

A Study of Tokamak Energy and Particle Transport,
Based on
Modulated Electron Cyclotron Resonance Heating

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There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.

William Shakespeare, *The Tragedy of Hamlet, Prince of Denmark*, I.v.174–175.

Στους γονείς μου,
με αγάπη και ευχαριστίες.

Abstract

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A dynamical technique for the study of tokamak energy and particle transport has been developed. The plasma in the medium-sized DITE tokamak was perturbed by the application of modulated electron cyclotron resonance heating, with wave-launching from the high-field side. These experiments were carried out with absorption at various distances from the plasma centre, over a range of densities. Energy transport through the electron channel was dominant, and the variations in electron temperature and density were measured using the soft X-ray, electron cyclotron emission and microwave interferometer diagnostics. Analysis in the frequency domain enabled the propagation of the thermal wave to be followed. The observed behaviour was generally indicative of diffusive propagation of the thermal perturbation. Further observations indicated a modulation of the horizontal plasma shifts, diffusive propagation to the edge and a low modulation level of line-averaged density. In some atypical cases, the observed behaviour was qualitatively different; this type of behaviour was accompanied by a pronounced sawtooth oscillation locked with the modulation.

Two models have been employed for the interpretation of these results. The first model, based on the diffusive thermal transport of the perturbation, has led to results in good agreement with the experimental data. Values of the electron thermal diffusivity were deduced, in good agreement with those obtained from the alternative techniques of power balance analysis and sawtooth heat pulse propagation analysis; such agreement has not been universally obtained in similar experiments. The width of the absorption region has emerged as an important consideration in this analysis. A more complex model, including non-linear, coupled equations of particle and energy balance, has produced results in partial agreement with the experimental data, supporting, to some extent, the presence of coupled transport. It has been demonstrated how perturbation techniques can afford a useful means of testing transport models.

Contents

Abstract	iii
Contents	iv
Acknowledgements	vii
Introduction	viii
1 Nuclear Fusion and Tokamak Research	1
1.1 Nuclear fusion	1
1.2 Plasma confinement	4
1.2.1 Lawson criterion for fusion	4
1.2.2 Confinement techniques	7
1.3 Tokamaks	8
1.4 Tokamak equilibrium	11
1.4.1 Grad-Shafranov equation	11
1.4.2 Magnetohydrodynamic stability	13
1.4.3 Particle orbits	15
1.5 Additional heating	17
1.6 Diagnostics	21
2 Particle and Energy Confinement in Tokamaks	25
2.1 Global confinement times	25
2.2 Classical transport	29
2.2.1 Coulomb collisions	29
2.2.2 Fluid equations	31
2.2.3 Braginskii equations	33
2.3 Neoclassical transport	35
2.3.1 Basic considerations	35
2.3.2 Neoclassical transport regimes	36
2.4 Anomalous transport	38
2.4.1 Experimental observations	38
2.4.2 Turbulent fluctuations and transport	40
2.4.3 Anomalous transport models	43
2.5 Modelling of transport	47

	Introduction	47
	Transport equations	48
	Fluxes of particles and energy	49
	Sources and sinks of particles and energy	51
2.6	Measurement of transport	56
	Equilibrium methods	57
	Dynamical methods	58
	Comparison of methods	59
2.7	Review of dynamical measurements of transport	60
2.7.1	Sawtooth pulse propagation	60
2.7.2	Modulation of heating power	64
2.7.3	Modulation of gas feed	66
2.7.4	Pellet injection	67
3	Experiments on DITE with Modulated ECRH	68
3.1	Description of experiments	68
3.1.1	The DITE tokamak	68
3.1.2	Parameters of discharges with modulated ECRH	69
3.2	Electron cyclotron resonance heating	72
3.2.1	Theory of propagation and absorption of ECRH waves	72
3.2.2	Gyrotrons	77
3.2.3	Results of ECRH experiments	79
3.2.4	Implementation of high-field-side ECRH on DITE	80
3.2.5	Results of high-field-side ECRH experiments on DITE	83
3.3	Soft X-ray measurements	89
3.3.1	Detection of soft X-ray emission	89
	Spectral emissivity	89
	Detected soft X-ray emissivity	90
	Modulation index and chordal integration	92
	Experimental and numerical analysis of emission	93
3.3.2	The DITE soft X-ray (SXR) diagnostic	96
3.4	Electron cyclotron emission measurements	99
3.4.1	Detection of electron cyclotron emission	99
3.4.2	The DITE electron cyclotron emission (ECE) diagnostic	102
3.5	Electron density measurements	105
3.5.1	Microwave interferometry	105
3.5.2	The DITE microwave interferometer (MWI) diagnostic	107
3.6	Spectral analysis of modulated signals	110
3.6.1	Background: the discrete Fourier transform (DFT)	110
3.6.2	Computational implementation: the fast Fourier transform (FFT)	111
4	Thermal Transport Analysis of Data from Modulated ECRH	118
4.1	Analytical solutions of diffusion equation	118
4.1.1	Slab geometry	118
4.1.2	Cylindrical geometry	121

4.2	Results of experiments with modulated ECRH	123
4.2.1	Diffusive behaviour	123
4.2.1.1	Preliminary observations	123
4.2.1.2	Thermal diffusivity	131
4.2.1.3	Modulation of the horizontal plasma shift	132
4.2.1.4	Response of the plasma periphery	134
4.2.1.5	Modulation of the line-averaged density	139
4.2.2	Indications of non-diffusive behaviour	142
4.3	Numerical modelling of diffusive thermal transport	151
4.3.1	Dynamical thermal transport model	151
4.3.2	The MTRANSP1 computer code	155
4.4	Results from modelling of diffusive thermal transport	157
4.4.1	Application of transport model	157
4.4.2	Results of transport model	160
	Model fits	160
	Conclusions	173
4.5	Results from alternative analyses of thermal transport	175
4.5.1	Power balance	175
4.5.2	Sawtooth heat pulse propagation	177
4.5.3	Juxtaposition of techniques	183
5	Coupled Transport Analysis of Data from Modulated ECRH	187
5.1	Evidence for the presence of coupled transport	187
5.2	Analytical investigation of coupled transport	189
5.3	Numerical modelling of generalized coupled transport	194
5.3.1	Coupled transport model	194
5.3.2	MHD shifts of flux surfaces	197
5.3.3	The MTRANSP2 computer code	199
5.4	Results from modelling of generalized coupled transport	204
5.4.1	Assessment of coupled transport	204
5.4.2	Assessment of some transport models	219
	Dissipative-trapped-electron (DTE) mode	219
	Woods 'second-order' classical model	220
6	Concluding Comments	232
6.1	Summary	232
	Experiments	232
	Interpretation: diffusive thermal transport	233
	Interpretation: generalized coupled transport	234
	Final conclusions	235
6.2	Recommendations for future experiments and interpretation	235
	Table of Symbols	238
	Bibliography	242

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N. D.

Introduction

The extent to which the energy and particles of a tokamak plasma can be confined is of fundamental importance to the attainment of the conditions that are needed for the generation of power by nuclear fusion. Extensive investigations are being carried out in both experimental and theoretical fronts, in order to unravel the transport mechanisms that are largely responsible for this confinement. The purpose of the present work was to apply a dynamical technique for the measurement of thermal transport, which involves a perturbation of the energy balance and possibly of the particle balance, and, by interpreting the results on the basis of transport theory, to shed light on some aspects of the energy transport and its possible interaction with the particle transport. In chapter 1, an introduction to nuclear fusion can be found, followed by a description of the salient features of tokamak devices and their equilibrium, additional heating schemes and diagnostic techniques. In chapter 2, various aspects of the energy and particle transport are treated, and the background against which the present work has been carried out is outlined. The theories of classical, neoclassical and anomalous transport, and associated experimental observations, are briefly introduced; the numerical modelling and measurement of transport are subsequently covered in some detail; a review of previous work in the field of dynamical measurement of transport is finally presented. Chapter 3 deals with the experimental aspects of the present work. The DITE tokamak and the experiments with modulated electron cyclotron resonance heating are described; the various aspects of this heating scheme, including the associated apparatus, are outlined, and the DITE implementation is examined; the diagnostic techniques for the measurement of temperature by detection of the soft X-ray radiation and electron cyclotron emission, and of density by microwave interferometry, are then examined in some detail, covering both the pertinent principles and the DITE systems, and devoting considerable attention to the soft X-ray measurements, these forming the main set of data; finally, the fast Fourier transform techniques used in the analysis of data in the frequency domain are presented. Chapter 4 presents the experimental results and their interpretation on the basis of diffusive thermal transport. An analytical treatment of diffusive wave propagation is first given; the various aspects of the experimental results are considered; a numerical model of diffusive thermal transport and its results are then presented; finally, alternative measurements of diffusive thermal transport, based on power balance and sawtooth heat pulse propagation, are introduced, and the results of all measurements are compared. Chapter 5 addresses the question of coupling between energy and particle transport. A qualitative discussion and analytical treatment are first given; subsequently, a numerical model of coupled transport is detailed, some applications are described and conclusions are drawn. In chapter 6, the experiments and their interpretation are summarized, and some suggestions are offered for future investigations in the field.

Chapter 1

Nuclear Fusion and Tokamak Research

1.1 Nuclear fusion

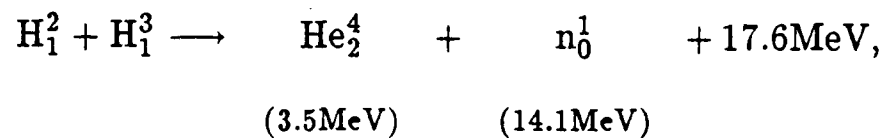
The motivation for nuclear fusion research arises from the need of an abundant and reliable source of energy. Nuclear fusion has the additional advantages of negligible atmospheric pollution and of posing only a comparatively minor radioactivity problem. It is therefore desirable to attain the conditions that are required for nuclear fusion in a power-producing reactor, this being the predominant object in nuclear fusion research, which has been pursued since the 1950's.

In contrast to nuclear fission, the splitting of heavy nuclei, nuclear fusion involves reactions between light nuclei which combine to form heavier ones that are more strongly bound, thus releasing energy.¹ Because the strong nuclear attraction which is responsible for fusion reactions has a very short range in comparison with the electrostatic repulsion of the nuclei,² energy has to be provided in order to initiate and sustain the process by overcoming the electrostatic barrier. The energy released per fusion reaction is typically of the order of 10MeV, whilst the cross-sections of such reactions peak at about 100keV —owing to quantum mechanical tunnelling,

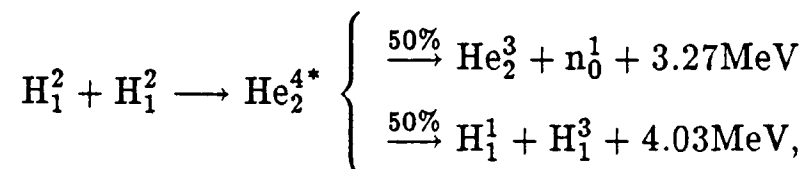
¹for more details on nuclei and nuclear reactions, see, e.g., Burcham (1973)

²the range of the strong nuclear force is of the order of the nuclear radius or smaller, that of the electrostatic force being infinite

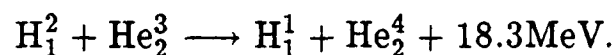
this is lower than the energy of the electrostatic barrier. One therefore expects fusion reactions, the mechanism responsible for energy production in stellar interiors, to be useful as the basis of a power generation system. The fusion reactions that are of particular interest in this connection, because of their comparatively favourable cross-sections, are the D-T reaction,



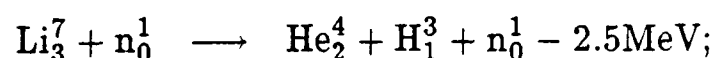
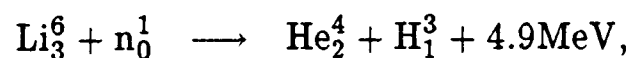
the D-D reaction,



and the D-He³ reaction,



One notes the high energies produced by the D-T and D-He³ reactions, which arise from the strong binding of the helium-4 nucleus. The D-He³ reaction is interesting in that it does not produce neutrons. The cross-sections for these reactions are shown in figure 1.1. The D-T reaction has the highest cross-section of all fusion reactions, peaking at an ion energy of about 100keV, and its sustainment at a sufficient rate is therefore the goal of current fusion experiments. Deuterium is stable and occurs naturally, but tritium is radioactive (decaying to helium-3 by β^- -emission, with a half-life of about 12.3 years) and will have to be produced from lithium using the fusion-generated neutrons in the reactions



the relative abundances of the light and heavy isotopes of lithium are 7% and 93% respectively (some form of processing of the lithium will probably be needed, to minimize the neutron energy lost in breeding tritium from lithium).

The fusion reactions will take place in a gas of ions, and the reaction rate per unit volume can be calculated from the reaction cross-section by an integration over ion velocities:

$$R = \int \int \sigma(|\mathbf{v}_1 - \mathbf{v}_2|) |\mathbf{v}_1 - \mathbf{v}_2| f_1(\mathbf{v}_1) f_2(\mathbf{v}_2) d^3\mathbf{v}_1 d^3\mathbf{v}_2 = n_1 n_2 \langle \sigma v \rangle, \quad (1.1)$$

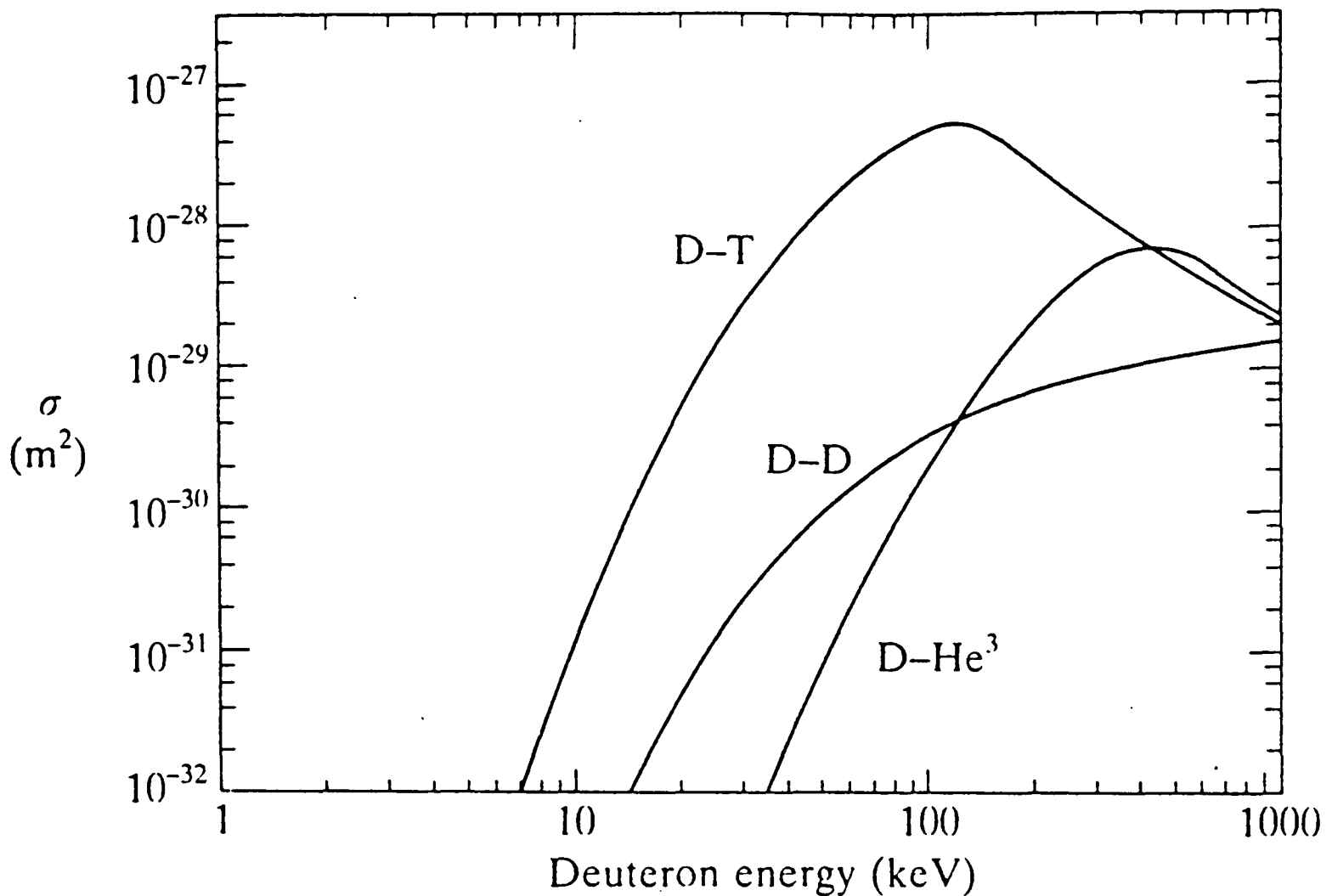


Figure 1.1: Cross-sections for D-T, D-D and D-He³ reactions, as functions of deuteron energy (from Wesson, 1987).

where σ is the fusion reaction cross-section, $f(v)$ denotes an ion velocity distribution function and n is an ion number density. The reaction rates for the D-T and D-D reactions, for Maxwellian distributions of ions, are given in figure 1.2. For a Maxwellian distribution of deuterium and tritium ions, the reaction rate of the D-T reaction peaks at a mean energy of about 70 keV. This energy is equivalent to a temperature of some 8×10^8 K, the reacting matter being in the plasma state.

Deuterium constitutes about 0.015% by mass of all natural hydrogen, a very large quantity for the purposes of energy generation by nuclear fusion. Because deuterium and lithium are both abundant and the products of fusion reactions are non-radioactive, nuclear fusion based on the D-T reaction should have considerable environmental and economic advantages. As the radioactive tritium will be contained within the fusion system, the only source of radiation will be the neutron-activated structural materials. It is envisaged that it will become feasible, in the more distant future, to exploit the D-D reaction (coupled with the D-He³ reaction), thereby gaining access to a virtually inexhaustible source of energy. Several different fusion schemes are

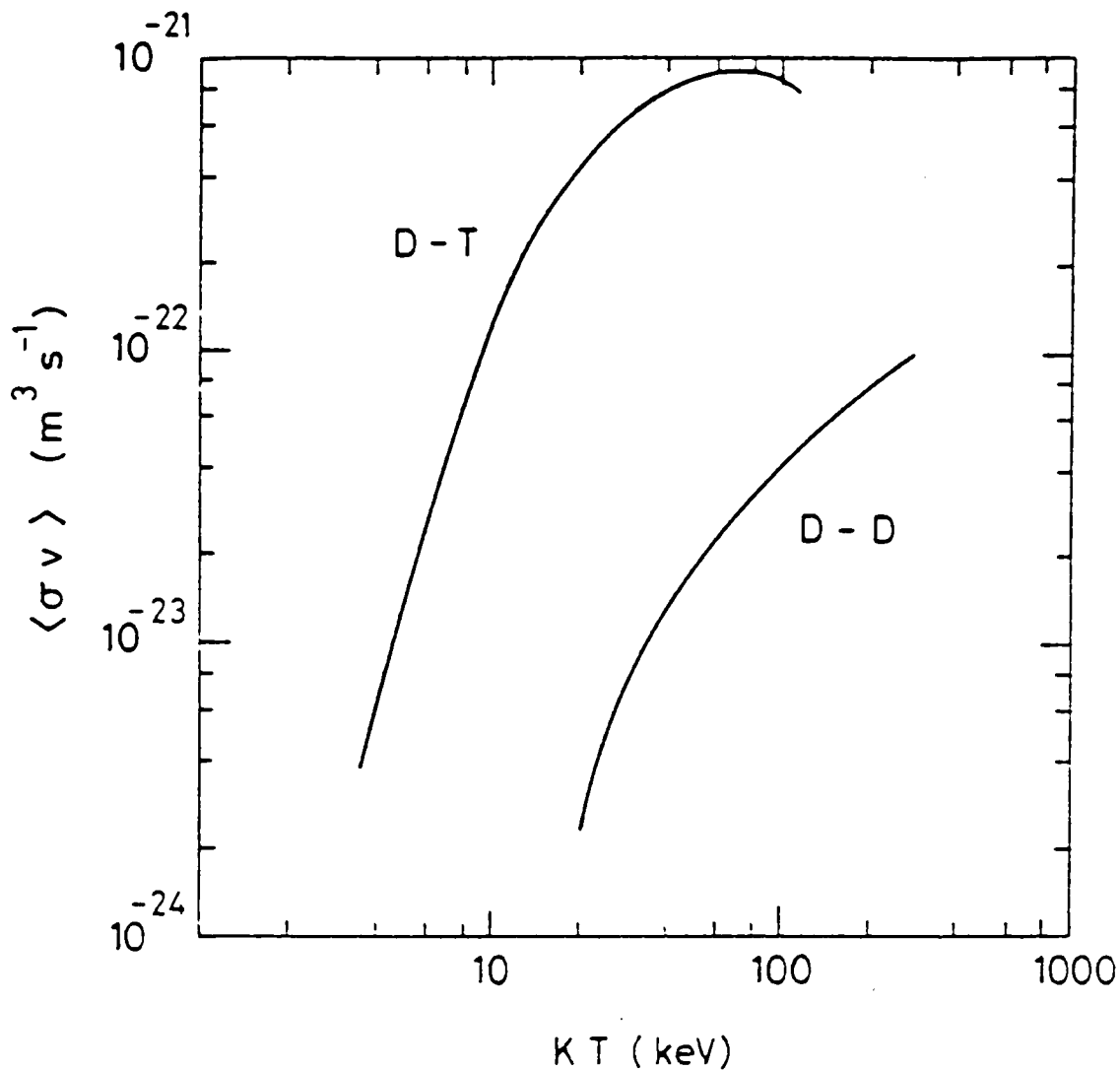


Figure 1.2: Reaction rates for D-T and D-D reactions, for Maxwellian distributions, as functions of ion temperature (from Hugill, 1981).

being studied, including “aneutronic” and hybrid fusion-fission processes.

1.2 Plasma confinement

1.2.1 Lawson criterion for fusion

A system generating net power from fusion reactions will contain a volume of reacting plasma, from which energy will be lost through transport processes and radiation. The energy generated by the fusion reactions, in addition to that lost, will be converted to electrical energy; a certain fraction of that energy will have to be used in sustaining the plasma conditions against the losses, whilst the remaining fraction will be useful energy. If any net power is to be generated by such a nuclear fusion system, then the total power being produced by the plasma, that is, the thermonuclear power plus the losses through energy transport and radiation, should, after

conversion, be in excess of the power losses from the plasma, i.e.

$$\eta \left[\frac{n^2}{4} \langle \sigma v \rangle \mathcal{E} + \frac{2\frac{3}{2}nT}{\tau_E} + \alpha n^2 T^{1/2} \right] > \left[\frac{2\frac{3}{2}nT}{\tau_E} + \alpha n^2 T^{1/2} \right],$$

where η is the conversion efficiency, $\mathcal{E}$ is the energy released per reaction, n is the total ion number density (also the electron density), T is the temperature (in energy units), τ_E is the energy confinement time³ and α is a parameter for bremsstrahlung.⁴ The first term in the left-hand side is the fusion power, the second represents the transport losses and the third is the power lost through bremsstrahlung.⁵ This condition leads to a requirement of the form

$$n\tau_E > \frac{12T}{\frac{\eta}{1-\eta} \langle \sigma v \rangle \mathcal{E} - 4\alpha T^{1/2}}; \quad (1.2)$$

the right-hand side is a function of temperature, which, for the D-T reaction and taking $\eta = \frac{1}{3}$, has a minimum value

$$n\tau_E > 0.6 \times 10^{20} \text{m}^{-3} \text{s}, \quad T = 25 \text{keV}$$

—this is the *Lawson criterion for breakeven* (Lawson, 1957).

Furthermore, ignition will occur when the fraction of the thermonuclear power that is retained in the plasma in the form of charged particles is sufficient to balance the power losses,⁶ i.e.

$$\frac{n^2}{4} \langle \sigma v \rangle \mathcal{E}_\alpha > \frac{2\frac{3}{2}nT}{\tau_E},$$

where $\mathcal{E}_\alpha$ is the charged particle energy per reaction. This condition leads to a requirement which is similar in form to, but more stringent than, the breakeven criterion, i.e.

$$n\tau_E > \frac{12T}{\langle \sigma v \rangle \mathcal{E}_\alpha}; \quad (1.3)$$

for the D-T reaction, this has a minimum value

$$n\tau_E > 1.5 \times 10^{20} \text{m}^{-3} \text{s}, \quad T = 30 \text{keV}$$

³corrected for the radiation loss

⁴ $\alpha = 9.55 \times 10^{-20} Z^2 \text{eV}^{1/2} \text{s}^{-1} \text{m}^3$, where Z is the ion charge

⁵bremsstrahlung is expected to be the dominant radiative process at high temperatures or in the absence of impurities

⁶the radiation loss is ignored in this case, as it is not expected to be dominant

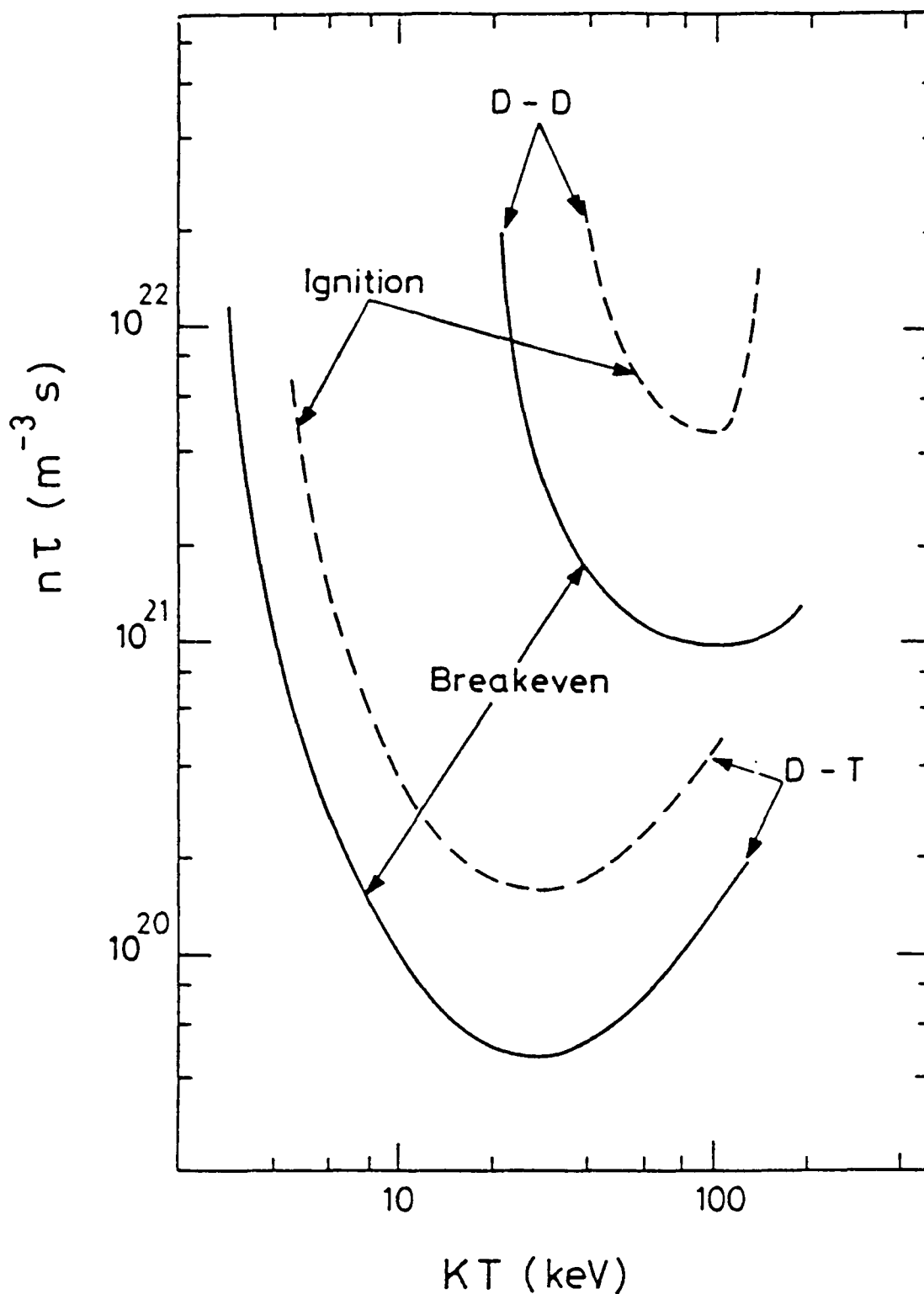


Figure 1.3: Breakeven and ignition criteria for the D-T and D-D reactions, as functions of ion temperature (from Hugill, 1981).

—this is the criterion for *ignition*. The dependences of the breakeven and ignition criteria on temperature are shown in figure 1.3. It should be emphasized that both of these criteria are strongly sensitive to the presence of impurities in the plasma,⁷ and that they become much more stringent as the impurity content increases.

From these results it can be seen that the optimization of the energy confinement time on the one hand, and of the density of the plasma that can be successfully contained on the other hand, are two goals of fundamental importance in nuclear fusion research.

⁷through the bremsstrahlung parameter α (proportional to $\overline{Z^2}$) and the ratio of reacting ions to total ions

1.2.2 Confinement techniques

A thermonuclear plasma has to be confined over a time-scale dependent on its density, if net power is to be generated. There exist two general approaches to the containment of thermonuclear plasma, namely those of *magnetic* and *inertial* confinement, operating in quite different regimes of plasma density and energy confinement time. Inertial confinement techniques⁸ involve radiation or particle beams of high power density (produced by lasers or accelerators) that are used to cause ablation of the outer layer of a fuel pellet, resulting in the compression of the core to a very high density, comparable to that in a stellar core, the confinement time being correspondingly short, of the order of nanoseconds; this is limited by the size of the pellet, of the order of millimetres. Magnetic confinement techniques, on the other hand, are characterized by a moderate plasma density (the pressure being of a similar order of magnitude as the atmospheric pressure) and a relatively long confinement time, from a few milliseconds to about one second.

The basic principle underlying magnetic confinement is the fact that charged particles in a uniform magnetic field will follow helical orbits around magnetic field lines. If there be no collisions, the particles will be constrained against motion in the directions across the magnetic field lines, but will be free to move along them. Consequently, it is possible to use magnetic fields to contain the plasma and effectively to isolate it from the walls of the vacuum vessel which contains it, in order to maintain its temperature. Several magnetic field configurations have been or are being investigated. These fall into two categories, namely (a) *open geometry* and (b) *closed geometry* systems, the distinction being that in the first category the field lines emerge from the plasma confinement region, whereas in the second category the field lines close on themselves *within* the plasma region; the first category includes linear devices (e.g. magnetic mirrors, linear pinches), while the second includes (topologically) toroidal devices (e.g. toroidal pinches, stellarators, tokamaks).

⁸for an introduction to inertial confinement, see, e.g., Pert (1981)

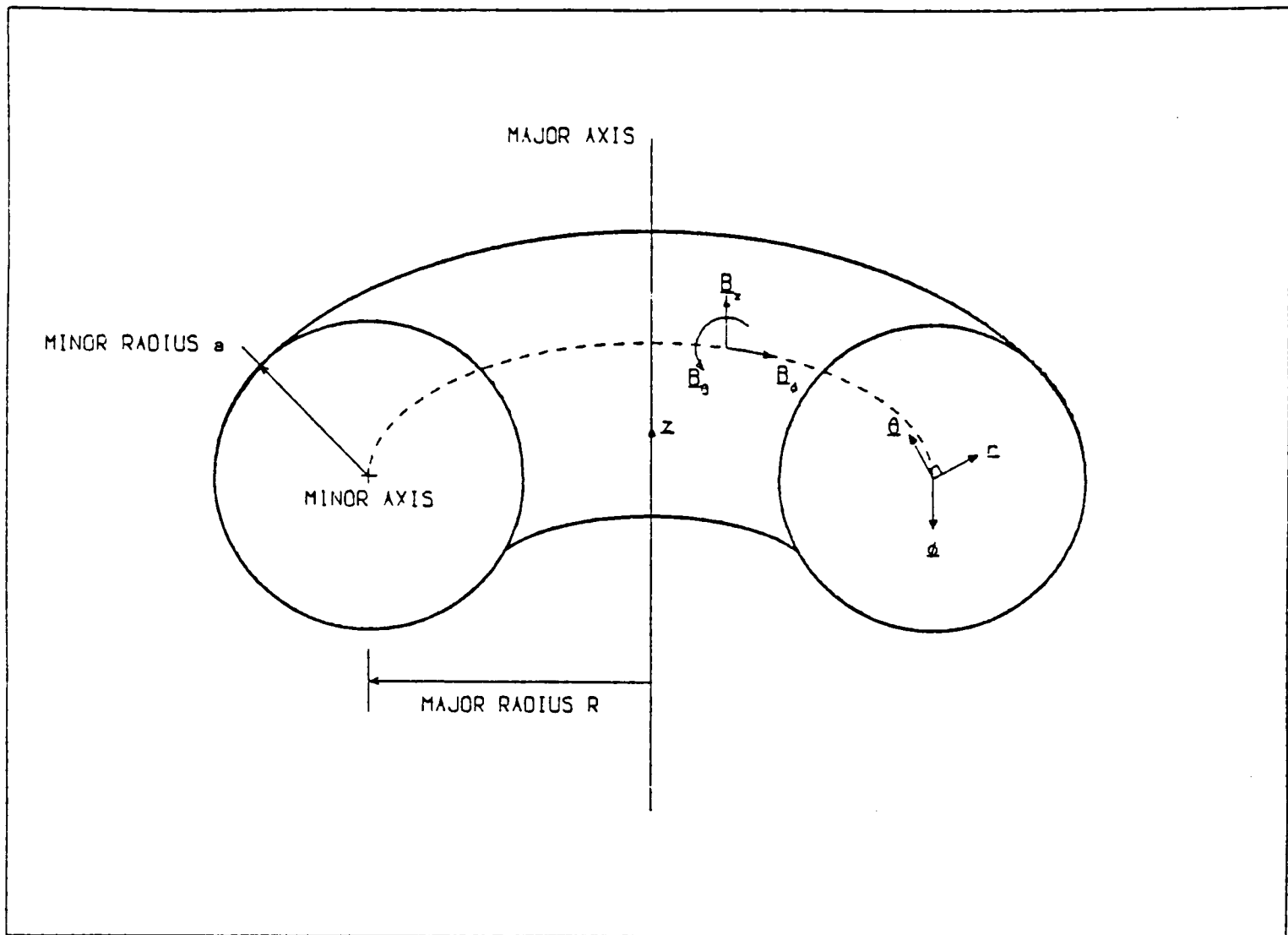


Figure 1.4: Basic geometry and associated nomenclature of a tokamak device.

1.3 Tokamaks

A tokamak is an axisymmetric toroidal magnetic confinement device (with symmetry along the toroidal axis), developed in the USSR in the late 50's and early 60's —the name derives from the Russian acronyms for “toroidal magnetic chamber”. The early progress of tokamak experiments, including their construction, basic physics and experimental results, has been reviewed by Artsimovich (1972) and Furth (1975), and later again by Furth (1981). The basic magnetic geometry of a tokamak device is shown in figure 1.4 and comprises (a) a *toroidal field* B_ϕ , around the major axis, which is generated by external toroidal field coils; (b) a *poloidal field* B_θ , around the minor axis, which is produced by the plasma current induced by the changing current in the poloidal field coils, by transformer action;⁹ and (c) a *vertical field* B_z , parallel to the major axis, which is produced by the poloidal field or special coils. In a tokamak, in contrast to a toroidal pinch, the toroidal field is much stronger than the poloidal field (by a factor of 10 or

⁹non-inductive current drive is possible

so), this being the principal distinguishing feature of this concept. The resulting magnetic field lines in a tokamak are therefore toroidal with a twist in the poloidal direction, which varies over the minor cross-section. Moreover, in contrast to a stellarator, the poloidal field is generated by the plasma current, rather than by external helical windings. In connection with the components of a vector, the terms ‘parallel’ (subscript $\parallel$) and ‘perpendicular’ (subscript $\perp$) refer to the orientation with respect to the equilibrium magnetic field.

The gradient and curvature of the equilibrium magnetic field, both inevitably present in a toroidal confinement device, give rise to vertical drifts of the plasma ions and electrons in opposite directions (see, e.g., Cairns, 1985), the drift velocity for a particle of mass m and charge e being

$$v_d = \frac{m}{eRB_\phi} \left(\frac{1}{2}v_\perp^2 + v_\parallel^2 \right), \quad (1.4)$$

where v is the particle velocity —the first and second terms in the brackets correspond to the gradient and curvature drifts respectively. These drifts tend to produce a vertical separation of electrons and ions in the plasma, and, in the absence of any twist in the magnetic field lines, the resulting electric field would cause a horizontal $\mathbf{E} \times \mathbf{B}$ drift; the poloidal field is therefore necessary in order to twist the magnetic field lines, thus cancelling the electric field. Whilst the toroidal field provides the basic confinement, the poloidal field is most important for the stability, as well as in the transport processes, of tokamak plasmas. The vertical field is required to prevent the expansion of the current-carrying plasma ring; it was originally produced by image toroidal currents induced in a highly conductive shell surrounding the vacuum vessel, but current tokamak devices use coils and feedback stabilization systems.

The tokamak concept has so far proved to be most promising as the basis of a future fusion reactor. Since its inception, it has led to a steady improvement in the achieved temperature, density and confinement time. The four large experimental magnetic confinement machines that are currently in operation around the world, concentrating on conditions approaching those of power reactor operation, namely¹⁰ JET (Europe), TFTR (USA), JT-60 (Japan) and T-15 (USSR), are all tokamak devices. The European JET device has recently achieved conditions

¹⁰in order of decreasing size

close to breakeven for a D-T plasma. The salient features and some results of several past and current tokamak experiments have been described by Hugill (1987). Extensive descriptions of tokamak experiments and their results can also be found in the Nuclear Fusion Anniversary Issue (1985).

Tokamak sizes and aspect ratios¹¹ vary widely, and cross-sections can be circular or elongated (usually D-shaped). The general trend is towards fairly tight aspect ratios, about 3, and vertically elongated cross-sections, such cross-sections tending to enhance the plasma stability. The densities of tokamak plasmas are typically 10^{19} – 10^{21}m^{-3} , and the energy confinement times range from a few milliseconds for the smaller tokamaks, to about one second for the large ones. Ion and electron temperatures are between 1keV and 30keV, the pressures being roughly atmospheric. Magnetic fields vary between 1T and 10T, and plasma currents are from 100kA to 5MA or more.

As the plasma densities are lower than the atmospheric density by factors of 10^5 – 10^6 , vacuum vessels operating at a very low base pressure are used. The performance of the confinement system is seriously impaired by the presence of impurities. Impurity ions are normally introduced into the plasma through desorption from the vessel walls (this can be limited by baking or discharge cleaning), and sputtering, arcing and evaporation releasing the material of the walls themselves. One distinguishes between light impurities (e.g. oxygen) and heavy impurities (e.g. iron), depending on whether they are fully ionized in the centre of the plasma. Impurities are detrimental to the energy confinement, because they increase the radiation loss and dilute the working gas ions; it is consequently important to limit their entry into the plasma. The separation of the plasma from the vacuum vessel is effected by using either material limiters to define the plasma edge and localize the interactions with the plasma, or some sort of magnetic divertor (see, e.g., McCracken, 1987).

The physics problems currently being addressed in tokamak research are related, on the one hand to the energy confinement time, and on the other hand to the maximum density and plasma pressure (or specific energy) that can be successfully contained; and further, to

¹¹the aspect ratio is defined as the ratio of the major to minor radius

plasma heating techniques additional to the Ohmic heating. The energy confinement time is determined by conduction, convection and radiation; all these mechanisms, being extremely complex and almost certainly varying qualitatively within the plasma, are only inadequately understood at present. The density and plasma pressure ('beta') limits are governed by the magnetohydrodynamic stability of the plasma.

A fusion reactor based on a tokamak will have superconducting toroidal field coils to limit the power consumption of the confinement system. The primary vessel will be surrounded by a blanket to absorb the neutrons and convert their energy into heat, and to breed tritium from lithium. If the reactor is to be operated in a steady state, this being advantageous from the technological point of view, in order to minimize thermal stressing fatigue, some form of non-Ohmic current drive will be needed. The structural materials of the reactor will, of course, be activated as a result of irradiation by the fusion neutrons, and it will therefore be important to develop suitable materials with the required mechanical strength and low activation properties. Further problems affecting the economics of a reactor are related to the wall loading and the efficiency of utilization of the magnetic field (beta value). Several technological and ultimately political problems will have to be solved, in order successfully to develop a large tokamak device and to integrate it in a power generating system.

1.4 Tokamak equilibrium

1.4.1 Grad-Shafranov equation

A confined plasma is in magnetohydrodynamic (MHD) equilibrium when the force exerted by the magnetic field balances that due to the pressure gradient, as expressed by the single fluid equation of motion for a stationary plasma,¹²

$$\mathbf{j} \times \mathbf{B} = \nabla p, \quad (1.5)$$

where $\mathbf{j}$ is the current density, $\mathbf{B}$ is the magnetic field and p is the plasma pressure. It follows that both current density and magnetic field vectors are normal to the pressure gradient vector

¹²obtained from (2.16) with $\mathbf{u} = 0$ and an isotropic pressure

and thus lie in surfaces of constant pressure, called *flux surfaces*. The magnetic flux through a minor cross-section of such a surface can be used as an effective radial co-ordinate in many problems of transport analysis. The magnetic field is related to the plasma current, through Ampère's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}; \quad (1.6)$$

the magnetic field produced by the plasma current causes the plasma column to contract radially, this being the '*pinch*' effect. By eliminating $\mathbf{j}$ between (1.5) and (1.6) and taking the radial component in a local cylindrical geometry (with the z -axis in the toroidal direction), one obtains the approximate pressure balance equation

$$-\frac{dp}{dr} = \frac{B_\theta}{\mu_0 r} \frac{d}{dr} (r B_\theta) + \frac{d}{dr} \left(\frac{B_\phi^2}{2\mu_0} \right). \quad (1.7)$$

It follows that the pressure associated with a magnetic field is $B^2/2\mu_0$, this being the maximum pressure that can be contained by the field. In practice, however, the plasma pressure p will be a fraction of the magnetic field pressure $B^2/2\mu_0$. This fraction, the *beta value* β , is a measure of the efficiency with which the magnetic field is utilized and is clearly an important parameter in the design of a fusion reactor.¹³

The equilibrium of an axisymmetric confinement system such as a tokamak, with $\frac{\partial}{\partial \phi} = 0$ for all variables, is described by the *Grad-Shafranov equation* (see, e.g., Cairns, 1985; Woods, 1987; Wesson, 1987) which can be readily obtained in the cylindrical co-ordinate system (R, ϕ, z) of figure 1.4, from the radial component of (1.5) using all components of (1.6):

$$R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \psi}{\partial R} \right) + \frac{\partial^2 \psi}{\partial z^2} = -\mu_0 R^2 \frac{dp}{d\psi} - \mu_0^2 f \frac{df}{d\psi}, \quad (1.8)$$

where ψ is the poloidal magnetic flux, defining the flux surface, $p=p(\psi)$ is the plasma pressure and $f=f(\psi)$ is a current flux function.¹⁴ Typical solutions including the equilibrium flux

¹³ values of β in excess of 5% are thought to be necessary for an economically viable reactor

¹⁴ The flux functions ψ and f are related to the magnetic field and current density by the following relations:

$$\begin{aligned} B_R &= -\frac{1}{R} \frac{\partial \psi}{\partial z}, & j_R &= -\frac{1}{R} \frac{\partial f}{\partial z}, \\ B_z &= \frac{1}{R} \frac{\partial \psi}{\partial R}, & j_z &= \frac{1}{R} \frac{\partial f}{\partial R}, \\ B_\phi &= \frac{\mu_0 f}{R}; & j_\phi &= -\frac{1}{\mu_0 R} \left(R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \psi}{\partial R} \right) + \frac{\partial^2 \psi}{\partial z^2} \right). \end{aligned}$$

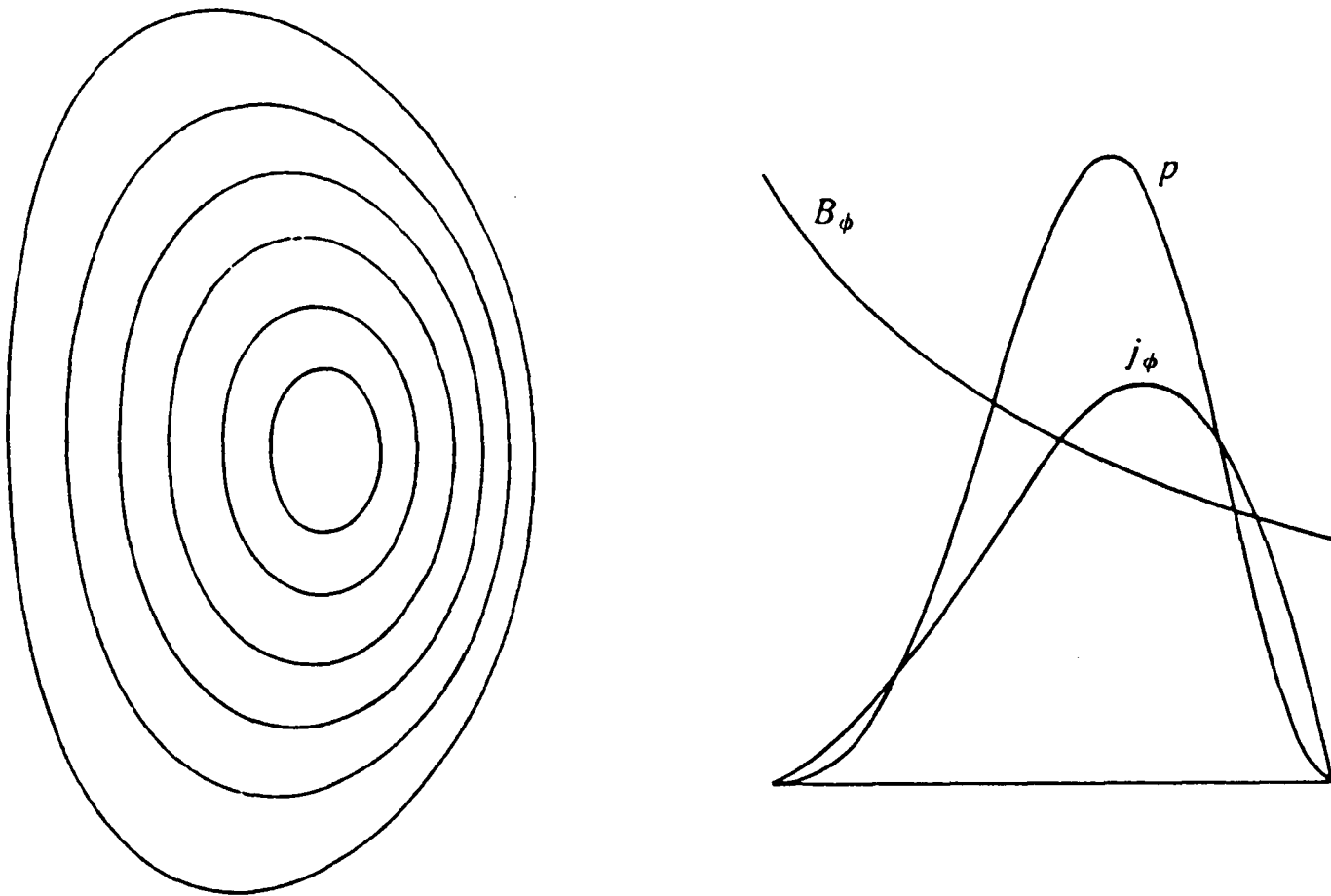


Figure 1.5: Solutions of Grad-Shafranov equation: poloidal cross-sections of flux surfaces and radial plots of toroidal magnetic field, toroidal current density and pressure.

surfaces, magnetic field, current density and pressure are indicated in figure 1.5.

1.4.2 Magnetohydrodynamic stability

For a toroidal confinement device, one defines the so-called *safety factor* by

$$q = \frac{\Delta\phi}{2\pi} = \frac{1}{2\pi} \oint \frac{1}{R} \frac{B_\phi}{B_\theta} ds \simeq \frac{B_\phi}{R} \frac{r}{B_\theta}, \quad (1.9)$$

where $\Delta\phi$ represents the change in the toroidal angle that will produce a change of 2π in the poloidal angle for a given field line —the integral is around a flux surface, and the approximation is valid for a circular cross-section. The safety factor for a tokamak is greater than unity at the plasma edge and throughout most of the plasma volume, except possibly near the magnetic axis. The variable $q(r)$ is important in controlling the MHD stability of the plasma, hence its name: higher values of q will generally enhance stability; q also plays an important role in transport theory through geometrical effects. The MHD stability of a tokamak plasma against ideal (non-resistive¹⁵) MHD instabilities is determined by the *Kruskal-Shafranov condition*, $q > 1$.

¹⁵with $\eta = 0$ in Ohm's law (2.20)

Resistive instabilities characterized by a wave vector $\mathbf{k}$ can arise on surfaces satisfying $\mathbf{k} \cdot \mathbf{B} = 0$, where $\mathbf{k} = \frac{m}{r} \hat{\theta} + \frac{n}{R} \hat{\phi}$, m and n being, respectively, the poloidal and toroidal mode numbers of the instability. Such surfaces have $q = m/n$ and are called *rational surfaces*; of particular importance are the instabilities arising at the surfaces with $q = 1, 3/2$ and 2 . A concise introduction to MHD instabilities in tokamaks is given by Wesson (1987). Briefly, there are three types of instability that are most commonly observed in tokamaks:

- The sawtooth oscillation (von Goeler et al., 1974; Jahns et al., 1978) is commonly observed in several signals from tokamak diagnostics. It is thought to arise from an instability with $m=1$ and $n=1$, leading to a periodic collapse of the temperature profile inside the $q=1$ surface, accompanied by a corresponding increase outside. The shape of the sawtooth oscillation becomes inverted beyond a certain radius, which is taken as the position of the $q=1$ surface.
- Mirnov oscillations, occurring close to the plasma surface, are $q=m$ tearing modes.¹⁶
- Disruptions involve an abrupt loss of plasma current and are accompanied by precursor oscillations. They would clearly be undesirable in reactors, as they would lead to a loss of confinement and impose unacceptably large wall loadings. The mechanisms leading to disruptions are not well understood. It is thought that they are predominantly due to an $m=2$ tearing mode, radiative cooling possibly playing a role. There appears to be an upper limit on the plasma density that can be confined without a disruption at a given plasma current (*Hugill limit*), and hence using a given toroidal field for a fixed safety factor. Disruptions further impose an upper limit on the beta value. These are clearly important considerations in assessing the performance of tokamak reactors. Attempts are being made to stabilize disruption precursors by localized heating and feedback cancellation of the $m=2$ mode.

¹⁶A tearing mode involves the breaking up and reconnection of magnetic field lines, leading to the formation of a “magnetic island”.

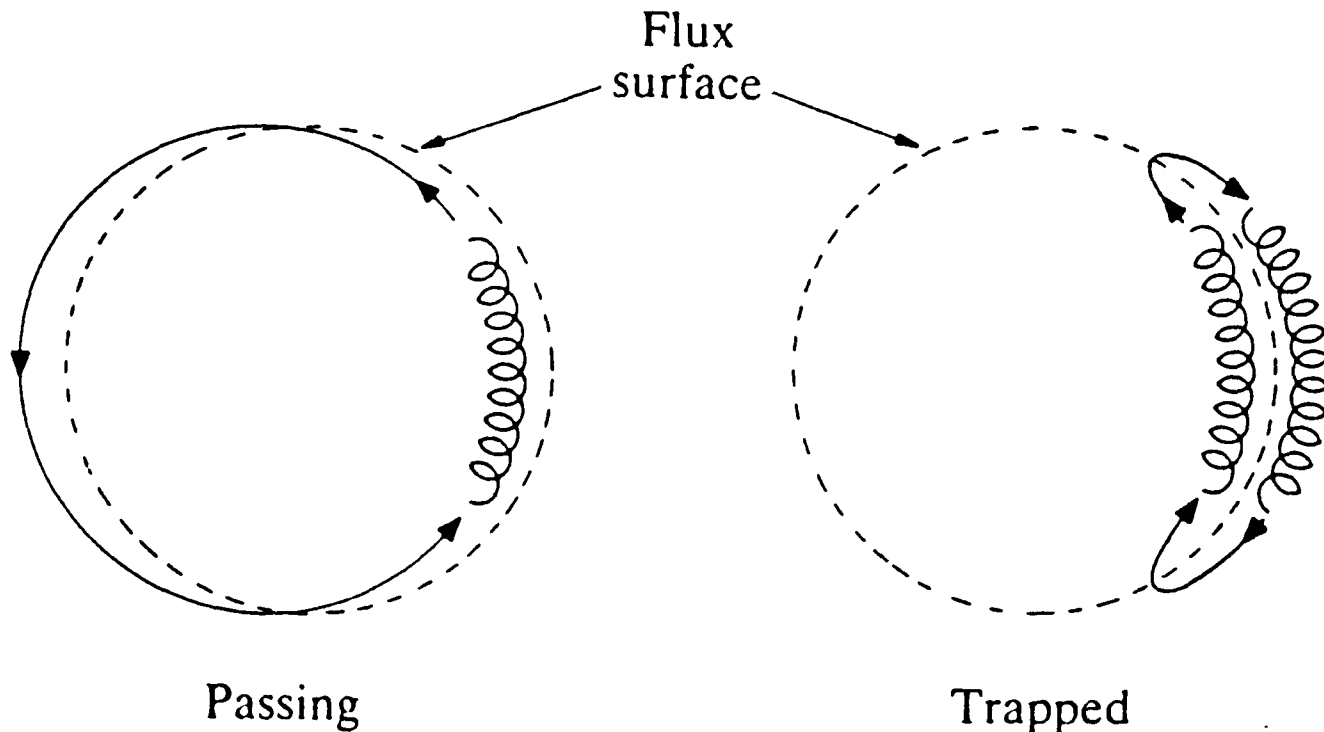


Figure 1.6: Poloidal-plane projections of the orbits of (a) passing and (b) trapped particles.

1.4.3 Particle orbits

An important aspect of the confinement of plasma particles in a toroidal magnetic geometry is the fact that the variation of the magnetic field with major radius, or with poloidal angle around a flux surface, as described by

$$B = B_0 \left(1 + \frac{r}{R_0} \cos \theta \right)^{-1}, \quad (1.10)$$

leads to the trapping of a certain class of the particles in the outer region of weaker magnetic field, and thus to two different types of particle orbits. The orbits follow from the conservation of magnetic moment $\mu \propto (v_\perp^2/B)$, and the conservation of total energy $W \propto (v_\perp^2 + v_\parallel^2)$ (see, e.g., Wesson, 1987), and are shown in figure 1.6; the following discussion applies to circular flux surfaces:

1. *Passing particles* have a sufficiently large parallel velocity to follow magnetic field lines around the torus without being reflected. Their trajectories arise from the interaction of the rotation in the poloidal plane, with an angular frequency given by

$$\omega = \frac{B_\theta}{B_\phi} \frac{v_\parallel}{r} = \frac{v_\parallel}{qR}, \quad (1.11)$$

and the vertical drift velocity v_d given by (1.4). The poloidal projection of the trajectory of the guiding centre (the instantaneous centre of gyration around the field line) is a circle

that is (inwardly) displaced from the flux surface by a distance given, in the strongly passing limit ($v_{\parallel} \gg v_{\perp}$), by

$$\Delta r = -\frac{v_d}{\omega} = -q \frac{v_{\parallel}}{\omega_c}, \quad (1.12)$$

where $\omega_c = eB_{\phi}/m$ is the cyclotron frequency in the toroidal field.

2. *Trapped particles*, on the other hand, have a sufficiently small ratio of parallel to perpendicular velocities to be reflected as they move along field lines in the direction of increasing magnetic field. The condition for trapping is that the particle velocity should be inside the trapping cone in phase space, given by

$$\frac{v_{\parallel 0}}{v_{\perp 0}} \leq \left(2 \frac{r}{R}\right)^{1/2} = \sqrt{2\varepsilon}, \quad (1.13)$$

where the subscript 0 refers to the outermost point on the poloidal cross-section of the flux surface, R is the major radius at the centre of that cross-section, and $\varepsilon = r/R$ is the inverse aspect ratio. For a Maxwellian distribution, one can readily show that the fraction of particles that are thus trapped is $\sqrt{2\varepsilon}/(1 + \varepsilon)$.¹⁷ The poloidal projection of the guiding centre trajectory is banana-shaped; the bounce frequency (the frequency of the guiding centre motion in its closed trajectory) is given, in the strongly trapped limit ($v_{\perp} \gg v_{\parallel}$), by

$$\omega_b = \frac{v_{\perp}}{2qR} \sqrt{2\varepsilon}. \quad (1.14)$$

The half-width of the trapped particle trajectory, given by

$$\Delta r = \frac{B_{\phi}}{B_{\theta}} \frac{v_{\parallel}}{\omega_c}, \quad (1.15)$$

is greater than the displacement of a passing particle from the flux surface, by a factor R/r , this feature being important in considerations of transport. Collisional detrapping (escape from the trapping cone) occurs at the detrapping frequency, given in terms of the collision frequency by

$$\nu_{dt} = \frac{\nu}{2\varepsilon}. \quad (1.16)$$

¹⁷One can show that the probability of $|v_{\parallel}| < \alpha|v_{\perp}|$ for a Maxwellian distribution is $(1 + \alpha^{-2})^{-1/2}$; for small α , this is $\sim \alpha/(1 + \frac{1}{2}\alpha^2)$.

When the detrapping frequency exceeds the bounce frequency, the particles are detrapped before they complete a banana orbit.

1.5 Additional heating

A tokamak plasma is subject to Ohmic heating by the induced current; Ohmic heating is, however, limited, both because the plasma resistivity decreases with increasing temperature, and because a continuously changing current in the poloidal field coils, which is limited by the “volt-seconds” capability of the transformer arrangement, is needed to induce a current in the plasma. Auxiliary heating systems are therefore commonly employed, some of which may additionally enable the plasma current to be driven continuously, eliminating the need for pulsed operation. These are briefly outlined here:

1. Neutral beam heating involves the absorption of energetic beams of neutral particles (see, e.g., Hemsworth, 1981; Wesson, 1987). A typical neutral beam production system includes an ion source, an electrostatic accelerator, a gas neutralizing chamber and a deflector magnet which removes any ions that are not neutralized. The beam particles are, of course, faster than the plasma ions, but slower than the electrons. The energetic neutral particles penetrate the magnetic field and travel within the plasma until they are absorbed by (a) charge exchange, (b) ionization by collisions with ions or (c) ionization by collisions with electrons; absorption by these processes depends on the beam energy and plasma density. This heating method is characterized by relatively straightforward absorption mechanisms and is very effective in increasing the ion temperature, albeit usually with a degradation of the energy confinement; current drive using neutral beam injection is possible. However, the production of neutral beams is an inefficient process and negative ion sources will be needed to alleviate this problem; also, strong absorption is expected in high density plasmas, leading to edge heating.
2. Non-oscillatory and low-frequency heating schemes rely on the conservation of flux or magnetic moment during relatively slow processes in order to increase the energy of the plasma. Two such schemes have been used on tokamak devices. Adiabatic toroidal compression

(see, e.g., Fielding, 1981) involves moving the plasma to a region of higher toroidal field by increasing the vertical field, leading to a decrease in radius by conservation of magnetic flux, and to a corresponding increase in pressure; the characteristic time for this process is chosen to be longer than the equilibration time, so that it will be reversible, but shorter than the energy confinement time, so that it will also be adiabatic. Transit-time magnetic pumping (see, e.g., Riviere, 1981), on the other hand, is an irreversible process involving modulation of the magnetic field with a time-scale appropriate to the transit time through the modulation region (the frequencies involved are 10–500kHz); heating arises from the temporary conservation of the magnetic moment $\mu = W_{\perp}/B$, leading to a higher energy, and the subsequent collisional dissipation. One can also include in this class of low frequency heating schemes the excitation of Alfvén waves, using frequencies of 1–10MHz.

3. High-frequency heating schemes are based on the various resonances of electromagnetic waves in a magnetized plasma. Of importance in such schemes, in connection with the propagation and absorption of the waves, are the *cyclotron frequency* (the frequency of gyration in the magnetic field)

$$\omega_{cs} = \frac{e_s B}{m_s}, \quad (1.17)$$

and the *plasma frequency* (the frequency of electrostatic plasma oscillations)

$$\omega_{ps} = \left(\frac{n e_s^2}{\epsilon_0 m_s} \right)^{1/2}; \quad (1.18)$$

clearly, these frequencies differ significantly for different plasma species. Absorption occurs when the Doppler-shifted wave frequency is near one of the plasma species resonance frequencies or one of their harmonics, i.e.

$$\omega - k_{\parallel} v_{\parallel} = \ell \omega_{cs}.$$

The absorption is by Landau damping for $\ell=0$, and by cyclotron damping for $\ell \neq 0$; both damping processes are collisionless, so they are not subject to the limitation of collisional heating. Cold plasma theory may be used to describe the propagation of the electromagnetic wave for a particular scheme, including the cut-offs, but fails near the resonances

which can be fully predicted only by warm plasma theory. A sufficiently fast relaxation process is needed if the plasma is to remain Maxwellian, but the creation of superthermal populations is useful in current drive schemes.

- (a) Ion cyclotron resonance heating (ICRH; see, e.g., Fielding, 1981; Riviere, 1981; Wesson, 1987) is based on resonances at the ion cyclotron frequency ω_{ci} and its harmonics, and at the two-ion hybrid frequency given by

$$\omega_{12}^2 = \frac{\omega_{c1}^2 \omega_{p2}^2 + \omega_{c2}^2 \omega_{p1}^2}{\omega_{p1}^2 + \omega_{p2}^2}.$$

The propagation of these waves is by the fast magnetosonic (compressional Alfvén) wave which propagates above ω_{ci} ; there is an evanescent region near the antenna because of the low density cut-off. The absorption at the fundamental frequency is small because the polarization of the fast magnetosonic wave is opposite to the ion gyration; however, at the second harmonic, or at the fundamental frequency with a minority species, a significant fraction of the perpendicular electric field possesses the required polarization. At a low minority concentration, the two-ion resonance is close to that of the minority ions and the wave is launched from the low-field side; at a higher minority concentration, conversion to an ion cyclotron wave occurs and the wave is launched from the high-field side. The absorption region is a vertical chord, but in large tokamaks the power can be focused along that chord. RF generators with high efficiencies can be readily obtained for ICRH frequencies (25–100MHz), but the antennae required (coils) are complex and must be in close proximity to the plasma.

- (b) Lower hybrid resonance heating (LHRH; see references above) is based on the lower hybrid resonance of the X-mode, approximately given (for $\omega_{pi} \gg \omega_{ci}$) by

$$\omega_{LH}^2 \simeq \frac{\omega_{pi}^2}{1 + \omega_{pe}^2 / \omega_{ce}^2}.$$

It is not possible freely to vary the location of the resonance. Only the slow wave has a resonance, this being evanescent near the antenna; the Stix-Golant accessibility condition requires that the parallel wavenumber should exceed a critical value for evanescence farther away to be avoided. A grill antenna is used, with a suitable

phase difference between adjacent elements in order to produce the required parallel wavenumber and to excite mainly the slow wave, which travels several times around the torus before being absorbed. Both ions and electrons are heated with this scheme, ion heating resulting from conversion to ion Bernstein waves, and electron heating being produced by Landau damping of the parallel electric field component (the latter need not be confined to the resonance region). A superthermal population of electrons may be produced, leading to the possibility of lower hybrid current drive (LHCD), this having been demonstrated experimentally. As with ICRH, efficient generators (klystrons) are obtainable for the LHRH frequencies (1–5GHz) and transmission is straightforward, but the absorption processes are complicated and the absorption is not easily controllable.

- (c) Electron cyclotron resonance heating (ECRH; see references above; Cairns, 1985)¹⁸ involves absorption by electrons at the cyclotron frequency and its harmonics, and at the upper hybrid frequency given by

$$\omega_{UH}^2 = \omega_{ce}^2 + \omega_{pe}^2.$$

Warm plasma theory predicts resonances at these frequencies for both the O-mode and the X-mode. The O-mode will propagate above the cut-off frequency, while the X-mode is characterized by a low- and a high-density cut-off. The fundamental and second harmonic resonances are most suitable at the temperatures of interest, with waves launched from either the low- or the high-field side. This scheme is characterized by the simplest propagation and absorption mechanisms and the localization of the power can be readily controlled by changing the toroidal field; transmission and launching of the power are straightforward, employing waveguides and antennae; the latter do not have to be in close proximity to the plasma, a major advantage of this scheme. Gyrotron generators have been developed to produce the required microwave power (30–200GHz), being themselves based on the electron cyclotron

¹⁸more details on ECRH, the technique that has been used in this work, can be found in chapter 3

resonance; however, these are inefficient (about 30%) and much more development will be needed before it becomes feasible to use ECRH for reactor plasma heating.

In addition to these heating schemes, it is expected that the charged fusion products (e.g., alpha particles) produced by fusion reactions will be contained in a thermonuclear plasma and will contribute to its heating; no external heating will be needed at ignition.

1.6 Diagnostics

The parameters that are most commonly measured by tokamak diagnostic techniques are briefly presented here.¹⁹ Most plasma diagnostic techniques have been reviewed comprehensively, covering both the underlying theory and the experimental implementations, by Equipe TFR (1978);²⁰ a more recent review with more emphasis on diagnostics for large tokamaks is by Orlinskij and Magyar (1988); most of the underlying principles are covered in the book by Hutchinson (1987).

1. Magnetic diagnostics, which include a variety of loops and coils around the tokamak in proximity to the plasma, are used to determine the plasma current, loop voltage (the electric potential around the torus) and plasma position, as well as selected magnetic oscillations. By invoking MHD theory, one can calculate the shape of the outermost plasma flux surface from magnetic measurements. The plasma diamagnetism (or paramagnetism)²¹ may also be measured, enabling the total energy content of the plasma to be determined. The current density and pressure profiles may also be inferred from magnetic field measurements. These diagnostics are also used to provide feedback signals for the equilibrium control systems.
2. The electron density is usually obtained from measurements of the plasma refractive index, by microwave or far-infrared interferometry. The O-mode wave with propagation perpen-

¹⁹the diagnostic techniques that have been used in this work will be described in greater detail in chapter 3

²⁰this somewhat dated review article does cover most features of present diagnostic systems, although some of the performances quoted have been improved upon

²¹The plasma kinetic pressure always tends to decrease the toroidal magnetic field, but the poloidal magnetic field pressure tends to increase it (see (1.7)); both diamagnetic and paramagnetic effects are therefore possible.

dicular to the magnetic field is used, as in this case the refractive index depends only on the density. However, interferometry can also be used to obtain the (average) magnetic field strength along the line of sight, by measurements of the Faraday rotation of the plane of polarization.

3. The electron temperature and energy distribution are determined by the active technique of Thomson scattering, which additionally enables the density to be measured. A high power visible or near-infrared laser is typically used, the short wavelength, in comparison with the Debye screening length of the plasma, leading to incoherent scattering on individual electrons; the Doppler broadening of the scattered radiation provides a measure of the electron temperature, while the total scattered power is proportional to the electron density. This technique is limited by the low repetition rate of the laser.

The passive technique of detection of the electron cyclotron emission (ECE) in the microwave region, using Michelson, Fabry-Perot or heterodyne detection systems, is also widely employed and provides good spatial and temporal resolution. The electron cyclotron emission arises from the acceleration of the electrons as they gyrate around the magnetic field lines. The emission and subsequent re-absorption of electron cyclotron radiation is generally complex; however, for an optically thick Maxwellian plasma, the detected emission is black-body radiation, being proportional to the electron temperature and independent of the density, thus providing a direct measurement of temperature under these conditions. The variation with major radius of the toroidal magnetic field (and hence of the electron cyclotron frequency) is employed to provide spatial resolution.

High resolution soft X-ray spectroscopy is also employed in assessing the electron temperature, based on the plasma emission arising through various atomic processes. Finally, of mainly historical interest, one can deduce a 'conductivity temperature' from the measured resistance of the plasma (by assuming an ionization state), and a 'diamagnetic temperature' from the small reduction of the toroidal field, arising from the diamagnetic nature of the plasma and being proportional to the plasma energy density.

4. The ion temperature and energy distribution can be obtained from the energy spectra of neutral particles, produced by charge exchange processes in the plasma, which penetrate the magnetic field; this method can be enhanced by injecting a neutral beam into the plasma, in order to obtain greater sensitivity and spatial resolution.

Other techniques include the detection of neutrons of thermonuclear origin, which is strongly sensitive to the presence of a high-energy tail in the ion energy distribution; far-infrared laser scattering (collective scattering on the electrons surrounding the ions); and measurement of the Doppler broadening of emission lines.

5. The plasma impurity content and state of ionization are investigated by measuring the plasma resistivity and comparing this with the expected (neoclassical) resistivity for a pure hydrogenic plasma, in order to deduce an effective ion charge and thereby to obtain a simple averaged measure of the type and concentration level of the impurity ions. Visible, vacuum ultraviolet and X-ray spectroscopic techniques are also widely employed to measure the recombination and line radiation from the impurity ions. Measurements of the $H\alpha$ radiation provide an important measure of the density of neutral particles and hence of the source of charged particles.
6. Fluctuations in plasma density, throughout its volume, are measured by collective electromagnetic wave scattering on density fluctuations. This technique enables frequency and wavenumber spectra to be obtained, subject to certain geometrical limitations on accessibility. Microwave and far-forward infrared scattering can be used. Heavy ion beam probes have also been used to measure density and electric potential fluctuations; in this technique the particles produced along the heavy ion beam are detected with spatial resolution. No satisfactory techniques have been developed so far for measuring, throughout the plasma, fluctuations of other plasma parameters (in particular temperature and electromagnetic fluctuations).
7. MHD instabilities are studied by using the plasma soft X-ray (SXR) emission; good spatial and temporal resolutions can be obtained with systems of SXR cameras comprising many collimated detectors. The signals obtained represent chordal integrals along the lines of

sight of the total SXR emissivity (integrated over a range of photon energies); numerical tomographic reconstruction techniques enable one to compute the local SXR emissivity. The plasma SXR emission, generally arising from bremsstrahlung, and from recombination and line radiation, is sensitive to the electron temperature, density and plasma impurity content. This provides an effective and experimentally simple technique for following temperature changes with good spatial and temporal resolution.

8. The total radiated power over the entire energy spectrum is measured by bolometric detectors. This technique is difficult accurately to calibrate, but nevertheless provides a better measurement of the radiative loss than spectroscopic techniques.
9. Electrostatic and electromagnetic probes are used to investigate several parameters at and near the plasma edge, such as electron density and temperature, electric and magnetic fields, and the fluctuations in these parameters. Electrostatic (Langmuir) probes produce a voltage-current characteristic, from which the plasma density and the electron temperature can be readily inferred. Electromagnetic probes (coils) measure magnetic field changes. Measurements using material probes are, of course, limited by the small penetration of the probes into the bulk of the plasma (the limit on penetration is set by the plasma temperature —the heat would destroy the probe and release impurities into the plasma).

Chapter 2

Particle and Energy Confinement in Tokamaks

2.1 Global confinement times

It has been seen that the energy confinement time is an important parameter in the achievement of a net power output from a thermonuclear plasma. The definition of the global energy confinement time, under conditions of equilibrium, is

$$\tau_E = W/P = \int_V \frac{3}{2}(n_e T_e + n_i T_i) d\tau \bigg/ \int_V Q d\tau, \quad (2.1)$$

where the first integral is the total energy content of the plasma (n and T denote, respectively, the number densities and temperatures of electrons and ions), and the second is the total input power required to sustain it (Q is here the sum of all energy inputs).¹ When the input power is reduced by the measured radiation loss, the resulting energy confinement time is denoted by τ_E^* and provides a measure of the transport losses. One can further define, in a similar way, electron and ion energy confinement times, τ_{Ee} and τ_{Ei} respectively.

¹The general definition of the energy confinement time, covering non-stationary cases, is

$$\tau_E = W/(P - \dot{W}).$$

A particle confinement time is defined, in an analogous manner to the energy confinement time, by

$$\tau_P = \int_V n \, d\tau \bigg/ \int_V S \, d\tau, \quad (2.2)$$

where the first integral is the total number of particles and the second is the particle input rate arising from recycling at the wall and the gas feed processes; as this is difficult accurately to calculate, measurements of the particle confinement time are not as extensive as those of the energy confinement time. The particle confinement time is typically longer than the energy confinement time by up to an order of magnitude, suggesting that the energy losses are mainly due to conduction and radiation rather than convection.²

The mechanisms governing the dependence of the energy and particle confinement times on the various tokamak and plasma parameters, which are of crucial importance in the design of future tokamak reactors, are extremely complex and so far only partially understood.

In a cylindrically symmetric, magnetically confined plasma, particle and energy losses arise from the Coulomb scattering of the charged particles, a process known as *classical transport*. In a tokamak or other toroidal device, the transport is increased (by up to two orders of magnitude in a tokamak) because of the reduced symmetry of the configuration, a process known as *neoclassical transport*; this depends qualitatively on the collisionality of the plasma and on the magnetic geometry (neoclassical transport is greater for non-axisymmetric devices, such as stellarators).

The observed transport in tokamaks is significantly different from that predicted by neoclassical transport theory. It has transpired that both electron thermal and particle transport greatly exceed neoclassical predictions, typically by one or two orders of magnitude, while ion thermal transport is most probably also in excess of such predictions. These higher transport rates are called *anomalous transport* and are still insufficiently understood, despite the considerable experimental and theoretical effort that is currently being expended on these problems. The global energy confinement time is determined mainly by losses through the electron channel, as the anomalous electron thermal transport is faster than the corresponding ion transport, at

²It has been pointed out by Hugill (1983) that the particle confinement time would be shorter than the energy confinement time if the energy transport were convective.

least for low-density, Ohmic-heated plasmas. However, losses through the ion channel become increasingly important and may become dominant as the plasma density is increased (or for low-temperature plasmas) leading to an increasingly strong coupling between the electron and ion energies; this is also the case when the ion component is directly heated and the ion energy approaches or exceeds the electron energy. Transport appears to be of a diffusive nature,³ with particle and heat fluxes determined by density and temperature gradients; however, non-diffusive, inward components (pinches) are present in the particle flux and possibly also in the thermal flux.

Further, non-diffusive energy losses from a tokamak plasma arise from the radiation loss from electrons and the charge exchange loss from ions. The radiation loss is due to bremsstrahlung, and to recombination and line radiation; there is a contribution from the electron cyclotron emission but this is normally insignificant because of re-absorption. The radiation loss increases dramatically with an increasing impurity level. This fact, and the dilution of the working gas ions between which fusion reactions can occur, mean that the issues of impurity content and transport are of considerable importance. The charge exchange loss is small.

Several empirical *scaling laws* have been determined experimentally from data accumulated from several tokamaks, or with the help of a mixture of theoretical arguments and experimental results, in order to describe the dependence of both energy and particle confinement times on various plasma parameters for both Ohmic- and additionally-heated tokamak plasmas. The most commonly accepted scaling of the energy confinement time is a linear one in average density, which saturates at high densities. This saturation can readily be attributed to the increasing interaction between electrons and ions as the density increases, thereby introducing the ion contribution to the thermal transport, and possibly to a nearly neoclassical ion transport, which increases with increasing density, becoming comparable to, and eventually exceeding, the anomalous electron transport, which decreases with density. Another widely observed scaling involves the degradation of the energy confinement with increasing heating power for additionally-heated tokamak plasmas. An operational regime of improved confinement has

³this has been confirmed by perturbation measurements of transport, to be described later

been obtained under certain conditions (see subsection 2.4.1); this has been termed ‘H-mode’, in contrast to the standard, low confinement ‘L-mode’. Global and local experimental observations of tokamak transport, including scaling results and transport modelling, have been reviewed by Hugill (1983).

Amongst the most commonly used L-mode scaling laws are the following (Coppi and Mazzucato, 1979; Merezhkin and Mukhovatov, 1981; Goldston, 1984; Kaye and Goldston, 1985; Kaye, 1985 (review); see also Liewer, 1985; Wesson, 1987), where $\bar{n}$ is the line-averaged electron density, R and a are the major and minor radii respectively, $\langle T \rangle$ is the volume-averaged temperature, P is the total heating power, I is the plasma current, B is the toroidal magnetic field, q_a is the edge safety factor, κ is the elongation for non-circular cross-sections, A is the ion atomic mass and Z_{eff} is the ion effective charge⁴ —the energy confinement in the H-mode appears to be similar to that in the L-mode, but with an enhancement factor of between 2 and 3:

INTOR:
$$\tau_E/s = 5 \times 10^{-2} (\bar{n}/10^{19} \text{m}^{-3}) (a/\text{m})^2;$$

Neo-Alcator:
$$\tau_E/s = 1.9 \times 10^{-2} (\bar{n}/10^{19} \text{m}^{-3}) (a/\text{m})^{1.04} (R/\text{m})^{2.04};$$

Coppi-Mazzucato:
$$\tau_E/s = 3.2 \times 10^{-2} (\bar{n}/10^{19} \text{m}^{-3})^{0.8} (a/\text{m})^2 \\ \times (\langle T \rangle / \text{keV}) (B/\text{T})^{-1} q_a A^{0.5} Z_{\text{eff}}^{-0.2};$$

Merezhkin-Mukhovatov:
$$\tau_E/s = 1.1 \times 10^{-3} (\bar{n}/10^{19} \text{m}^{-3}) (a/\text{m})^{0.25} (R/\text{m})^{2.75} \\ \times (\langle T \rangle / \text{keV})^{-0.5} q_a \kappa^{0.125} A^{0.5};$$

Goldston (L-mode):
$$\tau_E = \left(\tau_{\text{OH}}^{-2} + \tau_{\text{AUX}}^{-2} \right)^{-1/2},$$

with
$$\tau_{\text{OH}}/s = 1.0 \times 10^{-2} (\bar{n}/10^{19} \text{m}^{-3}) (a/\text{m})^{1.04} (R/\text{m})^{2.04} q_a^{0.5},$$

and
$$\tau_{\text{AUX}}/s = 3.8 \times 10^{-2} (a/\text{m})^{-0.37} (R/\text{m})^{1.75} \\ \times (I/\text{MA}) (P/\text{MW})^{-0.5} \kappa^{0.5};$$

Kaye-Goldston:
$$\tau_E/s = 3.04 \times 10^{-2} (a/\text{m})^{-0.49} (R/\text{m})^{1.65} (I/\text{MA})^{1.24} \\ \times (P/\text{MW})^{-0.58} (\bar{n}/10^{19} \text{m}^{-3})^{0.26} (B/\text{T})^{-0.09} \kappa^{0.28} (A/1.5)^{0.5}.$$

Such scaling laws are often extrapolated in attempts to predict the performance of future tokamak reactors, but the uncertainties involved are clearly substantial. These scaling laws are in

⁴these parameters cannot all be independently varied

good agreement for the current experimental parameters, but diverge dramatically for projected parameters.

2.2 Classical transport

2.2.1 Coulomb collisions

Classical transport in a plasma arises from the Coulomb scattering of the charged particles, effectively an electrostatic interaction with infinite range. The *kinetic equation*, which governs the behaviour in phase space of the distribution function of a given plasma species, is used in describing many phenomena with short length- and time-scales, including classical transport:⁵

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{e_s}{m_s} (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\partial f_s}{\partial t} \right)_{\text{coll.}}; \quad (2.3)$$

$f_s(\mathbf{r}, \mathbf{v}, t)$ is the distribution function for species s in the phase space $(\mathbf{r}, \mathbf{v}, t)$, and $\mathbf{E}$ and $\mathbf{B}$ are the *macroscopic* electric and magnetic fields. The right-hand side is the collision term, which includes *microscopic* scale interactions and can be expressed by a Taylor expansion as

$$\left(\frac{\partial f_s}{\partial t} \right)_{\text{coll.}} = -\frac{\partial}{\partial \mathbf{v}_s} \cdot \left(\frac{\langle \Delta \mathbf{v}_s \rangle}{\Delta t} f_s \right) + \frac{1}{2} \frac{\partial^2}{\partial \mathbf{v}_s \partial \mathbf{v}_s} : \left(\frac{\langle \Delta \mathbf{v}_s \Delta \mathbf{v}_s \rangle}{\Delta t} f_s \right), \quad (2.4)$$

where the averages in angular brackets involve integration over all scattering angles and velocities and a summation over all species; this result is the *Fokker-Planck collision term*. With the assumption of Coulomb collisions between the species s and s' , this collision term becomes

$$\left(\frac{\partial f_s}{\partial t} \right)_{\text{coll.}} = \sum_{s'} \frac{Z_s^2 Z_{s'}^2 e^4 \ln \Lambda}{4\pi \epsilon_0^2 m_s^2} \left\{ -\frac{\partial}{\partial \mathbf{v}_s} \cdot \left(\frac{\partial H_{s'}(\mathbf{v}_s)}{\partial \mathbf{v}_s} f_s(\mathbf{v}_s) \right) + \frac{1}{2} \frac{\partial^2}{\partial \mathbf{v}_s \partial \mathbf{v}_s} : \left(\frac{\partial^2 G_{s'}(\mathbf{v}_s)}{\partial \mathbf{v}_s \partial \mathbf{v}_s} f_s(\mathbf{v}_s) \right) \right\}, \quad (2.5)$$

where $H_{s'}(\mathbf{v}_s)$ and $G_{s'}(\mathbf{v}_s)$ are the *Rosenbluth potentials*, given by

$$H_{s'}(\mathbf{v}_s) = \left(1 + \frac{m_s}{m_{s'}} \right) \int \frac{f_{s'}(\mathbf{v})}{|\mathbf{v}_s - \mathbf{v}|} d^3 \mathbf{v} \quad (2.6)$$

$$\text{and} \quad G_{s'}(\mathbf{v}_s) = \int f_{s'}(\mathbf{v}) |\mathbf{v}_s - \mathbf{v}| d^3 \mathbf{v}. \quad (2.7)$$

⁵the equation given here is, strictly, the kinetic equation incorporating an appropriate expression for the particle acceleration due to the macroscopic fields (Lorentz force)

In these results, Z_s is the atomic charge number, and $\ln \Lambda = \ln(\lambda_D/\lambda_{\min})$ is the *Coulomb logarithm* which arises from the integration over the scattering angle or impact parameter, the upper limit on the impact parameter being the Debye screening length λ_D ,⁶ with the lower limit $\lambda_{\min}$ being the quantum mechanical (de Broglie) wavelength.⁷

The Fokker-Planck collision term leads to the establishment of a *Maxwellian distribution* of particle speeds, given by

$$f(v) = \frac{n}{(2\pi T/m)^{3/2}} \exp\left(-\frac{\frac{1}{2}mv^2}{T}\right). \quad (2.8)$$

The characteristic relaxation times of a plasma, that is, the times required for the establishment of equilibrium Maxwellian distributions, are obtained from the collision term by considering a “test particle” with the average thermal velocity in a plasma with a Maxwellian distribution. This procedure leads to an electron-ion collision time

$$\tau_e = 3(2\pi)^{3/2} \frac{\varepsilon_0^2 m_e^{1/2} T_e^{3/2}}{n_i Z^2 e^4 \ln \Lambda_e}, \quad (2.9)$$

and an ion-ion collision time

$$\tau_i = 12\pi^{3/2} \frac{\varepsilon_0^2 m_i^{1/2} T_i^{3/2}}{n_i Z^4 e^4 \ln \Lambda_i}; \quad (2.10)$$

furthermore, the time for heat exchanges between ions and electrons is given by

$$\tau_{ei} = \frac{m_i}{2m_e} \tau_e, \quad (2.11)$$

reflecting the small fraction of the electron energy of the order of (m_e/m_i) transferred to the ions per collision. The ratios of these characteristic times are $\tau_e : \tau_i : \tau_{ei} \sim 1 : (m_i/m_e)^{1/2} : (m_i/m_e)$, so that the possibility arises of the electron and ion components having different temperatures. Concise expositions of Coulomb collisions in a plasma can be found in Cairns (1985), Woods (1987) and Wesson (1987).

⁶the Debye screening length $\lambda_D = (\varepsilon_0 T_e / n_e e^2)^{1/2}$ is the distance over which separation of charges can occur or the effective range of Coulomb scattering

⁷the quantum mechanical wavelength must be replaced by the impact parameter for a 90° scatter, when the latter is larger than the former (under tokamak conditions, this is the case for ions but not for electrons)

2.2.2 Fluid equations

A distribution function can be described in terms of its velocity moments; the most important of these are the density, mean velocity and pressure,⁸ which can provide a complete description of the species considered, if it follows a Maxwellian distribution. One can consequently obtain velocity moments of the kinetic equation that describe, in terms of such macroscopic variables, the transport of particles, momentum and energy. These are applicable, provided that the time-scales of the phenomena under consideration be sufficiently long compared with the collision times, and that their length-scales be sufficiently large compared with the mean free paths, or to the Larmor radii for the directions transverse to the magnetic field. These results are the single species fluid equations for each component of the plasma. The results given here have been obtained on the assumption that the collisional operator conserves particles *separately* (i.e. there is no ionization or recombination), total momentum and total kinetic energy. The particle transport equation is

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_s \cdot \nabla \right) n_s + n_s \nabla \cdot \mathbf{u}_s = 0, \quad (2.12)$$

where n_s is the density and $\mathbf{u}_s$ is the mean velocity. The momentum transport equation is

$$n_s m_s \left(\frac{\partial}{\partial t} + \mathbf{u}_s \cdot \nabla \right) \mathbf{u}_s = -\nabla \cdot \mathbf{p}_s + n_s e_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \mathbf{R}_s, \quad (2.13)$$

where $\mathbf{p}_s$ is the momentum flux tensor (pressure tensor) and $\mathbf{R}_s$ is the rate of transfer of momentum from the other species. The energy transport equation is

$$\frac{3}{2} n_s \left(\frac{\partial}{\partial t} + \mathbf{u}_s \cdot \nabla \right) T_s = -\nabla \cdot \mathbf{q}_s - \mathbf{p}_s : \nabla \mathbf{u}_s + Q_s, \quad (2.14)$$

⁸The definitions of these velocity moments are as follows:

density	$n = \int f d^3v,$
mean velocity	$\mathbf{u} = \frac{1}{n} \int \mathbf{v} f d^3v,$
pressure tensor	$\mathbf{p} = \int m(\mathbf{v} - \mathbf{u})(\mathbf{v} - \mathbf{u}) f d^3v.$

where T , is the temperature,⁹ $\mathbf{q}$, is the heat flux vector and Q , is the rate of energy transfer, which in general incorporates equipartition, heating and radiation terms.¹⁰ The temperature is defined in terms of the diagonal components of the pressure tensor, by $T = \frac{1}{3} \text{tr } \mathbf{p} / n$. When the distribution is Maxwellian and isotropic, the temperature defined in this way is the same as the characteristic temperature of the distribution. For anisotropic distributions, one can define parallel and perpendicular temperatures.

