RADIATIVE CORRECTIONS

TO

$e^+e^- \rightarrow \mu^+\mu^-$

IN THE

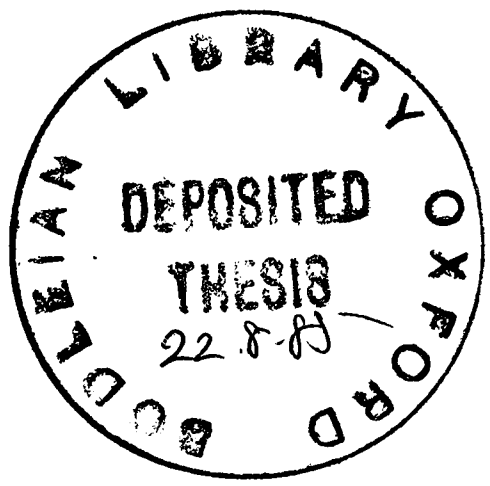
GLASHOW-SALAM-WEINBERG

MODEL

R. G. Stuart

Thesis submitted for the degree of

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Hilary Term

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Oxford

There was from the beginning no need for a struggle between the finite and the infinite, the peace we are seeking so eagerly after has been there all the time.

D. T. Suzuki

Abstract

Radiative corrections to $e^+e^- \rightarrow \mu^+\mu^-$ in the Glashow-Salam-Weinberg model.

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A complete renormalization scheme for the Glashow-Salam-Weinberg model is presented. As input parameters the scheme uses the fine structure constant, α, the muon decay constant, G_μ and the Z° mass, M_Z which are known or will be measured to high accuracy in the near future. These are used along with the Higgs and fermion masses to obtain the W mass, M_W. The full one-loop weak radiative corrections to the longitudinal polarization asymmetry, A_{pol}, and the forward-backward asymmetry, A_{fb}, in $e^+e^- \rightarrow \mu^+\mu^-$ are then calculated both on and off resonance. On resonance the results depend extremely sensitively on the Z° mass, M_Z, and to a lesser extent on the top quark mass, m_t, and the Higgs mass, M_H, showing that this is a good place to test the standard model at the one loop level. The results are displayed over the full range of the parameters allowed by experimental and theoretical constraints. It is suggested that sufficiently accurate experiments may be able to set an upper bound on one of m_t or M_H if the other is known. It is noted that on resonance A_{pol} is only weakly dependent on the beam pipe and on the nature of the outgoing fermions. Comparison is made with other calculations of A_{fb} and the agreement is found to be good. The predictions are also compared with measurements of A_{fb} off resonance and are found in all cases to lie within the experimental errors. Previous analyses that indicated a possible discrepancy are shown to be incorrect.

Acknowledgements

I wish to record my sincere gratitude to Dr B. W. Lynn whose timely arrival in Oxford so dramatically changed the course of my research and whose enthusiasm, support, encouragement and prodding have brought this work to fruition. Thanks too go to Professor P. G. H. Sandars, whose work it was that brought me to Oxford, for his support and supervision in my first year and a half.

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1. Introduction

The discovery of the W and Z^0 bosons by the UA1 and UA2 collaborations at CERN in 1983 marked yet another triumph for the standard Glashow-Salam-Weinberg (GSW) model of electroweak interactions. To investigate the detailed structure of the model and in particular that feature which makes gauge theories stand apart from non-renormalizable theories, namely the radiative corrections, requires measurements to a level that current experiments can only begin to approach.

The coming of the new generation of e^+e^- colliders, the SLC at SLAC and LEP1 at CERN will allow the exploitation of the Z^0 resonant enhancement to improve event rates vastly. In its first year of running the SLC is expected to produce 30,000 $e^+e^- \rightarrow \mu^+\mu^-$ events. In addition, the lowest order photon exchange contribution is strongly suppressed on Z^0 resonance, thereby isolating the more interesting Z^0 exchange part.

A definitive test of the model will require not only a high degree of experimental accuracy but also reliable theoretical predictions. For the latter, one is generally restricted to leptonic processes which are largely free from strong interaction uncertainties, although it is sometimes possible to construct quantities in which strong interaction effects cancel to the required accuracy. Calculations must be done, in the context of some model, using the most accurately known quantities as input data. For the standard model these are the fine structure constant, α, the muon decay coupling constant, G_μ, (or equivalently the muon lifetime, τ_μ) and the mass of the Z^0 boson, M_Z. The last of these will be measured to better than 0·1% early in the lives of the SLC and LEP. This scheme has come to be known as the Weinberg-Salam scheme as

opposed to the Glashow scheme, which uses α, $\sin^2\theta_W$ from neutrino scattering and M_Z as input data.

Sufficient accuracy in the theoretical predictions is obtained only when the complete one-loop, and in some cases two-loop, electro-weak radiative corrections are taken into account. From bitter experience, it is found that partial calculations are not to be trusted, as radiative corrections have a tendency to cancel.

Because of the experimental difficulties associated with obtaining beams of polarized positrons, all cross sections used to define the asymmetries will be expressed in terms of polarized electrons on unpolarized positrons. Left (right) hand polarized electrons will be denoted e^-_L (e^-_R) and f is a generic fermion. Thus following Böhm and Hollik 1982,

$$\sigma_L = \frac{d\sigma}{d\Omega}\left(e^+e^-_L \rightarrow f\bar{f}\right) \qquad (1\cdot1\cdot1)$$

$$\sigma_R = \frac{d\sigma}{d\Omega}\left(e^+e^-_R \rightarrow f\bar{f}\right) \qquad (1\cdot1\cdot2)$$

$$\sigma_u = \sigma_L + \sigma_R \qquad (1\cdot1\cdot3)$$

In the centre of mass frame, θ is the angle between incoming electron and outgoing fermion. The detector solid angle is restricted to lie outside the beampipe, $|\cos\theta| \leq x$. ϕ is the azimuthal angle. The longitudinal polarization asymmetry at 4-momentum transfer, q^2, is

$$A_{pol}(q^2;x) = \frac{\int_0^{2\pi} d\phi \int_{-x}^{x} d(\cos\theta)\,(\sigma_L-\sigma_R)}{\int_0^{2\pi} d\phi \int_{-x}^{x} d(\cos\theta)\,(\sigma_L+\sigma_R)} \qquad (1\cdot1\cdot4)$$

and the forward-backward asymmetry is

$$A_{fb}(q^2;x) = \frac{\int_0^{2\pi} d\phi \left[\int_0^{x} - \int_{-x}^{0}\right] d(\cos\theta)\,\sigma_u}{\int_0^{2\pi} d\phi \left[\int_0^{x} + \int_{-x}^{0}\right] d(\cos\theta)\,\sigma_u} \qquad (1\cdot1\cdot5)$$

For two-fermion ($f\bar{f}$) final states the forward-backward asymmetry is equal to the charge asymmetry.

On resonance, the tree level A_{pol} and A_{fb} are suppressed by factors of $v_\theta = (4\sin^2\theta_W - 1)$ and v_θ^2 respectively which is small because $\sin^2\theta_W \sim \frac{1}{4}$. Hence the effect of radiative corrections on the asymmetries is expected to be large, thus providing a good place to test the model and to search for deviations due to new physics. As well as the three parameters mentioned previously the predictions depend on the presently unknown Higgs mass, M_H, and top quark mass, m_t. Sufficiently accurate measurements of the asymmetries might give an upper bound on one if the other were known.

The aim of this work, then, is to calculate the purely weak corrections to the longitudinal polarization asymmetry, A_{pol}, and the forward-backward asymmetry, A_{fb}, both on and off Z^0 resonance in $e^+e^- \to \mu^+\mu^-$. The gauge invariant set of QED corrections (Figure 4·6) has been calculated by others.

Throughout the work an attempt has been made to systematize the calculation and to keep formulas and computer programs as general as possible so that they may be applied after straightforward modification to other processes and to extensions of the standard model (extra fermion doublets, supersymmetry etc.). To achieve this, a complete renormalization prescription for the GSW model at the Green's function level is developed. This work contains the first complete calculation of the weak radiative corrections to A_{pol} and serves to confirm and clarify existing and subsequent calculations of the corrections to A_{fb}.

On the experimental side, it is found that the introduction of polarization enhances the accuracy obtainable by providing an avenue for exploring systematic errors and by making available an additional set of observables. Longitudinal polarization is a property that is difficult to maintain in circular colliders and consequently will

probably not be available at LEP. It allows direct control of the weak isospin t_3 components of the incoming beam.

The longitudinal polarization asymmetry A_{pol}, is intrinsically less sensitive to detector systematics than A_{fb} because it is not necessary to tag the μ^-. One is then involved with a pure counting experiment and there is no need to reconstruct the shape of the tracks as when identification is required.

It is possible to define two further quantities that may prove of interest. The first is the polarized beam forward-backward asymmetries

$$A_{fb}^{L,R}(q^2;x) = \frac{\int_0^{2\pi} d\phi \left[\int_0^{x} - \int_{-x}^{0} \right] d(\cos\theta)\, \sigma_{L,R}}{\int_0^{2\pi} d\phi \left[\int_0^{x} + \int_{-x}^{0} \right] d(\cos\theta)\, \sigma_{L,R}} \qquad (1\cdot1\cdot6)$$

The difference $A_{fb}^{L} - A_{fb}^{R}$ is expected to be less sensitive to systematics than is A_{fb} (SLAC 1982, p.36).

The second quantity is essentially A_{pol} but with the ratios of polarized to unpolarized cross sections formed bin by bin and then integrated

$$A'_{pol}(q^2;x) = \int_0^{2\pi} d\phi \int_{-x}^{x} d(\cos\theta) \left[\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \right] \qquad (1\cdot1\cdot7)$$

thereby eliminating systematics due to angular dependence in detector sensitivity.

Because the angular integrations are done numerically in the computer programs developed here, $A_{FB}^{L,R}$ and A'_{pol} can be generated straightforwardly.

The largest systematic uncertainty in A_{pol} at the SLC lies in the determination of the beam polarization, P. The uncertainty can be shown to be

$$\frac{\delta A_{pol}}{A_{pol}} = -\frac{\delta P}{P} \qquad (1\cdot1\cdot8)$$

With the beam polarization technology now available one expects

$$\frac{\delta P}{P} = \pm 0{\cdot}05$$

and further improvements make $\pm$ 0·01 a reasonable goal for the SLC.

On resonance $0{\cdot}04 \leq A_{pol} \leq 0{\cdot}5$ so that the absolute error is small. Overall the experimental uncertainty in A_{fb} is expected to be $\pm$ 0·01 and in A_{pol} $\pm$ 0·005 or possibly as low as $\pm$ 0·003.

The layout of the thesis is as follows. Chapter 2 gives a complete renormalization prescription for the GSW model, including the Higgs sector. Here and in the appendices lies the basic ground work but subsequent chapters are intelligible without having read Chapter 2. The reader whose primary interest is in the asymmetries is advised to skip to Chapter 3.

Chapter 3 shows how the W boson mass, M_W, can be eliminated as an input parameter in favour of τ_μ and how the hadronic uncertainty in the contribution of fermion loops to the photon vacuum polarization can be dealt with.

Chapter 4 discusses the radiative corrections to the asymmetries and presents the results in tabular and graphical form along with a comparison of the predictions for A_{fb} with previous and subsequent calculations and with experimental results.

Chapter 5 briefly describes the use and capabilities of the computer program that was developed for this work.

The conclusions are presented in Chapter 6.

Appendix A summarizes the standard model and the conventions used for this work.

Appendices B and C contain calculational details for propagator and vertex corrections respectively.

Appendix D shows how heavy box diagrams were calculated.

Appendix E contains the derivation of a general expression for the cross sections that was used for calculation of the asymmetries.

2. The Renormalization Scheme

2·1 Introduction

To begin it may be helpful to review some of the general features of renormalization. In order to obtain a finite theory it is necessary to make a distinction between the bare Lagrangian L^0, which is the fundamental Lagrangian of the theory, and the renormalized Lagrangian, L^R. The former is naturally expressed in terms of, possibly infinite, bare fields, masses and coupling constants which are denoted by the superscript (^{0}). The renormalized Lagrangian is obtained by replacing the bare quantities in L^0 by the corresponding renormalized ones. The difference between the two Lagrangians is called the counter-term Lagrangian δL. Hence

$$L^0 = L^R + \delta L \qquad (2\cdot1\cdot1)$$

The set of Feynman rules generated by δL are referred to as the counter-terms. No counter-term Lagrangian can have an imaginary part for this would violate unitarity. A theory is described as renormalizable provided the finite number of counter-term parameters so produced are sufficient to yield a finite physical theory. A theory will be renormalizable provided that there is no coupling constant with dimensions $[\text{mass}]^n$ where $n < 0$ and provided that boson propagators behave like $(q^2)^{-1}$ and fermion propagators like $(q^2)^{-\frac{1}{2}}$ for large $|q^2|$. These conditions ensure that the number of types of divergent integral does not go up indefinitely with increasing orders of perturbation theory.

In linear gauges the gauge fixing Lagrangian is not renormalized to one loop. In such gauges the divergences are proportional to the transverse tensor $q^2\delta_{\mu\nu} - q_\mu q_\nu$ and $M^2\delta_{\mu\nu}$. This is not true in non-linear gauges where divergences proportional to the longitudinal

$q_\mu q_\nu$, in general, will appear. The Ward identities play a crucial role in proving unitarity of the S-matrix. The bare Lagrangian is the true Lagrangian of the theory and so is gauge invariant under some local group G. Because renormalization is essentially a reordering of terms in perturbation theory, enforcing gauge invariance order-by-order requires that the renormalized Lagrangian be invariant under the same group G. The transformations may be different but each must be a representation of G. The proof that this can be arranged can be found in Taylor 1979. The counter-term Lagrangian need not be invariant under any set of local transformations. Renormalized Ward identities may be derived to prove unitarity of the renormalized Lagrangian. In practice it is clearly advantageous to use a regularization method, such as dimensional regularization (see Appendix A), which does not break gauge invariance and hence does not violate the Ward identities.

One must provide renormalization conditions for all superficially divergent 1PI Green's functions. These conditions are arbitrary as long as they are compatible with all Ward identities. They correspond to different finite parts of the renormalization constants and each choice defines a scheme of renormalization.

Baulieu 1984 has emphasized the convenience of on-shell renormalization in order to provide a consistent particle interpretation of the theory. That is, one must be able to impose the requirements that the poles of the physical 2-point Green's functions are located at the same places as the zeroes of the 1PI self energies, the physical mass, and the coupling constant is defined as an on-shell vertex with external physical particles. The prescription adopted here manifestly satisfies this requirement.

In closing this section it should be pointed out that the need for renormalization is not a fiction or a theoretical drawback, but a reflection of the fact that there are no absolute scales in physics. Only dimensionless ratios of quantities can be calculated. With regard to the S-matrix, this means that the parameters appearing in the bare Lagrangian have no physical content. To make contact with the real world, what is effectively done in practice is to calculate a sufficient number of 'standard' matrix elements in terms of the bare parameters and relate these to experimental results via renormalization conditions. For the case of pure QED these standard matrix elements relate the physical mass and charge with the parameters of the bare Lagrangian. Any further matrix element can be expressed in terms of the standard matrix elements and the parameters of the bare Lagrangian eliminated yielding finite results. The bare Lagrangian, then, must be regarded as a means of relating a sufficient number of experimental results to each other. The result of any one experiment is not predicted.

Renormalization of the GSW model

In the following subsections are given details of a complete renormalization prescription for the standard model at the Green's function level in the bosonic, Higgs and fermionic sectors. The first complete Green's function treatment was given by Lynn 1982, 1984a.

From a practical point of view Green's function renormalization entails no more work for the production of physical predictions than does S-matrix renormalization (see section 2·6) and allows the suppression of classes of contributions. This is often necessary when comparing numerical predictions with those from other calculations or with experimental results.

The details of the notation used and a summary of the GSW model appear in Appendix A. For the bosonic sector the bare Lagrangian is

$$\mathcal{L}^{0}_{\substack{\text{gauge}\\ \text{fields}}} = -\tfrac{1}{4}\left|\partial_\mu B^{0}_\nu - \partial_\nu B^{0}_\mu\right|^2 - \tfrac{1}{4}\left|\partial_\mu \underset{\sim}{W}^{0}_\nu - \partial_\nu \underset{\sim}{W}^{0}_\mu + g_0 \underset{\sim}{W}^{0}_\mu \times \underset{\sim}{W}^{0}_\nu\right|^2 \qquad (2\cdot1\cdot2)$$

The bare Higgs potential is

$$V^{0}_{\phi} = \tfrac{1}{2}\lambda_0 \left(\phi^{0\dagger}\phi^{0} - E_0^2\right)^2 \qquad (2\cdot1\cdot3)$$

for a Higgs isodoublet

$$\phi^{0} = \begin{pmatrix} F_0 \\ 0 \end{pmatrix} + \begin{pmatrix} \dfrac{H^{0} + i\phi_Z^{0}}{\sqrt{2}} \\ i\phi^{0+} \end{pmatrix} \qquad (2\cdot1\cdot4)$$

The gauge boson coupling to a fermion field is described by

$$\mathcal{L}^{0}_{\substack{\text{gauges}\\ \text{fermions}}} = -\bar{L}^{0}\left(\partial_\mu - \frac{i}{2} g_0 \underset{\sim}{\tau}\cdot\underset{\sim}{W}^{0}_\mu - \frac{i}{2} g'_0 y B^{0}_\mu\right)\gamma_\mu L^{0} - \bar{R}^{0}\left(\partial_\mu - \frac{i}{2} g'_0 y B^{0}_\mu\right)\gamma_\mu R^{0} \qquad (2\cdot1\cdot5)$$

where L^0 represents the left-hand fermion isodoublet and R^0 is a right-handed fermion singlet.

The gauge covariant bare Lagrangian for the Higgs field coupling to a fermion with isospin t_3 is

$$\mathcal{L}^{0}_{\substack{\text{fermions}\\ \text{Higgs}}} = \begin{cases} -\dfrac{m^{0}_f}{F_0}\left[\bar{L}^{0}\phi^{0}R^{0} + \bar{R}^{0}\phi^{0\dagger}L^{0}\right]; & t_3 = \tfrac{1}{2} \\ -\dfrac{m^{0}_f}{F_0}\left[\bar{L}^{0}\tilde{\phi}^{0}R^{0} + \bar{R}^{0}\tilde{\phi}^{0\dagger}L^{0}\right]; & t_3 = -\tfrac{1}{2} \end{cases} \qquad (2\cdot1\cdot6)$$

where $\tilde{\phi}^{0} = -i\sigma_2\phi^{0*}$ and m^{0}_f is interpreted as the fermion's bare mass.

2·2 The Bosonic Sector

The free Feynman propagator for the A, Z and W fields will be denoted $G^F_{\mu\nu}$ and in 't Hooft's R_ξ gauge ('t Hooft 1971) is

$$(2\pi)^4 i\, G^F_{\mu\nu} = \frac{1}{q^2+M^2-i\varepsilon}\, T_{\mu\nu} + \frac{\xi}{q^2+\xi M^2}\, L_{\mu\nu} \qquad (2\cdot2\cdot1)$$

where

$$T_{\mu\nu} = \delta_{\mu\nu} - q_\mu q_\nu/q^2 \qquad (2\cdot2\cdot2)$$

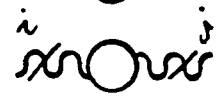
is the transverse tensor and

$$L_{\mu\nu} = q_\mu q_\nu/q^2 \qquad (2\cdot2\cdot3)$$

is the longitudinal tensor. M is the mass of the corresponding particle. Note that no iε prescription needs to be given for the term proportional to $L_{\mu\nu}$ because the unphysical pole cancels against similar contributions from the unphysical Goldstone bosons $\phi^\pm$, ϕ_Z ('t Hooft 1971). Given any two fields i(x) and j(x), G^{ij} will denote the complete two point Green's function and Π^{ij} is the 1PI self energy of the fields. Diagramatically

G^{ij} = 

(2·2·4)

Π^{ij} = 

(2·2·5)

where the / in (2·2·5) indicates that external legs have been amputated.

G^{ij} and Π^{ij} may carry one, two or zero Lorentz indices depending on the Lorentz structure of the fields i and j. For i and j vector fields write,

$$G^{ij}_{\mu\nu} = G^{ij}_T\, T_{\mu\nu} + G^{ij}_L\, L_{\mu\nu} \qquad (2\cdot2\cdot6)$$

$$\Pi^{ij}_{\mu\nu} = \Pi^{ij}_T\, T_{\mu\nu} + \Pi^{ij}_L\, L_{\mu\nu} \qquad (2\cdot2\cdot7)$$

$\tilde{\Pi}^{ij}$ will be used to denote the 1PI self energy excluding the counter-term contribution.

Closely following Ross and Taylor 1973, the renormalized fields in the GSW Lagrangian given in Appendix A are related to the bare fields by the rescalings

$$W^0_\mu = Z_W^{\frac{1}{2}} W_\mu \quad ; \quad Z_W = 1 + \delta Z_W \qquad (2\cdot2\cdot8)$$

$$B^0_\mu = Z_B^{\frac{1}{2}} B_\mu \quad ; \quad Z_B = 1 + \delta Z_B \qquad (2\cdot2\cdot9)$$

To one loop, no distinction needs to be made between the bare and renormalized ghost fields since these only occur in closed loops.

The coupling constants are rescaled as

$$g_0 = g\left(1 + \frac{\delta g}{g}\right) \qquad (2\cdot2\cdot10)$$

$$g'_0 = g'\left(1 + \frac{\delta g'}{g'}\right) \qquad (2\cdot2\cdot11)$$

and the masses of the W and Z bosons as

$$M_W^{0\,2} = M_W^2 \left(1 + \frac{\delta M_W^2}{M_W^2}\right) \qquad (2\cdot2\cdot12)$$

$$M_Z^{0\,2} = M_Z^2 \left(1 + \frac{\delta M_Z^2}{M_Z^2}\right) \qquad (2\cdot2\cdot13)$$

where is is to be remembered that the masses and mass counter-terms can be written in terms of couplings to the Higgs field.

In the on-shell renormalization scheme adopted here the renormalized and physical masses are equal and it is convenient to define

$$s_\theta^2 \equiv \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \qquad (2\cdot2\cdot14)$$

and hence

$$c_\theta^2 \equiv \cos^2\theta_W = \frac{M_W^2}{M_Z^2} \qquad (2\cdot2\cdot15)$$

$$v_\theta = 4s_\theta^2 - 1 \qquad (2\cdot2\cdot16)$$

exactly to all orders of perturbation theory. Note that the above is merely a shorthand device and the whole theory can be formulated

without mention of the weak mixing angle, θ_W. The physical mass eigenstate Z and A fields are then

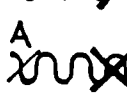
$$W_3 = c_\theta Z + s_\theta A \qquad (2\cdot2\cdot17)$$

$$B = c_\theta A - s_\theta Z \qquad (2\cdot2\cdot18)$$

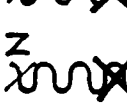
The two point counter-terms thus obtained are to one loop,



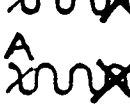
$$= -\delta Z_W (q^2\delta_{\mu\nu}-q_\mu q_\nu) - \delta M_W^2\delta_{\mu\nu} - \delta Z_W M_W^2\delta_{\mu\nu} \qquad (2\cdot2\cdot19)$$



$$= -\tfrac{1}{2}\,\delta Z_A (q^2\delta_{\mu\nu}-q_\mu q_\nu) \qquad (2\cdot2\cdot20)$$



$$= -\tfrac{1}{2}\,\delta Z_Z (q^2\delta_{\mu\nu}-q_\mu q_\nu) - \tfrac{1}{2}\,\delta M_Z^2 - \tfrac{1}{2}M_Z^2\delta Z_Z\delta_{\mu\nu} \qquad (2\cdot2\cdot21)$$



$$= -\delta Z_{ZA} (q^2\delta_{\mu\nu}-q_\mu q_\nu) - \tfrac{1}{2}\,M_Z^2\delta Z_{ZA}\,\delta_{\mu\nu} - 2M_Z^2 s_\theta c_\theta \delta_-\delta_{\mu\nu} \qquad (2\cdot2\cdot22)$$

where

$$\delta Z_A = \delta Z_W s_\theta^{\,2} + \delta Z_B c_\theta^{\,2} \qquad (2\cdot2\cdot23)$$

$$\delta Z_Z = \delta Z_W c_\theta^{\,2} + \delta Z_B s_\theta^{\,2} \qquad (2\cdot2\cdot24)$$

$$\delta Z_{ZA} = s_\theta c_\theta (\delta Z_W - \delta Z_B) \qquad (2\cdot2\cdot25)$$

$$\delta_- = \tfrac{1}{2}\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right) \qquad (2\cdot2\cdot26)$$

Note that because of the unbroken $U(1)_{em}$ symmetry masslessness of the photon is guaranteed and there is no photon mass renormalization counter-term.

The general structure of the counter-terms (2·2·19), (2·2·20) and (2·2·21) is as expected to one loop from a Lagrangian of the form

$$-\tfrac{1}{2}\left|\partial_\mu W_\nu - \partial_\nu W_\mu\right|^2 - M_W^2\, W_\mu^+\, W_\mu^- \qquad (2\cdot2\cdot27)$$

as the gauge fixing Lagrangian goes unrenormalized. Writing (2·2·19) as

$$-\delta Z_W \left[(q^2 + M_W^2)\,\delta_{\mu\nu} - q_\mu q_\nu\right] - \delta M_W^2\,\delta_{\mu\nu} \qquad (2\cdot2\cdot28)$$

it is clear that on mass shell $-\delta M_W^2$ is sufficient to cancel all divergences proportional to $\delta_{\mu\nu}$ in the self energy. Off mass-shell δZ_W is required to cancel all divergences.

Although mixing between the physical vector boson and unphysical Goldstone bosons is eliminated in lowest order by the adoption of the 't Hooft's R_ξ gauge, it still occurs in higher orders. One is then forced to consider sets of simultaneous linear Dyson equations in order to obtain expressions for full two-point Green's functions in terms of 1PI self energies (Baulieu and Coquereaux 1982). For the present purposes it is sufficient to consider the $\{W^{\pm}, \phi^{\pm}\}$ and $\{A, Z, \phi_Z\}$ systems given in Figures (2·2·1) and (2·2·2). The physical Higgs does not appear in the second of these systems because of the CP invariance of the model. Moreover, in light of the earlier comment that in linear gauges the divergences are proportional to the transverse tensor (2·2·2), only the transverse part of the full Green's functions are required. These are easily obtained by acting on the systems of equations with transverse projection operator (2·2·2) with the added bonus that the scalars decouple leading to

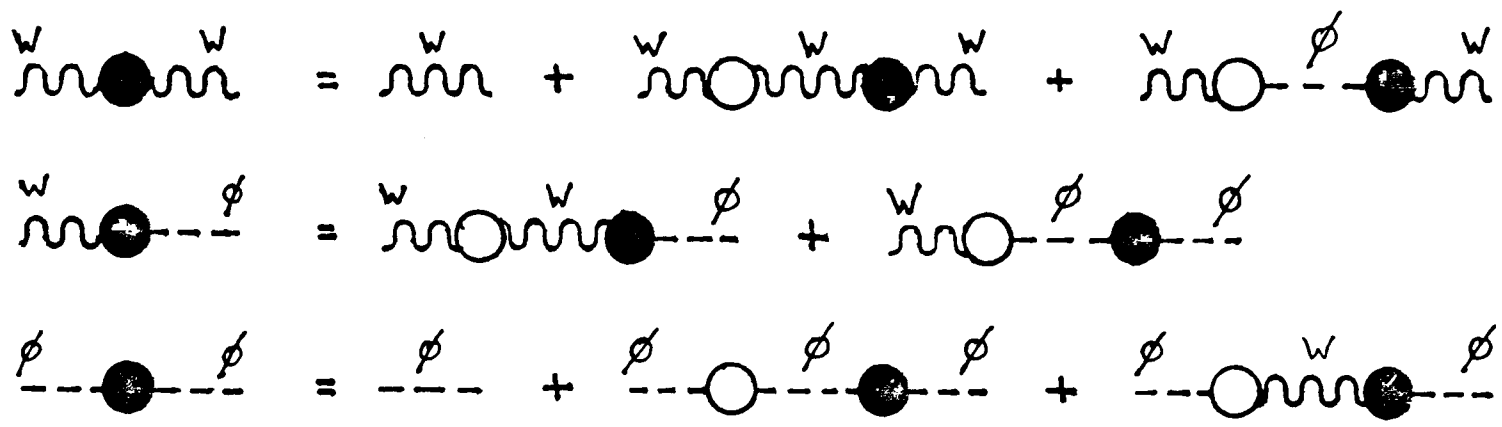
$$G_T^{AA} = \frac{1}{[q^2 - \Pi_T^{AA} - i\varepsilon] - \dfrac{[\Pi_T^{ZA}]^2}{[q^2 + M_Z^2 - \Pi_T^{ZZ} - i\varepsilon]}} \qquad (2\cdot2\cdot29)$$

$$G_T^{ZZ} = \frac{1}{[q^2 + M_Z^2 - \Pi_T^{ZZ} - i\varepsilon] - \dfrac{[\Pi_T^{ZA}]^2}{[q^2 - \Pi_T^{AA} - i\varepsilon]}} \qquad (2\cdot2\cdot30)$$

$$G_T^{ZA} = \frac{\Pi_T^{ZA}}{[q^2 - \Pi_T^{AA} - i\varepsilon][q^2 + M_Z^2 - \Pi_T^{ZZ} - i\varepsilon] - [\Pi^{ZA}]^2} \qquad (2\cdot2\cdot31)$$

$$G_T^{WW} = \frac{1}{q^2 + M_W^2 - \Pi_T^{WW} - i\varepsilon} \qquad (2\cdot2\cdot32)$$

In the GSW model there are sufficient 'natural' parameters available to define a 'physical' on-shell renormalization scheme. This is in contradistinction to QCD where a light quark Green's function


W
W
=
+
+
W
W
W

cannot be defined on-shell because of the infra-red (confinement) problem. The renormalization conditions adopted for this work are given below

(1) Setting the Z^0 boson mass to it's 'experimental' value M_Z to one loop gives

$$\text{Real } \tilde{\Pi}_T^{ZZ}(-M_Z^2) = \delta M_Z^2 \qquad (2\cdot2\cdot33)$$

(2·2·33) follows directly form (2·2·30) remembering that all 1PI self energies are $O(\alpha)$. The generalization of this condition to higher orders is straightforward.

(2) Setting the W boson mass to its 'experimental' value, M_W, gives to one loop

$$\text{Real } \tilde{\Pi}_T^{WW}(-M_W^2) = \delta M_W^2 \qquad (2\cdot2\cdot34)$$

In practice the experimental value of M_W will be calculated from the muon lifetime τ_μ (see section 3·1) so as to make use of the best available experimental data.

(3) Setting the residue at the pole $q^2 = 0$ of the photon propagator to 1, thereby assuring that external photons go uncorrected gives

$$\tilde{\Pi}'_{AA}(0) \equiv \left.\frac{\partial \tilde{\Pi}_T^{AA}}{\partial q^2}\right|_{q^2 = 0} \equiv \delta Z_A \qquad (2\cdot2\cdot35)$$

(4) Requiring that Z-A mixing vanishes for on-shell photons ($q^2 = 0$) gives

$$2\delta_- + \tfrac{1}{2}(\delta Z_W - \delta Z_B) - \frac{1}{M_Z^2 s_\theta c_\theta}\tilde{\Pi}_T^{ZA}(0) = 0 \qquad (2\cdot2\cdot36)$$

Expressing the conditions (2·1·33) - (2·1·36) in terms of 1PI self energies allows their application to any gauge theory based on the gauge group SU(2) x U(1) in which only Higgs doublets get vacuum expectation values. As an example take GSW with extra fermion generations or supersymmetric GSW. The actual expressions for $\bar{\Pi}^{ij}$ will be modified by the presence of extra particles. For the standard model

$$\frac{1}{M_Z^2\, s_\theta c_\theta}\ \bar{\Pi}_T^{ZA}(0) \;=\; \frac{-2g^2}{16\pi^2}\ (\Delta - \ell n\ M_W^2) \qquad (2\cdot2\cdot37)$$

where the meaning of the quantity $\Delta - \ell n\ M_W^2$ is discussed in Appendices A and B.

By considering the formulas (2·3·18) and (2·3·19) one obtains to one loop

$$4s_\theta^2\delta_- \;=\; \frac{\delta M_W^2}{M_W^2} \;-\; \frac{\delta M_Z^2}{M_Z^2} \qquad (2\cdot2\cdot38)$$

Electric charge renormalization provides a fifth condition that is sufficient to completely determine the bosonic sector. This will be discussed later in section 2·5.

2·3 The Higgs Sector

The parameters of the bare Higgs field are related to those of the renormalized field by

$$\phi^0 = Z_\phi^{\frac{1}{2}}\,\phi \qquad (2\cdot3\cdot1)$$

$$\lambda_0 = \lambda\left(1 + \frac{\delta\lambda}{\lambda}\right) \qquad (2\cdot3\cdot2)$$

$$E_0^2 = E^2 + \delta E^2 \qquad (2\cdot3\cdot3)$$

$$F_0^2 = F^2 + \delta F^2 \qquad (2\cdot3\cdot4)$$

but note that from the form of (A·2·5)

$$\delta F^2 = \delta Z_\phi\, F^2 \qquad (2\cdot3\cdot5)$$

With these rescalings the relation

$$M_H^{02} = 2\lambda_0 F_0^2 + \lambda_0(F_0^2 - E_0^2) \qquad (2\cdot3\cdot6)$$

leads to

$$\delta M_H^2 = 2\delta\lambda F^2 + \delta\lambda\,(F^2-E^2) + 2\lambda\delta F^2 + \lambda(\delta F^2-\delta E^2) \qquad (2\cdot3\cdot7)$$

The Higgs potential exhibits a term proportional to one power of the physical Higgs field,

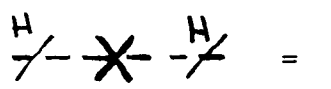
$$\sqrt{2}\,FH\,(\beta + \delta\beta) \qquad (2\cdot3\cdot8)$$

where

$$\beta = \lambda(F^2 - E^2) \qquad (2\cdot3\cdot9)$$

$$\delta\beta = (\delta\lambda - \lambda\delta Z_\phi)\,(F^2 - E^2) + \lambda(\delta Z_\phi F^2 - \delta E^2)\,. \qquad (2\cdot3\cdot10)$$

The 2-point Higgs counter-term may then be written using (2·3·6) and (2·3·10) as



$$= -(q^2 + M_H^2)\delta Z_\phi - \delta M_H^2 \qquad (2\cdot3\cdot11)$$

In deriving this result it is convenient to write

$$\delta M_H^2 = 2\delta\lambda F^2 + \delta\beta + \delta Z_\phi M_H^2 + 2\lambda\delta Z_\phi (E^2 - F^2) \qquad (2\cdot3\cdot12)$$

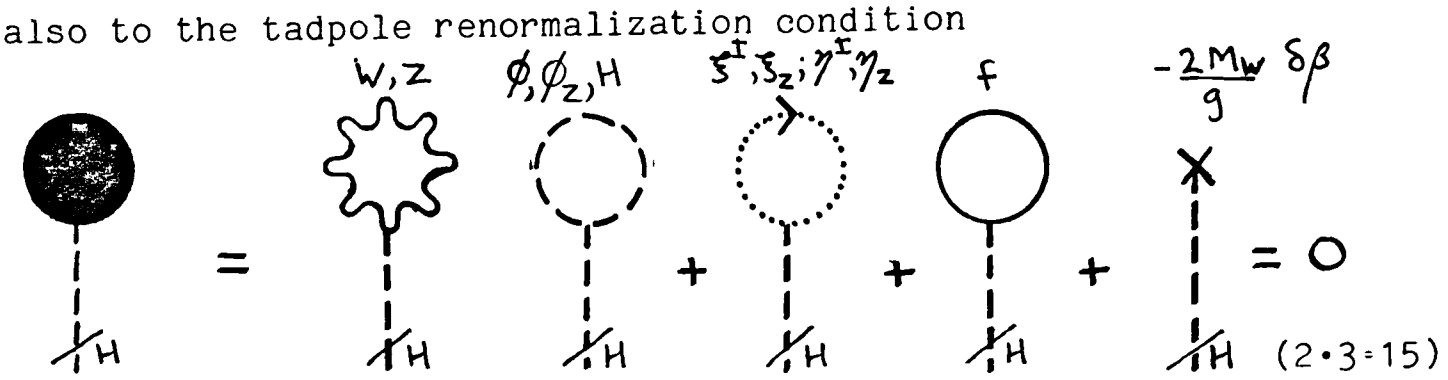
Requiring that the physical Higgs, H, has zero vacuum expectation value order by order in perturbation theory leads to the condition

$$E^2 = F^2 \qquad (2\cdot3\cdot13)$$

although

$$E_0^2 \neq F_0^2 \qquad (2\cdot3\cdot14)$$

and also to the tadpole renormalization condition



(2·3·15)

where the relation

$$M_W = \frac{Fg}{\sqrt{2}} \qquad (2\cdot3\cdot16)$$

has been used in the counter-term.

2) Setting the mass of the Higgs to its 'experimental' value yields

$$\text{Real}\ \tilde{\Pi}^{HH}(-M_H^2) = \delta M_H^2 \qquad (2\cdot3\cdot17)$$

The quadratic divergence in the Higgs self energy is cancelled by a similar divergence in $\delta\beta$.

The rescaling (2·3·1) - (2·3·4) applied to (A·2·21) and (A·2·22) gives

$$\frac{\delta M_W^2}{M_W^2} = \left(\frac{2\delta g}{g} + \frac{\delta F^2}{F^2}\right) \qquad (2\cdot3\cdot18)$$

$$\frac{\delta M_Z^2}{M_Z^2} = \left(2c_\theta^2 \frac{\delta g}{g} + 2s_\theta^2 \frac{\delta g'}{g'} + \frac{\delta F^2}{F^2}\right) \qquad (2\cdot3\cdot19)$$

and hence

$$\delta Z_{\phi} = \frac{\delta F^2}{F^2} = \left(\frac{\delta M_W^2}{M_W^2} - \frac{2\delta g}{g}\right) \qquad (2\cdot3\cdot20)$$

which can be used in evaluating (2·3·11).

2·4 The Fermion Sector

In the standard model, left-handed fermion wavefunctions

$$L = \gamma_+ L \tag{2·4·1}$$

and right-handed fermion wave functions

$$R = \gamma_- R \tag{2·4·2}$$

renormalize differently.

The bare and renormalized wavefunctions are related by

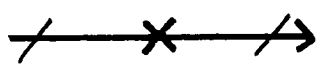
$$L^0 = Z_L^{\frac{1}{2}} L \tag{2·4·3}$$

$$R^0 = Z_R^{\frac{1}{2}} R \tag{2·4·4}$$

Writing

$$Z = 1 + \delta Z \tag{2·4·5}$$

for both left-handed and right-handed rescalings leads to



$$= - i\not{p} \left[\delta Z_L \gamma_+ + \delta Z_R \gamma_-\right] - \delta m \tag{2·4·6}$$

The counter-term δm contains both δZ_L and δZ_R as well as contributions from the Higgs sector.

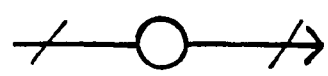
The full Feynman propagator for fermions will be denoted

$$S(p) = \tag{2·4·7}$$



and the 1PI self energy without external legs as

$$(2\pi)^4 i\, \Sigma(p) = \tag{2·4·8}$$



and one may write

$$\Sigma(p) = i\not{p} \left(A\,(p^2)\, \gamma_+ + B\,(p^2)\, \gamma_-\right) + mC(p^2). \tag{2·4·9}$$

Again the / in (2·4·8) indicates that external legs are not included. With this notation one may write the linear Dyson equation



(2·4·10)

which has the solution

$$(2\pi)^4 i\, S(p) = \left(i\not{p} + m - \Sigma(p)\right)^{-1} \qquad (2\cdot4\cdot11)$$

Consider now a pair of fermions whose left-handed components form a weak isospin doublets under $SU(2)_L$. The subscripts 'u' and 'd' on form factors indicate reference to the $t_3 = \frac{1}{2}$ and $t_3 = -\frac{1}{2}$ components respectively. The corresponding fermion masses are m_u and m_d. The form factors will be required expanded about the pole of the propagator for the physical particle for which $p^2 = -m^2$. Thus, for example,

$$A(p^2) = \underline{A} + (p^2 + m^2)\, A' + (p^2 + m^2)^2\, A^R \qquad (2\cdot4\cdot12)$$

with

$$\underline{A} = A(p^2 = -m^2) \qquad (2\cdot4\cdot13)$$

$$A' = \frac{\partial A}{\partial p^2}\bigg|_{p^2 = -m^2}. \qquad (2\cdot4\cdot14)$$

As for the boson sector a tilde ($\tilde{}$) over the form factors will indicate the omission of the counter-term contribution.

A natural set of physical renormalization conditions is adopted. The conditions and consequences for one loop are listed below

(1) Setting the masses of the fermions in the doublet to their experimental values by adjusting the poles of propagators gives

$$\tfrac{1}{2}\,(\underline{A}_u + \underline{B}_u) = \underline{C}_u \qquad (2\cdot4\cdot15)$$

$$\tfrac{1}{2}\,(\underline{A}_d + \underline{B}_d) = \underline{C}_d \qquad (2\cdot4\cdot16)$$

(2) Requiring that left and right handed residues at the poles are equal for each of the two fermions' 2-point Green's functions gives

$$\underline{A}_u = \underline{B}_u \qquad (2\cdot4\cdot17)$$

$$\underline{A}_d = \underline{B}_d \qquad (2\cdot4\cdot18)$$

As a consequence of the gauge symmetry it is not possible to arrange that the residues of the 2-point Green's functions for both fermions to be 1 at their respective poles.

