Communication, Construction and Community:
Learning Addition in Primary Classrooms.

Alison J. Price

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Abstract

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Alison J. Price                                      D. Phil.
Kellogg College                                        Trinity 2000

This study examines the teaching and early learning of addition in primary classrooms. The relationship between teaching and learning is examined at the level of classroom interaction, in the completion of mathematical tasks. The mathematics lessons of two classes in each of two schools were observed over a period of six months, involving four teachers and the 4, 5 and 6 year old children in their classes. The mathematical focus of the study was the learning of addition, one of the first formal mathematical concepts taught in school. This formed a basis for exploring the factors involved in the teaching of mathematics to young children, and their learning.

The methodology is qualitative, with participant observation the main method of data collection. Detailed fieldnotes were taken of all mathematics lessons observed; short unstructured interviews with teachers were carried out before and after the lessons. The children's understanding of number concepts and addition was assessed at both the beginning and the end of the observation period.

The data was analysed using a grounded theory approach, which produced patterns of recurring variables. Analysis of these variables, influenced by the theoretical perspectives of the researcher, provided analytical pictures of teaching and learning, from which the findings emerged.

The study highlights the complexity of the classroom for teachers and young children, where curriculum considerations, understanding of the mathematics concepts, social interaction and integration into the community of the classroom, vie for attention. It indicates that children are more likely to make sense of mathematics when the number curriculum is taught with a view to its complexity, rather than broken down into simple steps; the problems young children have learning to use mathematical symbols; and that the use of story is important in helping especially the youngest children understand mathematics.

This was a small scale study, but provides a 'thick description' of teaching and early learning of addition, which can form a basis for future studies.

Key Words: addition, early mathematical development, primary school, constructivism, socio-cultural theory, situated cognition, symbols, real world scripts, narrative.
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Chapter One

Introduction to the study

1.1 Introduction

This study is concerned with the teaching and early learning of addition in primary school. In particular it examines the world of the classroom, the ways that the addition curriculum is presented and represented to the children and the meaning that they make of this. The study takes place in primary school classrooms observing the teaching and learning of children in their Reception year (aged 4-5) and Year One (aged 5-6), reflecting my experience and interest in children at the very start of their formal schooling in mathematics. It examines the way that the curriculum for mathematics is structured, the way that teachers and children represent addition and the influence of the wider, social and cultural world of the classroom on the children's learning.

The study is located within a wide range of research in mathematics education at all levels of schooling, studies in the philosophy of mathematics education, more general studies of teaching and learning in primary schools, and a body of work within the psychology of mathematics education relating to the development of children's understanding of mathematics.

Chapter 1 examines how the study emerged from my own personal experiences and interests, and outlines the overall structure of the thesis. Chapter 2 discusses the literature relating to theories of cognition and the influence of these on early mathematics education, while Chapter 3 describes some of the historical background of the teaching and learning of primary mathematics in England, highlighting issues that are particularly relevant to this present research.

1.2 Personal Context

The research arises out of my own experience as a learner of mathematics, as a parent watching my own children struggle to make sense of school mathematics, as a teacher of young children and as a teacher educator striving to help the next generation of teachers to understand how mathematics is learnt.

As a learner of mathematics in school, I was aware that my own view of what was important about learning mathematics was not the same as that valued by the teachers. I needed to understand the concepts rather than accept procedures and learn them by rote. Geometric proofs, which I reconstructed from first principles because of my poor memory skills, were marked down, while those of my friends, who learnt them by rote but did not understand them,
were given full marks. I did not enjoy mathematics at school even though I was successful at it. It is only as an adult that I have come to enjoy and be enchanted by it.

As a parent, watching the way that my children learnt mathematics, I became fascinated by the differences between children. One was happy to accept the rules and use them, and succeeded at mathematics gaining A’s at A-level, while his sister, who started with the same teacher in the same school, found it dull and boring. Later she blossomed with a different teacher who introduced her to the creativity and wonder of mathematics. The third, by copying her older siblings, could calculate and record addition and subtractions to 20 before starting school. She, like me, was unhappy to accept new mathematical ideas until she could understand why they worked. Even allowing for individual differences between children it left me wondering what was it about mathematics and approaches to teaching it that made such a difference. It was working with my own children and observing the way that they learnt that led me into teaching.

As a teacher of young children (the majority aged 5 to 7 years) I was again challenged to consider how children learnt mathematics and how my teaching effected their learning. Reflection on my own teaching caused me to realise that the way that I taught mathematics was not the same as the way that I taught language. In teaching children to read and write I was open to the children’s creative ideas, allowing them to develop their emerging skills, while acting as a guide and model to bring these skills closer to the socially accepted norms. By contrast, in mathematics the starting points were social norms, and children were not expected to deviate from this. In spelling, praise was given for being ‘nearly right’ and the right spelling looked at for comparison. In mathematics, answers were either right or wrong. In writing the child was taught to value communication over presentation, in mathematics correct presentation was expected. This disparity of practice led to my master’s dissertation, an Action Research study entitled “Developing a concept of number at Key Stage One: Can the principles of developmental writing help?” (Price 1993). This showed that a more child-centred approach was possible, starting from the child’s existing understanding and moving towards socially accepted mathematical practices.

Finally, as a teacher educator I have continued with the desire to develop and improve practice, both my own and that of the students and teachers with whom I work. This new role has offered opportunities to observe and work with teachers, trainee teachers and pupils in a range of schools. Some of these opportunities have confirmed previously held views, others have challenged them. The complexity of concepts involved in learning even the early stages of arithmetic, the national and international arguments over the ‘best’ ways to teach mathematics, the apparent variability in both achievement and attitude in mathematics between different children, different adults, different countries, and the variety of theories which attempt to explain these have continued to fascinate me and form the background to this research.
1.3 Rationale

The motivation for the research was therefore an interest in young children and their learning of mathematics in the classroom. The study aims to explore, and add to, the already existing understanding of how teachers teach mathematics and how young children learn mathematics with the ultimate, though not immediate, intention to improve practice.

The focus is threefold.

- In terms of mathematical content the focus is on addition because it is the earliest identified teaching of arithmetic in school and there is already a considerable range of research on the development of children’s understanding of early addition concepts, though not on how this understanding is achieved.

- In terms of context the focus is on the interaction between teaching and learning that occurs in primary classrooms.

- In terms of process the focus is on the development of children’s skills and understanding, relative to the content and context.

Through these three foci, I aim to develop a description of how children learn early addition in the context of the classroom. The study’s originality relates to these three foci.

In relation to content, much previous research into children’s learning of addition consists of psychological developmental snapshots of children’s skills as their understanding develops e.g. Nunes et al. 1996. It has not addressed the means by which the understanding came into being.

In relation to context, other studies have looked at the teaching of older children (Yackel et al. 1990; Jaworski 1994; Boaler 1997) or focused more on the role of the teacher than on the children’s understanding (Desforges et al. 1987; Aubrey 1997).

In relation to process, the focus is on the way that children develop understanding and skills in such ‘ordinary’ classrooms, while much of the current research in primary mathematics education takes the form of curriculum or teaching intervention and evaluation of effect (Wood et al. 1990; Cobb et al. 1991). My aim was to look at naturalistic settings, disturbed as little as possible by my presence, to find out what is happening in the classroom for ‘ordinary’ children in ‘ordinary’ mathematical lessons in ‘ordinary’ schools.

1.4 An outline of the study

The study takes the form of a preliminary stage and a main stage of data collection and analysis, each carried out in primary classrooms. The preliminary study was carried out in a single school and charted the progress of a small group of children as they moved from the nursery to
Chapter 1  Introduction to the study

the reception class. This allowed the researcher to gain experience in the process of participant observation, data collection and analysis and clarify the research methods for the main study. The preliminary study enabled the identification of key ideas that would be developed in the main study, helping to define the research question more clearly. A range of issues were identified as influencing children’s learning relating to the curriculum for number and addition, the use of representations in communicating mathematics and the influence of the wider social and cultural practices of the classroom. In particular the preliminary study resulted in a clearer focus on the children’s learning. I found that I was interested in teaching only as it related to the children, and not in the motives and intentions of the teachers themselves. Finally, the preliminary study gave opportunity for practice and refinement of the research methods used, participant observation and a grounded theory approach to data analysis. So the main study evolved from the preliminary one in terms of methodology and focus.

Diagram 1.1 overleaf identifies the progress of the research and the content development of the thesis.

Note:

In common with the majority of teachers of young children, all the teachers in the study were female; the pronouns she, her and hers will therefore be used. The teachers and children have been given alternative names, consistent with their gender and, as far as possible, ethnic origin. The schools have also been given alternative names.
Figure 1.1 Overview of research showing chapter connections
Chapter Two

Theoretical Perspectives on Learning

2.1 Introduction

This chapter contains a review of the literature which underpins, and is developed through, this research study. Three key areas of literature are discussed which relate to:

- theories of cognition which contribute to explanations of the children’s mathematics learning;
- the role of representations in the teaching of mathematics which will be used to analyse the interactions between teacher and children; and
- the development of children’s understanding of addition which provides the mathematical background of the study.

Chapter Three will then consider primary education and primary mathematics education in the light of these theoretical perspectives.

The diagram overleaf provides an overview of how these aspects of literature relate to one another and feed into the study.
Diagram 2.1 How the theoretical perspectives and literature reviewed in Chapters 2 and 3 interrelate

2.2 Theories of Cognition
- constructivism
- socio-cultural theory
- social practice theory
- cognitive theory

2.3 Representations in mathematics
- artefacts
- socio-cultural tools
- representations

2.4 Addition
- curriculum content
- teaching and learning interface

3 Primary Education
- teaching mathematics in primary schools
- discovery learning
- child-centred learning
- scaffolding

Learning early addition in primary classrooms
2.2 Theories of Cognition

This section addresses theories of cognition which are current in the study of mathematics education. Application of such theories to education centres around a continuum which describes learning as an individual or as a social act (Ernest 1995). In mathematics education three perspectives characterise this: constructivism, socio-cultural theory and social practice theory. These perspectives respectively situate learning in the individual learner, in social interaction and in social practice. For the purpose of this study I cannot address all aspects of these theories which have been extensively discussed elsewhere (Wertsch 1985b; Lave 1988; Walkerdine 1988; Glasersfeld 1989; 1991; Lave and Wenger 1991; Confrey 1994; Ernest 1994; Steffe and Kieren 1994; Confrey 1995; Glasersfeld 1995a; Lerman 1998). Each of the three perspectives has its origins in a different academic discipline, and in each case their application to education is not simple. I will concentrate in this chapter on those aspects which have informed, and are developed through the study; therefore this section will consider each of these theories in turn with reference to young children learning mathematics.

Traditional views of learning

Two theoretical ideas underlie traditional views of teaching and learning as experienced in British classrooms. Empiricism is founded on an ontology of naive realism, that there is an ultimate reality, and an objectivist epistemology, that true knowledge of this real world is possible (Richardson 1985; Ernest 1995). This has led to a transmission view of teaching and pedagogy, that the teacher can give knowledge to the children, and a passive-receptive view of learning. The child is seen as a clean slate, tabula rasa, on which knowledge can be written. Failure to learn rests on the child for not attending, for failing to receive, or for being a leaky vessel which does not retain that which s/he has learnt.

The second traditional view is that of behaviourism, which sees teaching in terms of training (Richardson 1985; Glasersfeld 1995b). Skinner (1968) described learning as consisting of three main elements: stimulus, behaviour, reinforcement. The process of learning was seen as passive, individuals experiencing a stimulus from the environment around them, altering their behaviour in the light of this stimulus and when this was encountered several times (reinforced) the altered behaviour became automatic and therefore 'learnt'. Glasersfeld argues that behaviourism is not concerned with concepts of 'meaning, representation and thought' since response is automatic (Glasersfeld 1987). It does not help us to explain the application of existing knowledge to solve new problems and, as such, has limited application to mathematics education.

Each of these theories has influenced the teaching of mathematics in the past (Cockcroft 1982) and residual effects are evident in teachers today (perhaps as a result of the way that they themselves were taught) and in political understanding of education (Ernest 1995). They each
have an underlying perspective of learning as a passive act. In contrast, the three theories discussed below, constructivism, socio-cultural theory and social practice theory emphasise an active role to learning.

2.2.1. Constructivism

Constructivism can be seen as having its origins in the work of Vico and Kant in the 18th century, but came to the fore in the twentieth century through the work of Popper and Kuhn in the philosophy of science, and in education through the extensive work and writings of Piaget (Glasersfeld 1984; Richardson 1985; Phillips 1995). More recently it has been developed by those studying the psychology of child development (e.g. Donaldson 1978; Hughes 1986) and science education (e.g. (Driver & Oldham 1986; 1995; Richie 1995), as well as its application to mathematics education both theoretically (Glasersfeld 1984; Glasersfeld 1995b) and in practical application (Steffe 1991; Cobb & Steffe 1983; Yackel et al. 1990; Cobb 1994).

Glasersfeld notes that "it is difficult to glean a coherent theory of cognitive development from Piaget’s enormous body of work" (1995a, p. 53). However two key ideas run through Piaget’s work reflecting his interest in epistemology and human development. The first addresses the way that knowledge is acquired by the individual by a process of cognitive adaptation and built up through a network of schemes (constructivism), the second looks at the development of learning, observed through experimentation with children (stage theory). Each of these ideas have had a significant influence on primary education and mathematics education.

Cognitive development

Piaget describes the acquisition of knowledge as active construction through a process of organisation and cognitive adaptation, as a result of the assimilation and accommodation of new experiences (Piaget 1966; 1980). Each new experience is compared with already existing schemes of knowledge and assimilated if the new experience is compatible with the existing understanding for, as Piaget says;

...no behaviour, even if it is new to the individual, constitutes an absolute beginning. It is always grafted onto previous schemes and therefore amounts to assimilating new elements to already constructed structures (innate as reflexes are, or previously acquired). (Piaget 1976, p.17)

If assimilation is not possible, because the new experience is incompatible with the existing knowledge, perturbation results and, through reflective abstraction, the existing understanding is altered to accommodate the new experience.

This study is based on the premise that learning is a process of active construction, that children ‘make sense’ of their experiences in the classroom and is therefore founded within a constructivist perspective.
Constructivism has affected primary education both in the emphasis on the individual and in an emphasis on learning by doing. In Chapter Three I will show how this led to the development of child-centred and active learning approaches to primary education. In early primary mathematics education, the focus on experience led to an emphasis on learning through practical activity, "I do and I understand" (Nuffield Foundation 1967) and Dienes concept of multiple embodiment (Dienes 1964) which held that children would generalise a concept through experiences with a variety of physical representations of the same concept. For example, the base ten structure of number would be generalised through experience of Dienes blocks, money, and sets of other objects collected into tens and hundreds. Emphasis on multiple representations became commonplace in both mathematics schemes and textbooks for teachers (e.g. Williams and Shuard 1982). A wide range of research into the use of representations in learning mathematics, is further considered in section 2.3 below.

**Stages of development**

The second key idea in primary education, developed from Piaget's work, has been that of stages of development. Piaget asserted that children pass through clear stages of development which enable (or perhaps restrict) their understanding and abilities (Donaldson 1978). Piaget identified four stages through which children grow. These are described as:

- The sensori-motor period (0 to 2 years);
- the pre-operational period (2-7 years);
- and the operational period which has two divisions into:
  - concrete operations (7-11 years);
  - formal operations (11 years to adulthood).

Piaget claimed that teaching in advance of the child's stage of development will be at best unproductive and may be harmful. In primary education the concept of 'readiness for learning', and the dangers of 'moving children on too fast' are predicated on this stage theory (Walkerdine 1984). Recognition that inappropriate instruction may even be harmful to the child's development has resulted in the teacher's 'fear of telling' which Jaworski refers to as the teacher's dilemma (1989).

However this dilemma may be built on insecure foundations for, while the theory of cognitive development has persisted within the constructivism movement, the idea of developmental stages is less secure. Donaldson and others challenged Piaget's assertion that young children were unable to think logically. They acknowledged that Piaget's results could generally be replicated, indicating a lack of logical thought in young children. However, by recontextualising some of the experiments they found that the children were able to think
logically when they understood the language, the purpose and context of the task (Donaldson 1978; Hughes 1986). Brown and Desforges (1979) have also challenged Piaget’s findings, questioning his clinical research methods and his approach to the relationship between language and thought, which did not allow for alternative interpretations of meaning.

Confrey observes that, as a result of Piaget’s work on stages of development, “many constructivists assume an incremental view of knowledge construction” (p.6) which results in the assumption that there are clear pathways through the development of mathematical knowledge. Case (1978) describes how Piaget’s theory of cognitive development has led to attempts to teach children to complete Piagetian tasks (e.g. Gelman and Gallistel 1978) but argues that it cannot lead to a theory of teaching. Resnick and Ford reiterating Case’s argument, discuss how:

the theory is structural rather than functional. Piaget’s protocols of children performing tasks offer us still snapshots of cognitive structures at various levels of functioning. They do not offer the kind of detailed psychological model of the acquisition and development of structures that is needed for designing instruction (Resnick & Ford, 1984).

Despite these doubts, stage theory has resulted in very structured primary mathematics schemes, a feature of mathematics teaching and learning which I will address in this study.

**Radical constructivism**

Within mathematics education, Glasersfeld and others have built on Piaget’s work to develop a radical constructivist perspective. Glasersfeld defines radical constructivism as having two main tenets:

- “knowledge is not passively received but built up by the cognizing subject;
- the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality.”

(1995a, p. 18)

Constructivism starts from the premise that “there is no way of transferring knowledge - every knower has to build it up for him - or herself” p.612 (Glasersfeld 1982). Learning is an individual and active process, in which the learner constructs meaning from his/her experience of the world. In describing the building up of knowledge Spivey (1995) uses the metaphors of carpentry and architecture to describe the building of understanding, though she goes on to note that the product is not a rigid, static one but dynamic. But Ernest points out that the building is not composed of ready made pieces of knowledge but that “[t]he process is recursive (Kieren and Pirie 1991) and so the ‘building blocks’ of understanding are the product of previous acts of construction” (Ernest 1995, p.461).

The second tenet, that cognition is adaptive, leads Glasersfeld to the notion of viability and fit rather than match (1984). He suggests that the concept of match assumes an external reality
against which the match can be measured, while a concept of fit allows for the new knowledge to be compared with that which already exists for the knower, through a process of reflective abstraction, with assimilation or accommodation of the new knowledge as an outcome. Ideas which are found to fit are considered viable; this is not to say that they are in any way judged to be 'true' since this requires an external source for comparison, but viable in that they work within the existing constructed model of experience.

It is the second tenet that creates radical constructivism, for Glasersfeld explains that constructivism does not set out to explain the real world. It is radical because

... it breaks with convention and develops a theory of knowledge in which knowledge does not reflect an "objective" ontological reality, but exclusively an ordering and organization of the world constituted by our experience" (1984, p.24).

Constructivism has been accused of solipsism: if all knowledge is constructed by the individual, can we believe whatever we want, and how do we account for a body of mathematical knowledge (Lewin 1995; Phillips 1995)? But radical constructivism does not deny reality, it is ambivalent about its existence, seeking only to assert that if there is such a reality we can only know it through experience and it is therefore not possible to match our experience to any external criteria. Ernest describes this as being ontologically neutral (1995).

From a personal perspective, I hold strongly the tenet that knowledge, at least that mathematical knowledge, is constructed, for it accords with my own experiences of teaching and learning, but I am less convinced of the second tenet. The idea of adaption allows me to understand the apparent inconsistencies of understanding seen in the classroom, to realise that what the teacher 'sees' and what the child 'sees' may not be the same, but I do not need the second tenet, for inconsistencies in understanding can also be explained in terms of variation in previous knowledge. To use Ernest's metaphor, can we expect two buildings to be the same, if we are building with different sets of bricks? Furthermore, the assertion that we cannot know whether there is a reality may be philosophically logical, but I find it difficult to embrace. I believe that it might make constructivism internally inconsistent for, if we cannot know whether there is a reality, how can we be so certain that we learn through experience?

A constructivist perspective offers us understanding of how individuals learn. However it has been argued that it takes no account of the role of language, communication, feelings, concerns and values that are a part of the context of cognitive learning (Donaldson 1978; Ernest 1995). As Lerman points out, "[w]hen an action gains significance for a child, becoming bound up with goals, aims and needs and associated with purpose, it is a social event" (1996, p.136). As a theory of learning, constructivism does not regard language, social interaction and social context, as central to teaching and learning. For this I will look to socio-cultural theory.
2.2.2 Socio-cultural theory

Origins of socio-cultural theory

Socio-cultural theory has its origins in the writings of Lev Vygotsky, a lawyer, philologist and psychologist working in the Soviet Union in the 1920s and 30s, who died at the age of 38 (Wertsch 1985b). His work was heavily influenced by the writings of Marx and Engels which pointed to the importance of the society rather than the individual. Since his death there have been significant developments within psychology and education which build on his writing. In this section I will address only those aspects which relate to this study.

Vygotsky set out to examine 'the relation between human beings and their environment' and 'the relation between the use of tools and the development of speech' (Vygotsky 1978, p.19). This examination resulted in theories of conceptual development, and of mediated activity and role of psychological tools. Underlying these is the central tenet that social factors are the basis of learning. Meaning, attention, conception are all social in origin, originating in the social dimension before being developed by the individual.

Conceptual development in a social context

For Vygotsky, development results from a child's internalization of social interactions. His general genetic law of cultural development states:

> (e)very function in the child's cultural development appears twice: first on the social level, and later, on the individual level; first between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals (Vygotsky 1978 p.57)

In an attempt to analyse the relationship between development and learning, Vygotsky formulated his theory of the zone of proximal development (ZPD), which Wertsch (1985b) describes as a 'special case' of the general genetic law. In a problem solving situation children are able to do some things unaided, which he termed their actual developmental level: "the level of development of a child's mental functions that has been established as a result of certain already completed developmental cycles" (Vygotsky 1978, p.85). However, with the help of a more experienced 'other', children are able to do far more than they are unaided and this Vygotsky describes as their potential developmental level. The zone of proximal development is therefore described as:

> ... the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more able peers (p.86).
It is within this zone that learning takes place, as Vygotsky explains “what a child can do with assistance today she will be able to do by herself tomorrow” (p.87). Furthermore, Vygotsky maintains that it is only within the zone that learning occurs; learning will not aid the child’s development if it addresses the actual developmental level of the child, for “the only ‘good learning’ is that which is in advance of development” (p.89).

The concept of the ZPD was a late Vygotskian idea, which he was unable to develop before his death, on which others, especially those working with young children, have continued to build. Wood, Bruner and Ross (1976) extend the idea of a ZPD back into infancy arguing that “tutorial interactions are, in short, a crucial feature of infancy and childhood” (p.89). They use the metaphor of ‘scaffolding’ to describe the process whereby the adult supports the children’s learning to enable them to achieve more than they could alone.

Confrey observes that scaffolding has been most closely studied within the context of problem solving, applying skills and knowledge, rather than teaching and learning (Confrey 1995b), though I believe that this observation may arise from a constructivist division between problem solving (application) and learning (construction). In the school context Hobsbaum, Peters and Sylva (1996) claim in Reading Recovery programmes scaffolding can occur only when teachers work individually with a single child. Bliss, Askew and Macrae (1996) claim that scaffolding is rarely seen in the context of the classroom. Indeed a teacher of a class of 30 children, each with a different ‘zone of proximal development’, may find the theory no more helpful that that of Piaget’s stage theory. How does she scaffold the learning of each child (Jaworski 1994)?

But for Vygotsky the idea of a zone of proximal development helped him to clarify his original aims about the relation between development and learning, for he concludes that learning is not development but that mental development comes as a result of learning (Vygotsky 1978). However he does accept that:

our hypothesis establishes the unity but not the identity of learning processes and internal development processes. It presupposes that the one is converted into the other. Therefore it becomes an important concern of psychological research to show how external knowledge and abilities in children become internalized” (p. 91).

This issue of internalization he addresses through the theory of mediated action.

Mediated activity and role of psychological tools

Vygotsky differentiates between the natural behaviour of the child and the higher psychological (or mental) functions which arise from social interaction, “processes such as deduction and understanding, evolution about notions of the world, interpretation of physical causality, and mastery of logical forms of thought and abstract logic” (Vygotsky 1978, p.79). He theorises that these higher functions arise through the mediation of tools and signs. Physical tools allow
us to do more than we can unaided and Vygotsky explains that this is also a characteristic of sign systems, including language, writing and number systems (Bruner 1966; Confrey 1995b).

Vygotsky observed that young children would often accompany their actions with a ‘running commentary’, and argued that this was not, as Piaget had claimed, mere egocentric speech, but the child talking themselves through the activity in order to enable them to carry it out (Vygotsky 1978). In support of this argument he notes that such speech increases as the child finds the task more challenging. He therefore describes language as having an intrapersonal use which arises out of the interpersonal language development. At first the language must accompany action but as the child becomes more skilled in the use of language s/he is able to use it preceding action in order to make plans. Such intrapersonal speech develops internally to become thought, the process of internalization.

So, language is used as a psychological tool, allowing the child to carry out tasks which could not be done without it, which leads to the development of higher psychological functions, thought and reasoning. Just as physical tools are directed at the experiential world, leading to changes in objects, psychological tools or signs are directed at the internal world, leading to changes in behaviour and thought. The use of tools and signs and their eventual internalization are the means by which the interpersonal becomes intrapersonal.

Wertsch, in developing Vygotsky’s ideas, believes that internalization involves both mastery and appropriation (Wertsch 1998). First, mastery involves learning how to use the tool in the context of its social practice, for example the number system in the context of mathematics lessons. This is a social act and I believe can also be seen from a social practice perspective as legitimate peripheral participation (Lave and Wenger 1991, see section 2.3.3 below). Secondly, Wertsch argues that the tool must be appropriated by the learner, who must make it his/her own and use it. Wertsch maintains that both these aspects of internalization, mastery and appropriation, arise from action, action mediated through the use of tools.

So, socio-cultural theory offers explanations of the social nature of learning, the role of tools, signs and especially language in mediating understanding, and the role of the adult or more able peer in moving the child’s thinking forward.

Socio-cultural perspectives on teaching and learning

Vygotskian ideas have been developed in many forms within education. In this section I want to look in more detail at four authors who discuss socio-cultural perspectives on primary schooling.

Teaching as Assisted Performance

Tharp and Gallimore (1988) draw on ‘neoVygotskian’ theory (including Rogoff and Bruner) in an attempt to devise a “unified theory of education” (p.12). Building on the concepts of the
ZPD and internalization, they propose a model of teaching as assisted performance, through which the children are guided to 'reinvent' socially accepted knowledge and practice. Such teaching involves a combination of modelling, contingency management (rewards and punishments to reward behaviour), feedback, instruction, questioning and cognitive structuring (making explicit the links between ideas). Their work, situated in multicultural North American schools, focuses mainly on the teaching of language skills. To date, I am unaware of any work in English primary school mathematics which is based on the principles of 'assisted performance'.

**Teaching as guided participation**

In a similar way to Tharp and Gallimore, Mercer draws on socio-cultural theory to explain "people helping other people to learn" (p. 6), proposing 'guided participation' as a theory of learning.

According to this theory, talk is used to construct knowledge. This is a social and historical process in the sense that the talk creates its own context and continuity, so that the knowledge that is created carries with it echoes of the conversations in which it was generated (Mercer 1995, p.84).

Mercer uses the concept of scaffolding to describe how teachers, and others, can help children to learn more than they could do alone. He recognises that scaffolding in a school setting may take different forms from that in parent-child dyads. He describes classrooms as 'communities of educated discourse' in which teacher and children develop language forms which are specific to that setting, the teacher scaffolding learning by guiding the children in the use of that discourse. This enables the child to 'talk and think like the teacher', a characteristic of school learning.

**Learning through narrative**

Bruner has published a wide range of writings which, amongst other things, chart his personal development from constructivism towards an understanding of learning as social and cultural. As he says:

"I have come increasingly to recognise that most learning in most settings is a communal activity, a sharing of the culture. It is not just that the child must make his knowledge his own, but that he must make it his own in a community of those who share his sense of belonging to a culture." (Bruner 1986, p.127)

I have already considered Bruner's involvement in the development of 'scaffolding'. Two other areas of work are particularly relevant to this current study. The first is his understanding of the use of signs as psychological tools, and the relevance of this to mathematics education (Bruner 1966), which will be included in a discussion of representations in section 2.3 below.

Secondly, in his later work on the culture of education, Bruner uses a socio-cultural view of learning to explain the role of narrative in children's learning (Bruner 1996). He argues that
human culture is framed by narratives; our personal as well as our collective histories are told as narrative, stories which have a common structure even though the detail and the interpretations may change. He summarises:

it seems evident, then, that narrative construction and narrative understanding is crucial to constructing our lives and a “place” for ourselves in the possible world we will encounter.” (p. 40)

Through narrative, young children learn about their personal and cultural histories; their understanding of the social world and the world of language is developed not only through personal experience but also through the telling and hearing of narrative stories. Bruner asserts that:

understanding is the outcome of organizing and contextualizing essentially contestable, incompletely verifiable propositions in a disciplined way. One of our principle means of doing so is through narrative: by telling a story of what something is about” (p. 90).

Bruner identifies nine “universals of narrative realities” (p. 133 ff):

- The structure of committed time - that narratives describe actions happening over time;
- generic particularity - that narratives are about particular cases but reflect general ‘truths’;
- actions have reasons - what people do in narratives is motivated by beliefs, desires, theories, values;
- hermeneutic composition - narratives have more than one interpretation;
- implied canonicity - the interest of the story is that it runs counter to expectations, which are often not explicit but implied;
- ambiguity of reference - what the story is about is open to question;
- the centrality of trouble - “stories worth telling, ..., are typically born in trouble” (p.142);
- inherent negotiability - though there may be different ‘readings’ of the text, these can be negotiated with reference to the story; and
- the historical extensibility of narrative - stories have beginnings and endings but imply a time before the beginning and after the end.

Krummheuer (2000) has drawn on this theory of narrative to describe how children talk about the mathematics they have carried out. In this study I will argue that narrative can be used to explain the role of stories and real world contexts in the learning of mathematics (Chapter Eight).
2.2.3 Social Practice Theory

The third theory of cognition, current within mathematics education, is that of social practice theory, or situated cognition. While constructivism arose from cognitive psychology and Vygotskian theory from Marxist dialectic, social practice theory has its origins in social anthropology, although it also draws on socio-cultural theory. It is heavily based on the work of Jean Lave (Lave 1988; Lave and Wenger 1991; Lave 1996). Lave set out to examine the notion of formal and informal learning as postulated by Scribner and Cole (1973), a development of Vygotskian theory. Through her studies of apprentice tailors in Liberia, Lave came to realise that all learning has a social origin for “there is no activity that is not situated” and learning is a process whereby “agent, activity and the world mutually constitute each other” (Lave and Wenger 1991, p.33).

For Lave, the starting point for learning is not the individual but the social practice in which the community is engaged. In Lave and Wenger's words:

... a theory of social practice emphasises the relational interdependency of agent and world, activity, meaning cognition, learning and knowing. It emphasises the inherently socially negotiated character of meaning and the interested, concerned character of the thought and actions of persons-in activity .... In a theory of practice, cognition and communication in and with the social world are situated in the historical development of ongoing activity. (p.50-51)

Lave describes learning as situated in 'communities of practice' and describes how 'newcomers' to the community become more knowledgeable in the practice through a process of 'legitimate peripheral participation', a bridge between the individual and the community of practice. Through such participation the individual learns to become a full member of the community of practice, to reach full participation and become an 'old-timer'. Lave equates legitimate peripheral participation to an interpretation of Vygotsky's ZPD, as a more able other guides an apprentice (1991, p.47). Adler (1998) notes that full participation is not “an endpoint in learning/knowing all there is to know about the practice” (p.165). Full participation involves mastery - having control over the resources (physical and mental) present in the practice. Learning within the community of practice requires these resources to be accessible to the learner. Lave and Wenger describe two ways in which this mastery can develop. The first is through the concept of transparency:

The significance of artefacts in the full complexity of their relations with the practice can be more or less transparent to learners. Transparency in its simplest form may imply that the inner workings of an artefact are available for the learner's inspection .. transparency refers to the way in which using artefacts and understanding their significance interact to become one learning process (1991, p.102-3)

In its simplest form, transparency could be described as seeing not only what to do but why, and with this knowledge mastery, rather than imitation, is achieved. The transparency or the
lack of it can "enable, obstruct or even deny peripheral participation and hence access to the practice" (Adler 1998, p.167).

Secondly, legitimate participation involves "learning how to talk (and be silent) in the manner of full participants" (p.167). Lave and Wenger distinguish between talking within and talking about the practice (1991, p.109). Participants in the practice must learn how to talk as members of the community. "For newcomers then the purpose is not to learn from talk as a substitute for legitimate peripheral participation; it is to learn to talk as a key to legitimate peripheral participation" (1991, p.105, original italics).

**Social practice perspectives on teaching and learning in school**

Lave studied how adults learnt to use mathematics; does her theory have relevance to teaching children? Nunes (1991) describes how children, who had competent strategies for calculation when selling fruit in the street markets, were unable to make the same calculations in the school setting, resorting to half learnt formal algorithms. She concludes that the children held different understandings of the mathematics, that were tied to the situation in which they occurred, and had little or no transference. This appears to confirm understanding as situated in practice, and poses a challenge for teachers, since the main purpose of school learning is its application outside the classroom.

Hughes and Greenhough (1998) found that children were able to make links between similar activities carried out at home and at school. The children played games with the teacher at school, and with parents at home, each time being observed by the researcher. Two different forms of the game were used which contained the same mathematical ideas but set in a different context. They found that the children could recognise the mathematics and use the same mathematical strategies to play the game, and conclude that transfer from one community of practice to another is possible. I question whether these could be considered separate, rather than overlapping, communities of practice since the activities shared similar purpose and goals, and the researcher and child were present in each, the teacher and school being replaced by the parent and home. However, we already know that young children can make links between home and school knowledge, otherwise they would have to learn to count again when they started school. The situatedness of school knowledge requires further consideration.

Adler observes that social practice theory faces its greatest challenge when we try to apply it to the practice of teaching and learning in schools (Adler 1998). For if we are to describe the mathematics classroom as a community of practice what is the nature of the practice - teaching, learning, mathematics? Who are the old-timers - teachers or older pupils - and what are the newcomers learning to become? We cannot say that they are learning to become teachers since this will apply to few of them. Few, also will become mathematicians, even if teachers see themselves as mathematicians, yet mathematics is what they are doing. And what is the
product of the community of school mathematics learners? Adler sums these questions up in asking "what might constitute legitimate peripheral participation in the mathematics classroom (p.169).

In this sense it is difficult to see how the mathematics classroom can be equated to the work, or even the social, contexts of adults. The children are not there by choice (Ernest 1998) and if they share the aims of the teachers these are seldom made explicit. The newcomer/old-timer, peripheral and full participation models seem to break down. Does this indicate that social practice theory has nothing to say to the teaching of mathematics in school?

Adler argues that:

Lave and Wenger have, nevertheless, constructed concepts that could provoke insights into learning and teaching mathematics in school. Specifically, access and sequestration, the availability of learning resources, transparency and the distinction between talking within and about a practice are easily read into the pedagogical relation in mathematics teaching in school (p.170, original italics).

In particular she notes the relevance of transparency to the use of resources (physical and mental) for learning mathematics in the classroom, where the child may find the resource opaque and so focus on the resource rather than on the underlying mathematics. This opacity of resource can lead to alienation from the practice. Adler relates talking within and about mathematics to the difference between children using language to help solve a problem and that used to explain it to others. With reference to Edwards and Mercer (1987) she asks whether the implicit ‘educational ground rules’ which are an aspect of the culture of the classroom could be made explicit in order to allow participation by all the children.

Finally, in an attempt to explain the sense in which the classroom is a community she draws on the idea of discourse, “language as it is used to carry out the social and intellectual life of a community” (Mercer 1995, p.79). Adler argues that the discourse of the classroom is not that of the mathematician, nor the apprentice, nor the everyday, but that it is a distinctive discourse “a social practice with specific time-space relations, activities and discursive practices. School mathematics is a distinct practice. A hybrid where there are recontextualisations from the discipline of mathematics and its applications into the curriculum.” (p.173)

So, Adler and others would argue that social practice theory has clear relevance to the classroom; but the mathematics education community may still need to make it its own (though see Watson 1998 for further discussion). To date, there has been little published research on the application of social practice theory to early mathematics education.

This study, especially the findings in Chapter Nine, draws on social practice theory to explain teaching and learning early mathematics.
2.2.4 Unity or diversity

Three theories of cognition have been summarised and their relevance to teaching and learning discussed. In this section I want to address the question of whether it is possible to adopt all three, or whether a choice needs to be made. Many researchers in this field have recognised the need for theory to address teaching and learning from a social as well as an individual, psychological point of view. Jaworski, who started her doctoral study from a radical constructivist perspective, recognised the need for consideration of the social world (Jaworski 1994). Bruner, having started from a constructivist viewpoint, affirms both the role of the individual and the role of culture when he remarks:

"I have come increasingly to recognise that most learning in most settings is a communal activity, a sharing of the culture. It is not just that the child must make his knowledge his own, but that he must make it his own in a community of those who share his sense of belonging to a culture." (Bruner 1986, p.127)

For these writers acknowledgement of the social as well as the individual aspect of learning is important.

However, Lerman argues strongly that it is not possible to combine socio-cultural theories and radical constructivism, since they have “fundamentally different orientations, the former placing the social life as primary and the latter placing the individual as primary” (1996, p.133). The major issue here is that of intersubjectivity. Radical constructivism sees knowledge as individually constructed and is ontologically neutral and so cannot easily account for shared knowledge. Indeed Goldin asserts that “radical constructivism does not in principle ever permit us to conclude that two individuals have ‘the same’ knowledge” (Goldin 1990, p.39). For Lerman, from a socio-cultural perspective, it is within social interaction and language that psychological phenomena “actually exist” (1996, p.134), agreeing with Vygotsky’s claim that “the true direction of the development of thinking is not from the individual to the socialised, but from the social to the individual” (Vygotsky 1986, p.32). The two views, constructivism and socio-cultural theory, are therefore inconsistent. However, Lerman does not address the other side of the argument, how learners construct individual understandings which are not direct matches of the socially shared understanding. To speak of an “interplay between sociocultural origins and individuality” (p.147) is to accept the role of the individual but does not explain this role.

Constructivists differ in how they address this issue. Piaget saw social interaction with others as just part of the environment of the child, and Glasersfeld claims that ‘others’ that populate the child’s world do not differ from other physical objects (Glasersfeld 1995a). Confrey (1994) and Steffe (1995) claim that constructivism does address social interaction, speaking of a process of model building and the construction of others which allows the radical constructivist to describe how, through a process of building a model of the ‘other’ person
(Steffe refers to these as second order models), knowledge can be intersubjective and not just individual. Others, Bauersfeld (1992), Krummheuer (2000), and Cobb and Yackel (1998) draw on symbolic interactionism (Blumer 1969) to explain how social interaction enables individual construction. Confrey (1994) recognises the problems in combining constructivist and socio-cultural theories too simplistically, and avoids this by asserting that radical constructivism does acknowledge social interaction, socio-cultural theory is not needed.

Ernest began (1989) by proposing a theory of social constructivism, based on constructivism but including the social nature of both mathematics and learning, and drawing on Vygotsky’s work. However later (1994) he questioned whether this merger was possible and advocated a form of social constructivism which emphasised the role of the social world much more strongly than that of the individual.

More recently, Steffe and Johnson have reopened the argument (2000), arguing that there is much of Vygotsky’s theory that is compatible with radical constructivism and that social interaction is an essential element of the radical constructivist perspective. In doing this they equate social interaction with Lerman’s concept of intersubjectivity. Lerman (2000) responds that the social element of radical constructivism is a weak one which addresses the role of others for the individual, but does not address the essential paradigm problem he posed earlier, that “culture and meanings are on an external plane and must be internalised by the child: they cannot be created by the child” (p.213). Lerman argues for a ‘thick notion’ of social which takes into consideration “different social, economic and cultural situations” (p. 212), and concludes that “what is needed is a more complex theoretical framework for the study of learning than can be offered by cognitive studies” (p. 221).

Jaworski (personal communication) develops the metaphor of lenses to describe how it is sometimes useful to ‘see’ the situation from a constructivist perspective and at other times to focus in from a socio-cultural or social practice theory perspective. While this is consistent with Lerman’s (1996) language as he speaks of “the gaze of the psychologist” (p.136) or “the focus of study” (p.137), it does not address the ontological incompatibility set out above.

Outside of mathematics education there seems to be less debate. Pollard (Pollard 1996) in his study of primary schools and Wood (Wood 1998) from a child psychology perspective, both acknowledged a social constructivist position, which draws on socio-cultural theory without leaving constructivism behind. Driver and Oldham (1986) describe three constructivist approaches to science education ‘children’s personal constructions’, ‘the interpersonal construction of knowledge’ and ‘the construction of science as public knowledge’.

Opinion is therefore still divided as to whether any one theory can be used to explain how mathematics is learnt, whether it is possible to develop a unified theory of learning mathematics or whether each has some merit within particular practical or theoretical contexts. Adler (1998 p.176) summarises her thoughts by saying that “perhaps learning is, after all, not a unitary
phenomenon, and thus not amenable to one all-embracing theory”. Or perhaps we have yet to find the ‘holy grail’, agreeing with Bruner that “a theory of development is *par excellence* a future topic … For the time being we will have modest theories, local in concern, free of grand concepts of future possibility” (Bruner 1986, p.148).

**A personal view of cognition**

I began this study, happy to describe myself as a social constructivist, drawing, like Pollard and Wood, and others involved in early primary education, on constructivism and aspects of Vygotskian ideas: the role of language, of social interaction and scaffolding. Since opinion is so divided and I found it difficult to identify a coherent account of a social constructivist perspective from the literature, I have decided to leave aside social constructivism as a perspective. In the study that follows I will draw on each of the three theoretical perspectives discussed above: constructivism, socio-cultural theory and social practice theory, individually. In Chapter Ten I will return to discussion of an all embracing theory to describe young children’s learning of mathematics.
2.3 Representations in Mathematics

2.3.1 The role of representations in the teaching and learning of mathematics

One of the ways to look at teaching and learning in mathematics is to consider how the mathematics is represented by the teacher, as evidence of teaching, and by the children as evidence of learning. A wide range of practical equipment, as well as pictures and symbols, are used in primary mathematics classrooms. As a teacher and teacher educator I was aware of models of teaching and learning, particularly those developed by Liebeck (1984) and Haylock and Cockburn (1997), discussed below, found in the professional literature on primary mathematics education. In this section I will explore the literature on representations in mathematics, as it relates to primary mathematics education.

Research into the use of representations in mathematics has mainly arisen within a constructivist paradigm (Cobb et al. 1992; Presmeg 1998; Lesh et al 1987a), drawing on ideas of a fit or viability between the concept and the representation (Glasersfeld 1987), and it was from this perspective that I viewed representations at the start of this study. However the use of representations are also discussed from socio-cultural and social practice theoretical perspectives. From a Vygotskian perspective Vergnaud (1998) discusses the role of language and symbols, Jones (2000) considers artifacts as socio-cultural tools, and Nunes (1997) draws on Luria’s ideas of mediation through systems of signs. The meaning of such socio-cultural tools must be negotiated through social interaction and internalised by the individual. Meira (1998), and Adler (1998), use Lave’s concept of transparency to ‘make sense of instructional devices’. Such devices are opaque to the uninitiated, their attention drawn to the surface features, while through mastery of the practice they become transparent, providing insight into the underlying meaning. Goldin (1998) identifies himself with “none of the previously mentioned philosophical movements or belief systems” (p.140) claiming that the “notions of representations and representational systems” (p.141) are fundamental to the goal of the development of a unified theory of learning. It is therefore possible to consider the use of representations within each of these paradigms as a way of analysing teaching and learning in the classroom. Since much of the literature is based in constructivism I shall use the word ‘representation’ rather than ‘tool’, ‘artefact’ or ‘systems of sign’, except where I am referring to a particular theoretical perspective.

Representations defined

The word representation is used to describe a range of external and internal (mental) ways to describe and manipulate mathematical ideas. Glasersfeld observes that the English language is limited in having only one word, ‘represent’, while in German there are four different words
with different meanings: to depict; to form a mental image; to act or substitute for (a person); and to stand for, signify, or denote (Glasersfeld 1987). He claims that representations in mathematics education are used for three of these purposes - to depict (pictures, graphs etc.), to form a mental image (of a numberline, say), and to stand for, signify, denote (words, symbols and manipulatives). It is therefore important to bear in mind this range of meaning when using the word representation. Pimm (1995) suggests that we should consider the function of the representation - why it is being used in a particular context.

There is considerable research and discussion of the use of representations in mathematics teaching and learning much of it relating to mathematics at a secondary school level involving algebraic and geometric concepts. Work at primary level has mainly concentrated on the use of structured manipulative materials, especially Dienes Multibase Activity Blocks (MABs) in making sense of standard written algorithms for the four operations (+,-, x, ÷), which will be discussed below (section 2.4.2) (Janvier 1987; Goldin et al. 1998). Goldin summarises the work of the PME¹ Working Group on Representations by defining four forms of representations in mathematics²:

A. External and Physical embodiments, ... e.g. (1) a number line, drawn and labelled, illustrating order relationships among numbers; (2) a configuration of pegs on a pegboard providing an array model for multiplication ...; (3) a calculator or computer-based environment...

B. External linguistic embodiments ...(including) verbal, syntactic and related semantic aspects of the commonly shared language in which mathematical problems are posed and mathematics is discussed.

C. Formal mathematical constructs ... e.g. (1) state-space representations of problems or games such as the Tower of Hanoi, Nim etc.; (2) representations of mathematical entities such as groups, rings, functions etc. ...

D. Internal cognitive representations ... internal cognitive configurations of learners and problem solvers. (1998, p.285)

Kaput (1998) challenges the notion of internal and external representations since any external manifestation is only seen as representative by the individual and therefore is in a sense internal. Elsewhere Goldin alludes to this when he writes of the “essential presence of ambiguities in representational systems” (1992, p.241, original emphasis).

Goldin (1998) also draws on the work of Saussure who describes signs in terms of signifier and signified - the signifier being that which carries the sign (the representation) and the signified being that which is carried (the meaning). This description helps distinguish between representation and meaning, acknowledging that while use of the representation may be shared between individuals, between teacher and child, its meaning may not shared. Skemp (1979)

¹ PME Psychology of Mathematics Education
² this quotation is a summary of a longer section. (...) denotes further explanation which has been omitted
uses the idea of surface and deep structure, arguing that in order to use representations successfully the learner must get beyond the surface characteristics of the representation (the signifier) to the deep structural meaning (that which is signified) or, in Lave's terms, the representation must become transparent. From a variety of theoretical perspectives, therefore, writers agree that representations of mathematics do not simply carry knowledge but must be interpreted by those who use them. Through interaction (Blumer 1969), mediation (Nunes 1997), or participation in practice (Adler 1998) the meaning of the representation must be negotiated.

**Models of representations examined**

Mathematics can be represented in the primary classroom in a range of different ways. Practical materials, real life contexts, words, pictures and symbols are all used to represent mathematical concepts such as number and number operations. A variety of models to explain mathematical learning, mathematical action and concept development have arisen which draw on work by Bruner (1966). Bruner proposed a framework for concept learning to explain "how the child gets free of present stimuli and conserves past experience in a model, and the rules that govern storage and retrieval of information from this model" (p.10). He suggests three ways that this freedom is accomplished. First the concept is experienced through an *enactive* form, through experience and interaction with the environment. Secondly, it may be represented in an *iconic* form, an image that can be called to mind which represents the absent object or concept. Finally the concept may be represented in spoken word or written symbols - a *symbolic* form. Here the word or symbol will call to mind the concept directly. Echoing Vygotsky, Bruner argues that language gives children power to control their actions; which perhaps shows something of Bruner's move from constructivism towards socio-cultural theory. While recognising the uniqueness of 'compact' written mathematical symbols (p12), he does not differentiate between the spoken and written symbolic forms, a omission which I believe may be significant for those who work with young children.

My first reading of Bruner's framework was that the enactive/iconic/symbolic forms are about how concepts are constructed (learning). However, he then goes on to look at the implications of this for teaching (instruction).

> If it is true that the usual course of intellectual development moves from enactive through iconic to symbolic representation of the world, it is likely that an optimum sequence [for instruction] will progress in the same direction. (p. 49)

Developments in Bruner's framework, in the teaching and learning of mathematics, fall into three separate strands. The first I will call the sequential strand since the key idea is that the

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3 This section draws on a paper I presented at the British Congress for Mathematics Education (Price 1999)
three modes of representation are a sequence: enactive → iconic → symbolic, with the symbolic stage an end point. In Liebeck’s model for mathematics teaching, developed for use in early childhood mathematics education, the symbolic form of spoken language is separated from that of written symbols and given a higher priority in young children’s learning (Liebeck 1984). Using the framework: Experience → Language → Pictures → Symbols (known by the acronym ELPS), Liebeck encourages teachers to provide activities which develop the experience and language elements, before more abstract pictures and then symbols are introduced. Liebeck acknowledges drawing on Piaget, Bruner's early work, Skemp and Dienes for the development of her model. The Open University (1982a) model: Do, Talk, Record (DTR), also describes a progression towards symbolic representation. However the two differ in that ELPS is about teaching and the slow progression to the more abstract symbolic form, while DTR is about learning mathematics, with the assumption that all three representations may be used within a single mathematical act.

The models which constitute the second strand are connective models. The first of these is described by Haylock and Cockburn (1997) and builds on constitutive elements similar to ELPS, emphasising the connections between these representations, and involving:

“the building up of a network of cognitive connections between the four types of experience that we have identified above: concrete experiences, symbols, language and pictures. Any of the arrows in [the figure below] represents a possible connection between experiences that might form part of the understanding of a mathematical concept. (Haylock and Cockburn 1997, p3.)

![Figure 2.2 Relationship between representations, based on Haylock and Cockburn 1997](image)

Haylock and Cockburn assert that learning which develops from understanding of the movement between each of the representations will be relational (conceptual), rather than instrumental (procedural or rote) learning (Skemp 1971; Hiebert1986).

The second connection model is that of Lesh, Post and Behr (1987a, p.33), developed in the context of problem solving, similar to Haylock and Cockburn but including the element of real scripts - sentences or stories describing real world contexts for mathematics, such as shopping. Again the connections between these, the ability to move between one representation and another when solving problems, are a key feature.
Both these connection models have been developed within a primarily constructivist paradigm.

The final strand, exemplified by Mason (1987), relates Bruner's framework to concept development in the mind. Again there is an emphasis on individual, cognitive construction of knowledge. Mason speaks of a spiral which describes an inner movement of increasing confidence:

- from confidently manipulable objects/symbols,
- through their use to gain a 'sense of' some idea involving a full range of imagery but at an inarticulate level,
- through a symbolic record of that sense,
- to a confidently manipulable use of the new symbols. (p. 74,5)

Like Liebeck and the Do, Talk, Record model, Mason sees symbols as an end point; however here he does not just value the ability to read or recognise the symbol but the ability to use it in a meaningful way; "the symbols are not mere marks on the paper but indicate, or speak to entities that are almost palpable, almost substantial. When this happens, the symbols are no longer abstractly symbolic" (p. 74). Symbols are seen as an end point for the learning of a concept but also as the beginning of a new cycle of learning. Mastery of the symbol allows the learner to use it as a new object on which to act.

I have shown how Bruner's enactive, iconic, symbolic framework has been developed into a range of models of teaching and learning. Whether as a sequence, interconnected network, or spiral, each model contains some or all of the following elements: physical materials, spoken language, pictures, written symbols, and real world scripts, which will be discussed individually in more detail in the following sections.
2.3.2 Manipulable Materials

The use of manipulative materials as representations is a daily occurrence in many primary classrooms (Dufour-Janvier et al. 1987) much of it based on the work of Dienes (1964) relating to multidigit calculation. Dienes’ idea of ‘multiple embodiment’ asserted that if children experienced the same concept in a range of manipulable materials, for example place value encountered in base ten blocks, straws and bundles of ten straws, money etc., they would abstract the underlying concept.

However, while the manipulable materials have become commonplace in the teaching of mathematics especially in primary schools, the purposes of the materials have been questioned (Lesh et al. 1987b; Hall 1998). Boulton-Lewis (1998) observes that practical materials are “used by teachers in the belief that they will facilitate the construction, understanding and retrieval of mathematical concepts” (p. 220). She goes on to describe how work by herself and Halford (Halford & Boulton-Lewis 1992) concluded that manipulatives:

- reduce learning effort and serve as memory aids; provide a means of verifying the truth; increase flexibility of thinking; facilitate retrieval of information from memory; mediate transfer between tasks and situations; indirectly facilitate transition to higher levels of abstraction and can be used generatively to predict unknown facts. (Boulton-Lewis 1998, p. 221)

But she warns that this process is not automatic, that if the mathematical meaning behind the manipulative is not understood working with manipulatives can produce more, rather than less, cognitive load in the pupil, as they struggle to understand not just the concepts but the manipulative itself. For example, in the context of multidigit addition and subtraction, Dienes Multibase Arithmetic Blocks (MABs) were only of use when the children already had a good understanding of single digit addition and place value, without which the children’s understanding of multidigit number was made worse by instruction with MABs.

A further problem with manipulative materials is that they can result in situated learning, with children associating the learning with the materials and not generalising the concept. Dufour-Janvier, Bednarz, and Belanger (1987) comment that teachers “expect that the learner should perceive these conventional representations as mathematical tools” (p. 111) which can be selected and used as necessary. Instead they found that pupils would see tasks as specific to the representation used, and not be surprised if different representations produced different results for the same calculation. The task had become grounded in the materials. They conclude that it may be better to allow the children to develop their own representations rather than use adult structured ones.

