Counting Partial Objects

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Abstract

In this thesis, I develop a theory of counting. Chapter 1 introduces the topic and explains why it is important to give a theory of counting whole *and* partial objects. Chapter 2 shows that giving a theory of counting is surprisingly difficult because counting has five special features: counting is kind sensitive, source sensitive, context dependent, graded in felicity, and underspecified. Based on the observation that counting is tightly connected with the notion of *mentally merging* objects, I develop a theory of counting in Chapter 3 that accounts for all these features. Roughly speaking, we can count objects with respect to a predicate P iff we can merge them such that they form objects that are sufficiently similar to whole Ps and at most one partial P. Chapter 4 argues that this theory of counting performs better than its competitors. Chapter 5 refines the proposed theory in three ways, while Chapter 6 replies to six objections against the theory. Chapter 7 concludes these discussions by exploring potential future research questions.

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Chapter 1

Introduction

The main goal of this thesis is to develop a theory of counting. In our daily lives, we count all the time. I am sure you already counted a variety of things today. Whether that was to determine if you have enough oranges to make jam or to estimate how long you will be in line at Tesco, counting is ubiquitous and essential to many tasks we perform.

Counting also has broad philosophical significance. In metaphysics, counting is of importance concerning theories of persistence (cf. Hawley, 2002; Lewis, 1986, 1976; Moss, 2012; Moyer, 2008; Viebahn, 2013; Sider et al., 2001, chapter 5.8), arbitrary reference (cf. Breckenridge & Magidor, 2012, p. 390), presentism (Szabó, 2006), the debate between necessitists and contingentists (cf. Fritz & Goodman, 2017; Rosefeldt, 2017; Tomasetta, 2010; Williamson, 1998, 2000a,c, 2013), and the problem of the many (cf. Johnston, 1992; Lewis, 1993; Liebesman, 2020; Noonan, 1993; Unger, 1980). In epistemology, philosophers refer to counting within the debate on how to measure knowledge (cf. Treanor, 2013, 2018; Williams, 2001, p. 131). Further, counting is not only of importance within theoretical philosophy but also within ethics, e.g. for theories of value aggregation (cf. Broome, 1992; Fehige, 1998; Holtug, 2015), and, similarly, within decision theory (Steele & Stefánsson, 2020; Resnik, 1987). In the following, I briefly highlight five debates in which counting is relevant to illustrate its philosophical significance.

In relation to theories of persistence, counting is, for instance, relevant in the debate between worm theorists and stage theorists. Worm theorists believe that ordinary objects are fusions of instantaneous temporal stages, while stage theorists think that ordinary objects are instantaneous temporal stages. It is generally believed that stage theorists can account easily for counting in cases that involve fission or fusion, but worm theorists cannot (cf. Sider et al., 2001, chapter 5.8). Imagine that there is exactly one car in Baker street which is about to undergo fission, i.e. will split up symmetrically into exactly two cars. Before the car undergoes fission, the following count is true: "Currently, there is exactly one car in Baker street." This count is predicted by stage theorists, as we count ordinary objects and at any given time before the car undergoes fission, there is exactly one instantaneous car stage in Baker street. By contrast, worm theorists predict that there are two cars even before the car undergoes fission. For according to worm theory, there are two distinct fusions of temporal stages, which spatially coincide before the fission.

While stage theorists easily account for counting in cases that involve fission and fusion, they struggle to account for diachronic counting (Hawley, 2002, p. 63). Imagine that Chris had exactly one tree in his living room for the past month. Then it is true to say "There has been exactly one tree in Chris' living room in the last week." Stage theory, however, predicts that infinitely many trees have been in Chris' living room in the past week, for infinitely many instantaneous tree stages have been in Chris' living room. In contrast, worm theorists do not struggle to predict the right count, for only one fusion of instantaneous tree stages (assuming the tree will not undergo fission in the future) has been in Chris' living room.

As a second example, the problem of the many is inherently related to counting: the boundaries of Mount Everest are not sharp because there are many stones for which it is undetermined whether they are part of Mount Everest or not. Thus, we can think of different objects all consisting of the core of Mount Everest plus a selection of the stones at its boundaries. Although it seems correct to call all these objects "Mount Everest", we count exactly one Mount Everest and not an infinite number.

As a third example, in epistemology philosophers refer to counting within the debate about how to measure knowledge. For example, Treanor (2013, 2018) argues that we cannot measure how much a person knows by counting all their true justified beliefs, and Williams (2001) argues that it is impossible for us to count how many beliefs we have, for we lack clear criteria to individuate them.

My fourth and fifth examples are that counting plays a key role in ethical theories of aggregation, e.g. for utilitarianism, and in decision theory. Utilitarianists think that we ought to maximize some overall good. That overall good must be derived from many individual goods, plausibly by counting the individual goods together in some way.[1,2] The situation is similar for decision theory. If one thinks that the best action is the one that maximizes expected utility, and expected utility is a weighted average of the utilities of the possible outcomes (Steele & Stefánsson, 2020; Resnik, 1987), then counting is relevant to determine how one should act.

As counting arguments play a role in many areas of philosophy, understanding counting promises to shed light on, and help us clarify, a number of philosophical debates.

At first glance, it seems easy to understand counting. Consider counting two oranges on a table: according to the simple account, exactly two oranges are on the table iff there are two things, which are both on the table, both oranges and not identical, and everything else is either not on the table or not an orange.[3] This

[1] Bentham (1996, ch. 4), for example, writes "Sum up the values of all the pleasures on one side and of all the pains on the other. If the balance is on the side of pleasure, that is the over-all good tendency of the act with respect to the interests of that person; if on the side of pain, its over-all bad tendency." Hence, counting values together is clearly important for Bentham's utilitarianism.

[2] Cf. Harsanyi (1955, p. 314) or Holtug (2015, p. 268) for formal representations of utilitarianism that show that utilitarianism is based on some sort of counting.

[3] Put formally a sentence "$NxFx$" (read: "There are exactly n objects x such that Fx") is true iff $\exists x_1 ... \exists x_n[(\bigwedge_{i\in\{1,...,n\}\wedge j\in\{1,...,n\}\wedge i\neq j}(x_i \neq x_j)) \wedge (\bigwedge_{i\in\{1,...,n\}} Fx_i) \wedge \forall z(Fz \rightarrow (\bigvee_{i\in\{1,...,n\}}(z = x_i))]$. Note, however, that this formalization would not work for all quantifiers as Kaplan (1966a,b) has proved that it does not work for "most". One could instead define "$NxFx$" in Barwise & Cooper's

simple account seems to work well for cases where we count whole objects. However, it does not make the right predictions when partial objects are considered (Salmon, 1997).

Imagine there are two oranges on a table. Titus eats half of one of the two oranges and puts the remaining half back on the table. Presumably, in this scenario the sentence "One and a half oranges are on the table" is true. This is problematic for the simple account. By law of the excluded middle the half orange either is an orange or it is not an orange. If it is an orange, the simple account predicts that "There are exactly two oranges on the table" is true, for there are two different things which are oranges and on the table (the whole orange and the orange half) and everything else is either not an orange or not on the table. If the orange half is not an orange, the simple account predicts that "There is exactly one orange on the table" is true, for there is one thing, which is an orange and on the table (the whole orange) and everything else is either not an orange or not on the table. Both predictions are not compatible with the intuitively correct reply "One and a half oranges are on the table".

As we often count partial objects, the problem raised for the simple account serves as a prima facie reason to develop a new theory for counting partial objects. But there is a more pressing reason to do so, for the problem generalizes to affect the truth conditions of counting sentences that involve only natural numbers. If there are one and a half oranges on the table, the sentences "There are exactly two oranges on the table" and "There is exactly one orange on the table" are both false. On the simple account one of them is true: if one thinks that half oranges are oranges, "There are exactly two oranges on the table" is true. If one thinks half oranges are not oranges, "There is exactly one orange on the table" is true. This generalization of the problem can only be avoided if we restrict counting situations such that they only involve whole objects.

(1981) framework.

Thus, the simple account makes false predictions for counting sentences that involve non-natural numbers or count objects that include partial objects. A different account is needed. As the truth conditions of counting sentences that include only natural numbers and counting sentences that include non-natural numbers are connected, we should aim for a unified theory of counting that makes the right predictions for both counting partial and whole objects.

But developing a theory of counting that makes the right predictions for counting partial objects is surprisingly difficult. Suppose Titus sits at a table. In front of him is a plate with three top halves of oranges. It is then natural to say that one and a half oranges are on Titus' plate, which demonstrates the ease with which we count partial objects. But counting partial objects is as mysterious as it is effortless. Suppose Bjarne collects old car parts. At the moment, he has three half cars in his garage. Each of the halves is the front half of a car. Unlike with Titus' oranges, it is at least not obvious that we can count the car halves in Bjarne's garage as one and a half cars. This observation is puzzling since the two cases are structurally analogous; their only difference is that the objects counted are of different kinds. Thus, the kind of the counted objects appears to influence whether they can be counted together. None of the current theories of counting partial objects (Liebesman, 2016, 2015; Snyder & Barlew, 2019; Snyder, forthcoming) predict that counting is kind sensitive; hence, they cannot explain the difference between counting top halves of oranges and counting front halves of cars.

In this thesis, I discuss five intriguing features of counting partial objects. Counting is not only kind sensitive, but also source sensitive, context dependent, graded in felicity, and underspecified. I then set out a general theory of counting which accounts for these features better than its competitors. Roughly speaking, the central assumption of my theory is MENTAL MERGE:

> MENTAL MERGE: We can count objects as Ps iff we can *mentally merge* them to form objects that are sufficiently similar to whole Ps and at most

one partial P.

According to MENTAL MERGE, we can count objects with respect to a predicate P iff we can form merges of the objects, such that at most one merge is sufficiently similar to a partial P, and all other merges are sufficiently similar to whole Ps.

The structure of this thesis is as follows: the second Chapter examines the five surprising characteristics of counting. The third Chapter proposes a new theory of counting: objects can be counted iff their merges are sufficiently similar to whole objects and at most one partial object. The new theory explains the five features of counting. Chapter 4 argues that other theories of counting partial objects fail to do justice to the five characteristics of counting and that they are problematic for independent reasons. Chapter 5 improves the new theory of counting with three refinements, and Chapter 6 defends the theory against six objections. The seventh Chapter summarizes these discussions and explores potential avenues for future research.

Chapter 2

Counting Is Mysterious

This Chapter explains why counting, although effortless, is also mysterious. It does so by discussing surprising data concerning counting from which I deduce five features of counting. These features are that counting is kind sensitive, source sensitive, context dependent, graded in felicity, and underspecified. I will start with a preliminary remark about my usage of notions like “partial object”, “half an object”, and “half of an object”. Then I explain why counting is kind sensitive, source sensitive, context dependent, graded in felicity, and underspecified.

2.1 Parthood and Partialhood

In the following, when I speak of a partial object, half an object or a half of an object, e.g. of an orange, I intend this to involve the parthood relation, i.e. that a partial orange, an orange half, or half of an orange is a part of an orange. Parthood is an extensively researched relation between objects (Kearns, 2011; Simons, 1987; Varzi, 2019). Liebesman (ms) recently introduced the notion “partialhood”, to talk about a relation between kinds and objects which he takes to be different from parthood. While an unfinished house is not a part of a house, it is a partial house. Conversely, while the right mirror of a car is not a partial car, it is a part of a car. Since we can count unfinished partial houses as houses, but we cannot count right mirrors of cars as cars, it is plausible that the relevant notion for counting is partialhood. What I will say can be translated to partialhood, if this should turn out to be the relevant relation.

2.2 Counting Is Kind Sensitive

To show that counting is kind sensitive, I introduce three counting cases that differ in felicity. I argue that they differ in felicity because they count objects of different kinds.

> ORANGE TOPS: Titus has a plate with three top halves of oranges on it. Titus says: "One and a half oranges are on the plate."

> BAGEL BOTTOMS: Martha has a plate with three bagel bottoms (she thinks the bottom of a bagel is crispier). She asserts: "One and a half bagels are on my plate."

> CAR FRONTS: There are three front halves of cars in Bjarne's garage. He says: "One and a half cars are in the garage."

Note that in ORANGE TOPS, BAGEL BOTTOMS and CAR FRONTS, there are always three halves of Ps, which are all the same part of a P. For example, in ORANGE TOPS, all three orange halves are top halves. In each scenario, these three halves are counted as one and a half Ps. In this respect, counting in the three cases is very similar. However, while Titus' utterance in ORANGE TOPS is perfectly felicitous, Martha's and Bjarne's utterances in BAGEL BOTTOMS and CAR FRONTS, respectively, appear infelicitous or at least less felicitous than Titus' utterance in ORANGE TOPS. As the main difference between the three cases is the kind of the counted objects, they show that counting is kind sensitive: if we count objects of different kinds, F and G, such that for every object that is an nth of a F there is exactly one object that is an nth of a G and vice versa, it is possible that while we can count the Fs, we cannot count the Gs.

This observation does not entail that partial cars and partial bagels can never be counted. For there are felicitous cases of counting partial bagels or partial cars. Consider Mixed Bagels and Mixed Cars:

> Mixed Bagels: Johanna prefers to eat bagel tops over bagel bottoms. Martha and Johanna are eating from the same plate on which there are two bagel bottoms and one bagel top. Johanna utters "One and a half bagels are on our plate."

> Mixed Cars: There are two front halves of cars and one back half in Bjarne's garage. He says: "One and a half cars are in the garage."

Both utterances in Mixed Bagels and Mixed Cars appear to be felicitous or at least more felicitous than the utterances in Bagel Bottoms and Car Fronts. Still, the original cases importantly show that counting is kind sensitive.

2.3 Counting Is Source Sensitive

Counting is also source sensitive. Even if two groups of objects are intrinsically qualitatively identical[1], sometimes only the objects in one of the two groups can be counted. The object's source is relevant in this regard. To see why, consider the following cases:

> Sphinx Car: Imagine that in addition to a front half of a Ferrari, Bjarne has a special car called "Beetlemobile". Beetlemobile's front half is the same as the front half of a Volkswagen Beetle, while its back half is the same as that of a pick-up truck. Bjarne cuts Beetlemobile down the

[1]Two objects are intrinsically qualitatively identical iff they share all their intrinsic qualitative properties (Lewis, 1983; Noonan & Curtis, 2018).

> middle, such that there is now the front half of a Ferrari, the front half of a Beetle, and the back half of a pick-up truck in his garage. Bjarne says: "One and a half cars are in my garage."
>
> ORDINARY CARS: In Bjarne's garage, there is a front half of a Ferrari, a whole Volkswagen Beetle, and a whole pick-up truck. After cutting his whole cars down the middle, Bjarne gets rid of the back half of the Beetle and of the front half of the pick-up truck. Being left with the front half of a Ferrari, the front half of a Beetle, and the back half of a pick-up truck, Bjarne says: "One and a half cars are in my garage."

It seems as if in SPHINX CAR, Bjarne's utterance is felicitous, but in ORDINARY CARS his utterance appears infelicitous or at least less felicitous than his utterance in SPHINX CAR.

But the car halves in SPHINX CAR and ORDINARY CARS are intrinsically qualitatively identical. In both cases, there is a front half of a Ferrari, a front half of a Beetle, and a back half of a pick-up truck in Bjarne's garage. The only difference is that in SPHINX CAR, but not in ORDINARY CARS, the front half of the beetle and the back half of the pick-up truck originate from the same car, viz., the Beetlemobile. Therefore, SPHINX CAR and ORDINARY CARS show that counting is source sensitive, where the source refers to the whole object from which a partial object originated.

2.4 Counting Is Context Dependent

This subsection argues that counting is also context dependent. Think again of CAR FRONTS. In CAR FRONTS, Bjarne has three front halves of cars in his garage. It seems like we cannot count them as one and a half cars.

Interestingly, if we keep fixed the objects we are counting, e.g. the three front halves, but change the context, "one and a half" can be a correct count of the three front halves of cars. JUNKYARD illustrates this:

> JUNKYARD: Imagine Bjarne wants to recycle his three front halves of cars. To do so, he brings them to a junkyard. The director of the junkyard, Leonie, only cares about the amount of metal in Bjarne's cars. Leonie takes a look at the front halves and asks "You want to recycle these one and a half cars?"

In JUNKYARD, Leonie's utterance appears felicitous or at least more felicitous than Bjarne's utterance in CAR FRONTS. Thus, we seem to be able to count three front halves of cars as one and a half cars in JUNKYARD, but not in CAR FRONTS. This discrepancy illustrates the context dependence of counting: the same objects can be counted together in some contexts, but not in others.

2.5 Counting Is Graded in Felicity

Different assertions of counts vary in their felicity. While it is felicitous to assert that there are one and a half oranges in ORANGE TOPS, it still seems more felicitous to assert that there are one and a half oranges if there is a whole orange and an orange half on a plate. Similarly, while it seems better to sum together three front halves of cars in JUNKYARD than to sum them together in CAR FRONTS, the count in JUNKYARD still does not feel as natural as counting one whole car plus one car half as one and a half cars. So different counts vary in how natural they feel. Accordingly, assertions of counts are graded in felicity.

2.6 Counting Is Underspecified

As Frege (1996, §22, §46)[2] noted, counting is underspecified; that is, in many scenarios, there is more than one correct count. If Titus has a whole orange, it is true

[2] Cf. Geach (1962, p. 63), Irvine (2010), Lowe (1989), Thomasson (2007, chapter 6), and Yourgrau (1997) for discussion.

to say that he has one orange. But it is equally true to say that he has two orange halves. Importantly, the two counts compete with each other: while we can say that Titus has one orange, or that he has two orange halves, we cannot say that he has one orange and two orange halves.

Similarly, if Titus has three halves of oranges, we can count them together. It is felicitous to say that Titus has one and a half oranges. However, it is equally felicitous to count the orange halves and say that Titus has three orange halves. These two counts also compete with each other: we cannot say that Titus has one and a half oranges and three orange halves.

The reason why there are multiple correct counts is that we are counting different things. If we count how many oranges Titus has, the count is one and a half; if we count how many orange halves he has, the count is three. Thus, we do not count objects *simpliciter* (Frege, 1996, §22) but relative to a property, which provides us with the relevant individuation conditions.

2.7 Conclusion

This Chapter demonstrated that counting has five interesting characteristics. Counting is kind sensitive, source sensitive, context dependent, graded in felicity, and underspecified. A good theory of counting should account for these five features of counting.

