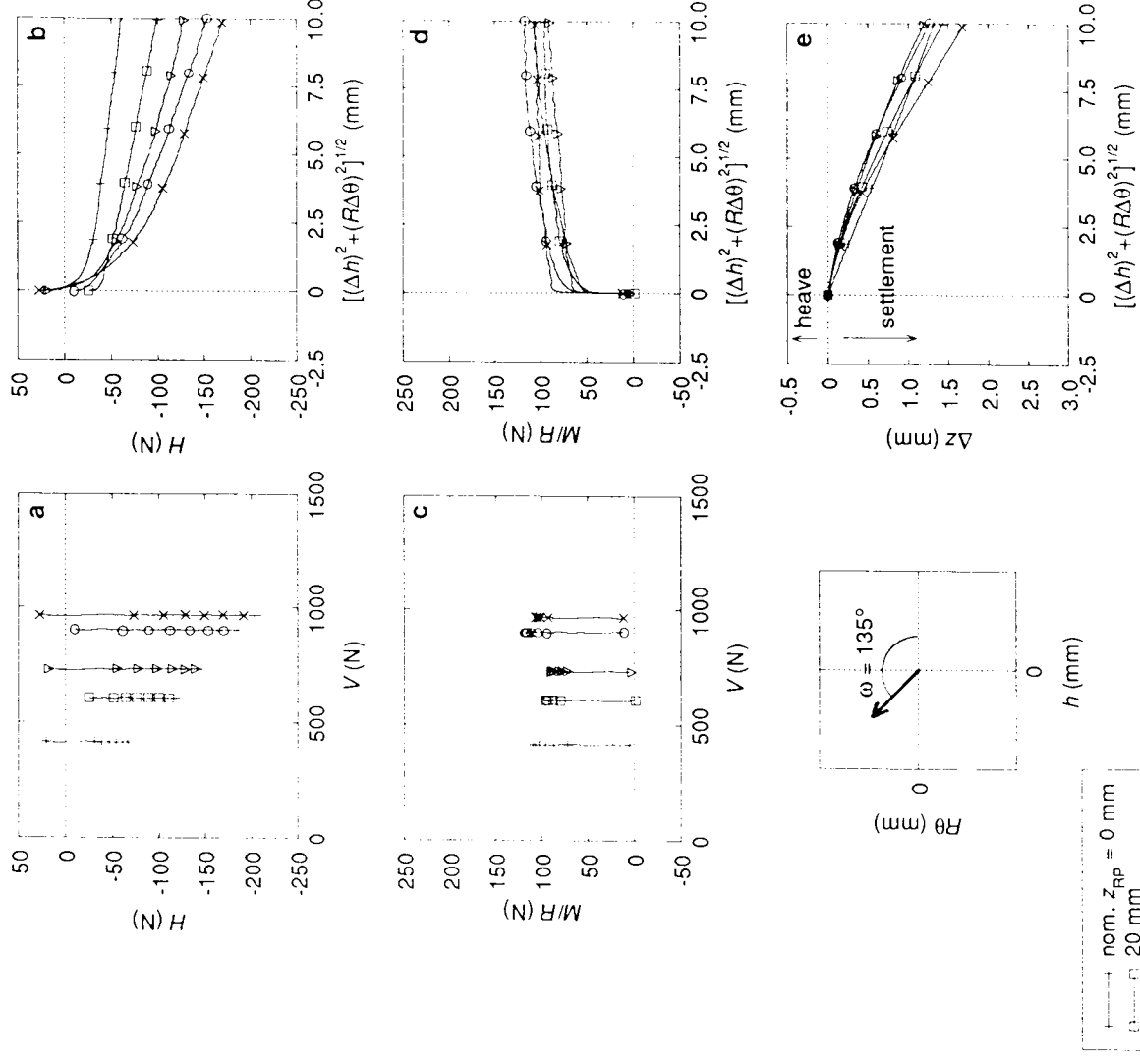


FIGURE 4.37 Results of probing events in T103
($\omega = 135^\circ$, $V/V_0 = 0.75$)

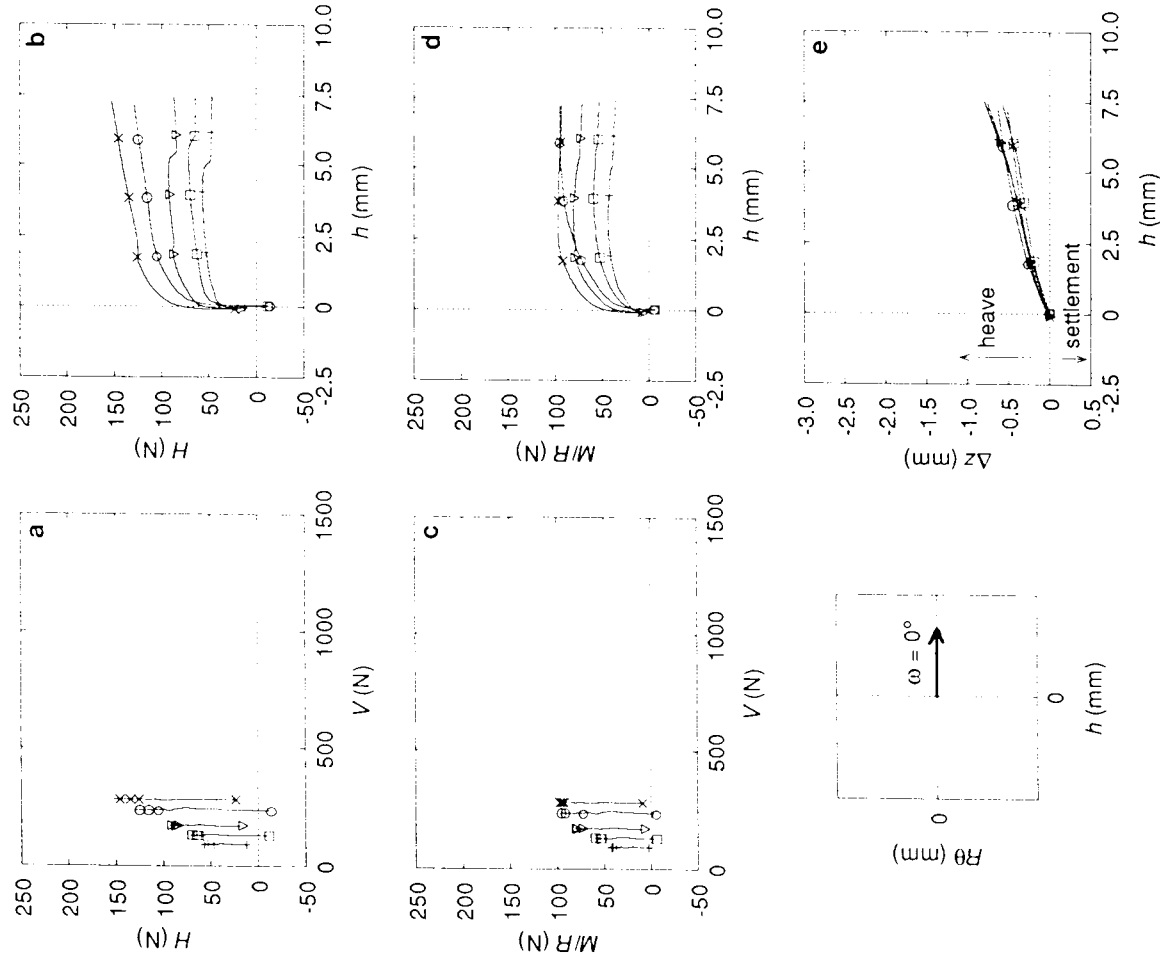


FIGURE 4.38 Results of probing events in T091
($\omega = 0^\circ$, $V/V_0 = 0.2$)

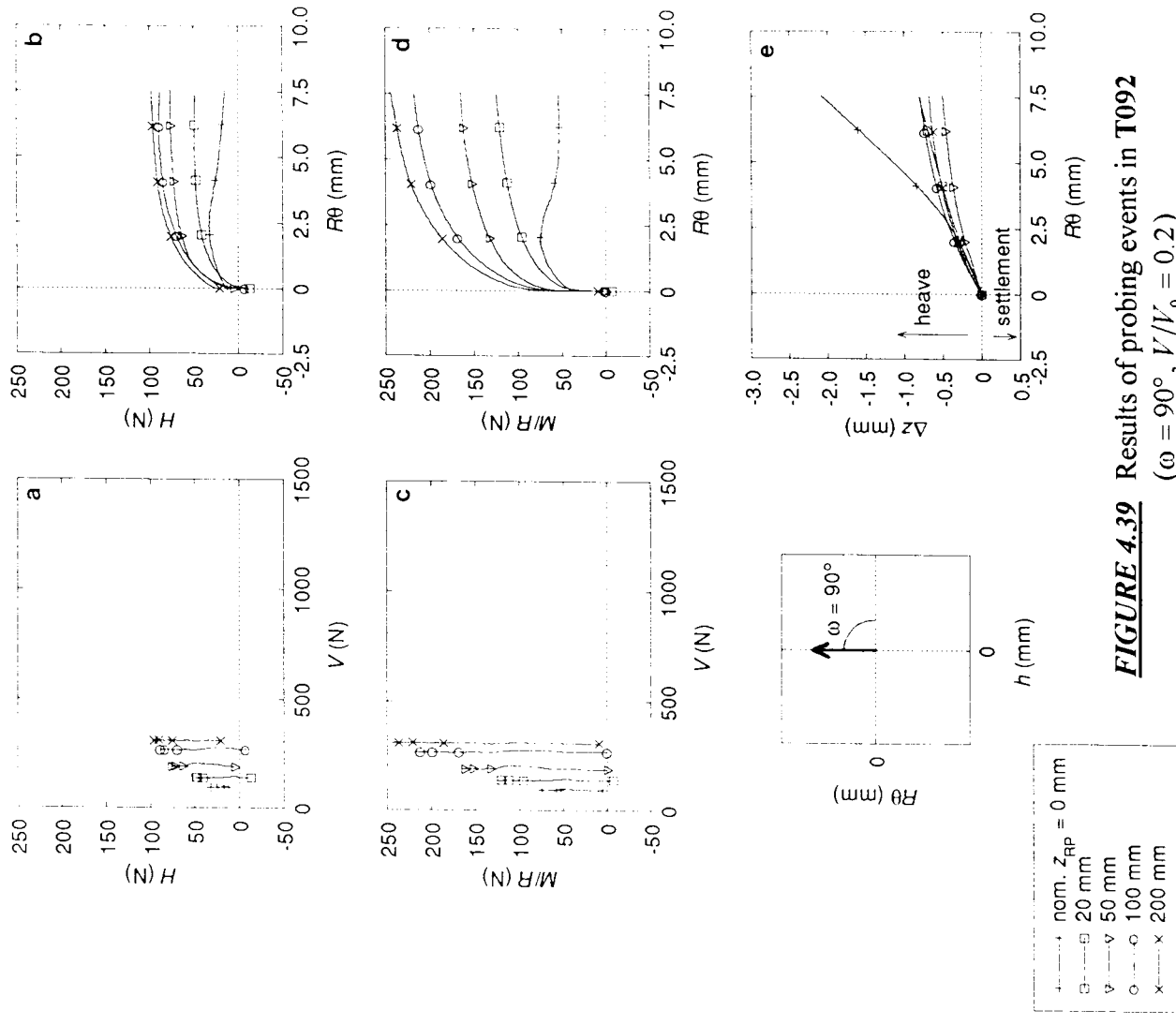
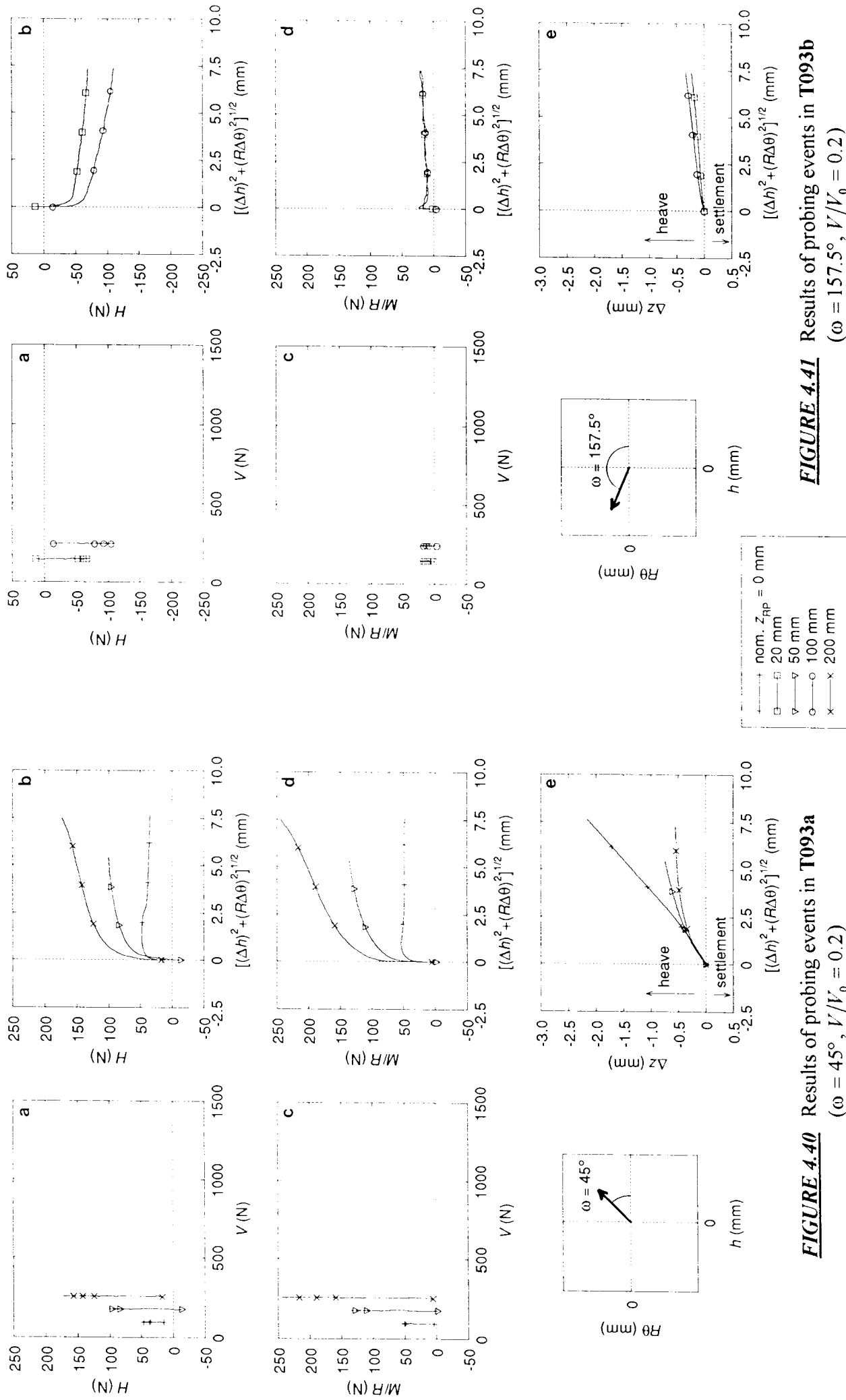


FIGURE 4.39 Results of probing events in T092
($\omega = 90^\circ$, $V/V_0 = 0.2$)



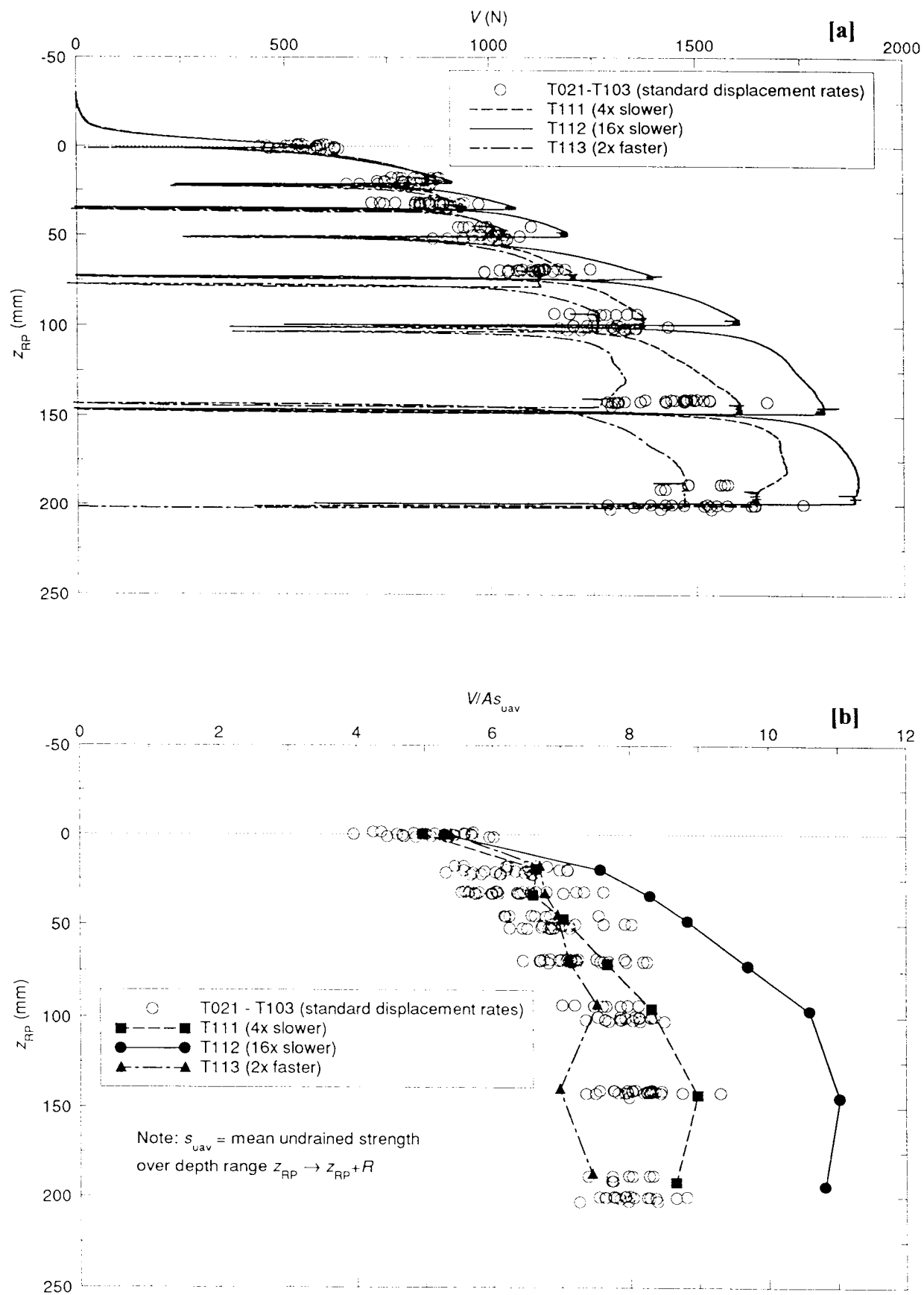


FIGURE 4.42 Effect of displacement rate on vertical load:penetration curve
[a] actual measured vertical loads; **[b]** normalised vertical loads

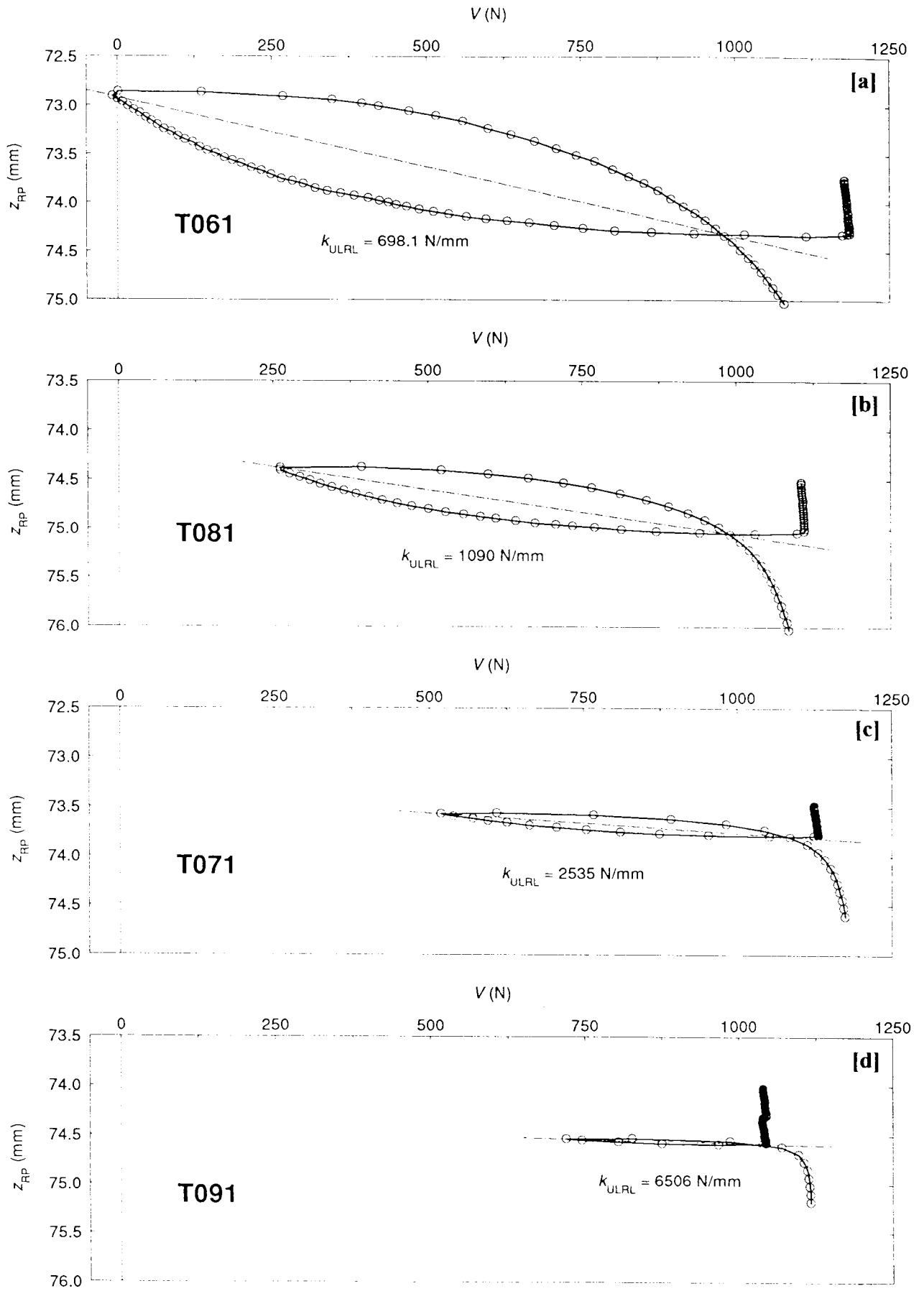


FIGURE 4.43 Dependence of chord stiffness on unload-reload loop amplitude

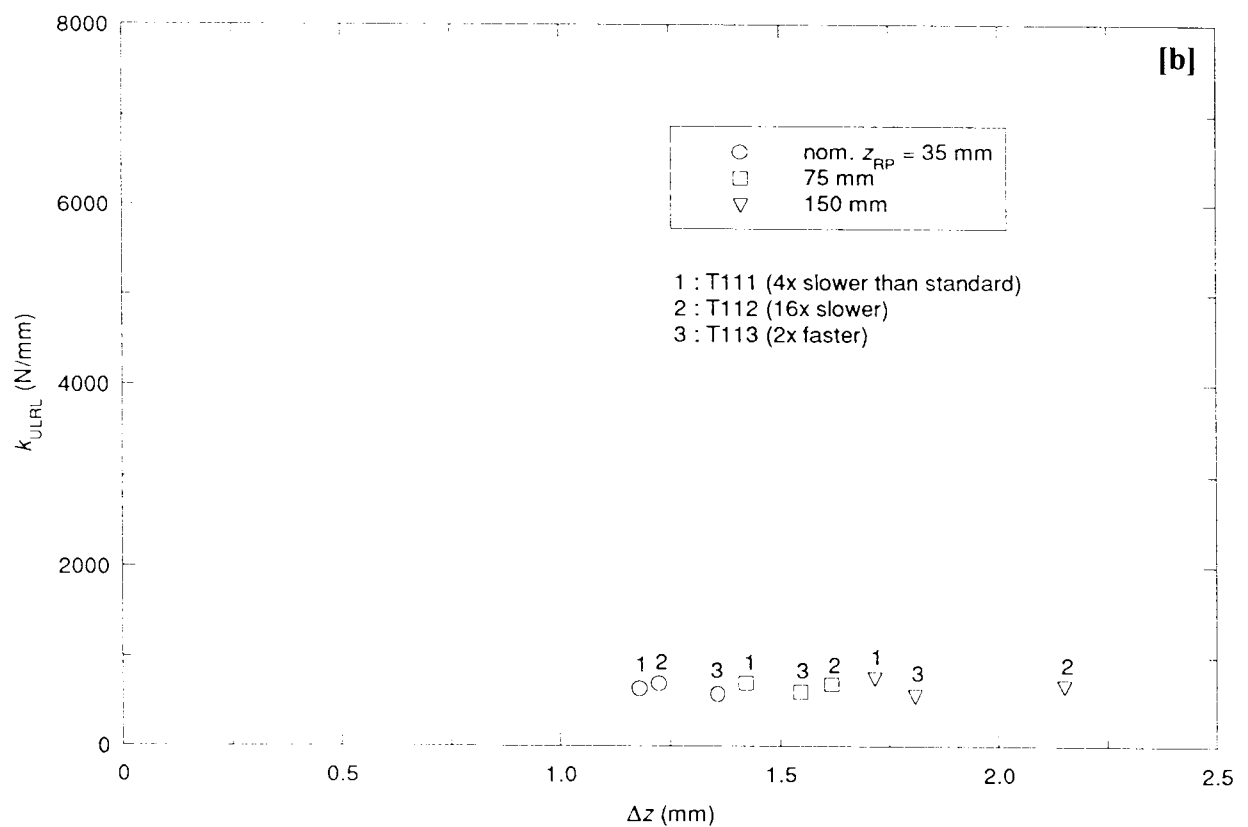
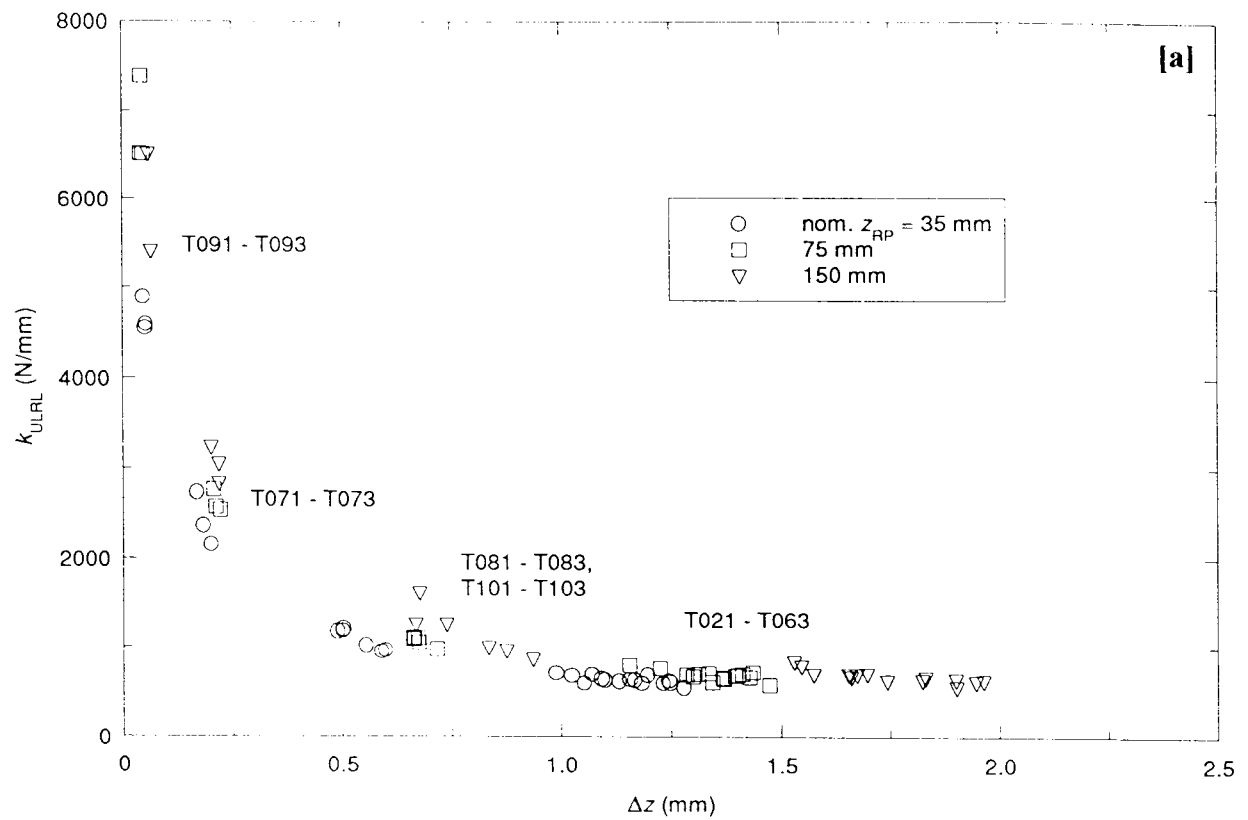


FIGURE 4.44 Summary of vertical unload-reload loop results
(chord stiffness *versus* displacement amplitude)
[a] standard displacement rates;
[b] Batch 11 displacement rates

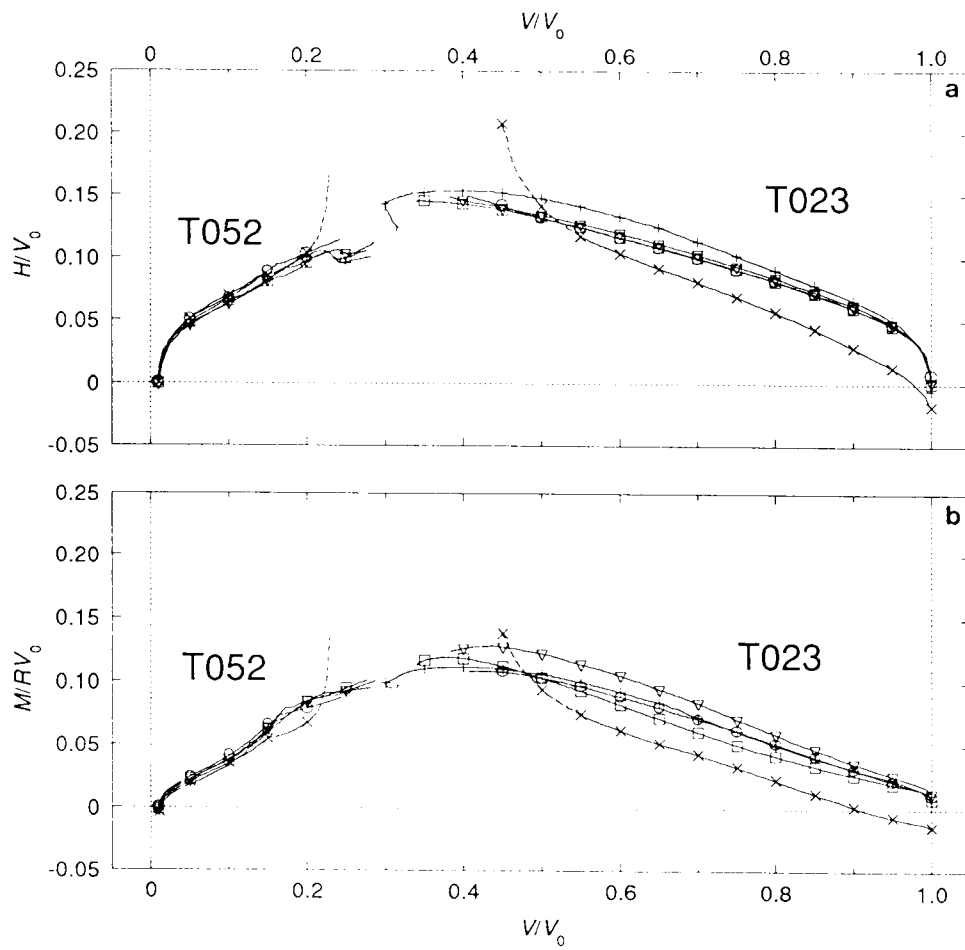


FIGURE 5.1
Tests T023 and T052:
normalised tracking
event results

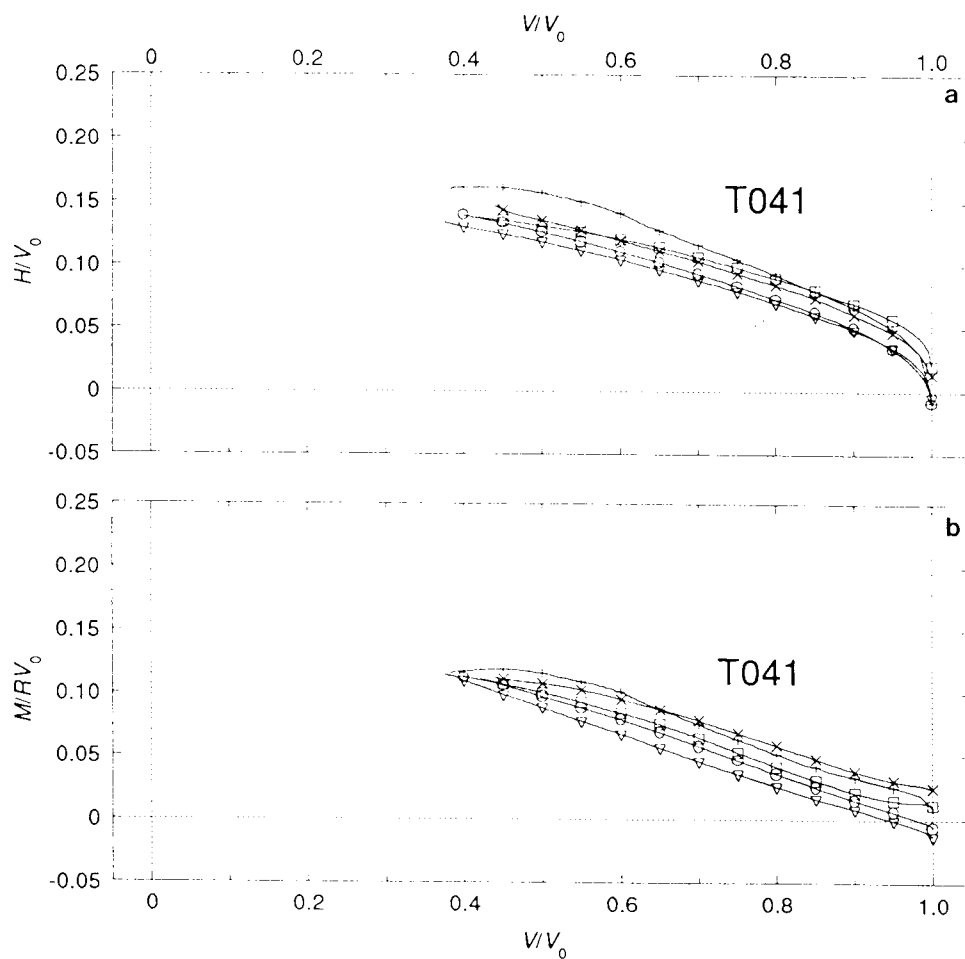


FIGURE 5.2
Test T041:
normalised tracking
event results

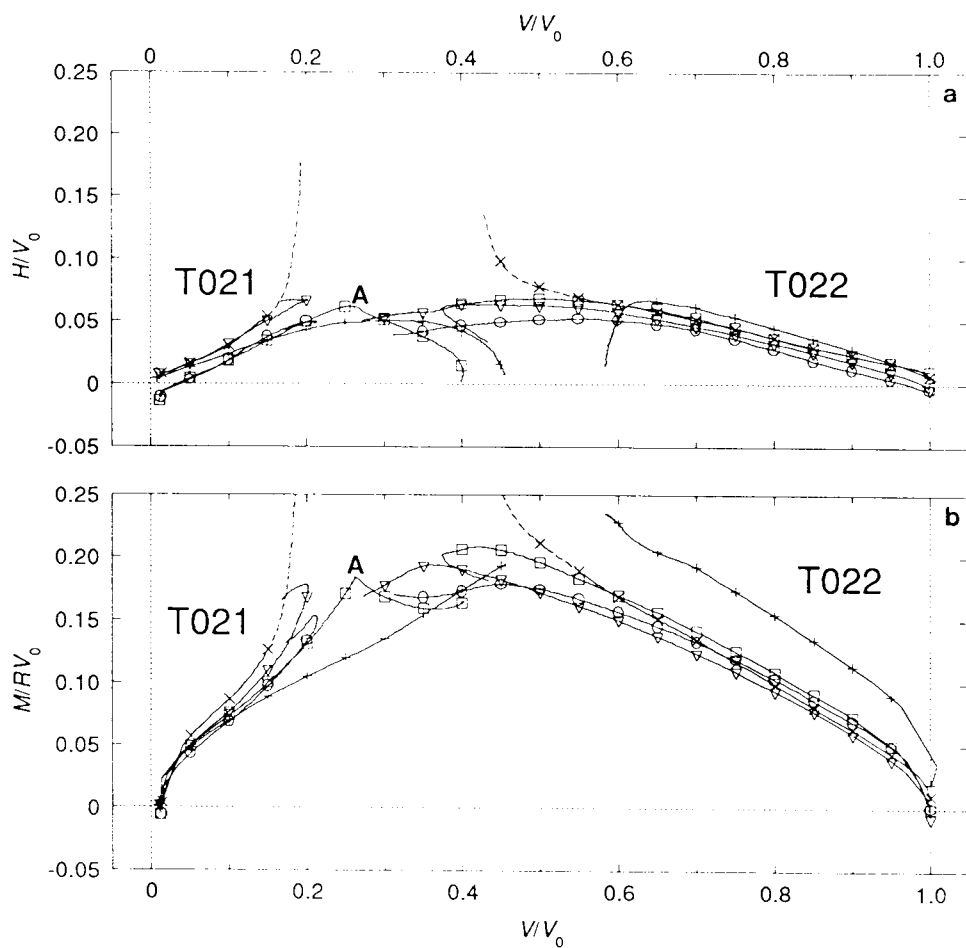


FIGURE 5.3
Tests T022 and T021:
normalised tracking
event results

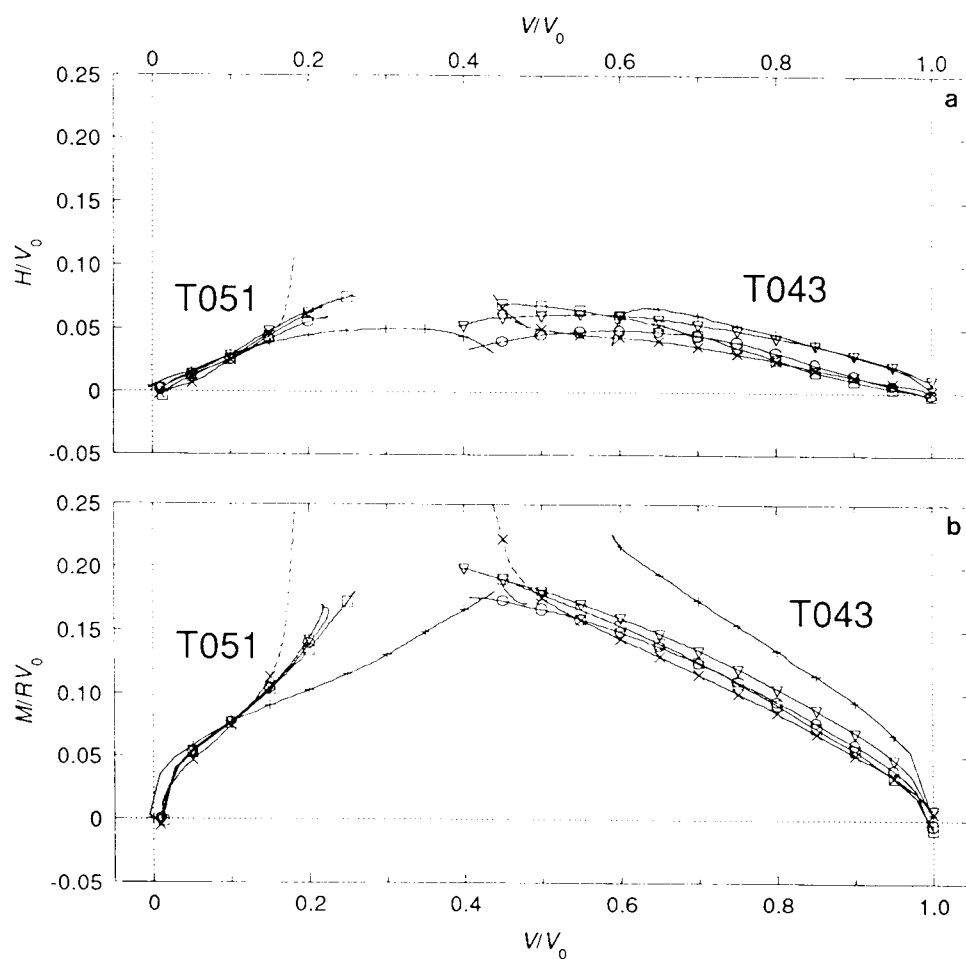


FIGURE 5.4
Tests T043 and T051:
normalised tracking
event results

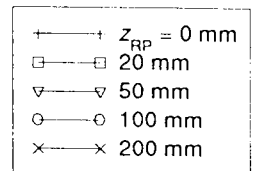
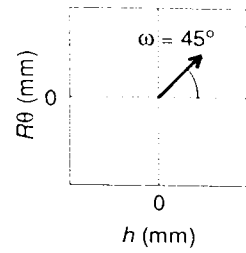
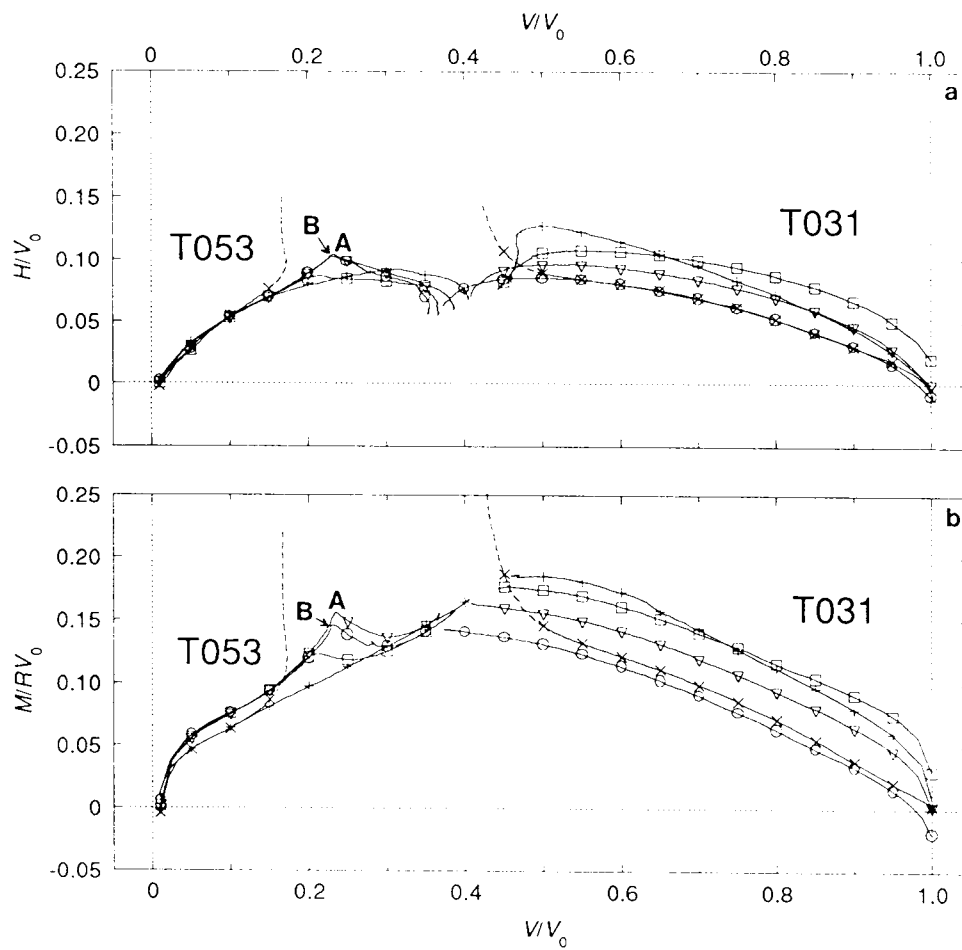