(3) Setting the residue of the Green's functions of, say, the 'u' fermion to 1 leads to

$$\tfrac{1}{2}(\underline{A}_u + \underline{B}_u) + m_u^2[2C'_u - (A'_u + B'_u)] = 0. \qquad (2\cdot4\cdot19)$$

The wavefunctions of this fermion go uncorrected when they are attached to external on-shell legs in Feynman diagrams.

When the 'd' fermion of the doublet is attached to an external on shell leg, its wavefunction must be corrected by a factor

$$(1 - \tfrac{1}{2}K_d) \qquad (2\cdot4\cdot20)$$

where $1 + K_d$ is the residue of its propagator at the pole $i\not{p} + m_d = 0$. When this fermion is massless the counter-terms will be IR divergent on mass shell.

If the gauge symmetry were absent both residues could be set equal to 1 because there would be six independent counter-terms available. It is a remarkable fact that in the present case the five counter-terms

$$\delta Z_L,\ \delta Z_R^u,\ \delta Z_R^d,\ \delta m^u,\ \delta m^d \qquad (2\cdot4\cdot21)$$

are sufficient to render the theory finite. Moreover, the final result does not depend on which fermion's residue was set equal to unity. It can be easily shown that the infinite part of the counter-term δZ_L does not depend on this choice. The charge, Q, and 3-component of weak isospin, t_3, of the fermions are related by

$$t_3 \rightarrow -t_3 \qquad (2\cdot4\cdot22)$$

$$Q \rightarrow Q - 2t_3 \qquad (2\cdot4\cdot23)$$

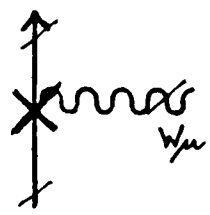
By explicit calculation using (B·41) - (B·43), (B·28) and dropping term $O(G_F\, m_f^2/M_W^2)$ gives for the infinite part

$$\frac{-g^2}{16\pi^2}\ \frac{1}{c_\theta^2}\ \left\{t_3^2 + s_\theta^2\ (Q - 2t_3)Q\right\}\ \Delta \qquad (2\cdot4\cdot24)$$

which is invariant under the substitutions (2·4·22) and (2·4·23). The logarithmically divergent quantity Δ is defined in (A·3·10).

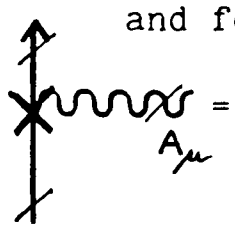
2·5 Charge Renormalization

The rescaling outlined in the previous section produces the vertex counter-terms



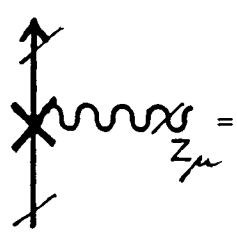
$$= \frac{ig}{\sqrt{2}} \gamma_\mu \gamma_+ \left(\delta Z_L + 2\delta k\right) \qquad (2\cdot5\cdot1)$$

and for a fermion of charge Q and 3-component of weak isospin, t_3



$$= igs_\theta \gamma_\mu \{Q(\delta Z_L \gamma_+ + \delta Z_R \gamma_-) + 2t_3 \delta k \gamma_+\}$$

$$+ igs_\theta \gamma_\mu \left(\tfrac{1}{2}\delta Z_B + \frac{\delta g'}{g'}\right) \{(t_3 - Q)\gamma_+ - Q\gamma_-\} \qquad (2\cdot5\cdot2)$$



$$= ig \gamma_\mu \left\{\left(\frac{4t_3 - 4Q s_\theta^2}{4c_\theta}\right) \delta Z_L \gamma_+ + \left(\frac{-4Q s_\theta^2}{4c_\theta}\right) \delta Z_R \gamma_- + 2t_3 c_\theta^2 \delta k \gamma_+\right\}$$

$$+ \frac{ig}{c_\theta} \gamma_\mu s_\theta^2 \left(\tfrac{1}{2}\delta Z_B + \frac{\delta g'}{g'}\right) \{(t_3 - Q)\gamma_+ - Q\gamma_-\}$$

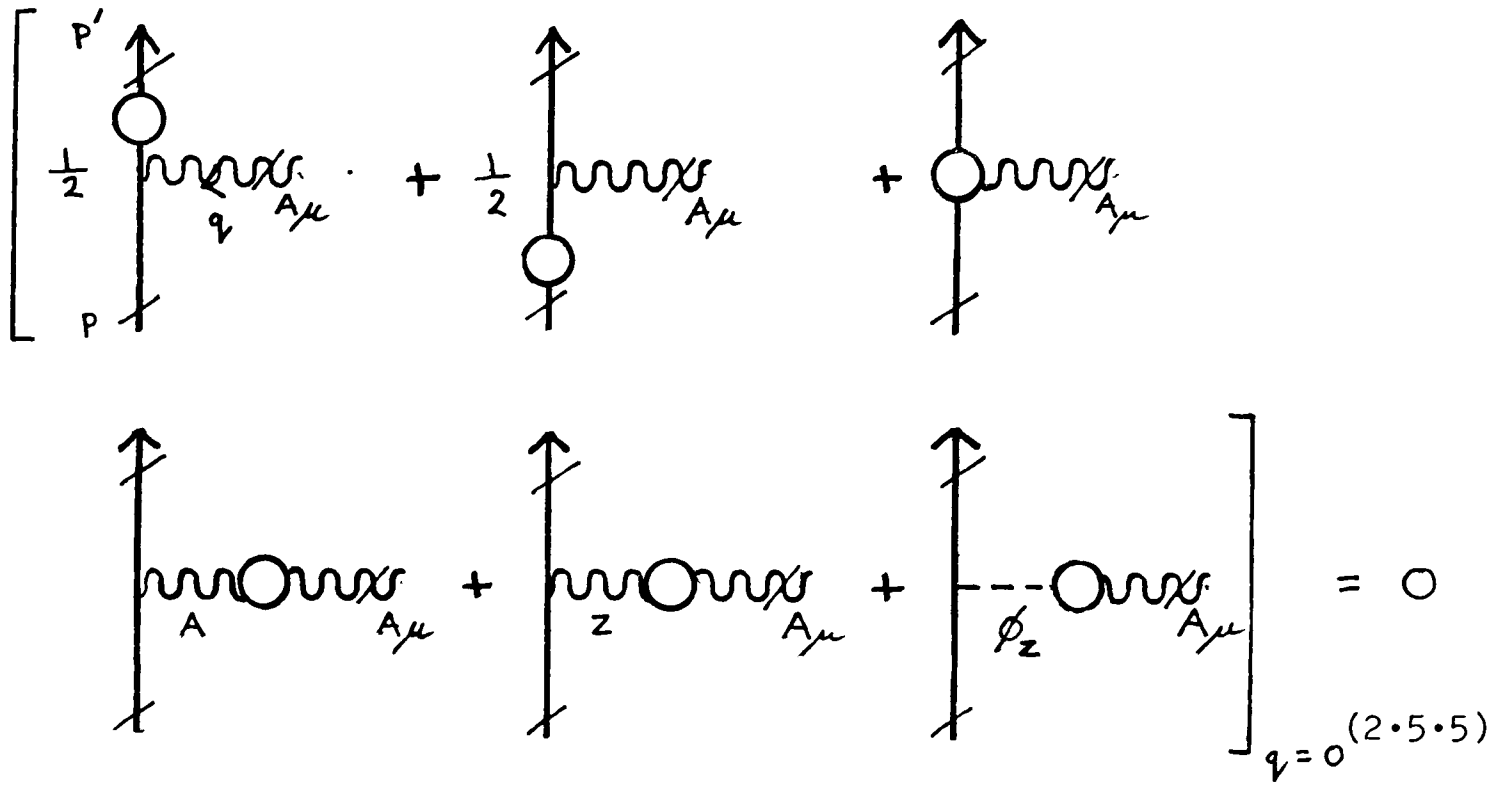
$$(2\cdot5\cdot3)$$

Here

$$\delta k = \tfrac{1}{2}\frac{\delta g}{g} + \tfrac{1}{4}\delta Z_W \qquad (2\cdot5\cdot4)$$

For the purposes of renormalization, the electric charge is defined to be the photon-charged fermion vertex coupling constant for Thomson scattering in the limit that the 4-momentum carried by the photon, $q \to 0$. Note that the W is also charged and it could have been used to define the renormalization.

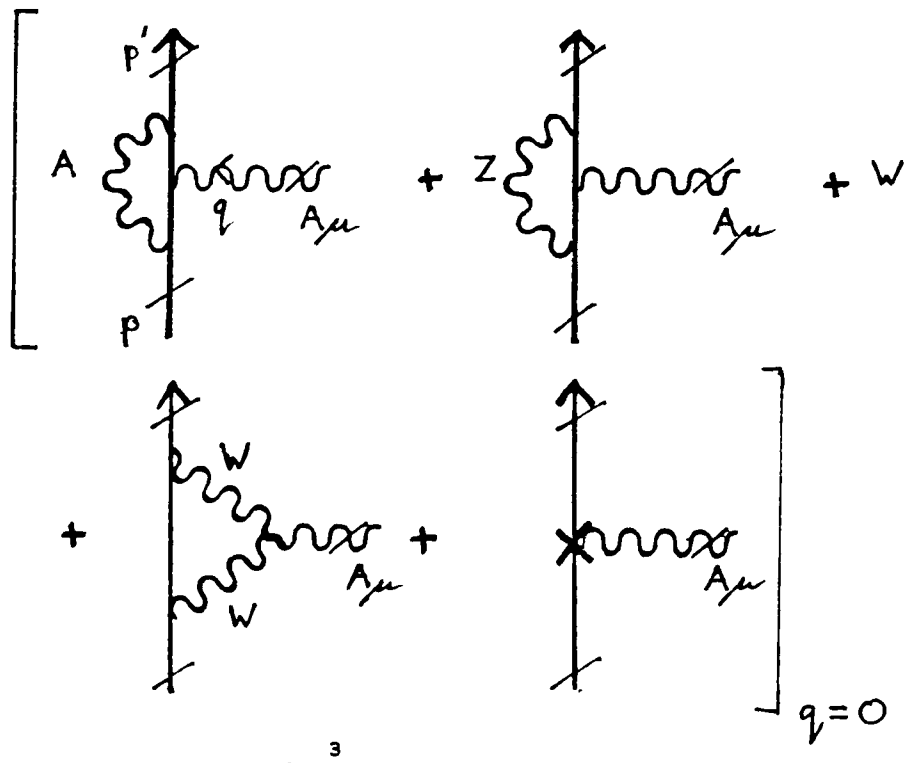
For a fermion of mass m the requirement is



(2·5·5)

The work is reduced and no generality is lost if it is assumed that the external fermion has been renormalized in such a way that the residue at the pole of its propagator is 1. In consequence the external fermion legs go uncorrected and the first two diagrams of the l.h.s. vanish. The sixth diagram on the l.h.s. is zero because ϕ_Z - A mixing is proportional to q. The fourth and fifth diagrams are zero because of (2·2·35) and (2·2·36) respectively.

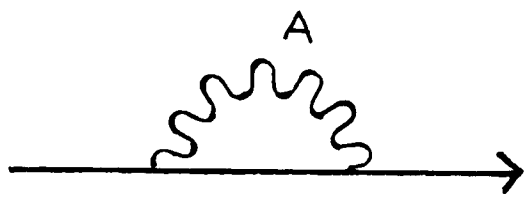
By explicit calculation using (2·5·2) it is possible to show that



$$= \frac{ig^3 s_\theta}{16\pi^2} \gamma_\mu \gamma_+ 2t_3 (\Delta - \ell n\, M_W^2) + ig s_\theta \gamma_\mu \gamma_+ 2t_3 \delta k$$

$$+ ig s_\theta \gamma_\mu \left(\tfrac{1}{2}\delta Z_B + \frac{\delta g'}{g'}\right) \{(t_3 - Q)\gamma_+ - Q\gamma_-\} \qquad (2\cdot5\cdot6)$$

To obtain this result note that the first graph on the l.h.s. of (2·5·6) cancels with the photonic part of the counter-term graph which contributes to δZ_L and δZ_R via the graph



This cancellation of photonic parts amongst themselves takes place courtesy of the well-known QED Ward identity.

Hence imposing (2·5·5) and using (2·2·35) yields

$$\tfrac{1}{2}\,\delta Z_B + \frac{\delta g'}{g'} = 0 \qquad (2\cdot5\cdot7)$$

This relation is analogous to the U(1) Ward identity for pure QED,

$$\tfrac{1}{2}\,\delta Z_A + \frac{\delta e}{e} = 0 \qquad (2\cdot5\cdot8)$$

Using (2·2·36) and (2·5·7) it follows,

$$\tfrac{1}{2}\,\delta Z_W + \frac{\delta g}{g} - \frac{1}{M_Z^2 s_\theta c_\theta}\,\tilde{\Pi}_T^{ZA}(0) = 0 \qquad (2\cdot5\cdot9)$$

This completes the description of the renormalizable framework. Combining (2·5·7) with (2·2·33) - (2·2·36) yields

$$\delta Z_W = \tilde{\Pi}'_{AA}(0) - 4c_\theta^2\left[\delta_- - \frac{1}{2s_\theta c_\theta M_Z^2}\,\tilde{\Pi}_T^{ZA}(0)\right] \qquad (2\cdot5\cdot10)$$

$$\delta Z_A = \tilde{\Pi}'_{AA}(0) \qquad (2\cdot5\cdot11)$$

$$\delta Z_Z = \tilde{\Pi}'_{AA}(0) + 4(s_\theta^2 - c_\theta^2)\left[\delta_- - \frac{1}{2s_\theta c_\theta M_Z^2}\,\tilde{\Pi}_T^{ZA}(0)\right] \qquad (2\cdot5\cdot12)$$

$$\delta Z_{ZA} = -4s_\theta c_\theta\left[\delta_- - \frac{1}{2s_\theta c_\theta M_Z^2}\,\tilde{\Pi}_T^{ZA}(0)\right] \qquad (2\cdot5\cdot13)$$

$$\delta_- = \frac{1}{4s_\theta^2}\,\text{Real}\left[\frac{\tilde{\Pi}_T^{WW}(-M_W^2)}{M_W^2} - \frac{\tilde{\Pi}_T^{ZZ}(-M_Z^2)}{M_Z^2}\right] \qquad (2\cdot5\cdot14)$$

2·6 Sirlin's Renormalization Scheme

The purpose of this section is to demonstrate the equivalence between schemes in which wavefunction renormalization is carried out and those in which it is not. Moreover when contributions including counter-terms are expressed as sums of 1PI self-energies exactly the same number of terms appear in either scheme. It does not follow then that it is more economical and less error prone to do only coupling constant renormalization but not wavefunction renormalization, although it may be helpful in avoiding double counting of graphs. The only real difference is that wavefunction renormalization automatically arranges the contributions in such a way as to make the individual Green's functions finite and making cancellations easier to see. In Sirlin's scheme cancellations occur at the S-matrix level between contributions from many different classes of diagrams. This could be a disadvantage for the present calculation on Z^0 resonance because the lowest order Z^0 exchange blows up alone on resonance making it necessary to combine it algebraically with the graphs involving first order corrections to the Z^0 propagator. The terms which cancel the infinite part in this first order contribution must be extracted from their various sources and combined with the result.

What follows is concerned with the work of Sirlin 1980 except that the conventions and notation consistent with usage of this work will be adopted. In particular the vertex counter-terms will not be separated into photon and Z^0 boson currents coupling to the photon and Z^0 which is allowable provided one only deals with leptons. There seems to be no real advantage in not defining F to be exactly the vacuum expectation value of the Higgs field. This will be taken as done (see section 2·3) with the consequence that no tadpole terms appear.

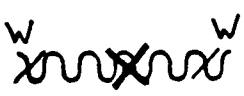
Sirlin's renormalization scheme is applicable only to on-shell S-matrix elements, not Green's functions (see comment following (2·2·28)), and as will be shown explicitly, later, for Z-A mixing, essentially combines vertex and wavefunction counter-terms into a single counter-term. (Sakakibara 1984, section 8.) The counter-terms are obtained by the rescalings

$$g^0 \rightarrow g + \delta g \qquad (2\cdot6\cdot1)$$

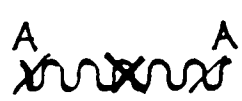
$$g^{0\prime} \rightarrow g' + \delta g' \qquad (2\cdot6\cdot2)$$

$$F^{02} \rightarrow F^2 + \delta F^2 \qquad (2\cdot6\cdot3)$$

in the bare Lagrangian and after application of renormalization conditions (1) and (2) of section 2·2 lead to



$$= -\,\text{Real}\,\tilde{\Pi}_T^{WW}(-M_W^2) \qquad (2\cdot6\cdot4)$$



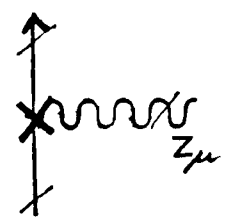
$$= 0 \qquad (2\cdot6\cdot5)$$



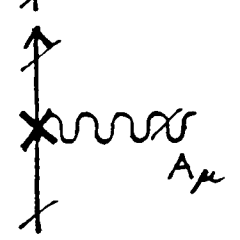
$$= -\tfrac{1}{2}\,\text{Real}\,\tilde{\Pi}_T^{ZZ}(-M_Z^2) \qquad (2\cdot6\cdot6)$$



$$= -\tfrac{1}{2} M_Z^2 \frac{c_\theta}{s_\theta}\,\text{Real}\left[\frac{\tilde{\Pi}_T^{WW}(-M_W^2)}{M_W^2} - \frac{\tilde{\Pi}_T^{ZZ}(-M_Z^2)}{M_Z^2}\right] \qquad (2\cdot6\cdot7)$$

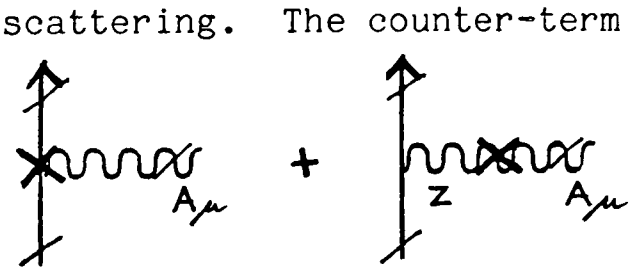


$$= ig\gamma_\mu\left\{\frac{\delta g'}{g'}\left[\left(\frac{4t_3 - 4s_\theta^2 Q}{4c_\theta}\right)\gamma_+ + \left(\frac{-4s_\theta^2 Q}{4c_\theta}\right)\gamma_-\right] + 2t_3 c_\theta \delta_-\gamma_+\right\} \qquad (2\cdot6\cdot8)$$



$$= ig\, s_\theta \gamma_\mu \left\{Q\,\frac{\delta g'}{g'} + 2t_3\,\delta_-\,\gamma_+\right\} \qquad (2\cdot6\cdot9)$$

The electric charge is defined as in section 2·5 from the q = 0 limit of Thomson scattering. The counter-term contribution is



$$= i\,\delta e\, Q\,\gamma_\mu \qquad (2\cdot6\cdot10)$$

where

$$\frac{\delta e}{e} = \frac{\delta g'}{g'} + 2s_\theta^2\,\delta_- \qquad (2\cdot6\cdot11)$$

δ_- is fixed from mass renormalization considerations analogous to conditions (1) and (2) of section 2·2 and $\delta g'/g'$ is adjusted to cancel the contribution from the four classes of graphs,

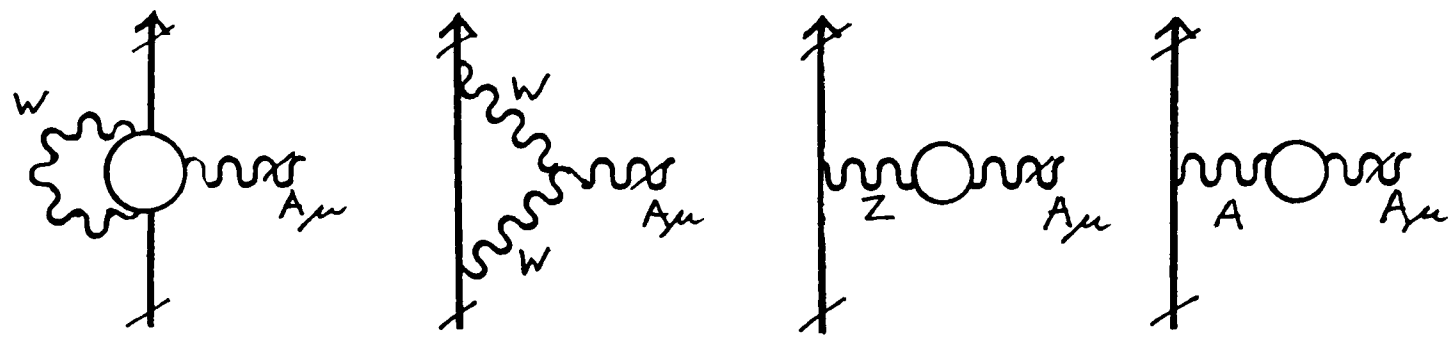
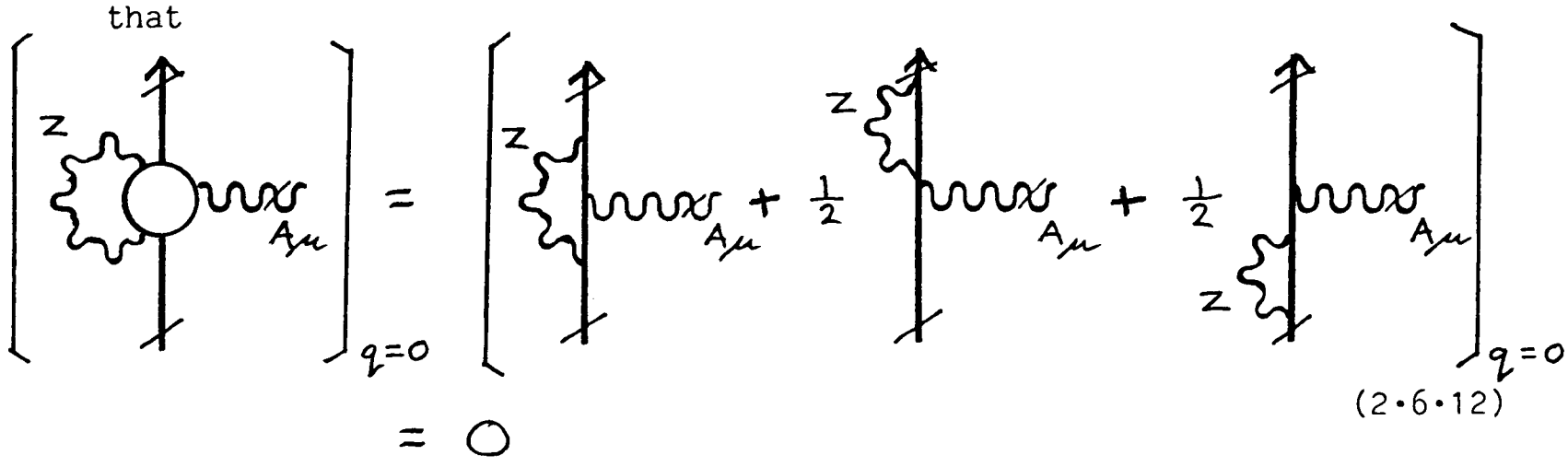


Figure 2·6·1

The blobs in Figure 2·6·1 now indicate that the virtual propagators are attached to the fermion lines in all possible ways. Note in particular that



(2·6·12)

where, as usual, the factor of $\frac{1}{2}$ is associated with renormalization of external on-shell fermions. One obtains

$$\tfrac{1}{2}\tilde{\Pi}'_{AA}(0) + \frac{\delta e}{e} + \frac{2g^2 s_\theta^2}{16\pi^2}\left(\Delta - \ell n\, M_W^2\right) = 0 \qquad (2\cdot6\cdot13)$$

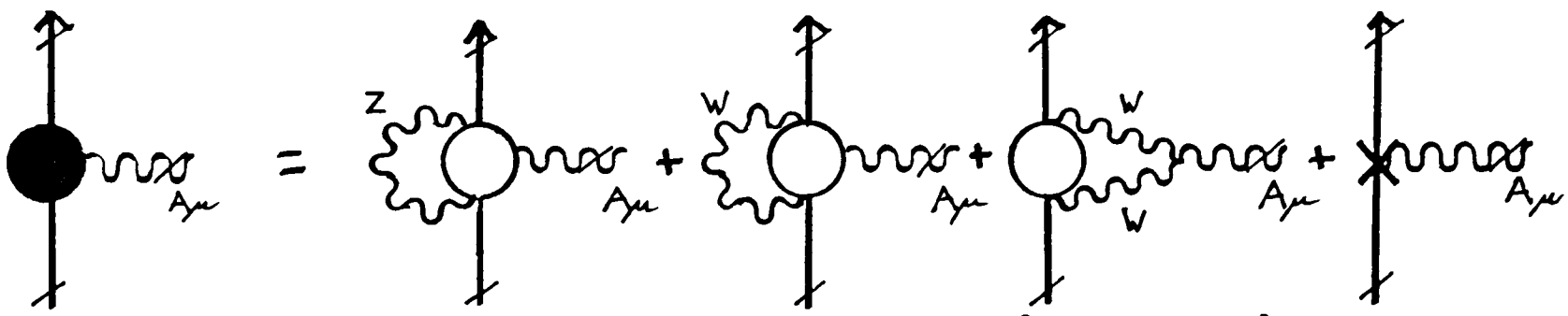
Defining for the GSW model

$$\Xi = \delta_- + \frac{2g^2}{16\pi^2}\left(\Delta - \ell n\, M_W^2\right) \qquad (2\cdot6\cdot14)$$

and using

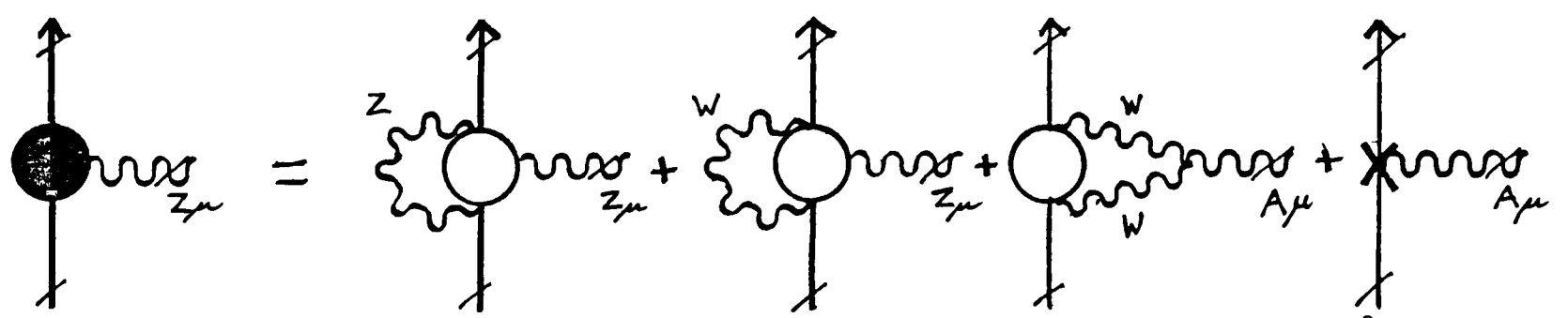
$$\frac{\delta g'}{g'} = -\tfrac{1}{2}\tilde{\Pi}'_{AA}(0) - \frac{2g^2 s_\theta^2}{16\pi^2}\left(\Delta - \ell n\, M_W^2\right) - 2s_\theta^2\,\delta_- \qquad (2\cdot6\cdot15)$$

one may write the complete one particle reducible (1PR) vertices at q = 0 as



$$= igs_\theta Q\gamma_\mu \{-\tfrac{1}{2}\tilde{\Pi}'_{AA}(0)\} + ig\gamma_\mu 2s_\theta c_\theta \Xi \{(\frac{4t_3 - 4s_\theta^2 Q}{4c_\theta})\gamma_+ + (\frac{-4s_\theta^2 Q}{4c_\theta})\gamma_-\}$$

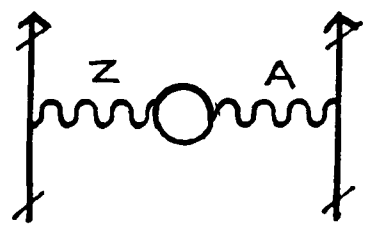
(2·6·16)



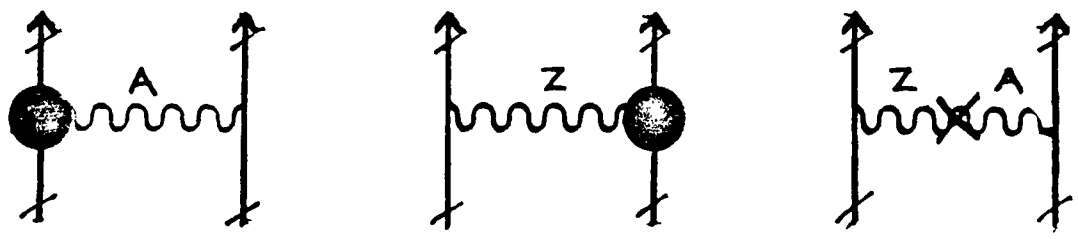
$$= igs_\theta Q\gamma_\mu \{2s_\theta c_\theta \Xi\} + ig\gamma_\mu \{(-\tfrac{1}{2}\tilde{\Pi}'_{AA}(0) - 2(s_\theta^2 - c_\theta^2)\Xi)\,[(\frac{4t_3 - 4s_\theta^2}{4c_\theta})\gamma_+$$

$$+ (\frac{-4s_\theta Q}{4c_\theta})\gamma_-] \qquad (2·6·17)$$

These conditions are independent of q^2 and it is now a simple matter by comparing with expressions for the counter-terms in section 2·2, that are known to give finite results at the Green's function level, to see how the cancellations occur. Consider for example the Z-A mixing graph



In the scheme of sections 2·2 - 2·5 these infinites are cancelled by a single Z-A mixing counter-term but here one must appeal to the graphs.



The sum of the relevant parts of the first two graphs is proportional to

$$\frac{1}{q^2 + M_Z^2} + \frac{1}{q^2} = \frac{2q^2 + M_Z^2}{q^2(q^2 + M_Z^2)} \qquad (2\cdot6\cdot18)$$

the numerator of the r.h.s. appears as a factor in the Z-A counter-term (2·2·22).

3 Accuracy and uncertainties in the renormalization scheme

3·1 Calculation of M_W in terms of the muon lifetime

The renormalization scheme described in the previous chapter possesses a number of drawbacks that serve to limit the accuracy attainable in a practical calculation. This chapter deals with ways in which these problems may be alleviated.

The first of the difficulties arises from the use of input data in the renormalization conditions that are not known to high accuracy from current experiments. A second, the hadronic uncertainty, is dealt with in section 3·2. The W and Z masses required by conditions (1) and (2) of section 2·2 and elsewhere through the definition of $\sin^2\theta_W$ (A·2·25) have been measured by the UA1 collaboration to be (Arnison et al. 1983)

$$M_W = 80{\cdot}9 \pm 1{\cdot}5 \pm 1{\cdot}2 \text{ GeV} \qquad (3{\cdot}1{\cdot}1)$$

$$M_Z = 95{\cdot}6 \pm 1{\cdot}4 \pm 1{\cdot}4 \text{ GeV} \qquad (3{\cdot}1{\cdot}2)$$

The presently unknown Higgs mass and top quark mass whose value is not yet well established (Arnison et al. 1984) are also required as input parameters. The results of calculations will therefore need to be displayed over the full range of possible values. As new experimental data comes to light sections of the tables can be excluded. The Higgs mass is bounded below by ~ 10 GeV (Weinberg 1976, Linde 1976) and the calculation is valid only in the region where the three and four Higgs vertices, proportional to M_H^2/M_W^2, remain perturbative.

The Z^0 mass is expected to be measured to an accuracy of better than 0·1% by the new generation of e^+e^- colliders; the SLC at SLAC and LEP1 at CERN, before measurement of the e^+e^- annihilation asymmetries is undertaken. On the other hand a very precise value for M_W will not be available until the W boson is produced in pairs.

The introduction of the muon lifetime τ_μ, measured at

$$\tau_\mu = 2{\cdot}19695 \pm 0{\cdot}00006\ \mu s \qquad (3{\cdot}1{\cdot}3)$$

(Giovanetti et al. 1984) provides a means of circumventing the problem. A complete calculation of radiative corrections to one loop has been undertaken by Sirlin 1980. The W mass may be expressed as a function of M_Z, τ_μ, α plus fermion and Higgs masses. One obtains

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 - \frac{8m_e^2}{m_\mu^2}\right)\left[1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2} + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{2\alpha}{3\pi}\ln\frac{m_\mu}{m_e}\right)\right] \qquad (3{\cdot}1{\cdot}4)$$

where m_e and m_μ are the electron and muon masses respectively. The $O(\alpha)$ contributions are traditional QED corrections and were first obtained by Kinoshita and Sirlin 1959 for a four fermion Hamiltonian evaluated with a cut off M_W. Comparison with the standard model gives

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1 + \Delta r) \qquad (3{\cdot}1{\cdot}5)$$

where Δr contains the effect of purely weak radiative corrections and those parts of the QED corrections not included in (3·1·4).

Using the relation

$$e = g\sin\theta_W \qquad (3{\cdot}1{\cdot}6)$$

and (A·2·26) gives at once

$$M_W^2 = \left(\frac{\pi\alpha}{\sqrt{2}G_\mu}\right)\frac{(1 + \Delta r)}{s_\theta^2} \qquad (3{\cdot}1{\cdot}7)$$

or

$$M_W^2 = M_Z^2\left[\tfrac{1}{2} + \tfrac{1}{2}\left(1 - \frac{4A_0^2}{M_Z^2}(1 + \Delta r)\right)^{\frac{1}{2}}\right] \qquad (3{\cdot}1{\cdot}8)$$

where

$$A_0 = \left(\frac{\pi\alpha}{\sqrt{2}G_\mu}\right)^{\frac{1}{2}} \qquad (3\cdot1\cdot9)$$

$$= 37\cdot281 \text{ GeV}$$

is obtained from τ_μ. The replacement

$$(1 + \Delta r) \rightarrow \left(1 - \Delta r - 16\pi^2\alpha^2 \text{Real } \Pi'^{(2)}_{AA}(-M_Z^2)\right)^{-1} \qquad (3\cdot1\cdot10)$$

in (3·1·7) and (3·1·8) takes account of all corrections $O(\alpha^n \ell n^n m_f^2/M_W^2)$ for $n \geq 2$ and $O(\alpha^2 \ell n\, m_f^2/M_W^2)$ where m_f is a generic fermion mass (Sirlin 1984, Marciano 1979) through Renormalization Group Equations (RGE). Here $16\pi^2\alpha^2 \text{Real } \Pi'^{(2)}_{AA}(q^2)$ denotes the $O(\alpha^2)$ contribution to the renormalized photon vacuum polarization function. In practice, to one loop, the third term on the r.h.s. of (3·1·10) can be neglected.

Δr is given by (Sirlin 1980)

$$\Delta r = \frac{g^2}{16\pi^2}\left\{4(\Delta - \ell n\, M_Z^2) + 6 + \left(4 + c_\theta^2 - \frac{6c_\theta^2}{s_\theta^2}\right)\ell n\, \frac{1}{c_\theta^2}\right\}$$

$$+ \text{Real } \frac{c_\theta^2}{s_\theta^2}\left[\frac{\tilde{\Pi}_T^{WW}(-M_W^2)}{M_W^2} - \frac{\tilde{\Pi}_T^{ZZ}(-M_Z^2)}{M_Z^2}\right]$$

$$+ \frac{2\delta e}{e}$$

$$+ \frac{1}{M_W^2}\left[\tilde{\Pi}_T^{WW}(0) - \text{Real } \Pi_T^{WW}(-M_W^2)\right]$$

$$+ \frac{g^2}{16\pi^2}\,\frac{5c_\theta^4 - 3s_\theta^2}{2s_\theta^2}\,\ell n\, \frac{1}{c_\theta^2} \qquad (3\cdot1\cdot11)$$

where the first term comes from vertex graphs,

the second from counter-terms

$$2\frac{\delta e}{e} = -\tilde{\Pi}'_{AA}(0) - \frac{g^2 s_\theta^2}{4\pi^2}(\Delta - \ell n\, M_W^2) \qquad \text{in the GSW}$$

the fourth term comes from the W self energy

and the fifth comes from heavy box diagrams.

Δr is naturally divided into three parts

$$\Delta r = \Delta r^{(1)} + \Delta r^{(2)} + \Delta r^{(3)} \qquad (3\cdot1\cdot12)$$

The quantity $\Delta r^{(1)}$ represents the contribution of the radiative corrections from heavy bosons, ghosts and Higgs particles to the muon decay matrix element. $\Delta r^{(2)}$ is the contribution from virtual lepton loops in the boson self energies of (3·1·11) and for $m_\ell^2 << M_Z^2$ is

$$\Delta r^{(2)} = \frac{g^2 s_\theta^2}{12\pi^2}\left[\sum_{\ell=e,\mu,\tau} \ell n\left(\frac{M_Z^2}{m_\ell^2}\right) - 5 + \frac{3}{4}\,\frac{(s_\theta^2 - c_\theta^2)}{s_\theta^4}\,\ell n\, c_\theta^2\right]$$

(3·1·13)

$\Delta r^{(3)}$ is the contribution from quark loops to the radiative corrections and as such involves the quantity $\tilde{\Pi}'^{(h)}_{AA}(0)$ which is the hadronic contribution to $\tilde{\Pi}'_{AA}(0)$ that arises courtesy of condition (3) in section 2·3. The evaluation of this contribution is problematic and subject to uncertainty. It is discussed in the next section. $\Delta r^{(2)}$ and $\Delta r^{(3)}$ are the dominant contributions by virtue of the large logarithms in (3·1·13) and the equivalent in the free field expansion of $\Delta r^{(3)}$.

From a practical point of view (3·1·8) does not give M_W in closed form because Δr depends on $\sin^2\theta_W$ which is related to M_W through the definition (A·2·26). However, it is found that when (3·1·8) is iterated, it quickly converges to give a self-consistent M_W for sensible starting values.

The result of an actual calculation with M_Z = 94 GeV and top quark mass, m_t = 30 GeV gives

$$\Delta r = \begin{cases} 0{\cdot}0650 \ ; \ M_H = 10 \text{ GeV} \\ 0{\cdot}0697 \ ; \ M_H = 100 \text{ GeV} \\ 0{\cdot}0789 \ ; \ M_H = 1000 \text{ GeV} \end{cases}$$

Other fermion masses are as given in point 3) in section 4·1.

In the case of M_H = 100

$$\Delta r^{(1)} = 0{\cdot}0031$$

$$\Delta r^{(2)} = 0{\cdot}0333$$

$$\Delta r^{(3)} = 0{\cdot}0333$$

Table 4·1 lists the results of a calculation of $\sin^2\theta_W$ as described in this section, for 90 GeV $\leqq M_Z \leqq$ 98 GeV, 30 GeV $\leqq m_t \leqq$ 230 GeV and 10 GeV $\leqq M_H \leqq$ 1000 GeV. This requires special attention to the hadronic uncertainty as discussed in the following section.

Table 4·2 gives the corresponding values for M_W.

A similar calculation has been carried out by Consoli et al. 1983 who have displayed the results graphically.

3·2 Hadronic contributions

Hadronic contributions are bound to arise from quark loops in boson self energies when these are evaluated for small $|q^2| \leq \Lambda_{QCD}$, the QCD scale. This represents a further difficulty for the renormalization scheme of section 2 and occurs because the electric charge is defined at $q = 0$.

In the calculation of the muon lifetime, corrections to the W propagator produce a term $\tilde{\Pi}_T^{WW}(0)$. The hadronic part is however $0(\alpha\, m_f^2/M_W^2)$, for m_f a generic hadron mass, and can be safely neglected. With exception of $\tilde{\Pi}'_{AA}(0)$, all other self energies are evaluated at large invariant momenta and can presumably be treated using perturbative QCD. In the calculation, the quantity, $\tilde{\Pi}_T^{ZA}(0)$, also appears for condition (4) in section 2·2 but its contribution from quarks vanishes because $\tilde{\Pi}_T^{ZA}(q^2) \propto q^2$.

According to (2·5·10) - (2·5·12), $\tilde{\Pi}'_{AA}(0)$ appears in the same way in all of the counter-terms δZ_W, δZ_A and δZ_Z. To evaluate the hadronic contribution one writes

$$\tilde{\Pi}'^{(h)}_{AA}(0) = \tilde{\Pi}'^{(h)}_{AA}(\hat{q}^2) - [\tilde{\Pi}'^{(h)}_{AA}(\hat{q}^2) - \tilde{\Pi}'^{(h)}_{AA}(0)] \qquad (3\cdot2\cdot1)$$

where $\hat{q}^2$ is some space-like invariant momentum ($\hat{q}^2 > 0$) so that $\tilde{\Pi}'^{(h)}_{AA}$ is real.