Chapter 3

Demystifying Counting

We have seen that counting has several interesting characteristics: kind sensitivity, source sensitivity, context dependence, gradedness in felicity, and underspecification. This Chapter presents a novel theory that explains all these surprising features of counting and thereby demystifies counting. I will begin working towards my theory by discussing the relevance of symmetry for explaining the data. Then I present my own theory and show how it explains the various features of counting.

3.1 Symmetry and Mental Merging

Working towards a theory of counting, I begin with a very robust intuition: partial objects of kind F can be counted iff whole Fs are symmetric. One may think that this intuition explains the disanalogy between counting cars, oranges, and bagels: oranges are, intuitively speaking, closer to being symmetric than bagels or cars.[1] This is why we can count partial oranges, but not partial bagels or cars.

However, symmetry is not sufficient to demystify counting. Imagine that you have one half of a very big orange, one half of a medium-sized orange, and one half of a small orange. Even though oranges are (roughly) symmetric, in this example it

[1] Of course bagels and cars show bilateral symmetry. Bagels even show rotational symmetry. However, there is still a sense in which oranges are more symmetric, for they have more axes of symmetry than bagels and cars.

would be somewhat strange to say that you have one and a half oranges.

Nonetheless, there is something important about the intuition that we can count perfectly symmetric objects. For we can usually count perfectly symmetric objects because *mentally merging* perfectly symmetric objects is easier than mentally merging asymmetric objects.[2] By mentally merging objects, I mean something really intuitive and loose, viz., the process of assembling objects in our head to form other objects. The basic intuition behind my account is that the counting and the mental merging of objects are tightly connected:

> MENTAL MERGE: We can count objects with respect to a predicate P iff we can mentally merge them such that they form objects that are sufficiently similar to whole Ps and at most one partial P.

While we can mentally merge two similar-sized top halves of oranges to form an object sufficiently similar to a whole orange, we cannot mentally merge two bagel bottoms or two front halves of cars to form an object that is sufficiently similar to a whole bagel or a whole car, respectively. Hence, we can count the two top halves of oranges as one orange, but we cannot count two front halves of cars as one car. Equally, we cannot mentally merge orange halves of very different sizes to be sufficiently similar to a whole orange, and thus cannot count such halves together. I will now give a semantics that underwrites the intuition that we can count objects iff we can mentally merge them. My explanation will be informal; in the Appendix A, I state my theory formally.

3.2 A Novel Theory of Counting

To determine whether the objects in a set X can be merged to form whole Ps and at most one partial P, and thus can be counted with respect to a predicate P, we

[2]More generally, parts of perfectly symmetric objects of kind F can always be mentally merged to a whole or partial F, provided that they have the right size and together do not count for more than one.

have to look at different partitional groupings of the objects in X. For example, one partitional grouping of the set {orange quarter, orange quarter, orange eighth, orange half} is {{orange quarter, orange quarter}, {orange eighth, orange half}}; here, the two orange quarters are grouped together and the orange eighth is grouped with the orange half.

For each element S of a possible partitional grouping, we look at the *merge* associated with S. The merge of a set of objects is the object which, out of all possible objects that are composed of all and only members of S, is the most similar to a possible P (when I say "possible", I mean metaphysically possible, i.e. possible in the broadest sense).[3] For example, the merge of two quarter oranges is similar to a half orange (importantly, a merge of a set of objects is not the same as their mereological sum. In contrast to mereological sums, merges fulfill requirements on how the objects are arranged, i.e. such that they are as similar to a P as possible. I discuss this in section 3.4).

With this at hand, the COUNTING CONSTRAINT is the following:

> COUNTING CONSTRAINT: A set of objects X can be counted with respect to a predicate P iff there exists a partitional grouping of X satisfying the following two requirements:
>
> - The merge of every element of the grouping is sufficiently similar to a possible whole or partial P.
> - At most one element of the grouping has a merge that is not sufficiently similar to any possible whole P (assuming that whole Ps are not partial Ps).[4]

Together, these two requirements entail that the merge of maximally one element of the grouping is sufficiently similar to a partial P and all other elements have

[3] There may be more than one such object. In those cases, S has several potential merges.

[4] As elements can have more than one potential merge, we need to quantify over all the merges an element has. This will be done in detail in the Appendix A, where I give a formalization of the presented theory.

merges that are sufficiently similar to a possible whole P.[5,6] In cases in which the COUNTING CONSTRAINT is satisfied, I will say that the objects in X can be merged in a way that leaves at most one partial P.

3.2.1 Counting Constraint in Orange Tops

What does the COUNTING CONSTRAINT predict for ORANGE TOPS? Remember that in ORANGE TOPS, Titus has three top halves of oranges. The set we are counting is the set of Titus' Oranges:

(1) TITUS' ORANGES = {top half$_1$, top half$_2$, top half$_3$}

One grouping of TITUS' ORANGES is given in (2). It divides Titus' oranges into two subsets, the set of top half$_1$ and top half$_2$ (set A) and the set that only contains top half$_3$ (set B):

(2) GROUPING OF TITUS' ORANGES = {{top half$_1$, top half$_2$}, {top half$_3$}}

Two orange top halves can be merged to form an object that is sufficiently similar to a whole orange (assuming the halves have a similar size). Accordingly, the merge of set A is similar enough to a possible whole orange. Further, the merge of set B, which consists of one top half, is just that top half, which is trivially similar enough to a possible partial orange.

As all merges of elements of GROUPING OF TITUS' ORANGES are similar enough to possible whole or partial oranges, the first constraint is satisfied. Since only the merge of the grouping of set B is similar enough to a possible partial orange, the second constraint is satisfied. Hence, the account correctly predicts that Titus can count his oranges because they can be merged in a way that leaves at most one partial orange.

[5]It is not surprising that similarity plays a role in the semantics of counting, for similarity has been thought to play a role in other parts of semantics, e.g., in the semantics of counterfactuals (Lewis, 1973; Stalnaker, 1968; Sprigge, 1970).

[6]For simplicity, I am assuming that no object is sufficiently similar to both a partial and a whole P.

The next section shows how, given that Titus can count his oranges, the correct count of his oranges is predicted.

3.2.2 The Right Count

To calculate the count of a set of objects X with respect to a predicate P in cases that satisfy the Counting Constraint, we look at a grouping that satisfies the Counting Constraint. For every element of that grouping, we look at its merge[7] and select an element in P to which the merge is sufficiently similar. For each selected P we determine how much of a whole P it is.[8] For example, a half orange counts for 0.5 with respect to "orange" as half oranges are half of a whole orange. Then we sum the values we get for the individual selected Ps together. Thereby, we end up with the count of the objects in X with respect to P.

3.2.3 The Right Count in Orange Tops

Does the theory predict that Titus' orange halves count as one and a half oranges in Orange Tops? I have already shown that the grouping of Titus' Oranges given in (2) satisfies the Counting Constraint. Set A consists of two orange halves. The merge of A is sufficiently similar to a possible whole orange, which counts for 1. Set B consists of one orange half. Accordingly, its merge is sufficiently similar to a possible half orange, which counts for 0.5. As $1 + 0.5$ equals 1.5, the theory correctly predicts that Titus has one and a half oranges in Orange Tops.

3.3 Explaining the Various Features

Thinking of counting in terms of merging objects not only makes the right predictions for Orange Tops. It also accounts for the various features discussed in

[7]If an element has more than one merge, we select one of them.

[8]In some cases, a partial P can count for different amounts of a whole P. Such cases are not covered by the presented theory. They will be discussed in Chapter 5 and the theory will be defined accordingly.

Chapter 2. This section shows how.

3.3.1 Kind Sensitivity

Just like symmetry, kind sensitivity is a symptom of a more general feature of counting, viz., *mergeability.* A group of objects is mergeable with respect to a predicate P iff the objects can be grouped such that the merge of every subgroup is sufficiently similar to a possible whole or partial P and at most one merge is sufficiently similar to a possible partial P (i.e. iff there is a grouping that satisfies the COUNTING CONSTRAINT).

That mergeability is a feature of counting entails that kind sensitivity is also a feature of counting. Parts of objects of kinds that are usually symmetric, like oranges, can be merged independently of which part of an orange they are. For the merge of two orange top halves is (provided that they are roughly the same size) quite similar to a possible orange. This mergeability does not hold for cars and bagels. The merge of two bagel bottoms or two front halves of cars is not similar enough to a possible whole bagel or a possible whole car, respectively.

However, one might think that there are possible cars that consist of two front halves. Then the account would wrongly predict that two front halves of cars are mergeable, for their merge would be similar enough to a possible car. Rather, the difference between a merge composed of two top halves and a merge composed of two front halves, is that the first merge is sufficiently similar to a *normal* orange, while the second merge is not sufficiently similar to a *normal* car.[9] I will strengthen my account by assuming that merges have to be similar enough to a partial or whole P that is sufficiently normal, where the given context c guides us in evaluating which

[9]A theory of counting according to which only what is normal counts as possible makes the same predictions as the proposed theory without introducing normality as an additional requirement (on such a theory, normality is already entailed by merges being similar to a possible P).

possible *P*s are sufficiently normal.[10,11]

Possible normal cars in a context *c* are those which are normal in *c* with respect to the standards for cars of *c*. In most contexts, a car composed of two front halves is not normal. While an orange composed of two top halves is usually also not normal, the merge of two orange top halves is still sufficiently similar to a normal orange, i.e. an orange that consists of a top and a bottom half. This condition explains why two orange top halves are, but two front halves of cars are not, mergeable; therefore, it also explains why counting is kind sensitive.

3.3.2 Source Sensitivity

If mergeability is a feature of counting, source sensitivity is a feature of counting, too. If objects have the same source, and their source is a whole *P*, then, in most cases, the objects' source and parts of it are sufficiently normal possible partial or whole *P*s. For usually the source of objects is contextually salient and counts as a normal *P*. Then, if objects originate from the same source, they can usually be merged to objects that are similar enough to a sufficiently normal possible whole or partial *P*.

Remember that in Sphinx Car, Bjarne cuts his Beetlemobile, a car whose front half is a Volkswagen Beetle and whose back half is a pick-up truck, down the middle. The front half of a Beetle and the back half of a pick-up truck can then be merged to an object that is similar enough to the Beetlemobile which is a sufficiently normal car in this context, for it was once a car Bjarne owned.

[10] While often worlds are ordered with respect to their normality (Goodman, 2013; Smith, 2017), I use the same notion to order individuals. In a similar vein, (Goodman & Salow, 2018, p. 191) define normality to order possible states of the world, Carter (forthcoming) defines normality to order propositions, and Loets (ms) defines normality to order individuals.

[11] One problem with such an assumption is that we can also count two halves of a very abnormal orange as one orange. I will assume that in such contexts, the orange is normal in the context, but abnormal globally. This is slightly problematic for we can truthfully assert that the orange is abnormal, even in the local context. One way to circumvent these problems is to use possibility instead of normality: the difference between a merge of two orange top halves and a merge of two front car halves could also be that the former is sufficiently similar to an object that is more possible than the object the latter merge is sufficiently similar to.

In Ordinary Cars, Bjarne is also counting the front half of a Ferrari, the front half of a Beetle, and the back half of a pick-up truck. But in contrast to Sphinx Car, all the car halves have different sources. Importantly, in ordinary contexts, strange cars like the Beetlemobile are not sufficiently normal. Accordingly, in Ordinary Cars, the merge of the two car halves is not similar enough to a sufficiently normal possible car.

Hence, the presented account predicts both kind and source sensitivity. It further explains why counting has those two features: whether objects can be counted depends on their mergeability and the mergeability of objects depends on their kind and their source.

3.3.3 Context Dependence

In section 2.4, we saw that counting is context dependent. In some contexts, e.g. on a junkyard, several front halves of cars can be summed together.

But if front halves of cars cannot be merged to an object which is similar enough to a sufficiently normal possible car, front halves of cars are not mergeable. It then seems as if the current account cannot predict that there are scenarios in which several front halves of cars can be counted together.

Importantly, in cases like Junkyard, a whole car consisting of two front halves counts as sufficiently normal. In Junkyard, only the amount of scrap metal matters and, with respect to the amount of scrap metal, a car composed of two front halves is normal.

As normality is a context sensitive notion, it can thus account for the context dependence of counting. The merges of subgroups of a set of objects X can be similar enough to a sufficiently normal possible partial or whole P in some contexts, but not others. Accordingly, counting is context dependent.[12]

[12]Alternatively, one might think of similarity as context dependent (Douven & Decock, 2010). Two objects are then sufficiently similar iff they are sufficiently similar in their relevant properties. This could be modelled in a similarity model on conceptual spaces (Denby, 2001; Douven et al.,

3.3.4 Grades of Felicity

Counts differ in how natural they feel. Correspondingly, assertions of counts differ in felicity. To see how degrees of felicity are predicted by the developed theory, we first have to rethink the way normality was introduced.

So far, I have assumed that objects are either sufficiently normal or not sufficiently normal. However, that does not seem quite right. While the Beetlemobile is a sufficiently normal car in Sphinx Car, it is still less normal than a whole Volkswagen Beetle. Thus, normality is graded (Carter, forthcoming; Smith, 2010, footnote 7) and, correspondingly, mergeability is graded, too.

Moreover, mergeability is not only graded because normality is graded, but also because similarity is graded. The merge of two orange halves is usually sufficiently similar to a whole orange, but still not as similar to a whole orange as the merge of a whole orange which is even identical to a whole orange.[13]

Interestingly, differences in degree of mergeability correspond to differences in felicity. Counts feel very natural if all merges are very similar to very normal possible Ps. They feel less natural if the merges are less similar to less normal possible Ps. Even though both counts feel natural, counting a whole orange plus a half orange as one and a half oranges is more natural than counting three half oranges as one and a half oranges, for the merge of a whole orange is more similar to a whole orange and, thus, more mergeable than the merge of two orange halves. Equivalently, while a car composed of two front halves is more normal in contexts like Junkyard than in contexts like Front Halves, it is still less normal than a car composed of a front and a back half. This discrepancy explains why summing the three front halves in Junkyard is not as natural as summing two front halves and one back half and, thus, why assertions of the count "One and a half cars" are graded in felicity.

2013; Gärdenfors, 2004).

[13]Similarity is probably graded in a second way. Objects that are similar in many properties are more similar than objects that are similar in fewer properties.

3.3.5 Underspecification

The theory of counting presented herein is sensitive to the predicate P with respect to which a set of objects is counted. By focusing on different predicates, we can get different counts for the same set of objects. For example, if we count three orange halves with respect to "orange", they count (provided they have a similar size) for one and a half oranges. If, however, we count them with respect to the predicate "half of an orange", they count for three halves of oranges. These counts are mutually exclusive, for we can *either* count a whole orange with respect to the predicate "orange" *or* with respect to "half of an orange".[14]

As every whole orange is composed of two halves, we can always count those halves and thereby predict that if somebody has an orange, it is true that they have two orange halves.

Nevertheless, in most cases it is misleading to say of somebody who has an orange that they have two orange halves and vice versa. In Chapter 6, I argue that this misleadingness is due to pragmatics and not semantics.

3.4 Mereological Sums Are Not Merges

Importantly, the merge of a set is not the same as its fusion or mereological sum. First, what merge a set has depends on the predicate with respect to which we form the merge. For the merge of a set X is the object that is composed of all the objects in X and as similar to a normal P as possible. Hence, in contrast to mereological sums, merges fulfill requirements on how the objects are arranged, i.e. such that they are as similar to a normal P as possible.

Further, fusions are thought to be unique (Varzi, 2014), while merges are not. We can form two merges that are equally similar to a normal partial orange out of an orange half and an orange quarter, for we can attach the orange quarter either

[14] As it stands, my theory predicts that there are scenarios in which we double count objects. In Chapter 5, I refine my theory to avoid double counting.

on the right side of the orange half or on the left side. Hence, the merge of a set is not unique.[15] In contrast, the fusion of the orange half and the orange quarter is unique: it is not specified whether the orange half is on the right or the left of the orange quarter in the fusion. To be the fusion of a set X, it is sufficient to contain all the elements of X and nothing else.

3.5 Merging and Mental Merging

I have argued that we can count objects iff the COUNTING CONSTRAINT is satisfied. However, the COUNTING CONSTRAINT alone does not capture our basic intuition MENTAL MERGE.

> MENTAL MERGE: We can count objects as Ps iff we can *mentally merge* them to form objects that are sufficiently similar to whole Ps and at most one partial P.

According to the COUNTING CONSTRAINT, we can count the objects of a set X with respect to a predicate P iff there is a partitional grouping G of X such that maximally one element of G has a merge that is similar enough to a partial normal P, and the merges of all other elements of G are similar enough to a whole normal P. Crucially, nothing in the COUNTING CONSTRAINT has anything to do with what we *mentally* do when we count.

However, together with an additional premise, the COUNTING CONSTRAINT can capture MENTAL MERGE. Plausibly, semantics is connected to the mental processes taking place when we investigate whether a given sentence is true. The idea is that when we evaluate whether a sentence is true, we check whether its semantic truth conditions are fulfilled. In this way, semantics is connected to mental processes.

[15]Importantly, in this case, both merges count as 0.75 oranges. Hence, that a set does not have a unique merge usually does not prevent the count of the objects from being unique. I discuss cases in which sets of objects do not have a unique count in Chapter 5.

With this at hand, we can explain the relation between the COUNTING CONSTRAINT and MENTAL MERGE: we can evaluate counting sentences as true with respect to a predicate P, iff we can mentally merge the objects in question such that at most one of the merges is sufficiently similar to a partial normal P. Thus, MENTAL MERGE follows from the COUNTING CONSTRAINT and my assumption because to check whether the COUNTING CONSTRAINT is satisfied, we mentally merge the objects in question to evaluate whether they can be merged in a way that leaves at most one partial normal P. Hence, while the developed semantics of counting is not psychological in the first place, it has an indirect impact on the psychology of counting if we assume that when we count, we evaluate the truth of counting sentences.

Note that the assumption that semantics is related to psychology in the described way is not uncommon. Kratzer (2013), for example, thinks that factual domain projection is "a mechanism that relates to a very basic cognitive ability: a creature's ability to map a part of its own world to a range of worlds representing possible ways that part could be 'extended' to or 'grow into' a complete world" (Kratzer, 2013, p. 192). Similarly, one could say that the COUNTING CONSTRAINT relates to a creature's ability to map a part of its own world - the counted objects - to a range of worlds representing possible ways in which that part of the world could "grow into" complete objects of the relevant kind.

3.6 Conclusion

We started with the observation that, although counting is often effortless, counting data is surprisingly hard to systematize. Systematizing counting data is difficult because of five special features of counting: kind sensitivity, source sensitivity, context dependence, gradedness in felicity, and underspecification.