FIGURE 5.5
Tests T031 and T053:
normalised tracking
event results

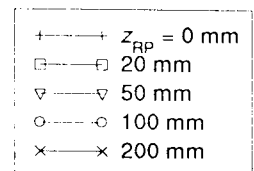
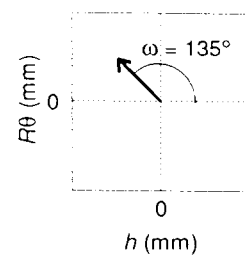
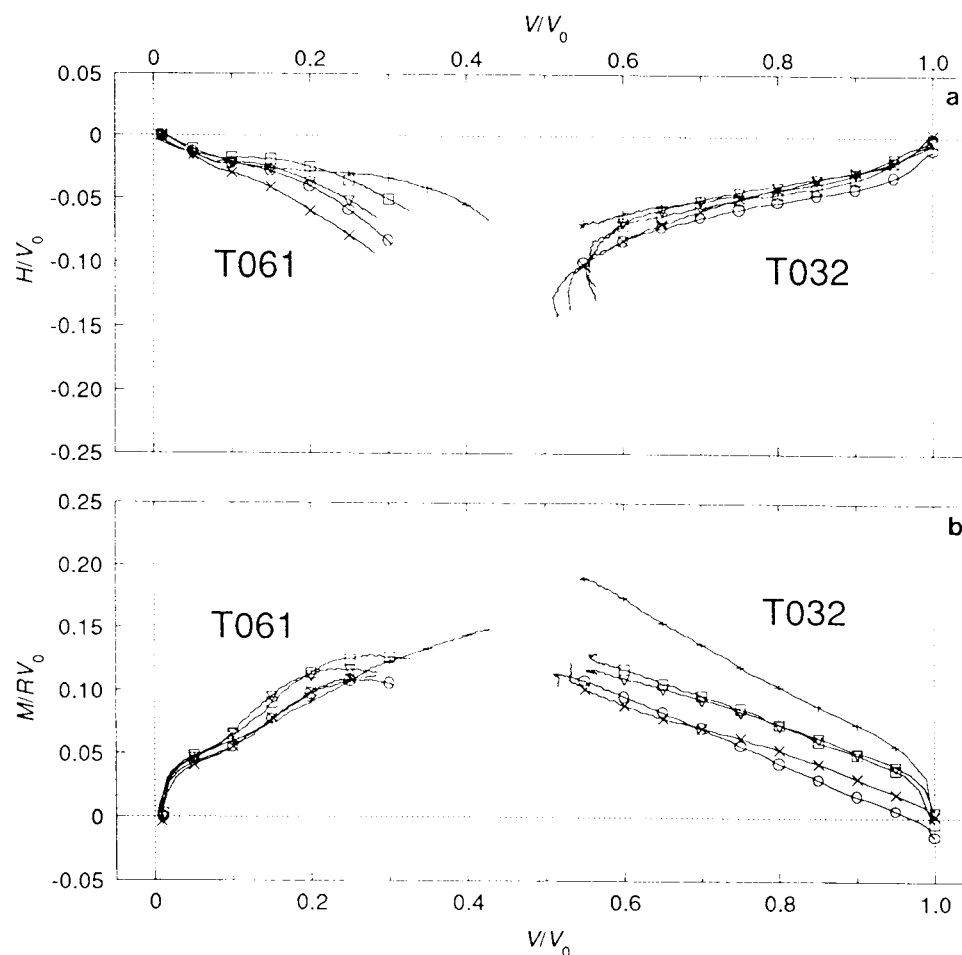


FIGURE 5.6
Tests T032 and T061:
normalised tracking
event results

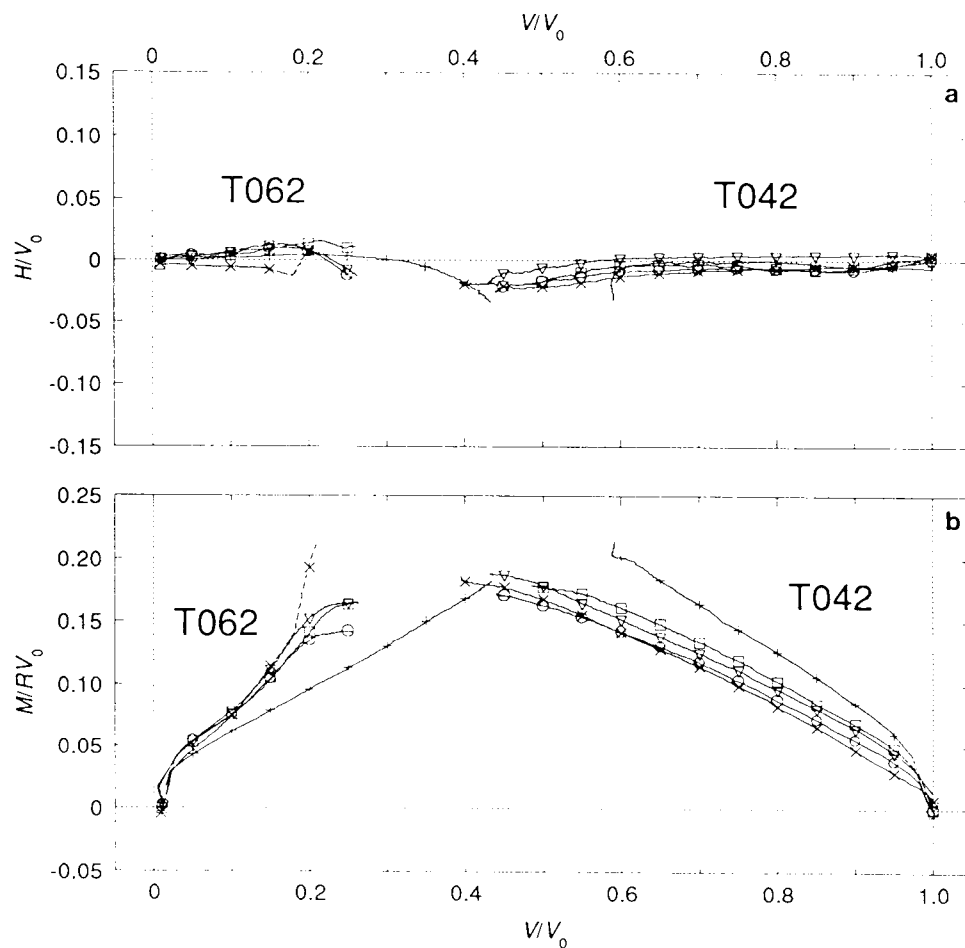


FIGURE 5.7
Tests **T042** and **T062**:
normalised tracking
event results

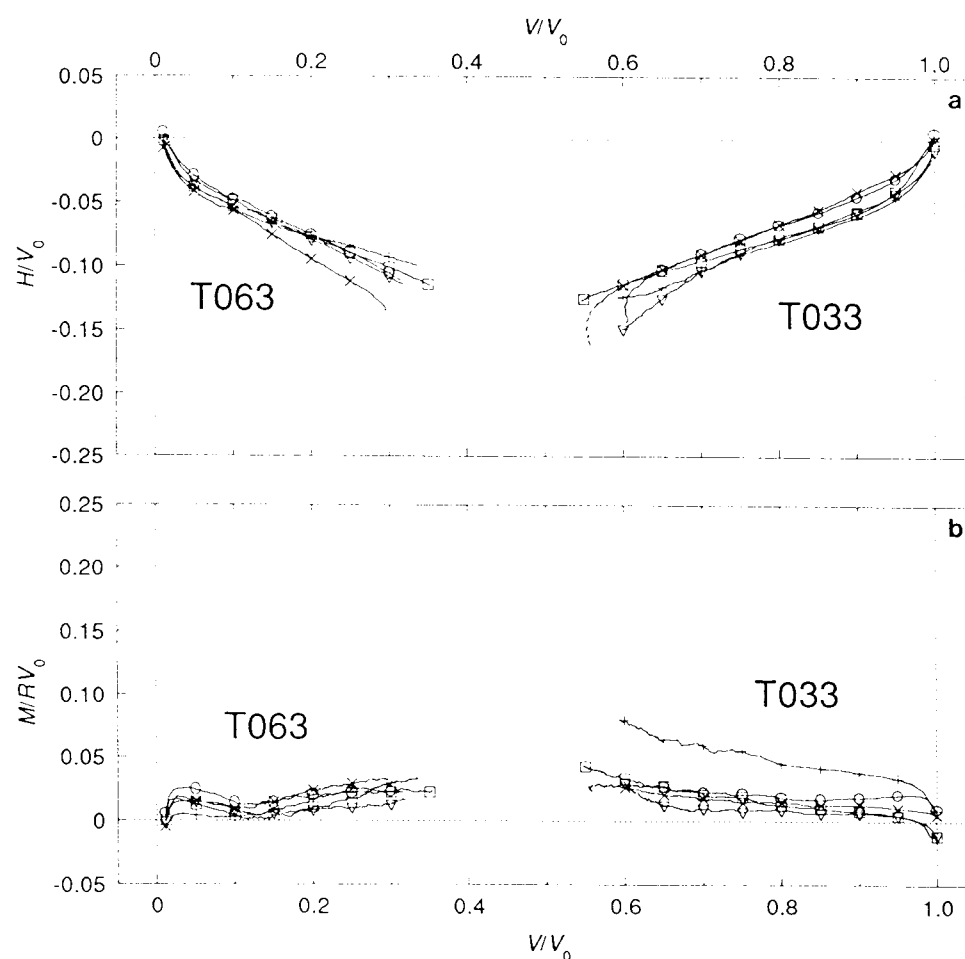
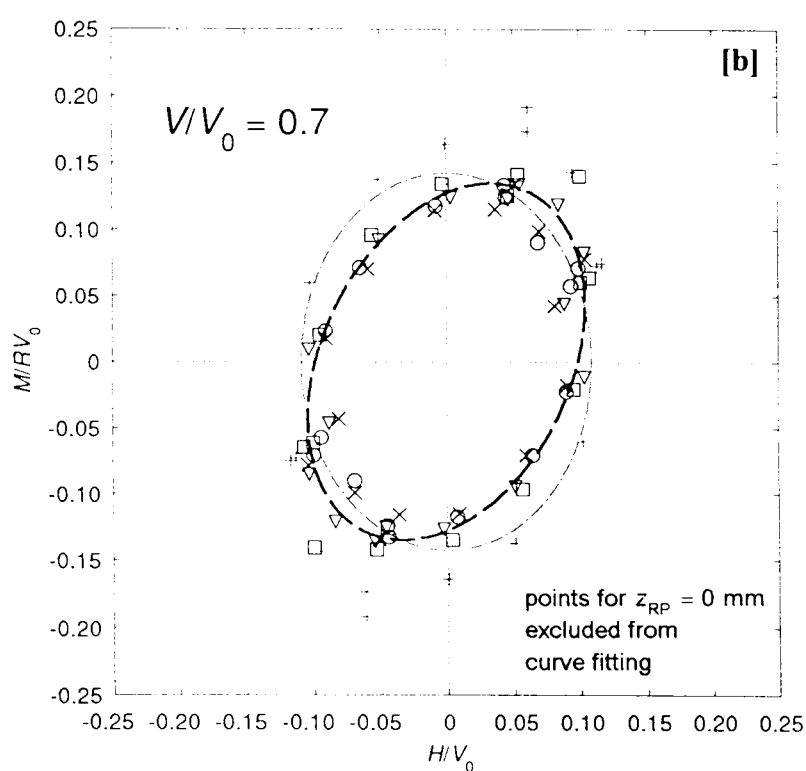
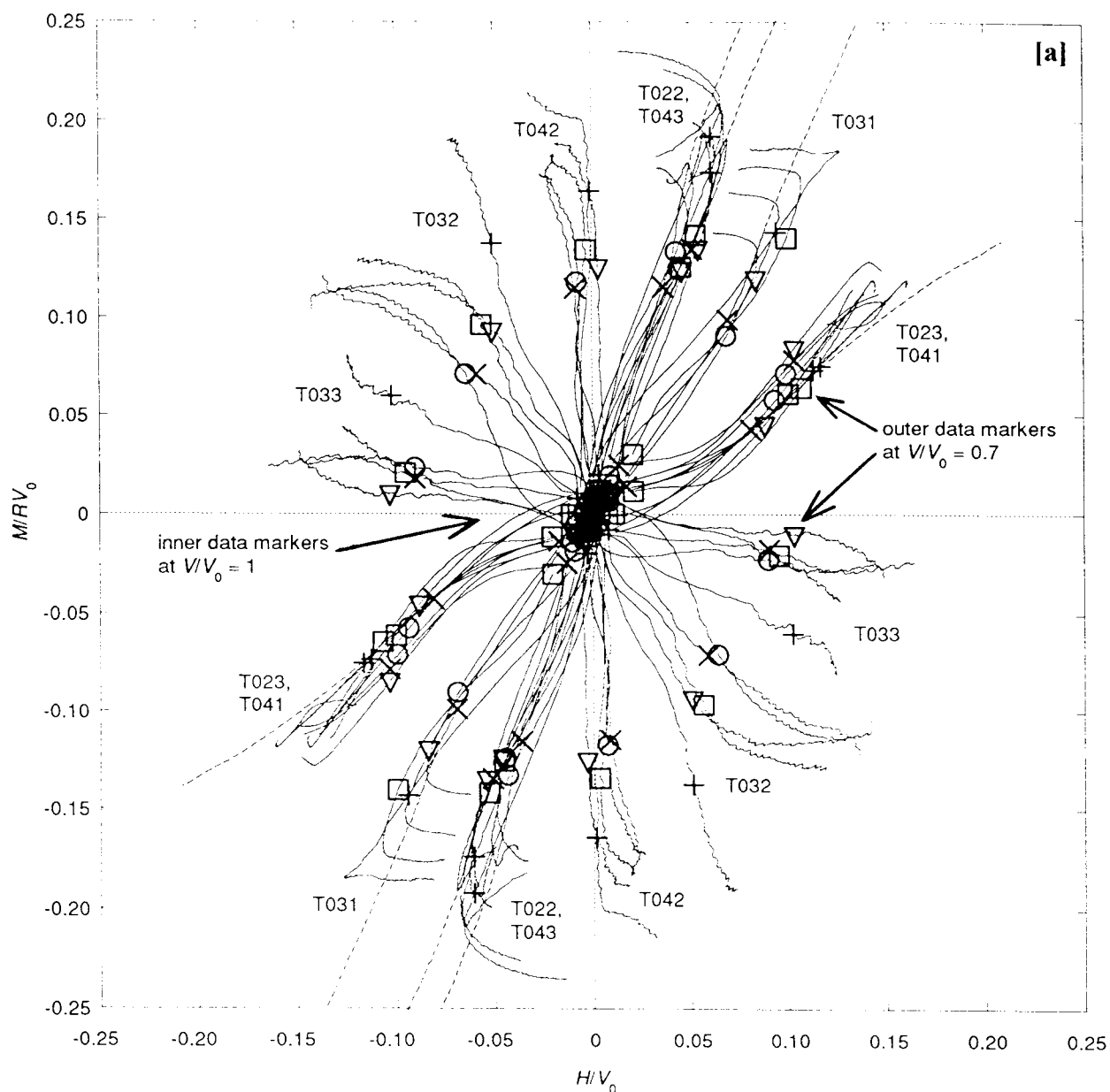


FIGURE 5.8
Tests **T033** and **T063**:
normalised tracking
event results



+	nom. $z_{RP} = 0$ mm
□	20 mm
▽	50 mm
○	100 mm
×	200 mm

FIGURE 5.9
Tracking events commencing from $V/V_0 = 1$
[a] H/V_0 : M/RV_0 load paths;
[b] H/V_0 : M/RV_0 points at which $V/V_0 = 0.7$

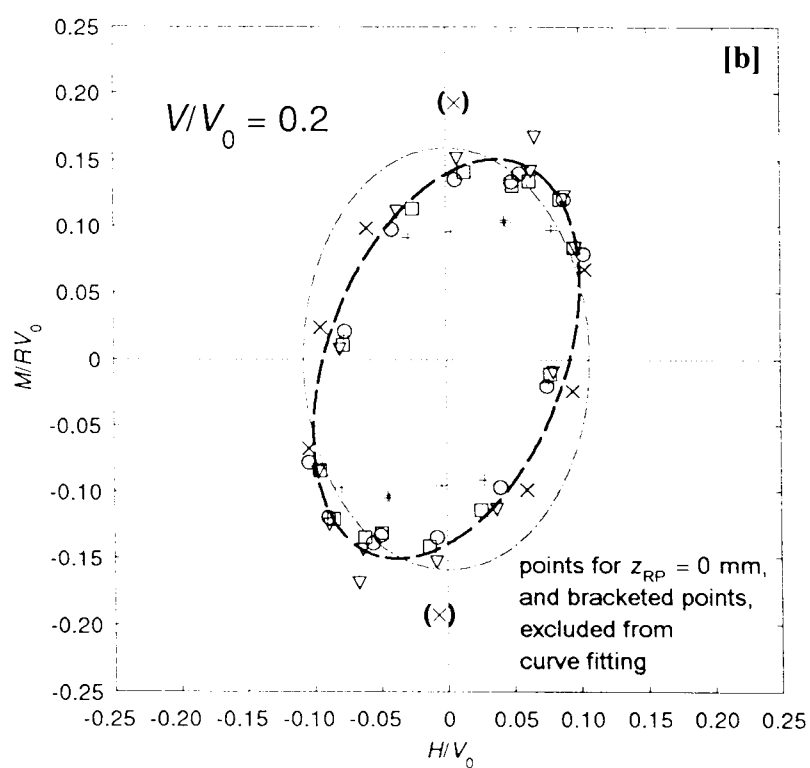
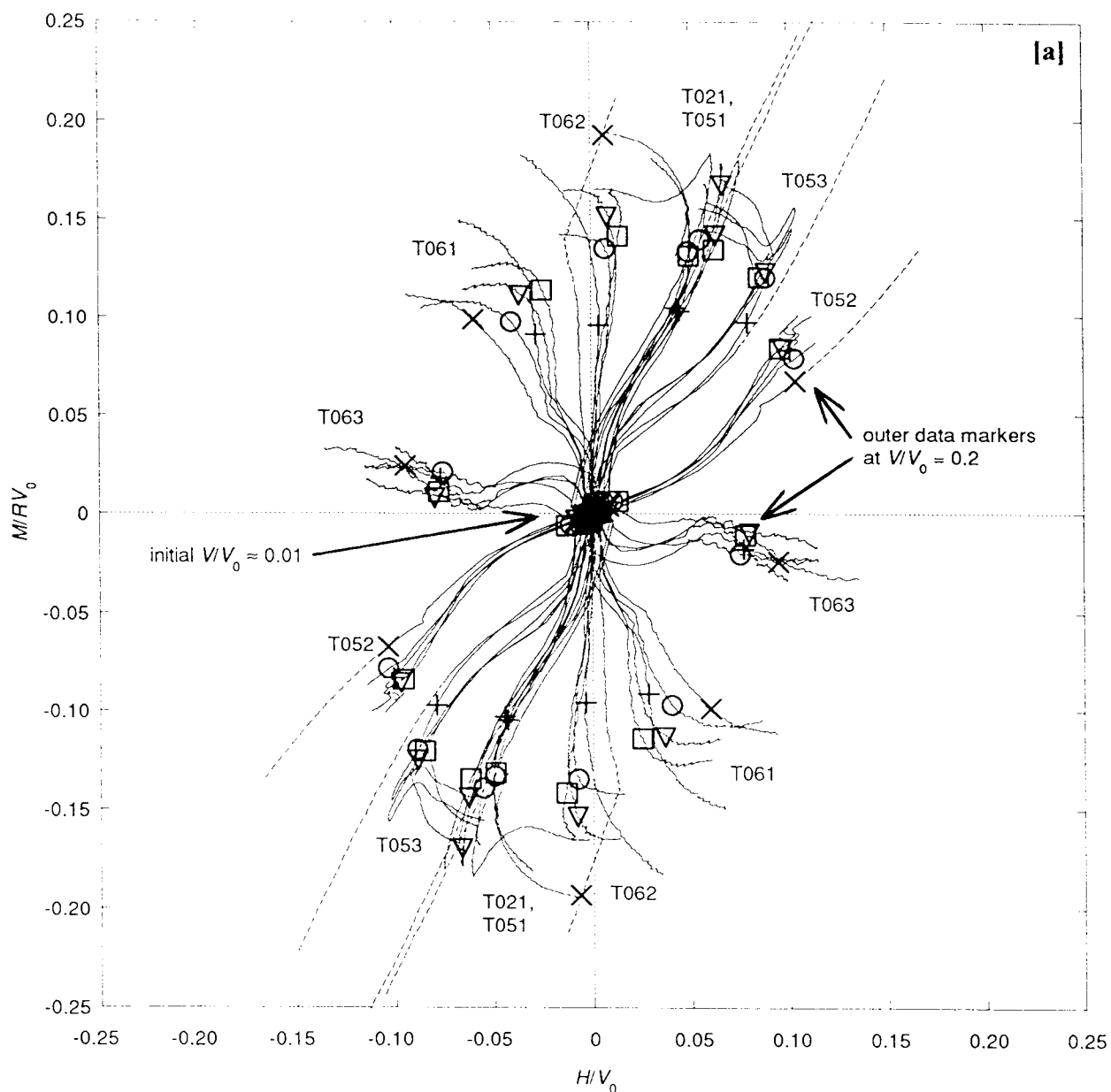


FIGURE 5.10
Tracking events commencing from $V/V_0 \approx 0.01$
[a] H/V_0 : M/RV_0 load paths;
[b] H/V_0 : M/RV_0 points at which $V/V_0 = 0.2$

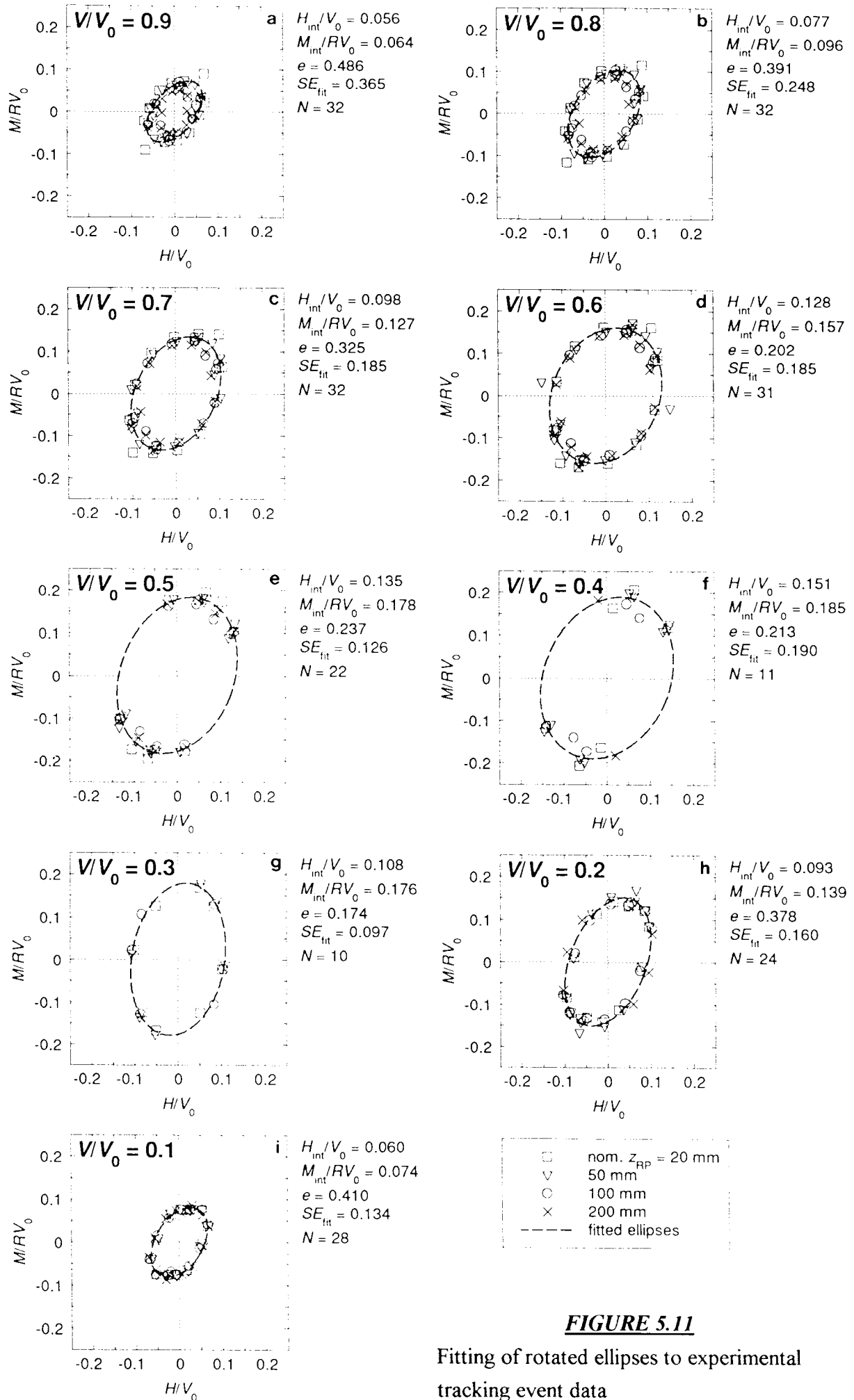


FIGURE 5.11

Fitting of rotated ellipses to experimental tracking event data

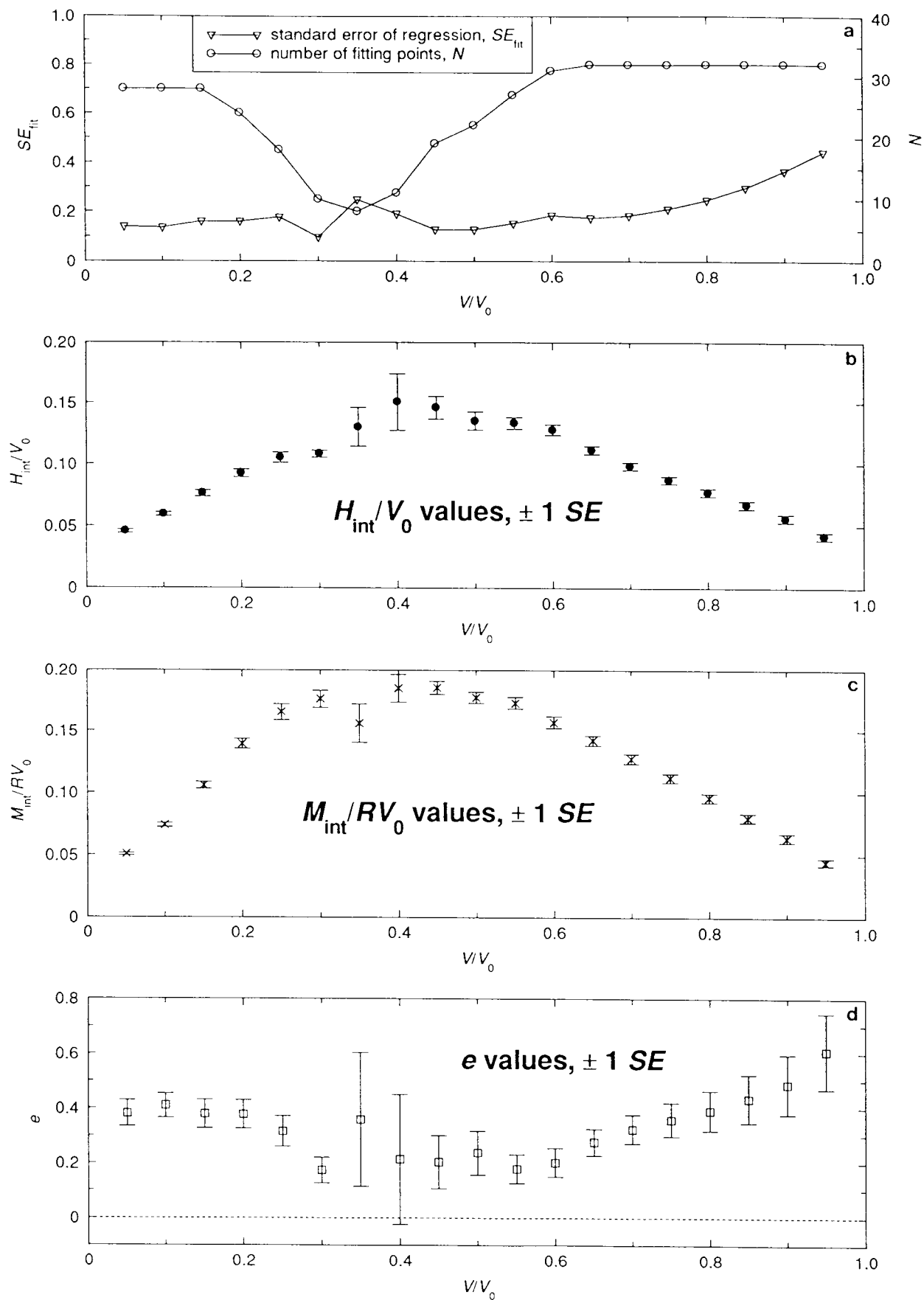


FIGURE 5.12 Summary of rotated ellipse regressions at $V/V_0 = 0.05, 0.10, \dots, 0.95$

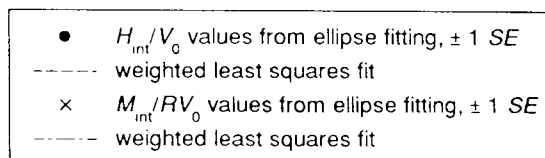
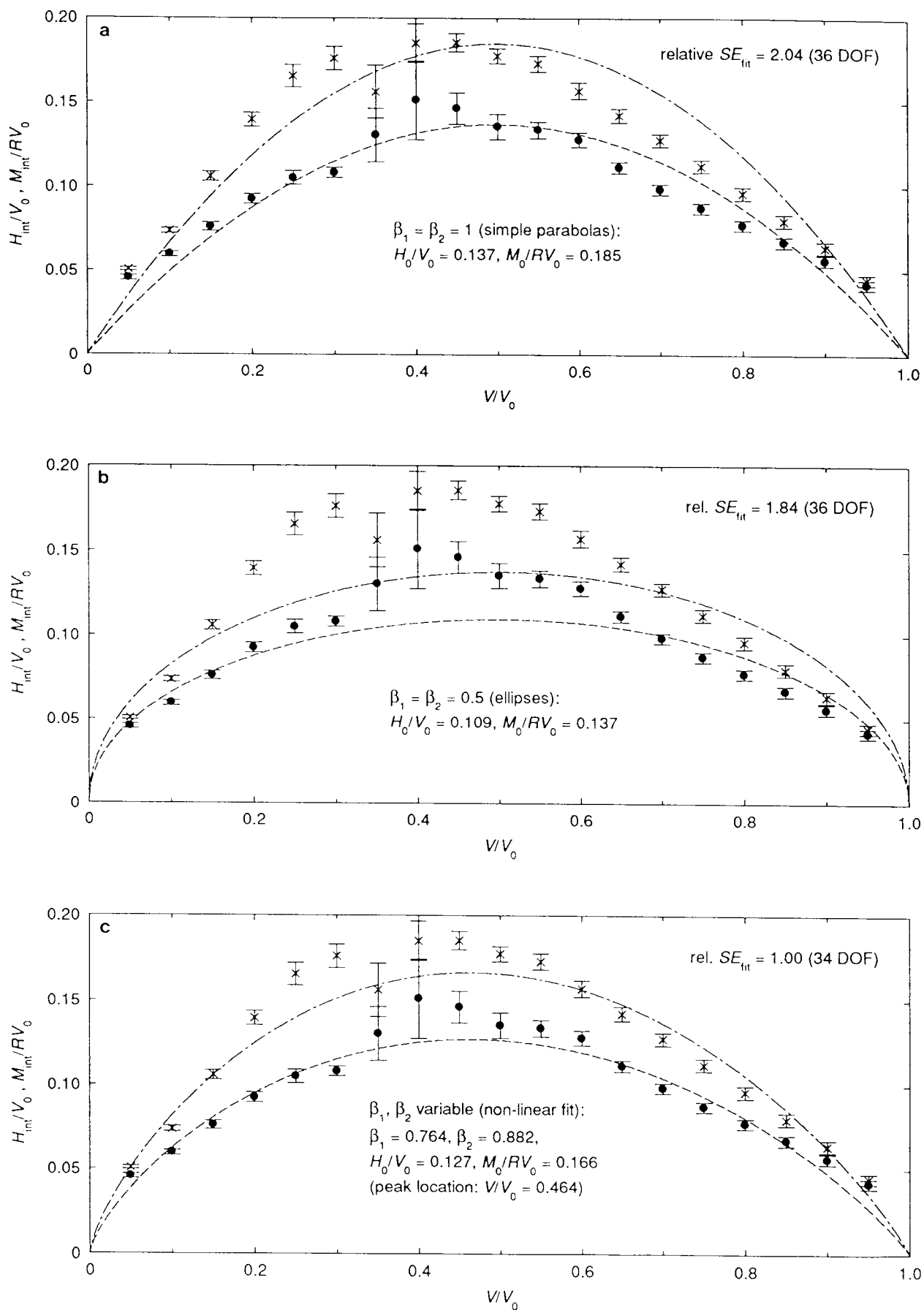


FIGURE 5.13

Curve fitting to H_{int}/V_0 and M_{int}/RV_0 values obtained from rotated ellipse regressions at $V/V_0 = 0.05, 0.1, \dots, 0.95$

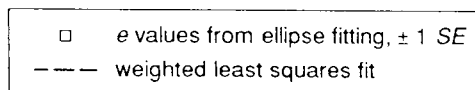
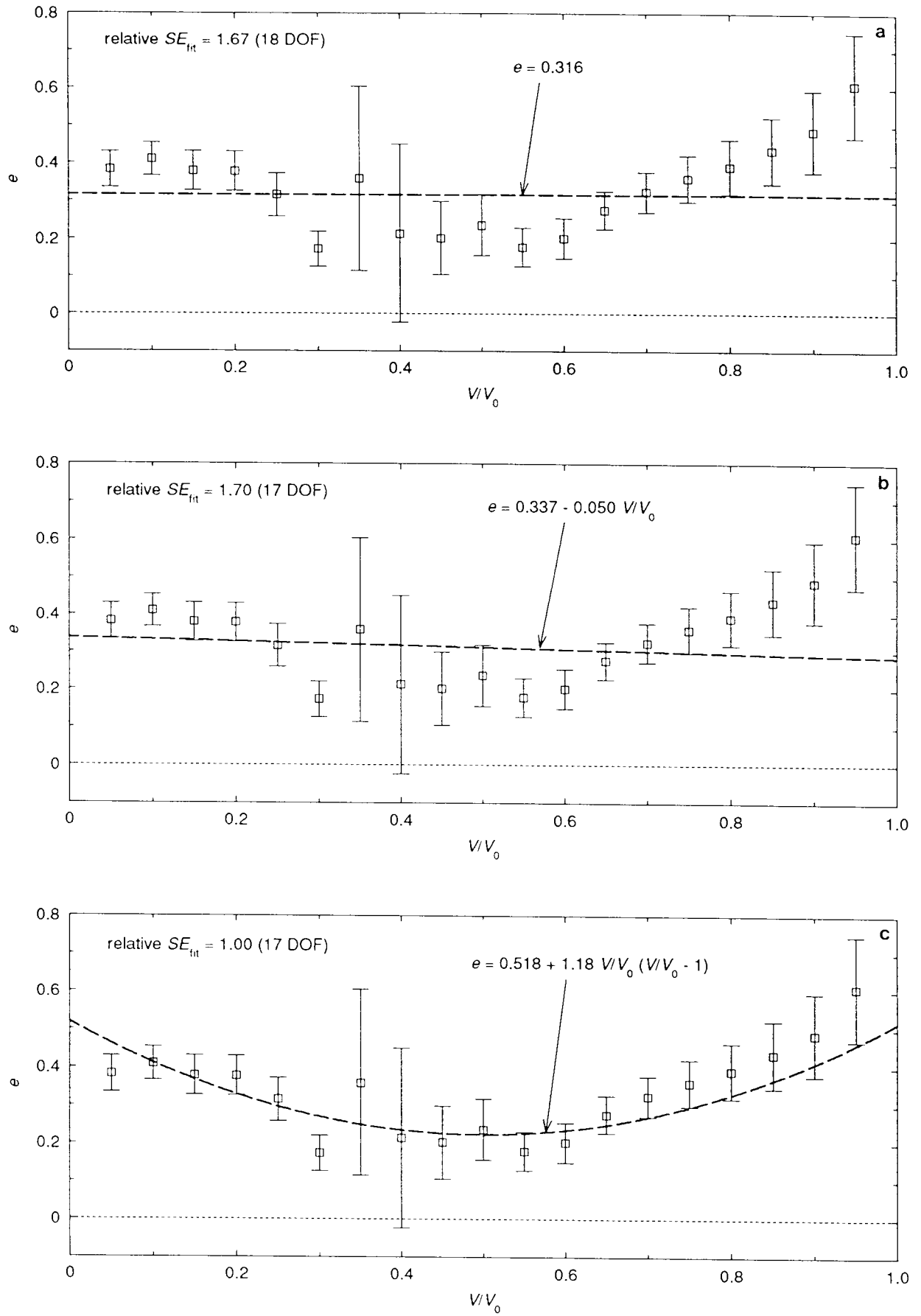


FIGURE 5.14

Curve fitting to e (eccentricity parameter) values obtained from rotated ellipse regressions at $V/V_0 = 0.05, 0.1, \dots, 0.95$