Appealing to asymptotic freedom, the first term on the r.h.s. of (3·2·1) can be evaluated using free field theory and assuming three colours

$$\tilde{\Pi}'^{(h)}_{AA}(\hat{q}^2) = \frac{g^2 s_\theta^2}{16\pi^2}\, 4 \sum_q Q_q^2 \left[6B_3(\hat{q};\, m_q,\, m_q) + \ell n\, 2\right] \qquad (3\cdot2\cdot2)$$

where the sum is over individual quarks. The form factor B_3 is defined in Appendix B.

Note in particular that the terms involving light quarks in (3·2·2) are by (B·31c)

$$\frac{g^2 s_\theta^2}{16\pi^2}\, 4 \sum_{q=u,d,s} \left(-\Delta + \ell n\, \hat{q}^2 - \tfrac{5}{3} + \ell n\, 2\right) + \text{terms} \propto \frac{m_q^2}{\hat{q}^2} \qquad (3\cdot2\cdot3)$$

and the ambiguities associated with light quark masses are thus alleviated.

The bracketted term in (3·2·1) can be evaluated by appealing to experimental data and the subtracted dispersion relation,

$$\tilde{\Pi}'_{AA}(q^2) - \tilde{\Pi}'_{AA}(0) = \frac{-q^2}{4\pi^2\alpha} \int_{-\infty}^{-4\mu^2} \frac{\sigma(q'^2)}{(q'^2 - q^2 - i\varepsilon)}\, dq'^2 \qquad (3\cdot2\cdot4)$$

where σ is the cross section for $e^+e^- \to$ hadrons and μ is the π^0 mass.

The data has been examined (Paschos 1979, Wetzel 1981) and gives

$$\tilde{\Pi}'^{(h)}_{AA}(100\ \text{GeV}^2) - \tilde{\Pi}'^{(h)}_{AA}(0) = 0{\cdot}01474 \qquad (3\cdot2\cdot5)$$

for a top quark mass of 18 GeV.

Assuming that the contributions from the heavy quarks can be calculated perturbatively even at $q^2 = 0$ (Novikov et al. 1978, section 5) this value may be readily corrected for other top quark masses. In particular, one finds that the contribution from u, d, c, s and b quarks only is

$$\left[\tilde{\Pi}'^{(h)}_{AA}(100\ \text{GeV}^2) - \tilde{\Pi}'^{(h)}_{AA}(0)\right]_{u,d,c,s,b} = 0{\cdot}01472 \qquad (3\cdot2\cdot6)$$

The e^+e^- → hadrons data was incomplete between the two pion and four pion thresholds and an error of ± 0·002 has been given by Sirlin 1980.

Caprini et al. 1984 have succeeded in placing bounds on this quantity from purely theoretical arguments by consideration of its analytic structure within the context of QCD subject to reasonable assumptions about its derivatives near the onset of asymptotic freedom

and its contribution to the muon anomalous magnetic moment. They obtain including only u, d, s, and c quarks

$$0{\cdot}0112 \leq \bar{\Pi}'^{(h)}_{AA}(79\ \mathrm{GeV}^2) - \bar{\Pi}'^{(h)}_{AA}(0) \leq 0{\cdot}0154 \qquad (3{\cdot}2{\cdot}7)$$

thus giving a theoretical error in $\Delta r \pm 0{\cdot}0021$ corresponding to an uncertainty of $\pm 0{\cdot}0006$ in $\sin^2\theta_W$. This will give an uncertainty of $\pm 0{\cdot}005$ in A_{pol} on resonance.

Penso and Verzegnassi 1985 have recently reanalysed the experimental data for $e^+e^- \rightarrow$ hadrons and have succeeded in reducing the error in the Δr. They find

$$\tilde{\Pi}'^{(h)}_{AA}(79\ \mathrm{GeV}^2) - \tilde{\Pi}'^{(h)}_{AA}(0) = 0{\cdot}0145 \pm 0{\cdot}0013$$

which would give an uncertainty of $\pm 0{\cdot}0004$ in $\sin^2\theta_W$ and $\pm 0{\cdot}003$ in A_{pol} on resonance.

4·1 Asymmetries in e^+e^- annihilation

On Z^0 resonance

This section discusses the features of the longitudinal polarization asymmetry, A_{pol}, and the forward-backward asymmetry, A_{fb}, that on resonance make them stringent tests of the standard model. To begin, general expressions for A_{pol} and A_{fb} in terms of effective vertex coupling constants are derived. Noting definitions (1·1·4) and (1·1·5), a general matrix element with electron-boson vertex

$$i\gamma_\mu (\beta_+^e \gamma_+ + \beta_-^e \gamma_-) \qquad (4\cdot1\cdot1)$$

outgoing fermion-boson vertex

$$i\gamma_\mu (\beta_+^f \gamma_+ + \beta_-^f \gamma_-) \qquad (4\cdot1\cdot2)$$

and boson propagator $\hat{\chi}$ as shown in Figure 4·1 leads to the expressions

$$A_{pol} = \frac{\beta_+^{e2} - \beta_-^{e2}}{\beta_+^{e2} + \beta_-^{e2}} \qquad (4\cdot1\cdot3)$$

$$A_{fb} = \frac{x}{1 + x^2/3} \frac{(\beta_+^{f2} - \beta_-^{f2})}{(\beta_+^{f2} + \beta_-^{f2})} \frac{(\beta_+^{e2} - \beta_-^{e2})}{(\beta_+^{e2} + \beta_-^{e2})} \qquad (4\cdot1\cdot4)$$

for real $\beta_\pm$.

On resonance and considering only the dominant Z^0 exchange graphs gives,

$$A_{pol} = \frac{-2v_\theta}{1 + v_\theta^2} . \qquad (4\cdot1\cdot5)$$

When the outgoing fermions are charged leptons other than electrons

$$A_{fb} = \frac{x}{1 + x^2/3} A_{pol}^2 \qquad (4\cdot1\cdot6)$$

For out-going up-type quarks,

$$A_{fb} = \frac{x}{1 + x^2/3} \frac{3(1 - v_\theta)}{(5 - 2v_\theta + 2v_\theta^2)} A_{pol} \qquad (4\cdot1\cdot7)$$

and out-going down-type quarks,

$$A_{fb} = \frac{x}{1 + x^2/3} \frac{6(1 - v_\theta)}{(13 - 4v_\theta + v_\theta^2)} A_{pol} \qquad (4\cdot1\cdot8)$$

For the case of quark final states one assumes that fragmentation into hadrons can be considered as a second step occurring after pair creation and not affecting the cross-sections.

Note that A_{pol} is independent of the beampipe, the identity of the outgoing fermions and the exchange propagator. These properties persist to some extent in first order and may be understood as follows;

The product of matrix elements for left (right) hand polarized incoming electrons from (E.3) is of the form

$$\int_{-x}^{x} MM'^{\dagger}\, d(\cos\theta) = \tfrac{2}{3} x\,(3 + x^2)\,(\beta_+^f \bar{\beta}_+^{'f} + \beta_-^f \bar{\beta}_-^{'f})\,\beta_{+(-)}^e \bar{\beta}_{+(-)}^{'e}\,\chi\chi'^{\dagger} \qquad (4\cdot1\cdot9)$$

where $\beta_\pm^{'}$, χ' refer to the matrix element M' and $\chi = q^2\hat{\chi}$, $\chi' = q^2\hat{\chi}'$ for 4-momentum transfer q. The $\beta_\pm$ are taken to be independent of θ.

For the moment we assume that the outgoing particle vertex is described by local vector and axial vector effective coupling. Expression (4·1·9) may be used in (1·1·4) to form A_{pol}. The beampipe factor cancels out provided the $\beta_\pm$ are independent of θ. This is the case for all but the box diagrams which however are relatively small because they contain no resonant Z^0 propagator. It will be seen that $\beta_\pm^e$ and $\beta_\pm^{'e}$ are real and the same for all terms with the exception of those arising from interference of the lowest order Z^0 exchange with the graphs in Figure 4·2, heavy box diagrams and corrected photon exchange as well as interference between lowest order photon exchange and corrected Z^0 exchange. Note that the interference between the lowest order Z^0 exchange and photon exchange vanishes on resonance.

The boxes are small for the reasons just mentioned and terms involving a photon graph are $O(\alpha\Gamma_Z/M_Z)$ where Γ_Z is the decay width of the Z^0. The contributions of the graphs of Figure 4·2 may be absorbed as $O(\alpha)$ corrections to the effective couplings, $\beta_{\pm}^{e}$. Thus the cancellation of the propagator and outgoing fermion dependence goes through to $O(\alpha)$ with the exception of the suppressed photon interference terms. This is true for Bhabha scattering only when the angle θ is large enough that the t channel contribution is negligible.

For this reason it is only necessary to know the decay width of the Z^0, Γ_Z to $O(\alpha)$ to obtain 1% accuracy in the asymmetries. A two-loop calculation would be required, however, to predict the absolute cross-sections on resonance to 1%. In this case, the $O(\alpha_s)$ gluonic corrections to Γ_Z should be included as well (Hayashi and Katsuura 1984). From (2·2·30) one has

$$\text{Im}\left[G_T^{ZZ}(-M_Z^2)\right]^{-1} = \text{Im}\left\{\Pi_T^{ZZ} - \frac{\left[\Pi_T^{ZA}\right]^2}{\Pi_T^{AA} + M_Z^2}\right\} \qquad (4\cdot1\cdot10)$$

or to one loop

$$\Gamma_Z = \frac{1}{M_Z}\,\text{Im}\,\Pi_T^{ZZ} \qquad (4\cdot1\cdot11)$$

where the r.h.s's of (4·1·10) and (4·1·11) are evaluated at $q^2 = -M_Z^2$. Note that any large logarithms in the second term of (4·1·10) are at least $O(\alpha^3 \ell n\, m_f^2/M_Z^2)$ and hence (4·1·11) is an adequate approximation.

When the outgoing fermions are quarks the situation is potentially more complicated. QCD corrections to vertices can produce a magnetic moment type term, $\sigma_{\mu\nu}q_\nu\gamma_{\pm}$. Naive perturbative QCD indicates that such terms lead to contributions proportional to $\frac{m_f^2}{M_Z^2}\,\ell n\,\frac{m_f^2}{M_Z^2}$ or $\frac{m_f^2}{M_Z^2}\ell n^2\,\frac{m_f^2}{M_Z^2}$ and are likely to be negligible (Itzykson and Zuber 1980).

This aside, the foregoing discussion can be repeated for the general case where the vertex associated with the outgoing fermions is

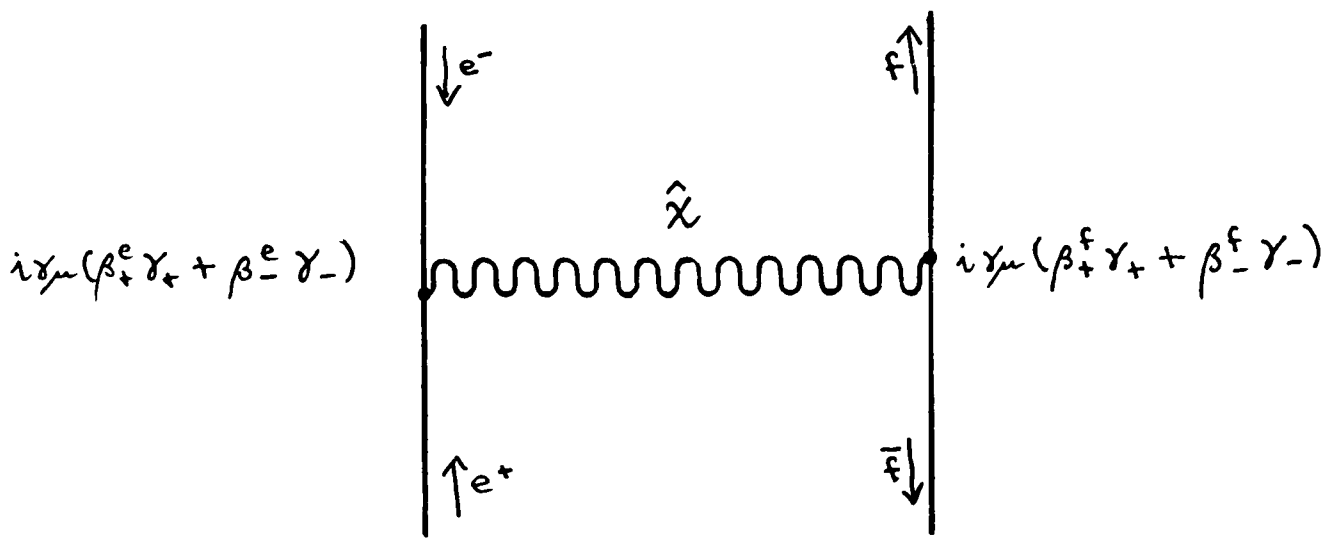


Figure 4·1

The general matrix element.

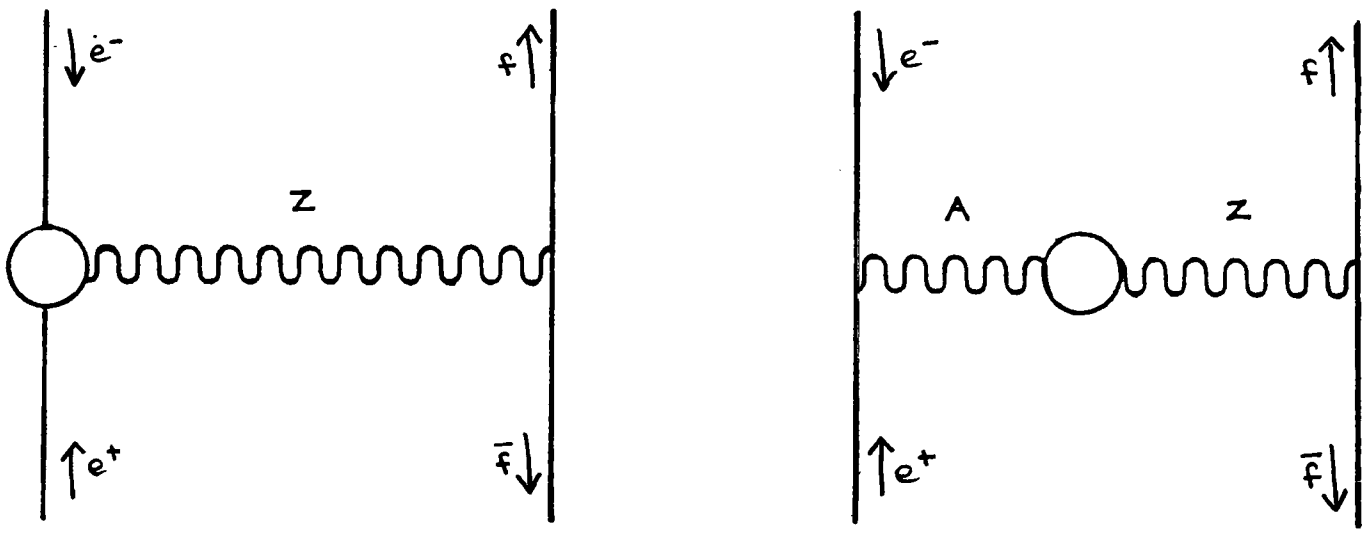


Figure 4·2

Diagrams modifying the effective electron-Z^0 vertex.

expressible in terms of local vector, axial-vector, tensor and pseudo-tensor effective couplings. For simplicity it will be assumed that the tensor and pseudo-tensor couplings are not present in lowest order. An expression similar to (4·1·9) arises. It is proportional to $\beta^{e}_{\mp(-)}\overline{\beta}'^{e}_{+(-)}$ but the dependence on beampipe is different for those terms involving (axial) vector couplings and those involving (pseudo) tensors. The beampipe dependence then persists in the latter type in A_{pol}.

A cancellation of the propagator and outgoing particle dependence now occurs as for the pure (axial) vector case. From the previous discussion, then, it is clear that QCD corrections to A_{pol} on resonance only occur in interference with photon exchange graphs. Again these are suppressed by at least a factor Γ_Z/M_Z.

One may conclude that higher order QCD corrections are likely to be negligible on resonance. However should this turn out to be not the case once calculated, their inclusion into the computer program would be a straight forward matter.

In the case of A_{fb} similar considerations lead to the conclusion that the $O(\alpha)$ corrected result is independent of the exchange propagator but dependent on the nature of the outgoing fermions. QCD effects thus appear when these are quarks and have been calculated to first order in α_s by Jersák et al. 1981.

If s^2_θ is calculated from M_Z and τ_μ as described in section 3·1, then, because of the suppression by the factor v_θ in (4·1·5) - (4·1·8), the asymmetries change by an order of magnitude over the range of M_Z currently allowed by UA1 data (3·1·2). For μ pair production A_{fb} changes by almost two orders. This makes the measurement of the asymmetries on resonance an extremely sensitive test of the predictions of the standard model and a good place to search for deviations.

Off Z^0 resonance

Below resonance the photon exchange and the interference between Z^0 exchange and photon exchange dominate and it is in this region that, because of available accelerator energies, most calculations of A_{fb} have concentrated. Here the sensitivity to M_Z, m_t and M_H falls off and the box diagrams increase in relative importance. There is still no need to know

$$\mathrm{Im}\left(\Pi_T^{ZZ}(q^2)\right) \qquad (4\cdot1\cdot12)$$

to better than $O(\alpha)$ because now it is swamped by $q^2 + M_Z^2$ in the Z^0 propagator, $(q^2 + M_Z^2 - iM_Z\Gamma_Z - i\varepsilon)^{-1}$. A_{pol} remains weakly dependent on the beampipe at the one loop level. From an experimental point of view, the statistics are comparatively poorer.

Details of the calculation

This work is concerned only with purely weak radiative corrections and not the QED corrections of Figure 4·6. Fermion loop corrections to the photon propagator are included amongst these because of the care needed in evaluating the hadronic corrections. This strategy also isolates large logarithm contributions to the propagators in the weak corrections where they cancel in forming the asymmetries both on and off resonance. These large logarithm contributions are $\propto \ell n\,(m_f^2/q^2)$ and arise from $\tilde{\Pi}'_{AA}(0)$ in the boson wave-function renormalization counter-terms. The cancellation may be quickly seen from (2·5·10) - (2·5·12) by noting that $\tilde{\Pi}'_{AA}(0)$ enters in the same way for each counter-term. The effect is a consequence of the fact that the SU(2) and U(1) coupling constants g and g' are renormalized in the same way by the dominant internal fermion loop contributions (but not the

bosons), in the RGE which sums automatically the terms proportional to $\alpha^{n+1}\ \ell n^n\ (m_f^2/M_Z^2)$ in 4-fermion matrix elements.

There is one exception to this cancellation of large logs which has assumed $m_f << M_Z$. For top quark mass $m_t >> M_Z$ the global flavour SU(2) symmetry in the quark mass matrix is quite badly broken since $m_b << M_Z$. The corrections to the asymmetries themselves and to $\sin^2\theta_W$ can be substantial, blowing up quadratically with m_t as

$$A_{pol}\ (-M_Z^2) \sim \frac{12\alpha}{\pi}\ \frac{s_\theta{}^2}{(1 + v_\theta{}^2)^2}\ \frac{m_t^2}{M_Z^2} \qquad (4\cdot1\cdot13)$$

No further large logarithms occur in the weak correction to the asymmetries and those from the photonic vertex and bremsstrahlung graphs cancel amongst themselves in the asymmetries.

The only place that large logarithms now enter is in the calculation of M_W in terms of M_Z and τ_μ. All other radiative corrections are strictly $O(\alpha)$ and therefore small.

The calculational details for self energies are to be found in Appendix B and for vertices are in Appendix C. Details for the evaluation of the heavy box diagrams form the contents of Appendix D and Appendix E contains formulas for the relevant cross sections. Integrations of cross sections over solid angle are carried out numerically.

Because the free Z^0 propagator proportional to $(q^2 + M_Z^2 - i\varepsilon)^{-1}$ blows up on resonance, it is necessary to combine the lowest, first and a subset of higher order corrections to the propagator by substituting $(q^2 + M_Z^2 - \Pi_T^{ZZ}(q^2) - i\varepsilon)^{-1}$ wherever it occurs. The full q^2 dependence of the imaginary part of $\Pi_T^{ZZ}(q^2)$ is thus correctly accounted for off resonance instead of just representing it as a constant (2·5 GeV) M_Z as

has often been done previously. One may calculate to one loop that for M_Z = 94 GeV, m_t = 30 GeV and M_H = 100 GeV.

$$\frac{1}{M_Z}\,\mathrm{Im}\,\Pi_T^{ZZ}(q^2) = \begin{cases} 0{\cdot}235\ \text{GeV} & ;\quad q^2 = -(29\ \text{GeV})^2 \\ 0{\cdot}334\ \text{GeV} & ;\quad q^2 = -(34{\cdot}5\ \text{GeV})^2 \\ 0{\cdot}546\ \text{GeV} & ;\quad q^2 = -(44\ \text{GeV})^2 \\ 1{\cdot}02\ \text{GeV} & ;\quad q^2 = -(60\ \text{Gev})^2 \\ 2{\cdot}67\ \text{GeV} & ;\quad q^2 = -M_Z^2 \end{cases} \qquad (4{\cdot}1{\cdot}14)$$

Note that these values do not include QCD corrections to internal quark loops. They can be allowed for, at large $|q^2|$, by multiplying the quark contribution by the factor

$$(1 + \frac{\alpha_S}{\pi})$$

with

$$\frac{\alpha_S}{\pi} = \frac{12}{(33 - 4N)\ \ell n\ (-q^2/\Lambda^2_{QCD})} \qquad (4{\cdot}1{\cdot}15)$$

Here N is the number of generations and Λ is the QCD scale,

$$\Lambda_{QCD} \sim 0{\cdot}4\ \text{GeV}$$

(Hayashi and Katsuura 1984, Chang et al. 1982). Typically these corrections increase the width by roughly 5%.

Further points to note about the calculation are:

(1) In accord with the Feynman rules of Appendix A, the calculation is done in 't Hooft $R_\xi(\xi=1)$ gauge.

(2) Kobayashi - Maskawa mixing is neglected.

(3) The fermion masses used were (all in GeV)

$$m_e = 0{\cdot}511 \times 10^{-3}$$

$$m_\mu = 0{\cdot}105$$

$$m_\tau = 1{\cdot}78$$

$$m_\nu = 0$$

$$m_u = m_d = 0{\cdot}3$$

$$m_s = 0{\cdot}7$$

$$m_c = 1{\cdot}5$$

$$m_b = 4{\cdot}5$$

(4) It is assumed that $m_f^2 << |q^2|, M_Z^2$ where m_f is the mass of the external fermions or internal fermions in vertex and box diagrams.

(5) From (4) it follows that q^2 is above production threshold for most fermions and need not be below threshold for the heavy bosons and a careful treatment of the imaginary parts of the various contributions is therefore required. This has been done and the results in Appendices B, C and D are correct for all q^2 satisfying the criterion of point (4). In particular there is no upper limit for $|q^2|$ in the formulas obtained.

From previous discussion, certain checks can be made as to the correctness of the calculation

(1) When an arbitrary scale Λ is inserted into the dimensional logarithms of the B form factors, the numerical result for the asymmetries and Δr must not depend on the value of Λ. This indicates both the cancellation of dimensions in logarithms and of the divergent Δ's.

(2) All masses and A_0 of (3·1·9) and $(\hat{q}^2)^{\frac{1}{2}}$ of (3·2·1) can be scaled by a common arbitrary factor without changing the result for the dimensionless Δr and asymmetries.

(3) To eliminate the possibility of double counting of graphs, the results were recalculated using Sirlin's renormalization scheme (section 2·6). In this scheme the divergences cancel only when all graphs are taken together and consequently it can only be applied straightforwardly off resonance.

(4) All large logarithms should have cancelled on resonance so the results for the asymmetries should be close to the tree level results of (4·1·5) - (4·1·8).

These checks were all performed and found to be satisfied. In addition, the values of Δr produced were compared with those given by Sirlin 1980 and agreed for appropriate choice of M_Z.

Numerical results

The numerical results are given in Tables 4·1 - 4·6. All tables list results over a range; $90 \leqq M_Z \leqq 98$, $30 \leqq m_t \leqq 230$ and, with the exception of Table 4·3, $10 \leqq M_H \leqq 1000$. The beampipe for Tables 4·3 - 4·6 is $|\cos\theta| \leqq 1$.

Table 4·1 lists $\sin^2\theta_W$ calculated as described in section 3·1 and Table 4·2 gives the corresponding values for M_W.

Table 4·3 lists A_{fb} for $q^2 = -(29\ \text{GeV})^2$, $-(34{\cdot}5\ \text{GeV})^2$, $-(44\ \text{GeV})^2$ and $-(60\ \text{GeV})^2$. The variation in A_{fb} over the range $10 \leqq M_H \leqq 1000$ is at most 0·0006.

Table 4·4 gives A_{fb} on resonance.

Table 4·5 lists A_{pol} for $q^2 = -(60\ \text{GeV})^2$ and displays a high degree of sensitivity to the input parameters. Thus if longitudinal polarization were available at TRISTAN detailed studies of electro-weak theory at the one-loop level could be carried out.

Table 4·6 gives A_{pol} on resonance.

Tables 4·3 - 4·6 do not include contributions from the graphs of Figure 4·6.

The tables bear out the previously discussed sensitivity of the results on M_Z with $\sin^2\theta_W$ on resonance. LEP1 should give the Z^0 mass to better than 0·1% so that a large part of the tables could be excluded. Note that A_{pol} on resonance can depend dramatically on m_t for large top quark mass as a result of (4·1·13) which blows up quadratically for $m_t >> M_Z$. The dependence on large Higgs mass is less dramatic blowing up logarithmically for $M_H >> M_Z$. Thus an experiment capable of measuring A_{pol} to ± 0·01 on Z^0 resonance could set upper bounds for one of these masses if the other were known. The dependence of A_{pol} on M_H for M_Z = 94 GeV and m_t = 30 GeV is shown in Figure 4·3a. A_{pol} as a function of m_t for M_Z = 94 GeV and M_H = 100 GeV is displayed in Figure 4·3b.

The forward-backward asymmetry, A_{fb}, and the longitudinal polarization asymmetry, A_{pol}, in $e^+e^- \to \mu^+\mu^-$ are displayed in Figures 4·4a,b and Figures 4·5a,b respectively over the range 20 GeV $\leqq \sqrt{-q^2} \leqq$ 120 GeV with and without radiative corrections. Reasonable experimental error bars are also indicated. The solid lines include all radiative corrections except those of Figure 4·6 for m_t = 30 GeV and M_H = 100 GeV. The dashed lines include only tree graphs for μ decay and $e^+e^- \to \mu^+\mu^-$. From the figures it can be seen that the best place to look for the effects of radiative corrections is near resonance and of the two asymmetries A_{pol} is the more sensitive. The dotted lines of Figure 4·5b give the polarization asymmetry including only the renormalization of g^2, the $SU(2)_L$ muon decay coupling constant, by fermion loops. The observation of this (dominant) contribution would indicate that $g^2/4\pi$ is renormalized in the same way as α by the large logarithms and therefore that some sort of unification picture is correct. It is possible to imagine a

scenario in which muon decay is unrelated to QED and in that case this large shift in the polarization asymmetry, and to a smaller extent in the forward-backward asymmetry, would not appear.

It might be possible to reduce the range of M_Z in the tables by consideration of semi-leptonic and neutrino scattering processes (Marciano and Sirlin 1980, 1981, Llewellyn-Smith and Wheater 1981, Wheater and Llewellyn-Smith 1982, Sarantakos et al. 1983) especially in conjunction with the results of ν_μ - e scattering from the CHARM II collaboration (Busi et al. 1983).

Table 4·1

$\sin^2\theta_W$ for various values of M_Z, M_H, m_t (all masses in GeV)

$$\sin^2\theta_W = 1 - M_W^2/M_Z^2$$

	M_Z	$M_H = 10$	$M_H = 100$	$M_H = 1000$
	90	.2418	.2434	.2466
	92	.2271	.2287	.2318
$m_t = 30$	94	.2141	.2155	.2185
	96	.2023	.2038	.2067
	98	.1917	.1931	.1959
	90	.2420	.2436	.2468
	92	.2276	.2291	.2322
$m_t = 60$	94	.2147	.2162	.2192
	96	.2032	.2046	.2075
	98	.1927	.1941	.1969
	90	.2382	.2398	.2431
	92	.2237	.2253	.2284
$m_t = 90$	94	.2108	.2123	.2153
	96	.1993	.2008	.2037
	98	.1889	.1903	.1932
	90	.2337	.2353	.2386
	92	.2191	.2207	.2239
$m_t = 130$	94	.2061	.2076	.2107
	96	.1944	.1959	.1989
	98	.1839	.1853	.1883
	90	.2267	.2284	.2318
	92	.2122	.2138	.2171
$m_t = 180$	94	.1991	.2007	.2039
	96	.1874	.1889	.1920
	98	.1767	.1782	.1813
	90	.2176	.2194	.2229
	92	.2030	.2047	.2081
$m_t = 230$	94	.1899	.1916	.1949
	96	.1781	.1797	.1830
	98	.1672	.1688	.1721

Table 4·2

M_W for various values of M_Z, M_H, m_t, (all masses in GeV)

$$M_W = M_Z \cos\theta_W$$

	M_Z	$M_H = 10$	$M_H = 100$	$M_H = 1000$
	90	78.37	78.29	78.12
	92	80.88	80.80	80.64
$m_t = 30$	94	83.33	83.26	83.10
	96	85.74	85.66	85.51
	98	88.11	88.03	87.88
	90	78.36	78.27	78.11
	92	80.86	80.78	80.62
$m_t = 60$	94	83.30	83.22	83.06
	96	85.69	85.62	85.46
	98	88.05	87.98	87.82
	90	78.55	78.47	78.30
	92	81.06	80.98	80.81
$m_t = 90$	94	83.50	83.43	83.27
	96	85.90	85.82	85.67
	98	88.26	88.18	88.03
	90	78.79	78.70	78.53
	92	81.30	81.22	81.05
$m_t = 130$	94	83.75	83.67	83.51
	96	86.16	86.08	85.92
	98	88.53	88.45	88.29
	90	79.14	79.06	78.88
	92	81.66	81.57	81.40
$m_t = 180$	94	84.12	84.04	83.87
	96	86.54	86.46	86.29
	98	88.92	88.84	88.67
	90	79.61	79.52	79.34
	92	82.13	82.04	81.87
$m_t = 230$	94	84.61	84.52	84.34
	96	87.03	86.95	86.77
	98	89.43	89.34	89.17

Table 4·3

Forward-backward asymmetry, A_{fb}, off resonance for M_H = 100 GeV and various values of q^2, M_Z and m_t (all masses in GeV)

A_{fb} (q^2, $|\cos\theta| \leq 1$)

q^2	M_Z	m_t = 30	m_t = 90	m_t = 230
	90	-.0602	-.0604	-.0612
	92	-.0599	-.0601	-.0609
- (29 GeV)^2	94	-.0596	-.0598	-.0606
	96	-.0594	-.0595	-.0604
	98	-.0591	-.0592	-.0601
	90	-.0893	-.0895	-.0907
	92	-.0886	-.0888	-.0900
- (34.5 GeV)^2	94	-.0880	-.0881	-.0894
	96	-.0874	-.0876	-.0888
	98	-.0868	-.0870	-.0883
	90	-.1612	-.1615	-.1637
	92	-.1590	-.1593	-.1615
- (44 GeV)^2	94	-.1570	-.1573	-.1595
	96	-.1552	-.1555	-.1577
	98	-.1535	-.1537	-.1559
	90	-.3856	-.3855	-.3905
	92	-.3740	-.3739	-.3787
- (60 GeV)^2	94	-.3635	-.3634	-.3680
	96	-.3539	-.3537	-.3582
	98	-.3452	-.3450	-.3494

Table 4·4

Forward-backward asymmetry, A_{fb}, on Z^0 resonance for various values of M_Z, M_H, m_t (all masses in GeV)

A_{fb} ($q^2 = -M_Z^2$, $|\cos\theta| \leq 1$)

	M_Z	$M_H = 10$	$M_H = 100$	$M_H = 1000$
	90	.0046	.0039	.0028
	92	.0255	.0236	.0209
$m_t = 30$	94	.0596	.0570	.0532
	96	.1011	.0979	.0934
	98	.1457	.1422	.1374
	90	.0044	.0037	.0025
	92	.0256	.0237	.0208
$m_t = 60$	94	.0599	.0572	.0533
	96	.1014	.0982	.0936
	98	.1460	.1425	.1375
	90	.0047	.0041	.0033
	92	.0269	.0251	.0226
$m_t = 90$	94	.0619	.0593	.0558
	96	.1039	.1008	.0965
	98	.1487	.1454	.1407
	90	.0046	.0040	.0036
	92	.0282	.0265	.0245
$m_t = 130$	94	.0643	.0618	.0587
	96	.1069	.1039	.1001
	98	.1522	.1489	.1447
	90	.0032	.0028	.0030
	92	.0292	.0276	.0263
$m_t = 180$	94	.0668	.0645	.0620
	96	.1105	.1077	.1045
	98	.1564	.1533	.1498
	90	-.0010	-.0012	-.0001
	92	.0279	.0266	.0261
$m_t = 230$	94	.0676	.0655	.0640
	96	.1126	.1101	.1078
	98	.1595	.1566	.1539

Table 4·5

Longitudinal polarization asymmetry, A_{pol}, at $q^2 = -(60\ GeV)^2$ for various values of M_Z, M_H, m_t (all masses in GeV)

A_{pol} $(q^2 = -(60\ GeV)^2,\ |\cos\theta| \leq 1)$

	M_Z	$M_H = 10$	$M_H = 100$	$M_H = 1000$
	90	-.0113	-.0098	-.0075
	92	-.0323	-.0309	-.0288
$m_t = 30$	94	-.0504	-.0492	-.0472
	96	-.0662	-.0650	-.0631
	98	-.0800	-.0789	-.0771
	90	-.0121	-.0106	-.0083
	92	-.0332	-.0318	-.0297
$m_t = 60$	94	-.0513	-.0500	-.0480
	96	-.0670	-.0658	-.0640
	98	-.0808	-.0797	-.0779
	90	-.0136	-.0121	-.0098
	92	-.0346	-.0332	-.0331
$m_t = 90$	94	-.0526	-.0513	-.0493
	96	-.0683	-.0671	-.0652
	98	-.0820	-.0809	-.0791
	90	-.0160	-.0144	-.0121
	92	-.0368	-.0354	-.0332
$m_t = 130$	94	-.0548	-.0534	-.0514
	96	-.0704	-.0691	-.0672
	98	-.0841	-.0829	-.0811
	90	-.0199	-.0183	-.0159
	92	-.0405	-.0391	-.0368
$m_t = 180$	94	-.0583	-.0570	-.0549
	96	-.0738	-.0726	-.0706
	98	-.0875	-.0862	-.0844
	90	-.0252	-.0236	-.0211
	92	-.0456	-.0441	-.0418
$m_t = 230$	94	-.0632	-.0618	-.0596
	96	-.0787	-.0773	-.0752
	98	-.0922	-.0910	-.0889

Table 4·6

Longitudinal polarization asymmetry, A_{pol}, on Z^0 resonance for various values of M_Z, m_t, M_H (all masses in GeV)

$$A_{pol}\ (q^2 = -M_Z^2,\ |\cos\theta| \leq 1)$$

	M_Z	$M_H = 10$	$M_H = 100$	$M_H = 1000$
	90	.0613	.0536	.0423
	92	.1762	.1691	.1590
$m_t = 30$	94	.2756	.2692	.2602
	96	.3615	.3557	.3475
	98	.4354	.4302	.4229
	90	.0640	.0563	.0451
	92	.1786	.1716	.1615
$m_t = 60$	94	.2777	.2714	.2624
	96	.3633	.3575	.3495
	98	.4370	.4318	.4246
	90	.0703	.0626	.0513
	92	.1844	.1773	.1672
$m_t = 90$	94	.2830	.2767	.2676
	96	.3681	.3624	.3543
	98	.4414	.4362	.4290
	90	.0805	.0728	.0616
	92	.1936	.1866	.1765
$m_t = 130$	94	.2914	.2851	.2760
	96	.3757	.3699	.3619
	98	.4482	.4430	.4358
	90	.0975	.0898	.0786
	92	.2087	.2018	.1918
$m_t = 180$	94	.3048	.2986	.2897
	96	.3876	.3819	.3741
	98	.4589	.4537	.4467
	90	.1193	.1117	.1007
	92	.2280	.2212	.2115
$m_t = 230$	94	.3219	.3158	.3072
	96	.4026	.3971	.3895
	98	.4723	.4671	.4604

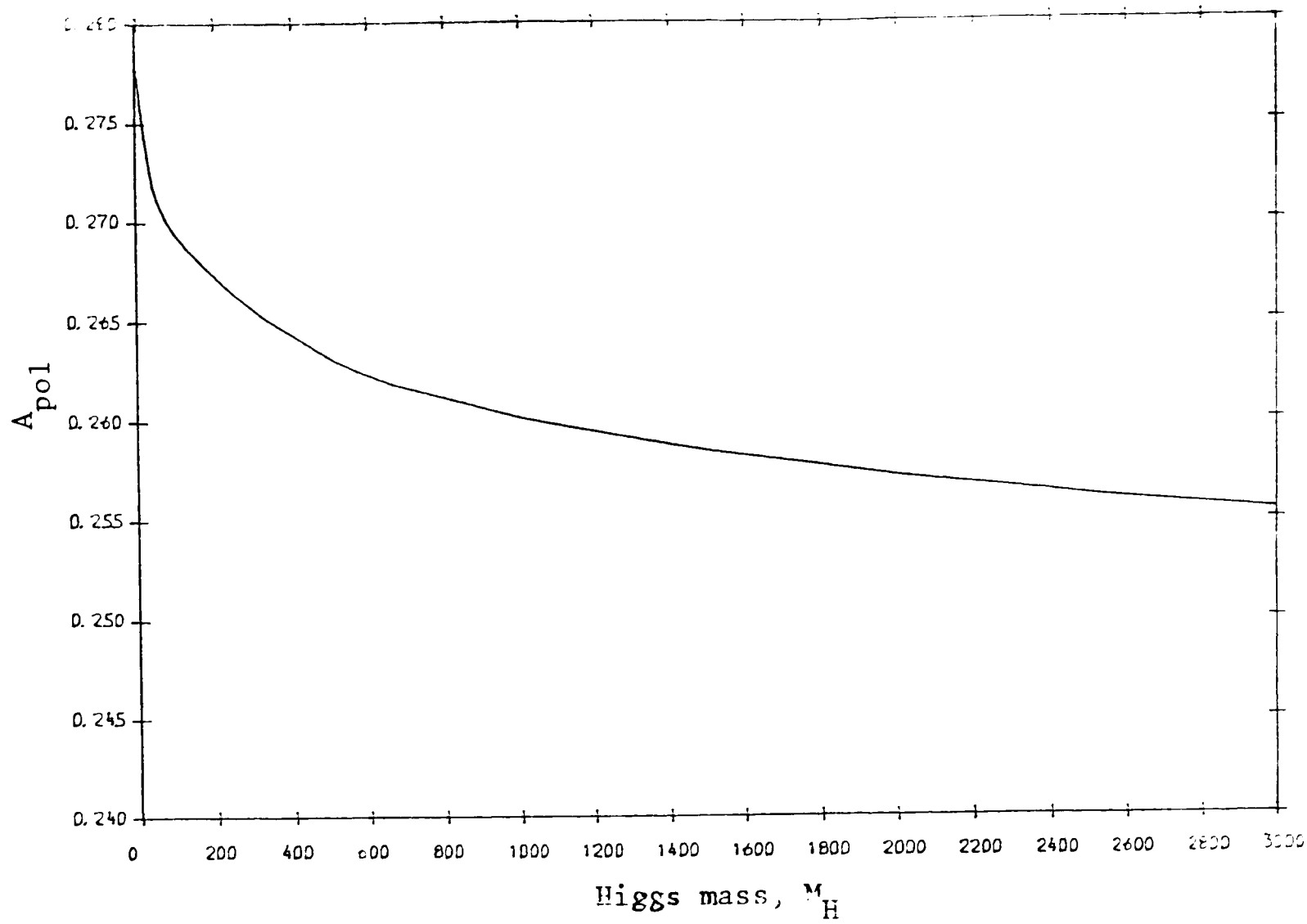


Figure 4·3a

Dependence of A_{pol} ($q^2 = -M_Z^2$) on M_H; M_Z = 94 GeV; m_t = 30 GeV.