When the phenomena under consideration are sufficiently slow, so that the spatial separation of electrons and ions can be ignored, the plasma can be treated as a single fluid, this being the magnetohydrodynamic (MHD) description of the plasma. The appropriate equations, obtained from the multiple fluid equations above, are the equation of continuity

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \rho + \rho \nabla \cdot \mathbf{u} = 0, \quad (2.15)$$

and the equation of motion

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla \cdot \mathbf{p} + \mathbf{j} \times \mathbf{B}, \quad (2.16)$$

where ρ is the mass density, $\mathbf{u}$ is the bulk fluid velocity, $\mathbf{p}$ is the pressure tensor ($\nabla \cdot \mathbf{p}$ can be replaced by the gradient of the isotropic pressure, ∇p , for small viscosity), $\mathbf{j}$ is the plasma current density and $\mathbf{B}$ is the magnetic field. One also uses the adiabatic law (i.e. conservation of entropy),

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) (p \rho^{-\gamma}) = 0, \quad (2.17)$$

where γ is the adiabatic constant. In the MHD description of the plasma, these single-fluid

⁹The usual convention in plasma physics of expressing temperature in energy units, i.e. incorporating the Boltzmann constant in $k_B T$, is followed throughout this thesis. The temperature is defined in terms of the kinetic energy density by

$$\frac{3}{2} n T = \int \frac{1}{2} m (\mathbf{v} - \mathbf{u})^2 f d^3 v = \frac{1}{2} \text{tr } \mathbf{p}.$$

¹⁰that is, terms that describe interactions with the other species and the electromagnetic field

equations are complemented by Maxwell's equations¹¹

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}, \quad (2.18)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \quad (2.19)$$

and a simplified expression of Ohm's law

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \mathbf{E}^* = \eta_{\parallel} \mathbf{j}_{\parallel} + \eta_{\perp} \mathbf{j}_{\perp}, \quad (2.20)$$

where $\mathbf{E}^*$ is the local electric field in a co-ordinate system moving with the fluid velocity $\mathbf{u}$; the assumption $\eta = 0$ is made in the ideal or non-resistive MHD description of the plasma.

2.2.3 Braginskii equations

The set of moment equations is not closed, as each equation involves higher-order moments. The Chapman-Enskog method may be used to consider a perturbation to an equilibrium Maxwellian distribution, arising from density and temperature gradients, and hence to deduce the so-called *transport coefficients* which relate the moments of the distribution function to "thermodynamic forces". For example, the heat conduction tensor links the heat flux vector to the temperature gradient; the viscosity tensor links the pressure tensor to the velocity gradient; the diffusion tensor links the particle flux vector to the density gradient; and the conductivity tensor links the current density to the electric field. The transport coefficients that pertain to a magnetically confined plasma are clearly anisotropic, as the magnetic field tends to inhibit motions of charged particles in a perpendicular direction but not in the parallel direction.

The classical transport coefficients have been calculated by Braginskii from the Fokker-Planck collision operator (Braginskii, 1965 —the standard work on classical transport theory), and are briefly reproduced here, in a slightly simplified form that is appropriate to a magnetically confined plasma for which the cyclotron frequency is much greater than the collision frequency, i.e. $\omega_c \tau \gg 1$, for a plasma of electrons and singly charged ions ($n=n_e=n_i$). The classical resistivity, first calculated by Spitzer, is

$$\eta_{\parallel} = 0.51 \frac{m_e}{n e^2 \tau_e} = 0.51 \frac{m_e^{1/2} e^2 \ln \Lambda_e}{3(2\pi)^{3/2} \epsilon_0^2 T_e^{3/2}} \quad (2.21)$$

¹¹the displacement current in (2.18) is often ignored for slow phenomena

$$\text{and} \quad \eta_{\perp} = \frac{m_e}{ne^2\tau_e} = 1.98\eta_{\parallel}. \quad (2.22)$$

It is seen that the resistivity is very weakly dependent on density (through $\ln \Lambda_e$) and is proportional to $T_e^{-3/2}$, decreasing as the temperature increases; it is also clear that most of the current is carried by the electrons because of their higher mobility, so that only the electrons are directly heated in an Ohmic-heated plasma.

The rate of transfer of momentum from ions to electrons is

$$\mathbf{R} = \mathbf{R}_u + \mathbf{R}_T, \quad (2.23)$$

where the frictional transfer rate is

$$\mathbf{R}_u = -\frac{m_e n}{\tau_e} (0.51 \mathbf{u}_{\parallel} + \mathbf{u}_{\perp}) = ne(\eta_{\parallel} \mathbf{j}_{\parallel} + \eta_{\perp} \mathbf{j}_{\perp}), \quad (2.24)$$

and the thermal transfer rate is

$$\mathbf{R}_T = -n \left(0.71 \nabla_{\parallel} T_e + \frac{3}{2} \frac{1}{|\omega_{ce}| \tau_e} \hat{\mathbf{b}} \times \nabla T_e \right); \quad (2.25)$$

$\hat{\mathbf{b}}$ is the magnetic field direction unit vector; and $\mathbf{u} = \mathbf{u}_e - \mathbf{u}_i$.

The electron heat flux vector is

$$\mathbf{q}^e = \mathbf{q}_u^e + \mathbf{q}_T^e, \quad (2.26)$$

$$\text{with} \quad \mathbf{q}_u^e = nT_e \left(0.71 \mathbf{u}_{\parallel} + \frac{3}{2} \frac{1}{|\omega_{ce}| \tau_e} \hat{\mathbf{b}} \times \mathbf{u} \right), \quad (2.27)$$

$$\text{and} \quad \mathbf{q}_T^e = -nT_e \frac{\tau_e}{m_e} \left(3.16 \nabla_{\parallel} T_e + 4.66 \frac{1}{\omega_{ce}^2 \tau_e^2} \nabla_{\perp} T_e + \frac{5}{2} \frac{1}{|\omega_{ce}| \tau_e} \hat{\mathbf{b}} \times \nabla T_e \right); \quad (2.28)$$

the ion heat flux vector is

$$\mathbf{q}^i = -nT_i \frac{\tau_i}{m_i} \left(3.91 \nabla_{\parallel} T_i + 2 \frac{1}{\omega_{ci}^2 \tau_i^2} \nabla_{\perp} T_i - \frac{5}{2} \frac{1}{\omega_{ci} \tau_i} \hat{\mathbf{b}} \times \nabla T_i \right). \quad (2.29)$$

The heat exchange rate from electrons to ions is

$$Q_i = \frac{3m_e}{m_i} \frac{n}{\tau_e} (T_e - T_i) = \tau_{ei}^{-1} \frac{3}{2} (n_e T_e - n_i T_i); \quad (2.30)$$

the heat transfer rate to electrons is

$$Q_e = -\mathbf{R} \cdot \mathbf{u} - Q_i = \eta_{\parallel} j_{\parallel}^2 + \eta_{\perp} j_{\perp}^2 + \frac{1}{ne} \mathbf{j} \cdot \mathbf{R}_T - \frac{3m_e}{m_i} \frac{n}{\tau_e} (T_e - T_i), \quad (2.31)$$

where the first term in the right-hand side represents the Ohmic heating.

One notes that the classical diffusivities for conduction *across* the magnetic field lines are of the form

$$D_{\perp}^{cl} \sim \frac{1}{\tau} \frac{v_T^2}{\omega_c^2} = \frac{\rho^2}{\tau}, \quad (2.32)$$

where $v_T = (2T/m)^{1/2}$ is the perpendicular thermal velocity of a particle and $\rho = v_T/\omega_c$ is the Larmor radius of its particle orbit. This form can be obtained from an argument based on the random walk of a particle, with a step length equal to the Larmor radius ρ and a characteristic time given by the collision time τ .

2.3 Neoclassical transport

2.3.1 Basic considerations

In a toroidal confinement device, energy and particle transport depends mainly on the effects of toroidal geometry, being substantially faster than that obtained in cylindrical geometry. In a fluid description of the plasma, a combination of the equation of equilibrium

$$\mathbf{j} \times \mathbf{B} = \nabla p$$

with the perpendicular component of Ohm's law

$$\mathbf{E}_{\perp} + \mathbf{u}_{\perp} \times \mathbf{B} = \eta_{\perp} \mathbf{j}_{\perp},$$

leads to the plasma flow velocity across the magnetic field

$$\mathbf{u}_{\perp} = -\eta_{\perp} \frac{\nabla p}{B^2} + \frac{\mathbf{E}_{\perp} \times \mathbf{B}}{B^2}, \quad (2.33)$$

where the first term represents classical transport with a diffusivity given by

$$D_{\perp} \sim \frac{\eta_{\perp} n T}{B^2} = \frac{1}{\tau} \frac{v_T^2}{\omega_c^2} = \frac{\rho^2}{\tau},$$

whilst the second term corresponds to the neoclassical enhancement due to the $\mathbf{E} \times \mathbf{B}$ drift caused by the poloidal electric field. In a particle description of the plasma, the neoclassical enhancement arises from the increased step-lengths of the particles in toroidal geometry.

Some further features of neoclassical transport theory include (a) the coupling of particle and energy diffusion (the particle and energy fluxes being driven by both density and temperature gradients), (b) an inward particle pinch velocity, called Ware pinch, and (c) a so-called bootstrap current driven by the density gradient. These effects have to some extent been identified in experimental results, although pinch velocities deduced from experiments are often much greater than the neoclassical Ware pinch. Furthermore, the electrical conductivity of the plasma is modified from its classical (Spitzer) value in neoclassical theory, because of the existence of trapped particles, which do not carry any parallel current, and the collisional friction between passing and trapped particles; this important neoclassical enhancement of resistivity has been experimentally confirmed. Standard treatments of neoclassical transport theory are those by Hinton and Hazeltine (1976), Galeev and Sagdeev (1979), and Hirshman and Sigmar (1981); introductory accounts are given by Laing (1981) and Wesson (1987).

2.3.2 Neoclassical transport regimes

The degree of collisionality, which decreases with increasing temperature, is an important factor in neoclassical transport theory. Another important factor in the case of weak collisionality is the presence of trapped particles which follow orbits characterized by greater excursions from the flux surfaces (see subsection 1.4.3). One identifies three transport regimes, based on the collision frequency $\nu = \tau^{-1}$, and on the local magnetic field geometry, as described by the safety factor $q = (B_\phi/B_\theta)(r/R)$ and the inverse aspect ratio $\epsilon = r/R$:

1. In the *collisional* (Pfirsch-Schlüter) regime, a fluid description is valid, as the mean free path of a particle is shorter than the connection length, the latter being the distance along the field line from the inside to the outside of the torus; i.e. $v_T \tau < Rq$, or

$$\frac{\nu Rq}{v_T} > 1. \quad (2.34)$$

The neoclassical enhancement to the particle transport arises from $\mathbf{E} \times \mathbf{B}$ drifts averaged over flux surfaces. In the case of energy transport, the enhancement is due to the temperature variation in a magnetic surface, arising from the finite Larmor radii. The enhancements of both particle and energy transport are of the order of q^2 , typically representing

one order of magnitude above classical transport, i.e.

$$D_{\perp}^n \sim q^2 \left(\nu \frac{v_T^2}{\omega_c^2} \right). \quad (2.35)$$

2. In the *collisionless* (“banana”) regime, particles that are trapped in the outer region of weak magnetic field, describing orbits with banana-shaped projections in a minor cross-section, complete such orbits before being detrapped by Coulomb collisions; i.e. $\omega_b > \nu_{dt}$ (see subsection 1.4.3), or

$$\frac{\nu R q}{v_T} < \varepsilon^{3/2}. \quad (2.36)$$

Both trapped and passing particles play a role in transport. A kinetic approach is necessary in this regime, which can be based on gyro-kinetic equations —these are averages of the kinetic equations over the rapidly varying particle gyrophase and describe the behaviour of the guiding centres of the particle orbits. The enhancement arising from the trapped particles may be obtained from a random-walk calculation, based on the width of the banana orbit Δr and the detrapping frequency ν_{dt} , and taking the fraction $\sqrt{2\varepsilon}$ of trapped particles into account (see, e.g., Kadomtsev and Pogutse, 1971):

$$D_{\perp}^n \sim \varepsilon^{1/2} \nu_{dt} \Delta r^2 = \varepsilon^{-3/2} q^2 \left(\nu \frac{v_T^2}{\omega_c^2} \right); \quad (2.37)$$

the additional enhancement over Pfirsch-Schlüter transport is typically of one order of magnitude.

3. In the *intermediate* (“plateau”) regime, the collisionality is between those of the collisional and collisionless regimes, i.e.

$$1 > \frac{\nu R q}{v_T} > \varepsilon^{3/2}. \quad (2.38)$$

Passing particles can complete transits before being scattered, but trapped particles cannot. The transport is dominated by particles with small parallel velocities, such that they complete a single transit while undergoing a small-angle scattering. The diffusivity in this regime is independent of collisionality and matches those of the other two regimes at their respective limits, i.e.

$$D_{\perp}^n \sim \frac{q v_T}{R} \frac{v_T^2}{\omega_c^2}. \quad (2.39)$$

Numerical expressions for the particle and energy fluxes are given by, for instance, Hinton and Hazeltine (1976). In all neoclassical transport regimes, the ion energy transport is greater than the electron energy transport by a factor $(m_i/m_e)^{1/2}$ (as seen from the mass dependence in (2.35), (2.37) and (2.39)), as is the case in classical transport, and the particle transport is determined by the slower electron transport. Like classical transport, neoclassical transport is expected to increase linearly with density, through the collisional frequency being proportional to density. The neoclassical enhancement of transport for non-axisymmetric toroidal devices, such as stellarators, is much greater than that for tokamaks and often sufficient to account for the observed transport.

A further geometrical enhancement of transport in tokamaks is that due to the ripple effect of the toroidal field which is generated by discrete toroidal coils. This ripple, which is superimposed on the $1/R$ variation of the field, leads to modifications of particle orbits in the low-collisionality regimes and to the existence of a new class of toroidally trapped particles (with very small parallel velocities) that become localized in the regions between toroidal field coils. These effects are significant, even for very small ripple variations. The effect of the helical field ripple is responsible for the particularly large neoclassical transport in stellarators.

2.4 Anomalous transport

2.4.1 Experimental observations

At an early stage of tokamak experimentation, it transpired that neoclassical theory is inadequate in explaining the observed confinement: in particular, the electron transport is grossly underestimated by neoclassical theory, typically by one or two orders of magnitude, while it has also become evident that the ion transport significantly deviates from neoclassical predictions. Furthermore, the observed transport decreases with an increasing plasma density, instead of increasing as would be predicted by neoclassical theory. Moreover, the scaling of the confinement time with heating power, as deduced from experiments with additional heating, also differs appreciably from that predicted by neoclassical theory. Both particle and thermal diffusivities increase markedly towards the plasma periphery. The observed energy and particle losses that

cannot be attributed to classical and neoclassical effects are termed *anomalous losses*. Anomalous electron thermal conduction is the dominant energy loss mechanism, but anomalous ion thermal conduction and particle transport are also important. In addition to this enhanced transport, all measurements of fluctuations of plasma parameters that have been carried out on various tokamaks reveal the existence of *microturbulence*, that is, random fluctuations of plasma parameters, characterized by spatial scales much smaller than the plasma minor radius, and by incoherent spectra. It is widely, albeit not universally, thought that this microturbulence is responsible for the observed anomalous transport. There exists a growing bulk of theories that purport to predict the nature of these fluctuations and to explain their role in anomalous transport. Whilst many theoretical predictions of anomalous transport are consistent with some of the experimental observations of microturbulence and transport, no theory has so far succeeded in satisfactorily describing anomalous transport in tokamaks over a wide range of parameters. It seems likely that the microturbulence is due to a large number of different instabilities, depending critically on the plasma parameters, and that the observed transport is their total effect. The overall energy confinement time further includes the effects of radiation and charge exchange losses. One would therefore expect empirical scalings of the global confinement in tokamaks, in terms of various globally averaged parameters, to be inadequate as a means of understanding transport. It is clearly of great importance fully to understand the role of turbulence in anomalous transport. First, this will enable the performance of proposed tokamak experiments and of future tokamak reactors to be assessed with a reasonable degree of certainty. Second, it will indicate how the microturbulence and the resulting transport losses might be reduced, for example by suitable control of the radial profiles, which can be accomplished by localized additional heating and/or plasma fuelling by pellet injection. Experimental measurements of microturbulence and anomalous transport in tokamaks and corresponding theoretical models have been reviewed by Liewer (1985).

Two notable improvements in confinement have been observed experimentally under certain conditions. First, a new operating regime of improved confinement, the so-called *H-mode*, was discovered on the ASDEX tokamak in 1982 (Wagner et al., 1982), the confinement of

additionally-heated plasma having been observed to be as good as that of the Ohmic-heated plasma, despite its degradation with the application of additional heating power, as normally observed in the usual, low-confinement *L-mode*. In the H-mode, a small increase of the heating power seems to produce a much larger increase in the total plasma energy, leading to a longer confinement time and a higher beta value. This high confinement mode seems to be obtainable by employing a divertor configuration or the so-called X-point operation, both approaches introducing magnetic field null points. It has been successfully reproduced on larger experiments with substantially better confinement than the low-confinement L-mode (typically by a factor of between 2 and 3), thus recovering the confinement time of the Ohmic heating phase, albeit still with an unfavourable scaling with heating power. The H-mode is characterized by flatter density and temperature profiles with steep gradients near the edge, and is accompanied by a large reduction in the fluctuation activity near the edge. It is not yet clear whether it is due mainly to edge effects or to changes in the nature of the transport processes, or whether it is due to a reduction in the electron or ion thermal transport.

Second, significantly improved confinement was observed on the Alcator-C tokamak in 1983 (Greenwald et al., 1984) when pellet injection was used to fuel high-density plasmas. Pellets of solid hydrogen were injected into the plasma using a pneumatic device; these can penetrate the plasma almost reaching the magnetic axis before they are ionized, thereby leading to more peaked density profiles than those obtained with the usual fuelling technique of gas feed at the edge. This effect appears to be due to a reduction in the ion thermal transport. Again, this improvement in confinement has been reproduced on larger experiments.

2.4.2 Turbulent fluctuations and transport

Fluctuations of the poloidal electric and radial magnetic fields, associated with electrostatic and electromagnetic turbulence, will both produce a fluctuating radial velocity which can be written as

$$\tilde{v}_r \simeq \frac{\tilde{E}_\theta}{B} + v_{||} \frac{\tilde{B}_r}{B}, \quad (2.40)$$

where the first term is the $\tilde{\mathbf{E}} \times \mathbf{B}$ drift due to the fluctuating poloidal electric field, and the second is a radial component of the parallel particle velocity produced by the fluctuating radial magnetic field. Clearly, such a fluctuation in just one plasma parameter would not produce any net transport; the transport depends not just on a single fluctuation level, but rather on the correlations between various fluctuations, being a second or higher order effect. The turbulent particle flux, for example, arises from the correlation between fluctuations in density and radial velocity:

$$\Gamma^t = \langle \tilde{n} \tilde{v}_r \rangle = \frac{\langle \tilde{n} \tilde{E}_\theta \rangle}{B} - \frac{\langle \tilde{j}_\parallel \tilde{B}_r \rangle}{eB}, \quad (2.41)$$

where the angular brackets denote an average over all frequencies and wavenumbers of the fluctuation. The turbulent thermal flux due to an electrostatic fluctuation, including both conduction and convection, can be shown to be (Ross, 1988)

$$q^t = \frac{3}{2} \frac{\langle \tilde{p} \tilde{E}_\theta \rangle}{B}. \quad (2.42)$$

In the presence of magnetic field fluctuations, a substantial increase in the turbulent thermal flux is expected, because a component of the large parallel conductivity in a collisional plasma, or of the large parallel velocity in a collisionless plasma, will be directed radially.

Only density fluctuations have so far been measured experimentally throughout the plasma volume, including the hot region away from the edge which controls the energy confinement; such fluctuations are measured using microwave or far-forward infrared scattering techniques, employing collective scattering (Equipe TFR, 1978; Liewer, 1985; Hutchinson, 1987). Techniques relying on heavy ion beam probes have more recently provided measurements of density and electric potential fluctuations in the core (Hallock et al., 1986). Measurements of fluctuations in density, temperature, and electric and magnetic fields near the edge of the plasma have also been carried out using material probes, but fluctuations in ion temperature have still not been measured. One can readily compute the auto-correlations and cross-correlations of these edge fluctuations, thus obtaining estimates of anomalous particle and energy transport. Such estimates can then be compared with the measured transport near the edge.

Fluctuation measurements from several tokamaks display a similarity in features, which suggests that the nature of microturbulence is fairly universal. In general, microturbulence in

tokamaks is characterized by broadband frequency spectra. It therefore appears that the fluctuation modes are incoherent (with $\Delta\omega \sim \omega$), as a result of strong coupling. Macroscopic MHD modes, on the other hand, are coherent and may be seen superimposed as a line spectrum on the broadband spectrum of the microturbulence; there is no correlation between the amplitude of these modes and the observed transport. However, coherent modes have been seen at much higher frequencies than those of the MHD modes, in H-mode confinement. The wavenumber spectra are also broad, as are the frequency spectra, with most of the spectral energy concentrated in the region $k_{\perp}\rho_s < 1$,¹² where $\rho_s = c_s/\omega_{ce}$.¹³ Density fluctuation levels increase towards the edge, often reaching 100%. Although they are independent of the density for Ohmic-heated plasmas, they do increase as expected with an increasing additional heating power. The radial and poloidal (perpendicular) wavenumbers are similar and much greater than the toroidal (parallel) wavenumbers. Magnetic field fluctuation levels near the edge are from 10^{-5} to 10^{-4} , apparently increasing farther into the bulk of the plasma. Fluctuation levels, especially edge magnetic field fluctuations, have been observed to decrease dramatically at the onset of H-mode confinement.

Finally, some positive results have been obtained by correlating fluctuations in the edge region and comparing these with the observed transport (see, e.g., Liewer et al., 1986), suggesting that the transport can be attributed and related to the fluctuations. The strong electrostatic turbulence at the edge appears to be responsible for the particle flux, but fails sufficiently to account for the thermal flux. Magnetic fluctuations, on the other hand, appear to be too small to contribute to the observed transport. Nevertheless, their amplitude being a fraction β of that of the density fluctuations, they become much more important at the high beta values achieved with additional heating, in which case they often lead to better agreement with the measured thermal transport. Magnetic fluctuations may also be a significant cause of transport within the plasma core. It should be noted that, if electrostatic turbulence be dominant in determining both particle and energy transport, the particle and thermal diffusivities should be comparable. In practice,

¹²the approximation $k_{\theta}\rho_s \sim k_r\rho_s \sim 0.3$ is often valid for the mean perpendicular wavenumbers

¹³ $c_s = (T_e/m_i)^{1/2}$ is the sound speed

the measured thermal diffusivities are typically higher than the particle diffusivities (by almost one order of magnitude; see, e.g., Gondhalekar et al., 1989), and this favours electromagnetic rather than electrostatic turbulence as the mechanism responsible for thermal transport in the confinement region of the plasma. It is also possible that stationary distortions, rather than fluctuations, of the magnetic field produce the observed thermal transport.

2.4.3 Anomalous transport models

A large body of anomalous transport theories has been developing since the first experimental observations of transport and microturbulence. There are three stages in all theories attempting to explain microturbulence and the resulting anomalous transport: first, an instability mechanism is postulated (this may be linear or non-linear); second, a non-linear saturation mechanism, consistent with the strong coupling that is expected, is invoked to predict the fluctuation levels and their frequency and wavenumber spectra; third, the resulting transport is calculated from the correlations of the fluctuations. The sources of free energy that can drive various instabilities are the gradients in density, temperature, current density and magnetic fields; such gradients are inevitably present in a confined plasma. The saturation is through various energy dissipation mechanisms and non-linear coupling between the different modes that can arise. The transport follows from the statistical properties of the fluctuations. Non-localized coupling between modes can lead to non-diffusive transport. The resulting transport, in conjunction with the sources and sinks of particles and energy, in turn determines the profiles through the energy and particle balance; closure is thus obtained. An important aspect of anomalous confinement is that of *profile consistency*, first introduced by Coppi (1980): the observed resilience of the temperature profile, outside the $q=1$ radius, against changes in the additional heating, suggests that a transport mechanism imposes a strong constraint on this profile.

The microinstabilities that are considered to be most important in determining anomalous transport, because of their more universal occurrence, are generally low frequency modes, up to 1MHz, with $\omega \ll \omega_{ci}$. Broadly, there exist two types of turbulence, involving electrostatic and electromagnetic fluctuations. The well-known ion acoustic modes with $\omega = \pm k_{\parallel} c_s$, and the

shear Alfvén modes with $\omega = \pm k_{\parallel} c_A$,¹⁴ are modified for an inhomogeneous plasma (one in which gradients exist) and become, respectively, the electron and ion drift waves, and the drift Alfvén waves. The condition for the electron drift wave instability, related to the density gradient, is

$$\omega_{*e} = k_{\perp} v_{de} = k_{\perp} \frac{T_e}{e B L_n} \gtrsim \omega, \quad (2.43)$$

where ω_{*e} is the electron diamagnetic drift frequency, v_{de} is the diamagnetic drift velocity and $L_n = |\nabla \ln n|^{-1}$ is the density scale length. Thus, there are electron drift wave instabilities (collisional, collisionless and trapped); ion drift waves; trapped-ion modes; and electromagnetic drift-Alfvén waves. Instabilities associated with electrostatic fluctuations will enhance the transport through $\tilde{\mathbf{E}} \times \mathbf{B}$ drifts. On the other hand, electromagnetic fluctuations can enhance the transport by the interaction of the radial magnetic field fluctuations with the fast parallel transport. Furthermore, tearing or micro-tearing modes can cause interactions between magnetic islands, thus leading to magnetic field ergodicity and turbulent transport.

A large number of instabilities leading to microturbulence have been proposed and pursued theoretically. Some agreement has often been claimed with experimental results, namely measurements of fluctuation levels and spectra on the one hand, and measurements of global confinement scaling and local transport on the other hand. A further experimental test is provided by the transition from L- to H-mode confinement. Unfortunately, the experimental evidence available does not point unambiguously towards a single one of these models, and experimental results are often mutually inconsistent. The density fluctuations in the plasma bulk, which have been observed to propagate in the electron diamagnetic direction, appear to be electron drift waves, and good agreement with global energy confinement has often been demonstrated (see, e.g., Romanelli, Tang and White, 1986). The onset of fluctuations propagating in the ion diamagnetic direction, when a certain condition for the ion-gradient (η_i -) mode is satisfied,¹⁵ provides experimental support for this mode (see, e.g., Lee and Diamond, 1986).

¹⁴ $c_s = (T_e/m_i)^{1/2}$ and $c_A = B/(\mu_0 m_i n)^{1/2}$ are the sound and Alfvén speeds respectively (see, e.g., Cairns, 1985)

¹⁵The relevant instability condition is

$$\eta_i = \frac{\partial \ln T_i}{\partial \ln n_i} > \eta_c$$

There is still no explanation for the increase of the particle and thermal diffusivities towards the edge, although such an explanation may be sought in neoclassical-like effects connected with the changing magnetic field geometry. Many more questions regarding experimental observations remain unresolved. A treatment of microinstabilities has been presented by Tang (1978). The reader is also referred to the comparative review by Liewer (1985) and the experimental accounts by Robinson (1986 and 1989) and Wootton et al. (1988); possible connections between transport models and scaling laws have been pursued by Romanelli, Tang and White (1986); Tang (1986); and Romanelli (1989).

There are several instability mechanisms depending on the mode, and the amplitudes grow until the modes become non-linear and strongly coupled. Although several techniques have been used in the past to obtain the saturation levels, such as quasi-linear and weak coupling theories, in view of the strongly turbulent (incoherent) nature of the fluctuations that are observed in practice, only two approaches can provide one with realistic solutions, namely renormalization theories and numerical solutions. *Renormalization* theories concentrate on a macroscopic description of the plasma turbulence, attempting to calculate various statistical averages. There are two classes of renormalization theories: in one-point renormalization, coherent normal modes are considered ($\Delta\omega \ll \omega$), but in two-point renormalization, the incoherent parts of the fluctuations are also taken into account, leading to experimentally relevant results. Such theories tend to be extremely complex and yet still inadequate, as they typically concentrate on only some of the possible instability and saturation mechanisms that can lead to turbulence. Finally, one can calculate the anomalous transport resulting from the *measured* electrostatic or electromagnetic fluctuations, for comparisons with the corresponding measured transport. In particular, the problem of calculating the thermal transport resulting from stochastic magnetic fields has received considerable attention.

Taking a simplified analysis of an electron drift wave mode as an example of the derivation of anomalous transport coefficients from turbulent fluctuations, one starts from the electron

—as η_i decreases when the density profile is made more peaked, thereby enhancing stability, the η_i -mode may provide an explanation for the aforementioned improvement in energy confinement with pellet fuelling.

density fluctuation associated with the perturbed Maxwellian distribution of a typical electron drift wave instability:

$$\tilde{n}_k = n \frac{e\tilde{\phi}_k}{T_e} \left(1 - i \frac{\gamma_k}{\omega_{*e}} \right), \quad (2.44)$$

where γ_k is the linear growth rate, with the subscript k representing a Fourier component, and $\omega_{*e} = k_\perp v_{de}$, v_{de} being the electron diamagnetic drift velocity. This perturbation will lead to an anomalous particle flux

$$\Gamma^t = \langle \tilde{n} \tilde{v}_r \rangle = \frac{\langle \tilde{n} \tilde{E}_\theta \rangle}{B} = \frac{\langle \tilde{n}_k i k_\perp \tilde{\phi}_k \rangle_k}{B}, \quad (2.45)$$

using (2.44) to express $\tilde{n}_k$ in terms of $\tilde{\phi}_k$, this can be written as

$$\Gamma^t = n \frac{T_e}{e B v_{de}} \left\langle \gamma_k \left(\frac{e\tilde{\phi}_k}{T_e} \right)^2 \right\rangle_k = n L_n \left\langle \gamma_k \left(\frac{e\tilde{\phi}_k}{T_e} \right)^2 \right\rangle_k. \quad (2.46)$$

This result may be used to compare the measured fluxes with the experimentally determined values of the fluctuation levels. A heuristic estimate of the fluctuation level may be arrived at, by postulating that the saturation occurs when the perturbed density gradient balances the equilibrium gradient that drives the instability (producing a local flattening), i.e. $k_\perp \tilde{n} \sim n/L_n$; including the Boltzmann relation $\tilde{n}/n \sim e\tilde{\phi}/T$, this leads to

$$\frac{e\tilde{\phi}}{T} \sim \frac{\tilde{n}}{n} \sim \frac{1}{k_\perp L_n}. \quad (2.47)$$

At this fluctuation level, the so-called *mixing length level*, the diamagnetic drift velocity v_{de} is equal to the $\tilde{\mathbf{E}} \times \mathbf{B}$ drift velocity. The result for the density fluctuation level (also known as the Kadomtsev limit) is usually in good agreement with experimental observations, but the results concerning the applicability or otherwise of the Boltzmann relation are more ambiguous. From (2.46) and (2.47), one obtains an anomalous diffusivity from

$$D_\perp^t = \Gamma^t / (n/L_n) \sim \frac{\gamma_k}{k_\perp^2} \Big|_{\max}. \quad (2.48)$$

This result for the diffusivity is of the random-walk type, with a step length $k_\perp^{-1}$ and a characteristic time γ_k^{-1} .

In the context of models for turbulent transport, it should be mentioned that Woods has lately proposed an alternative theory of anomalous transport, which does not rely on micro-turbulence to predict the observed confinement, but rather on an alternative, 'second-order'

development of classical transport. In the conventional approach to classical transport, criticized by Woods, one ignores the second and higher orders of ϵ , the ratio of the collisional time-scale to the macroscopic time-scale, whereas in this development, the second-order terms are retained and transpire to be dominant in tokamak transport (the theory is expounded in the book *Principles of Magnetoplasma Dynamics* by Woods, 1987). This theory, applied to tokamak transport, has enjoyed considerable success in explaining certain aspects of the observed transport, in particular the scaling of the energy confinement time, and may reflect an important aspect of the underlying transport mechanisms. Nevertheless, this fact does not diminish the strong experimental evidence for the presence of microturbulence and its most probable role in transport.

2.5 Modelling of transport

Introduction Particle and energy transport in a tokamak is clearly a most important factor underlying its performance as a plasma confinement system, the other important factors being radiation¹⁶ and MHD behaviour. However, it has been seen that the various mechanisms involved are still rather poorly understood. Considerable experimental effort and theoretical analysis are currently being undertaken in order to unravel the exact nature of transport. Computer codes for the modelling of transport are a most important tool in comparing theoretical results against experimental observations, as well as in assessing the performance of proposed tokamak systems.

Transport codes are developed around the particle and energy balance equations, incorporating suitable fluxes, sources and sinks of particles and energy, solved with appropriate boundary conditions. MHD equilibrium solutions, typically evolving on slower time-scales than those of the particle and energy transport, are often included. As the radial transport perpendicular to the magnetic field is of greatest practical importance and is expected to be much slower than transport in the toroidal and poloidal directions, one-dimensional radial transport codes are usu-

¹⁶radiation is not normally regarded as a transport mechanism in the context of a tokamak plasma, because re-absorption is insignificant

ally employed, often in conjunction with two-dimensional calculations of the MHD equilibrium solutions.

Tokamak diagnostic techniques typically measure only the radial and temporal variation of the parameters of interest, the assumption of toroidal and poloidal symmetries being implicit. It has been seen that the equilibrium pressure is constant on a flux surface and is a function of a radial variable; in general, one uses the poloidal flux variable ψ , this approach being adequate for non-circular as well as for circular minor cross-sections. It is often reasonable to assume that other parameters of interest are also flux surface variables, at least in the limit of large aspect ratio. However, this assumption may be inadequate near the edge of the plasma, because of the asymmetry of the limiter or divertor configurations that are generally employed: some of the sources, notably the particle source and the sources of additional heating, are not poloidally (circularly) or toroidally symmetric; furthermore, there might be some toroidal asymmetries, due to the small toroidal variation of the magnetic field. Nevertheless, one can in all cases use averages of all quantities of interest over the flux surfaces, thus exploiting the fast transport in the poloidal and toroidal directions compared with that in the radial direction.

Transport codes typically evaluate the radial dependence and temporal evolution of various plasma parameters such as density, temperature and radiation, which can be compared with corresponding experimental measurements. Such comparisons may entail substantial processing of experimental data, for instance, tomographic reconstruction of line-integrated data.

Transport equations The following coupled partial differential equations for transport in a 1-D cylindrical geometry are solved, representing for the most general case particle balance, and electron and ion energy balance. It is important to note that when the dependences of all the transport coefficients, sources and sinks are included, these partial differential equations are non-linear and strongly coupled.

$$\frac{\partial n}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r\Gamma) + S \quad (2.49)$$

$$\frac{3}{2} \frac{\partial}{\partial t} nT_e = -\frac{1}{r} \frac{\partial}{\partial r} (rq_e) + Q_\Omega + Q_{Ae} - Q_{exc} - Q_{rad} \quad (2.50)$$

$$\frac{3}{2} \frac{\partial}{\partial t} n T_i = -\frac{1}{r} \frac{\partial}{\partial r} (r q_i) + Q_{Ai} + Q_{exc} - Q_{cex} \quad (2.51)$$

In these equations, n is the density,¹⁷ T_e and T_i are the electron and ion temperatures, Γ is the particle flux, q_e and q_i are the total electron and ion thermal fluxes (including conduction and convection), S is the particle source (including any sinks), and Q represents the various sources and sinks of energy (to be described below). The transport equations are supplemented by Maxwell's equations for the poloidal magnetic field B_θ and the toroidal electric field E_ϕ , namely Ampère's law

$$\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \mu_0 j_\phi, \quad (2.52)$$

and Faraday's law

$$\frac{\partial E_\phi}{\partial r} = \frac{\partial B_\theta}{\partial t}; \quad (2.53)$$

taking $E_\phi = \eta_\parallel j_\phi$ and eliminating B_θ , these give a diffusion-convection equation for the parallel current density,

$$\frac{\partial j_\phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} \left(\frac{\eta_\parallel}{\mu_0} j_\phi \right) \right]. \quad (2.54)$$

Fluxes of particles and energy The particle flux, the electron and ion energy fluxes, and the parallel current density are related to the gradients of density, electron and ion temperatures, and to the toroidal electric field by 16 transport coefficients: most generally, one writes¹⁸

$$\Gamma = -n \left(L_{11} \frac{n'}{n} + L_{12} \frac{T_e'}{T_e} + L_{13} \frac{T_i'}{T_i} \right) + L_{14} E_\phi + n V_1, \quad (2.55)$$

$$q_e = -n T_e \left(L_{21} \frac{n'}{n} + L_{22} \frac{T_e'}{T_e} + L_{23} \frac{T_i'}{T_i} \right) + L_{24} E_\phi + \frac{3}{2} n T_e V_2 + \frac{5}{2} \Gamma T_e, \quad (2.56)$$

¹⁷By quasineutrality, i.e. $n_e = n_i \bar{Z}$, electrons and ions are expected to follow the same density distribution, so that only one density variable is used; yet the profiles of electron and ion density will generally be somewhat different, so that, strictly, one should use separate equations for each plasma species (cf. also discussion of impurity ion transport below).

¹⁸It should be noted that different conventions are adopted by various authors; the nomenclature found in the literature also varies —e.g., the total energy fluxes are often denoted by Q , their conductive parts being denoted by q , i.e. $Q = q + \frac{5}{2} \Gamma T$.

$$q_i = -nT_i \left(L_{31} \frac{n'}{n} + L_{32} \frac{T_e'}{T_e} + L_{33} \frac{T_i'}{T_i} \right) + L_{34} E_\phi + \frac{3}{2} nT_i V_3 + \frac{5}{2} \Gamma T_i, \quad (2.57)$$

$$j_\phi = L_{41} \frac{n'}{n} + L_{42} \frac{T_e'}{T_e} + L_{43} \frac{T_i'}{T_i} + L_{44} E_\phi. \quad (2.58)$$

For classical processes, but not necessarily in anomalous transport results, the off-diagonal transport coefficients L_{ij} are related through Onsager's symmetry (see, e.g., Woods, 1975). The diagonal transport coefficients in the equations above are more commonly referred to as the particle diffusivity D (L_{11}), the electron and ion thermal diffusivities χ_e and χ_i (L_{22} and L_{33}), and the parallel conductivity $\sigma_{||}$ (L_{44}). The terms containing the velocities V_1 , V_2 and V_3 represent non-diffusive components in the particle and energy fluxes and are negative for inward pinches; it is usual for these terms to incorporate the terms in E_ϕ . It should be pointed out that, in the present definition, the energy fluxes q_e and q_i are expressed in a laboratory frame of reference, rather than in a convected one, and thus include the convective contributions $\frac{5}{2} \Gamma T_e$ and $\frac{5}{2} \Gamma T_i$ respectively —the factor $\frac{5}{2}$ is the product of the factors $\frac{3}{2}$ (for the 3 degrees of freedom) and $\frac{5}{3}$ (the ratio of heat capacities for an adiabatic process), but a factor $\frac{3}{2}$ is often used instead; the thermal flux diffusivities, however, are those in the convected frame. Considerable care should be exercised in ascertaining how a thermal flux and its associated diffusivities are defined in particular theoretical applications and experimental measurements. Nevertheless, all theories of anomalous transport calculate the particle and total thermal fluxes explicitly, so that these issues affect only the nomenclature and the definitions of the diffusivities. Finally, it is important to note that all the transport coefficients may depend on several plasma parameters, and in particular on density, temperature and their gradients, thus introducing an additional mode of transport coupling. In this case, the distinctions between the various transport coefficients will be to some extent arbitrary. The forms of the transport equations and fluxes have been discussed by Ross (1988).

In modelling plasma transport, it is often useful to distinguish between three regions of the minor cross-section, of qualitatively different transport processes. The central region, within the sawtooth inversion radius, is probably dominated by magnetohydrodynamic instabilities and may be characterized by fast transport. The intermediate or confinement region is characterized

by some anomalous transport mechanism(s). Finally, transport in the edge region is controlled by further mechanisms involving atomic phenomena. The boundary conditions that one uses in connection with the differential equations representing plasma transport are related to these atomic processes at the edge. In particular, the edge boundary condition for the particle balance equation depends on the recycling of the outward particle flux and on the gas feed and ionization, while the edge boundary conditions for the energy balance equations depend on radiation and charge exchange loss processes at the edge. Such boundary conditions are, of course, dependent on the edge geometry (limiters and/or divertors) and discharge conditions, and can be strongly localized in the poloidal and toroidal directions.

Sources and sinks of particles and energy The source terms in the particle and energy balance equations are modelled with the help of a mixture of theoretical results and experimental measurements.

- The particle source term S is difficult to model and can be asymmetric; it depends on the ionization and recombination processes, and on the density distribution of neutral particles. As most elements are fully ionized over the central region of a tokamak plasma, the particle source is strongly localized near the edge.
- The Ohmic heating term Q_Ω is typically modelled by using the Spitzer parallel resistivity with neoclassical corrections due to the trapped particles, in order to obtain the current density distribution. The resistivity of a plasma containing electrons and ions of charge Z has a Z^2 dependence through the collision frequency τ_e^{-1} , and a further Z^{-1} dependence due to the multiplication of the electron population. One therefore defines, for an impure plasma containing several ion species, an *effective ion charge* by taking an average over all elements and ionization states, including the working gas:

$$Z_{\text{eff}} = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i} = \frac{\overline{Z^2}}{\overline{Z}}. \quad (2.59)$$

This effective ion charge is calculated from the relative abundances of the ionization states of the impurity ions (as discussed below in connection with the radiation term). The neoclassical resistivity for a plasma of effective charge state Z_{eff} can then be found by

using the following formulae (e.g., Düchs et al., 1977; but cf. Woods, 1987):

$$\eta_{\text{neo}}(Z_{\text{eff}}) = \eta_{\text{cl}}(1) \gamma(Z_{\text{eff}}) / (1 - f_{\text{tr}}), \quad (2.60)$$

$$\text{where } \eta_{\text{cl}}(1) = \frac{m_e}{n_e e^2 \tau_e} = \frac{m_e^{1/2} e^2 \ln \Lambda_e}{3(2\pi)^{3/2} \varepsilon_0^2 T_e^{3/2}}, \quad (2.61)$$

$$\ln \Lambda_e = 9.4 - \frac{1}{2} \ln(n_e / 10^{19} \text{m}^{-3}) + \ln(T_e / \text{eV}), \quad (2.62)$$

$$\gamma(Z) = Z \left(0.295 + \frac{0.39}{0.85 + Z} \right), \quad (2.63)$$

$$\text{and } f_{\text{tr}} = \frac{1.95\varepsilon^{1/2} - 0.95\varepsilon}{1 + \nu_e^*} \quad (2.64)$$

$$\text{with } \nu_e^* = \frac{\nu_{\text{dt}}}{\omega_b} = \frac{qR_0}{(T_e/m_e)^{1/2} \varepsilon^{3/2} \tau_e}, \quad (2.65)$$

where $\eta_{\text{cl}}(1)$ is the classical resistivity for a pure hydrogenic plasma as given by (2.21); $\gamma(Z)$ is the Spitzer modification factor,¹⁹ which reflects the effect of electron-electron collisions and is approximately proportional to Z as explained above (and equal to 0.51 for $Z=1$); and the factor $1/(1 - f_{\text{tr}})$ represents the neoclassical enhancement due to the presence of trapped particles, ν_e^* being the normalized detrapping frequency (the neoclassical correction is derived from a consideration of the reciprocal fraction of passing particles, the friction between passing and trapped particles and the collisional detrapping of the trapped particles; cf. subsection 1.4.3, (1.16) and (1.14)).

A complete description of the Ohmic heating power density requires a knowledge of the impurity content of the plasma, and of the current density distribution $j_\phi(r)$, as obtained from the solution of the current density equation (2.54), but the assumption of full field penetration can often be made, when the transformer action is on a longer time-scale than the skin time, leading to a radially uniform parallel electric field E_ϕ , which can be directly related to the plasma loop voltage. The assumption of a uniform parallel electric field is equivalent, through Faraday's law (2.53), to a time-independent current density (or poloidal field) distribution. This can be justified by the associated diffusivity $\eta_{||}/\mu_0$ being small compared with the particle and thermal diffusivities; however, the current

¹⁹ the function given here is related to the Spitzer gamma by $0.295Z/\gamma(Z) = \gamma_E(Z)$

density profile may be perturbed on a short time-scale in the central region, because of the sawtooth oscillation or other MHD activity. In the simple case, the current density profile can be calculated from the resistivity by taking the trivial solution of (2.54),

$$j_\phi(r) = E_\phi / \eta_{||}(r) = E_\phi \sigma_{||}(r); \quad (2.66)$$

E_ϕ can be subsequently determined from the normalization of the current density profile, using the known total plasma current, i.e.

$$I_p = \int_0^a j_\phi(r) 2\pi r dr = E_\phi \int_0^a \sigma_{||}(r) 2\pi r dr; \quad (2.67)$$

the loop voltage is then obtained from

$$V_l = 2\pi R_0 E_\phi. \quad (2.68)$$

The Ohmic heating power density is finally obtained from

$$Q_\Omega(r) = E_\phi j_\phi(r) = \eta_{||}(r) j_\phi^2(r) \propto j_\phi(r) \propto \sigma_{||}(r). \quad (2.69)$$

Alternatively, if it can be further assumed that the effective charge is radially uniform, one can first calculate the current density profile, by using the hydrogenic conductivity in (2.66) and (2.67), and subsequently determine the Ohmic heating power density and the effective charge from the measured loop voltage.

- The additional heating terms Q_{Ae} and Q_{Ai} can be reliably calculated for some additional heating schemes, such as ECRH or ICRH (using ray-tracing computer codes), but can be more complicated for other schemes, such as neutral beam heating. These source terms generally have to be treated using particle rather than fluid descriptions, as the equilibrium Maxwellian distributions can be significantly perturbed by additional plasma heating.
- The electron-ion equipartition term Q_{exc} is usually assumed to be classical (given by (2.30)) and is particularly important at high densities, for which the ion thermal transport may be comparable to, or exceed, the electron thermal transport. It should be noted that this term is also dependent upon the impurities and their ionization state.

- The radiation term Q_{rad} includes energy losses due to bremsstrahlung (free-free transitions), recombination radiation (free-bound transitions) and atomic line radiation (bound-bound transitions), integrated over all photon energies (refer to subsection 3.3.1 for a discussion of the energy spectra), and is extremely difficult fully to model, depending sensitively and qualitatively on several plasma parameters, including the impurity content and transport. The radiation loss can be a substantial fraction of the total power losses in a tokamak (up to 80%) and typically peaks near the edge. The emissivities for all three radiative processes are proportional to the electron and impurity densities, and depend on the electron temperature and on the ionization state of the impurity ions; line radiation also depends on the excitation state of the ions. The ionization and excitation states depend, of course, on the electron temperature (and less strongly on the density) through the ionization, recombination, excitation and de-excitation rates. The processes that control the ionization and excitation states of atoms and ions, and the resulting emission of radiation, are described by McWhirter (1981); a further account is given by Hutchinson (1987).

The radiation term is applied only to the electron energy balance, because energy is transferred from electrons by inelastic collisions with ions and immediately radiated —the ion internal energy is not included in the ion energy balance. Tokamak plasmas are optically thin to all radiation but microwaves, and therefore, the radiation does not lead to any thermal transport. The contribution from the electron cyclotron emission to radiation is negligible because of re-absorption. However, superthermal emission in the core accompanied by re-absorption in the outer regions can lead to radiative energy transport. The steady-state approximation is often made in calculating the radiation from a tokamak plasma, but it should be pointed out that, whereas the radiation itself is virtually instantaneous, the processes controlling the ionization and excitation states are characterized by time-scales that can be comparable to those of transport.

The calculation of the total radiated power proceeds in four steps: first, the abundances of the various impurity elements are determined; second, the ionization states and exci-

tation levels that are present for each element are determined (the former also controls the plasma collisionality and resistivity), in terms of fractional abundances; third, the radiation spectra for the three processes are calculated; fourth, the spectra are integrated over all photon energies. Collisions between ions and electrons in a plasma lead to collisional excitation and ionization, and radiation is produced by spontaneous de-excitation, radiative recombination and the acceleration of electrons in the electric fields of the ions (bremsstrahlung). Under conditions of coronal equilibrium, the ionization and recombination rates for each excitation state are balanced, and the average ionization state is only a function of temperature. In the coronal approximation, which pertains to tenuous plasmas, all ionizations and excitations are collisional (because of the low radiation density) and all recombinations and de-excitations are radiative (because of the low electron density). Post et al. (1977) have calculated the charge state parameters $\bar{Z}$ and $\bar{Z}^2$, and the total radiated power, as functions of electron temperature, for all common impurity ions under coronal conditions.

However, a coronal description is not always applicable to tokamak plasmas. The impurity transport plays an important role in modifying the radiated power, even under steady-state conditions. The balance of the ionization states for each impurity element present is generally described by a system of particle balance equations of the form

$$\frac{\partial n_Z}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_Z) + n_e [S_{Z-1} n_{Z-1} - S_Z n_Z - \alpha_Z n_Z + \alpha_{Z+1} n_{Z+1}], \quad (2.70)$$

$$\text{with } \Gamma_Z = -D_Z \frac{\partial n_Z}{\partial r}, \quad (2.71)$$

where S_Z and α_Z are, respectively, the ionization and recombination rates²⁰ for the ionization state Z ; D_Z is an impurity diffusivity which may be neoclassical or anomalous, and may depend on Z . Equations of this form, for all impurities and ionization states, strictly, have to be solved with the energy balance equations. Using this impurity transport model and the emission coefficients for the various processes involved, Roberts (1981) has calculated the total radiation as a function of the electron temperature and density, average

²⁰these are strongly dependent on the electron temperature and weakly dependent on the electron density

effective charge and impurity confinement time. It transpires from these calculations that the impurity radiation under conditions of finite impurity confinement is higher than the coronal equilibrium level, and that its reduction with increasing temperature is weaker; also, the dependence on density is weaker than quadratic. It also follows that, for a finite impurity confinement time, the local radiated power is not simply a function of the local temperature and density, but depends on their radial profiles. It is finally shown that, for most tokamak discharges, coronal equilibrium is not applicable.

Because of the strong dependence of radiation on the impurity content, the generation and transport of impurity ions generally has to be included in the modelling, but the assumption of an impurity content profile is often sufficient. A further simplification arises from the fact that, in most cases, the radiation is dominated by only few impurities.

- Finally, the charge exchange term Q_{cex} includes losses due to several processes between ions and neutral atoms (the same processes that govern the particle source S) and is also difficult to model; however, the charge exchange loss is small compared with other losses.

2.6 Measurement of transport

Two broad classes of techniques exist for the measurement of transport processes, namely methods relying on pseudo-stationary phenomena and those relying on transient ones. In both cases, one typically uses *effective* diffusivities in the expressions for the particle and thermal fluxes, thus ignoring the off-diagonal transport coefficients: the particle flux is written as

$$\Gamma = -D \frac{\partial n}{\partial r} + Vn, \quad (2.72)$$

and the electron and ion thermal fluxes are written as

$$q = -n\chi \frac{\partial T}{\partial r} + \frac{5}{2}\Gamma T. \quad (2.73)$$

D and χ are, respectively, the effective particle and thermal diffusivities; the velocity V is negative, representing an inward particle pinch, which is needed to balance the finite density gradients in the central region of the plasma where the particle source is negligibly small —the pinch term

produces an effective source. Such effective diffusivities can, of course, be defined regardless of whether or not the fluxes are driven solely by the gradients included in the expressions above.

Equilibrium methods In the first class of techniques, one studies the particle and energy balance under conditions of equilibrium, that is, over time-scales that are long compared with the energy and particle confinement times, such that the temporal derivatives in the transport equations can be set to zero; the spatial derivatives are then balanced by the sources and sinks. At the simplest level, one can employ a *global power balance* analysis in order to obtain the global energy confinement time and its dependence on global plasma parameters (such as averaged density, temperature or input power). One can express an average diffusivity in terms of an appropriate confinement time, by solving a simplified steady-state diffusion equation of the form

$$0 = \bar{\chi} \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\frac{3}{2}T}{\tau};$$

taking the plasma minor radius as the first zero of the Bessel function solution, one obtains, e.g.,

$$\bar{\chi}_e = \frac{a^2}{3.9\tau_{Ee}^*}. \quad (2.74)$$

At a higher level of complexity, one can apply a *local power balance* analysis, based on the measured profiles of density and temperature at equilibrium, and on obtaining the required fluxes from a knowledge of the profiles of the relevant sources and sinks. The local fluxes can be calculated by an integration of the relevant sources and sinks over the volume bounded by a flux surface. One obtains the thermal flux from the sum of the heat sources and sinks,

$$q(r) = \frac{1}{r} \int_0^r \sum Q(r) r dr. \quad (2.75)$$

From an analogous result, it follows that the particle flux $\Gamma(r)$ is negligible over much of the plasma cross-section, where the particle source is negligible. Therefore, the total thermal flux which is measured in the power balance approach, is approximately equal to the conductive part of the thermal flux, except near the edge; it follows that the effective thermal diffusivity determined from the measured thermal flux should *not* include the convective contributions of the particle flux diffusivities, because the latter are almost exactly cancelled by the particle pinch, except in the outer region of finite ionization where the effect of convection is important.

This procedure for calculating the fluxes, and hence the effective diffusivities, is most straightforward for the Ohmic input power, but more complicated for the other sources and sinks. One notes that these approaches are usually subject to large experimental errors because of the uncertainties involved both in the measurement of the profile gradients and in the determination of the sources and sinks. Statistical methods, using data from several similar discharges, are often employed in measurements of this type. Examples of recent results of this type of analysis, explaining the methods used and the assumptions typically made, have been presented by Christiansen et al. (1988, JET), Zarnstorff et al. (1989, TFTR) and Sing et al. (1989, TEXT).

Dynamical methods In the second class of techniques, one uses a *perturbation* on one or more plasma parameters, which either occurs naturally (the sawtooth pulse) or is externally induced (such as modulated ECRH or pellet injection). In this case, one studies the evolution of the perturbation and compares the results with an appropriate transport model; the spatial derivatives are now balanced by the temporal ones, outside the region where the sources and sinks are perturbed. The advantages of such techniques are that they can provide one with more direct measurements of the transport and that they can, at least in principle, reveal certain dependences of the transport coefficients on various perturbed plasma parameters. Another potential advantage is that, when the applied perturbation leads to a linearized partial differential equation, a detailed knowledge of all the sources might not be necessary, although the perturbations of these sources, if any, will have to be included. One can distinguish between an impulse-like perturbation (an instantaneous modification of parameter profiles, such as the sawtooth collapse or pellet injection), and a continuously applied one (a variation of a source, such as modulated heating or gas feed). In experiments with an initial perturbation, no detailed knowledge of the sources is necessary for the transport analysis, but the signal-to-noise ratio of the data may be limited by the availability of only a transient response, the sawtooth pulse being, of course, an exception. On the other hand, in measurements using a continuously applied perturbation, one can obtain a stationary response, thus improving the reliability of the technique. Experiments based on dynamical methods will be reviewed in the following section.

Comparison of methods The principal difference between measurements of stationary and transient transport phenomena is that, in the former case, one obtains, for example, effective thermal diffusivities involving the equilibrium fluxes and gradients, of the form

$$\chi_0 = -\frac{q}{nT'}, \quad (2.76)$$

whereas, in the latter case, one obtains “incremental” diffusivities involving the perturbed fluxes and gradients, of the form

$$\chi_1 = -\frac{\Delta q}{n\Delta T'}. \quad (2.77)$$

These two values will be equal, *only if* (i) the thermal flux contain no terms other than that due to the temperature gradient, *and* (ii) the underlying diffusivity χ be independent of the parameters that are perturbed in a transient measurement, and in particular of the gradients, which can possibly change sign as a result of a perturbation. In this connection, one should distinguish between, on the one hand, an *indirect* dependence on a parameter, which will manifest itself as a radial variation, being independent of a perturbation with a short time-scale, and, on the other hand, a *direct* dependence, which will lead to a modified value when a perturbation is applied. The non-diffusive part of the particle flux may be perturbed in a different way from the diffusive part, even if the relevant transport coefficients be constant, thus leading to a finite perturbation of the particle flux and hence to an additional perturbation of the thermal flux. Similarly, if the thermal flux contain a net steady-state pinch term, this will affect the stationary effective value of the diffusivity more than the transient value, because the perturbation of the temperature gradient will be typically greater than the perturbation of the temperature.

It can be seen from this brief description that it should be possible, at least in principle, to obtain much information regarding the particle and thermal fluxes from equilibrium and dynamical measurements, by using different perturbation mechanisms and/or by varying the perturbation level. An important final consideration in all transport measurements is whether the technique that is used leads to explicit results for the transport coefficient(s), which can be used on a routine basis.

2.7 Review of dynamical measurements of transport

Dynamical techniques relying on naturally or externally excited perturbations of the plasma equilibrium, such as the sawtooth oscillations arising from the MHD activity of the plasma, modulation of the additional heating power input, modulation of the gas feed and pellet injection, provide one with a useful and versatile means of exploring transport and of testing theoretical results. Several investigations into energy and particle transport have been carried out on various tokamaks in the past, using dynamical techniques, and a mainly qualitative review is presented in this section, of the most notable experiments of this type and their results. Some of the quantitative aspects of these measurements, in particular those pertaining to the ECRH modulation experiments carried out on DITE, will be discussed in subsequent chapters.

2.7.1 Sawtooth pulse propagation

Sawtooth oscillations are caused by MHD activity inside the $q=1$ surface (see subsection 1.4.2). They involve a periodic collapse of the temperature profile inside the $q=1$ surface, with a corresponding rise of the profile outside that surface. The radius of the $q=1$ surface is known as the *inversion* radius, reflecting the inversion in the sawtooth shape observed at that point. The sawtooth profile collapse is limited to a certain radius known as the *mixing* radius. Beyond the mixing radius, the perturbation in temperature due to the sawtooth collapse appears to propagate diffusively, and one can use the sawtooth pulse propagation to determine a characteristic diffusivity, usually referred to as the *heat pulse value* of the thermal diffusivity. Sawtooth oscillations affect the density profile in a similar way as the temperature profile, leading to the possibility of determining a *density pulse value* of the particle diffusivity.

Callen and Jahns (1977), and Soler and Callen (1979) have introduced and developed the technique for the determination of the thermal diffusivity from the sawtooth heat pulse propagation. A simple electron heat diffusion equation is used to describe the propagation of the temperature profile perturbation. The initial perturbation, that is, the sawtooth collapse, is approximated by a pair of delta functions, a negative one within the inversion radius and a positive one between the inversion and mixing radii, such that the integrated perturbation is

zero. Simple formulae are obtained for the diffusivity in terms of the heat pulse delay time. An alternative and more complete development is based on fitting to the data the complete surface of the radial and temporal variation of the perturbation. These techniques were applied to data from the ORMAK tokamak, leading to diffusivity values in agreement with those from power balance analysis.

Bell et al. (1984) used broadly the same technique in conjunction with data from the ISX-B tokamak, but using a transport code to obtain a more general solution of the perturbation equation and simulating the line-integrated SXR signals. Again, the diffusivity values from heat pulse propagation and from power balance were similar.

Fredrickson et al. (1986) measured the heat pulse propagation on the TFTR tokamak, invoking a simple solution of the perturbed energy transport equation. Both SXR and ECE data were used, and the longer time-scales of this large tokamak were exploited to carry out several tests of the diffusive model. Solutions were obtained using the dipole perturbation and compared with the experimental pulse delay times and pulse shapes. A Fourier technique was also used to extract the phase of the perturbation as a function of radius, and a simple expression of the diffusivity in terms of the gradient of this phase profile was applied. Different rates of diffusion were observed for the first sawtooth pulse in a discharge and for subsequent ones, the latter apparently diffusing outwards faster than the former. All the results obtained were consistent with diffusive transport, but the values obtained for the diffusivity from this heat pulse propagation analysis were larger, typically by a factor of 5, than those deduced from a power balance analysis. In order to explain the discrepancy between the stationary and transient values of the diffusivity, the authors suggested that either (i) a perturbed diffusivity, arising from a parametric dependence and leading to an increased heat pulse value, or (ii) an inward thermal pinch term, leading to a reduced stationary effective value, might be responsible.

Goedheer (1986) has considered the assumptions previously made by other authors in calculating values of the diffusivity from the heat pulse propagation. Analytical and numerical solutions are used to investigate the effects of a non-uniform diffusivity, different initial perturbation shapes, a finite extent of the plasma and perturbed heat sources and sinks. It transpires

that simplified diffusion models that do not include these effects are likely to lead to an *overestimation* of the diffusivity by a factor of up to 2. Lopes Cardozo et al. (1988) have adopted these results and have derived a formula for the calculation of the diffusivity, from the measured variation with minor radius of the peaking time and peak amplitude of the heat pulse. A numerical solution is obtained, of an equation for the temperature perturbation, incorporating a sink term that represents the effect of the variations in the local Ohmic heating and in the radiation and equipartition losses; this solution is used to obtain the formula for the diffusivity.

Tubbing et al. (1987) measured the heat pulse propagation on the JET tokamak, their aim being to explain the discrepancy, also observed on this device, between the diffusivity values obtained from the analyses of stationary and transient phenomena. The level of the additional heating power was varied, and it was observed that the discrepancy between the two thermal diffusivities decreased with increasing power. Both global and local analyses of power balance that were carried out on similar discharges (reported by Christiansen et al., 1988) indicated that there was an off-set linear relationship between the plasma energy content and the input power, involving a constant *incremental confinement time* τ_{inc} , i.e.

$$W = \tau_{\text{inc}} P + W_0; \quad (2.78)$$

this led to an energy confinement time decreasing with power (consistent with the usual scaling laws), i.e.

$$\tau_E = \tau_{\text{inc}} + W_0/P. \quad (2.79)$$

This result then suggested that the effective diffusivity χ_e increased with an increasing input power, approaching the higher *incremental diffusivity* χ_{inc} . The thermal flux measured in the local power balance analysis was of the form

$$q = -n\chi_{\text{inc}} \frac{\partial T}{\partial r} + q_p, \quad (2.80)$$

where $q_p < 0$, corresponding to an inward thermal pinch; this term produces an extra temperature pedestal, associated with a stored energy W_0 . The incremental diffusivity χ_{inc} was found to be inversely proportional to the plasma current, but independent of the density and the temperature gradient. The diffusivity was measured by heat pulse propagation, using the theoretical

results reported by Lopes Cardozo et al. (1988). (Formulae for the transformation from elliptical to circular minor cross-sections were used in this application.) The diffusivity values obtained from the analysis of the heat pulse propagation were independent of the input power and similar to the incremental value χ_{inc} . This led the authors to propose a temperature-gradient model for the diffusivity χ_e , which postulates a value increasing with the temperature gradient, when the latter exceeds a critical gradient, and approaching the incremental value χ_{inc} , of the form

$$\chi_e = \chi_0 + (\chi_{\text{inc}} - \chi_0) \left(1 - \frac{(T'_e)_{\text{crit}}}{T'_e}\right) H(|T'_e| - |(T'_e)_{\text{crit}}|), \quad (2.81)$$

where $\chi_0 < \chi_{\text{inc}}$ and $H(x)$ is a unit step function. This definition of the diffusivity clearly leads to a thermal flux consistent with (2.80), the critical temperature gradient giving the inward thermal pinch

$$q_p = -n(\chi_{\text{inc}} - \chi_0) |(T'_e)_{\text{crit}}| H(|T'_e| - |(T'_e)_{\text{crit}}|). \quad (2.82)$$

Callen et al. (1987) have considered the temperature profiles and confinement scaling relations arising, for a variety of heating power profiles, from two thermal flux models: (i) the constant heat pinch model of (2.80) and (ii) a non-linear diffusivity model (without a heat pinch) which could also be used to fit the local power balance results. Comparisons with the data from JET suggested to the authors that the model of constant heat pinch was more appropriate. These results have led to the formulation of the critical-temperature-gradient model of transport, by Rebut, Lallia and Watkins (1988).

Kim et al. (1988) used the sawtooth oscillation for simultaneous determinations, on the TEXT tokamak, of the particle diffusivity from the density pulse propagation, and of the electron thermal diffusivity from the heat pulse propagation. The particle and thermal diffusivities were found to scale in similar ways with the line-averaged density and the edge safety factor, the dependence on the latter being stronger. The authors therefore suggested that the density and heat pulse propagation might be coupled. A further experiment was carried out on JET by Sips et al. (1989), using the inwardly propagating density perturbation caused by the sawtooth heat pulse reaching the limiter, in order to determine the particle diffusivity.

2.7.2 Modulation of heating power

In this technique, one excites a thermal wave in the plasma, by locally modulating some additional heating source. The propagation of the thermal wave, when measured with spatial and temporal resolution, affords one with a way of determining the associated thermal diffusivity.

The first experiments of this kind were carried out by Jahns et al. (1986), on the Doublet D-III tokamak. The ECRH power was modulated with central power deposition, and the resulting perturbation in the electron temperature was monitored using the SXR diagnostic; only a small density perturbation was observed. A particularly simple model was used to describe the propagation of the temperature perturbation outside the power deposition region, based on pure diffusion (without convection) and flat profiles of density and diffusivity. A Fourier technique was used to extract the relative phase profile of the perturbation, which was compared with numerical solutions (with the assumption that the phase of the perturbation of the line-integrated SXR signals was the same as that of the local temperature perturbation), in order to obtain a value for the diffusivity. A rather high value was deduced, this being higher than the value obtained from the power balance analysis. This was most probably an overestimate because of the simplifications made: the perturbation on the Ohmic heating term was not included, and the chordal-integral nature of the SXR data was not taken into account.²¹

Hartfuss et al. (1986) carried out similar experiments on the Wendelstein VII-A stellarator using partially modulated ECRH power; magnetic shear was introduced externally to avoid modification of transport due to instabilities. ECE data were used to monitor the temperature perturbation; averages of the signals over several similar discharges were taken and the time delays of the perturbation were measured. It was found impossible to make such measurements at low magnetic fields, as the diffusivity was then very high. A simple diffusive model based on a Taylor expansion of the electron energy balance equation was used. The values obtained for the thermal diffusivity were similar to, but slightly higher than, those obtained from the power balance analysis.

Joye et al. (1987 and 1988) and Dudok de Wit et al. (1989(a) and (b)) have described

²¹these effects will be discussed in more detail in the following chapters

experiments based on the modulation of Alfvén wave heating on the TCA tokamak. This particular heating scheme is characterized by a complex absorption condition, dependent on the local values of the toroidal field, safety factor and mass density, which can lead to the presence of more than one resonant layers within the plasma. The power deposition was off-axis. A modulation of the period of the sawtooth oscillation was observed in addition to the modulation of the signal. SXR data were used and the signals were fitted using a sinusoidal wave superimposed on a linear drift, with time-dependent amplitudes. A very small density modulation was observed, which was ignored in the analysis. Several sets of results were obtained with different modulation frequencies and safety factors (the latter being adjusted by varying the plasma current). These experiments were characterized by a behaviour that was fundamentally different from that observed in other modulation experiments. In particular, it was not consistent with diffusive transport, since relatively flat profiles of perturbation amplitude and relative phase were obtained. The relative phase was found to be constant with radius within the $q=1$ surface, and to exhibit a 180° -shift at a radius closely corresponding to the $q=1$ radius; the position of this phase shift moved when the q -profile was changed, following the changes in the $q=1$ radius. On the other hand, the dependences of the amplitude and relative phase of the perturbation on the modulation frequency were compatible with diffusive transport (the phase lag increased as the modulation frequency was increased). Very similar plasma responses were obtained when radially resolved time delays were measured for various externally induced transient phenomena, such as gas feed modulation (in Ohmic-heated discharges), heating power turn-off, heating power pulses, and crossing of the Alfvén resonance (introducing an additional resonant layer) by changing the density or the RF frequency. Furthermore, an experiment with “counter-modulated” gas feed and RF heating, such as to minimize the density modulation, led to an almost complete cancellation of the SXR oscillation. The authors concluded that (a) the directly driven temperature perturbation (through the modulated heating, rather than through the density perturbation) was small, and (b) an insulating layer was present near the $q=1$ surface.

Tibone et al. (1988) and Start et al. (1988) have reported measurements with modulated

ion cyclotron heating on the JET tokamak. These have relied on the presence of stabilized (“monster”) sawteeth of particularly long duration (Campbell et al., 1988) —earlier modulation experiments in discharges with normal sawteeth had not yielded useful results. The electron temperature perturbation was measured with the ECE diagnostic, and the analysis was again based on a simple diffusive model, taking into account both direct electron heating and collisional heat transfer from the minority ions. Experiments were carried out with partially modulated RF power, at a suitably low frequency, consistent with the long energy confinement time of this tokamak; the thermalization and modulation times were comparable in these experiments. The ICRH scheme employed was hydrogen minority heating in a helium-3 plasma; under these conditions, the direct electron heating was dominant. The electron thermal transport analysis, based on a radially constant diffusivity, led to values $2\text{--}4\text{m}^2\text{s}^{-1}$, consistent with those obtained from the heat pulse propagation analysis, and higher than those obtained from the power balance analysis. It was also concluded that the RF power deposition profile was very narrow.

2.7.3 Modulation of gas feed

This method can be used to obtain values for the particle transport coefficients, namely the particle diffusivity and inward pinch velocity; the technique has been described by Gentle et al. (1987). A particle balance equation, with a flux defined in terms of the diffusivity and pinch velocity, is used to describe the perturbation of the density caused by the modulated gas feed; the particle source term can be taken to be zero for the central region of the discharge and an appropriate boundary condition can be used to produce the modulation. Vasin et al. (1984) have reported measurements of particle diffusion on the T-10 tokamak using this technique, in addition to that using an instantaneous gas feed to produce a transient response; a particle diffusion model has been used, incorporating profiles of neutral particle density and particle diffusivity. Further results of this type of experiment, carried out with ECRH discharges on the TEXT tokamak, have been presented by Gentle et al. (1989).²² It is shown that, when the ECRH power was applied, the particle diffusivity increased and the inward pinch velocity

²²cf. also the results of the experiments on the TCA tokamak above

decreased over the outer half of the minor radius. This is consistent with the observed decrease in the central density, but difficult to explain in terms of any changes in plasma parameters in the region where the transport coefficients change.

2.7.4 Pellet injection

The injection of a fuel pellet into the plasma can be used to produce a transient response in both density and temperature, by initiating perturbations of the corresponding profiles; in particular, the density is increased and the temperature is slightly decreased near the pellet deposition radius. A transport analysis on JET including the use of this technique has been reported by Gondhalekar et al. (1989). These experimenters analysed JET discharges using (a) the inward propagation of the density and temperature perturbations following the pellet injection; (b) the outward propagation of the density and temperature pulses following the sawtooth collapse, as described above; and (c) the evolution of the profiles of non-stationary plasmas. The transport model used in all cases was based on simple particle and energy fluxes (each including diffusive and pinch terms), and led to a ratio of thermal to particle diffusivities $\chi_e/D_e \sim 7$ in the confinement region.²³

As an important general remark on the measurements of transport employing dynamical techniques, one can point out that, whilst their results are in general mutually consistent, they lead to higher values for the thermal diffusivity than the equilibrium techniques do, with the possible exception of the smaller, more collisional tokamaks; these values probably also scale differently with the plasma parameters. There is so far no commonly accepted theory explaining this discrepancy by either a thermal pinch term or a diffusivity with an appropriate parametric dependence, whilst the possibility also exists of coupling between particle and energy transport.

²³This result suggested to the authors that magnetic field stochasticity, rather than electrostatic turbulence, was responsible for thermal transport (see subsection 2.4.2).

Chapter 3

Experiments on DITE with Modulated ECRH

3.1 Description of experiments

3.1.1 The DITE tokamak

DITE (Divertor and Injection Tokamak Experiment)¹ is a medium-sized tokamak with a large aspect ratio and a circular cross-section. It was originally designed for operation with a bundle divertor and tangential neutral beam injection, but these features were not in current use during the period of the ECRH experiments. The vacuum vessel has a major radius of 117cm and a minor radius of 26cm. An iron-cored transformer design is employed and 16 cryogenic (but non-superconducting) toroidal field coils are used. A conventional feedback system controls the plasma position. An electron cyclotron resonance heating system has been installed, comprising three gyrotron lines and antennae on the high-field (inner) side. An impurity control limiter (ICL) on the outer side of the torus was in use during the first experiments with modulated ECRH, having a concave surface facing the plasma and flat surfaces facing the torus wall at an angle of 35° to the toroidal field; its purpose was to divert sputtered impurities away from the

¹Some of the DITE results have been summarized by Paul (1985); for the engineering details of the machine and its major upgrade, see Bayes et al. (1974(a) and (b)), Bell et al. (1976) and Bayes et al. (1980).

plasma and towards the torus wall, thus lowering the impurity content of the plasma (Matthews et al., 1988). The impurity control limiter was later replaced by an arrangement comprising a pumped limiter on the outer side, with a variable aperture and a separate pumping chamber, and a bumper limiter on the inner side (Johnson et al., 1989). All limiters were made of graphite. The DITE apparatus has recently been used for studies of current drive, high-field-side electron cyclotron resonance heating, lower hybrid resonance heating and current drive, edge plasma physics, transport, and feedback control of disruption precursor oscillations.

3.1.2 Parameters of discharges with modulated ECRH

The ECRH modulation experiments on DITE were carried out during two experimental campaigns in 1987 and 1988/89, employing two different limiter configurations (the impurity control limiter was replaced by the pumped limiter arrangement between the two campaigns), with torus major radii of 117cm and 119cm respectively, and limiter minor radii of 21cm and 24cm respectively, using first helium and later hydrogen or deuterium as working gases. The principal purpose of these experiments was to study electron heat transport, this being the dominant energy loss mechanism in a tokamak with the size and plasma parameters of DITE. The launching of the 60GHz, X-mode ECRH wave was from the high-field (inner) side of the discharge. Experiments were performed using the fundamental resonance with a toroidal field about 2T and a plasma current of 100kA; and the second harmonic resonance with a toroidal field about 1T and a plasma current of 60kA; the values of the safety factor at the edge were about 5 and 4 respectively. The measured loop voltage was about 2.0V for 100kA discharges and 2.5V for 60kA discharges. The line-averaged electron density was varied between $0.5 \times 10^{19} \text{m}^{-3}$ and $3.4 \times 10^{19} \text{m}^{-3}$.² The central electron temperature before the ECRH was about 800eV for 100kA discharges (as determined by Thomson scattering on similar discharges), and about 600eV for 60kA discharges

²the designations 'low', 'medium' and 'high density', used in the remainder of this thesis, refer to this density range

(as estimated from the measured loop voltage³). The values of various plasma parameters of interest under these conditions are given in table 3.1.

The electron cyclotron wave was launched at an angle of 45° to the toroidal field for fundamental resonance, and at 60° for second harmonic resonance, but the antenna orientation was changed for some of the discharges. Experiments were carried out with the ECRH wave launched in both the parallel and anti-parallel directions to the plasma current. The duration of the modulated ECRH varied from 56ms to 147ms, with a total discharge duration about 700ms; the modulated ECRH power was switched on during the steady-state (flat-top) period of the discharge, which was attained by Ohmic heating. The Ohmic power input was about 200kW for 100kA discharges, and 150kW for 60kA discharges, and the maximum ECRH power input was 150kW (only one of the three gyrotron lines was used). ECRH modulation periods of 7ms (143Hz) and 3ms (333Hz) were used, comparable to or smaller than the global energy confinement time, the purpose being adequately to resolve transport phenomena by observation of the plasma response to the modulated heating. These modulation periods were much longer than the electron relaxation time (3–10μs at the centre), so that the heating occurred on a much longer time-scale than the ensuing thermalization. The modulation was effected by a square wave signal of full depth (the modulation patterns were 4/3ms and 2/1ms on/off). The number of cycles, over which the response to the modulated ECRH was obtained, was 8–21 for the 7ms period, and about 50 for the 3ms period. In some cases, however, system faults shortened the duration of the modulated ECRH.

The major radius of the unshifted resonance position was varied between 110cm and 140cm by changing the toroidal magnetic field (2.0–2.5T on the torus axis, for fundamental resonance),

³A simple derivation, omitting the neoclassical enhancement factor, gives

$$\frac{V_l}{R_0} \propto \frac{I_p \overline{Z_{\text{eff}}}}{T_0^{3/2} r_T^2},$$

where V_l is the loop voltage, I_p is the plasma current, T_0 is the central electron temperature, r_T is the half-width of the temperature profile and $\overline{Z_{\text{eff}}}$ is a mean effective charge. The temperature profiles were observed to be narrower, and the effective charge was probably lower, for the low-current discharges; these observations, combined with the measured plasma resistance V_l/I_p , lead to a value for the ‘conductivity temperature’.

Table 3.1: Central values of plasma parameters for two DITE discharges typical of modulation experiments with fundamental and 2nd harmonic ECRH (H^1 ions; $\bar{Z} \sim 1.2$, $Z_{\text{eff}} \sim 2.4$).

Minor radius	a	24cm	
Major radius	R_0	119cm	
Electron density	$n_e(0)$	$6.5 \times 10^{19} \text{m}^{-3}$	$1.2 \times 10^{19} \text{m}^{-3}$
Electron temperature	$T_e(0)$	800eV	600eV
Ion temperature	$T_i(0)$	$\sim 200 \text{eV}$	$\sim 150 \text{eV}$
Toroidal field	$B_{\phi 0}$	2.2T	1.1T
Plasma current	I_p	100kA	60kA
Loop voltage	V_l	2.0V	2.5V
Energy confinement time	τ_E	12ms	2ms
Beta value (averaged/central)	β	0.06/0.52%	0.02/0.29%
Poloidal beta value	β_p	0.44/3.7	0.11/1.4
Coulomb logarithm (electrons)	$\ln \Lambda_e$	15.2	15.7
Electron plasma frequency	$\omega_{pe}/2\pi$	72GHz	31GHz
Electron cyclotron frequency	$\omega_{ce}/2\pi$	62GHz	31GHz
Electron collision frequency	ν_e	300kHz	90kHz
Electron Larmor radius	ρ_e	43 μm	75 μm
Electron mean free path	λ_e	39m	110m
Debye length	λ_D	26 μm	53 μm
Plasma parameter	$n_e \lambda_D^3$	1.2×10^6	1.7×10^6
Resistivity	$\eta_{ }$	$6.9 \times 10^{-8} \Omega\text{m}$	$11 \times 10^{-8} \Omega\text{m}$
Electron-ion exchange time	τ_{ei}	3.0ms	10ms
Coulomb logarithm (ions)	$\ln \Lambda_i$	15.1	15.5
Ion plasma frequency	$\omega_{pi}/2\pi$	1.5GHz	0.66GHz
Ion cyclotron frequency	$\omega_{ci}/2\pi$	34MHz	17MHz
Ion collision frequency	ν_i	230kHz	67kHz
Ion Larmor radius	ρ_i	0.93mm	1.6mm
Ion mean free path	λ_i	0.6m	2m

and experiments with both on-axis and off-axis heating were performed. No systematic scans of plasma parameters have been possible during the modulation experiments, other than those of the resonance position and line-averaged density, because of experimental constraints. Lack of experimental time made it impossible to investigate the variation of other parameters, and in particular the effects of different power levels of Ohmic and additional heating. Approximately 150 discharges with modulated ECRH were initiated and investigated, 50 during the first campaign and 100 during the second.

Some signals from the basic magnetic diagnostics are shown in figures 3.1 for a medium-density discharge with fundamental resonance, and 3.2 for a low-density discharge with second harmonic resonance. The modulation in various plasma parameters, driven by the modulated ECRH, is evident. The variation in the Ohmic power will be discussed in subsection 4.3.1, and the modulated horizontal shift will be treated in subsection 5.3.2.

3.2 Electron cyclotron resonance heating

3.2.1 Theory of propagation and absorption of ECRH waves

Electron cyclotron resonance heating is characterized by relatively simple propagation and absorption processes. In a plasma in thermal equilibrium (with a Maxwellian distribution), wave damping involves interactions with electrons that experience a Doppler-shifted frequency which is close to a harmonic of the electron cyclotron frequency. The resonance condition, incorporating the Doppler and relativistic shifts, is

$$\ell\omega_{ce} = \gamma(\omega - k_{\parallel}v_{e\parallel}), \quad (3.1)$$

where ω is the wave frequency, $k_{\parallel}$ is the wave vector component parallel to the equilibrium magnetic field, $v_{e\parallel}$ is the parallel electron thermal velocity, ω_{ce} is the (non-relativistic) electron cyclotron frequency, $\gamma = (1 - v_e^2/c^2)^{-1/2}$ is the relativistic mass factor and ℓ is an integer (the harmonic number). The resonance obtained for $\ell=0$ is due to Landau damping, whilst the absorption of energy from the wave at the cyclotron frequency and its harmonics is due to cyclotron damping. The resonance (or absorption) is controlled by the electron cyclotron

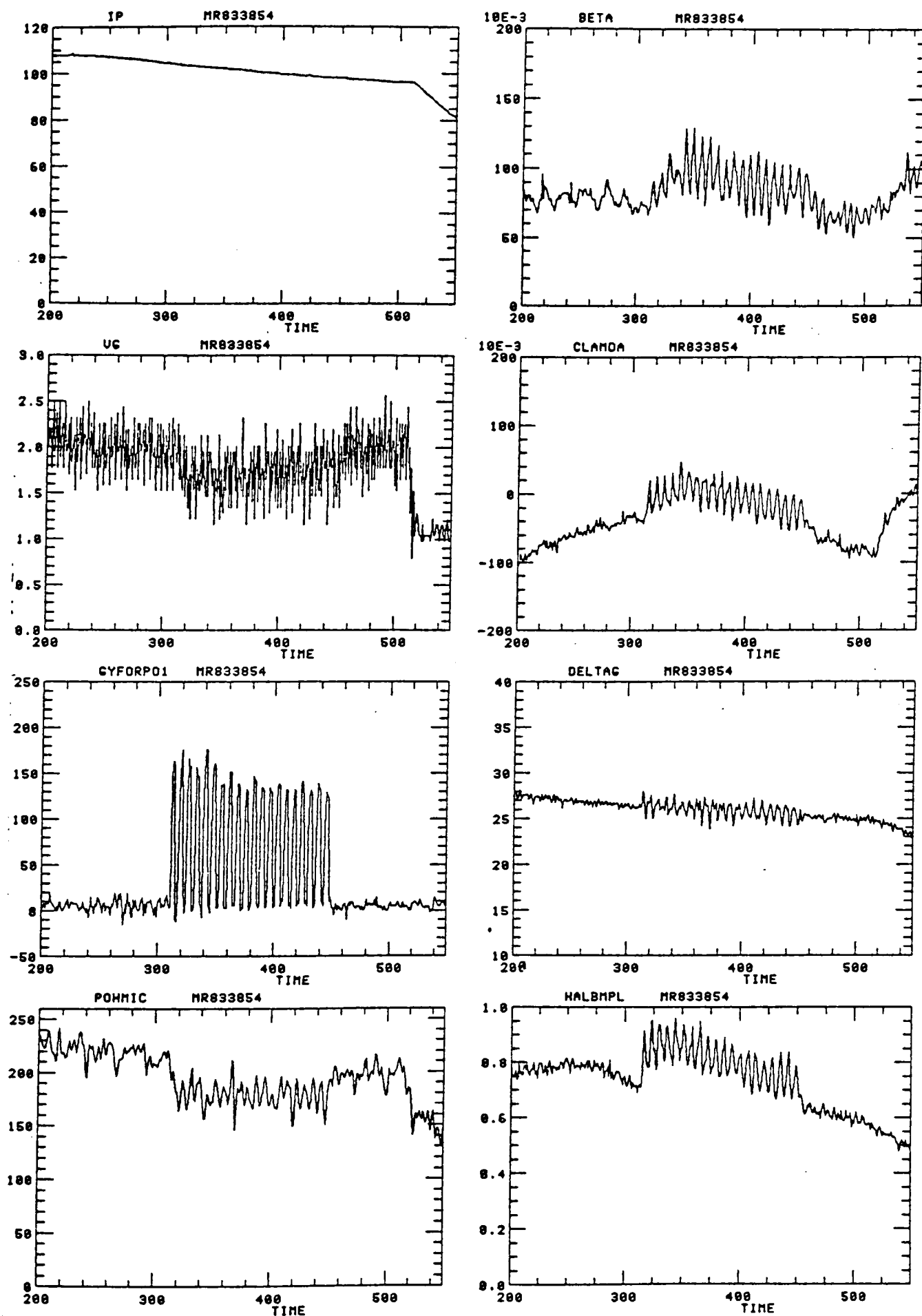


Figure 3.1: Signals of basic diagnostics from medium-density modulated ECRH discharge with fundamental resonance: plasma current in kA (IP); loop voltage in V (VG); gyrotron power (GYFORP01); (smoothed) Ohmic power in kW (POHMIC); poloidal beta value (BETA); Shafranov lambda (CLAMDA); horizontal magnetic axis shift in mm (DELTAG); bumper limiter H α emission (HALBMPL).