Hall stresses the importance of developing links between the use of materials and language since “concrete representation systems have the pedagogical advantage of being easier to talk about, to describe and to analyse than language-based solutions” (p. 35). These links were
emphasised by Fuson and Briars (1990) who used base-ten blocks successfully with children by linking the manipulation with written symbols and much verbalisation to teach place value and the addition and subtraction of multidigit numbers. It would seem that the links between materials, language and symbols, advocated by Haylock and Cockburn above, are important for conceptual development.

Wood, Cobb and Yackel (1995) observe that “adults are able to recognize place value in the blocks simply because they have already constructed a relatively sophisticated understanding of place-value numeration” (p. 405) and therefore assume that the children will ‘see’ it too. However, they acknowledge the usefulness of material representations “provided we reconceptualize their function in the social situation of the classroom. In particular we stress that the ways these materials are interpreted and acted on must necessarily be negotiated by the teacher and students” (p. 406).

As I reported earlier, most of the literature on the use of manipulative materials in primary schools focuses on the use of multibase materials. I found no discussion of the use of single unit manipulable materials. Since the children in my research concentrated mainly on single digit addition, the use of practical materials may be different. Multibase materials require interpretation to link them to the place value structure of number; is this also true of manipulable materials with a one-to-one numerical relationship, such as bricks and counters?

### 2.3.3 Pictures

Although pictures are commonly used to represent mathematics in primary schools, both as a way of recording mathematics and as a way to present work to children, there is little discussion of the use of pictures in the mathematics literature. (Voigt 1998) discusses the ambiguity of arithmetic presented in pictures since the picture can be interpreted in a range of different ways. In particular, the picture is static, while the mathematics involves a state of change. He argues that work presented to children in the form of pictures, requires negotiation of interpretation.

Harries and Sutherland (1999) compared the way representations of mathematics were used in the textbooks aimed at primary school children, across a range of different countries. They concluded that in Singapore, Hungary and France, representations were used “to draw attention to mathematical structure, mathematical objects and mathematical processes” (p. 63) and colour was used in pictures “as an analytical tool to support learning” (p. 64). In English and American texts coloured pictures served to make the text more attractive, but often drew attention away from the mathematics.

Consideration of the way that pictures are used in the teaching of addition to young children, analysed through this study, may add to this limited range of research.
2.3.4 Spoken Language

Kaput (1987) describes spoken language as the representation that is learned "virtually from birth and thus the means by which most other symbol systems are interpreted" (p.186). However this very 'naturalness' can deter from the realisation that the meaning of words must be negotiated (whether in terms of fit or through social interaction). Much of the work on the meaning of spoken language refers to literary and philosophical discussions of the use of language, since all speaking and listening must be seen in terms of ideas that are translated into language and language interpreted as ideas by the listener (Bruner 1983; Glasersfeld 1984; Goldin 1987). Others draw on the idea of signifier/signified to distinguish between the word itself and its meaning (Brown 1994; Steinbring 2000). While language has a wider use in the mathematics classroom as a means of developing understanding, as a means of developing language and social skills (Mercer 1995, from a socio-cultural perspective), and as a means of assessment (Brissenden 1988; Durkin & Shire 1991), in this section I have chosen to look primarily at language as a representation of mathematical concepts (a constructivist perspective).

Children begin to use the language of number before entering school (Durkin and Shire 1991; Fuson 1991) and also begin to learn about aspects of arithmetic in everyday contexts, but rarely use formal mathematical language to describe these. Indeed the use of formal mathematical language in test situations has been shown to underestimate young children's understanding of the mathematics (Donaldson 1978; Davis 1991). Even when they have been in school for some time pupils will tend to use informal language when describing their mathematics and one of the roles of the teacher is to model and support the use of more formal mathematical language (Liebeck 1984; Pimm 1987).

A complication of mathematical language is that it uses everyday words for specific mathematical meanings. Consider for example the meanings of 'a number smaller than 8' (8?), the difference between 8 and 4 (eight is curly and 4 is straight) and the request to produce a table of your findings (coffee table?), while it also uses single words or phrases (for example add) to describe a wide range of contexts (Haylock and Cockburn 1997). In Mercer's terms mathematics has a discourse of its own, which the children must learn (Mercer 1995). From a social practice perspective, Adler describes acquisition of this discourse as learning to 'talk within' the community of practice (Adler 1998). So, mathematical language must be considered in the context of other representations and contexts in which the mathematics occurs.

2.3.5 Written symbols

The use of signs and symbols is crucial to mathematics for, as Gelman and Gallistel (1978) observe, "formal arithmetic as such is a symbolic game. Certain symbols - whose meaning, denotation, or reference need not be considered at all - are manipulated according to certain
rules - the laws of arithmetic” (p. 192). Mathematics educators might argue the extent to which the ‘meaning, denotation or reference’ needs to be considered, but it is possible to manipulate symbols without reference to their meaning. The introduction and use of written symbols is a crucial issue for teachers working with younger children (Whitebread 1995; Williams 1996).

That children have difficulty in understanding mathematics symbols has been shown in the work of Martin Hughes (1986) who found that preschool children had a good command of early mathematical concepts but many were not able to use mathematical symbols in a meaningful way, although they could use pictures and tallies to record quantities. Even the school children he studied did not use arithmetic signs (+, -, =) to indicate changes in quantity, though they were familiar with these in the context of school arithmetic. More recently, Munn (1997) found that preschool children needed both an understanding of the mathematical concept and of the concept of using symbols to communicate, before they could use symbols. By the time the children reached this stage of understanding they were usually familiar enough with standard numerals, without having to be taught them explicitly.

Children entering primary school have to understand both the symbols of writing and of mathematics (Price 1999). At the same time as they are learning about the concept of number, they must also learn the spoken work ‘three’ can be written both as ‘t h r e e’ and as 3, and many writers note the problem that this can cause. Moreover, the symbol on its own is insufficient for meaning since its size, position, relationship to other numbers etc. influence its contextual meaning; consider the meaning of 3 in the following: 3, 32, 1/3, 3a, 4^3 (Skemp 1971). Mathematical symbols are more difficult to understand than spoken language, being a more condensed form of language, a kind of ‘verbal shorthand’ (Atkinson 1992; Pimm 1995). The form of the symbol bears no resemblance to the meaning (Bruner 1966). Nunes speaks of the compression of symbols both in their appearance, for example many children will initially represent 123 as 100203, and in their meaning, for example the cardinality of 5 as representing the whole set (Nunes 1997). It is this ‘compression of meaning’ that allows the symbol to be manipulated as an object without consideration of the context (Mason 1987). Glasersfeld observes that “any sound or mark on paper becomes a symbol only when it is deliberately associated with a conceptual meaning” (1994, p. 5), so lack of understanding of the relationship between the surface and deep structure of symbols can lead to children carrying out the ‘manipulation of empty symbols’ (Skemp 1989).

Several writers discuss the progression from informal spoken language to formal written symbols. Pimm (1991) describes how this progression can be developed, either through the development of more formal spoken language (as in the Open University: Do; Talk; Record model), or through the development of informal written recording (Atkinson 1992; Whitebread 1995). I see these developments as alternate tracks from informal language to formal symbolisation, as shown in the following diagram:
2.3.6 Real World Scripts

Real world scripts are probably the most difficult category of representations to identify, since there is much argument over the 'real world' element of any school mathematics. In the context of problem solving Lesh, Post and Behr (1987a) use the expression 'real scripts' to identify those "in which knowledge is organized around 'real world' events that serve as general contexts for interpreting and solving other kinds of problem situations" (p.33). Verschaffel and De Corte (1997) summarise a considerable body of research into different kinds of word problems and their relative complexities as children try to identify the mathematics within the context. In their summary, the emphasis is on the problems children have in interpreting the language of the problems into mathematics. Little consideration is given in the research to the relevance of the word problems to the children's lives.

Greer and Harel (1998) comment on the lack of transfer between abstract and contextualised tasks which are structurally related, quoting Johnson-Laird, Legrenzi and Legrenzi's research (1972). Johnson-Laird et al. found adults were less successful in solving an abstract task than a task containing the same mathematics calculation, but written in the context of working in a post office. But, they found a difference in success between those adults for whom the post office situation was familiar, and those for whom, because of youth or experience of a different post office system, the task was still outside their experience. Johnson-Laird et al. concluded that real world contexts will only enable understanding of the task if the contexts are familiar, social contexts for an individual.

This work has parallels with that of Donaldson and colleagues (Donaldson 1978), who talk of the need for the task to make 'human sense' for young children. Chassapis' study of Greek elementary school textbooks (Chassapis 1997) found that most contextual mathematics problems could be defined as "artificially and socially indefinite situations coming out of a supposedly child-like and ostensibly abstract natural or social world" (p. 25) or "financial, and especially commercial, situations devoid of any pertinent social relationships" (p. 26). Neither of these categories appear to relate to the 'real world' of the children.

Finally, Cooper and Dunne (1998) analysing differences in the way children answered word problem questions in National Curriculum mathematics tests, show how word problems that
are relevant to the lives of some children create difficulties for children from a different social class. They found that the difficulty arose because the 'working class' children were more likely to make social sense of the situation, while 'service class' children were more likely to consider the task a mathematical one and ignore its social sense. If the social sense and the mathematical sense did not agree, then the 'working class' children failed to reach the correct answer.

Word problems relating to the 'real world' have generally been seen as ways of applying already learnt concepts, yet Donaldson, and Cooper and Dunne, have shown this to be problematic. If teachers teach with little reference to the real world, is it reasonable to expect children to be able to apply their knowledge? Social practice theory suggests that transfer of knowledge from one social practice to another is difficult; knowledge is situated in practice. Researchers from the Freudenthal Institute (Streefland 1991) in the Netherlands have worked with the idea of introducing mathematical concepts through “activities in which they move back and forth between the real world and the world of symbols” (van den Brink 1991, p. 83). They use the terms horizontal and vertical mathematization to distinguish between the mathematization of the real world (horizontal) and developing reasoning within mathematics (vertical). Real world activities require the real world of the child, rather than the adult world into which they are growing, and van den Brink concludes that

by taking children’s playing and creative potential into account and making use of it in education, learning addition and subtraction can take place faster and with more insight. (...) Arithmetic, in this design, need not be a dry, isolated activity but can become an integrated part of the children’s life. (p. 91)

The use of real world scripts as context for mathematics is therefore a contentious issue. Several issues have been discussed, raising the following questions.

- Can children be expected to apply mathematics taught in an abstract way in the classroom, to real world problems outside?

- Are the contexts of real world problems relevant to the lives of the children?

- Are the contexts of real world problems relevant to the lives of all the children taking into account social class, gender and ethnicity?

Most of the research in this area has been on the application of mathematics in real world contexts and with older children than those in my study; there is little discussion, in the literature, of the use of real world scripts in the teaching of mathematics to young children. The findings of this study, in particular Chapter Eight, discuss the use of real world scripts in teaching young children mathematics.
2.3.7 Summary of literature review on representations

This section has looked at the use of representations in the teaching and learning of mathematics. Work in defining representations, on models of representations emphasising interaction between them, and on individual representations has been considered. All representations are found to be problematic; teachers may assume that the children will 'see' the underlying concept within the representation, while the representation may remain opaque for the child; the child may see only the surface structure and not the deep mathematical structure. While the literature is diverse, emerging from a range of different theoretical perspectives, it shows the importance of how mathematics is represented in the classroom by both teacher and pupils, for pupils' learning.

I have therefore chosen the study of representations, and systems of representations, as a significant way of describing the teaching and learning of mathematics in this study. Consideration of the representations used by the teacher to teach, the children's reactions to these, and the way the children themselves use representations to explain and carry out mathematics, will offer essential insight into classroom actions and interactions.
2.4 Understanding the Mathematics of Addition

This section will set out the underlying theory of the development of children's concept of number and addition strategies. While the literature on learning addition forms a foundation for the research study, it is not a key area related to analysis of the data. The issues are little debated in the literature and little challenged by my previous teaching experience or by this current research. One of the reasons for the selection of early addition as the mathematical context for the research, was that the development of early addition strategies is clearly defined and agreed from a psychological perspective, identifying the stages through which children progress. Addition therefore offers me stable mathematical content on which to base a study of teaching and learning in mathematics.

**2.4.1 Preschool arithmetic**

While addition is the first area of mathematics to be taught explicitly in the primary school, most children come to school with a understanding of numbers, of quantity and of the use of counting to ascertain the number of a small set of objects. Many children also have elementary addition strategies, which allow them to add small numbers to an existing set by counting. These form the foundations of the development of a concept of addition taught in the early years of schooling. So, when we come to teach children addition we do not start with a clean slate.

Wynn (1998) has shown how children as young as 5 months show number awareness. They are able to subitise (recognise without counting) quantities up to three, and to recognise when they find there are more or fewer objects than they expected to see. In their second year they begin to learn the counting words both in context (two biscuits) and as a rhyme of sounds (one, two, three, four, five...). Gelman and Gallistel (1978) describe the counting strategies of preschool children and the complexity of: saying the number words in the correct order, matching one word to one object, keeping track of which objects have already been counted, knowing that the order in which the objects are counted is not important and knowing that the final count word represents the set.

Hughes (1986) and others (Carpenter, T. et al. 1983) have shown how preschool children are often able to add small numbers together by counting, especially if the objects can be seen or represented in some way, for example with bricks or fingers. But it must be noted that Rittle-Johnson and Siegler (1998) describe simple addition skills as a characteristic of 'middle-class' American children, and schooling in America typically starts a year later than in Britain. Addition strategies cannot be assumed of all children starting school in Britain.
2.4.2 Early arithmetic in school

Drawing on work by Carpenter, Romberg and Moser (Carpenter and Moser 1983; Carpenter et al. 1992), Nunes and Bryant (1996) summarise the stages that children move through as they develop addition strategies. The first of these, and the most naive, involves counting out each set (e.g. 1,2,3, then 1,2,3,4,5) combining the sets and counting all of the new, larger set (1,2,3,4,5,6,7,8), and is referred to as counting-all (or the sum procedure). With practice, children develop in their understanding and confidence of number so that they move from needing real concrete experience of the things to be counted, to representing them with other objects (including fingers), to being able to visualise them and point to the imagined objects in order to count them and finally to imagining them in their heads and counting what they can 'see'.

At some stage the children realise that they do not need to recount the first set of objects, but can count on from this set (3...4, 5, 6, 7, 8). Initially they will often count on from the number of the first set but eventually realise that it does not matter which set they start with (addition being commutative) and will start to count on from the larger number (5...6, 7, 8 - known also as the 'min' procedure). Baroody and Ginsberg (1986) showed that this counting-on strategy might be 'invented' by children, while Fuson and Fuson (1992) developed ways to teach it. At the same time children begin to learn some of the number facts, especially the double facts (e.g. 5 + 5 are always 10) (Gray 1997), and then to use these facts to derive others (if 5 + 5 are 10 then 5 + 6 are 11) (Thompson 1997). Further developments in addition strategies require an understanding of place value to solve multidigit addition (Nunes and Bryant 1996).

These addition strategies can be summarised as follows:

- counting all;
- using apparatus;
- representing unknown on fingers or something else;
- unknown imagined visually with pointing to count;
- unknown imagined in head; leading to
  - counting on from first (with similar representations as above)
  - counting on from highest;
  - knowing the answer;
  - working out the answer from known facts;
  - using knowledge of place value to solve addition of larger numbers.

These stages can be seen from a Piagetian perspective as developmental; however children do not all develop along a similar path. Resnick and Ford (1981, p. 54) note that identification of hierarchies of learning should not indicate "necessary or optimal sequences of instruction" since...
children learn specific skills in different orders and some miss out stages altogether. In my own experience, some children will have mental images of the numbers to count while still using a counting-all strategy, while others will have developed a counting-on strategy while still relying on fingers to represent the second addend. Some may discover commutativity before the counting-on procedure and therefore go straight from counting-all to counting-on from highest. Different children remember the number facts at different rates and therefore may know some answers, for example, $2 + 2$ is $4$, before learning to count-on.

In learning addition, children not only learn increasingly effective strategies for calculating, but also develop an understanding of the uses of addition. They must be able to recognise situations where addition is the required calculation, which is not only when combining two sets of objects (3 marbles and 5 marbles are 8 marbles altogether) but also situations of equivalence (3 cm of string and 5 cm of string are together the same length as 8 cm of string), of change (I had three marbles and someone gave me five more, now I have 8) and of growth (my plant was 3 cm tall, it grew 5 cm taller and is now 8 cm tall (Lesh et al. 1982; Carpenter and Moser 1983).

Addition of single digit numbers is only the start of addition but its importance cannot be overstated. Addition of larger numbers is dependent on an understanding of single digit addition and of the numeration system (place value). Nunes (cited in Nunes and Bryant 1996) found that such understanding of place value was also dependent on the development of addition skills - the ability to use counting-on seems necessary to understand the additive composition of ‘tens and units’. Addition of multidigit numbers is beyond the scope of this study.

As with the literature on representations, the research on addition has mainly been conducted within a cognitive/constructivist framework. A study of children’s learning within the classroom may provide opportunities to consider alternative perspectives.

2.5 Summary

This chapter has summarised and analysed literature which forms the basis for the thesis, and on which it will build. It has addressed underlying theories of learning in mathematics education, research into the use of representations in mathematics, and the development of children’s understanding of addition. Chapter Three will discuss literature relating more generally to teaching and learning in the primary classroom.
Chapter Three

Teaching and Learning Mathematics in Primary School

3.1 Introduction

Chapter Two has situated the study in the literature relating to learning, to learning mathematics and, in particular, to learning addition. This chapter sets out to situate the study in the primary classroom. It considers the historical background to primary education, and looks at the way theories of learning and political ideologies have influenced education in primary schools. Finally it looks at the way that wider influences on primary education have influenced mathematics teaching. Throughout there will be particular reference to the early years of schooling.

3.2 English Primary Schools

This section will look at primary school teaching first from a historical perspective and then at the current state of teaching in English primary schools. It aims to set the teaching of mathematics in the wider context of primary teaching, and considers the influence of theories of cognition, especially constructivist and Piagetian views, on this phase of education.

3.2.1 Historical perspective

The history of primary education since its development from elementary schools in the 1930s to the present day is one of apparent conflict between traditional and progressive educational ideologies. The contrast is greatest between instrumental teaching in elementary schools at the beginning of the twentieth century (based on a transmission model of teaching and learning) and the progressive innovations of the 1960s and early 70s. The former could be recognised in the classroom by rows of desks facing the teacher at the front, whole class teaching with an emphasis on the rote learning of facts and skills, and frequent testing to ensure retention. Progressive methods were based on a child-centred approach (Walkerdine 1984). The classroom was reorganised, so that areas of the room could be used for different purposes, with tables grouped for small group work. The teacher moved around the classroom, or sat with a group while other children work independently. Different groups of children would often be working on different aspects of the curriculum at the same time. The emphasis was on
play and active learning, and on understanding, as the Nuffield Mathematics Scheme showed in its motto: *I do and I understand* (Nuffield Foundation 1967).

However such a contrast is a misreading of history, for it is clear that a child-centred approach to the primary curriculum was evident in the Hadow Report of 1931 (Hadow Report 1931) which reformed the elementary system into the primary/secondary system. The report argued against prescriptive methods, designed:

> to secure that children acquired a minimum standard of proficiency in reading, writing and arithmetic, subjects in which their attainments were assessed annually by quantitative standards" (Introduction)

and asserted that primary schooling should be seen as a phase in its own right, where "the curriculum is to be thought of in terms of activity and experience rather than of knowledge to be acquired and facts to be stored" (Hadow Report 1933, p. 121).

The tensions between 'starting from the child' and 'preparing the child for the future demands of school and employment' are already evident in a subsequent 'Handbook of Suggestions for Teachers' published by the Board for Education in 1937 but the authors are clearly on the side of the child, arguing that "there is every reason why the aim of the Junior School should be set out in terms of the nature of its pupils rather than exclusively in terms of subjects and standards of achievement" (Board for Education 1937, p. 100).

At the time of the Hadow reports, educationalists were already influenced by Piaget's work on developmental psychology, Froebel's (1782-1852) work on spontaneous play and Montessori's (1870-1952) work with structured didactic equipment (as well as the work of Issacs, McMillan and Steiner). The influence is much clearer in the Plowden report (1967) which is more generally held to have introduced progressive methods into primary education.

The Plowden report summarises its philosophy of education by stating:

> School is not merely a teaching shop, it must transmit values and attitudes. It is a community in which children learn to live first and foremost as children and not as future adults. In family life children learn to live with people of all ages. The school sets out deliberately to devise the right environment for children, to allow them to be themselves and to develop in the way and at the pace appropriate to them. It tries to equalise opportunities and to compensate for handicaps. It lays special stress on individual discovery, on first-hand experience and on opportunities for creative work. It insists that knowledge does not fall into neatly separate compartments and that work and play are not opposite but complementary. A child brought up in such an atmosphere at all stages of his education has some hope of becoming a balanced and mature adult and of being able to live in, to contribute to, and to look critically at the society of which he forms a part. Not all primary schools correspond to this picture, but it does represent a general and quickening trend. (pp. 187-8)

The ideas inherent in the Plowden report, and implemented in many primary schools, centre around expressions such as child centred, individual discovery, active learning, learning through play, etc. The report highlighted the need for an integrated curriculum and many
schools also introduced an integrated day where there were no clear lessons and not all the children were working on the same subject at the same time. Such ‘progressive’ ideas are summarised by Blenkin and Kelly (1981) as “those of development, growth, activity, individualism and, in general, child centeredness” (p. 33).

The child as an individual, with individual needs and interests, is central to the Plowden Report which recognised that it built on the ideas of the Hadow report concluding:

that the Hadow emphasis on the individual was right though we would wish to take it further. Whatever form of organisation is adopted, teachers will have to adapt their methods to individuals within the class or school. Only in this way can the needs of gifted and slow learning children and all those between the extremes be met (Plowden, 1967, para. 1229).

Blenkin and Kelly note that “the theory and practice of primary schooling has been greatly influenced by the work of those developmental psychologists who have been concerned to persuade us of the necessity of fitting our educational provision to the developmental level of the individual child” (1981, p. 15). So, we have a primary school system based on Piagetian developmental theory despite the fact that as I noted above, such theory was not directly applicable to teaching. The approach emphasises teaching the individual, with little recognition of the role of culture and community in the education process.

There is considerable evidence that many primary schools did not adopt progressive teaching methods wholeheartedly. The DES report ‘Primary Schooling in England: a survey by HMI’ (DES 1978) observed that progressivism was not widespread in schools. Alexander (1984) agrees, describing the gap between the progressive methods that are thought to be used and what actually happens as “the myth of progressivism” (p. 11). Studies from a sociological perspective, showed that teachers were often unable to change their practice from traditionalism to progressivism, demonstrating a ‘reality’ of traditionalism despite a ‘rhetoric’ of progressivism (King 1978). Classrooms would be reorganised, and children sitting in groups, and an emphasis on individual learning, and yet many teachers held to a transmission model of learning. Pluckrose (1987) speaks of the perceived polarisation of traditional and child-centred (progressive) teaching yet notes that “teachers are rarely confident or certain enough to present themselves as ‘progressives’ or ‘traditionalists’” (p.54).

In 1976, Neville Bennett (1976) published research showing that progressive, activity orientated learning methods, used well with clear goals and organisation, were more effective than formal whole class teaching methods. However Blenkin and Kelly (1981) observe that many press reports of the study claimed that children did better with formal teaching, reflecting public disquiet with ‘progressive’ teaching methods, and a perceived lack of skills in school leavers. As a result, Prime Minister Callahan’s speech at Ruskin College, Oxford on 18th November of the same year called for a public debate on education. The DES report that followed, ‘Education in Schools: A Consultative Document’ (DES 1977), observed that:
The Speech was made against a background of strongly critical comments in the Press and elsewhere on educational standards. Children’s standards of performance in their school work were said to have declined. The curriculum, it was argued, paid too little attention to the basic skills of reading, writing and arithmetic, and was overloaded with fringe subjects. ... Underlying all this was the feeling that the educational system was out of touch with the fundamental need for Britain to survive economically in a highly competitive world through the efficiency of its industry and commerce. (Introduction 1.2)

Blenkin and Kelly admit that much of this criticism is fair, there was a gap between the world of the school and the world of work, and lack of awareness about the needs of industry, but there was no evidence of a decline in standards. The report called for an understanding of the ways that pupils progress, ways for identifying progress, clarification of progression across classes, coherence between the curriculum of individual schools, and emphasis on literacy and numeracy. The DES report stressed standards in primary schools, reminiscent of the traditional “minimum standard of proficiency in reading, writing and arithmetic, subjects in which their attainments were assessed annually”, which the Hadow report had tried to abandon, and in contrast to Plowden’s view of school as a “community in which children learn to live first and foremost as children and not as future adults”.

In 1981 Blenkin and Kelly could still talk of “the relative weakness of external constraints on the curriculum of the primary school” (1981, p. 15), but the debate was opened and the resultant Education Reform Act (1988) followed by the introduction, and several revisions, of the National Curriculum (1989, 1991, 1995, 2000) and Key Stage Statutory Assessment tasks are now more recent history. The primary curriculum, far from being integrated, fractured into ten subject areas defined by the secondary school curriculum, with external government control.

Thus the model for teaching has, over the past century, moved from one based on the passing on of knowledge (transmission), to one based on the developmental needs of the child (child centred progressivism), to one based on the perceived needs of society (utilitarian). What effect has this had on current primary school teaching?

The issues highlighted in this section, concerning the relationship between an understanding of children’s mathematical development, an understanding of the curriculum and teaching and learning methods will be addressed in the findings of this study (Chapter Seven).

3.2.2 Current Issues

3.2.2 Current Issues

I have shown how trends in primary education have swung between an emphasis on child-centred progressive approaches (allowing children to be children) and the more utilitarian demands of standards and testing (preparing children for the world of work). Anning (1997) summarises well the current situation for the early primary teacher:
Thus infant teachers in the 1990s find themselves caught between the relentless currents of child-centred progressivism and utilitarian demands to teach the basic skills. Marooned in such an uncomfortable position, it will not be surprising if they feel vulnerable and confused as to how they should set about reaching the shore. Their uncertainty has been further compounded by legislation which prescribed National Curriculum core and foundation subject content and assessment procedures for 5-7-year-olds mainly on the basis of a secondary school curriculum system backed down into infant classrooms. (p. 19)

She goes on to show how this can result in conflict for the teacher; “on the one hand teachers see their role as being responsible for teaching children the kind of knowledge that is deemed desirable by society and, on the other, they see their role as guiding children through a voyage of discovery towards their own personal knowledge” (p. 50). Teachers are torn between what they perceive to be the needs of the children and what they are told to be the needs of society. This was my own position as a teacher in the early 1990s, and I can remember feeling angry that there was no one to recognise the dilemma in which I found myself. Just as a previous generation of teachers found it difficult to reconcile progressive teaching methods with their own traditionalist view of teaching, so my generation had to reconcile utilitarian methods with child centred views.

While I do not believe that this study fully answers this question, some of the conflicts are addressed in Chapter Nine, and in the implications section of the final chapter.

3.2.3 Educating three to five year olds

Consideration of teaching and learning in primary schools must also give consideration to the youngest children in school. In 1870 the Houses of Parliament debated the starting age of children for compulsory schooling and decided that despite the claim that “five was a tender age for compulsory attendance” (quoted in Whitbread 1972), “the difficulty was to obtain education without encroaching on the time for gaining a living; beginning early and ending early would present a solution” (National Education Union 1870). Thus the age for starting school was set at five, younger than in most other countries across the world, for economic rather than educational reasons. In the elementary schools of the nineteenth century, children as young as three would often be present experiencing a formal drill and skill curriculum. The Hadow report identified clearly defined stages of schooling with primary schools serving children from the age of five and younger children attending nursery schools or classes, with emphasis on play and social development, though provision was not compulsory and tended to be patchy across the country.

Since the introduction of the National Curriculum in 1989, children entering primary school early have increasingly been introduced to a more formal curriculum. The National Curriculum (NC) became a legal requirement for all children of statutory school age (the term following
their fifth birthday), while the Nursery Desirable Outcomes for Children's Learning introduced in 1996 (NDOs, SCAA 1996) identified what children leaving nursery school should be able to achieve. Since the introduction of the NDOs also heralded the issuing of Nursery vouchers, entitling the payment of money from the government to early years settings, increasingly children were admitted to mainstream schooling at the age of four. Teachers in Nursery and reception classes were unsure of the appropriate curriculum for four and five year olds, in some cases applying the NDOs until the child's fifth birthday and then moving to the NC. The recent publication of the Early Learning Goals (ELGs, QCA 1999), to be implemented in the 2000-2001 academic year, has clarified the curriculum to be covered by children aged three to five, establishing a foundation stage of learning, but at the beginning of this study the situation was far from clear. While there has been some research into the experiences of young children in formal school settings there has been little research into the effect of starting school at the age of four on their development of mathematical understanding.

3.3 Learning Mathematics in Primary Schools

3.3.1 Background

While the study of education in primary schools shows a history of attempts to change primary education during the twentieth century, the effect of these changes on mathematics education is less clear. Hilary Shuard summed up the first half of the century saying:

Two important aspects of the primary school curriculum remained virtually unchanged until about five years ago - science and mathematics. Each, in its own field, has often suffered from rigidity of approach, from being divorced from the happenings of everyday life and consequently from lack of vitality (DES 1969, p. 2).

The 'until five years ago' referred to the introduction of the 'new maths' curriculum with its emphasis on logic and sets, statistical ideas and modelling mathematics with practical apparatus. Brown (1999) maintains that the 'new maths' curriculum resulted in significant changes in the way some teachers worked, supported by inservice training, innovative teaching projects, such as the Nuffield Mathematics Teaching Project, and the creation of LEA mathematics teacher's centres. However the momentum was not maintained, the Nuffield project reluctantly published its own mathematics scheme and "translated into text, some of the practical and investigatory spirit was inevitably lost" (Brown 1999, p. 7).

While the Nuffield scheme resulted in a broader mathematics curriculum with an emphasis on understanding and less time spent on the practice of algorithms, Brown observes that many teachers "chose to stick closely to the books, and often let children work through them on their own". This she attributes both to teacher's lack of confidence in mathematics, and in particular
the modern mathematics curriculum, and to the importance of pupil autonomy encouraged by the Plowden report.

In the early 1980s teachers were still finding the teaching of mathematics challenging. Blenkin and Kelly (1981), discussing the integrated approach to the primary curriculum encouraged by Plowden, observed that problems arose "when teachers feel that their own understanding is particularly weak (as is the case with mathematics)" (p. 183). Such teachers found it hard to integrate mathematics and it became isolated from the rest of the curriculum. Continued concerns from government and industry about standards in mathematics, initiated the Cockcroft Report (1982). For the youngest children in school, Cockcroft emphasised the importance of practical work and of language. While the emphasis on practical work and the discussion of children moving through stages of progression at different rates (para. 289) indicate a Piagetian view of learning, the underlying ideas of how children learn are mixed. A primary head teacher who submitted evidence to the committee is quoted as saying that "there is a need for more talking time ... ideas and findings are passed on through language and developed through discussion for it is this discussion after the activity that finally sees the point home" (para. 306, italics mine). The role of language is seen in terms of transmission, rather than supporting children's own construction (constructivism), or providing an environment, community or culture in which learning occurs (socio-cultural, social practice theories).

The report, with its emphasis on the use of a range of teaching styles, has been a significant influence on mathematics teaching as a profession, but I would argue that it effected little change at the primary level. Six years after its publication Hilary Shuard noted that:

It is still the case that the major model of teaching employed in mathematics classes is that of transmission. The teacher has mathematical knowledge and his or her task is to convey it to the pupils (Shuard 1988, p. 146).

During this time, Desforges and Cockburn's study (1987) aimed "to try and understand teachers in the way they taught mathematics" (p. 56). They found that the main emphasis was on workcards or workbooks which the children carried out with "pleasure and industry" (p. 57) but which emphasised set procedures, even where the children had other strategies with which they were more confident. Many of the teachers studied did little direct mathematics teaching. They administered worksheets, marked work and helped individual children with problems. Practical work was limited and teachers lacked confidence in whole class discussions. The children in these classes often spent long time queuing to see the teacher. There was little evidence of the influences of the Cockcroft report. Desforges and Cockburn concluded that the complexity of the role of primary teacher was underestimated, and recommended a slimming down of the curriculum to allow for more depth of study to develop children's understanding.
However the stage was already set for the 1989 National Curriculum which, far from slimming down the curriculum, made statutory the teaching of ten National Curriculum subjects throughout the primary school.

The underlying messages about learning are mixed in the National Curriculum too. The attainment targets with their multi-levels through which children are expected to progress remind me again of Piaget's developmental stage theory. Yet the Non-statutory Guidelines which accompany them, point out that mathematics should not be seen as a linear process but that progression "is to do with the way in which teachers explore, make sense of and construct pathways through a network of ideas (N.C.C. 1989, b C2). Isaacson (1992) contrasts the creativeness of the non-statutory guidance with the utilitarianism of the statements of attainment, as if creative maths is only a means to a more utilitarian end. The guidelines also speak of mathematics as "a way of viewing and making sense of the world. It is used to analyse and communicate information and ideas and to tackle a range of practical tasks and real life problems" (2.1:A2), as if in response to Alexander's observation, written five years earlier, that "few subjects are less like the way children subjectively make sense of experience, or are more 'artificial' than mathematics: the 'truths' appealed to are beyond the child, absolute and unchanging..." (Alexander 1984, p. 67).

But these views of mathematics as 'making sense of the world' and of progression as 'pathways' required primary teachers to have a sound understanding of both mathematics and children's learning in order to construct the pathways for the children to follow. Although the introduction of the National Curriculum surely did lead to changes in the teaching of mathematics in primary schools the changes were not immediately obvious. Anning sums this up well:

The NCC prescriptions sounded very fine, but the realities for teachers of being exhorted to teach mathematics through practical activities, and to base their interventions on a close understanding of the particular pathways to understanding each child might take, were complex and demanding. Extracts from teaching and learning episodes ... demonstrate that skilful teachers can and do cope with such complexities. The temptation to which many teachers succumb, however, is to rely on a maths scheme to keep children 'occupied', with each child working individually at their own pace (Anning 1997, p. 111).

Like the teachers in the 1960s and 70s, at the end of the Nuffield project innovations, and in Desforges and Cockburn's study, teachers resorted to the familiar security of individualised maths schemes, with little or no whole class or group teaching. Anning's summary is confirmed by a series of Ofsted reports following the introduction of the NC. The 1991-2 Report (Ofsted 1993a) found that : "the majority of schools based their course around commercial texts and found these supportive. The quality of work varied according to the way the teachers used these texts" (p. 19). They noted that good practice involved using the texts selectively supported by other materials while "the rigid adherence to one scheme did not adequately meet the range of mathematical ability in the year group or provide the content
required by the National Curriculum” (p. 20). On teaching styles they noted that “overall, too few classes achieved a good mix of whole class, small group and individual teaching”. “In KS1, teachers were more likely to introduce new work to a small group of pupils while in KS2 the pupils frequently learnt new topics largely on their own from written texts” (Ofsted 1993b, p. 9). More lessons at KS1 were considered to be satisfactory than at KS2.

3.3.2 Current issues in mathematics education

The state of primary mathematics teaching and achievement of pupils in mathematics tests is still a major educational issue at the time of writing. The time between the inception of this study and its reporting has seen the introduction of the National Numeracy Strategy. One of the major factors in this new government initiative was the results of tests used to compare achievement with other countries. The Numeracy Task Force consultation document ‘Numeracy Matters’ (Numeracy Task Force 1998) notes “studies comparing England’s performance in mathematics with other countries show this country to be performing relatively poorly...”, though the evidence for this is contested (Brown 1998). There is a tendency to find simplistic solutions for this perceived underachievement, e.g. whole class teaching or rote learning, harking back to dissatisfaction with Plowden and progressive teaching methods; although often these solutions are selected with little evidence that they are any more influential than less politically attractive factors such as higher pay for teachers, a greater proportion of ‘non-contact’ time or smaller pupil:teacher ratios.

As we have seen above, this concern with mathematical achievement is not new. As we move into the twenty-first century the importance of mathematical skills is unlikely to diminish.

Much of the recent discussion of mathematics teaching assumes that teaching mathematics to secondary pupils is the same as teaching primary mathematics and, within the primary phase, that teaching ten year olds is the same as five year olds. For example a study of children’s ability to read mathematics (1994) is summarised in the paper entitled ‘Reading to Learn Mathematics in the Primary Age Range’ yet studied children only in the age range 7 to 11 years. In contrast, studies of child development show that children develop and mature with age and I believe it naive to assume KS1 and KS2 age phases are the same, agreeing with the writers of the Hadow report that the infant phase (KS1) should be seen as distinct, a bridge between infancy and childhood. At the end of the previous section, I noted that Ofsted reported KS1 mathematics teaching as generally better than KS2, yet the supposed failure of teaching at KS2 and beyond has led to demands for changes at both key stages. In many of the countries used for comparative studies of achievement, children of 5 or 6 years of age are not yet in school. Whitburn (2000) describes how the young children in Japanese kindergartens are

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1 KS1 Key stage One, children aged 5 to 7 years, KS2 Key Stage Two, children aged 7-11 years.
being prepared for their social integration into school rather than beginning formal learning. Her comparative study shows that, while the Japanese children know less mathematics than British children at the age of 6, they soon overtake them in achievement. It cannot therefore be right to assume that teaching and learning methods used with older children in such countries can be automatically used as a model for teaching our young children successfully.

3.3.3 Preschool Mathematics

Before the introduction of the Nursery Desirable Outcomes (SCAA 1996), preschool mathematics emphasised 'pre-number skills': sorting, ordering, matching, and comparing, with number rhymes and games providing opportunities for learning the early number names in order (Thompson 1997). There was little discussion of larger numbers or of number operations. Aspects of shape, space and comparative measures were learnt through play and through real world activities such as cooking. However, the introduction of the Nursery Desirable Outcomes signalled the first standard mathematics targets for these young children, with the emphasis moving from pre-number skills to an expectation that the children will be taught number and arithmetic skills including the use of symbols. The curriculum can be seen as imposed from above in that the National Curriculum identified the primary mathematics required to underpin the secondary curriculum, while the NDOs identified the mathematics required to underpin the primary curriculum, with little consideration of what it was appropriate for the children to learn or the way that it should be introduced. Confident early years practitioners have been able to incorporate the curriculum requirements into their early learning ethos, while those less confident have tended to move to more formal teaching reminiscent of their own primary schooling.

This study of four to six year olds sets out to consider these questions about the appropriate curriculum for four to five year olds and the needs of young children learning mathematics.

3.4 Summary

The literature reviewed has raised many issues relating to the teaching and learning of mathematics in early primary classrooms. These issues are rooted in the history of primary schooling and of mathematics teaching and learning, and formed the impetus for this study of young children learning mathematics.

Chapter Four will identify the research question, discuss the methodology selected for the study and describe the preliminary study.
Chapter Four

The Preliminary Study, Discussion of Research Perspective and Methodology

4.1 Introduction

This chapter describes the preliminary study undertaken to clarify the research question and methods. It sets out the methodology and research design in detail as they applied to both the preliminary and the main study (Chapter Five will consider developments in methodology made in the light of the preliminary study). It describes the setting up and implementation of the preliminary study, analysis of the data collected, and the findings, and looks at the implications of these for the main study.

4.2 Nature of the Research Problem

Chapter Two discussed theories of cognition, representations of mathematics used to communicate mathematical ideas, and children's development of understanding in addition, while Chapter Three grounded the study in the context of primary education. Little of the evidence that supports the theories of cognition and development of understanding is based on studies of early years classrooms.

At the beginning of my research, the research question took the form:

What is the relationship between what the teacher teaches and what the child learns during mathematics lessons in primary classrooms?

At the beginning of my research the focus of the study was tentative. I was clear that there must be a relationship between teaching and learning, otherwise why do we teach, but I was unsure what I meant by relationship. As I come towards the end of the study I can re-articulate the question of teaching and learning relationships in different ways. What actions, language, interactions might define the relationship and how can it be described? Can the relationship be described in different ways at different levels of analysis - curriculum, discourse, scaffolding? However at the beginning of the study I was much less clear about the characteristics of these phenomena - teaching and learning. One of the main purposes for the preliminary study was to define the relationship more clearly.
4.3 The Research Perspective

4.3.1 The nature of methodological decisions

I can describe my methodological decision making in two ways. The first is to say that it was an obvious choice. The nature of the research question demanded a qualitative, ethnographic research method through observation. The choice was made for me as soon as I decided to look at the teaching and learning of mathematics. The second way is to describe the decision in terms of an understanding of my epistemological and ontological position (described below).

Yet essentially these are to say the same thing. While at the beginning my decision may have been instinctive, it was still based on epistemological and ontological understandings even if these were, at that stage, subconscious. The origins of these understandings are in my experience as a human being, as a learner, as a parent, as a teacher and as a teacher educator. The instinctive decision was based on my growing understanding of how children learn, the constructed nature of knowledge and mathematics, and the social interactive nature of education in terms of teaching and learning. Since I wished to understand these complex situations, it was necessary to look in detail at teachers and children in the classroom.

What I will set out here is therefore articulated in the language of philosophy and research as a development of, but not in opposition to, these previously inarticulate understandings. Three major areas influence methodological thinking in mathematics education research: ontological perspectives, epistemological perspectives, and ideas on the nature of mathematics itself. These are not clear cut categories but interrelate; each leading to implications for the others.

4.3.2 The effect of ontological perspectives on methodology

Ontology can be defined as the nature of being. The nature of the phenomena under consideration must first be identified before it can be studied. The ontological basis of my research is that the social phenomenon under discussion, the mathematics classroom, is not an objective reality; rather it is subjective, that which is experienced by the participants. This arises out of both a constructivist view of learning which is ontologically neutral and a social view of learning that sees 'reality' as shared and subjective experience. From both perspectives the social, educational and mathematical experience can only be known in the actions, reactions and behaviours of the participants, whether the 'knowing' is seen as an individual or shared construct. The methodology must therefore reflect this complexity and subjectivity.
4.3.3 The effect of epistemological perspectives on methodology

Epistemology is concerned with a theory of knowledge, and the process of coming to know. Having established a view of subjective experience, how can we come to know and understand such experience?

The theories of learning considered in Chapter Two see learning as an individual act and as social interaction within a community of practice. Ernest (1998) argues that constructivist theories of learning are probably the major influence on the development of qualitative research in mathematics education. Similarly, Cole and Scribner, in their introduction to Vygotsky’s Mind in Society (Vygotsky 1978), claim that a qualitative research methodology is an implication of a Vygotskian approach to understanding learning arguing that “detailed descriptions, based on careful observation, will constitute an important part of experimental findings” (p.14); while social practice theory, arising out of anthropology, adopts an ethnographic methodology.

I require a methodology that considers all three aspects, the individual learner, the social interaction between participants and the wider social context of the community. My position, based on my personal experience in the classroom, identifies teaching and learning as complex interactions between teacher, curriculum, process, child and context, with communication and language an essential part of this interaction.

A researcher holding this position on the nature of knowledge must also be aware that she has only her own perception on what is happening in the classroom; others may see it differently from their individual perspective and alternative ‘readings’ of the situation may be equally valid. This has implications for the way that the research is presented and claims for its validity, issues that will be discussed later (Chapter Five, section 5.7).

4.3.4 Beliefs in the nature of mathematics

Mathematics is just one form of knowledge identified in western society but it is deserving of separate consideration, since it is often seen in a different way from other forms of knowledge. It is easy to see art, literature and music as socially constructed since they differ between societies, but it is more difficult to see mathematics as socially constructed; after all there is only one answer to $24 \times 32$. However the answer is fixed only within the socially and logically developed framework which we call mathematics, and there may be many different ways in which that answer is reached. While a traditional view of mathematics is as absolute, logical and static, and looks for evidence of mathematical learning in correct answers and logical arguments, the social constructivist can see it as fallibilistic, creative and dynamic and look for evidence in actions and communication (Ernest 1991). From a fallibilistic perspective, mathematics is no longer seen as a product, with the aim of teaching being to produce the correct answer, but instead as a process of
coming to know that which more knowledgeable, more experienced mathematicians already know, a socio-cultural rather than a constructivist perspective. The selected methodology therefore requires a way to look at the process of mathematics teaching and learning, the ways that this knowledge comes together.

### 4.3.5 Summary of the research perspective

To summarise this section therefore, I believe the phenomena being researched are complex requiring knowledge of people and social interaction, characterised by phenomena of social situations, teaching and learning, and the nature of mathematics itself. It assumes that we can know something of the experience of others in three interrelated ways, each of which will influence methodology. The first is that to know what the participants are experiencing we need to be present with them in that experience, which requires research methods based on participation. The second is that we can know something of their experiences through observing their actions and behaviours, which requires research methods based primarily on observation, and identifies something about the nature of the data to be collected. The third is that we can understand something of their experiences through their communication and language, which further identifies the necessary data. While the details of methodology are still to be determined, the essential nature, a qualitative research perspective is established.

### 4.4 Methodological Implications

The research sets out to give an account of, and to interpret, ordinary, everyday classroom teaching and learning in early primary mathematics classes, where the children are learning the early stages of mathematics. Rather than change the situation in any way, I wanted to identify what was happening at present. As Jaworski observes “I needed to study teaching closely, but not to prescribe what I was looking for” (Jaworski 1994, p. 61).

Such research requires a qualitative research method which is ethnographic in style, and where the data is collected through close observation over a period of time. Hammersley and Atkinson describe how ethnography:

> "... involves the ethnographer participating, overtly or covertly, in people’s daily lives for an extended period of time, evaluating what happens, listening to what is said, asking questions - in fact collecting whatever data are available to throw light on the issues that are the focus of the research." (1995, p.1)

The research is ethnographic since it is concerned with close observation of the social interaction concerned with early mathematics teaching and learning, although it does not consider the school
as a society in its wider sense. The data from this research will be that seen through my own eyes since as Burgess, speaking of qualitative research in natural setting, observes that “in this context the main research instrument is the researcher who attempts to obtain a participant’s account of the situation under study.” (Burgess 1985, p.8)

The research arises from what Malinowski (1922) terms ‘foreshadowed problems,” rather than preconceived ideas. Hammersley and Atkinson expand on this to explain how:

“the aim of the prefieldwork phase and the early stages of data collection is to turn the foreshadowed problems into a set of questions to which an answer can be given, whether this be a narrative description of a sequence of events, a generalised account of the perspectives and practices of a particular group of actors, or a more abstract theoretical formulation (1995, p. 29).

Such research methodology is not without its critics. Cohen and Manion note that data collected through participant observation is often described as “subjective, biased, impressionistic, idiosyncratic and lacking in the quantifiable measures that are the hallmark of survey research and experimentation” (1994, p.129). Ethnography does not set out to be quantifiable and accounts will of necessity be subjective. The process of sharing such accounts with the teachers observed, discussing them with colleagues and submitting articles for peer review avoids some of the pitfalls of subjectivity, and the theory as it emerges is tested against new situations to provide validity (see Chapter Five, section 5.7 for further discussion of validity).

4.5 Research design and data collection

This section describes in more detail the methods used in both the preliminary and main studies including discussion of participant observation, research ethics, access and data collection.

4.5.1 Participant Observation

The role of the researcher was that of a participant observer, participant not in the sense of being involved in the everyday teaching and learning of the classroom but because I am a teacher educator with experience of primary teaching and a current role of observing student teachers. I question whether it is possible to be a truly non-participant observer in classroom research unless the researcher is not present in the classroom, agreeing with Rossman and Rallins (1997, p.163) that “no matter how unobtrusive you try to be, you cannot help but become part, however small, of the setting”. I did not attempt to maintain a high level of detachment, which King (1978) found especially difficult when observing young children. In describing his experience in infant classes he explains: “I rapidly learnt that children in an infant’s classroom define any adult as another
teacher or teacher-surrogate" (p. 4), and observed how difficult it was not to get involved with the children in order to maintain his observation role, devising strategies such as remaining standing so as to be on a different level to the children, not showing any interest or talking to them, avoiding eye contact and, at one point, hiding away in the ‘Wendy house’. While his aim was to remain as far as possible a non-participant observer, so that he would not influence the setting, I find it hard to understand how the children would find this less distracting than his participation!

My previous experience as an infant teacher meant that I would not feel uncomfortable in the situation; the children would be more likely to accept me as just another adult in the classroom. However, this familiarity would not necessarily be an advantage since it could mean that I could lose the role of researcher and get too involved in the day to day running of the classroom, which could further influence the situation which I was there to study. I was aware that it would not be possible to be present yet have no effect on the situation, so it would be necessary to consider the effect of my participation during the preliminary study.

4.5.2 Observing the Familiar

Observing that which is already familiar can be at the same time an advantage and a disadvantage. I already had a shared understanding both of the situation and of the language used, so there would be no element of the 'culture shock' (Hammersley and Atkinson 1995) found in more traditional ethnographic situations. But familiarity also has its pitfalls. Galton and Delamont found that with their researchers: "everyone ... had been to school, many of them had been teachers, and thus classrooms were 'so familiar'" (1985, p. 177). This could result in observations being restricted only to what is expected or only to what unusual in the situation. Research into classroom situations is peppered with discussions of the problems of observing in familiar settings (Atkinson 1984; Cummings 1985; Galton and Delamont 1985; Griffiths 1985; Measor et al. 1991; Hammersley and Atkinson 1995). Hammersley and Atkinson describe how it is "difficult to suspend ones preconceptions ...".

Also, entering into classroom research involves a change of role. So Measor and Woods, working on the Changing Schools project found that Lynda Measor

"had some difficulty initially. Her background was in training teachers, her experience of watching lessons from the back of the classroom, judging the performance of student teachers. This was something she had to stop doing. She had to 'wash her mind clean' of this and discover what the research was really looking for” (Measor and Woods 1991, p. 70).

which could also be a problem for me.
Atkinson (1984) summarises the problem for the researcher: "Ethnographers are currently confronted by problems of 'familiarity' and 'strangeness'. We seek to render the familiar strange and the strange familiar" (p.169). It would not be possible for me to carry out my intended study in any setting other than familiar classrooms, so I knew that I would need to work hard at identifying what was happening during mathematics lessons without bringing too much of my own experience and expectations to bear on observation and interpretation, to develop a critical edge to my observations and interpretations.

In fact, once I had focused my study mainly on the children I found that this problem was not as acute as I had feared. It was possible initially to get personally involved in the role of the teacher, but once I started to concentrate on the children I found that this was not familiar. A teacher with a class of 30 does not have time to watch her children in this way. The world of children in the classroom and their learning was indeed strange.

4.5.3 Access

The first goal of the ethnographer is to gain access to the area for research. Access is not a problem for some researchers since many social situations are open to the public; but schools and classrooms are not. In both the preliminary and the main phases of data collection, access to classes was negotiated through existing contacts. For the preliminary study I had previously worked in the school and knew the teachers and catchment area. These relationships allowed me to concentrate on the role of researcher without having to build up new relationships. However, the setting was perhaps too familiar (see above); it is harder to be objective with friends and colleagues, and would not be ideal for the main study. Also, the school was not a 'typical' school, having a much higher number of children with special needs and behavioural difficulties than most. For the main study I would need to consider the choice of schools more closely.

4.5.4 Data Collection

The main recording of data was through fieldnotes for, as Hammersley and Atkinson (1995, p. 175) note, "fieldnotes are the traditional means in ethnography for recording observational data". For the preliminary study some of the sessions were also recorded on audiotape but this proved difficult to transcribe. The children observed had quiet voices when working with the teacher and infant classrooms are very active places with considerable movement and noise. Those lessons that were audio-taped proved to provide no extra information to the field notes taken at the same time. Some documentation was also gathered, including school mathematics policies, teachers' lesson plans and examples of children's recordings, which supported the fieldnotes. Assessment
of the children's understanding of number and addition was carried out in an informal way as outlined below, using games and activities with individuals and groups of children, with fieldnotes used to record the data.

The unit of data was defined as a teaching episode. Sometimes that was the whole of the mathematics lesson and at other times only part. Many of the lessons would contain a whole class session, often involving mental and oral arithmetic, followed by the children, organised in groups, working on differentiated tasks. Sometimes the group work followed on from the whole class session in which case this lesson was seen as a single episode. At other times the subject of the two parts was different in which case they were treated as separate episodes. The two parts were not necessarily both related to 'addition' in which case notes were made of both but they were not always used in the analysis. Sometimes a teaching episode would not seem to be directly related to the teaching of addition but the subject matter would require the children to carry out addition tasks which gave further insight into their understanding of addition. For this reason fieldnotes were made of all the sessions observed and decisions about their relevance made afterwards.

For each teaching episode data was collected through:

- a prior discussion with the teacher on the intended learning outcomes of the lesson;
- observation of the teacher
  - what is being taught
  - how it is taught - actions and language;
- observation of the children
  - their involvement, participation and attention
  - what they say - language and meaning
  - what they do and record;
- individual discussions with the children about what they are doing;
- discussion with the teacher about her teaching and the children's learning.

Later, ideally straight after each lesson, the notes were edited while the episodes were still fresh in my mind, for as Hammersley and Atkinson observe:

"Even if it proves possible to make fairly extensive note in the field, they - like brief jottings - will need to be worked on, expanded on and developed... There is no advantage in observing social action over extended periods of time if inadequate time is allowed for the preparation of notes. The information will quickly trickle away, and the effort will be wasted" (1995, p. 178).
At this stage, the fieldnotes became more than just direct observations of the episodes and included comments from my own initial reflections. These comments were clearly marked, enclosed in square brackets, to identify them. Hammersley and Atkinson describe how:

"in ethnography the analysis of data is not a distinct stage of the research. In many ways, it begins in the prefieldwork phase, in the formulation and clarification of research problems, and continues through the process of writing reports, articles and books." (1995)

Initial reflections, while the episode is still fresh in the mind, are particularly valuable. As the distance in time between data collection and reflections increases, subsequent observations and background reading will produce different interpretations. Both immediate and subsequent reflections have their own but different values.