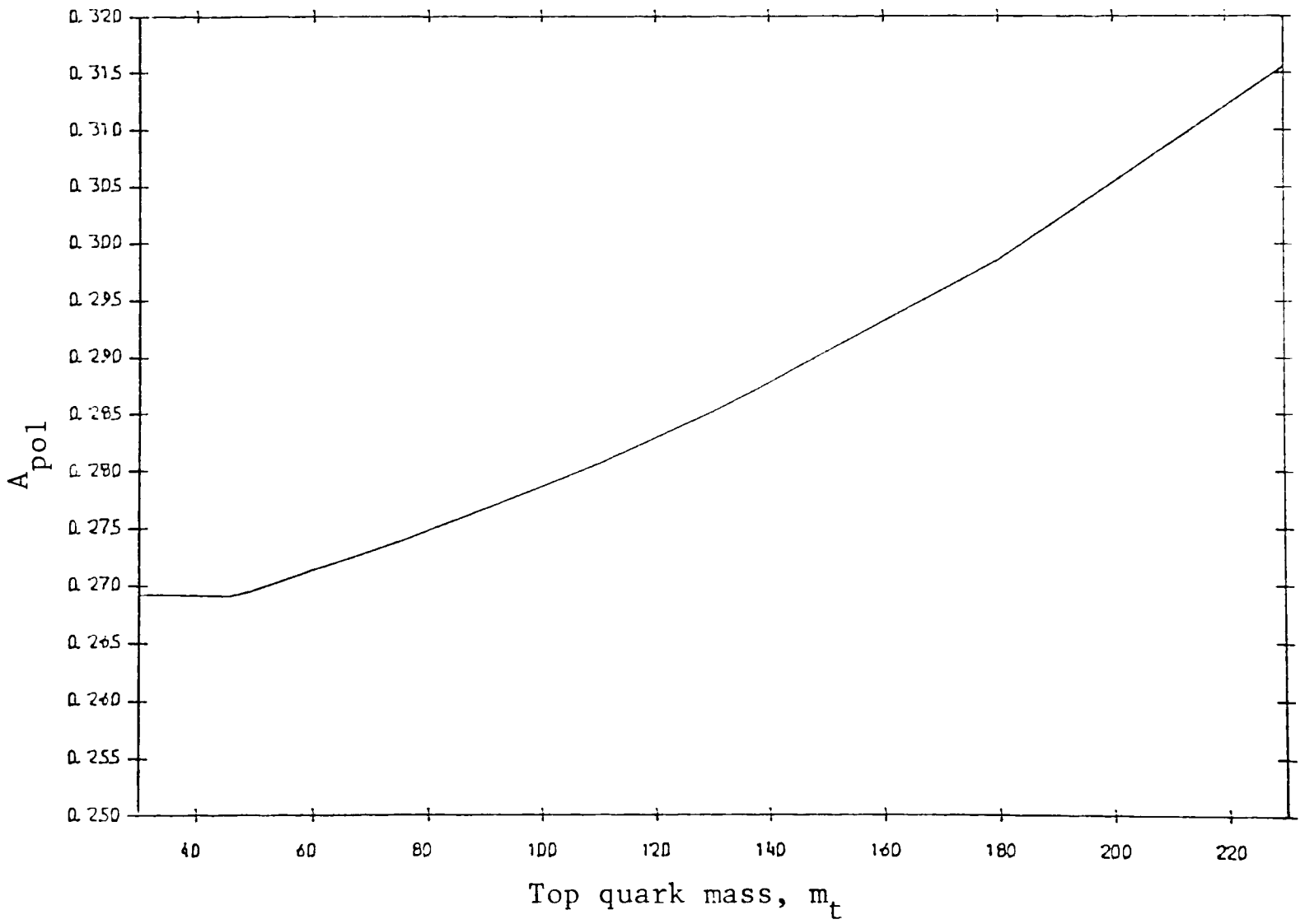


Figure 4·3b

Dependence of A_{pol} ($q^2 = -M_Z^2$) on m_t; M_Z = 94 GeV; M_H = 100 GeV.

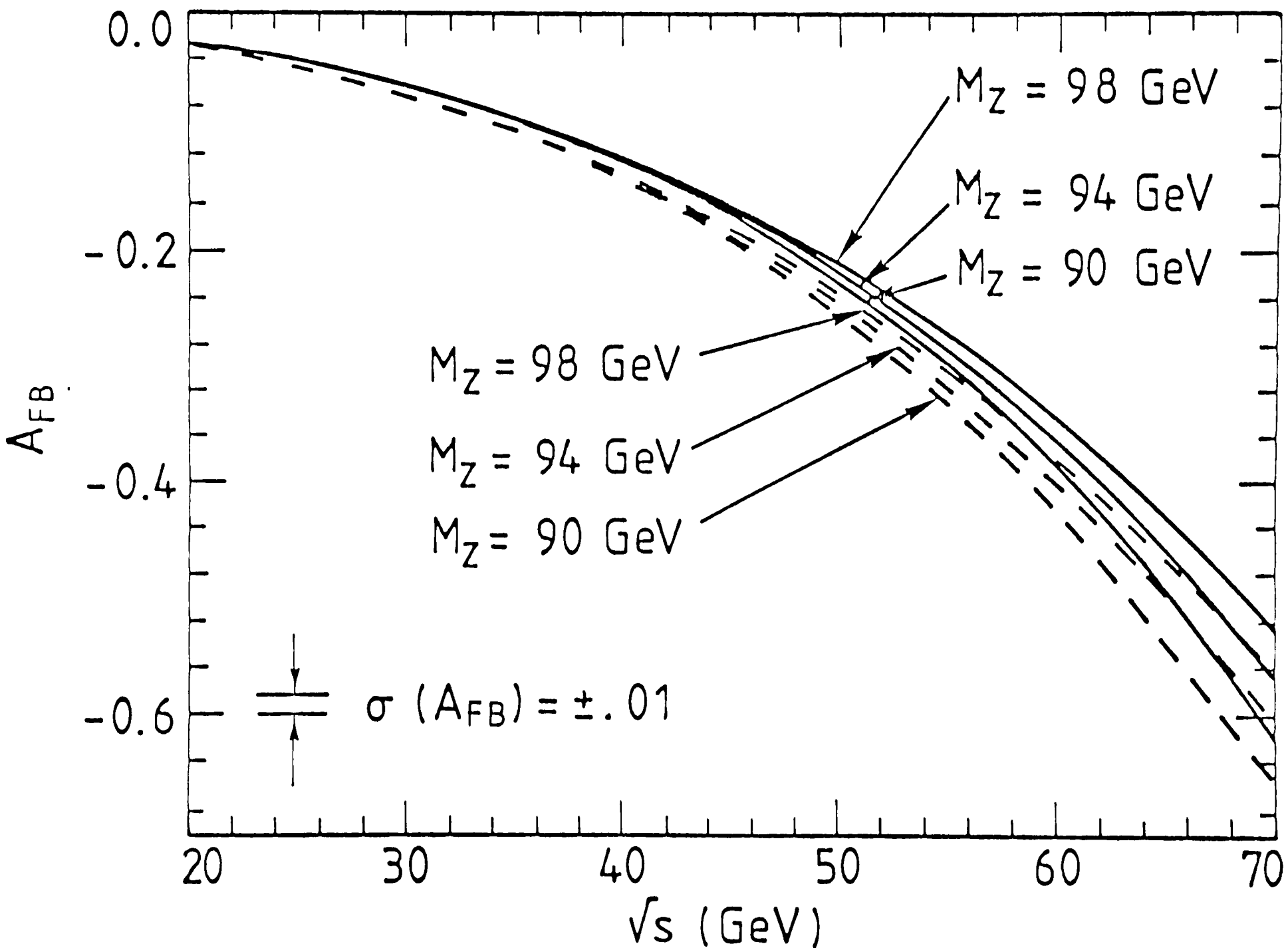

A_{FB} with and without radiative corrections
0.0
-0.2
-0.4
-0.6
A_{FB}
M_Z = 98 GeV
M_Z = 94 GeV
M_Z = 90 GeV
M_Z = 98 GeV
M_Z = 94 GeV
M_Z = 90 GeV
σ (A_{FB}) = ±.01
20
30
40
50
60
70
$\sqrt{s}$ (GeV)

Fig. 4.4a

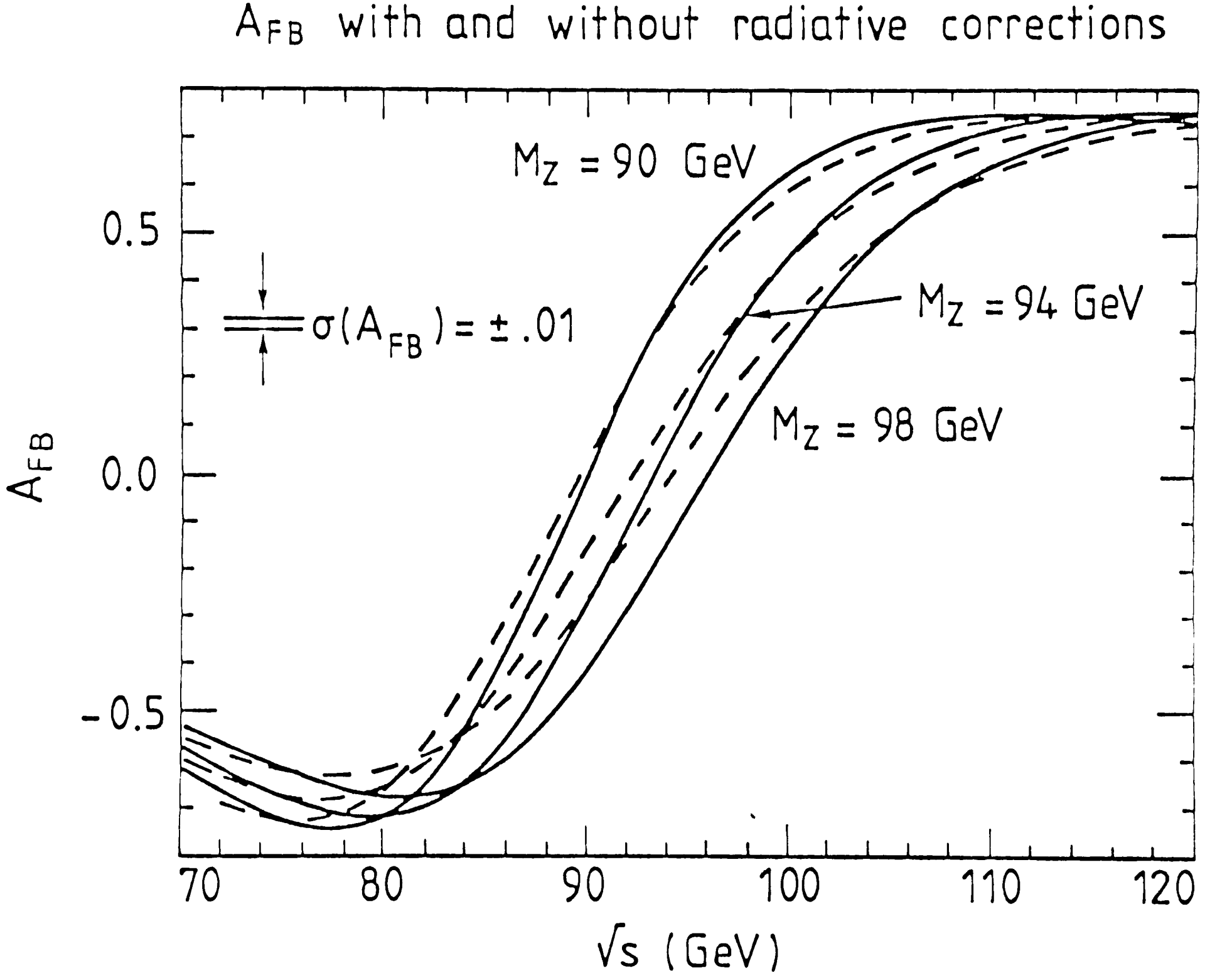

A_{FB} with and without radiative corrections
M_Z = 90 GeV
M_Z = 94 GeV
M_Z = 98 GeV
$\sigma(A_{FB}) = \pm .01$
A_{FB}
0.5
0.0
-0.5
70
80
90
100
110
120
$\sqrt{s}$ (GeV)

Fig. 4.4 b

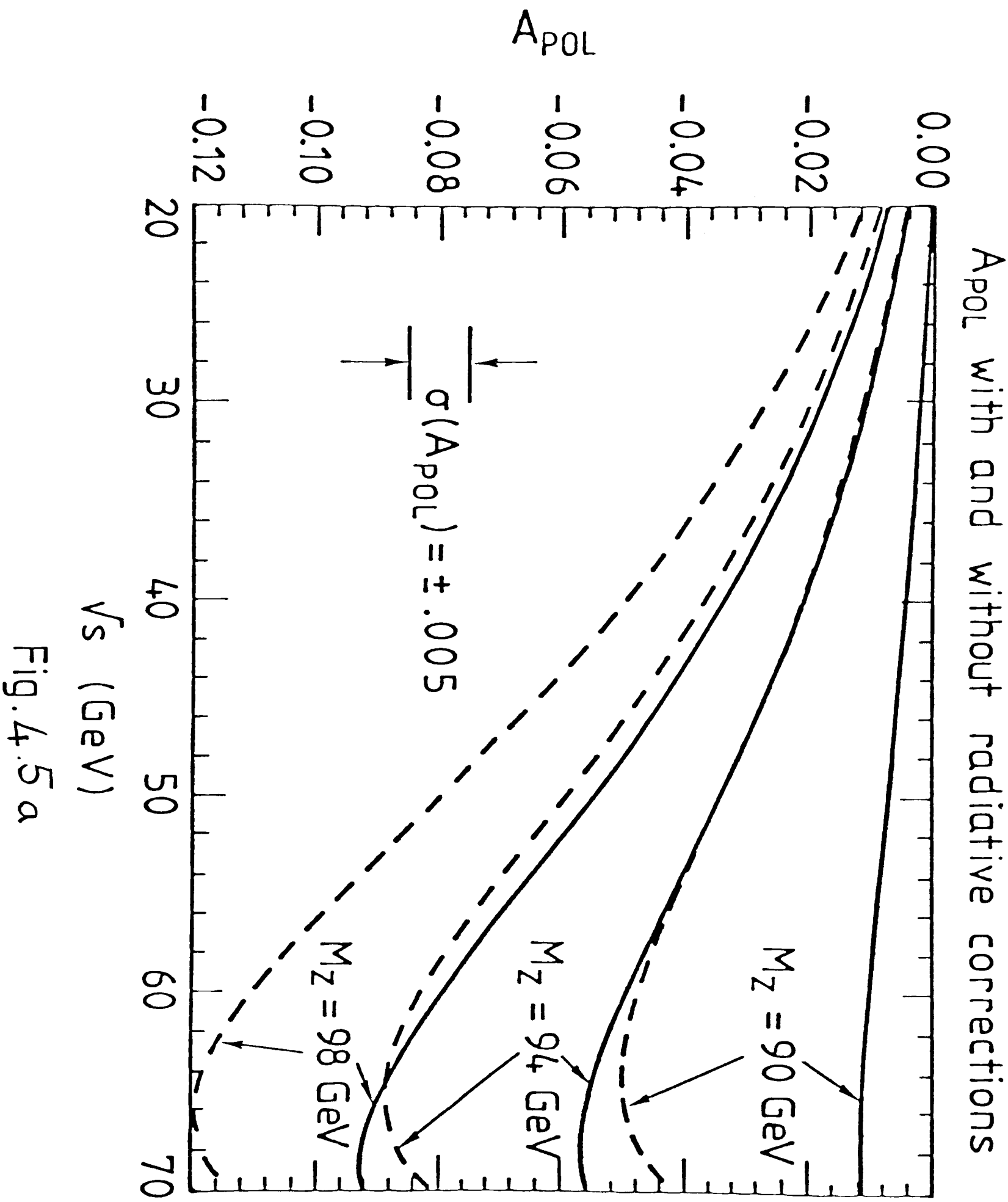

A_{POL}
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
A_{POL} with and without radiative corrections
$\sigma(A_{POL}) = \pm .005$
$M_Z = 98$ GeV
$M_Z = 94$ GeV
$M_Z = 90$ GeV
20
30
40
50
60
70
$\sqrt{s}$ (GeV)

Fig. 4.5a

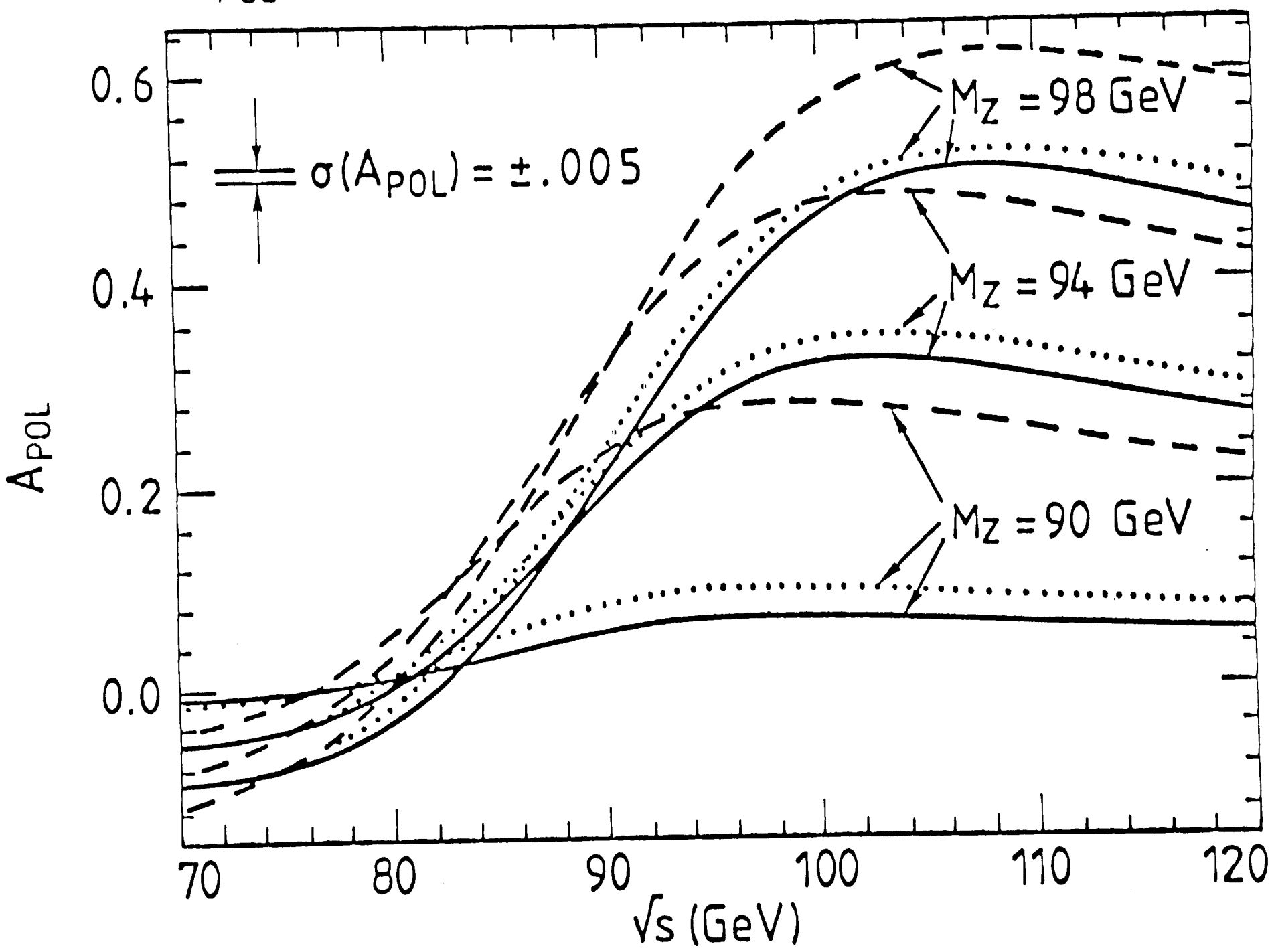
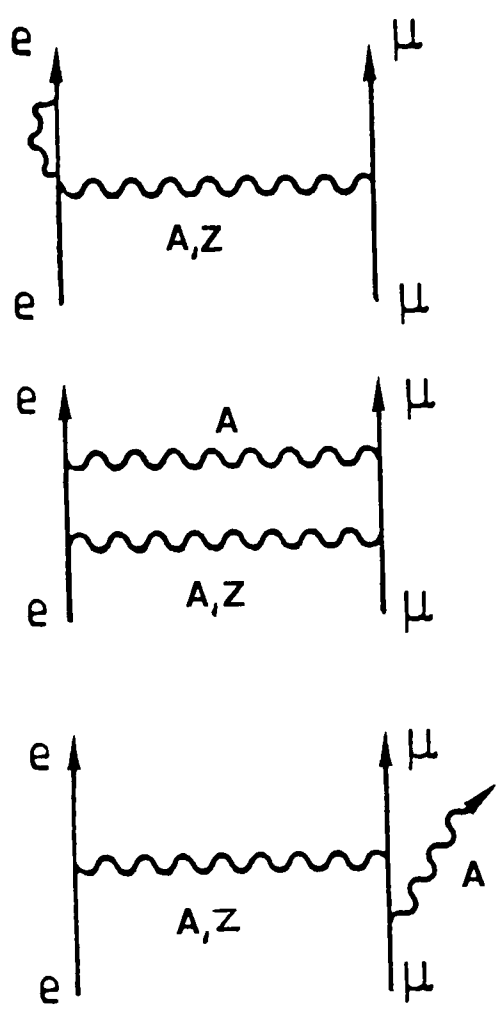
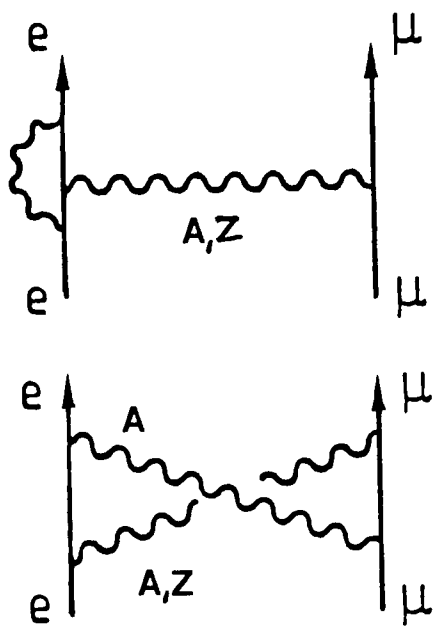

A_POL with and without radiative corrections
σ(A_POL) = ±.005
M_Z = 98 GeV
M_Z = 94 GeV
M_Z = 90 GeV
A_POL
0.6
0.4
0.2
0.0
70
80
90
100
110
120
√s (GeV)

Fig. 4.5b





PLUS ALL PERMUTATIONS

Figure 4·6

QED corrections not included in Tables 4·3 - 4·6.

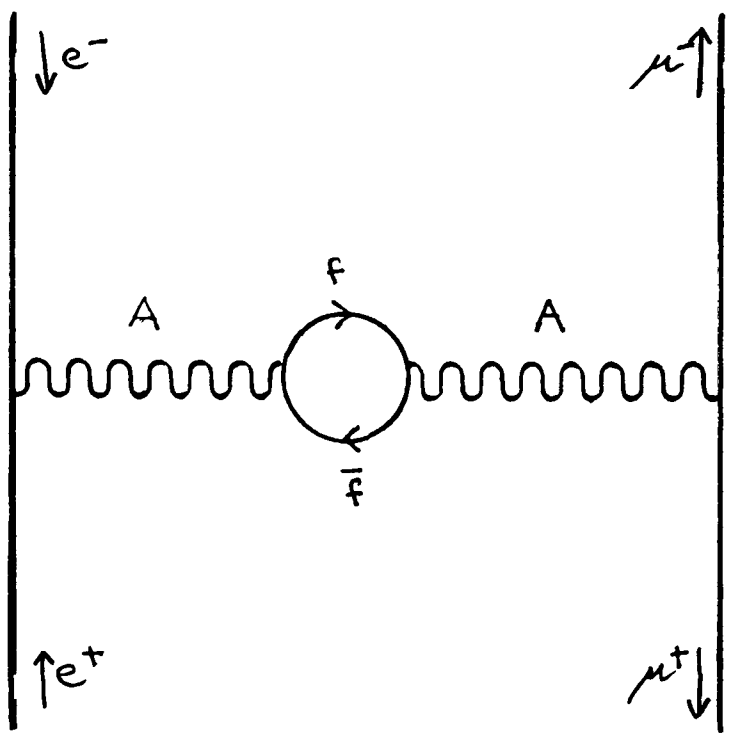


Figure 4·7

Contribution of fermion loops to photon vacuum polarization.

4·2 Comparison with other calculations

It is now appropriate to compare the predictions of this work with other one-loop calculations of A_{fb}, in particular around $q^2 = -(34{\cdot}5\ \mathrm{GeV})^2$, and with the experimental results available for this asymmetry.

Certain authors (Wetzel 1983a, Böhm and Hollik 1984a) list their numerical results by sequentially adding to the Born term, the QED corrections and weak corrections. In such cases, in order to make comparison, it is assumed that A_{fb} is linear in the corrections.

Most authors include the contribution of fermion loops to the photon vacuum polarization (Figure 4·7) in their QED corrections. For this reason, when a comparison is made in the following section the result quoted for this work does not include the contribution of this graph. This allows a more direct comparison with currently available experimental results (see Section 4·3)

Wetzel 1983a has calculated the full electroweak corrections to A_{fb} using G_μ as an input parameter. He does not calculate $\sin^2\theta_W$ from G_μ for use in the lowest order matrix elements but rather introduces an additional and unnecessary parameter of $\sin^2\theta_W$ as determined from experiment after radiative corrections have been included. Because the lowest order Z^0 exchange matrix element, proportional to $(q^2 + M_Z^2)^{-1}$, blows up on resonance, Wetzel has chosen to combine with it the $O(\alpha)$ corrections to the Z^0 self energy as is done here. The 'Born term' of Wetzel's Table 1 contains this contribution and is large compared with the Born terms of other calculations. Note that this is not a consequence of having used G_μ as input (the Weinberg-Salam scheme) but rather of requiring that formulas are well behaved near resonance. Wetzel notes that, "corrections tend to diminish the asymmetry". This is true for all but the Z^0 self energy correction whose effect is not

displayed separately in Table 1. For M_Z = 93 GeV, m_t = 30 GeV, M_H = 100 GeV and $q^2 = -(34\ \text{GeV})^2$ the Born term is -8·97% to which must be added +0·04% from weak corrections. The result of -8·93% agrees exactly with the value obtained using the methods described here when these parameters are used as input.

Cole 1983 has carried out a calculation of the purely weak corrections to A_{fb} excluding the contribution of fermion loops to the photon vacuum polarization (Figure 4·8) and those of Figure 4·7. However, he appears to use too many renormalization prescription conditions to define his input data parameters. Cole chooses α, M_W and M_Z as input parameters by forcing the relevant vector self energies to vanish at $q^2 = -M_W^2$ and $q^2 = -M_Z^2$ respectively. Sirlin 1980 makes it clear that the only definition of $\sin^2\theta_W = e^2/g^2$ compatible with this choice is in terms of the boson masses (A·2·25). The point being that $\sin^2\theta_W$ is now well defined and enters only as a shorthand convention. Cole then goes on to give an extra renormalization prescripton condition by defining $\sin^2\theta_W$ from electromagnetic and neutral currents. The two definitions are equivalent at tree level but differ by terms of $O(\alpha)$ at one loop. Thus, he is in danger of over-constraining the theory, violating one-loop Ward identities and yielding a self-inconsistent calculation. Indeed, Cole observes, "The two definitions give results which differ numerically by only small amounts as long as the mass of the top quark is small compared with M_W. The Higgs mass dependence is also different in the two cases." Of course, it is still possible to use the ratio of currents as an input parameter but one must calculate this ratio in terms of the other parameters and eliminate, say, M_W in favour of the experimentally measured ratio of currents, much as M_W was eliminated in favour of G_μ here. Cole's numerical result of A_{fb} = -9·3% (from his Figure 6) at $q^2 = -(34{\cdot}5\ \text{GeV})^2$ agrees with that of the present calculation for small

m_t and M_H (see Table 4·7). The values of M_Z, m_t and M_H that he used have not been specified.

Brown et al. 1984 correctly observe that there are large logs $\sim \alpha \ln m_f^2/M_Z^2$ in the Z^0 self energy due to internal fermion loops for $m_f^2 << M_Z^2$ and ignoring contributions from all other sources obtain $A_{fb} = -9{\cdot}3\%$ at $q^2 = -(34{\cdot}4\ \text{GeV})$, $M_Z = 91{\cdot}9$ GeV, $M_W = 80{\cdot}8$ GeV. This approach is, however, slightly misleading since Böhm and Hollik (1982) observe similar large logs in the photon self energy that are included in their analysis of the QED corrections to A_{fb}. As noted earlier, these contributions cancel each other in the complete one-loop calculation presented here and so the charge asymmetry has no such large logs for $m_f^2 << M_Z^2$ except via corrections to $\sin^2\theta_W$ from the muon lifetime. This emphasizes the danger in relying on partial calculations of the one-loop electroweak corrections. The value obtained using the results of this work give $A_{fb} = -9{\cdot}21\%$ for $q^2 = -(34{\cdot}4\ \text{GeV})^2$, $M_Z = 91{\cdot}9$ GeV ($M_W = 80{\cdot}7$ GeV), $m_t = 30$ GeV and $M_H = 100$ GeV. Note that in this case M_W has been calculated from α, M_Z and G_μ.

Böhm and Hollik 1984a have calculated the full one-loop corrections including QED corrections. For $M_Z = 93$ GeV, $M_W = 82{\cdot}1$ GeV, $m_t = 30$ GeV and $M_H = 100$ GeV they obtain a Born term $A_{fb}^B = -7{\cdot}62\%$ for $q^2 = -(34{\cdot}5\ \text{GeV})^2$ and $|\cos\theta| < 0{\cdot}8$ which corresponds to $A_{fb}^B = -8{\cdot}67\%$ at $|\cos\theta| < 1$. To this must be added the corresponding weak correction from their Table 2 of $-0{\cdot}70\%$ with the result A_{fb} ($\sqrt{s} = 34{\cdot}5$ GeV) $= -9{\cdot}37\%$. The method used here yields $A_{fb} = -9{\cdot}23\%$, ($M_W = 82{\cdot}0$).

Wetzel (1983b) has calculated the forward-backward asymmetry at LEP energies using α, G_μ and M_Z as input parameters but he has not displayed the dramatic variation of A_{fb} near resonance with M_Z or the less dramatic variation with m_t shown in Table 4·7. Rather he chooses

to fix M_Z = 93 GeV, m_t = 30 GeV and M_H = 100 GeV. His on-resonance result of $A_{fb} \approx 0{\cdot}04$ (from his Figure 5a) which includes a contribution from fermion loops in the photon vacuum polarization (Figure 4·8) is in good agreement with the results in Table 4·4.

After the present calculation was completed, Hollik 1984 published the results of a calculation of purely weak corrections to A_{fb} and A_{pol} in $e^+e^- \rightarrow \mu^+\mu^-$ around the Z^0 resonance. Consistent with his calculation of QED corrections (Böhm and Hollik 1982) he does not include the contribution of fermion loops to photon vacuum polarization (Figure 4·8). Compared to the predictions of the present work, with the graph of Figure 4·8 excluded, his results seem to be somewhat larger in magnitude. Note that this is also the case for his off resonance prediction for A_{fb} (Böhm and Hollik 1984a) which is the largest of the several predictions available.

Bellucci 1984, has calculated the $O(\alpha)$ and $O(\alpha^2 \ell n M_W^2)$ corrections to A_{fb} but his results are presented in a way which does not allow a straightforward numerical comparison to be made.

4·3 Comparison with Experiment

In this neutral current process the radiative corrections due to virtual photons (Figure 4·6) form a gauge invariant set which may be unambiguously separated off. An extensive and expanding literature now exists (M. Greco et al. 1980, Berends and Kleiss 1981, Böhm and Hollik 1982, 1984b) on the subject. Although there is general agreement over the formulas the numerical evaluation of the soft bremsstrahlung tail of the resonance has proved non-trivial. (Berends et al. 1982). Soft bremsstrahlung corrections are, of course, related to the experimental resolution and hard bremsstrahlung corrections are dependent on the characteristics of the detector, requiring specific Monte Carlo simulation. In order to make sense of raw data, experimenters must subtract out detector-dependent hard and soft bremsstrahlung effects. Since the bremsstrahlung graphs of Figure 4·6, cancel I.R. divergences in the QED vertex, box and self-energy graphs these must be subtracted as well. In order to avoid confusion, experimenters are urged to subtract all of the gauge invariant set of graphs in Figure 4·6 but not the contribution of fermion loops to the photon vacuum polarization (Figure 4·7) because it contains large logarithms which cancel against similar large logs in the Z^0 self energy. There is also a hadronic uncertainty, associated with this graph for quark loops (see Section 3·2), that will presumably decrease as further theoretical refinements are made or as better experimental data becomes available for use in the dispersion relation (3·2·4).

Note that the QED corrections can be quite sensitive to changes in M_Z and hence $\sin^2\theta_W$. This is especially true on resonance so that the data of Table 4·1 must be used in their evaluatiuon. Strictly one should use the q^2 dependent $\mathrm{Im}\Pi_T^{ZZ}(q^2)$ rather than the constant (2·5 GeV) M_Z.

Until recently, most experimental groups subtracted out only, the so-called, reduced QED corrections from the data (i.e. those graphs of Figures 4·6 and 4·7 containing photons but no Z^0) using the Monte Carlo program of Berends and Kleiss 1981. The interference terms of the QED graphs with the lowest order Z^0 exchange were not subtracted.

Now, however, experimental data is in some cases presented with all the graphs of Figures 4·6 and 4·7 subtracted along with interference with the lowest order photon and Z^0 exchange graphs using programs of Berends et al. 1982. This is still not universally done and care is required in determining just what has been subtracted before comparison with theoretical predictions can be made. Failure to do so (Saxon 1984) reduces the level of agreement between theory and experiment.

Because all experimental data available have the contribution of fermion loops to the photon vacuum polarization (Figure 4·7) subtracted, direct comparison with Table 4·3 is not possible. In order to make comparison the contribution of this diagram and appropriate counter-term were removed from the computer program used here and the predictions recalculated. In practice this is a simple matter because the renormalization has been done at the Green's function level. The resulting predictions are given in Table 4·7 for M_Z = 94 GeV, m_t = 30 GeV, M_H = 100 GeV.

To reiterate, the experimental results and theoretical predictions given in Table 4·7 for A_{fb} do not include (Althoff et al. 1984) the graphs of Figures 4·6 and 4·7.

Of the errors quoted for the measured value of A_{fb}, the first is statistical and the second is systematic.

It is clear that the predictions of the GSW model and the results of experiment, as currently presented, agree well.

As a result of the work reported here, certain logical inconsistencies have been discovered in the analysis of experimental data as performed for the TASSO experiment. They are presumably present for other experiments as well and are the subject of a forthcoming paper (Cashmore et al. 1985). Rather than dwell on the details of the inconsistencies, which are somewhat outside the scope of the present work, suffice it to say that the results presented here make possible an unambiguous comparison between experiment and the predictions of the GSW model. As stated earlier, care must be taken to use the appropriate value of $\sin^2\theta_W$ from Table 4·1 in the calculation of the QED corrections to the Z^0 exchange.

In the context of other models based on SU(2) x U(1), the currently accepted practice of removing QED corrections to Z^0 exchange from raw experimental data requires that some particular value of $\sin^2\theta_W$ be assumed in order to evaluate the fermion couplings. The data has been biassed by this external input and is useless for comparison with the predictions of other models, especially those based on other gauge groups for which $\sin^2\theta_W$ may not be meaningful. The hadronic uncertainty in the photon propagator is incorporated into raw data in a similar way.

Ideally physicists should compare raw data with predictions that contain all relevant radiative corrections. For the case in question this would include a Born term + QED corrections + weak corrections. As a consequence of this work the TASSO experiment has been provided with a computer program which will make this possible within the context of the GSW model.

Table 4·7

Experiment	$\sqrt{s}$	A_{fb} (%)	Theory (one-loop)	Reference
MAC	29·0	-7·9 ± 1·8 ± 0·1	-6·23	Fernandez 1983
TASSO	34·5	-9·8 ± 2·3 ± 0·5	-9·20	Althoff 1984
MARK J	34·6	-10·6 ± 2·3 ± 1·0	-9·26	Adeva 1982
PLUTO	34·7	-13·4 ± 3·1 ± 1·0	-9·32	Berger 1983

5 The Computer Program

The computer program used to carry out the calculation was written in FORTRAN to make it widely comprehensible. As far as practicable it is constructed in modular form to facilitate the use of its component parts in other applications. In particular the following quantities are calculated by function subprograms:

The B form factors

1PI self energies (excluding counter-terms)

The Spence function

Box diagrams

The 1PI self energy subprograms receive the fermion masses and charges, boson masses, α, $\sin^2\theta_W$ and $\cos^2\theta_W$ through a COMMON block. The argument q^2 and the arbitrary scale of dimensional regularization for use in the B form factors are passed as explicit arguments. This allows one to check that the physical results are independent of scale.

The program begins by calculating M_W in terms of a given M_Z by iterating the formula (3·1·8) until a self consistent value is found. $\sin^2\theta_W$ is then calculated.

Using this value of $\sin^2\theta_W$ the coupling constants $\beta_{\pm}^e$, $\beta_{\pm}^f$ and effective propagator $\hat{\chi}$ (see notation of section 4·1) are calculated, stored in an array and finally combined to form the differential cross sections. This has the advantage that only expressions for matrix elements are required so that a lot of tedious algebra is eliminated and a high degree of generality is attained. The cross sections are obtained by numerical integration.

Quark masses squared and charges and lepton masses squared and charges are read from external files. Control data for the program is read from a third file.

The control data includes

(i) The number of points to be calculated for the angular integration of the differential cross sections.

(ii) The number of lowest order and first order matrix elements to be used in forming the differential cross sections.

(iii) The t_3 component of weak isospin and charge of the incoming fermion.

(iv) The t_3 component of weak isospin and charge of the outgoing fermion.

(v) An initial estimate of the W mass.

(vi) The range and step size over which the Z mass is to be varied.

(vii) The Higgs mass.

(viii) A variable that indicates whether the calculation is to be done on resonance and if not the value of q^2 to be used.

(ix) The beampipe factor $(|\cos\theta| < x)$ to be used.

This set of input detail makes the program very flexible.

Individual classes of diagrams can be suppressed either by adjusting the values in (ii) or by explicitly setting them equal to zero in the program.

The program outputs $\sin^2\theta_W$, A_{pol} and A_{fb} as a function of M_Z over the range specified by (vi).

6 Conclusions

A complete renormalization prescription for the GSW model of electroweak interactions at the Green's function level, including the Higgs sector, has been described and used to calculate the full one-loop weak radiative corrections to A_{fb} and A_{pol} for $e^+e^- \rightarrow \mu^+\mu^-$ both on and off resonance.

The calculations were done in a scheme which has α, G_μ and M_Z as input parameters along with the Higgs and top quark masses. On resonance the results vary by one or two orders of magnitude for M_Z in the range allowed by UA1 data. This dramatic variation is due to suppression at tree level of the asymmetries by a factor $v_\theta = 4\sin^2\theta_W - 1$ or v_θ^2. A weaker dependence on M_H and m_t is observed except when $m_t >> M_Z$.

The effect of radiative corrections is greatest for A_{pol} on resonance and are due mainly to large logarithms ($\ell n\ m_f^2/M_Z^2$) in the formula that relates G_μ to $\sin^2\theta_W = 1 - M_W^2/M_Z^2$. This is thus the best place to look for deviations due to new physics.

The remaining weak radiative corrections to A_{pol} are strictly $O(\alpha)$ because the large logarithms, arising from fermion loops in the corrections to the photon and Z^0 propagators, cancel in the asymmetries.

The result for A_{fb} was found to agree numerically with other calculations. The predictions were also compared with experimental results and it was concluded that there is no discrepancy between the one-loop calculation and the experimental results at $q^2 \approx -(34{\cdot}5\ \text{GeV})^2$.

Throughout this work, the formulas derived, and computer programs produced, were kept as general as practicable in order to allow easy modification to other processes. They have already been applied to the calculation of weak radiative corrections to neutrino-electron

scattering and Bhabha scattering. In the former case there is an additional hadronic uncertainty arising from Z-A mixing that must be considered but can be related to $\sigma(e^+e^- \rightarrow \text{hadrons})$ in a similar fashion to that explained in section 3·2. The Bhabha scattering calculation is only valid to $O(\alpha)$ away from resonance or when the scattering angle is small. Elsewhere a two-loop calculation of the Z^0 width, Γ_Z, is required.

The computer program developed here is currently on-line at SLAC and has been used by others (Lynn 1984b, Lynn and Peskin 1985) in a modified form to calculate the radiative corrections to the asymmetries in supersymmetric and technicolour models and GSW with extra fermion generations. As a result of this work the TASSO experiment has been supplied with a version of the program that is being used in conjunction with the QED program of Berends et al. 1983 for the first logically consistent analysis of A_{fb} data.

Some of the results of this work are available in the literature (Lynn and Stuart 1985, Cashmore et al. 1985).

The first experimental results from the SLC and LEP are awaited with eager anticipation.