Interestingly, an investigation into counting demonstrated that counting is tightly connected to the concept of merging objects. We can count objects with respect to a predicate P iff they are mergeable, i.e. iff they can be merged such that at most one partial normal P remains. The proposed account is built around a context dependent, predicate sensitive, and graded notion of mergeability and successfully explains all the intriguing characteristics of counting. It thereby demystifies counting. The formally minded reader will find a more rigorous presentation of the developed theory in the Appendix A.

3.7 Formal Appendix A

I have laid out my theory of counting, which is that objects can be counted iff they can be merged in a way that leaves at most one partial object. However, I have done so quite informally. Thus, in the following I present a formal version of the proposed theory.

3.7.1 The Counting Constraint Formalized

The Counting Constraint is a constraint on when we can count objects of a set with respect to a predicate P. Consider a set of objects $X = \{x_1, x_2, x_3, x_4, ..., x_n\}$.

A partition G_Y of X is a set of non-empty pairwise disjoint subsets of X whose union is X. That is $\forall g_1, g_2 \in G_Y : g_1 \neq g_2 \rightarrow g_1 \cap g_2 = \varnothing$ and $\cup G_Y = X$. Let $B_G(X)$ be the set of all partitions of X, indexed by the first m positive integers, where m is the Bell number B_n: $B_G(X) = \{G_1, G_2, G_3, ..., G_m\}$.

To arrive at the set of potential merges of an element S of a partition G_Y of X, I define the function C that maps S to the set of all possible objects that are composed of all and only parts that are intrinsically qualitatively identical to members of S.

Let D be the domain of all possible individuals. I assume for simplicity that objects do not exist in multiple possible worlds, though the proposed theory could be formulated without this assumption.[16]

I also assume that two objects are intrinsically qualitatively identical ("$\cong$") iff they share all their intrinsic qualitative properties. Two sets S and S' are intrinsically qualitatively identical iff there exists a bijection f from S to S' such that $f(z)$ is intrinsically qualitatively identical to z, i.e. $S \cong S'$ iff $\exists f : S \xrightarrow{\sim} S'$ such that $\forall z(z \cong f(z))$.

The function C is then given in (3). $C(S)$ takes any set S such that $S \cong S'$ and returns the set of all possible objects that are composed exactly of members of S', where $\oplus S$ is the generalized sum of a set S, i.e. the object which contains every element of S and whose parts each overlap with an element of S.[17]

(3) $C(S) = \{y \in D : \exists S'(S' \cong S \wedge \oplus S' = y\}$

In order to satisfy the Counting Constraint, $C(S)$ has to have an element that is sufficiently similar to a possible P. The set of whole Ps is given in (4) and the set of partial Ps is given in (5) (I assume that whole Ps are not partial Ps). For matters of simplicity, I assume that partial Ps are also Ps and, therefore, that the set of possible Ps, given in (6), is the union of possible whole and possible partial Ps.

(4) $\boldsymbol{whole\ P} = \cup_{w \in W} [\![whole\ P]\!]^w$

(5) $\boldsymbol{partial\ P} = \cup_{w \in W} [\![partial\ P]\!]^w$

(6) $\boldsymbol{P} = \cup_{w \in W} [\![P]\!]^w$

As argued in section 3.3.1, the most similar merges not only need to be similar enough to a P, they need to be similar enough to a sufficiently normal$_c$ P. We define a normality structure as follows: a normality structure is a pair $\langle S, \leq^N_{c,P} \rangle$, where S is a non-empty set and $\leq^N_{c,P}$ is a reflexive and transitive relation on S. S is a set of

[16]Without this assumption, we would have to look at pairs of worlds and objects. To see why, consider a case in which we count the objects in X with respect to P. Their merge is similar enough to the object y at world w_1. Assume also that y exists in w_2 too, in which it is a whole P. But then, in order to count the objects in X as one P, y also needs to be a whole P at world w_1.

[17]For a formal definition of the generalized sum of a set see Champollion (2017, pp. 13-14).

objects and $s \leq^N_{c,P} t$ reads "s is at least as normal with respect to P as t given the standards of normality for Ps in c".

A context c provides us with an object y, such that $\forall z$ iff $z \leq^N_{c,P} y$ then z is sufficiently normal in c. $\boldsymbol{Normal_c\ P}$ contains x iff $x \in \boldsymbol{P}$ such that $x \leq^N_{c,P} y$. With this at hand, we restrict the sets $\boldsymbol{whole\ P}$, $\boldsymbol{partial\ P}$, and $\boldsymbol{P}$, to $\boldsymbol{normal_c\ whole\ P}$, $\boldsymbol{normal_c\ partial\ P}$, and $\boldsymbol{normal_c\ P}$.

Similarly, we define a similarity structure as follows: a similarity structure is a pair $\langle T, \leq^S \rangle$, where T is a set of pairs and $\leq^S$ is a reflexive and transitive relation on S. "$\langle r, s\rangle \leq^S \langle t, u\rangle$" reads "$r$ is at least as similar to s as t is similar to u". A context c provides us with a pair $\langle t, u\rangle \in S$, such that $\forall \langle r, s\rangle \in S$ if $\langle r, s\rangle \leq^S \langle t, u\rangle$ then r is sufficiently similar to s in c.

Objects in a set X can then be summed with respect to a predicate P iff the Counting Constraint given in (7) is satisfied, where "$x \sim_c y$" reads "x is sufficiently similar to y in c":[18]

(7) $\exists G_Y \in B_G(X)$:

1. $\forall S \in G_Y \exists x \in C(S) \exists y \in \boldsymbol{normal_c\ P}(x \sim_c y)$
2. $|\{S \in G_Y : \exists x \in C(S) \exists y \in \boldsymbol{normal_c\ partial\ P}(x \sim_c y)\}| \leq 1$

That is, objects can be counted if two things are given. First, there must exist a partition G_Y of X such that for each element S of G_Y, there is an element in $C(S)$ that is sufficiently similar to a sufficiently normal$_c$ element in $\boldsymbol{P}$ (remember that $\boldsymbol{P}$ includes whole and partial Ps). Second, there must be at most one element R of G_Y for which there is an element in $C(R)$ that is sufficiently similar to a sufficiently normal$_c$ possible partial P.[19]

[18] I am assuming here that if for $S \in G_Y : \exists x \in C(S) \exists y \in \boldsymbol{normal_c\ partial\ P}(x \sim_c y)$ then $\neg \exists z \in C(S) \exists y \in \boldsymbol{normal_c\ whole\ P}(z \sim_c y)$. That is if an element of $C(S)$ is sufficiently similar to a normal$_c$ possible partial P, then no element of $C(S)$ is sufficiently similar to a normal$_c$ possible whole P.

[19] Similarity could also be modelled to be interest or purpose relative. This could be done in the same way in which Fara (2000, 2008) argues that "significantly" is interest relative.

If the COUNTING CONSTRAINT is satisfied, we can merge the objects in X to form objects that are similar enough to whole normal$_c$ Ps and maximally one partial normal$_c$ P.

3.7.2 The Counting Function Formalized

To calculate the count with respect to a predicate P in cases that satisfy the COUNTING CONSTRAINT, we need a function that tells us the degree a specific object x is of an object in the set $\boldsymbol{P}$. $d(x,\boldsymbol{P})$ is such a function:[20]

(8) $d(x,\, \boldsymbol{P}) = r$, where r is the fraction x is of an element in $\boldsymbol{P}$

If x is a whole P, $d(x,\, \boldsymbol{whole\ P}) = 1$, for x is then itself a P. If x is a partial P, $d(x,\, \boldsymbol{whole\ P})$ is the fraction x is of a whole P.[21] For example, half an orange is 0.5 of a whole orange.

The counting function for a set X where the partition G_Y satisfies the COUNTING CONSTRAINT is given in (9). Sim_{G_Y} is the set of normal Ps such that for every element S of G_Y there is exactly one y in Sim_{G_Y} such that $\exists x \in C(S) : x \sim_c y$ and no other elements are in Sim_{G_Y}. Intuitively, Sim_{G_Y} is the set of normal$_c$ Ps to which the merges of G_Y are sufficiently similar.

(9) $\#(X,P)_c = \sum_{x \in Sim_{G_Y}} d(x, \boldsymbol{normal_c\ whole\ P})$

The count of a set X with respect to a P is the sum of the degrees to which the objects in Sim_{G_Y} are normal$_c$ whole possible Ps.

[20] $d(x,\boldsymbol{P})$ is underspecified in cases in which an object z as well as a part of z are in $\boldsymbol{P}$. Here, I am assuming that before we count with respect to P, we restrict $\boldsymbol{P}$ such that no object in $\boldsymbol{P}$ is part of another object in $\boldsymbol{P}$. In Chapter 5, I refine the theory to make the right predictions in underspecified cases.

[21] Work by Ionin et al. (2006) and Ionin & Matushansky (2018) pertaining to the semantics of fractions could be used to obtain the right value of $d(x,\, \boldsymbol{P})$.

Chapter 4

Counterevidence to Other Accounts

To date, there are two well-developed theories of counting partial objects: Liebesman's (2015; 2016) and Snyder & Barlew's (2019; forthcoming). Both accounts were developed to answer the puzzle of counting partial objects raised in the introduction: Salmon (1997) showed that counting sentences cannot simply quantify over objects, for then the truth conditions of counting statements would not respect partial objects.[1] This Chapter discusses both Liebesman's and Snyder & Barlew's theories.

The first section introduces Liebesman's theory and investigates to what extent it can explain the data introduced in the second Chapter. The second section introduces and discusses Snyder & Barlew's theory. I ultimately argue that both theories cannot do justice to the various features of counting. The third section argues that both theories of counting are problematic for reasons independent of the five features of counting.

[1]Salmon's own tentative solution is not spelled out in great detail, hence, I will not discuss it here. My solution implements Salmon's proposal that when we count pluralities, we count different objects for different amounts.

4.1 Liebesman's Account

Liebesman thinks that when we count a set of objects X with respect to a property P, the count of X equals the sum of two summands, where the first summand counts whole oranges and the second summand counts partial oranges.

The first summand is equivalent to the cardinality of the set that contains all the objects in X that instantiate the property P in our world. As Liebesman thinks that only whole Ps instantiate the property P (Liebesman, 2016, 2,4), this cardinality equals the number of whole Ps in X.

The second summand is itself a sum. For every object that is in X but does not instantiate the property P, we look at the value that a partiality measuring function (supplied by context) gives with respect to P for that object and sum those values together. A partiality-measuring function takes pairs of properties and objects and maps them to a number. For example, it maps the pair of a particular half orange and the property of being a whole orange to 0.5, as a half orange is 0.5 of a whole orange.

Liebesman presupposes that we can only count a set of objects if the sum of the values the partiality-measuring function gives us for the partial objects is strictly smaller than one. If this presupposition is satisfied, we can count the two summands defined above together and arrive at the count of X with respect to a property P.[2]

Liebesman's account predicts that we can count a whole orange and a half orange as one and a half oranges. There is one object that instantiates the property of being

[2]Put formally, Liebesman defines the counting function # as a function from pairs of pluralities and properties to numbers. It is given in (1).

(1) $\#(F, xx) = |\{xx \in F\downarrow\}| +$ the $n < 1$ such that $\sum_{y \in \{xx\} \setminus F\downarrow}^{\infty} f(F, y)$ is n

(1) is a sum consisting of two summands. The first part of the sum in (1) captures whole oranges, the second part captures partial oranges. $\downarrow$ is a function that takes a property to the set of objects instantiating that property in a given world (Liebesman, 2016, 8).

The first part of the sum in (1) is the cardinality of the set that contains all the xx instantiating the property F in our world.

The second part is a sum that sums the value f gives with respect to the property F for all objects that are part of xx but do not instantiate the property F. f is a partiality-measuring function: it takes pairs of properties and objects and maps them to a number. The second summand must be strictly smaller than one.

an orange (the whole orange) and for the remaining object (the half orange) the partiality-measuring function gives the value 0.5 with respect to being an orange. As 0.5 is strictly smaller than 1, we can sum the two values, and arrive at a count of one and a half oranges.

In this way, Liebesman gives a semantics of counting that integrates partial objects into the truth conditions of counting.[3]

4.1.1 Liebesman and the Data

How can Liebesman account for the five features of counting? I will start by investigating whether Liebesman can account for kind and source sensitivity. Looking at the two summands of Liebesman's counting function, it is obvious that Liebesman cannot predict kind and source sensitivity: there is no parameter that can distinguish between counting objects of different kinds or counting objects of different sources. Think of two different sets, X and Y. We count X with respect to the property F, and Y with respect to the property G. Now if for every object in X that counts for r with respect to F, there is exactly one object in Y that counts for r with respect to G and vice versa, Liebesman predicts the same count for them. Accordingly, Liebesman makes the same predictions for Orange Tops and Car Fronts, and for Sphinx Car and Ordinary Car respectively.[4] Thus, Liebesman's theory cannot account for kind sensitivity or source sensitivity.

Similarly, Liebesman's theory cannot account for differences in degrees of felicity. Liebesman predicts that assertions of counts are either felicitous or infelicitous, but not that they can vary in their degree of felicity. Additionally, that his theory cannot account for kind sensitivity and source sensitivity entails that it cannot account for differences in degrees of felicity connected to these two features, e.g. that it is more natural to count a back half and a front half that originate from the same car as

[3]Cf. Marshall (2017) and Nicolas (2016) for detailed discussion of Liebesman's account.

[4]In Sphinx Car and Ordinary Car the two sets are counted with respect to the same property.

one car than to count a back half and front half from different cars as one car.

Further, Liebesman's account has no means of accounting for context dependence. A front quarter of a car has to count for 0.25 of a car since it is a quarter of a car. Thus, the partiality-measuring function has to yield 0.25 for front quarters of cars. But then, Liebesman predicts that when we count two front quarters, we can count them as half a car regardless of whether we count the quarters in a scenario like JUNKYARD or FRONT CARS. Thus, Liebesman cannot account for the context dependence of counting.

On the positive side, Liebesman can account for underspecification: according to Liebesman, the correct count of a group of objects depends on the predicate with respect to which we count. Both whether an object instantiates a predicate P in our world and, if not, for how much it measures according to the partiality-measurement is dependent on the predicate P with respect to which we count. For example, half oranges do not count for one orange if we count with respect to "orange", for they do not instantiate the property of being an orange. However, they do count for one orange with respect to "half orange", for they instantiate the property of being a half orange. Similarly, while quarter oranges measure for 0.25 with respect to the predicate "orange" as they are 0.25 of a whole orange, they measure for 0.5 with respect to "half orange" for they are half of a half orange. Thus, Liebesman correctly predicts that the count of a set of objects depends on the predicate P with respect to which we count.

In summary, Liebesman's theory can account for the underspecification of counting, but neither for the dependence of counting on context and on the objects' kind and source, nor for the fact that counting is graded in felicity.

4.2 Snyder & Barlew's Account

According to Snyder and Barlew, counting statements are ambiguous between a measurement and an individuating reading.[5] The distinction between measurement and individuating readings was introduced by Rothstein (2009, 2010a,b, 2017) appealing to examples such as these:

(10) I bring two glasses of water for my guests.

(11) There are two glasses of water in the soup.

In (10), "glasses" has the individuating reading: the speaker brings her guests two different glasses filled with water. In (11), "glasses" has the measurement reading: the quantity of water in the soup is twice the quantity a standard glass of water contains.

Snyder & Barlew (2019) think that counting statements like "Two oranges are on Titus' plate" are also ambiguous between a measurement and an individuating reading. When we count oranges in measurement contexts, we measure them based on an ad-hoc orange unit and then count the measures of the individual oranges together (Snyder & Barlew, 2019, p. 10): if there are many small pieces of oranges in the soup, we measure each of them against an ad-hoc orange unit provided by context and then sum together the values that these measures yield. If the small pieces together measure for one and a half oranges, we can say that there are one and a half oranges in the soup.

In individuating contexts, however, we count the number of objects that are specific parts of an orange together, e.g. we count the whole orange, the halves of oranges, the thirds of oranges, etc. (Snyder & Barlew, 2019, p. 15). Each of the oranges counts for one, with respect to the part of an orange it is. When one whole orange and one half of an orange are counted in an individuating context, Snyder &

[5]It is controversial whether there is enough evidence for this ambiguity. In order to consider Snyder & Barlew's theory, I will assume that there is an ambiguity between individuating readings and measurement readings, that is different from the mass/count distinction.

Barlew predict that there is one orange and one half orange. They do not predict that there are one and a half oranges, however, as counts in individuating contexts can only contain whole numbers.

4.2.1 Snyder & Barlew and the Data

Can Snyder & Barlew account for the features of counting that were introduced in Chapter 2?

Snyder & Barlew's account suggests two ways in which the kind or source of objects could influence counts. First, they could influence whether a context is a measurement or individuating context. Second, in measurement contexts, the kind or source of objects could indicate the salient ad-hoc unit against which the parts are measured. Differences in ad-hoc units, however, only influence *what* the count of objects is and not *whether* they can be counted. That the latter is precisely what the kind and source of objects influences is shown by the difference between ORANGE TOPS and FRONT CARS: while the halves can be counted in ORANGE TOPS they cannot be counted in FRONT CARS. Hence, only the distinction between individuating and measurement contexts can potentially account for kind and source sensitivity.

The salient reading in ORDINARY CARS and BAGEL BOTTOMS would then have to be the individuating reading. For on the individuating reading, it is true to say that there are three bagel bottoms or three front halves, but not true to say that there are one and a half bagels or cars. This result matches the intuition in ORDINARY CARS and BAGEL BOTTOMS.

To get the desired contrast that the objects in ORANGE TOPS, SPHINX CAR, and MIXED BAGELS can count for one and a half, respectively, the salient reading in those scenarios must be the measurement reading. On the measurement reading, the relevant objects get measured against an ad-hoc unit of an orange, bagel, or car, and count for one and a half if they measure for one and a half relative to this unit. This reasoning would then explain why counting is possible in ORANGE TOPS, SPHINX CAR, and MIXED BAGELS, but not in ORDINARY CAR and BAGEL BOTTOMS.

However, given the similarity of the described scenarios, it is unclear, why ORANGE TOPS, SPHINX CAR, and MIXED BAGELS should be measurement contexts, while ORDINARY CARS and BAGEL BOTTOMS are individuating contexts. In ORDINARY CARS and SPHINX CAR, the counted objects are intrinsically qualitatively identical; this situation makes it particularly implausible that the former is an individuating and the latter a measurement context. Furthermore, replacing a bagel bottom with a top could change a context from an individuating to a measurement context, as this is the only difference between BAGEL BOTTOMS and MIXED BAGELS.