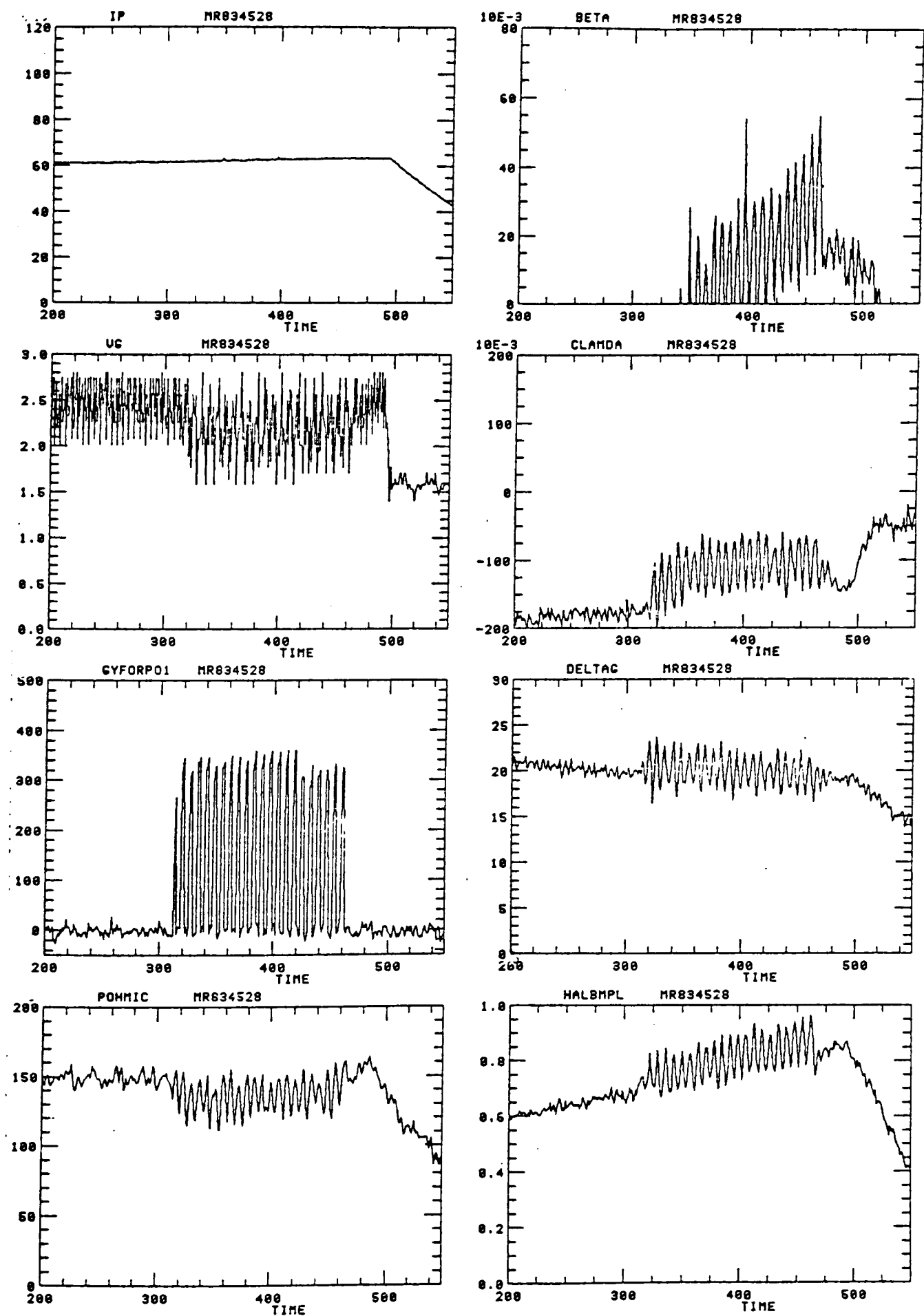


Figure 3.2: Signals of basic diagnostics from low-density modulated ECRH discharge with 2nd harmonic resonance.

frequency $\omega_{ce} = eB/m_e$, and the cut-off by the plasma frequency $\omega_{pe} = (n_e e^2 / \epsilon_0 m_e)^{1/2}$; these frequencies and their spatial variations are of particular importance to the heating scheme. As the magnetic field and consequently the electron cyclotron frequency vary with major radius in a tokamak, the absorption of a wave of given frequency will be localized. The broadening of the resonance region in the direction of the major radius generally arises from both Doppler and relativistic effects. Doppler broadening is the dominant effect in the DITE experiments, with plasmas of relatively low electron temperature and a finite wave vector component parallel to the field;⁴ this broadening is given by

$$\Delta R \simeq R \frac{\Delta \omega}{\ell \omega_{ce}} \simeq R \frac{(2\pi)^{1/2} v_{e\parallel}}{c} \frac{k_{\parallel}}{k}. \quad (3.2)$$

The small deposition width is a feature of ECRH which, in addition to the fact that the heating power is directly coupled only to the electron component of the plasma, is of considerable importance to the experimental technique that is described here. It should be noted that broadening in the vertical direction arises from the intersection of the resonant layer with the antenna radiation pattern—the latter is, of course, modified by refraction, especially near the resonance region, and particularly so for high-density plasmas. An integration of the absorption pattern over the poloidal angle gives the radial profile of the ECRH power density (including the geometrical effect that leads to a higher power density, with the same total power, for absorption nearer the axis).

The cold plasma approximation may be invoked, in order adequately to describe the wave propagation in the plasma. The cold plasma dispersion relation (see, e.g., Fielding, 1981; Cairns, 1985) is obtained from the linearized plasma fluid equations with a zero plasma pressure.⁵ For propagation parallel to the magnetic field, this predicts a resonance for the right-hand circularly polarized wave at the electron cyclotron frequency ω_{ce} ; for propagation perpendicular to the magnetic field, this wave becomes the extraordinary mode (X-mode) which has an electric

⁴the condition for Doppler broadening to be dominant is $ck_{\parallel}/\omega > v_e/c$

⁵The cold plasma dispersion relation is derived from the linearized versions of the momentum equation (2.16), in the limit of zero plasma pressure, and Maxwell's equations (2.18) and (2.19): a dielectric tensor is first obtained, which leads to a dispersion relation of the form $f(\mathbf{k}, \omega, \omega_{ce}, \omega_{pe}) = 0$.

field perpendicular to the equilibrium magnetic field, with a resonance at the upper hybrid frequency $(\omega_{ce}^2 + \omega_{pe}^2)^{1/2}$; no resonance is predicted for the ordinary mode (O-mode) which has an electric field parallel to the equilibrium magnetic field. The wave propagation is described by the refractive index vector,

$$\mathbf{n} = \frac{c\mathbf{k}}{\omega}. \quad (3.3)$$

The cut-off conditions, obtained from cold plasma theory by setting $n_{\perp} = 0$, are

$$\omega = \omega_{pe} \quad (3.4)$$

for the O-mode (giving the usual plasma cut-off above a certain density); and

$$\left(\frac{\omega_{pe}}{\omega}\right)^2 = (1 - n_{\parallel}^2) \left(1 \pm \left(\frac{\omega_{ce}}{\omega}\right)\right) \quad (3.5)$$

for the X-mode (giving low- and high-density cut-offs). The accessibility of the resonances to the launched wave is determined by the variations over the minor cross-section of the plasma frequency $\omega_{pe} \propto n_e(r)^{1/2}$, and of the electron cyclotron frequency $\omega_{ce} \propto B(R)$.

However, the cold plasma approximation, which requires that the wave phase velocity should be much larger than the electron thermal velocity (i.e. $\omega/k \gg v_e$), breaks down at resonance. Warm plasma wave theory predicts for both modes the resonances given by (3.1), in addition to the resonance at the upper hybrid frequency, all with distinct absorption coefficients. An extensive treatment of plasma wave theory, for both cold and warm plasmas, can be found in the book by Stix (1962); the theory of absorption is covered in a review by Bornatici et al. (1983). The absorption (attenuation) coefficients and optical depths from warm plasma theory for the O- and X-modes are given by Bornatici (1982), as functions of electron density and temperature, and wave launching angle. In practice, for fusion plasmas, only the fundamental and second harmonic resonances are suitable for ECRH.

The accessibility of an electron cyclotron resonance to a given wave is studied with the help of the high-frequency part of the CMA (Clemmow-Mullaly-Allis) diagram shown in figure 3.3. With the fundamental resonance, one can employ either O-mode heating with a large angle of incidence (relative to the equilibrium magnetic field direction), or X-mode heating with a small angle of incidence, in order to obtain maximum absorption. The CMA diagram shows that

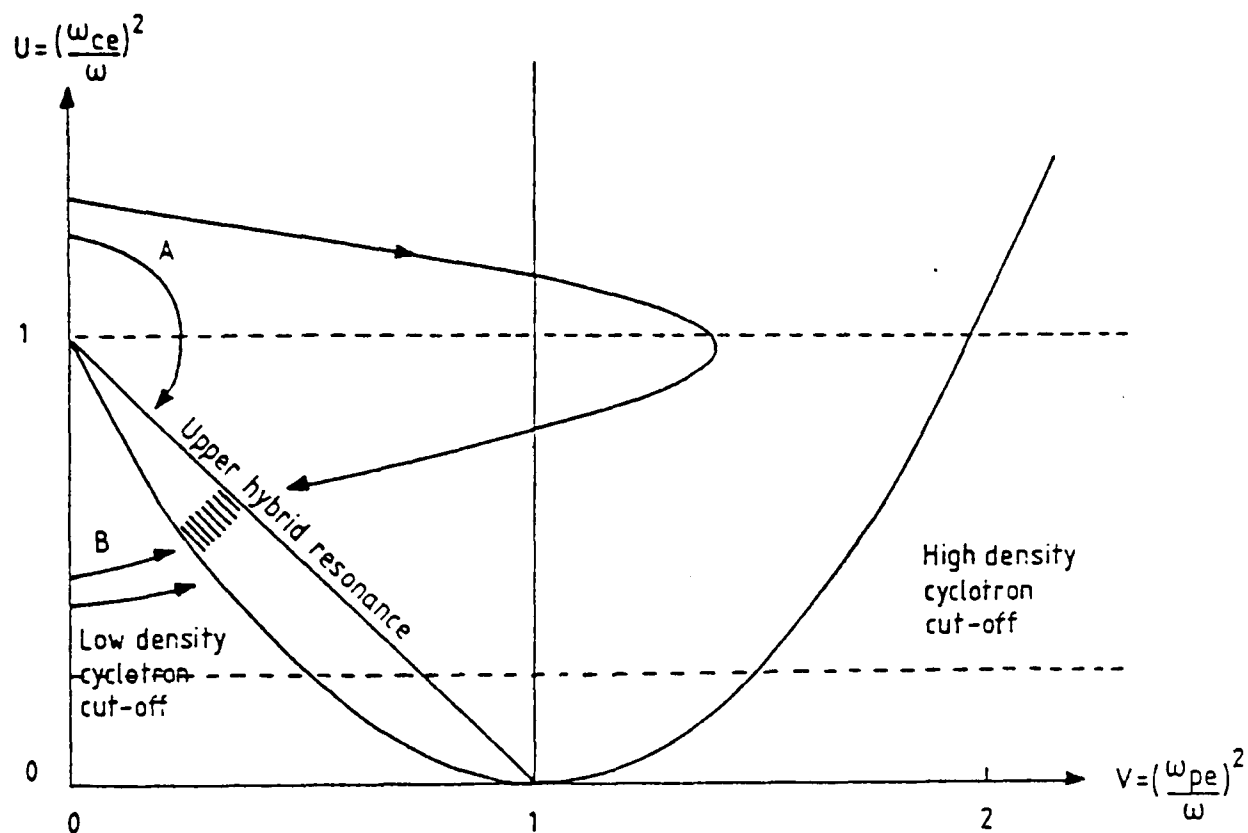


Figure 3.3: High frequency part of CMA diagram for electron cyclotron resonance heating; for a given wave frequency ω , the V-axis represents the electron density $(\omega_{pe}/\omega)^2 \propto n_e(r)$ and the U-axis represents the square of the magnetic field $(\omega_{ce}/\omega)^2 \propto B(R)^2$. The cut-offs are shown for the O-mode (vertical line) and the X-mode (solid curve). The fundamental and 2nd harmonic resonances are indicated (dotted lines), as is the upper hybrid resonance. Curves A and B show X-mode waves propagating from the high- and low-field sides respectively (from Fielding, 1981).

the O-mode is subject only to the plasma cut-off at the critical density (for which the plasma frequency is equal to the wave frequency), while the X-mode has to be launched from the high-field side to avoid the low-density cut-off. With the second harmonic resonance, only X-mode heating produces significant absorption; in this case, the wave can be launched from either side.

3.2.2 Gyrotrons

The high electromagnetic power at millimetre-wavelengths that is needed for ECRH experiments is generated by gyrotron oscillators. The usual cavity resonators (such as klystrons) are, of course, limited by the problem of heat dissipation on the cavity walls, aggravated by the cavity size which has to decrease with increasing frequency. A gyrotron (see, e.g., Alikaev, 1978; Jory, 1978; Smith, 1981), schematically shown in figure 3.4, is essentially a free electron maser. A hollow electron beam is injected into a resonant cavity with a uniform longitudinal magnetic field, such that the cyclotron frequency is nearly but not exactly equal to that of the cavity

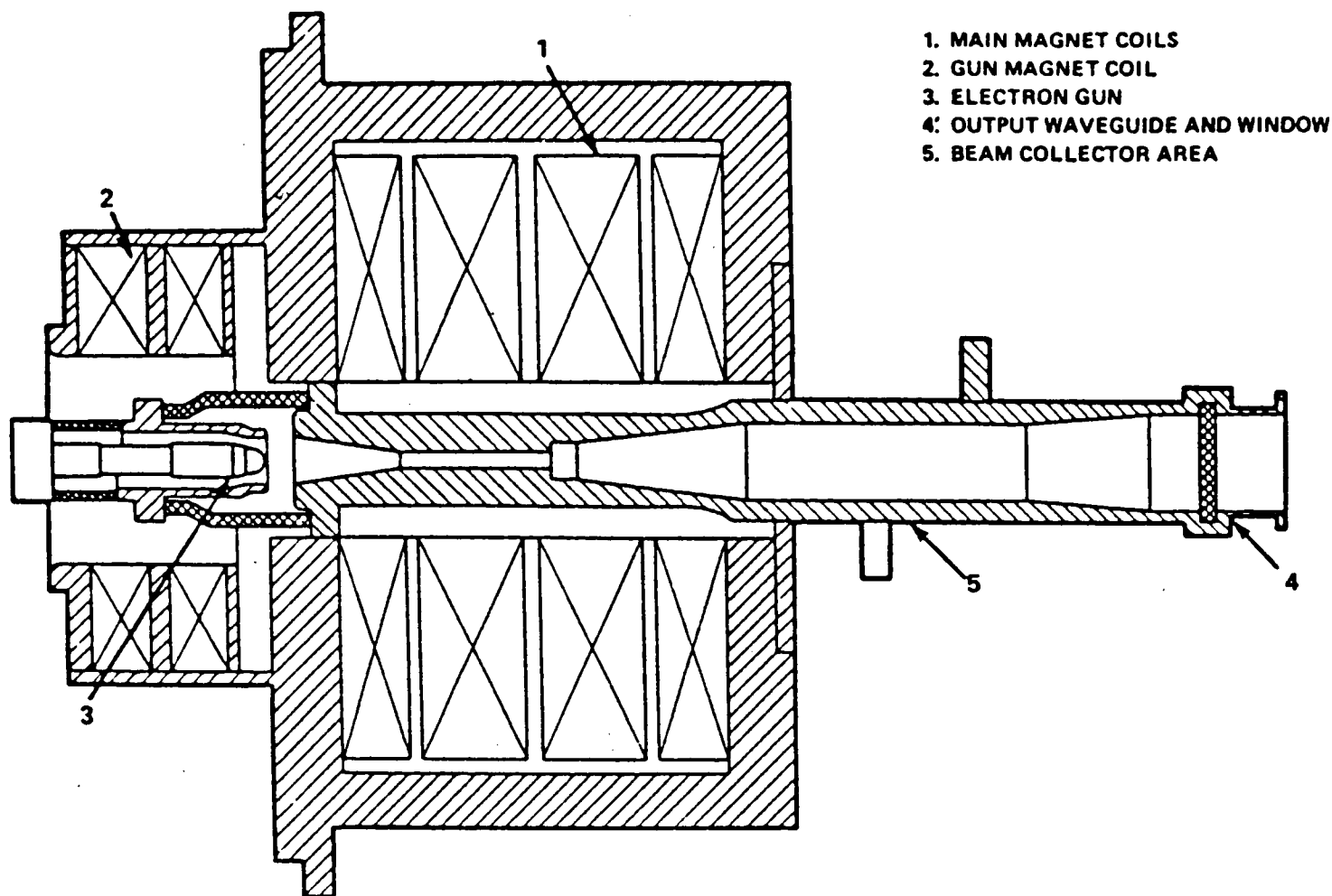


Figure 3.4: Schematic diagram of a gyrotron oscillator.

mode. In the case of the relativistic electrons of the beam (typical beam energies being of the order of 100keV), the cyclotron frequency is modified by the relativistic mass factor γ , i.e.

$$\omega_{ce}(v_e) = \frac{eB}{\gamma m_e}; \quad (3.6)$$

the desired frequency is arranged to be between the relativistic and the (higher) non-relativistic cyclotron frequency. In the region of uniform magnetic field, the transverse electric field of the wave accelerates or decelerates the gyrating electrons: accelerated electrons have their orbit periods increased and lag in phase, whilst decelerated ones have their orbit periods decreased and lead in phase, so that eventually all electrons acquire the same phase with respect to that of the wave (this process is referred to as *azimuthal phase bunching*). In the region of decreasing magnetic field, past the main cavity, energy is transferred from the electron beam to the resonant wave by a suitable adjustment of the phase of the bunched electrons with respect to that of the wave. “Population inversion” occurs when the perpendicular electron energy exceeds the parallel energy, a cyclotron instability develops and the power of the electron beam is converted to electromagnetic field power, the typical conversion efficiency being 30–40%. The

frequency of operation depends, in principle, only on the magnetic field and not on size, and the cavity diameter can be made much larger than the wavelength, thus allowing a sufficiently high microwave power to be produced. In practice, however, the size must be limited in order that the required mode purity may be obtained, and the power obtainable is then limited by the heat dissipation capability of the cavity. A circular transverse electric wave mode (TE_{011} or TE_{021}) is used in the cavity, as its electric field has no angular dependence (the output mode being TE_{01} or TE_{02} respectively).

3.2.3 Results of ECRH experiments

Some results from tokamak and stellarator experiments with electron cyclotron resonance heating and current drive have been presented by Riviere (1986), and more recently by Prater (1989). Measurements of particle and thermal transport in such experiments (including some of the results of the present work) have been reviewed by Cox and Robinson (1988) and Prater (1989). The principal features emerging from such experiments are outlined here.

At medium and high densities, the energy confinement time with ECRH falls below its Ohmic value, tending to decrease with increasing heating power. However, in low density discharges, a strong non-thermal electron population can be created, leading to a dramatically increased energy confinement time. On-axis ECRH absorption leads to more peaked temperature profiles but broader density profiles. The line-averaged density drops markedly at the start of ECRH, despite the observed increase in the particle source, indicating a decrease in the particle confinement time; however, this decrease can be cancelled or reversed in discharges with strong recycling at the edge (for instance, with helium plasmas, or with unsatisfactory wall conditioning leading to increased desorption). The increases in both particle and thermal transport appear to be correlated with increases in the edge fluctuation levels of density, and electric and magnetic fields (see, e.g., Vayakis, Mantica and Matthews, 1988, for DITE measurements).

Localized ECRH can be used to modify the temperature profile, and hence the current density profile, giving rise to the possibility of controlling certain MHD instabilities, including the sawtooth oscillation and disruption precursors. On-axis heating produces a more peaked

current density profile, causing the safety factor to decrease near the centre, whereas off-axis ECRH produces an opposite, stabilizing effect. This has been demonstrated in experiments where off-axis ECRH led to the suppression of the sawtooth activity, whereas on-axis ECRH enhanced it.

3.2.4 Implementation of high-field-side ECRH on DITE

The ECRH scheme that has been chosen for DITE is X-mode resonance at 60GHz, with wave launching from the high-field (inner) side of the torus, with a finite parallel component of the wave vector. The advantage of high-field side wave launching, compared with low-field side launching, is that it allows wave propagation to higher plasma densities. Three ECRH lines have been installed, each with a 60GHz Varian gyrotron generator rated at 200kW and capable of pulses up to 200ms. The TE_{02} mode produced by the gyrotrons is converted to the TE_{01} mode for efficient transmission by waveguides over a distance of about 40m, and finally to the TE_{11} mode with a linearly polarized electric field, for launching into the torus. The main problem of wave launching from the high-field side is that an arc breakdown will occur at the point where the waveguide crosses the resonance surface, if its pressure be near the torus pressure, leading to absorption of ECRH power. Two designs of wave launching antennae, adopting different solutions to this problem, have been used on DITE:

1. Two Culham antennae (gyrotron lines #2 and #3), as shown in figure 3.5, use a vacuum window located inside the torus on the high-field side, at a position within the resonance surface (i.e. at a higher field), and a waveguide filled with nitrogen at atmospheric pressure. The antennae are mounted at a poloidal angle of 45° above the equatorial plane. The launching mirrors rotate axially with the waveguide. These antennae are used with the hybrid, linearly polarized HE_{11} mode. Details of these antennae and of the transmission waveguides are given by Dellis et al. (1987).
2. A General Atomic antenna (gyrotron line #1), as shown in figure 3.6, uses a remote vacuum window (in the horizontal section of the waveguide) and an evacuated, curved waveguide following the inner torus wall, which is diametrically split and insulated, and electrically

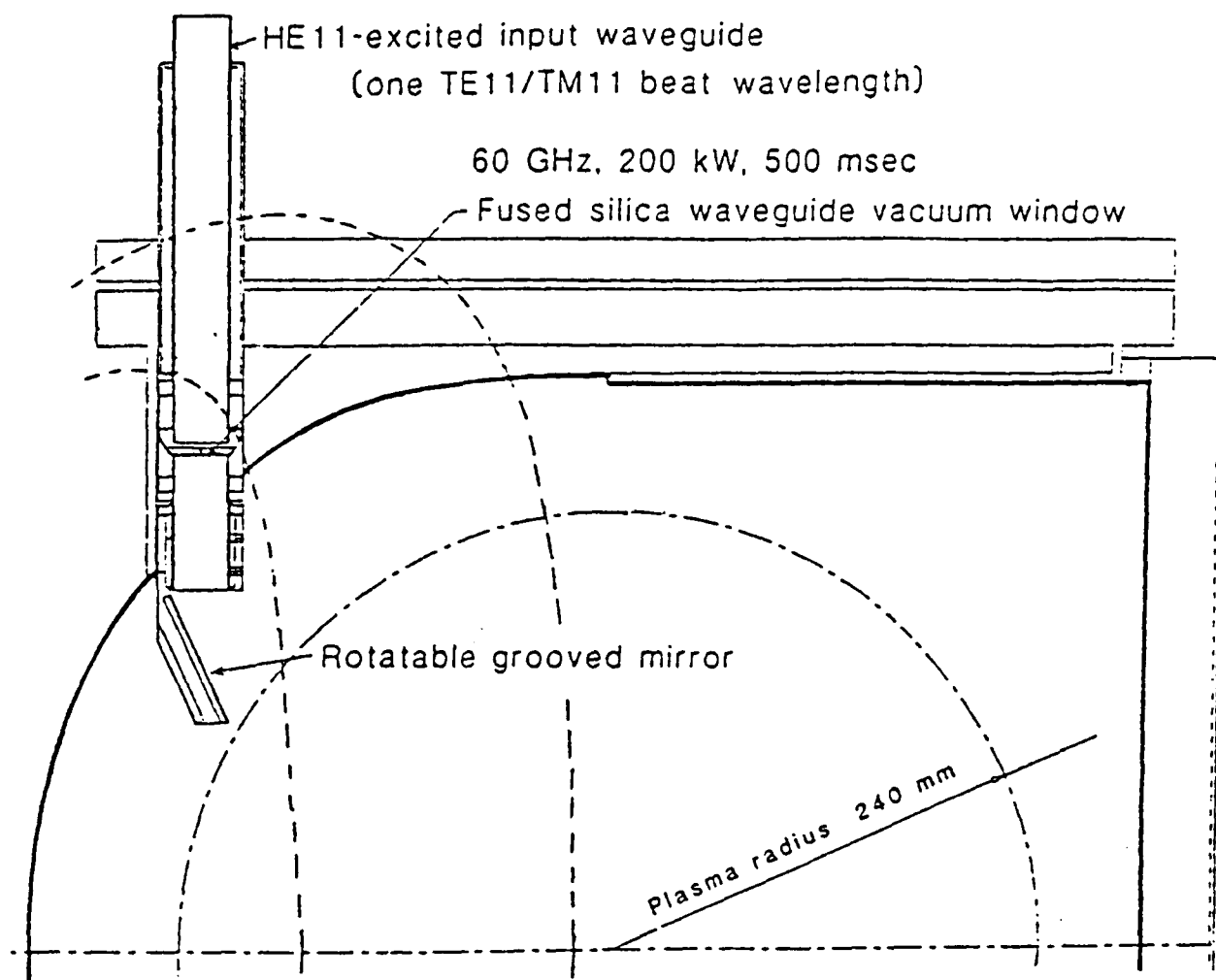


Figure 3.5: Arrangement of a Culham ECRH antenna on DITE; the dotted curves are contours of equilibrium magnetic field (from Dellis et al., 1987).

biased (at 100V DC), in order that any electrical carriers that can initiate a breakdown may be removed. The launching mirror is separate and located on the equatorial plane.

Details of this antenna are given by Moeller et al. (1987) and Dellis et al. (1987).

Both designs of antenna incorporate grooved launching mirrors, in order to produce an elliptical polarization mode and match the X-mode polarization. The General Atomic antenna was used for the experiments with modulated ECRH; its vacuum radiation pattern has an angular half-width of 19.7° at the 5% power level, which, combined with refraction, can lead to substantial vertical broadening of the ECRH absorption profile, particularly at higher plasma densities.

The square wave modulation signal was applied to the anode of the electron gun in the gyrotron. The ECRH power was measured by microwave detectors mounted on the transmission lines, measuring the forward and reverse waves, and by others located inside the torus.

The unshifted resonance position is calculated from

$$R_{\text{res}} = \frac{B_0 R_0}{B_{\text{res}}}, \quad (3.7)$$

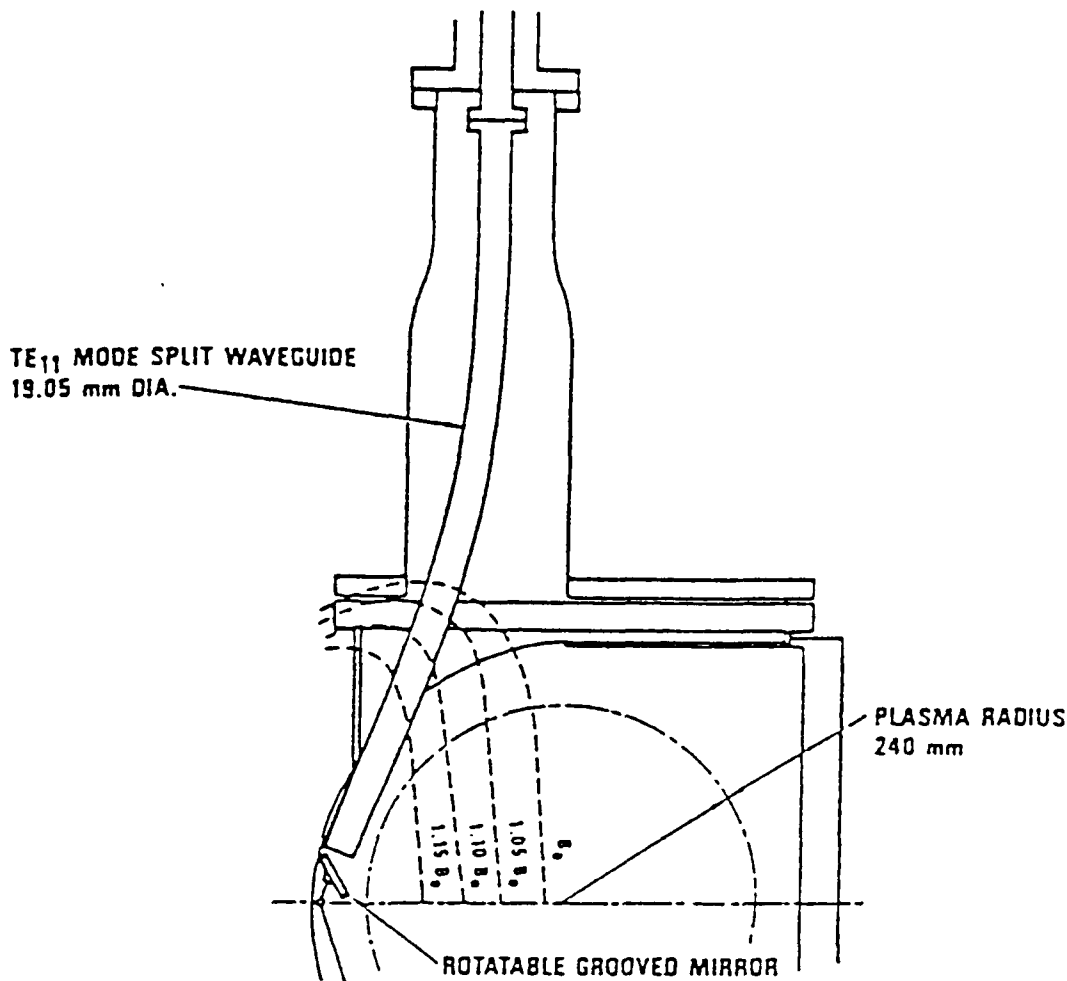


Figure 3.6: Arrangement of the General Atomic ECRH antenna on DITE; the dotted curves are contours of equilibrium magnetic field (from Moeller et al., 1987).

where the resonance field B_{res} is 2.143T for fundamental resonance at 60GHz, and 1.072T for second harmonic resonance; B_0 is taken as the vacuum toroidal magnetic field on the torus axis, at the major radius R_0 .⁶

The propagation and absorption of the ECRH wave has been modelled with the help of a ray tracing computer code (O'Brien, 1989). This employs the cold plasma dispersion relation to predict the propagation of the wave, and warm plasma theory to evaluate the absorption. These calculations take into account the vacuum radiation pattern of the antenna and its orientation with respect to the toroidal field. The antenna is modelled as a point source, but this should not significantly affect the results, the radiation pattern typically being the most important effect. The ray tracing is based on given electron density and temperature profiles, to which it is fairly sensitive. The broadening in the horizontal direction, due to the Doppler and relativistic effects, and in the vertical direction, due to the antenna radiation pattern, can be assessed. Typical

⁶the vacuum magnetic field was calculated from the current in the toroidal field coils, using a multiplicative factor

results from this code, based on three DITE discharges with X-mode ECRH waves launched with the General Atomic antenna, are shown in figures 3.7 for on-axis fundamental resonance in a medium-density plasma, 3.8 for off-axis fundamental resonance in a high-density plasma, and 3.9 for second harmonic resonance in a low-density plasma. These figures show the predicted absorption region in the minor cross-section, and the poloidally-averaged ECRH power expressed as a function of minor radius. The Doppler shift towards the inner side of the resonance is evident in all cases. The vertical broadening is also apparent and is most pronounced in the case of high density. One notes that these absorption profiles are fairly broad, this being a feature that somewhat curtails the power of the modulated heating technique.

3.2.5 Results of high-field-side ECRH experiments on DITE

The results of experiments on DITE with steady-state and modulated ECRH have been reported by Riviere et al. (1987, 1988) and Ashraf et al. (1988(b)). Briefly, absorption at the lower densities ($\bar{n}_e = 1.3\text{--}1.6 \times 10^{19} \text{m}^{-3}$) was found to be most efficient when the unshifted resonance position was well towards the low-field side; at higher densities ($\bar{n}_e = 2.5\text{--}3.1 \times 10^{19} \text{m}^{-3}$), the absorption was optimized when the unshifted resonance position was closer to the magnetic axis (see figure 3.10). These results indicate strongly the presence of a Doppler “down-shift”, that is, an inward shift of the resonance zone towards a greater magnetic field and electron cyclotron frequency. These shifts are in agreement with results from ray tracing, and have furthermore been confirmed by the ECRH modulation experiments that are described in this thesis. Such a comparison of the unshifted resonance position with the inwardly shifted absorption zone, as calculated by the ray tracing code and deduced from the modelling of the modulated ECRH results (to be described in the following chapter), is presented in figure 3.11. The Doppler shifts of the resonance zone, arising from the finite parallel component of the wave vector, were observed for the first time in these experiments on DITE. A small fraction of the plasma current was driven by the ECRH, in the low-current, low-density discharges with second harmonic heating. The high-density cut-off was observed, as predicted, at a line-averaged density about $\bar{n}_e = 4.0 \times 10^{19} \text{m}^{-3}$ (the critical density for the O-mode would have been $4.5 \times 10^{19} \text{m}^{-3}$).

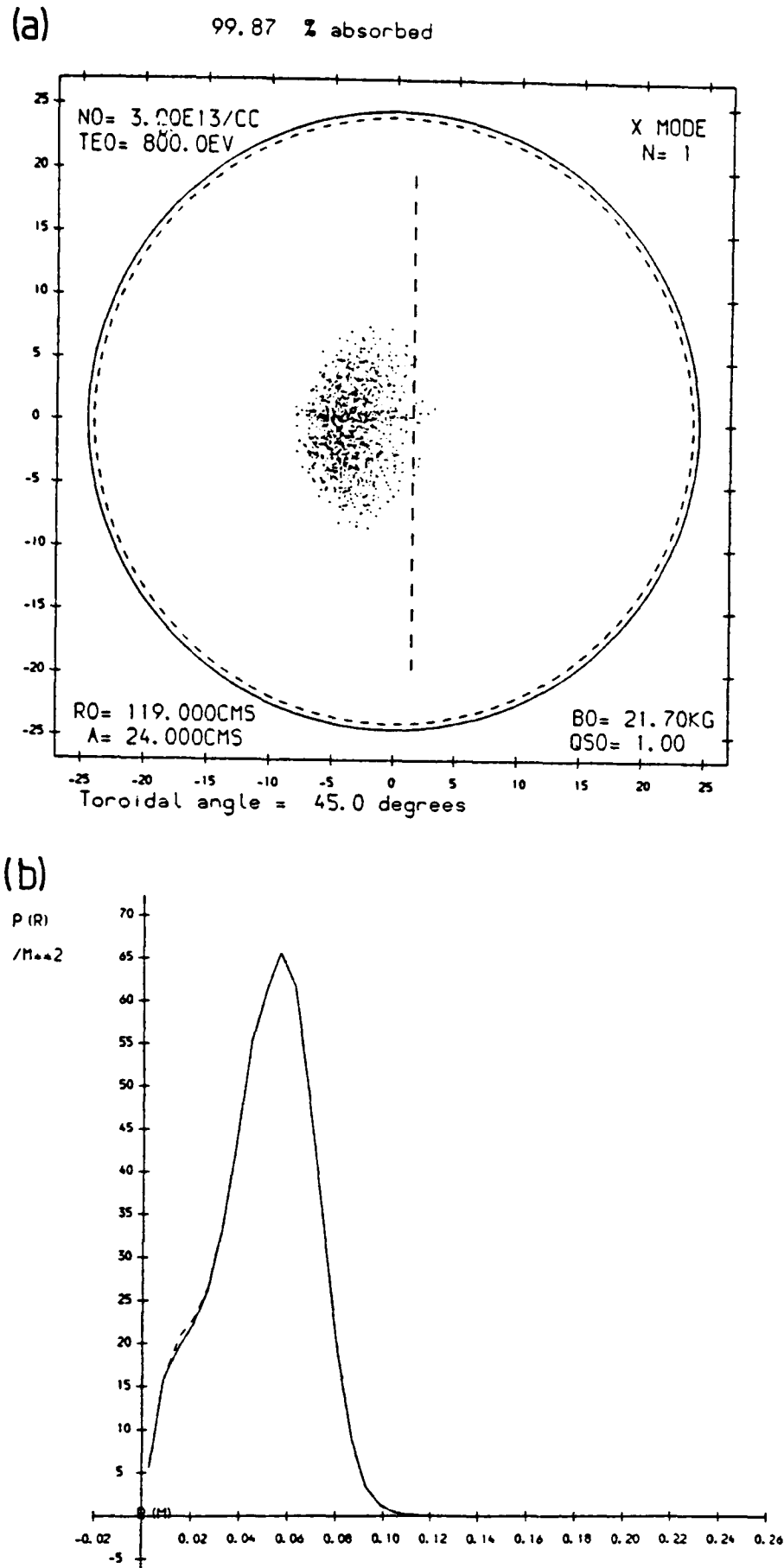


Figure 3.7: Ray tracing predictions for DITE discharge with medium density $\bar{n}_e = 1.6 \times 10^{19} \text{m}^{-3}$, on-axis fundamental resonance and launching at 45° to the toroidal field (GA antenna). (a) Absorption zone in minor cross-section, vertical dotted line indicating unshifted resonance, inner-side antenna being on the left of diagram; (b) poloidally-averaged, fractional ECRH power absorbed per unit area (averaged over poloidal angle), as a function of minor radius; the dotted sections represent the additions due to Bernstein mode absorption. The Doppler shift to the inside of the resonance is evident.

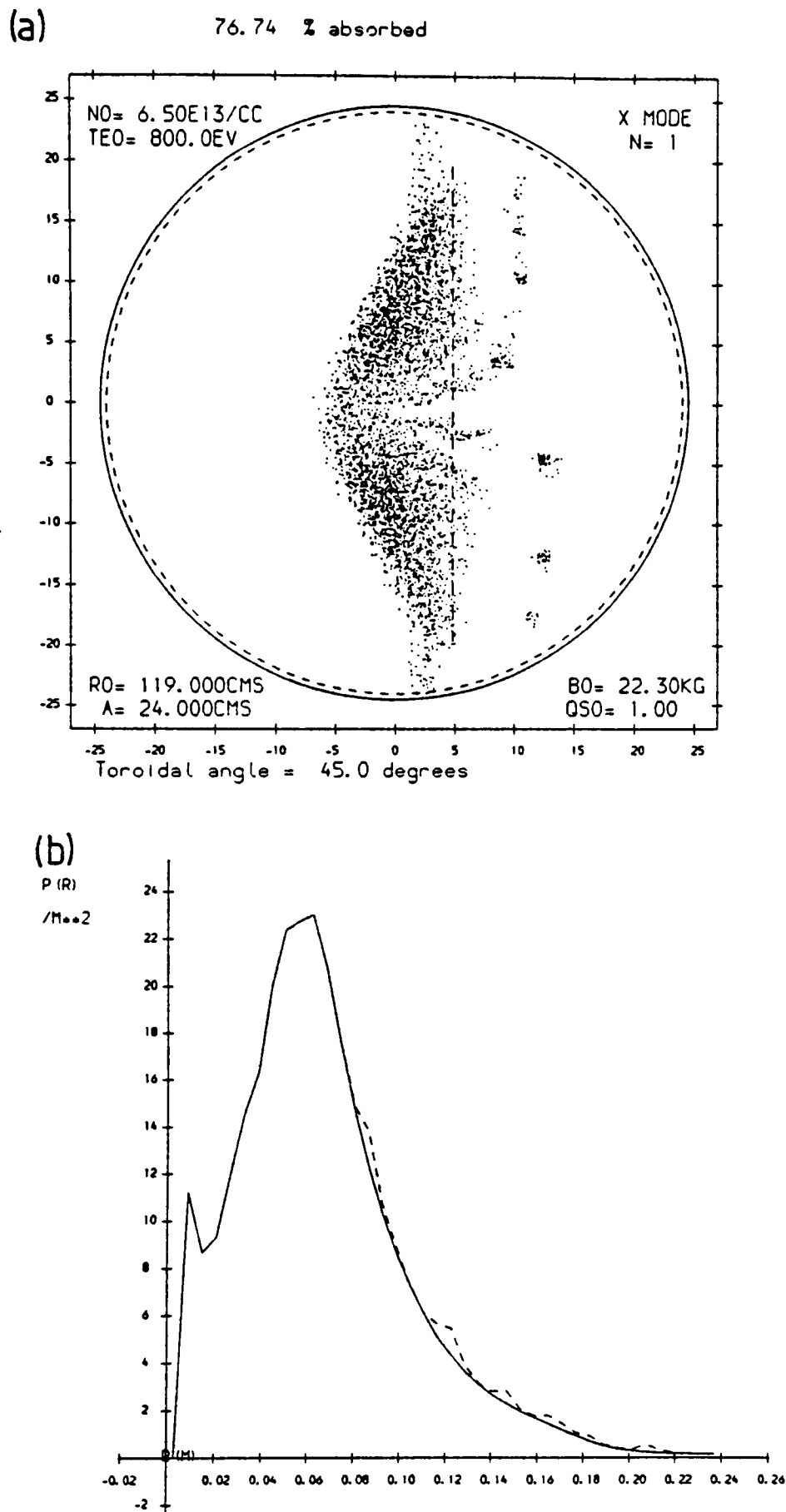


Figure 3.8: Ray tracing predictions for DITE discharge with high density $\bar{n}_e=3.2\times10^{19}\text{m}^{-3}$, off-axis fundamental resonance and launching at 45° to the toroidal field (GA antenna). The vertical broadening is pronounced, because of the high density.

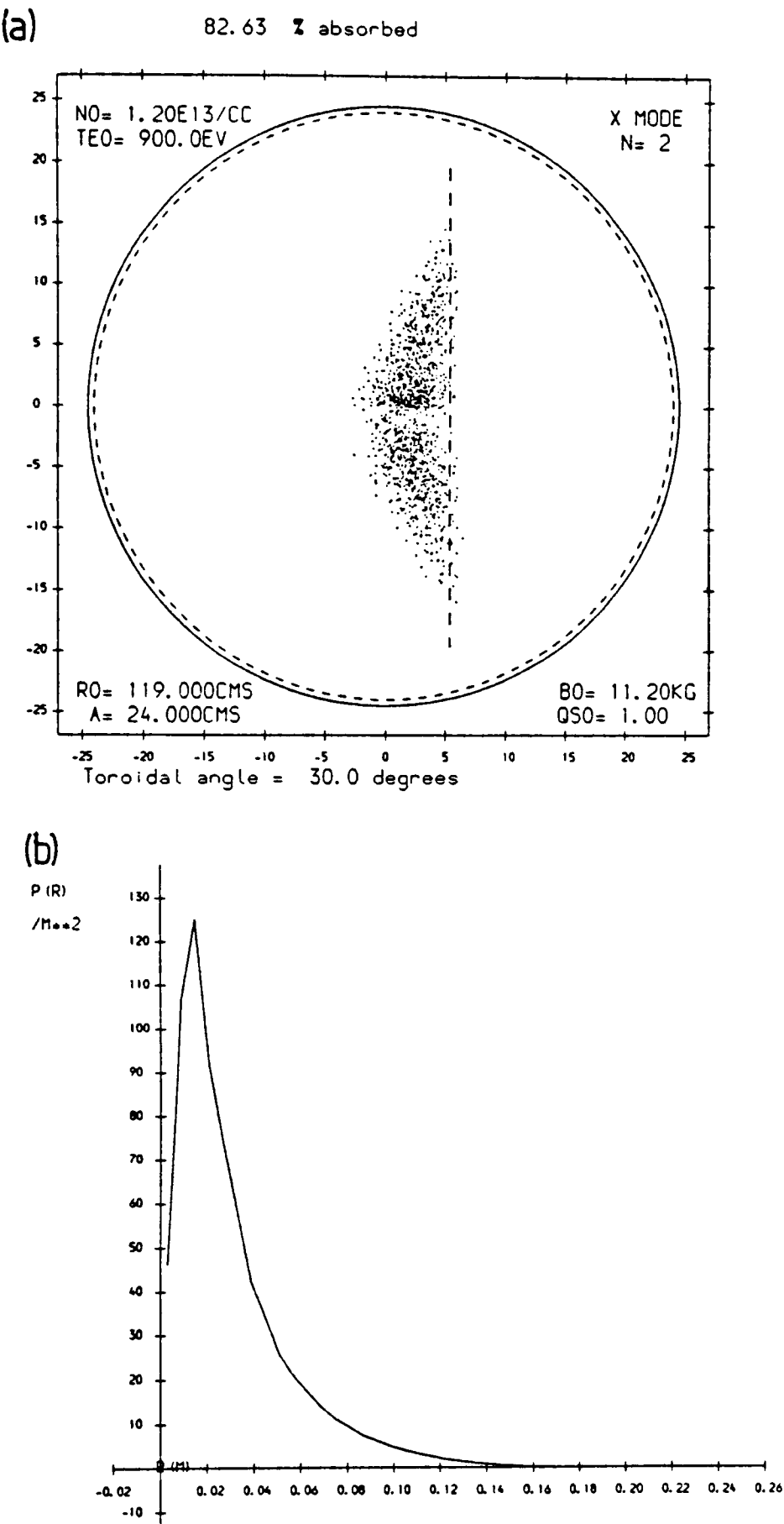


Figure 3.9: Ray tracing predictions for DITE discharge with low density $\bar{n}_e=0.5 \times 10^{19} \text{m}^{-3}$, 2nd harmonic resonance and launching at 60° to the toroidal field (GA antenna).

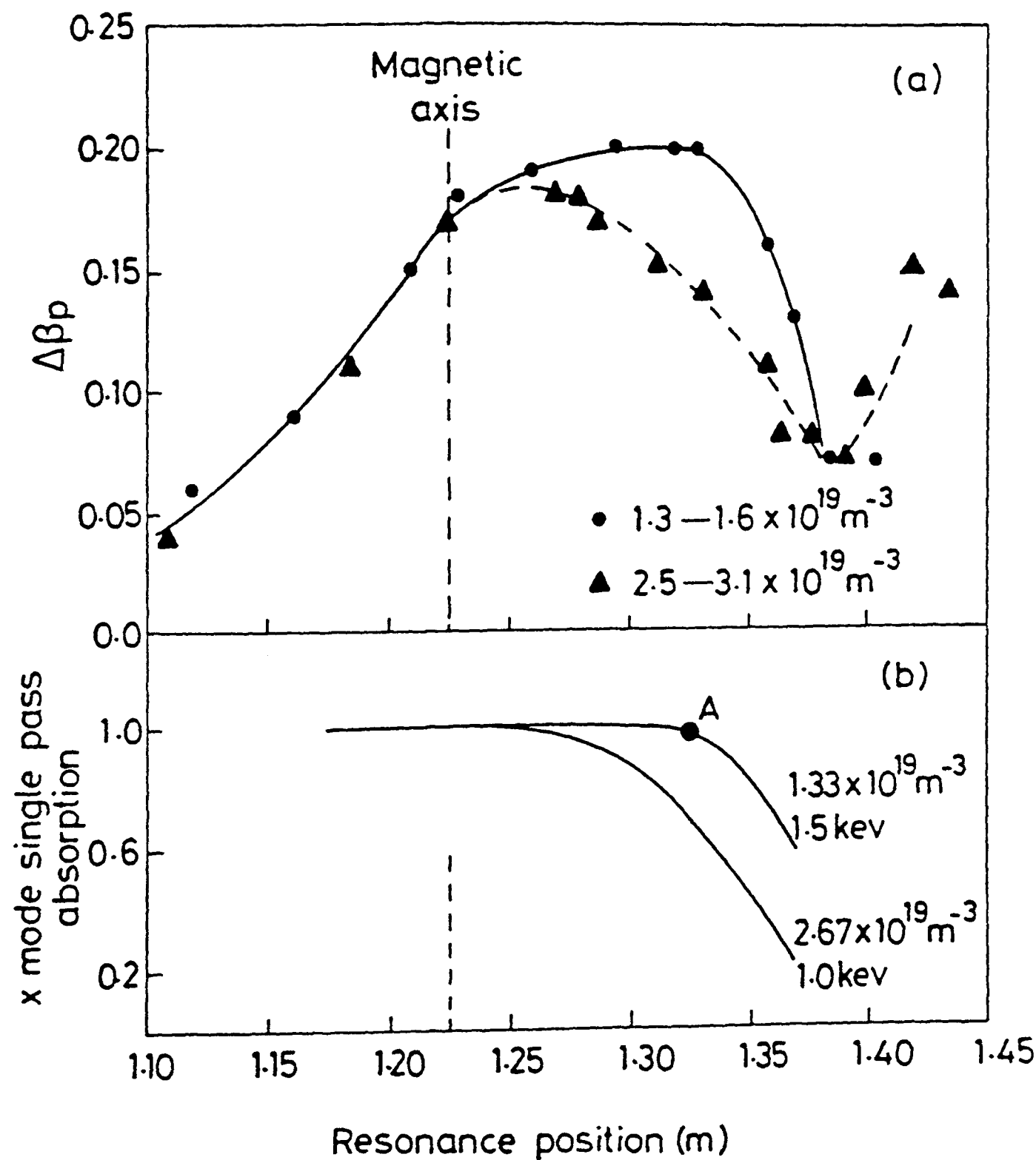


Figure 3.10: (a) Variation of the increase in plasma energy content ($\Delta\beta_p$) during ECRH and (b) single pass absorption fraction from ray tracing, plotted against unshifted ECH resonance position for various densities (from Ashraf et al., 1988(b)).

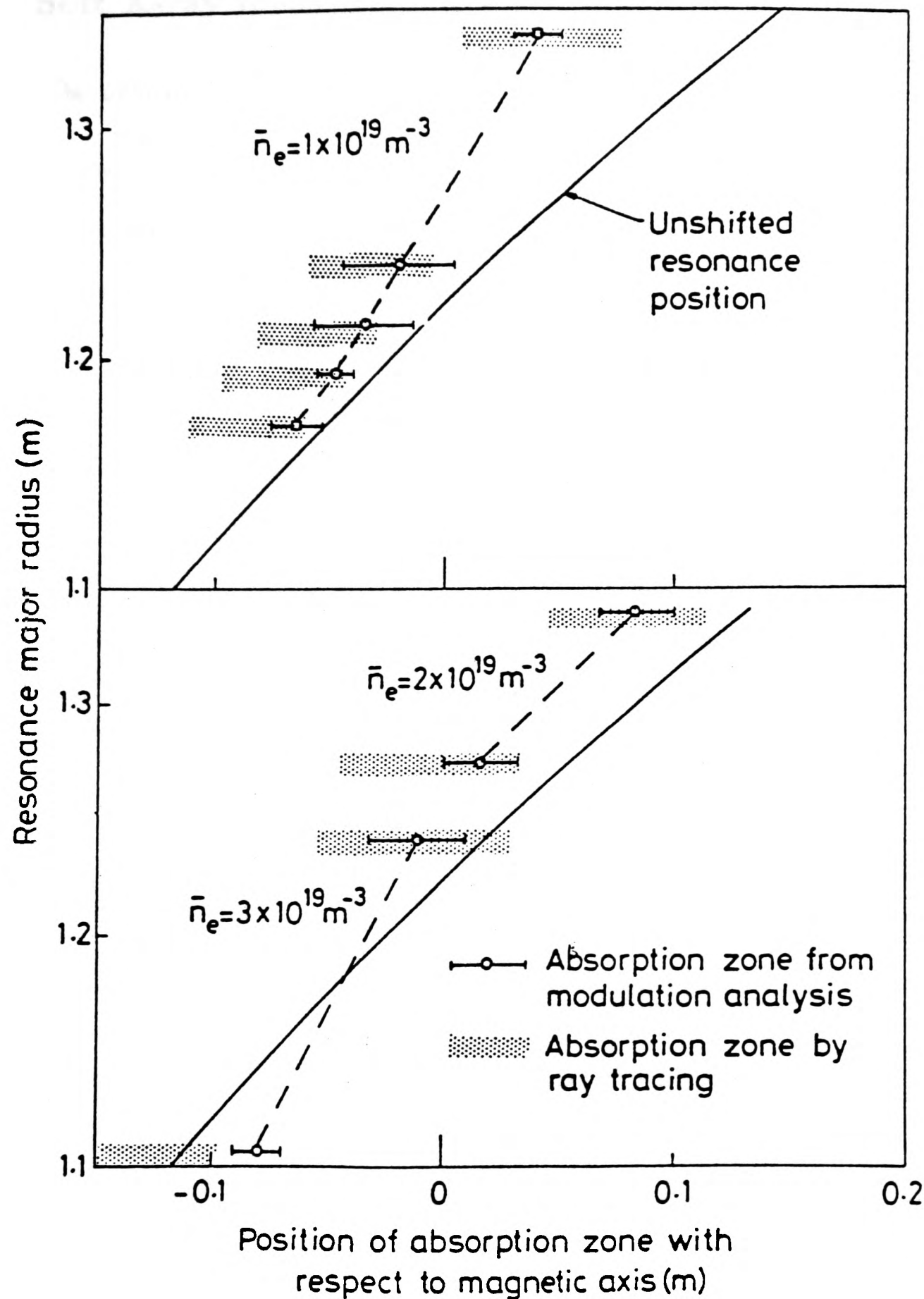


Figure 3.11: Major radius of unshifted ECH resonance compared with the actual Doppler down-shifted absorption zone, as predicted by ray tracing (shaded areas) and estimated from the results of modulation experiments (open circles). The unshifted resonance values include a small correction for the poloidal field and diamagnetism. The positions and widths of the ECRH absorption zones are indicated for three different densities (from Ashraf et al., 1988(b)).

3.3 Soft X-ray measurements

3.3.1 Detection of soft X-ray emission

The signal measured by a soft X-ray detector is an integral of the spectral emissivity over a certain photon energy range, above a cut-off energy defined by the detector filter. The emission arises from bremsstrahlung, and recombination and atomic line radiation, each having a different dependence on electron temperature (see, e.g., McWhirter, 1981; von Goeler et al., 1975; von Goeler, 1978). The problem of calculating the detected SXR emissivity is, of course, similar to that of determining the total electron radiation loss as discussed in section 2.5, the only difference arising from the limits of the integration over photon energy. Nevertheless, this difference in the energy range is significant, because, for photon energies of the order of or greater than the electron temperature (which typically corresponds to energies in the SXR region), the SXR emissivity has a strong, monotonically increasing dependence on the temperature, thus providing a sensitive measure thereof.

Spectral emissivity The combined spectral emissivity for bremsstrahlung and recombination processes is given by

$$\frac{dQ}{d\mathcal{E}} = C n_e n_i T_e^{-1/2} \exp\left(-\frac{\mathcal{E}}{T_e}\right) \sum_{\text{El}} f_{\text{El}} Z_{\text{El}}^2 [1 + \gamma_{\text{El}}(T_e)], \quad (3.8)$$

where dQ is the radiated power per unit volume in all directions,⁷ in the photon energy range $(\mathcal{E}, \mathcal{E} + d\mathcal{E})$, f_{El} is the fractional concentration of ions of element El, and the summation is over all the elements present in the plasma; γ_{El} is the *enhancement factor* for element El, defined as the ratio of recombination to bremsstrahlung emission, which can be expressed as

$$\begin{aligned} \gamma_{\text{El}}(T_e) = & \sum_Z f_Z(T_e) \frac{Z^2}{Z_{\text{El}}^2} \\ & \times \frac{1}{T_e} \left\{ \frac{\xi}{\mu^3} \chi_\mu \exp\left(\frac{\chi_\mu}{T_e}\right) + \sum_{\nu=\mu+1}^{\infty} \frac{2}{\nu} (Z^2 \chi_H / \nu^2) \exp\left(\frac{Z^2 \chi_H / \nu^2}{T_e}\right) \right\}, \end{aligned} \quad (3.9)$$

⁷the value of the constant C is $1.53 \times 10^{-38} \text{ W m}^3 \text{ eV}^{-1/2}$ (for emission over all solid angles, i.e. including the factor 4π)

where f_Z is the (temperature-dependent) relative abundance⁸ of ionization state Z , and the summation is over all ionization states of the element; the first term corresponds to recombination to the ground state (the lowest unfilled shell), with principal quantum number μ , number of vacancies⁹ ξ and ionization potential χ_μ , while the second term is a sum over excited states (all the higher shells), assumed to be hydrogen-like, having an ionization potential $Z^2\chi_H/\mu^2$. The appropriate Gaunt factors in the expressions above are close to unity and have been omitted, but it should be borne in mind that an ionization state will contribute to the recombination radiation only at photon energies above its ionization potential $Z^2\chi_H/\mu^2$, thus leading to the existence of steps or *recombination edges* in the spectrum; for light impurities (e.g. O, C) these will occur at photon energies below the system cut-off energy, but this will not be true of heavy impurities (e.g. Fe). Furthermore, if heavy impurities be present, one will obviously have to include those atomic radiation lines, which are in the SXR region of the spectrum.

The problem of determining the relative abundances of the ionization states (and excitation states if necessary) has been discussed in section 2.5. One recalls that, in the general case of finite impurity confinement and non-applicability of the coronal equilibrium model, the impurity transport introduces a dependence of the ionization states on density, thus weakening the quadratic density dependence of the emissivity, and modifies the temperature dependence; furthermore, the calculation will have to be carried out for given density and temperature profiles, and a specified impurity transport model.

Finally, the fractional concentration of impurity elements in the plasma has to be inferred from measurements of the effective ion charge, and of the total, SXR and frequency-resolved radiation.

Detected soft X-ray emissivity The *local* emissivity per unit volume in all directions that can be potentially detected by a soft X-ray diode system is obtained from an integration of the spectral emissivity over the system acceptance energy range, as determined mainly by the

⁸normalized with respect to the total number of ions of element El

⁹as there are ξ vacancies instead of the maximum $2\mu^2$, the appropriate quantum mechanical weight is $\xi/2\mu^2$

filter used, i.e.

$$X = \int_{\mathcal{E}_c}^{\infty} T(\mathcal{E}) \frac{dQ}{d\mathcal{E}} d\mathcal{E}, \quad (3.10)$$

where $\mathcal{E}_c$ is a lower cut-off energy limit for the filter and $T(\mathcal{E})$ is a characteristic filter transmission function. The evaluation of the energy integral above is straightforward when all the recombination edges are below the cut-off limit, as in this case the energy dependence of the spectral emissivity $\frac{dQ}{d\mathcal{E}}$ is only through the exponential $\exp\left(-\frac{\mathcal{E}}{T_e}\right)$, and not through the enhancement factor which is a function of temperature (and possibly of density) only. With $T(\mathcal{E}) = 1$, one obtains by integrating (3.8)

$$X = C n_e n_i T_e^{1/2} \exp\left(-\frac{\mathcal{E}_c}{T_e}\right) \sum_{\text{El}} f_{\text{El}} Z_{\text{El}}^2 [1 + \gamma_{\text{El}}(T_e)]. \quad (3.11)$$

It transpires that the filter cut-off energy and transmission function play an important role in determining the dependence of the detected soft X-ray emissivity on electron temperature, particularly when the cut-off energy is comparable to or larger than the electron temperature.

In order to obtain the *total* detected SXR power for an arbitrary viewing geometry, one has to consider the volume integral over the viewing cone of the local emissivity above, considering the position-dependent solid angle subtended at the point of emission by the entrance area of the detection system; writing the volume integral as a line integral along the axis of the viewing cone, one obtains

$$I = \int_L \frac{X(l)}{4\pi} \Omega(l) A(l) dl, \quad (3.12)$$

where X is the local emissivity per unit volume over all solid angles, as defined above, Ω is the solid angle subtended at the point of emission by the entrance area of the emission system, and A is the area of the intersection of the viewing cone with the plasma volume; the geometrical parameter ΩA is called the *étendue* of the collecting system. In the important special case, however, of a narrow viewing cone filled with plasma, with a focus near the entrance area, one has $\Omega \propto 1/l^2$ and $A \propto l^2$, so that the étendue is independent of the distance l from the entrance area, and the detected power is simply proportional to the line integral of the local emissivity along the viewing chord.¹⁰ Several techniques exist for the *tomographic reconstruction*

¹⁰the relevant constant of proportionality is $S^2/4\pi h^2$, where S is the entrance area and h is its distance to the focus (assumed to be much smaller than its distance to the plasma)

of the local emission $X(r, \theta)$, given a set of line integrals $I(p, \phi)$, where p is the chordal radius and ϕ is the angle of the normal to the chord in the poloidal plane (refer to figure 5.1). Such techniques introduce a series representation for the local emissivity, and attempt to optimize the coefficients in that series from the measured line integrals. When there is no angular dependence, the reconstruction procedure is known as *Abel inversion*. A method, based on Tschebycheff polynomials for the radial variation and sinusoidal functions for the angular dependence, has been developed by Cormack (1963 and 1964) and applied on tokamak SXR data by Granetz and Camacho (1985); further techniques have been presented by Sauthoff and von Goeler (1979).

Modulation index and chordal integration In order to model the dependence of the detected SXR emissivity on electron temperature, one introduces an incremental logarithmic quotient, or *modulation index*, which is defined as follows:

$$\alpha(T_e) = \frac{\partial \ln X}{\partial \ln T_e}; \quad (3.13)$$

an integration over temperature, using the value at a reference temperature T_C , yields

$$X(T_e) = X(T_C) \exp \left[- \int_{T_e}^{T_C} \frac{\alpha(T)}{T} dT \right]; \quad (3.14)$$

the dependence on electron density is assumed to be quadratic, that is, one takes $n_e n_i \propto n_e^2$, the temperature-dependent ratio of the electron to ion densities (i.e. $\bar{Z}$) being incorporated in the function $\alpha(T_e)$. A Taylor expansion of (3.14) yields the local SXR perturbation caused by a temperature perturbation, as

$$\frac{\bar{X}}{X_0} = \alpha \frac{\bar{T}}{T_0} + \frac{1}{2}(\alpha^2 - \alpha + \alpha' T_0) \left(\frac{\bar{T}}{T_0} \right)^2 + \dots \quad (3.15)$$

Hence, the *local* SXR perturbation will be in phase with the corresponding temperature perturbation, if the latter be sufficiently small; large periodic temperature perturbations (having discrete Fourier representations) cannot affect the local SXR phase at the fundamental frequency through the non-linear terms in the expansion. On the other hand, the amplitude of the SXR perturbation will be sensitive to the magnitude of the modulation index and its dependence on the electron temperature (and effectively its variation over the minor radius). The effects due to

changes in the modulation index on the *line-integrated* SXR perturbation are more complicated; these can be considered more conveniently by rewriting the equation for the SXR perturbation above in the form

$$\tilde{X} \simeq \alpha X_0 \frac{\tilde{T}}{T_0}.$$

An overall increase in the modulation index α will lead to a greater decrease of the average emissivity X_0 (because of the inverse exponential dependence of the latter on the former) and thus to a net decrease of the product αX_0 . For a line of sight near the edge of the discharge, this will not produce an effect in the phase as only few points will be involved, but for a line of sight near the centre, the contribution of the cooler outer region will be diminished, thus weakening the effect of the chordal integration and leading to a more localized integrated perturbation. This feature is of considerable experimental importance, since by manipulating the modulation index, for instance through the introduction of impurities into the plasma or by an appropriate selection of filter, one can adjust the response of the detected SXR signals, so that a localized measurement may be obtained. A numerical investigation was carried out, of the effect of the modulation index on the line-integrated SXR signals, for given profiles of electron density n_e , steady-state temperature T_0 and complex temperature modulation amplitude $\tilde{T}$ (the latter was computed from the Bessel solution of the perturbed diffusion equation, as described in subsection 4.1.2). The results of this analysis, which confirm the qualitative argument above, are presented in figure 3.12; it is clear that, depending on the modulation index, the chordal integration of the local emissivity modifies the modulation amplitudes significantly, as it does the steady-state amplitudes, and can also affect the modulation phase, typically reducing the gradients somewhat, even when the phase of the emissivity is locally equal to that of the temperature.

Experimental and numerical analysis of emission An analysis of the detected soft X-ray emissivity on DITE has been carried out by Fielding (1988), based on the bremsstrahlung and recombination spectra of hydrogen and possible impurities; energy integrals of these spectra have been obtained above a detection cut-off energy $\mathcal{E}_c$ (1keV for the DITE system). It is shown that, for emission dominated by bremsstrahlung from a pure hydrogen plasma or a high-

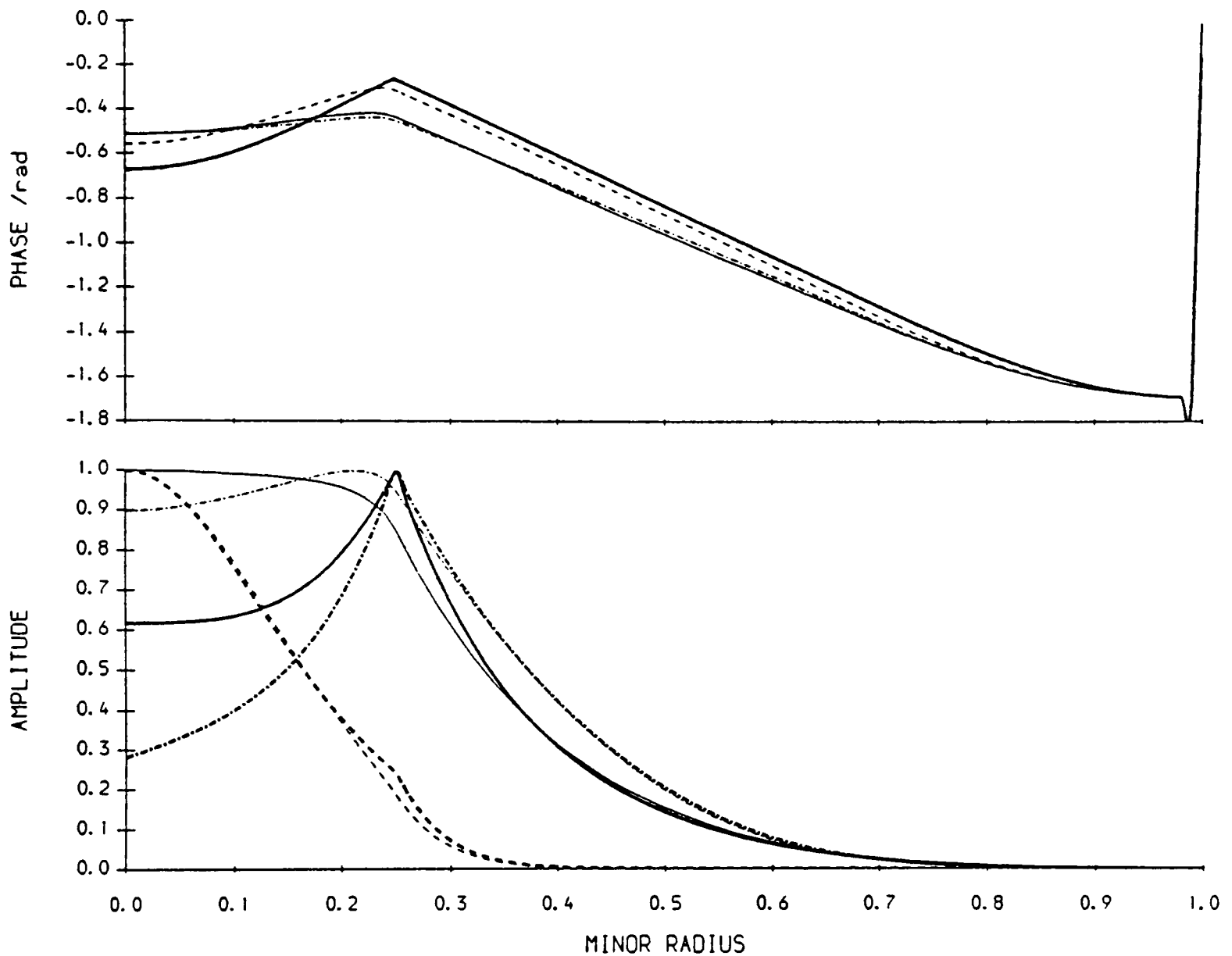


Figure 3.12: (a) Phase and (b) amplitude variations of *local* SXR modulation $\tilde{X}$ (bold curves) compared with those of the corresponding *line-integrated* modulation $\tilde{I}$ (thin curves), as obtained for given profiles of density n_e , steady-state temperature T_0 and complex temperature modulation amplitude $\tilde{T}$ (from solution of diffusion equation with off-axis deposition of the modulated heating power). The effects of different modulation index dependences are indicated: $\alpha=0.4$ (solid curves); $\alpha=6.0$ (broken curves); $\alpha(r)=0.25 \rightarrow 6.0$ (dash-dot curves). The local phase is independent of the modulation index for linear perturbations.

temperature helium plasma, the incremental exponent is given by

$$\alpha(T_e) = \frac{1}{2} + \frac{\mathcal{E}_c}{T_e}; \quad (3.16)$$

for emission dominated by recombination involving only transitions to the ground level of fully ionized species,

$$\alpha(T_e) = -\frac{1}{2} + \frac{\mathcal{E}_c - \chi_\mu}{T_e}, \quad (3.17)$$

leading to the possibility of the incremental exponent being negative.

An inspection of the data from modulation experiments was carried out to estimate the modulation index by comparing the relative modulation levels of the SXR and ECE signals—these tended to be fairly uniform near the centre of the discharge. This analysis led to central values of about 0.3 for hydrogen discharges and 0.5 for helium discharges, which are consistent with the SXR emissivity being dominated by recombination radiation from fully ionized carbon and oxygen impurities¹¹ (these being the principal impurities in most tokamaks), at concentration levels of about 2% each. It was found more difficult to estimate the modulation index in this way from data farther away from the centre, as the relative modulation levels for the outer region increased with radius and were not reproducible; it was nevertheless evident that the modulation index increased rapidly with decreasing temperature, since the relative modulation level of the SXR emissivity rose much more rapidly with radius than that of the temperature (see figures 4.2 and 4.3 respectively).

A radiation computer code was written, based on the equilibrium of the impurity ionization states,¹² in order to evaluate the radiation of a hydrogen or helium plasma containing carbon and oxygen impurities. The code determines the relative concentrations of the various impurity ionization states, evaluates the spectral emissivity, and calculates its integrals (a) over the entire energy range to obtain the total radiated power (including the line radiation) and (b) over the

¹¹the ionization potentials for C⁵⁺ and O⁷⁺ are 490eV and 870eV respectively

¹²A steady-state version of (2.70) was used, incorporating a simplified impurity flux in order that the finite impurity confinement might be accounted for; atomic data for carbon and oxygen, including ionization and recombination rates and total line radiation coefficients, were taken from tables; (3.8) and (3.9) were used for the spectral emissivity of bremsstrahlung and recombination radiation.

energy range pertinent to the SXR detection system using an appropriate filter function; the response of the SXR detection system to modulation of the electron temperature is expressed in terms of the modulation index $\alpha(T_e)$. These parameters are calculated as functions of electron temperature, for given impurity ion concentrations that are assumed to be constant throughout the plasma volume. The carbon and oxygen concentrations were varied to match the central values of the modulation index and effective ion charge, as observed. The modulation index and effective ion charge thus calculated are plotted as functions of electron temperature in figure 3.13, which also shows the detected SXR emissivity and the total radiated emissivity.

3.3.2 The DITE soft X-ray (SXR) diagnostic

The DITE soft X-ray diagnostic consists of two arrays of diode detectors, mounted vertically and horizontally on the torus at the same toroidal position; the vertical and horizontal arrays comprise 21 and 19 detectors respectively. The geometry and nomenclature of the system are shown in figure 3.14. The spatial resolution of this diagnostic is about 11–19mm. Each array incorporates a pinhole and two parallel rows of diode detectors (in order to improve the spatial resolution) in a vacuum box. The detectors are silicon barrier diodes, producing a current proportional to the detected radiated power. The filters, located at the two pinholes, are made from an aluminium layer of thickness $0.1\mu\text{m}$ on a mylar substrate of thickness $6\mu\text{m}$; these provide a low-energy cut-off calculated to be about 1keV, with 1% transmission at 780eV. The signals correspond to line integrals of the local soft X-ray radiation emitted over a range of photon energies.

The data acquisition system includes analogue amplifiers, analogue-to-digital converters (ADC's), digital buffer memory units and a local computer. A DC bias is applied to each diode detector to optimize its response and recovery time, and each signal is filtered by an RC circuit to reduce the noise and aliasing effects. The amplifier gain is adjustable and was set according to the level of the corresponding signal; the sampling rate was $90\mu\text{s}$. Two sets of samples were recorded before and after each discharge, allowing a zero level to be calculated and subtracted from the signal, in order to compensate for the amplifier drift that is normally

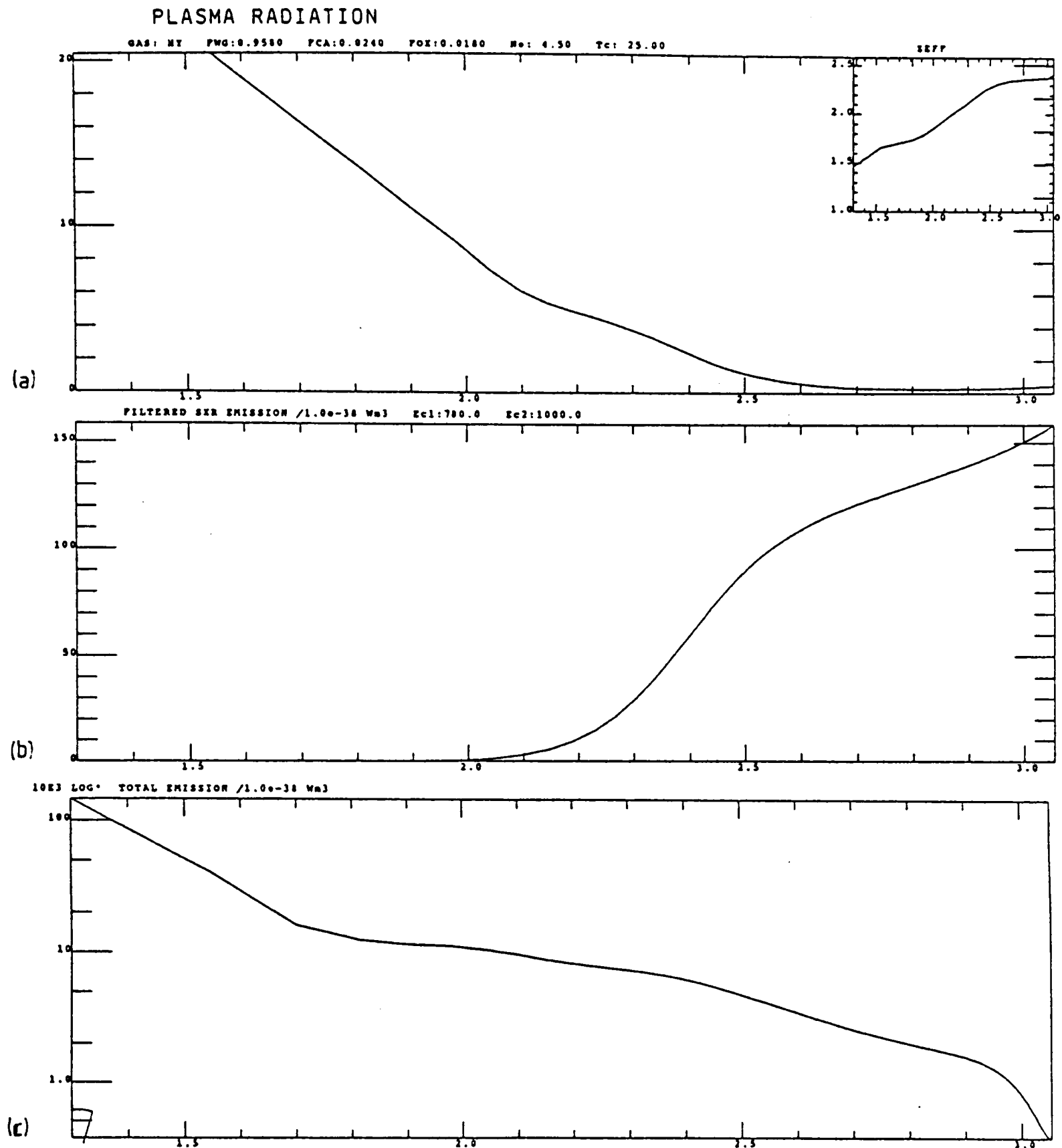


Figure 3.13: (a) Modulation index $\alpha(T_e)$ of detected SXR emissivity (DITE system) and effective ion charge $Z_{eff}(T_e)$ (inset); (b) detected SXR emissivity $X(T_e)$, (c) total radiated emissivity $Q_{rad}(T_e)$ (multiply by n_e^2 to obtain power densities): calculated as functions of electron temperature ($\log T_e/\text{eV}$), under steady-state conditions with a finite impurity confinement time (25ms), for a hydrogen plasma with 2.4% carbon and 1.8% oxygen impurities.

ALL DIMENSIONS IN mm

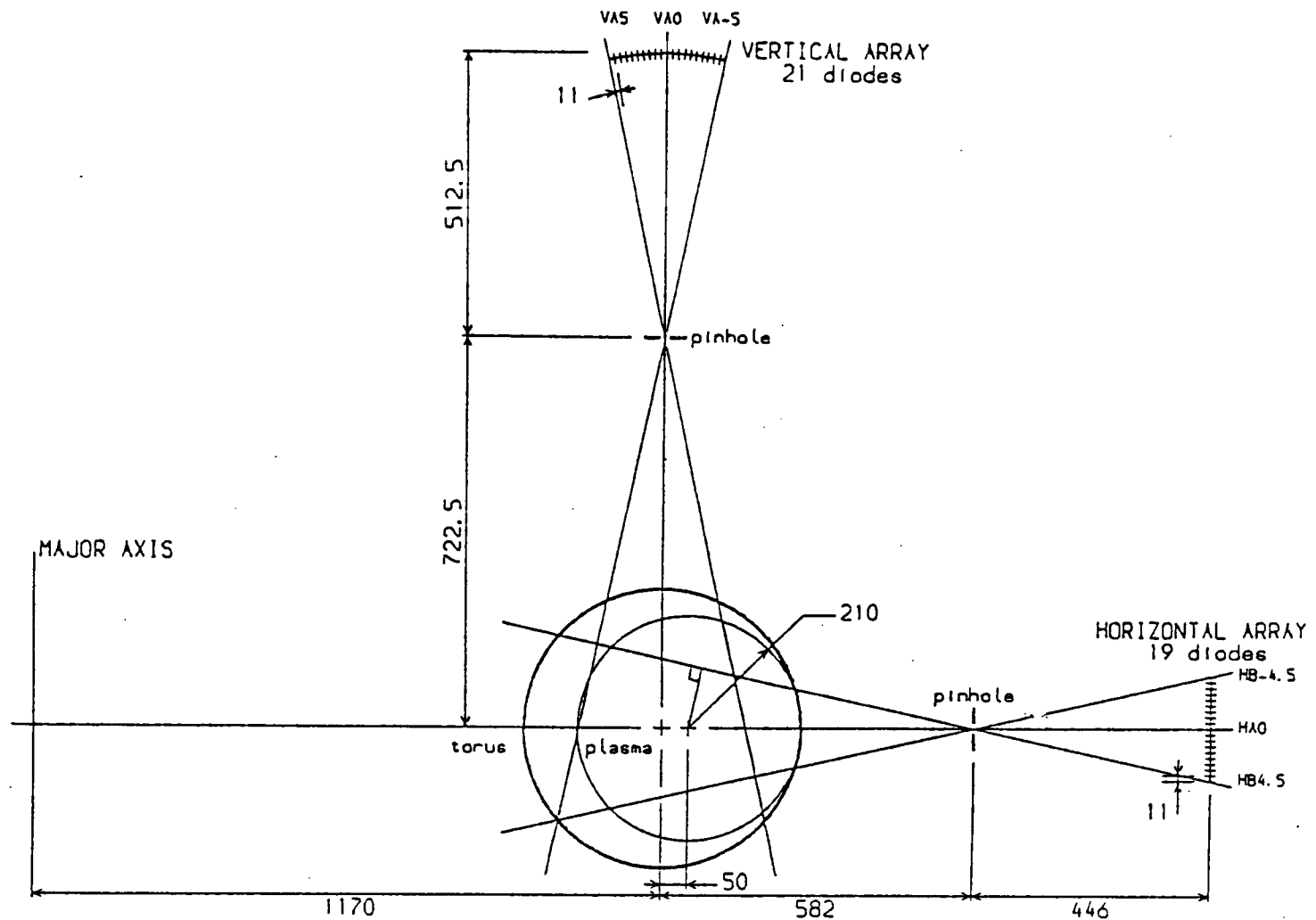


Figure 3.14: Geometry and nomenclature of the SXR diode array detection system on DITE.

present. Each signal was scaled using a calibration factor and the amplifier gain; the latter was calculated from the recorded amplifier response, to a step voltage following a geometric progress, which was applied to the amplifiers after each discharge. The amplifier saturation level was arranged to be outside that of the ADC, so that any saturation in the recorded signals could be readily detected. The data comprising the time records, including the gyrotron power reference signal from the microwave detector on the transmission waveguide, were recorded in compressed data files and transferred to a mainframe computer for analysis.

This diagnostic provided signals that were amenable to an analysis using fast Fourier transform techniques, whose purpose was to extract the amplitude and relative phases (with respect to the ECRH modulation) of the signals, as functions of chordal radius. The detectors in the system were subject to a differential geometrical error due to their alignment with the pinholes (vignetting), and had varying sensitivities. A further error in the measured SXR emissivity profiles arose from the geometrical variation of the effective filter thickness, depending on the

obliqueness of the line of sight, the maximum thickness variation being about 2.5% for both detector arrays. These defects of the SXR system have not been adequately corrected and result in some discontinuities in the measured profiles and also in discrepancies between the two detector arrays. It was found impossible to apply tomographic reconstruction techniques to these data, mainly because of the poor quality of the amplitude profiles. It has been observed, in the course of analysing the data from the modulation experiments, that the SXR signals obtained during apparently identical discharges occasionally differed in amplitude and radial dependence; it appears that these discrepancies were due to a changing plasma impurity state. Typical SXR signals recorded during a modulated discharge are shown in figure 3.15.

3.4 Electron cyclotron emission measurements

3.4.1 Detection of electron cyclotron emission

Electron cyclotron emission is a form of bremsstrahlung which arises from the acceleration of the electrons as they gyrate around the magnetic field lines, and occurs in the microwave region, at the electron cyclotron frequency and its harmonics (subject, as with ECRH, to Doppler and relativistic shifts). The calculation of the emission and re-absorption of the cyclotron radiation is in general extremely complicated, being particularly sensitive to the electron energy distribution. However, for a Maxwellian plasma and for harmonics and wave modes for which the plasma is *optically thick*, re-absorption occurs in the plasma such that the resulting equilibrium emission intensity is given by the Planck expression for black-body radiation,¹³ in which the radiative

¹³Let the emission intensity be $j(\omega)$ and the absorption coefficient be $\alpha(\omega)$; the radiation intensity $I(\omega)$ is governed by

$$\frac{dI}{ds} = j(\omega) - I\alpha(\omega);$$

an integration along a ray through the plasma using the integrating factor $\exp(\int \alpha ds)$ gives

$$I = \frac{j}{\alpha}(1 - e^{-\tau}),$$

where $\tau = \int_L \alpha ds$ is the optical depth, the measurement of optical thickness (reflection on the surrounding walls has been ignored); for $\tau \rightarrow \infty$, one obtains Kirchhoff's law $I = j/\alpha$, thus leading to the black-body result.

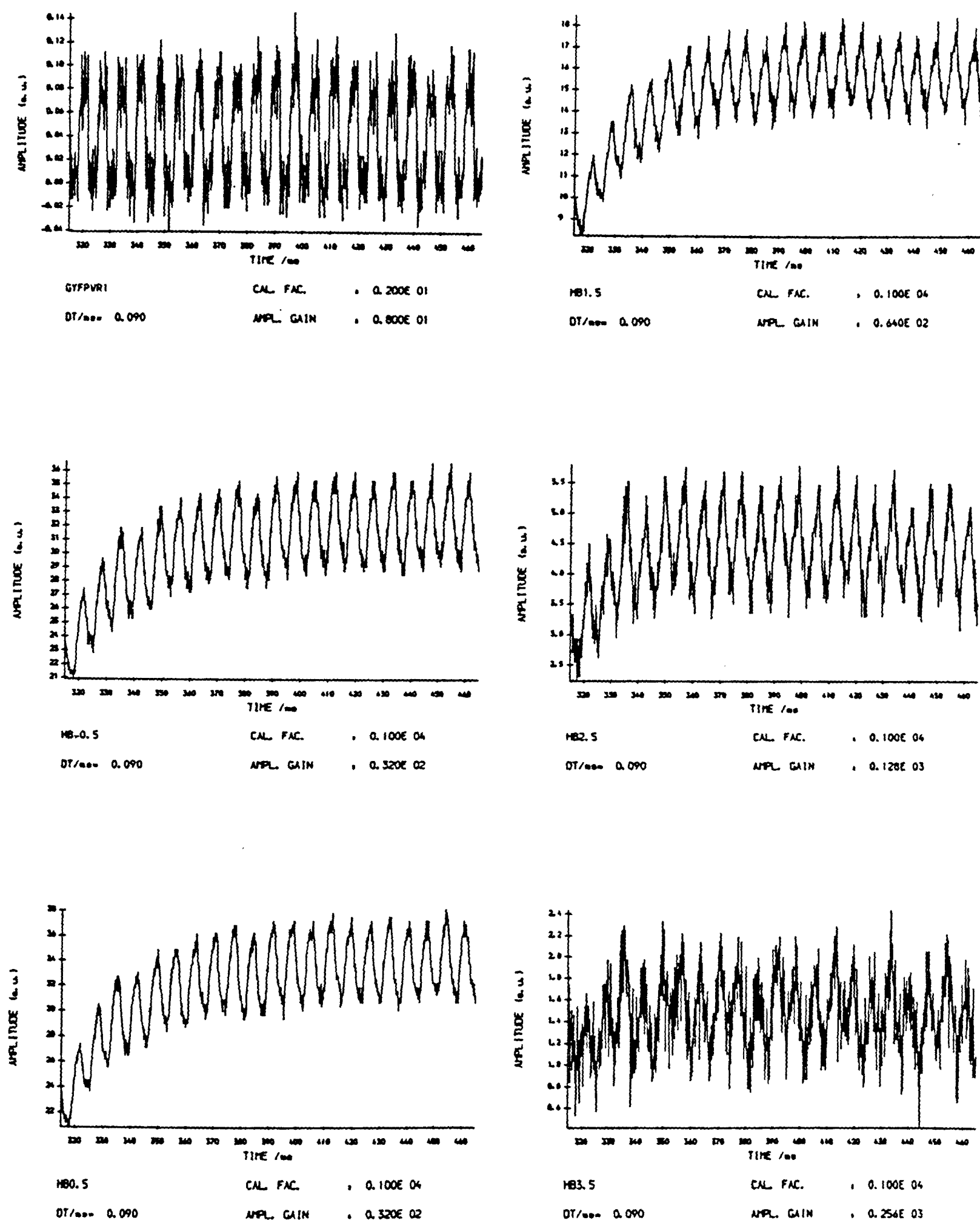


Figure 3.15: Horizontal array SXR signals from low-density discharge with 2nd harmonic ECRH (same discharge as in figure 3.2); the reference signal (GYFPVR1) is from a waveguide-mounted detector, indicating the forward power; the signals are identified by the respective chord codes (HB-0.5-HB3.5, from the centre towards the top). The modulation period is 7ms (4/3ms on/off).

temperature provides one with a good estimate of the electron temperature:

$$I(\omega) = \frac{\hbar\omega^3}{8\pi^3c^2} \frac{1}{\exp(\hbar\omega/T) - 1} \simeq \frac{\omega^2 T_e(R(\omega))}{8\pi^3c^2}; \quad (3.18)$$

the first result is the Planck formula and the second the Rayleigh-Jeans approximation which is valid in the classical limit $\hbar\omega \ll T$ (this is, of course, a very good approximation in the case of ECE radiation which is in the microwave region). On the other hand, for harmonics and wave modes for which the plasma is *optically thin*, the detected emission is proportional to $n_e(R)T_e(R)^\ell$, where ℓ is the harmonic number.

The major radius $R(\omega)$ at which the emission occurs at the frequency ω , is obtained from the resonance condition

$$\omega = \ell\omega_{ce}(R) = \ell \frac{eB_0}{m_e} \frac{R_0}{R(\omega)}. \quad (3.19)$$

It can be seen that harmonic overlap is possible in a tight-aspect-ratio tokamak. The propagation of the ECE waves is analogous to that of the ECRH waves (see CMA diagram, figure 3.3). With observation from the low-field side, the O-mode is used at the fundamental frequency, and the X-mode at the second harmonic frequency (the X-mode is subject to the low-density cut-off at the fundamental frequency). The horizontal broadening of the observation profile arises from the Doppler and relativistic broadening of the emission lines, as is the case for ECRH absorption,¹⁴ and, of course, from the finite frequency resolution of the measuring instrumentation. In practice, the resolution along the direction of observation is limited to 1% of the major radius, the resolution perpendicular to this direction being determined by the antenna radiation pattern.