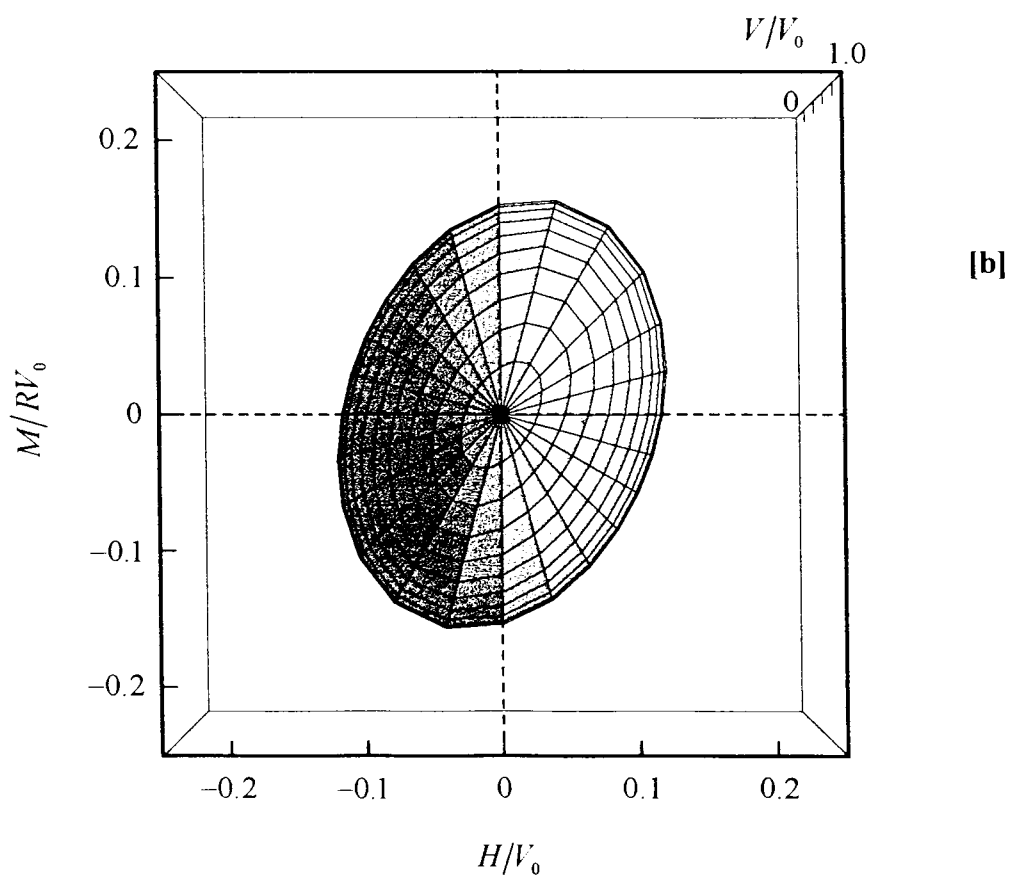
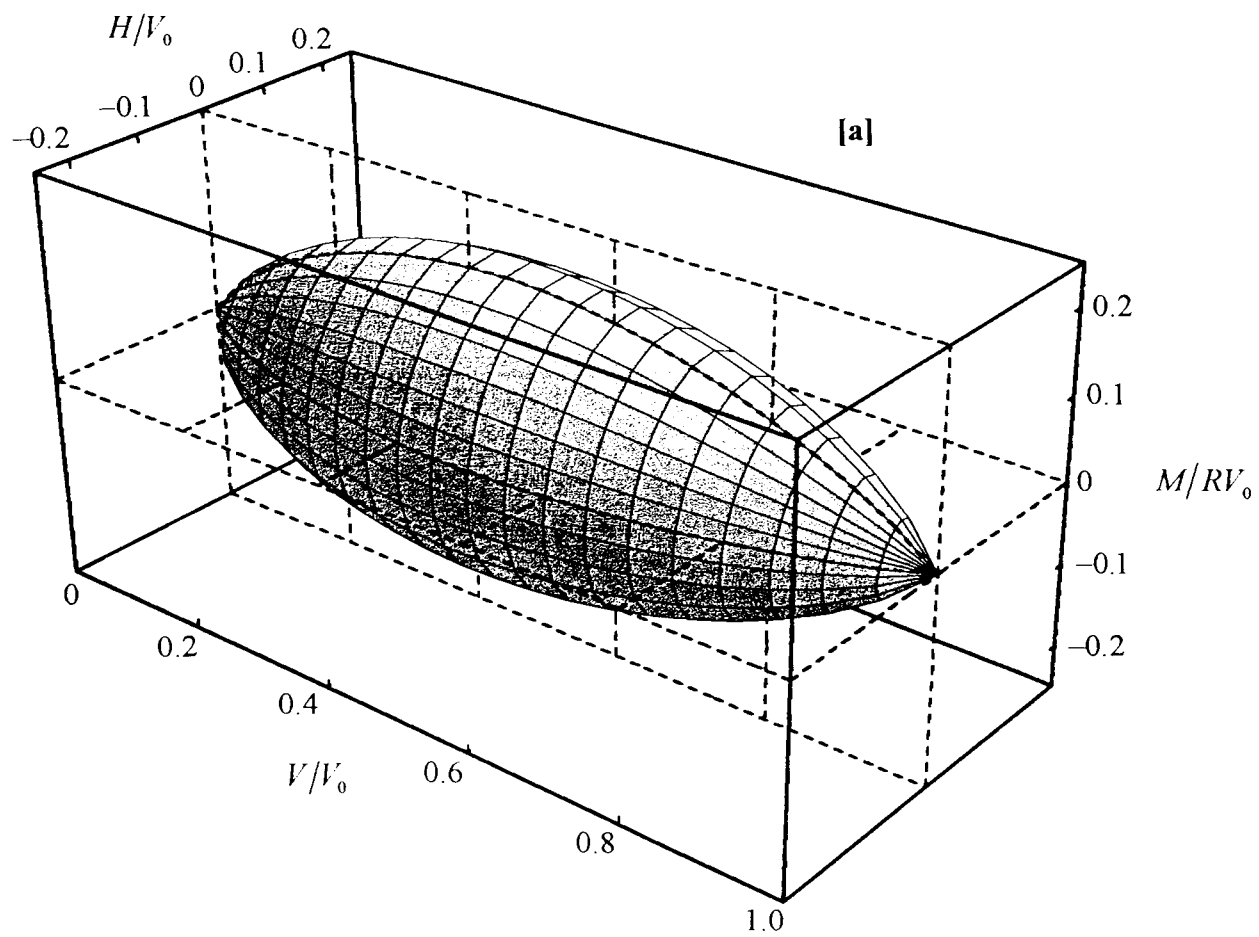
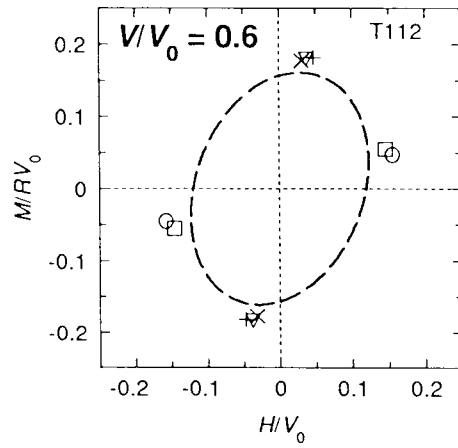
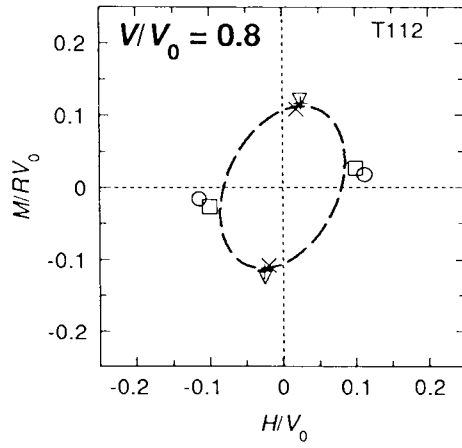
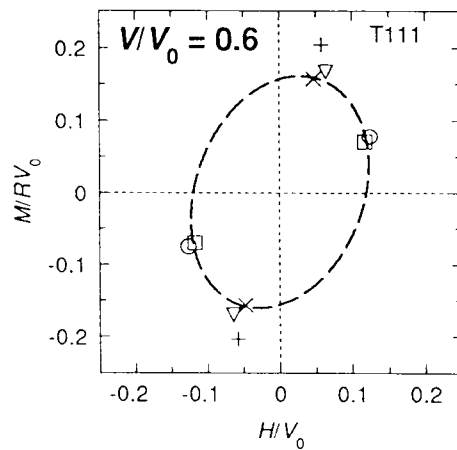
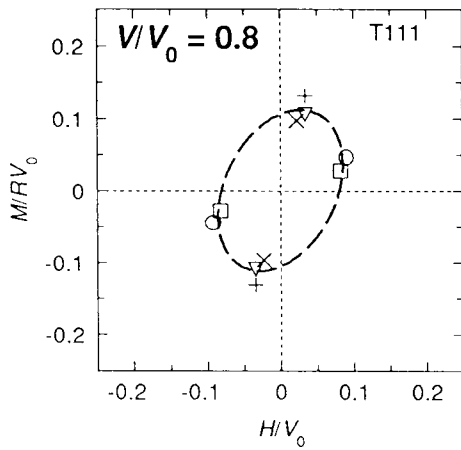


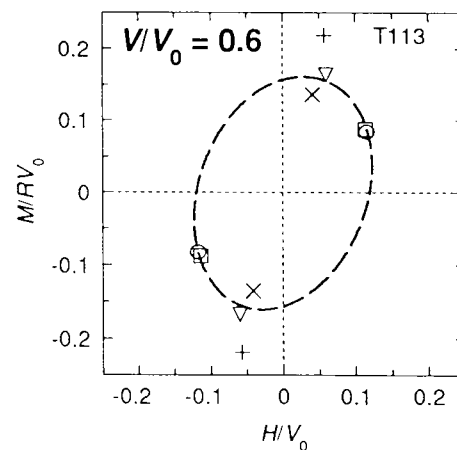
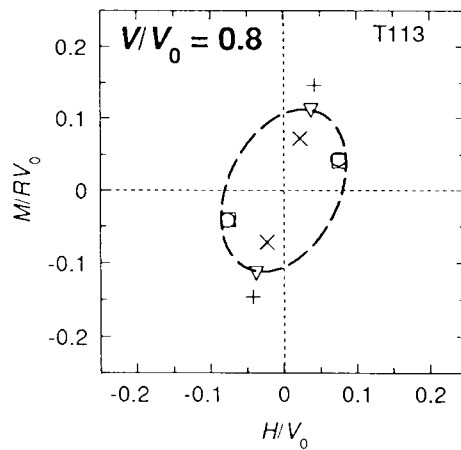
FIGURE 5.15 Yield surface fitted to experimental tracking event load paths (Eqns 5.15)
[a] perspective view; **[b]** view down V/V_0 axis



[a]



[b]



[c]

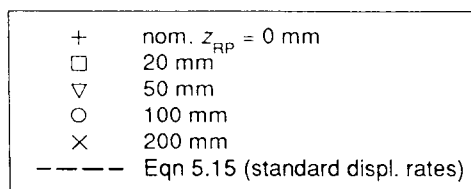


FIGURE 5.16

Effect of displacement rate on
normalised tracking event results

[a] T112 (16 times slower than standard)

[b] T111 (4 times slower than standard)

[c] T113 (2 times faster than standard)

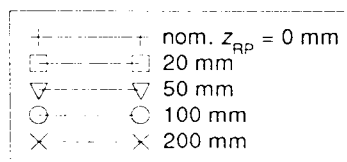
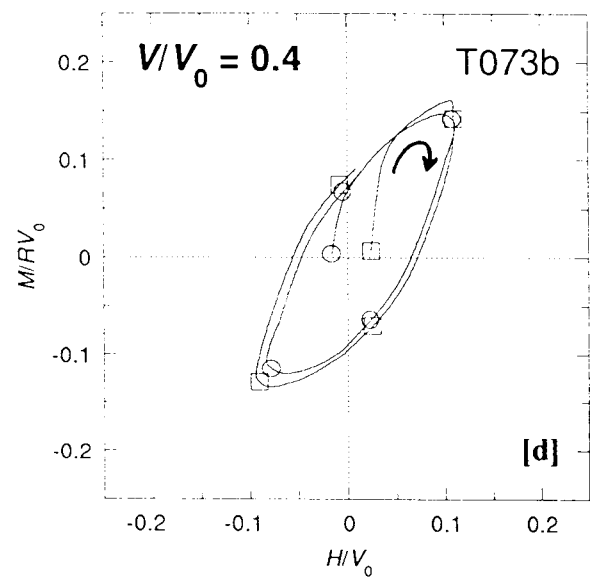
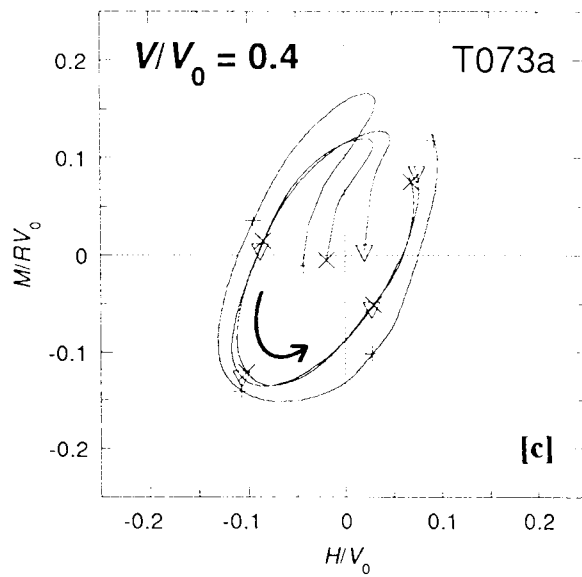
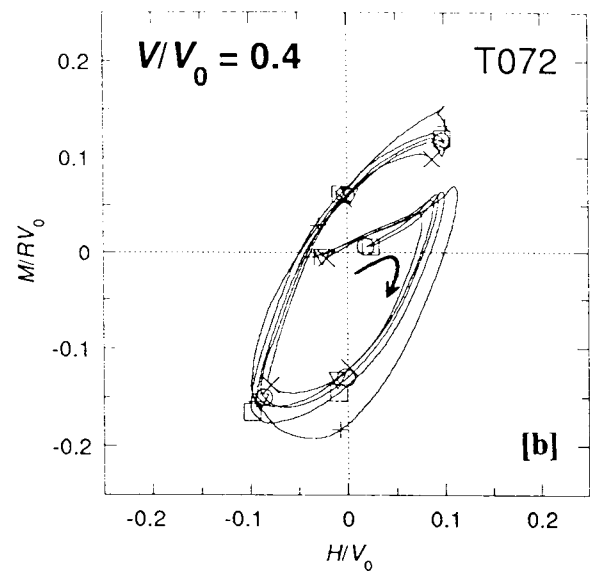
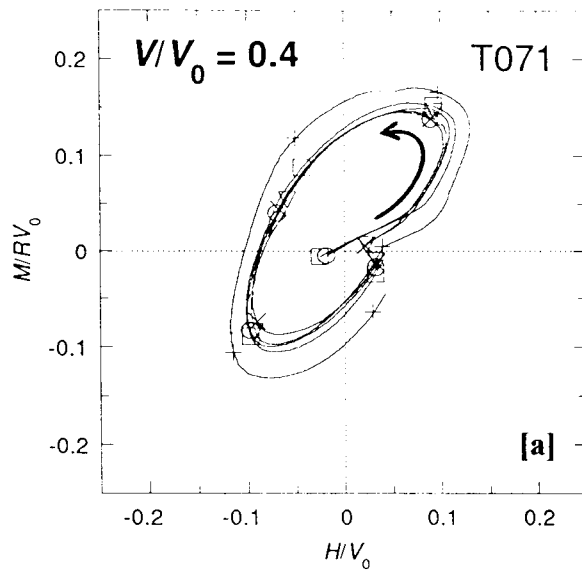


FIGURE 5.17

$H/V_0 : M/RV_0$ load paths from Batch 7 tests
(loop events at $V/V_0 = 0.4$)

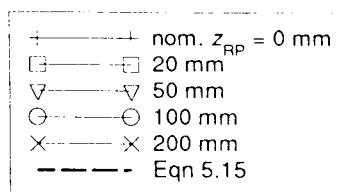
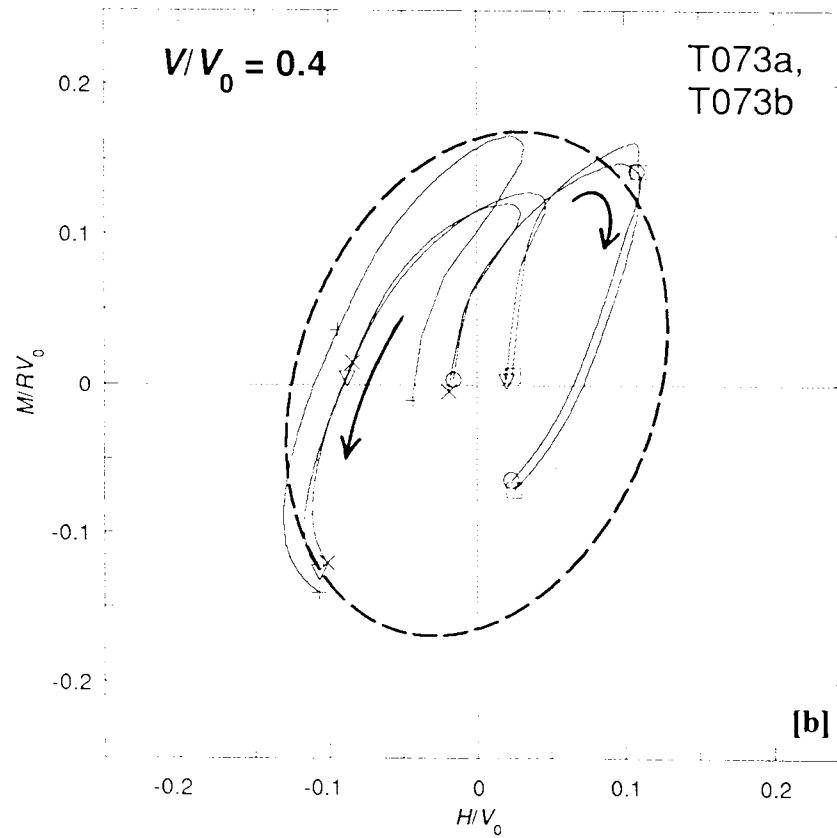
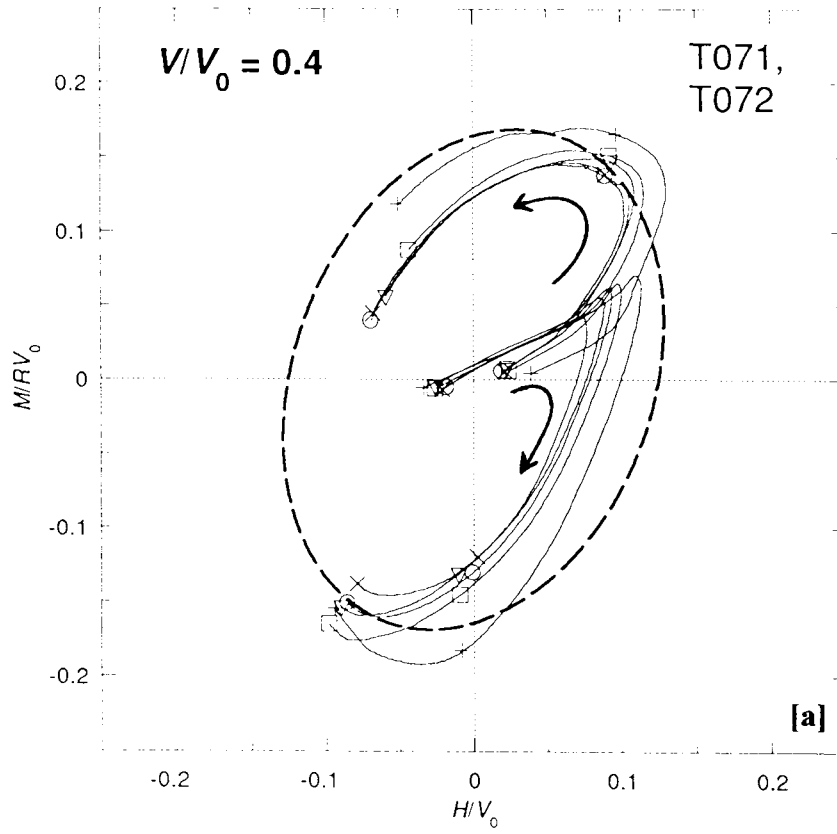


FIGURE 5.18

First halves of the $H/V_0 : M/RV_0$ load paths followed in the loop events of
[a] T071 and T072; [b] T073a and T073b

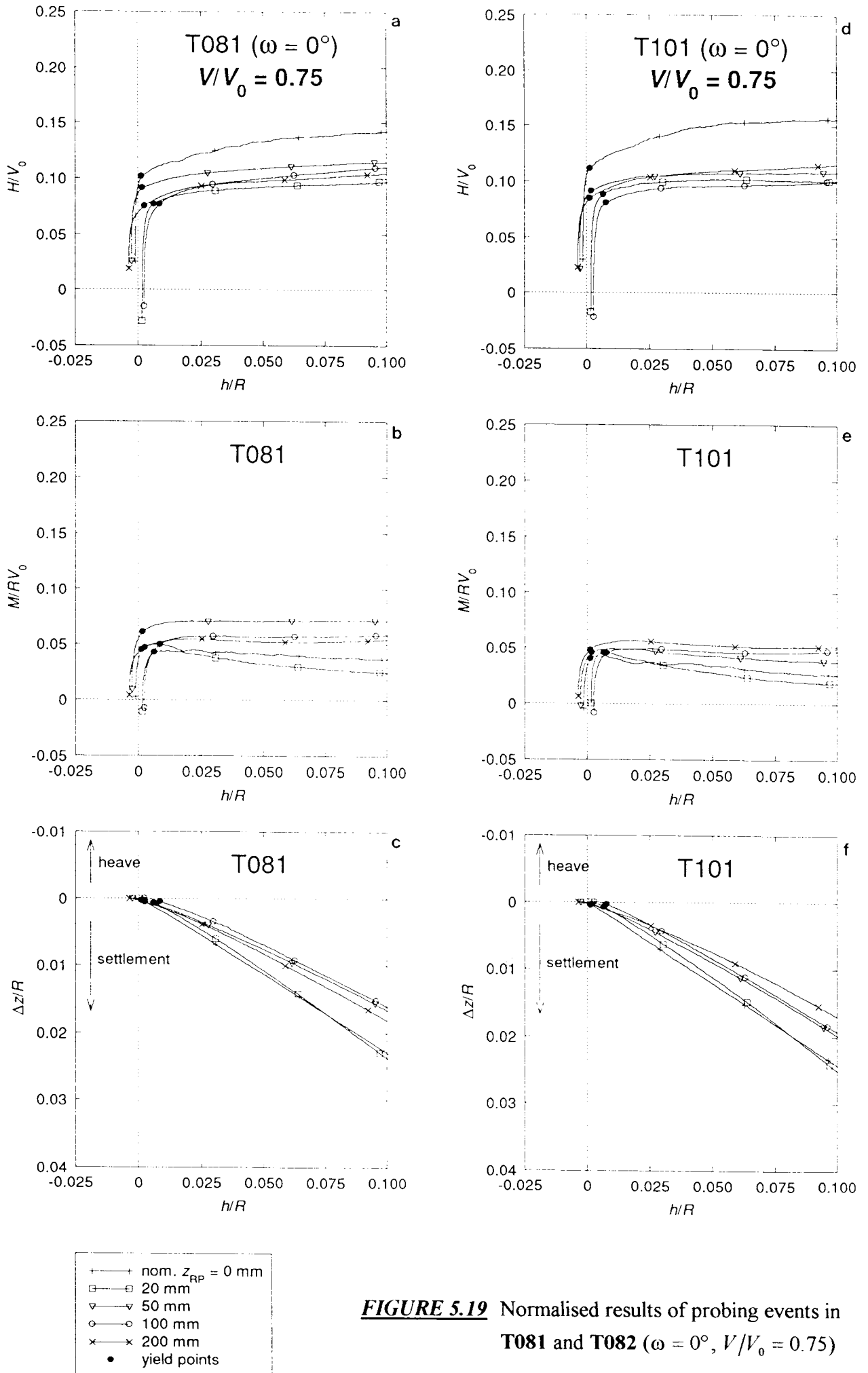


FIGURE 5.19 Normalised results of probing events in T081 and T082 ($\omega = 0^\circ$, $V/V_0 = 0.75$)

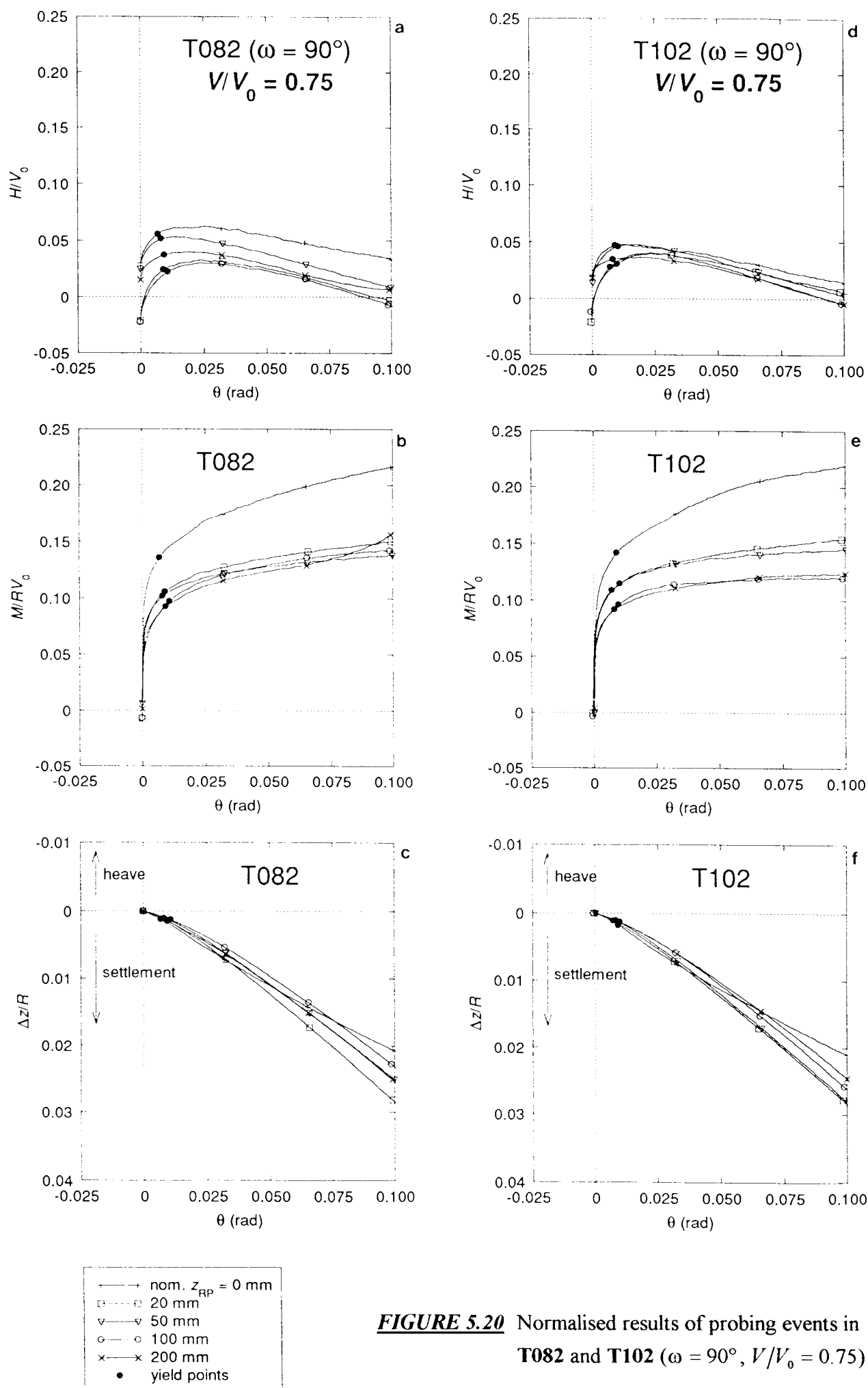


FIGURE 5.20 Normalised results of probing events in **T082** and **T102** ($\omega = 90^\circ$, $V/V_0 = 0.75$)

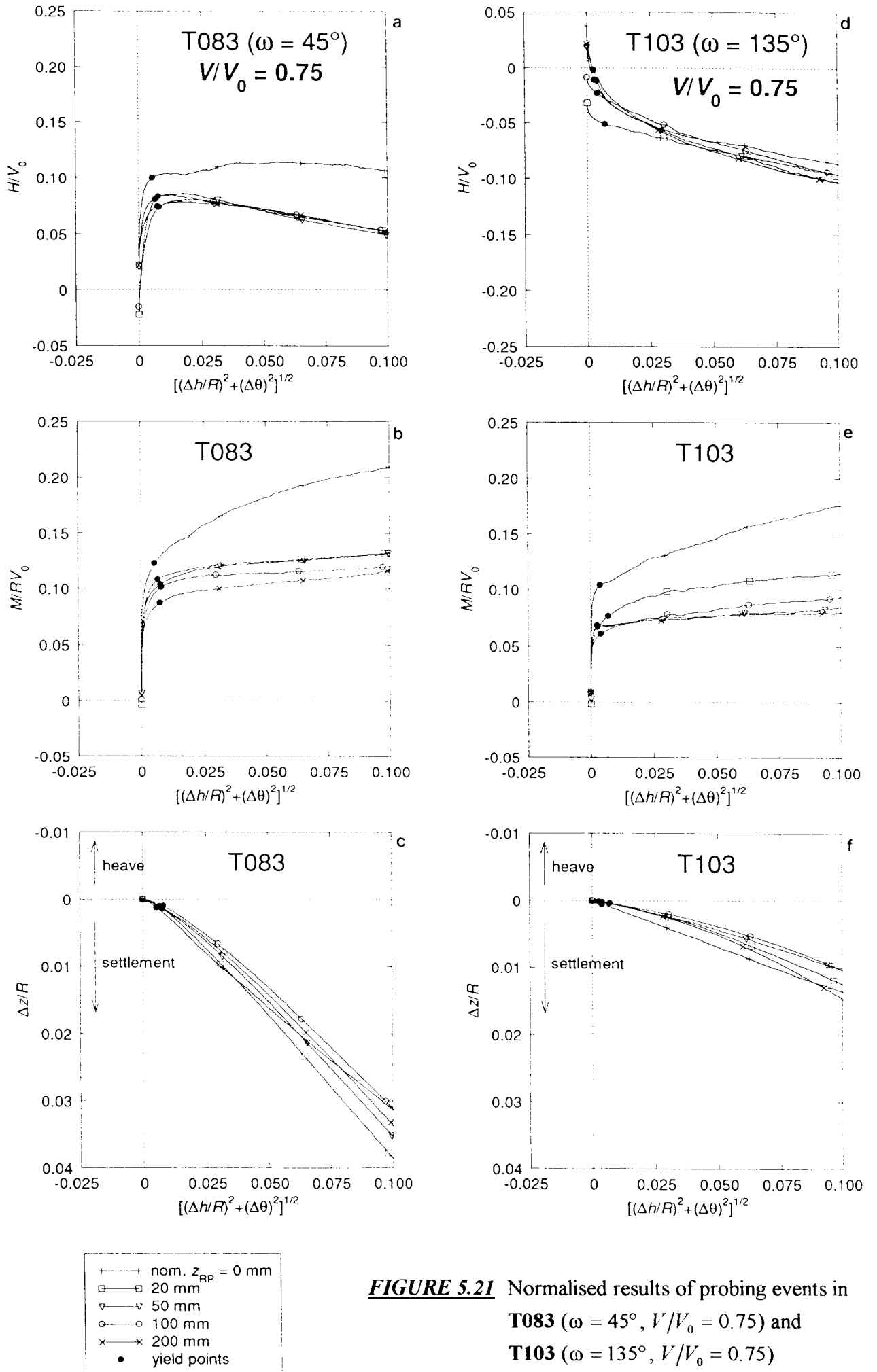


FIGURE 5.21 Normalised results of probing events in
T083 ($\omega = 45^\circ$, $V/V_0 = 0.75$) and
T103 ($\omega = 135^\circ$, $V/V_0 = 0.75$)

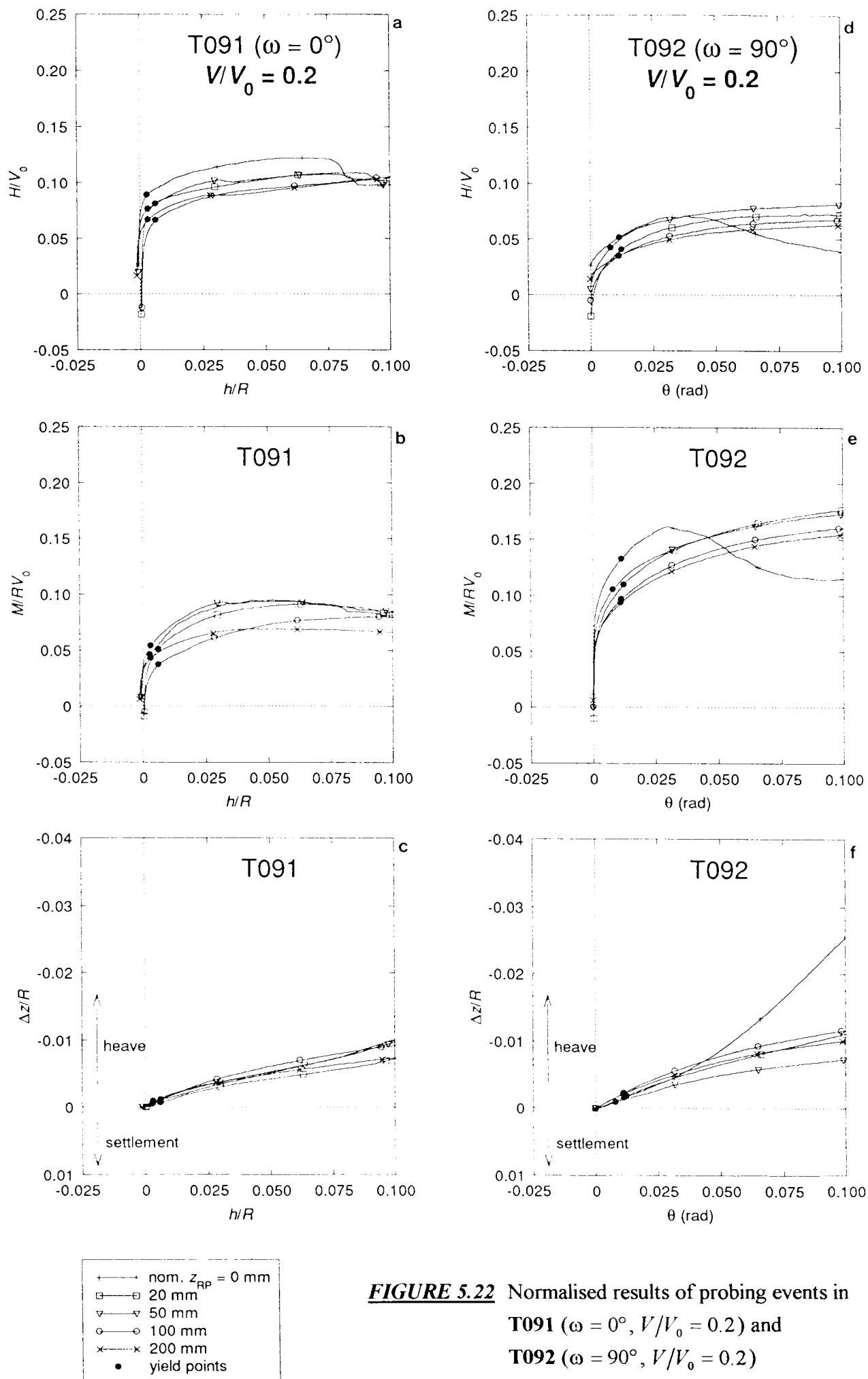


FIGURE 5.22 Normalised results of probing events in
T091 ($\omega = 0^\circ$, $V/V_0 = 0.2$) and
T092 ($\omega = 90^\circ$, $V/V_0 = 0.2$)

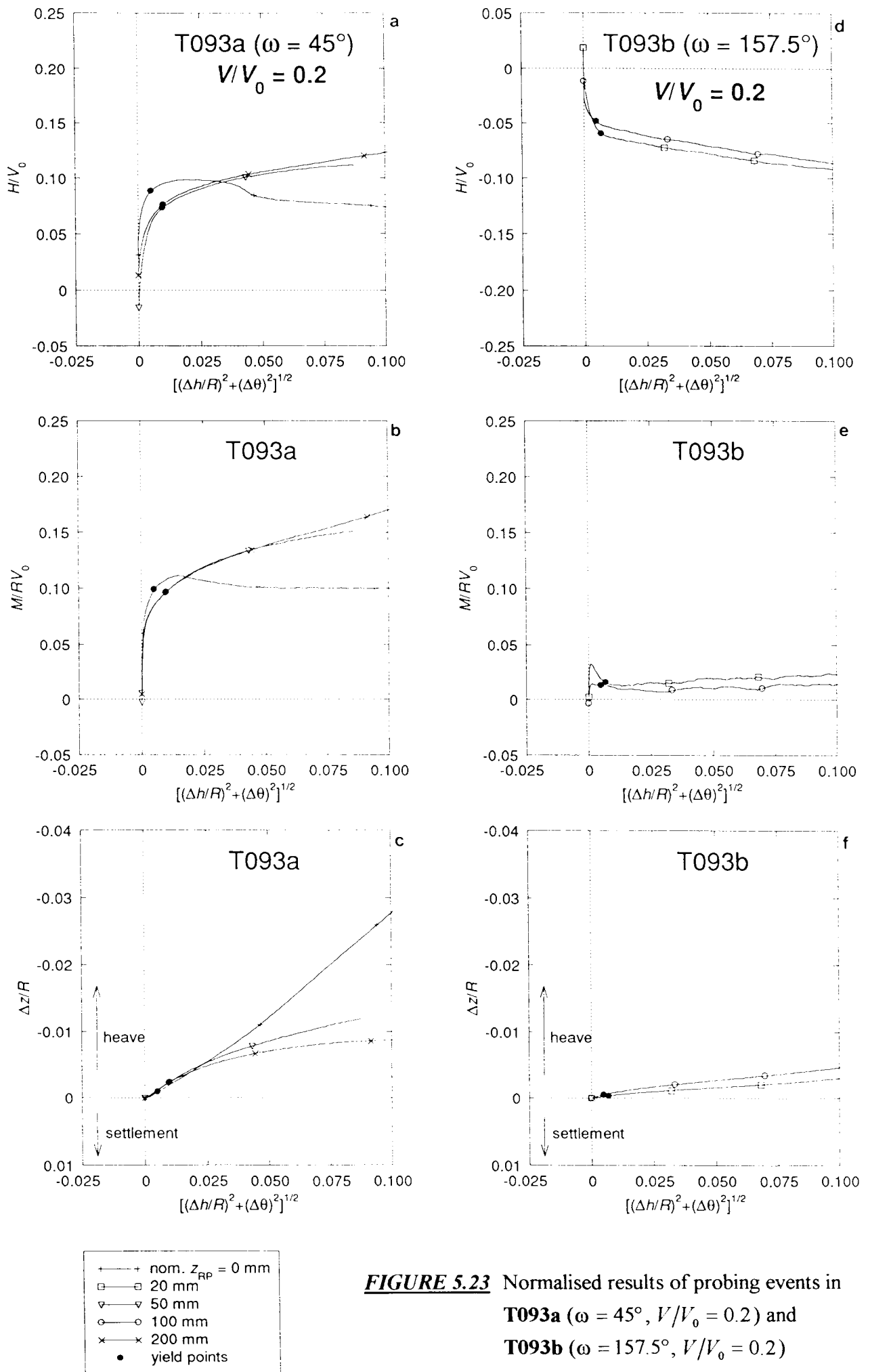


FIGURE 5.23 Normalised results of probing events in **T093a** ($\omega = 45^\circ$, $V/V_0 = 0.2$) and **T093b** ($\omega = 157.5^\circ$, $V/V_0 = 0.2$)

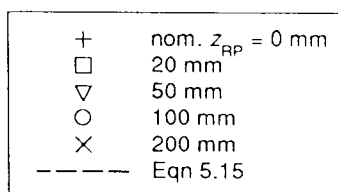
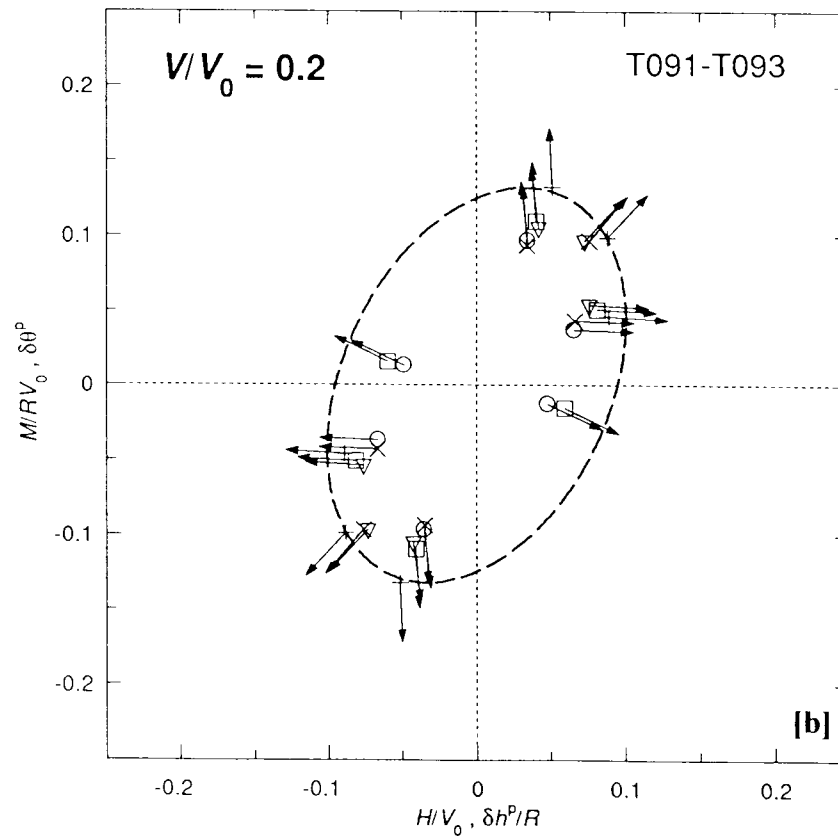
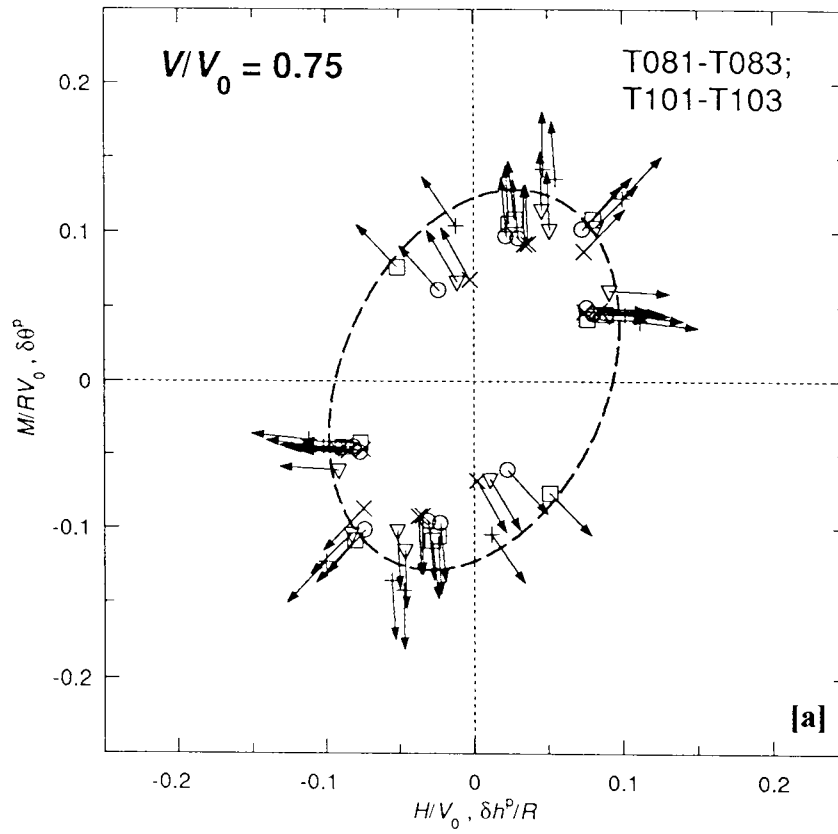