Assuming that all contexts in which we can count partial objects are measurement contexts leads to another problem. Remember that we cannot count three orange halves if they are of very different sizes (e.g., small, medium, and big orange halves), but we can count them if two orange halves are of sufficiently similar size (two small orange halves and one medium orange halve). As we can count the orange halves in the latter case, it has to be a measurement context. Then, plausibly, the former case is also a measurement context. Snyder & Barlew could then potentially account for the infelicity of counting in the case of three different sizes by appealing to the ad-hoc orange unit. If objects differ in size, the appropriate ad-hoc unit may be unclear. This ambiguity, in turn, could lead to inability to count.

However, this ambiguity would carry over to cases with only two different sizes, for in those it is equally unclear which ad-hoc unit is appropriate. It does not help to claim that the ad-hoc unit is a small orange because there are *two* small oranges but only *one* medium orange. This reasoning is problematic because the three orange halves would then measure for more than one and a half: relative to a small orange, a medium orange counts for more than 0.5. Thus, the distinction between measurement and individuating contexts cannot explain kind and source sensitivity.

Further, Snyder & Barlew's theory does not account for degrees of felicity. All counts which they predict to be accessible have the same degree of felicity. Accordingly, they make the incorrect prediction that referring to three front halves of cars with "one and a half cars" (on a junkyard) is equally felicitous as referring to one orange and one half of an orange with "one and a half oranges".[6]

Like Liebesman, Snyder & Barlew can account for underspecification. In measurement contexts, the ad-hoc unit differs depending on the predicate with respect to which we count. When counting with respect to "half oranges", the ad-hoc unit is usually different from the ad-hoc unit when counting with respect to "oranges". This is even more evident when we consider completely different predicates: clearly, the ad-hoc unit for counting with respect to "orange" differs from the ad-hoc unit for counting with respect to "banana". In individuating contexts, we count the number of objects that are specific parts of the predicate with respect to which we count together. Hence, also in individuating contexts counts are only well defined if it is clear with respect to which predicate a set of objects is counted.

In summary, Snyder & Barlew can account for kind sensitivity, source sensitivity, and context dependence. However, this is conditional on BAGEL BOTTOMS being an individuating context and MIXED BAGELS being a measurement context. This distinction seems arbitrary. Assuming an argument can be made for the distinction, Snyder & Barlew still struggle to account for contrasts in counting orange halves of different sizes.

[6] Of course, Snyder & Barlew could supplement their account with an element that predicts gradedness in felicity. However, as gradedness of felicity is linked to kind and source sensitivity, it is not obvious how Snyder & Barlew can predict the *right* differences in felicity without accounting for kind and source sensitivity, too.

4.3 Independent Problems

Liebesman's and Snyder & Barlew's theories both cannot account for all of the five features of counting discussed in Chapter 2. However, both accounts also have independent problems. This section discusses four such problems. First, Liebesman's theory does not account for cases in which we count partial objects that sum to more than one. Second, Liebesman and Snyder & Barlew both rely on the problematic assumption that partial *P*s are not *P*s. Third, Snyder & Barlew's alleged measurement contexts all test as individuating contexts.

4.3.1 Liebesman

In most of the counting scenarios I introduced, the partial objects together count for more than one. For example, Titus has three half oranges, hence three partial oranges, which together count for one and a half oranges.

Importantly, Liebesman's theory cannot account for counts where the partial objects sum to one or more. Liebesman thinks that the sum of the values the partiality-measuring function gives us has to be strictly smaller than one. He, thus, thinks that counting a plurality presupposes that the partial objects in the plurality sum to less than one.[7] If we look at ORANGE TOPS, MIXED BAGELS and MIXED CARS, Liebesman therefore incorrectly predicts that we cannot count in any of these scenarios, for in all of them the second summand of the counting function is one and a half, which is greater than one. All partial objects are counted in the second summand and in all three cases, there are three halves of *P*s that are summed together to one and a half *P*s.

Besides, Liebesman predicts that adding a small part of an object can result in the inability to count the relevant objects, for it can cause a presupposition failure. If Titus has 99/100 of an orange he can count them, however, if he has 2/100 more,

[7] Liebesman only posits this presupposition for count readings, and not for measurement readings (Liebesman, 2016, footnote 5).

he cannot count the orange pieces anymore. This prediction is not intuitive and it is also due to the assumption that partial objects cannot be counted if their sum is ≥ 1.

My guess is that Liebesman introduces this assumption in order to predict that we cannot count several partial Ps as one P. Indeed, in many cases it is odd to refer to partial Ps with "one P". However, I do not think that this strangeness should be explained with an assumption as the one that Liebesman posits. In Chapter 6, I argue that the strangeness of counting two orange halves as one orange results from pragmatics, not semantics.

4.3.2 Are Partial Oranges Oranges?

This subsection makes an additional observation: Liebesman (2016) and Snyder & Barlew (2019) all think that partial Ps are not Ps. Moreover, Liebesman's and Snyder & Barlew's accounts make false predictions if partial Ps are Ps.[8] Based on observations about how we use "orange" and expressions denoting partial oranges like "half orange", I argue that there is some evidence that partial Ps are Ps. The theory of counting developed in the last Chapter is independent of whether partial Ps are Ps, which is an additional advantage of the theory.[9]

Liebesman Thinks Partial Oranges Are Not Oranges

Liebesman states clearly that he thinks that partial Ps are not Ps as he writes "[...] since half-bagels are not bagels [...]" (Liebesman, 2016, p. 2) and "So, are half-

[8]Salmon's (2018) preferred solution to the counting problem does not depend on the assumption that partial oranges are not oranges. Very roughly, his solution is that we count pluralities and not classes with sentences like "Two and a half orange are on the table". While he thinks that half oranges are not oranges, he still thinks that they are among the plurality of oranges on the table. Within this plurality, some objects count for less than one, for example the half orange counts for 0.5. This claim is independent of whether partial oranges are oranges, as in both cases partial oranges can be among the plurality of oranges on the table.

[9]Many of the arguments in this subsection are based on my submitted essay (2020) [reference omitted for marking anonymity reasons] in which I argued that partial Ps are Ps. The claim defended in this section, however, is weaker than the claim defended in that essay. Here, I only claim that a theory that makes the right predictions independently of whether or not partial Ps are Ps is favourable.

bagels in the extension of 'bagel'? I don't have an argument here, but my strong sense is that they aren't" (Liebesman, 2016, p. 4).

Further, Liebesman's account of counting would make different predictions if partial Ps were Ps. Remember that on Liebesman's account, the correct count of a set of objects X with respect to a predicate P is the sum of two summands: the cardinality of the set of Ps and the sum of the values the partiality-measurement function gives us for objects that are not P, but instantiate the property P. Now if partial P were P, the sum in the second part would always sum over nothing, while the first part of the sum would count each partial P for one, as the partial P would then be in the set of Ps. This would lead to the false prediction that in a scenario in which there is one whole and one half orange on a table, "Two oranges are on the table" is true, as there are two objects that are oranges (the whole orange and the orange half) on the table.

Snyder & Barlew Think Partial Oranges Are Not Oranges

Snyder & Barlew (2019, p. 17) write explicitly, "A half orange is definitely not an orange." Further, Snyder & Barlew not only think that partial oranges are not oranges, but their semantics also makes false predictions if partial oranges are oranges. For if an orange half was an orange, their account would predict that "Two and a half oranges are on the table" is true in scenarios in which there is one half orange and one whole orange on a table, for there would then be two things on the table that are oranges (the orange and the orange half) and one thing that is an orange half (the orange half).

Partial Oranges Are Oranges

This section argues that there is some evidence that partial Ps are Ps. This evidence is based on observations about how we use expressions denoting partial Ps and whole Ps. I start by giving three examples that support the claim that partial Ps are Ps, for they show that we can refer to partial Ps with "P".

Consider the conversation in (12).

(12) Q: "Do you still have oranges, Titus?" R: "Yes, I still have oranges."

It seems as if Titus should reply "yes" to (12) even if he only has half of an orange.[10] This suggests that partial Ps are Ps, because speakers can refer to partial Ps with "P".

Now imagine we are at a market where the speaker is pointing at a basket that contains a whole unripe papaya and half a ripe papaya. Somebody utters (13):

(13) "One of the papayas is ripe."

With the term "papayas", the speaker refers to both the whole papaya and the partial papaya. This is shown by the fact that an utterance of "All the papayas are ripe" is false in (13), but true in contexts in which there is only one ripe papaya.

Finally, consider (14), uttered in front of a table on which there is a honeydew melon and one half of a watermelon.

(14) "Some fruits on the table are green."

The speaker in (14) also refers to the partial watermelon with "fruit" when uttering (14). As (12)-(14) show that we can sometimes refer to partial Ps with "P", they suggest that partial Ps are Ps.

Objections

I now consider some objections concerning the data presented in favour of partial Ps being Ps. First, one might object that (12)-(14) do not have an influence on whether partial oranges are oranges, because metaphysical questions are not about language

[10] Note that it is already interesting that a speaker who has exactly one orange should reply "yes" to this question, given that "oranges" is a bare plural. However, the reply in (12) seems infelicitous if a speaker has one orange, but felicitous if they have one and a half oranges. Hence, the plural in the reply seems to require more than one oranges. That having an additional half is sufficient to make assertions of the plural felicitous further indicates that an orange half is an orange.

in any distinctive sense. However, I will assume that it is good to preserve as much of common sense as possible (Lewis, 1986; Daly, 2010, p. 15). Other things being equal, a theory that does not presuppose that partial Ps are Ps is then preferable to Liebesman's or Snyder & Barlew's, as it preserves more of common sense.

Other objections to the argument have a common form: They argue that examples in which we refer to partial Ps with the term "P" all involve a specific semantic phenomenon. Because of this phenomenon, "P" can refer to partial Ps in a specific context. There is no need to assume that partial Ps are Ps, as the phenomenon explains why in specific contexts we can refer to partial Ps with "P".

In the following, I discuss two such phenomena, the mass/count distinction and the measurement/individuating reading distinction. I argue that none of them can explain why it is possible to refer to partial Ps with "P" in (12)-(14).

Many nouns are ambiguous between a mass reading and a count reading. Roughly, on the mass reading terms like "orange"$_{MASS}$ can be thought as standing for "orange stuff". It is very plausible that partial Ps are in the extension of "orange"$_{MASS}$. If in (12)-(14) the nouns had the mass reading rather than the count reading, partial Ps could still not be in the extension of "P"$_{COUNT}$.

However, the mass reading normally requires the noun to be in the singular as in (15) (Rothstein, 2010a, $346-347$).

(15) "There is bagel all over the floor."

(16) "There are bagels all over the floor."

(15) is true if a dog spit out chewed bagels on the floor.[11] (16), however, is not true in this scenario, but seems to require that each partial bagel can be identified as a unique object.[12] Because in (12)-(14) the bare plural (henceforth "BP") is in the singular, (12)-(14) do not have the mass reading, but the count reading.

As introduced earlier, Rothstein has recently argued for a distinct ambiguity of

[11]This example is due to Liebesman (Liebesman, 2016, 4).

[12]Some might claim that (16) is only true if there are only whole bagels, I do not think that this needs to be the case, as long as the partial bagels are not too small fractions of a bagel.

nouns: the ambiguity between individuating and measurement readings (Rothstein, 2009, 2010a,b, 2017). Remember that "glasses" is thought to have this ambiguity, as shown by (17) and (18).

(17) "I bring two glasses of water for my guests."

(18) "There are two glasses of water in the soup."

In (17), "glasses" has the individuating reading: it says that the speaker brings her guests two different glasses filled with water. In (18), "glasses" has the measurement reading: the quantity of water in the soup is twice the quantity a standard glass of water contains.

On the measurement reading, partial *P*s are plausibly part of the extension of "*P*". So if in (12)-(14) "oranges" is "oranges"$_{MEAS}$, it could still be that partial oranges are not part of the extension of "oranges"$_{INDIV}$.

I will introduce two tests that indicate whether a BP has the measurement or individuating reading.[13] The first test is that it is possible to insert adjectives in front of the noun if the noun has the individuating reading, but not if it has the measurement reading (Liebesman, 2015, 34). An utterance of (19) is felicitous, while an utterance of (20) is not.

(19) "I bring two (expensive, heavy) glasses of water for my guests."

(20) # "There are two (expensive, heavy) glasses of water in the soup."

Similarly to (17), we can insert an adjective in utterances of (21)-(23):

(21) "Do you still have (delicious, ripe) oranges?"

(22) "One of the (beautiful, expensive) papayas is ripe."

(23) "Some (delicious, ripe) fruits on the table are green."

[13]There are more tests that Rothstein proposes for the distinction (Rothstein, 2010b), I will introduce two that apply nicely to (12)-(14), though according to the other tests, (12)-(14) also do not test as having the measurement reading.

According to this test then, (12)-(14) have the individuating reading. The second test is that it is only possible to insert "each" on the individuating reading (Rothstein, 2010b, 4). While an utterance of (24) is felicitous, an utterance of (25) is not.[14]

(24) "The two glasses of wine on the tray cost two Pounds each."

(25) #"The two glasses of wine in this soup cost two Pounds each."

In (12)-(14) it is possible to insert "each", as (26)-(28) are felicitous. So also according to the second test, (12)-(14) have the individuating reading.

(26) "The oranges I have are each delicious" (Uttered in ORANGE TOPS.)

(27) "How ripe are these papayas each?"

(28) "Green fruits on the table cost two Pounds each."

So (12)-(14) test as including the individuating and not the measurement reading. This is crucial as most people would allow that partial *P*s are in the extension of "P"$_{MASS}$, but claim that partial *P*s are not in the extension of "P"$_{INDIV}$ (Liebesman, 2016; Salmon, 1997; Snyder & Barlew, 2019). As (12)-(14) tested as having the individuating meaning, also on the individuating reading partial *P*s seem to be *P*s.[15]

The New Theory and Partial Oranges

The theory of counting developed in the last Chapter does not make the assumption that partial *P*s are not *P*s. On the developed theory, we can count a set of objects with respect to a predicate P iff we can merge the objects such that they are sufficiently similar to whole normal *P*s and at most one partial normal P. Independently of whether partial *P*s are *P*s, they certainly are not whole *P*s. Hence, my account

[14] I am unsure whether (25) really is infelicitous. As Rothstein thinks it is, I will assume so as well.

[15] If partial oranges are oranges, the sentence "All oranges are round," would be false because half oranges are not round. However, I think we can invoke quantifier domain restriction or ellipsis to preserve the truth of "All oranges are round" even if partial oranges are oranges.

makes the right predictions while staying neutral with respect to whether partial Ps are Ps.

As the linguistic evidence in favour of partial Ps being Ps is strong, but not conclusive, it is desirable that a theory of counting remains neutral on whether partial Ps are Ps until further evidence is gathered. While Liebesman's and Snyder & Barlew's theories could be refined in order to make the right predictions if partial Ps should turn out to be Ps, the developed theory already makes the right predictions independently of whether partial Ps are Ps.

4.3.3 Snyder & Barlew

I introduced two tests to distinguish between measurement and individuating readings when discussing the data in favour of partial Ps being Ps. Interestingly, these tests also serve as an additional argument against Snyder & Barlew's account.

Remember that in order to account for kind and source sensitivity on Snyder & Barlew's account, all scenarios in which we can sum partial objects together have to be measurement contexts. Hence, Orange Tops, Mixed Bagels, and Mixed Cars all have to be measurement contexts.

But these scenarios test as having the individuating reading. The two introduced tests were that we can only insert adjectives in front of nouns in individuating contexts and that we can only refer to the objects with "each" in individuating contexts. In examples like Orange Tops, Mixed Bagels or Mixed Cars, it seems as if we can do both, as the following can be uttered felicitously in Mixed Bagels:

(29) "On Martha's and Johanna's plates, there are one and a half delicious bagels. Each of the bagels will be eaten in a bit."

This suggests that Orange Tops, Mixed Bagels, and Mixed Cars are individuating contexts. If that is right, Snyder & Barlew do not predict the accessible count that there are one and a half bagels on the plate in Mixed Bagels and the analogous counts in Orange Tops and Mixed Cars.

4.4 Conclusion

This Chapter showed that other theories of counting, i.e. Liebesman's and Snyder & Barlew's theories of counting, cannot account for the data introduced in Chapter 2. It further showed that both theories have independent problems. While Liebesman wrongly predicts that we can never count partial objects that sum to more than one, Snyder & Barlew's distinction between measurement and individuating contexts seems unsupported by tests that help to distinguish such contexts. Finally, both accounts would make wrong predictions if partial Ps turn out to be Ps, in support of which there is some linguistic data. Taken together, these problems serve as a good reason to favour the developed theory of counting over Liebesman's and Snyder & Barlew's theories.

Chapter 5

Refinements of the Theory

Chapter 3 introduced a new theory of counting. We can count objects with respect to a predicate P iff we can *merge* them to form objects that are sufficiently similar to whole Ps and at most one partial P. I showed that this account explains the five characteristics brought out in Chapter 2. I also showed, in the last Chapter, that my theory performs better than its competitors. However, there are cases in which the proposed theory makes incorrect predictions and which, therefore, call for refinements of the theory. This Chapter introduces three such cases and suggests how the proposed theory can be refined in order to make the right predictions.

The structure of this Chapter is as follows: first, I discuss cases in which my theory predicts double counting and propose how the theory is to be refined to avoid double counts. Second, I explain why counting is even more source sensitive and underspecified than my theory predicts and suggest how the theory could be refined to do justice to these findings. In the third section, I show that apart from ordinary counts, we sometimes perform combinatorial counts and that the presented theory already offers a framework to account for them.

5.1 Double Counting

Sometimes, we count a group of objects with respect to different predicates. For example, when we count the different fruits in a fruit bowl:

> FRUIT BOWL: Susanna is a world-famous circus artist. She is currently staying at the Randolph in Oxford. In her hotel room is a bowl with different fruits. There are three bananas, four grapefruits, and two kiwis in the fruit bowl. However, to prepare for her next performance, Susanna needs three oranges to practice the fruit-juggling part. Susanna calls her manager and complains: "As usual, the Randolph did not give me any oranges. There are only three bananas, four grapefruits, and two kiwis in my fruit bowl."