The most important advantage of the detection of electron cyclotron emission, over the measurement of soft X-ray emission, is the potentially direct measurement of electron temperature. However, an important consideration is the presence of non-thermal electrons in the energy distribution, as in this case the black-body result above is not valid, and absorption and emission must be calculated independently; even a small fraction of superthermal electrons can significantly change the result. Because of the predominantly relativistic shift, the contribution of superthermal electrons to the emission is shifted towards the low frequencies and will consequently distort the data on the low-field side. Details of the underlying theory and of practical

¹⁴in the case of ECE detection with perpendicular viewing, the relativistic broadening is dominant

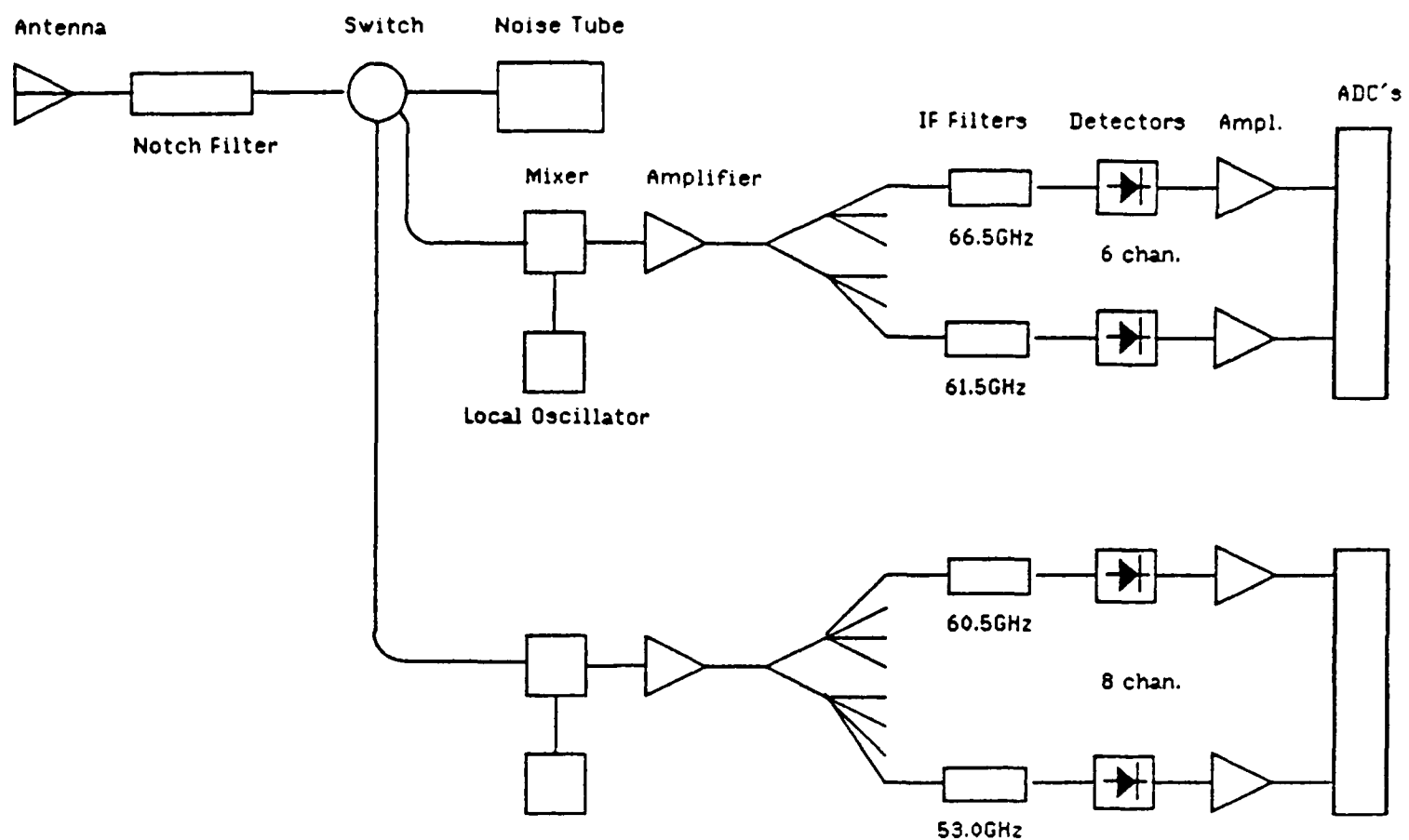


Figure 3.16: Schematic diagram of the ECE heterodyne detection system on DITE.

detection systems have been presented by Equipe TFR (1978) and Costley (1982); the theory is treated in the extensive review by Bornatici et al. (1983) and by Hutchinson (1987).

3.4.2 The DITE electron cyclotron emission (ECE) diagnostic

The DITE electron cyclotron emission diagnostic systems are a Fourier transform Michelson interferometer and a heterodyne detection system; the latter was used in the present experiments. Heterodyne detection systems are characterized by good temporal and spatial (frequency) resolutions and a high sensitivity. The DITE heterodyne system comprises 14 channels at frequencies between 53GHz and 66.5GHz, and is shown schematically in figure 3.16. The ECE radiation from the plasma is collected by a rectangular, tapered horn antenna (60×35mm with a 250mm tapered length), which is optimized for X-mode second harmonic frequency detection and located on the outer equatorial plane, and is channeled through an angled waveguide. Four notch filters with a 30MHz bandwidth are used to reject the 60GHz gyrotron radiation. A noise tube is provided for the calibration of the radiometer. The radiometer system is split into two heterodyne receivers, one having 8 channels in the range 53.0–60.5GHz, and one having 6 channels in the range 61.5–66.5GHz. The signal from the antenna is split, passed through appropriate filters

and fed into the two receiver systems. Each heterodyne receiver has a fixed-frequency local oscillator and a broad-bandwidth mixer. The IF (intermediate frequency) signals are selected for detection by band-pass filters at 1GHz intervals. The IF detectors are Schottky barrier diodes followed by amplifiers.

The data acquisition system is similar to that of the SXR diagnostic already described, but the duration of sampling at $100\mu\text{s}$ was limited by the memory capacity to about 150ms. The ECE signals were also analysed using fast Fourier transform techniques. The channels of this diagnostic were relatively but not absolutely calibrated, because of certain experimental constraints and the lack of electron temperature data from the Thomson scattering diagnostic. The relative calibration has been carried out with the help of a series of similar discharges with a scanned toroidal field, such that the ECE channels were located at the same major radius in succession; averaged calibration factors were then obtained for all the channels. The spatial resolution of this diagnostic is 16–21mm along the line of sight, and about 70mm perpendicular to it, for O-mode detection. The major radii of the observation points are calculated for each channel from the vacuum magnetic field, using (3.19) and a small correction for the estimated poloidal magnetic field and plasma diamagnetism. The 60.5GHz channel was effectively a gyrotron power monitor in the experiments with ECRH; an additional gyrotron power signal, from a microwave detector or “sniffer probe” inside the torus, was also available on this diagnostic for some of the discharges.

The optical depth at the central electron temperature for a typical DITE plasma is about 6.5 for the second harmonic X-mode, but only about 1.1 for the fundamental O-mode, as shown in figure 3.17. This suggests that the radiative temperature measured in the former case will be a good measurement of the electron temperature over most of the profile, but that measured in the latter case will be a reliable estimate only close to the profile centre. However, the density was low in the low-field discharges with second harmonic ECRH and ECE detection, leading to increased superthermal electron effects.

The data obtained from the outer (low-frequency) channels are subject to a modification of the simple proportionality to temperature, because of the shifted contribution of the superther-

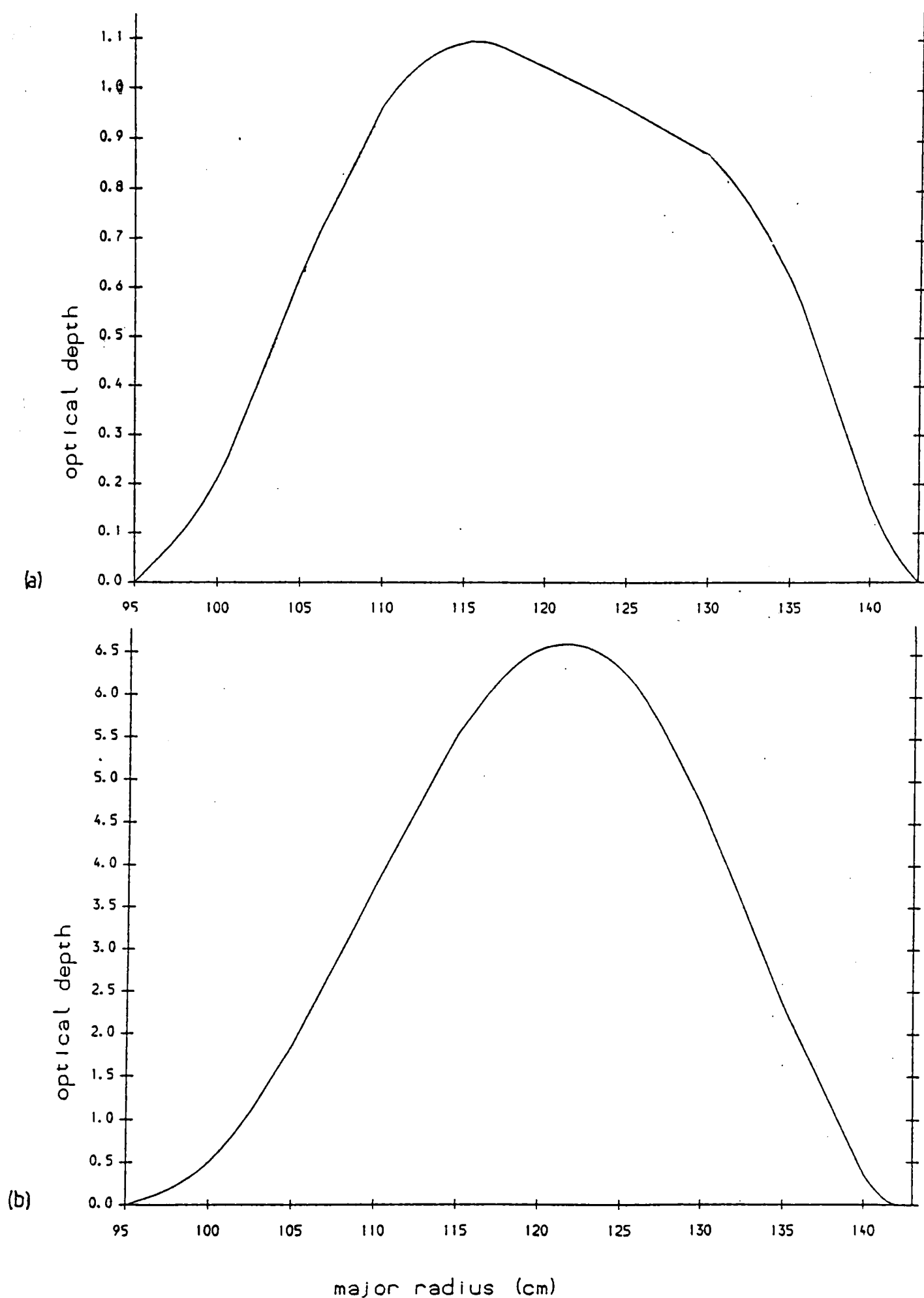


Figure 3.17: Optical depth profiles for ECE detection on DITE (a) at the fundamental frequency O-mode and (b) at the second harmonic frequency X-mode, calculated for perpendicular viewing, on the assumption of parabolic density and temperature profiles (Simonetto, 1988).

mal electrons, and do not therefore yield a direct measure of electron temperature; this usually makes them unsuitable for analysis. Further problems were associated with unstable amplifier gains and drifts. The signal levels that were measured by this diagnostic, of both steady and modulated components, often changed drastically between successive discharges, without there being any obvious changes in the plasma parameters. Some of the individual channels often produced spurious signal levels. Somewhat better results were obtained, as expected, in experiments with second harmonic X-mode detection. In all cases, however, this diagnostic yielded rather unsatisfactory temperature modulation data which could not be independently used in the analysis. Typical ECE signals recorded during a modulated discharge are shown in figure 3.18.

3.5 Electron density measurements

3.5.1 Microwave interferometry

The measurement of electron density using microwave interferometry is based on the density dependence of the refractive index for the O-mode —perpendicular incidence of the wave is necessary to ensure that this mode will depend only on the plasma frequency and not on the cyclotron frequency. One obtains, for $\omega \gg \omega_{pe}$, the following approximation for the refractive index of a plasma of electron density n_e :

$$n_{\perp} = \frac{ck}{\omega} = \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)^{1/2} \simeq 1 - \frac{n_e e^2}{2\epsilon_0 m_e \omega^2}; \quad (3.20)$$

hence, the phase shift between a beam passing through the plasma along a line of sight L and a reference beam is given by

$$\Delta\phi = \frac{2\pi}{\lambda_0} \int_L (1 - n_{\perp}) dl = \frac{e^2}{2\epsilon_0 c m_e \omega} \int_L n_e(l) dl. \quad (3.21)$$

In a microwave system, the phase difference $\Delta\phi$ is measured as follows (see, e.g., Equipe TFR, 1978): a microwave source is frequency-modulated by a sawtooth signal, and two (or more) beams are produced, one propagating along a reference path and one (or more) propagating through the plasma; the phase difference between the reference and plasma beams is thus

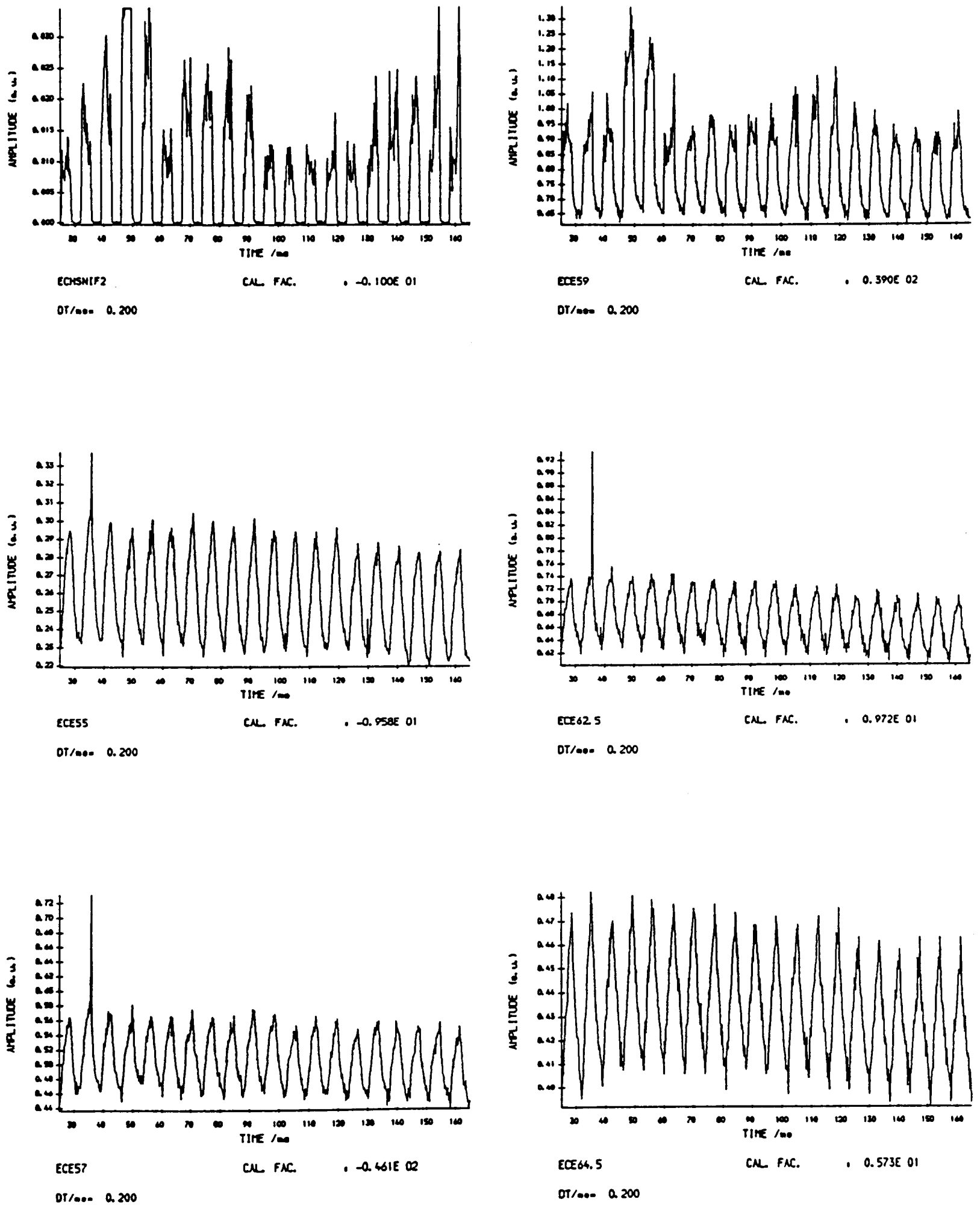


Figure 3.18: ECE signals from low-density discharge with 2nd harmonic ECRH (same discharge as in figure 3.15); the reference signal (ECHSNIF2) is from a detector inside the torus, monitoring the gyrotron power; the signals are identified by the respective channel frequencies (ECE55–ECE64.5).

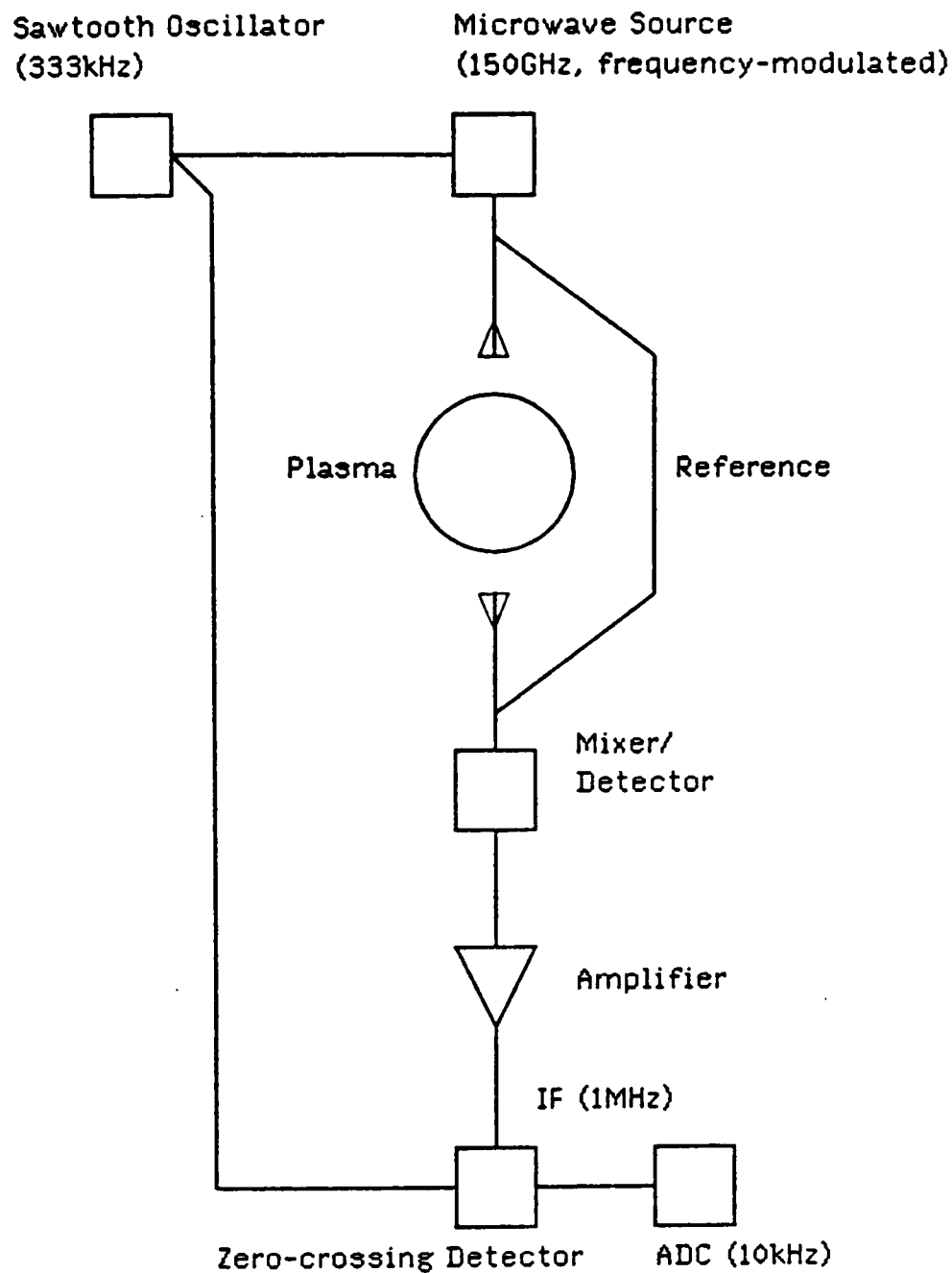


Figure 3.19: Schematic diagram of the density microwave interferometer (MWI) system on DITE (only one chord is shown).

converted to a frequency difference; when the beams are mixed and detected, a signal at the intermediate frequency is obtained; finally, a zero-crossing detector is used to sample the modulating sawtooth signal at every cycle of the intermediate frequency signal, to produce pulses proportional to the phase difference and hence to the line-integrated density; the final sampling is at a much lower rate than the sawtooth and intermediate frequencies; by making the intermediate frequency greater than the sawtooth frequency, a pattern of several parallel traces is obtained, separated by one fringe (a phase difference of 2π).

3.5.2 The DITE microwave interferometer (MWI) diagnostic

The DITE density diagnostic is a 2mm (150GHz) microwave interferometer¹⁵ system; a phase difference of one fringe corresponds to a line-averaged density of about $0.22 \times 10^{19} \text{m}^{-3}$ (on the central chord). The system, schematically shown in figure 3.19, comprises a 4-channel arrangement and a single-channel one, each using one microwave oscillator. Both systems have parallel, vertically viewing, movable chords. The chord of the single-channel system passes near the plasma centre; this provided the measurement of the line-averaged plasma density. The phase shift in each chord is measured by the frequency-modulation technique, as described above. The sawtooth and intermediate (fringe) frequencies are 333kHz and 1MHz respectively, the sampling interval being $100\mu\text{s}$ (10kHz); the data therefore consist of three parallel traces, separated by one fringe. Channels with faster sampling rates, but limited in time range, were provided for the purpose of counting the fringes during the density rise and decay phases of the discharge, in order to obtain the steady-state density. The 4-channel interferometer was characterized by a poor signal-to-noise ratio, because all its channels were fed by a single microwave source. The amplifiers had unstable gains and drifts which further deteriorated the accuracy. The time sequences produced by this diagnostic were not synchronized with those of the other diagnostics, as no reference signals were available to it. Its usefulness in the modulation experiments was consequently severely limited, and it has been possible only to obtain the line-averaged density modulation levels by fitting sinusoidal oscillations with quadratic trends to the raw data, using a least-squares optimization; because of the low signal-to-noise ratio, it has been impossible to obtain the relative phase of the density modulation. The density signals obtained in a modulated discharge with second harmonic heating¹⁶ are shown in figure 3.20.

¹⁵the abbreviation MWI will be used hereafter

¹⁶the line-averaged density signals from low density discharges with second harmonic resonance showed the largest relative modulation amplitudes that were observed (see following chapter)

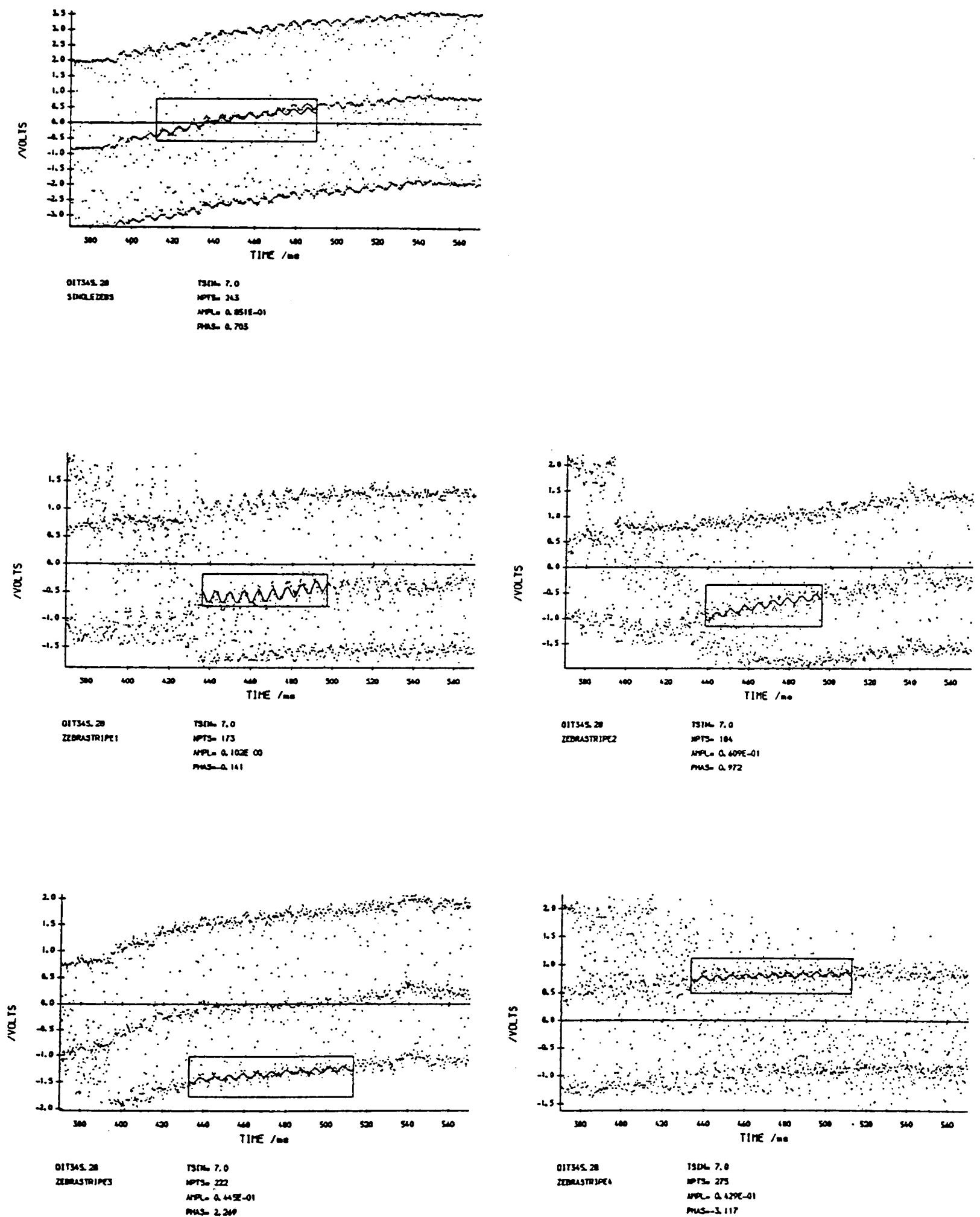


Figure 3.20: Line-averaged density signals from low-density discharge with 2nd harmonic ECRH (same discharge as in figure 3.15); the signals from both single- and multi-channel systems are shown (SINGLEZEBS, ZEBRASTRIPE1-ZEBRASTRIPE4). The small modulations of the signals are fitted by sinusoidal functions with quadratic trends.

3.6 Spectral analysis of modulated signals

3.6.1 Background: the discrete Fourier transform (DFT)

The soft X-ray and electron cyclotron emission signals that were recorded during the modulated ECRH experiments exhibit an oscillating component due to the modulation. Radial transport in the plasma manifests itself in the form of a time delay between the oscillations of the response and reference signals; the time delay increases, and the oscillation amplitude decreases, with an increasing radial distance from the absorption zone.

The analysis of these experimental data in the frequency domain is clearly advantageous, because the modulated part of each signal can be readily extracted and higher harmonics can be analysed; for a linear system, the harmonics provide an indication of the response at higher modulation frequencies. Details of the principles and methods of spectral analysis of signals using discrete and fast Fourier transforms can be found in Bendat and Piersol (1980); Oppenheim and Willsky (1983); and Bracewell (1965, 1986). The discrete Fourier transform (DFT) effects a transformation from the time to the frequency domain, that is, it enables the frequency spectrum of a signal to be calculated. The discrete Fourier transform of a temporal series f_t is defined as the complex number sequence

$$\hat{f}_k = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} f_t \exp\left(-i \frac{2\pi k}{N} t\right), \quad \text{for } k = 0, 1, \dots, N-1, \quad (3.22)$$

where N is the number of samples in the temporal series. If the sampling interval for the temporal series be Δt , then the corresponding frequency interval of the spectrum is

$$\Delta f = \frac{1}{N \Delta t}; \quad (3.23)$$

in general there are $N-1$ such intervals, but because of the symmetry properties, only $[N/2] + 1$ independent values exist for real temporal series; hence, the maximum frequency that can be resolved is the Nyquist frequency

$$f_{\text{Nyq}} = \frac{1}{2\Delta t}. \quad (3.24)$$

In order to study the relation between two signals, one of them being a reference signal g_t and the other a response signal f_t , one uses their cross-correlation; in the frequency domain,

this is given by the convolution theorem,

$$\hat{c}_k = \sqrt{N} \hat{f}_k \hat{g}_k^*. \quad (3.25)$$

The relative phase spectrum (cross-phase spectrum) of the response signal with respect to the reference signal is given by

$$\phi_k = \arg(\hat{c}_k) = \begin{cases} \arccos\left(\frac{\Re(\hat{c}_k)}{|\hat{c}_k|}\right), & \text{if } \Im(\hat{c}_k) \geq 0 \\ 2\pi - \arccos\left(\frac{\Re(\hat{c}_k)}{|\hat{c}_k|}\right), & \text{if } \Im(\hat{c}_k) < 0. \end{cases} \quad (3.26)$$

The amplitude spectrum of the response signal is given by

$$|\hat{f}_k| = \frac{1}{\sqrt{N}} \frac{|\hat{c}_k|}{|\hat{g}_k|}. \quad (3.27)$$

When a tomographic reconstruction technique is being employed to recover the local response from line-integrated data, it should be possible *first* to apply the Fourier transformation to the line-integrated signals and *subsequently* to perform the reconstructions of the resulting (line-integrated) real and imaginary parts, in order finally to obtain the local modulation amplitude and phase. This approach, based on the linearity of the two processes, is straightforward and economical in computation, and its applicability was confirmed by numerical simulation; it was, however, found impossible to apply it on SXR data from the DITE diagnostic.

3.6.2 Computational implementation: the fast Fourier transform (FFT)

Four FORTRAN programmes have been developed to process the experimental SXR and ECE data, employing fast Fourier transform (FFT) algorithms to compute the cross-amplitude and cross-phase spectra between the response and reference signals. The FFT is an algorithm for the evaluation of a DFT, which is exact, but considerably more economical in computation. Two different techniques for the spectral analysis of the signals have been implemented:

1. An FFT algorithm¹⁷ is used to compute a DFT for each signal, after a quadratic trend is subtracted from it. The coefficients of the quadratic trend are calculated from the average

¹⁷the NAG routine C06FAF (FFT) is used (NAG, 1984, vol. 1)

values of the signal in three successive time ranges of equal duration. The amplitude and relative phase are then obtained from the two DFT's of the reference and response signals, by calculating their cross-correlation. In this case, the "spectral leakage" has to be minimized by making the duration of the analysed segment of the signal equal to an integer multiple of the modulation period, or equivalently, by making the frequency interval a divisor of the fundamental frequency. This is necessary in order that the information in the spectra may be prevented from leaking into several frequency samples, thus leading to indistinct spectral peaks and distorted results.

2. Spectral smoothing of the cross-correlation spectra by a trapezium window is used. This method uses a running, trapezium-weighted average of the unsmoothed correlation spectrum, as calculated from two FFT operations. A linear trend correction and a cosine taper are applied in the time domain.¹⁸

The results obtained from these two methods were virtually identical and not sensitive to changes in the various parameters used for the spectral analysis (frequency intervals, averaging window widths etc.). The FFT results were further confirmed by fitting sinusoidal oscillations with quadratic trends to the signals in the time domain, using a least-squares optimization. In most cases, the results were also found to be insensitive to the choice of the time interval for the signal analysis, pointing to the important conclusion that the response to the modulated heating was *time-invariant*. Typically, the response signals were analysed over the entire duration of the ECRH modulation.

The correlation spectra that were obtained from the SXR signals of figure 3.15 are shown in figure 3.21; the corresponding cross-phase spectra are shown in figure 3.22. The FFT techniques were extremely effective in extracting the modulated component, even in cases where only weak signals with a relatively high noise content, covering as few as 4 modulation periods, were available for analysis. These codes were used to extract the modulation amplitudes $|\hat{f}|$ and relative phases ϕ of the signals, as well as the average amplitudes $\langle f \rangle$ over the analysed time interval, and the relative modulation amplitudes $|\hat{f}|/\langle f \rangle$ (i.e. the absolute modulation ampli-

¹⁸the NAG routines G13CBF (auto-correlation) and G13CDF (cross-correlation) are used (NAG, 1984, vol. 6)

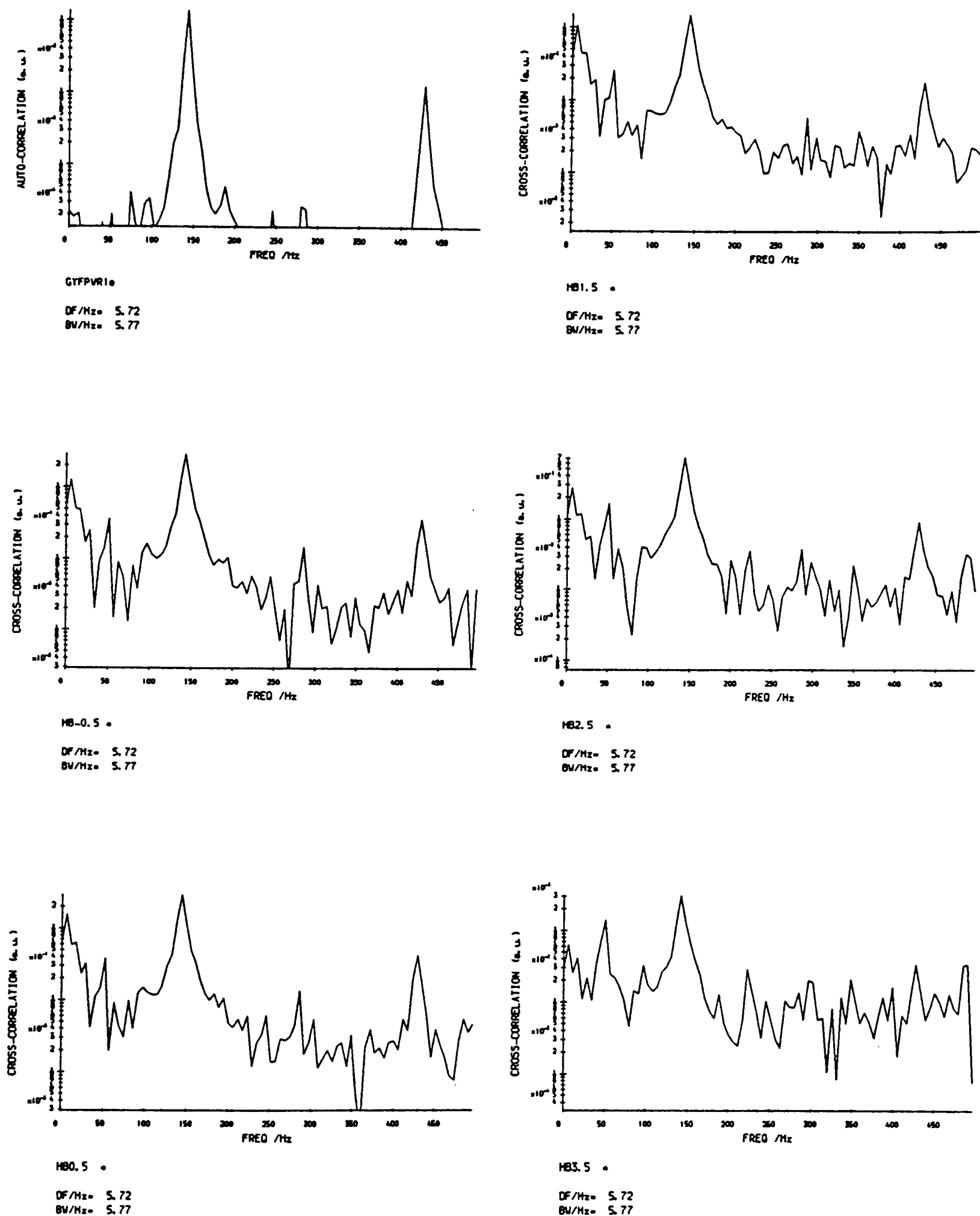


Figure 3.21: Auto-correlation spectrum of reference signal and (normalized) cross-amplitude spectra of SXR signals of figure 3.15 (the FFT analysis has been carried out over the time range of that figure). The modulation frequency is 143Hz: the peaks corresponding to the fundamental, second and third harmonic components are clearly discernible (note the higher amplitude of the third harmonic). The frequency interval is 5.72Hz and the bandwidth is 5.77Hz.

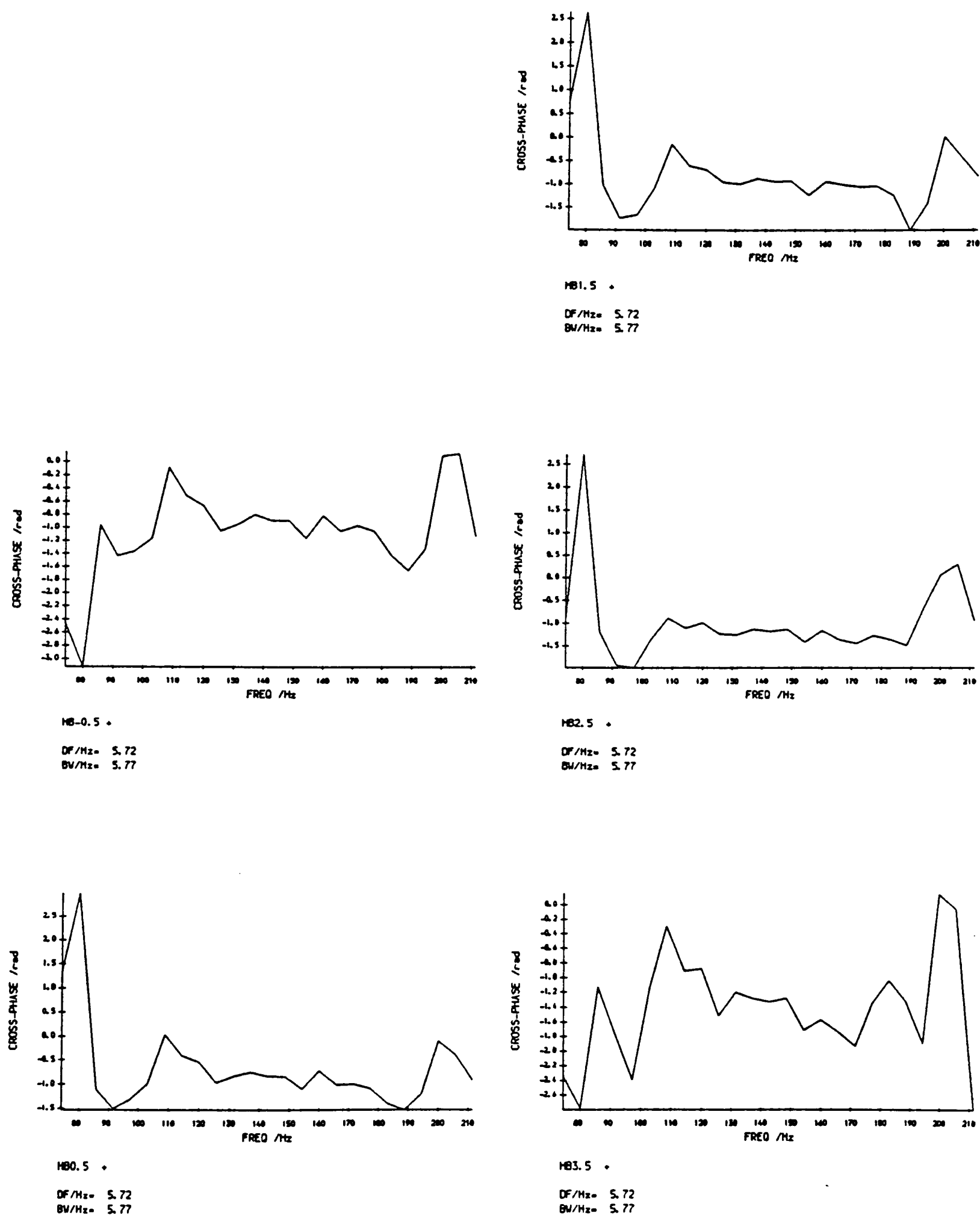


Figure 3.22: Cross-phase spectra of SXR signals of figure 3.15. 12 spectral points are shown on either side of the modulation frequency; the flat segment of the phase spectrum near the modulation frequency is evident.

tudes normalized with respect to the average signal levels). The relative phases and the relative modulation amplitudes are clearly insensitive to calibration errors in the channels, but different temperature dependences between channels (as might occur, for instance, because of different SXR filter transmission characteristics) will have an effect, even when the equilibrium profiles are corrected by multiplicative factors. These modulation parameters were calculated at the fundamental modulation frequency and also at its higher harmonics. The relative amplitudes of the harmonic components of the square wave modulating signal depend on the ratio of the “on” time to the period. Most modulation experiments were carried out using a square wave with a 7ms period and a 4ms “on” time;¹⁹ the fundamental and third harmonic components of the resulting modulated signals were used in the transport analysis.

It is possible, in principle, to obtain estimates of the *relative* errors in the phase results as follows: the cross-phase spectrum between each signal and the reference is expected to be relatively flat on either side of each peak (see figure 3.22), and the weighted mean and standard deviation can be calculated for a small set of three or five spectral samples centred on the modulation peak of the cross-amplitude spectrum. This procedure has been employed for some of the experimental data, leading to error estimates for the relative phase profiles; however, it often resulted in unrealistically large error bars, except when the frequency interval was made very small. The profiles of amplitude and relative phase, including error bars for the phase, extracted from the spectra of the SXR data of figure 3.15, are shown in figure 3.23. It is clear that the phases of all but the weakest signals can be obtained with good accuracy.

The parameters that were obtained for each set of signals were plotted against the chordal radius in the case of the SXR signals, or against the minor radius in the case of the ECE signals. The chordal radii of the SXR data points were calculated by taking an estimated magnetic axis as the origin, as indicated in figure 3.14. The minor radii of the ECE data were obtained by subtracting the major radius of the magnetic axis from the major radii of the channels, as obtained from the vacuum toroidal magnetic field. In both cases, the magnetic axis was taken

¹⁹For this square wave, the amplitudes of the fundamental and 2nd, 3rd and 4th harmonic components are in the ratios 1:0.223:0.267:0.200; when the $\omega^{-1/2}$ dependence of the response amplitude is included, these components are in the ratios 1:0.157:0.154:0.100.

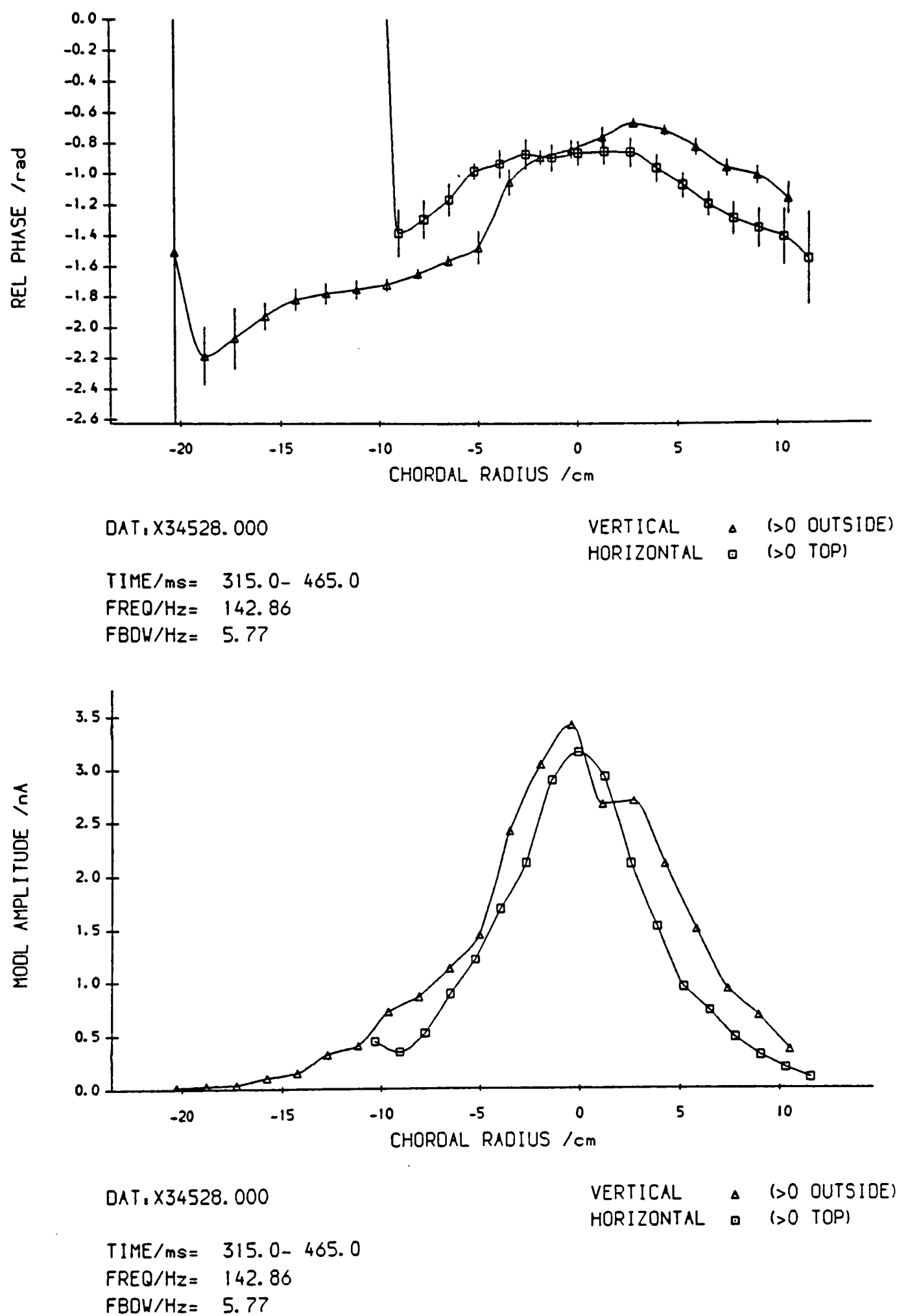


Figure 3.23: Profiles of (a) relative phase and (b) amplitude of fundamental component of modulation, extracted from the spectra of figures 3.21 and 3.22. Data from both vertical and horizontal SXR arrays are shown. The error estimates for the phase data points have been obtained using a set of five spectral points, covering an interval of 22.88Hz and centred on the modulation frequency, 143Hz.

to be at a major radius of 122cm.

Important note: It should finally be emphasized that *all* modulation amplitudes that are henceforth quoted in this thesis, both absolute and relative, refer to the amplitudes of the Fourier components, and are therefore equal to the *half* peak-to-peak amplitudes.

Chapter 4

Thermal Transport Analysis of Data from Modulated ECRH

4.1 Analytical solutions of diffusion equation

In order to illustrate the response of a diffusive thermal model to modulated heating, and to investigate certain effects, it will be helpful analytically to solve a parabolic partial differential equation with a localized, oscillating source, describing the propagation of a temperature perturbation (see section 4.3.1). It will be implicitly assumed that the underlying energy balance equation can be linearized, so that the perturbed part may be treated in isolation from the steady-state part. Uniform, time-independent density and diffusivity and a strongly localized, sinusoidally varying excitation will be used, in order to facilitate analytical progress. It will then be outlined how the response to an excitation of arbitrary spatial dependence may be obtained in terms of Green's function for the system. The response to an excitation of general temporal dependence will finally be obtained from a Fourier representation of the temporal part.

4.1.1 Slab geometry

The diffusion equation to be solved in this case, describing a temperature perturbation $\tilde{T}(x, t)$, is

$$\frac{\partial \tilde{T}}{\partial t} = D \frac{\partial^2 \tilde{T}}{\partial x^2} + P \delta(x) \exp(i\omega t), \quad (4.1)$$

where $\frac{3}{2}D$ is the thermal diffusivity, and $\frac{3}{2}nP$ is the heating power per unit area of slab, localized at $x=0$. The solution for the temperature perturbation is of the form

$$\tilde{T}(x, t) = f(x) \exp(i\omega t), \quad (4.2)$$

where the equation satisfied by the spatial part $f(x)$ is¹

$$i\omega f = Df'' + P\delta(x). \quad (4.3)$$

For $x \neq 0$ the spatial part is of the form²

$$f(x) = ae^{\alpha x} + be^{-\alpha x}, \quad (4.4)$$

$$\text{with } \alpha = \left(\frac{i\omega}{D}\right)^{1/2} = \left(\frac{\omega}{2D}\right)^{1/2} (1 + i) = s(1 + i). \quad (4.5)$$

The boundary condition is

$$f(l) = 0, \quad (4.6)$$

and the delta-function driving term gives (by integrating (4.3) from $x=-\varepsilon$ to $x=+\varepsilon$ and taking the limit $\varepsilon \rightarrow 0$)

$$f'(0^+) = -\frac{P}{2D}. \quad (4.7)$$

The evaluation of the amplitudes a and b now leads to the spatial dependence of the perturbation,

$$f(x) = \frac{P}{2D\alpha} \frac{1}{e^{\alpha l} + e^{-\alpha l}} \left(e^{\alpha(l-x)} - e^{-\alpha(l-x)} \right), \quad (4.8)$$

with a relative phase dependence

$$\phi(x) = \arg(f(x)) = -\frac{\pi}{4} - \arctan[\tanh(sl) \tan(sl)] + \arctan[\coth(s(l-x)) \tan(s(l-x))]. \quad (4.9)$$

With the assumption of a medium of infinite extent, i.e. $|\alpha|l \gg 1$ or $l \gg (2D/\omega)^{1/2}$, one obtains a solution independent of the boundary condition,

$$f(x) = \frac{P}{2D\alpha} e^{-\alpha x}, \quad (4.10)$$

¹Note that, with a temporal dependence $\exp(i\omega_0 t)$, the Fourier transform of the temperature perturbation $\tilde{T}(x, t)$ is given by $\hat{T}(x, \omega) = f(x) \sqrt{2\pi} \delta(\omega - \omega_0)$.

²the branch $x > 0$ is considered here, assuming symmetry about $x = 0$

with a relative phase dependence

$$\phi(x) = \arg(f(x)) = -\frac{\pi}{4} - sx. \quad (4.11)$$

Hence, the phase gradient outside the heating region is constant and given in terms of the diffusivity by the formula

$$\left(\frac{d\phi}{dx}\right)^{-2} = (-\Im(\alpha))^{-2} = \frac{2D}{\omega}, \quad (4.12)$$

and the logarithmic amplitude gradient is given by

$$\left(\frac{d \ln |f|}{dx}\right)^{-2} = (-\Re(\alpha))^{-2} = \frac{2D}{\omega}. \quad (4.13)$$

One notes that the phase and amplitude gradients are larger for smaller diffusivities, and increase as the modulation frequency is increased; one therefore expects a modulation frequency comparable to the inverse confinement time, i.e. $\omega \sim D/l^2$, to provide enhanced sensitivity in fitting experimental modulation data using a transport model. One finally notes that, at the point of excitation, the perturbation amplitude takes its maximum value, proportional to $\omega^{-1/2}$, and the phase lag has its minimum value, $\pi/4$.

We now turn to a consideration of the effect of a damping term added to the diffusion equation (4.1) —this can arise through the variation with temperature of the steady-state heat sources and sinks (see section 4.3.1):

$$\frac{\partial \tilde{T}}{\partial t} = D \frac{\partial^2 \tilde{T}}{\partial x^2} - \frac{\tilde{T}}{\tau} + P\delta(x) \exp(i\omega t); \quad (4.14)$$

τ is a characteristic time constant for the damping. The solution of (4.14) is of the same form as that of (4.1), given by (4.8), but the complex spatial frequency α is now given by

$$\alpha = \left(\frac{i\omega + \tau^{-1}}{D}\right)^{1/2}. \quad (4.15)$$

The phase gradient of the solution, outside the heating region, is now given in terms of the diffusivity by

$$\left(\frac{d\phi}{dx}\right)^{-2} = (-\Im(\alpha))^{-2} = \frac{2D}{\omega} \left[1 + (\omega\tau)^{-2}\right]^{-1/2} \frac{1}{2} \csc^2 \left[\frac{1}{2} \arctan(\omega\tau)\right]. \quad (4.16)$$

For $\tau \rightarrow \infty$, (4.16) reduces to (4.12), but for $\omega\tau = 3.6$ (a typical value for the DITE experiments), the modifying factor in the right-hand side of (4.16) is 1.32 —this means that, if (4.12) were used instead in this case, the diffusivity would be overestimated by 32%.

4.1.2 Cylindrical geometry

The diffusion equation to be solved in this case is

$$\frac{\partial \tilde{T}}{\partial t} = D \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \tilde{T}}{\partial r} \right) + \frac{P}{2\pi r_0} \delta(r - r_0) \exp(i\omega t), \quad (4.17)$$

where $\frac{3}{2}nP$ is now the heating power per unit length of cylinder, localized at $r=r_0$.³ The solution for the temperature perturbation is of the form

$$T(r, t) = f(r) \exp(i\omega t), \quad (4.18)$$

where the equation satisfied by the spatial part is

$$i\omega f = D \left(f'' + \frac{1}{r} f' \right) + \frac{P}{2\pi r_0} \delta(r - r_0). \quad (4.19)$$

The spatial part is of the form

$$f(r) = \begin{cases} f_1(r) = a_1 I_0(\alpha r) + b_1 K_0(\alpha r) & r < r_0 \\ f_2(r) = a_2 I_0(\alpha r) + b_2 K_0(\alpha r) & r > r_0, \end{cases} \quad (4.20)$$

where the complex spatial frequency α is given by (4.5), or by (4.15) when a damping term is included as before; I_0 and K_0 are the zeroth order hyperbolic Bessel functions of the first and second kind respectively (see, e.g., Arfken, 1970). From the matching of the two branches, f_1 and f_2 , at the discontinuity due to the delta-function driving term at $r=r_0$, one obtains

$$f_2(r_0^+) - f_1(r_0^-) = 0, \quad (4.21)$$

$$f_2'(r_0^+) - f_1'(r_0^-) = -\frac{P}{2\pi r_0 D}. \quad (4.22)$$

The boundary conditions, representing continuity at the centre and a zero perturbation amplitude at the edge, are

$$f_1'(0) = 0, \quad (4.23)$$

$$f_2(a) = 0. \quad (4.24)$$

³Note that there would now be a loss of generality if the heating were localized at $r=0$. The factor $1/(2\pi r_0)$ normalizes the delta function $\delta(r - r_0)$ in polar co-ordinates.

The radial dependence, obtained by evaluating the amplitudes a_1 , b_1 , a_2 and b_2 , is

$$f(r) = \begin{cases} \frac{P}{2\pi D} \frac{1}{I_0(\alpha a)} [I_0(\alpha a)K_0(\alpha r_0) - K_0(\alpha a)I_0(\alpha r_0)] I_0(\alpha r) & r < r_0 \\ \frac{P}{2\pi D} \frac{1}{I_0(\alpha a)} [I_0(\alpha a)K_0(\alpha r) - K_0(\alpha a)I_0(\alpha r)] I_0(\alpha r_0) & r > r_0. \end{cases} \quad (4.25)$$

In the infinite-medium approximation, i.e. $|\alpha|a \gg 1$, this simplifies to

$$f(r) = \begin{cases} \frac{P}{2\pi D} K_0(\alpha r_0) I_0(\alpha r) & r < r_0 \\ \frac{P}{2\pi D} I_0(\alpha r_0) K_0(\alpha r) & r > r_0. \end{cases} \quad (4.26)$$

We now turn to the response to a heating input $g(r) \exp(i\omega t)$ of arbitrary radial dependence; this can be obtained by expressing the source in terms of an infinite set of delta functions using the filtering theorem, i.e.

$$g(r) = \int_0^a g(r') \delta(r - r') dr', \quad 0 \leq r \leq a. \quad (4.27)$$

As the diffusion equation (4.17) is linear, the response corresponding to such a linear superposition of delta functions can be obtained by forming an identical linear superposition of the individual responses to the delta-function excitations. We now write $\hat{G}(r, r_0; \omega)$ to denote the solution of (4.19) with a unit delta-function source at $r = r_0$, namely *Green's function* of the system (see, e.g., Arfken, 1970); it is given by (4.25) omitting the factor $P/(2\pi r_0)$.⁴ With this definition, the spatial part $f(r)$ of the system response to the general excitation $g(r)$ is given by the superposition integral

$$f(r) = \int_0^a g(r') \hat{G}(r, r'; \omega) dr'. \quad (4.28)$$

An important point transpiring from this analysis is that, in the limit of the modulation frequency ω tending to infinity ($|\alpha| \rightarrow \infty$), Green's function $\hat{G}(r, r_0; \omega)$ tends to the delta function $\omega^{-1/2} \delta(r - r_0)$, so that the radial dependence of the response approaches that of the excitation. Therefore, the radial dependence of a suitably high harmonic of the response, provided of course that this can be reliably extracted, will be almost independent of the transport and

⁴Note that the complementary function $G(r, r_0; t)$ in the Fourier transform pair is the response to the unit impulse $\delta(r - r_0) \sqrt{2\pi} \delta(t)$.

identical to the profile of the heating input. Further generalizing to an excitation $\tilde{Q}(r, t)$ of arbitrary radial and temporal dependence, we consider the frequency-space representation obtained from the Fourier transform

$$\hat{Q}(r, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{Q}(r, t) \exp(-i\omega t) dt; \quad (4.29)$$

using the linearity of (4.17) as before, we obtain the system response from the Fourier synthesis integral of the single-frequency solution, i.e.

$$\tilde{T}(r, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_0^a \hat{Q}(r', \omega') \hat{G}(r, r'; \omega') dr' \exp(i\omega' t) d\omega', \quad (4.30)$$

The amplitude and relative phase of the radial dependence $f(r)$ of the perturbation (for off-axis heating power deposition and values typical of DITE experiments) are indicated in figure 4.1, where they are compared with the corresponding results of the slab geometry case. One notes that the behaviour of the cylindrical model is almost identical to that of the slab model outside the heating region, except near the centre and the edge—in particular, the phase gradients are equal. It follows that the simple formulae (4.12) and (4.16) can be used to determine the diffusivity from experimental data outside the heating region, provided that the phase gradient be not measured either too near the centre where the effects of the cylindrical geometry are important, or too near the edge where the boundary conditions modify the solutions. In connection with the SXR signals, the reader is referred to subsection 3.3.1 where it has already been demonstrated how the temperature response of the SXR emissivity, combined with the chordal integration, can significantly modify the amplitude and phase profiles.

4.2 Results of experiments with modulated ECRH

4.2.1 Diffusive behaviour

4.2.1.1 Preliminary observations

In the majority of discharges with modulated ECRH that were analysed in the frequency domain, as described in section 3.6, a behaviour consistent with diffusive electron energy transport was readily observed. An example of such a typical discharge with off-axis absorption of the ECRH

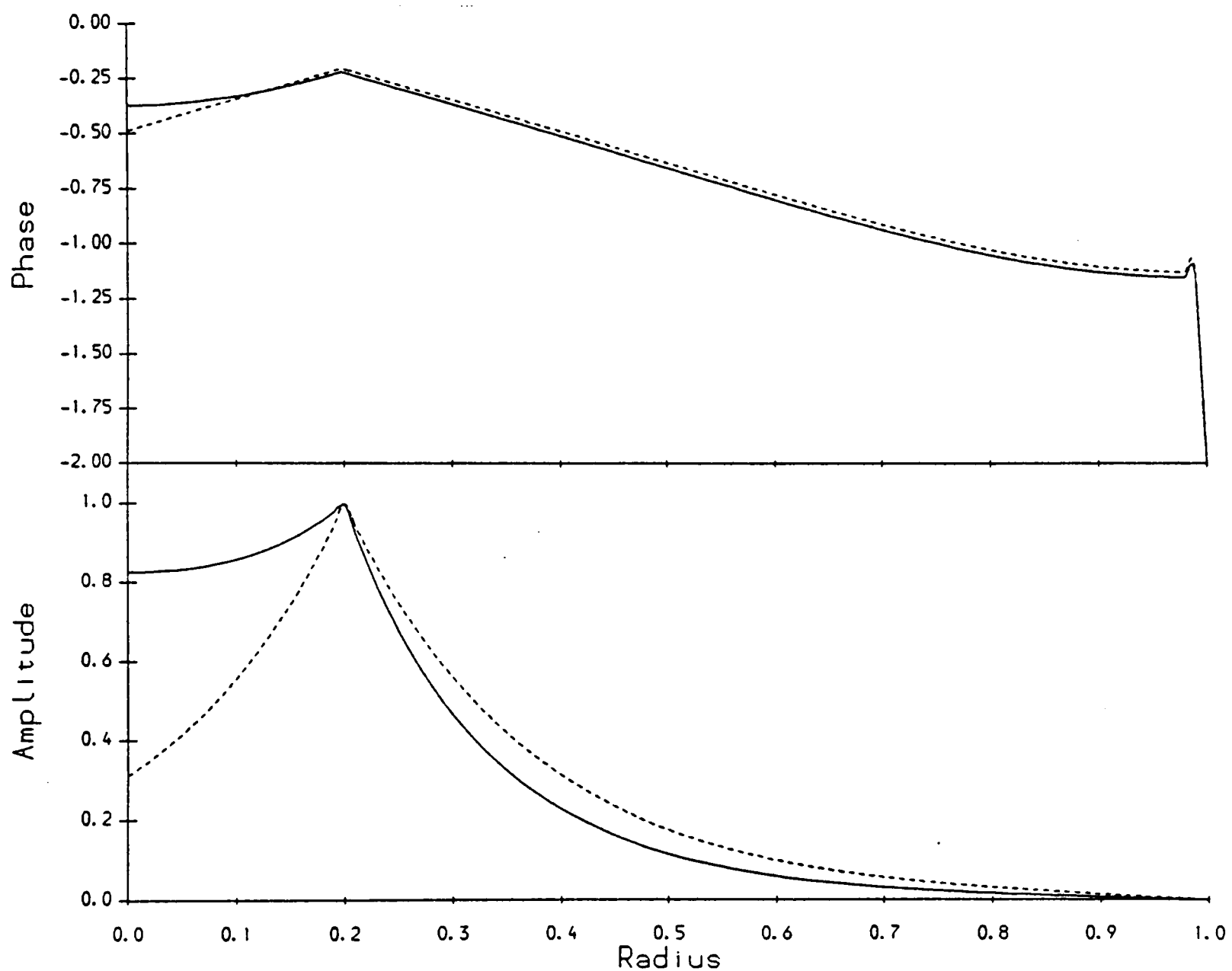


Figure 4.1: Green's function of diffusive model: radial variations of (a) phase ($/\pi$ rad) and (b) amplitude of the solution of diffusion equation with delta-function excitation: cylindrical geometry (solid curves), compared to slab geometry (broken curves). Typical DITE values are used: $a = 24\text{cm}$, $\omega = 2\pi 143\text{s}^{-1}$, $D = 1\text{m}^2\text{s}^{-1}$, $\tau = 4\text{ms}$, $r_0 = 4.8\text{cm}$.

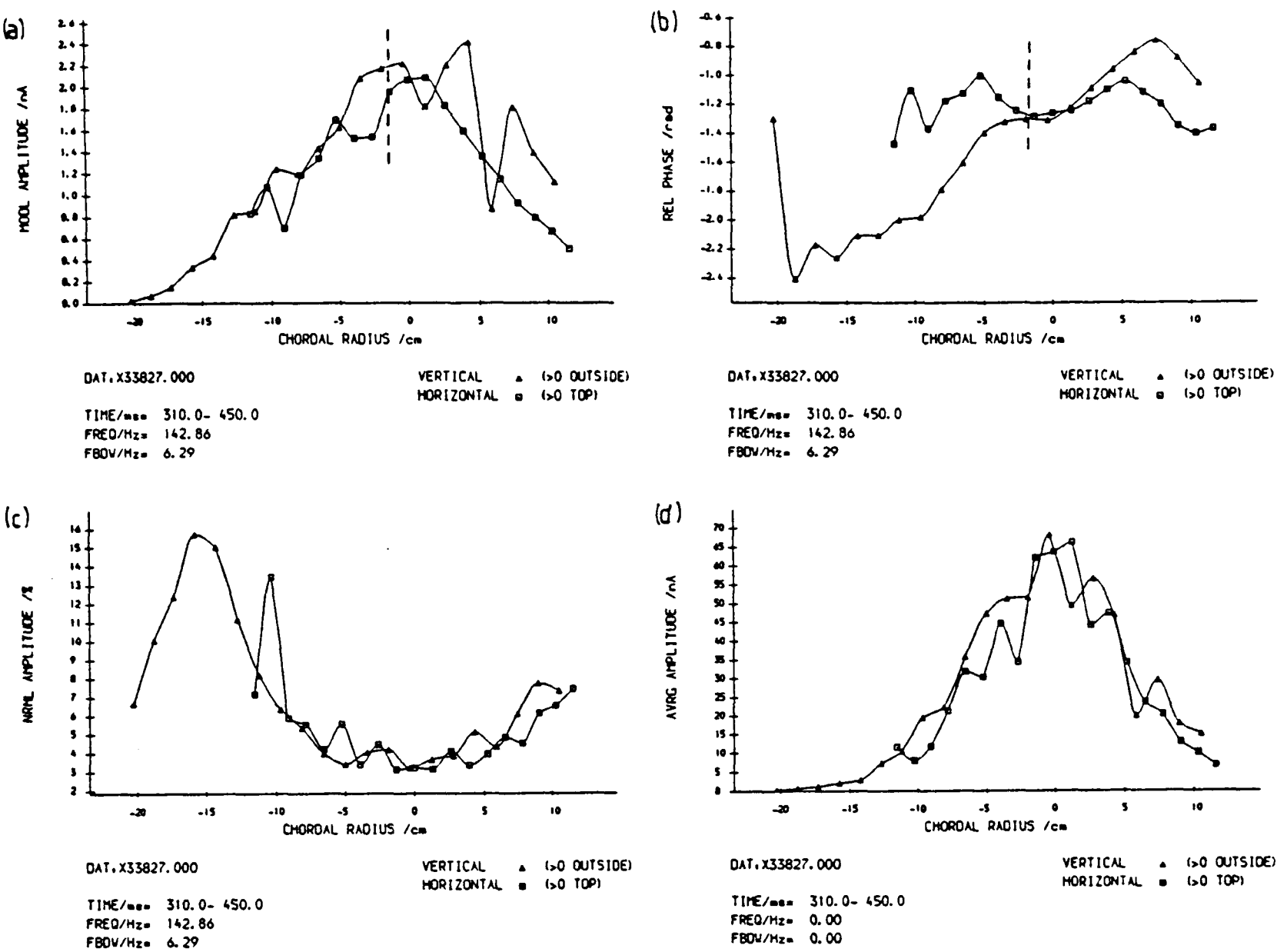


Figure 4.2: Analysed SXR data for a typical H discharge with off-axis ECRH, showing diffusive behaviour. Chordal profiles: (a) absolute modulation level; (b) relative phase; (c) relative modulation level; (d) average signal level. Data from both the horizontal and vertical SXR arrays are plotted (squares and triangles respectively). The dotted line shows the unshifted resonance position.

power is presented in figures 4.2 and 4.3 which show, for the SXR and ECE signals respectively, the profiles of absolute and relative modulation levels, relative phase and average level; a further example of a discharge with on-axis absorption is shown in figure 4.4. An example of a discharge with heating at the second harmonic of the cyclotron frequency is shown in figures 4.5 and 4.6, for the SXR and ECE signals respectively.

The absolute modulation amplitude decreases with increasing radial distance from the resonance region, while the relative phase lag (corresponding to the time delay) increases. It has also been observed that the oscillations of the signals become increasingly sinusoidal in form with increasing distance, as the higher harmonics of the square wave modulation are attenuated more strongly. The relative modulation level increases rapidly towards the edge, particularly

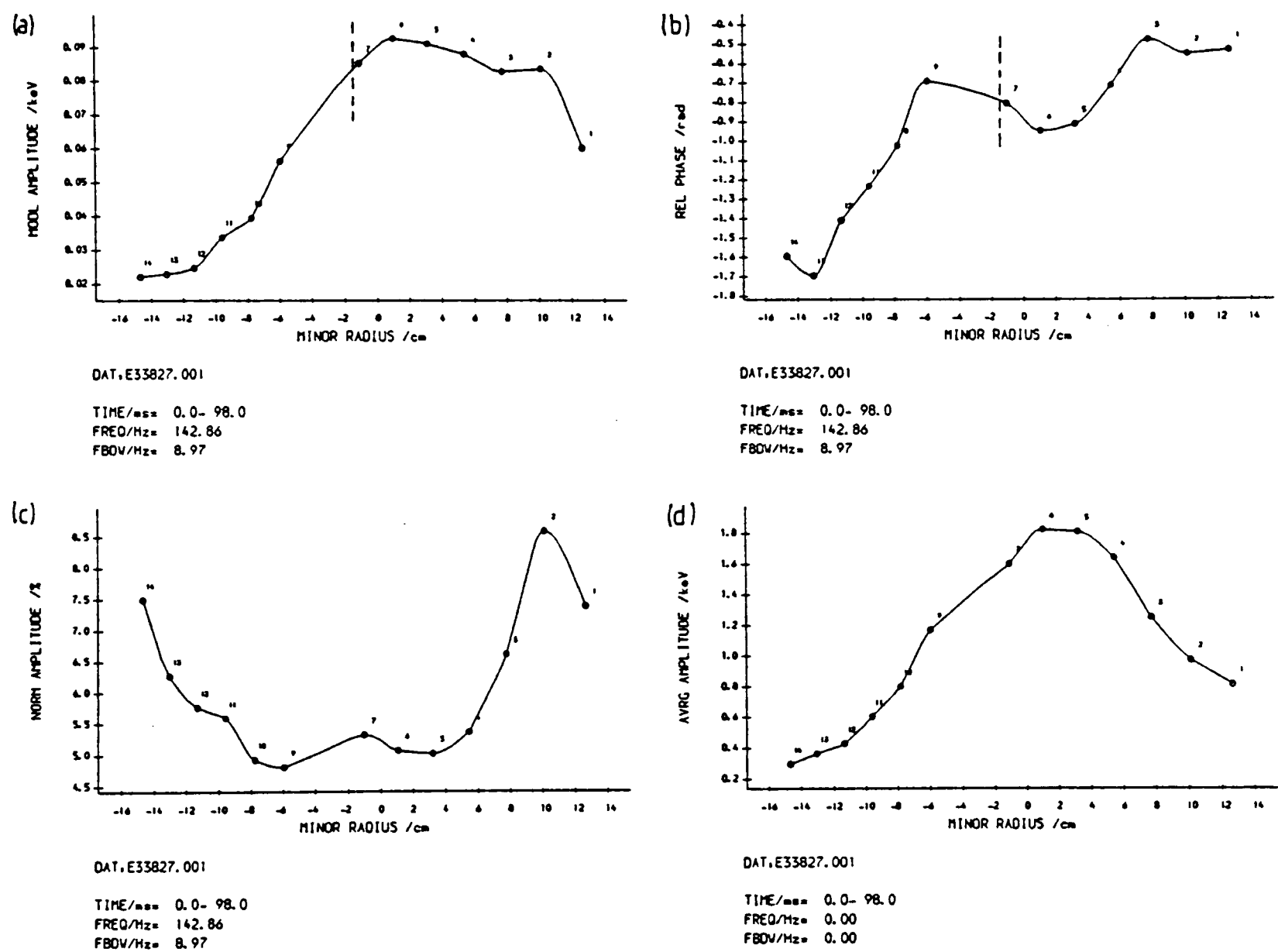


Figure 4.3: Analysed ECE data for the discharge of figure 4.2, showing diffusive behaviour. Radial profiles: (a) absolute modulation level; (b) relative phase; (c) relative modulation level; (d) average signal level. The dotted line shows the unshifted resonance position. The distortion on the outer (low-field) side is due to the shifted contribution of superthermal electrons.

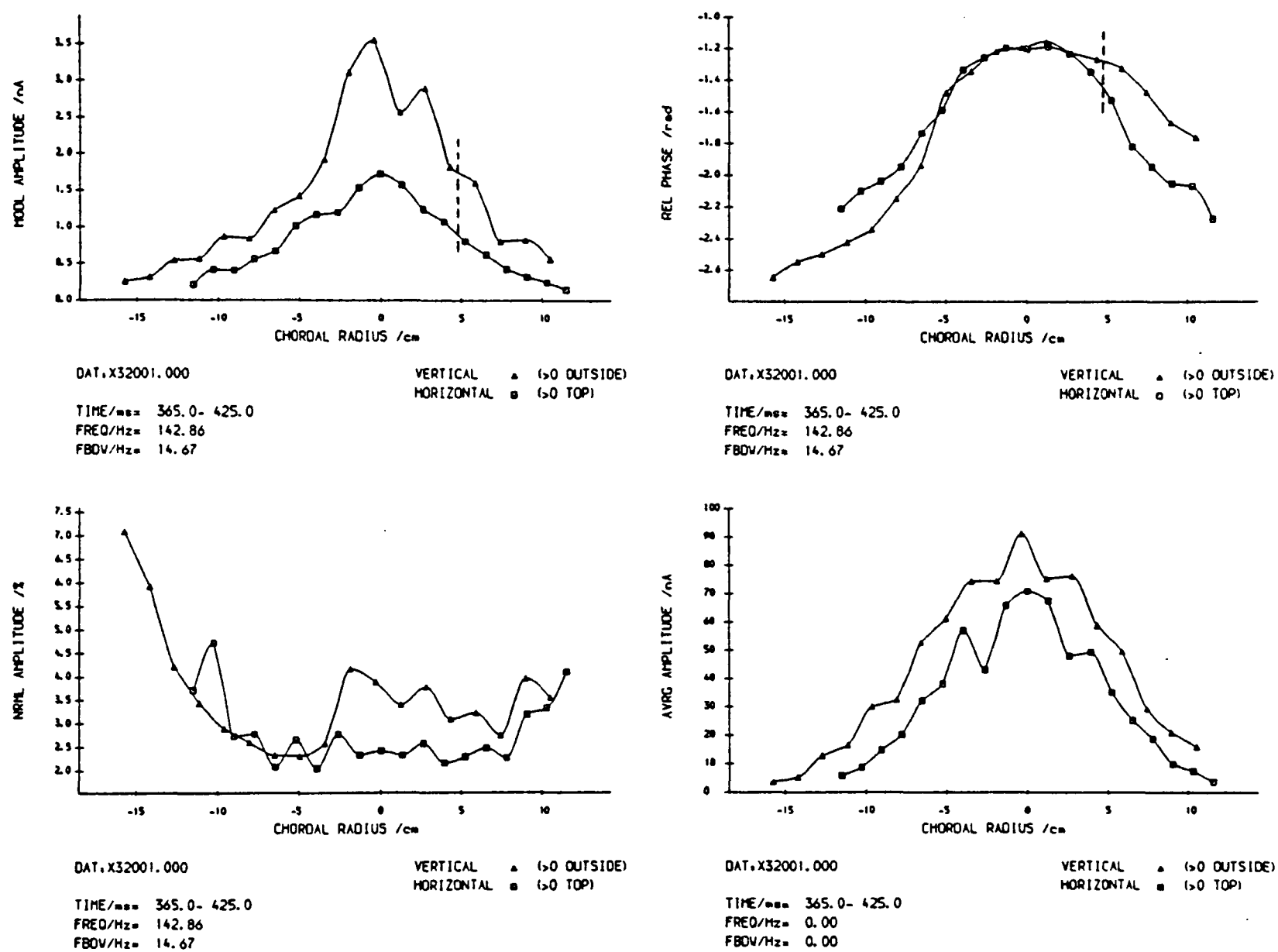


Figure 4.4: Analysed SXR data for a typical He discharge with on-axis ECRH, showing diffusive behaviour.

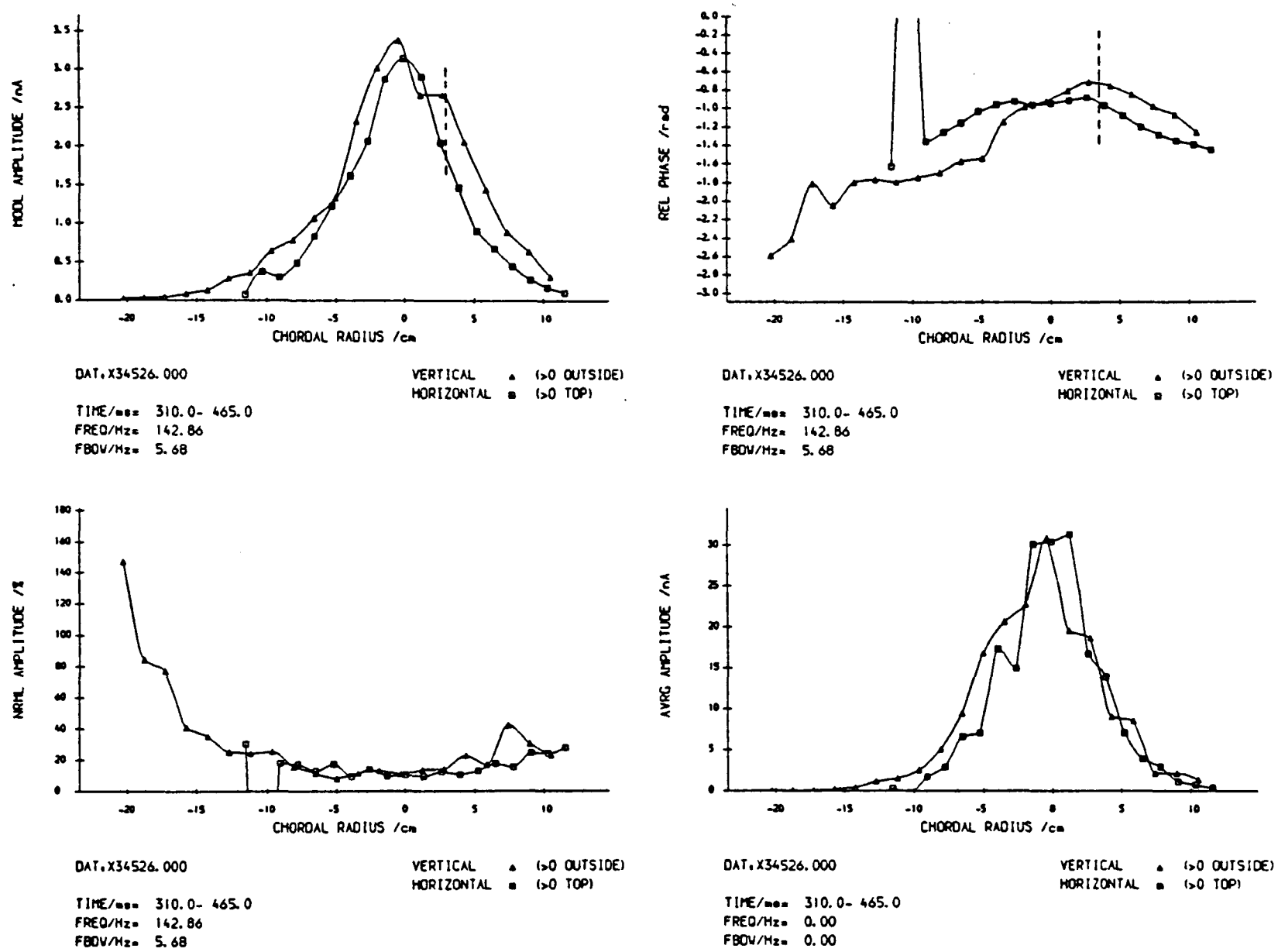
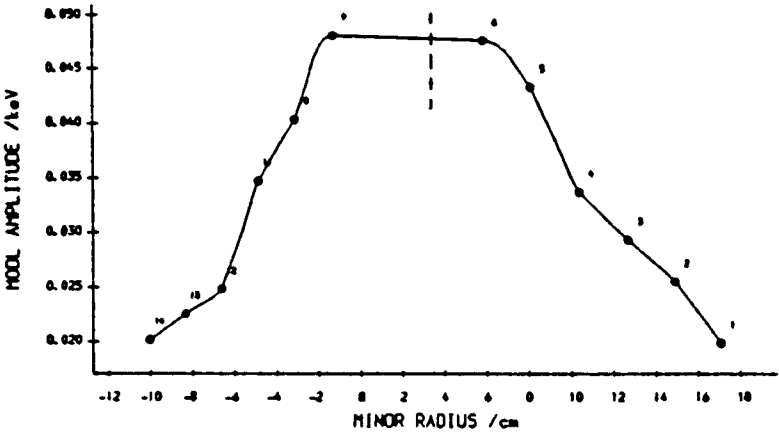
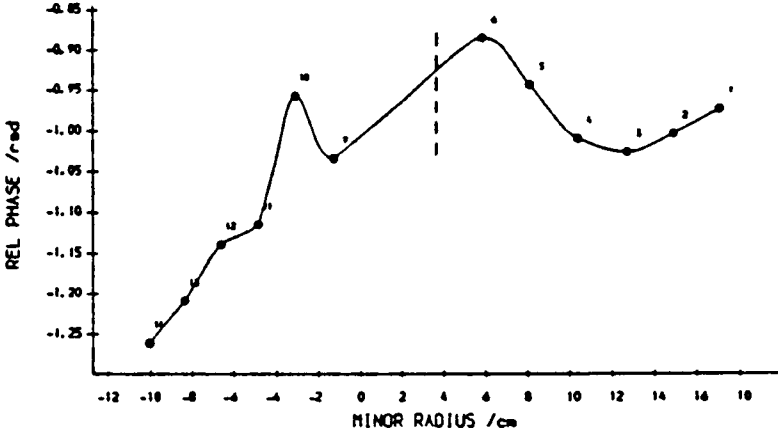


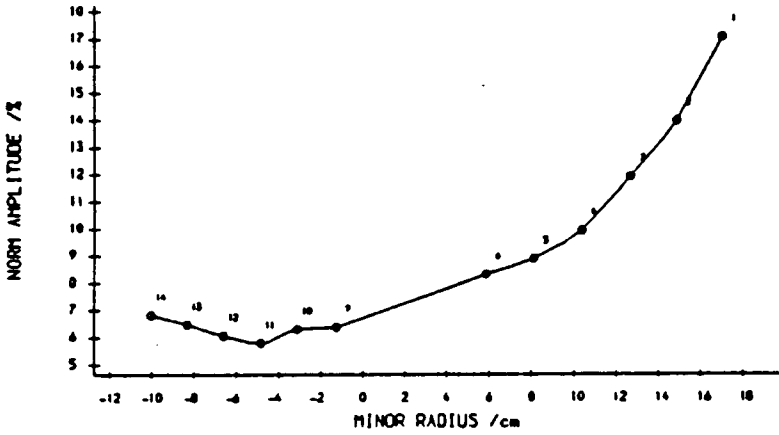
Figure 4.5: Analysed SXR data for a typical H discharge with 2nd harmonic ECRH, showing diffusive behaviour.



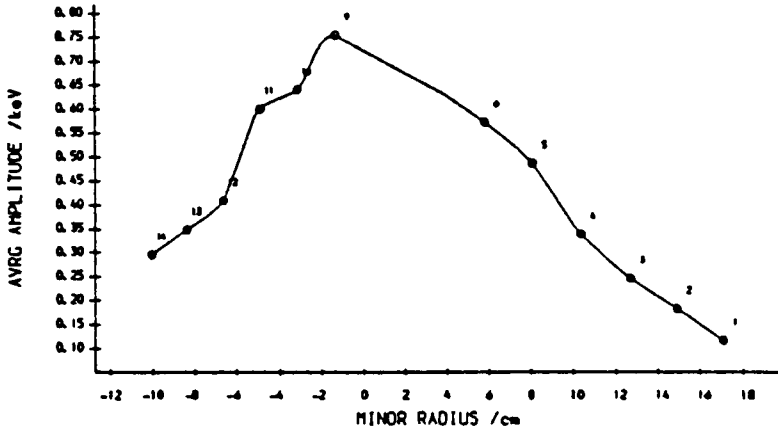
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FREQ/MHz= 142.86
FBDV/MHz= 5.49



DAT.E34526.001
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FREQ/MHz= 142.86
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TIME/msec= 15.0- 165.0
FREQ/MHz= 142.86
FBDV/MHz= 5.49



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Figure 4.6: Analysed ECE data for the discharge of figure 4.5, showing diffusive behaviour.

so for the SXR signals, being approximately constant over the central region. The minimum phase lag corresponds to the centre of the ECRH absorption zone and allows one to establish whether there is a Doppler or relativistic shift of the resonance, by comparing these results with the unshifted resonance position calculated from the known spatial variation of the toroidal field (using (3.7)). This comparison, indicated in figures 4.2–4.6, has led to clear indications of shifted resonance, towards the high-field, inner side of the torus, as detailed in subsection 3.2.5 —these somewhat empirical results were further confirmed by the numerical modelling of the modulated data.

Even though the heating is localized around a particular major radius, poloidal transport proceeding on a faster time-scale than that of radial transport should lead to a circularly symmetric deposition region in the minor cross-section. Toroidal transport proceeding on an even faster time-scale ensures that the circularly symmetric deposition region is also axisymmetric. The symmetry of the response profiles of the horizontal array SXR signals shows merely that there is no vertical asymmetry, this being insufficient to confirm the assumption of poloidal symmetry (vertical symmetry would have been expected even in the absence of circular symmetry, as the ECRH antenna was mounted on the equatorial plane and the wave launching geometry was vertically symmetric). A stronger, albeit less obvious, confirmation of poloidal symmetry is afforded by the response profiles of the vertical array SXR signals: although these were not symmetric, their asymmetry can be attributed to a horizontal plasma shift modulation (as will be discussed later) and was independent of the resonance position. A partial confirmation of toroidal symmetry is indirectly provided by the response profiles of the ECE signals, the ECE and SXR diagnostics being located at different toroidal angles: the ECE and SXR data could be fitted with the same diffusion model.

The modulation data from several sets of different discharges with the same parameters (and resonance radius) have been compared and found to be similar; therefore, the response of similar discharges to modulated ECRH was generally repeatable. As it has already been pointed out, the response to the modulated ECRH has also been found to be *time-invariant* during a single discharge.

It is evident that the reliability of the SXR data is superior to that of the ECE data; the quality of the latter was usually poor. This somewhat unfortunate circumstance, arising from the contribution of superthermal electrons and instrumentation problems, prevented the extensive use of the ECE data, which should have afforded one with a more direct measurement of the local electron temperature and its modulation. The amplitude profiles of the SXR signals displayed certain irregularities which could not be corrected by the use of calibration factors alone; this made it impossible successfully to implement various techniques for the inversion of the data, in order directly to obtain the local emissivity. As it has transpired that the SXR diagnostic uses filters at the two pinholes rather than individual ones in front of the detectors, as it was originally thought, the interpretation of these discrepancies remains somewhat problematic. On the other hand, the relative phase profiles of the SXR signals were of a good quality and formed the basis of most of the analysis that is described here.