FIGURE 5.24

Summary of $H/V_0 : M/RV_0$ yield points and incremental plastic displacement vectors observed in probing events at
[a] $V/V_0 = 0.75$; **[b]** $V/V_0 = 0.2$

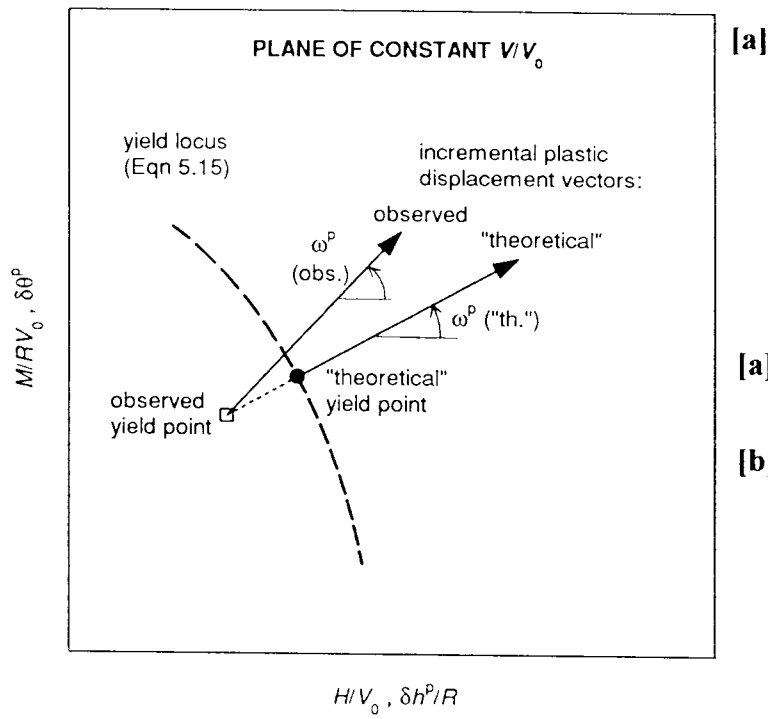
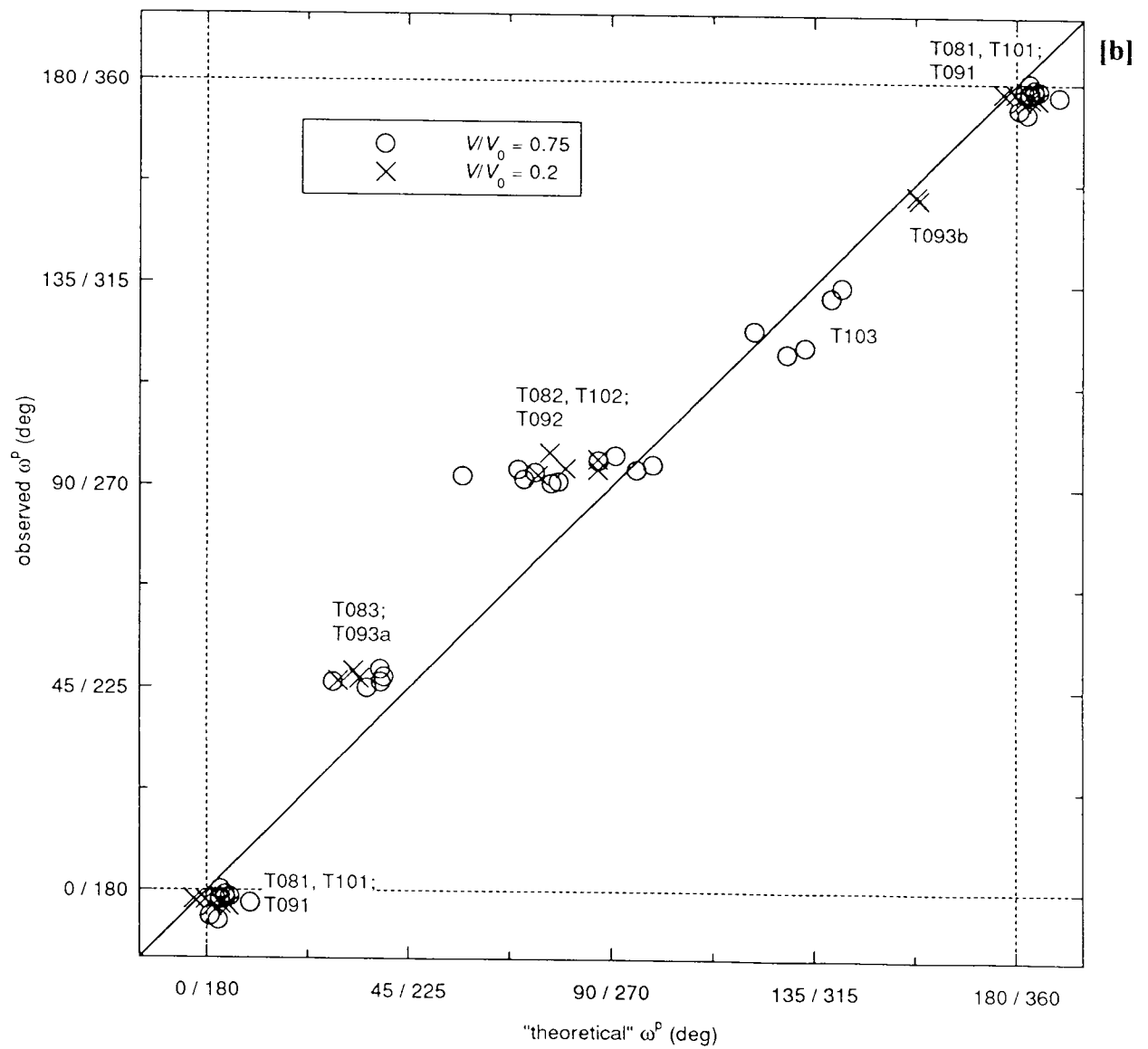


FIGURE 5.25

- [a]** definition of observed and "theoretical" directions ω^p ;
- [b]** plot of observed ω^p against "theoretical" ω^p



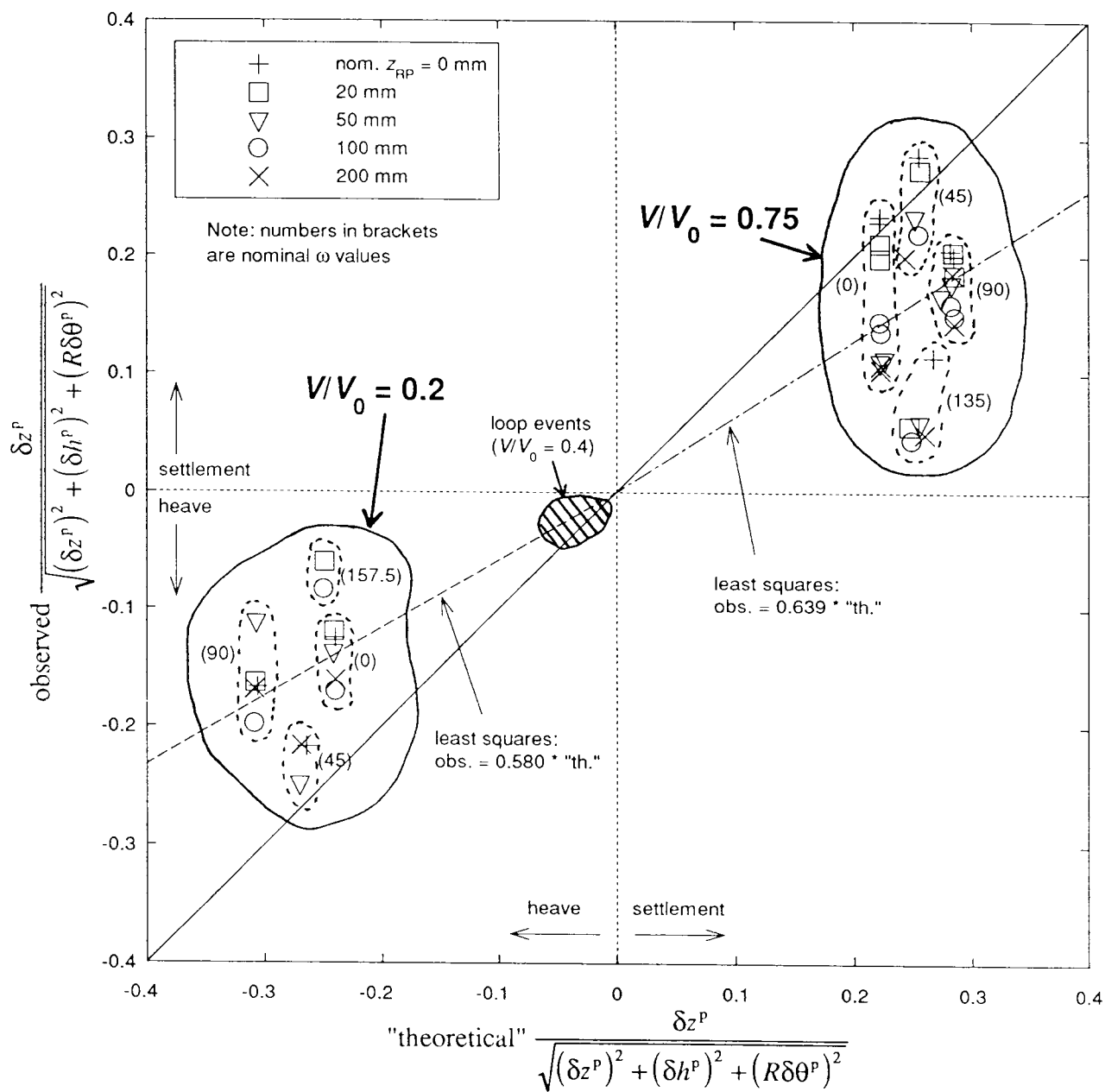


FIGURE 5.26 Observed and "theoretical" vertical plastic displacements at yield

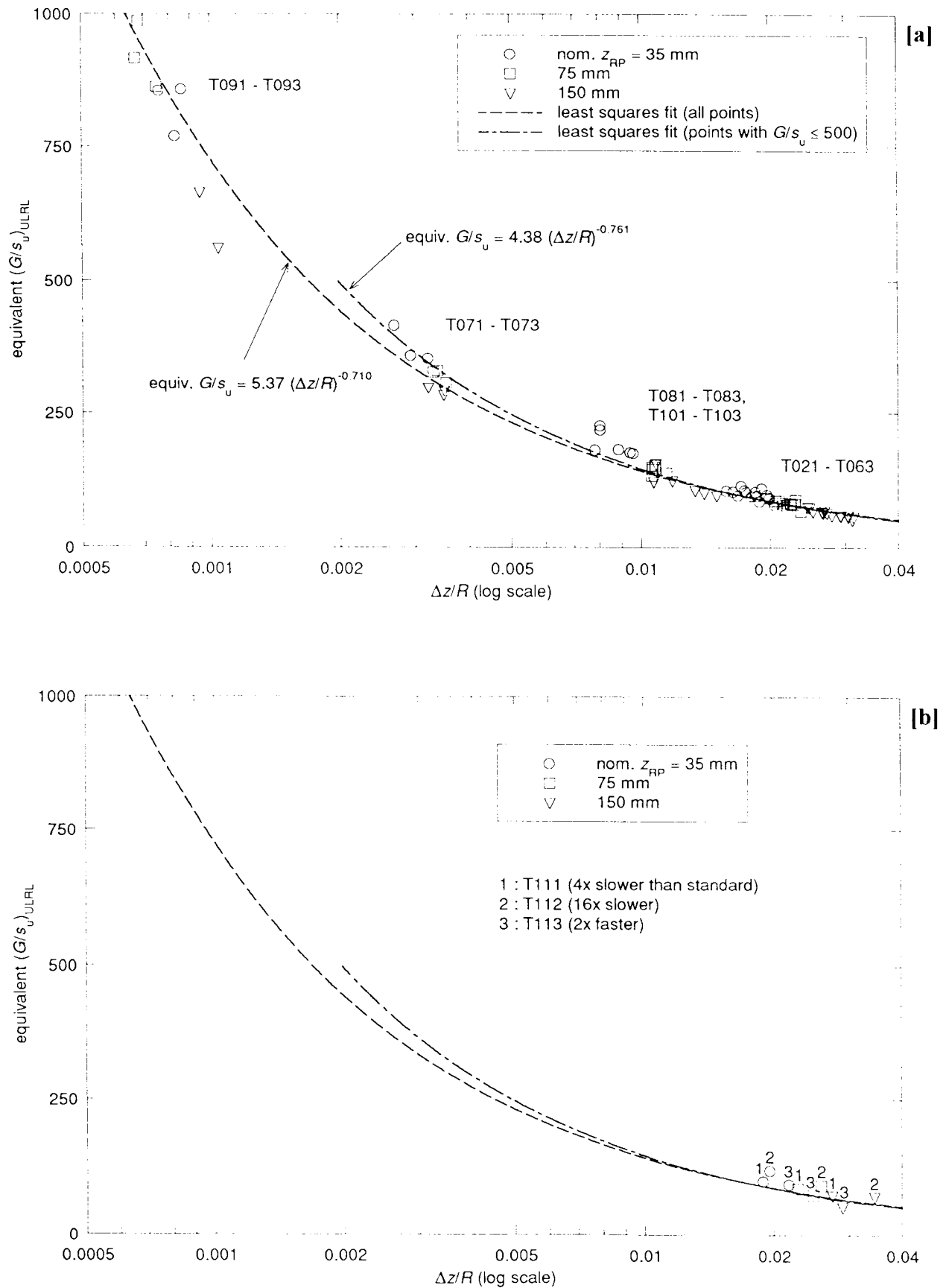


FIGURE 5.27 Summary of equivalent rigidity indices inferred from vertical unload-reload loops using **[a]** standard displacement rates; **[b]** Batch 11 displacement rates

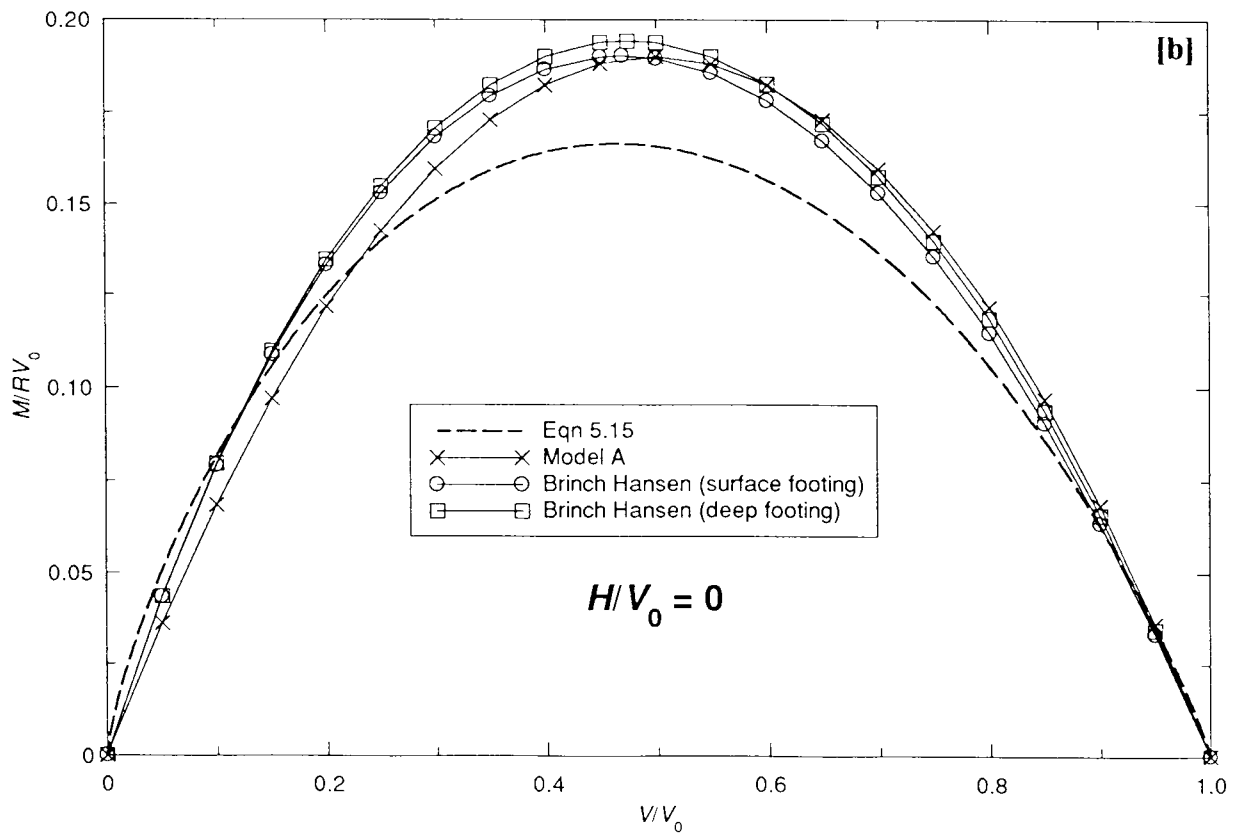
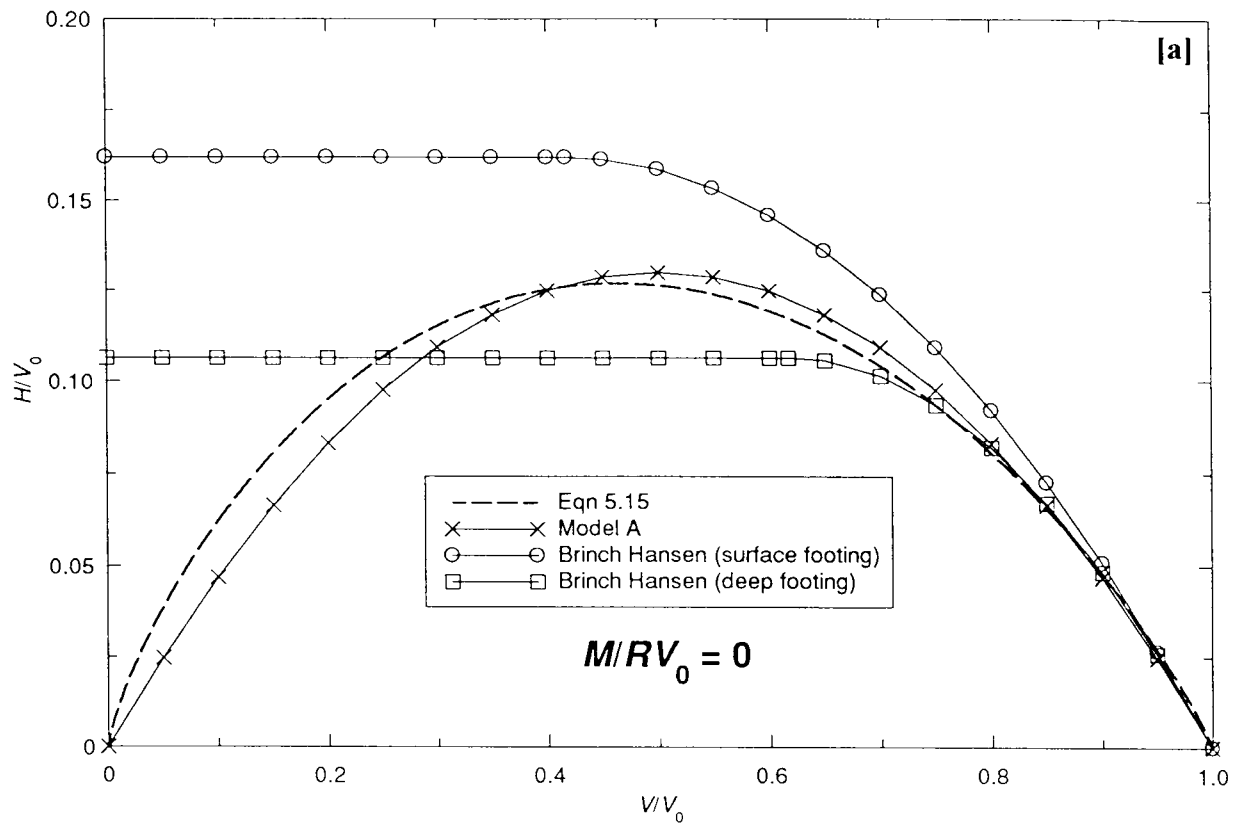


FIGURE 5.28 Bearing capacity interaction diagrams for circular footings on clay
[a] V/V_0 : H/V_0 plane; **[b]** V/V_0 : M/RV_0 plane

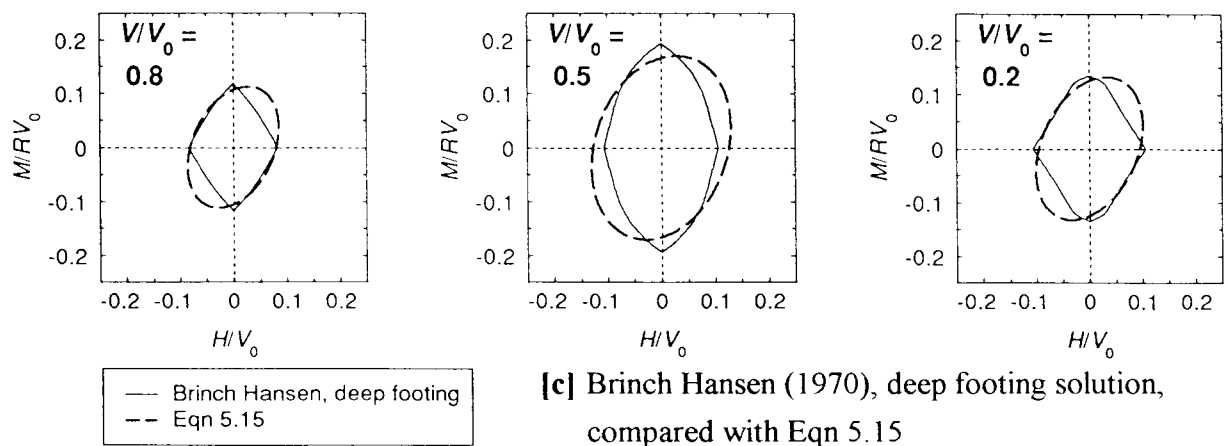
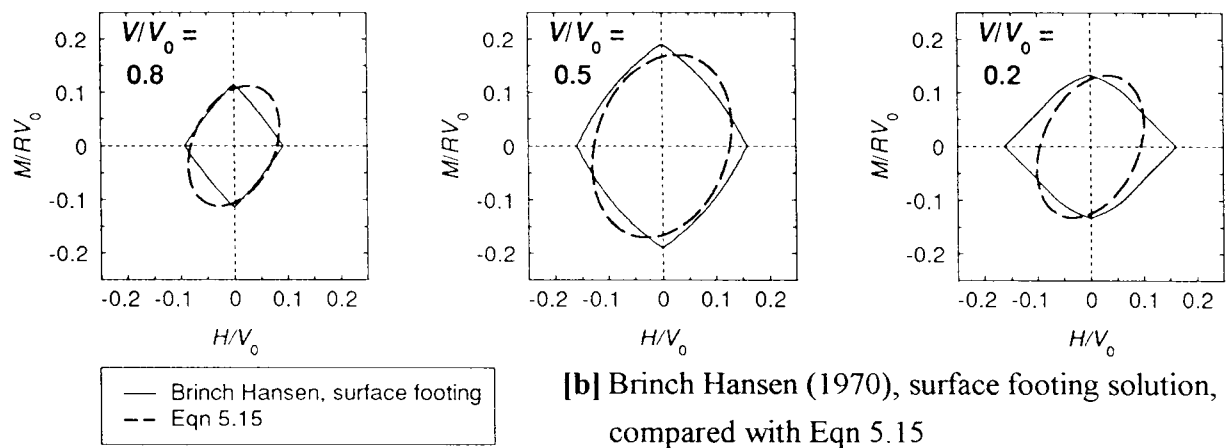
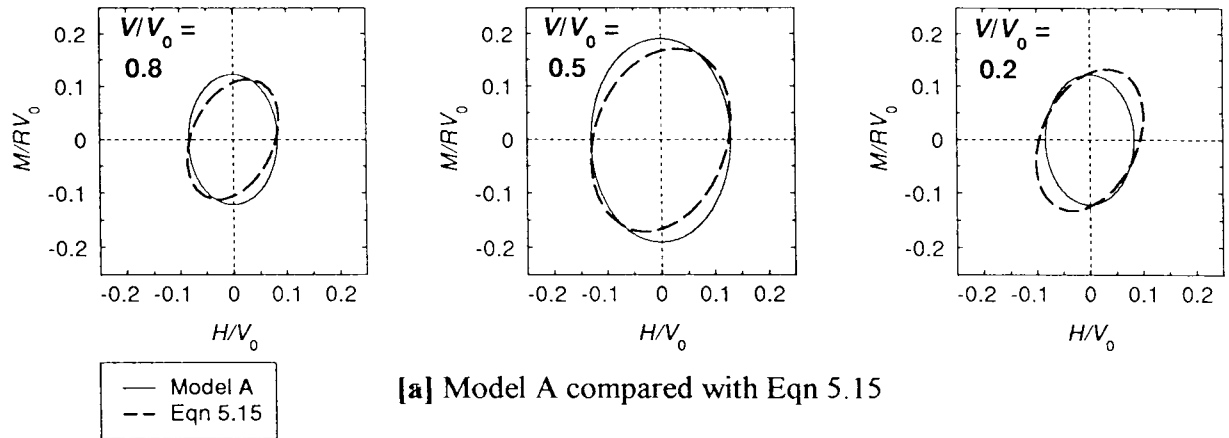


FIGURE 5.29 $H/V_0 : M/RV_0$ interaction diagrams for circular footings on clay

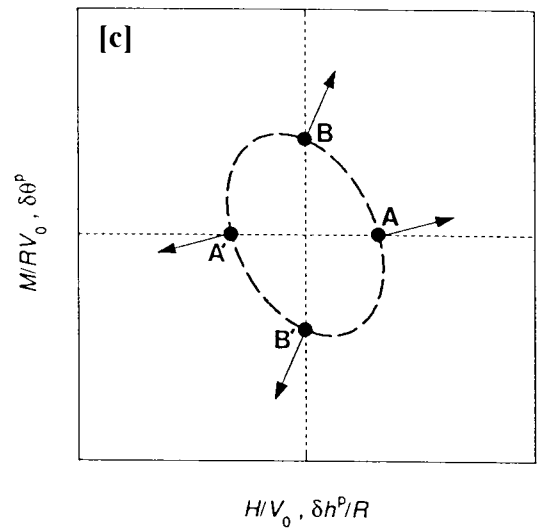
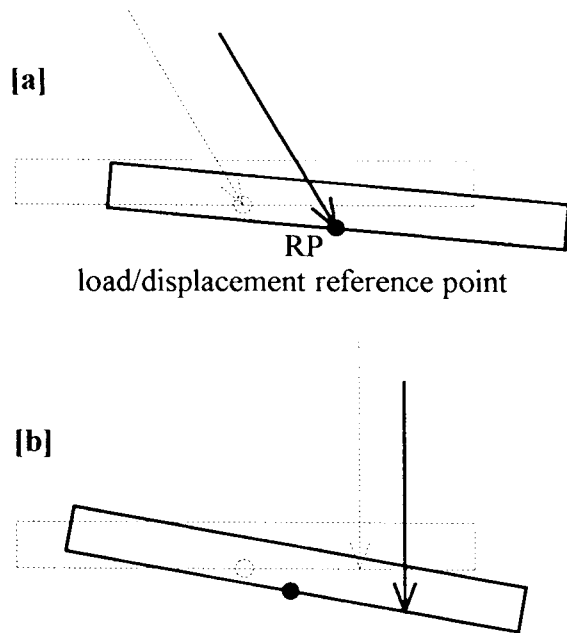


FIGURE 5.30

[a], [b] Combined loading tests of 100 mm diameter flat circular footing on sand
[c] Anticipated cross-sectional shape of yield surface, assuming associated flow

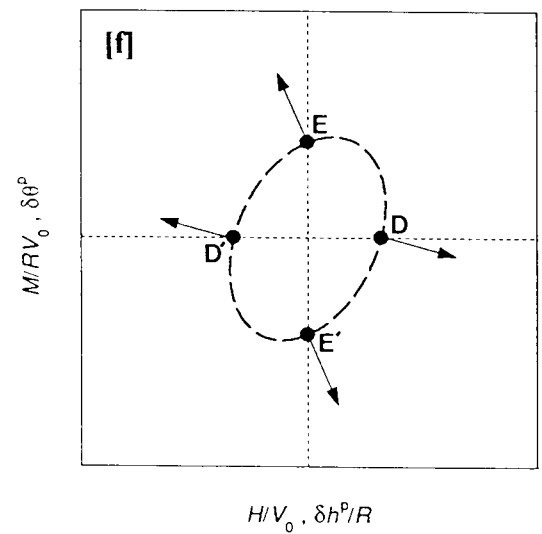
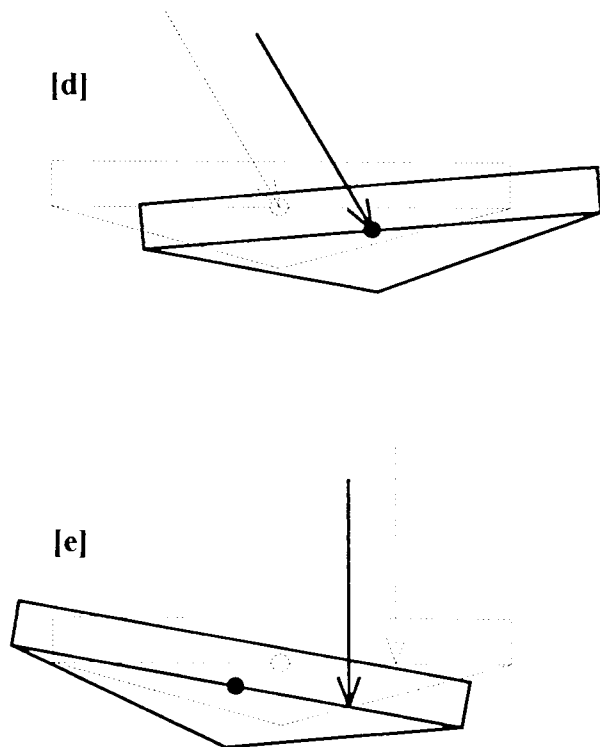


FIGURE 5.30

[d], [e] Combined loading tests of 100 mm diameter 150° conical footing on sand
[f] Anticipated cross-sectional shape of yield surface, assuming associated flow

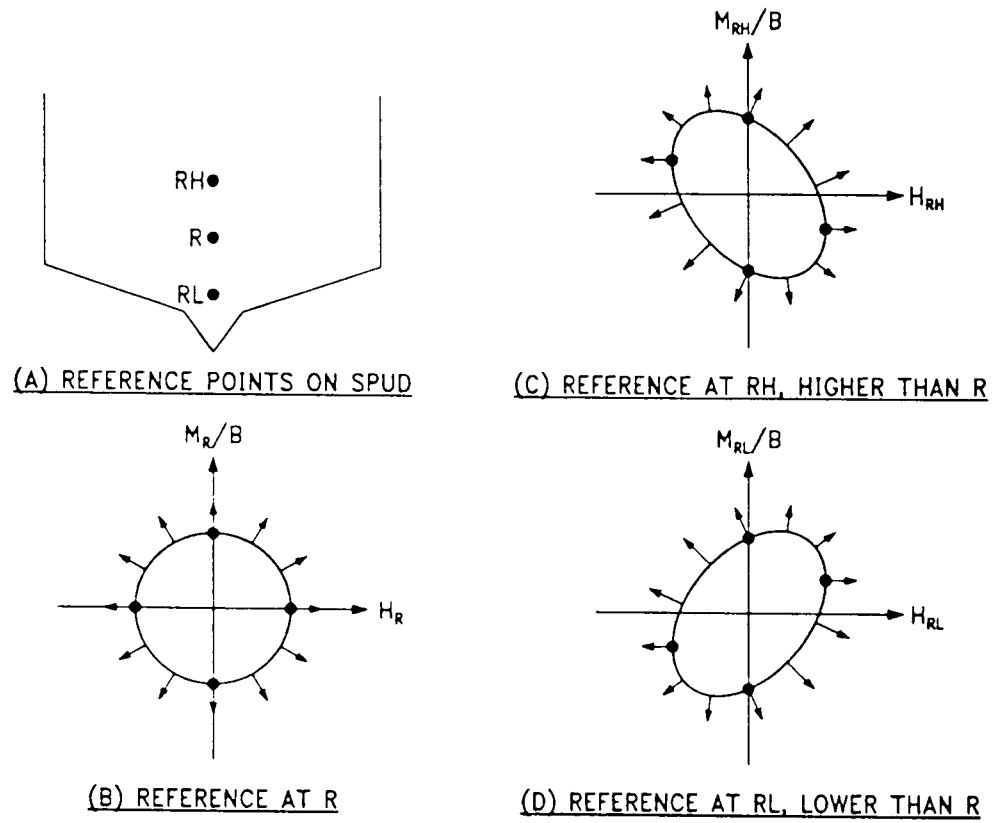


FIGURE 5.31 Effect of moving load/displacement reference point up or down on footing centreline (after Dean *et al.*, 1992)

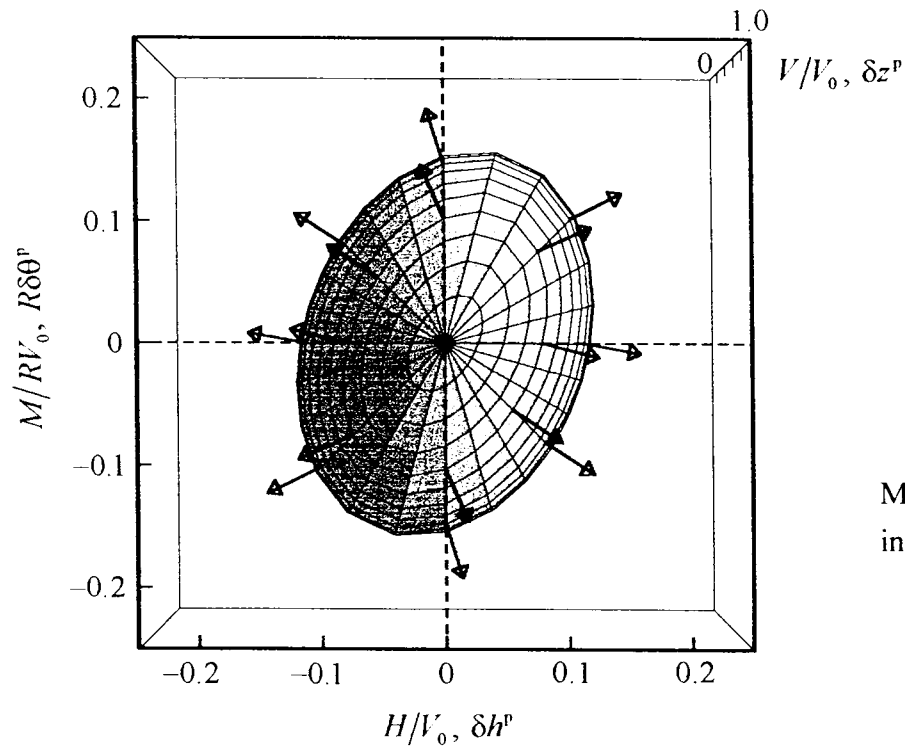


FIGURE 6.1
Model B: associated flow
in the $H:M/R$ plane

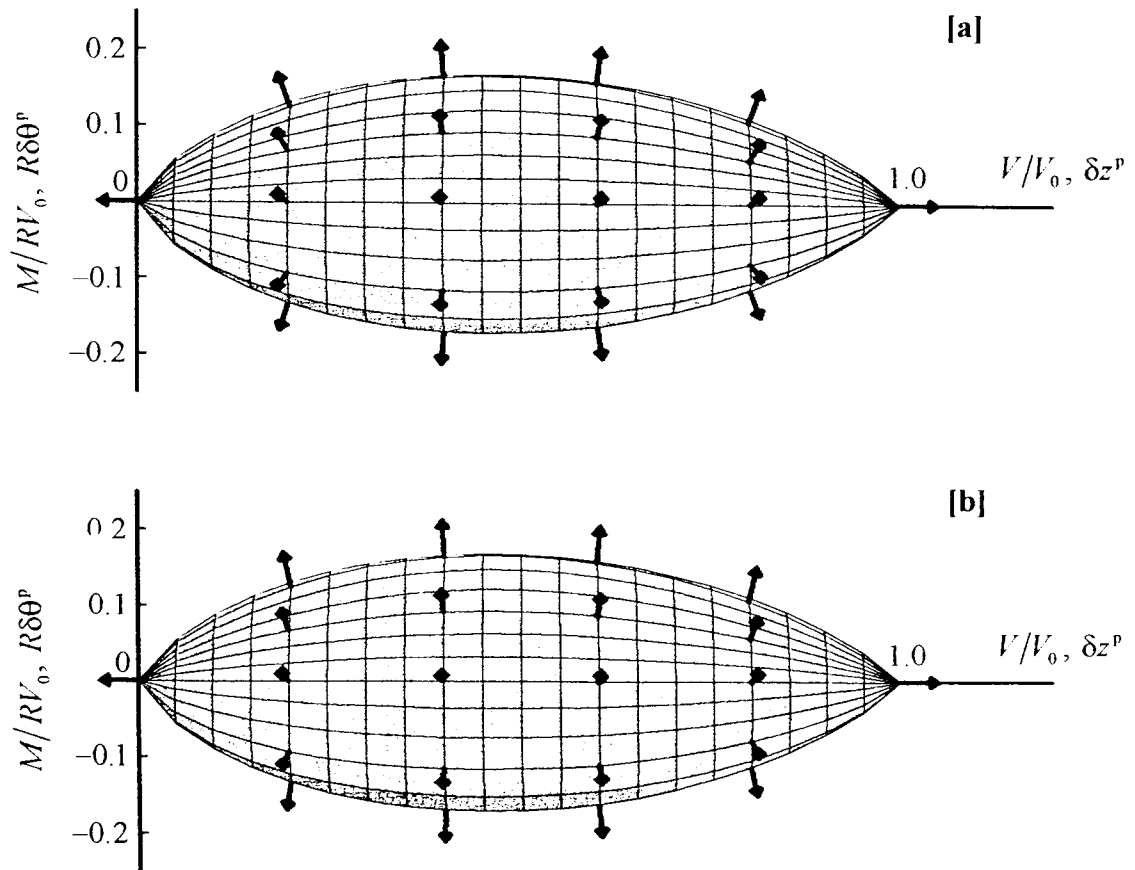


FIGURE 6.2 Model B: incremental plastic displacement vectors with
[a] $\zeta_s = 1$; [b] ζ_s given by Eqn 6.7b