Predicting the correct count, i.e. three bananas, four grapefruits, and two kiwis, in FRUIT BOWL is straightforward.[1] If we count the bananas, grapefruits, and kiwis separately, we end up with the correct individual counts, which can then be conjoined to count all fruits in the bowl.

This explanation, however, does not work for similar scenarios in which we count with respect to a predicate P and a predicate T, where Ts are partial Ps.

> BOWL CONTAINING PARTIAL FRUITS: In the meantime the Randolph has got wind that Susanna wants oranges in her fruit bowl. But the hotel employees continue to be ignorant of the fact that Susanna does not want to eat the oranges, but wants to juggle with them. They place a fruit bowl with a whole orange, four half oranges, and two quarter oranges in Susanna's room. Susanna is not happy and complains: "This time, I got three and a half oranges, seven half oranges, and fourteen quarter oranges."

[1] Of course we could also count the fruits in FRUIT BOWL as nine fruits, as predicted by the developed theory.

Susanna's utterance in Bowl containing Partial Fruits is false. But my theory of counting predicts that if we count the objects in Susanna's fruit bowl with respect to "oranges", we count them as three and a half oranges. With respect to "half oranges" we count seven half oranges, and with respect to "quarter oranges" we count fourteen quarter oranges. If we would combine these counts (like I suggested we do in Fruit Bowl), we would wrongly predict that Susanna's fruit bowl contains three and a half oranges, seven half oranges, and fourteen quarter oranges. The problem with such a count seems to be that we count some objects twice (the orange halves) and some thrice (the orange quarters).

It is generally accepted that counting an object more than once in one count is not permissible (Berkey, 2015; Lewis, 1986, 1991; Liebesman, 2020; Varzi, 2014; Resnik, 1987, p. 51). Lewis (1986, p. 252), for example, writes: "It reeks of double counting to say that here we have a dishpan, and we also have a dishpan-shaped bit of plastic that is just where the dishpan is, weighs just what the dishpan weighs (why don't the two together weigh twice as much?), and so on."

5.1.1 Non-boolean Conjunction

But how do we prevent double (or triple) counting in examples such as Bowl containing Partial Fruits? I suggest that the "and" in both Fruit Bowl and Bowl Containing Partial Fruits is the collective, non-boolean "and" (Hoeksema, 1988, 1983b; Krifka, 1990). The following discussion shows why. If in Bowl Containing Partial Fruits we would ask Susanna how many oranges she has, her reply "I have three and a half oranges" is felicitous. If we then ask her how many half oranges she has, it seems equally fine for Susanna to say that she has seven orange halves, and if, finally, we ask how many quarter oranges she has, she can felicitously reply that she has fourteen quarter oranges. Hence, taken individually, assertions of "There are three and a half oranges", "There are seven half oranges", and "There are fourteen quarter oranges" are felicitous in Bowl

Containing Partial Fruits. As assertions of these sentences are all felicitous in Bowl Containing Partial Fruits, we should be able to assert their boolean conjunction too. Since asserting their conjunction is infelicitous, the conjunction is non-boolean.

That there are non-boolean conjunctions that link noun phrases is widely acknowledged (Champollion, 2013; Heycock & Zamparelli, 2005; Link, 1984; Winter, 2002). From sentences such as "Hannah and Larissa are carrying a piano" the following two sentences do not follow: "Hannah is carrying a piano" and "Larissa is carrying a piano." Instead of conjoining these two sentences, the "and" in "Hannah and Larissa are carrying a piano" conjoins the noun phrases "Larissa" and "Hannah" to form the collection of Hannah and Larissa. It is this plurality that is carrying the piano, and not one of its individual parts.

Similarly, Susanna states with "I have three and a half oranges, seven half oranges, and fourteen quarter oranges" that she has a collection of objects, consisting of three and a half oranges, seven half oranges, and fourteen quarter oranges. But this is false: the collection of objects only consists of one orange, four half oranges, and two quarter oranges. In contrast, it would be true for her to say "I have one orange, four orange halves, and two quarter oranges", for the collection of objects in her bowl consists of one orange, four orange halves, and two orange quarters. Equally, it is felicitous to assert any of "I have three and a half oranges", "I have seven half oranges", and "I have fourteen quarter oranges", for these are all ways in which we can count Susanna's collection of oranges.

Krifka (1990, p. 164), however, thinks that collection is indempotent, i.e. that the collection of Hannah and Hannah is equivalent to the collection of Hannah. Based on this, one may think that the collection of Hannah, Larissa, and Hannah's arm is the same as the collection of Hannah and Larissa. But if that was the case, Susanna's utterance "I have three and a half oranges, seven half oranges, and fourteen quarter oranges" should be felicitous as well, for her collection consists of three and a half oranges, seven orange halves (overlapping with the oranges), and fourteen orange quarters (overlapping with the oranges and the orange halves). Luckily, there is independent reason to think that the collection of overlapping parts is not equivalent to the collection of the non-overlapping parts and that, hence, coreferential NPs cannot always be conjoined by non-boolean "and": if Hannah and Larissa are lifting a piano together, we cannot say "Hannah, Larissa, and Hannah are lifting a piano." However, if the collection of Hannah, Hannah, and Larissa is equivalent to the collection of Larissa and Hannah, we should be able to felicitously assert the sentence, for we can felicitously say "Hannah and Larissa are lifting the piano."[2]

5.1.2 Counting Collections

Given that the "and" in Susanna's utterance is non-boolean, how do we predict that she has one orange, four half oranges, and two quarter oranges? Crucially, if we count a collection with respect to different predicates, we look at a partition of the objects in the collection, where for each different predicate with respect to which we count, we look at one cell of the partition. Since an object can be maximally in one cell, we count every object exactly once and thus prevent counting an object multiple times.

Put formally, if we count a set of objects X with respect to different predicates $P_1, P_2, ..., P_n$, we first look at a partition G of the objects in X such that: $\forall S \in G((\exists x \in S(x\#P_y)) \rightarrow (\forall z \in S(z\#P_y)))$, where $x\#P_y$ means that we count an object x with respect to the predicate P_y. Then we count the elements of G with respect to their associated predicates and conjoin the counts of the different predicates with the non-boolean "and".

[2] Hoeksema (1988, p. 30) remarks that this may not be true for coreferential NPs like "Cicero" and "Tullio", for we can say "Cicero and Tullio are one and the same person." But in other cases, we cannot conjoin "Cicero" and "Tullio". Imagine Cicero is lifting a piano by himself. Then we can say "Cicero is lifting a piano," or "Tullio is lifting a piano," but not "Cicero and Tullio are lifting a piano."

5.1.3 Pragmatically Infelicitous Counts of Collections

This suggestion, however, would license the count "One and a half oranges and four half oranges are in the fruit bowl" of objects in Susanna's fruit bowl. For the count is licensed if we divide the fruits in two subsets (one containing the whole orange and the orange quarters, and one containing only the orange halves) and count the first subset with respect to "oranges" and the second subset with respect to "half oranges". While such a count appears true, it feels unnatural. The reason for the strangeness of the count is due to counting the orange quarters as whole oranges, but the half oranges as half oranges: if we count orange halves and whole oranges with respect to different predicates, we should also count whole oranges and orange quarters with respect to different predicates.

Thus it seems as if normally, when we count Fs and Ps separately, where Fs are partial Ps and could, thus, be counted with the Ps, we also count other predicates that could be counted with the Ps separately. If we count oranges and orange halves separately, we tend to also count orange quarters separately. I will now discuss how we can account for this observation pragmatically.

Mental Merging Is Costly

Plausibly, mentally merging objects is more costly than not mentally merging them both in terms of mental processing and also in terms of coordinating communication. We argued earlier that plausibly, when we evaluate counts, we evaluate whether and how a set of objects satisfies the Counting Constraint. Sometimes, this includes mentally merging objects. But if we are given the option not to mentally merge, that option is psychologically less costly, for it saves the step of evaluating whether objects can be merged to form an object that is sufficiently similar to a normal P. Additionally, mentally merging objects requires more coordination, for it requires speakers to coordinate on normality. What Ps are normal depends on context. Hence, to assert a count that requires mental merging, speakers have to additionally assume that everybody involved in the conversation agrees on what Ps are normal. This is not the case with counts that do not rely on mental merging, for usually salient Ps count as normal.

The costliness of mental merging can explain the above contrast as follows: if we count the objects in Susanna's fruit bowl as "One and a half oranges and four half oranges are in the fruit bowl", we introduce the option to not mentally merge objects, for the four half oranges are not mentally merged, even though they could be. Given the option of saving the cost of mentally merging, we should also count the oranges and quarter oranges separately, for this would be even more cost efficient. All of this, however, is pragmatic, for it only has to do with our psychological processes and not with the semantics of counting. Hence, while "One and a half oranges and four half oranges are in the fruit bowl", "One orange, four half oranges, and two quarter oranges are in the fruit bowl", and "Three and a half oranges are in the fruit bowl" are all true counts of Susanna's fruit, the first count is not cost efficient and, hence, pragmatically infelicitous.

But the costliness explanation has a significant shortcoming. For it cannot explain why it is better (though still weird) to count Susanna's fruits as one orange and five half oranges than to count them as one and a half oranges and four half oranges. From a cost efficiency perspective, both counts have the same cost efficiency, for both involve the mental merging of objects even though the context introduces the option not to mentally merge. Hence, the costliness of mental merging cannot explain the difference in felicity.

Levels of Granularity

To explain the contrast between counting Susanna's fruits as one orange and five half oranges and counting them as one and a half oranges and four half oranges we note that similar cases occur when we use different levels of granularity.

Granularity frameworks are linked to the pragmatics of loose talk. We can often assert that we have one kilogram of flour, even if we have one kilogram and two grams. To explain this, many philosophers and linguists appeal to granularity frameworks (Thomas & Deo, 2020; Gyarmathy, 2017; Kennedy & McNally, 2005; Krifka, 2007; Sauerland & Stateva, 2011; Solt, 2015a).[3] The general thought is that we can use scales with different levels of granularity. Using "one kilogram" to refer to one kilogram and two grams is often felicitous, for we may be using a scale that has no minimal size unit for one kilogram and two grams. Thus, "one kilogram" may be the closest unit to one kilogram and two grams on the scale we are using.

However, if we say that we have one kilogram and one gram of butter, we cannot refer to one kilogram and two grams of flour with "one kilogram" anymore. The reason for this is that using "one kilogram and one gram" to measure butter implies that we are using a scale that is quite fine grained. If we use such a scale to measure butter, we should use the same scale to measure the flour.

Importantly, similar contrasts in felicity as the one between counting Susanna's fruits as one orange and five half oranges and counting them as one and a half oranges and four half oranges occur with respect to granularity scales. If we have 1 kilogram and 750 grams of flour, it is true but strange to say that we have 1.2 kilograms and 550 grams. Normal counts of the flour in such scenarios are "1.75 kilograms" and "1 kilogram and 750 grams". Thus, we seem to either count everything together and have a non-natural numbers for the kilograms of flour, or split things up in grams and kilograms and use natural numbers for both. Equally, we prefer referring to Susanna's fruits with one orange and five half oranges for then we use natural numbers for both the whole and the half oranges we are counting. This is not the case if we count Susanna's fruits as one and a half oranges and four half oranges, where we have a non natural number counting the oranges and a natural number counting the orange halves. Hence, whatever is going on in the granularity case seems to explain the contrast in felicity between the two counts of Susanna's fruits.

[3] Alternatively, such cases of loose talk can be explained using pragmatic halos (Lasersohn, 1999; Morzycki, 2011).

But this explanation cannot account for the strangeness of counting twelve orange quarters as one orange and four half oranges. For in such a count, both the number of whole oranges and the number of half oranges are natural numbers. Still, it seems more natural to count the orange quarters as three oranges or as six orange halves. This difference in felicity can be explained by yet another analogy to levels of granularity. It is strange to refer to $2,250$ grams of flour with "1 kilogram and $1,250$ grams". Instead, we can felicitously refer to $2,250$ grams with "$2,250$ grams" or "2 kilograms and 250 grams". If we are counting kilograms and grams separately, and we can convert $1,000$ grams to 1 kilogram, we should do so. Similarly, if we count twelve orange quarters as one orange and four half oranges, we should also convert the four half oranges to oranges.[4]

However, we still need to supplement this explanation with the finding that mental merging is costly, for otherwise we cannot explain why it is felicitous to count one whole orange and two orange halves as one orange and two orange halves. In such a scenario, it seems fine not to count the two halves as one orange. The reason is the costliness of mental merging: given that we have the option of not merging objects, it is felicitous not to do so and to count the whole oranges and orange halves separately.

5.2 Counting Cucumbers

So far, we have assumed that all partitions that satisfy the Counting Constraint for a set of objects X have the same associated count. This assumption, however, is not fully justified, because there are cases where several partitions with different associated counts satisfy the Counting Constraint. Additionally, sometimes several counts are associated with the same partition. In this section, I discuss problems related to such cases and propose three ways in which we need to refine the

[4]Counting 13 orange quarters as six and a half oranges is odd, too. Similarly, it is odd to count $1,750$ kilograms of flour as 3.5 times 500 grams of flour.

developed theory to account for them. First, partitions that satisfy the COUNTING CONSTRAINT are ordered with respect to the similarity of their merges to normal Ps. Second, to calculate the count of a set of objects, we have to sum the degrees to which normal Ps that are *maximally* similar to the merges (and not only *sufficiently* similar) are whole normal Ps. The count of a set is the count that is associated with the best, i.e. highest ordered, partition. Third, if several partitions are best or the best partition has different counts associated with it, it is vague which count is correct: counts are then only true if we restrict the set of possible Ps to a well-defined subset, but not true in general.

To get a first idea of the problem, consider CUCUMBER PIECES:

> CUCUMBER PIECES: Sherlock Holmes loves posing riddles. Today, he leads you to a table with five pieces of cucumbers on it. All of the pieces have exactly the same diameter d and the same length l. Two of the cucumber pieces are end pieces of a cucumber and three cucumber pieces are middle pieces. Sherlock asks you: "How many cucumbers are on the table?"

What is the correct count in CUCUMBER PIECES? As cucumbers differ in their length, there are possible cucumbers with diameter d of many different lengths, including $1l$,$2l$, $3l$, $4l$, and $5l$. But then, Sherlock's cucumber pieces can be merged to be sufficiently similar to a whole cucumber of length $5l$, but also to be sufficiently similar to a whole cucumber of length $4l$ and a partial cucumber of length l. Accordingly, it seems as if there are many ways of counting the cucumber pieces. We could, for example, count them as 1, or as 1.25 cucumbers.

However, if all of Sherlock's cucumber pieces came from the same $5l$ cucumber, the only correct answer to Sherlock's question is "One cucumber is on the table" and not, e.g., "1.25 cucumbers are on the table". Conversely, if four pieces came from the same $4l$ cucumber and the fifth piece came from a second $4l$ cucumber, the only correct count of the five cucumber pieces is "1.25 cucumbers", and not "1 cucumber".

Our current theory of counting does not predict this additional source sensitivity, since the COUNTING CONSTRAINT does not take the object's source into account. Independently of their source, the cucumber pieces can be merged in different ways, among them merges that are similar to one $5l$ cucumber as well as merges that are similar to a $4l$ cucumber and a quarter of a $4l$ cucumber.[5]

5.2.1 Counting Is Very Source Sensitive

What explains the additional source sensitivity is that the merge of a group of objects with the same source (a whole P) is always maximally similar to that source or a part thereof, for they are almost identical.[6] In addition, sources usually count as normal Ps. Therefore, the merge of objects of the same source is more similar to its source or a part thereof than to all other normal Ps.

Remember that the developed theory predicts that counts feel more natural the more similar the relevant merges are to normal Ps. Hence, the theory already predicts that the most felicitous count is associated with the partition that groups objects of the same source together, for in that way we arrive at merges that are maximally similar to normal Ps.[7]

But predicting differences in felicity is not sufficient to account for the additional source sensitivity. Counting five pieces of the same $5l$ cucumber as one cucumber is not only more felicitous than counting them as 1.25 cucumbers, but is in fact the only available account. I propose that contrary to my earlier assumption that every

[5]One might think that the additional source sensitivity shows that the developed theory of counting should be replaced by a theory according to which we can count objects iff they have the same source. However, there are many scenarios in which we count objects that are obviously of different sources. Imagine Titus, Bjarne, and Johanna all brought one orange half to their picnic. Despite it being obvious that the halves originate from different oranges, it is felicitous to say that they have 1.5 oranges. Thus, a theory according to which we can count objects iff they have the same source is not favourable to the presented theory.

[6]The merge of only one object is even identical to its source or a part thereof.

[7]If the source of the partial objects is an abnormal P, it seems odd to think that the sources of salient partial objects are always normal. This indicates that possibility, and not normality, might be the relevant notion: sources are more possible than other Ps, for they were once an actual P that consisted of the salient partial Ps.

partition that satisfies the COUNTING CONSTRAINT has an associated felicitous count, we give precedence to partitions with merges that are maximally similar to normal Ps. To do so, we order partitions with respect to similarity. The highest ordered partitions are those with merges that are maximally similar to normal Ps. The count that is associated to the highest ordered partitions is the correct count (in the Appendix B, I formalize this proposal).[8]

However, sometimes, a highest ordered partition still has more than one associated count. Remember that when we introduced the count associated with a grouping G, for every element S of G we selected a P that was sufficiently similar to the merge of S. The count associated with G was the sum of the degrees to which the selected Ps are whole Ps. But if we count five cucumber pieces of which four originated from a $4l$ cucumber and one originated from a second $4l$ cucumber, the merge of the single cucumber piece is not only sufficiently similar to a quarter of a $4l$ cucumber, but also to a fifth of a $5l$ cucumber. Importantly, depending on whether the cucumber fourth or the cucumber fifth is selected, we arrive at different counts. The correct count, however, is only the one that counts the single piece for 0.25 of a cucumber. This is because the single piece is maximally similar to a quarter of its source, because they are identical. Hence, instead of selecting a P that is sufficiently similar to the merge of S, we need to select the P that is maximally similar to S. Then we only select the quarter of a $4l$ cucumber, but not the fifth of a $5l$ cucumber and thereby arrive at the right count (a refined formal version of the counting function can be found in the Appendix B).

Giving precedence to partitions with merges that are maximally similar to normal Ps and redefining the counting function such that it selects maximally similar Ps explains the additional source sensitivity of counting. If Sherlock's five pieces originate from the same $5l$ cucumber, the count of the cucumber pieces is "one cucumber". This is because the partition that groups all five pieces together has a

[8]There are different ways to derive a similarity ordering of sets from a similarity ordering of individuals, which yield different results. I discuss these in the Appendix B.

merge that is maximally similar to a normal object, i.e. the source of the five pieces, which counts for one cucumber.