The intention of the initial discussion was to identify the mathematical content and structure of the episode so that I had some idea of what to expect. In the post-lesson discussion I would try to identify the teacher’s view of how the lesson had gone, whether there were aspects of it that had not progressed as expected or responses from the children that had surprised her. These were used both to give another point of view from my own and to confirm, or challenge, my observations.

Through analysis of the data I hoped to produce what Lutz describes as:

"a thick description of the interactive process involving the discovery of important and recurring variables in the society as they relate to one another, under specified conditions, and as they affect or produce certain results and outcomes in the society" (Lutz 1981, p. 119),

in the hope that such ‘important and recurring variables’ may help me to identify the factors affecting the relationship between teaching and learning addition. In the preliminary study my understanding of the way in which this analysis would occur was limited, with the intention that lessons would be learnt to use on the main study. Analysis of the observation data consisted of detailed reading and noting key issues that appeared to emerge from the data. On reflection this analysis was probably the least effective part of the preliminary study as a more rigorous analysis could have prepared me better for the analysis of the data in the main study. However the analysis did fulfi l the aims of the preliminary study, in highlighting key issues in the teaching and learning of mathematics that could be developed in the main study, and in highlighting the need for a clearer method of analysis for the main study.

**Assessment Activities**

In addition to the observations carried out in the classroom I carried out assessment activities with the children at the beginning and the end of the observation period. In the preliminary study the purpose of these assessments had been to see what the children could do at the end of the
observation period that they could not do at the start. It was hoped that the results could be compared with what they had been taught in order to identify a relationship.

The assessments comprised questions about the children's counting competences, their ability to read and write numerals, and understanding of simple addition and subtraction (±1 or 2). The children were assessed in groups of not more than five, informally either in the classroom or a part of the school where they were often withdrawn to work in small groups with another adult, so that they did not feel that they were being tested in a formal sense or treated in a different way from normal.

4.6 Description of the preliminary study

At this stage the research question was expressed as:

*What is the relationship between what the teacher teaches and what the child learns during mathematics lessons in the primary classroom?*

Four supplementary questions were used in the analysis of the data in order to further clarify the focus on teaching and learning. These were:

1. *What did the teacher teach?*
2. *What did the children learn?*
3. *Did the children learn what the teacher taught?*
4. *What other factors affected their learning?*

The preliminary study was undertaken in an attempt to clarify the research question and the methodology. The aims of the preliminary study were expressed as:

a) to identify key issues in the teaching and learning of early mathematics which would focus the main research;

b) to practice and refine the skills of data collection, in particular participant observation, and analysis.

To fulfil these aims I completed a period of participant observation in a reception classroom over a period of one term. I carried out an informal assessment of a group of ten children in Nursery school who were due to start school after Easter, observed these children in their first term in school and further assessed their learning at the end of term.
4.6.1 The setting

The preliminary study was carried out in a first school (4-9 years) on a city housing estate that I shall call Denton. The school comprised a Nursery class (4-5 yrs), a Reception class (5 yrs), two parallel year 1 and 2 classes (5-7 yrs), a year three class (7-8 yrs) and a year four class (8-9 yrs). Ten children formed the focus of my study as they moved from the nursery to the reception class: Gordon, Laura, Adam, Colin, Dan, Rickie, Len, John, Mary and Jane (not their real names). These ten children had birthdays between February and March and had all just turned five years old (the usual age for school entry in this local authority). They joined a reception class already containing eleven children; two who were Year One but had learning and/or behavioural problems, nine in their second term in school, these ten new children bringing the total to twenty-one.

The class teacher (Ann) had taught the reception class at the school for nearly five years. The school policy for mathematics was based on the HBJ Mathematics scheme (Kerslake et al. 1990), which has a topic based approach. Each half term the mathematics is focused around activities with a topic or theme. Many of these will have a ‘real world’ context. The teacher would choose the mathematical theme to fit with the class topic for the term, which would form the basis for their other subject areas - especially science, history and geography. This term the overall class topic was Growth (science), and the mathematics topic was based around Animals. The class teacher supplemented the mathematics scheme with additional activities for number, some of which were related to animals.

4.6.2 Duration

The research consisted of five visits to the nursery in the spring term, followed by twelve, weekly visits for observation in the reception class in the summer term 1996. I spent one day a week in the classroom observing the children doing mathematics and acting as a classroom assistant during the other lessons in the day. This allowed me to give something back to the class and teacher in return for allowing me to observe, as well as providing an opportunity to observe mathematics related activities happening outside of mathematics lessons. I also received detailed lesson plans from the class teacher of the mathematics that the children did when I was not there, since, as a part-time researcher, it was not possible to spend every day in the classroom.

4.6.3 Data collection

As identified in 4.5.4 above the data consisted of notes on unstructured discussions with the teacher before and after the lessons, detailed observation field notes of the twelve lessons observed, notes on some informal discussions with children about their work, lesson plans for
those lessons for which I was not present, and data from assessment activities carried out at the beginning and the end of the data collection period.

**Assessment data**

Data from the assessment activities carried out in the nursery formed the basis of my knowledge of the children's mathematical understanding. I spent two days in the Nursery class getting to know the children and then sat with each child individually for about half an hour asking questions and playing games. The children were asked to estimate the number of a small array of bricks, say the count words in order as far as they could go, count a collection of up to thirty objects, and read the numerals 0 to 10. A game similar to Martin Hughes' Tins Game (Hughes 1986) was played in which the children had to record the number of cubes in the plastic pot by drawing or writing something on a post-it label, in order to see if they could use symbols in a meaningful way to record quantity and whether they could read these symbols a short while later. Finally, they were asked to give the number one or two, more or less than a given number within the range 1 to 5. These activities were similar to assessments carried out by Aubrey (1993) as well as Hughes. The assessment activities were carried out again at the end of the observation period in the reception class.

**Observation data**

Evidence of teaching and learning in the reception class was then collected through participant observation. For the first two lessons I took paper and pen and tried to write down everything that I saw that related to mathematics. This allowed me to identify the kinds of issues on which I might wish to focus. Since I wanted to research teaching and learning, I kept detailed notes on episodes where the teacher was interacting with a group of children, keeping briefer notes when the children were working on individual tasks. In an attempt to structure the observations, I then developed an observation schedule based on a lesson plan format. This was not an attempt to carry out 'systematic observation' (Croll 1986), where the frequency of clearly defined variables is noted, but a structured way of ordering my observations. I soon discovered that this was not always useful as many of the 'lessons' did not have a formal 'introduction, activity, conclusion' structure. I therefore changed the schedule to make it more adaptable and this was used for the rest of the preliminary study. Even this form of recording was not ideal and I would often use the back of the sheet and supplementary paper for a more detailed explanation, sometimes with pictures, of the children's activity, and to record the children's successes and problems. An area at the bottom of the sheet entitled 'Comment' allowed me to note anything that occurred to me that was not a direct observation, and questions to ask the teacher at the end of the lesson. In this way the focus was
widened from the format of the lessons to as much of what was happening in the classroom as I was able to notice and record.

**Teacher interviews**

Talking to the teacher about the lessons was one of the most difficult aspects of the data collection. Primary school teachers are busy people, they have no 'free periods' and spend most of the non-teaching time (break, lunch, before and after school) sorting out children, tidying up and preparing for the next lesson or day. Discussions with the teacher were therefore generally snatched conversations rather than formal interviews. At the start of the day she would explain what mathematics she was doing with the children that day. After the lesson we would discuss specific issues that had arisen in the lesson, though we were not able to have in depth analyses of the lessons. I noted that if this data proved to be an important element for the main study I would need to reorganise the way in which the information was collected.

**Informal discussions with children**

The second aspect of data collection that proved difficult was that of talking to the children about the work that they were doing. I had thought that this would elucidate their actions. However I found that these young children were unable to reflect upon their learning sufficiently. Responses to questions such as “Can you tell me what you are doing?” would elicit replies like ‘We’re playing with the bricks” without any reference to the mathematics involved. When asked why they were doing a particular activity they would reply for example “Because the teacher says so”, or “It’s our work”. While these give useful insight into the children’s understanding of the social world of the classroom they did not help in my understanding of their mathematics learning. Again, in the main study I would need to consider the extent to which such information was important and how to collect it.

**4.6.4 Data Analysis of Preliminary Study**

The data consisted of assessment data on the children both before and at the end of the observation period, fieldnotes from a series of lesson observations, notes from discussions with the teacher and (to some extent) the children about the lesson, and written lesson plans for those lessons at which I had not been present during the observation period. This was supplemented by my own initial comments and thoughts, recorded following the lesson.

Initial analysis was carried out through a process of;
• reading the data collected and making notes of my reflections;

• highlighting significant aspects of both data and notes relating to teaching and learning; and

• identifying to what extent these significant aspects could be justified with reference to the rest of the data, and begin to answer the research question(s).

Analysis of this process would also inform the data analysis of the main study.
4.7 Research Findings from the Preliminary Study

The findings of the preliminary study relate to the supplementary research questions identified in 4.6. above. These were:

1. What mathematics did the teacher teach?
2. What mathematics did the children learn?
3. Did the children learn what the teacher taught?
4. What other factors affected their learning?

This section will address each of these questions in turn and summarise them; the following section will then look at the implications for the main study.

4.7.1 What did the teacher teach

I found that this question could be answered in many different ways. What the teacher taught could be described as 'what mathematics did the teacher teach?'; 'how did the teacher decide which children to teach which mathematics to?'; and 'how was this mathematics presented to the children?'. Each of these interpretations provided valuable insights into the teaching.

What mathematics did the teacher teach?

- curriculum

At the beginning of term there was a strong emphasis on using the activities set down in the HBJ Teacher’s Handbook. Gradually, as the term progressed Ann identified areas in which she felt the children needed more experience and selected activities from a range of sources that would provide practice in these areas. On at least two occasions she invented activities since she could not find ones that fulfilled the perceived need.

The number activities can be described as fitting into one of three types according to a cardinal, ordinal and nominal use of number. For this purpose I identified any activity that concentrated on 'how many' objects as cardinal, any that concentrated on counting and number order as ordinal, and activities which concentrated on recognition of numerals, without reference to their quantity or position, as nominal.

An example of a cardinal number activity was making towers of Unifix cubes of different height and then counting the cubes and labelling each tower with the appropriate numeral. A typical ordinal number activity was the use of a number 'washing line' on which hung cards with
numerals. The teacher would alter the line in some way and the class were asked which numerals were missing that day or whether they were all in the right order. Nominal number was encountered in discussion about bus or telephone numbers but also when playing a game like Beetle where a throw of the die results in the children being required to colour in part of the beetle - 6 for the body, 3 for a leg etc..

Of all the activities I observed (and those in lesson plans provided by Ann) the majority used cardinal number; only two or three related to ordinal number and these concentrated on the relative position of numbers. The games that used nominal number were used to assess the children's understanding of matching the dots on the dice to the numerals. This use of nominal number tended to confuse the children who were unclear about the different uses of number. In the beetle game several children thought that if they scored three they should pick up three legs rather than just one leg that had been labelled with a 3 in a nominal way. In the assessment activities (discussed more fully below) most of the children had already shown that they could use numbers in different ways, using ordinal counting words, responding to the question 'how many' by giving final count word and reading numerals. I concluded that the activity did not provide sufficient clues for the children to make 'human sense' of the way that number was being used. Observation of this activity highlighted a significant issue in relation to the children's understanding of number that was found to be consistent with the rest of the data. Since moving from a cardinal to an ordinal model of number is needed to move from addition by counting-all to counting-on, this was to prove a key idea in the main study.

The children did no addition activities in the mathematics lessons. Ann expressed the belief that they needed “a good understanding of number first, before they can start adding up” (Fieldnotes: Lesson 6). However I observed that a large number of whole class activities which developed the children's understanding of number, counting and data handing in real contexts, happened in the classroom at other times. These activities included counting the number of children present at registration, counting and calculating when collecting up reading books, sorting crisps at break time, singing and saying number rhymes like 'Five Currant Buns in the Bakers Shop'. In the context of these activities the language of addition and subtraction was used and the children were expected to join the teacher in simple calculations.

For the days that I did not carry out observations, Ann provided me with lesson plans for the mathematics tasks but made no mention of these extra activities. It was also possible that similar activities took place in school outside of the classroom, not to mention at home. Since I could not spend all day, everyday, with the children, I identified that it was going to be impossible to identify all the mathematics that the children had encountered.
How did the teacher decide what mathematics to teach to which children?

- differentiation

After the first week of term, which she spent getting to know the children, Ann had organised the target children into two groups. This was a generic grouping across all subject areas and based on 'ability' - although through further discussion I identified that 'ability' was mostly about the children's ability to settle down and get on with work. Usually the two target groups did the same activity, though adaptations were made for the 'poorer' (Green) group. For instance when playing the Gingerbread game the Green group used only the numerals 1 to 4 while the Red group used 1-6. Occasionally the Green group would do the same work but stop before the others. Early in the term the children used sorting trays with the numeral 1 in the centre and were asked to put one toy into each partition. The Red group then went on to do the number 2 as well. The Green group stopped after 1 and were allowed to choose something else to do. I asked whether this differentiation had been planned, and Ann explained "I made the decision not to go on. They had succeeded at '1' and I wanted to stop while they were succeeding". I asked if she thought that they could have done 2 and she answered "Some of them could, Adam and John, but the others ... I think it was good for them to stop". She would continue with the work at a later date.

The curriculum experienced by the target children was therefore not uniform, but differentiated according to the teacher's perception of their understanding and learning needs. As the term progressed, the gap widened so that, by the end of term, one group had only covered numbers 1 to 5 in detail while the other had learnt about numbers to 10. I noted that Laura had already been able to do most of the number work set before she left the nursery (as evidenced through my initial assessment) and would perhaps have been better working with the older children in the class for mathematics.

These observations raised questions about how and why to use grouping as a form of differentiation. Individual learning programmes are not practical in a classroom where the children cannot read tasks for themselves. Whole class teaching was carried out with these children at other times - the number rhymes, counting games, data handling and practical problem solving related to everyday tasks noted above; but whole class teaching may not be practicable all the time when working with a class containing children who have been in school for different lengths of time.

Group work can seem the most useful way of organising the class for tasks requiring management of a limited supply of equipment and teacher time.

Ann used groups in this way, concentrating on teaching one group while others were working with the classroom assistant or working alone on a task not needing much adult input. Croll and Hastings (1996) observed that most children working in groups were in fact just organised into groups for management purposes and worked as individuals, concluding:
All of the studies show that teachers use groups extensively as a strategy for organising the classroom and, often, the curriculum but that relatively little group interaction takes place. Children sit in groups but the teacher does not typically work with them as groups (p. 18).

This organisational grouping did sometimes happen (for example when the children were sorting toys into counting trays they worked as individuals with little interaction) but in the main the children observed worked as a group, directed by Ann, talking about their work so that they were developing the important language element of the activity, and sometimes playing games.

Grouping children of mixed ability has the advantage of the children being able to support one another's learning, but children must be taught to do this. But we can see how grouping children by perceived ability can run the risk of limiting their experience to the teacher's expectations and therefore stifling the children's development. Grouping by ability also assumes a linear curriculum where what is taught next depends upon what is already learnt. Yet there is evidence (for example see Nunes and Bryant 1996) that children can have a limited understanding of a concept before so called earlier concepts are fully understood. Nunes and Bryant cite the example of children who can share objects using one by one distribution (an early understanding of division) before they are able to count. Or again, children can do addition with small numbers of objects, as many of the target group were able to do in the end of term assessment, before they can count and understand larger numbers. Apart from occasional number rhymes such as Five Currant Buns, the target children did no addition or subtraction activities. Could a wider curriculum for all children, not just the 'more able' have benefited their learning?

Therefore, in relation to the research question, teaching involved control over access to the curriculum by differentiation through a 'linear' curriculum.

How was the mathematics presented to the children?
- representation

The third element of teaching related to what sorts of activities the teacher chose to develop the children's mathematical understanding. The majority of the activities carried out in mathematics lessons were practical activities that involved the children in counting and in recognising symbols for numbers 1 to 10 (or 1 to 5 for some children). The emphasis was on the children gaining experience of quantities, numerals and counting in a variety of contexts. Analysis of the activities using the ELPS framework (Experience, Language, Pictures, Symbols; Liebeck 1984) showed that most involved practical experience and language. In some activities the teacher used representations of number as picture (of things to count) and numeral symbols. There was almost no written recording. The children used numeral cards or Unifix hats to label sets and towers, though they did learn to write numerals as part of their handwriting curriculum. Mathematical
language was emphasised whenever Ann worked with the children. She rehearsed appropriate language with them and corrected mistakes that they made. The children picked up the new language quickly. For example, at the beginning of an activity when asked “who has more cubes (than the teacher)”, Dan answered "I've got lots" and was corrected "yes, you've got more." By the end of the lesson he was able to use 'more' correctly.

The way that Ann presented mathematics to the children therefore fitted with Liebeck’s model for young children, with an emphasis on practical experience and language. However, analysis of this aspect of teaching highlighted that there were lessons in which I had not recorded a sufficient level of detail to identify how the individual participants used these elements. In the main study it would be necessary to record more detail at this level of observation.

4.7.2 What mathematics did the children learn?

The purpose of the initial assessments, carried out while the children were still in the nursery class, was to provide a baseline for their mathematical understanding before the observation began. Section 4.6.3 outlined the assessment activities. The same assessment activities were carried out at the end of the preliminary study period of observation, providing evidence of what the children had learnt during the time of the study period. Here I want to provide a brief overview of the children’s apparent learning.

Nursery Assessments

The nursery assessments indicated the children’s understanding of number. All the children except Dan were able to give a number related response when asked to estimate “how many bricks” in a collection of ten. Dan described the colours, indicating limited understanding of the language of number. All they children knew some count words and eight children were able to use these to count objects. Dan and Jane understood how to count but were not very good at matching one, and only one, counting word to each object, or at remembering the count words in the right order. These same two children recognised no numerals; the others varied, with from two to nineteen symbols being recognised.

When asked to make labels to show how many cubes were inside a plastic pot four children refused the task. Of the others, five attempted to use symbols and Adam used tally marks. Adam and Colin could read back all their symbols or tallies, while Laura and Rickie read some correctly. In response to the question “what is one more (or one less) than x ”, seven of the children were able to add or subtract 1 brick mentally (without any external evidence of counting), but none was able to add or subtract two. These results showed a range of mathematical understanding. When
compared with the results of Carol Aubrey's research (Aubrey 1993; 1997) the children were performing at a similar level to Aubrey's sample, but her children had an average age of 4 years 4 months whereas the average age of the target children in this research was 5 years 1 month, adding weight to the observation, in section 4.5.3 above, that the school has a much higher number of children with special needs than most.

**End of term assessment**

Six children's ability to estimate had improved: Adam was happy to estimate rather than count, Colin and Gordon gave numerical answers (10 and 8) rather than general quantity words ('loads', 'lots') and Dan who now gave a quantity related response ('a lot') rather than referring to colour. Colin could now count to 39 and Adam to 66 (both 29 previously) while Jane's counting had improved from 5 to 20. The other children's counting skills had not changed significantly.

The ability to recognise numerals had improved for all the children, with the children in the Red group showing greater competence than those in the Green group. All the children attempted to use numerals to record quantities in the pots game, and eight children could now read back all their labels. Jane could read some of hers, while Dan was unable to read back any. Of the three children who had not previously been able to add or subtract one, John and Jane could do so now, Dan was now the only child who could not. Five of the other children could now add or subtract 2, which none of the children had been able to do previously.

**Evidence of learning during teaching episodes**

While assessment gave a summative account of the children's learning I was aware at the analysis stage that I had little data on what the children learnt during the lessons I observed, in part due to the difficulty of accessing this information. The children produced little written work, were unable to discuss their work in detail and were limited in their mathematical language. What evidence I did have generally arose from their conversations or their actions as they worked, for instance Dan's ability to use the word 'more' in context, or the children's confusion over whether to colour one beetle leg or three when the die showed 3. However I was aware that this level of detail had not always been recorded, which would need to be considered in the main study.
4.7.3 Did the children learn what the teacher taught?

As can be seen from the above summary the main area of improvement for all the children was their ability to read and use numerals to record quantity. This improvement relates well to the concentration of work on counting and recognising the related numerals noted in lessons. It is also noticeable that the children in the Red group: John, Len, Adam, Rickie, Laura and Colin, could recognise and use bigger numbers than those in the Green group with the exception of Gordon. The differentiated curriculum appeared to result in a differentiated outcome.

All of the children now attempted to use numerals to record quantities and all but Dan could read back their own labels. I had no evidence of the children being taught to write numerals in order to record quantities so they appeared to have connected their ability to write numerals (gained from 'handwriting' practice) with the recording of quantities using numeral cards or Unifix hats. Estimation had not been a focus of any of the teaching I had observed and the children's ability to estimate had changed little, apart from Dan's ability to give a number related answer. So far the children appeared to have learnt from what they had been taught.

However, the final assessment also showed the children having developed other mathematical skills and understanding. Colin and Adam had improved considerably in their knowledge of count words. I had no evidence that count words greater than 30 were taught in the classroom; I certainly saw no teaching of numbers greater than 30, and teens and twenties were used only in non-lesson contexts such as the register. Both these children come from families with older children in the school so this may have been learned at home.

All the children except Dan could now add and subtract one unit in context. This had not been taught directly though there were instances of counting where 'and one more makes ...' was discussed, and rhymes such as 'Five Currant Buns' encouraged 'take away one'. Five of the children, all from the Red group, could now add or subtract two or more objects in context though I was aware of no direct teaching in this.

It was therefore difficult to explain the relationship between teaching and learning of mathematics in terms of relating what they had been taught to what they had learnt. Even when it was possible to gain access to the children's ongoing learning, it was difficult to account for all the teaching they had received in school, and there were indications that they were also learning mathematics outside of school. While this finding was not a surprise, it indicated that characterising the teaching / learning relationship in terms of whether the children had learnt what the teacher taught, would not prove possible. The focus of the main research must address this difficulty.
4.7.4 What other factors affected their learning?

Analysis of the data also showed that the children had learned more than mathematics from their lessons. At the start of term I noted that some of the children had difficulties in playing games. They did not understand the idea of taking turns and John in particular could not cope with losing. By contrast, at the end of June I recorded that ‘the children really do have much more idea about playing games, taking turns, passing the dice around, helping one another, monitoring who is winning’ (Fieldnotes: comment Lesson 10). This social, rather than mathematical learning was also having a positive effect on their mathematical learning and will stand them in good stead for future activities in the context of games. It was a reminder of the range of learning that goes on in a reception class, and of the inter-relationship between learning and social development.

Another noticeable feature during the final assessment period was the change in attitude of the children. All the children were happy to work with me on the assessment which took about 30 minutes per child and none refused a task or said that they could not do it, whereas in the initial assessment, carried out in the nursery, several of the children lost concentration or refused tasks they thought looked difficult. The children's attitude, application and concentration had improved with their term in school. The socialisation of the children into the role of pupils is a key feature of teaching in the early years of schooling and cannot be separated from the subject teaching. In mathematics lessons they had been taught and learnt more than just mathematics.

This finding seemed to indicate a more socio-cultural aspect to learning mathematics than I had previously acknowledged. Mathematics learning was not social only in so far as it happened through social interaction and language but was being influenced by, and in turn influenced, the wider social and cultural development of the children. The extent to which radical constructivism addressed the social aspects of learning, acknowledging interaction and the role of language, may not be sufficient to explain classroom mathematics learning. Further analysis of this relationship in the main study could give insight into the theories of learning described in Chapter Two.

4.7.5 Summary of Findings

The findings show a range of different relationships between what was taught to the children and what they learnt. The children were given a range of mathematical tasks, chosen by the teacher in accordance with the mathematics scheme, and differentiated according to the teacher's perception of their ability. The majority of these tasks related to an understanding of early number concepts which most of these children had not previously developed in the preschool phase. The teaching followed the ELPS framework with an emphasis on practical experience and spoken language. Direct comparison of what they were taught with what they had learnt proved difficult, since
assessment showed improvement both in areas they had been taught and in other areas not covered in the curriculum. In general the children in the 'lower' group had made less progress than those in the 'higher' group, though whether this lack of progress was due to the differentiated curriculum or to individual 'ability' factors was not evident. In addition, the children's social skills, concentration and attitudes to work had all improved, which would influence their subsequent mathematics learning.

4.8 Outcomes from Preliminary study and implications

The aims of the preliminary study were expressed in 4.6 as:

a) to identify key issues in the teaching and learning of early mathematics which would focus the main research;

b) to practice and refine the skills of data collection, in particular participant observation, and analysis.

The findings of the preliminary study outlined above have both clarified the research focus and analysed the methodological process. This section will address the implications of these findings for the main study.

4.8.1 Development of Research Design and Refining of Research Tools

The findings from each of the supplementary questions provided evidence of use in the formulation of the main study. With reference to what the teacher taught, I found that at the analysis stage I was less focused on the teacher, concentrating on what the mathematics was and how it was represented to the children. This discovery highlighted the fact that, although the focus was on teaching and learning, it was the effect that teaching has on the children’s learning that I wished to study, rather than a study of how teachers teach. This was reflected in the material chosen for the literature review, which draws on paradigms to explain children's mathematical learning. The main study would therefore focus on teaching and learning at the classroom interaction level.

It had proved difficult to find a direct relationship between what was taught and what was learnt, since all the teaching and learning could not be identified. A match between what was taught and what was learnt would not be attempted in the main study. But the initial assessment tasks also gave insight into the children’s existing mathematical understanding that proved useful during the observation phase. With this evidence it was possible to explain, say, Dan’s poor counting, as a
result of the lack of one to one correspondence rather than lack of number words. The initial assessments would therefore be continued into the main study. However, I noted in section 4.8.2 that the children in the preliminary study had poorer mathematical skills than others of their age. The initial assessment tasks would be retained in the main study, but they may need refining since the children chosen may have a wider understanding in mathematics. Also, children with a higher level of achievement in mathematics would be more likely to be working on addition rather than basic number skills, which had been the preferred mathematical focus.

With reference to the original aims the study provided the opportunity to practice data collection, and in particular participant observation. I was able to modify my methods of data collection as I went along and collected a range of useful data. Participant observation served to provide a range of information about both teaching and learning. Insight into the children’s learning during the teaching episodes, and into how the teacher was presenting the mathematics to the children, was possible when the data was sufficiently detailed in terms of what was done and said. Structured observation schedules had proved ineffective for recording such data and more open field notes highlighted more detailed social interactions between the participants. Participant observation also provided evidence of a range of social development in the children, which had an effect on their learning of mathematics. This had not been an expected outcome of the study, yet clearly related to questions raised by the literature review into learning paradigms. I was interested to continue to gather evidence of the wider socialisation in the classroom and its effect on mathematics learning. This level of analysis would require more consistently detailed observations of the children’s actions and language in the main study.

Discussion with the teacher before and after the sessions had proved difficult but useful in clarifying the aims of the session, answering questions afterwards and getting another person’s viewpoint on what had happened. These discussions would be continued into the main study as a means of confirming data and providing background to the children’s learning. Since the focus was no longer on the teacher *per se* I would not need to find ways to improve this method of data collection. It had not proved possible to gather much information from individual discussions with the children and these would be discontinued. While it proved impossible to make a direct comparison between the learning as evidenced by improved results in assessment and the teaching observed, the assessment results did allow me insight into the children’s understanding and were continued into the main study for this purpose. I was able to gain a greater understanding of the children’s mathematics, which illuminated the observational data.

The data analysis phase identified the need for a clearer methodology for data analysis. Further reading identified the suitability of a grounded theory approach (Glaser and Strauss 1967) which will be discussed in detail in Chapter 6. The analysis also showed that the research questions
needed a clear focus. The process of analysis had been broadly successful but it had taken a long time to identify what aspects of teaching and learning I wanted to look at. The preliminary study had helped identify these aspects so that more clearly focused questions could be formulated for the main study, enabling more focused data analysis.

4.8.2 Focusing of Research Questions

The preliminary study both reinforced and clarified the nature of the overall research question. The relationship between teaching and learning of addition in the classroom was still the central theme. However the relationship had been further defined as involving what mathematics was being taught, how the mathematics was communicated to the children and how the wider social context of the classroom influenced their learning. My focus was clearly on the children's learning, on the teaching only in so far as it impacted on the learning. The research question could therefore be simplified to:

*How do young children learn addition in primary classrooms?*

Three supplementary research questions were identified in order to clarify the focus still further. These are:

- **Q1** *How does the way that the addition curriculum is planned and implemented influence children's learning of addition?*
- **Q2** *How does the way that addition is represented to the children influence children's learning of addition?*
- **Q3** *How does the wider social context of the classroom influence children's learning of addition?*

These three questions would form the basis of the main study.

4.9 Summary

This chapter has described the preliminary study undertaken to clarify the research question and methods. It set out the methodology and research design in detail as they applied to both the preliminary and the main study. It described the setting up and implementation of the preliminary study, analysis of the data collected, and the findings, and looked at the implications of these for the main study, resulting in the formulation of new research questions. Chapter Five will expand on the methodology of the main study and describe the study in detail.
Chapter Five

The Main Study: Research Questions and Method

*How do young children learn addition in primary classrooms?*

5.1 Introduction

This chapter describes the progress of the main study, which evolved from the experience and findings of the preliminary study in the previous chapter. It considers the nature of the research problem and the formulation of new research questions. The significance of the study is then discussed and finally the methodological considerations of the focused study in terms of data collection and analysis are considered.

5.2 Nature of the Research Problem and Formulation of New Research Questions

The research problem remained essentially unchanged after the preliminary study; the overall focus was still on the relationship between teaching and learning. However the preliminary study served to clarify ways to characterise this relationship and the data required to do this. As the literature review in Chapters 2 and 3 indicated much of the theoretical study of children's learning is founded on experimental data in laboratory settings (Piaget, Vygotsky) or experimental changes in classroom practice (Cobb, Yackel and Wood, Tharp and Gallimore) rather than on observation in naturalistic settings. The preliminary study, by contrast, looked at what was already happening in primary classrooms as the children started to learn formal mathematics.

From insights derived from the data collected in the preliminary study and from the literature reviewed it was possible to narrow the research focus of the main study. I realised that it was the children's learning on which I wished to concentrate. This was not to say that the teaching was incidental to the study. Teaching, the actions, language and intentions of the teachers, was a crucial part of the context in which the children learnt. However, to investigate the sense that the children were making of teaching, and their developing ideas about mathematics, would require me to focus on the children, rather than on the teachers.
Chapter 5 The Main Study: Research Questions and Method

My focus on children learning mathematics in the early years of formal schooling arose from personal interests and the literature reviewed, and was reinforced by the preliminary study: the way that children make links between their preschool and school mathematics, the specific needs of such children learning to be pupils in school for the first time and the sense that they are making of this strange new world. The specific focus on addition was strengthened by observation in the preliminary study that this was the core arithmetic curriculum for children at this stage of learning. However addition was not the research focus of the study but provided a background of mathematics, based on the considerable research findings from the literature review.

This narrowing of focus from 'the relationship between teaching and learning' to 'how do young children learn addition' was therefore in response to the outcomes of the preliminary study and the literature review. It was also essential in order to limit the research to the time and availability of the researcher to complete the study.

The preliminary study also highlighted aspects of the research process needed for the main study. As a result of the preliminary study analysis and literature review, I noted that:

- the study of a single teacher, classroom and school did not allow for differences in practice to be identified, observations might be specific to this class, so a wider sample would be needed for the main study;

- It was not possible to observe all the children in the class so a group of target children should be identified to allow more focused observation;

- Since the focus was to be children in the early years of schooling and the early stages of addition, classes containing children in the first two years of school would be studied allowing identification of how their learning needs developed as they grew more confident in the school setting;

- Since children in different LEAs start formal schooling at different ages it would be helpful to identify more than one LEA within which to work.

Within the time and availability of the researcher to carry out the research I therefore decided that the study of Reception and Year One children in two schools from different LEAs would give a range of settings from which to gather data. There was no intention that this would be a representative sample producing generalisable results but the range of settings would avoid the specificity of a single classroom study.
So, the main focus of the research is children in the first two years of formal schooling learning addition in the classroom with their class teachers. The new research question, identified in the previous chapter, centres around the relationship between teaching and learning, focusing on the ways this relationship is seen in the classroom and in particular on the children’s learning.

*How do young children learn addition in primary classrooms?*

Three areas were identified: the way that the curriculum is structured, the way that addition is represented to the children and the wider social context of the classroom. This led to the formulation of three, more detailed, research questions to address in the main study.

These are:

1. **Q1** How does the way that the addition curriculum is planned and implemented influence children’s learning of addition?
2. **Q2** How does the way that addition is represented to the children influence children’s learning of addition?
3. **Q3** How does the wider social context of the classroom influence children’s learning of addition?

The main study would therefore attempt to find answers to these questions and by doing so present a broad picture of teaching and learning early addition that would address the overall research problem. It is necessary first to justify the significance of this study within the field of mathematics education.

### 5.3 Significance of the Study

The study is important for three main reasons:

- it investigates the learning of addition in early years classrooms, relating a psychological understanding of the development of addition in children, with what is actually happening in classrooms;
- it analyses the way that mathematics is represented to and by young children, drawing on studies of the use of mathematical representations in older primary and secondary aged children;
- it explores the context in which learning is embedded, the socio-cultural setting of the classroom.
The issues emerge both from the preliminary study and the literature review. This section will consider each of these in detail.

### 5.3.1 The learning of addition in early years classrooms

Three key ideas are embedded in ‘the learning of addition in early years classrooms’: learning, addition, and early years classrooms. Chapters Two and Three identified current research in these areas. While research addresses each of these elements, I have found none with a focus on all three. Studies of children learning addition concentrate on identification of the developmental stages through which children progress (Nunes and Bryant 1999; Carpenter and Moser 1983; Carpenter et al. 1992), or on older children (7+) using Dienes multibase materials to learn multidigit addition (Dienes 1964; Wood et al. 1995; Fuson and Briars 1990) or interpreting word problems (Vershaffel and De Corte 1997). How do younger children learn addition?

Desforges and Cockburn (1987) carried out an in depth study of first school teachers but this concentrated on the teaching rather than the learning, and the study is now rather dated. More recently, Aubrey (1997) has looked at young children starting school, their existing mathematical understanding and how teachers structure their teaching to develop the children’s mathematical understanding, but this does not focus on the children’s learning except as a product. What can we say about the processes of children’s learning?

Studies of learning have tended to concentrate on individual achievement rather than learning in the classroom e.g. (Carpenter and Moser 1983; Geary 1994; Nunes & Bryant 1999). What can we learn from in situ study? From a constructivist and interactionist perspective, Cobb, Wood and Yackel (1991) in the United States have implemented changes in teaching elementary mathematics. What will a study of existing practice in England reveal?

A study of learning addition in the early years will therefore build on and extend existing research, of interest to researchers and practitioners of early primary mathematics teaching.

### 5.3.2 The way that mathematics is represented to and by young children

The literature on representation in mathematics, and particularly that on the use of symbols is generally focused on older children especially those learning algebra. In the early years, studies of children’s use of symbols, e.g. (Hughes 1986; Matthews 1989; Munn 1997), look at how they use symbols rather than how children learn to use them. Much of the work on manipulative materials concentrates on the introduction of place value and addition and subtraction of multi-digit numbers rather than their use in early arithmetic. And Lesh, Post and
Behr's work on representations (Lesh et al. 1987a) looks at problem solving in secondary school rather than with young primary aged children.

So, while there is a range of relevant research in this area, none of it has a focus on teaching and learning of addition in early years classrooms proposed in this study, aimed at analysing the way that teachers represent addition to the children, the sense that the children make of such representations, and the way that the children begin to use representations for themselves to calculate and communicate addition.

5.3.3 The context in which learning is embedded
The move from constructivism to social constructivism, described in Chapter Two as a model for looking at learning in the primary school, reflects an increasing interest, among the research community, in the affect of the social context of the classroom on children's learning. A change of theoretical perspective challenges me not to see children as recipients of transferred knowledge, or as isolated individuals constructing knowledge for themselves, but as part of the society that makes up the classroom and school. Consideration of the wider affect of the classroom context on children's learning of mathematics is therefore a current issue in mathematics education that will be further illuminated by the results of this study.

5.4 Methodological Considerations
The methodology of the main study remains essentially the same as that of the preliminary study, though the methods have been refined in the light of this experience. The preliminary study confirmed the methodology as appropriate to an exploratory study of children learning. Since much of the methodology has been extensively discussed in Chapter Four, I will not reproduce it here, concentrating only on improvements identified as a result of the preliminary study. With reference to data collection, I recognised a need for more detailed recording of what the children said and did. Since the children in the main study would have a wider age range (4-6) than the preliminary study children (all just turned 5) I would need to adapt the assessment tasks, in order to assess addition as well as number concepts. Data analysis in the preliminary study indicated that a more rigorous approach would be required for the main study, where a greater quantity of data would be gathered. In the following sections I will outline how these issues have been addressed.
5.4.1 Data Collection

Participant observation
As in the preliminary study, three methods were used for data collection. The main method was participant observation, the details of which were discussed in full in Chapter Four. In the preliminary study I noted that the data collected was sometimes not detailed enough to show clearly what the children had done and said. In the main study it would be necessary to collect data in more detail. To this end a target group of up to six children would be selected from each class, in consultation with each teacher. While all the class would be observed at times, these groups would be the focus of attention when the class split into group or individual work. This would allow for more detailed observation of fewer children.

Selection would be based on a variety of criteria:

- the mathematics that the children were doing, in order to focus on early addition;
- the confidence of the children, especially with the younger children who might be upset by being observed;
- other issues within the classroom that might affect the selection such as provision for children with special needs or English as an Additional Language (EAL).

There was no suggestion that the children would be specially selected for their level of achievement. Observations would be recorded through unstructured field notes and written up as soon as possible after the lesson.

Discussions with teachers
The second way of gathering data was through conversations with the teachers before and after the lessons. Although difficult to find time for, these had proved useful in the preliminary study and would be continued. They helped me to understand the teacher’s objectives for the lesson I was about to observe, building on the information I had from written lesson plans, and provided an opportunity afterwards to discuss what had happened in the lesson. Sometimes the teacher would want to discuss things that she had noticed during the lesson and at other times I would ask questions to illuminate my observations.

However, I did not continue with informal discussions with the children during the lessons since these had provided little information in the preliminary study. Either the children were not able to reflect on what they were doing and why, or they were unable to explain it to me. More
careful observation and noting of what they did and said would have to be relied on, to give insight into their thinking.

Assessment Activities
The final area was that of initial and final assessments of the children's understanding of number and addition. These followed a similar pattern to those used in the preliminary study (Section 4.6.3) but were extended since some of the children were older and would be able to achieve more. These assessment activities had originally been designed to use as a pre- and post-study measure of what the children had learnt, in order to compare this with what they had been taught. In analysing the preliminary study I noted that although it had not been possible to do such a direct comparison, I had found the assessment data useful in interpreting the children's responses during the observation period. I would therefore continue with assessment of the children as part of the data collection for the main study.

The assessment activities were adapted to comprise questions about the children's counting and number writing competencies, and a dice game to ascertain their addition competence and strategies. The children were assessed informally, in groups of not more than five children, either in the classroom or in a part of the school where they were often withdrawn to work in small groups with another adult.

Saying the Counting Words
The children were individually asked to count out loud as far as they could. When they stopped I would ask them if they knew what number came after their last count word which sometimes prompted them to go on further. I recorded the highest count word and noted any inaccuracies in order. This was repeated again later in the same session to identify persistent errors. This identified errors the children might be making in counting rather than addition. For example a child who normally counts 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12 when solving 7 add 4 might give the answer 12 indicating a correct addition strategy but a counting error. For a child who counts correctly, an answer of 12 would indicate an addition error.

Reading numerals to ten
A set of cards with numerals to ten was used to assess the children's ability to read single digit numerals out of order. Children are often able to produce the count words in order and match these to a set of ordered numerals, without necessarily recognising the numerals, so the cards were shuffled and turned over one by one. If the child could read all of these numerals, random two digit numbers were made from the cards to see if they could read larger numbers. I would
then be able to identify whether a child writing $5 + 5 = 4$, was doing so because they confused
the symbols 5 and 2 (a common error amongst young children), rather than because they did not
understand the addition.

**Writing numbers to 10**

The Reception children were asked to write down the numbers 1 to 10 in order. This was to
identify any inaccuracies in their number recording that might affect the observed problems with
addition. For example a child recording the total of 2 and 3 as 2 might not understand the
addition process, or on the other hand they may have calculated the sum correctly but not yet
mastered the difference between writing 2 and 5. The older, Year 1, children were asked to
write down as many numbers in order as they could. Rather than expect them to go on writing
for ever, those who reached 31 without errors were then asked to write a few specified higher
numbers, since they had usually reached the stage of understanding the pattern of written, and
spoken number.

**The Dice Game**

In this game the children were required to throw two dice, to add the totals and remember this.
After they had all had a turn, the child with the highest total won a counter and the first child to
win five counters won the game. The children were encouraged to show how they carry out the
addition by talking aloud. Initially the game was played with two dice with dots representing
quantity. The children could count the dots if they had no other addition strategies.

After some time one of the dice was replaced by a die with numerical symbols on it. The game
proceeded as before and the way that the children responded to this change was noted. Children
used a range of addition strategies, from needing to represent the symbol as a quantity on their
fingers in order to count all; counting on from the number represented by the symbol to reach
the total; treating both representations as numbers and counting on from the highest; to knowing
the answer as a learnt fact. The game therefore gave insight into the children’s understanding
of, and ability to solve, simple addition.

**Written addition sums**

The children were shown a written sum, $2 + 3 = \square$ and asked them if they has seen anything
like this and did they know what it meant. The reception children at Ashburne may not have
experienced such written sums in school, but the reception children at St. David's were more
likely to have encountered their use in the classroom since they worked alongside the older,
Year One children. The Year One children in both schools would have experienced such sums
in their mathematics lessons. This would show their understanding of written mathematical symbols as well as understanding of addition.

**Summary of data collection methods**

For each lesson field notes were collected on:

- a prior discussion with the teacher on the intended learning outcomes of the lesson;
- observation of the teacher
  - what is being taught
  - how it is taught - materials, actions and language;
- observation of the children
  - their involvement, participation and attention
  - what they say - language and meaning
  - what they do and record;
- discussion with the teacher about her teaching and the children's learning.

Later, ideally straight after each lesson, the field notes were written up in full and word processed. In addition the target children were assessed at the beginning and end of the data collection period.

**5.4.2 Data Analysis**

Data analysis was based on a grounded theory approach, described by Glaser, Strauss and Corbin (Glaser & Strauss 1967; Strauss & Corbin 1990) as a qualitative research method suitable for social situations. Strauss (1987) asserts that a grounded theory approach:

> is not really a specific method or technique. Rather it is a style of doing qualitative analysis that includes a number of distinct features, and certain methodological guidelines ... to ensure conceptual development and density. (p.5)

**Why use Grounded Theory for this study?**

The data from this study is qualitative. The study is exploratory, designed to build theory on how children learn mathematics in early primary classrooms, rather than to test theory or evaluate change. I am aware of the danger that the process of analysis will be influenced by my own biases and assumptions, resulting in finding only what I want to find. I therefore need a method of analysis that will enable me to make sense of a wealth of unstructured data in a systematic and rigorous way.
Strauss explains that grounded theory arises out of the "deepest convictions that social phenomena are complex phenomena" (p. 6), which reflects my belief expressed in Chapter Four that 'the phenomenon being researched is a complex one requiring knowledge of people and social interaction'. Strauss and Corbin (1990) summarise the analytical procedures of Grounded Theory as designed to:

1) build rather than test theory;
2) give the research process the rigor necessary to make the theory "good" science;
3) help the analyst to break through the biases and assumptions brought to, and that can develop during, the research process;
4) provide the grounding, build the density and develop the sensitivity and integration needed to generate a rich, tightly woven explanatory theory that closely approximates the reality it represents." (p. 57)

Grounded theory serves to analyse, interpret and theorise qualitative data in a systematic and rigorous manner. It can therefore provide the rigor required for analysis of my data, in order to "generate a rich, tightly woven explanatory theory that closely approximates the reality" of the classroom.

The Grounded Theory Process

The process requires successive stages of coding data, using codes to develop concepts and categories, and consideration of these categories in context to produce a statement of what is happening. Such statements are then verified against the data and, through a process of successive memo writing and the making of diagrams, order and theory should emerge from the initial chaos of data.

To talk of order and theory 'emerging' can suggest that theory springs from nowhere. Approaches to qualitative analysis may be seen on a continuum between the deductive, analysed according to pre-existing categories, and the inductive, with theory emerging from the data. Miles and Huberman (1994) argue that these extremes should be "dialectical rather than mutually exclusive"(p.134). I recognise that theory is developed both from the data and from the theoretical perspectives of the researcher, and to this extent is both inductive and deductive. Figure 5.1 shows how the background and the theoretical perspectives of the researcher are influential throughout the study, in the selection of the setting, the collection of data, the analysis of this data, and therefore influence the final findings.
It is not possible to eliminate these influences but it is necessary to be explicit about the researcher's background and theoretical perspectives so that the reader can judge the findings in the light of these influences.

Lofland and Lofland (1995) comment that the first two stages of analysis start before coding. These require the researcher to recognise that the data is framed within a social science and therefore aims to develop theory, hypothesis, themes, story lines etc. rather than elicit proof. Secondly they acknowledge that researchers must overcome the natural "fear and anxiety" that theory will not emerge from the data. They suggest therefore that grounded theory provides a structure for such analysis, with the confidence that theory will develop.

5.4.3 Summary of methodological considerations

In summary, the primary methodological features of the main study are:

- a qualitative research perspective;
- a study of four classes across two schools and two Local Education Authorities, containing Reception and/or Year One children;
- data collection through participant observation, recorded as field notes, supported by assessments of the children and discussions with the teachers;
- a grounded theory approach to data analysis, grounded in the data and in the theoretical perspectives of the researcher.
5.5 Selecting the Schools

Choosing the schools to work with and obtaining consent is a crucial part of the educational research process. Within the time constraints of the research I had decided to study four classes in two schools from two different Local Education Authorities. Since at the start of the study I was working in one LEA and living in another, I chose to work in these areas. The next task was to obtain access to two schools.

5.5.1 Access

I obtained access to the two main phase research schools in very different ways. In the first school, which I shall call Ashburne, I gained access through one of the teachers. Beth was recommended to me by a fellow teacher trainer, as someone interested in mathematics teaching, and who had recently attended an in-service training course at the institution at which I taught. She was teaching a reception class. When I contacted her she suggested that I also involve her colleague, Chris, who was the mathematics co-ordinator in the school and taught the Y1 class. I met with them and discussed my teaching background, my research interest, what I wanted to look at in the classroom and the commitment and time scale involved. Beth was very enthusiastic from the start and keen to be involved. She offered to discuss the situation with the head teacher, who was absent from the school that day but had known that I was coming, and I followed this up with a phone call to check that all was well. Chris was less sure at first, but agreed to try it out and I arranged to start visiting the school the following term. After the first few visits I spoke with Chris and ascertained that she was happy to continue participating in the research.

The second school, St David’s, was recommended to me by a fellow student, and I first approached the head teacher. This access route required a higher level of negotiation with the individual teachers before they accepted me into their classrooms. Access granted by the head teacher does not necessarily mean that the individual teachers are in agreement with having a researcher in the class (Measor and Woods 1991). At St David’s there was some reluctance to working with researchers, in part as a result of the school’s involvement with researchers from the university psychology department who would remove individual children from the classroom for testing, causing disruption of normal lessons. The fact that I was seen as a fellow teacher, I had previously taught in schools in the area, and that I wanted to spend time in the classroom observing what would normally be happening, helped me to establish a good working relationship with these teachers. In all the classes I spent some time in the classroom, acting as a ‘classroom helper’ and talking to the teachers and children, before starting data collection, so that I was a familiar figure to the children and ancillary staff.
5.5.2 The Schools

The classes were therefore essentially an opportunistic sample. However, the two schools were in different Local Education Authorities and different areas; Ashburne is a semi-rural large commuter village, while St David’s is in an inner-city area, giving a range of background factors. Ashburne school has very few children with special needs or English as an additional language, while St David’s has a very wide social, academic and cultural mix. One teacher in each school, Beth at Ashburne and Debbie at St David’s, expressed a particular interest in teaching mathematics. However in neither school were the teachers particularly recommended to me as ‘good’ teachers of mathematics. It was the focus of the study to see what was happening in ordinary classrooms, in so far as these can be defined, rather than looking at innovatory practice. The practice that I subsequently observed was very similar to the practice I was used to seeing in many other classrooms across several counties in my role as teacher educator, confirming that the classes were in no way particularly unusual. They were therefore suitable for the study in this respect.

Ashburne County Primary School was a medium sized primary school in a commuter village close to a large town. The buildings were modern, the classrooms set around a central school hall. The two reception classes were side by side with a wide, shared passageway between them. A door from this passageway led to the playground area outside. Beth’s classroom had a carpet area in the corner near the windows and a sink and ‘wet’ area in the opposite corner. The rest of the classroom had tables and chairs in groups. The classroom was light and airy and decorated with displays and children’s work. Beth had a classroom assistant who worked with her much of the time, and a student on a ‘Specialist Teaching Assistant’ (STA) course who came in part time.

Chris’s classroom seemed smaller, though this may have been due to the presence of up to 36 children and at least three adults whenever I visited. One child in Chris’ class had severe hearing problems necessitating a welfare assistant. In addition Chris had a qualified STA (Mary) who worked to support the teaching of this large class. But the class had access to a neighbouring, unoccupied classroom in which some of the children would often work with the STA so that the children had more room to work. Both rooms were light and airy and well equipped.

St David’s is a city first school housed in a Victorian school building. Debbie and Eve worked in classes next door to one another but with no access from one to the other. The classrooms opened onto a hall that was used for assemblies and for group work, while the hall in another building where the older children had their classrooms, was used for PE and school dinners. Doors into this hall space were the only way out of the classrooms, there was no direct access to
Chapter 5  The Main Study: Research Questions and Method

the outside. The classrooms felt enclosed, with high ceilings and windows high on the wall so it was not possible to see out. The walls were decorated with displays and children’s work. Debbie and Eve had no regular help in the classroom, though at times a classroom assistant would come in to work particularly with children who had English as an additional language.

5.5.3 Informed Consent
I explained to the teachers, as far as was possible at this stage, what the purpose of my research would be. Hammersley and Atkinson (1995) describe some of the difficulties in obtaining consent that is truly 'informed', observing that often the researchers themselves do not know from the outset what information will be collected or what use will be made of the resultant data, and this was true here. The teachers knew that I wanted to look at the teaching and learning that occurred normally in the classroom, that the mathematics focus was addition and that I wanted to focus in on a group of children to observe since it would not be possible for me to observe the whole class at the same time. In all the classes the children worked for some of the time in groups and I negotiated with the teacher which group to observe, based on their knowledge of which children were at an early stage of learning addition. In the reception class at Ashburne, where all the children were at a similar stage, the teacher chose a group that were more confident, able to explain what they were doing and least likely to be put off by being observed. The children were told only that I was coming to see how they learnt maths and they accepted this without comment. This seemed to me, and to the teachers involved, to be sufficient since 'informed consent' is difficult with young children. In all cases the head teachers agreed that it was not necessary for parents to be informed since there was no significant change in the children’s education as a result of my presence in the classroom.

5.6 Chronology of the Study
Data was collected over a period of six months. During this time I visited each school for one day a week, observing one mathematics lesson in each classroom. Because of the relative difficulty in obtaining access to St David’s School, data collection started slightly later here than at Ashburne. In practice this staggered start was an asset since it allowed me to get to know the first school before starting on the second. At the beginning I spent time getting to know the children and letting them get to know me, working as if a classroom assistant so that I would not be seen an unusual in the classroom. During this time I identified with the class teacher which children I would use as the target group. I then carried out initial assessments of the children as described in 5.4.1 above. All subsequent mathematics lessons that happened on that day in the week were then observed, although there were a few days when I was unable to see
any mathematics due to other occurrences in the school such as visiting speakers or visits out of school.

In all I collected data on a total of almost sixty teaching episodes. Such teaching episodes did not necessarily comprise a single lesson. Often there would be a whole class starting activity and the children would then split into groups and I would collect data on the starter activity and the target group’s work. Where the mathematics of group activity followed on clearly from the whole class activity this lesson would be categorised as a single teaching episode. If, however, the focus of the group activity was different from that of the whole class activity then I would collect data on both but categorise these as separate teaching episodes. A teaching episode can therefore be defined as an activity or set activities, carried out by the teacher and target children during a lesson, and with a single mathematical focus.

From the start, field notes were collected in school and these were written up in full after the lesson. At this early stage additional notes were made consisting of initial reflections on the data. Some of these notes were procedural, reminders to look at aspects of the class that I had not collected data on, ask questions of the teachers or to collect copies of the children’s finished work. Other notes were the beginnings of data analysis, reflecting on the teaching and learning seen.

At the end of the data collection period the final assessments were carried out. These took exactly the same form as the initial assessments in order to measure the children’s increased understanding of number and addition. Diagram 5.2 shows a time line of the research process.

Because I have been studying part-time, and changed jobs in the summer of 1997, the analysis and writing up has taken more time than it would for a full-time student.
5.7 Validity and Reliability of the Study

Many writers acknowledge the difficulty of validity and generalisability in qualitative research (Cohen and Manion 1980, Hammersley and Atkinson 1983). Woods (1996) summarises a wide range of qualitative studies and describes how the authors argue against the use of 'validity', 'reliability' and 'generalisability' preferring to use other terms including 'understanding', 'fidelity', 'trustworthiness', 'plausibility', 'credibility' and 'authenticity'. These move away from the positivist view of the world as real and measurable towards a subjective, or intersubjective, ontology.