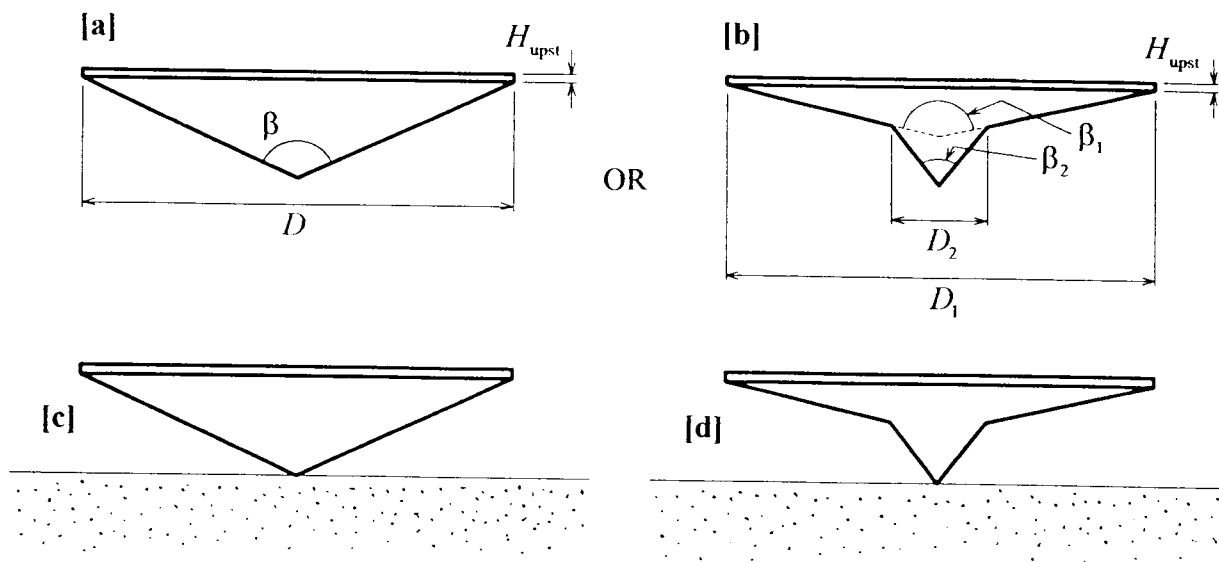


FIGURE 6.3 Conical and spudcan footings in program *OXSPUD*

[a], [b] dimensions required as input data;

[c], [d] initial footing locations

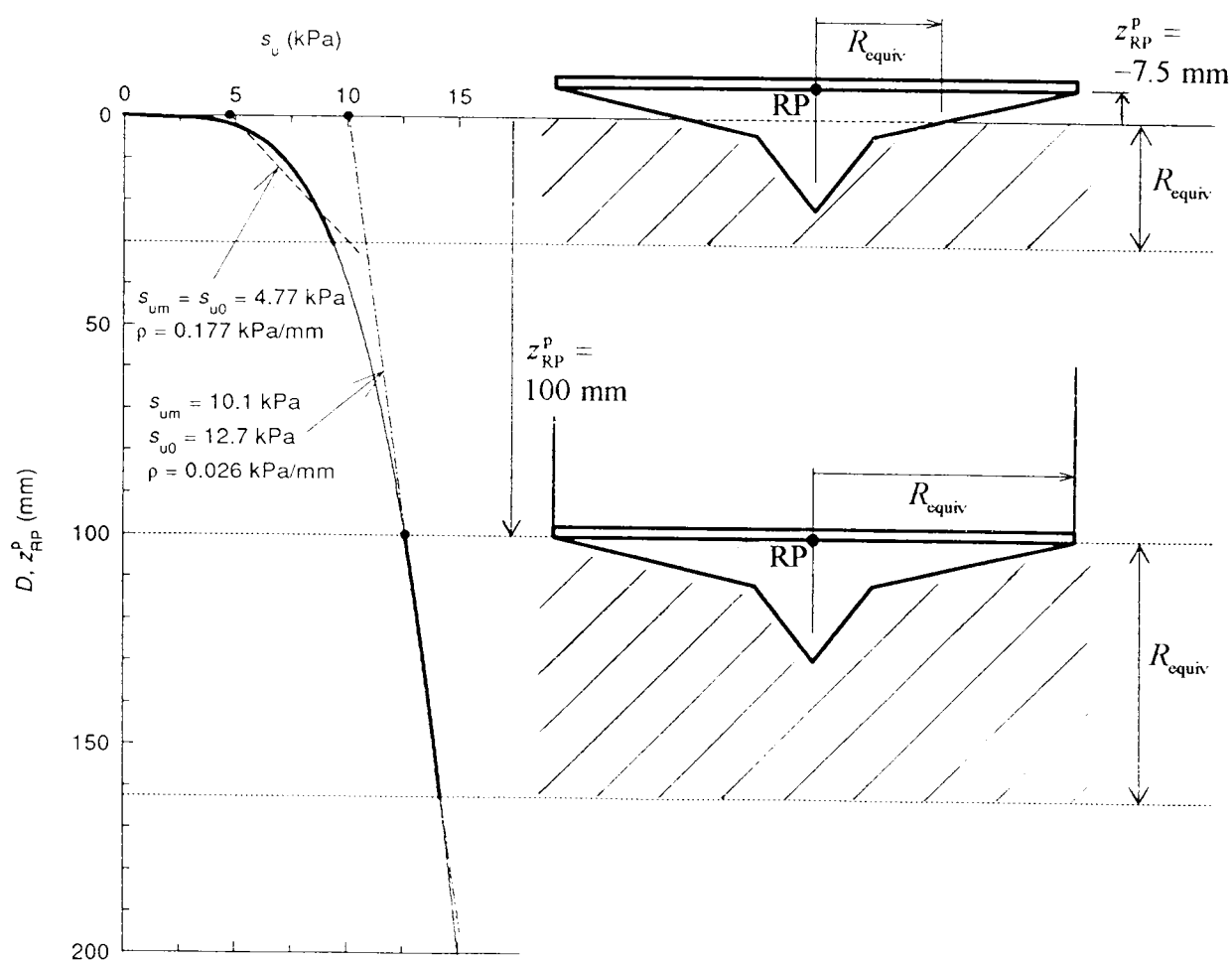


FIGURE 6.4 Calculation of vertical bearing capacity V_0 in subroutine *VOCAL*

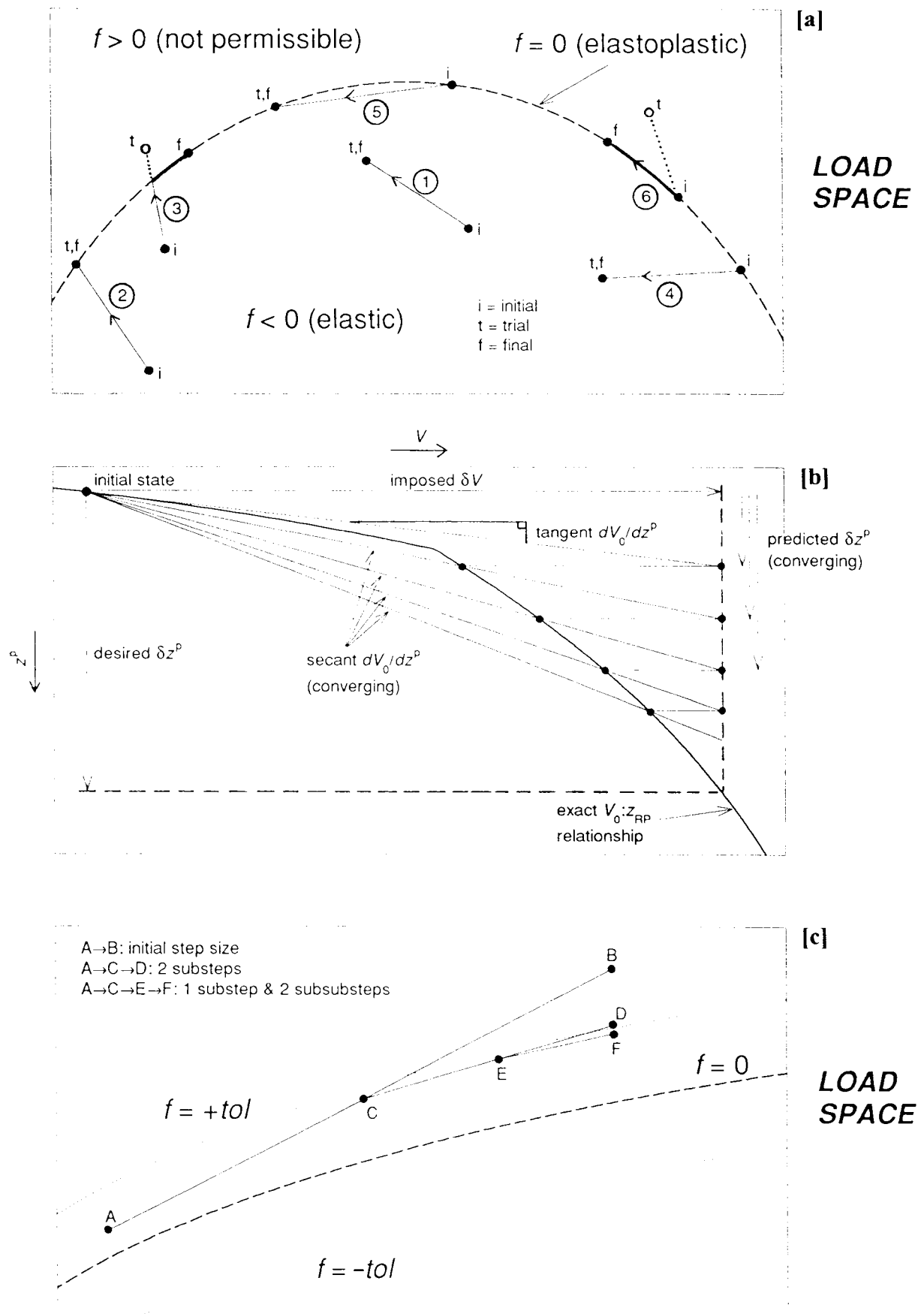


FIGURE 6.5 Solution control in program *OXSPUD*

- [a]** Possible increment types; **[b]** Iteration until convergence of δz^p ;
[c] Automatic step size division procedure

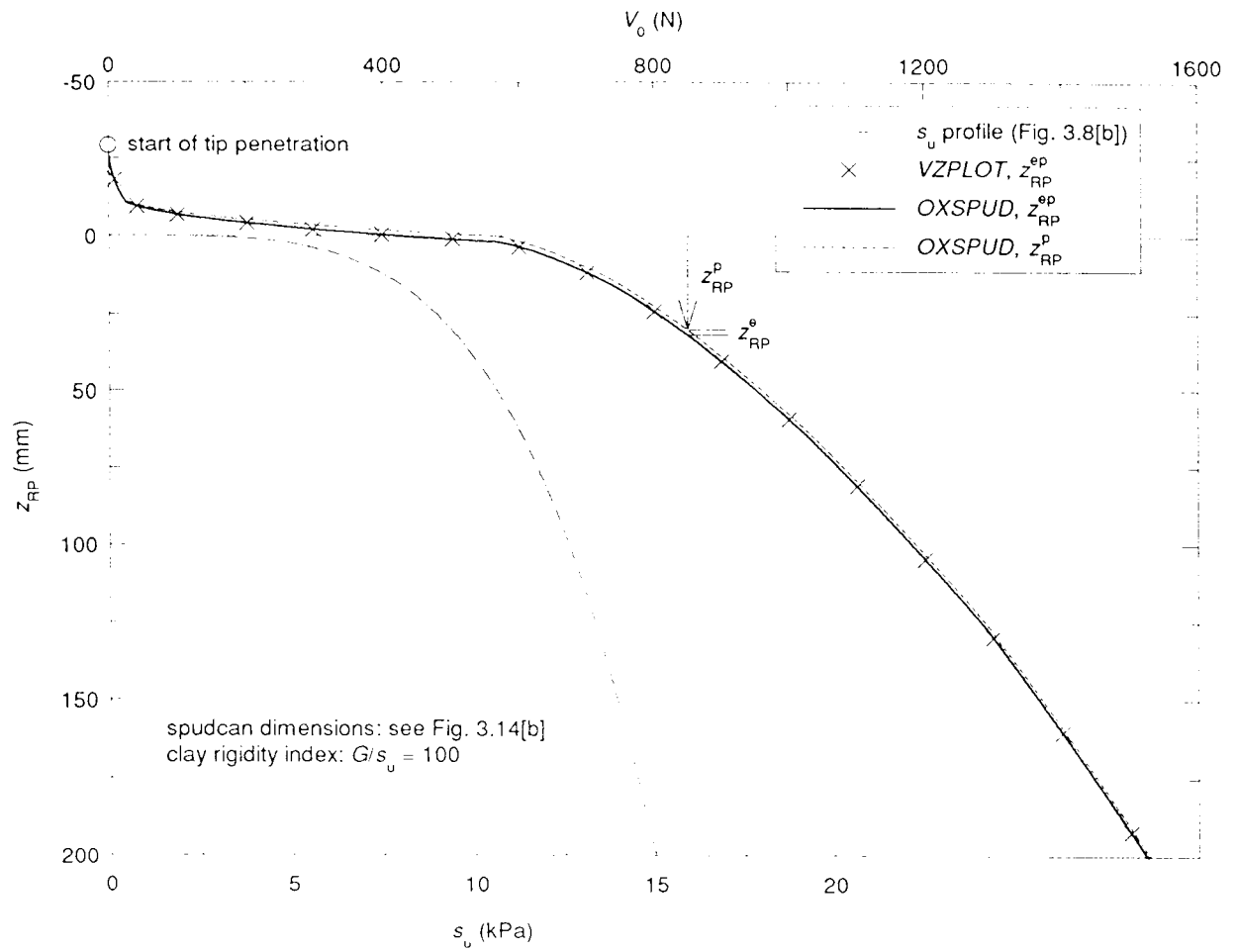


FIGURE 6.6 Vertical load:penetration curves calculated with *VZPLOT* and *OXSPUD*

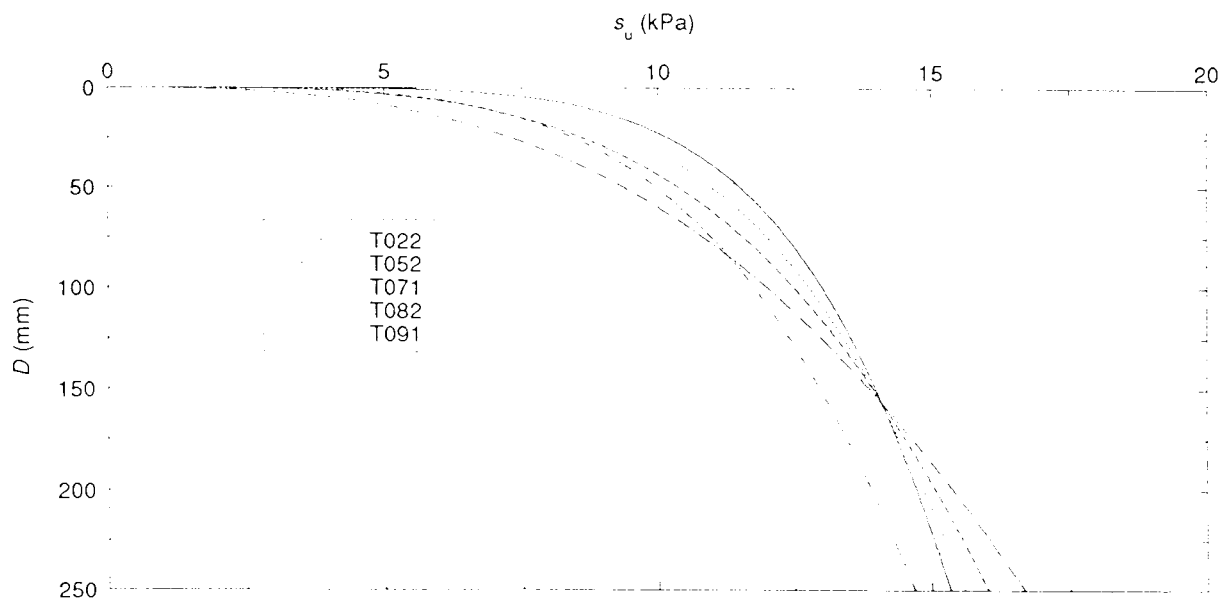


FIGURE 6.7 Fitted variations of undrained strength with depth (used for numerical predictions of test results)

FIGURE 6.8

T022: experimental results
(data markers at 2.0 mm intervals of $R\theta$)

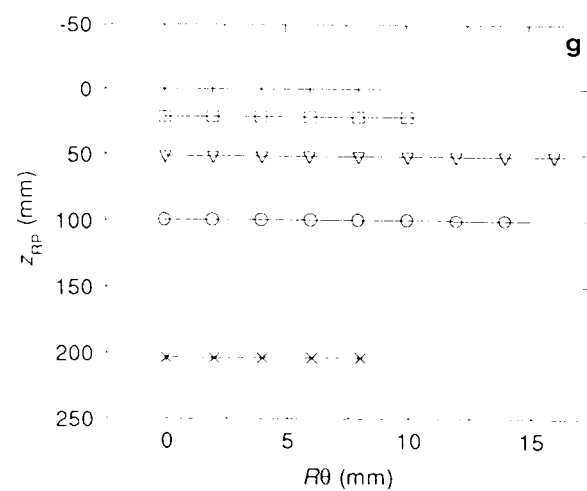
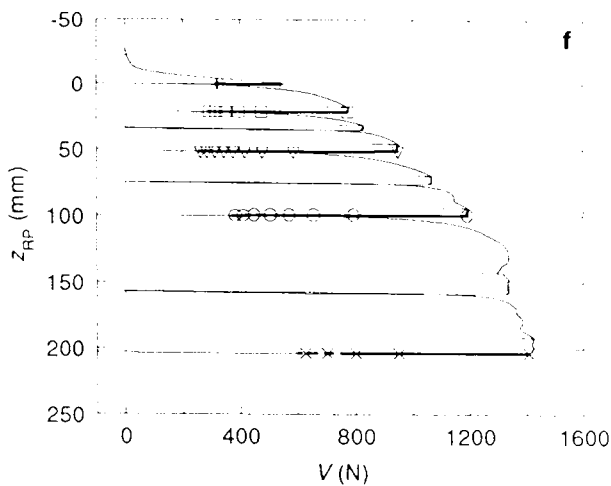
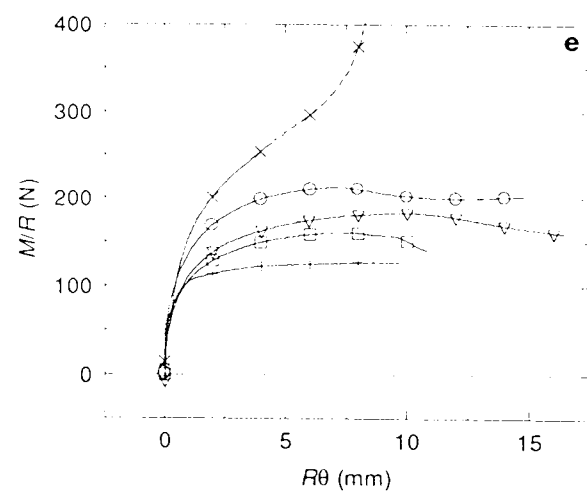
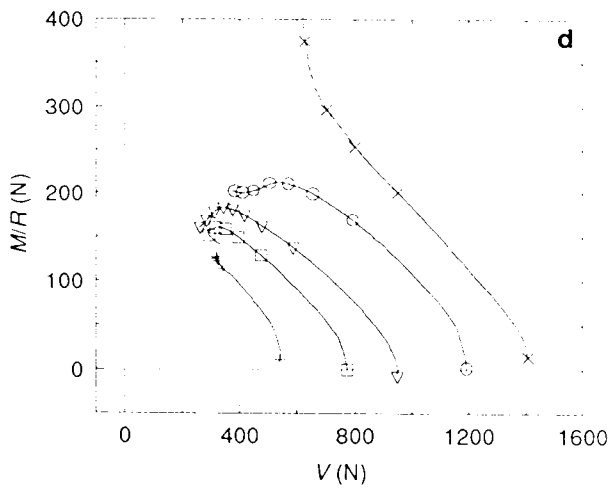
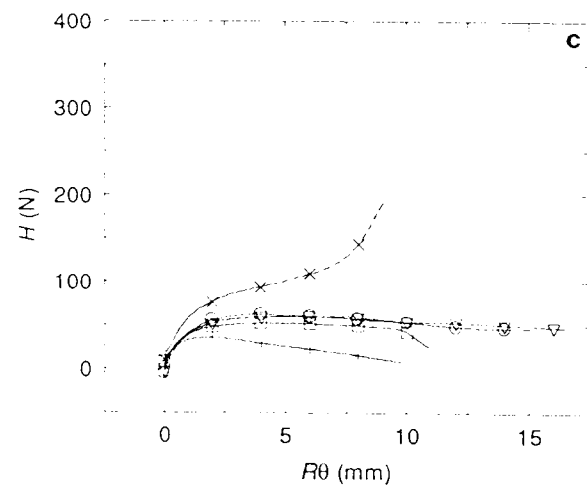
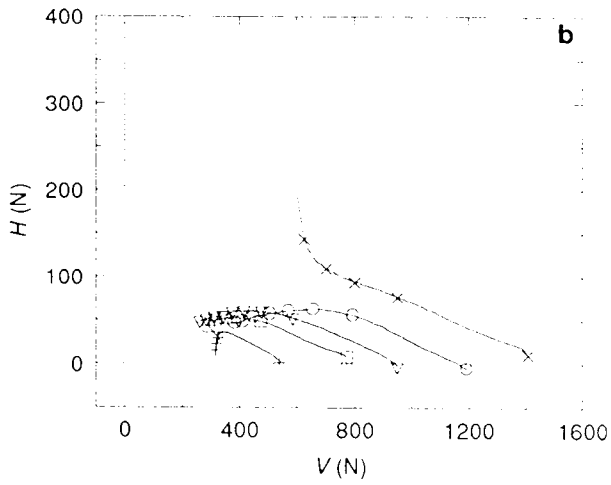
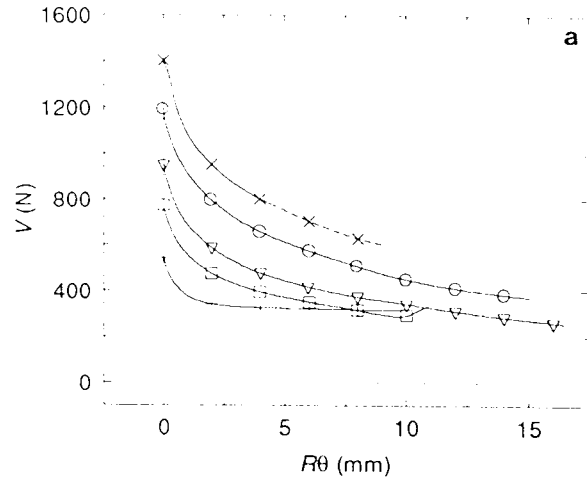
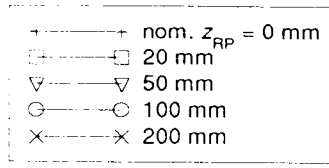


FIGURE 6.9

T022: OXSPUD prediction
(data markers at 2.0 mm
intervals of $R\theta$; rigidity
index $G/s_u = 100$)

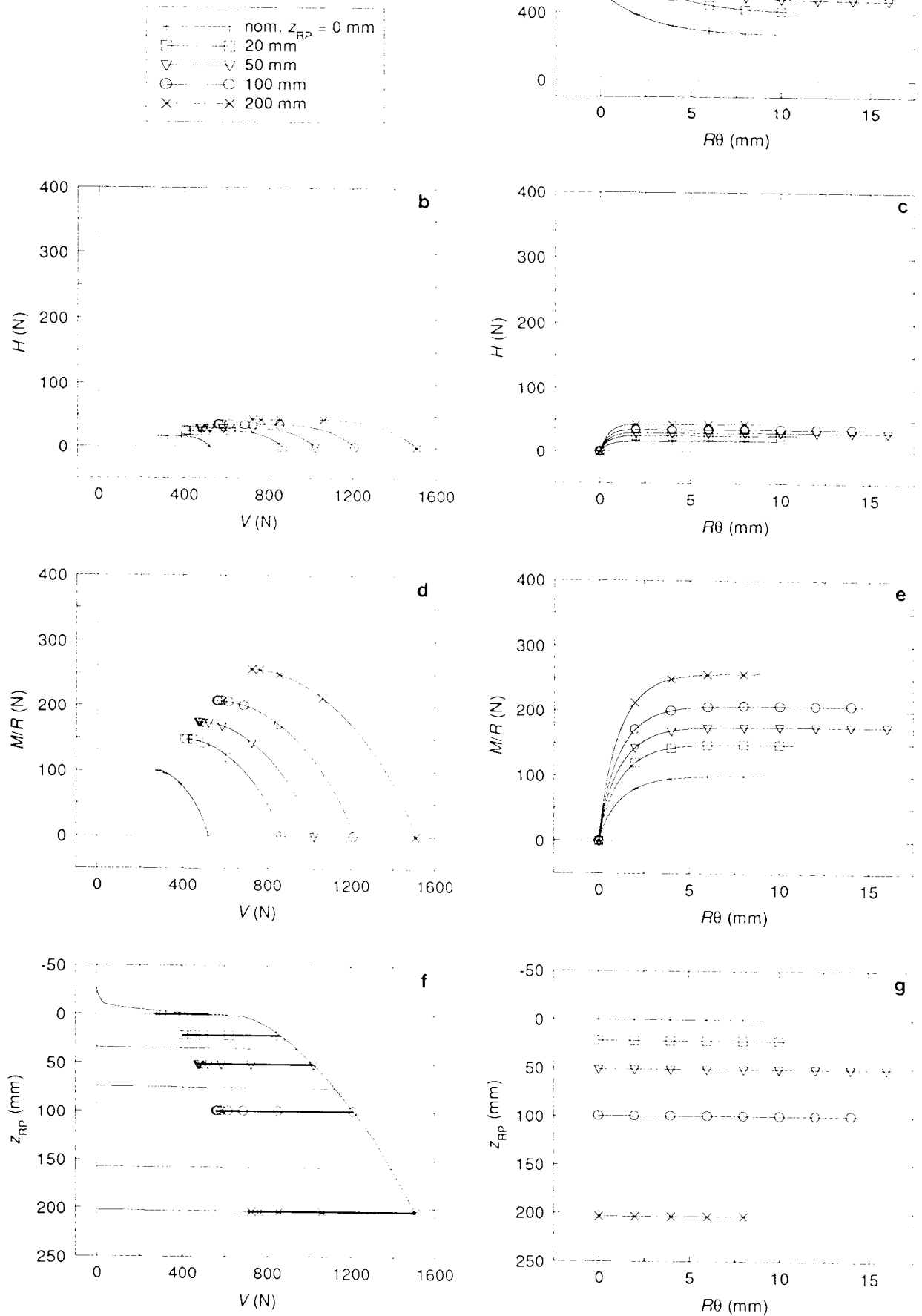


FIGURE 6.10

T052: experimental results
(data markers at 2.0 mm
intervals of h)

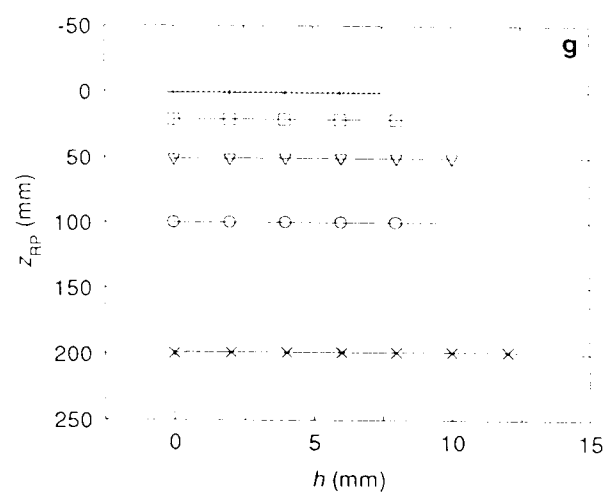
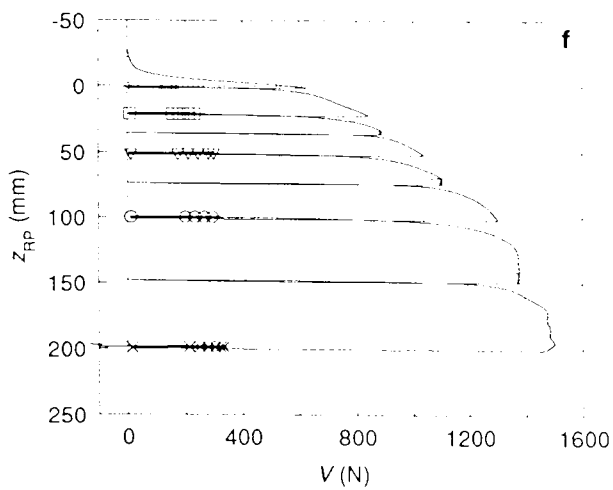
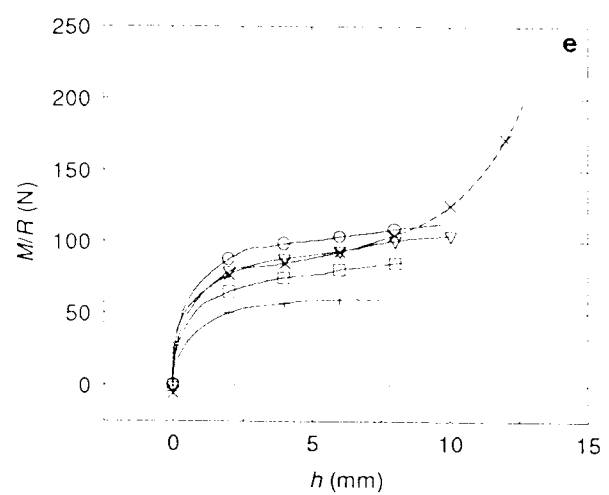
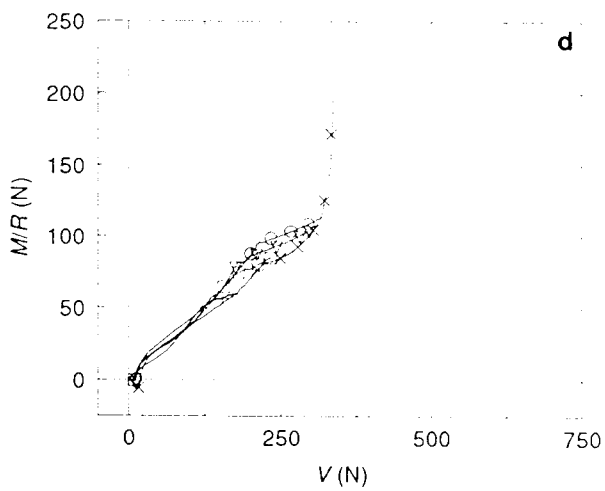
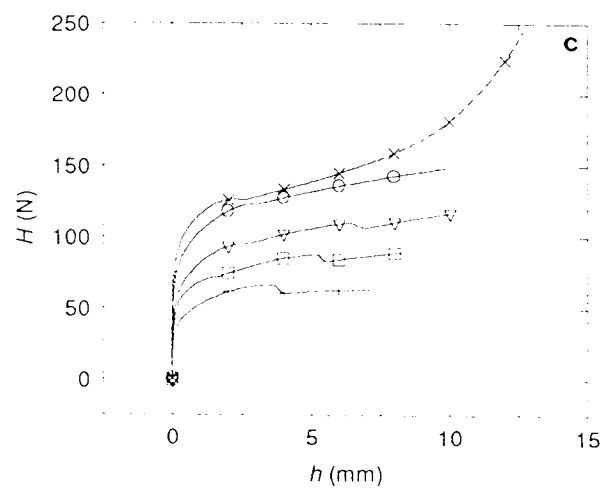
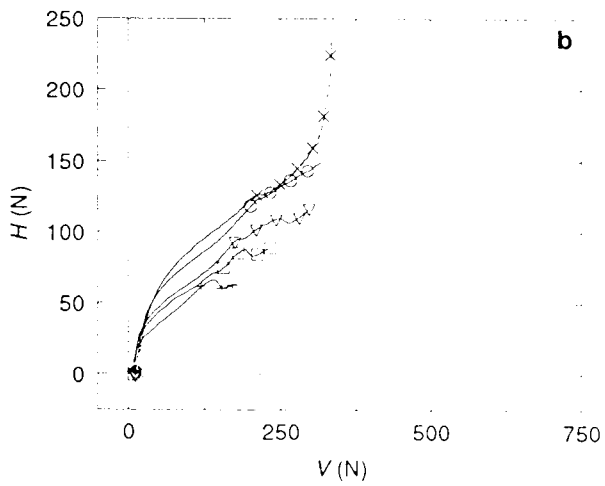
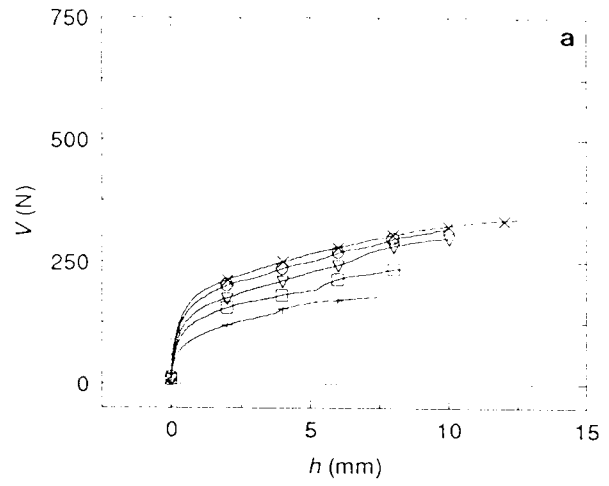
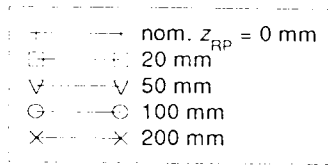


FIGURE 6.11

T052: OXSPUD prediction
(data markers at 2.0 mm
intervals of h ; rigidity
index $G/s_u = 100$)

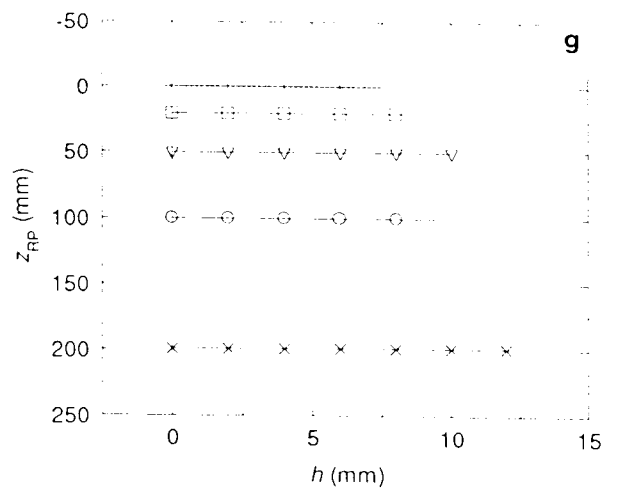
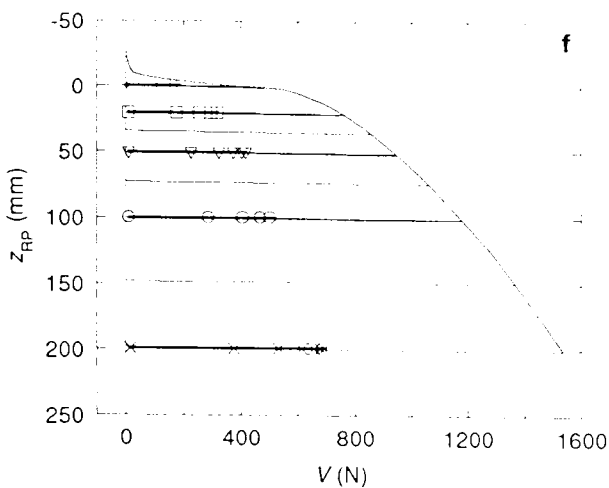
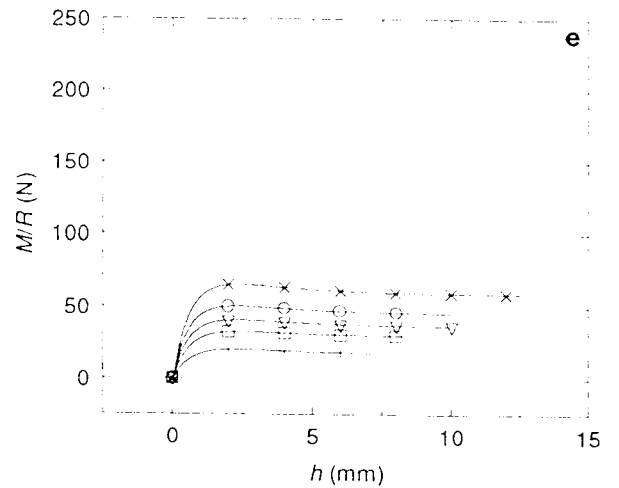
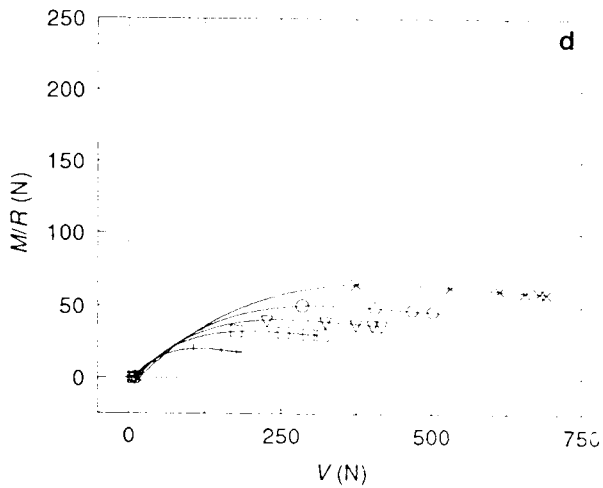
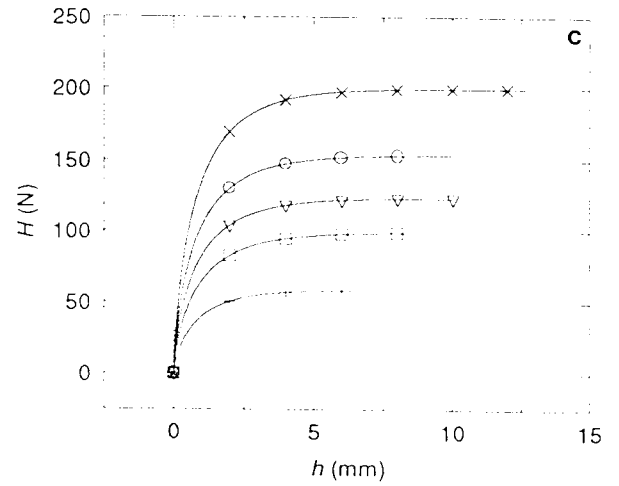
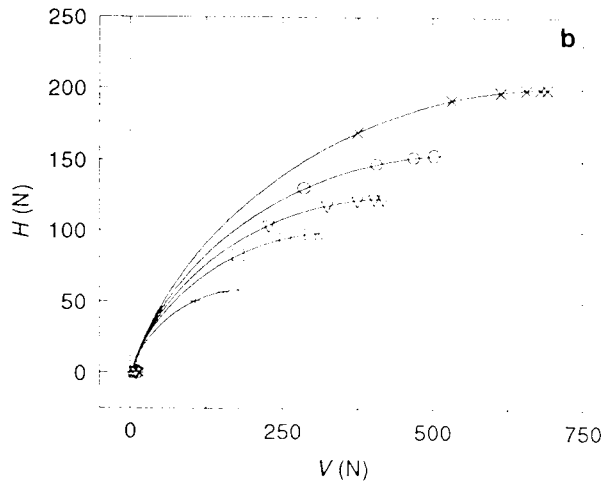
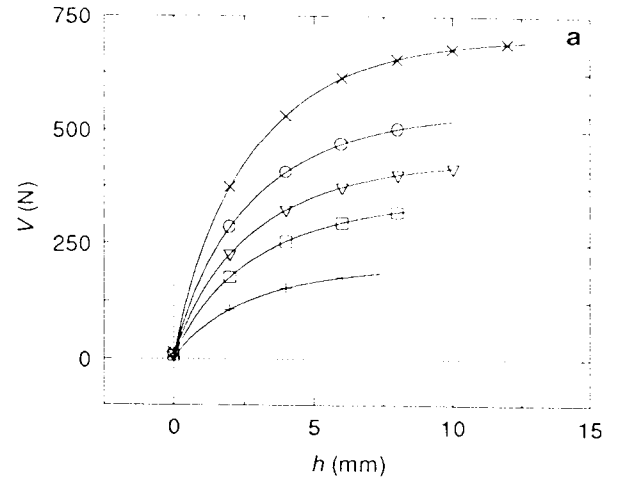
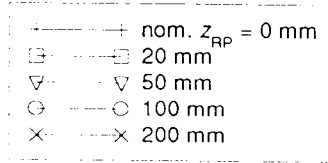


FIGURE 6.12

T082: experimental results
(data markers at 2.0 mm
intervals of $R\theta$)