Had four of the pieces originated from a $4l$ cucumber and one of them had originated from a different $4l$ cucumber, the count of the cucumber pieces would be "1.25 cucumbers". In this case, the partition with merges that are maximally similar to normal Ps is the one that groups the four pieces that originated from the same cucumber together. The remaining piece is grouped with no other pieces. As the merge of the four cucumber pieces is maximally similar to their source, they count for one cucumber. The merge of the remaining cucumber piece is maximally similar to a quarter of its source and, hence, counts for 0.25. Thus, we predict the count of 1.25 if four of Sherlock's cucumber pieces had originated from the same $4l$ cucumber and the remaining piece originated from a different $4l$ cucumber.

5.2.2 Counting Can Be Highly Underspecified

But how would we count the pieces in Cucumber pieces had they originated from different cucumbers? In cases like Orange Tops, it is clear that we count Titus' three top halves of oranges as one and a half oranges even if they originated from three different oranges. But there are various ways to count the cucumber pieces of five different cucumbers, e.g. as 1 or as 1.25 cucumbers. This is not a case of underspecification as discussed in previous Chapters. In those cases, counting is only underspecified regarding the predicate with respect to which we count. In Cucumber Pieces, however, we end up with different counts with respect to the same predicate, i.e. "cucumber". Thus, counting is not only more source sensitive, but also more underspecified than we thought!

This underspecification seems to be caused by two factors. First, there is more than one best partition with more than one associated count. For example, the partition that groups all objects together and the partitions that group four objects together and one alone are comparable in the similarity of their merges to normal

Ps. Second, even if we select one partition, the refined measurement function does not give us a unique count. If, e.g., we look at a partition that groups four objects together and one alone, the merge of the single piece is very similar to a third of a normal $3l$ cucumber, but also comparably similar to a fourth of a normal $4l$ cucumber, or a fifth of a normal $5l$ cucumber. Importantly, depending on the length of the cucumber to which the single piece is similar, we arrive at different counts, e.g. $1.\overline{3}$, 1.25, or 1.2. Hence, as the maximally similar Ps yield different counts, there is no unique count of the cucumber pieces.

Both of these factors occur because the set of whole cucumbers contains both an object x and a proper part thereof and, contrary to source cases, there is no best way of merging and counting the objects. This explanation is accurate, for when we restrict the set of whole possible cucumbers to the set of $4l$ cucumbers, we arrive at a unique count, as illustrated by $4l$ Cucumbers:

> $4l$ Cucumbers: Sherlock Holmes leads you to a table with five pieces of cucumbers on top. All of the pieces have exactly the same diameter d and the same length l. Two of the cucumber pieces are end pieces of a cucumber and three cucumber pieces are middle pieces. Sherlock asks you: "Assuming cucumbers are $4l$ long, how many cucumbers are on the table?"

In $4l$ Cucumbers, we count the cucumber pieces with respect to cucumbers that are of length $4l$. As all possible whole $4l$ cucumbers have the same length, i.e. $4l$, all groupings of the five cucumber pieces that satisfy the Counting Constraint form one merge that is sufficiently similar to a whole possible cucumber and one merge that is sufficiently similar to a quarter of a possible cucumber. Further, both merges are measured with respect to $4l$ cucumbers. Thus, we arrive at the unique count "1.25 cucumbers" of the cucumber pieces with respect to the predicate "$4l$ cucumbers".

I propose that in highly underspecified cases, it is vague which well-defined restriction of the predicate gives us the right count. A restriction is well defined iff it restricts a set of objects X to a subset S such that for no x in S, S contains a proper part of x, too. To account for the involved vagueness, I use van Fraassen's (1966) theory of supervaluations. According to van Fraassen, partially defined semantic interpretations correspond to classes of completely defined evaluations. A sentence is supertrue only if it is true under all evaluations. Accordingly, we can think of the set of possible whole Ps which contains both an object x and a proper part of x as the class of all subsets S of the set of possible whole Ps such that for no x in S, a proper part of x is in S, too.

As Cucumber Pieces is underspecified (assuming the pieces originate from different cucumbers), we treat the set of whole cucumbers as the class of all its subsets that do not contain both an object x and a proper part thereof. That class then contains the subsets of $5l$ cucumbers, $4l$ cucumbers, $3l$ cucumbers etc. Importantly, if we evaluate the count of the cucumber pieces with respect to those subsets, we arrive at unique counts. For the subset of $4l$ cucumbers, we end up with a count of 1.25 cucumbers, which is different from the count we obtain with respect to $5l$ cucumbers, i.e. "1 cucumber".[9] When we count in underspecified cases, we do not predict a true count, for it is not the case that all well-defined subsets yield the same count. Still, the classes of completely defined evaluations correspond to ways in which the set of possible whole Ps could be restricted in order to deliver a true count.[10]

[9]Plausibly, some restrictions do not count as normal. For example, cucumbers of length 100 rarely count as normal. Hence, a restriction to $100l$ cucumbers does not provide us with an accessible count. As normality is context dependent, the context also influences what cucumber lengths count as normal.

[10]Note that it follows from this discussion that we can only count with respect to cucumbers of different lengths in source cases.

5.3 Combinatorial Counts

Sometimes, we count objects in a way that the developed theory does not predict, for we seem to count possible combinations of objects. Such combinatorial counts have been discussed in the debate about contingentism and necessitism (Fritz & Goodman, 2017; Rosefeldt, 2017; Tomasetta, 2010; Williamson, 2013, 2000c,a, 1998, cf.). In the following, I will not discuss the implications of combinatorial counting for metaphysics. Instead, I will only be concerned with capturing their truth conditions.

Consider Elia's Wardrobe:

> Elia's Wardrobe: Elia owns two t-shirts and two trousers. Lea asks Elia "How many outfits do you own?"

Intuitively, the answer to Lea's question is four, since Elia can combine his t-shirts and trousers to the following four outfits: T-shirt$_1$ + Trouser$_1$, T-shirt$_1$ + Trouser$_2$, T-shirt$_2$ + Trouser$_1$, and T-shirt$_2$ + Trouser$_2$.

However, my theory of counting predicts that Elia has two outfits. For the partition $\{\{$T-shirt$_1$+ Trouser$_1\}, \{$T-shirt$_2$+ Trouser$_2\}\}$ of the set of Elia's clothes satisfies the Counting Constraint: the merges of its elements are both sufficiently similar to possible whole normal outfits. Thus, we predict that Elia has two outfits.

Note first that in some contexts, "two" is a correct count of Elia's outfits. One such context is Masquerade Party:

> Masquerade Party: Lea, Rolf, and Beatriz want to dress up as Elia. To do so, they ask whether they can all wear his clothes. Elia tells them that he only has two outfits.

Masquerade Party shows that the developed theory does not make an incorrect prediction for counting Elia's outfits, because "two" is a correct count of his outfits. Instead, there seem to be two different ways in which we can count objects with respect to a predicate P. The first one tells us how many Ps we can form of the objects in X, whereas the second one tells us how many different Ps *could* be formed out of the objects in X. My theory predicts the correct count for the first way of counting. This way of counting is arguably more common than the second way, which I will call combinatorial counting. Note that in contrast to the ordinary way of counting, combinatorial counts are always whole numbers.

In the following, I discuss Rosefeldt's and Williamson's accounts of combinatorial counts. I then show that an account of combinatorial counts is also derivable from the developed theory of counting. Finally, I consider cases in which combinatorial counts are strange and which, thus, cast doubt on whether combinatorial counting is an additional way of counting objects.

5.3.1 Rosefeldt's Suggestion for Combinatorial Counts

One may think that ELIA'S WARDROBE does not suggest an ambiguity in how we count, but is instead an instance of a well-researched ambiguity of nouns. Nouns can denote both kinds and individual objects belonging to that kind. The difference between combinatorial counts and normal counts could then be that we count individual objects in normal counts and kinds in combinatorial counts. Indeed, Rosefeldt (2017) suggests that what is happening in cases like ELIA'S WARDROBE is that we count kinds and not natural objects. In ELIA'S WARDROBE, we quantify over kinds of outfits. Hence, when we count four outfits, we say that there are four kinds of outfits consisting of t-shirt$_1$, t-shirt$_2$, trouser$_1$, and trouser$_2$. Rosefeldt finds this approach particularly promising for it does not force him to accept the existence of possible objects, but only to there being infinitely many kinds.

However, assume that for something to be a kind K, it has to be the case that if an object x is a K, all y such that y is intrinsically qualitatively identical to x are also Ks. Together with this assumption about kinds, Rosefeldt cannot account for all combinatorial counts. To see why, consider ONE TROUSER:

> One Trouser: Elia has only one trouser (trouser$_1$), and two intrinsically qualitatively identical t-shirts (t-shirt$_1$, and t-shirt$_2$).

There are two ways to count how many outfits Elia has in One Trouser. We can count one outfit, for both possible outfits are intrinsically qualitatively identical, but we can also count two outfits, for there are two possible combinations: trouser$_1$ + t-shirt$_1$, and trouser$_1$ + t-shirt$_2$. If my assumption about kinds is correct, Rosefeldt can only account for the first count. As t-shirt$_1$ and t-shirt$_2$ are intrinsically qualitatively identical, they necessarily are of the same kind. But then, the outfits trouser$_1$ + t-shirt$_1$, and trouser$_1$ + t-shirt$_2$ are also of the same kind and, accordingly, we only count one kind. As there is only one kind, Rosefeldt cannot predict one of the correct counts, i.e. "two outfits", in One Trouser.

There are two replies to this objection. First, plausibly no two non-numerically identical t-shirts can be intrinsically qualitatively identical. Then a case like One Trouser is impossible. Second, assuming that the first reply is not effective, the assumption we made may be problematic. If one thinks of kinds as a purely theoretical construct such that for every property there exists a corresponding kind, then it may well be that there are kinds which have x as a member but not y despite y being intrinsically qualitatively identical to x.

5.3.2 Williamson's Suggestion for Combinatorial Counts

Let us now examine whether Williamson's (1998; 2000a; 2000c; 2013) suggestion for the truth conditions of combinatorial counts can account for all cases of combinatorial counting. Williamson thinks that when we perform combinatorial counts, we count possible objects. Setting metaphysical worries aside (Fritz & Goodman, 2017; Rosefeldt, 2017), there seems to be a linguistic problem with Williamson's solution, for there are cases in which the solution appears not to deliver the right counts. In particular, as argued by Rosefeldt (2017), there are scenarios in which the theory *undercounts*.[11] To see why, consider TRENDSETTER:

> TRENDSETTER: Elia is a trendsetter and owns two t-shirts and two trousers. Recently, he has started to wear some of his clothes inside out. When Elia composes a new outfit, for each piece of clothing, Elia can either wear it inside out or wear it traditionally.

In TRENDSETTER, Elia has sixteen outfits. For every combination of the four combinations of pieces of clothing (t-shirt$_1$ + trouser$_1$, t-shirt$_1$ + trouser$_2$, t-shirt$_2$ + trouser$_1$, and t-shirt$_2$ + trouser$_2$), there are four ways in which Elia could wear it. He could wear both his t-shirt and trousers inside out, he could wear both in the traditional way, he could wear only his t-shirt inside out, or he could wear only his trousers inside out. Hence, Elia seems to have $4 * 4$ outfits, i.e. sixteen. But if one assumes that no two different possible objects can consist of exactly the same objects (Williamson, 1998, p. 267), Williamson cannot predict "sixteen" as a count of Elia's outfits, but only the count of four outfits. For the four possibilities of wearing the same combination, e.g. the combination of t-shirt$_1$ + trouser$_1$, are all instances of the same possible object, i.e. the object consisting of t-shirt$_1$ and trouser$_1$.

To avoid this objection, it seems plausible to abandon the assumption that no two different possible objects can consist of exactly the same objects. For TRENDSETTER just seems to show that more than one possible outfit can be made of the same t-shirt and trouser:[12] Elia *actually* has sixteen outfits. As everything that is actual is possible, he also has sixteen possible outfits.

[11] Rosefeldt (2017) argues that there are also cases in which Williamson overcounts. This seems incorrect, for plausibly, Williamson can appeal to quantifier domain restriction in those cases. Rosefeldt himself appeals to quantifier domain restriction to save his own account from overcounting.

[12] Williamson (1998, p. 267) introduces the assumption that at most one possible outfit can be made out of one t-shirt and one trouser as follows: "Suppose, for simplicity, that a suit consists of a jacket and a pair of trousers, that necessarily they make a suit just in case they are originally hung together, and that at most one possible suit can be made of a given jacket and pair of trousers." Hence, it may well be that Williamson thinks that the assumption that at most one possible outfit can be made of a given jacket and pair of trousers is mainly helpful to simplify things and not true in general.

5.3.3 Combinatorial Counts and Possible Merges

Both Williamson's and Rosefeldt's account seem to be in a good position to account for the truth conditions of combinatorial counts. However, there is an alternative account of combinatorial counts that is derivable from my theory of counting:[13] what happens in ELIA'S WARDROBE is that we count all the different ways in which the objects in X can be merged to be sufficiently similar to a possible whole normal outfit, where two merges are different iff they are not sufficiently similar. In ELIA'S WARDROBE, there are four different merges, i.e. those which are sufficiently similar to t-shirt$_1$ + trouser$_1$, t-shirt$_1$ + trouser$_2$, t-shirt$_2$ + trouser$_1$, and t-shirt$_2$ + trouser$_2$, respectively. The combinatorial count of a set X with respect to P is just the cardinality of the set of merges of objects in X that are sufficiently similar to possible whole normal Ps and different from each other (in the Appendix C, I state the proposed theory of combinatorial counting formally). As the cardinality of the set {T-shirt$_1$ + Trouser$_1$, T-shirt$_1$ + Trouser$_2$, T-shirt$_2$ + Trouser$_1$, T-shirt$_2$ + Trouser$_2$} is 4, the combinatorial count of Elia's outfits in ELIA'S WARDROBE is "four".

This account of combinatorial counts also explains why combinatorial counts are always natural numbers: a combinatorial count is the cardinality of a set and cardinalities of sets are always natural numbers (at least the cardinalities of sets with finitely many elements).

Trendsetter and Possible Merges

This account of combinatorial counting also makes the right predictions in TRENDSETTER: the merge of t-shirt$_1$ and trouser$_1$, where both are worn inside out, is not sufficiently similar to the merge of t-shirt$_1$ and trouser$_1$, worn traditionally. Hence, we predict that Elia has sixteen outfits in TRENDSETTER.

[13] I will remain silent regarding the metaphysical implications of this account of combinatorial counting, and only argue that it accounts for the linguistic data.

But if outfits that are worn inside out differ from traditional outfits, how do we predict that Elia has four outfits in ELIA'S WARDROBE? There are two mechanisms that explain restrictions of counts. First, in many contexts, outfits that are worn inside out are not *normal* outfits, and hence only count in contexts in which they are normal. Second, not only normality is graded, but also similarity. Moreover, similarity is graded in two ways. First, similarity is graded such that where x and y are both similar to z, but x is *more* similar to z, x has a higher grade of similarity than y. Second, depending on context, we are only interested in the similarity of certain contextually salient properties. For example, in ELIA'S WARDROBE, outfits only need to consist of different parts, whereas in TRENDSETTER, the outfits also have to be worn in the same way. Importantly, to predict the combinatorial count, we look at the cardinality of the set of possible merges, where all merges need to be different, i.e. are not sufficiently similar. As sufficient similarity is context dependent, two merges can can be different in one, but not another context.

There is a further reason to think that which merges count as sufficiently similar can vary by context. Imagine Elia has one trouser (trouser$_1$) and two t-shirts (t-shirt$_1$ and t-shirt$_2$). T-shirt$_1$ is blue and has a v-neck, t-shirt$_2$ is blue and has a round neck. Apart from the different type of t-shirt neckline, the two t-shirts are the same. Now it seems as if we can say both that Elia has 1 outfit and that Elia has two outfits, depending on whether we consider two outfits that vary only in having t-shirts with different necklines as different. If similarity is context dependent, we can account for both counts in a straightforward way.

One Trouser and Possible Merges

But what about the problem we raised against Rosefeldt, i.e. that we can sometimes count intrinsically qualitatively identical outfits for more than one? If we use a graded notion of similarity, such that numerical identical objects are predicted to be more similar than objects which are intrinsically qualitatively identical, but not numerical identical, we can explain why Elia has two outfits in One Trouser: while the two merges are intrinsically qualitatively identical, they are not numerically identical. Hence, there are contexts in which we can count them as two outfits.

5.3.4 Combinatorial Counts of Orange Halves

If we can count a set of objects in two ways, i.e. in the normal and in the combinatorial way, it is surprising that there are cases in which it is hard to get a true reading of combinatorial counts. If Titus has three half oranges, it it is false to say that he has three oranges. But "three oranges" is a combinatorial count of his halves, for the set of possible merges is {orange half_1 + orange half_2, orange half_1 + orange half_3, orange half_2 + orange half_3}.

Note that this is not only a problem for my account of combinatorial counting, but also for Williamson's and Rosefeldt. They also have to explain, why in some contexts, it is strange to count possible objects or kinds.

I think what might explain the strangeness of combinatorial counts in many contexts is that all the merges of orange halves are normally considered to be sufficiently similar, i.e. are not different enough. What makes this more plausible is that we can get a true reading of "There are three oranges" if we modify the above scenario. Imagine a future where genetic engineering has unlocked many new possibilities. For example, it has made possible that Titus has one orange half that tastes like a banana, one that has a cherry taste, and one that actually tastes like an orange. Then it is easier to get a true reading of Titus having three oranges, for those potential oranges have important differences: one tastes like cherry and banana, one like orange and banana, and one like orange and cherry.

With this explanation, however, it is still strange that in the normal case, it is hard to get a true reading of "Titus has one orange", which we would predict as a true reading if all the potential merges are sufficiently similar.

5.4 Conclusion

This Chapter discussed several scenarios that put pressure on the developed theory of counting. In response, I refined and expanded the developed theory in three ways. First, to avoid double counting when counting with respect to different predicates, we partition the objects in X before counting them. Then we count different elements of this partition with respect to different predicates. Second, we saw that counting is more source sensitive and underspecified than we thought. We showed that we can account for these features by ordering partitions, redefining the counting function, and supplementing the theory with a theory of vagueness. Third, we can sometimes perform combinatorial counts. A combinatorial count of a set X with respect to P is the cardinality of the set of different merges of objects in X that are sufficiently similar to possible whole normal Ps. The following appendix offers more rigorous presentations of my solutions to the last two problems.