4.2.1.2 Thermal diffusivity

The gradient of the relative phase curve *outside* the ECRH deposition region provides a rough estimate of the effective thermal diffusivity, based on the following simple relation, obtained from (4.12) with $\chi = 3D/2$; this is valid for a slab model, or away from the centre for a cylindrical model, assuming an infinite medium with no boundary effects:

$$\chi = \frac{3}{4}\omega \left(\frac{d\phi}{dr} \right)^{-2}. \quad (4.31)$$

The values obtained from this formula must generally be corrected for the effect of the modulation of the Ohmic heating power, that is, (4.16) should be used. The phase gradient decreases towards the edge, pointing to a higher diffusivity than near the centre of the cross-section, but it should be remembered that there might be other effects that give rise to such a saturation in the phase lag, in particular the boundary conditions and, in the case of the SXR signals, the radial variation of the modulation index, as seen in subsection 3.3.1. More importantly, a broader than anticipated ECRH deposition profile will also have such an effect, this being a general weakness of the modulation technique. A further weakness is the strong dependence of the diffusivity on the phase gradient, or conversely, the weak dependence of the phase gradient on the diffusivity,

limiting the sensitivity of the response profiles to transport. It should be borne in mind, when applying the formula above to the response profiles of SXR signals, that these data are in fact chordal integrals rather than simple measurements at a single radius. Chordal integration of a circularly symmetric profile with a central peak will clearly produce a less peaked profile with smaller gradients, and a direct application of the formula above would lead to an overestimate for the diffusivity (refer to subsection 3.3.1 and figure 3.12). The effect of chordal integration will, of course, be most important in the case of hollow radial profiles obtained with off-axis ECRH absorption; in such cases, chordal integration leads to a much smaller central depression in the SXR response profile than that seen on the ECE response profile. The application of the formula above to the SXR and ECE modulation data has led to thermal diffusivity values $\chi_e \sim 0.8\text{--}1.5\text{m}^2\text{s}^{-1}$. Naturally, a more detailed, numerical analysis is necessary, taking several effects into account, namely the profiles of the density, diffusivity and ECRH deposition, and the chordal integration of the SXR signals; this approach will be pursued in the two following sections.

4.2.1.3 Modulation of the horizontal plasma shift

The SXR data from the horizontal array appear to be vertically symmetric about the centre of the minor cross-section, but those from the vertical array display a clear horizontal asymmetry (see, e.g., figure 4.2). This is attributed to a horizontal oscillatory movement of the plasma column, following the modulated ECRH power. A horizontal, *outward* shift of the plasma column, with a smaller phase lag with respect to the modulated ECRH power than that of the temperature response, leads to a *decreased* phase lag being observed on the *outer* vertical chords and to an *increased* phase lag on the *inner* chords. This modulation of the horizontal plasma shift has been observed with the magnetic diagnostics, the relevant signals being included in figures 3.1 and 3.2. It arises mainly from the modulation of plasma pressure, a treatment of this effect being presented in subsection 5.3.2. Such a plasma movement would have led to a modulation of the observed signals, even in the absence of a local, intrinsic modulation, simply because of the presence of gradients in all plasma parameters: an outward movement of the plasma

column leads to an increase of a measurement on the outside of the corresponding profile and to a decrease on the inside, without affecting the measured value near the profile peak where the gradient is zero. Measurements along the nearly horizontal viewing chords are only slightly affected by the horizontal plasma movement.

In practice, the modulated plasma shifts are not confined to the horizontal shift of the entire plasma column, but include horizontal shifts of the flux surfaces with respect to one another. We consider a modulated plasma parameter F , such as temperature, density or SXR emissivity, with a dependence of the form

$$F(R, t) = F_0(R) + F_1(R) \exp[i\omega t + i\phi(R)]; \quad (4.32)$$

we now assume that a modulated flux surface shift is present, of the form

$$\delta(R, t) = \delta_0(R) + \delta_1(R) \exp[i\omega t + i\Phi(R)]; \quad (4.33)$$

if the parameter F be observed at the major radius R , the measured modulation $\tilde{F}$ can be expressed, to first order in the modulation amplitudes, as

$$\tilde{F}(R, t) = \Delta F(R - \delta(R, t), t) \simeq F_1(R) \exp[i\omega t + i\phi(R)] - \frac{dF_0}{dR} \delta_1(R) \exp[i\omega t + i\Phi(R)]. \quad (4.34)$$

The effect of the modulated flux surface shift on the resultant signal will depend on the relative amplitudes and phases of the two terms in this expression; it will be greatest approximately half-way between the centre and the edge, where the underlying profile gradient is greatest. Therefore, the peaks seen in the vertical array amplitude and phase profiles correspond to the point of maximum underlying gradient, to a greater extent than they do to the point of the local extremum in the perturbation (i.e. the ECRH deposition region).

The DITE feedback system controls the vertical magnetic field to maintain a constant plasma column position, but its response time is not significantly shorter than the ECRH modulation period. It has been estimated, with the help of a profile-fitting algorithm (Millar, 1988), that the form of the data from the vertical and horizontal arrays can be attributed to a modulated horizontal shift of the plasma column of 1.5–2mm; the horizontal shift measured with the magnetic diagnostics showed a modulation amplitude of about 2mm (see figure 3.1). A greater modulation amplitude will obtain for discharges with a lower plasma current, since the modulation

level of the poloidal beta value will be greater (see subsection 5.3.2), and the effectiveness of the feedback stabilization system will be impaired. This increase in the modulation amplitude of the plasma column position was in fact observed with the magnetic diagnostics, for discharges with second harmonic resonance, with a lower plasma current; the measured modulation amplitude for such discharges was about 5mm (see figure 3.2), and the modulation profiles obtained from the vertical array were often significantly distorted.

The effect of the modulated horizontal plasma shift is clearly a characteristic of all diagnostic techniques with a similar geometry to that of the vertical SXR array, relying on vertical observation, and in particular of the ECE and density diagnostics; it is, however, expected to be greatest in the case of the SXR signals, because the profile of the SXR emissivity is more peaked than those of the temperature and density. In extreme cases, an example of which is shown in figure 4.7, the modulation of the plasma position can dominate the observed modulation in all viewing directions, including the nearly horizontal ones that are normally unaffected (their amplitude and phase variations become flat, concealing the transport effects); this will occur when the density and temperature profiles are narrow (i.e. the emissivity profile has a large gradient) and/or the movement of the plasma column is large (e.g. when the current density is low). Clearly, the modulated plasma shift can render the extraction of information relevant to transport difficult or impossible. The stability of the plasma column is consequently of vital importance in all measurements of the response to modulated heating, with a view to obtaining the local, transport-related response.

4.2.1.4 Response of the plasma periphery

Further evidence supporting diffusive thermal transport was provided by measurements carried out near the plasma periphery by Mantica, Pitts and Vayakis (1988), using edge probes to observe the electromagnetic and electrostatic activity; in addition, detectors were used to observe the edge $H\alpha$ radiation. The following edge diagnostics were used:

- Two sets of 16 Mirnov coils each, equally spaced poloidally and separated by 90° toroidally, at a radial distance of 1cm beyond the plasma edge, were used to measure fluctuations in

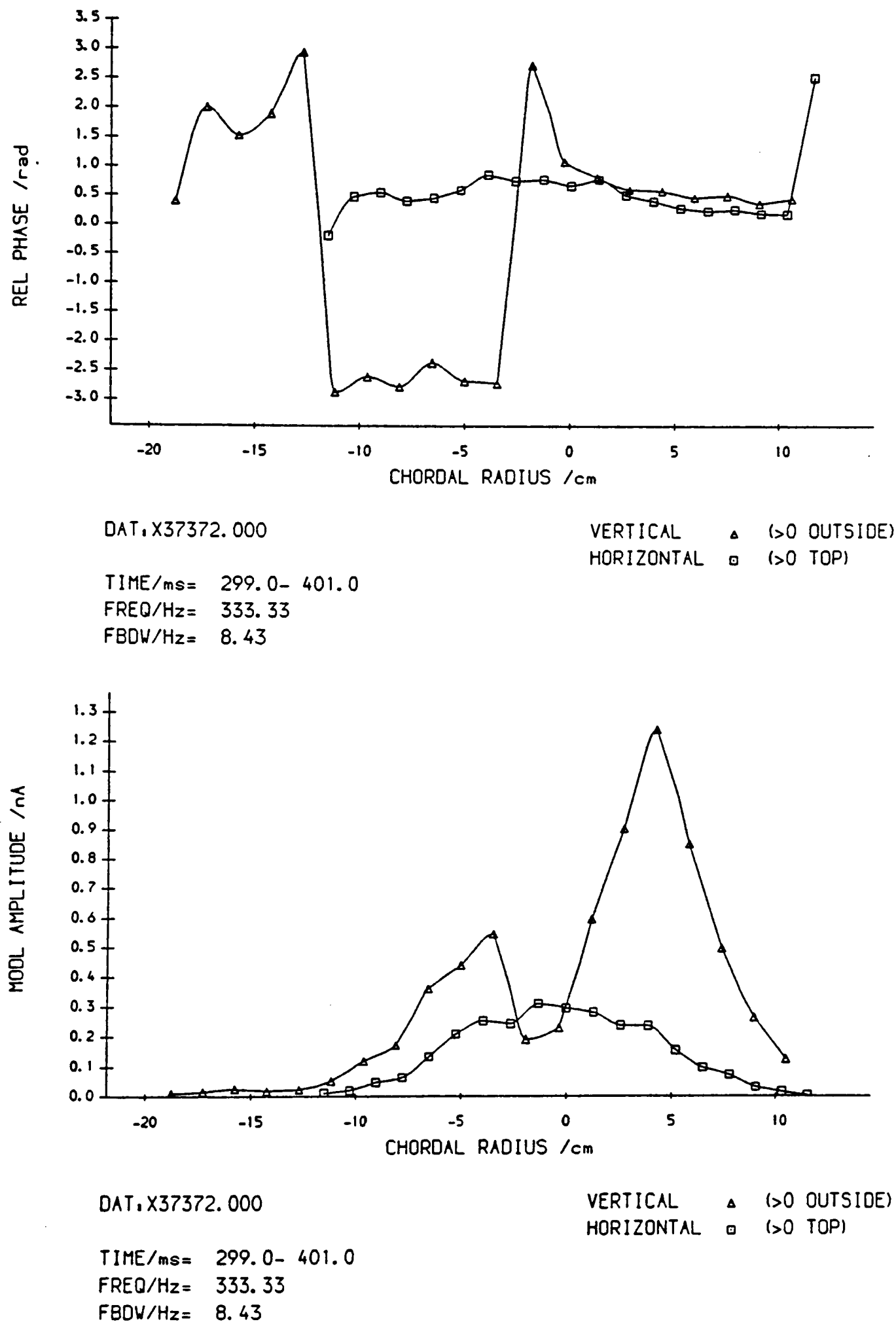


Figure 4.7: Analysed SXR data for a low-density H discharge with 2nd harmonic ECRH, showing the dominant effect of a relatively large plasma movement combined with narrow density and temperature profiles. The pronounced phase shifts and amplitude peaks occur near the location of the maximum in the emissivity gradient.

the poloidal magnetic field $\tilde{B}_\theta$.

- A reciprocating drive carrying a pick-up coil, 10° above the outer equatorial plane, was used to measure fluctuations in the radial magnetic field $\tilde{B}_r$, at 1cm from the plasma edge.
- 4 Langmuir probes on the same reciprocating drive, 10° below the outer equatorial plane, were used to measure the floating potential V_f and its fluctuations (corresponding to fluctuations in the poloidal electric field $\tilde{E}_\theta$), and the fluctuations in the ion saturation current $\tilde{j}_{\text{sat}}$ (the relevant probe was driven to permanent ion saturation). Ion saturation obtains when the probe is negatively biased, such that electrons are repelled and ions form a sheath around it; the value of the ion saturation current depends on the negative potential required to repel all the electrons and is therefore a measure of electron temperature and plasma density, being proportional to $n_e T_e^{1/2}$.
- 8 Langmuir probes embedded in the pumped limiter, at various poloidal angles above and below the outer equatorial plane, were used to measure the steady ion saturation current j_{sat} .
- 8 similar Langmuir probes embedded in the bumper limiter were used.
- 4 Langmuir probes in a deposition probe assembly, near the top of the periphery, were used to measure the steady ion saturation current j_{sat} , at radial distances between 1cm and 3.5cm from the plasma edge.
- 3 visible light detectors with interference filters, viewing the bumper and pumped limiters and the torus wall, were used to monitor both the $\text{H}\alpha$ and HeII lines of the neutral particle emission.

The responses to the modulated ECRH of the steady-state values and of the RMS components of the fluctuating values were analysed, using cross-correlation techniques similar to those already described, to yield modulation amplitudes and relative phases (with respect to the ECRH reference signal from the microwave detector inside the torus). Such measurements were carried out in modulation experiments where the ECRH absorption radius was scanned at a fixed density,

mostly with second harmonic resonance heating; and in other experiments where the density was scanned with a fixed ECRH absorption radius.

A typical example of the results of the experiments with a scanned ECRH absorption radius is shown in figure 4.8, which includes data for the fluctuations in the ion saturation current and the radial magnetic field rate-of-change. All the results obtained were consistent with diffusive thermal transport. In particular, the modulation amplitudes had minimum values and the phase lags had maximum values, when the ECRH absorption was on-axis. Furthermore, all phase lags of the periphery signals were quite compatible with the corresponding ones of the SXR and ECE signals, when the latter were extrapolated to near the plasma edge (refer to figure 4.10), considering of course the relatively large errors involved. These results suggest that there was no direct interaction of the ECRH wave at the edge, and that the response of the plasma periphery arose solely from the outwardly propagating thermal wave, causing a modification of both electrostatic and electromagnetic fluctuations at the plasma edge. The measurements of the responses of edge fluctuations indicated no toroidal asymmetries, but did reveal certain poloidal asymmetries (similar for all measurements); the asymmetry between the inside and outside measurements can probably be attributed to the modulated plasma column position, but there was also some vertical asymmetry whose causes have not been investigated. The poloidal variations of the relative phases of all the edge signals, from one of the discharges in a sequence with a scanned ECRH radius, are shown in figure 4.9. The responses of the magnetic signals were similar to those of the electric signals (most notably to that of the ion saturation current, that of the floating potential being somewhat different), indicating that electromagnetic and electrostatic fluctuations were linked (cf. also Vayakis, Mantica and Matthews, 1988, where a link with edge particle transport is further established). Finally, a large increase of the ion saturation current with ECRH relative to that during Ohmic heating (up to 100%) was observed.

The density scans indicated no definite trends, except, possibly, a decrease of the phase delay with increasing density, which was not consistent with the decreasing transport, but which might be explained by the direct interaction at the edge becoming stronger at higher densities. This is shown in figure 4.10, where the relative phases of the fluctuations of poloidal magnetic field

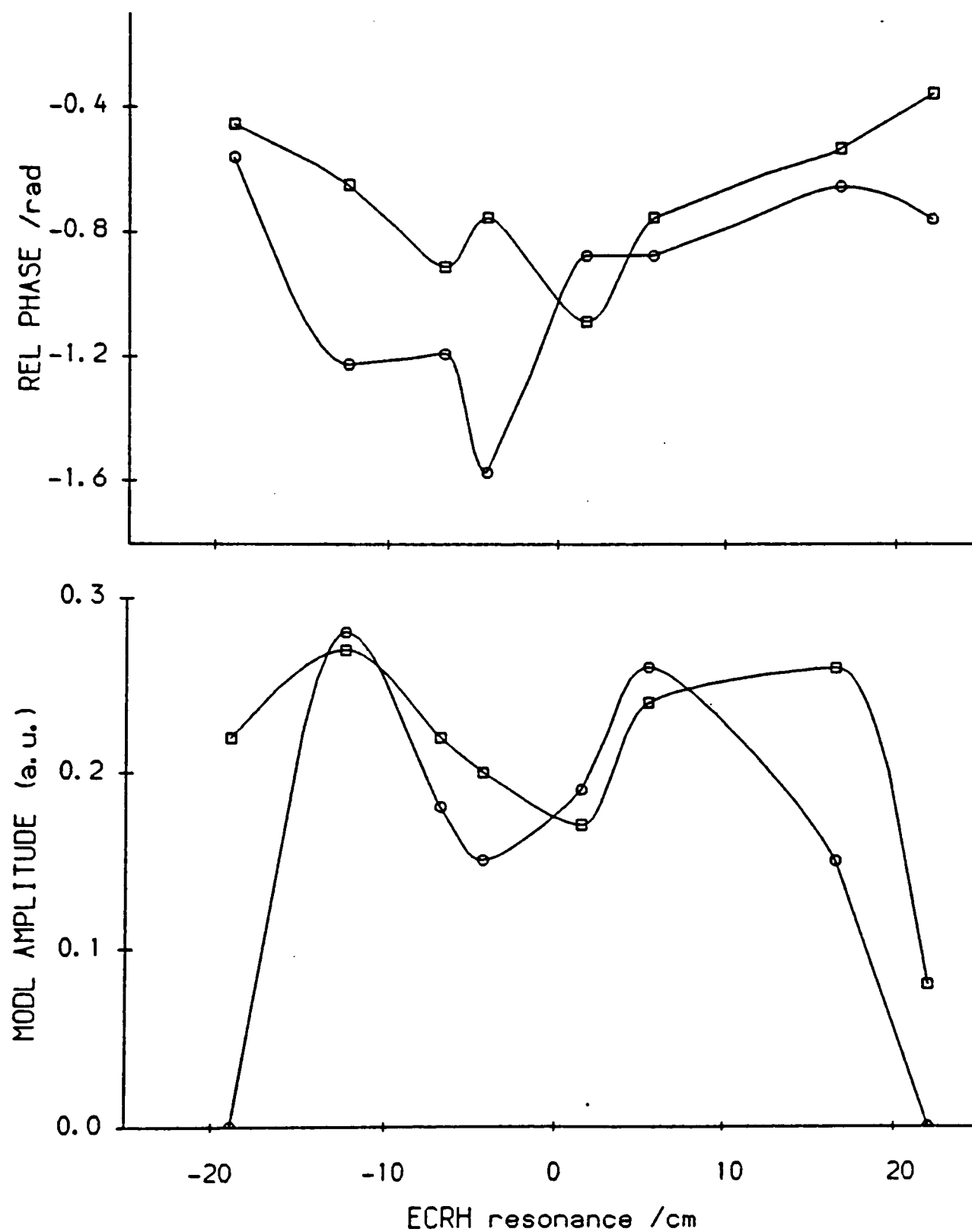


Figure 4.8: Relative phases and modulation amplitudes of fluctuations of ion saturation current $\tilde{j}_{\text{sat}}$ (circles) and radial magnetic field rate-of-change $\tilde{B}_r$ (squares), plotted against unshifted ECRH resonance radius for a sequence of low-density discharges with 2nd harmonic heating.

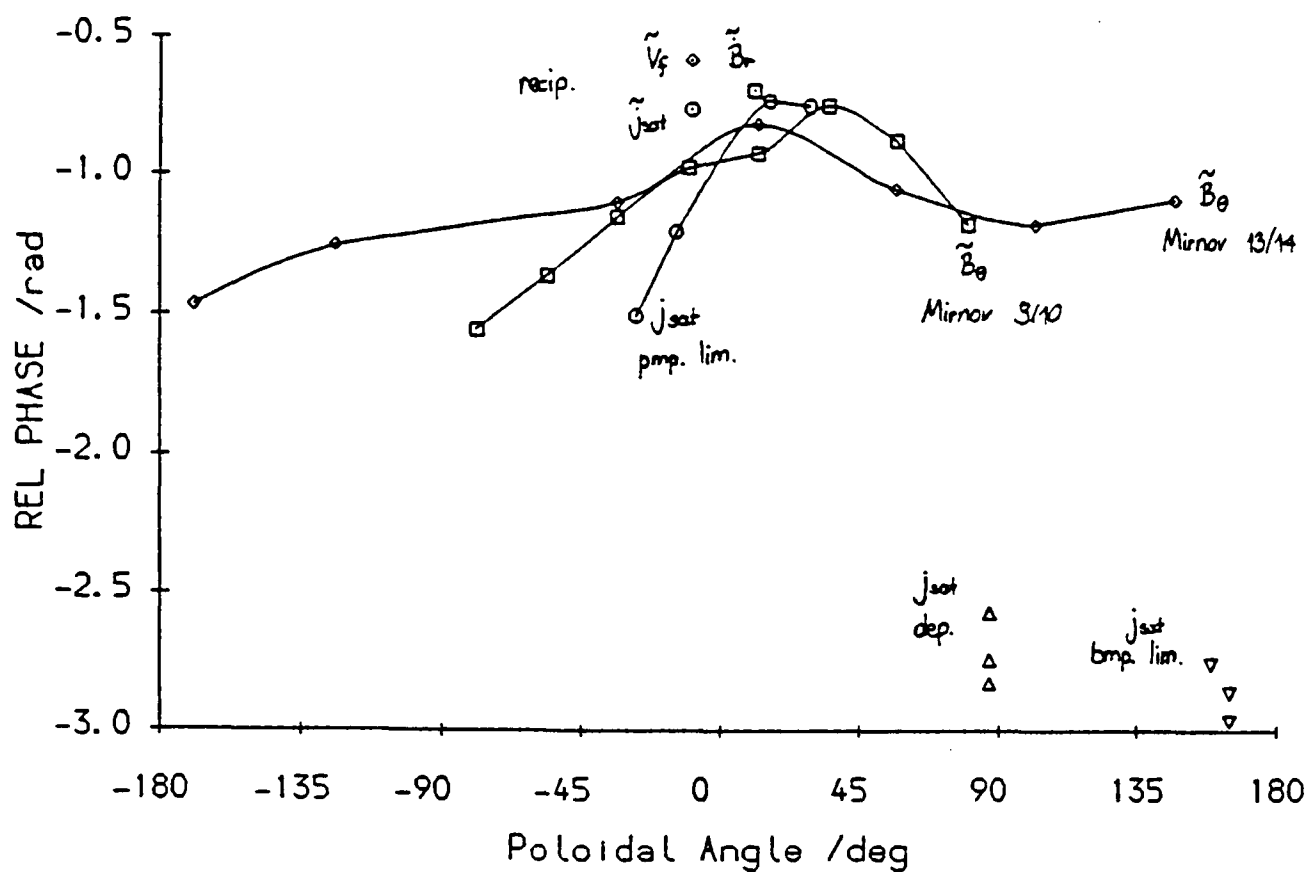


Figure 4.9: Relative phases of all edge signals, plotted against poloidal angle (measured upwards of outside equatorial plane), for one of the discharges in the sequence of figure 4.8 with resonance at +6cm.

are additionally compared with the extrapolated phases of the SXR signals.

The signals from the $H\alpha$ detector viewing the bumper limiter showed smaller phase lags ($\sim 1\text{rad}$) than the phases of the other edge signals and the extrapolated SXR phases. The signal from the detector viewing the torus wall was in phase with the ECRH signal, whilst the signal from the detector viewing the pumped limiter showed an intermediate response, between those of the other two detectors. These results indicate the presence of a prompt response in the $H\alpha$ emission, depending on the area viewed, which implicates a certain direct interaction of the ECRH wave at the edge (direct microwave ionization). The general conclusion that follows from the periphery results is that, whilst the edge effect of the modulated ECRH is predominantly a delayed response due to the diffusively propagating thermal wave, a prompt response arising from direct edge absorption is also present.

4.2.1.5 Modulation of the line-averaged density

The modulation levels of the line-averaged density signals that have been observed in the ECRH modulation experiments, with the help of the microwave interferometer diagnostic, were low in comparison with the modulation levels of the ECE and SXR signals (an example of the line-

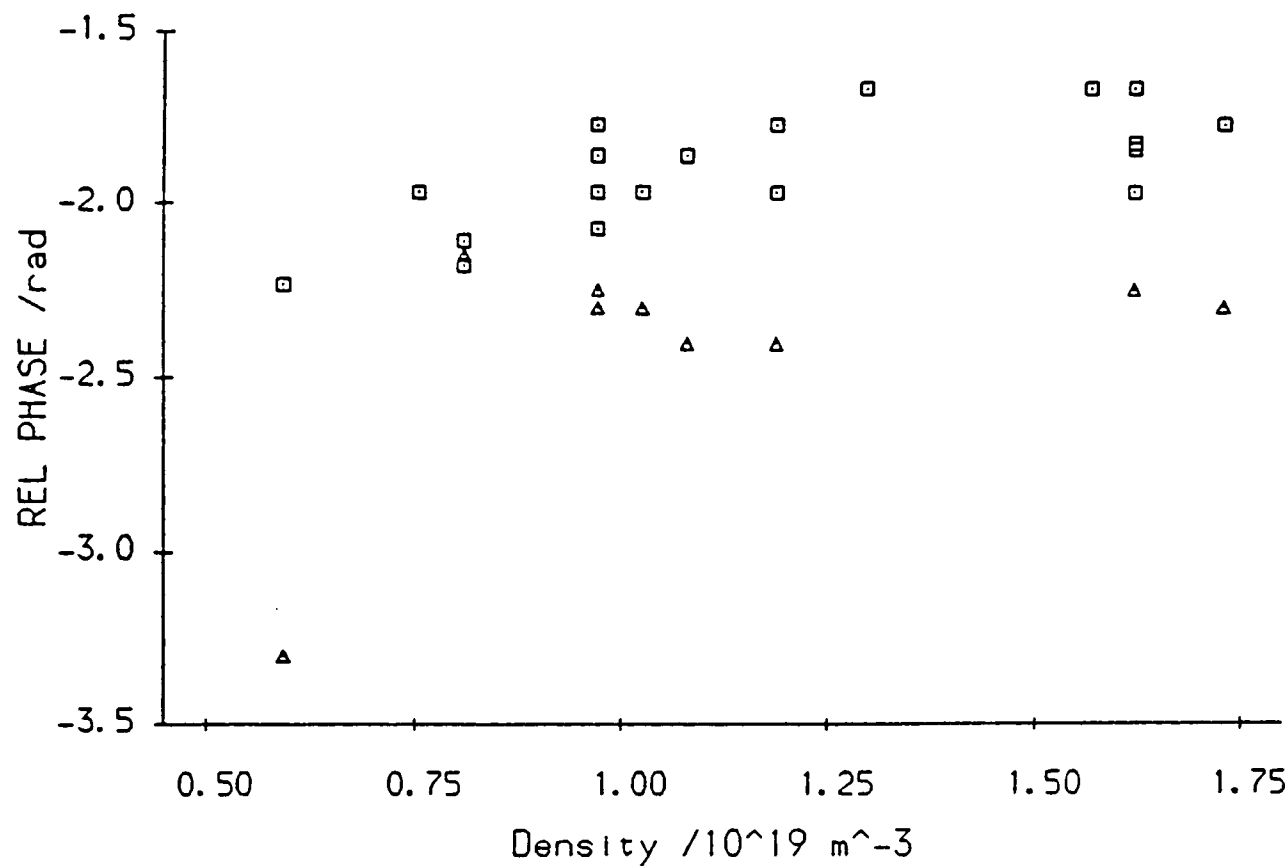


Figure 4.10: Relative phases of fluctuations in poloidal magnetic field $\tilde{B}_\theta$ (squares), compared with extrapolated phases of SXR signals (triangles, error $\pm 0.2\text{rad}$), plotted against line-averaged density, for discharges with fundamental resonance at +1.5cm.

Table 4.1: Relative modulation levels of line-averaged density signals ($\tilde{n}_e/\bar{n}_e$), in central and outer chord MWI channels (half peak-to-peak amplitudes).

Working gas	ECH resonance	Central chord	Outer chord
He	fundamental	$\lesssim 0.2\%$	$\lesssim 0.5\%$
H/D	fundamental	$\lesssim 0.5\%$	$\lesssim 2\%$
H/D	2nd harmonic	$\lesssim 1.5\%$	$\lesssim 5.5\%$

averaged density signals is shown in figure 3.20). The observed relative modulation levels of the line-averaged density, in helium and hydrogen/deuterium discharges, and with fundamental and second harmonic electron cyclotron heating, are shown in table 4.1. In all cases, upper limits on the modulation levels are given, since the modulation was only barely detectable above the noise level. These relative modulation levels are to be compared with the corresponding modulation levels over the central region, of the ECE (temperature) signals, 5–12%, and of the SXR signals, $\lesssim 5\%$. It can be seen that, in all cases, the relative modulation level of the density signals increases towards the edge (by a factor of 3), as it is also observed for the ECE and SXR

signals. The density modulation levels are lower with helium as the working gas, than with hydrogen or deuterium. This difference can be attributed to the smaller edge effects present in helium plasmas —for such plasmas, the edge recycling rate is higher, producing a greater damping of any density changes, and also less gas is stored in the vacuum vessel walls, thus decreasing any effects of particle desorption by the direct interaction of the ECRH wave or by the interaction of the propagating thermal wave. This observation is consistent with results from other experiments with ECRH in well-conditioned discharges, where it is found that the density changes are smaller in helium than in hydrogen plasmas. Coupling between energy and particle transport, being possibly different in helium and hydrogen plasmas, may be invoked as a further, possible cause of this difference in the density modulation levels. Finally, the density modulation level is higher in discharges with second harmonic electron cyclotron heating. This increase in the observed modulation is almost completely due to the lower plasma current of these discharges leading to an increased modulation amplitude of the plasma column position, as it has already been discussed, and is consequently unrelated to the transport processes.

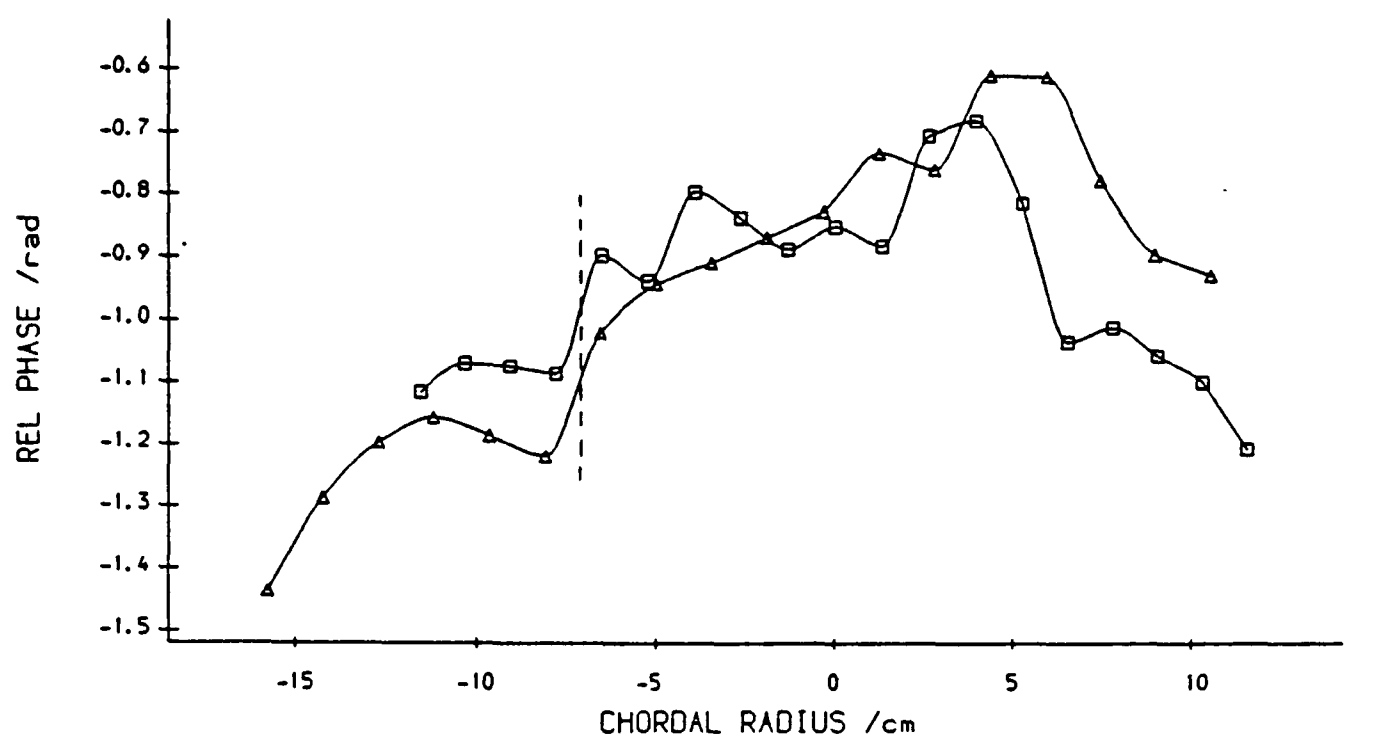
The analysis of the MWI signals is outlined in subsection 3.5.2. It has been found impossible to measure the phase of the line-averaged density modulation with meaningful accuracy, because of their high noise content. The possible causes and effects of the density modulation will be examined in more detail in the following chapter. For the purposes of the analysis described in this chapter, however, it has been assumed, on the basis of the relative modulation levels of density being much smaller than those of the ECE and SXR signals, that the effect of the density modulation on the thermal transport is negligible. It should, nevertheless, be noted that the effect of a *local* density perturbation⁵ on the SXR emissivity is expected to be greater than that of a temperature perturbation of the same size, at least in the hot central region of the discharge, where the modulation index for the temperature perturbation is well below 2, the corresponding value for the density perturbation. It is also conceivable that the density modulation level observed in the line-averaged signals may be smaller than that of the local variable, owing to

⁵i.e. one that is due to a change in the transport or in the particle source, as opposed to one due to a change in the column position

the possible cancellation of oscillations with short scale lengths. Even bearing these factors in mind, however, the effect of the observed density modulation on the SXR emissivity is expected to be small.

4.2.2 Indications of non-diffusive behaviour

Whilst the majority of discharges in the present ECRH modulation experiments displayed a behaviour broadly consistent with diffusive processes, as described in the previous subsection, a number of discharges exhibited a behaviour that was qualitatively different. The first type of this behaviour, shown in figure 4.11, is characterized by flat amplitude profiles, and unusually small phase lags with a profile minimum well away from the expected unshifted resonance radius. These effects have manifested themselves with ECRH absorption well towards the high field side. Further analysis revealed that such “anomalous” discharges were accompanied by an increased MHD activity and that the sawtooth oscillations were locked in period and phase with the modulated part of the signal; this implies that the sawtooth period was modified at the onset of the modulated ECRH. This effect has not been observed in the modulated ECRH discharges that were associated with the “normal”, diffusive behaviour, for which the sawtooth oscillation was in general relatively weak. The sawtooth locking was revealed by a coherent addition (or so-called “box-car”) procedure, involving the superposition of successive segments of a signal, of duration equal to one modulation period, following the elimination of a quadratic trend from the signal. In this way, any oscillation which is not coherent with the modulation is averaged out, but an oscillation which is locked in phase with the modulation is enhanced and becomes readily apparent. The result of this analysis on the SXR signals, for the “anomalous” discharge of figure 4.11, is shown in figure 4.12. The same analysis carried out on a similar discharge with on-axis heating, characterized by the “normal”, diffusive behaviour, does *not* reveal any sawtooth locking effects, as shown in figure 4.13. Whilst the relation between sawtooth locking and non-diffusive behaviour has become apparent, it is not clear how these effects arise, as such “anomalous” behaviour could not be readily reproduced; nevertheless, off-axis heating appears to be a contributing factor. Analysis of the sawtooth signals separated from the underlying



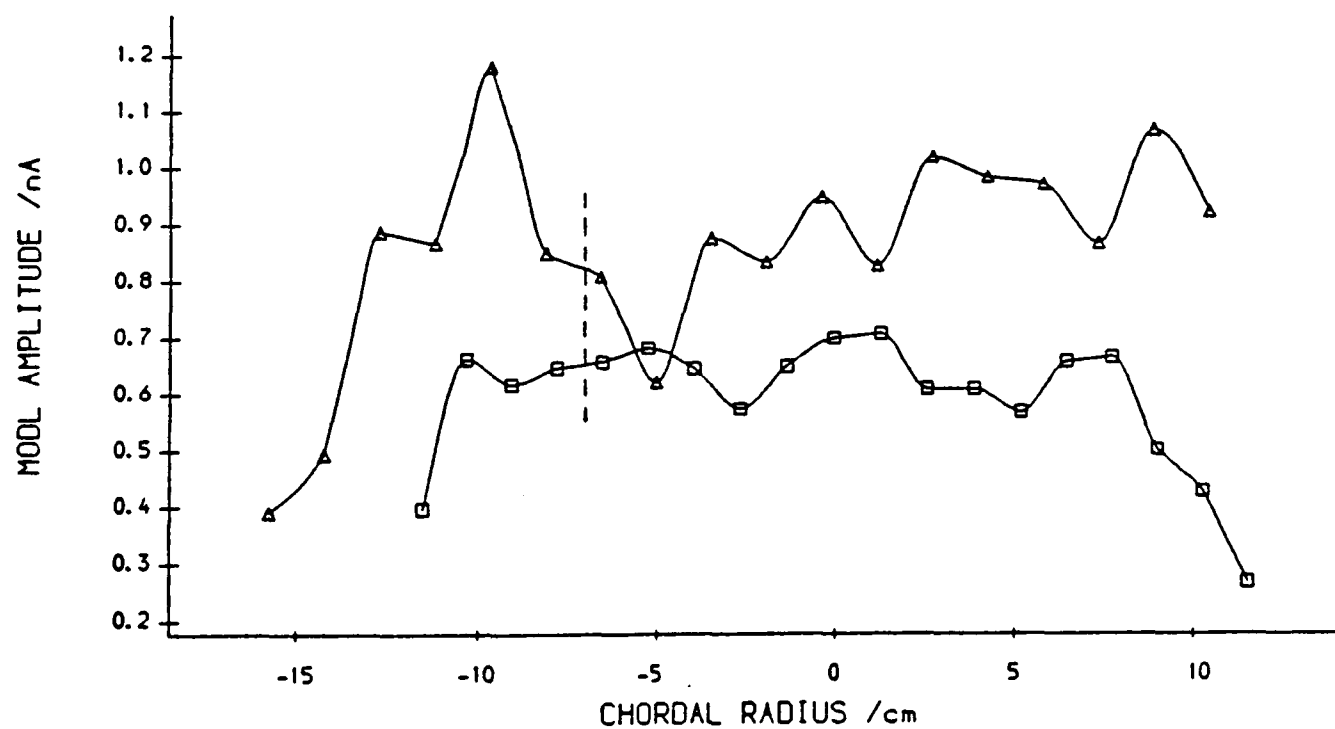
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VERTICAL $\triangle$ (>0 OUTSIDE)HORIZONTAL $\square$ (>0 TOP)

TIME/ms= 365.0- 425.0

FREQ/Hz= 142.86

FBDW/Hz= 13.75



DAT: X31996.000

VERTICAL $\triangle$ (>0 OUTSIDE)HORIZONTAL $\square$ (>0 TOP)

TIME/ms= 365.0- 425.0

FREQ/Hz= 142.86

FBDW/Hz= 13.75

Figure 4.11: Analysed SXR data for atypical low-density He discharge (type-I anomaly), showing small phase lags with a profile minimum well away from the unshifted resonance position (indicated by the dotted line) and a flat amplitude profile.

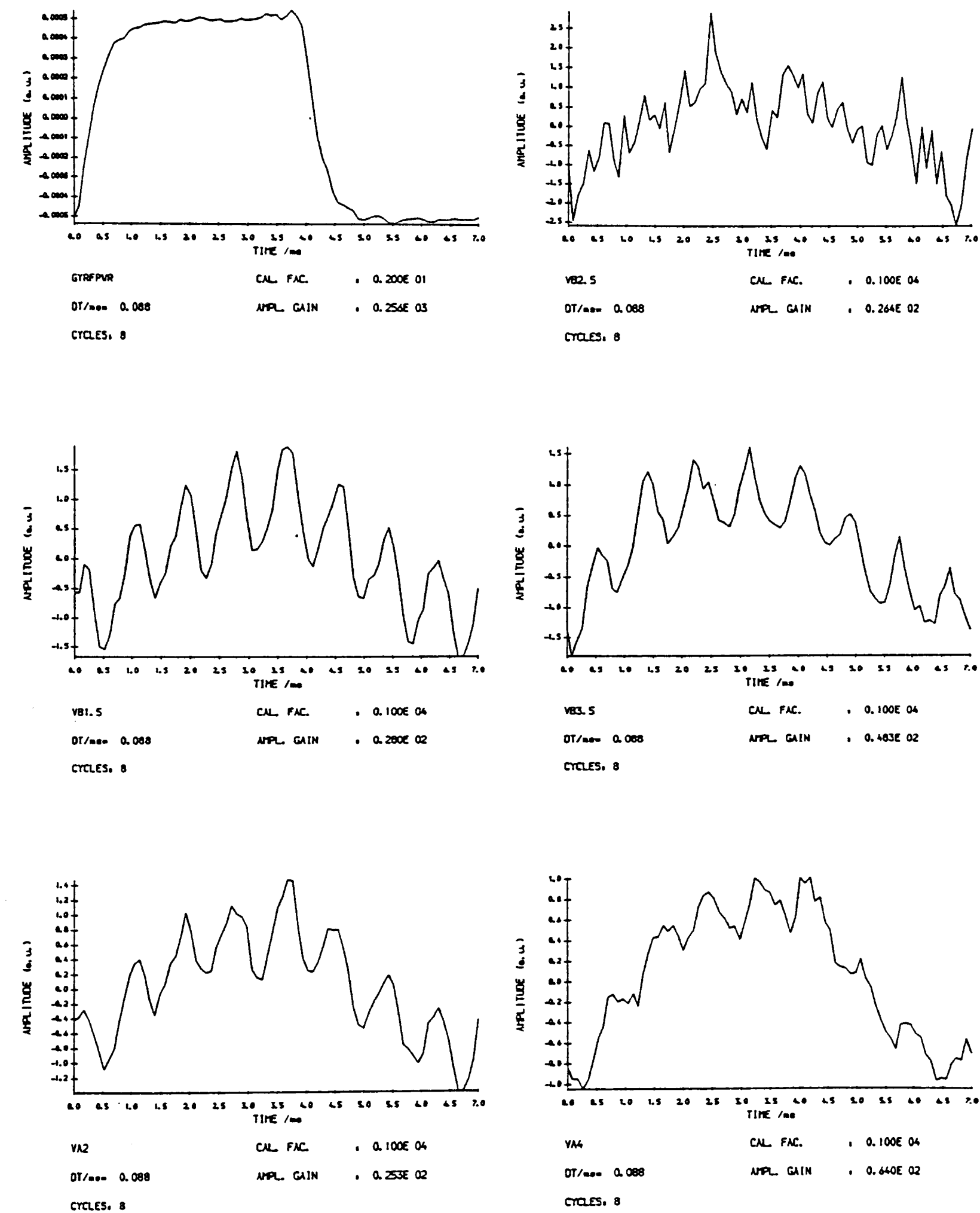


Figure 4.12: Vertical array SXR signals from the “anomalous” discharge of figure 4.11, as analysed by the coherent addition procedure, exhibit sawtooth locking. Signals both inside and outside the inversion radius are shown.

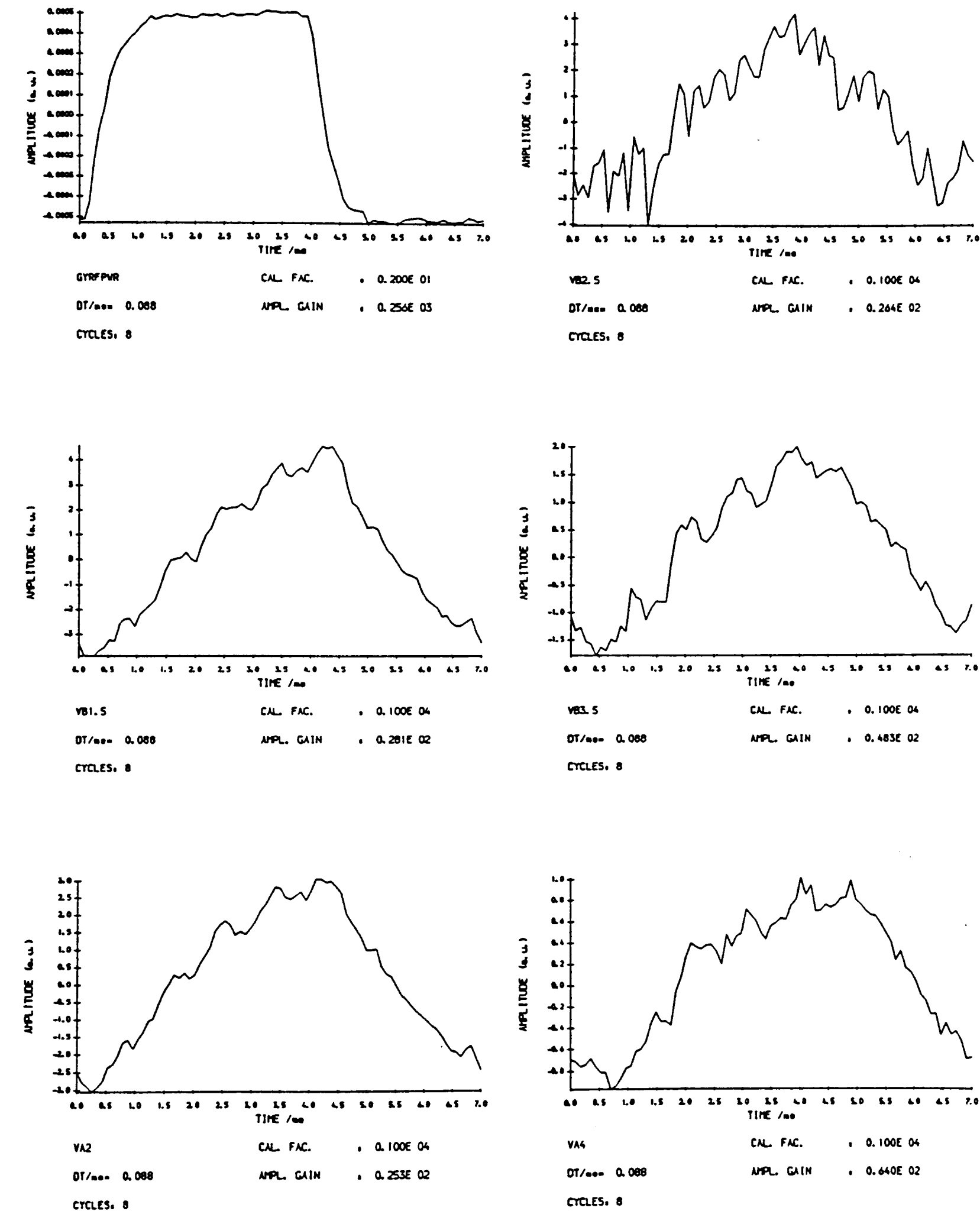


Figure 4.13: Vertical array SXR signals from a discharge with the “normal”, diffusive behaviour, as analysed by the coherent addition procedure, do not show a clear sawtooth locking effect. This discharge has the same parameters as that of figure 4.12, but with on-axis ECRH absorption.

modulation signals indicated a negligible contribution at the fundamental modulation frequency. One thinks that this non-diffusive behaviour is due to an internal MHD instability which leads to energy transfer. This might be driven unstable by changes in the resistivity that are brought about by the modulation of temperature. Such MHD instabilities will in general both modify the local transport, thus leading to changes in the local temperature and density, and distort the flux surfaces, thus further modifying the line-integrated SXR emissivity.

A further type of “anomaly” has manifested itself, somewhat less infrequently than the first type, in this case with moderately off-axis heating, in the form of an abnormally large phase lag in the central region, as shown in figure 4.14. This type of behaviour, usually observed at higher densities, was also commonly accompanied by sawtooth locking, an example of which is shown in figure 4.15. In this particular case, the amplitude of the sawtooth oscillation is greater than that of the oscillation at the modulation frequency, by a factor of about 2; the modulation oscillation was not visually identifiable in the response signals. One therefore expects that the sawtooth oscillation will have a substantial effect on the relative phase at the modulation frequency, if the amplitude or period of the sawtooth oscillation change within one modulation period. Although this type of behaviour is not entirely incompatible with transport of a diffusive nature, and the parts of the amplitude and relative phase curves outside the power deposition radius were fitted satisfactorily using a diffusive transport model, the central region of abnormally large phase lag has been quite impossible to explain in terms of the diffusive model. A further example of this general type of behaviour is shown in figure 4.16, where large phase gradients suggest the presence of a layer of unusually low thermal conductivity (about $0.4\text{m}^2\text{s}^{-1}$ or less) with heating inside that layer; such large phase gradients, frequently associated with extrema of small phase lag, have been observed at minor radii of between 4cm and 6cm. The position of this feature in the phase profile is comparable to the location of the $q=1$ surface, at an approximate minor radius of 5cm. Other experimenters (Joye et al., 1987; see subsection 2.7.2) have reported analogous behaviour in experiments with modulated Alfvén wave heating on the TCA tokamak (similar to DITE), as well as with other forms of perturbation: a large phase gradient and a shift of 180° at the $q=1$ radius, accompanied by only a small phase variation at other radii, were

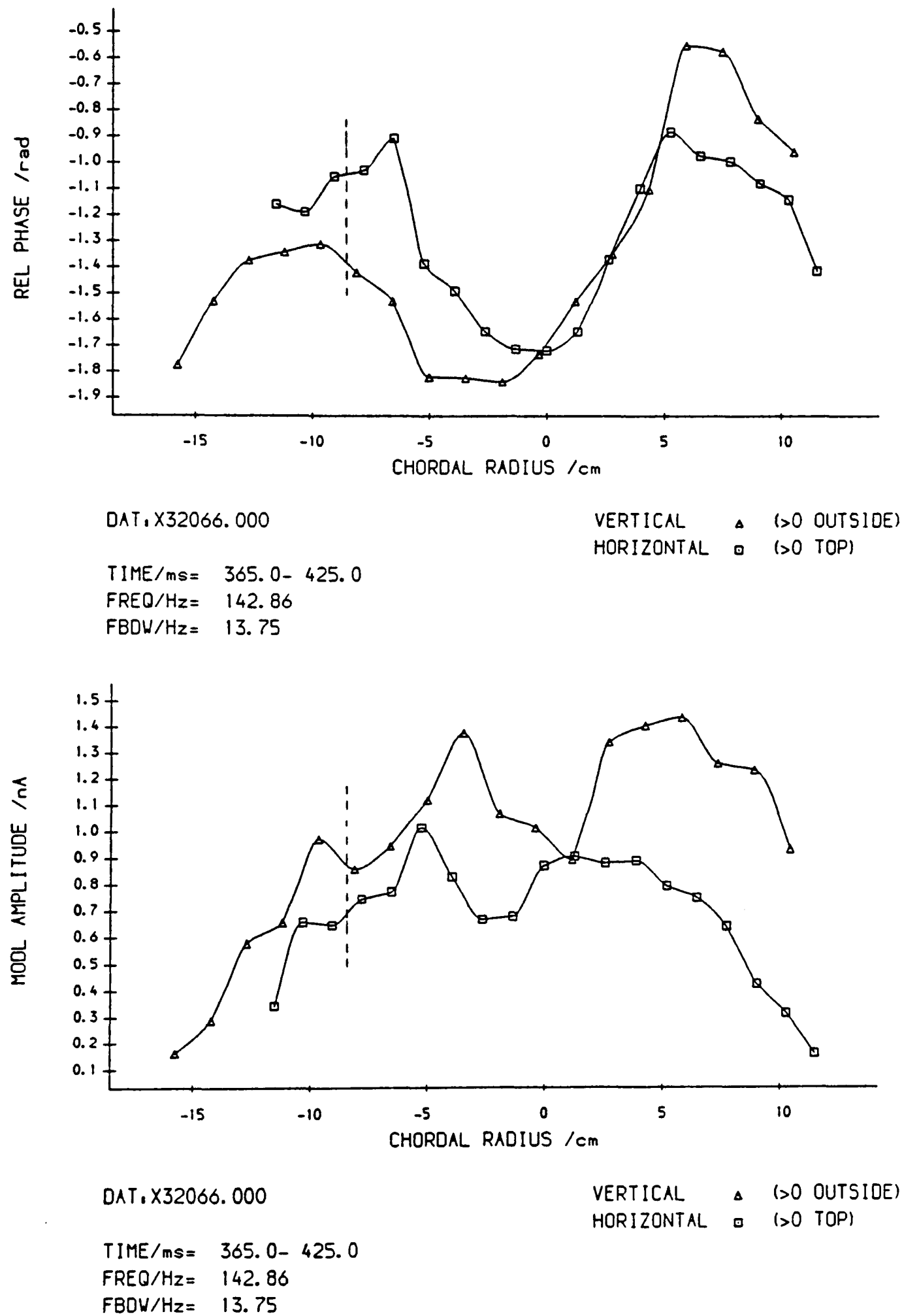


Figure 4.14: Analysed SXR data for atypical high-density He discharge (type-II anomaly), showing a large phase lag in the central region.

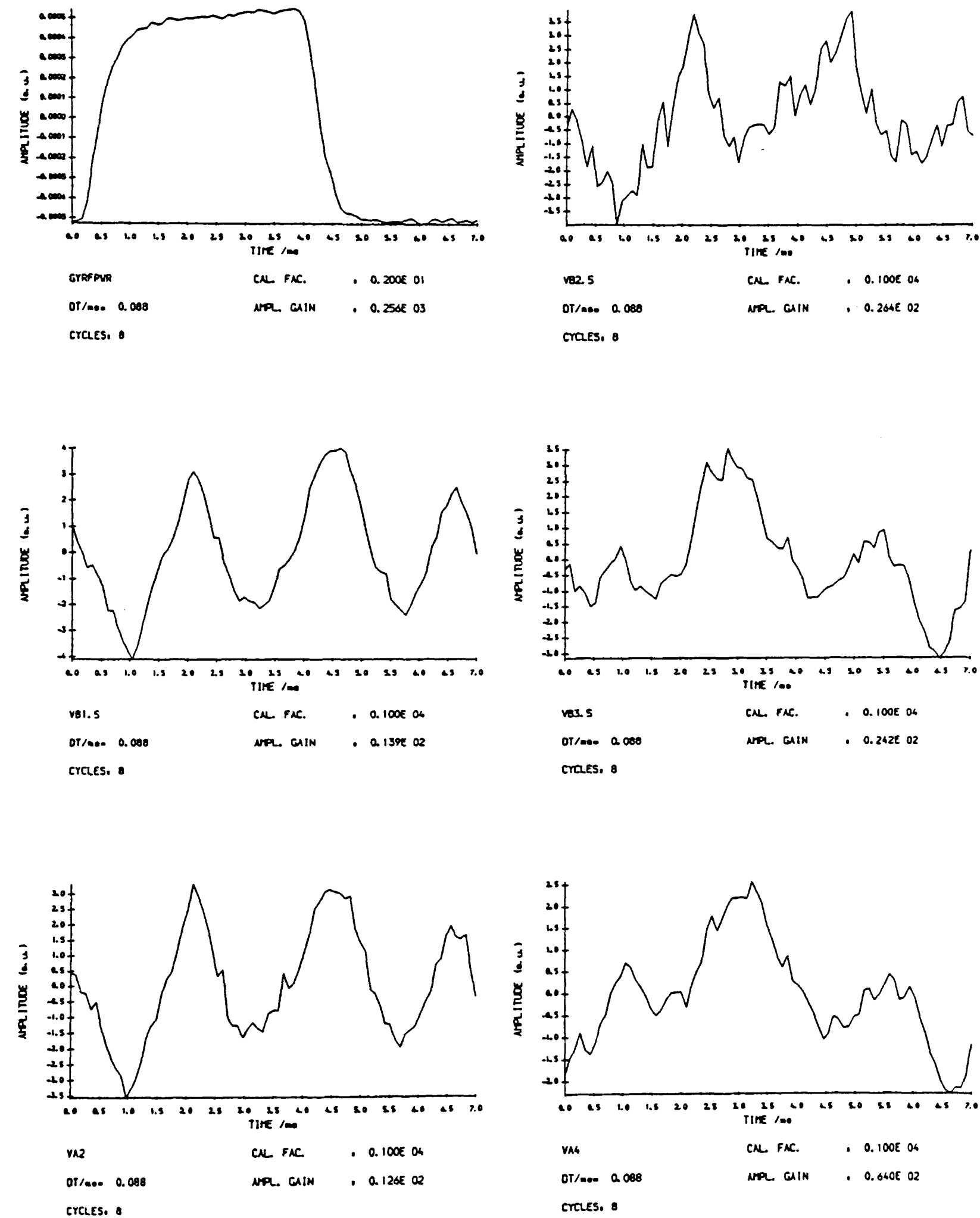


Figure 4.15: Vertical array SXR signals from the “anomalous” discharge of figure 4.14, as analysed by the coherent addition procedure, exhibit sawtooth locking. In this case, the sawtooth oscillation is dominant over the oscillation at the modulation frequency (cf. the amplitude of the latter in figure 4.14).

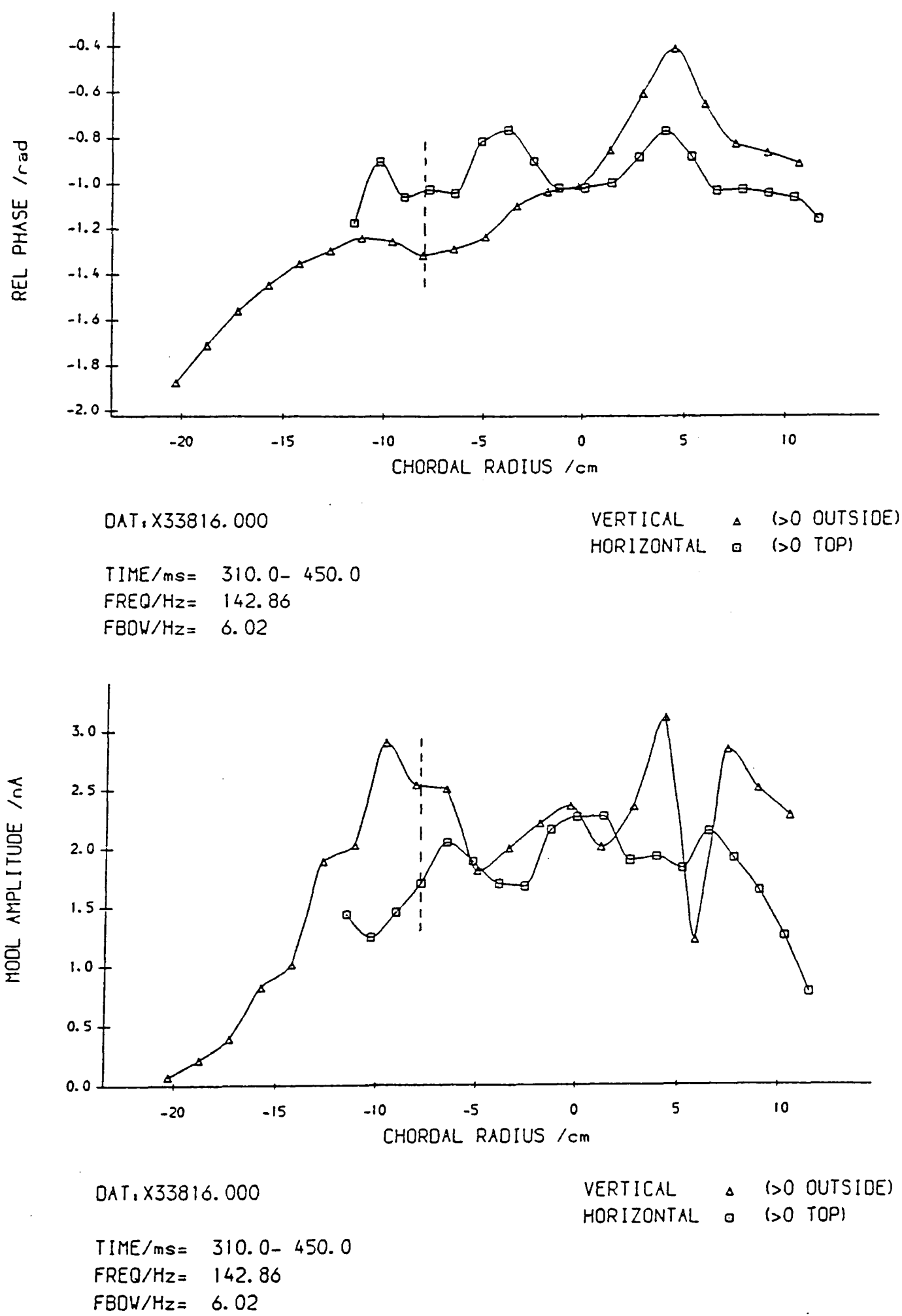


Figure 4.16: Analysed SXR data for atypical medium-density H discharge (type-II anomaly), showing a narrow region of large phase gradient and extrema of small phase lag, suggesting the presence of an insulating layer near $r=4\text{cm}$.

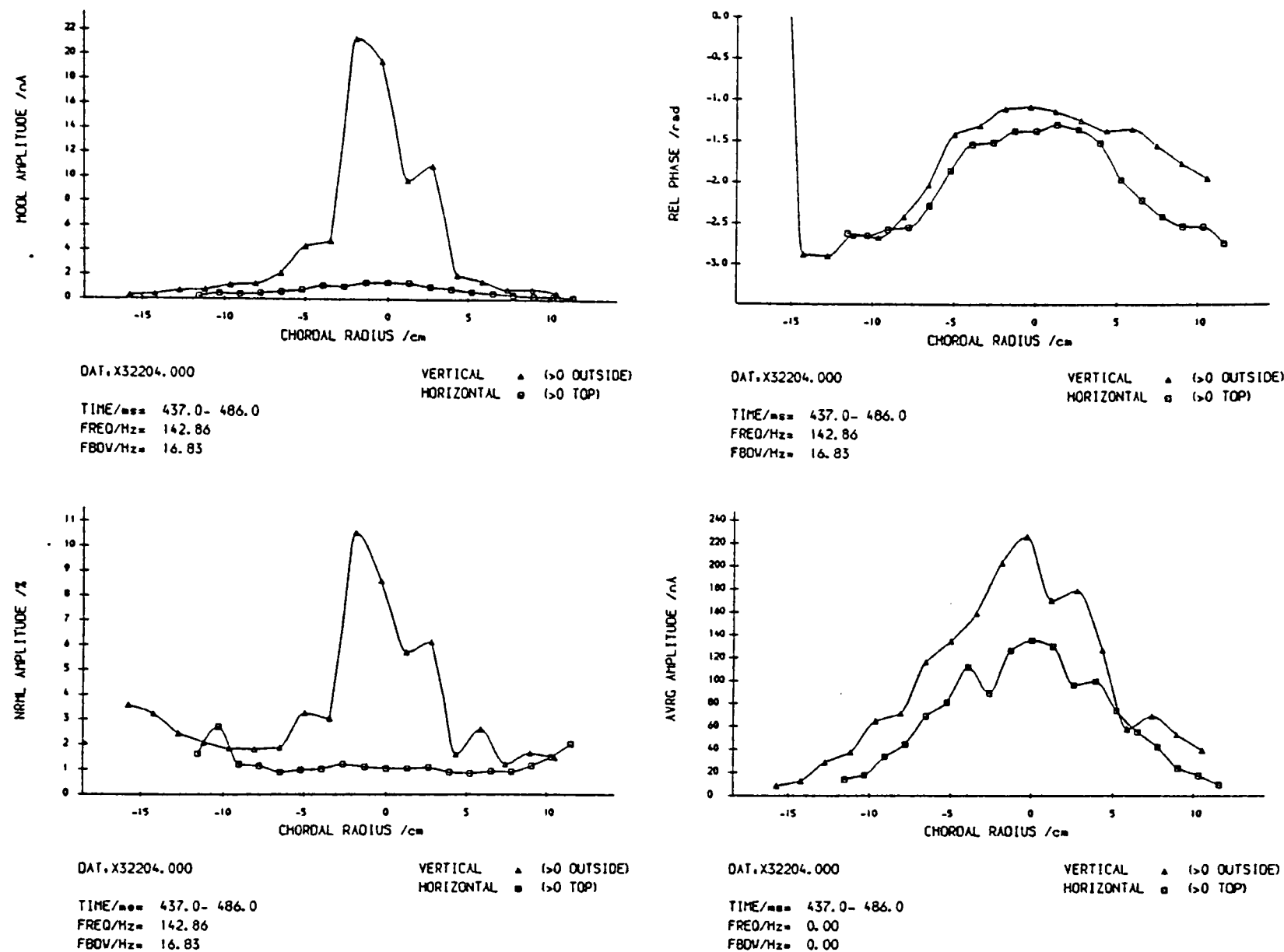


Figure 4.17: Analysed SXR data for atypical low-density He discharge (type-III anomaly), showing a large modulation amplitude detected by the vertical array.

attributed to the presence of an insulating layer at the inversion radius surrounding the heated area. Those results were, however, more pronounced in their persistence and incompatibility with diffusive behaviour.

A final case of atypical behaviour occurred in certain low-density discharges, where the modulation amplitudes of the vertical array SXR signals were dramatically increased (by factors of up to 10) above those of the horizontal array signals, an example being presented in figure 4.17. This can be attributed to vertically drifting ripple-trapped electrons striking the torus walls and producing radiation detected by the vertical array. Nevertheless, the response measured by the horizontal array was normal, and in agreement with the results of the diffusive model.

In conclusion, it should be emphasized that such “anomalous” discharges were *not* typical of the DITE modulation experiments. It seems likely that, for some of the modulation experiments, the conditions were near the transition between diffusive-like and “anomalous” behaviour, the

transition itself being dependent upon small changes in the conditions. Many such cases of atypical behaviour were observed only in the earlier experiments of the first campaign, with helium as the working gas. The ECE diagnostic was not available for those experiments, thus making any interpretation more speculative. The possibility cannot therefore be ruled out that some of the observed effects in SXR signals were related to (a) the density modulation level and/or (b) the level and behaviour of impurities —impurity transport can be very different from, or possibly opposite to, thermal transport. A further factor contributing to some of the observed anomalies might be the presence of the locked sawteeth in itself, which could, in some of the cases, have led to a distortion of the data extracted at the modulation frequency (this effect will be discussed in subsection 4.5.2), although such a contribution has not been conclusively observed by independent analysis of the locked sawteeth components of the signals.

4.3 Numerical modelling of diffusive thermal transport

4.3.1 Dynamical thermal transport model

For the purposes of a numerical study of the evolution of the electron temperature perturbation excited by the modulated ECRH, and in order (i) to obtain estimates of the effective electron thermal diffusivity χ_e , and (ii) to ascertain the location and width of the ECRH absorption region, a simplified version of the electron energy balance equation was invoked. In particular, the terms included in the model were those corresponding to heat diffusion driven solely by a temperature gradient, electron cyclotron resonance heating and a perturbation of the Ohmic heating. A circularly symmetric solution for the electron temperature perturbation $\tilde{T}_e(r, t)$ was obtained,⁶ using cylindrical co-ordinates in 1-D. A fluid approach is justified by the modulation time being much longer than the thermalization time (see subsection 3.1.2). The linearized form of the perturbed electron energy balance equation is

$$\frac{3}{2} n_e(r) \frac{\partial}{\partial t} \tilde{T}_e(r, t) = \frac{1}{r} \frac{\partial}{\partial r} \left[r n_e(r) \chi_e(r) \frac{\partial}{\partial r} \tilde{T}_e(r, t) \right] + \tilde{Q}_{ech}(r, t) + \tilde{Q}_\Omega(r, t). \quad (4.35)$$

⁶note that the perturbation is on the equilibrium, rather than on the average, temperature

It is assumed here that the density $n_e(r)$ and the electron thermal diffusivity $\chi_e(r)$ are time-independent. $\tilde{Q}_{ech}$ is the modulated electron cyclotron heating power density, the driving term, and $\tilde{Q}_\Omega$ is the variation of the Ohmic heating power density, which is caused by the temperature perturbation through the temperature dependence of the resistivity, a damping term. It is further assumed that the equipartition and radiation terms are not affected by the temperature perturbation.

A cubic form is used as an approximation to the profile of the heat conduction coefficient, i.e.

$$\kappa_e(r) = n_e(r)\chi_e(r) = n_e(0)\chi_e(0) \left(1 + K \left(\frac{r}{a}\right)^3\right), \quad (4.36)$$

this being consistent with corresponding profiles from power balance analysis (Hugill, 1988; refer to subsection 4.5.1); it should be noted, however, that the exponent of 3 has been varied somewhat in the process of fitting the model to the experimental data.

The density is assumed to follow a Gaussian profile of the form

$$n_e(r) = n_e(0) \left\{1 - C \left[1 - \exp\left(-G \left(\frac{r}{a}\right)^2\right)\right]\right\}, \quad (4.37)$$

where the constant G represents a measure of the profile gradient, the half-width being $a/\sqrt{G}$, and the constant C is fixed by the profile edge-to-centre ratio.

The power density of the electron cyclotron heating is taken as the product of a spatial term representing the deposition profile, and a temporal term representing the modulation. On the basis of the results from ray tracing (see subsection 3.2.4), the deposition profile is approximated by two Gaussian functions, a secondary one being added to the primary one in order to represent the broadening of the deposition zone:

$$\tilde{Q}_{ech}(r, t) = Q_{ech}(r)S(t) = Q_0 [G(r; r_0, r_W) + fG(r; r'_0, r'_W)] S(t), \quad (4.38)$$

where the functions $G(r)$ are folded Gaussian functions that satisfy continuity at $r=0$, i.e.

$$G(r; r_0, r_W) = \exp\left(-\left(\frac{r - r_0}{r_W}\right)^2\right) + \exp\left(-\left(\frac{r + r_0}{r_W}\right)^2\right), \quad (4.39)$$

and $S(t)$ is a square-wave modulating function (with finite rise and fall times); r_0 is the minor radius at the centre of the ECRH deposition profile and r_W is the half-width (at the e^{-1} point),

f is the fraction of the broader secondary profile, which is centred at r'_0 with a half-width r'_W , and Q_0 is a power density determined from the total ECRH power absorbed in the plasma, P_{ech} , by the normalization

$$P_{ech} = 2\pi R_0 \int_0^a Q_{ech}(r) 2\pi r dr. \quad (4.40)$$

In several applications of this model of the deposition profile, two distinct half-widths were used for the inner ($r < r_0$) and outer ($r > r_0$) branches of the primary Gaussian function, in order more closely to approximate the results obtained from ray tracing (the outer width being typically somewhat larger than the inner width).

The perturbation to the Ohmic heating power density is modelled on the assumption of a time-independent current density profile,⁷ by considering the linearized perturbation to the classical resistivity ($\eta \propto T_e^{-3/2}$), giving

$$\begin{aligned} \bar{Q}_\Omega(r, t) &= \bar{\eta} j^2 \simeq -\frac{3}{2} \eta j^2 \frac{\tilde{T}_e}{T_e} \simeq -\frac{3}{2} \frac{\frac{3}{2} n_e T_e}{\tau_{Ee}} \frac{\tilde{T}_e}{T_e} = \\ &= -\frac{n_e}{4\tau_{Ee}/9} \tilde{T}_e(r, t). \end{aligned} \quad (4.41)$$

It should be noted that the local ratio of the plasma energy density to the Ohmic power density, that is, the local energy confinement time $\frac{3}{2} n_e T_e / Q_\Omega$, is assumed here to be constant over the minor radius and therefore equal to the global electron energy confinement time τ_{Ee} . Making the usual assumptions of radially uniform electric field and effective ion charge (refer to section 2.5), and also ignoring the neoclassical correction to the resistivity, it follows that the Ohmic power density $Q_\Omega(r)$ is proportional to $T_e(r)^{3/2}$; the density $n_e(r)$ is nearly proportional to $T_e(r)^{1/2}$ in typical DITE discharges, so that the approximation of a uniform local confinement time is a reasonable one. The neoclassical resistivity enhancement factor has a relatively weak dependence on temperature and has been omitted here. The negative perturbation of the Ohmic power, linked to the temperature perturbation, is evident in figures 3.1 and 3.2; the negative perturbation of the Ohmic power is about 18% (peak-to-peak) in figure 3.1, corresponding to an effective temperature modulation level about 6%, in good agreement with the ECE relative modulation levels.

⁷the safety factor profile is conserved in practice, rather than the current density profile, but the approximation made here is sufficiently good

By using an appropriate time constant τ in the place of $-4\tau_{Ee}/9$ in the damping term (4.41), it is possible to include the perturbations of the equipartition and radiation loss terms. This time constant is given by

$$\tau = n_e \left(\frac{\partial Q}{\partial T_e} \right)^{-1}, \quad (4.42)$$

where $Q = Q_\Omega - Q_{exc} - Q_{rad}$ is the total source term (Ohmic heating, and equipartition and radiation losses).

We first address the effect of the source perturbation arising from the electron-ion equipartition term $-Q_{exc}$. Using the result $\tau_{ei} \propto \tau_e \propto T_e^{3/2}$ for the equipartition time, and ignoring any perturbation to the ion energy, one obtains

$$\begin{aligned} -\bar{Q}_{exc} &= -\left[\bar{\tau}_{ei}^{-1} \frac{3}{2} (n_e T_e - n_i T_i) + \tau_{ei}^{-1} \frac{3}{2} n_e \bar{T}_e \right] \\ &\simeq \tau_{ei}^{-1} \left(\frac{3}{4} - \frac{9}{4} \frac{p_i}{p_e} \right) n_e \bar{T}_e; \end{aligned} \quad (4.43)$$

from this result, it follows that the characteristic frequency for the equipartition term perturbation (a driving term when $p_i/p_e < 1/3$) is

$$\nu_{exc} = +\tau_{ei}^{-1} \left(\frac{3}{4} - \frac{9}{4} \frac{p_i}{p_e} \right), \quad (4.44)$$

to be compared with the corresponding frequency for the Ohmic term perturbation

$$\nu_\Omega = -\tau_{Ee}^{-1} \frac{9}{4}. \quad (4.45)$$

For the low-density conditions of table 3.1, one has $\nu_\Omega = -1.1\text{kHz}$ and $\nu_{exc} = 19\text{Hz}$, whilst, for the high-density conditions, one has $\nu_\Omega = -190\text{Hz}$ and $\nu_{exc} = 63\text{Hz}$ (taking $p_i/p_e = 0.25$). Clearly then, the effect of the perturbation of the equipartition term is usually small or negligible, at least in the central region or for low and medium densities, even when the steady-state electron-ion exchange time is comparable to or shorter than the energy confinement time. The ratio of the ion to electron pressures in the present experiments was near $1/3$, the value required for the complete cancellation of the perturbation, so that the corresponding term cannot be calculated with confidence. The confinement time that is used in the calculation should probably be increased for high-density applications, in order that the reduced characteristic frequency may be taken into account.

The effect of the source perturbation arising from the radiation term Q_{rad} is difficult analytically to evaluate, but one notes that the radiation loss is smaller than the Ohmic power in DITE discharges, and has a weaker dependence on electron temperature, at least in the central region of the plasma. It therefore appears that this perturbation term can also be safely ignored. The foregoing discussion of the source perturbations is, of course, applicable to small temperature perturbations that do not violate linearity, the experimental temperature modulation levels being $\lesssim 12\%$ over most of the minor radius.

Returning to the diffusion of the temperature perturbation, the initial condition for (4.35) is

$$\tilde{T}_e(r, 0) = 0, \quad (4.46)$$

and the boundary conditions are

$$\tilde{T}'_e(0, t) = 0 \quad (4.47)$$

$$\text{and} \quad \tilde{T}'_e(a, t) = -A\tilde{T}_e(a, t). \quad (4.48)$$

The first boundary condition arises from continuity at the centre, and the second one at the plasma edge from a Neumann-type cooling law, which also characterizes the equilibrium temperature profile from which the constant A is obtained. Whilst there is no experimental evidence either for or against the edge boundary condition used here, it is a physically plausible one and has only a small effect on most of the results of the transport modelling (virtually identical results have been obtained with the assumption of a vanishing perturbation at the edge). Furthermore, it allows a finite temperature perturbation at the edge, this being consistent with experimental observations. The edge boundary condition is not important in this model, because the condition for the infinite-medium approximation is satisfied for the radii of the SXR and ECE data, which cover only half of the major radius.

4.3.2 The MTRANSP1 computer code

The FORTRAN programme MTRANSP1 has been written to simulate the results of the ECRH modulation experiments, as processed in the frequency domain, on the basis of the simple heat diffusion model described in the previous subsection. The code first solves numerically the

perturbed electron energy balance equation,⁸ obtaining a solution for the electron temperature perturbation over the minor radius (using 51 mesh points) and over several periods of the modulated input (typically 5 cycles with 40 samples per cycle). The computation is carried out for given profiles of density, thermal diffusivity and ECRH power density.

The SXR and ECE signals are subsequently calculated from the solution obtained for the electron temperature perturbation. The ECE signals are obtained from the local temperature variations, whereas the SXR signals are obtained from chordal integrals of the local emissivity. The SXR emissivity is calculated on the basis of a modulation index expressed as a function of electron temperature; a Gaussian profile is assumed for the equilibrium temperature, similar to that of the density, in order to calculate the local SXR emissivity and the associated modulation, using (3.14). Early versions of this code were used with a fixed, temperature-independent modulation index about 0.5. For more recent versions, a temperature-dependent modulation index was estimated with the help of the radiation code described in subsection 3.3.1 (refer to figure 3.13).

Finally, a DFT procedure is used to extract the modulation amplitude and relative phase⁹ of the computed signals, at the modulation frequency and at its second and third harmonics. These computed results are subsequently compared with the experimental ones, and the model parameters are interactively optimized in successive runs of the code. In later versions of the code, some further changes were made to the model profiles. A heat conduction coefficient $\kappa_e(r)$ with a flexible radial dependence was investigated: a variable exponent was used in (4.36); also, provisions were made to simulate the effects of a narrow insulating layer of low heat conductance, surrounding a region of high conductance; finally, the resultant thermal diffusivity, which increases rapidly towards the edge, was not allowed to exceed a certain limit. For cases with off-axis heating, the ECRH power density $Q_{ech}(r)$ had a Gaussian profile with different inner and outer half-widths in (4.38), in addition to the secondary, broad Gaussian profile, in order better to approximate the corresponding profiles computed by ray tracing (the outer

⁸A NAG routine, D03PBF, based on Gear's method is used for the numerical solution (NAG, 1984, vol. 2); this routine is intended for simple parabolic partial differential equations.

⁹steps of 2π can often appear in the relative phase profiles and these are detected and eliminated

half-width being typically greater than the inner).

The experimental data from the horizontal SXR array were used for this analysis, as these were almost unaffected by the horizontal plasma shift. The analysis was supplemented by the ECE data when these were available, but it should be noted that in most cases only a few channels on the inner side could be used for these purposes. The centres of the SXR and ECE model profiles were aligned with the plasma magnetic axis of the corresponding experimental profiles; this was facilitated by the symmetry of the horizontal array SXR data, but there was some uncertainty in the case of the ECE data. In comparing the model data with the experimental ones, the model modulation amplitude profiles were *scaled*, and the model relative phase profiles were *shifted* by a constant phase, in order to match the corresponding experimental profiles—a positive phase shift implies that the phase lag predicted by the model had to be decreased to match the experimental phase lag. In most cases, however, virtually null phase shifts were required. A relative RMS error is calculated for the fit of the amplitude data points, and a χ^2 -error is obtained for the fit of the phase data points. The model average amplitude profiles were further compared with the corresponding experimental ones, with the same scaling as that used for the modulation amplitude, as a check on the model profiles of temperature and SXR emissivity.

4.4 Results from modelling of diffusive thermal transport

4.4.1 Application of transport model

Before we commence a discussion of the results from the modelling of diffusive thermal transport, we shall consider those aspects of the model as applied to the DITE modulation experiments, which remain somewhat uncertain owing to the lack of sufficient measurements, and discuss how these could affect the results of the analysis:

1. The *density profile* was not known with any accuracy, as the density diagnostic was not reliable and produced only four profile points. The density profile was estimated as follows.
An approximate electron temperature profile was first established from the ECE time-

averaged data (the ECE temperature profiles were often distorted and did not indicate the central temperature). A density profile was subsequently obtained on the assumption that the profile constant $\eta_e = \frac{T_e'/T_e}{n_e'/n_e}$ was about 1.5. The density and temperature profiles were finally checked for consistency with certain experimental parameters, namely the loop voltage and the expected central safety factor, both of which are determined by the current density profile and are therefore sensitive, through the resistivity, to the central value and width of the temperature profile; and the beta value which is a measure of the total energy corresponding to the density and temperature profiles. These concepts have already been introduced in the first two chapters, and the calculations were carried out using the equilibrium section of the general transport code described in the following chapter. The gradient constants of the Gaussian density and temperature profiles (see (4.37)), as estimated for discharges with fundamental ECRH, were 3.5 and 5.0 respectively, corresponding to half-widths 53% and 45% of the minor radius respectively, with edge levels (pedestals) about 8% of the central values; narrower profiles were estimated for the low-current discharges with second harmonic ECRH, the corresponding half-widths being 45% and 37% respectively. These profiles provided reasonably good agreement with the unusually narrow profiles of line-integrated SXR emissivity. The effect of the density profile on the model results is through the energy balance (a broader density profile leading to a flatter phase variation), with a further obvious effect on the SXR emissivity profile (this can also affect the phase variation of the line-integrated emissivity). The effect of the temperature profile is only on the SXR emissivity profile and on the radial variation of the SXR modulation index.

2. The *thermal diffusivity profile* (or, given the density profile, the heat conduction coefficient profile) was not known and was estimated in accordance with the results of a simple power balance analysis, including the universally observed increase towards the edge. Minor adjustments of the heat conduction coefficient profile were made in fitting the model, in order to obtain agreement with data at some distance from the axis, but these did not affect the central value of the thermal diffusivity.

3. The most important effect on the modelling is that due to the *ECRH power density profile*.

This can be reliably calculated with a ray tracing code, as shown in subsections 3.2.4 and 3.2.5, at least in the case of the DITE experiments for which near-field effects and fast particle absorption were not important, but the results are sensitive to the choice of density and temperature profiles. In many cases, the ray tracing predicted rather broad absorption profiles, which were less localized than one might expect. The predicted broadening was particularly pronounced at high densities, owing to the vertical refractive broadening of the antenna radiation pattern. These broad profiles were modelled using a linear combination of two Gaussian profiles (see (4.38)), the broad profile having an amplitude 5–20% of the localized one and accounting for up to 65% of the total power (for high densities). The half-widths were typically 1.5–2.5cm for the narrow profile and 5–7cm for the broad profile. A broader deposition profile in the model clearly leads to a broader phase variation; this change entails a *reduction* of the model thermal diffusivity, for the consistency with the experimental data to be maintained; although this reduction is mainly in the diffusivity profile, the effect on the central value is not insignificant. As these corrections are more important at higher densities, the resulting reduction in diffusivity leads to better agreement of the diffusivity values deduced from the ECRH modulation, with those estimated from the global energy confinement time (which increases with density) and from local power balance analysis. It was not possible independently to ascertain the extent of the profile broadening from the ECRH modulation data, without the predictions of the ray tracing code. Nevertheless, the use of the broader profiles resulted in better model fits to the experimental data at both the fundamental and the third harmonic modulation components (the latter affords a more direct indication of the deposition profile, as shown in subsection 4.1.2; it was, however, impossible to use any higher harmonics). The broader deposition profiles also contributed in eliminating most of the phase shifts, which were needed in earlier analyses, between model and experimental results.

4. The temperature dependence of the *SXR modulation index* was difficult reliably to establish from the radiation code described in subsection 3.3.1, as the type and concentration of

impurities were not experimentally determined. Nevertheless, an approximate dependence was obtained, this being of mainly qualitative value and showing the rapid increase of the index with decreasing temperature (refer to figure 3.13); this modulation index was used in the present modelling and was varied in order to obtain reasonable agreement of the code results with the experimental profiles of the steady-state and modulated components of the line-integrated SXR emissivity. The effects of the modulation index on the local and line-integrated amplitude and phase of the SXR emissivity have already been discussed. It will suffice here to remind the reader that the SXR phase variation, and particularly its gradient, are probably (but not necessarily) affected slightly by the choice of the modulation index, whilst the SXR amplitude is significantly modified. Only approximate agreement of the model was obtained with the SXR modulation amplitude, and the SXR phase was used as the main criterion for the determination of the diffusivity.

5. The *electron energy confinement time* used in the Ohmic damping term (see (4.41)) has a significant effect on the model, but was known with reasonable accuracy from magnetic measurements (for DITE discharges, the electron energy confinement time is close to the overall energy confinement time). The inclusion of this damping term, which also leads to a reduction in the model thermal diffusivity, is more important for lower densities. The time constant that was used for higher densities was taken to be larger, by up to 20%, than the energy confinement time, in order to compensate for the partial cancellation of the Ohmic damping term by the equipartition driving term (refer to subsection 4.3.1, (4.41) and (4.43)).

4.4.2 Results of transport model

Model fits The modulation data from a large number of discharges that were characterized by the usual, diffusive behaviour were fitted using the thermal transport code. Good agreement of the simple model of dynamical thermal transport has been obtained with the experimental data, the various aspects of the model being as outlined in the previous subsection.

The electron thermal diffusivity $\chi_e(r)$ obtained from the modelling of medium-density modu-

Table 4.2: Thermal diffusivities obtained from dynamical transport modelling of low-, medium- and high-density discharges with modulated ECRH.

$\chi_e/\text{m}^2\text{s}^{-1}$	low density $0.5 \times 10^{19}\text{m}^{-3}$	medium density $1.6 \times 10^{19}\text{m}^{-3}$	high density $2.8 \times 10^{19}\text{m}^{-3}$
$r = 0$	1.8	1.3	0.9
$r = a/3$	3.5	2.0	1.3
$r = a/2$	8.0	3.6	2.1

lation data was $(1.3 \pm 0.2)\text{m}^2\text{s}^{-1}$ at the centre, around $2.0\text{m}^2\text{s}^{-1}$ at the radius $a/3$, and $3.6\text{m}^2\text{s}^{-1}$ at the radius $a/2$; this diffusivity was higher for low-density data and lower for high-density data, as shown in table 4.2. However, the dependence of this diffusivity on density was weaker than the reciprocal dependence which one might expect from the corresponding, linear scaling of the energy confinement time. The low-density results were obtained from discharges with second harmonic resonance, and the higher diffusivity values could, to some extent, be attributed to the narrower profiles and the lower toroidal magnetic field and plasma current. The profiles that were used for the heat conduction coefficient $\kappa_e(r)$ had exponents of between 2.5 and 3.5 (see (4.36)), and the edge values were greater than the central values by factors of between 1.5 and 3, but, within the experimental errors, all the profiles were similar. The density profile typically dropped to 8% of its central value at the edge. The edge values of the resulting thermal diffusivity $\chi_e(r)$, however, were limited to levels 20–30 times higher than the central values, in order to maintain consistency with experimental estimates of the edge thermal transport. The radial dependence of the diffusivity that was obtained from the model is not considered to be reliable outside the radius $a/2$.

An example of a comparison of the diffusive thermal transport model with experimental results, from a typical, repeatable discharge of medium density, with fundamental, off-axis heating, is presented in figures 4.18 and 4.19, showing the fundamental and third harmonic components of the SXR data, and in figures 4.20 and 4.21, showing the fundamental and third harmonic components of the ECE data. It will be noted that the ECE data were limited in usefulness,

and that the SXR data consequently provided the most important fitting criteria. Nevertheless, there was reasonably good agreement of the model with the ECE data. The ECE data were most useful in determining the temperature modulation level (about 8% in the case shown). The phase delays of the ECE signals, being smaller than those of the SXR signals, provided a useful additional confirmation of the model results. Another difference arose from the localized nature of the ECE signals, compared with the line-integrated SXR signals: with off-axis ECRH absorption, the ECE data are characterized by much more hollow profiles than the SXR data; this aspect of the data was also predicted by the model. An advantage of off-axis deposition is that the central, hollow part of the phase profile allows one to fit a section of the model phase profile which is relatively insensitive to the chosen profiles of density, diffusivity and SXR modulation index. For the present case, figure 4.22 shows the model radial profiles of the ECRH power density $Q_{ech}(r)$, density $n_e(r)$, heat conduction coefficient $\kappa_e(r)$, thermal diffusivity $\chi_e(r)$ and SXR modulation index $\alpha(T_e)$; and the radial and temporal variation of the computed electron temperature perturbation $\tilde{T}_e(r, t)$.