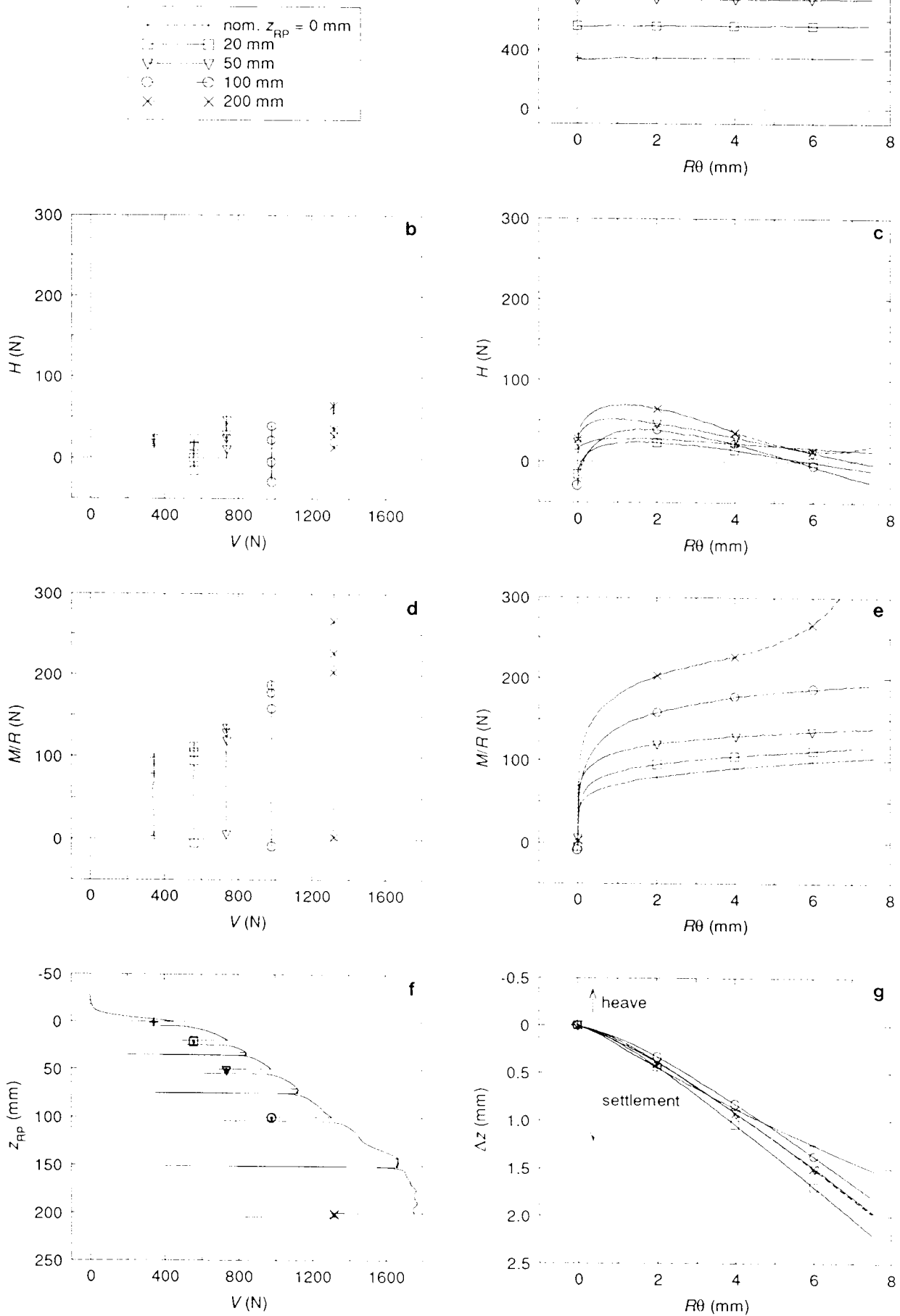


FIGURE 6.13

T082: OXSPUD prediction
(data markers at 2.0 mm
intervals of $R\theta$; rigidity
index $G/s_u = 100$)

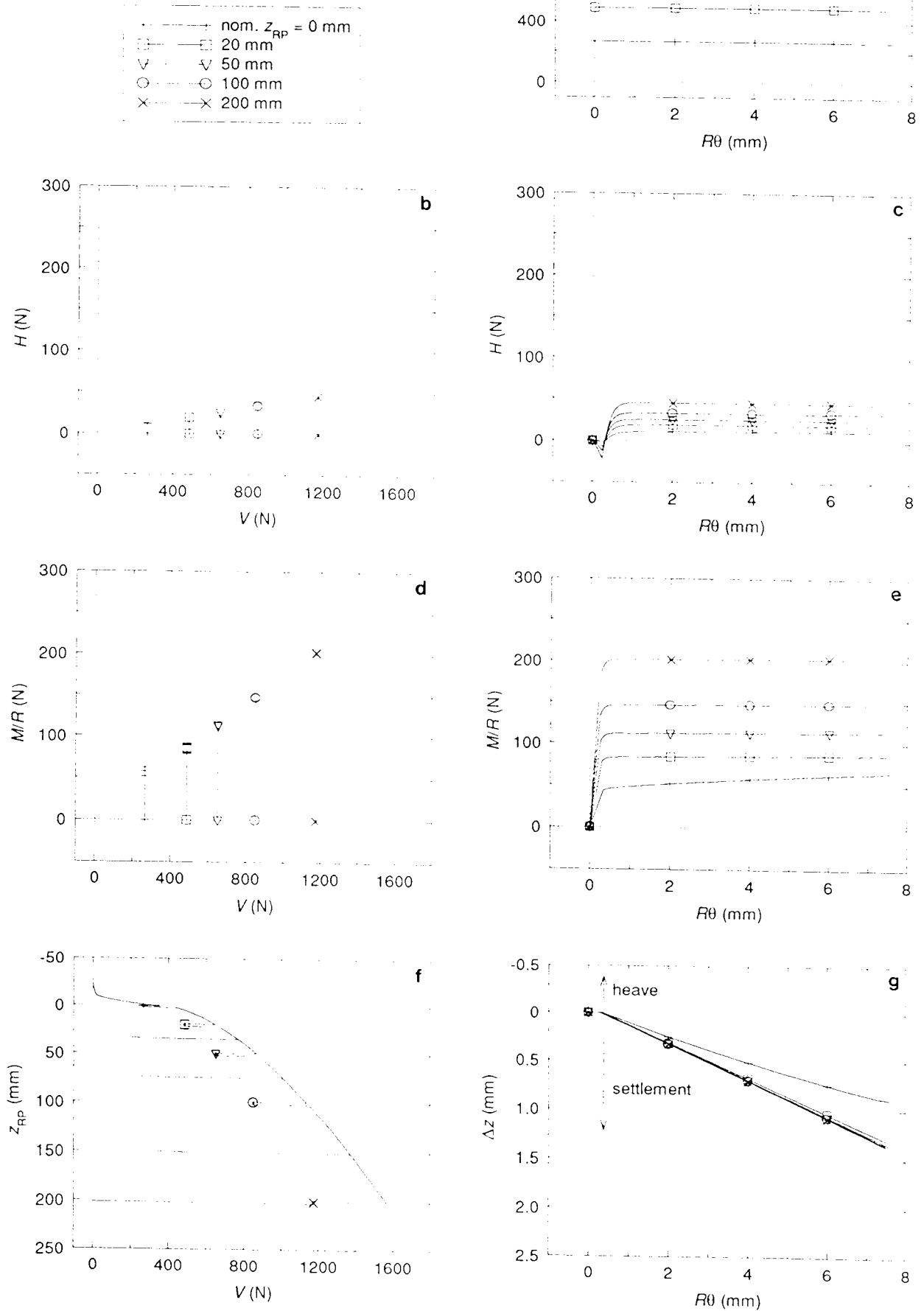


FIGURE 6.14

T091: experimental results
(data markers at 2.0 mm intervals of h)

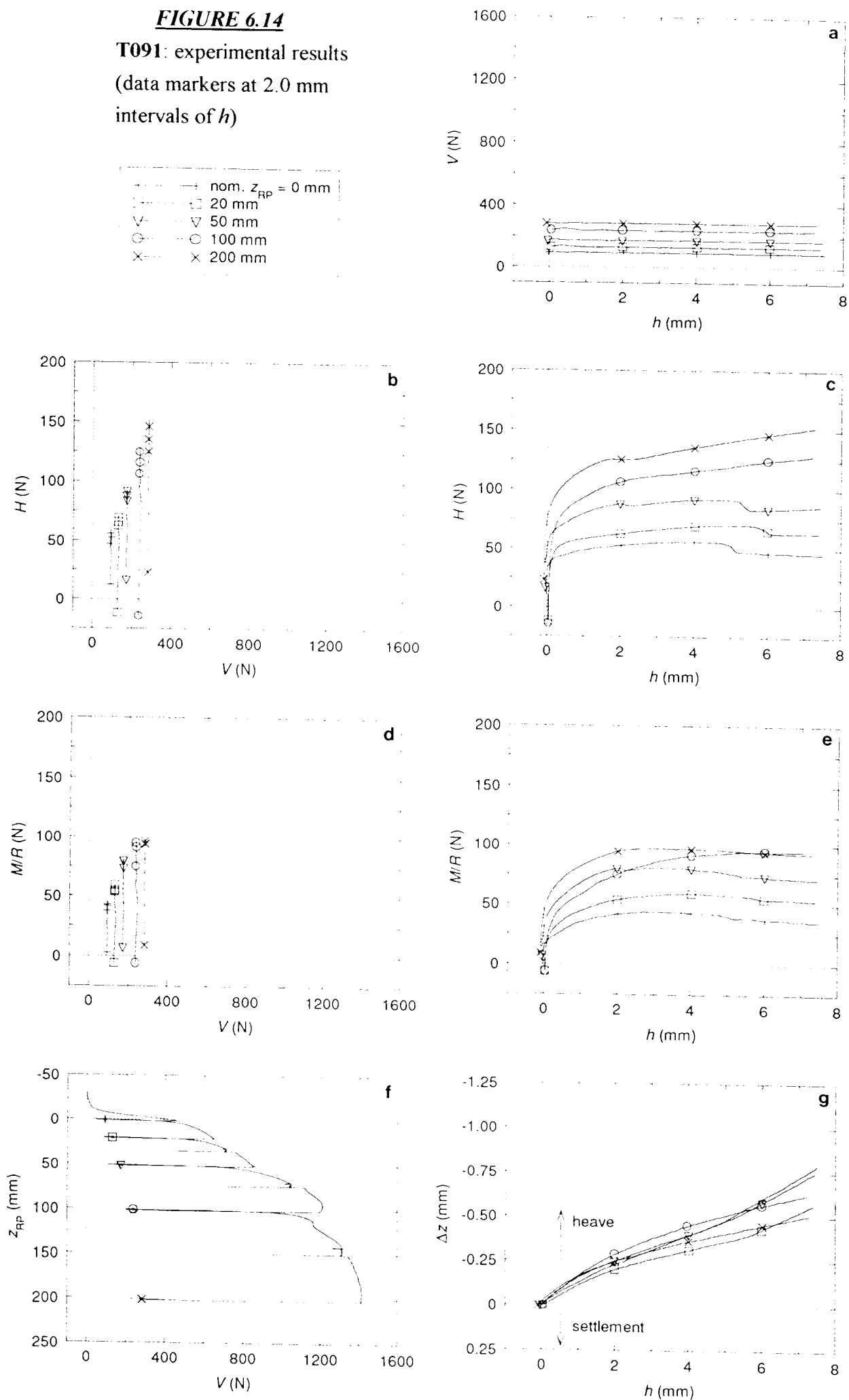
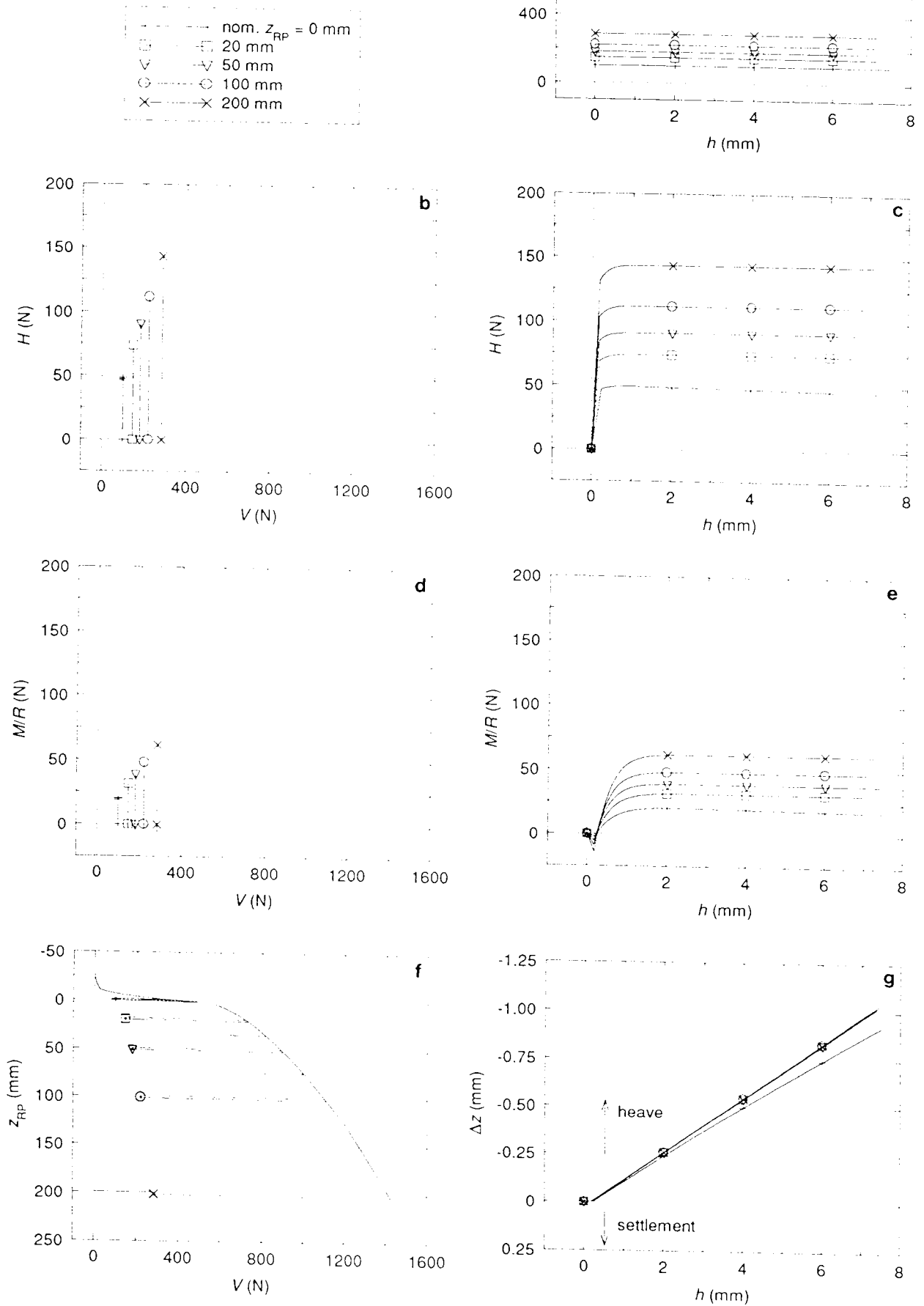


FIGURE 6.15

T091: OXSPUD prediction
(data markers at 2.0 mm
intervals of h ; rigidity
index $G/s_u = 100$)



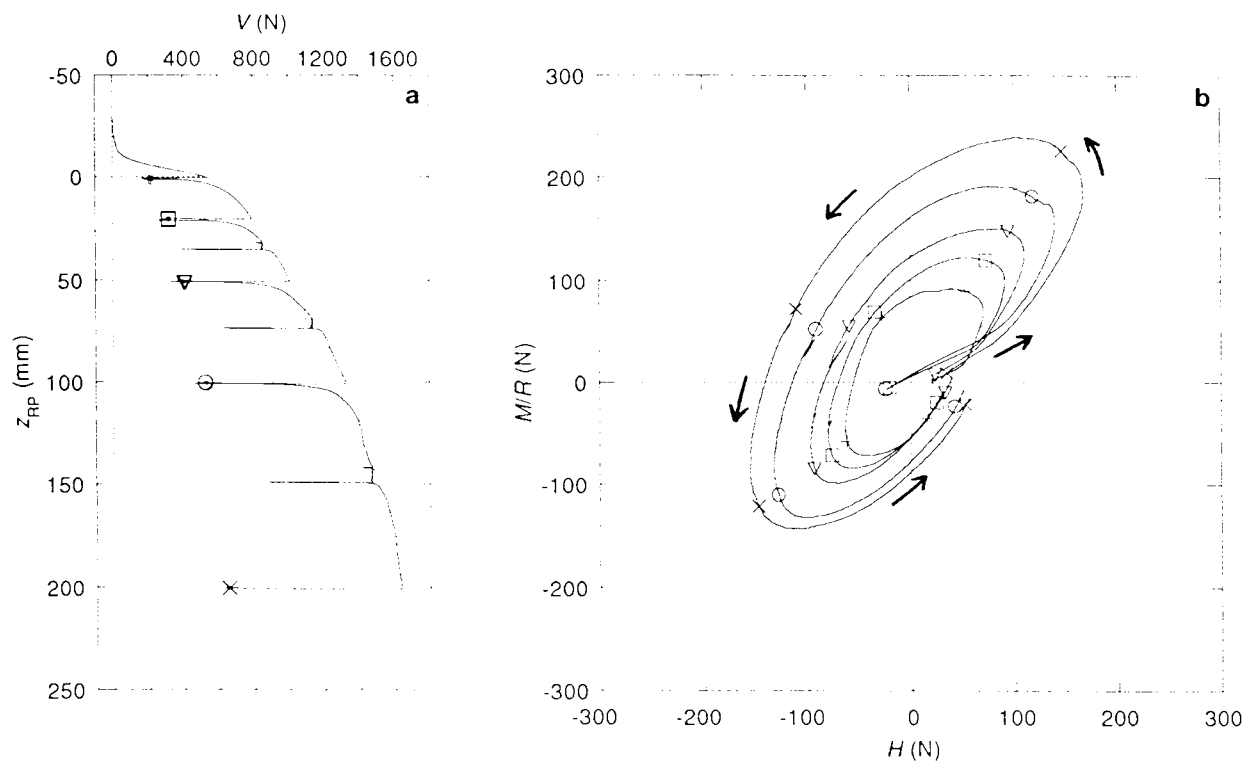


FIGURE 6.16

T071: experimental results

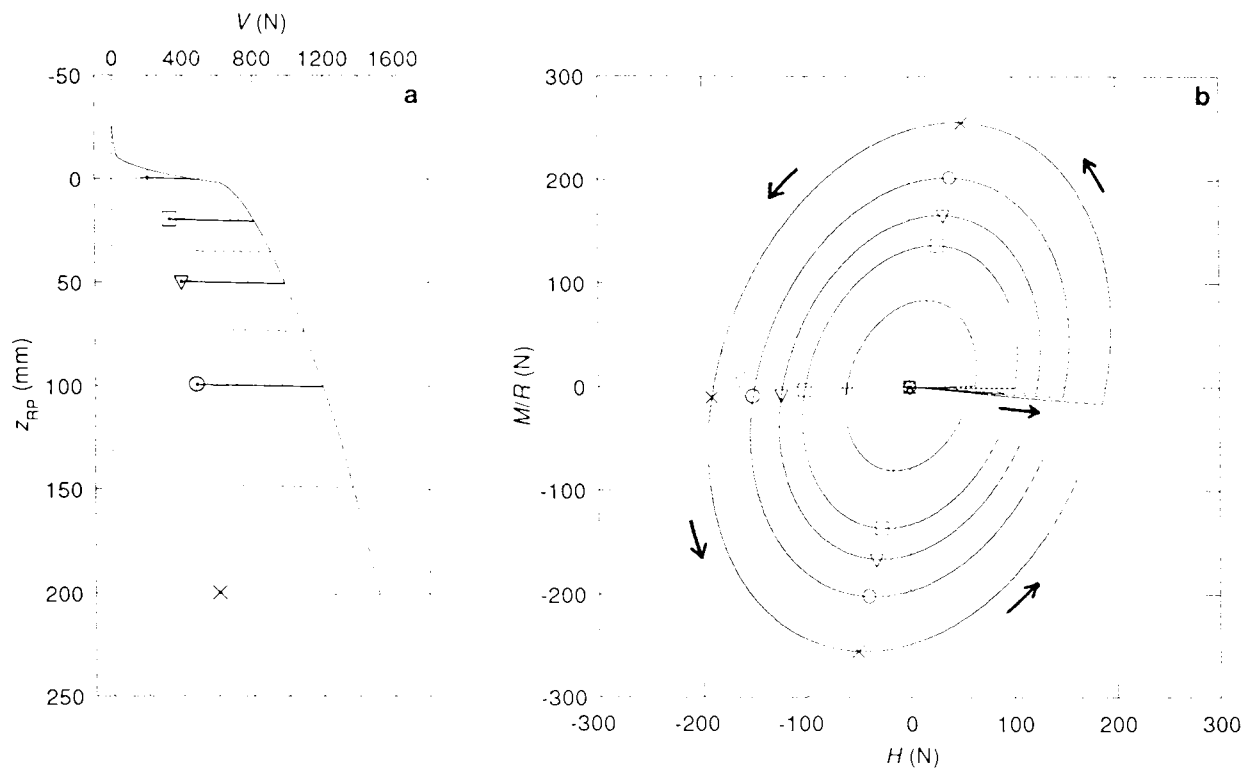
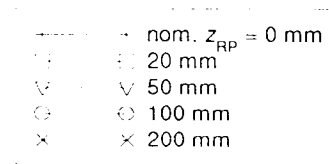
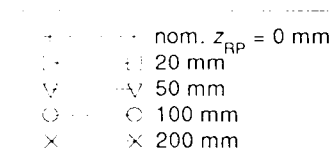


FIGURE 6.17

T071: OXSPUD prediction
(rigidity index $G/s_u = 100$)



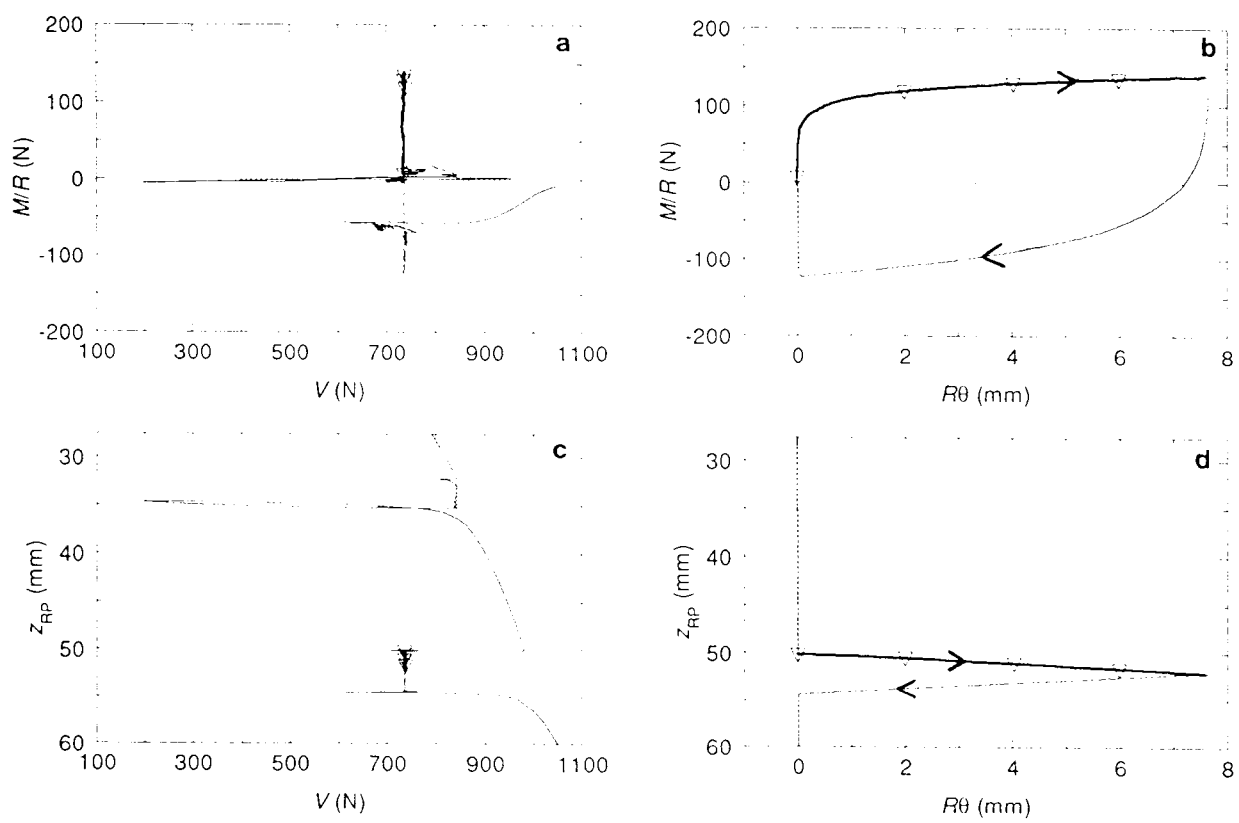


FIGURE 6.18

Extract from **T082**: experimental results

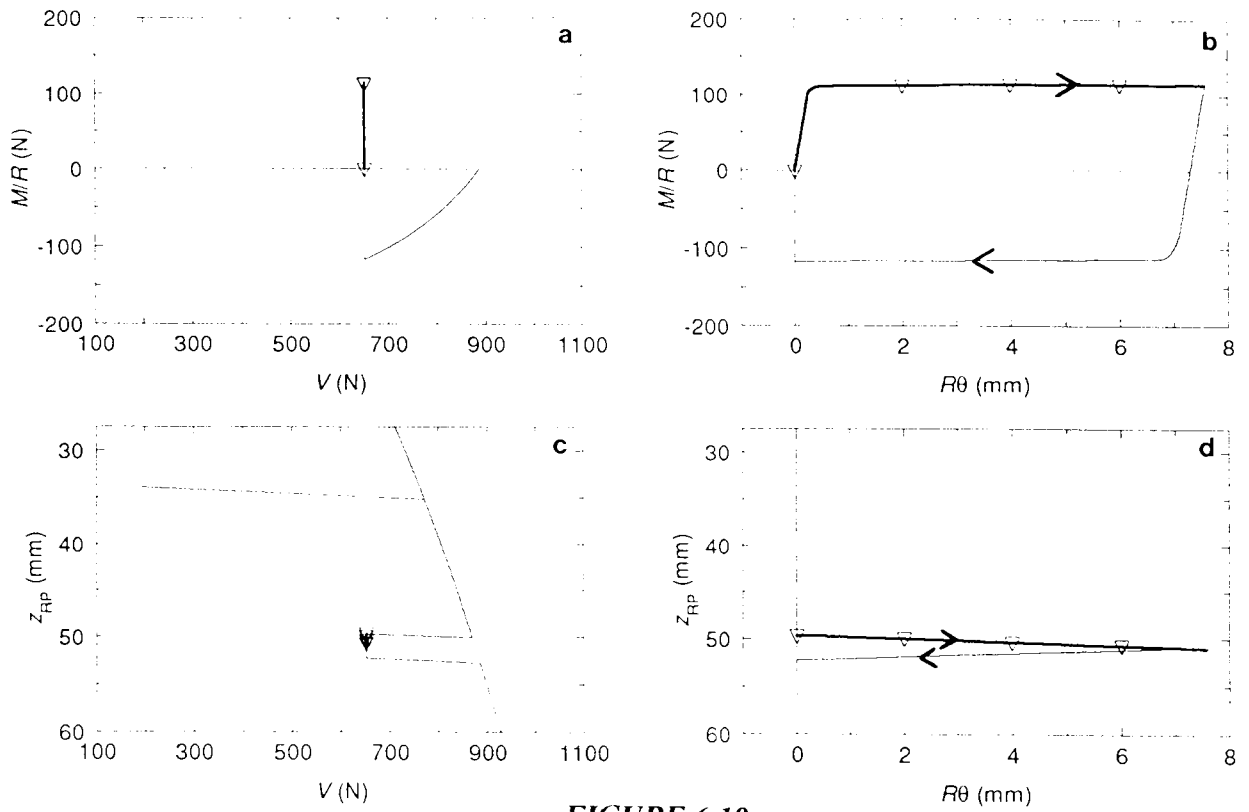


FIGURE 6.19

Extract from **T082**: *OXSPUD* prediction
(rigidity index $G/s_u = 100$)

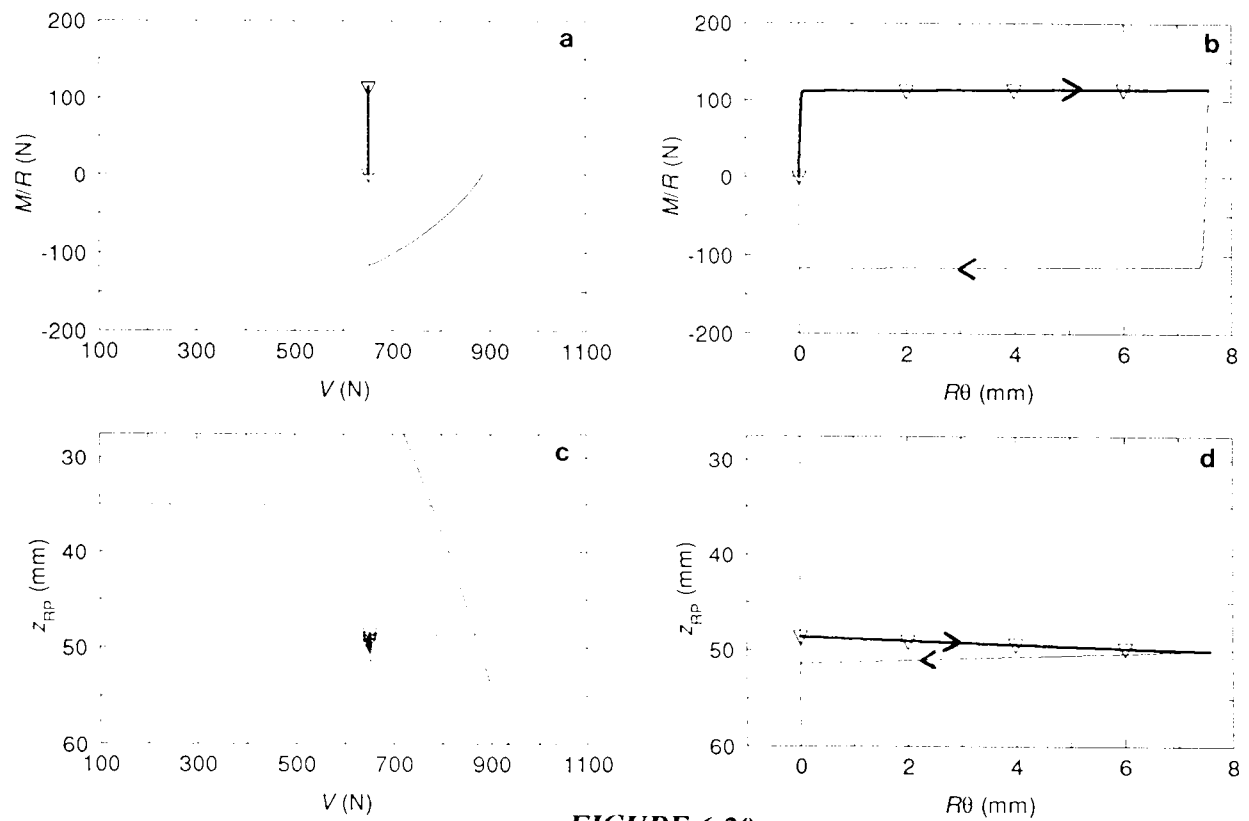


FIGURE 6.20

Extract from **T082**: *OXSPUD* prediction
(rigidity index $G/s_u = 500$)

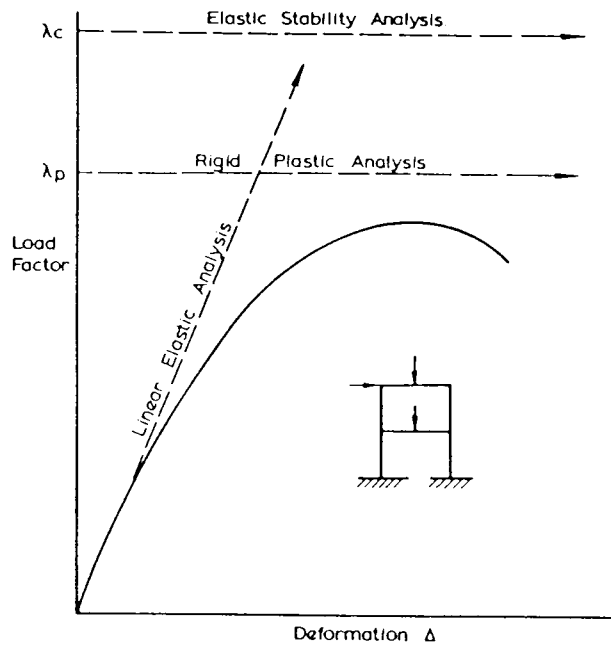
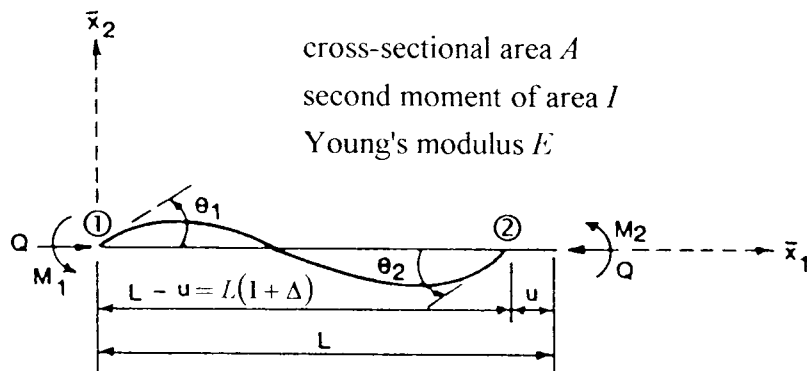
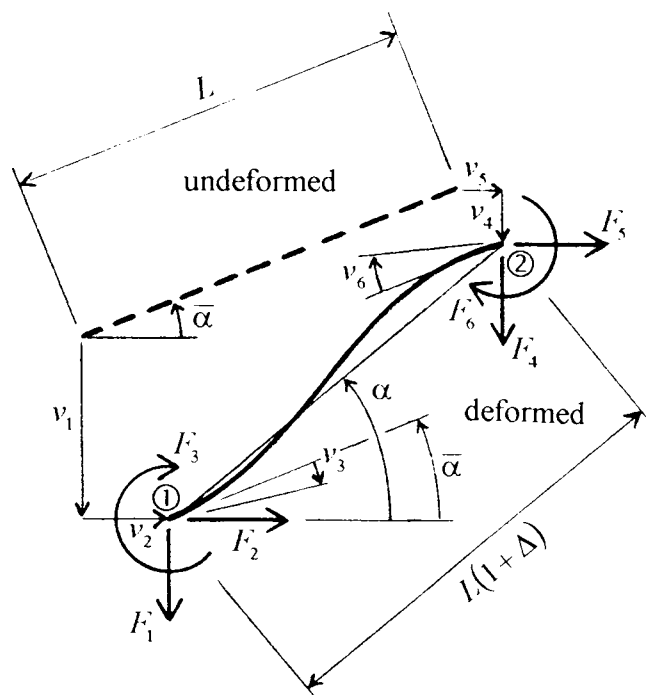


FIGURE 7.1
Mathematical models of structural behaviour (after Harrison, 1973)



[a] local coordinates



[b] global coordinates

FIGURE 7.2 Beam-column element forces and deformations (after Kassimali, 1983)

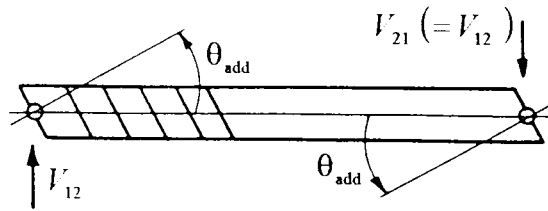


FIGURE 7.3

Effect of shear strains on member end rotations (after Harrison, 1980)

$$\text{shear stress} = \frac{V_{12}}{A_s} = G\gamma; \gamma = \frac{V_{12}}{A_s G}$$

shear modulus G

shear strain γ

area of section resisting shear A_s

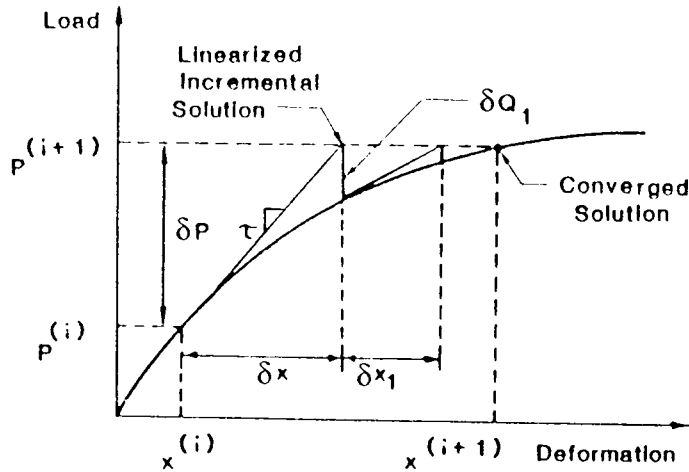


FIGURE 7.4

Newton-Raphson iteration strategy (after Kassimali, 1983)

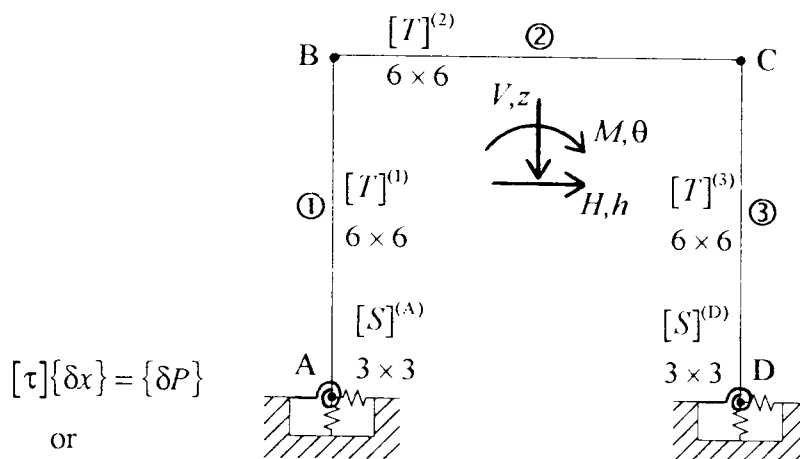
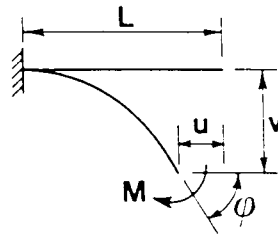


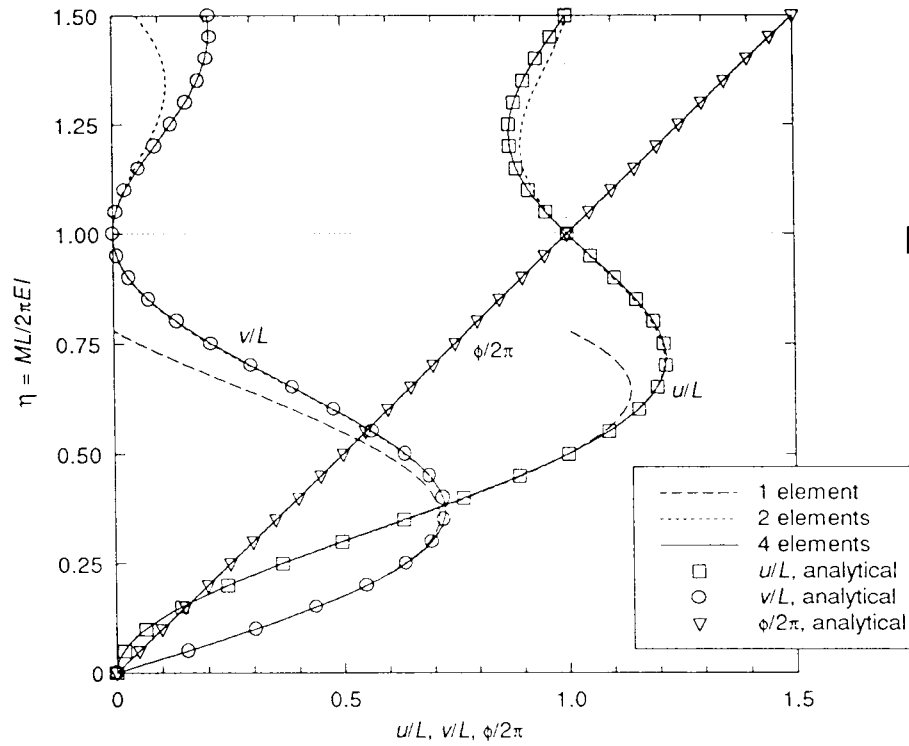
FIGURE 7.5

Incorporation of foundation stiffness matrices into global tangent stiffness matrix