5.5 Formal Appendix B

In section 5.2, I have explained informally how we can account for the additional source sensitivity of counting. I proposed that we order partitions with respect to their similarity. This appendix shows how this proposal can be integrated into the formal account of the developed theory of counting. I discuss three ways in which we can derive a similarity ordering for sets from the similarity ordering introduced in the Appendix A. I also refine the counting function to select maximally similar normal Ps.

5.5.1 A Similarity Structure for Sets

Remember that we introduced a similarity structure $\langle T, \leq^S \rangle$ in the Appendix A, where T is a non-empty set of pairs and $\leq^S$ is a reflexive and transitive relation on T. "$\langle r, s \rangle \leq^S \langle t, u \rangle$" reads "$r$ is at least as similar to s as t is similar to u".

To order partitions with respect to similarity, we derive an ordering of sets of pairs from the introduced ordering of pairs. The sets of pairs that we need to order are sets of pairs of all the merges of a partition G_Y and the normal$_c$ P they are most similar to. We define the set of merges M_{G_Y}, where for every element S in G_Y, the x in $C(S)$ such that x is maximally similar to y, where y is a normal$_c$ P, is an element of M_{G_Y} and nothing else is an element of M_{G_Y}.[14] For every x in M_{G_Y}, there is exactly one corresponding pair $\langle x, y\rangle$ in the set of pairs T_{G_Y}, where y is the object in the set of normal$_c$ P that is most similar to x.[15]

There are many ways to derive an ordering of sets of pairs from the ordering of pairs I introduced in the Appendix A. I discuss three of them.

Incomplete Ordering

First, one might define an ordering of sets of pairs as follows: $X \leq^{S_{sets}} Y$ iff $\forall\langle x, y\rangle \in X, \forall\langle z_1, z_2\rangle \in Y, \langle x, y\rangle \leq^S \langle z_1, z_2\rangle$.

The problem with such an ordering is that it is incomplete, i.e. it does not give us an ordering of two sets S and R such that $\exists\langle x_1, y_1\rangle \in S, \exists\langle x_2, y_2\rangle \in R$ $(\langle x_1, y_1\rangle \leq^S \langle x_2, y_2\rangle)$ and $\exists\langle x_3, y_3\rangle \in R, \exists\langle x_4, y_4\rangle \in S$ $(\langle x_3, y_3\rangle \leq^S \langle x_4, y_4\rangle)$. Hence, we will look at other options.

Maximum Ordering

If we order sets of pairs with respect to their maxima, i.e. the pairs with maximal symmetry, we obtain a complete ordering. According to such an ordering, $X \leq^{S_{sets}} Y$ iff $max(X) \leq^S max(Y)$, where $max(X)$ gives us an object in x such that $\forall z \in X(x \leq^S z)$.

However, such an ordering would rank a partition with one extremely similar pair and ten sufficiently similar pairs higher than a partition with ten highly (but

[14]In cases where for a set S there are several elements of $C(S)$ that are maximally similar to a normal P, we select one of those elements.

[15]Again, if there are several normal$_c$ Ps that are maximally similar, we select one of them.

not extremely) similar pairs. This is not accurate, for the latter partition seems more similar than the former.

Ordering Average Similarity

If we assign numerical degrees of similarity to pairs, we can order partitions with respect to their average similarity. To do so, we define a function n that takes pairs to their degree of similarity. $n(\langle x, y\rangle)$ is the degree to which x is similar to y. Then we define a similarity structure of pairs as follows: a similarity structure of sets of pairs is a pair $\langle R, \leq^{S_{sets}}\rangle$, where R is a non-empty set of sets of pairs and $\leq^{S_{sets}}$ is a reflexive and transitive relation on S. $T \leq^{S_{sets}} U$ iff $\frac{\sum_{\langle x,y\rangle \in T} n(\langle x,y\rangle)}{|T|} \leq^{S} \frac{\sum_{\langle x,y\rangle \in U} n(\langle x,y\rangle)}{|U|}$.

This, I think, is the right way to order partitions with respect to similarity for it provides us with a complete ordering that weighs the similarity of each pair equally. However, it does require more structure than the other two proposed orderings as we need to assign numerical degrees of similarity to pairs.

5.5.2 The Counting Function Refined

We also noted in section 5.2 of this Chapter that we have to refine the counting function in order to sum the degrees to which normal$_c$ Ps that are *maximally* similar to merges are whole normal$_c$ Ps.

Accordingly, we redefine Sim_{G_Y} to be the set of Ps such that for every element x of M_{G_Y} there is exactly one y in Sim_{G_Y} such that y is maximally similar to x and no other elements are in Sim_{G_Y}. Intuitively, Sim_{G_Y} is the set of normal$_c$ Ps to which the merges of G_Y are maximally similar.

(30) $\#(X, P)_c = \sum_{x \in Sim_{G_Y}} d(x, \boldsymbol{normal_c\ whole\ P})$

The count of a set X with respect to P is the sum of the degrees to which the objects in Sim_{G_Y} are normal$_c$ whole possible Ps. Importantly, the count is only well defined if the degree function d assigns a unique degree to every x in Sim_{G_Y}.

5.6 Formal Appendix C

Formally, the combinatorial count $\#_c$ of a set of objects X that is counted with respect to the predicate P is as follows:

(31) $\#_c X = |PC|$, where the set of possible merges PC is such that :

1. $\forall x, y \in PC(x \neq y \rightarrow \neg(x \sim_c y))$
2. $\forall x \in PC \exists y \in \boldsymbol{normal_c whole\ P}(x \sim_c y)$
3. $\forall x \in PC \exists S \in \wp(X) \exists y \in C(S){:}\ y = x$

The combinatorial count is the cardinality of the set of possible merges PC of X, where all objects in PC are different from each other (i.e. not sufficiently similar), all objects in PC are sufficiently similar to a normal$_c$ whole P, and every object in PC is a merge of an element of the powerset of X ($\wp(X)$).

Chapter 6

Objections to the Theory

I introduced a theory of counting in Chapter 3 and refined this theory in the last Chapter. This Chapter discusses six objections against the proposed theory. These objections share a common structure. They all claim that the developed theory yields incorrect predictions for some counts. In response to the first five objections, I argue that my theory either does not make the disputed predictions or that the allegedly incorrect predictions are in fact correct. My response to the sixth objection remains more speculative.

First, I discuss whether counting is more interest relative than my theory predicts. Second, I examine whether we can count one orange as two orange halves and vice versa. In the third section, I look at the difference between counting animated and inanimated objects. Fourth, I discuss scenarios in which we appear to *mentally slice* objects before counting them. The fifth section discusses counting small pieces of a P as Ps, while the sixth section investigates why we can sometimes count n Ps for more than n Ps.

6.1 Interest Relativity

In Chapter 2 we noted that while we usually cannot count three front halves of a car together, we can do so in special contexts, e.g. on a junkyard. Hence, JUNKYARD showed that counting is context dependent: intrinsically qualitatively identical objects can sometimes be counted in one, but not in another context. The reason for this asymmetry is that what counts as normal is context dependent. Further, JUNKYARD illustrated that counting is interest relative. The reason why a car composed out of two front halves is sufficiently normal in JUNKYARD is that on a junkyard, people are mainly interested in the amount of metal of a car. Given this interest, a car composed of two front halves is normal. Hence, we can count two front halves as a car.

However, one might think that counting is more interest relative than the proposed account predicts. To see why, consider PICKY JUGGLER:

> PICKY JUGGLER: Susanna is preparing for her circus performance tomorrow. To practice, she needs three oranges for the fruit-juggling part of her performance. Hence, she asks for three oranges at the hotel reception of the Randolph. When six half oranges that originate from three whole oranges are brought to her, Susanna is furious.

PICKY JUGGLER suggests that whether we can count two orange halves as an orange depends on Susanna's interests. Since Susanna needs whole oranges for juggling, we cannot count partial oranges together in PICKY JUGGLER. In contrast to the interest relativity in JUNKYARD, my theory cannot account for interest relativity in PICKY JUGGLER. To see why, consider an orange which we slice in two halves. The merge of the two orange halves is similar to the same possible orange as the original whole orange. Plausibly, it is also sufficiently similar to a possible whole orange, as the two halves have the same size and, thus, fit on each other perfectly. But then, both the original whole orange and the merge of its two halves can be counted as one orange iff the possible orange they are similar to is sufficiently normal. My account cannot distinguish between an object and the merge of all its parts, for they are usually sufficiently similar to the same object. Thus, we predict that also in PICKY

JUGGLER, six halves of oranges count as three oranges, for they originate from three whole oranges.

However, I do not think that a theory of counting should account for the the interest relativity demonstrated by PICKY JUGGLER. This gets evident if we consider an analogous example:

> JUGGLING EGGS: As Susanna did not get whole oranges from the hotel, she decides to juggle with eggs. She calls the reception and orders three eggs. When three fried eggs are brought to her, Susanna is again incensed.

Surely, three fried eggs are three eggs. Susanna is not angry because she did not receive three eggs. Rather, Susanna is angry because she thought it was clear that she needed whole eggs in order to juggle with them. Thus, the interest relativity of JUGGLING EGGS is pragmatic, and not semantic. PICKY JUGGLER is an instance of the same pragmatic phenomenon: Susanna thought it was evident that she needed whole oranges and is furious, because she thinks the hotel is being uncooperative.

While JUNKYARD demonstrates that counting is inherently interest relative, some instances of interest relativity in counting scenarios, e.g. PICKY JUGGLER, are not due to the semantics of counting but are due to more general pragmatic phenomena.

6.2 Counting One Orange as Two Halves and Vice Versa

If Titus has a whole orange, it would be strange if Bjarne asked him "Could you pass me one of your two orange halves?" More generally, it often seems highly misleading to say of somebody who has an orange that they have two orange halves. My theory of counting, however, predicts such utterances to be felicitous. Every whole orange consists of two orange halves. Hence, if we count an orange with respect to "orange half", it counts for two orange halves.

Even worse, it is also usually misleading to say of somebody who has two orange halves that they have one orange. Consider a case in which there are two orange halves on Titus' table. It would be strange if Titus asserted "One orange is on the table." The proposed theory of counting predicts this utterance to be true, because the merge of two orange halves is usually sufficiently similar to a whole orange.

I think that the infelicity of both "One orange is on the table" and "Could you pass me one of your two orange halves?" in the described scenarios is no evidence for the falsehood of the proposed theory of counting. The reason for the infelicity of both utterances, I take it, is pragmatic. This is shown by there being exceptions in which we can felicitously refer to two orange halves with "one orange" and vice versa.

Imagine that Titus wants to make fresh orange juice for breakfast for which he needs one orange. After a long search of the kitchen, he finds two orange halves in the fridge and says relieved, "Hooray, I do have an orange after all, enough to make orange juice." Analogously, there are contexts in which we can refer to a whole orange with "two orange halves". If, e.g., you get one dollar for every half orange you have, it is natural to say that you have two half oranges, when in fact you have one whole orange.

Hence, referring to a whole orange with "two orange halves" or to two orange halves with "one orange" is only pragmatically infelicitous. Pragmatic infelicity can still explain the potential to mislead of such assertions, for hearers usually assume (if compatible with their evidence) that the speaker is in a situation where their assertion is not pragmatically infelicitous.[1] This potential is, thus, not due to the

[1]It has been discussed extensively whether it is possible to mislead with false conversational implicatures. Plausibly, the general mechanism for misleading with pragmatically infelicitous assertions is similar. Pace Adler (1997); Horn (2009, 2010, 2016, 2017); Mahon (2016); and Saul (2012b), Meibauer (2014) and Viebahn (2017, 2019) think that speakers can also lie with false implicatures. Should this turn out to be the case, the speakers in the discussed examples could be accountable not only for misleading, but also for lying. It is still a matter of ongoing discussion,

semantics of counting but due to pragmatics.

6.3 Merging Animated Objects

One may think that the presented theory only predicts the right counts for counting *inanimated* objects, but not for counting animated objects. To see why, think of a modified version of the story "Judgement of Solomon" from the Bible:

> Judgement of Solomon: Two women come to Solomon. They claim to be the mother of the same child and ask Solomon for advice. Solomon cuts the baby in two halves and passes one half to each mother. He says "Now we have one baby and two happy mothers."

Plausibly, we can merge the two baby halves to form a whole baby. My theory would then predict that we can count the two baby halves as one baby. But it seems like we cannot count the two halves as one baby. Most certainly, the two mothers would object to Solomon's utterance.

Further, that we are unable to count the two halves together is not due to the pragmatic infelicity of counting two half Ps for one P which we discussed in the last section. Imagine that Solomon already had one half baby. It seems as if we cannot count the half baby and the two baby halves as one and a half babies, whereas we can count half a baby and an untouched baby as one and a half babies. Hence, one may think that my theory only delivers the right predictions for counting inanimated objects, but not for counting animated objects.

But upon consideration, my theory does not predict that we can count the two baby halves. Remember that we can count a set X with respect to a predicate P, iff we can mentally merge the objects in X to form objects that are sufficiently similar to whole *normal* Ps and maximally one partial *normal* P. Crucially, in Judgement

however, whether there is an ethical difference between lying and misleading. See Adler (1997); Berstler (2019); Chisholm & Feehan (1977); Pepp (2019); Strudler (2010) and Webber (2013) for arguments for an ethical difference and (Saul, 2012a,b) and Williams (2002) for arguments against.

of Solomon and in most contexts, a baby merged out of two halves is not a normal baby. One important property of normal babies is that they are alive. A merged baby, however, is not alive. Still, there are contexts in which a merged baby is a normal baby. Imagine, for example, that cannibals are talking about what they had for dinner. If they ate two halves of a baby, it is perfectly felicitous for them to say that they ate one baby. Similarly, if we think of a baby elevator that can carry the weight of up to two babies, it seems as if the elevator's capacity is reached no matter whether two babies or four baby halves are in the elevator.

While Judgement of Solomon does not show that my theory of counting only works for inanimated objects, it demonstrates two interesting points. First, objects of the same source (a whole P) cannot always be merged to form a whole or partial P. While the baby halves have the same source, we cannot merge them to form a whole baby. Second, parts of perfectly symmetric objects cannot always be merged. For the sake of the argument, assume that babies were perfectly symmetric. That they are symmetric would not change the judgement concerning Solomon's utterance in Judgement of Solomon: two half babies would still not count as one baby.

The reason for those two points seems to be the following: in some contexts, whole normal Ps have emergent properties, i.e. properties that are over and above the properties of their parts. In such contexts, merges are not sufficiently similar to whole normal Ps even if they all originated from the same P or are parts of perfectly symmetric objects, because merges of partial Ps cannot have emergent properties.

This observation provides us with an important insight into merging objects. One might have thought that when we merge cucumber pieces that originate from the same cucumber, we turn back time to undo cutting the cucumber into pieces. But then, merges would also have the emergent properties of whole normal Ps. Instead, merging objects is more alike to a mechanic mechanism such as sticking the cucumber pieces together with glue. This understanding of merging objects explains why merges do not have the emergent properties that their sources have.

6.4 Mental Slicing

Sometimes, we can count objects even though we cannot mentally merge them to form objects that are sufficiently similar to whole normal Ps and maximally one partial normal P. Consider THREE QUARTER CAKES:

> THREE QUARTER CAKES: The philosophy faculty is hosting a party with lots of cakes for their hungry students. When the party is over, there are three quarters of a round cheesecake and three quarters of a round red velvet cake left. Lea decides to take the leftovers home. She calls her flatmates and says "I am bringing one and a half cakes for dinner!"

Interestingly, we can count the cakes in THREE QUARTER CAKES as one and a half cakes, even though it seems like we cannot merge them to form objects that are sufficiently similar to a whole and a partial normal cake. Thus, as it stands, my theory does not predict Lea's utterance in THREE QUARTER CAKES to be felicitous.

There are two ways forward. First, one might think that in THREE QUARTER CAKES, "cakes" has a measurement or mass reading, which explains why we can sum two three quarter cakes to one and a half cakes. For both measurement readings and mass readings, it is plausible that we measure objects with respect to an ad-hoc unit, rather than count them. On such readings, we can thus count partial objects even if they do not satisfy the COUNTING CONSTRAINT. Second, we could try to explain the scenario within the developed theory: sometimes, we can *mentally slice* objects before we mentally merge them.

To see whether "cakes" could have the mass reading in THREE QUARTER CAKES, remember the test we introduced in Chapter 4: the mass reading normally requires the noun to be in the singular (Rothstein, 2010a, 346 – 347). As "cakes" is not in the singular but in the plural in Lea's utterance, "cakes" does not have the mass reading.

Could the relevant reading of "cakes" in Three Quarter Cakes be a measurement reading? Remember that on the measurement reading we cannot insert adjectives before the noun. However, had Lea said instead that she is bringing one and a half delicious (or fresh, vegan, etc.) cakes, her utterance would still have been perfectly felicitous. Thus, neither the mass/count reading distinction nor the measurement/individuating reading distinction can help us to explain why we can count the partial cakes in Three Quarter Cakes.

However, the intuitive explanation for why we can count the pieces in Three Quarter Cakes is that if we would have cut a quarter of one of the cakes of, we could have mentally merged this quarter with the other three quarter cake to form a whole cake. Together with the remaining half of a cake, we would have ended up with one and a half cakes.

I think that we sometimes *mentally slice* objects, i.e. divide an object x into two parts, which together form x, if that would result in being able to count them. Importantly, my account already allows for slicing objects. Up until now, when we discussed whether a set of objects can be counted, all partitions that could potentially satisfy the Counting Constraint shared a common feature: they all grouped spatially continuous objects in the same subgroup. When we partitioned a set X, we never split an object $x \in X$ up. However, if we abandon this assumption, we can account for the count of one and a half cakes in Three Quarter Cake in a straightforward way: the Counting Constraint is satisfied by partitions that have as one element the set containing a quarter of the cheesecake and the three-quarter red velvet cake, and as their second element the set containing the remaining half of the cheesecake. The count of this partition is one and a half cakes, for the first element is sufficiently similar to a whole normal cake and the second element is sufficiently similar to a half normal cake. Importantly, in most contexts spatially continuous objects are grouped together, for if the Counting Constraint can be satisfied in that fashion there is no need to slice objects up.

There seem to be two constraints on slicing objects. First, how natural counts that involve slicing are depends on how many times an object has to be sliced. Second, it matters how we divide objects. Slicing objects feels most natural with cuts that are easy to grasp, and it feels less normal if we slice objects in more complicated ways.