5.7.1 Internal Validity

Internal validity is grounded in the methodology and some issues have already been addressed. Pollard (1996) observes that “validity is strengthened to the extent that ‘natural’ social processes are undisturbed” (p.302), although as I noted in section 4.5.1, it is not possible to carry out participant observation without influencing the situation under study. I aimed to minimise this affect by familiarising myself with the situation first and remaining, as a far as possible, in observation mode. In section 3.3.3 I wrote that a researcher must be “aware that she has only her own perception on what is happening in the classroom, others may see it differently from their individual perspective and alternative ‘readings’ of the situation may be equally valid.” Qualitative research is of necessity subjective, but detailed and well kept observations, ‘thick descriptions’, allow me to return to the data collected to check my findings against the original observations. This allows me to look for examples that do not fit the theory that emerges as well as those that confirm it, providing a rigorous analysis. This process will be further described in the discussion of data analysis in Chapter Six. In sections 4.4 and 5.4.2, I observed that, far from findings emerging from nowhere, interpretation of data would be grounded in my own personal background and theoretical perspectives and I have tried to be explicit about this in the early chapters of the thesis. So, at each stage, presence in the classroom, data collection, and data analysis I am aware of the threats to validity and have designed ways to minimise these.

5.7.2 Reliability

Reliability of research traditionally addresses the issue of replicability of findings (Guba & Lincoln 1981). If the research were carried out in the same way by another researcher, would the same findings be produced? For an ethnographer this is an difficult interpretation since the researcher herself is the primary research instrument. The researcher cannot observe everything
in the setting; consciously or subconsciously she will notice some things and ignore others. This noticing will be based on individual, personal, previous experiences and interests. Another researcher in the same classroom is unlikely to notice the same things. Furthermore, the same is true of the process of data analysis, which draws on both the data and the experience and theoretical framework of the researcher. The reliability of ethnographic research is therefore based on the production of a thick description of the setting and clarity in the process of data analysis, so that readers can assess for themselves whether the research process has been reliable and whether the findings are logical and plausible, and it is such a thick description that I have set out to produce.

5.7.3 External validity
With respect to external validity (generalisability) the schools chosen were not expected to be representative of all primary schools. Mason (1996) explains that such empirical generalisability has no place in qualitative research. However rigour of analysis and sensitive selection of research contexts can provide theoretical generalisability.

There was no expectation that the classes I chose for the main study would be representative of a larger population. But these classes where not significantly unusual compared with those in which I have previously taught, or observed student teachers. Teachers and teacher educators with whom I have shared my findings have not challenged their ‘normality’. The description and analysis is that of ‘normal’ classrooms and as such may be compared with other classrooms to test the plausibility and credibility of the findings.

Lessons learnt from one situation will illuminate another, even if the two are found to differ. This will be work both for myself and for others who build on the findings. I note Glaser and Strauss’ warning (1967), that researchers must ensure that theory produced as a result of Grounded Theory fits the situation and the data, rather than distorting the data to fit the theory, and that it must be understandable and make sense to others working in similar situations. This can be ensured through the process of testing the findings against further data and by looking for counter examples (as described later in section 7.5.3).

Lincoln and Guba (1985) argue that the development of a thick description of the situation supported by open explanation of analysis allows the reader “to make a reasoned judgement about the degree of transferability possible” (p. 247). The onus is on the researcher to provide adequate information rather than claim generalisability, and on the reader to judge its relevance to their own and other situations.
5.8 Ethical Considerations

Research ethics involve a tension between the right of the researcher to knowledge and the individual rights of those being researched. The first consideration is that of consent which has been discussed in section 5.5.3 above in the context of access. The second issue is that of dissemination of the findings. Cohen and Manion (1994) assert that there are no absolutes when discussing research ethics but suggest three areas which ought to be considered: the sensitivity of the information, the degree of openness of the setting observed and the question of how the findings are disseminated with attention to anonymity and confidentiality.

5.8.1 The sensitivity of the information

I did not consider the data I was collecting to require a high level of confidentiality since the information is not highly personal or threatening and the setting is relatively open. In analysing the data I tried to analyse what happened in the classrooms without criticising the teaching. This proved a problem when I began sharing observations and findings with the wider research community when others have wished me to condemn teachers for what they did or said. While I can agree that in a particular situation I might have wished them to say or do something different, I am not willing to criticise the teachers. Given the pressures on a primary school teacher in the classroom with 30 or more children it is possible to make a decision on what to say or do that would not be made given more time and thought, as Desforges and Cockburn (1987) show. Criticisms of individual teachers and their actions would pose a personal threat that I considered to be unethical.

5.8.2 The setting being observed

Participant observation can be carried out in a range of settings within a continuum of public to private. While primary schools are not open to the public in the same way as say railway stations, neither are they essentially private. Primary schools, especially infant classes, invite parents and others to help with practical tasks, hearing readers etc.. We have all experienced school from a pupil’s point of view at some stage, so there is not an issue of confidentiality in the day to day running of the classroom. I would be careful to keep confidential any personal details about the children which arose while I was there, for it is accepted in any educational situation that some information, particularly about the children’s personal lives, is confidential.
5.8.3 Dissemination of the information - anonymity and confidentiality

Consideration must be given concerning the information that can be shared within the school during the data collection, and how the research is to be published in order to maintain confidentiality. I decided that I would not discuss the teaching I observed with anyone within the school, including the head teacher.

When publishing research on members of the same community of practice there is a clear overlap between those being researched and those reading the research. From the start I have used pseudonyms for both schools and participants in an attempt to maintain anonymity from the wider readership, but this will not extend to those involved, and others who were aware of my data collection, who may be able to identify schools and even teachers as a result of their own insider knowledge (Burgess 1985). This is especially a problem in qualitative research where a small number of cases are investigated, whereas in quantitative research the individual can more easily be ‘hidden’ within the statistics of the whole (Walford 1991). Since it may therefore not be possible to ensure anonymity, I have also tried to maintain a non-judgmental stance with regard to findings, the data being described and analysed but not criticised.

5.9 Summary

This chapter has discussed how the main study evolved from the preliminary one with reference to the focus of the research and methodology, resulting in the formulation of three key research questions and a clearer understanding of the research tools and data analysis. The research method, selection of classes for the study, data collection, and chronology of the study were described, and methodological implications of validity, reliability, generalisability and research ethics discussed.

Chapter Six will consider how the collected data was analysed in the study, and Chapters Seven, Eight, and Nine will present detailed findings from the research, which will be summarised and synthesised in Chapter Ten.
Chapter Six

Methods of Analysis

6.1 Introduction
This chapter will examine the way that the data has been analysed. It will summarise a grounded theory approach to data analysis, discuss how grounded theory techniques have been used in this study, and explain how the analysis resulted in the findings presented in the following chapters.

6.2 Grounded Theory
In Chapter Five I introduced the grounded theory approach to data analysis.

The stages of detailed analysis were:

- orientation and familiarisation with the data through repeated reading;
- open coding;
- axial coding (the development of second order constructs through code grouping);
- analysis of critical incidents;
- examination of relationships and coding for process;
- theoretical sampling;
- verification of statements against the data and the search for counter examples; and
- the location of phenomena in context through the development of a matrix.

Throughout this process the developing theory was recorded using memos and diagrams. Strauss and Corbin define memos as “written records of analysis relating to the formulation of theory” (p. 197). Such memos began with my comments written within the fieldnotes during data collection (see section 4.5.4). Other examples included code notes - written during coding; theoretical notes - noting developing ideas as they emerge; and operational notes - indicating what may need to be done, tried or discussed next. Tables and diagrams provided visual representations of relationships within the data. The codes, memos and diagrams themselves became constructs on which the findings were based.
The following sections (6.3 - 6.9) will discuss the stages of analysis in greater detail with reference to the data.

6.3 Getting to know the data

The first stage of data analysis consisted of repeated reading of the data produced when the field notes were written up. The reading enabled me to become familiar with detail within the data. Notes were made on two aspects of the data: a complete list of teaching episodes was made with a brief description of the content of the episode, for use as an index to the data; and I noted any particular teaching episodes which contained critical incidents; incidents which appeared significant in some way as I read them through (see section 6.5 below for discussion of how these were used).

6.4 Coding

6.4.1 Open Coding

The second stage of analysis was to annotate the data using a process of open coding. Each line of data was examined and code words given that identified key aspects of the content. These codes could be words or phrases from the text itself (in vivo code) or be assigned by the researcher. In addition, whole sentences were considered and coded, and larger sections were also considered in order to identify answers to the question ‘What is happening here?’

In order to exemplify the process, the coding of one teaching episode will be shown and discussed in the following section.

6.4.2 Coding of Lesson Beth 10

The table below shows the coded data from one teaching episode, chosen because it exemplifies many of the analysis stages. Codes are inserted to the left of the data. Between each section of text a memo may be inserted which is comment on the larger section in response to the question - what is happening here? Finally a memo on the whole episode raises issues that arise out of the reading of the whole episode.

The teaching episode is taken from the data collected at Ashburne School in Beth’s reception class. The target group of five children (4-5 years old) are working with Beth.
The group sat around a table with a box of Multilink and a pot of coloured pencils in the centre. They each had a sheet looking something like:

<table>
<thead>
<tr>
<th>Coding</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group Multilink worksheet</td>
<td>The group sat around a table with a box of Multilink and a pot of coloured pencils in the centre. They each had a sheet looking something like:</td>
</tr>
<tr>
<td>8</td>
<td>Beth I want you to choose eight Multilink cubes and put them in your top set.</td>
</tr>
<tr>
<td>multilink set instructions</td>
<td>Beth I want you to choose eight Multilink cubes and put them in your top set.</td>
</tr>
</tbody>
</table>

Section memo: Beth gives clear instructions for what she wants the children to do - procedure. There is no introduction to the task - what they are going to do and learn. Note: Why should an introduction seem important?

| Count words strategies compliance accurate count set procedure problem strategies compliance | Angela and Charles counted out the cubes singly saying the number words as they went, Emma took them one at a time but said nothing aloud, Ian and Jacob each put out a handful, counted them and then added the required extra cubes to make eight.  
Beth Can you put some of the eight in this set and some of them in this set, [indicating the appropriate places on the sheet].  
There was a problem here since more than three cubes would not fit into the set rectangle drawn on the sheet.  
Angela fixed hers together in a 'square', Charles tried to squash his in and the other children let them spill out. All the children placed 4 in each set except Emma who had 3 and 5. |

Section memo: Procedural instructions - I need to explore this. Problem not acknowledged by teacher. Children all devised strategies to overcome / bypass problem.

| Number story hesitant scaffolding altog. make imitation altog. make mistake | Beth Now I am going to go round the group and everyone can tell me a number story, Jacob.  
Jacob Four and four.....(hesitated, pause)  
Beth altogether make  
Jacob altogether make eight.  
Ian Four and four altogether make eight.  
Charles Four and four altogether make ten. (sic - not noticed) |
<table>
<thead>
<tr>
<th>Section memo:</th>
<th>Emphasis on procedural language used in previous lesson. Beth scaffolds Jacob’s language - helps but does not do it all for him. What is the difference between scaffolding and prompting? All others able to use accepted language form.</th>
</tr>
</thead>
<tbody>
<tr>
<td>altog. make</td>
<td>Beth So, everybody except Emma made four and four altogether make eight and Emma made five and three altogether make eight. Now... I want you to change your number story now but to a different number story in the bottom sets.</td>
</tr>
<tr>
<td>different</td>
<td>Jacob made 6 and 2 and read it back correctly, Ian made 5 and 3 and read it back, and Charles made 5 and 3 and read it as 3 and 5. Beth corrected him and insisted he read it from left to right.</td>
</tr>
<tr>
<td>number story</td>
<td>Angela still had 4 and 4 since she had moved each set of four to the diagonally opposite boxes.</td>
</tr>
<tr>
<td>sets</td>
<td>Beth It’s still 4 and 4. Can you break it up and make it different?</td>
</tr>
<tr>
<td>compliance</td>
<td>Angela removed one cube from the left hand set and hid it in her hand.</td>
</tr>
<tr>
<td>procedure</td>
<td>Angela Three and four together make eight (very hesitantly. She looked at the extra cube and put it back on the three but this time to make an 'l shape' rather than a 'square') Four and four together make eight.</td>
</tr>
<tr>
<td>L → R reading</td>
<td>Beth Can you change the bricks? (Angela looked puzzled.) Have a look at some of the others to see if it will give you a clue.</td>
</tr>
<tr>
<td>compliance/</td>
<td>Angela repeated her procedure of removing one and counting the rest (?)</td>
</tr>
<tr>
<td>Misunderstand</td>
<td>Ian She needs one more. (to teacher)</td>
</tr>
<tr>
<td>ing¹ (misU)</td>
<td>Beth Charles, can you help her? Angela, if you have 4 and 3 and it makes seven, how many more do you need?</td>
</tr>
<tr>
<td>re-explain</td>
<td>Section memo: Angela complies with Beth’s instructions but not with Beth’s intentions. She breaks it up, she makes it different, she changes it.</td>
</tr>
<tr>
<td>different</td>
<td>Angela shows clear evidence of misunderstanding. Beth is unwilling to ‘tell’ her how to do it (teacher’s dilemma) and resorts to getting Charles to help - why is this preferable?</td>
</tr>
<tr>
<td>compliance</td>
<td>Beth then went round the table repeating the number stories from each of the other children. Charles helped Angela to make 6 and 2.</td>
</tr>
<tr>
<td>procedure</td>
<td>Angela Six and two altogether makes eight.</td>
</tr>
<tr>
<td>counting</td>
<td>Beth Is that right, have you got eight this time?</td>
</tr>
<tr>
<td>help</td>
<td>Angela (counting) 1,2,3,4,5,6,7,8.</td>
</tr>
</tbody>
</table>

¹ I have italicised the *mis* in misunderstanding to emphasis that the child is not just ‘getting it wrong’ but forming a logical understanding of her own which is not the same as that of the teacher.
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Section memo:  Charles has no hesitation in showing Angela what to do. Angela able to use accepted language form. When challenged, she still needed to count to be sure.

| assessment | Beth Angela, can you show me any more ways of making eight with your cubes? |
| compliance no misunderstanding | Angela (rearranged cubes instantly and said) I’ve done it. Three and five. |

Section memo:  Beth checks Angela’s understanding - perhaps she has not understood what Charles did. Angela is quickly able to comply showing understanding.

Lesson Beth 10:  Critical incident:

Why is there this misunderstanding on Angela’s part? None of the other children have this problem. Perhaps the fact that she puts the cubes together at the beginning so that they fit into the ‘box’ has confused her - but why does she not take them apart when she sees what the other children have done. It is as if the more confused she gets the more closely she attends to the language Beth uses and complies with the instructions without trying to understand what Beth wants from her. The sense is missing and only the words are left.

From detailed coding of all of Beth’s teaching episodes, and subsequently each of the other teacher’s, it was possible to produce a comprehensive list of codes. The purpose of the codes is to break down the data in order to examine and compare. The next stage required me to make some sense of these.

6.4.3 Axial coding

In the next stage of analysis the code words were considered and grouped together to form second order constructs, characteristics that began to describe the situation under analysis. Eventually these groups themselves were also gathered together to form distinct categories on which the findings are based. Strauss and Corbin (1990) describe this process as ‘axial coding’. For example the code words, which indicate physical representations of the mathematics, cubes, counters, Dienes blocks etc., are grouped together to as manipulable materials. These can be further aggregated with language and symbols under the category heading representations. (Throughout this section, words will be italicised when used as code words.) It was at this stage that I became aware of the way in which my background reading and experience were highly influential in the way that the grouping of codes developed. While initial coding had identified elements of the teaching episode relating to how the teacher represented the mathematics to the children, at the stage of grouping these I found myself using the categories
manipulable materials, language, picture and symbols defined in the literature. So, the analysis of representations became more deductive, analysed according to pre-existing categories, than emergent (inductive).

However, the other categories identified were less reliant on the literature. A second group related to code words such as *procedure* and *compliance* indicated something about the way that the teaching was carried out. Beth was directing the children in what she wanted them to do and say. Generally the children complied with her wishes except when Jacob could not remember the form of words to describe his number sentence and when Angela was unable to make sense of the instructions. Non-compliance required Beth to use some other teaching strategy, *scaffolding* or *re-explanation* in these cases, in order that the children could eventually comply. At this stage these were grouped under the heading *controlling*, although they later became subsumed within a wider category labelled *atomistic*, because of the way the task was broken down into small pieces that the children were expected to learn. Set *procedures* and the expected *compliance* of the children were one aspect of an *atomistic* approach to the curriculum. So, the construct *atomistic* arose out of the group *controlling*, and other group characteristics (*small numbers, cardinal representation, developmental stages, repetition*) to categorise one set of data, in contrast to the construct labelled *holistic* which summarised contrasting group characteristics (*large numbers, pattern, relationships, ordinal representation, exploring*). These two categories, *atomistic* and *holistic*, provided overarching descriptions of the mathematics curriculum, which defined differences between schools and teachers, leading to one of the main findings of the research and will be further explained in Chapter Seven.

Grouping therefore began to make sense of the data by grouping code words to form second order constructs. Each construct was given a label that helped define its common characteristics. Groups were then further aggregated to form wider categories, which served to describe key ideas within the data.

### 6.5 The Role of ‘Critical Incidents’

The fourth stage of analysis involved consideration of the ‘critical incidents’ (Lerman 1994) identified at stage one. Lerman defines critical incidents as

ones that provide insight into classroom learning and the role of the teacher, ones that in fact challenge our opinions and beliefs and our notions of what learning and teaching mathematics are all about (p. 54).
For me, critical incidents were occasions where something happened which was out of the ordinary. Sometimes these arose out of reading the episode itself, and at other times were highlighted in the post-lesson discussion with the classteacher. These critical incidents were analysed with respect to the second order constructs emerging from the coded data. Significant differences in coding between the teaching episode containing the critical incident and other, similar but non-critical teaching episodes, led to new insights into the emerging constructs.

In the teaching episode described above, the problems that Angela was having in understanding the task was highlighted as one such critical incident. Both the teacher and I had commented on Angela’s misunderstanding as uncharacteristic. It was necessary to try to identify what made this teaching episode ‘critical’ while others, outwardly very similar, contained no similar misunderstanding.

Comparison of this teaching episode with one from a previous, very similar, lesson where Angela showed no evidence of misunderstanding, was at first unproductive. Apart from the absence of the code misunderstanding, the coding proved very similar. The teacher was controlling in both episodes, there was use of manipulative materials, language and symbols in both. The number used in the critical incident episode (8) was bigger than that in the previous lesson, so perhaps Angela was having trouble with larger numbers, but assessment at the start of the research period showed that she was competent with bigger numbers (she could count to 30 and then in tens to 90). It was therefore difficult to see any significant difference.

Further analysis showed that in some of the lessons the mathematics was framed in the context of a story or real world example. The teaching episode used for comparison had involved a context in which beetles (cut out from card) were moving around on leaves. A hypothesis was formed that for Angela the context (whether it related to the world outside the mathematics lesson) influenced her understanding. Re-coding of all the data with an additional code for real world scripts (after Lesh et al. 1987a) showed that the context in which the mathematics was embedded was significant, especially for the youngest children. As a result of this discovery it was necessary to include a real world script code into the representation group codes in order to differentiate the difference between such teaching episodes. It was at this stage that the literature on representations was identified as significant and further reading carried out (although because of the linear nature of thesis writing, this literature is included in Chapter Two). The significance of real world contexts became one of the findings of the thesis, discussed in Chapter Eight.
6.6 Examining for Relationships and Connections

Further examination of relationships and connections formed the fifth stage of analysis, which relied heavily on the formation of diagrams to map relationships. Such relationships both strengthened and clarified the emergent findings.

Two examples of such relationships will be described in this section. The first shows analysis of the relationships between elements within a category, and the second shows how this relationship is then used to map for progression within the data.

6.6.1 Relationships

As described above, a key area of coding related to the way mathematics was represented to and by the children. Five ways of representing mathematics were eventually identified: manipulative materials, language, picture, symbols and real world scripts (though the characteristics of this later code were still hazy). All teaching episodes included more than one representation so it was necessary to look at how these interrelated. Perhaps the presence of one representation was in itself significant and the others superfluous. Or perhaps the language used was not about the mathematics at all. Each teaching episode was therefore analysed using a pentagonal diagram that contained these elements.

![Figure 6.1 Pentagonal model showing interrelationships between mathematical representations](based on Lesh et al. 1987a)

The particular aspects of each representation were added (which manipulative material was used, what key mathematical language was being used etc.) and the relationships mapped. The following diagram shows the mapping for the teaching episode coded above:
The diagram shows that language was being used to describe the mathematics modelled by the cubes (a), the symbol 8 was interpreted as spoken language ("eight") (b), and physically by the cubes \( \text{multilink cubes} \) (c), while real world scripts and pictures were not used in this teaching episode. It was clear that not only were three representations being used in the teaching episode but also that they were not used separately but interrelated.

It was subsequently possible to analyse the representations used by the teacher and those used by the individual child separately. Consideration of the relationships between these two diagrams, which Strauss and Corbin (1990) describe as 'coding for process' enabled further identification of the learning process.

6.6.2 Coding for Process

While coding and group coding allows for patterns to be seen within the data and mapping relationships enables interaction between representations to be identified, they do not show how these patterns and characteristics develop over time. For example coding may identify that one child is able to use materials, language and symbols to carry out her mathematics but not identify that her use of symbol develops later than the use of spoken language.

In order to identify such changes it was necessary to compare data from different teaching episodes across time. The following example, from analysis of an earlier teaching episode in Beth's class, models how one child, Emma, observes the representations of the teacher (teacher's presentation), begins to imitate these (child's representation), and then in a
subsequent lesson is able to use the representations unaided to carry out a mathematical task (child’s presentation).

This interrelationship is shown in Figure 6.3 below. For clarity the interconnecting arrows have not all been included in this model and representations are given only initial letters: Manipulative Materials, Pictures, Language, Symbols and Real World Scripts.

![Figure 6.3: Progression in the use of representations across more than one lesson.](image)

At (i) Emma is seen to be able to imitate the teacher in the use of manipulative materials, language and symbols in this lesson. In the subsequent lesson she is able to express her mathematics in terms of manipulative materials and language (ii) but not yet able to use symbols unaided. Subsequent lessons could then be examined to identify when, and in what contexts, Emma was able to use symbols unaided.

This stage in the analysis allowed identification of progression both in the teaching and in the children’s learning. Analysis of Emma’s learning in this way identified that she found symbols more difficult to learn than any of the other mathematics representations, an observation which is reflected in the findings in Chapter Eight.
6.7 Theoretical Sampling, the verification of statements against the data and the search for counter examples

Two related processes are used to develop and confirm the validity of emerging theory. The first is that of theoretical sampling which Glaser and Strauss describe as a process whereby during analysis the researcher “decides which data to collect next” (1967, p. 45), returning to the field to collect more data. However, Strauss and Corbin (1990) extend this idea to include theoretical sampling from within the data itself. Analysis of part of the data generates theoretical constructs, and the research “returns to the data themselves” (p. 209). In this study I have used this alternative definition. Constructs emerging from analysis of one teaching episode or class were used to examine as yet unanalysed data.

Once a construct had been built it was necessary to examine whether it was consistent with the whole data, or whether counter examples could be seen which would require further analysis (negative case analysis).

For example, the detection of two main approaches to the curriculum and teaching described above (section 6.4.3) and labelled atomistic and holistic initially arose out of examination of the teaching of one teacher from each school (Beth from Ashburne and Debbie from St David’s). Analysis of the data from the other two teachers (Chris and Eve) showed that these approaches could be validated from this data too, and there appeared to be a consistent approach within each school. Nevertheless, before defining these as characteristics of the teachers in a particular school, it was necessary to look for counter examples in each teacher’s practice. It might be that these characteristics had been identified in only a few teaching episodes and that these had been thought to be representative of the whole. Detailed analysis of each teaching episode to look for counter examples yielded further information. Two of the teachers, one in each school, showed a consistent pattern of an atomistic or a holistic approach to the curriculum with no counter examples. With the other two teachers there was a generally consistent pattern; however a few teaching episodes from each of these two teachers showed counter examples. This observation led to further analysis to identify how and when these counter examples occurred which provided further development of the findings, which would have been missed if counter examples had not been sought.

6.8 Locating a phenomenon in context

Strauss and Corbin (1998) describe how, in order to build theory, it is necessary to “understand as much as possible about the phenomenon under investigation. This means locating a
phenomenon contextually..." (p. 181). The process of locating, results in the construction of a conditional/consequential matrix, which attempts to put together the micro and macro conditions of the phenomenon. Strauss and Corbin argue that “events that occur “out there” are not just interesting background material. When they emerge from the data as relevant, they too should be brought into the analysis. Sorting all this out is where the matrix is helpful” (p. 183).

During analysis of the data from this study the wider social world of the child in and out of school was seen to impinged on the child’s mathematical learning. The micro conditions of learning mathematics (teaching and learning within the mathematics lesson) was affected by the child’s understanding and knowledge of the wider world. This led to the development of a diagram\(^2\) to describe how the mathematical learning of the children in the classroom was influenced by and influenced the wider social world of the children within and outside the classroom was developed through this analysis. This relationship will be further described and analysed in Chapter 9, section 9.2.

\[\text{Figure 6.4 The world of the child}\]

\(^2\) since in mathematical terms this is not a matrix I have chosen to describe it as a diagram.
6.9 Reporting research findings

Although the process has been described in consecutive stages, it was in fact a complex spiral; stages were returned to and re-analysed as the research findings were developed. At each stage of thesis writing, and in the writing of conference papers based on the research, new ideas were developed which required returning to the data. It would be easy to develop theory from the products of the analysis - group codes, relationship diagrams, memos, etc. - which were not in fact based on the data itself. It is therefore important that the eventual findings are also checked against the original data to ensure that error has not crept into the process. This cross checking is the final stage of the process, which provides validity to the findings. Strauss and Corbin (1990) explain that “validating ones theory against the data completes its grounding” (p. 133).

6.10 Summary

This chapter has discussed in detail the methods by which the research data were analysed. The analysis of data followed a grounded theory procedure of coding, axial coding, analysis of critical incidents, and the mapping of relationships both between mathematical representations and across lessons. Theoretical sampling of previously unanalysed data developed and confirmed emerging constructs; findings were further validated through the search for further examples and counterexamples, and the process of locating the phenomenon in context provided a diagram to explore the wider social influences on teaching and learning mathematics. The analysis provided a detailed description of the teaching and learning of addition in the classrooms studied, with clear categories with which to describe the findings of the research.

The analysis resulted in the building of theory which is discussed in the following three chapters, as it relates to the three research questions identified in Chapter Five. Chapter Seven will look at the mathematics of early addition, the way that the curriculum was planned and implemented, and the effect this had on the children’s learning. Chapter Eight addresses the use of the representations by teachers and children, and in particular the role of real world scripts in the teaching of mathematics to young children. Chapter Nine then looks at the classroom as a social and mathematical context in which learning takes place.
Chapter Seven

The Mathematics Curriculum for Early Addition

7.1 Introduction

This chapter analyses the way that the teachers structured the mathematics curriculum and the effect that this had on the children's learning. It sets out to answer the first of the three questions identified in Chapter Five, following the preliminary study:

Q1 How does the way that the addition curriculum is planned and implemented influence children's learning of addition?

Analysis of the data showed that there were clear differences between the teaching in terms of its approach to the curriculum, which had an effect on the children's learning of addition. Two different schools had been selected for the study in order to broaden the data set. They were not chosen in contrast to one another and I had not been expected to find significant differences between teaching approaches at the two. However the findings in this chapter centre around a contrast between what I have come to define as an atomistic approach to the curriculum, with the curriculum broken down into small steps and taught as a sequence, which emerged from the data collected at Ashburne School, and a holistic approach which emerged from the St David's School data.

I will describe how the mathematics curriculum, relating to the teaching of addition, was identified, coded and categorized (section 7.2), look in detail at the types of curriculum experienced by the children in each of the two schools (sections 7.3 to 7.5) and finally show how the analysis of these curriculum types led to the development of the atomistic and holistic definitions (sections 7.6 to 7.8).

7.2 Classification of Mathematical Content

As described in Chapter Two, the mathematics of early addition has been explored from a cognitive perspective in terms of what children can do and understand. A clear progression has been defined from counting-all through to having a complex range of addition strategies including the use of known facts, counting-on, and place value knowledge, though every child does not necessarily pass through each of these stages in the same order. These addition strategies build on children's preschool understanding of number and are summarized as:
• counting all;
  • using apparatus;
  • representing unknown on fingers or something else;
  • unknown imagined visually with pointing to count;
  • unknown imagined in head; leading to
  • counting on from first (with similar representations as above)
  • counting on from highest;
  • knowing the answer;
  • working out the answer from known facts;
  • using knowledge of place value to solve addition of larger numbers.

The Programme of Study for Number in the National Curriculum for Mathematics (DfEE 1995) requires that children at Key Stage One (KS1, 5–7 years) are taught to:

3c: know addition and subtraction facts to 20, and develop a range of mental methods for finding, from known facts, those they cannot recall;

3d: develop a variety of methods for adding and subtracting, including using the fact that addition is the inverse of subtraction;

4a: understand the operations of addition and subtraction, recognise situations to which they apply and use them to solve problems with whole numbers, including situations involving money.

It therefore seemed clear to me at the start of the study that I could identify addition when it was being taught in the classroom. This confidence was reinforced by my initial observations both in the preliminary study at Denton School, and in the main study at Ashburne. The mathematical content of these classes fell clearly into such categories as teaching children to:
understand basic number concepts; ‘count-all’; use a number track to ‘count-on’; and understand place value through the use of apparatus such as Dienes, in order to develop an understanding of numbers greater than twenty.

However when observing and analysing the teaching at St David’s School, identification of the mathematics as addition was far more complex. In only six out of the twenty-seven lessons observed at St David’s did the target children carry out tasks which could be clearly identified as related to children’s development of addition. The majority of teaching episodes concentrated on an understanding of the number system, using numberlines, hundred squares,
and oral counting, and looking at patterns in these. Within these lessons there might be some small element of addition but not as an explicit part of the intended teaching. Yet in the few lessons that did require the children to do addition, and in the dice game assessments I carried out with the target children, it became evident that their understanding of addition and of more complex addition strategies was developing, including adding-on strategies, knowledge of known facts and understanding of adding-on in tens and in units.

This led me to look more closely at how the mathematics curriculum was structured in order to enable the children’s learning.

7.3 Addition at Ashburne School

7.3.1 Introduction - The Atomistic Curriculum

Analysis of the addition curriculum at Ashburne School identified key characteristics of the way that the mathematics was broken down and taught to the children, which I have defined as ‘atomistic’. Atomism is defined as ‘any doctrine or theory which propounds or implies the existence of irreducible constituent units’ (SOED, Brown 1993). The most significant characteristic of the atomistic approach to the teaching of addition was that the curriculum was broken down into elements which were taught in a set order, which related to the findings on children’s development of arithmetic strategies listed above. There was an assumption that if research has found that children develop increasingly complex addition strategies, then the teaching of these strategies in a set order would result in learning. Other characteristics were found also to be related to this approach.

The characteristics of this atomistic approach to the curriculum were identified as:

A1. an initial emphasis on small numbers [small numbers],
A2. teaching addition in isolation from subtraction [isolation];
A3. emphasis on procedures rather than patterns and relationships [procedural];
A4. the use of physical and predominantly cardinal representations of number [cardinal];
A5. progression from one ‘stage of development’ to the next [developmental];
A6. the repetition of similar activities in order to reinforce the procedure [repetition].

In the following section I shall describe how these characteristics emerged from the data with reference to specific teaching episodes. The section is structured to reflect the sequential nature of the teaching, since this was the overarching characteristic of an atomistic approach to the curriculum. The other key characteristics emerge throughout and will be highlighted by their
reference number and key word e.g. [A6. repetition]. In the final section I shall draw together the ideas to show how these characteristics define the curriculum at Ashburne.

Since the children at Ashburne School were divided into classes by age, the first sections we will consider were observed in Beth’s Reception class and the final sections in Chris’ Year One class. The curriculum was determined to some extent by the Nelson Mathematics Scheme adopted by the school (Domoney and Harrison 1995), although in both classes this was adapted by the teacher to meet the perceived needs of the children in the light of their achievements. This adaptation of the curriculum was most evident in Chris’ class, where Chris relied on her long experience of teaching mathematics in primary schools to plan her own activities. At times Beth adapted the activities given by the maths scheme but the new activity always addressed the same area of mathematics in a similar way; the changes related to the materials used rather than the mathematics.

Analysis of the data shows a clear development in the mathematics curriculum for these children which can be seen in table 7.1.

<table>
<thead>
<tr>
<th>Developmental stages in addition</th>
<th>Class at Ashburne School</th>
<th>Mathematical Content in chronological order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-addition number skills</td>
<td>Reception</td>
<td>Confidence with numbers to 10 and their numerals</td>
</tr>
<tr>
<td>Counting-all</td>
<td>Reception</td>
<td>Partitioning and re-aggregation of numbers to 10</td>
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<tr>
<td></td>
<td>Year 1</td>
<td>Solving number ‘sums’ up to 20 with the help of practical apparatus</td>
</tr>
<tr>
<td>Counting-on</td>
<td>Year 1</td>
<td>Using the numbertrack to ‘add-on’</td>
</tr>
<tr>
<td>Place Value Strategies</td>
<td>Year 1</td>
<td>Introduction to place value materials</td>
</tr>
</tbody>
</table>

Table 7.1 Relationship between developmental stages and curriculum at Ashburne

Each of these stages will be considered using typical examples from the study data, to show how the teaching developed and how the children responded.
7.3.2 Confidence with numbers to 10 and their numerals

As in Ann's class in the preliminary study, the children in Beth's class spent many of their early mathematics lessons building on their preschool knowledge of small numbers [A1. small numbers]. Each week the mathematics lessons concentrated on the next counting number which was the focus for counting and counting out activities, reinforcement of the spoken word and introduction to the numeral and written name. Whilst this was not teaching about addition, it formed the basis for the addition work on small numbers. When the children were confident with the numbers and numerals to five, they continued to learn more about the numbers six to ten, and at the same time started to learn about addition with numbers up to five [A5. developmental].

Learning about 8

The following extract illustrates this form of teaching, when the target children were learning about the number eight. Beth was working with the whole class as an introduction to the lesson.

Beth We're going to do quite a lot of work with eight today and tomorrow. Charles can you point out number eight on the numberline.

Charles counts up to eight pointing at each numeral.

Beth Can you tell me a number smaller than eight?
Children 1, 2, 7, 5, 4, 3. (These are accepted)

David 9

Beth Is that smaller than eight?

David (shakes head)

Beth So all the numbers up to eight are smaller than eight.

Charles I can draw a number eight.

Beth Charles thinks that he can draw a number eight. Shall we see if he can.

Charles draws an eight, formed correctly but a bit skewed.

Beth So he started at the top and made an S shape and then went back to the beginning. Sometimes we make an 8 shape with the train tracks or the road track ...

Beth Let's see if we can do it in the air. Use your hand you write with David. (David is using his left hand and is not left handed). All the way down and round and back to the beginning.

Beth goes on to explain the activities she wants the children to do.

(Beth 10)1

In the preliminary study I highlighted that understanding both ordinal and cardinal number was important for the development of a range of addition strategies. The introduction described above addresses ordinal aspects of the number, focusing on locating the number and on its

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1 Transcripts are labelled with the name of the teacher and the sequential number of the lesson observed.
relationship with the numbers 1 to 7 which have been studied in the previous lessons, as well as the formation of the numeral 8. The activities which followed concentrated on practice of writing the numeral and on cardinal number, counting out objects into sets of eight [A4. cardinal]. Beth explains the activities the children will do.

Beth With the plasticine - roll out into a long thin sausage or a long thin line and then see if you can change it into a number 8.

... On the carpet we have got a little tray of sand. We don't usually have sand on the carpet do we? Be careful with it. The sand must stay on the blue tray, and you can use your pointing finger to make a lovely number eight in the sand (demonstrating.) Obviously not a lot of people can crowd around at once so you have to take turns.
Also on the carpet are quite a few activities to choose from. With beads I want you to make a bead pattern with eight. Thread on a number eight (card) and then make your pattern, eight of each colour, and I want you to put your name on it too.
You might want to do a sorting activity, with a friend choose a set ring and a number eight card and I want you to put a set of eight things in the set circle that are the same in some way.

Emma Same colour?

Beth It's up to you. You might want to collect a set of things that are the same colour, or the same shape, or, it is up to you

Beth We've also got number puzzles, you know how to do these.

(Beth 10)

By the end of the week the children would each have completed all the tasks, as well as working with Beth on an activity involving addition bonds to eight. The children experienced a range of activities focused on counting out sets of objects and on forming the numeral. There were no individual, or small group, activities which emphasized ordinal number and relationships.

Subsequent observations of the target children showed that they had no difficulty in selecting objects or using the numerals to record within the range of numbers they had worked on in class. It would seem that the activities had been successful. Although they were able to use written numerals to label sets of objects correctly, they were unable to use the numerals as a representation of quality on which to act. When playing the dice game (see section 5.4.1 for details) with one dotted and one numeral die, they could not interpret the numeral as a quantity. For example, if they threw:
They either ignored the 4 altogether, claiming their score as the number of dots on the other die (3), or counted the symbol as one, whatever it represented, and counted-on the dots on the other die (1; 2, 3, 4). Despite time spent on learning about small numbers and practising the numerals, while they understood that the numeral could be used to represent a physical quantity of items, they could not interpret a numeral as a quantity of unknown items.

**Ordinal number**

Other activities also addressed the relationship between small numbers, for example the class sang the number rhyme ‘Five currant buns in the baker’s shop’, using five picture cards of buns labelled with the numerals 1 to 5 which had to be ordered before the rhyme began. These activities fulfilled what Beth thought important for the children at that stage of learning:

> We're mainly looking at ordering and relationships between numbers at the moment. Numbers one to five. The children are beginning to say ‘that's one more' or the difference between them. Just beginning to do some 'add' in rhymes etc.. I want to go slowly because I've found in Y2 (the class she taught before) that if they haven't got this relationship they get lost. So, I'm going very slow really. (Beth 2)

The teaching episode described below followed this observation from Beth and illustrates her approach. The children were given number tracks labelled 0 to 5 and multilink bricks which they had to make into towers.

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<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
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</tbody>
</table>

**Beth** Can you make a tower to match each one? Put your tower next to the right number. Why are there none here? (Pointing to the zero on the track)

**Charles** It's none. They look like stairs.

**Angela** We can go upstairs and downstairs (moving her finger up and down the 'stairs'.)

**Beth** Which is the most, the biggest number...

**Emma** five

**Beth** and if we look at our towers that's the tallest. Who can tell me which is the smallest number?

**Jacob** One

**Beth** Which number comes after two?

**Jacob** Three
Despite Beth's acknowledgement of the importance of ordering and relationships between numbers, there were few other lessons which had this as the focus. As in Ann's lessons in the preliminary study, most of the subsequent lessons concentrated on cardinal rather than ordinal aspects of number and addition [A4. Cardinal], as we will see in section 7.3.3.

In the context of towers and number tracks the children were able to talk about the relationship between numbers and their position in the counting sequence, however, when Beth carried out an assessment of their mathematical knowledge as part of the Baseline Assessment\(^2\) towards the end of my data collection period, none of the children in the target group were able to give an answer to "which number comes before ...", in the range 1 to 10, when there was no number track or sets of objects to which to refer. The children were experiencing a curriculum that does not appear to concentrate sufficiently on ordering and relationships between numbers for them to develop a mental representation of ordinal number and number relationships.

**Summary of confidence with numbers and their numerals**

In both the examples given above, the curriculum experienced by the children appears appropriate: the activities are related to learning objectives, the activities are carried out successfully and the learning outcomes indicate success. As we have seen, their learning about number appears to be closely related to the context of the activity and not generalizable, or internalized mentally [A3. procedural]. The target children could read and write numerals and use them to label a given set, but not to use the symbol as representative of an unknown set. They were able to say the number before or after a given number when using number tracks and towers, but not in their Baseline assessment task where there were no external representations to which to refer.

Analysis of teaching episodes involving confidence with number to ten and their numerals showed significant atomistic characteristics. There was, understandably, a concentration on small numbers [A1 small numbers] in sequential order [A5 developmental]. Learning was related to the context of the activity and not generalizable [A3 procedural]. While a few activities addressed ordinal number, most used physical, cardinal representations of number

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\(^2\) Baseline assessment tests are carried out on children in the term after they reach statutory school age and are used to assess progress when compared with Key Stage One SATs, carried out at the end of Year 2.
A core of the teaching and learning of addition in Beth’s reception classroom during the period of the study, centred around the idea of partitioning. Partitioning can be defined as the separation of a number into two or more, not necessarily equal, sets. This is the inverse of aggregation which combines two or more sets to find a total, and partitioning can therefore be seen as a form of subtraction. The way in which Beth used partitioning was to concentrate on the additive nature of the subsets (two red beads and three blue beads together making five) rather than on the difference between the original number and one of the subsets (five beads, two of them are red, how many blue?) In other words having partitioned the quantity into subsets, these were re-aggregated, introducing the children to the concept of addition as aggregation.

The Nelson mathematics scheme takes each of the counting numbers in turn and recommends activities which emphasize the act of partitioning and re-aggregation. The teaching episodes observed on partitioning and re-aggregation consisted of:

- Animals in fields: partitioning 3 plastic animals across two fields;
- Lucky Dip: partitioning a set of 4 bricks according to colour;
- Beetles and leaves: partitioning 5 beetles on either side of a leaf;
- Multilink cubes: partitioning a set of 8 cubes into two sets
- Children on the bus: partitioning 10 children upstairs and downstairs on a bus.

The teaching episodes followed a similar format. The teacher, Beth, worked with a group of five or six children. Some form of practical apparatus was used and each child had the target quantity of apparatus and partitioned this quantity to form all the possible number bonds of the target number [A1. Small numbers, A6. repetition]. The ‘Animals in fields’ lesson was the first of the partitioning lessons for these children and set the teacher’s expectations in terms of the way that the group would interact, the way that language was used in describing the mathematics and the way that the mathematics was to be recorded [A3. procedural].
Animals in fields

The teaching episode started with the children around the table with Beth. On the table were sets of three farm animals and a card for each child with two irregular ‘fields’ drawn on.

Beth What have we got today?
All Animals, Farm animals
Jacob We’ve got three pigs and three cows.
Ian Three cows.
Emma Three sheep.
Beth That’s odd isn’t it? All in sets of three. I’ll give you a set each, all got a little set of three ... stand them up.

Beth went around the circle saying e.g. ‘So, Emma’s got ... three sheep’.
Beth Now we’ve all got a little picture of two fields. Can you put your animals in the fields. Let’s have a look at our animals. Ian, how many in that field
Ian 3
Beth and in that field?
Ian None
Beth How many altogether?
Ian Three
Beth So you started with three and you’ve still got three in all your fields.
Beth Jacob, tell me about your fields.
Jacob Got 2 pigs in that field and 1 pig in that field.
Beth How many altogether?
James Three
Beth You’ve still got three. What about you Emma?
Emma I’ve got two and one but altogether still got 3.
Beth What about you others? Angela?
Angela Two and one more is three.
Charles Two in that field and one in that.
Beth How many altogether?
Charles Three

All the children accepted the instruction to place the animals in the ‘fields’ and the use of the animals to partition. Emma readily took up the teacher’s language to describe her mathematics [A3. procedural]. Angela was able to include the necessary information in her own words, while James and Charles needed prompting to recognising the importance of the total and in constructing the relevant sentence.

Beth Now, can we all make our sets look like Ian’s? (Ian has 3 and 0) Can you tell me a little number sentence about what you’ve got here?
Angela We’ve got three and none (Charles interrupts with zero) and altogether got three.
Beth Now, all the cows walk into the other field. Tell me a different number sentence.
Ian We’ve got zero and three and altogether got three.
Beth Can you make one of your animals walk into another field?
What have you got now?
Jacob Got one in that field and two in that one.
Beth So, one and two altogether make three. What if another one
walks across?
Angela Two and one
Beth (prompting) altogether
Angela Altogether makes three.

(Beth 4)

Once all the number combinations of three had been rehearsed, the children were required to
record their work in their mathematics books. Beth had already drawn two sets of two fields in
their books and the children had to draw animals in the fields and write the number sentence
(e.g. \(3 + 0 \rightarrow 3\)) underneath. The drawing they did unaided, and they were able to write the
appropriate numeral under each set, but they needed help from Beth to write the other symbols.

Learning addition - aggregation

The main focus of the teaching episodes was the use of appropriate mathematical language and
the recording of addition, related to the use of manipulable materials [A4. cardinal].
Observation of the teaching episode showed the children learning to use materials, the language
and written recording chosen by the teacher [A3. procedural] in order to represent early
addition, with apparent success. Their learning is procedural [A3.], developing skills rather
than concepts and strategies. In playing the dice game at the start of the observation period they
showed that they could already combine two sets, represented on dotted dice, to give a total, by
counting all. The same was true at the final assessment at the end of the data collection period,
the only difference being that they could now use the symbol on the number die to represent a
quantity. After six months they had learnt more about the use of numerals, to be able to
represent the numeral on their fingers in order to count all, a small step further forward in their
understanding of addition.

In learning addition, the children were experiencing a curriculum with similar characteristics to
their learning of number. Small steps [A5] were used as they look at the partitioning and re-
aggregation of each small number in turn [A1], using practical equipment [A4]. Despite the
opportunity to discuss the relationship between addition and subtraction afforded by the
partitioning (-) and re-aggregation (+) model, the children were taught addition in isolation [A2
isolation]. The children were tightly controlled in order to reproduce the teacher's set
procedures of actions, language and symbols [A3], and this was reinforced by the repetition of
similar activities, language and formal symbols. There were some activities which did not
conform to these characteristics and these will be considered in section 7.5 below; but most of
the lessons in Beth’s classroom shared these characteristics.
Were these characteristics of Beth's class alone? Analysis of the lessons in Chris' class would answer this.

7.3.4 Solving number 'sums' up to 20 with the help of practical apparatus

The children in Chris' class had already had the kinds of experience described in Beth's class and were moving on to addition with larger numbers. My initial assessment of the target group showed that they were able to read numbers to 10, most of them to 30 and beyond, and that they were still using counting all as their only strategy for addition. This was confirmed in the first two lessons I observed.

The focus of these two lessons was the solution of written number 'sums', using numbers to 20 [A1. small numbers], presented in two different forms, in the first lesson as equations with a missing addend and a total of ten e.g.

\[ 4 + \Box = 10 \]

and in the second lesson as mapping diagrams

\[ 12 +3 \rightarrow \Box \]

The children were encouraged to use fingers, counters or other apparatus to help them and were told at the start of the lesson that they would be expected to explain to Chris how they had solved them when presenting their work for marking. All the children solved the problems correctly, the majority of the children describing a partitioning strategy for the missing addends and a counting all strategy for the mapping sums, representing the numbers with practical equipment or fingers [A4. cardinal]. Of the target group, only Geoff was seen to use counting-on (using his fingers to represent the addend and counting-on from the augend giving each finger the next count word) but when explaining to Chris how he had carried out the task, he said that he had used counting-on, but could not describe procedure.

These written number 'sums' were given with no expectation of relationships or patterns between the answers [A3. procedures]. The numbers given had no regularity, which could emphasize commutativity or sequential patterns. After completing the missing addend sums the children were given 10 by 10 peg boards and asked to make a pattern in two colours, to show all the combinations of ten. The children did not understand this instruction so Chris showed them how to start off the first two rows. The children completed the pattern visually without counting any of the pegs (figure 7.1, overleaf).
When the children showed their completed patterns to Chris she made no link to the ‘missing addend to 10’ problems they had just completed, and the children made no reference to it. We will see later how pattern was an integral part of the mathematics at St David’s but here Chris, having planned the pattern task to follow the calculations, appears aware of the relationship, but does not make the relationship explicit to the children.

![Figure 7.1 Peg board showing number bonds to 10](image)

There was little direct teaching at this stage. The children were expected to carry out skill based tasks and, while Chris modelled adding-on in her introduction to the second lesson, she encouraged the use of manipulatives to solve the tasks [A4. cardinal].

<table>
<thead>
<tr>
<th>Chris</th>
<th>So, this is the sheet we are doing. You can see the sign here, what does it say</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>Add</td>
</tr>
<tr>
<td>Chris</td>
<td>So, were doing some addition. So, what do you think you are going to have to do?</td>
</tr>
<tr>
<td>Helen</td>
<td>Use counters</td>
</tr>
<tr>
<td>Chris</td>
<td>Yes, what sort of sum?</td>
</tr>
<tr>
<td>Will</td>
<td>adding</td>
</tr>
<tr>
<td>Chris</td>
<td>I want you to work out the sum and write in the equals sign by the side of the two numbers and outside I want you to write the answer. (Draws on the board 3+6 =) What is the answer?</td>
</tr>
<tr>
<td>Children</td>
<td>(shouting out various answers) 12, 10, 8, 9</td>
</tr>
<tr>
<td>Chris</td>
<td>(when she hears 9) Yes because it is 6 in your head (touches head) and 7, 8, 9</td>
</tr>
</tbody>
</table>

(Chris 2)

Chris modelled a counting-on procedure but there was no further discussion of it, and no expectation that the children would use it. The message that seemed to be taken by the children was from the interchange with Helen “use counters”, “Yes”, since when competing the task all the children needed some form of counters or used fingers to count-all, except Geoff who again used his fingers to count-on.
As a result of the children's individual discussion of their strategies with the teacher, Chris decided that they were able to solve addition problems using manipulative materials and needed to be taught to count-on [A5. developmental] for, as she observed to me:

They have to use the number line but still need lots of practice to add-on. (Chris 2)

### 7.3.5 Augmentation and the Number track

As we saw in Chapter Two, the move from counting-all to counting-on is a key idea in the development of addition concepts and skills. The key strategy that enables the confident mathematician to move on to the addition of larger numbers is that of adding-on or augmentation.

Use of a number track to solve addition problems appears to be a kind of halfway house between, or perhaps a synthesis of, cardinal and ordinal number. The track is divided into squares, each labelled with a counting number in order. Since the squares can be counted by the numbers labelling them, it is usual not to include an initial square labelled 0. Some teachers have an initial square labelled 'start', as might be present on a game board, in order to allow the children somewhere to place their counters before carrying out the first movement.

<table>
<thead>
<tr>
<th>Start</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
</table>

I will call these number tracks, although different teachers and mathematics schemes sometimes refer to them as number strips or even number lines, an expression I reserve for a single line with points along it labelled with the numbers, and which can be used to indicate the relative position of all real numbers:

\[
\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & \cdots
\end{array}
\]

Chris used number tracks with her Year One class specifically in order to encourage the children to count-on. As we saw from her comment above, she saw 'adding-on' as an important step in addition in which the children required more practice. Chris therefore decided that the children needed to work on the number track. The four teaching episodes that followed focused on the number track, the first three of which were planned by Chris but taught to the target group by the Specialist Teaching Assistant, Mary, who worked regularly with the
children, supporting the teaching of the 36 children in Chris' class (see Chapter Five, section 5.5.2).

**Number track games**

In the first of these the children had made a 1 to 20 number track which they used to play a game [A1. small numbers]. The children took turns to throw a die and put that number of counters on the track. On second and subsequent turns they were required to use the language of adding-on.

Barry threw five and placed five counters, one each on the squares marked 1 to 5. The next time it was Barry's turn he threw another five.

Mary So, count out five to one side first of all, and then count-on from that number and put the cubes down one at a time.

Barry 5, 6, 7, 8, 9, 10. (Chris 3)

This game formed a link between the use of discrete objects e.g. counters to model addition and the use of the number track, between cardinal and ordinal number. In the next lesson Mary introduced the children to the use of the number track with a single counter to emphasize ordinal number. Here the number is represented by the final count only so that 3 add 4 would be shown by placing the counter on the three and counting 4, 5, 6, 7 as the counter is moved to add on 4. The counting-on procedure requires the child to keep track of both the addend (1, 2, 3, 4) and the count words (4, 5, 6, 7) at the same time.

The game was played in the same way. The children were encouraged to count-on (i.e. say 4, 5, 6, 7 rather than 1, 2, 3, 4 as they add on four) and Mary reinforced the language of counting-on [A3 procedural]. For example, Barry is on 7 and throws a three; he moves along his track:

Barry 8, 9, 10
Mary So, 7 count-on 3 is 10
Barry 10 (Chris 3)

**Frog on the track**

The third of these lessons was also taught by Mary, the STA, and this time the number track activity was introduced in the form of a story.
Mary: This is Freddy Frog. He is very good at jumping but he is not very good at his numbers. If he is adding he starts at the left and goes to the right. We're going to do some adding with little stories. So watch carefully. We've got five flowers in a vase and someone comes and puts in three more. Where is Freddy frog going to start? (5) and how many jumps (3) so how many altogether? (8)

The children take it in turns to use Freddy Frog and the number track to solve their problems:

Mary: Geoff, eleven boys playing football and two more join in.
Geoff: Thirteen

Mary: Joanne, fourteen girls skipping and three more come and join them. How many girls can Freddy see skipping?
Joanne: Seventeen

Mary: Helen, twelve cows in a field and two come and join them.
Helen: Fourteen

Each of these children is able to use the frog and track to find the answer, placing the frog on the first number given and moving it in jumps to the answer.

(Chris 5)

By this third lesson each of the children understood that they had to place the frog on the first number and then count-on. They all solved the problem questions correctly except Dipesh who refused to try. Since Dipesh had English as an additional language, it was difficult to identify whether he did not understand the language, the social complexity of the task, or the mathematics. I had identified in my initial assessment that Dipesh was uncertain of the teens number names, which may have caused his reluctance to try the counting-on task. Most of the children placed the frog on the first number and counted-on along the track until they got to the answer. However, they were not encouraged to articulate the counting-on as they had previously been. I noted that the children were solving the problems by counting the addend along the track e.g. counting 1, 2, 3 instead of 6, 7, 8 when solving 5 + 3, and reading the answer from the number track. While this is the easiest way to deal with both the addend and the total when using the number track, giving the correct answer, it does not model the counting-on strategy needed for mental arithmetic when no number track is present. If the child mentally counts only the addend they do not get a total. In the previous two lessons Mary had encouraged the children to use the more difficult counting-on strategy which they had problems with. Now they are not being encouraged to do this, they appear to have reverted to the more simple, number track dependent strategy [A4. procedural].