Appendix A

A·1 Metric and Notation

Throughout this work the Euclidean metric is used with 4 vectors written as

$$v_\mu = (\underset{\sim}{v}, v_4)$$
$$v_4 = iv_0 \qquad (A\cdot1\cdot1)$$

so that on mass shell

$$p^2 = \underset{\sim}{p}^2 + p_4^2 = -m^2. \qquad (A\cdot1\cdot2)$$

The γ matrices used are

$$\underset{\sim}{\gamma} = \begin{pmatrix} 0 & -i\underset{\sim}{\sigma} \\ i\sigma & 0 \end{pmatrix} \quad ; \quad \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad (A\cdot1\cdot3)$$

with $\underset{\sim}{\sigma}$ the usual 2 x 2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (A\cdot1\cdot4)$$

It follows that

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \qquad (A\cdot1\cdot5)$$

and for convenience one defines

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \qquad (A\cdot1\cdot6)$$

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \qquad (A\cdot1\cdot7)$$

$$\gamma_\pm = \tfrac{1}{2}(1 \pm \gamma_5) \qquad (A\cdot1\cdot8)$$

The following identity is often useful

$$\gamma_\alpha\gamma_\beta\gamma_\lambda = \delta_{\alpha\beta}\gamma_\lambda - \delta_{\alpha\lambda}\gamma_\beta + \delta_{\beta\lambda}\gamma_\alpha - \varepsilon_{\alpha\beta\lambda\nu}\gamma_\nu\gamma_5 \qquad (A\cdot1\cdot9)$$

where $\varepsilon_{\rho\alpha\sigma\beta}$ is the completely antisymmetric tensor with

$$\varepsilon_{1234} = 1$$

and it may be shown that

$$\varepsilon_{\alpha\beta\lambda\mu}\,\varepsilon_{\alpha\beta\nu\sigma} = 4\delta_{\lambda\nu}\,\delta_{\mu\sigma} - 4\delta_{\lambda\sigma}\delta_{\nu\mu} \qquad (A\cdot1\cdot10)$$

$$\varepsilon_{\alpha\beta\lambda\mu}\,\varepsilon_{\alpha\beta\lambda\nu} = 6\delta_{\mu\nu} \qquad (A\cdot1\cdot11)$$

Also one has

$$\mathrm{Tr}\left[\gamma_\alpha\gamma_\beta\gamma_\lambda\gamma_\mu\gamma_\pm\right] = \frac{2}{2}^{n/2}\left\{\delta_{\alpha\beta}\delta_{\lambda\mu} - \delta_{\alpha\lambda}\delta_{\beta\mu} + \delta_{\alpha\mu}\delta_{\beta\lambda} \pm \varepsilon_{\alpha\beta\lambda\mu}\right\} \qquad (A\cdot1\cdot12)$$

$$\mathrm{Tr}\left[\gamma_\alpha\gamma_\beta\gamma_\pm\right] = \frac{2}{2}^{n/2}\delta_{\alpha\beta} \qquad (A\cdot1\cdot13)$$

where n is the number of space-time dimensions.

In the Euclidean metric the Dirac equation becomes

$$\left(i\not{p} + m\right)\psi^a = 0 \qquad (A\cdot1\cdot14)$$

where

$$\not{p} = \gamma_\mu p_\mu = \sum_{i=1}^{4}\gamma_i p_i \qquad (A\cdot1\cdot15)$$

The superscript 'a' indexes the four linearly independent solutions of (A·1·14) and

$$\psi^a = u^a(p)\; e^{-i\varepsilon_a p_\mu x_\mu} \qquad (A\cdot1\cdot16)$$

with

$$\varepsilon_a = \begin{cases} 1 & ;\quad a = 1,\ 2 \\ -1 & ;\quad a = 3,\ 4 \end{cases} \qquad (A\cdot1\cdot17)$$

The u^a's satisfy the equation

$$\left(i\not{p} + \varepsilon_a m\right) u^a(p) = 0 \qquad (A\cdot1\cdot18)$$

and will be chosen to have normalization such that

$$\sum_{\alpha=1}^{4} u_\alpha^{j*}(p)\; u_\alpha^{i}(p) = \delta_{ij} \qquad (A\cdot1\cdot19)$$

Spin summations then give

$$\sum_{i=1}^{4} u_{\beta}^{i}(p)\,\bar{u}_{\alpha}^{i}(p) = \frac{1}{2p_0}\,(-i\not{p} + m)_{\beta\alpha} \qquad (A\cdot1\cdot20)$$

$$\sum_{i=3}^{4} u_{\beta}^{i}(p)\,\bar{u}_{\alpha}^{i}(p) = -\frac{1}{2p_0}\,(i\not{p} + m)_{\beta\alpha}$$

where $\bar{u} = u^{+}\gamma_4$.

The energy projection operators are from (A·1·18)

$$\Lambda_{+}(p) = \frac{-i\not{p} + m}{2p_0} \qquad (A\cdot1\cdot21)$$

$$\Lambda_{-}(p) = \frac{i\not{p} + m}{2p_0} \qquad (A\cdot1\cdot22)$$

The spin projection operator is

$$\sum(s) = \frac{1 + \gamma_5 i\not{s}}{2} \qquad (A\cdot1\cdot23)$$

where $s^2 = 1$ and $s.p = 0$.

As $p \to \infty$ then $s_\mu \to \pm p_\mu/m$ and

$$\sum(s) \to \gamma_{\mp} \qquad (A\cdot1\cdot24)$$

In forming cross-sections the following relations prove useful;

$$[\bar{u}'\,\Gamma\,u]^{\dagger} = \bar{u}\,\bar{\Gamma}\,u' \qquad (A\cdot1\cdot25)$$

where

$$\bar{\Gamma} = \gamma_4\Gamma^{\dagger}\gamma_4$$

It is a simple matter to obtain

$$\bar{\gamma}_{\mu} = -(-1)^{\delta_{4\mu}}\,\gamma_{\mu} \qquad (A\cdot1\cdot26)$$

$$\overline{\sigma_{\mu\nu}q_{\nu}} = (-1)^{\delta_{4\mu}}\sigma_{\mu\nu}q_{\nu} \qquad (A\cdot1\cdot27)$$

A·2 The Glashow-Salam-Weinberg Model

The particle zoo of the standard model encompasses three generations of fermions in left-handed weak iso-spin $(SU(2)_L)$ doublets and right-handed singlets

Leptons

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L \quad e_R\ \mu_R\ \tau_R \tag{A·2·1}$$

Quarks

$$\begin{pmatrix}u\\ d\end{pmatrix}_L \quad \begin{pmatrix}c\\ s\end{pmatrix}_L \quad \begin{pmatrix}t\\ b\end{pmatrix}_L \quad u_R\ d_R\ c_R\ s_R\ t_R\ b_R \tag{A·2·2}$$

There is a triplet of gauge bosons associated with $SU(2)_L$ and a singlet B_μ associated with the U(1) symmetry. Couplings of the gauge bosons to the fermions and Higgses are obtained by the substitution of the covariant derivative in kinetic terms in the Lagrangian.

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - \frac{i}{2} g\, \underset{\sim}{\sigma}.\underset{\sim}{W}_\mu - \frac{i}{2} g' y B_\mu \tag{A·2·3}$$

The scalar Higgs field ϕ forms an isodoublet and has vacuum expectation value

$$\langle 0|\phi|0\rangle = \begin{pmatrix}F\\ 0\end{pmatrix} \tag{A·2·4}$$

and may be conveniently expressed in the form

$$\phi = \begin{pmatrix}F\\ 0\end{pmatrix} + \begin{pmatrix}\frac{H + i\phi_Z}{\sqrt{2}}\\ i\,\phi^+\end{pmatrix} \tag{A·2·5}$$

where H is the physical Higgs particle and $\phi^\pm$, ϕ_Z are unphysical Goldstone modes or Higgs-Kibble ghosts. The Higgs potential of the bare Lagrangian is

$$V_\phi^0 = \tfrac{1}{2} \lambda_0 (\phi^{0\dagger}\phi^0 - E_0^2)^2 \tag{A·2·6}$$

where the '0' sub and superscripts denote bare quantities. The consequences of (A·2·6) for the renormalized and counter-term Lagrangian are discussed in section 2·3.

The weak isospin, hypercharge (t_3, y) of the various particles are given in Table A·1. For fermions only the first generation is listed.

Note that

$$Q = t_3 + y/2 \qquad (A\cdot2\cdot7)$$

In order to be able to calculate, the renormalized fields must be constrained by the addition of a gauge fixing Lagrangian.

$$\mathcal{L}_{g.f.} = \sum_{a=1,2,3,B} - \frac{1}{2\xi} \left| F^a(\underset{\sim}{W}_\mu, B_\mu, \phi) \right|^2 \qquad (A\cdot2\cdot8)$$

The corresponding ghost Lagrangian must be included to prevent the propagation of unphysical longitudinal modes and is obtained by the infinitesimal gauge transformation

$$\underset{\sim}{W}_\mu \rightarrow \underset{\sim}{W}_\mu + \partial_\mu \underset{\sim}{\eta} - g\, \underset{\sim}{\eta} \times \underset{\sim}{W}_\mu \qquad (A\cdot2\cdot9)$$

$$B_\mu \rightarrow B_\mu + \partial_\mu \eta_B \qquad (A\cdot2\cdot10)$$

$$\phi \rightarrow \phi + \frac{i}{2} (g\, \underset{\sim}{\sigma}\cdot\underset{\sim}{\eta} + g' y \eta_B)\, \phi \qquad (A\cdot2\cdot11)$$

and finding the variation in the gauge fixing function

$$F^a(\underset{\sim}{W}_\mu, B_\mu, \phi) \rightarrow F^a(\underset{\sim}{W}_\mu, B_\mu, \phi) + \Delta F(\underset{\sim}{W}_\mu, B_\mu, \phi, \underset{\sim}{\eta}, \eta_B) \qquad (A\cdot2\cdot12)$$

The Faddeev - Popov ghost Lagrangian is then

$$\mathcal{L}_{ghost} = \sum_{a=1,2,3,B} \xi^a \Delta F^a(\underset{\sim}{W}_\mu, B_\mu, \phi, \underset{\sim}{\eta}, \eta_B) \qquad (A\cdot2\cdot13)$$

and $\underset{\sim}{\eta}$, η_B, $\underset{\sim}{\xi}$, ξ_B are anti-commuting scalar fields.

In order to express the Lagrangian in terms of physical particles one writes

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \pm i\, W^2_\mu) \qquad (A\cdot2\cdot14)$$

$$\phi^\pm = \frac{1}{\sqrt{2}} (\phi_1 \pm i\, \phi_2) \qquad (A\cdot2\cdot15)$$

Table A•1

Particle zoo of the GSW model (t_3, y)

ν_e $(\frac{1}{2}, -1)$	e_L $(-\frac{1}{2}, -1)$	e_R $(0, -2)$	
u_L $(\frac{1}{2}, \frac{1}{3})$	d_L $(-\frac{1}{2}, \frac{1}{3})$	u_R $(0, \frac{4}{3})$	d_R $(0, -\frac{2}{3})$
B_μ $(0, 0)$			
W^+_μ $(1, 0)$	W^-_μ $(-1, 0)$	$W_{3\mu}$ $(0, 0)$	
H $(-\frac{1}{2}, 1)$	$\phi^\pm$ $(\pm\frac{1}{2}, \pm 1)$	ϕ_Z $(-\frac{1}{2}, 1)$	

$$\eta^{\pm} = \frac{1}{\sqrt{2}} (\eta_1 \pm i\, \eta_2) \qquad (A\cdot2\cdot16)$$

$$\xi^{\pm} = \frac{1}{\sqrt{2}} (\xi_1 \pm i\, \xi_2) \qquad (A\cdot2\cdot17)$$

and defines the mass eigenstates Z_μ and A_μ as

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = U \begin{pmatrix} W^3_\mu \\ A_\mu \end{pmatrix} \qquad (A\cdot2\cdot18)$$

and

$$U = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \qquad (A\cdot2\cdot19)$$

is chosen to diagonalize the mass matrix of the physical (renormalised) particles. It is natural to write,

$$\begin{pmatrix} \xi_Z \\ \xi_A \end{pmatrix} = U \begin{pmatrix} \xi_3 \\ \xi_B \end{pmatrix} \quad ; \quad \begin{pmatrix} \eta_Z \\ \eta_A \end{pmatrix} = U \begin{pmatrix} \eta_3 \\ \eta_B \end{pmatrix} \qquad (A\cdot2\cdot20)$$

Note that $\{\xi^{\pm}, \xi_Z, \xi_A\}$ are not anti-particles of $\{\eta^{\pm}, \eta_Z, \eta_A\}$.

Calculations will be performed in 't Hooft $R_\xi(\xi=1)$ gauge with

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} \left\{ 2 \left| \partial_\mu W^+_\mu - \xi M_W \phi^+ \right|^2 + (\partial_\mu Z_\mu - \xi M_Z \phi_Z)^2 + (\partial_\mu A_\mu)^2 \right\}$$

which has the effect of cancelling the mixing between gauge bosons and unphysical Goldstone modes in lowest order.

In what follows ψ_f will be taken to denote the wavefunction of a general fermion of mass m_f, isospin t_3, and charge Q. Where wavefunctions ψ_f, ψ'_f of two different fermions appear in a given term in the Lagrangian, ψ_f will be taken to have $t_3 = -\frac{1}{2}$ and ψ'_f has $t_3 = +\frac{1}{2}$.

Defining

$$M_W = \frac{gF}{\sqrt{2}} \tag{A·2·21}$$

$$M_Z = \frac{F}{\sqrt{2}}\sqrt{g^2 + g'^2} \tag{A·2·22}$$

$$M_H^2 = 2\lambda F^2 \tag{A·2·23}$$

$$s_\theta^2 \equiv \sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{e^2}{g^2} \tag{A·2·24}$$

and one deduces the following useful relationships

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \tag{A·2·25}$$

$$M_W = M_Z \cos\theta_W \tag{A·2·26}$$

Ignoring Kobayashi-Maskawa mixing the Lagrangian of the GSW model becomes

$$\begin{aligned}
\mathcal{L}^R = {} & \mathcal{L}_{\text{free}} + \mathcal{L}_{\substack{\text{gauge}\\\text{field}}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\substack{\text{gauges}\\\text{Higgs}}} + \mathcal{L}_{\substack{\text{gauges}\\\text{ghosts}}} + \mathcal{L}_{\substack{\text{ghosts}\\\text{Higgs}}} + \mathcal{L}_{\substack{\text{fermions}\\\text{gauges}}} + \mathcal{L}_{\substack{\text{fermions}\\\text{Higgs}}} \\
\mathcal{L}_{\text{free}} = {} & -\tfrac{1}{2}(\partial_\nu A_\mu)\partial_\nu A_\mu - \tfrac{1}{2}(\partial_\nu Z_\mu)\partial_\nu Z_\mu - \tfrac{1}{2}M_Z^2 Z_\mu Z_\mu \\
& - (\partial_\nu W_\mu^+)\partial_\nu W_\mu^- - M_W^2 W_\mu^+ W_\mu^- \\
& - \tfrac{1}{2}(\partial_\mu H)\partial_\mu H - \tfrac{1}{2}M_H^2 H^2 - \tfrac{1}{2}(\partial_\mu \phi_Z)\partial_\mu \phi_Z - \tfrac{1}{2}M_Z^2 \phi_Z^2 \\
& - (\partial_\mu \phi^+)\partial_\mu \phi^- - M_W^2 \phi^+ \phi^- \\
& - \bar{\psi}_f(\not{\partial} + m_f)\,\psi_f \\
& + \xi_A \partial^2 \eta_A + \xi^-(\partial^2 - M_W^2)\eta^- + \xi^+(\partial^2 - M_W^2)\eta^+ \\
& + \xi_Z(\partial^2 - M_Z^2)\eta_Z
\end{aligned} \tag{A·2·27}$$

$$\mathcal{L}_{\substack{\text{gauge}\\\text{fields}}} = -igc_\theta\{\partial_\nu Z_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu)$$

$$+ Z_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^-_\mu)\} - igs_\theta\{\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu)$$

$$- A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)\}$$

$$-\frac{g^2}{2}(W^+_\mu W^-_\mu W^+_\nu W^-_\nu) + \frac{g^2}{2}(W^+_\mu W^-_\nu W^+_\mu W^-_\nu)$$

$$+ g^2 c^2_\theta\{Z_\mu W^+_\mu Z_\nu W^-_\nu - Z_\mu Z_\mu W^+_\nu W^-_\nu\}$$

$$+ g^2 s^2_\theta\{A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu\}$$

$$+ g^2 s_\theta c_\theta\{A_\mu Z_\nu(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu) - 2A_\mu Z_\mu W^+_\nu W^-_\nu\} \qquad (A\cdot2\cdot28)$$

$$\mathcal{L}_{\text{Higgs}} = -g\frac{M^2_H}{4M_W}(H^2 + \phi^2_Z + 2\phi^+\phi^-)H$$

$$-\frac{g^2M^2_H}{32M^2_W}\{H^4 + \phi^4_Z + 4\phi^+\phi^-\phi^+\phi^- + 4\phi^2_Z\phi^+\phi^- + 4H^2\phi^+\phi^- + 2H^2\phi^2_Z\}$$

$$(A\div2\cdot29)$$

$$\mathcal{L}_{\substack{\text{gauges}\\\text{Higgs}}} = -gM_W H W^+_\mu W^-_\mu - \frac{g}{2}\frac{M_W}{c^2_\theta} H Z_\mu Z_\mu$$

$$+\frac{g}{2c_\theta} Z_\mu(H\partial_\mu\phi_Z - \phi_Z\partial_\mu H)$$

$$-\frac{ig}{2}\{W^+_\mu(\phi_Z\partial_\mu\phi^- - \phi^-\partial_\mu\phi_Z) - W^-_\mu(\phi_Z\partial_\mu\phi^+ - \phi^+\partial_\mu\phi_Z)$$

$$+\frac{g}{2}\{W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\}$$

$$-\frac{igs^2_\theta}{c_\theta} M_W Z_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+)$$

$$-\frac{ig}{2c_\theta}(s^2_\theta - c^2_\theta) Z_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+)$$

$$- igs_\theta A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{g^2}{4} W_\mu^+W_\mu^-(H^2+\phi_Z^2+2\phi^+\phi^-)$$

$$- \frac{g^2}{8c_\theta^2} Z_\mu Z_\mu \left(H^2 + \phi_Z^2 + 2(s_\theta^2 - c_\theta^2)^2\ \phi^+\phi^-\right)$$

$$- g^2 s_\theta^2\ A_\mu A_\mu \phi^+\phi^- - \frac{g^2}{2}\frac{s_\theta^2}{c_\theta} Z_\mu\phi_Z\ (W_\mu^+\phi^- + W_\mu^-\phi^+)$$

$$+ \frac{ig}{2} s_\theta A_\mu H\ (W_\mu^+\phi^- - W_\mu^-\phi^+)$$

$$- \frac{ig}{2}\frac{s_\theta^2}{c_\theta} Z_\mu H\ (W_\mu^+\phi^- - W_\mu^-\phi^+)$$

$$+ \frac{g^2}{2} s_\theta A_\mu\phi_Z\ (W_\mu^+\phi^- + W_\mu^-\phi^+)$$

$$+ \frac{gs_\theta}{c_\theta}(s_\theta^2 - c_\theta^2)\ Z_\mu A_\mu\phi^+\phi^- + igs_\theta A_\mu\ (W_\mu^+\phi^- - W_\mu^-\phi^+) \qquad (A\text{-}2\cdot30)$$

$$\mathcal{L}_{\substack{\text{gauges}\\\text{ghosts}}} = -igW_\mu^+\ (c_\theta\eta^-\partial_\mu\xi_Z - s_\theta\eta_A\partial_\mu\xi^+ - c_\theta\eta_Z\partial_\mu\xi^+)$$

$$+ igW_\mu^-\ (c_\theta\eta^+\partial_\mu\xi_Z - s_\theta\eta_A\partial_\mu\xi^- - c_\theta\eta_Z\partial_\mu\xi^-)$$

$$- ig\ c_\theta Z_\mu(\eta^+\partial_\mu\xi^+ - \eta^-\partial_\mu\xi^-)$$

$$- ig\ s_\theta A_\mu(\eta^+\partial_\mu\xi^+ - \eta^-\partial_\mu\xi^-)$$

$$- ig\ s_\theta(\partial_\mu\xi_A)\ (W_\mu^-\eta^+ - W_\mu^+\eta^-) \qquad (A\text{-}2\cdot31)$$

$$\mathcal{L}_{\substack{\text{ghosts}\\ \text{Higgs}}} = -\frac{g}{2} M_W H \left(\xi^+\eta^+ + \xi^-\eta^- + \frac{1}{c_\theta^2}\xi_Z\eta_Z\right)$$

$$+ \frac{ig}{2c_\theta}\left(s_\theta^2 - c_\theta^2\right) M_W \eta_Z \left(\phi^-\xi^- - \phi^+\xi^+\right)$$

$$- \frac{ig}{2} M_W \phi_Z \left(\xi^+\eta^+ - \xi^-\eta^-\right)$$

$$- igs_\theta M_W \eta_A \left(\phi^+\xi^+ - \phi^-\xi^-\right)$$

$$+ \frac{ig}{2c_\theta} M_W \xi_Z \left(\eta^+\phi^- - \eta^-\phi^+\right) \qquad (A\cdot2\cdot32)$$

$$\mathcal{L}_{\substack{\text{fermions}\\ \text{gauges}}} = igs_\theta A_\mu \bar{\psi}_f \gamma_\mu Q \psi_f$$

$$+ \frac{ig}{4c_\theta} Z_\mu \bar{\psi}_f \gamma_\mu \left\{\left(4t_3 - 4s_\theta^2 Q\right)\gamma_+ + \left(-4s_\theta^2 Q\right)\gamma_-\right\} \psi_f$$

$$+ \frac{ig}{\sqrt{2}} W_\mu^+ \bar{\psi}_f \gamma_\mu \gamma_+ \psi_f' + \frac{ig}{\sqrt{2}} W_\mu^- \bar{\psi}_f' \gamma_\mu \gamma_+ \psi_f \qquad (A\cdot2\cdot33)$$

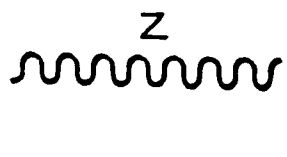
$$\mathcal{L}_{\substack{\text{fermions}\\ \text{Higgs}}} = -\frac{g}{2}\frac{m_f}{M_W} H \bar{\psi}_f \psi_f + ig \frac{m_f}{M_W} t_3 \phi_Z \bar{\psi}_f \gamma_5 \psi_f$$

$$+ \frac{ig}{\sqrt{2}M_W} \phi^+ \bar{\psi}_f \left\{m_f \gamma_+ - m_f' \gamma_-\right\} \psi_f'$$

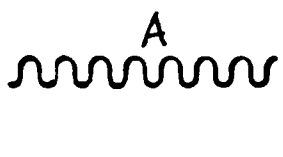
$$+ \frac{ig}{\sqrt{2}M_W} \phi^- \bar{\psi}_f' \left\{m_f' \gamma_+ - m_f \gamma_-\right\} \psi_f \qquad (A\cdot2\cdot34)$$

Feynman Rules

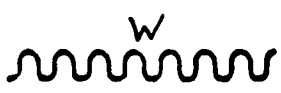
The corresponding propagators are



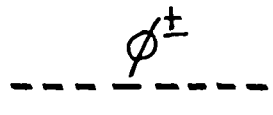
$$\frac{\delta_{\mu\nu}}{p^2 + M_Z^2 - i\varepsilon}$$



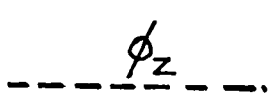
$$\frac{\delta_{\mu\nu}}{p^2 - i\varepsilon}$$



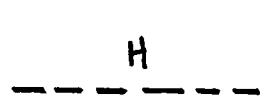
$$\frac{\delta_{\mu\nu}}{p^2 + M_W^2 - i\varepsilon}$$



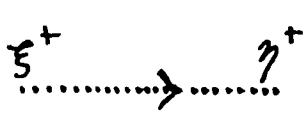
$$\frac{1}{p^2 + M_W^2 - i\varepsilon}$$



$$\frac{1}{p^2 + M_Z^2 - i\varepsilon}$$



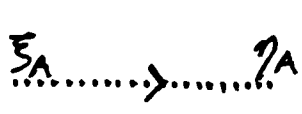
$$\frac{1}{p^2 + M_H^2 - i\varepsilon}$$



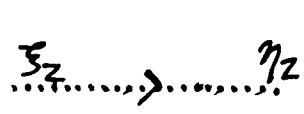
$$\frac{1}{p^2 + M_W^2 - i\varepsilon}$$



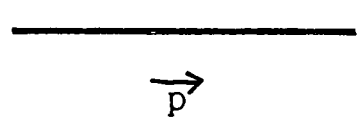
$$\frac{1}{p^2 + M_W^2 - i\varepsilon}$$



$$\frac{1}{p^2 - i\varepsilon}$$

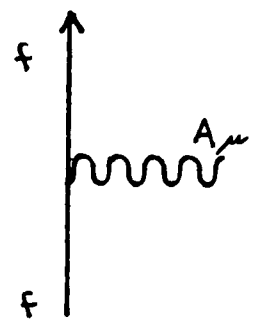


$$\frac{1}{p^2 + M_Z^2 - i\varepsilon}$$

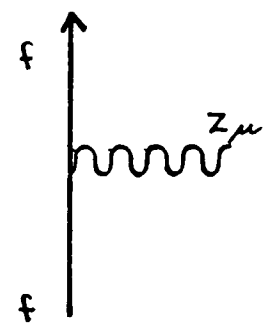


$$\frac{-i\not{p} + m}{p^2 + m^2 - i\varepsilon}$$

The Lagrangian leads to 69 primitive vertices. These are given without combinatoric factors.



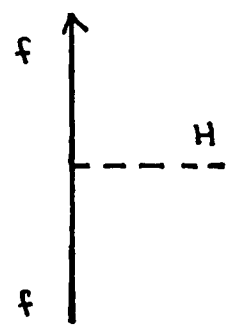
$ig s_\theta Q \gamma_\mu$



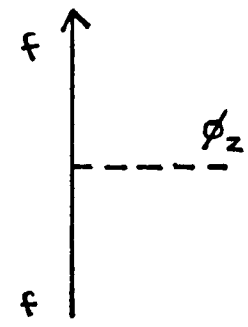
$\frac{ig\gamma_\mu}{4c_\theta}[(4t_3 - 4s_\theta^2 Q)\gamma_+ + (-4s_\theta^2 Q)\gamma_-]$



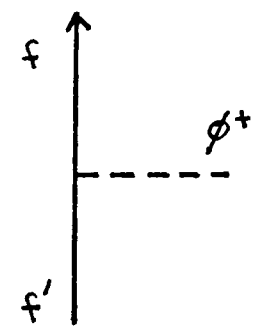
$\frac{ig}{\sqrt{2}}\gamma_\mu\gamma_+$



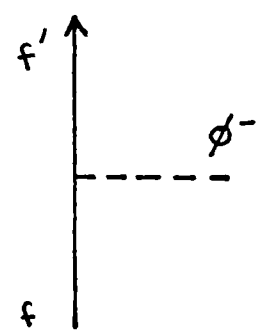
$-\frac{g}{2}\frac{m_f}{M_W}$



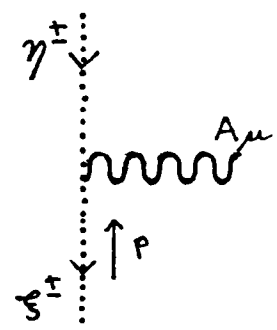
$igt_3\frac{m_f}{M_W}\gamma_5$



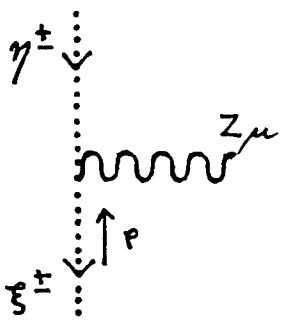
$\frac{ig}{\sqrt{2}M_W}[m_f\gamma_+ - m_{f'}\gamma_-]$



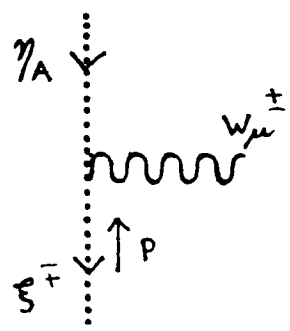
$\frac{ig}{\sqrt{2}M_W}[m_{f'}\gamma_+ - m_f\gamma_-]$



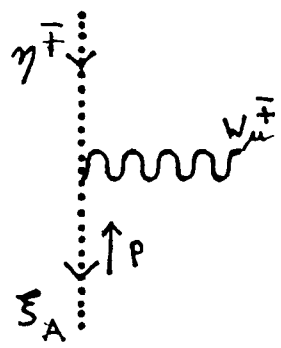
$\pm g s_\theta p_\mu$



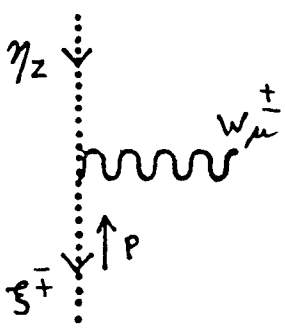
$\pm g c_\theta p_\mu$



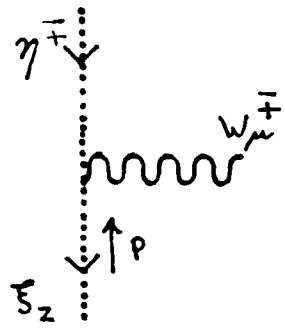
$\pm g s_\theta p_\mu$



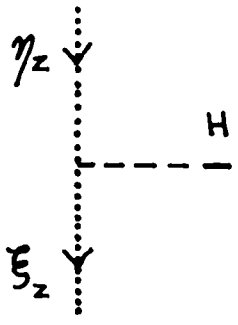
$\pm g s_\theta p_\mu$



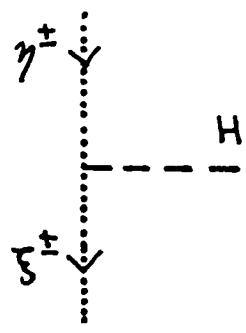
$\pm g c_\theta p_\mu$



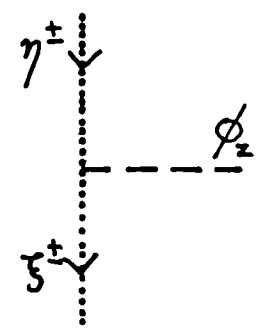
$\pm g c_\theta p_\mu$



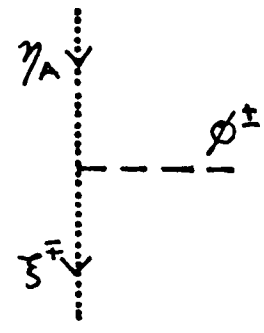
$$\frac{-g}{2c_\theta^2} M_W$$



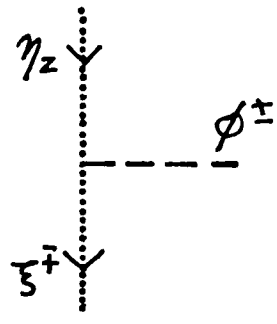
$$\frac{-g}{2} M_W$$



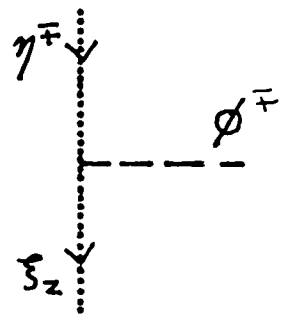
$$\pm\frac{ig}{2} M_W$$



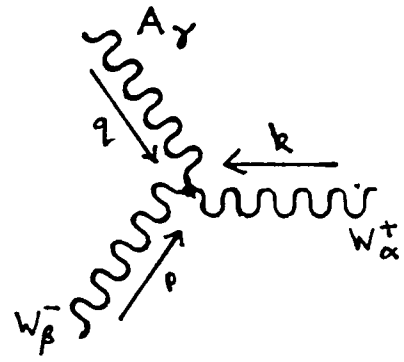
$$\pm ig s_\theta M_W$$



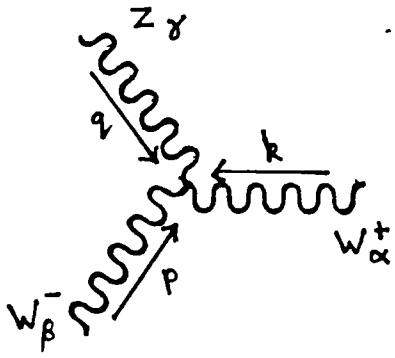
$$\pm\frac{ig}{2c_\theta}(s_\theta^2 - c_\theta^2) M_W$$



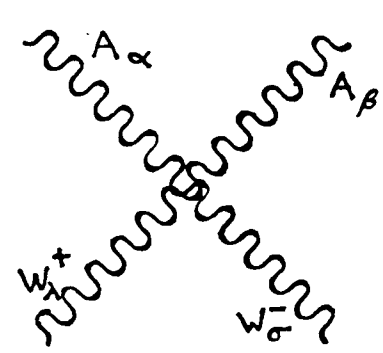
$$\pm\frac{ig}{2c_\theta} M_W$$



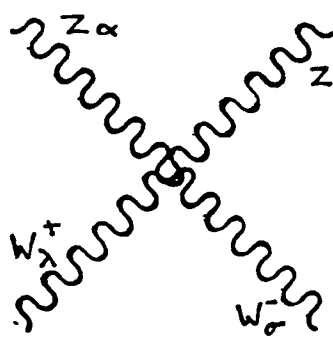
$$g s_\theta [\delta_{\alpha\beta}(k-p)_\gamma + \delta_{\beta\gamma}(p-q)_\alpha + \delta_{\gamma\alpha}(q-k)_\beta]$$



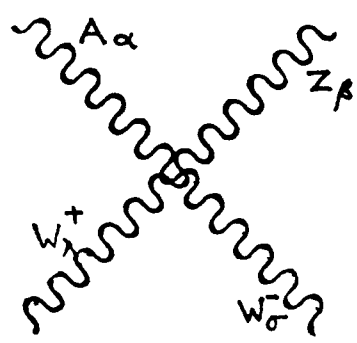
$$g c_\theta [\delta_{\alpha\beta}(k-p)_\gamma + \delta_{\beta\gamma}(p-q)_\alpha + \delta_{\gamma\alpha}(q-k)_\beta]$$



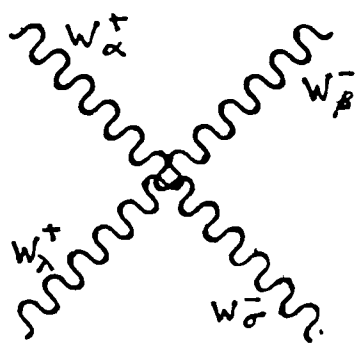
$$g^2 s_\theta^2 [\delta_{\alpha\sigma}\delta_{\beta\lambda} - 2\delta_{\alpha\beta}\delta_{\lambda\sigma} + \delta_{\alpha\lambda}\delta_{\beta\sigma}]$$



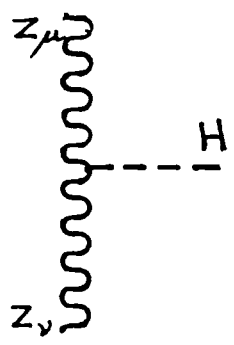
$$g^2 c_\theta^2 [\delta_{\alpha\sigma}\delta_{\beta\lambda} - 2\delta_{\alpha\beta}\delta_{\lambda\sigma} + \delta_{\alpha\lambda}\delta_{\beta\sigma}]$$



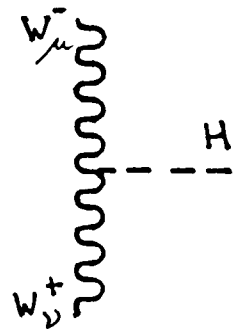
$$g^2 s_\theta c_\theta [\delta_{\alpha\sigma}\delta_{\beta\lambda} - 2\delta_{\alpha\beta}\delta_{\lambda\sigma} + \delta_{\alpha\lambda}\delta_{\beta\sigma}]$$



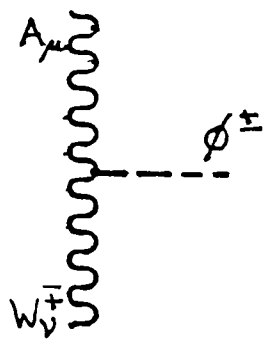
$$-g^2 [\delta_{\alpha\sigma}\delta_{\beta\lambda} - 2\delta_{\alpha\beta}\delta_{\lambda\sigma} + \delta_{\alpha\lambda}\delta_{\beta\sigma}]$$



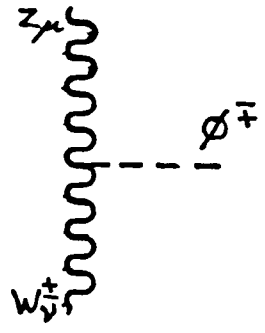
$$-\frac{g}{2c_\theta^2} M_W \delta_{\mu\nu}$$



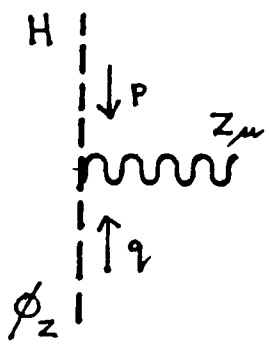
$$-g M_W \delta_{\mu\nu}$$



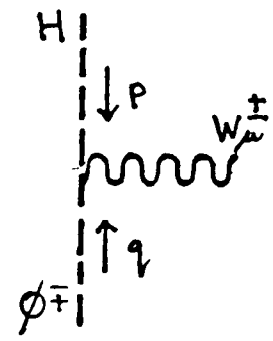
$$\pm ig s_\theta M_W \delta_{\mu\nu}$$



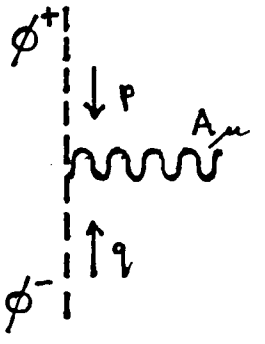
$$\pm i g M_W \frac{S_\theta^2}{C_\theta} \delta_{\mu\nu}$$



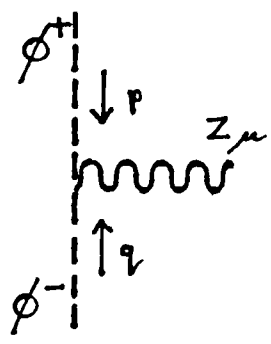
$$\frac{ig}{2C_\theta}(q-p)_\mu$$



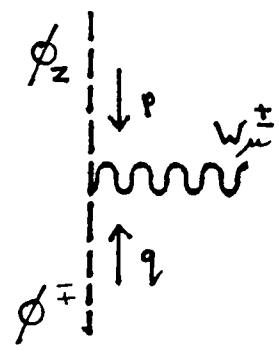
$$\frac{ig}{2}(q-p)_\mu$$



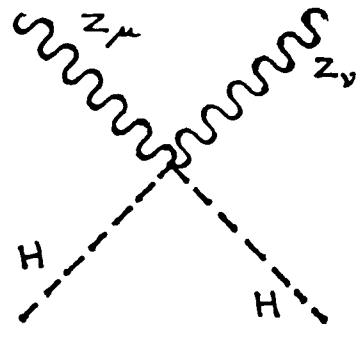
$$g S_\theta (q-p)_\mu$$



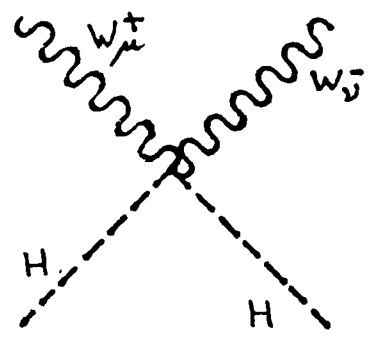
$$\frac{g}{2C_\theta}(S_\theta^2 - C_\theta^2)(q-p)_\mu$$



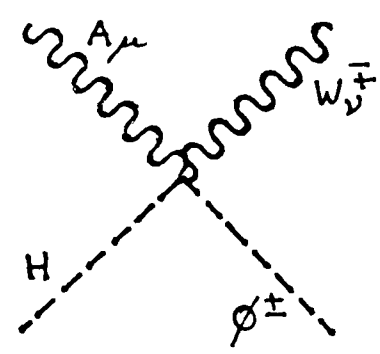
$$\pm \frac{g}{2}(q-p)_\mu$$



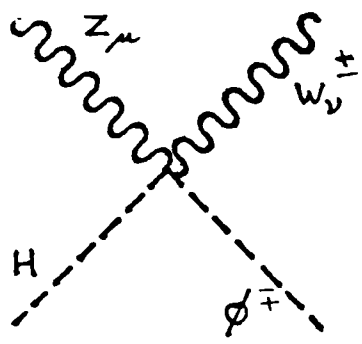
$$-\frac{g^2}{8C_\theta^2}\delta_{\mu\nu}$$



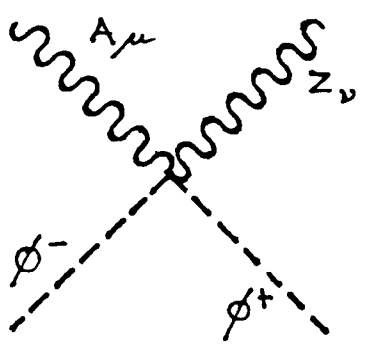
$$-\frac{g^2}{4}\delta_{\mu\nu}$$



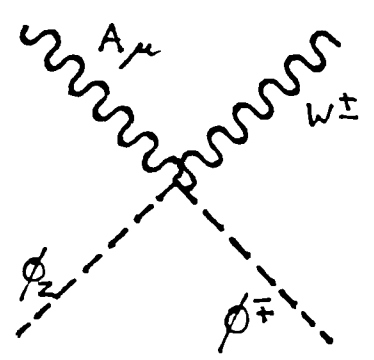
$$\pm \frac{i g^2 S_\theta}{2}\delta_{\mu\nu}$$