Further examples of the diffusive transport modelling (showing only fundamental-component SXR data) are presented in figures 4.23 for a medium-density discharge with fundamental, on-axis heating; 4.24 for a low-density discharge with second harmonic, off-axis heating; 4.25 for a medium-density helium discharge with fundamental, off-axis heating and modulation at the higher frequency; and 4.26 for a high-density helium discharge with fundamental, on-axis heating.

An interesting special case deserves a special mention: when the diffusion model was used to fit the data from a discharge with strongly off-axis ECRH absorption, as shown in figure 4.27, the unusually low value of $0.6\text{m}^2\text{s}^{-1}$ was obtained for the central thermal diffusivity (with a radial profile similar to those obtained in other fits). Unfortunately, however, very few discharges of this type were available for analysis, and it has been impossible further to investigate this behaviour.¹⁰ In this case, the low thermal diffusivity pertains to an *inward* propagation of the

¹⁰The available discharges with strongly off-axis ECRH had modulation lasting for only a few cycles, so that accurate phase profiles were difficult to compute —nevertheless, the phases have been checked by three different analysis techniques (see section 3.6) and appear to be accurate.

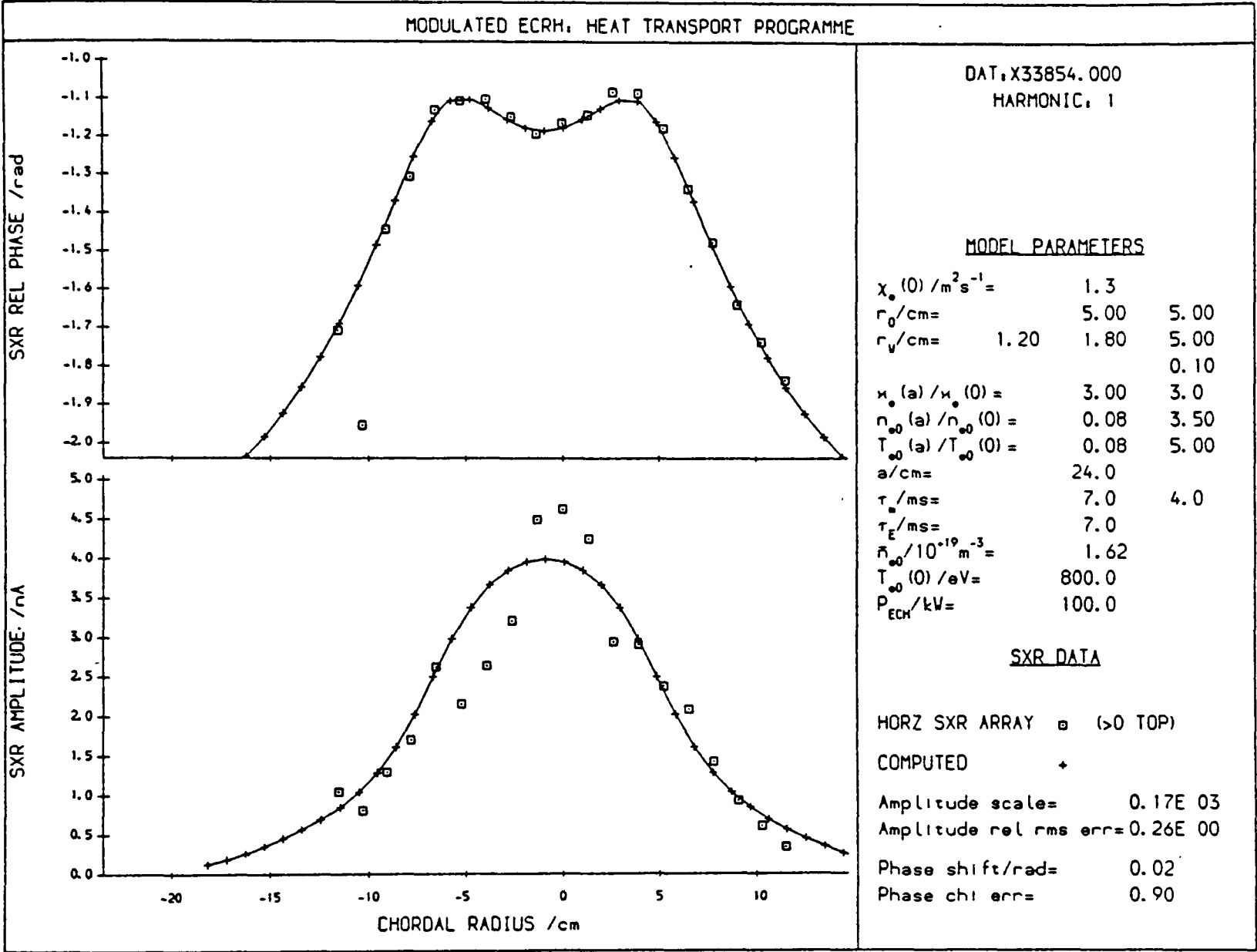


Figure 4.18: SXR relative phases and modulation amplitudes of fundamental component, fitted using dynamical thermal transport model (the vertical array data are not shown). H discharge, medium density, fundamental off-axis heating. Hollow part of phase profile facilitates fit.

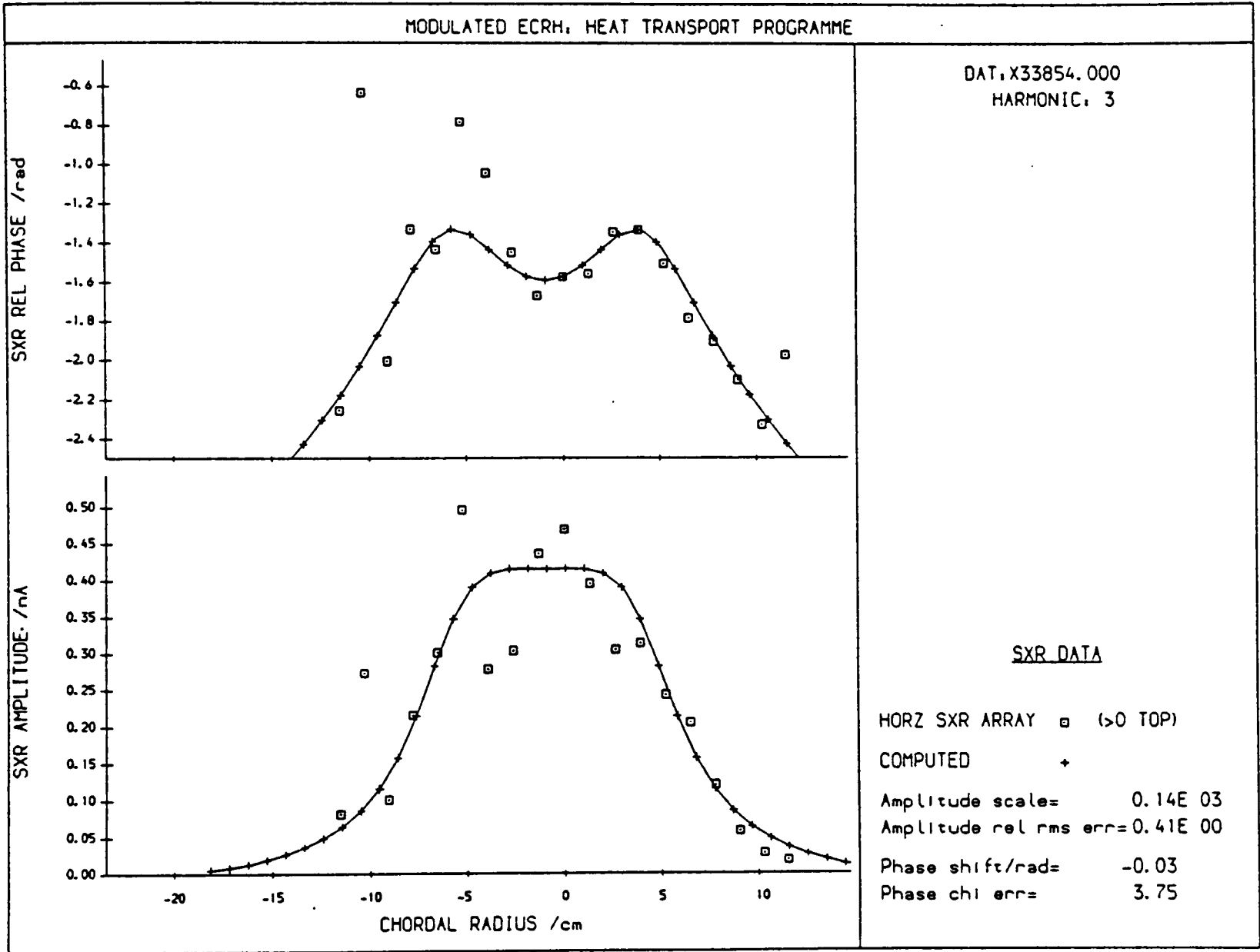


Figure 4.19: SXR relative phases and modulation amplitudes of 3rd harmonic component of data in figure 4.18, with model fit.

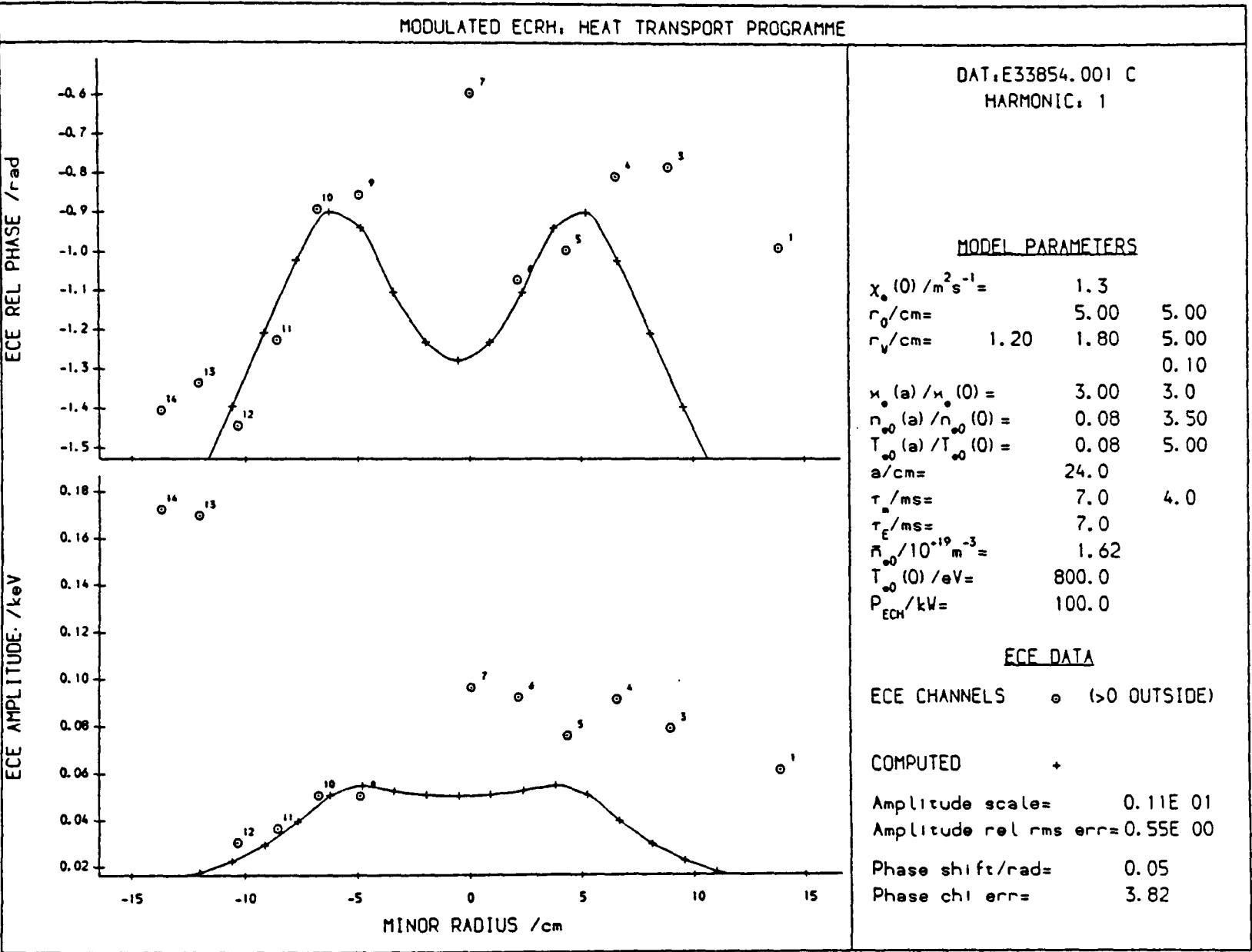


Figure 4.20: ECE relative phases and modulation amplitudes of fundamental component, fitted using dynamical thermal transport model. Discharge of figure 4.18. Poor quality of data makes fit uncertain, but phase gradient and minimum phase lag are in agreement with model.

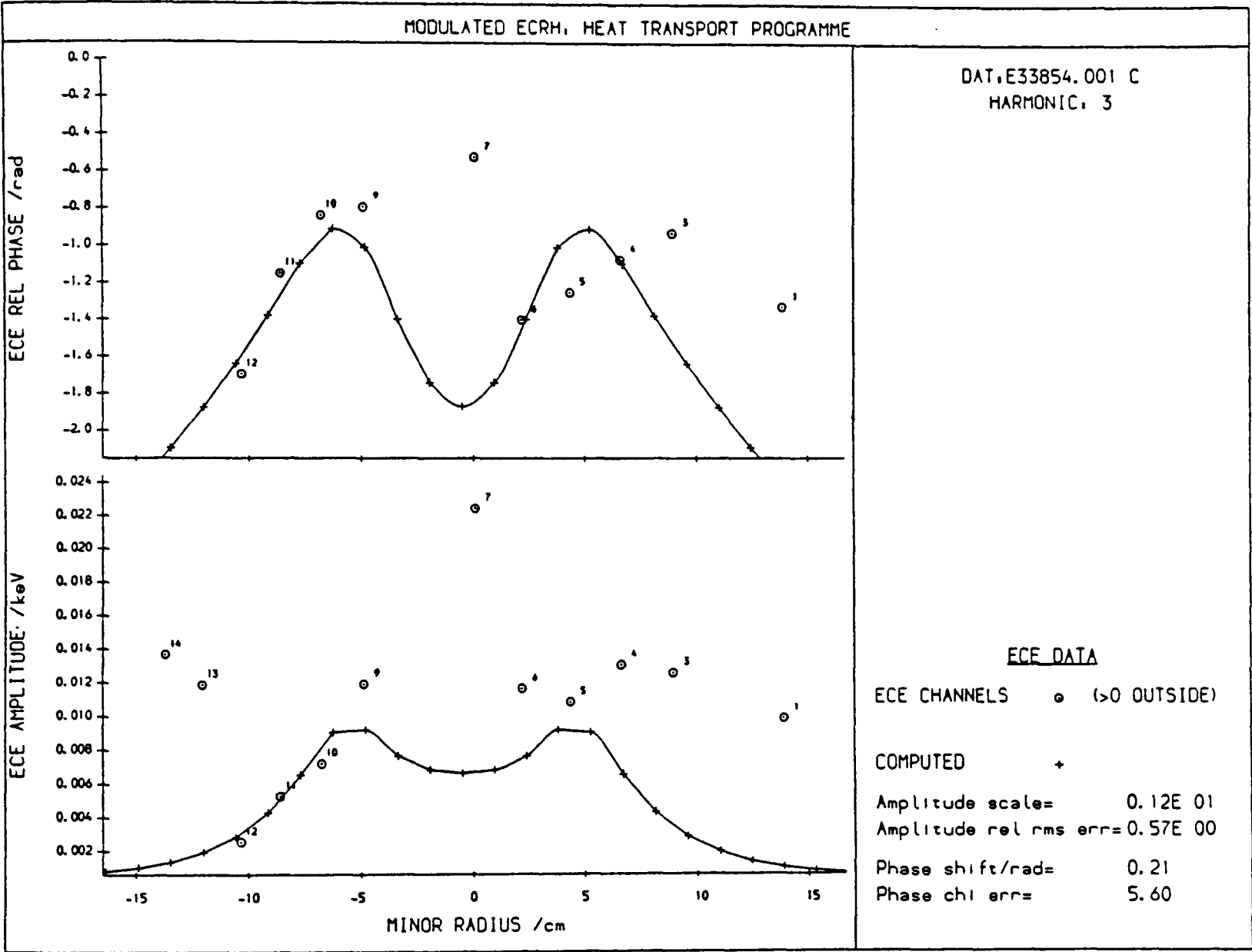


Figure 4.21: ECE relative phases and modulation amplitudes of 3rd harmonic component of data in figure 4.20, with model fit.

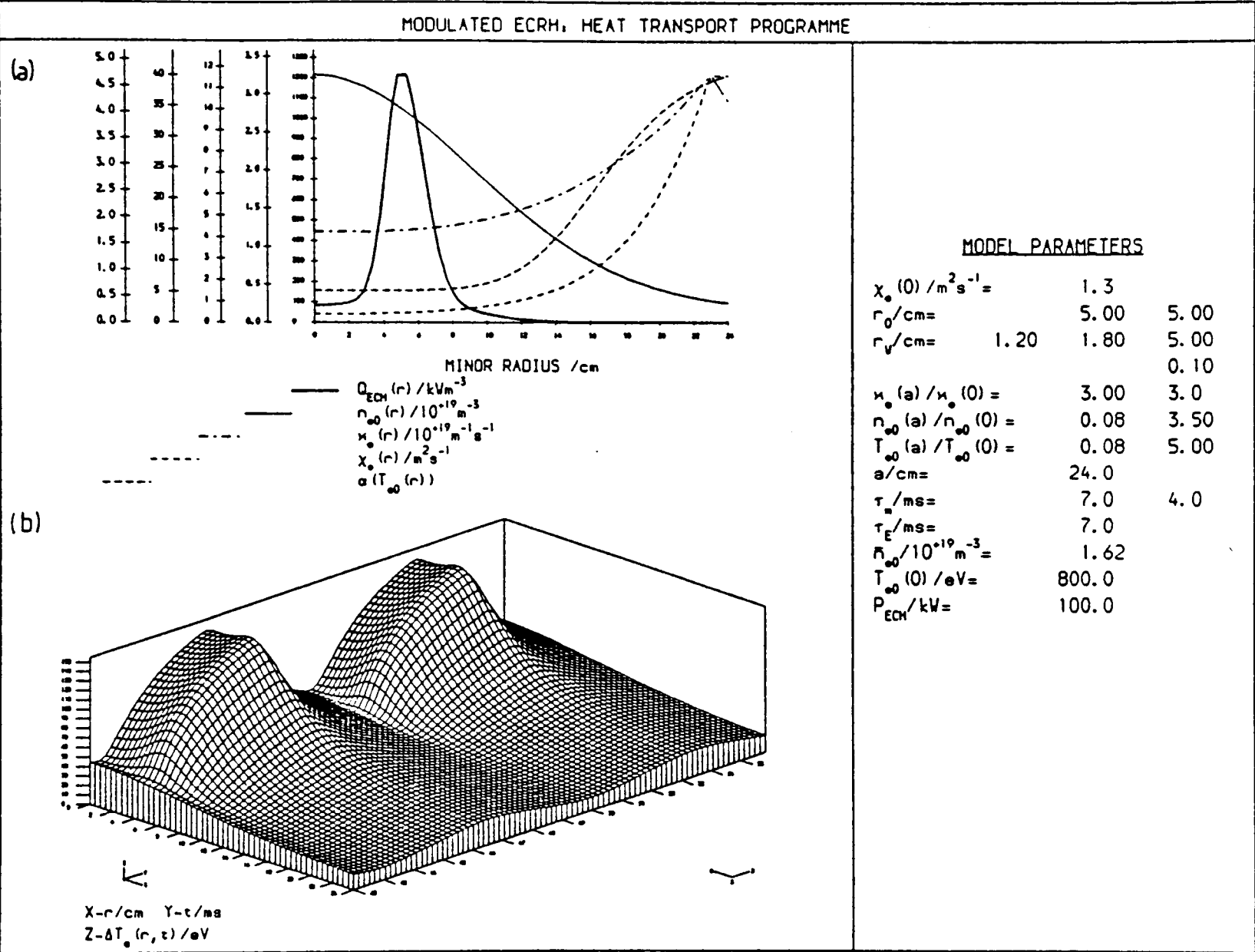


Figure 4.22: Dynamical thermal transport model: (a) radial profiles of ECRH power density Q_{ech} , density n_e , heat conduction coefficient κ_e , thermal diffusivity χ_e , and SXR modulation index α ; (b) electron temperature perturbation $\tilde{T}_e$, plotted as function of minor radius r and time t . Model of figures 4.18–4.21.

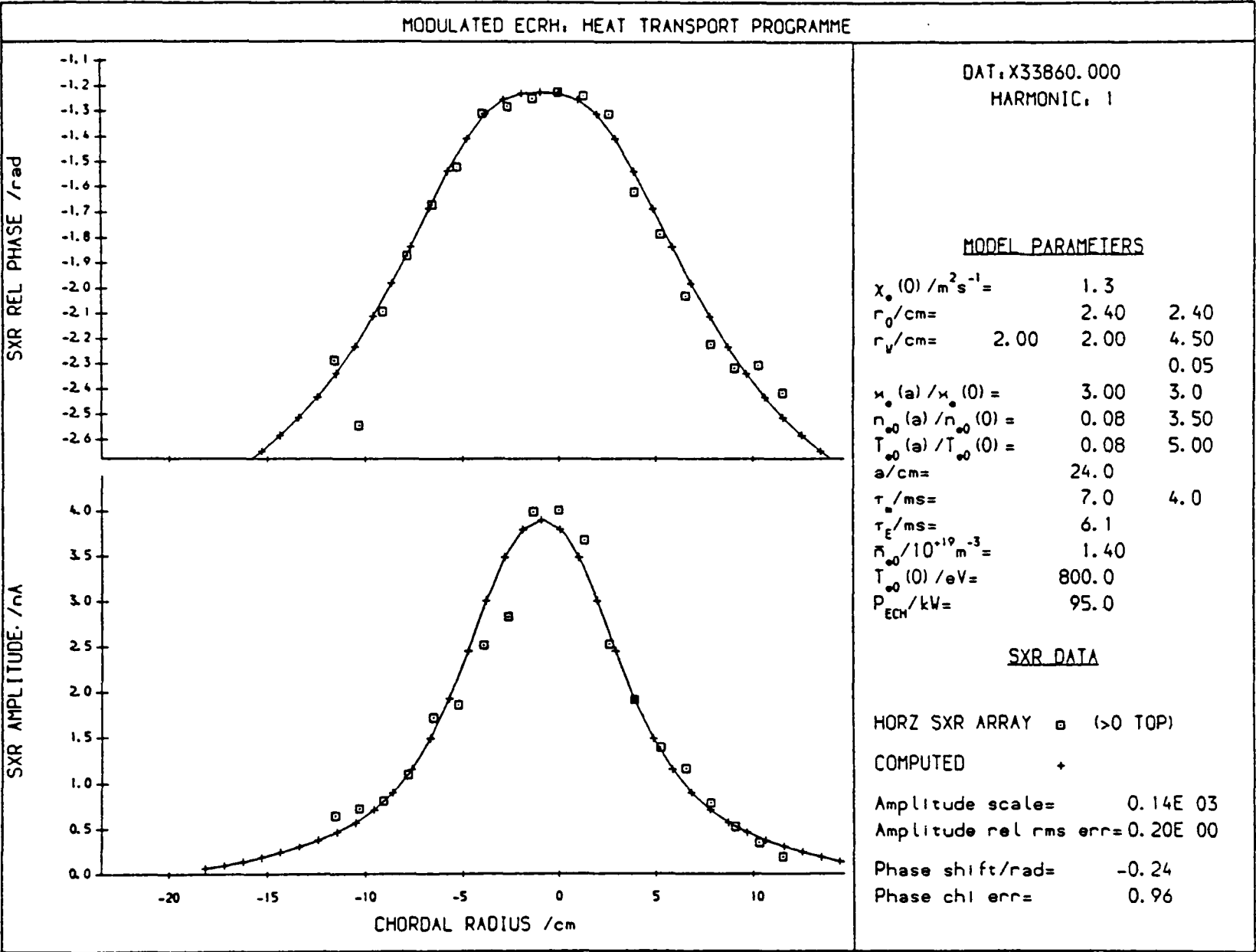


Figure 4.23: SXR relative phases and modulation amplitudes of fundamental component, fitted using dynamical thermal transport model. H discharge, medium density, fundamental on-axis heating. Negative phase shift had to be applied to model.

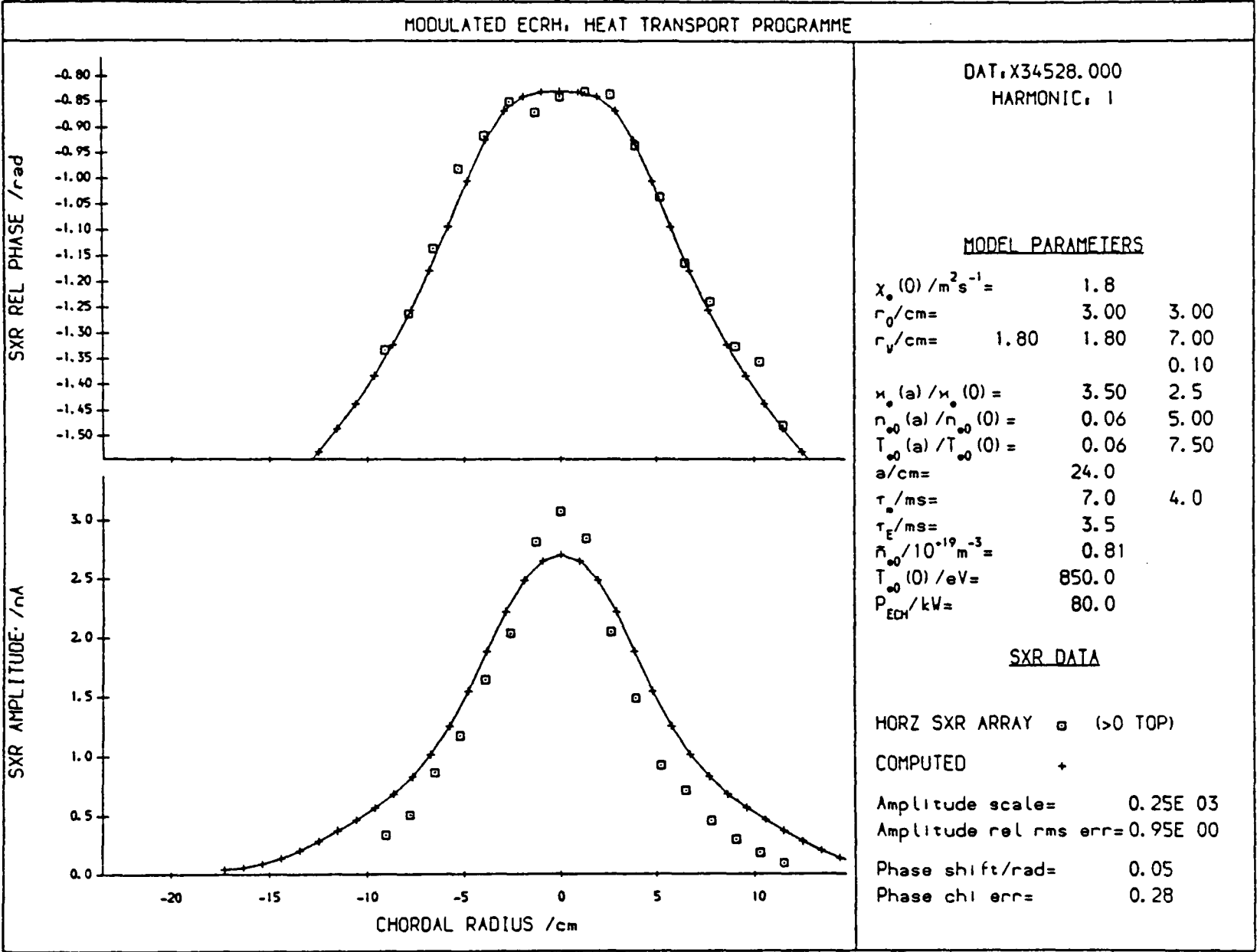


Figure 4.24: SXR relative phases and modulation amplitudes of fundamental component, fitted using dynamical thermal transport model. H discharge, low density, 2nd harmonic off-axis heating. The lower plasma current leads to narrower profiles; the high diffusivity is due to the low density and plasma current.

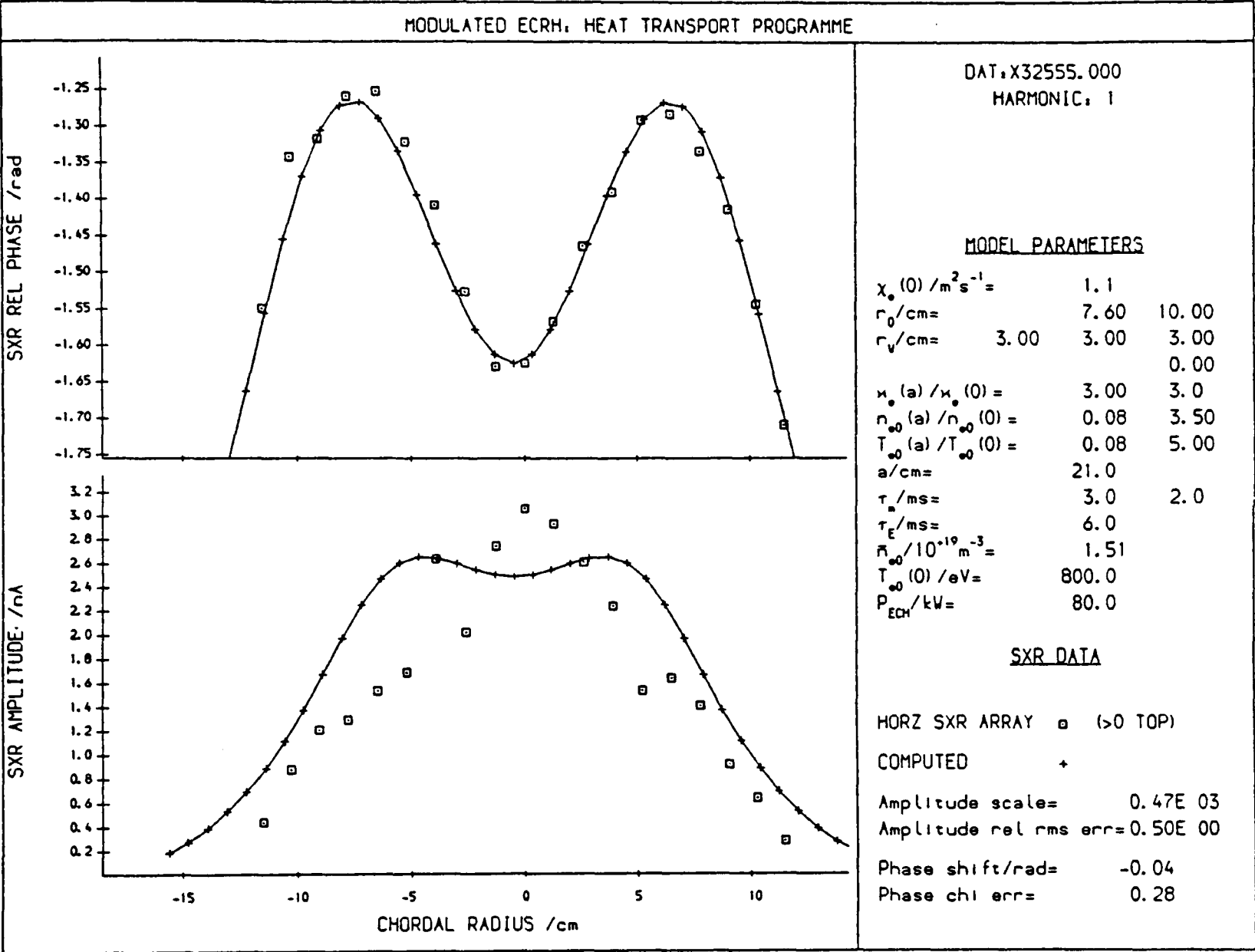
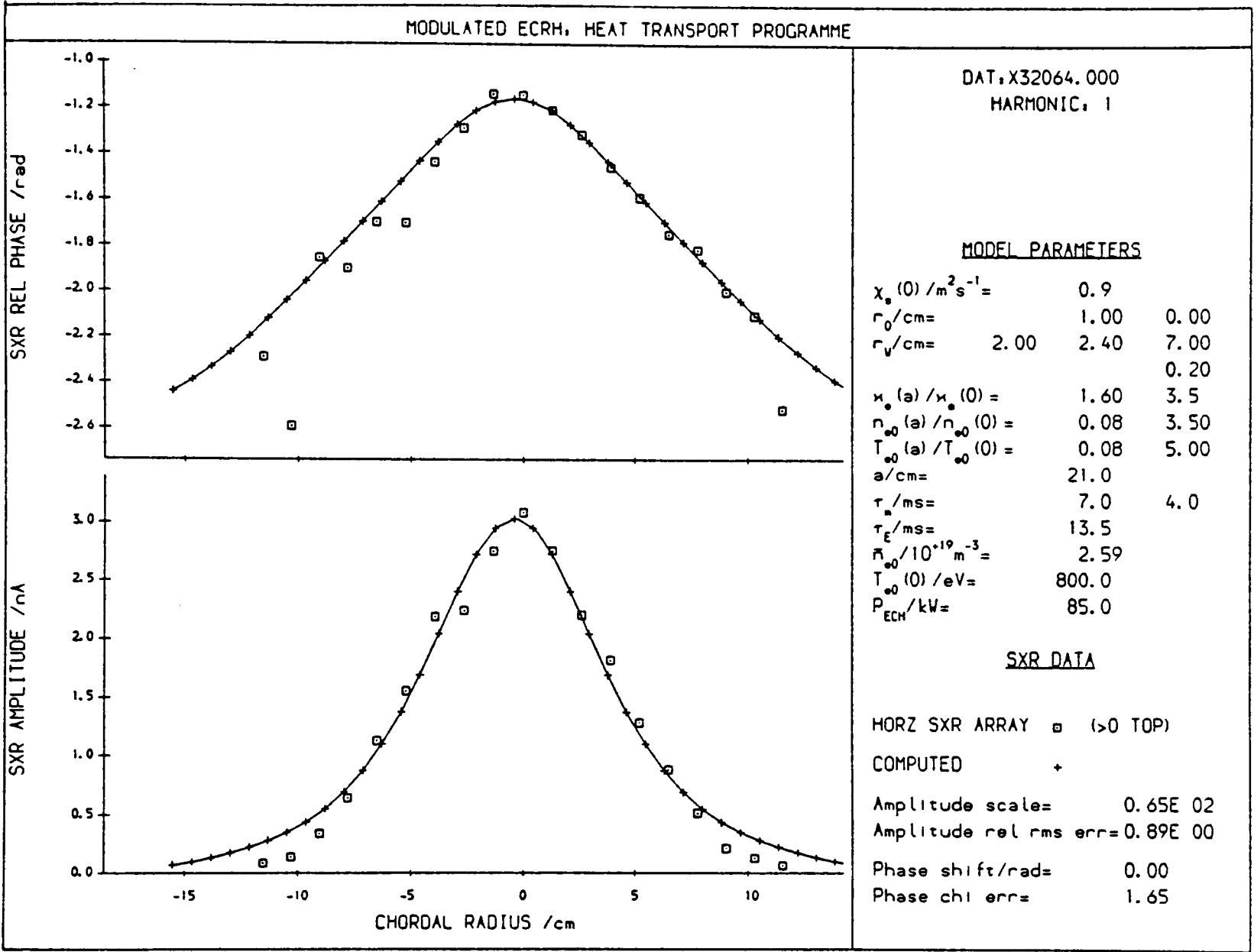


Figure 4.25: SXR relative phases and modulation amplitudes of fundamental component, fitted using dynamical thermal transport model. He discharge, medium density, fundamental off-axis heating. Pronounced phase gradients arise from higher modulation frequency (333Hz).



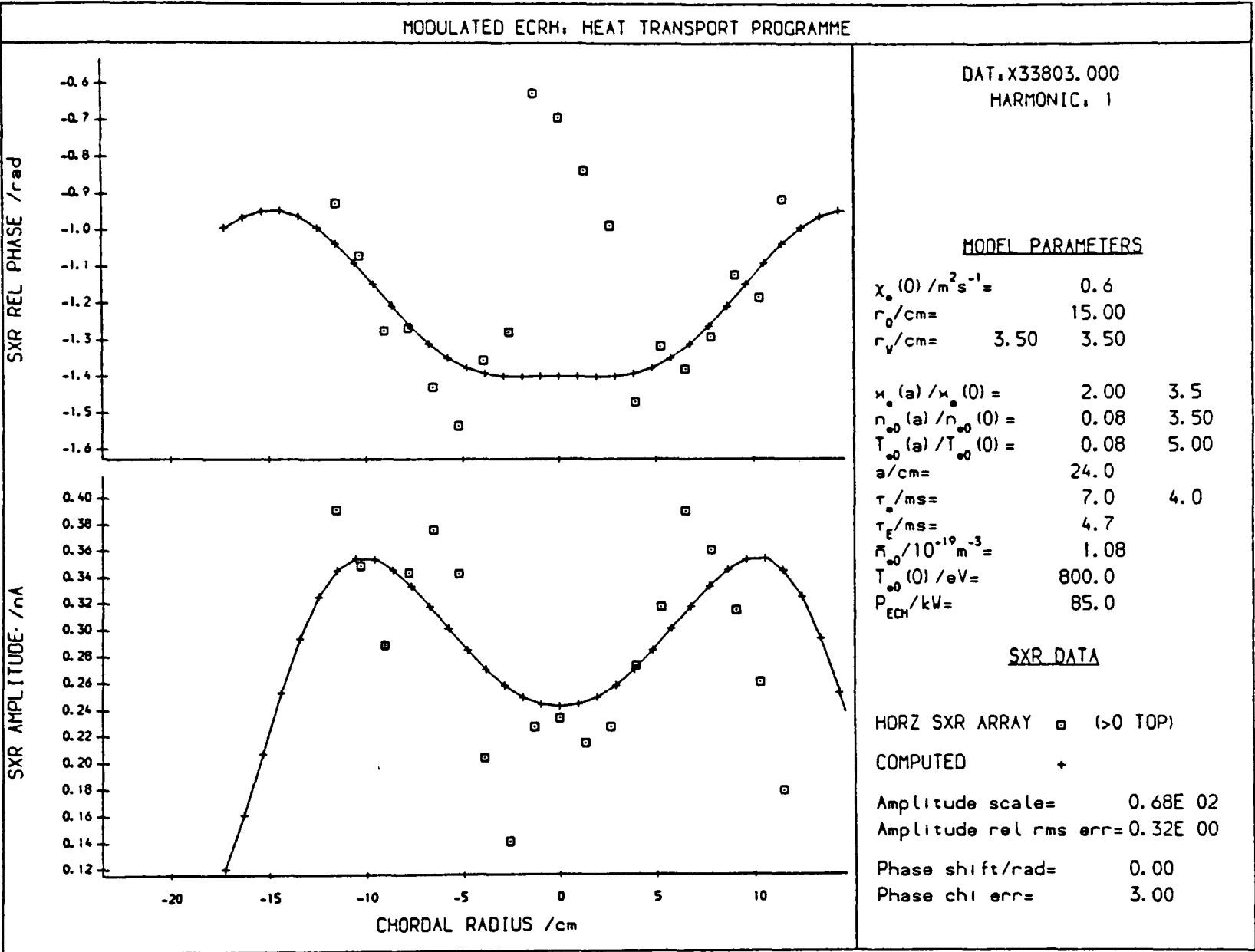


Figure 4.27: SXR modulation amplitudes and relative phases of fundamental component, fitted using dynamical thermal transport model. He discharge, low-medium density, strongly off-axis heating. Unusually low diffusivity value ($0.6\text{m}^2\text{s}^{-1}$) is obtained for inward propagation.

thermal perturbation. One might therefore argue that a term in the perturbed thermal flux that has not been taken into account, for instance a thermal pinch, is responsible for the apparent discrepancy between the effective diffusivities for outward and inward propagation of the thermal wave, and that this term is small near the centre, so that it does not affect the results obtained from discharges with moderately off-axis heating. It is important further to pursue this effect in future dynamical measurements.

Conclusions In earlier work on the simulation of modulated heating, where the damping to the Ohmic heating perturbation term was not included, the thermal diffusivity was overestimated by about 40% (Deliyanakis, 1987). In the same work, the central thermal diffusivity was further overestimated by about 50%, because a flat profile was used for the heat conduction coefficient κ_e . On the other hand, the different edge boundary condition¹¹ that was used in that work had virtually no effect on the results obtained, as the experimental results were fitted at a sufficient distance from the edge, so that the infinite-medium approximation applied. More recent work on the simulation (Hugill et al., 1988; Ashraf et al., 1988(a)) was still based on relatively narrow ECRH absorption profiles; as the broadening associated with the higher densities was not adequately treated, the dependence of the thermal diffusivity on the plasma line-averaged density was found to be weak and consequently not clear, the small changes that were noted being within the experimental errors. This situation has since been rectified with the inclusion of the broader deposition profiles, leading to improved model fits to the modulation data, and to lower thermal diffusivities for the higher densities, the latter being consistent, albeit only qualitatively, with the well-established density scaling of the global confinement time (see figure 4.32).

The main remaining weakness of this model is the need occasionally to use a phase shift in order to obtain full agreement. In most cases, however, the phase shift was comparable to the errors in the experimental phase lags and the computational error.¹² In general, the more noticeable negative shifts, implying that the model phase lags had to be increased, were

¹¹the modulation was assumed to vanish at the edge

¹²As 40 time steps per cycle were used, a time step was equivalent to a phase of 0.16rad, this being much larger than most of the phase shifts needed ($\lesssim 0.05$ rad).

required in cases of on-axis ECRH absorption (up to 0.2rad); much smaller shifts, both positive and negative, were required in cases of off-axis absorption (up to 0.05rad).

The model additionally enables one to ascertain the position and, with less confidence, the width of the ECRH power deposition region. These results were found to be in good agreement with those predicted by ray tracing, and have unambiguously indicated the presence of Doppler shifts of the absorption region, towards the inner, high-field side (refer to figure 3.11). It has further been possible to estimate the amount of ECRH power actually being absorbed in the plasma. The relative modulation level near the centre of the discharge was obtained from the ECE data (5–12%) and, by estimating a value for the central temperature (800eV), the temperature modulation level was calculated and compared to the corresponding result from the model. This procedure led to total ECRH power levels of about 85–100kW, to be compared with the gyrotron power of 150kW. The temperature modulation level predicted by the model was considerably higher for on-axis absorption than for off-axis absorption of a given ECRH input power, at a given density, mostly because of the geometrical effects that are involved in the ECRH power density profile and in transport in the cylindrical geometry. This aspect of the model was also in good agreement with the temperature modulation levels deduced from the ECE data.¹³ As the ECRH absorption zone was moved outwards, for a given density, the temperature modulation level decreased, leading to a roughly constant modulation level of the total plasma energy.

No significant differences were found between the diffusivities deduced for helium and hydrogen/deuterium discharges with similar densities. This suggests that no mass (or “isotope”) effects are present. In this connection, it is commonly supposed that the thermal transport scales with the ion atomic mass number as $A^{-1/2}$; such a dependence, which would have given a factor of 2 between the thermal diffusivity values for helium and hydrogen, has not been observed (see figure 4.32). It should, of course, be borne in mind that all of the results presented here pertain to densities that are sufficiently low for the ion thermal transport to play only a

¹³For the discharges modelled in figures 4.18ff (off-axis ECRH) and 4.23 (on-axis ECRH), the central ECE modulation levels were 8% and 12% respectively; these modulation levels were predicted by the diffusive model, with similar ECRH power levels in the two cases.

minor role (the scaling of the energy confinement time is linear in density, up to the highest density investigated); any isotope effects would therefore arise from the possible dependence of the electron thermal transport on ion mass, for instance, through the sound speed.

4.5 Results from alternative analyses of thermal transport

It has already been demonstrated that a simple diffusive model is able adequately to describe most, if not all, aspects of the temperature response to the modulated ECRH. This, however, does not necessarily confirm the original assumptions that the thermal transport is purely diffusive and that the thermal flux is driven only by the temperature gradient. Estimates of the electron thermal diffusivity may be obtained from analyses of (a) power balance and (b) sawtooth propagation. Such analyses have been carried out using data from DITE discharges, with a view to comparing their results with those of the modulated ECRH experiments.

4.5.1 Power balance

The first technique of power balance analysis uses the equilibrium temperature and density profiles, in order to estimate the thermal diffusivity profile. In the simplest case, one makes similar assumptions to those described above for the perturbed electron energy balance equation and obtains, for the equilibrium,

$$0 = \frac{1}{r} \frac{d}{dr} \left[r n_e(r) \chi_e(r) \frac{dT_e}{dr} \right] + E_\phi j(r), \quad (4.49)$$

where it is assumed that the toroidal electric field E_ϕ has fully penetrated and is therefore uniform. Taking the volume integral of this equation, and expressing the toroidal electric field in terms of the plasma loop voltage, i.e. $E_\phi = V_l/2\pi R$, one obtains the following expression for the effective thermal diffusivity from power balance, in terms of the temperature profile:

$$\chi_{\text{eff}}^{PB}(r) = \frac{V_l I(r)}{2\pi R 2\pi r n_e(r) |T'_e(r)|}, \quad (4.50)$$

where $I(r)$, the plasma current within a flux surface of radius r , is obtained from

$$I(r) = \int_0^r j(r) 2\pi r dr; \quad (4.51)$$

the current density distribution is calculated from the plasma current, using the proportionality $j(r) \propto T_e(r)^{3/2}$, which follows from the dependence of the Spitzer resistivity on temperature, ignoring the neoclassical correction, with the implicit assumption of a flat effective charge profile (refer to section 2.5).

Only incomplete information on temperature and density profiles was available from DITE experiments, and the power balance analysis was consequently carried out using a model based on simple Gaussian profiles (Hugill, 1988). Information on the safety factor profile was used to impose a constraint on the resultant current density profile. By writing the current density profile as a Gaussian of half-width r_j ,

$$j(r) = j(0) \exp\left(-\frac{r^2}{r_j^2}\right), \quad (4.52)$$

one obtains a safety factor profile

$$q(r) = \frac{2B_{\phi 0}}{R_0 \mu_0 j(0)} \frac{r^2/r_j^2}{\left(1 - \exp(-r^2/r_j^2)\right)};$$

$$\text{hence, } \frac{a}{r_j} = \sqrt{\frac{q(a)}{q(0)}}. \quad (4.53)$$

Having determined r_j , one obtains the temperature profile half-width $\sqrt{3/2} r_j$ and, by assuming a value for the profile parameter $\eta_e = \frac{T_e'/T_e}{n_e'/n_e}$, the corresponding density profile half-width $\sqrt{3\eta/2} r_j$. These profiles, used in (4.50) and (4.51), lead to a thermal diffusivity of the form

$$\chi_{\text{eff}}^{PB}(r) = \frac{3V_l I_p}{16\pi^2 R n_e(0) T_e(0)} f\left(\frac{r}{r_j}\right), \quad (4.54)$$

$$\text{with } f(x) = \frac{1 - \exp(-x^2)}{x^2 \exp\left(-\frac{2}{3}(1 + \eta^{-1})x^2\right)}. \quad (4.55)$$

The function $f(x)$, that is, the radial dependence of the power balance thermal diffusivity, is plotted in figure 4.28. As the pedestals of the profiles have been ignored for analytical simplicity, this form grossly overestimates the diffusivity as the edge is approached.

There are several important weaknesses inherent in this approach. First, the density and temperature profiles were not known with any accuracy and have been estimated in a simple way from some knowledge of the safety factor, in particular ignoring the neoclassical resistivity correction. Second, the radiation and equipartition losses were ignored, clearly leading to some

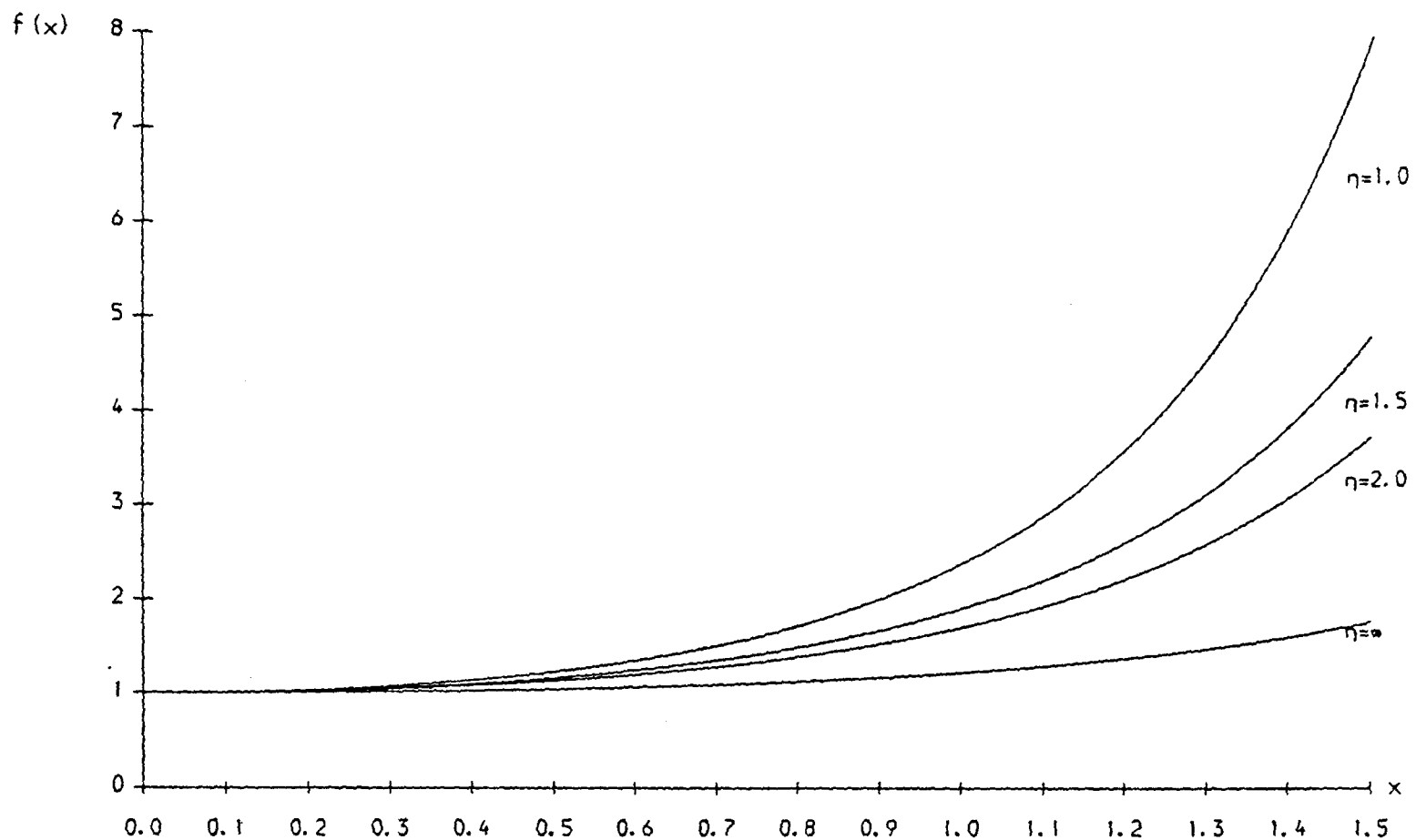


Figure 4.28: Profile of thermal diffusivity from simple model of power balance: $f(x)$, as given by (4.55), plotted against $x = r/r_j = r/a \sqrt{q(a)/q(0)}$, for various values of η . The curve for $\eta \rightarrow \infty$ corresponds to the profile of the heat conduction coefficient.

overestimation of the thermal diffusivity; however, these losses were reasonably small compared with the Ohmic heating (up to 30%). Third, it has been implicitly assumed that the confinement during the modulated ECRH is similar to that during the Ohmic phase —no adequate justification can be produced for this assumption.

4.5.2 Sawtooth heat pulse propagation

The second technique of sawtooth propagation analysis is similar to the analysis of the ECRH modulation, but uses the periodic collapse of the temperature profile caused by the sawtooth instability as a natural perturbation (refer to subsections 1.4.2 and 2.7.1 for more details). One basically measures the time delay of the peak of the temperature pulse, as a function of radius, in order to obtain information similar in nature to the results of the modulated ECRH experiments. In the frequency domain, one calculates the effective thermal diffusivity from heat pulse propagation by the following formula, obtained from (4.12):

$$\chi_{\text{eff}}^{HP}(r) = \frac{3}{4} \frac{2\pi}{\tau_{\text{st}}} \left(\frac{d\phi}{dr} \right)^{-2}, \quad (4.56)$$

where τ_{st} is the sawtooth period, and $d\phi/dr$ is the sawtooth phase gradient outside the mixing radius. In the time domain, one uses the following formula, obtained from a solution of the simple diffusion equation for the sawtooth perturbation (see, e.g., Soler and Callen, 1979):

$$\chi_{\text{eff}}^{HP}(r) = \frac{1}{8} \frac{r^2 - r_{\text{mix}}^2}{\tau_p(r)}, \quad (4.57)$$

where τ_p is the sawtooth pulse peak time at the radius r , and r_{mix} is the mixing radius, where the heat pulse originates.

The sawtooth propagation analysis was carried out in the frequency domain, using the data from the horizontal SXR array. The sawtooth frequency was in the range 250Hz–1.3kHz, depending on the density (the sawtooth period increases as the density is increased). Typical sawtooth oscillations, observed in a high-density discharge with modulated ECRH, are shown in figure 4.29; in this particular case, the sawtooth oscillation was dominant, and the perturbation at the ECRH modulation frequency could only be detected by an FFT analysis at that frequency. The spectral analysis of the sawtooth oscillation was carried out with the help of the same FFT codes that were used for the modulation analysis.¹⁴ The sawtooth frequency for a discharge is not clearly defined, as the sawtooth period fluctuates somewhat, but the results obtained from the FFT analysis of the sawtooth oscillation were not critically sensitive to the choice of sawtooth frequency (at least within the bandwidth of the analysis). An example of the results of this analysis, showing the phase and normalized amplitude of the sawtooth perturbation that was extracted from the Fourier spectrum of the SXR data at the appropriate sawtooth frequency, is shown in figure 4.30. The inversion radius can be readily located at the phase shift of approximately 2π (where, additionally, the normalized amplitude shows a minimum). The mixing radius, beyond which the transport of the sawtooth perturbation is diffusive, appears to be just outside the inversion radius. It can be seen that the data from the horizontal and vertical arrays almost coincide, there being only a small discrepancy due to the flux surface shifts. The flat phase within the inversion radius, implying a very high diffusivity associated with the sawtooth instability, should finally be noted; such a fast transport, however, did not appear to be present in the modulation analysis of discharges with the “normal”, diffusive behaviour.

¹⁴a sinusoidal function of arbitrary phase was used as a reference signal

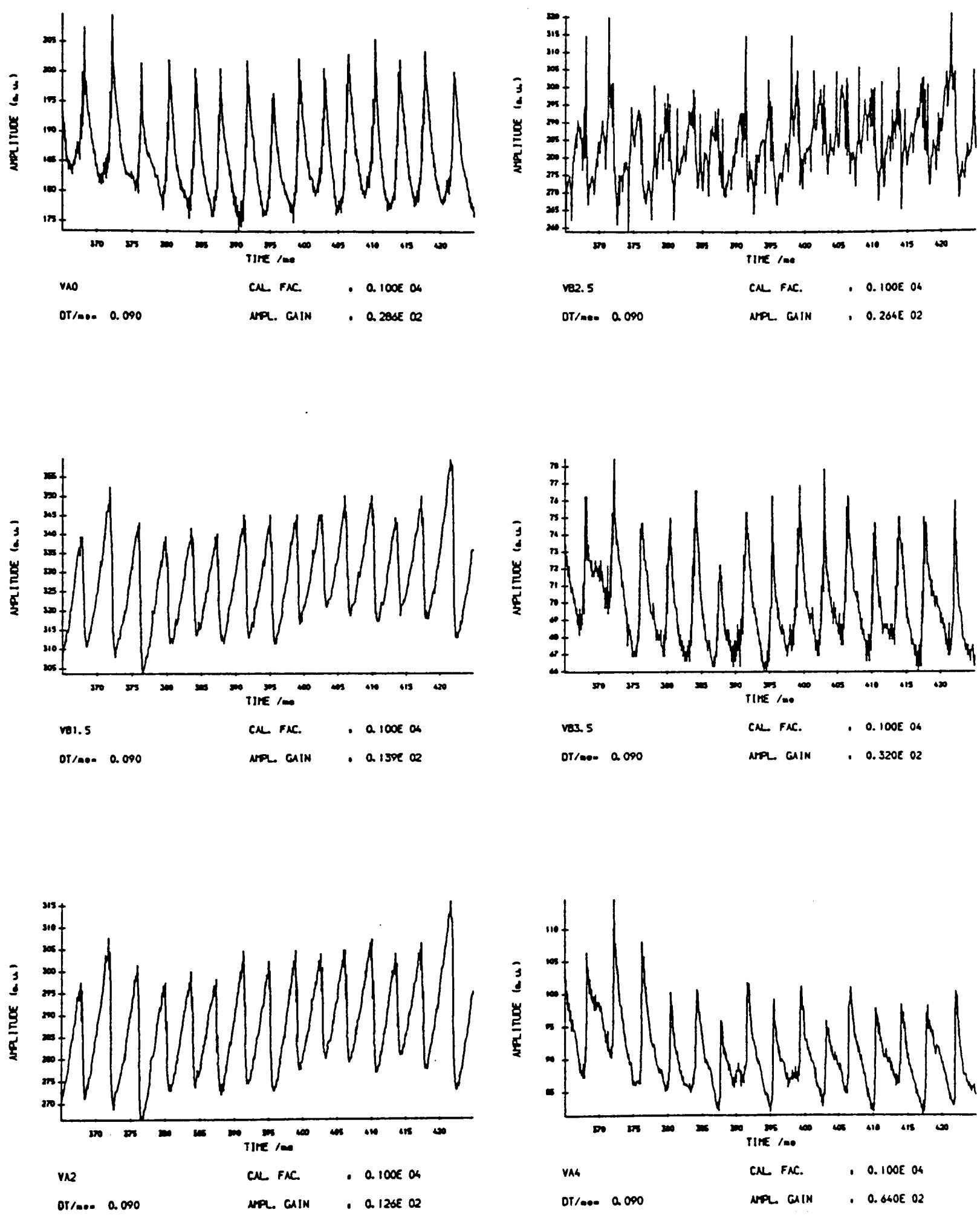


Figure 4.29: Vertical array SXR signals from high-density He discharge, for which the sawtooth oscillation was dominant over that due to the modulation. Signals both inside and outside the inversion radius are shown.

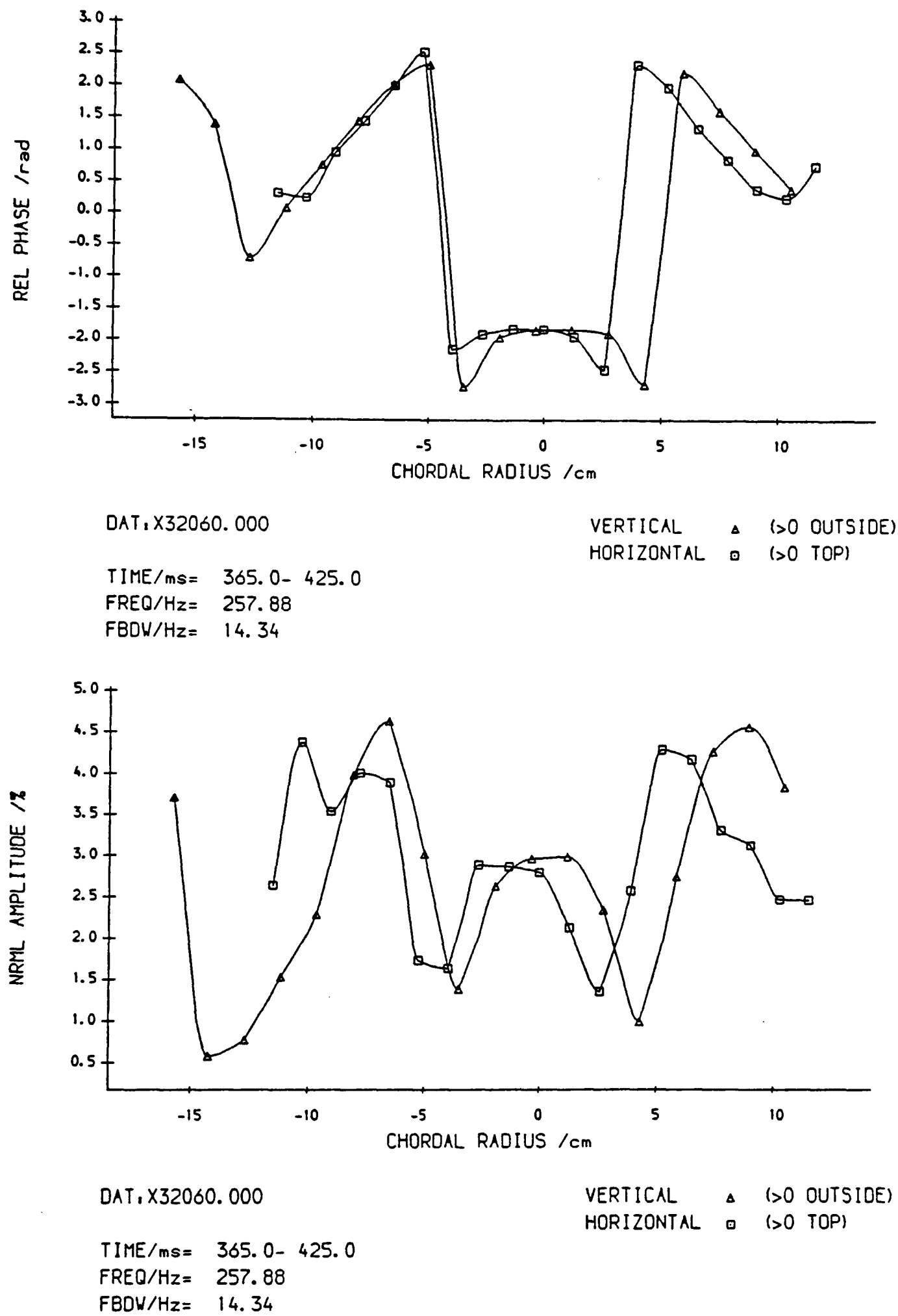


Figure 4.30: Analysed SXR data, showing phase and normalized amplitude of sawtooth oscillation in signals of figure 4.29 (obtained from FFT analysis of signals at sawtooth frequency). The location of the inversion point is clear, and the mixing point appears to be just outside it.

The phase gradient was obtained from the data points outside the mixing radius. It should be recalled that the accuracy of this technique is limited by the weak dependence of the phase gradient on the thermal diffusivity. In sawtooth pulse analysis the perturbation frequency is high compared with the energy confinement time, and the overestimation of the thermal diffusivity due to the omission of the Ohmic damping correction in (4.16) is only about 4% (independent of density) and can be safely ignored. Any modulated plasma motion will only slightly affect the data at the sawtooth frequency. However, a more important effect in measurements of SXR perturbation profiles is the spatially varying response of the SXR emissivity to temperature, as has been described in subsection 3.3.1 and in this chapter in connection with the modelling of the response to the modulated ECRH. The SXR temperature modulation index and its radial variation, and the effect of the chordal integration, were not included in the analysis of the SXR sawtooth pulse and this somewhat limits the confidence that can be placed on the measurements; at the same time, it was impossible to use ECE data for sawtooth analysis.

We now address the possible effect of the sawtooth oscillation on the computation of the amplitude and phase of the response to modulated ECRH, in cases where the former is locked with the latter. This can be illustrated by considering the inverse effect, namely the influence on the locked sawtooth oscillation of a harmonic component of the modulated response, an example of which is shown in figure 4.31. In this case, the frequency of the sawtooth oscillation coincides with the third harmonic of the modulation frequency (refer to figure 4.15). It is clear, from a comparison with figure 4.30, that the third harmonic component of the modulation does indeed have an effect on the extracted sawtooth response, within the central region: the phase is decreased and the usual shift of the vertical array data due to the modulated plasma position is apparent. Outside that region, however, where there are no sources of perturbation due to either the modulated ECRH or the sawtooth collapse, there will be no effect other than a modification of the amplitude —the phase gradient will still provide a useful measure of thermal diffusion. Returning to the original question, the locked sawtooth oscillation can *possibly* have an effect on the fundamental component of the modulation (even though its frequency is a multiple of the fundamental frequency of the modulation), if its amplitude or period be modified within a

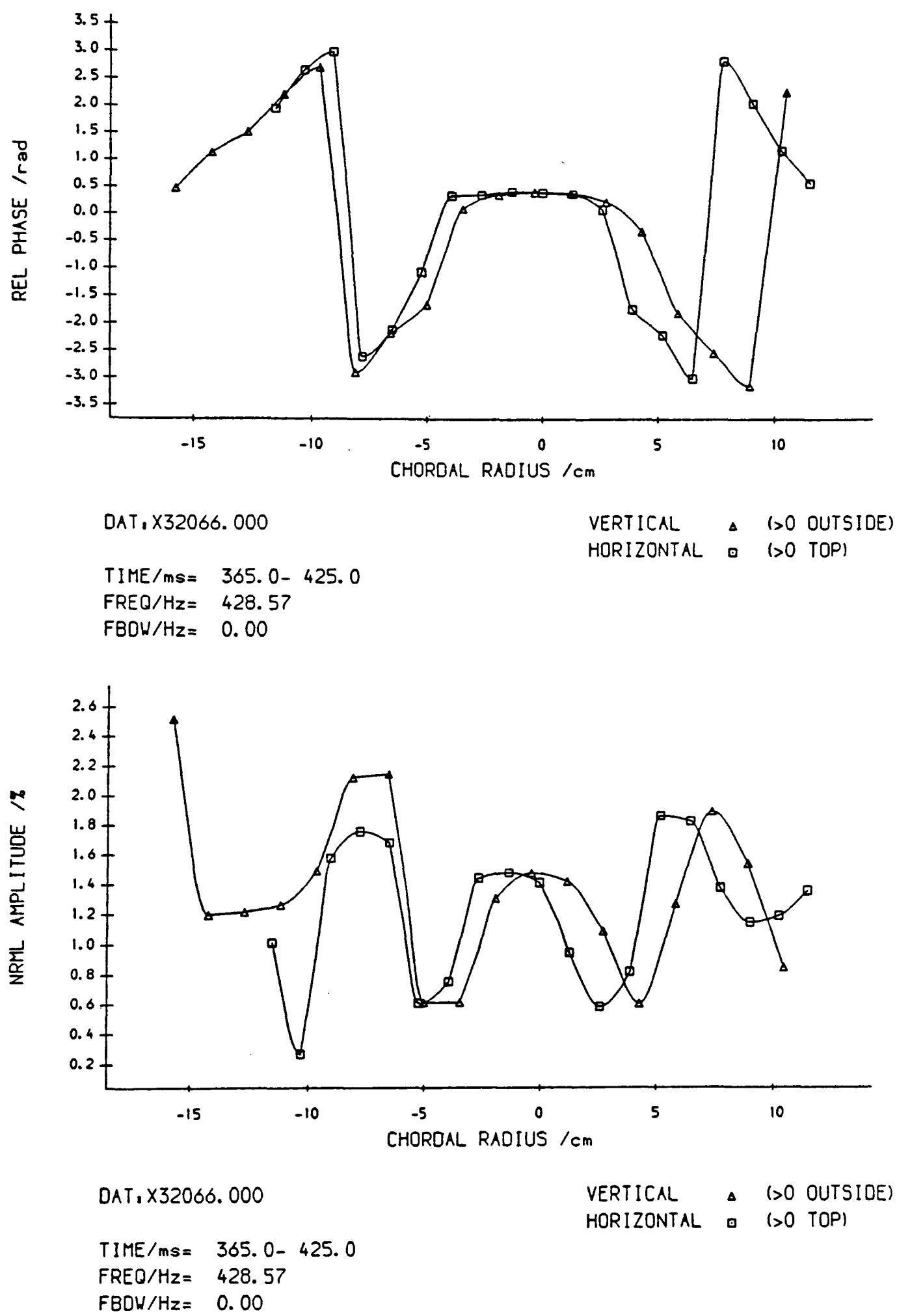


Figure 4.31: Analysed SXR data, showing phase and normalized amplitude of locked sawtooth oscillation, of frequency equal to the 3rd harmonic of the modulation frequency, in signals of figure 4.15.

modulation period. Such an effect might be one of the underlying factors of the “anomalous” behaviour that was discussed earlier, although its presence has not been conclusively observed.

Other analyses of the sawtooth heat pulse propagation in the time domain (Ashraf, 1988) have revealed that the thermal diffusivity did not change appreciably at the onset of the modulated ECRH, nor was it different during the ECRH “on” and “off” time intervals.

4.5.3 Juxtaposition of techniques

A comparison of the electron thermal diffusivity values that were inferred from the ECRH modulation experiments with those that were deduced from the alternative analyses based on power balance and sawtooth heat pulse propagation, carried out on the same or similar discharges, is presented in figure 4.32, where the thermal diffusivity at the radius $r = a/3$ is plotted against line-averaged density.

In the original approach to the modelling of the ECRH modulation, when the high-density broadening of the ECRH deposition profiles was not being taken into account, the thermal diffusivity values from the modulation analysis were transpiring to be noticeably higher than those obtained from the power balance analysis. Furthermore, the values from the modulation analysis depended only weakly on density, whereas the latter were observed to fall with increasing density; at the lower densities, the values from both techniques were in agreement.

The values of χ_e obtained from the sawtooth propagation analysis are in broad agreement with those obtained from the power balance analysis. This might be considered to be an unusual result, in the light of the results of similar analyses of heat pulse propagation (outlined in subsection 2.7.1), which typically yield higher effective thermal diffusivity values than those obtained from equilibrium measurements. A further issue is, of course, the difference between the effective thermal diffusivities measured by the two dynamical methods, sawtooth heat pulse propagation and ECRH modulation, these being of the same nature. The underlying differences between these two perturbations are as follows:

1. The frequency of the perturbation is higher for sawtooth pulses than for ECRH modulation.

However, the effect of this is probably negligible, as indicated by the good agreement

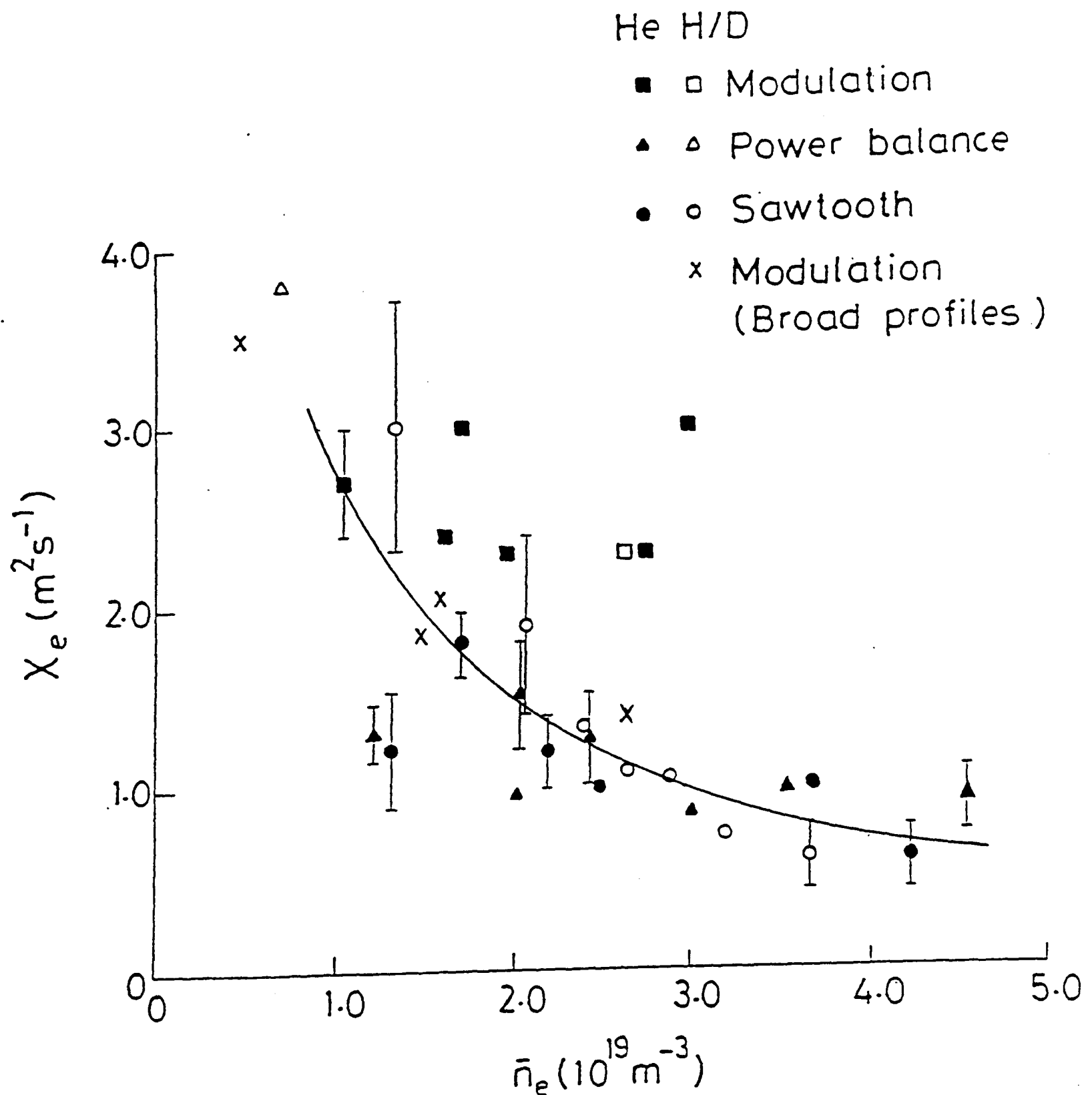


Figure 4.32: Comparison of representative values of effective electron thermal diffusivity χ_{eff} as deduced from (a) ECRH modulation analysis (squares), (b) power balance analysis (triangles) and (c) sawtooth propagation analysis (circles), at the radius $a/3$, plotted against line-averaged density. Measurements both for helium discharges (solid symbols) and for hydrogen/deuterium discharges (open symbols) are shown. The values deduced from the more recent ECRH modulation analysis (crosses), with broader ECRH deposition profiles, are in better agreement with those from the other two techniques. The curve indicates a reciprocal dependence of the diffusivity on density, following the corresponding, linear scaling of the confinement time.

between the results obtained from the modelling of the modulated ECRH discharges at the two modulation frequencies of 143Hz and 333Hz (the second frequency being near the sawtooth frequencies).

2. The temperature modulation levels are different; this can produce an effect if a transport coefficient with a temperature or temperature-gradient dependence be perturbed in different ways in the two techniques.
3. The associated density modulation levels are different; this can modify the power balance, or produce effects through transport coefficients with a density dependence. The relatively large density perturbation associated with sawtooth activity may have a considerable effect on the SXR emissivity, greater than that of the temperature perturbation—this point may help to explain the non-diffusive behaviour found in some of the modulated ECRH discharges.
4. Most importantly, the forms of the corresponding sources of perturbation are different, almost certainly leading to different spatial scales and gradients. The sawtooth perturbation is a transient one and takes the form of an initial condition, whereas that due to the modulated ECRH takes the form of a time-dependent energy source. If the particle and energy transport be coupled, these differences are very likely to lead to distinct combinations of temperature and density perturbations, with different effective thermal diffusivities—this point will be pursued in the following chapter.

It is not clear how these differences might affect the underlying transport processes or the observed behaviour.

However, the width of the ECRH deposition profile has emerged as an important consideration in the analysis of the response to the modulated ECRH. Many of the foregoing conclusions have had to be modified in the light of the more recent analysis of the ECRH modulation data, where the high-density broadening of the deposition profile was introduced into the modelling. This more recent analysis produced better fits of the model results to the experimental data than those obtained previously, and further led to lower thermal diffusivity values, in better agreement with those obtained from the other two techniques, as is shown in figure 4.32. The

scaling of all the thermal diffusivity data with line-averaged density is close to the reciprocal dependence that is expected from the energy confinement time being proportional to density. It is finally noted that an analogous partial agreement between different techniques has in fact been reported in past dynamical and equilibrium measurements of transport that have been carried out on the smaller tokamaks similar to DITE. Nevertheless, the diffusivity values determined from the modulated ECRH may still be somewhat higher, at higher densities, than those deduced from the other two techniques, and a certain element of doubt remains as to the exact form of the ECRH deposition profile.

Chapter 5

Coupled Transport Analysis of Data from Modulated ECRH

5.1 Evidence for the presence of coupled transport

It has already been seen in the previous chapter that the observed modulation of line-averaged density was small in comparison with the temperature modulation, throughout the discharge, and that it was therefore reasonable to assume that the coupling between the energy transport, which was directly perturbed in the ECRH modulation experiments, and the particle transport was weak, so that the latter could be regarded as being unperturbed; that is, any terms in the energy and particle balance equations giving rise to coupling of the temperature and density variations were assumed to be small. This approach led to good agreement of the dynamical diffusion model with the experimental results. However, it has also been seen that there may be discrepancies between the values of electron thermal diffusivity obtained from the ECRH modulation analysis on the one hand, and those deduced from the power balance analysis and the sawtooth heat pulse propagation analysis on the other hand, though this observation has been considerably weakened by the final results of the thermal transport analysis that were presented in the previous chapter. A general explanation for this effect is that the modulation of the density caused by the modulated ECRH, which was considered to be negligible, may affect the propagation of the heat wave and hence the effective thermal diffusivity deduced from it. In the

case of the power balance analysis, a different effective thermal diffusivity is generally obtained, because of the omission of the coupling terms. A further effective thermal diffusivity is generally obtained from the sawtooth heat pulse propagation analysis, when the perturbation associated with the sawtooth collapse and that driven by the modulated ECRH have different forms. It has been pointed out by Hossain et al. (1987) that, in the presence of coupling between particle and energy transport, the temperature is *not* an eigenfunction of the coupled system of partial differential equations, nor is the thermal conductivity an eigenvalue, the actual eigenvalues being combinations of the diagonal and off-diagonal elements of the appropriate transport matrix. Further discrepancies can arise when the transport coefficients are functions of the perturbed variables and their gradients, this approach having been pursued by Gentle (1988).

There are three distinct mechanisms that can give rise to a modulation of the observed line-integrated density in a discharge with a modulated energy input; the interaction of these mechanisms may produce either a reinforcement or a cancellation of the modulation:

1. Off-diagonal diffusivities in the energy and particle balance equations, as predicted by neoclassical theory as well as by most models of anomalous transport, will lead to a non-linear coupling of the particle and energy balance. Further coupling may, of course, result from the presence of convective and non-diffusive terms in the thermal flux. In general, this coupling should involve both electron and ion energy transport, as well as the variation of the current density, but these additional effects (ion transport and magnetic field diffusion) can often be negligible in a perturbation analysis because of their longer time-scales. One can point out that coupling of energy and particle transport might always be expected, because of the presence of the convection term introducing the density gradient into the energy flux; this term is cancelled by an equal and opposite non-diffusive flux, under conditions of power balance (see section 2.6), but this will not necessarily be the case when the fluxes are perturbed. The coupling may be different in helium and hydrogen/deuterium plasmas, this being a possible cause of the differences between the observed density modulation levels in discharges with different gases.

2. The neutral particle density near the edge will be modulated, leading to a corresponding

modulation in the edge particle source, because of

- the direct absorption of the ECRH wave at the edge;
- the interaction of the propagating thermal wave at the edge; and
- the modulated flux surface and plasma column shifts (arising from the modulation of pressure).

This edge modulation has in fact been observed in the $H\alpha$ -radiation signals, as outlined in the previous chapter. An examination of these signals typically reveals a response with two time-scales: a prompt response attributed to the direct absorption of the ECRH wave, and a delayed response attributed to the interaction of the propagating heat wave (and also to the plasma shifts). Other edge measurements, also presented in the previous chapter, reveal only a delayed response. Recycling is expected to reduce the density modulation; as recycling is stronger in helium than in hydrogen/deuterium plasmas, this may contribute to the differences between the observed density modulation levels.

3. The modulation of the total plasma kinetic pressure will produce a modulated, differential, horizontal movement of the flux surfaces, leading to a modulation of the observed line-integrated density (as well as to similar contributions to the vertical SXR and the ECE signals), as discussed in the previous chapter.

5.2 Analytical investigation of coupled transport

Some insight into the nature and effects of transport coupling mechanisms, and the way in which discrepancies can arise between the values of the effective diffusivities obtained from equilibrium and dynamical measurements, can be gained with the help of analytical solutions of a simplified problem.

The following coupled equations describing particle and energy transport are taken as the basis of this analysis, where the assumption of a single radial dependence for all the diffusivities is made, and any non-diffusive terms are incorporated in the source terms:

$$\frac{\partial n}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r n f(r) \left(L_{11} \frac{n'}{n} + L_{12} \frac{T'}{T} \right) \right] + S(r, t) \quad (5.1)$$

$$\frac{3}{2} \frac{\partial(nT)}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r n T f(r) \left((L_{21} + \frac{5}{2} L_{11}) \frac{n'}{n} + (L_{22} + \frac{5}{2} L_{12}) \frac{T'}{T} \right) \right] + Q(r, t) \quad (5.2)$$

(the diffusivities L_{21} and L_{22} do not include the convective contributions from L_{11} and L_{12} , which have been explicitly included—refer to the complete definitions in section 2.5 and the discussion in section 2.6).

Some analytic progress is rendered possible by linearizing and simplifying these equations by assuming small perturbations, such that $n\tilde{n}' \gg n'\tilde{n}$ and $T\tilde{T}' \gg T'\tilde{T}$. Hence, one obtains the perturbed diffusion equations,

$$\frac{\partial}{\partial t} \left(\frac{\tilde{n}}{n} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left[r f(r) \left(M_{11} \left(\frac{\tilde{n}}{n} \right)' + M_{12} \left(\frac{\tilde{T}}{T} \right)' \right) \right] + \tilde{S}(r, t)/n, \quad (5.3)$$

$$\frac{\partial}{\partial t} \left(\frac{\tilde{T}}{T} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left[r f(r) \left(M_{21} \left(\frac{\tilde{n}}{n} \right)' + M_{22} \left(\frac{\tilde{T}}{T} \right)' \right) \right] + \frac{2}{3} \tilde{Q}(r, t)/nT - \tilde{S}(r, t)/n, \quad (5.4)$$

with the following elements of the dynamical diffusivity matrix M :

$$M_{11} = L_{11}, \quad M_{12} = L_{12}, \quad M_{21} = \frac{2}{3}(L_{21} + L_{11}), \quad M_{22} = \frac{2}{3}(L_{22} + L_{12}). \quad (5.5)$$

These coupled equations can be written as

$$\frac{\partial}{\partial t} \begin{pmatrix} \tilde{n}/n \\ \tilde{T}/T \end{pmatrix} = \frac{1}{r} \frac{\partial}{\partial r} \left\{ [r f(r) M] \cdot \frac{\partial}{\partial r} \begin{pmatrix} \tilde{n}/n \\ \tilde{T}/T \end{pmatrix} \right\} + \begin{pmatrix} \tilde{S}/n \\ \frac{2}{3} \tilde{Q}/nT - \tilde{S}/n \end{pmatrix}. \quad (5.6)$$

The linearized system of equations can now be solved by diagonalizing the diffusivity matrix M , by applying the transformation

$$U^{-1} M U = \Lambda, \quad (5.7)$$

or equivalently

$$M \cdot \mathbf{u}_{\pm} = \lambda_{\pm} \mathbf{u}_{\pm}, \quad (5.8)$$

where Λ is the diagonal matrix of the eigenvalues $\lambda_{\pm}$, and U is the matrix of the eigenvectors $\mathbf{u}_{\pm}$, of the matrix M :

$$\Lambda = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}; \quad (5.9)$$

$$U = \begin{pmatrix} 1 & 1 \\ \mathbf{u}_+ & \mathbf{u}_- \end{pmatrix}. \quad (5.10)$$

The eigenvalues of the diffusivity matrix are given by

$$\lambda_{\pm} = \frac{1}{2} \left\{ (M_{11} + M_{22}) \pm \left[(M_{11} - M_{22})^2 + 4M_{12}M_{21} \right]^{1/2} \right\}, \quad (5.11)$$

and the corresponding eigenvector elements are given by

$$u_{\pm} = \frac{\lambda_{\pm} - M_{11}}{M_{12}} = \frac{M_{21}}{\lambda_{\pm} - M_{22}}. \quad (5.12)$$

The coupled solutions are then expressed in terms of the decoupled eigenmode solutions by the transformation

$$\begin{pmatrix} \tilde{n}/n \\ \tilde{T}/T \end{pmatrix} = U \cdot \begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix}. \quad (5.13)$$

Substituting (5.13) in and applying (5.7) to (5.6), one obtains the decoupled diffusion equations, in the region where the sources are not perturbed,

$$\frac{\partial \xi_{\pm}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r f(r) \lambda_{\pm} \frac{\partial \xi_{\pm}}{\partial r} \right], \quad (5.14)$$

where the eigenvalues λ_+ and λ_- are the diffusivities of the eigenmodes $\xi_+(r, t)$ and $\xi_-(r, t)$ respectively. The coupled solutions for the density and temperature oscillations are linear combinations of the eigenfunctions, defined by (5.13),

$$\begin{pmatrix} \tilde{n} \\ n \end{pmatrix} = \xi_+ + \xi_-, \quad (5.15)$$

$$\begin{pmatrix} \tilde{T} \\ T \end{pmatrix} = u_+ \xi_+ + u_- \xi_-. \quad (5.16)$$

It can be seen that the original diffusion coefficients are not eigenvalues of the system, nor are the density and temperature its eigenfunctions. For a uniform profile $f(r)$, the solutions of (5.14) are given in terms of Bessel functions (as in subsection 4.1.2). For clarity, we take here $f(r) = cr^{-2}$, and introduce $x = \frac{1}{2}r^2$, giving

$$\frac{\partial \xi_{\pm}}{\partial t} = c \lambda_{\pm} \frac{\partial^2 \xi_{\pm}}{\partial x^2}; \quad (5.17)$$

the solutions are

$$\xi_{\pm}(x, t) = \exp(-s_{\pm}x) \left[A_{\pm} \sin(\omega t - s_{\pm}x) + B_{\pm} \cos(\omega t - s_{\pm}x) \right], \quad (5.18)$$

$$\text{with } s_{\pm} = \left(\frac{\omega}{2c\lambda_{\pm}} \right)^{1/2}. \quad (5.19)$$

We first consider the problem of the density and temperature responses to modulated ECRH and introduce a modulated, localized excitation of the energy balance at $x=0$, i.e. $\tilde{S} = 0$ and $\tilde{Q} = P\delta(x)\cos(\omega t)$: from the null particle flux perturbation, i.e.

$$\left(M_{11} \left(\frac{\tilde{n}}{n} \right)' + M_{12} \left(\frac{\tilde{T}}{T} \right)' \right) \Big|_{x=0} = 0, \quad (5.20)$$

we obtain

$$A_+ = B_+, \quad (5.21)$$

$$A_- = B_- \quad (5.22)$$

$$\text{and } \frac{A_+}{A_-} = - \left(\frac{\lambda_+}{\lambda_-} \right)^{1/2} \frac{M_{11} + M_{12}u_-}{M_{11} + M_{12}u_+}, \quad (5.23)$$

leading to a minimum phase lag of $\pi/4$; the amplitudes of A_+ and A_- can be determined from the finite energy flux perturbation. It is important to note that a different perturbation, for instance one due to the sawtooth collapse, will generally lead to a different combination of eigenfunctions, and thereby to different density and temperature perturbations, generally associated with a distinct effective diffusivity. We therefore consider the density and temperature perturbations caused by the sawtooth collapse: as this is a convective instability, we assume density and temperature perturbations with equal scale lengths at $x=0$, i.e.

$$\left(\left(\frac{\tilde{n}}{n} \right)' - \left(\frac{\tilde{T}}{T} \right)' \right) \Big|_{x=0} = 0; \quad (5.24)$$

we still obtain (5.21) and (5.22), but (5.23) becomes

$$\frac{A_+}{A_-} = - \left(\frac{\lambda_+}{\lambda_-} \right)^{1/2} \frac{1 - u_-}{1 - u_+}. \quad (5.25)$$

The ‘near-field’ solution, for $x \rightarrow 0$, gives a ratio of density and temperature relative modulation levels

$$\frac{\tilde{n}/n}{\tilde{T}/T} = \frac{(A_+/A_-) + 1}{u_+(A_+/A_-) + u_-}, \quad (5.26)$$

which depends on the form of the perturbation; the ‘far-field’ solution, for $x \gg 0$, is dominated by the eigenfunction with the larger eigenvalue λ_+ , associated with the weaker attenuation, and gives a ratio

$$\frac{\tilde{n}/n}{\tilde{T}/T} = \frac{1}{u_+}. \quad (5.27)$$

We now address the problem of the discrepancy between the *effective* thermal diffusivities, as obtained from analyses of the perturbed system described by (5.3) and (5.4) and from measurements on the equilibrium solutions of (5.1) and (5.2).¹ On the one hand, the effective thermal diffusivity χ_{eff}^{HP} obtained from the dynamical measurement of a certain perturbation is related to the spatially varying phase of the associated heat pulse; the effective phase gradient, as measured, is given by

$$s_{\text{eff}} = -\frac{d}{dx} \arctan \left[\frac{u_+ A_+ \exp(-s_+ x) \sin(-s_+ x) + u_- A_- \exp(-s_- x) \sin(-s_- x)}{u_+ A_+ \exp(-s_+ x) \cos(-s_+ x) + u_- A_- \exp(-s_- x) \cos(-s_- x)} \right]. \quad (5.28)$$

On the other hand, the effective thermal diffusivity χ_{eff}^{PB} obtained from equilibrium measurements is a linear combination of the two energy diffusivities L_{21} and L_{22} in the *unperturbed* energy balance equation. In the ‘near-field’ approximation $x \rightarrow 0$, the ratio of the two diffusivities is

$$\frac{\chi_{\text{eff}}^{HP}}{\chi_{\text{eff}}^{PB}} = \frac{\frac{3}{2}}{L_{21}\eta^{-1} + L_{22}} \left(\frac{u_+(A_+/A_-)\lambda_+^{-1/2} + u_-\lambda_-^{-1/2}}{u_+(A_+/A_-) + u_-} \right)^{-2}; \quad (5.29)$$

in the ‘far-field’ approximation, the ratio is

$$\frac{\chi_{\text{eff}}^{HP}}{\chi_{\text{eff}}^{PB}} = \frac{\frac{3}{2}\lambda_+}{L_{21}\eta^{-1} + L_{22}}; \quad (5.30)$$

the profile parameter $\eta = \frac{T'/T}{n'/n}$ is a characteristic of the equilibrium solution that depends on the diffusivities and sources.