The apparent success of this lesson, in terms of the children being able to solve the word problems using the frog and the number track, does not demonstrate an increased understanding of the counting-on procedure. The children appear to have learnt to use the number track to help solve addition problems successfully, but not how to add-on mentally. At the end of this lesson Mary tried to explain the need to work mentally.
Mary: Now, imagine that you've got no number lines and no equipment whatsoever. So what could we do (to solve addition problems)?

Geoff: We could use our fingers.

Mary: But how many fingers have you got? (10) So, we can't do big sums. What else could we use?

Joanne: Our heads.

Mary: If we use our heads what will we do?

Geoff: Think.

Mary: If you've got five sweets and add five more are we going to go 1,2,3,4,5; 6,7,8,9,10? No? What shall I start with? (5)

So I go 5: 6, 7, 8, 9, 10. In order to help us count-on using our heads and our fingers we are going to do it like this:

Six boats on a lake and two more come.

I'm going to put that number six in my head, 6 (touches head) 7, 8 (counts on fingers.) Geoff, eight and three?

Geoff: 8; 9, 10.

Mary: Add three

Geoff: 9, 10, 11.

(Chris 5)

The children were now required to count-on without the number track to help them. None of the children successfully used a counting-on strategy. Geoff, who had previously been able to use counting-on successfully, tried and immediately ran into the problem of trying to keep track of the addend while counting-on to the total, and is unable to solve the problem unaided.

In the fourth and final lesson, Chris gave the target children a selection of written ‘sums’ to solve using the number track. They were all able to do so within the range of numbers to 20 but only using the number track. None of the children was able to solve them by counting-on mentally for the addition of more than two.

**Learning to count-on**

The lessons on using the number track to count-on did not appear to have moved the target children on in their addition strategies. Counting-on appears to be a difficult skill to teach by using a number track; the mechanics of counting-on, of keeping track of more than one count at the same time, confuses the children.

These lessons appear to be trying to do two things:

- teach the children the skill of using the number track to model addition; and

- provide the children with a model of number which will enable them to count-on mentally.
The teaching has emphasized the first of these [A4 procedural]. The children can use the number track to model a counting-on procedure and find a correct answer. But it does not seem to have been successful at achieving the second, the ultimate aim. The children had learnt how to use a number track to count-on, but not learnt to use a counting-on strategy as a way of solving addition problems. This observation reinforces other, similar observations from my own teaching and that of other classes I have observed. As Lewis notes, “the child may learn the apparatus rather than the concept and not be able to generalise the mathematics” (1996, p. 134).

However, Chris’ observation was that they could now use an adding-on strategy successfully and were ready to move on to an understanding of place value [A5. developmental].

7.3.6 Place Value

Chris considered that once the children were secure in the addition of small numbers they were ready to be introduced to the concept of place value so that their addition strategies could be extended to numbers greater than 20 (though initially, as we will see, Chris still uses relatively small numbers). So far, I had noted no mathematics involving numbers greater than 20 at Ashburne except outside of mathematics lessons, for example, in Beth’s class the number of children present at school was often discussed, involving the use of numbers to 30. I did not think it a coincidence that in my assessment of Beth’s children they were all able to count more or less reliably to 30 since this reflected the number of children in the class. In Chris’ class I did not observe any discussion of larger numbers. Although they were a year older than Beth’s children, the target group children in Chris’ class were also all found to be able to count orally, to at least 30, except for Dipesh who was seen to have difficulties counting orally beyond 14, at least in English, and could not read the numerals beyond this. In neither class were there large numbers on display.

The introduction to place value was again based on a cardinal view of number and consisted of three activities carried out with Chris and the target group. These took the form of a game, the only difference being that in the first two lessons the numbers were modelled using straws and bundles of ten straws [A4. cardinal], while in the third lesson units and ten sticks of Dienes apparatus were used [A6. repetition]. The following observation was made during the second lesson.

Each child was given a base board on which to work and a card in the teens or early twenties as their target number [A1. small numbers]. The target numbers were all different; the magnitude of the number was selected for the individual child.
A pot containing straws and a box with bundles of ten straws was placed in the centre of the table. Chris would bang a tin and the children were required to pick up one straw for each bang, and place it in the units section of the base board. When they had ten single straws they had to exchange them for a bundle of ten and place it in the tens section. When they had accumulated sufficient straws to match their target number they stopped. Although this was described as a game there was no element of competition or way of winning.

Chris Right you should all have a number and the word for it. You need singles and these (indicating the bundles of 10). We are all making a number. Are they all the same?

Children No

Chris I hit the tin with a beater and each time you hear the sound you take one straw. But when you get to ten you exchange ten ones for a bundle of ten.

Chris hits the tin and the children take a straw.

Chris So, where do you put that one? (The children point to the units box.) In the box, good. ...

When the children all had ten straws Chris waited.

Chris How many have you got

Dipesh 10

Chris You should have ten in there (indicating units box.) I thought we did something last time.

Geoff Exchanged

Chris Yes, you don't leave ten in the units box.

The children all changed their units for a bundle of ten.

Chris Now remember what number you are making and you've got one ten which is that one at the beginning (of the numeral on the numeral card).

The game carried on as before, each time one straw being taken.

Dipesh got to 13, his target number, and stopped. The play went on to 14.

Chris Dipesh, how many have you got

Dipesh 4 (counting the bundle of ten as a single unit)

Chris You've got ten, eleven, twelve, thirteen (indicating the bundle of ten, then counting-on the units).

When all finished Chris gave them each another card related to their target number. For example Dipesh's card read:
The children had to ask for the correct tens card, 10 or 20, and the correct units card. They placed these in the empty rectangle so that the units covered the 0 of the tens card.

The aim of these lessons was to develop an understanding that numbers greater than ten consist of a number of tens and units. This is the convention for writing these numbers and is one of
the strengths of the number system. For numbers greater than twenty this is mirrored in the way that the words are said but for the teens numbers this is not so. In fact, this makes restricting the numbers to teens and early twenties a less than useful strategy [A1. small numbers]. However, as noted above, the target children were not confident with the number names greater than 30. The children were therefore learning the symbols and the use of a range of manipulative materials to model place value [A4 cardinal]. While they learned to exchange ten units for one ten and place these on the correct part of the base board, as with learning to use the number track, they seemed to be learning how to use the apparatus, rather than understanding the concept of place value [A3. procedural].

7.3.7 Summary of addition curriculum at Ashburne School

Analysis of the teaching in the two classes at Ashburne school led to characteristics which could be identified in both classes. I chose to describe the approach used as 'atomistic'; that is the curriculum area had been fragmented into its constituent parts as identified from the literature on children's understanding of addition. This approach assumes a direct correlation between that which is taught, the element of the curriculum, and that which will be learnt, the next stage of development. The teaching approach based on this atomistic approach to the curriculum I have elsewhere described as 'Arrow' teaching (Coles & Price 1997). The teacher identifies the next stage of development through analysis of the children's achievement, and directs the teaching to that area, as if by an arrow directly to the target.

The atomistic approach aims to identify the children's learning needs and target these with the activities which address the next stage of development, but it would appear that to break the mathematical content down into such small pieces may only teach the children how to complete that task in an instrumental way, to comply with the teacher's accepted procedures, losing the larger picture of how these ideas interrelate (Skemp 1971; Hiebert 1986). While recognising the problems in this approach to teaching, I do not want to indicate that Beth and Chris were in any way poor teachers. The approach they were using is a common one in infant classes, the focus of many mathematics schemes, and the way that I was trained to teach mathematics. In Chapter Three (section 3.3.1) I noted that Ofsted inspectors found that "In KS1, teachers were more likely to introduce new work to a small group of pupils while in KS2 the pupils frequently learnt new topics largely on their own from written texts" (Ofsted 1993b, p. 9), as a result of which more lessons at KS1 were considered to be satisfactory than at KS2. Nothing is said in the Ofsted report about the way the curriculum was broken up, but in terms of teaching approaches, introducing work to small groups rather than individualised work, the teaching at Ashburne appears to conform with Ofsted approved methods. The advantages, and disadvantages of atomistic approaches will be discussed towards the end of this chapter (section 7.7).
We have seen how the data shows the following characteristics of this atomistic approach to the curriculum:

A1. an initial emphasis on small numbers [small numbers] - teaching about individuals numbers to ten; addition to 10, to 20 etc.

A2. teaching addition in isolation from subtraction [isolation] - despite the link between partitioning and aggregation;

A3. emphasis on procedures rather than patterns and relationships [procedural] - procedures for use of the number track and base ten materials, procedural language forms and formal recording;

A4. the use of physical and predominantly cardinal representations of number [cardinal] - animals, bricks, counters, base ten materials, with little encouragement to see beyond the concrete to the abstract (opaque);

A5. progression from one ‘stage of development’ to the next [developmental] - number, single digit addition (counting-all), counting-on, place value;

A6. the repetition of similar activities in order to reinforce the procedure [repetition] - series of lessons on partitioning and re-aggregation, number track activities, place value activities.

These characteristics were not only seen in the lessons observed; they were also evident in the school’s planning documents for mathematics at KS1, showing that it was a curriculum, rather than individual teachers’, approach.

How do these compare with the curriculum approaches at St David’s School?
Chapter 7  The Mathematics Curriculum for Early Addition

7.4 Addition at St David’s School

7.4.1 Introduction - The Holistic Curriculum

Analysis of the addition curriculum at St David’s School identified very different characteristics in the way that the mathematics was broken down and taught to the children, which I have defined as ‘holistic’. The most significant characteristic of the holistic approach to the addition curriculum was an emphasis on the number system, with calculation seen as one element of the relationship between numbers within the overall system. Further analysis showed other characteristics were related to this approach.

The characteristics of this holistic approach to the curriculum were identified as:

H1. an emphasis on large numbers as well as small ones [large numbers],
H2. an emphasis on patterns in number and pattern spotting [pattern];
H3. discussion of the inverse relationship between addition and subtraction [inverse];
H4. emphasis on relationships rather than procedures [relationship];
H5. emphasis on ordinal representations of number and the development of mental imagery [ordinal];
H6. a eclectic curriculum in which the children are immersed in a wide range of activities, with little apparent sequence [immersion].

Debbie and Eve taught in parallel classes, both containing Reception and Year One children, aged 5 to 6: because of the admissions policy in the county, children did not start school until they had reached statutory school age (5). The school had its own scheme of work for mathematics which drew on a range of published scheme materials, and on ideas from the National Numeracy Project, introduced through the LEA mathematics team. Analysis of the teaching content in the classes at St David’s shows how little sense of developmental progression is evident compared with that at Ashburne. The activities can be seen to contribute to the children’s developing understanding of addition strategies, but were not being taught in sequence nor with the explicit intent that the children would learn to move on to the next stage of development. Apart from a series of lessons on Data Handling in Eve’s class, which I will discuss later, there was no sense of working through consecutive numbers or building on previous lessons in the light of evaluation of the children’s perceived learning as could be seen at Ashburne. Table 7.2 summarizes the teaching episodes observed at St David’s School.
The Mathematics Curriculum for Early Addition

Chapter 7

<table>
<thead>
<tr>
<th>Developmental stages in addition</th>
<th>Class at St David’s School</th>
<th>Related activities, not chronological</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-addition counting skills</td>
<td>Debbie and Eve</td>
<td>Oral counting in units and larger steps, forwards and backwards</td>
</tr>
<tr>
<td>Counting-all</td>
<td>Debbie and Eve</td>
<td>Sometimes used by the children but not taught</td>
</tr>
<tr>
<td>Counting-on</td>
<td>Debbie and Eve</td>
<td>Ordinal number in counting, hundred square, but no overt teaching of counting-on</td>
</tr>
<tr>
<td>Place Value Strategies</td>
<td>Debbie and Eve</td>
<td>Analysis of the hundred square, no cardinal representations</td>
</tr>
</tbody>
</table>

Table 7.2 Relationship between developmental stages and curriculum at St David’s

It was possible to divide the arithmetic curriculum into four areas:

- Counting - oral counting in units and in larger steps;
- Locating - finding the position of numbers in the number sequence;
- Pattern spotting;
- Exploring addition.

Analysis of the data with respect to these categories helped to define the characteristics of a holistic approach to teaching addition, as the following sections will show. The sections will describe typical activities within each of these categories. However they do not provide the depth of detail seen in the analysis of teaching episodes from Ashburne School. I believe that this is due to three, interrelated factors. Firstly, the eclectic nature of the curriculum, a characteristic of the holistic approach at St David’s, which had made identification of addition difficult, also made it difficult to show how the curriculum developed over time. I had collected many individual teaching episodes which could all have been included since they were all different, but this would not benefit the reader. Selection of a few teaching episodes which show the kinds of mathematics being addressed will, I believe, provide an overview of the curriculum.

The second reason is that this method of teaching provided me with fewer critical incidents than the curriculum at Ashburne. Because there was no expectation that the children would learn a particular piece of mathematics, there was no surprise if they did not do so. At Ashburne, the
expectation was that the children would comply with the teacher’s expectations, while failure to
do so often resulted in a 'critical incident'. But at St David's, the expectation was that the
children would 'take part'. Evidence of their learning came from increased participation and
from my assessment of the target children, which will be discussed in section 7.4.6. Thirdly,
most of the interactive teaching at St David's was whole class and detailed field notes were
taken of all of these. When the children worked independently from the teacher they would
work in different groups, as pairs, and as individuals according to the task. It was therefore
more difficult to track a single group of children in the way that I had done at Ashburne.

7.4.2 Counting

In both classes there was a heavy emphasis on counting, especially in the whole class
introduction to the mathematics lessons. This was not the counting of objects that had been
seen in Beth's class in learning about small numbers, nor counting along the number track to
encourage counting-on in Chris' class, but oral repetition of the number words in units and in
larger steps, counting in twos, fives, tens etc., forwards and backwards.

In the first lesson I observed in Debbie's class, the whole class were sitting together on the
carpet.

Debbie Hands on heads. Close your eyes. Count in your heads as high as you can go ... Stop. How many Samira?
Samira 20
Debbie Right let's all count together to 20 starting at zero
Class³ 0, 1, 2, ..., 20, 19, 18, ..., 0
Debbie Right close your eyes again and start counting ... stop. Mark?
Mark 11
Debbie Count on in twos from 11
Class 11, 13, 15, 17, 19, 21.
Debbie Right close your eyes again and start counting ... stop. Henry?
Henry Two
Debbie Count in twos from 2
Class 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.
Debbie Can we have a go at counting back in twos? That is really clever.
Debbie and some children 20, 18, 16, ..., 0.

At the end of the lesson Debbie explained that asking the children to
put their hands on their heads had three effects. First it settled the
children, it is difficult to move around much with hands on heads,

³ 'Class' here indicates that the teacher and children joined in with the counting. Not necessarily all the children
said every word.
secondly it stopped them from using their fingers to count and calculate and thirdly it emphasizes thinking in your head. (Debbie 1)

This I coded as 'counting' and as such was not obviously part of the study of early addition. However, further analysis showed that it was teaching essential skills towards the development of mental addition strategies where numbers are added by counting-on in units or larger steps. The children were developing mental and oral, ordinal representations of number which will help them count-on later [H5. ordinal]. Debbie's explanation of 'hands on heads' indicated the importance she placed on developing mental images rather than relying on physical representations. The children were also learning to count on from a number greater than one. By asking the children to start counting, then stopping them and asking where individual children had reached in their counting, Debbie generated a new number each time to use as the start for a subsequent count sequence. The children were therefore practising the skill of counting-on, apart from a context of addition.

The counting was not restricted to small numbers. During a lesson on the hundred square [H5. ordinal], Debbie had highlighted the multiples of ten and she and the children counted in tens:

Debbie Shall we .. er .. count with me and we'll go all the way down* to one hundred and then see if we can keep going, 10, 20, 30, ... 100, 110, 120, 130, ... 190, 200 ... Some children are still counting with Debbie on two hundred, others saying a hundred and twenty, all picked up again at 210. They continued counting to 800.

Debbie I am absolutely amazed. Give yourselves a big clap. Can anyone tell me what happens after 970, 980, 990?

Zeb 991, 992
William A thousand
Rita it goes on and on
Sarah And then it is minus 1, before that it's -2 and before that -3 ...

(Debbie 7)

Many of the children were not confident in keeping track of the hundreds as they counted and they relied on Debbie to provide the next hundred number, but were very good at the pattern of tens. They were praised and felt good about counting with such high numbers [H1. large numbers]. The counting sparked a discussion which showed the children's wider understanding of the number system including negative numbers [H4. relationships]. Sarah's comment could be read as indicating that she thought negative numbers came after a thousand, but I believe that her comment was an observation on the structure of the number system, sparked off by consideration of larger numbers, and not a direct continuation of the increasing number pattern on which the class was working.

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*I believe that Debbie talked of counting 'down' rather than the more usual counting 'up' to 100, since the number square showed the tens increasing down the right hand column.
As a result of these lessons spent counting the children were developing confidence in the number system, including counting in steps [H2. pattern], that would be useful in solving addition problems and counting-on from a higher number. There was no overt mention of the relevance to addition; Debbie did not say why they are counting or relate it to their addition strategies. Yet the children showed that they were able to use these skills in addition. For example, later in the same lesson, Fergus was required to count in units and twos in order to calculate the total of a set of coins.

Debbie shows the children the page in the pupil’s workbook which requires them to calculate the total amount of money represented by a group of coins needed to pay for goods.

Debbie: Who remembers how to do these? Fergus, two count on one, count on one, count on two, count on two, count on one (pointing to the coins 2p, 1p, 1p, 2p, 2p, 1p in the book).

Fergus: 3, 4, 6, 8, 9. (Debbie 7)

Fergus showed confidence in counting in units and twos, saying the words without the sort of hesitation that would indicate that he was adding singly in his head and only saying the significant totals. The counting practice seems to have provided Fergus with counting-on strategies which he can use to solve addition problems.

Individual and small group activities reinforced this counting. For example, the children were required to identify and colour in even numbers on a 1 - 100 number track, by counting on in twos. In a similar lesson, in Eve’s class, the children practised counting in tens.

The children had placed a set of 1 to 100 number tiles on a magnetic board to form the 100 square. They then counted in tens from 10 to 100 and back to 10; in tens starting in turn at two, three and five. Each time Eve pointed to the 100 square tiles as a focus. Though some of the children did not appear to be watching, they all joined in the counting, as if following the patterns in the words. Afterwards the children recorded multiples of 10 in their books, as far as they could go.

(Eve 1)

During a later lesson in Eve’s class (Eve 6), which focused on data handling, the children were using tallying to collect information. I observed that many of the children were able to count in fives and ones in order to reckon the tallies.

```
5 10 15 20 21,22,23
```

The children, apparently drawing on the practice of counting in fives, were using counting-on to find the total.
So, counting was a key area of the mathematics curriculum at St David's, which developed the children's confidence in larger numbers and counting patterns within the number system, their mental and oral ordinal representations of number and their skills in counting-on. Children counted backwards and forwards, singly and in steps, using different starting numbers. Whereas, at Ashburne, counting concentrated on cardinal counting of small sets of objects.

**7.4.3 Locating**

Most of the activities I have identified as 'locating' focus on locating numbers on the hundred square. While oral counting reinforces the ordinal pattern of the counting words, locating extends this to finding individual numbers within the system. The activities relate to an understanding of the number system and the relationship between numbers [H1. large numbers, H4. relationships, H5. ordinal].

In both classes at St David's, the hundred square was used to talk about the location of individual numbers. Debbie's class were looking at patterns in the hundred square.

Debbie  Samira, what number comes after 13?
Samira  14
Debbie  What number after 2, Seleka?
Seleka  3
Debbie  What number before 17, Leone?
Leone  16
Debbie  After 29, Emma?
Emma  30
Debbie  Before 61, Eliza?
Eliza  60
Debbie  Well done, that one is difficult because it means working back.

(Debbie indicates on the board that going from 61 to 60 involves moving up a row and to the other end.)

\[
\begin{array}{cccccccc}
51 & 52 & 53 & 54 & 55 & 56 & 57 & 58 & 59 & 60 \\
61 & 62 & 63 & 64 & 65 & 66 & 67 & 68 & 69 & 70 \\
\end{array}
\]

I am reminded of Beth's lesson with the towers where she was asking the children similar questions, but only in the range 1 to 5 and using cardinal representations of brick towers as well as the number track. Debbie's children were working with bigger numbers and place value, as well as simple counting relationships. They have only an ordinal number representation to help them, the hundred square, and must understand its structure to solve the task [H1 larger numbers, H3. pattern, H4. relationships, H5. ordinal].

In Eve's class the children also worked on the hundred square. In one activity, which I coded as 'locating', they worked with Eve using only the numbers 1 to 20. She gave individual
children a number tile and asked them to place it on the hundred square board. Initially the children used counting to find the position, but as more tiles were in place they began to use the units already placed to locate the position of the teens numbers, for example placing the 18 below the 8.

Eve checked that they were all correctly placed and the whole class counted from one to twenty with her. Eve then introduced a game that used these numbers.

Eve Let's just count the numbers, 1, 2, 3, ...20. Right now close you eyes, all of you, don't look while I change the numbers. (Eve changed over a pair of numbers, changing the 8 and the 12.) Right I have muddled the numbers up. Now I am going to ask somebody to choose two numbers, change two numbers round and put them in the right order.

Faith 8 and er... 12 (checked 12 by mentally counting-on from 10, moving her head as she kept track of the count.)

Faith approaches the board and changes them over. Eve then repeats the activity several times, each time a child answers and changes the tiles back ...

Eve Is it right now? Let's check 1, 2, 3, ... 20. (Children join in.)

Eve Close your eyes and I'll do it one more time. You can have a go at playing this on your own when we have finished. (This time she changes round a lot of tiles at once.)

Rhonna 17 and 10
Sara 7 and 3 (these were both wrong but changing them over did not make them correct, the 7 was now in the eighth space.)

Rita 12 and 16.
Faith 17 and 15.

In fact the 17 was already in its correct place but it was not below the 7 which was in the eighth space. Faith seemed to be using her knowledge of place value, 17 being placed below the 7, to carry out the task.

(Eve 7)

By learning to locate the numbers within the number system the children were offered a model of how the number system is formed, its base ten place value and the numbers that are one more and less, and ten more and less, than a given number, those around it on the number square [H3. Inverse].

<table>
<thead>
<tr>
<th></th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

Locating activities develop familiarity with the structure of the number system and provide a model of number which can be used to solve addition and subtraction problems involving tens.
and units. Since the numbers were not already placed, the children must use a mental image and logical thinking to decide where to put them. The emphasis is on ordinal [H5] number. During the observation period of the study the 100 square was not used for calculations, but this served to prepare the children for the addition of two digit numbers in the future.

Individual and group activities involving 'locating' included a group of children constructing a 'washing line' with numeral cards 1 to 20, string and pegs, another group constructing the 100 square from number tiles, and individual children making a 100 square for themselves by writing the numbers on squared paper, as a personal resource.

7.4.4 Pattern Spotting

The counting and locating activities observed above often involved elements of pattern [H2], which was further reinforced by activities that specifically addressed looking for patterns. The following is a continuation of the 'counting in tens' episode from Debbie's class at which we looked in section 7.4.2.

Debbie Well done, now lets look at the number square, can anybody spot a pattern?

Esther 1, 1, 1, 1, 1, (pointing to the tens in the teens row)

John 1, 1, 1, 1, 1, ; 2, 2, 2, 2, 2, .... 9, 9, 9, 9, (indicating the units in the vertical columns.)

Debbie Can you see what John means? He is saying that in the ones row there are all ones. What is your pattern Ruth?

Ruth It goes 1, 2, 3, 4, 5, 6, 7, 8, 9, (indicating tens in the 1 to 91 column.)

Debbie Does it do that in every row (column)?

Ruth Yes

Debbie So, as it goes down (indicating the 1-91 column) it goes up in steps of ...?

Ellen 2

Debbie Do you agree with her? If you increase in steps of two ...

Ruth Ten times table

Debbie So it's jumping up in steps of ...?

Ruth Ten

(Debbie 6)

The children were being asked to identify patterns and did this successfully. In a similar lesson in Eve's class, Eve constructed a rectangle with the numeral tiles 1 to 20. The lesson started by counting in fives, up to and then beyond twenty.
Eve: Let's count in fives: 5, 10, 15, 20. What next?
Aisha: 25
Eve: Let's go on: 25, 30, 25, 40 ...100
Who can see a pattern in that now?
Rose: They go 5, oh, 5 oh.
Eve: What happens here? (together they read the units down each column: 4, 9, 4, 9.; 3, 8, 3, 8; 2, 7, 2, 7; 1, 6, 1, 6.)
Eve: Is it the tens that are changing or the units?
Shaun: the units.
Eve: That's right.

In both these examples the pattern spotting was related to locating, yet other activities showed that pattern was an important part of mathematics, not only with the hundred square. In the next example Ruth, a Year One child (six year old) in Debbie's class, was involved in an activity making increasing squares on a peg board and looking at the numbers produced.

Ruth came to show me (the researcher) that she had found that the number of pegs, required to make the next square in the sequence, increased by 8 each time (from 4 to 12 and from 12 to 20) so she had calculated that the pattern was + 8. The next one would be 28.

(Debbie 1)
While such pattern spotting is not an explicit element of the addition curriculum, it develops a flexible understanding of the relationships within number that could enable the children to develop flexible addition strategies [H4. Relationship]. Ruth used addition and subtraction to analyse the relationships within her pattern. In the following section I will show how the children used pattern in relation to addition.

### 7.4.5 Exploring addition

While the emphasis at St David’s, during the observation period, was on teaching the children to understand the number system, there were also occasions when addition was discussed by the whole class or carried out in group tasks.

#### The answer is seven

One whole class activity in Debbie’s class centred around the number 7. Before the lesson Debbie talked to me about what she was going to do.

> Some days we just look at a number and see how many different ways we can make sums that have that answer. Some of the children will just be able to do simple addition or subtraction, some might get into patterns [H4]. I hope that all the children will be able at least to try. I think it is important for them to look at sums this way.

I asked her what she meant by ‘this way’.

> Like there can be lots of different sums with the same answer, you can get there by doing different things.

(Debbie 4 - pre-lesson discussion)

Debbie wrote a large number 7 in the centre of the board.

Debbie Who can think of a way to make seven?

Rob Three and four.

At each suggestion Debbie wrote the calculation in symbols on left-hand side of the board (3 + 4).

Nozumo Four and three.

Ruth Seven and zero.

Lucia Twelve take away three ... er ... five.

Subtractions were recorded on the right-hand side of the board.

John Twenty take twelve.

Debbie Nearly

John Twenty take thirteen.

Nozumo Two add five.

Ellen Six and one.

Zeb Five and two.

John Twenty-one take fourteen.

Twenty-two take fifteen.

Ruth Twenty-three take away sixteen.
Ellen  Twenty-four minus seventeen.
Nozumo  One and six.
John   Seven add seven take seven.
Ruth  Twenty-five take away eighteen (back to the pattern.)
       Twenty-six take away nineteen.
Beaty one add one add one add one add one add one add one, and 10 take away 3.
John   Seven take seven add seven
       eleven take four
       twenty-seven take twenty
Beaty  twenty-eight take away twenty one
Ruth   eight take away one, twenty-nine take away twenty-two.
       (Debbie 4)

I noted at the time the way that the children were using pattern [H2] to generate more examples, applying their knowledge of pattern to addition. Nozumo (one of the new reception children) seemed aware of commutativity, providing 4 + 3 after Rob gave 3 + 4, and later 1 + 6 when 6 + 1 was already on the board [H5 relationship]. John uses inverse (7+7-7) [H3. inverse] and equal addition. The children were working mentally and at their own level. Nozumo, Zeb and Rob (reception children) were able to concentrate on simple one step addition, while experiencing the subtraction and more complex multi-step calculations offered by the others [H6. immersion].

Addition to ten

Eve’s class carried out a similar activity with the number ten: the task involved only addition since it led into an activity on making combinations of coins that made 10p.

Eve started the lesson by writing in the centre of the board:

```
+  
10
```

Eve   We need to find different ways to make ten. It’s adding though, addition. Different ways of making ten, Rose?
Rose nine and one
Iona five add five
Anne six add six
Eve   No. Ellen?
Ellen six and one
Eve   No, what is 6 add 1? 1, 2, 3, 4, 5, 6 add 1 makes 7
       (children join in counting).
Mohammed 6 and 4
Alan   7 and 3
Eve   Can you come and write it, Alan, write the sum on the board?

Several more children are chosen to come up and write their sums on the board - just two numbers and an + sign without = 10.
Eve (reads from board) 6 and 4, we’ve got 6 and 4, what could you do to make it a different way? (no response) You could turn it round, what would it make?

Ellen 4 add 6
Eve What about using zero?
Paul 8 add 2
Mike 2 add 8
Naomi 10 add a nought
Eve Good
Faith Zero add ten
Eve Anyone else?
Iona 1 and 9
Alan 3 and 7
Mohammed 2 and 8

(Eve 5)

Not all of the children were successful; Anne and Ellen offered incorrect answers. But, they were given immediate feedback and the opportunity to hear correct answers from their peers.

Eve suggested the use of commutativity to generate new combinations and several of the children took this up. Commutativity is an important element of addition since it allows strategy development from ‘counting-on from the first number’ to ‘counting-on from the highest’. Yet at this stage it is not being taught to the children as a method of developing their addition strategies but as part of the relationship between numbers [H4. relationship].

Calculations

Two other lessons in Debbie’s class had a specific emphasis on addition. In the first the children were using a calculator to solve ‘sums’, though in fact, the focus of this teaching episode was on use of a calculator, rather than on the sums themselves.

The workbook page showed a series of calculations

e.g. 2 + 4 = □ or 3 count on 4 = □

and instructed the children to use a calculator.

Debbie Right now we are going to use the calculator to do these sums. What is first
Rob 2
Debbie What next?
Ellen 4
Debbie No, there’s something in the middle
Rob Plus
Debbie Yes, you need to press the addition key. Can you all see that. What next? Yes the equals sign and that gives you the answer.
Debbie So what does this sign mean (+)?
Ellen It adds up, counts on some more, add, and, it’s plus, counts on.
Debbie What about this (=)?
Rob It’s equals
Debbie What does that mean?
Naomi It adds things up. When you are doing a sum and you put equals in it adds up how many you want it to and then when you press the button (=) it tells you the answer.
Debbie What if you were doing take aways?
Emma It does the sum, whatever you tell it.

Debbie took the opportunity to find out about the children’s understanding of the symbols which gives us insight into the children’s understanding of the language of addition. There is no requirement to conform to the teacher’s set language “5 and 3 altogether makes 8”. A variety of ways to talk about addition is evident but interpretation of the equals sign is less secure. Talking about the meaning of symbols was a common occurrence in Debbie’s class. Sometimes this formed the focus of a planned whole class discussion, and at other times Debbie would take opportunities as they occurred. In the following lesson a group of children had to solve missing number problems.

Debbie Now we are going to be number detectives today.
(Writes in the board $2 + \Box = 7$)
If the answer is going to be 7 what number would be in the box?
Ruth 5
Debbie What about $\Box + 3 = 4$?
Ellen 1
Debbie So where there is a space you have to be a number detective and find out what is missing. But be careful because some of them have this sign (write + on board) and some of them have this sign (-). What does this sign mean (+)?
Zeb Plus
Debbie and this sign (-)
Saresh equals
Debbie No equals is this (point to = in previous sum). What is that?
Saresh Take away
Debbie What else?
Children minus, subtract, count back.

The children were given no instructions as to how to do the mathematics for these problems. But because these children do not do very much written mathematics Debbie discussed the symbols with them. She did not discuss solution strategies and at the end of the lesson she discussed how the children had worked them out.

Naomi shows how she used a 1-10 number line counting-on or back to find the difference. Ruth counted-on using her fingers. Zeb ‘just knew’. Ellen knows her number bonds and was able to use them for the inverse:
Ellen I just know 5 and 3 makes 8 so this one (8 - □ = 5) must be 3.

(Debbie 6)

Ellen was able to use her knowledge of the inverse nature of addition and subtraction [H3] and explain it to Debbie. In each of these activities, which involved more formal addition tasks, the emphasis was not explicitly on teaching addition but on offering activities which allowed the children to engage in addition, to use their own developing strategies and to talk about their understanding [H6. immersion]. This was in direct contrast to the lessons at Ashburne which were focused on teaching specific procedures for understanding and carrying out addition.

7.4.6 Assessing children’s learning of addition at St David’s

As discussed earlier, it was difficult to identify where the children at St David’s were being taught addition, and I have shown how the activities they did carry out were related to the learning of addition strategies. In this section I will summarise the evidence that they were able to apply their understanding of the number system, counting, locating and number patterns, to addition contexts.

There was some evidence that the children were developing their understanding of addition: Fergus could count on in ones and twos to calculate the total value of coins, Eve’s children could count in fives and ones to reckon the tally (section 7.4.2); Ruth was able to use addition and subtraction to find the pattern of her peg squares (section 7.4.4); and the children used patterns to generate addition and subtraction bonds to 7 and 10, discussed the meaning of symbols, and used a range of strategies to solve missing number problems (section 7.4.5).

In the assessment at the start of the observation period, the children chosen as the target groups in Debbie’s and Eve’s classes were at a similar stage, in their understanding of addition, to those in Chris’ class. The children were a similar age (Year 1), though they had been in formal schooling for less time. During the dice game they could count-all, using their fingers to represent the quantity shown by the numeral die, though two children could also count-on when the addend was 1 or 2. They could all read and write numerals to thirty and beyond, and all but Moni (who was not yet confident with spoken English) could say the count words to at least 100. In the final assessment the target children had all moved on in their addition strategies. All the children except Moni could now use counting-on to solve the numeral/dotted dice task. But they also used a wider range of strategies, including known facts (‘I just knew’), and deriving from known facts (6 + 5 is eleven, ‘cos 5 and 5 are ten’). Moni could count-on 1 or 2 but was less secure with bigger numbers and resorted to counting all.

It would therefore seem that, despite the lack of specific teaching of more advanced addition strategies, the children’s increased understanding of the structure of, and relationships within,
the number system through activities involving counting, locating and pattern spotting, had led to the development of these strategies.

7.4.7 Summary of Teaching Addition at St David’s

In contrast to the atomistic approach to the curriculum identified at Ashburne School the curriculum at St David’s concentrates on understanding of the number system and relationships between numbers with addition only one small part of the overall picture. I have chosen to call this a ‘holistic’ approach since it views the number curriculum as a whole rather than focusing on its constituent parts and developmental stages.

Characteristics of this holistic approach to the curriculum can be identified as:

H1. an emphasis on large numbers as well as small ones - especially with numbers to 100,

H2. an emphasis on patterns in number and pattern spotting - within the 100 square and algebraic patterns;

H3. discussion of the inverse relationship between addition and subtraction [inverse];

H4. emphasis on relationships rather than procedures - e.g. commutativity;

H5. emphasis on ordinal representations of number and the development of mental imagery [ordinal] - especially the 100 square, to see ‘through’ the representation to the meaning beyond (transparent);

H6. an eclectic curriculum in which the children are immersed in a wide range of activities, with little apparent sequence [immersion].

In contrast to the curriculum approach at Ashburne which I described as Atomistic, resulting in a targeted, arrow approach to teaching, the curriculum at St David’s is Holistic. Elsewhere I have described this as a ‘shotgun’ approach to teaching (Coles and Price 1997). The curriculum offers a wide range of ideas rather than a narrow focus and the children take from this what ‘hits them’ - what makes sense or is within their capabilities at the time. I am now unhappy with this analogy and would want to change it to an ‘immersion’ curriculum. The children are immersed in a wide variety of potentially complex tasks from which to make sense. So we saw how, in the ‘answer is 7’ activity, Nozumo was able to work with small addition sums while John worked with subtraction and more complex multi-step calculations. Not only did this allow all the children to contribute, but the younger, or lower achieving, children were experiencing more complex mathematics, without the pressure to achieve. There was no assumption that all the children will learn what is taught, nor learn the same as each other. Debbie’s own words sum it up well:
Some of the children will just be able to do simple addition or subtraction, some might get into patterns. I hope that all the children will be able at least to try. (Debbie 4)

There is an assumption that all of the children will be able to learn something from what is on offer and there were no lessons I observed where children seemed completely lost.

Before I considered the relative advantages and disadvantages of these two approaches I looked for counter examples in each individual teacher’s practice.

### 7.5 Exceptions to the atomistic and holistic categories

The atomistic and holistic categories described above, characterize the curriculum experienced by the children in the classes observed at the two schools, yet at times I found that teachers would change the way that they worked. This was particularly true of Beth at Ashburne and Eve at St David’s.

**Beth**

We saw earlier how Beth expressed her belief in the importance of relationships between numbers, a holistic characteristic.

> We’re mainly looking at ordering and relationships between numbers at the moment. Numbers one to five. The children are beginning to say ‘that’s one more’ or the difference between them. Just beginning to do some add in rhymes etc. I want to go slowly because I’ve found in Y2 (the class she taught before) that if they haven’t got this relationship they get lost. So, I’m going very slow really. (Beth 2)

Yet the activities she offered which looked at relationships, focused only on the relationships between small numbers using cardinal representations or a number track. The children were not encouraged to see the big picture, to have a holistic view of number. Restricted by the school mathematics curriculum, Beth seemed unaware of how to achieve her aim of teaching the children to understand relationships between numbers.

When Beth moved away from the activities prescribed by the mathematics scheme, a less atomistic view of the number curriculum could be seen. Since money was the focus of some of the mathematics scheme plans, Beth set up the ‘home corner’ as a café. This was well stocked with tables, chairs, cutlery and crockery, plus a range of cakes and biscuits made from salt dough and painted. There were menus on the table and dressing up clothes so that the customers could be well dressed with handbag, purse and money, while the waiters wore
aprons complete with order pads and pencils. The children took it in turns to play in the café. The waiters were required to take the orders, get the food and add up the bill for their customers. Emma (the customer) and Angela (the waiter) were observed playing together. Emma places her order:

Emma A jam tart
Angela draws a jam tart on her order pad.
Emma (and) a biscuit, a rectangular one.
Angela draws a biscuit and goes to get her order, and brings them to Emma.
Emma Can I have a orange juice?
Angela adds this to the list which now has three drawings and 2p, 2p, 1p on it.
Beth How much is Emma going to pay?
Angela Five p.

(Beth 9)

This activity provided opportunity for the children to use addition in a less formal way than in the mathematics lessons. There was no assumption that the formal language of addition was used, and the children could record it in their own way on the order pad, and use whatever addition strategy they wished. I would not characterize this as leading to a holistic view of the number curriculum, but it gave the children a less atomistic experience of addition.

In Chapter Three I quoted Anning (1997) describing the conflicts inherent in adopting the National Curriculum in infant schools: "on the one hand teachers see their role as being responsible for teaching children the kind of knowledge that is deemed desirable by society and, on the other, they see their role as guiding children through a voyage of discovery towards their own personal knowledge" (p. 50), and I commented that 'just as a previous generation of teachers found it difficult to reconcile progressive teaching methods with their own traditionalist view of teaching, so this generation must reconcile utilitarian methods with child centred views' (section 3.2.2). I believe that, in Beth, I can see just such a conflict between the atomistic curriculum of the mathematics scheme and a child centred view of learning which is evident when she is able to plan activities for herself.

Eve

In Eve’s class it was also when she moved outside of the planned curriculum that her teaching and curriculum approach changed. Here, Eve became more atomistic. Data handling was a planned part of the curriculum for her children for a later date but while completing a science topic on living things, and reading a story book entitled 'The Very Hungry Caterpillar'. Eve decided to combine the data handling mathematics with the science. The children carried out data collection and graphing of information, initially on the children's favourite fruit. In the process, the children were expected to record their data collection as tallies. However, few of them used tallying correctly: some made lists, others used tally marks without collecting the
fives together. As a result, the focus of the lessons moved from handling the data to the mechanics of tallying and drawing graphs.

Eve: So, you started to find out about other things, Mohammed and Lisa did this one (holding up the children's tally chart) and went around the class asking the children about vegetables. You did tallying 1, 2, 3, 4, and what did you do with 5?

Lisa: Put a cross through.

Eve: So, here you had five (pointing to /////) and three more altogether liked potatoes.

Mohammed: Eight

(Eve 6)

The children then selected another area of information that they wanted to collect data on and went round asking questions of other members of the class.

Jade went around asking people which playground games they liked, e.g. skipping, and wrote the name of the game down each time. When she asked Peter, he told her she was doing it wrong. She made no further attempt to engage with the activity but in the general melee of people asking questions she managed just to mill about.

(Eve 6)

Jade, one of the youngest children in the class, failed to understand the tallying and therefore learnt little about data handling. The curriculum, being atomistic, required a level of differentiation which was not present. The approach used was an arrow approach for which the target was not appropriate for all the children. As I noted earlier the curriculum at St David's was being influenced by the National Numeracy Project (section 7.4.1). Prior to that, the mathematics curriculum had been predominantly based on atomistic mathematics schemes. When provided with activities through the school mathematics plans, Eve was able to work in a holistic way, but left to her own ideas she resorted to an atomistic curriculum.

This is in accord with the findings of Askew et al.'s (1997) investigation into effective teachers of numeracy in primary schools. They found that teaching was influenced by the teacher's own beliefs about

- what it means to be numerate;
- the relationship between teaching and pupil's learning of mathematics;
- presentation and intervention strategies (p. 1).

The most effective teachers were able to influence their less effective colleagues' beliefs if they were able to work "closely with them in planning and evaluating detailed teaching approaches, and work together in the classroom (p. 4). I believe that Eve would benefit from intensive work alongside an effective colleague to influence her own beliefs; being provided with a curriculum from which to work was insufficient to change her practice.
7.6 The Relationship between Curriculum Content, Differentiation and Teaching Strategies

In discussing atomistic and holistic curriculum approaches, I have at the same time mentioned other factors which could have influenced the children’s learning including differentiation and organisational strategies: whole class, group and individual work. It was possible that the effect on learning which I have explained as a difference in curriculum approach, is in fact due to such other factors. It is difficult to separate out these constituent elements of teaching and learning.

First, I considered whether the characterisation of curriculum approach was in fact determined by the degree of differentiation, required as a result of the way that the children were allocated to classes. At Ashburne the children are in single age group classes, Beth’s class being all Reception age and Chris’ Year One. At St David’s the two classes I observed had children who were Reception and Year One in both classes. The level of differentiation needed for a mixed age class would be different from that on a single age group. A logical conclusion would be that the classes with the widest age range would need a more, rather than less, differentiated curriculum; yet this was not so. But it was the classes at Ashburne, and especially Chris’ class, where the children were clearly grouped and experienced a differentiated curriculum, and these were the classes with a single age range. In Chris’ class the children were ability grouped and experienced a differentiated curriculum, where there was little whole class teaching and little discussion with the whole class about an individual group’s work. In the first week of observation I noted that after explaining the work to the group the children were sent off with the instruction:

Chris OK, so you all need to be able to explain how you did them. OK, any queries? Off you go. Remember not to sit next to anyone else doing maths. (Chris 1)

I observed that this was a general rule in the classroom so that the children did not copy from one another; the work was differentiated and the expectation was that it would be completed individually.

There was little differentiation in Beth’s class, instead the children experienced a restricted curriculum, as Beth herself commented:

The children are in groups roughly by mathematics achievement but not much differentiated. I am probably holding back some of the brighter ones but I want to start them off together, learning how to record not just their numbers. (Beth 1)
So rather than differentiating the curriculum, Beth restricted the whole class to what she felt the least high achieving children could manage.

At St David’s there was some differentiation when the children carried out group or individual work, yet the majority of the teaching happened in a whole class setting and was undifferentiated, except by outcome, despite the classes having a mixed age range. This raises the question as to whether the significant factor is neither differentiation nor the approach to the curriculum but the organisational strategies in terms of whole class, group or individual work.

Table 7.3 shows the differences between the four classes in each of these three respects; curriculum, differentiation and teaching strategies.

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<thead>
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<th>Class</th>
<th>Ashburne School</th>
<th>St David’s School</th>
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<tbody>
<tr>
<td>Age Group</td>
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<td>Chris</td>
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</table>

Table 7.3 Characteristics of the four classes studied

This table shows that the observed differences, between the way mathematics was taught at Ashburne and St David’s, appear primarily in the approach to the curriculum, which is the only element that is a characteristic of the school. Whole class teaching and undifferentiated curriculum are predominant both at St David’s and in Beth’s class at Ashburne. Yet the approach to the curriculum is different. This indicates that it is the curriculum approach that characterizes the teaching rather than the degree of differentiation, the predominant teaching approach or the wider age range of children in the class.

7.7 Atomism and Holism Explored

Through analysis of the data I have built a theoretical explanation of the differences between the two school’s approaches to the curriculum. These I have categorized as Atomistic and Holistic, and I have justified why I feel that the overall differences are primarily as a result of the
curriculum approach, rather than differentiation or organisational issues. While these
differences were found between the two schools, I have also shown how they may be
influenced by the teachers' own beliefs about mathematics and learning.

In 7.3.3 I observed that, despite considerable emphasis on early number and addition in their
curriculum, Beth's children had not progressed in their addition strategies when these were
assessed at the end of the observation period apart from their ability to interpret the numeral as
representing quantity. Similarly, at the end of section 7.3.5 I noted that Chris' children had
not developed further addition strategies, despite the emphasis on counting-on in their
curriculum. All the target children, except Geoff who could already use counting-on at the start
of the observation period, used only the counting-all strategy at the final assessment. In
contrast to this, the target children at St David's had developed a wide range of addition
strategies. In section 7.4.6 I observed that all the children except Moni could now use
counting-on to solve the numeral/dotted dice task. But they also used a wider range of
strategies, including known facts ('I just knew'), and derivation from known facts (6 + 5 is
eleven, 'cos 5 and 5 are ten'). Moni could count-on 1 or 2 but was less secure with bigger
numbers and resorted to counting-all. This evidence would indicate a relationship between a
holistic curriculum approach and the development of a range of more advanced addition
strategies. The theory is tentative, being built from a limited data set, derived from the target
children in two classes, in each of two schools, over a six month period.

In the section that follows, I want to draw on three existing areas of theory to explore why a
holistic approach may help children to develop more advanced addition strategies.

**7.7.1 Additive composition of number**

The first of these relates to the structure of the number system itself and, in particular, the
additive composition of number. Nunes and Bryant (1996) define this as "any number \( n \) can
be decomposed into two others that come before it in the ordinal list of numbers, in such a way
that these two add up exactly to \( n \)" (p. 46). A special, and important, case is that of place
value, since larger numbers can be decomposed into their constituent multiples of ten and units.
Nunes and Bryant found that an understanding of this characteristic of number is important not
only for addition of large numbers, but is also essential for children to learn to count-on. It is
the basis for understanding that a number can be added to another number directly, without
them having to be reduced to their unitary elements and counted singly. Nunes and Bryant
emphasize the importance not only of counting, but also of "understanding the relative value of
counting units and their additive composition" (p. 51). It therefore follows that teaching
children to count-on using a number track is likely to be ineffective if they do not understand
this additive structure of number.
It is possible that children in other countries who start school later than British children, may already have a greater understanding of the additive composition of number, which can be learnt through regularities in the counting words and experiences of monetary systems as well as formal mathematics. This greater understanding of the additive composition of number could explain their ability to learn early mathematical concepts more quickly than British children (Whitburn 2000, see Chapter Three, section 3.2.2). It also demonstrates why direct teaching of small areas of arithmetic do not necessarily result in the learning of those particular areas (Denvir & Brown 1986).

The concept of additive composition of number provides insight into the advantages of a holistic curriculum. Such a curriculum allows an understanding of the relationships between numbers and patterns within numbers, which characterize the additive composition of number, and are essential for more complex addition strategies than counting-all. This is more than an argument that place value should be taught in advance of addition (an atomistic approach). It is to argue that addition must be seen as an integral part of the number structure rather than a mathematical topic in its own right.

7.7.2 Mathematical reasoning

A second reason why a holistic approach to the curriculum is preferable to an atomistic one is related to the nature of mathematics and learning. A holistic curriculum gives children greater opportunities to mathematize, to act in a mathematical way. Patterns and relationships are an essential part of mathematics (NC, AT1) which enables the development of conceptual structures (Skemp 1971; Hiebert 1986). Procedural learning, identified as a characteristic of an atomistic curriculum, can result in limited understanding. Children are able to carry out the procedure but not understand why, which Skemp calls instrumental understanding. When the procedure has been forgotten, or the necessary materials are not to hand, the learner has no way to reconstruct it. I have shown how breaking the curriculum down into very small pieces which are learnt in isolation (atomism), can result in such instrumental learning. Teaching children the ‘bigger picture’ first, an understanding of the way that the number system fits together (holism), offers them an overall structure in which constituent sub-topics within arithmetic, such as addition, can be defined. This finding resonates with Boaler’s (1997) study of teaching and learning in secondary school mathematics classrooms, which found that the children experiencing a ‘progressive’ curriculum learnt better than those taught in a ‘traditional’ way. Boaler also found that where children where taught in ‘mixed-ability’ groups for mathematics they experienced a wider curriculum and did better than those children put into differentiated sets, which is also in accord with my findings.
7.7.3 Complexity

For the third and final explanation of the advantages of a holistic curriculum I want to appeal to the wider human experience of learning. Chomsky (1980) proposed a language acquisition device (LAD) to explain how children learn to speak. He reasoned that, since language was extremely complex, young children must have some special way to make sense of it. They appear to learn not only by copying the language that they hear, but by constructing their own logical rules. For example, young children will often generalize regular forms of the past tense to include goed (went), seed (saw) or buyed (bought). Chomsky therefore argued for a specialized area of the brain, specific to language learning and complete with a LAD. Bruner argued that such a LAD was in fact culturally influenced, offering instead a LASS (language acquisition support system, Bruner 1986).

I want to argue that language acquisition is not a unique part of learning. All children’s learning in the world outside school is as an experience of immersion, in language and in culture. It is in the nature of children to make sense of the world around them. Not that they will always get it right, because of limited experience. It would seem logical to assume that, to teach mathematics through immersion into the number system, is to take advantage of the way that children learn. To break it down into ‘bite sized pieces’ is analogous to teaching children to speak using only nouns first, or allowing them only to relate socially to one other human being because more than one may confuse them. In Chapter Ten I will look at how this view relates to the theoretical perspectives on cognition. Here, I am appealing not to a broad range of theoretical knowledge, but to observation and common sense, based on the findings of this chapter. This ‘theory of complexity’ does not set out to explain how holistic learning takes place but to explain why.

7.7.4 Summary

The theory built in this chapter is that different approaches to the curriculum, which I have described as atomistic and holistic, have a different effect on the children’s learning. The data indicate that a holistic approach may help children to develop more complex strategies for addition calculations. In this section I have considered three ways to help explain why a holistic approach to the curriculum may help the children develop these strategies. The explanations are based on an understanding of the additive composition of the number system, an understanding of mathematical reasoning, and a personal hypothesis about children’s learning.
7.8 Conclusion and Implications

One of the ways in which there is a relationship between the teaching and learning of early addition is therefore the way that the curriculum is experienced by the children. I have described this as atomistic or holistic. An atomistic approach to the curriculum breaks the addition curriculum into its developmental stages and teachers target teaching to the next stage according to the teacher's perception of the children's needs. Activities address these stages being designed to teach counting-all, counting-on, and place value; starting from cardinal representations of number. This is the predominant curriculum experience at Ashburne School. A holistic approach to the curriculum sees addition as part of the relationships within the number system. Teachers offer children a range of activities which develop facility with number including counting and locating, and patterns in number as well as more formal addition tasks. This is the predominant way of presenting the curriculum at St David's.

I have shown how this way of describing contrasting approaches to the mathematics curriculum emerged from the data collected in this study, and finally given some explanations to support the tentative evidence that a holistic approach may be preferable to an atomistic approach to developing young children's addition strategies. Analysis of the data for counter examples showed that these approaches were also influenced by the individual understanding of the teacher. I believe that the implications of these findings are twofold.

The first, and strongest, implication is that children will learn early addition more easily if they experience a holistic curriculum. More complex addition strategies may not be developed through direct teaching of these strategies but through an understanding of the complexities of the number system. Children, offered a broad range of activities which explore the number system in its complexity, will develop understanding and skills which are specific to addition.

The second implication is that, in order to teach in a holistic way, teachers may have to change their views of mathematics, addition and how children learn. Offering teachers a holistic curriculum will not prevent them from reverting to an atomistic approach if this is all that they understand.
Chapter Eight

Representing Mathematics in the Classroom and in the Real World

8.1 Introduction

While the previous chapter looked at teaching and learning at the curriculum level, this chapter focuses in more closely to the interaction between teacher and children and, in particular, on representations of mathematics. It discusses findings related to the second of the research questions identified in Chapter Five, following the preliminary study, which asked:

Q2 How does the way that addition is represented to the children influence children's learning of addition?

It draws on the theories of representation described in Chapter Two, and analyses the use of such representations in the classroom, both by the teacher to communicate mathematics to the children, and by the children as evidence of their understanding of the mathematics they are learning. While it is also possible to describe representations of mathematics as socio-cultural tools, or artefacts, according to theoretical perspectives on cognition (see Chapter Two, section 2.4.1), I recognise that my decision to use the term 'representation' reflects the main body of research in this area which is undertaken largely from a constructivist position (Janvier 1987; Goldin and Janvier 1998). At the beginning of the research, and throughout the data collection period I was unaware of how much my thinking on representations was influenced by this constructivist view. With the exception of 'language', which I recognised as an important socio-cultural tool within my conception of social-constructivism (see Chapter Two, section 2.2.4), I had not yet understood the use of pictures, symbols and especially manipulable materials, in any but a constructivist way. This chapter therefore charts both my findings from the data and my own 'making sense' of representations within mathematics from a wider perspective.