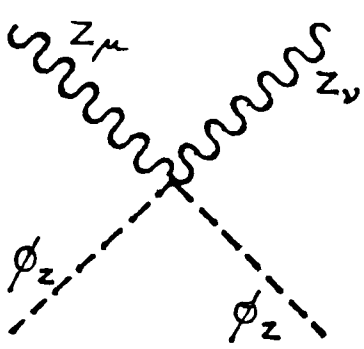
$$\pm \frac{i g^2}{2}\frac{S_\theta^2}{C_\theta}\delta_{\mu\nu}$$



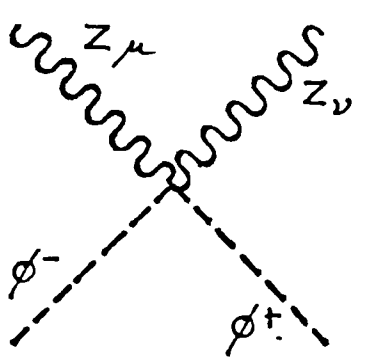
$$g^2\frac{S_\theta}{C_\theta}(S_\theta^2 - C_\theta^2)\delta_{\mu\nu}$$



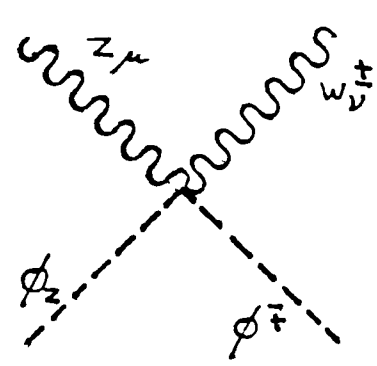
$$\frac{g^2 S_\theta}{2}\delta_{\mu\nu}$$



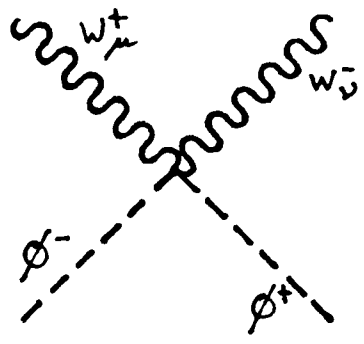
$$\frac{-g^2}{8C_\theta^2}\delta_{\mu\nu}$$



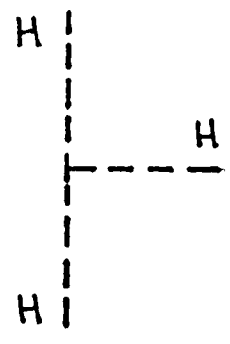
$$-\frac{g^2}{4}\frac{(S_\theta^2 - C_\theta^2)^2}{C_\theta^2}\delta_{\mu\nu}$$



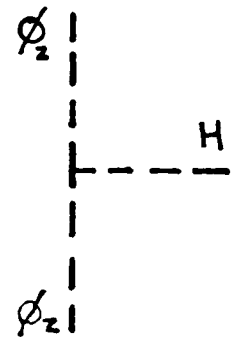
$$-\frac{g^2}{2}\frac{S_\theta^2}{C_\theta}\delta_{\mu\nu}$$



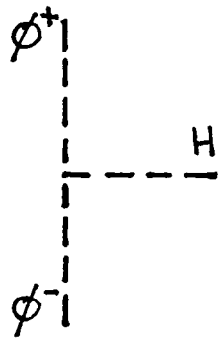
$$-\frac{g^2}{2}\delta_{\mu\nu}$$



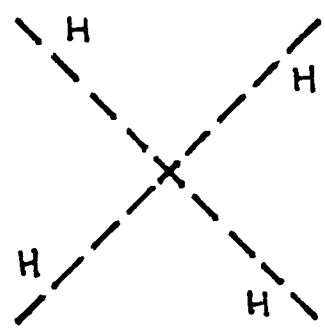
$$-\frac{g}{4}\frac{M_H^2}{M_W}$$



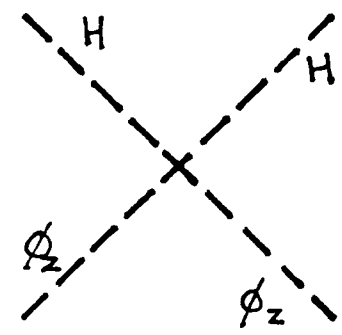
$$-\frac{g}{4}\frac{M_H^2}{M_W}$$



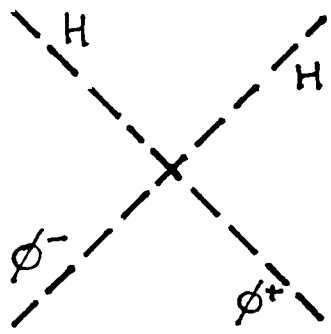
$$-\frac{g}{2}\frac{M_H^2}{M_W}$$



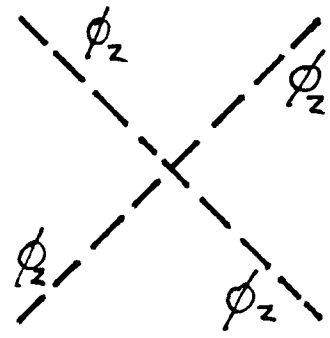
$$-\frac{g^2 M_H^2}{32 M_W^2}$$



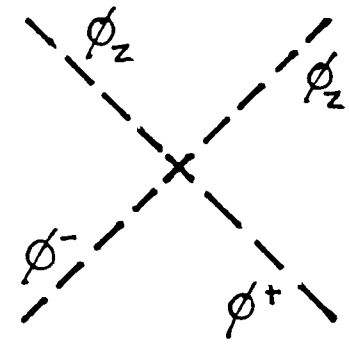
$$-\frac{g^2 M_H^2}{16 M_W^2}$$



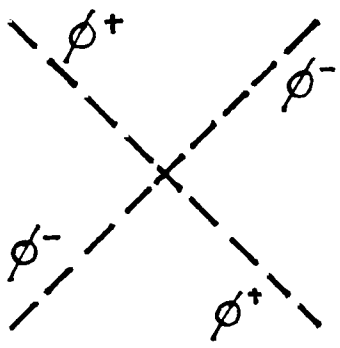
$$-\frac{g^2 M_H^2}{8 M_W^2}$$



$$-\frac{g^2 M_H^2}{32 M_W^2}$$



$$-\frac{g^2 M_H^2}{32 M_W^2}$$



$$-\frac{g^2 M_H^2}{8 M_W^2}$$

To these vertices and propagators must be added the rules

(1) a factor of -1 for each closed fermion loop

(2) a factor of -1 for each ghost loop

(3) a factor of $\frac{1}{(2\pi)^4 i}$ for each propagator

(4) a factor of $(2\pi)^4 i$ for each vertex

(5) 4 - momentum conservation at the vertices

(6) a factor $\int d^n k$ for loop momentum integration

(7) $[\xi^+, \eta^+]$ and $[\xi^-, \eta^-]$ contribute separately since the two pairs are <u>not</u> antiparticles of each other

(8) an overall factor $\frac{1}{(2\pi)^4 i}$ for the matrix element

(9) to external lines are associated

$u^a(p)\sqrt{2p^0}$; a = 1, 2 for incoming particle

$-\bar{u}^a(p)\sqrt{2p^0}$; a = 3, 4 for incoming antiparticle

$\bar{u}^a(p)\sqrt{2p^0}$; a = 1, 2 for outgoing particle

$u^a(p)\sqrt{2p^0}$; a = 3, 4 for outgoing antiparticle

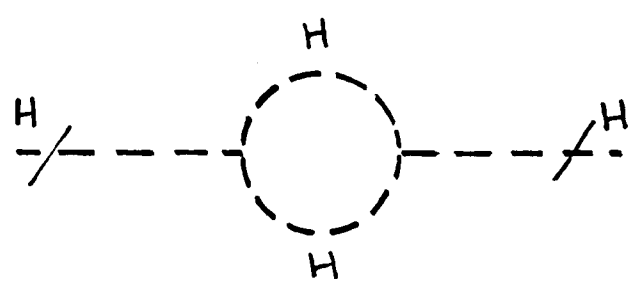
where the momentum is directed inward for incoming (anti) particles and outward for outgoing (anti) particles.

Combinatoric Factors

(1) With a diagram containing r_i vertices of type i is associated a combinatoric factor

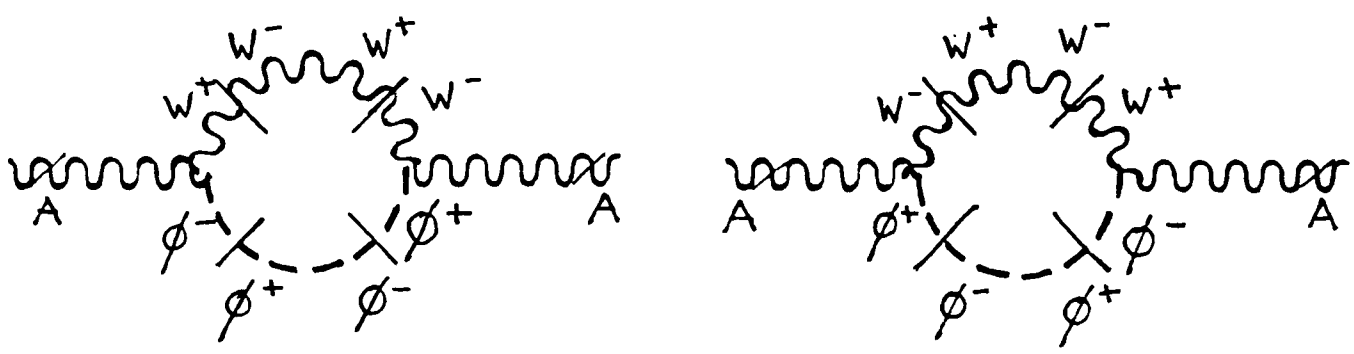
$$(r_1!\ r_2!\ r_3!\ \ldots\ )^{-1} \qquad (A{\cdot}2{\cdot}35)$$

Further factors arise for the possible ways the available propagators and vertices can be connected together to form the required topologically distinct diagram. For example the diagram



gains a factor of 18 by this means. There is also a factor of 1/2! from (A·2·35) but in this work a factor of $\frac{1}{2}$ is extracted from such neutral particle propagators and not considered as part of the diagram. This is because when the diagram is connected to external wavefunctions or internal propagators an additional factor of 2 appears.

(2) Consider the two distinct diagrams



The vertices concerned come from the term in the Lagrangian

$$igs_\theta A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+)$$

and the diagrams must be formed as shown, i.e. containing one of each type of vertex in order to obtain the correct overall sign. Combinatoric factors are calculated accordingly.

A-3 Dimensional Regularization

Dimensional regularisation is now a well established method for taming divergent integrals.

It possesses the important property that it does not destroy gauge invariance and consequently does not lead to a violation of the Ward-Takahashi-Taylor-Slavnov identities. In dimensional regularization, the usual 4-dimensional momentum integral which arises from loop diagrams is generalized to n dimensions.

$$\int d^4k f(k) \rightarrow \int d^nk f(k) = \int f(r) r^{n-1} dr \sin^{n-2}\theta_{n-1} d\theta_{n-1} \sin^{n-3}\theta_{n-2} d\theta_{n-2} \ldots d\theta_1 \qquad (A.3.1)$$

with for f a function only of $r^2 = k_1^2 + \ldots + k_n^2$ and $0 \leq \theta_i \leq \pi$, except $0 \leq \theta_1 \leq 2\pi$.

The angular integrals always converge and may thus be analytically continued to any dimension without difficulty using formulas like

$$\int_0^\pi \sin^m\theta d\theta = \sqrt{\pi} \frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma\left(\frac{m+2}{2}\right)} \qquad (A\cdot3\cdot2)$$

that for (A·3·1) leads to

$$\int d^nk\ f(k) = \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} \int f(r)\ r^{n-1}\ dr \qquad (A\cdot3\cdot3)$$

for which

$$\int_0^\infty dx \frac{x^\beta}{(x^2 + M^2)^\alpha} = \frac{1}{2} \frac{\Gamma\left(\frac{\beta+1}{2}\right) \Gamma\left(\alpha - \frac{\beta+1}{2}\right)}{\Gamma(\alpha)\ (M^2)^{\alpha-(\beta+1)/2}} \qquad (A\cdot3\cdot4)$$

is useful.

To obtain an expression for

$$\int d^nk \frac{1}{(k^2 + 2k.p + m^2)^\alpha} \qquad (A\cdot3\cdot5)$$

the obvious way to proceed is to complete the square in the denominator and shift the integrand. It may be shown on quite general grounds that in the dimensional regularization prescription, shift terms arising from divergent integrals do not appear. One formally obtains by differentiation with respect to p_μ

$$\int d^n k \frac{1}{(k^2 + 2k.p + m^2)^\alpha} = \frac{i\pi^{n/2}}{(m^2 - p^2)^{\alpha-n/2}} \frac{\Gamma(\alpha - \frac{n}{2})}{\Gamma(\alpha)} \qquad (A\cdot3\cdot6)$$

$$\int d^n k \frac{k_\mu}{(k^2 + 2k.p + m^2)^\alpha} = \frac{i\pi^{n/2}}{(m^2 - p^2)^{\alpha-n/2}} \frac{\Gamma(\alpha - \frac{n}{2})}{\Gamma(\alpha)} (-p_\mu) \qquad (A\cdot3\cdot7)$$

$$\int d^n k \frac{k^2}{(k^2 + 2k.p + m^2)^\alpha} = \frac{i\pi^{n/2}}{(m^2 - p^2)^{\alpha-n/2}} \frac{1}{\Gamma(\alpha)}$$

$$x \left\{\Gamma(\alpha - \tfrac{n}{2})p^2 + \Gamma(\alpha - \tfrac{n}{2}) \tfrac{n}{2} (m^2-p^2)\right\} \qquad (A\cdot3\cdot8)$$

$$\int d^n k \frac{k_\mu k_\nu}{(k^2 + 2k.p + m^2)^\alpha} = \frac{i\pi^{n/2}}{(m^2 - p^2)^{\alpha-n/2}} \frac{1}{\Gamma(\alpha)}$$

$$x \left\{\Gamma(\alpha - \tfrac{n}{2})k^2 + \Gamma(\alpha - 1 - n) \tfrac{1}{2} \delta_{\mu\nu}(m^2-p^2)\right\} \qquad (A\cdot3\cdot9)$$

For the definition of the form factors to follow in Appendices B and C, it is convenient to define

$$\Delta = \pi^{n/2 - 2}\Gamma(2 - \tfrac{n}{2}) \qquad (A\cdot3\cdot10a)$$

$$= \frac{1}{2 - \frac{n}{2}} - \gamma - \ell n\pi \qquad (A\cdot3\cdot10b)$$

with the (A·3·10b) valid in a neighbourhood of n = 4. Consistent with the generalization to n dimensions

$$\gamma_\mu \gamma_\mu = n \qquad (A\cdot3\cdot11)$$

$$\mathrm{Tr}\ I = 2^{n/2} \qquad (A\cdot3\cdot12)$$

That is the vectors of integration have n components but spinors have $2^{n/2}$ components. Note in consequence that

$$\delta_{\mu\mu} = n \qquad (A\cdot3\cdot13)$$

when it arises from bosonic fields and

$$\delta_{\mu\mu} = 2^{n/2} \qquad (A\cdot3\cdot14)$$

from fermions. The standard model is free from γ_5 anomalies so that the generalization of γ_5 does not lead to difficulties.

Using (A·3·10) one has

$$(n - a)\Delta = (4 - a)\Delta - 2 \qquad (A\cdot3\cdot15)$$

$$2^{n/2}\Delta = 4\Delta - 4\ell n2 \qquad (A\cdot3\cdot16)$$

It is important to keep all finite constant terms generated by (A·3·15) in order to preserve properties such as photon transversality that arise from the symmetry of the theory. The $\ell n2$ terms from (A·3·16) all cancel within the fermionic sector and it is found that any prescription in which the number of spinor components tends to four at n = 4 will do (Taylor 1979).

Dimensional regularization has also been applied to control infrared divergences (Gastmans and Meuldermans 1973, Marciano and Sirlin 1975).

Appendix B

Two-point form factors

In gauge theories treated to one loop, there is a limited number of types of integrands that can appear and it is convenient to systematize their nomenclature and evaluation.

In this work the notation of Passarino and Veltman 1979 is adopted;

$$A(m) = \int \frac{d^n k}{i\pi^2} \frac{1}{k^2 + m^2 - i\varepsilon} \qquad (B\cdot 1)$$

$$= \frac{\pi^{n/2 - 2}}{(m^2 - i\varepsilon)^{1 - n/2}} \Gamma\left(1 - {}^{n}/_{2}\right) \qquad (B\cdot 1a)$$

$$\{B_0, B_\mu, B_{\mu\nu}\}(p; m_1, m_2) = \int \frac{d^n k}{i\pi^2} \frac{\{1, k_\mu, k_\mu k_\nu\}}{[k^2 + m_1^2 - i\varepsilon][(k + p)^2 + m_2^2 - i\varepsilon]} \qquad (B\cdot 2)$$

From Lorentz covariance it follows that

$$B_\mu(p; m_1, m_2) = p_\mu B_1(p; m_1, m_2) \qquad (B\cdot 3)$$

$$B_{\mu\nu}(p; m_1, m_2) = p_\mu p_\nu B_{21}(p; m_1, m_2) + \delta_{\mu\nu} B_{22}(p; m_1, m_2) \qquad (B\cdot 4)$$

which serves as a definition of the quantities, $B_{i(j)}$. These form factors are functions only of the squares of their arguements.

All the integrals are divergent and are tamed by dimensional regularization which has the property that it preserves the gauge symmetry and hence does not lead to a violation of the Ward identities of the theory.

A and B_{22} exhibit their quadratic divergence through poles at integer values of $n \geq 2$. The remaining form factors are logarithmically divergent with poles at integer $n \geq 4$.

With the exception of tadpole and Higgs self-energy diagrams, no quadratic divergences appear in the GSW model. The relations

$$(2 - n)B_{22} = -p^2 B_{21} + (m_1^2 - p^2 - m_2^2)B_1 + m_1^2 B_0 \qquad (B\cdot5)$$

$$4B_{22} - A(m_1^2) - A(m_2^2) = -4p^2 B_{21} + 2(m_1^2 - 2p^2 - m_2^2)B_1 + (-p^2 - m_2^2 + m_1^2)B_0 \qquad (B\cdot6)$$

will be required to express apparent quadratic divergences in terms of logarithmically divergent form factors.

In evaluating these form factors particular care must be taken of the imaginary parts generated by the $i\varepsilon$ prescription. This point was overlooked by Passarino and Veltman 1979 in their original formulation. The following preliminary results are required;

Result 1

If x_1 and x_2 are the roots of the equation

$$ax^2 + bx + c = 0$$

with a, b, c real and Real$(x_1) \geqq$ Real (x_2) then the roots of

$$ax^2 + bx + c - i\varepsilon = 0$$

are $\tilde{x}_1 = x_1 + i\varepsilon/a$ and $\tilde{x}_2 = x_2 - i\varepsilon/a$. When the roots are complex the $i\varepsilon$ is ignored.

Result 2

It is shown by t'Hooft and Veltman 1979 that for A and B real and ε' of the same sign as ε

$$\ell n\,(AB - i\varepsilon) = \ell n\,(A - i\varepsilon') + \ell n\,(B - i\varepsilon/A) \qquad (B\cdot7)$$

Combining the results leads to

$$\ln (ax^2 + bx + c - i\varepsilon) = \ln (a - i\varepsilon') + \ln (x - x_1 - i\varepsilon/a) + \ln (x - x_2 + i\varepsilon/a) \qquad (B\cdot8)$$

By means of the usual Feynman parameterization and the formulas of t'Hooft and Veltman 1979,

$$B_0 = \Delta - \int_0^1 dx \ln [m_1^2(1 - x) + (p^2 + m_2^2)x - p^2x^2 - i\varepsilon] \qquad (B\cdot9)$$

$$B_1 = -\tfrac{1}{2}\Delta + \int_0^1 dx\, x \ln [m_1^2(1 - x) + (p^2 + m_2^2)x - p^2x^2 - i\varepsilon] \qquad (B\cdot10)$$

$$B_{21} = \tfrac{1}{3}\Delta - \int_0^1 dx\, x^2 \ln [m_1^2(1 - x) + (p^2 + m_2^2)x - p^2x^2 - i\varepsilon] \qquad (B\cdot11)$$

where Δ is defined in (A·3·10). Note that the equations (B·9) - (B·11) contain the logarithm of a quantity in the mass scale of dimension 2. Such 'dimensionful logarithms' occur only in the intermediate steps but are necessarily absent in the final 'physical' result. An aesthetically more satisfactory treatment may be obtained by (Itzykson and Zuber 1980; p.378) scaling the coupling constants that appear by an arbitrary factor $\Lambda^{n/2\ -2}$, where Λ has dimension 1 in the mass scale. This has the effect of keeping them dimensionless as n is continued away from 4 and the arguments for all dimensionful logarithms appear divided by Λ^2. Because an observable quantity must be independent of such an arbitrary scale, its introduction can provide a method for checking a calculation numerically.

Carrying out the Feynman integrations by exploiting (B·8) one obtains

$$B_0 = \Delta - \left[\ell n\,(-p^2 - i\varepsilon) + \sum_{j=1}^{2} \{\ell n\,(1 - \tilde{x}_j) + F(1, \tilde{x}_j)\}\right] \qquad (B\cdot12)$$

$$B_1 = -\tfrac{1}{2}\,\Delta + \tfrac{1}{2}\left[\ell n\,(-p^2 - i\varepsilon) + \sum_{j=1}^{2} \{\ell n\,(1 - \tilde{x}_j) + F(2, \tilde{x}_j)\}\right] \qquad (B\cdot13)$$

$$B_{21} = \tfrac{1}{3}\,\Delta - \tfrac{1}{3}\left[\ell n\,(-p^2 - i\varepsilon) + \sum_{j=1}^{2} \{\ell n\,(1 - \tilde{x}_j) + F(3, \tilde{x}_j)\}\right] \qquad (B\cdot14)$$

F(n,x) is defined by

$$\int_0^1 dx\; x^n\, \ell n\,(x - x_1) = \frac{1}{n+1}\{\ell n\,(1 - x_1) + F(n+1, x_1)\} \qquad (B\cdot15)$$

and $\sum_{j=1}^{2}$ sums over the roots $\tilde{x}_j$ of the equation

$$-p^2 x^2 + (p^2 + m_2^2 - m_1^2)\, x + m_1^2 - i\varepsilon = 0 \qquad (B\cdot16)$$

It is a simple matter to obtain

$$F(n, x) = -x^n\, \ell n\left(\frac{x-1}{x}\right) - x^{n-1} - \tfrac{1}{2}\, x^{n-2} \ldots - \frac{1}{n} \qquad (B\cdot17)$$

In practice this may be difficult to evaluate numerically for x large and the formula

$$F(n, x) = \frac{1}{n+1}\,\frac{1}{x} + \frac{1}{n+2}\,\frac{1}{x^2} + \ldots \qquad (B\cdot18)$$

was used for $|x| > 4$.

Note also that near x = 1 there is a strong cancellation of the large logarithms in (B·12) - (B·14)

$$\ell n\,(1 - x) - x^n\, \ell n\left(\frac{x-1}{x}\right) \qquad (B\cdot19)$$

and these are better written as

$$(1 - x^n)\, \ell n\,(1 - x) + x^n\, \ell n\,(-x) \qquad (B\cdot20)$$

From the above formulas it follows that

$$F(n, 1) = -\sum_{i=1}^{n} \frac{1}{i} \tag{B·21}$$

It will be useful to describe in detail how the imaginary parts arise in (B·12) - (B·14). The special case of $m = m_1 = m_2$ will be calculated since it may be anticipated that the imaginary parts appear for timelike $p^2 < -4m^2$.

The roots of (B·16) are

$$\tilde{x}_{1,2} = \tfrac{1}{2} \pm \left\{ \tfrac{1}{2} \sqrt{1 + \frac{4m^2}{p^2}} - \frac{i\varepsilon}{p^2} \right\}$$

Equations (B·12) - (B·14) lead to the expression of the form

$$\ell n\,(-p^2 - i\varepsilon) + \sum_{j=1}^{2} \left\{ \ell n\,(1 - \tilde{x}_j) - \tilde{x}_j^n\, \ell n \left(\frac{\tilde{x}_j - 1}{\tilde{x}_j} \right) + P(n-1, \tilde{x}_j) \right\} \tag{B·22}$$

where $P(n-1, \tilde{x}_j)$ is a polynomial of degree $n - 1$ in $\tilde{x}_j$. Henceforth this will be discarded because $P(n-1, \tilde{x}_1) + P(n-1, \tilde{x}_2)$ is real. Equation (B·22) can be written

$$\begin{aligned} &\ell n\,(-p^2 - i\varepsilon) + (1 - \tilde{x}_1^n)\, \ell n\ \tilde{x}_2 + \tilde{x}_2^n\, \ell n\,(-\tilde{x}_2) \\ &+ (1 - \tilde{x}_2^n)\, \ell n\, \tilde{x}_1 + \tilde{x}_1^n\, \ell n\,(-\tilde{x}_1) \end{aligned} \tag{B·23}$$

where use has been made of the relation

$$\tilde{x}_1 + \tilde{x}_2 = 1 \tag{B·24}$$

There are three regions in p^2 that need to be considered.

$\underline{p^2 > 0}$

The roots lie in the region $x_{1,2}\varepsilon(-\infty, 0)U(1, \infty)$ and are of opposite sign. One of each of the pairs $\ell n(\tilde{x}_1)$, $\ell n(-\tilde{x}_1)$ and $\ell n(\tilde{x}_2)$, $\ell n(-\tilde{x}_2)$ contributes $\pm i\pi$ but the imaginary parts of the terms proportional to $\tilde{x}_1^n$ cancel. The remaining $i\pi$ cancels with a $-i\pi$ from $\ell n(-p^2 - i\varepsilon)$ and the overall expression is real.

$\underline{-4m^2 < p^2 < 0}$

The roots are imaginary with $\tilde{x}_1 = \tilde{x}_2^*$. There is no imaginary part overall because $\ell n(-p^2 - i\varepsilon)$ is real and

$$\left[(1 - \tilde{x}_1^n)\ \ell n\ \tilde{x}_2 + \tilde{x}_2^n\ \ell n\ (-\tilde{x}_2)\right]^* = (1 - \tilde{x}_2^n)\ \ell n\ \tilde{x}_1 + \tilde{x}_1^n \ell n\ (-\tilde{x}_1)$$

$\underline{p^2 < -4m^2}$

The roots lie in the interval $x_{1,2}\varepsilon(0, 1)$ and because they are of the same sign there is no longer a cancellation of imaginary parts proportional to x_1^n. The expression (B·22) now develops a negative imaginary part $\pi(\tilde{x}_2^n - \tilde{x}_1^n)$.

For notational convenience, one defines

$$B_3 = B_{21} + B_1 \qquad \text{(B·25a)}$$

$$B_4 = B_{21} + \tfrac{1}{2}B_1 + B_0 \qquad \text{(B·25b)}$$

$$B_5 = B_1 + \tfrac{1}{2}B_0 \qquad \text{(B·25c)}$$

Formulas for recurring sets of arguments are given below

$\underline{B\ (0;\ m_1\ m_2)}$ $\qquad \alpha = \dfrac{m_1^2 - i\epsilon}{m_1^2 - m_2^2}$

$$B_0 = \Delta - \ell n\ m_2^2 - F(1\ ,\ \alpha) \tag{B·26a}$$

$$B_1 = -\tfrac{1}{2}\left[\Delta - \ell n\ m_2^2 - F(2\ ,\ \alpha)\right] \tag{B·26b}$$

$$B_{21} = \tfrac{1}{3}\left[\Delta - \ell n\ m_2^2 - F(3\ ,\ \alpha)\right] \tag{B·26c}$$

$\underline{B\ (0;\ m,\ m)}$

$$B_0 = \Delta - \ell n\ m^2 \tag{B·27a}$$

$$B_1 = -\tfrac{1}{2}\left[\Delta - \ell n\ m^2\right] \tag{B·27b}$$

$$B_3 = -\tfrac{1}{6}\left[\Delta - \ell n\ m^2\right] \tag{B·27c}$$

$\underline{B\ (0;\ 0,\ m)}$

$$B_0 = \Delta - \ell n\ m^2 + 1 \tag{B·28a}$$

$$B_1 = -\tfrac{1}{2}\left[\Delta - \ell n\ m^2\right] - \tfrac{1}{4} \tag{B·28b}$$

$\underline{B\ (p;\ m,\ m)}$, $m^2 >> |p^2|$

$$B_0 = \Delta - \ell n\ m^2 - \tfrac{1}{6}\,\frac{p^2}{m^2} \tag{B·29a}$$

$$B_1 = -\tfrac{1}{2}\left[\Delta - \ell n\ m^2\right] + \tfrac{1}{12}\,\frac{p^2}{m^2} \tag{B·29b}$$

$$B_3 = -\tfrac{1}{6}\left[\Delta - \ell n\ m^2\right] + \tfrac{1}{30}\,\frac{p^2}{m^2} \tag{B·29c}$$

$\underline{B\ (p;\ 0,\ m),\ p^2 + m^2 = 0}$

$$B_0 = \Delta - \ell n\ m^2 + 2 \tag{B·30a}$$

$$B_1 = -\tfrac{1}{2}\left[\Delta - \ell n\, m^2\right] - \tfrac{1}{2} \qquad \text{(B·30b)}$$

$$B_3 = -\tfrac{1}{6}\left[\Delta - \ell n\, m^2\right] - \tfrac{5}{18} \qquad \text{(B·30c)}$$

<u>B (m^2; 0, 0)</u>

$$B_0 = \Delta - \ell n\, m^2 + 2 \qquad \text{(B·31a)}$$

$$B_1 = -\tfrac{1}{2}\left[\Delta - \ell n\, m^2\right] - 1 \qquad \text{(B·31b)}$$

$$B_3 = -\tfrac{1}{6}\left[\Delta - \ell n\, m^2\right] - \tfrac{5}{18} \qquad \text{(B·31c)}$$

It is found that the quadratic divergences within certain subsets of diagrams cancel or reduce to logarithmic divergences. Ignoring possible multipliative constants such a set is given in Figure B·1 summing these and using (B·6) gives

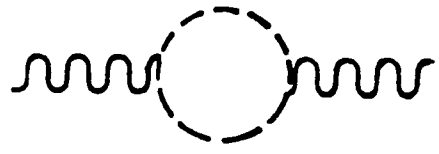
$$(p^2\delta_{\mu\nu} - p_\mu p_\nu)(-4B_3 - B_0) + (m_1^2 - m_2^2)(2B_1 + B_0)\,\delta_{\mu\nu} \qquad \text{(B·32)}$$

A second set is given in Figure B·2 and after summing and using (B·5) and (B·6) leads to

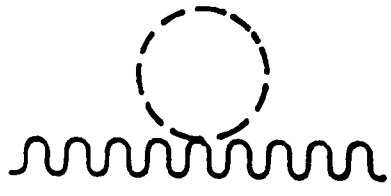
$$(p^2\delta_{\mu\nu} - p_\mu p_\nu)\left[-8B_3 + 2B_0 + \tfrac{2}{3}\right] + (m_1^2 - m_2^2)(4B_1 + B_0)\,\delta_{\mu\nu} - 2m_2^2 B_0 \delta_{\mu\nu} \qquad \text{(B·33)}$$

In both cases the coefficient of the B form factors are $B(p;\, m_1,\, m_2)$. Motivated by this one may define two new form factors $4B_7$ and B_8 to be the coefficients of $\delta_{\mu\nu}$ in (B·32) and (B·33) respectively. This greatly reduces the work in finding expressions for the one particle irreducible (1PI) one loop self energies for the photon, Z, W and Z-A mixing. Note in passing that B_0, B_3, B_7 and B_8 are symmetric under interchange of m_1 and m_2 but that

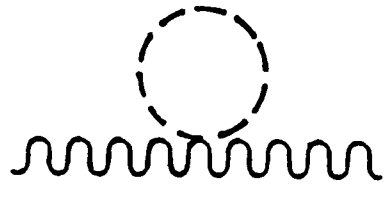
$$B_1(p;\, m_2,\, m_1) = -B_1(p;\, m_1,\, m_2) - B_0(p;\, m_1,\, m_2) \qquad \text{(B·34)}$$



$$4B_{22}\delta_{\mu\nu} + p_\mu p_\nu (4B_1 + B_0)$$

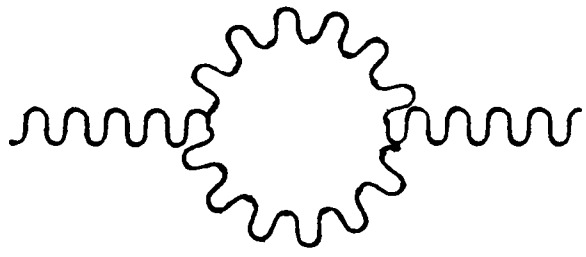


$$-\delta_{\mu\nu} A(m_1)$$

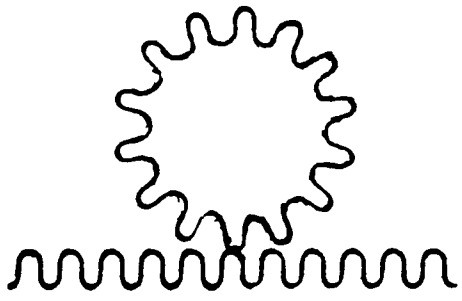


$$-\delta_{\mu\nu} A(m_2)$$

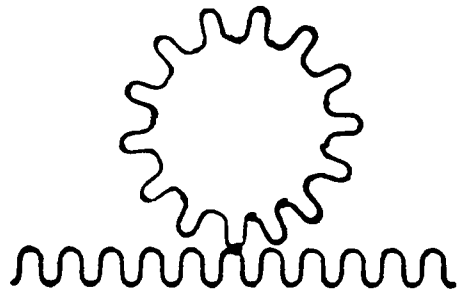
Figure B.1



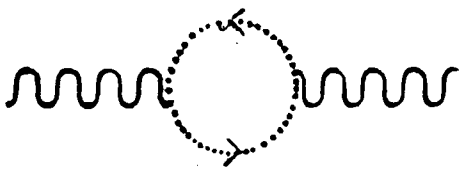
$$6(n-1)\delta_{\mu\nu}B_{22} + 2[\delta_{\mu\nu}p^2 + (2n-3)p_\mu p_\nu]B_3 + [5p^2\delta_{\mu\nu} + (n-6)p_\mu p_\nu]B_0$$



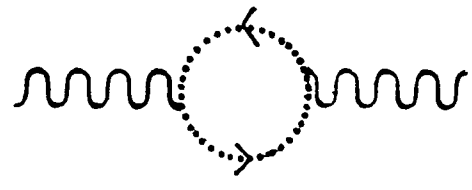
$$\delta_{\mu\nu}(1-n)A(m_1)$$



$$\delta_{\mu\nu}(1-n)A(m_2)$$



$$-(p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22})$$



$$-(p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22})$$

Figure B.2

Vector boson self-energies

With obvious notation the transverse parts (parts proportional to $\delta_{\mu\nu}p^2 - p_\mu p_\nu$) of the 1PI vector boson self energies are given below. The tilda indicates that the counter-term diagram is not included. In all cases, tadpole renormalization is assumed to have been carried out $\sum_f$ denotes the sum over fermions with mass m_f, 3 component of weak isospin $t_{3,f}$ and charge Q_f. $\sum_D$ is the sum over $SU(2)_L$ doublets composed of fermions of mass m_1^D and m_2^D.

$$\tilde{\Pi}_T^{AA}(p^2) = \frac{g^2 s_\theta^2}{16\pi^2} p^2 \{-12B_3(p; M_W, M_W) + B_0(p; M_W, M_W) + \frac{2}{3}\}$$

$$+ \frac{8g^2 s_\theta^2}{16\pi^2} p^2 \sum_f Q_f^2 [B_3(p; m_f, m_f) + \frac{1}{6} \ell n\, 2] \qquad (B\cdot35)$$

$$\tilde{\Pi}_T^{ZZ}(p^2) = \frac{g^2}{16\pi^2} \{c_\theta^2 B_8(p; M_W, M_W) + \frac{(s_\theta^2 - c_\theta^2)^2}{c_\theta^2} B_7(p; M_W, M_W)$$

$$+ \frac{1}{c_\theta^2} B_7(p; M_Z, M_H) + 2M_W^2 \frac{s_\theta^4}{c_\theta^2} B_0(p; M_W, M_W) + \frac{M_W^2}{c_\theta^4} B_0(p; M_Z, M_H)\}$$

$$+ \frac{g^2}{16\pi^2} \frac{1}{2c_\theta^2} \sum_f \{[1 + (2t_{3,f} - 4Q_f s_\theta^2)^2] p^2 (B_3(p; m_f, m_f) + \frac{1}{6} \ell n 2)$$

$$- m_f^2 (B_0(p; m_f, m_f) - \ell n\, 2)\}$$

(B-36)

$$\tilde{\Pi}_T^{ZA}(p^2) = \frac{g^2}{16\pi^2} \{s_\theta c_\theta B_8(p; M_W, M_W) - \frac{2s_\theta^3}{c_\theta} M_W^2 B_0(p; M_W, M_W)$$

$$- \frac{2s_\theta(s_\theta^2 - c_\theta^2)}{c_\theta} B_7(p; M_W, M_W)\}$$

$$+ \frac{g^2}{16\pi^2} \frac{2s_\theta}{c_\theta} p^2 \sum_f (2t_{3,f} - 4s_\theta^2 Q_f) Q_f [B_3(p^2, m_f, m_f) + \frac{1}{6} \ell n 2]$$

(B·37)

$$\bar{\Pi}_T^{WW}(p^2) = \frac{g^2}{16\pi^2}\{c_\theta^2 B_8(p; M_W, M_Z) + s_\theta^2 B_8(p; M_W, 0) + B_7(p; M_W, M_H)$$

$$+ B_7(p; M_W, M_Z) + M_W^2 \frac{s_\theta^4}{c_\theta^2} B_0(p; M_W, M_Z) + M_W^2 B_0(p; M_W, M_H)$$

$$+ s_\theta^2 M_W^2 B_0(p; M_W, 0)\} + \frac{g^2}{16\pi^2}\sum_D \{2p^2[B_3(p; m_1^D, m_2^D) + \tfrac{1}{6}\ell n2]$$

$$+ (m_1^D)^2 B_1(p; m_1^D, m_2^D) + (m_2^D)^2 B_1(p; m_2^D, m_1^D) + \tfrac{1}{2}(m_1^2 + m_2^2)\ell n2\} \quad (B\text{-}38)$$

Higgs self energy

In the case of the Higgs 1PI self energy it is convenient to sum all one loop diagrams given and include the quadratically divergent $\delta\beta$ from the counter-term to produce a logarithmically divergent result. Thus

$$\bar{\Pi}_{HH}(p^2) - \delta\beta = \frac{g^2}{16\pi^2}\frac{1}{4}\{[-6p^2 B_1 + p^2 B_0 + 12M_W^2 B_0^2 + \frac{M_H^4}{M_W^2} B_0](p; M_W, M_W)$$

$$+ \frac{1}{2c_\theta^2}[-6p^2 B_1 + p^2 B_0 + 12M_Z^2 B_0 + \frac{M_H^4}{M_Z^2} B_0](p; M_Z, M_Z)$$

$$+ \frac{9}{2}\frac{M_H^4}{M_W^2} B_0(p; M_H, M_H) - 8M_W^2 - \frac{4M_Z^2}{C_\theta^2}\}$$

$$- \frac{2g^2}{16\pi^2}\sum_f \frac{m_f^2}{M_W^2}(p^2 + 4m_f^2)[B_0(p; m_f, m_f) - \ell n2] \quad (B\cdot 39)$$

Fermion self energies

The general one loop self energy excluding counter-term contributions for a fermion of charge Q, 3-component of weak isospin t_3, mass m_1 and having $SU(2)_L$ doublet partner of mass m_2 is after summing the diagrams is

$$\sum(p) = i\not{p}\,[A(p^2)\gamma_+ + B(p^2)\gamma_-] + m_1\, C(p^2) \qquad (B\cdot40)$$

with

$$A = \frac{g^2}{16\pi^2}\,[B_1(p;\, m_2,\, M_W) + \tfrac{1}{2}] + \frac{2g^2}{16\pi^2}\left(\frac{4t_3 - 4s_\theta^2 Q}{4c_\theta}\right)^2 [B_1(p;\, m_1,\, M_W) + \tfrac{1}{2}]$$

$$+ \frac{g^2 s_\theta^2}{16\pi^2}\,[B_1(p;\, m_1,\, 0) + \tfrac{1}{2}] + \frac{g^2}{16\pi^2}\,\frac{m_1^2}{4M_W^2}\,B_1(p;\, m_1,\, M_H)$$

$$+ \frac{g^2}{16\pi^2}\,\frac{m_1^2}{4M_W^2}\,B_1(p;\, m_1,\, M_Z) \qquad (B\cdot41)$$

$$B = \frac{2g^2}{16\pi^2}\left(\frac{-4s_\theta^2 Q}{4c_\theta}\right)^2 [B_1(p;\, m_1,\, M_Z) + \tfrac{1}{2}] + \frac{g^2 s_\theta^2}{16\pi^2}\,[B_1(p;\, m_1,\, 0) + \tfrac{1}{2}]$$

$$+ \frac{g^2}{16\pi^2}\,\frac{m_1^2}{4M_W^2}\,B_1(p;\, m_1,\, M_H) + \frac{g^2}{16\pi^2}\,\frac{m_1^2}{2M_W^2}\,B_1(p;\, m_2,\, M_W)$$

$$+ \frac{g^2}{16\pi^2}\,\frac{m_1^2}{4M_W^2}\,B_1(p;\, m_1,\, M_Z) \qquad (B\cdot42)$$

$$C = -\frac{g^2}{16\pi^2}\left(\frac{4t_3 - 4s_\theta^2 Q}{4c_\theta}\right)\left(\frac{-4s_\theta^2 Q}{4c_\theta}\right)\left[4B_0(p;\, m_1,\, M_Z) - 2\right]$$

$$-\frac{g^2 s_\theta^2}{16\pi^2}\left[4B_0(p;\, m_1,\, m_2) - 2\right] + \frac{g^2}{16\pi^2}\frac{m_1^2}{4M_W^2}B_0(p;\, m_1,\, M_H)$$

$$-\frac{g^2}{16\pi^2}\frac{m_1^2}{4M_W^2}B_0(p;\, m_1,\, M_Z) \qquad (B\cdot43)$$

In general, however, throughout this work terms containing a factor $\frac{m^2}{M_W^2}$ are neglected. Equations (B·28) can then be used to provide explicit expressions for B_0 and B_1 in the remaining terms provided $M_W^2 \gg |p^2|$.