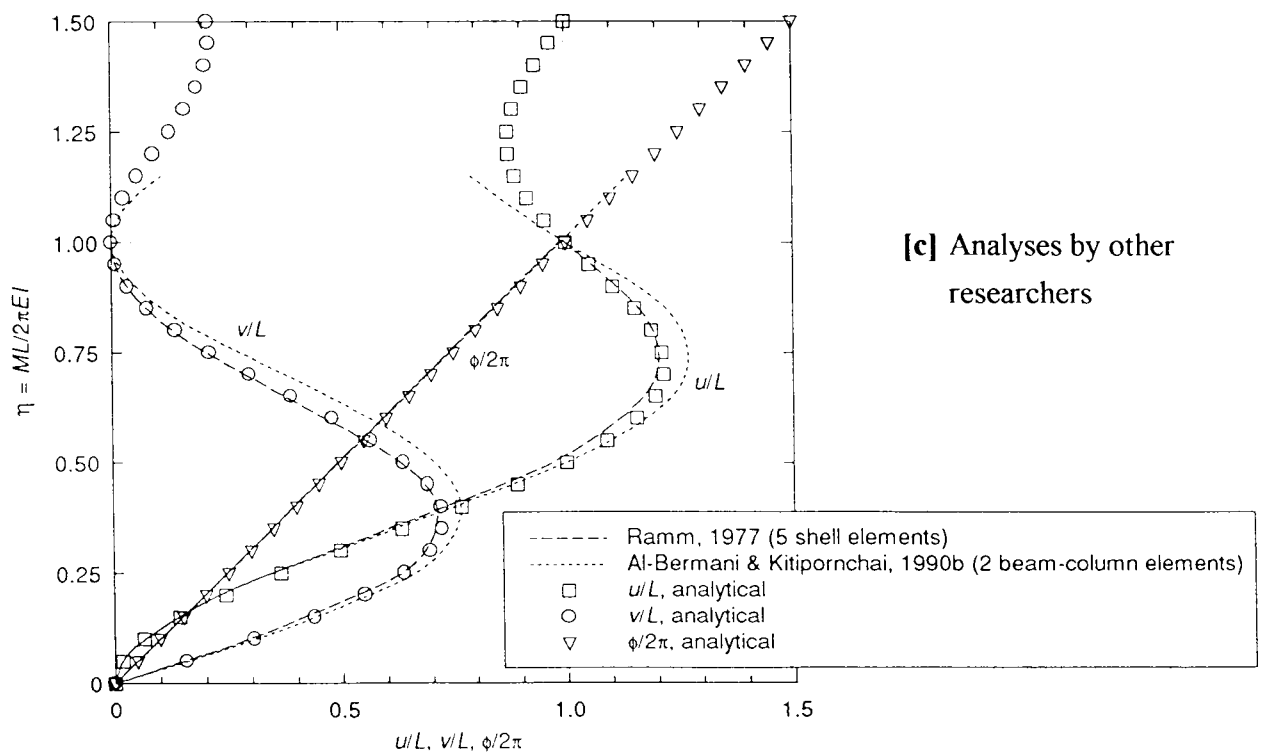
$$\begin{bmatrix}
 T_{11}^{(1)} + S_{11}^{(A)} & T_{12}^{(1)} + S_{12}^{(A)} & T_{13}^{(1)} + S_{13}^{(A)} & T_{14}^{(1)} & T_{15}^{(1)} & T_{16}^{(1)} & 0 & \dots \\
 T_{21}^{(1)} + S_{21}^{(A)} & T_{22}^{(1)} + S_{22}^{(A)} & T_{23}^{(1)} + S_{23}^{(A)} & T_{24}^{(1)} & T_{25}^{(1)} & T_{26}^{(1)} & 0 & \dots \\
 T_{31}^{(1)} + S_{31}^{(A)} & T_{32}^{(1)} + S_{32}^{(A)} & T_{33}^{(1)} + S_{33}^{(A)} & T_{34}^{(1)} & T_{35}^{(1)} & T_{36}^{(1)} & 0 & \dots \\
 T_{41}^{(1)} & T_{42}^{(1)} & T_{43}^{(1)} & T_{44}^{(1)} + T_{11}^{(2)} & T_{45}^{(1)} + T_{12}^{(2)} & T_{46}^{(1)} + T_{13}^{(2)} & T_{14}^{(2)} & \dots \\
 T_{51}^{(1)} & T_{52}^{(1)} & T_{53}^{(1)} & T_{54}^{(1)} + T_{21}^{(2)} & T_{55}^{(1)} + T_{22}^{(2)} & T_{56}^{(1)} + T_{23}^{(2)} & T_{24}^{(2)} & \dots \\
 T_{61}^{(1)} & T_{62}^{(1)} & T_{63}^{(1)} & T_{64}^{(1)} + T_{31}^{(2)} & T_{65}^{(1)} + T_{32}^{(2)} & T_{66}^{(1)} + T_{33}^{(2)} & T_{34}^{(2)} & \dots \\
 0 & 0 & 0 & T_{41}^{(2)} & T_{42}^{(2)} & T_{43}^{(2)} & T_{44}^{(2)} + T_{11}^{(3)} & \dots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
 \end{bmatrix}
 \begin{Bmatrix}
 \delta z_A \\
 \delta h_A \\
 \delta \theta_A \\
 \delta z_B \\
 \delta h_B \\
 \delta \theta_B \\
 \delta z_C \\
 \vdots
 \end{Bmatrix}
 =
 \begin{Bmatrix}
 \delta V_A \\
 \delta H_A \\
 \delta M_A \\
 \delta V_B \\
 \delta H_B \\
 \delta M_B \\
 \delta V_C \\
 \vdots
 \end{Bmatrix}$$



[a] Problem definition



[b] Analyses with
program *2DJUAN*



[c] Analyses by other
researchers

FIGURE 7.6 Large deformation analysis of cantilever subjected to end moment

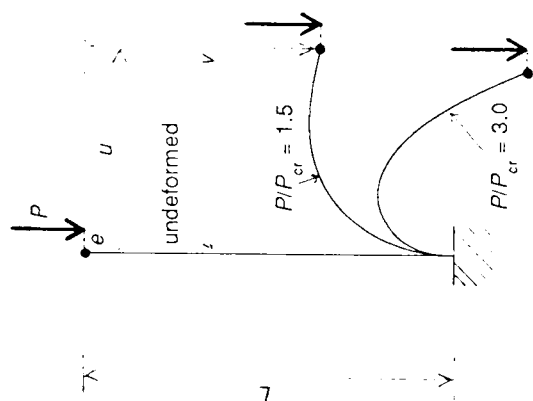
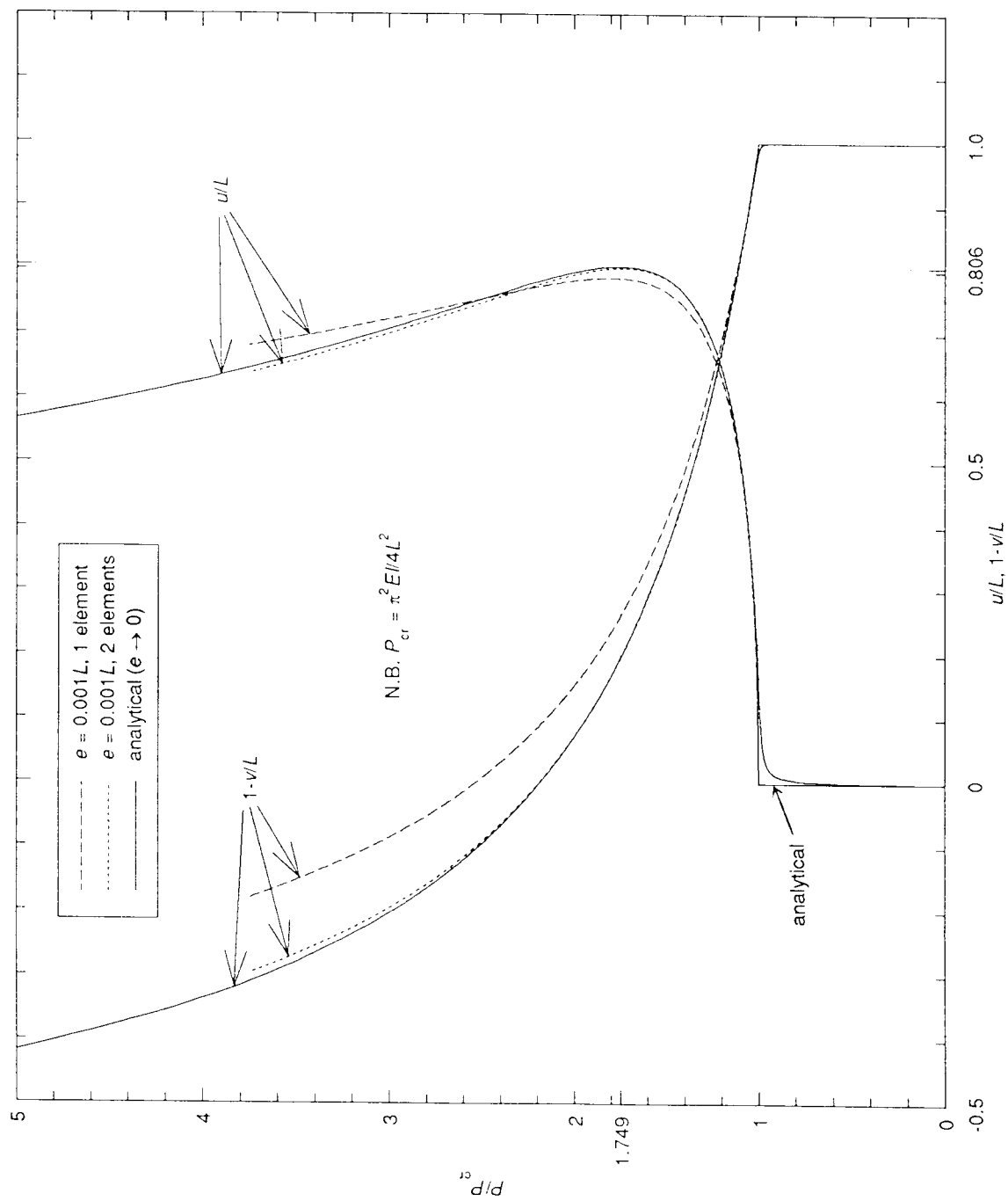


FIGURE 7.7 Large deformation analysis of axially loaded strut with program 2D/UAN

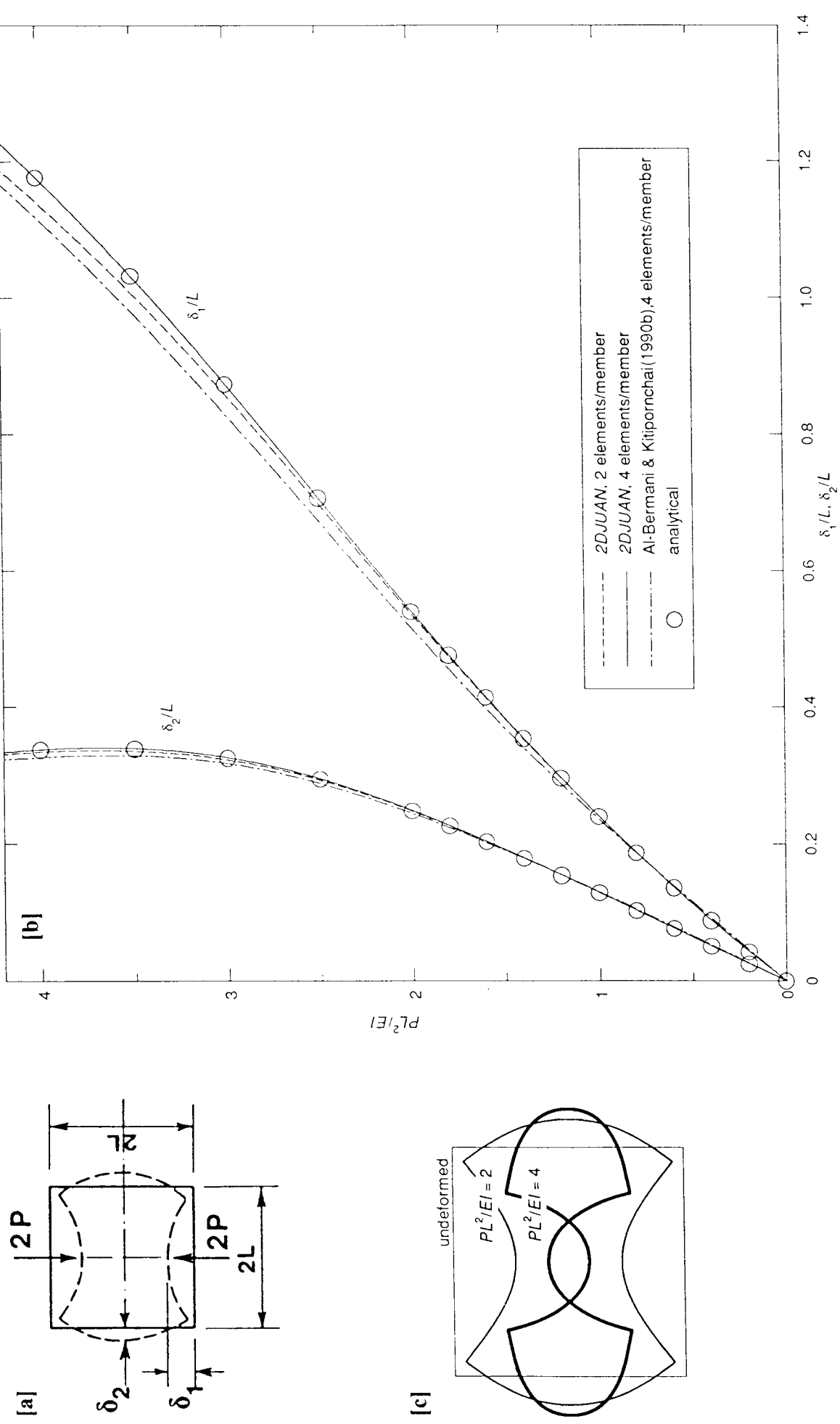
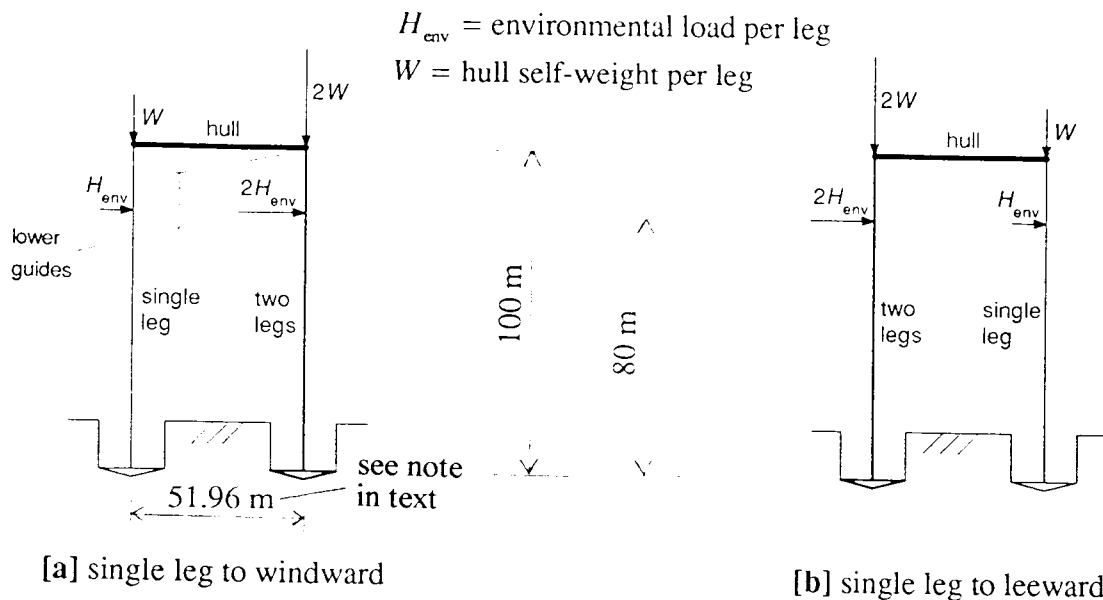


FIGURE 7.8 Large deformation analysis of square frame in compression



spudcan diameter	20 m
single leg equivalent I	7.5 m^4
single leg equivalent A	0.6 m^2
single leg equivalent A_s	0.04 m^2
hull equivalent I	$10 \times \text{single leg } I$
hull equivalent A	$50 \times \text{single leg } A$
hull equivalent A_s	$50 \times \text{single leg } A_s$

E of structure	200000 MPa
G of structure	80000 MPa
total preload weight $3W$	300 MN
total operating weight $3W$	150 MN

Clay seabed properties:

undrained strength at mudline s_{um}	10 kPa
strength increase with depth ρ	2 kPa/m
rigidity index G/s_u	100

FIGURE 7.9

Idealised two dimensional jack-up
for analysis with 2DJUAN

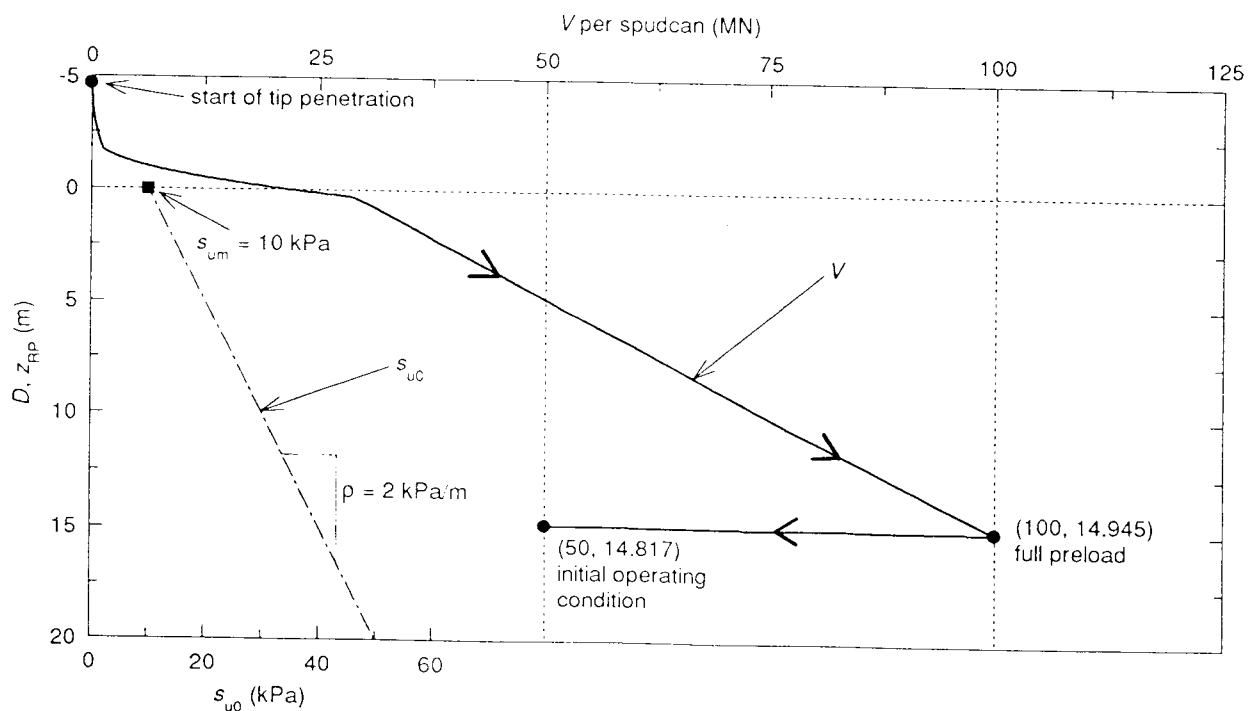


FIGURE 7.10 Prediction of jack-up footing penetrations during installation

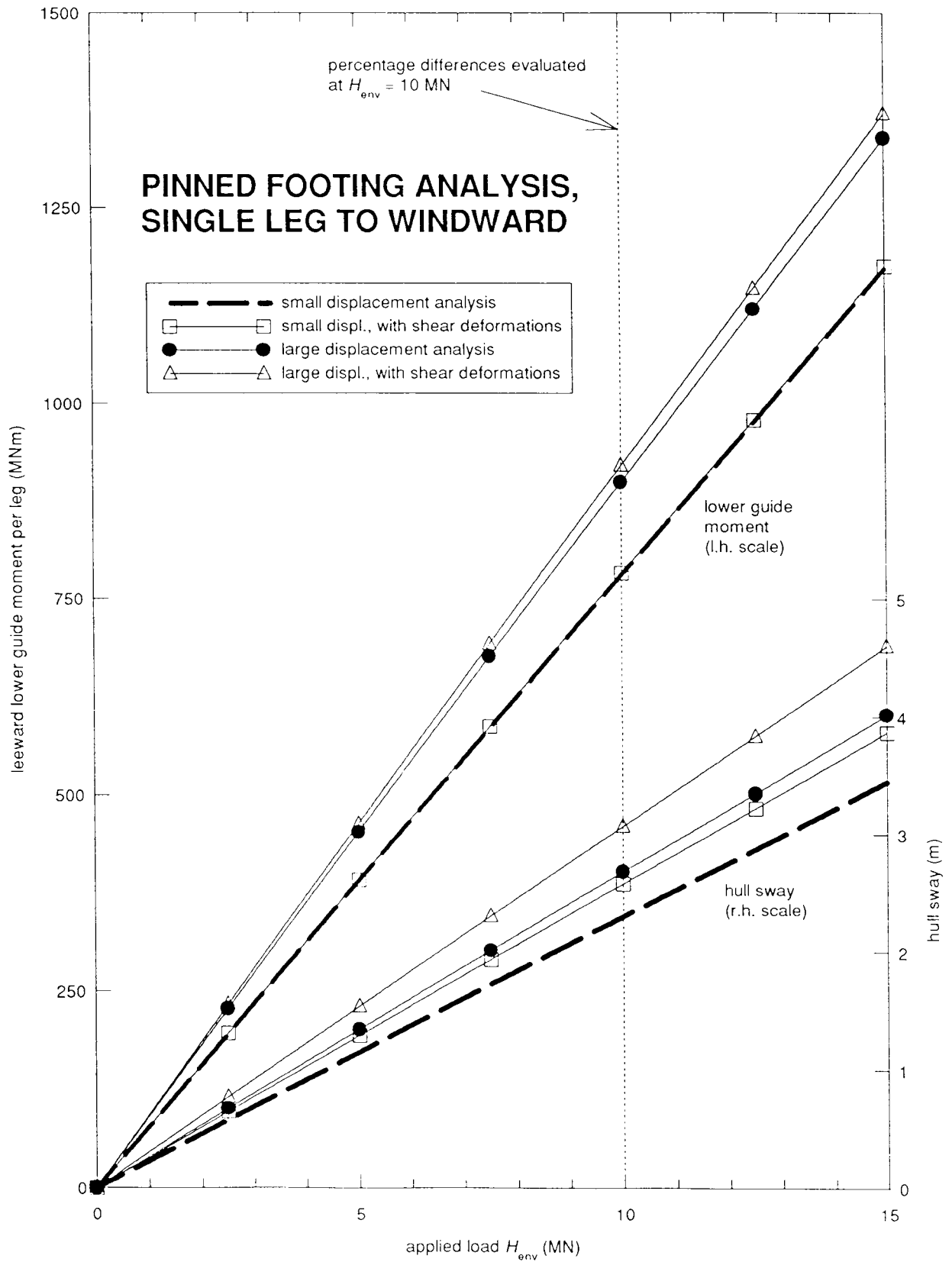


FIGURE 7.11 Effect of including shear deformations and large displacements in *2DJUAN* analysis of typical jack-up (pinned footings, single leg to windward)

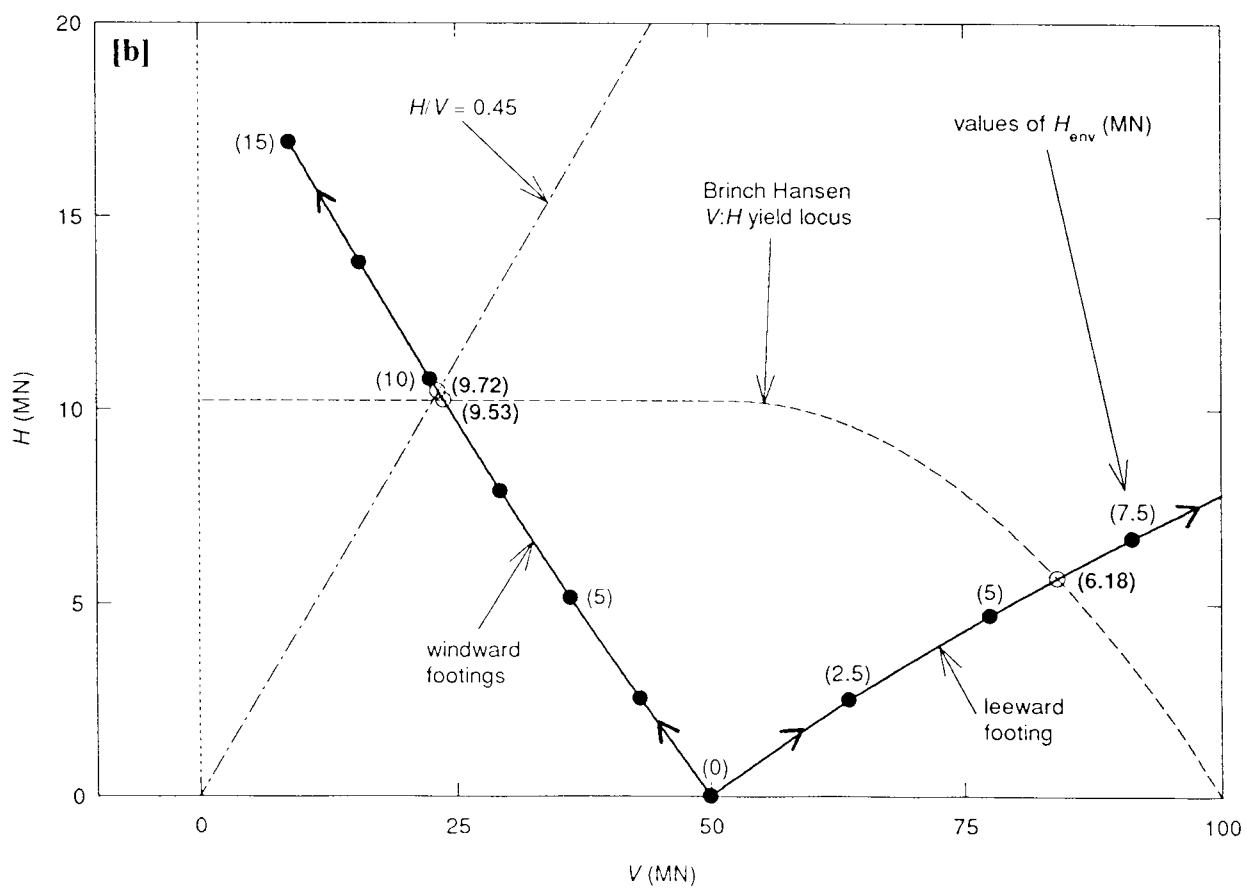
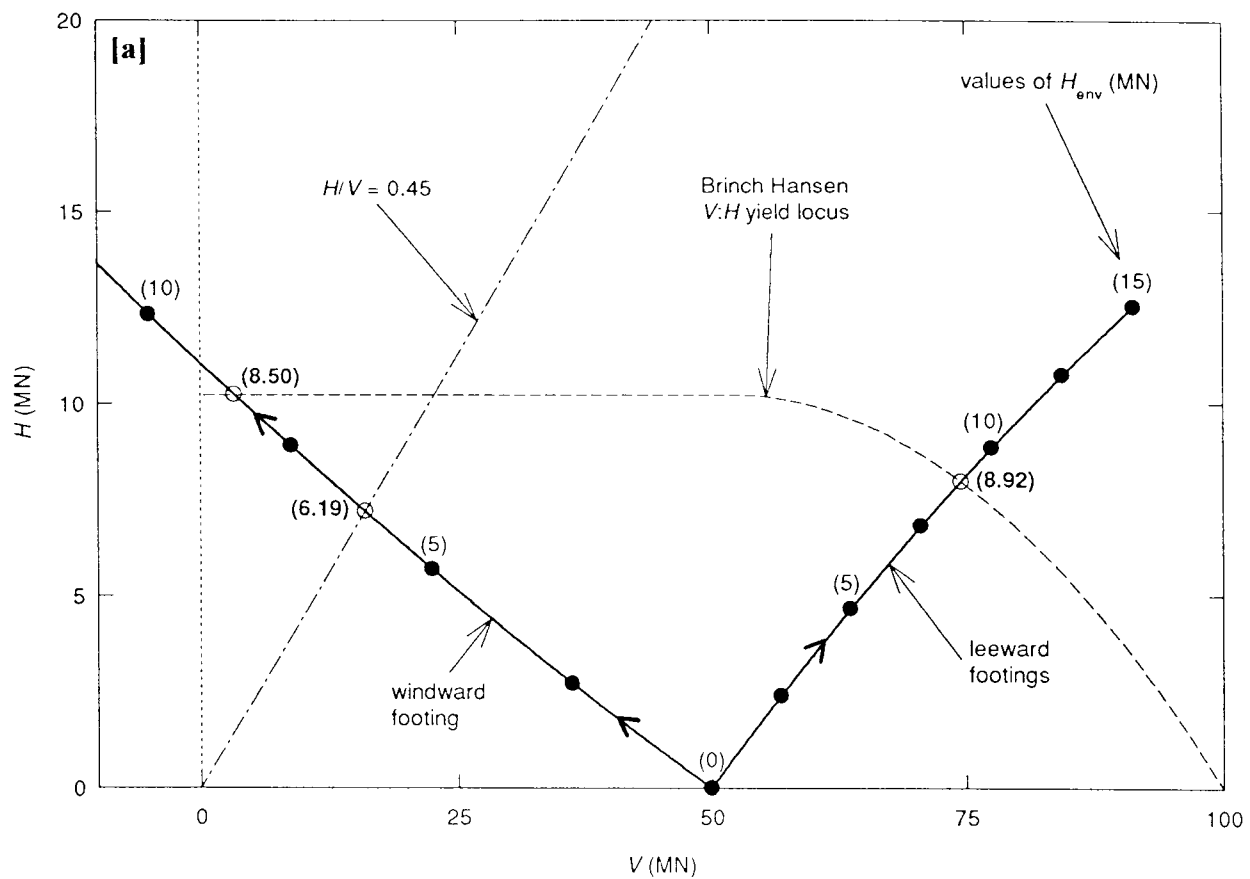


FIGURE 7.12 Spudcan reactions from 2DJUAN pinned footing analysis, with
[a] single leg to windward; **[b]** single leg to leeward

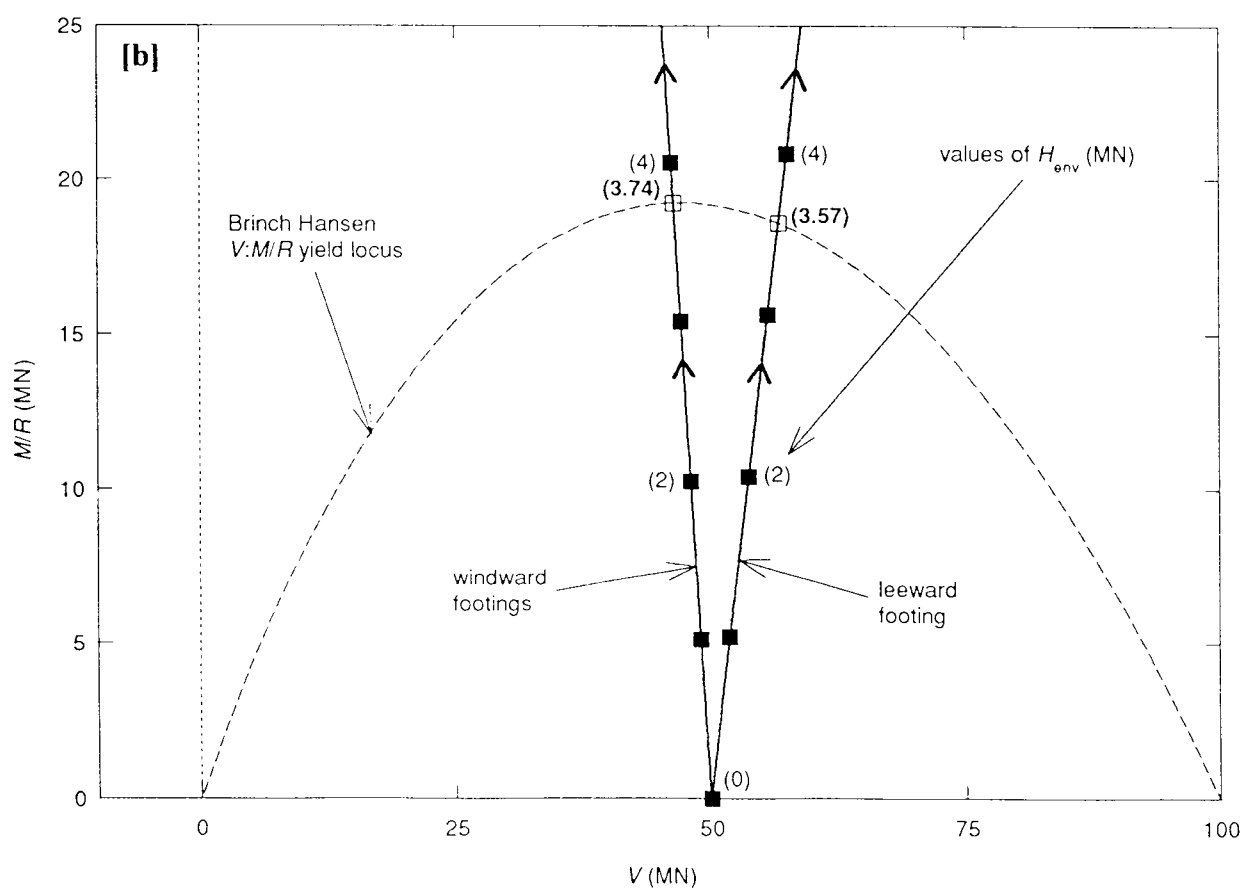
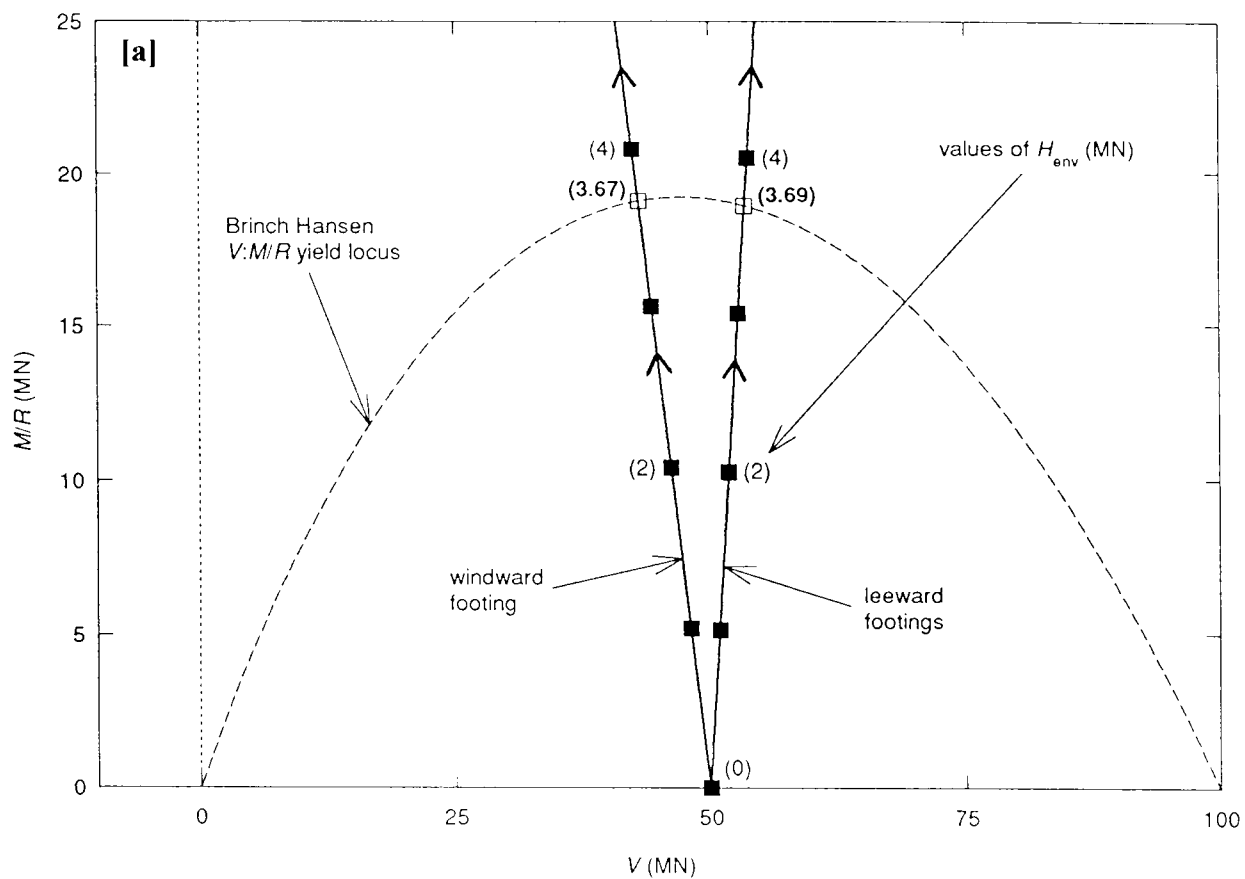


FIGURE 7.13 Spudcan reactions from 2DJUAN encasté footing analysis, with
[a] single leg to windward; **[b]** single leg to leeward

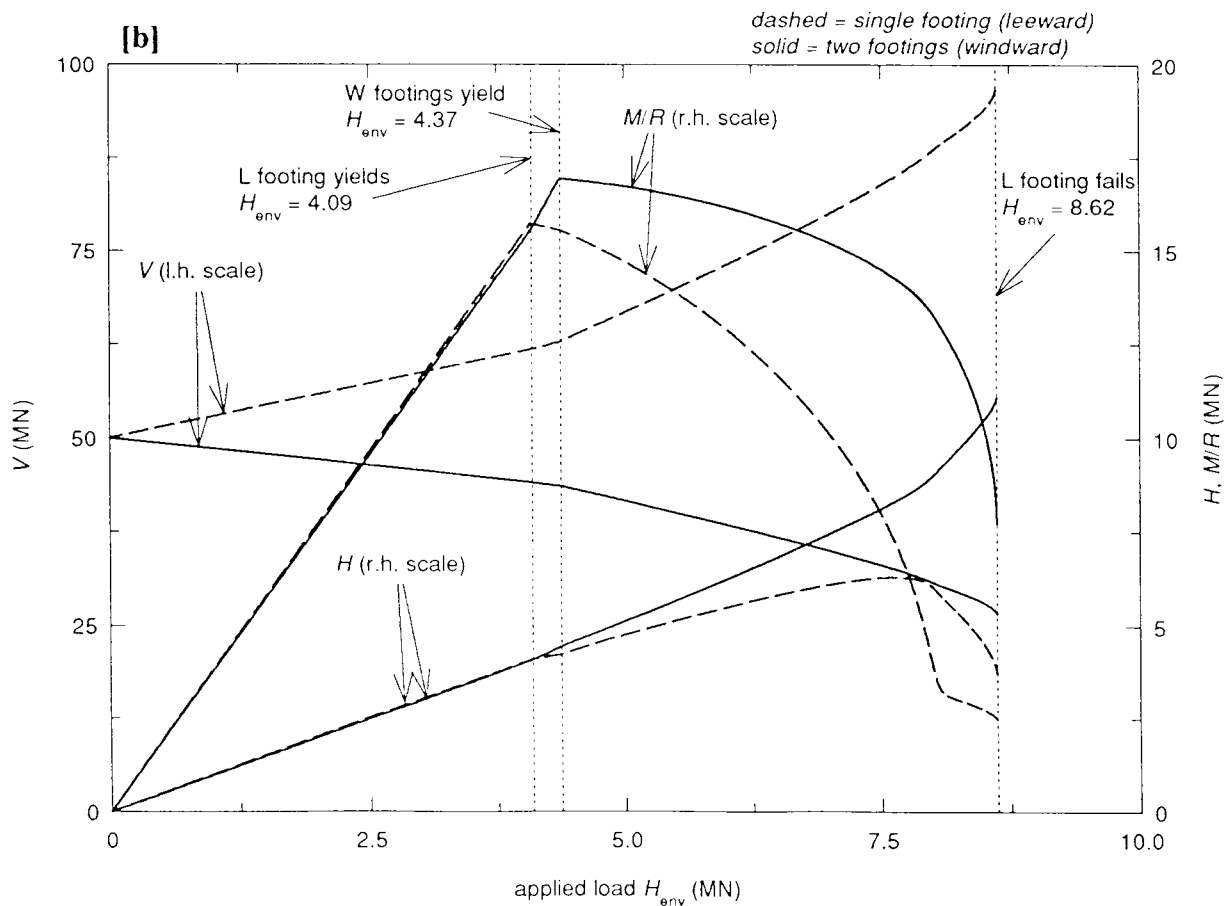
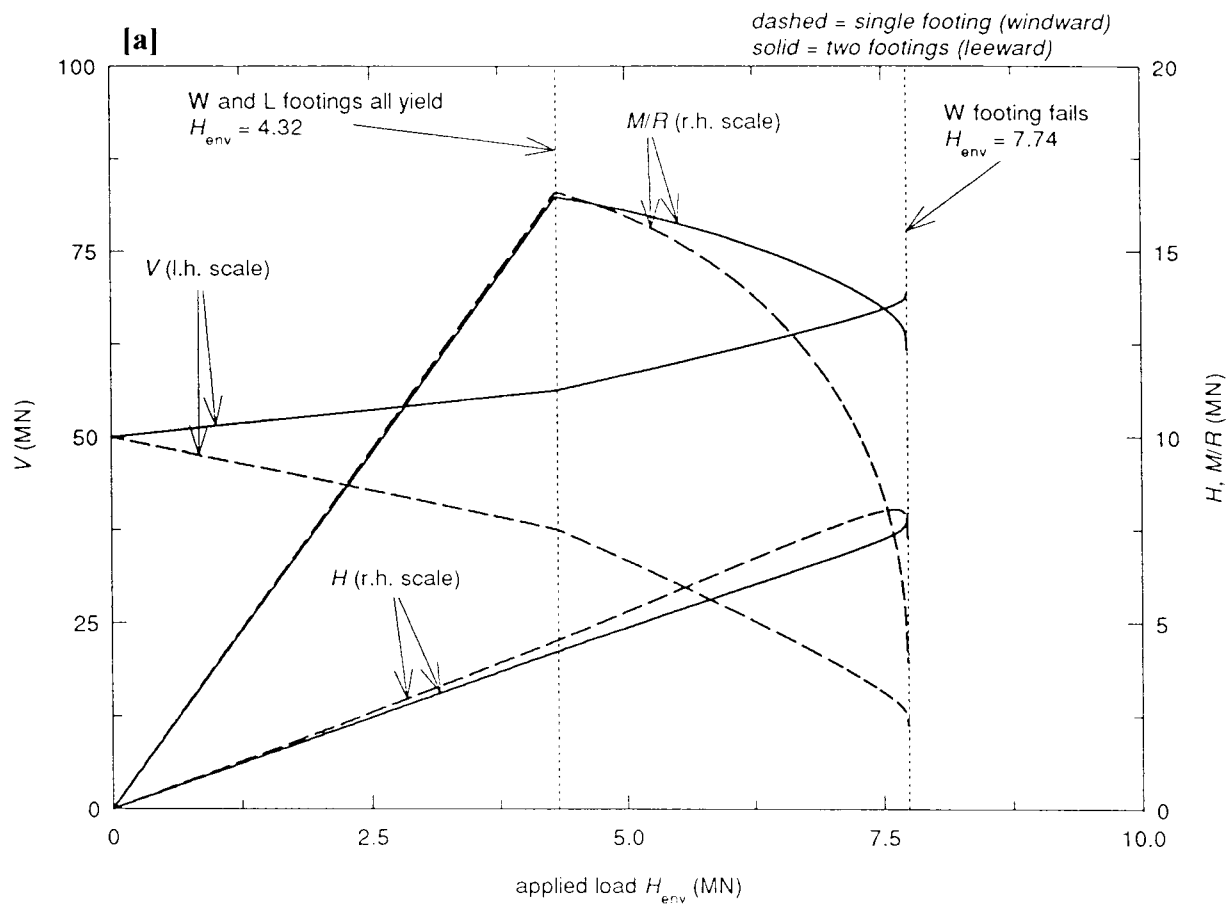