Chapter Six has shown how the process of data analysis defined and identified areas of representation as a key element of the teaching and learning process. This chapter describes the results of the analysis. First it looks at the purposes for which representations were used in the classroom by both teachers and children. Secondly it addressing how children adopted and made use of mathematical representations for themselves, particularly as they learnt to use symbols. Finally, it shows how the category of real world scripts was identified, defined and came to be seen as significant for children’s learning, and discusses this significance with reference to work by Jerome Bruner (1996) on the use of narrative.
8.2 Representing Mathematics in the Classroom

As we saw in Chapter Two, mathematics can be represented in enactive (being able to carry
out), iconic (being able to represent as an image) and symbolic (spoken or written language and
mathematics symbols) modes (Bruner 1966). Bruner's idea has been adapted to produce
models for teaching (Liebeck 1984, ELPS; Haylock and Cockburn 1997), for learning
(Open University 1982, DTR), and for analysing problem solving (Lesh et al. 1987a). The
initial analysis of the data was carried out using elements of the Haylock and Cockburn model,
which emphasises the relationship between the four elements:

![Figure 8.1 Relationships between representations (based on Haylock and Cockburn 1997)](image)

However, I found that this model was inadequate to explain the apparent difference between
similar episodes, identified through critical incident analysis (see Chapter Six, section 6.4).
The significance of a real world representation was required to fully describe the data. An
additional category was therefore added to the model to include real world scripts, and I have
adopted a pentagonal model based on that of Lesh et al.

![Figure 8.2 Pentagonal model showing interrelationships between mathematical representations
(after Lesh, et al 1987a)](image)
The constitutive elements of Lesh's model have therefore been used here to analyse the teaching and learning of addition, resulting in the formulation of the following findings:

- children were given very few opportunities to choose, or discuss the use of, different representations, tending to use what the teacher suggested or supplied;
- there was a difference between the way that manipulable materials were used at Ashburne following Dienes' idea of multiple embodiment, a constructivist concept, and the use of the 100 square at St David's as a socio-cultural tool to discuss the number system;
- the children's showed varying levels of confidence in the understanding, and use, of different representations within mathematics, especially with regard to symbols; and
- the children's social understanding of the representation affected their mathematical understanding, especially when real world scripts were used.

In the following sections I will describe how these findings relate to the data.

### 8.3 The Use of Representations in Teaching Addition

Across the lessons studied, all five representations: manipulable materials, spoken language, pictures, symbols and real world scripts, were used by both teachers and children. These were not used uniformly across the lessons. In the lessons with the younger children (4-5) there was an emphasis on unstructured manipulable materials (toys, bricks, counters etc.), spoken language and, in some cases, real world scripts. With the older children (5-6) the materials used tended to be mathematically structured - number lines, hundred squares and Dienes and there was more emphasis on symbols. Pictures were little used in any of the classes except occasionally in workbooks, and where younger children were encouraged to use them as a way of recording their work. In all the lessons more than one representation was used and with the younger children, in Beth's class, manipulable materials and spoken language were used first and picture and symbols used at the end of the lesson to record the work done (following Liebeck's model: practical experience→ language→ pictures→ symbols).

Analysis showed that teachers used different representations in different ways and for different purposes. Teachers' use could be described as:

- interactive teaching: the major representation used here was spoken language but in almost all cases manipulable materials, pictures, symbols or real world scripts were used as a focus for the discussion.
- task setting: pictures and symbols were used to set tasks for children to carry out independently, and to develop their skills in understanding and carrying out mathematical exercises. For the youngest children, tasks would also include manipulable materials. In
neither school did the teachers rely heavily on pupil workbooks for setting work. At Ashburne the children occasionally used photocopied sheets from the Nelson mathematics scheme, and at St David's they occasionally used the Cambridge children's workbooks. In the majority of lessons in both schools, activities were prepared by the teacher.

- **display**: displays were used to celebrate the children's work, to reinforce symbols (numerals) and as mathematical contexts for exploration (interactive displays). In none of the classrooms were mathematical symbols other than numerals (operation and equivalence signs) evident in display.

In almost all the lessons seen, children used the same representations as the teachers, which were introduced through the interactive teaching or a set task. Children used representations to:

- explain their mathematical understanding: when working with the teacher, children would use language, often associated with other representations to explain their thinking,
- aid calculation: when carrying out individual or group tasks children would use pictures and symbols and manipulable materials and
- record their work: usually through pictures and symbols.

On only a few occasions were children seen to choose to use a particular form of representation without being directed. Beth's children used pictures and symbols to record items bought when playing in the café (see section 8.6.2 below). Both Chris and Debbie, setting tasks which involved written calculations, allowed the children to choose what form of manipulable materials they would use to help them. All but one of Chris's children chose unstructured manipulable materials, while Debbie's mostly used mental calculation strategies. One child in each class chose to use a number track. In Chris's class the children were required to explain to Chris how they had worked on the calculation. This they did individually. By contrast, Debbie discussed with the groups together the different strategies used.

It is possible that the lack of choice is due to the age of the children, the need to teach them about using representation before they are able to choose for themselves. However, I believe that more opportunity to choose, and to talk about the use of different representations and strategies would benefit even young children. Unless they can see the relative merits of other strategies they are most likely to stick with the ones with which they already feel secure, rather than moving on.

The following sections will look in detail at the use of individual representational forms.
8.4 Manipulative Materials

The majority of lessons observed used some form of manipulable material. Such materials could be divided into unstructured materials (cubes, counters, toys) and structured materials including number lines and squares. These materials were used by both teachers and children in order to model mathematical situations and aid calculation. At Ashburne school Beth used unstructured materials for counting and to model simple addition from the start, while the older children in Chris’s class (Y1) were introduced to more structured apparatus as they moved to counting on and learning about place value. At St David’s structured materials were used throughout.

8.4.1 Unstructured Materials

Unstructured materials included toys, counters, and cubes. At Ashburne School these were used to demonstrate simple contexts for addition and to aid calculation. Beth’s class used toy animals, cubes and small, cut out pictures, to model addition. The children were expected to use their own set of materials to model different number combinations. The manipulable materials were being used as a focus to develop the children’s understanding of addition and in particular to develop the associated language.

In this example the target children are working with Beth. Each of the children has a card with a picture of a leaf on it, with a clearly defined central vein. On the table are sets of small, coloured beetles, cut from card.

Beth What have we got here?
Children Ladybirds
Beth Are they?
Children Beetles
Beth I’m going to do a number 5 on the top (drew 5 on the top of the sheet). Emma, can you find a set of blue beetles? [repeated - Angela/orange, Charles/yellow, Ian/brown, Jacob/pink] Have you all got five?. Can you check? I’m just going to check that I have got five. 1, 2, 3, 4, 5. Now, if you look at the leaf you will see that you have got a line down the middle. It divides the leaf into two halves. So, some of the beetles are going to go on one side of the leaf and some on the other side of the leaf. Can you do that? I’m going to put that one there and that one there, and ... [the children copy with their beetles]
We’ll wait for Jacob. Oh, that one is in the middle which side is it going?
Emma can you tell us about your leaf?
Emma Two on that side and three on that side makes five altogether.
[All the children take a turn at explaining what their leaf ‘says’. Jacob is unable to initiate ‘altogether makes’ without help.]
Beth Now some of the beetles like to go for a walk across the leaf. Can you make some of them...?

[The children move their beetles and read out the sums. Jacob is reluctant to move any of his but eventually moved all of them changing 4 and 1 to 1 and 4. Most of the children had changed only one beetle resulting in from 2 and 3 to 3 and 2 but they still read it as two and three ignoring which side it was on.]

Beth Ian can you read my leaf for me? [it read 5 and 0]
Ian 1, 2, 3, 4, 5.
Beth And how many on this side?
Ian Five
Beth And this side?
Ian Zero
Beth So what is the sum?
Ian Five and zero altogether makes five.
Beth Good, can you make a five and zero sum?
[Ian does and the others do to]
Beth Good, can you make another five and zero sum that looks different from this one?
[After a pause the children move all their beetles across the leaf]
Beth Good, OK let's see who can make a little number sum that has one beetle on one side and four on the other. [All the children do so] So, one and four altogether make five. But look at Ian. He has got his four on the other side so four and one altogether make five as well. It's just turning the numbers round the other way. Can you make a two and three sum? So we've got...

Children Two and three altogether make five.

The children were then left to record their work in pictures and symbols.

(Beth 6)

Where the numbers were small, up to three or four, there was no evidence of the children using the materials to count, since they could subitize the quantities without counting. But as children worked with large numbers they will often need to count out unstructured manipulable materials to describe or help solve calculations. On my second visit to Chris's class, the children were asked to choose how to solve addition 'sums' written in their exercise books. Geoff chose to use a number track, but the rest of the children used fingers or counters. The field notes observed:

Joanne counted on her fingers. Natalie, Donna, Will and Dipesh used counters, counting out two sets and combining. Barry used counters and a pencil to divide the two sets.

(Chris 2)

However at St David's school none of the children was observed to use unstructured manipulable materials in either of these two ways which I believe to be an aspect of their holistic approach to teaching mathematics, discussed in the previous chapter. Much more emphasis was placed from the start on structured materials, recitation of counting words, mental calculations, and the counting of real objects in context e.g. the number of children who
liked apples in the context of making a graph. Occasionally the children at St David's were seen to use their fingers but the children were also encouraged to use mental methods. For example, the first lesson I observed in Debbie's class started with whole class rote counting.

Debbie Hands on heads. Close your eyes. Count in your heads as high as you can go ... Stop. How many Sarah?

After the lesson I asked Debbie about the 'hands on heads' instruction. She explained:

"Asking the children to put their hands on their heads has three effects. First it settles the children, it is difficult to move around much with hands on heads, secondly it stops them from using their fingers to count and calculate, and thirdly it emphasises thinking in your head."

(Debbie 1)

So, at Ashburne School, unstructured manipulable materials, counters cubes and toys, were used to model addition for the youngest children and to aid calculation with small numbers, a characteristic of the atomistic curriculum described in Chapter Seven. I will return to discuss these unstructured manipulable materials further, in the context of real world scripts (section 8.4 below). But unstructured materials are less useful when carrying out calculation with larger numbers, which was where a variety of structured materials came into play.

8.4.2 Structured Materials

Again there was a clear difference between the two schools in the use of structured materials. At Ashburne the emphasis was on the number track and base ten materials, while at St David's the 100 square was the most common structured representation used. During data analysis, when grouping codes, I was at first unsure whether to code number tracks and 100 squares as manipulable materials or as pictures. They seemed to fit into neither category very easily. However the way that the number track was used, involving the manipulation of counters along it to model addition, and the way that the 100 square was used at St David's with individual tiles that could be removed and reordered, indicated that they were more than just static representations of number relationships. In Bruner's terms they were used at the 'enactive' stage and I therefore included them as structured manipulable materials.

We have seen in Chapter Seven how Chris's class used number tracks to learn about counting on and how the classes at St David's used the 100 square to learn about the relationship between numbers and place value and I will not reproduce that evidence here. A few children in both schools were seen to use number tracks or lines to help solve calculations in the number range 1-20.

One further representation was identified as a structured manipulable material. Money can be interpreted as a real world context and as such was used in both Beth's and Eve's classes. However it was also used in Chris and Debbie's class as a structured material to encourage counting-on in steps greater then one, and as a base ten material.
Debbie: These children have already done some money so they recognise the coins but I want them to add up totals. Sometimes they only count the number of coins instead of the value. I want to see what they can do now. First we are going to do some work with multiples of 10. ... they will need to be able to count in tens to do the harder money questions later one.

(Debbie 7)

But in neither class was money used sufficiently, during the observation period, to identify much about the children's learning.

8.4.3 Summary of the use of manipulative materials

Some form of manipulative materials was therefore used in all the classes observed in almost all the lessons. Such materials could be unstructured and used for counting tasks or simple addition calculations by counting-all, or could be structured to represent relationships between numbers (ordinal and base 10 number structure). In all these lessons, manipulative materials were used as a focus for developing spoken language and symbolic representational forms.

As we saw in Chapter Seven, children tended to become dependent on manipulable materials with a one-to-one relationship with number (counters, base 10 apparatus, and number tracks used for counting-on), needing to use the materials to solve addition problems, while use of the 100 square offered children a mental model with which to work. However, it is difficult to say whether this is due to the nature of the materials, or a wider effect of the holistic and atomistic approached to the curriculum. Teaching children how to use the manipulable materials to model mathematics, did not encourage them to move from the concrete to the abstract: the materials remained opaque; the focus was on the meaning of the material rather than the underlying mathematics (see Chapter Seven, section 7.3). Using the 100 square to discuss relationships seemed to help the children to abstract the structure of the number system; the material became transparent. The children were not just learning 'what' to do, but 'why' (Adler 1998). They were encouraged to 'think' maths, "hands on heads, close your eyes", rather than teaching procedures for using equipment, could also encourage independence from manipulable materials.

The use of manipulable materials at Ashburne reflected Dienes' idea of multiple embodiment. Drawing on work by Piaget and others he suggests that:

a certain series of artificial exercises in the form of problems arising out of specially devised personal experience can lead young children through the logico-mathematical development of the concepts of mathematics (1965, p. 14).

Repeated experience with a range of similar manipulable materials, one of the characteristics of the Atomistic curriculum at Ashburne, can therefore be seen as a teaching approach which arises from a Piagetian/constructivist perspective.
I now believe that the problems that I had identifying numberlines and 100 squares as manipulable materials when I began to analyse the data was due to my own constructivist perspective on representations. I was familiar with ideas of the use of manipulable materials and multiple embodiment, which rely heavily on cardinal representation. The number track, while representing ordinal number, still retains its links with cardinal 1:1 correspondence (see Chapter Seven, section 7.3.5). However numberlines and 100 squares are predominantly ordinal, moving away from concrete 1:1 representation, to represent structure within the number system. The use of numberlines and 100 squares, can more clearly be seen from a socio-cultural perspective. Viewed as tools, they allow the teacher and children to talk about, and develop mental images of, the number system, which itself can be seen as a socio-cultural tool since it has a socially imposed structure which cannot be abstracted from the real world (hence Dienes' need for 'artificial exercises').

8.5 Language as a representation of mathematics

Spoken language is, of course, the least substantial representation of mathematics and yet the most common in the classrooms seen. In its simplest form it consisted of the everyday language of children and teachers applied to the context of mathematics, usually in order to talk about some external representation - manipulative materials or symbols, or about a real world script. Mathematics also has specific mathematical language associated with its concepts both as specific words (e.g. addition, equals) and as specific sentence forms (e.g. four and three altogether make seven) which children learn in the mathematics classroom. This section will consider how the children developed competence in language use.

8.5.1 Talking about mathematics

Across all the classrooms studied the children were encouraged to talk about their mathematics in most of the lessons. Talk centred around manipulative materials, real world scripts and symbols. The teachers often used everyday language to explain the mathematics. Here Debbie is talking to the class about halves and quarters. She comments that the diagram she has drawn on the board (a square divided into quarters) reminds her of a Battenburg cake.

Debbie: When I went to the shop I couldn't get a Battenburg cake because they had sold out. So I had to get a walnut cake instead (produces walnut cake from bag.) How many different ways are there to cut that one in half.

Gus: That way and that way (indicates vertical cut down centre and horizontal cut across).

Debbie: If we cut it that way and that way what would we have?

Mia: Quarters

Debbie: And how many quarters?

Chris: 4
Debbie If we just cut it that way what would we have?
Chris Half
Debbie So how many quarters is the same as a half
Ruth Two, 'cos that quarter goes with that quarter and that quarter goes with that quarter so they are halves.
Debbie Good. 

Debbie seemed happy to use Gus's expression "that way and that way" rather than introduce the mathematical terms horizontal and vertical. The children were not yet familiar with these terms and it seems possible that she did not want to confuse them with new mathematical terms which were not the focus of the lesson (fractions).

Beth would also use everyday language to help the children remember specific mathematical skills. When teaching them how to write the numeral 5 she explained:

Beth Now we are going to practice writing the number 5. (Beth writes 5 on the blackboard and again on the whiteboard.) You can practice writing 5 on the blackboard and on the whiteboard, if you have a spare minute you can just come and practice. Let's practice together
straight line down first
then that lovely hook
then we put the hat on

(the children draw 5s in the air).
How are we going to write the word? F, i, v, e, five, if you are not five already you soon will be. 

(Beth 1)

Here, Beth relates their new knowledge (how to draw the number five) to existing social experiences outside the classroom (hooks, hats and birthdays). This use of socio-cultural experience will be further discussed in Chapter Nine.

The children's language was often imprecise but accepted in context. In the following excerpt Debbie is talking with the class about the 100 square.

Debbie Can anyone tell me a pattern in the tens?
Florina 1, 2, 3, 4, ... (pointing to the tens digits)
Debbie Well done, anything else that is the same?
Naomi oh, oh, oh, ... (units digits) 

(Debbie 7)

Debbie did not insist that the children use more specific mathematical language to explain (the tens digit increases by one; the units digit is always zero), and by referring to the 100 square, Florina could demonstrate her meaning. The use of less precise, everyday language allowed both teacher and children to talk about mathematics in context. However there was also an expectation that children would begin to use more precise language.
8.5.2 Specific mathematical language

In Chapter Seven we saw how a variety of mathematically specific language can relate to a single mathematical concept. While children can talk about their mathematics using everyday language, clarity of thinking and explanation require clarity of language. At times therefore there was more emphasis on using specific mathematical language, usually in the context of talking about another representation. Debbie was introducing a task to a group of children, that required the use of a calculator. Since the children had not used calculators recently, the discussion centred around the meaning of the symbols and the order in which the keys should be pressed. The first calculation on the worksheet was $2 + 4$.

Debbie: Right, now we are going to use the calculator to do these sums. What is first?
Ellen: 2
Debbie: What next?
Sarita: 4
Debbie: No, there's something in the middle.
John: Plus
Debbie: Yes, you need to press the **addition** key. Can you all see that. What next? Yes the **equals** sign and that gives you the answer.
Debbie: So what does this sign mean (+)?
Ellen: It **adds up**, **counts on** some more, **add**, and, **it's plus**, **counts on**.
Debbie: What about this (=)?
Sarita: It's **equals**
Debbie: What does that mean?
John: It **adds things up**. When you are doing a sum and you put **equals** in it **adds up** how many you want it to and then when you press the button (=) it tells you the answer.
Debbie: What if you were doing take aways?
Ellen: It does the sum, whatever you tell it.

(Debbie 5)

The emphasis here is on the language and meaning related to the symbols used. The children show that they are aware of a range of words, and related meaning, about addition. John's interpretation of the equals sign appears limited to addition (it adds things up, tells you the answer) although Ellen can relate it to subtraction when asked. Such discussions of meaning were always shared within the class or group. This form of mathematical language learning was seen in both classes at St David's, and in Beth's class at Ashburne. Language was used by the teacher and increasingly by the children in the way that children naturally learn language. If the children made mistakes the teacher would repeat the sentence but substitute the correct word. During whole class and group discussions, the children would also use and learn from one another (see Chapter Nine, section 9.4.1 for an example of this).

A constructivist view of language as a representation of mathematics offers a focus on specific words as they relate to other representations, and will be further developed in the discussion of
symbols (section 8.6.2). However, a socio-cultural perspective shows teachers and children establishing meaning through negotiation and this will be further discussed in Chapter Nine.

8.5.3 Procedural language forms

The calculator example from Debbie's class shows the emphasis on a range of language forms which was common in both classes at St David's. But at Ashburne School the teachers would commonly expect the children to use a specific form of language to talk about their mathematics. I have called this procedural language, as the spoken language is given as a set procedure, in the same way as we saw the activities following such procedures in Chapter Seven. So, Beth taught her children to use the language form "x and y altogether makes z" as an articulation of all addition contexts. The language form was introduced in the second lesson I observed, when the children were being formally introduced to addition for the first time. They had made towers of multilink bricks to represent the numbers to five.

Beth Now I'm going to put some of these together. If I put the 2 and the 1 towers together how many altogether?
Emma 3
Beth The 1 tower and the 5 tower
Ian 6
Beth The 2 and the 3 towers
Charles 5
Beth so, 2 and 3 altogether make 5.

(Beth 2)

As Beth introduced the idea of combining two sets she also introduced one specific way of talking about addition. At the end of the lesson, Beth revised the work they had been doing. She made two towers and placed one on each of her index fingers, and asked "Four and one, how many altogether?" Each time Beth accepted a single word answer from the children but restated it as a number sentence 'so, four and one altogether makes five', modelling the required language structure. After three turns, some of the children began to state the sentence for themselves using this procedural language.

However it took time for the children to remember and use this form of language for themselves. Two weeks later, in the context of addition with toy animals, Beth asked:

Beth Jacob, tell me about your fields.
Jacob Got 2 pigs in that field and 1 pig in that field.
Beth How many altogether?
Jacob Three
Beth You've still got three. What about you Emma?
Emma I've got two and one but altogether still got 3.
Beth Angela?
Angela Two and one more is three.
Beth What about you Charles?
Angela and Emma are able to talk about their toy animals in everyday language that shows their understanding of addition, but Charles and Jacob state only the numbers in the two sets, and not the addition total. Beth scaffolds their language by offering prompts in the form of a question “how many altogether?”.

The following week the activity was to pick four cubes from a bag containing red and blue cubes and describe what they had picked. Emma picked hers first.

Beth Can you put them into colours and tell me a number story?
Emma Three blue and one red makes four.
(Jacob picks out four cubes exactly the same as Emma.)
Jacob It's the same.
Beth Put them back and see if you can get something different.
(Jacob puts them back then draws out four blue ones.)
Beth Tell me a number story about yours.
Jacob There's four blues.
Beth So, four blues and no reds altogether makes ...
Jacob Four
(Charles takes his turn.)
Beth Charles, what have you got there?
Charles Four reds
Beth and how many blues?
Charles None
Beth So, can you tell me a number story?
Charles Four reds and no more altogether makes four.

Emma is nearly able to use Beth’s language (she omits ‘altogether), while Jacob needs help in interpreting his bricks as addition, perhaps because he only has one colour. Charles can now reproduce the sentence form when prompted. It is several weeks before Jacob is observed to use the form correctly unaided. Here Beth is working with the whole class doing some examples which some of the children are going to work on in their groups. The worksheet contains pictures of a set containing some counters which have been partitioned by a line. She had drawn one on the board and asks Jacob to help.

Jacob There’s two there and two there altogether makes two (sic).
Beth How many altogether?
Jacob Four
Beth Can you write it for us?
(Jacob writes 2 + 2)
Beth Altogether makes...
Jacob writes → 4 and Beth gives it a tick. The class is asked to give him a clap which they do. (Beth 9)

Jacob manages to get the right form of words but the wrong answer. It is as if he has too much to think about and the effort of getting the words right overwhelms the calculation. Rather than helping Jacob to understand addition, the procedural form of words seem to be a stumbling block between his understanding of addition and his ability to talk about it. Just as we saw how manipulable materials could be either opaque, focusing the child on the materials themselves, or transparent, allowing the child to see the underlying structure of the number system, so it seems that procedural language forms may also be opaque, emphasising the words rather than the meaning behind them.

However, Jacob had finally mastered this language structure and continued to use it correctly in subsequent lessons.

The teaching of procedural language was also a feature of Chris's class where children were expected to reproduce forms such as “six count on four is ten” in the context of addition using the number track (Chris 5), and “one ten and two units is twelve” (Chris 8) as an interpretation of how many straws the children had collected. I found no such examples in the data from St David's School, a further indication of the atomistic/holistic approach to the curriculum (discussed in Chapter Seven). At St David's the holistic approach meant that children were encouraged to express their understanding in their own words and use a range of language forms. At Ashburne the atomistic approach resulted in the teaching of procedures (for example for counting-on) and procedural language. Teachers supported the learning of these language forms through a scaffolding process which involved modelling, and prompting, until the children could reproduce the teacher's language form for themselves.

**8.5.4 Summary of the use of language**

Spoken language was used in all the classes by both teacher and children in order to talk about their mathematics. Both teachers and children would use a mixture of everyday language and more specific mathematical language. Teachers tended to reinforce the formal language relating to the concept under discussion, using more informal language for explanation of related mathematical concepts which the children had not yet met. Children were able to use everyday language and they increasingly used a wide range of mathematical terms. These terms were used by the teachers, and more knowledgeable peers, and adopted by the children in a natural way. At St David's teachers encouraged a range of terms to describe a symbol or concept. At Ashburne the children were sometimes taught to use specific language forms to talk about their mathematics. These procedural language forms were always related to the procedural use of apparatus and, as such, were a further characteristic of the atomistic approach at Ashburne.
However in both schools the children's language learning can be seen from a socio-cultural perspective. Teachers scaffolded the children's language learning, through cues and prompts, and it took many weeks before some of the children could use the required form with confidence. Teachers and children negotiated meaning, through discussion. Mercer (1995) speaks of communities of discourse which have their own forms of language and ways of speaking. He argues that pupils need to engage in, and practice being users of, educated discourse. I would argue that what I have seen in the classrooms studied is the development of educated mathematical discourse, which is not uniform but may vary from school to school or classroom to classroom. The discourse uses mathematically specific words and specific language forms, which the children are encouraged to understanding and use. The idea of communities of discourse will be further developed in Chapter Nine.

8.6 Recording mathematics in pictures and symbols

As discussed in section 8.3, teachers used pictures and symbols in all the classes observed as a focus for discussion during interactive teaching, as an introduction to the task and as a way of recording mathematical tasks and in display. In this section I want to look the way that the children understood the use of pictures and symbols in learning about mathematics. Children have first to understand that tasks carried out practically can be recorded, which in many early years classrooms is done first using pictures, following Liebeck's ELPS (either explicitly by the teacher, or through the intermediary of the mathematics scheme).

8.6.1 The use of pictures as recordings

Despite representation through pictures being an integral part of many of the models for teaching and learning discussed in Chapter Two, pictures were not greatly in evidence in Chris, Debbie or Eve's classes. In Beth's class learning to communicate on paper was still in its early stages as the children were only just beginning to learn to write. The first lesson in which the children were asked to record their mathematics was the lesson where they used toy animals, partitioned across two fields, and described in terms of addition. After modelling addition with the toy animals, and talking about their work, the children are taught to record using pictures and symbols.

Beth Very good, now can you get out your books. Let's draw a little picture of what we did.
Jacob What we did on holiday?
Beth No, no, you'll find a picture of fields.
Children I've found it!
Beth Now let's have a look. Here we've got a field, it's a bit like a set ring, and a little arrow to tell us how many things we've got altogether. How many?

Children Three

Beth Can you remember how to draw a three? Draw one at the top for me.

(All the children draw a 3 at the top of the page. Ian starts to draw a backwards 3 and then corrects himself. The others have no trouble.)

Beth Well done. I'm going to let you choose how to put your animals. Either 2 and 1 or like Angela's 3 all in one field. Just see if you can draw your little animals just like they are on your fields. (To Ian) How many are you going to put in there?

Ian 2

Beth Good, don't worry about the shape, just a body and head and four little legs - have they all got four legs those animals?

(Beth 4)

The lesson was in the week following the half term holiday. Earlier in the week the children had drawn a picture and written about what they did in the holiday. For Jacob, the memory of holiday pictures seemed to be triggered by the phrase 'what we did'. He had not yet learnt that what we learn in mathematics can also be recorded by drawing 'a little picture'. Beth taught him that this picture is a mathematical representation, the numbers matter (even the number of legs on each animal) but this was not an art lesson and the drawing need only be approximate. Following this lesson, the reception children in Beth's class regularly recorded their mathematics as pictures in a similar way, and in the following section we will see the children choosing to use pictures in the context of a café activity.

However, learning to use symbols was more problematic and also highlighted unforeseen problems in relation to the use of pictures to record addition.

8.6.2 Learning to use symbols

Two types of symbols were used by the children to record addition: numerals and operational and relational signs (+, =). In Beth's an arrow (→) was initially used instead of the equals sign, following the use in the mathematics scheme. The older children were confident with both numerals and operation signs, although their interpretation of the purpose of these signs was less secure as can be seen from the discussion of symbols on the calculator in 8.6.2 above. The younger children were generally confident in the use of numerals to record and were taught to use operational and relational signs. For this reason the notable findings reported in this section draw on observations from Beth's class where the children were youngest (4-5 years) and therefore making sense of early encounters with formal symbols. Chapter Two summarised findings by Hughes (1986) and others which showed that children often do not
understand the reason for using symbols, even when they are able to interpret and manipulate them in mathematics lessons, and this study produces similar findings.

**Numerals**

Children used numerals throughout the study first to represent quantities of objects, and later as objects in their own right. Unlike some of the children in the preliminary study (see Chapter Four), all the target children in the main study could read the numerals to six when I began observing them. Even the youngest children in the main study had been in school for longer than the preliminary study children and fewer of them came from socially deprived backgrounds. The children were able to read the numerals to six, write the numerals in a recognisable form as a record of ‘how many’ (although Jacob could only write 1 to 4), and respond to a given numeral by counting out an appropriate number of cubes.

When playing the dice game (described in Chapter Five, section 5.4.1) the five target children in Beth’s class all counted the dots to find the total on their two dice when both showed dots. But, when one die with numerals was substituted, none of the children was able to use the numeral to represent a quantity to which to add the dots on the other die. The purpose of this substitution is to encourage the children to use a counting-on procedure. Counting-on would be too advanced a strategy for the children in Beth’s class. Children who cannot count-on will usually find some way of representing the number signified by the numeral in order to count all, which might involve the use of fingers or imagining an array of objects which are then counted. The children in Beth’s class were unable to use the numeral in this way. Ian and Charles ignored the symbol die altogether, counting only the dots on the remaining die to ascertain their score. The other three children (Angela, Emma and Jacob) counted the symbol as representing one, whatever the numeral, and continued to count the dots on the other die. They could not make the connection from the written symbol to a mental or physical representation in order to carry out the addition. It would seem that they were unable to see the symbol as representing a quantity in the context of the dice game (see Chapter Seven, section 7.3.2 for further discussion of this). In the game the children are required to do more than just move from one form of representation of number to another (word or objects to symbol, or symbol to objects), as they had been seen to do in the past. They are required to use the number represented by the symbol and operate upon it. When faced with the numeral 3 in a calculation it is no longer relevant to ask ‘three what?’. This symbol becomes an object in its own right upon which to operate.

This illustrates what is probably the earliest example of the children’s mathematics reaching the final stage of Mason’s spiral described in Chapter Two:

- an inner movement of increasing confidence:
  - from confidently manipulable objects/symbols,
through their use to gain a ‘sense of’ some idea involving a full range of imagery but at an inarticulate level,

通过一个符号性的记录，这种感觉

通过一个自信地操作使用的新的符号。

(1987, p.74,5)

The children here were able to use numerals as a ‘symbolic record’ of the quantity in a set represented by objects or pictures, but not yet ‘confidently manipulate’ the new symbols in the context of the dice game.

Over the observation period the children began to make sense of numerals in a range of addition contexts. By the final assessment all the children could interpret the die symbol as representing a quantity (most commonly on their fingers) and add to it. All the children could use numerals to record their work when asked and in the following extract we can see Angela and Emma choosing to use symbols in the context of play. Four of the children were playing in the class ‘café’. This was set up in a corner of the classroom, with tables and chairs, dressing up clothes, cutlery and crockery, food made from salt dough and painted, and a menu with pictures of the food and prices.

Jacob and Angela put aprons on to be waiters and then Emma and Charles dressed up as customers with a handbag each containing money.

Jacob started off with a gruff voice “What do you want?”

Angela got a pad and pencil to be a waitress.

Beth Is the tea shop open? (Angela changes the ‘closed’ sign to ‘open’) Oh, good you’ve got the open sign on. Now when you come to the shop, when the customers come in they want to be served so you need to come over and see what they want, have you got a pad and pencil? And you two need to decide what you want.

Emma A jam tart
Beth How much
Emma Two p.

Angela draws a jam tart on her list and writes 2p.

Jacob is very keen to get the jam tart but Beth stops him.

Beth What else does she want?

Emma A biscuit, a rectangular one. (The biscuits came in several shapes to reinforce shape language.)

Angela draws a biscuit and goes to get her order.

Jacob comes to take Chris’ order.

Jacob What do you want, er?

Charles A biscuit please, a round one.

Emma (to Angela who has brought her cakes) Can I have a orange juice?

Angela adds this to the list which now has drawings and 2p, 2p, 1p on it.

Beth How much is Emma going to pay?

Angela Five

Emma pays her, then Angela and Emma change roles.

Beth then left them to play on their own. The girls wrote prices and drew pictures very clearly and knew what to charge. The boys were less clear in
their drawing and had to try and remember how much to charge and often got it wrong.

(Beth 9)

Perhaps influenced by the format of the menu, all of the children used pictures to record what had been ordered. Angela has seen a purpose for recording the amount of money as well as the picture on her pad and is able to use this record to charge her 'customer'. Emma was also able to do so, while Jacob and Charles did not yet seem to realise the purpose for recording numerals. The children are beginning to see a purpose for symbols as a record of number.

**Operational and Relational Signs**

While the children grew increasingly confident at using numerals to record quantity, they found recording complete number sentences in addition more of a problem. It should be noted that, unlike numerals, I found no examples of operational or relational signs in classroom displays, nor are such symbols evident in the everyday world outside the classroom. The children do not therefore have opportunity to learn about the use of these symbols except in mathematics lessons. Analysis of what they did do, related to the other forms of representation which they used, gave insight into their understanding of symbols.

In Beth's class the children were working with the number four, choosing four cubes at random from a bag containing red and blue cubes. Beth then taught them to record this as a number sentence.

Beth  
So, each time we make four we have a different number story. Do you remember when we did our number earlier, we used this little sign (Beth writes + under the first line of four cubes) What does it mean?

Angela  
Altogether (hesitantly)

Charles  
Add

Beth  
Yes, Altogether, add or and. So we can use this little sign again (completes the number sentence 2 + 2 → 4). So, what would we write under this one? (1 + 3) The children take it in turns to write the numbers below the cubes and put in the + and → sign.

Beth  
Well, done. Now Angela and Charles and Emma, in your maths books you can draw me a picture of three different ways of making 4, just using red and blue colours and your writing pencils. Jacob, can you just start by drawing one of the ways to make four.

(Beth 5)

Emma drew three blue cubes and one red cube in her book and wrote 1+1+1 underneath the blue cubes. She hesitated, counted all the cubes and, finding that the red cube gave a total of four, wrote 4 below it. She looked at her work for a little longer and then added an arrow before the four.
Emma’s symbols seemed to represent her thinking about the mathematics. She was aware of the symbols to use for recording addition (+, \(\rightarrow\)) and associated these with her elementary addition strategy which is to count all. Since the cardinal number of the set is also the count word for the final cube (the cardinality principle, Gelman and Gallistel 1978), the red cube is represented by both. She knows that the arrow sign is associated with the final count word. Although she is able to represent her addition calculation correctly in spoken language, “three and one altogether make four”, she does not seem to make the link between spoken and written representations and instead tries to represent her actions in symbols. In terms of addition, her model is counting-all and therefore the final count word is associated both with the last cube and with the set, which she shows in her recording.

Following this lesson, Beth introduced the children to the equals sign instead of the arrow sign used in the earlier lessons, because Charles was having trouble drawing the arrow sign and, as she observed to me at the end of the lesson:

“It would be a good idea to use the equals sign now wouldn’t it because it is easier.” (Beth 5)

The same group of children some weeks later were using the story about a bus to discuss addition. In the story that day, there were ten children on a double decker bus with some children upstairs and others downstairs, which could then be represented as an addition sum. Ian drew a bus with nine children upstairs and one child downstairs. He told the teacher that he had “one up and nine down and ten altogether”. He was asked to write a ‘number sentence’ underneath. Ian wrote 1 9 10 and looked for a while. He then wrote an equals sign between the 1 and the 9 giving 1 = 9 10.

Again, Ian seemed to be making a record of the addition process rather than trying to interpret in symbols that which he could articulate in words. Although Ian’s recording is perhaps even less recognisable as the formal recording for addition than Emma’s, it seems to show a better understanding of the process of addition. He records that 1 and 9 are ‘joined together’ in order to make ten, indicating an understanding of addition as the combining of two sets. Although he is using a standard symbol, he is using it in his own way to represent the addition. Ian is able to translate the story into a picture, reinterpret that in the language of addition and to represent the process of addition as the union of two sets in symbols. He appears ready to recognise the usefulness of symbols and needs to have the addition sign reinforced in this context. He is not yet making a relationship between his spoken language and written symbols.

Emma and Ian seem to understand that operation signs are used to record what they have done. But the problem of interpreting a static iconic representation (picture) in symbols does not help
the children to understand the use of operational and relational signs. The signs in \( 1 + 9 = 10 \) show a process by which 1 and 9 are combined, resulting in a new set (10). But the new set is not represented in the picture.

\[
\begin{array}{cccccc}
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1 & + & 9 & = & 10
\end{array}
\]

While we might expect the children to attempt to record their spoken language symbolically, to move from language to symbols as they do when writing in words, instead the children attempted to represent their actions in symbols, moving straight from enactive to symbolic mode, based on their current enactive understanding of addition.

While it seems a great step from enactive to symbolic mode, getting children to record their actions first in pictures may not help move them towards symbols either. The static nature of the picture does not help to represent the dynamic addition operation, or the relationships between the numbers that the written symbols represent.

### 8.6.3 Summary of recording mathematics in pictures and symbols

This section shows the problems that children can have in understanding the representations used for recording mathematics. The use of pictures is less problematic once the children have understood their use in mathematics. The children were then able to use pictures independently. The use of numerals to record quantity was already understood by the children at the start of the study. But this understanding did not mean that they were able to use a numeral as representative of an unseen set as shown in the dice game. The children required more experience of numerals before this abstraction was mastered.

Learning about operation signs seemed most difficult for the children. Unlike numerals, these signs appear exclusively in mathematics lessons and the children will have little experience of them before entering school; while many people carry out addition in their everyday life and work they seldom use signs to record it. But recording using pictures did not appear to help the children to understand the use of symbols for recording. The children tended to show understanding of the enactive process of addition (counting-all and combining sets) in their use of signs, rather than making links with pictures or spoken language. We have seen that such links are in themselves problematic, since neither manipulable materials, nor pictures, nor spoken language, represents the mathematics in a form that matches the formal symbolic representation. Although this is only one specific example of learning mathematics, this finding appears to challenge the use of Bruner’s enactive, iconic, symbolic framework as applied to physical representations for mathematics teaching.
The model of representations, derived from Lesh et al., and derived from a constructivist perspective has not helped me to understand the problems that the children are having in learning to use symbols. I believe that to see symbols as a socio-cultural tool which the children are learning to master, is a more powerful perspective. This perspective would allow teachers to recognise the children's developing mastery, rather than seeing non-compliance (1+1+1= 4) as incorrect or as misunderstanding.

8.7 Real World Scripts

8.7.1 What is a Real World Context?

The fifth and final category of representations identified was that of 'real world script'. As identified in Chapter Two, the category 'real world script' was an uncertain one with reference to the literature. It was chosen to identify where the mathematics was represented in the form of a real world context, or in the form of a story. Lesh et al. (1987a) use the term 'real script' to speak of "experienced based 'scripts' - in which knowledge is organized around 'real world' events that serve as general contexts" (p. 33), but I have included the word 'world' to remind the reader that the context is outside the mathematics classroom.

This category of representation included a range of contexts which I found could be further subdivided into real world contexts, stories, and socio-dramatic play where the children acted out situations which could be seen in the real world. Indeed, I was attracted by the term 'script', rather than the more familiar 'context', as it related well to the use of story and play, as well as contexts from the real world. We will look briefly at these in turn.

Real World Contexts

Few mathematical contexts arising out of the real world emerged from the data. Real world contexts in mathematics textbooks are often related to the world of work and finances. The world of the classroom seemed a long way from the world of work both geographically and in time; these children had twelve or more years in school ahead of them. However, as in the preliminary study, real mathematical contexts for the teacher and school did sometimes emerge. Discussion of how many children were present, were absent that day, were allowed to do a particular activity etc., occurred throughout the day in all classes. Sometimes these discussions involved addition: for example, the teacher would ask the children to add together the number of children who had requested school dinners and the number of children who had brought a packed lunch, in order to check that all the children present were included on the dinner register. The significant aspect of such a real world context was that it often required a higher level of mathematical thinking, or dealing with larger numbers than the children used in their mathematics lessons. On my first visit to Beth's class just such a calculation was discussed.
Beth used a pictograph to record what the children were having for lunch. On the left-hand side pictures showed a lunch box, a plate of food, and a house for those going home to lunch. Small badges, each with the name of a child in the class (represented by O in the diagram below), were used to record what sort of lunch the children would have. A row of numerals were written below the badges to encourage the children to read off the total, instead of counting the badges.

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Sixteen packed lunches and eight dinners had been recorded. Beth How many more packed lunches, Charles? Charles (counted silently, pointing to the badges to the right of 8 on the packed lunch row) eight Beth So, 16 is 8 more than 8. Charles and eight and eight are sixteen. Beth Good boy.

At this stage the children were working on the numbers 1 to 5 in their mathematics lessons and had not yet started formal addition. Charles shows competence with his understanding of addition of much larger numbers. Perhaps the pictograph helped, but perhaps the fact that it was about real data arising out of the everyday world of the classroom also helped him to make sense of it.

**Stories as contexts**

Much more common than real world contexts was the use of stories to represent mathematics. Some of these were based on published stories (e.g. The Very Hungry Caterpillar), or nursery rhymes (e.g. Five currant buns in the baker’s shop), while some used toys to model imaginary situations (e.g. animals in fields, in the lesson described in section 8.5.3 above). In one sense none of these were really real world contexts, caterpillars are not known for eating sweets (as the one in the story does), modern children may not have been to a baker’s shop and cows are rarely free to move from field to field at will. But it is closer to the world of the child; playing with toys, reading books, chanting nursery rhymes and watching television, than the real world of the adult is. I will consider the effect of these stories on the children’s learning in section 8.8.2 below.
Socio-dramatic play contexts

The third category of real world script was that of socio-dramatic play - a psychological term used in early education settings for context-based role play, during which "children learn that people play different roles in our society and can come to terms with mastering skills and competence" (Curtis 1998, p. 98). Often these play contexts were set up as contexts in which the children could learn about money and carry out simple calculations. We have already seen Beth's children playing in the café in this way. In Eve’s class the children were learning about growing things in the science curriculum and the corner of the classroom was set up as a flower shop. The children had made flowers from sticks, wire and tissue paper. They took it in turns to 'play' in the shop and were given instructions as to how to play.

Eve  Right, I am going to have a little group of you playing in the shop. Can you look at the prices and try and see if you can buy and sell the flowers, not just playing about. Who hasn’t had a go in the shop?

(Eve 5)

The children took turns to act as shopkeeper and customers and used plastic money and a toy till. They checked one another’s calculations and the money and change offered. While the shop is still not a real context (toy flowers in the classroom) it is giving the children an approximation to the real world. The children are representing adults and acting as adults.

We saw in Chapter Three, how learning through play was a key foundation of both the Hadow and the Plowden reports but not of the National Curriculum. In play the children do more than carry out teacher directed tasks. They act out their understanding of what adults do in the situation, as can be seen in Jacob's role as waiter in the café.

Jacob and Angela put aprons on for the shop keepers and the others dressed up as customers and got a bag each.
Jacob started off with a gruff voice “what do you want?”

(Beth 9)

Jacob's experience of male waiters in cafés was that they had gruff voices so he would too (although he has not yet understood the concept of politeness!). The role playing therefore gives the children the opportunity not just to do real world mathematics but to act as adults carrying out mathematics in the real world. Far from 'just playing' they are 'learning to be' and in 'learning to be' they are also learning to be mathematical.
8.7.2 The significance of 'real world scripts'

As identified in Chapter Six, the category of real world scripts arose out of the analysis of a critical incident from the data and I want to look more closely here at how real world scripts emerged. Three lessons, two of which will already be familiar to readers of this thesis, will help in this explanation, although these are representative of others observed. Each of the lessons presented started with a whole class introduction, following which the teacher, Beth, worked with a group of five children: Emma (5), Angela (5), Charles (4), Ian (5), and Jacob (5).

Farm Animals in Fields

The lesson started with the children around the table with Beth.

On the table are sets of three farm animals and a card for each child with two irregular fields drawn on.

Beth  Now we've all got a little picture of two fields. Can you put your animals in the fields. Let's have a look at our animals. Ian, how many in that field (3) and in that field? (none) How many altogether? (3) So you started with three and you've still got three in all your fields.

Beth  Jacob, tell me about your fields.

Jacob  Got 2 pigs in that field and 1 pig in that field.

Beth  How many altogether? (3) You've still got three. What about you Emma?

The children each explain sets of animals in similar language.

Beth  Now, can we all make our sets look like Ian's? Can you tell me a little number sentence about what you've got here?

Angela  We've got three and none (Charles interrupts with zero) and altogether got three.

Beth  Now, all the cows walk into the other field. Tell me a different number sentence.

Ian  We've got zero and three and altogether got three.

Beth  Can you make one of your animals walk into another field? What have you got now?

Jacob  Got one in that field and two in that one...

(Beth 4)

The initial analysis of this lesson showed elements of spoken language, manipulative materials (animals) and later, as the children recorded their work, pictures and number symbols. The lesson went as planned with no apparent evidence of misunderstanding. It is included here as a contrast to the next lesson we will look at, which was used in Chapter Six as an example of coding, and identified the code: misunderstanding.

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1 This section draws on work published as (Price 2000)
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Partitioning Eight Cubes

The children started with a worksheet containing rectangles and were asked to select eight multilink cubes which they placed in the top section.

Beth Can you put some of the eight in this set and some of them in this set? (indicating the two rectangles on the middle row.)

There is a problem here since no more than three cubes will fit into each rectangle drawn on the sheet. Angela fixes hers together in a square, Charles tries to squash his in, while the others let them spill out over the lines. All the children place 4 in each subset except Emma who has 3 and 5.

The children took turns to describe their cubes as a number sentence.

Beth So, everybody except Emma made 'four and four altogether make eight' and Emma made 'five and three altogether make eight'. Now... I want you to change your number story now, but to a different number story in the bottom sets.

All the children create a different number sentence with their cubes except Angela who still has 4 and 4, since she had merely transposed the cubes to the opposite boxes.

Beth It’s still 4 and 4. Can you break it up and make it different?

Angela removes one cube from the left hand set and hides it in her hand.

Angela Three and four together make eight (very hesitantly, looked at the extra cube and put it back in the three but this time to make an L shape rather than a square) Four and four together make eight.

Beth Can you change the cubes? (Angela looks puzzled.) Have a look at some of the others to see if it will give you a clue.

Angela repeats her procedure of removing one and counting the rest “1,2,3,4,5,6,7.”

(Beth 10)

As in Chapter 6, I have italicised the mis in misunderstanding to emphasis that the child is not just 'getting it wrong' but forming a logical understanding of her own, which is not the same as that of the teacher - a constructivist perspective.
In terms of elements of communication, this part of the cubes lesson consists of manipulative materials used to model the mathematics and spoken language used to describe it. The teacher has started from the mathematics and used the multilink cubes as a model. Superficially there seems little difference between this lesson and the farm animals lesson in terms of mathematics. The numbers used are greater but Angela can confidently count out objects to thirty. So what causes Angela such problems?

First the cubes will link together and, once linked, the mathematical model is changed in a subtle way. For Angela the shape of the resulting structure seems to take over. The cubes are no longer seen as individual items but as an entity. When asked to change them she can no longer focus on the cubes being the same numerically, since they are so obviously different with respect to shape.

\[
\begin{array}{c}
\begin{array}{c}
\text{is not the same as}
\end{array}
\end{array}
\]

The multilink cubes, which seem socially neutral compared with the animals and should therefore be less distracting, cause more distractions; their meaning becomes opaque and Angela can no longer see the mathematics that they are being used to represent. Are the cubes more distracting than the animals because they have characteristics of colour and are able to be fixed together? But there are also characteristics in the animals that could have distracted the children - they are smaller than real animals and made out of plastic, plastic animals cannot walk, and the fields have no grass in them. The children questioned none of these, perhaps realising that because it was a story, some characteristics were representative rather than real. In the case of the cubes, Angela does not know what the cubes are meant to represent and seems unable to express her frustration, while Beth is unable to clarify it for her. Communication fails.

The teacher sees the mathematics as situated in the manipulation of cubes from which generalisations about number pairs that make eight can be generated. But Angela wants the task to make sense in her social, experiential world and is confused when it does not seem to. After the lesson, Beth expressed her frustration at not being able to understand what Angela was thinking. It was this critical incident that highlighted the possible significance of a real world context. Did misunderstandings arise when real world scripts were not used and not when they were?

The third lesson takes us back to a social context, but here Charles is having problems.

**The Bus Lesson**

The bus lesson starts with the whole class of twenty-seven children sitting together with their classteacher. The teacher, Beth, introduces the lesson with a rhyme. Each time the rhyme is
said, different numbers of children are said to get on the bus, creating pairs of numbers for addition.

The teacher takes a situation, children travelling on a double-decker bus, which can be used mathematically to exemplify the concept of partitioning ten into two subsets (upstairs and downstairs) and which can then be explained in the language of addition to re-aggregate the subsets. In this example the teacher uses pictures, of children and the bus, to model the situation and uses spoken language and later symbols to describe the situation in mathematical terms. But for the children, the mathematics is also situated in a real world script, that of children travelling on a bus, an experience which they have shared as a class. The teacher is therefore able to relate the mathematics to their shared social experience.

Beth Here comes the bus it soon will stop,
Hurry up children in you pop,
5 inside and 5 on top,
How many altogether?

The children raise their hands and one is chosen to answer. The numbers vary but each time the total is ten.

Beth Well done ...Now, can we all see the board?

Beth draws a bus outline on the board. She has also prepared ten circles of card with children's faces drawn on them, and with blu-tack on the back so that they will stick onto the board.

Beth Let's see if we have got ten children (counts the faces with the class). Now, all the children want to go on the bus. Do you think they are going to the Garden Centre, or perhaps they are going swimming.

Beth Now, let's put some on the bus (puts five faces onto the top deck counting) 1, 2, 3, 4, 5. So, how many inside (places rest of faces onto lower deck counting) 1, 2, 3, 4, 5.

Charles What about the bus driver? (this is ignored)

Emma Five add five altogether make ten. ...

Charles (again) What about the driver?

Beth We're not worried about the driver. I think he has gone for a cup of coffee. Now, while he was having his coffee some of the children started to run about, so ... she went upstairs ... and so did he ... and so did she (moving a face each time.)

Beth Who can tell me something about the numbers now?

Ian It still makes ten

Beth Very good. Ian, can you tell me about this? How many on top?

Ian Eight.

Beth And how many downstairs?

Ian Two

Beth So, can you tell me anything else?

Ian Eight add two altogether makes ten. ... (Beth 12)

The fact that the mathematics is situated within the context of a bus journey does not mean that it is unambiguous. Charles is concerned with the driver, perhaps because there should be another person on board, making 11, or perhaps because he is concerned about how the bus
will move. But he is able to use his knowledge of bus journeys to articulate his concern and Beth is able to use the story element to deal with his concerns, which is in contrast to Angela’s problem in the previous excerpt, where she was unable to articulate her problem so Beth was unable to recognise and deal with it.

8.7.3 Real World Scripts Identified

While this theory emerged from analysis of these three teaching episodes, verification against all the teaching episodes from Beth’s class, showed that where the lesson contained elements of story, grounded in the real world experience of the child, then the children were more able to make sense of the mathematics and more able to talk about problems which did arise. This story element I categorised as a ‘real world script’. The scripts identified a shared social and cultural world, in which the mathematics could be communicated through shared meaning. Because the children had been in school for only a short time they had not yet built up, amongst themselves and with the teacher, a shared mathematical world in which to work. They relied on a shared understanding of the wider world. In the language of the Freudenthal Institute (discussed in Chapter Two, section 2.3.6) the real world script allows the introduction of addition through horizontal mathematization (from the real world) and, as their mathematical knowledge develops, they will be better able to work within the mathematics of addition itself (vertical mathematization).

Further analysis of the data showed that this effect of real world scripts was not just a characteristic of Beth’s class. There were occasions in each of the other classes where a real world script element enabled the children to carry out a task they found difficult otherwise. For example, the children in Debbie’s class were having difficulty, as I have observed many children having, with missing number problems of the form:

\[ 7 + \Box = 12 \]

The activity was introduced in the context of detectives. A number thief had stolen some of the numbers leaving empty boxes and the children’s task was to find out which numbers he had stolen. This ‘story’ (however implausible) acted as a scaffold, enabling the children to understand the symbols and carry out the task.

What, then, are the characteristics of a real world script which scaffold the children’s understanding?

8.7.4 Characteristics of Real World Scripts

In the lessons containing a real world script I identified five common factors: animation, integration of the mathematics, motivation, shared culture and interpretation.
Chapter 8 Representing Mathematics in the Classroom and in the Real World

Animation

Firstly the objects (people, animals, beetles etc.) were all animate objects. In many mathematical situations, and particularly in addition, the mathematics describes change or movement. In Martin Hughes’ (1986) study of children’s use of symbols to record addition and subtraction, none of the children used formal addition and subtraction signs (+, -) but several of the children drew a hand effecting a change in the number of cubes present, while one child used pictures of soldiers marching on (+) or off (-) the page. Animate objects are capable of change through movement in a way that inanimate objects are not.

So, Beth is able to instruct the children:

Beth Now, all the cows walk into the other field. Tell me a different number sentence.

Or

Beth Now, while he (the driver) was having his coffee some of the children started to run about, so ... she went upstairs ... and so did he ... and so did she (moving a face each time.)

 Whereas in the cubes lesson the instructions have no animate meaning; the emphasis is on the child to effect the change.

Beth Now... I want you to change your number story now, but to a different number story in the bottom sets...

Beth It’s still 4 and 4. Can you break it up and make it different?