Derivatives of two-point form-factors

When considering photonic contributions to fermion renormalization, derivatives of the B form factors are needed.

$$\frac{\partial B_0}{\partial p^2} = \frac{-1}{p^2(\tilde{x}_1 - \tilde{x}_2)}\left\{(1 - \tilde{x}_1)\,F(1, \tilde{x}_1) - (1 - \tilde{x}_2)\,F(1, \tilde{x}_2)\right\} \qquad (B\cdot44)$$

$$\frac{\partial B_1}{\partial p^2} = \frac{1}{p^2(\tilde{x}_1 - \tilde{x}_2)}\left\{(1 - \tilde{x}_1)\,F(2, \tilde{x}_1) - (1 - \tilde{x}_2)\,F(2, \tilde{x}_2)\right\} \qquad (B\cdot45)$$

For completeness

$$\frac{\partial B_1}{\partial p^2}(0;\, m_1,\, m_2) = \frac{-1}{m_1^2 - m_2^2}\left\{\tfrac{1}{2} + \frac{m_2^2}{m_2^2 - m_1^2}F\left(1, \frac{m_1^2 - i\varepsilon}{m_1^2 - m_2^2}\right)\right\} \qquad (B\cdot46)$$

$$\frac{\partial B_1}{\partial p^2}(0;\, m_1,\, m_2) = \frac{1}{m_1^2 - m_2^2}\left\{\tfrac{1}{3} + \frac{m_2^2}{m_2^2 - m_1^2}F\left(2, \frac{m_1^2 - i\varepsilon}{m_1^2 - m_2^2}\right)\right\} \qquad (B\cdot47)$$

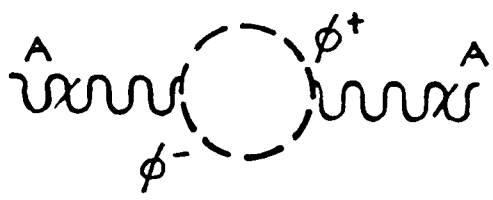
and

$$\frac{\partial B_0}{\partial p^2}(0;\, m,\, m) = \frac{-1}{6m^2} \qquad (B\cdot48)$$

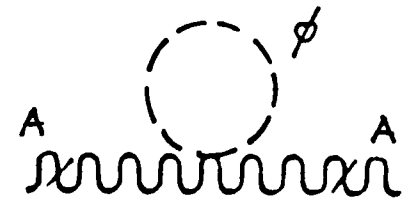
$$\frac{\partial B_1}{\partial p^2}(0;\, m,\, m) \;=\; \frac{1}{12m^2} \qquad (B\cdot 49)$$

Graphs involving a heavy internal boson have derivatives proportional to $\frac{m^2}{M_W^2}$ and hence are neglected.

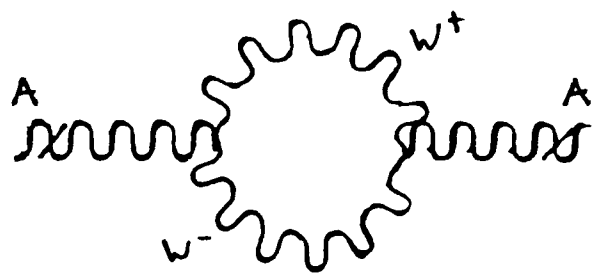
Photon self-energy diagrams



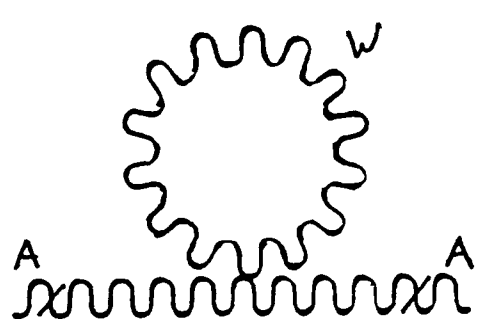
$\frac{1}{16\pi^2} g^2 s_\theta^2 \{4 B_{22} \delta_{\mu\nu} + p_\mu p_\nu (4B_3 + B_2)\}(p; M_W, M_W)$



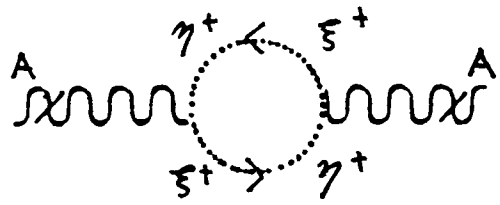
$-\frac{1}{16\pi^2} g^2 2 s_\theta^2 \delta_{\mu\nu} A(M_W)$



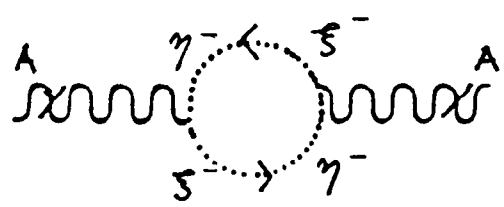
$\frac{1}{16\pi^2} g^2 s_\theta^2 \{6(n-1)\delta_{\mu\nu} B_{22} + 2[p^2 \delta_{\mu\nu} + (2n-3) p_\mu p_\nu] B_3 + [5p^2 \delta_{\mu\nu} + (n-6) p_\mu p_\nu] B_2\}(p; M_W, M$



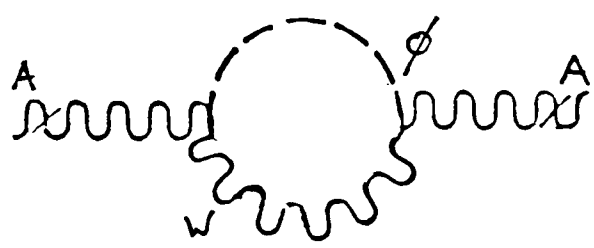
$\frac{1}{16\pi^2} g^2 2 s_\theta^2 \delta_{\mu\nu} (1-n) A(M_W)$



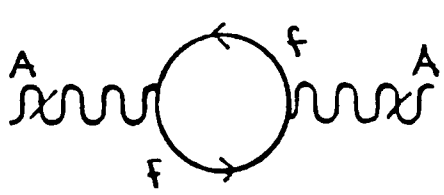
$-\frac{1}{16\pi^2} g^2 s_\theta^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p; M_W, M_W)$



$-\frac{1}{16\pi^2} g^2 s_\theta^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p; M_W, M_W)$

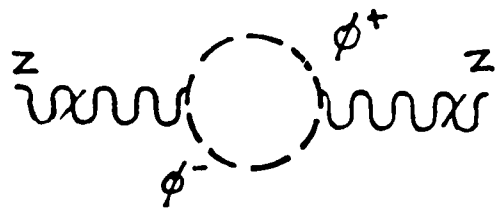


$\frac{1}{16\pi^2} g^2 s_\theta^2 2 M_W^2 \delta_{\mu\nu} B_0(p; M_W, M_W)$

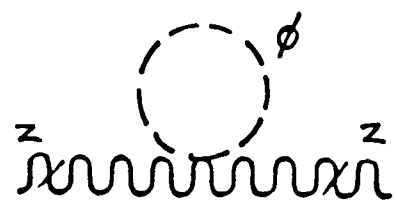


$\frac{1}{16\pi^2} g^2 s_\theta^2 2^{n/2} 2 Q^2 (p^2 \delta_{\mu\nu} - p_\mu p_\nu) B_3(p; m_f, m_f)$

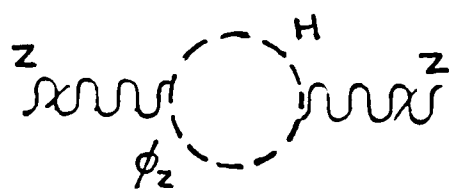
Z^0 self-energy diagrams



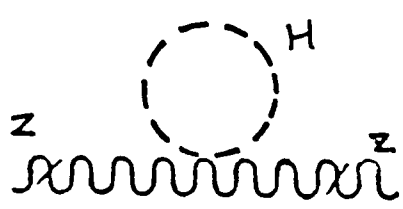
$$\frac{1}{16\pi^2} g^2 \left(\frac{S_\theta^2 - C_\theta^2}{2C_\theta}\right)^2 \{4\delta_{\mu\nu} B_{22} + p_\mu p_\nu (4B_3 + B_0)\}(p; M_W, M_W)$$



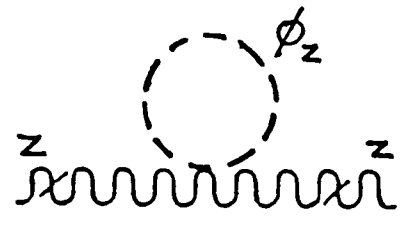
$$\frac{-1}{16\pi^2} g^2 \frac{1}{2} \left(\frac{S_\theta^2 - C_\theta^2}{C_\theta}\right)^2 A(M_W)$$



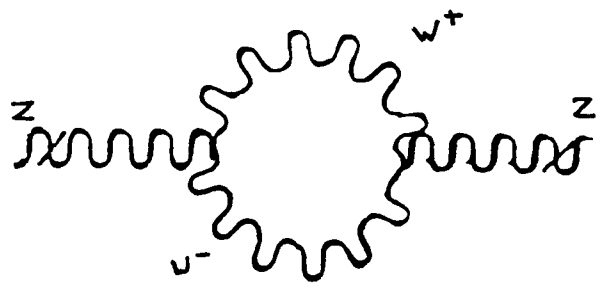
$$\frac{1}{16\pi^2} g^2 \frac{1}{4C_\theta^2} \{4 B_{22} \delta_{\mu\nu} + p_\mu p_\nu (4B_3 + B_0)\}(p; M_Z, M_H)$$



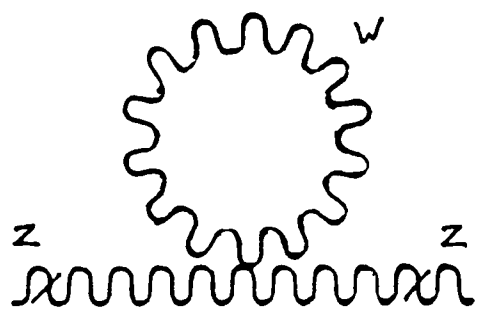
$$\frac{-1}{16\pi^2} g^2 \frac{1}{4C_\theta^2} \delta_{\mu\nu} A(M_H)$$



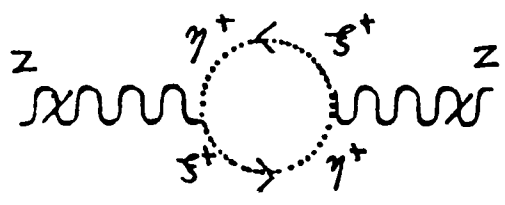
$$\frac{-1}{16\pi^2} g^2 \frac{1}{4C_\theta^2} \delta_{\mu\nu} A(M_Z)$$



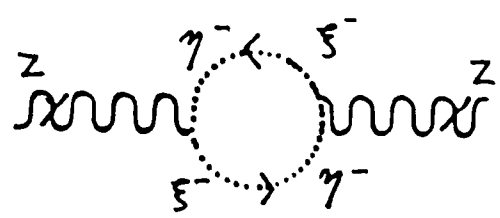
$$\frac{1}{16\pi^2} g^2 C_\theta^2 \{6(n-1)\delta_{\mu\nu} B_{22} + 2[p^2 \delta_{\mu\nu} + (2n-3) p_\mu p_\nu] B_3 + [5p^2 \delta_{\mu\nu} + (n-6) p_\mu p_\nu] B\}(p; M_W, M_$$



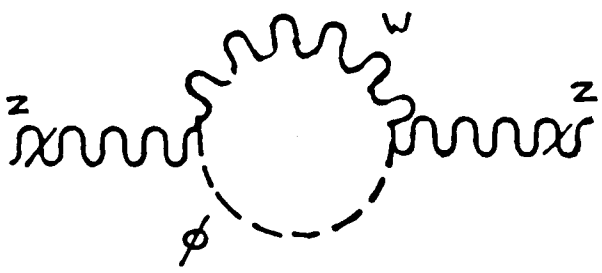
$$\frac{1}{16\pi^2} g^2 2C_\theta^2 \delta_{\mu\nu} (1-n) A(M_W)$$



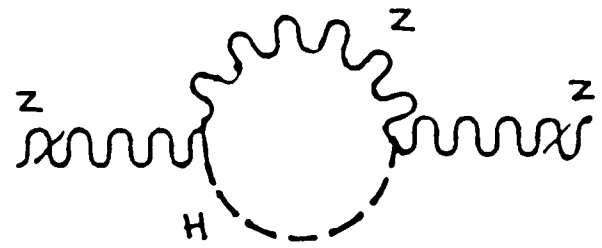
$$\frac{-1}{} g^2 C_\theta^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p; M_W, M_W)$$



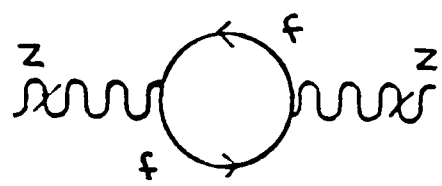
$$\frac{-1}{16\pi^2} g^2 C_\theta^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p; M_W, M_W)$$



$$\frac{1}{16\pi^2} g^2 2M_W^2 \frac{S_\theta^4}{C_\theta^2} \delta_{\mu\nu} B_0(p; M_W, M_W)$$

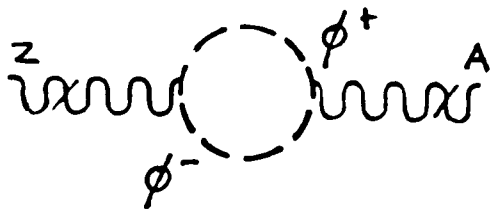


$$\frac{1}{16\pi^2} g^2 M_Z^2 \frac{1}{C_\theta^2} \delta_{\mu\nu} B_0(p; M_Z, M_H)$$

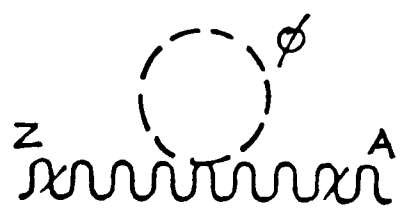


$$\frac{1}{16\pi^2} g^2 \frac{2^{n_f}}{8 C_\theta^2} \{[(2t_3)^2 + (2t_3 - 4Q_f S_\theta^2)^2](p^2\delta_{\mu\nu} - p_\mu p_\nu) B_3 - m_f^2 (2t_3)^2 \delta_{\mu\nu} B_0\}(p; m_f, m$$

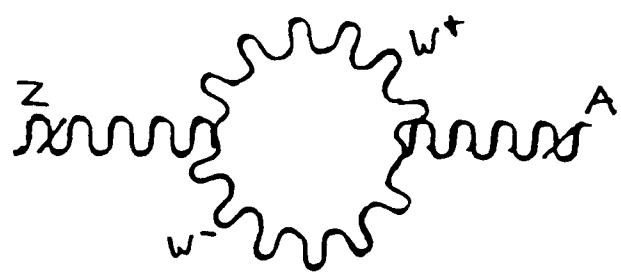
Z-A mixing diagrams



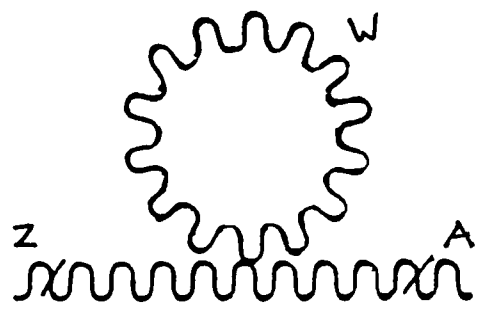
$$-\frac{1}{16\pi^2} g^2 s_\theta \frac{(s_\theta^2 - c_\theta^2)}{2c_\theta} \{4\,\delta_{\mu\nu} B_{22} + p_\mu p_\nu (4B_3 + B_2)\}(p\,; M_W, M_W)$$



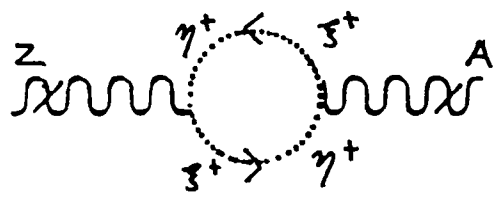
$$\frac{1}{16\pi^2} g^2 \frac{s_\theta}{c_\theta}(s_\theta^2 - c_\theta^2)\delta_{\mu\nu} A(M_W)$$



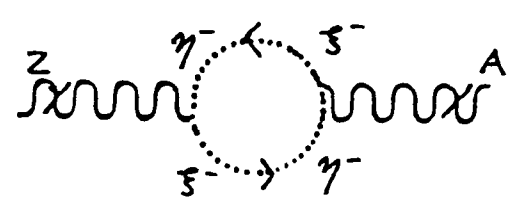
$$\frac{1}{16\pi^2} g^2 s_\theta c_\theta \{6(n-1)\,\delta_{\mu\nu} B_{22} + 2[p^2 \delta_{\mu\nu} + (2n-3) p_\mu p_\nu] B_3 + [5p^2 \delta_{\mu\nu} + (n-6) p_\mu p_\nu] B_2\}(p\,; M_W, M$$



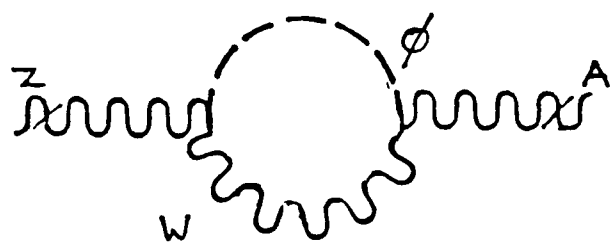
$$\frac{1}{16\pi^2} g^2 2 s_\theta c_\theta\, \delta_{\mu\nu}(1-n) A(M_W)$$



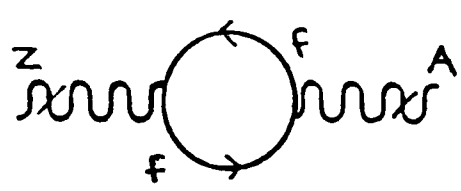
$$-\frac{1}{16\pi^2} g^2 s_\theta c_\theta \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p\,; M_W, M_W)$$



$$\frac{-1}{16\pi^2} g^2 s_\theta c_\theta \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p\,; M_W, M_W)$$

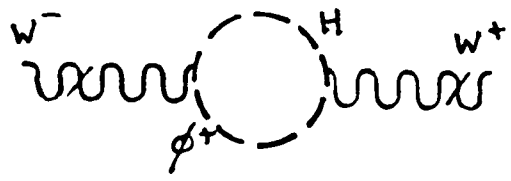


$$-\frac{1}{16\pi^2} g^2 \frac{2 s_\theta^3}{c_\theta} M_W^2\, \delta_{\mu\nu} B_0(p\,; M_W, M_W)$$

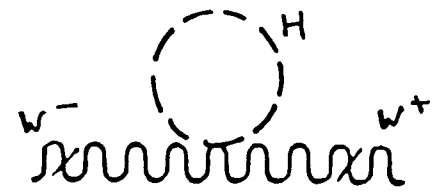


$$\frac{1}{16\pi^2} g^2 \frac{s_\theta}{c_\theta} \frac{2^{\frac{n}{2}}}{2} (2t_3 - 4 s_\theta^2 Q) Q (p^2 \delta_{\mu\nu} - p_\mu p_\nu) B_3(p\,; m_f, m_f)$$

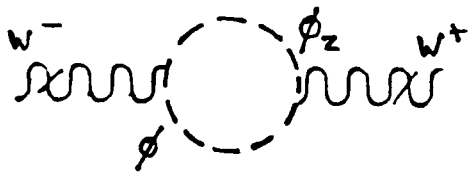
W self-energy diagrams



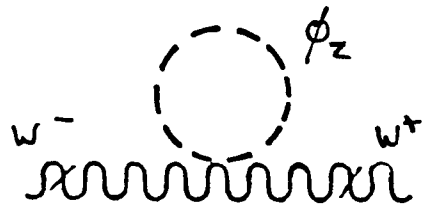
$$\frac{1}{16\pi^2} g^2 \frac{1}{4} \{4 B_{22} \delta_{\mu\nu} + p_\mu p_\nu (4B_3 + B_0)\}(p ; M_W, M_H)$$



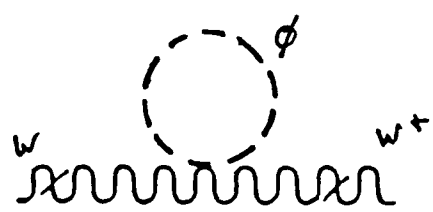
$$-\frac{1}{16\pi^2} g^2 \frac{1}{4} \delta_{\mu\nu} A(M_H)$$



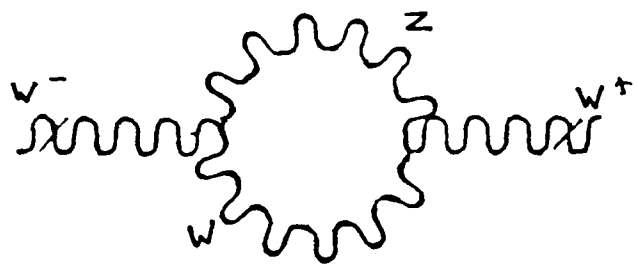
$$\frac{1}{16\pi^2} g^2 \frac{1}{4} \{4 B_{22} \delta_{\mu\nu} + p_\mu p_\nu (4B_3 + B_0)\}(p ; M_W, M_Z)$$



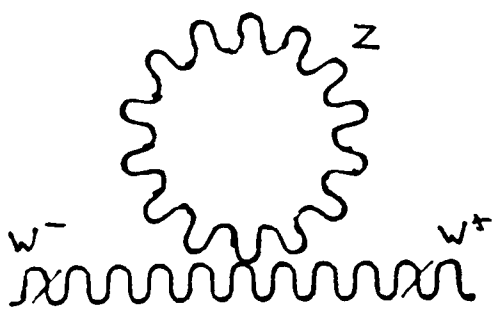
$$-\frac{1}{16\pi^2} g^2 \frac{1}{4} \delta_{\mu\nu} A(M_Z)$$



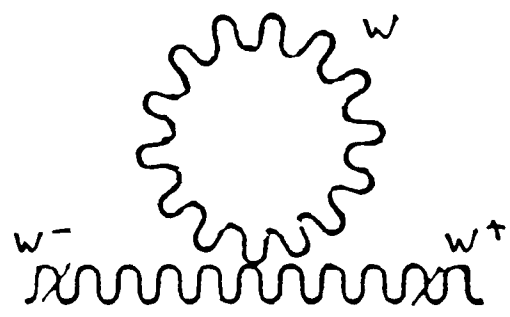
$$-\frac{1}{16\pi^2} g^2 \frac{1}{2} \delta_{\mu\nu} A(M_W)$$



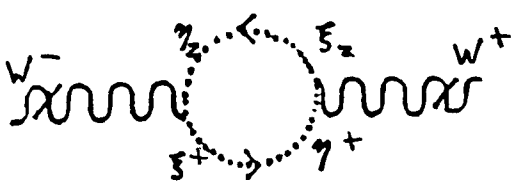
$$\frac{1}{16\pi^2} g^2 c_0^2 \{6(n-1)\delta_{\mu\nu} B_{22} + 2[p^2 \delta_{\mu\nu} + (2n-3) p_\mu p_\nu] B_3 + [5p^2 \delta_{\mu\nu} + (n-6) p_\mu p_\nu] B_0\}(p ; M_W,$$



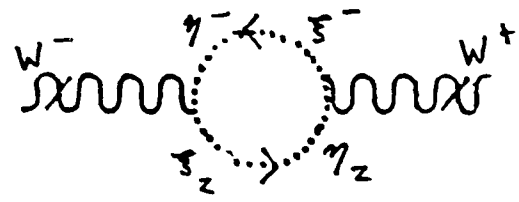
$$\frac{1}{16\pi^2} g^2 c_0^2 \delta_{\mu\nu} (1-n) A(M_Z)$$



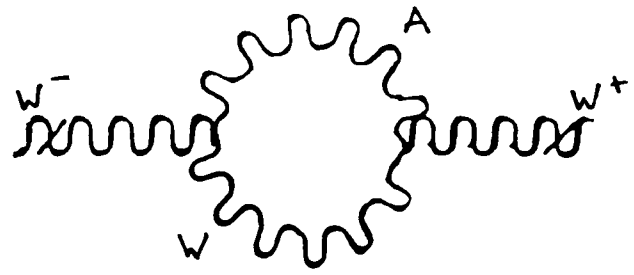
$$\frac{1}{16\pi^2} g^2 \delta_{\mu\nu} (1-n) A(M_W)$$



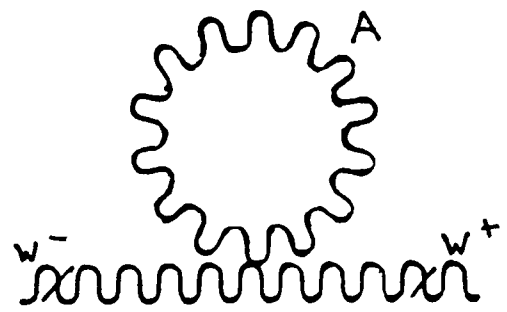
$$-\frac{1}{16\pi^2} g^2 c_0^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p ; M_W, M_Z)$$



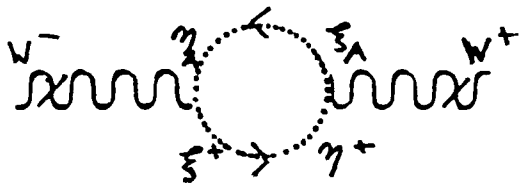
$$-\frac{1}{16\pi^2} g^2 c_0^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p ; M_W, M_Z)$$



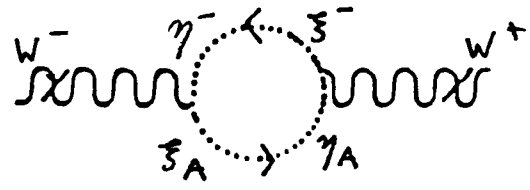
$$\frac{1}{16\pi^2} g^2 s_\theta^2 \{6(n-1)\delta_{\mu\nu} B_{22} + 2[p^2 \delta_{\mu\nu} + (2n-3) p_\mu p_\nu] B_3 + [5p^2 \delta_{\mu\nu} + (n-6) p_\mu p_\nu] B_0\}(p; M_W,$$



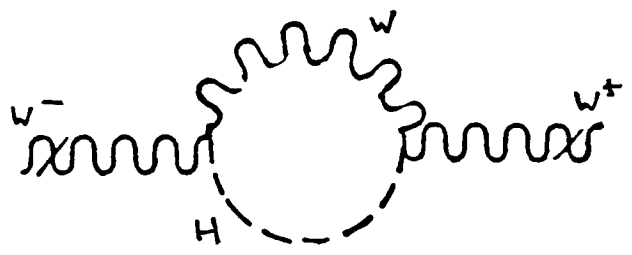
$$\frac{1}{16\pi^2} g^2 s_\theta^2 \delta_{\mu\nu} (1-n) A(0)$$



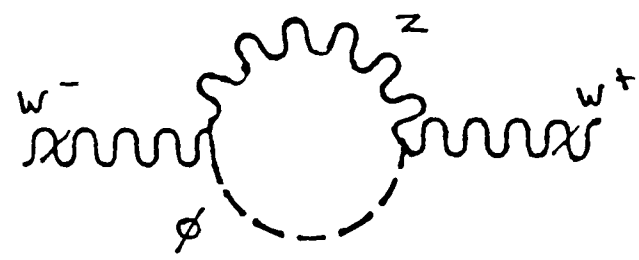
$$-\frac{1}{16\pi^2} g^2 s_\theta^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p; M_W, 0)$$



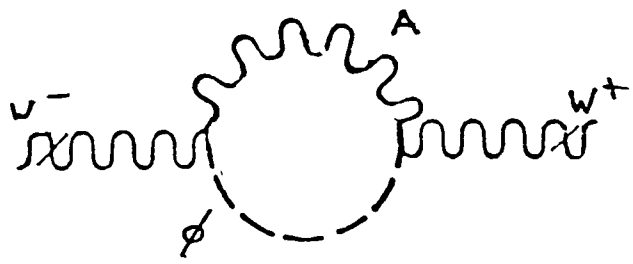
$$-\frac{1}{16\pi^2} g^2 s_\theta^2 \{p_\mu p_\nu B_3 + \delta_{\mu\nu} B_{22}\}(p; M_W, 0)$$



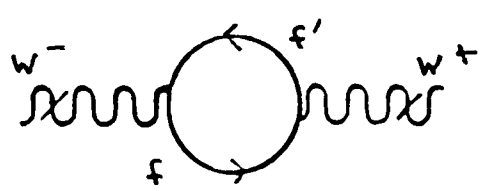
$$\frac{1}{16\pi^2} g^2 M_W^2 \delta_{\mu\nu} B_0(p; M_W, M_H)$$



$$\frac{1}{16\pi^2} g^2 M_Z^2 s_\theta^4 \delta_{\mu\nu} B_0(p; M_W, M_Z)$$

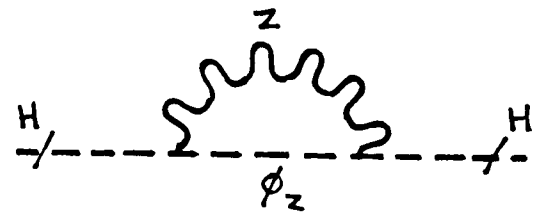


$$\frac{1}{16\pi^2} g^2 s_\theta^2 M_W^2 \delta_{\mu\nu} B_0(p; M_W, 0)$$

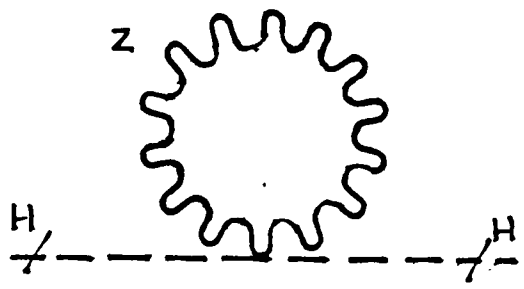


$$\frac{1}{16\pi^2} g^2 \frac{2^2}{4} \{2(p^2 \delta_{\mu\nu} - p_\mu p_\nu) B_3 + \delta_{\mu\nu} (m_f^2 - m_{f'}^2) B_1 - \delta_{\mu\nu} m_{f'}^2 B_0\}(p; m_{f'}, m_f)$$

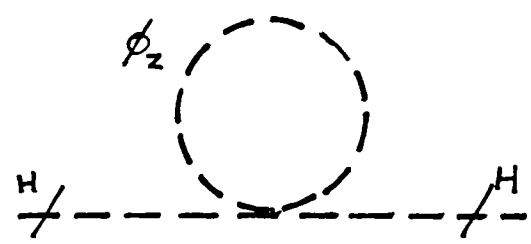
Higgs' self-energy diagrams



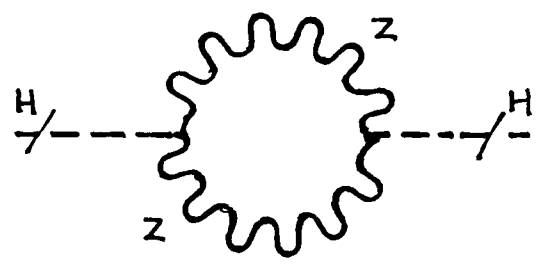
$$\frac{1}{16\pi^2} g^2 \frac{1}{4C_\theta^2} \{ n B_{22} + p^2 (B_{21} - 2B_1 + B_0) \} (p; M_z, M_z)$$



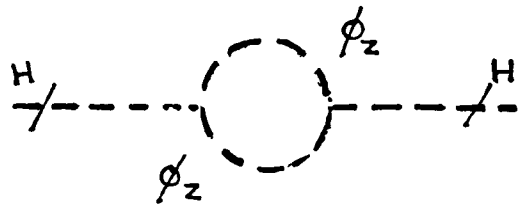
$$\frac{-1}{16\pi^2} g^2 \frac{n}{4C_\theta^2} A(M_z)$$



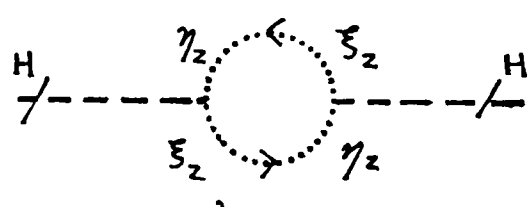
$$\frac{-1}{16\pi^2} g^2 \frac{1}{8} \frac{M_H^2}{M_W^2} A(M_z)$$



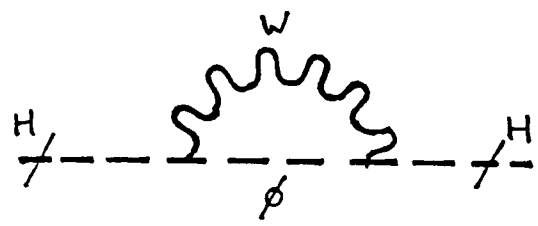
$$\frac{1}{16\pi^2} g^2 M_W^2 \frac{n}{2C_\theta^4} B_0(p; M_z, M_z)$$



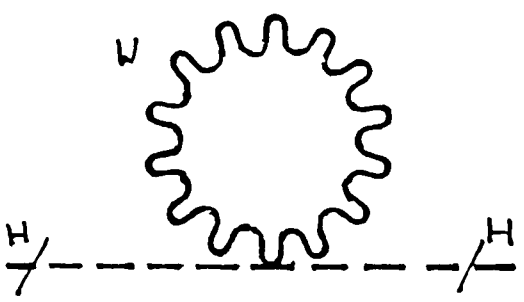
$$\frac{1}{16\pi^2} g^2 \frac{1}{8} \frac{M_H^4}{M_W^2} B_0(p; M_z, M_z)$$



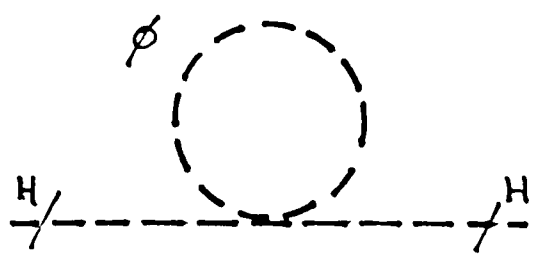
$$\frac{-1}{16\pi^2} g^2 \frac{M_W^2}{4C_\theta^4} B_0(p; M_z, M_z)$$



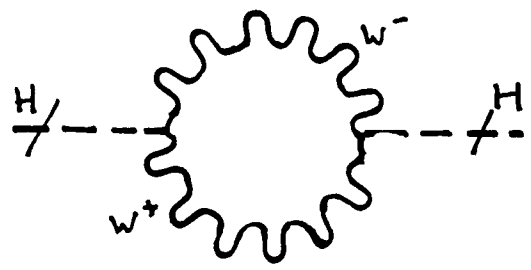
$$\frac{1}{16\pi^2} g^2 \frac{1}{2} \{ n B_{22} + p^2 (B_{21} - 2B_1 + B_0) \} (p; M_w, M_w)$$



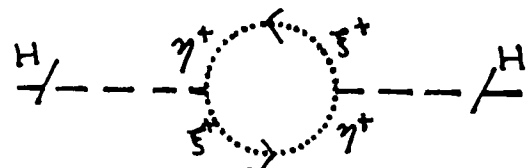
$$\frac{-1}{16\pi^2} g^2 \frac{n}{2} A(M_W)$$



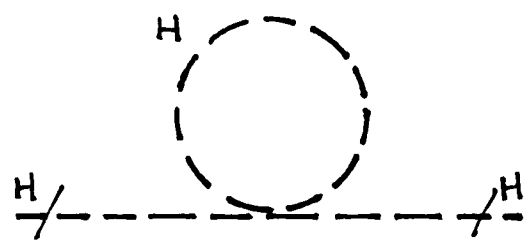
$$\frac{-1}{16\pi^2} g^2 \frac{1}{4} \frac{M_H^2}{M_W^2} A(M_W)$$



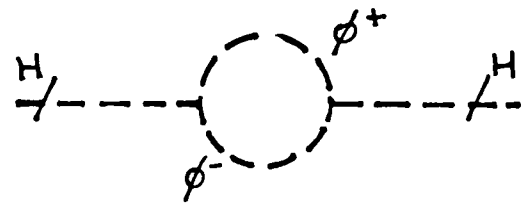
$$\frac{1}{16\pi^2} g^2 M_W^2 \, n \, B_0(p; M_W, M_W)$$



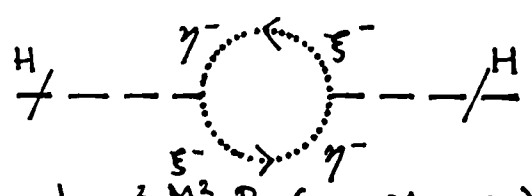
$$-\frac{1}{16\pi^2} g^2 \frac{M_W^2}{4} B_0(p; M_W, M_W)$$



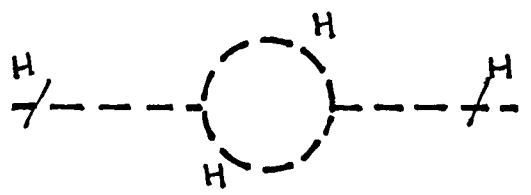
$$\frac{-1}{16\pi^2} g^2 \frac{3}{8} \frac{M_H^2}{M_W^2} A(M_H)$$



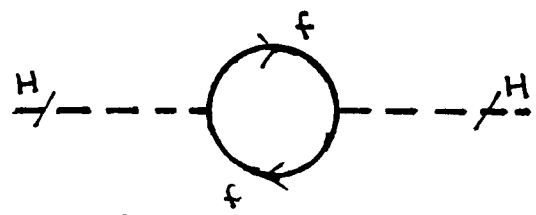
$$\frac{1}{16\pi^2} g^2 \frac{1}{4} \frac{M_H^4}{M_W^2} B_0(p; M_W, M_W)$$



$$-\frac{1}{16\pi^2} g^2 \frac{M_W^2}{4} B_0(p; M_W, M_W)$$



$$\frac{1}{16\pi^2} \frac{g^2}{4} \frac{9}{2} \frac{M_H^4}{M_W^2} B_0(p; M_H, M_H)$$



$$\frac{1}{16\pi^2} g^2 \frac{2^{\frac{n}{2}}}{4} \frac{m_f^2}{M_W^2} \{n B_{22} + p^2 B_3 - m_f^2 B_0\}(p; m_f, m_f)$$

Appendix C

Three-point form factors

The 3-point form factors are again defined following Passarino and Veltman 1977 as

$$\{C_0;\ C_\mu;\ C_{\mu\nu}\}\ (p,\ q;\ m_1,\ m_2,\ m_3)$$

$$= \int \frac{d^n k}{i\pi^2} \frac{\{1,\ k_\mu;\ k_\mu k_\nu\}}{[k^2 + m_1^2 - i\varepsilon]\ [(k+p)^2 + m_2^2 - i\varepsilon]\ [(k+p+q)^2 + m_3^2 - i\varepsilon]} \qquad (C\cdot1)$$

Lorentz covariance demands

$$C_\mu = p_\mu C_{11} + q_\mu C_{12} \qquad (C\cdot2)$$

$$C_{\mu\nu} = p_\mu p_\nu C_{21} + q_\mu q_\nu C_{22} + (p_\mu q_\nu + q_\mu p_\nu)\ C_{23} + \delta_{\mu\nu}\ C_{24} \qquad (C\cdot3)$$

These last defined form factors C_{ij} are all finite except C_{24} which is logarithmically divergent.

Using the standard Feynman parameterization and the formulas of 't Hooft and Veltman 1974

$$C_{i(j)}\ (p,\ q,\ m_1,\ m_2,\ m_3)$$

$$= \int_0^1 dz_1 \int_0^{1-z_1} dz_2 \frac{f_{i(j)}(z_1,\ z_2)}{[(1-z_1-z_2)m_1^2 + (p^2+m_1^2)z_1 + (p'^2+m_2^2)z_2 - (pz_1+p'z_2)^2 - i\varepsilon]^{\alpha - n/2}} \qquad (C\cdot4)$$

where $p' - p = q$

and with $\alpha = 3$; $f_0 = 1$, $f_{11} = -z_1 - z_2$, $f_{12} = z_2$, $f_{22} = z_2^2$, $f_{23} = z_2(z_1 + z_2)$.