As an illustration of the effects of parametric dependences of the equilibrium diffusivities on the dynamical response, we now consider a case where all the diffusivities of the equilibrium system have a functional dependence on, say, the density gradient. We take a particle flux

$$\Gamma = -nf(r) \left(\frac{n'}{n} \right)^\ell \left(L_{11} \frac{n'}{n} + L_{12} \frac{T'}{T} \right); \quad (5.31)$$

the linearized perturbed particle flux, obtained after a Taylor expansion and some algebraic manipulation (with the same assumptions as those used for the linearization of the transport equations), is

$$\tilde{\Gamma} = -nf(r) \left(\frac{n'}{n} \right)^\ell \left[\left((\ell + 1)L_{11} + \ell\eta L_{12} \right) \left(\frac{\tilde{n}}{n} \right)' + L_{12} \left(\frac{\tilde{T}}{T} \right)' \right], \quad (5.32)$$

¹In the dynamical case, one measures the perturbed thermal flux $\tilde{q} = -n\chi^{HP}\tilde{T}'$, whereas in the equilibrium case, one measures the steady-state thermal flux $q = -n\chi^{PB}T'$ (see section 2.6).

with a similar result for the thermal flux; one notes that the perturbed diffusivities now depend on the equilibrium solutions through the parameter η . The subsequent treatment proceeds in the same way as above, the expressions for the diffusivities in the perturbed fluxes being modified in (5.5), using the substitutions $L_{11} \rightarrow (\ell + 1)L_{11} + \ell\eta L_{12}$ and $L_{21} \rightarrow (\ell + 1)L_{21} + \ell\eta L_{22}$.

Bishop and Connor (1988) have investigated the effects of coupling in three transport models, namely the collisionless and collisional neoclassical models (Hinton and Hazeltine, 1976), and the anomalous dissipative-trapped-electron (DTE) mode (Horton, 1976), one of the reasonably good candidates for anomalous transport; the latter predicts diffusivities with a cubic dependence on the density gradient, as treated above. The different perturbations and effective thermal diffusivities were evaluated for these models, using the formulae derived here. The results of these calculations, corresponding to the typical profile parameter $\eta=1.5$, are shown in table 5.1 and are broadly consistent with experimental observations; in particular, the *dynamical* values of the effective thermal diffusivity are *higher* than the corresponding *equilibrium* values, and the density perturbations are often smaller in relative amplitude than the temperature perturbations.

The present treatment provides a qualitative understanding of coupled transport processes, and indicates that large discrepancies are possible, between the effective thermal diffusivities determined from steady-state and distinct transient measurements. In practice, the transport coefficients will have much more complicated functional forms which can introduce further discrepancies through additional terms in the perturbed diffusivities. Furthermore, there may be a perturbation of the particle source, in addition to those of the energy sources and sinks; such a perturbation could enhance or reduce the density modulation.

5.3 Numerical modelling of generalized coupled transport

5.3.1 Coupled transport model

For the purposes of a numerical study of the observed responses of the electron density and temperature to the ECRH perturbation, and in order (i) to assess the extent to which coupled transport can be reconciled with experimental data, and (ii) to investigate the applicability or otherwise of theoretical transport models, a system of coupled partial differential equations was

Table 5.1: Results from analytical investigation of coupled transport systems (relative magnitudes of the steady-state coefficients are shown): ratio of density to temperature perturbations and effective thermal diffusivities, as determined from modulation, saw-tooth and power balance measurements.

Model	L_{11}	L_{12}	L_{21}	L_{22}	$\frac{\tilde{n}/n}{T/T}$		$\frac{\chi_{eff}^{HP}}{\chi_{eff}^{PB}} \eta = 1.5$			
					near field		far field	near field		far field
					mod	s/t		mod	s/t	
collisionless neoclassical	1.04	-0.36	-1.40	1.65	0.19	1.04	-1.66	1.59	1.31	2.63
collisional neoclassical	0.33	0.06	-0.27	0.71	-0.08	0.83	0.31	1.43	1.58	1.49
anomalous DTE ($\eta = 1.5$)	2.0	6.0	1.0	15.0	-0.14	0.53	0.47	0.14	12.7	4.56

used. These describe the transport of particles and electron energy and govern the variables of electron density $n_e(r, t)$ and electron temperature $T_e(r, t)$; they include generalized terms for the fluxes of particles and energy:

$$\frac{\partial n_e}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r n_e \left(L_{11} \frac{n'_e}{n_e} + L_{12} \frac{T'_e}{T_e} - V_1 \right) \right] + S(r, t), \quad (5.33)$$

$$\begin{aligned} \frac{3}{2} \frac{\partial (n_e T_e)}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} \left[r n_e T_e \left(L_{21} \frac{n'_e}{n_e} + L_{22} \frac{T'_e}{T_e} - \frac{3}{2} V_2 \right) + \frac{5}{2} r n_e T_e \left(L_{11} \frac{n'_e}{n_e} + L_{12} \frac{T'_e}{T_e} - V_1 \right) \right] \\ & + Q_\Omega(r, t) - Q_{exc}(r, t) - Q_{rad}(r, t) + Q_{ech}(r, t). \end{aligned} \quad (5.34)$$

In the present formulation, L_{22} corresponds to the electron thermal diffusivity χ_e , L_{11} corresponds to the particle diffusivity D , and the convective contribution of the particle flux to the thermal flux has been included explicitly; the velocities V_1 and V_2 appear in the non-diffusive terms and are negative for inward transport; S is the particle source, Q_Ω is the Ohmic heating source, Q_{exc} is the electron-ion exchange sink, Q_{rad} is the radiation sink and Q_{ech} is the ECRH source. This system of equations is, in general, impossible to linearize or diagonalize. Both steady-state and perturbed parts of the density and temperature variables are therefore considered.

Approximate equilibrium profiles of density and temperature are used as initial conditions; the boundary conditions are

$$n'_e(0, t) = 0, \quad (5.35)$$

$$T'_e(0, t) = 0; \quad (5.36)$$

$$\Gamma(a, t) = -n_e \left(L_{11} \frac{n'_e}{n_e} + L_{12} \frac{T'_e}{T_e} - V_1 \right) \Big|_{r=a} = F(t), \quad (5.37)$$

$$T'_e(a, t) = -AT_e(a, t). \quad (5.38)$$

The boundary conditions for the centre arise from symmetry; those for the edge postulate a time-dependent particle flux (through which the effect of a modulated particle source may be introduced) and a Neumann-type cooling law, and can be established from the density and temperature equilibrium profiles.

The transport coefficients L_{11} , L_{12} , L_{21} , L_{22} , V_1 and V_2 are, in general, functions of several plasma variables, such as n_e , n'_e , T_e , T'_e , and also r , ε , q and q' , depending on the transport model, and will consequently be modulated. Many transport models predict fluxes with complex and sensitive dependences on these parameters, and are therefore difficult not only conclusively to compare with experimental results, but also numerically to evaluate. In the simplest case, however, a simple radial dependence is assumed for all the diffusivities, increasing towards the edge in order to maintain consistency with the results from power balance analysis. It should be mentioned, incidentally, that a determination of all diffusivities in the coupled transport equations is impossible from an analysis of power balance alone, except, possibly, when the density and temperature gradients can be varied independently, for example, by varying the additional heating power.

The transport of ion energy has not been included in this analysis, because, at the relatively low densities and ion temperatures at which the present modulation experiments were carried out, energy transport through the ion channel should be small, and the ion temperature gradients should be substantially smaller than the electron temperature gradients. No data directly relating to the ion component were available from these experiments. The effect of electron-ion equipartition on the oscillating response should be small (see derivation in subsection 4.3.1),

although a less insignificant effect on the response over several cycles can be expected. For higher densities or higher ion energy densities, however, the inclusion of the ion energy balance equation would be necessary. The approach adopted in the present analysis consists in retaining the equipartition term, despite the non-inclusion of the ion energy balance, and in assuming a time-independent ion pressure (or energy density) profile, so that the modulation of the equipartition term is only due to the combined variations of the electron pressure and the exchange time.

The most difficult aspect of this approach to the modelling of perturbed coupled transport, given a transport model, is the determination of the particle and energy sources and sinks. It is clear that all the sources will be modulated. In the simplest, albeit not necessarily realistic, approach, one can determine the *effective* sources that are required to sustain the measured or assumed equilibrium profiles indefinitely, subject to the particular set of transport coefficients, and use these as the steady-state sources; one then has only to introduce the modulated ECRH source and the perturbation of the steady-state Ohmic source, ignoring all other source perturbations. An inspection of the effective sources so determined will provide an additional check on the transport model used, although it should be noted that these sources will be strongly sensitive to the form of the equilibrium profiles (depending on radial derivatives up to the second order), which cannot be determined with any accuracy in the present experiments.

5.3.2 MHD shifts of flux surfaces

In order to include an estimate for the modulated, differential shifts of the flux surfaces, an analysis based on MHD equilibrium in the large-aspect-ratio limit is incorporated in the present model. The underlying theory has been developed by Shafranov (1966), and Mukhovatov and Shafranov (1971). This postulates differential, horizontal shifts of the centres of the flux surfaces which remain circular. These differential shifts are modulated because of the modulation of the electron component of the plasma pressure; the ion pressure is taken to be constant (at least over the modulation time-scale) and equal to some fraction of the equilibrium electron pressure, reflecting the low ion temperature (the ion density being also smaller than the electron density).

It has been assumed that the current density distribution and hence the profile of the poloidal magnetic field (and also that of the safety factor) remain unchanged; consequently, a differential equation describing the current density evolution is not included in the present analysis. Experimentally, no variation of the plasma current was observed, the modulation of the Ohmic heating term being connected solely to the variation of the loop voltage. This observation is consistent with the long magnetic field diffusion time, of the order of 1s, following from the low value of the associated diffusivity (equal to $\eta_{||}/\mu_0$). Naturally, a modulation of the current density profile, such that the total plasma current remains constant, is possible and can be produced by the sawtooth oscillation or other MHD activity, on a small length-scale and a correspondingly short time-scale.

Returning to the MHD equilibrium, the horizontal, outward displacement of a flux surface of minor radius r , with respect to the outermost flux surface of minor radius a , is given by

$$\delta(r) = \int_r^a \frac{r'}{R_0} (\Lambda(r') + 1) dr', \quad (5.39)$$

$$\begin{aligned} \text{with } \Lambda(r) &= \beta_p(r) - \beta_{p0}(r) + \frac{1}{2}l_i(r) - 1 = \\ &= \left[\frac{1}{\pi r^2} \int_0^r \left(p(r') - p(r) + \frac{1}{2} \frac{B_\theta^2(r')}{2\mu_0} \right) 2\pi r' dr' \right] / \left(\frac{B_\theta^2(r)}{2\mu_0} \right) - 1; \end{aligned} \quad (5.40)$$

p is the plasma pressure, $\beta_p(r)$ is the poloidal beta using the area-averaged pressure inside the radius r (this being the usual definition of the poloidal beta), $\beta_{p0}(r)$ is the poloidal beta using the local pressure, and $l_i(r)$ is the dimensionless internal inductance; $\Lambda(r)$ is an asymmetry factor called the *Shafranov lambda*. These results have corrections of the order $(r/R_0)^2$. The poloidal magnetic field is given in terms of the current density distribution, by

$$B_\theta(r) = \frac{\mu_0 I(r)}{2\pi r} = \frac{\mu_0}{2\pi r} \int_0^r j(r) 2\pi r dr. \quad (5.41)$$

The (time-independent) current density profile is calculated from the classical (Spitzer) conductivity with a neoclassical correction for trapped particles (see section 2.5 and (2.60)–(2.65)), i.e.

$$j(r) \propto \sigma_{||}(T_e(r), n_e(r), r),$$

and is normalized using the known plasma current, leading to a value for the loop voltage. The safety factor profile is given by

$$q(r) = \frac{B_{\phi 0}}{R_0} \frac{r}{B_{\theta}(r)} = \frac{B_{\phi 0}}{R_0} \frac{2\pi r^2}{\mu_0} \bigg/ \int_0^r j(r) 2\pi r dr, \quad (5.42)$$

its central value being

$$q(0) = \frac{B_{\phi 0}}{R_0} \frac{2}{\mu_0 j(0)}.$$

The shifts of the flux surfaces are not taken into account in the solution of the transport equations. We now assess the possible effect of this simplification. We consider two flux surfaces of radii r_1 and r_2 , with $r_2 > r_1 \geq 0$; the centres of these flux surfaces are separated by $\Delta\delta < r_2 - r_1$. The distance between the flux surfaces, as a function of the poloidal angle, is

$$\begin{aligned} \Delta r(\theta) &= \left[r_2^2 + \Delta\delta^2 - 2r_2\Delta\delta \cos\theta \right]^{1/2} - r_1 \\ &= r_2 - r_1 - \Delta\delta \cos\theta + \frac{1}{2} \frac{\Delta\delta^2}{r_2} \sin^2\theta + O\left(\frac{\Delta\delta^3}{r_2^2}\right); \end{aligned} \quad (5.43)$$

taking an average over the poloidal angle gives

$$\langle \Delta r \rangle \simeq r_2 - r_1 + \frac{\pi}{2} \frac{\Delta\delta^2}{r_2}. \quad (5.44)$$

The modification due to the differential shift is less than 1.5% and can be easily neglected.

Furthermore, a modulated, horizontal shift is possible, of the entire plasma column, of amplitude comparable to the internal MHD shifts of the flux surfaces. The response of the plasma column to the applied modulated power depends not only on the plasma equilibrium, but also (and mainly) on the response of the feedback stabilization system. For a complete description, this column shift should be applied to the outermost flux surface.

5.3.3 The MTRANSP2 computer code

The FORTRAN programme MTRANSP2 has been developed to implement the general model of coupled transport described above, with a view to evaluating a variety of transport models. A numerical technique is used for the solution of the system of coupled partial differential equations for particle and energy transport, (5.33) and (5.34), subject to the boundary conditions, (5.35)–

(5.38).² The perturbation is driven by a modulated ECRH power density term, as given by (4.38). The solutions are obtained on a mesh over the minor radius, and all variables, transport coefficients and source terms are, of course, averages over flux surfaces. The mesh-point spacing is variable and is typically chosen to be smaller near the centre and the ECRH power deposition radius, and larger near the edge, in order to enhance the accuracy of the solution. The mesh is defined by a mesh-point density function of the form

$$G(r) = \left[1 + N_1 \left(1 - \left(\frac{r}{a} \right)^N \right) \right] \left[1 + N_2 \exp \left(- \left(\frac{r - r_0}{r_W} \right)^2 \right) \right], \quad (5.45)$$

where the parameters N_1 , N_2 and N can be freely varied; typically, there are 61 mesh points. The solutions are advanced in time, over several periods of the modulation, typically with 40 samples per cycle, subject to the partial differential equations which generally have variable transport coefficients as well as variable source terms.

The code produces solutions for the electron density n_e and pressure $p_e = n_e T_e$, from which the electron temperature T_e is calculated. The plasma pressure, used in the calculation of the flux surface shifts, is the sum of the electron and ion pressures; the latter is calculated on the assumption of a steady-state ion temperature profile—in DITE discharges the ion temperature is some small fraction of the electron temperature, typically 0.25 at the centre and approaching 1.0 towards the edge, so that the form of the ion temperature profile and its possible temporal variation are not important. A local soft X-ray emissivity is also calculated from the electron density and temperature, using the results presented in subsection 3.3.1; in the case of coupled transport, the SXR density modulation index will be of importance: the value of 2 that is used may not reflect the true variation of the local SXR emissivity with density, depending on the impurity confinement (refer to section 2.5). The time-dependent Shafranov lambda factor (5.40) and horizontal flux surface shift (5.39) are computed concurrently, using numerical

²A NAG routine, D03PGF, based on Gear's method is used (NAG, 1984, vol. 2); this routine specifically solves systems of parabolic (diffusive-type) partial differential equations, and allows considerable flexibility in the choice of the parametric dependences of the transport coefficients, sources and boundary conditions. The diffusive terms in the fluxes are introduced explicitly, whereas the non-diffusive ones are introduced as effective sources. Dimensionless variables are used throughout the numerical solution.

integration over the minor radius to implement the results presented in subsection 5.3.2. These horizontal shifts introduce the only source of poloidal asymmetry in the computation; their temporal dependence arises only from the modulation of the plasma pressure, the poloidal magnetic field profile being time-independent. A provision has been made for the inclusion of the modulated column shift, using a sinusoidal variation of appropriate amplitude and phase. The time-dependent horizontal shifts are important in the calculation of the simulated diagnostic signals, both local and line-integrated. They are also used in calculating the shift in minor radius of the ECRH power deposition profile, as well as the major radius change in the safety factor and inverse aspect ratio, but these effects are small.

Separate programme modules, which can be readily modified, are used in the code to calculate the instantaneous radial profiles of the transport coefficients, from the transport model under consideration, and those of the sources, using the computed instantaneous profiles of density, electron temperature and flux surface shift; the time-independent profiles of safety factor and inverse aspect ratio can also be used. These calculations of the transport coefficients and source terms are effected between successive time steps. The relevant profiles are smoothed and expressed in terms of cubic splines, the latter being used to obtain all the required derivatives. Computational errors can often lead to unstable solutions. Several problems of numerical nature can be encountered in calculating the fluxes of particular transport models, and in solving the corresponding system of equations.

Two different approaches have been implemented for the calculation of the particle and energy sources. In the first and simplest approach, effective sources are calculated, which are required to sustain the equilibrium profiles, given the transport model, as explained above, and only the modulated ECRH source is separately included. In the second approach, more realistic energy sources are used for the Ohmic heating, and for the electron-ion exchange and radiation losses: these energy source terms have been discussed in section 2.5, where the relevant expressions are given. A neoclassical resistivity with detrapping corrections has been used in the code. The perturbation of the Ohmic heating term is produced from the time-dependent resistivity, using the equilibrium current density distribution. The equipartition power density is calculated

from the steady-state ion energy density. The radiation code described in subsection 3.3.1 is used to assess the effective ion charge, the radiation power density and the SXR power density and (temperature) modulation index, given the concentration levels of carbon and oxygen impurity ions —the calculated radiation loss near the edge is probably an underestimate. However, an effective particle source is still used, as required to sustain the equilibrium density profile, because a suitable calculation of the particle source was not available.

The code computes equilibrium solutions for the electron density and temperature profiles under conditions of Ohmic heating (supplemented by steady-state ECRH, where applicable), subject to the specified transport model and to the particle and energy sources. These solutions are obtained by allowing the system to evolve from the initial conditions, without a perturbation and over a time-scale comparable to the energy confinement time, until the changes in the profiles between successive time steps are sufficiently small; this approach, however, usually fails to yield unique results, independent of the initial profiles, because of the accumulation of computational errors. For the equilibrium solution, the code further computes the volume integrals of the particle and heat sources and those of the electron, plasma and magnetic field energy densities; this provides a further assessment of the validity of the transport model, based on the equilibrium, in addition to the subsequent study of the dynamical response. The loop voltage is also calculated, this being sensitively dependent on the temperature profile and the resistivity model used, and subject, of course, to the effective ion charge calculated by the radiation code. The modulated ECRH is initiated after the equilibrium solutions have been obtained.

Simulated signals of line-integrated soft X-ray emissivity, electron cyclotron emission and line-integrated density are finally generated over the duration of the perturbation, taking the shifts of the flux surfaces into account. The required integration chords and the local sampling points can be arbitrarily specified. SXR signals from both horizontal and vertical lines of sight are typically simulated. The geometry of the shifted, circular flux surfaces is shown in figure 5.1, where the origin is at the centre of the unshifted outer flux surface. The calculation of a chordal integral is effected by numerical integration, using the intersections of the flux surfaces of the

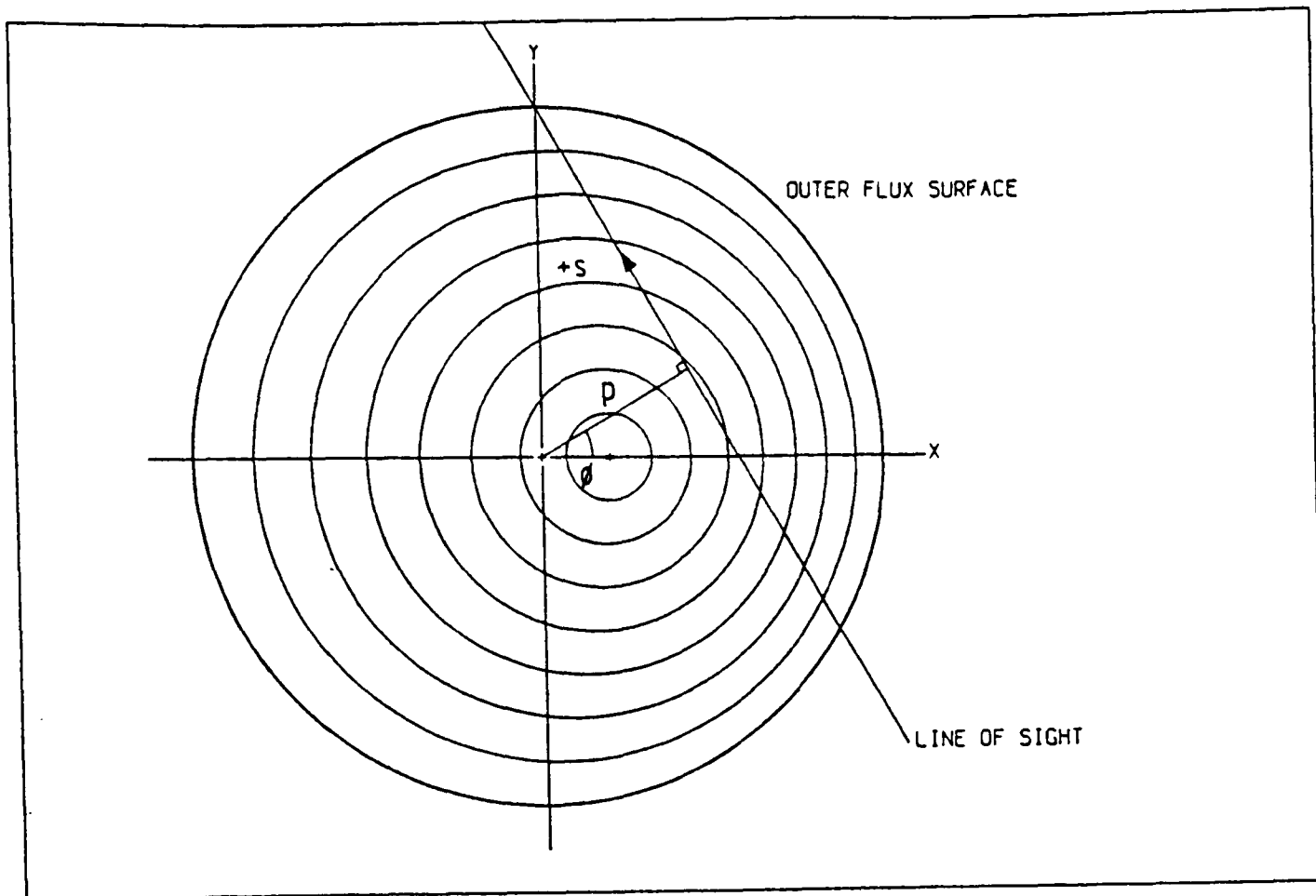


Figure 5.1: Geometry of horizontally shifted, circular flux surfaces used in coupled transport code, showing an integration chord (line-of-sight).

mesh with the chord, as defined by the chordal radius p and the angle of the normal ϕ . In the geometry of figure 5.1, the values of the chordal co-ordinate s at the intersections of the flux surface of minor radius r and horizontal shift $\delta(r)$, at points $x = p \cos \phi - s \sin \phi$, $y = p \sin \phi + s \cos \phi$ of the cross-section, are the solutions of the quadratic equation

$$p^2 + s^2 + \delta(r)^2 - 2\delta(r)(p \cos \phi - s \sin \phi) = r^2. \quad (5.46)$$

The calculation of a local value at the point $x = p \cos \phi$, $y = p \sin \phi$ of the cross-section is by interpolation, with a radius r calculated by using a linear iteration method to solve

$$p^2 + \delta(r)^2 - 2p\delta(r) \cos \phi = r^2. \quad (5.47)$$

These calculations are carried out at each time step.

A DFT procedure, over a selected number of cycles, is used to obtain the profiles of absolute modulation amplitude, relative phase, average level and relative modulation amplitude, in order to effect a direct comparison with the corresponding experimental results. In comparing the experimental with the corresponding model data, the absolute modulation amplitude *and* average level profiles of the model are *scaled*, thus leaving the relative amplitude profile unchanged; and

the relative phase profile is *shifted*. The comments in section 4.3.2, concerning the calculation and fits of the amplitude and phase profiles, also apply in this context. The code can generate output data in various forms: each local variable can be plotted as a surface representing the radial and temporal variation of the solution obtained, its perturbation (with respect to the corresponding equilibrium profile) or its relative (percentage) modulation; furthermore, the temporal variation of a local variable at any mesh point can be plotted; finally, the equilibrium profiles of the electron density and temperature, current distribution and safety factor, transport coefficients, sources and sinks can be plotted, additionally showing the extrema (over the duration of the modulated heating) of the time-dependent transport coefficients, sources and sinks.

5.4 Results from modelling of generalized coupled transport

5.4.1 Assessment of coupled transport

This subsection describes some applications of the coupled transport model, whose purpose was to assess the compatibility of particle and energy fluxes containing finite off-diagonal diffusivities with the analysed data from the modulated ECRH experiments. Early results of this investigation were reported by Bishop et al. (1989). The equilibrium profiles of density and temperature, which were used in this analysis, were similar to those estimated for the model of diffusive thermal transport, as described in the previous chapter. The particle and energy sources were obtained as described in subsection 5.3.3: an effective, time-independent particle source was used, as required to sustain the equilibrium density profile; on the other hand, all the energy sources and sinks, namely the Ohmic heating, and the equipartition and radiation losses, were time-dependent, and calculated on the basis of the instantaneous profiles of density and temperature (the driving ECRH source term was the same as that used in the diffusive thermal transport model). The present investigation was based on the simplest possible coupled transport model, that is, one based on a time-independent diffusivity matrix with a single radial profile (consistent with the power balance results); particle and energy pinch terms, with another radial profile, could be included. The diffusivity matrix used was that of the collisionless

neoclassical model (for $Z_i=1$; refer to Hinton and Hazeltine, 1976; and section 5.2, table 5.1); the entire diffusion matrix was scaled to a level appropriate to anomalous transport, and its radial variation was taken to be similar to that of the thermal diffusivity in the diffusive thermal transport model (cf. (4.36)). There is, of course, no justification in the use of this particular model of coupled transport, since neoclassical theory fails to predict the observed tokamak transport. Nevertheless, it provides one with a means of testing the basic aspects of coupled transport: it has already been seen that simple analytical solutions indicate a finite density modulation on the one hand, and a discrepancy between the equilibrium and dynamical values of the effective thermal diffusivity on the other hand.

The practical aspects of the application of the diffusive thermal transport model to the results of the DITE modulation experiments, as discussed in subsection 4.4.1, are also pertinent to the application of the present coupled transport model; several of the considerations that were regarded there, in particular the density and temperature profiles, are more important in this case. As the SXR data provide the most extensive information from the DITE modulation experiments, the dependence of the SXR emissivity on both electron temperature and electron density is an important aspect of the model. The complexity of the present model means that the experimental uncertainties are compounded by computational errors, particularly so, when the transport model involves sensitive dependences on the varying plasma parameters. It is therefore impossible conclusively to assess the validity or otherwise of a given transport model, using the present experimental data and model, and only qualitative importance should be ascribed to the tentative results that were obtained from this analysis.

An example of a comparison of the coupled diffusion model with the modulation data from a medium-density discharge with fundamental, off-axis heating, is presented in figures 5.2 and 5.3, showing the SXR and ECE data respectively. For this case, figure 5.4 shows the model equilibrium profiles, including those of electron density and temperature, plasma pressure and current density; figure 5.5 shows the profile of the diffusivity matrix; figure 5.6 shows the profiles of the particle and energy sources, with the extrema of the energy sources during the ECRH modulation; figures 5.7 and 5.8 show the profile evolution of the perturbed components of the electron

density and temperature variables, respectively, expressed as absolute and relative (percentage) quantities; figure 5.9 shows the profile evolution of the horizontal flux surface shift; and figure 5.10 shows the modulation data for the simulated, line-integrated density signals (the latter could not be compared with corresponding experimental data from the MWI diagnostic). A further example, with modulation data from a medium-density discharge with fundamental, on-axis heating, is presented in figures 5.11–5.15, showing similar information as above.

The main conclusions from this exercise were as follows:

1. The computed temperature perturbation was broadly similar, in form and magnitude, to that obtained from the simpler diffusive thermal transport analysis (cf. figure 4.22).
2. The computed density perturbation, which was entirely due to the diffusion coupling (there being no modulated particle source term), was small under some conditions, in particular with off-axis heating, and its scale length was shorter than that of the temperature perturbation. The shorter scale lengths of both amplitude and phase variations led to a significant cancellation of the modulation amplitude on calculating the line-integrated values; in the case with on-axis heating, the modulation level of the local density was about 3% at the centre, but that of the line-integrated signal was $\lesssim 1\%$; in the case with off-axis heating, the modulation levels were lower and the cancellation was still significant. The model results were therefore compatible with the low values of the measured relative modulation levels of the line-integrated density signals; the associated relative phases could not be compared. It should be noted that the measured density modulation levels were *not* predicted by the model in the absence of a local density modulation, that is, from the horizontal plasma motion alone (see figure 5.26).
3. Some agreement was obtained in fitting the computed temperature and SXR emissivity responses to those measured experimentally. However, the agreement is clearly less satisfactory than that obtained from the simple model of diffusive thermal transport model, as described in the previous chapter. In particular, the SXR data were fitted satisfactorily, but the corresponding ECE data were not. This highlights the importance, in this model of coupled transport, of the SXR modulation indices for both density and temperature;

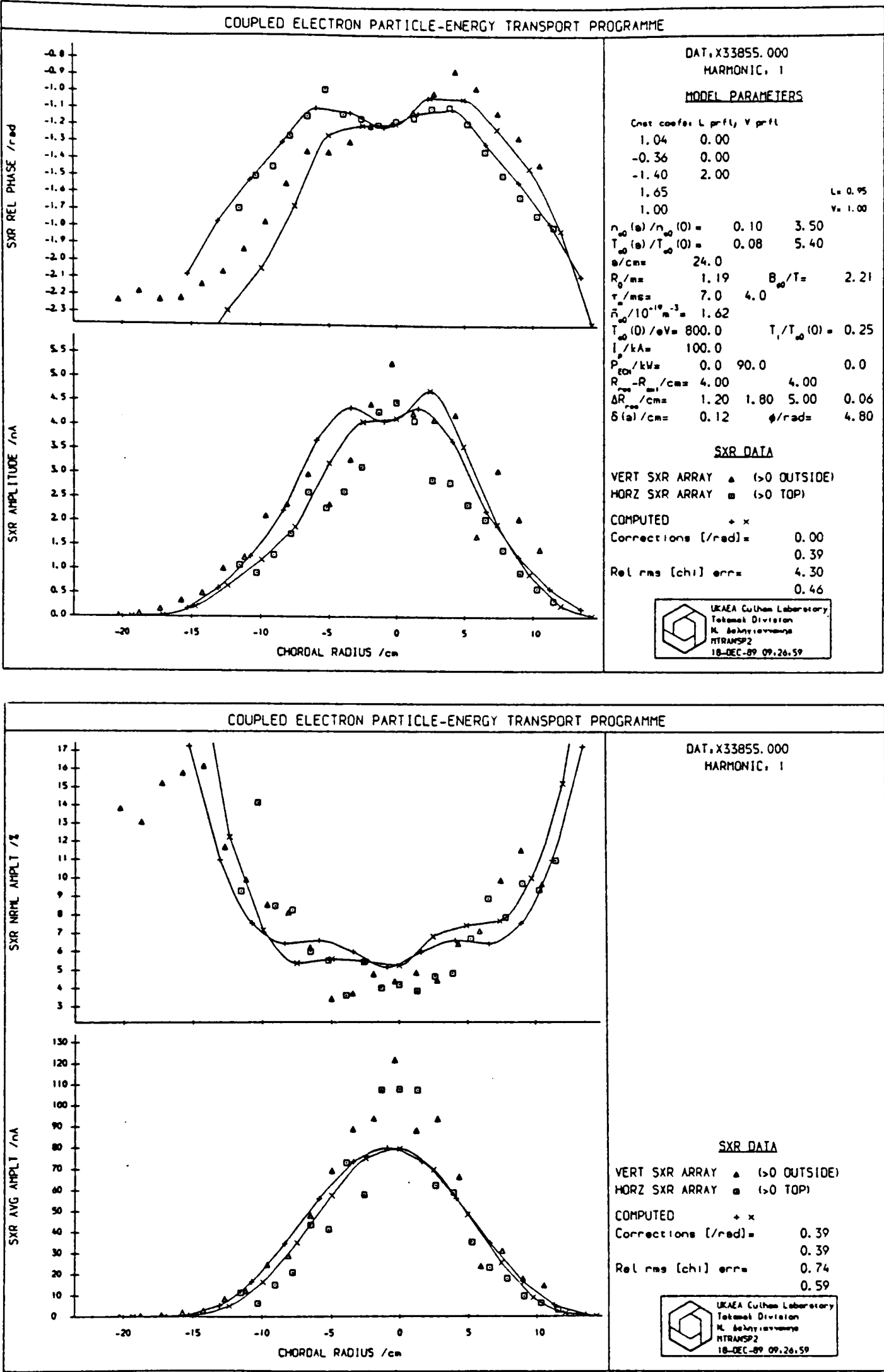


Figure 5.2: SXR modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), fitted with coupled transport model. H discharge, medium density, fundamental off-axis heating.

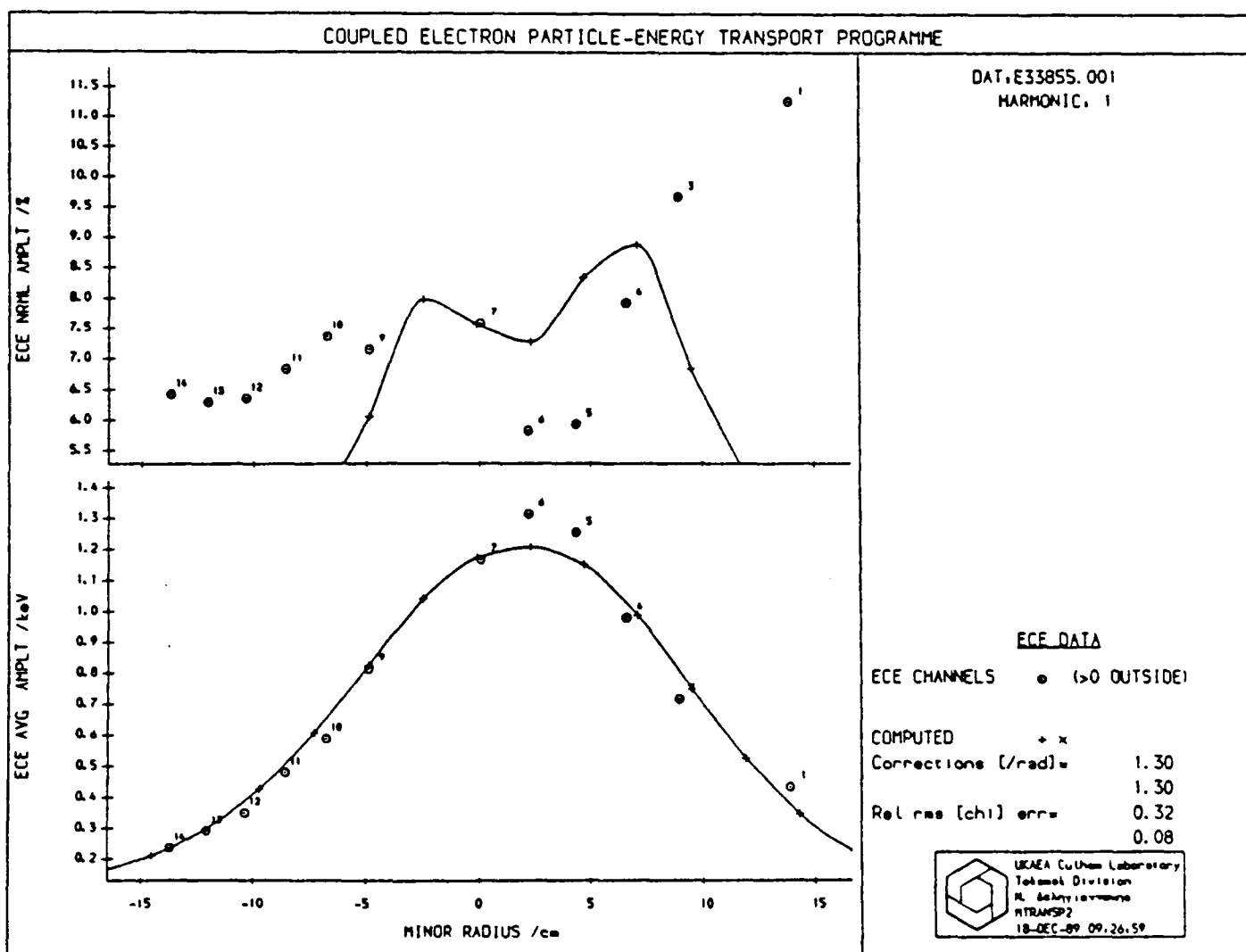


Figure 5.3: ECE modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), fitted with coupled transport model. Discharge of figure 5.2.

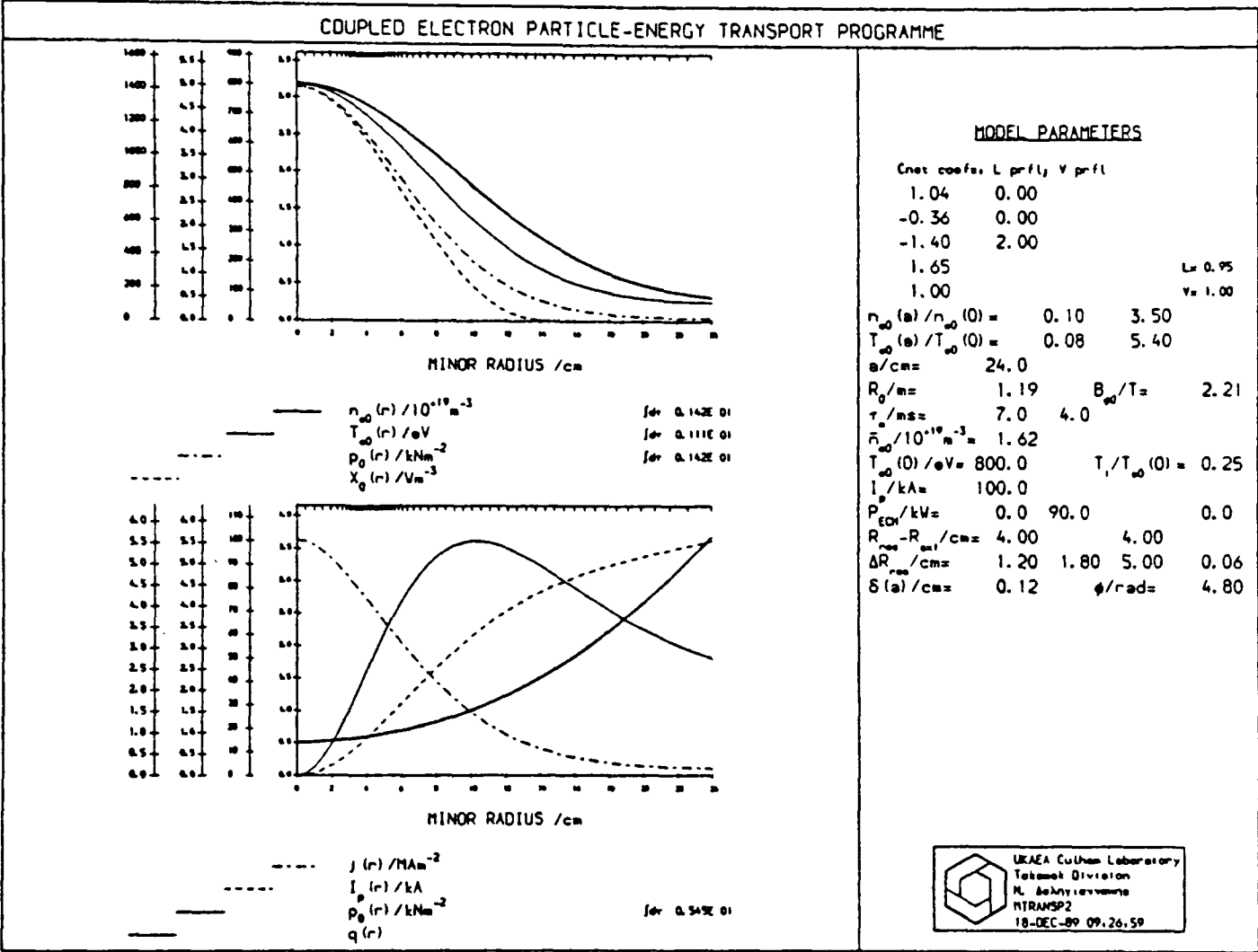


Figure 5.4: Coupled transport model: equilibrium profiles of density n_e , temperature T_e , plasma pressure p , local SXR power density X ; time-independent profiles of current density j , current I within a flux surface, poloidal magnetic field pressure p_θ and safety factor q . The radial mesh is also indicated. Model of figures 5.2–5.3.

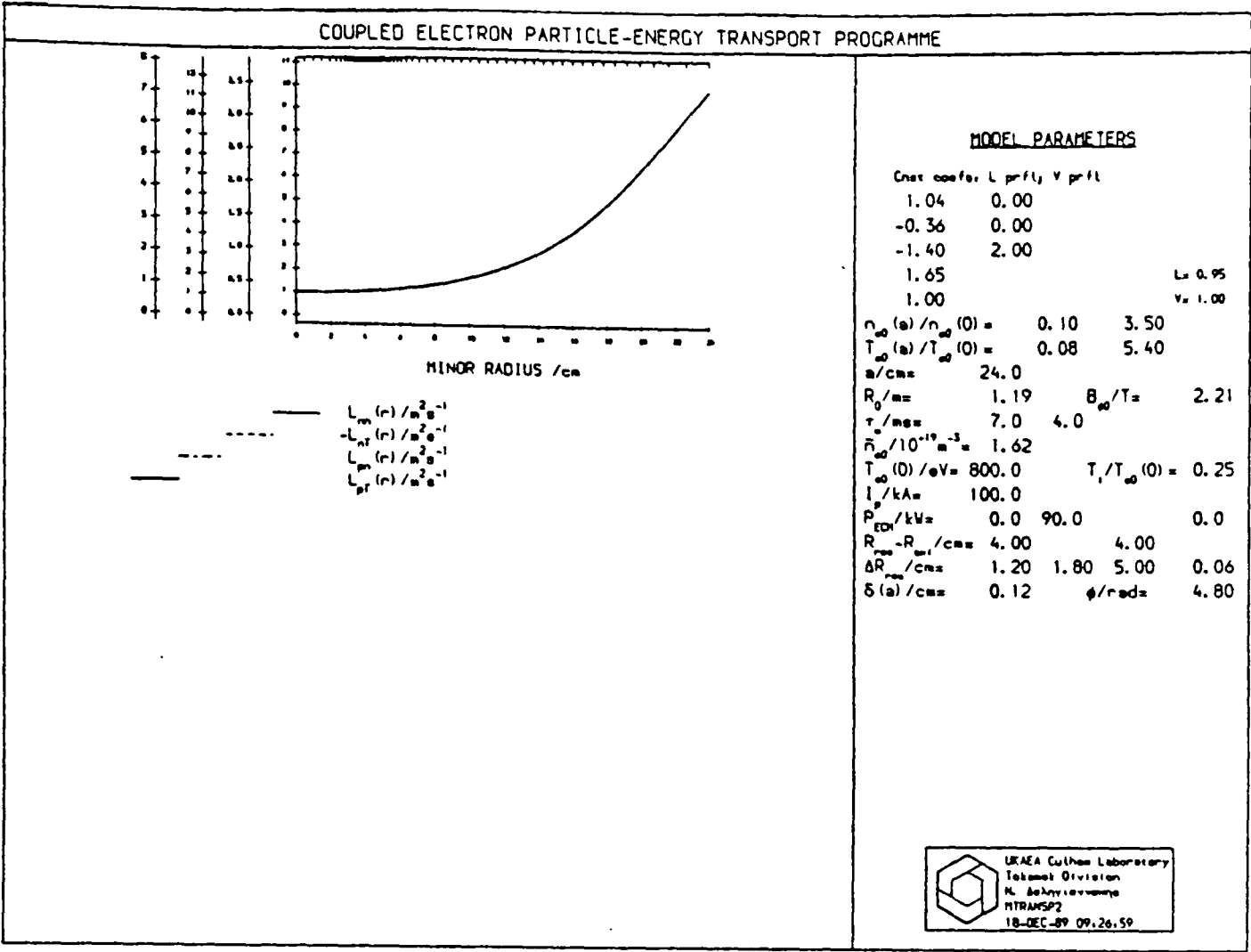


Figure 5.5: Coupled transport model: time-independent profile of diffusion matrix L. Model of figures 5.2–5.3.

it has already been explained how these indices can depend on the impurity confinement. In the present analysis, the modulation index for density had the fixed value of 2, this being the natural choice, but the real value could be nearer 1; nevertheless, it was found that, under the present conditions, the modulation of temperature still had a dominant effect on that of the line-integrated SXR emissivity (only a small change of the predicted SXR modulation data was observed when the density modulation was eliminated from the calculation). There is a clear advantage in using both SXR and ECE data in applications of complex transport models, as these data impose two different constraints on the model solutions.

4. The off-diagonal transport coefficients can be comparable in magnitude to the diagonal ones, without necessarily leading to a large density modulation; in the case with on-axis heating, the modulation levels of central density and temperature were 3% and 15% respectively. The behaviour of the coupled transport model is, of course, sensitive to the choice of diffusivity matrix: whilst the diffusivity matrix used in the present analysis

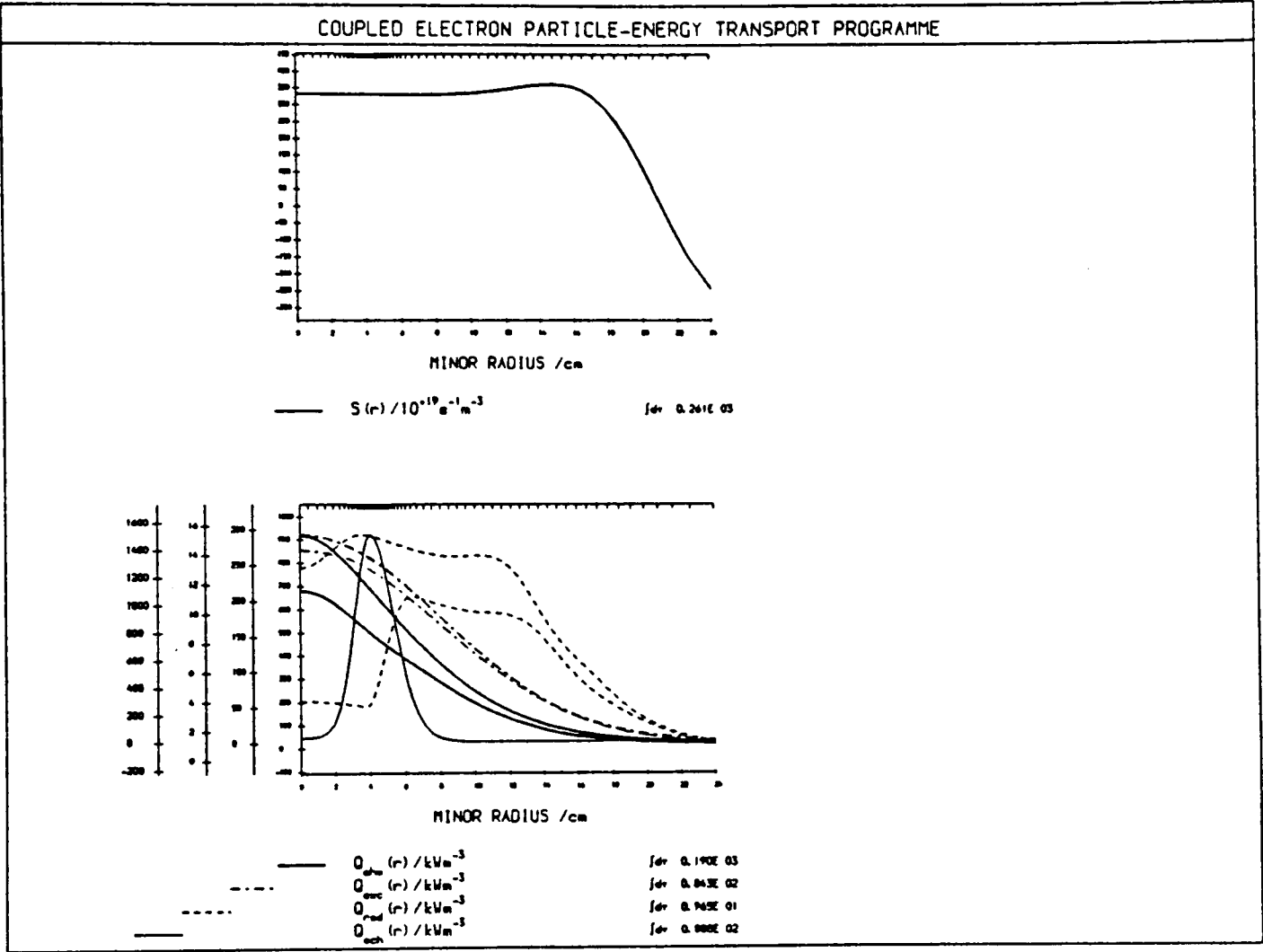


Figure 5.6: Coupled transport model: time-independent profile of effective particle source S ; equilibrium profiles and extrema of Ohmic heating power density Q_{oh} , equipartition loss power density Q_{exc} , radiation loss power density Q_{rad} and ECRH power density Q_{ech} . Model of figures 5.2–5.3.

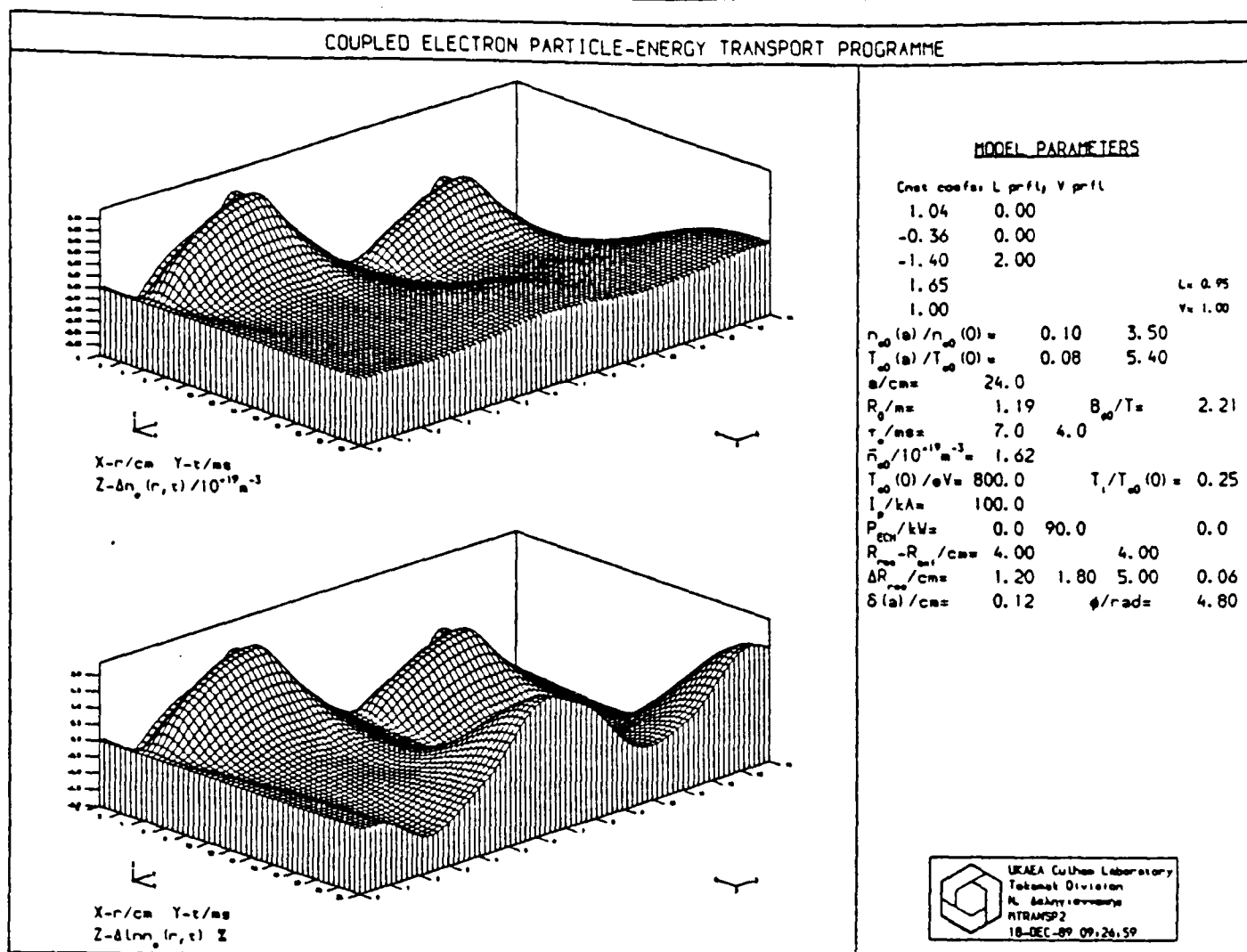


Figure 5.7: Coupled transport model: electron density perturbation $\tilde{n}_e$ and percentage perturbation $\tilde{n}_e/n_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.2–5.3.

did not lead to good agreement with all the experimental data, it appears likely that a different matrix may yield results in closer agreement. The effective power balance thermal diffusivity ($0.9\text{m}^2\text{s}^{-1}$ at the radius $a/3$), as calculated from the code profiles and energy sources, was in good agreement with the corresponding experimental value.

5. The explicit inclusion of a particle pinch term in both particle and thermal fluxes led to a small modification of the modulation results, because of the finite density modulation: the phase gradients became somewhat larger, for the same diffusivity values —the effective particle source included an implicit particle pinch which was time-independent.
6. The modulation of the MHD shifts of the flux surfaces led to the expected asymmetries, the results being consistent both with the experimental shifts, as measured with the magnetic diagnostics, and with the experimental modulation data. A column shift was applied, with an appropriate amplitude (about 1mm) and phase lag (about 4.8rad); this led to a closer agreement for the signals of the vertical SXR chords.

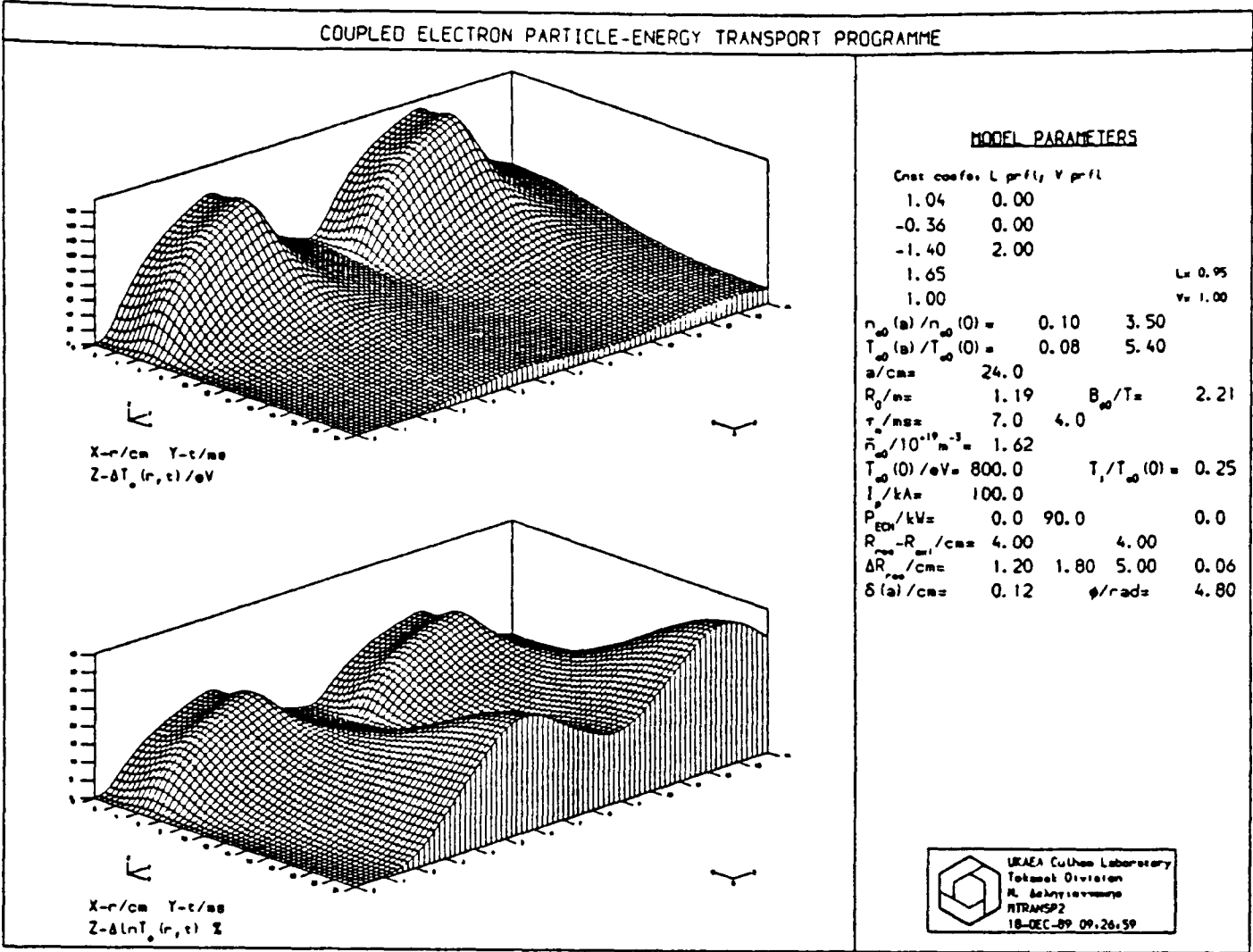


Figure 5.8: Coupled transport model: electron temperature perturbation $\bar{T}_e$ and percentage perturbation $\bar{T}_e/T_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.2–5.3.

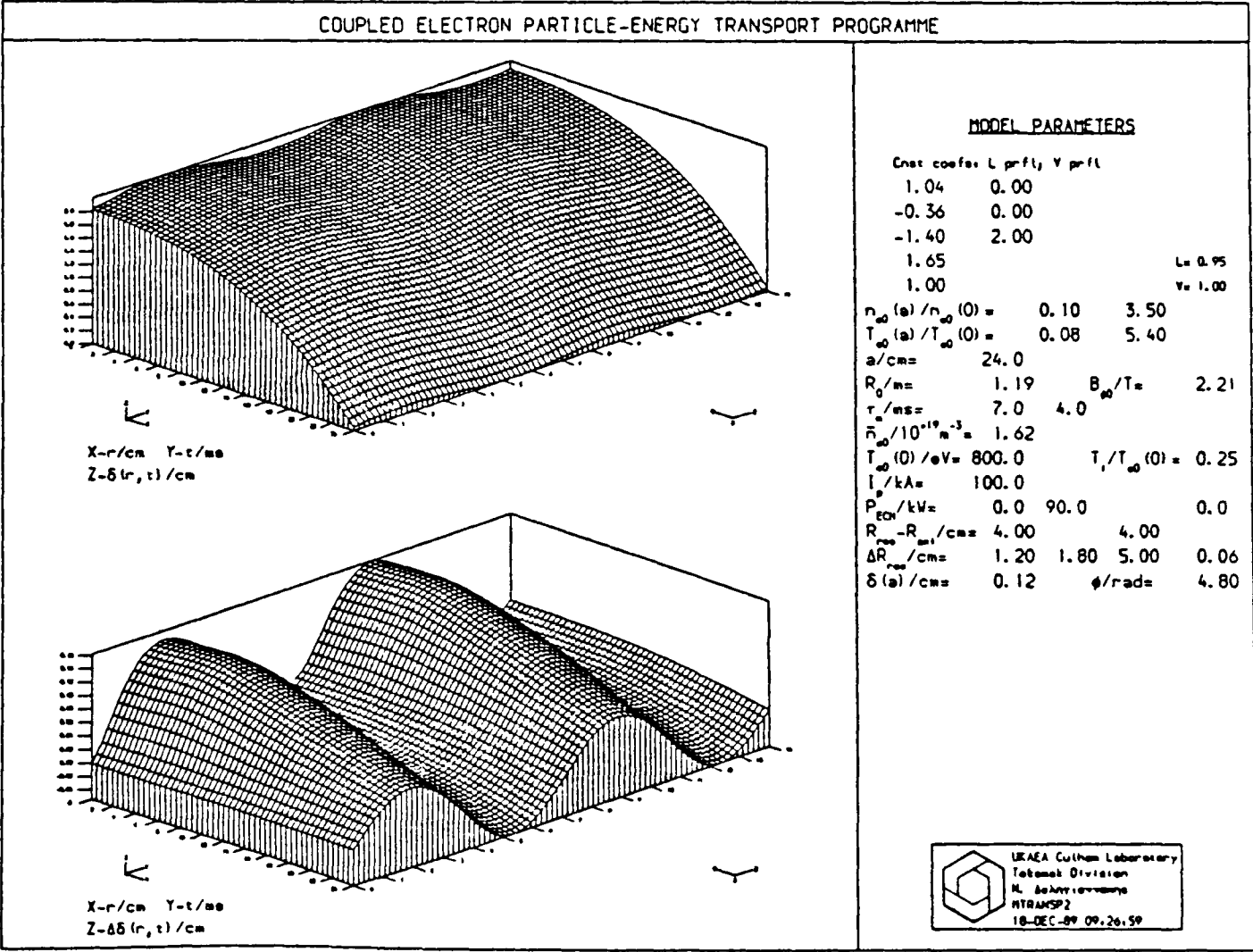


Figure 5.9: Coupled transport model: horizontal flux surface shift δ and perturbation $\bar{\delta}$, plotted as function of minor radius r and time t . Model of figures 5.2–5.3.

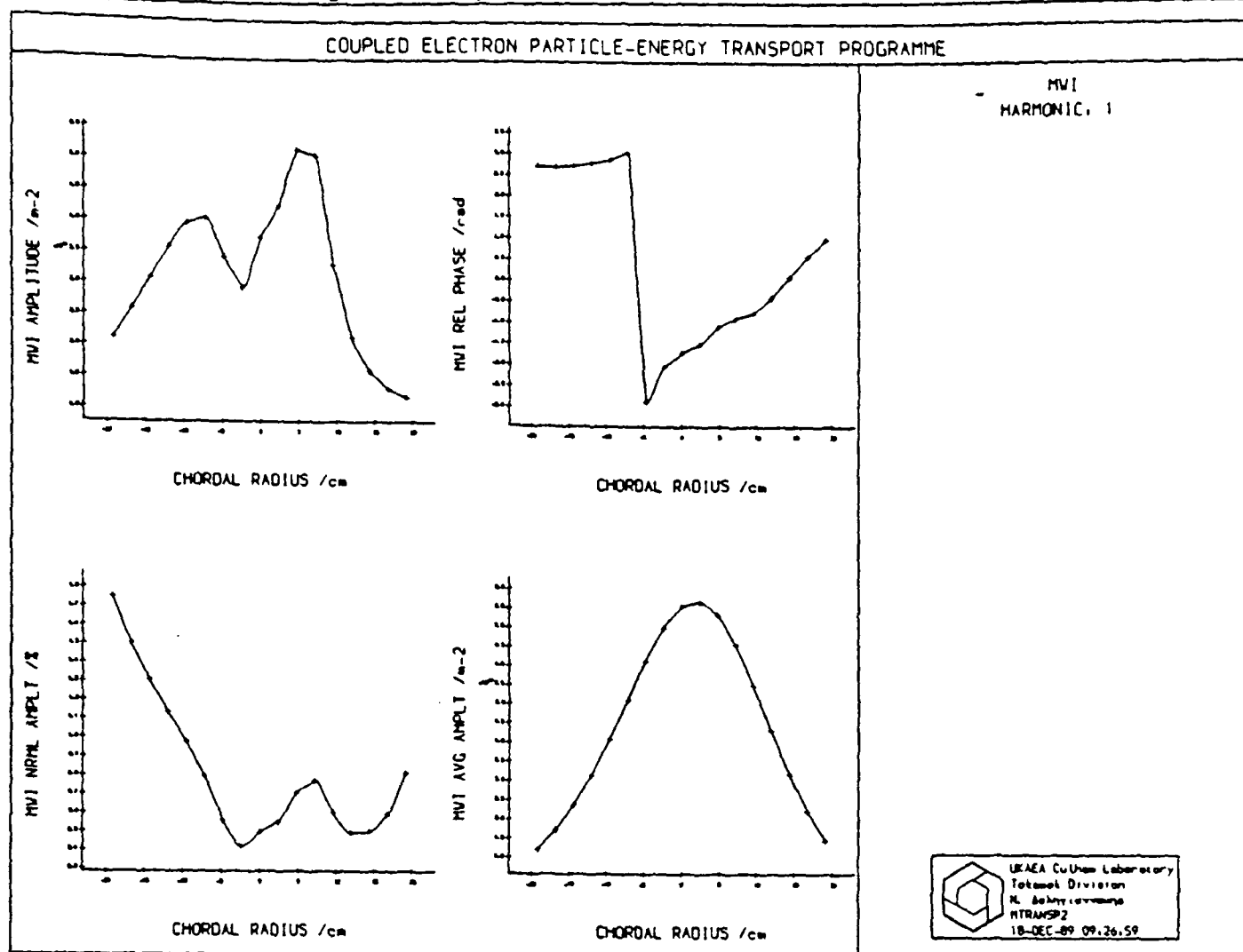
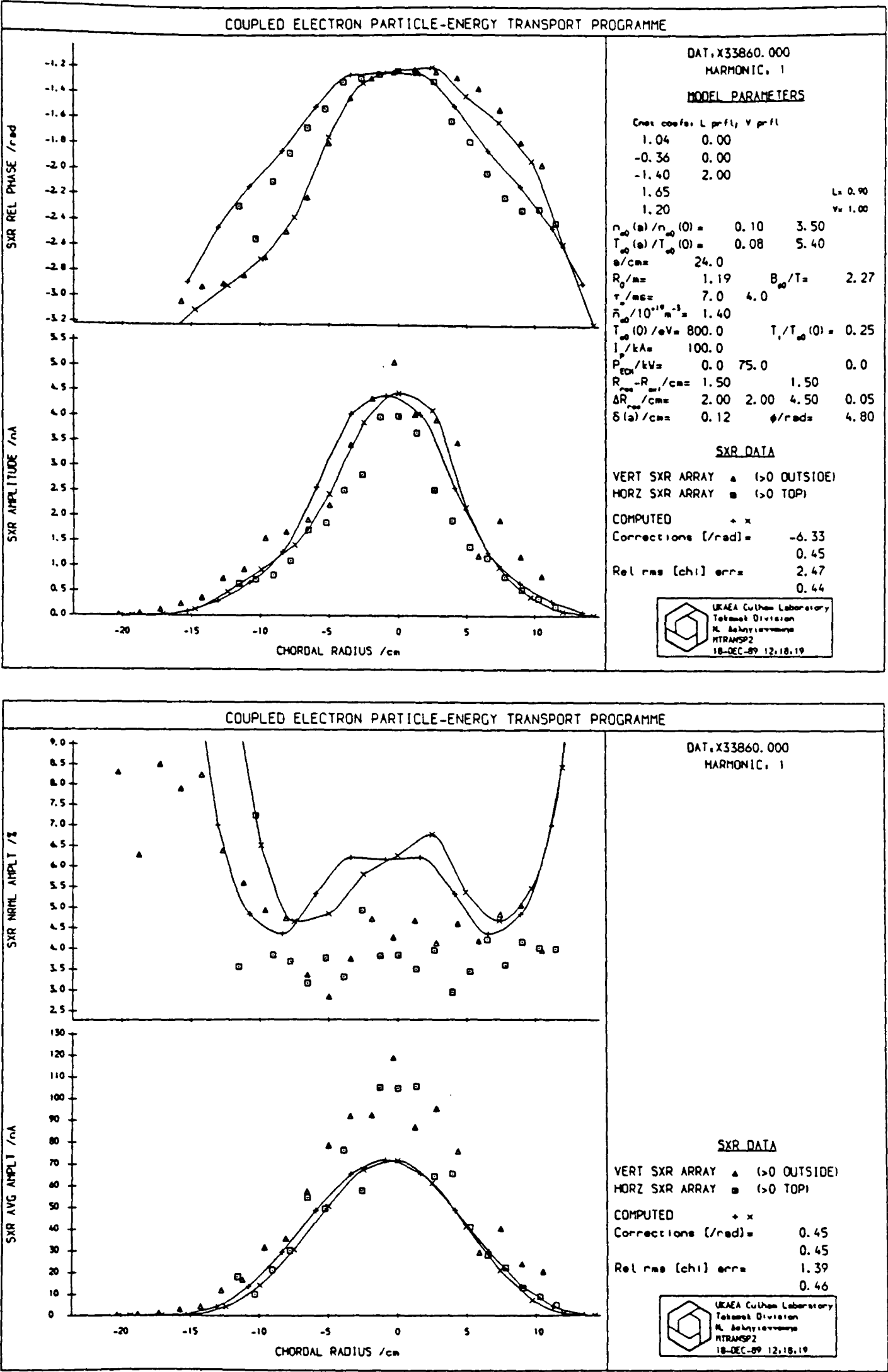


Figure 5.10: Coupled transport model: MWI (line-integrated density) modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), as simulated. Model of figures 5.2–5.3.

7. All the results of the coupled transport model were strongly dependent on the equilibrium profiles of density and temperature, as well as on the form of the transport model. The temperature profile that was used was within an average deviation of 10% from that obtained by allowing the model solution to approach an energy balance, with the calculated energy sources. As a suitable calculation of the particle source was not implemented, such a test could not be carried out on the density profile.
8. It should finally be noted that the present model of coupled transport, when used with zero off-diagonal diffusivities, yielded results that were not identical with those of the diffusive thermal transport model. The discrepancies arose from the flux surface shifts which were not included in the early analysis, and, more importantly, from the different perturbations of the energy sources and sinks. It follows that a more complete analysis of the energy source perturbations would have somewhat modified the results of the simple model of diffusive thermal transport.



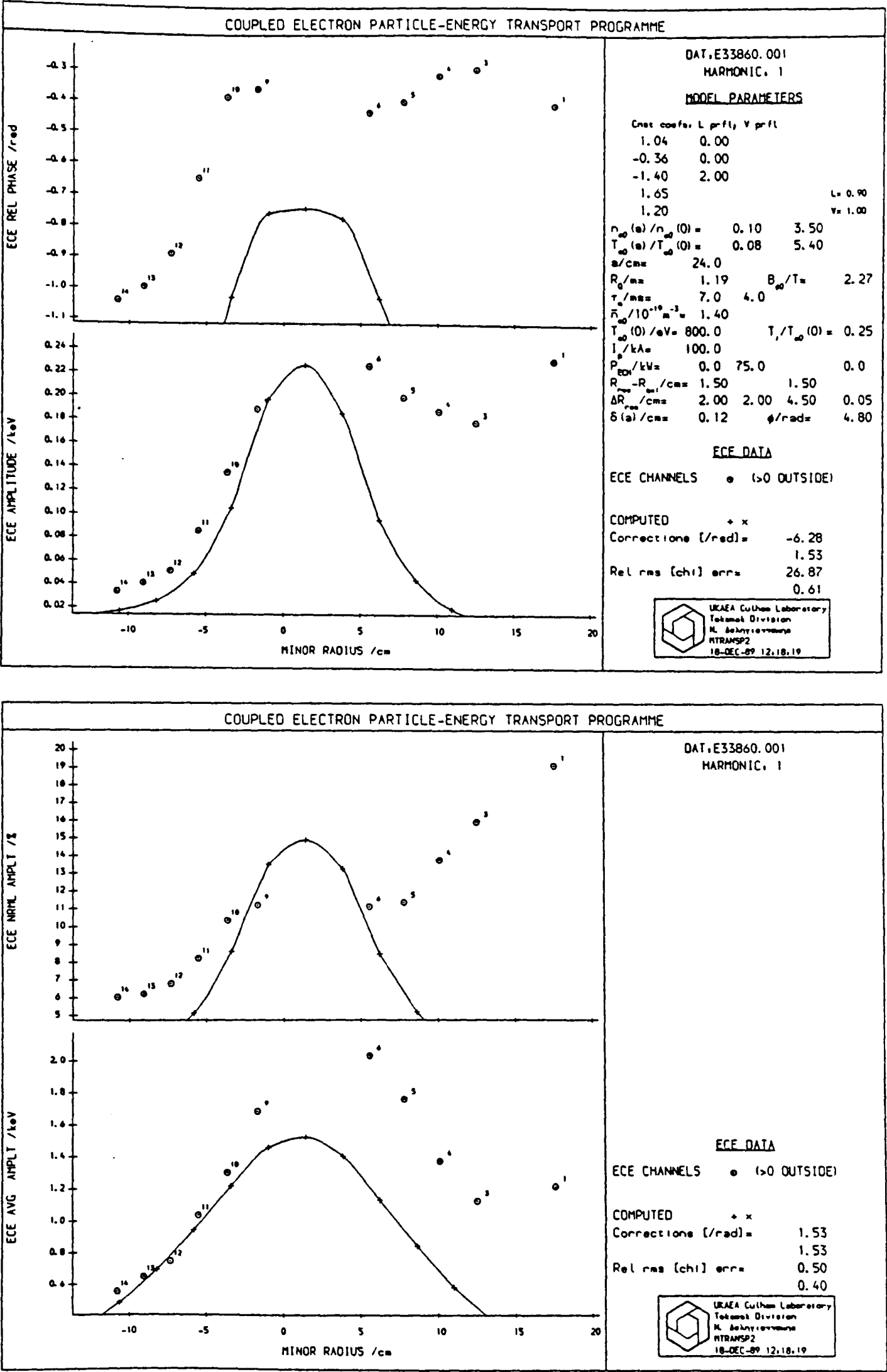


Figure 5.12: ECE modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), fitted with coupled transport model. Discharge of figure 5.11.

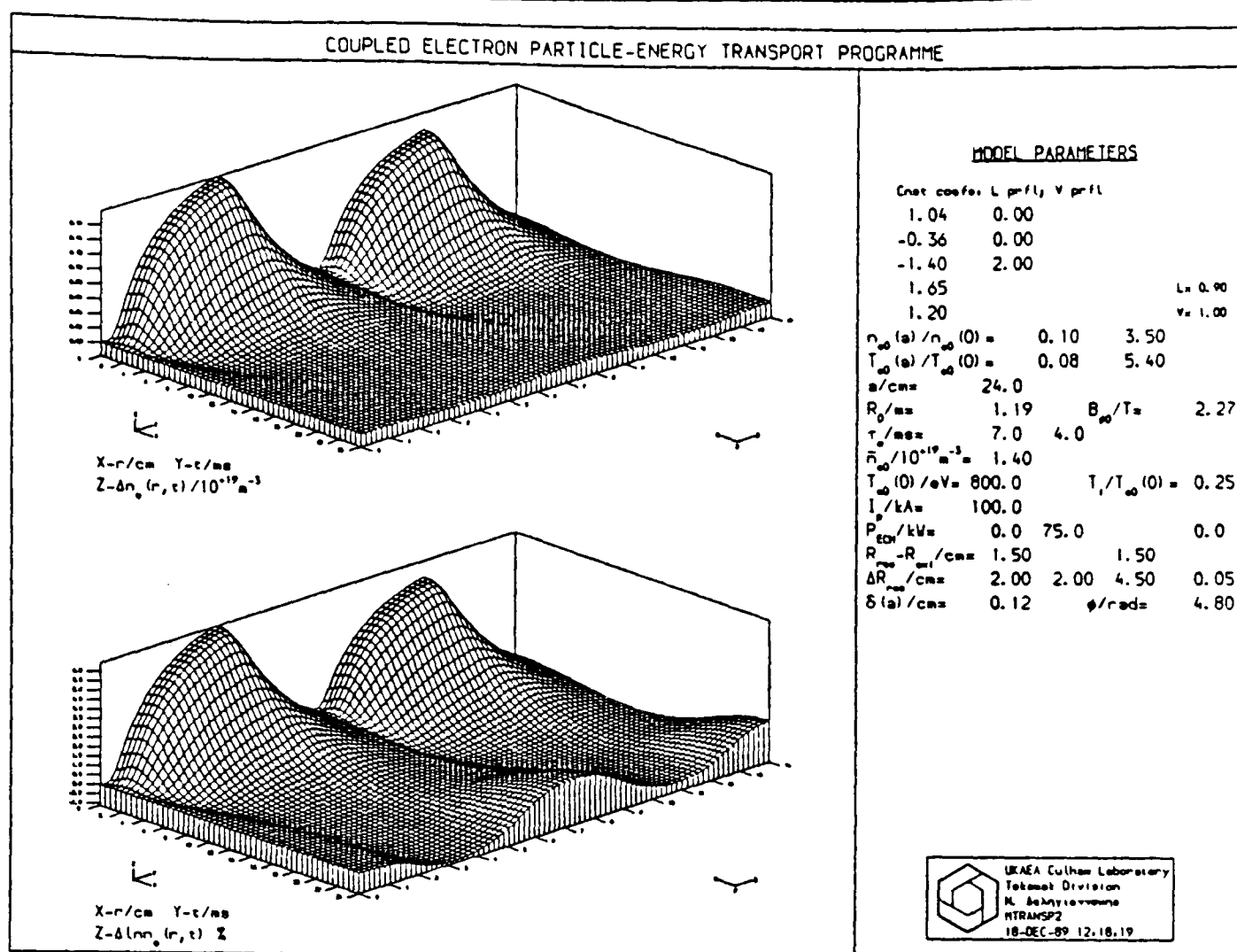


Figure 5.13: Coupled transport model: electron density perturbation $\bar{n}_e$ and percentage perturbation $\bar{n}_e/n_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.11–5.12.

The main weaknesses of this analysis are (a) the time-independent, effective particle source and (b) the parameter-independent diffusivities. In connection with the second point, most theories of transport predict dependences of the transport coefficients on the local plasma parameters from which the radial dependences, as obtained from analyses of power balance, arise. However, in the presence of a perturbation in some of the plasma parameters, these dependences will lead to corresponding perturbations in the transport coefficients and to a modification of the behaviour that would have been observed with constant coefficients. It has been suggested (Gentle, 1988) that these effects can account for the discrepancies between values of transport coefficients obtained from equilibrium measurements on the one hand, and from perturbation analyses on the other hand. In a linearized perturbation approximation, a perturbed transport coefficient will produce an effect by acting on the corresponding equilibrium gradient, thus leading to a modified effective transport coefficient. A further effect could be that due to the presence of modulated non-diffusive terms in the particle and/or energy fluxes (even when the

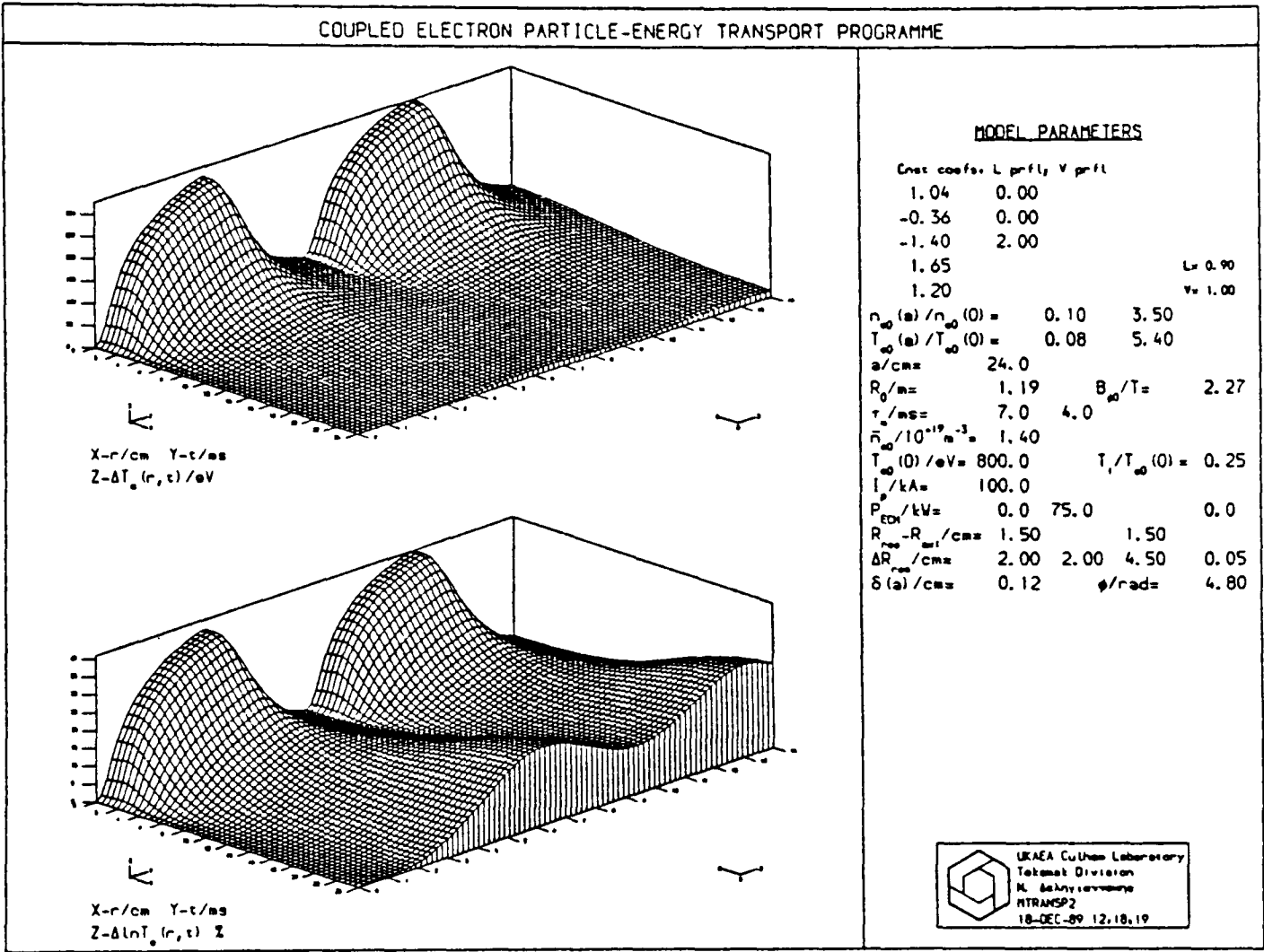


Figure 5.14: Coupled transport model: electron temperature perturbation $\tilde{T}_e$ and percentage perturbation $\tilde{T}_e/T_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.11–5.12.