Animation, the first characteristic of a real world script, gives meaning to the condition of change which can be described mathematically.

Mathematics is an integral part of the story

Secondly, because there is animation in the action, the mathematics can be seen as an integral part of the real world script, the real world script not merely added on to the mathematics for cosmetic purposes, to make the mathematics more interesting. The mathematical concept is contained within the story. Children moving around on the bus, represent different ways in which they can be partitioned. So too do animals moving from field to field. The movement brings about the changed mathematical state, a different combination of numbers which make the total. Representation of mathematics through real world scripts requires not just a story, but a story where the mathematics is an integral part of the story line.

This can be explained by looking at a counter example where the mathematics was not integral to a story context. In Chris’s class the children were working on addition and subtraction calculations. To make them more interesting, and to relate them to the class topic on growing things, Chris had designed a worksheet with a picture of a rabbit. Around the rabbit were a large number of carrots each containing a calculation. The children were required to do the calculations first. At the bottom of the sheet was a number-to-colour key and the children had
to use this to colour in the carrots, according to the answer to the calculation (e.g. all the calculations with an answer of 12 were coloured blue). I recorded that Dipesh was having trouble with colouring the carrots, because he wanted to colour them all orange! Here the ‘story’ about rabbits and carrots was decorative, and possibly motivational, but the mathematics was not contained within the story, and the story element did not give meaning to the mathematics.

**Motivation**

Thirdly, animate objects are considered to be *motivated* to change. There can be a good reason for an animal to want to move from one field to another. Children enjoy running up and down stairs, especially if they are not supposed to be doing so. For, as Beth exclaims later in the lesson" 

**Beth** They are a bit naughty these children running up and down the stairs, (Beth 12)

Inanimate objects are not mobile, nor are they motivated. Why should multilink cubes be partitioned in different ways except to represent the mathematics? Cubes are not sentient, they have no motivation.

**Shared Culture**

The fourth characteristic of real world scripts is that they allow the teacher and children to have a shared understanding of the context. The contexts chosen arose out of the *shared culture* of the classroom and society. Examples may not reflect actual experience; it is possible that Jacob may not have seen a live pig, farms around the school tending to keep cows or sheep, but pigs are part of the shared culture, in stories and on the television. In the bus story Beth refers directly to their shared experiences:

**Beth** Now, all the children want to go on the bus. Do you think they are going to the Garden Centre, or perhaps they are going swimming? (Beth 12)

The children had recent experience of travelling in buses. The previous week the class had been on a visit to a large garden centre as part of their science work on plant growth. They also went swimming once a week during the summer term. For each of these they were taken in a bus. So the *shared culture* behind the story helps the children to understand the mathematics contained within it. This seems a good example of Bruner’s observation that

“most learning in most settings is a communal activity, a sharing of the culture. It is not just that the child must make his knowledge his own, but that he must make it his own in a community of those who share his sense of belonging to a culture.” (1986, p. 127).

Mathematics is not just an abstract subject encountered in the classroom but relates to the real, shared, cultural world of the children and teacher.
Open to interpretation

Finally, real world scripts allow the teacher and children to negotiate text, e.g. to discuss the absence of a driver, since they allow for interpretation. Charles interprets the situation as lacking a driver, Beth is able to reinterpret it to explain the naughty children. This interpretation is in contrast to the cubes lesson, where the teacher was unable to understand why Angela was having trouble understanding her instruction “Can you break it up and make it different”, because there was no context in which to interpret the instruction. Angela makes a literal interpretation, breaks the cubes and makes them look different. It is the nature of real world contexts to allow for interpretation, but without a real world script Beth sees the cubes as representing the mathematics and cannot understand why Angela cannot see it too. I am reminded of Paul Cobb’s work on the use of manipulatives in teaching mathematics (Cobb 1987). The teacher sees the mathematics as situated in the manipulative materials, for example sees Dienes apparatus as representing place value, but the children see only the apparatus and have no way of interpreting it because of their lack of place value knowledge. Cobb concludes that the use of manipulatives in teaching place value is therefore problematic. In Chapter Two, I questioned whether findings about the use of manipulative was relevant to the teaching of simple arithmetic, which the literature did not address. This study has therefore confirmed that the lack of shared meaning (Cobb), or opacity of artefacts(Adler 1998), is a problem for single digit representations as well as materials used to teach place value and more complex mathematics. But real world scripts help to provide transparency.

Each of these factors: animation, integration of the mathematics, motivation, shared cultural knowledge and interpretation, which seem to effect successful learning in mathematics, can be seen as characteristics of real world scripts. They appear to scaffold the children’s learning. But they do not explain why such characteristics should be valuable in teaching addition. Such an explanation may be found in Bruner’s theory of narrative.

8.7.5 Real World Scripts as examples of narrative

In Chapter Two, I discussed Bruner’s theory of narrative, arising from a socio-cultural perspective of learning, in which he argues that human culture is framed by narratives (1996). Our personal as well as our collective histories are told as narrative, stories which have a common structure even though the detail and the interpretations may change. He argues that

[i]t seems evident, then, that narrative construction and narrative understanding is crucial to constructing our lives and a “place” for ourselves in the possible world we will encounter.” (p. 40)

Through narrative, young children learn about their personal and cultural histories; their understanding of the social word, and the world of language, is developed not only through personal experience but also through the telling and hearing of narrative stories. Bruner asserts that:
Understanding is the outcome of organizing and contextualizing essentially contestable, incompletely verifiable propositions in a disciplined way. One of our principle means of doing so is through narrative: by telling a story of what something is about". (1996 p. 90)

Recently, Krummheuer (2000) has related this idea to argumentation in the mathematics class, encouraging children to ‘tell the story’ about how they understood and carried out a task. Does it also apply it to learning addition through stories and real world scripts?

If we see mathematics as about fixed truths then this may not fit with Bruner’s ‘contestable, incompletely verifiable propositions’. We may then see narrative as applicable to the learning of the social sciences but not to mathematics. The nature of mathematics has been discussed at length (Ernest 1991) and it is beyond the scope of this chapter to rehearse these arguments. However young children have not yet learnt that society may see some disciplines as ‘factual’ and others as ‘relative’. Children, constructing knowledge for themselves, will test all new knowledge in the light of their existing understanding. So, the learning of mathematics through narrative becomes a viable proposition. Is this, then, a description of what is happening in the use of real world scripts?

Bruner identifies nine “universals of narrative realities” which were discussed more fully in Chapter Two (section 2.2.2).

- The structure of committed time;
- generic particularity;
- actions have reasons;
- hermeneutic composition;
- implied canonicity;
- the centrality of trouble;
- ambiguity of reference;
- inherent negotiability; and

The real world scripts described above have a simplicity when compared to the complex narratives of literature or history, animals moving around fields do not make an exciting story, but do the characteristics of real world scripts have resonance with Bruner’s universals? If we accept with Bruner that narrative is one of the principle ways in which we make sense of the world, then relating the characteristics of real world scripts to those of narrative may help to explain the effectiveness of real world scripts in teaching early addition.

**Animation and the structure of committed time**

Bruner talks of the “structure of committed time” as a characteristic of narrative explaining this in terms of “time that is bounded not simply by clocks but by the humanly relevant actions that occur within its limits” (p. 133). ‘Humanly relevant actions’ can be equated to the concept of animation which I identified as a characteristic of a real world script. It is not just a description
of a static situation but contains action within time. The numbers of children up and down stairs, numbers of animals in the fields alter with time, while, without animation, the cubes remain the same over time.

Integration of the mathematics and generic particularity

Narratives, according to Bruner, contain particular examples which contain generic truths. So literary narratives can be categorised within genres, while historic narratives can help us to form principles which guide our future action. Mathematical contexts can also be seen as containing the general, the generic concept, within the particular. The stories of animals in fields or children on the bus contain the mathematics of partitioning and reaggregation within the particularity of the story. So, real world scripts which carry integrally a mathematical concept can be seen as having generic particularity within mathematics.

Motivation and actions have reasons

"What people do in narratives is never by chance, nor is it strictly determined by cause and effect; it is motivated by beliefs, desires, theories, values or other "intentional states". Narrative actions imply intentional states." (p. 136). So, Bruner identifies motivation as a narrative element; motivation is both a characteristic of real world scripts and of narrative.

Interpretation and hermeneutic composition, ambiguity of reference, inherent negotiability and the historical extensibility of narrative.

Each of these four characteristics of narrative, that it requires interpretation, is ambiguous, that its meaning is inherently negotiable and that the narrative is only part of an extended story, all reflect the view that one of the characteristics of real world scripts is that they allow for interpretation, and as such meaning can be negotiated. So, Beth and Charles were happy to negotiate their reading of the absence of a driver, Beth could use Charles's problem to extend the story. But she was unable to understand why Angela did not interpret the situation with the cubes in the same way as Beth herself did, since there was no story, nothing to negotiate. The class has yet to establish the 'normative patterns of interaction and discourse' of the classroom (Wood 2000) which will allow a shared understanding of more abstract mathematics apart from the 'real world'.

Shared cultural knowledge and implied canonicity, the centrality of trouble.

Bruner identifies that narratives often breach implied rules about how life and society work and it is this that makes them interesting. The implied canonicity assumes a shared cultural knowledge, rules about how life and society work. There is a shared knowledge about behaviour on buses. The 'naughtiness' of the children breaches implied rules about how to behave, causing trouble. The children also know that the total number of children on the bus
does not alter as they run up and down stairs, which allows Ian to conserve number in a way
that he was not able to in the context of cubes:

Beth Now, while he was having his coffee some of the children
started to run about, so ... she went upstairs ... and so did
he ... and so did she (moving a face each time.)
Who can tell me something about the numbers now?

Ian It still makes ten.

Which contrasts strongly with Angela trying to change the number sentence represented by the
cubes:

Beth It's still 4 and 4. Can you break it up and make it different?

Angela removes one cube from the left hand set and hides it in her hand.

Angela Three and four together make eight (very hesitantly, looked at
the extra cube and put it back in the three but this time to
make an L shape rather than a square) Four and four together
make eight.

Angela has no expectations of how the cubes should behave and seems surprised when the
number sentence does not turn out as she expects. She has ‘broken it up’ and ‘made it
different’ as asked, but she knows that the number sentence is supposed to still make eight.
Chris’s children showed similar lacks of understanding when using the base ten materials,
materials with which they were unfamiliar and could give no meaning to.

8.7.6 Implications and conclusions on the use of real world scripts
Characteristics of real world scripts therefore can be related to Bruner’s nine universals of
narrative. Real world scripts, as defined in the teaching of early addition, can be identified as
narrative. So, Bruner’s assertion that one of the principle ways of understanding is through
narrative can help us to see how learning mathematics through real world scripts supports
children’s understanding. I am not arguing that mathematics need always be taught through the
use of real world scripts. As children gained more experience of learning mathematics they
developed a shared mathematical culture within the classroom to which to relate their new
learning. But at the beginning of school mathematics learning the children had no such
experience. Perhaps it would be appropriate to introduce each new concept through the use of
real world scripts. The introduction could be situated in the social experience of the children,
allowing horizontal mathematization, while subsequent work could build on existing
mathematical understanding, vertical mathematization, and the developing mathematical culture
of the classroom (Bauersfeld 1992; Cobb and Yackel 1998; Seeger et al. 1998; Krummheuer
2000). The identification of the characteristics of real world scripts, carried out in this study,
could further aid the planning of appropriate activities.

Mathematics is only one way of looking at the world into which children need to be inducted,
and stories, narratives, real world scripts can effect this induction.
8.8 Conclusion and implications on the use of representations

Overall the study shows the children’s increasing competence with representations within the mathematics of addition. The children were given few opportunities to select and use representations for themselves; in general the decision was made for them by the teacher, which seemed to situate the representation within the activity, rather than allowing the children to see it as a tool for learning and calculating. However, at St David’s the discussion about the 100 square and encouragement to calculate mentally, moved the children away from manipulable materials towards a range of mental strategies.

For the youngest (Reception) children, competence with manipulable materials, spoken language, pictures and symbols appeared to follow the order indicated by Liebeck’s ELPS model. Manipulable materials formed a focus for discussion and modelled mathematical situations. Spoken language was used in a range of informal and more mathematical forms with teacher scaffolding the use of specific procedural language. The children found the use of symbols and especially operational and relational signs, the most difficult to understand. While this difficulty could be due to the abstract nature of such symbols, and the lack of their use in everyday life, analysis of the relationship between symbols and the other mathematical representations showed that they was little direct relationship. Recording their work using pictures, did not seem to help them to learn to record using symbols, as the structure of the two representations was not the same. Pictures show only one set of the total number of objects while the symbol number sentence requires both the subsets and the total. Neither did the children see the symbolic sentence structure as similar to the spoken language structure, where the syntax of the spoken sentence is not directly analogous to that of the symbolic formula. Children’s own early recording more closely reflected their actions in the manipulable materials, counting, combining.

The use of real world scripts was significant, especially teaching the youngest children mathematics. They allowed a shared, social understanding of the situation under discussion which was not always present when a more abstract, mathematical context was used. As the children learnt more mathematics they and the teacher built up a shared mathematical culture on which to base their understanding but there were still occasions when the introduction of a real world contexts ‘scaffolded’ a difficult task for them.
Chapter Nine

The Social World and the Mathematical World of the Child

9.1 Introduction

This chapter considers the relationship between the social world of the children and the children’s learning. It sets out to answer the third of the research questions identified in Chapter Five, following the preliminary study:

\[ Q3 \quad \text{How does the wider social context of the classroom influence children’s learning of addition?} \]

It explores the relationships between different areas of that world, the wider social world, the world of the classroom, the world of learning, and the learning of mathematics. It reinforces findings from Chapter Eight, that opportunities to bring mathematics contexts from the wider world into the classroom not only enable the children to understand the mathematics but also help them to make links between school mathematics and their own everyday mathematics. It shows how the social world and the children’s learning of addition complement one another, how the learning demands of one subject area can also affect others and how the social world and learning may make conflicting demands on teacher and child. I have drawn on Mercer’s ideas of a ‘community of discourse’ (1995), mentioned in Chapter Eight with reference to language, and Lave and Wenger’s communities of practice (1991), to suggest that the classroom consists of multiple local communities of practice, each with their own local practices and discourse.

In comparison with the findings in Chapters Seven and Eight, the findings in this chapter are tentative, often arising from individual incidents. Yet these were critical incidents, relating to the wider findings of the study, and deserved attention. The findings therefore indicate areas for future research.

9.2 The World of the Child and the World of Mathematics

For the child, making sense of addition in mathematics lessons is only part of the wider task of making sense of the world. In the preliminary study, comparison of the children’s initial and final assessment tasks suggested that children had developed areas of mathematical knowledge which they had not been taught in mathematics lessons (see Chapter Four, section 4.8.3 for discussion of these findings), raising the question of how the wider social world of the classroom affected the children’s learning of addition. In addressing this question I want to
look not just at the social world of the classroom but also at indications of how the wider world outside school influenced the children's learning, and how these elements of the child's life interact and interrelate. In order to structure the chapter, I will describe the child's experience as a series of nested 'worlds', shown in Figure 9.1 below. The outer region is that of the child's experiential and social world, much of which happens outside the classroom. The world of the child consists of such things as family and friends; shops, libraries and cafés; exploration of the natural and man-made environment; play, games, sport, watching TV and celebrating birthdays and in section 9.3.2 I will consider the extent to which this world of the child is social and culturally constructed. In this study I had no access to the outer world of the child except in so far as it was brought into the classroom through what the child said and did.

As the child enters the classroom there is an inner region which represents the social world of the classroom, characterised by a different sort of social structure from that of family groupings in the world outside. Here children are organised for learning, in whole class situations, groups or individually, with different sets of rules for different settings. Behaviour is managed and modified through rules and reminders. Children are expected to take turns, not all demand...
attention at once; to put up their hands if they want to talk; to ask if they can go to the toilet. In addition to the teacher led structure of the classroom there is also a peer structure where children have to find their niche (Pollard 1996). Children make new friends and begin to identify themselves as individuals, apart from their role in the family at home.

The youngest children in the study were having to learn a new world of behaviour and relationships at the same time as they learnt addition.

The purpose for this organised social structure in the classroom is the third region, that of school learning. For these children in the early years of schooling, although there are ten National Curriculum subjects to contend with, mastering the basics of reading, writing and mathematics are often considered the most important by parents and government (hence the current National Numeracy and Literacy Strategies). But the children have yet to learn the way that adults differentiate parts of the curriculum into subject areas. For them, learning mathematics starts as an undifferentiated part of what happens at school.

The final, inner region for the purpose of this study is that of the mathematics lesson. Here the children not only learn to do mathematics, but also to develop attitudes towards this area of the curriculum and to see themselves as good, or not so good, mathematicians. However, learning about addition does not just happen in the mathematics lesson but happens in all areas of the child’s life. In the following sections I will look at how the outer layer worlds effect and affect the child’s mathematics learning and at how the mathematics learning influences the wider world of the child.

The diagram does not set out to show mathematics learning as a clearly defined area within these ‘nested worlds’. Mathematics permeates everyday life inside and outside the classroom as well as being the focus of mathematics lessons, while other areas of knowing and learning are the focus of other elements of everyday life and the life of the classroom (social relationships, learning to read and write, for example) but also permeate the mathematics lesson. In the following sections I will try to tease out these relationships as shown in the data collected for this study.
9.3 The relationship between the wider social world and mathematics learning

![Figure 9.2 Relationship between the wider social world and mathematics learning](image)

9.3.1 Using the wider social world to exemplify mathematics

The way that the wider social world of the child is used by teachers to exemplify contexts for addition has been discussed in detail in Chapter Eight in the context of real world scripts. Such examples appeared to help the children to make sense of the mathematics, and both children and teachers were able to talk about addition within real world contexts more easily than within more abstract mathematical contexts. Bringing examples from the wider social world into the mathematics classroom, had a positive effect on the children's learning.

9.3.2 Children's use of the wider social world in mathematics lessons

A second relationship between mathematics and the wider social and experiential world of the children was observed when the children themselves made reference to mathematical examples from the real world. There were very few examples, within the mathematics classroom, of the children expressing their knowledge of mathematics in the wider world, in contrast to lessons in other subject areas where they would often refer to things that had happened at home, out of school or on the television.

In Chris' class (at Ashburne), and in both the classes at St David's, little reference was made to the world outside the classroom during mathematics lessons. But, as well as using real world scripts, Beth often made passing reference to social use of number. For example when discussing the number five she commented:

> How are we going to write the word?  f, i v, e, five, if you are not five already you soon will be. (Beth 1)

Perhaps as a result, the children would occasionally make reference to the world outside the classroom in relation to their mathematics. In the lesson which followed, Beth demonstrated to
the children how to complete a worksheet containing six boxes each labelled with a number from 0 to 5. The children were asked to draw the right number of objects in each box. They could chose which objects to draw in which box. Beth asked Clare what she might draw in the box labelled 5.

Beth So what shall I draw here?
Clare Five candles on my cake.

(Beth 2)

Clare had recently had her fifth birthday and made the link between the social world use of number and the mathematics lesson.

On another occasion Jacob was working with Beth on the bus task, expressing how many children there were upstairs and downstairs on the bus as an addition sum. That day the total was always ten.

Beth Jacob, read your sentence out to me.
Jacob Eight and two altogether makes ten. We're learning about ten.
Beth What do you know about ten?
Jacob If you are ten you can run faster than me, because you know Martin who comes and sees me sometimes, he's ten and soon he is going to be eleven.

(Beth 12)

At the time I noted surprise at Jacob’s unsolicited observation “we’re learning about ten”. Since he could also have said ‘we’re learning about children’ or ‘we’re learning about buses’ this was a perceptive comment. He had come a long way from the child who did not know how to draw a picture of his mathematics and thought he should draw one of what he did on holiday, his previous experience of drawing a picture of ‘what he did’ (see 8.6.1 for a fuller description of this incident). It marked for me a change in Jacob’s thinking about his learning; he seemed to be beginning to identify number learning as a clearly differentiated part of the curriculum. This incident might be seen as the beginning, for Jacob, of mathematization, recognising the abstraction of mathematics as a particular area of his learning experience.

Afterwards Beth also expressed surprise, but was even more surprised by his answer to her question “What do you know about ten”. She was expecting something like ‘you can make ten with lots of different numbers’ (the focus of the lesson), but Jacob’s answer relates to his knowledge of the number ten in the world outside, in the context of age “If you are ten you can run faster than me...” Ten is a significant age for Jacob as he looks up to his older and more able friend. And he also knows that eleven comes next. Jacob is able to identify the relevance of number in the lesson and to relate it to his wider experience of mathematics in the world.

These are isolated examples and I do not want to read too much into them but both the children use age as a real world context for number. Age is important for the children both in terms of having birthdays and in terms of relative age, older, younger than me. It can relate ordinal
("he's ten and soon he is going to be eleven") and cardinal ("five candles on my cake") number. Perhaps, as teachers, we could make more of this real world context of the children to help them make sense of number, which would require us to talk to the children about both their life outside the classroom and their understanding of mathematics. For Jacob, the context of the mathematics had been one of children travelling on a bus. Beth had related it to the children's own experiences of buses.

Beth Now, all the children want to go on the bus. Do you think they are going to the Garden Centre, or perhaps they are going swimming.

(The previous week the class had been on a visit to a large garden centre as part of their science work on how plants grow. They also go swimming once a week in the summer term. For each of these they are taken in a bus.)

(Beth 12)

I believe that it was not a coincidence that this lesson, which used a real world example for teaching addition, firmly rooted in the children's own experience, elicited an example from Jacob's own real world experience of number. If we encouraged children to make more links between their social world and the world of the mathematics classroom, would they be better able to make mathematical sense of the world outside the classroom?

For mathematics learning to be effective, horizontal mathematization must work both ways: abstracting the mathematics from real situations and applying the mathematics to new real situation. Nunes (1991) study of street mathematics and school mathematics showed how children do not make links between what goes on in and outside school. Lave (1988) goes further and argues that cognitive skills are situated; they cannot be disembedded from their contexts and transferred to new tasks. Yet such an argument could lead to the disestablishment of schools altogether; if no transfer is possible then how can we equip children for the future? An alternative argument is that in order to enable learning transfer, teachers need to make explicit links between these different contexts. This present study suggests that it is possible to make links between the mathematics of the classroom and the mathematics of the world outside, enabling the children to make such links for themselves. If these, the youngest children in the primary school are able to make these links, there seems no logical reason why older children cannot do so too. If we want children to learn to make sense of mathematics outside the classroom, then we need to be explicit about the relationship between school mathematics and everyday mathematics.

Recently, Cooper and Dunne (2000) have published research which shows the problems that some children have in interpreting 'realistic' word problems in KS2 and 3 SATs tests. They found that "compared with service-class children, working-class and intermediate-class children performed less well in 'realistic' items in comparison with 'esoteric' items" in the tests (p. 199). These children demonstrated a conflict between social and mathematical
understandings of the problems set, and were more likely to give a socially appropriate but mathematically inappropriate responses. This finding appears to support the theory that learning is situated, the children could give mathematical answers to the ‘esoteric’ mathematical questions but not to the ‘realistic’ questions, which placed the mathematics in real world contexts. Cooper and Dunne conclude that “there are considerable grounds for concern over the use of ‘realistic’ items in the context of timed paper and pencil tests” (p. 200).

While, on the surface, this may seem to challenge my assertions that children are able to make links successfully between the world of mathematics and the world outside the classroom, I believe that it may, in fact, support my hypothesis. The children in Cooper and Dunne’s study are unlikely to have experienced the sort of mathematics curriculum I am advocating, where horizontal mathematization, learning mathematics from real world examples and applying mathematics to the real world, had high priority. I am not therefore surprised that they found applying mathematics to ‘realistic’ test questions difficult. I would assert that more time spent in mathematics lessons on mathematization of the real world and application to the real world, with discussion of how the problems can be solved both socially and mathematically, and under what conditions a choice should be made, could improve both attainment in ‘realistic’ test questions, and transfer of mathematical knowledge outside the classroom. Such discussion could especially help both working class children, and children from ethnic minority groups (a group not considered in the Cooper and Dunne research), who may have different social and cultural understandings from ‘service-class’ children.

Recognition of differences between classroom and everyday mathematics seems analogous to teaching children in English lessons that there are different forms of English, standard and dialect, which have relative strengths, weaknesses and purposes, which is acknowledged in the National Curriculum for English.
9.4 The relationship between the world of the classroom and mathematics learning

We saw in the preliminary study (section 4.8.1), and in Chapter Eight, how occasions arose in the classroom to carry out mathematical tasks, such as calculations of children staying to dinner, providing opportunities to mathematize real contexts arising out of the everyday life of the classroom. Teachers were able to take real examples of mathematical problems and work on them with the children. The children showed that their understanding of the real context enabled understanding of addition. In the context of working out the number of children staying to school dinners or eating packed lunch Charles was able to deduce that "eight and eight are sixteen" despite the fact that he was unable yet to do such calculations in the mathematics lesson.

The social world of the classroom was also seen to influence the mathematics learning. The way the teacher managed and interacted with the children influenced the children's understanding, and their motivation and attitudes to mathematics, as I will show in the following sections.

9.4.1 The influence of social interaction in the classroom on mathematics learning

Comparison of the different classroom situations shows that way that the children were organised for learning influenced their mathematics learning. Whole class, group and individual work did not only provide different mathematical experiences but developed different relationships between the teacher and the children and between the children themselves, which gave the children different messages about their mathematics.

Working individually

In Chris' class, at the beginning of the study, the children did most of their mathematics individually. After explaining the task to the children, Chris sent them off with the instruction:
Chris  OK, any queries? Off you go. Remember not to sit next to anyone else doing maths.

(Chris 1)

The children knew that they were expected to work alone, sitting next to children from other groups who were working on another subject area. They were not expected to talk about their work except to the teacher.

When I took the target children to carry out the assessment tasks Chris told them that they were going to do some mathematics with me. Several of the children expressed displeasure and Sam asked if it was "going to be hard sums". Having carried out the assessment tasks including the dice game, I said that it was time to go back into the main classroom. The children were surprised as they felt that they had not yet done any mathematics, their view of mathematics being to work individually on worksheets or in their mathematics books. On my next visit to the classroom Sam asked if we could go and "do them games instead of maths".

Like the children in Desforge and Cockburn's study (1987) these children saw mathematics tasks as being work that needed to be completed, which was not open for discussion. The work was competitively motivated by finishing first and getting more right answers, the slower, or less able children, always losing out. There was no discussion between the children about what they were doing and the teacher was seen as the mathematical authority within the class. Interactions between Chris and individual children about their work concentrated on discussions of "how did you do them?" Responses concentrated on the mechanical "used counters", "used a numberline", or "in my head", rather than on reasoning.

Working in groups

After I had been observing for several weeks in Chris' class, the target group began to work together, initially with the Specialist Teaching Assistant, Mary, and later with Chris herself. The other groups in the class continued to do mostly individual work and the change to group teaching may have been as a result of my presence observing the target group. Initially there was little difference in the way that the children interacted when working as a group; they continued to work individually and to defer to the teacher. However over time they began to work more co-operatively and help one another.

As we have seen in the many examples from Beth's class the target children would often work as a group along with the teacher. (Beth would work with each of the groups over the course of the week.) Although the interactions tended to be teacher led, Beth asking questions and individual children answering, there was evidence that the children were also listening and learning from one another. In the following extract the children are describing how they have partitioned their three animals across two field. Ian has three in one field and none in the other.
Beth Now we've all got a little picture of two fields. Can you put your animals in the fields. Let's have a look at our animals. Ian, how many in that field
Ian Three
Beth and in that field?
Ian none
Beth How many altogether?
Ian Three
Beth So you started with three and you've still got three in all your fields.

Beth asks the other children to describe their animals.
Beth Now, can we all make our sets look like Ian's? Can you tell me a little number sentence about what you've got here?
Angela We've got three and none (Charles interrupts with zero) and altogether got three.

Ian initially uses the word 'none' to describe how many in his empty field. I had previously recorded only Charles using the word 'zero' in mathematics lesson, even the teacher tended to use 'none', and I can only assume that Charles had learnt to use the word outside of the classroom, at home or playgroup. When Charles appears to correct Angela's use of 'none', suggesting 'zero', Ian adopts the word for himself in his subsequent sentence. In subsequent mathematics lessons Ian was heard to use the word regularly, in context, and the other children and the teacher adopted its use. While working in a group the children are therefore not just learning from the teacher but listening to and learning from one another.

Working together as a whole class

In Debbie's class much of the mathematics was carried out with the whole class group. There was a heavy emphasis on language, the children being given the opportunity to talk about the mathematics. For example, in this lesson the class are working with Debbie and a 100 square.

Debbie Start counting until I say stop. How many did you get to?
(Lots of responses about 20, Rosie 88, Fergus 100!)
Debbie Choose a number between 20 and 30
Saresh 20
Debbie OK lets start counting from 0 all the way up to 20 and back again. 1, 2, 3, ...20, 19, ...0
(Debbie points to the numerals on the board as she counts and the children join in.)
Debbie Choose a number between 15 and 20
Chloe 15
Debbie Let's count in twos from 1 to 15 and back again. Oh no that's not going to work! Oh yes it will, count in twos from 1 to 15
and back again. 1, 3, 5, ... 15, 13, 11, ... 1 (the children join in counting.)

Debbie  Well done, now lets look at the number square, can anybody spot a pattern?

Naomi  1, 1, 1, 1, 1, (pointing to the tens in the teens row)

John  1, 1, 1, 1, 1, ; 2, 2, 2, 2, 2, .... 9, 9, 9, 9, (indicating the units in the vertical rows.)

Debbie  Can you see what John means? He is saying that in the ones row there are all ones. What is your pattern Ruth?

Ruth  It goes 1, 2, 3, 4, 5, 6, 7, 8, 9, (indicating tens in the 1 to 91 column.)

Debbie  Does it do that in every row? (Column)

Ruth  Yes.

Debbie  So, as it goes down (indicating the 1-91 column) it goes up in steps of ...?

Helen  2

Debbie  Do you agree with her? If you increase in steps of two ...

Ruth  Ten times table

Debbie  So it's jumping up in steps of ...?

Ruth  Ten

(Debbie 6)

This is a typical example of how Debbie and the children interacted. The questioning was open ("can anyone spot a pattern?") and she urged the whole class to think about incorrect ideas ("do you agree with her?), enabling the children to develop the mathematical skills of spotting patterns and reasoning within a mathematical context. When working in groups, the children would continue to discuss their work together, comparing and justifying answers. As a result, the teacher was not seen as the arbiter of truth but she encouraged the children to listen to one another and to reason for themselves, encouraging mathematical reasoning and exploratory talk, which Mercer defines as one of the distinguishing features of discourse in the classroom (1995).

A further element of Debbie's teaching, also present in Eve's class, was that of counting together. As I noted in Chapter Seven, there was no expectation that every child would be able to do everything. Many of the children, especially the younger children, would soon get lost in the counting backwards and in larger steps and would begin to 'mourn the words' or opt out altogether. However they were still listening to and experiencing the counting sequences, building on their experience of number. What is happening here is much more than using language to communicate - one of the ways in which I had acknowledged the need for a social aspect to constructivism in Chapter Two. Here the children are participating in a social act, joining in as far as possible, learning from one another and from the teacher. In Lave and Wenger's terms they could be seen as involved in legitimate peripheral participation; participation in the very localised, social practice of counting, a practice in which the teacher is
'master' and the children 'apprentices' at differing stages of becoming full participants (1991). (See Chapter Ten, section 10.2.2 for further discussion of this issue.)

**Summary**

The differing forms of social interaction and levels of participation within the classroom, were therefore influencing the children’s attitudes and the way that they developed their mathematics. Chris’ class seemed to see mathematics as an individual act, with work to be ‘got through’ and the teacher as the only judge of success, while in Beth and Debbie’s classes mathematics there is a social and shared act requiring participation, reasoning and justification. The social interactions within the classroom are influencing both the children’s understanding and their attitudes to mathematics. However, sometimes the social demands on the child may conflict with the mathematical ones.

When playing the dice game with the target group from Beth’s class as part of the initial assessment activities I noted that Jacob insisted that he had won the round, despite having a low score.

Jacob seemed unsure about which number was bigger e.g. 10 or 7, but when I asked if he would rather have ten sweets or seven sweets he quickly said ten. He may have been reluctant to admit defeat - the 7 were his!

(Beth Initial Assessment)

The social demands of winning the game seemed to override his knowledge of numbers. Walkerdine (1988) found that children are often unwilling to lose face and will therefore give mathematically incorrect answers rather than admit defeat, which seems to describe what Jacob was doing. He was prepared to assert that seven was bigger than ten in order to win the game. At this stage Jacob’s social understanding of winning the game overrides his understanding that the mathematics is important, and he responds in a non-mathematical way.

**9.4.2 Further social influences on the children’s understanding, motivation and attitudes to mathematics**

In addition to the way that the mathematics learning was organised, I noted further influences on the children’s understanding, motivation and attitudes to mathematics, within the world of the classroom, which arose from the teacher’s attitude to and management of the children. Where teachers used positive management techniques with the children, using praise and encouragement, the children’s mathematical learning benefited. The observation was not unexpected (Pollard et al. 1992) but these examples contrast with Bruner’s observation on motivation (Bruner 1974) that the children “develop rote abilities and depend upon being able to give back what is expected rather than to make it something that relates to the rest of their cognitive life” (p. 406). Most of the children were developing positive attitudes and working at making sense of the mathematics, as shown in the following sections.
Dealing with errors

In the teaching episode considered in the previous section (9.4.1) I noted how Debbie encouraged the children to extend and consider the responses of others. In this way the class built up a reflective, interactive way of working with mathematics. I found no incidents where children were ever told that they were wrong in Debbie’s class. If an incorrect answer was given she would ask the children to consider it ("Do you agree with her?") or she would often indicate that it was not correct using the word ‘nearly’, which seemed to communicate to the children that they should self correct and they would often produce the correct answer, as we can see in the following example. The children were offering number calculations with an answer of seven. After several children had given addition examples, John tried subtraction.

John Twenty take twelve.
Debbie Nearly
John Twenty take thirteen.
Ho Two add five.
Ellis Six and one.
Zeb Five and two.
John Twenty-one take fourteen.
Twenty-two take fifteen.
(Debbie 4)

(Mathematically, John’s answer is wrong, twenty minus twelve is not seven. But Debbie seems to recognise that John has made a good attempt to widen the mathematical ‘conversation’ to include subtraction from larger numbers, and to say that it was wrong might discourage further attempts. From Debbie’s response “nearly”, John recognises that he has made a mistake, corrects his answer and then has the confidence to build on his corrected answer. Debbie and the children are building a class ethos where it is OK to ‘have a go’ and to make mistakes. The ethos has a positive effect on the children’s attitudes and motivation in mathematics.

In Beth’s classroom any attempt to record the mathematics using symbols was praised, even if attention was then drawn to the correct form of the numeral. However, Chris believed that being able to record the mathematics was as important as being able to calculate the answers. After sending a group off to complete some calculations she explained to me:

They look at the number line to find the number (numeral). I don’t mark it right if they’ve got it the wrong way round. I say to go away and see how to write it.
(Chris 2)

As a result the children were not always sure whether it was the answer or the numeral that needed correction. A child moving through the school from Beth’s class to Chris’ class could have found the change in rules difficult. Individual classes therefore build up individual classroom cultures which influence how mathematics is carried out and thought about.)
Chris' increased expectations may also be due to the pressures from outside the school which increase as the children grow older. In the following year Chris' children will have to sit standardised tests in mathematics, and for these tests the correct use of symbols is important. Many of the rules about how to record mathematics: how to form the symbols, the order in which these occur in a written calculation, writing from left to right, are conventions and not an immutable part of the mathematics. Concentration on 'correct' recording may detract from the mathematics itself. But correct recording is valued by parents, and a requirement of external tests, and therefore an important part of teaching mathematics.

**Teacher's attitudes to children and mathematics**

In Beth's class the children were encouraged through the use of praise. I noted twenty-one occasions when Beth used the word 'good', sometimes as 'very good', 'good girl', 'good boy' etc... She would also describe the children as 'clever'. For example, at the end of the 'bus lesson', when all the children completed their recording, Beth turned to mental arithmetic based on number bonds to ten, the focus of the session. This lesson was the first time I had observed her giving them mental calculations without any social or manipulable context.

Beth: Now close your books and let's play a little game. Put your hands up. What is five and five? ...

The 'game' continued with all the combinations of ten being named and all the children putting up their hands to answer. Beth targeted children to answer e.g. she asked Charles to answer $10 + 0$ as this calculation was one he had been unsure of earlier. All the children replied correctly.

Beth: Goodness me you are clever children.

(Beth 12)

First, Beth describes the task as "a little game" which puts the children at their ease and leads them to expect something enjoyable. She then uses praise to build up their self esteem. Woods (1990) speaks of the dangers of external motivation that can result in the desire to please the teacher rather than to learn, but while Beth's praise is external it encouraged the children to work and created positive attitudes to mathematics learning. In a previous lesson, when the children were asked to record what they had found out in their books, I noted that:

Charles worked very fast commenting on his cleverness!

Charles: I'm very clever at doing these.

(Beth 11)

Charles (not yet 5 years old) is already building an image of himself as a competent mathematician, the teacher's praise resulting in an internal motivation and sense of achievement, 'I can do it - I'm clever'.

Analysis of Eve's lessons showed an attitude of which I was not aware while observing in the classroom. She would often ask a question of the children and follow it up with the expression
"this is difficult". Sometimes she would elaborate; in the context of talking about the findings from a graph of favourite fruit, Eve asked:

Eve Twelve add thirteen apples, that's a difficult one. Can you do that one? I know Mosim knows, Laura can you do 12 add 13? No, Mosim?
Mosim 25

(Eve 6)

I noted that none of the children tried to use the fruit graph to work out the answer. They seemed to respond to the instruction 'that's a difficult one', by opting out. Mosim, whose mathematics was more secure that that of the rest of the class, regularly carried out calculations correctly and the children appeared to defer to him. By suggesting that the mathematics was difficult, Eve seemed to discourage the children from 'having a go', in contrast to Debbie and Beth's encouragement. I am sure that discouragement was not the effect Eve would have wanted. She was perhaps the least mathematically confident of the teachers studied and appeared unsure about asking challenging questions of a mixed age class; she recognised that it might be difficult for them and did not want to put the younger children off. Nevertheless, her lack of confidence seemed to discourage them rather than raising their expectations of themselves as learners of mathematics.

The social world of the classroom, including the attitudes of the teachers to the children and to mathematics, was seen to influence the children's motivation and attitudes to learning mathematics, their ability to do mathematics and their image of themselves as mathematicians.

9.4.3 Social skills developed during mathematics lessons

In the preliminary study, I had found examples where the mathematics learning would contribute to the development of the children's social skills in the classroom, through interaction and the playing of mathematical games (Chapter Four, section 4.8.4). At the start of term I noted that some of the children had difficulties in playing games. They did not understand the idea of taking turns and John in particular could not cope with losing. By contrast, at the end of June I recorded that 'the children really do have much more idea about playing games, taking turns, passing the dice around, helping one another, monitoring who is winning' (Preliminary Study Fieldnotes: comment, Lesson 10).

The main study did not provide sufficient evidence to show that mathematics learning played a specific role in social development. Evidence of the children's developing social skills was noted several times towards the end of the study. The children had become more competent at listening to one another, taking turns, playing games, explaining their ideas etc.. I believe that this social development was, at least in part, due to their experience of learning mathematics. However, a study of the children in and out of mathematics lessons would be required to show whether mathematics learning played a unique part in the process or was only one aspect of
classroom experience. I have therefore chosen not to explore the relationship between mathematics learning and the development of wider, social skills in more detail here.

9.4.4 Summary of the relationship between the world of the classroom and children's mathematics learning

The world of the classroom offers opportunities for the mathematization of real contexts such as dinner registers, taking mathematics out of the mathematics lessons and relating it to the everyday world of the classroom and offering the children real contexts on which to act mathematically. The way that the social world of the classroom is organised for learning and the social interaction between teacher and children and between groups of children influences the children's attitudes towards their mathematical learning and towards themselves as mathematicians. Such influences were generally positive though there was also the potential for the development of negative attitudes. The evidence that mathematics learning developed the children's social skills was inconclusive.

So, the world of the classroom could be seen to influence the children's mathematical learning, and in particular their understanding, motivation, and attitudes towards mathematics.
9.5 The relationship between school learning and mathematics learning

In this section I want to look at the relationship between different aspects of the children's learning in the classroom. I have already shown how the teacher might use other subject areas explicitly as a context for learning mathematics, for example Eve's use of science as a context for data handling (Chapter Seven, section 7.5). Data handling is an area of mathematics which lends itself to addressing cross curricular topics. However, there were also occasions when other subjects impinged on the mathematics lessons in less structured ways. Beth used mathematics to consolidate learning, especially of reading and writing; which at times could get in the way of the mathematics.

9.5.1 Using mathematics lessons to support the learning of other subjects

Although Beth planned separate lessons for different subject areas, she took every opportunity to reinforce other subject areas during the mathematics lessons. Many of these related to literacy skills. Here, Beth is explaining how to do some of the tasks she had prepared for the children to work at in groups while she teaches the target group.

Beth Make a eight with the plasticine. Roll it out into a long thin sausage or a long thin line and then see if you can change it into a number eight.

Beth tries to demonstrate this with the plasticine but it is back to front for the children.

Charles It's a bit tricky.

Beth Let's see if we can do it in the air. Use your hand you write with David. (David is using his left hand and is not left handed). All the way down and round and back to the beginning.
Beth reminded David that he is to use his writing hand even though, as a mathematics task, it is perhaps the shape of the number that is most important. Writing may not be the focus of the lesson but Beth took the opportunity within the mathematics lesson to reinforce writing skills.

As the children's reading skills developed Beth wrote more instructions in their mathematics books in order to give them opportunities to practice their reading skills in context. Towards the end of the 'bus' lesson the children were required to record their work.

Beth Right, if you look in your books you will see the bus. What does it say?
Beth (slowly) Draw ten children on the bus. (The children join in reading.)
Charles I can read that.
Beth I thought you could. You are getting very good at reading.

The opportunity to join in reading for a purpose allowed the children to develop positive attitudes to their emerging reading skills. It is unlikely that Charles could have read the sentence unaided and out of context. But the clear relationship between the words and the mathematics that the children had been doing, combined with the participation in joint reading as a group with the teacher, scaffolded his reading skills to such an extent that he felt a sense of achievement.

So, mathematics lessons were used to reinforce language skills. The reinforcement was seen most often in Beth's class where she worked with the youngest children (reception, aged 4-5 years). Perhaps as the children grow older, or more experienced in school, there is less need to reinforce the basics, or perhaps teachers aim for a clear differentiation between areas of the curriculum. However it is also possible that moving towards a more clearly differentiated mathematics lesson will further isolate the mathematics from the rest of the curriculum and the everyday world.

9.5.2 Conflicting demands of different subject areas

However sometimes it might seem that there is conflict between the demands of one subject and another. Working with sets of eight multilink cubes arranged in two sets, Beth asked the children to interpret their manipulable materials as a spoken sentence.

Jacob made 6 and 2 and read it back correctly, Ian made 5 and 3 and read it back, Charles made 5 and 3 and read it as 3 and 5. Beth corrected him and insisted he read it from left to right.
Beth insists that Charles reads his ‘number sentence’ from left to right, which is the way that the children read writing, and the way that he will be required to record when he has drawn a picture of the two sets.

\[
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
5
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
3
\end{array}
\end{array}
\end{array}
\end{array}
\end{array} = 8
\end{array}
\]

However, mathematically five add three and three add five are the same since addition is commutative, and Beth’s insistence on interpreting the materials in a particular order does not help Charles to understand this. He may well think that it does matter which number he starts with. So, Beth’s correction of his interpretation may have corrected his reading and writing skills but could confuse his mathematical understanding. There appears to be a conflict here between the process of socialization into the way that mathematics is recorded and the mathematization of the situation modelled by the bricks.

Similarly in Eve’s class Mosin was having trouble reading multidigit numerals. Eve observed that the problem had arisen recently, since he started learning to read the Koran which is read from right to left. The problem here arises from Mosin’s own conflict in learning different writing systems. Recognition of this conflict enabled Eve to discuss it with Mosin which seemed to clear up the problem.

Adler (1998) asks whether the implicit ‘educational ground rules’ which are an aspect of the culture of the classroom could be made explicit in order to allow participation by all the children (see Chapter Two, section 2.3.3). However, this seems to imply that there is only one set of such rules. In 8.6.1 I showed how Jacob’s understanding of drawing a picture of ‘what we did’ was interpreted as ‘what we did on holiday’, based on his experience of drawing and writing about what he did on holiday in an earlier English lesson. Recording ‘what we did’ in mathematics requires a specific mathematical interpretation of the task. In another context a picture of children gathered round the table with their teacher, and a written description could fulfil the request, but ‘what we did’ in mathematics requires recording of manipulable materials and symbols, which is an example of socialization into a specific social practice. Children must learn that ‘in this situation we do this’, in mathematics lessons we record the mathematics, rather than drawing a picture of the participants involved in the task.

Different subject areas therefore can have different rules, which may confuse the children. Some of these differences may not be noticed by the teacher and many are not made explicit to the children. Children, at the same time as they are making sense of the mathematics, are also required to make sense of the situation at many different levels. They may be unsure which is the important level on which to focus and children’s misunderstanding or lack of compliance may arise from such conflicting demands on the child’s attention. As the children develop more experience in the situation they can begin to differentiate which social practice they are working in, for a particular task. Within the primary classroom, even though the same teacher
and children work together for the most part of the day, there appear to be multiple local communities of practice since different subject areas have different practices; a further example of the complexity of the learning situation in which these young children find themselves. The children must be socialised into each of these communities of practice, learning to act appropriately in each.

9.6 Conclusions and Implications

This chapter has considered the relationship between the wider social world of the children and their mathematics learning. Identification of the relationships seen between different areas of that world, the wider social world, the world of the classroom, the world of learning, and the learning of mathematics have been explored. Two key ideas have been discussed.

1. Opportunities to bring mathematics contexts from the wider world into the classroom not only enable the children to understand the mathematics but also help them to make links between school mathematics and their own everyday mathematics. These links were most explicit when working with the youngest children. I propose that continuation of such opportunities with the children as they progress through the mathematics curriculum could sustain these links.

2. From the perspective of the classroom as a community of practice, I have identified that, although the children and teacher are together throughout the day, working on different areas of the curriculum, a local community of mathematics practice appears to exist with its own rules, discourses and practices. The rules were seldom explicit, were different across classes even within the same school, and yet were a key part of the children’s mathematical learning.

The chapter concludes the main findings of the study. Chapter Ten will bring together the findings from chapters 7-9 and finally Chapter Eleven will address the implications of these findings.
Chapter Ten

Making Sense of Addition

10.1 Introduction

This chapter brings together the research findings of Chapters 7, 8 and 9 in order to examine the central focus of this study - children learning addition.

Chapters 7, 8 and 9 described the main findings of the study, developed through the analysis of the data in response to the three research questions identified following the preliminary study:

Q1 How does the way that the addition curriculum is planned and implemented influence children's learning of addition?

Q2 How does the way that addition is represented to the children influence children's learning of addition?

Q3 How does the wider social context of the classroom influence children's learning of addition?

Chapter Seven showed how the differences between an atomistic and a holistic approach to the curriculum for addition, affected the sense that the children made of their work. The characteristics of an atomistic and a holistic approach to the curriculum were summarised and reasons why the holistic approach may be more effective were discussed.

Chapter Eight discussed the way that the mathematics was represented by both teachers and children. Two findings were found that were particularly relevant to working with young children (4-5 years olds) and which contribute to previous research. First, analysis of the use of representations showed that young children found learning to use symbols the most difficult representation of mathematics at this stage of their learning, the children making few links between the symbols and pictures or spoken language. Secondly, it showed that for the youngest children the use of 'real world scripts', contexts with reference to the social world in which the children live, was more effective in enabling understanding than more abstract contexts, even where the abstract context involved the manipulation of physical objects. As the teachers and children developed a shared experience and understanding within the classroom, more abstract contexts could be used.

Chapter Nine considered the effect of the wider social world on the children's mathematics learning. The findings here are intricate: they show a complex picture of children who are developing mathematical understanding within a social experiential world, inside and outside the classroom. The social interaction between teachers and children, and between groups of
children, which differed from class to class; the children’s understanding of the world outside the classroom and the social world within it; and the sometimes conflicting demands of social understanding, mathematical understanding and understanding in other subject areas, are all part of the experience of a child trying to make sense of addition in the classroom. I have suggested that multiple ‘communities of practice’ exist in the primary classroom, a community of mathematics practice having its own rules, discourse and practices.

In bringing these findings together, this chapter addresses the overarching question with which this main study began:

\textit{How do young children learn addition in primary classrooms?}

The three research questions defined in Chapter Five, and reproduced above, set out to focus the overall research question. The analysis of data, related to each of these questions, yielded findings relevant to the overall question. My next task is to consider how these three sets of findings relate to one another, in order to provide a ‘coherent story’ of how young children learn addition in primary school. I will not assert that this is the only story that could be told. As I noted in Chapter Five (section 5.2.3), at every stage the research process is influenced by my personal experiences and theoretical perspectives. The story is therefore one story, my story, of how children learn addition in primary school, grounded in the data collected.

Three sections will be presented. Section 10.2 will discuss the findings from Chapters 7, 8 and 9, section 10.3 relates these findings to the literature on theories of cognition, and section 10.4 will draw the story together.

\section*{10.2 Summary of findings}

\subsection*{10.2.1 The mathematics of early addition}

In Chapter Seven, I analysed the way that the curriculum for addition was structured and presented with reference to the cognitive research on the development of addition discussed in Chapter Two. Analysis showed that the two schools studied used contrasting ways in which to structure the curriculum, and that this had an effect on the way that the children understood the mathematics.

At Ashburne School the addition curriculum tended to be ‘Atomistic’, fragmented into its constituent parts, and related to the literature on children’s understanding of addition. This approach assumed a direct correlation between that which is taught, the element of the curriculum, and that which would be learnt, the next stage of development. The teaching approach, based on an atomistic approach to the curriculum I have described as ‘Arrow’ teaching. The teacher identified the next stage of development through assessment of the
children's achievement, and directed the teaching to that area, as if by an arrow directly to the target. While the atomistic approach aimed to identify the children's learning needs, and target these with the activities which address the next stage of development, it would appear that to break the mathematical content down into such small pieces may only teach the children how to complete that task in an instrumental way, to comply with the teacher's accepted procedures, losing the larger picture of how these ideas interrelate.

Characteristics of this atomistic approach to the curriculum were identified as:

A1. an initial emphasis on small numbers;
A2. teaching addition in isolation from subtraction;
A3. emphasis on procedures rather than patterns and relationships;
A4. the use of physical and predominantly cardinal representations of number;
A5. progression from one ‘stage of development’ to the next;
A6. the repetition of similar activities in order to reinforce the procedure.

In contrast, the curriculum at St David's concentrated on understanding of the number system and relationships between numbers, with addition as only one small part of the overall picture. I have chosen to call this a 'Holistic' approach since it views the number curriculum as a whole rather than focusing on its constituent parts and developmental stages.

Characteristics of this holistic approach to the curriculum were identified as:

H1. an emphasis on large numbers as well as small ones;
H2. an emphasis on patterns in number and pattern spotting;
H3. discussion of the inverse relationship between addition and subtraction;
H4. emphasis on relationships rather than procedures;
H5. emphasis on ordinal representations of number and the development of mental imagery;
H6. an eclectic curriculum in which the children are immersed in a wide range of activities, with little apparent sequence.

I have described the teaching approach as an 'Immersion' approach. The curriculum offers immersion in a wide range of ideas, rather than a narrow focus, and the children learn what makes sense, or is within their capabilities, at the time. There is no assumption that all the children will learn what is taught, or even that there is, recognisably, something taught, nor that they will learn the same as each other.

The findings indicate that children make sense of mathematics through immersion into the number system in its complexity, rather than through step by step instruction. The 'making sense' is an active process; immersion should not imply that children will learn if not engaged with the mathematics. The deficiency of targeted teaching is consistent with Denvir and Brown's (1986a; 1986b) work with low attaining junior school pupils where they found that
targeted teaching often resulted in a child learning facts, skills or concepts other than the ones taught.

Boaler (1997), in her study of teaching and learning in secondary school mathematics classrooms, found that where the pupils were taught in ‘mixed-ability’ groups for mathematics they experienced a wider curriculum than those children put into differentiated sets, which is also in accord with my findings. She also found that pupils who experienced a ‘progressive’ curriculum were more successful at completing conceptual test questions, which examined their underlying understanding of the mathematics rather than their ability to carry out procedures. She claims that these pupils “developed a mathematical understanding that they are more able to make use of” in a “flexible and adaptable” way (p. 81). In contrast, those pupils who were taught a ‘traditional’ curriculum developed “broad knowledge of mathematical facts, rules and procedures that they demonstrated in their textbook questions, but they found it difficult remembering these over any length of time” (p. 81).

Boaler’s findings seem very close to my findings on atomistic and holistic curriculum approaches. The children in my study who experienced the holistic curriculum were able to use their developing knowledge of the number system to choose strategies in order to solve addition tasks, while the children who experienced an atomistic curriculum learnt to use procedures that appeared tied to the situation in which they were learnt.

10.2.2 Representing Mathematics in the Classroom and in the Real World

Chapter Eight looked at the way that mathematics was represented to and by the children and the sense that the children made of these representations. Overall the study shows the children’s increasing competence with representations within the mathematics of addition. Competence with manipulable materials, spoken language, pictures and symbols appeared to follow the order indicated by Liebeck’s ELPS model (1984). Manipulable materials formed a focus for discussion and modelled mathematical situations. Spoken language was used in a range of informal and more mathematical forms with teachers scaffolding the use of specific procedural language. The children learnt to interpret simple addition represented in pictorial form and to record their use of manipulable materials in pictures. However, they found the use of symbols, especially operational and relational signs, difficult to learn. The order in which the children learnt to use representations may have been due to the order in which the representations were presented to the children; however I believe that the specific difficulties that the children experienced in learning to use symbols, indicates how these, the most abstract and ‘dense’ of the representations, are more difficult to master.