With $\alpha = 2$; $f_{24} = \frac{1}{2}\pi^{n/2}\ ^{-2}\Gamma(2-n/2)$

The Spence Function

In deriving specific expressions for the C form factors that appear in the calculation of vertex corrections, the Spence or dilogarithm function

$$Sp(x) = -\int_0^1 \frac{\ell n\ (1 - xt)}{t}\, dt \qquad (C\cdot 5a)$$

$$= -\int_0^x \frac{\ell n\ (1 - t)}{t}\, dt \qquad (C\cdot 5b)$$

frequently occurs. The properties of this function are well documented in the literature (Kolbig et al. 1970, Lewin 1958, Barbieri et al. 1972) but of immediate application here are the formulas

$$Sp(x) = -Sp\ (1 - x) + \tfrac{1}{6}\pi^2 - \ell n(x)\ \ell n(1- x) \qquad (C\cdot 6)$$

$$Sp(x) = -Sp\ (\tfrac{1}{x}) - \tfrac{1}{6}\pi^2 - \tfrac{1}{2}\ \ell n^2(-x) \qquad (C\cdot 7)$$

$$Sp(1) = \frac{\pi^2}{6} \qquad (C\cdot 8)$$

The function can be most easily evaluated numerically by making the substitution $t = 1 - e^{-u}$ in (C·5b) to obtain

$$Sp(x) = -\int_0^{-\ell n\ (1-x)} \frac{u}{e^u - 1}\, du \qquad (C\cdot 9)$$

The integrand may be expanded as a series and writing $z = -\ell n(1-x)$ gives

$$Sp(x) = \int_0^z du \sum_{n=0}^{\infty} B_n \frac{u^n}{n!} = \sum_{n=0}^{\infty} B_n \frac{z^{n+1}}{(n+1)!} \qquad (C\cdot 10)$$

where the coefficients B_n are the Bernoulli numbers

$$B_0 = 1 \qquad B_1 = -\frac{1}{2} \qquad B_2 = \frac{1}{6} \qquad B_4 = -\frac{1}{30}$$

$$B_6 = \frac{1}{42} \qquad B_8 = -\frac{1}{30} \qquad B_{10} = \frac{5}{66} \qquad B_{12} = -\frac{691}{2730}$$

$$B_{14} = \frac{7}{6} \qquad B_{16} = -\frac{3617}{510} \qquad B_{18} = \frac{43867}{798}$$

In practice the formulas (C·6) and (C·7) can be used to bring the argument of Sp(x) into the domain $|x| < 1$, Real $(x) < \frac{1}{2}$ before the series is evaluated.

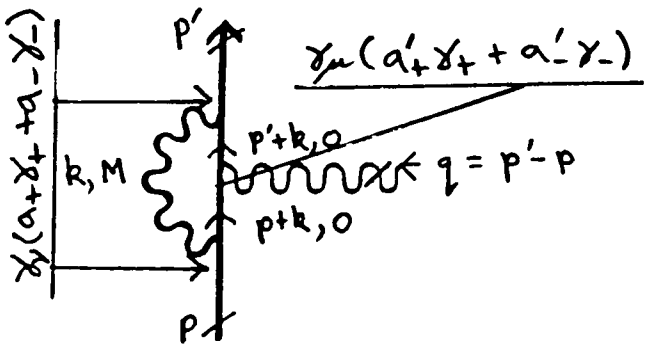
The Spence function possesses a branch cut lying along the real axis from 1 to ∞. Using (C·7) one can easily show that

$$\mathrm{Sp}(a \pm i\varepsilon) = \begin{cases} \mathrm{Sp}(a); & a \leq 1 \\ -\mathrm{Sp}\left(\frac{1}{a}\right) + \frac{1}{3}\pi^2 - \frac{1}{2}\ln^2 a \pm i\pi \ln a; & a > 1 \end{cases} \qquad (C\cdot 11)$$

This result is of importance in investigating the analytic behaviour of vertex corrections.

Vertex corrections for A_{pol} and A_{fb}

In the approximation that terms containing a factor $\frac{m_f^2}{M_W^2}$ (m_f an external fermion mass) are neglected, two classes of diagrams contribute to the vertex corrections in the purely weak (non-photonic) sector. In generic form, with internal vertices and particle masses as indicated, the first is



$$= \frac{1}{16\pi^2}\,\gamma_\mu\,(a_+^2 a'_+\gamma_+ + a_-^2 a'_-\gamma_-)\,\{2 - B_0(q;0,0) + M^2 C_0 + (2q^2 - M^2)(C_0 + C_{11})\}$$

(C·12)

The arguments for the C form factors are C(p, q; M, 0, 0) and the results

$$q^2(C_{23} - C_{22}) + M^2 (C_0 + C_{11}) = \tfrac{1}{2} \qquad (C\cdot13)$$

derived form (C·4) and

$$q^2 (C_{22} - C_{23}) + nC_{24} + M^2 C_0 = B_0(q; 0, 0) \qquad (C\cdot14)$$

have been used to obtain this expression.

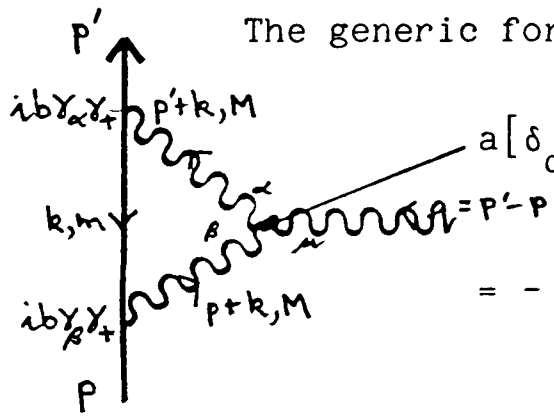
With $\alpha = \frac{q^2}{M^2}$ and taking due care of the $i\varepsilon$ prescription, the Feynman integral in the expressions for the C form factors in (C·12) may be evaluated to yield

$$C_0 = \frac{1}{q^2} \left\{\frac{\pi^2}{6} - Sp\ (1 - \alpha + i\varepsilon)\right\} \qquad (C\cdot15)$$

$$C_0 + C_{11} = \frac{1}{q^2} \left\{2[1 - \ell n\ (\alpha - i\varepsilon)] + \left(1 - \frac{2}{\alpha}\right) \left[\frac{\pi^2}{6} - Sp\ (1 - \alpha + i\varepsilon)\right]\right\} \qquad (C\cdot16)$$

Note from (C·11) that these expressions develop imaginary parts when $q^2 < 0$ which is consistent with the production of the (approximately massless) internal fermions.

The generic form of the second class of contribution is



$$a[\delta_{\alpha\mu}(-p-k-2q)_\beta + \delta_{\alpha\beta}(q+2p+2k)_\mu + \delta_{\mu\beta}(q-p-k)_\alpha]$$

$$= -\frac{i}{16\pi^2} ab^2\ \gamma_\mu\gamma_+ \{-3B_0(q;M,M) + (2q^2-M^2)\ C_{11}(p,q;0,M,M)\} \qquad (C\cdot17)$$

The results

$$q^2(C_{23} - C_{22}) - M^2 C_{11} = \tfrac{1}{2} \qquad (C\cdot18)$$

derived from (C·4) and

$$q^2 (C_{22} - C_{23}) + n\ C_{24} = B_0\ (q;M,M) \qquad (C\cdot19)$$

have been used.

The C_{11} form factor is now rather more difficult to evaluate than in the previous case. After the first integration

$$C_{11} = -\frac{2}{M^2}\int_0^1 dz_1 \frac{z_1'}{(\alpha z_1 + 1)} \{\ell n\,(1 + \alpha z_1(1 - z_1) - i\varepsilon) - \ell n\,(z_1 - i\varepsilon)\} \qquad (C\cdot20)$$

Because $z(1 - z) < \frac{1}{4}$ for $0 < z < 1$, this expression develops imaginary parts when $q^2 < -4M^2$ which is consistent with the production of the internal heavy bosons. The details of the final integration will not be given here as it is somewhat long. Suffice it to say that when adding and subtracting logarithms, careful consideration of the formula ('t Hooft and Veltman 1979)

$$\ell n\,(ab) = \ell n\, a + \ell n\, b + \eta\,(a,b) \qquad (C\cdot21)$$

$$\eta\,(a,b) = 2\pi i\,\{\theta(-Ima)\,\theta(-Imb)\,\theta(Imab) - \theta(Ima)\,\theta(Imb)\,\theta(-Imab)\} \qquad (C\cdot22)$$

is required. Equation (B·7) is a special case of this. The result is

$$C_{11}(p,q,;\,0,M,M) = \frac{2}{M^2\alpha^2}\{a\,\alpha\,\ell n\left[\frac{a-1}{a} - \frac{i\varepsilon}{\alpha}\right] + b\,\alpha\,\ell n\left[\frac{b-1}{b} + \frac{i\varepsilon}{\alpha}\right]$$

$$+ Sp\left(\frac{1+i\varepsilon}{A}\right) - Sp\left(\frac{1+\alpha+i\varepsilon}{A}\right) + Sp\left(\frac{1-i\varepsilon}{B}\right) - Sp\left(\frac{1+\alpha-i\varepsilon}{B}\right)$$

$$- \frac{\pi^2}{6} + Sp\,(1+\alpha-i\varepsilon\alpha)\} \qquad (C\cdot23)$$

where

$$A = 1 + \alpha a$$

$$B = 1 + \alpha b$$

and a,b are the roots of

$$-\alpha z^2 + \alpha z + 1 = 0$$

with Real a > Real b.

As before $\alpha = \frac{q^2}{M^2}$.

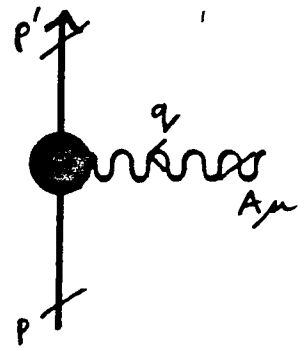
To obtain expressions for the $O(\alpha)$ purely weak corrections to the fermion-photon and fermion-Z vertices, it is convenient to define the finite functions

$$\Lambda(M) = 3[\Delta - \ell n\, M^2 - B_0(q;\, M,\, M)] + (2q^2 - M^2)\, C_{11}(p,\, q;\, 0,\, M,\, M) - \tfrac{1}{2} \tag{C·24}$$

and

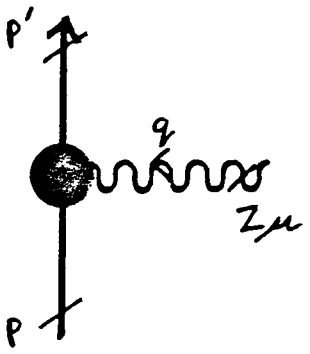
$$\rho(M) = B_0(q;\, 0,\, 0) - \Delta + \ell n\, M^2 - M^2\, C_0 + (M^2 - 2q^2)\,(C_0 + C_{11}) - \tfrac{3}{2} \tag{C·25}$$

The arguments of the C form factors in (C·25) are $C(p,\, q;\, M,\, 0,\, 0)$. Now, from previous results of this appendix as well as expressions for the vertex counter-terms (2·5·2) and (2·5·3) one obtains for on-shell fermions



$$= \frac{1}{16\pi^2}\, ig^3 s_\theta \gamma_\mu \left\{\gamma_+\left[Q\left(\frac{4t_3 - 4s_\theta^2 Q}{4c_\theta}\right)^2 \rho\,(M_Z) + \tfrac{1}{2}\,(Q - 2t_3)\,\rho\,(M_W)\right.\right.$$

$$\left.\left. - t_3\,\Lambda\,(M_W)\right] + \gamma_-\left[Q\left(\frac{-4s_\theta^2 Q}{4c_\theta}\right)^2 \rho\,(M_Z)\right]\right\} \tag{C·26}$$



$$= \frac{1}{16\pi^2}\, ig^3 \gamma_\mu \left\{\gamma_+\left[\left(\frac{4t_3 - 4s_\theta^2 Q}{4c_\theta}\right)^3 \rho\,(M_Z) - t_3\, c_\theta \Lambda\,(M_W)\right.\right.$$

$$\left.\left. + \left(\frac{4t_3(s_\theta^2 - c_\theta^2) - 4s_\theta^2 Q}{8c_\theta}\right) \rho\,(M_W)\right] + \gamma_-\left[\left(\frac{-4s_\theta^2 Q}{4c_\theta}\right)^3 \rho\,(M_Z)\right]\right\} \tag{C·27}$$

Here Q and t_3 are the charge and 3-component of weak isospin of the external fermion. In deriving these expressions, use has been made of (2·5·7) and the relation

$$\delta k = -\frac{g^2}{16\pi^2}\,(\Delta - \ell n\, M_W^2) \tag{C·28}$$

in the standard model that follows from (2·2·37).

Appendix D

Four-point form factors

The 4-point form factors that arise in this calculation are defined by

$$\{D_0;\ D_\mu;\ D_{\mu\nu}\}\ (p,\ q,\ r;\ m_1,\ m_2,\ m_3,\ m_4)$$

$$= \int \frac{d^n k}{i\pi^2} \frac{\{1;\ k_\mu;\ k_\mu k_\nu\}}{[k^2+m_1^2]\ [(k+p)^2 + m_2^2]\ [(k+p+q)^2 + m_3^2]\ [(k+p+q+r)^2 + m_4^2]} \tag{D·1}$$

As for the B and C form factors these may be decomposed to

$$D_\mu = p_\mu D_{11} + q_\mu D_{12} + r_\mu D_{13} \tag{D·2}$$

$$D_{\mu\nu} = p_\mu p_\nu D_{21} + q_\mu q_\nu D_{22} + r_\mu r_\nu D_{23} + (p_\mu q_\nu + p_\nu q_\mu)\ D_{23}$$

$$+ (p_\mu r_\nu + r_\mu p_\nu)\ D_{25} + (q_\mu r_\nu + q_\nu r_\mu)\ D_{26} + \delta_{\mu\nu} D_{27} \tag{D·3}$$

In this appendix, the notation will be adopted that C(2,3,4) is the C form factor derived from (D·1) by omitting the first factor in the denominator of the integral. Similarly C(1,3,4) is obtained by omitting the second factor. For example, then, $C_{12}(1,3,4)$ is the form factor C_{12} of the expression

$$C_\mu(1,3,4) = \int \frac{d^n k}{i\pi^2} \frac{k_\mu}{[k^2+m_1^2]\ [(k+p+q)^2 + m_3^2]\ [(k+p+q+r)^2 + m_4^2]} \tag{D·4}$$

Before deriving a general expression for the heavy box diagram a number of results are required. As in the text p_e, $-p'_e$ are the momenta of the incoming electron and positron of mass m_e and $-p_f$, p'_f are the momenta of the outgoing fermion and anti-fermion each having mass, m_f. Throughout what follows, terms containing a factor $\frac{m_e^2}{M_W^2}$ or $\frac{m_f^2}{M_W^2}$ will be dropped without comment.

The 4-momentum transferred from the electron-positron pair to the outgoing fermion pair in the annihilation is

$$q = p_f' - p_f \tag{D·5a}$$

$$= p_e - p_e' \tag{D·5b}$$

This yields immediately

$$2p_e \cdot p'_e = 2p_f \cdot p'_f = -q^2 \tag{D·6}$$

$$2p_e \cdot p'_f = 2p'_e \cdot p_f = q^2 + 2p_e \cdot p_f \tag{D·7}$$

$$2p'_e \cdot p'_f = 2p_e \cdot p_f \tag{D·8}$$

An expression will be needed for

$$[\not{p}_f]_e [\not{p}_e]_f$$

where $[\ ]_e$ ($[\ ]_f$) indicates that the enclosed operator acts on electron (outgoing fermion) wavefunctions.

Defining

$$P^e = p_e + p'_e \tag{D·9}$$

$$P^f = p_f + p'_f \tag{D·10}$$

the required expression becomes after some straightforward manipulations

$$\tfrac{1}{4} [\gamma_\alpha]_e [P^e_\beta]_e [\gamma_\beta]_f [P^f_\alpha]_f \tag{D·11}$$

The relation

$$i\not{p}'\sigma_{\mu\nu} + i\sigma_{\nu\mu}\not{p} = (p' + p)_\nu \gamma_\mu - (p' + p)_\mu \gamma_\nu + \varepsilon_{\mu\nu\alpha\beta}\gamma_\beta\gamma_5(p'_\alpha - p_\alpha) \tag{D·12}$$

with

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

can now be used to give,

$$[\not{p}_e]_f[\not{p}_f]_e = \tfrac{1}{4} q^2[\gamma_\alpha\gamma_5]_e[\gamma_\alpha\gamma_5]_f + \tfrac{1}{4}(q^2+4p_e\cdot p_f)[\gamma_\alpha]_e[\gamma_\alpha]_f \quad (D\cdot13)$$

and hence

$$[\not{p}_e\gamma_\pm]_f[\not{p}_f\gamma_\pm]_e = \tfrac{1}{2}(q^2+2p_e\cdot p_f)\,[\gamma_\alpha\gamma_\pm]_e[\gamma_\alpha\gamma_\pm]_f \quad (D\cdot14)$$

$$[\not{p}_e\gamma_\pm]_f[\not{p}_f\gamma_\mp]_e = p_e\cdot p_f\,[\gamma_\alpha\gamma_\pm]_e[\gamma_\alpha\gamma_\mp]_f \quad (D\cdot15)$$

The following results will also prove of use in this application. (Sirlin 1981)

$$[\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\pm]_e\,[\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\pm]_f = 4\delta_{\rho\sigma}[\gamma_\alpha\gamma_\pm]_e\,[\gamma_\alpha\gamma_\pm]_f \quad (D\cdot16)$$

$$[\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\pm]_e\,[\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\mp]_f = 4[\gamma_\rho\gamma_\pm]_e\,[\gamma_\sigma\gamma_\mp]_f \quad (D\cdot17)$$

$$[\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\pm]_e\,[\gamma_\nu\gamma_\rho\gamma_\mu\gamma_\pm]_f = 4[\gamma_\rho\gamma_\pm]_e\,[\gamma_\sigma\gamma_\pm]_f \quad (D\cdot18)$$

$$[\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\pm]_e\,[\gamma_\nu\gamma_\rho\gamma_\mu\gamma_\mp]_f = 4\delta_{\rho\sigma}[\gamma_\alpha\gamma_\pm]_e\,[\gamma_\alpha\gamma_\mp]_f \quad (D\cdot19)$$

In all of the heavy box diagrams which occur in the annihilation, the internal bosons are either both Z's or both W's. Moreover, whereas there are both direct and crossed Z boxes, only one W box occurs (Figure D·2). The most general expression for the box diagram in the present case has the form (Figure D·1)

$$\frac{1}{(2\pi)^4 i}\int d^n k\,\frac{\begin{array}{r}[\gamma_\mu(a_+^e\gamma_+ + a_-^e\gamma_-)(i\not{k})\gamma_\nu(a_+^e\gamma_+ + a_-^e\gamma_-)]_e \\ \times\,[\gamma_\mu(a_+^f\gamma_+ + a_-^f\gamma_-)(-i(\not{k}+\not{p}_e+\not{p}_f)\gamma_\nu(a_+^f\gamma_+ + a_-^f\gamma_-)]_f\end{array}}{k^2[(k+p_e')^2+M^2][(k+p_e'+q)^2+M^2](k+p_e'+q+p_f)^2} \quad (D\cdot20)$$

Observe first that in this zero fermion mass limit there are no terms to mix left-handed and right-handed components of the external fermion wave functions. If follows that magnetic moment terms are

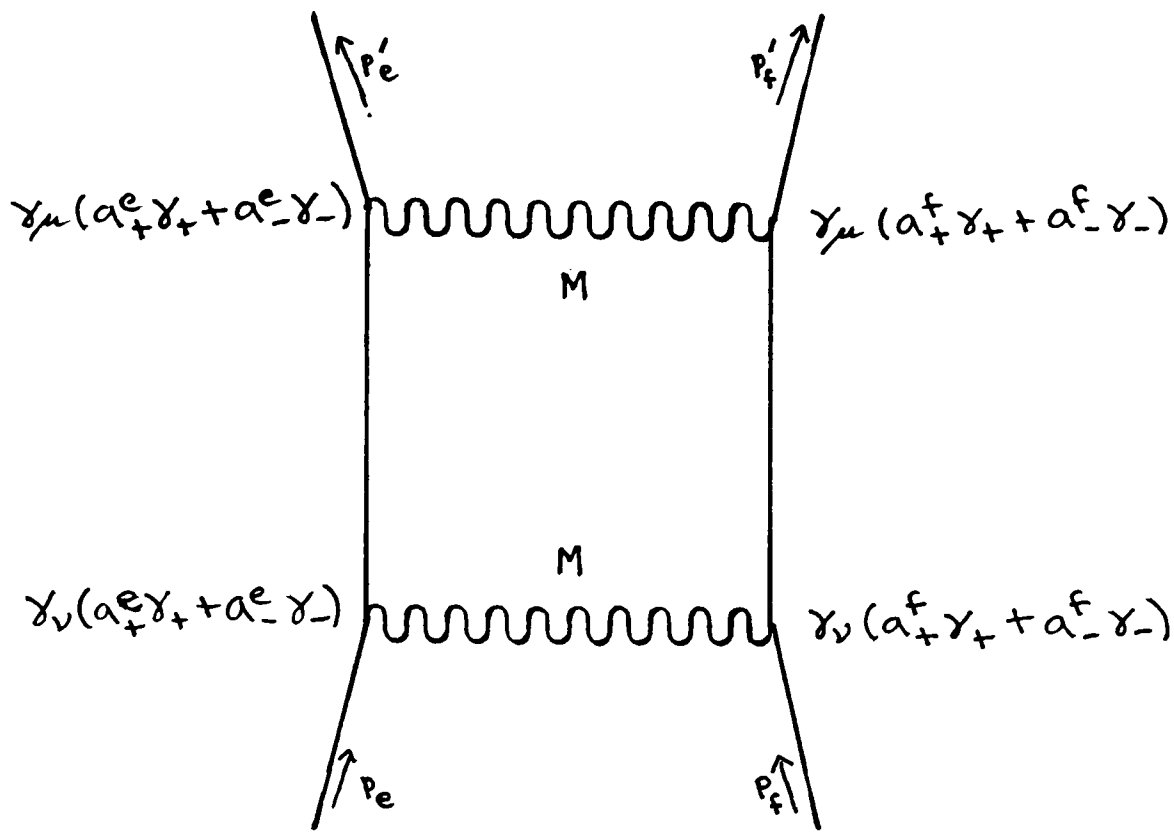


Figure D·1

The general heavy box diagram.

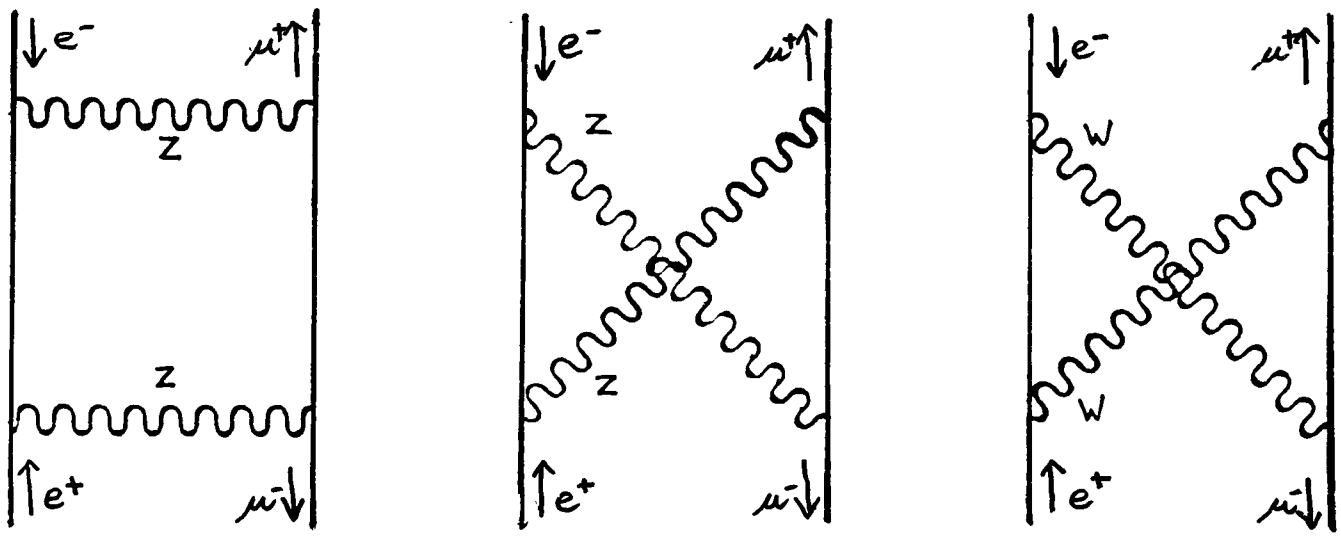


Figure D·2

Heavy box diagrams that occur in $e^+e^- \to \mu^+\mu^-$.

absent since these have this property. The only contributions are of the type (axial) vector - (axial) vector.

Expressed in terms of the D form factors

$$\frac{1}{4\pi^2}\{(\delta_{\mu\nu}D_{\mu\nu} + (p_e+p_f)_\mu D_\mu)(a_+^{e2}a_+^{f2}[\gamma_\alpha\gamma_+]_e[\gamma_\alpha\gamma_+]_f + a_-^{e2}a_-^{f2}[\gamma_\alpha\gamma_-]_e[\gamma_\alpha\gamma_-]_f)$$

$$+ (p_e\cdot p_f D_{25}+D_{27}+p_e\cdot p_f D_{11})(a_+^{e2}a_-^{f2}[\gamma_\alpha\gamma_+]_e[\gamma_\alpha\gamma_-]_f + a_-^{e2}a_+^{f2}[\gamma_\alpha\gamma_-]_e[\gamma_\alpha\gamma_+]_f)\} \qquad (D\cdot21)$$

The arguments are $D(p_e',q,p_f;0,M,M,0)$.

The D_0 form factor that arises in the direct boxes may be written with the aid of the Feynman parameters as

$$D_0(p_e', q, p_f; 0, M, M, 0) =$$

$$\int_{-\infty}^{\infty} dz_1dz_2dz_3dz_4 \frac{\delta(\sum_i z_i - 1)\prod_i \theta(z_i)}{[M^2(z_2+z_3) + q^2z_2z_3 + \rho^2z_1z_4 - i\varepsilon]^2} \qquad (D\cdot22)$$

where θ is the usual step function and $\rho^2 = 2p_e\cdot p_f$. Making the substitution

$$z_1 = y-z \qquad (D\cdot23a)$$

$$z_2 = z \qquad (D\cdot23b)$$

$$z_3 = x-y \qquad (D\cdot23c)$$

$$z_4 = 1-x \qquad (D\cdot23d)$$

this eventually yields

$$\int_0^1 dy \frac{1}{q^2\rho^2y(1-y)+M^2(\rho^2-M^2)-i\varepsilon(q^2+\rho^2)}\{\ell n\left[1 + \frac{q^2}{M^2}y(1-y)-i\varepsilon\right]+\ell n\left[\frac{\rho^2}{M^2}-i\varepsilon\right]\} \qquad (D\cdot24)$$

The integral, which will be denoted I, has no poles in the interval (0,1) provided that the quantity

$$-\frac{4M^2}{q^2}\left(1-\frac{M^2}{\rho^2}\right) \qquad (D\cdot25)$$

does not lie in the interval (0,1). For $\rho^2 < 0$ this requires that $q^2 > -4M^2$. When $\rho^2 > 0$ the residue of I at the poles is zero for all q^2. There is no pole contribution to the integral for $q^2 > -4M^2$, Moreover, the only contribution to the imaginary part comes from the second logarithm. In the case when $p^2 < 0$ and $q^2 < -4M^2$ there is a complicated interplay between the infinitesimal imaginary parts in the denominator and those in the logarithms. The expression (D·4) can be rewritten in the form

$$\int_0^1 dy \left(\frac{1}{q^2\rho^2}\right)\frac{1}{(a-b)}\left[\frac{1}{y-a+i\delta}-\frac{1}{y-b-i\delta}\right]$$

$$x\ \{\ell n\ (-\alpha-i\varepsilon)+\ell n(\frac{\rho^2}{M^2}-i\varepsilon)+\ell n(y-c+i\varepsilon/\alpha)+\ell n(y-d-i\varepsilon/\alpha)\} \qquad (D\cdot26)$$

with Re c > Re d; Ra a > Re b, $\alpha = \frac{q^2}{M^2}$ and $\delta = \frac{(q^2+\rho^2)}{q^2\rho^2}\varepsilon$.

This can be expressed in terms of the general integral

$$\int_0^1 dy\ \frac{1}{y-a\pm i\varepsilon}\ \ell n\ (y-c+i\delta)\ . \qquad (D\cdot27)$$

which after some algebra leads to

$$\left[\ell n(a-c)+\eta(a-c,\frac{1-c+i\delta}{a-c})+F(\frac{1-a\pm i\varepsilon}{c-a},\frac{-a\pm i\varepsilon}{c-a})\right]\ell n\ (\frac{1-a\pm i\varepsilon}{x_0^+(c-a)})+Sp(x_0^+)$$

$$-Sp(\frac{1-a+i\delta}{c-a})-\left[\ell n\ (a-c)+\eta(a-c,\frac{-c+i\delta}{a-c})\right]\ell n\ (\frac{-a\pm i\varepsilon}{x_0^-(c-a)})+Sp(\frac{-a-i\delta}{c-a})$$

$$-Sp(x_0^-)\ . \qquad (D\cdot28)$$

The function η is defined in (C·22). The quantity $\ell n(a-c) + \eta(a-c)$ is defined for real a-c by adding an imaginary infinitesimal of arbitary sign to a-c. x_0 the point where a line connecting $(\frac{-a}{c-a})$ and $(\frac{1-a}{c-a})$ on the complex plane crosses the real axis.

$$x_0 = \frac{a^*(a-1) - a(a^*-1)}{(c-a)^* - (c-a)}$$

and

$$x_0^+ = x_0 + \varepsilon' \left(\frac{1-a\pm i\varepsilon}{c-a}\right) \qquad (D\cdot 29a)$$

$$x_0^- = x_0 + \varepsilon' \left(\frac{1-a\pm i\varepsilon}{c-a}\right) \qquad (D\cdot 29b)$$

where ε' is a positive infinitesimal.

The function F(a,b) is non-zero only when a straight line connecting a and b crosses the negative real axis between two points. That is

$$F(a,b) = -2\pi i \text{ sign } (\text{Im}b)\ \theta\left(\frac{a^*(a-1) - a(a^*-1)}{(c-a) - (c-a)^*}\right) \qquad (D\cdot 30)$$

This function is present to ensure that, if the path of integration after transformation of the integral crosses a branch cut, the integral varies smoothly without interference from η.

When a is real and $0 < a < 1$ there must be added to (D·6) a pole contribution

$$\mp i\pi\ \ell n\ (a - c + i\delta) \qquad (D\cdot 31)$$

Note that in this case $x_0 = 0$ and arguments of the logarithms containing $x_0^{\pm}$ are positive real numbers.

It can easily be shown, by a similar method to that used to obtain (D·24)

$$D_{12} = \int_0^1 yIdy \qquad (D\cdot 32)$$

Notice that I is invariant under the substitution $y \to 1 - y$. It follows immediately that

$$D_0 = -2D_{12} \tag{D·33}$$

The Feynman parameter method can be used to show also that

$$D_{11} + D_{13} = 2D_{12} \tag{D·34}$$

and

$$D_{27} = \tfrac{1}{2}\{C_0(2,3,4) + M^2(D_{11} - D_{13}) + 2p_e.p_f D_{13}\} \tag{D·35}$$

We now turn to the form factors that appear in the expression for the general box (D·21). Consider

$$\delta_{\mu\nu} D_{\mu\nu}{}^{+(p_e+p_f)}{}_{\mu}\, D_{\mu} = \int \frac{d^n k}{i\pi^2} \frac{k.(k+p_e+p_f)}{k^2[(k+p_e')^2+M^2]\,[(k+p_e'+q)^2+M^2]\,(k+p_e'+q+p_f)^2} \tag{D·36}$$

by writing the numerator of the integrand as

$$(k + p_e + p_f)^2 - k.(p_e + p_f) - (p_e + p_f)^2 \tag{D·37}$$

and using (D·34) this becomes

$$C_0\,(p_e', q; 0, M, M) - p_e.p_f\, D_0 \tag{D·38}$$

A similar approach may be used to obtain after some algebra

$$(q^2 + 2p_e.p_f)D_{11} = -\tfrac{1}{2}\,(q^2 + 4p_e.p_f - 2M^2)D_0 - C_0(1,3,4) + C_0(2,3,4) \tag{D·39}$$

$$(q^2 + 2p_e.p_f)D_{13} = -\tfrac{1}{2}\,(q^2 + 2M^2)D_0 + C_0(1,3,4) - C_0(2,3,4) \tag{D·40}$$

$$2p_e\cdot p_f D_{25} + 2D_{27} = \left[-2p_e\cdot p_f\left(C_{12}(1,3,4) + C_{12}(1,2,4)\right) - q^2 C_{12}(2,3,4)\right.$$
$$+ q^2 C_0(2,3,4) + M^2 q^2 (D_{11} - D_{13})$$
$$\left. + 2(M^2 - 2p_e\cdot p_f)\, 2p_e\cdot p_f D_{13}\right]\left[2(q^2 + 2p_e\cdot p_f)\right]^{-1} \qquad (D\cdot 41)$$

Note that these expressions apparently blow up as $(q^2+2p_e\cdot p_f) \to 0$ or $p_f \to p_e$. In fact this does not occur because it can be shown, by the same technique as was used to derive these expressions, that the numerator tends to zero as well. This, then, provides a powerful check on the numerical evaluation of the integrals.

Again, using Feynman parameters to perform the integrations gives

$$C_0(2,3,4) = C_0(1,2,3) = I_1\left(\frac{q^2}{M^2}, M^2\right) \qquad (D\cdot 42)$$

$$C_0(1,3,4) = C_0(1,2,4) = I_2\left(\frac{2p_e\cdot p_f}{M^2}, M^2\right) \qquad (D\cdot 43)$$

$$C_{12}(2,3,4) = C_{12}(1,2,3) = I_3\left(\frac{q^2}{M^2}, M^2\right) \qquad (D\cdot 44)$$

$$C_{12}(1,3,4) = C_{12}(1,2,4) = I_4\left(\frac{2p_e\cdot p_f}{M^2}, M^2\right) \qquad (D\cdot 45)$$

and

$$I_1(\alpha, M^2) = \frac{1}{M^2\alpha}\left\{Sp\left(\frac{1+i\varepsilon}{A}\right) - Sp\left(\frac{1+\alpha+i\varepsilon}{A}\right) + Sp\left(\frac{1-i\varepsilon}{B}\right) - Sp\left(\frac{1+\alpha-i\varepsilon}{B}\right)\right.$$
$$\left. - \frac{\pi^2}{6} + Sp\,(1+\alpha-i\varepsilon\alpha)\right\} \qquad (D\cdot 47)$$

$$I_2(\alpha, M^2) = \frac{1}{M^2\alpha}\left\{\frac{\pi^2}{6} - Sp\,(1-\alpha+i\varepsilon)\right\}. \qquad (D\cdot 48)$$

$$I_3(\alpha, M^2) = -\frac{1}{M^2\alpha}\left\{2a\ \ell n\left[\frac{a-1}{a} - \frac{i\varepsilon}{\alpha}\right] + 2b\ \ell n\left[\frac{b-1}{b} + \frac{i\varepsilon}{\alpha}\right] + 2\right.$$

$$+ (1 + 2/\alpha)\left[Sp\left(\frac{1+i\varepsilon}{A}\right) - Sp\left(\frac{1+\alpha+i\varepsilon}{A}\right) + Sp\left(\frac{1-i\varepsilon}{B}\right)\right.$$

$$\left.\left. - Sp\left(\frac{1+\alpha-i\varepsilon}{B}\right) - \frac{\pi^2}{6} + Sp\,(1+\alpha-i\varepsilon\alpha)\right]\right\} \qquad (D\cdot49)$$

$$I_4(\alpha, M^2) = \frac{1}{M^2\alpha^2}\left[\alpha - \alpha\ \ell n\,(\alpha - i\varepsilon) + Sp\,(1 - \alpha + i\varepsilon) - \frac{\pi^2}{6}\right] \qquad (D\cdot50)$$

In the above a and b are the roots of the equation $-\alpha x^2+\alpha x+1 = 0$ with Real a $\geqq$ Real b and $A = 1 + \alpha a$, $B = 1 + \alpha b$. As mentioned before, the corresponding expressions for the crossed boxes Figure D·2b,c may be obtained by the substitution

$$2p_e\cdot p_f \rightarrow -(q^2 + 2p_e\cdot p_f)\ .$$

in all expressions obtained in this appendix including the arguments of the I_i form factors.

Appendix E

Cross-sections for A_{pol} and A_{fb}

Consider two matrix elements of the form

$$M = [i\gamma_\rho(\beta_+^e\gamma_+ + \beta_-^e\gamma_-)]_e \hat{\chi} [i\gamma_\rho(\beta_+^f\gamma_+ + \beta_-^f\gamma_-)]_f \qquad (E\text{-}1a)$$

$$M' = [i\gamma_\lambda(\beta_+^{'e}\gamma_+^e + \beta_-^{'e}\gamma_-)]_e \hat{\chi}' [i\gamma_\lambda(\beta_+^{'f}\gamma_+ + \beta_-^{'f}\gamma_-)]_f \qquad (E\cdot1b)$$

The quantities β_+, β_- are vertex coupling constants and $\hat{\chi}$ and $\hat{\chi}'$ are propagators. The operators enclosed in $[\ ]_e$ act on the electron-positron wave functions and those enclosed in $[\ ]_f$ act on the outgoing fermion wavefunctions. At high energy the spin projection operator

$$\sum(s) = \frac{1 + \gamma_s i\not{s}}{2} \qquad (E\cdot2)$$

is approximated by the helicity operator $\gamma_\pm$. Specifically γ_- projects out positive energy states with spin parallel to the direction of the motion. The product

$$\sum MM'^+$$

in which the sum is over outgoing fermion spins is, for incoming electrons in spin eigenstates of γ_i and positions in eigenstates of γ_j (i, j = $\pm$) and momenta as given by

$$\{(1+c^2)\ (\beta_+^f\bar{\beta}_+^{'f} + \beta_-^f\bar{\beta}_-^{'f}) + 2jc(\beta_+^f\bar{\beta}_+^{'f} - \beta_-^f\bar{\beta}_-^{'f})\}\ \beta_i^e\bar{\beta}_j^{'e}\delta_{ij}\chi\chi'^+ \qquad (E\text{-}3)$$

Here $\chi = q^2\hat{\chi}$, $\chi' = q^2\hat{\chi}'$ for 4-momentum transfer q and $c = \cos\theta$ is the cosine of the angle between the outgoing fermion and the beam. This

result, after the inclusion of phase space factors appropriate to the centre of mass system leads to

$$\frac{d\sigma_p}{d\Omega} = \frac{1}{16\pi^2 s}\{(1+c)^2(\beta_+^{e\bar{-}'e}\beta_+^{f\bar{-}'f} - \beta_-^{e\bar{-}'e}\beta_-^{f\bar{-}'f}) + (1-c)^2(\beta_+^{e\bar{-}'e}\beta_-^{f\bar{-}'f} - \beta_-^{e\bar{-}'e}\beta_+^{f\bar{-}'f})\}\chi\chi'^{+} \qquad (E\cdot4)$$

$$\frac{d\sigma_u}{d\Omega} = \frac{1}{16\pi^2 s}\{(1+c)^2(\beta_+^{e\bar{-}'e}\beta_+^{f\bar{-}'f} + \beta_-^{e\bar{-}'e}\beta_-^{f\bar{-}'f}) + (1-c)^2(\beta_+^{e\bar{-}'e}\beta_-^{f\bar{-}'f} + \beta_-^{e\bar{-}'e}\beta_+^{f\bar{-}'f})\}\chi\chi'^{+} \qquad (E\cdot5)$$

where $\sigma_p = \sigma_L - \sigma_R$, $\sigma_u = \sigma_L + \sigma_R$ and σ_L, σ_R are defined in (1·1·1) - (1·1·3).

These expressions are to be integrated over a solid angle appropriate to the beam pipe. Accordingly

$$\int_{-x}^{x}(1 \pm c)^2\,dc = \tfrac{2}{3}x(3 + x^2) \qquad (E\cdot6)$$

$$\int_{0}^{x}(1 \pm c)^2\,dc = x \pm x^2 + \tfrac{1}{3}x^3 \qquad (E\cdot7a)$$

$$\int_{-x}^{0}(1 \pm c)^2\,dc = x \mp x^2 + \tfrac{1}{3}x^3 \qquad (E\cdot7b)$$

Note that all the matrix elements required for this work can be written in the form (E·1); even the boxes. This is a direct consequence of treating the fermions as massless.

The formulas given here allow the cross section to be formed directly from the matrix elements by computer, thereby avoiding a lot of tedious algebra.

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