The proposed account by itself does not explain this. But think again of the connection between psychology and semantics, i.e. that when we assess the truth of counting sentences, we check whether they can be mentally merged. In the same way in which merging objects is connected to mentally merging objects, slicing objects is connected to mentally slicing objects. To evaluate whether objects can be sliced to fulfill the COUNTING CONSTRAINT, we mentally slice them. With this at hand, we can explain the two constraints on mental slicing. If objects need to be sliced in a complicated way or very often to fulfill the COUNTING CONSTRAINT, it is very hard for us to realize that they can be mentally sliced to fulfill the constraint. Hence, in such contexts, it is difficult for us to work out the right count of objects. This fact does not mean that there is no true count of the relevant objects, but only that we cannot evaluate their true count due to our limited cognitive capacities. The easiest counts are those for which we do not have to mentally slice any object, which also explains why such counts feel the most natural. Counts that involve mental slicing are also cognitively more costly, for they require more effort to check whether we can mentally merge the relevant objects.

6.5 Counting Small Pieces

So far, my theory has placed no restriction on how small partial objects can be in order to still be counted. SMALL PIECES suggests that we cannot count a large amount of very small parts of a P as whole or partial Ps:

SMALL PIECES: Sherlock has yet another riddle for you. He leads you to

> a plate with small cucumber pieces on it. Unbeknownst to you, Sherlock had cut one cucumber and one half of a cucumber into 1000 pieces and placed all of them on the plate. He asks you: "How many cucumbers are on the plate?"

The cucumber pieces in Small Pieces can be merged to form objects that are sufficiently similar to one and a half normal cucumbers. Thus, my theory makes the prediction that one and a half cucumbers are on the plate. This is surprising for it does not seem as if we can count the cucumber pieces in Small Pieces at all. Note first that this problem is not unique to the proposed theory. Liebesman (2016) also predicts that $1,000$ pieces of a cucumber consisting of $2,000$ pieces, can be counted as half a cucumber.[2]

6.5.1 Knowledge and Assertion

How can we explain the tension between the prediction my theory makes and the intuitively correct answer? I take it that while "one and a half cucumbers" is a correct count in Small Pieces, we are not in a position to assert that count, because we cannot know (without counting the pieces) that the $1,000$ cucumber pieces can be merged to form objects that are sufficiently similar to one whole and one half normal cucumber. Hence, we are unable to evaluate the truth of the count. Counting in Small Pieces is similar to counting in Jar of Marbles:

> Jar of Marbles: Unbeknownst to you, Sherlock has 1734 marbles. He puts all of them in a jar and asks: "How many marbles are in the jar?"

While it is true that there are $1,734$ marbles in Sherlock's jar, we are not in a position to know that because we cannot rule out that there are, e.g., $1,735$ marbles instead (unless we count all the marbles). More generally, Jar of Marbles is

[2]On Liebesman's account, even $1,000$ end pieces of $1,000$ different $2,000$-piece cucumbers can be counted as half a cucumber.

an instance of margin for error cases as discussed by (among others) Williamson (2000b, chapter 5), Egré (2006), and Graff (2002). If Sherlock has n marbles, then you cannot know that Sherlock does not have $n+1$ marbles. Hence, you cannot know that Sherlock has n marbles for you cannot exclude for every $m \neq n$ that Sherlock has m marbles.

Similarly, while it is true that there are one and a half cucumbers in Small Pieces, we are not in a position to know that the pieces can be merged to form objects sufficiently similar to one whole and one half normal cucumber, which is why we cannot know their count. Analogously to Jar of Marbles we cannot exclude that the pieces can be merged to be sufficiently similar to, e.g. 1.75 cucumbers, because we cannot (without counting the pieces) know whether there are $1,000$ or $1,166$ cucumber pieces on Sherlock's plate. Furthermore, even if we knew the number of the cucumber pieces on the plate we cannot easily know whether the pieces can form merges that are sufficiently similar to one whole and one half normal cucumber, because we cannot know (without checking) whether we have the *right* pieces, and not, e.g., only pieces of the cucumber shell. However, if we would glue the cucumber pieces together as if they were a $1,000$ pieces cucumber $3D$ puzzle, we can come to know that the cucumber pieces can be assembled to be sufficiently similar to one whole and one half normal cucumber.

Hence, if one assumes a knowledge norm of assertion, as proposed among others by Unger (1978), Moore (1962), and Williamson (1996, 2000b), according to which one should only assert p if one knows that p, we can explain why it is infelicitous to assert that there are one and a half cucumbers in Small Pieces: usually, we cannot know that the $1,000$ pieces can be merged to be sufficiently similar to one whole and one half normal cucumber. Thus, even though the pieces can be merged to be sufficiently similar to one whole and one half normal cucumber, it is normally infelicitous to assert that there are one and a half cucumbers in Small pieces.[3]

[3]The smaller the parts get, the more difficult it is to count them together. While it also gets more difficult to know whether the parts in question can be merged in a way that satisfies the

6.6 The Iberian Lady

THE IBERIAN LADY is a sculpture by Manolo Valdés. The sculpture is situated in Valencia and depicts the silhouette of the Lady of Elche (the Lady of Elche is an Iberian artifact from the 4th century BC). It is itself made out of $22,000$ smaller sculptures of the silhouette of the Lady of Elche. When looking at THE IBERIAN LADY, we seem to be able to count $22,001$ silhouettes of the Lady of Elche.

However, the developed theory does not predict this count. To satisfy the COUNTING CONSTRAINT, we partition the set of $22,000$ sculptures, such that every sculpture is in one cell of the partition. Every cell of the partition then has a merge that is sufficiently similar to a normal sculpture. Hence, we predict a count of $22,000$ Ladies of Elche, but not of $22,001$. Moreover, combinatorial counting also does not help us to predict the right count. For there are many more possible merges than $22,001$. For example, we do not count the possible merge of 500 small sculptures that is sufficiently similar to a medium-sized sculpture of the Lady of Elche. In general, cases like THE IBERIAN LADY occur when we count n Ps, which together compose more, e.g. m, Ps.[4] The right count in such scenarios seems to be $n + m$.[5]

Cases like THE IBERIAN LADY are interesting, for they allow us to double count (we count an object both as a whole P and as part of a different whole P) but leave less freedom for double counting than combinatorial counts do. While I do not yet have a solution for how to integrate counts like THE IBERIAN LADY into the developed theory of counting, I would like to make two remarks about such cases.

First, many philosophers (Berkey, 2015; Lewis, 1986, 1991; Liebesman, 2020; Varzi, 2014; Resnik, 1987, p. 51) think that double counting is bad (and as we have

COUNTING CONSTRAINT, one may want to have an additional explanation for this difficulty. One option is that we count objects with respect to P iff they are partial P. Hence, we cannot count objects with respect to P that are parts of Ps, but not partial Ps. While a molecule of a cucumber is a part of a P, it is usually not a partial P. This could explain why we can count small pieces of cucumbers (that are still partial cucumbers) as cucumbers, while we cannot count molecules of cucumbers as cucumbers.

[4] There are of course conceivable scenarios of higher order, i.e. cases in which whole Ps compose whole Ps, which again compose whole Ps, and so on.

[5] Varzi (2014, p. 52) discusses such cases with respect to fusions.

seen in Chapter 5, it often is). Importantly, everybody who thinks double counting is bad has to explain why we can double count in THE IBERIAN LADY. Second, the main reason for why we can double count in THE IBERIAN LADY lies in the way the small sculptures are arranged: we could not count $22,001$ sculptures if the $22,000$ small sculptures were not arranged in a way that forms an additional, larger sculpture. These cases seem comparable to logic riddles, in which one is asked to count the number of triangles or squares in a picture. Importantly, the arrangement matters in such cases: we can only count merges of small squares as squares if they are *visible* without rearranging any objects. Hence, what might explain such cases is that we can count merges of merges iff they are already visible. Building on this insight, future research will hopefully help to shed more light on cases in which we can count n Ps for more than n Ps.

6.7 Conclusion

In this Chapter, I replied to six objections against the developed theory of counting. First, I showed that while counting is inherently interest relative, some instances of interest relativity in counting scenarios are not due to the semantics of counting but due to more general pragmatic phenomena. Second, counting two orange halves as one orange or one orange as two orange halves can be misleading because assertions of such counts are often pragmatically infelicitous. Third, my theory predicts that we usually cannot merge parts of P to form a normal P if P is an animated object. This observation provided us with a valuable insight into merging objects: when we merge objects to form a whole P, we cannot arrive at emergent properties of normal whole Ps. In the fourth section, I noted that we can sometimes place parts of the same object x in different cells of a partition to satisfy the COUNTING CONSTRAINT. Accordingly, we sometimes *mentally slice* objects before counting them. Fifth, assuming a knowledge norm of assertion, we often cannot felicitously assert counts of very small pieces because we cannot *know* whether they can be merged in a way that satisfies the COUNTING CONSTRAINT. Lastly, we can sometimes count n Ps for more than n Ps. While my theory does not yet capture such cases, the arrangement of the counted object seems critical in such cases.

Chapter 7

Conclusions

In this thesis, I have proposed and defended a novel theory of counting partial objects. In the Introduction, I discussed the philosophical relevance of counting, and why we should aim for a unified theory of counting that yields the right predictions both for counting partial and for counting whole objects. In the second Chapter I observed that, although counting is often effortless, counting data is surprisingly hard to systematize. Systematizing counting data is difficult because of five special features of counting: kind sensitivity, source sensitivity, context dependence, gradedness in felicity, and underspecification. Based on these five features, I developed a novel theory of counting in the third Chapter. I argued that counting is tightly connected to the concept of *merging* objects. We can count objects with respect to a predicate P iff they are mergeable, i.e. iff they can be merged to be sufficiently similar to whole normal Ps and at most one partial normal P. The proposed account is built around a context-dependent, predicate-sensitive, and graded notion of mergeability and successfully explains all the intriguing characteristics of counting. Together with an assumption about the relation between psychology and semantics, the importance of mental merging can be derived from my theory: when we count objects, we mentally merge them. In the fourth Chapter, I showed that current theories of counting partial objects (Liebesman, 2016; Snyder & Barlew, 2019) cannot account for the five features of counting. Further, I remarked that Liebesman's and Snyder & Barlew's theories are problematic for independent reasons. Collectively, these findings suggested that my theory of counting is preferable to both theories. The fifth and sixth Chapter defended the developed theory. Chapter 5 introduced three refinements of the theory: I showed how the theory can be refined to avoid double counting, how it can account for the additional source sensitivity and underspecification of counting, and how it can predict combinatorial counts. Chapter 6 replied to six objections, which concluded the defense of the developed theory.

There is much more work to do. The proposed theory could, for example, be integrated into formal semantics to define the lexical entries of cardinal numbers, fractions, and quantity words like "many" and "most". Most linguists use a counting function denoted by "#" to define the semantic entries of numerals.[1] For example, one proposal is that numerals like "four" are similar to adjectives [2] and have semantic entries as in (32):

(32) $[\![\text{four}]\!] = \lambda x.\#x = 4$

According to (32), "four" denotes the set of plural entities that count for four. Sentences like "Four oranges are on the table" are true iff the plurality of objects on the table contains objects that are oranges and count for (at least) four. In the literature, the counting function is often taken to count the *atomic entities* in a plurality, e.g. the individual oranges. Importantly, the insights developed in this thesis show that such an analysis is bound to fail: we do not only count oranges, but also orange halves. Accordingly, not every entity in a plurality counts for the same

[1] One of the few approaches that does not involve "#" treats numerals like determiners. On such a view, the denotation of "four" is as follows:

(1) $[\![\text{four}]\!] = \lambda A \lambda B.|A \cap B| = 4$

Such a semantics of numerals, however, fails for reasons discussed in Bylinina (2016); Bylinina & Nouwen (2020); Link (1987). In addition, this thesis presents an independent reason to reject such a semantics, for it cannot predict that we have four oranges in cases where we have three oranges and two half oranges.

[2] See Bartsch (1973); Chierchia (1985); Landman (2003); Rothstein (2017) and Rothstein (2013) for such a view.

amount. This problem can be avoided if we use the counting function developed in this thesis. As most theories of cardinals already involve an operator to count objects ("#"), this task is very simple: we only have to use the developed counting function as a counting operator.[3]

Also proposals for the semantics of quantity words like "many", "much", "little", and "few" include counting and measuring operators.[4,5] For example, Rett (2018) recently suggested that quantity words denote intervals and that, hence, the semantics entries of quantity words like "much" are as follows:[6]

(33) $[\![\text{much}]\!] = \lambda D \lambda d.\ d$ is the size/quantity of D

According to such a semantics, "John is much taller than Sue" is predicted to be true iff the interval between the height of Sue and John is above a contextually fixed threshold. Importantly, to derive the start and end points of this interval, we often need a counting function. If we say "Titus has much more oranges than Bjarne has cars", we need the right count of Titus' oranges and of Bjarne's cars in order to calculate the length of the relevant interval. Hence, a proposal such as Rett's (and most other proposals for the semantics of quantity words[7]) depends on a counting function, and thus represents yet another region of linguistics in which the developed theory of counting could be used.

Independently of the assumption I made about the relation between semantics and psychology, there is evidence that the developed theory could be integrated within psychology and philosophy of mind to explain what happens in our mind

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[3]Similarly, we could use the developed counting function for the semantics of fractions.

[4]Though see Pietroski et al. (2009) for an analysis of "most" that does not necessarily rely on counting.

[5]Like for numerals, my theory presents an argument against determiner analyses of quantity words. If A has one half apple, she has fewer apples than B, who has one whole apple. If C has one whole apple, she has fewer apples than D who was 3 apple halves. This is not explained by simply comparing the cardinalities of the sets of As and Bs apples. A and B have the same amount of relevant objects and C has more whole apples than D. See Rett (2018) for different arguments against determiner analyses of quantity words.

[6]For discussion see Rett (2006, 2008, 2014) and Solt (2009, 2015b).

[7]See Bresnan (1973); Cresswell (1976); Hoeksema (1983a); Hackl (2000); Grosu & Landman (1998); Partee (1988), and Romero (1998) for other proposals for the semantics of quantity words that include a counting function.

when we count. First, work by Finks et al. (1989) and Shepard & Cooper (1986) suggests that we sometimes have mental representations of merges of objects. For example, when people imagine the letter "D" rotated by 90° and the letter "J" underneath it, they report *seeing* an umbrella (Finks et al., 1989). This suggests, that we can have mental representations of merges, for the umbrella that people report to *see* is a merge of the letters "D" and "J".

Further, it is widely accepted within cognitive sciences that we can *mentally rotate* objects.[8] This was first noted by Shepard & Metzler (1971), who observed that when we evaluate whether two three-dimensional or two-dimensional objects are intrinsically qualitatively identical, the time we take to make a positive identity judgement is a linearly increasing function of the angular orientations of the two objects in relation to each other. This suggests that we mentally rotate the objects to check whether they are identical. Hence, a process akin to mental merging explains how we make identity judgements, which prima facie seem highly connected to counting judgements.

In addition, the way that study participants described mental rotation is very similar to how potential explanations of mental merging could be described. For example, one participant reported "I take the inverted monkey and try to picture what it would look like if I rotated it to the same position as the first monkey"(Estes, 1998), and another participant (a child) described mental rotating as follows: "Pretend your mind put them right side up. I turn this one around in my head"(Estes, 1998). Similarly, we could imagine people describing mental merging as follows: "I take the two orange halves and try to picture whether I can put them together to form a whole orange," or "Pretend your mind disconnected the orange halves. I put them back together in my mind."[9]

Finally, a recent study by Guan & Firestone (2019) provides evidence that we

[8]See Cooper (1975); Marmor (1977); Tarr & Pinker (1989); Wexler et al. (1998) and Zacks (2008) for discussion.

[9]This is of course very speculative and would need to be confirmed by future experiments about counting partial objects.

tend to mentally merge partial objects. Guan & Firestone (2019) presented participants of their study with a target that was either a circle or a square. Participants were then asked to press a button (within a time limit), if they saw the target. Crucially, participants also pressed the button when presented with disconnected parts of their target. Hence, they confused parts that could together form the targeted object with the targeted object. This suggests that we *see* the potential, or the possibility, of parts of objects, i.e. how they could be mentally merged to form the relevant target. Such a cognitive ability seems highly analogous to the psychological account that corresponds to the developed theory of counting: when we count, we count with respect to a predicate (the target). When seeing disconnected parts that could together form an instance of the predicate, we recognize the potential of the parts to form a predicate, and hence count them as one. Interestingly, Guan & Firestone (2019, p. 1) argue that we can recognize such possibilities in a very immediate, reflexive, and visual way. This ability could explain why counting is effortless for us even though (upon closer inspection) it is difficult to systematize.

Collectively, these data suggest that it is very likely that merging objects not only helps us to properly understand the semantics of counting but also explains what we do when we count! Of course, to support this claim, many more experiments need to be done. For example, no experiments have been performed to date about *seeing* the possibilities of disconnected objects to form more complex shapes than squares and circles. As we often count objects with complex shapes, mental merging is only a plausible explanation for what we do when we count if we also see the possibilities of more complex objects than circles and squares.

Apart from such broader-sense research questions in linguistics and cognitive science, the developed account generates more avenues of thought within philosophy. Future work could, for example, explore applications of more sophisticated theories of counting to theories of persistence. For instance, worm theorists could use the developed theory to give an account of how and whether partial worms (i.e. fusions of temporal instantaneous stages that do not include a stage for *every* time) can be counted. Such an account could be particularly helpful to develop a theory of diachronic counting.

Besides, there is a usage of "half a P" and other words denoting partial objects that I have not been concerned with in this thesis. If, for example, Elia owns two outfits and a trash bag, which he could conceivably wear, we might say something like "Elia has two and a half outfit." Similarly, if in a specific context Xanita has two believes and some inconclusive evidence that supports the belief that R, she might say something like "There are two and a half things that I believe to be true." Plausibly, the developed theory can account for such usages, as the measurement function could measure the trash bag as 0.5 of an outfit and the belief that R as 0.5 of a belief Xanita has.

In summary, this thesis demonstrated that counting is of importance for everyday practice and for philosophy. It showed that counting is puzzling, especially if we take the need for a theory of counting partial objects seriously. My goal was to develop a plausible theory of counting; what has emerged is that counting is a point of fruitful contact, not only between philosophy of language and metaphysics, but also between philosophy, linguistics, and cognitive science more broadly.

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