The children’s difficulty in learning to use symbols could be due to the abstract nature of symbols, and the lack of their use in everyday life. However, analysis at the level of
representations of addition showed that the children did not easily make links between symbols and other mathematical representations. While the younger children could record their work with manipulable materials using pictures, this did not directly help them to learn to record using symbols. The structure of the two representations is not the same: pictures show only one set of the total number of objects while the symbol number sentence requires both the subsets and the total.

\[
\begin{array}{c}
\begin{array}{c}
\text{I I I I I I I I I} \\
\downarrow \\
3
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{I I} \\
\downarrow
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{? } \\
\downarrow
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{} \\
\updownarrow
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{5}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{= } \\
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\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{8}
\end{array}
\end{array}
\end{array}
\]

Enactively, in order to produce the total set (8), the two subsets (3 and 5) have to be destroyed. Since the picture is a static representation it can represent either the two subsets or the total but not both, since at any one time there are only eight objects. Furthermore, the process of destroying the two subsets and combining them to form a single, total set is an active one which cannot be shown in pictures. The signs + and = therefore have no counterpart in the picture. The children therefore found it difficult to make links between picture and symbols when recording their work.

Neither could the children relate the symbolic sentence to their spoken language, for the syntax of the spoken sentence was not directly analogous to that of the symbolic equation.

\[
\begin{array}{c}
\begin{array}{c}
\text{Three} \\
\downarrow
\end{array}
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\begin{array}{c}
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\begin{array}{c}
\text{and} \\
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\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{five} \\
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\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{altogether} \\
\downarrow
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{make} \\
\downarrow
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{eight} \\
\downarrow
\end{array}
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{3} \\
\downarrow
\end{array}
\end{array}
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\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{+} \\
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\end{array}
\end{array}
\end{array}
\begin{array}{c}
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\begin{array}{c}
\text{5} \\
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\begin{array}{c}
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\text{?} \\
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\end{array}
\begin{array}{c}
\begin{array}{c}
\text{=} \\
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\begin{array}{c}
\begin{array}{c}
\text{8}
\end{array}
\end{array}
\end{array}
\]

This is in contrast to the development of writing, a skill the children were also learning, where there is a direct order correspondence between the spoken and written word, and further clues in the phonic interpretation of the symbols used, which help to identify their meaning.

Children's own early recording (symbolic) was found to reflect more closely their actions in the manipulable materials (enactive), carried out to find the total, such as counting and combining (see Chapter 8, section 8.6.2, for further discussion of this issue). Bruner's framework, which indicates that an iconic stage helps movement from enactive to symbolic, was not confirmed in this context. It is possible that the children were developing a mental iconic representation to which I had no access, but neither the physical, pictorial stage, nor the particular use of spoken language, was seen to help link the enactive and symbolic stages. I believe that, children should be encouraged in their own recording, rather as Emma and Charles misused the formal symbols to record their work (in Chapter Eight, section 8.6.2), in order to understand the use for symbols. If the teacher also models formal recording with the children, the children will gradually come to use the formal symbols for themselves.
The use of real world contexts for addition was found to be influential, especially when teaching the youngest children (4-5 year olds). These contexts allowed a shared, social understanding of the mathematics under discussion, which was not always present when a more abstract, mathematical context was used. The term 'real world script' arose from Lesh et al.'s work on problem solving (1987a), which, like much of the literature on representations, has a constructivist perspective. In Chapter Eight, I redefined real world scripts within a socio-cultural framework. The characteristics of real world scripts were identified, and the reason for their success in enabling children's understanding was discussed with reference to Bruner's theory of narrative. As the children learnt more mathematics they and the teacher built up a shared mathematical culture on which to base their understanding, but there were still occasions when the introduction of real world contexts 'scaffolded' a difficult task for them. Aspects of the use of real world scripts are further discussed in the section below, with reference to the relationship between the social world and the mathematical world of the child.

10.2.3 The Social World and the Mathematical World of the Child

In Chapter Nine I considered the relationship between the wider social world of the children and their mathematics learning. Identification of the relationships seen between different areas of that world, the wider social world, the world of the classroom, the world of learning, and the learning of mathematics have been explored. Two key findings emerged.

First, opportunities to bring mathematics contexts from the wider world into the classroom not only enabled the children to understand the mathematics, as detailed above, but also helped them to make links between school mathematics and their own everyday mathematics. These links were most explicit when working with the youngest children. I have suggested that continuation of such opportunities with the children, as they progress through the mathematics curriculum, could sustain these links, preventing the disjunction between everyday mathematics and school mathematics described by Nunes (1991). As the children move towards a more formal mathematics curriculum, the differences between mathematics as used in the real world and the academic study of mathematics in the classroom, two different 'communities of practice', could also be explicitly addressed. This might prevent the problems identified by Cooper and Dunne (1998) who found that working class children's everyday, social knowledge of situations overrode their mathematical interpretations, causing them to offer socially appropriate but mathematically inappropriate answers in mathematics test situations. These are tentative proposals which require further research.

Secondly, I found that although the children and teacher are together throughout the day, working on different areas of the curriculum, a particular mathematics community of practice appears to exist in the classroom, with its own rules about ways of working, its own discourse and its own practices. The rules were seldom explicit, were different across classes even within the same school, were different across different subject areas within the same class, and
yet were a key part of the children’s mathematical learning. Rather than there being a single community of practice in the primary classroom, multiple communities of local practice appear to exist (as discussed in Chapter Nine, section 9.5.2). As with the links between everyday mathematics and school mathematics, I have suggested that explicit discussion, in the classroom, of the different communities of practice may help children clarify the underlying expectations of each community.

Overview of findings
I have considered the learning of addition in the classroom in terms of curriculum, representations and participation in the world of the classroom. Each of these has offered insights into the way that children understand and learn addition. However to develop theory, rather than remain a collection of isolated findings I want to relate these more specifically to existing theoretical ideas of cognition.

10.3 Towards an understanding of learning addition
I want first to tell the story of a young child, I will call her Anna, entering primary school at the age of 4, and embarking on learning about school mathematics. I hope that the reader will recognise Anna for what she is, an archetype based on the summarised findings of this study.

Anna comes to school as an experienced learner. She has learnt, for example, to walk, feed and dress herself, and how to relate to people and experiences in and out of her home. She has learnt to communicate, to talk in complete sentences, how to construct the past tense of regular verbs. Anna already has an understanding of number, in relation to small quantities of countable objects; to ages, birthdays and candles on cakes; to symbols on the television buttons (or remote control) that change channels, on buses, house doors, and telephones. She can subitize small numbers of objects, read some numerals, count a small set of objects and knows that the last count word represents the total. Anna knows that numbers increase: as people get older, as she is given more toys, and decrease: when the biscuits get eaten or toys get lost. She knows that there are big numbers that she cannot yet count. How then has Anna learnt all of these things? She has not learnt through attending ‘growing-up lessons’, “today we are going to learn how to use a knife and fork”, but through participation in the social world around her, through interaction with parents, wider family members and other children with whom she plays.

As Anna enters the school for the first time she enters a new social world organised for learning. Here there are rules for how to behave, for where to put her coat, for how to get the teacher’s attention, in fact for just about everything. Some of these rules are confusing: when she sits on the carpet with the rest of the class she has to put up her hand, but when she is
working with the teacher at a table she doesn’t. Gradually, Anna comes to realise that there are different lessons with names like literacy, mathematics and art. Each of these has its own rules. In literacy lessons “draw a picture” means communicate something she wants to say about the situation under discussion, from her own experience or from a story. In art it means attention to detail, colour and form. In mathematics it means to draw the equipment being used with attention to quantity and sometimes to size and shape. How does Anna come to learn all of these things? She has learnt through participation in the social world around her, through interaction with her teacher, other adults in the classroom and other children with whom she works and plays.

As Anna begins to differentiate one lesson from another she realises that mathematics lesson are about number. In mathematics lessons she is learning addition. She is learning that numbers can be combined to make larger numbers, that this process has special words associated with it, and can be written down using special symbols. Anna learns that the symbols which she has come to know as numbers are not just labels, showing how many in a set or which bus to catch, but objects in their own right which can be taken apart and joined together. She also learns about larger numbers and the way that the number names, and symbols, are joined together in patterns so that they go on for ever. Anna is actively involved in her learning, making sense of addition and of symbols with reference to what she already knows and understanding about mathematics, her own actions and the social world in which she lives. And how does Anna come to learn all of these things? Well …

As I come to consider the findings of this study in the light of the theories of cognition summarised in Chapter Two, I want to keep Anna in mind, to consider how theories of cognition might tell us something about teaching and learning which can help Anna’s teacher to teach her better, and to help me to teach her next teacher, and the one after that, to be better teachers of mathematics to children like Anna. For I believe that for theories of cognition to be relevant to the teaching of mathematics in primary classroom they must address not only ‘the learning of mathematics in the classroom’ but how children, children like Anna, learn.

I began this study with three central theories of cognition, constructivism, socio-cultural theory and social practice theory, current in mathematics education literature (Chapter Two). I also considered the effects that constructivism, in particular, has had on primary and mathematics education (Chapter Three), and the way that constructivism, and ideas on the zone of proximal development and scaffolding, derived from socio-cultural perspectives, have been interpreted in early childhood studies. Throughout, I have made reference to how aspects of these theories help to explain the findings of this study into how children learn addition. Here I want to consider each theory in relation to the findings.

Cobb and Yackel (1998) argue for a relationship between theory and practice that is “reflexive in that the theory guides practice, which feeds back to inform theory” (p. 159). I therefore
want to consider not only to what extent the theory is seen to guide practice in this study of mathematics education, but also whether the findings expand our understanding of the theories of cognition themselves.

10.3.1 Constructivism

A constructivist perspective of cognition focuses on learning as an individual act, building a viable model of experience through a process of assimilation and accommodation of new experiences. Throughout this study I have, at times, focused on the individual learner and so found much that can be explained in constructivist terms. Indeed the very title of this chapter “making sense of addition” itself implies a constructive approach to learning and throughout I have offered examples from the classroom that show children trying to ‘make sense’ of their mathematics. I have used the expression misunderstanding, clearly distinguished by italics, to indicate that, for the child, the understanding is a meaningful one, drawn logically from observation and in the light of their individual previous experiences, even if these do not fit with the teacher’s construction. I have used models of representations, derived from constructivist views of cognition, to analyse the relationship between teaching and learning at the level of communication. And, as a learner myself, I know that constructivism accords with my personal experiences of learning mathematics, accumulating knowledge through a wide variety of experiences and being challenged at times to reconstruct my understanding to accommodate new ideas. Active construction, rather than passive reception, accord with my personal view of learning and with what I have seen in the classrooms in this study.

Has constructivism also provided me with a tool to analyse teaching and learning, which may offer insights into how children learn?

Constructivism, atomism and holism

When I consider the teaching and learning of addition at a curriculum level, discussed in Chapter Seven, a constructivist perspective reveals an apparent incongruity. An Atomistic approach to the curriculum arose from a Piagetian view of learning. Walkerdine notes how the work of the Nuffield Mathematics Project, which was based on Piaget’s work on children’s development, led to “the first and most influential curriculum intervention into primary school mathematics in the 1960s” (Walkerdine 1984, p. 555) and most other primary mathematics schemes, including that used at Ashburne, can be seen to draw on the Nuffield innovations. Yet this study shows the approach can result in disjointed, procedural learning.

However, the merits of a Holistic approach to the curriculum can also be explained from a constructivist perspective. A Holistic approach increases the opportunity for children to experience learning at their own level of prior understanding, and to assimilate that information,
those skills, which most closely link with existing skills and knowledge. The Atomistic approach, has the potential to be poorly targeted, through the teacher’s lack of detailed assessment (a difficult task when teaching up to 36 children), and therefore ‘miss the mark’ and be ineffective; in accordance with Piaget’s claim that teaching in advance of the child’s stage of development will be at best unproductive, and may be harmful. Analysis of the data shows children experiencing developmentally inappropriate mathematics at times. For example, the children in Chris’s class being taught to use the number track to count-on, are unable to do so (section 7.3.5), probably as a result of their lack of understanding of the additive composition of number (see section 7.7.1 for discussion of this.) Learning to count-on is therefore unsuccessful and the children resort to learning how to use a number track - procedural rather than relational understanding. But, if targeted teaching can ‘miss the mark’ and may be harmful how is it that experience as a result of immersion in complexity is not also harmful? The child is being taught a wide range of mathematics much of which, for the least experienced child in the class, will be ‘in advance of development”. Why is this too, not either potentially harmful, or ineffective, leading to procedural learning or even no learning at all?

In Chapter Two I described how Piaget’s work arose from his study of both epistemology and human development. I believe that this dual focus can help explain the apparent inconsistency that has arisen. While constructivism, and in particular radical constructivism, is founded on Piaget’s work on epistemology, the construction of knowledge, the Atomistic approach to the curriculum draws on Piaget’s work on stages of development, particularly children’s development of logical thinking. Piaget’s observation that teaching in advance of development of the child’s stage of development will be at best unproductive, was also based on his work on developmental stages. The following table sets out the relationship between Piaget’s two areas of thinking and the developments I have observed in primary mathematics.

<table>
<thead>
<tr>
<th>Constructivism</th>
<th>Developmental stage theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>experiential learning (learning through ‘doing’)</td>
<td>developmental learning (learning readiness)</td>
</tr>
<tr>
<td>which led to</td>
<td>which led to</td>
</tr>
<tr>
<td>use of manipulable materials, multiple embodiment etc.</td>
<td>Atomistic curriculum and individualised learning</td>
</tr>
</tbody>
</table>

But, as I noted in Chapter Two, much of the developmental stage theory has been challenged, since it took little account of the children’s understanding of the language and social context of the experimental tasks on which the findings were based. An approach to the curriculum based on Piagetian stages of development, an atomistic curriculum, is therefore unlikely to succeed, and I believe that this is shown by this study. The developments from constructivism which
relate to the use of manipulable materials and multiple embodiment will be discussed in section 10.3.2 below.

A constructivist perspective can allow insights into children’s learning of addition but does not seem to offer guidance on approaches to the curriculum. Piagetian curriculum approaches have arisen from developmental stage theory and not from constructivism. In the light of these findings, I would agree with Confrey that “constructivism may lack an adequate theory of instruction” (1994).

**Constructivism and representations**

As I described in Chapter Two, most of the writers on representations were themselves working from a constructivist perspective. The key discussion on representations centres around how teachers and children share the underlying meanings attached to representations. A constructivist perspective explains misunderstanding as an individual construction of the situation, which does not match the teacher’s personal construction. As I focus in on children learning to use symbols, the child finds the learning of symbols difficult because she cannot take on the teacher’s understanding of symbols, and must construct an understanding for herself. However because the child has had insufficient experience of the use of symbols in recording mathematics, personal construction is difficult. This lack of experience is exacerbated by the lack of use of operational and relational signs in the world outside the classroom. Informal recordings, which make sense to the individual child, gradually come to match the more formal recording of the teacher, as the children strive to match their own efforts with those they experience.

The use of real world scripts in learning mathematics can also be explained in terms of the children constructing knowledge and skills by making links with their previous cognitive schemes. Real world scripts create an arena in which the teacher and children can discuss the mathematics in the context of a shared experience, which enables conflicting constructs to reach agreement. For the child just starting school most of this previous understanding is grounded in experience of the world outside the classroom, and so addition situated in real world contexts can be more easily assimilated. But while the work on real world scripts has its foundations in constructivism, in Chapter Eight I have drawn on Bruner’s socio-cultural work on narratives to elucidate this aspect of the representations.

**Constructivism and the social world of the child**

A constructivist perspective considers the social world of the child as part of the experiential world the child inhabits and must therefore construct for him or herself. Steffe (1996), and Cobb and Yackel (1998) describe knowledge in the classroom from a constructivist perspective as being built up through individual construction of shared experience which can be ‘taken as shared’ within the group. This serves to clarify how individuals can come to develop shared
Chapter 10 Making Sense of Addition

constructs but does not fully explain the role of social interaction in mathematics learning. I believe that socio-cultural and social practice perspectives allow me to understand the social world of the child more clearly than constructivism allows.

10.3.2 Socio-cultural perspectives

A socio-cultural perspective sees learning as a social act where knowledge and skills are first shared in the social realm and then internalized by the individual. At the start of the study, my understanding of a socio-cultural perspective was based on its application to early years education as reflected in the literature (Chapter Two, section 2.2.4). First, in contrast to constructivism, socio-cultural theory addressed the role of social interaction and the importance of language in the learning process, and secondly, it drew on scaffolding, derived from Vygotsky's theory of the zone of proximal development, to explain the role of the 'tutor' in helping young children to learn. As the study arose from my own perspectives on learning, data collection and analysis relied heavily on both a constructivist view of learning (making sense of addition) and on the socio-cultural role of language and social interaction. My methodology was based on this premise (as described in Chapter Four, section 4.3.6). Almost all the data used in the findings chapters, to exemplify the developing argument, rely on language and social interaction between teacher and children.

However, throughout the study I have come increasingly to realise that mine was a naive reading of socio-cultural theory, which addresses mediated action through the use of socio-cultural tools. In this section I will first explore scaffolding, and then consider the role of representations from the perspective of cultural tools.

Scaffolding

From a socio-cultural perspective, the finding that a holistic curriculum contributes to children's learning of mathematics more effectively than an atomistic one, required me to look more closely at the concept of scaffolding. For, a simple reading of scaffolding seemed to indicate that to break down the curriculum into small parts, could help the teacher target the zone of proximal development of an individual child, scaffolding their learning. Moreover, I believed with Confrey that scaffolding was used within the context of problem solving (1995), and interpreted this as applying skills and knowledge, rather than learning new ideas. I discovered this was both to misinterpret the concept of scaffolding, and a socio-cultural view of learning. Mercer (1995) quotes Bruner's description of scaffolding as:

the steps taken to reduce the degrees of freedom in carrying out some task so that the child can concentrate on the difficult skill she is in the process of acquiring.

(Bruner 1978, p. 16)
This definition allows me to make sense of the relationship between scaffolding and approaches to the curriculum. The effect of an atomistic curriculum is to reduce the task of learning mathematics to a series of unrelated procedures; children are not able to 'concentrate on the difficult skill' because the overall task is no longer visible. An atomistic curriculum does not therefore accord with Bruner's view of scaffolding. A holistic curriculum allows children to see 'the whole', while the teacher scaffolds their learning by her active involvement in the task, acting as their 'vicarious consciousness' (Bruner et al 1976). Learning can be seen as a communal, social act in which the children join.

For example, in Chapter Seven we saw how the children joined in counting activities with their teacher, Debbie. As many of the children were less secure with the tens words (twenty, thirty, forty ...) these were supplied by Debbie, and those children who did know them, and the other children continued counting on in units. In this way the children were able to count to 900, a task beyond the capabilities of most of the individual children. I believe that this illustrates Vygotsky's view that development is present first on the social plane (intermental) and then on the internal plane (intramental) (Vygotsky 1978). Counting, which starts as a shared, social act, in which the teacher scaffolds the counting task so that the children can succeed, becomes internalized as an individual skill.

I have also used scaffolding to explain how real world scripts are used to help children understand addition. The mathematics is not simplified for the child but, by placing it in a real world context, the teacher enables the child to make sense of it more easily. Because the child is familiar with the context, 'the degrees of freedom' in interpretation of the context are reduced: with the use of more abstract, mathematical contexts, this familiarity disappears and the child is left confused. The scaffold is used as a tool by the teacher to bring together the two worlds of the child, the social world outside the classroom and the mathematical world within it, to enable learning to take place. I believe that I have only just begun to identify how teachers can scaffold children's learning through a holistic approach to the curriculum. More research would be needed to explore my conjectures presented here.

The majority of research on scaffolding is, as Confrey says, based on small scale problem solving tasks, often with adult child dyads. In Chapter Two I noted that Hobsbaum, Peters and Sylva (1996) claim that, in Reading Recovery programmes, scaffolding can occur only when teachers work individually with a single child. However, I believe that may be to view scaffolding from a constructivist perspective on learning. From a socio-cultural perspective, problem solving is not about applying already learnt skills to problems, but it is through the completion of problem solving tasks that learning occurs, and learning is not about abstraction from experience, but activity within a social context.
Further re-examination of scaffolding, in the context of the early learning mathematics could, I believe, offer important perspectives on teaching and learning in whole class and small group situations.

**Mediation through cultural tools**

The second aspect of socio-cultural theory I want to consider is that of mediation through cultural tools. To interpret representations as cultural tools is to see them from a different perspective, a socio-cultural perspective. What light does this perspective throw on the study of representation in early addition? Socio-cultural theory offers the view that since representations are cultural tools, learning to use them is a social act. Children cannot be expected to 'see' an external meaning embodied within them but must join in their use, and through such activity, the meaning will become transparent.

Constructivism's emphasis on manipulable materials as representations, has resulted in Dienes' idea of multiple embodiment (see Chapter Eight, section 8.4.3), with the expectation that children will abstract the meaning by experience with a wide range of representations of the same concept. Such manipulable materials are seen as teaching aids. But from a socio-cultural perspective the number system is not seen as an attribute of the experiential world but as a tool to allow us to manage calculations with numbers greater than we can comfortably count. From this perspective, multiple embodiment is superfluous. For most purposes in life one tool is sufficient: the only advantage of more tools is if they allow us to do something we cannot do with the one we already have, or to do it more efficiently, as in the development of logarithms and the electronic calculator. The use of counting and the 100 square enables children to develop a mental image, a mental tool, to understand the number system and to effect calculations, whereas a wide range of manipulable materials is superfluous and may cause 'cognitive overload'.

As I consider the ways in which place value was introduced to the children in this study, there is a sharp contrast between the use of tens and units materials in Chris's class (straws, Dienes' apparatus and money) and the use of the 100 square at St David's. For many of the children, Chris's approach resulted in procedural learning of how to use the base 10 apparatus, rather than the children seeing 'through' the materials to the underlying concept. The 100 square, by contrast, allowed the children to discuss, not the material itself, but the inherent pattern of number in words and symbols. The 100 square offered a tool to both develop understanding of the number system, and to use, as a physical or mental model, for calculation. From a socio-cultural perspective, tools are mastered through social interaction in practice. So the children used and developed their understanding through interactive teaching based around the 100 square model and counting, over time. In contrast, the children in Chris's class had to learn to use a wider range of tools, and did not show evidence of mastery in any.
From a constructivist perspective, instrumental understanding of facts and skills through rote learning is considered as subordinate to conceptual understanding. However I believe that a socio-cultural perspective challenges me to re-evaluate this distinction. My discussion of understanding the number system through consideration of the 100 square, indicates that through the rote practices of counting in tens and ones, a conceptual understanding of place value is being developed. This conjecture about the relationship between rote learning and concept formation may require further research in the future.

A socio-cultural perspective has provided key insights into the findings of this study. Socio-cultural theory has challenged me to reconsider the way that children learn within the context of the classroom, in terms of the teacher’s role in scaffolding learning, the use of socio-cultural tools, and the difference between instrumental and relational understanding. In each case I believe that further research could be done to explore early mathematics education from a socio-cultural perspective.

Since the findings which discuss the effects of the wider social world of the child on children’s learning, relate to those aspects of socio-cultural theory that have been expanded within social practice theory, I will leave discussion of the social world of the child to section 10.3.3 which follows.

10.3.3 Social Practice Theory

Social practice theory, arising out of anthropological research, emphasises learning as an integral part of the community of practice in which it is situated (Lave and Wenger 1991). Throughout the analysis I have drawn on aspects of social practice theory that have provided insight into the findings of this study, and I will discuss these aspects further in the following sections. However, I will argue that the premise of social practice theory, of learning as a process of moving from apprentice to master of a particular practice, is less easy to relate to the classroom.

Legitimate peripheral participation

The concept of immersion, one of the key characteristics of a holistic curriculum, resonates strongly with social practice theory ideas of legitimate peripheral participation in a social practice. In Chapter Nine (section 9.4.1) I argued that participation in social acts such as rote counting could be expressed in terms of legitimate peripheral participation, as children’s knowledge of the counting words developed through participation. In attempting to define legitimate peripheral participation, Lave and Wenger seek to redefine the nature of a concept:
“Until recently, the notion of a concept was viewed as something for which clarity, precision, simplicity and maximum definition seemed commendable. We have tried ... to reconceive it in interconnected, relational terms” (p. 121).

I would argue that this is not only true of the concept of legitimate peripheral participation but also of mathematical concepts. Instead of the apparent ‘clarity, precision, simplicity and maximum definition’ which characterises the atomistic curriculum, holism provides an ‘interconnected and relational’ view of mathematics, with apparent benefits to the children.

Communities of practice

In Chapter Nine, I described the world of the classroom in terms of a community into which children are socialized, with its own culture defined by language, rules and ways of working. I have gone further to propose that multiple local communities of practice arise, as the learning demands of different subject areas include subtle changes in practice. I have described these as local communities of practice since the practice of teaching and learning is peculiar to the classroom, and specific to a particular combination of teacher and children. Like participation in other communities of practice, participation in mathematics classroom practice involves legitimate peripheral participation (playing games, joining in the counting chant), involving transparency of artefacts (as the teacher’s meaning of manipulable materials, language, pictures and symbols is shared with the children), and discourse, ways of talking about and within the practice (as children move from using everyday language to specific mathematical language in talking about what they do in the classroom and relating it to the outside world) (Adler 1998).

Aspects of social practice theory have therefore been used to give insight into the findings of this study: however two problems remain in relating the findings to social practice theory.

The first was highlighted in discussion of social practice theory in Chapter Two, where I asked ‘what is the product of the community of school mathematics learners?’ According to Lave and Wenger (1991), cognition is situated in the practice of the community, which has the purpose of production (tailoring etc.) and where the apprentice participant has the goal of becoming a master. In the classroom the purpose of the community is less clear and the goal is not one of turning the pupil into a teacher. Neither is the purpose of learning at school the same as learning at home. In the world outside the classroom, children will learn aspects of counting through participation in social contexts, ‘how many cakes do we need to buy for the party?’, ‘have we got enough chairs?’ etc., the ‘community of practice’ of the home. The counting is not the focus of the social activity but a means to an end - finding out ‘how many’ for a real purpose. In the classroom, learning to count is itself the aim. However it may be that this distinction is an adult one. The children are at school to learn, but is that how they would see it? For the young child, participation in whatever social practice she finds herself, involves joining in and, in joining in, learning new things. As with parent-child interaction, where the
child is not learning to be a parent but a more able member of the family group, so the child in
the classroom is working towards becoming a more educated member of society, though I do
not see these as conscious aims. I believe that by redefining 'practice' to include what people
do in many different social situations, and redefining the terms 'master' and 'apprentice' to
include any situation with significantly more and less able participants with the intention to
share knowledge, it is possible see the classroom, and indeed the home, as communities of
practice.

The second problem in relating social practice theory to teaching and learning, lies in discussion
of the transfer of knowledge from one community of practice to another. Drawing on the idea
of situated cognition, social practice theory has been used to explain the lack of application of
knowledge learned in one community of practice to another, for example how the Brazilian
street children in Nunes' study were using different mathematics in school and in the street
markets (1991). I believe that this present study of children's learning gives insight into the
problem of transfer of knowledge. It indicates, but by no means confirms, that increased
discussion in the classroom about the mathematics in the 'real world', and explicit recognition
of the differences between the mathematics of the real world and school mathematics, might
overcome problems such as those of the Brazilian street children. Recognition that particular
communities have particular practices is not often discussed in the classroom. The lack of
transfer of knowledge from one situation to another could therefore be explained, not in terms
of situated cognition, but by failure to acknowledge that practices differ.

10.3.4 Summary

In this section I have sought a coherent theory of learning that is consistent with the findings of
my study. I have found that each of the three theoretical perspectives currently used within the
mathematics education community can be used to illuminate many of the findings of the study.
While constructivism provides a focus on the individual which has allowed insight into
understandings and misunderstandings of children, socio-cultural and social practice theories
provide a theoretical framework within which the findings on teaching and learning can be
explained, a framework on which further study in this area can be built. To return to Anna, my
archetypal four year old, 'how does Anna come to learn all of these things?'

The answer has to be 'through participation in the social world around her, through interaction
with her teacher, other adults in the classroom and other children with whom she works and
plays'. The study has identified key aspects of this participation and interaction indicating the
Anna learns through:

- the construction of knowledge (making sense) of the physical and social world she inhabits;
- the teacher's role in scaffolding her learning and language, to enable her to 'practice'
  mathematics while still holding on to the complexity of the holistic curriculum;
• active participation in using cultural tools, the use of which is modelled by the teacher, and coming to appropriate them for herself;

• narratives which enable her to mathematize examples from the real world, and through which she can learn to mathematize the world for herself;

• identifying the particular community of practice in which she is working. This identification might be made easier by more explicit discussion, in the classroom, of the differences between communities of practice inside and outside the classroom, just as her parents might discuss the different behaviours expected when Anna is at home or visiting an elderly aunt.

10.4 The whole story or the story so far

This research has addressed the teaching and learning of addition in primary classrooms from three points of view: curriculum, representations and the culture of the classroom, with reference to three perspectives on cognition: constructivism, socio-cultural theory and social practice theory. It set out to provide a ‘thick description’ of the process of teaching and learning in the classroom and has produced the findings described above. These findings have been discussed in relation to the theories of cognition, and a story of learning addition produced in early years classrooms.

I am not claiming that this is the whole story. For I am not sure that there will ever be a complete account of how unique individuals, in ever changing social and cultural groups, make sense of addition. However I have provided what I believe is a more in-depth story about the learning of addition than has been previously told.
Chapter Eleven

Conclusions and Implications for Further Study

11.1 Introduction

This study has examined the teaching and learning of addition in early years classrooms. It has focused on both individual construction of meaning and the social context in which that construction takes place, drawing on theories of cognition, of representations in mathematics and on the psychological study of children's development of addition. The preliminary study identified three areas of mathematical activity that influenced the learning of addition: the structure of the curriculum, representations of mathematics and the wider, social world of the child. The research process has been described as involving participant observation in two classes, in each of two schools from different local education authorities. A grounded theory approach to data analysis resulted in findings related to each of these three areas of mathematical activity. In the previous chapter I have summarised the findings and related them to the theories of cognition, in order to offer a coherent story of young children learning addition in primary classrooms. In this final chapter I will discuss the implications of the findings and describe how these relate to the National Numeracy Strategy, which has influenced the teaching and learning of primary mathematics since I began my data collection, in section 11.2. Section 11.3 will consider future research which arises from the findings and in section 11.4, I will critique the methodology of the study. I end with a personal reflection on learning.

11.2 Implications of the study, with reference to the National Numeracy Strategy

In this section I will look in turn at the findings of the study and at the implications of each for the teaching of early primary mathematics. Since the study's conception and data collection, primary mathematics education has changed substantially with the introduction of the National Numeracy Strategy (DfEE 1999). When considering the implications I will therefore also show how my findings relate to the teaching of primary mathematics recommended in the strategy.

11.2.1 A brief overview of the National Numeracy Strategy

The strategy has three key features to which I will refer in the following section and which I will explain here: the framework, the three part lesson structure and the training of teachers.
Framework for teaching mathematics: Reception to Year 6

The framework is a document which serves two purposes. First it discusses the principles of the strategy and answers the sorts of questions that teacher might ask in endeavouring to implement the strategy in their school. This section includes a detailed description of the three part lesson structure.

Secondly, the curriculum is subdivided into four areas of mathematics:

- Counting and recognising numbers
- Calculations
- Solving problems
- Measures, shape and space.

Within these headings, the framework sets out the learning objectives for each year group, which are divided into key objectives, which the majority of the children are expected to achieve at the end of that school year, and other objectives which are designed to build their understanding and knowledge for future work. Termly planning documents are provided so that teachers can plan when to teach to each objective and there is further exemplification of the objectives in terms of what sort of evidence would be expected from a child to show that s/he had reached the objective.

The three part daily mathematics lesson

Within the strategy, teachers are recommended to use the three part lesson structure. This consists of:

- **Oral and mental calculation** (about 5 to 10 minutes)
  the whole class work to rehearse, sharpen and develop mental and oral skills

- **The main teaching activity** (about 30 to 40 minutes)
  teaching input and pupil activities
  work as a whole class, in groups, in pairs or as individuals

- **a plenary** to round of the lesson (about 10 to 15 minutes)
  work with the whole class to sort out misconceptions and identify progress, to summarise key facts and ideas and what to remember, to make links to other work and discuss the next steps, and to set work to do at home. (DfEE 1999 1:13)

Training

The strategy has a training programme so that all teachers will be trained in how to teach the three part lesson, how to use the framework for planning, and, increasingly, extra training to improve teachers' subject knowledge. Numeracy consultants work at LEA level to train
headteachers and mathematics co-ordinators, who then disseminate this training to teachers in their schools. The numeracy consultants also support head teachers and co-ordinators in school, where necessary.

11.2.2 Central Conclusions of the Thesis

Chapters 7, 8 and 9 described the findings which provide the main conclusions for this thesis:

- That children make sense of mathematics through immersion into the number system in its complexity (a holistic approach to the curriculum) more easily than through an atomistic, step by step approach to the curriculum, which can lead to procedural, rather than relational, understanding.

- Consideration of the way that mathematics was represented in the classroom shows that the youngest children in the study (4 and 5 years) had the most difficulty understanding and using symbols, especially those which indicate operations and relationships between numbers (+, =).

- For these young children, representation of the mathematics as real world scripts, stories or contexts which related to the world outside the classroom, were found to help initial understanding of addition concepts. As the children learnt more mathematics they and the teacher built up a shared mathematical culture on which to base their understanding, but there were still occasions when the introduction of a real world context ‘scaffolded’ a difficult task for them.

- Furthermore, opportunities to bring mathematics contexts from the wider world into the classroom, not only enabled the children to understand the mathematics, but also helped them to make links between school mathematics and their own everyday mathematics.

- Although the teacher and children are together throughout the day, working on different areas of the curriculum, a particular mathematics community of practice appears to exist in the classroom with its own rules about ways of working. These rules were seldom explicit, were different across classes even within the same school, were different across different subject areas within the same class, and yet were a key part of the children’s mathematical learning.

What, then, are the implications of these findings?

11.2.3 Holistic or Atomistic Curriculum

The first finding concluded that children respond better to a holistic approach to the curriculum, with emphasis on an overview of the number system and links between one part of the number curriculum and the rest, which has implications for the teaching of mathematics. An atomistic
curriculum, broken down into small, manageable steps, tends to result in procedural rather than relational understanding. However this is the traditional way that the curriculum has been structured for young children: the Nelson Mathematics Scheme used at Ashburne School is just one of many similar schemes. Indeed I find it difficult to imagine written curriculum materials, designed to guide primary school teachers in their mathematics teaching, which can provide a holistic approach to mathematics and, at the same time, break the curriculum down into units to be taught to children of different ages.

How does the National Numeracy Strategy address this? The message is a mixed one which, as we saw in Chapter Three (section 3.4.1), was true for the National Curriculum itself. The training materials (used to train teachers in using the NNS) and elements of the Numeracy Framework (used by teachers for planning) emphasise the importance of a wide range of activities including the use of number lines and 100 squares, to teach about the number system, the effectiveness of alternative calculation strategies, and relationships between the four arithmetic operations. These emphases are seen as important not only for the development of children’s calculation strategies but also for laying the foundations of algebra (DfEE 1999, Section 1, p. 9-10). The holistic way that the teachers were working at St David’s was, in part, due to the influence of the Numeracy Project, the precursor to the NNS. However, the way that the curriculum is broken down into small, detailed learning objectives for each year group (DfEE 1999, Section 3), could lead teachers to teach to these objectives, providing an atomistic curriculum. Use of the more detailed examples, given at the end of the framework document to further exemplify learning outcomes (DfEE 1999, Sections 4 to 6), could exacerbate this atomistic approach.

Since teaching a holistic curriculum may rely on the teacher’s own understanding of mathematics as a whole, rather than isolated facts and skills, as well as their previous mathematics experiences and dispositions to learning mathematics, the success of the strategy may depend on the quality of training rather than effective use of the planning framework. Askew et al. (1997) postulate that the most effective training may consist of good teachers working with others, both to help with planning and by working alongside them in the classroom.

### 11.2.4 Learning to use Symbols

The problems children have learning to use symbols, discussed as a finding of this study, is not new. Since Martin Hughes’ study, if not before, teachers of young children have recognised that teaching symbols is both important and difficult (Hughes 1986). Some mathematics schemes have introduced a range of intermediate symbols (e.g. brackets and mapping diagrams with arrows); however, as we saw with the children at Ashburne School, this can result in the children having to learn and relearn a series of symbols (for example →
then =) instead of only one. The focus of the lesson can then become learning to use yet another symbolic form, rather than the underlying mathematical meaning. Children need time in order to develop their understanding and use of formal notation. I have shown that the use of pictures, providing an iconic representation, may not be helpful in moving young children from enactive experience to symbolic recording. Addition is an operation of change which is not easily recorded as static pictures. While numerals are already part of the child’s experience when they enter the classroom, operational and relational signs are peculiar to the mathematics classroom. Formal symbolic recording is a convention of the mathematics community, and children need to be given time to experience the use of symbols in context. The teacher needs to model how to record mathematics in context, and discuss with the children the meanings of individual symbols, as we saw in Debbie’s class, in order to help the children to develop a wider understanding of symbol use.

The NNS requires that children are able to read and write numerals to 10 in Reception, to 20 in Year 1 and to 100 in Year 2, a very atomistic approach. However, the strategy recommends that children are not introduced to formal written algorithms until year 3 (8-9 year olds) and the correct use of operational and relational signs is not a key objective until Year 4 (DfEE 1999, Section 2, p. 4). The emphasis on mental and oral methods of calculation before written methods, aims to ensure that children are confident in their understanding of the operation of addition before being expected to use formal recording.

A more detailed look at the yearly teaching objectives reveals that children are to begin to use +, - and = signs to record mental calculations in Year 1. But, since the end of Key Stage One assessment (SATs) requires that children are able to read and interpret these symbols at the end of Year 2, it is only to be expected that teachers will continue to see correct use of these symbols as an important target for Key Stage One. Teachers, used to giving pages of calculations to children, will need training to understand how to introduce symbols, discuss their meanings and to demonstrate how to use them to record mental calculation.

My study would recommend that children are encouraged to use their own symbols, while the teacher continues to use the mathematically accepted forms, until the children adopt these for themselves. Less emphasis on written operational and relational signs in KS1 tests, might help more children to demonstrate their mathematical understanding.

11.2.5 The Use of Real World Scripts

The findings showed that, for the youngest children, examples of addition which used contexts from the real world, outside and inside the classroom, from stories and rhymes, and from play contexts, allowed the children better opportunities to enter into the activity, to talk about it to the teacher and to understand the mathematics. The implications of this finding are that real world contexts are important when introducing formal mathematics to children and that this could be a
feature, not only of introducing addition to young children, but of introducing any new mathematical content in the primary school. This approach would be similar to the Realistic Mathematics Education approach used in the Netherlands (Streefland 1991).

The NNS guidance for Reception teaching, acknowledges the importance of stories, rhymes and play for children as they start formal schooling (DfEE 1999, Section 1, p. 27). It also recommends the use of real everyday classroom mathematics, such as taking the attendance register, as opportunities to reinforce mathematical ideas. Examples of early addition activities in the strategy use everyday social objects; cakes, cars, fingers; rather than mathematical equipment.

I feel that it is a shame that once the child reaches Year 1, apart from a passing reference to the continued use of number rhymes in counting, almost all reference to the real world is removed from the ‘calculations’ objectives, to the ‘solving problems’ section which requires children to “solve simple word problems set in ‘real life’ contexts and explain how the problem was solved” (DfEE 1999, Section 5, p. 66). The examples given for understanding and learning about addition in Year 1 are now devoid of context, ‘3 add 1’, ‘how many are 3 and 5 altogether?’, while the role of real world examples has moved from a role in teaching children in reception classes, to the more traditional role of getting the children to apply the context free mathematics they are learning in school, to the ‘real world’ outside the classroom. Similarly, in Year 2, when multiplication and addition are introduced for the first time, there is initial reference to real world examples of multiplication, e.g. “how many wheels are there on four cars?”, and of division by sharing “six sweets shared equally between two children”, but thereafter the examples quickly move away from real world contexts. The message the strategy gives is that vertical mathematization (developing reasoning within mathematics) is important, while horizontal mathematization (the relationship between mathematics and the world) is unidirectional, from the classroom out to the real world (using and applying), rather than as a context for learning (starting from the real world).

The findings of this study indicate that more attention could be given, in the strategy, to the use of real world scripts in teaching, not just using and applying, arithmetic.

11.2.6 The Culture of the Mathematics Classroom

The study concluded that the primary classroom is a complex culture with local communities of practice, specific to the study of different subject areas. A mathematics community of practice has its own rules, discourses and practices, and I have suggested that the more explicit teachers are about the specifics of such practices, the less confused the children will become as they move from the study of one subject to another.
One of the strengths of the structured daily mathematics lesson may be that it clearly identifies for the children in which community of practice they are to work, the oral and mental starter marking the beginning of a clearly defined mathematics lesson. Much of the introductory section of the framework document addresses the way that the children should be organised for learning; the role of the teacher, focusing on direct teaching rather than providing experiences; and the structure and pace of the lesson to maintain concentration and motivation. In reception classes, the children are not expected to have a complete mathematics lesson in one sitting; the teacher is advised to break up mathematics activities into shorter periods, with some whole class, group and plenary time during each day. At the same time, the aim is for the teacher to,

prepare children, by the end of Reception, for the dedicated mathematics lesson of about 45 minutes that will be part of each day in Year 1. For example, you will need to help them to learn how to listen, how to show and talk about what they have been doing in front of other children, how to find and use the equipment that they need, how to take turns, and so on (DfEE 1999, Section 1, p. 27),

recognising the need to socialise children into the mathematics classroom. However I believe that the findings of this study indicate that all teachers, not just those in Reception, would be advised to enter into discussion, with one another and with the children, about what characterises the culture of the mathematics classroom, in order to make explicit to the children they are teaching, the particular practices, rules and discourse of learning mathematics.

11.3 Further Study

If teachers are not to be subject to swings of the political pendulum (Brown 1999) and the application of theories from related, academic disciplines (Walkerdine 1984), then mathematics education needs deep, considered, forceful and reflective theoretical ideas of its own. For primary teachers, this needs to be grounded in a wider theory of teaching young children, with the differences across curriculum subjects openly discussed.

The field is not a static one. Aubrey (1997) found teacher’s practices in primary mathematics to be similar to those of Desforge and Cockburn’s findings (1987) ten years earlier. My own study has shown the remnants of such practices, but significant changes in the teaching of at least one school. The NNS will have had a more significant influence on a wider range of schools as I write, requiring follow up study and analysis. The findings of this study indicate that further study which concentrates on the social aspects of learning, viewing teaching and learning as scaffolding and legitimate peripheral participation in shared activity using socio-cultural tools, could redress the balance of previous, heavily constructivist, approaches. In particular, four areas arise out of my study:
1. A study of primary teaching in the light of my findings on atomism and holism to identify the effect of the mixed messages in the NNS: a holistic approach to teaching number and number calculations with an atomistic approach to objectives;

2. A study of the role of the teacher from a socio-cultural perspective to identify ways in which teachers scaffold children’s learning and could do so more effectively;

3. A re-evaluation of the constructivist ideas of procedural and conceptual understanding, in the light of questions I have raised about the role of rote learning from a socio-cultural perceptive;

4. Further consideration of the role of real world scripts in teaching early mathematics, perhaps leading to the evaluation of curriculum innovations in the early years of schooling.

11.4 Critique of Methodology

In the research and writing for this study I have learnt a great deal both about the subject under consideration and also about the research process. This section sets out to consider what aspects of the study I might have done differently, if I were starting out again.

11.4.1 Research design

In Chapter Four I explained how, since the situation under study was a complex one requiring consideration of both individual learning and the social situation of the classroom, participant observation in the classroom was chosen as the method of data collection which could best provide the data to analyse this situation. I believe that the level of detail in the data, which resulted in the findings that I have presented in this thesis, justify that original decision. Following the preliminary study, I decided that it was necessary to look at more than just one classroom, not to provide a generalisable sample but in order to broaden the data set. Since children start school in different ages in different LEAs, and since the focus was on children starting to learn formal arithmetic in school, I arranged access to two classes in each of two different schools in different LEAs. Had I decided instead to spend more time in a single classroom the findings would have been different. Similarly, I was limited by the part-time nature of my research to the observation of only one lesson a week in each class. Had I watched every mathematics lesson, I would have gathered additional data which could have resulted in different findings. But that is not to invalidate the findings that I have presented; a different sample, a different depth of focus would have resulted in the telling of a different part of ‘the story’. Detailed evidence from the data collected, explicit description of my own theoretical perspectives and previous experiences, as they influenced the research process, and a clear description of the methodology and findings, allow the reader to judge for themselves
the trustworthiness and credibility of my story and the relevance of the findings to their own situation.

However, there are issues which I believe I could have addressed differently, with reference to particular stages in the research process, which I will discuss in the following sections.

11.4.2 Researcher-practitioner relationships

First, I want to consider the relationship between myself as researcher and the teachers who generously gave me access to their classrooms. I was challenged by reading Wagner's 'framework for reconsidering researcher-practitioner co-operation' (Wagner 1997), which questions the relationship between researcher and practitioner for all situations of educational research. Wagner offers three views of researcher-practitioner relationships: data extraction agreements in which the practitioner is seen as part of the setting; a clinical partnership where researcher and practitioner are working together, though the research is essentially the initiative and property of the researcher; and co-learning agreements, where the research is a joint work of mutual benefit. I was uncomfortable to identify the essential relationship between me and the teachers with whom I worked, as a data extraction agreement. The research was being done 'on' the practitioners, who had little say in the research question, the process design, the ownership of the data or the use to which the findings will be put. The research question was essentially mine; though it was shared with the teachers and, since teaching addition is a key element in mathematics teaching at key stage one, they also saw it as an important question, one in which they would be happy to participate. The initial research design was also originated by me. My own intentions within the research were to "inform educational policy and [ultimately] improve instruction" (a data extraction agreement criterion). Such a data extraction agreement has ethical considerations, treating teachers as objects of research rather than fellow practitioners, which I regret.

I also believe that such research has a limited affect on teaching. The research produced is often seen as irrelevant by teachers, and soon forgotten by the participants. Involving teachers in the joint analysis of their practice is more likely to result in reflective practitioners who will continue to reflect on their practice long after the collaboration is finished. Involvement of the teachers in the analysis and findings of my research could have developed their own practice, and produced findings which were not only relevant to my own aims of exploration, but also provided me with more insight into the applicability of the findings to the classroom.

Wagner's article therefore challenged me to reflect on the researcher-practitioner relationship and will influence how I see, and plan for, co-operation in future research projects.
11.4.3 The effect of theoretical perspectives on data collection

In the process of writing up this study, I became increasingly aware of the influence of my personal and theoretical perspectives at all stages of the research. However, in this section I want to consider one issue with which I have struggled. In the writing of Chapter Seven on the difference between curriculum approaches at the two schools, I found that it was easier to write about Ashburne School in depth. The data was richer in evidence of individual construction and it was therefore easier to see the effect of the curriculum on the individual child. When looking at the data from St David's, I found it more difficult to find that depth of analysis and I have addressed this in section 7.4.1. Reflection on the data collection process has led me to see that one of the reasons for the difference in depth of data, was that at Ashburne the practice I was seeing was more familiar, more recognisable from my own experience as a teacher of young children, grounded in constructivism and child-centred practice. I could recognise the familiar more readily than the strange. At St David's the teaching was less familiar, there was less evidence of individual construction and I may well have missed data, which I would now notice, as a result of my changed perceptions. This is not to say that I would have set out to collect data in a different way, but that the data that I noticed and the data of which I was not aware, might have been different. For example when observing Debbie's class involved in rote counting, I noted that not all the children joined in all of the time. I did not think to watch particular children and see how their participation increased over time. The significance of participation in rote counting emerged through analysis of the data and I would now observe the children more closely. In participant observation the researcher is also the research instrument, and data will therefore always be only that which is noticed. But recognition of that fact will enable me, when carrying out future research, to be more aware of the limitations, as well as the strengths, of the research process.

11.4.4 Building theory through a grounded theory methodology

In Chapter 5, section 5.3 I described the significance of this study in developing an understanding of teaching and learning mathematics in primary classrooms. It has personal significance for me as a teacher and teacher educator, and academic significance as an in-depth exploration of early mathematics learning in situ. In Chapter 5, section 5.4.2 I identified a Grounded Theory approach to data analysis as appropriate to this study, since it required a rigorous analysis of ethnographic data derived from a complex social setting. I noted that "the study is exploratory, designed to build theory on how children learning mathematics in early primary classrooms, rather than to test theory or evaluate change." Here I will summarise the theory that I have built through this process.

With respect to approaches to the mathematical curriculum, this study has a built a theory to describe different approaches, which I have termed atomistic and holistic, and which I have characterised in Chapter 7. This theory emerged from the data rather than being built from
previous published research. While the evidence that a holistic approach is preferable to an atomistic one is tentative, based on a small in-depth study of four classes, the characterisation of these two approaches provides theory which others can develop.

The findings in Chapter 8 relating to young children's use of symbols build on the theory of others, including Hughes (1986) and Mason (1987). However the significant finding of this chapter relate to the use of Real World Scripts in the teaching of mathematics to young children. Here I have built a theoretical argument both for the usefulness of such scripts in the classroom, drawing on the data collected, and for the reasons why such scripts may be useful, by identifying the characteristics of RWS and drawing on Bruner's work on narrative from outside the field of mathematics education.

Finally, in Chapter 10 I have attempted to build from the data a view of the place of addition learning in the wider social world of the child both in and outside school. This is the most tentative of the theoretical constructs developed from the data, the one which will require the most further work to verify the relationships which I have attempted to determine. Verification was beyond the limits of this doctoral thesis as it would require considerably more data collection, both in and out of the classroom.

A grounded theory approach to data analysis has therefore successfully enabled me to make sense of the data collected from the complex social phenomenon of teaching and learning in early primary classrooms and to build theory from the constructs which emerged.

### 11.4.5 Validity and generalisability of the findings

I discussed generalisability and validity in detail in Chapter Five, section 5.7, and will not rehearse the issues discussed there. One further decision was made relating to the analysis of the data, which I believe to be relevant to the validity of the findings and which I want to address here. I made the decision not to involve the participating teachers in the process of analysis and development of findings. One of the reasons for this was the length of time between data collection and analysis, caused by my move to a different job at the end of the data collection period. By the time the data was analysed, I was no longer in contact with either school, and two of the teachers had moved on, to early retirement and to a new teaching post, so contacting them would have been difficult. The second reason was that I was not sure what the purpose of such 'member checking' would be. If the teachers challenged points of fact there would be no evidence for appeal, other than my own documentation. If they challenged points of interpretation then the interpretation was mine, grounded as I have said in my own background and perspectives and I would not expect theirs to be the same.

I now realise that discussion with the teachers could have provided me with alternative perspectives on the data, making me aware of my own perspectives and serving to validate the findings. This issue is a further aspect of the researcher-practitioner relationship, discussed in
section 11.4.2 above, and I now believe that a much greater involvement of the teachers at every level would have been of benefit to them as well as to me. The story would be a shared one.

Ernest (1995), discussing the reflexivity of constructivism, says:

"Fallibilistic epistemology requires humility in knowledge claims at every level, including both educational research and mathematics. This must be recognised in the methodology employed, with its limited aspirations for lasting knowledge." (p. 486)

The study presented is essentially mine: it is my story of the process of research and the findings are my story of what I have found. But I believe that it provides findings, clearly grounded in existing research, methodology and practice, on which others can build.

11.5 Personal perspectives

I started the study with the aim of looking at how children learn mathematics, and took up the idea of the child having to 'make sense' of their learning in the classroom. The research was permeated by two assumptions: active learning and the importance of social interaction and language. I held a personal and professional view of learning which could best be described as social constructivist, which took account both of individual and social aspects of learning. Exploration of the literature on mathematics learning showed that others found this synthesis problematic. If mathematics is individually constructed, then the social world is only part of the child's experiential world; the constructivist would say that we can only know what we can experience, we cannot know whether there is any reality outside of ourselves. But if learning mathematics is primarily a social act in which knowledge begins on the social plane and is internalized by the learner, then we are assuming a reality. So, Lerman (1996) argues that the two ideas are therefore ontologically incompatible. One of the personal, underlying purposes of the study was to explore for myself how these theories of cognition related to what happened in the classroom.

Each of the theories of cognition which I have considered; constructivism, socio-cultural theory and social practice theory; offer perspectives from which to look at teaching and learning, yet none has provided 'the whole story'. Each can help answer the question "what is happening here?" from a different perspective, which allows insight into the situation. At the end of the study I find that I have developed my own understanding of each of the theories as they relate to the early learning of mathematics and no longer seek a unified theory. One of the main findings of the study has been my personal realisation that all theories, however well debated in the research community, are other peoples' constructions rather than 'fact' - a constructivist perspective; that all theories have arisen through interaction in social contexts - a socio-cultural perspective; and that all theories have arisen from the personal and theoretical background of
the researcher and the community of practice in which they work - a social practice perspective. I have found my position as a researcher, with a background as a teacher of young children and a teacher educator, in a world of psychologists, philosophers, sociologists, mathematicians and early childhood experts a challenging one at times; rather like trying to walk along multiple tightropes at once. I know that I have not always reflected the views of others, which are founded in their own theoretical perspectives and practices.

In Chapter Ten I expressed the doubt that a ‘whole story’ would ever be told, but I do believe that there has been such an emphasis on constructivism and developmental stage theory in early years mathematics education, and that more research from socio-cultural and social practice theoretical perspectives, could help redress the balance.
Bibliography


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