FIGURE 7.14 Spudcan reactions from 2DJUAN analysis using Model B footings:
[a] single leg to windward; **[b]** single leg to leeward

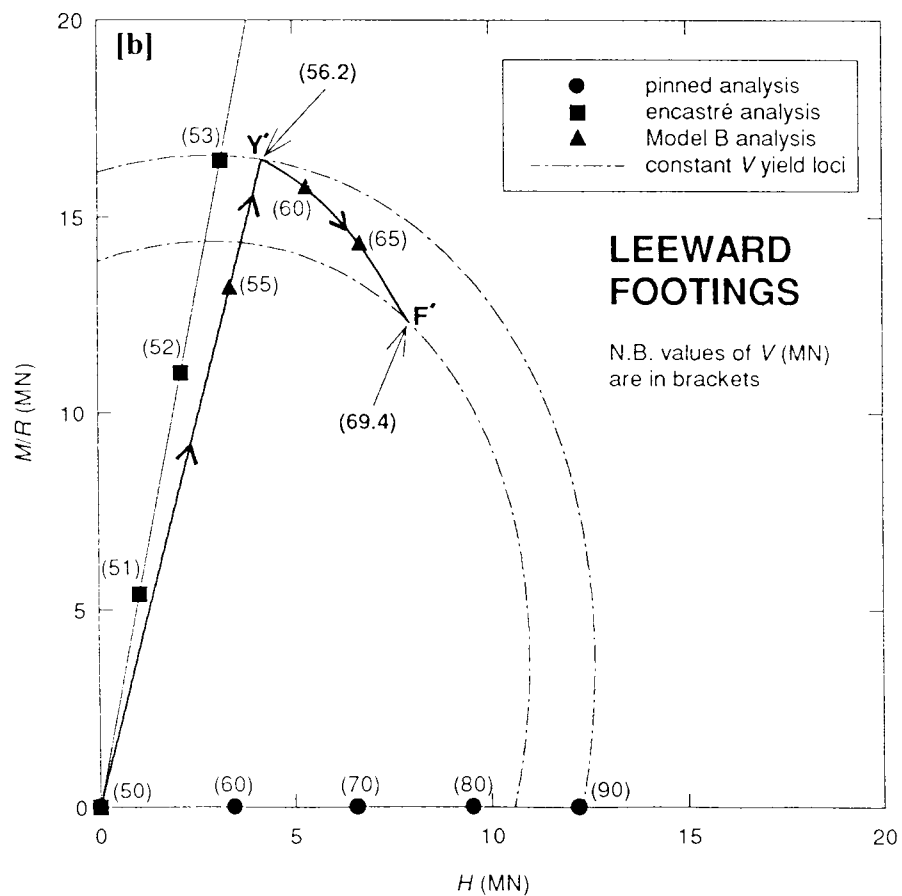
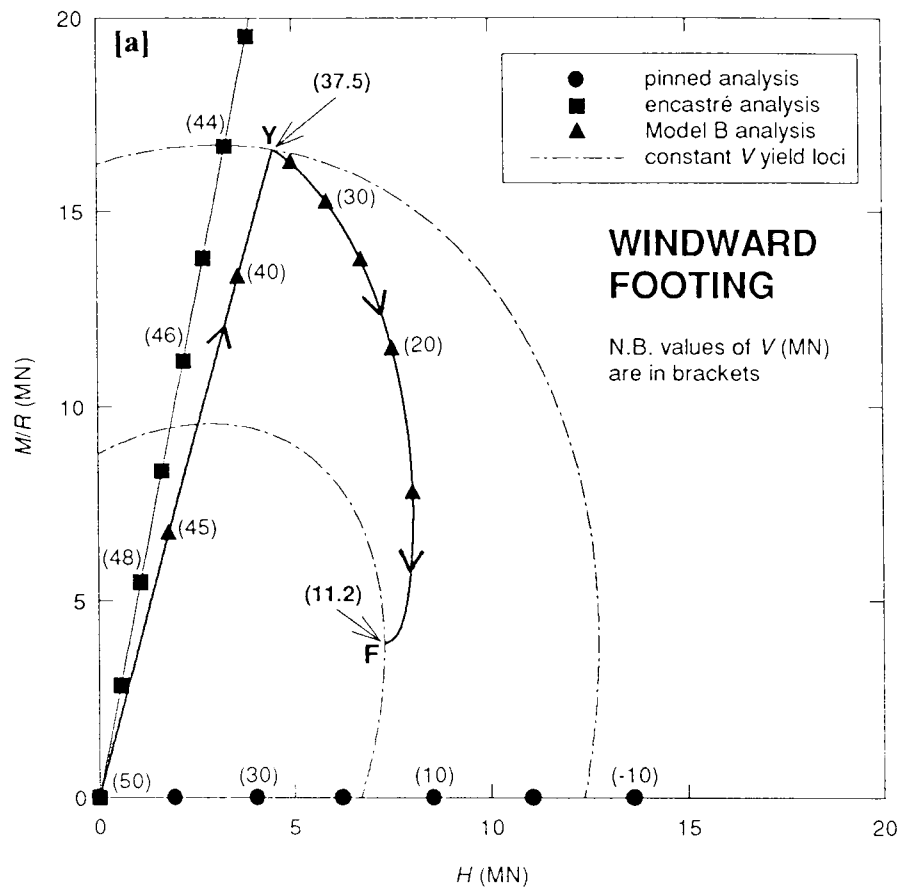


FIGURE 7.15 Spudcan load paths from various *2DJUAN* analyses, single leg to windward

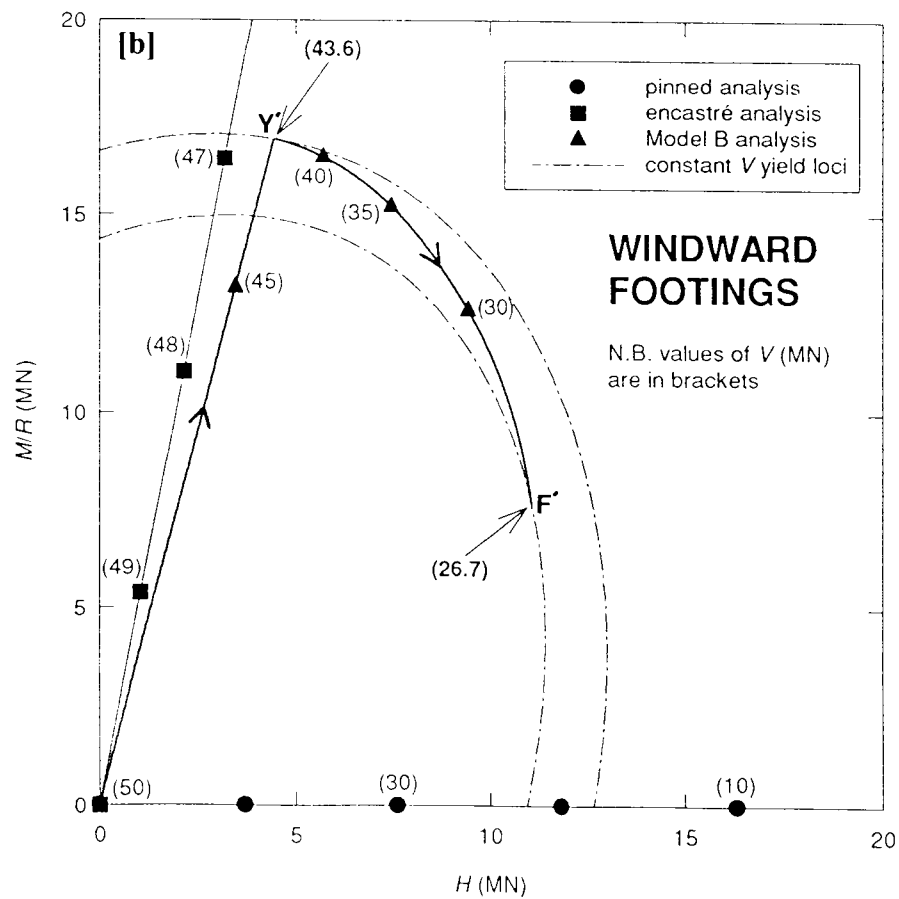
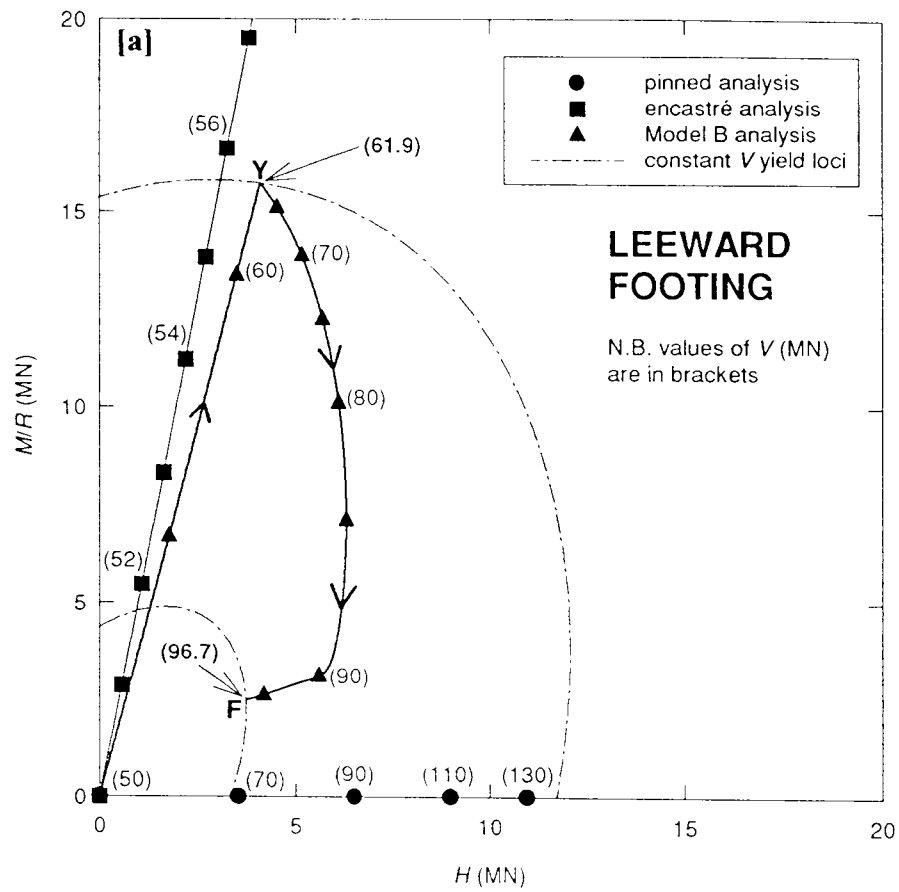


FIGURE 7.16 Spudcan load paths from various 2DJUAN analyses, single leg to leeward

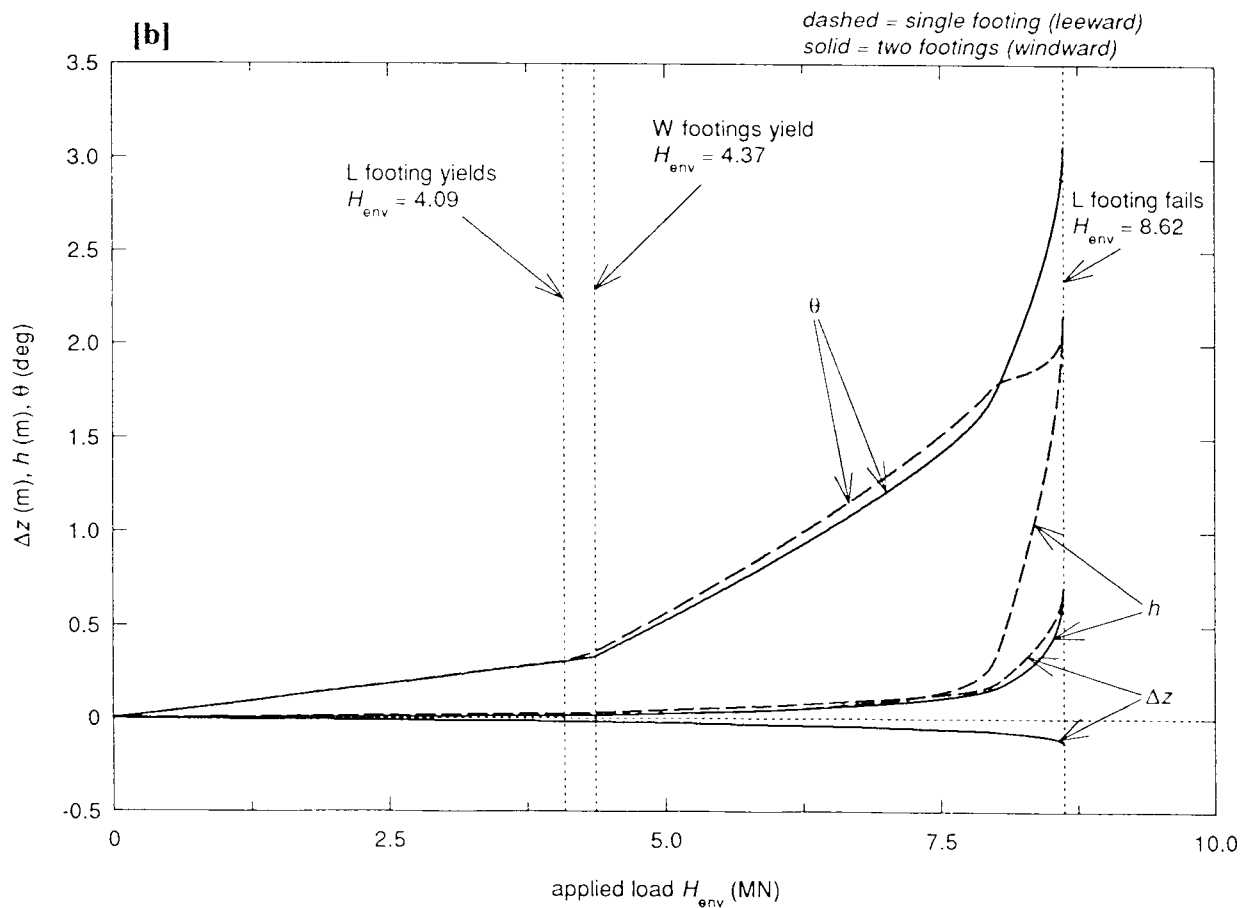
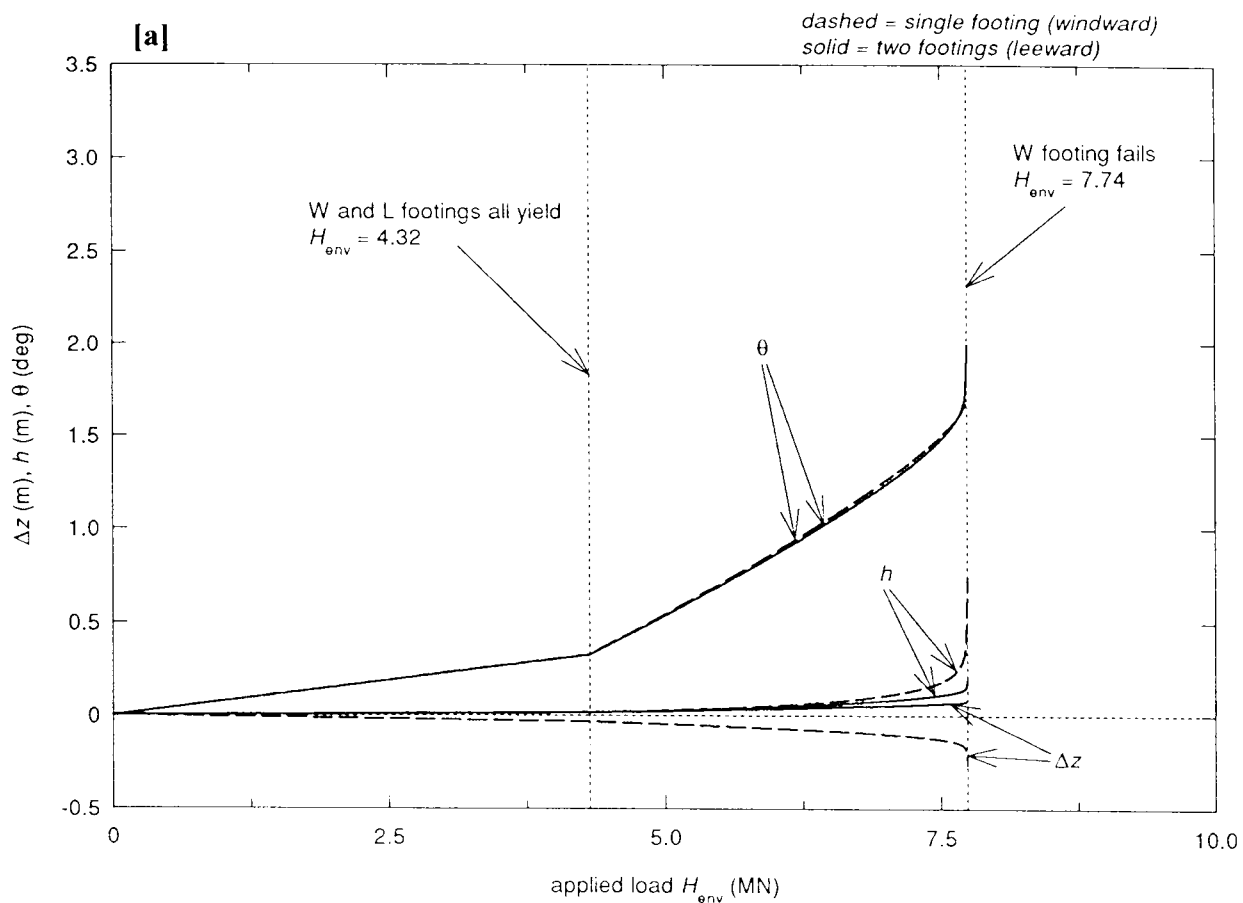


FIGURE 7.17 Spudcan displacements from 2DJUAN analysis using Model B footings:
[a] single leg to windward; [b] single leg to leeward

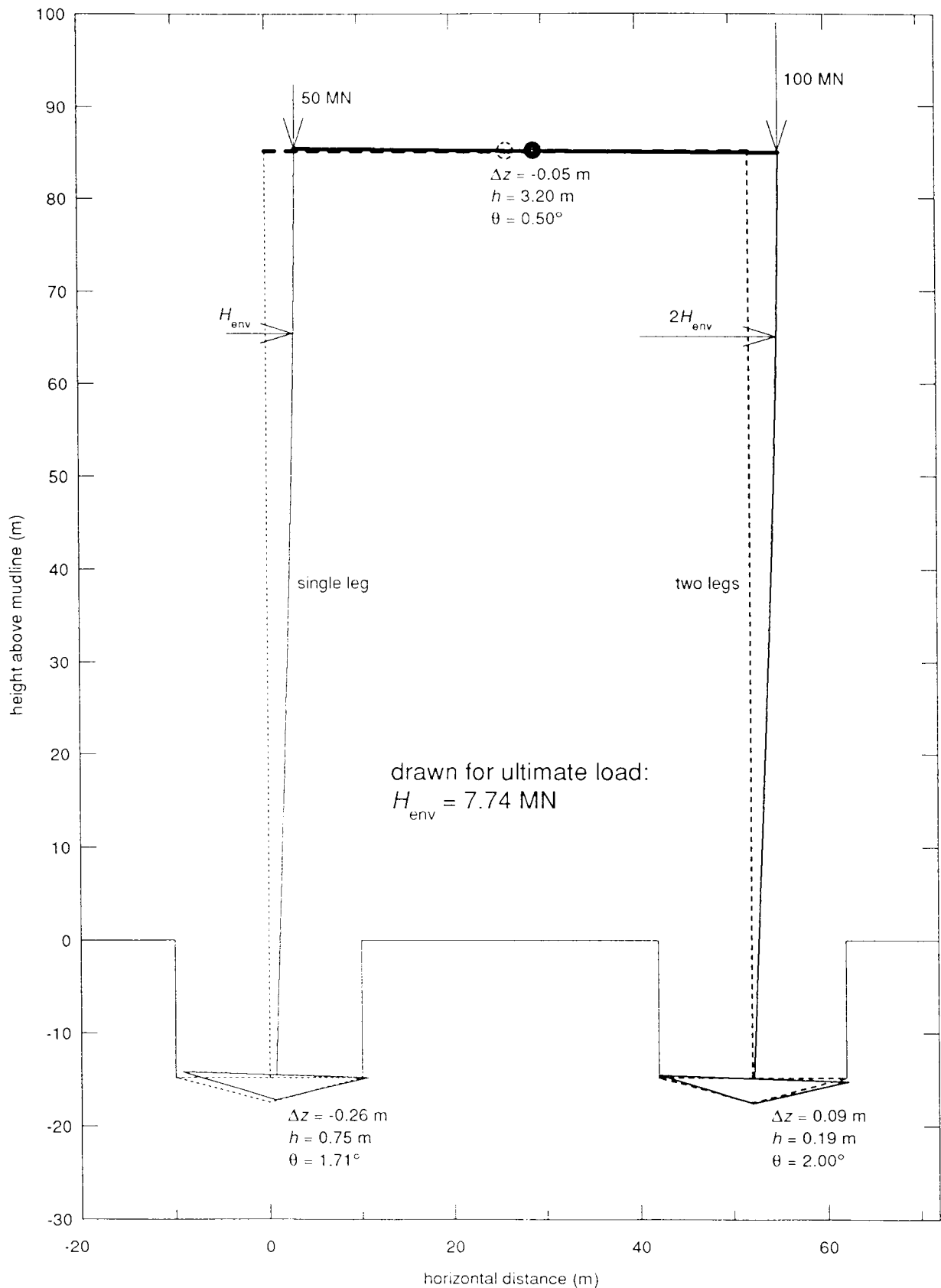


FIGURE 7.18 2DJUAN analysis using Model B footings, single leg to windward:
deflected shape at ultimate load (failure of windward footing)

N.B. all deflections are drawn to scale;
for simplicity spudcans are shown as conical footings;
spudcan deflections refer to the footing/leg intersection point.

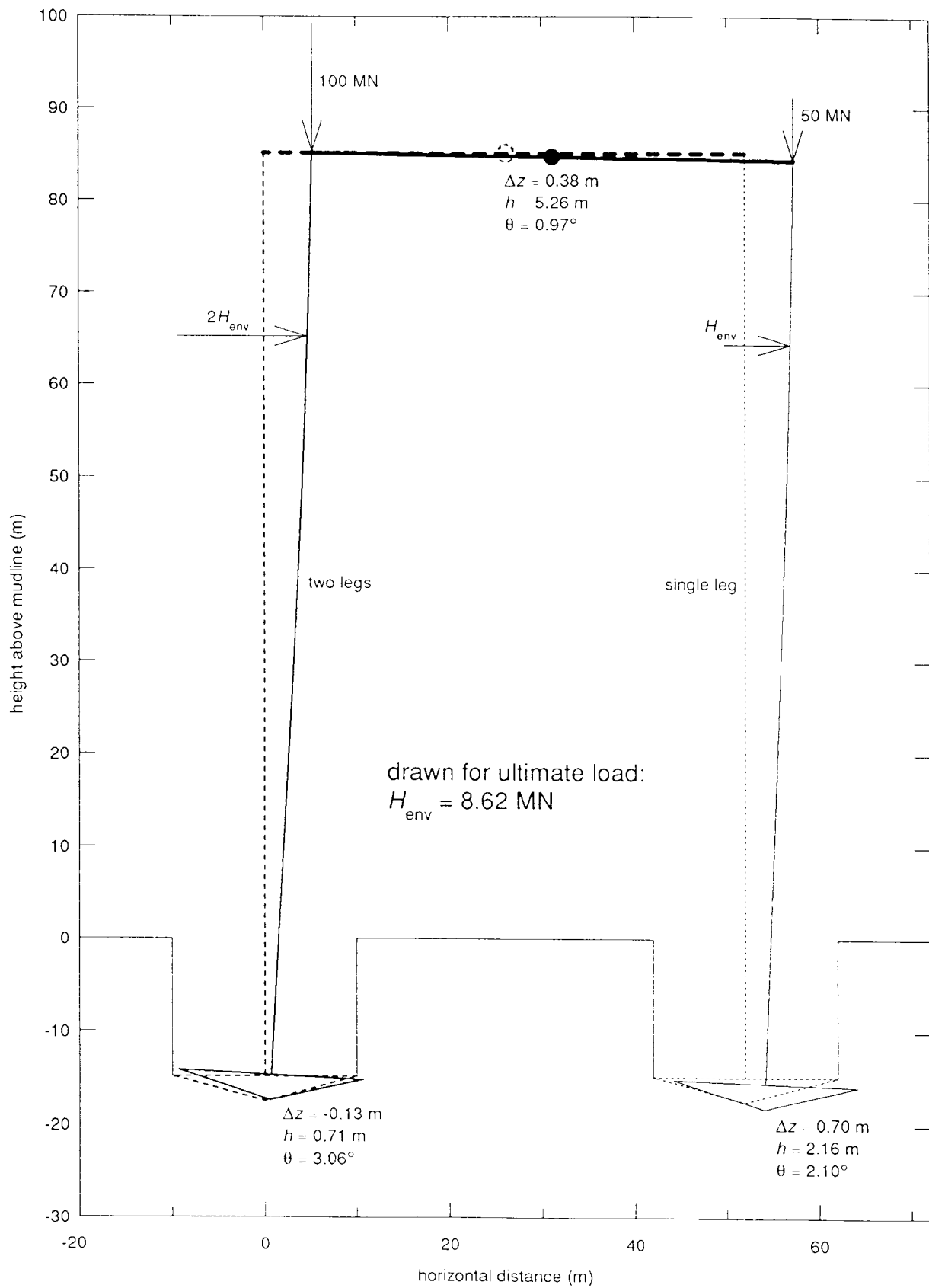


FIGURE 7.19 2DJUAN analysis using Model B footings, single leg to leeward:
deflected shape at ultimate load (failure of leeward footing)

N.B. all deflections are drawn to scale;
for simplicity spudcans are shown as conical footings;
spudcan deflections refer to the footing/leg intersection point.

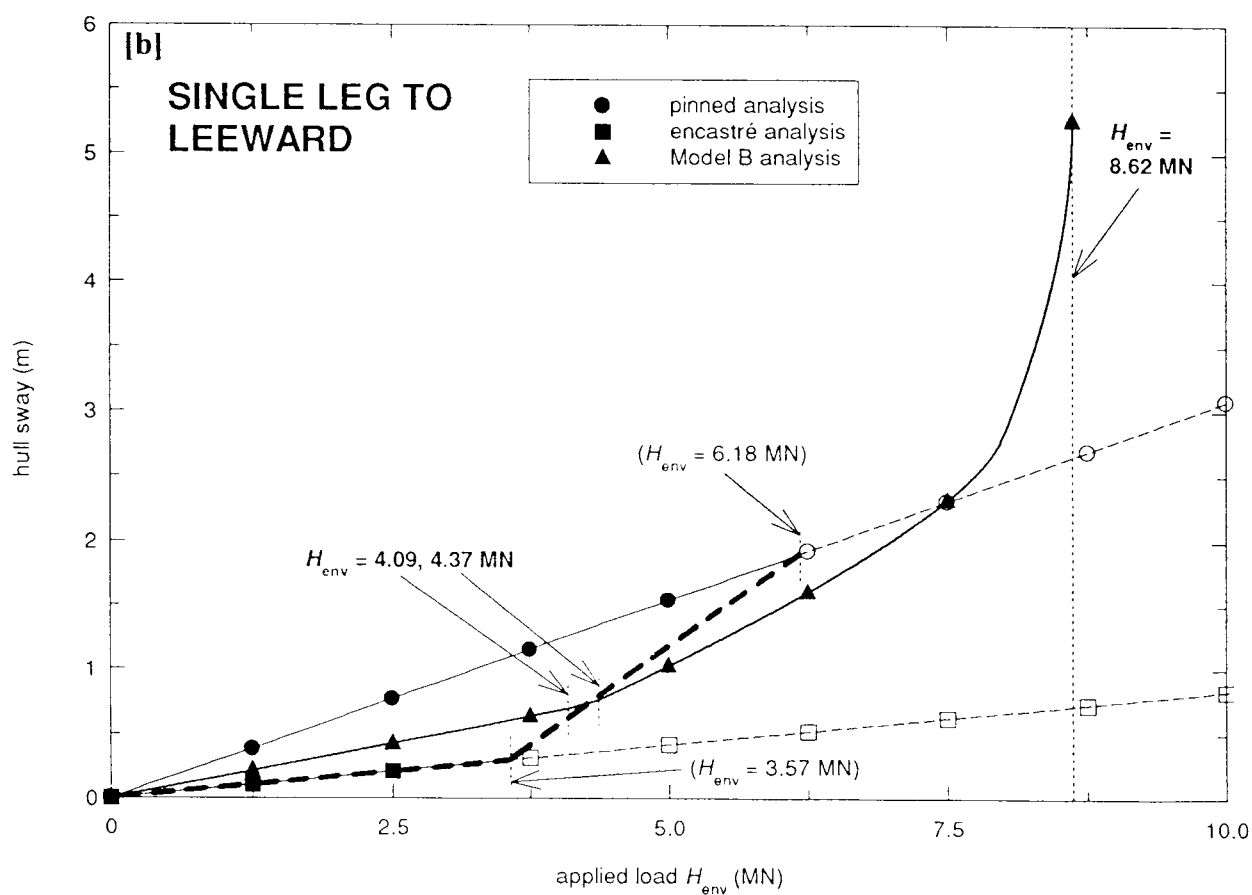
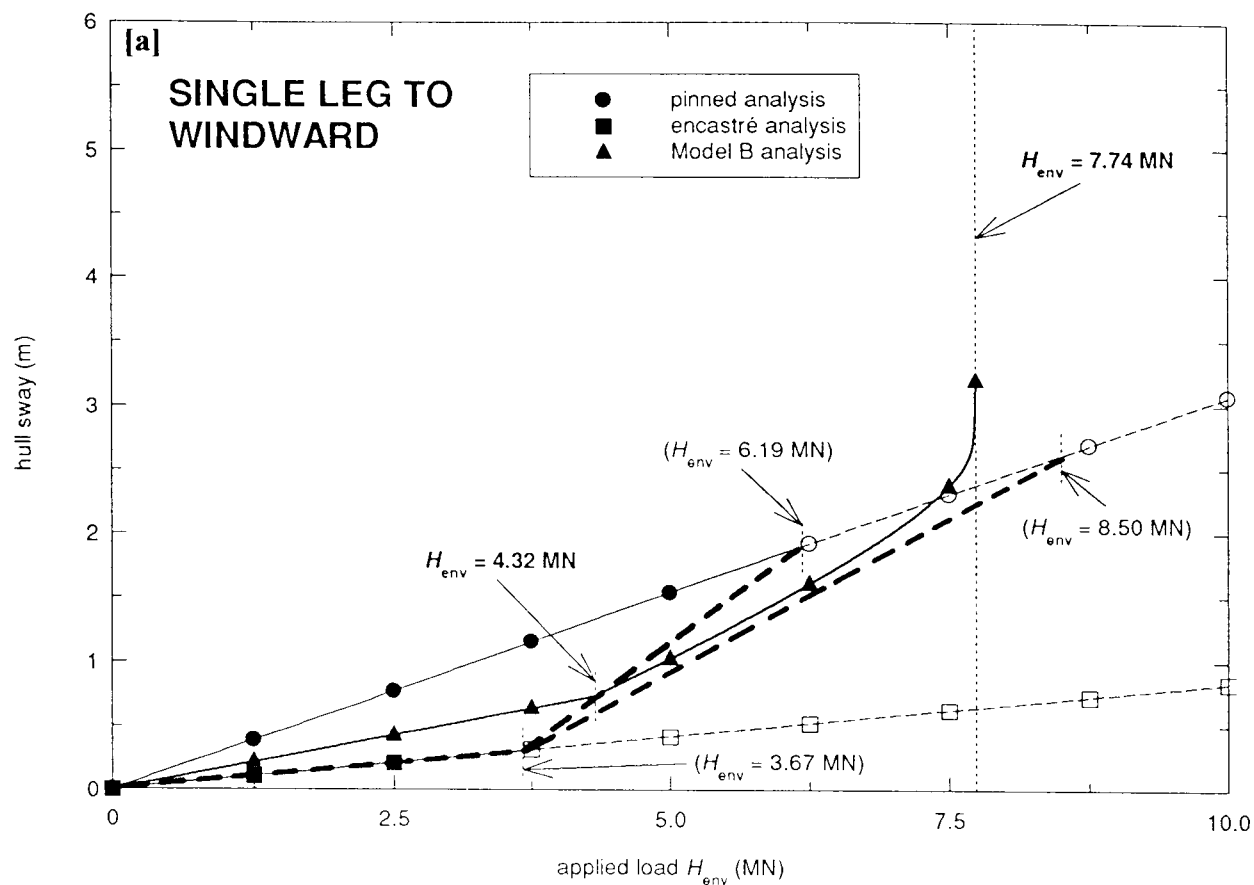


FIGURE 7.20 Predictions of hull sway from various *2DJUAN* analyses, with
[a] single leg to windward; **[b]** single leg to leeward

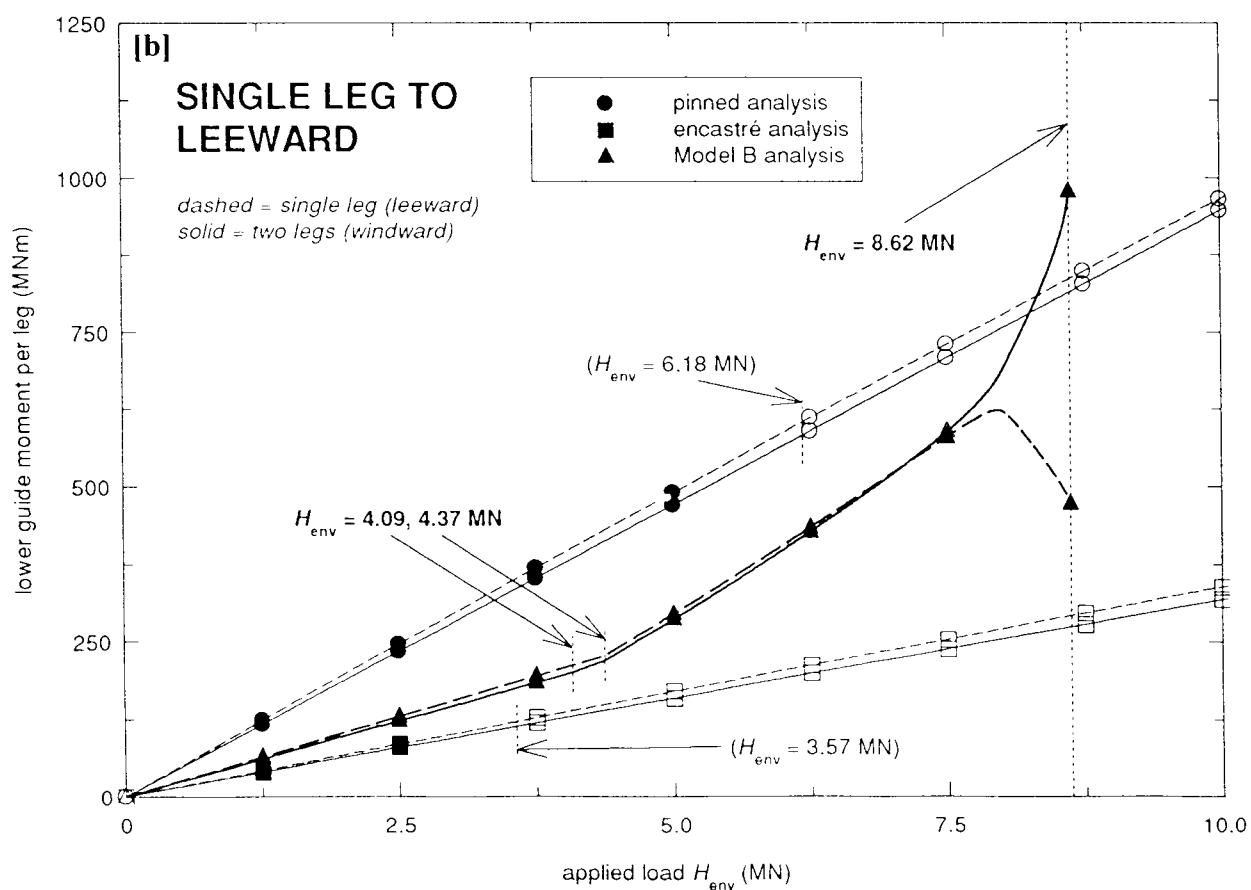
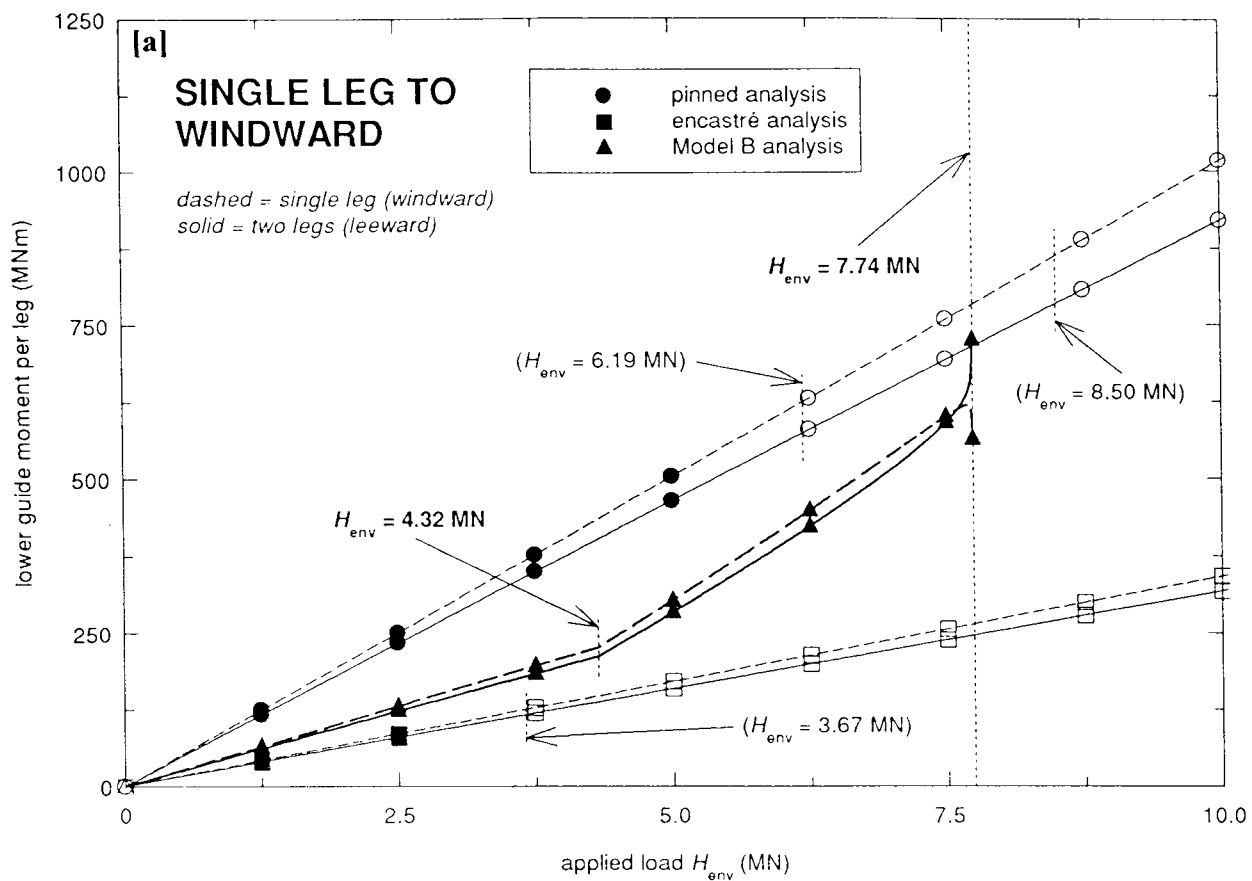


FIGURE 7.21 Predictions of lower guide bending moments from various 2DJUAN analyses
[a] single leg to windward; **[b]** single leg to leeward