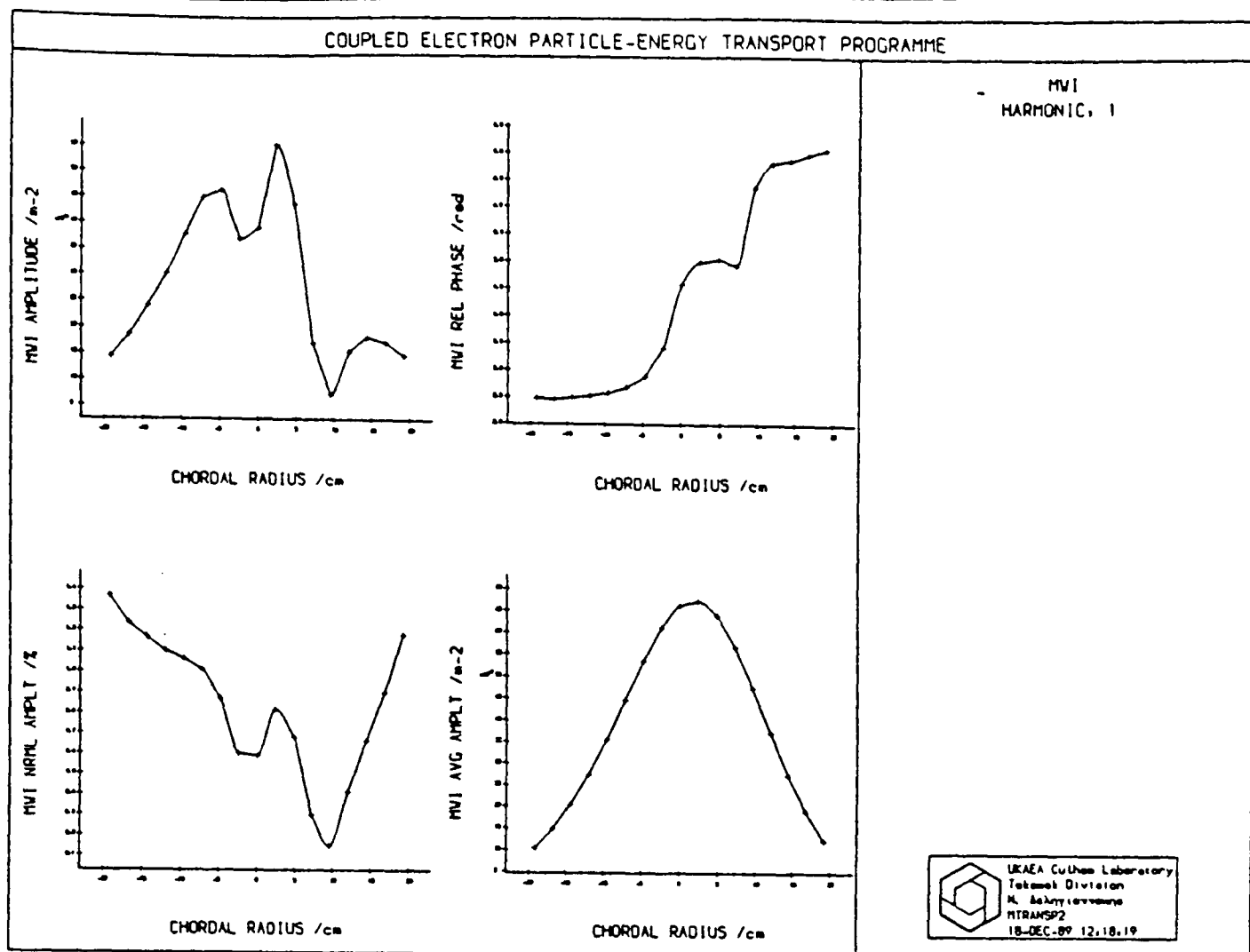


Figure 5.15: Coupled transport model: MWI (line-integrated density) modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), as simulated. Model of figures 5.11–5.12.

relevant velocities are constant, the non-diffusive fluxes will be modulated).

5.4.2 Assessment of some transport models

This subsection presents assessments of two transport models, using the analysis technique developed for the study of generalized coupled transport. In view of the somewhat insufficient experimental data, as well as of the uncertainties in some aspects of the model, it must be stressed that these results should be regarded as tentative: they are presented with every reservation, merely to illustrate the analysis technique that has been developed.

Dissipative-trapped-electron (DTE) mode This transport model (Horton, 1976) is based on the microturbulence associated with the dissipative-trapped-electron instability, a form of drift wave. It predicts coupled transport of particles and energy, with diffusivities that all have the same dependence on the density gradient (see section 5.2, table 5.1 and (5.31)). The other dependences predicted by this model were not investigated, and a radial profile was assumed

for the equilibrium diffusivities, which were scaled to produce transport in agreement with the power balance estimates; the dependence on the density gradient then led to a perturbation of the diffusivities. The results of the code, including a comparison with modulation data from a medium-density discharge with fundamental, off-axis heating, are presented in figures 5.16–5.21. The agreement with the modulation data is poor, and, although a sufficiently small density modulation is predicted, the density builds up near the centre during the modulation. It follows that this formulation of transport can be ruled out.

Woods ‘second-order’ classical model This transport model, proposed by Woods (1987), is based on a ‘second-order’ development of classical transport (see subsection 2.4.3). This model predicts a particle flux given in terms of a radial fluid velocity by

$$\Gamma/n = V_r = -\frac{E_\phi B_\theta}{B_\phi^2} + \frac{0.73m_e}{2e^4 B_\phi^2 n_e} \frac{1}{r^2} \left[r^2 p_e \tau_e \left(\frac{j_\phi}{n_e} \right)^{\prime 2} \right]', \quad (5.48)$$

and an electron thermal flux given by

$$q_e/(nT_e) = -\chi_e \frac{T_e'}{T_e} + \frac{5}{2} V_r, \quad (5.49)$$

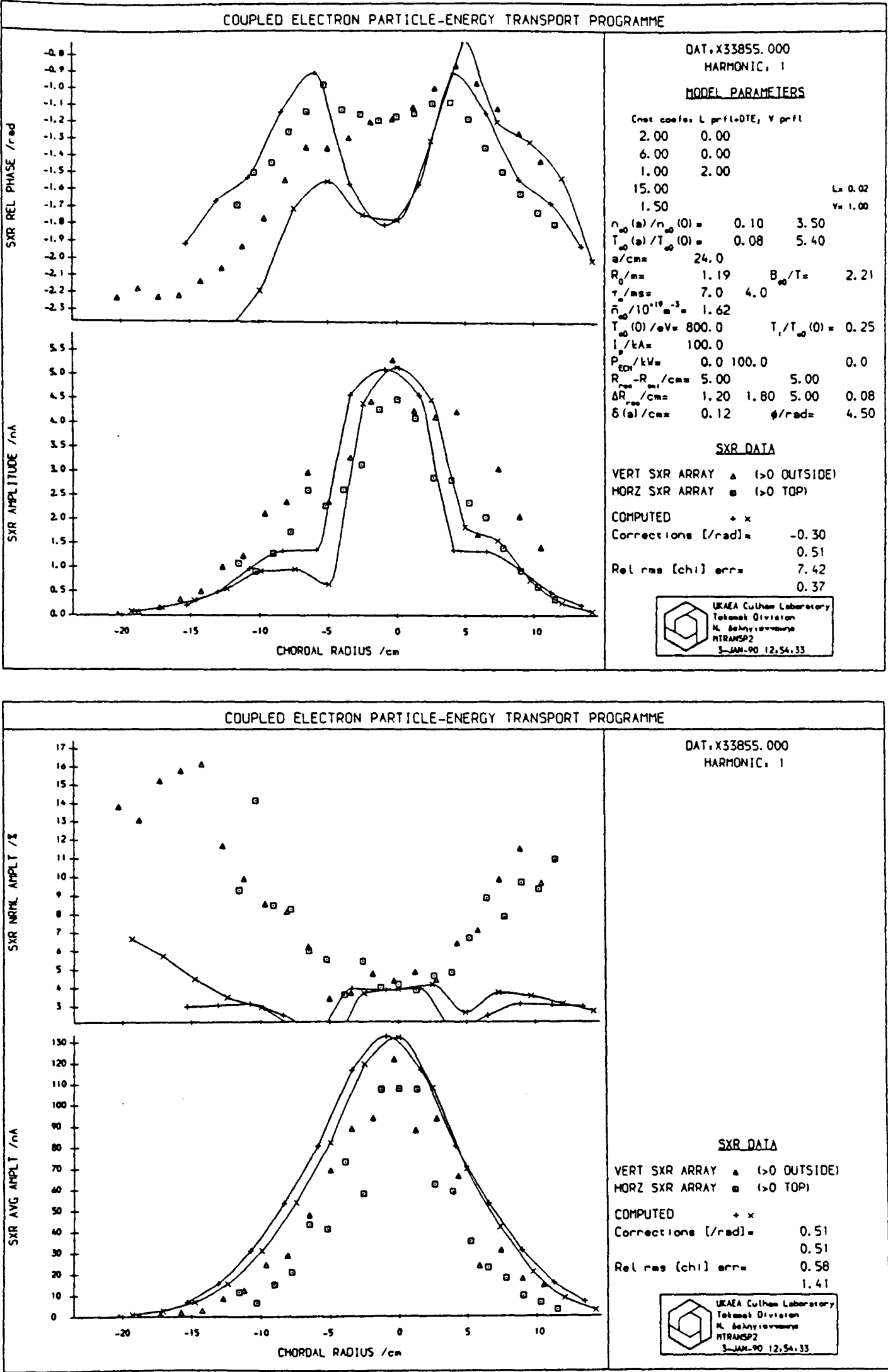
$$\text{with } \chi_e = \frac{5m_e^{1/2}}{2\sqrt{2}e^2} \frac{T_e^{1/2} q}{B_\phi^2 n_e} \left\{ \mu_0 j_\phi r \left(\frac{I(r)}{\pi r^2} - j_\phi \right) - n_e r^2 \left(\frac{p_t'}{r n_e} \right)' \right\}; \quad (5.50)$$

the electron energy balance equation has the extra source term

$$Q_{u\nabla p} = -V_r p_i'. \quad (5.51)$$

The predictions of this model for the electron thermal transport have been implemented in the transport code. The model gave a vanishing thermal diffusivity at the centre, and consequently a lower limit was imposed. Owing to numerical problems, the particle balance perturbation was not included, and an empirical particle diffusivity was used instead (the latter did not affect the results for the energy perturbation). The results of the code, including a comparison with modulation data from a medium-density discharge with fundamental, off-axis heating, are presented in figures 5.22–5.26. The predicted thermal transport had to be scaled by a factor of 3 to obtain agreement with the power balance estimates; this increased transport then led to some agreement with the modulated data, and in particular with the phase gradients. Some

agreement with the SXR data has been obtained, but the ECE data have indicated that the predicted thermal transport is too low near the centre. Despite the complex dependence of this model on the various profiles, it has transpired that the transport parameters (the fluid velocity and the thermal diffusivity) were not strongly sensitive to changes in the profiles, within plausible bounds.



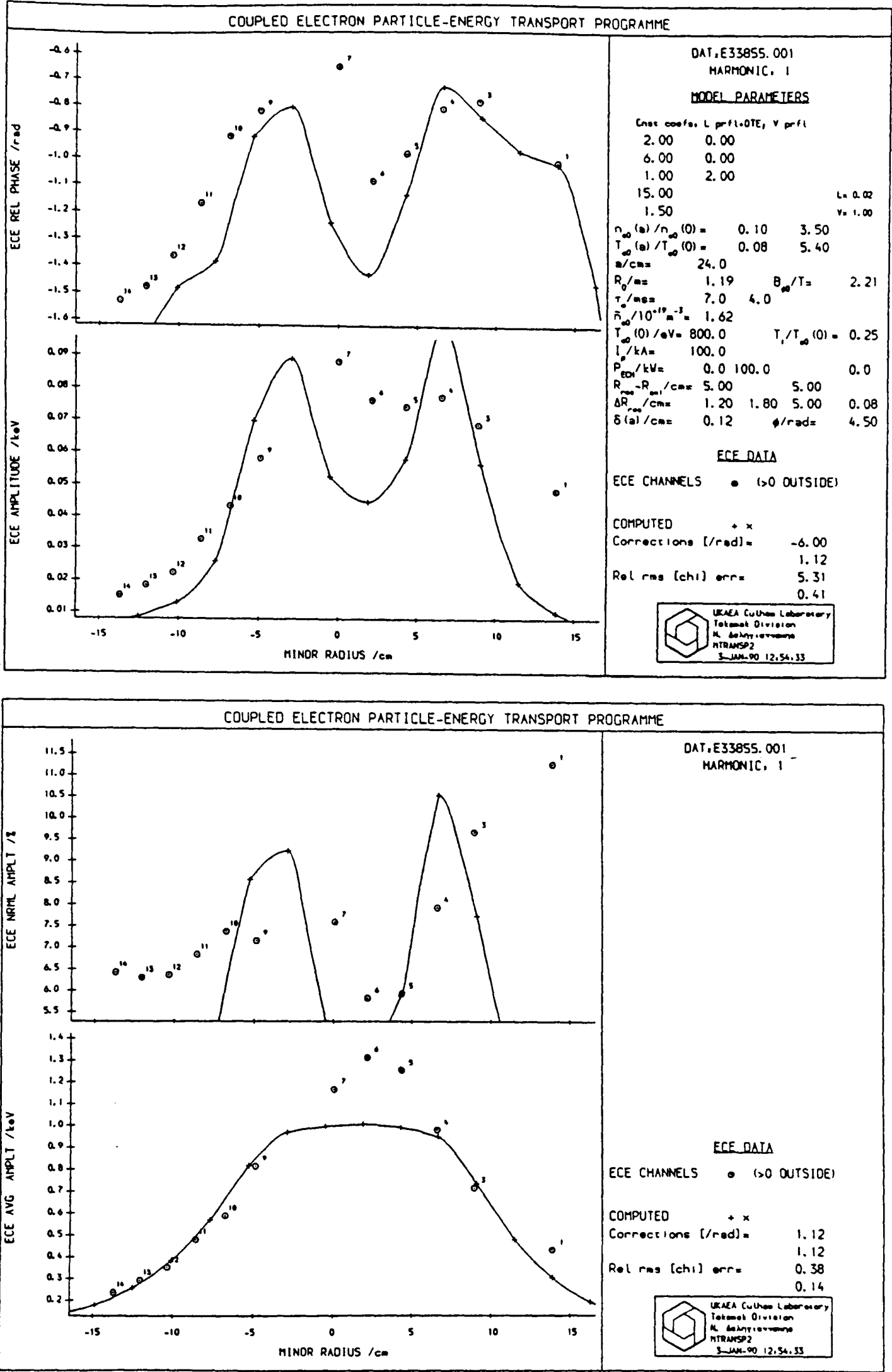


Figure 5.17: ECE modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), compared with DTE transport model. Discharge of figure 5.16.

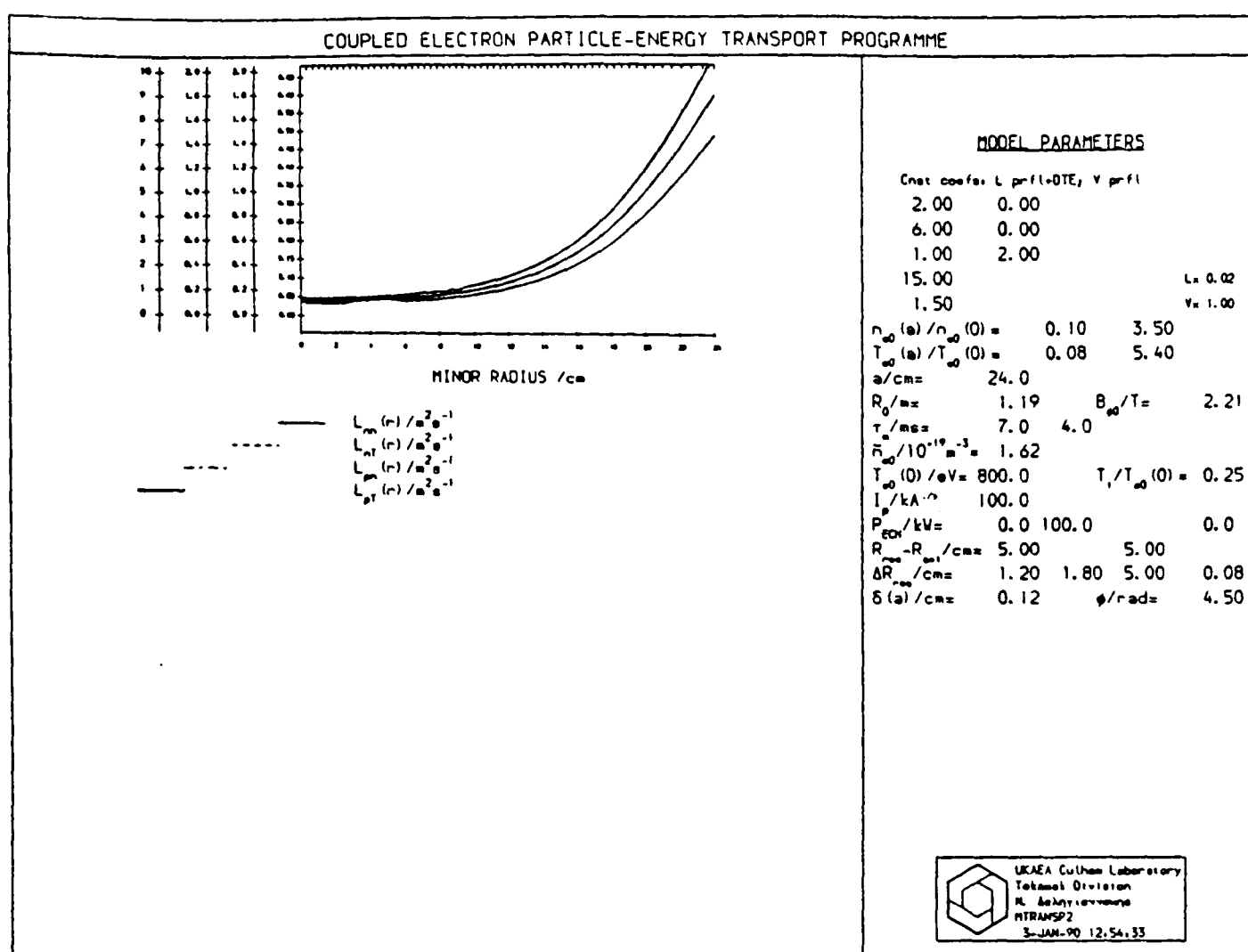


Figure 5.18: DTE transport model: profile of diffusivity matrix L , showing extrema during modulation. Model of figures 5.16–5.17.

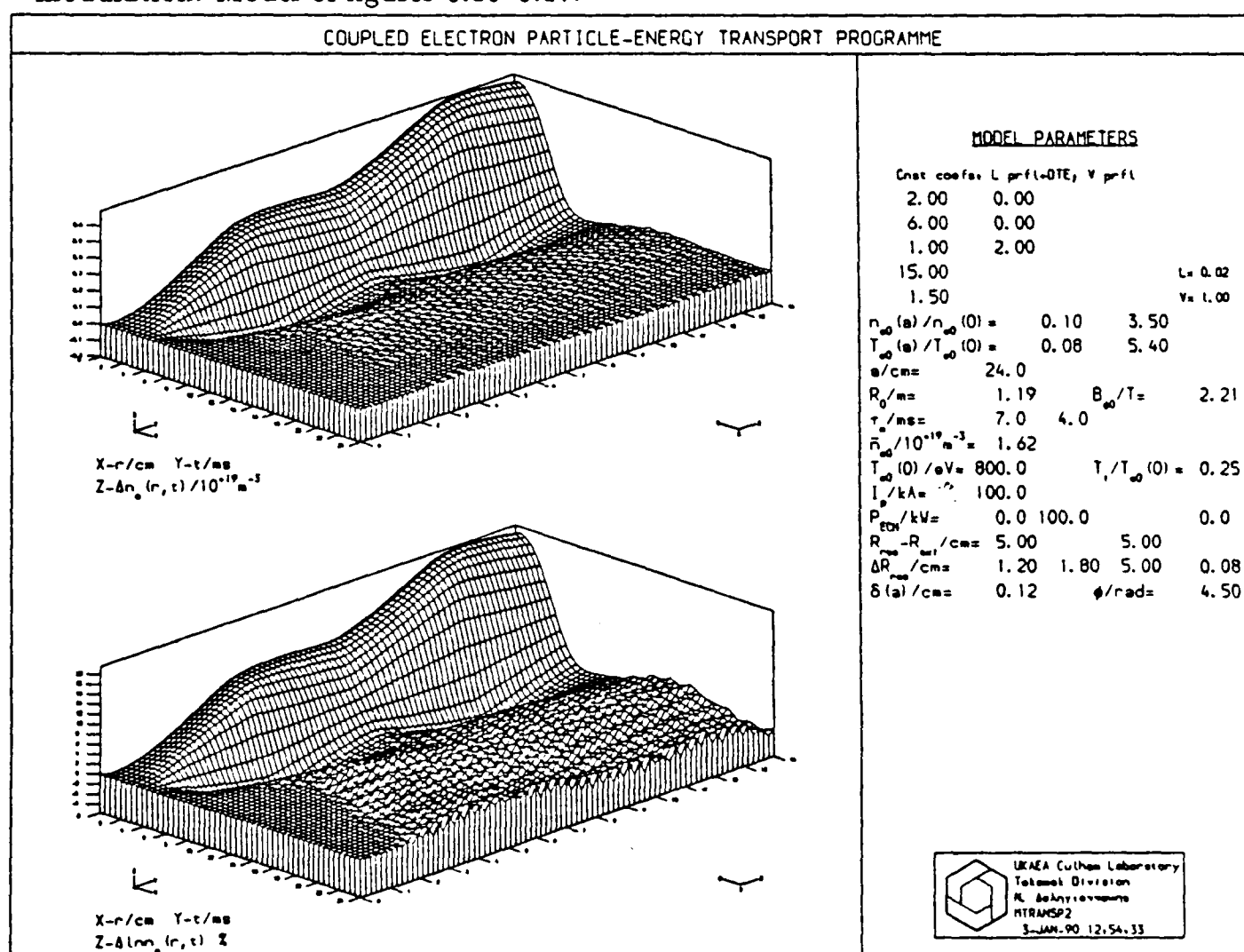


Figure 5.19: DTE transport model: electron density perturbation $\tilde{n}_e$ and percentage perturbation $\tilde{n}_e/n_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.16–5.17.

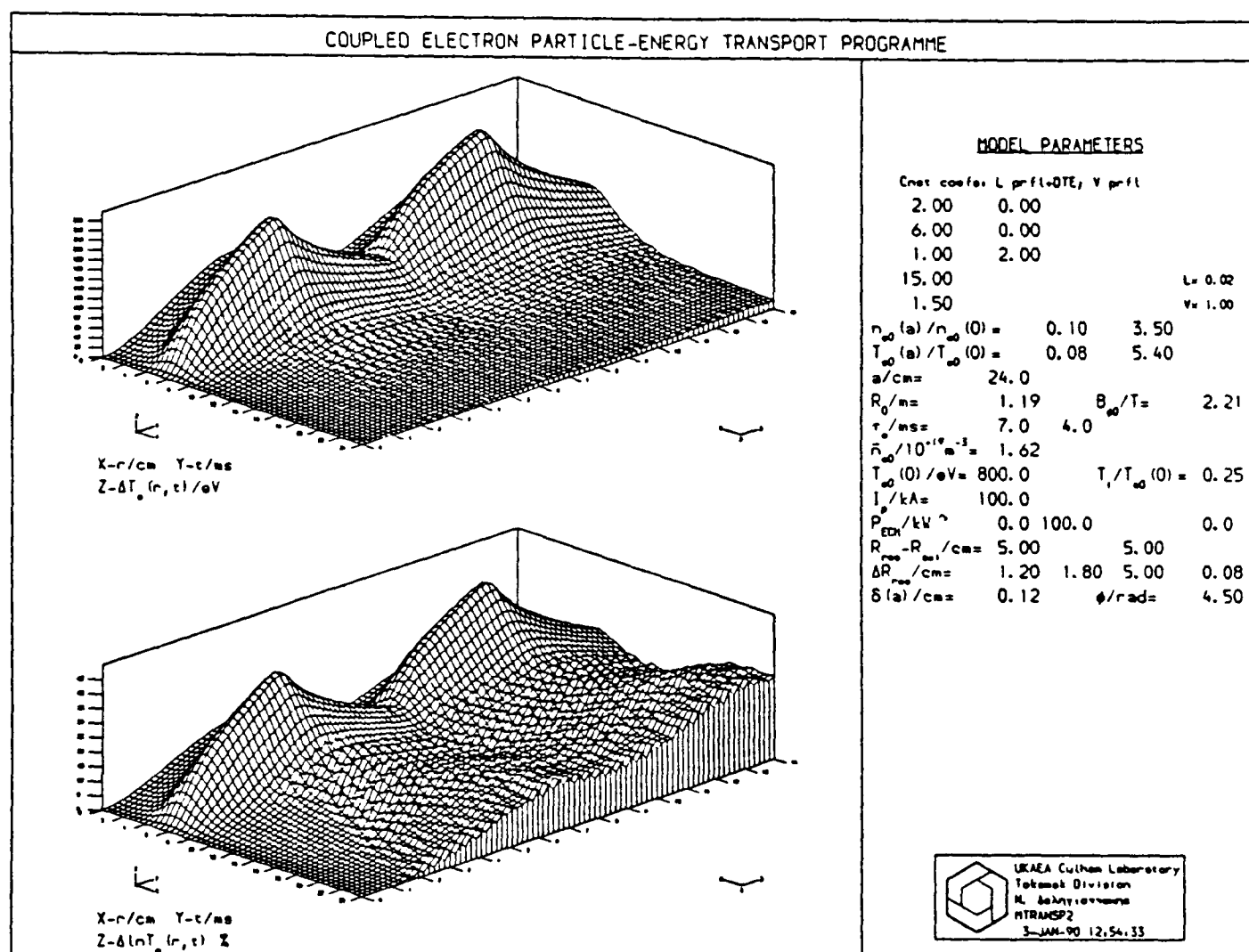


Figure 5.20: DTE transport model: electron temperature perturbation $\tilde{T}_e$ and percentage perturbation $\tilde{T}_e/T_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.16–5.17.

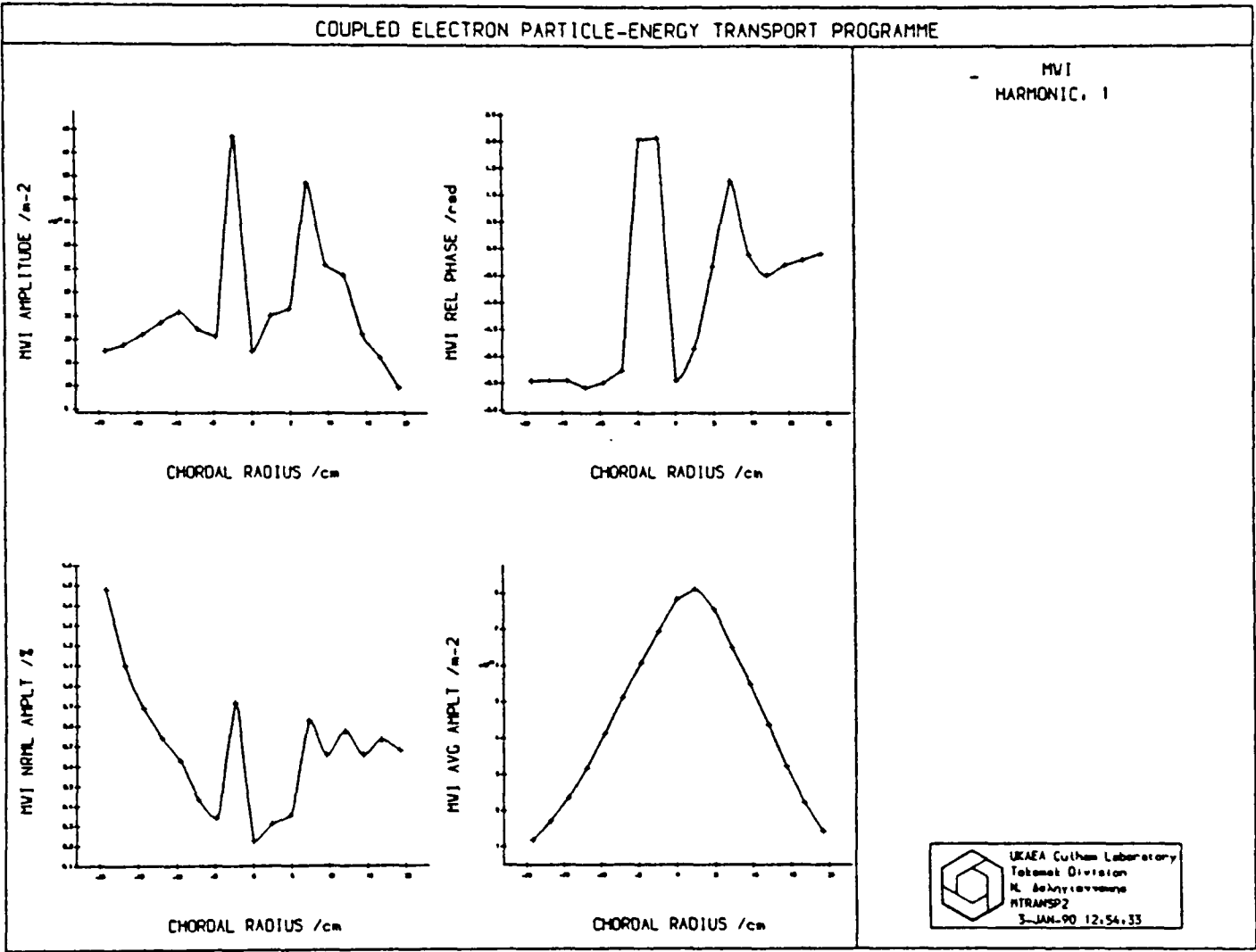


Figure 5.21: DTE transport model: MWI (line-integrated density) modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), as simulated. Model of figures 5.16–5.17.

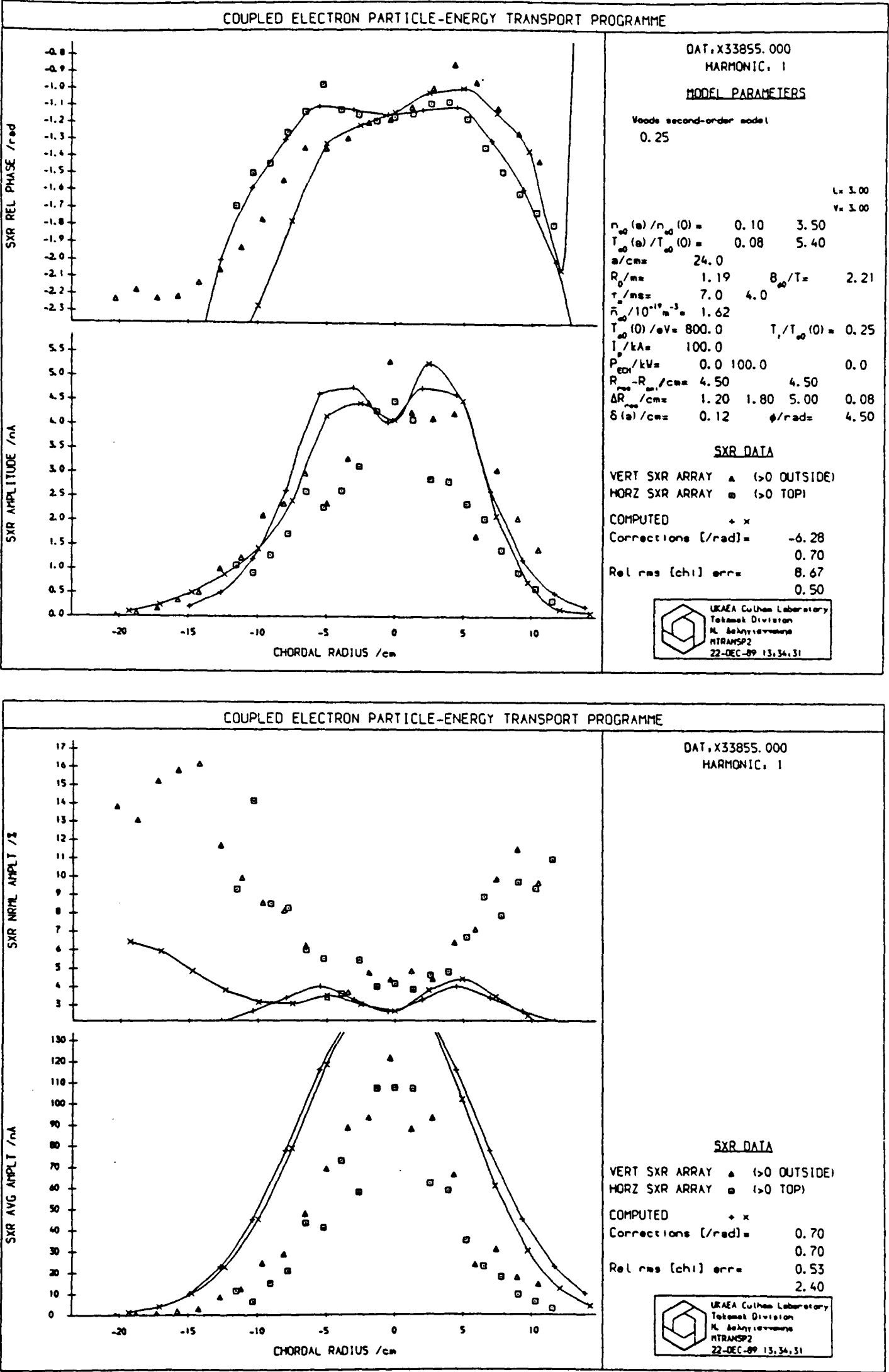
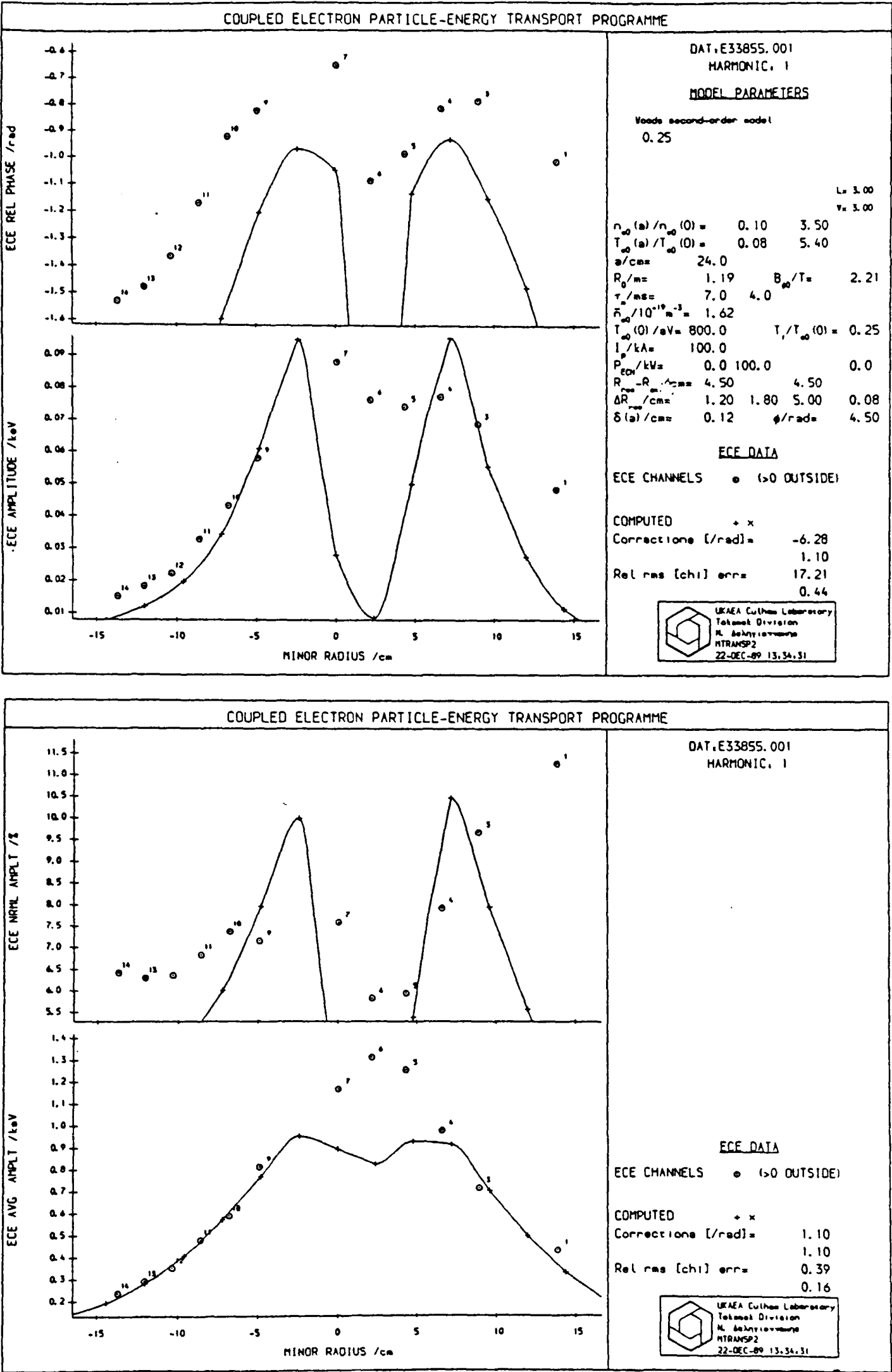


Figure 5.22: SXR modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), compared with Woods classical transport model. Discharge of figure 5.2.



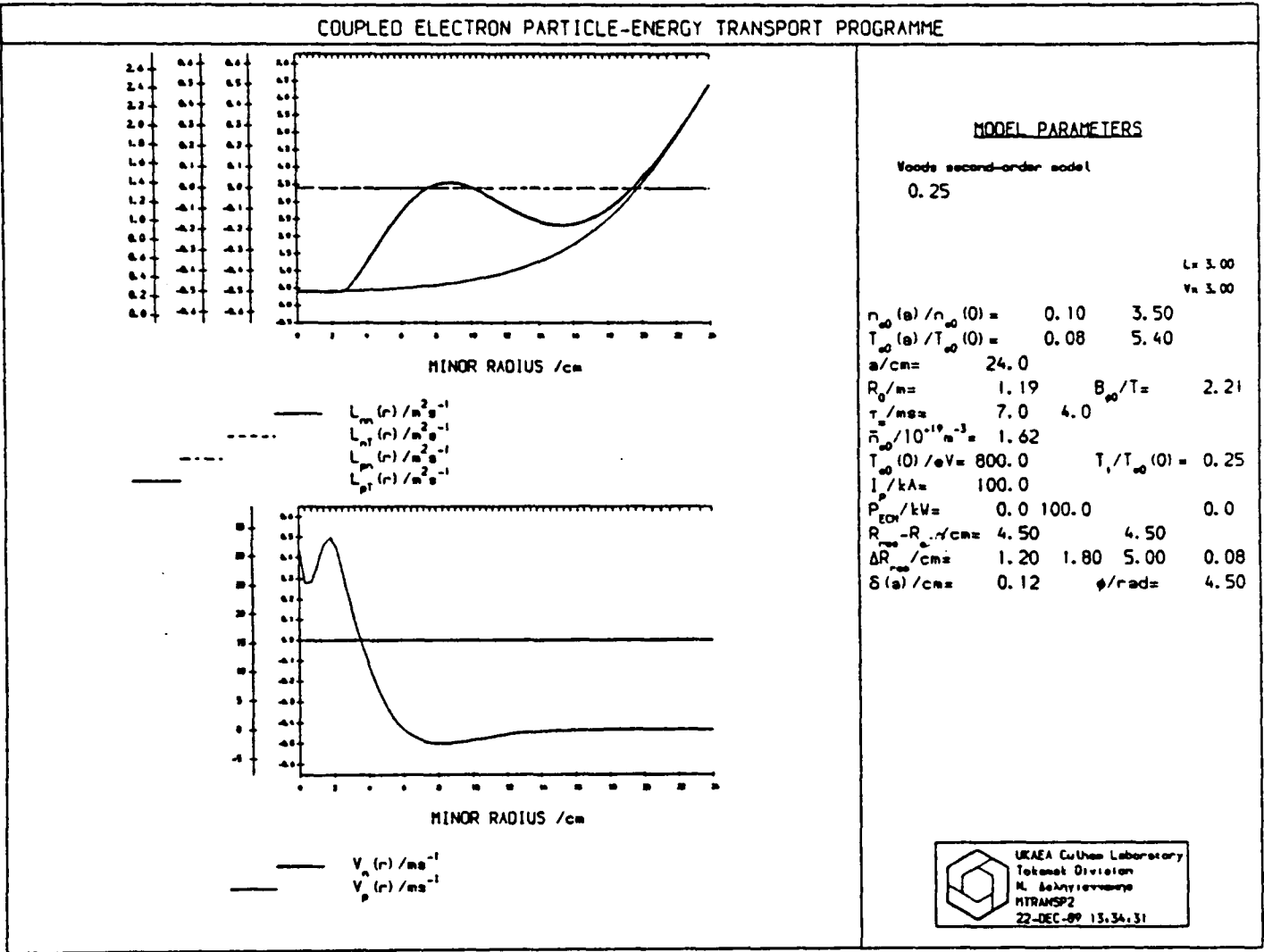


Figure 5.24: Woods classical transport model: profiles of particle diffusivity D (L_{nn}), thermal diffusivity χ_e (L_{pT}) and convection velocity $\frac{5}{2}V_r$; an empirical particle diffusivity has been used, and a lower limit has been imposed on the thermal diffusivity. Model of figures 5.22–5.23.

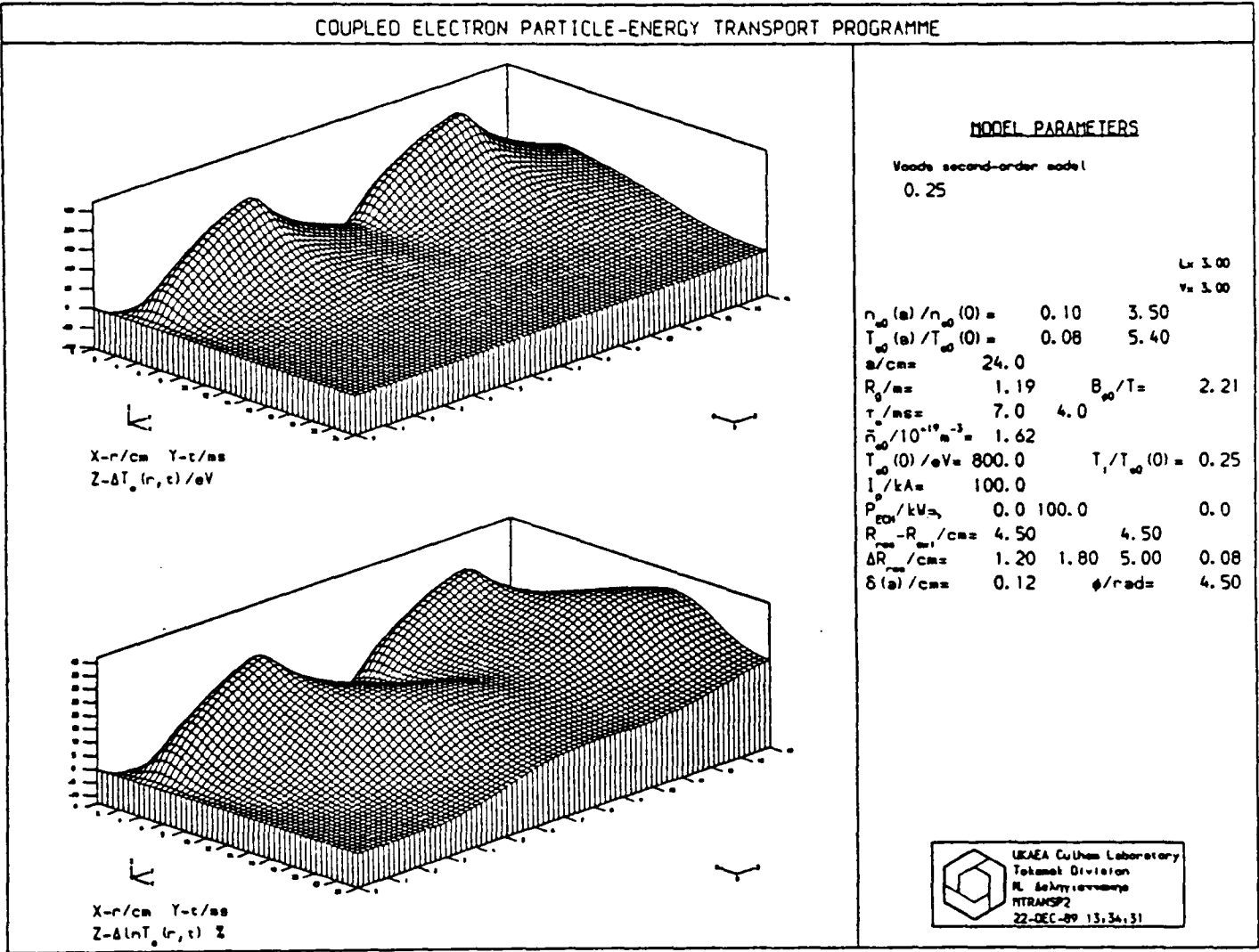


Figure 5.25: Woods classical transport model: electron temperature perturbation $\tilde{T}_e$ and percentage perturbation $\tilde{T}_e/T_{e0}$, plotted as function of minor radius r and time t . Model of figures 5.22–5.23.

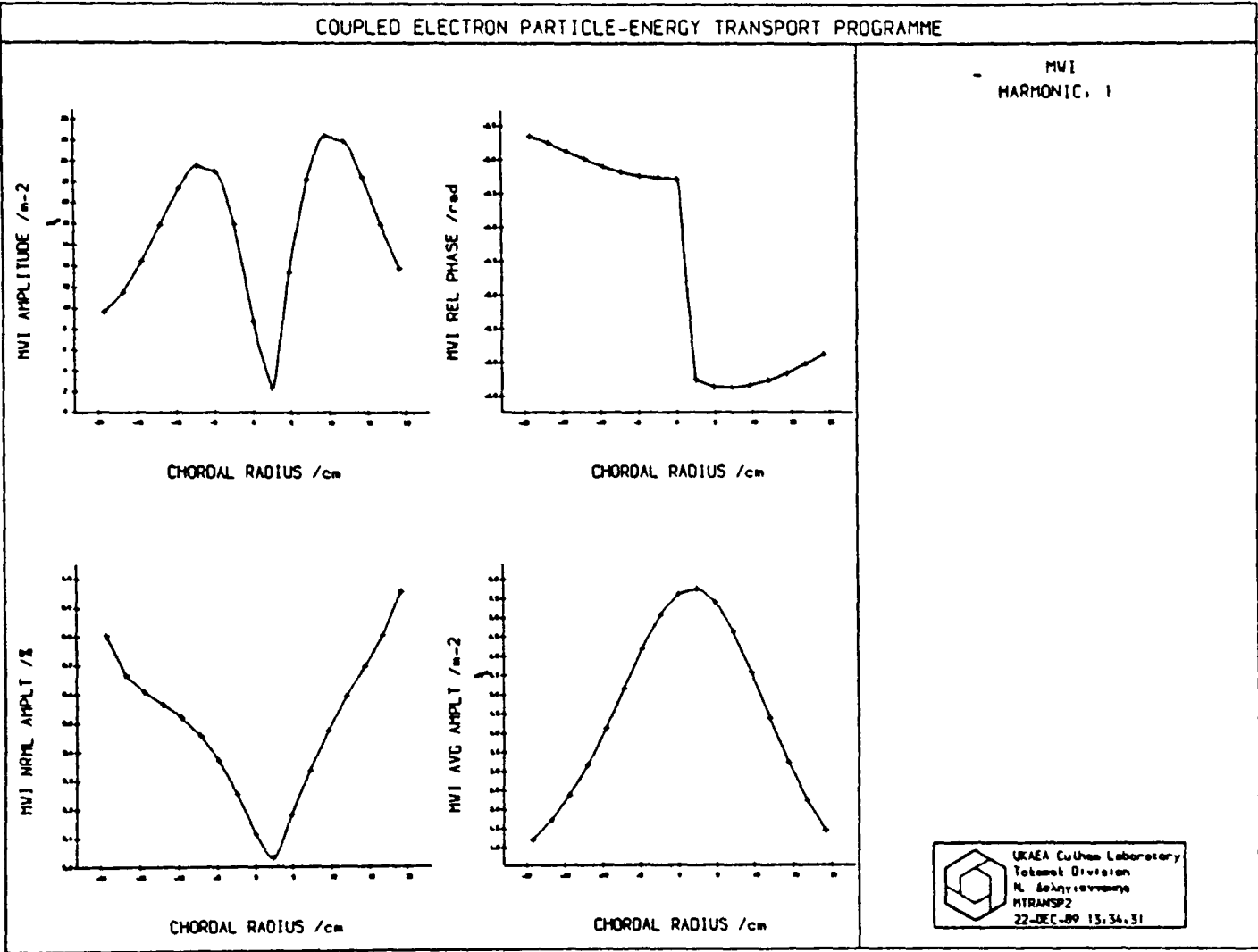


Figure 5.26: Woods classical transport model: MWI (line-integrated density) modulation data (relative phase, absolute/relative modulation amplitude and average amplitude), as simulated. In this case, the modulation arises only from the plasma motion, as the particle balance perturbation was not considered. Model of figures 5.22–5.23.

Chapter 6

Concluding Comments

6.1 Summary

Experiments This thesis has dealt with the experimental and theoretical aspects of a dynamical technique for the study and measurement of tokamak thermal transport, based on the propagation in the plasma of a thermal wave, which is excited by the modulation of a localized heating input. The experimental technique and associated measurements have been examined. The experiments, carried out on the medium-sized DITE tokamak, used modulation of electron cyclotron resonance heating (ECRH), with wave-launching from the high-field side. This scheme of plasma heating is generally characterized by a strongly localized and readily controlled absorption of the wave power, this feature being of particular importance to the modulation technique; however, the launching antenna used for the present experiments on DITE had a relatively broad radiation pattern which led to correspondingly broad power deposition profiles. Only the electron component of the plasma is directly heated, and the thermal transport on this tokamak is predominantly through the electron channel. The modulation experiments that were carried out concentrated on the scanning in minor radius of the ECRH absorption zone and on the variation of the plasma density; both helium and hydrogen/deuterium were used as working gases. The thermal wave due to the modulated ECRH was measured with spatial and temporal resolution using the soft X-ray (SXR) and electron cyclotron emission (ECE) diagnostics. On the DITE tokamak, the SXR system provided better spatial resolution and

presented fewer instrumentation problems than the ECE system; the former therefore provided the main experimental results, and the characteristics of SXR emission and detection have been studied in some detail. In general, however, the technique of ECE detection should be preferable to that of SXR detection, as it affords one with a more direct measurement of the local electron temperature, at least in the absence of a superthermal electron population. SXR measurements, on the other hand, introduce several complicating factors, namely the dependence of the local SXR emissivity on the electron density, and on the concentration levels and transport processes of the impurity ions, and also the chordal integration of the local emissivity. The density variation was measured by the microwave interferometer (MWI) diagnostic, although this was limited by instrumentation problems. The data were examined and analysed in the frequency domain, using fast Fourier transform techniques.

Interpretation: diffusive thermal transport The interpretation of the data that were obtained from the modulation experiments is the main issue that has been addressed in the thesis. The analysed data from both SXR and ECE diagnostics readily showed a behaviour that was indicative of diffusive thermal transport. The asymmetric data from the vertically viewing detectors of the SXR diagnostic were interpreted as being due to modulated horizontal shifts in the plasma. The diffusive nature of thermal transport was confirmed by data from various electric and magnetic edge diagnostics. A low-level modulation of the line-averaged density was also observed, in addition to the larger one of the electron temperature. In a few atypical cases, however, the observed behaviour was qualitatively different, and the possible causes of this effect have been investigated; this “anomalous” behaviour was associated with the locking of the sawtooth oscillation with the modulation.

Two transport models and associated computer codes have been developed and applied to the detailed investigation of the data. The first model is based on the perturbation of the underlying, diffusive thermal transport; it has been extensively applied to the experimental data and its results have been found to be in good agreement with the experimental data. Notwithstanding some uncertainties concerning the application of this model to the DITE experimental results, it has been possible to use it both to measure the electron thermal diffusivity and to estimate the

position (in minor radius) and width of the ECRH absorption zone. The width of the heated zone has emerged as an important aspect of the model: ray tracing calculations have been used to assess the ECRH absorption, and the relatively broad deposition profiles predicted have led to better fits, and to lower estimates of thermal transport, at the higher densities. In an important special case, a perturbation applied at a particularly large distance from the centre was found to propagate inwards with an exceptionally low diffusivity. The values for the effective electron thermal diffusivity were in good agreement with those obtained from the alternative techniques of power balance analysis and sawtooth heat pulse propagation analysis, which have been implemented for the purposes of this comparison; it is noteworthy that such agreement has not been universally obtained in similar experiments. The values obtained for helium and hydrogen/deuterium plasmas were similar.

Interpretation: generalized coupled transport The second model is a more complex one, which is based on equations of coupled particle and energy balance, its main purpose being to shed light upon the causes and effects of the observed density modulation. For the purposes of using this model, realistic assessments have been carried out of the energy sources and of the flux surface shifts, and a generalized description of the particle and energy fluxes has been formulated. Bearing in mind the considerable difficulties and uncertainties associated with the various aspects of this model, both experimental and theoretical, some qualitative agreement of the model with experimental data has been obtained, so that the possibility of coupled transport has not been discounted. The main conclusion of this exercise is that, under some circumstances, the density modulation arising from this coupling can appear to be small and at the same time affect the propagation of the thermal wave. Tentative assessments of two transport models have been presented, namely the DTE mode and the 'second-order' classical model proposed by Woods. It has been demonstrated that the sensitivity of the modulation procedure, with its associated interpretation, should be useful in tests of transport models. The advantage of using both SXR and ECE data to measure the temperature modulation has become apparent.

Final conclusions An existing technique for the dynamical measurement of transport has been developed and applied extensively to the measurement and interpretation of electron energy and particle transport. The experimental aspects have been examined. Some important aspects of the modelling have been addressed and included in the analysis. A simple analysis of thermal diffusion has satisfactorily accounted for the experimental data and has yielded reliable estimates of thermal transport, in agreement with those deduced from alternative analyses of power balance and sawtooth heat pulse propagation. An analysis of generalized coupled transport has been developed, in order further to study the nature of transport, and in particular the question of coupling between particle and energy transport. Whilst it has not been possible conclusively to establish the nature of the underlying transport mechanisms, it has been shown that some form of coupling, as predicted by most theories of transport, is indeed possible. It has been demonstrated that modulated heating, possibly combined with equilibrium measurements and alternative perturbations of the equilibrium, affords one with a powerful technique for the assessment of proposed transport models, subject, of course, to some developments in the experimental measurements and theoretical descriptions.

6.2 Recommendations for future experiments and interpretation

It has been seen that dynamical techniques constitute a promising means of unravelling the nature of transport in tokamaks. Several improvements should be possible on the experimental technique as used in this work. The most important of these are (a) a more strongly localized deposition of the ECRH power, and (b) more reliable measurements of the temperature and density modulation. It has been seen how both of these improvements can potentially enhance the reliability of the results obtained from the modelling of the modulated data. The basic measurements of electron temperature and density can be supplemented by further measurements of ion temperature, total radiation and spectroscopic and magnetic measurements. For the purposes of a complete transport analysis, it will be important to obtain reliable measurements of the steady-state, as well as of the modulated, components of the various parameters; the density

and temperature gradients, for both electron and ion components, are of particular importance in connection with transport models. It has been seen that ECE data should provide a better assessment of the electron temperature than SXR data. Nevertheless, SXR measurements can yield reliable information and should be retained as a potentially useful diagnostic technique, especially as an alternative measurement of the temperature in tests of complex transport models. In this connection, it will be important (a) to apply tomographic reconstruction techniques, which can yield information on the poloidal structure of the thermal wave propagation; and (b) to establish a reliable description of the impurities present in the plasma and of their confinement, so that the SXR emissivity and its dependence on temperature and density can be assessed with confidence —this development should further provide one with a technique for an alternative assessment of the density modulation. The experiments should be extended to other tokamak devices, with different plasma parameters. It is important to use different methods of perturbation on similar discharges: such methods include modulation of heating power, modulation of gas feed, pellet injection and the naturally excited sawtooth pulse. As these perturbations are qualitatively different, their combined results should be particularly useful in assessing models of coupled transport. Perturbation experiments can be carried out under different conditions; in particular the variation of the steady-state input power (Ohmic, ECRH or other) should modify the profiles and the transport, and can possibly reveal some of the dependences of the particle and thermal fluxes on the plasma parameters, when such fluxes are measured by both equilibrium and dynamical methods. Furthermore, the point of application of the perturbation can be varied for some of these experiments, thus providing information about both the outward and inward propagation. Such experiments, carried out systematically, will generate a large volume of data, which will be of considerable importance in assessing the validity or otherwise of proposed transport models.

There is ample scope for refinement of the interpretation methods. The modelling of energy and particle balance should be extended to include (a) a two-fluid formulation, ion thermal transport being generally important, (b) the transport of impurity ions, as this affects the radiation loss and the SXR emission, (c) MHD effects, in particular the evolution on a slow

time-scale of the current density distribution, this playing an important role in Ohmic heating and in many transport models, (d) more complete descriptions of the sources, in particular the particle source and the radiation loss, and (e) more complete boundary conditions. Transport codes with these features are, of course, available, but versions should be developed specifically for the study of perturbations on relatively short time-scales, comparable to, or shorter than, the energy confinement time. A very important aspect of a transport code is the numerical solution of the coupled partial differential equations; in view of the complex dependences on many variables, which are introduced by most transport models, the development of accurate solutions can be a formidable task; the computation required is substantial, and this will have to be performed by more powerful computers than the one used for the present work. For a given description of transport, covering the fluxes and sources of particles and energy, solutions will have to be obtained for different perturbations of the equilibrium solutions; these can be initial conditions (impulse-like perturbations) or additional sources (modulated perturbations). A further important development that is needed in the implementation of complex transport models is an improvement in the optimization of the parameters of the model: as many parameters as possible, and in particular the profiles of density and temperature, should be assessed experimentally, and the remaining ones should be optimized. Comparisons of the model with the corresponding experimental results can be carried out in the time or frequency domain, or both. The critical-temperature-gradient model (Rebut, Lallia and Watkins, 1988) and the ion-temperature-gradient model (Lee and Diamond, 1986) have lately received considerable attention and should be carefully assessed using these techniques. The 'second-order' classical model (Woods, 1987) is also of considerable interest, as it is based on an entirely different approach.

It is hoped that this work will encourage, and will be an aid in, further investigations, both experimental and theoretical, in the field of dynamical measurement of tokamak transport.

Table of Symbols

a	limiter minor radius (m)
A	ion atomic mass number; area (m^2)
B	magnetic field (T)
c	speed of light ($2.998 \times 10^8 \text{ms}^{-1}$)
c_A	Alfvén speed (ms^{-1})
c_s	sound speed (ms^{-1})
ce_x	(subscript) charge exchange loss
C	carbon
$d\tau$	volume element (m^3)
D	particle (or unspecified) diffusivity (m^2s^{-1})
D	deuterium
DFT	discrete Fourier transform
e	electronic charge ($1.602 \times 10^{-19}\text{C}$); particle charge (C); (subscript) electron
E	electric field (Vm^{-1})
$\mathcal{E}$	photon energy, fusion energy per particle (eV)
ech	(subscript) electron cyclotron heating
exc	(subscript) electron-ion exchange
ECE	electron cyclotron emission
ECRH	electron cyclotron resonance heating
f	distribution function (m^{-6}s^3); relative concentration fraction
FFT	fast Fourier transform
$\hbar$	Planck's constant ($1.055 \times 10^{-34}\text{Js}$)
H	hydrogen
He	helium
i	(subscript) ion
I	current (A); emission intensity (W)
I_p	total plasma current (A)
j	current density (Am^{-2})
k	wavenumber (m^{-1})
l	length, distance (m)
l_i	internal inductance
ℓ	harmonic number
L	general diffusivity in steady-state flux (m^2s^{-1}); profile scale length (m)

Li	lithium
m	particle mass (kg); poloidal mode number
m_e	electron mass ($9.109 \times 10^{-31} \text{kg}$)
M	general diffusivity in perturbed flux ($\text{m}^2 \text{s}^{-1}$)
MHD	magnetohydrodynamic
MWI	microwave interferometer
n	number density (m^{-3}); toroidal mode number
$n_{\parallel}, n_{\perp}$	refractive index components
n	neutron
O	oxygen
p	pressure (Nm^{-2}); chordal radius of line of sight (m)
P	power (input or loss) (W)
q	thermal flux (conduction, convection) (Wm^{-2}); safety factor
Q	power density (source or sink) (Wm^{-3})
r	minor radius variable (m); (subscript) radial direction
R	major radius variable (m)
R_0	magnetic axis (or torus) major radius (m)
rad	(subscript) radiation loss
s	distance along line of sight (m); (subscript) species designation
S	particle source ($\text{s}^{-1} \text{m}^{-3}$)
SXR	soft X-ray
t	temporal variable (s)
T	temperature (including Boltzmann constant k_B) ($1 \text{eV} = 1.602 \times 10^{-19} \text{J} = k_B 1.160 \times 10^4 \text{K}$)
T	tritium
u	mean (fluid) velocity (ms^{-1})
v	particle velocity (ms^{-1})
v_d	drift velocity (ms^{-1})
V	velocity in non-diffusive particle or thermal flux (ms^{-1})
V_l	loop voltage (V)
W	particle energy, energy content (J)
X	SXR power density (Wm^{-3})
z	vertical co-ordinate (m); (subscript) vertical direction
Z	ion charge number
Z_{eff}	effective ion charge
α	SXR modulation index (logarithmic SXR/temperature derivative); complex spatial frequency (m^{-1})

β	beta value (fraction of plasma to magnetic field pressure)
β_p	poloidal beta value, using area-averaged pressure
β_{p0}	poloidal beta value, using local pressure
γ	growth/decay rate (s^{-1}); enhancement factor (ratio of recombination radiation to bremsstrahlung); relativistic mass factor; adiabatic constant
Γ	particle flux ($s^{-1}m^{-2}$)
δ	flux surface shift (m)
ε	inverse aspect ratio
ε_0	vacuum permittivity ($8.854 \times 10^{-12} Fm^{-1}$)
η	profile parameter (logarithmic temperature/density radial derivative); electrical resistivity (Ωm); conversion efficiency
θ	poloidal angle variable; (subscript) poloidal direction
κ	heat conduction coefficient ($m^{-1}s^{-1}$)
λ	wavelength (m); mean free path (m)
λ_D	Debye screening length (m)
Λ	Shafranov lambda; argument of Coulomb logarithm
μ_0	vacuum permeability ($4\pi \times 10^{-7} Hm^{-1}$)
ν	collision frequency (Hz)
ν_{dt}	detrapping frequency (Hz)
ν^*	normalized detrapping frequency (detrapping/bounce frequency)
ρ	Larmor radius (m); mass density (kgm^{-3})
σ	reaction cross-section (m^2) electrical conductivity ($\Omega^{-1}m^{-1}$)
τ	collision time (s); optical depth
τ_E	energy confinement time (s)
τ_E^*	energy confinement time, corrected for radiation loss (s)
τ_p	particle confinement time (s)
ϕ	toroidal angle variable; (subscript) toroidal direction; angle of normal to line of sight; phase angle; electric potential (V)
χ	thermal diffusivity (m^2s^{-1})
χ_μ	ionization potential (eV)
ψ	poloidal magnetic flux (flux surface variable) (Wb)
ω	wave frequency (s^{-1})

ω_b	bounce frequency (s^{-1})
ω_c	cyclotron frequency (s^{-1})
ω_p	plasma frequency (s^{-1})
ω_*	diamagnetic drift frequency (s^{-1})
Ω	solid angle; (subscript) Ohmic
$\parallel$	(subscript) parallel to equilibrium magnetic field
$\perp$	(subscript) perpendicular to equilibrium magnetic field
$\sim$	fluctuation, perturbation or modulation component (units of variable)
$\wedge$	temporal Fourier transform (units of variable $\times s$); discrete Fourier transform (units of variable)
$'$	gradient (units of variable $\times m^{-1}$)
$\cdot$	rate of change (units of variable $\times s^{-1}$)

Bibliography

The publications marked by ¶ have been (co-)written by the author of this thesis.

- Alikaev, V.V., et al., *Gyrotrons for Electron Cyclotron Plasma Heating in Large Tokamaks*, Proceedings of 1st Joint Varenna-Grenoble International Symposium on Heating in Toroidal Plasmas, CEC/Pergamon (1978), vol. II, 339.
- Arfken, G., *Mathematical Methods for Physicists*, 2nd ed., Academic Press, New York (1970).
- Artsimovich, L.A., *Tokamak Devices*, Nuclear Fusion, **12** (1972), 215.
- Ashraf, M., private communication (1988).
- Ashraf, M., et al., *Thermal Wave Investigation of Anomalous Transport by Modulated ECRH and Sawteeth*, Proceedings of 12th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, Paper IAEA-CN-50/A-V-2-1 (1988(a)) ¶.
- Ashraf, M., et al., *Inside Launch Electron Cyclotron Heating and Current Drive on DITE*, Proceedings of 12th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, Paper IAEA-CN-50/E-I-3 (1988(b)) ¶.
- Bayes, D.V., et al., *The Engineering Design and Construction of DITE*, Proceedings of 8th Symposium on Fusion Technology, Noordwijkerhout, CEC (1974(a)), 127.
- Bayes, D.V., et al., *The Vertical and Radial Field Systems of the DITE Tokamak and their Controlled Power Supplies*, Proceedings of 8th Symposium on Fusion Technology, Noordwijkerhout, CEC (1974(b)), 161.
- Bayes, D.V., et al., *DITE Status and Upgrade Proposals*, Proceedings of 11th Symposium on Fusion Technology, Oxford, CEC/Pergamon (1980), vol. I, 669.
- Bell, D., et al., *Design and Operation of the DITE Power Supplies and Control Systems*, Proceedings of 9th Symposium on Fusion Technology, Garmisch-Partenkirchen, CEC/Pergamon (1976), 685.
- Bell, J.D., et al., *Measurement of χ_e in ISX-B Beam-Heated Discharges by Heat Pulse Propagation*, Nuclear Fusion, **24** (1984), 997.
- Bendat, J.S., Piersol, A.G., *Engineering Applications of Correlation and Spectral Analysis*, Wiley, New York (1980).
- Bishop, C.M., Connor, J.W., private communication (1988); *Heat Pulse Propagation and the Role of Density Perturbations*, Culham plasma physics note, PPN89/13 (1989).
- Bishop, C.M., et al., *Density Modulation During Modulated ECRH in DITE*, Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics, Venice, **13B** (1989), 1131 ¶.
- Bornatici, M., *Theory of Electron Cyclotron Absorption of Magnetized Plasmas*, Plasma Physics, **24** (1982), 629.
- Bornatici, M., Cano, R., De Barbieri, O., Engelmann, F., *Electron Cyclotron Emission and Absorption In Fusion Plasmas*, Nuclear Fusion, **23** (1983), 1153.

- Bracewell, R.N., *The Fourier Transform and its Applications*, McGraw-Hill, New York (1965); 2nd ed. (1986).
- Braginskii, S.I., *Transport Processes in a Plasma*, in *Reviews of Plasma Physics* (ed. Leontovich, M.A.), Consultants Bureau, New York, vol. I (1965), 205.
- Burcham, W.E., *Nuclear Physics: an Introduction*, 2nd ed., Longman, London (1973).
- Cairns, R.A., *Plasma Physics*, Blackie, Glasgow (1985).
- Callen, J.D., Christiansen, J.P., Cordey, J.G., Thomas, P.R., Thomsen, K., *Modelling of Temperature Profiles and Transport Scaling in Auxiliary Heated Tokamaks*, *Nuclear Fusion*, **27** (1987), 1857.
- Callen, J.D., Jahns, G.L., *Experimental Measurement of Electron Heat Diffusivity in a Tokamak*, *Physical Review Letters*, **38** (1977), 491.
- Campbell, D.J., et al., *Stabilization of Sawteeth with Additional Heating in the JET Tokamak*, *Physical Review Letters*, **60** (1988), 2148.
- Christiansen, J.P., Callen, J.D., Cordey, J.G., Thomsen, K., *Analysis of Local Heat Flux in JET*, *Nuclear Fusion*, **28** (1988), 817.
- Coppi, B., *Non-classical Transport and the "Principle of Profile Consistency"*, *Comments on Plasma Physics and Controlled Fusion*, **5** (1980), 261.
- Coppi, B., Mazzucato, E., *Transport of Electron Thermal Energy in High Temperature Plasmas*, *Physics Letters*, **71A** (1979), 337.
- Cormack, A.M., *Representation of a Function by its Line Integrals, with some Radiological Applications*, pt I, *Journal of Applied Physics*, **34** (1963), 2722; pt II, *ibid.*, **35** (1964), 2908.
- Costley, A.E., *Electron Cyclotron Emission from Magnetically Confined Plasmas: I-Diagnostic Potential*, *Proceedings of Course on Diagnostics for Fusion Reactor Conditions*, Varenna, CEC (1982), vol. I, 129; *II-Instrumentation and Measurements*, *ibid.* (1982), vol. I, 149.
- Cox, M., Robinson, D.C., *Review of Transport Investigations with ECRH*, *Plasma Physics and Controlled Fusion*, **30** (1988), 1431.
- Deliyanakis, N., *A Study of Radial Thermal Transport on the DITE Tokamak by Modulated ECRH and a Synchronous Detection Technique*, MSc Thesis, University of Oxford (1987); *Studies of Radial Thermal Transport on the DITE Tokamak by Modulated ECRH and a Synchronous Detection Technique*, *Proceedings of 6th Joint Workshop on ECE and ECRH*, Oxford, CLM-ECR (1987), 247 ¶.
- Dellis, A.N., et al., *Experience with High-Field-Side ECRH Antennae on DITE*, *Proceedings of 6th Joint Workshop on ECE and ECRH*, Oxford, CLM-ECR (1987), 347.
- Düchs, D.F., Post, D.E., Rutherford, P.H., *A Computer Model of Radial Transport in Tokamaks*, *Nuclear Fusion*, **17** (1977), 565.
- Dudok de Wit, Th., Duval, B.P., Joye, B., Lister, J.B., Moret, J.-M., *Investigation of the Energy Transport Mechanism in the TCA Tokamak by Studying the Plasma Dynamical Response*, *Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics*, Venice, **13B**

- (1989(a)), 59.
- Dudok de Wit, Th., Joye, B., Lister, J.B., Moret, J.-M., *Effects of the Alfvén Wave Heating in the TCA Tokamak Deduced from the the Plasma Dynamical Response*, Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics, Venice, **13B** (1989(b)), 1195.
- Equipe TFR, *Tokamak Plasma Diagnostics*, Nuclear Fusion, **18** (1978), 647.
- Fielding, P.J., *The Theory of Radio Frequency Plasma Heating*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 477.
- Fielding, S.J., *The Temperature Sensitivity of X-Ray Diode Array Measurements in ECRH Modulation Experiments*, Culham tokamak group note, TGN(88)10 (1988).
- Fredrickson E.D., et al., *Heat Pulse Propagation Studies in TFTR*, Nuclear Fusion, **26** (1986), 849.
- Furth, H.P., *Tokamak Research*, Nuclear Fusion, **15** (1975), 487.
- Furth, H.P., *The Tokamak*, in *Fusion* (ed. Teller, E.), Academic Press, New York (1981), vol. I, pt A, 123.
- Galeev, A.A., Sagdeev, R.Z., *Theory of Neoclassical Diffusion*, in *Reviews of Plasma Physics* (ed. Leontovich, M.A.), Consultants Bureau, New York, vol. VII (1979), 257.
- Gentle, K.W., *Dependence of Heat Pulse Propagation on Transport Mechanisms: Consequences of Non-constant Transport Coefficients*, Physics of Fluids, **31** (1988), 1105.
- Gentle, K.W., Richards, B., Waelbroek, F., *A Measurement of Hydrogen Ion Transport Parameters in Tokamak Discharges*, Plasma Physics and Controlled Fusion, **29** (1987), 1077.
- Gentle, K.W., et al., *Effect of ECRH on Particle Transport in the TEXT Tokamak*, Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics, Venice, **13B** (1989), 159.
- Goedheer, W.J., *Inference of Electron Heat Conductivity from the Propagation of a Temperature Perturbation in the Outer Confinement Region of a Tokamak*, Nuclear Fusion, **26** (1986), 1043.
- Goldston, R.J., *Energy Confinement Scaling in Tokamaks: Some Implications of Recent Experiments with Ohmic and Strong Auxiliary Heating*, Plasma Physics and Controlled Fusion, **26** (1984), 87.
- Gondhalekar, A., et al., *Simultaneous Measurements of Electron Thermal and Particle Transport in JET*, Plasma Physics and Controlled Fusion, **31** (1989), 805.
- Granetz, R.S., Camacho, J.F., *Soft-X-Ray Tomography on Alcator-C*, Nuclear Fusion, **25** (1985), 727.
- Greenwald, M., et al., *Energy Confinement of High-Density Pellet-Fuelled Plasmas in the Alcator-C Tokamak*, Physical Review Letters, **53** (1984), 352.
- Hallock, G.A., et al., *Space Potential Distribution in the ISX-B Tokamak*, Physical Review Letters, **56** (1986), 1248.
- Hartfuss, H.J., Maassberg, H., Tutter, M., et al., *Evaluation of the Local Heat Conductivity Coefficient by Power-Modulated Electron Cyclotron Heating in the Wendelstein VII-A Stellarator*,

- Nuclear Fusion, **26** (1986), 678.
- Hemsworth, R.S., *Neutral Injection Plasma Heating*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 455.
- Hinton, F.L., Hazeltine, R.D., *Theory of Plasma Transport in Toroidal Confinement Systems*, Reviews of Modern Physics, **48** (1976), 239.
- Hirshman, S.P., Sigmar, D.J., *Neoclassical Transport of Impurities in Tokamak Plasmas*, Nuclear Fusion, **21** (1981), 1079.
- Horton, W., *Drift Wave Stability of Inverted Gradient Profiles in Tokamaks*, Physics of Fluids, **19** (1976), 711.
- Hossain, M., Kress, M., Hu, P.N., Blank, A.A., Grad, H., *Anomalous Transport and the Coupling of Plasma Diffusion and Heat Flow*, Physical Review Letters, **58** (1987), 487.
- Hugill, J., *Nuclear Fusion Research*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 33.
- Hugill, J., *Transport in Tokamaks — a Review of Experiment*, Nuclear Fusion, **23** (1983), 331.
- Hugill, J., *[Tokamak] Experiments*, in *Tokamaks* (Wesson, J.A.), Oxford University Press (1987), 265.
- Hugill, J., *Calculations of χ_e from Incomplete Data*, Culham tokamak group note, TGN(88)27 (1988).
- Hugill, J., et al., *Transport Studies in the DITE Tokamak with Modulated ECRH*, Proceedings of 15th EPS Conference on Controlled Fusion and Plasma Physics, Dubrovnik, **12B** (1988), 807 ¶.
- Hutchinson, I.H., *Principles of Plasma Diagnostics*, Cambridge University Press (1987).
- Jahns, G.L., et al., *Internal Disruptions in Tokamaks*, Nuclear Fusion, **18** (1978), 609.
- Jahns, G.L., Wong, S.K., Prater, R., Lin, S.H., Ejima, S., *Measurement of Thermal Transport by Synchronous Detection of Modulated Electron Cyclotron Heating in the Doublet-III Tokamak*, Nuclear Fusion, **26** (1986), 226.
- Johnson, P.C., et al., *Results from the DITE Pump-Limiter Experiment*, Journal of Nuclear Materials, **162–164** (1989), 686.
- Jory, H.R., *Development of Gyrotron Power Sources in the Millimetre Wavelength Range*, Proceedings of 1st Joint Varenna-Grenoble International Symposium on Heating in Toroidal Plasmas, CEC/Pergamon (1978), vol. II, 351.
- Joye, B., Lister, J.B., Moret, J.-M., *Effects of the Alfvén Wave Heating on the TCA Plasma Studied by the Dynamical Response*, Proceedings of 15th EPS Conference on Controlled Fusion and Plasma Physics, Dubrovnik, **12B** (1988), 924.
- Joye, B., Lister, J.B., Moret, J.-M., Pochelon, A., Simm, C.W., *Dynamical Plasma Response to Additional Heating*, CRPP Lausanne report, LRP 330/87 (1987); Plasma Physics and Controlled Fusion, **30** (1988), 743.
- Kadomtsev, B.B., Pogutse O.P., *Trapped Particles in Toroidal Magnetic Systems*, Nuclear Fu-

- sion, **11** (1971), 67.
- Kaye, S.M., *A Review of Energy Confinement and Local Transport Scaling Results in Neutral-Beam-Heated Tokamaks*, *Physics of Fluids*, **28** (1985), 2327.
- Kaye, S.M., Goldston, R.J., *Global Energy Confinement Scaling for Neutral-Beam-Heated Tokamaks*, *Nuclear Fusion*, **25** (1985), 65.
- Kim, S.K., et al., *Coupling of Particle and Heat Transport Measured via Sawtooth Induced Pulse Propagation*, *Proceedings of 12th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, Paper IAEA-CN-50/A-V-2-2* (1988).
- Laing, E.W., *Transport Theory*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 155.
- Lawson, J.D., *Some Criteria for a Power Producing Thermonuclear Reactor*, *Proceedings of the Physical Society*, **B70** (1957), 6.
- Lee, G.S., Diamond, P.H., *Theory of Ion-Temperature-Gradient-Driven Turbulence in Tokamaks*, *Physics of Fluids*, **29** (1986), 3291.
- Liewer, P.C., *Measurements of Microturbulence in Tokamaks and Comparisons with Theories of Turbulence and Anomalous Transport*, *Nuclear Fusion*, **25** (1985), 543.
- Liewer, P.C., McChesney, J.M., Zweben, S.J., Gould, R.W., *Temperature Fluctuations and Heat Transport in the Edge Regions of a Tokamak*, *Physics of Fluids*, **29** (1986), 309.
- Lopes Cardozo, N.J., Tubbing, B.J.D., Tibone, F., Taroni, A., *Heat Pulse Propagation: Diffusive Models Checked Against Full Transport Calculations*, *Nuclear Fusion*, **28** (1988), 1173.
- McCracken, G.M., *Plasma-Surface Interactions*, in *Tokamaks* (Wesson, J.A.), Oxford University Press (1987), 201.
- McWhirter, R.W.P., *Plasma Radiation*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 235.
- Mantica, P., Pitts, R.A., Vayakis, G., *Response of the DITE Boundary to Modulated ECRH*, Culham tokamak group note, TGN(88)30 (1988).
- Matthews, G.F., et al., *The Impurity Control Limiter Experiment*, *Nuclear Fusion*, **28** (1988), 2209.
- Merezhkin, V.G., Mukhovatov, V.S., *Electron Thermal Conductivity and Diffusion in a Tokamak*, *JETP (Journal of Experimental and Theoretical Physics) Letters*, **33** (1981), 446.
- Millar, W., private communication (1988).
- Moeller, C.P., et al., *A High-Field Launch for Electron Cyclotron Heating using an External Window*, *Proceedings of 6th Joint Workshop on ECE and ECRH*, Oxford, CLM-ECR (1987), 355.
- Mukhovatov, V.S., Shafranov, V.D., *Plasma Equilibrium in a Tokamak*, *Nuclear Fusion*, **11** (1971), 605.
- NAG (Numerical Algorithms Group), *The NAG Manual, Mark 11*, 6 vols, Oxford (1984); *The NAG Manual, Mark 13*, 7 vols, Oxford (1988).

- Nuclear Fusion Anniversary Issue (many authors), *Nuclear Fusion*, **25** (1985), 1011–1194.
- O'Brien, M.R., private communication (1989).
- Oppenheim, A.V., Willsky, A.S., *Signals and Systems*, Prentice-Hall, New Jersey (1983).
- Orlinskij, D.V., Magyar, G., *Plasma Diagnostics on Large Tokamaks*, *Nuclear Fusion*, **28** (1988), 611.
- Paul, J.W.M., *The DITE Experiment*, *Nuclear Fusion*, **25** (1985), 1097.
- Pert, G.J., *Inertial Confinement*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 571.
- Post, D.E., et al., *Steady-State Radiative Cooling Rates for Low-Density, High-Temperature Plasmas*, *Atomic and Nuclear Data Tables*, **20** (1977), 397.
- Prater, R., *Recent Results on the Application of Electron Cyclotron Heating to Tokamaks*, General Atomics report, GA-A19788 (1989).
- Rebut, P.H., Lallia, P.P., Watkins, M.L., *The Critical Temperature Gradient Model of Plasma Transport: Applications to JET and Future Tokamaks*, Proceedings of 12th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, Paper IAEA-CN-50/D-IV-1 (1988).
- Riviere, A.C., *Radio Frequency Plasma Heating Experiments*, in *Plasma Physics and Nuclear Fusion Research* (ed. Gill, R.D.), Academic Press, London (1981), 501.
- Riviere, A.C., *A Review of ECRH Experiments*, *Plasma Physics and Controlled Fusion*, **28** (1986), 1263.
- Riviere, A.C., et al., *Results from High-Field-Side Launch ECRH Experiments on DITE*, Proceedings of 6th Joint Workshop on ECE and ECRH, Oxford, CLM-ECR (1987), 237.
- Riviere, A.C., et al., *Down-Shifted Electron Cyclotron Heating Experiments in a Near-Thermal Plasma*, Proceedings of 15th EPS Conference on Controlled Fusion and Plasma Physics, Dubrovnik, **12B** (1988), 811.
- Roberts, D.E., *Total Impurity Radiation Power Losses from Steady-State Tokamak Plasmas*, *Nuclear Fusion*, **21** (1981), 215.
- Robinson, D.C., *Confinement and Electrostatic and Electromagnetic Fluctuations*, Proceedings of International Workshop on Small Scale Turbulence and Anomalous Transport in Magnetized Plasmas, Cargèse (1986), 21.
- Robinson, D.C., *Electrostatic Turbulence and Anomalous Transport*, Proceedings of Cadarache Workshop on Electrostatic Turbulence, Cadarache (1989), 1.
- Romanelli, F., *Models of Plasma Transport Based on Microturbulence*, *Plasma Physics and Controlled Fusion*, **31** (1989), 1535.
- Romanelli, F., Tang, W.M., White, R.B., *Anomalous Thermal Confinement in Ohmically Heated Tokamaks*, *Nuclear Fusion*, **26** (1986), 1515.
- Ross, D.W., *On Standard Forms for Transport Equations and Fluxes*, Comments on Plasma

- Physics and Controlled Fusion, **12** (1988), 155.
- Sauthoff, N.R., von Goeler, S., *Techniques for the Reconstruction of Two-Dimensional Images from Projections*, IEEE Transactions on Plasma Science, **7** (1979), 141.
- Shafranov, V.D., *Plasma Equilibrium in a Magnetic Field*, in Reviews of Plasma Physics (ed. Leontovich, M.A.), Consultants Bureau, New York, vol. II (1966), 103.
- Simonetto, A., private communication (1988).
- Sing, D.C., et al., *Thermal Transport During Electron Cyclotron Heating in the TEXT Tokamak*, Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics, Venice, **13B** (1989), 155.
- Sips, A.C.C., et al., *Measurements of the Electron Particle Diffusion Coefficient with the JET Multichannel Reflectometer*, Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics, Venice, **13B** (1989), 99.
- Smith, M.J., *The Gyrotron*, Electronics and Power, **27** (1981), 389.
- Soler, M., Callen, J.D., *On Measuring the Electron Heat Diffusion Coefficient in a Tokamak from Sawtooth Oscillation Observations*, Nuclear Fusion, **19** (1979), 703.
- Start, D.F.H., et al., *Experimental and Theoretical Studies of Ion Cyclotron Heating on JET*, Proceedings of 12th IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, Paper IAEA-CN-50/E-II-3 (1988).
- Stix, T.H., *The Theory of Plasma Waves*, McGraw-Hill, New York (1962).
- Tang, W.M., *Microinstability Theory in Tokamaks*, Nuclear Fusion, **18** (1978), 1089.
- Tang, W.M., *Microinstability-Based Model for Anomalous Thermal Confinement in Tokamaks*, Nuclear Fusion, **26** (1986), 1605.
- Tibone, F., et al., *Predictions for ICRF Power Deposition in JET and Modulation Experiments During Sawtooth-Free Periods*, Proceedings of 15th EPS Conference on Controlled Fusion and Plasma Physics, Dubrovnik, **12B** (1988), 709.
- Tubbing, B.J.D., Lopes Cardozo, N.J., van der Wiel, M.J., *Tokamak Heat Transport — A Study of Heat Pulse Propagation in JET*, Nuclear Fusion, **27** (1987), 1843.
- Vasin, N.L., Vershkov, V.A., Zhuravlev, V.A., *Study of Plasma Diffusion in the T-10 Tokamak by Deuterium Puffing and by Periodic Modulation of the Deuterium Influx into the Plasma*, Soviet Journal of Plasma Physics, **10** (1984), 645.
- Vayakis, G., Mantica, P., Matthews, G.F., *Broadband Fluctuations and Particle Transport in the DITE Boundary Plasma*, Culham tokamak group note, TGN(88)29 (1988).
- von Goeler, S., *Soft X-Ray Measurements*, Proceedings of Course on Diagnostics for Fusion Experiments, Varenna, CEC/Pergamon (1978), 79.
- von Goeler, S., Stodiek, W., Sauthoff, N., *Studies of Internal Disruptions and $m=1$ Oscillations in Tokamak Discharges with Soft-X-Ray Techniques*, Physical Review Letters, **33** (1974), 1201.
- von Goeler, S., et al., *Thermal X-Ray Spectra and Impurities in the ST Tokamak*, Nuclear Fusion,

15 (1975), 301.

Wagner, F., et al., *Regime of Improved Confinement and High Beta in Neutral-Beam-Heated Divertor Discharges of the ASDEX Tokamak*, Physical Review Letters, **49** (1982), 1408.

Wesson, J.A., *Tokamaks*, Oxford University Press (1987).

Woods, L.C., *The Thermodynamics of Fluid Systems*, Oxford University Press (1975).

Woods, L.C., *Principles of Magnetoplasma Dynamics*, Oxford University Press (1987).

Wootton, A.J., et al., *Fluctuations and Anomalous Transport (in Tokamaks, particularly TEXT)*, Plasma Physics and Controlled Fusion, **30** (1988), 1479.

Zarnstorff, M.C., et al., *Transport Analysis of TFTR Super-Shots*, Proceedings of 16th EPS Conference on Controlled Fusion and Plasma Physics, Venice, **13B** (1989), 35.

